Notes

- “Stephen’s contributions to black hole physics and cosmology find a synthesis in his semi-classical treatment of de Sitter space. Gibbons and Hawking (1977) demonstrated that the de Sitter horizon, like a black hole, is endowed with entropy and temperature. Thus, the quantum properties of black holes will extend to the universe as a whole, if the vacuum energy is positive. “ - Adventures in de Sitter space by Raphael Bousso: https://arxiv.org/pdf/hep-th/0205177.pdf ✔

- de Sitter space is a maximally symmetric solution to Einstein’s field equations in a vacuum with a positive cosmological constant, lambda. ✔

- de Sitter space is positively curved space-time (like a sphere) with a positive cosmo constant and energy density whereas adS is negatively curved space-time (a hyperboloid/saddle) with negative cosmo constant and energy density. Minkowski space is flat space. Our universe is spatially flat (just in the 3 spatial dimensions) but it's curved when we include the time dimension. A de Sitter universe is considered to be empty of mass so our Universe isn't wholly de Sitter, but the vacuum of space in our Universe is. Although, since our Universe is expanding, it is tending towards a de Sitter universe. ✔
    - One could say our Universe is de Sitter in the future, not the present.
    - http://www.quantumfieldtheory.info/dS_and_AdS_spaces.pdf

- In regards to black holes, the Wiki article says, "In general relativity, the **de Sitter–Schwarzschild** solution describes a black hole in a causal patch of de Sitter space." So this means the solution describes a black hole that is in the light cone of de sitter space i.e. the black hole exists in a Universe that is tending towards de sitter (aka our Universe). This one I'm not too sure if I've got it right.

- "Unlike a flat-space black hole, there is a largest possible de Sitter black hole, which is the **Nariai spacetime**. The Nariai limit has no singularities, the cosmological and black hole horizons have the same area, and they can be mapped to each other by a discrete reflection symmetry in any causal patch." I'm not totally sure how to interpret this piece.

- Since de Sitter space is defined to have a cosmological constant which is static everywhere in the Universe (via Einstein's field equations), then the curvature of said Universe is homogeneous throughout. In other words, de Sitter space is maximally symmetric. http://www.quantumfieldtheory.info/dS_and_AdS_spaces.pdf ✔

- "Roughly speaking, the Bekenstein-Hawking entropy of empty de Sitter space is the largest entropy attainable in any asymptotically de Sitter spacetime (Fischler, 2000a,b; Banks, 2000)." - https://arxiv.org/pdf/hep-th/0205177.pdf

- The Schwarzschild metric is an example of a highly symmetric spacetime. It has continuous symmetries in space (under rotation) and in time (under translation in time). In addition, it has discrete symmetries under spatial reflection and time reversal. https://phys.libretexts.org/Bookshelves/Relativity/Book%3A_General_Relativity_(Crowell)/07%3A_Symmetries/7.01%3A_Killing_Vectors

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All of the below is a direct copy and paste from Adventures in de Sitter Space, https://arxiv.org/pdf/hep-th/0205177.pdf , except for the first bullet point about the Nariai solution.

- Nariai solution - largest possible black hole in de Sitter space

- Globally, de Sitter space can be written as a closed FRW universe:

$$\frac{ds^2}{l^2} = -d\tau^2 + cosh^2\tau d\Omega_3^2$$

- In the static coordinate system,

$$\frac{ds^2}{l^2} = -V(r)dt^2 + \frac{1}{V(r)}dr^2 + r^2 d\Omega_2^2$$

where, $V(r) = 1 - r^2$.

- "In short, the cosmological horizon acts like a black hole "surrounding" the observer. Note that the symmetry of space-time implies that the location of the cosmological horizon is observer-dependent."

