Aspects of Quantum Gravity in de Sitter Spaces

Article *in* Journal of Cosmology and Astroparticle Physics · July 2004

DOI: 10.1088/1475-7516/2004/11/006 · Source: arXiv

Luciano Vanzo
Università degli Studi di Trento

IFUM-757-FT
UTF-454
hep-th/0407255

Aspects of Quantum Gravity in de Sitter Spaces

Dietmar Klemm[1*] **and Luciano Vanzo**[2†]

[1] Dipartimento di Fisica dell'Università di Milano,
and
INFN, Sezione di Milano,
Via Celoria 16, I-20133 Milano.

[2] Dipartimento di Fisica dell'Università di Trento,
and
INFN, Gruppo Collegato di Trento,
Via Sommarive 14, I-38050 Trento.

Abstract

In these lectures we give a review of recent attempts to understand quantum gravity on de Sitter spaces. In particular, we discuss the holographic correspondence between de Sitter gravity and conformal field theories proposed by Hull and by Strominger, and how this may be reconciled with the finite-dimensional Hilbert space proposal by Banks and Fischler. Furthermore we review the no-go theorems that forbid an embedding of de Sitter spaces in string theory, and discuss how they can be circumvented. Finally, some curious issues concerning the thermal nature of de Sitter space are elucidated.

*dietmar.klemm@mi.infn.it
†vanzo@science.unitn.it

1 Introduction and Motivation

Recent astrophysical data coming from type Ia supernovae [1] indicate that the cosmic expansion is accelerating and point towards a small but nonvanishing positive cosmological constant (cf. figure 1). This means that our universe might currently be in a de Sitter (dS) phase.

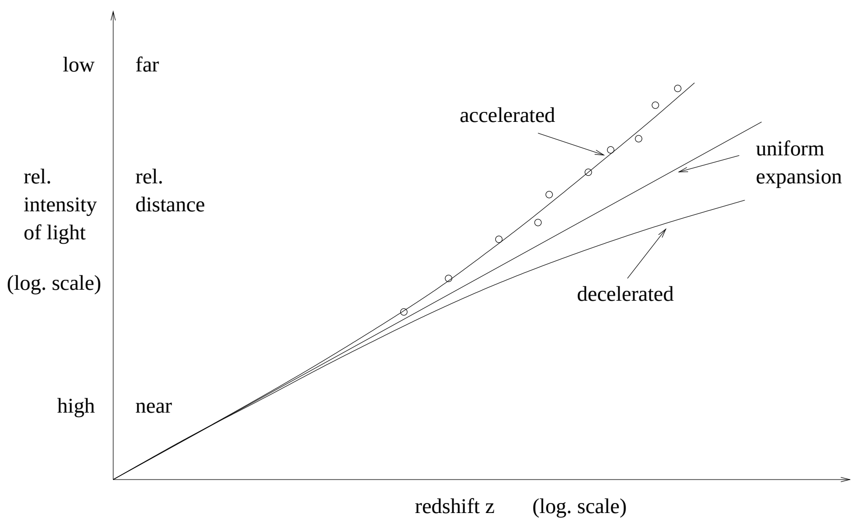

Figure 1: Relative intensity of light (corresponding to relative distance) of type Ia supernovae measured as a function of the redshift (corresponding to relative velocity) [1]. The measurements indicate that type Ia supernovae are about 25% dimmer than forecast, which means that the cosmic expansion is accelerating.

Independent evidence for this comes from measurements of the cosmic microwave background (CMB) anisotropy (COBE, Boomerang, WMAP), from which we learned that the universe is spatially flat. But in order to have a flat geometry, one needs a total energy density much larger than the total density of ordinary matter observed in our universe. This indicates that the major part of the energy in the universe is a kind of "dark energy". The simplest and most convincing model for dark energy is a small, positive cosmological constant Λ. According to recent WMAP data, about 73% of the energy density of the universe is in a dark energy sector (with $\sim$ 22% dark matter and $\sim$ 4.4% baryons) [2]. Current estimates for the energy density associated to Λ yield

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} \leq \left(10^{-3}\mathrm{eV}\right)^4 \simeq 10^3 \ \mathrm{eV} \cdot \mathrm{cm}^{-3} \,, \tag{1.1}$$

corresponding to a mass density $10^{-29}\mathrm{g/cm}^3$. Note that this value is by a factor of about 10^{123} too small with respect to the vacuum energy of the fields in the standard model, if we take the Planck scale as a cutoff. This is the so-called cosmological constant problem [3]. It is tempting to envoke supersymmetry in order to solve this problem, due to the cancellation of the vacuum fluctuations of bosons and fermions. However, some kind of fine-tuning must cancel only virtual-particle energies to 123 decimal places but leave the 124-th untouched–a precision seen nowhere else in nature. We shall not discuss the cosmological constant problem in detail in these lectures, and refer the reader instead to the numerous reviews [3, 4].

Apart from the current accelerating cosmic expansion, further motivation for the interest in de Sitter gravity comes from the inflationary era, during which one assumes that the universe was also described by a de Sitter phase.

A less phenomenological and more academic problem is the realization of the holographic principle [5] for spaces more general than anti-de Sitter (AdS) [6]. For AdS gravity, we know now that there exists a dual description in terms of conformal field theories in one dimension lower [7]. The best-understood example of this is the correspondence between type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N} = 4$ super Yang-Mills theory [8]. It would of course be desirable to know further concrete realizations of the holographic principle, for spacetimes different from AdS. As we will see below, de Sitter spaces represent another interesting test ground for holography, which motivates to study dS gravity under this point of view.

A full understanding of holography for de Sitter gravity seems to require an embedding of dS spaces in string theory. However, as we will see below, there are severe obstacles to this, resulting in no-go theorems [9, 10]. Yet, being the most serious candidate for a complete quantum theory of gravity, string theory should admit dS vacua, if the astrophysical data that indicate that our universe is currently in a de Sitter phase, are interpreted correctly. In this sense, dS gravity represents a big challenge for string theory.

Last but not least, dS space possesses a cosmological event horizon, to which one can associate a temperature and entropy [11]. As is the case with black holes [12], one would like to reproduce this gravitational entropy by a microstate counting[1].

The reasons described above led to an increasing activity on the theoretical side, with the general aim to shed some light on quantum gravity on de Sitter spaces [15], and to embed de Sitter space in string theory. Despite considerable effort, these points are far from being well-understood. Different approaches do even appear to clash with each other; for instance the proposal that de Sitter gravity in D dimensions is dual to a Euclidean conformal field theory in one dimension lower [16] seems to contradict the claim by Banks [17] and Fischler [18] that the Hilbert space of quantum gravity on de Sitter spaces is finite-dimensional.

This review was written down while we tried to understand these various approaches to quantum gravity on de Sitter spaces, and how they might be reconciled.

Our paper does not pretend to be exhaustive, rather the intention was to elucidate some selected issues that seemed particularly interesting to us. For a review of accelerating universes in string theory and de Sitter holography, that is to some extent complementary to the material presented here, we refer the reader to [19].

Nevertheless, we still would like to mention the perturbative results that have been obtained so far, with our apologies for any possible omission. On the one hand these are

[1]For steps in this direction cf. [13, 14].

interesting in their own, on the other they help understanding de Sitter more.
Past researches on de Sitter space were directed first to uncover quantum field theory on a fixed dS background and semiclassical Einstein gravity, the scheme whereby quantum fields act as source of gravity via the expectation value of the stress tensor in a conventional vacuum state. Certainly the problem of a choice of vacuum, the need to understand particle detection and the stability properties of de Sitter space were all present in the early investigations. We discuss these in turn.
The richer vacuum structure of de Sitter space was immediately recognized. While in Minkowski spacetime there is only one Poincaré invariant vacuum, for scalar fields in de Sitter space there is a two parameter family of de Sitter invariant vacua [20], presumably distinguished by a superselection parameter[2] α. These were studied by several authors [21–24] in the early days, and also more recently in [25, 26]; one of these vacua is the so called Euclidean vacuum, which is uniquely selected by a number of properties such as covariance and analyticity of the Wightman functions, or the fact that it reduces to the ordinary Poincaré vacuum as Λ goes to zero. The other vacuum states came to be known as the MA-states, or α-states, after Mottola and Allen discovered the Bogoliubov transformation relating them [22, 23]. They play a role especially in relation to initial perturbations in inflationary scenarios. None of these vacua can be obtained by analytic continuation from AdS [25].
Massless fields are more problematic; for a while it was thought that they break de Sitter invariance [27], and in fact it was shown that there exists no de Sitter invariant Fock representation; instead, a two parameter family of O(4)-invariant states were found [28]. As has been clarified by Ford [29], the physical origin of this fact is an infrared divergence in the two-point function for such states, and this was used by him to infer a quantum instability of de Sitter space. However, there exists another representation of the canonical commutation relations leading to a de Sitter invariant vacuum, now known as the Kirsten-Garriga vacuum [30].
The existence of many de Sitter invariant vacua brings us to the problem of particle detection and stability. One has to distinguish between what an "Unruh detector" sees from the relationships among asymptotic vacuum states at future/past infinity [31]. With one exception, the response of a monopole detector in the α-states shows an excitation rate which does not satisfy the principle of detailed balance, but equilibrate nevertheless [24,25]. The exception is the Euclidean vacuum, whose quantum noise is perfectly thermal and satisfies detailed balance.
For special values of the parameter α there exist states which can be interpreted as asymptotic vacua at timelike infinity. It is interesting that in odd-dimensional de Sitter space the asymptotic vacua actually coincide [25]. But apart from this case, the incoming de Sitter vacuum for scalar particles will decay driving a perturbative instability of the space [22], since the created particles will tend to reduce the effective cosmological constant if the scalar mass parameter satisfies the inequality $m^2+12\xi H^2 > 9H^2/4$, $H^2 = \Lambda/3$ being the Hubble constant of de Sitter space and ξ the coupling of the scalar particle to the

[2] It seems that no observable can change α.

curvature. To our knowledge there are no analogous results for fermions.
Another kind of instability was discussed by Myhrvold [32] in an interacting $\lambda\phi^4$ theory; in this theory the spacetime curvature can make a particle to decay into three[3], each one of which can decay into three more, and so on in a runaway process. The presence of interaction is essential for this argument to work; the back reaction due to conformally invariant free quantum fields always gives a semiclassically stable de Sitter solution [33].
The instabilities generated by interacting matter fields are only one face of the problem, the other being quantum gravity itself. In a series of remarkable papers Tsamis and Woodard [34] analyzed the structure of perturbative quantum gravity in de Sitter space in inflationary coordinates. They showed that single gravitons can decay into two and the vacuum into three, so neither the vacuum nor the one graviton states are stable. The instability is enhanced by infrared divergences due to the exponential rate of expansion, which enormously red shift all momenta to zero. They argue that, as a result, the cosmological "constant" is driven to smaller values, thereby reducing the inflation expansion rate. Another source of instability come from the existence of the Schwarzschild-de Sitter black hole; Ginsparg and Perry [35] argued that this solution is a saddle point of the Euclidean functional integral for gravity, thereby inferring a semiclassical instability of de Sitter space against formation of black holes. Once formed they presumably evaporate away on a time scale $t \simeq \Lambda^{-3/2}/G$ much shorter than that required for another nucleation[4].
What can we learn from all this is hard to say, also because *exact* de Sitter space is only an approximation to the real world. We only want to note that a richer vacuum structure is an additional degree of freedom at our disposal (for example to incorporate inflation) and that the various instabilities may not be a disaster in a gravitational setting, where the instability time scale is of order of the universe lifetime. After all, we owe stars and galaxies to gravitational clumping.

The lectures are organized as follows: In order to be self-contained, we briefly review the dS geometry in the next section. In section 3 we explain how to define conserved charges associated to conformal Killing vectors for asymptotically dS spaces. This nice idea goes back to Kastor and Traschen [38], and circumvents the drawback that de Sitter space has no globally defined timelike Killing vector to which one could associate a positive energy. In section 4 we describe the no-go theorems that prevent us from embedding dS spaces in string theory, and discuss how these theorems can be circumvented. The correspondence between de Sitter gravity and Euclidean conformal field theories is reviewed in section 5. In 6, we try to explain the motivations that led Banks and Fischler to the proposal that the Hilbert space of quantum gravity on dS spaces is finite-dimensional, and we indicate how this might be reconciled with the dS/CFT correspondence. Finally, we discuss some issues related to the thermal nature of dS in section 7.

[3]It is important that "particle" here is defined relative to the Euclidean vacuum.

[4]The instability was not seen by Abbott and Deser in their classical stability analysis [46] because they missed the Nariai solution.

