Chapter 1

Problem Solutions

1.1

(a) fcc: 8 corner atoms $\times 1/8 = 1$ atom
 6 face atoms $\times 1/2 = 3$ atoms
 Total of 4 atoms per unit cell

(b) bcc: 8 corner atoms $\times 1/8 = 1$ atom
 1 enclosed atom $= 1$ atom
 Total of 2 atoms per unit cell

(c) Diamond: 8 corner atoms $\times 1/8 = 1$ atom
 6 face atoms $\times 1/2 = 3$ atoms
 4 enclosed atoms $= 4$ atoms
 Total of 8 atoms per unit cell

1.2

(a) Simple cubic lattice: $a = 2r$

Unit cell vol $= a^3 = (2r)^3 = 8r^3$

1 atom per cell, so atom vol $= (1)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{\left(\dfrac{4\pi r^3}{3}\right)}{8r^3} \times 100\% = 52.4\%$$

(b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow a = \frac{d}{\sqrt{2}} = 2\sqrt{2} \cdot r$$

Unit cell vol $= a^3 = \left(2\sqrt{2} \cdot r\right)^3 = 16\sqrt{2} \cdot r^3$

4 atoms per cell, so atom vol $= (4)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(4)\left(\dfrac{4\pi r^3}{3}\right)}{16\sqrt{2} \cdot r^3} \times 100\% = 74\%$$

(c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}} \cdot r$$

Unit cell vol $= a^3 = \left(\dfrac{4}{\sqrt{3}} \cdot r\right)^3$

2 atoms per cell, so atom vol $= (2)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(2)\left(\dfrac{4\pi r^3}{3}\right)}{\left(\dfrac{4r}{\sqrt{3}}\right)^3} \times 100\% = 68\%$$

(d) Diamond lattice

Body diagonal $= d = 8r = a\sqrt{3} \Rightarrow a = \dfrac{8}{\sqrt{3}} \cdot r$

Unit cell vol $= a^3 = \left(\dfrac{8r}{\sqrt{3}}\right)^3$

8 atoms per cell, so atom vol $= (8)\left(\dfrac{4\pi r^3}{3}\right)$

Then

$$\text{Ratio} = \frac{(8)\left(\dfrac{4\pi r^3}{3}\right)}{\left(\dfrac{8r}{\sqrt{3}}\right)^3} \times 100\% = 34\%$$

1.3

(a) $a = 5.43\ \mathring{A}$; From Problem 1.2d,

$$a = \frac{8}{\sqrt{3}} \cdot r$$

Then $r = \dfrac{a\sqrt{3}}{8} = \dfrac{(5.43)\sqrt{3}}{8} = 1.176\ \mathring{A}$

Center of one silicon atom to center of nearest neighbor $= 2r = 2.35\ \mathring{A}$

(b) Number density

$$= \frac{8}{\left(5.43 \times 10^{-8}\right)^3} = 5 \times 10^{22}\ \text{cm}^{-3}$$

(c) Mass density

$$= \rho = \frac{N(At.Wt.)}{N_A} = \frac{\left(5 \times 10^{22}\right)(28.09)}{6.02 \times 10^{23}}$$

$\Rightarrow \rho = 2.33$ grams/cm^3

1.4

(a) 4 Ga atoms per unit cell

$$\text{Number density } = \frac{4}{\left(5.65\times10^{-8}\right)^3}$$

$\Rightarrow$ Density of Ga atoms $= 2.22\times10^{22}\ \text{cm}^{-3}$

4 As atoms per unit cell

$\Rightarrow$ Density of As atoms $= 2.22\times10^{22}\ \text{cm}^{-3}$

(b) 8 Ge atoms per unit cell

$$\text{Number density } = \frac{8}{\left(5.65\times10^{-8}\right)^3}$$

$\Rightarrow$ Density of Ge atoms $= 4.44\times10^{22}\ \text{cm}^{-3}$

1.5

From Figure 1.15

(a) $d = \left(\frac{a}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = (0.4330)a$

$= (0.4330)(5.65) \Rightarrow d = 2.447\ \mathring{A}$

(b) $d = \left(\frac{a}{2}\right)\sqrt{2} = (0.7071)a$

$= (0.7071)(5.65) \Rightarrow d = 3.995\ \mathring{A}$

1.6

$$\sin\left(\frac{\theta}{2}\right) = \frac{\frac{a}{2}\sqrt{2}}{\frac{a}{2}\sqrt{3}} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\theta}{2} = 54.74^\circ$$

$\Rightarrow \theta = 109.5^\circ$

1.7

(a) Simple cubic: $a = 2r = 3.9\ \mathring{A}$

(b) fcc: $a = \frac{4r}{\sqrt{2}} = 5.515\ \mathring{A}$

(c) bcc: $a = \frac{4r}{\sqrt{3}} = 4.503\ \mathring{A}$

(d) diamond: $a = \frac{2(4r)}{\sqrt{3}} = 9.007\ \mathring{A}$

1.8

(a) $2(1.035)\sqrt{2} = 2(1.035) + 2r_B$

$r_B = 0.4287\ \mathring{A}$

(b) $a = 2(1.035) = 2.07\ \mathring{A}$

(c) A-atoms: # of atoms $= 8\times\frac{1}{8} = 1$

$$\text{Density } = \frac{1}{\left(2.07\times10^{-8}\right)^3}$$

$= 1.13\times10^{23}\ \text{cm}^{-3}$

B-atoms: # of atoms $= 6\times\frac{1}{2} = 3$

$$\text{Density } = \frac{3}{\left(2.07\times10^{-8}\right)^3}$$

$= 3.38\times10^{23}\ \text{cm}^{-3}$

1.9

(a) $a = 2r = 4.5\ \mathring{A}$

# of atoms $= 8\times\frac{1}{8} = 1$

$$\text{Number density } = \frac{1}{\left(4.5\times10^{-8}\right)^3}$$

$= 1.097\times10^{22}\ \text{cm}^{-3}$

$$\text{Mass density } = \rho = \frac{N(At.Wt.)}{N_A}$$

$$= \frac{\left(1.0974\times10^{22}\right)(12.5)}{6.02\times10^{23}}$$

$= 0.228\ \text{gm/cm}^3$

(b) $a = \frac{4r}{\sqrt{3}} = 5.196\ \mathring{A}$

# of atoms $8\times\frac{1}{8} + 1 = 2$

$$\text{Number density } = \frac{2}{\left(5.196\times10^{-8}\right)^3}$$

$= 1.4257\times10^{22}\ \text{cm}^{-3}$

$$\text{Mass density } = \rho = \frac{\left(1.4257\times10^{22}\right)(12.5)}{6.02\times10^{23}}$$

$= 0.296\ \text{gm/cm}^3$

1.10

From Problem 1.2, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

Volume = 0.74 cm^3

1.11

(b) $a = 1.8 + 1.0 = 2.8\ \overset{o}{A}$

(c) Na: Density $= \dfrac{(1/2)}{(2.8\times10^{-8})^3}$

$= 2.28\times10^{22}\ \text{cm}^{-3}$

Cl: Density $= 2.28\times10^{22}\ \text{cm}^{-3}$

(d) Na: At. Wt. = 22.99
Cl: At. Wt. = 35.45

So, mass per unit cell

$$= \frac{\left(\frac{1}{2}\right)(22.99)+\left(\frac{1}{2}\right)(35.45)}{6.02\times10^{23}} = 4.85\times10^{-23}$$

Then mass density

$$\rho = \frac{4.85\times10^{-23}}{(2.8\times10^{-8})^3} = 2.21\ \text{grams/cm}^3$$

1.12

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8\ \overset{o}{A}$

Then $a = 4.62\ \overset{o}{A}$

Density of A:

$$= \frac{1}{(4.62\times10^{-8})^3} = 1.01\times10^{22}\ \text{cm}^{-3}$$

Density of B:

$$= \frac{1}{(4.62\times10^{-8})^3} = 1.01\times10^{22}\ \text{cm}^{-3}$$

(b) Same as (a)
(c) Same material

1.13

$$a = \frac{2(2.2)+2(1.8)}{\sqrt{3}} = 4.619\ \overset{o}{A}$$

(a) For 1.12(a), A-atoms

Surface density $= \dfrac{1}{a^2} = \dfrac{1}{(4.619\times10^{-8})^2}$

$= 4.687\times10^{14}\ \text{cm}^{-2}$

For 1.12(b), B-atoms: $a = 4.619\ \overset{o}{A}$

Surface density $= \dfrac{1}{a^2} = 4.687\times10^{14}\ \text{cm}^{-2}$

For 1.12(a) and (b), Same material

(b) For 1.12(a), A-atoms; $a = 4.619\ \overset{o}{A}$

Surface density

$$= \frac{1}{a^2\sqrt{2}} = 3.315\times10^{14}\ \text{cm}^{-2}$$

B-atoms;
Surface density

$$= \frac{1}{a^2\sqrt{2}} = 3.315\times10^{14}\ \text{cm}^{-2}$$

For 1.12(b), A-atoms; $a = 4.619\ \overset{o}{A}$

Surface density

$$= \frac{1}{a^2\sqrt{2}} = 3.315\times10^{14}\ \text{cm}^{-2}$$

B-atoms;
Surface density

$$= \frac{1}{a^2\sqrt{2}} = 3.315\times10^{14}\ \text{cm}^{-2}$$

For 1.12(a) and (b), Same material

1.14

(a) Vol. Density $= \dfrac{1}{a_o^3}$

Surface Density $= \dfrac{1}{a_o^2\sqrt{2}}$

(b) Same as (a)

1.15

(i) (110) plane
(see Figure 1.10(b))

(ii) (111) plane
(see Figure 1.10(c))

(iii) (220) plane $\Rightarrow \left(\frac{1}{2}, \frac{1}{2}, \infty\right) \Rightarrow (1,1,0)$

Same as (110) plane and [110] direction

(iv) (321) plane $\Rightarrow \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{1}\right) \Rightarrow (2,3,6)$

Intercepts of plane at
$p = 2, q = 3, s = 6$

[321] direction is perpendicular to
(321) plane

1.16

(a)

$$\left(\frac{1}{1},\frac{1}{3},\frac{1}{1}\right)\Rightarrow(313)$$

(b)

$$\left(\frac{1}{4},\frac{1}{2},\frac{1}{4}\right)\Rightarrow(121)$$

1.17

Intercepts: 2, 4, 3 $\Rightarrow\left(\frac{1}{2},\frac{1}{4},\frac{1}{3}\right)\Rightarrow$

(634) plane

1.18

(a) $d = a = 5.28\ \overset{o}{A}$

(b) $d = \dfrac{a\sqrt{2}}{2} = 3.734\ \overset{o}{A}$

(c) $d = \dfrac{a\sqrt{3}}{3} = 3.048\ \overset{o}{A}$

1.19

(a) Simple cubic

(i) (100) plane:

$$\text{Surface density} = \frac{1}{a^2} = \frac{1}{\left(4.73\times10^{-8}\right)^2}$$

$$= 4.47\times10^{14}\ \text{cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{1}{a^2\sqrt{2}}$$

$$= 3.16\times10^{14}\ \text{cm}^{-2}$$

(iii) (111) plane:

$$\text{Area of plane} = \frac{1}{2}bh$$

where $b = a\sqrt{2} = 6.689\ \overset{o}{A}$

Now

$$h^2 = \left(a\sqrt{2}\right)^2 - \left(\frac{a\sqrt{2}}{2}\right)^2 = \frac{3}{4}\left(a\sqrt{2}\right)^2$$

So $h = \dfrac{\sqrt{6}}{2}(4.73) = 5.793\ \overset{o}{A}$

Area of plane

$$= \frac{1}{2}\left(6.68923\times10^{-8}\right)\left(5.79304\times10^{-8}\right)$$

$$= 19.3755\times10^{-16}\ \text{cm}^2$$

$$\text{Surface density} = \frac{3\times\frac{1}{6}}{19.3755\times10^{-16}}$$

$$= 2.58\times10^{14}\ \text{cm}^{-2}$$

(b) bcc

(i) (100) plane:

$$\text{Surface density} = \frac{1}{a^2} = 4.47\times10^{14}\ \text{cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2\sqrt{2}}$$

$$= 6.32\times10^{14}\ \text{cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density} = \frac{3\times\frac{1}{6}}{19.3755\times10^{-16}}$$

$$= 2.58\times10^{14}\ \text{cm}^{-2}$$

(c) fcc

(i) (100) plane:

$$\text{Surface density} = \frac{2}{a^2} = 8.94\times10^{14}\ \text{cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2\sqrt{2}}$$

$$= 6.32\times10^{14}\ \text{cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density} = \frac{3\times\frac{1}{6}+3\times\frac{1}{2}}{19.3755\times10^{-16}}$$

$$= 1.03\times10^{15}\ \text{cm}^{-2}$$

1.20

(a) (100) plane: - similar to a fcc:

$$\text{Surface density} = \frac{2}{\left(5.43\times10^{-8}\right)^2}$$

$$= 6.78\times10^{14}\ \text{cm}^{-2}$$

(b) (110) plane:

$$\text{Surface density} = \frac{4}{\sqrt{2}\left(5.43\times10^{-8}\right)^2}$$

$$= 9.59\times10^{14}\ \text{cm}^{-2}$$

(c) (111) plane:

$$\text{Surface density} = \frac{2}{\left(\sqrt{3}/2\right)\left(5.43\times10^{-8}\right)^2}$$
$$= 7.83\times10^{14}\ \text{cm}^{-2}$$

1.21

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.37)}{\sqrt{2}} = 6.703\ \overset{o}{A}$$

(a) $$\#/\text{cm}^3 = \frac{8\times\frac{1}{8}+6\times\frac{1}{2}}{a^3} = \frac{4}{\left(6.703\times10^{-8}\right)^3}$$
$$= 1.328\times10^{22}\ \text{cm}^{-3}$$

(b) $$\#/\text{cm}^2 = \frac{4\times\frac{1}{4}+2\times\frac{1}{2}}{a^2\sqrt{2}}$$
$$= \frac{2}{\left(6.703\times10^{-8}\right)^2\sqrt{2}}$$
$$= 3.148\times10^{14}\ \text{cm}^{-2}$$

(c) $$d = \frac{a\sqrt{2}}{2} = \frac{(6.703)\sqrt{2}}{2} = 4.74\ \overset{o}{A}$$

(d) $$\#\text{ of atoms} = 3\times\frac{1}{6}+3\times\frac{1}{2} = 2$$

Area of plane: (see Problem 1.19)

$$b = a\sqrt{2} = 9.4786\ \overset{o}{A}$$
$$h = \frac{\sqrt{6}a}{2} = 8.2099\ \overset{o}{A}$$

Area

$$= \frac{1}{2}bh = \frac{1}{2}\left(9.4786\times10^{-8}\right)\left(8.2099\times10^{-8}\right)$$
$$= 3.8909\times10^{-15}\ \text{cm}^2$$

$$\#/\text{cm}^2 = \frac{2}{3.8909\times10^{-15}}$$
$$= 5.14\times10^{14}\ \text{cm}^{-2}$$
$$d = \frac{a\sqrt{3}}{3} = \frac{(6.703)\sqrt{3}}{3} = 3.87\ \overset{o}{A}$$

1.22

Density of silicon atoms $= 5\times10^{22}\ \text{cm}^{-3}$ and 4 valence electrons per atom, so

Density of valence electrons $= 2\times10^{23}\ \text{cm}^{-3}$

1.23

Density of GaAs atoms

$$= \frac{8}{\left(5.65\times10^{-8}\right)^3} = 4.44\times10^{22}\ \text{cm}^{-3}$$

An average of 4 valence electrons per atom, So

Density of valence electrons

$$= 1.77\times10^{23}\ \text{cm}^{-3}$$

1.24

(a) $$\frac{5\times10^{17}}{5\times10^{22}}\times100\% = 10^{-3}\%$$

(b) $$\frac{2\times10^{15}}{5\times10^{22}}\times100\% = 4\times10^{-6}\%$$

1.25

(a) Fraction by weight

$$\cong \frac{\left(2\times10^{16}\right)(10.82)}{\left(5\times10^{22}\right)(28.06)} = 1.542\times10^{-7}$$

(b) Fraction by weight

$$\cong \frac{\left(10^{18}\right)(30.98)}{\left(5\times10^{22}\right)(28.06)} = 2.208\times10^{-5}$$

1.26

$$\text{Volume density} = \frac{1}{d^3} = 2\times10^{16}\ \text{cm}^{-3}$$

So $d = 3.684\times10^{-6}\ \text{cm} \Rightarrow d = 368.4\ \overset{o}{A}$

We have $a_o = 5.43\ \overset{o}{A}$

Then $$\frac{d}{a_o} = \frac{368.4}{5.43} = 67.85$$

1.27

$$\text{Volume density} = \frac{1}{d^3} = 4\times10^{15}\ \text{cm}^{-3}$$

So $d = 6.30\times10^{-6}\ \text{cm} \Rightarrow d = 630\ \overset{o}{A}$

We have $a_o = 5.43\ \overset{o}{A}$

Then $$\frac{d}{a_o} = \frac{630}{5.43} = 116$$

Chapter 2

2.1

Sketch

2.2

Sketch

2.3

Sketch

2.4

From Problem 2.2, phase $= \dfrac{2\pi x}{\lambda} - \omega t$

$= \text{constant}$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = +\omega\left(\frac{\lambda}{2\pi}\right)$$

From Problem 2.3, phase $= \dfrac{2\pi x}{\lambda} + \omega t$

$= \text{constant}$

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = -\omega\left(\frac{\lambda}{2\pi}\right)$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90 \text{ eV} = (4.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(4.90)(1.6\times10^{-19})} = 2.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.254\,\mu$m

Cesium: $E = 1.90 \text{ eV} = (1.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(1.90)(1.6\times10^{-19})} = 6.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.654\,\mu$m

2.6

(a) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{550\times10^{-9}}$

$= 1.205\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^{3} \text{ m/s}$$

or $\upsilon = 1.32\times10^{5}$ cm/s

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{440\times10^{-9}}$

$= 1.506\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^{3} \text{ m/s}$$

or $\upsilon = 1.65\times10^{5}$ cm/s

(c) Yes

2.7

(a) (i)

$$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.2)(1.6\times10^{-19})}$$

$= 5.915\times10^{-25}$ kg-m/s

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{5.915\times10^{-25}} = 1.12\times10^{-9} \text{ m}$$

or $\lambda = 11.2\,\overset{o}{A}$

(ii) $p = \sqrt{2(9.11\times10^{-31})(12)(1.6\times10^{-19})}$

$= 1.87\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{1.8704\times10^{-24}} = 3.54\times10^{-10} \text{ m}$$

or $\lambda = 3.54\,\overset{o}{A}$

(iii) $p = \sqrt{2(9.11\times10^{-31})(120)(1.6\times10^{-19})}$

$= 5.915\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{5.915\times10^{-24}} = 1.12\times10^{-10} \text{ m}$$

or $\lambda = 1.12\,\overset{o}{A}$

(b)

$$p = \sqrt{2(1.67\times10^{-27})(1.2)(1.6\times10^{-19})}$$
$$= 2.532\times10^{-23}\ \text{kg-m/s}$$
$$\lambda = \frac{6.625\times10^{-34}}{2.532\times10^{-23}} = 2.62\times10^{-11}\ \text{m}$$

or $\lambda = 0.262\ \overset{o}{A}$

2.8

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right)(0.0259) = 0.03885\ \text{eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$
$$= \sqrt{2(9.11\times10^{-31})(0.03885)(1.6\times10^{-19})}$$

or

$$p_{avg} = 1.064\times10^{-25}\ \text{kg-m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{1.064\times10^{-25}} = 6.225\times10^{-9}\ \text{m}$$

or

$$\lambda = 62.25\ \overset{o}{A}$$

2.9

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$
$$= \frac{2(9.11\times10^{-31})(3\times10^8)^2}{100}$$
$$= 1.64\times10^{-15}\ \text{J} = 10.25\ \text{keV}$$

2.10

(a) $$p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}}{85\times10^{-10}}$$
$$= 7.794\times10^{-26}\ \text{kg-m/s}$$
$$\upsilon = \frac{p}{m} = \frac{7.794\times10^{-26}}{9.11\times10^{-31}} = 8.56\times10^4\ \text{m/s}$$

or $\upsilon = 8.56\times10^6$ cm/s

$$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}(9.11\times10^{-31})(8.56\times10^4)^2$$
$$= 3.33\times10^{-21}\ \text{J}$$

or $$E = \frac{3.334\times10^{-21}}{1.6\times10^{-19}} = 2.08\times10^{-2}\ \text{eV}$$

(b) $$E = \frac{1}{2}(9.11\times10^{-31})(8\times10^3)^2$$
$$= 2.915\times10^{-23}\ \text{J}$$

or $$E = \frac{2.915\times10^{-23}}{1.6\times10^{-19}} = 1.82\times10^{-4}\ \text{eV}$$

$$p = m\upsilon = (9.11\times10^{-31})(8\times10^3)$$
$$= 7.288\times10^{-27}\ \text{kg-m/s}$$
$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-35}}{7.288\times10^{-27}} - 9.09\times10^{-8}\ \text{m}$$

or $\lambda = 909\ \overset{o}{A}$

2.11

(a) $$E = h\nu = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34})(3\times10^8)}{1\times10^{-10}}$$
$$= 1.99\times10^{-15}\ \text{J}$$

Now

$$E = e\cdot V \Rightarrow V = \frac{E}{e} = \frac{1.99\times10^{-15}}{1.6\times10^{-19}}$$
$$V = 1.24\times10^4\ \text{V} = 12.4\ \text{kV}$$

(b) $$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.99\times10^{-15})}$$
$$= 6.02\times10^{-23}\ \text{kg-m/s}$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{6.02\times10^{-23}} = 1.10\times10^{-11}\ \text{m}$$

or

$$\lambda = 0.11\ \overset{o}{A}$$

2.12

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-6}}$$
$$= 1.054\times10^{-28}\text{ kg-m/s}$$

2.13

(a) (i) $\Delta p \Delta x = \hbar$

$$\Delta p = \frac{1.054\times10^{-34}}{12\times10^{-10}} = 8.783\times10^{-26}\text{ kg-m/s}$$

(ii) $$\Delta E = \frac{dE}{dp}\cdot\Delta p = \frac{d}{dp}\left(\frac{p^2}{2m}\right)\cdot\Delta p$$
$$= \frac{2p}{2m}\cdot\Delta p = \frac{p\Delta p}{m}$$

Now $p = \sqrt{2mE}$
$$= \sqrt{2\left(9\times10^{-31}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 2.147\times10^{-24}\text{ kg-m/s}$$

so $$\Delta E = \frac{\left(2.1466\times10^{-24}\right)\left(8.783\times10^{-26}\right)}{9\times10^{-31}}$$
$$= 2.095\times10^{-19}\text{ J}$$

or $$\Delta E = \frac{2.095\times10^{-19}}{1.6\times10^{-19}} = 1.31\text{ eV}$$

(b) (i) $\Delta p = 8.783\times10^{-26}$ kg-m/s

(ii) $$p = \sqrt{2\left(5\times10^{-28}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 5.06\times10^{-23}\text{ kg-m/s}$$
$$\Delta E = \frac{\left(5.06\times10^{-23}\right)\left(8.783\times10^{-26}\right)}{5\times10^{-28}}$$
$$= 8.888\times10^{-21}\text{ J}$$

or $$\Delta E = \frac{8.888\times10^{-21}}{1.6\times10^{-19}} = 5.55\times10^{-2}\text{ eV}$$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-2}} = 1.054\times10^{-32}\text{ kg-m/s}$$
$$p = m\upsilon \Rightarrow \Delta\upsilon = \frac{\Delta p}{m} = \frac{1.054\times10^{-32}}{1500}$$
$$\Delta\upsilon = 7\times10^{-36}\text{ m/s}$$

2.15

(a) $\Delta E \Delta t = \hbar$

$$\Delta t = \frac{1.054\times10^{-34}}{(0.8)\left(1.6\times10^{-19}\right)} = 8.23\times10^{-16}\text{ s}$$

(b) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{1.5\times10^{-10}}$$
$$= 7.03\times10^{-25}\text{ kg-m/s}$$

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$
$$+V(x)\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1(x,t)+\Psi_2(x,t)\right]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t)+\Psi_2(x,t)$ is also a solution.

(b) If $\Psi_1(x,t)\cdot\Psi_2(x,t)$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1\cdot\Psi_2\right] + V(x)\left[\Psi_1\cdot\Psi_2\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1\cdot\Psi_2\right]$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1\frac{\partial^2\Psi_2}{\partial x^2} + \Psi_2\frac{\partial^2\Psi_1}{\partial x^2} + 2\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x)\left[\Psi_1\cdot\Psi_2\right] = j\hbar\left[\Psi_1\frac{\partial\Psi_2}{\partial t} + \Psi_2\frac{\partial\Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1\cdot\Psi_2$, we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + \frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial\Psi_1}{\partial t}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we have

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we have

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\int_{-1}^{+3}A^2\cos^2\left(\frac{\pi x}{2}\right)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(\pi x)}{2\pi}\right]\Bigg|_{-1}^{+3}=1$$

$$A^2\left[\frac{3}{2}-\left(\frac{-1}{2}\right)\right]=1$$

so $A^2=\dfrac{1}{2}$

or $|A|=\dfrac{1}{\sqrt{2}}$

2.18

$$\int_{-1/2}^{+1/2}A^2\cos^2(n\pi x)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(2n\pi x)}{4n\pi}\right]\Bigg|_{-1/2}^{+1/2}=1$$

$$A^2\left[\frac{1}{4}-\left(-\frac{1}{4}\right)\right]=1=A^2\left(\frac{1}{2}\right)$$

or $|A|=\sqrt{2}$

2.19

Note that $\int_0^\infty\Psi\cdot\Psi^*dx=1$

Function has been normalized.

(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P=(-1)\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields

$P=0.393$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P=(-1)\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields

$P=0.239$

(c)

$$P=\int_0^{a_o}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

$$=(-1)[\exp(-2)-1]$$

which yields

$P=0.865$

2.20

$$P=\int\left|\psi(x)\right|^2 dx$$

(a) $$\int_0^{a/4}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{2}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_0^{a/4}$$

$$=\left(\frac{2}{a}\right)\left[\frac{\left(\frac{a}{4}\right)}{2}+\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{8}+\frac{(1)(a)}{4\pi}\right]$$

or $P=0.409$

(b) $$P=\int_{a/4}^{a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\frac{a}{8}-\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$=2\left[\frac{1}{4}+0-\frac{1}{8}-\frac{1}{4\pi}\right]$$

or $P=0.0908$

(c) $$P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\left(\frac{-a}{4}\right)-\frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)}\right]$$

or $P=1$

2.21

(a) $$P=\int_0^{a/4}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_0^{a/4}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{8}-\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(b) $$P=\int_{a/4}^{a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{a}{8}\right)+\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(c) $$P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{-a}{4}\right)+\frac{\sin(-2\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=1$

2.22

(a) (i) $$\upsilon_p=\frac{\omega}{k}=\frac{8\times10^{12}}{8\times10^{8}}=10^4\text{ m/s}$$

or $\upsilon_p=10^6$ cm/s

$$\lambda=\frac{2\pi}{k}=\frac{2\pi}{8\times10^{8}}=7.854\times10^{-9}\text{ m}$$

or $\lambda = 78.54\,\overset{o}{A}$

(ii) $p = m\upsilon = \left(9.11\times10^{-31}\right)\left(10^{4}\right)$

$= 9.11\times10^{-27}$ kg-m/s

$E = \dfrac{1}{2}m\upsilon^2 = \dfrac{1}{2}\left(9.11\times10^{-31}\right)\left(10^{4}\right)^2$

$= 4.555\times10^{-23}$ J

or $E = \dfrac{4.555\times10^{-23}}{1.6\times10^{-19}} = 2.85\times10^{-4}$ eV

(b) (i) $\upsilon_p = \dfrac{\omega}{k} = \dfrac{1.5\times10^{13}}{-1.5\times10^{9}} = -10^4$ m/s

or $\upsilon_p = -10^6$ cm/s

$\lambda = \dfrac{2\pi}{|k|} = \dfrac{2\pi}{1.5\times10^{9}} = 4.19\times10^{-9}$ m

or $\lambda = 41.9\,\overset{o}{A}$

(ii) $p = -9.11\times10^{-27}$ kg-m/s

$E = 2.85\times10^{-4}$ eV

2.23

(a) $\Psi(x,t) = Ae^{-j(kx+\omega t)}$

(b) $E = (0.025)\left(1.6\times10^{-19}\right) = \dfrac{1}{2}m\upsilon^2$

$= \dfrac{1}{2}\left(9.11\times10^{-31}\right)\upsilon^2$

so $|\upsilon| = 9.37\times10^{4}$ m/s $= 9.37\times10^{6}$ cm/s

For electron traveling in $-x$ direction,

$\upsilon = -9.37\times10^{6}$ cm/s

$p = m\upsilon = \left(9.11\times10^{-31}\right)\left(-9.37\times10^{4}\right)$

$= -8.537\times10^{-26}$ kg-m/s

$\lambda = \dfrac{h}{|p|} = \dfrac{6.625\times10^{-34}}{8.537\times10^{-26}} = 7.76\times10^{-9}$ m

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{7.76\times10^{-9}} = 8.097\times10^{8}$ m^{-1}

$\omega = k\cdot|\upsilon| = \left(8.097\times10^{8}\right)\left(9.37\times10^{4}\right)$

or $\omega = 7.586\times10^{13}$ rad/s

2.24

(a) $p = m\upsilon = \left(9.11\times10^{-31}\right)\left(5\times10^{4}\right)$

$= 4.555\times10^{-26}$ kg-m/s

$\lambda = \dfrac{h}{p} = \dfrac{6.625\times10^{-34}}{4.555\times10^{-26}} = 1.454\times10^{-8}$ m

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.454\times10^{-8}} = 4.32\times10^{8}$ m^{-1}

$\omega = k\upsilon = \left(4.32\times10^{8}\right)\left(5\times10^{4}\right)$

$= 2.16\times10^{13}$ rad/s

(b) $p = \left(9.11\times10^{-31}\right)\left(10^{6}\right)$

$= 9.11\times10^{-25}$ kg-m/s

$\lambda = \dfrac{6.625\times10^{-34}}{9.11\times10^{-25}} = 7.27\times10^{-10}$ m

$k = \dfrac{2\pi}{7.272\times10^{-10}} = 8.64\times10^{9}$ m^{-1}

$\omega = \left(8.64\times10^{9}\right)\left(10^{6}\right) = 8.64\times10^{15}$ rad/s

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2.25

$E_n = \dfrac{\hbar^2 n^2\pi^2}{2ma^2} = \dfrac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(75\times10^{-10}\right)^2}$

$E_n = n^2\left(1.0698\times10^{-21}\right)$ J

or

$E_n = \dfrac{n^2\left(1.0698\times10^{-21}\right)}{1.6\times10^{-19}}$

or $E_n = n^2\left(6.686\times10^{-3}\right)$ eV

Then

$E_1 = 6.69\times10^{-3}$ eV

$E_2 = 2.67\times10^{-2}$ eV

$E_3 = 6.02\times10^{-2}$ eV

2.26

(a) $E_n = \dfrac{\hbar^2 n^2\pi^2}{2ma^2} = \dfrac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(10\times10^{-10}\right)^2}$

$= n^2\left(6.018\times10^{-20}\right)$ J

or $E_n = \dfrac{n^2\left(6.018\times10^{-20}\right)}{1.6\times10^{-19}} = n^2(0.3761)$ eV

Then

$E_1 = 0.376$ eV

$E_2 = 1.504$ eV

$E_3 = 3.385$ eV

(b) $\lambda = \dfrac{hc}{\Delta E}$

$\Delta E = (3.385 - 1.504)\left(1.6\times10^{-19}\right)$

$= 3.01\times10^{-19}$ J

$$\lambda = \frac{(6.625\times 10^{-34})(3\times 10^{8})}{3.01\times 10^{-19}}$$
$$= 6.604\times 10^{-7}\text{ m}$$
or $\lambda = 660.4\text{ nm}$

2.27

(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$15\times 10^{-3} = \frac{n^2 (1.054\times 10^{-34})^2 \pi^2}{2(15\times 10^{-3})(1.2\times 10^{-2})^2}$$
$$15\times 10^{-3} = n^2 (2.538\times 10^{-62})$$
or $n = 7.688\times 10^{29}$

(b) $E_{n+1} \cong 15\text{ mJ}$

(c) No

2.28

For a neutron and $n=1$:
$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{(1.054\times 10^{-34})^2 \pi^2}{2(1.66\times 10^{-27})(10^{-14})^2}$$
$$= 3.3025\times 10^{-13}\text{ J}$$
or
$$E_1 = \frac{3.3025\times 10^{-13}}{1.6\times 10^{-19}} = 2.06\times 10^{6}\text{ eV}$$
For an electron in the same potential well:
$$E_1 = \frac{(1.054\times 10^{-34})^2 \pi^2}{2(9.11\times 10^{-31})(10^{-14})^2}$$
$$= 6.0177\times 10^{-10}\text{ J}$$
or
$$E_1 = \frac{6.0177\times 10^{-10}}{1.6\times 10^{-19}} = 3.76\times 10^{9}\text{ eV}$$

2.29

Schrodinger's time-independent wave equation
$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$
We know that
$$\psi(x) = 0 \text{ for } x \ge \frac{a}{2} \text{ and } x \le \frac{-a}{2}$$
We have
$$V(x) = 0 \text{ for } \frac{-a}{2} < x < \frac{+a}{2}$$
so in this region
$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$
The solution is of the form
$$\psi(x) = A\cos kx + B\sin kx$$
where
$$k = \sqrt{\frac{2mE}{\hbar^2}}$$
Boundary conditions:
$$\psi(x) = 0 \text{ at } x = \frac{-a}{2},\ x = \frac{+a}{2}$$
First mode solution:
$$\psi_1(x) = A_1 \cos k_1 x$$
where
$$k_1 = \frac{\pi}{a} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$
Second mode solution:
$$\psi_2(x) = B_2 \sin k_2 x$$
where
$$k_2 = \frac{2\pi}{a} \Rightarrow E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$$
Third mode solution:
$$\psi_3(x) = A_3 \cos k_3 x$$
where
$$k_3 = \frac{3\pi}{a} \Rightarrow E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$$
Fourth mode solution:
$$\psi_4(x) = B_4 \sin k_4 x$$
where
$$k_4 = \frac{4\pi}{a} \Rightarrow E_4 = \frac{16\pi^2\hbar^2}{2ma^2}$$

2.30

The 3-D time-independent wave equation in cartesian coordinates for $V(x,y,z) = 0$ is:
$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$
Use separation of variables, so let
$$\psi(x,y,z) = X(x)Y(y)Z(z)$$
Substituting into the wave equation, we obtain

$$YZ\frac{\partial^2 X}{\partial x^2}+XZ\frac{\partial^2 Y}{\partial y^2}+XY\frac{\partial^2 Z}{\partial z^2}$$
$$+\frac{2mE}{\hbar^2}XYZ=0$$

Dividing by XYZ and letting $k^2=\frac{2mE}{\hbar^2}$, we find

(1) $\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}+k^2=0$

We may set

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}=-k_x^2\Rightarrow\frac{\partial^2 X}{\partial x^2}+k_x^2X=0$$

Solution is of the form

$$X(x)=A\sin(k_x x)+B\cos(k_x x)$$

Boundary conditions: $X(0)=0\Rightarrow B=0$

and $X(x=a)=0\Rightarrow k_x=\frac{n_x\pi}{a}$

where $n_x=1,2,3\ldots.$

Similarly, let

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}=-k_y^2 \text{ and } \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}=-k_z^2$$

Applying the boundary conditions, we find

$$k_y=\frac{n_y\pi}{a},\quad n_y=1,2,3\ldots.$$

$$k_z=\frac{n_z\pi}{a},\quad n_z=1,2,3\ldots$$

From Equation (1) above, we have

$$-k_x^2-k_y^2-k_z^2+k^2=0$$

or

$$k_x^2+k_y^2+k_z^2=k^2=\frac{2mE}{\hbar^2}$$

so that

$$E\to E_{n_xn_yn_z}=\frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2+n_y^2+n_z^2\right)$$

2.31

(a) $\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial y^2}+\frac{2mE}{\hbar^2}\cdot\psi(x,y)=0$

Solution is of the form:

$$\psi(x,y)=A\sin k_x x\cdot\sin k_y y$$

We find

$$\frac{\partial\psi(x,y)}{\partial x}=Ak_x\cos k_x x\cdot\sin k_y y$$

$$\frac{\partial^2\psi(x,y)}{\partial x^2}=-Ak_x^2\sin k_x x\cdot\sin k_y y$$

$$\frac{\partial\psi(x,y)}{\partial y}=Ak_y\sin k_x x\cdot\cos k_y y$$

$$\frac{\partial^2\psi(x,y)}{\partial y^2}=-Ak_y^2\sin k_x x\cdot\sin k_y y$$

Substituting into the original equation, we find:

(1) $-k_x^2-k_y^2+\frac{2mE}{\hbar^2}=0$

From the boundary conditions,

$A\sin k_x a=0$, where $a=40\overset{o}{A}$

So $k_x=\frac{n_x\pi}{a}$, $n_x=1,2,3,\ldots$

Also $A\sin k_y b=0$, where $b=20\overset{o}{A}$

So $k_y=\frac{n_y\pi}{b}$, $n_y=1,2,3,\ldots$

Substituting into Eq. (1) above

$$E_{n_xn_y}=\frac{\hbar^2}{2m}\left(\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{b^2}\right)$$

(b) Energy is quantized - similar to 1-D result. There can be more than one quantum state per given energy - different than 1-D result.

2.32

(a) Derivation of energy levels exactly the same as in the text

(b) $\Delta E=\frac{\hbar^2\pi^2}{2ma^2}\left(n_2^2-n_1^2\right)$

For $n_2=2, n_1=1$

Then

$$\Delta E=\frac{3\hbar^2\pi^2}{2ma^2}$$

(i) For $a=4\overset{o}{A}$

$$\Delta E=\frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(4\times10^{-10}\right)^2}$$
$$=6.155\times10^{-22}\text{ J}$$

or $\Delta E=\frac{6.155\times10^{-22}}{1.6\times10^{-19}}=3.85\times10^{-3}$ eV

(ii) For $a = 0.5$ cm

$$\Delta E = \frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(0.5\times10^{-2}\right)^2}$$

$$= 3.939\times10^{-36}\text{ J}$$

or

$$\Delta E = \frac{3.939\times10^{-36}}{1.6\times10^{-19}} = 2.46\times10^{-17}\text{ eV}$$

2.33

(a) For region II, $x > 0$

$$\frac{\partial^2\psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2\exp(jk_2x) + B_2\exp(-jk_2x)$$

where

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave and term with A_2 represents reflected wave.

Region I, $x < 0$

$$\frac{\partial^2\psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

General form of the solution is

$$\psi_1(x) = A_1\exp(jk_1x) + B_1\exp(-jk_1x)$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave and the term involving A_1 represents reflected wave: but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1\exp(-jk_1x)$$
$$\psi_2(x) = A_2\exp(jk_2x) + B_2\exp(-jk_2x)$$

(b)

Boundary conditions:

(1) $\psi_1(x=0) = \psi_2(x=0)$

(2) $\left.\frac{\partial\psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_2}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$
$$k_2A_2 - k_2B_2 = -k_1B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\cdot B_2$$

$$B_1 = \left(\frac{2k_2}{k_2 + k_1}\right)\cdot B_2$$

The reflection coefficient is

$$R = \frac{A_2A_2^*}{B_2B_2^*} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4k_1k_2}{(k_1 + k_2)^2}$$

2.34

$$\psi_2(x) = A_2\exp(-k_2x)$$

$$P = \frac{|\psi(x)|^2}{A_2A_2^*} = \exp(-2k_2x)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\right)(3.5 - 2.8)\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

$$k_2 = 4.286\times10^9\text{ m}^{-1}$$

(a) For $x = 5\,\overset{o}{A} = 5\times10^{-10}$ m

$$P = \exp(-2k_2x)$$
$$= \exp\left[-2\left(4.2859\times10^9\right)\left(5\times10^{-10}\right)\right]$$
$$= 0.0138$$

(b) For $x = 15\,\overset{o}{A} = 15\times10^{-10}$ m

$$P = \exp\left[-2\left(4.2859\times10^9\right)\left(15\times10^{-10}\right)\right]$$
$$= 2.61\times10^{-6}$$

(c) For $x = 40\,\overset{o}{A} = 40\times10^{-10}$ m

$$P = \exp\left[-2\left(4.2859\times10^9\right)\left(40\times10^{-10}\right)\right]$$
$$= 1.29\times10^{-15}$$

2.35

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1 - \frac{E}{V_o}\right)\exp(-2k_2a)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\right)(1.0 - 0.1)\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

or $\quad k_2 = 4.860\times10^{9}\ \text{m}^{-1}$

(a) For $a = 4\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976\times10^{9}\right)\left(4\times10^{-10}\right)\right]$$

$$= 0.0295$$

(b) For $a = 12\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976\times10^{9}\right)\left(12\times10^{-10}\right)\right]$$

$$= 1.24\times10^{-5}$$

(c) $J = N_t e\upsilon$, where N_t is the density of transmitted electrons.

$$E = 0.1\,\text{eV} = 1.6\times10^{-20}\ \text{J}$$

$$= \frac{1}{2}m\upsilon^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\upsilon^2$$

$$\Rightarrow \upsilon = 1.874\times10^{5}\ \text{m/s} = 1.874\times10^{7}\ \text{cm/s}$$

$$1.2\times10^{-3} = N_t\left(1.6\times10^{-19}\right)\left(1.874\times10^{7}\right)$$

$$N_t = 4.002\times10^{8}\ \text{electrons/cm}^3$$

Density of incident electrons,

$$N_i = \frac{4.002\times10^{8}}{0.0295} = 1.357\times10^{10}\ \text{cm}^{-3}$$

2.36

$$T \cong 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp\left(-2k_2 a\right)$$

(a) For $m = (0.067)m_o$

$$k_2 = \sqrt{\frac{2m\left(V_O - E\right)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)\left(9.11\times10^{-31}\right)(0.8-0.2)\left(1.6\times10^{-19}\right)}{\left(1.054\times10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 1.027\times10^{9}\ \ \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$
$$\times\exp\left[-2\left(1.027\times10^{9}\right)\left(15\times10^{-10}\right)\right]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$

$$k_2 = \left\{\frac{2(1.08)\left(9.11\times10^{-31}\right)(0.8-0.2)\left(1.6\times10^{-19}\right)}{\left(1.054\times10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 4.124\times10^{9}\ \ \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$
$$\times\exp\left[-2\left(4.124\times10^{9}\right)\left(15\times10^{-10}\right)\right]$$

or

$$T = 1.27\times10^{-5}$$

2.37

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1-\frac{E}{V_o}\right)\exp\left(-2k_2 a\right)$$

where $k_2 = \sqrt{\dfrac{2m\left(V_o - E\right)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(1.67\times10^{-27}\right)(12-1)\times10^{6}\times\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

$$= 7.274\times10^{14}\ \text{m}^{-1}$$

(a)

$$T \cong 16\left(\frac{1}{12}\right)\left(1-\frac{1}{12}\right)\exp\left[-2\left(7.274\times10^{14}\right)\left(10^{-14}\right)\right]$$

$$= 1.222\exp\left[-14.548\right]$$

$$= 5.875\times10^{-7}$$

(b)

$$T = (10)\left(5.875\times10^{-7}\right)$$
$$= 1.222\exp\left[-2\left(7.274\times10^{14}\right)a\right]$$

$$2\left(7.274\times10^{14}\right)a = \ln\left(\frac{1.222}{5.875\times10^{-6}}\right)$$

or $\quad a = 0.842\times10^{-14}$ m

2.38

Region I $(x<0)$, $V = 0$;

Region II $(0<x<a)$, $V = V_O$

Region III $(x>a)$, $V = 0$

(a) Region I:

$$\psi_1(x) = A_1\exp\left(jk_1x\right) + B_1\exp\left(-jk_1x\right)$$

(incident) (reflected)

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:

$$\psi_2(x) = A_2 \exp(k_2 x) + B_2 \exp(-k_2 x)$$

where

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

Region III:

$$\psi_3(x) = A_3 \exp(jk_1 x) + B_3 \exp(-jk_1 x)$$

(b)

In Region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into Region III, there will not be a reflected wave so that $B_3 = 0$.

(c) Boundary conditions:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow$$

$$jk_1 A_1 - jk_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(k_2 a) + B_2 \exp(-k_2 a) = A_3 \exp(jk_1 a)$$

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$k_2 A_2 \exp(k_2 a) - k_2 B_2 \exp(-k_2 a) = jk_1 A_3 \exp(jk_1 a)$$

The transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4k_1 k_2} \{(k_2^2 - k_1^2)[\exp(k_2 a) - \exp(-k_2 a)] - 2jk_1 k_2 [\exp(k_2 a) + \exp(-k_2 a)]\} \times \exp(jk_1 a)$$

We then find

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \{(k_2^2 - k_1^2)[\exp(k_2 a) - \exp(-k_2 a)]^2 + 4k_1^2 k_2^2 [\exp(k_2 a) + \exp(-k_2 a)]^2\}$$

We have

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

If we assume that $V_O >> E$, then $k_2 a$ will be large so that

$$\exp(k_2 a) >> \exp(-k_2 a)$$

We can then write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \{(k_2^2 - k_1^2)[\exp(k_2 a)]^2 + 4k_1^2 k_2^2 [\exp(k_2 a)]^2\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} (k_2^2 + k_1^2) \exp(2k_2 a)$$

Substituting the expressions for k_1 and k_2, we find

$$k_1^2 + k_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$k_1^2 k_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O - E)(E)$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O)\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^* \left(\frac{2mV_O}{\hbar^2}\right)^2 \exp(2k_2 a)}{16\left[\left(\frac{2m}{\hbar^2}\right)^2 V_O \left(1 - \frac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)}$$

Finally,

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)$$

2.39

Region I: $V = 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0 \Rightarrow$$

$$\psi_1(x) = A_1 \exp(jk_1 x) + B_1 \exp(-jk_1 x)$$

incident reflected

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: $V = V_1$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2(x) = 0 \Rightarrow$$

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

transmitted reflected

where

$$k_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$$

Region III: $V = V_2$

$$\frac{\partial^2 \psi_3(x)}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3(x) = 0 \Rightarrow$$

$$\psi_3(x) = A_3 \exp(jk_3 x)$$

transmitted

where

$$k_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$$

There is no reflected wave in Region III.
The transmission coefficient is defined as:

$$T = \frac{\upsilon_3}{\upsilon_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{k_3}{k_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From the boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow$$

$$k_1 A_1 - k_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jk_2 a) + B_2 \exp(-jk_2 a) = A_3 \exp(jk_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$k_2 A_2 \exp(jk_2 a) - k_2 B_2 \exp(-jk_2 a) = k_3 A_3 \exp(jk_3 a)$$

But $k_2 a = 2n\pi \Rightarrow$

$$\exp(jk_2 a) = \exp(-jk_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the boundary condition equations, we find

$$T = \frac{k_3}{k_1} \cdot \frac{4k_1^2}{(k_1 + k_3)^2} = \frac{4k_1 k_3}{(k_1 + k_3)^2}$$

2.40

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1(x) = B_1 \exp(k_1 x)$$

$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$$

$$\psi_3(x) = 0$$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right) B_2$$

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right) B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right) \tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.41

$$E_n = \frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \text{ (J)}$$

$$= \frac{-m_o e^3}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \text{ (eV)}$$

$$= \frac{-(9.11\times10^{-31})(1.6\times10^{-19})^3}{[4\pi(8.85\times10^{-12})]^2 2(1.054\times10^{-34})^2 n^2}$$

or

$$E_n = \frac{-13.58}{n^2} \text{ (eV)}$$

$n = 1 \Rightarrow E_1 = -13.58 \text{ eV}$

$n = 2 \Rightarrow E_2 = -3.395 \text{ eV}$

$n = 3 \Rightarrow E_3 = -1.51 \text{ eV}$

$n = 4 \Rightarrow E_4 = -0.849 \text{ eV}$

2.42

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^*$$

$$= 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3} \left\{ \left(\frac{-2}{a_o}\right)(r^2) \exp\left(\frac{-2r}{a_o}\right) + 2r \exp\left(\frac{-2r}{a_o}\right) \right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow r = a_o$$

or $r = a_o$ is the radius that gives the greatest probability.

2.43

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}(E - V(r))\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi\epsilon_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

Then

$$\frac{\partial \psi_{100}}{\partial r} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left(\frac{-1}{a_o}\right) \exp\left(\frac{-r}{a_o}\right)$$

so

$$r^2 \frac{\partial \psi_{100}}{\partial r} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

We then obtain

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi_{100}}{\partial r}\right) = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \times \left[2r \exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right) \exp\left(\frac{-r}{a_o}\right)\right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right)-\frac{r^2}{a_o}\exp\left(\frac{-r}{a_o}\right)\right]$$
$$+\frac{2m_o}{\hbar^2}\left[E+\frac{\hbar^2}{m_o a_o r}\right]$$
$$\times\left(\frac{1}{\sqrt{\pi}}\right)\cdot\left(\frac{1}{a_o}\right)^{3/2}\exp\left(\frac{-r}{a_o}\right)=0$$

where

$$E=E_1=\frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2}=\frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]\left\{\frac{-1}{r^2 a_o}\left[2r-\frac{r^2}{a_o}\right]\right.$$
$$\left.+\frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o}+\frac{\hbar^2}{m_o a_o r}\right)\right\}=0$$

or

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]$$
$$\times\left\{\frac{-2}{a_o r}+\frac{1}{a_o^2}+\left(\frac{-1}{a_o^2}+\frac{2}{a_o r}\right)\right\}=0$$

which gives 0 = 0 and shows that ψ_{100} is indeed a solution to the wave equation.

2.44

All elements are from the Group I column of the periodic table. All have one valence electron in the outer shell.

Chapter 3

3.1

If a_o were to increase, the bandgap energy would decrease and the material would begin to behave less like a semiconductor and more like a metal. If a_o were to decrease, the bandgap energy would increase and the material would begin to behave more like an insulator.

3.2

Schrodinger's wave equation is:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x)\cdot\Psi(x,t) = j\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Assume the solution is of the form:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Region I: $V(x)=0$. Substituting the assumed solution into the wave equation, we obtain:

$$\frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\left\{ jku(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right] + \frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$= j\hbar\left(\frac{-jE}{\hbar}\right)\cdot u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

which becomes

$$\frac{-\hbar^2}{2m}\left\{ (jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right] + 2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right] + \frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$= +Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation may be written as

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2}+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_1(x)$ for region I, the equation becomes:

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-\left(k^2-\alpha^2\right)u_1(x)=0$$

where

$$\alpha^2=\frac{2mE}{\hbar^2}$$ Q.E.D.

In Region II, $V(x)=V_O$. Assume the same form of the solution:

$$\Psi(x,t)=u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

Substituting into Schrodinger's wave equation, we find:

$$\frac{-\hbar^2}{2m}\left\{ (jk)^2u(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right] + 2jk\frac{\partial u(x)}{\partial x}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right] + \frac{\partial^2 u(x)}{\partial x^2}\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]\right\}$$

$$+V_Ou(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

$$= Eu(x)\exp\left[j\left(kx-\left(\frac{E}{\hbar}\right)t\right)\right]$$

This equation can be written as:

$$-k^2u(x)+2jk\frac{\partial u(x)}{\partial x}+\frac{\partial^2u(x)}{\partial x^2} -\frac{2mV_O}{\hbar^2}u(x)+\frac{2mE}{\hbar^2}u(x)=0$$

Setting $u(x)=u_2(x)$ for region II, this equation becomes

$$\frac{d^2u_2(x)}{dx^2}+2jk\frac{du_2(x)}{dx} -\left(k^2-\alpha^2+\frac{2mV_O}{\hbar^2}\right)u_2(x)=0$$

where again

$$\alpha^2=\frac{2mE}{\hbar^2}$$ Q.E.D.

3.3

We have

$$\frac{d^2u_1(x)}{dx^2}+2jk\frac{du_1(x)}{dx}-(k^2-\alpha^2)u_1(x)=0$$

Assume the solution is of the form:

$$u_1(x)=A\exp[j(\alpha-k)x]$$
$$+B\exp[-j(\alpha+k)x]$$

The first derivative is

$$\frac{du_1(x)}{dx}=j(\alpha-k)A\exp[j(\alpha-k)x]$$
$$-j(\alpha+k)B\exp[-j(\alpha+k)x]$$

and the second derivative becomes

$$\frac{d^2u_1(x)}{dx^2}=[j(\alpha-k)]^2A\exp[j(\alpha-k)x]$$
$$+[j(\alpha+k)]^2B\exp[-j(\alpha+k)x]$$

Substituting these equations into the differential equation, we find

$$-(\alpha-k)^2A\exp[j(\alpha-k)x]$$
$$-(\alpha+k)^2B\exp[-j(\alpha+k)x]$$
$$+2jk\{j(\alpha-k)A\exp[j(\alpha-k)x]$$
$$-j(\alpha+k)B\exp[-j(\alpha+k)x]\}$$
$$-(k^2-\alpha^2)\{A\exp[j(\alpha-k)x]$$
$$+B\exp[-j(\alpha+k)x]\}=0$$

Combining terms, we obtain

$$\left[-(\alpha^2-2\alpha k+k^2)-2k(\alpha-k)-(k^2-\alpha^2)\right]$$
$$\times A\exp[j(\alpha-k)x]$$
$$+\left[-(\alpha^2+2\alpha k+k^2)+2k(\alpha+k)-(k^2-\alpha^2)\right]$$
$$\times B\exp[-j(\alpha+k)x]=0$$

We find that

$0=0$ Q.E.D.

For the differential equation in $u_2(x)$ and the proposed solution, the procedure is exactly the same as above.

3.4

We have the solutions

$$u_1(x)=A\exp[j(\alpha-k)x]$$
$$+B\exp[-j(\alpha+k)x]$$

for $0<x<a$ and

$$u_2(x)=C\exp[j(\beta-k)x]$$
$$+D\exp[-j(\beta+k)x]$$

for $-b<x<0$.

The first boundary condition is

$$u_1(0)=u_2(0)$$

which yields

$$A+B-C-D=0$$

The second boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=0}=\left.\frac{du_2}{dx}\right|_{x=0}$$

which yields

$$(\alpha-k)A-(\alpha+k)B-(\beta-k)C$$
$$+(\beta+k)D=0$$

The third boundary condition is

$$u_1(a)=u_2(-b)$$

which yields

$$A\exp[j(\alpha-k)a]+B\exp[-j(\alpha+k)a]$$
$$=C\exp[j(\beta-k)(-b)]$$
$$+D\exp[-j(\beta+k)(-b)]$$

and can be written as

$$A\exp[j(\alpha-k)a]+B\exp[-j(\alpha+k)a]$$
$$-C\exp[-j(\beta-k)b]$$
$$-D\exp[j(\beta+k)b]=0$$

The fourth boundary condition is

$$\left.\frac{du_1}{dx}\right|_{x=a}=\left.\frac{du_2}{dx}\right|_{x=-b}$$

which yields

$$j(\alpha-k)A\exp[j(\alpha-k)a]$$
$$-j(\alpha+k)B\exp[-j(\alpha+k)a]$$
$$=j(\beta-k)C\exp[j(\beta-k)(-b)]$$
$$-j(\beta+k)D\exp[-j(\beta+k)(-b)]$$

and can be written as

$$(\alpha-k)A\exp[j(\alpha-k)a]$$
$$-(\alpha+k)B\exp[-j(\alpha+k)a]$$
$$-(\beta-k)C\exp[-j(\beta-k)b]$$
$$+(\beta+k)D\exp[j(\beta+k)b]=0$$

3.5

(b) (i) First point: $\alpha a=\pi$

Second point: By trial and error,

$$\alpha a=1.729\pi$$

(ii) First point: $\alpha a=2\pi$

Second point: By trial and error,

$$\alpha a=2.617\pi$$

3.6

(b) (i) First point: $\alpha a = \pi$

Second point: By trial and error,

$$\alpha a = 1.515\pi$$

(ii) First point: $\alpha a = 2\pi$

Second point: By trial and error,

$$\alpha a = 2.375\pi$$

3.7

$$P' \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

Let $ka = y$, $\alpha a = x$

Then

$$P' \frac{\sin x}{x} + \cos x = \cos y$$

Consider $\frac{d}{dy}$ of this function.

$$\frac{d}{dy}\left\{\left[P' \cdot (x)^{-1} \sin x\right] + \cos x\right\} = -\sin y$$

We find

$$P'\left\{(-1)(x)^{-2} \sin x \cdot \frac{dx}{dy} + (x)^{-1} \cos x \cdot \frac{dx}{dy}\right\}$$

$$-\sin x \frac{dx}{dy} = -\sin y$$

Then

$$\frac{dx}{dy}\left\{P'\left[\frac{-1}{x^2}\sin x + \frac{\cos x}{x}\right] - \sin x\right\} = -\sin y$$

For $y = ka = n\pi$, $n = 0, 1, 2, \ldots \Rightarrow \sin y = 0$

So that, in general,

$$\frac{dx}{dy} = 0 = \frac{d(\alpha a)}{d(ka)} = \frac{d\alpha}{dk}$$

And

$$\alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

So

$$\frac{d\alpha}{dk} = \frac{1}{2}\left(\frac{2mE}{\hbar^2}\right)^{-1/2}\left(\frac{2m}{\hbar^2}\right)\frac{dE}{dk}$$

This implies that

$$\frac{d\alpha}{dk} = 0 = \frac{dE}{dk} \quad \text{for } k = \frac{n\pi}{a}$$

3.8

(a) $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{\pi^2 \hbar^2}{2m_o a^2} = \frac{(\pi)^2 \left(1.054 \times 10^{-34}\right)^2}{2\left(9.11 \times 10^{-31}\right)\left(4.2 \times 10^{-10}\right)^2}$$

$$= 3.4114 \times 10^{-19} \text{ J}$$

From Problem 3.5

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 \left(1.054 \times 10^{-34}\right)^2}{2\left(9.11 \times 10^{-31}\right)\left(4.2 \times 10^{-10}\right)^2}$$

$$= 1.0198 \times 10^{-18} \text{ J}$$

$$\Delta E = E_2 - E_1$$

$$= 1.0198 \times 10^{-18} - 3.4114 \times 10^{-19}$$

$$= 6.7868 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{6.7868 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.24$ eV

(b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 \left(1.054 \times 10^{-34}\right)^2}{2\left(9.11 \times 10^{-31}\right)\left(4.2 \times 10^{-10}\right)^2}$$

$$= 1.3646 \times 10^{-18} \text{ J}$$

From Problem 3.5,

$$\alpha_4 a = 2.617\pi$$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}} \cdot a = 2.617\pi$$

$$E_4 = \frac{(2.617\pi)^2 \left(1.054 \times 10^{-34}\right)^2}{2\left(9.11 \times 10^{-31}\right)\left(4.2 \times 10^{-10}\right)^2}$$

$$= 2.3364 \times 10^{-18} \text{ J}$$

$$\Delta E = E_4 - E_3$$

$$= 2.3364 \times 10^{-18} - 1.3646 \times 10^{-18}$$

$$= 9.718 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{9.718 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.07$ eV

3.9

(a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19} \text{ J}$$

At $ka = 0$, By trial and error,

$$\alpha_o a = 0.859\pi$$

$$E_o = \frac{(0.859\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 2.5172 \times 10^{-19} \text{ J}$$

$$\Delta E = E_1 - E_o$$

$$= 3.4114 \times 10^{-19} - 2.5172 \times 10^{-19}$$

$$= 8.942 \times 10^{-20} \text{ J}$$

or $\Delta E = \dfrac{8.942 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.559$ eV

(b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18} \text{ J}$$

At $ka = \pi$. From Problem 3.5,

$$\alpha_2 a = 1.729\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.729\pi$$

$$E_2 = \frac{(1.729\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.0198 \times 10^{-18} \text{ J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646 \times 10^{-18} - 1.0198 \times 10^{-18}$$

$$= 3.4474 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{3.4474 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.15$ eV

3.10

(a) $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 3.4114 \times 10^{-19} \text{ J}$$

From Problem 3.6, $\alpha_2 a = 1.515\pi$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 7.830 \times 10^{-19} \text{ J}$$

$$\Delta E = E_2 - E_1$$

$$= 7.830 \times 10^{-19} - 3.4114 \times 10^{-19}$$

$$= 4.4186 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{4.4186 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.76$ eV

(b) $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.3646 \times 10^{-18} \text{ J}$$

From Problem 3.6, $\alpha_4 a = 2.375\pi$

$$\sqrt{\frac{2m_o E_4}{\hbar^2}} \cdot a = 2.375\pi$$

$$E_4 = \frac{(2.375\pi)^2 (1.054 \times 10^{-34})^2}{2(9.11 \times 10^{-31})(4.2 \times 10^{-10})^2}$$

$$= 1.9242 \times 10^{-18} \text{ J}$$

$$\Delta E = E_4 - E_3$$

$$= 1.9242 \times 10^{-18} - 1.3646 \times 10^{-18}$$

$$= 5.597 \times 10^{-19} \text{ J}$$

or $\Delta E = \dfrac{5.597 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.50$ eV

3.11

(a) At $ka = \pi$, $\alpha_1 a = \pi$

$$\sqrt{\frac{2m_o E_1}{\hbar^2}} \cdot a = \pi$$

$$E_1 = \frac{(\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 3.4114\times10^{-19}\text{ J}$$

At $ka = 0$, By trial and error,

$$\alpha_o a = 0.727\pi$$

$$\sqrt{\frac{2m_o E_o}{\hbar^2}} \cdot a = 0.727\pi$$

$$E_o = \frac{(0.727\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.8030\times10^{-19}\text{ J}$$

$$\Delta E = E_1 - E_o$$

$$= 3.4114\times10^{-19} - 1.8030\times10^{-19}$$

$$= 1.6084\times10^{-19}\text{ J}$$

or $\Delta E = \dfrac{1.6084\times10^{-19}}{1.6\times10^{-19}} = 1.005\text{ eV}$

(b) At $ka = 2\pi$, $\alpha_3 a = 2\pi$

$$\sqrt{\frac{2m_o E_3}{\hbar^2}} \cdot a = 2\pi$$

$$E_3 = \frac{(2\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-31})(4.2\times10^{-10})^2}$$

$$= 1.3646\times10^{-18}\text{ J}$$

At $ka = \pi$, From Problem 3.6,

$$\alpha_2 a = 1.515\pi$$

$$\sqrt{\frac{2m_o E_2}{\hbar^2}} \cdot a = 1.515\pi$$

$$E_2 = \frac{(1.515\pi)^2 (1.054\times10^{-34})^2}{2(9.11\times10^{-34})(4.2\times10^{-10})^2}$$

$$= 7.830\times10^{-19}\text{ J}$$

$$\Delta E = E_3 - E_2$$

$$= 1.3646\times10^{-18} - 7.830\times10^{-19}$$

$$= 5.816\times10^{-19}\text{ J}$$

or $\Delta E = \dfrac{5.816\times10^{-19}}{1.6\times10^{-19}} = 3.635\text{ eV}$

3.12

For $T = 100$ K,

$$E_g = 1.170 - \frac{(4.73\times10^{-4})(100)^2}{636+100} \Rightarrow$$

$$E_g = 1.164\text{ eV}$$

$T = 200$ K, $E_g = 1.147$ eV

$T = 300$ K, $E_g = 1.125$ eV

$T = 400$ K, $E_g = 1.097$ eV

$T = 500$ K, $E_g = 1.066$ eV

$T = 600$ K, $E_g = 1.032$ eV

3.13

The effective mass is given by

$$m^* = \left(\frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\right)^{-1}$$

We have

$$\frac{d^2E}{dk^2}(curve\ A) > \frac{d^2E}{dk^2}(curve\ B)$$

so that $m^*(curve\ A) < m^*(curve\ B)$

3.14

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2}\cdot\left|\frac{d^2E}{dk^2}\right|\right)^{-1}$$

We have that

$$\left|\frac{d^2E}{dk^2}\right|(curve\ A) > \left|\frac{d^2E}{dk^2}\right|(curve\ B)$$

so that $m_p^*(curve\ A) < m_p^*(curve\ B)$

3.15

Points A,B: $\dfrac{dE}{dk} < 0 \Rightarrow$ velocity in -x direction

Points C,D: $\dfrac{dE}{dk} > 0 \Rightarrow$ velocity in +x direction

Points A,D: $\dfrac{d^2E}{dk^2} < 0 \Rightarrow$

negative effective mass

Points B,C: $\dfrac{d^2E}{dk^2} > 0 \Rightarrow$

positive effective mass

3.16

For A: $E = C_i k^2$

At $k = 0.08\times10^{+10}\ \text{m}^{-1}$, $E = 0.05\,\text{eV}$

Or $E = (0.05)(1.6\times10^{-19}) = 8\times10^{-21}\ \text{J}$

So $8\times10^{-21} = C_1(0.08\times10^{10})^2$

$\Rightarrow C_1 = 1.25\times10^{-38}$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{(1.054\times10^{-34})^2}{2(1.25\times10^{-38})}$

$= 4.44\times10^{-31}\ \text{kg}$

or $m^* = \dfrac{4.4437\times10^{-31}}{9.11\times10^{-31}}\cdot m_o$

$m^* = 0.488\,m_o$

For B: $E = C_i k^2$

At $k = 0.08\times10^{+10}\ \text{m}^{-1}$, $E = 0.5\,\text{eV}$

Or $E = (0.5)(1.6\times10^{-19}) = 8\times10^{-20}\ \text{J}$

So $8\times10^{-20} = C_1(0.08\times10^{10})^2$

$\Rightarrow C_1 = 1.25\times10^{-37}$

Now $m^* = \dfrac{\hbar^2}{2C_1} = \dfrac{(1.054\times10^{-34})^2}{2(1.25\times10^{-37})}$

$= 4.44\times10^{-32}\ \text{kg}$

or $m^* = \dfrac{4.4437\times10^{-32}}{9.11\times10^{-31}}\cdot m_o$

$m^* = 0.0488\,m_o$

3.17

For A: $E - E_\upsilon = -C_2 k^2$

$-(0.025)(1.6\times10^{-19}) = -C_2(0.08\times10^{10})^2$

$\Rightarrow C_2 = 6.25\times10^{-39}$

$m^* = \dfrac{-\hbar^2}{2C_2} = \dfrac{-(1.054\times10^{-34})^2}{2(6.25\times10^{-39})}$

$= -8.8873\times10^{-31}\ \text{kg}$

or $m^* = \dfrac{-8.8873\times10^{-31}}{9.11\times10^{-31}}\cdot m_o$

$m^* = --0.976\,m_o$

For B: $E - E_\upsilon = -C_2 k^2$

$-(0.3)(1.6\times10^{-19}) = -C_2(0.08\times10^{10})^2$

$\Rightarrow C_2 = 7.5\times10^{-38}$

$m^* = \dfrac{-\hbar^2}{2C_2} = \dfrac{-(1.054\times10^{-34})^2}{2(7.5\times10^{-38})}$

$= -7.406\times10^{-32}\ \text{kg}$

or $m^* = \dfrac{-7.406\times10^{-32}}{9.11\times10^{-31}}\cdot m_o$

$m^* = -0.0813\,m_o$

3.18

(a) (i) $E = h\nu$

or $\nu = \dfrac{E}{h} = \dfrac{(1.42)(1.6\times10^{-19})}{6.625\times10^{-34}}$

$= 3.429\times10^{14}\ \text{Hz}$

(ii) $\lambda = \dfrac{hc}{E} = \dfrac{c}{\nu} = \dfrac{3\times10^{10}}{3.429\times10^{14}}$

$= 8.75\times10^{-5}\ \text{cm} = 875\,\text{nm}$

(b) (i) $\nu = \dfrac{E}{h} = \dfrac{(1.12)(1.6\times10^{-19})}{6.625\times10^{-34}}$

$= 2.705\times10^{14}\ \text{Hz}$

(ii) $\lambda = \dfrac{c}{\nu} = \dfrac{3\times10^{10}}{2.705\times10^{14}}$

$= 1.109\times10^{-4}\ \text{cm} = 1109\,\text{nm}$

3.19

(c) Curve A: Effective mass is a constant
Curve B: Effective mass is positive around $k = 0$, and is negative around $k = \pm\dfrac{\pi}{2}$.

