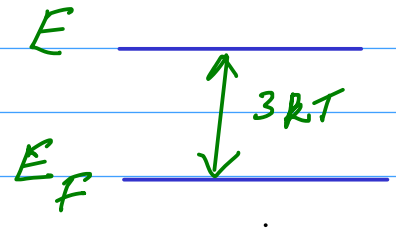
Let $T = 300$ K. Determine the probability that an energy level $3kT$ above the Fermi energy is occupied by an electron.

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{3kT}{kT}\right)}$$

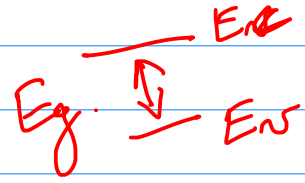


$$f_F(E) = \frac{1}{1 + 20.09} = 0.0474 = 4.74\%$$

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

$n_i = ?$ @ $T = 250$ K $n_i = ?$ @ $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.



$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$N_c = (N_c)_{300} \left(\frac{T}{300}\right)^{3/2} \qquad N_v = (N_v)_{300} \left(\frac{T}{300}\right)^{3/2}$$

$$n_i^2 = N_c N_v \left(\frac{T}{300}\right)^3 \exp\left[\frac{-E_g}{kT}\right]$$

$kT = 0.0259$ at 300 K. $kT = (kT)_{300} \left(\frac{T}{300}\right)$

$T = 250$ K

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right]$$

$$= 4.90 \times 10^{15}$$

$$n_i = 7.0 \times 10^7 \text{ cm}^{-3}$$

$T = 400$ K

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(400/300)}\right]$$

$$= 5.67 \times 10^{24}$$

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$N_v \propto T^{3/2}$

$N_c \propto T^{3/2}$

$$N_{v1} = c\,T_1^{3/2}$$

$$N_{v2} = c\,T_2^{3/2}$$

$$\frac{N_{v1}}{N_{v2}} = \frac{T_1^{3/2}}{T_2^{3/2}} = \left(\frac{T_1}{T_2}\right)^{3/2}$$

$$N_{v2} = N_{v1}\left(\frac{T_2}{T_1}\right)^{3/2}$$

illy $N_{c2} = N_{c1}\left(\frac{T_2}{T_1}\right)^{3/2}$

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Repeat for GaAs.

Repeat for Ge

TYU 4.3 Calculate the intrinsic concentration in silicon at (*a*) $T = 200$ K and (*b*) $T = 450$ K. (*c*) Determine the ratio of n_i at $T = 450$ K to that at $T = 200$ K.

EXAMPLE 4.2

Calculate the thermal-equilibrium hole concentration in silicon at $T = 400$ K.

Assume that the Fermi energy is 0.27 eV above the valence-band energy. The value of N_v for silicon at $T = 300$ K is $N_v = 1.04 \times 10^{19}$ cm^{-3}. (See Appendix B)

$N_v = (N_v)_{300} \left(\frac{T}{300}\right)^{3/2}$

$T = 400$ K

$$N_v = (1.04 \times 10^{19}) \left(\frac{400}{300}\right)^{3/2} = 1.60 \times 10^{19} \text{ cm}^{-3}$$

$kT = (kT)_{300} \left(\frac{T}{300}\right)$

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453 \text{ eV}$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (1.60 \times 10^{19}) \exp\left(\frac{-0.27}{0.03453}\right)$$

$$p_0 = 6.43 \times 10^{15} \text{ cm}^{-3}$$

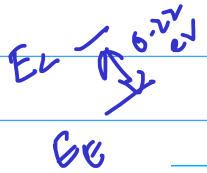
Table B.3 | Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{+23}$ atoms per gram molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$e = 1.60 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s
	$\frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.998 \times 10^{10}$ cm/s
Thermal voltage ($T = 300$ K)	$V_t = \frac{kT}{e} = 0.0259$ V
	$kT = 0.0259$ eV

Table B.4 | Silicon, gallium arsenide, and germanium properties ($T = 300$ K)

Property	Si	GaAs	Ge
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV) — Eg	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3}) n_i	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass $\left(\frac{m^*}{m_0}\right)$			
Electrons	$m_l^* = 0.98$	0.067	1.64
	$m_t^* = 0.19$		0.082
Holes	$m_{lh}^* = 0.16$	0.082	0.044
	$m_{hh}^* = 0.49$	0.45	0.28
Density of states effective mass			
Electrons $\left(\frac{m_{dn}^*}{m_o}\right)$	1.08	0.067	0.55
Holes $\left(\frac{m_{dp}^*}{m_o}\right)$	0.56	0.48	0.37
Conductivity effective mass			
Electrons $\left(\frac{m_{cn}^*}{m_o}\right)$	0.26	0.067	0.12
Holes $\left(\frac{m_{cp}^*}{m_o}\right)$	0.37	0.34	0.21

TYU 4.1 Calculate the thermal equilibrium electron and hole concentration in silicon at $T = 300$ K for the case when the Fermi energy level is 0.22 eV below the conduction-band energy E_c. The value of E_g is given in Appendix B.4.



$E_g = 1.12$ eV

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$N_c = 2.8 \times 10^{19}$ cm^{-3}

$kT = 0.0259$ eV

$T = 300$ K

$n_0 = 2.8 \times 10^{19} \exp\left[\frac{-0.22}{0.0259}\right]$

$n_0 = 5.73 \times 10^{15}$ cm^{-3}

$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$

$E_g = 1.12$ eV (Si) | $T = 300$k

$= 2.8 \times 10^{19} \times 1.04 \times 10^{19} \times \exp\left[\frac{-1.12}{0.0259}\right]$

$n_i^2 = 4.829 \times 10^{19}$

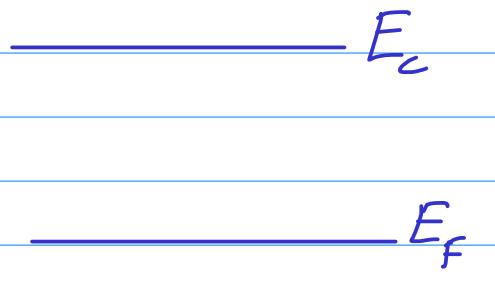
$n_0 p_0 = n_i^2$

$p_0 = \frac{n_i^2}{n_0} = 8.43 \times 10$ cm^{-3}

TYU 4.2

Determine the thermal equilibrium electron and hole concentration in GaAs at $T = 300$ K for the case when the Fermi energy level is 0.30 eV above the valence-band energy E_v. The value of E_g is given in Appendix B.4.

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$



$p_0 = 6.53 \times 10^{13}$ cm^{-3}

$n_0 = 0.0779 \times 10^{-3}$ cm^{-3}

EXAMPLE 4.4 Objective: Calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at $T = 300$ K.

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ and $m_p^* = 0.56m_0$.

The intrinsic Fermi level with respect to the center of the bandgap is

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.56}{1.08}\right)$$

or

$$E_{Fi} - E_{\text{midgap}} = -0.0128 \text{ eV} = -12.8 \text{ meV}$$

EXAMPLE 4.5 Objective: Calculate the thermal equilibrium concentrations of electrons and holes for a given Fermi energy.

Consider silicon at $T = 300$ K so that $N_c = 2.8 \times 10^{19}$ cm^{-3} and $N_v = 1.04 \times 10^{19}$ cm^{-3}. Assume that the Fermi energy is 0.25 eV below the conduction band. If we assume that the bandgap energy of silicon is 1.12 eV, then the Fermi energy will be 0.87 eV above the valence band.

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

E_c n

0.25 eV

E_F

$$n_0 = (2.8 \times 10^{19}) \exp\left(\frac{-0.25}{0.0259}\right) = 1.8 \times 10^{15} \text{ cm}^{-3}$$

E_F p

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

0.87 eV

E_v

$$p_0 = (1.04 \times 10^{19}) \exp\left(\frac{-0.87}{0.0259}\right) = 2.7 \times 10^4 \text{ cm}^{-3}$$

3.4 A Si wafer is doped with 10^{16} P atoms/cm^3. Calculate the equilibrium hole concentration p_0 at 300 K. How is E_F located relative to E_i? Sketch the resulting band diagram. Take $n_i = 1.5 \times 10^{10}$ cm^{-3}.

