

Recursive Function

Session 15

Recursive algorithms

- In *recursive problem solving*, a **problem** is **repeatedly broken down into similar sub problems**, until the sub problems can be directly solved without further breakdown.

What Is a Recursive Function?

- A **recursive function** is often defined as “a function that calls itself.”

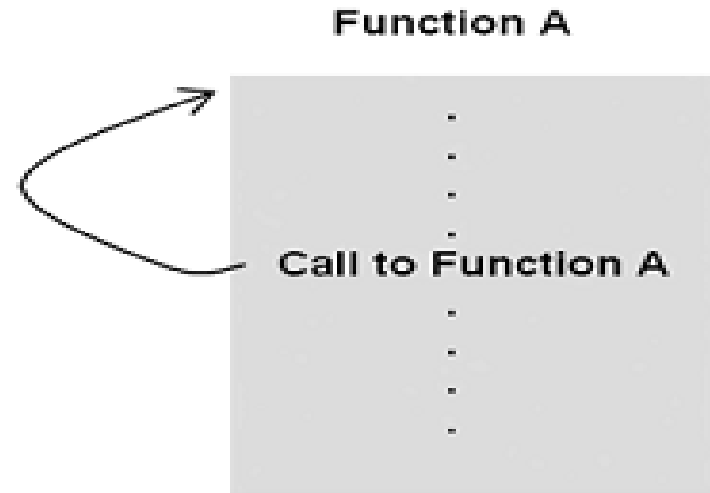
```
def myFun(x)
    if (x>0)
        print(x)
        myFun(x-1)
```

```
myFun(10)
```

```
10
```

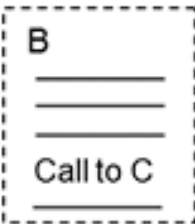
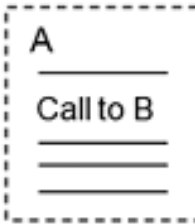
```
9
```

```
8
```

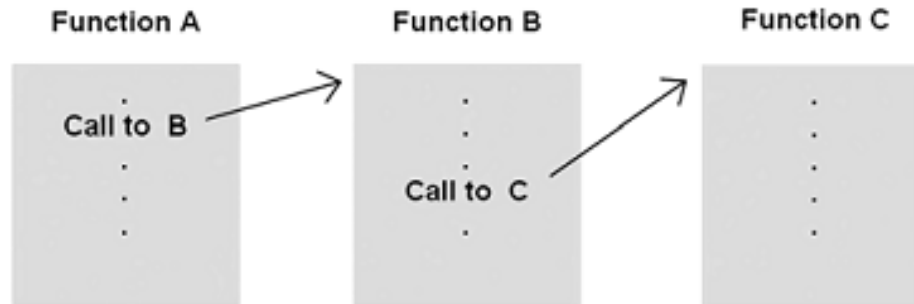


General mechanism of non-recursive function

Function Definitions



Function Instances

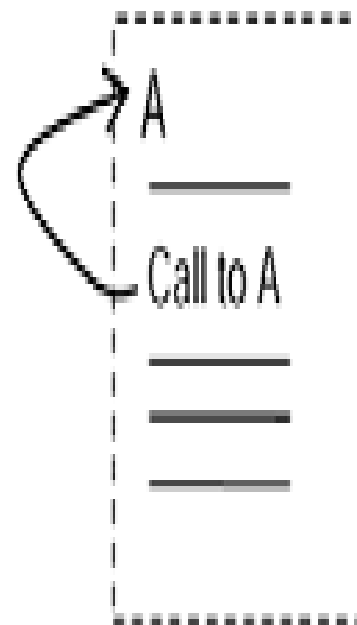


Recursive function execution instances

- Execution of a series of recursive function instances is similar to the execution of series of non-recursive instances, except that the execution instances are “clones” of each other (that is, of the same function definition).

Execution Instances

Function Definition



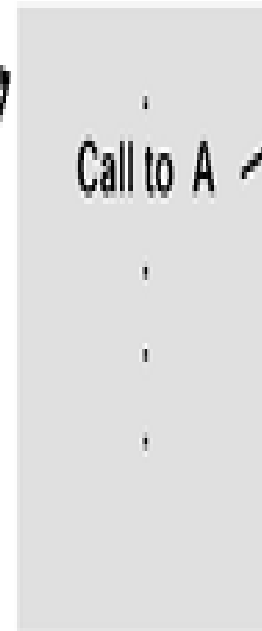
Function A1



Function A2



Function A3



...

LET'S TRY IT

From the Python Shell, enter the following and observe the results.

```
>>> def rfunc(n):  
    print(n)  
    if n > 0:  
        rfunc(n - 1)
```

```
>>> rfunc(4)
```

```
???
```

```
>>> rfunc(0)
```

```
???
```

```
>>> rfunc(100)
```

```
???
```

```
>>> def rfunc(n):  
    if n == 1:  
        return 1  
    else:  
        return n + rfunc(n - 1)
```

```
>>> rfunc(1)
```

```
???
```

```
>>> rfunc(3)
```

```
???
```

```
>>> rfunc(100)
```

```
???
```

Example: Factorial

- Problem:

The factorial function is an often-used example of the use of recursion. The computation of the factorial of 4 is given as,

$$\text{factorial}(4) = 4 * 3 * 2 * 1 = 24$$

In general, the computation of the factorial of any (positive, nonzero) integer n is,

$$\text{factorial}(n) = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1$$

The one exception is the factorial of 0, defined to be 1.

logic

- The factorial of n can be defined as n times the factorial of $n - 1$

$$\text{factorial}(n) = n \cdot \underbrace{(n - 1) \cdot (n - 2) \cdot \dots \cdot 1}_{\text{factorial}(n - 1)}$$

Thus, the complete definition of the factorial function is,

$$\begin{aligned} \text{factorial}(n) &= 1, && \text{if } n = 0 \\ &= n \cdot \text{factorial}(n - 1), && \text{otherwise} \end{aligned}$$

A Recursive Factorial Function Implementation

```
def factorial(n):  
    if n == 0:  
        return 1  
    else:  
        return n * factorial(n-1)
```

Input

Factorial(4)

Factorial Recursive Instance Calls

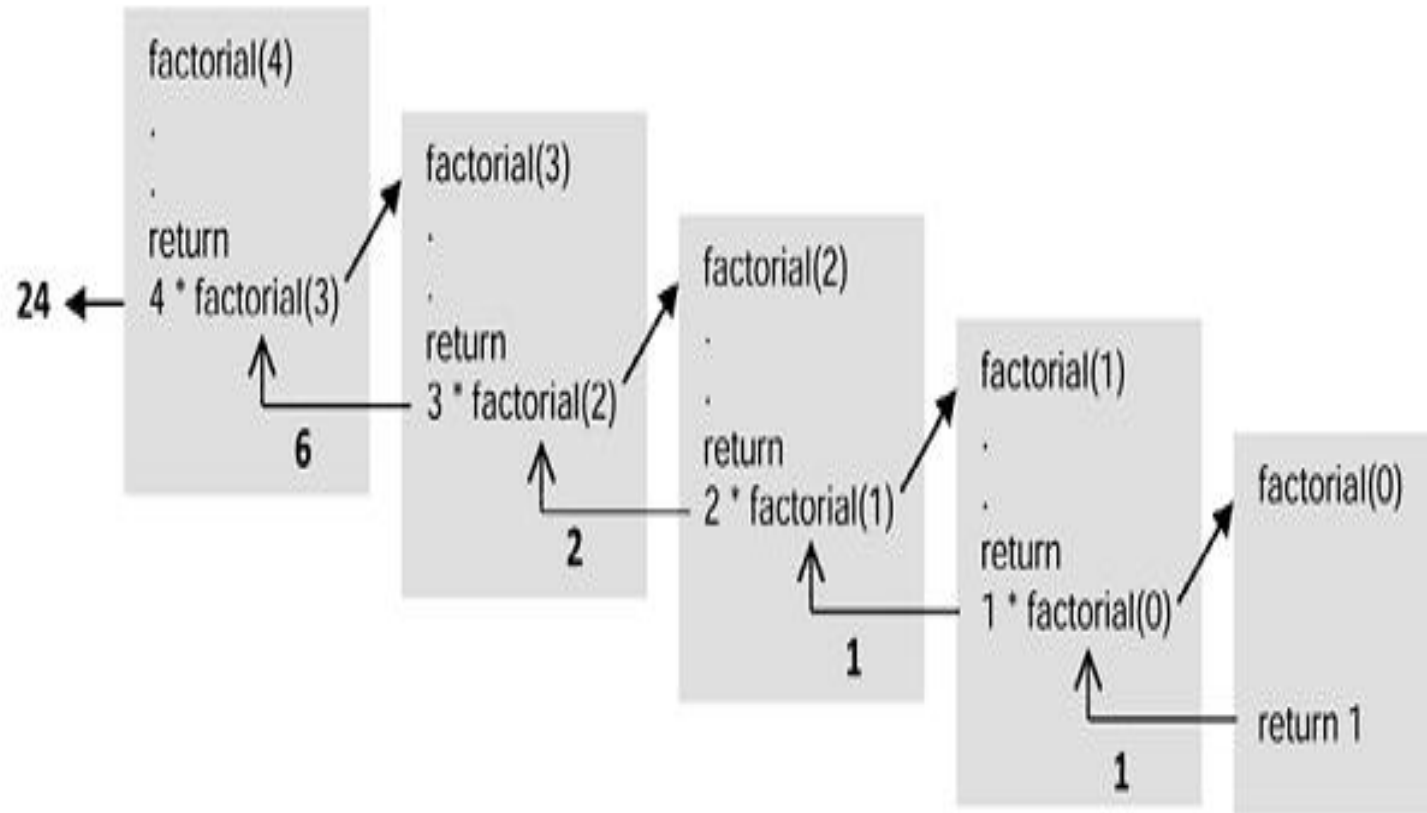


FIGURE 11-6 Factorial Recursive Instance Calls

LET'S TRY IT

From the Python Shell, enter the following and observe the results.

```
>>> def factorial(n):  
    if n == 0:  
        return 1  
    return n * factorial(n - 1)
```

```
>>> factorial(4)  
???
```

```
>>> factorial(0)  
???
```

```
>>> factorial(100)  
???
```

```
>>> factorial(10000)  
???
```

```
>>> def ifactorial(n):  
    result = 1  
    if n == 0:  
        return result  
  
    for k in range(n, 0, -1):  
        result = result * k  
  
    return result
```

```
>>> ifactorial(0)  
???
```

