Data Structure and Algorithms

Session-13

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Polynomials

- A ***polynomial*** is an expression that contains more than two terms. A term is made up of coefficient and exponent. An example of polynomial is

$$P(x) = 4x^3+6x^2+7x+9$$

Polynomials [cont…]

- A polynomial thus may be represented using arrays or linked lists.
- Array representation assumes that the exponents of the given expression are arranged from 0 to the highest value (degree), which is represented by the subscript of the array beginning with 0. The coefficients of the respective exponent are placed at an appropriate index in the array. The array representation for the above polynomial expression is given below:

arr	9	7	6	4	(coefficients)
	0	1	2	3	(exponents)

Polynomials [cont...]

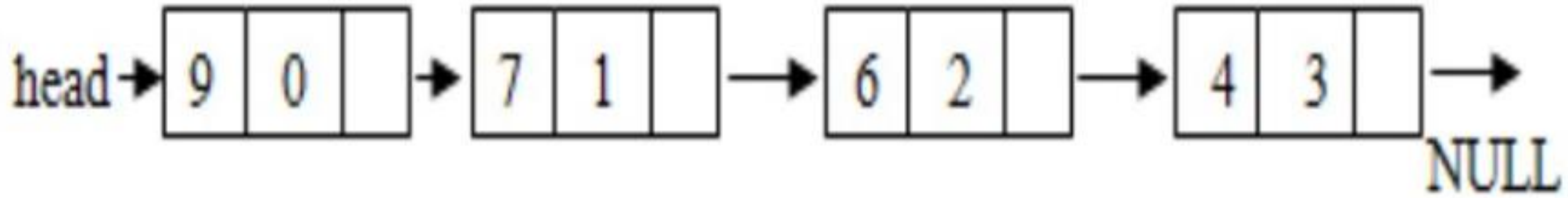
- A polynomial may also be represented using a linked list. A structure may be defined such that it contains two parts- one is the coefficient and second is the corresponding exponent. The structure definition may be given as shown below:

coef	expon	link

-
```
struct polynomial
{
int coefficient;
int exponent;
struct polynomial *next;
};
```

Polynomials [cont...]

- Thus the above polynomial may be represented using linked list as shown below:



Polynomials [cont…]

- **Addition of two Polynomials:**
  - For adding two polynomials using arrays is straightforward method, since both the arrays may be added up element wise beginning from 0 to n-1, resulting in addition of two polynomials.
  - Addition of two polynomials using linked list requires comparing the exponents, and wherever the exponents are found to be same, the coefficients are added up. For terms with different exponents, the complete term is simply added to the result thereby making it a part of addition result.

$$3x^2 + 2x + 1$$

$$5x^2 - x + 2$$

- **Multiplication of two Polynomials:**
  - Multiplication of two polynomials however requires manipulation of each node such that the exponents are added up and the coefficients are multiplied.
  - After each term of first polynomial is operated upon with each term of the second polynomial, then the result has to be added up by comparing the exponents and adding the coefficients for similar exponents and including terms as such with dissimilar exponents in the result.

Addition

Let say we have two polynomials

$$3x^2 + 2x + 1 \quad \text{———} \quad (1)$$

$$5x^2 - x + 2 \quad \text{———} \quad (2)$$

Addition of two polynomials involves combining like terms present in the two polynomials.

means adding terms having same variables and same exponents.

$$
\begin{array}{r}
3x^2 + 2x + 1 \\
5x^2 - x + 2 \\
\hline
8x^2 + x + 3
\end{array}
$$

In these two polynomials, $3x^2$ and $5x^2$ are like terms. Similarly, $2x$ and x are like terms.

Addition of two polynomials becomes easier if the terms are arranged in descending order of their exponents.

PROCEDURE FOR ADDING TWO POLYNOMIALS

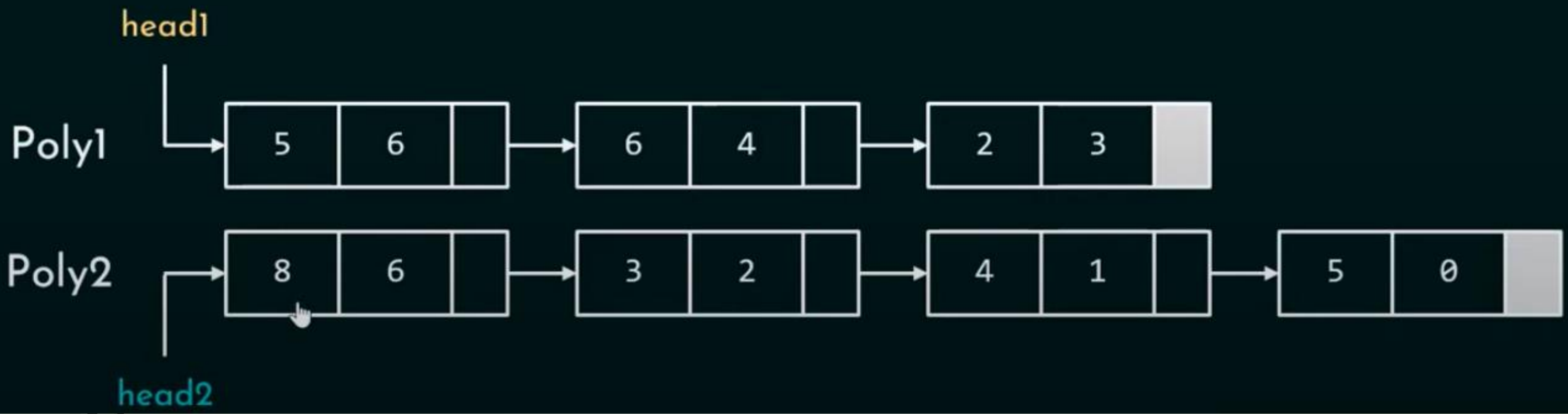
Let say we have the following two polynomials and our job is to add them.

$$5x^6 + 6x^4 + 2x^3$$

$$8x^6 + 3x^2 + 4x + 5$$

Terms are arranged in descending order of their exponents.

Let's represent the above two polynomials using linked lists.



Adding two polynomials means adding their like terms.

The only thing we have to do is to compare their exponents.

$$
\begin{array}{lr}
\boxed{5x^{6}} + 6x^{4} + 2x^{3} & \text{(Poly 1)} \\
\boxed{8x^{6}} + 3x^{2} + 4x + 5 & \text{(Poly 2)} \\
\hline
13x^{6} &
\end{array}
$$

Compare the exponents of $5x^{6}$ and $8x^{6}$

$$6 = 6$$

Adding two polynomials means adding their like terms.

