Carrier Transport Phenomena

Carrier Transport

- The net flow of the electrons and holes in a semiconductor will generate currents.

- The process by which these charged particles move is called *transport.*

- two basic transport mechanisms in a semiconductor crystal: drift and diffusion

- drift— the movement of charge due to electric fields

- diffusion—the flow of charge due to density gradients.

CARRIER DRIFT

- An electric field applied to a semiconductor will produce a force on electrons and holes so that they will experience a net acceleration and net movement, provided there are available energy states in the conduction and valence bands.

- This net movement of charge due to an electric field is called drift.

- The net drift of charge gives rise to a drift current.

Drift Current Density

- If we have a positive volume charge density moving at an average drift velocity v_d, the drift current density is given by $$J_{drf} = \rho v_d$$

- If the volume charge density is due to positively charged holes, then $$J_{p|drf} = (ep)v_{dp}$$

- where $J_{p|drf}$ is the drift current density due to holes and v_{dp} is the average drift velocity of the holes.

- The equation of motion of a positively charged hole in the presence of an electric field is $$F = m_{cp}^* a = e\mathrm{E}$$

- where e is the magnitude of the electronic charge, a is the acceleration, E is the electric field, and m_{cp}^* is the conductivity effective mass of the hole.

Drift Current Density

- charged particles in a semiconductor are involved in collisions with ionized impurity atoms and with thermally vibrating lattice atoms.

- These collisions, or scattering events, alter the velocity characteristics of the particle.

- As the hole accelerates in a crystal due to the electric field, the velocity increases.

- When the charged particle collides with an atom in the crystal, for example, the particle loses most, or all, of its energy.

- The particle will again begin to accelerate and gain energy until it is again involved in a scattering process. This continues over and over again.

- Throughout this process, the particle will gain an average drift velocity which, for low electric fi elds, is directly proportional to the electric field $v_{dp} = \mu_p \mathrm{E}$ μ_p is hole mobility

Drift Current Density

- The mobility describes how well a particle will move due to an electric field.

- the drift current density due to holes

$$J_{p|drf} = (ep)v_{dp} = e\mu_p p\mathrm{E}$$

- the drift current density due to electrons

$$J_{n|drf} = \rho v_{dn} = (-en)v_{dn}$$

$$v_{dn} = -\mu_n\mathrm{E}$$

$$J_{n|drf} = (-en)(-\mu_n\mathrm{E}) = e\mu_n n\mathrm{E}$$
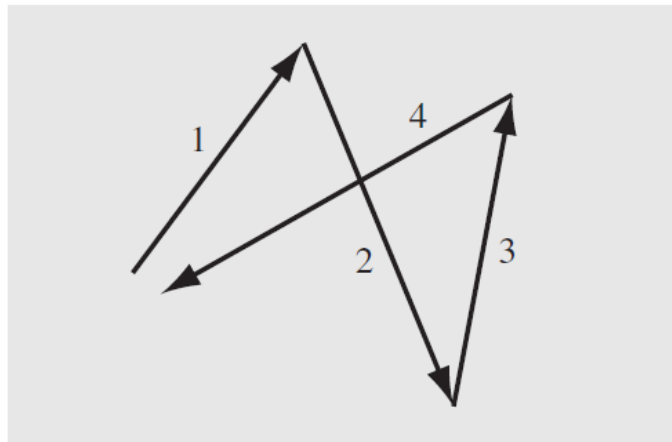
μ_n electron mobility

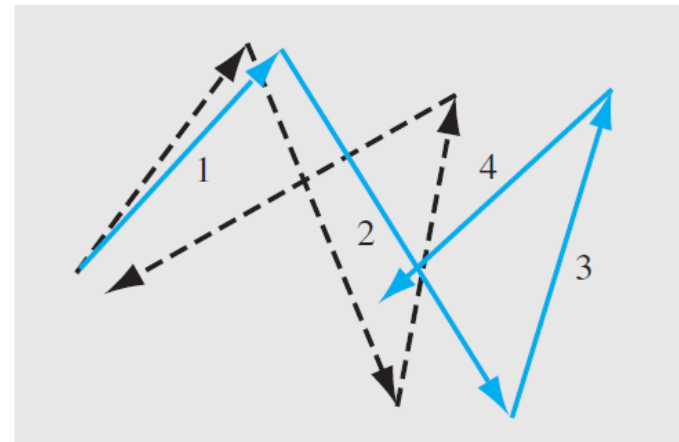- the total *drift current density*

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}$$

Mobility Effects

$$F = m_{cp}^* \frac{dv}{dt} = e\mathrm{E} \qquad \int \frac{dv}{dt} = \int \frac{e\mathrm{E}}{m_{cp}^*} \qquad v = \frac{e\mathrm{E}t}{m_{cp}^*}$$

E field

(a)   (b)

$p \rightarrow$ $J \rightarrow$

$e \leftarrow$

Figure 5.1 | Typical random behavior of a hole in a semiconductor (a) without an electric field and (b) with an electric field.

mean time between collisions - τ_{cp}

Mobility Effects

- If a small electric field (E-field) is applied, there will be a net drift of the hole in the direction of the E-field, and the net drift velocity will be a small perturbation on the random thermal velocity.

- the mean peak velocity just prior to a collision or scattering event is

$$v_{d|\text{peak}} = \left(\frac{e\tau_{cp}}{m_{cp}^*}\right) \text{E}$$

$$\upsilon = \frac{e \, Et}{m_{cp}^*}$$

$$\upsilon_{d|y} = \frac{e \, \tau_{cp}}{m_{cp}^*} \, E$$

- The average drift velocity is one half the peak value

$$\langle v_d \rangle = \frac{1}{2}\left(\frac{e\tau_{cp}}{m_{cp}^*}\right) \text{E}$$

Mobility Effects

- Accurate model – statistical in nature (no ½)

$$\mu_p = \frac{v_{dp}}{E} = \frac{e\tau_{cp}}{m^*_{cp}}$$

$$\mu_n = \frac{e\tau_{cn}}{m^*_{cn}}$$

Lattice and ionized impurity scattering

- There are two collision or scattering mechanisms that dominate in a semiconductor and affect the carrier mobility: phonon or lattice scattering, and ionized impurity scattering.

- The atoms in a semiconductor crystal have a certain amount of thermal energy at temperatures above absolute zero that causes the atoms to randomly vibrate about their lattice position within the crystal.

- The thermal vibrations cause a disruption of the potential function, resulting in an interaction between the electrons or holes and the vibrating lattice atoms. This *lattice scattering* is also referred to as *phonon scattering.*

Lattice and ionized impurity scattering

- lattice scattering is related to the thermal motion of atoms

- the rate at which the scattering occurs is a function of temperature.

$$\mu_L \propto T^{-3/2}$$

- In lightly doped semiconductors, lattice scattering dominates and the carrier mobility decreases with temperature

Lattice and ionized impurity scattering

- The second interaction mechanism affecting carrier mobility is called ionized impurity scattering.