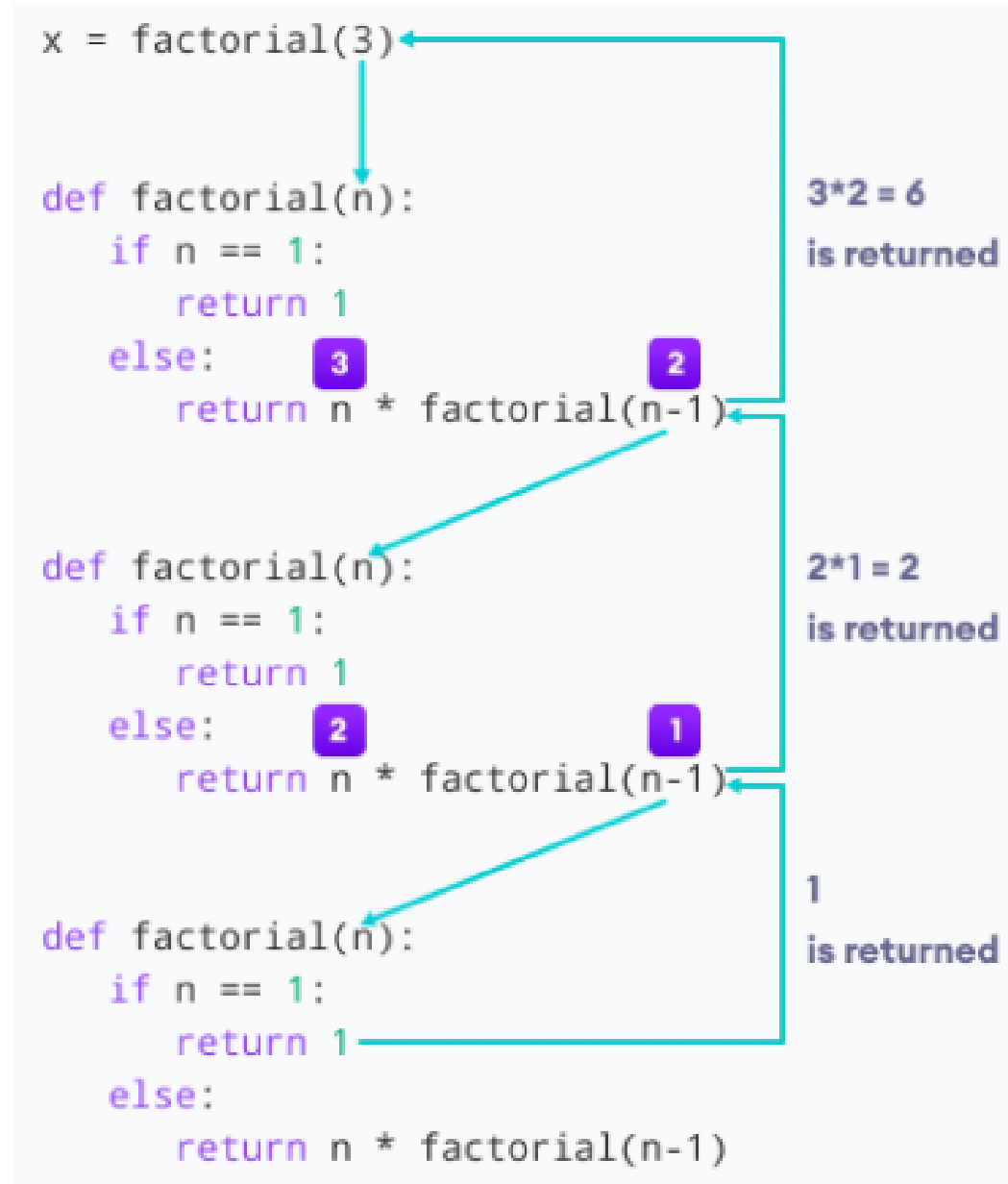
```
>>> ifactorial(100)  
???
```

```
>>> ifactorial(10000)  
???
```

Working of a recursive factorial function

factorial(3)	# 1st call with 3
3 * factorial(2)	# 2nd call with 2
3 * 2 * factorial(1)	# 3rd call with 1
3 * 2 * 1	# return from 3rd call as number=1
3 * 2	# return from 2nd call
6	# return from 1st call

Working of a recursive factorial function



Python **Lambda Function**

- A lambda function is a small anonymous function.
- A lambda function can take any number of arguments, but can only have one expression.
- Syntax

lambda arguments : expression

The expression is executed and the result is returned

Lambda Function

Add 10 to argument a, and return the result:

```
x = lambda a : a + 10  
print(x(5))
```

Output:

15

Lambda Function

- Lambda functions can take any number of arguments:

Example - Multiply argument a with argument b and return the result:

```
x = lambda a, b : a * b
```

```
print(x(5, 6))
```

Output:

30

```
x = lambda a, b, c : a + b + c
```

```
print(x(5, 6, 2))
```

Output:

13

Lambda function inside another function

Use the function definition (myfunc()) to make a function that always doubles the number you send in.

```
def myfunc(n):  
    return lambda a : a * n
```

```
mydoubler = myfunc(2)
```

```
print(mydoubler(11))
```

Output:

22

Lambda function

```
def myfunc(n):  
    return lambda a : a * n
```

```
mydoubler = myfunc(2)  
mytripler = myfunc(3)
```

```
print(mydoubler(11))  
print(mytripler(11))
```

Output:

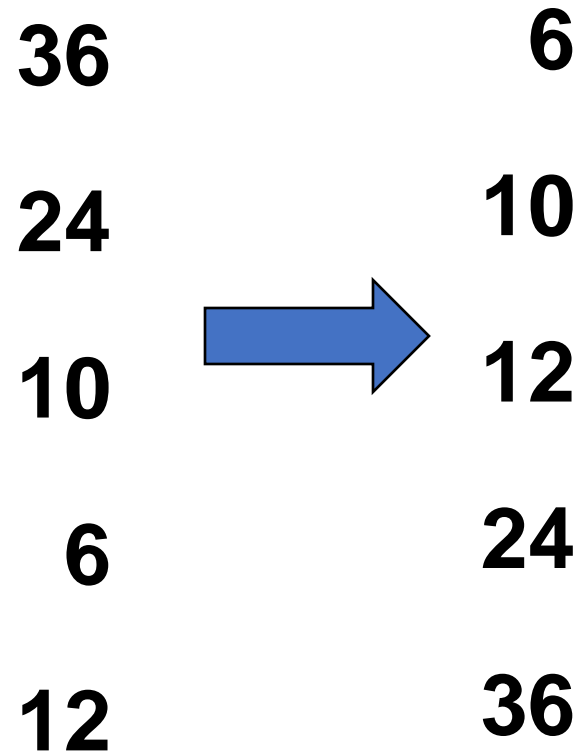
22

33

Sorting

Sorting means . . .

- Sorting rearranges the elements into either ascending or descending order within the array (we'll use ascending order).



Sorting also means...

- There are several sorting algorithms available like bubble sort, selection sort, insertion sort, quick sort, merge sort, radix sort etc.
- Sorting operation is performed in many applications to provide the output in desired order.
- For example listing all the product in the increasing order of their names or decreasing order of supplier names
- Searching will be easier in a sorted collection of elements
- List containing exam scores sorted from lowest to highest or vice versa
- We study **Bubble Sorting, Selection Sorting** and **Insertion Sorting** in this lab course

Bubble Sort

values	[0]	36
	[1]	24
	[2]	10
	[3]	6
	[4]	12

- Compares neighboring pairs of array elements, starting with the last/first array element, and swaps neighbors whenever they are not in correct order.
- On each pass, this causes the smallest element to “bubble up” to its correct place in the array.