$N_D = 10^6$

$n_0 = 10^6$

$$n_0 \approx N_D \qquad \text{when } N_D \gg n_i$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25 \times 10^{20}}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$n_0 = N_c e^{-(E_c - E_F)/kT} = \left(N_c e^{-(E_c - E_i)/kT}\right) e^{-(E_F - E_i)/kT}$$

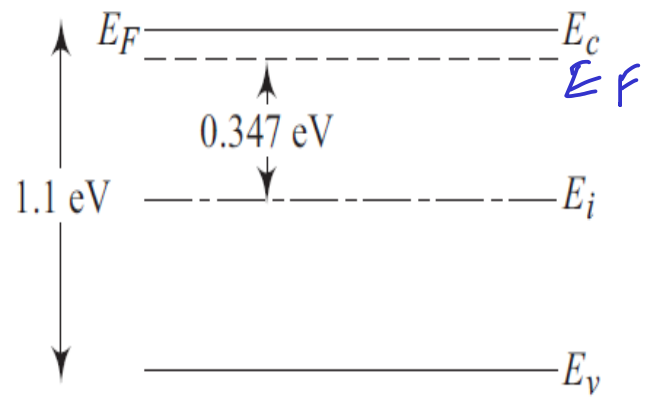
$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT \ln \frac{n_0}{n_i}$$

$$E_F - E_i = 0.0259 \ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right) = 0.347 \text{ eV}$$

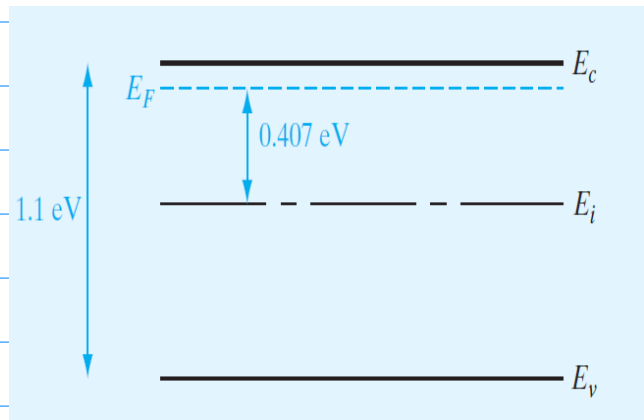
Si =
Ge =



A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration p_0 at 300 K? Where is E_F relative to E_i?

Since $N_d \gg n_i$, we can approximate $n_0 = N_d$

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$



$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT \ln \frac{n_0}{n_i} = 0.0259 \ln \frac{10^{17}}{1.5 \times 10^{10}} = 0.407 \text{ eV}$$

Objective: Calculate the probability that a quantum state in the conduction band at $E = E_c + kT/2$ is occupied by an electron, and calculate the thermal-equilibrium electron concentration in silicon at $T = 300$ K.

Assume the Fermi energy is 0.25 eV below the conduction band. The value of N_c for silicon at $T = 300$ K is $N_c = 2.8 \times 10^{19}$ cm^{-3} (see Appendix B).

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \cong \exp\left[\frac{-(E - E_F)}{kT}\right] = \exp\left[\frac{-(E_c + (kT/2) - E_F)}{kT}\right]$$

$$f_F(E) = \exp\left[\frac{-(0.25 + (0.0259/2))}{0.0259}\right] = 3.90 \times 10^{-5}$$

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = (2.8 \times 10^{19}) \exp\left[\frac{-0.25}{0.0259}\right]$$

$$n_0 = 1.80 \times 10^{15} \text{ cm}^{-3}$$

EXAMPLE 4.7 Determine the fraction of total electrons still in the donor states at $T = 300$ K.

Consider phosphorus doping in silicon, for $T = 300$ K, at a concentration of $N_d = 10^{16}$ cm^{-3}.

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d} \exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

$$\frac{n_d}{n_0 + n_d} = \frac{1}{1 + \frac{2.8 \times 10^{19}}{2(10^{16})} \exp\left(\frac{-0.045}{0.0259}\right)} = 0.0041 = 0.41\%$$

Comment

This example shows that there are very few electrons in the donor state compared with the conduction band. Essentially all of the electrons from the donor states are in the conduction band and, since only about 0.4 percent of the donor states contain electrons, the donor states are said to be completely ionized.

EXAMPLE 4.8 Objective: Determine the temperature at which 90 percent of acceptor atoms are ionized.

Consider p-type silicon doped with boron at a concentration of $N_a = 10^{16}$ cm^{-3}.

$$\frac{p_a}{p_0 + p_a} = \frac{1}{1 + \frac{N_v}{4N_a} \cdot \exp\left[\frac{-(E_a - E_v)}{kT}\right]}$$

For 90 percent ionization,

$$\frac{p_a}{p_0 + p_a} = 0.10 = \frac{1}{1 + \frac{(1.04 \times 10^{19})\left(\frac{T}{300}\right)^{3/2}}{4(10^{16})} \cdot \exp\left[\frac{-0.045}{0.0259\left(\frac{T}{300}\right)}\right]}$$

EXAMPLE 4.9

Objective: Determine the thermal-equilibrium electron and hole concentrations in silicon at $T = 300$ K for given doping concentrations. (*a*) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (*b*) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}.

Recall that $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at $T = 300$ K.

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

(*a*) From Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 10^{16} \text{ cm}^{-3}$$

The minority carrier hole concentration is found to be

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

(*b*) Again, from Equation (4.60), the majority carrier electron concentration is

$$n_0 = \frac{5 \times 10^{15} - 2 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15} - 2 \times 10^{15}}{2}\right)^2 + (1.5 \times 10^{10})^2} \cong 3 \times 10^{15} \text{ cm}^{-3}$$

The minority carrier hole concentration is

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{15}} = 7.5 \times 10^4 \text{ cm}^{-3}$$

4.62 Silicon atoms, at a concentration of 7×10^{15} cm^{-3}, are added to gallium arsenide. Assume that the silicon atoms act as fully ionized dopant atoms and that 5 percent of the concentration added replace gallium atoms and 95 percent replace arsenic atoms. Let $T = 300$ K. (*a*) Determine the donor and acceptor concentrations. (*b*) Is the material n type or p type? (*c*) Calculate the electron and hole concentrations. (*d*) Determine the position of the Fermi level with respect to E_{Fi}.

EXAMPLE 6.7

Objective: Calculate the quasi-Fermi energy levels.

Consider an n-type semiconductor at $T = 300$ K with carrier concentrations of $n_0 = 10^{15}\ \text{cm}^{-3}$, $n_i = 10^{10}\ \text{cm}^{-3}$, and $p_0 = 10^5\ \text{cm}^{-3}$. In nonequilibrium, assume that the excess carrier concentrations are $\delta n = \delta p = 10^{13}\ \text{cm}^{-3}$.