- We have seen that impurity atoms are added to the semiconductor to control or alter its characteristics.

- These impurities are ionized at room temperature so that a coulomb interaction exists between the electrons or holes and the ionized impurities.

- This coulomb interaction produces scattering or collisions and also alters the velocity characteristics of the charge carrier.

$$\mu_I \propto \frac{T^{+3/2}}{N_I} \qquad N_I = N_d^+ + N_a^-$$

N_I total ionized impurity concentration

- As the impurity concentration increases, the number of impurity scattering centers increases, thus reducing mobility.

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

Conductivity

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}$$

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma\mathrm{E}$$

$$\sigma = e(\mu_n n + \mu_p p)$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

$$\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p$$

$$\sigma \approx e\mu_\rho N_a \approx \frac{1}{\rho}$$
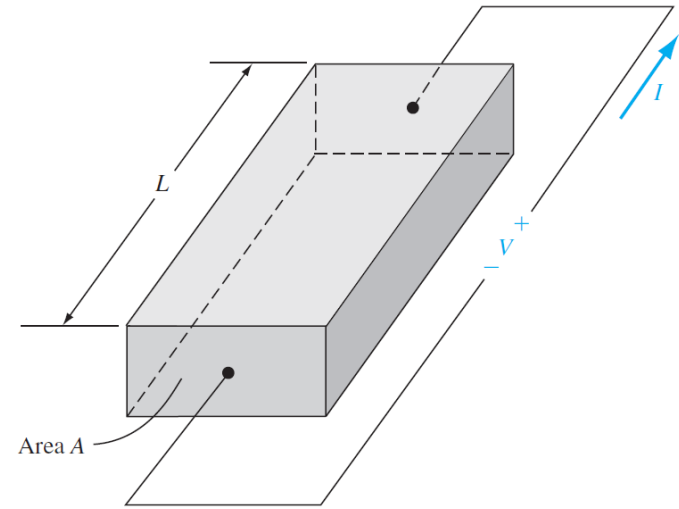
$$\sigma_i = e(\mu_n + \mu_p)\, n_i$$

$$V = IR$$

$$R = \frac{\rho L}{A}$$

$$J = \frac{I}{A}$$

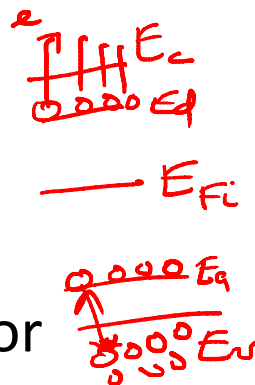$$J = \frac{V}{RA} = \frac{V}{\frac{\rho L}{A} A} = \frac{1}{\rho} \cdot \frac{V}{L}$$

$$= \sigma E$$

$$J = \sigma E$$

P semiconductor

Complete ionization

Intrinsic Semiconductor

$$p = N_a$$

$$\sigma \simeq e \mu_n N_d$$

$$n = p = n_i$$

$$J_T = J_{df} + J_{ds}$$

?)

CARRIER DIFFUSION

- *Diffusion* is the process whereby particles flow from a region of high concentration toward a region of low concentration.

- If the gas molecules were electrically charged, the net flow of charge would result in a *diffusion current.*
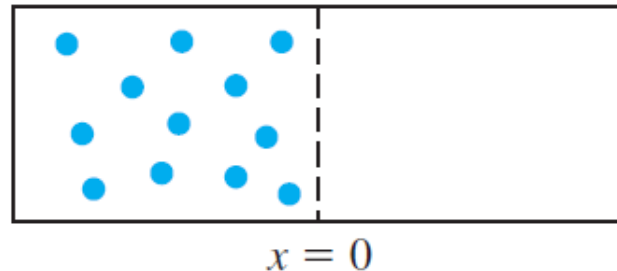

$x = 0$

Figure 5.9 | Container divided by a membrane with gas molecules on one side.
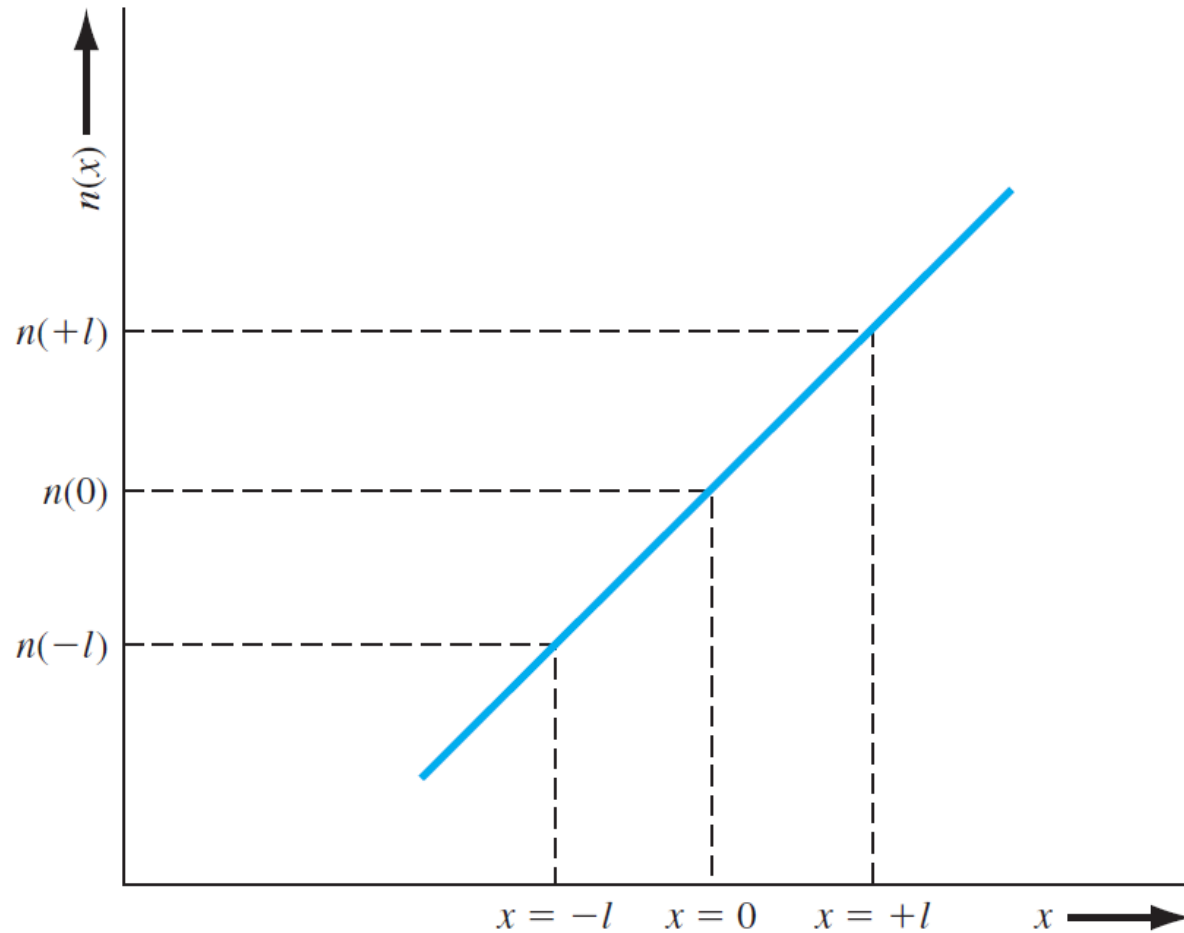
Diffusion Current Density