Algorithm

```
begin BubbleSort(list)
  for all elements of list
    if list[i] > list[i+1]
      swap(list[i], list[i+1])
    end if
  end for

  return list

end BubbleSort
```

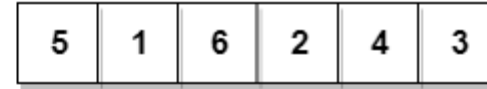

Bubble Sort Pseudo Code

```
def bubbleSort(lyst):  
    n = len(lyst)  
    while n > 1:                                # Do n - 1 bubbles  
        i = 1                                    # Start each bubble  
        while i < n:  
            if lyst[i] < lyst[i - 1]:           # Exchange if needed  
                swap(lyst, i, i - 1)  
            i += 1  
        n -= 1
```

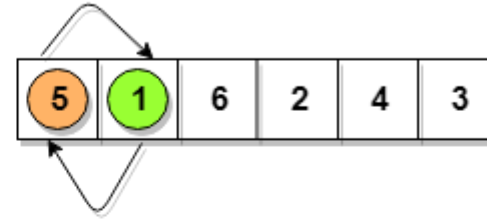
Let's consider an array with values

{5, 1, 6, 2, 4, 3}

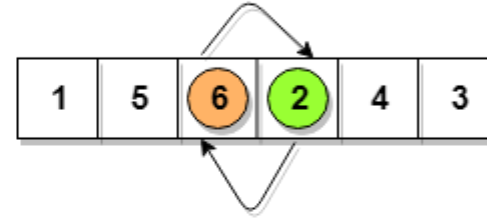
$5 > 1$
so interchange



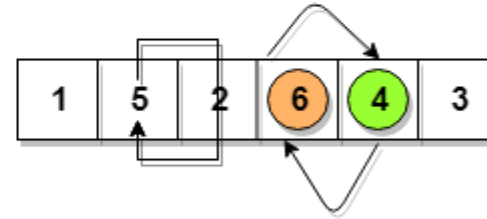
$5 < 6$
No swapping



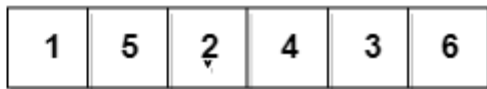
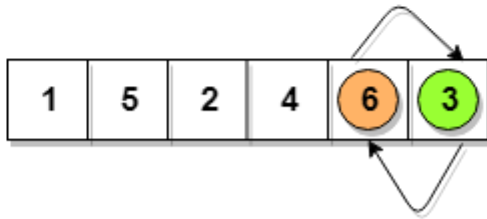
$6 > 2$
so interchange