2 De Sitter Geometry

Consider $(D+1)$-dimensional Minkowski space $\mathbb{R}_1^{D+1}$ with coordinates X^A, where $A = 0, \ldots, D$, and metric $\eta_{AB} = \mathrm{diag}(-1, 1, \ldots, 1)$. dS_D can then be represented as the hypersurface

$$\eta_{AB} X^A X^B = \ell^2 \,, \tag{2.1}$$

where the constant ℓ is essentially the curvature radius of de Sitter space. The hyperboloid (2.1) is shown in figure 2.

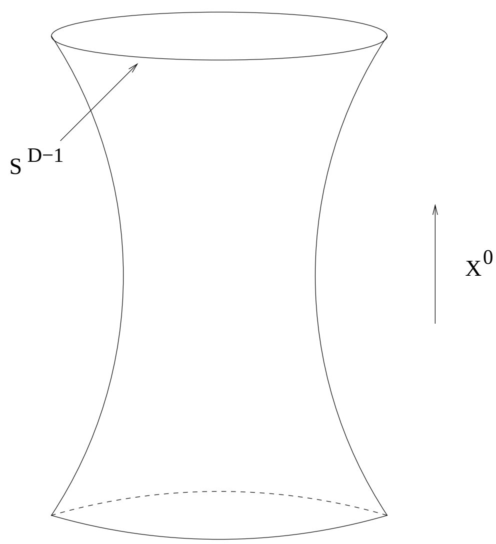

Figure 2: De Sitter space dS_D as a hypersurface in $(D+1)$-dimensional Minkowski spacetime.

The "time" X^0 runs vertically, while slices of constant X^0 are $(D-1)$-dimensional spheres S^{D-1}. De Sitter space is thus a sphere S^{D-1} that contracts to reach a minimal radius, and then reexpands. For the metric $g_{\mu\nu}$ on dS_D we take the induced metric from the embedding space $\mathbb{R}_1^{D+1}$. D-dimensional de Sitter space is then a space of constant curvature, which implies that it is also Einstein, i. e. , the Einstein tensor $G_{\mu\nu}$ satisfies

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \,, \tag{2.2}$$

with a cosmological constant given by

$$\Lambda = \frac{(D-2)(D-1)}{2\ell^2} \,. \tag{2.3}$$

By construction it is clear that the isometry group of dS_D is $\mathrm{O}(D, 1)$. Note that this is also the Euclidean conformal group in $D-1$ dimensions. This will become important in section 5.
We shall now introduce some coordinate systems that will be useful later. If we set

$$X^0 = \ell \sinh \frac{\tau}{\ell} \,, \tag{2.4}$$

we get from (2.1)

$$(X^1)^2 + \ldots + (X^D)^2 = \ell^2 \cosh^2 \frac{\tau}{\ell}\,, \tag{2.5}$$

i. e. , the coordinates $X^1, \ldots, X^D$ range over a $(D-1)$-sphere with radius $\ell \cosh \frac{\tau}{\ell}$. The induced metric on (2.1) takes the form

$$ds^2 = -d\tau^2 + \ell^2 \cosh^2 \frac{\tau}{\ell} d\Omega^2_{D-1}\,, \tag{2.6}$$

where $d\Omega^2_{D-1}$ denotes the standard line element on the unit S^{D-1}. The coordinates in (2.6) are global, they cover the whole manifold. From (2.6) it is also evident that dS_D is a contracting and then reexpanding $(D-1)$-sphere.
Future inflationary coordinates (t, x^i), $i = 1, \ldots, D-1$, are defined by

$$\begin{aligned} X^0 &= \ell \sinh \frac{t}{\ell} + \frac{\vec{x}^2}{2\ell} e^{t/\ell}\,, \\ X^i &= x^i e^{t/\ell}\,, \\ X^D &= \ell \cosh \frac{t}{\ell} - \frac{\vec{x}^2}{2\ell} e^{t/\ell}\,. \end{aligned} \tag{2.7}$$

This leads to the metric

$$ds^2 = -dt^2 + e^{2t/\ell} d\vec{x}^2\,, \tag{2.8}$$

where $d\vec{x}^2$ is the $(D-1)$-dimensional flat line element. Inflationary coordinates cover the upper right triangle of the Carter-Penrose diagram (cf. figure 3), with $t = \infty$ and $t = -\infty$ corresponding to future infinity and the past horizon respectively.
In order to obtain the metric in static coordinates, fix

$$(X^0)^2 - (X^D)^2 = r^2 - \ell^2\,. \tag{2.9}$$

One has then

$$(X^1)^2 + \ldots + (X^{D-1})^2 = r^2\,, \tag{2.10}$$

so the coordinates $X^1, \ldots, X^{D-1}$ range over S^{D-2} with radius r. If we parametrize the hyperbola (2.9) of fixed r by

$$X^0 = \sqrt{\ell^2 - r^2} \sinh \frac{t}{\ell}\,, \qquad X^D = \sqrt{\ell^2 - r^2} \cosh \frac{t}{\ell}\,, \tag{2.11}$$

then the induced metric on the hypersurface (2.1) is given by

$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right) dt^2 + \left(1 - \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega^2_{D-2}\,, \tag{2.12}$$

where $d\Omega^2_{D-2}$ is the round metric on the unit S^{D-2}. The line element (2.12) becomes singular for $r = \ell$, which is a cosmological event horizon to which one can associate a temperature and entropy [11]

$$T = \frac{1}{2\pi\ell}\,, \qquad S = \frac{A_{\mathrm{Hor}}}{4G}\,. \tag{2.13}$$

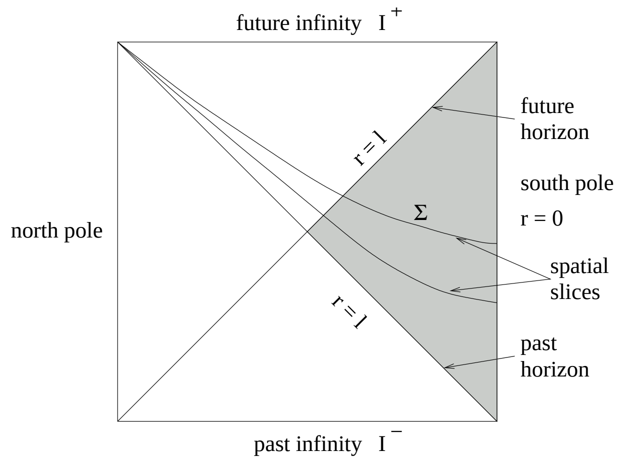

Figure 3: Carter-Penrose diagram for de Sitter space [41]. The future and past horizon of a static observer sitting at the south pole $r = 0$ (with r being the radial coordinate in (2.12)) of the spatial $(D-1)$-sphere are shown. The static patch is shaded. Two spatial slices Σ of constant time in inflationary coordinates are indicated.

Further useful parametrizations can be found in [39, 40].
The Carter-Penrose diagram of de Sitter space is shown in figure 3.
We noted above that de Sitter space has topology $\mathbb{R}\times\ \mathrm{S}^{D-1}$ and the noncompact isometry group $\mathrm{O}(D,1)$. If one requires dS physics to be maximally symmetric then this and elliptic de Sitter space[5] are the only possibilities. As was observed by Witten [15], maximal symmetry with non-compact groups is not welcome in a theory whose content is restricted to a finite-dimensional Hilbert space, and it was also strongly suggested [43] that the $O(D,1)$ symmetry must be spontaneously broken in the vacuum. The case for less symmetric versions of de Sitter (and anti-de Sitter) space was then proposed and defended in an elegant manner by McInnes [44, 45].
Having de Sitter spaces with less symmetry is therefore a good avenue for quantum gravity, and a way to achieve this is taking quotients by discrete groups of isometries. In fact, if a space M with symmetry group $\mathcal{G}$ is quotiented by a discrete subgroup Γ, acting without fixed points, then the isometries of M that are also isometries of M/Γ are those in the normalizer $N(\Gamma)$ of Γ, that is those $g \in \mathcal{G}$ such that $g\gamma g^{-1} \in \Gamma$ for any $\gamma \in \Gamma$. But since Γ acts trivially on M/Γ, the true isometry group is actually $N(\Gamma)/\Gamma$, which is even smaller than $N(\Gamma)$, and in general far smaller than the original group. In this way the symmetry is broken.
How much symmetry must be broken is unknown, but it seems fair enough to assume that a generic FLRW-like symmetry should remain, namely the universe should be spatially isotropic and homogeneous. This singles out the group $\Gamma = \mathbb{Z}_2$ generated by the antipodal map $(X^0, X^1, \ldots, X^D) \to (X^0, -X^1, \ldots, -X^D)$, which gives a de Sitter universe with

[5]This is obtained from dS through the antipodal identification $X^A \to -X^A$ of the embedding coordinates, and is a maximally symmetric, non time-orientable manifold with isometry group $\mathrm{SO}(D,1)$ [42].

spatial topology $\mathbb{RP}^{D-1}$. The metric remains the same, given by (2.6). For $D-1=2k$ this space in not orientable, and $\mathbb{RP}^{1+4k}$ is not a spin manifold, so dS($\mathbb{RP}^3$) and dS($\mathbb{RP}^7$) seem to be the only two available alternatives to a spherically based de Sitter space dS(S^n), at least for sufficiently low dimensions. The universe with $\mathbb{RP}^3$ spatial section was actually the preferred choice for de Sitter himself, for reasons that he left unexplained.
From the general rule given above, it is not difficult to see that the isometry group of projective de Sitter space is the compact group $\mathbb{Z}_2 \times \mathrm{O}(4)/\mathbb{Z}_2$, where the first $\mathbb{Z}_2$ factor is generated by the matrix $\mathrm{diag}(-1,1,1,1,1)$ and the second $\mathbb{Z}_2$ by the matrix $\mathrm{diag}(1,-1,-1,-1,-1)$. Apart from a discrete symmetry exchanging the future with the past, we see that the space is spatially homogeneous and isotropic with a compact symmetry group, and admits now finite-dimensional unitary representations. In view of these facts, it seems not easy to decide which is the "correct version" of de Sitter space (see [44] for a detailed discussion).

3 Conserved Charges

In order to make sense of a (still to be discovered) theory of quantum gravity on de Sitter spaces, or even of dS gravity classically, one would like to have a definition of energy in de Sitter spacetime, and this energy should be positive definite. Usually, a definition of energy is provided by the Abbott and Deser construction [46], where one considers spacetimes that asymptote at infinity to a fixed background, which has a certain number of Killing vectors. For each background Killing vector there exists then a conserved current, and the corresponding conserved charge can be expressed as a boundary integral at spatial infinity. Energy is then the charge associated to the time translation Killing vector. If the considered background is Minkowski or AdS space, one can use supersymmetry to show that this energy is positive definite [47, 48]. This is however not possible for dS space, as the latter is not a supersymmetric vacuum state of ordinary supergravity theories[6]. Another, related reason, why one does not expect a positive energy theorem to hold for dS gravity is the absence of a globally defined timelike Killing vector field. For instance the Killing vector ∂_t of the metric (2.12) is timelike inside the horizon $r=\ell$, but becomes spacelike for $r>\ell$. Correspondingly, the conserved charge associated to ∂_t receives negative contributions from matter or gravitational fluctuations outside the horizon [46]. This is related to the fact that actually ∂_t generates a boost in the isometry group, and a boost always has fixed points. More generally, all conjugacy classes[7] in the de Sitter groups $\mathrm{O}(4k,1)$ are ambivalents, which means that every group element is in the same class as its inverse. It then follows that every infinitesimal generator H can be transformed into its negative, that is, a group element G can be found such that $GHG^{-1}=-H$. In any unitary representation no positive definite operators can then be found, and it turns

[6]Note that de Sitter superalgebras exist [49, 50], but they do not have unitary highest weight representations. Accordingly, the corresponding supergravity theories (that admit supersymmetric dS vacua) have ghosts [49].