- Using Bogolubov transformation techniques, Hawking demonstrated that black holes emit radiation by a quantum process, at a temperature,

$$T_{hor} = \frac{\kappa}{2\pi}$$

where $\kappa = surface\ gravity,\ playing\ the\ role\ of\ temperature$. For a Schwarzschild black hole of mass M, κ = (4M) −1 . Via the first law of thermodynamics,

$$\frac{1}{T_{hor}} = \frac{\delta S_{hor}}{\delta M}$$

- Its numerical coefficient:

$$S_{hor} = \frac{A}{4} \qquad \textbf{Eqn. 3.3}$$

- In spacetimes which are asymptotically de Sitter in the future, any observer is surrounded by an event horizon. At late times, its area is given by,

$$A_0 = 4\pi l^2$$

where $l = \sqrt{\frac{3}{\Lambda}}$ is the curvature radius and Λ is the cosmological constant

- Using Euclidean techniques, Gibbons and Hawking (1977) demonstrated that an observer in de Sitter space does detect thermal radiation at a temperature,

$$T_{dS} = \frac{1}{2\pi l}$$

- Up to an additive constant, which is taken to vanish, Eq. (3.3) thus applies both to cosmological and to black hole horizons. In particular, the total entropy of empty de Sitter space is given by its horizon entropy, which is,

$$S_0 = \frac{A_0}{4} = \frac{3\pi}{\Lambda} \quad \textbf{Eqn. 3.9}$$

- Thus, Gibbons and Hawking (1977) showed that the de Sitter horizon is endowed with the same quantum properties as a black hole horizon: a temperature and an entropy. They noted that the de Sitter horizon, unlike a black hole horizon, is observer dependent.

Below is direct quote from
https://inspirehep.net/files/dd379b79a6b1266fcbaba060386d64c6,

- The two positive zeros of V (r) will be denoted by r+ and r++. These are the positions of the black hole and the cosmological event horizons, respectively. For M > 0 it follows from the form of V (r) that the value of r+ is larger than that of the Schwarzschild radius in Minkowski space and that r++ is smaller than the de Sitter radius of a pure dSd space. Further as M increases they will approach each other until the two horizons touch. Such a black hole is called a Nariai black hole.

https://arxiv.org/pdf/gr-qc/0301090.pdf

- The difference between SdS and just dS or just Schwarzschild: SdS has two event horizons - the cosmological one and the black hole one - whereas the theories apart have only 1 horizon.

- The conclusions of the above works are that the temperature of a black hole is proportional to the surface gravity and that the area of its event horizon plays the role of its entropy. https://arxiv.org/pdf/1108.1337.pdf
- With the birth of string theory [6], as a candidate for quantum gravity and loop quantum gravity [7], a new window was opened to the problem of black hole radiation. This was because the nature of black hole radiation is such that quantum gravity effects cannot be neglected [8]. According to all of the above remarkable works, it is believed that a black

hole is a quantum mechanical system and thus like any other quantum system its physical states can be described by a wavefunction. https://arxiv.org/pdf/1108.1337.pdf

- For example, the coordinate transformation t ↔ r converts the Schwarzschild metric to a dynamical model which can be identified with its inside space-time. Indeed, at the horizon of a Schwarzschild black hole the light cone tips over and for r < rs, ∂/∂r become time-like while ∂/∂t become spacelike. This means that inside the black hole the metric has time-dependent coefficients. https://arxiv.org/pdf/1108.1337.pdf
- A naked singularity is one without an event horizon between the observer and the singularity?
    - Since the cosmic censorship conjecture says every singularity has an event horizon, shielding it from view.

https://arxiv.org/pdf/1811.11528.pdf

- “In a quantum theory with a gravity dual, this state is dual to an eternal black hole. Black holes remain poorly understood; it is a matter of debate whether an observer falling into a large black hole falls freely through the horizon, as predicted by the equivalence principle, or encounters a ‘firewall’ at the horizon.

  Despite many papers on this topic, a consensus has not yet been reached. The primary difficulty is that the notion of a firewall depends on experiences of observers localized near an event horizon. However, local observables are not believed to exist in quantum gravity. In principle, all we should discuss is the S-matrix, but, from this data alone it is essentially impossible to decipher the experiences of the brave soul who sailed into the black hole and was long ago scrambled into Hawking radiation.

  A key step forward was taken in [1, 2] where it was realized that by applying a simple perturbation coupling the two sides of an eternal AdS black hole one may make the wormhole traversable. This, in principle, allows us to probe behind the horizon without dealing with issues of bulk locality - all we need to do is send an observer from one side to the other and ask them how they felt. Susskind has predicted that we will be able to perform experiments of this type ‘within the next decade or two’ [3]. The eternal AdS black hole is dual to the thermofield double state of the two boundary CFTs [4, 5]. The thermofield double state is also of interest beyond the context of black holes, in the study of thermal field theories”