The Fermi level for thermal equilibrium can be determined from Equation (6.84a). We have

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982\ \text{eV} \qquad n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

We can use Equation (6.85a) to determine the quasi-Fermi level for electrons in nonequilibrium. We can write

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_0 + \delta n}{n_i}\right) = 0.2984\ \text{eV} \qquad n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Equation (6.85b) can be used to calculate the quasi-Fermi level for holes in nonequilibrium. We can write

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_0 + \delta p}{n_i}\right) = 0.179\ \text{eV} \qquad p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

An n-type silicon sample contains a donor concentration of $N_d = 10^{16}$ cm^{-3}. The minority carrier hole lifetime is found to be $\tau_{p0} = 20$ μs. (*a*) What is the lifetime of the majority carrier electrons? (*b*) Determine the thermal-equilibrium generation rate for electrons and holes in this material. (*c*) Determine the thermal-equilibrium recombination rate for electrons and holes in this material.

$$n(t) = n_0 + \delta n(t)$$
$$p(t) = p_0 + \delta p(t)$$

(a) Recombination rates are equal

$$R_{n0} = R_{p0}$$

$\tau_{n0} = ?$

$$\frac{n_o}{\tau_{nO}} = \frac{p_o}{\tau_{pO}}$$

$$n_o = N_d = 10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$$

Then

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25\times10^4}{20\times10^{-6}}$$

which yields

$$\tau_{nO} = 8.89\times10^{+6} \text{ s}$$

(b) Generation rate = recombination rate

Then

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

$$G = \frac{2.25\times10^4}{20\times10^{-6}} = 1.125\times10^9 \text{ cm}^{-3}\text{ s}^{-1}$$

$$G = \frac{p_o}{\tau_{p0}} \text{ (n-type)}$$

(c)

$$R = G = 1.125\times10^9 \text{ cm}^{-3}\text{ s}^{-1}$$

$$G = \frac{n_o}{\tau_{n0}} \text{ (p-type)}$$

An n-type silicon semiconductor, doped at $N_d = 4 \times 10^{16}$ cm^{-3}, is steadily illuminated such that $g' = 2 \times 10^{21}$ cm^{-3} s^{-1}. Assume $\tau_{n0} = 10^{-6}$ s and $\tau_{p0} = 5 \times 10^{-7}$ s.
(*a*) Determine the thermal-equilibrium value of $E_F - E_{Fi}$. (*b*) Calculate the quasi-Fermi levels for electrons and holes with respect to E_{Fi}. (*c*) What is the difference (in eV) between E_{Fn} and E_F ?

(a) $$E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$$

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

$n_0 = N_d = 4 \times 10^{16}$ cm^{-3}

$$= (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.383225 \text{ eV}$$

(b) $$\delta n = \delta p = g'\tau_{p0} = (2\times10^{21})(5\times10^{-7})$$

$$= 10^{15} \text{ cm}^{-3}$$

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$= (0.0259)\ln\left(\frac{4\times10^{16} + 10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.383865 \text{ eV}$$

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

$$= (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.28768 \text{ eV}$$

(c) $$E_{Fn} - E_F = 0.383865 - 0.383225$$

$$= 0.000640 \text{ eV}$$

or $$= 0.640 \text{ meV}$$

Consider a p-type silicon semiconductor at $T = 300$ K doped at $N_a = 5 \times 10^{15}$ cm^{-3}. (*a*) Determine the position of the Fermi level with respect to the intrinsic Fermi level. (*b*) Excess carriers are generated such that the excess carrier concentration is 10 percent of the thermal-equilibrium majority carrier concentration. Determine the quasi-Fermi levels with respect to the intrinsic Fermi level. (*c*) Plot the Fermi level and quasi-Fermi levels with respect to the intrinsic level.

(a) p-type

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

$$p_0 = N_a = 5\times10^{15}\ \text{cm}^{-3}$$

or

$$E_{Fi} - E_F = 0.3294 \text{ eV}$$

(b)

$$\delta n = \delta p = 5\times10^{14} \text{ cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} = 4.5\times10^4 \text{ cm}^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4.5\times10^4 + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fn} - E_{Fi} = 0.2697 \text{ eV}$$

and

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15} + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_{Fp} = 0.3318 \text{ eV}$$

(*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) N_a, (*ii*) N_d, (*iii*) x_n, (*iv*) x_p, and (*v*) $|E_{max}|$. (*b*) Repeat part (*a*) for a GaAs pn junction with $V_{bi} = 1.180$ V.

$$x_n = 0.25W = 0.25(x_n + x_p)$$

$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$

$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $N_d = 3N_a$

(a) $$V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{(1.5\times10^{10})^2}\right]$$

or $$3N_a^2 = (1.5\times10^{10})^2 \exp\left(\frac{0.710}{0.0259}\right)$$

which yields $N_a = 7.766\times10^{15}$ cm^{-3}

$N_d = 2.33\times10^{16}$ cm^{-3}

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2}$$

$\Rightarrow x_n = 9.93\times10^{-6}$ cm

or $x_n = 0.0993\ \mu$ m

$$x_p = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2}$$

$= 2.979\times10^{-5}$ cm

or $x_p = 0.2979\ \mu$ m

Now

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6\times10^{-19})(2.33\times10^{16})(0.0993\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

$= 3.58\times10^4$ V/cm

$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$\epsilon_s = \epsilon_0 \epsilon_r$$

$$\epsilon_s = (8.85\times10^{-14})(11.7)$$

An abrupt silicon pn junction at $T = 300$ K has impurity doping concentrations of $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}. Calculate (*a*) V_{bi}, (*b*) W at (*i*) $V_R = 0$ and (*ii*) $V_R = 5$ V, and (*c*) $|E_{max}|$ at (*i*) $V_R = 0$ and (*ii*) $V_R = 5$.

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{16})(10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.6767 \text{ V}$$

$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

(b) $$W = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right\}^{1/2}$$

$$W = \left\{\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a+N_d}{N_a N_d}\right]\right\}^{1/2}$$

(i) For $V_R = 0$,

$$W = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.6767)}{1.6\times10^{-19}} \times\left[\frac{5\times10^{16}+10^{15}}{(5\times10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 9.452\times10^{-5} \text{ cm}$$

or $W = 0.9452\ \mu$ m

(c) $$|E_{max}| = \frac{2(V_{bi}+V_R)}{W}$$

$$E_{max} = \frac{-2(V_{bi}+V_R)}{W}$$

(i) For $V_R = 0$,

$$|E_{max}| = \frac{2(0.6767)}{0.9452\times10^{-4}} = 1.43\times10^4 \text{ V/cm}$$

(ii) For $V_R = 5$ V,

$$|E_{max}| = \frac{2(0.6767+5)}{2.738\times10^{-4}} = 4.15\times10^4 \text{ V/cm}$$

Consider two p^+n silicon junctions at $T = 300$ K reverse biased at $V_R = 5$ V. The impurity doping concentrations in junction A are $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}, and those in junction B are $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. Calculate the ratio of the following parameters for junction A to junction B: (*a*) W, (*b*) $|E_{max}|$, and (*c*) C_j'.

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in_s (V_{biA}+V_R)}{e}\left(\dfrac{N_a+N_{dA}}{N_a N_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in_s (V_{biB}+V_R)}{e}\left(\dfrac{N_a+N_{dB}}{N_a N_{dB}}\right)\right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}\cdot\frac{(N_a+N_{dA})}{(N_a+N_{dB})}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find

$$V_{biA} = (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.7543 \text{ V}$$

$$V_{biB} = (0.0259)\ln\left[\frac{(10^{18})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.8139 \text{ V}$$

We find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right)\times\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or

$$\frac{W(A)}{W(B)} = 3.13$$

(b)

$$\frac{E(A)}{E(B)} = \frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}} = \frac{W(B)}{W(A)}\cdot\frac{V_{biA}+V_R}{V_{biB}+V_R}$$

$$= \left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or

$$\frac{E(A)}{E(B)} = 0.316$$

(c)

$$\frac{C_j'(A)}{C_j'(B)} = \frac{\left[\dfrac{\in_s N_a N_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in_s N_a N_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or

$$\frac{C_j'(A)}{C_j'(B)} = 0.319$$

(*a*) A uniformly doped silicon p^+n junction at $T = 300$ K is to be designed such that, at a reverse-biased of $V_R = 10$ V, the maximum electric field is limited to $|E_{max}| = 2.5 \times 10^5$ V/cm. Determine the maximum doping concentration in the n region. (Use an approximate value for V_{bi}.) (*b*) Repeat part (*a*) if the maximum electric field is limited to $|E_{max}| = 10^5$ V/cm.

$$|E_{max}| \cong \left[\frac{2e(V_{bi}+V_R)N_d}{\epsilon_s}\right]^{1/2} \leftarrow E_{max} = -\left\{\frac{2e(V_{bi}+V_R)}{\epsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right\}^{1/2}$$

$N_a >> N_d$

Let $V_{bi} \cong 0.75$ V

(a) $(2.5\times10^5)^2$

$$= \left[\frac{2(1.6\times10^{-19})(0.75+10)N_d}{(11.7)(8.85\times10^{-14})}\right]$$

$$\Rightarrow N_d = 1.88\times10^{16}\ \text{cm}^{-3}$$

(b) $(10^5)^2$

$$= \left[\frac{2(1.6\times10^{-19})(0.75+10)N_d}{(11.7)(8.85\times10^{-14})}\right]$$

$$\Rightarrow N_d = 3.01\times10^{15}\ \text{cm}^{-3}$$

Consider a silicon n^+p junction diode. The critical electric field for breakdown in silicon is approximately $E_{crit} = 4 \times 10^5$ V/cm. Determine the maximum p-type doping concentration such that the breakdown voltage is (*a*) 40 V and (*b*) 20 V.