3.20

$E = E_O - E_1\cos[\alpha(k - k_O)]$

Then

$\dfrac{dE}{dk} = (-E_1)(-\alpha)\sin[\alpha(k - k_O)]$

$= +E_1\alpha\sin[\alpha(k - k_O)]$

and

$\dfrac{d^2E}{dk^2} = E_1\alpha^2\cos[\alpha(k - k_O)]$

Then

$$\frac{1}{m^*}=\frac{1}{\hbar^2}\cdot\frac{d^2E}{dk^2}\bigg|_{k=k_o}=\frac{E_1\alpha^2}{\hbar^2}$$

or

$$m^*=\frac{\hbar^2}{E_1\alpha^2}$$

3.21

(a) $m_{dn}^*=4^{2/3}\left[(m_t)^2 m_l\right]^{1/3}$

$$=4^{2/3}\left[(0.082m_o)^2(1.64m_o)\right]^{1/3}$$

$$m_{dn}^*=0.56m_o$$

(b) $\frac{3}{m_{cn}^*}=\frac{2}{m_t}+\frac{1}{m_l}=\frac{2}{0.082m_o}+\frac{1}{1.64m_o}$

$$=\frac{24.39}{m_o}+\frac{0.6098}{m_o}$$

$$m_{cn}^*=0.12m_o$$

3.22

(a) $m_{dp}^*=\left[(m_{hh})^{3/2}+(m_{lh})^{3/2}\right]^{2/3}$

$$=\left[(0.45m_o)^{3/2}+(0.082m_o)^{3/2}\right]^{2/3}$$

$$=[0.30187+0.02348]^{2/3}\cdot m_o$$

$$m_{dp}^*=0.473m_o$$

(b) $m_{cp}^*=\frac{(m_{hh})^{3/2}+(m_{lh})^{3/2}}{(m_{hh})^{1/2}+(m_{lh})^{1/2}}$

$$=\frac{(0.45)^{3/2}+(0.082)^{3/2}}{(0.45)^{1/2}+(0.082)^{1/2}}\cdot m_o$$

$$m_{cp}^*=0.34m_o$$

3.23

For the 3-dimensional infinite potential well, $V(x)=0$ when $0<x<a$, $0<y<a$, and $0<z<a$. In this region, the wave equation is:

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2}+\frac{\partial^2\psi(x,y,z)}{\partial y^2}+\frac{\partial^2\psi(x,y,z)}{\partial z^2}+\frac{2mE}{\hbar^2}\psi(x,y,z)=0$$

Use separation of variables technique, so let

$$\psi(x,y,z)=X(x)Y(y)Z(z)$$

Substituting into the wave equation, we have

$$YZ\frac{\partial^2X}{\partial x^2}+XZ\frac{\partial^2Y}{\partial y^2}+XY\frac{\partial^2Z}{\partial z^2}+\frac{2mE}{\hbar^2}\cdot XYZ=0$$

Dividing by XYZ, we obtain

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2}+\frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2}+\frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2}+\frac{2mE}{\hbar^2}=0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2X}{\partial x^2}=-k_x^2\Rightarrow\frac{\partial^2X}{\partial x^2}+k_x^2X=0$$

The solution is of the form:

$$X(x)=A\sin k_xx+B\cos k_xx$$

Since $\psi(x,y,z)=0$ at $x=0$, then $X(0)=0$ so that $B=0$.

Also, $\psi(x,y,z)=0$ at $x=a$, so that $X(a)=0$. Then $k_xa=n_x\pi$ where $n_x=1,2,3,\ldots$

Similarly, we have

$$\frac{1}{Y}\cdot\frac{\partial^2Y}{\partial y^2}=-k_y^2 \quad\text{and}\quad \frac{1}{Z}\cdot\frac{\partial^2Z}{\partial z^2}=-k_z^2$$

From the boundary conditions, we find

$$k_ya=n_y\pi \quad\text{and}\quad k_za=n_z\pi$$

where

$$n_y=1,2,3,\ldots \quad\text{and}\quad n_z=1,2,3,\ldots$$

From the wave equation, we can write

$$-k_x^2-k_y^2-k_z^2+\frac{2mE}{\hbar^2}=0$$

The energy can be written as

$$E=E_{n_xn_yn_z}=\frac{\hbar^2}{2m}\left(n_x^2+n_y^2+n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

3.24

The total number of quantum states in the 3-dimensional potential well is given (in k-space) by

$$g_T(k)dk=\frac{\pi k^2dk}{\pi^3}\cdot a^3$$

where

$$k^2=\frac{2mE}{\hbar^2}$$

We can then write

$$k=\frac{\sqrt{2mE}}{\hbar}$$

Taking the differential, we obtain

$$dk = \frac{1}{\hbar}\cdot\sqrt{2m}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{E}}\cdot dE = \frac{1}{\hbar}\cdot\sqrt{\frac{m}{2E}}\cdot dE$$

Substituting these expressions into the density of states function, we have

$$g_T(E)dE = \frac{\pi a^3}{\pi^3}\left(\frac{2mE}{\hbar^2}\right)\cdot\frac{1}{\hbar}\cdot\sqrt{\frac{m}{2E}}\cdot dE$$

Noting that

$$\hbar = \frac{h}{2\pi}$$

this density of states function can be simplified and written as

$$g_T(E)dE = \frac{4\pi a^3}{h^3}(2m)^{3/2}\cdot\sqrt{E}\cdot dE$$

Dividing by a^3 will yield the density of states so that

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\cdot\sqrt{E}$$

3.25

For a one-dimensional infinite potential well,

$$\frac{2m_n^* E}{\hbar^2} = \frac{n^2\pi^2}{a^2} = k^2$$

Distance between quantum states

$$k_{n+1} - k_n = (n+1)\left(\frac{\pi}{a}\right) = (n)\left(\frac{\pi}{a}\right) = \frac{\pi}{a}$$

Now

$$g_T(k)dk = \frac{2\cdot dk}{\left(\frac{\pi}{a}\right)}$$

Now

$$k = \frac{1}{\hbar}\cdot\sqrt{2m_n^* E}$$

$$dk = \frac{1}{\hbar}\cdot\frac{1}{2}\cdot\sqrt{\frac{2m_n^*}{E}}\cdot dE$$

Then

$$g_T(E)dE = \frac{2a}{\pi}\cdot\frac{1}{2\hbar}\cdot\sqrt{\frac{2m_n^*}{E}}\cdot dE$$

Divide by the "volume" a, so

$$g(E) = \frac{1}{\hbar\pi}\cdot\sqrt{\frac{2m_n^*}{E}}$$

So

$$g(E) = \frac{1}{(1.054\times10^{-34})(\pi)}\cdot\frac{\sqrt{2(0.067)(9.11\times10^{-31})}}{\sqrt{E}}$$

$$g(E) = \frac{1.055\times10^{18}}{\sqrt{E}}\ \text{m}^{-3}\ \text{J}^{-1}$$

3.26

(a) Silicon, $m_n^* = 1.08m_o$

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

$$g_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2kT}\sqrt{E-E_c}\cdot dE$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(E-E_c)^{3/2}\Big|_{E_c}^{E_c+2kT}$$

$$= \frac{4\pi(2m_n^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= \frac{4\pi[2(1.08)(9.11\times10^{-31})]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= (7.953\times10^{55})(2kT)^{3/2}$$

(i) At $T = 300$ K, $kT = 0.0259$ eV

$$= (0.0259)(1.6\times10^{-19})$$

$$= 4.144\times10^{-21}\ \text{J}$$

Then $g_c = (7.953\times10^{55})[2(4.144\times10^{-21})]^{3/2}$

$$= 6.0\times10^{25}\ \text{m}^{-3}$$

or $\quad g_c = 6.0\times10^{19}\ \text{cm}^{-3}$

(ii) At $T = 400$ K, $kT = (0.0259)\left(\frac{400}{300}\right)$

$$= 0.034533\ \text{eV}$$

$$= (0.034533)(1.6\times10^{-19})$$

$$= 5.5253\times10^{-21}\ \text{J}$$

Then

$$g_c = (7.953\times10^{55})[2(5.5253\times10^{-21})]^{3/2}$$

$$= 9.239\times10^{25}\ \text{m}^{-3}$$

or $\quad g_c = 9.24\times10^{19}\ \text{cm}^{-3}$

(b) GaAs, $m_n^* = 0.067m_o$

$$g_c = \frac{4\pi[2(0.067)(9.11\times10^{-31})]^{3/2}}{(6.625\times10^{-34})^3}\cdot\frac{2}{3}\cdot(2kT)^{3/2}$$

$$= (1.2288\times10^{54})(2kT)^{3/2}$$

(i) At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_c = \left(1.2288\times10^{54}\right)\left[2\left(4.144\times10^{-21}\right)\right]^{3/2}$$
$$= 9.272\times10^{23}\ \text{m}^{-3}$$

or $g_c = 9.27\times10^{17}$ cm^{-3}

(ii) At $T = 400$ K, $kT = 5.5253\times10^{-21}$ J

$$g_c = \left(1.2288\times10^{54}\right)\left[2\left(5.5253\times10^{-21}\right)\right]^{3/2}$$
$$= 1.427\times10^{24}\ \text{m}^{-3}$$
$$g_c = 1.43\times10^{18}\ \text{cm}^{-3}$$

(i)At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_\upsilon = \left(2.3564\times10^{55}\right)\left[3\left(4.144\times10^{-21}\right)\right]^{3/2}$$
$$= 3.266\times10^{25}\ \text{m}^{-3}$$

or $g_\upsilon = 3.27\times10^{19}$ cm^{-3}

(ii)At $T = 400$ K, $kT = 5.5253\times10^{-21}$ J

$$g_\upsilon = \left(2.3564\times10^{55}\right)\left[3\left(5.5253\times10^{-21}\right)\right]^{3/2}$$
$$= 5.029\times10^{25}\ \text{m}^{-3}$$

or $g_\upsilon = 5.03\times10^{19}$ cm^{-3}

3.27

(a) Silicon, $m_p^* = 0.56m_o$

$$g_\upsilon(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_\upsilon - E}$$

$$g_\upsilon = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_\upsilon-3kT}^{E_\upsilon}\sqrt{E_\upsilon - E}\cdot dE$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_\upsilon - E\right)^{3/2}\Big|_{E_\upsilon-3kT}^{E_\upsilon}$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[-\left(3kT\right)^{3/2}\right]$$

$$= \frac{4\pi\left[2\left(0.56\right)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= \left(2.969\times10^{55}\right)\left(3kT\right)^{3/2}$$

(i)At $T = 300$ K, $kT = 4.144\times10^{-21}$ J

$$g_\upsilon = \left(2.969\times10^{55}\right)\left[3\left(4.144\times10^{-21}\right)\right]^{3/2}$$
$$= 4.116\times10^{25}\ \text{m}^{-3}$$

or $g_\upsilon = 4.12\times10^{19}$ cm^{-3}

(ii)At $T = 400$ K, $kT = 5.5253\times10^{-21}$ J

$$g_\upsilon = \left(2.969\times10^{55}\right)\left[3\left(5.5253\times10^{-21}\right)\right]^{3/2}$$
$$= 6.337\times10^{25}\ \text{m}^{-3}$$

or $g_\upsilon = 6.34\times10^{19}$ cm^{-3}

(b) GaAs, $m_p^* = 0.48m_o$

$$g_\upsilon = \frac{4\pi\left[2\left(0.48\right)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\left(\frac{2}{3}\right)\left(3kT\right)^{3/2}$$

$$= \left(2.3564\times10^{55}\right)\left(3kT\right)^{3/2}$$

3.28

(a) $$g_c(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_c}$$

$$= \frac{4\pi\left[2\left(1.08\right)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\sqrt{E - E_c}$$

$$= 1.1929\times10^{56}\sqrt{E - E_c}$$

For $E = E_c$; $g_c = 0$

$E = E_c + 0.1$ eV; $g_c = 1.509\times10^{46}$ m^{-3} J^{-1}

$E = E_c + 0.2$ eV; $= 2.134\times10^{46}$ m^{-3} J^{-1}

$E = E_c + 0.3$ eV; $= 2.614\times10^{46}$ m^{-3} J^{-1}

$E = E_c + 0.4$ eV; $= 3.018\times10^{46}$ m^{-3} J^{-1}

(b) $$g_\upsilon = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_\upsilon - E}$$

$$= \frac{4\pi\left[2\left(0.56\right)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\sqrt{E_\upsilon - E}$$

$$= 4.4541\times10^{55}\sqrt{E_\upsilon - E}$$

For $E = E_\upsilon$; $g_\upsilon = 0$

$E = E_\upsilon - 0.1$ eV; $g_\upsilon = 5.634\times10^{45}$ m^{-3} J^{-1}

$E = E_\upsilon - 0.2$ eV; $= 7.968\times10^{45}$ m^{-3} J^{-1}

$E = E_\upsilon - 0.3$ eV; $= 9.758\times10^{45}$ m^{-3} J^{-1}

$E = E_\upsilon - 0.4$ eV; $= 1.127\times10^{46}$ m^{-3} J^{-1}

3.29

(a) $$\frac{g_c}{g_\upsilon} = \frac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\frac{1.08}{0.56}\right)^{3/2} = 2.68$$

(b) $$\frac{g_c}{g_\upsilon} = \frac{\left(m_n^*\right)^{3/2}}{\left(m_p^*\right)^{3/2}} = \left(\frac{0.067}{0.48}\right)^{3/2} = 0.0521$$

3.30

Plot

3.31

(a) $W_i = \dfrac{g_i!}{N_i!(g_i - N_i)!} = \dfrac{10!}{(7!)(10-7)!}$

$= \dfrac{(10)(9)(8)(7!)}{(7!)(3!)} = \dfrac{(10)(9)(8)}{(3)(2)(1)} = 120$

(b) (i) $W_i = \dfrac{12!}{(10!)(12-10)!} = \dfrac{(12)(11)(10!)}{(10!)(2)(1)}$

$= 66$

(ii) $W_i = \dfrac{12!}{(8!)(12-8)!} = \dfrac{(12)(11)(10)(9)(8!)}{(8!)(4)(3)(2)(1)}$

$= 495$

3.32

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

(a) $E - E_F = kT$, $f(E) = \dfrac{1}{1+\exp(1)} \Rightarrow$

$f(E) = 0.269$

(b) $E - E_F = 5kT$, $f(E) = \dfrac{1}{1+\exp(5)} \Rightarrow$

$f(E) = 6.69\times10^{-3}$

(c) $E - E_F = 10kT$, $f(E) = \dfrac{1}{1+\exp(10)} \Rightarrow$

$f(E) = 4.54\times10^{-5}$

3.33

$$1 - f(E) = 1 - \frac{1}{1+\exp\left(\dfrac{E-E_F}{kT}\right)}$$

or

$$1 - f(E) = \frac{1}{1+\exp\left(\dfrac{E_F-E}{kT}\right)}$$

(a) $E_F - E = kT$, $1 - f(E) = 0.269$

(b) $E_F - E = 5kT$, $1 - f(E) = 6.69\times10^{-3}$

(c) $E_F - E = 10kT$, $1 - f(E) = 4.54\times10^{-5}$

3.34

(a) $f_F \cong \exp\left[\dfrac{-(E-E_F)}{kT}\right]$

$E = E_c$; $f_F = \exp\left[\dfrac{-0.30}{0.0259}\right] = 9.32\times10^{-6}$

$E_c + \dfrac{kT}{2}$; $f_F = \exp\left[\dfrac{-(0.30+0.0259/2)}{0.0259}\right]$

$= 5.66\times10^{-6}$

$E_c + kT$; $f_F = \exp\left[\dfrac{-(0.30+0.0259)}{0.0259}\right]$

$= 3.43\times10^{-6}$

$E_c + \dfrac{3kT}{2}$; $f_F = \exp\left[\dfrac{-(0.30+3(0.0259/2))}{0.0259}\right]$

$= 2.08\times10^{-6}$

$E_c + 2kT$; $f_F = \exp\left[\dfrac{-(0.30+2(0.0259))}{0.0259}\right]$

$= 1.26\times10^{-6}$

(b) $1 - f_F = 1 - \dfrac{1}{1+\exp\left[\dfrac{E-E_F}{kT}\right]}$

$\cong \exp\left[\dfrac{-(E_F-E)}{kT}\right]$

$E = E_\upsilon$; $1 - f_F = \exp\left[\dfrac{-0.25}{0.0259}\right] = 6.43\times10^{-5}$

$E_\upsilon - \dfrac{kT}{2}$; $1 - f_F = \exp\left[\dfrac{-(0.25+0.0259/2)}{0.0259}\right]$

$= 3.90\times10^{-5}$

$E_\upsilon - kT$; $1 - f_F = \exp\left[\dfrac{-(0.25+0.0259)}{0.0259}\right]$

$= 2.36\times10^{-5}$

$E_\upsilon - \dfrac{3kT}{2}$;

$1 - f_F = \exp\left[\dfrac{-(0.25+3(0.0259/2))}{0.0259}\right]$

$= 1.43\times10^{-5}$

$E_\upsilon - 2kT$;

$1 - f_F = \exp\left[\dfrac{-(0.25+2(0.0259))}{0.0259}\right]$

$= 8.70\times10^{-6}$

3.35

$$f_F = \exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_c + kT - E_F)}{kT}\right]$$

and

$$1 - f_F = \exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$= \exp\left[\frac{-(E_F - (E_\upsilon - kT))}{kT}\right]$$

So $\exp\left[\frac{-(E_c + kT - E_F)}{kT}\right]$

$$= \exp\left[\frac{-(E_F - E_\upsilon + kT)}{kT}\right]$$

Then $E_c + kT - E_F = E_F - E_\upsilon + kT$

Or $E_F = \dfrac{E_c + E_\upsilon}{2} = E_{midgap}$

3.36

$$E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

For $n = 6$, Filled state

$$E_6 = \frac{(1.054\times10^{-34})^2 (6)^2 (\pi)^2}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$= 1.5044\times10^{-18}$ J

or $E_6 = \dfrac{1.5044\times10^{-18}}{1.6\times10^{-19}} = 9.40$ eV

For $n = 7$, Empty state

$$E_7 = \frac{(1.054\times10^{-34})^2 (7)^2 (\pi)^2}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$= 2.048\times10^{-18}$ J

or $E_7 = \dfrac{2.048\times10^{-18}}{1.6\times10^{-19}} = 12.8$ eV

Therefore $9.40 < E_F < 12.8$ eV

3.37

(a) For a 3-D infinite potential well

$$\frac{2mE}{\hbar^2} = \left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

For 5 electrons, the 5th electron occupies the quantum state $n_x = 2, n_y = 2, n_z = 1$; so

$$E_5 = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi}{a}\right)^2$$

$$= \frac{(1.054\times10^{-34})^2 (\pi)^2 \left(2^2 + 2^2 + 1^2\right)}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$= 3.761\times10^{-19}$ J

or $E_5 = \dfrac{3.761\times10^{-19}}{1.6\times10^{-19}} = 2.35$ eV

For the next quantum state, which is empty, the quantum state is $n_x = 1, n_y = 2, n_z = 2$. This quantum state is at the same energy, so

$E_F = 2.35$ eV

(b) For 13 electrons, the 13th electron occupies the quantum state $n_x = 3, n_y = 2, n_z = 3$; so

$$E_{13} = \frac{(1.054\times10^{-34})^2 (\pi)^2 \left(3^2 + 2^2 + 3^2\right)}{2(9.11\times10^{-31})(12\times10^{-10})^2}$$

$= 9.194\times10^{-19}$ J

or $E_{13} = \dfrac{9.194\times10^{-19}}{1.6\times10^{-19}} = 5.746$ eV

The 14th electron would occupy the quantum state $n_x = 2, n_y = 3, n_z = 3$. This state is at the same energy, so

$E_F = 5.746$ eV

3.38

The probability of a state at $E_1 = E_F + \Delta E$ being occupied is

$$f_1(E_1) = \frac{1}{1 + \exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)}$$

The probability of a state at $E_2 = E_F - \Delta E$ being empty is

$$1 - f_2(E_2) = 1 - \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1 + \exp\left(\frac{-\Delta E}{kT}\right)} = \frac{\exp\left(\frac{-\Delta E}{kT}\right)}{1 + \exp\left(\frac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)}$$

so $f_1(E_1) = 1 - f_2(E_2)$ Q.E.D.

3.39

(a) At energy E_1, we want

$$\frac{\dfrac{1}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}{\dfrac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}}=0.01$$

This expression can be written as

$$\frac{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}{\exp\left(\dfrac{E_1-E_F}{kT}\right)}-1=0.01$$

or

$$1=(0.01)\exp\left(\frac{E_1-E_F}{kT}\right)$$

Then

$$E_1=E_F+kT\ln(100)$$

or

$$E_1=E_F+4.6kT$$

(b)

At $E=E_F+4.6kT$,

$$f(E_1)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}=\frac{1}{1+\exp(4.6)}$$

which yields

$$f(E_1)=0.00990\cong 0.01$$

3.40

(a)

$$f_F=\exp\left[\frac{-(E-E_F)}{kT}\right]=\exp\left[\frac{-(5.80-5.50)}{0.0259}\right]$$

$$=9.32\times10^{-6}$$

(b) $kT=(0.0259)\left(\dfrac{700}{300}\right)=0.060433\text{ eV}$

$$f_F=\exp\left[\frac{-0.30}{0.060433}\right]=6.98\times10^{-3}$$

(c) $1-f_F\cong\exp\left[\dfrac{-(E_F-E)}{kT}\right]$

$$0.02=\exp\left[\frac{-0.25}{kT}\right]$$

or $\exp\left[\dfrac{+0.25}{kT}\right]=\dfrac{1}{0.02}=50$

$$\frac{0.25}{kT}=\ln(50)$$

or

$$kT=\frac{0.25}{\ln(50)}=0.063906=(0.0259)\left(\frac{T}{300}\right)$$

which yields $T=740\text{ K}$

3.41

(a)

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.0259}\right)}=0.00304$$

or 0.304%

(b) At $T=1000\text{ K}$, $kT=0.08633\text{ eV}$

Then

$$f(E)=\frac{1}{1+\exp\left(\dfrac{7.15-7.0}{0.08633}\right)}=0.1496$$

or 14.96%

(c) $f(E)=\dfrac{1}{1+\exp\left(\dfrac{6.85-7.0}{0.0259}\right)}=0.997$

or 99.7%

(d)

At $E=E_F$, $f(E)=\dfrac{1}{2}$ for all temperatures

3.42

(a) For $E=E_1$

$$f(E)=\frac{1}{1+\exp\left(\dfrac{E_1-E_F}{kT}\right)}\cong\exp\left[\frac{-(E_1-E_F)}{kT}\right]$$

Then

$$f(E_1)=\exp\left(\frac{-0.30}{0.0259}\right)=9.32\times10^{-6}$$

For $E=E_2$, $E_F-E_2=1.12-0.30=0.82\text{ eV}$

Then

$$1-f(E)=1-\frac{1}{1+\exp\left(\dfrac{-0.82}{0.0259}\right)}$$

or

$$1-f(E)\cong 1-\left[1-\exp\left(\frac{-0.82}{0.0259}\right)\right]$$
$$=\exp\left(\frac{-0.82}{0.0259}\right)=1.78\times10^{-14}$$

(b) For $E_F - E_2 = 0.4$ eV,
$$E_1 - E_F = 0.72\text{ eV}$$

At $E = E_1$,
$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-0.72}{0.0259}\right)$$
or
$$f(E)=8.45\times10^{-13}$$
At $E = E_2$,
$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$
$$=\exp\left(\frac{-0.4}{0.0259}\right)$$
or
$$1-f(E)=1.96\times10^{-7}$$

3.43

(a) At $E = E_1$
$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-0.30}{0.0259}\right)$$
or
$$f(E)=9.32\times10^{-6}$$
At $E = E_2$, $E_F - E_2 = 1.42 - 0.3 = 1.12$ eV
So
$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$
$$=\exp\left(\frac{-1.12}{0.0259}\right)$$
or
$$1-f(E)=1.66\times10^{-19}$$

(b) For $E_F - E_2 = 0.4$,
$$E_1 - E_F = 1.02\text{ eV}$$

At $E = E_1$,
$$f(E)=\exp\left[\frac{-(E_1-E_F)}{kT}\right]=\exp\left(\frac{-1.02}{0.0259}\right)$$
or
$$f(E)=7.88\times10^{-18}$$
At $E = E_2$,
$$1-f(E)=\exp\left[\frac{-(E_F-E_2)}{kT}\right]$$
$$=\exp\left(\frac{-0.4}{0.0259}\right)$$
or $1-f(E)=1.96\times10^{-7}$

3.44

$$f(E)=\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^{-1}$$
so
$$\frac{df(E)}{dE}=(-1)\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^{-2}$$
$$\times\left(\frac{1}{kT}\right)\exp\left(\frac{E-E_F}{kT}\right)$$
or
$$\frac{df(E)}{dE}=\frac{\left(\frac{-1}{kT}\right)\exp\left(\frac{E-E_F}{kT}\right)}{\left[1+\exp\left(\frac{E-E_F}{kT}\right)\right]^2}$$

(a) At $T = 0$ K, For
$$E<E_F \Rightarrow \exp(-\infty)=0\Rightarrow \frac{df}{dE}=0$$
$$E>E_F \Rightarrow \exp(+\infty)=+\infty\Rightarrow \frac{df}{dE}=0$$
At $E=E_F \Rightarrow \dfrac{df}{dE}=-\infty$

(b) At $T = 300$ K, $kT = 0.0259$ eV

For $E << E_F$, $\dfrac{df}{dE}=0$

For $E >> E_F$, $\dfrac{df}{dE}=0$

At $E = E_F$,
$$\frac{df}{dE}=\frac{\left(\frac{-1}{0.0259}\right)(1)}{(1+1)^2}=-9.65\,(\text{eV})^{-1}$$

(c) At $T = 500\,\text{K}$, $kT = 0.04317\text{ eV}$

For $E << E_F$, $\dfrac{df}{dE} = 0$

For $E >> E_F$, $\dfrac{df}{dE} = 0$

At $E = E_F$,

$$\frac{df}{dE} = \frac{\left(\frac{-1}{0.04317}\right)(1)}{(1+1)^2} = -5.79\,(\text{eV})^{-1}$$

3.45

(a) At $E = E_{midgap}$,

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12\,\text{eV}$,

$$f(E) = \frac{1}{1+\exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$$f(E) = 4.07\times10^{-10}$$

Ge: $E_g = 0.66\,\text{eV}$

$$f(E) = \frac{1}{1+\exp\left[\frac{0.66}{2(0.0259)}\right]}$$

or

$$f(E) = 2.93\times10^{-6}$$

GaAs: $E_g = 1.42\,\text{eV}$

$$f(E) = \frac{1}{1+\exp\left[\frac{1.42}{2(0.0259)}\right]}$$

or

$$f(E) = 1.24\times10^{-12}$$

(b) Using the results of Problem 3.38, the answers to part (b) are exactly the same as those given in part (a).

3.46

(a) $f_F = \exp\left[\dfrac{-(E-E_F)}{kT}\right]$

$$10^{-8} = \exp\left[\frac{-0.60}{kT}\right]$$

or $\dfrac{0.60}{kT} = \ln(10^{+8})$

$$kT = \frac{0.60}{\ln(10^8)} = 0.032572\,\text{eV}$$

$$0.032572 = (0.0259)\left(\frac{T}{300}\right)$$

so $T = 377\,\text{K}$

(b) $10^{-6} = \exp\left[\dfrac{-0.60}{kT}\right]$

$$\frac{0.60}{kT} = \ln(10^{+6})$$

$$kT = \frac{0.60}{\ln(10^6)} = 0.043429$$

$$0.043429 = (0.0259)\left(\frac{T}{300}\right)$$

or $T = 503\,\text{K}$

3.47

(a) At $T = 200\,\text{K}$,

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267\text{ eV}$$

$$f_F = 0.05 = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

$$\exp\left(\frac{E-E_F}{kT}\right) = \frac{1}{0.05} - 1 = 19$$

$$E - E_F = kT\ln(19) = (0.017267)\ln(19) = 0.05084\,\text{eV}$$

By symmetry, for $f_F = 0.95$,

$$E - E_F = -0.05084\,\text{eV}$$

Then $\Delta E = 2(0.05084) = 0.1017\text{ eV}$

(b) $T = 400\,\text{K}$, $kT = 0.034533\text{ eV}$

For $f_F = 0.05$, from part (a),

$$E - E_F = kT\ln(19) = (0.034533)\ln(19) = 0.10168\,\text{eV}$$

Then $\Delta E = 2(0.10168) = 0.2034\text{ eV}$

Chapter 4

4.1

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

$$= N_{cO} N_{\upsilon O}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

where N_{cO} and $N_{\upsilon O}$ are the values at 300 K.

(a) Silicon

T (K)	kT (eV)	n_i (cm^{-3})
200	0.01727	7.68×10^{4}
400	0.03453	2.38×10^{12}
600	0.0518	9.74×10^{14}

(b) Germanium (c) GaAs

T (K)	n_i (cm^{-3})	n_i (cm^{-3})
200	2.16×10^{10}	1.38
400	8.60×10^{14}	3.28×10^{9}
600	3.82×10^{16}	5.72×10^{12}

4.2

Plot

4.3

(a) $n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$

$$\left(5\times10^{11}\right)^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$2.5\times10^{23} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 367.5\text{ K}$

(b)

$$n_i^2 = \left(5\times10^{12}\right)^2 = 2.5\times10^{25}$$

$$= \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error, $T \cong 417.5\text{ K}$

4.4

At $T = 200\text{ K}$, $kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267\text{ eV}$

At $T = 400\text{ K}$, $kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533\text{ eV}$

$$\frac{n_i^2(400)}{n_i^2(200)} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^{2}\right)^2} = 3.025\times10^{17}$$

$$= \frac{\left(\frac{400}{300}\right)^3}{\left(\frac{200}{300}\right)^3} \times \frac{\exp\left[\frac{-E_g}{0.034533}\right]}{\exp\left[\frac{-E_g}{0.017267}\right]}$$

$$= 8\exp\left[\frac{E_g}{0.017267} - \frac{E_g}{0.034533}\right]$$

$$3.025\times10^{17} = 8\exp\left[E_g(57.9139 - 28.9578)\right]$$

or

$$E_g(28.9561) = \ln\left(\frac{3.025\times10^{17}}{8}\right) = 38.1714$$

or $E_g = 1.318\text{ eV}$

Now

$$\left(7.70\times10^{10}\right)^2 = N_{co} N_{\upsilon o}\left(\frac{400}{300}\right)^3 \times\exp\left(\frac{-1.318}{0.034533}\right)$$

$$5.929\times10^{21} = N_{co} N_{\upsilon o}(2.370)\left(2.658\times10^{-17}\right)$$

so $N_{co} N_{\upsilon o} = 9.41\times10^{37}\text{ cm}^{-6}$

4.5

$$\frac{n_i(B)}{n_i(A)} = \frac{\exp\left(\dfrac{-1.10}{kT}\right)}{\exp\left(\dfrac{-0.90}{kT}\right)} = \exp\left(\frac{-0.20}{kT}\right)$$

For $T = 200\,\text{K}$, $kT = 0.017267$ eV
For $T = 300\,\text{K}$, $kT = 0.0259$ eV
For $T = 400\,\text{K}$, $kT = 0.034533$ eV

(a) For $T = 200\,\text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.017267}\right) = 9.325\times10^{-6}$$

(b) For $T = 300\,\text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.0259}\right) = 4.43\times10^{-4}$$

(c) For $T = 400\,\text{K}$,

$$\frac{n_i(B)}{n_i(A)} = \exp\left(\frac{-0.20}{0.034533}\right) = 3.05\times10^{-3}$$

4.6

(a) $$g_c f_F \propto \sqrt{E - E_c}\,\exp\left[\frac{-(E - E_F)}{kT}\right]$$

$$\propto \sqrt{E - E_c}\,\exp\left[\frac{-(E - E_c)}{kT}\right]$$

$$\times\exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

Let $E - E_c = x$

Then $g_c f_F \propto \sqrt{x}\,\exp\left(\dfrac{-x}{kT}\right)$

To find the maximum value:

$$\frac{d(g_c f_F)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right)$$

$$-\frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right) = 0$$

which yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

The maximum value occurs at

$$E = E_c + \frac{kT}{2}$$

(b)

$$g_\upsilon(1 - f_F) \propto \sqrt{E_\upsilon - E}\,\exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$\propto \sqrt{E_\upsilon - E}\,\exp\left[\frac{-(E_\upsilon - E)}{kT}\right]$$

$$\times\exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

Let $E_\upsilon - E = x$

Then $g_\upsilon(1 - f_F) \propto \sqrt{x}\,\exp\left(\dfrac{-x}{kT}\right)$

To find the maximum value

$$\frac{d[g_\upsilon(1 - f_F)]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\,\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2}$$

or

$$E = E_\upsilon - \frac{kT}{2}$$

4.7

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_c}\,\exp\left[\dfrac{-(E_1 - E_c)}{kT}\right]}{\sqrt{E_2 - E_c}\,\exp\left[\dfrac{-(E_2 - E_c)}{kT}\right]}$$

where

$$E_1 = E_c + 4kT \quad \text{and} \quad E_2 = E_c + \frac{kT}{2}$$

Then

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\dfrac{kT}{2}}}\exp\left[\frac{-(E_1 - E_2)}{kT}\right]$$

$$= 2\sqrt{2}\,\exp\left[-\left(4 - \frac{1}{2}\right)\right] = 2\sqrt{2}\,\exp(-3.5)$$

or

$$\frac{n(E_1)}{n(E_2)} = 0.0854$$

4.8

Plot

4.9
Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56 m_o$, $m_n^* = 1.08 m_o$

$$E_{Fi} - E_{midgap} = -0.0128 \text{ eV}$$

Germanium: $m_p^* = 0.37 m_o$, $m_n^* = 0.55 m_o$

$$E_{Fi} - E_{midgap} = -0.0077 \text{ eV}$$

Gallium Arsenide: $m_p^* = 0.48 m_o$,

$$m_n^* = 0.067 m_o$$

$$E_{Fi} - E_{midgap} = +0.0382 \text{ eV}$$

4.11

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_\upsilon}{N_c}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04\times10^{19}}{2.8\times10^{19}}\right) = -0.4952(kT)$$

T (K)	kT (eV)	($E_{Fi} - E_{midgap}$)(eV)
200	0.01727	-0.0086
400	0.03453	-0.0171
600	0.0518	-0.0257

4.12

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{0.70}{1.21}\right)$$

$$\Rightarrow -10.63 \text{ meV}$$

(b) $$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.75}{0.080}\right)$$

$$\Rightarrow +43.47 \text{ meV}$$

4.13
Let $g_c(E) = K = \text{constant}$
Then

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= K \int_{E_c}^{\infty} \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\cong K \int_{E_c}^{\infty} \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_c}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E_c - E_F) + (E - E_c)$$

so that

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_c-E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c-E_F)}{kT}\right] \int_0^{\infty} \exp(-\eta) d\eta$$

which becomes

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

4.14
Let $g_c(E) = C_1(E - E_c)$ for $E \ge E_c$
Then

$$n_o = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$= C_1 \int_{E_c}^{\infty} \frac{(E-E_c)}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\cong C_1 \int_{E_c}^{\infty} (E-E_C)\exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_c}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E - E_c) + (E_c - E_F)$$

Then

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right] \times \int_{E_c}^{\infty} (E - E_c)\exp\left[\frac{-(E - E_c)}{kT}\right] dE$$

or

$$n_o = C_1 \exp\left[\frac{-(E_c - E_F)}{kT}\right] \times \int_0^{\infty} (kT)(\eta)[\exp(-\eta)](kT)\,d\eta$$

We find that

$$\int_0^{\infty} \eta \exp(-\eta)\,d\eta = \exp(-\eta)(-\eta - 1)\Big|_0^{\infty} = +1$$

So

$$n_o = C_1 (kT)^2 \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

4.15

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For germanium, $\in_r = 16$, $m^* = 0.55 m_o$

Then

$$r_1 = (16)\left(\frac{1}{0.55}\right)a_o = (29)(0.53)$$

or

$$r_1 = 15.4\ \mathring{A}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6)\,\text{eV}$$

$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow E = 0.029\ \text{eV}$$

4.16

We have $\dfrac{r_1}{a_o} = \in_r \left(\dfrac{m_o}{m^*}\right)$

For gallium arsenide, $\in_r = 13.1$,

$$m^* = 0.067 m_o$$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53) = 104\ \mathring{A}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or

$$E = 0.0053\,\text{eV}$$

4.17

(a) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{7\times10^{15}}\right)$$

$$= 0.2148\ \text{eV}$$

(b) $E_F - E_\upsilon = E_g - (E_c - E_F)$

$$= 1.12 - 0.2148 = 0.90518\ \text{eV}$$

(c) $p_o = N_\upsilon \exp\left[\dfrac{-(E_F - E_\upsilon)}{kT}\right]$

$$= (1.04\times10^{19})\exp\left[\frac{-0.90518}{0.0259}\right]$$

$$= 6.90\times10^3\ \text{cm}^{-3}$$

(d) Holes

(e) $E_F - E_{Fi} = kT \ln\left(\dfrac{n_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{7\times10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.338\ \text{eV}$$

4.18

(a) $E_F - E_\upsilon = kT \ln\left(\dfrac{N_\upsilon}{p_o}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{2\times10^{16}}\right)$$

$$= 0.162\ \text{eV}$$

(b) $E_c - E_F = E_g - (E_F - E_\upsilon)$

$$= 1.12 - 0.162 = 0.958\ \text{eV}$$

(c) $n_o = (2.8\times10^{19})\exp\left(\dfrac{-0.958}{0.0259}\right)$

$$= 2.41\times10^3\ \text{cm}^{-3}$$

(d) $E_{Fi} - E_F = kT \ln\left(\dfrac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.365\text{ eV}$$

4.19

(a) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{2\times10^{5}}\right)$$

$$= 0.8436\text{ eV}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F)$$

$$= 1.12 - 0.8436$$

$$E_F - E_\upsilon = 0.2764\text{ eV}$$

(b) $p_o = (1.04\times10^{19})\exp\left(\dfrac{-0.27637}{0.0259}\right)$

$$= 2.414\times10^{14}\text{ cm}^{-3}$$

(c) p-type

4.20

(a) $kT = (0.0259)\left(\dfrac{375}{300}\right) = 0.032375\text{ eV}$

$$n_o = (4.7\times10^{17})\left(\frac{375}{300}\right)^{3/2}\exp\left[\frac{-0.28}{0.032375}\right]$$

$$= 1.15\times10^{14}\text{ cm}^{-3}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.28$$

$$= 1.14\text{ eV}$$

$$p_o = (7\times10^{18})\left(\frac{375}{300}\right)^{3/2}\exp\left[\frac{-1.14}{0.032375}\right]$$

$$= 4.99\times10^{3}\text{ cm}^{-3}$$

(b) $E_c - E_F = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{1.15\times10^{14}}\right)$

$$= 0.2154\text{ eV}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.42 - 0.2154$$

$$= 1.2046\text{ eV}$$

$$p_o = (7\times10^{18})\exp\left[\frac{-1.2046}{0.0259}\right]$$

$$= 4.42\times10^{-2}\text{ cm}^{-3}$$

4.21

(a) $kT = (0.0259)\left(\dfrac{375}{300}\right) = 0.032375\text{ eV}$

$$n_o = (2.8\times10^{19})\left(\frac{375}{300}\right)^{3/2}\exp\left[\frac{-0.28}{0.032375}\right]$$

$$= 6.86\times10^{15}\text{ cm}^{-3}$$

$$E_F - E_\upsilon = E_g - (E_c - E_F) = 1.12 - 0.28$$

$$= 0.840\text{ eV}$$

$$p_o = (1.04\times10^{19})\left(\frac{375}{300}\right)^{3/2}\exp\left[\frac{-0.840}{0.032375}\right]$$

$$= 7.84\times10^{7}\text{ cm}^{-3}$$

(b) $E_c - E_F = kT \ln\left(\dfrac{N_c}{n_o}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6.862\times10^{15}}\right)$$

$$= 0.2153\text{ eV}$$

$$E_F - E_\upsilon = 1.12 - 0.2153 = 0.9047\text{ eV}$$

$$p_o = (1.04\times10^{19})\exp\left[\frac{-0.904668}{0.0259}\right]$$

$$= 7.04\times10^{3}\text{ cm}^{-3}$$

4.22

(a) p-type

(b) $E_F - E_\upsilon = \dfrac{E_g}{4} = \dfrac{1.12}{4} = 0.28\text{ eV}$

$$p_o = N_\upsilon \exp\left[\frac{-(E_F - E_\upsilon)}{kT}\right]$$

$$= (1.04\times10^{19})\exp\left[\frac{-0.28}{0.0259}\right]$$

$$= 2.10\times10^{14}\text{ cm}^{-3}$$

$$E_c - E_F = E_g - (E_F - E_\upsilon)$$

$$= 1.12 - 0.28 = 0.84\text{ eV}$$

$$n_o = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= (2.8\times10^{19})\exp\left[\frac{-0.84}{0.0259}\right]$$

$$= 2.30\times10^{5}\text{ cm}^{-3}$$

4.23

(a) $n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$

$= (1.5\times10^{10})\exp\left[\frac{0.22}{0.0259}\right]$

$= 7.33\times10^{13}\ \text{cm}^{-3}$

$p_o = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$

$= (1.5\times10^{10})\exp\left[\frac{-0.22}{0.0259}\right]$

$= 3.07\times10^{6}\ \text{cm}^{-3}$

(b) $n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$

$= (1.8\times10^{6})\exp\left[\frac{0.22}{0.0259}\right]$

$= 8.80\times10^{9}\ \text{cm}^{-3}$

$p_o = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$

$= (1.8\times10^{6})\exp\left[\frac{-0.22}{0.0259}\right]$

$= 3.68\times10^{2}\ \text{cm}^{-3}$

4.24

(a) $E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{15}}\right)$

$= 0.1979\ \text{eV}$

(b) $E_c - E_F = E_g - (E_F - E_\upsilon)$

$= 1.12 - 0.19788 = 0.92212\ \text{eV}$

(c) $n_o = (2.8\times10^{19})\exp\left[\frac{-0.92212}{0.0259}\right]$

$= 9.66\times10^{3}\ \text{cm}^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$

$= 0.3294\ \text{eV}$

4.25

$kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533\ \text{eV}$

$N_\upsilon = (1.04\times10^{19})\left(\frac{400}{300}\right)^{3/2}$

$= 1.601\times10^{19}\ \text{cm}^{-3}$

$N_c = (2.8\times10^{19})\left(\frac{400}{300}\right)^{3/2}$

$= 4.3109\times10^{19}\ \text{cm}^{-3}$

$n_i^2 = (4.3109\times10^{19})(1.601\times10^{19})$

$\times\exp\left[\frac{-1.12}{0.034533}\right]$

$= 5.6702\times10^{24}$

$\Rightarrow n_i = 2.381\times10^{12}\ \text{cm}^{-3}$

(a) $E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.034533)\ln\left(\frac{1.601\times10^{19}}{5\times10^{15}}\right)$

$= 0.2787\ \text{eV}$

(b) $E_c - E_F = 1.12 - 0.27873 = 0.84127\ \text{eV}$

(c) $n_o = (4.3109\times10^{19})\exp\left[\frac{-0.84127}{0.034533}\right]$

$= 1.134\times10^{9}\ \text{cm}^{-3}$

(d) Holes

(e) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$= (0.034533)\ln\left(\frac{5\times10^{15}}{2.381\times10^{12}}\right)$

$= 0.2642\ \text{eV}$

4.26

(a) $p_o = (7\times10^{18})\exp\left[\frac{-0.25}{0.0259}\right]$

$= 4.50\times10^{14}\ \text{cm}^{-3}$

$E_c - E_F = 1.42 - 0.25 = 1.17\ \text{eV}$

$n_o = (4.7\times10^{17})\exp\left[\frac{-1.17}{0.0259}\right]$

$= 1.13\times10^{-2}\ \text{cm}^{-3}$

(b) $kT = 0.034533$ eV

$$N_\upsilon = \left(7\times10^{18}\right)\left(\frac{400}{300}\right)^{3/2}$$

$$= 1.078\times10^{19}\ \text{cm}^{-3}$$

$$N_c = \left(4.7\times10^{17}\right)\left(\frac{400}{300}\right)^{3/2}$$

$$= 7.236\times10^{17}\ \text{cm}^{-3}$$

$$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$$

$$= (0.034533)\ln\left(\frac{1.078\times10^{19}}{4.50\times10^{14}}\right)$$

$$= 0.3482\ \text{eV}$$

$$E_c - E_F = 1.42 - 0.3482 = 1.072\ \text{eV}$$

$$n_o = \left(7.236\times10^{17}\right)\exp\left[\frac{-1.07177}{0.034533}\right]$$

$$= 2.40\times10^{4}\ \text{cm}^{-3}$$

4.27

(a) $$p_o = \left(1.04\times10^{19}\right)\exp\left[\frac{-0.25}{0.0259}\right]$$

$$= 6.68\times10^{14}\ \text{cm}^{-3}$$

$$E_c - E_F = 1.12 - 0.25 = 0.870\ \text{eV}$$

$$n_o = \left(2.8\times10^{19}\right)\exp\left[\frac{-0.870}{0.0259}\right]$$

$$n_o = 7.23\times10^{4}\ \text{cm}^{-3}$$

(b) $kT = 0.034533$ eV

$$N_\upsilon = \left(1.04\times10^{19}\right)\left(\frac{400}{300}\right)^{3/2}$$

$$= 1.601\times10^{19}\ \text{cm}^{-3}$$

$$N_c = \left(2.8\times10^{19}\right)\left(\frac{400}{300}\right)^{3/2}$$

$$= 4.311\times10^{19}\ \text{cm}^{-3}$$

$$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{p_o}\right)$$

$$= (0.034533)\ln\left(\frac{1.601\times10^{19}}{6.68\times10^{14}}\right)$$

$$= 0.3482\ \text{eV}$$

$$E_c - E_F = 1.12 - 0.3482 = 0.7718\ \text{eV}$$

$$n_o = \left(4.311\times10^{19}\right)\exp\left[\frac{-0.77175}{0.034533}\right]$$

$$= 8.49\times10^{9}\ \text{cm}^{-3}$$

4.28

(a) $$n_o = \frac{2}{\sqrt{\pi}}N_cF_{1/2}(\eta_F)$$

For $E_F = E_c + kT/2$,

$$\eta_F = \frac{E_F - E_c}{kT} = \frac{kT/2}{kT} = 0.5$$

Then $F_{1/2}(\eta_F) \cong 1.0$

$$n_o = \frac{2}{\sqrt{\pi}}\left(2.8\times10^{19}\right)(1.0)$$

$$= 3.16\times10^{19}\ \text{cm}^{-3}$$

(b) $$n_o = \frac{2}{\sqrt{\pi}}N_cF_{1/2}(\eta_F)$$

$$= \frac{2}{\sqrt{\pi}}\left(4.7\times10^{17}\right)(1.0)$$

$$= 5.30\times10^{17}\ \text{cm}^{-3}$$

4.29

$$p_o = \frac{2}{\sqrt{\pi}}N_\upsilon F_{1/2}(\eta_F')$$

$$5\times10^{19} = \frac{2}{\sqrt{\pi}}\left(1.04\times10^{19}\right)F_{1/2}(\eta_F')$$

So $F_{1/2}(\eta_F') = 4.26$

We find $\eta_F' \cong 3.0 = \dfrac{E_\upsilon - E_F}{kT}$

$$E_\upsilon - E_F = (3.0)(0.0259) = 0.0777\ \text{eV}$$

4.30

(a) $$\eta_F = \frac{E_F - E_c}{kT} = \frac{4kT}{kT} = 4$$

Then $F_{1/2}(\eta_F) \cong 6.0$

$$n_o = \frac{2}{\sqrt{\pi}}N_cF_{1/2}(\eta_F)$$

$$= \frac{2}{\sqrt{\pi}}\left(2.8\times10^{19}\right)(6.0)$$

$$= 1.90\times10^{20}\ \text{cm}^{-3}$$

(b) $n_o = \frac{2}{\sqrt{\pi}}\left(4.7\times10^{17}\right)(6.0)$

$= 3.18\times10^{18}\ \text{cm}^{-3}$

4.31

For the electron concentration

$n(E) = g_c(E) f_F(E)$

The Boltzmann approximation applies, so

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E-E_c} \times \exp\left[\frac{-(E-E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_c-E_F)}{kT}\right] \times \sqrt{kT}\sqrt{\frac{E-E_c}{kT}}\exp\left[\frac{-(E-E_c)}{kT}\right]$$

Define

$$x = \frac{E-E_c}{kT}$$

Then

$n(E) \to n(x) = K\sqrt{x}\exp(-x)$

To find maximum $n(E) \to n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x) + x^{1/2}(-1)\exp(-x)\right]$$

or

$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2}-x\right]$$

which yields

$$x = \frac{1}{2} = \frac{E-E_c}{kT} \Rightarrow E = E_c + \frac{1}{2}kT$$

For the hole concentration

$p(E) = g_\upsilon(E)\left[1-f_F(E)\right]$

Using the Boltzmann approximation

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_\upsilon - E} \times \exp\left[\frac{-(E_F-E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_F-E_\upsilon)}{kT}\right] \times \sqrt{kT}\sqrt{\frac{E_\upsilon-E}{kT}}\exp\left[\frac{-(E_\upsilon-E)}{kT}\right]$$

Define

$$x' = \frac{E_\upsilon - E}{kT}$$

Then

$p(x') = K'\sqrt{x'}\exp(-x')$

To find maximum value of $p(E) \to p(x')$, set $\frac{dp(x')}{dx'} = 0$ Using the results from above,

we find the maximum at

$$E = E_\upsilon - \frac{1}{2}kT$$

4.32

(a) Silicon: We have

$$n_o = N_c\exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

We can write

$E_c - E_F = (E_c - E_d) + (E_d - E_F)$

For

$E_c - E_d = 0.045$ eV and $E_d - E_F = 3kT$ eV

we can write

$$n_o = \left(2.8\times10^{19}\right)\exp\left[\frac{-0.045}{0.0259} - 3\right]$$

$$= \left(2.8\times10^{19}\right)\exp(-4.737)$$

or

$n_o = 2.45\times10^{17}\ \text{cm}^{-3}$

We also have

$$p_o = N_\upsilon\exp\left[\frac{-(E_F-E_\upsilon)}{kT}\right]$$

Again, we can write

$E_F - E_\upsilon = (E_F - E_a) + (E_a - E_\upsilon)$

For

$E_F - E_a = 3kT$ and $E_a - E_\upsilon = 0.045$ eV

Then

$$p_o = \left(1.04\times10^{19}\right)\exp\left[-3 - \frac{0.045}{0.0259}\right]$$

$$= \left(1.04\times10^{19}\right)\exp(-4.737)$$

or

$p_o = 9.12\times10^{16}\ \text{cm}^{-3}$

(b) GaAs: assume $E_c - E_d = 0.0058$ eV

Then

$$n_o = \left(4.7\times10^{17}\right)\exp\left[\frac{-0.0058}{0.0259} - 3\right]$$

$$= \left(4.7\times10^{17}\right)\exp(-3.224)$$

or

$$n_o = 1.87\times10^{16}\ \text{cm}^{-3}$$

Assume $E_a - E_\upsilon = 0.0345$ eV

Then

$$p_o = \left(7\times10^{18}\right)\exp\left[\frac{-0.0345}{0.0259} - 3\right]$$

$$= \left(7\times10^{18}\right)\exp(-4.332)$$

or

$$p_o = 9.20\times10^{16}\ \text{cm}^{-3}$$

4.33

Plot

4.34

(a) $p_o = 4\times15 - 10^{15} = 3\times10^{15}\ \text{cm}^{-3}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{15}} = 7.5\times10^{4}\ \text{cm}^{-3}$$

(b) $n_o = N_d = 3\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5\times10^{3}\ \text{cm}^{-3}$$

(c) $n_o = p_o = n_i = 1.5\times10^{10}\ \text{cm}^{-3}$

(d) $$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{375}{300}\right)^3 \times\exp\left[\frac{-(1.12)(300)}{(0.0259)(375)}\right]$$

$$\Rightarrow n_i = 7.334\times10^{11}\ \text{cm}^{-3}$$

$$p_o = N_a = 4\times10^{15}\ \text{cm}^{-3}$$

$$n_o = \frac{\left(7.334\times10^{11}\right)^2}{4\times10^{15}} = 1.34\times10^{8}\ \text{cm}^{-3}$$

(e) $$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{450}{300}\right)^3 \times\exp\left[\frac{-(1.12)(300)}{(0.0259)(450)}\right]$$

$$\Rightarrow n_i = 1.722\times10^{13}\ \text{cm}^{-3}$$

$$n_o = \frac{10^{14}}{2} + \sqrt{\left(\frac{10^{14}}{2}\right)^2 + \left(1.722\times10^{13}\right)^2}$$

$$= 1.029\times10^{14}\ \text{cm}^{-3}$$

$$p_o = \frac{\left(1.722\times10^{13}\right)^2}{1.029\times10^{14}} = 2.88\times10^{12}\ \text{cm}^{-3}$$

4.35

(a) $p_o = N_a - N_d = 4\times10^{15} - 10^{15}$

$$= 3\times10^{15}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^{6}\right)^2}{3\times10^{15}} = 1.08\times10^{-3}\ \text{cm}^{-3}$$

(b) $n_o = N_d = 3\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{\left(1.8\times10^{6}\right)^2}{3\times10^{16}} = 1.08\times10^{-4}\ \text{cm}^{-3}$$

(c) $n_o = p_o = n_i = 1.8\times10^{6}\ \text{cm}^{-3}$

(d) $$n_i^2 = \left(4.7\times10^{17}\right)\left(7.0\times10^{18}\right)\left(\frac{375}{300}\right)^3 \times\exp\left[\frac{-(1.42)(300)}{(0.0259)(375)}\right]$$

$$\Rightarrow n_i = 7.580\times10^{8}\ \text{cm}^{-3}$$

$$p_o = N_a = 4\times10^{15}\ \text{cm}^{-3}$$

$$n_o = \frac{\left(7.580\times10^{8}\right)^2}{4\times10^{15}} = 1.44\times10^{2}\ \text{cm}^{-3}$$

(e) $$n_i^2 = \left(4.7\times10^{17}\right)\left(7.0\times10^{18}\right)\left(\frac{450}{300}\right)^3 \times\exp\left[\frac{-(1.42)(300)}{(0.0259)(450)}\right]$$

$$\Rightarrow n_i = 3.853\times10^{10}\ \text{cm}^{-3}$$

$$n_o = N_d = 10^{14}\ \text{cm}^{-3}$$

$$p_o = \frac{\left(3.853\times10^{10}\right)^2}{10^{14}} = 1.48\times10^{7}\ \text{cm}^{-3}$$

4.36

(a) Ge: $n_i = 2.4\times10^{13}\ \text{cm}^{-3}$

(i) $$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{2\times10^{15}}{2} + \sqrt{\left(\frac{2\times10^{15}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$$

or

$$n_o \cong N_d = 2\times10^{15}\ \text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(2.4\times10^{13}\right)^2}{2\times10^{15}}$$

$$= 2.88\times10^{11}\ \text{cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 10^{16} - 7\times10^{15}$

$$= 3\times10^{15}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.4\times10^{13}\right)^2}{3\times10^{15}}$$

$$= 1.92\times10^{11}\ \text{cm}^{-3}$$

(b) GaAs: $n_i = 1.8\times10^{6}\ \text{cm}^{-3}$

(i) $n_o \cong N_d = 2\times10^{15}$ cm

$$p_o = \frac{\left(1.8\times10^{6}\right)^2}{2\times10^{15}} = 1.62\times10^{-3}\ \text{cm}^{-3}$$

(ii) $p_o \cong N_a - N_d = 3\times10^{15}\ \text{cm}^{-3}$

$$n_o = \frac{\left(1.8\times10^{6}\right)^2}{3\times10^{15}} = 1.08\times10^{-3}\ \text{cm}^{-3}$$

(c) The result implies that there is only one minority carrier in a volume of $10^3\ \text{cm}^3$.

4.37

(a) For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1+\frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\frac{n_d}{N_d} = 8.85\times10^{-4}$$

(b) We have

$$f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_c) + (E_c - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1+\exp\left(1+\frac{0.245}{0.0259}\right)}$$

or

$$f_F(E) = 2.87\times10^{-5}$$

4.38

(a) $N_a > N_d \Rightarrow$ p-type

(b) Silicon:

$$p_o = N_a - N_d = 2.5\times10^{13} - 1\times10^{13}$$

or

$$p_o = 1.5\times10^{13}\ \text{cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{13}} = 1.5\times10^{7}\ \text{cm}^{-3}$$

Germanium:

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5\times10^{13}}{2}\right) + \sqrt{\left(\frac{1.5\times10^{13}}{2}\right)^2 + \left(2.4\times10^{13}\right)^2}$$

or

$$p_o = 3.26\times10^{13}\ \text{cm}^{-3}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.4\times10^{13}\right)^2}{3.264\times10^{13}} = 1.76\times10^{13}\ \text{cm}^{-3}$$

Gallium Arsenide:

$$p_o = N_a - N_d = 1.5\times10^{13}\ \text{cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^{6}\right)^2}{1.5\times10^{13}} = 0.216\ \text{cm}^{-3}$$

4.39

(a) $N_d > N_a \Rightarrow$ n-type

(b) $n_o \cong N_d - N_a = 2\times10^{15} - 1.2\times10^{15}$

$$= 8\times10^{14}\ \text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{8\times10^{14}} = 2.81\times10^{5}\ \text{cm}^{-3}$$

(c) $p_o \cong (N_a' + N_a) - N_d$

$4\times10^{15} = N_a' + 1.2\times10^{15} - 2\times10^{15}$

$\Rightarrow N_a' = 4.8\times10^{15}\text{ cm}^{-3}$

$$n_o = \frac{(1.5\times10^{10})^2}{4\times10^{15}} = 5.625\times10^4\text{ cm}^{-3}$$

4.40

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{2\times10^5} = 1.125\times10^{15}\text{ cm}^{-3}$$

$n_o > p_o \Rightarrow$ n-type

4.41

$$n_i^2 = (1.04\times10^{19})(6.0\times10^{18})\left(\frac{250}{300}\right)^3 \times\exp\left[\frac{-0.66}{(0.0259)(250/300)}\right]$$

$= 1.8936\times10^{24}$

$\Rightarrow n_i = 1.376\times10^{12}\text{ cm}^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{n_i^2}{4n_o} \Rightarrow n_o^2 = \frac{1}{4}n_i^2$$

$$\Rightarrow n_o = \frac{1}{2}n_i$$

So $n_o = 6.88\times10^{11}\text{ cm}^{-3}$,

Then $p_o = 2.75\times10^{12}\text{ cm}^{-3}$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$\left(2.752\times10^{12} - \frac{N_a}{2}\right)^2 = \left(\frac{N_a}{2}\right)^2 + 1.8936\times10^{24}$$

$$7.5735\times10^{24} - (2.752\times10^{12})N_a + \left(\frac{N_a}{2}\right)^2 = \left(\frac{N_a}{2}\right)^2 + 1.8936\times10^{24}$$

so that $N_a = 2.064\times10^{12}\text{ cm}^{-3}$

4.42

Plot

4.43

Plot

4.44

Plot

4.45

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$1.1\times10^{14} = \frac{2\times10^{14} - 1.2\times10^{14}}{2} + \sqrt{\left(\frac{2\times10^{14} - 1.2\times10^{14}}{2}\right)^2 + n_i^2}$$

$(1.1\times10^{14} - 4\times10^{13})^2 = (4\times10^{13})^2 + n_i^2$

$4.9\times10^{27} = 1.6\times10^{27} + n_i^2$

so $n_i = 5.74\times10^{13}\text{ cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{3.3\times10^{27}}{1.1\times10^{14}} = 3\times10^{13}\text{ cm}^{-3}$$

4.46

(a) $N_a > N_d \Rightarrow$ p-type

Majority carriers are holes

$p_o = N_a - N_d = 3\times10^{16} - 1.5\times10^{16}$

$= 1.5\times10^{16}\text{ cm}^{-3}$

Minority carriers are electrons

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{1.5\times10^{16}} = 1.5\times10^4\text{ cm}^{-3}$$

(b) Boron atoms must be added

$p_o = N_a' + N_a - N_d$

$5\times10^{16} = N_a' + 3\times10^{16} - 1.5\times10^{16}$

So $N_a' = 3.5\times10^{16}\text{ cm}^{-3}$

$$n_o = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 4.5\times10^3\text{ cm}^{-3}$$

4.47

(a) $p_o << n_i \Rightarrow$ n-type

(b) $p_o = \dfrac{n_i^2}{n_o} \Rightarrow n_o = \dfrac{n_i^2}{p_o}$

$$n_o = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^4} = 1.125\times10^{16}\ \text{cm}^{-3}$$

$\Rightarrow$ electrons are majority carriers

$p_o = 2\times10^4\ \text{cm}^{-3}$

$\Rightarrow$ holes are minority carriers

(c) $n_o = N_d - N_a$

$1.125\times10^{16} = N_d - 7\times10^{15}$

so $N_d = 1.825\times10^{16}\ \text{cm}^{-3}$

4.48

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$$

For Germanium

T (K)	kT (eV)	n_i (cm^{-3})
200	0.01727	2.16×10^{10}
400	0.03453	8.60×10^{14}
600	0.0518	3.82×10^{16}

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$ and

$N_a = 10^{15}\ \text{cm}^{-3}$

T (K)	p_o (cm^{-3})	$(E_{Fi} - E_F)$(eV)
200	1.0×10^{15}	0.1855
400	1.49×10^{15}	0.01898
600	3.87×10^{16}	0.000674

4.49

(a) $E_c - E_F = kT\ln\left(\dfrac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

For $10^{14}\ \text{cm}^{-3}$, $E_c - E_F = 0.3249\ \text{eV}$

$10^{15}\ \text{cm}^{-3}$, $E_c - E_F = 0.2652\ \text{eV}$

$10^{16}\ \text{cm}^{-3}$, $E_c - E_F = 0.2056\ \text{eV}$

$10^{17}\ \text{cm}^{-3}$, $E_c - E_F = 0.1459\ \text{eV}$

(b) $E_F - E_{Fi} = kT\ln\left(\dfrac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

For $10^{14}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.2280\ \text{eV}$

$10^{15}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.2877\ \text{eV}$

$10^{16}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.3473\ \text{eV}$

$10^{17}\ \text{cm}^{-3}$, $E_F - E_{Fi} = 0.4070\ \text{eV}$

4.50

(a) $n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$n_o = 1.05N_d = 1.05\times10^{15}\ \text{cm}^{-3}$

$$\left(1.05\times10^{15} - 0.5\times10^{15}\right)^2 = \left(0.5\times10^{15}\right)^2 + n_i^2$$

so $n_i^2 = 5.25\times10^{28}$

Now

$$n_i^2 = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$5.25\times10^{28} = \left(2.912\times10^{38}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-12972.973}{T}\right]$$

By trial and error, $T = 536.5\ \text{K}$

(b) At $T = 300\ \text{K}$,

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$$

$$= 0.2652\ \text{eV}$$

At $T = 536.5\ \text{K}$,

$$kT = (0.0259)\left(\frac{536.5}{300}\right) = 0.046318\ \text{eV}$$

$$N_c = \left(2.8\times10^{19}\right)\left(\frac{536.5}{300}\right)^{3/2}$$

$$= 6.696\times10^{19}\ \text{cm}^{-3}$$

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_o}\right)$$

$$E_c - E_F = (0.046318)\ln\left(\frac{6.696\times10^{19}}{1.05\times10^{15}}\right)$$

$$= 0.5124\text{ eV}$$

then $\Delta(E_c - E_F) = 0.2472$ eV

(c) Closer to the intrinsic energy level.