The only thing we have to do is to compare their exponents.

$$
\begin{array}{r}
5x^6 + \boxed{6x^4} + 2x^3 \quad \text{(Poly 1)} \\
8x^6 + \boxed{3x^2} + 4x + 5 \quad \text{(Poly 2)} \\
\hline
13x^6 \qquad\qquad\qquad\qquad\qquad
\end{array}
$$

Compare the exponents of $6x^4$ and $3x^2$

$4 > 2$

Adding two polynomials means adding their like terms.

The only thing we have to do is to compare their exponents.

$$5x^6 + 6x^4 + 2x^3 \quad \text{(Poly 1)}$$

$$8x^6 + 3x^2 + 4x + 5 \quad \text{(Poly 2)}$$

$$13x^6 + 6x^4 + 2x^3$$

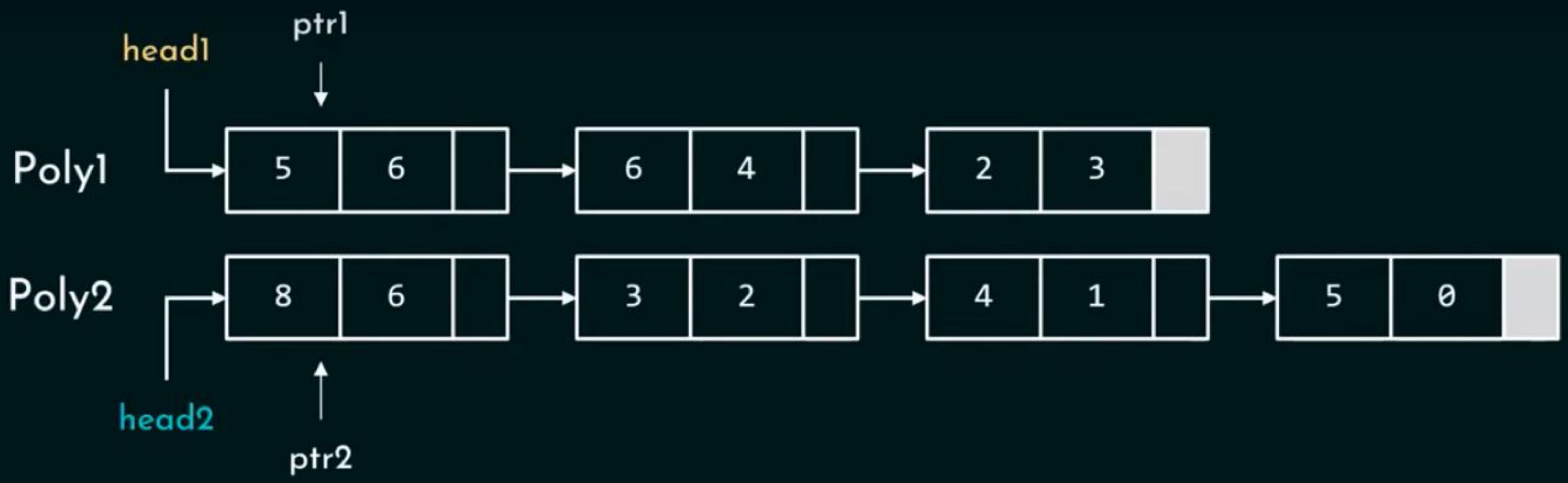
Terms of Poly 1 are finished.

Put all the remaining terms of Poly 2 in the resultant polynomial

Adding two polynomials means adding their like terms.

The only thing we have to do is to compare their exponents.

$$
\begin{array}{r}
5x^{6} + 6x^{4} + 2x^{3} \quad \text{(Poly 1)} \\
8x^{6} + 3x^{2} + 4x + 5 \quad \text{(Poly 2)} \\
\hline
13x^{6} + 6x^{4} + 2x^{3} + 3x^{2} + 4x + 5
\end{array}
$$



Algorithm:

Repeat the following until ptr1 or ptr2 becomes NULL

if(ptr1->expo == ptr2->expo)

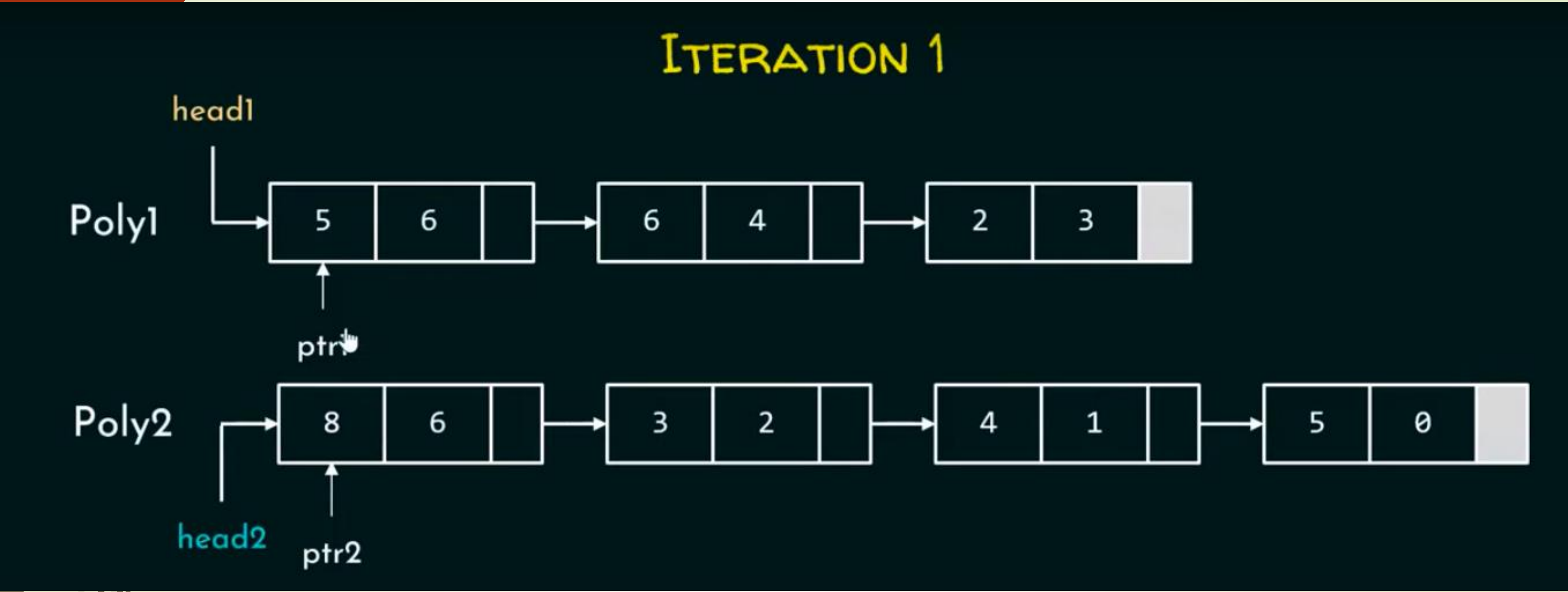
Add the coefficients and insert the newly created node in the resultant linked list and make ptr1 and ptr2 point to the next nodes.