(a) $V_B = \dfrac{\epsilon_s E_{crit}^2}{2eN_B}$ $\qquad V_B = \dfrac{\epsilon_s E_{crit}^2}{2eN_B}$

$V_B = 40V$

or

$$N_B = \frac{\epsilon_s E_{crit}^2}{2eV_B} = \frac{(11.7)(8.85\times10^{-14})(4\times10^5)^2}{2(1.6\times10^{-19})(40)}$$

Then $N_B = N_a = 1.294\times10^{16}\ \text{cm}^{-3}$

(b) $N_B = \dfrac{(11.7)(8.85\times10^{-14})(4\times10^5)^2}{2(1.6\times10^{-19})(20)}$

$V_B = 20V$

Or $N_B = N_a = 2.59\times10^{16}\ \text{cm}^{-3}$

A silicon pn junction has impurity doping concentrations of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentrations at the edges of the space charge region for (*a*) $V_a = 0.45$ V, (*b*) $V_a = 0.55$ V, and (*c*) $V_a = -0.55$ V.

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{8\times10^{15}} = 2.8125\times10^4 \text{ cm}^{-3}$$

$$n_{p0} \approx \frac{n_i^2}{N_a}$$ P

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{2\times10^{15}} = 1.125\times10^5 \text{ cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d}$$ N

$$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

$$p_n(x_n) = p_{n0}\exp\left(\frac{eV_a}{kT}\right)$$

$$n_p(-x_p) = n_{po}\exp\left(\frac{V_a}{V_t}\right)$$

$$n_p(-x_p) = n_{p0}\exp\left(\frac{eV_a}{kT}\right)$$

(a) $V_a = 0.45$ V,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.45}{0.0259}\right)$$

$$= 3.95\times10^{12} \text{ cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.45}{0.0259}\right)$$

or

$$n_p(-x_p) = 9.88\times10^{11} \text{ cm}^{-3}$$

(b) $V_a = 0.55$ V,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 1.88\times10^{14} \text{ cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 4.69\times10^{13} \text{ cm}^{-3}$$

(c) $V_a = -0.55$ V

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

A one-sided p^+n silicon diode has doping concentrations of $N_a = 5 \times 10^{17}$ cm^{-3} and $N_d = 8 \times 10^{15}$ cm^{-3}. The minority carrier lifetimes are $\tau_{n0} = 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 2 \times 10^{-4}$ cm^2. Calculate the (*a*) reverse-biased saturation current, and (*b*) the forward-bias current at (*i*) $V_a = 0.45$ V, (*ii*) $V_a = 0.55$ V, and (*iii*) $V_a = 0.65$ V.

$|E_{max}| = ?$ when $V_R = 7V$

(a) $$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

$$= (1.6\times10^{-19})(1.5\times10^{10})^2$$

$$\times\left[\frac{1}{5\times10^{17}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{8\times10^{15}}\sqrt{\frac{10}{8\times10^{-8}}}\right]$$

$$J_s = 5.145\times10^{-11}\ \text{A/cm}^2$$

$D_n = 25\ cm^2/s$

$D_p = 10\ cm^2/s$.

$$I_s = AJ_s = (2\times10^{-4})(5.145\times10^{-11})$$

$$= 1.029\times10^{-14}\ \text{A}$$

$J = \frac{I}{A}$

$I_s = J_s A$

$$|E_{max}| = \left[\frac{2e(V_{bi}+V_R)N_d}{\epsilon_s}\right]^{1/2}$$

(b) $$I = I_s \exp\left(\frac{V_a}{V_t}\right)$$

(i) $$I = (1.029\times10^{-14})\exp\left(\frac{0.45}{0.0259}\right)$$

$V_a = 0.45V$

$$= 3.61\times10^{-7}\ \text{A}$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$\epsilon_s = \epsilon_0 \epsilon_r$

$= (8.85\times10^{-14})(11.7)$

(ii) $$I = (1.029\times10^{-14})\exp\left(\frac{0.55}{0.0259}\right)$$

$V_a = 0.55V$

$$= 1.72\times10^{-5}\ \text{A}$$

(iii) $$I = (1.029\times10^{-14})\exp\left(\frac{0.65}{0.0259}\right)$$

$V_a = 0.65$

$$= 8.16\times10^{-4}\ \text{A}$$

For a silicon pn junction at $T = 300$ K, assume $\tau_{p0} = 0.1\tau_{n0}$ and $\mu_n = 2.4\,\mu_p$. The ratio of electron current crossing the depletion region to the total current is defined as the electron injection efficiency. Determine the expression for the electron injection efficiency as a function of (*a*) N_d/N_a and (*b*) the ratio of n-type conductivity to p-type conductivity.

(a)

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{eD_n n_{po}}{L_n}}{\dfrac{eD_n n_{po}}{L_n} + \dfrac{eD_p p_{no}}{L_p}}$$

$$= \frac{\sqrt{\dfrac{D_n}{\tau_{no}} \cdot \dfrac{n_i^2}{N_a}}}{\sqrt{\dfrac{D_n}{\tau_{no}} \cdot \dfrac{n_i^2}{N_a}} + \sqrt{\dfrac{D_p}{\tau_{po}} \cdot \dfrac{n_i^2}{N_d}}}$$

$$= \frac{1}{1 + \sqrt{\dfrac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n} = \frac{\mu_p}{\mu_n} = \frac{1}{2.4} \quad \text{and} \quad \frac{\tau_{no}}{\tau_{po}} = \frac{1}{0.1}$$

so

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + \sqrt{\dfrac{1}{2.4} \cdot \dfrac{1}{0.1}} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

or

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + (2.04)\left(\dfrac{N_a}{N_d}\right)}$$

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$L_n = \sqrt{D_n \tau_{n0}}$ $\quad n_{p0} = \dfrac{n_i^2}{N_a}$

$L_p = \sqrt{D_p \tau_{p0}}$ $\quad p_{n0} = \dfrac{n_i^2}{N_d}$

$D_n = \mu_n \quad D_p = \mu_p$

(b) Using Einstein's relation, we can write

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a} + \dfrac{e\mu_p}{L_p} \cdot \dfrac{n_i^2}{N_d}}$$

$$= \frac{e\mu_n N_d}{e\mu_n N_d + \dfrac{L_n}{L_p} \cdot e\mu_p N_a}$$

We have

$$\sigma_n = e\mu_n N_d \quad \text{and} \quad \sigma_p = e\mu_p N_a$$

Also

$$\frac{L_n}{L_p} = \sqrt{\frac{D_n \tau_{no}}{D_p \tau_{po}}} = \sqrt{\frac{2.4}{0.1}} = 4.90$$

Then

$$\frac{J_n}{J_n + J_p} = \frac{(\sigma_n / \sigma_p)}{(\sigma_n / \sigma_p) + 4.90}$$

EXAMPLE 5.1 | **Objective:** Calculate the drift current density in a semiconductor for a given electric field.

