Performance Characteristics of Sensors

Module 2

Contents

- Static characteristics: accuracy, precision, resolution, sensitivity, linearity, span and range
- Dynamic characteristics, Mathematical model of transducer: zero, first and second, Response to impulse, step, ramp and sinusoidal inputs, Selection criteria of sensor.

What are Static Characteristics?

- Static characteristics of a sensor refer to the characteristics of the system when the input is either held constant or varying very slowly. Some of them are broadly classified as:
  - Non Linearity
  - Sensitivity
  - Resolution
  - Accuracy
  - Precision
  - Hysteresis
  - Repeatability
  - Range or Span or Full Scale Input

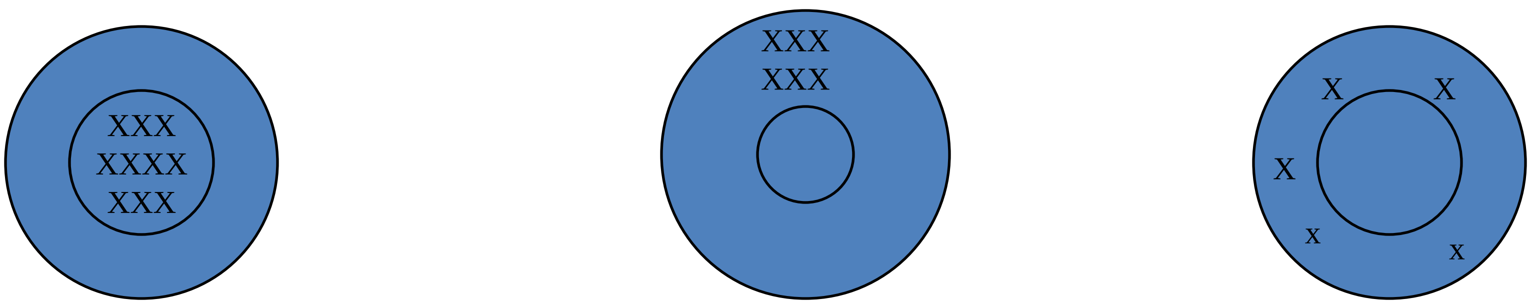
ACCURACY and PRECISION

- Accuracy indicates the closeness of the measured value with the actual or true value, and is expressed in the form of the maximum error (= measured value – true value) *as a percentage of* full scale reading.
- Precision indicates the repeatability or reproducibility of a sensor. If a sensor is used to measure the same input, but at different instants, the output from the sensor should be same.

ACCURACY (vs) PRECISION

Example :

X : result and Centre circle : true value



High accuracy, high precision	Low accuracy, high precision	Low accuracy, low precision

- The difference between precision and accuracy needs to be understood carefully
- Precision means repetition of successive readings, but it does not guarantee accuracy; successive readings may be close to each other, but far from the true value
- On the other hand, an accurate instrument has to be precise also, since successive readings must be close to the true value.

NUMERICAL

Example :

Two pressure gauges (pressure gauge A and B) have a full scale accuracy of ± 5%. Sensor A has a range of 0-1 bar and Sensor B 0-10 bar. Which gauge is more suitable to be used if the reading is 0.9 bar?

Answer :

Sensor A :

Equipment max error $= \pm \frac{5}{100} \times 1 \text{ bar} = \pm 0.05 \text{ bar}$

Equipment Error %

@ 0.9 bar (in %) $= \pm \frac{0.05 \text{ bar}}{0.9 \text{ bar}} \times 100 = \pm 5.6\%$

Sensor B :

$$\text{Equipment max error} = \pm \frac{5}{100} \times 10 \text{ bar} = \pm 0.5 \text{ bar}$$

Equipment Error %

$$@\ 0.9 \text{ bar (in \%)} = \pm \frac{0.5 \text{ bar}}{0.9 \text{ bar}} \times 100 = \pm 55\%$$

Conclusion :

Sensor A is more suitable to use at a reading of 0.9 bar because the error percentage (± 5.6%) is smaller compared to the percentage error of Sensor B (± 55%).

Q1) The volume of a liquid is 26 mL. A student measures the volume and finds it to be 26.2 mL, 26.1 mL, 25.9 mL, and 26.3 mL in the first, second, third, and fourth trial, respectively. Which of the following statements is true for his measurements?

a. They are neither precise nor accurate.

b. They have poor accuracy.

c. They have good precision.

d. They have poor precision.

Answer: They have good precision.

Q2) The volume of a liquid is 20.5 mL. Which of the following sets of measurement represents the value with good accuracy?

18.6 mL, 17.8 mL, 19.6 mL, 17.2 mL

19.2 mL, 19.3 mL, 18.8 mL, 18.6 mL

18.9 mL, 19.0 mL, 19.2 mL, 18.8 mL

20.2 mL, 20.5 mL, 20.3 mL, 20.1 mL

- **Example:** A thermistor is used to measure temperature between –30 and +80 °C and produce an output voltage between 2.8V and 1.5V. Because of errors, the accuracy in sensing is ±0.5°C. so the measured value may be high than or lower than by 0.5 °C

(i)Find the percentage of Error.
(ii)If the accuracy in sensing is shifted to $\pm 0.059V$, in terms of output, then what is the updated percentage of Error?

Answer: (i)

Percentage of error (w.r.t. Temperature)
= [0.5 / (80+30)] * 100 =0.454%

Answer: (ii)

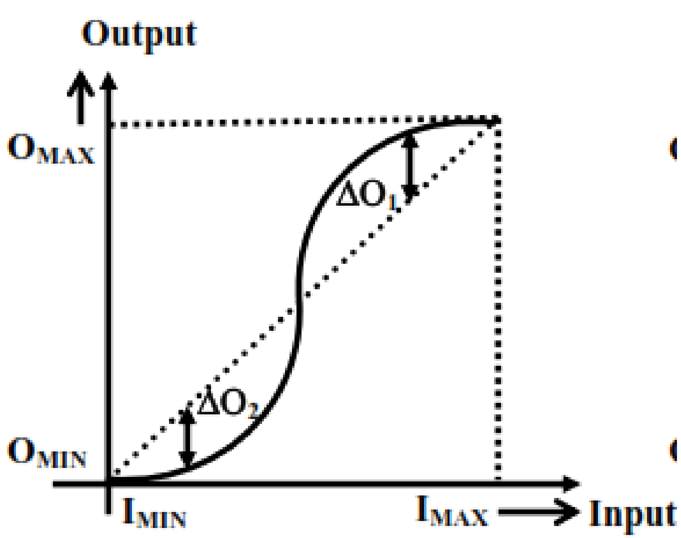
Percentage of error (w.r.t. V)
= [0.059 / (2.8-1.5)] * 100 =4.53%

$$Linearity = \frac{\Delta O}{O_{max} - O_{min}}$$

where, $\Delta O = \max(\Delta O_1, \Delta O_2)$.

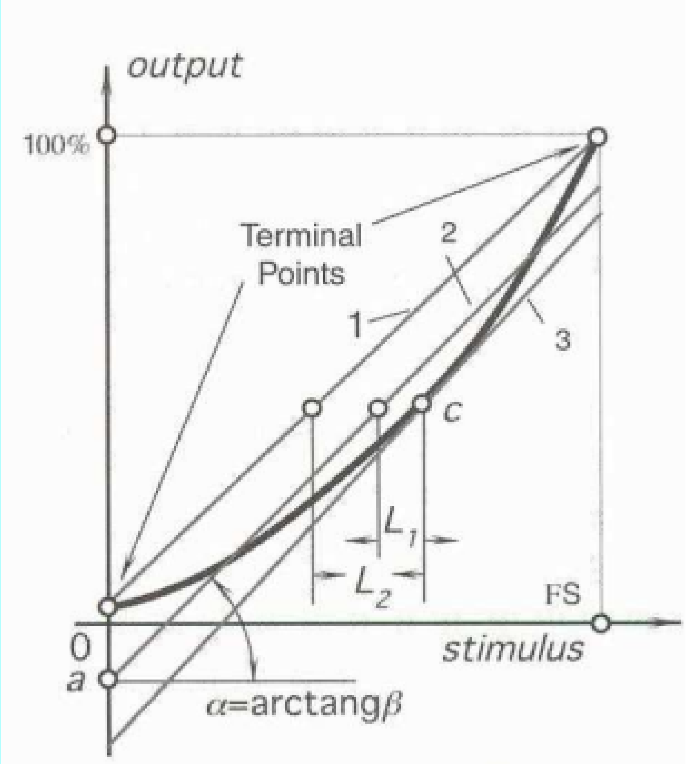
NON-LINEARITY

- The non-linearity is defined as the maximum deviation from the linear characteristics as a percentage of the full scale output.



Nonlinearity

- Nonlinearity is defined as the maximum deviation from the ideal linear transfer function.
- Nonlinearity must be deduced from the actual transfer function or from the calibration curve
- A few methods to do so:
- a. by use of the range of the sensor
  - Pass a straight line between the range points (line 1)
- b. use a linear best fit (least squares) through the points of the curve (line 2)



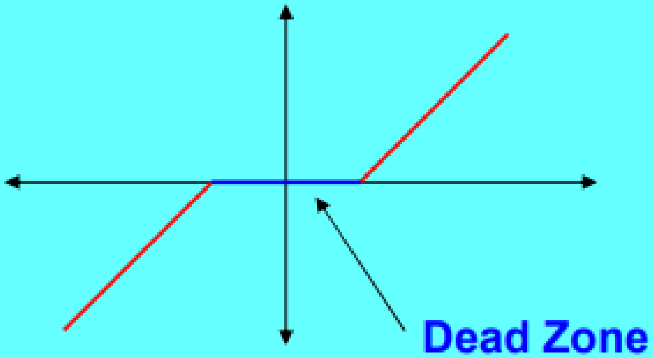
- c. use the tangent to the curve at some point on the curve

  Take a point in the middle of the range of interest

  -Draw the tangent and extend to the range of the curve (line 3)

Deadband

- **Deadband:** the lack of response or insensitivity of a device over a specific range of the input.
- In this range which may be small, the output remains constant.
- A device should not operate in this range unless this insensitivity is acceptable.