if(ptr1->expo > ptr2->expo)

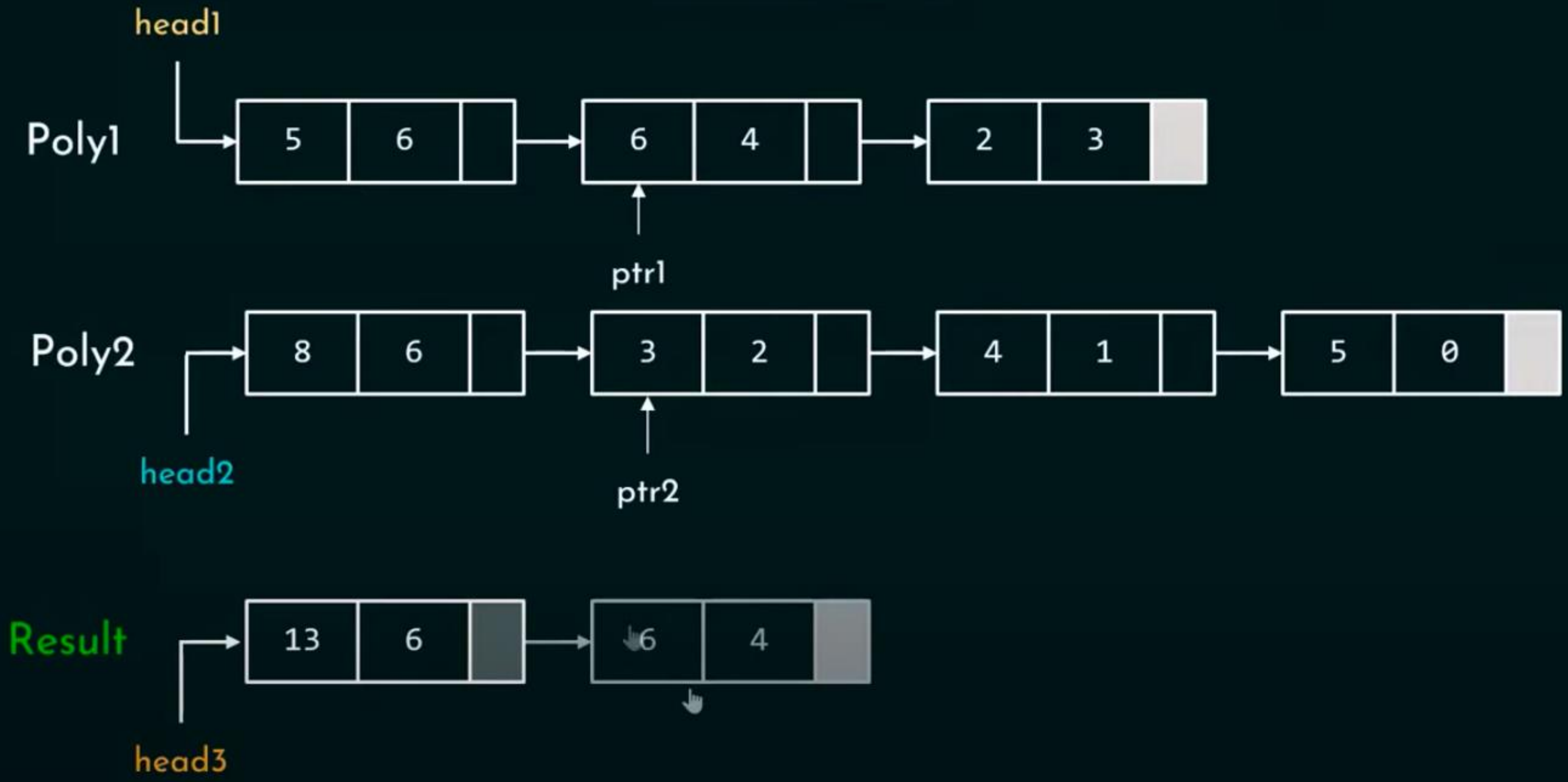
Insert the node pointed by ptr1 in the resultant linked list and make ptr1 point to the next node.

if(ptr1->expo < ptr2->expo)

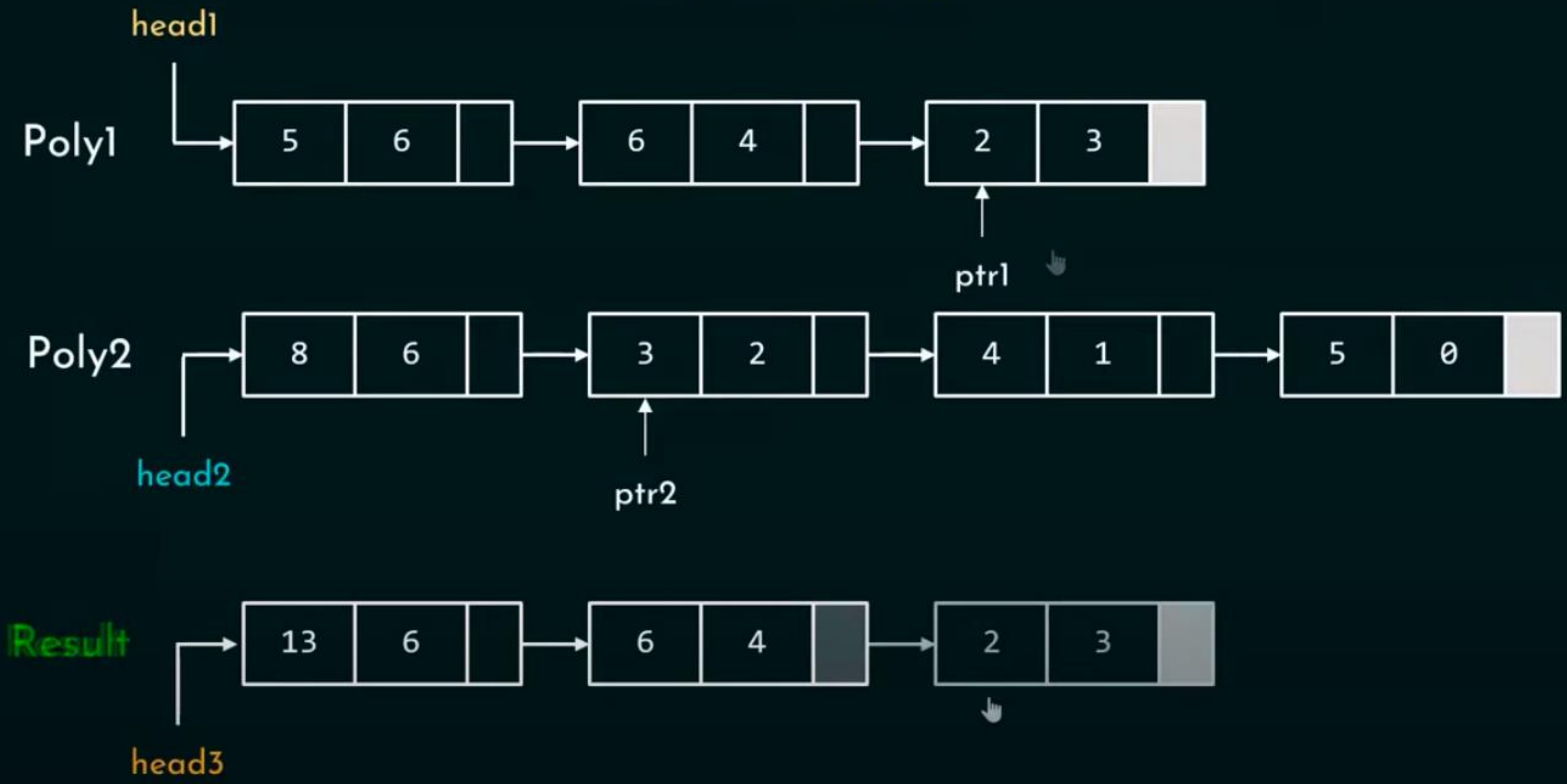
Insert the node pointed by ptr2 in the resultant linked list and make ptr2 point to the next node.