Figure 5.10 | Electron concentration versus distance.

Diffusion Current Density

- If the distance l is less than the mean-free path of an electron, that is, the average distance an electron travels between collisions $\quad l < v_{th}\,\tau_{cn}$

- The net rate of electron flow, F_n, in the +x direction at x=0 is given by

$$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}[n(-l) - n(+l)]$$

- expand the electron concentration in a Taylor series about $x = 0$ keeping only the first two terms

$$F_n = \frac{1}{2}v_{th}\left\{\left[n(0) - l\frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right\}$$

$$F_n = -v_{th}\,l\,\frac{dn}{dx}$$

$$f(c) + f'(c)(x-c) + \frac{f^{(2)}(c)}{2!}(x-c)^2 + \frac{f^{(3)}(c)}{3!}(x-c)^3 + \ldots$$

Diffusion Current Density

- The diffusion of electrons from a region of high concentration to a region of low concentration produces a fl ux of electrons flowing in the negative *x* direction

$$J = -eF_n = +ev_{th}l\frac{dn}{dx}$$

$$J_{nx|dif} = eD_n\frac{dn}{dx}$$

$$J_{px|dif} = -eD_P\frac{dp}{dx}$$

$$D_n = v_{th}\,l$$

$$D_p = v_{th}\,l.$$

- where D_n is called the *electron diffusion coefficient*
- *Where D_p is called the hole diffusion coefficient*

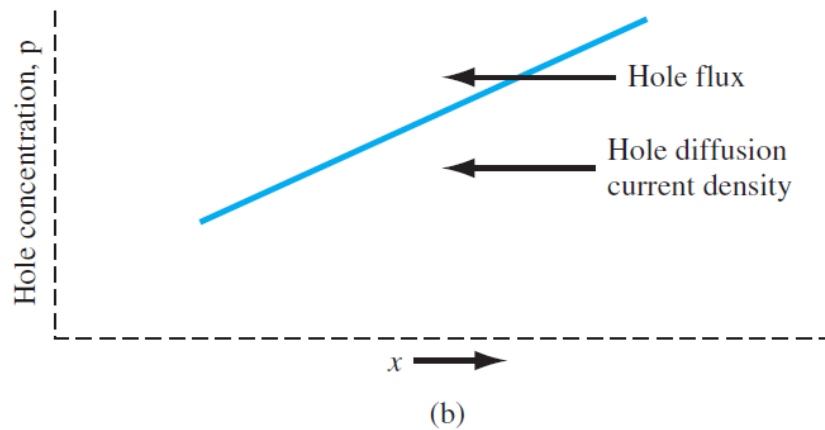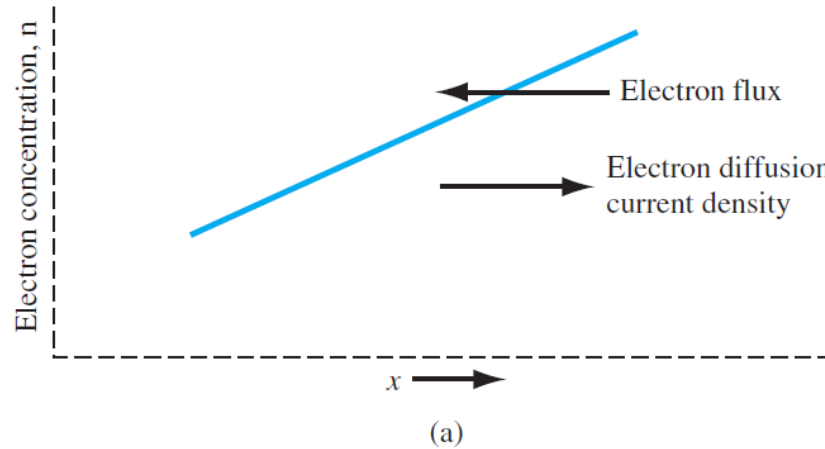$$J_{d6} = e\,D_n\frac{dn}{dx} - e\,D_p\frac{dp}{dx}$$

Diffusion Current Density

Figure 5.11 | (a) Diffusion of electrons due to a density gradient. (b) Diffusion of holes due to a density gradient.

Total Current Density

$$J = J_{df} + J_{ds}$$

$$J = en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

$$J = en\mu_n E + ep\mu_p E + eD_n \nabla n - eD_p \nabla p$$

- The electron mobility gives an indication of how well an electron moves in a semiconductor as a result of the force of an electric field.

- The electron diffusion coefficient gives an indication of how well an electron moves in a semiconductor as a result of a density gradient.