4.51

$$E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$$

At $T = 200$ K, $kT = 0.017267$ eV
$T = 400$ K, $kT = 0.034533$ eV
$T = 600$ K, $kT = 0.0518$ eV

At $T = 200$ K,

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{200}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.017267}\right]$$

$\Rightarrow n_i = 7.638\times10^4\text{ cm}^{-3}$

At $T = 400$ K,

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{400}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.034533}\right]$$

$\Rightarrow n_i = 2.381\times10^{12}\text{ cm}^{-3}$

At $T = 600$ K,

$$n_i^2 = (2.8\times10^{19})(1.04\times10^{19})\left(\frac{600}{300}\right)^3 \times\exp\left[\frac{-1.12}{0.0518}\right]$$

$\Rightarrow n_i = 9.740\times10^{14}\text{ cm}^{-3}$

At $T = 200$ K and $T = 400$ K,

$$p_o = N_a = 3\times10^{15}\text{ cm}^{-3}$$

At $T = 600$ K,

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= \frac{3\times10^{15}}{2} + \sqrt{\left(\frac{3\times10^{15}}{2}\right)^2 + (9.740\times10^{14})^2}$$

$$= 3.288\times10^{15}\text{ cm}^{-3}$$

Then, $T = 200$ K, $E_{Fi} - E_F = 0.4212$ eV
$T = 400$ K, $E_{Fi} - E_F = 0.2465$ eV
$T = 600$ K, $E_{Fi} - E_F = 0.0630$ eV

4.52

(a)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{N_a}{1.8\times10^6}\right)$$

For $N_a = 10^{14}\text{ cm}^{-3}$, $E_{Fi} - E_F = 0.4619$ eV
$N_a = 10^{15}\text{ cm}^{-3}$, $E_{Fi} - E_F = 0.5215$ eV
$N_a = 10^{16}\text{ cm}^{-3}$, $E_{Fi} - E_F = 0.5811$ eV
$N_a = 10^{17}\text{ cm}^{-3}$, $E_{Fi} - E_F = 0.6408$ eV

(b)

$$E_F - E_\upsilon = kT\ln\left(\frac{N_\upsilon}{N_a}\right) = (0.0259)\ln\left(\frac{7.0\times10^{18}}{N_a}\right)$$

For $N_a = 10^{14}\text{ cm}^{-3}$, $E_F - E_\upsilon = 0.2889$ eV
$N_a = 10^{15}\text{ cm}^{-3}$, $E_F - E_\upsilon = 0.2293$ eV
$N_a = 10^{16}\text{ cm}^{-3}$, $E_F - E_\upsilon = 0.1697$ eV
$N_a = 10^{17}\text{ cm}^{-3}$, $E_F - E_\upsilon = 0.1100$ eV

4.53

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln(10)$$

or

$E_{Fi} - E_{midgap} = +0.0447$ eV

(b) Impurity atoms to be added so
$E_{midgap} - E_F = 0.45$ eV
(i) p-type, so add acceptor atoms
(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947$ eV

Then

$$p_o = n_i\exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= (10^5)\exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_o = N_a = 1.97\times10^{13}\text{ cm}^{-3}$$

4.54

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5\times10^{15} + \left(2.8\times10^{19}\right)\exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5\times10^{15} + 6.95\times10^{15}$$

or

$$N_d = 1.2\times10^{16}\ \text{cm}^{-3}$$

4.55

(a) Silicon

(i) $E_c - E_F = kT\ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6\times10^{15}}\right) = 0.2188\ \text{eV}$$

(ii) $E_c - E_F = 0.2188 - 0.0259 = 0.1929\ \text{eV}$

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= \left(2.8\times10^{19}\right)\exp\left[\frac{-0.1929}{0.0259}\right]$$

$$N_d = 1.631\times10^{16}\ \text{cm}^{-3} = N_d' + 6\times10^{15}$$

$\Rightarrow N_d' = 1.031\times10^{16}\ \text{cm}^{-3}$ Additional donor atoms

(b) GaAs

(i) $E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{15}}\right)$

$$= 0.15936\ \text{eV}$$

(ii) $E_c - E_F = 0.15936 - 0.0259 = 0.13346\ \text{eV}$

$$N_d = \left(4.7\times10^{17}\right)\exp\left[\frac{-0.13346}{0.0259}\right]$$

$$= 2.718\times10^{15}\ \text{cm}^{-3} = N_d' + 10^{15}$$

$\Rightarrow N_d' = 1.718\times10^{15}\ \text{cm}^{-3}$ Additional donor atoms

---

4.56

(a) $E_{Fi} - E_F = kT\ln\left(\frac{N_\upsilon}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{2\times10^{16}}\right) = 0.1620\ \text{eV}$$

(b) $E_F - E_{Fi} = kT\ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{2\times10^{16}}\right) = 0.1876\ \text{eV}$$

(c) For part (a);

$$p_o = 2\times10^{16}\ \text{cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}}$$

$$= 1.125\times10^{4}\ \text{cm}^{-3}$$

For part (b):

$$n_o = 2\times10^{16}\ \text{cm}^{-3}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}}$$

$$= 1.125\times10^{4}\ \text{cm}^{-3}$$

4.57

$$n_o = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$= \left(1.8\times10^{6}\right)\exp\left[\frac{0.55}{0.0259}\right]$$

$$= 3.0\times10^{15}\ \text{cm}^{-3}$$

Add additional acceptor impurities

$$n_o = N_d - N_a$$

$$3\times10^{15} = 7\times10^{15} - N_a$$

$$\Rightarrow N_a = 4\times10^{15}\ \text{cm}^{-3}$$

4.58

(a) $E_{Fi} - E_F = kT\ln\left(\frac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3\times10^{15}}{1.5\times10^{10}}\right) = 0.3161\ \text{eV}$$

(b) $E_F - E_{Fi} = kT\ln\left(\frac{n_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\ \text{eV}$$

(c) $E_F = E_{Fi}$

(d) $E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$

$= (0.0259)\left(\frac{375}{300}\right)\ln\left(\frac{4\times10^{15}}{7.334\times10^{11}}\right)$

$= 0.2786$ eV

(e) $E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$

$= (0.0259)\left(\frac{450}{300}\right)\ln\left(\frac{1.029\times10^{14}}{1.722\times10^{13}}\right)$

$= 0.06945$ eV

4.59

(a) $E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$

$= (0.0259)\ln\left(\frac{7.0\times10^{18}}{3\times10^{15}}\right) = 0.2009$ eV

(b) $E_F - E_\upsilon = (0.0259)\ln\left(\frac{7.0\times10^{18}}{1.08\times10^{-4}}\right)$

$= 1.360$ eV

(c) $E_F - E_\upsilon = (0.0259)\ln\left(\frac{7.0\times10^{18}}{1.8\times10^{6}}\right)$

$= 0.7508$ eV

(d) $E_F - E_\upsilon = (0.0259)\left(\frac{375}{300}\right)$

$\times \ln\left[\frac{(7.0\times10^{18})(375/300)^{3/2}}{4\times10^{15}}\right]$

$= 0.2526$ eV

(e) $E_F - E_\upsilon = (0.0259)\left(\frac{450}{300}\right)$

$\times \ln\left[\frac{(7.0\times10^{18})(450/300)^{3/2}}{1.48\times10^{7}}\right]$

$= 1.068$ eV

4.60

n-type

$E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$

$= (0.0259)\ln\left(\frac{1.125\times10^{16}}{1.5\times10^{10}}\right) = 0.3504$ eV

4.61

$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$

$5.08\times10^{15} = \frac{5\times10^{15}}{2}$

$+ \sqrt{\left(\frac{5\times10^{15}}{2}\right)^2 + n_i^2}$

$(5.08\times10^{15} - 2.5\times10^{15})^2$

$= (2.5\times10^{15})^2 + n_i^2$

$6.6564\times10^{30} = 6.25\times10^{30} + n_i^2$

$\Rightarrow n_i^2 = 4.064\times10^{29}$

$n_i^2 = N_c N_\upsilon \exp\left[\frac{-E_g}{kT}\right]$

$kT = (0.0259)\left(\frac{350}{300}\right) = 0.030217$ eV

$N_c = (1.2\times10^{19})\left(\frac{350}{300}\right)^2 = 1.633\times10^{19}$ cm^{-3}

$N_\upsilon = (1.8\times10^{19})\left(\frac{350}{300}\right)^2 = 2.45\times10^{19}$ cm^{-3}

Now

$4.064\times10^{29} = (1.633\times10^{19})(2.45\times10^{19})$

$\times \exp\left[\frac{-E_g}{0.030217}\right]$

So

$E_g = (0.030217)\ln\left[\frac{(1.633\times10^{19})(2.45\times10^{19})}{4.064\times10^{29}}\right]$

$\Rightarrow E_g = 0.6257$ eV

4.62

(a) Replace Ga atoms $\Rightarrow$ Silicon acts as a donor

$$N_d = (0.05)(7\times10^{15}) = 3.5\times10^{14} \text{ cm}^{-3}$$

Replace As atoms $\Rightarrow$ Silicon acts as an acceptor

$$N_a = (0.95)(7\times10^{15}) = 6.65\times10^{15} \text{ cm}^{-3}$$

(b) $N_a > N_d \Rightarrow$ p-type

(c) $p_o = N_a - N_d = 6.65\times10^{15} - 3.5\times10^{14}$

$$= 6.3\times10^{15} \text{ cm}^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8\times10^6)^2}{6.3\times10^{15}} = 5.14\times10^{-4} \text{ cm}^{-3}$$

(d) $E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{6.3\times10^{15}}{1.8\times10^6}\right) = 0.5692 \text{ eV}$$

Chapter 5

5.1

(a) $\rho = \dfrac{1}{e\mu_n N_d} = \dfrac{1}{(1.6\times10^{-19})(1300)(10^{15})}$

$= 4.808\,\Omega\text{-cm}$

(b) $\sigma = \dfrac{1}{\rho} = \dfrac{1}{4.8077} = 0.208\,(\Omega\text{-cm})^{-1}$

5.2

$\sigma = e\mu_p N_a$

or $N_a = \dfrac{\sigma}{e\mu_p} = \dfrac{1.80}{(1.6\times10^{-19})(380)}$

$= 2.96\times10^{16}\ \text{cm}^{-3}$

5.3

(a) $\sigma = e\mu_n N_d$

$10 = (1.6\times10^{-19})\mu_n N_d$

From Figure 5.3, for $N_d = 6\times10^{16}\ \text{cm}^{-3}$ we find $\mu_n \cong 1050\ \text{cm}^2/\text{V-s}$ which gives

$\sigma = (1.6\times10^{-19})(1050)(6\times10^{16})$

$= 10.08\,(\Omega\text{-cm})^{-1}$

(b) $\rho = \dfrac{1}{e\mu_p N_a}$

$0.20 = \dfrac{1}{(1.6\times10^{-19})\mu_p N_a}$

From Figure 5.3, for $N_a = 10^{17}\ \text{cm}^{-3}$ we find $\mu_p \cong 320\ \text{cm}^2/\text{V-s}$ which gives

$\rho = \dfrac{1}{(1.6\times10^{-19})(320)(10^{17})} = 0.195\ \Omega\text{-cm}$

5.4

(a) $\rho = \dfrac{1}{e\mu_p N_a}$

$0.35 = \dfrac{1}{(1.6\times10^{-19})\mu_p N_a}$

From Figure 5.3, for $N_a = 8\times10^{16}\ \text{cm}^{-3}$ we find $\mu_p \cong 220\ \text{cm}^2/\text{V-s}$ which gives

$\rho = \dfrac{1}{(1.6\times10^{-19})(220)(8\times10^{16})}$

$= 0.355\ \Omega\text{-cm}$

(b) $\sigma = e\mu_n N_d$

$120 = (1.6\times10^{-19})\mu_n N_d$

From Figure 5.3, for $N_d = 2\times10^{17}\ \text{cm}^{-3}$, then $\mu_n \cong 3800\ \text{cm}^2/\text{V-s}$ which gives

$\sigma = (1.6\times10^{-19})(3800)(2\times10^{17})$

$= 121.6\,(\Omega\text{-cm})^{-1}$

5.5

$R = \dfrac{\rho L}{A} = \dfrac{L}{\sigma A} = \dfrac{L}{(e\mu_n N_d)A}$

or $\mu_n = \dfrac{L}{(eN_d)RA}$

$= \dfrac{2.5}{(1.6\times10^{-19})(2\times10^{15})(70)(0.1)}$

$= 1116\ \text{cm}^2/\text{V-s}$

5.6

(a) $n_o = N_d = 10^{16}\ \text{cm}^{-3}$

and

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4}\ \text{cm}^{-3}$

(b)

$J = e\mu_n n_o \mathrm{E}$

For GaAs doped at $N_d = 10^{16}\ \text{cm}^{-3}$,

$\mu_n \cong 7500\ \text{cm}^2/\text{V-s}$

Then

$J = (1.6\times10^{-19})(7500)(10^{16})(10)$

or

$J = 120\ \text{A/cm}^2$

(b) (i) $p_o = N_a = 10^{16}\ \text{cm}^{-3}$

$n_o = \dfrac{n_i^2}{p_o} = 3.24\times10^{-4}\ \text{cm}^{-3}$

(ii) For GaAs doped at $N_a = 10^{16}\ \text{cm}^{-3}$,

$\mu_p \cong 310\ \text{cm}^2/\text{V-s}$

$$J = e\mu_p p_o \mathrm{E}$$
$$= \left(1.6\times10^{-19}\right)(310)\left(10^{16}\right)(10)$$
or
$$J = 4.96\ \mathrm{A/cm}^2$$

5.7

(a) $V = IR \Rightarrow 10 = (0.1)R$

or

$R = 100\,\Omega$

(b)

$$R = \frac{L}{\sigma A} \Rightarrow \sigma = \frac{L}{RA}$$

or

$$\sigma = \frac{10^{-3}}{(100)\left(10^{-3}\right)} = 0.01\ (\Omega\text{-cm})^{-1}$$

(c) $\sigma \cong e\mu_n N_d$

or

$$0.01 = \left(1.6\times10^{-19}\right)(1350)N_d$$

Then

$$N_d = 4.63\times10^{13}\ \mathrm{cm}^{-3}$$

(d) $\sigma \cong e\mu_p p_o$

or

$$0.01 = \left(1.6\times10^{-19}\right)(480)p_o$$

Then

$$p_o = 1.30\times10^{14}\ \mathrm{cm}^{-3} = N_a - N_d$$

So

$$N_a = 1.30\times10^{14} + 10^{15} = 1.13\times10^{15}\ \mathrm{cm}^{-3}$$

Note: For the doping concentrations obtained, the assumed mobility values are valid.

5.8

(a) $R = \dfrac{L}{\sigma A} = \dfrac{L}{\left(e\mu_p N_a\right)A}$

For $N_a = 2\times10^{16}\ \mathrm{cm}^{-3}$, then

$$\mu_p \cong 400\ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$$R = \frac{(0.075)}{\left(1.6\times10^{-19}\right)(400)\left(2\times10^{16}\right)\left(8.5\times10^{-4}\right)}$$
$$= 68.93\,\Omega$$

$$I = \frac{V}{R} = \frac{2}{68.93} = 0.0290\ \mathrm{A}$$

or $I = 29.0\ \mathrm{mA}$

(b) $R \propto L \Rightarrow R = (68.93)(3) = 206.79\,\Omega$

$$I = \frac{V}{R} = \frac{2}{206.79} = 0.00967\ \mathrm{A}$$

or $I = 9.67\ \mathrm{mA}$

(c) $J = ep_o \upsilon_d$

For (a), $J = \dfrac{29.0\times10^{-3}}{8.5\times10^{-4}} = 34.12\ \mathrm{A/cm}^2$

Then $\upsilon_d = \dfrac{J}{ep_o} = \dfrac{34.12}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}$

$$= 1.066\times10^4\ \mathrm{cm/s}$$

For (b), $J = \dfrac{9.67\times10^{-3}}{8.5\times10^{-4}} = 11.38\ \mathrm{A/cm}^2$

$$\upsilon_d = \frac{11.38}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}$$
$$= 3.55\times10^3\ \mathrm{cm/s}$$

5.9

(a) For $N_d = 2\times10^{15}\ \mathrm{cm}^{-3}$, then

$$\mu_n \cong 8000\ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$$R = \frac{V}{I} = \frac{5}{25\times10^{-3}} = 200\,\Omega$$

$$R = \frac{L}{\left(e\mu_n N_d\right)A}$$

or $L = \left(e\mu_n N_d\right)RA$

$$= \left(1.6\times10^{-19}\right)(8000)\left(2\times10^{15}\right)(200)\left(5\times10^{-5}\right)$$
$$= 0.0256\ \mathrm{cm}$$

(b) $J = \dfrac{I}{A} = en_o\upsilon_d$

or $\upsilon_d = \dfrac{I}{A\left(en_o\right)}$

$$= \frac{25\times10^{-3}}{\left(5\times10^{-5}\right)\left(1.6\times10^{-19}\right)\left(2\times10^{15}\right)}$$
$$= 1.56\times10^6\ \mathrm{cm/s}$$

(c) $I = \left(en_o\upsilon_d\right)A$

$$= \left(1.6\times10^{-19}\right)\left(2\times10^{15}\right)\left(5\times10^6\right)\left(5\times10^{-5}\right)$$
$$= 0.080\ \mathrm{A}$$

or $I = 80\ \mathrm{mA}$

5.10

(a) $\mathrm{E} = \dfrac{V}{L} = \dfrac{3}{1} = 3$ V/cm

$$\upsilon_d = \mu_n \mathrm{E} \Rightarrow \mu_n = \frac{\upsilon_d}{\mathrm{E}} = \frac{10^4}{3}$$

or

$$\mu_n = 3333 \text{ cm}^2/\text{V-s}$$

(b)

$$\upsilon_d = \mu_n \mathrm{E} = (800)(3)$$

or

$$\upsilon_d = 2.4 \times 10^3 \text{ cm/s}$$

5.11

(a) Silicon: For $\mathrm{E} = 1$ kV/cm,

$$\upsilon_d = 1.2 \times 10^6 \text{ cm/s}$$

Then

$$t_t = \frac{d}{\upsilon_d} = \frac{10^{-4}}{1.2 \times 10^6} = 8.33 \times 10^{-11} \text{ s}$$

For GaAs: $\upsilon_d = 7.5 \times 10^6$ cm/s

Then

$$t_t = \frac{d}{\upsilon_d} = \frac{10^{-4}}{7.5 \times 10^6} = 1.33 \times 10^{-11} \text{ s}$$

(b) Silicon: For $\mathrm{E} = 50$ kV/cm,

$$\upsilon_d = 9.5 \times 10^6 \text{ cm/s}$$

Then

$$t_t = \frac{10^{-4}}{9.5 \times 10^6} = 1.05 \times 10^{-11} \text{ s}$$

For GaAs: $\upsilon_d = 7 \times 10^6$ cm/s

Then

$$t_t = \frac{10^{-4}}{7 \times 10^6} = 1.43 \times 10^{-11} \text{ s}$$

5.12

$$\rho = \frac{1}{e\mu_n n_o + e\mu_p p_o} = \frac{1}{e(\mu_n + \mu_p)n_i}$$

(a) $N_a = N_d = 10^{14}$ cm^{-3}

$$\Rightarrow \mu_n \cong 1350 \text{ cm}^2/\text{V-s}$$

$$\mu_p \cong 480 \text{ cm}^2/\text{V-s}$$

$$\rho = \frac{1}{(1.6 \times 10^{-19})(1350 + 480)(1.5 \times 10^{10})}$$

$$= 2.28 \times 10^5 \ \Omega\text{-cm}$$

(b) $N_a = N_d = 10^{16}$ cm^{-3}

$$\Rightarrow \mu_n \cong 1250 \text{ cm}^2/\text{V-s}$$

$$\mu_p \cong 410 \text{ cm}^2/\text{V-s}$$

$$\rho = \frac{1}{(1.6 \times 10^{-19})(1250 + 410)(1.5 \times 10^{10})}$$

$$= 2.51 \times 10^5 \ \Omega\text{-cm}$$

(c) $N_a = N_d = 10^{18}$ cm^{-3}

$$\Rightarrow \mu_n \cong 290 \text{ cm}^2/\text{V-s}$$

$$\mu_p \cong 130 \text{ cm}^2/\text{V-s}$$

$$\rho = \frac{1}{(1.6 \times 10^{-19})(290 + 130)(1.5 \times 10^{10})}$$

$$= 9.92 \times 10^5 \ \Omega\text{-cm}$$

5.13

(a) GaAs:

$$\sigma \cong e\mu_p p_o \Rightarrow 5 = (1.6 \times 10^{-19})\mu_p p_o$$

From Figure 5.3, and using trial and error, we find

$$p_o \cong 1.3 \times 10^{17} \text{ cm}^{-3} \quad \text{and}$$

$$\mu_p \cong 240 \text{ cm}^2/\text{V-s}$$

Then

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{1.3 \times 10^{17}} = 2.49 \times 10^{-5} \text{ cm}^{-3}$$

(b) Silicon:

$$\sigma = \frac{1}{\rho} \cong e\mu_n n_o$$

or

$$n_o = \frac{1}{\rho e \mu_n} = \frac{1}{(8)(1.6 \times 10^{-19})(1350)}$$

which gives

$$n_o = 5.79 \times 10^{14} \text{ cm}^{-3}$$

and

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5 \times 10^{10})^2}{5.79 \times 10^{14}} = 3.89 \times 10^5 \text{ cm}^{-3}$$

Note: For the doping concentrations obtained in part (b), the assumed mobility values are valid.

5.14

$$\sigma_i = en_i(\mu_n + \mu_p)$$

Then

$$10^{-6} = (1.6\times10^{-19})(1000+600)n_i$$

or

$$n_i\,(300\text{ K}) = 3.91\times10^{9}\text{ cm}^{-3}$$

Now

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

or

$$E_g = kT\ln\left(\frac{N_c N_\upsilon}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{19})^2}{(3.91\times10^{9})^2}\right]$$

which gives

$$E_g = 1.122\text{ eV}$$

Now

$$n_i^2\,(500\text{K}) = (10^{19})^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right]$$

$$= 5.15\times10^{26}$$

or

$$n_i\,(500\text{ K}) = 2.27\times10^{13}\text{ cm}^{-3}$$

Then

$$\sigma_i = (1.6\times10^{-19})(2.27\times10^{13})(1000+600)$$

which gives

$$\sigma_i\,(500\text{ K}) = 5.81\times10^{-3}\ (\Omega\text{-cm})^{-1}$$

5.15

(a) (i) Silicon: $\sigma_i = en_i(\mu_n + \mu_p)$

$$\sigma_i = (1.6\times10^{-19})(1.5\times10^{10})(1350+480)$$

or

$$\sigma_i = 4.39\times10^{-6}\,(\Omega\text{-cm})^{-1}$$

(ii) Ge:

$$\sigma_i = (1.6\times10^{-19})(2.4\times10^{13})(3900+1900)$$

or

$$\sigma_i = 2.23\times10^{-2}\,(\Omega\text{-cm})^{-1}$$

(iii) GaAs:

$$\sigma_i = (1.6\times10^{-19})(1.8\times10^{6})(8500+400)$$

or

$$\sigma_i = 2.56\times10^{-9}\,(\Omega\text{-cm})^{-1}$$

(b) $R = \dfrac{L}{\sigma A}$

(i) Si:

$$R = \frac{200\times10^{-4}}{(4.39\times10^{-6})(85\times10^{-8})} = 5.36\times10^{9}\ \Omega$$

(ii) Ge:

$$R = \frac{200\times10^{-4}}{(2.23\times10^{-2})(85\times10^{-8})} = 1.06\times10^{6}\ \Omega$$

(iii) GaAs:

$$R = \frac{200\times10^{-4}}{(2.56\times10^{-9})(85\times10^{-8})} = 9.19\times10^{12}\ \Omega$$

5.16

(a) $\sigma = e\mu_n N_d$

$$0.25 = (1.6\times10^{-19})\mu_n N_d$$

From Figure 5.3, for $N_d = 1.2\times10^{15}\text{ cm}^{-3}$,

then $\mu_n \cong 1300\text{ cm}^2/\text{V-s}$

So $\sigma = (1.6\times10^{-19})(1300)(1.2\times10^{15})$

$$= 0.2496\,(\Omega\text{-cm})^{-1}$$

(b) Using Figure 5.2,

(i) For $T = 250\text{ K}$ ($-23°$ C),

$$\Rightarrow \mu_n \cong 1800\text{ cm}^2/\text{V-s}$$

$$\sigma = (1.6\times10^{-19})(1800)(1.2\times10^{15})$$

$$= 0.346\,(\Omega\text{-cm})^{-1}$$

(ii) For $T = 400\text{ K}$ ($127°$ C),

$$\Rightarrow \mu_n \cong 670\text{ cm}^2/\text{V-s}$$

$$\sigma = (1.6\times10^{-19})(670)(1.2\times10^{15})$$

$$= 0.129\,(\Omega\text{-cm})^{-1}$$

5.17

$$\sigma_{avg} = \frac{1}{t}\int_0^t \sigma(x)dx = \frac{1}{t}\int_0^t \sigma_o \exp\left(\frac{-x}{d}\right)dx$$

$$= \frac{\sigma_o}{t}(-d)\exp\left(\frac{-x}{d}\right)\Big|_0^t$$

$$= \frac{-\sigma_o d}{t}\left[\exp\left(\frac{-t}{d}\right)-1\right]$$

$$= \frac{(20)(0.3)}{(1.5)}\left[1-\exp\left(\frac{-1.5}{0.3}\right)\right]$$

$$= 3.97\,(\Omega\text{-cm})^{-1}$$

5.18

(a) $\mathrm{E} = \dfrac{V}{L} = \dfrac{2}{150\times10^{-4}} = 133.3 \text{ V/cm}$

(b) $\sigma(x) = e\mu_n N_d(x)$

$$\sigma_{avg} = e\mu_n \cdot \frac{1}{T}\int_0^T \left(2\times10^{16}\right)\left(1-\frac{x}{1.111T}\right)dx$$

$$= \frac{e\mu_n\left(2\times10^{16}\right)}{T}\left[x-\frac{x^2}{2(1.111T)}\right]\Bigg|_0^T$$

$$= \frac{e\mu_n\left(2\times10^{16}\right)}{T}\left[T-\frac{T^2}{2(1.111T)}\right]$$

$$= e\mu_n\left(2\times10^{16}\right)(0.55)$$

$$= \left(1.6\times10^{-19}\right)(750)\left(2\times10^{16}\right)(0.55)$$

$$\sigma_{avg} = 1.32\,(\Omega\text{-cm})^{-1}$$

(c) $I = \dfrac{\sigma_{avg} A}{L}\cdot V = \dfrac{(1.32)\left(7.5\times10^{-4}\right)\left(10^{-4}\right)}{150\times10^{-4}}\cdot 2$

$= 1.32\times10^{-5}$ A

or $I = 13.2\ \mu$A

(d) Top surface;

$$\sigma = \left(1.6\times10^{-19}\right)(750)\left(2\times10^{16}\right)$$

$$= 2.4\,(\Omega\text{-cm})^{-1}$$

$$J = \sigma\mathrm{E} = (2.4)(133.3) = 320 \text{ A/cm}^2$$

Bottom surface:

$$\sigma = \left(1.6\times10^{-19}\right)(750)\left(2\times10^{15}\right)$$

$$= 0.24\,(\Omega\text{-cm})^{-1}$$

$$J = \sigma\mathrm{E} = (0.24)(133.3 = 32)\text{A/cm}^2$$

5.19

Plot

5.20

(a) $\mathrm{E} = 10$ V/cm

so

$$\upsilon_d = \mu_n\mathrm{E} = (1350)(10) = 1.35\times10^4 \text{ cm/s}$$

or

$$\upsilon_d = 1.35\times10^2 \text{ m/s}$$

Then

$$T = \frac{1}{2}m_n^*\upsilon_d^2$$

$$= \frac{1}{2}(1.08)\left(9.11\times10^{-31}\right)\left(1.35\times10^2\right)^2$$

or

$$T = 8.97\times10^{-27}\text{ J} \Rightarrow 5.60\times10^{-8} \text{ eV}$$

(b) $\mathrm{E} = 1$ kV/cm

$$\upsilon_d = (1350)(1000) = 1.35\times10^6 \text{ cm/s}$$

or

$$\upsilon_d = 1.35\times10^4 \text{ m/s}$$

Then

$$T = \frac{1}{2}(1.08)\left(9.11\times10^{-31}\right)\left(1.35\times10^4\right)^2$$

or

$$T = 8.97\times10^{-23}\text{ J} \Rightarrow 5.60\times10^{-4} \text{ eV}$$

5.21

(a) $n_i^2 = N_c N_\upsilon \exp\left(\dfrac{-E_g}{kT}\right)$

$$= \left(2\times10^{19}\right)\left(1\times10^{19}\right)\exp\left(\frac{-1.10}{0.0259}\right)$$

$$= 7.18\times10^{19}$$

or

$$n_i = 8.47\times10^9 \text{ cm}^{-3}$$

For $N_d = 10^{14}\text{ cm}^{-3} >> n_i \Rightarrow n_o = 10^{14}\text{ cm}^{-3}$

Then

$$J = \sigma\mathrm{E} = e\mu_n n_o\mathrm{E}$$

$$= \left(1.6\times10^{-19}\right)(1000)\left(10^{14}\right)(100)$$

or

$$J = 1.60 \text{ A/cm}^2$$

(b) A 5% increase is due to a 5% increase in electron concentration, so

$$n_o = 1.05\times10^{14} = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

which becomes

$$\left(1.05\times10^{14} - 5\times10^{13}\right)^2 = \left(5\times10^{13}\right)^2 + n_i^2$$

and yields

$$n_i^2 = 5.25\times10^{26}$$

$$= \left(2\times10^{19}\right)\left(1\times10^{19}\right)\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

or

$$2.625\times10^{-12} = \left(\frac{T}{300}\right)^3 \exp\left[\frac{-1.10}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$T = 456 \text{ K}$$

5.22

(a) $\sigma = e\mu_n n_o + e\mu_p p_o$ and $n_o = \dfrac{n_i^2}{p_o}$

Then

$$\sigma = \frac{e\mu_n n_i^2}{p_o} + e\mu_p p_o$$

To find the minimum conductivity, set

$$\frac{d\sigma}{dp_o} = 0 = \frac{(-1)e\mu_n n_i^2}{p_o^2} + e\mu_p$$

which yields

$$p_o = n_i\left(\frac{\mu_n}{\mu_p}\right)^{1/2} \quad \text{(Answer to part (b))}$$

Substituting into the conductivity expression

$$\sigma = \sigma_{\min} = \frac{e\mu_n n_i^2}{\left[n_i(\mu_n/\mu_p)^{1/2}\right]} + e\mu_p\left[n_i(\mu_n/\mu_p)^{1/2}\right]$$

which simplifies to

$$\sigma_{\min} = 2en_i\sqrt{\mu_n\mu_p}$$

The intrinsic conductivity is defined as

$$\sigma_i = en_i(\mu_n + \mu_p) \Rightarrow en_i = \frac{\sigma_i}{\mu_n + \mu_p}$$

The minimum conductivity can then be written as

$$\sigma_{\min} = \frac{2\sigma_i\sqrt{\mu_n\mu_p}}{\mu_n + \mu_p}$$

5.23

(a) n-type: $n_o = N_d = 5\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.5\times10^{10})^2}{5\times10^{16}}$$

$$= 4.5\times10^3\ \text{cm}^{-3}$$

p-type: $p_o = N_a = 2\times10^{16}\ \text{cm}^{-3}$

$$n_o = \frac{(1.5\times10^{10})^2}{2\times10^{16}} = 1.125\times10^4\ \text{cm}^{-3}$$

compensated: $n_o = N_d - N_a$

$$= 5\times10^{16} - 2\times10^{16}$$

$$= 3\times10^{16}\ \text{cm}^{-3}$$

$$p_o = \frac{(1.5\times10^{10})^2}{3\times10^{16}} = 7.5\times10^3\ \text{cm}^{-3}$$

(b) From Figure 5.3,

n-type: $\mu_n \cong 1100\ \text{cm}^2/\text{V-s}$

p-type: $\mu_p \cong 400\ \text{cm}^2/\text{V-s}$

compensated: $\mu_n \cong 1000\ \text{cm}^2/\text{V-s}$

(c) n-type: $\sigma = e\mu_n n_o$

$$= (1.6\times10^{-19})(1100)(5\times10^{16})$$

$$= 8.8\ (\Omega\text{-cm})^{-1}$$

p-type: $\sigma = e\mu_p p_o$

$$= (1.6\times10^{-19})(400)(2\times10^{16})$$

$$= 1.28\ (\Omega\text{-cm})^{-1}$$

compensated: $\sigma = e\mu_n n_o$

$$= (1.6\times10^{-19})(1000)(3\times10^{16})$$

$$= 4.8\ (\Omega\text{-cm})^{-1}$$

(d) $J = \sigma \mathrm{E} \Rightarrow \mathrm{E} = \dfrac{J}{\sigma}$

n-type: $\mathrm{E} = \dfrac{120}{8.8} = 13.6\ \text{V/cm}$

p-type: $\mathrm{E} = \dfrac{120}{1.28} = 93.75\ \text{V/cm}$

compensated: $\mathrm{E} = \dfrac{120}{4.8} = 25\ \text{V/cm}$

5.24

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}$$

$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{500}$$

$$= 0.00050 + 0.000667 + 0.0020$$

or

$$\frac{1}{\mu} = 0.003167$$

Then

$$\mu = 316\ \text{cm}^2/\text{V-s}$$

5.25

$$\mu_n = (1300)\left(\frac{T}{300}\right)^{-3/2} = (1300)\left(\frac{300}{T}\right)^{+3/2}$$

(a) At $T = 200\ \text{K}$,

$$\mu_n = (1300)\left(\frac{300}{200}\right)^{3/2} = 2388\ \text{cm}^2/\text{V-s}$$

(b) At $T = 400\ \text{K}$, $\mu_n = 844\ \text{cm}^2/\text{V-s}$

5.26

$$\frac{1}{\mu}=\frac{1}{\mu_1}+\frac{1}{\mu_2}=\frac{1}{250}+\frac{1}{500}=0.006$$

Then

$$\mu = 167 \text{ cm}^2/\text{V-s}$$

5.27

Plot

5.28

Plot

5.29

$$J_n = eD_n\frac{dn}{dx}=eD_n\left(\frac{5\times10^{14}-n(0)}{0.01-0}\right)$$

$$0.19=\left(1.6\times10^{-19}\right)(25)\left(\frac{5\times10^{14}-n(0)}{0.010}\right)$$

Then

$$\frac{(0.19)(0.010)}{\left(1.6\times10^{-19}\right)(25)}=5\times10^{14}-n(0)$$

which yields

$$n(0)=0.25\times10^{14}\text{ cm}^{-3}$$

5.30

$$J_n = eD_n\frac{dn}{dx}=eD_n\frac{\Delta n}{\Delta x}$$

$$J_n=\left(1.6\times10^{-19}\right)(27)\left[\frac{2\times10^{16}-5\times10^{15}}{0-0.012}\right]$$

$$J_n=-5.4\text{ A/cm}^2$$

5.31

(a) $$J_n = eD_n\frac{dn}{dx}=eD_n\frac{\Delta n}{\Delta x}$$

$$-2=\left(1.6\times10^{-19}\right)(30)\left[\frac{10^{15}-n(x_1)}{0-20\times10^{-4}}\right]$$

$$4\times10^{-3}=4.8\times10^{-3}-4.8\times10^{-18}\,n(x_1)$$

which yields

$$n(x_1)=1.67\times10^{14}\text{ cm}^{-3}$$

(b) $$-2=\left(1.6\times10^{-19}\right)(230)\left[\frac{10^{15}-n(x_1)}{0-20\times10^{-4}}\right]$$

$$4\times10^{-3}=3.68\times10^{-2}-3.68\times10^{-17}\,n(x_1)$$

$$n(x_1)=8.91\times10^{14}\text{ cm}^{-3}$$

5.32

$$J_p=-eD_p\frac{dp}{dx}=-eD_p\frac{d}{dx}\left[10^{16}\left(1+\frac{x}{L}\right)^2\right]$$

$$=-eD_p\cdot\frac{10^{16}}{L}\cdot2\left(1+\frac{x}{L}\right)$$

(a) For $x=0$,

$$J_p=\frac{-\left(1.6\times10^{-19}\right)(10)\left(10^{16}\right)(2)}{12\times10^{-4}}$$

$$=-26.7\text{ A/cm}^2$$

(b) For $x=-6\,\mu$m,

$$J_p=\frac{-\left(1.6\times10^{-19}\right)(10)\left(10^{16}\right)(2)\left(1-\frac{6}{12}\right)}{12\times10^{-4}}$$

$$=-13.3\text{ A/cm}^2$$

(c) For $x=-12\,\mu$m,

$$J_p=0$$

5.33

For electrons:

$$J_n=eD_n\frac{dn}{dx}=eD_n\frac{d}{dx}\left[10^{15}e^{-x/L_n}\right]$$

$$=\frac{-eD_n\left(10^{15}\right)e^{-x/L_n}}{L_n}$$

At $x=0$,

$$J_n=\frac{-\left(1.6\times10^{-19}\right)(25)\left(10^{15}\right)}{2\times10^{-3}}=-2\text{ A/cm}^2$$

For holes:

$$J_p=-eD_p\frac{dp}{dx}=-eD_p\frac{d}{dx}\left[5\times10^{15}e^{+x/L_p}\right]$$

$$=\frac{-eD_p\left(5\times10^{15}\right)e^{+x/L_p}}{L_p}$$

For $x=0$,

$$J_p=\frac{-\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)}{5\times10^{-4}}=-16\text{ A/cm}^2$$

$$J_{Total}=J_n(x=0)+J_p(x=0)$$

$$=-2+(-16)=-18\text{ A/cm}^2$$

5.34

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dx}\left[5\times10^{15} e^{-x/L_p}\right]$$

$$= \frac{eD_p\left(5\times10^{15}\right)e^{-x/L_p}}{L_p}$$

(a) (i) $J_p = \dfrac{\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)}{50\times10^{-4}}$

$= 1.6 \text{ A/cm}^2$

(ii) $J_p = \dfrac{\left(1.6\times10^{-19}\right)(48)\left(5\times10^{15}\right)}{22.5\times10^{-4}}$

$= 17.07 \text{ A/cm}^2$

(b) (i) $J_p = \dfrac{\left(1.6\times10^{-19}\right)(10)\left(5\times10^{15}\right)e^{-1}}{50\times10^{-4}}$

$= 0.589 \text{ A/cm}^2$

(ii) $J_p = \dfrac{\left(1.6\times10^{-19}\right)(48)\left(5\times10^{15}\right)e^{-1}}{22.5\times10^{-4}}$

$= 6.28 \text{ A/cm}^2$

5.35

$$J_n = e\mu_n n\mathrm{E} + eD_n \frac{dn}{dx}$$

or

$$-40 = \left(1.6\times10^{-19}\right)(960)\left[10^{16}\exp\left(\frac{-x}{18}\right)\right]\mathrm{E}$$
$$+\left(1.6\times10^{-19}\right)(25)\left(10^{16}\right)$$
$$\times\left(\frac{-1}{18\times10^{-4}}\right)\exp\left(\frac{-x}{18}\right)$$

Then

$$-40 = (1.536)\left[\exp\left(\frac{-x}{18}\right)\right]\mathrm{E} - 22.22\exp\left(\frac{-x}{18}\right)$$

We find

$$\mathrm{E} = \frac{(22.22)\exp\left(\frac{-x}{18}\right) - 40}{(1.536)\exp\left(\frac{-x}{18}\right)}$$

or

$$\mathrm{E} = 14.5 - (26.0)\exp\left(\frac{+x}{18}\right)$$

5.36

(a) $J_n = eD_n \dfrac{dn}{dx} = eD_n \dfrac{d}{dx}\left[2\times10^{15} e^{-x/L}\right]$

$= \dfrac{-eD_n\left(2\times10^{15}\right)e^{-x/L}}{L}$

$= \dfrac{-\left(1.6\times10^{-19}\right)(27)\left(2\times10^{15}\right)e^{-x/L}}{15\times10^{-4}}$

$= -5.76e^{-x/L}$

(b) $J_p = J_{Total} - J_n = -10 - \left(-5.76e^{-x/L}\right)$

$= \left[5.76e^{-x/L} - 10\right] \text{ A/cm}^2$

(c) We have $J_p = \sigma\mathrm{E} = \left(e\mu_p p_o\right)\mathrm{E}$

$5.76e^{-x/L} - 10 = \left(1.6\times10^{-19}\right)(420)\left(10^{16}\right)\mathrm{E}$

So $\mathrm{E} = \left[8.57e^{-x/L} - 14.88\right]$ V/cm

5.37

(a) $J = e\mu_n n(x)\mathrm{E} + eD_n \dfrac{dn(x)}{dx}$

We have $\mu_n = 8000 \text{ cm}^2$/V-s, so that

$D_n = (0.0259)(8000) = 207 \text{ cm}^2\text{/s}$

Then

$$100 = \left(1.6\times10^{-19}\right)(8000)(12)n(x)$$
$$+\left(1.6\times10^{-19}\right)(207)\frac{dn(x)}{dx}$$

which yields

$$100 = \left(1.536\times10^{-14}\right)n(x) + \left(3.312\times10^{-17}\right)\frac{dn(x)}{dx}$$

Solution is of the form

$$n(x) = A + B\exp\left(\frac{-x}{d}\right)$$

so that

$$\frac{dn(x)}{dx} = \frac{-B}{d}\exp\left(\frac{-x}{d}\right)$$

Substituting into the differential equation, we have

$$100 = \left(1.536\times10^{-14}\right)\left[A + B\exp\left(\frac{-x}{d}\right)\right]$$
$$-\frac{\left(3.312\times10^{-17}\right)}{d}B\exp\left(\frac{-x}{d}\right)$$

This equation is valid for all x, so

$$100 = \left(1.536\times10^{-14}\right)A$$

or

$$A = 6.51\times10^{15}$$

Also

$$1.536\times10^{-14}B\exp\left(\frac{-x}{d}\right)$$
$$-\frac{\left(3.312\times10^{-17}\right)}{d}B\exp\left(\frac{-x}{d}\right)=0$$

which yields

$$d=2.156\times10^{-3}\text{ cm}$$

At $x=0$, $e\mu_n n(0)\mathrm{E}=50$

so that

$$50=\left(1.6\times10^{-19}\right)(8000)(12)(A+B)$$

which yields

$$B=-3.255\times10^{15}$$

Then

$$n(x)=6.51\times10^{15}-3.255\times10^{15}\exp\left(\frac{-x}{d}\right)\text{cm}^{-3}$$

(b)

At $x=0$, $n(0)=6.51\times10^{15}-3.255\times10^{15}$

Or

$$n(0)=3.26\times10^{15}\text{ cm}^{-3}$$

At $x=50\,\mu$m,

$$n(50)=6.51\times10^{15}-3.255\times10^{15}\exp\left(\frac{-50}{21.56}\right)$$

or

$$n(50)=6.19\times10^{15}\text{ cm}^{-3}$$

(c)

At $x=50\,\mu$m,

$$J_{drf}=e\mu_n n(50)\mathrm{E}$$
$$=\left(1.6\times10^{-19}\right)(8000)\left(6.19\times10^{15}\right)(12)$$

or

$$J_{drf}(x=50)=95.08\text{ A/cm}^2$$

Then

$$J_{diff}(x=50)=100-95.08$$

or

$$J_{diff}(x=50)=4.92\text{ A/cm}^2$$

5.38

$$n=n_i\exp\left(\frac{E_F-E_{Fi}}{kT}\right)$$

(a) $E_F-E_{Fi}=ax+b$, $b=0.4$

$$0.15=a\left(10^{-3}\right)+0.4$$

which yields

$$a=-2.5\times10^{2}$$

Then

$$E_F-E_{Fi}=0.4-2.5\times10^{2}x$$

so

$$n=n_i\exp\left(\frac{0.4-2.5\times10^{2}x}{kT}\right)$$

(b) $J_n=eD_n\dfrac{dn}{dx}$

$$=eD_n n_i\left(\frac{-2.5\times10^{2}}{kT}\right)\exp\left(\frac{0.4-2.5\times10^{2}x}{kT}\right)$$

Assume $T=300$ K, so $kT=0.0259$ eV and $n_i=1.5\times10^{10}\text{ cm}^{-3}$

Then

$$J_n=\frac{-\left(1.6\times10^{-19}\right)(25)\left(1.5\times10^{10}\right)\left(2.5\times10^{2}\right)}{(0.0259)}$$
$$\times\exp\left(\frac{0.4-2.5\times10^{2}x}{0.0259}\right)$$

or

$$J_n=-5.79\times10^{-4}\exp\left(\frac{0.4-2.5\times10^{2}x}{0.0259}\right)$$

(i) At $x=0$, $J_n=-2.95\times10^{3}\text{ A/cm}^2$

(ii) At $x=5\,\mu$m, $J_n=-23.7\text{ A/cm}^2$

5.39

(a) $J_n=e\mu_n n\mathrm{E}+eD_n\dfrac{dn}{dx}$

$$-80=\left(1.6\times10^{-19}\right)(1000)\left(10^{16}\right)\left(1-\frac{x}{L}\right)\mathrm{E}$$
$$+\left(1.6\times10^{-19}\right)(25.9)\left(\frac{-10^{16}}{L}\right)$$

where $L=10\times10^{-4}=10^{-3}$ cm

We find

$$-80=(1.6)\mathrm{E}-(1.6)\left(\frac{x}{10^{-3}}\right)\mathrm{E}-41.44$$

or

$$80=(1.6)\left(\frac{x}{L}-1\right)\mathrm{E}+41.44$$

Solving for the electric field, we find

$$\mathrm{E}=\frac{24.1}{\left(\frac{x}{L}-1\right)}\text{ V/cm}$$

(b) For $J_n = -20\ \text{A/cm}^2$

$$20 = (1.6)\left(\frac{x}{L} - 1\right)\text{E} + 41.44$$

Then

$$\text{E} = \frac{13.3}{\left(1 - \frac{x}{L}\right)}\ \text{V/cm}$$

5.40

(a) $$\text{E}_X = -\left(\frac{kT}{e}\right)\cdot\frac{1}{N_d(x)}\cdot\frac{dN_d(x)}{dx}$$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}}\cdot\frac{d}{dx}\left[N_{do}e^{-x/L}\right]$$

$$= \frac{-(0.0259)}{N_{do}e^{-x/L}}\cdot\left(\frac{-1}{L}\right)N_{do}e^{-x/L}$$

$$= \frac{0.0259}{L} = \frac{0.0259}{10\times10^{-4}}$$

or $\text{E}_X = 25.9\ \text{V/cm}$

(b) $$\phi = -\int_0^L \text{E}_X\,dx = -(25.9)(L - 0)$$

$$= -(25.9)(10\times10^{-4}) = -0.0259\ \text{V}$$

or $\phi = -25.9\ \text{mV}$

5.41

From Example 5.6

$$\text{E}_x = \frac{(0.0259)(10^{19})}{(10^{16} - 10^{19}x)} = \frac{(0.0259)(10^3)}{(1 - 10^3 x)}$$

$$V = -\int_0^{10^{-4}} \text{E}_x\,dx$$

$$= -(0.0259)(10^3)\int_0^{10^{-4}} \frac{dx}{(1 - 10^3 x)}$$

$$= -(0.0259)(10^3)\left(\frac{-1}{10^3}\right)\ln\left[1 - 10^3 x\right]\Big|_0^{10^{-4}}$$

$$= (0.0259)\left[\ln(1 - 0.1) - \ln(1)\right]$$

or

$$V = -2.73\ \text{mV}$$

5.42

$$\text{E}_x = -\left(\frac{kT}{e}\right)\cdot\frac{1}{N_d(x)}\cdot\frac{dN_d(x)}{dx}$$

For $N_d(x) = N_{do}e^{-x/L}$

So $\text{E}_X = \dfrac{0.0259}{L} = 500\ \text{V/cm}$

Which yields $L = 5.18\times10^{-5}\ \text{cm}$

5.43

(a) We have

$$J_{diff} = eD_n\frac{dn}{dx} = eD_n\frac{dN_d(x)}{dx}$$

$$= \frac{eD_n}{(-L)}\cdot N_{do}\exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (6000)(0.0259)$$

or

$$D_n = 155.4\ \text{cm}^2/\text{s}$$

Then

$$J_{diff} = \frac{-(1.6\times10^{-19})(155.4)(5\times10^{16})}{(0.1\times10^{-4})}\exp\left(\frac{-x}{L}\right)$$

or

$$J_{diff} = -1.243\times10^5\exp\left(\frac{-x}{L}\right)\ \text{A/cm}^2$$

(b)

$$0 = J_{drf} + J_{diff}$$

Now

$$J_{drf} = e\mu_n n\text{E}$$

$$= (1.6\times10^{-19})(6000)(5\times10^{16})\left[\exp\left(\frac{-x}{L}\right)\right]\text{E}$$

or

$$J_{drf} = (48)\left[\exp\left(\frac{-x}{L}\right)\right]\text{E}$$

We have

$$J_{drf} = -J_{diff}$$

so

$$(48)\left[\exp\left(\frac{-x}{L}\right)\right]\text{E} = 1.243\times10^5\exp\left(\frac{-x}{L}\right)$$

which yields

$$\text{E} = 2.59\times10^3\ \text{V/cm}$$

5.44

Plot

5.45

(a) (i) $D_n = (0.0259)(1150) = 29.8 \text{ cm}^2/\text{s}$

(ii) $D_n = (0.0259)(6200) = 160.6 \text{ cm}^2/\text{s}$

(b) (i) $\mu_p = \dfrac{8}{0.0259} = 308.9 \text{ cm}^2/\text{V-s}$

(ii) $\mu_p = \dfrac{35}{0.0259} = 1351 \text{ cm}^2/\text{V-s}$

5.46

$L = 10^{-1}\text{ cm}, \quad W = 10^{-2}\text{ cm}, \quad d = 10^{-3}\text{ cm}$

(a)

$$V_H = \frac{-I_X B_Z}{ned} = \frac{-(1.2\times10^{-3})(5\times10^{-2})}{(2\times10^{22})(1.6\times10^{-19})(10^{-5})}$$

$= -1.875\times10^{-3}$ V

or $V_H = -1.875$ mV

(b)

$$E_H = \frac{V_H}{W} = \frac{-1.875\times10^{-3}}{10^{-2}} = -0.1875 \text{ V/cm}$$

5.47

(a) $V_H = \dfrac{-I_x B_z}{ned}$

$$= \frac{-(250\times10^{-6})(5\times10^{-2})}{(5\times10^{21})(1.6\times10^{-19})(5\times10^{-5})}$$

or

$V_H = -0.3125$ mV

(b)

$$E_H = \frac{V_H}{W} = \frac{-0.3125\times10^{-3}}{2\times10^{-2}}$$

or

$E_H = -1.56\times10^{-2}$ V/cm

(c)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(250\times10^{-6})(10^{-3})}{(1.6\times10^{-19})(5\times10^{21})(0.1)(2\times10^{-4})(5\times10^{-5})}$$

or

$\mu_n = 0.3125 \text{ m}^2/\text{V-s} = 3125 \text{ cm}^2/\text{V-s}$

5.48

(a) $V_H < 0 \Rightarrow$ n-type

(b) $$n = \frac{-I_X B_Z}{edV_H} = \frac{-(0.50\times10^{-3})(0.10)}{(1.6\times10^{-19})(10^{-5})(-5.2\times10^{-3})}$$

$= 6.01\times10^{21} \text{ m}^{-3}$

or $n = 6.01\times10^{15} \text{ cm}^{-3}$

(c) $\mu_n = \dfrac{I_X L}{enV_X Wd}$

$$= \frac{(0.5\times10^{-3})(10^{-3})}{(1.6\times10^{-19})(6.01\times10^{21})(15)(10^{-4})(10^{-5})}$$

$= 0.03466 \text{ m}^2/\text{V-s}$

or $\mu_n = 346.6 \text{ cm}^2/\text{V-s}$

5.49

(a) $V_H = E_H W = -(16.5\times10^{-3})(5\times10^{-2})$

or

$V_H = -0.825$ mV

(b) $V_H =$ negative $\Rightarrow$ n-type

(c) $n = \dfrac{-I_x B_z}{edV_H}$

$$= \frac{-(0.5\times10^{-3})(6.5\times10^{-2})}{(1.6\times10^{-19})(5\times10^{-5})(-0.825\times10^{-3})}$$

or

$n = 4.924\times10^{21} \text{ m}^{-3} = 4.924\times10^{15} \text{ cm}^{-3}$

(d)

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$= \frac{(0.5\times10^{-3})(0.5\times10^{-2})}{(1.6\times10^{-19})(4.924\times10^{21})(1.25)(5\times10^{-4})(5\times10^{-5})}$$

or

$\mu_n = 0.1015 \text{ m}^2/\text{V-s} = 1015 \text{ cm}^2/\text{V-s}$

5.50

(a) V_H = negative $\Rightarrow$ n-type

(b) $n = \dfrac{-I_x B_z}{edV_H}$

$$= \frac{-\left(2.5\times10^{-3}\right)\left(2.5\times10^{-2}\right)}{\left(1.6\times10^{-19}\right)\left(0.01\times10^{-2}\right)\left(-4.5\times10^{-3}\right)}$$

or

$n = 8.68\times10^{20}\ \text{m}^{-3} = 8.68\times10^{14}\ \text{cm}^{-3}$

(c) $\mu_n = \dfrac{I_x L}{enV_x Wd}$

$$= \left[\frac{\left(2.5\times10^{-3}\right)\left(0.5\times10^{-2}\right)}{\left(1.6\times10^{-19}\right)\left(8.68\times10^{20}\right)(2.2)}\right]$$

$$\times\left[\frac{1}{\left(0.05\times10^{-2}\right)\left(0.01\times10^{-2}\right)}\right]$$

or

$\mu_n = 0.8182\ \text{m}^2/\text{V-s} = 8182\ \text{cm}^2/\text{V-s}$

(d) $\sigma = \dfrac{1}{\rho} = e\mu_n n$

$$= \left(1.6\times10^{-19}\right)(8182)\left(8.68\times10^{14}\right)$$

or

$\rho = 0.88\,(\Omega\text{-cm})$

Chapter 6

6.1

$n_o = N_d = 5\times10^{15}$ cm^{-3}

$$p_o = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{15}} = 4.5\times10^4 \text{ cm}^{-3}$$

(a) Minority carrier hole lifetime is a constant.

$$\tau_{pt} = \tau_{p0} = 2\times10^{-7} \text{ s}$$

$$R_{po} = \frac{p_o}{\tau_{p0}} = \frac{4.5\times10^4}{2\times10^{-7}} = 2.25\times10^{11} \text{ cm}^{-3}\text{ s}^{-1}$$

(b) $$R'_{po} = \frac{p_o + \delta p}{\tau_{p0}} = \frac{4.5\times10^4 + 10^{14}}{2\times10^{-7}}$$

$$= 5\times10^{20} \text{ cm}^{-3}\text{ s}^{-1}$$

6.2

$p_o = N_a = 2\times10^{16}$ cm^{-3}

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8\times10^6\right)^2}{2\times10^{16}} = 1.62\times10^{-4} \text{ cm}^{-3}$$

(a) $$R' = \frac{\delta n}{\tau_{n0}} = \frac{5\times10^{14}}{5\times10^{-7}} = 10^{21} \text{ cm}^{-3}\text{ s}^{-1}$$

(b) $$R_p = \frac{p_o}{\tau_{pt}} = \frac{n_o}{\tau_{nt}} = \frac{n_o}{\tau_{n0}}$$

$$\tau_{pt} = \frac{p_o}{n_o}\cdot\tau_{n0} = \frac{\left(2\times10^{16}\right)}{\left(1.62\times10^{-4}\right)}\cdot\left(5\times10^{-7}\right)$$

$$= 6.17\times10^{13} \text{ s}$$

6.3

(a) Recombination rates are equal

$$\frac{n_o}{\tau_{nO}} = \frac{p_o}{\tau_{pO}}$$

$n_o = N_d = 10^{16}$ cm^{-3}

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$$

Then

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25\times10^4}{20\times10^{-6}}$$

which yields

$$\tau_{nO} = 8.89\times10^{+6} \text{ s}$$

(b) Generation rate = recombination rate
Then

$$G = \frac{2.25\times10^4}{20\times10^{-6}} = 1.125\times10^9 \text{ cm}^{-3}\text{ s}^{-1}$$

(c)

$$R = G = 1.125\times10^9 \text{ cm}^{-3}\text{ s}^{-1}$$

6.4

(a) $$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625\times10^{-34}\right)\left(3\times10^8\right)}{6300\times10^{-10}}$$

or

$E = 3.15\times10^{-19}$ J; energy of one photon

Now

1 W = 1 J/s $\Rightarrow 3.17\times10^{18}$ photons/s

Volume = (1)(0.1) = 0.1 cm^3

Then

$$g = \frac{3.17\times10^{18}}{0.1}$$

$= 3.17\times10^{19}$ e-h pairs/cm^3-s

(b)

$$\delta n = \delta p = g\tau = \left(3.17\times10^{19}\right)\left(10\times10^{-6}\right)$$

or

$$\delta n = \delta p = 3.17\times10^{14} \text{ cm}^{-3}$$

6.5

We have

$$\frac{\partial p}{\partial t} = -\nabla\bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p p\mathrm{E} - eD_p\nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p p\mathrm{E} - D_p\nabla p$$

Now

$$\nabla\bullet F_p^+ = \mu_p\nabla\bullet\left(p\mathrm{E}\right) - D_p\nabla\bullet\nabla p$$

We can write

$$\nabla\bullet\left(p\mathrm{E}\right) = \mathrm{E}\bullet\nabla p + p\nabla\bullet\mathrm{E}$$

and

$$\nabla\bullet\nabla p = \nabla^2 p$$

so

$$\nabla\bullet F_p^+ = \mu_p\left(\mathrm{E}\bullet\nabla p + p\nabla\bullet\mathrm{E}\right) - D_p\nabla^2 p$$

Then

$$\frac{\partial p}{\partial t} = -\mu_p (\mathrm{E} \bullet \nabla p + p \nabla \bullet \mathrm{E})$$

$$+ D_p \nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p \nabla^2 p - \mu_p (\mathrm{E} \bullet \nabla p + p \nabla \bullet \mathrm{E})$$

$$+ g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

6.6

From Equation (6.18),

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \bullet F_p^+ + g_p - R_p$$

For a one-dimensional case,

$$\frac{dF_p^+}{dx} = g_p - R_p = 10^{20} - 2 \times 10^{19}$$

or

$$\frac{dF_p^+}{dx} = 8 \times 10^{19} \ \mathrm{cm}^{-3} \mathrm{s}^{-1}$$

6.7

From Equation (6.18),

$$0 = -\frac{dF_p^+}{dx} + 0 - 2 \times 10^{19}$$

or

$$\frac{dF_p^+}{dx} = -2 \times 10^{19} \ \mathrm{cm}^{-3} \mathrm{s}^{-1}$$

6.8

We have the continuity equations

(1) $D_p \nabla^2 (\delta p) - \mu_p [\mathrm{E} \bullet \nabla (\delta p) + p \nabla \bullet \mathrm{E}]$

$$+ g_p - \frac{p}{\tau_p} = \frac{\partial (\delta p)}{\partial t}$$

and

(2) $D_n \nabla^2 (\delta n) + \mu_n [\mathrm{E} \bullet \nabla (\delta n) + n \nabla \bullet \mathrm{E}]$

$$+ g_n - \frac{n}{\tau_n} = \frac{\partial (\delta n)}{\partial t}$$

By charge neutrality,

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla (\delta n) = \nabla (\delta p)$$

and

$$\nabla^2 (\delta n) = \nabla^2 (\delta p) \ \text{ and } \ \frac{\partial (\delta n)}{\partial t} = \frac{\partial (\delta p)}{\partial t}$$

Also

$$g_n = g_p \equiv g, \quad \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we have

(1) $D_p \nabla^2 (\delta n) - \mu_p [\mathrm{E} \bullet \nabla (\delta n) + p \nabla \bullet \mathrm{E}]$

$$+ g - R = \frac{\partial (\delta n)}{\partial t}$$

and

(2) $D_n \nabla^2 (\delta n) + \mu_n [\mathrm{E} \bullet \nabla (\delta n) + n \nabla \bullet \mathrm{E}]$

$$+ g - R = \frac{\partial (\delta n)}{\partial t}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and add the two equations.

We find

$$(\mu_n n D_p + \mu_p p D_n) \nabla^2 (\delta n)$$

$$+ \mu_n \mu_p (p - n) \mathrm{E} \bullet \nabla (\delta n)$$

$$+ (\mu_n n + \mu_p p)(g - R)$$

$$= (\mu_n n + \mu_p p) \frac{\partial (\delta n)}{\partial t}$$

Divide by $(\mu_n n + \mu_p p)$, then

$$\left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \right) \nabla^2 (\delta n)$$

$$+ \left[\frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p} \right] \mathrm{E} \bullet \nabla (\delta n)$$

$$+ (g - R) = \frac{\partial (\delta n)}{\partial t}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p (n + p)}{D_n n + D_p p}$$

and

$$\mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Then we have

$$D' \nabla^2 (\delta n) + \mu' \mathrm{E} \bullet \nabla (\delta n) + (g - R) = \frac{\partial (\delta n)}{\partial t}$$

Q.E.D.