ITERATION 1
head1
Poly1
5
6
6
4
2
3
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2

ITERATION 2


head1
Poly1
5
6
6
4
2
3
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2
Result
13
6
6
4
head3

ITERATION 3

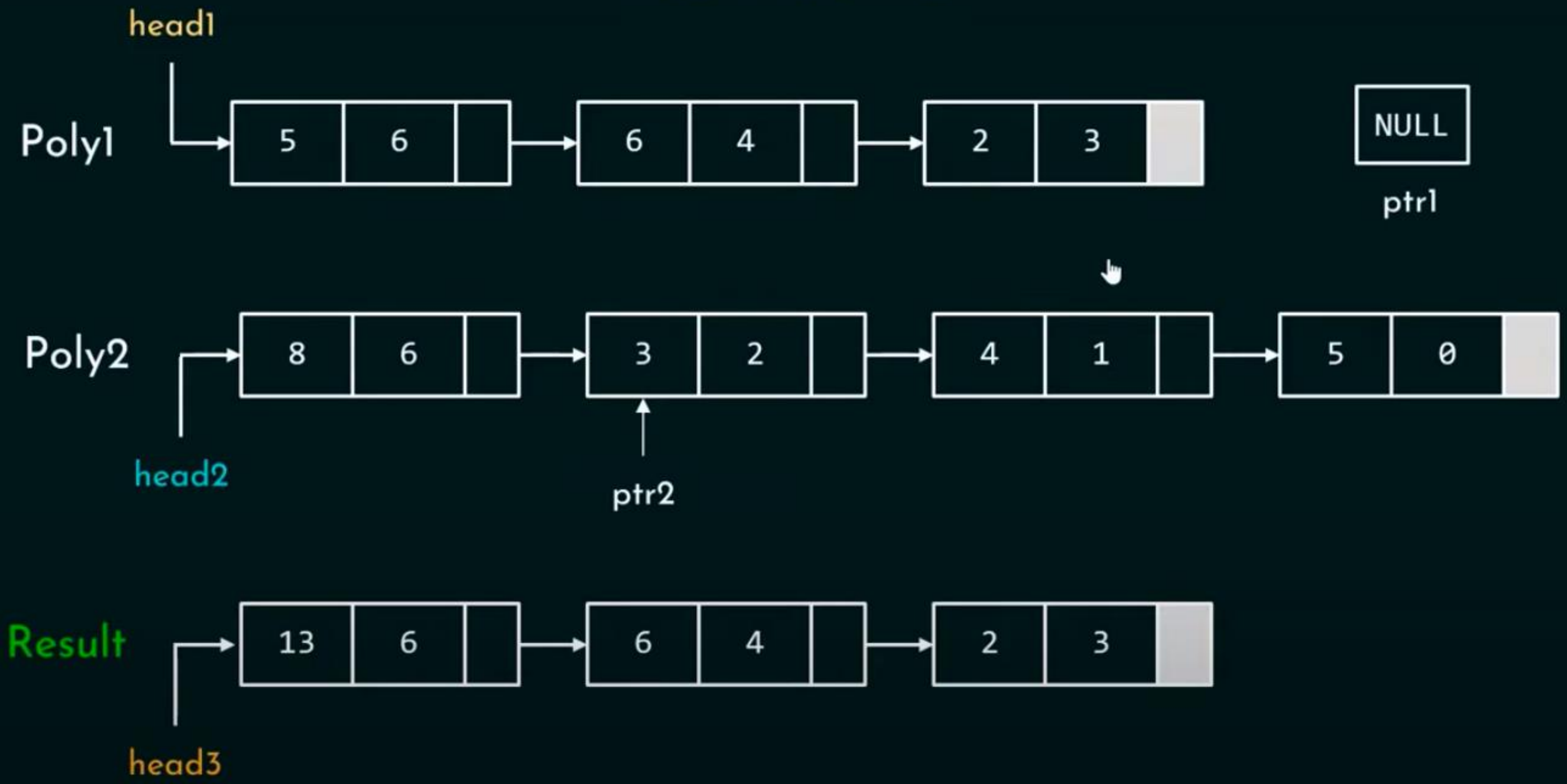

head1
Poly1
5
6
6
4
2
3
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2
Result
13
6
6
4
2
3
head3

After iteration 3, ptr1 becomes NULL.
Now, there is nothing to compare with ptr2->expo.