Consider a gallium arsenide sample at $T = 300$ K with doping concentrations of $N_a = 0$ and $N_d = 10^{16}$ cm^{-3}. Assume complete ionization and assume electron and hole mobilities given in Table 5.1. Calculate the drift current density if the applied electric field is E = 10 V/cm.

Sol

Since $N_d > N_a$, the semiconductor is n type and the majority carrier electron concentration, from Chapter 4 is given by

$n_i = 1.8 \times 10^6$ @ $T = 300$K

$$n = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \approx 10^{16} \text{ cm}^{-3}$$

The minority carrier hole concentration is

$np = n_i^2$

$$p = \frac{n_i^2}{n} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

For this extrinsic n-type semiconductor, the drift current density is

$n = N_d$ (complete ionization)

$$J_{drf} = e(\mu_n n + \mu_p p)\text{E} \approx e\mu_n N_d \text{E}$$

$= e\mu_n n E = e\mu_n N_d E$

Then

$$J_{drf} = (1.6 \times 10^{-19})(8500)(10^{16})(10) = 136 \text{ A/cm}^2$$

Table 5.1 | Typical mobility values at $T = 300$ K and low doping concentrations

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

Ex 5.1 A drift current density of $J_{drf} = 75$ A/cm² is required in a device using p-type silicon when an electric field of E = 120 V/cm is applied. Determine the required impurity doping concentration to achieve this specification. Assume that electron and hole mobilities given in Table 5.1 apply. (Ans. $N_a = 8.14 \times 10^{15}$ cm^{-3})

Sol

$J_{drf} = 75$ A/cm^2 $\quad E = 120$ V/cm

$J_{drf} = e(\mu_n n + \mu_p p)E \simeq e\mu_p p E \quad p = N_a$

$N_a = J_{drf} / e\mu_p E = 8.14 \times 10^{15}$ cm^{-3} $\simeq e\mu_p N_a E$

Objective: Determine the doping concentration and majority carrier mobility given the type and conductivity of a compensated semiconductor.

EXAMPLE 5.3

Consider compensated n-type silicon at $T = 300$ K, with a conductivity of $\sigma = 16\ (\Omega\text{-cm})^{-1}$ and an acceptor doping concentration of $10^{17}\ \text{cm}^{-3}$. Determine the ~~donor concentration~~ and the electron mobility. $N_d \approx 3.5 \times 10^{17}\ \text{cm}^{-3}$

Sol

For n-type silicon at $T = 300$ K, we can assume complete ionization; therefore the conductivity, assuming $N_d - N_a \gg n_i$, is given by

$$\sigma \approx e\mu_n n = e\mu_n (N_d - N_a)$$

We have that

$$16 = (1.6 \times 10^{-19})\mu_n(N_d - 10^{17})$$

$$\mu_n \approx 400\ \text{cm}^2/\text{V-s}$$

Ex 5.3 A compensated p-type silicon material at $T = 300$ K has impurity doping concentrations of $N_a = 2.8 \times 10^{17}\ \text{cm}^{-3}$ and $N_d = 8 \times 10^{16}\ \text{cm}^{-3}$. Determine the (*a*) ~~hole mobility~~, (*b*) conductivity, and (*c*) resistivity.

[Ans. (*a*) $\mu_p \cong 200\ \text{cm}^2/\text{V-s}$; (*b*) $\sigma = 6.4\ (\Omega\text{-cm})^{-1}$, (*c*) $\rho = 0.156\ (\Omega\text{-cm})$]

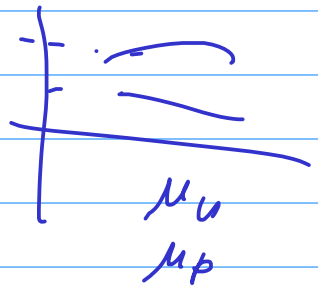
Sol

$$\sigma = e\mu_p p = e\mu_p (N_a - N_d)$$

(a) For $N_I = N_a + N_d = 2.8\times10^{17} + 8\times10^{16}$

$= 3.6\times10^{17}\ \text{cm}^{-3}$,

$\Rightarrow \mu_p = 200\ \text{cm}^2/\text{V-s}$



(b) $\sigma = e\mu_p(N_a - N_d)$

$= (1.6\times10^{-19})(200)(2\times10^{17})$

$\sigma = 6.4\ (\Omega\text{-cm})^{-1}$

(c) $\rho = \dfrac{1}{\sigma} = \dfrac{1}{6.4} = 0.156\ \Omega\text{-cm}$

EXAMPLE 5.5

Objective: Calculate the diffusion current density given a density gradient.

Assume that, in an n-type gallium arsenide semiconductor at $T = 300$ K, the electron concentration varies linearly from 1×10^{18} to 7×10^{17} cm^{-3} over a distance of 0.10 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 225$ cm^2/s.

■ **Solution**

The diffusion current density is given by

$$J_{n|dif} = eD_n \frac{dn}{dx} \approx eD_n \frac{\Delta n}{\Delta x}$$

$$= (1.6 \times 10^{-19})(225)\left(\frac{1 \times 10^{18} - 7 \times 10^{17}}{0.10}\right) = 108 \text{ A/cm}^2$$

EXAMPLE 5.8

Objective: Determine the majority carrier concentration and mobility, given Hall effect parameters.

Consider the geometry shown in Figure 5.13. Let $L = 10^{-1}$ cm, $W = 10^{-2}$ cm, and $d = 10^{-3}$ cm. Also assume that $I_x = 1.0$ mA, $V_x = 12.5$ V, $B_z = 500$ gauss $= 5 \times 10^{-2}$ tesla, and $V_H = -6.25$ mV.

A negative Hall voltage for this geometry implies that we have an n-type semiconductor. Using Equation (5.56), we can calculate the electron concentration as

$$n = \frac{-(10^{-3})(5 \times 10^{-2})}{(1.6 \times 10^{-19})(10^{-5})(-6.25 \times 10^{-3})} = 5 \times 10^{21} \text{ m}^{-3} = 5 \times 10^{15} \text{ cm}^{-3}$$

$$n = -\frac{I_x B_z}{edV_H}$$

The electron mobility is then determined from Equation (5.60) as

$$\mu_n = \frac{(10^{-3})(10^{-3})}{(1.6 \times 10^{-19})(5 \times 10^{21})(12.5)(10^{-4})(10^{-5})} = 0.10 \text{ m}^2\text{/V-s}$$

or

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$\mu_n = 1000 \text{ cm}^2\text{/V-s}$$

5.1 The concentration of donor impurity atoms in silicon is $N_d = 10^{15}$ cm^{-3}. Assume an electron mobility of $\mu_n = 1300$ cm^2/V-s and a hole mobility of $\mu_p = 450$ cm^2/V-s. (*a*) Calculate the resistivity of the material. (*b*) What is the conductivity of the material?

Sol

$$\rho = \frac{1}{\sigma} = \frac{1}{e\mu_n N_d}$$

(a) $$\rho = \frac{1}{e\mu_n N_d} = \frac{1}{(1.6\times10^{-19})(1300)(10^{15})}$$
$$= 4.808\ \Omega\text{-cm}$$

(b) $$\sigma = \frac{1}{\rho} = \frac{1}{4.8077} = 0.208\ (\Omega\text{-cm})^{-1}$$

5.5 A silicon sample is 2.5 cm long and has a cross-sectional area of 0.1 cm^2. The silicon is n type with a donor impurity concentration of $N_d = 2 \times 10^{15}$ cm^{-3}. The resistance of the sample is measured and found to be 70 Ω. What is the electron mobility? (μ_n)

Sol

$$\sigma = e\mu_n N_d$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n N_d)A}$$

or $$\mu_n = \frac{L}{(eN_d)RA}$$

$$= \frac{2.5}{(1.6\times10^{-19})(2\times10^{15})(70)(0.1)}$$

$$= 1116\ \text{cm}^2/\text{V-s}$$

5.23 Consider three samples of silicon at $T = 300$ K. The n-type sample is doped with arsenic atoms to a concentration of $N_d = 5 \times 10^{16}$ cm^{-3}. The p-type sample is doped with boron atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. The compensated sample is doped with both the donors and acceptors described in the n-type and p-type samples. (*a*) Find the equilibrium electron and hole concentrations in each sample, (*b*) determine the majority carrier mobility in each sample, (*c*) calculate the conductivity of each sample, (*d*) and determine the electric field required in each sample to induce a drift current density of $J = 120$ A/cm^2.