Induced Electric Field

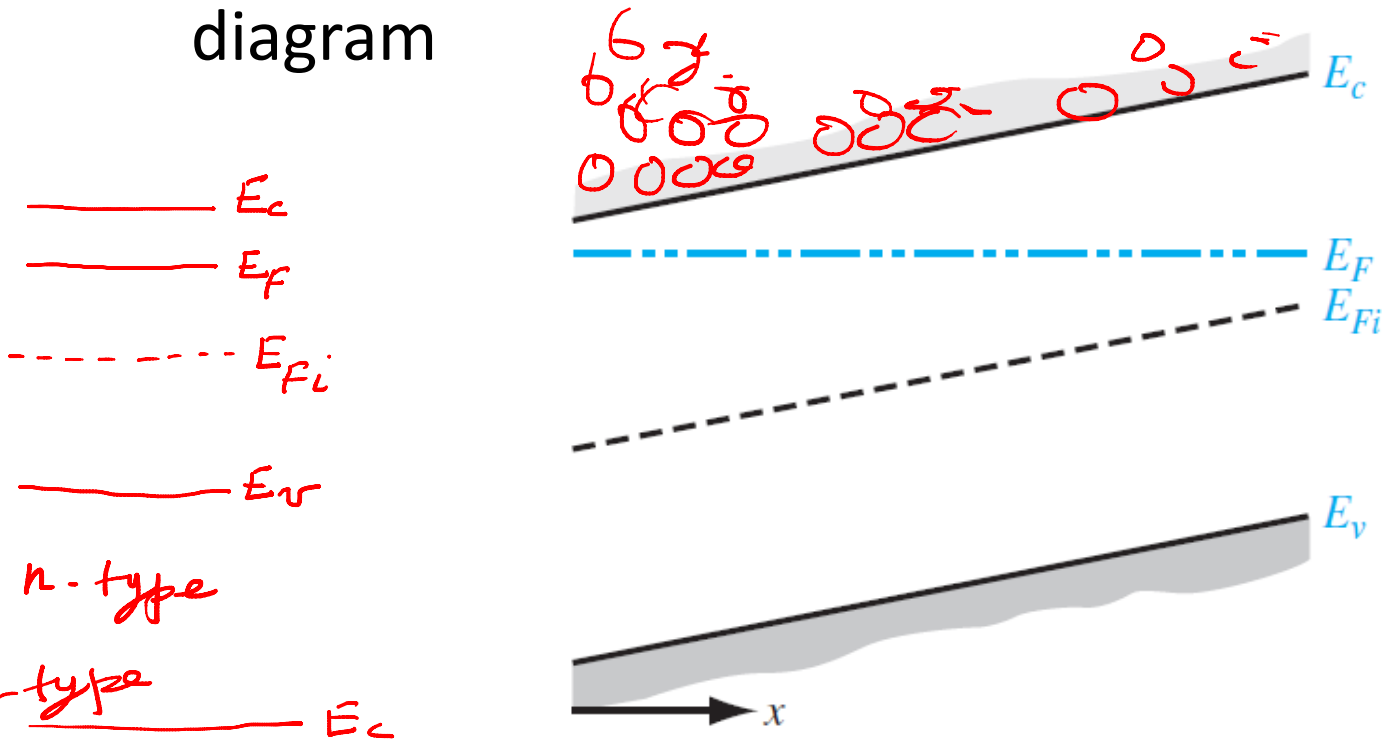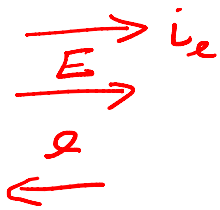- If we consider the nonuniformly doped semiconductor represented by the energy band diagram

Figure 5.12 | Energy-band diagram for a semiconductor in thermal equilibrium with a nonuniform donor impurity concentration.

Induced Electric Field

- The doping concentration decreases as *x* increases.

- There will be a diffusion of majority carrier electrons from the region of high concentration to the region of low concentration, which is in the *x* direction.

- The flow of negative electrons leaves behind positively charged donor ions.

- The separation of positive and negative charge induces an electric field that is in a direction to oppose the diffusion process.

Induced Electric Field

- Electric Potential

$$\Delta PE = q\Delta V$$

$$\Delta V = \frac{1}{q}\Delta PE$$

w.r.t (-e)

$$\phi = -\frac{1}{e}(E_{Fi} - E_F)$$

$$\phi = +\frac{1}{e}(E_F - E_{Fi})$$

$$\mathrm{E}_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

- If we assume a quasi-neutrality condition in which the electron concentration is almost equal to the donor impurity concentration

$$n_0 \simeq N_d(x)$$

$$n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] = N_d(x)$$

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d(x)}{n_i}\right)$$

Induced Electric Field

- E_F – Fermi level is constant for thermal equilibrium

$$-\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)} \frac{dN_d(x)}{dx}$$

$$E_x = -\left(\frac{kT}{e}\right) \frac{1}{N_d(x)} \frac{dN_d(x)}{dx}$$

The Einstein Relation

- Assume there are no electrical connections so that the semiconductor is in thermal equilibrium, then the individual electron and hole currents must be zero

$$J_n = 0 = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx}$$

$$n \approx N_d(x)$$

$$J_n = 0 = e\mu_n N_d(x)\mathrm{E}_x + eD_n \frac{dN_d(x)}{dx}$$

$$\mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx}$$

The Einstein Relation

$$\frac{D_n}{\mu_n} = \frac{kT}{e}$$

$$\frac{D_p}{\mu_p} = \frac{kT}{e}$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

IIIly $J_p = 0$

- As the mobilities are strong functions of temperature because of the scattering processes, the diffusion coefficients are also strong functions of temperature.

Non equilibrium Excess Carriers

- When a voltage is applied or a current exists in a semiconductor device, the semiconductor is operating under non equilibrium conditions.

- Excess electrons in the conduction band and excess holes in the valence band may exist in addition to the thermal-equilibrium concentrations if an external excitation is applied to the semiconductors.

- Excess electrons and excess holes do not move independently of each other. These excess carriers diffuse, drift, and recombine with the same effective diffusion coefficient, drift mobility, and lifetime. This phenomenon is called ambipolar transport.

CARRIER GENERATION AND RECOMBINATION

- generation is the process whereby electrons and holes are created.

- recombination is the process whereby electrons and holes are annihilated.

- A sudden increase in temperature, for example, will increase the rate at which electrons and holes are thermally generated so that their concentrations will change with time until new equilibrium values are reached.

- An external excitation, such as light (a flux of photons), can also generate electrons and holes, creating a nonequilibrium condition.

The Semiconductor in Equilibrium

- In thermal equilibrium, electrons and holes concentrations are independent of time.