RESOLUTION

- *Resolution* indicates the minimum change in input variable that is detectable (or) Resolution describes the smallest increments of stimulus which can be sensed by a sensor.

Resolution

- **Resolution:** the minimum increment in stimulus to which the sensor can respond. It is the magnitude of the input change which results in the smallest observable output.
- Example: a digital voltmeter with resolution of 0.1V is used to measure the output of a sensor. The change in input (temperature, pressure, etc.) that will provide a change of 0.1V on the voltmeter is the resolution of the sensor/voltmeter system.
- In digital systems generally, resolution may be specified as $1/2^N$ (N is the number of bit.)

SENSITIVITY

- It can be defined as the ratio of the incremental output and the incremental input. *While defining* the sensitivity, we assume that the input-output characteristic of the instrument is approximately linear in the range.
- Example: sensitivity of a spring balance can be expressed as 25 mm/kg (say),indicating additional load of 1 kg will cause additional displacement of the spring by 25mm.

Sensitivity

- **Sensitivity** of a sensor is defined as the change in output for a given change in input, usually a unit change in input. Sensitivity represents the slope of the transfer function.
- Also is used to indicate sensitivity to other environment that is not measured.
- Example: sensitivity of resistance measurement to temperature change

$$\boxed{\frac{d}{dR}(aT+b)=1 \quad \rightarrow \quad \frac{dR}{dT}=a \quad \left[\frac{\Omega}{°C}\right]}$$

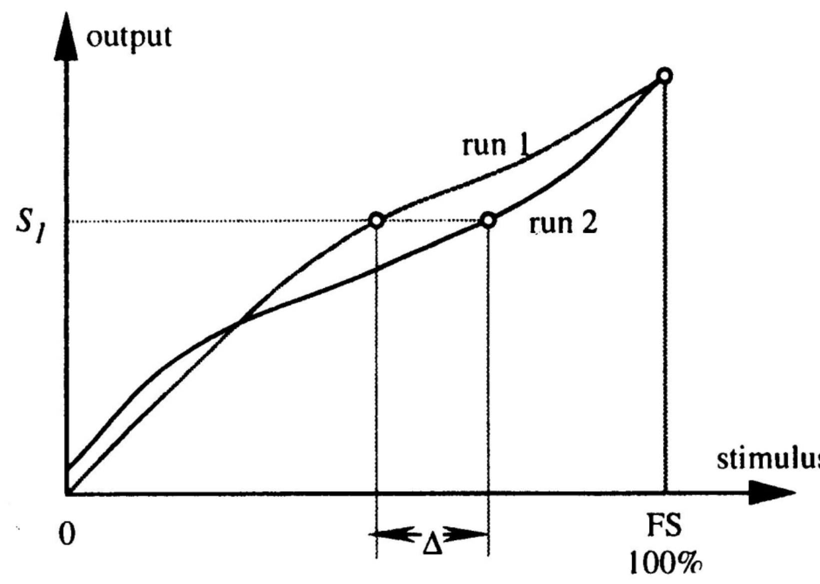
REPEATABLITY

- Repeatability error of a sensor is its **inability to represent the same value under identical** conditions. It is expressed as the maximum difference between output readings as determined by two measurement cycles.

  $\delta_r = [\Delta/FS * 100\%]$,

where ,

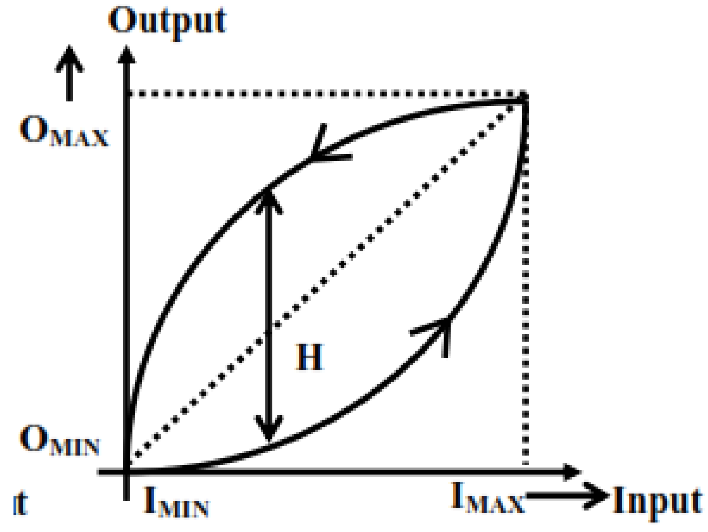
  FS=Full scale of sensor



HYSTERESIS

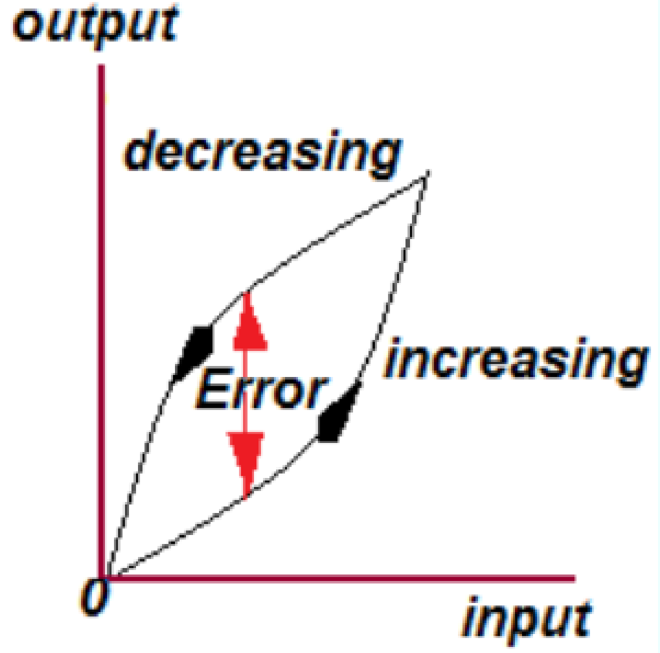
- It is the difference between output readings for the same measurand, when approached while increasing from the minimum value and the other while decreasing from the peak value.

$$Hysteresis = \frac{H}{O_{max} - O_{min}} X100.$$



Hysteresis

- **Hysteresis** is the deviation of the sensor's output at any given point when approached from two different directions
- Caused by electrical or mechanical systems
  - Magnetization
  - Thermal properties
  - Loose linkages



- If temperature is measured, at a rated temperature of 50°C, the output might be 4.95V when temperature increases but 5.05V when temperature decreases.
- This is an error of ±0.5% (for an output full scale of 10V in this idealized example).

RANGE or FULL SCALE INPUT

- A dynamic range of stimuli which may be converted by a sensor is called Span of Full Scale.
- It represents the highest possible input value that can be applied to a sensor without causing an unacceptably large inaccuracy.
- It defines the maximum and minimum values of the inputs or the outputs for which the instrument is recommended to use.

Range and Span (Example)

- Example: a sensors is designed for: –30 °C to +80 °C to output 2.5V to 1.2V
- Range: –30°C and +80 °C
- Span: 80– (–30)=110 °C
- Input full scale = 110 °C
- Output full scale = 2.5V-1.2V=1.3V

- Full-scale output
  - algebraic difference between the electrical output signals measured with maximum input stimulus and the lowest input stimulus applied
  - E.g. LM35
- Output Impedance
  – The output impedance *Zout* is important to know how to interface a sensor with the electronic circuit
  – A current generating sensor should have an output impedance as high as possible and the circuit's input impedance should be low
  – For the voltage connection, a sensor is preferable with lower *Zout* and the circuit should have Zin as high as practical

Numerical on Static characteristics

Static Error

- The most important characteristic of an instrument or measurement system is its accuracy, which is the agreement of the instrument reading with the true value of quantity being measured. The accuracy of an instrument is measured in terms of its error.

$$\delta A = A_m - A_t$$

where δA = error,

A_m = measured value of quantity,

and A_t = true value of quantity.

δA is also called the absolute static error of quantity A.

We have $\epsilon_0 = \delta A$

where ϵ_0 = absolute static error of quantity A (under measurement).

- The absolute value of A does not indicate precisely the accuracy of measurements
- As an Example, an error of + 2 A is negligible when the current being measured is of the order of 1000 A

while the same error of ±2 A may be regarded as intolerable when the current under measurement is 10 A or so.

- Thus the quality of measurement is provided by the relative static error, i.e., the ratio of absolute static error δA to the true value A_t of the quantity under measurement. The relative static error Er is given by :

$$\epsilon_r = \frac{\text{absolute error}}{\text{true value}} = \frac{\delta A}{A_t} = \frac{\epsilon_0}{A_t}$$

Percentage static error $\% \epsilon_r = \epsilon_r \times 100$

We have
$$A_t = A_m - \delta A$$
$$= A_m - \epsilon_0 = A_m - \epsilon_r A_t$$
$$= \frac{A_m}{1+\epsilon_r}$$

Static Correction

It is the difference between the true value and the measured value of the quantity, or

$$\delta C = A_t - A_m$$

Where, δC=static correction= $-\delta A$

Example #1

A meter reads 127.50 V and the true value of the voltage is 127.43 °C V. Determine (a) static error and (b) static correction for this instrument.

Solution:

(a) The static error is $\delta A = A_m - A_{t=} 127.5 - 127.43 = 0.07V$

(b) Static Correction $\delta C = -\delta A = -0.07V$

Example #2

A Thermometer reads 95.45 °C and the static correction given in the correction curve is -0.08 °C. Determine the true value of the temperature.