[7]A conjugacy class in a group is a set of elements of the form $g\gamma g^{-1}$. Different conjugacy classes are necessarily disjoint as they define an equivalence relation in the group.

out that if H represents the energy operator, the element G reversing the sign is the rotation sending a point to its antipode.
A further difficulty consists in the fact that in global coordinates (2.6) the spatial slices of dS_D are closed $(D-1)$-spheres, so they have no spatial infinity at which to define an ADM-like expression for the mass. In spite of these objections, Kastor and Traschen [38] were able to prove a positive energy theorem for asymptotically de Sitter spaces. The difficulties stated above are thereby circumvented as follows: First of all, energy is defined as the conserved charge associated to a globally timelike *conformal* Killing vector rather than to the time translation Killing vector ∂_t of (2.12) (which, as we said, is not globally timelike). Similar to Witten's proof of positive energy for asymptotically flat spaces [47], one can then use a spinor construction to show that this charge is positive[8]. Thereby the supercovariant derivative

$$\hat{\nabla}_\mu = \nabla_\mu + \frac{i}{2\ell}\gamma_\mu \,, \tag{3.1}$$

that appears in the dS supergravities of [49], plays an essential role. This means that dS supergravity, though facing non-unitary problems when quantized (at least perturbatively), is nevertheless useful for deriving classical results. Of course one must specify an asymptotic region where the mass is to be defined. As explained above, this is problematic in de Sitter space, where the spatial slices are compact (if we use global coordinates), and thus there is no spatial infinity. To overcome this, the authors of [38] considered metrics that approach asymptotically (in the far future) that of dS spacetime in inflationary coordinates (2.8). The spatial slices, two of which are shown in the Carter-Penrose diagram 3, are then planes, and the point at the upper left hand corner of the diagram can be regarded as spatial infinity.
Let us describe slightly more in detail the construction of [38], which is for $D=4$, so we specialize to this case in the rest of this section. The generalization to arbitrary dimension is straightforward. The isometry group of dS_4 is SO(4,1), and the conformal group SO(4,2). The five additional generators, i. e. , the conformal Killing vectors, are simply the projections of the translational Killing vectors $\xi^{(A)} = \partial/\partial X^A$ of the embedding space onto the hyperboloid (2.1). The particular linear combination $\zeta = \xi^{(0)} - \xi^{(4)}$ reads

$$\zeta = e^{t/\ell}\partial_t \,, \tag{3.2}$$

and is globally timelike and future directed. It is the conserved charge Q_ζ associated with ζ that is non-negative. For a general spacetime that asymptotes to (2.8) in the far future, Q_ζ can be calculated in the following way: Denote the metric of the general, asymptotically dS spacetime by $g_{\mu\nu}$, and that of the background (2.8) by $\tilde{g}_{\mu\nu}$. Define the deviation

$$h_{\mu\nu} = g_{\mu\nu} - \tilde{g}_{\mu\nu} \,, \tag{3.3}$$

[8]Shiromizu et al. [51] arrive independently at the association of a positive conserved charge with a globally timelike conformal Killing vector, but their line of reasoning is different from that of Kastor and Traschen: Positivity of the charge is derived starting from the ordinary positive energy theorem in a conformally related asymptotically flat spacetime.

which is small near spatial infinity. Define furthermore

$$\begin{aligned}
H^{\mu\nu} &= h^{\mu\nu} - \frac{1}{2}\tilde{g}^{\mu\nu}h \,, \\
K^{\alpha\beta\gamma\delta} &= \frac{1}{2}[\tilde{g}^{\alpha\delta}H^{\beta\gamma} + \tilde{g}^{\beta\gamma}H^{\alpha\delta} - \tilde{g}^{\alpha\gamma}H^{\beta\delta} - \tilde{g}^{\beta\delta}H^{\alpha\gamma}] \,, \\
\mathcal{B}^{\alpha\beta} &= (\tilde{\nabla}_\gamma K^{\alpha\beta\delta\gamma})\zeta_\delta - K^{\alpha\beta\delta\gamma}\tilde{\nabla}_{[\gamma}\zeta_{\delta]} \,,
\end{aligned} \tag{3.4}$$

where $\tilde{\nabla}$ denotes the connection of the de Sitter background. If one chooses a spatial surface Σ of the kind discussed above, with boundary $\partial\Sigma$, the conserved charge associated to ζ reads

$$Q_\zeta = \frac{1}{8\pi G}\int_{\partial\Sigma} \mathcal{B}^{\mu\nu} dS_{\mu\nu} \,, \tag{3.5}$$

where $dS_{\mu\nu}$ denotes the background area element. As an example, consider the Reissner-Nordström-de Sitter solution

$$ds^2 = -F(R)dT^2 + \frac{dR^2}{F(R)} + R^2 d\Omega_2^2 \,, \qquad F(R) = 1 - \frac{2m}{R} + \frac{q^2}{R^2} - \frac{R^2}{\ell^2} \,. \tag{3.6}$$

In inflationary-type coordinates, (3.6) reads[9]

$$ds^2 = -V(ar)dt^2 + a^2 U(ar)(dr^2 + r^2 d\Omega_2^2) \,, \tag{3.7}$$

where $a = \exp(t/\ell)$ and

$$V(ar) = \frac{\left[1 - \frac{m^2-q^2}{4a^2r^2}\right]^2}{\left[\left(1+\frac{m}{2ar}\right)^2 - \frac{q^2}{4a^2r^2}\right]^2} \,, \qquad U(ar) = \left[\left(1 + \frac{m}{2ar}\right)^2 - \frac{q^2}{4a^2r^2}\right]^2 \,. \tag{3.8}$$

The Carter-Penrose diagrams as well as a detailed study of this metric in relation to the dS/CFT duality can be found in the interesting paper [52]. Using (3.5), one obtains for the charge associated to the background conformal Killing vector $\zeta = a(t)\partial_t$,

$$Q_\zeta = a(t)\frac{m}{G} \,. \tag{3.9}$$

One might wonder about the time-dependence of Q_ζ, since, as was explained above, it is a conserved charge. The reason is that there is a nonzero flux of the spatial components of the conserved current $\mathcal{C}^\alpha = \tilde{\nabla}_\beta \mathcal{B}^{\alpha\beta}$ through spatial infinity.

We close this section with the remark that the existence of a conserved charge associated to a conformal Killing vector, and the fact that this charge can be shown to be positive semidefinite using the covariant derivative of dS supergravity seems to suggest that there are some residues of conformal supersymmetry in de Sitter.

[9] The coordinate transformation connecting (3.6) and (3.7) is given by $R = \sqrt{U(r')}r'$, $T = t - \int W(r')dr'$, $W(r') = \frac{r'}{\ell}U(r')/(\frac{r'^2}{\ell^2}U(r') - V(r'))$, $r' = a(t)r$.

4 De Sitter Space in String Theory

There are several reasons that make it desirable to embed de Sitter spaces in string theory. First, a full understanding of holography for dS gravity seems to require such an embedding. Second, being the most serious candidate for a complete quantum theory of gravity, string theory should admit dS vacua, if the astrophysical data that indicate that our universe is currently in a de Sitter phase, are interpreted correctly. However, dS vacua seem to be forbidden in string theory, due to some no-go theorems [9, 10]. In this section we will review them and discuss some possible ways in which these theorems can be circumvented.

4.1 No-Go Theorems

The first no-go theorem is due to Gibbons [9]. (For a more recent review cf. [53]). He considered warped compactifications $M = X \times_w Y$, where M and X denote N- and d-dimensional Lorentzian manifolds respectively, and Y is a compact $(N-d)$-dimensional Euclidean space. In local coordinates x^M which split as x^μ for X and y^m for Y, the metric on M reads

$$g_{MN}\, dx^M dx^N = w^2(y)\, g_{\mu\nu}\, dx^\mu dx^\nu + g_{mn}\, dy^m dy^n \,, \tag{4.1}$$

where $w(y)$ is the warp factor. The form of the line element (4.1) is restricted by the fact that for all pure supergravity models the bosonic energy-momentum tensor T_{MN} satisfies the strong energy condition, i. e. ,

$$\left(T_{MN} - \frac{1}{N-2} g_{MN} T^L{}_L\right) V^M V^N \geq 0 \tag{4.2}$$

for all non-spacelike vectors V^M. If we use the Einstein equations in M,

$$R_{MN} = 8\pi G_N \left(T_{MN} - \frac{1}{N-2} g_{MN} T^L{}_L\right) \,, \tag{4.3}$$

we obtain

$$R_{MN} V^M V^N \geq 0 \,, \tag{4.4}$$

or

$$R_{00} \geq 0 \tag{4.5}$$

in local coordinates. Physically this means that locally gravity is attractive. Applied to cosmology, it implies that the acceleration of the universe is always negative. This can be seen as follows: If Y is compact without boundary and $w(y)$ is smooth and nowhere vanishing, (4.5) implies ${}^X R_{00} \geq 0$, where ${}^X R_{\mu\nu}$ denotes the Ricci tensor of X [53]. Now take e. g. X to be Einstein, ${}^X R_{\mu\nu} = \lambda g_{\mu\nu}$. One has then $\lambda \leq 0$, so that de Sitter space is excluded.

The second no-go theorem goes back to Maldacena and Nuñez [10]. They also consider warped compactifications from N to d dimensions. The assumptions that go into the theorem are

- Higher curvature corrections (like those coming from integrating out massive string modes) are absent in the gravitational action
- The scalar potential is nonpositive
- Massless fields come with positive kinetic terms (note that this assumption is violated in Hull's II* theories [54])
- The d-dimensional effective Newton constant is finite
- The only possible singularities are such that the warp factor $w(y)$ goes to zero at the singularity

These assumptions imply that there is no compactification to de Sitter space [10].

4.2 How to circumvent them

Of course a no-go theorem is no better than the assumptions that go into it. There are several ways to circumvent the above theorems, e. g. by

- Considering non-compact internal manifolds [55], like e. g. hyperbolic spaces. "Compactification" of ten- or eleven-dimensional supergravity on such spaces gives rise to lower-dimensional supergravities with non-compact gaugings, which indeed admit dS vacua. The problem here is that the d-dimensional Newton constant G_d, which is related to the Newton constant G_N in N dimensions by $G_d = G_N/V_Y$, is zero, because the volume V_Y of the internal manifold is infinite. This indicates that gravity is not localized in the large extra dimensions of the internal space.
- Coupling supergravity to super-matter [56]. Generically, in such matter-coupled theories the strong-energy condition is violated, and the scalar potential can be positive, so that both Gibbon's theorem and that by Maldacena/Nuñez can be circumvented, and dS solutions are possible. It is however not clear how to embed these solutions in ten-or eleven-dimensional supergravity.
- Considering Hull's II* theories [54]. The IIB* theory, for instance, admits a $\mathrm{dS}_5 \times \mathrm{H}^5$ vacuum. Unfortunately, it is not clear if these theories are well-defined, because the kinetic terms of all RR fields have the wrong sign, which might lead to ghosts.
- Including localized sources with negative tension [57]. Note that such sources are present in string theory (e. g. orientifold planes). The no-go theorem of [10] states that in the absence of localized sources there can be no NS or RR fluxes, which are necessary for a nonconstant warp factor. (A constant warp factor does not allow dS compactifications [10]).
- Including α' or quantum corrections to the leading order supergravity Lagrangian [58]. The authors of [58] consider IIB compactifications with nontrivial NS and

RR three-form fluxes, and thus, as was explained above, they also need localized sources. Furthermore, they include nonperturbative quantum corrections to the superpotential for the Calabi-Yau moduli, which is generated in the presence of nonzero fluxes.

- Including the tachyon. This violates the strong energy condition. The rolling of the tachyon down its potential, away from the false (perturbative) vacuum, has been proposed as a mechanism for inflation. An accelerating universe is possible in such a scenario, but there are several shortcomings as a mechanism for inflation. A review of tachyon cosmology can be found in [53].