- the net carrier concentrations are independent of time in thermal equilibrium, the rate at which electrons and holes are generated and the rate at which they recombine must be equal.
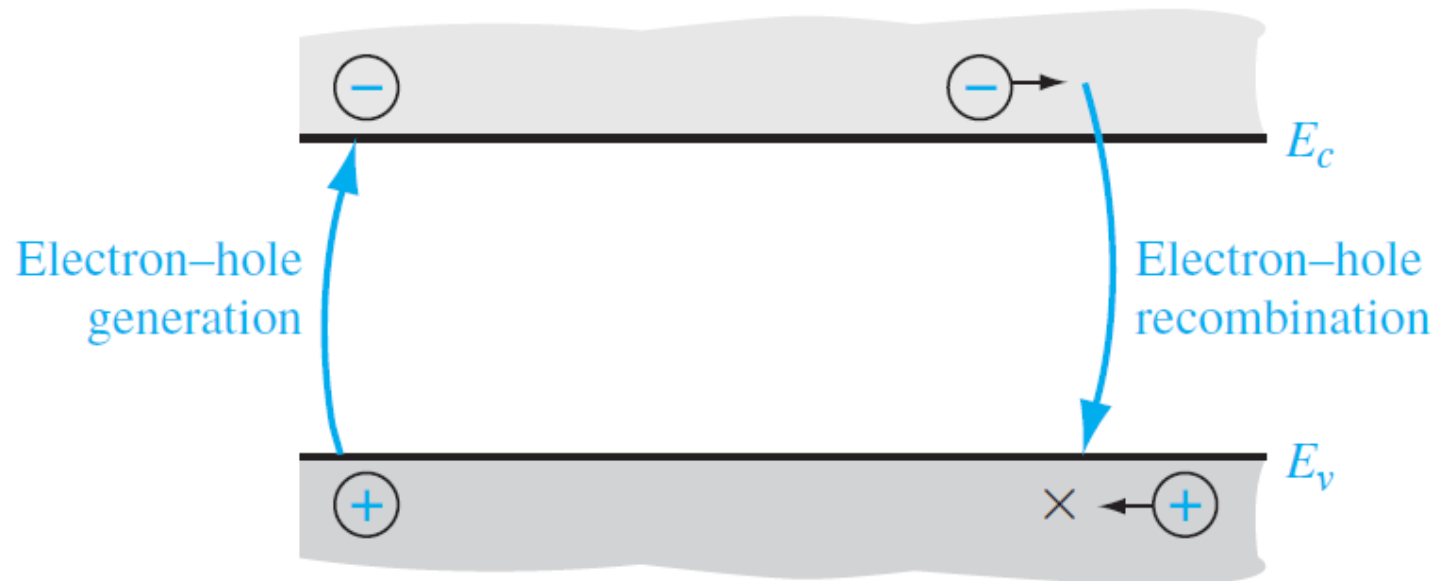
The Semiconductor in Equilibrium

- Let G_{n0} and G_{p0} be the thermal-generation rates of electrons and holes, respectively.

- For the direct band-to-band generation, the electrons and holes are created in pairs.

$$G_{n0} = G_{p0}$$

- Let R_{n0} and R_{p0} be the recombination rates of electrons and holes, respectively.

- In direct band-to-band recombination, electrons and holes recombine in pairs

$$R_{n0} = R_{p0}$$

- In thermal equilibrium, the concentrations of electrons and holes are independent of time; therefore, the generation and recombination rates are equal

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$
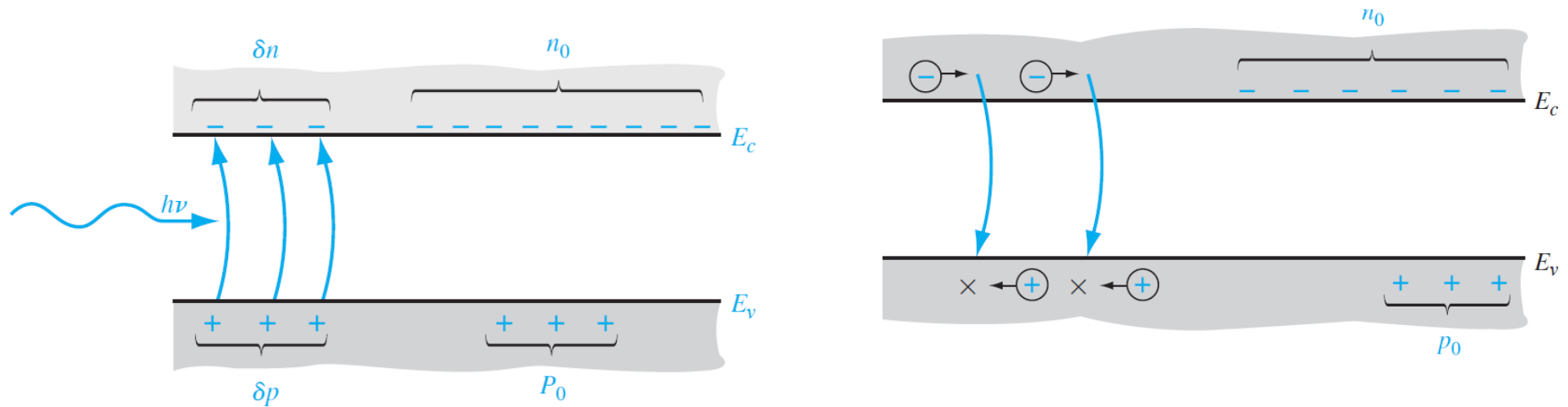
Excess Carrier Generation and Recombination

- The excess electrons and holes are generated by an external force at a particular rate.

- Let g'_n be the generation rate of excess electrons and g'_p be that of excess holes.

$$g'_n = g'_p$$

- When excess electrons and holes are created, the concentration of electrons in the conduction band and of holes in the valence band increase above their thermal equilibrium value.

$$n = n_0 + \delta n$$

$$p = p_0 + \delta p$$

- where n_0 and p_0 are the thermal-equilibrium concentrations, and δn and δp are the excess electron and hole concentrations.

Excess Carrier Generation and Recombination

$$np \neq n_0 p_0 = n_i^2$$

Non equilibrium condition

Excess Carrier Generation and Recombination

- The recombination rate for excess electrons is denoted by R'_n and for excess holes by R'_p.

$$R'_n = R'_p$$

- In the direct band-to-band recombination that we are considering, the recombination occurs spontaneously.

- The probability of an electron and hole recombining is constant with time.

- The rate at which electrons recombine must be proportional to the electron concentration and must also be proportional to the hole concentration.

Excess Carrier Generation and Recombination

- The net rate of change in the electron concentration can be written as

Generation

Recombination

$$\frac{dn(t)}{dt} = \alpha_r \left[n_i^2 - n(t)p(t) \right]$$

$$n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

$$\frac{d(\delta n(t))}{dt} = \alpha_r \left[n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t)) \right]$$

$$\delta n(t) = \delta p(t)$$

$$= -\alpha_r \delta n(t)[(n_0 + p_0) + \delta n(t)]$$

Excess Carrier Generation and Recombination

- Equation can easily be solved if we impose the condition of low-level injection.

- Low-level injection puts limits on the magnitude of the excess carrier concentration compared with the thermal-equilibrium carrier concentrations.

- Low-level injection means that the excess carrier concentration is much less than the thermal-equilibrium majority carrier concentration.

- high-level injection occurs when the excess carrier concentration becomes comparable to or greater than the thermal-equilibrium majority carrier concentrations.