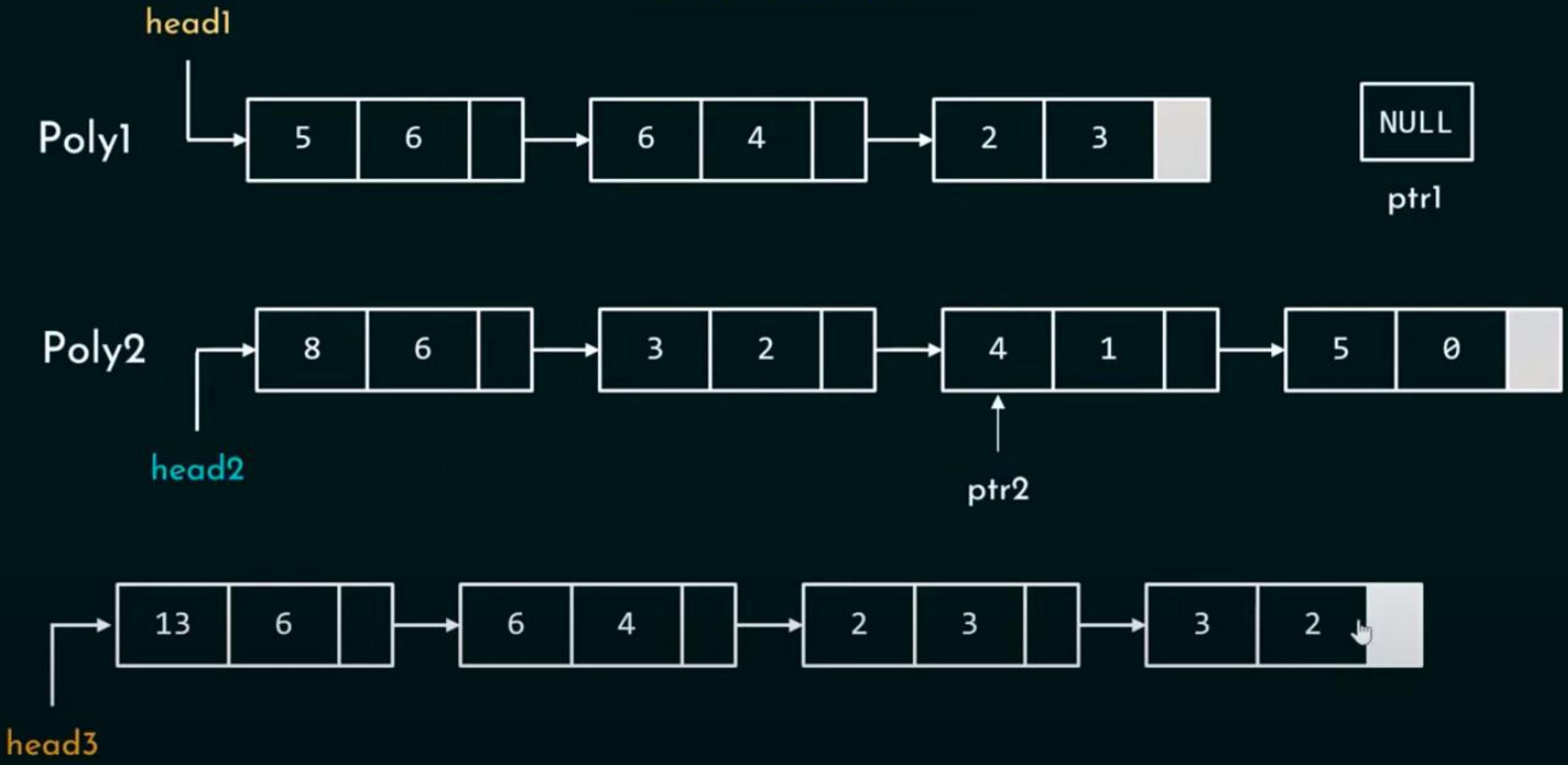
At this stage, we must add all the remaining nodes of the second linked list at the end of the resultant linked list.

```
repeat until ptr2 != NULL
        Insert the remaining nodes
repeat until ptr1 != NULL
        Insert the remaining nodes
```