6.9

p-type material;
minority carriers are electrons

(a) $\mu' = \mu_n$

From Figure 5.3, $\mu_n \cong 1300 \text{ cm}^2\text{/V-s}$

(b) $D' = D_n = \left(\dfrac{kT}{e}\right)\cdot \mu_n = (0.0259)(1300)$

$= 33.67 \text{ cm}^2\text{/s}$

(c) $\tau_{nt} = \tau_{n0} = 10^{-7}$ s

$p_o = N_a = 7\times 10^{15} \text{ cm}^{-3}$

$n_o = \dfrac{n_i^2}{N_a} = \dfrac{(1.5\times 10^{10})^2}{7\times 10^{15}}$

$= 3.21\times 10^4 \text{ cm}^{-3}$

$\dfrac{n_o}{\tau_{nt}} = \dfrac{p_o}{\tau_{pt}}$

$\dfrac{3.214\times 10^4}{10^{-7}} = \dfrac{7\times 10^{15}}{\tau_{pt}}$

so $\tau_{pt} = 2.18\times 10^4$ s

6.10

For Ge: $n_i = 2.4\times 10^{13} \text{ cm}^{-3}$

$n_o = \dfrac{N_d}{2} + \sqrt{\left(\dfrac{N_d}{2}\right)^2 + n_i^2}$

$= \dfrac{4\times 10^{13}}{2} + \sqrt{\left(\dfrac{4\times 10^{13}}{2}\right)^2 + (2.4\times 10^{13})^2}$

$= 5.124\times 10^{13} \text{ cm}^{-3}$

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{(2.4\times 10^{13})^2}{5.124\times 10^{13}} = 1.124\times 10^{13} \text{ cm}^{-3}$

(a) We have:

$\mu_n = 3900 \text{ cm}^2\text{/V-s}, \quad D_n = 101 \text{ cm}^2\text{/s}$

$\mu_p = 1900 \text{ cm}^2\text{/V-s}, \quad D_p = 49.2 \text{ cm}^2\text{/s}$

For very, very low injection,

$D' = \dfrac{D_n D_p (n+p)}{D_n n + D_p p}$

$= \dfrac{(101)(49.2)(5.124\times 10^{13} + 1.124\times 10^{13})}{(101)(5.124\times 10^{13}) + (49.2)(1.124\times 10^{13})}$

$= 54.2 \text{ cm}^2\text{/s}$

and

$\mu' = \dfrac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$

$= \dfrac{(3900)(1900)(1.124\times 10^{13} - 5.124\times 10^{13})}{(3900)(5.124\times 10^{13}) + (1900)(1.124\times 10^{13})}$

$= -1340 \text{ cm}^2\text{/V-s}$

(b) For holes, $\tau_{pt} = \tau_{p0} = 2\times 10^{-6}$ s

For electrons,

$\dfrac{n}{\tau_{nt}} = \dfrac{p}{\tau_{p0}}$

$\dfrac{5.124\times 10^{13}}{\tau_{nt}} = \dfrac{1.124\times 10^{13}}{2\times 10^{-6}}$

$\Rightarrow \tau_{nt} = 9.12\times 10^{-6}$ s

6.11

$\sigma = e\mu_n n + e\mu_p p$

With excess carriers

$n = n_o + \delta n$ and $p = p_o + \delta p$

For an n-type semiconductor, we can write

$\delta n = \delta p \equiv \delta p$

Then

$\sigma = e\mu_n (n_o + \delta p) + e\mu_p (p_o + \delta p)$

or

$\sigma = e\mu_n n_o + e\mu_p p_o + e(\mu_n + \mu_p)(\delta p)$

so

$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$

In steady-state, $\delta p = g'\tau_{pO}$

So that

$\Delta\sigma = e(\mu_n + \mu_p)(g'\tau_{pO})$

6.12

(a) $p_o = N_a = 10^{16} \text{ cm}^{-3}$

$n_o = \dfrac{n_i^2}{p_o} = \dfrac{(1.5\times 10^{10})^2}{10^{16}} = 2.25\times 10^4 \text{ cm}^{-3}$

$\sigma = e\mu_n (n_o + \delta n) + e\mu_p (p_o + \delta p)$

$\cong e\mu_p p_o + e(\mu_n + \mu_p)\delta n$

Now $\delta n = \delta p = g'\tau_{n0}\left(1 - e^{-t/\tau_{n0}}\right)$

$= (8\times 10^{20})(5\times 10^{-7})\left(1 - e^{-t/\tau_{n0}}\right)$

$= 4\times 10^{14}\left(1 - e^{-t/\tau_{n0}}\right) \text{ cm}^{-3}$

Then $\sigma = (1.6\times10^{-19})(380)(10^{16})$
$+(1.6\times10^{-19})(900+380)$
$\times(4\times10^{14})(1-e^{-t/\tau_{n0}})$

$\sigma = 0.608 + 0.0819(1-e^{-t/\tau_{n0}})$ $(\Omega\text{-cm})^{-1}$

(b) (i) $\sigma(0) = 0.608$ $(\Omega\text{-cm})^{-1}$

(ii) $\sigma(\infty) = 0.690$ $(\Omega\text{-cm})^{-1}$

6.13

(a) For $0 \le t \le 10^{-6}$ s,

$$\delta n = \delta p = g'\tau_{p0}(1-e^{-t/\tau_{p0}})$$
$$= (4\times10^{21})(5\times10^{-8})(1-e^{-t/\tau_{p0}})$$
$$= (2\times10^{14})(1-e^{-t/\tau_{p0}})\ \text{cm}^{-3}$$

At $t = 10^{-6}$ s,

$$\delta p(10^{-6}) = (2\times10^{14})(1-e^{-10^{-6}/5\times10^{-8}})$$
$$= 2\times10^{14}\ \text{cm}^{-3}$$

Then for $t \ge 10^{-6}$ s,

$$\delta p = (2\times10^{14})e^{-(t-10^{-6})/\tau_{p0}}\ \text{cm}^{-3}$$

(b) $n_o = 5\times10^{15}$ cm^{-3}

$$\sigma = e\mu_n n_o + e(\mu_n+\mu_p)\delta p$$

For $0 \le t \le 10^{-6}$ s,

$$\sigma = (1.6\times10^{-19})(7500)(5\times10^{15})$$
$$+(1.6\times10^{-19})(7500+310)$$
$$\times(2\times10^{14})(1-e^{-t/\tau_{p0}})$$
$$= 6.0 + 0.250(1-e^{-t/\tau_{p0}})\ (\Omega\text{-cm})^{-1}$$

For $t \ge 10^{-6}$ s,

$$\sigma = 6.0 + 0.250e^{-(t-10^{-6})/\tau_{p0}}\ (\Omega\text{-cm})^{-1}$$

6.14

$$I = \frac{V}{R};\quad R = \frac{L}{\sigma A}$$

$$\Rightarrow I = \frac{\sigma A}{L}\cdot V$$

For $N_I = N_d + N_a = 8\times10^{15} + 2\times10^{15}$
$= 10^{16}$ cm^{-3}

Then, $\mu_n \cong 1300$ cm^2/V-s
$\mu_p \cong 400$ cm^2/V-s

$$\sigma \cong e\mu_n n_o + e(\mu_n+\mu_p)\delta p$$

where $\delta p = g'\tau_{p0}e^{-t/\tau_{p0}}$

$$= (8\times10^{20})(5\times10^{-7})e^{-t/\tau_{p0}}$$
$$= 4\times10^{14}e^{-t/\tau_{p0}}\ \text{cm}^{-3}$$

$$\sigma = (1.6\times10^{-19})(1300)(8\times10^{15}-2\times10^{15})$$
$$+(1.6\times10^{-19})(1300+400)$$
$$\times(4\times10^{14})e^{-t/\tau_{p0}}$$

$$\sigma = 1.248 + 0.109e^{-t/\tau_{p0}}$$

$$I = \frac{\left[1.248+0.109e^{-t/\tau_{p0}}\right](10^{-5})(10)}{0.05}$$
$$= 2.496\times10^{-3} + 2.18\times10^{-4}e^{-t/\tau_{p0}}\ \text{A}$$

or $I = 2.496 + 0.218e^{-t/\tau_{p0}}$ mA

6.15

$$p_o = N_a - N_d = 2\times10^{16} - 6\times10^{15}$$
$$= 1.4\times10^{16}\ \text{cm}^{-3}$$

(a) $\delta n = \delta p = g'\tau_{n0}$

$5\times10^{14} = 2\times10^{21}\tau_{n0}$

$\Rightarrow \tau_{n0} = 2.5\times10^{-7}$ s

(b) $\delta n = \delta p = g'\tau_{n0}(1-e^{-t/\tau_{n0}})$

$$= 5\times10^{14}(1-e^{-t/\tau_{n0}})$$

$$R' = \frac{\delta n}{\tau_{n0}} = \frac{5\times10^{14}}{2.5\times10^{-7}}(1-e^{-t/\tau_{n0}})$$
$$= 2\times10^{21}(1-e^{-t/\tau_{no}})\ \text{cm}^{-3}\,\text{s}^{-1}$$

(c)

(i) $\left(\frac{1}{4}\right)(5\times10^{14}) = 5\times10^{14}(1-e^{-t/\tau_{n0}})$

$t = \tau_{n0}\ln(1.3333) = 7.19\times10^{-8}$ s

(ii) $\left(\frac{1}{2}\right)(5\times10^{14}) = 5\times10^{14}(1-e^{-t/\tau_{n0}})$

$t = \tau_{n0}\ln(2) = 1.73\times10^{-7}$ s

(iii) $\left(\frac{3}{4}\right)(5\times10^{14}) = 5\times10^{14}(1-e^{-t/\tau_{n0}})$

$t = \tau_{n0}\ln(4) = 3.47\times10^{-7}$ s

(iv) $(0.95)(5\times10^{14}) = 5\times10^{14}(1-e^{-t/\tau_{n0}})$

$t = \tau_{n0}\ln(20) = 7.49\times10^{-7}$ s

6.16

$n_o = N_d - N_a = 8\times10^{15} - 2\times10^{15}$
$= 6\times10^{15}$ cm^{-3}

$p_o = \dfrac{n_i^2}{n_o} = \dfrac{\left(1.8\times10^6\right)^2}{6\times10^{15}} = 5.4\times10^{-4}$ cm^{-3}

(a) $R_o = \dfrac{p_o}{\tau_{p0}} \Rightarrow 4\times10^4 = \dfrac{5.4\times10^{-4}}{\tau_{p0}}$

so $\tau_{p0} = 1.35\times10^{-8}$ s

(b) $\delta p = g'\tau_{p0} = \left(2\times10^{21}\right)\left(1.35\times10^{-8}\right)$
$= 2.7\times10^{13}$ cm^{-3}

(c) $\tau = \tau_{p0} = 1.35\times10^{-8}$ s

6.17

(a) (i)For $0 \le t \le 5\times10^{-7}$ s

$\delta p(t) = g'\tau_{p0}\left(1 - e^{-t/\tau_{p0}}\right)$
$= \left(5\times10^{20}\right)\left(5\times10^{-7}\right)\left(1 - e^{-t/\tau_{p0}}\right)$
$= 2.5\times10^{14}\left(1 - e^{-t/\tau_{p0}}\right)$ cm^{-3}

At $t = 5\times10^{-7}$ s,

$\delta p = 2.5\times10^{14}\left(1 - e^{-1/1}\right)$
$= 1.58\times10^{14}$ cm^{-3}

For $t \ge 5\times10^{-7}$ s

$\delta p(t) = 1.58\times10^{14}\, e^{-\left(t - 5\times10^{-7}\right)/\tau_{pO}}$ cm^{-3}

(ii) $\delta p\left(5\times10^{-7}\right) = 1.58\times10^{14}$ cm^{-3}

(b) (i) For $0 \le t \le 2\times10^{-6}$ s

$\delta p(t) = 2.5\times10^{14}\left(1 - e^{-t/\tau_{p0}}\right)$ cm^{-3}

At $t = 2\times10^{-6}$ s,

$\delta p = 2.5\times10^{14}\left(1 - e^{-\left(2\times10^{-6}\right)/\left(5\times10^{-7}\right)}\right)$
$= 2.454\times10^{14}$ cm^{-3}

For $t \ge 2\times10^{-6}$ s,

$\delta p(t) = 2.454\times10^{14}\, e^{-\left(t - 2\times10^{-6}\right)/\tau_{pO}}$ cm^{-3}

(ii) $\delta p\left(2\times10^{-6}\right) = 2.454\times10^{14}$ cm^{-3}

6.18

(a) For $0 \le t \le 2\times10^{-6}$ s

$\delta n(t) = g'\tau_{n0} e^{-t/\tau_{n0}}$
$= \left(10^{21}\right)\left(5\times10^{-7}\right)e^{-t/\tau_{n0}}$
$= 5\times10^{14}\, e^{-t/\tau_{n0}}$ cm^{-3}

At $t = 2\times10^{-6}$ s,

$\delta n_1 = 5\times10^{14}\, e^{-\left(2\times10^{-6}\right)/\left(5\times10^{-7}\right)}$
$= 9.16\times10^{12}$ cm^{-3}

For $t \ge 2\times10^{-6}$ s

$\delta n = \left(5\times10^{14} - 9.16\times10^{12}\right)\left(1 - e^{-t/\tau_{n0}}\right)$
$+ 9.16\times10^{12}$
$= 4.908\times10^{14}\left(1 - e^{-t/\tau_{n0}}\right) + 9.16\times10^{12}$ cm^{-3}

(b) (i) $\delta n(0) = 5\times10^{14}$ cm^{-3}

(ii) $\delta n\left(2\times10^{-6}\right) = 9.16\times10^{12}$ cm^{-3}

(iii) $\delta n(\infty) = 5\times10^{14}$ cm^{-3}

6.19

p-type; minority carriers - electrons

$D_n = \left(\dfrac{kT}{e}\right)\mu_n = (0.0259)(1200)$
$= 31.08$ cm^2/s

$L_n = \sqrt{D_n\tau_{n0}} = \left[(31.08)\left(10^{-6}\right)\right]^{1/2}$
$= 5.575\times10^{-3}$ cm

(a) $\delta n(x) = \delta p(x) = 2\times10^{14}\, e^{-x/L_n}$ cm^{-3}

(b) $J_n = eD_n \dfrac{d(\delta n)}{dx} = eD_n \dfrac{d}{dx}\left[2\times10^{14}\, e^{-x/L_n}\right]$

$= \dfrac{-eD_n}{L_n}\left(2\times10^{14}\right)e^{-x/L_n}$

$= \dfrac{-\left(1.6\times10^{-19}\right)(31.08)\left(2\times10^{14}\right)}{\left(5.575\times10^{-3}\right)} e^{-x/L_n}$

$J_n = -0.1784 e^{-x/L_n}$ A/cm^2

Holes diffuse at same rate as minority carrier electrons, so

$J_p = +0.1784 e^{-x/L_n}$ A/cm^2

6.20

(a) p-type; $p_{pO} = 10^{14}$ cm^{-3}

and

$n_{pO} = \dfrac{n_i^2}{p_{pO}} = \dfrac{\left(1.5\times10^{10}\right)^2}{10^{14}} = 2.25\times10^6$ cm^{-3}

(b) Excess minority carrier concentration

$\delta n = n_p - n_{pO}$

At $x = 0$, $n_p = 0$ so that

$\delta n(0) = 0 - n_{pO} = -2.25\times10^6$ cm^{-3}

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where } L_n^2 = D_n \tau_{nO}$$

The general solution is of the form

$$\delta n = A \exp\left(\frac{-x}{L_n}\right) + B \exp\left(\frac{+x}{L_n}\right)$$

For $x \to \infty$, δn remains finite, so $B = 0$.
Then the solution is

$$\delta n = -n_{pO} \exp\left(\frac{-x}{L_n}\right)$$

6.21

$$\delta n(x) = 5\times10^{14} e^{-x/L_n} \text{ cm}^{-3}$$

where $L_n = \sqrt{D_n \tau_{n0}} = \left[(25)(10^{-6})\right]^{1/2}$

$$= 5\times10^{-3} \text{ cm}$$

$$J_n = eD_n \frac{d(\delta n)}{dx} = eD_n \frac{d}{dx}\left[(5\times10^{14})e^{-x/L_n}\right]$$

$$= -\frac{eD_n}{L_n}(5\times10^{14})e^{-x/L_n}$$

$$= -\frac{(1.6\times10^{-19})(25)(5\times10^{14})}{(5\times10^{-3})} e^{-x/L_n}$$

$$J_n = -0.4 e^{-x/L_n} \text{ A/cm}^2$$

(a) For $x = 0$,

$$\delta n(0) = 5\times10^{14} \text{ cm}^{-3}$$
$$J_n(0) = -0.4 \text{ A/cm}^2$$
$$J_p(0) = +0.4 \text{ A/cm}^2$$

(b) For $x = L_n = 5\times10^{-3}$ cm,

$$\delta n(L_n) = (5\times10^{14})e^{-1} = 1.84\times10^{14} \text{ cm}^{-3}$$
$$J_n(L_n) = -0.4e^{-1} = -0.147 \text{ A/cm}^2$$
$$J_p(L_n) = +0.4e^{-1} = +0.147 \text{ A/cm}^2$$

(c) For $x = 15\times10^{-3} \text{ cm} = 3L_n$

$$\delta n(3L_n) = (5\times10^{14})e^{-3} = 2.49\times10^{13} \text{ cm}^{-3}$$
$$J_n(3L_n) = -0.4e^{-3} = -0.020 \text{ A/cm}^2$$
$$J_p(3L_n) = +0.4e^{-3} = +0.020 \text{ A/cm}^2$$

6.22

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx} - \mu_p \mathrm{E}_o \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{pO}} = 0$$

Assume the solution is of the form

$$\delta p = A \exp(sx)$$

Then

$$\frac{d(\delta p)}{dx} = As\exp(sx), \quad \frac{d^2(\delta p)}{dx^2} = As^2 \exp(sx)$$

Substituting into the differential equation

$$D_p As^2 \exp(sx) - \mu_p \mathrm{E}_o As\exp(sx) - \frac{A\exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p \mathrm{E}_o s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p, we have

$$s^2 - \frac{\mu_p \mathrm{E}_o}{D_p} s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2}\left[\frac{\mu_p}{D_p}\mathrm{E}_o \pm \sqrt{\left(\frac{\mu_p}{D_p}\mathrm{E}_o\right)^2 + \frac{4}{L_p^2}}\right]$$

which can be rewritten as

$$s = \frac{1}{L_p}\left[\frac{\mu_p L_p \mathrm{E}_o}{2D_p} \pm \sqrt{\left(\frac{\mu_p L_p \mathrm{E}_o}{2D_p}\right)^2 + 1}\right]$$

Define

$$\beta \equiv \frac{\mu_p L_p \mathrm{E}_o}{2D_p}$$

Then

$$s = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

In order that $\delta p = 0$ as $x \to +\infty$, use the minus sign for $x > 0$ and the plus sign for $x < 0$. Then the solution is

$$\delta p = A\exp(s_- x) \quad \text{for } x > 0$$
$$\delta p = A\exp(s_+ x) \quad \text{for } x < 0$$

where

$$s_\pm = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

6.23

Plot

6.24

(a) From Equation (6.55)

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \mathrm{E}_o \frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n} \mathrm{E}_o \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n} \mathrm{E}_o = \frac{\mathrm{E}_o}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'} \cdot \frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

The solution is of the form

$$\delta n = \delta n(0) \exp(-\alpha x) \quad \text{where } \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \quad \text{and} \quad \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we find

$$\alpha^2(\delta n) + \frac{1}{L'}[-\alpha(\delta n)] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n}\left[\frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'}\right)^2 + 1}\right]$$

We may note that if $\mathrm{E}_o = 0$, then $L' \to \infty$ and $\alpha = \dfrac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n \tau_{nO}} \quad \text{where } D_n = \mu_n\left(\frac{kT}{e}\right)$$

so

$$D_n = (1200)(0.0259) = 31.1 \text{ cm}^2/\text{s}$$

and

$$L_n = \sqrt{(31.1)(5\times10^{-7})} = 39.4\times10^{-4} \text{ cm}$$

or

$$L_n = 39.4\ \mu\text{m}$$

For $\mathrm{E}_o = 12$ V/cm, then

$$L' = \frac{(kT/e)}{\mathrm{E}_o} = \frac{0.0259}{12} = 21.6\times10^{-4} \text{ cm}$$

and

$$\alpha = 5.75\times10^2 \text{ cm}^{-1}$$

(c) Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

6.25

p-type so the minority carriers are electrons and

$$D_n \nabla^2(\delta n) + \mu_n \mathrm{E} \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that $\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{nO} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \quad \text{which gives } \delta n = g't + C_1$$

For $t \le 0$, $\delta n = 0 \Rightarrow C_1 = 0$

Then

$$\delta n = G_o' t \quad \text{for } 0 \le t \le T$$

For $t > T$, $g' = 0$ so that $\dfrac{d(\delta n)}{dt} = 0$

And

$$\delta n = G_o' T \quad \text{(no recombination)}$$

6.26

n-type, so minority carriers are holes and

$$D_p \nabla^2(\delta p) - \mu_p \mathrm{E} \bullet \nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $\mathrm{E} = 0$, and $\dfrac{\partial(\delta p)}{\partial t} = 0$ (steady-state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \quad \text{or} \quad \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G_o' =$ constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G_o'}{D_p} x + C_1$$

and

$$\delta p = -\frac{G'_o}{2D_p}x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_3 \text{ and}$$

$$\delta p = C_3 x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = C_5 \text{ and}$$

$$\delta p = C_5 x + C_6$$

The boundary conditions are:

(1) $\delta p = 0$ at $x = +3L$

(2) $\delta p = 0$ at $x = -3L$

(3) δp continuous at $x = L$

(4) δp continuous at $x = -L$

(5) $\dfrac{d(\delta p)}{dx}$ continuous at $x = L$

(6) $\dfrac{d(\delta p)}{dx}$ continuous at $x = -L$

Applying the boundary conditions, we find

$$\delta p = \frac{G'_o}{2D_p}\left(5L^2 - x^2\right) \text{ for } -L < x < +L$$

$$\delta p = \frac{G'_o L}{D_p}(3L - x) \text{ for } L < x < 3L$$

$$\delta p = \frac{G'_o L}{D_p}(3L + x) \text{ for } -3L < x < -L$$

6.27

$$\mathrm{E}_0 = \frac{V}{L} = \frac{8}{0.4} = 20 \text{ V/cm}$$

$$\mu_p = \frac{d}{\mathrm{E}_0 t_0} = \frac{0.25}{(20)\left(32\times10^{-6}\right)}$$

$$= 390.6 \text{ cm}^2\text{/V-s}$$

$$D_p = \frac{\left(\mu_p \mathrm{E}_0\right)^2 (\Delta t)^2}{16 t_0}$$

$$= \frac{[(390.6)(20)]^2 \left(9.35\times10^{-6}\right)^2}{16\left(32\times10^{-6}\right)}$$

$$D_p = 10.42 \text{ cm}^2\text{/s}$$

We find

$$\frac{D_p}{\mu_p} = \frac{10.42}{390.6} = 0.02668 \text{ V}$$

This value is very close to 0.0259 for $T = 300$ K.

6.28

(a)

Assume that $f(x,t) = (4\pi Dt)^{-1/2}\exp\left(\dfrac{-x^2}{4Dt}\right)$

is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\frac{\partial^2 f}{\partial x^2} = (4\pi Dt)^{-1/2}\left[\left(\frac{-2x}{4Dt}\right)^2\exp\left(\frac{-x^2}{4Dt}\right) + \left(\frac{-2}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)\right]$$

Also

$$\frac{\partial f}{\partial t} = (4\pi Dt)^{-1/2}\left(\frac{-x^2}{4D}\right)\left(\frac{-1}{t^2}\right)\exp\left(\frac{-x^2}{4Dt}\right) + (4\pi D)^{-1/2}\left(\frac{-1}{2}\right)t^{-3/2}\exp\left(\frac{-x^2}{4Dt}\right)$$

Substituting the expressions for $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial f}{\partial t}$ into the differential equation, we find $0 = 0$.

Q.E.D.

(b)

Consider

$$\int_{-\infty}^{+\infty}\exp\left(\frac{-x^2}{4Dt}\right)dx$$

Let $u = x^2$, then $du = 2x\cdot dx$ or

$$dx = \frac{du}{2x} = \frac{du}{2\sqrt{u}}$$

Let $a = \dfrac{1}{4Dt}$

Now

$$\int_{-\infty}^{+\infty} \exp\left(\frac{-x^2}{4Dt}\right) dx = 2\int_0^{\infty} \exp\left(\frac{-x^2}{4Dt}\right) dx$$

$$= 2\int_0^{\infty} \frac{1}{2\sqrt{u}} \exp(-au) du = \int_0^{\infty} \frac{1}{\sqrt{u}} \exp(-au) du$$

$$= \sqrt{\frac{\pi}{a}} = \sqrt{4\pi Dt}$$

Then

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{4\pi Dt}} \exp\left(\frac{-x^2}{4Dt}\right) dx = \frac{\sqrt{4\pi Dt}}{\sqrt{4\pi Dt}} = 1$$

6.29

Plot

6.30

(a) $E_F - E_{Fi} = kT \ln\left(\frac{n_o}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.383225 \text{ eV}$$

(b) $\delta n = \delta p = g'\tau_{p0} = (2\times10^{21})(5\times10^{-7})$

$$= 10^{15} \text{ cm}^{-3}$$

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{4\times10^{16} + 10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.383865 \text{ eV}$$

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$\cong (0.0259) \ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.28768 \text{ eV}$$

(c) $E_{Fn} - E_F = 0.383865 - 0.383225$

$$= 0.000640 \text{ eV}$$

or $= 0.640 \text{ meV}$

6.31

(a) p-type

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.3294 \text{ eV}$$

(b)

$$\delta n = \delta p = 5\times10^{14} \text{ cm}^{-3}$$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} = 4.5\times10^4 \text{ cm}^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_o + \delta n}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{4.5\times10^4 + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fn} - E_{Fi} = 0.2697 \text{ eV}$$

and

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_o + \delta p}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5\times10^{15} + 5\times10^{14}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_{Fp} = 0.3318 \text{ eV}$$

6.32

(a) For n-type,

$$E_{Fn} - E_F = (E_{Fn} - E_{Fi}) - (E_F - E_{Fi})$$

$$= kT \ln\left(\frac{n_o + \delta n}{n_i}\right) - kT \ln\left(\frac{n_o}{n_i}\right)$$

$$= kT \ln\left(\frac{n_o + \delta n}{n_o}\right)$$

So $0.00102 = (0.0259) \ln\left(\frac{5\times10^{15} + \delta n}{5\times10^{15}}\right)$

$$5\times10^{15} + \delta n = 5\times10^{15} \exp\left(\frac{0.00102}{0.0259}\right)$$

Which yields $\delta n \cong 2\times10^{14} \text{ cm}^{-3}$

(b) $E_{Fn}-E_{Fi}=kT\ln\left(\frac{n_o+\delta n}{n_i}\right)$

$=(0.0259)\ln\left(\frac{5\times10^{15}+2\times10^{14}}{1.5\times10^{10}}\right)$

$=0.33038\text{ eV}$

(c) $E_{Fi}-E_{Fp}\cong kT\ln\left(\frac{\delta p}{n_i}\right)$

$=(0.0259)\ln\left(\frac{2\times10^{14}}{1.5\times10^{10}}\right)$

$=0.2460\text{ eV}$

6.33

(a) $E_{Fn}-E_{Fi}\cong kT\ln\left(\frac{\delta n}{n_i}\right)$

or $\delta n=n_i\exp\left[\frac{E_{Fn}-E_{Fi}}{kT}\right]$

$=(1.5\times10^{10})\exp\left[\frac{0.270}{0.0259}\right]$

$=5.05\times10^{14}\text{ cm}^{-3}$

(b) $E_{Fi}-E_{Fp}=kT\ln\left(\frac{p_o+\delta p}{n_i}\right)$

$=(0.0259)\ln\left(\frac{6\times10^{15}+5.05\times10^{14}}{1.5\times10^{10}}\right)$

$=0.33618\text{ eV}$

(c) (i) $E_F-E_{Fp}=(E_{Fi}-E_{Fp})-(E_{Fi}-E_F)$

$=kT\ln\left(\frac{p_o+\delta p}{n_i}\right)-kT\ln\left(\frac{p_o}{n_i}\right)$

$=kT\ln\left(\frac{p_o+\delta p}{p_o}\right)$

(ii) E_F-E_{Fp}

$=(0.0259)\ln\left(\frac{6\times10^{15}+5.05\times10^{14}}{6\times10^{15}}\right)$

$=2.093\times10^{-3}\text{ eV}$

or $=2.093\text{ meV}$

6.34

(a) (i) $E_{Fn}-E_{Fi}=kT\ln\left(\frac{n_o+n}{n_i}\right)$

$=(0.0259)\ln\left(\frac{(1.02)(10^{16})}{1.8\times10^{6}}\right)$

$=0.58166\text{ eV}$

(ii) $E_{Fi}-E_{Fp}\cong kT\ln\left(\frac{\delta p}{n_i}\right)$

$=(0.0259)\ln\left(\frac{0.02\times10^{16}}{1.8\times10^{6}}\right)$

$=0.47982\text{ eV}$

(b) (i) $E_{Fn}-E_{Fi}=(0.0259)\ln\left(\frac{1.1\times10^{16}}{1.8\times10^{6}}\right)$

$=0.58361\text{ eV}$

(ii) $E_{Fi}-E_{Fp}=(0.0259)\ln\left(\frac{0.1\times10^{16}}{1.8\times10^{6}}\right)$

$=0.52151\text{ eV}$

6.35

Quasi-Fermi level for minority carrier electrons:

$E_{Fn}-E_{Fi}=kT\ln\left(\frac{n_o+\delta n}{n_i}\right)$

$n_o=\frac{n_i^2}{p_o}=\frac{(1.8\times10^{6})^2}{10^{16}}=3.24\times10^{-4}\text{ cm}^{-3}$

We have

$\delta n=(10^{14})\left(\frac{x}{50}\right)$

Then

$E_{Fn}-E_{Fi}=kT\ln\left[\frac{3.24\times10^{-4}+(10^{14}x/50)}{1.8\times10^{6}}\right]$

We find

x (μ m)	($E_{Fn}-E_{Fi}$) (eV)
0	-0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

Quasi-Fermi level for holes: we have

$$E_{Fi}-E_{Fp}=kT\ln\left(\frac{p_o+\delta p}{n_i}\right)$$

We have $p_o=10^{16}\text{ cm}^{-3}$ and $\delta n=\delta p$.
We find

x (μ m)	($E_{Fi}-E_{Fp}$) (eV)
0	+0.58115
50	+0.58140

6.36

(a) We can write

$$E_{Fi}-E_F=kT\ln\left(\frac{p_o}{n_i}\right)$$

and

$$E_{Fi}-E_{Fp}=kT\ln\left(\frac{p_o+\delta p}{n_i}\right)$$

so that

$$\left(E_{Fi}-E_{Fp}\right)-\left(E_{Fi}-E_F\right)=E_F-E_{Fp}$$
$$=kT\ln\left(\frac{p_o+\delta p}{n_i}\right)-kT\ln\left(\frac{p_o}{n_i}\right)$$

or

$$E_F-E_{Fp}=kT\ln\left(\frac{p_o+\delta p}{p_o}\right)=(0.01)kT$$

Then

$$\frac{p_o+\delta p}{p_o}=\exp(0.01)=1.010$$

or

$$\frac{\delta p}{p_o}=0.010\Rightarrow \text{low injection, so that}$$

$$\delta p=5\times10^{12}\text{ cm}^{-3}$$

(b)

$$E_{Fn}-E_{Fi}\cong kT\ln\left(\frac{\delta p}{n_i}\right)$$
$$=(0.0259)\ln\left(\frac{5\times10^{12}}{1.5\times10^{10}}\right)$$

or

$$E_{Fn}-E_{Fi}=0.1505\text{ eV}$$

6.37

Plot

6.38

(a) $E_{Fi}-E_{Fp}\cong kT\ln\left(\dfrac{\delta p}{n_i}\right)$

$$=(0.0259)\ln\left(\frac{\delta p}{1.5\times10^{10}}\right)$$

δp	$E_{Fi}-E_{Fp}$
$\delta p=10^{11}\text{ cm}^{-3}$,	$E_{Fi}-E_{Fp}=0.04914$ eV
10^{12}	0.10877
10^{13}	0.16841
10^{14}	0.22805
10^{15}	0.28768

(b) $E_{Fn}-E_{Fi}=kT\ln\left(\dfrac{n_o+\delta n}{n_i}\right)$

$$=(0.0259)\ln\left(\frac{2\times10^{16}+\delta n}{1.5\times10^{10}}\right)$$

δn	$E_{Fn}-E_{Fi}$
$\delta n=10^{11}\text{ cm}^{-3}$,	$E_{Fn}-E_{Fi}=0.365273$ eV
10^{12}	0.365274
10^{13}	0.365286
10^{14}	0.365402
10^{15}	0.366536

6.39

(a)

$$R=\frac{C_nC_pN_t\left(np-n_i^2\right)}{C_n\left(n+n'\right)+C_p\left(p+p'\right)}$$
$$=\frac{\left(np-n_i^2\right)}{\tau_{pO}\left(n+n'\right)+\tau_{nO}\left(p+p'\right)}$$

Let $n'=p'=n_i$. For $n=p=0$

$$R=\frac{-n_i^2}{\tau_{pO}n_i+\tau_{nO}n_i}=\frac{-n_i}{\tau_{pO}+\tau_{nO}}$$

(b) We had defined the net generation rate as

$$g-R=g_o+g'-\left(R_o+R'\right)$$

where $g_o=R_o$ since these are the thermal equilibrium generation and recombination rates.

If $g'=0$, then $g-R=-R'$ and

$$R'=\frac{-n_i}{\tau_{pO}+\tau_{nO}}$$

so that $g-R=+\dfrac{n_i}{\tau_{pO}+\tau_{nO}}$

Thus a negative recombination rate implies a net positive generation rate.

6.40

We have that

$$R = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n\left(n + n'\right) + C_p\left(p + p'\right)}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}\left(n + n_i\right) + \tau_{nO}\left(p + n_i\right)}$$

If $n = n_o + \delta n$ and $p = p_o + \delta n$, then

$$R = \frac{\left(n_o + \delta n\right)\left(p_o + \delta n\right) - n_i^2}{\tau_{pO}\left(n_o + \delta n + n_i\right) + \tau_{nO}\left(p_o + \delta n + n_i\right)}$$

$$= \frac{n_o p_o + \delta n\left(n_o + p_o\right) + \left(\delta n\right)^2 - n_i^2}{\tau_{pO}\left(n_o + \delta n + n_i\right) + \tau_{nO}\left(p_o + \delta n + n_i\right)}$$

If $\delta n << n_i$, we can neglect $(\delta n)^2$: also

$n_o p_o = n_i^2$

Then

$$R = \frac{\delta n\left(n_o + p_o\right)}{\tau_{pO}\left(n_o + n_i\right) + \tau_{nO}\left(p_o + n_i\right)}$$

(a) For n-type; $n_o >> p_O$, $n_o >> n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7}\ \text{s}^{-1}$$

(b) For intrinsic, $n_o = p_o = n_i$

Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO}\left(2n_i\right) + \tau_{nO}\left(2n_i\right)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO} + \tau_{nO}} = \frac{1}{10^{-7} + 5\times 10^{-7}} \Rightarrow$$

$$\frac{R}{\delta n} = 1.67\times 10^{+6}\ \text{s}^{-1}$$

(c) For p-type; $p_o >> n_o$, $p_o >> n_i$

Then

$$\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5\times 10^{-7}} = 2\times 10^{+6}\ \text{s}^{-1}$$

6.41

(a) From Equation (6.56)

$$D_p \frac{d^2\left(\delta p\right)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g'\tau_{pO} + A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right)$$

At $x = +\infty$, $\delta p = g'\tau_{pO}$ so that $B = 0$,

Then

$$\delta p = g'\tau_{pO} + A\exp\left(\frac{-x}{L_p}\right)$$

We have

$$D_p \left.\frac{d\left(\delta p\right)}{dx}\right|_{x=0} = s\left(\delta p\right)\Big|_{x=0}$$

We can write

$$\left.\frac{d\left(\delta p\right)}{dx}\right|_{x=0} = \frac{-A}{L_p} \quad \text{and} \quad \left(\delta p\right)\Big|_{x=0} = g'\tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s\left(g'\tau_{pO} + A\right)$$

Solving for A, we find

$$A = \frac{-sg'\tau_{pO}}{\dfrac{D_p}{L_p} + s}$$

The excess concentration is then

$$\delta p = g'\tau_{pO}\left[1 - \frac{s}{\left(D_p/L_p\right) + s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

where

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(10)\left(10^{-7}\right)} = 10^{-3}\ \text{cm}$$

Now

$$\delta p = \left(10^{21}\right)\left(10^{-7}\right) \times\left[1 - \frac{s}{\left(10/10^{-3}\right) + s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

or

$$\delta p = 10^{14}\left[1 - \frac{s}{10^4 + s}\cdot\exp\left(\frac{-x}{L_p}\right)\right]$$

(i) For $s = 0$,

$$\delta p = 10^{14}\ \text{cm}^{-3}$$

(ii) For $s = 2000$ cm/s,

$$\delta p = 10^{14}\left[1 - 0.167\exp\left(\frac{-x}{L_p}\right)\right]$$

(iii) For $s = \infty$,

$$\delta p = 10^{14}\left[1 - \exp\left(\frac{-x}{L_p}\right)\right]$$

(b)
(i) For $s=0$,
$\delta p(0)=10^{14}\text{ cm}^{-3}$
(ii) For $s=2000$ cm/s,
$\delta p(0)=0.833\times10^{14}\text{ cm}^{-3}$
(iii) For $s=\infty$,
$\delta p(0)=0$

6.42

$$L_n=\sqrt{D_n\tau_{nO}}=\sqrt{(25)(5\times10^{-7})}$$
$$=35.4\times10^{-4}\text{ cm}$$

(a) At $x=0$,
$$g'\tau_{nO}=(2\times10^{21})(5\times10^{-7})=10^{15}\text{ cm}^{-3}$$
or
$$\delta n(0)=g'\tau_{nO}=10^{15}\text{ cm}^{-3}$$
For $x>0$
$$D_n\frac{d^2(\delta n)}{dx^2}-\frac{\delta n}{\tau_{nO}}=0\Rightarrow\frac{d^2(\delta n)}{dx^2}-\frac{\delta n}{L_n^2}=0$$
The solution is of the form
$$\delta n=A\exp\left(\frac{-x}{L_n}\right)+B\exp\left(\frac{+x}{L_n}\right)$$
At $x=0$,
$$\delta n=\delta n(0)=A+B$$
At $x=W$,
$$\delta n=0=A\exp\left(\frac{-W}{L_n}\right)+B\exp\left(\frac{+W}{L_n}\right)$$
Solving these two equations, we find
$$A=\frac{-\delta n(0)\exp(+2W/L_n)}{1-\exp(+2W/L_n)}$$
and
$$B=\frac{\delta n(0)}{1-\exp(+2W/L_n)}$$
Substituting into the general solution, we find
$$\delta n=\frac{\delta n(0)}{\left[\exp\left(\frac{+W}{L_n}\right)-\exp\left(\frac{-W}{L_n}\right)\right]}$$
$$\times\left\{\exp\left[\frac{+(W-x)}{L_n}\right]-\exp\left[\frac{-(W-x)}{L_n}\right]\right\}$$
which can be written as
$$\delta n=\frac{\delta n(0)\sinh\left[\frac{W-x}{L_n}\right]}{\sinh\left[\frac{W}{L_n}\right]}$$
where
$\delta n(0)=10^{15}\text{ cm}^{-3}$ and $L_n=35.4\ \mu\text{m}$

(b) If $\tau_{nO}=\infty$, we have
$$\frac{d^2(\delta n)}{dx^2}=0$$
so the solution is of the form
$$\delta n=Cx+D$$
Applying the boundary conditions, we find
$$\delta n=\delta n(0)\left[1-\frac{x}{W}\right]$$

6.43

For $\tau_{pO}=\infty$, we have
$$\frac{d^2(\delta p)}{dx^2}=0$$
So the solution is of the form
$$\delta p=Ax+B$$
At $x=W$
$$-D_p\left.\frac{d(\delta p)}{dx}\right|_{x=W}=s(\delta p)\Big|_{x=W}$$
or
$$-D_pA=s(AW+B)$$
which yields
$$B=\frac{-A}{s}(D_p+sW)$$
At $x=0$, the flux of excess holes is
$$10^{19}=-D_p\left.\frac{d(\delta p)}{dx}\right|_{x=0}=-D_pA$$
so that
$$A=\frac{-10^{19}}{10}=-10^{18}\text{ cm}^{-4}$$
and
$$B=\frac{10^{18}}{s}(10+sW)=10^{18}\left(\frac{10}{s}+W\right)$$
The solution is now
$$\delta p=10^{18}\left(W-x+\frac{10}{s}\right)$$

(a) For $s = \infty$,

$\delta p = 10^{18}\left(20\times10^{-4} - x\right)$ cm^{-3}

Then

$$J_p = -eD_p \frac{d(\delta p)}{dx}$$

$$= -\left(1.6\times10^{-19}\right)(10)\left(-10^{18}\right)$$

or

$$J_p = 1.6 \text{ A/cm}^2$$

(b) For $s = 2\times10^3$ cm/s,

$\delta p = 10^{18}\left(70\times10^{-4} - x\right)$ cm^{-3}

Also

$$J_p = 1.6 \text{ A/cm}^2$$

6.44

For $-W < x < 0$

$$D_n \frac{d^2(\delta n)}{dx^2} + G_o' = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G_o'}{D_n}x + C_1$$

and

$$\delta n = -\frac{G_o'}{2D_n}x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so that

$$\delta n = C_3 x + C_4$$

The boundary conditions are

(1) $s = 0$ at $x = -W$ so that

$$\left.\frac{d(\delta n)}{dx}\right|_{x=-W} = 0$$

(2) $s = \infty$ at $x = +W$ so that

$$\delta n(W) = 0$$

(3) δn continuous at $x = 0$

(4) $\frac{d(\delta n)}{dx}$ continuous at $x = 0$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G_o'W}{D_n} \text{ and } C_2 = C_4 = +\frac{G_o'W^2}{D_n}$$

Then for $-W < x < 0$

$$\delta n = \frac{G_o'}{2D_n}\left(-x^2 - 2Wx + 2W^2\right)$$

and for $0 < x < +W$

$$\delta n = \frac{G_o'W}{D_n}(W - x)$$

6.45

Plot

6.48

(a) GaAs:

$$R = \frac{V}{I} = \frac{2}{2\times10^{-6}} = 10^6\ \Omega$$

$$R = \frac{L}{(\Delta\sigma)A} \text{ and } \Delta\sigma = e\left(\mu_n + \mu_p\right)\delta p$$

$$\delta p = g'\tau_{p0} = \left(10^{21}\right)\left(5\times10^{-8}\right) = 5\times10^{13} \text{ cm}^{-3}$$

For $N_d = 10^{16}$ cm^{-3}, from Figure 5.3,

$\mu_n \cong 7000$ cm^2/V-s, $\mu_p \cong 310$ cm^2/V-s

$$\Delta\sigma = \left(1.6\times10^{-19}\right)(7000 + 310)\left(5\times10^{13}\right)$$

$$= 0.05848\ (\Omega\text{-cm})^{-1}$$

Let $W = 20\ \mu$m

Then $A = Wd = \left(20\times10^{-4}\right)\left(4\times10^{-4}\right)$

$$= 80\times10^{-8} \text{ cm}^2$$

So $R = 10^6 = \dfrac{L}{(0.05848)\left(80\times10^{-8}\right)}$

Which yields $L = 4.68\times10^{-2}$ cm

(b) Silicon:

$R = 10^6\ \Omega$, $\delta p = 5\times10^{13}$ cm^{-3}

For $N_d = 10^{16}$ cm^{-3}, from Figure 5.3,

$\mu_n \cong 1300$ cm^2/V-s, $\mu_p \cong 410$ cm^2/V-s

$$\Delta\sigma = \left(1.6\times10^{-19}\right)(1300 + 410)\left(5\times10^{13}\right)$$

$$= 0.01368\ (\Omega\text{-cm})^{-1}$$

Let $W = 20\ \mu$m

Then $A = Wd = \left(20\times10^{-4}\right)\left(4\times10^{-4}\right)$

$$= 80\times10^{-8} \text{ cm}^2$$

So $R = 10^6 = \dfrac{L}{(0.01368)\left(80\times10^{-8}\right)}$

Which yields $L = 1.09\times10^{-2}$ cm

Chapter 7

7.1

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

(a)

$$\text{(i)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.611\text{ V}$$

$$\text{(ii)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.671\text{ V}$$

$$\text{(iii)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(2\times10^{17})}{(1.5\times10^{10})^2}\right]$$

$$= 0.731\text{ V}$$

(b)

$$\text{(i)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.731\text{ V}$$

$$\text{(ii)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.790\text{ V}$$

$$\text{(iii)}\ V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{17})}{(1.5\times10^{10})^2}\right]$$

$$= 0.850\text{ V}$$

7.2

Si: $n_i = 1.5\times10^{10}\text{ cm}^{-3}$

Ge: $n_i = 2.4\times10^{13}\text{ cm}^{-3}$

GaAs: $n_i = 1.8\times10^{6}\text{ cm}^{-3}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \text{ and } V_t = 0.0259\text{ V}$$

(a) $N_d = 10^{14}\text{ cm}^{-3}$, $N_a = 10^{17}\text{ cm}^{-3}$,

Then Si: $V_{bi} = 0.635\text{ V}$

Ge: $V_{bi} = 0.253\text{ V}$

GaAs: $V_{bi} = 1.10\text{ V}$

(b) $N_d = 5\times10^{16}\text{ cm}^{-3}$, $N_a = 5\times10^{16}\text{ cm}^{-3}$

Si: $V_{bi} = 0.778\text{ V}$

Ge: $V_{bi} = 0.396\text{ V}$

GaAs: $V_{bi} = 1.25\text{ V}$

(c) $N_d = 10^{17}\text{ cm}^{-3}$, $N_a = 10^{17}\text{ cm}^{-3}$

Si: $V_{bi} = 0.814\text{ V}$

Ge: $V_{bi} = 0.432\text{ V}$

GaAs: $V_{bi} = 1.28\text{ V}$

7.3

(a) Silicon ($T = 300$ K)

$$V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$$

For $N_a = N_d = 10^{14}\text{ cm}^{-3}$; $V_{bi} = 0.4561\text{ V}$

$= 10^{15}$; $= 0.5754\text{ V}$

$= 10^{16}$; $= 0.6946\text{ V}$

$= 10^{17}$; $= 0.8139\text{ V}$

(b) GaAs ($T = 300$ K)

$$V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.8\times10^{6})^2}\right]$$

For $N_a = N_d = 10^{14}\text{ cm}^{-3}$; $V_{bi} = 0.9237\text{ V}$

$= 10^{15}$; $= 1.043\text{ V}$

$= 10^{16}$; $= 1.162\text{ V}$

$= 10^{17}$; $= 1.282\text{ V}$

(c) Silicon (400 K), $kT = 0.034533$

$n_i = 2.38\times10^{12}\text{ cm}^{-3}$

For $N_a = N_d = 10^{14}\text{ cm}^{-3}$; $V_{bi} = 0.2582\text{ V}$

$= 10^{15}$; $= 0.4172\text{ V}$

$= 10^{16}$; $= 0.5762\text{ V}$

$= 10^{17}$; $= 0.7353\text{ V}$

GaAs(400 K), $n_i = 3.29\times10^{9}\text{ cm}^{-3}$

For $N_a = N_d = 10^{14}\text{ cm}^{-3}$; $V_{bi} = 0.7129\text{ V}$

$= 10^{15}$; $= 0.8719\text{ V}$

$= 10^{16}$; $= 1.031\text{ V}$

$= 10^{17}$; $= 1.190\text{ V}$

7.4

(a) n-side

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3294\ \text{eV}$$

p-side

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.4070\ \text{eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$V_{bi} = 0.7364\ \text{V}$$

(c)

$$V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{17})(5\times10^{15})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.7363\ \text{V}$$

(d)

$$x_n = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.736)}{1.6\times10^{-19}} \times\left(\frac{10^{17}}{5\times10^{15}}\right)\left(\frac{1}{10^{17}+5\times10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 0.426\times10^{-4}\ \text{cm} = 0.426\ \mu\text{m}$$

Now

$$x_p = \left[\frac{2(11.7)(8.85\times10^{-14})(0.736)}{1.6\times10^{-19}} \times\left(\frac{5\times10^{15}}{10^{17}}\right)\left(\frac{1}{10^{17}+5\times10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 0.0213\times10^{-4}\ \text{cm} = 0.0213\ \mu\text{m}$$

We have

$$\left|\mathrm{E}_{\max}\right| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(5\times10^{15})(0.426\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$$\left|\mathrm{E}_{\max}\right| = 3.29\times10^4\ \text{V/cm}$$

7.5

(a) n-side

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3653\ \text{eV}$$

p-side

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.3653\ \text{eV}$$

(b)

$$V_{bi} = 0.3653 + 0.3653$$

or

$$V_{bi} = 0.7306\ \text{V}$$

(c)

$$V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(2\times10^{16})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.7305\ \text{V}$$

(d)

$$x_n = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$=\left[\frac{2(11.7)(8.85\times10^{-14})(0.7305)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{2\times10^{16}}{2\times10^{16}}\right)\left(\frac{1}{2\times10^{16}+2\times10^{16}}\right)\right]^{1/2}$$

or

$$x_n = 0.154\times10^{-4}\text{ cm} = 0.154\ \mu\text{m}$$

By symmetry

$$x_p = 0.154\times10^{-4}\text{ cm} = 0.154\ \mu\text{m}$$

Now

$$|\mathrm{E}_{max}| = \frac{eN_d x_n}{\in_s}$$
$$= \frac{(1.6\times10^{-19})(2\times10^{16})(0.1537\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$$|\mathrm{E}_{max}| = 4.75\times10^4\text{ V/cm}$$

7.6

(b) $N_d = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$

$$= (1.5\times10^{10})\exp\left(\frac{0.365}{0.0259}\right)$$

or $N_d = 1.98\times10^{16}\text{ cm}^{-3}$

$$N_a = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$
$$= (1.5\times10^{10})\exp\left(\frac{0.330}{0.0259}\right)$$

or $N_a = 5.12\times10^{15}\text{ cm}^{-3}$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{(5.12\times10^{15})(1.98\times10^{16})}{(1.5\times10^{10})^2}\right]$$
$$= 0.695\text{ V}$$

7.7

200 K; $kT = 0.017267$; $n_i = 1.38\text{ cm}^{-3}$

300 K; $kT = 0.0259$; $n_i = 1.8\times10^6\text{ cm}^{-3}$

400 K; $kT = 0.034533$; $n_i = 3.28\times10^9\text{ cm}^{-3}$

For 200 K;

$$V_{bi} = (0.017267)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.38)^2}\right]$$
$$= 1.257\text{ V}$$

For 300 K;

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.8\times10^6)^2}\right]$$
$$= 1.157\text{ V}$$

For 400 K;

$$V_{bi} = (0.034533)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(3.28\times10^9)^2}\right]$$
$$= 1.023\text{ V}$$

7.8

$$x_n = 0.25W = 0.25(x_n + x_p)$$
$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$
$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $N_d = 3N_a$

(a) $V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{(1.5\times10^{10})^2}\right]$$

or $3N_a^2 = (1.5\times10^{10})^2 \exp\left(\frac{0.710}{0.0259}\right)$

which yields $N_a = 7.766\times10^{15}\text{ cm}^{-3}$

$N_d = 2.33\times10^{16}\text{ cm}^{-3}$

$$x_n = \left\{\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$
$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{1}{3}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2}$$

$\Rightarrow x_n = 9.93\times10^{-6}\text{ cm}$

or $x_n = 0.0993\ \mu\text{m}$

$$x_p = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{3}{1}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2}$$
$$= 2.979\times10^{-5}\text{ cm}$$

or $x_p = 0.2979\ \mu\text{m}$

Now

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6\times10^{-19})(2.33\times10^{16})(0.0993\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

$$= 3.58\times10^4 \text{ V/cm}$$

(b) From part (a), we can write

$$3N_a^2 = (1.8\times10^6)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127\times10^{15}$ cm^{-3}

$N_d = 2.438\times10^{16}$ cm^{-3}

$$x_n = \left\{\frac{2(13.1)(8.85\times10^{-14})(1.180)}{1.6\times10^{-19}} \times\left(\frac{1}{3}\right)\left[\frac{1}{4(8.127\times10^{15})}\right]\right\}^{1/2}$$

$$= 1.324\times10^{-5} \text{ cm}$$

or $x_n = 0.1324\ \mu$m

$$x_p = \left\{\frac{2(13.1)(8.85\times10^{-14})(1.180)}{1.6\times10^{-19}} \times\left(\frac{3}{1}\right)\left[\frac{1}{4(8.127\times10^{15})}\right]\right\}^{1/2}$$

$$= 3.973\times10^{-5} \text{ cm}$$

or $x_p = 0.3973\ \mu$m

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6\times10^{-19})(2.438\times10^{16})(0.1324\times10^{-4})}{(13.1)(8.85\times10^{-14})}$$

$$= 4.45\times10^4 \text{ V/cm}$$

7.9

(a) $V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5\times10^{10})^2}\right]$

or

$V_{bi} = 0.635$ V

(b)

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.6350)}{1.6\times10^{-19}} \times\left(\frac{10^{16}}{10^{15}}\right)\left(\frac{1}{10^{16}+10^{15}}\right)\right]^{1/2}$$

or

$x_n = 0.8644\times10^{-4}$ cm $= 0.8644\ \mu$m

Now

$$x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.6350)}{1.6\times10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{16}+10^{15}}\right)\right]^{1/2}$$

or

$x_p = 0.08644\times10^{-4}$ cm $= 0.08644\ \mu$m

(c)

$$|E_{max}| = \frac{eN_d x_n}{\epsilon_s}$$

$$= \frac{(1.6\times10^{-19})(10^{15})(0.8644\times10^{-4})}{(11.7)(8.85\times10^{-14})}$$

or

$|E_{max}| = 1.34\times10^4$ V/cm

7.10

(a) $V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$

$= 0.80813$ V

(b) V_{bi} increases as temperature decreases

At $T = 300$ K, we can write

$$n_i^2 = (1.5\times10^{10})^2$$

$$= K(2.8\times10^{19})(1.04\times10^{19})\exp\left(\frac{-1.12}{0.0259}\right)$$

$$\Rightarrow K = 4.659$$

At $T = 287$ K, $kT = 0.024778$ eV

$$n_i^2 = K(2.8\times10^{19})(1.04\times10^{19})\left(\frac{287}{300}\right)^3 \times\exp\left(\frac{-1.12}{0.024778}\right)$$

$$= (4.659)(2.5496\times10^{38})(2.3404\times10^{-20})$$

So $n_i^2 = 2.780\times10^{19}$

Then

$$V_{bi} = (0.024778)\ln\left[\frac{(2\times10^{17})(4\times10^{16})}{2.780\times10^{19}}\right]$$

$$= 0.82494 \text{ V}$$

We find

$$\frac{V_{bi}(287)-V_{bi}(300)}{V_{bi}(300)}\times 100\%$$

$$= \frac{0.82494-0.80813}{0.80813}\times 100\% = 2.08\%$$

$$\cong 2\%$$

7.11

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$0.550 = (0.0259)\left(\frac{T}{300}\right)\ln\left[\frac{(4\times10^{16})(2\times10^{15})}{n_i^2}\right]$$

Using the procedure from Problem 7.10, we can write, for $T = 300$ K,

$$n_i^2 = (1.5\times10^{10})^2$$

$$= K(2.8\times10^{19})(1.04\times10^{19})\exp\left(\frac{-1.12}{0.0259}\right)$$

$\Rightarrow K = 4.659$

At $T = 300$ K,

$$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.68886 \text{ V}$$

For $V_{bi} = 0.550$ V, $\Rightarrow T > 300$ K

At $T = 380$ K, $kT = 0.032807$ eV

Also

$$n_i^2 = (4.659)(2.8\times10^{19})(1.04\times10^{19})\left(\frac{380}{300}\right)^3 \times\exp\left(\frac{-1.12}{0.032807}\right)$$

$$= 4.112\times10^{24}$$

Then

$$V_{bi} = (0.032807)\ln\left[\frac{(4\times10^{16})(2\times10^{15})}{4.112\times10^{24}}\right]$$

$$= 0.5506 \text{ V} \quad \cong 0.550 \text{ V}$$

7.12

(b) For $N_d = 10^{16}$ cm^{-3},

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3473 \text{ eV}$$

For $N_d = 10^{15}$ cm^{-3}

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.2877 \text{ eV}$$

Then

$$V_{bi} = 0.34732 - 0.28768$$

or

$$V_{bi} = 0.0596 \text{ V}$$

7.13

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(10^{12})(10^{16})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.456 \text{ V}$$

(b)

$$x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(0.456)}{1.6\times10^{-19}} \times\left(\frac{10^{12}}{10^{16}}\right)\left(\frac{1}{10^{12}+10^{16}}\right)\right]^{1/2}$$

or

$$x_n = 2.43\times10^{-7} \text{ cm}$$

(c)

$$x_p = \left[\frac{2(11.7)(8.85\times10^{-14})(0.456)}{1.6\times10^{-19}} \times\left(\frac{10^{16}}{10^{12}}\right)\left(\frac{1}{10^{12}+10^{16}}\right)\right]^{1/2}$$

or

$$x_p = 2.43\times10^{-3} \text{ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_n}{\in_s}$$
$$= \frac{(1.6\times10^{-19})(10^{16})(2.43\times10^{-7})}{(11.7)(8.85\times10^{-14})}$$

or

$$|E_{max}| = 3.75\times10^2 \text{ V/cm}$$

7.14

Assume silicon, so

$$L_D = \left(\frac{\in_s kT}{e^2 N_d}\right)^{1/2}$$
$$= \left[\frac{(11.7)(8.85\times10^{-14})(0.0259)(1.6\times10^{-19})}{(1.6\times10^{-19})^2 N_d}\right]^{1/2}$$

or

$$L_D = \left(\frac{1.676\times10^5}{N_d}\right)^{1/2}$$

(a) $N_d = 8\times10^{14}$ cm^{-3}, $L_D = 0.1447\ \mu$m

(b) $N_d = 2.2\times10^{16}$ cm^{-3}, $L_D = 0.02760\ \mu$m

(c) $N_d = 8\times10^{17}$ cm^{-3}, $L_D = 0.004577\ \mu$m

Now

(a) $V_{bi} = 0.7427$ V

(b) $V_{bi} = 0.8286$ V

(c) $V_{bi} = 0.9216$ V

Also

$$x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(V_{bi})}{1.6\times10^{-19}} \times\left(\frac{8\times10^{17}}{N_d}\right)\left(\frac{1}{8\times10^{17}+N_d}\right)\right]^{1/2}$$

Then

(a) $x_n = 1.096\ \mu$m

(b) $x_n = 0.2178\ \mu$m

(c) $x_n = 0.02730\ \mu$m

Now

(a) $\dfrac{L_D}{x_n} = 0.1320$

(b) $\dfrac{L_D}{x_n} = 0.1267$

(c) $\dfrac{L_D}{x_n} = 0.1677$

7.15

$$|E_{max}| = \left\{\frac{2eV_{bi}}{\in_s}\left(\frac{N_a N_d}{N_a + N_d}\right)\right\}^{1/2}$$

We find

$$\frac{2e}{\in_s} = \frac{2(1.6\times10^{-19})}{(11.7)(8.85\times10^{-14})} = 3.0904\times10^{-7}$$

(a)

(i) For $N_a = 10^{17}$, $N_d = 10^{14}$; $V_{bi} = 0.6350$ V

(ii) $= 10^{15}$; $= 0.6946$ V

(iii) $= 10^{16}$; $= 0.7543$ V

(iv) $= 10^{17}$; $= 0.8139$ V

(i) For $N_a = 10^{17}$,
$N_d = 10^{14}$; $|E_{max}| = 0.443\times10^4$ V/cm

(ii) $= 10^{15}$; $= 1.46\times10^4$ V/cm

(iii) $= 10^{16}$; $= 4.60\times10^4$ V/cm

(iv) $= 10^{17}$; $= 11.2\times10^4$ V/cm

(b)

(i) For $N_a = 10^{14}$, $N_d = 10^{14}$; $V_{bi} = 0.4561$ V

(ii) $= 10^{15}$; $= 0.5157$ V

(iii) $= 10^{16}$; $= 0.5754$ V

(iv) $= 10^{17}$; $= 0.6350$ V

(i) For $N_a = 10^{14}$,
$N_d = 10^{14}$; $|E_{max}| = 0.265\times10^4$ V/cm

(ii) $= 10^{15}$; $= 0.381\times10^4$ V/cm

(iii) $= 10^{16}$; $= 0.420\times10^4$ V/cm

(iv) $= 10^{17}$; $= 0.443\times10^4$ V/cm

(c) $|E_{max}|$ increases as the doping increases, and the electric field extends further into the low-doped side of the pn junction.

7.16

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{16})(10^{15})}{(1.5\times10^{10})^2}\right]$$
$$= 0.6767 \text{ V}$$

(b) $$W = \left\{\frac{2\in_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right\}^{1/2}$$

(i) For $V_R = 0$,

$$W = \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.6767)}{1.6\times10^{-19}} \times \left[\frac{5\times10^{16} + 10^{15}}{(5\times10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 9.452\times10^{-5}\ \text{cm}$$

or $W = 0.9452\ \mu\text{m}$

(ii) For $V_R = 5$ V,

$$W = \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.6767+5)}{1.6\times10^{-19}} \times \left[\frac{5\times10^{16} + 10^{15}}{(5\times10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$= 2.738\times10^{-4}\ \text{cm}$$

or $W = 2.738\ \mu\text{m}$

(c) $$\left|\mathrm{E}_{\max}\right| = \frac{2(V_{bi}+V_R)}{W}$$

(i)For $V_R = 0$,

$$\left|\mathrm{E}_{\max}\right| = \frac{2(0.6767)}{0.9452\times10^{-4}} = 1.43\times10^4\ \text{V/cm}$$

(ii)For $V_R = 5$ V,

$$\left|\mathrm{E}_{\max}\right| = \frac{2(0.6767+5)}{2.738\times10^{-4}} = 4.15\times10^4\ \text{V/cm}$$

7.17

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.8081\ \text{V}$$

(b)

$$x_n = \left\{ \frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.8081+2.5)}{1.6\times10^{-19}} \times \left(\frac{2\times10^{17}}{4\times10^{16}}\right)\left(\frac{1}{2\times10^{17}+4\times10^{16}}\right) \right\}^{1/2}$$

$$= 0.2987\times10^{-4}\ \text{cm}$$

or $x_n = 0.2987\ \mu\text{m}$

$$x_p = \left\{ \frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.8081+2.5)}{1.6\times10^{-19}} \times \left(\frac{4\times10^{16}}{2\times10^{17}}\right)\left(\frac{1}{2\times10^{17}+4\times10^{16}}\right) \right\}^{1/2}$$

$$= 5.97\times10^{-6}\ \text{cm}$$

or $x_p = 0.0597\ \mu\text{m}$

$$W = \left\{ \frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.8081+2.5)}{1.6\times10^{-19}} \times \left[\frac{2\times10^{17}+4\times10^{16}}{(2\times10^{17})(4\times10^{16})}\right] \right\}^{1/2}$$

$$= 0.3584\times10^{-4}\ \text{cm}$$

or $W = 0.3584\ \mu\text{m}$

Also $W = x_n + x_p = 0.3584\ \mu\text{m}$

(c) $$\left|\mathrm{E}_{\max}\right| = \frac{2(V_{bi}+V_R)}{W} = \frac{2(0.8081+2.5)}{0.3584\times10^{-4}}$$

$$= 1.85\times10^5\ \text{V/cm}$$

(d) $$C = A\left\{ \frac{e\in_s N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)} \right\}^{1/2}$$

$$= (2\times10^{-4})\left\{ \frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})}{2(0.8081+2.5)} \times \left[\frac{(2\times10^{17})(4\times10^{16})}{2\times10^{17}+4\times10^{16}}\right] \right\}^{1/2}$$

$$= 5.78\times10^{-12}\ \text{F}$$

or $C = 5.78\ \text{pF}$

7.18

(a) $$V_{bi} = V_t \ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$= V_t \ln\left(\frac{80N_d^2}{n_i^2}\right)$$

We find

$$80N_d^2 = n_i^2 \exp\left(\frac{V_{bi}}{V_t}\right)$$

$$= (1.5\times10^{10})^2 \exp\left(\frac{0.740}{0.0259}\right)$$

$$= 5.762\times10^{32}$$

$$\Rightarrow N_d = 2.684\times10^{15}\ \text{cm}^{-3}$$
$$N_a = 2.147\times10^{17}\ \text{cm}^{-3}$$

(b)
$$x_n = \left\{\frac{2\in_s(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$
$$=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.740+10)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{80}{1}\right)\left(\frac{1}{2.147\times10^{17}+2.684\times10^{15}}\right)\right\}^{1/2}$$
$$=2.262\times10^{-4}\ \text{cm}$$
or $x_n = 2.262\ \mu\text{m}$
$$x_p = \left\{\frac{2\in_s(V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$
$$=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.740+10)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{1}{80}\right)\left(\frac{1}{2.147\times10^{17}+2.684\times10^{15}}\right)\right\}^{1/2}$$
$$=2.83\times10^{-6}\ \text{cm}$$
or $x_p = 0.0283\ \mu\text{m}$

(c) $$|\text{E}_{\max}| = \frac{2(V_{bi}+V_R)}{W}$$
$$=\frac{2(0.740+10)}{(2.262+0.0283)\times10^{-4}}$$
$$=9.38\times10^4\ \text{V/cm}$$

(d) $$C' = \left\{\frac{e\in_s N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$
$$=\left\{\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})}{2(0.740+10)}\right.$$
$$\left.\times\left[\frac{(2.147\times10^{17})(2.684\times10^{15})}{2.147\times10^{17}+2.684\times10^{15}}\right]\right\}^{1/2}$$
$$C' = 4.52\times10^{-9}\ \text{F/cm}^2$$

7.19

(a) $V_{bi}(3N_a)-V_{bi}(N_a)$
$$=V_t\ln\left[\frac{N_d(3N_a)}{n_i^2}\right]-V_t\ln\left[\frac{N_dN_a}{n_i^2}\right]$$
$$=V_t\left\{\ln(3)+\ln\left[\frac{N_dN_a}{n_i^2}\right]\right\}-V_t\ln\left[\frac{N_dN_a}{n_i^2}\right]$$
$$=V_t\ln(3)=(0.0259)\ln(3)$$
$$=0.02845\ \text{V}$$

(b) $$C' \cong \left\{\frac{e\in_s N_a}{2(V_{bi}+V_R)}\right\}^{1/2}$$

So $$\frac{C'(3N_a)}{C'(N_a)} = \left\{\frac{3N_a}{N_a}\right\}^{1/2} = \sqrt{3} = 1.732$$

(c) For a larger doping, the space charge width narrows which results in a larger capacitance.