(a) n-type: $n_o = N_d = 5 \times 10^{16}$ cm^{-3}

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}}$$

$$= 4.5 \times 10^3 \text{ cm}^{-3}$$

p-type: $p_o = N_a = 2 \times 10^{16}$ cm^{-3}

$$n_o = \frac{(1.5 \times 10^{10})^2}{2 \times 10^{16}} = 1.125 \times 10^4 \text{ cm}^{-3}$$

compensated: $n_o = N_d - N_a$

$$= 5 \times 10^{16} - 2 \times 10^{16}$$

$$= 3 \times 10^{16} \text{ cm}^{-3}$$

$$p_o = \frac{(1.5 \times 10^{10})^2}{3 \times 10^{16}} = 7.5 \times 10^3 \text{ cm}^{-3}$$

n-type: $\mu_n \cong 1100$ cm^2/V-s

p-type: $\mu_p \cong 400$ cm^2/V-s

compensated: $\mu_n \cong 1000$ cm^2/V-s

(c) n-type: $\sigma = e\mu_n n_o$

$$= (1.6 \times 10^{-19})(1100)(5 \times 10^{16})$$

$$= 8.8 \ (\Omega\text{-cm})^{-1}$$

p-type: $\sigma = e\mu_p p_o$

$$= (1.6 \times 10^{-19})(400)(2 \times 10^{16})$$

$$= 1.28 \ (\Omega\text{-cm})$$

compensated: $\sigma = e\mu_n n_o$

$$= (1.6 \times 10^{-19})(1000)(3 \times 10^{16})$$

$$= 4.8 \ (\Omega\text{-cm})^{-1}$$

(d) $J = \sigma E \Rightarrow E = \dfrac{J}{\sigma}$

n-type: $E = \dfrac{120}{8.8} = 13.6 \text{ V/cm}$

p-type: $E = \dfrac{120}{1.28} = 93.75 \text{ V/cm}$

compensated: $E = \dfrac{120}{4.8} = 25 \text{ V/cm}$

5.29 Consider a sample of silicon at $T = 300$ K. Assume that the electron concentration varies linearly with distance, as shown in Figure P5.29. The diffusion current density is found to be $J_n = 0.19$ A/cm^2. If the electron diffusion coefficient is $D_n = 25$ cm^2/s, determine the electron concentration at $x = 0$.

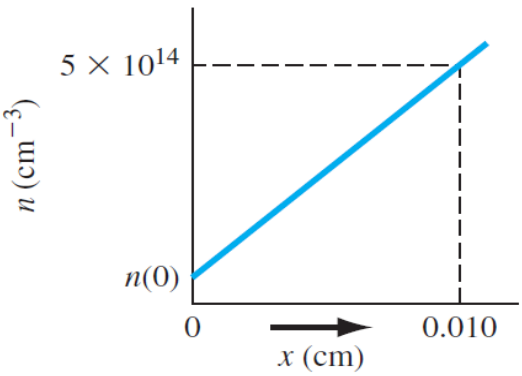


Figure P5.29 | Figure for Problem 5.29.

$$J_n = eD_n \frac{dn}{dx} = eD_n \left(\frac{5 \times 10^{14} - n(0)}{0.01 - 0} \right)$$

$$0.19 = (1.6 \times 10^{-19})(25) \left(\frac{5 \times 10^{14} - n(0)}{0.010} \right)$$

Then

$$\frac{(0.19)(0.010)}{(1.6 \times 10^{-19})(25)} = 5 \times 10^{14} - n(0)$$

which yields

$$n(0) = 0.25 \times 10^{14} \text{ cm}^{-3}$$

5.35 The electron concentration in silicon at $T = 300$ K is given by

$$n(x) = 10^{16} \exp\left(\frac{-x}{18}\right) \text{ cm}^{-3}$$

where x is measured in μm and is limited to $0 \le x \le 25$ μm. The electron diffusion coefficient is $D_n = 25$ cm²/s and the electron mobility is $\mu_n = 960$ cm²/V-s. The total electron current density through the semiconductor is constant and equal to $J_n = -40$ A/cm². The electron current has both diffusion and drift current components. Determine the electric field as a function of x which must exist in the semiconductor.

$$J_n = e\mu_n n\text{E} + eD_n \frac{dn}{dx}$$

or

$$-40 = (1.6\times10^{-19})(960)\left[10^{16}\exp\left(\frac{-x}{18}\right)\right]\text{E} + (1.6\times10^{-19})(25)(10^{16}) \times\left(\frac{-1}{18\times10^{-4}}\right)\exp\left(\frac{-x}{18}\right)$$

$$\frac{dn(x)}{dx} = \frac{d}{dx}\left[10^{16}\exp\left(\frac{-x}{18}\right)\right]$$

$$= 10^{16} \times \frac{-1}{18\times10^{-4}} \exp\left(\frac{-x}{18}\right)$$

Then

$$-40 = (1.536)\left[\exp\left(\frac{-x}{18}\right)\right]\text{E} - 22.22\exp\left(\frac{-x}{18}\right)$$

We find

$$\text{E} = \frac{(22.22)\exp\left(\frac{-x}{18}\right) - 40}{(1.536)\exp\left(\frac{-x}{18}\right)}$$

or

$$\text{E} = 14.5 - (26.0)\exp\left(\frac{+x}{18}\right)$$

5.36 The total current in a semiconductor is constant and equal to $J = -10$ A/cm^2. The total current is composed of a hole drift current and electron diffusion current. Assume that the hole concentration is a constant and equal to 10^{16} cm^{-3} and assume that the electron concentration is given by $n(x) = 2 \times 10^{15} e^{-x/L}$ cm^{-3} where $L = 15$ μm. The electron diffusion coefficient is $D_n = 27$ cm^2/s and the hole mobility is $\mu_p = 420$ cm^2/V-s. Calculate (*a*) the electron diffusion current density for $x > 0$, (*b*) the hole drift current density for $x > 0$, and (*c*) the required electric field for $x > 0$.

(a) $$J_n = eD_n \frac{dn}{dx} = eD_n \frac{d}{dx}\left[2\times 10^{15} e^{-x/L}\right]$$

$$= \frac{-eD_n\left(2\times 10^{15}\right)e^{-x/L}}{L}$$

$$= \frac{-\left(1.6\times 10^{-19}\right)(27)\left(2\times 10^{15}\right)e^{-x/L}}{15\times 10^{-4}}$$

$$= -5.76e^{-x/L}$$

(b) $$J_p = J_{Total} - J_n = -10 - \left(-5.76e^{-x/L}\right)$$

$$= \left[5.76e^{-x/L} - 10\right] \text{ A/cm}^2$$

(c) We have $J_p = \sigma \mathrm{E} = \left(e\mu_p p_o\right)\mathrm{E}$

$$5.76e^{-x/L} - 10 = \left(1.6\times 10^{-19}\right)(420)\left(10^{16}\right)\mathrm{E}$$