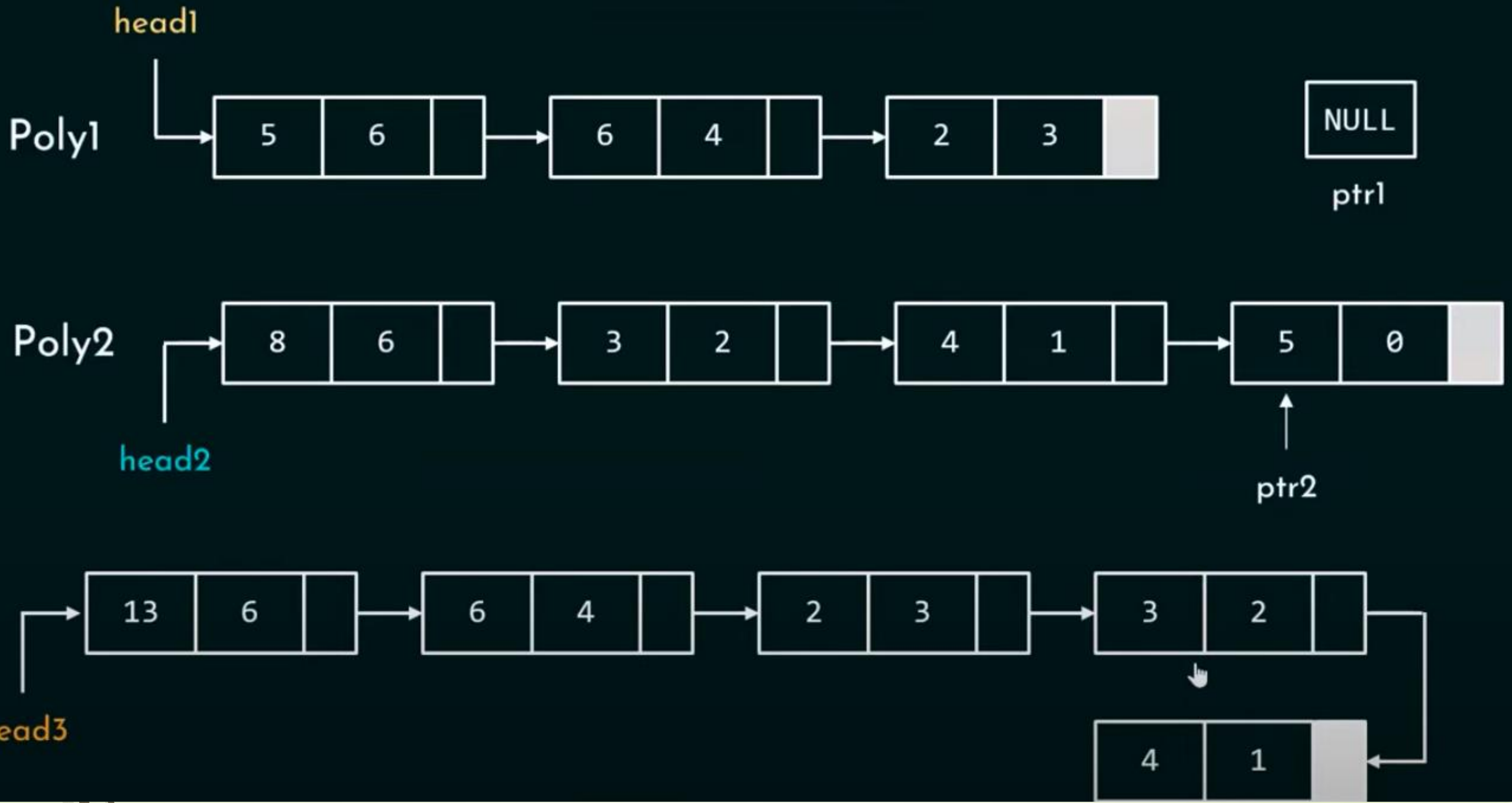
ITERATION 3


head1
Poly1
5
6
6
4
2
3
NULL
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2
Result
13
6
6
4
2
3
head3

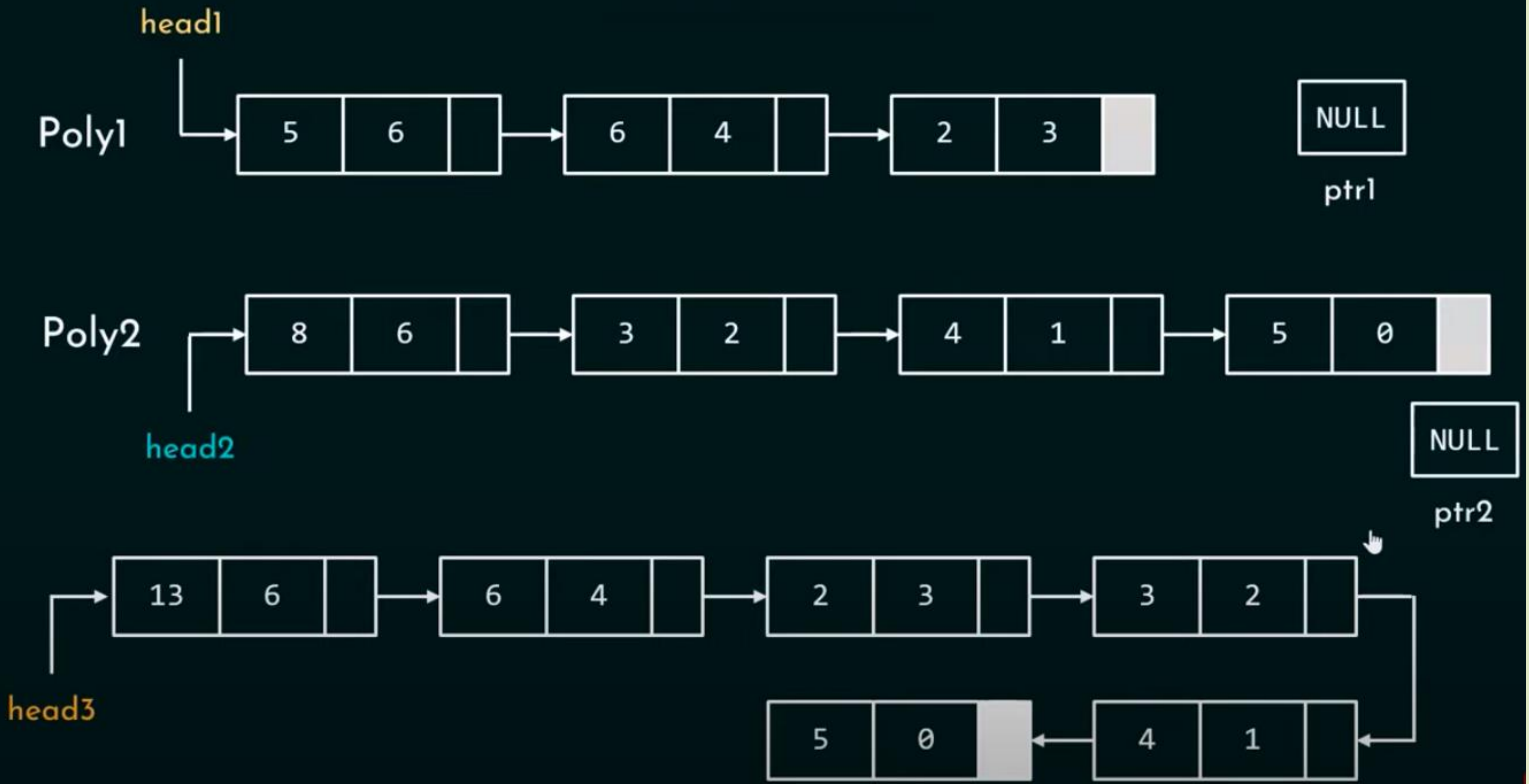
ITERATION 4


head1
Poly1
5
6
6
4
2
3
NULL
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2
13
6
6
4
2
3
3
2
head3

ITERATION 5


head1
Poly1
5
6
6
4
2
3
NULL
ptr1
Poly2
8
6
3
2
4
1
5
0
head2
ptr2
13
6
6
4
2
3
3
2
4
1
head3

ITERATION 6



head1
Poly1
5 6
6 4
2 3
NULL
ptr1
Poly2
8 6
3 2
4 1
5 0
head2
NULL
ptr2
13 6
6 4
2 3
3 2
head3
5 0
4 1

Multiplication

Consider the following polynomials

$$4x^3 + 3x^2 + 1 \text{ —— } (1)$$

$$5x^3 + 7x + 5 \text{ —— } (2)$$

Each term of the polynomial (1) must be multiplied with each term of the polynomial (2)