Solution:

True Value of the temperature is:

$$A_t = A_m + \delta C$$

$$= 95.45 - 0.08$$

$$= 95.37 \ °C$$

Example #3

A Voltmeter shows a true value of 1.5 V. An analog indicating instrument with a scale range of 0-2.5 V shows a voltage of 1.46 V. What are the values of absolute error and correction. Express the error as a fraction of the true value and the full scale deflection.

Solution:

Absolute Error : $\delta A = A_m - A_t = 1.46 - 1.50 = -0.04$

Absolute Correction : $\delta C = -\delta A = +0.04\ V$

Relative Error : $\varepsilon_r = \frac{\delta A}{A_t} = \frac{-0.04}{1.50}$ x 100 = -2.66 %

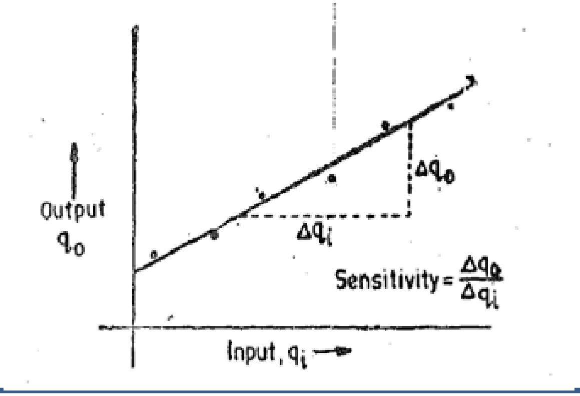
Relative Error ((expressed as a percentage of Full Scale) = $\frac{-0.04}{2.5}$ x 100 = - 1.60%

Static Sensitivity

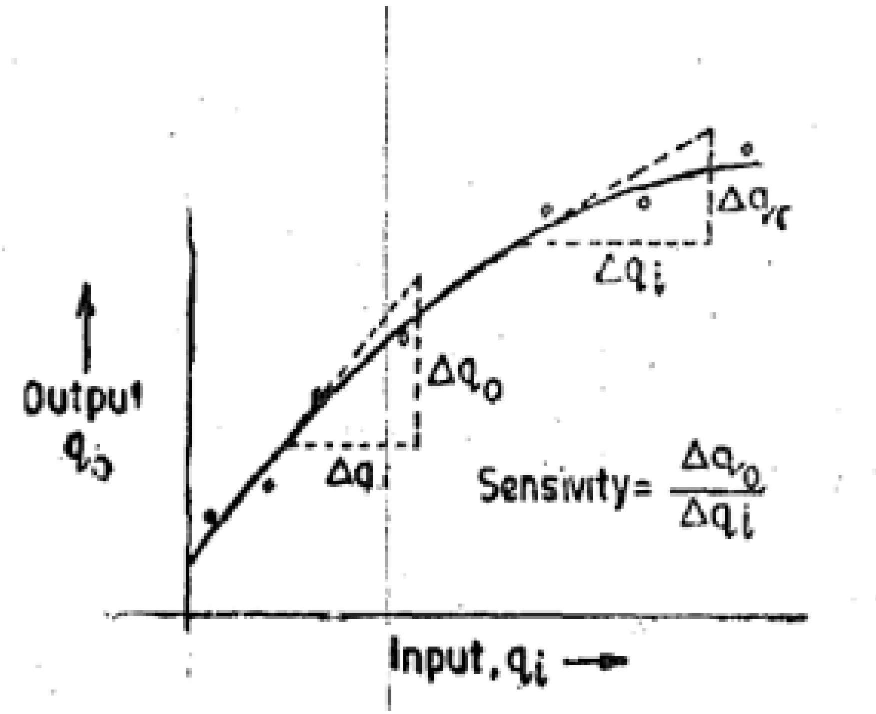
- The static sensitivity of an instrument or an instrumentation system is the ratio of the magnitude of the output signal or response to the magnitude of input signal or the quantity being measured.

- Its unit depends on the type of the input and the output

- When a calibration curve is linear, the sensitivity of the instrument can be defined as in slope of the calibration curve. For this case, the sensitivity is constant over the entire range of the instrument.



- If the curve is not nominally a straight line, the sensitivity varies with the input. The sensitivity in this case varies.



In general, the static sensitivity at the operating point is defined as :

Static Sensitivity = $\frac{infinistesimal\ change\ in\ output}{infinistesimal\ change\ in\ input} = \frac{\Delta q_o}{\Delta q_i}$

Similarly,

The inverse sensitivity or deflection factor = $\frac{\Delta q_i}{\Delta o}$

Example #4

A Wheatstone bridge requires a change of 7Ω in the unknown arm of the bridge to produce
a change in deflection of 3mm of the galvanometer. Determine the sensitivity.
Also determine the deflection factor.

$=\frac{3\ mm}{7\ \Omega}$ $=0.429\ mm/\Omega$

Solution: Sensitivity = $\frac{Magnitude\ of\ output\ response}{Magnitude\ of\ input}$

$=\frac{7\ \Omega}{3\ mm}$ $=2.33\ \Omega/mm$

Inverse sensitivity or scale factor = $\frac{Magnitude\ of\ the\ input}{MAgnitude\ of\ the\ output\ response}$

Example #5

A mercury thermometer has a capillary tube of 0.25 mm diameter. If the bulb is made of a zero expansion
material what volume Must it have if a sensitivity of 2.55 mm/$°C$ is desired? Assume that the operating temperature
is 20$°C$ and the coefficient of volumetric Expansion of mercury is $0.181x\ 10^{-3}/$ °C.

Solution:

Let $\boldsymbol{L_c}$ be the length of the capillary tube which would be occupied by mercury contained in the bulb when it is
Not heated in mm

$\boldsymbol{L_c} + \Delta \boldsymbol{L_c}$ be the length of the capillary tube which would be occupied by mercury contained in the bulb when
heated in mm

$\boldsymbol{A_c}$ be the area of the capillary tube in mm^2

α_v be the coefficient of volumetric expansion mm^2/°C

ΔT be the change in temperature, °C

It should be noted that there will be only a change in length of mercury column since it is given that
the bulb has a zero expansion material and hence there will be no changes in its area and length.
This is true for capillary tube as well.

$$Sensitivity\ S = \frac{\Delta q_0}{\Delta q_i} = \frac{(L_c + \Delta L_c) - L_c}{\Delta T} = \frac{\Delta L_c}{\Delta T} = 2.5\ mm/°C$$

Now, $A_c\ (L_c + \ \Delta L_c) = A_c\ (L_c + \alpha_v\ \Delta L_c)$

The Length of the capillary tube is

$$L_c = \frac{1}{\alpha_v}\frac{\Delta L_c}{\Delta T} = \frac{1}{0.181\ X\ 10^{-3}}\ x\ 2.5 = 13.8\ x\ 10^3\ mm = 13.8\ m$$

Hence, the volume of the bulb $= A_c L_c = \frac{\pi}{4}(0.25)^2\ x\ 13.8\ x\ 10^3$

$$= 680\ mm^3$$

What are Dynamic Characteristics?

- Dynamic characteristics refer to the performance of the instrument when the input variable is changing rapidly with time.
- The dynamic performance of an instrument is normally expressed by a differential equation relating the input and output quantities.
- It is always convenient to express the input-output dynamic characteristics in form of a linear differential equation.

The dynamic characteristics of any measurement system are:

(i) **Speed of response and Response time**

Speed of Response is defined as the rapidity with which an instrument or measurement system responds to changes in measured quantity.

Response Time is the time required by instrument or system to settle to its final steady position after the application of the input.

(ii) **Lag**

An instrument does not react to a change in input immediately. The delay in the response of an instrument to a change in the measured quantity is known as *measuring lag.*

(iii) **Fidelity**

Fidelity of a system is defined as the ability of the system to reproduce the output in the same form as the input.

(iv) **Dynamic error**

The dynamic error is the difference between the true value of the quantity changing with time and the value indicated by the instrument if no static error is assumed

- So, often a nonlinear mathematical model is linearized and expressed in the form as below:

$$a_n \frac{d^n x_0}{dt^n} + a_{n-1} \frac{d^{n-1} x_0}{dt^{n-1}} + \cdots + a_1 \frac{dx_0}{dt} + a_0 x_0 = b_m \frac{d^m x_i}{dt^m} + b_{m-1} \frac{d^{m-1} x_i}{dt^{m-1}} + \cdots + b_1 \frac{dx_i}{dt} + b_0 x_i$$

Where,

- X_o and X_i are output and input variables respectively.
- The above expression can also be expressed in terms of a transfer function, as:

$$G(s) = \frac{x_0(s)}{x_i(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} \cdots + b_1 s + b_0}{a_n s^n + b_{n-1} s^{n-1} \cdots + a_1 s + a_0}$$

- Normally $m<n$ and ‘n’ is called the order of the system. Commonly available sensor characteristics can usually be approximated as either zero-th order, first order or second order dynamics.

- The order of the system is defined by the total number of energy storage components in the system.

    E.g. Energy-Storing Elements

  Inertial Elements ⟶Mass, Inductance,...

  Capacitance Elements⟶ Electric, Thermal, Fluid

- It also has an influence on the speed of the response, which is defined as, a delay between the applied input and the corresponding output is irrelevant from the measurement point of view.
- The dynamic error is the difference between the indicated value and the true value for the measured quantity, when the static error is zero. It describes the difference between a sensor's response to the same input magnitude, depending on whether the input is constant or variable with time.

Steps to derive Transfer function:

- To determine the dynamic characteristics of a sensor, we must apply a variable quantity to its input. This input can take many different forms, but it is usual to study the response to transient inputs (impulse, step, ramp).