7.20

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{15})(4\times10^{17})}{(1.5\times10^{10})^2}\right]$$
or
$$V_{bi} = 0.766\ \text{V}$$
Now
$$|\text{E}_{\max}| = \left[\frac{2e(V_{bi}+V_R)}{\in_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right]^{1/2}$$
or
$$(3\times10^5)^2 = \left[\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})}\times\frac{(4\times10^{15})(4\times10^{17})}{4\times10^{15}+4\times10^{17}}\right]$$
or
$$9\times10^{10} = 1.224\times10^9(V_{bi}+V_R)$$
so that
$$(V_{bi}+V_R) = 73.53\ \text{V}$$
which yields
$$V_R = 72.8\ \text{V}$$

(b) $$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(4\times10^{17})}{(1.5\times10^{10})^2}\right]$$
or
$$V_{bi} = 0.826\ \text{V}$$
We have
$$(3\times10^5)^2 = \left[\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})}\times\frac{(4\times10^{16})(4\times10^{17})}{4\times10^{16}+4\times10^{17}}\right]$$
so that
$$(V_{bi}+V_R) = 8.008\ \text{V}$$

which yields
$V_R = 7.18$ V

(c) $V_{bi} = (0.0259)\ln\left[\dfrac{(4\times10^{17})(4\times10^{17})}{(1.5\times10^{10})^2}\right]$

or
$V_{bi} = 0.886$ V

We have

$$(3\times10^5)^2 = \left[\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})} \times \frac{(4\times10^{17})(4\times10^{17})}{4\times10^{17}+4\times10^{17}}\right]$$

so that
$(V_{bi}+V_R) = 1.456$ V

which yields
$V_R = 0.570$ V

7.21

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in_s(V_{biA}+V_R)}{e}\left(\dfrac{N_a+N_{dA}}{N_aN_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in_s(V_{biB}+V_R)}{e}\left(\dfrac{N_a+N_{dB}}{N_aN_{dB}}\right)\right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}\cdot\frac{(N_a+N_{dA})}{(N_a+N_{dB})}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find

$$V_{biA} = (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.7543\text{ V}$$

$$V_{biB} = (0.0259)\ln\left[\frac{(10^{18})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.8139\text{ V}$$

We find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right) \times\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or

$$\frac{W(A)}{W(B)} = 3.13$$

(b)

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)} = \frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}} = \frac{W(B)}{W(A)}\cdot\frac{V_{biA}+V_R}{V_{biB}+V_R}$$

$$= \left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)} = 0.316$$

(c)

$$\frac{C'_j(A)}{C'_j(B)} = \frac{\left[\dfrac{\in_s N_aN_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in_s N_aN_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or

$$\frac{C'_j(A)}{C'_j(B)} = 0.319$$

7.22

(a) We have

$$\frac{C'_j(0)}{C'_j(10)} = \frac{\left[\dfrac{\in_s N_aN_d}{2(V_{bi})(N_a+N_d)}\right]^{1/2}}{\left[\dfrac{\in_s N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}}$$

or

$$\frac{C'_j(0)}{C'_j(10)} = 3.13 = \left(\frac{V_{bi}+V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10$ V, we find

$$(3.13)^2V_{bi} = V_{bi} + 10$$

or
$V_{bi} = 1.137$ V

(b)
$x_p = 0.2W = 0.2(x_p + x_n)$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

so

$$1.137 = (0.0259)\ln\left[\frac{0.25N_a^2}{(1.8\times10^6)^2}\right]$$

We can then write

$$N_a = \frac{1.8\times10^6}{\sqrt{0.25}}\exp\left[\frac{1.137}{2(0.0259)}\right]$$

which yields

$$N_a = 1.23\times10^{16} \text{ cm}^{-3}$$

and

$$N_d = 3.07\times10^{15} \text{ cm}^{-3}$$

7.23

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{16})(5\times10^{15})}{(1.8\times10^6)^2}\right]$$

$$= 1.162 \text{ V}$$

$$C' \propto \frac{1}{\sqrt{V_{bi}+V_R}}$$

So $$\frac{C'(V_{R1})}{C'(V_{R2})} = \sqrt{\frac{V_{bi}+V_{R2}}{V_{bi}+V_{R1}}}$$

$$1.50 = \sqrt{\frac{1.162+V_{R2}}{1.162+0.5}}$$

$$(1.50)^2 = \frac{1.162+V_{R2}}{1.662}$$

which yields $V_{R2} = 2.58$ V

7.24

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.6889 \text{ V}$$

$$C = AC' = A\left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= (5\times10^{-4})\left\{\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})}{2(0.6889+V_R)} \times \frac{(2\times10^{15})(4\times10^{16})}{(2\times10^{15}+4\times10^{16})}\right\}^{1/2}$$

$$C = \frac{6.2806\times10^{-12}}{\sqrt{0.6889+V_R}}$$

(i) For $V_R = 0$,

$C = 7.567$ pF

(ii) For $V_R = 5$ V,

$C = 2.633$ pF

(b) $$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(4\times10^{16})}{(1.8\times10^6)^2}\right]$$

$$= 1.157 \text{ V}$$

$$C = AC' = A\left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= (5\times10^{-4})\left\{\frac{(1.6\times10^{-19})(13.1)(8.85\times10^{-14})}{2(1.157+V_R)} \times \frac{(2\times10^{15})(4\times10^{16})}{(2\times10^{15}+4\times10^{16})}\right\}^{1/2}$$

$$C = \frac{6.6457\times10^{-12}}{\sqrt{1.157+V_R}}$$

(i) For $V_R = 0$,

$C = 6.178$ pF

(ii) For $V_R = 5$ V,

$C = 2.678$ pF

7.25

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(5\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.7543 \text{ V}$$

(a) $$C = AC' = A\left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= (8\times10^{-4})\left\{\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})}{2(0.7543+10)} \times \frac{(2\times10^{17})(5\times10^{15})}{(2\times10^{17}+5\times10^{15})}\right\}^{1/2}$$

$$C = 4.904\times10^{-12} \text{ F}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{C(2\pi f)^2}$$

$$L = \frac{1}{\left(4.904\times10^{-12}\right)\left[2\pi\left(1.25\times10^{6}\right)\right]^2}$$
$$= 3.306\times10^{-3}\ \text{H} = 3.306\ \text{mH}$$

(b)

(i) For $V_R = 1$ V, $C = 12.14$ pF

$$f = \frac{1}{2\pi\left[\left(3.306\times10^{-3}\right)\left(12.14\times10^{-12}\right)\right]^{1/2}}$$
$$= 7.94\times10^{5}\ \text{Hz} = 0.794\ \text{MHz}$$

(ii) For $V_R = 5$ V, $C = 6.704$ pF

$$f = \frac{1}{2\pi\left[\left(3.306\times10^{-3}\right)\left(6.704\times10^{-12}\right)\right]^{1/2}}$$
$$= 1.069\times10^{6}\ \text{Hz} = 1.069\ \text{MHz}$$

7.26

$$\left|\mathrm{E}_{\max}\right| \cong \left[\frac{2e\left(V_{bi}+V_R\right)N_d}{\epsilon_s}\right]^{1/2}$$

Let $V_{bi} \cong 0.75$ V

(a) $\left(2.5\times10^{5}\right)^2$

$$= \left[\frac{2\left(1.6\times10^{-19}\right)\left(0.75+10\right)N_d}{\left(11.7\right)\left(8.85\times10^{-14}\right)}\right]$$

$\Rightarrow N_d = 1.88\times10^{16}\ \text{cm}^{-3}$

(b) $\left(10^{5}\right)^2$

$$= \left[\frac{2\left(1.6\times10^{-19}\right)\left(0.75+10\right)N_d}{\left(11.7\right)\left(8.85\times10^{-14}\right)}\right]$$

$\Rightarrow N_d = 3.01\times10^{15}\ \text{cm}^{-3}$

7.27

$$x_p = (0.20)W = (0.20)\left(x_n + x_p\right)$$
$$(0.8)x_p = (0.2)x_n$$
$$x_n = 4x_p$$
$$N_a x_p = N_d x_n = N_d\left(4x_p\right)$$
$$\Rightarrow N_a = 4N_d$$

(a) $V_{bi} = V_t \ln\left[\dfrac{N_a N_d}{n_i^2}\right]$

$$= (0.0259)\ln\left[\frac{4N_d^2}{\left(1.8\times10^{6}\right)^2}\right]$$

$$C = AC' = A\left\{\frac{e\epsilon_s N_a N_d}{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}\right\}^{1/2}$$

$$= \left(10^{-4}\right)\left\{\frac{\left(1.6\times10^{-19}\right)\left(13.1\right)\left(8.85\times10^{-14}\right)}{2\left(V_{bi}+2\right)} \times \frac{\left(4N_d^2\right)}{\left(5N_d\right)}\right\}^{1/2}$$

$$0.6\times10^{-12} = 2.724\times10^{-20}\sqrt{\frac{N_d}{V_{bi}+2}}$$

By trial and error,

$N_d = 1.504\times10^{15}\ \text{cm}^{-3}$,

$N_a = 6.016\times10^{15}\ \text{cm}^{-3}$,

$V_{bi} = 1.10$ V

(b) From part (a),

$$0.6\times10^{-12} = 2.724\times10^{-20}\sqrt{\frac{N_d}{V_{bi}+5}}$$

By trial and error,

$N_d = 2.976\times10^{15}\ \text{cm}^{-3}$,

$N_a = 1.19\times10^{16}\ \text{cm}^{-3}$,

$V_{bi} = 1.135$ V

7.28

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{\left(5\times10^{15}\right)\left(10^{14}\right)}{\left(1.5\times10^{10}\right)^2}\right]$

or

$V_{bi} = 0.5574$ V

(b)

$$x_p = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2\left(11.7\right)\left(8.85\times10^{-14}\right)\left(0.5574\right)}{1.6\times10^{-19}} \times \left(\frac{10^{14}}{5\times10^{15}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\right]^{1/2}$$

or

$x_p = 5.32\times10^{-6}$ cm

Also

$$x_n = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$=\left[\frac{2(11.7)(8.85\times10^{-14})(0.5574)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{5\times10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 2.66\times10^{-4}\text{ cm}$$

(c) For $x_n = 30\,\mu$m, we have

$$30\times10^{-4} = \left[\frac{2(11.7)(8.85\times10^{-14})(V_{bi}+V_R)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{5\times10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\right]^{1/2}$$

which becomes

$$9\times10^{-6} = 1.269\times10^{-7}(V_{bi}+V_R)$$

We find

$$V_R = 70.4\text{ V}$$

7.29

An n^+p junction with $N_a = 10^{14}\text{ cm}^{-3}$,

(a) A one-sided junction and assume $V_R >> V_{bi}$. Then

$$x_p \cong \left[\frac{2\in_s V_R}{eN_a}\right]^{1/2}$$

or

$$(50\times10^{-4})^2 = \frac{2(11.7)(8.85\times10^{-14})V_R}{(1.6\times10^{-19})(10^{14})}$$

which yields

$$V_R = 193\text{ V}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p\left(\frac{N_a}{N_d}\right)$$

so

$$x_n = (50\times10^{-4})\left(\frac{10^{14}}{10^{16}}\right)$$
$$= 0.50\times10^{-4}\text{ cm} = 0.50\,\mu\text{m}$$

(c)

$$|\mathrm{E}_{max}| \cong \frac{2V_R}{W} = \frac{2(193.15)}{50.5\times10^{-4}}$$

or

$$|\mathrm{E}_{max}| = 7.65\times10^4\text{ V/cm}$$

7.30

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$
$$= 0.7305\text{ V}$$

(b) $$C = AC' \cong A\cdot\left[\frac{e\in_s N_d}{2(V_{bi}+V_R)}\right]^{1/2}$$
$$= (10^{-5})\left\{\frac{(1.6\times10^{-19})}{2(V_{bi}+V_R)}\right.$$
$$\left.\times(11.7)(8.85\times10^{-14})(2\times10^{15})\right\}^{1/2}$$
$$C = \frac{1.287\times10^{-13}}{\sqrt{V_{bi}+V_R}}$$

(i) For $V_R = 1$ V, $C = 9.783\times10^{-14}$ F

(ii) For $V_R = 3$ V, $C = 6.663\times10^{-14}$ F

(iii) For $V_R = 5$ V, $C = 5.376\times10^{-14}$ F

7.31

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(8\times10^{16})N_d}{(1.8\times10^6)^2}\right] = 1.20$$
$$(8\times10^{16})N_d = (1.8\times10^6)^2\exp\left(\frac{1.20}{0.0259}\right)$$
$$\Rightarrow N_d = 5.36\times10^{15}\text{ cm}^{-3}$$

(b) $$C = AC' = A\left\{\frac{e\in_s N_aN_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$
$$1.10\times10^{-12} = A\left\{\frac{(1.6\times10^{-19})}{2(1.20+1.0)}\right.$$
$$\left.\times\frac{(13.1)(8.85\times10^{-14})(8\times10^{16})(5.36\times10^{15})}{(8\times10^{16}+5.36\times10^{15})}\right\}^{1/2}$$
$$\Rightarrow A = 7.56\times10^{-5}\text{ cm}^2$$

(c) $$0.80\times10^{-12} = (7.56\times10^{-5})\left\{\frac{(1.6\times10^{-19})}{2(V_{bi}+V_R)}\right.$$
$$\left.\times\frac{(13.1)(8.85\times10^{-14})(8\times10^{16})(5.36\times10^{15})}{(8\times10^{16}+5.36\times10^{15})}\right\}^{1/2}$$
$$1.0582\times10^{-8} = \frac{2.1585\times10^{-8}}{\sqrt{V_{bi}+V_R}}$$
$$\Rightarrow V_{bi}+V_R = 4.161 = 1.20+V_R$$
$$V_R = 2.96\text{ V}$$

7.32

Plot

7.33

(a) $V_{bi} = V_t \ln\left(\dfrac{N_{aO} N_{dO}}{n_i^2}\right)$

(c) p-region

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\in_s} = -\frac{eN_{aO}}{\in_s}$$

or

$$\mathrm{E} = -\frac{eN_{aO}\,x}{\in_s} + C_1$$

We have

$$\mathrm{E} = 0 \text{ at } x = -x_p \Rightarrow C_1 = -\frac{eN_{aO}x_p}{\in_s}$$

Then for $-x_p < x < 0$

$$\mathrm{E} = -\frac{eN_{aO}}{\in_s}\left(x + x_p\right)$$

n-region, $0 < x < x_O$

$$\frac{d\mathrm{E}_1}{dx} = \frac{\rho(x)}{\in_s} = \frac{eN_{dO}}{2\in_s}$$

or

$$\mathrm{E}_1 = \frac{eN_{dO}\,x}{2\in_s} + C_2$$

n-region, $x_O < x < x_n$

$$\frac{d\mathrm{E}_2}{dx} = \frac{\rho(x)}{\in_s} = \frac{eN_{dO}}{\in_s}$$

or

$$\mathrm{E}_2 = \frac{eN_{dO}\,x}{\in_s} + C_3$$

We have $\mathrm{E}_2 = 0$ at $x = x_n$

$$\Rightarrow C_3 = -\frac{eN_{dO}x_n}{\in_s}$$

so that for $x_O < x < x_n$, we have

$$\mathrm{E}_2 = -\frac{eN_{dO}}{\in_s}\left(x_n - x\right)$$

We also have $\mathrm{E}_2 = \mathrm{E}_1$ at $x = x_O$

Then

$$\frac{eN_{dO}x_O}{2\in_s} + C_2 = -\frac{eN_{dO}}{\in_s}\left(x_n - x_O\right)$$

which gives

$$C_2 = -\frac{eN_{dO}}{\in_s}\left(x_n - \frac{x_O}{2}\right)$$

Then for $0 < x < x_O$ we have

$$\mathrm{E}_1 = \frac{eN_{dO}x}{2\in_s} - \frac{eN_{dO}}{\in_s}\left(x_n - \frac{x_O}{2}\right)$$

7.34

(a) $\dfrac{d^2\phi(x)}{dx^2} = -\dfrac{\rho(x)}{\in_s} = -\dfrac{d\mathrm{E}(x)}{dx}$

For $-2 < x < -1\,\mu$m, $\rho(x) = +eN_d$

So

$$\frac{d\mathrm{E}}{dx} = \frac{eN_d}{\in_s} \Rightarrow \mathrm{E} = \frac{eN_d\,x}{\in_s} + C_1$$

At $x = -2\,\mu\text{m} \equiv -x_O$, $\mathrm{E} = 0$

So

$$C_1 = \frac{eN_d\,x_O}{\in_s}$$

Then

$$\mathrm{E} = \frac{eN_d}{\in_s}\left(x + x_O\right)$$

At $x = 0$, $\mathrm{E}(0) = \mathrm{E}(x = -1)$, so

$$\mathrm{E}(0) = \frac{eN_d}{\in_s}(-1+2)\times 10^{-4}$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)}{(11.7)\left(8.85\times10^{-14}\right)}\left(1\times10^{-4}\right)$$

or

$$\mathrm{E}(0) = 7.726\times10^4 \text{ V/cm}$$

(c) Magnitude of potential difference is

$$|\phi| = \int \mathrm{E}dx = \frac{eN_d}{\in_s}\int\left(x + x_O\right)dx$$

$$= \frac{eN_d}{\in_s}\left(\frac{x^2}{2} + x_O\cdot x\right) + C_2$$

Let $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{eN_d}{\in_s}\left(\frac{x_O^2}{2} - x_O^2\right) + C_2 \Rightarrow C_2 = \frac{eN_d\,x_O^2}{2\in_s}$$

Then we can write

$$|\phi| = \frac{eN_d}{2\in_s}\left(x + x_O\right)^2$$

At $x = -1\,\mu$m

$$|\phi_1| = \frac{\left(1.6\times10^{-19}\right)\left(5\times10^{15}\right)}{2(11.7)\left(8.85\times10^{-14}\right)}\left[(-1+2)\times10^{-4}\right]^2$$

or

$$|\phi_1| = 3.863 \text{ V}$$

Potential difference across the intrinsic region

$$|\phi_2| = \mathrm{E}(0)\cdot d = (7.726\times10^4)(2\times10^{-4})$$

or

$$|\phi_2| = 15.45\ \mathrm{V}$$

By symmetry, the potential difference across the p-region space-charge region is also 3.863 V. The total reverse-bias voltage is then

$$V_R = 2(3.863) + 15.45 = 23.2\ \mathrm{V}$$

7.35

(a) $$V_B = \frac{\in_s \mathrm{E}_{crit}^2}{2eN_B}$$

or

$$N_B = \frac{\in_s \mathrm{E}_{crit}^2}{2eV_B} = \frac{(11.7)(8.85\times10^{-14})(4\times10^5)^2}{2(1.6\times10^{-19})(40)}$$

Then $N_B = N_a = 1.294\times10^{16}\ \mathrm{cm}^{-3}$

(b) $$N_B = \frac{(11.7)(8.85\times10^{-14})(4\times10^5)^2}{2(1.6\times10^{-19})(20)}$$

Or $N_B = N_a = 2.59\times10^{16}\ \mathrm{cm}^{-3}$

7.36

$$N_a = \frac{\in_s \mathrm{E}_{crit}^2}{2eV_B} = \frac{(11.7)(8.85\times10^{-14})(4\times10^5)^2}{2(1.6\times10^{-19})(80)}$$

$$= 6.47\times10^{15}\ \mathrm{cm}^{-3}$$

7.37

(a) For $N_d = 10^{16}\ \mathrm{cm}^{-3}$, from Figure 7.15,
$V_B \cong 75\ \mathrm{V}$

(b) For $N_d = 10^{15}\ \mathrm{cm}^{-3}$,
$V_B \cong 450\ \mathrm{V}$

7.38

(a) From Equation (7.36),

$$|\mathrm{E}_{\max}| = \left\{\frac{2e(V_{bi}+V_R)}{\in_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right\}^{1/2}$$

Set $|\mathrm{E}_{\max}| = \mathrm{E}_{crit}$ and $V_R = V_B$

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{16})(2\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$$= 0.7305\ \mathrm{V}$$

Then

$$4\times10^5 = \left\{\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})}\right.$$

$$\left.\times\left[\frac{(2\times10^{16})(2\times10^{16})}{2\times10^{16}+2\times10^{16}}\right]\right\}^{1/2}$$

$$\Rightarrow V_{bi} + V_B = 51.77\ \mathrm{V}$$

So $V_B = 51.04\ \mathrm{V}$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{15})(5\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.6587\ \mathrm{V}$$

Then

$$4\times10^5 = \left\{\frac{2(1.6\times10^{-19})(V_{bi}+V_R)}{(11.7)(8.85\times10^{-14})}\right.$$

$$\left.\times\left[\frac{(5\times10^{15})(5\times10^{15})}{5\times10^{15}+5\times10^{15}}\right]\right\}^{1/2}$$

$$\Rightarrow V_{bi} + V_R = 207.1$$

So $V_R \cong 206\ \mathrm{V}$

7.39

For a silicon p^+n junction with $N_d = 5\times10^{15}\ \mathrm{cm}^{-3}$ and $V_B \cong 100$ V, then, neglecting V_{bi} we have

$$x_n \cong \left[\frac{2\in_s V_B}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(100)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$$

or

$$x_n(\min) = 5.09\times10^{-4}\ \mathrm{cm} = 5.09\ \mu\mathrm{m}$$

7.40

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(10^{18})}{(1.5\times10^{10})^2}\right] = 0.933\ \mathrm{V}$$

Now

$$\mathrm{E}_{\max} = \frac{eN_d x_n}{\in_s}$$

so

$$10^6 = \frac{\left(1.6\times10^{-19}\right)\left(10^{18}\right)x_n}{(11.7)\left(8.85\times10^{-14}\right)}$$

which yields

$$x_n = 6.47\times10^{-6}\ \text{cm}$$

Now

$$x_n = \left[\frac{2\in_s\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

Then

$$\left(6.47\times10^{-6}\right)^2 = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)}{1.6\times10^{-19}} \times\left(V_{bi}+V_R\right)\left(\frac{10^{18}}{10^{18}}\right)\left(\frac{1}{10^{18}+10^{18}}\right)\right]$$

which yields

$$V_{bi}+V_R = 6.468\ \text{V}$$

or

$$V_R = 5.54\ \text{V}$$

7.41

Assume silicon: For an n^+p junction

$$x_p = \left[\frac{2\in_s\left(V_{bi}+V_R\right)}{eN_a}\right]^{1/2}$$

Assume $V_{bi} << V_R$

(a) For $x_p = 75\ \mu\text{m}$

$$\left(75\times10^{-4}\right)^2 = \frac{2(11.7)\left(8.85\times10^{-14}\right)V_R}{\left(1.6\times10^{-19}\right)\left(10^{15}\right)}$$

which yields

$$V_R = 4.35\times10^3\ \text{V}$$

(b) For $x_p = 150\ \mu\text{m}$

$$\left(150\times10^{-4}\right)^2 = \frac{2(11.7)\left(8.85\times10^{-14}\right)V_R}{\left(1.6\times10^{-19}\right)\left(10^{15}\right)}$$

which yields

$$V_R = 1.74\times10^4\ \text{V}$$

Note: From Figure 7.15, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

7.42

Impurity gradien

$$a = \frac{2\times10^{18}}{2\times10^{-4}} = 10^{22}\ \text{cm}^{-4}$$

From Figure 7.15, $V_B \cong 15\ \text{V}$

7.43

(a) For the linearly graded junction

$$\rho(x) = eax$$

Then

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\in_s} = \frac{eax}{\in_s}$$

Now

$$\mathrm{E} = \int \frac{eax}{\in_s}dx = \frac{ea}{\in_s}\cdot\frac{x^2}{2} + C_1$$

At $x = +x_O$ and $x = -x_O$, $\mathrm{E} = 0$

So

$$0 = \frac{ea}{\in_s}\left(\frac{x_O^2}{2}\right) + C_1 \Rightarrow C_1 = -\frac{ea}{\in_s}\left(\frac{x_O^2}{2}\right)$$

Then

$$\mathrm{E} = \frac{ea}{2\in_s}\left(x^2 - x_O^2\right)$$

(b)

$$\phi(x) = -\int \mathrm{E}dx = -\frac{ea}{2\in_s}\left[\frac{x^3}{3} - x_O^2\cdot x\right] + C_2$$

Set $\phi = 0$ at $x = -x_O$, then

$$0 = -\frac{ea}{2\in_s}\left[\frac{-x_O^3}{3} + x_O^3\right] + C_2 \Rightarrow C_2 = \frac{eax_O^3}{3\in_s}$$

Then

$$\phi(x) = -\frac{ea}{2\in_s}\left(\frac{x^3}{3} - x_O^2\cdot x\right) + \frac{eax_O^3}{3\in_s}$$

7.44

We have that

$$C' = \left[\frac{ea\in_s^2}{12\left(V_{bi}+V_R\right)}\right]^{1/3}$$

Then

$$\left(7.2\times10^{-9}\right)^3 = \left[\frac{a\left(1.6\times10^{-19}\right)\left[(11.7)\left(8.85\times10^{-14}\right)\right]^2}{12(0.7+3.5)}\right]$$

which yields

$$a = 1.1\times10^{20}\ \text{cm}^{-4}$$

7.45

(a) $C_j = AC' \cong A \cdot \left[\dfrac{e \in_s N_a}{2(V_{bi} + V_R)} \right]^{1/2}$

Let $N_a = 5 \times 10^{15}\ \text{cm}^{-3} << N_d$

Then $V_{bi} = (0.0259) \ln \left[\dfrac{(3 \times 10^{17})(5 \times 10^{15})}{(1.5 \times 10^{10})^2} \right]$

$= 0.7648\ \text{V}$

Now

$$C_j = 0.45 \times 10^{-12} = A \left\{ \frac{(1.6 \times 10^{-19})}{2(0.7648 + 5)} \right.$$

$$\left. \times (11.7)(8.85 \times 10^{-14})(5 \times 10^{15}) \right\}^{1/2}$$

$$0.45 \times 10^{-12} = A(8.476 \times 10^{-9})$$

$\Rightarrow A = 5.31 \times 10^{-5}\ \text{cm}^2$

(b) $$C_j = (5.309 \times 10^{-5}) \left\{ \frac{(1.6 \times 10^{-19})}{2(V_{bi} + V_R)} \right.$$

$$\left. \times (11.7)(8.85 \times 10^{-14})(5 \times 10^{15}) \right\}^{1/2}$$

$$C_j = \frac{1.0805 \times 10^{-12}}{\sqrt{V_{bi} + V_R}}$$

(i) For $V_R = 2.5\ \text{V}$, $C_j = 0.598\ \text{pF}$

(ii) For $V_R = 0$, $C_j = 1.24\ \text{pF}$

Chapter 8

8.1

In forward bias

$$I_f \cong I_S \exp\left(\frac{eV}{kT}\right)$$

Then

$$\frac{I_{f1}}{I_{f2}} = \frac{I_S \exp\left(\frac{eV_1}{kT}\right)}{I_S \exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\left(\frac{e}{kT}\right)(V_1 - V_2)\right]$$

or

$$V_1 - V_2 = \left(\frac{kT}{e}\right)\ln\left(\frac{I_{f1}}{I_{f2}}\right)$$

(a)

For $\frac{I_{f1}}{I_{f2}} = 10$, then

$$V_1 - V_2 = (0.0259)\ln(10)$$

or

$$V_1 - V_2 = 59.6 \text{ mV} \cong 60 \text{ mV}$$

(b)

For $\frac{I_{f1}}{I_{f2}} = 100$, then

$$V_1 - V_2 = (0.0259)\ln(100)$$

or

$$V_1 - V_2 = 119.3 \text{ mV} \cong 120 \text{ mV}$$

8.2

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{8\times10^{15}} = 2.8125\times10^4 \text{ cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{2\times10^{15}} = 1.125\times10^5 \text{ cm}^{-3}$$

$$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

$$n_p(-x_p) = n_{po}\exp\left(\frac{V_a}{V_t}\right)$$

(a) $V_a = 0.45$ V,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.45}{0.0259}\right)$$

$$= 3.95\times10^{12} \text{ cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.45}{0.0259}\right)$$

or

$$n_p(-x_p) = 9.88\times10^{11} \text{ cm}^{-3}$$

(b) $V_a = 0.55$ V,

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 1.88\times10^{14} \text{ cm}^{-3}$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{0.55}{0.0259}\right)$$

$$= 4.69\times10^{13} \text{ cm}^{-3}$$

(c) $V_a = -0.55$ V

$$p_n(x_n) = (1.125\times10^5)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

$$n_p(-x_p) = (2.8125\times10^4)\exp\left(\frac{-0.55}{0.0259}\right)$$

$$\cong 0$$

8.3

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.8\times10^6)^2}{4\times10^{16}} = 8.1\times10^{-5} \text{ cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.8\times10^6)^2}{10^{16}} = 3.24\times10^{-4} \text{ cm}^{-3}$$

(a) $V_a = 0.90$ V,

$$p_n(x_n) = (3.24\times10^{-4})\exp\left(\frac{0.90}{0.0259}\right)$$

$$= 4.0\times10^{11} \text{ cm}^{-3}$$

$$n_p(-x_p) = (8.1\times10^{-5})\exp\left(\frac{0.90}{0.0259}\right)$$

$$= 10.0\times10^{10} \text{ cm}^{-3}$$

(b) $V_a = 1.10$ V

$$p_n(x_n) = (3.24\times10^{-4})\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 9.03\times10^{14} \text{ cm}^{-3}$$

$$n_p(-x_p) = (8.1\times10^{-5})\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 2.26\times10^{14} \text{ cm}^{-3}$$

(c) $p_n(x_n) \cong 0$

$n_p(-x_p) \cong 0$

8.4

(a) $$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times 10^{10})^2}{5\times 10^{16}}$$
$$= 4.5\times 10^3 \text{ cm}^{-3}$$
$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times 10^{10})^2}{5\times 10^{15}}$$
$$= 4.5\times 10^4 \text{ cm}^{-3}$$

(i) $$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

or $$V_a = V_t \ln\left[\frac{p_n(x_n)}{p_{no}}\right]$$
$$= (0.0259)\ln\left[\frac{(0.1)(5\times 10^{15})}{4.5\times 10^4}\right]$$
$$= 0.599 \text{ V}$$

(ii) n-region - lower doped side

(b) $$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times 10^{10})^2}{7\times 10^{15}}$$
$$= 3.214\times 10^4 \text{ cm}^{-3}$$
$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times 10^{10})^2}{3\times 10^{16}}$$
$$= 7.5\times 10^3 \text{ cm}^{-3}$$

(i) $$V_a = V_t \ln\left[\frac{(0.1)N_a}{n_{po}}\right]$$
$$= (0.0259)\ln\left[\frac{(0.1)(7\times 10^{15})}{3.214\times 10^4}\right]$$
$$= 0.6165 \text{ V}$$

(ii) p-region - lower doped side

8.5

(a) $$J_n(-x_p) = \frac{eD_n n_{po}}{L_n}\exp\left(\frac{V_a}{V_t}\right)$$
$$= \frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}\cdot\exp\left(\frac{V_a}{V_t}\right)$$
$$= \frac{(1.6\times 10^{-19})(1.8\times 10^6)^2}{5\times 10^{16}}\sqrt{\frac{205}{5\times 10^{-8}}}$$
$$\times\exp\left(\frac{1.10}{0.0259}\right)$$
$$= 1.849 \text{ A/cm}^2$$
$$I_n = AJ_n(-x_p) = (10^{-3})(1.849) \text{ A}$$

or $I_n = 1.85$ mA

(b) $$J_p(x_n) = \frac{eD_p p_{no}}{L_p}\exp\left(\frac{V_a}{V_t}\right)$$
$$= \frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\cdot\exp\left(\frac{V_a}{V_t}\right)$$
$$= \frac{(1.6\times 10^{-19})(1.8\times 10^6)^2}{10^{16}}\sqrt{\frac{9.80}{10^{-8}}}$$
$$\times\exp\left(\frac{1.10}{0.0259}\right)$$
$$= 4.521 \text{ A/cm}^2$$
$$I_p = AJ_p(x_n) = (10^{-3})(4.521) \text{ A}$$

or $I_p = 4.52$ mA

(c) $I = I_n + I_p = 1.85 + 4.52 = 6.37$ mA

8.6

For an n^+p silicon diode
$$I_S = Aen_i^2\cdot\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}$$
$$= \frac{(10^{-4})(1.6\times 10^{-19})(1.5\times 10^{10})^2}{10^{16}}\sqrt{\frac{25}{10^{-6}}}$$

or
$$I_S = 1.8\times 10^{-15} \text{ A}$$

(a) For $V_a = 0.5$ V,
$$I_D \cong I_S\exp\left(\frac{V_a}{V_t}\right)$$
$$= (1.8\times 10^{-15})\exp\left(\frac{0.5}{0.0259}\right)$$

or
$$I_D = 4.36\times 10^{-7} \text{ A}$$

(b) For $V_a = -0.5$ V,
$$I_D = (1.8\times 10^{-15})\left[\exp\left(\frac{-0.5}{0.0259}\right) - 1\right]$$

or
$$I_D \cong -I_S = -1.8\times 10^{-15} \text{ A}$$

8.7

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

$$= \left(1.6\times10^{-19}\right)\left(2.4\times10^{13}\right)^2$$

$$\times\left[\frac{1}{4\times10^{15}}\sqrt{\frac{90}{2\times10^{-6}}} + \frac{1}{2\times10^{17}}\sqrt{\frac{48}{2\times10^{-6}}}\right]$$

$$J_s = 1.568\times10^{-4}\ \text{A/cm}^2$$

(a) $I = AJ_s \exp\left(\frac{V_a}{V_t}\right)$

$$= \left(10^{-4}\right)\left(1.568\times10^{-4}\right)\exp\left(\frac{0.25}{0.0259}\right)$$

$$= 2.44\times10^{-4}\ \text{A}$$

or $I = 0.244\ \text{mA}$

(b) $I = -I_s = -AJ_s = -\left(10^{-4}\right)\left(1.568\times10^{-4}\right)$

$$= -1.568\times10^{-8}\ \text{A}$$

8.8

(a) $J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$

$$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2$$

$$\times\left[\frac{1}{5\times10^{17}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{8\times10^{15}}\sqrt{\frac{10}{8\times10^{-8}}}\right]$$

$$J_s = 5.145\times10^{-11}\ \text{A/cm}^2$$

$$I_s = AJ_s = \left(2\times10^{-4}\right)\left(5.145\times10^{-11}\right)$$

$$= 1.029\times10^{-14}\ \text{A}$$

(b) $I = I_s \exp\left(\frac{V_a}{V_t}\right)$

(i) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.45}{0.0259}\right)$

$$= 3.61\times10^{-7}\ \text{A}$$

(ii) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.55}{0.0259}\right)$

$$= 1.72\times10^{-5}\ \text{A}$$

(iii) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.65}{0.0259}\right)$

$$= 8.16\times10^{-4}\ \text{A}$$

8.9

We have

$$I = I_S\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

or we can write this as

$$\frac{I}{I_S} + 1 = \exp\left(\frac{V}{V_t}\right)$$

so that

$$V = V_t \ln\left(\frac{I}{I_S} + 1\right)$$

In reverse bias, I is negative, so at

$\frac{I}{I_S} = -0.90$, we have

$$V = (0.0259)\ln(1 - 0.90)$$

or

$$V = -59.6\ \text{mV}$$

8.10

Case 1: $I = I_s \exp\left(\frac{V_a}{V_t}\right)$

$$0.50\times10^{-3} = I_s \exp\left(\frac{0.65}{0.0259}\right)$$

$$\Rightarrow I_s = 6.305\times10^{-15}\ \text{A} = 6.305\times10^{-12}\ \text{mA}$$

$$J_s = \frac{I_s}{A} = \frac{6.305\times10^{-12}}{2\times10^{-4}}$$

$$= 3.153\times10^{-8}\ \text{mA/cm}^2$$

Case 2: $I = I_s \exp\left(\frac{V_a}{V_t}\right)$

$$= \left(2\times10^{-12}\right)\exp\left(\frac{0.70}{0.0259}\right)$$

or $I = 1.093\ \text{mA}$

$$J_s = \frac{I_s}{A} = \frac{2\times10^{-12}}{1\times10^{-3}}$$

$$= 2\times10^{-9}\ \text{mA/cm}^2$$

Case 3: $I = AJ_s \exp\left(\frac{V_a}{V_t}\right)$

So $V_a = V_t \ln\left[\frac{I}{AJ_s}\right]$

$$= (0.0259)\ln\left[\frac{0.80}{\left(10^{-4}\right)\left(10^{-7}\right)}\right]$$

$$V_a = 0.6502\ \text{V}$$

Then

$$I_s = AJ_s = (10^{-4})(10^{-7}) = 10^{-11} \text{ mA}$$

Case 4: $$I_s = \frac{I}{\exp\left(\frac{V_a}{V_t}\right)} = \frac{1.20}{\exp\left(\frac{0.72}{0.0259}\right)}$$

$$I_s = 1.014 \times 10^{-12} \text{ mA}$$

$$A = \frac{I_s}{J_s} = \frac{1.014 \times 10^{-12}}{2 \times 10^{-8}}$$

$$= 5.07 \times 10^{-5} \text{ cm}^2$$

8.11

(a) $$\frac{J_n}{J_n + J_p} = \frac{\frac{eD_n n_{po}}{L_n}}{\frac{eD_n n_{po}}{L_n} + \frac{eD_p p_{no}}{L_p}}$$

$$= \frac{\sqrt{\frac{D_n}{\tau_{no}}} \cdot \frac{n_i^2}{N_a}}{\sqrt{\frac{D_n}{\tau_{no}}} \cdot \frac{n_i^2}{N_a} + \sqrt{\frac{D_p}{\tau_{po}}} \cdot \frac{n_i^2}{N_d}}$$

$$0.90 = \frac{1}{1 + \sqrt{\frac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\frac{N_a}{N_d}\right)}$$

$$\sqrt{\frac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\frac{N_a}{N_d}\right) = \frac{1}{0.90} - 1$$

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}} \left(\frac{1}{0.90} - 1\right)$$

$$= \sqrt{\frac{(25)(10^{-7})}{(10)(5 \times 10^{-7})}} (0.1111)$$

$$\frac{N_a}{N_d} = 0.07857 \text{ or } \frac{N_d}{N_a} = 12.73$$

(b) From part (a),

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}} \left(\frac{1}{0.20} - 1\right)$$

$$= \sqrt{\frac{(25)(10^{-7})}{(10)(5 \times 10^{-7})}} (4)$$

$$\frac{N_a}{N_d} = 2.828 \text{ or } \frac{N_d}{N_a} = 0.354$$

8.12

The cross-sectional area is

$$A = \frac{I}{J} = \frac{10 \times 10^{-3}}{20} = 5 \times 10^{-4} \text{ cm}^2$$

We have

$$J \cong J_S \exp\left(\frac{V_D}{V_t}\right) \Rightarrow 20 = J_S \exp\left(\frac{0.65}{0.0259}\right)$$

which yields

$$J_S = 2.522 \times 10^{-10} \text{ A/cm}^2$$

We can write

$$J_S = en_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

We want

$$\frac{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}}}{\frac{1}{N_a} \cdot \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \cdot \sqrt{\frac{D_p}{\tau_{pO}}}} = 0.10$$

or

$$\frac{\frac{1}{N_a} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}}}{\frac{1}{N_a}\sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5 \times 10^{-7}}}}$$

$$= \frac{7.071 \times 10^3}{7.071 \times 10^3 + \frac{N_a}{N_d}(4.472 \times 10^3)} = 0.10$$

which yields

$$\frac{N_a}{N_d} = 14.23$$

Now

$$J_S = 2.522 \times 10^{-10} = (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$

$$\times \left[\frac{1}{(14.23)N_d} \cdot \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{N_d} \cdot \sqrt{\frac{10}{5 \times 10^{-7}}}\right]$$

We find

$$N_d = 7.09 \times 10^{14} \text{ cm}^{-3}$$

and

$$N_a = 1.01 \times 10^{16} \text{ cm}^{-3}$$

8.13

Plot

8.14

(a)

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{eD_n n_{po}}{L_n}}{\dfrac{eD_n n_{po}}{L_n} + \dfrac{eD_p p_{no}}{L_p}}$$

$$= \frac{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{no}}} \cdot \dfrac{n_i^2}{N_a} + \sqrt{\dfrac{D_p}{\tau_{po}}} \cdot \dfrac{n_i^2}{N_d}}$$

$$= \frac{1}{1 + \sqrt{\dfrac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n} = \frac{\mu_p}{\mu_n} = \frac{1}{2.4} \quad \text{and} \quad \frac{\tau_{no}}{\tau_{po}} = \frac{1}{0.1}$$

so

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + \sqrt{\dfrac{1}{2.4} \cdot \dfrac{1}{0.1}} \cdot \left(\dfrac{N_a}{N_d}\right)}$$

or

$$\frac{J_n}{J_n + J_p} = \frac{1}{1 + (2.04)\left(\dfrac{N_a}{N_d}\right)}$$

(b) Using Einstein's relation, we can write

$$\frac{J_n}{J_n + J_p} = \frac{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n} \cdot \dfrac{n_i^2}{N_a} + \dfrac{e\mu_p}{L_p} \cdot \dfrac{n_i^2}{N_d}}$$

$$= \frac{e\mu_n N_d}{e\mu_n N_d + \dfrac{L_n}{L_p} \cdot e\mu_p N_a}$$

We have

$$\sigma_n = e\mu_n N_d \quad \text{and} \quad \sigma_p = e\mu_p N_a$$

Also

$$\frac{L_n}{L_p} = \sqrt{\frac{D_n \tau_{no}}{D_p \tau_{po}}} = \sqrt{\frac{2.4}{0.1}} = 4.90$$

Then

$$\frac{J_n}{J_n + J_p} = \frac{(\sigma_n / \sigma_p)}{(\sigma_n / \sigma_p) + 4.90}$$

8.15

(a) p-side;

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5 \times 10^{15}}{1.5 \times 10^{10}}\right)$$

or

$$E_{Fi} - E_F = 0.329 \text{ eV}$$

Also on the n-side;

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5 \times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.407 \text{ eV}$$

(b) We can find

$$D_n = (1250)(0.0259) = 32.4 \text{ cm}^2/\text{s}$$

$$D_p = (320)(0.0259) = 8.29 \text{ cm}^2/\text{s}$$

Now

$$J_S = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})^2$$

$$\times \left[\frac{1}{5 \times 10^{15}} \sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}} \sqrt{\frac{8.29}{10^{-7}}}\right]$$

or

$$J_S = 4.426 \times 10^{-11} \text{ A/cm}^2$$

Then

$$I_S = AJ_S = (10^{-4})(4.426 \times 10^{-11})$$

or

$$I_S = 4.426 \times 10^{-15} \text{ A}$$

We find

$$I = I_S \exp\left(\frac{V_D}{V_t}\right)$$

$$= (4.426 \times 10^{-15}) \exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I = 1.07 \times 10^{-6} \text{ A} = 1.07 \ \mu\text{A}$$

(c) The hole current is

$$I_p = en_i^2 A \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \exp\left(\frac{V_D}{V_t}\right)$$

$$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2\left(10^{-4}\right)\left(\frac{1}{10^{17}}\right)$$
$$\times\sqrt{\frac{8.29}{10^{-7}}}\exp\left(\frac{V_D}{V_t}\right)$$

or

$$I_p = 3.278\times10^{-16}\exp\left(\frac{V_D}{V_t}\right) \quad \text{(A)}$$

Then

$$\frac{I_p}{I} = \frac{J_p}{J_S} = \frac{3.278\times10^{-16}}{4.426\times10^{-15}} = 0.0741$$

8.16

(a) $I_{sp} = A\left(\frac{eD_p p_{no}}{L_p}\right) = eA\sqrt{\frac{D_p}{\tau_{po}}}\cdot\frac{n_i^2}{N_d}$

$$= \left(1.6\times10^{-19}\right)\left(5\times10^{-4}\right)\sqrt{\frac{10}{8\times10^{-8}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}}$$

$I_{sp} = 1.342\times10^{-14}$ A

(b) $I_{sn} = A\left(\frac{eD_n n_{po}}{L_n}\right) = eA\sqrt{\frac{D_n}{\tau_{no}}}\cdot\frac{n_i^2}{N_a}$

$$= \left(1.6\times10^{-19}\right)\left(5\times10^{-4}\right)\sqrt{\frac{25}{2\times10^{-7}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}}$$

$I_{sn} = 4.025\times10^{-15}$ A

(c) $V_{bi} = (0.0259)\ln\left[\frac{\left(5\times10^{16}\right)\left(1.5\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$

$= 0.746826$ V

$V_a = (0.8)V_{bi} = (0.8)(0.746826) = 0.59746$ V

$$p_n(x_n) = p_{no}\exp\left(\frac{V_a}{V_t}\right) = \frac{n_i^2}{N_d}\exp\left(\frac{V_a}{V_t}\right)$$
$$= \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}}\exp\left(\frac{0.59746}{0.0259}\right)$$
$$= 1.56\times10^{14}\ \text{cm}^{-3}$$

(d) $I_n(-x_p) = I_n(x_n) = I_{sn}\exp\left(\frac{V_a}{V_t}\right)$

$$= \left(4.025\times10^{-15}\right)\exp\left(\frac{0.59746}{0.0259}\right)$$
$$= 4.1981\times10^{-5}\ \text{A}$$

(e) $I_p(x_n) = I_{sp}\exp\left(\frac{V_a}{V_t}\right)$

$$= \left(1.342\times10^{-14}\right)\exp\left(\frac{0.59746}{0.0259}\right)$$
$$= 1.3997\times10^{-4}\ \text{A}$$
$$I_{Total} = I_n + I_p$$
$$= 4.1981\times10^{-5} + 1.3997\times10^{-4}$$
$$= 1.820\times10^{-4}\ \text{A}$$

Now

$$I_p\left(x_n + \frac{1}{2}L_p\right) = I_p(x_n)\exp\left(\frac{-(1/2)L_p}{L_p}\right)$$
$$= \left(1.3997\times10^{-4}\right)\exp\left(\frac{-1}{2}\right)$$
$$= 8.4896\times10^{-5}\ \text{A}$$

Then

$$I_n\left(x_n + \frac{1}{2}L_p\right) = I_{Total} - I_p\left(x_n + \frac{1}{2}L_p\right)$$
$$= 1.820\times10^{-4} - 8.4896\times10^{-5}$$
$$= 9.710\times10^{-5}\ \text{A}$$

8.17

(a) The excess hole concentration is given by

$$\delta p_n = p_n - p_{no}$$
$$= p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp\left(\frac{-x}{L_p}\right)$$

We find

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4\ \text{cm}^{-3}$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(8)\left(0.01\times10^{-6}\right)}$$
$$= 2.828\times10^{-4}\ \text{cm} = 2.828\ \mu\text{m}$$

Then

$$\delta p_n = \left(2.25\times10^4\right)\left[\exp\left(\frac{0.610}{0.0259}\right) - 1\right]$$
$$\times\exp\left(\frac{-x}{2.828\times10^{-4}}\right)$$

or

$$\delta p_n = \left(3.81\times10^{14}\right)\exp\left(\frac{-x}{2.828\times10^{-4}}\right)\ \text{cm}^{-3}$$

(b) We have

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}$$

$$= \frac{eD_p(3.808\times 10^{14})}{2.828\times 10^{-4}} \exp\left(\frac{-x}{2.828\times 10^{-4}}\right)$$

At $x = 3\times 10^{-4}$ cm,

$$J_p(3) = \frac{(1.6\times 10^{-19})(8)(3.808\times 10^{14})}{2.828\times 10^{-4}} \exp\left(\frac{-3}{2.828}\right)$$

or

$$J_p(3) = 0.5966 \text{ A/cm}^2$$

(c) We have

$$J_{no} = \frac{eD_n n_{po}}{L_n} \exp\left(\frac{V_a}{V_t}\right)$$

We can determine that

$$n_{po} = 4.5\times 10^3 \text{ cm}^{-3} \text{ and } L_n = 10.72\ \mu\text{m}$$

Then

$$J_{no} = \frac{(1.6\times 10^{-19})(23)(4.5\times 10^3)}{10.72\times 10^{-4}} \times \exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{no} = 0.2615 \text{ A/cm}^2$$

We can also find

$$J_{po} = 1.724 \text{ A/cm}^2$$

Then at $x = 3\ \mu$m,

$$J_n(3) = J_{no} + J_{po} - J_p(3)$$

$$= 0.2615 + 1.724 - 0.5966$$

or

$$J_n(3) = 1.39 \text{ A/cm}^2$$

8.18

(a) Problem 8.7

$$n_p = n_{po} \exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t \ln\left(\frac{n_p}{n_{po}}\right) = V_t \ln\left[\frac{(0.1)N_a}{n_i^2/N_a}\right]$

$$= V_t \ln\left[\frac{(0.1)N_a^2}{n_i^2}\right]$$

$$= (0.0259)\ln\left[\frac{(0.1)(4\times 10^{15})^2}{(2.4\times 10^{13})^2}\right]$$

$$= 0.205 \text{ V}$$

(b) Problem 8.8

$$p_n = p_{no} \exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t \ln\left[\frac{p_n}{p_{no}}\right] = V_t \ln\left[\frac{(0.1)N_d}{n_i^2/N_d}\right]$

$$= V_t \ln\left[\frac{(0.1)N_d^2}{n_i^2}\right]$$

$$= (0.0259)\ln\left[\frac{(0.1)(8\times 10^{15})^2}{(1.5\times 10^{10})^2}\right]$$

$$= 0.623 \text{ V}$$

8.19

The excess electron concentration is given by

$$\delta n_p = n_p - n_{po}$$

$$= n_{po}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot \exp\left(\frac{-x}{L_n}\right)$$

The total number of excess electrons is

$$N_p = A\int_0^\infty \delta n_p\, dx$$

We may note that

$$\int_0^\infty \exp\left(\frac{-x}{L_n}\right)dx = -L_n \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = L_n$$

Then

$$N_p = AL_n n_{po}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

We find that

$$D_n = 25 \text{ cm}^2\text{/s and } L_n = 50.0\ \mu\text{m}$$

Also

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5\times 10^{10})^2}{8\times 10^{15}} = 2.81\times 10^4 \text{ cm}^{-3}$$

Then

$$N_p = (10^{-3})(50.0\times 10^{-4})(2.8125\times 10^4) \times\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

or

$$N_p = (0.1406)\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

Then, we find the total number of excess electrons in the p-region to be:

(a) $V_a = 0.3$ V, $N_p = 1.51\times 10^4$

(b) $V_a = 0.4$ V, $N_p = 7.17\times 10^5$

(c) $V_a = 0.5$ V, $N_p = 3.40\times 10^7$

Similarly, the total number of excess holes in the n-region is found to be

$$P_n = AL_p p_{no}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$$

We find that

$D_p = 10.0\ \text{cm}^2/\text{s}$ and $L_p = 10.0\ \mu\text{m}$

Also

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times 10^{10}\right)^2}{10^{16}} = 2.25\times 10^4\ \text{cm}^{-3}$$

Then

$$P_n = \left(2.25\times 10^{-2}\right)\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$$

So

(a) $V_a = 0.3$ V, $P_n = 2.41\times 10^3$

(b) $V_a = 0.4$ V, $P_n = 1.15\times 10^5$

(c) $V_a = 0.5$ V, $P_n = 5.45\times 10^6$

8.20

$$I \propto n_i^2 \exp\left(\frac{V_a}{V_t}\right) \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_a}{kT}\right)$$

Then

$$I \propto \exp\left(\frac{eV_a - E_g}{kT}\right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{eV_{a1}-E_{g1}}{kT}\right)}{\exp\left(\frac{eV_{a2}-E_{g2}}{kT}\right)}$$

or

$$\frac{I_1}{I_2} = \exp\left(\frac{eV_{a1}-eV_{a2}-E_{g1}+E_{g2}}{kT}\right)$$

We then have

$$\frac{10\times 10^{-3}}{10\times 10^{-6}} = \exp\left(\frac{0.255-0.32-0.525+E_{g2}}{0.0259}\right)$$

or

$$10^3 = \exp\left(\frac{E_{g2}-0.59}{0.0259}\right)$$

Then

$$E_{g2} = 0.59 + (0.0259)\ln\left(10^3\right)$$

or

$$E_{g2} = 0.769\ \text{eV}$$

8.21

(a) We have

$$I_S = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

which can be written in the form

$$I_S = C'n_i^2$$

$$= C'N_{cO}N_{\upsilon O}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

or

$$I_S = CT^3\exp\left(\frac{-E_g}{kT}\right)$$

(b) Taking the ratio

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3\cdot\frac{\exp\left(\frac{-E_g}{kT_2}\right)}{\exp\left(\frac{-E_g}{kT_1}\right)}$$

$$= \left(\frac{T_2}{T_1}\right)^3\cdot\exp\left[+E_g\left(\frac{1}{kT_1}-\frac{1}{kT_2}\right)\right]$$

For $T_1 = 300$ K, $kT_1 = 0.0259$, $\dfrac{1}{kT_1} = 38.61$

For $T_2 = 400$ K, $kT_2 = 0.03453$, $\dfrac{1}{kT_2} = 28.96$

(i) Germanium: $E_g = 0.66$ eV

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3\exp\left[(0.66)(38.61-28.96)\right]$$

or $\dfrac{I_{S2}}{I_{S1}} = 1383$

(ii) Silicon: $E_g = 1.12$ eV

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3\exp\left[(1.12)(38.61-28.96)\right]$$

or $\dfrac{I_{S2}}{I_{S1}} = 1.17\times 10^5$

8.22

Plot

8.23

First case:

$$\left|\frac{I_f}{I_s}\right| = \exp\left(\frac{V_a}{V_t}\right)$$

or $V_t = \dfrac{V_a}{\ln\left|\dfrac{I_f}{I_s}\right|} = \dfrac{0.50}{\ln\left(2\times10^4\right)} = 0.05049$ V

Now $0.05049 = (0.0259)\left(\dfrac{T}{300}\right)$

$\Rightarrow T = 584.8$ K

Second case:

$$I_s = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]$$

$$1.2\times10^{-6} = \left(5\times10^{-4}\right)\left(1.6\times10^{-19}\right)\left(n_i^2\right) \times\left[\frac{1}{4\times10^{15}}\sqrt{\frac{25}{5\times10^{-7}}} + \frac{1}{2\times10^{17}}\sqrt{\frac{10}{10^{-7}}}\right]$$

or $n_i^2 = 8.2519\times10^{27}$

Now

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

$$\left(8.2519\times10^{27}\right) = \left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\left(\frac{T}{300}\right)^3 \times\exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

$$2.8337\times10^{-11} = \left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error,

$T \cong 502$ K

The reverse-bias current is limiting factor.

8.24

$$L_p = \sqrt{D_p\tau_{po}} = \sqrt{(10)\left(10^{-7}\right)} = 10^{-3} \text{ cm}$$

or $L_p = 10\,\mu$m; $\Rightarrow W_n << L_p$

(a) $J_p(x_n) = \dfrac{eD_p p_{no}}{W_n}\exp\left(\dfrac{V_a}{V_t}\right)$

(i) $p_n(x_n) = (0.1)N_d = p_{no}\exp\left(\dfrac{V_a}{V_t}\right)$

$$= \left(\frac{n_i^2}{N_d}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t \ln\left[\dfrac{(0.1)N_d^2}{n_i^2}\right]$

$$= (0.0259)\ln\left[\frac{(0.1)\left(2\times10^{15}\right)^2}{\left(1.5\times10^{10}\right)^2}\right]$$

$V_a = 0.5516$ V

(ii) $I_p = \dfrac{AeD_p}{W_n}\left(\dfrac{n_i^2}{N_d}\right)\exp\left(\dfrac{V_a}{V_t}\right)$

$$= \frac{\left(10^{-3}\right)\left(1.6\times10^{-19}\right)(10)\left(1.5\times10^{10}\right)^2}{\left(0.7\times10^{-4}\right)\left(2\times10^{15}\right)} \times\exp\left(\frac{0.5516}{0.0259}\right)$$

$I_p = 4.565\times10^{-3}$ A

$$I_n = \frac{AeD_n n_{po}}{L_n}\exp\left(\frac{V_a}{V_t}\right)$$

$$= Ae\sqrt{\frac{D_n}{\tau_{no}}}\left(\frac{n_i^2}{N_a}\right)\exp\left(\frac{V_a}{V_t}\right)$$

$$= \left(10^{-3}\right)\left(1.6\times10^{-19}\right)\sqrt{\frac{25}{5\times10^{-7}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{2\times10^{17}} \times\exp\left(\frac{0.5516}{0.0259}\right)$$

$I_n = 2.26\times10^{-6}$ A

$I = I_n + I_p$

$= 2.26\times10^{-6} + 4.565\times10^{-3}$

$= 4.567\times10^{-3}$ A

or $I = 4.567$ mA

(b) (i) $n_p(-x_p) = (0.1)N_a = n_{po}\exp\left(\dfrac{V_a}{V_t}\right)$

$$= \left(\frac{n_i^2}{N_a}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or $V_a = V_t\ln\left[\dfrac{(0.1)N_a^2}{n_i^2}\right]$

$$= (0.0259)\ln\left[\frac{(0.1)\left(2\times10^{15}\right)^2}{\left(1.5\times10^{10}\right)^2}\right]$$

$V_a = 0.5516$ V

(ii) $I_p = \dfrac{AeD_p}{W_n}\left(\dfrac{n_i^2}{N_d}\right)\exp\left(\dfrac{V_a}{V_t}\right)$

$$= \frac{(10^{-3})(1.6\times10^{-19})(10)(1.5\times10^{10})^2}{(0.7\times10^{-4})(2\times10^{17})} \exp\left(\frac{0.5516}{0.0259}\right)$$

$I_p = 4.565\times10^{-5}$ A

$$I_n = Ae\sqrt{\frac{D_n}{\tau_{no}}}\left(\frac{n_i^2}{N_a}\right)\exp\left(\frac{V_a}{V_t}\right)$$

$$= (10^{-3})(1.6\times10^{-19})\sqrt{\frac{25}{5\times10^{-7}}}\cdot\frac{(1.5\times10^{10})^2}{2\times10^{15}} \times\exp\left(\frac{0.5516}{0.0259}\right)$$

$I_n = 2.2597\times10^{-4}$ A

$I = I_n + I_p$

$= 2.2597\times10^{-4} + 4.565\times10^{-5}$

$= 2.716\times10^{-4}$ A

or $I = 0.2716$ mA

8.25

(a) We can write for the n-region

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The general solution is of the form

$$\delta p_n = A\exp\left(\frac{+x}{L_p}\right) + B\exp\left(\frac{-x}{L_p}\right)$$

The boundary condition at $x = x_n$ gives

$$\delta p_n(x_n) = p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= A\exp\left(\frac{+x_n}{L_p}\right) + B\exp\left(\frac{-x_n}{L_p}\right)$$

and the boundary condition at $x = x_n + W_n$ gives

$$\delta p_n(x_n + W_n) = 0$$

$$= A\exp\left(\frac{x_n + W_n}{L_p}\right) + B\exp\left(\frac{-(x_n + W_n)}{L_p}\right)$$

From this equation, we have

$$A = -B\exp\left[\frac{-2(x_n + W_n)}{L_p}\right]$$

Then, from the first boundary condition, we obtain

$$p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= -B\exp\left[\frac{-(x_n + 2W_n)}{L_p}\right] + B\exp\left(\frac{-x_n}{L_p}\right)$$

$$= B\exp\left(\frac{-x_n}{L_p}\right)\left[1 - \exp\left(\frac{-2W_n}{L_p}\right)\right]$$

We then obtain

$$B = \frac{p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\exp\left(\frac{-x_n}{L_p}\right)\left[1 - \exp\left(\frac{-2W_n}{L_p}\right)\right]}$$

which can be written as

$$B = \frac{p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp\left[\frac{x_n + W_n}{L_p}\right]}{\exp\left(\frac{W_n}{L_p}\right) - \exp\left(\frac{-W_n}{L_p}\right)}$$

We can also find

$$A = \frac{-p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp\left[\frac{-(x_n + W_n)}{L_p}\right]}{\exp\left(\frac{W_n}{L_p}\right) - \exp\left(\frac{-W_n}{L_p}\right)}$$

The solution can now be written as

$$\delta p_n = \frac{p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{2\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left\{\exp\left[\frac{x_n + W_n - x}{L_p}\right] - \exp\left[\frac{-(x_n + W_n - x)}{L_p}\right]\right\}$$

or finally

$$\delta p_n = p_{no}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\frac{\sinh\left(\frac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)}$$

(b)

$$J_p = -eD_p \left.\frac{d(\delta p_n)}{dx}\right|_{x=x_n}$$

$$= \frac{-eD_p p_{no}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]}{\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left.\left(\frac{-1}{L_p}\right)\cosh\left(\frac{x_n+W_n-x}{L_p}\right)\right|_{x=x_n}$$

Then

$$J_p = \frac{eD_p p_{no}}{L_p}\coth\left(\frac{W_n}{L_p}\right)\cdot\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$$

8.26

$$I_D \propto n_i^2 \exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \le T \le 320$ K, neglect the change in N_c and N_υ.

Then

$$I_D \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_D}{kT}\right)$$

$$\propto \exp\left[\frac{-(E_g - eV_D)}{kT}\right]$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\frac{-(E_g - eV_{D1})}{kT_1}\right]}{\exp\left[\frac{-(E_g - eV_{D2})}{kT_2}\right]}$$

We then have

$$\frac{E_g - eV_{D1}}{kT_1} = \frac{E_g - eV_{D2}}{kT_2}$$

We have

$T = 300$ K, $V_{D1} = 0.60$ V and

$$kT_1 = 0.0259\text{ eV},\ \frac{kT_1}{e} = 0.0259\text{ V}$$

$T = 310$ K,

$$kT_2 = 0.02676\text{ eV},\ \frac{kT_2}{e} = 0.02676\text{ V}$$

$T = 320$ K,

$$kT_3 = 0.02763\text{ eV},\ \frac{kT_3}{e} = 0.02763\text{ V}$$

For $T = 310$ K,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D2}}{0.02676}$$

which yields

$$V_{D2} = 0.5827\text{ V}$$

For $T = 320$ K,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D3}}{0.02763}$$

which yields

$$V_{D3} = 0.5653\text{ V}$$

8.27

(a) We can write

$$I_D = C\cdot n_i^2\cdot\exp\left(\frac{eV_a}{kT}\right)$$

where C is a constant, independent of temperature.

As a first approximation, neglect the variation of N_c and N_υ with temperature over the range of interest. We can then write

$$I_D = C_1\cdot\exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{V_a}{V_t}\right)$$

$$= C_1\cdot\exp\left[\frac{-(E_g - eV_a)}{kT}\right]$$

where C_1 is another constant, independent of temperature. We find

$$\frac{E_g - eV_a}{kT} = \ln\left(\frac{C_1}{I_D}\right)$$

or

$$V_a = E_g - \left(\frac{kT}{e}\right)\cdot\ln\left(\frac{C_1}{I_D}\right)$$

8.28

(a) $$I_s = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right]$$

$$= \left(10^{-4}\right)\left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2$$

$$\times\left[\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}}\right]$$

$$I_s = 2.323\times10^{-15}\text{ A}$$

(b) $$I_{gen} = \frac{Aen_iW}{2\tau_0}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$
$$= 0.7665 \text{ V}$$

and

$$W = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right\}^{1/2}$$
$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.7665+5)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left[\frac{4\times10^{16}+4\times10^{16}}{(4\times10^{16})(4\times10^{16})}\right]\right\}^{1/2}$$
$$W = 6.109\times10^{-5} \text{ cm}$$

Then

$$I_{gen} = \frac{(10^{-4})(1.6\times10^{-19})(1.5\times10^{10})(6.109\times10^{-5})}{2(10^{-7})}$$
$$= 7.331\times10^{-11} \text{ A}$$

(c) $\dfrac{I_{gen}}{I_s} = \dfrac{7.331\times10^{-11}}{2.323\times10^{-15}} = 3.16\times10^4$

8.29

(a) Set $I_S = I_{gen}$,

$$Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] = \frac{Aen_iW}{2\tau_0}$$
$$n_i\left[\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}}+\frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}}\right]$$
$$= \frac{W}{2\tau_0} = \frac{6.109\times10^{-5}}{2(10^{-7})}$$

so $n_i = \dfrac{3.0545\times10^2}{3.9528\times10^{-13}+2.50\times10^{-13}}$

$$= 4.734\times10^{14} \text{ cm}^{-3}$$

Then

$$n_i^2 = 2.2407\times10^{29}$$
$$= (2.8\times10^{19})(1.04\times10^{19})\left(\frac{T}{300}\right)^3$$
$$7.6947\times10^{-10} = \left(\frac{T}{300}\right)^3 \exp\left[\frac{-(1.12)(300)}{(0.0259)(T)}\right]$$

By trial and error,

$$T \cong 567 \text{ K}$$

We have

$$I_s = I_{gen} = \frac{Aen_iW}{2\tau_0}$$
$$= \frac{(10^{-4})(1.6\times10^{-19})(4.734\times10^{14})(6.109\times10^{-5})}{2(10^{-7})}$$

Then

$$I_s + I_{gen} = 2.314\times10^{-6} \text{ A}$$

or $I_s = I_{gen} = 2.314\ \mu\text{A}$

(b) From Problem 8.28

$$I_s = 2.323\times10^{-15} \text{ A}$$
$$I_{gen} = 7.331\times10^{-11} \text{ A}$$

So $I = I_s \exp\left(\dfrac{V_a}{V_t}\right) = I_{gen}\exp\left(\dfrac{V_a}{2V_t}\right)$

$$(2.323\times10^{-15})\exp\left(\frac{V_a}{V_t}\right)$$
$$= (7.331\times10^{-11})\exp\left(\frac{V_a}{2V_t}\right)$$
$$\frac{\exp\left(\frac{V_a}{V_t}\right)}{\exp\left(\frac{V_a}{2V_t}\right)} = \frac{7.331\times10^{-11}}{2.323\times10^{-15}}$$
$$\exp\left(\frac{V_a}{2V_t}\right) = 3.1558\times10^4$$
$$V_a = 2V_t\ln(3.1558\times10^4)$$
$$= 0.5366 \text{ V}$$

8.30

$$D_n = \left(\frac{kT}{e}\right)\cdot\mu_n = (0.0259)(5500)$$
$$= 142.5 \text{ cm}^2/\text{s}$$
$$D_p = (0.0259)(220) = 5.70 \text{ cm}^2/\text{s}$$

(a)

(i) $I_s = Aen_i^2\left[\dfrac{1}{N_a}\sqrt{\dfrac{D_n}{\tau_{n0}}}+\dfrac{1}{N_d}\sqrt{\dfrac{D_p}{\tau_{p0}}}\right]$

$$= (2\times10^{-4})(1.6\times10^{-19})(1.8\times10^6)^2$$
$$\times\left[\frac{1}{7\times10^{16}}\sqrt{\frac{142.5}{2\times10^{-8}}}+\frac{1}{7\times10^{16}}\sqrt{\frac{5.70}{2\times10^{-8}}}\right]$$
$$I_s = 1.50\times10^{-22} \text{ A}$$

(ii) $I_D = I_s \exp\left(\dfrac{V_a}{V_t}\right)$

$= \left(1.50\times10^{-22}\right)\exp\left(\dfrac{0.6}{0.0259}\right)$

$= 1.726\times10^{-12}$ A

(iii) $I_D = \left(1.50\times10^{-22}\right)\exp\left(\dfrac{0.8}{0.0259}\right)$

$= 3.896\times10^{-9}$ A

(iv) $I_D = \left(1.50\times10^{-22}\right)\exp\left(\dfrac{1.0}{0.0259}\right)$

$= 8.795\times10^{-6}$ A

(b) $I_{gen} = \dfrac{Aen_iW}{2\tau_0}$

$$V_{bi} = (0.0259)\ln\left[\frac{\left(7\times10^{16}\right)\left(7\times10^{16}\right)}{\left(1.8\times10^{6}\right)^2}\right]$$

$= 1.263$ V

$$W = \left\{\frac{2(13.1)\left(8.85\times10^{-14}\right)(1.263+3)}{1.6\times10^{-19}}\times\left[\frac{7\times10^{16}+7\times10^{16}}{\left(7\times10^{16}\right)\left(7\times10^{16}\right)}\right]\right\}^{1/2}$$

$= 4.201\times10^{-5}$ cm

(i)Then

$$I_{gen} = \frac{\left(2\times10^{-4}\right)\left(1.6\times10^{-19}\right)\left(1.8\times10^{6}\right)\left(4.201\times10^{-5}\right)}{2\left(2\times10^{-8}\right)}$$

$= 6.049\times10^{-14}$ A

(ii) $I_{rec} = I_{ro}\exp\left(\dfrac{V_a}{2V_t}\right)$

$= \left(6\times10^{-14}\right)\exp\left(\dfrac{0.6}{2(0.0259)}\right)$

$= 6.436\times10^{-9}$ A

(iii) $I_{rec} = \left(6\times10^{-14}\right)\exp\left(\dfrac{0.8}{2(0.0259)}\right)$

$= 3.058\times10^{-7}$ A

(iv) $I_{rec} = \left(6\times10^{-14}\right)\exp\left(\dfrac{1.0}{2(0.0259)}\right)$

$= 1.453\times10^{-5}$ A

8.31

Using results from Problem 8.30, we find

$V_a = 0.4$ V, $I_d = 7.64\times10^{-16}$ A,
$I_{rec} = 1.35\times10^{-10}$ A, $I_T \cong 1.35\times10^{-10}$ A

$V_a = 0.6$ V, $I_d = 1.73\times10^{-12}$ A
$I_{rec} = 6.44\times10^{-9}$ A, $I_T \cong 6.44\times10^{-9}$ A

$V_a = 0.8$ V, $I_d = 3.90\times10^{-9}$ A
$I_{rec} = 3.06\times10^{-7}$ A, $I_T = 3.10\times10^{-7}$ A

$V_a = 1.0$ V, $I_d = 8.80\times10^{-6}$ A
$I_{rec} = 1.45\times10^{-5}$ A, $I_T = 2.33\times10^{-5}$ A

$V_a = 1.2$ V. $I_d = 1.99\times10^{-2}$ A
$I_{rec} = 6.90\times10^{-4}$ A, $I_T = 2.06\times10^{-2}$ A

8.32

Plot

8.33

Plot

8.34

We have that

$$R = \frac{np - n_i^2}{\tau_{pO}(n+n') + \tau_{nO}(p+p')}$$

Let $\tau_{pO} = \tau_{nO} \equiv \tau_O$ and $n' = p' = n_i$

We can write

$$n = n_i \exp\left(\frac{E_{Fn}-E_{Fi}}{kT}\right)$$

and

$$p = n_i \exp\left(\frac{E_{Fi}-E_{Fp}}{kT}\right)$$

We also have

$$\left(E_{Fn}-E_{Fi}\right) + \left(E_{Fi}-E_{Fp}\right) = eV_a$$

so that

$$\left(E_{Fi}-E_{Fp}\right) = eV_a - \left(E_{Fn}-E_{Fi}\right)$$

Then

$$p = n_i \exp\left[\frac{eV_a - \left(E_{Fn}-E_{Fi}\right)}{kT}\right]$$

$$= n_i \exp\left(\frac{eV_a}{kT}\right)\cdot\exp\left[\frac{-\left(E_{Fn}-E_{Fi}\right)}{kT}\right]$$

Define

$$\eta_a = \frac{eV_a}{kT} \text{ and } \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Then the recombination rate can be written as

$$R = \frac{\left(n_i e^{\eta}\right)\left(n_i e^{\eta_a} \cdot e^{-\eta}\right) - n_i^2}{\tau_O\left[n_i e^{\eta} + n_i + n_i e^{\eta_a} \cdot e^{-\eta} + n_i\right]}$$

or

$$R = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O} \cdot \frac{d}{d\eta}\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^{-1}$$

or

$$0 = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O} \cdot (-1)\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^{-2} \times \left[e^{\eta} - e^{\eta_a} \cdot e^{-\eta}\right]$$

which simplifies to

$$0 = \frac{-n_i\left(e^{\eta_a} - 1\right)}{\tau_O} \cdot \frac{\left[e^{\eta} - e^{\eta_a} \cdot e^{-\eta}\right]}{\left[2 + e^{\eta} + e^{\eta_a} \cdot e^{-\eta}\right]^2}$$

The denominator is not zero, so we have

$$0 = e^{\eta} - e^{\eta_a} \cdot e^{-\eta}$$

or

$$e^{2\eta} = e^{\eta_a} \Rightarrow \eta = \frac{\eta_a}{2}$$

Then the maximum recombination rate becomes

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a} \cdot e^{-\eta_a/2}\right]}$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a/2}\right]}$$

or

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{2\tau_O\left(e^{\eta_a/2} + 1\right)}$$

which can be written as

$$R_{\max} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau_O\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

If $V_a >> (kT/e)$, then we can neglect the (-1) term in the numerator and the (+1) term in the denominator, so we finally have

$$R_{\max} = \frac{n_i}{2\tau_O}\exp\left(\frac{eV_a}{2kT}\right)$$

Q.E.D.

8.35

We have

$$J_{gen} = \int_0^W eG\,dx$$

In this case, $G = g' = 4 \times 10^{19}\ \text{cm}^{-3}\,\text{s}^{-1}$ and is a constant through the space charge region.