4.3 Spacelike Branes

There is yet another way to overcome the above no-go theorems, namely by allowing the size of the internal manifold to change in time. (Note that the metric (4.1) is static). This idea has been used by Townsend and Wohlfarth [59] to obtain FLRW universes in string theory that show a transient phase of acceleration. This makes these solutions phenomenologically interesting, although they are not really dS spaces[10]. Actually the accelerating cosmologies presented in [59] are a special case of so-called spacelike branes (S-branes) [64]. Before we come to a closer description of this type of branes, let us present the general idea following [65]. We start from the higher-dimensional action

$$I = \frac{1}{16\pi G_{4+n}} \int d^{4+n}x\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_{[4]}^2\right) , \tag{4.6}$$

and let the geometry be a warped product of a four-dimensional spacetime and an internal compact Einstein manifold $Y_{\sigma,n}$ with ${}^{Y}R_{mn} = \sigma(n-1)g_{mn}$, $\sigma = 0, \pm 1$,

$$ds^2 = e^{-n\psi(x)}g_{\mu\nu}(x)\, dx^\mu dx^\nu + e^{2\psi(x)}g_{mn}\, dy^m dy^n \, . \tag{4.7}$$

The field strength is taken as $*F_{[4]} = b\,\mathrm{vol}(Y_{\sigma,n})$. This leads upon reduction on $Y_{\sigma,n}$ to the four-dimensional action

$$I = \frac{1}{16\pi G_4} \int d^4x\sqrt{-g}\left(R - \frac{n(n+2)}{2}(\nabla\psi)^2 - V(\psi)\right) , \tag{4.8}$$

with the potential

$$V(\psi) = \frac{b^2}{2}e^{-3n\psi} - \sigma n(n-1)e^{-(n+2)\psi} \, . \tag{4.9}$$

A particularly interesting case appears for $n = 7$, which corresponds to a compactification of eleven-dimensional supergravity down to four dimensions. For vacuum solutions without flux ($b = 0$), one needs $\sigma < 0$ in order to have a positive potential (which is necessary to get an accelerated cosmic expansion in four dimensions). Thus, for $b = 0$,

[10] After the Townsend and Wohlfarth paper, other interesting accelerating cosmologies in this vein were found in [60–63].

the internal manifold $Y_{\sigma,n}$ must have negative scalar curvature. In particular, it could be a space of constant negative curvature, obtained by identifying hyperbolic n-space under the action of a freely acting discrete subgroup of its $\mathrm{SO}(1,n)$ isometry group. This is the case considered in [59]. For flux compactifications, $b \neq 0$, there is always a positive contribution to the potential, and also $\sigma \geq 0$ is possible.
The equations of motion following from (4.8) admit the FLRW solutions [65]

$$g_{\mu\nu}\, dx^\mu dx^\nu = -a^6(t)\, dt^2 + a^2(t)\, d\vec{x}_3^{\,2}\,, \tag{4.10}$$

where $d\vec{x}_3^{\,2}$ denotes the three-dimensional flat metric, and the scale factor is given by

$$a(t) = e^{B+n\psi/2}\,, \tag{4.11}$$

with

$$\begin{aligned} B(t) &= -\frac{1}{3}\log\left(b\sqrt{\frac{n-1}{6(n+2)}}\cosh 3(t-t_0)\right)\,, \\ \psi(t) &= \frac{1}{n-1}\log\gamma(t) - \frac{3}{n-1}B(t)\,, \\ \gamma(t) &= \begin{cases} \beta\,\mathrm{cosech}[(n-1)\beta|t|]\,, & \sigma = -1\,, \\ \exp[(n-1)\beta t]\,, & \sigma = 0\,, \\ \beta\,\mathrm{sech}[(n-1)\beta t]\,, & \sigma = +1\,, \end{cases} \end{aligned} \tag{4.12}$$

and

$$\beta = \frac{1}{n-1}\sqrt{\frac{3(n+2)}{n}}\,.$$

Here, t_0 denotes an integration constant. The proper time τ of a four-dimensional observer is defined by $d\tau = a^3(t)\, dt$. A closer analysis of the above solution shows that the condition for acceleration, $d^2a/d\tau^2 > 0$, is satisfied near the turning point of the radion ψ, which starts at $\psi \to +\infty$ with large kinetic energy and runs up the potential [65]. Note that, in this model, dark energy would be represented by the radion potential. A detailed study of (4.8) (with $n = 7$ and added dark matter) from a phenomenological point of view was presented in [66]. There, it was found that the model might not be phenomenologically viable, because the Compton wavelengths m_{KK}^{-1} of the Kaluza-Klein particles are of the same order as the size of the observable part of the universe, so that our universe would be effectively eleven-dimensional.
If we lift (4.10) to $4+n$ dimensions using (4.7), we obtain

$$ds^2 = -e^{6B+2n\psi}dt^2 + e^{2B}d\vec{x}_3^{\,2} + e^{2\psi}g_{mn}\, dy^m dy^n\,. \tag{4.13}$$

This solution, which was found in [67], is an example of a spacelike brane (if g_{mn} is the metric on a space of constant negative curvature). In particular, for $n = 7$, (4.13) represents an S2-brane of M-theory (SM2-brane). An S-brane is a topological defect

for which all of its longitudinal dimensions are spacelike, and therefore exists only for a moment of time. In the example above, the metric on the brane is given by $d\vec{x}_3^2$. The symmetry group of an Sp-brane in N dimensions is $\mathrm{ISO}(p+1) \times \mathrm{SO}(1, N-p-2)$, where $\mathrm{ISO}(p+1)$ is the group of motions on the Euclidean world volume of the brane, and $\mathrm{SO}(1, N-p-2)$ represents the isometry group of the hyperbolic $(N-p-2)$-manifold, into which the space transverse to the brane is sliced. Similar to Dp-branes, which are hypersurfaces where open strings can end, there exist SDp-branes, in which the time coordinate obeys a Dirichlet boundary condition.
For further literature on spacelike branes, we refer the reader to [68] and references therein.

5 The dS/CFT Correspondence

First evidence for a correspondence between de Sitter gravity in D dimensions and conformal field theories in $D-1$ dimensions was given by Hull [54][11], who considered so-called IIA* and IIB* string theories, which are obtained by T-duality on a timelike circle from the IIB and IIA theories respectively. The type IIB* theory admits E4-branes, which are the images of D4-branes under T-duality along the time coordinate of the brane. The E4-branes interpolate between Minkowski space at infinity and $\mathrm{dS}_5 \times \mathrm{H}^5$ near the horizon, where H^5 denotes hyperbolic space. The effective action describing E4-brane excitations is a Euclidean $D = 4$, $\mathcal{N} = 4$ $\mathrm{U}(N)$ super Yang-Mills theory, which is obtained from SYM in ten dimensions by reduction on a six-torus with one timelike circle. This leads to a duality between type IIB* string theory on $\mathrm{dS}_5 \times \mathrm{H}^5$ and the mentioned Euclidean SYM theory [54]. Unfortunately this example is pathological, because both theories have ghosts[12].
Later Strominger proposed a more general holographic duality relating quantum gravity on dS_D to a conformal field theory residing on one of the conformal boundaries of dS_D [16]. One of the reasons that lead to this proposal is the fact that the isometry group $\mathrm{O}(D,1)$ of dS_D coincides with the Euclidean conformal group in $D-1$ dimensions. If physical states of quantum gravity form a nontrivial representation of this group, this suggests that quantum gravity on dS_D is equivalent to a conformal field theory in one dimension lower. Strominger argued that in general this CFT may be non-unitary, with operators having complex conformal weights, if the dual bulk fields are sufficiently massive. We mention that an important check of the suggested dS/CFT correspondence came from the calculation of conformal anomalies [71], from applications to the problem of quantum creation of de Sitter universes [72] and from black hole physics [73].
Further support for a dS/CFT correspondence comes from three-dimensional de Sitter gravity, where more quantitative predictions can be made. In the absence of matter, dS_3

[11] For related work cf. [14, 69].

[12] Note however that Euclidean super Yang-Mills theory can be twisted to obtain a well-defined topological field theory in which the physical states are the BRST cohomology classes [70]. According to [54], this should correspond to a twisting of the type IIB* string theory, with a topological gravity limit.

gravity can be written as an SL$(2,\mathbb{C})$ Chern-Simons theory [74], with action

$$I = \frac{is}{4\pi}\int \mathrm{Tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A) - \frac{is}{4\pi}\int \mathrm{Tr}(\bar{A}\wedge d\bar{A} + \frac{2}{3}\bar{A}\wedge \bar{A}\wedge \bar{A})\,, \tag{5.1}$$

where

$$s = -\frac{\ell}{4G}\,, \tag{5.2}$$

and

$$A = A^a\tau_a = \left(\omega^a + \frac{i}{\ell}e^a\right)\tau_a\,, \qquad \bar{A} = \bar{A}^a\tau_a = \left(\omega^a - \frac{i}{\ell}e^a\right)\tau_a\,, \quad a = 0,1,2\,, \tag{5.3}$$

e^a denoting the dreibein, $\omega^a = \frac{1}{2}\epsilon^{abc}\omega_{bc}$ the spin connection, and the τ_a are SL$(2,\mathbb{R})$ generators.
Chern-Simons theory is known to reduce to a WZNW model in presence of a boundary [75][13]. The boundary conditions for asymptotically past de Sitter spaces [16] provide then the constraints for a Hamiltonian reduction from the WZNW model to Liouville field theory, leading to the action [76, 77]

$$I = \frac{1}{4\pi}\int \sqrt{g}d^2x\left[\frac{1}{2}g^{ij}\partial_i\Phi\partial_j\Phi + \frac{\lambda}{2\gamma^2}\exp(\gamma\Phi) + \frac{Q}{2}R\Phi\right]\,, \tag{5.4}$$

where the "cosmological constant" λ, the coupling constant γ and the background charge Q are given by $\lambda = 16\ell^{-2}$, $\gamma = \sqrt{8G/\ell}$ and $Q = \sqrt{\ell/2G}$ respectively. The Liouville model (5.4) is defined on the past conformal boundary $\mathcal{I}^-$ of dS$_3$. As is well-known, Liouville theory has a classical central charge $c = 12/\gamma^2$, which reproduces correctly the central charge $c = 3\ell/2G$ that appears in the asymptotic symmetry algebra of dS$_3$ gravity [16,39].
One can compute the Liouville field corresponding to the Schwarzschild-de Sitter solution[14]

$$ds^2 = -\left(\mu - \frac{r^2}{\ell^2}\right)dt^2 + \left(\mu - \frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2d\phi^2\,, \tag{5.5}$$

with the result [77]

$$e^{\gamma\Phi} = \mu\left[e^{\frac{i}{2}\sqrt{\mu}(z-\bar{z})} - e^{-\frac{i}{2}\sqrt{\mu}(z-\bar{z})}\right]^{-2}\,, \tag{5.6}$$

where $z = \phi + it/\ell$. Now remember that the asymptotic boundary of the spacetime (5.5) has the topology of a cylinder. One can pass to the plane (with coordinates $w, \bar{w}$) by the transformation $w = e^{iz}$, leading to

$$e^{\gamma\Phi}\,dzd\bar{z} = \frac{\mu}{(w\bar{w})^{1-\sqrt{\mu}}\left[1-(w\bar{w})^{\sqrt{\mu}}\right]^2}\,dwd\bar{w}\,, \tag{5.7}$$

[13] As we saw in section 2, de Sitter space has two conformal boundaries rather than one. In the reduction from the Chern-Simons theory (5.1) to a WZNW model carried out in [76], only the past boundary is considered. This might be motivated for instance if one is interested in spacetimes that are asymptotically dS in the past but not necessarily in the future. This occurs e. g. for configurations that finish in a big crunch, in which case there exists no future boundary. Below we will comment on this more in detail.

[14] We focus for the moment on the case $\mu > 0$. The case $\mu < 0$ will be considered later.

which is the standard classical elliptic solution of Liouville theory [78]. Semiclassically, the Liouville vertex operators $e^{\alpha\Phi}$ appear as sources of curvature in the classical equation of motion and lead to solutions with local elliptic monodromy with $\sqrt{\mu} = 1 - \gamma\alpha$ [78]. From this we obtain

$$\alpha = \frac{1 - \sqrt{\mu}}{\gamma}\,, \tag{5.8}$$

i. e. , a relation between the mass parameter μ of the dS_3 solution and the parameter α of the vertex operator. In the classical theory, $e^{\alpha\Phi}$ has conformal dimension

$$\Delta_{\mathrm{class}}(e^{\alpha\Phi}) = \frac{\alpha}{\gamma} = \frac{1 - \sqrt{\mu}}{\gamma^2} = \frac{\ell}{8G}(1 - \sqrt{\mu})\,. \tag{5.9}$$

Now the Schwarzschild-de Sitter solution contains a pair of conical defects at antipodal points on the spatial two-sphere, so only one of these is inside the cosmological horizon. The bare mass of a conical defect is [77, 79]

$$m_{\mathrm{class}} = \frac{1 - \sqrt{\mu}}{4G}\,. \tag{5.10}$$

Comparing this with (5.9), one obtains

$$\ell m_{\mathrm{class}} = \Delta_{\mathrm{class}} + \bar{\Delta}_{\mathrm{class}}\,, \tag{5.11}$$

so the classical conformal weights reproduce exactly the mass of the classical point particle in dS_3 that causes the conical defect.
The quantum dimension of the operator $e^{\alpha\Phi}$ is given by [78]

$$\Delta(e^{\alpha\Phi}) = \frac{\alpha}{\gamma} - \frac{1}{2}\alpha^2 = \frac{1 - \mu}{2\gamma^2} = \frac{c}{24}(1 - \mu) = \frac{l}{16G}(1 - \mu)\,. \tag{5.12}$$

Using $\bar{\Delta} = \Delta$ (the vertex operators are scalars), and $\tilde{c} = c$, we obtain

$$\Delta + \bar{\Delta} = \ell M + \frac{c + \tilde{c}}{24}\,, \tag{5.13}$$

where $M = -\mu/8G$ denotes the mass of the Schwarzschild-de Sitter solution (5.5) [40]. This means that the sum of the *quantum* conformal dimensions $(\Delta + \bar{\Delta})/\ell$ coincides (modulo a constant shift of $(c+\tilde{c})/24\ell$ coming from the transformation from the cylinder to the plane) with the mass M associated to the *classical* geometry (5.5). This mass includes the contribution of the gravitational field, and should not be confused with the bare mass (5.10) of the particles that cause the conical defects.
Eq. (5.8) establishes thus a quantitative correspondence between vertex operators in the boundary CFT and Schwarzschild-de Sitter solutions in the bulk. A further point that can be checked is the equality of the Liouville stress tensor for elliptic solutions [78] and the Brown-York energy-momentum tensor of (5.5) [77].