  Other types of inputs →Periodic (Sinusoidal) & Random (White noise)

- Selection of above inputs depends on type of sensor used

  E.g. Temperature (Step, ~~Sinusoidal~~)

  Acceleration (Impulse, ~~Step~~)

- To mathematically describe the behavior of a sensor, we assume that its input and output are related through a constant-coefficient linear differential equation.
- Then the relation between sensor output and input can be expressed in a simple form, as a quotient, by taking the Laplace transform of each signal and the transfer function of the sensor

The Laplace Transform (review)

- **The Laplace transform of a time signal y(t) is denoted by**
  - $L[y(t)] = Y(s)$
    - The s variable is a complex number $s=\sigma+j\omega$
      - The real component σ defines the real exponential behavior
      - The imaginary component defines the frequency of oscillatory behavior
- **The fundamental relationship is the one that concerns the transformation of differentiation**

$$L\left[\frac{d}{dt}y(t)\right] = sY(s) - f(0)$$

- **Other useful relationships are**

$$\text{Impulse}: \quad L[\delta(t)] = 1$$

$$\text{Step}: \quad L[u(t)] = \frac{1}{s}$$

$$\text{Ramp}: \quad L[r(t)] = \frac{1}{s^2}$$

$$\text{Decay}: \quad L[\exp(at)] = (s-a)^{-1}$$

$$\text{Sine}: \quad L[\sin(\omega t)] = \frac{\omega}{s^2+\omega^2}$$

$$\text{Cosine}: \quad L[\cos(\omega t)] = \frac{s}{s^2+\omega^2}$$

Zero Order Systems

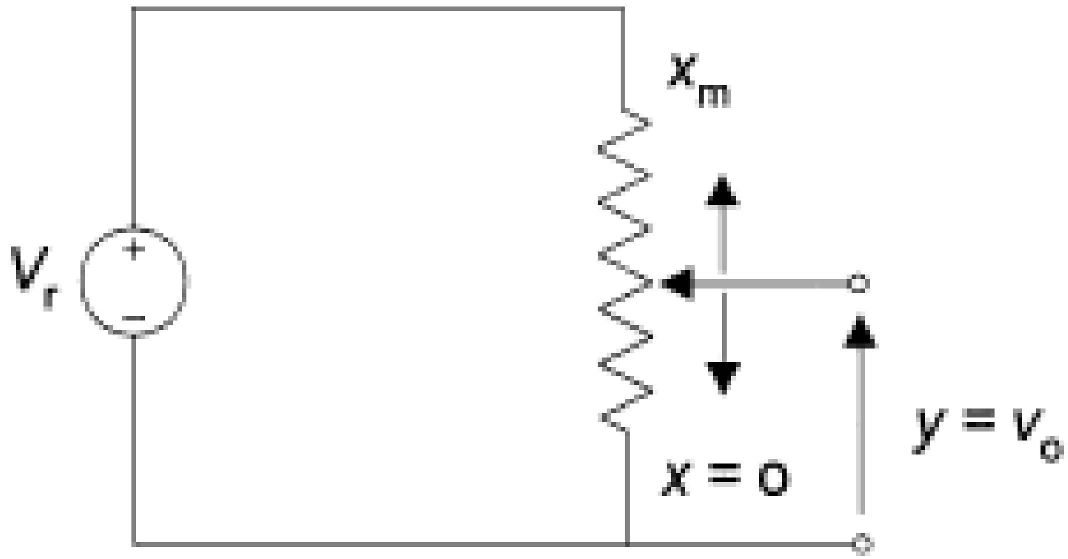
- The output of a zero-order sensor is related to its input through an equation of the type

$$y(t) = k \cdot x(t)$$

- Its behavior is characterized by its static sensitivity *k* and remains constant regardless of input frequency. Hence, its dynamic error and its delay are both zero.
- Infinite bandwidth
- The sensor only changes the amplitude of the input signal
- An input-output relationship such as that in above equation requires that the sensor does not include any energy-storing element.

- For E.g. Linear Potentiometer as a Position sensor

$$y = V_r \frac{x}{x_m}$$



$$k = V_r / x_m$$

First Order Systems

- In a first-order sensor there is an element that stores energy and another one that dissipates it. The relationship between the input $x(t)$ and the output $y(t)$ is described by a differential equation with the form

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 \; x(t)$$

- After applying a Laplace transformation to the above equation, we can transform it as:

$$\frac{y(s)}{x(s)} = \frac{b_0/a_0}{\left(\frac{a_1}{a_0} s + 1\right)} = \frac{K}{(\tau s + 1)}$$

$K = b_0 / a_0 =$ static sensitivity

$\tau = a_1 / a_0 =$ time constant

Impulse response of Transducer

Let y(s) be the Laplace transform of the response. Then y(s), in general, is given as

$$y(s) = \frac{K}{1 + s\tau} x(s)$$

where u(s) is the Laplace transform of the input.

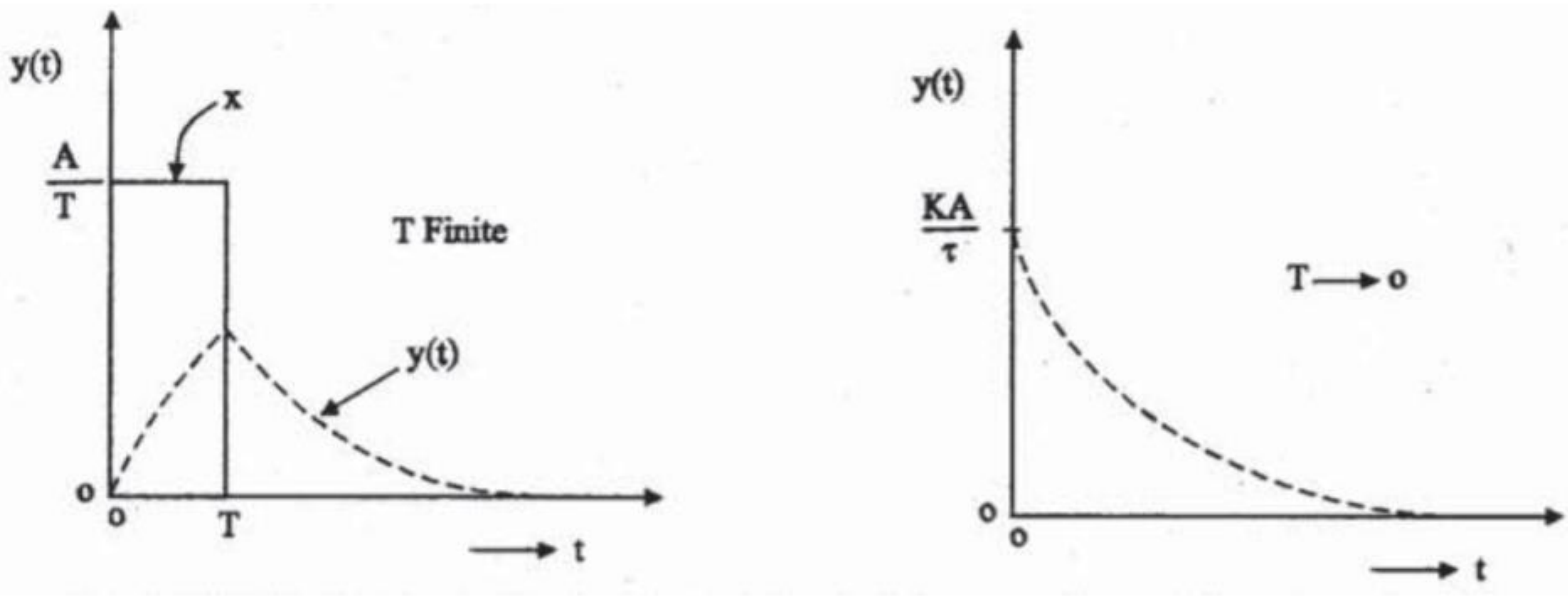
If the input is assumed to be an unit impulse, then $x(s) = 1$. Therefore

$$y(s) = \frac{K}{1 + s\tau} 1$$

The time response can be determined by taking the Laplace inverse of y(s)

$$L^{-1} y(s) = y(t) = L^{-1} \frac{K}{1 + s\tau} = \frac{K}{\tau} e^{-t/\tau}$$

Impulse response



Response of first-order system (a) for a prolonged impulse - input; (b) for an ideal impulse input.

