① $$\lim_{n\to\infty}\frac{3n^2-n+2}{5n^2+2}=\left[\frac{\infty}{\infty}\right]=\lim_{n\to\infty}\frac{3-\frac{1}{n}+\frac{2}{n^2}}{5+\frac{2}{n^2}}=\frac{3}{5}$$

$$(a-b)(a+b)=a^2-b^2$$

② $$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n-1}\right)=[\infty-\infty]=\lim_{n\to\infty}\frac{\left(\sqrt{n+1}-\sqrt{n-1}\right)\left(\sqrt{n+1}+\sqrt{n-1}\right)}{\sqrt{n+1}+\sqrt{n-1}}=$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt{n+1}\right)^2-\left(\sqrt{n-1}\right)^2}{\sqrt{n+1}+\sqrt{n-1}}=\lim_{u\to\infty}\frac{n+1-(n-1)}{\sqrt{n+1}+\sqrt{n-1}}=\lim_{n\to\infty}\frac{2}{\sqrt{n+1}+\sqrt{n-1}}=$$

$$=2\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n-1}}=2\cdot 0=0$$

③ $$\lim_{n\to\infty}\frac{\sqrt{n^3}}{\sqrt{n}+1}=\left[\frac{\infty}{\infty}\right]=\lim_{n\to\infty}\frac{1}{\frac{\sqrt{n}}{\sqrt{n^3}}+\frac{1}{\sqrt{n^3}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{1}{n^2}}+\frac{1}{\sqrt{n^3}}}=\lim_{n\to\infty}\frac{1}{\frac{1}{n}+\frac{1}{\sqrt{n^3}}}=\infty$$

④ $$\lim_{n\to\infty}\frac{3^{n+1}-2\cdot 4^n}{4^{n+1}+5}=\lim_{n\to\infty}\frac{3\cdot 3^n-2\cdot 4^n}{4\cdot 4^n+5}=\left[\frac{\infty}{\infty}\right]=\lim_{n\to\infty}\frac{\frac{3\cdot 3^n-2\cdot 4^n}{4^n}}{\frac{4\cdot 4^n+5}{4^n}}=\lim_{n\to\infty}\frac{3\cdot\left(\frac{3}{4}\right)^n-2}{4+\frac{5}{4^n}}=-\frac{2}{4}=-\frac{1}{2}$$

⑤ $$\lim_{n\to\infty}\frac{(3n+1)!+(3n+2)!}{(3n+3)!}=\left[\frac{\infty}{\infty}\right]=$$

$n!=1\cdot 2\cdot 3\cdot\ldots\cdot n$

$5!=1\cdot 2\cdot 3\cdot 4\cdot 5$
$(3n+3)!=1\cdot\ldots\cdot(3n+3)$
$(3n+3)!=1\cdot\ldots\cdot(3n+2)(3n+3)$
$(3n+3)!=1\cdot\ldots\cdot 3n(3n+1)(3n+2)(3n+3)$

$$=\lim_{n\to\infty}\frac{1\cdot 2\cdot 3\cdot\ldots\cdot 3n(3n+1)+1\cdot 2\cdot 3\cdot\ldots\cdot 3n(3n+1)(3n+2)}{1\cdot 2\cdot 3\cdot\ldots\cdot 3n(3n+1)(3n+2)(3n+3)}=\lim_{n\to\infty}\frac{\cancel{1\cdot 2\cdot 3\cdot\ldots\cdot 3n(3n+1)}[1+(3n+2)]}{\cancel{1\cdot 2\cdot 3\ldots 3n(3n+1)}(3n+2)(3n+3)}$$

$$=\lim_{n\to\infty}\frac{1+3n+2}{(3n+2)(3n+3)}=\lim_{n\to\infty}\frac{\cancel{3n+3}}{(3n+2)\cancel{(3n+3)}}=\lim_{n\to\infty}\frac{1}{3n+2}=0$$