Then

$$J_{gen} = eg'W$$

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(5 \times 10^{15}\right)\left(5 \times 10^{15}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.659\ \text{V}$$

Also

$$W = \left[\frac{2 \in_s \left(V_{bi} + V_R\right)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.659 + 10)}{1.6 \times 10^{-19}} \times \left(\frac{5 \times 10^{15} + 5 \times 10^{15}}{\left(5 \times 10^{15}\right)\left(5 \times 10^{15}\right)}\right)\right]^{1/2}$$

or

$$W = 2.35 \times 10^{-4}\ \text{cm}$$

Then

$$J_{gen} = \left(1.6 \times 10^{-19}\right)\left(4 \times 10^{19}\right)\left(2.35 \times 10^{-4}\right)$$

or

$$J_{gen} = 1.5 \times 10^{-3}\ \text{A/cm}^2$$

8.36

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2\left[\frac{1}{3\times10^{16}}\sqrt{\frac{18}{10^{-7}}} + \frac{1}{10^{18}}\sqrt{\frac{6}{10^{-7}}}\right]$$

or

$$J_S = 1.638\times10^{-11}\ \text{A/cm}^2$$

Now

$$J_D = J_S \exp\left(\frac{V_D}{V_t}\right)$$

We want

$$J = 0 = J_G - J_D$$

or

$$0 = 25\times10^{-3} - 1.638\times10^{-11}\exp\left(\frac{V_D}{V_t}\right)$$

which can be written as

$$\exp\left(\frac{V_D}{V_t}\right) = \frac{25\times10^{-3}}{1.638\times10^{-11}} = 1.526\times10^9$$

We find

$$V_D = V_t \ln\left(1.526\times10^9\right)$$

or

$$V_D = 0.548\ \text{V}$$

8.37

(a) $$r_d = \frac{V_t}{I_{DQ}} = \frac{0.0259}{1.2\times10^{-3}} = 21.6\ \Omega$$

$$C_d = \frac{I_{DQ}\tau_0}{2V_t} = \frac{\left(1.2\times10^{-3}\right)\left(0.5\times10^{-6}\right)}{2(0.0259)}$$

$$= 1.16\times10^{-8}\ \text{F}$$

or $C_d = 11.6\ \text{nF}$

(b) $$r_d = \frac{0.0259}{0.12\times10^{-3}} = 216\ \Omega$$

$$C_d = \frac{\left(0.12\times10^{-3}\right)\left(0.5\times10^{-6}\right)}{2(0.0259)}$$

$$= 1.16\times10^{-9}\ \text{F}$$

or $C_d = 1.16\ \text{nF}$

8.38

(a) $C_d = \dfrac{\Delta Q}{\Delta V}$, For $I_D = 1.2\ \text{mA}$

$$\Delta Q = C_d \cdot \Delta V = \left(1.158\times10^{-8}\right)\left(50\times10^{-3}\right)$$

$$= 5.79\times10^{-10}\ \text{C}$$

(b) For $I_D = 0.12\ \text{mA}$

$$\Delta Q = C_d \cdot \Delta V = \left(1.158\times10^{-9}\right)\left(50\times10^{-3}\right)$$

$$= 5.79\times10^{-11}\ \text{C}$$

8.39

For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t},\quad C_d = \frac{I_{DQ}\tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86\times10^{-2}\ \text{S}$$

and

$$C_d = \frac{\left(10^{-3}\right)\left(10^{-7}\right)}{2(0.0259)} = 1.93\times10^{-9}\ \text{F}$$

We have

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

where $\omega = 2\pi f$

We obtain

$f = 10\ \text{kHz}$, $Z = 25.9 - j0.0814$
$f = 100\ \text{kHz}$, $Z = 25.9 - j0.814$
$f = 1\ \text{MHz}$, $Z = 23.6 - j7.41$
$f = 10\ \text{MHz}$, $Z = 2.38 - j7.49$

8.40

Reverse bias

$$C_j = AC' = A\cdot\left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5\times10^{17}\right)\left(8\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.790\ \text{V}$$

$$C_j = \left(2\times10^{-4}\right)\left\{\frac{\left(1.6\times10^{-19}\right)(11.7)\left(8.85\times10^{-14}\right)}{2(V_{bi}+V_R)} \times\left[\frac{\left(5\times10^{17}\right)\left(8\times10^{15}\right)}{5\times10^{17}+8\times10^{15}}\right]\right\}^{1/2}$$

$$C_j = \frac{5.1078\times10^{-12}}{\sqrt{V_{bi}+V_r}} \text{ F}$$

V_R (V)	C_j (pF)
10	1.555
5	2.123
3	2.624
1	3.818
0	5.747
–0.20	6.650
–0.40	8.179

Forward bias

For $N_a >> N_d \Rightarrow I_{po} >> I_{no}$

Then

$$C_d = \frac{I_{po}\tau_{p0}}{2V_t} = \frac{I_{po}\left(8\times10^{-8}\right)}{2(0.0259)} = \left(1.544\times10^{-6}\right)I_{po}$$

$$I_{po} = Ae\sqrt{\frac{D_p}{\tau_{p0}}}\cdot\left(\frac{n_i^2}{N_d}\right)\cdot\exp\left(\frac{V_a}{V_t}\right)$$

$$= \left(2\times10^{-4}\right)\left(1.6\times10^{-19}\right)\sqrt{\frac{10}{8\times10^{-8}}}\cdot\frac{\left(1.5\times10^{10}\right)^2}{8\times10^{15}} \times\exp\left(\frac{V_a}{V_t}\right)$$

$$I_{po} = \left(1.006\times10^{-14}\right)\exp\left(\frac{V_a}{V_t}\right) \text{ A}$$

V_a (V)	C_d (F)	+ C_j (F)	= C_{Total} (F)
0.20	3.51×10^{-17}	$+6.650\times10^{-12}$	$\cong 6.650\times10^{-12}$
0.40	7.92×10^{-14}	$+8.179\times10^{-12}$	$= 8.258\times10^{-12}$
0.60	1.79×10^{-10}	$+\ldots$	$\cong 1.79\times10^{-10}$

8.41

For a p^+n diode, $I_{pO} >> I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t}\right)\left(I_{pO}\tau_{pO}\right)$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5\times10^{-6} \text{ F/A}$$

Then

$$\tau_{pO} = 2(0.0259)\left(2.5\times10^{-6}\right)$$

or

$$\tau_{pO} = 1.3\times10^{-7} \text{ s}$$

At 1 mA,

$$C_d = \left(2.5\times10^{-6}\right)\left(10^{-3}\right)$$

or

$$C_d = 2.5\times10^{-9} \text{ F}$$

8.42

(a) $N_a >> N_d \Rightarrow I_{po} >> I_{no}$

(i) $$C_d = \frac{I_{po}\tau_{p0}}{2V_t}$$

or $$I_{po} = \frac{2V_t(C_d)}{\tau_{p0}} = \frac{2(0.0259)\left(10^{-9}\right)}{10^{-7}}$$

$$= 5.18\times10^{-4} \text{ A}$$

or $I_{po} = 0.518$ mA

(ii) $$I_{po} = Ae\sqrt{\frac{D_p}{\tau_{p0}}}\cdot\frac{n_i^2}{N_d}\exp\left(\frac{V_a}{V_t}\right)$$

$$0.518\times10^{-3} = \left(5\times10^{-4}\right)\left(1.6\times10^{-19}\right)\sqrt{\frac{10}{10^{-7}}} \times\frac{\left(1.5\times10^{10}\right)^2}{8\times10^{15}}\exp\left(\frac{V_a}{V_t}\right)$$

$$V_a = (0.0259)\ln\left(\frac{0.518\times10^{-3}}{2.25\times10^{-14}}\right)$$

$$= 0.618 \text{ V}$$

(iii) $$r_d = \frac{V_t}{I_D} = \frac{0.0259}{0.518\times10^{-3}} = 50\,\Omega$$

(b)

(i) $$I_{po} = \frac{2V_t(C_d)}{\tau_{p0}} = \frac{2(0.0259)\left(0.25\times10^{-9}\right)}{10^{-7}}$$

$$= 1.295\times10^{-4} \text{ A}$$

or $I_{po} = 0.1295$ mA

(ii) $$V_a = (0.0259)\ln\left(\frac{0.1295\times10^{-3}}{2.25\times10^{-14}}\right)$$

$$= 0.5821 \text{ V}$$

(iii) $$r_d = \frac{0.0259}{0.1295\times10^{-3}} = 200\,\Omega$$

8.43

(a) p-region:

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{(e\mu_p N_a)A}$$

so

$$R_p = \frac{0.2}{(1.6\times10^{-19})(480)(10^{16})(10^{-2})}$$

or

$$R_p = 26\,\Omega$$

n-region:

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{(e\mu_n N_d)A}$$

so

$$R_n = \frac{0.10}{(1.6\times10^{-19})(1350)(10^{15})(10^{-2})}$$

or

$$R_n = 46.3\,\Omega$$

The total resistance is

$$R = R_p + R_n = 26 + 46.3$$

or

$$R = 72.3\,\Omega$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

which yields

$$I = 1.38\text{ mA}$$

8.44

$$R = \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)}$$

$$= \frac{(0.2)(10^{-2})}{2\times10^{-5}} + \frac{(0.1)(10^{-2})}{2\times10^{-5}}$$

or

$$R = 150\,\Omega$$

We can write

$$V = I_D R + V_t \ln\left(\frac{I_D}{I_S}\right)$$

(a) (i) For $I_D = 1$ mA,

$$V = (10^{-3})(150) + (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or $V = 0.567$ V

(ii) For $I_D = 10$ mA,

$$V = (10^{-2})(150) + (0.0259)\ln\left(\frac{10^{-2}}{10^{-10}}\right)$$

or $V = 1.98$ V

(b) Set $R = 0$

(i) For $I_D = 1$ mA,

$$V = (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or $V = 0.417$ V

(ii) For $I_D = 10$ mA,

$$V = (0.0259)\ln\left(\frac{10^{-2}}{10^{-10}}\right)$$

or $V = 0.477$ V

8.45

(a) $$r_d = \frac{V_t}{I_D} \Rightarrow I_D = \frac{V_t}{r_d} = \frac{0.0259}{32}$$

or $I_D = 8.09375\times10^{-4}$ A

$$V_a = V_t \ln\left(\frac{I_D}{I_s}\right)$$

$$= (0.0259)\ln\left(\frac{8.09375\times10^{-4}}{5\times10^{-12}}\right)$$

$$V_a = 0.4896\text{ V}$$

(b) $$I_D = \frac{V_t}{r_d} = \frac{0.0259}{60} = 4.3167\times10^{-4}\text{ A}$$

$$V_a = (0.0259)\ln\left(\frac{4.3167\times10^{-4}}{5\times10^{-12}}\right)$$

$$= 0.4733\text{ V}$$

8.46

(a) $$\frac{1}{r_d} = \frac{dI_D}{dV_a} = I_S\left(\frac{1}{V_t}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$r_d = 1.2\times10^{11}\ \Omega$$

(b)

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{-0.02}{0.0259}\right)$$

which yields

$$r_d = 5.6\times10^{11}\ \Omega$$

8.47

(a) If $\dfrac{I_R}{I_F} = 0.2$

Then we have

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = \frac{I_F}{I_F + I_R} = \frac{1}{1 + \dfrac{I_R}{I_F}} = \frac{1}{1 + 0.2}$$

or

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\frac{t_s}{\tau_{pO}}} = 0.978 \Rightarrow \frac{t_s}{\tau_{pO}} = 0.956$$

(b) If $\dfrac{I_R}{I_F} = 1.0$, then

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = \frac{1}{1+1} = 0.50$$

which yields

$$\frac{t_s}{\tau_{pO}} = 0.228$$

8.48

(a) $$\text{erf}\sqrt{\frac{t_s}{\tau_p}} = \frac{I_F}{I_F + I_R}$$

$$\text{erf}\sqrt{0.3} = \text{erf}(0.5477)$$
$$\cong \text{erf}(0.55) = 0.56332$$

Then $$0.56332 = \frac{1}{1 + \dfrac{I_R}{I_F}}$$

$$\frac{I_R}{I_F} = \frac{1}{0.56332} - 1 = 0.775$$

(b) $$\text{erf}\sqrt{\frac{t_2}{\tau_{p0}}} + \frac{\exp\left(\dfrac{-t_2}{\tau_{p0}}\right)}{\sqrt{\pi\left(\dfrac{t_2}{\tau_{p0}}\right)}} = 1 + (0.1)\left(\frac{I_R}{I_F}\right)$$

$$= 1 + (0.1)(0.775)$$
$$= 1.0775$$

By trial and error, $\dfrac{t_2}{\tau_{p0}} \cong 0.80$

8.49

$C_j = 18\text{ pF at } V_R = 0$

$C_j = 4.2\text{ pF at } V_R = 10\text{ V}$

We have

$\tau_{nO} = \tau_{pO} = 10^{-7}\text{ s}$, $I_F = 2\text{ mA}$

and

$$I_R \cong \frac{V_R}{R} = \frac{10}{10} = 1\text{ mA}$$

So

$$t_s \cong \tau_{pO}\ln\left(1 + \frac{I_F}{I_R}\right) = \left(10^{-7}\right)\ln\left(1 + \frac{2}{1}\right)$$

or

$t_s = 1.1\times10^{-7}\text{ s}$

Also

$$C_{avg} = \frac{18 + 4.2}{2} = 11.1\text{ pF}$$

The time constant is

$$\tau_S = RC_{avg} = \left(10^4\right)\left(11.1\times10^{-12}\right)$$
$$= 1.11\times10^{-7}\text{ s}$$

Now, the turn-off time is

$$t_{off} = t_s + \tau_S = (1.1 + 1.11)\times10^{-7}$$

or

$$t_{off} = 2.21\times10^{-7}\text{ s}$$

8.50

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5\times10^{19}\right)^2}{\left(1.5\times10^{10}\right)^2}\right] = 1.136\text{ V}$$

We find

$$W = \left[\frac{2\in_s\left(V_{bi} - V_a\right)}{e}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)(1.136 - 0.40)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{5\times10^{19} + 5\times10^{19}}{\left(5\times10^{19}\right)^2}\right)\right]^{1/2}$$

which yields

$$W = 6.17\times10^{-7}\text{ cm} = 61.7\ \overset{o}{A}$$

8.51

Sketch

8.53

From Figure 7.15, $N_d \cong 9\times10^{15}\ \text{cm}^{-3}$

Let $N_a = 5\times10^{17}\ \text{cm}^{-3}$

$$p_n(x_n) = (0.1)N_d = 9\times10^{14} = p_{no}\exp\left(\frac{V_a}{V_t}\right)$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5\times10^{10})^2}{9\times10^{15}} = 2.5\times10^4\ \text{cm}^{-3}$$

Then $V_a = (0.0259)\ln\left(\dfrac{9\times10^{14}}{2.5\times10^4}\right) = 0.6295\ \text{V}$

$$I_s = \frac{I}{\exp\left(\dfrac{V_a}{V_t}\right)} = \frac{50\times10^{-3}}{\exp\left(\dfrac{0.6295}{0.0259}\right)}$$

$$= 1.389\times10^{-12}\ \text{A}$$

$$I_s \cong \frac{AeD_p p_{no}}{L_p} = \frac{Aen_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}$$

$$1.389\times10^{-12}$$

$$= \frac{A(1.6\times10^{-19})(1.5\times10^{10})^2}{9\times10^{15}}\sqrt{\frac{10}{2\times10^{-7}}}$$

$$1.389\times10^{-12} = A(2.828\times10^{-11})$$

or $A = 4.91\times10^{-2}\ \text{cm}^2$

Chapter 9

9.1

(a) We have

$$e\phi_n = eV_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right) = 0.206 \text{ eV}$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\phi_{BO} = 0.27 \text{ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064 \text{ V}$$

Also

$$x_d = \left[\frac{2\in_s V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.064)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 9.1\times10^{-6} \text{ cm}$$

Then

$$|\mathrm{E}_{\max}| = \frac{eN_d x_d}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(10^{16})(9.1\times10^{-6})}{(11.7)(8.85\times10^{-14})}$$

or

$$|\mathrm{E}_{\max}| = 1.41\times10^4 \text{ V/cm}$$

(d)

Using the figure, $\phi_{Bn} = 0.55$ V

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344 \text{ V}$$

We then find

$$x_n = 2.11\times10^{-5} \text{ cm}$$

and

$$|\mathrm{E}_{\max}| = 3.26\times10^4 \text{ V/cm}$$

9.2

(a) $V_{bi} = \phi_{B0} - \phi_n$

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{5\times10^{15}}\right)$$

$=0.2235$ V

$$V_{bi} = 0.65 - 0.2235 = 0.4265 \text{ V}$$

(b) $\phi_n = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right)$

$= 0.2056$ V

$$V_{bi} = 0.65 - 0.2056 = 0.4444 \text{ V}$$

V_{bi} increases, ϕ_{B0} remains constant

(c) $\phi_n = (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{15}}\right)$

$= 0.2652$ V

$$V_{bi} = 0.65 - 0.2652 = 0.3848 \text{ V}$$

V_{bi} decreases, ϕ_{B0} remains constant

9.3

(a) $\phi_{B0} = \phi_m - \chi = 5.1 - 4.01 = 1.09$ V

(b) $V_{bi} = \phi_{B0} - \phi_n$

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{10^{16}}\right) = 0.2056 \text{ V}$$

$$V_{bi} = 1.09 - 0.2056 = 0.8844 \text{ V}$$

(c) $x_n = \left[\frac{2\in_s (V_{bi}+V_R)}{eN_d}\right]^{1/2}$

(i) $x_n = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8844+1)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$

$= 4.939\times10^{-5}$ cm

or $x_n = 0.4939\ \mu$m

$$|\mathrm{E}_{\max}| = \frac{eN_d x_n}{\in_s}$$

$$= \frac{(1.6\times10^{-19})(10^{16})(4.939\times10^{-5})}{(11.7)(8.85\times10^{-14})}$$

$= 7.63\times10^4$ V/cm

(ii) $x_n = \left[\dfrac{2(11.7)(8.85\times10^{-14})(0.8844+5)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$

$= 8.727\times10^{-5}$ cm

or $x_n = 0.8728\ \mu$m

$|E_{max}| = \dfrac{(1.6\times10^{-19})(10^{16})(8.727\times10^{-5})}{(11.7)(8.85\times10^{-14})}$

$= 1.35\times10^{5}$ V/cm

9.4

(a) $\phi_{B0} = \phi_m - \chi = 5.1 - 4.07 = 1.03$ V

(b) $\phi_n = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{5\times10^{15}}\right) = 0.1177$ V

(c) $V_{bi} = 1.03 - 0.1177 = 0.9123$ V

(d)

(i) $x_n = \left[\dfrac{2(13.1)(8.85\times10^{-14})(0.9123+1)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$

$= 7.445\times10^{-5}$ cm

or $x_n = 0.7445\ \mu$m

$|E_{max}| = \dfrac{(1.6\times10^{-19})(5\times10^{15})(7.445\times10^{-5})}{(13.1)(8.85\times10^{-14})}$

$= 5.14\times10^{4}$ V/cm

(ii) $x_n = \left[\dfrac{2(13.1)(8.85\times10^{-14})(0.9123+5)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$

$= 1.309\times10^{-4}$ cm

or $x_n = 1.309\ \mu$m

$|E_{max}| = \dfrac{(1.6\times10^{-19})(5\times10^{15})(1.309\times10^{-4})}{(13.1)(8.85\times10^{-14})}$

$= 9.03\times10^{4}$ V/cm

9.5

(b) $\phi_n = 0.1177$ V

(c) $V_{bi} = 0.88 - 0.1177 = 0.7623$ V

(d)

(i) $x_n = \left[\dfrac{2(13.1)(8.85\times10^{-14})(0.7623+1)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$

$= 7.147\times10^{-5}$ cm

or $x_n = 0.7147\ \mu$m

$|E_{max}| = \dfrac{(1.6\times10^{-19})(5\times10^{15})(7.147\times10^{-5})}{(13.1)(8.85\times10^{-14})}$

$= 4.93\times10^{4}$ V/cm

(ii) $x_n = \left[\dfrac{2(13.1)(8.85\times10^{-14})(0.7623+5)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$

$= 1.292\times10^{-4}$ cm

or $x_n = 1.292\ \mu$m

$|E_{max}| = \dfrac{(1.6\times10^{-19})(5\times10^{15})(1.292\times10^{-4})}{(13.1)(8.85\times10^{-14})}$

$= 8.92\times10^{4}$ V/cm

9.6

(a) $C' = \left[\dfrac{e\in_s N_d}{2(V_{bi}+V_R)}\right]^{1/2}$

We have $\phi_{B0} = 0.88$ V

$\phi_n = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{10^{15}}\right) = 0.265$ V

$V_{bi} = 0.88 - 0.265 = 0.615$ V

(i)

$C = (10^{-4})\left[\dfrac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{15})}{2(0.615+1)}\right]^{1/2}$

$= 7.16\times10^{-13}$ F

or $C = 0.716$ pF

(ii)

$C = (10^{-4})\left[\dfrac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{15})}{2(0.615+5)}\right]^{1/2}$

$= 3.84\times10^{-13}$ F

or $C = 0.384$ pF

(b) $\phi_n = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{10^{16}}\right) = 0.206$ V

$V_{bi} = 0.88 - 0.206 = 0.674$ V

(i)

$C = (10^{-4})\left[\dfrac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{16})}{2(0.674+1)}\right]^{1/2}$

$= 2.22\times10^{-12}$ F

or $C = 2.22$ pF

(ii)

$C = (10^{-4})\left[\dfrac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{16})}{2(0.6745+5)}\right]^{1/2}$

$= 1.21\times10^{-12}$ F

or $C = 1.21$ pF

9.7

(a) From the figure, $V_{bi} = 0.90$ V

(b) We find

$$\frac{\Delta\left(\frac{1}{C'}\right)^2}{\Delta V_R} = \frac{3\times 10^{15} - 0}{2-(-0.90)} = 1.034\times 10^{15}$$

and

$$1.034\times 10^{15} = \frac{2}{e\in_s N_d}$$

We can then write

$$N_d = \frac{2}{\left(1.6\times 10^{-19}\right)(13.1)\left(8.85\times 10^{-14}\right)\left(1.034\times 10^{15}\right)}$$

or

$$N_d = 1.04\times 10^{16}\ \text{cm}^{-3}$$

(c)

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7\times 10^{17}}{1.04\times 10^{16}}\right)$$

or

$$\phi_n = 0.0986\ \text{V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0986$$

or

$$\phi_{Bn} = 0.9986\ \text{V}$$

9.8

From Figure 9.5, $\phi_{BO} \cong 0.63$ V

(a) $$\phi_n = (0.0259)\ln\left(\frac{2.8\times 10^{19}}{5\times 10^{15}}\right) = 0.224\ \text{V}$$

$$V_{bi} = \phi_{B0} - \phi_n = 0.63 - 0.224 = 0.406\ \text{V}$$

(i) $$x_n = \left[\frac{2(11.7)\left(8.85\times 10^{-14}\right)(0.406+1)}{\left(1.6\times 10^{-19}\right)\left(5\times 10^{15}\right)}\right]^{1/2}$$

$$= 6.033\times 10^{-5}\ \text{cm}$$

or $x_n = 0.6033\ \mu$m

$$\left|\mathrm{E}_{\max}\right| = \frac{\left(1.6\times 10^{-19}\right)\left(5\times 10^{15}\right)\left(6.033\times 10^{-5}\right)}{(11.7)\left(8.85\times 10^{-14}\right)}$$

$$= 4.66\times 10^4\ \text{V/cm}$$

(ii) $$x_n = \left[\frac{2(11.7)\left(8.85\times 10^{-14}\right)(0.406+5)}{\left(1.6\times 10^{-19}\right)\left(5\times 10^{15}\right)}\right]^{1/2}$$

$$= 1.183\times 10^{-4}\ \text{cm}$$

or $x_n = 1.183\ \mu$m

$$\left|\mathrm{E}_{\max}\right| = \frac{\left(1.6\times 10^{-19}\right)\left(5\times 10^{15}\right)\left(1.183\times 10^{-4}\right)}{(11.7)\left(8.85\times 10^{-14}\right)}$$

$$= 9.14\times 10^4\ \text{V/cm}$$

(b)

(i) $$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi\in_s}}$$

$$= \left[\frac{\left(1.6\times 10^{-19}\right)\left(4.66\times 10^4\right)}{4\pi(11.7)\left(8.85\times 10^{-14}\right)}\right]^{1/2}$$

$$= 0.0239\ \text{V}$$

$$x_m = \sqrt{\frac{e}{16\pi\in_s \mathrm{E}}}$$

$$= \left[\frac{\left(1.6\times 10^{-19}\right)}{16\pi(11.7)\left(8.85\times 10^{-14}\right)\left(4.66\times 10^4\right)}\right]^{1/2}$$

or

$$x_m = 2.57\times 10^{-7}\ \text{cm}$$

(ii) $$\Delta\phi = \left[\frac{\left(1.6\times 10^{-19}\right)\left(9.14\times 10^4\right)}{4\pi(11.7)\left(8.85\times 10^{-14}\right)}\right]^{1/2}$$

$$= 0.0335\ \text{V}$$

$$x_m = \left[\frac{1.6\times 10^{-19}}{16\pi(11.7)\left(8.85\times 10^{-14}\right)\left(9.14\times 10^4\right)}\right]^{1/2}$$

$$= 1.83\times 10^{-7}\ \text{cm}$$

9.9

We have

$$-\phi(x) = \frac{-e}{16\pi\in_s x} - \mathrm{E}x$$

or

$$e\phi(x) = \frac{e^2}{16\pi\in_s x} + \mathrm{E}ex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi\in_s x^2} + \mathrm{E}e$$

Solving for x, we find

$$x = x_m = \sqrt{\frac{e}{16\pi\in_s \mathrm{E}}}$$

Substituting this value of $x = x_m$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi\in_s\sqrt{\frac{e}{16\pi\in_s \mathrm{E}}}} + \mathrm{E}\sqrt{\frac{e}{16\pi\in_s \mathrm{E}}}$$

which yields

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in_s}}$$

9.10

From Figure 9.5, $\phi_{BO} \cong 0.88$ V

(a) $\phi_n = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{16}}\right) = 0.0997$ V

$$V_{bi} = \phi_{B0} - \phi_n = 0.88 - 0.0997 \cong 0.780 \text{ V}$$

$$x_n = \left[\frac{2(13.1)(8.85\times10^{-14})(0.780)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

$$= 3.362\times10^{-5} \text{ cm}$$

or $x_n = 0.3362\ \mu$m

$$|\mathrm{E}_{\max}| = \frac{(1.6\times10^{-19})(10^{16})(3.362\times10^{-5})}{(13.1)(8.85\times10^{-14})}$$

$$= 4.64\times10^4 \text{ V/cm}$$

(b) $\Delta\phi = (0.05)(0.88) = 0.044$ V

$$= \sqrt{\frac{e\mathrm{E}}{4\pi \in_s}}$$

$$(0.044)^2 = \frac{(1.6\times10^{-19})\mathrm{E}}{4\pi(13.1)(8.85\times10^{-14})}$$

$$\mathrm{E} = \frac{(0.044)^2(4\pi)(13.1)(8.85\times10^{-14})}{1.6\times10^{-19}}$$

$$= 1.763\times10^5 \text{ V/cm}$$

Now

$$\mathrm{E} = 1.763\times10^5 = \frac{(1.6\times10^{-19})(10^{16})x_n}{(13.1)(8.85\times10^{-14})}$$

$$\Rightarrow x_n = 1.277\times10^{-4} \text{ cm}$$

And

$$x_n^2 = (1.277\times10^{-4})^2$$

$$= \frac{2(13.1)(8.85\times10^{-14})(0.780+V_R)}{(1.6\times10^{-19})(10^{16})}$$

$$\Rightarrow V_R = 10.5 \text{ V}$$

9.11

Plot

9.12

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$$\phi_{BO} = 1.13 \text{ V}$$

(b) We have

$$(E_g - e\phi_O - e\phi_{Bn})$$

$$= \frac{1}{eD_{it}}\sqrt{2e \in_s N_d(\phi_{Bn} - \phi_n)}$$

$$- \frac{\in_i}{eD_{it}\delta}[\phi_m - (\chi + \phi_{Bn})]$$

which becomes

$$e(1.43 - 0.60 - \phi_{Bn})$$

$$= \frac{1}{e\left(\frac{10^{13}}{e}\right)}\left[2(1.6\times10^{-19})(13.1)(8.85\times10^{-14})\right.$$

$$\left.\times(10^{16})(\phi_{Bn} - 0.10)\right]^{1/2}$$

$$- \frac{(8.85\times10^{-14})}{e\left(\frac{10^{13}}{e}\right)(25\times10^{-8})}[5.2 - (4.07 + \phi_{Bn})]$$

or

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221(1.13 - \phi_{Bn})$$

We find

$$\phi_{Bn} = 0.858 \text{ V}$$

(c)

If $\phi_m = 4.5$ V, then

$$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$$

or

$$\phi_{BO} = 0.43 \text{ V}$$

From part (b), we have

$$0.83 - \phi_{Bn}$$

$$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221[4.5 - (4.07 + \phi_{Bn})]$$

We then find

$$\phi_{Bn} = 0.733 \text{ V}$$

With interface states, the barrier height is less sensitive to the metal work function.

9.13

We have that

$$(E_g - e\phi_O - e\phi_{Bn})$$

$$= \frac{1}{eD_{it}}\sqrt{2e \in_s N_d(\phi_{Bn} - \phi_n)}$$

$$- \frac{\in_i}{eD_{it}\delta}[\phi_m - (\chi + \phi_{Bn})]$$

Let $eD_{it} = D'_{it}$ (cm^{-2} eV^{-1})

Then we can write

$$e(1.12-0.230-0.60)$$
$$=\frac{1}{D'_{it}}\left[2\left(1.6\times10^{-19}\right)(11.7)\left(8.85\times10^{-14}\right)\right.$$
$$\left.\times\left(5\times10^{16}\right)(0.60-0.164)\right]^{1/2}$$
$$-\frac{\left(8.85\times10^{-14}\right)}{D'_{it}\left(20\times10^{-8}\right)}\left[4.75-(4.01+0.60)\right]$$

We then find

$$D'_{it}=4.97\times10^{11}\ \text{cm}^{-2}\,\text{eV}^{-1}$$

9.14

(a) $\phi_n=(0.0259)\ln\left(\frac{2.8\times10^{19}}{5\times10^{15}}\right)=0.224\ \text{V}$

(b) $V_{bi}=\phi_{Bn}-\phi_n=0.89-0.224=0.666\ \text{V}$

(c) $J_{sT}=A^*T^2\exp\left(\frac{-e\phi_{Bn}}{kT}\right)$

$$=(120)(300)^2\exp\left(\frac{-0.89}{0.0259}\right)$$
$$J_{sT}=1.29\times10^{-8}\ \text{A/cm}^2$$

(d)

$$V_a=V_t\ln\left(\frac{J}{J_{sT}}\right)=(0.0259)\ln\left(\frac{5}{1.29\times10^{-8}}\right)$$
$$V_a=0.512\ \text{V}$$

9.15

(a) $\phi_{B0}\cong0.63\ \text{V}$

$$J_{sT}=(120)(300)^2\exp\left(\frac{-0.63}{0.0259}\right)$$
$$=2.948\times10^{-4}\ \text{A/cm}^2$$
$$I_{sT}=\left(10^{-4}\right)\left(2.948\times10^{-4}\right)=2.948\times10^{-8}\ \text{A}$$

(i) $V_a=V_t\ln\left(\frac{I}{I_{sT}}\right)$

$$=(0.0259)\ln\left(\frac{10\times10^{-6}}{2.948\times10^{-8}}\right)$$
$$=0.151\ \text{V}$$

(ii) $V_a=(0.0259)\ln\left(\frac{100\times10^{-6}}{2.948\times10^{-8}}\right)$

$$=0.211\ \text{V}$$

(iii) $V_a=(0.0259)\ln\left(\frac{10^{-3}}{2.948\times10^{-8}}\right)$

$$=0.270\ \text{V}$$

(b) $kT=(0.0259)\left(\frac{350}{300}\right)=0.030217\ \text{eV}$

$$I_{sT}=\left(10^{-4}\right)(120)(350)^2\exp\left(\frac{-0.63}{0.030217}\right)$$
$$=1.296\times10^{-6}\ \text{A}$$

(i) $I=I_{sT}\left[\exp\left(\frac{V_a}{V_t}\right)-1\right]$

$$V_a=(0.030217)\ln\left[\frac{10\times10^{-6}}{1.296\times10^{-6}}+1\right]$$
$$=0.0654\ \text{V}$$

(ii) $V_a=(0.030217)\ln\left[\frac{100\times10^{-6}}{1.296\times10^{-6}}+1\right]$

$$=0.1317\ \text{V}$$

(iii) $V_a\cong(0.030217)\ln\left(\frac{10^{-3}}{1.296\times10^{-6}}\right)$

$$=0.201\ \text{V}$$

9.16

(a) $\phi_{Bn}\cong0.88\ \text{V}$

(b) $J_{sT}=(1.12)(300)^2\exp\left(\frac{-0.88}{0.0259}\right)$

$$=1.768\times10^{-10}\ \text{A/cm}^2$$

(c) $V_a=(0.0259)\ln\left(\frac{10}{1.768\times10^{-10}}\right)$

$$=0.641\ \text{V}$$

(d) $\Delta V_a=V_t\ln(2)=(0.0259)\ln(2)$

$$=0.0180\ \text{V}$$

9.17

Plot

9.18

From the figure, $\phi_{Bn}=0.68\ \text{V}$

$$J_{ST}=A^*T^2\exp\left(\frac{-\phi_{Bn}}{V_t}\right)\cdot\exp\left(\frac{\Delta\phi}{V_t}\right)$$
$$=(120)(300)^2\exp\left(\frac{-0.68}{0.0259}\right)\cdot\exp\left(\frac{\Delta\phi}{V_t}\right)$$

or

$$J_{ST}=4.277\times10^{-5}\exp\left(\frac{\Delta\phi}{V_t}\right)$$

We have

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi \in_s}}$$

Now

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times 10^{19}}{10^{16}}\right) = 0.2056 \text{ V}$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.2056 = 0.4744 \text{ V}$$

(a) We find for $V_R = 2$ V,

$$x_d = \left[\frac{2\in_s (V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times 10^{-14})(2.4744)}{(1.6\times 10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.566\times 10^{-4} \text{ cm} = 0.566\ \mu\text{m}$$

Then

$$|\mathrm{E}_{\max}| = \frac{eN_d x_d}{\in_s}$$

$$= \frac{(1.6\times 10^{-19})(10^{16})(0.566\times 10^{-4})}{(11.7)(8.85\times 10^{-14})}$$

or

$$|\mathrm{E}_{\max}| = 8.745\times 10^4 \text{ V/cm}$$

Now

$$\Delta\phi = \left[\frac{(1.6\times 10^{-19})(8.745\times 10^4)}{4\pi(11.7)(8.85\times 10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0328 \text{ V}$$

Then

$$J_{ST1} = (4.277\times 10^{-5})\exp\left(\frac{0.0328}{0.0259}\right)$$

or

$$J_{ST1} = 1.52\times 10^{-4} \text{ A/cm}^2$$

For $A = 10^{-4}$ cm^2, we find

$$I_{R1} = 1.52\times 10^{-8} \text{ A}$$

(b) For $V_R = 4$ V, then

$$x_d = \left[\frac{2(11.7)(8.85\times 10^{-14})(4.4744)}{(1.6\times 10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.761\times 10^{-4} \text{ cm} = 0.761\ \mu\text{m}$$

Also

$$|\mathrm{E}_{\max}| = \frac{(1.6\times 10^{-19})(10^{16})(0.761\times 10^{-4})}{(11.7)(8.85\times 10^{-14})}$$

or

$$|\mathrm{E}_{\max}| = 1.176\times 10^5 \text{ V/cm}$$

and

$$\Delta\phi = \left[\frac{(1.6\times 10^{-19})(1.176\times 10^5)}{4\pi(11.7)(8.85\times 10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.03803 \text{ V}$$

Then

$$J_{ST2} = (4.277\times 10^{-5})\exp\left(\frac{0.03803}{0.0259}\right)$$

or

$$J_{ST2} = 1.86\times 10^{-4} \text{ A/cm}^2$$

Finally,

$$I_{R2} = 1.86\times 10^{-8} \text{ A}$$

9.19

We have that

$$J_{s\to m}^{-} = \int_{E_c}^{\infty} \upsilon_x \, dn$$

The incremental electron concentration is

$$dn = g_c(E) f_F(E) dE$$

where

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}$$

and assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E-E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c} \times \exp\left[\frac{-(E-E_F)}{kT}\right]dE$$

If the energy above E_c is kinetic energy, then

$$\frac{1}{2} m_n^* \upsilon^2 = E - E_c$$

We can then write

$$\sqrt{E-E_c} = \upsilon\sqrt{\frac{m_n^*}{2}}$$

and

$$dE = \frac{1}{2} m_n^* \cdot 2\upsilon d\upsilon = m_n^* \upsilon d\upsilon$$

We can also write

$$E - E_F = (E - E_c) + (E_c - E_F) = \frac{1}{2} m_n^* \upsilon^2 + e\phi_n$$

so that

$$dn = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right) \times \exp\left(\frac{-m_n^*\upsilon^2}{2kT}\right) \cdot 4\pi\upsilon^2 d\upsilon$$

We can write

$$\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2$$

The differential volume element is

$$4\pi\upsilon^2 d\upsilon = d\upsilon_x d\upsilon_y d\upsilon_z$$

The current is due to all x-directed velocities that are greater than υ_{Ox} and for all y- and z- directed velocities. Then

$$J_{s\to m}^- = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right) \times \int_{\upsilon_{Ox}}^{\infty} \upsilon_x \exp\left(\frac{-m_n^*\upsilon_x^2}{2kT}\right) d\upsilon_x \times \int_{-\infty}^{\infty} \exp\left(\frac{-m_n^*\upsilon_y^2}{2kT}\right) d\upsilon_y \times \int_{-\infty}^{\infty} \exp\left(\frac{-m_n^*\upsilon_z^2}{2kT}\right) d\upsilon_z$$

We can write

$$\frac{1}{2} m_n^* \upsilon_{Ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^*\upsilon_x^2}{2kT} = \alpha^2 + \frac{2(V_{bi} - V_a)}{kT}$$

or

$$\upsilon_x^2 = \frac{2kT}{m_n^*}\left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT}\right]$$

Taking the differential, we find

$$\upsilon_x d\upsilon_x = \left(\frac{2kT}{m_n^*}\right) \alpha d\alpha$$

We may note that when $\upsilon_x = \upsilon_{Ox}$, $\alpha = 0$.

We may define other change of variables,

$$\frac{m_n^*\upsilon_y^2}{2kT} = \beta^2 \Rightarrow \upsilon_y = \left(\frac{2kT}{m_n^*}\right)^{1/2} \cdot \beta$$

$$\frac{m_n^*\upsilon_z^2}{2kT} = \gamma^2 \Rightarrow \upsilon_z = \left(\frac{2kT}{m_n^*}\right)^{1/2} \cdot \gamma$$

Substituting the new variables, we have

$$J_{s\to m}^- = 2\left(\frac{m_n^*}{h}\right)^3 \cdot \left(\frac{2kT}{m_n^*}\right)^2 \cdot \exp\left(\frac{-e\phi_n}{kT}\right) \times \exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right] \cdot \int_0^{\infty} \alpha \exp(-\alpha^2) d\alpha \times \int_{-\infty}^{\infty} \exp(-\beta^2) d\beta \cdot \int_{-\infty}^{\infty} \exp(-\gamma^2) d\gamma$$

9.20

For the Schottky diode,

$$0.80\times10^{-3} = (10^{-4})(6\times10^{-8})\exp\left(\frac{V_a}{V_t}\right)$$

(a) $V_a(SB) = (0.0259)\ln\left[\frac{0.80\times10^{-3}}{(10^{-4})(6\times10^{-8})}\right] = 0.4845 \text{ V}$

Then

$$V_a(pn) = 0.4845 + 0.285 = 0.7695 \text{ V}$$

(b) $0.80\times10^{-3} = A_{pn}(10^{-11})\exp\left(\frac{0.7695}{0.0259}\right)$

$$\Rightarrow A_{pn} = 0.998\times10^{-5} \cong 10^{-5} \text{ cm}^2$$

9.21

For the pn junction,

$$I_s = (8\times10^{-4})(8\times10^{-13}) = 6.4\times10^{-16} \text{ A}$$

(a) $V_a = (0.0259)\ln\left(\frac{150\times10^{-6}}{6.4\times10^{-16}}\right) = 0.678 \text{ V}$

(b) $V_a = (0.0259)\ln\left(\frac{700\times10^{-6}}{6.4\times10^{-16}}\right) = 0.718 \text{ V}$

(c) $V_a = (0.0259)\ln\left(\frac{1.2\times10^{-3}}{6.4\times10^{-16}}\right) = 0.732 \text{ V}$

For the Schottky junction,

$$I_{sT} = (8\times10^{-4})(6\times10^{-9}) = 4.8\times10^{-12}\text{ A}$$

(a) $V_a = (0.0259)\ln\left(\dfrac{150\times10^{-6}}{4.8\times10^{-12}}\right)$

$= 0.447\text{ V}$

(b) $V_a = (0.0259)\ln\left(\dfrac{700\times10^{-6}}{4.8\times10^{-12}}\right)$

$= 0.487\text{ V}$

(c) $V_a = (0.0259)\ln\left(\dfrac{1.2\times10^{-3}}{4.8\times10^{-12}}\right)$

$= 0.501\text{ V}$

9.22

(a) (i) $I = 0.80$ mA in each diode

(ii)

$$V_a(SB) = (0.0259)\ln\left[\frac{0.8\times10^{-3}}{(8\times10^{-4})(6\times10^{-9})}\right]$$

$= 0.490\text{ V}$

$$V_a(pn) = (0.0259)\ln\left[\frac{0.8\times10^{-3}}{(8\times10^{-4})(8\times10^{-13})}\right]$$

$= 0.721\text{ V}$

(b) Same voltage across each diode

$$I = 0.8\times10^{-3} = I_{SB} + I_{pn}$$

$$= (8\times10^{-4})(6\times10^{-9})\exp\left(\frac{V_a}{V_t}\right)$$

$$+ (8\times10^{-4})(8\times10^{-13})\exp\left(\frac{V_a}{V_t}\right)$$

$$= (4.8\times10^{-12} + 6.4\times10^{-16})\exp\left(\frac{V_a}{V_t}\right)$$

Then

$$V_a = (0.0259)\ln\left[\frac{0.8\times10^{-3}}{4.8\times10^{-12} + 6.4\times10^{-16}}\right]$$

$$V_a = 0.49032\text{ V}$$

$$I_{SB} = (4.8\times10^{-12})\exp\left(\frac{0.49032}{0.0259}\right)$$

$$\Rightarrow I_{SB} = 0.7998\text{ mA}$$

$$I_{pn} = (6.4\times10^{-16})\exp\left(\frac{0.49032}{0.0259}\right)$$

$$\Rightarrow I_{pn} \cong 0.107\ \mu\text{A}$$

9.23

(a) For $I = 0.8$ mA, we find

$$J = \frac{0.8\times10^{-3}}{7\times10^{-4}} = 1.143\text{ A/cm}^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

For the pn junction diode,

$$V_a = (0.0259)\ln\left(\frac{1.143}{3\times10^{-12}}\right) = 0.6907\text{ V}$$

For the Schottky diode,

$$V_a = (0.0259)\ln\left(\frac{1.143}{4\times10^{-8}}\right) = 0.4447\text{ V}$$

(b) For the pn junction diode,

$$J_S \propto n_i^2 \propto \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\frac{J_S(400)}{J_S(300)} = \left(\frac{400}{300}\right)^3$$

$$\times\exp\left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259}\right]$$

$$= 2.37\exp\left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453}\right]$$

or

$$\frac{J_S(400)}{J_S(300)} = 1.17\times10^5$$

Now

$$I = (7\times10^{-4})(1.17\times10^5)(3\times10^{-12})$$

$$\times\exp\left(\frac{0.6907}{0.03453}\right)$$

or

$$I = 120\text{ mA}$$

For the Schottky diode,

$$J_{ST} \propto T^2 \exp\left(\frac{-e\phi_{BO}}{kT}\right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)} = \left(\frac{400}{300}\right)^2$$

$$\times\exp\left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259}\right]$$

$$= 1.778\exp\left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453}\right]$$

or

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.856\times10^{3}$$

Then

$$I = \left(7\times10^{-4}\right)\left(4.856\times10^{3}\right)\left(4\times10^{-8}\right) \times \exp\left(\frac{0.4447}{0.03453}\right)$$

or

$$I = 53.3\,\text{mA}$$

9.24

Plot

9.25

(a) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-3}} = 0.1\,\Omega$

(b) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-4}} = 1\,\Omega$

(c) $R = \dfrac{R_c}{A} = \dfrac{10^{-4}}{10^{-5}} = 10\,\Omega$

9.26

(a) $R = \dfrac{R_c}{A} = \dfrac{5\times10^{-5}}{10^{-5}} = 5\,\Omega$

(i) $V = IR = (1)(5) = 5\,\text{mV}$

(ii) $V = IR = (0.1)(5) = 0.5\,\text{mV}$

(b) $R = \dfrac{5\times10^{-5}}{10^{-6}} = 50\ \Omega$

(i) $V = IR = (1)(50) = 50\,\text{mV}$

(ii) $V = IR = (0.1)(50) = 5\,\text{mV}$

9.27

$$R_c = \frac{V_t \exp\left(\dfrac{\phi_{Bn}}{V_t}\right)}{A^* T^2}$$

or $\phi_{Bn} = V_t \ln\left[\dfrac{R_c A^* T^2}{V_t}\right]$

(a) $\phi_{Bn} = (0.0259)\ln\left[\dfrac{\left(5\times10^{-5}\right)(120)(300)^2}{0.0259}\right]$

$= 0.258\,\text{V}$

(b) $\phi_{Bn} = (0.0259)\ln\left[\dfrac{\left(5\times10^{-6}\right)(120)(300)^2}{0.0259}\right]$

$= 0.198\,\text{V}$

9.28

(b) We need $\phi_n = \phi_m - \chi = 4.2 - 4.0 = 0.20\,\text{V}$

And

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

or

$$0.20 = (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

which yields

$$N_d = 1.24\times10^{16}\ \text{cm}^{-3}$$

(c)

Barrier height = 0.20 V

9.29

We have that

$$\mathrm{E} = \frac{-eN_d}{\in_s}\left(x_n - x\right)$$

Then

$$\phi = -\int \mathrm{E}dx = \frac{eN_d}{\in_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$, so

$$\phi = \frac{eN_d}{\in_s}\left(x_n \cdot x - \frac{x^2}{2}\right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\in_s}\cdot\frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2\in_s V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right)$$

Now for

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35\,\text{V}$$

we have

$$0.35 = \frac{\left(1.6\times10^{-19}\right)N_d}{(11.7)\left(8.85\times10^{-14}\right)}\left[x_n\left(50\times10^{-8}\right) - \frac{\left(50\times10^{-8}\right)^2}{2}\right]$$

or

$$0.35 = 7.73\times10^{-14}\,N_d\left(x_n - 25\times10^{-8}\right)$$

We have

$$x_n = \left[\frac{2(11.7)\left(8.85\times10^{-14}\right)V_{bi}}{\left(1.6\times10^{-19}\right)N_d}\right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error, we find

$$N_d = 3.5\times10^{18}\text{ cm}^{-3}$$

9.30

(b) $\phi_{BO} = \phi_p = V_t \ln\left(\frac{N_\upsilon}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{16}}\right)$$

or

$$\phi_{BO} = 0.138\text{ V}$$

9.31

Sketches

9.32

Sketches

9.33

Electron affinity rule

$$\Delta E_c = e\left(\chi_n - \chi_p\right)$$

For GaAs, $\chi = 4.07$ and for AlAs, $\chi = 3.5$.

If we assume a linear extrapolation between GaAs and AlAs, then for

$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As} \Rightarrow \chi = 3.90$

Then

$$\left|\Delta E_c\right| = 4.07 - 3.90 = 0.17\text{ eV}$$

9.34

Consider an n-P heterojunction in thermal equilibrium. Poisson's equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\in} = -\frac{d\mathrm{E}}{dx}$$

In the n-region,

$$\frac{d\mathrm{E}_n}{dx} = \frac{\rho(x)}{\in_n} = \frac{eN_{dn}}{\in_n}$$

For uniform doping, we have

$$\mathrm{E}_n = \frac{eN_{dn}x}{\in_n} + C_1$$

The boundary condition is

$\mathrm{E}_n = 0$ at $x = -x_n$, so we obatin

$$C_1 = \frac{eN_{dn}x_n}{\in_n}$$

Then

$$\mathrm{E}_n = \frac{eN_{dn}}{\in_n}\left(x + x_n\right)$$

In the P-region,

$$\frac{d\mathrm{E}_p}{dx} = -\frac{eN_{aP}}{\in_P}$$

which gives

$$\mathrm{E}_P = -\frac{eN_{aP}x}{\in_P} + C_2$$

We have the boundary condition that

$\mathrm{E}_P = 0$ at $x = x_P$, so that

$$C_2 = \frac{eN_{aP}x_P}{\in_P}$$

Then

$$\mathrm{E}_P = \frac{eN_{aP}}{\in_P}\left(x_P - x\right)$$

Assuming zero surface charge density at $x = 0$, the electric flux density D is continuous, so $\in_n \mathrm{E}_n(0) = \in_P \mathrm{E}_P(0)$, which yields

$$N_{dn}x_n = N_{aP}x_P$$

We can determine the electric potential as

$$\phi_n(x) = -\int \mathrm{E}_n\,dx$$

$$= -\left[\frac{eN_{dn}x^2}{2\in_n} + \frac{eN_{dn}x_n x}{\in_n}\right] + C_3$$

Now

$$V_{bin} = \left|\phi_n(0) - \phi_n(-x_n)\right|$$

$$= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{dn}x_n^2}{\in_n}\right]$$

or

$$V_{bin} = \frac{eN_{dn}x_n^2}{2\in_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_P^2}{2\in_P}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{aP}x_P^2}{2\in_P}$$

We can write

$$x_P = x_n\left(\frac{N_{dn}}{N_{aP}}\right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e\in_P N_{dn}N_{aP} + e\in_n N_{dn}^2}{2\in_n\in_P N_{aP}}\right]\cdot x_n^2$$

Solving for x_n, we have

$$x_n = \left[\frac{2\in_n\in_P N_{aP}V_{bi}}{eN_{dn}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

Similarly on the P-side, we have

$$x_P = \left[\frac{2\in_n\in_P N_{dn}V_{bi}}{eN_{aP}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_P$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2\in_n\in_P V_{bi}\left(N_{aP} + N_{dn}\right)}{eN_{dn}N_{aP}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

Chapter 10

10.1

(a) p-type; inversion
(b) p-type; depletion
(c) p-type; accumulation
(d) n-type; inversion

10.2

(a) (i) $\phi_{fp} = V_t \ln\left(\dfrac{N_a}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{7\times10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.3381\,\mathrm{V}$$

$$x_{dT} = \left[\frac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85\times10^{-14})(0.3381)}{(1.6\times10^{-19})(7\times10^{15})}\right]^{1/2}$$

$$= 3.54\times10^{-5}\,\mathrm{cm}$$

or $x_{dT} = 0.354\,\mu\mathrm{m}$

(ii) $\phi_{fp} = (0.0259)\ln\left(\dfrac{3\times10^{16}}{1.5\times10^{10}}\right)$

$$= 0.3758\,\mathrm{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3758)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$$= 1.80\times10^{-5}\,\mathrm{cm}$$

or $x_{dT} = 0.180\,\mu\mathrm{m}$

(b) $kT = (0.0259)\left(\dfrac{350}{300}\right) = 0.03022\,\mathrm{V}$

$$n_i^2 = N_c N_\upsilon \exp\left(\frac{-E_g}{kT}\right)$$

$$= (2.8\times10^{19})(1.04\times10^{19})\left(\frac{350}{300}\right)^3 \times\exp\left(\frac{-1.12}{0.03022}\right)$$

$$= 3.71\times10^{22}$$

so $n_i = 1.93\times10^{11}\,\mathrm{cm}^{-3}$

(i) $\phi_{fp} = (0.03022)\ln\left(\dfrac{7\times10^{15}}{1.93\times10^{11}}\right)$

$$= 0.3173\,\mathrm{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3173)}{(1.6\times10^{-19})(7\times10^{15})}\right]^{1/2}$$

$$= 3.43\times10^{-5}\,\mathrm{cm}$$

or $x_{dT} = 0.343\,\mu\mathrm{m}$

(ii) $\phi_{fp} = (0.03022)\ln\left(\dfrac{3\times10^{16}}{1.93\times10^{11}}\right)$

$$= 0.3613\,\mathrm{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3613)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$$= 1.77\times10^{-5}\,\mathrm{cm}$$

or $x_{dT} = 0.177\,\mu\mathrm{m}$

10.3

(a) $|Q'_{SD}(\max)| = eN_d x_{dT} = eN_d\left[\dfrac{4\in_s \phi_{fn}}{eN_d}\right]^{1/2}$

$$= \left[(eN_d)(4\in_s \phi_{fn})\right]^{1/2}$$

1st approximation: Let $\phi_{fn} = 0.30\,\mathrm{V}$

Then

$$(1.25\times10^{-8})^2 = \left[(1.6\times10^{-19})(N_d)(4)(11.7)(8.85\times10^{-14})(0.30)\right]$$

$\Rightarrow N_d = 7.86\times10^{14}\,\mathrm{cm}^{-3}$

2nd approximation:

$$\phi_{fn} = (0.0259)\ln\left(\frac{7.86\times10^{14}}{1.5\times10^{10}}\right) = 0.2814\,\mathrm{V}$$

Then

$$(1.25\times10^{-8})^2 = \left[(1.6\times10^{-19})(N_d)(4)(11.7)(8.85\times10^{-14})(0.2814)\right]$$

$\Rightarrow N_d = 8.38\times10^{14}\,\mathrm{cm}^{-3}$

(b) $\phi_{fn} = (0.0259)\ln\left(\dfrac{8.38\times10^{14}}{1.5\times10^{10}}\right) = 0.2831\,\mathrm{V}$

$$\phi_s = 2\phi_{fn} = 2(0.2831) = 0.566\,\mathrm{V}$$

10.4

p-type silicon

(a) Aluminum gate

$$\phi_{ms} = \left[\phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)\right]$$

We have

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{6\times10^{15}}{1.5\times10^{10}}\right) = 0.334 \text{ V}$$

Then

$$\phi_{ms} = [3.20 - (3.25 + 0.56 + 0.334)]$$

or

$$\phi_{ms} = -0.944 \text{ V}$$

(b) n^+ polysilicon gate

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp}\right) = -(0.56 + 0.334)$$

or

$$\phi_{ms} = -0.894 \text{ V}$$

(c) p^+ polysilicon gate

$$\phi_{ms} = \left(\frac{E_g}{2e} - \phi_{fp}\right) = (0.56 - 0.334)$$

or

$$\phi_{ms} = +0.226 \text{ V}$$

10.5

$$\phi_{fp} = (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right) = 0.3832 \text{ V}$$

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)$$

$$= 3.20 - (3.25 + 0.56 + 0.3832)$$

$$\phi_{ms} = -0.9932 \text{ V}$$

10.6

(a) $N_d \cong 2\times10^{17} \text{ cm}^{-3}$

(b) Not possible - ϕ_{ms} is always positive.

(c) $N_d \cong 2\times10^{15} \text{ cm}^{-3}$

10.7

From Problem 10.5, $\phi_{ms} = -0.9932$ V

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

(a) $$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}}$$

$$= 1.726\times10^{-7} \text{ F/cm}^2$$

$$V_{FB} = -0.9932 - \frac{(5\times10^{10})(1.6\times10^{-19})}{1.726\times10^{-7}}$$

$$= -1.040 \text{ V}$$

(b) $$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{80\times10^{-8}}$$

$$= 4.314\times10^{-7} \text{ F/cm}^2$$

$$V_{FB} = -0.9932 - \frac{(5\times10^{10})(1.6\times10^{-19})}{4.314\times10^{-7}}$$

$$= -1.012 \text{ V}$$

10.8

(a) $\phi_{ms} \cong -0.42$ V

$V_{FB} = \phi_{ms} = -0.42$ V

(b)

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}} = 1.726\times10^{-7} \text{ F/cm}^2$$

(i) $$\Delta V_{FB} = -\frac{Q'_{ss}}{C_{ox}} = -\frac{(4\times10^{10})(1.6\times10^{-19})}{1.726\times10^{-7}}$$

$$= -0.0371 \text{ V}$$

(ii) $$\Delta V_{FB} = -\frac{(10^{11})(1.6\times10^{-19})}{1.726\times10^{-7}}$$

$$= -0.0927 \text{ V}$$

(c) $V_{FB} = \phi_{ms} = -0.42$ V

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}} = 2.876\times10^{-7} \text{ F/cm}^2$$

(i) $$\Delta V_{FB} = -\frac{(4\times10^{10})(1.6\times10^{-19})}{2.876\times10^{-7}}$$

$$= -0.0223 \text{ V}$$

(ii) $$\Delta V_{FB} = -\frac{(10^{11})(1.6\times10^{-19})}{2.876\times10^{-7}}$$

$$= -0.0556 \text{ V}$$

10.9

$$\phi_{ms} = \phi'_m - \left(\chi' + \frac{E_g}{2e} + \phi_{fp}\right)$$

where

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.365 \text{ V}$$

Then

$$\phi_{ms} = 3.20 - (3.25 + 0.56 + 0.365)$$

or

$$\phi_{ms} = -0.975 \text{ V}$$

Now

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

or

$$Q'_{ss} = (\phi_{ms} - V_{FB})C_{ox}$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{450\times10^{-8}}$$

or

$$C_{ox} = 7.67\times10^{-8} \text{ F/cm}^2$$

So now

$$Q'_{ss} = [-0.975 - (-1)]\cdot(7.67\times10^{-8})$$
$$= 1.92\times10^{-9} \text{ C/cm}^2$$

or

$$\frac{Q'_{ss}}{e} = 1.2\times10^{10} \text{ cm}^{-2}$$

10.10

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3653)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$
$$= 2.174\times10^{-5} \text{ cm}$$

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$
$$= (1.6\times10^{-19})(2\times10^{16})(2.174\times10^{-5})$$
$$= 6.958\times10^{-8} \text{ C/cm}^2$$

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}} = 2.301\times10^{-7} \text{ F/cm}^2$$

$$V_{TN} = \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$
$$= \frac{6.958\times10^{-8} - (7\times10^{10})(1.6\times10^{-19})}{2.301\times10^{-7}} + \phi_{ms} + 2(0.3653)$$
$$= 0.9843 + \phi_{ms}$$

(a) n$^+$ poly gate on p-type: $\phi_{ms} \cong -1.12$ V

$$V_{TN} = 0.9843 - 1.12 = -0.136 \text{ V}$$

(b) p$^+$ poly gate on p-type: $\phi_{ms} \cong +0.28$ V

$$V_{TN} = 0.9843 + 0.28 = +1.26 \text{ V}$$

(c) Al gate on p-type: $\phi_{ms} \cong -0.95$ V

$$V_{TN} = 0.9843 - 0.95 = +0.0343 \text{ V}$$

10.11

$$\phi_{fn} = (0.0259)\ln\left(\frac{3\times10^{15}}{1.5\times10^{10}}\right) = 0.3161 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3161)}{(1.6\times10^{-19})(3\times10^{15})}\right]^{1/2}$$
$$= 5.223\times10^{-5} \text{ cm}$$

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$
$$= (1.6\times10^{-19})(3\times10^{15})(5.223\times10^{-5})$$
$$= 2.507\times10^{-8} \text{ C/cm}^2$$

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}} = 2.301\times10^{-7} \text{ F/cm}^2$$

$$V_{TP} = -\left[\frac{|Q'_{SD}(\max)| + Q'_{ss}}{C_{ox}}\right] + \phi_{ms} - 2\phi_{fn}$$
$$= -\left[\frac{2.507\times10^{-8} + (1.6\times10^{-19})(7\times10^{10})}{2.301\times10^{-7}}\right] + \phi_{ms} - 2(0.3161)$$

$$V_{TP} = -0.7898 + \phi_{ms}$$

(a) n$^+$ poly gate on n-type: $\phi_{ms} \cong -0.41$ V

$$V_{TP} = -0.7898 - 0.41 = -1.20 \text{ V}$$

(b) p$^+$ poly gate on n-type: $\phi_{ms} \cong +1.0$ V

$$V_{TP} = -0.7898 + 1.0 = +0.210 \text{ V}$$

(c) Al gate on n-type: $\phi_{ms} \cong -0.29$ V

$$V_{TP} = -0.7898 - 0.29 = -1.08 \text{ V}$$

10.12

$$\phi_{fp} = (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right) = 0.3294\text{ V}$$

The surface potential is

$$\phi_s = 2\phi_{fp} = 2(0.3294) = 0.659\text{ V}$$

We have

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}} = -0.90\text{ V}$$

Now

$$V_T = \frac{|Q'_{SD}(\max)|}{C_{ox}} + \phi_s + V_{FB}$$

We obtain

$$x_{dT} = \left[\frac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85\times10^{-14})(0.3294)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.413\times10^{-4}\text{ cm}$$

Then

$$|Q'_{SD}(\max)| = (1.6\times10^{-19})(5\times10^{15})(0.413\times10^{-4})$$

or

$$|Q'_{SD}(\max)| = 3.304\times10^{-8}\ \text{C/cm}^2$$

We also find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{400\times10^{-8}}$$

or

$$C_{ox} = 8.629\times10^{-8}\text{ F/cm}^2$$

Then

$$V_T = \frac{3.304\times10^{-8}}{8.629\times10^{-8}} + 0.659 - 0.90$$

or

$$V_T = +0.142\text{ V}$$

10.13

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{220\times10^{-8}}$$

$$= 1.569\times10^{-7}\text{ F/cm}^2$$

$$Q'_{ss} = (1.6\times10^{-19})(4\times10^{10})$$

$$= 6.4\times10^{-9}\text{ C/cm}^2$$

By trial and error, let $N_a = 4\times10^{16}\text{ cm}^{-3}$.

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.3832\text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3832)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

$$= 1.575\times10^{-5}\text{ cm}$$

$$|Q'_{SD}(\max)|$$

$$= (1.6\times10^{-19})(4\times10^{16})(1.575\times10^{-5})$$

$$= 1.008\times10^{-7}\text{ C/cm}^2$$

$$\phi_{ms} \cong -0.94\text{ V}$$

Then

$$V_{TN} = \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{1.008\times10^{-7} - 6.4\times10^{-9}}{1.569\times10^{-7}} - 0.94 + 2(0.3832)$$

Then $V_{TN} = 0.428\text{ V} \cong 0.45\text{ V}$

10.14

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$

$$= 1.9175\times10^{-7}\text{ F/cm}^{-3}$$

$$Q'_{ss} = (1.6\times10^{-19})(4\times10^{10})$$

$$= 6.4\times10^{-9}\text{ C/cm}^2$$

By trial and error, let $N_d = 5\times10^{16}\text{ cm}^{-3}$

Now

$$\phi_{fn} = (0.0259)\ln\left(\frac{5\times10^{16}}{1.5\times10^{10}}\right)$$

$$= 0.3890\text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3890)}{(1.6\times10^{-19})(5\times10^{16})}\right]^{1/2}$$

$$= 1.419\times10^{-5}\text{ cm}$$

$$|Q'_{SD}(\max)|$$

$$= (1.6\times10^{-19})(5\times10^{16})(1.419\times10^{-5})$$

$$= 1.135\times10^{-7}\text{ C/cm}^{-3}$$

$$\phi_{ms} \cong +1.10\text{ V}$$

Then

$$V_{TP}=-\frac{\left(\left|Q'_{SD}(\max)\right|+Q'_{ss}\right)}{C_{ox}}+\phi_{ms}-2\phi_{fn}$$

$$=-\frac{\left(1.135\times10^{-7}+6.4\times10^{-9}\right)}{1.9175\times10^{-7}}$$
$$+1.10-2(0.3890)$$

Then $V_{TP}=-0.303$ V, which is within the specified value.

10.15

We have $C_{ox}=1.569\times10^{-7}\ \text{F/cm}^2$

$Q'_{ss}=6.4\times10^{-9}\ \text{C/cm}^2$

By trial and error, let $N_d=5\times10^{14}\ \text{cm}^{-3}$

Now

$$\phi_{fn}=(0.0259)\ln\left(\frac{5\times10^{14}}{1.5\times10^{10}}\right)$$
$$=0.2697\ \text{V}$$

$$x_{dT}=\left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.2697)}{\left(1.6\times10^{-19}\right)\left(5\times10^{14}\right)}\right]^{1/2}$$
$$=1.182\times10^{-4}\ \text{cm}$$

$\left|Q'_{SD}(\max)\right|$
$$=\left(1.6\times10^{-19}\right)\left(5\times10^{14}\right)\left(1.182\times10^{-4}\right)$$
$$=9.456\times10^{-9}\ \text{C/cm}^2$$

$\phi_{ms}\cong-0.33$ V

Then

$$V_{TP}=-\frac{\left(\left|Q'_{SD}(\max)\right|+Q'_{ss}\right)}{C_{ox}}+\phi_{ms}-2\phi_{fn}$$

$$=-\left(\frac{9.456\times10^{-9}+6.4\times10^{-9}}{1.569\times10^{-7}}\right)$$
$$-0.33-2(0.2697)$$
$$=0.970\ \text{V}$$

Then $V_{TP}=-0.970\ \text{V}\cong-0.975$ V which meets the specification.

