NUMERICALS
Module 4

It can be demonstrated through the use of solid-state physics that the general characteristics of a semiconductor diode can be defined by the following equation, referred to as Shockley's equation, for the forward- and reverse-bias regions:

$$\boxed{I_D = I_s(e^{V_D/nV_T} - 1)} \quad \text{(A)} \qquad (1.2)$$

where I_s is the reverse saturation current
V_D is the applied forward-bias voltage across the diode
n is an ideality factor, which is a function of the operating conditions and physical construction; it has a range between 1 and 2 depending on a wide variety of factors ($n = 1$ will be assumed throughout this text unless otherwise noted).

The voltage V_T in Eq. (1.1) is called the *thermal voltage* and is determined by

$$\boxed{V_T = \frac{kT_K}{q}} \quad \text{(V)} \qquad (1.3)$$

where k is Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K
T_K is the absolute temperature in kelvins $= 273 +$ the temperature in °C
q is the magnitude of electronic charge $= 1.6 \times 10^{-19}$ C

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Given $I_o = 0.3 \times 10^{-6}$ A and $V_F = 0.15$ V.

The current flowing through the *PN* diode under forward bias is

$$I = I_o\,(e^{40\,V_F} - 1)$$

$$= 0.3 \times 10^{-6}\,(e^{40 \times 0.15} - 1)$$

$$= 120.73\ \mu\text{A}$$

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Solution Given

$$V_F = 0.6\text{ V},\ T = 273 + 25 = 298\text{ K}$$

$$I_o = 10\ \mu\text{A} = 1 \times 10^{-5}\text{ A and } \eta = 2 \text{ for silicon.}$$

The volt-equivalent of the temperature (T) is

$$V_T = \frac{T}{11{,}600} = \frac{298}{11{,}600} = 25.7 \times 10^{-3}\text{ V}$$

Therefore, the diode current, $I = I_o\left(e^{\frac{V_F}{\eta V_T}} - 1\right)$

$$I = 10^{-5}\left(e^{\frac{0.6}{2 \times 25.7 \times 10^{-3}}} - 1\right) = 1.174\text{ A}$$

The diode current is 0.6 mA when the applied voltage is 400 mV, and 20 mA when the applied voltage is 500 mV. Determine η. Assume $\frac{kT}{q} = 25\,\text{mV}$.

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Solution

The diode current,
$$I = I_o\left(e^{\frac{qV}{\eta kT}} - 1\right)$$

Therefore,
$$0.6 \times 10^{-3} = I_o\left(e^{\frac{qV}{\eta kT}} - 1\right) = I_o\, e^{\frac{qV}{\eta kT}}$$
$$= I_o \cdot e^{\frac{400}{25\eta}} = I_o \cdot e^{\frac{16}{\eta}}$$

Also,
$$20 \times 10^{-3} = I_o \cdot e^{\frac{500}{25\eta}} = I_o \cdot e^{\frac{20}{\eta}}$$

Dividing Eq. (2) by Eq. (1), we get

$$\frac{20 \times 10^{-3}}{0.6 \times 10^{-3}} = \frac{I_o \cdot e^{\frac{20}{\eta}}}{I_o \cdot e^{\frac{16}{\eta}}}$$

Therefore, $$\frac{100}{3} = e^{\frac{4}{\eta}}$$

Taking natural logarithms on both sides, we get

$$\log_e \frac{100}{3} = \frac{4}{\eta}$$

$$3.507 = \frac{4}{\eta}$$

$$\eta = \frac{4}{3.507} = 1.14$$

Determine the ideal reverse saturation current density in a silicon *PN* junction at $T = 300$ K. Consider the following parameters in the silicon *PN* junction:
$N_A = N_D = 10^{16}\ \text{cm}^{-3}$, $n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$, $D_n = 25\ \text{cm}^2/\text{s}$, $T_{po} = T_{no} = 5 \times 10^{-7}$ s, $D_p = 10\ \text{cm}^2/\text{s}$, $\varepsilon_r = 11.7$.
Comment on the result.

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Solution Given, $T = 300$ K, $N_A = N_D = 10^{16}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $D_n = 25$ cm^2/s, $T_{po} = T_{no} = 5 \times 10^{-7}$ s, $D_p = 10$ cm^2/s

The reverse saturation current is given by

$$I_o = Aq\left[\frac{D_p}{L_p \cdot N_D} + \frac{D_n}{L_n \cdot N_D}\right]n_i^2$$

We know that, $J_o = \dfrac{I_o}{A}$

Therefore, reverse saturation current density is,

$$J_o = q\left[\frac{D_p}{L_p \cdot N_D} + \frac{D_n}{L_n \cdot N_D}\right]n_i^2$$

$N_D = N_A = 10^{16}/cm^3$

$= 10^{22}/m^3$

$$L_p = \sqrt{D_p \cdot T_{po}} = \sqrt{10 \times 10^{-4} \times 5 \times 10^{-7}} = 2.236 \times 10^{-5}$$

$$L_n = \sqrt{D_n \cdot T_{no}} = \sqrt{25 \times 10^{-4} \times 5 \times 10^{-7}} = 3.535 \times 10^{-5}$$

$$J_o = 1.602 \times 10^{-19} \left[\frac{10 \times 10^{-4}}{2.236 \times 10^{-5} \times 10^{22}} + \frac{25 \times 10^{-4}}{3.535 \times 10^{-5} \times 10^{22}} \right] (1.5 \times 10^{-16})^2$$

$= 0.416\ \mu A/\cancel{?}$

$$\boxed{J_o = 0.416\ \mu A/\underline{m^2}}$$

Determine the forward resistance of a PN junction diode, when the forward current is 5 mA at $T = 300$ K. Assume silicon diode.