$6 > 4$
so interchange



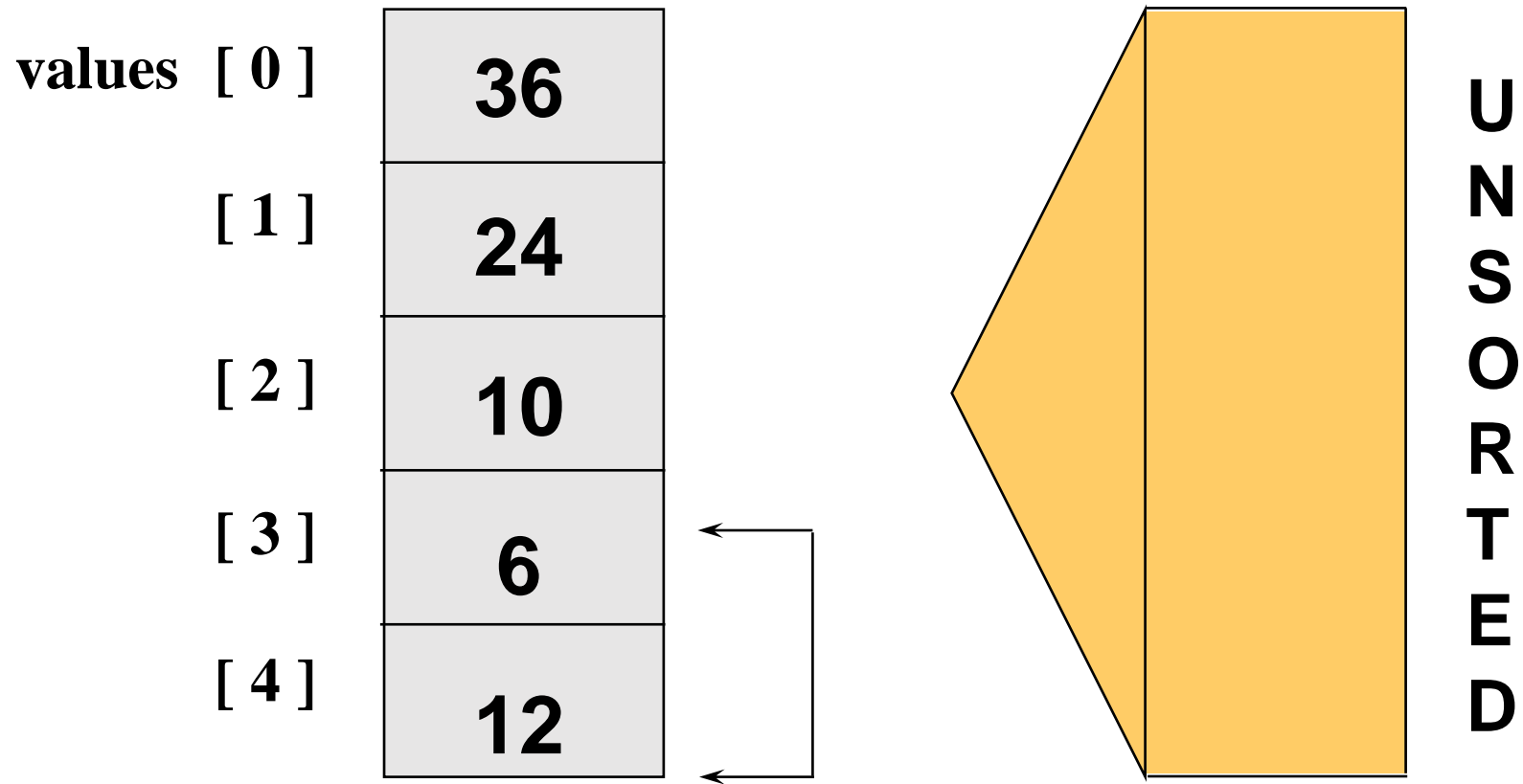
$6 > 3$
so interchange



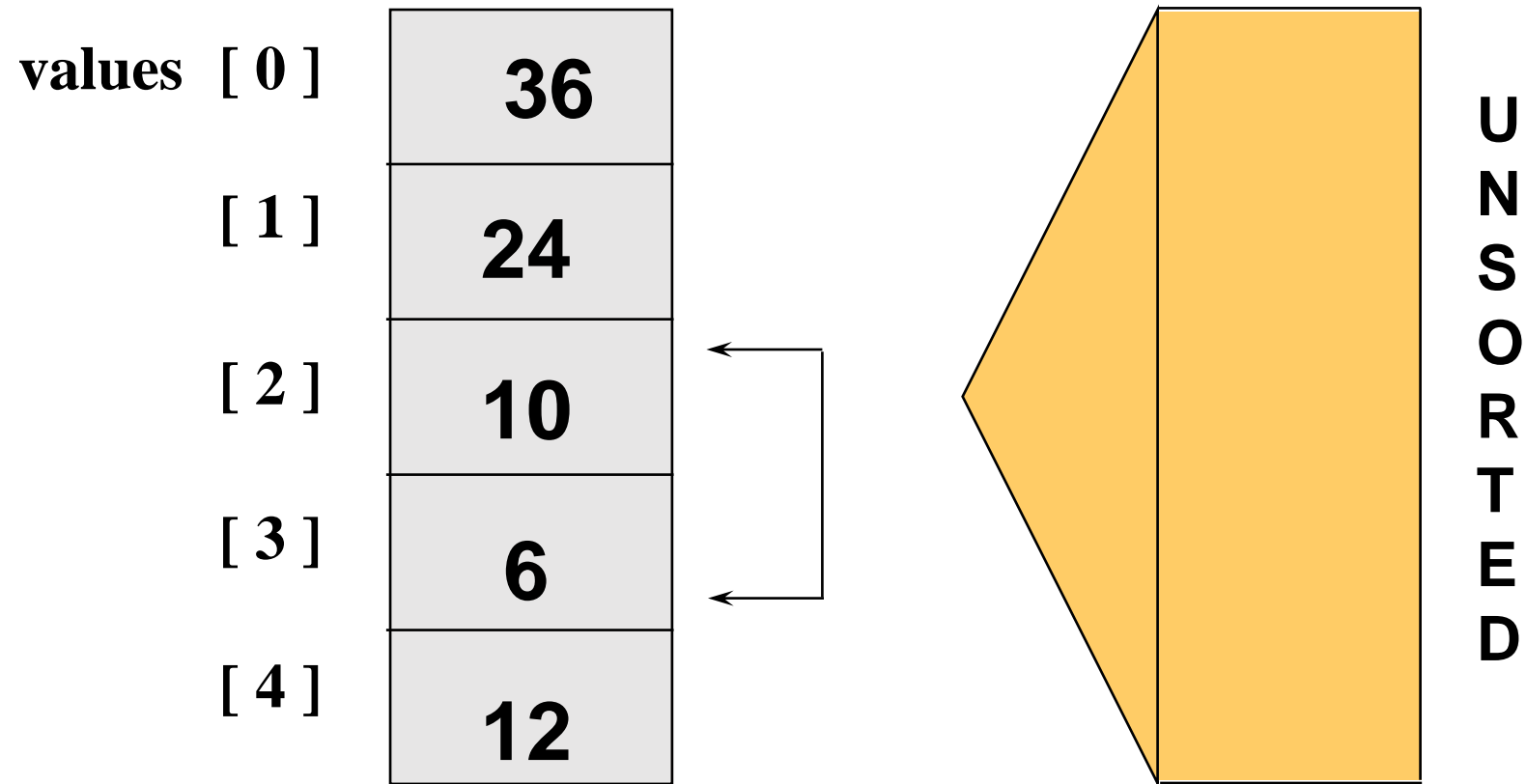
This is first insertion

similarly, after all the iterations, the array gets sorted

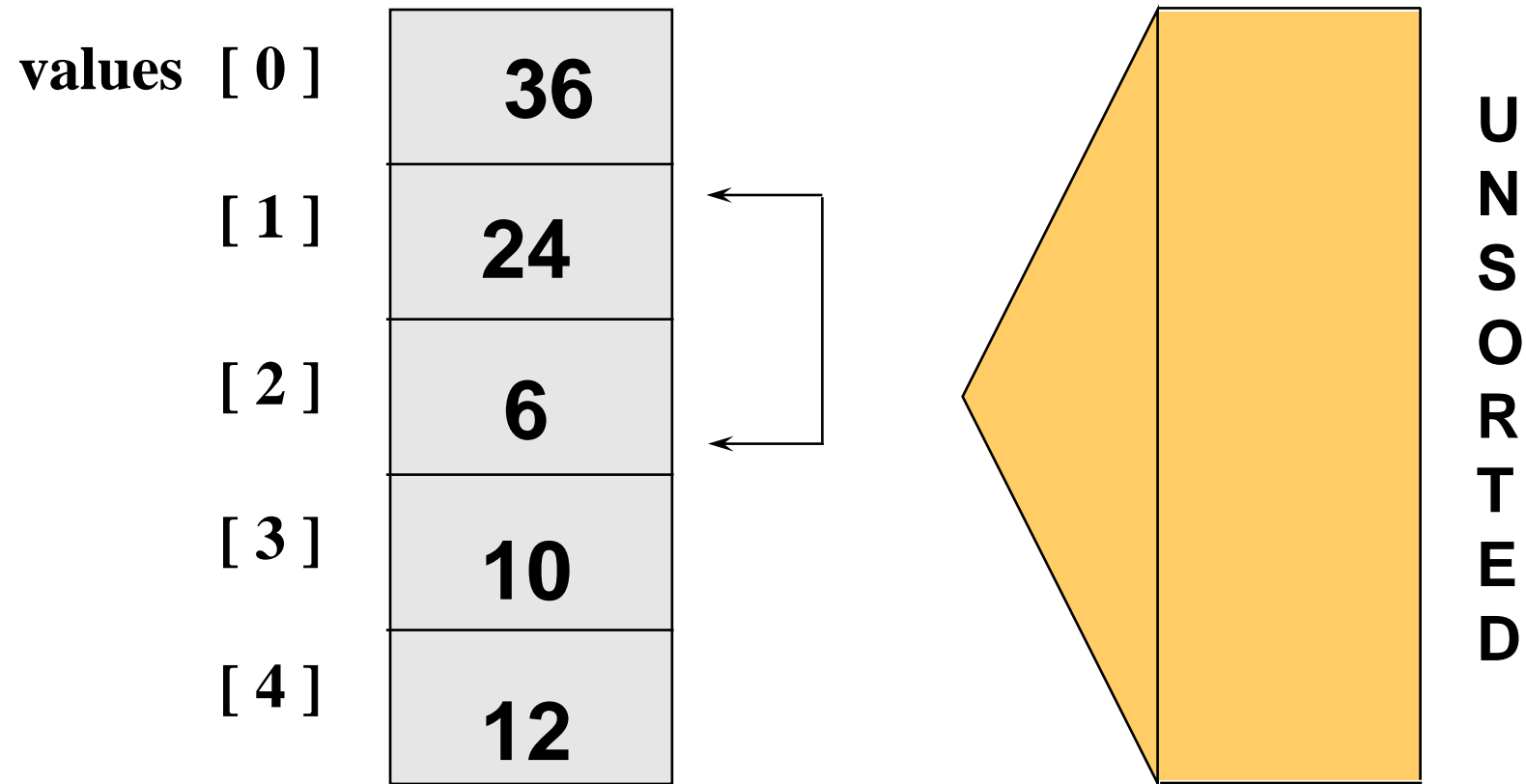
Bubble Sort: Pass One



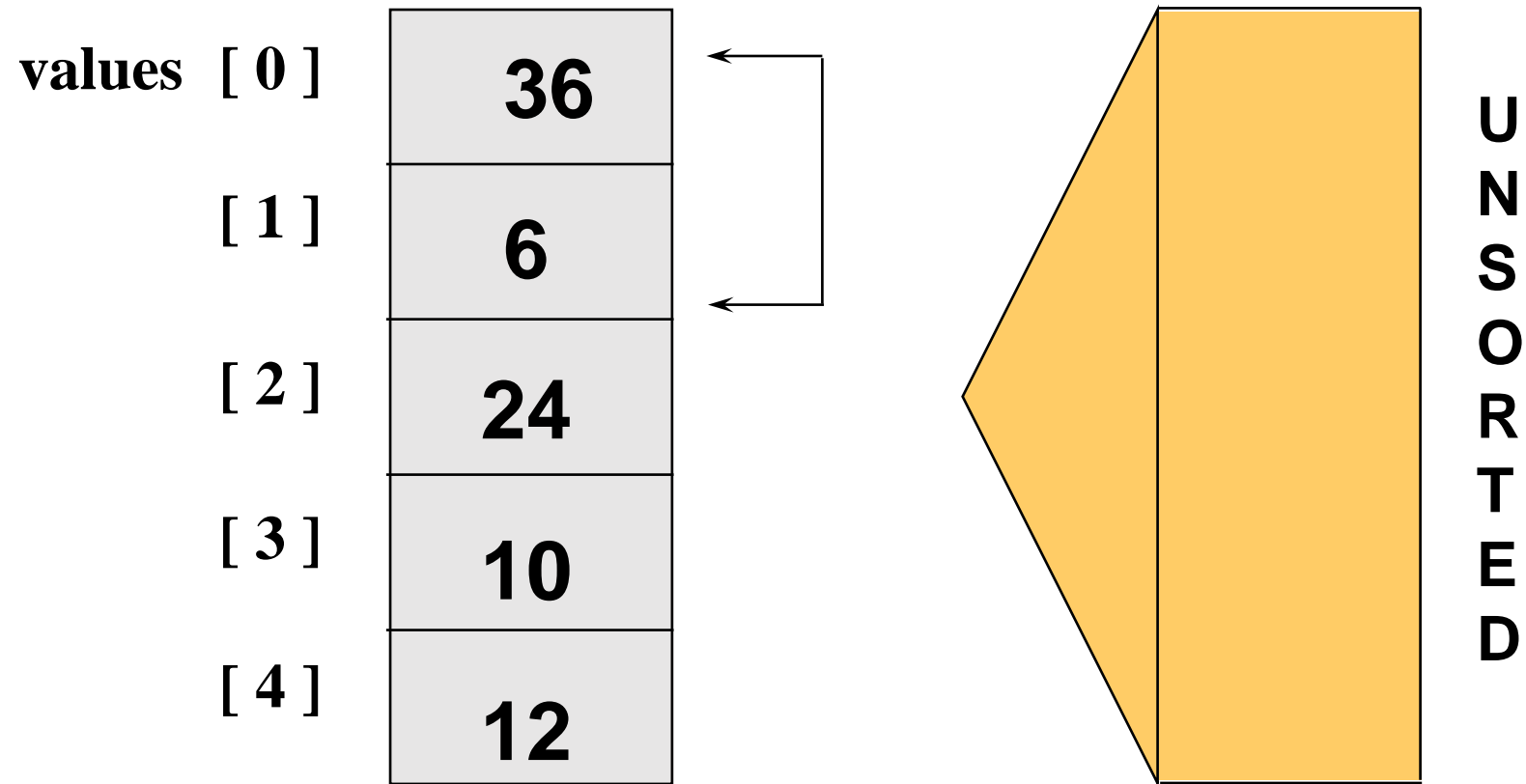
Bubble Sort: Pass One



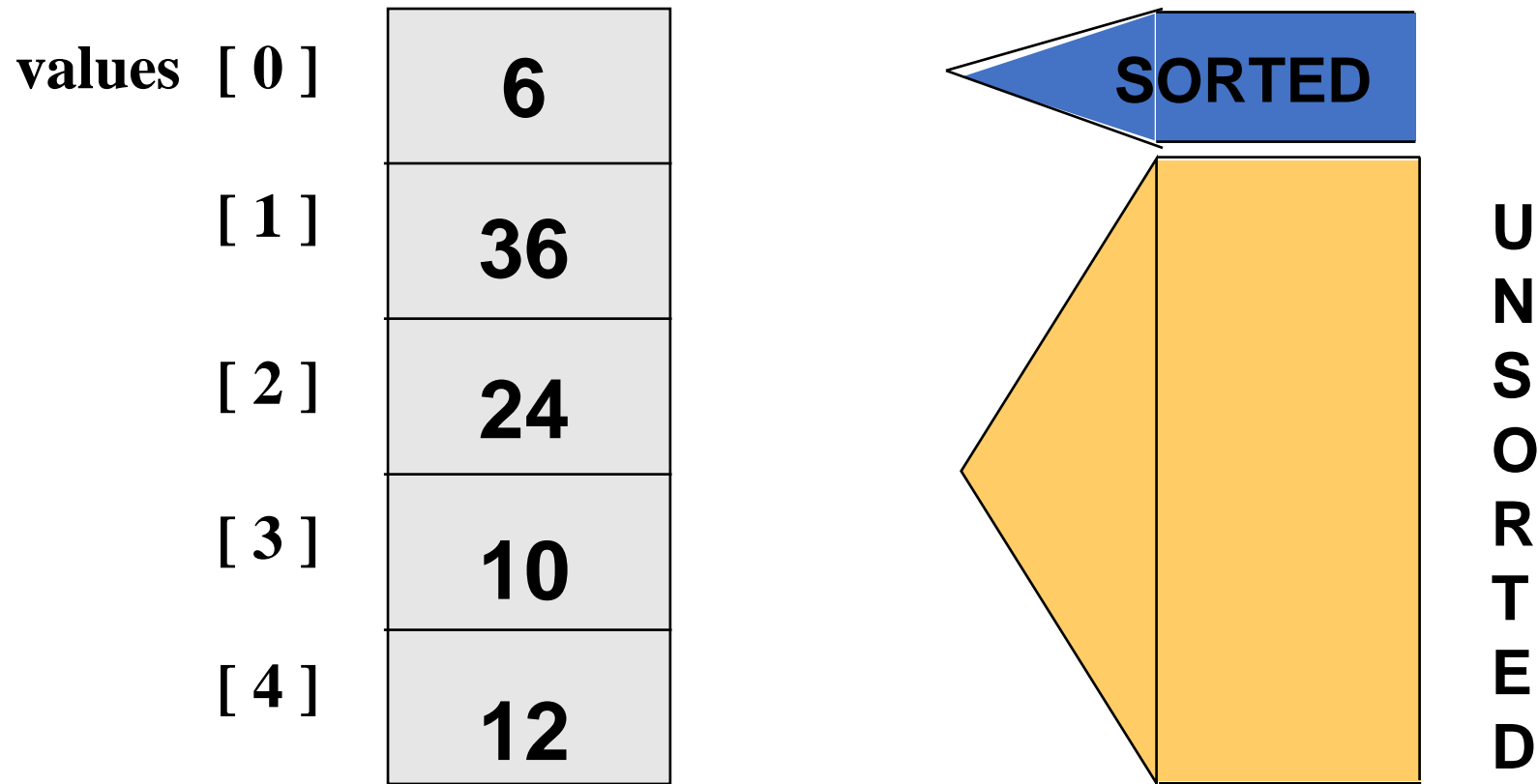
Bubble Sort: Pass One



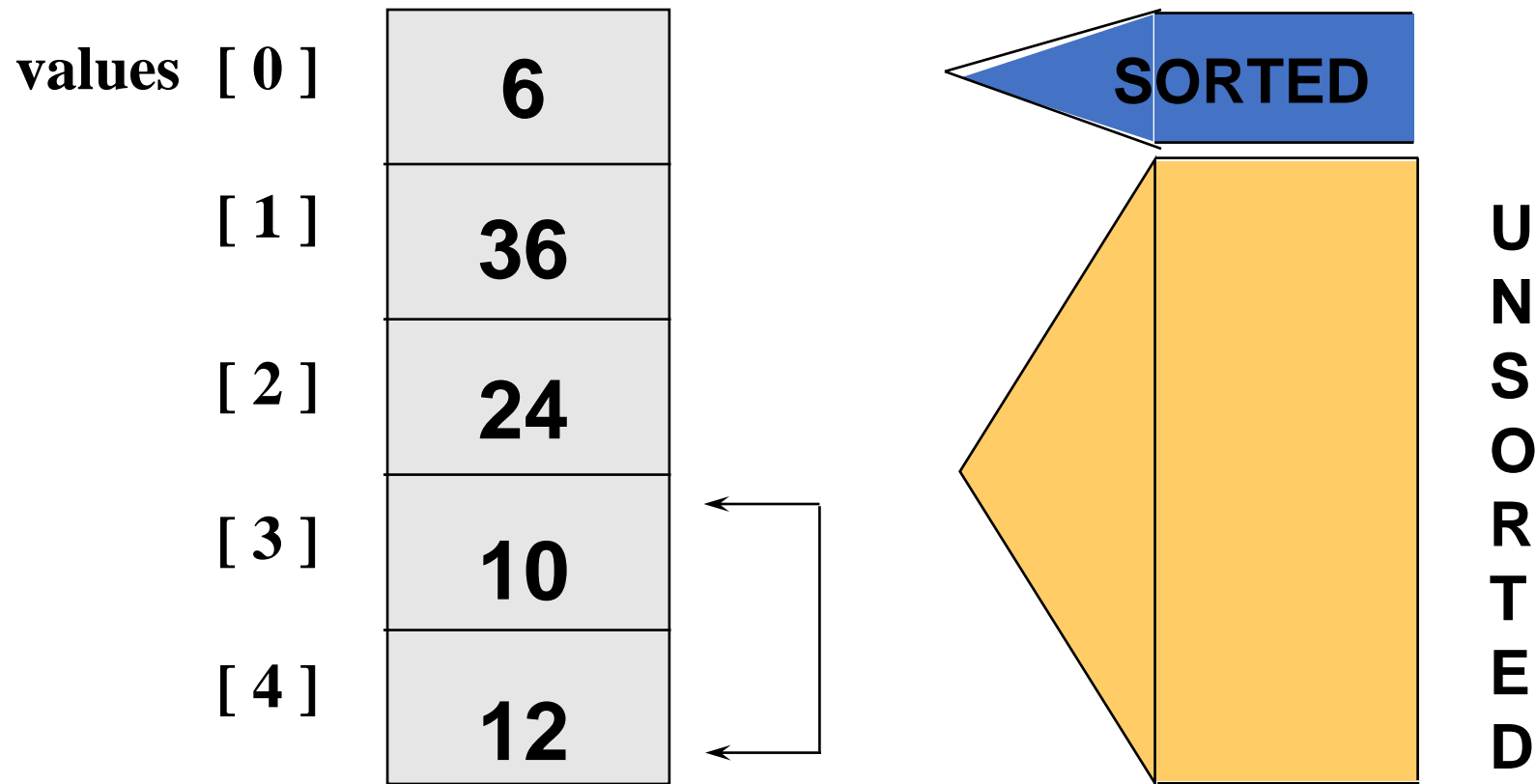
Bubble Sort: Pass One