Source: https://yidnekachew.files.wordpress.com/2013/04/transducer-engineering-by-nagaraj-1.pdf as on 19-02-2021

Step response of Transducer

When a first-order transducer is excited by a unit-step input,

$$x(s) = 1/s$$

Now

$$y(s) = \frac{K}{s(1 + s\tau)}$$

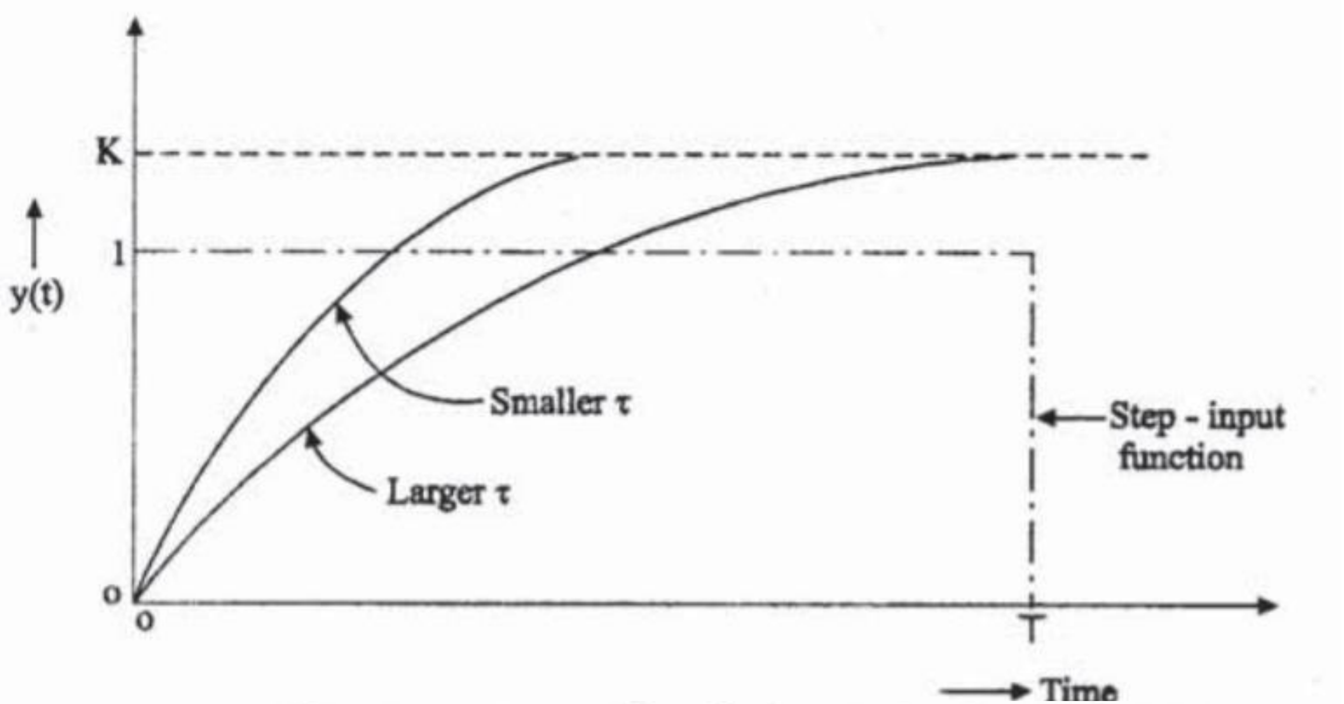
$$y(t) = L^{-1} y(t)$$

$$= L^{-1} \frac{K}{s(1 + s\tau)}$$

From the Laplace transform table,

$$y(t) = K(1 - e^{-t/\tau})$$

Unit Step Response



step response of a first - order transducer

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Ramp response of Transducer

Consider a first-order transducer subjected to a ramp-input given by

$$x(t) = R\,t \quad \text{for} \quad t \geq 0$$
$$= 0 \quad \text{for} \quad t < 0$$

where R is the slope of the ramp.

$$x(s) = \frac{R}{s^2}$$

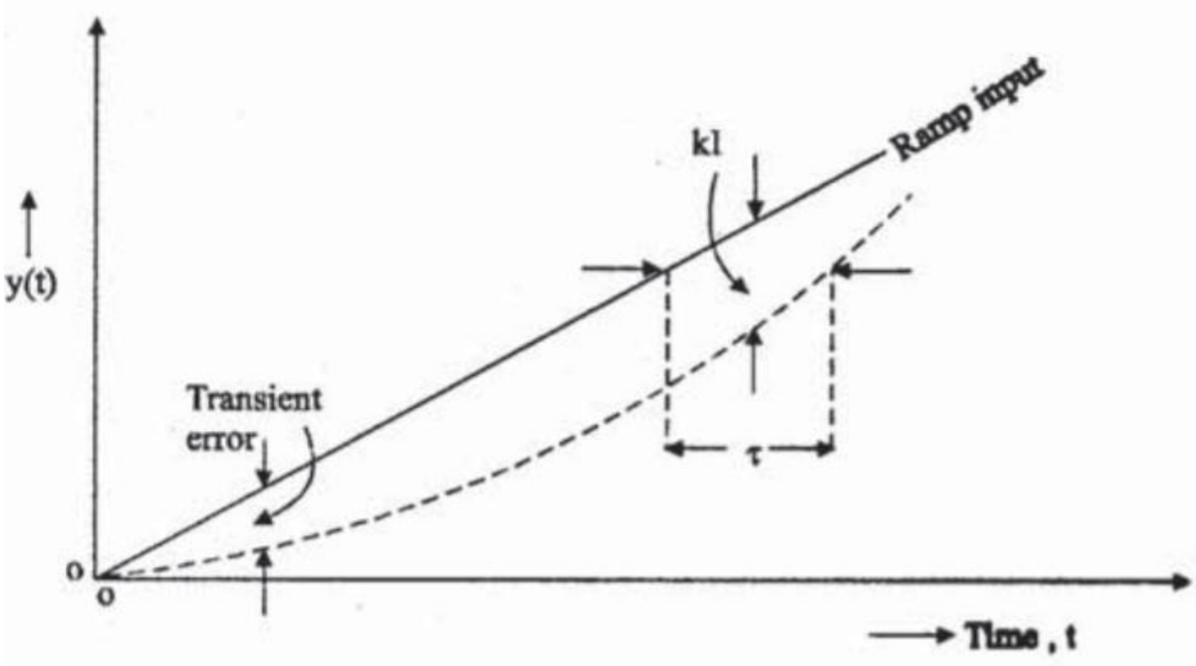
$$y(s) = \frac{K}{1 + s\tau} \cdot \frac{R}{s^2}$$

$$y(t) = L^{-1} \frac{KR}{s^2(1 + s\tau)}$$

$$= KR\,(\tau\, e^{-t/\tau} + t - \tau)$$

for an unit ramp, R=1 and $y(t) = (\tau\, e^{-t/\tau} + t - \tau)$

Ramp response



Ramp of first-order transducer

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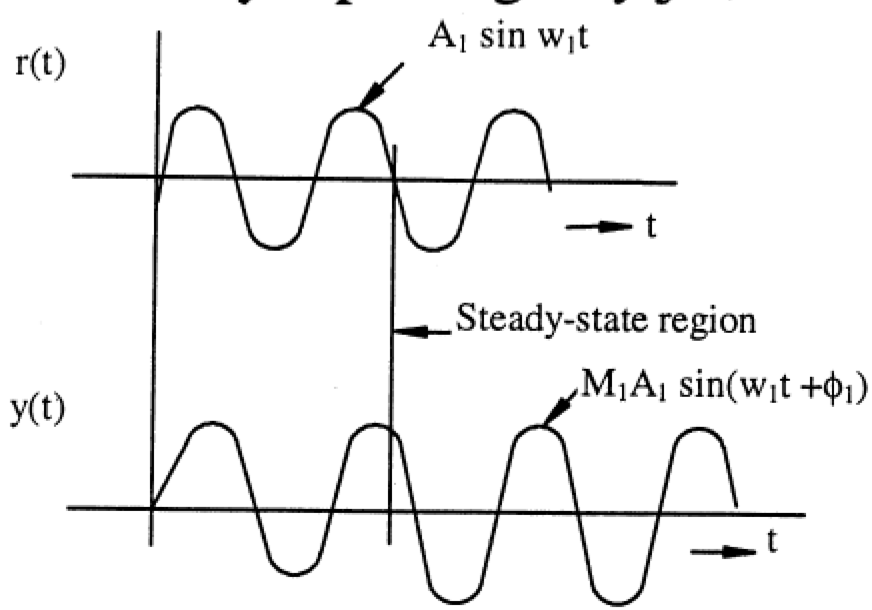
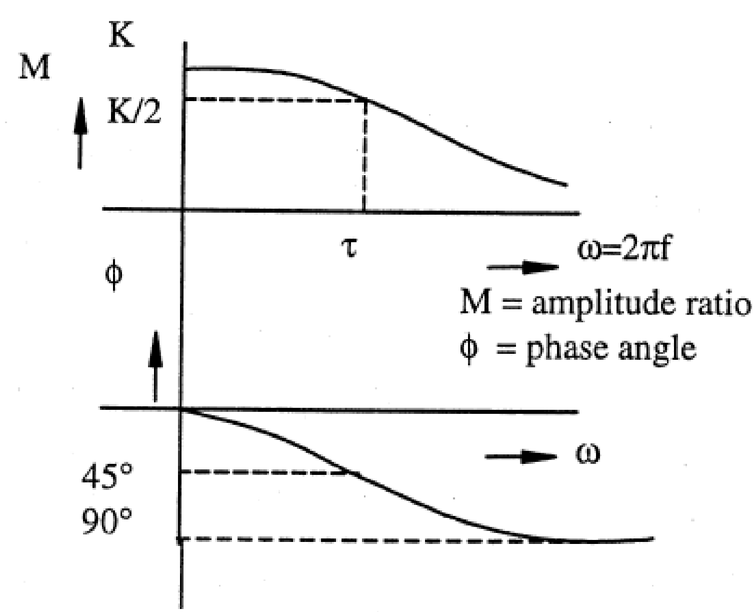
Frequency response of Transducer

Frequency response is thus defined as the steady-state output of a transducer when it is excited with sinusoidal input.

The frequency response of any system can be obtained from the frequency transfer function. Frequency transfer function is obtained by replacing s by jw, in the laplace transfer function.