So $\mathrm{E} = \left[8.57e^{-x/L} - 14.88\right]$ V/cm

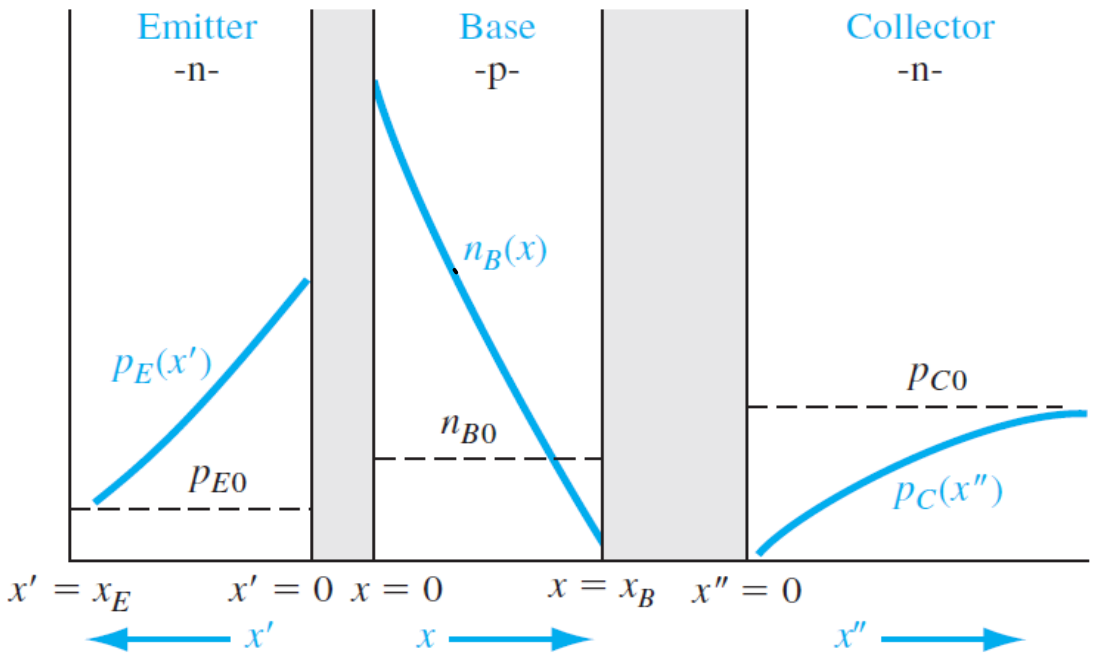


Figure 12.14 | Minority carrier distribution in an npn bipolar transistor operating in the forward-active mode.

Table 12.2 | Taylor expansions of hyperbolic functions

Function	Taylor expansion
sinh (x)	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$
cosh (x)	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$
tanh (x)	$x - \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$

TYU 12.1 The emitter and base of a silicon npn bipolar transistor are uniformly doped at impurity concentrations of 10^{18} cm^{-3} and 10^{16} cm^{-3}, respectively. A forward-bias B–E voltage of $V_{BE} = 0.610$ V is applied. The neutral base width is $x_B = 2$ μm and the minority carrier diffusion length in the base is $L_B = 10$ μm. Calculate the excess minority carrier concentration in the base at (*a*) $x = 0$ and (*b*) $x = x_B/2$. (*c*) Determine the ratio of the actual minority carrier concentration at $x = x_B/2$ [Equation (12.15a)] to that in the ideal case of a linear minority carrier distribution [Equation (12.15b)].

[Ans. (*a*) $n_B(0) = 3.81 \times 10^{14}$ cm^{-3}; (*b*) $\delta n_B(x_B/2) \approx n_B(x_B/2) = 1.8947 \times 10^{14}$ cm^{-3}; (*c*) Ratio $= (1.8947 \times 10^{14}/1.9042 \times 10^{14}) = 0.9950$]

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}}$$

$$= 2.25\times10^4 \text{ cm}^{-3}$$

$n_p = n_i^2$

$n_{B0} N_B = n_i^2$

$n_B(x) = n_{B0} + \delta n_B(x)$

$\delta n_B(x) = n_B(x) - n_{B0}$

(a) $$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= 2.25\times10^4 \exp\left(\frac{0.61}{0.0259}\right)$$

$$= 3.81\times10^{14} \text{ cm}^{-3}$$

$$\delta n_B(0) = n_B(x=0) - n_{B0} = n_{B0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

$\delta n_B(0) = n_{B0} \exp\left(\frac{eV_{BE}}{kT}\right) - n_{B0}$

$\Downarrow$

$n_B(0)$

(b) Using Equation (12.15a), we find

At $x = \dfrac{x_B}{2}$, $\dfrac{x_B/2}{L_B} = \dfrac{2/2}{10} = 0.10$

Then

$\sinh(0.10) = 0.100167$

$\sinh(0.20) = 0.20134$

So

$$\delta n(x = x_B/2) = \frac{(2.25\times10^4)}{(0.20134)}$$

$$\times\left\{\left[\exp\left(\frac{0.61}{0.0259}\right) - 1\right](0.100167) - (0.100167)\right\}$$

or

$$\delta n(x = x_B/2) = 1.8947\times10^{14} \text{ cm}^{-3}$$

$$\delta n_B(x) = \frac{n_{B0}\left\{\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}}{\sinh\left(\frac{x_B}{L_B}\right)}$$

$x = x_B/2$.

(c) For an ideal linear function

$$\delta n_B(x = x_B/2) = \frac{3.8084\times10^{14}}{2}$$

$$= 1.9042\times10^{14} \text{ cm}^{-3}$$

$$\text{Ratio} = \frac{1.8947}{1.9042} = 0.9950$$

TYU 12.2 Consider a silicon npn bipolar transistor with emitter and base regions uniformly doped at concentrations of 10^{18} cm^{-3} and 10^{16} cm^{-3}, respectively. A forward-bias B–E voltage of $V_{BE} = 0.610$ V is applied. The neutral emitter width is $x_E = 4$ μm and the minority carrier diffusion length in the emitter is $L_E = 4$ μm. Calculate the excess minority carrier concentration in the emitter at (*a*) $x' = 0$ and (*b*) $x' = x_E/2$.

[Ans. (*a*) 3.808×10^{12} cm^{-3}; (*b*) 1.689×10^{12} cm^{-3}]

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{10^{18}} = 2.25\times10^2 \text{ cm}^{-3}$$

(a) $$\delta p_E(0) = p_{EO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (2.25\times10^2)\exp\left(\frac{0.61}{0.0259}\right)$$

$$= 3.808\times10^{12} \text{ cm}^{-3}$$

$$\delta p_E(0) = p_E(x'=0) - p_{E0} = p_{E0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

(b) We have

$$\sinh\left(\frac{x_E/2}{L_E}\right) = \sinh(0.5) = 0.5211$$

and

$$\sinh(1) = 1.1752$$

Then, using Equation (12.21a),

$$\delta p_E(x' = x_E/2) = \frac{(3.808\times10^{12})(0.5211)}{1.1752}$$

$$= 1.689\times10^{12} \text{ cm}^{-3}$$

$$\delta p_E(x') = \frac{p_{E0}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]\sinh\left(\frac{x_E - x'}{L_E}\right)}{\sinh\left(\frac{x_E}{L_E}\right)}$$

TYU 12.3 Consider the collector region of an npn bipolar transistor biased in the forward-active region. At what value of x'', compared to L_C, does the magnitude of the minority carrier concentration reach 95 percent of the thermal-equilibrium value? (Ans. $x''/L_C \approx 3$)

TYU 12.3

$$p_C(x'') = p_{CO} + \delta p_C = p_{CO}\left(1 - \exp\left(\frac{-x''}{L_C}\right)\right)$$

Then

$$0.95 = 1 - \exp\left(\frac{-x''}{L_C}\right)$$

or

$$\exp\left(\frac{-x''}{L_C}\right) = 0.05 \Rightarrow \exp\left(\frac{+x''}{L_C}\right) = \frac{1}{0.05}$$

so

$$\frac{x''}{L_C} = \ln\left(\frac{1}{0.05}\right) \cong 3$$

12.9 A uniformly doped silicon npn bipolar transistor at $T = 300$ K is biased in the forward-active mode. The doping concentrations are $N_E = 8 \times 10^{17}$ cm^{-3}, $N_B = 2 \times 10^{16}$ cm^{-3}, and $N_C = 10^{15}$ cm^{-3}. (*a*) Determine the thermal-equilibrium values p_{E0}, n_{B0}, and p_{C0}. (*b*) For $V_{BE} = 0.640$ V, calculate the values of n_B at $x = 0$ and p_E at $x' = 0$. (*c*) Sketch the minority carrier concentrations through the device and label each curve.