10.16

(a) $\phi_{ms}\cong-1.03$ V

$$C_{ox}=\frac{(3.9)\left(8.85\times10^{-14}\right)}{180\times10^{-8}}$$
$$=1.9175\times10^{-7}\ \text{F/cm}^2$$

Now

$$V_{FB}=\phi_{ms}-\frac{Q'_{ss}}{C_{ox}}$$
$$=-1.03-\frac{\left(1.6\times10^{-19}\right)\left(6\times10^{10}\right)}{1.9175\times10^{-7}}$$

$V_{FB}=-1.08$ V

(b) $$\phi_{fp}=(0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)$$
$$=0.2877\ \text{V}$$

$$x_{dT}=\left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.2877)}{\left(1.6\times10^{-19}\right)\left(10^{15}\right)}\right]^{1/2}$$
$$=8.630\times10^{-5}\ \text{cm}$$

$\left|Q'_{SD}(\max)\right|$
$$=\left(1.6\times10^{-19}\right)\left(10^{15}\right)\left(8.630\times10^{-5}\right)$$
$$=1.381\times10^{-8}\ \text{C/cm}^2$$

Now

$$V_{TN}=\frac{\left|Q'_{SD}(\max)\right|}{C_{ox}}+V_{FB}+2\phi_{fp}$$
$$=\frac{1.381\times10^{-8}}{1.9175\times10^{-7}}-1.08+2(0.2877)$$

or $V_{TN}=-0.433$ V

10.17

(a) We have n-type material under the gate, so

$$x_{dT}=t_C=\left[\frac{4\in_s\phi_{fn}}{eN_d}\right]^{1/2}$$

where

$$\phi_{fn}=(0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right)=0.288\ \text{V}$$

Then

$$x_{dT}=\left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.288)}{\left(1.6\times10^{-19}\right)\left(10^{15}\right)}\right]^{1/2}$$

or

$$x_{dT}=t_C=0.863\times10^{-4}\ \text{cm}=0.863\ \mu\text{m}$$

(b)

$$V_T=-\left(\left|Q'_{SD}(\max)\right|+Q'_{ss}\right)\left(\frac{t_{ox}}{\in_{ox}}\right)+\phi_{ms}-2\phi_{fn}$$

For an n^+ polysilicon gate,

$$\phi_{ms}=-\left(\frac{E_g}{2e}-\phi_{fn}\right)=-(0.56-0.288)$$

or

$\phi_{ms} = -0.272$ V

Now

$$|Q'_{SD}(\max)| = \left(1.6\times10^{-19}\right)\left(10^{15}\right)\left(0.863\times10^{-4}\right)$$

or

$$|Q'_{SD}(\max)| = 1.38\times10^{-8} \text{ C/cm}^2$$

We have

$$Q'_{ss} = \left(1.6\times10^{-19}\right)\left(10^{10}\right) = 1.6\times10^{-9} \text{ C/cm}^2$$

We now find

$$V_T = \frac{-\left(1.38\times10^{-8} + 1.6\times10^{-9}\right)}{(3.9)\left(8.85\times10^{-14}\right)}\left(500\times10^{-8}\right) - 0.272 - 2(0.288)$$

or

$$V_T = -1.07 \text{ V}$$

10.18

(b) $\phi_{ms} = \phi'_m - \left(\chi' + \dfrac{E_g}{2e} + \phi_{fp}\right)$

where

$$\phi'_m - \chi' = -0.20 \text{ V}$$

and

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.3473 \text{ V}$$

Then

$$\phi_{ms} = -0.20 - (0.56 + 0.3473)$$

or

$$\phi_{ms} = -1.107 \text{ V}$$

(c) For $Q'_{ss} = 0$

$$V_{TN} = |Q'_{SD}(\max)|\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

We find

$$x_{dT} = \left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.3473)}{\left(1.6\times10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.30\times10^{-4} \text{ cm} = 0.30\ \mu\text{m}$$

Now

$$|Q'_{SD}(\max)| = \left(1.6\times10^{-19}\right)\left(10^{16}\right)\left(0.30\times10^{-4}\right)$$

or

$$|Q'_{SD}(\max)| = 4.797\times10^{-8} \text{ C/cm}^2$$

Then

$$V_T = \frac{\left(4.797\times10^{-8}\right)\left(300\times10^{-8}\right)}{(3.9)\left(8.85\times10^{-14}\right)} - 1.107 + 2(0.3473)$$

or

$$V_T = +0.00455 \text{ V} \cong 0 \text{ V}$$

10.19

Plot

10.20

Plot

10.21

Plot

10.22

Plot

10.23

(a) For $f = 1$ Hz (low freq),

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85\times10^{-14}\right)}{120\times10^{-8}}$$

$$= 2.876\times10^{-7} \text{ F/cm}^2$$

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right)\sqrt{\dfrac{V_t \in_s}{eN_a}}}$$

$$= \frac{(3.9)\left(8.85\times10^{-14}\right)}{120\times10^{-8} + \left(\dfrac{3.9}{11.7}\right)\sqrt{\dfrac{(0.0259)(11.7)\left(8.85\times10^{-14}\right)}{\left(1.6\times10^{-19}\right)\left(10^{16}\right)}}}$$

$$C'_{FB} = 1.346\times10^{-7} \text{ F/cm}^2$$

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\dfrac{\in_{ox}}{\in_s}\right)\cdot x_{dT}}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.3473 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.3473)}{\left(1.6\times10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

$$= 3.00\times10^{-5} \text{ cm}$$

Then

$$C'_{\min} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)(3.00\times10^{-5})}$$

$$= 3.083\times10^{-8} \text{ F/cm}^2$$

$$C'(\text{inv}) = C_{ox} = 2.876\times10^{-7} \text{ F/cm}^2$$

(b) $f = 1\text{MHz}$ (high freq),

$$C_{ox} = 2.876\times10^{-7} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'_{FB} = 1.346\times10^{-7} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'_{\min} = 3.083\times10^{-8} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'(\text{inv}) = C'_{\min} = 3.083\times10^{-8} \text{ F/cm}^2$$

(c) $V_{FB} = \phi_{ms} \cong -1.10 \text{ V}$

$$V_{TN} = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6\times10^{-19})(10^{16})(3.00\times10^{-5})$$

$$= 4.80\times10^{-8} \text{ C/cm}^2$$

$$V_{TN} = \frac{4.80\times10^{-8}}{2.876\times10^{-7}} - 1.10 + 2(0.3473)$$

$$V_{TN} = -0.2385 \text{ V}$$

10.24

(a) $f = 1\text{Hz}$ (low freq),

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}}$$

$$= 2.876\times10^{-7} \text{ F/cm}^2$$

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\sqrt{\frac{V_t \in_s}{eN_a}}}$$

$$= \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259)(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(5\times10^{14})}}}$$

$$C'_{FB} = 4.726\times10^{-8} \text{ F/cm}^2$$

$$C'_{\min} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\cdot x_{dT}}$$

Now

$$\phi_{fn} = (0.0259)\ln\left(\frac{5\times10^{14}}{1.5\times10^{10}}\right) = 0.2697 \text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.2697)}{(1.6\times10^{-19})(5\times10^{14})}\right]^{1/2}$$

$$= 1.182\times10^{-4} \text{ cm}$$

Then

$$C'_{\min} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8} + \left(\frac{3.9}{11.7}\right)(1.182\times10^{-4})}$$

$$= 8.504\times10^{-9} \text{ F/cm}^2$$

$$C'(\text{inv}) = C_{ox} = 2.876\times10^{-7} \text{ F/cm}^2$$

(b) $f = 1\text{MHz}$ (high freq),

$$C_{ox} = 2.876\times10^{-7} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'_{FB} = 4.726\times10^{-8} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'_{\min} = 8.504\times10^{-9} \text{ F/cm}^2 \quad \text{(unchanged)}$$

$$C'(\text{inv}) = C'_{\min} = 8.504\times10^{-9} \text{ F/cm}^2$$

(c) $V_{FB} = \phi_{ms} \cong 0.95 \text{ V}$

$$V_{TP} = -\frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} - 2\phi_{fn}$$

Now

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$

$$= (1.6\times10^{-19})(5\times10^{14})(1.182\times10^{-4})$$

$$= 9.456\times10^{-9} \text{ C/cm}^2$$

Then

$$V_{TP} = -\frac{9.456\times10^{-9}}{2.876\times10^{-7}} + 0.95 - 2(0.2697)$$

$$V_{TP} = +0.378 \text{ V}$$

10.25

The amount of fixed oxide charge at x is

$\rho(x)\Delta x$ C/cm^2

By lever action, the effect of this oxide charge on the flatband voltage is

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\left(\frac{x}{t_{ox}}\right)\rho(x)\Delta x$$

If we add the effect at each point, we must integrate so that

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx$$

10.26

(a) We have $\rho(x) = \dfrac{Q'_{SS}}{\Delta t}$

Then

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx$$

$$\approx -\frac{1}{C_{ox}}\int_{t_{ox}-\Delta t}^{t_{ox}} \left(\frac{t_{ox}}{t_{ox}}\right)\left(\frac{Q'_{SS}}{\Delta t}\right)dx$$

$$= -\frac{1}{C_{ox}}\left(\frac{Q'_{SS}}{\Delta t}\right)\left[t_{ox} - \left(t_{ox} - \Delta t\right)\right] = -\frac{Q'_{SS}}{C_{ox}}$$

or

$$\Delta V_{FB} = -Q'_{SS}\left(\frac{t_{ox}}{\in_{ox}}\right)$$

$$= \frac{-\left(1.6\times10^{-19}\right)\left(8\times10^{10}\right)\left(200\times10^{-8}\right)}{(3.9)\left(8.85\times10^{-14}\right)}$$

or

$$\Delta V_{FB} = -0.0742 \text{ V}$$

(b)

We have

$$\rho(x) = \frac{Q'_{SS}}{t_{ox}} = \frac{\left(1.6\times10^{-19}\right)\left(8\times10^{10}\right)}{200\times10^{-8}}$$

$$= 6.4\times10^{-3} = \rho_O$$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{x\rho(x)}{t_{ox}}dx = -\frac{\rho_O}{C_{ox}t_{ox}}\int_0^{t_{ox}} x\,dx$$

or

$$\Delta V_{FB} = -\frac{\rho_O t_{ox}^2}{2\in_{ox}}$$

$$= \frac{-\left(6.4\times10^{-3}\right)\left(200\times10^{-8}\right)^2}{2(3.9)\left(8.85\times10^{-14}\right)}$$

or

$$\Delta V_{FB} = -0.0371 \text{ V}$$

(c)

$$\rho(x) = \rho_O\left(\frac{x}{t_{ox}}\right)$$

We find

$$\frac{1}{2}t_{ox}\rho_O = Q'_{SS} \Rightarrow \rho_O = \frac{2\left(1.6\times10^{-19}\right)\left(8\times10^{10}\right)}{200\times10^{-8}}$$

or $\rho_O = 1.28\times10^{-2}$

Now

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{t_{ox}} \frac{1}{t_{ox}}\cdot x\cdot\rho_O\left(\frac{x}{t_{ox}}\right)dx$$

$$= -\frac{1}{C_{ox}}\cdot\frac{\rho_O}{t_{ox}^2}\int_0^{t_{ox}} x^2\,dx$$

which becomes

$$\Delta V_{FB} = -\frac{1}{\left(\dfrac{\in_{ox}}{t_{ox}}\right)}\cdot\frac{\rho_O}{t_{ox}^2}\cdot\frac{x^3}{3}\Bigg|_0^{t_{ox}} = -\frac{\rho_O t_{ox}^2}{3\in_{ox}}$$

Then

$$\Delta V_{FB} = \frac{-\left(1.28\times10^{-2}\right)\left(200\times10^{-8}\right)^2}{3(3.9)\left(8.85\times10^{-14}\right)}$$

or $\Delta V_{FB} = -0.0494$ V

10.27

Sketch

10.28

Sketch

10.29

(b)

$$V_{FB} = -V_{bi} = -V_t\ln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$= -(0.0259)\ln\left[\frac{\left(10^{16}\right)\left(10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

or

$$V_{FB} = -0.695 \text{ V}$$

(c) Apply $V_G = -3$ V, $\left|V_{ox}\right| \cong 3$ V

For $V_G = +3$ V,

$$\frac{d\mathrm{E}}{dx} = -\frac{\rho}{\in_s}$$

n-side: $\rho = eN_d$

$$\frac{d\mathrm{E}}{dx} = -\frac{eN_d}{\in_s} \Rightarrow \mathrm{E} = -\frac{eN_d x}{\in_s} + C_1$$

$\mathrm{E}=0$ at $x=-x_n$, then $C_1=-\frac{eN_d x_n}{\in_s}$

so

$$\mathrm{E}=-\frac{eN_d}{\in_s}(x+x_n) \text{ for } -x_n \le x \le 0$$

In the oxide, $\rho=0$, so

$\frac{d\mathrm{E}}{dx}=0 \Rightarrow \mathrm{E}=$ constant. From the boundary conditions, in the oxide

$$\mathrm{E}=-\frac{eN_d x_n}{\in_s}$$

In the p-region,

$$\frac{d\mathrm{E}}{dx}=-\frac{\rho}{\in_s}=+\frac{eN_a}{\in_s} \Rightarrow \mathrm{E}=\frac{eN_a x}{\in_s}+C_2$$

$\mathrm{E}=0$ at $x=(t_{ox}+x_p)$, then

$$\mathrm{E}=-\frac{eN_a}{\in_s}\left[(t_{ox}+x_p)-x\right]$$

At $x=t_{ox}$, $\mathrm{E}=-\frac{eN_a x_p}{\in_s}=-\frac{eN_d x_n}{\in_s}$

So that $N_a x_p=N_d x_n$

Since $N_a=N_d$, then $x_n=x_p$

The potential is

$$\phi=-\int \mathrm{E}dx$$

For zero bias, we can write

$$V_n+V_{ox}+V_p=V_{bi}$$

where V_n, V_{ox}, V_p are the voltage drops across
the n-region, the oxide, and the p-region, respectively. For the oxide:

$$V_{ox}=\mathrm{E}\cdot t_{ox}=\frac{eN_d x_n t_{ox}}{\in_s}$$

For the n-region:

$$V_n(x)=\frac{eN_d}{\in_s}\left(\frac{x^2}{2}+x_n\cdot x\right)+C'$$

Arbitrarily, set $V_n=0$ at $x=-x_n$, then

$C'=\frac{eN_d x_n^2}{2\in_s}$ so that

$$V_n(x)=\frac{eN_d}{2\in_s}(x+x_n)^2$$

At $x=0$, $V_n=\frac{eN_d x_n^2}{2\in_s}$ which is the voltage drop across the n-region. Because of symmetry, $V_n=V_p$. Then for zero bias, we have

$$2V_n+V_{ox}=V_{bi}$$

which can be written as

$$\frac{eN_d x_n^2}{\in_s}+\frac{eN_d x_n t_{ox}}{\in_s}=V_{bi}$$

or

$$x_n^2+x_n t_{ox}-\frac{V_{bi}\in_s}{eN_d}=0$$

Solving for x_n, we obtain

$$x_n=-\frac{t_{ox}}{2}+\sqrt{\left(\frac{t_{ox}}{2}\right)^2+\frac{\in_s V_{bi}}{eN_d}}$$

If we apply a voltage V_G, then replace V_{bi} by $V_{bi}+V_G$, so

$$x_n=x_p=-\frac{t_{ox}}{2}+\sqrt{\left(\frac{t_{ox}}{2}\right)^2+\frac{\in_s(V_{bi}+V_G)}{eN_d}}$$

We find

$$x_n=x_p=-\frac{500\times10^{-8}}{2}+\sqrt{\left(\frac{500\times10^{-8}}{2}\right)^2+\frac{(11.7)(8.85\times10^{-14})(3.695)}{(1.6\times10^{-19})(10^{16})}}$$

which yields

$$x_n=x_p=4.646\times10^{-5}\text{ cm}$$

Now

$$V_{ox}=\frac{eN_d x_n t_{ox}}{\in_s}=\frac{(1.6\times10^{-19})(10^{16})(4.646\times10^{-5})(500\times10^{-8})}{(11.7)(8.85\times10^{-14})}$$

or

$$V_{ox}=0.359\text{ V}$$

We also find

$$V_n=V_p=\frac{eN_d x_n^2}{2\in_s}=\frac{(1.6\times10^{-19})(10^{16})(4.646\times10^{-5})^2}{2(11.7)(8.85\times10^{-14})}$$

or

$$V_n=V_p=1.67\text{ V}$$

10.30

(a) n-type

(b) We have

$$C_{ox}=\frac{200\times10^{-12}}{2\times10^{-3}}=1\times10^{-7}\text{ F/cm}^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{1\times10^{-7}}$$

or

$$t_{ox} = 3.45\times10^{-6}\ \text{cm} = 34.5\ \text{nm} = 345\ \overset{o}{A}$$

(c)

$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$

or

$$-0.80 = -0.50 - \frac{Q'_{ss}}{10^{-7}}$$

which yields

$$Q'_{ss} = 3\times10^{-8}\ \text{C/cm}^2 = 1.875\times10^{11}\ \text{cm}^{-2}$$

(d)

$$C'_{FB} = \frac{\in_{ox}}{t_{ox} + \left(\frac{\in_{ox}}{\in_s}\right)\sqrt{\left(\frac{kT}{e}\right)\left(\frac{\in_s}{eN_d}\right)}}$$

$$= \left[(3.9)(8.85\times10^{-14})\right] \div \left[3.45\times10^{-6} + \left(\frac{3.9}{11.7}\right)\sqrt{\frac{(0.0259)(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(2\times10^{16})}}\right]$$

which yields

$$C'_{FB} = 7.82\times10^{-8}\ \text{F/cm}^2$$

or

$$C_{FB} = 156\ \text{pF}$$

10.31

(a) Point 1: Inversion
2: Threshold
3: Depletion
4: Flat-band
5: Accumulation

10.32

We have

$$Q'_n = -C_{ox}\left[(V_{GS} - V_x) - (\phi_{ms} + 2\phi_{fp})\right] - (Q'_{ss} + Q'_{SD}(\max))$$

Now let $V_x = V_{DS}$, so

$$Q'_n = -C_{ox}\left\{(V_{GS} - V_{DS}) + \left[\frac{|Q'_{SD}(\max)| + Q'_{ss}}{C_{ox}} - (\phi_{ms} + 2\phi_{fp})\right]\right\}$$

For a p-type substrate, $Q'_{SD}(\max)$ is a negative value, so we can write

$$Q'_n = -C_{ox}\left\{(V_{GS} - V_{DS}) - \left[\frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}\right]\right\}$$

Using the definition of threshold voltage V_T, we have

$$Q'_n = -C_{ox}\left[(V_{GS} - V_{DS}) - V_T\right]$$

At saturation

$$V_{DS} = V_{DS}(sat) = V_{GS} - V_T$$

which then makes Q'_n equal to zero at the drain terminal.

10.33

(a) $I_D = \frac{k'_n}{2}\cdot\frac{W}{L}\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$

$$= \left(\frac{0.18}{2}\right)(8)\left[2(0.8-0.4)(0.2) - (0.2)^2\right]$$

$$= 0.0864\ \text{mA}$$

(b) $I_D = \frac{k'_n}{2}\cdot\frac{W}{L}(V_{GS} - V_T)^2$

$$= \left(\frac{0.18}{2}\right)(8)(0.8-0.4)^2$$

$$= 0.1152\ \text{mA}$$

(c) Same as (b), $I_D = 0.1152$ mA

(d) $I_D = \frac{k'_n}{2}\cdot\frac{W}{L}(V_{GS} - V_T)^2$

$$= \left(\frac{0.18}{2}\right)(8)(1.2-0.4)^2$$

$$= 0.4608\ \text{mA}$$

10.34

(a) $I_D = \frac{k'_p}{2}\cdot\frac{W}{L}\left[2(V_{SG} + V_T)V_{SD} - V_{SD}^2\right]$

$$= \left(\frac{0.10}{2}\right)(15)\left[2(0.8-0.4)(0.25) - (0.25)^2\right]$$

$$I_D = 0.103\ \text{mA}$$

(b) $I_D = \frac{k'_p}{2}\cdot\frac{W}{L}(V_{SG} + V_T)^2$

$$= \left(\frac{0.10}{2}\right)(15)(0.8-0.4)^2$$

$$= 0.12\ \text{mA}$$

(c) $I_D = \frac{k'_p}{2} \cdot \frac{W}{L}(V_{SG} + V_T)^2$

$= \left(\frac{0.10}{2}\right)(15)(1.2 - 0.4)^2$

$= 0.48\,\text{mA}$

(d) Same as (c), $I_D = 0.48\,\text{mA}$

10.35

(a) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L}(V_{GS} - V_T)^2$

$1.0 = \left(\frac{0.6}{2}\right)\left(\frac{W}{L}\right)(1.4 - 0.8)^2$

$\Rightarrow \frac{W}{L} = 9.26$

(b) $I_D = \left(\frac{0.6}{2}\right)(9.26)(1.85 - 0.8)^2$

$= 3.06\,\text{mA}$

(c) $I_D = \frac{k'_n}{2} \cdot \frac{W}{L}\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$

$= \left(\frac{0.6}{2}\right)(9.26)\left[2(1.2 - 0.8)(0.15) - (0.15)^2\right]$

$= 0.271\,\text{mA}$

10.36

(a) Assume biased in saturation region

$I_D = \frac{k'_p}{2} \cdot \frac{W}{L}(V_{SG} + V_T)^2$

$0.10 = \left(\frac{0.12}{2}\right)(20)(0 + V_T)^2$

$\Rightarrow V_T = +0.289\,\text{V}$

Note: $V_{SD} = 1.0\,\text{V} > V_{SG} + V_T = 0 + 0.289\,\text{V}$

So the transistor is biased in the saturation region.

(b) $I_D = \left(\frac{0.12}{2}\right)(20)(0.4 + 0.289)^2$

$= 0.570\,\text{mA}$

(c) $I_D = \left(\frac{0.12}{2}\right)(20)\left[2(0.6 + 0.289)(0.15) - (0.15)^2\right]$

or

$I_D = 0.293\,\text{mA}$

10.37

$C_{ox} = \frac{(3.9)(8.85 \times 10^{-14})}{110 \times 10^{-8}} = 3.138 \times 10^{-7}\ \text{F/cm}^2$

$K_n = \frac{\mu_n C_{ox} W}{2L} = \frac{(425)(3.138 \times 10^{-7})(20)}{2(1.2)}$

$= 1.111 \times 10^{-3}\ \text{A/V}^2 = 1.111\ \text{mA/V}^2$

(a) $V_{GS} = 0$, $I_D = 0$

$V_{GS} = 0.6$ V, $V_{DS}(sat) = 0.15$ V,

$I_D(sat) = (1.111)(0.6 - 0.45)^2$

$= 0.025\,\text{mA}$

$V_{GS} = 1.2$ V, $V_{DS}(sat) = 0.75$ V,

$I_D(sat) = (1.111)(1.2 - 0.45)^2$

$= 0.625\,\text{mA}$

$V_{GS} = 1.8$ V, $V_{DS}(sat) = 1.35$ V,

$I_D(sat) = (1.111)(1.8 - 0.45)^2$

$= 2.025\,\text{mA}$

$V_{GS} = 2.4$ V, $V_{DS}(sat) = 1.95$ V,

$I_D(sat) = (1.111)(2.4 - 0.45)^2$

$= 4.225\,\text{mA}$

(c) $I_D = 0$ for $V_{GS} \le 0.45\,\text{V}$

$V_{GS} = 0.6$ V,

$I_D = (1.111)\left[2(0.6 - 0.45)(0.1) - (0.1)^2\right]$

$= 0.0222\,\text{mA}$

$V_{GS} = 1.2$ V,

$I_D = (1.111)\left[2(1.2 - 0.45)(0.1) - (0.1)^2\right]$

$= 0.156\,\text{mA}$

$V_{GS} = 1.8$ V,

$I_D = (1.111)\left[2(1.8 - 0.45)(0.1) - (0.1)^2\right]$

$= 0.289\,\text{mA}$

$V_{GS} = 2.4$ V,

$I_D = (1.111)\left[2(2.4 - 0.45)(0.1) - (0.1)^2\right]$

$= 0.422\,\text{mA}$

10.38

$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{110 \times 10^{-8}}$

$= 3.138 \times 10^{-7}\ \text{F/cm}^2$

$K_p = \frac{\mu_p C_{ox} W}{2L}$

$= \frac{(210)(3.138 \times 10^{-7})(35)}{2(1.2)}$

$= 9.61 \times 10^{-4}\ \text{A/V}^2 = 0.961\ \text{mA/V}^2$

(a) $V_{SG} = 0$, $I_D = 0$

$V_{SG} = 0.6$ V, $V_{SD}(sat) = 0.25$ V

$$I_D(sat) = (0.961)(0.6-0.35)^2$$
$$= 0.060 \text{ mA}$$

$V_{SG} = 1.2$ V, $V_{SD}(sat) = 0.85$ V

$$I_D(sat) = (0.961)(1.2-0.35)^2$$
$$= 0.694 \text{ mA}$$

$V_{SG} = 1.8$ V, $V_{SD}(sat) = 1.45$ V

$$I_D(sat) = (0.961)(1.8-0.35)^2$$
$$= 2.02 \text{ mA}$$

$V_{SG} = 2.4$ V, $V_{SD}(sat) = 2.05$ V

$$I_D(sat) = (0.961)(2.4-0.35)^2$$
$$= 4.04 \text{ mA}$$

(c) $I_D = 0$ for $V_{SG} \le 0.35$ V

$V_{SG} = 0.6$ V

$$I_D = (0.961)\left[2(0.6-0.35)(0.1)-(0.1)^2\right]$$
$$= 0.0384 \text{ mA}$$

$V_{SG} = 1.2$ V

$$I_D = (0.961)\left[2(1.2-0.35)(0.1)-(0.1)^2\right]$$
$$= 0.154 \text{ mA}$$

$V_{SG} = 1.8$ V

$$I_D = (0.961)\left[2(1.8-0.35)(0.1)-(0.1)^2\right]$$
$$= 0.269 \text{ mA}$$

$V_{SG} = 2.4$ V

$$I_D = (0.961)\left[2(2.4-0.35)(0.1)-(0.1)^2\right]$$
$$= 0.384 \text{ mA}$$

10.39

(a) From Problem 10.37, $K_n = 1.111 \text{ mA/V}^2$

For $V_{GS} = -0.8$ V, $I_D = 0$

$V_{GS} = 0$, $V_{DS}(sat) = 0.8$ V

$$I_D(sat) = (1.111)(0+0.8)^2$$
$$= 0.711 \text{ mA}$$

$V_{GS} = +0.8$ V, $V_{DS}(sat) = 1.6$ V

$$I_D(sat) = (1.111)(0.8+0.8)^2$$
$$= 2.84 \text{ mA}$$

$V_{GS} = 1.6$ V, $V_{DS}(sat) = 2.4$ V

$$I_D(sat) = (1.111)(1.6+0.8)^2$$
$$= 6.40 \text{ mA}$$

10.40

Sketch

10.41

Sketch

10.42

We have

$$V_{DS}(sat) = V_{GS} - V_T = V_{DS} - V_T$$

so that

$$V_{DS} = V_{DS}(sat) + V_T$$

Since $V_{DS} > V_{DS}(sat)$, the transistor is always biased in the saturation region. Then

$$I_D = K_n(V_{GS} - V_T)^2$$

where, from Problem 10.37,

$K_n = 1.111 \text{ mA/V}^2$ and $V_T = 0.45$ V

Then

$V_{DS} = V_{GS}$	I_D (mA)
0	0
1	0.336
2	2.67
3	7.22
4	14.0
5	23.0

10.43

From Problem 10.38, $K_p = 0.961 \text{ mA/V}^2$

$$I_D = K_p\left[2(V_{SG} + V_T)(V_{SD}) - V_{SD}^2\right]$$

$$g_d = \left.\frac{\partial I_D}{\partial V_{SD}}\right|_{V_{SD}\to 0} = 2K_p(V_{SG} + V_T)$$

For $V_{SG} \le 0.35$ V, $g_d = 0$

For $V_{SG} > 0.35$ V,

$$g_d = 2(0.961)(V_{SG} - 0.35)$$

For $V_{SG} = 2.4$ V,

$$g_d = 2(0.961)(2.4-0.35)$$
$$= 3.94 \text{ mA/V}$$

10.44

(a) $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$

$= \dfrac{\partial}{\partial V_{GS}} \left\{ K_n \left[2(V_{GS} - V_T)(V_{DS}) - V_{DS}^2 \right] \right\}$

$= K_n (2V_{DS})$

$1.25 = K_n (2)(0.05)$

$\Rightarrow K_n = 12.5 \text{ mA/V}^2$

(b) $I_D = (12.5)\left[2(0.8 - 0.3)(0.05) - (0.05)^2 \right]$

$= 0.594 \text{ mA}$

(c) $I_D = (12.5)(0.8 - 0.3)^2$

$= 3.125 \text{ mA}$

10.45

We find that $V_T \cong 0.2$ V

Now

$$\sqrt{I_D(sat)} = \sqrt{\frac{W\mu_n C_{ox}}{2L}} \cdot (V_{GS} - V_T)$$

where

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{425 \times 10^{-8}}$$

or

$C_{ox} = 8.12 \times 10^{-8} \text{ F/cm}^2$

We are given $W/L = 10$. From the graph, for $V_{GS} = 3$ V, we have

$\sqrt{I_D(sat)} \cong 0.033$,

then

$$0.033 = \sqrt{\frac{W\mu_n C_{ox}}{2L}} \cdot (3 - 0.2)$$

or

$$\frac{W\mu_n C_{ox}}{2L} = 0.139 \times 10^{-3}$$

or

$$\frac{1}{2}(10)\mu_n (8.12 \times 10^{-8}) = 0.139 \times 10^{-3}$$

which yields

$\mu_n = 342 \text{ cm}^2\text{/V-s}$

10.46

(a)

$V_{DS}(sat) = V_{GS} - V_T$

or

$4 = V_{GS} - 0.8 \Rightarrow V_{GS} = 4.8 \text{ V}$

(b)

$I_D(sat) = K_n (V_{GS} - V_T)^2 = K_n V_{DS}^2(sat)$

so

$2 \times 10^{-4} = K_n (4)^2$

which yields

$K_n = 12.5\ \mu \text{A/V}^2$

(c)

$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2 \text{ V}$

so $V_{DS} > V_{DS}(sat)$

$I_D(sat) = (1.25 \times 10^{-5})(2 - 0.8)^2$

or

$I_D(sat) = 18\ \mu \text{A}$

(d)

$V_{DS} < V_{DS}(sat)$

$I_D = K_n \left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2 \right]$

$= (1.25 \times 10^{-5})\left[2(3 - 0.8)(1) - (1)^2 \right]$

or

$I_D = 42.5\ \mu \text{A}$

10.47

(a) $C_{ox} = \dfrac{(3.9)(8.85 \times 10^{-14})}{180 \times 10^{-8}}$

$= 1.9175 \times 10^{-7} \text{ F/cm}^2$

(i) $k_n' = \mu_n C_{ox} = (450)(1.9175 \times 10^{-7})$

$= 8.629 \times 10^{-5} \text{ A/V}^2$

or $k_n' = 86.29\ \mu \text{A/V}^2$

(ii) $I_D(sat) = \left(\dfrac{k_n'}{2} \right)\left(\dfrac{W}{L} \right)(V_{GS} - V_T)^2$

$$0.8 = \left(\frac{0.08629}{2} \right)\left(\frac{W}{L} \right)(2 - 0.4)^2$$

$\Rightarrow \dfrac{W}{L} = 7.24$

(b) (i) $k_p' = \mu_p C_{ox} = (210)(1.9175 \times 10^{-7})$

$= 4.027 \times 10^{-5} \text{ A/V}^2$

or $k_p' = 40.27\ \mu \text{A/V}^2$

(ii) $I_D(sat) = \left(\dfrac{k_p'}{2} \right)\left(\dfrac{W}{L} \right)(V_{SG} + V_T)^2$

$$0.8 = \left(\frac{0.04027}{2} \right)\left(\frac{W}{L} \right)(2 - 0.4)^2$$

$\Rightarrow \dfrac{W}{L} = 15.5$

10.48

From Problem 10.37, $K_n = 1.111\ \text{mA/V}^2$

(a) $g_{mL} = \dfrac{\partial}{\partial V_{GS}}\left\{K_n\left[2(V_{GS}-V_T)(V_{DS})-V_{DS}^2\right]\right\}$

$= K_n(2V_{DS}) = (1.111)(2)(0.1)$

so $g_{mL} = 0.222\ \text{mA/V}$

(b) $g_{ms} = \dfrac{\partial}{\partial V_{GS}}\left\{K_n(V_{GS}-V_T)^2\right\}$

$= 2K_n(V_{GS}-V_T) = 2(1.111)(1.5-0.45)$

so $g_{ms} = 2.33\ \text{mA/V}$

10.49

From Problem 10.38, $K_p = 0.961\ \text{mA/V}^2$

(a) $g_{mL} = \dfrac{\partial}{\partial V_{SG}}\left\{K_p\left[2(V_{SG}+V_T)(V_{SD})-V_{SD}^2\right]\right\}$

$= K_p(2V_{SD}) = (0.961)(2)(0.1)$

or $g_{mL} = 0.192\ \text{mA/V}$

(b) $g_{ms} = \dfrac{\partial}{\partial V_{SG}}\left[K_p(V_{SG}+V_T)^2\right]$

$= 2K_p(V_{SG}+V_T) = 2(0.961)(1.5-0.35)$

or $g_{ms} = 2.21\ \text{mA/V}$

10.50

(a) $\gamma = \dfrac{\sqrt{2e\in_s N_a}}{C_{ox}}$

Now $C_{ox} = \dfrac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}}$

$= 2.301\times10^{-7}\ \text{F/cm}^2$

Then

$$\gamma = \frac{\sqrt{2(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(5\times10^{16})}}{2.301\times10^{-7}}$$

$\gamma = 0.5594\ \text{V}^{1/2}$

(b) $\phi_{fp} = (0.0259)\ln\left(\dfrac{5\times10^{16}}{1.5\times10^{10}}\right) = 0.3890\ \text{V}$

(i) $x_{dT} = \left[\dfrac{4(11.7)(8.85\times10^{-14})(0.3890)}{(1.6\times10^{-19})(5\times10^{16})}\right]^{1/2}$

$= 1.419\times10^{-5}\ \text{cm}$

$|Q'_{SD}(\max)|$

$= (1.6\times10^{-19})(5\times10^{16})(1.419\times10^{-5})$

$= 1.135\times10^{-7}\ \text{C/cm}^2$

$V_{TO} = \dfrac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$

$= \dfrac{1.135\times10^{-7}}{2.301\times10^{-7}} - 0.5 + 2(0.3890)$

$= 0.7713\ \text{V}$

$K_n = \dfrac{\mu_n C_{ox} W}{2L}$

$= \dfrac{(450)(2.301\times10^{-7})(8)}{2(1.2)}$

$= 3.452\times10^{-4}\ \text{A/V}^2$

or $K_n = 0.3452\ \text{mA/V}^2$

For $I_D = 0$, $V_{GS} = V_{TO} = 0.7713\ \text{V}$

For $I_D = 0.5 = (0.3452)(V_{GS}-0.7713)^2$

$\Rightarrow V_{GS} = 1.975\ \text{V}$

(c) (i) For $V_{SB} = 0$, $V_T = V_{TO} = 0.7713\ \text{V}$

(ii) $V_{SB} = 1\ \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389)+1} - \sqrt{2(0.389)}\right]$

$= 0.2525\ \text{V}$

$V_T = 0.7713 + 0.2525 = 1.024\ \text{V}$

(iii) $V_{SB} = 2\ \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389)+2} - \sqrt{2(0.389)}\right]$

$= 0.4390\ \text{V}$

$V_T = 0.7713 + 0.4390 = 1.210\ \text{V}$

(iv) $V_{SB} = 4\ \text{V}$,

$\Delta V_T = (0.5594)\left[\sqrt{2(0.389)+4} - \sqrt{2(0.389)}\right]$

$= 0.7294\ \text{V}$

$V_T = 0.7713 + 0.7294 = 1.501\ \text{V}$

10.51

$\phi_{fp} = (0.0259)\ln\left(\dfrac{10^{16}}{1.5\times10^{10}}\right) = 0.3473\ \text{V}$

$\Delta V_T = \gamma\left[\sqrt{2\phi_{fp}+V_{SB}} - \sqrt{2\phi_{fp}}\right]$

$= (0.12)\left[\sqrt{2(0.3473)+2.5} - \sqrt{2(0.3473)}\right]$

or

$\Delta V_T = 0.114\ \text{V}$

Now $V_T = V_{TO} + \Delta V_T$

$0.5 = V_{TO} + 0.114$

$\Rightarrow V_{TO} = 0.386\ \text{V}$

10.52

(a) $C_{ox} = \dfrac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}}$

$= 1.726\times10^{-7}\ \text{F/cm}^2$

$$\gamma = \frac{\sqrt{2e\in_s N_d}}{C_{ox}}$$

$$= \frac{\sqrt{2(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(5\times10^{15})}}{1.726\times10^{-7}}$$

$\gamma = 0.2358\ \text{V}^{1/2}$

(b) $\phi_{fn} = (0.0259)\ln\left(\dfrac{5\times10^{15}}{1.5\times10^{10}}\right) = 0.3294\ \text{V}$

$$\Delta V_T = -\gamma\left[\sqrt{2\phi_{fn} + V_{BS}} - \sqrt{2\phi_{fn}}\right]$$

$$-0.22 = -(0.2358)\left[\sqrt{2(0.3294) + V_{BS}} - \sqrt{2(0.3294)}\right]$$

$\Rightarrow V_{BS} = 2.39\ \text{V}$

10.53

(a) n^+ poly-to-p-type $\Rightarrow \phi_{ms} = -1.0\ \text{V}$

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = 0.288\ \text{V}$$

also

$$x_{dT} = \left[\frac{4\in_s \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85\times10^{-14})(0.288)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2}$$

or

$x_{dT} = 0.863\times10^{-4}\ \text{cm}$

Now

$|Q'_{SD}(\max)| = (1.6\times10^{-19})(10^{15})(0.863\times10^{-4})$

or

$|Q'_{SD}(\max)| = 1.38\times10^{-8}\ \text{C/cm}^2$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{400\times10^{-8}}$$

or

$C_{ox} = 8.63\times10^{-8}\ \text{F/cm}^2$

We find

$Q'_{ss} = (1.6\times10^{-19})(5\times10^{10}) = 8\times10^{-9}\ \text{C/cm}^2$

Then

$$V_T = \frac{|Q'_{SD}(\max)| - Q'_{ss}}{C_{ox}} + \phi_{ms} + 2\phi_{fp}$$

$$= \left(\frac{1.38\times10^{-8} - 8\times10^{-9}}{8.63\times10^{-8}}\right) - 1.0 + 2(0.288)$$

or

$V_T = -0.357\ \text{V}$

(b) For NMOS, apply V_{SB} and V_T shifts in a positive direction, so for $V_T = 0$, we want $\Delta V_T = +0.357\ \text{V}$.

So

$$\Delta V_T = \frac{\sqrt{2e\in_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

or

$$+0.357 = \frac{\sqrt{2(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{15})}}{8.63\times10^{-8}} \times\left[\sqrt{2(0.288) + V_{SB}} - \sqrt{2(0.288)}\right]$$

or

$0.357 = 0.211\left[\sqrt{0.576 + V_{SB}} - \sqrt{0.576}\right]$

which yields

$V_{SB} = 5.43\ \text{V}$

10.54

Plot

10.55

(a)

$$g_m = \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_T)$$

$$= \frac{W\mu_n \in_{ox}}{Lt_{ox}}(V_{GS} - V_T)$$

$$= \frac{(10)(400)(3.9)(8.85\times10^{-14})}{475\times10^{-8}}(5 - 0.65)$$

or

$g_m = 1.26\ \text{mS}$

Now

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = 0.8 = \frac{1}{1 + g_m r_s}$$

which yields

$$r_s = \frac{1}{g_m}\left(\frac{1}{0.8}-1\right) = \frac{1}{1.26}\left(\frac{1}{0.8}-1\right)$$

or

$$r_s = 0.198\,\text{k}\,\Omega$$

(b) For $V_{GS} = 3$ V, $g_m = 0.683$ mS

Then

$$g'_m = \frac{0.683}{1+(0.683)(0.198)} = 0.602 \text{ mS}$$

or

$$\frac{g'_m}{g_m} = \frac{0.602}{0.683} = 0.88$$

which is a 12% reduction.

10.56

(a) The ideal cutoff frequency for no overlap capacitance is,

$$f_T = \frac{g_m}{2\pi C_{gs}} = \frac{\mu_n (V_{GS}-V_T)}{2\pi L^2}$$

$$= \frac{(400)(4-0.75)}{2\pi(2\times10^{-4})^2}$$

or

$$f_T = 5.17\,\text{GHz}$$

(b) Now

$$f_T = \frac{g_m}{2\pi(C_{gsT}+C_M)}$$

where

$$C_M = C_{gdT}(1+g_m R_L)$$

We find

$$C_{gdT} = C_{ox}(0.75\times10^{-4})(20\times10^{-4})$$

$$= \frac{(3.9)(8.85\times10^{-14})}{500\times10^{-8}} \times(0.75\times10^{-4})(20\times10^{-4})$$

or

$$C_{gdT} = 1.035\times10^{-14}\,\text{F}$$

Also

$$g_m = \frac{W\mu_n C_{ox}}{L}(V_{GS}-V_T)$$

$$= \frac{(20\times10^{-4})(400)(3.9)(8.85\times10^{-14})}{(2\times10^{-4})(500\times10^{-8})} \times(4-0.75)$$

or

$$g_m = 0.8974\times10^{-3}\text{ S}$$

Then

$$C_M = (1.035\times10^{-14}) \times\left[1+(0.8974\times10^{-3})(10\times10^3)\right]$$

or

$$C_M = 1.032\times10^{-13}\text{ F}$$

Now

$$C_{gsT} = C_{ox}(L+0.75\times10^{-4})(W)$$

$$= \frac{(3.9)(8.85\times10^{-14})}{500\times10^{-8}} \times(2\times10^{-4}+0.75\times10^{-4})(20\times10^{-4})$$

or

$$C_{gsT} = 3.797\times10^{-14}\,\text{F}$$

We now find

$$f_T = \frac{g_m}{2\pi(C_{gsT}+C_M)}$$

$$= \frac{0.8974\times10^{-3}}{2\pi(3.797\times10^{-14}+1.032\times10^{-13})}$$

or

$$f_T = 1.01\,\text{GHz}$$

10.57

(a) For the ideal case

$$f_T = \frac{\upsilon_{ds}}{2\pi L} = \frac{4\times10^6}{2\pi(2\times10^{-4})}$$

or

$$f_T = 3.18\,\text{GHz}$$

(b) With overlap capacitance (using the values from Problem 10.56),

$$f_T = \frac{g_m}{2\pi(C_{gdT}+C_M)}$$

We find

$$g_m = WC_{ox}\upsilon_{ds}$$

$$= \frac{(20\times10^{-4})(3.9)(8.85\times10^{-14})(4\times10^6)}{500\times10^{-8}}$$

or

$$g_m = 0.5522\times10^{-3}\text{ S}$$

We have

$$C_M = C_{gdT}(1+g_m R_L)$$

$$= (1.035\times10^{-14}) \times\left[1+(0.5522\times10^{-3})(10\times10^3)\right]$$

or

$$C_M = 6.750\times10^{-14}\text{ F}$$

Then

$$f_T = \frac{0.5522\times10^{-3}}{2\pi\left(3.797\times10^{-14}+6.75\times10^{-14}\right)}$$

or

$$f_T = 0.833\ \text{GHz}$$

Chapter 11

11.1

(a)

$$I_D = 10^{-15}\exp\left(\frac{V_{GS}}{(2.1)V_t}\right)$$

For $V_{GS} = 0.5\text{ V}$,

$$I_D = 10^{-15}\exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$

$$I_D = 9.83\times10^{-12}\text{ A}$$

For $V_{GS} = 0.7\text{ V}$,

$$I_D = 3.88\times10^{-10}\text{ A}$$

For $V_{GS} = 0.9\text{ V}$,

$$I_D = 1.54\times10^{-8}\text{ A}$$

Then the total current is:

$$I_T = I_D\left(10^6\right)$$

For $V_{GS} = 0.5\text{ V}$, $I_T = 9.83\ \mu\text{A}$

For $V_{GS} = 0.7\text{ V}$, $I_T = 0.388\text{ mA}$

For $V_{GS} = 0.9\text{ V}$, $I_T = 15.4\text{ mA}$

(b)

Power: $P = I_T \cdot V_{DD}$

Then

For $V_{GS} = 0.5\text{ V}$, $P = 49.2\ \mu\text{W}$

For $V_{GS} = 0.7\text{ V}$, $P = 1.94\text{ mW}$

For $V_{GS} = 0.9\text{ V}$, $P = 77\text{ mW}$

11.2

$$\frac{I_{D2}}{I_{D1}} = \frac{\exp\left(\dfrac{V_{GS2}}{nV_t}\right)}{\exp\left(\dfrac{V_{GS1}}{nV_t}\right)} = \exp\left[\frac{\left(V_{GS2}-V_{GS1}\right)}{nV_t}\right]$$

$$V_{GS2}-V_{GS1} = nV_t\ln\left(\frac{I_{D2}}{I_{D1}}\right)$$

(a) $V_{GS2}-V_{GS1} = (0.0259)\ln(10)$
$= 0.0596\text{ V}$

(b) $V_{GS2}-V_{GS1} = (1.5)(0.0259)\ln(10)$
$= 0.0895\text{ V}$

(c) $V_{GS2}-V_{GS1} = (2.1)(0.0259)\ln(10)$
$= 0.125\text{ V}$

11.3

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\text{ V}$$

We find that

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)\left(8.85\times10^{-14}\right)}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}}$$

$$= 2.544\times10^{-5}\text{ cm/V}^{1/2}$$

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp}+V_{DS}} - \sqrt{\phi_{fp}+V_{DS}(sat)}\right]$$

(a) $V_{DS}(sat) = V_{GS}-V_T = 1.0-0.4 = 0.6\text{ V}$

$$\Delta L = \left(2.544\times10^{-5}\right)\times\left[\sqrt{0.3653+2}-\sqrt{0.3653+0.6}\right]$$

$$\Delta L = 1.413\times10^{-5}\text{ cm} = 0.1413\ \mu\text{m}$$

(b) $V_{DS}(sat) = V_{GS}-V_T = 1.0-0.4 = 0.6\text{ V}$

$$\Delta L = \left(2.544\times10^{-5}\right)\times\left[\sqrt{0.3653+4}-\sqrt{0.3653+0.6}\right]$$

$$\Delta L = 2.816\times10^{-5}\text{ cm} = 0.2816\ \mu\text{m}$$

(c) $V_{DS}(sat) = V_{GS}-V_T = 2.0-0.4 = 1.6\text{ V}$

$$\Delta L = \left(2.544\times10^{-5}\right)\times\left[\sqrt{0.3653+2}-\sqrt{0.3653+1.6}\right]$$

$$\Delta L = 3.461\times10^{-6}\text{ cm} = 0.0346\ \mu\text{m}$$

(d) $V_{DS}(sat) = V_{GS}-V_T = 2.0-0.4 = 1.6\text{ V}$

$$\Delta L = \left(2.544\times10^{-5}\right)\times\left[\sqrt{0.3653+4}-\sqrt{0.3653+1.6}\right]$$

$$\Delta L = 1.749\times10^{-5}\text{ cm} = 0.1749\ \mu\text{m}$$

11.4

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\text{ V}$$

We find that

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)\left(8.85\times10^{-14}\right)}{\left(1.6\times10^{-19}\right)\left(2\times10^{16}\right)}}$$

$$= 2.544\times10^{-5}\text{ cm/V}^{1/2}$$

$$V_{DS}(sat) = V_{GS}-V_T = 2.0-0.4 = 1.6\text{ V}$$

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp}+V_{DS}} - \sqrt{\phi_{fp}+V_{DS}(sat)}\right]$$

(a) $\Delta L = \left(2.544\times10^{-5}\right)$
$\times\left[\sqrt{0.3653+3}-\sqrt{0.3653+1.6}\right]$
$\Delta L = 1.10\times10^{-5}\text{ cm} = 0.110\ \mu\text{m}$

Now $\dfrac{\Delta L}{L} = 0.10 = \dfrac{0.110}{L}$
$\Rightarrow L = 1.10\ \mu\text{m}$

(b) $\Delta L = \left(2.544\times10^{-5}\right)$
$\times\left[\sqrt{0.3653+5}-\sqrt{0.3653+1.6}\right]$
$\Delta L = 2.326\times10^{-5}\text{ cm} = 0.2326\ \mu\text{m}$

Now $\dfrac{\Delta L}{L} = 0.10 = \dfrac{0.2326}{L}$
$\Rightarrow L = 2.326\ \mu\text{m}$

11.5

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85\times10^{-14}\right)}{120\times10^{-8}}$$
$$= 2.876\times10^{-7}\text{ F/cm}^2$$
$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$
$$= -0.5 - \frac{\left(4\times10^{10}\right)\left(1.6\times10^{-19}\right)}{2.876\times10^{-7}}$$
$$V_{FB} = -0.5223\text{ V}$$

Now
$$V_T = \frac{\left|Q'_{SD}(\max)\right|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

We find
$$\phi_{fp} = (0.0259)\ln\left(\frac{4\times10^{16}}{1.5\times10^{10}}\right) = 0.3832\text{ V}$$
$$x_{dT} = \left[\frac{4(11.7)\left(8.85\times10^{-14}\right)(0.3832)}{\left(1.6\times10^{-19}\right)\left(4\times10^{16}\right)}\right]^{1/2}$$
$$= 1.575\times10^{-5}\text{ cm}$$
$\left|Q'_{SD}(\max)\right|$
$$= \left(1.6\times10^{-19}\right)\left(4\times10^{16}\right)\left(1.575\times10^{-5}\right)$$
$$= 1.008\times10^{-7}\text{ C/cm}^2$$

So
$$V_T = \frac{1.008\times10^{-7}}{2.876\times10^{-7}} - 0.5223 + 2(0.3832)$$
$$= 0.595\text{ V}$$
$$V_{DS}(sat) = V_{GS} - V_T = 1.25 - 0.595 = 0.655\text{ V}$$

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)\left(8.85\times10^{-14}\right)}{\left(1.6\times10^{-19}\right)\left(4\times10^{16}\right)}}$$
$$= 1.799\times10^{-5}\text{ cm/V}^{1/2}$$

(a) $$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

(i) $\Delta L = \left(1.799\times10^{-5}\right)$
$\times\left[\sqrt{0.3832+0.655+1}-\sqrt{0.3832+0.655}\right]$
$\Delta L = 7.35\times10^{-6}\text{ cm} = 0.0735\ \mu\text{m}$

(ii) $\Delta L = \left(1.799\times10^{-5}\right)$
$\times\left[\sqrt{0.3832+0.655+2}-\sqrt{0.3832+0.655}\right]$
$\Delta L = 1.303\times10^{-5}\text{ cm} = 0.1303\ \mu\text{m}$

(iii) $\Delta L = \left(1.799\times10^{-5}\right)$
$\times\left[\sqrt{0.3832+0.655+4}-\sqrt{0.3832+0.655}\right]$
$\Delta L = 2.205\times10^{-5}\text{ cm} = 0.2205\ \mu\text{m}$

(b) $\dfrac{\Delta L}{L} = 0.12 = \dfrac{0.2205}{L}$
$L = 1.84\ \mu\text{m}$

11.6

$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\text{ V}$$
$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)\left(8.85\times10^{-14}\right)}{\left(1.6\times10^{-19}\right)\left(3\times10^{16}\right)}}$$
$$= 2.077\times10^{-5}\text{ cm/V}^{1/2}$$

(a) Ideal,
$$I_D = \frac{k'_n}{2}\cdot\frac{W}{L}\left(V_{GS}-V_T\right)^2$$
$$= \left(\frac{0.05}{2}\right)\left(\frac{15}{0.80}\right)(1.0-0.4)^2$$
$$= 0.16875\text{ mA}$$

(i) $V_{DS}(sat) = 1.0 - 0.4 = 0.6\text{ V}$
$\Delta L = \left(2.077\times10^{-5}\right)$
$\times\left[\sqrt{0.3758+2}-\sqrt{0.3758+0.6}\right]$
$= 1.150\times10^{-5}\text{ cm} = 0.115\ \mu\text{m}$
$$I'_D = \left(\frac{L}{L-\Delta L}\right)\cdot I_D$$

$$=\left(\frac{0.80}{0.80-0.115}\right)(0.16875)$$
$$=0.19708\text{ mA}$$

(ii) $\Delta L=\left(2.077\times10^{-5}\right)$
$$\times\left[\sqrt{0.3758+4}-\sqrt{0.3758+0.6}\right]$$
$$=2.293\times10^{-5}\text{ cm}=0.2293\,\mu\text{m}$$
$$I_D'=\left(\frac{L}{L-\Delta L}\right)\cdot I_D$$
$$=\left(\frac{0.80}{0.80-0.2293}\right)(0.16875)$$
$$=0.23655\text{ mA}$$

(b)
$$r_o=\left(\frac{\Delta I_D'}{\Delta V_{DS}}\right)^{-1}=\left[\frac{(0.23655-0.19708)\times10^{-3}}{4-2}\right]^{-1}$$
$$\Rightarrow r_o=5.07\times10^4\ \Omega=50.7\text{ k}\Omega$$

(c) $V_{DS}(sat)=V_{GS}-V_T=2.0-0.4=1.6$ V

(i) $\Delta L=\left(2.077\times10^{-5}\right)$
$$\times\left[\sqrt{0.3758+2}-\sqrt{0.3758+1.6}\right]$$
$$=2.819\times10^{-6}\text{ cm}=0.02819\,\mu\text{m}$$
$$I_D'=\left(\frac{L}{L-\Delta L}\right)\cdot I_D$$
$$=\left(\frac{0.80}{0.80-0.02819}\right)(0.16875)$$
$$=0.17491\text{mA}$$

(ii) $\Delta L=\left(2.077\times10^{-5}\right)$
$$\times\left[\sqrt{0.3758+4}-\sqrt{0.3758+0.6}\right]$$
$$=1.425\times10^{-5}\text{ cm}=0.1425\,\mu\text{m}$$
$$I_D'=\left(\frac{L}{L-\Delta L}\right)\cdot I_D$$
$$=\left(\frac{0.80}{0.80-0.1425}\right)(0.16875)$$
$$=0.20532\text{ mA}$$
$$r_o=\left(\frac{\Delta I_D'}{\Delta V_{DS}}\right)^{-1}$$
$$=\left[\frac{(0.20532-0.17491)\times10^{-3}}{4-2}\right]^{-1}$$
$$r_o=6.577\times10^4\ \Omega=65.77\text{ k}\Omega$$

11.7

(a)

(i) $I_D=\frac{k_n'}{2}\cdot\frac{W}{L}\left(V_{GS}-V_T\right)^2$
$$=\left(\frac{0.075}{2}\right)(10)(0.8-0.35)^2$$
$$=0.07594\text{ mA}=75.94\,\mu\text{ A}$$

(ii) $I_D'=I_D\left(1+\lambda V_{DS}\right)$
$$=(75.9375)[1+(0.02)(1.5)]$$
$$=78.22\,\mu\text{ A}$$

(iii) $r_o=\frac{1}{\lambda I_D}=\frac{1}{(0.02)(75.94)}$
$$=0.658\text{ M}\Omega=658\text{ k}\Omega$$

(b)

(i) $I_D=\left(\frac{0.075}{2}\right)(10)(1.25-0.35)^2$
$$=0.30375\text{ mA}$$

(ii) $I_D'=(0.30375)[1+(0.02)(1.5)]$
$$=0.3129\text{ mA}$$

(iii) $r_o=\frac{1}{(0.02)(0.30375)}=165\text{ k}\Omega$

11.8

Plot

11.9

(a) Assume $V_{DS}(sat)=1$ V. Then
$$\mathrm{E}_{sat}=\frac{V_{DS}(sat)}{L}$$

We find

L (μ m)	E_{sat} (V/cm)
3	3.33×10^3
1	1×10^4
0.5	2×10^4
0.25	4×10^4
0.13	7.69×10^4

(b)

Assume $\mu_n=500$ cm^2/V-s, we have
$$\upsilon=\mu_n\mathrm{E}_{sat}$$

Then

For $L=3\,\mu$m, $\upsilon=1.67\times10^6$ cm/s

For $L=1\,\mu$m, $\upsilon=5\times10^6$ cm/s

For $L\le0.5\,\mu$m, $\upsilon\cong10^7$ cm/s

11.10

$$k_n' = \mu_n C_{ox} = \frac{\mu_n \in_{ox}}{t_{ox}}$$
$$= \frac{(425)(3.9)(8.85\times10^{-14})}{110\times10^{-8}}$$
$$= 1.334\times10^{-4}\ \text{A/V}^2 = 0.1334\ \text{mA/V}^2$$
$$I_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$$
$$= \left(\frac{0.1334}{2}\right)\left(\frac{20}{1.2}\right)(0.8-0.45)^2$$
$$= 0.1362\ \text{mA}$$
$$\frac{1}{r_o} = \frac{\partial I_D'}{\partial V_{DS}} = \frac{\partial}{\partial V_{DS}}\left[\left(\frac{L}{L-\Delta L}\right)\cdot I_D\right]$$
$$= I_D L\cdot\frac{\partial}{\partial V_{DS}}[L-\Delta L]^{-1}$$
$$= I_D L(-1)(L-\Delta L)^{-2}\left(-\frac{\partial \Delta L}{\partial V_{DS}}\right)$$
$$= \frac{I_D L}{(L-\Delta L)^2}\cdot\frac{\partial \Delta L}{\partial V_{DS}}$$

Now

$$\Delta L = \sqrt{\frac{2\in_s}{eN_a}}\left[\sqrt{\phi_{fp}+V_{DS}} - \sqrt{\phi_{fp}+V_{DS}(sat)}\right]$$
$$\frac{\partial \Delta L}{\partial V_{DS}} = \sqrt{\frac{2\in_s}{eN_a}}\cdot\frac{1}{2}(\phi_{fp}+V_{DS})^{-1/2}$$

We find

$$\sqrt{\frac{2\in_s}{eN_a}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})(3\times10^{16})}}$$
$$= 2.077\times10^{-5}\ \text{cm/V}^{1/2}$$
$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\ \text{V}$$
$$V_{DS} = V_{DS}(sat) + \Delta V_{DS} = V_{GS} - V_T + \Delta V_{DS}$$
$$= 0.8 - 0.45 + 2 = 2.35\ \text{V}$$

Now

$$\frac{\partial \Delta L}{\partial V_{DS}} = \frac{(2.077\times10^{-5})}{2\sqrt{0.3758+2.35}} = 6.290\times10^{-6}\ \text{cm/V}$$
$$\Delta L = (2.077\times10^{-5})\left[\sqrt{0.3758+2.35} - \sqrt{0.3758+0.35}\right]$$
$$= 1.660\times10^{-5}\ \text{cm} = 0.166\ \mu\text{m}$$

(a)

$$\frac{1}{r_o} = \frac{I_D L}{(L-\Delta L)^2}\cdot\frac{\partial \Delta L}{\partial V_{DS}}$$
$$= \frac{(0.1362\times10^{-3})(1.2\times10^{-4})}{[(1.2-0.166)\times10^{-4}]^2}\cdot(6.290\times10^{-6})$$
$$= 9.615\times10^{-6}$$
$$\Rightarrow r_o = 1.04\times10^5\ \Omega = 104\ \text{k}\Omega$$

(b)

$$\frac{1}{r_o} = \frac{(0.1362\times10^{-3})(0.8\times10^{-4})}{[(0.8-0.166)\times10^{-4}]^2}\cdot(6.290\times10^{-6})$$
$$= 1.705\times10^{-5}$$
$$\Rightarrow r_o = 5.865\times10^4\ \Omega = 58.65\ \text{k}\Omega$$

11.11

(a)

$$I_D(sat) = \frac{W\mu_n C_{ox}}{2L}(V_{GS}-V_T)^2$$
$$= \left(\frac{10}{2}\right)(500)(6.9\times10^{-8})(V_{GS}-1)^2$$

or

$$I_D(sat) = 0.173(V_{GS}-1)^2\quad(\text{mA})$$

and

$$\sqrt{I_D(sat)} = \sqrt{0.173}(V_{GS}-1)\quad(\text{mA})^{1/2}$$

(b)

Let $\mu_{eff} = \mu_O\left(\frac{\text{E}_{eff}}{\text{E}_C}\right)^{-1/3}$

Where $\mu_O = 1000\ \text{cm}^2/\text{V-s}$ and

$\text{E}_C = 2.5\times10^4\ \text{V/cm}$

Let $\text{E}_{eff} = \frac{V_{GS}}{t_{ox}}$

We find

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow t_{ox} = \frac{\in_{ox}}{C_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{6.9\times10^{-8}}$$

or

$$t_{ox} = 500\ \mathring{A}$$

Then

V_{GS}	E_{eff}	μ_{eff}	$\sqrt{I_D(sat)}$
1	--	--	0
2	4×10^5	397	0.370
3	6×10^5	347	0.692
4	8×10^5	315	0.989
5	10×10^5	292	1.27

(c) The slope of the variable mobility curve is not constant, but is continually decreasing.

11.12
Plot

11.13

(a) $C_{ox} = \dfrac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}}$

$= 1.726\times10^{-7}$ F/cm^2

$K_n = \dfrac{\mu_n C_{ox} W}{2L}$

$= \dfrac{(475)(1.726\times10^{-7})(10)}{2(1.0)}$

$= 4.10\times10^{-4}$ A/V^2 $= 0.410$ mA/V^2

For $V_{GS} - V_T = 2$ V, $V_{DS}(sat) = 2$ V

(i) $I_D = (0.410)\left[2(2)(0.5) - (0.5)^2\right]$

$= 0.7175$ mA

(ii) $I_D = (0.410)\left[2(2)(1.0) - (1.0)^2\right]$

$= 1.23$ mA

(iii) $I_D = (0.410)\left[2(2)(1.25) - (1.25)^2\right]$

$= 1.409$ mA

(iv) $I_D = (0.410)\left[2(2)(2) - (2)^2\right]$

$= 1.64$ mA

(b) $I_D = WC_{ox}(V_{GS} - V_T)\upsilon_{ds}$

$= (10^{-3})(1.726\times10^{-7})(2)\upsilon_{ds}$

$= (3.452\times10^{-10})\upsilon_{ds}$ A

$= (3.452\times10^{-7})\upsilon_{ds}$ mA

(i) For $V_{DS} = 0.5$ V, $\upsilon_{ds} = \left(\dfrac{0.5}{1.25}\right)(4\times10^6)$

$= 1.6\times10^6$ cm/s

$I_D = (3.452\times10^{-7})(1.6\times10^6)$

$= 0.552$ mA

(ii) For $V_{DS} = 1.0$ V, $\upsilon_{ds} = \left(\dfrac{1.0}{1.25}\right)(4\times10^6)$

$= 3.2\times10^6$ cm/s

$I_D = (3.452\times10^{-7})(3.2\times10^6)$

$= 1.10$ mA

(iii) For $V_{DS} = 1.25$ V, $\upsilon_{ds} = 4\times10^6$ cm/s

$I_D = (3.452\times10^{-7})(4\times10^6)$

$= 1.38$ mA

(iv) For $V_{DS} = 2$ V, $\upsilon_{ds} = 4\times10^6$ cm/s

$I_D = 1.38$ mA

(c) For part (a), $V_{DS}(sat) = 2$ V

For part (b), $V_{DS}(sat) = 1.25$ V

11.14
Plot

11.15

(a) Non-saturation region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right]$$

We have

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} \Rightarrow \frac{C_{ox}}{k}$$

and

$W \Rightarrow kW$, $L \Rightarrow kL$

also

$V_{GS} \Rightarrow kV_{GS}$, $V_{DS} \Rightarrow kV_{DS}$

So

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right) \times\left[2(kV_{GS} - V_T)kV_{DS} - (kV_{DS})^2\right]$$

Then

$I_D \Rightarrow \cong kI_D$

In the saturation region,

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[kV_{GS} - V_T\right]^2$$

Then

$I_D \Rightarrow \cong kI_D$

(b)

$P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P$

11.16

$$I_D(sat) = WC_{ox}(V_{GS} - V_T)\upsilon_{sat}$$

$$\Rightarrow (kW)\left(\frac{C_{ox}}{k}\right)(kV_{GS} - V_T)\upsilon_{sat}$$

or

$$I_D(sat) \Rightarrow \cong kI_D(sat)$$

11.17

(a)

(i) $I_D(\max) = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$

$$= \left(\frac{0.15}{2}\right)\left(\frac{6}{1.2}\right)(3 - 0.45)^2$$

$$2.438\ \text{mA}$$

(ii) Scaled device:

$$V_D = V_{GS} = k(3) = (0.65)(3) = 1.95\ \text{V}$$

$$k_n' = \left(\frac{0.15}{k}\right) = \left(\frac{0.15}{0.65}\right) = 0.2308\ \text{mA/V}^2$$

$$L = k(1.2) = (0.65)(1.2) = 0.78\ \mu\text{m}$$

$$W = k(6) = (0.65)(6) = 3.90\ \mu\text{m}$$

Then

$$I_D(\max) = \left(\frac{0.2308}{2}\right)\left(\frac{3.9}{0.78}\right)(1.95 - 0.45)^2$$

$$= 1.298\ \text{mA}$$

(b) (i) $P(\max) = I_D(\max)V_D = (2.438)(3)$

$$= 7.314\ \text{mW}$$

(ii) $P(\max) = (1.298)(1.95)$

$$= 2.531\ \text{mW}$$

11.18

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}}$$

$$= 2.876\times10^{-7}\ \text{F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{5\times10^{16}}{1.5\times10^{10}}\right) = 0.3890\ \text{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3890)}{(1.6\times10^{-19})(5\times10^{16})}\right]^{1/2}$$

$$= 1.419\times10^{-5}\ \text{cm}$$

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left[\frac{r_j}{L}\left(\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right)\right]$$

$$= -\frac{(1.6\times10^{-19})(5\times10^{16})(1.419\times10^{-5})}{2.876\times10^{-7}}$$

$$\times\left[\frac{0.25}{0.80}\left(\sqrt{1 + \frac{2(0.1419)}{0.25}} - 1\right)\right]$$

$$\Delta V_T = -0.0569\ \text{V}$$

11.19

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{80\times10^{-8}}$$

$$= 4.314\times10^{-7}\ \text{F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\ \text{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3653)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$

$$= 2.174\times10^{-5}\ \text{cm}$$

$$\Delta V_T = -\frac{(1.6\times10^{-19})(2\times10^{16})(2.174\times10^{-5})}{4.314\times10^{-7}}$$

$$\times\left[\frac{0.30}{0.70}\left(\sqrt{1 + \frac{2(0.2174)}{0.30}} - 1\right)\right]$$

$$\Delta V_T = -0.0391\ \text{V}$$

$$V_T = V_{TO} + \Delta V_T$$

$$0.35 = V_{TO} - 0.0391$$

$$\Rightarrow V_{TO} = 0.389\ \text{V}$$

11.20

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}}$$

$$= 1.726\times10^{-7}\ \text{F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\ \text{V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3758)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$$= 1.80\times10^{-5}\ \text{cm}$$

$$\Delta V_T = -0.15$$
$$= -\frac{\left(1.6\times10^{-19}\right)\left(3\times10^{16}\right)\left(1.80\times10^{-5}\right)}{1.726\times10^{-7}}$$
$$\times\left[\frac{0.30}{L}\left(\sqrt{1+\frac{2(0.18)}{0.30}}-1\right)\right]$$
$$0.15 = (0.5006)\left(\frac{0.30}{L}\right)(0.4832)$$
$$\Rightarrow L = 0.484\ \mu\text{m}$$

11.21

We have
$$L' = L-(a+b)$$
and from the geometry
$$(1)\ \ (a+r_j)^2 + x_{dT}^2 = (r_j + x_{dS})^2$$
and
$$(2)\ \ (b+r_j)^2 + x_{dT}^2 = (r_j + x_{dD})^2$$
From (1)
$$(a+r_j)^2 = (r_j + x_{dS})^2 - x_{dT}^2$$
so that
$$a = \sqrt{(r_j + x_{dS})^2 - x_{dT}^2} - r_j$$
which can be written as
$$a = r_j\left[\sqrt{\left(1+\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$
or
$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\left(\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$
Define
$$\alpha^2 = \frac{x_{dS}^2 - x_{dT}^2}{r_j^2}$$
We can then write
$$a = r_j\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2} - 1\right]$$
Similarly from (2), we will have
$$b = r_j\left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2} - 1\right]$$
where
$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is
$$|Q_B'|\cdot L = eN_a x_{dT}\left(\frac{L+L'}{2}\right)$$
or
$$|Q_B'| = eN_a x_{dT}\left(\frac{L+L'}{2L}\right)$$
We can write
$$\frac{L+L'}{2L} = \frac{1}{2}+\frac{L'}{2L} = \frac{1}{2}+\frac{1}{2L}\left[L-(a+b)\right]$$
which is
$$\frac{L+L'}{2L} = 1-\frac{a+b}{2L}$$
Now, $|Q_B'|$ replaces $|Q_{SD}'(\max)|$ in the threshold equation. Then
$$\Delta V_T = \frac{|Q_B'|}{C_{ox}} - \frac{|Q_{SD}'(\max)|}{C_{ox}}$$
$$= \frac{eN_a x_{dT}}{C_{ox}}\left[1-\frac{a+b}{2L}\right] - \frac{eN_a x_{dT}}{C_{ox}}$$
or
$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{(a+b)}{2L}$$
Then substituting, we obtain
$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\cdot\frac{r_j}{2L}\left\{\left[\sqrt{1+\frac{2x_{dS}}{r_j}+\alpha^2}-1\right] + \left[\sqrt{1+\frac{2x_{dD}}{r_j}+\beta^2}-1\right]\right\}$$
Note that if $x_{dS} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

11.22

We have $L' = 0$, so Equation (11.27) becomes
$$\frac{L+L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2}$$
$$= \left\{1-\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$
or
$$\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right] = \frac{1}{2}$$
Then Equation (11.28) is

$$|Q'_B| = eN_a x_{dT}\left(\frac{1}{2}\right)$$

Then change in the threshold voltage is

$$\Delta V_T = \frac{|Q'_B|}{C_{oc}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

which becomes

$$\Delta V_T = -\frac{1}{2}\cdot\frac{eN_a x_{dT}}{C_{ox}}$$

11.23
Plot

11.24
Plot

11.25

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left\{\frac{kr_j}{kL}\left[\sqrt{1+\frac{2kx_{dT}}{kr_j}}-1\right]\right\}$$

or

$$\Delta V_T \Rightarrow k\Delta V_T$$

11.26

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{80\times10^{-8}}$$

$$= 4.314\times10^{-7}\text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{3\times10^{16}}{1.5\times10^{10}}\right) = 0.3758\text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3758)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$$= 1.80\times10^{-5}\text{ cm}$$

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

$$= \frac{(1.6\times10^{-19})(3\times10^{16})(1.80\times10^{-5})^2\left(\frac{\pi}{2}\right)}{(4.314\times10^{-7})(2.2\times10^{-4})}$$

$$\Delta V_T = +0.0257\text{ V}$$

11.27

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}}$$

$$= 2.876\times10^{-7}\text{ F/cm}^2$$

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5\times10^{10}}\right) = 0.3473\text{ V}$$

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3473)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

$$= 3.0\times10^{-5}\text{ cm}$$

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

In this case, $\xi = 1$
So

$$0.045 = \frac{(1.6\times10^{-19})(10^{16})(1.0)(3\times10^{-5})^2}{(2.876\times10^{-7})(W)}$$

$$\Rightarrow W = 1.11\,\mu\text{m}$$

11.28
Plot

11.29

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

Assume that ξ is a constant, then

$$\Delta V_T \Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left(\frac{\xi kx_{dT}}{kW}\right)$$

or

$$\Delta V_T \Rightarrow k\Delta V_T$$

11.30

(a)

(i) $\mathrm{E}_{ox} \cong \dfrac{V_G}{t_{ox}}$

$$6\times10^{6} = \frac{V_G}{200\times10^{-8}}$$

$$\Rightarrow V_G = 12 \text{ V}$$

(ii) $V_G = \dfrac{12}{3} = 4 \text{ V}$

(b)

(i) $6\times10^{6} = \dfrac{V_G}{80\times10^{-8}}$

$$\Rightarrow V_G = 4.8 \text{ V}$$

(ii) $V_G = \dfrac{4.8}{3} = 1.6 \text{ V}$

11.31

(a) $V_G = (8)(3) = 24 = \mathrm{E}_{ox} t_{ox} = (6\times10^{6})(t_{ox})$

$$t_{ox} = 4\times10^{-6} \text{ cm}$$

or $t_{ox} = 40 \text{ nm} = 400 \overset{o}{A}$

(b) $V_G = (12)(3) = 36 = \mathrm{E}_{ox} t_{ox} = (6\times10^{6})(t_{ox})$

$$t_{ox} = 6\times10^{-6} \text{ cm}$$

or $t_{ox} = 60 \text{ nm} = 600 \overset{o}{A}$

11.32

Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18)\log_{10}\left(\frac{I_O}{3\times10^{-9}}\right)$$

and

$$M = \frac{1}{1-\left(\dfrac{V_{CE}}{V_{BO}}\right)^{m}}$$

Let $V_{BO} = 15$ V and $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1-\left(\dfrac{V_{CE}}{15}\right)^{3}}$$

we can write this as

$$1-\left(\frac{V_{CE}}{15}\right)^{3} = \alpha \Rightarrow V_{CE} = 15\cdot\sqrt[3]{1-\alpha}$$

Now

I_D	α	V_{CE}
10^{-8}	0.0941	14.5
10^{-7}	0.274	13.5
10^{-6}	0.454	12.3
10^{-5}	0.634	10.7
10^{-4}	0.814	8.6
10^{-3}	0.994	2.7

11.33

One Debye length is

$$L_D = \left[\frac{\in_s (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85\times10^{-14})(0.0259)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

or

$$L_D = 4.09\times10^{-6} \text{ cm}$$

Six Debye lengths is then

$$6L_D = 0.246\times10^{-4} \text{ cm} = 0.246\ \mu\text{m}$$

From Example 11.5, we have

$x_{dO} = 0.336\ \mu$m, which is the zero-biased source-substrate junction width.