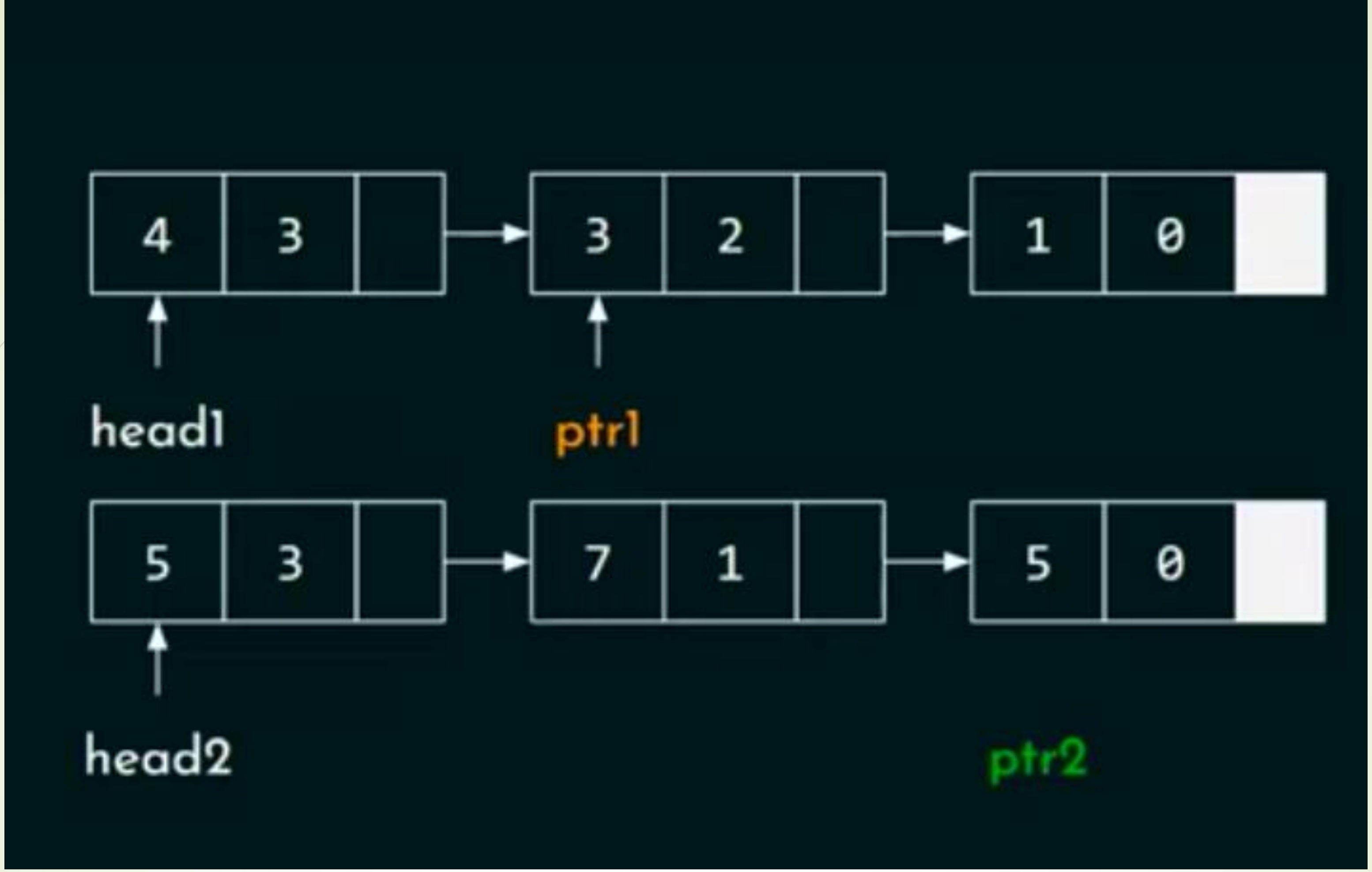
Multiplying each term means multiplying their coefficients and adding their exponents.

Consider the following polynomials

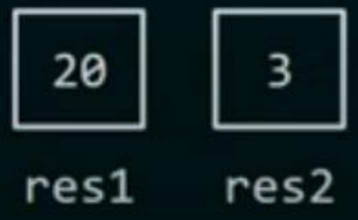
$$4x^3 + 3x^2 + 1 \text{ ——— } (1)$$

$$5x^3 + 7x + 5 \text{ ——— } (2)$$

$$(4\times5)x^{3+3} + (4\times7)x^{3+1} + (4\times5)x^{3+0} + (3\times5)x^{2+3} + (3\times7)x^{2+1} + (3\times5)x^{2+0} + (1\times5)x^{0+3} + (1\times7)x^{0+1} + (1\times5)x^{0+0}$$


4
3
3
2
1
0
head1
ptr1
5
3
7
1
5
0
head2
ptr2


```c
int res1, res2;
struct node* head3 = NULL;
while(ptr1 != NULL)
{
    ptr2 = head2;
    while(ptr2 != NULL)
    {
        res1 = ptr1->coeff * ptr2->coeff;
        res2 = ptr1->expo + ptr2->expo;
        head3 =insert(head3, res1, res2);
        ptr2 = ptr2->link;
    }
    ptr1 = ptr1->link;
}
```