(a) $$p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{8\times10^{17}}$$

$$= 2.8125\times10^{2}\ \text{cm}^{-3}$$

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{2\times10^{16}}$$

$$= 1.125\times10^{4}\ \text{cm}^{-3}$$

$$p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{15}}$$

$$= 2.25\times10^{5}\ \text{cm}^{-3}$$

(b) $$n_B(0) = n_{B0}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (1.125\times10^{4})\exp\left(\frac{0.640}{0.0259}\right)$$

$$= 6.064\times10^{14}\ \text{cm}^{-3}$$

$$p_E(0) = p_{E0}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (2.8125\times10^{2})\exp\left(\frac{0.640}{0.0259}\right)$$

$$= 1.516\times10^{13}\ \text{cm}^{-3}$$

12.10 A silicon pnp bipolar transistor at $T = 300$ K is uniformly doped and is biased in the forward-active mode. The doping concentrations are $N_E = 5 \times 10^{17}$ cm^{-3}, $N_B = 10^{16}$ cm^{-3}, and $N_C = 10^{15}$ cm^{-3}. (*a*) Find the thermal-equilibrium values n_{E0}, p_{B0}, and n_{C0}. (*b*) Determine the values of p_B at $x = 0$ and n_E at $x' = 0$ for $V_{EB} = 0.615$ V. (*c*) Sketch the minority carrier concentrations through the device and label each curve.

(a) $$n_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{5\times10^{17}}$$

$$= 4.5\times10^{2}\ \text{cm}^{-3}$$

$$p_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}}$$

$$= 2.25\times10^{4}\ \text{cm}^{-3}$$

$$n_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{15}}$$

$$= 2.25\times10^{5}\ \text{cm}^{-3}$$

(b) $$p_B(0) = p_{B0}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25\times10^{4})\exp\left(\frac{0.615}{0.0259}\right)$$

$$= 4.62\times10^{14}\ \text{cm}^{-3}$$

$$n_E(0) = n_{E0}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (4.5\times10^{2})\exp\left(\frac{0.615}{0.0259}\right)$$

$$= 9.24\times10^{12}\ \text{cm}^{-3}$$

Objective: Design the width of a MOSFET such that a specified current is induced for a given applied bias.

EXAMPLE 10.7

Consider an ideal n-channel MOSFET with parameters $L = 1.25\ \mu m$, $\mu_n = 650\ cm^2/V\text{-}s$, $C_{ox} = 6.9 \times 10^{-8}\ F/cm^2$, and $V_T = 0.65\ V$. Design the channel width W such that $I_D(sat) = 4\ mA$ for $V_{GS} = 5\ V$.

■ Solution

For the transition biased in the saturation region, we have, from Equation (10.66),

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS} - V_T)^2$$

or

$$4 \times 10^{-3} = \frac{W(650)(6.9 \times 10^{-8})}{2(1.25 \times 10^{-4})} \cdot (5 - 0.65)^2 = 3.39\ W$$

Then

$$W = 11.8\ \mu m$$

10.44 The transconductance of an n-channel MOSFET is found to be $g_m = \partial I_D/\partial V_{GS} = 1.25\ mA/V$ when measured at $V_{DS} = 50\ mV$. The threshold voltage is $V_T = 0.3\ V$. (*a*) Determine the conductance parameter K_n. (*b*) What is the current at $V_{GS} = 0.8\ V$ and $V_{DS} = 50\ mV$? (*c*) Determine the current at $V_{GS} = 0.8\ V$ and $V_{DS} = 1.5\ V$.

(a) $$g_m = \frac{\partial I_D}{\partial V_{GS}} \qquad I_D = K_n\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

$$= \frac{\partial}{\partial V_{GS}}\left\{K_n\left[2(V_{GS} - V_T)(V_{DS}) - V_{DS}^2\right]\right\}$$

$$= K_n(2V_{DS})$$

$$1.25 = K_n(2)(0.05)$$

$$\Rightarrow K_n = 12.5\ mA/V^2$$

(b) $$I_D = (12.5)\left[2(0.8 - 0.3)(0.05) - (0.05)^2\right] \qquad 0 \le V_{DS} \le V_{DS}(sat)$$

$$= 0.594\ mA \qquad V_{DS}(sat) = V_{GS} - V_T$$

(c) $$I_D = (12.5)(0.8 - 0.3)^2 \qquad V_{DS} \ge V_{DS}(sat)$$

$$= 3.125\ mA \qquad V_{DS}(sat) = V_{GS} - V_T$$

10.46 One curve of an n-channel MOSFET is characterized by the following parameters: $I_D(\text{sat}) = 2 \times 10^{-4}$ A, $V_{DS}(\text{sat}) = 4$ V, and $V_T = 0.8$ V.

(*a*) What is the gate voltage?

(*b*) What is the value of the conduction parameter?

(*c*) If $V_G = 2$ V and $V_{DS} = 2$ V, determine I_D.

(*d*) If $V_G = 3$ V and $V_{DS} = 1$ V, determine I_D.

(a)

$$V_{DS}(sat) = V_{GS} - V_T$$

or

$$4 = V_{GS} - 0.8 \Rightarrow V_{GS} = 4.8 \text{ V}$$

(b)

$$I_D(sat) = K_n(V_{GS} - V_T)^2 = K_n V_{DS}^2(sat)$$

so

$$2 \times 10^{-4} = K_n(4)^2$$

which yields

$$K_n = 12.5\ \mu \text{ A/V}^2$$

(c)

$$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2 \text{ V}$$

so $V_{DS} > V_{DS}(sat)$

$$I_D(sat) = (1.25 \times 10^{-5})(2 - 0.8)^2$$

or

$$I_D(sat) = 18\ \mu \text{ A}$$

(d)

$$V_{DS} < V_{DS}(sat)$$

$$I_D = K_n\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

$$= (1.25 \times 10^{-5})\left[2(3 - 0.8)(1) - (1)^2\right]$$

or

$$I_D = 42.5\ \mu \text{ A}$$

Consider the drain of an n – channel MOSFET is shorted to the gate with $V_T = 0.3$ V, Find the region of operation. If $I_D = 1.6$ mA for $V_{GS} = 2.6$ V, calculate the drain current and transconductance for $V_{GS} = 6$V.

Solution: $V_{GS} = V_{DS}$ $\quad I_D = 1.6\,mA$ for $V_{GS} = 2.6V$

$$V_{GS} = V_{DS} = 2.6V$$

$$V_{DS}(Sat) = V_{GS} - V_T = 2.6 - 0.3 = 2.3V \qquad V_{DS} > V_{DS}(Sat)$$

Saturation Region

$$I_D = k_n (V_{GS} - V_T)^2$$

$$1.6 = k_n (2.3)^2$$

$$k_n = 0.302\ mA/V^2$$

$V_{GS} = 6V$ $\quad V_{DS} = 6V$

$$V_{DS}(Sat) = V_{GS} - V_T = 6 - 0.3 = 5.7$$

$$V_{DS} > V_{DS}(Sat)$$

Saturation Region.

$$I_D = k_n (V_{GS} - V_T)^2$$

$$= 0.302 \times 10^{-3} (5.7)^2$$

$$I_D = 9.82\ mA$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2 k_n (V_{GS} - V_T)$$

$$= 3.442\ mA/V$$