Sinusoidal input and output of linear system



Frequency response of first-order transducer

The frequency transfer function of a first-order transducer is given by

$$\frac{Y(j\omega)}{R(j\omega)} = \frac{K}{1 + j\omega\tau}$$

The amplitude ratio is given by

$$\left|\frac{Y(j\omega)}{R(j\omega)}\right| = \left|\frac{K}{1 + j\omega\tau}\right|$$

$$= \frac{K}{\sqrt{1 + \omega^2 \tau^2}}$$

The phase shift between output and input is given by

$$\phi = \tan^{-1}\frac{0}{K} - \tan^{-1}\frac{\omega\tau}{1}$$

$$= 0 - \tan^{-1}\omega\tau$$

$$\phi = -\tan^{-1}\omega\tau$$

TABLE Dynamic Error and Delay for a First-Order Measurement System for Different Common Test Inputs

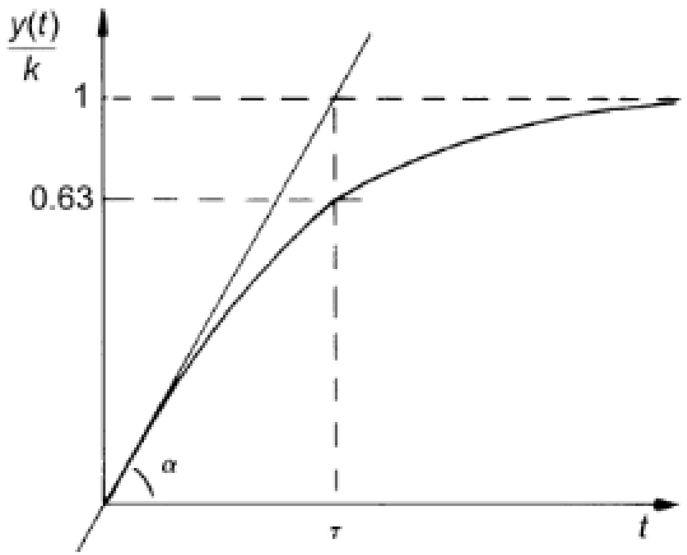
Input	Dynamic Error	Delay
Step $u(t)$	0	τ
Ramp Rt	$R[t + k(\tau - t)]$ or $R\tau$	τ
Sinusoid A, ω	$1 - \dfrac{1}{\sqrt{1 + \omega^2\tau^2}}$	$\dfrac{\arctan \omega\tau}{\omega}$

- The dynamic error for this kind of systems is defined as:

$$e_{\mathrm{d}} = y(t) - kx(t)$$

- The speed of response or transient characteristics of the system are as below:

Note: These characteristics are derived , when a step function is given as an input to the system.



Example of first-order Sensor → <u>THERMOMETER</u>

In Steady State

$$\text{(Heat in)} - \text{(Heat out)} = \text{Energy stored}$$

Assume sensor does not loose heat through its leads and there is negligible mass expansion, then

where 'Ti' ---Internal temperature
'Te' ---External temperature
'h' ---Heat transfer coeff.
'A' ---Heat transmission area
'M'--- Mass of object
'c'--- Specific heat

$$hA(T_e - T_i)\,dt - 0 = Mc\,dT_i$$

$$\frac{dT_i}{dt} = \frac{hA}{Mc}(T_e - T_i)$$

Taking Laplace transform and introducing $\tau = hA/Mc$,

$$\frac{T_i(s)}{T_e(s)} = \frac{1}{1 + \tau s}$$

Numerical 1

A temperature transducer with a time constant of 0.4 sec and a static sensitivity of 0.05 mv/°C is used to measure the temperature of a hot liquid medium which changes from 25°C to 65°C. The transducer is adjusted to read 0 at 25°C.

(a) Determine the time taken to read 80% of the final voltage value if the temperature changes as a step.

(b) Calculate the reading of the transducer at the end of 4 seconds if the temperature changes at a constant rate of 10° per sec from 25° to 65°C.

The given transducer is a I order instrument with transfer function

$$\frac{V(s)}{T(s)} = \frac{0.05 \times 10^{-3}}{1 + 0.4s}$$

where V(s) is the voltage output and T(s) is the temperature of the medium.

(a) then for a step change of 40°C

$r(t) = 40 \times u(t)$.

$v(t) = 40 \times 0.05 \times 10^{-3}\,(1 - e^{-t/0.4})$ volts

The steady state voltage value of the transducer $V_{ss} = 2$ mV

$\left(40 \times .05 \times 10^{-3}\right)$

80% of this final value is 1.6 mV

Time taken to read 1.6 mV is given by the equation

$$1.6 \times 10^{-3} = 2 \times 10^{-3}\left(1 - e^{-t/0.4}\right)$$

$$0.8 = 1 - e -^{t/0.4}$$

$$t = 0.64 \text{ sec}$$

(b) The input is ramp of slope 10°C/sec. r(t) = 10 t u(t) up to 6 sec and then it is terminated. For t < 6

$$V(t) = k \times 10\left(t - \tau + \tau e^{-t/\tau}\right)$$

$$= 0.05 \times 10\left(t - 0.4 + 0.4e^{-t/0.4}\right)$$

$$= 0.5\left(t - 0.4 + 0.4e^{-t/0.4}\right)$$

for t = 4 sec,

$$V(t) = 0.5\left(4 - 0.4 + 0.4e^{-4/0.4}\right)$$

$$= 0.5\left(3.6 + .4e^{-10}\right) \text{ mV} = 1.8 \text{ mV}$$

Numerical 2

The transfer function of a I order instrument with dead time is given by $e^{-1.5s}/(1+0.5s)$. Calculate the output of this system after 2 seconds for a unit step input.

The output can be calculated from the equation

$$\frac{y(s)}{x(s)} = \frac{e^{-1.5s}}{1+0.5s}$$

u(t) = unit step, . $x(s) = 1/s$

$$= \frac{e^{-1.5s}}{1+0.5\,s} \times \frac{1}{s}$$

The solution for this equation is

$$y(t) = 1 - e^{-\frac{t-1.5}{0.5}}$$

∴ output after 2 seconds will be

$$y(t)_{t=2} = \left(1 - e^{-1}\right) = 0.632$$

Numerical 3

A first order instrument must measure signals with frequency content upto 200Hz with an amplitude inaccuracy of 2 percent. What is the maximum allowable time constant ?. What will be the phase shift at 100 and 200 Hz for the cohsen time constant?

T.F of a I order instrument is

$$G(s) = \frac{K}{1 + s\tau}$$

The frequency T.F is

$$G(j\omega) = \frac{K}{1 + j\,\omega\,\tau}$$

$$M = |G(j\omega)| = \left|\frac{K}{1 + j\omega\tau}\right| = \frac{K}{\sqrt{1 + \omega^2\tau^2}}$$

Allowable accuracy at 200 Hz is 2% that means

$$.98K = \frac{K}{\sqrt{1 + (2\pi \times 200\,\tau)^2}}$$

$$\therefore\ (.98)^2\left[1 + (400\pi)^2\tau^2\right] = 1$$

$$(400\pi)^2\tau^2 = \frac{1 - .9604}{.9604}$$

$$15.775 \times 10^5\,\tau^2 = 0.412$$

$$\tau^2 = 0.26 \times 10^{-6}$$

$$\tau = 0.509 \times 10^{-3}\ \text{sec.}$$

Phase shift $= -\tan^{-1}(\omega T)$ radians

At 100 Hz / $\phi_{100} = -\tan^{-1} 2\pi \times 100 \times 0.509 \times 10^{-3}$

At 200 Hz / $\phi_{200} = -\tan^{-1} 2\pi \times 200 \times 0.509 \times 10^{-3}$

Second Order Systems

- A second-order sensor contains two energy-storing elements and one energy-dissipating element. Its input x(t) and output y(t) are related by a second-order linear differential equation of the form

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 u(t)$$

- The above equation can be converted in a transfer function after applic

$$\frac{Y(s)}{R(s)} = \frac{b_0/a_0}{\frac{a_2}{a_0} s^2 + \frac{a_1}{a_0} s + 1}$$

when all the initial conditions are zero.

$$\frac{Y(s)}{R(s)} = \frac{K}{\dfrac{s^2}{\omega_n^2} + \dfrac{2\zeta}{\omega_n} s + 1}$$

where $K = b_0/a_0$ = static sensitivity

$\omega_n = a_0/a_2$ = undamped natural frequency

$\zeta = \dfrac{a_1}{\sqrt{a_0 a_2}}$ = damping ratio

Hence, three parameters namely, K, ω_n and ζ characterise the second-order transducer.

- It can be noticed that all the parameters are related and if one is changed others are also varied.
- System behavior differs for

(i) $\zeta = 0 \longrightarrow$ Undamped case and system behaves like an oscillator

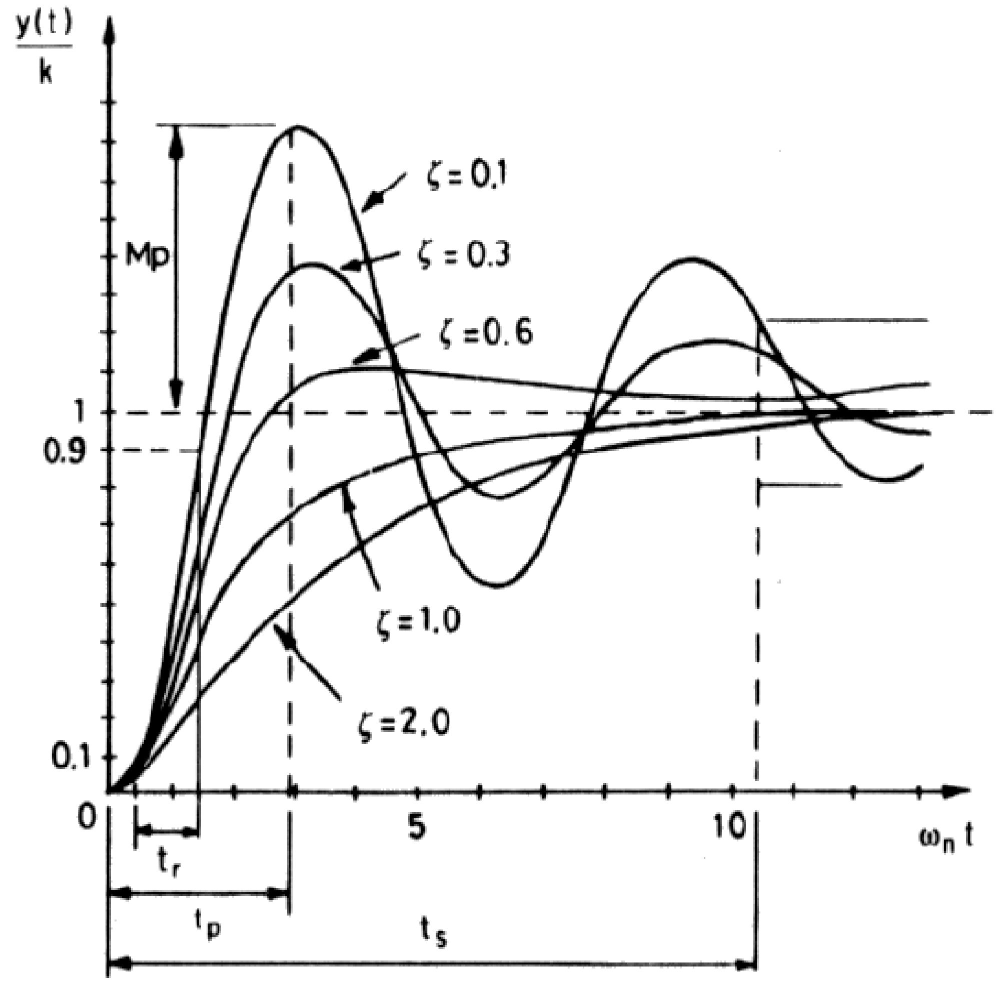
(ii) $0 < \zeta < 1 \longrightarrow$ Underdamped case-Oscillations will die down as time elapses-Damped oscillation