Excess Carrier Generation and Recombination

- If we consider a p-type material ($p_0 >> n_0$) under low-level injection ($\delta n(t) << p_0$)

$$\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t)$$

- The solution to the equation is an exponential decay from the initial excess concentration

$$\delta n(t) = \delta n(0) e^{-\alpha_r p_0 t} = \delta n(0) e^{-t/\tau_{n0}}$$

$$\tau_{n0} = (\alpha_r p_0)^{-1}$$

- τ_{n0} is often referred to as the *excess minority carrier lifetime*

Excess Carrier Generation and Recombination

- Recombination rate

$$R'_n = \frac{-d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}}$$

- p-type material

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

- n-type material

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{p0}}$$

CHARACTERISTICS OF EXCESS CARRIERS

- How the excess carriers behave with time and in space in the presence of electric fields and density gradients is of equal importance.

- What is the effective diffusion coefficient and what is the effective mobility that characterizes the behavior of these excess carriers?

- we must develop the continuity equations for the carriers and then develop the ambipolar transport equations.
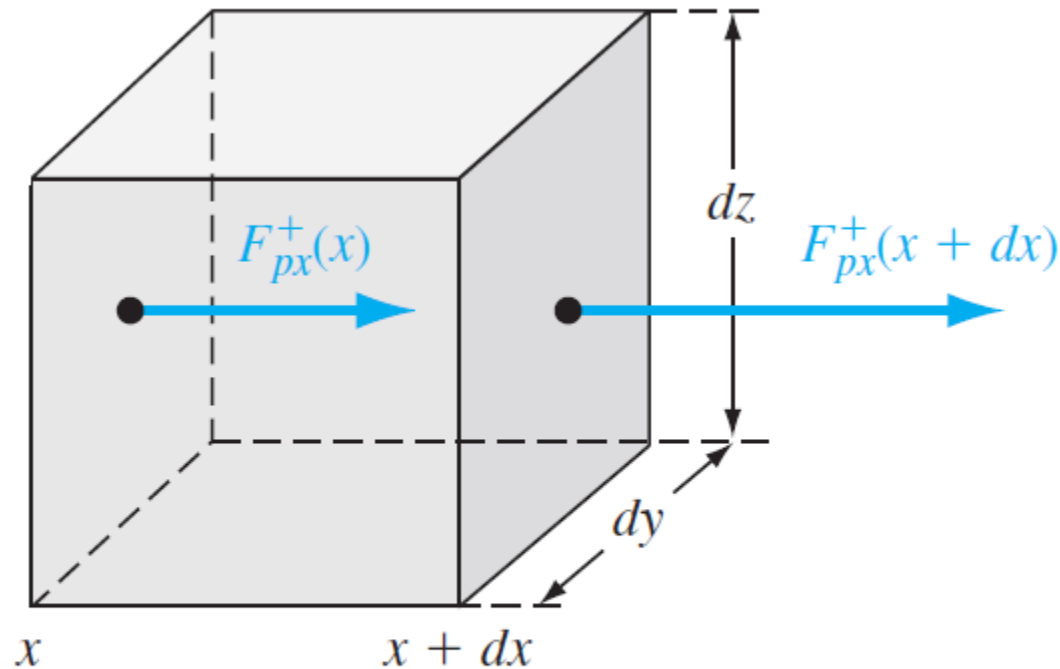
Continuity Equations

Figure 6.4 | Differential volume showing *x* component of the hole-particle flux.

Continuity Equations

- a differential volume element in which a one-dimensional hole particle flux is entering the differential element at *x* and is leaving the element at *x + dx*. The parameter F^+_{px} is the hole-particle flux, or flow

$$F^+_{px}(x + dx) = F^+_{px}(x) + \frac{\partial F^+_{px}}{\partial x} \cdot dx$$

Taylor Series➔ $f(x + h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) \dots$

- The net increase in the number of holes per unit time within the differential volume element due to the *x*-component of hole flux is given by

$$\frac{\partial p}{\partial t} dx\, dy\, dz = [F^+_{px}(x) - F^+_{px}(x + dx)]\, dy\, dz = -\frac{\partial F^+_{px}}{\partial x} dx\, dy\, dz$$

$$= \left[F^+_{px}(x) - F^+_{px}(x) - \frac{\partial F^+_{px}}{\partial x} dx \right] dy\, dz \quad \nearrow$$

Continuity Equations

- The net increase in the number of holes per unit time in the differential volume element is then given by

$\rightarrow$ Hole flux. $\rightarrow$ Generation $\rightarrow$ Recombination

$$\frac{\partial p}{\partial t}\, dx\, dy\, dz = -\frac{\partial F_p^+}{\partial x}\, dx\, dy\, dz + g_p\, dx\, dy\, dz - \frac{p}{\tau_{pt}}\, dx\, dy\, dz$$

- where p is the density of holes.

- The first term on the right side of Equation is the increase in the number of holes per unit time due to the hole flux.

- The second term is the increase in the number of holes per unit time due to the generation of holes.

- The last term is the decrease in the number of holes per unit time due to the recombination of holes.

Continuity Equations

- The continuity equation for holes.

$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}}$$

- The one-dimensional continuity equation for electrons

$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

- where F_n^- is the electron-particle flow, or flux

Time-Dependent Diffusion Equations

$$J_p = e\mu_p p\mathrm{E} - eD_p\frac{\partial p}{\partial x}$$

Hole and electron current densities

$$J_n = e\mu_n n\mathrm{E} + eD_n\frac{\partial n}{\partial x}$$

$$\frac{J_p}{(+e)} = F_p^+ = \mu_p p\mathrm{E} - D_p\frac{\partial p}{\partial x}$$

Flux

$$\frac{J_n}{(-e)} = F_n^- = -\mu_n n\mathrm{E} - D_n\frac{\partial n}{\partial x}$$

Taking the divergence of Flux Equations and, and substituting back into the continuity equations

$$\frac{\partial p}{\partial t} = -\mu_p\frac{\partial(p\mathrm{E})}{\partial x} + D_p\frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial n}{\partial t} = +\mu_n\frac{\partial(n\mathrm{E})}{\partial x} + D_n\frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

Time-Dependent Diffusion Equations

- expand the derivative of the product

$$\frac{\partial(p\mathrm{E})}{\partial x} = \mathrm{E}\frac{\partial p}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}$$

$$D_p\frac{\partial^2 p}{\partial x^2} - \mu_p\left(\mathrm{E}\frac{\partial p}{\partial x} + p\frac{\partial \mathrm{E}}{\partial x}\right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial p}{\partial t}$$

$$D_n\frac{\partial^2 n}{\partial x^2} + \mu_n\left(\mathrm{E}\frac{\partial n}{\partial x} + n\frac{\partial \mathrm{E}}{\partial x}\right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial n}{\partial t}$$

the time-dependent diffusion equations for holes and electrons

Time-Dependent Diffusion Equations

- The hole and electron concentrations are functions of both the thermal equilibrium and the excess values. The thermal equilibrium concentrations, n_0 and p_0, are not functions of time

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \left(\mathrm{E} \frac{\partial(\delta p)}{\partial x} + p \frac{\partial \mathrm{E}}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(\mathrm{E} \frac{\partial(\delta n)}{\partial x} + n \frac{\partial \mathrm{E}}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

For the special case of a homogeneous semiconductor, n_0 and p_0 are also independent of the space coordinates

AMBIPOLAR TRANSPORT

- The electric field term in Equations is then composed of the externally applied field plus the induced internal field.

$$E = E_{app} + E_{int}$$

- the internal E-field creates a force attracting the electrons and holes, this E-field will hold the pulses of excess electrons and excess holes together.