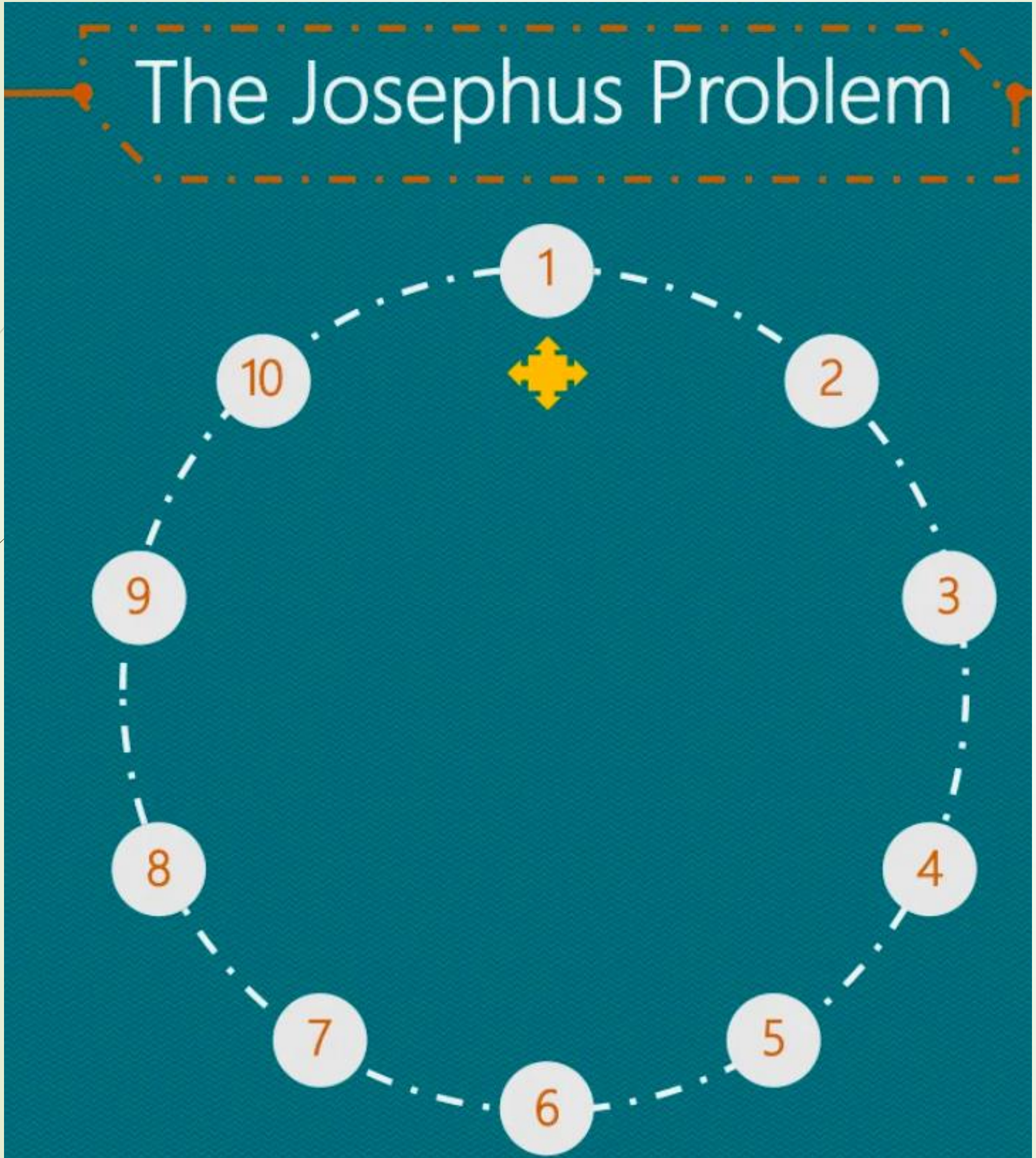
Josephus problem

Problem statement → Let n be the number of People numbered from 1 to n standing in a circle. Starting the count with person 1, we eliminate every second person until only one survivor is left.

↓

Determine the survivor

↳ Let the survivor number be denoted by $J(n)$


The Josephus Problem
1
2
3
4
5
6
7
8
9
10

Josephus problem

Let n be the no. of person's

case (i) if n is even

$$n = 2K$$

$$J(2K) = 2J(K) - 1$$

Base Case $J(1) = 1$

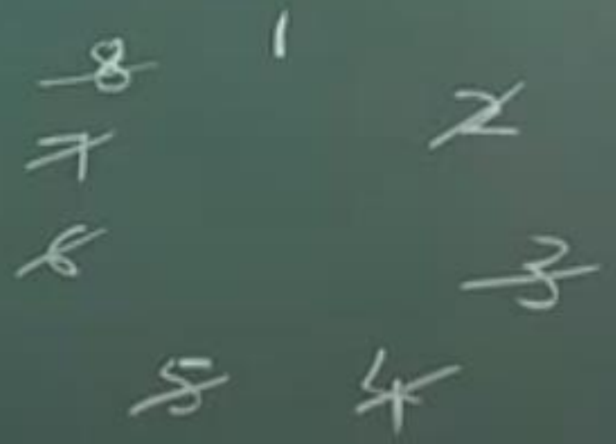
$n = 8 \quad K = 4$

$$J(8) = 2J(4) - 1$$
$$= 2[2J(2) - 1] - 1$$
$$= 4J(2) - 2 - 1$$
$$= 4J(2) - 3$$
$$= 4[2.J(1) - 1] - 3$$
$$= 8.J(1) - 4 - 3$$

Eg:-

$n = 8$

$J(8) = 1$



Case (ii) if n is odd

$$n = 2k+1$$

$$J(2k+1) = 2J(k)+1$$

Base case $J(1) = 1$

$$n = 7 \quad J(7) = 7$$

$$k = 3$$

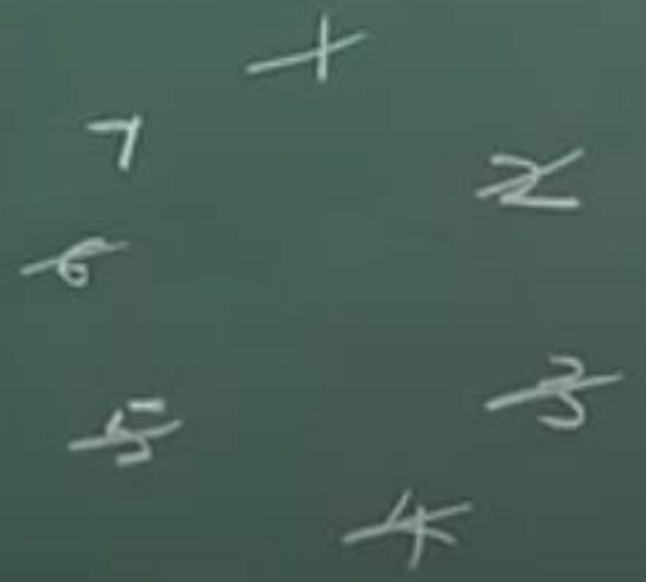
$$J(7) = 2J(3)+1$$

$$= 2\left[2J(1)+1\right]+1$$

$$= 4J(1)+2+1$$

$$= 4J(1)+3$$

$$n = 7 \quad J(7) = 7$$



The Josephus Problem

Recurrent equation

EVEN

$$\boxed{J(1) = 1;}$$

$$J(2n) = 2J(n) - 1; \quad \text{for } n \geq 1$$

$$J(2n+1) = 2J(n) + 1; \quad \text{for } n \geq 1$$

ODD

The Josephus Problem

Recurrent equation

EVEN

$\boxed{J(1) = 1;}$

$J(2n) = 2J(n) - 1;$ for $n \geq 1$

$J(2n+1) = 2J(n) + 1;$ for $n \geq 1$

ODD

$$
\begin{aligned}
J(41) &= 2\,J(20)+1 &&= 2 \times 9 + 1 &&= 19 \\
J(20) &= 2\,J(10)-1 &&= 2 \times 5 - 1 &&= 9 \\
J(10) &= 2\,J(5)-1 &&= 2 \times 3 - 1 &&= 5 \\
J(5) &= 2\,J(2)+1 &&= 2 \times 1 + 1 &&= 3 \\
J(2) &= 2\,J(1)-1 &&= 2 \times 1 - 1 &&= 1 \\
J(1) &= 1
\end{aligned}
$$

$$n = 7 \rightarrow 111 \qquad J(7) = J(111) = 111 = 7$$

$$n = 5 \rightarrow 101 \qquad J(5) = J(101) = 011 = 3$$


Thank you