Let us now consider the gravity solutions with $\mu < 0$. We will see that they also have a nice interpretation in Liouville theory. Defining E by $\sqrt{\mu} = iE\gamma$, one gets for the classical Liouville solution corresponding to (5.5) with $\mu < 0$

$$e^{\gamma\Phi} dz d\bar{z} = \frac{E^2\gamma^2}{4w\bar{w}\sin^2\left(\frac{E\gamma}{2}\ln w\bar{w}\right)} dw d\bar{w}\,, \tag{5.14}$$

which can also be obtained from (5.7) by analytical continuation. As is well-known (cf. e. g. [80] for a review), the hyperbolic solution (5.14) corresponds in the semiclassical limit $\gamma \to 0$ to the normalizable quantum states ψ_E with momentum E, i. e. , to the so-called macroscopic states. Formally, these states can be associated to vertex operators $e^{\alpha\Phi}$ with

$$\alpha = \frac{Q}{2} + iE\,. \tag{5.15}$$

If we use $\sqrt{\mu} = iE\gamma$ in our relation (5.8) that connects vertex operators and bulk solutions, we get exactly (5.15). Furthermore, since the quantum state ψ_E has energy $E^2/2 + Q^2/8$ [78], the sum of the energies of ψ_E (right-moving) and ψ_{-E} (left-moving) is

$$E^2 + \frac{Q^2}{4} = \frac{1-\mu}{\gamma^2} = \ell M + \frac{c+\tilde{c}}{24}\,, \tag{5.16}$$

which gives again the mass $M = -\mu/8G$.

Summarizing, one has thus the following picture: Gravity solutions that have a temperature (i. e. , with $\mu \geq 0$) correspond to vertex operators with $\alpha = (1-\sqrt{\mu})/\gamma$, i. e. , to non-normalizable or microscopic states. Solutions with $\mu < 0$ correspond to normalizable or macroscopic states with real momentum E, where E is given by $\sqrt{\mu} = iE\gamma$. Classical dS_3 gravity encodes therefore (at least some of) the quantum properties of Liouville theory. However, in the reduction from the $\mathrm{SL}(2,\mathbb{C})$ Chern-Simons theory (5.1) to the Liouville model (5.4) only one conformal boundary was taken into account. It would be interesting to see what happens if one considers both boundaries. Some discussion on this can be found in [81].

In general it is an unsettled question if the CFT dual to de Sitter gravity resides on one boundary or on both. By considering two-point correlators with one point on $\mathcal{I}^-$ and another on $\mathcal{I}^+$, Strominger [16] argues that one can identify the past and future boundaries of de Sitter space by identifying points connected by null geodesics, so that the holographic dual is a field theory on one $(D-1)$-dimensional boundary rather than two. On the other hand, the authors of [81] propose that the dual CFT should involve two disjoint, but possibly entangled factors. In order to clarify these points, one would like to have an explicit example of dS/CFT emerging directly from string theory. If one wants to mimic the argumentation that lead to the AdS/CFT correspondence [8], this requires the existence of brane solutions that interpolate between flat space and de Sitter space (times some internal manifold). But, apart from the fact that such brane solutions seem to be forbidden by the no-go theorems of section 4, dS vacua break all supersymmetries (in conventional supergravity theories). This makes it questionable how far one could trust a Maldacena-type argument in this case.

A further strong argument in favor of dS/CFT that we should mention comes from the study of quasi-normal modes, the decaying perturbations of de Sitter space. Remarkably enough, Abdalla et al. [82] showed that the quasi-normal modes of scalar perturbations of de Sitter space are contained in the spectrum of boundary CFT correlators. These quasi-normal mode frequencies are given by (see also [83])

$$\omega = -\frac{i}{\ell}(2n + l + h_{\pm}) \quad \text{or} \quad \omega = -\frac{i}{\ell}(2n - l - D + 3 + h_{\pm})$$

for natural n and l, where $h_{\pm} = (D-1 \pm \sqrt{(D-1)^2 - 4m^2\ell^2})/2$ are the conformal weights of the dual boundary operators. When the conformal weights $h_{\pm}$ are real, which is a necessary condition for the CFT to be unitary, the scalar perturbations do not propagate in the bulk. By contrast, well defined quasi-normal modes exist in de Sitter space for complex weights, when the CFT is non unitary. The frequencies appear as poles in the Fourier transform of the boundary correlator in static coordinates, see Eq. (5.18) below.
Let us finally comment on some criticisms concerning the existence of a dS/CFT correspondence that appeared in [84]. The authors of [84] considered a general finite closed system described by a thermal density matrix, and a thermal correlator

$$F(t) = < \mathcal{O}(0)\mathcal{O}(t) > \ . \tag{5.17}$$

It was then shown that the long time average of $F(t)F^*(t)$ is non-zero and positive, which leads to a contradiction with the dS/CFT result in static coordinates [39][15],

$$< \mathcal{O}(0,0)\mathcal{O}(t,\phi) > \sim \left[\cosh\frac{t}{\ell} - \cos\phi\right]^{-h} , \tag{5.18}$$

where $h = 1 + \sqrt{1 - m^2\ell^2}$. (5.18) is not the standard thermal correlator, rather it is the two-point function for dimension (h, h) operators on a cylinder, whose length (not circumference) is parametrized by the Euclidean time coordinate t. (5.18) behaves like

$$< \mathcal{O}(0,0)\mathcal{O}(t,\phi) > \sim e^{-ht/\ell} \tag{5.19}$$

for large t. Clearly the behaviour (5.19) would imply a zero long-time average. Obviously the apparent contradiction found in [84], which is based on the assumption that the dual CFT is described by a thermal density matrix, is resolved if the conformal field theory does not encode the thermal nature of de Sitter space. We would like to point out here some arguments in favour of this.
First of all, the concept of assigning a temperature to de Sitter space is well-defined only in the static patches. However, the past (and future-) boundary, where the CFT resides,

[15]We only consider the simple case of (2+1)-dimensional de Sitter space and operators $\mathcal{O}$ that couple to bulk scalars of mass m.

lies outside the static region. In particular, the local Tolman temperature of de Sitter space,

$$T(r) = \frac{1}{2\pi\ell\sqrt{1-\frac{r^2}{\ell^2}}} \tag{5.20}$$

formally becomes imaginary for $r > \ell$. It might thus be that the conformal field theory on $\mathcal{I}^-$ does not capture the thermal nature of de Sitter space. If this is true, and if the dual CFT nevertheless accounts somehow for de Sitter entropy, we do not expect this entropy to be thermal[16].

We will now argue that in the Liouville approach discussed above, dS entropy has indeed a non-thermal interpretation. Let us start with the KPZ equation for gravitational dressing of CFT operators with bare conformal weight Δ_0 by vertex operators $e^{\alpha\Phi}$ (cf. e. g. [80]),

$$\alpha - \frac{Q}{2} = -\sqrt{\frac{1}{4}Q^2 - 2 + 2\Delta_0}\,. \tag{5.21}$$

Setting $\Delta = 1 - \Delta_0$ yields

$$\alpha - \frac{Q}{2} = -\sqrt{\frac{1}{4}Q^2 - 2\Delta}\,, \tag{5.22}$$

which is of course the formula for the quantum conformal weights Δ of vertex operators $e^{\alpha\Phi}$. Now observe that the entropy of the Schwarzschild-dS$_3$ solution (5.5) is given by

$$S = \frac{\pi\ell\sqrt{\mu}}{2G} = \frac{4\pi\sqrt{\mu}}{\gamma^2}\,, \tag{5.23}$$

and, using (5.8) as well as the background charge $Q = 2/\gamma$, that

$$\alpha - \frac{Q}{2} = -\frac{\sqrt{\mu}}{\gamma} = -\frac{S\gamma}{4\pi}\,. \tag{5.24}$$

Inserting (5.24) into (5.22), one obtains

$$S = 2\pi\sqrt{2Q^2\left(\frac{Q^2}{8} - \Delta\right)}\,. \tag{5.25}$$

If we finally use the central charge $c = 3Q^2$ and $\bar{\Delta} = \Delta$, we get

$$S = 2\pi\sqrt{\frac{c}{6}\left(\frac{c}{24} - \Delta\right)} + 2\pi\sqrt{\frac{c}{6}\left(\frac{c}{24} - \bar{\Delta}\right)}\,. \tag{5.26}$$

(5.26) looks like the Cardy formula for the asymptotic level density of conformal field theories, but actually it is not, because the signs of the terms Δ and $c/24$ are interchanged.

[16] A non-thermal interpretation of de Sitter entropy in terms of a sort of Euclidean entanglement entropy was proposed in [85].

Rather, we saw that (5.26) is the KPZ equation. Looking at (5.24), one also sees what de Sitter entropy corresponds to in Liouville theory. Using

$$iE = \alpha - \frac{Q}{2} \tag{5.27}$$

for the Liouville momentum E (cf. (5.15)), one finally obtains

$$S = -\frac{4\pi i}{\gamma} E\,, \tag{5.28}$$

so that de Sitter entropy is essentially Liouville momentum. In particular, it has no statistical meaning in this approach. This is reminiscent of Wald's Noether charge interpretation of black hole entropy [86]. Note that the Schwarzschild-dS$_3$ solution with $\mu > 0$ corresponds to imaginary momentum E, so that S is real, as it should be.

6 Finite-dimensional Hilbert Space?

Banks and Fischler independently proposed [17,18] (see also [36] and [37]) that the Hilbert space of quantum gravity on de Sitter space is finite-dimensional, and argued that dS entropy is to be interpreted as the logarithm of the total number of states in the Hilbert space[17]. Let us briefly review some evidence for this proposal, which is of course at odds with a dS/CFT correspondence.

The first argument comes essentially from classical general relativity. In an unpublished work, Horowitz and Itzhaki showed that the classical phase space of four-dimensional general relativity with asymptotically de Sitter boundary conditions both in the past and in the future, is compact, if the energy-momentum tensor is that of homogeneous matter [87]. It is well-known that a compact phase space yields a finite-dimensional Hilbert space when quantized. The second argument in favour of a finite number of states goes as follows: Try to contradict the idea that asymptotically dS spaces have a finite number of degrees of freedom. Consider e. g. a spacetime which is dS$_D$ in the remote past. As the volume of space (a $(D-1)$-sphere) is very large, one can easily impose initial conditions that have a larger entropy than dS. However these initial conditions will not lead to an asymptotically dS solution in the remote future, rather the spacetime will finish in a big crunch [17]. Let us explain this in more detail: If we attribute to each source on the spacelike past boundary $\mathcal{I}^-$ a finite energy density, no matter how small, then for some finite number of sources, the resulting spacetime cannot be asymptotically dS in past and future. The energy density grows as we approach the point of maximal contraction, and at some point black holes form with radius larger than the putative radius of the dS sphere. In other words, we encounter a cosmological singularity and do not asymptote to dS space[18]. In the AdS/CFT correspondence, the boundary values Φ_0

[17]Goheer, Kleban and Susskind [43] assume only the weaker condition that the spectrum is discrete, which follows from finiteness of the entropy. Note that a finite entropy does not necessarily require a finite-dimensional Hilbert space.

[18]We are grateful to Tom Banks for clarifying correspondence on this point.

of bulk fields Φ are sources for CFT operators $\mathcal{O}$, but, according to what was said above, in dS we cannot put arbitrary sources. This seems thus to be a problem for a possible dS/CFT correspondence.

Apparently the Banks-Fischler argument does not take into account the gravitational backreaction of the sources which can be arbitrarily big. The classical example is that of a point mass in general relativity, where Arnowitt, Deser and Misner showed that for any bare mass gravity responds in a way that makes its clothed mass vanish [88]. Thus, in order to complete the argumentation by Banks and Fischler, one has to show that the negative gravitational interaction energy cannot accomodate the excitation of an arbitrarily large number of matter degrees of freedom.