- The negatively charged electrons and positively charged holes then will drift or diffuse together with a single effective mobility or diffusion coefficient. This phenomenon is called *ambipolar diffusion* or *ambipolar transport.*

AMBIPOLAR TRANSPORT

- the ambipolar transport equation for excess minority carrier holes in an n region

$$D_p \frac{\partial^2 (\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial (\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial (\delta p_n)}{\partial t}$$

- The ambipolar transport equation for a p-type semiconductor

$$D_n \frac{\partial^2 (\delta n)}{\partial x^2} + \mu_n E \frac{\partial (\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial (\delta n)}{\partial t}$$

g' generation rate for excess carriers

$$g'_n = g'_p \equiv g'$$

EXCESS CARRIER LIFETIME

- The rate at which excess electrons and holes recombine is an important characteristic of the semiconductor and influences many of the device characteristics

- the recombination rate is inversely proportional to the mean carrier lifetime.

- In a real semiconductor material, defects occur within the crystal and disrupt the perfect periodic-potential function.

- If the density of these defects is not too great, the defects will create discrete electronic energy states within the forbidden-energy band.

- These allowed energy states may be the dominant effect in determining the mean carrier lifetime.

- The mean carrier lifetime may be determined from the Shockley–Read–Hall theory of recombination
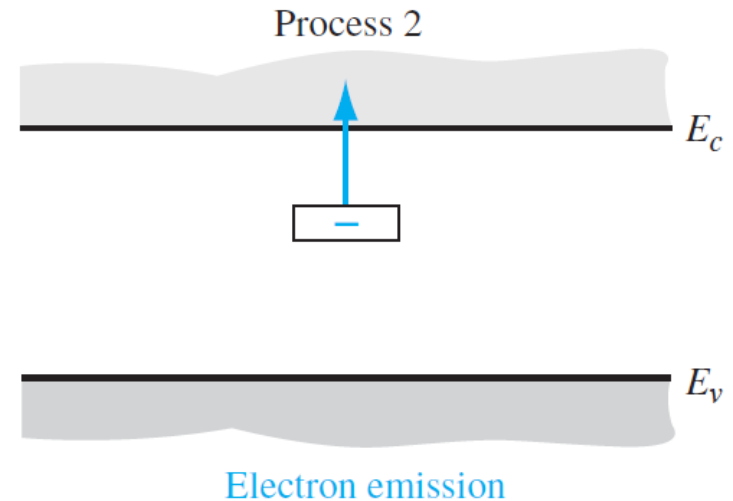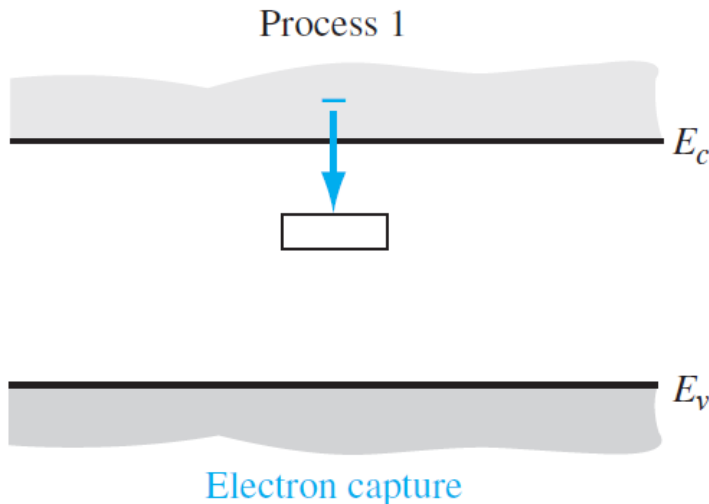
Shockley–Read–Hall Theory of Recombination

- An allowed energy state, also called a trap, within the forbidden bandgap may act as a recombination center, capturing both electrons and holes with almost equal probability.

- The Shockley–Read–Hall theory of recombination assumes that a single recombination center, or trap, exists at an energy E_t within the bandgap.

- There are four basic processes that may occur at this single trap.

- We will assume that the trap is an acceptor-type trap; that is, it is negatively charged when it contains an electron and is neutral when it does not contain an electron.
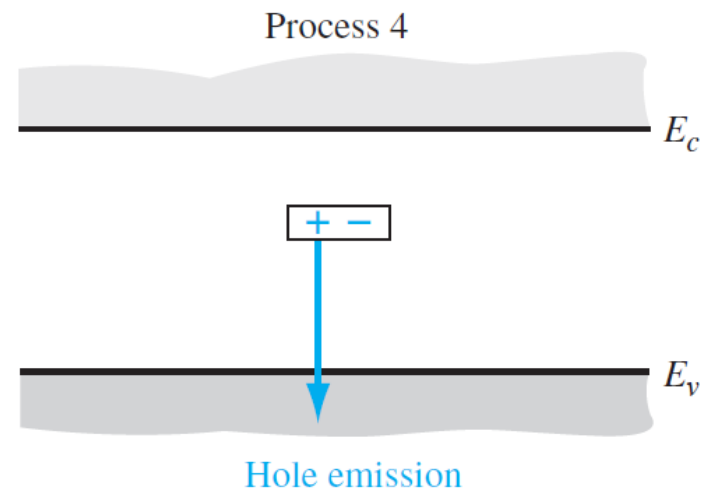
Shockley–Read–Hall Theory of Recombination

The four basic processes are as follows:

- Process 1: The capture of an electron from the conduction band by an initially neutral empty trap.

- Process 2: The inverse of process 1—the emission of an electron that is initially occupying a trap level back into the conduction band.

Process 1 — E_c — E_v — Electron capture

Process 2 — E_c — E_v — Electron emission

Shockley–Read–Hall Theory of Recombination

- Process 3: The capture of a hole from the valence band by a trap containing an electron. (Or we may consider the process to be the emission of an electron from the trap into the valence band.)

- Process 4: The inverse of process 3—the emission of a hole from a neutral trap into the valence band. (Or we may consider this process to be the capture of an electron from the valence band.)