(iii) $\zeta = 1 \longrightarrow$ Critically damped case-Response just becomes non-oscillatory

(iv) $\zeta > 1 \longrightarrow$ Overdamped case-Response similar to first order system without any oscillation

Second-order step response

- **Response types**
  - Underdamped (ζ<1)
  - Critically damped (ζ=1)
  - Overdamped (ζ>1)
- **Response parameters**
  - Rise time (t_r)
  - Peak overshoot (M_p)
  - Time to peak (t_p)
  - Settling time (t_s)



From [PAW91]

Response Parameters

Peak Overshoot

- Peak overshoot is defined as the deviation of the response at peak time from the final value of response. It is also called the **maximum overshoot**.

Peak Time

- It is the time required for the response to reach the **peak value** for the first time. It is demoted as $\mathbf{t}_p$

Rise Time

- It is the time required for the response to rise from **0% to 100% of its final value**. This is applicable for the **under-damped systems**. For the over-damped systems, consider the duration from 10% to 90% of the final value. Rise time is denoted by $\mathbf{t}_r$.

Settling time

- It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%.It is denoted by $\mathbf{t}_s$

Input--- Unit Step

For $\zeta > 1$ and $\zeta = 1$

$\longrightarrow$ No Overshoot or steady-state dynamic error

For $\zeta < 1$

$\longrightarrow$ Speed and Overshoot related

As Speed $\uparrow$, Overshoot $\uparrow$

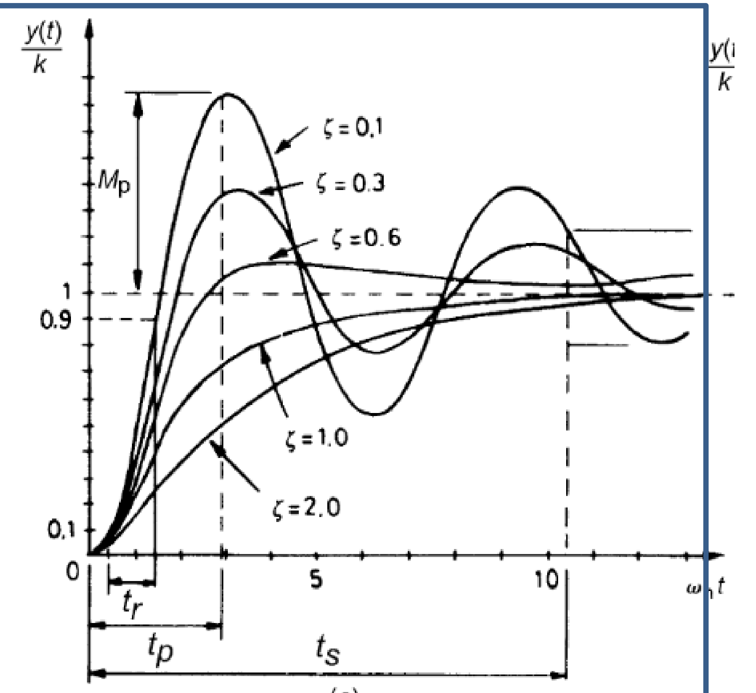
t_r – Rise time spent to increase from 10 % to 90% of its output value

$$t_r = \frac{\arctan(-\omega_d/\delta)}{\omega_d}$$

Where δ--- Attenuation

ω_d ---Natural damped frequency

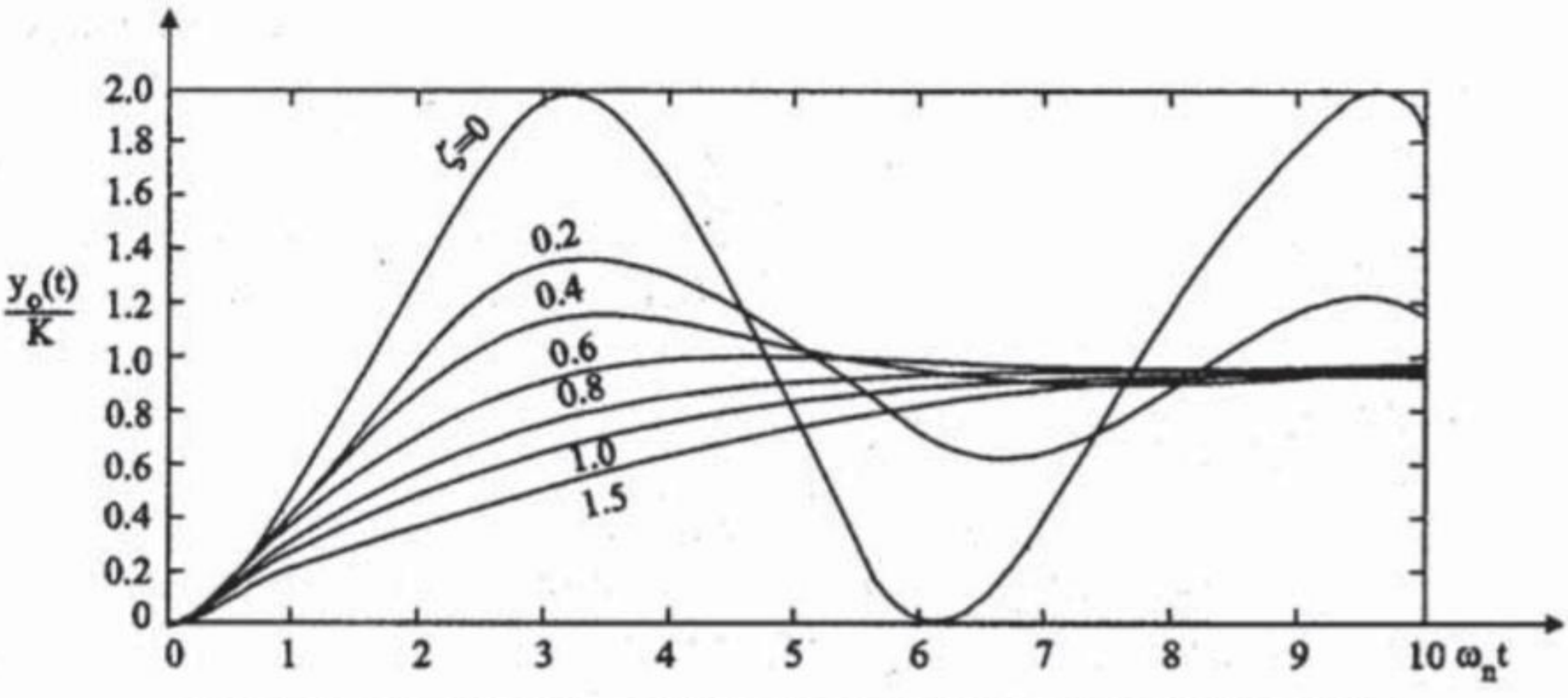
$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$



* The time transient response of the second order system, with a step input, is greatly influenced by the value of damping coefficient. It introduces HARMONICS in the system which are undesirable. So choosing proper a_0, a_1 and a_2 values for a system can reduce harmonics.