Further evidence for a finite-dimensional Hilbert space comes from the consideration of black holes in de Sitter space. Whereas in asymptotically flat or AdS spacetimes one can have black holes with arbitrarily large entropies, this is not possible in de Sitter spaces, where the entropy of a black hole is bounded from above by the entropy of the Nariai solution, which is the largest black hole that can fit within the cosmological horizon[19]. Thus one cannot have excitations with arbitrarily large entropy. We shall discuss this point further in section 7.

One might argue that a free matter quantum field theory on dS, renormalized such that the vacuum expectation value of the stress tensor is zero, has an infinite number of degrees of freedom, and the vacuum state would support de Sitter spacetime. However, the claim by Banks and Fischler is that the *combined* system matter plus gravity is described by a finite-dimensional Hilbert space. That free matter fields on dS give rise to a stable theory seems to be doubtful in view of the perturbative results described in the introduction. (Already a small perturbation $\lambda\phi^4$ of a free scalar leads to an exponential production of particles and thus to instabilities [32]).

Note that the isometry group $\mathrm{SO}(D,1)$ of dS_D has no nontrivial finite-dimensional unitary representations. If the Hilbert space $\mathcal{H}$ is finite-dimensional then the dS isometry group cannot act on $\mathcal{H}$ [15].

It is not clear how a finite-dimensional Hilbert space is compatible with a dS/CFT correspondence. Yet there are some possible loopholes that we will describe below.

First of all, the infinite dimension of the CFT Hilbert space might be cut down by a constraint such as $L_0 + \bar{L}_0 = 0$ [90], where L_0, $\bar{L}_0$ denote the Virasoro generators in the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence. After all, the total energy in dS is zero, because spatial sections are closed. (This is just the gravitational analogue of the usual Gauss law in electrodynamics).

The second way out of the conundrum is due to Witten [15]. In AdS, near the boundary $r \to \infty$, the metric behaves as

$$ds^2 \to dr^2 + e^{2r} d\vec{x}^2 \,, \tag{6.1}$$

[19]Something similar happens for black holes in Gödel-type universes [89], whose size is bounded from above by the velocity of light surface, beyond which closed timelike curves appear. This leads also to a maximal entropy.

where $d\vec{x}^2$ denotes the flat line element. This is generalized to

$$ds^2 \to dr^2 + e^{2r} g_{ij}^{(0)} dx^i dx^j \,, \tag{6.2}$$

with $g_{ij}^{(0)}$ being an arbitrary metric on the boundary, conformal to $d\vec{x}^2$. By considering dependence on $g_{ij}^{(0)}$, one gets correlation functions of the stress tensor in the boundary CFT,

$$< T_{i_1 j_1}(x_1) \ldots T_{i_m j_m}(x_m) >= \frac{\delta}{\delta g_{i_1 j_1}^{(0)}(x_1)} \cdot \ldots \cdot \frac{\delta}{\delta g_{i_m j_m}^{(0)}(x_m)} I(g_{ij}^{(0)}) \,, \tag{6.3}$$

where $I(g_{ij}^{(0)})$ is the bulk action for the field g_{ij} with boundary value $g_{ij}^{(0)}$. In de Sitter space, we have two boundaries $t \to \pm\infty$, where the metric approaches

$$ds^2 \to -dt^2 + e^{\pm 2t} d\Omega_{D-1}^2 \qquad \text{for} \quad t \to \pm\infty \,. \tag{6.4}$$

Here, $d\Omega_{D-1}^2$ denotes the round metric on S^{D-1}. To prepare an initial or final state $|i\rangle$ or $\langle f|$, pick a conformal metric $g^{(i)}$ or $g^{(f)}$ on the sphere and require

$$ds^2 \to \begin{cases} -dt^2 + e^{-2t} g_{ij}^{(i)} dx^i dx^j \,, & t \to -\infty \,, \\ -dt^2 + e^{+2t} g_{ij}^{(f)} dx^i dx^j \,, & t \to +\infty \,. \end{cases} \tag{6.5}$$

Then the path integral for metrics with this asymptotics gives an "observable" $\langle f|i\rangle$. (Witten calls this "metaobservable", because the formulation of $\langle f|i\rangle$ requires a global view of $\mathcal{I}^-$ and $\mathcal{I}^+$, and this is not available to any observer living in de Sitter space). $\langle f|i\rangle$ is an $\infty \times \infty$ matrix, but it may have finite rank. This finite rank is the dimension of the Hilbert space [15].

A further loophole that we should mention is the possibility that only a finite-dimensional subspace $\mathcal{H}$ of the CFT Hilbert space leads from a dS geometry in the remote past to a dS geometry in the asymptotic future, whereas states not contained in $\mathcal{H}$ might lead to a big crunch, in accordance with the discussion above.

Last but not least, there is the proposal by Güijosa and Lowe [91], who emphasized that dS/CFT should be formulated using unitary principal series representations of the de Sitter isometry group/conformal group, as opposed to the standard highest-weight representations usually considered in conformal field theory[20]. This avoids the problems associated with the non-unitarity of the highest-weight representations that appear in [16], but suffers from the drawback that the principal series representations of $\mathrm{SO}(D,1)$ are infinite-dimensional, and so do not account for the finite gravitational entropy of dS space in a natural way. For this reason the authors of [91] proposed to replace the classical isometry group by a q-deformed version, where q is a root of unity. This was carried out for dS_2 in [91] and for dS_3 in [92], and it was found that the unitary principal series representations deform to *finite-dimensional* unitary representations of the quantum group.

[20]This was also observed in [81].

In order to understand a little bit more in detail what is going on, let us ask the question how we would recognize a possible holographic dual of gravity on de Sitter spaces. The answer for AdS/CFT was initially through the symmetries, which exactly matched. Thus we might try to recognize the dS dual by looking at its symmetries, and requiring that they contain the $\mathrm{SO}(D,1)$ isometry group of dS_D. As this group coincides with the Euclidean conformal group in one dimension lower, this would suggest that the holographic dual is a Euclidean CFT in $D-1$ dimensions. The point is now that most of this group does not preserve the region that is causally accessible to an observer (the static region, shaded part of figure 3) [43]. This causal patch is preserved by the subgroup $\mathbb{R}\times\ \mathrm{SO}(D-1)$, where $\mathbb{R}$ corresponds to time translations (in the time t of (2.12)), and $\mathrm{SO}(D-1)$ is generated by rotations that leave the horizon invariant. Therefore the holographic dual associated with an *individual* observer should have the symmetry group $\mathbb{R}\times\ \mathrm{SO}(D-1)$ rather than $\mathrm{SO}(D,1)$, and is thus not a conformal field theory in $D-1$ dimensions. Banks constructed a toy model of such a "dS quantum mechanics" [93], by using fuzzy spheres, which allow to realize the spherical geometry of the horizon in a way that is compatible with a finite number of states[21]. His model was able to reproduce qualitatively the entropies of the cosmological and the black hole horizon.

In conclusion, the idea is that each *individual* observer has access only to a finite amount of degrees of freedom associated with the corresponding holographic region (her causal patch). There are *local*, observer-dependent holographic screens, which coincide with the observer's horizon [6]. Interestingly enough, however, Bousso's recipe to construct holographic screens yields a second possibility in the case of dS space, namely a *global* screen located either at $\mathcal{I}^-$ or at $\mathcal{I}^+$ [6]. Using such a global screen seems to lead to dS/CFT. Moreover, the idea that individual observers can access only a fraction of the total degrees of freedom is challenged by the semiclassical validity of the Reeh-Schlieder property in de Sitter space [96, 97]. Thus while it is true that the static vacuum has only the $\mathbb{R}\times\ \mathrm{SO}(D-1)$ symmetry, the geodesic observer can nevertheless explore the full Hilbert space by means of local operations performed on a de Sitter invariant vacuum in his/her causal patch.

7 De Sitter Thermodynamics

We conclude this review with a curiosity of de Sitter thermodynamics, namely the possible appearance of negative absolute temperatures. The discussion below is not to be intended as a proof, but rather as an alternative description of the same physics. Consider the Schwarzschild-dS_4 solution, which describes a black hole immersed in a dS background,

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega_2^2\,, \tag{7.1}$$

[21] Also Li [94] has utilized fuzzy spheres for a hypothetical description of dS quantum mechanics. Parikh and Verlinde [95] proposed as holographically dual theory a spin system which has finite dimension (being a representation of $\mathrm{SO}(D-1)$), but in which nevertheless there are $\mathrm{SO}(D,1)$-invariant probabilities.

with

$$V(r) = 1 - \frac{2m}{r} - \frac{r^2}{\ell^2}\,. \tag{7.2}$$

For $0 < m < m_N$, where

$$m_N = \frac{\ell}{3\sqrt{3}}\,, \tag{7.3}$$

there are two positive roots r_h and $r_c > r_h$ where $V(r)$ vanishes. r_h corresponds to the black hole event horizon, and r_c to the cosmological horizon. For $m = m_N$ both roots coalesce and we have the Nariai solution [98], which represents the largest black hole one can have in de Sitter space. For $m < 0$ the black hole disappears, and the spacetime describes a naked singularity in $r = 0$ surrounded by a cosmological horizon. Finally, for $m > m_N$ there is no static region. In this case the solution is asymptotically dS only in the far past (cf. the Carter-Penrose diagram, figure 4). If we want to consider only spacetimes that approach dS in both past and future, we have to discard the solutions with $m > m_N$.

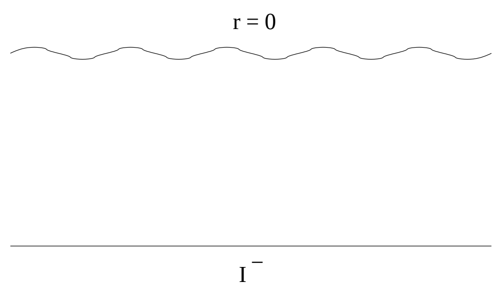

Figure 4: Carter-Penrose diagram for Schwarzschild-de Sitter space with mass parameter $m > m_N$. The solution starts from an asymptotically de Sitter geometry at past infinity $\mathcal{I}^-$ $(r = \infty)$, and finishes in a big crunch at $r = 0$ (curvature singularity). There is also a time inverted solution with a curvature singularity in the past.

The entropies associated to the black hole and the cosmological horizon are given by $S_h = A_h/4G = \pi r_h^2/G$ and $S_c = A_c/4G = \pi r_c^2/G$ respectively, where $A_{h,c}$ denote the horizon areas.
It is well-known that the imaginary time periods required to avoid conical singularities in the Euclidean section at both the black hole and the cosmological horizons do not match [11]. Physically this corresponds to the fact that the two horizons are not in thermal equilibrium. Strictly speaking the only thermal equilibrium state is given by the Nariai solution, with energy E proportional to m_N. Now start from this equilibrium state and subtract an infinitesimal amount of energy from the system. This results in a small separation of the black hole and the cosmological horizon. The black hole horizon shrinks slightly, whereas the cosmological horizon increases. Hence the entropy of the black hole decreases with decreasing energy, whereas the entropy of the cosmological horizon increases. As we have

$$\frac{1}{T} = \frac{\partial S}{\partial E}\,, \tag{7.4}$$

this behaviour implies that one should ascribe a negative temperature to the cosmological horizon.
At this point, let us open a parenthesis on negative absolute temperatures. Negative absolute temperatures [99] occur whenever the entropy of a thermodynamical system is not a monotonically increasing function of its internal energy. As T is defined by Eq. (7.4), we see that $T < 0$ if the entropy S decreases with increasing E. As a concrete example consider e. g. N atoms with spin $1/2$ on a one-dimensional wire in an external magnetic field pointing down (cf. figure 5). Suppose that spin-flip is the only degree of freedom. In the highest energy state all spins point up, and the entropy is zero (fig. 5i)). If we flip one spin, the energy is lowered, but the entropy increases, because there are N available microstates (one of which is shown in fig. 5ii)).