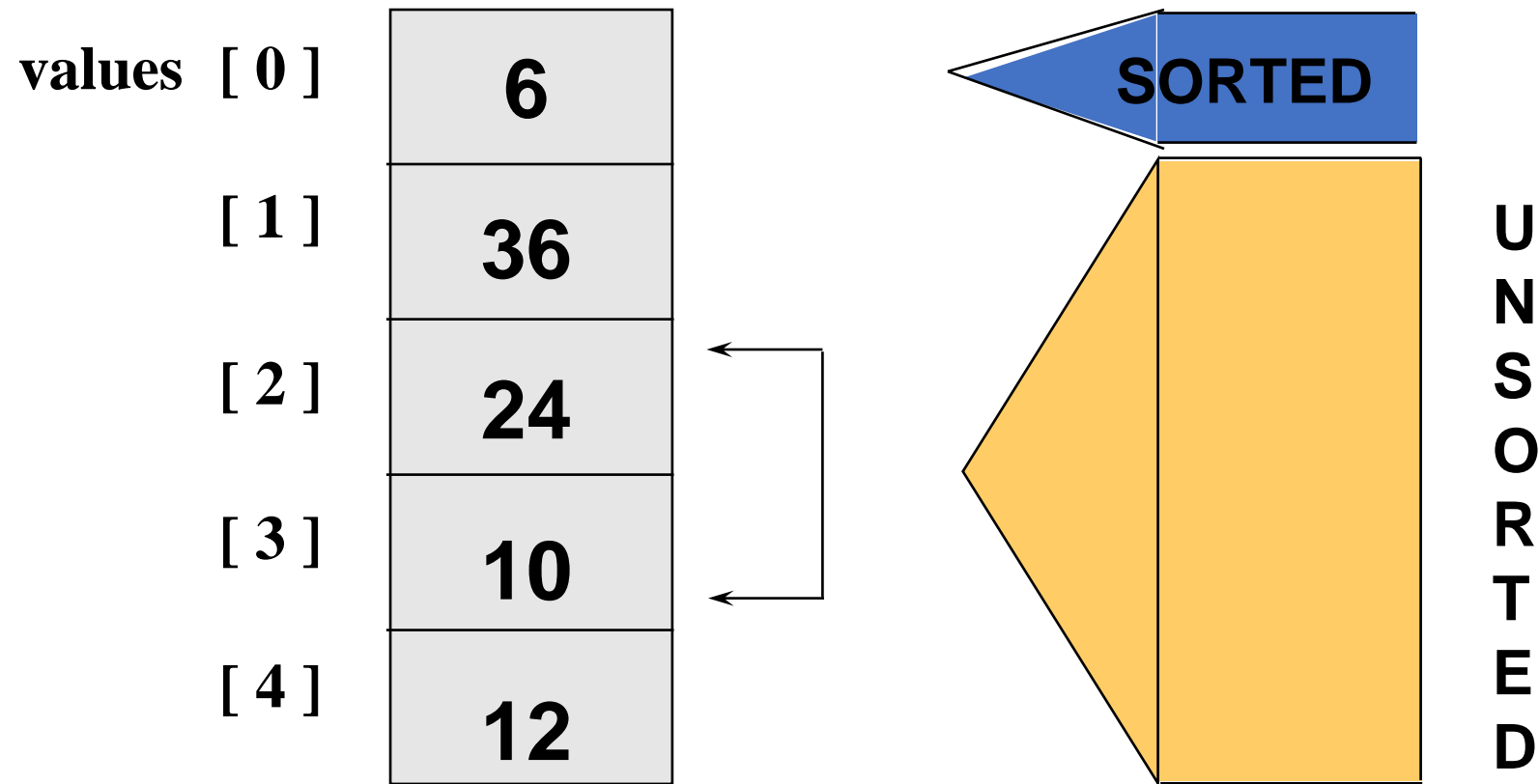
Bubble Sort: End Pass One



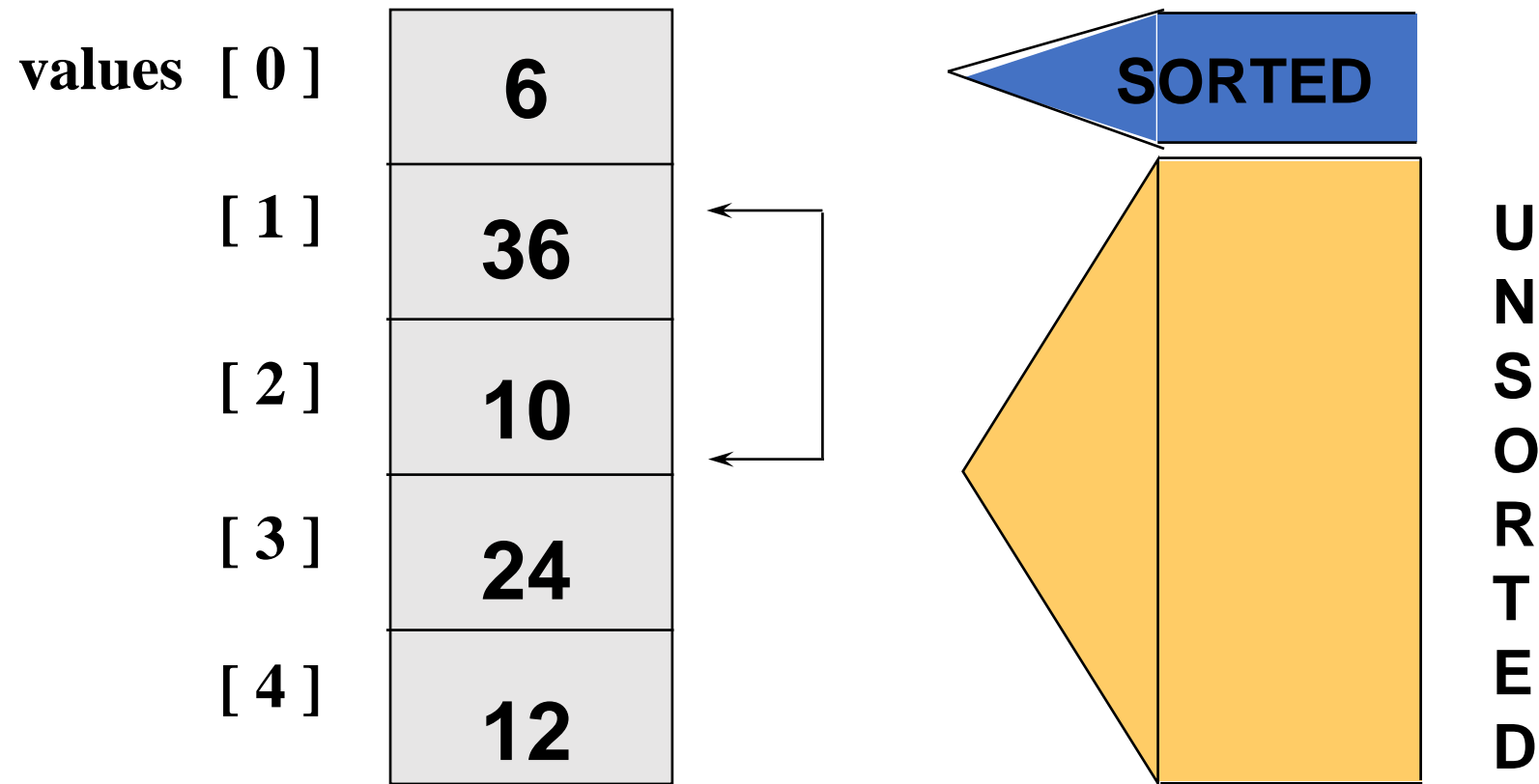
Bubble Sort: Pass Two



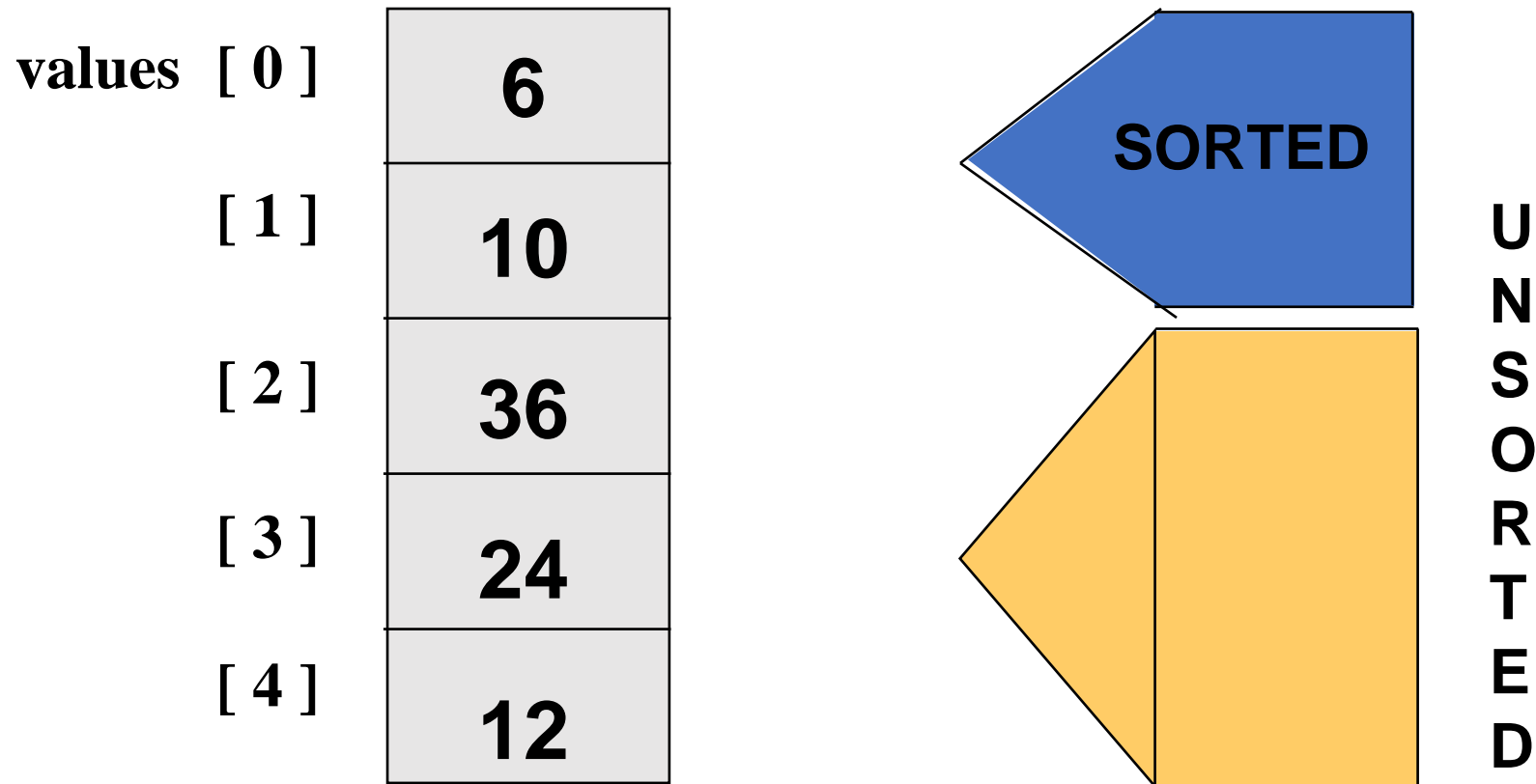
Bubble Sort: Pass Two



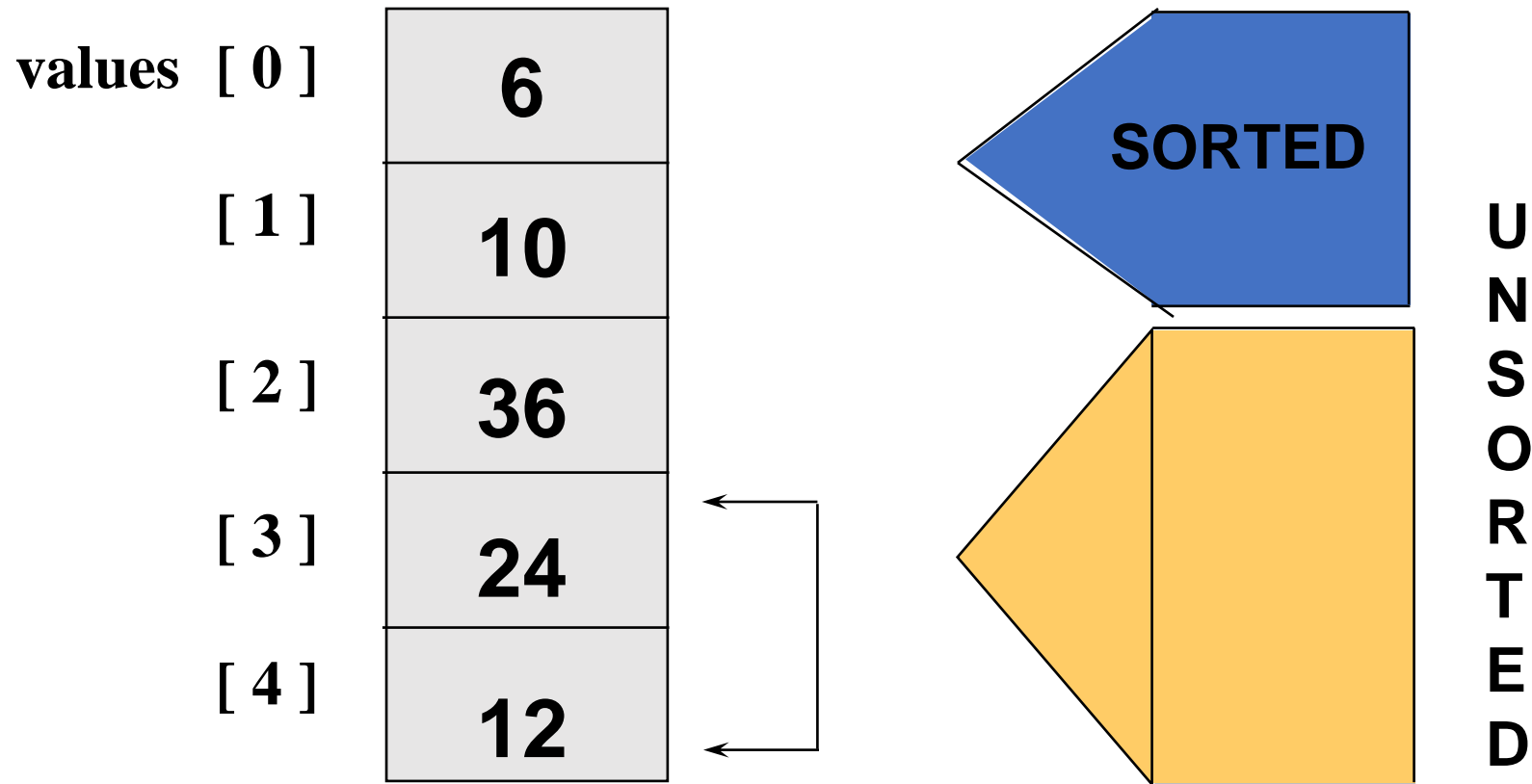
Bubble Sort: Pass Two



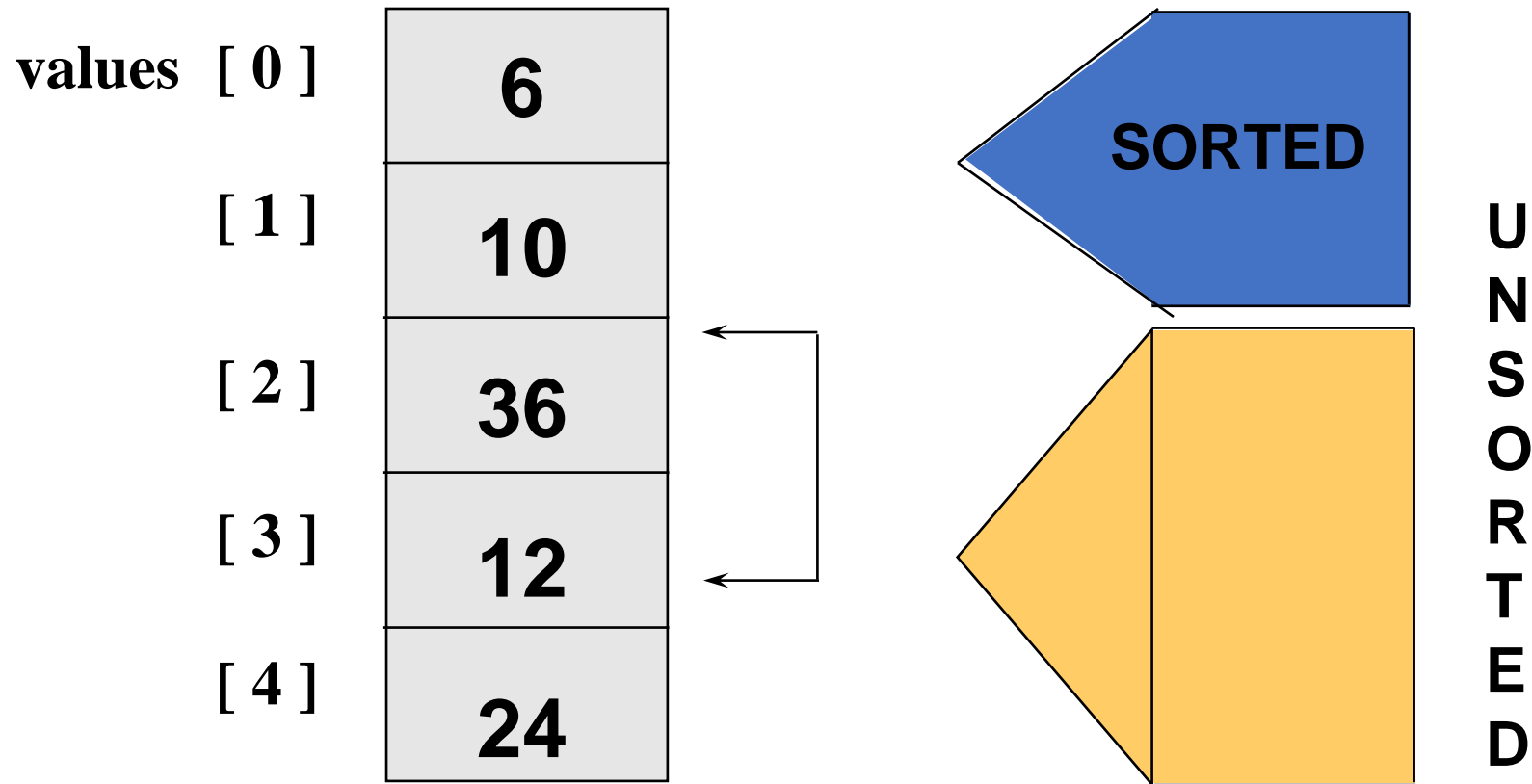
Bubble Sort: End Pass Two



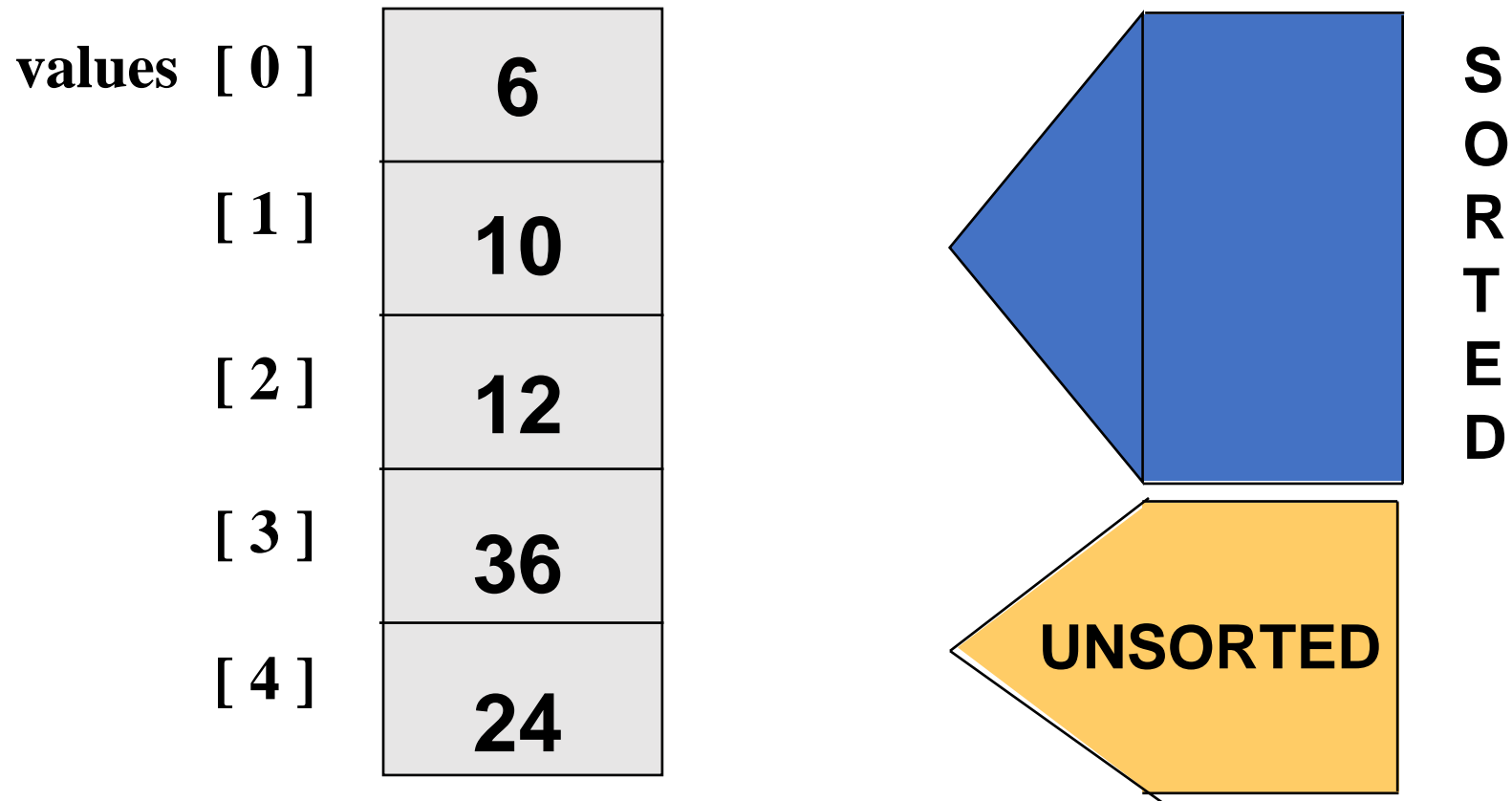
Bubble Sort: Pass Three



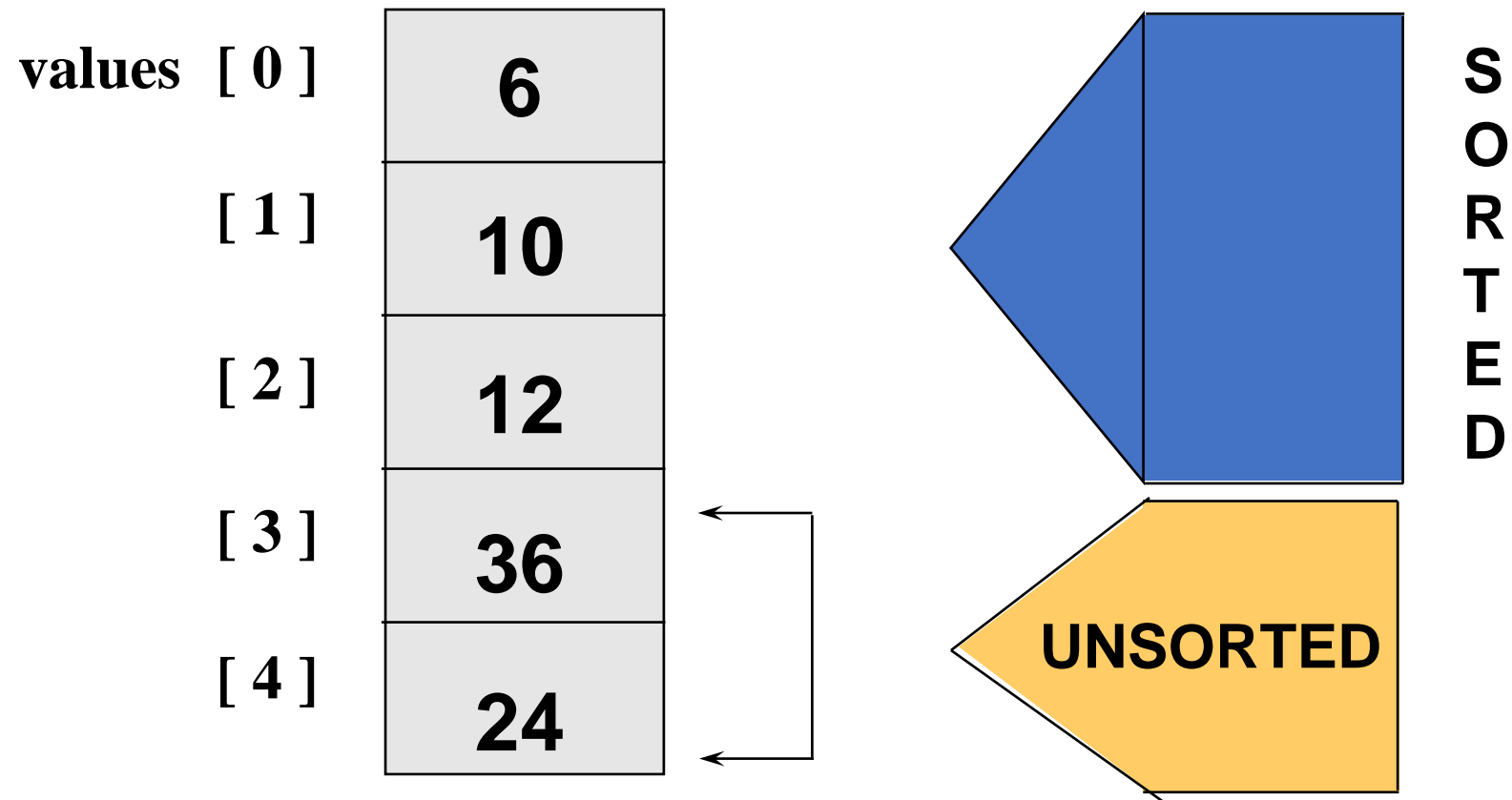
Bubble Sort: Pass Three



Bubble Sort: End Pass Three