Response of Second Order Transducer

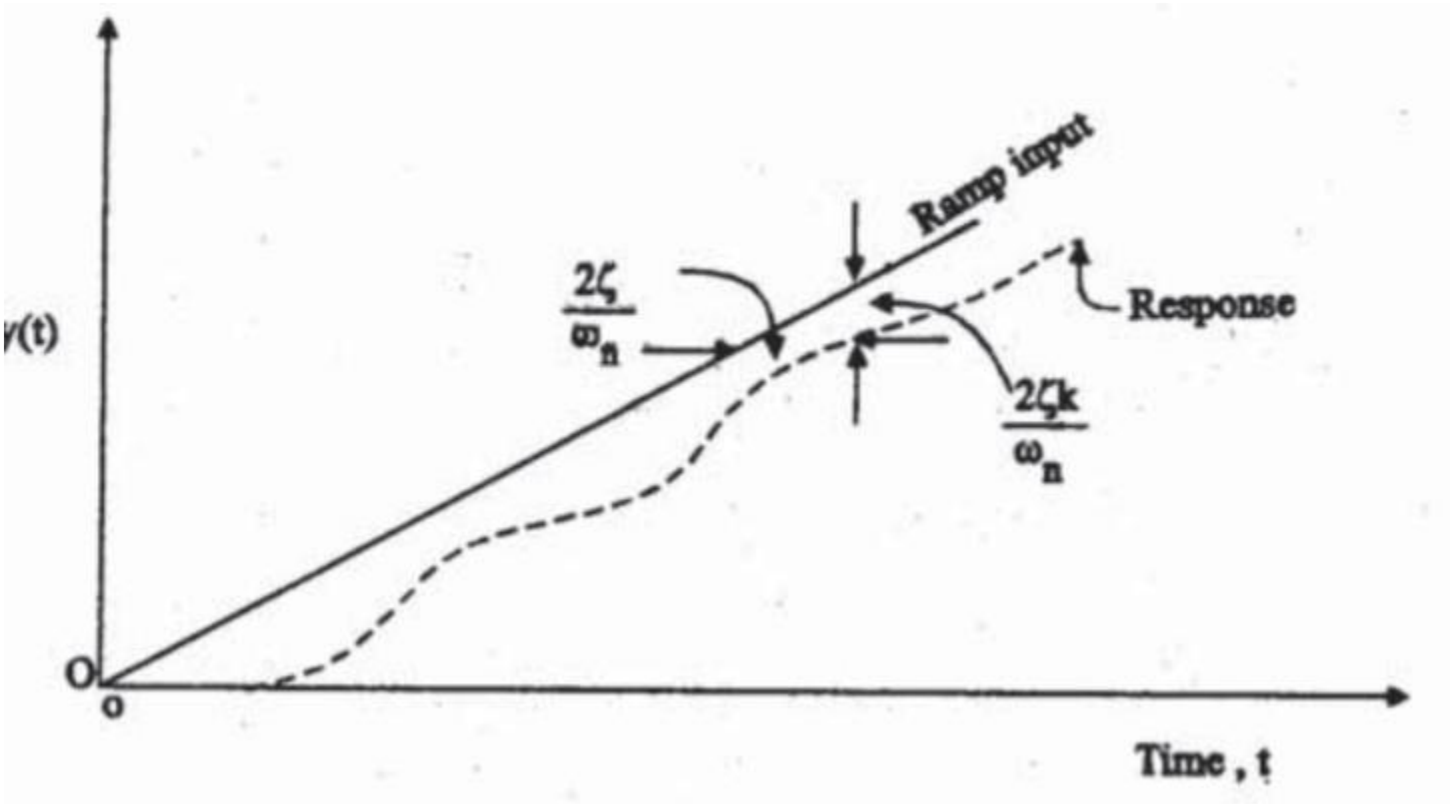
- Step Response



step response of a II - order transducer for various value of ξ.

Source: https://yidnekachew.files.wordpress.com/2013/04/transducer-engineering-by-nagaraj-1.pdf as on 19-02-2021

Ramp Response



Ramp response of a II order system

Source: https://yidnekachew.files.wordpress.com/2013/04/transducer-engineering-by-nagaraj-1.pdf as on 19-02-2021

Frequency Response

The frequency transfer function of a second-order transducer is given by replacing s by jw

$$\frac{Y(j\omega)}{R(j\omega)} = \frac{K}{\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1}$$

This can be put in the form

$$\frac{Y(j\omega)}{R(j\omega)} = \frac{K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \frac{4\zeta^2\omega^2}{\omega_n^2}}} \qquad \phi$$

where $\phi = -\tan^{-1}\left(\frac{2\zeta}{\frac{\omega_n}{\omega} - \frac{\omega}{\omega_n}}\right)$

TABLE Outputs of a Second-Order Measuring System for Different Common Test Inputs

Input	Output	
Unit step $u(t)$		
$0<\zeta<1$	$1-\dfrac{e^{-\delta t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\phi)$	$\delta=\zeta\omega_n$ $\omega_d=\omega_n\sqrt{1-\zeta^2}$ $\phi=\arcsin\dfrac{\omega_d}{\omega_n}$
$\zeta=1$	$1-e^{-\delta t}(1+\omega_n t)$	
$\zeta>1$	$1+\dfrac{\omega_n}{2\sqrt{\zeta^2-1}}\left(\dfrac{e^{-at}}{a}-\dfrac{e^{-bt}}{b}\right)$	$a=\omega_n(\zeta+\sqrt{\zeta^2-1})$ $b=\omega_n(\zeta-\sqrt{\zeta^2-1})$
Ramp Rt		
$0<\zeta<1$	$R\left\{t-\dfrac{2\zeta}{\omega_n}\left[1-\dfrac{e^{-\zeta\omega_n t}}{2\zeta\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\omega_n t+\phi)\right]\right\}$	$\phi=\arctan\left(\dfrac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}\right)$
$\zeta=1$	$R\left\{t-\dfrac{2\zeta}{\omega_n}\left[1-\left(1+\dfrac{\omega_n \tau}{2}\right)e^{-\omega_n t}\right]\right\}$	
$\zeta>1$	$R\left\{t-\dfrac{2\zeta}{\omega_n}\left[1+\dfrac{2\zeta(-\zeta-\sqrt{\zeta^2-1}+1)}{4\zeta\sqrt{\zeta^2-1}}e^{-at}+\dfrac{2\zeta(-\zeta-\sqrt{\zeta^2-1}-1)}{4\zeta\sqrt{\zeta^2-1}}e^{-bt}\right]\right\}$	
Sinusoid A,ω	$\dfrac{kA}{\sqrt{\left(1-\dfrac{\omega^2}{\omega_n^2}\right)^2+\left(\dfrac{2\zeta\omega}{\omega_n}\right)^2}}\sin(\omega t-\phi)$	$\phi=\arctan\dfrac{2\zeta\omega/\omega_n}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}$

Numerical 1

Determine the damping coefficient, natural frequency of oscillation for a second order transducer described by

$$\frac{d^2y(t)}{dt^2} + 2.8\frac{dy(t)}{dt} + 4y(t) = 11.7u(t)$$

where y(t) is the output and u(t) is the input.

SOLUTION

The generalised equation of a second order transducer is

$$\frac{d^2y}{dt^2} + 2\xi\omega_n\frac{dy}{dt} + \omega_n^2 y = K\omega_n^2 u(t)$$

where ω_n = natural frequency of oscillation

ξ = damping coefficient or ratio

K = static gain

Comparing the given equation with the standard generalised equation

$$\omega_n^2 = 4, \text{ so } \omega_n = 2, 2\xi\omega_n = 2.4, \text{ so } \xi = 0.7$$

damping coefficient is 0.7

natural frequency of oscillation is 2.

- *For reference (std Equation)*

$$\frac{Y(s)}{R(s)} = \frac{K}{\dfrac{s^2}{\omega_n^2} + \dfrac{2\zeta}{\omega_n} s + 1}$$

where $K = b_0/a_0$ = static sensitivity

$\omega_n = a_0/a_2$ = undamped natural frequency

$$\zeta = \frac{a_1}{\sqrt{a_0 a_2}}$$ = damping ratio

Hence, three parameters namely, K, ω_n and ζ characterise the second-order transducer.

Numerical 2

A transducer is described by Laplace transfer function

$$\frac{y}{x}(s) = \frac{K}{(1+0.1s)(1+0.5\,s)}$$

Find the amplitude inaccuracy and phase shift when x is a signal with frequency 100/2π Hz.

The frequency response of this transducer can be obtained by replacing s by jω.

$$\frac{y}{x}(j\omega) = \frac{K}{(1+0.1\,j\,\omega)(1+0.5\,j\,\omega)}$$

Amplitude ratio

$$M = \left|\frac{y}{x}(j\omega)\right| = \frac{K}{\sqrt{(1+(0.1\,\omega)^2}\quad\sqrt{1+(0.5\,\omega)^2}}$$

Phase angle between the output and the input signal is

$$\frac{y}{x}(j\,\omega) = \phi = -\tan^{-1}0.1\,\omega - \tan^{-1}0.5\,\omega$$

For a frequency of 100/2π, $\omega = 100$ $(\omega = 2\pi f)$

For $\omega = 100$,

Amplitude ratio

$$M = \frac{K}{\sqrt{1+10^2}\,\sqrt{1+50^2}} = \frac{K}{\sqrt{101}\,\sqrt{2501}} = \frac{K}{500}$$

If input magnitude is K output magnitude is K/500.

$$\therefore \ \% \text{ error} = \frac{K - \frac{K}{500}}{K} \times 100 = \frac{499}{500} \times 100 = 99.8\%$$

The output is just 0.2% of the input. So much attenuated

$$\phi = -\tan^{-1}(10) - \tan^{-1}(50) = -173.14^\circ$$

Other Sensor Characteristics

TABLE [illegible] Characteristics to Consider in Sensor Selection

Quantity to Measure[a]	Output Characteristics	Supply Characteristics	Environmental Characteristics	Other Characteristics
Span	Sensitivity	Voltage	Ambient temperature	Reliability
Target accuracy	Noise floor	Current	Thermal shock	Operating life
Resolution	Signal: voltage, current, frequency	Available power	Temperature cycling	Overload protection
Stability	Signal type: single ended, differential, floating	Frequency (ac supply)	Humidity	Acquisition cost
Bandwidth	Impedance	Stability	Vibration	Weight, size
Response time	Code, if digital		Shock	Availability
Output impedance			Chemical agents	Cabling requirements
Extreme values			Explosion risks	Connector type
Interfering quantities			Dirt, dust	Mounting requirements
Modifying quantities			Immersion	Installation time
			Electromagnetic environment	State when failing
			Electrostatic discharges	Calibration and testing cost
			Ionizing radiation	Maintenance cost
				Replacement cost

[a] Sensor static and dynamic characteristics must be compatible with the requirements of the quantity to measure.