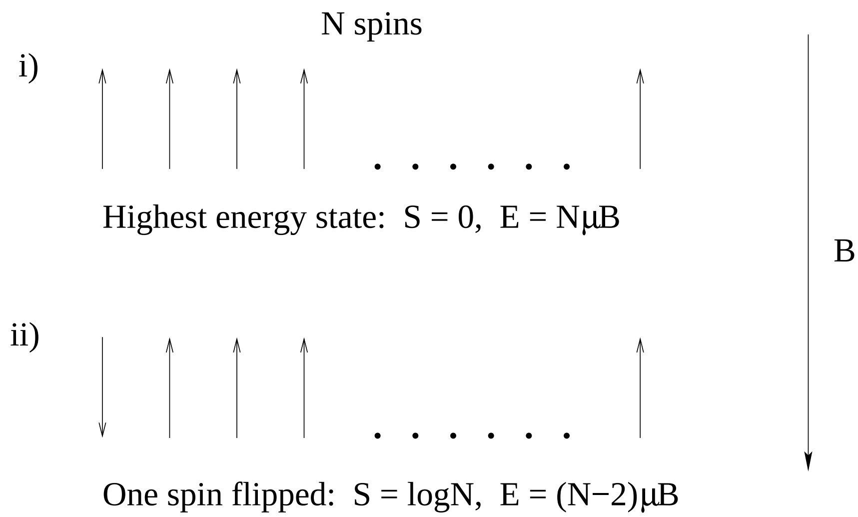

Figure 5: A typical system that exhibits negative temperatures: N atoms with spin $1/2$ and magnetic moment μ on a one-dimensional wire, in an external magnetic field B pointing down, with spin-flip the only degree of freedom.

The temperature is thus negative in this regime. Nuclear spin systems in pure LiF crystals do indeed realize negative temperatures experimentally [100]. In these systems, the spin-lattice relaxation times are as large as 5 minutes, whereas the spin-spin relaxation times are less than 10^{-5} seconds, so that the defintion of a spin temperature makes sense. Another well-known example displaying negative temperature is the laser (population inversion). A further class of problems that exhibit negative temperatures is the statistical mechanics of a vortex gas or Coulomb gas in two dimensions. Notice that any negative temperature is hotter than any positive temperature while for two temperatures of the same sign the one with the algebraically greater value is the hotter [99].
Note also that a necessary condition for the appearance of negative temperatures is an upper bound to the possible energy of the allowed states. To see this, consider the Boltzmann factor $\exp(-E_n/k_B T)$, which increases exponentially with increasing E_n for $T < 0$, so that the high-energy states are occupied more than the low-energy ones. (As

already mentioned above, this happens e. g. in a laser). Consequently, with no upper limit to the energy, negative temperatures could not be achieved with a finite energy.
But this is exactly what happens in de Sitter gravity, where the mass is bounded from above by the mass of the largest black hole that can fit within the cosmological horizon. Furthermore, negative temperatures typically occur in systems with finite-dimensional Hilbert spaces (like the spin system that we discussed). The observed thermodynamical behaviour fits thus with the claim by Banks and Fischler.
Actually, for the Schwarzschild-de Sitter black hole (7.1), even the *total* entropy of the black hole and the cosmological horizon decreases with increasing energy. This can be shown by using the result of Gibbons and Hawking [11], who integrated the Killing identity on a spacelike hypersurface Σ from r_h to r_c to get the Smarr-type formula

$$GM_c = \frac{\kappa_h A_h}{4\pi} + \frac{1}{4\pi}\int_\Sigma \Lambda K_\mu \, d\Sigma^\mu \,, \tag{7.5}$$

where $M_c = -\kappa_c A_c/4\pi$ denotes the total mass within the cosmological horizon and $\kappa_{h,c}$ are the surface gravities of the black hole and the cosmological horizon respectively. Furthermore, $K = \partial_t$ and $\Lambda = 3/\ell^2$ is the cosmological constant. One can interpret the first term on the rhs of Eq. (7.5) as the (positive) mass of the black hole, and the second term as the (negative) contribution of Λ to the total mass M_c within the cosmological horizon [11]. Evaluating (7.5) yields

$$GM_c = \frac{r_c}{2} - \frac{3r_c^3}{2\ell^2} \,. \tag{7.6}$$

Using the relation

$$r_h^2 + r_c^2 + r_h r_c = \ell^2 \,, \tag{7.7}$$

as well as $r_c \geq \ell/\sqrt{3}$ (the minimum value of r_c is obtained for the Nariai black hole), it is straightforward to show that M_c is a monotonically decreasing function of the total entropy $S_{tot} = S_h + S_c = \pi(r_h^2 + r_c^2)/G$. If we start from pure de Sitter space and form a black hole, then the total mass within the cosmological horizon increases, but this excitation has lower total entropy than dS space itself.
In three dimensions, the black hole horizon degenerates to a conical singularity in $r = 0$, and the Schwarzschild-de Sitter solution with mass E reads [79]

$$ds^2 = -\left(1 - 8GE - \frac{r^2}{\ell^2}\right) dt^2 + \left(1 - 8GE - \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\phi^2 \,. \tag{7.8}$$

The entropy of the cosmological horizon is $S_c = \pi r_c/2G$, where $r_c^2 = \ell^2(1 - 8GE)$. This leads to the thermodynamic fundamental relation

$$E(S_c) = \frac{1}{8G}[1 - (2GS_c/\pi\ell)^2] \,, \tag{7.9}$$

and thus $\partial E/\partial S_c$ yields minus the temperature normally assigned to dS space. In order to explain this, the authors of [40] argued that instead of (7.4) one should use

$$\frac{1}{T} = \frac{\partial S}{\partial(-E)} \tag{7.10}$$

to compute the dS temperature. The reason for this is that de Sitter entropy is supposed to correspond to the entropy of the degrees of freedom behind the horizon which cannot be observed. As the spatial sections of dS are spheres, putting something with positive energy on the north pole (where we assume that our observer sits) implies that necessarily there will be some negative energy on the south pole (i. e. , beyond the horizon of our observer), therefore the minus sign in Eq. (7.10). If the observer instead varies the entropy with respect to the energy $+E$ within her horizon then the usual laws of thermodynamics apply, but the price to pay is the introduction of a negative temperature.

Acknowledgements

This work was partially supported by INFN, MURST and by the European Commission RTN program HPRN-CT-2000-00131, in which D. K. is associated to the University of Torino. We would like to thank V. Balasubramanian, T. Banks and A. Strominger for useful discussions/correspondence.

References

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], "Observational evidence from supernovae for an accelerating universe and a cosmological constant," Astron. J. **116** (1998) 1009 [arXiv:astro-ph/9805201];
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], "Measurements of Omega and Lambda from 42 high-redshift supernovae," Astrophys. J. **517** (1999) 565 [arXiv:astro-ph/9812133].

[2] C. L. Bennett *et al.*, "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," Astrophys. J. Suppl. **148** (2003) 1 [arXiv:astro-ph/0302207].

[3] S. Weinberg, "The cosmological constant problem," Rev. Mod. Phys. **61** (1989) 1.

[4] S. M. Carroll, "The cosmological constant," Living Rev. Rel. **4** (2001) 1 [arXiv:astro-ph/0004075];
U. Ellwanger, "The cosmological constant," arXiv:hep-ph/0203252;
P. J. Peebles and B. Ratra, "The cosmological constant and dark energy," Rev. Mod. Phys. **75** (2003) 559 [arXiv:astro-ph/0207347].

[5] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026;
L. Susskind, "The World as a hologram," J. Math. Phys. **36** (1995) 6377 [arXiv:hep-th/9409089].

[6] R. Bousso, "Holography in general space-times," JHEP **9906** (1999) 028 [arXiv:hep-th/9906022].

[7] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2** (1998) 253 [arXiv:hep-th/9802150].

[8] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200].

[9] G. W. Gibbons, "Aspects of supergravity theories," Print-85-0061 (CAMBRIDGE) *Three lectures given at GIFT Seminar on Theoretical Physics, San Feliu de Guixols, Spain, Jun 4-11, 1984*

[10] J. M. Maldacena and C. Nuñez, "Supergravity description of field theories on curved manifolds and a no go theorem," Int. J. Mod. Phys. A **16** (2001) 822 [arXiv:hep-th/0007018].

[11] G. W. Gibbons and S. W. Hawking, "Cosmological event horizons, thermodynamics, and particle creation," Phys. Rev. D **15** (1977) 2738.

[12] A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," Phys. Lett. B **379** (1996) 99 [arXiv:hep-th/9601029].

[13] J. M. Maldacena and A. Strominger, "Statistical entropy of de Sitter space," JHEP **9802** (1998) 014 [arXiv:gr-qc/9801096];
M. Banados, T. Brotz and M. E. Ortiz, "Quantum three-dimensional de Sitter space," Phys. Rev. D **59** (1999) 046002 [arXiv:hep-th/9807216];
S. Hawking, J. M. Maldacena and A. Strominger, "DeSitter entropy, quantum entanglement and AdS/CFT," JHEP **0105** (2001) 001 [arXiv:hep-th/0002145];
M. Astorino, S. Cacciatori, D. Klemm and D. Zanon, "AdS_2 supergravity and superconformal quantum mechanics," Annals Phys. **304** (2003) 128 [arXiv:hep-th/0212096].

[14] M. I. Park, "Statistical entropy of three-dimensional Kerr-de Sitter space," Phys. Lett. B **440** (1998) 275 [arXiv:hep-th/9806119].

[15] E. Witten, "Quantum gravity in de Sitter space," arXiv:hep-th/0106109.

[16] A. Strominger, "The dS/CFT correspondence," JHEP **0110** (2001) 034 [arXiv:hep-th/0106113].

[17] T. Banks, "Cosmological breaking of supersymmetry or little Lambda goes back to the future. II," arXiv:hep-th/0007146.

[18] W. Fischler, "Taking de Sitter seriously", talk given at *Role of Scaling Laws in Physics and Biology (Celebrating the 60th Birthday of G. West)*, Santa Fe, December 2000.

[19] V. Balasubramanian, "Accelerating universes and string theory," Class. Quant. Grav. **21** (2004) S1337 [arXiv:hep-th/0404075].

[20] O. Nachtmann, "Quantum theory in de Sitter space," Commun. Math. Phys. **6**, 1 (1967);
N. A. Chernikov and E. A. Tagirov, "Quantum theory of scalar fields in de Sitter space-time," Annales Poincaré Phys. Theor. A **9** (1968) 109;
E. A. Tagirov, "Consequences of field quantization in de Sitter type cosmological models," Annals Phys. **76**, (1973) 561;
S. A. Fulling, "Scalar quantum field theory in a closed universe of constant curvature," Ph. D. Thesis, Princeton University, 1972 (unpublished);
R. Figari, R. Hoegh-Krohn and C. R. Nappi, "Interacting relativistic boson fields in the de Sitter universe with two space-time dimensions," Commun. Math. Phys. **44** (1975) 265.

[21] J. Géhéniau and C. Schlomblond, Acad. R. Belg. Bull. Cl. Sci. **54** (1968) 1147.

[22] E. Mottola, "Particle creation in de Sitter space," Phys. Rev. D **31** (1985) 754.

[23] B. Allen, "Vacuum states in de Sitter space," Phys. Rev. D **32** (1985) 3136.

[24] C. J. C. Burges, "The de Sitter vacuum," Nucl. Phys. B **247** (1984) 533.

[25] R. Bousso, A. Maloney and A. Strominger, "Conformal vacua and entropy in de Sitter space," Phys. Rev. D **65** (2002) 104039 [arXiv:hep-th/0112218].

[26] M. B. Einhorn and F. Larsen, "Interacting quantum field theory in de Sitter vacua," Phys. Rev. D **67** (2003) 024001 [arXiv:hep-th/0209159];
U. H. Danielsson, "On the consistency of de Sitter vacua," JHEP **0212** (2002) 025 [arXiv:hep-th/0210058];
H. Collins, R. Holman and M. R. Martin, "The fate of the alpha-vacuum," Phys. Rev. D **68** (2003) 124012 [arXiv:hep-th/0306028];
J. de Boer, V. Jejjala and D. Minic, "Alpha-states in de Sitter space," [arXiv:hep-th/0406217].

[27] A. D. Linde, "Scalar field fluctuations in expanding universe and the new inflationary universe scenario," Phys. Lett. B **116** (1982) 335;
A. A. Starobinsky, "Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations," Phys. Lett. B **117** (1982) 175;
A. Vilenkin and L. H. Ford, "Gravitational effects upon cosmological phase transitions," Phys. Rev. D **26** (1982) 1231;
L. H. Ford and L. Parker, "Infrared divergences in a class of Robertson-Walker universes," Phys. Rev. D **16** (1977) 245.

[28] B. Allen and A. Folacci, "The massless minimally coupled scalar field in de Sitter space," Phys. Rev. D **35** (1987) 3771.

[29] L. H. Ford, "Quantum instability of de Sitter space-time," Phys. Rev. D **31** (1985) 710.

[30] K. Kirsten and J. Garriga, "Massless minimally coupled fields in de Sitter space: O(4) symmetric states versus de Sitter invariant vacuum," Phys. Rev. D **48** (1993) 567 [arXiv:gr-qc/9305013];
A. J. Tolley and N. Turok, "Quantization of the massless minimally coupled scalar field and the dS/CFT correspondence," [arXiv:hep-th/0108119].