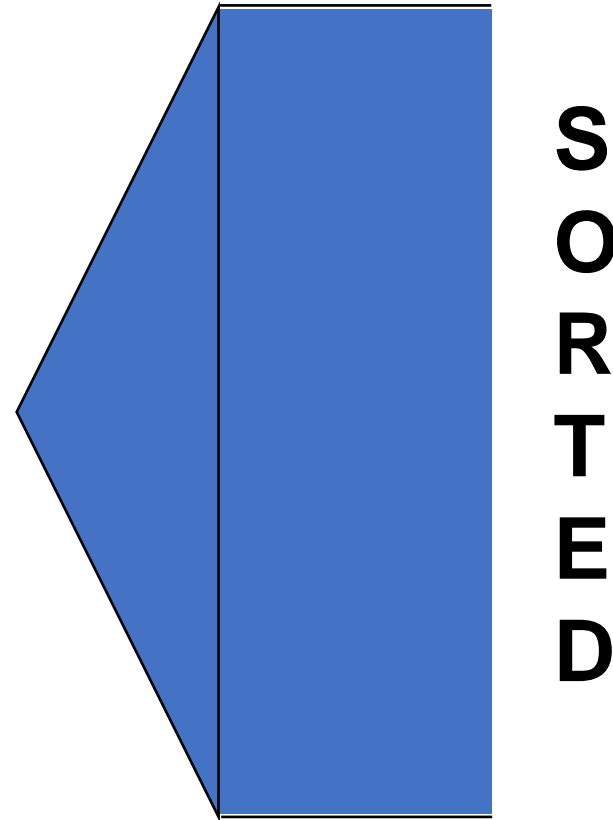


Bubble Sort: Pass Four



Bubble Sort: End Pass Four

values	[0]	6
	[1]	10
	[2]	12
	[3]	24
	[4]	36



Bubble sort in Python example

```
List = [29,8,7,6,5,4,3,2,1]
for i in range(0,len(List)-1):
    for j in range(0,len(List)-1-i):
        if List[j]>List[j+1]:
            List[j],List[j+1]=List[j+1],List[j]
print List
```

Bubble Sort (Recursive Version)

```
def bubbleSort(arr):
    n = len(arr)
    # Traverse through all list elements
    for i in range(n-1):
        # range(n) also work but outer loop will repeat one time more than needed.
        # Last i elements are already in place
        for j in range(0, n-i-1):
            # traverse the array from 0 to n-i-1
            # Swap if the element found is greater
            # than the next element
            if arr[j] > arr[j+1] :
                arr[j], arr[j+1] = arr[j+1], arr[j]

# Now the Program to test above function
arr = [64, 34, 25, 12, 22, 11, 90]
bubbleSort(arr)
print ("Sorted list is:")
for i in range(len(arr)):
    print (arr[i]),
```

Problem

- Results of VIT entrance exam has been released. Given the details of the students such as name, address and marks scored in entrance, write a program to sort the student details so that it will be convenient to call for counselling.

Test Case

Input

2 (No. of Students)

John (Name)

Chennai (Place)

99 (Score)

Mark (name)

Bangalore (Place)

80(score)

Output

[('John', 'Chennai', 99.0), ('Mark', 'Bangalore', 80.0)]