Process 3

E_c

$-$

$+$

E_v

Hole capture

Process 4

E_c

$+ -$

E_v

Hole emission
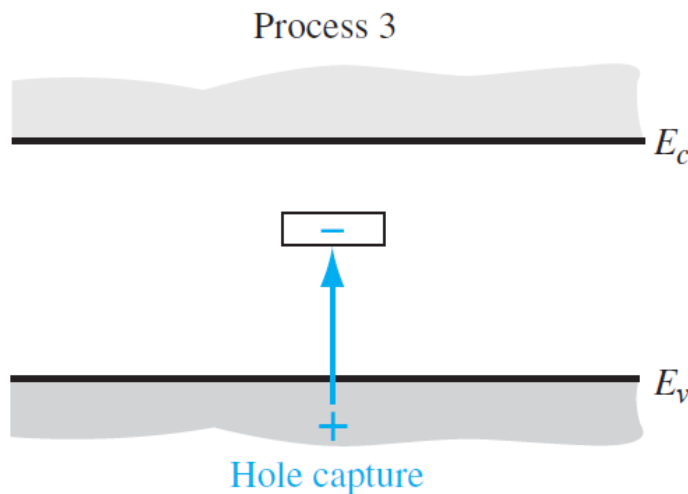
Shockley–Read–Hall Theory of Recombination

- In process 1, the rate at which electrons from the conduction band are captured by the traps is proportional to the density of electrons in the conduction band and proportional to the density of empty trap states.

$$R_{cn} = C_n N_t [1 - f_F(E_t)]n$$

R_{cn} = capture rate (#/cm³-s)

C_n = constant proportional to electron-capture cross section

N_t = total concentration of trapping centers

n = electron concentration in the conduction band

$f_F(E_t)$ = Fermi function at the trap energy

Shockley–Read–Hall Theory of Recombination

- The Fermi function at the trap energy is given by

$$f_F(E_t) = \frac{1}{1 + \exp\left(\dfrac{E_t - E_F}{kT}\right)}$$

- $f_F(E_t)$ - probability that a trap will contain an electron.
- The function $[1 - f_F(E_t)]$ is then the probability that the trap is empty.

Shockley–Read–Hall Theory of Recombination

- In process 2, the rate at which electrons are emitted from filled traps back into the conduction band is proportional to the number of filled traps.

$$R_{en} = E_n N_t f_F(E_t)$$

where

R_{en} = emission rate (#/cm^3-s)

E_n = constant

$f_F(E_t)$ = probability that the trap is occupied

Shockley–Read–Hall Theory of Recombination

- In thermal equilibrium, the rate of electron capture from the conduction band and the rate of electron emission back into the conduction band must be equal.

$$R_{en} = R_{cn} \qquad E_n N_t f_{F0}(E_t) = C_n N_t [1 - f_{F0}(E_t)] n_0$$

- where f_{F0} denotes the thermal-equilibrium Fermi function.

- In thermal equilibrium, the value of the electron concentration in the capture rate term is the equilibrium value n_0.

- Using the Boltzmann approximation for the Fermi function, we can find E_n in terms of C_n as

$$E_n = n' C_n$$

Shockley–Read–Hall Theory of Recombination

$$n' = N_c \exp\left[\frac{-(E_c - E_t)}{kT}\right]$$

- The parameter n' is equivalent to an electron concentration that would exist in the conduction band if the trap energy E_t coincided with the Fermi energy E_F.

- In nonequilibrium, excess electrons exist, so that the net rate at which electrons are captured from the conduction band is given by

$$R_n = R_{cn} - R_{en}$$

$$R_n = [C_n N_t (1 - f_F(E_t))n] - [E_n N_t f_F(E_t)]$$

- n is the total concentration, which includes the excess electron concentration.

$$E_n = n' C_n$$

$$R_n = C_n N_t [n(1 - f_F(E_t)) - n' f_F(E_t)]$$

Shockley–Read–Hall Theory of Recombination

- If we consider processes 3 and 4 in the recombination theory, the net rate at which holes are captured from the valence band is given by

$$R_p = C_p N_t [p f_F(E_t) - p'(1 - f_F(E_t))]$$

- where C_p is a constant proportional to the hole capture rate, and p' is given by

$$p' = N_v \exp\left[\frac{-(E_t - E_v)}{kT}\right]$$

- In a semiconductor in which the trap density is not too large, the excess electron and hole concentrations are equal and the recombination rates of electrons and holes are equal

Equate and Solve for $f_F(E_t)$

$$R_n = R_p \longrightarrow f_F(E_t) = \frac{C_n n + C_p p'}{C_n(n + n') + C_p(p + p')} \qquad n' p' = n_i^2.$$

Sub $f_F(E_t)$ back in $R_n = R_p = R$

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \equiv R$$

Shockley–Read–Hall Theory of Recombination

- Equation R is the recombination rate of electrons and holes due to the recombination center at $E = E_t$.

- If we consider thermal equilibrium, then $np = n_0 p_0 = n_i^2$, so that $R_n = R_p = 0$.

- R is the recombination rate of excess electrons and holes.

$$R = \frac{\delta n}{\tau}$$

- where δn is the excess carrier concentration and τ is the lifetime of the excess carriers

Limits of Extrinsic Doping and Low Injection

- Consider an n-type semiconductor under low injection

$$n_0 \gg p_0, \quad n_0 \gg \delta p, \quad n_0 \gg n', \quad n_0 \gg p'$$

- where δp is the excess minority carrier hole concentration. The assumptions of $n_0 \gg n'$ and $n_0 \gg p'$ imply that the trap level energy is near midgap so that n and p are not too different from the intrinsic carrier concentration.

$$R = C_p N_t \delta p \quad \longleftarrow \quad \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \equiv R$$

$$R = \frac{\delta n}{\tau} = C_p N_t \delta p \equiv \frac{\delta p}{\tau_{p0}}$$

$$\tau_{p0} = \frac{1}{C_p N_t}$$

- τ_{p0} is defined as the excess minority carrier hole lifetime.
- If the trap concentration increases, the probability of excess carrier recombination increases; thus, the excess minority carrier lifetime decreases.

Limits of Extrinsic Doping and Low Injection

- Similarly, if we have a strongly extrinsic p-type material under low iniection

$$p_0 \gg n_0, \quad p_0 \gg \delta n, \quad p_0 \gg n', \quad p_0 \gg p'$$

$$\tau_{n0} = \frac{1}{C_n N_t}$$

- for the n-type material, the lifetime is a function of C_p, which is related to the capture rate of the minority carrier hole.

- for the p-type material, the lifetime is a function of C_n, which is related to the capture rate of the minority carrier electron.

- The excess carrier lifetime for an extrinsic material under low injection reduces to that of the minority carrier.