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Solution Given, for a silicon diode, the forward current, $I = 5$ mA. $T = 300$ K.

Forward resistance of a PN junction diode, $r_f = \frac{\eta V_T}{I}$ where $V_T = \frac{T}{11{,}600}$ and $\eta = 2$ for silicon

Therefore,

$$r_f = \frac{2 \times \frac{T}{11{,}600}}{5 \times 10^{-3}} = \frac{2 \times 300}{11{,}600 \times 5 \times 10^{-3}} = 10.34\ \Omega$$

Calculate the dynamic forward and reverse resistance of a *PN* junction diode when the applied voltage is 0.25 V at $T = 300$ K given $I_o = 2\ \mu A$.

b)

Calculate the dynamic forward and reverse resistance of a PN junction diode when the applied voltage is 0.25 V at $T = 300$ K given $I_o = 2\ \mu A$.

$$I = I_0\left(e^{\left(\frac{0.25}{2\times 26\times 10^{-3}}\right)} - 1\right)$$

Solution Given $V = 0.25$ V, $T = 300$ K, $I_o = 2\ \mu A$.

At $T = 300$ K, $V_T = 26$ mV.

Assuming it to be silicon diode, $\eta = 2$

Therefore, $$I = I_o\left(e^{\frac{V}{\eta V_T}} - 1\right) = 2\times 10^{-6}\left(e^{\frac{0.25}{2\times 26\times 10^{-3}}} - 1\right) = 0.24 \text{ mA}$$

$I = 242.8\ \mu A$

$I = 0.24$ mA

$$r_f = \frac{\eta V_T}{I} = \frac{2\times 26\times \cancel{10^{-3}}}{0.24\times \cancel{10^{-3}}} = 216.67\ \Omega$$

HW

For germanium diode, $\boxed{\eta = 1.}$

$$I = I_o\left(e^{\frac{V}{\eta V_T}}-1\right) = 2 \times 10^{-6}\left(e^{\frac{0.25}{26\times 10^{-3}}}-1\right) = 0.03 \text{ A}$$

$$r_f = \frac{\eta V_T}{I} = \frac{26 \times 10^{-3}}{0.03} = 0.867\ \Omega$$

Reverse resistance

$$\boxed{\frac{V}{I_o} = \frac{0.25}{2 \times 10^{-6}} = 125\text{ k}\Omega}$$

Consider a silicon PN junction at $T = 300$ K so that $n_i = 1.5 \times 10^{10}$ cm^{-3}. The N type doping is 1×10^{10} cm^{-3} and a forward bias of 0.6 V is applied to the PN junction. Calculate the minority hole concentration at the edge of the space charge region.

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Solution Given $T = 300$K, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $n \times 1 \times 10^{10}$ cm^{-3} and $V_F = 0.6$ V.

Mass Action law given by Eq. (1.15) is

$$n \cdot p = n_i^2$$

Therefore, the concentration of holes, $p = \dfrac{n_i^2}{n}$

$$= \frac{(1.5 \times 10^{10})^2}{1 \times 10^{10}} = 2.25 \times 10^{10} \text{ cm}^{-3}$$

Find the conductivity of silicon (a) in intrinsic condition at a room temperature of 300 K, (b) with donor impurity of 1 in 10^8, (c) with acceptor impurity of 1 in 5×10^7 and (d) with both the above impurities present simultaneously. Given that n_i for silicon at 300 K is 1.5×10^{10} cm^{-3}, $\mu_n = 1300$ cm^2/V-s, $\mu_p = 500$ cm^2/V-s, number of Si atoms per cm$^3 = 5 \times 10^{22}$.

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Solution (a) In intrinsic condition, $n = p = n_i$

Hence,

$$\sigma_i = qn_i(\mu_n + \mu_p)$$

$$= (1.602 \times 10^{-19})(1.5 \times 10^{10})(1300 + 500)$$

$$= 4.32 \times 10^{-6} \text{ S/cm}$$

(b) Number of silicon atoms/cm^3 = 5×10^{22}

Hence, $N_D = \dfrac{5 \times 10^{22}}{10^8} = 5 \times 10^{14}\ \text{cm}^{-3}$

Further, $n \approx N_D$

Therefore,
$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D}$$
$$= \frac{(1.5 \times 10^{10})^2}{5 \times 10^{14}} = 0.46 \times 10^6\ \text{cm}^{-3}$$

Thus, $p << n$. Hence, p may be neglected while calculating the conductivity.

Hence,
$$\sigma = nq\mu_n = N_D q\mu_n$$
$$= (5 \times 10^{14})(1.602 \times 10^{-19})(1300)$$
$$= 0.104\ \text{S/cm}$$

$$N_A = \frac{5 \times 10^{22}}{5 \times 10^7} = 10^{15}\ \text{cm}^{-3}$$

(c)

Further, $p \approx N_A$

Hence, $n = \dfrac{n_i^2}{p} \approx \dfrac{n_i^2}{N_A}$

$$= \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3}$$

Thus, $p >> n$. Hence, n may be neglected while calculating the conductivity.

Hence, $\sigma = pq\mu_p = N_A q\mu_p$

$$= (10^{15} \times 1.602 \times 10^{-19} \times 500)$$

$$= 0.08 \text{ S/cm}$$

(d) With both types of impurities present simultaneously, the net acceptor impurity density is,

$$N'_A = N_A - N_D = 10^{15} - 5 \times 10^{14} = 5 \times 10^{14}\ \text{cm}^{-3}$$

Hence,

$$\sigma = N'_A\, q\mu_p$$

$$= (5 \times 10^{14})\,(1.602 \times 10^{-19})\,(500)$$

$$= 0.04\ \text{S/cm}$$