[31] D. W. Sciama, P. Candelas and D. Deutsch, "Quantum field theory, horizons and thermodynamics," Adv. Phys. **30** (1981) 327.

[32] N. P. Myhrvold, "Runaway particle production in de Sitter space," Phys. Rev. D **28** (1983) 2439.

[33] N. P. Myhrvold, "The existence and stability of semiclassical de Sitter and anti-de Sitter space-times," Phys. Lett. B **132** (1983) 308.

[34] N. C. Tsamis and R. P. Woodard, "Mode analysis and Ward identities for perturbative quantum gravity in de Sitter space," Phys. Lett. B **292** (1992) 269, "Relaxing the cosmological constant," Phys. Lett. B **301** (1993) 351, "The structure of perturbative quantum gravity on a de Sitter background," Commun. Math. Phys. **162** (1994) 217, "The physical basis for infrared divergences in inflationary quantum gravity," Class. Quant. Grav. **11** (1994) 2969, "Strong infrared effects in quantum gravity," Annals Phys. **238** (1995) 1, "Quantum gravity slows inflation," Nucl. Phys. B **474** (1996) 235 [arXiv:hep-ph/9602315].

[35] P. H. Ginsparg and M. J. Perry, "Semiclassical perdurance of de Sitter space," Nucl. Phys. B **222** (1983) 245.

[36] T. Banks and W. Fischler, "M-theory observables for cosmological space-times," arXiv:hep-th/0102077.

[37] R. Bousso, "Positive vacuum energy and the N-bound," JHEP **0011** (2000) 038 [arXiv:hep-th/0010252].

[38] D. Kastor and J. Traschen, "A positive energy theorem for asymptotically de Sitter spacetimes," Class. Quant. Grav. **19** (2002) 5901 [arXiv:hep-th/0206105].

[39] D. Klemm, "Some aspects of the de Sitter/CFT correspondence," Nucl. Phys. B **625** (2002) 295 [arXiv:hep-th/0106247].

[40] M. Spradlin, A. Strominger and A. Volovich, "Les Houches lectures on de Sitter space," arXiv:hep-th/0110007.

[41] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge Monographs On Mathematical Physics, Cambridge University Press (1973).

[42] M. K. Parikh, I. Savonije and E. Verlinde, "Elliptic de Sitter space: $dS/\mathbb{Z}_2$," Phys. Rev. D **67** (2003) 064005 [arXiv:hep-th/0209120].

[43] N. Goheer, M. Kleban and L. Susskind, "The trouble with de Sitter space," JHEP **0307** (2003) 056 [arXiv:hep-th/0212209].

[44] B. McInnes, "De Sitter and Schwarzschild-de Sitter according to Schwarzschild and de Sitter," JHEP **0309** (2003) 009 [arXiv:hep-th/0308022].

[45] B. McInnes, "Orbifold physics and de Sitter spacetime," arXiv:hep-th/0311055.

[46] L. F. Abbott and S. Deser, "Stability of gravity with a cosmological constant," Nucl. Phys. B **195** (1982) 76.

[47] E. Witten, "A simple proof of the positive energy theorem," Commun. Math. Phys. **80** (1981) 381.

[48] P. Breitenlohner and D. Z. Freedman, "Positive energy in anti-de Sitter backgrounds and gauged extended supergravity," Phys. Lett. B **115** (1982) 197; "Stability in gauged extended supergravity," Annals Phys. **144** (1982) 249;
G. W. Gibbons, C. M. Hull and N. P. Warner, "The stability of gauged supergravity," Nucl. Phys. B **218** (1983) 173.

[49] K. Pilch, P. van Nieuwenhuizen and M. F. Sohnius, "De Sitter superalgebras and supergravity," Commun. Math. Phys. **98** (1985) 105.

[50] D. G. C. McKeon and T. N. Sherry, "Supersymmetry in spaces of constant curvature," arXiv:hep-th/0301127.

[51] T. Shiromizu, D. Ida and T. Torii, "Gravitational energy, dS/CFT correspondence and cosmic no-hair," JHEP **0111** (2001) 010 [arXiv:hep-th/0109057].

[52] D. Astefanesei, R. Mann and E. Radu, "Reissner-Nordstroem-de Sitter black hole, planar coordinates and dS/CFT," JHEP **0401** (2004) 029 [arXiv:hep-th/0310273].

[53] G. W. Gibbons, "Thoughts on tachyon cosmology," Class. Quant. Grav. **20** (2003) S321 [arXiv:hep-th/0301117].

[54] C. M. Hull, "Timelike T-duality, de Sitter space, large N gauge theories and topological field theory," JHEP **9807** (1998) 021 [arXiv:hep-th/9806146].

[55] G. W. Gibbons and C. M. Hull, "De Sitter space from warped supergravity solutions," arXiv:hep-th/0111072.

[56] P. Fré, M. Trigiante and A. Van Proeyen, "Stable de Sitter vacua from N = 2 supergravity," Class. Quant. Grav. **19** (2002) 4167 [arXiv:hep-th/0205119].

[57] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66** (2002) 106006 [arXiv:hep-th/0105097].

[58] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].

[59] P. K. Townsend and M. N. R. Wohlfarth, "Accelerating cosmologies from compactification," Phys. Rev. Lett. **91**, 061302 (2003) [arXiv:hep-th/0303097].

[60] N. Ohta, "Accelerating cosmologies from S-branes," Phys. Rev. Lett. **91** (2003) 061303 [arXiv:hep-th/0303238], "A study of accelerating cosmologies from superstring/M theories," Prog. Theor. Phys. **110** (2003) 269 [arXiv:hep-th/0304172].

[61] S. Roy, "Accelerating cosmologies from M/string theory compactifications," Phys. Lett. B **567** (2003) 322 [arXiv:hep-th/0304084].

[62] C. M. Chen, P. M. Ho, I. P. Neupane, N. Ohta and J. E. Wang, "Hyperbolic space cosmologies," JHEP **0310** (2003) 058 [arXiv:hep-th/0306291].

[63] I. P. Neupane, "Accelerating cosmologies from exponential potentials," [arXiv:hep-th/0311071].

[64] M. Gutperle and A. Strominger, "Spacelike branes," JHEP **0204** (2002) 018 [arXiv:hep-th/0202210].

[65] R. Emparan and J. Garriga, "A note on accelerating cosmologies from compactifications and S-branes," JHEP **0305** (2003) 028 [arXiv:hep-th/0304124].

[66] M. Gutperle, R. Kallosh and A. Linde, "M/string theory, S-branes and accelerating universe," JCAP **0307** (2003) 001 [arXiv:hep-th/0304225].

[67] C. M. Chen, D. V. Gal'tsov and M. Gutperle, "S-brane solutions in supergravity theories," Phys. Rev. D **66**, 024043 (2002) [arXiv:hep-th/0204071].

[68] M. Kruczenski, R. C. Myers and A. W. Peet, "Supergravity S-branes," JHEP **0205** (2002) 039 [arXiv:hep-th/0204144];
S. Roy, "On supergravity solutions of space-like Dp-branes," JHEP **0208** (2002) 025 [arXiv:hep-th/0205198].
N. Ohta, "Intersection rules for S-branes," Phys. Lett. B **558** (2003) 213 [arXiv:hep-th/0301095].

[69] M. I. Park, "Symmetry algebras in Chern-Simons theories with boundary: Canonical approach," Nucl. Phys. B **544** (1999) 377 [arXiv:hep-th/9811033].

[70] P. de Medeiros, C. M. Hull, B. Spence and J. M. Figueroa-O'Farrill, "Conformal topological Yang-Mills theory and de Sitter holography," JHEP **0208** (2002) 055 [arXiv:hep-th/0111190].

[71] S. Nojiri and S. D. Odintsov, "Conformal anomaly from dS/CFT correspondence," Phys. Lett. B **519** (2001) 145 [arXiv:hep-th/0106191].

[72] S. Nojiri and S. D. Odintsov, "Quantum cosmology, inflationary brane-world creation and dS/CFT correspondence," JHEP **0112** (2001) 033 [arXiv:hep-th/0107134].

[73] M. Cvetič, S. Nojiri and S. D. Odintsov, "Black hole thermodynamics and negative entropy in deSitter and anti-deSitter Einstein-Gauss-Bonnet gravity," Nucl. Phys. B **628** (2002) 295 [arXiv:hep-th/0112045].

[74] E. Witten, "(2+1)-dimensional gravity as an exactly soluble system," Nucl. Phys. B **311** (1988) 46.

[75] S. Elitzur, G. W. Moore, A. Schwimmer and N. Seiberg, "Remarks on the canonical quantization of the Chern-Simons-Witten theory," Nucl. Phys. B **326** (1989) 108.

[76] S. Cacciatori and D. Klemm, "The asymptotic dynamics of de Sitter gravity in three dimensions," Class. Quant. Grav. **19** (2002) 579 [arXiv:hep-th/0110031].

[77] D. Klemm and L. Vanzo, "De Sitter gravity and Liouville theory," JHEP **0204** (2002) 030 [arXiv:hep-th/0203268].

[78] N. Seiberg, "Notes on quantum Liouville theory and quantum gravity," Prog. Theor. Phys. Suppl. **102** (1990) 319.

[79] S. Deser and R. Jackiw, "Three-dimensional cosmological gravity: Dynamics of constant curvature," Annals Phys. **153** (1984) 405.

[80] P. Ginsparg and G. W. Moore, "Lectures on 2-D gravity and 2-D string theory," [arXiv:hep-th/9304011].

[81] V. Balasubramanian, J. de Boer and D. Minic, "Exploring de Sitter space and holography," Class. Quant. Grav. **19** (2002) 5655 [Annals Phys. **303** (2003) 59] [arXiv:hep-th/0207245].

[82] E. Abdalla, K. H. C. Castello-Branco and A. Lima-Santos, "Support of dS/CFT correspondence from space-time perturbations," Phys. Rev. D **66** (2002) 104018 [arXiv:hep-th/0208065].

[83] D. P. Du, B. Wang and R. K. Su, "Quasinormal modes in pure de Sitter spacetimes," [arXiv:hep-th/0404047].

[84] L. Dyson, J. Lindesay and L. Susskind, "Is there really a de Sitter/CFT duality," JHEP **0208** (2002) 045 [arXiv:hep-th/0202163].

[85] D. Kabat and G. Lifschytz, "De Sitter entropy from conformal field theory," JHEP **0204** (2002) 019 [arXiv:hep-th/0203083].

[86] R. M. Wald, "Black hole entropy is Noether charge," Phys. Rev. D **48** (1993) 3427 [arXiv:gr-qc/9307038].

[87] G. T. Horowitz and N. Itzhaki, unpublished.

[88] R. Arnowitt, S. Deser and C. W. Misner, "The dynamics of general relativity," arXiv:gr-qc/0405109.

[89] E. G. Gimon and A. Hashimoto, "Black holes in Gödel universes and pp-waves," Phys. Rev. Lett. **91**, 021601 (2003) [arXiv:hep-th/0304181].

[90] A. Strominger, private communication.

[91] A. Guijosa and D. A. Lowe, "A new twist on dS/CFT," Phys. Rev. D **69**, 106008 (2004) [arXiv:hep-th/0312282].

[92] D. A. Lowe, "q-deformed de Sitter/conformal field theory correspondence", [arXiv:hep-th/0407188].

[93] T. Banks, "Some thoughts on the quantum theory of de Sitter space," [arXiv:astro-ph/0305037].

[94] M. Li, "Matrix model for de Sitter," JHEP **0204** (2002) 005, [arXiv:hep-th/0106184].

[95] M. K. Parikh and E. Verlinde, "De Sitter space with finitely many states: A toy story," arXiv:hep-th/0403140.

[96] J. Bros, H. Epstein and U. Moschella, "Analyticity properties and thermal effects for general quantum field theory on de Sitter space-time," Commun. Math. Phys. **196** (1998) 535 [arXiv:gr-qc/9801099].

[97] H. J. Borchers and D. Buchholz, "Global properties of vacuum states in de Sitter space," Annales Poincaré Phys. Theor. A **70** (1999) 23 [arXiv:gr-qc/9803036].

[98] H. Nariai, "On a new cosmological solution of Einstein's field equations of gravitation," Sci. Rep. Tohoku Univ. Ser. I **35** (1951) 62.

[99] N. F. Ramsey, "Thermodynamics and statistical mechanics at negative absolute temperature," Phys. Rev. **103** (1956) 20.

[100] E. M. Purcell and R. V. Pound, "A nuclear spin system at negative temperature," Phys. Rev. **81** (1951) 279.