At near punch-through, we will have

$$x_{dO} + 6L_D + x_d = L$$

where x_d is the reverse-biased drain-substrate junction width. Now

$$0.336 + 0.246 + x_d = 1.2$$

or

$$x_d = 0.618\ \mu\text{m}$$

Then, at near punch-through we have

$$x_d = \left[\frac{2\in_s (V_{bi}+V_{DS})}{eN_a}\right]^{1/2}$$

or

$$V_{bi}+V_{DS} = \frac{x_d^2 eN_a}{2\in_s}$$

$$= \frac{(0.618\times10^{-4})^2(1.6\times10^{-19})(10^{16})}{2(11.7)(8.85\times10^{-14})}$$

which yields

$$V_{bi}+V_{DS} = 2.95 \text{ V}$$

From Example 11.5, we have $V_{bi} = 0.874$ V, so

$V_{DS} = 2.08$ V

which is the near punch-through voltage. The ideal punch-through voltage was

$V_{DS} = 4.9$ V

11.34

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(3\times10^{16})}{(1.5\times10^{10})^2}\right] = 0.902 \text{ V}$$

The zero-biased source-substrate junction width is given by

$$x_{dO} = \left[\frac{2\in_s V_{bi}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$x_{dO} = 0.197\times10^{-4}$ cm $= 0.197\ \mu$m

The Debye length is

$$L_D = \left[\frac{\in_s (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85\times10^{-14})(0.0259)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$L_D = 2.36\times10^{-6}$ cm

so that

$6L_D = 0.142\times10^{-4}$ cm $= 0.142\ \mu$m

Now

$x_{dO} + 6L_D + x_d = L$

We have for $V_{DS} = 5$ V,

$$x_d = \left[\frac{2\in_s (V_{bi} + V_{DS})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+5)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$x_d = 0.505\times10^{-4}$ cm $= 0.505\ \mu$m

Then

$L = 0.197 + 0.142 + 0.505$

or

$L = 0.844\ \mu$m

11.35

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2\in_s (V_{bi} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+2)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$x_{dO} = 0.354\times10^{-4}$ cm $= 0.354\ \mu$m

We have $6L_D = 0.142\ \mu$m from the previous problem.

Now

$$x_d = \left[\frac{2\in_s (V_{bi} + V_{DS} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.902+5+2)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

or

$x_d = 0.584\times10^{-4}$ cm $= 0.584\ \mu$m

Then

$L = x_{dO} + 6L_D + x_d$

$= 0.354 + 0.142 + 0.584$

or

$L = 1.08\ \mu$m

11.36

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{120\times10^{-8}} = 2.876\times10^{-7} \text{ F/cm}^2$$

$$|\Delta V_T| = \frac{eD_I}{C_{ox}}$$

Implant acceptor ions for a positive threshold voltage shift.

$$D_I = \frac{\Delta V_T (C_{ox})}{e} = \frac{(0.80)(2.876\times10^{-7})}{1.6\times10^{-19}}$$

$= 1.438\times10^{12}$ cm^{-2}

11.37

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$

$= 1.9175\times10^{-7}$ F/cm^2

$$|\Delta V_T| = \frac{eD_I}{C_{ox}}$$

Implant donor ions for a negative threshold voltage shift.

$$D_I = \frac{|\Delta V_T| C_{ox}}{e} = \frac{(0.60)(1.9175\times10^{-7})}{1.6\times10^{-19}}$$
$$= 7.19\times10^{11}\ \text{cm}^{-2}$$

11.38

(a) $\phi_{ms} \cong -1.08$ V

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{150\times10^{-8}}$$
$$= 2.301\times10^{-7}\ \text{F/cm}^2$$
$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$
$$= -1.08 - \frac{(5\times10^{10})(1.6\times10^{-19})}{2.301\times10^{-7}}$$
$$= -1.115\ \text{V}$$
$$\phi_{fp} = (0.0259)\ln\left(\frac{6\times10^{15}}{1.5\times10^{10}}\right) = 0.3341\ \text{V}$$
$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3341)}{(1.6\times10^{-19})(6\times10^{15})}\right]^{1/2}$$
$$= 3.797\times10^{-5}\ \text{cm}$$
$$|Q'_{SD}(\max)|$$
$$= (1.6\times10^{-19})(6\times10^{15})(3.797\times10^{-5})$$
$$= 3.645\times10^{-8}\ \text{C/cm}^2$$
$$V_{TO} = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$
$$= \frac{3.645\times10^{-8}}{2.301\times10^{-7}} - 1.115 + 2(0.3341)$$
$$V_{TO} = -0.2884\ \text{V}$$

(b) For a positive threshold voltage shift, add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = 0.50 - (-0.2884)$$
$$= 0.788\ \text{V}$$

Then

$$D_I = \frac{(\Delta V_T) C_{ox}}{e} = \frac{(0.788)(2.301\times10^{-7})}{1.6\times10^{-19}}$$
$$= 1.13\times10^{12}\ \text{cm}^{-2}$$

11.39

(a) $\phi_{ms} \cong +1.08$ V

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{180\times10^{-8}}$$
$$= 1.9175\times10^{-7}\ \text{F/cm}^2$$
$$V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{ox}}$$
$$= +1.08 - \frac{(10^{11})(1.6\times10^{-19})}{1.9175\times10^{-7}}$$
$$= +0.9966\ \text{V}$$
$$\phi_{fn} = (0.0259)\ln\left(\frac{2\times10^{16}}{1.5\times10^{10}}\right) = 0.3653\ \text{V}$$
$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3653)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$
$$= 2.1744\times10^{-5}\ \text{cm}$$
$$|Q'_{SD}(\max)|$$
$$= (1.6\times10^{-19})(2\times10^{16})(2.1744\times10^{-5})$$
$$= 6.958\times10^{-8}\ \text{C/cm}^2$$
$$V_{TO} = -\frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} - 2\phi_{fn}$$
$$= -\frac{6.958\times10^{-8}}{1.9175\times10^{-7}} + 0.9966 - 2(0.3653)$$
$$V_{TO} = -0.0969\ \text{V}$$

(b) For a negative threshold voltage shift, add donor ions.

$$\Delta V_T = V_T - V_{TO} = -0.40 - (-0.0969)$$
$$= -0.3031\ \text{V}$$

Then $D_I = \frac{|\Delta V_T| C_{ox}}{e}$

$$= \frac{(0.3031)(1.9175\times10^{-7})}{1.6\times10^{-19}}$$
$$= 3.63\times10^{11}\ \text{cm}^{-2}$$

11.40

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{4\times10^{15}}{1.5\times10^{10}}\right) = 0.3236$ V

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{80\times10^{-8}}$$
$$= 4.314\times10^{-7}\ \text{F/cm}^2$$
$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.3236)}{(1.6\times10^{-19})(4\times10^{15})}\right]^{1/2}$$
$$= 4.576\times10^{-5}\ \text{cm}$$
$$|Q'_{SD}(\max)|$$
$$= (1.6\times10^{-19})(4\times10^{15})(4.576\times10^{-5})$$
$$= 2.929\times10^{-8}\ \text{C/cm}^2$$

$$V_{TO} = \frac{|Q'_{SD}(\max)|}{C_{ox}} + V_{FB} + 2\phi_{fp}$$

$$= \frac{2.929\times10^{-8}}{4.314\times10^{-7}} - 1.25 + 2(0.3236)$$

$$= -0.5349 \text{ V}$$

(b) For a positive threshold voltage shift, add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = 0.40 - (-0.5349)$$

$$= 0.9349 \text{ V}$$

Then $D_I = \dfrac{(\Delta V_T)C_{ox}}{e}$

$$= \frac{(0.9349)(4.314\times10^{-7})}{1.6\times10^{-19}}$$

$$= 2.52\times10^{12} \text{ cm}^{-2}$$

(c) Add acceptor ions.

$$\Delta V_T = V_T - V_{TO} = -0.40 - (-0.5349)$$

$$= 0.1349 \text{ V}$$

Then $D_I = \dfrac{(0.1349)(4.314\times10^{-7})}{1.6\times10^{-19}}$

$$= 3.64\times10^{11} \text{ cm}^{-2}$$

11.41

The total space charge width is greater than x_i, so from Chapter 10

$$\Delta V_T = \frac{\sqrt{2e\in_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{14}}{1.5\times10^{10}}\right) = 0.228 \text{ V}$$

and

$$C_{ox} = \frac{(3.9)(8.85\times10^{-14})}{500\times10^{-8}}$$

$$= 6.90\times10^{-8} \text{ F/cm}^2$$

Then

$$\Delta V_T = \frac{\left[2(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{14})\right]^{1/2}}{6.90\times10^{-8}} \times\left[\sqrt{2(0.228)+V_{SB}} - \sqrt{2(0.228)}\right]$$

or

$$\Delta V_T = 0.0834\left[\sqrt{0.456+V_{SB}} - \sqrt{0.456}\right]$$

Then

V_{SB} (V)	ΔV_T (V)
1	0.0443
3	0.0987
5	0.1385

11.42

(a)

$$\phi_{fn} = (0.0259)\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right) = 0.407 \text{ V}$$

and

$$x_{dT} = \left[\frac{4(11.7)(8.85\times10^{-14})(0.407)}{(1.6\times10^{-19})(10^{17})}\right]^{1/2}$$

$$= 1.026\times10^{-5} \text{ cm}$$

n^+ poly on n-type $\phi_{ms} = -0.32$ V

We have

$$|Q'_{SD}(\max)| = (1.6\times10^{-19})(10^{17})(1.026\times10^{-5})$$

$$= 1.64\times10^{-7} \text{ C/cm}^2$$

Now

$$V_{TP} = \left[-1.64\times10^{-7} - (1.6\times10^{-19})(5\times10^{10})\right] \times\frac{80\times10^{-8}}{(3.9)(8.85\times10^{-14})} - 0.32 - 2(0.407)$$

or

$$V_{TP} = -1.53 \text{ V} \quad \text{(Enhancement PMOS)}$$

(b) For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions.

$$\Delta V_T = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_I = \frac{(1.53)(3.9)(8.85\times10^{-14})}{(80\times10^{-8})(1.6\times10^{-19})}$$

or

$$D_I = 4.13\times10^{12} \text{ cm}^{-2}$$

11.43

The areal density of generated holes is

$$=\left(8\times10^{12}\right)\left(10^{5}\right)\left(750\times10^{-8}\right)=6\times10^{12}\ \text{cm}^{-2}$$

The equivalent surface charge trapped is

$$=(0.10)\left(6\times10^{12}\right)=6\times10^{11}\ \text{cm}^{-2}$$

Then

$$\Delta V_T=-\frac{Q'_{ss}}{C_{ox}}$$

$$=-\frac{\left(6\times10^{11}\right)\left(1.6\times10^{-19}\right)\left(750\times10^{-8}\right)}{(3.9)\left(8.85\times10^{-14}\right)}$$

or

$$\Delta V_T=-2.09\ \text{V}$$

11.44

The areal density of generated holes is $6\times10^{12}\ \text{cm}^{-2}$. Now

$$C_{ox}=\frac{\epsilon_{ox}}{t_{ox}}=\frac{(3.9)\left(8.85\times10^{-14}\right)}{750\times10^{-8}}$$

$$=4.6\times10^{-8}\ \text{F/cm}^2$$

Then

$$\Delta V_T=-\frac{Q'_{ss}}{C_{ox}}=-\frac{\left(6\times10^{12}\right)(x)\left(1.6\times10^{-19}\right)}{4.6\times10^{-8}}$$

where x is the fraction of holes that may be trapped. For $\Delta V_T=-0.50$ V we find

$$x=0.024\Rightarrow x=2.4\%$$

11.45

We have the areal density of generated holes as

$$=(g)(\gamma)(t_{ox})$$

where g is the generation rate and γ is the radiation dose. The equivalent charge trapped is

$$=xg\gamma t_{ox}$$

where x is the fraction of generated holes trapped.

Then

$$\Delta V_T=-\frac{Q'_{ss}}{C_{ox}}=-\frac{exg\gamma t_{ox}}{\left(\epsilon_{ox}/t_{ox}\right)}=-\left(\frac{exg\gamma}{\epsilon_{ox}}\right)(t_{ox})^2$$

or

$$\Delta V_T\propto-(t_{ox})^2$$

Chapter 12

12.1
Sketch

12.2
Sketch

12.3

(a) $I_S = \dfrac{eD_n A_{BE} n_{B0}}{x_B}$

$= \dfrac{(1.6\times10^{-19})(18)(5\times10^{-5})(4\times10^{3})}{0.80\times10^{-4}}$

$= 7.2\times10^{-15}$ A

(b) $I_C = I_S \exp\left(\dfrac{\upsilon_{BE}}{V_t}\right)$

(i) $I_C = (7.2\times10^{-15})\exp\left(\dfrac{0.58}{0.0259}\right)$

$= 3.827\times10^{-5}\text{ A} = 38.27\ \mu\text{A}$

(ii) $I_C = (7.2\times10^{-15})\exp\left(\dfrac{0.65}{0.0259}\right)$

$= 5.710\times10^{-4}\text{ A} = 0.571\text{ mA}$

(iii) $I_C = (7.2\times10^{-15})\exp\left(\dfrac{0.72}{0.0259}\right)$

$= 8.519\times10^{-3}\text{ A} = 8.519\text{ mA}$

12.4

(a) $|i_C| = \dfrac{eD_n A_{BE}}{x_B}\cdot n_{B0}\cdot\exp\left(\dfrac{\upsilon_{BE}}{V_t}\right)$

$2\times10^{-3} = \dfrac{(1.6\times10^{-19})(22)A_{BE}}{0.80\times10^{-4}}$

$\times(2\times10^{4})\exp\left(\dfrac{0.60}{0.0259}\right)$

$\Rightarrow A_{BE} = 1.975\times10^{-4}\text{ cm}^2$

(b) $5\times10^{-3} = \dfrac{(1.6\times10^{-19})(22)(1.975\times10^{-4})}{0.80\times10^{-4}}$

$\times(2\times10^{4})\exp\left(\dfrac{\upsilon_{BE}}{0.0259}\right)$

$5\times10^{-3} = (1.738\times10^{-13})\exp\left(\dfrac{\upsilon_{BE}}{0.0259}\right)$

Then

$\upsilon_{BE} = (0.0259)\ln\left(\dfrac{5\times10^{-3}}{1.738\times10^{-13}}\right)$

$\Rightarrow \upsilon_{BE} = 0.6237$ V

12.5

(a) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9850}{1-0.9850} = 65.7$

(b)

(i) For $I_C = 38.27\ \mu$A,

$I_B = \dfrac{I_C}{\beta} = \dfrac{38.27}{65.67} = 0.5828\ \mu\text{A}$

$I_E = \dfrac{I_C}{\alpha} = \dfrac{38.27}{0.9850} = 38.85\ \mu\text{A}$

(ii) For $I_C = 0.571$ mA,

$I_B = \dfrac{0.571}{65.67} = 0.008695\text{ mA}$

$= 8.695\ \mu\text{A}$

$I_E = \dfrac{0.571}{0.9850} = 0.5797\text{ mA}$

(iii) For $I_C = 8.519$ mA,

$I_B = \dfrac{8.519}{65.67} = 0.1297\text{ mA}$

$I_E = \dfrac{8.519}{0.9850} = 8.649\text{ mA}$

(c) $\beta = \dfrac{0.9940}{1-0.9940} = 165.7$

(i) For $I_C = 38.27\ \mu$A,

$I_B = \dfrac{38.27}{165.7} = 0.2310\ \mu\text{A}$

$I_E = \dfrac{38.27}{0.9940} = 38.50\ \mu\text{A}$

(ii) For $I_C = 0.571$ mA,

$I_B = \dfrac{0.571}{165.7} = 0.003446\text{ mA}$

$= 3.446\ \mu\text{A}$

$I_E = \dfrac{0.571}{0.9940} = 0.5744\text{ mA}$

(iii) For $I_C = 8.519$ mA,

$$I_B = \frac{8.519}{165.7} = 0.05141 \text{ mA}$$
$$= 51.41\ \mu\text{A}$$
$$I_E = \frac{8.519}{0.9940} = 8.570 \text{ mA}$$

12.6

(a) $\beta = \dfrac{I_C}{I_B} = \dfrac{0.625}{0.0042} = 148.8$

$$\alpha = \frac{\beta}{1+\beta} = \frac{148.8}{149.8} = 0.9933$$
$$I_E = \frac{I_C}{\alpha} = \frac{0.625}{0.9933} = 0.6292 \text{ mA}$$

(b) $\alpha = \dfrac{I_C}{I_E} = \dfrac{1.254}{1.273} = 0.9851$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9851}{1-0.9851} = 66.0$$
$$I_B = \frac{I_C}{\beta} = \frac{1.254}{66} = 0.0190 \text{ mA}$$
$$= 19.0\ \mu\text{A}$$

(c) $\alpha = \dfrac{150}{151} = 0.99338$

$$I_C = \beta I_B = (150)(0.065) = 9.75\ \mu\text{A}$$
$$I_E = (1+\beta)I_B = (151)(0.065) = 9.815\ \mu\text{A}$$

12.7

(c) For $i_B = 0.05$ mA,

$$i_C = \beta i_B = (100)(0.05)$$
or
$$i_C = 5 \text{ mA}$$
We have
$$\upsilon_{CE} = V_{CC} - i_C R = 10 - (5)(1)$$
or
$$\upsilon_{CE} = 5 \text{ V}$$

12.8

(a) For $V_{CE} = 3$ V, $I_C = 0$

(i) $V_{CE} = V_{CC} - I_C R_C$

$0.2 = 3 - I_C(25)$, $I_C = 0.112$ mA

(ii) For $V_{CB} = 0 \Rightarrow V_{CE} = V_{BE} = 0.65$ V

$0.65 = 3 - I_C(25)$, $I_C = 0.094$ mA

(b) For $V_{CE} = 3$ V, $I_C = 0$

(i) $V_{CE} = V_{CC} - I_C R_C$

$0.2 = 3 - I_C(10)$, $I_C = 0.28$ mA

(ii) For $V_{CB} = 0 \Rightarrow V_{CE} = V_{BE} = 0.65$ V

$0.65 = 3 - I_C(10)$, $I_C = 0.235$ mA

12.9

(a) $p_{E0} = \dfrac{n_i^2}{N_E} = \dfrac{(1.5\times10^{10})^2}{8\times10^{17}}$

$$= 2.8125\times10^{2}\ \text{cm}^{-3}$$
$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{2\times10^{16}}$$
$$= 1.125\times10^{4}\ \text{cm}^{-3}$$
$$p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{15}}$$
$$= 2.25\times10^{5}\ \text{cm}^{-3}$$

(b) $n_B(0) = n_{B0} \exp\left(\dfrac{V_{BE}}{V_t}\right)$

$$= (1.125\times10^{4})\exp\left(\frac{0.640}{0.0259}\right)$$
$$= 6.064\times10^{14}\ \text{cm}^{-3}$$
$$p_E(0) = p_{E0} \exp\left(\frac{V_{BE}}{V_t}\right)$$
$$= (2.8125\times10^{2})\exp\left(\frac{0.640}{0.0259}\right)$$
$$= 1.516\times10^{13}\ \text{cm}^{-3}$$

12.10

(a) $n_{E0} = \dfrac{n_i^2}{N_E} = \dfrac{(1.5\times10^{10})^2}{5\times10^{17}}$

$$= 4.5\times10^{2}\ \text{cm}^{-3}$$
$$p_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{16}}$$
$$= 2.25\times10^{4}\ \text{cm}^{-3}$$
$$n_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{15}}$$
$$= 2.25\times10^{5}\ \text{cm}^{-3}$$

(b) $p_B(0) = p_{B0} \exp\left(\frac{V_{EB}}{V_t}\right)$

$= (2.25\times10^4)\exp\left(\frac{0.615}{0.0259}\right)$

$= 4.62\times10^{14}\text{ cm}^{-3}$

$n_E(0) = n_{E0} \exp\left(\frac{V_{EB}}{V_t}\right)$

$= (4.5\times10^2)\exp\left(\frac{0.615}{0.0259}\right)$

$= 9.24\times10^{12}\text{ cm}^{-3}$

12.11

(a) $n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{2\times10^{16}}$

$= 1.125\times10^4\text{ cm}^{-3}$

Now

$n_B(0) = (0.1)(N_B) = 2\times10^{15}\text{ cm}^{-3}$

$= n_{B0}\exp\left(\frac{V_{BE}}{V_t}\right)$

Then $V_{BE} = (0.0259)\ln\left(\frac{2\times10^{15}}{1.125\times10^4}\right)$

$= 0.6709\text{ V}$

(b) $p_{E0} = \frac{n_i^2}{N_E} = \frac{(1.5\times10^{10})^2}{8\times10^{17}}$

$= 2.8125\times10^2\text{ cm}^{-3}$

$p_E(0) = p_{E0}\exp\left(\frac{V_{BE}}{V_t}\right) +$

$= (2.8125\times10^2)\exp\left(\frac{0.6709}{0.0259}\right)$

$= 5.0\times10^{13}\text{ cm}^{-3}$

12.12

We have

$$\frac{d(\delta n_B)}{dx} = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right.$$

$$\left.\times\left(\frac{-1}{L_B}\right)\cosh\left(\frac{x_B-x}{L_B}\right)-\frac{1}{L_B}\cosh\left(\frac{x}{L_B}\right)\right\}$$

At $x = 0$,

$$\left.\frac{d(\delta n_B)}{dx}\right|_{x=0} = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right.$$

$$\left.\times\cosh\left(\frac{x_B}{L_B}\right)+1\right\}$$

At $x = x_B$,

$$\left.\frac{d(\delta n_B)}{dx}\right|_{x=x_B} = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right.$$

$$\left.+\cosh\left(\frac{x_B}{L_B}\right)\right\}$$

Taking the ratio

$$\frac{\left.\frac{d(\delta n_B)}{dx}\right|_{x=x_B}}{\left.\frac{d(\delta n_B)}{dx}\right|_{x=0}}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]+\cosh\left(\frac{x_B}{L_B}\right)}{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\cosh\left(\frac{x_B}{L_B}\right)+1}$$

$$\cong \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}$$

(a) For $\frac{x_B}{L_B} = 0.1 \Rightarrow$ Ratio $= 0.9950$

(b) For $\frac{x_B}{L_B} = 1.0 \Rightarrow$ Ratio $= 0.648$

(c) For $\frac{x_B}{L_B} = 10 \Rightarrow$ Ratio $= 9.08\times10^{-5}$

12.13

In the base of the transistor, we have

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution to the differential equation is of the form

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

Also

$$\delta n_B(x_B) = A\exp\left(\frac{x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right)$$

$$= -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary condition, we find

$$B\left[\exp\left(\frac{x_B}{L_B}\right) - \exp\left(\frac{-x_B}{L_B}\right)\right]$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{x_B}{L_B}\right) + n_{BO}$$

Solving for *B*, we find

$$B = \frac{n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{x_B}{L_B}\right) + n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A = \frac{-n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right) - n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

12.14

In the base of the pnp transistor, we have

$$D_B\frac{d^2(\delta p_B(x))}{dx^2} - \frac{\delta p_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta p_B(x))}{dx} - \frac{\delta p_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta p_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\delta p_B(0) = A + B = p_B(0) - p_{BO}$$

$$= p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]$$

Also

$$\delta p_B(x_B) = A\exp\left(\frac{x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right)$$

$$= -p_{BO}$$

From the first boundary condition equation, we find

$$A = p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary equation, we obtain

$$B = \frac{p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{x_B}{L_B}\right) + p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A = \frac{-p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right) - p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for *A* and *B* into the general solution and collecting terms, we obtain

$$\delta p_B(x) = \frac{p_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)\right\}$$

12.15

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x) = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B - x}{x_B}\right)$$

The excess carrier concentration is

$$\delta n_B(x) = n_B(x) - n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B - x}{x_B}\right) - 1\right\}$$

At $x = \frac{1}{2}x_B$, we have

$$\delta n_{BO}\left(\frac{x_B}{2}\right) = n_{BO}\left\{\frac{1}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] - 1\right\}$$

For the actual case, we have

$$\delta n_B\left(\frac{x_B}{2}\right) = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\sinh\left(\frac{x_B}{2L_B}\right) - \sinh\left(\frac{x_B}{2L_B}\right)\right\}$$

(a) For $\frac{x_B}{L_B} = 0.10$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\frac{\delta n_{BO}\left(\frac{x_B}{2}\right) - \delta n_B\left(\frac{x_B}{2}\right)}{\delta n_{BO}\left(\frac{x_B}{2}\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right](0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$, then we find that the ratio is

$$= \frac{0.00063}{0.50} = 0.00126 \Rightarrow 0.126\%$$

(b) For $\frac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\frac{\delta n_{BO}\left(\frac{x_B}{2}\right) - \delta n_B\left(\frac{x_B}{2}\right)}{\delta n_{BO}\left(\frac{x_B}{2}\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BR}}{V_t}\right)\right](0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Again assume that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$. Then the ratio becomes

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow 11.32\%$$

12.16

(a) $\delta p_B(x_B) = -5\times10^3 = -p_{B0}$

$$\Rightarrow p_{B0} = 5\times10^3\ \text{cm}^{-3}$$

$$p_{B0} = \frac{n_i^2}{N_B}$$

$$\Rightarrow N_B = \frac{n_i^2}{p_{B0}} = \frac{(1.5\times10^{10})^2}{5\times10^3}$$

$$= 4.5\times10^{16}\ \text{cm}^{-3}$$

$$\delta p_B(0) \cong p_{B0}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$\Rightarrow V_{EB} = V_t \ln\left(\frac{\delta p_B(0)}{p_{B0}}\right)$$

$$= (0.0259)\ln\left(\frac{10^{15}}{5\times10^3}\right)$$

$$= 0.6740\ \text{V}$$

(b) Using the linear approximation,

$$|J| = eD_B\frac{d(\delta p_B(x))}{dx} \cong \frac{eD_B p_{B0}}{x_B}\exp\left(\frac{V_{EB}}{V_t}\right)$$

Since $x_B << L_B$, $|J|_{x=0} \cong |J|_{x=x_B}$

Then

$$|J| = \frac{(1.6\times10^{-19})(10)(5\times10^{3})}{0.8\times10^{-4}}\exp\left(\frac{0.674}{0.0259}\right)$$

$$= 20.0 \text{ A/cm}^2$$

(c) Using Equation (12.15a),

$$\frac{d(\delta p_B(0))}{dx} = \frac{-p_{B0}}{L_B\cdot\sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cosh\left(\frac{x_B-x}{L_B}\right)+\cosh\left(\frac{x}{L_B}\right)\right\}$$

Now

$$|J| = eD_B\frac{d(\delta p_B(x))}{dx} = \frac{eD_B p_{B0}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cosh\left(\frac{x_B-x}{L_B}\right)+\cosh\left(\frac{x}{L_B}\right)\right\}$$

For $x=0$, $\sinh(1)=1.1752$, $\cosh(1)=1.5431$

$\cosh(0)=1.0$

Then

$$|J|_{x=0} = \frac{(1.6\times10^{-19})(10)(5\times10^{3})}{(10\times10^{-4})(1.1752)}$$

$$\times\left\{\left[\exp\left(\frac{0.6740}{0.0259}\right)-1\right](1.5431)+(1.0)\right\}$$

$$|J|_{x=0} = 2.1042 \text{ A/cm}^2$$

For $x=x_B$,

$$|J|_{x=x_B} = \frac{(1.6\times10^{-19})(10)(5\times10^{3})}{(10\times10^{-4})(1.1752)}$$

$$\times\left\{\left[\exp\left(\frac{0.6740}{0.0259}\right)-1\right](1.0)+(1.5431)\right\}$$

$$|J|_{x=x_B} = 1.3636 \text{ A/cm}^2$$

(d) For part (b),

$$\frac{J\big|_{x=x_B}}{J\big|_{x=0}} = 1.0$$

For part (c),

$$\frac{J\big|_{x=x_B}}{J\big|_{x=0}} = \frac{1.3636}{2.1042} = 0.648$$

12.17

(a) For an npn transistor biased in saturation, the excess minority carrier electron concentration in the base is found from

$$D_B\frac{d^2(\delta n_B(x))}{dx^2}-\frac{\delta n_B(x)}{\tau_{BO}}=0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2}-\frac{\delta n_B(x)}{L_B^2}=0$$

where $L_B=\sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right)+B\exp\left(\frac{-x}{L_B}\right)$$

If $x_B \ll L_B$, then also $x \ll L_B$, so that

$$\delta n_B(x) \cong A\left(1+\frac{x}{L_B}\right)+B\left(1-\frac{x}{L_B}\right)$$

$$=(A+B)+(A-B)\left(\frac{x}{L_B}\right)$$

which can be written as

$$\delta n_B(x) = C + D\left(\frac{x}{L_B}\right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$

and

$$\delta n_B(x_B) = C + D\left(\frac{x_B}{L_B}\right) = n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

The coefficient D can be written as

$$D = \left(\frac{L_B}{x_B}\right)(n_{BO})\left\{\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(1-\frac{x}{x_B}\right)+\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{x}{x_B}\right)\right\}$$

(b) The electron diffusion current density is

$$J_n = eD_B \frac{d(\delta n_B(x))}{dx}$$

$$= eD_B n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(\frac{-1}{x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{1}{x_B}\right)\right\}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B}\left\{\exp\left(\frac{V_{BE}}{V_t}\right)-\exp\left(\frac{V_{BC}}{V_t}\right)\right\}$$

(c) The total excess charge in the base region is

$$Q_{nB} = -e\int_0^{x_B} \delta n_B(x)dx$$

$$= -en_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\left(x-\frac{x^2}{2x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\cdot\left(\frac{x^2}{2x_B}\right)\right\}\Bigg|_0^{x_B}$$

which yields

$$Q_{nB} = \frac{-en_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right] + \left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]\right\}$$

12.18

(a) Using the linear approximation, we can write

$$|J_n| = \frac{eD_B n_{B0}}{x_B}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-\exp\left(\frac{V_{BC}}{V_t}\right)\right]$$

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{5\times10^{16}}$$

$$= 4.5\times10^3 \text{ cm}^{-3}$$

Then

$$125 = \frac{(1.6\times10^{-19})(25)(4.5\times10^3)}{0.7\times10^{-4}} \times\left[\exp\left(\frac{0.70}{0.0259}\right)-\exp\left(\frac{V_{BC}}{V_t}\right)\right]$$

$$4.8611\times10^{11} = 5.4662\times10^{11} - \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$V_{BC} = (0.0259)\ln(6.051\times10^{10})$$

$$= 0.6430 \text{ V}$$

(b) $V_{CE}(sat) = V_{BE} - V_{BC} = 0.70 - 0.6430$

$$= 0.057 \text{ V}$$

(c) We have

$$\frac{|Q_n|}{e} = \frac{n_{B0}x_B}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)+\exp\left(\frac{V_{BC}}{V_t}\right)\right]$$

$$= \frac{(4.5\times10^3)(0.7\times10^{-4})}{2} \times\left[\exp\left(\frac{0.70}{0.0259}\right)+\exp\left(\frac{0.643}{0.0259}\right)\right]$$

$$= (0.1575)[5.466\times10^{11} + 6.052\times10^{10}]$$

$$\frac{|Q_n|}{e} = 9.56\times10^{10} \text{ cm}^{-2}$$

(d) In the collector,

$$\delta p_C(x'') \cong p_{C0}\left[\exp\left(\frac{V_{BC}}{V_t}\right)\right]\cdot\left[\exp\left(\frac{-x''}{L_C}\right)\right]$$

Now

$$\frac{|Q_p|}{e} = \int_0^\infty \delta p_C(x'')dx''$$

$$= p_{C0}\left[\exp\left(\frac{V_{BC}}{V_t}\right)\right](-L_C)\left[\exp\left(\frac{-x''}{L_C}\right)\right]\Bigg|_0^\infty$$

$$= p_{C0}L_C \exp\left(\frac{V_{BC}}{V_t}\right)$$

We find

$$p_{C0} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{15}}$$

$$= 2.25\times10^5 \text{ cm}^{-3}$$

Then

$$\frac{|Q_p|}{e} = (2.25\times10^5)(35\times10^{-4})\exp\left(\frac{0.643}{0.0259}\right)$$

$$= 4.77\times10^{13} \text{ cm}^{-2}$$

12.19

(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3 \text{ cm}^{-3}$$

and

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{7\times10^{15}} = 3.21\times10^4 \text{ cm}^{-3}$$

At $x = x_B$,

$$n_B(x_B) = n_{BO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= (2.25\times10^3)\exp\left(\frac{0.565}{0.0259}\right)$$

or $n_B(x_B) = 6.7\times10^{12} \text{ cm}^{-3}$

At $x'' = 0$,

$$p_C(0) = p_{CO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= (3.21\times10^4)\exp\left(\frac{0.565}{0.0259}\right)$$

or $p_C(0) = 9.56\times10^{13} \text{ cm}^{-3}$

(c) From the B-C space charge region,

$$V_{bi1} = (0.0259)\ln\left[\frac{(10^{17})(7\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.745 \text{ V}$$

Then

$$x_{p1} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.745-0.565)}{1.6\times10^{-19}} \times\left(\frac{7\times10^{15}}{10^{17}}\right)\left(\frac{1}{7\times10^{15}+10^{17}}\right)\right]^{1/2}$$

or $x_{p1} = 1.23\times10^{-6}$ cm

From the B-E space charge region

$$V_{bi2} = (0.0259)\ln\left[\frac{(10^{19})(10^{17})}{(1.5\times10^{10})^2}\right]$$

$$= 0.933 \text{ V}$$

Then

$$x_{p2} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.933+2)}{1.6\times10^{-19}} \times\left(\frac{10^{19}}{10^{17}}\right)\left(\frac{1}{10^{19}+10^{17}}\right)\right]^{1/2}$$

or

$$x_{p2} = 1.94\times10^{-5} \text{ cm}$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2}$$

$$= 1.20 - 0.0123 - 0.194$$

or

$$x_B = 0.994\ \mu\text{m}$$

12.20

Low-injection limit is reached when

$$p_C(0) = (0.10)N_C = (0.10)(5\times10^{14})$$

or

$$p_C(0) = 5\times10^{13} \text{ cm}^{-3}$$

We have

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{5\times10^{14}} = 4.5\times10^5 \text{ cm}^{-3}$$

Also

$$p_C(0) = p_{CO}\exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$V_{CB} = V_t \ln\left(\frac{p_C(0)}{p_{CO}}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{13}}{4.5\times10^5}\right)$$

or

$$V_{CB} = 0.48 \text{ V}$$

12.21

(a)

(i) $\gamma = \dfrac{1}{1+\dfrac{I_{pE}}{I_{nE}}} = \dfrac{1}{1+\dfrac{0.0035}{0.50}} = 0.99305$

(ii) $\alpha_T = \dfrac{I_{nC}}{I_{nE}} = \dfrac{0.495}{0.50} = 0.990$

(iii) $\delta = \dfrac{I_{nE}+I_{pE}}{I_{nE}+I_R+I_{pE}}$

$$= \frac{0.50+0.0035}{0.50+0.005+0.0035} = 0.990167$$

(iv) $\alpha = \delta\alpha_T\delta = (0.99305)(0.990)(0.990167)$

$= 0.97345$

(v) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.97345}{1-0.97345} = 36.7$

(b) For $\beta = 120 \Rightarrow \alpha = \dfrac{\beta}{1+\beta} = \dfrac{120}{121}$

$\alpha = 0.991736$

Then $\gamma = \alpha_T = \delta = 0.997238$

$$\alpha_T = 0.997238 = \frac{I_{nC}}{I_{nE}} = \frac{I_{nC}}{0.50}$$

$$\Rightarrow I_{nC} = 0.4986 \text{ mA}$$

$$\gamma = 0.997238 = \frac{1}{1+\dfrac{I_{pE}}{I_{nE}}} = \frac{1}{1+\dfrac{I_{pE}}{0.50}}$$

$$\Rightarrow I_{pE} = 0.00138\text{ mA} = 1.38\ \mu\text{A}$$

$$\delta = \frac{I_{nE} + I_{pE}}{I_{nE} + I_R + I_{pE}}$$

$$0.997238 = \frac{0.50 + 0.00138}{0.50 + I_R + 0.00138}$$

$$\Rightarrow I_R = 0.00139\text{ mA} = 1.39\ \mu\text{ A}$$

12.22

(a) Using Equation (12.37)

$$I_{nC} = \frac{eD_B p_{B0} A_{BE}}{L_B} \times \left\{ \frac{\left[\exp\left(\dfrac{V_{EB}}{V_t}\right) - 1\right]}{\sinh\left(\dfrac{x_B}{L_B}\right)} + \frac{1}{\tanh\left(\dfrac{x_B}{L_B}\right)} \right\}$$

Now

$$p_{B0} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}} = 2.25\times10^4\text{ cm}^{-3}$$

$$L_B = \sqrt{D_B \tau_{B0}} = \sqrt{(10)\left(5\times10^{-7}\right)} = 2.236\times10^{-3}\text{ cm}$$

We find

$$\sinh\left(\frac{x_B}{L_B}\right) = \sinh\left(\frac{0.70\times10^{-4}}{2.236\times10^{-3}}\right) = 0.03131$$

$$\tanh\left(\frac{x_B}{L_B}\right) = \tanh\left(\frac{0.70\times10^{-4}}{2.236\times10^{-3}}\right) = 0.03130$$

Then

$$I_{nC} = \frac{\left(1.6\times10^{-19}\right)(10)\left(2.25\times10^4\right)\left(5\times10^{-4}\right)}{2.236\times10^{-3}} \times \left\{ \frac{\left[\exp\left(\dfrac{0.550}{0.0259}\right) - 1\right]}{0.03131} + \frac{1}{0.03130} \right\}$$

$$I_{nC} = 4.29\times10^{-4}\text{ A} = 0.429\text{ mA}$$

(b)

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{x_B}{x_E}} = \frac{1}{1+\left(\dfrac{10^{16}}{5\times10^{17}}\right)\left(\dfrac{15}{10}\right)\left(\dfrac{0.7}{0.5}\right)} = 0.95969$$

$$\alpha_T = \frac{1}{\cosh\left(\dfrac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\dfrac{0.70\times10^{-4}}{2.236\times10^{-3}}\right)} = 0.99951$$

$$\alpha = \gamma\alpha_T\delta = (0.95969)(0.99951)(0.995) = 0.95442$$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.95442}{1-0.95442} = 20.94$$

Then

$$I_C = \beta I_B = (20.94)(0.80) = 16.75\ \mu\text{ A}$$

(c) $I_C = \alpha I_E = (0.95442)(125) = 119.3\ \mu\text{ A}$

12.23

(a) We have

$$J_{nE} = \frac{eD_B n_{BO}}{L_B}\left\{ \frac{\left[\exp\left(\dfrac{V_{BE}}{V_t}\right) - 1\right]}{\tanh\left(\dfrac{x_B}{L_B}\right)} + \frac{1}{\sinh\left(\dfrac{x_B}{L_B}\right)} \right\}$$

We find that

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3\text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)\left(5\times10^{-8}\right)} = 8.660\times10^{-4}\text{ cm}$$

Then

$$J_{nE} = \frac{\left(1.6\times10^{-19}\right)(15)\left(4.5\times10^3\right)}{8.66\times10^{-4}} \times \left\{ \frac{\exp\left(\dfrac{0.60}{0.0259}\right)}{\tanh\left(\dfrac{0.70}{8.66}\right)} + \frac{1}{\sinh\left(\dfrac{0.70}{8.66}\right)} \right\}$$

or

$$J_{nE} = 1.779\text{ A/cm}^2$$

We also have

$$J_{pE} = \frac{eD_E p_{EO}}{L_E}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\frac{1}{\tanh\left(\frac{x_E}{L_E}\right)}$$

Now

$$p_{EO} = \frac{n_i^2}{N_E} = \frac{\left(1.5\times10^{10}\right)^2}{10^{18}} = 2.25\times10^2\ \text{cm}^{-3}$$

and

$$L_E = \sqrt{D_E\tau_{EO}} = \sqrt{(8)\left(10^{-8}\right)}$$

$$= 2.828\times10^{-4}\ \text{cm}$$

Then

$$J_{pE} = \frac{\left(1.6\times10^{-19}\right)(8)\left(2.25\times10^2\right)}{2.828\times10^{-4}}$$

$$\times\left[\exp\left(\frac{0.60}{0.0259}\right)-1\right]\cdot\frac{1}{\tanh\left(\frac{0.8}{2.828}\right)}$$

or

$$J_{pE} = 0.04251\ \text{A/cm}^2$$

We can find

$$J_{nC} = \frac{eD_B n_{BO}}{L_B}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]}{\sinh\left(\frac{x_B}{L_B}\right)}+\frac{1}{\tanh\left(\frac{x_B}{L_B}\right)}\right\}$$

$$= \frac{\left(1.6\times10^{-19}\right)(15)\left(4.5\times10^3\right)}{8.66\times10^{-4}}$$

$$\times\left\{\frac{\left[\exp\left(\frac{0.60}{0.0259}\right)-1\right]}{\sinh\left(\frac{0.7}{8.66}\right)}+\frac{1}{\tanh\left(\frac{0.7}{8.66}\right)}\right\}$$

or

$$J_{nC} = 1.773\ \text{A/cm}^2$$

The recombination current density is

$$J_R = J_{ro}\exp\left(\frac{V_{BE}}{2V_t}\right)$$

$$= \left(3\times10^{-8}\right)\exp\left[\frac{0.60}{2(0.0259)}\right]$$

or

$$J_R = 3.218\times10^{-3}\ \text{A/cm}^2$$

(b) Using the calculated current densities, we find

$$\gamma = \frac{J_{nE}}{J_{nE}+J_{pE}} = \frac{1.779}{1.779+0.04251}$$

or

$$\gamma = 0.9767$$

We also find

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.773}{1.779}$$

or

$$\alpha_T = 0.9966$$

Also

$$\delta = \frac{J_{nE}+J_{pE}}{J_{nE}+J_R+J_{pE}}$$

$$= \frac{1.779+0.04251}{1.779+0.003218+0.04251}$$

or

$$\delta = 0.9982$$

Then

$$\alpha = \gamma\alpha_T\delta = (0.9767)(0.9966)(0.9982)$$

or

$$\alpha = 0.9716$$

Now

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9716}{1-0.9716}$$

or

$$\beta = 34.2$$

12.24

(a) We have

$$\gamma = \frac{1}{1+\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}} \cong 1-\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}$$

or

$$\gamma \cong 1-K\cdot\frac{N_B}{N_E}$$

(i) Now

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1-\frac{2N_{BO}}{N_E}\cdot K}{1-\frac{N_{BO}}{N_E}\cdot K}$$

$$\cong\left(1-\frac{2N_{BO}}{N_E}\cdot K\right)\left(1+\frac{N_{BO}}{N_E}\cdot K\right)$$

$$\cong 1-\frac{2N_{BO}}{N_e}\cdot K+\frac{N_{BO}}{N_E}\cdot K$$

or finally

$$\frac{\gamma(B)}{\gamma(A)} = 1 - \frac{N_{BO}}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}$$

(ii)

We have

$$\gamma \cong 1 - \frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E} = 1 - K'\cdot\frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K'\cdot\dfrac{x_{BO}}{2x_E}}{1 - K'\cdot\dfrac{x_{BO}}{x_E}}$$

$$\cong \left(1 - K'\cdot\frac{x_{BO}}{2x_E}\right)\left(1 + K'\cdot\frac{x_{BO}}{x_E}\right)$$

$$\cong 1 - K'\cdot\frac{x_{BO}}{2x_E} + K'\cdot\frac{x_{BO}}{x_E}$$

$$= 1 + K'\cdot\frac{x_{BO}}{2x_E}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 + \frac{N_{BO}}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_{BO}}{2x_E}$$

(b) (i) We find

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = \frac{\left(1 - \dfrac{1}{2}\cdot\left[\dfrac{(x_{BO}/2)}{L_B}\right]^2\right)}{\left(1 - \dfrac{1}{2}\cdot\left[\dfrac{x_{BO}}{L_B}\right]^2\right)}$$

$$= \frac{\left(1 - \dfrac{1}{8}\left[\dfrac{x_{BO}}{L_B}\right]^2\right)}{\left(1 - \dfrac{1}{2}\left[\dfrac{x_{BO}}{L_B}\right]^2\right)}$$

$$\cong \left(1 - \frac{1}{8}\left[\frac{x_{BO}}{L_B}\right]^2\right)\left(1 + \frac{1}{2}\left[\frac{x_{BO}}{L_B}\right]^2\right)$$

$$\cong 1 - \frac{1}{8}\left[\frac{x_{BO}}{L_B}\right]^2 + \frac{1}{2}\left[\frac{x_{BO}}{L_B}\right]^2$$

or finally

$$\frac{\alpha_T(C)}{\alpha_T(A)} \cong 1 + \frac{3}{8}\left[\frac{x_{BO}}{L_B}\right]^2$$

(c) Neglect any change in space charge width.

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{sO}}\cdot\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$\cong 1 - \frac{J_{rO}}{J_{sO}}\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right) = 1 - \frac{K}{J_{sO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \dfrac{K}{J_{sOB}}}{1 - \dfrac{K}{J_{sOA}}} \cong \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$

$$\cong 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now

$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so

$$\frac{\delta(B)}{\delta(A)} \cong 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then finally

$$\frac{\delta(B)}{\delta(A)} = 1 - \frac{J_{rO}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}{\left(\dfrac{eD_B n_{BO}}{x_B}\right)}$$

(ii) We find

$$\frac{\delta(C)}{\delta(A)} \cong 1 + \frac{J_{rO}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}{\left(\dfrac{eD_B n_{BO}}{x_B}\right)}$$

(d) Device C has the largest β. The emitter injection efficiency, base transport, and recombination factors all increase.

12.25

(a) We have

$$\gamma = \frac{1}{1 + \dfrac{N_B}{N_E}\cdot\dfrac{D_E}{D_B}\cdot\dfrac{x_B}{x_E}} = \frac{1}{1 + K\cdot\dfrac{N_B}{N_E}}$$

or

$$\gamma \cong 1 - K\cdot\frac{N_B}{N_E}$$

(i) Then

$$\frac{\gamma(B)}{\gamma(A)}=\frac{1-K\cdot\dfrac{N_B}{2N_{EO}}}{1-K\cdot\dfrac{N_B}{N_{EO}}}$$

$$\cong\left(1-K\cdot\frac{N_B}{2N_{EO}}\right)\left(1+K\cdot\frac{N_B}{N_{EO}}\right)$$

$$\cong 1-K\cdot\frac{N_B}{2N_{EO}}+K\cdot\frac{N_B}{N_{EO}}$$

$$=1+K\cdot\frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)}=1+\frac{N_B}{2N_{EO}}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}$$

(ii) Now

$$\gamma=\frac{1}{1+K'\cdot\dfrac{x_B}{x_E}}\cong 1-K'\cdot\frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)}=\frac{1-K'\cdot\dfrac{x_B}{(x_{EO}/2)}}{1-K'\cdot\dfrac{x_B}{x_{EO}}}$$

$$\cong\left(1-K'\cdot\frac{2x_B}{x_{EO}}\right)\left(1+K'\cdot\frac{x_B}{x_{EO}}\right)$$

$$\cong 1-2K'\cdot\frac{x_B}{x_{EO}}+K'\cdot\frac{x_B}{x_{EO}}$$

$$=1-K'\cdot\frac{x_B}{x_{EO}}$$

or finally

$$\frac{\gamma(C)}{\gamma(A)}=1-\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_{EO}}$$

(b) We have

$$\alpha_T=1-\frac{1}{2}\left(\frac{x_B}{L_B}\right)^2$$

(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)}=1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)}=1$$

(c) Neglect any change in space charge width.

$$\delta=\frac{1}{1+\dfrac{J_{rO}}{J_{sO}}\exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$=\frac{1}{1+\dfrac{K}{J_{sO}}}\cong 1-\frac{K}{J_{sO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)}=\frac{1-\dfrac{K}{J_{sOB}}}{1-\dfrac{K}{J_{sOA}}}\cong\left(1-\frac{K}{J_{sOB}}\right)\left(1+\frac{K}{J_{sOA}}\right)$$

$$\cong 1-\frac{K}{J_{sOB}}+\frac{K}{J_{sOA}}$$

Now

$$J_{sO}\propto\frac{1}{N_E x_E}$$

so

$$\frac{\delta(B)}{\delta(A)}=1-K'(2N_{EO})+K'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)}=1-K'\cdot N_{EO}$$

Recombination factor decreases

(ii) We have

$$\frac{\delta(C)}{\delta(A)}=1-K''\left(\frac{x_{EO}}{2}\right)+K''(x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)}=1+\frac{1}{2}K''\cdot x_{EO}$$

Recombination factor increases

12.26

(b)

$$n_{BO}=\frac{n_i^2}{N_B}=\frac{(1.5\times10^{10})^2}{10^{17}}=2.25\times10^3\ \text{cm}^{-3}$$

Then

$$n_B(0)=n_{BO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$=(2.25\times10^3)\exp\left(\frac{0.6}{0.0259}\right)$$

$$=2.59\times10^{13}\ \text{cm}^{-3}$$

Now

$$J_{nC} = \frac{eD_B n_B(0)}{x_B}$$
$$= \frac{(1.6\times10^{-19})(20)(2.59\times10^{13})}{10^{-4}}$$

or

$$J_{nC} = 0.828 \text{ A/cm}^2$$

Assuming a long collector

$$J_{pC} = \frac{eD_C p_{nO}}{L_C}\exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{cO} = \frac{n_i^2}{N_C} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4 \text{ cm}^{-3}$$

and

$$L_C = \sqrt{D_C\tau_{CO}} = \sqrt{(15)(2\times10^{-7})}$$
$$= 1.732\times10^{-3} \text{ cm}$$

Then

$$J_{pC} = \frac{(1.6\times10^{-19})(15)(2.25\times10^4)}{1.732\times10^{-3}} \times\exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359 \text{ A/cm}^2$$

The collector current is

$$I_C = (J_{nC} + J_{pC})\cdot A$$
$$= (0.828 + 0.359)(10^{-3})$$

or

$$I_C = 1.19 \text{ mA}$$

The emitter current is

$$I_E = J_{nC}\cdot A = (0.828)(10^{-3})$$

or

$$I_E = 0.828 \text{ mA}$$

12.27

(a)

$$\alpha_T = \frac{1}{\cosh(x_B/L_B)} \quad \text{and} \quad \beta = \frac{\alpha_T}{1-\alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	$\cong 0$

(b) For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1+(p_{EO}/n_{BO})} = \frac{1}{1+(N_B/N_E)}$$

and $\beta = \dfrac{\gamma}{1-\gamma}$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c) For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E << 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

12.28

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B\tanh(x_B/L_B)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^3 \text{ cm}^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})}$$
$$= 15.8\times10^{-4} \text{ cm}$$

Then

$$J_{sO} = \frac{(1.6\times10^{-19})(25)(2.25\times10^3)}{(15.8\times10^{-4})\tanh(0.7/15.8)}$$

or

$$J_{sO} = 1.287\times10^{-10} \text{ A/cm}^2$$

Now

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \frac{2\times10^{-9}}{1.287\times10^{-10}} \exp\left[\frac{-V_{BE}}{2(0.0259)}\right]}$$

or

(a)

$$\delta = \frac{1}{1 + (15.54)\exp\left(\frac{-V_{BE}}{0.0518}\right)}$$

and

(b)

$$\beta = \frac{\delta}{1-\delta}$$

Now

V_{BE}	δ	β
0.20	0.7535	3.06
0.40	0.99316	145
0.60	0.999855	6,902

(c) If $V_{BE} < 0.4$ V, the recombination factor is likely the limiting factor in the current gain.

12.29

$$\alpha = \frac{\beta}{1+\beta} = \frac{150}{151} = 0.993377$$

$$\alpha = \gamma\alpha_T\delta$$

$$0.993377 = (\gamma\alpha_T)(0.9975)$$

$$\Rightarrow \gamma\alpha_T = 0.995867$$

Let $x_B = 0.80\ \mu$m

$$L_B = \sqrt{D_B\tau_{B0}} = \sqrt{(23)(2\times10^{-7})}$$

$$= 2.145\times10^{-3}\ \text{cm}$$

Then

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = \frac{1}{\cosh\left(\frac{0.80\times10^{-4}}{2.145\times10^{-3}}\right)}$$

$$= 0.99930$$

Now

$$\gamma = \frac{\alpha}{\alpha_T\delta} = \frac{0.993377}{(0.99930)(0.9975)} = 0.99656$$

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}}$$

$$0.99656 = \frac{1}{1 + \left(\frac{2\times10^{16}}{N_E}\right)\left(\frac{8}{23}\right)\left(\frac{0.80}{0.35}\right)}$$

$$\Rightarrow N_E = 4.61\times10^{18}\ \text{cm}^{-3}$$

12.30

(a) We have $J_{rO} = 5\times10^{-8}\ \text{A/cm}^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5\times10^{10})^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})}$$

$$= 15.8\times10^{-4}\ \text{cm}$$

Then

$$J_{sO} = \frac{eD_Bn_{BO}}{L_B\tanh(x_B/L_B)}$$

$$= \frac{(1.6\times10^{-19})(25)(4.5\times10^3)}{(15.8\times10^{-4})\tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.139\times10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}}\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

For $T = 300$ K and $V_{BE} = 0.55$ V.

$$\delta = 0.995$$

$$= \frac{1}{1 + \left(\frac{5\times10^{-8}}{1.139\times10^{-11}}\right)\cdot\tanh\left(\frac{x_B}{L_B}\right)\cdot\exp\left(\frac{-0.55}{0.0518}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.0468$$

or

$$x_B = (0.0468)(15.8) = 0.739\ \mu\text{m}$$

(b) For $T = 400$ K and $J_{rO} = 5\times10^{-8}\ \text{A/cm}^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3 \cdot \frac{\exp\left[\frac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\frac{-E_g}{0.0259}\right]}$$

For $E_g = 1.12$ eV,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.175 \times 10^5$$

or

$$n_{BO}(400) = (1.175 \times 10^5)(4.5 \times 10^3)$$
$$= 5.29 \times 10^8 \text{ cm}^{-3}$$

Then

$$J_{sO} = \frac{(1.6 \times 10^{-19})(25)(5.29 \times 10^8)}{(15.8 \times 10^{-4})\tanh(0.739/15.8)}$$

or

$$J_{sO} = 2.865 \times 10^{-5} \text{ A/cm}^2$$

Finally

$$\delta = \frac{1}{1 + \dfrac{5 \times 10^{-8}}{2.865 \times 10^{-5}} \cdot \exp\left[\dfrac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\delta = 0.9999994$$

12.31
Plot

12.32
Plot

12.33
Plot

12.34
Plot

12.35

(a) $I_C = \dfrac{1}{r_o}(V_{CE} + V_A) \Rightarrow r_o = \dfrac{(V_{CE} + V_A)}{I_C}$

(i) $r_o = \dfrac{2 + 120}{1.2} = 101.67 \text{ k}\Omega$

(ii) $g_o = \dfrac{1}{r_o} = \dfrac{1}{101.67} = 0.00984 \text{ (k}\Omega\text{)}^{-1}$

$= 9.84 \times 10^{-6} (\Omega)^{-1}$

(iii) $I_C = \dfrac{4 + 120}{101.667} = 1.22 \text{ mA}$

(b)

(i) $r_o = \dfrac{V_{CE} + V_A}{I_C} = \dfrac{2 + 160}{0.25} = 648 \text{ k}\Omega$

(ii) $g_o = \dfrac{1}{r_o} = \dfrac{1}{648} = 0.00154 \text{ (k}\Omega\text{)}^{-1}$

$= 1.54 \times 10^{-6} (\Omega)^{-1}$

(iii) $I_C = \dfrac{4 + 160}{648} = 0.253 \text{ mA}$

12.36

$$r_o = \frac{\Delta V_{EC}}{\Delta I_C} \Rightarrow \Delta I_C = \frac{\Delta V_{EC}}{r_o} = \frac{5 - 2}{180}$$

$$\Delta I_C = 0.01667 \text{ mA} = 16.67 \ \mu\text{A}$$

12.37

$$x_{dB} = \left\{\frac{2\in_s (V_{bi} + V_{CB})}{e}\left[\frac{N_C}{N_B} \cdot \frac{1}{(N_B + N_C)}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_{CB})}{1.6 \times 10^{-19}}\right.$$
$$\left. \times \left[\frac{2 \times 10^{15}}{2 \times 10^{16}} \cdot \frac{1}{(2 \times 10^{15} + 2 \times 10^{16})}\right]\right\}^{1/2}$$

$$= \left\{(5.8832 \times 10^{-11})(V_{bi} + V_{CB})\right\}^{1/2}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_B N_C}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(2 \times 10^{15})(2 \times 10^{16})}{(1.5 \times 10^{10})^2}\right]$$

$$= 0.6709 \text{ V}$$

(i) For $V_{CB} = 4$ V, $x_{dB} = 0.1658 \ \mu$m

(ii) For $V_{CB} = 8$ V, $x_{dB} = 0.2259 \ \mu$m

(iii) For $V_{CB} = 12$ V, $x_{dB} = 0.2730 \ \mu$m

Neglecting the B-E space charge width,

(i) For $V_{CB} = 4$ V,

$$x_B = 0.85 - 0.1658 = 0.6842 \ \mu\text{m}$$

(ii) For $V_{CB} = 8$ V,

$$x_B = 0.85 - 0.2259 = 0.6241 \ \mu\text{m}$$

(iii) For $V_{CB} = 12$ V,

$$x_B = 0.85 - 0.2730 = 0.5770 \ \mu\text{m}$$

Now

$$J_C = \frac{eD_B n_{B0}}{x_B}\exp\left(\frac{V_{BE}}{V_t}\right)$$

where

$$n_{B0} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}}$$

$$=1.125\times10^4 \text{ cm}^{-3}$$

so

$$J_C = \frac{\left(1.6\times10^{-19}\right)(25)\left(1.125\times10^4\right)}{x_B}\exp\left(\frac{0.650}{0.0259}\right)$$

$$= \frac{3.5686\times10^{-3}}{x_B} \text{ A/cm}^2$$

(i)For $V_{CB} = 4$ V, $J_C = 52.16$ A/cm^2

(ii)For $V_{CB} = 8$ V, $J_C = 57.18$ A/cm^2

(iii)For $V_{CB} = 12$ V, $J_C = 61.85$ A/cm^2

(b) $$\frac{\Delta J_C}{\Delta V_{CE}} = \frac{J_C}{V_{CE}+V_A}$$

$$\frac{61.85-52.16}{12-4} = \frac{52.16}{4+0.650+V_A}$$

$$\Rightarrow V_A = 38.4 \text{ V}$$

12.38

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5\times10^3 \text{ cm}^{-3}$$

and

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$=\left(7.5\times10^3\right)\exp\left(\frac{0.7}{0.0259}\right)$$

or

$$n_B(0) = 4.10\times10^{15} \text{ cm}^{-3}$$

We have

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{\left(1.6\times10^{-19}\right)(20)\left(4.10\times10^{15}\right)}{x_B}$$

or

$$J = \frac{1.312\times10^{-2}}{x_B} \text{ A/cm}^2$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{\left(3\times10^{16}\right)\left(5\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.705 \text{ V}$$

Also

$$x_p = \left[\frac{2\in_s\left(V_{bi}+V_{CB}\right)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_B+N_C}\right)\right]^{1/2}$$

$$=\left[\frac{2(11.7)\left(8.85\times10^{-14}\right)\left(V_{bi}+V_{CB}\right)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{5\times10^{15}}{3\times10^{16}}\right)\left(\frac{1}{5\times10^{15}+3\times10^{16}}\right)\right]^{1/2}$$

or

$$x_p = \left[\left(6.163\times10^{-11}\right)\left(V_{bi}+V_{CB}\right)\right]^{1/2}$$

For $V_{CB} = 5$ V, $x_p = 0.1875\ \mu$m

For $V_{CB} = 10$ V, $x_p = 0.2569\ \mu$m

(a) For $x_{BO} = 1.0\ \mu$m

For $V_{CB} = 5$ V,

$$x_B = 1.0 - 0.1875 = 0.8125\ \mu\text{m}$$

Then

$$J = \frac{1.312\times10^{-2}}{0.8125\times10^{-4}} = 161.5 \text{ A/cm}^2$$

For $V_{CB} = 10$ V,

$$x_B = 1.0 - 0.2569 = 0.7431\ \mu\text{m}$$

and

$$J = \frac{1.312\times10^{-2}}{0.7431\times10^{-4}} = 176.6 \text{ A/cm}^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE}+V_A\right)$$

where

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6-161.5}{10-5}$$

$$= 3.02 \text{ A/cm}^2\text{/V}$$

Then

$$161.5 = (3.02)\left(5.7+V_A\right)$$

which yields

$$V_A = 47.8 \text{ V}$$

(b) For $x_{BO} = 0.80\ \mu\text{m}$

For $V_{CB} = 5\text{ V}$,

$x_B = 0.80 - 0.1875 = 0.6125\ \mu\text{ m}$

Then

$$J = \frac{1.312\times10^{-2}}{0.6125\times10^{-4}} = 214.2\text{ A/cm}^2$$

For $V_{CB} = 10\text{ V}$,

$x_B = 0.80 - 0.2569 = 0.5431\ \mu\text{ m}$

and

$$J = \frac{1.312\times10^{-2}}{0.5431\times10^{-4}} = 241.6\text{ A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{10-5}$$

$$= 5.48\text{ A/cm}^2\text{/V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

or

$$214.2 = (5.48)(5.7 + V_A)$$

which yields

$V_A = 33.4\text{ V}$

(c) For $x_{BO} = 0.60\ \mu\text{m}$

For $V_{CB} = 5\text{ V}$,

$x_B = 0.60 - 0.1875 = 0.4125\ \mu\text{ m}$

Then

$$J = \frac{1.312\times10^{-2}}{0.4125\times10^{-4}} = 318.1\text{ A/cm}^2$$

For $V_{CB} = 10\text{ V}$,

$x_B = 0.60 - 0.2569 = 0.3431\ \mu\text{ m}$

and

$$J = \frac{1.312\times10^{-2}}{0.3431\times10^{-4}} = 382.4\text{ A/cm}^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{10-5}$$

$$= 12.86\text{ A/cm}^2\text{/V}$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

or

$$318.1 = (12.86)(5.7 + V_A)$$

which yields

$V_A = 19.0\text{ V}$

12.39

(a)

$$x_{dB} = \left\{\frac{2\in_s\left(V_{bi}+V_{BC}\right)}{e}\left[\frac{N_C}{N_B}\cdot\frac{1}{\left(N_B+N_C\right)}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(V_{bi}+V_{BC})}{1.6\times10^{-19}}\right.$$

$$\left.\times\left[\frac{10^{15}}{10^{16}}\cdot\frac{1}{\left(10^{15}+10^{16}\right)}\right]\right\}^{1/2}$$

$$= \left\{\left(1.1766\times10^{-10}\right)\left(V_{bi}+V_{BC}\right)\right\}^{1/2}$$

Now

$$V_{bi} = V_t\ln\left(\frac{N_B N_C}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(10^{15}\right)\left(10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$

$$= 0.6350\text{ V}$$

For $V_{BC} = 1\text{ V}$, $x_{dB} = 0.1387\ \mu\text{ m}$

For $V_{BC} = 5\text{ V}$, $x_{dB} = 0.2575\ \mu\text{ m}$

Then $\Delta x_{dB} = 0.2575 - 0.1387 = 0.1188\ \mu\text{ m}$

(b) $$I_C = \frac{eD_B p_{B0} A_{BE}}{x_B}\exp\left(\frac{V_{EB}}{V_t}\right)$$

We find

$$p_{B0} = \frac{n_i^2}{N_B} = \frac{\left(1.5\times10^{10}\right)^2}{10^{16}}$$

$$= 2.25\times10^4\text{ cm}^{-3}$$

Then

$$I_C = \frac{\left(1.6\times10^{-19}\right)(10)\left(2.25\times10^4\right)\left(10^{-4}\right)}{x_B}$$

$$\times\exp\left(\frac{0.625}{0.0259}\right)$$

$$= \frac{1.0874\times10^{-7}}{x_B}\text{ A}$$

For $V_{BC} = 1\text{ V}$, $I_C = \dfrac{1.0874\times10^{-7}}{(0.70-0.1387)\times10^{-4}}$

$$= 1.937\times10^{-3}\text{ A} = 1.937\text{ mA}$$

For $V_{BC} = 5\text{ V}$, $I_C = \dfrac{1.0874\times10^{-7}}{(0.70-0.2575)\times10^{-4}}$

$$= 2.456\times10^{-3}\text{ A} = 2.456\text{ mA}$$

Then

$$\Delta I_C = 2.456 - 1.937 = 0.519\text{ mA}$$

(c) $\dfrac{\Delta I_C}{\Delta V_{BC}} = \dfrac{I_C}{V_{EC}+V_A}$

$$\frac{0.519\times10^{-3}}{5-1} = \frac{1.937\times10^{-3}}{1+0.625+V_A}$$

$V_A = 13.3$ V

(d) $r_o = \dfrac{V_{EC}+V_A}{I_C} = \dfrac{1.625+13.3}{1.937\times10^{-3}}$

$$= 7.705\times10^3\ \Omega = 7.705\ \mathrm{k}\Omega$$

12.40

Let $x_B = x_E$, $L_B = L_E$, $D_B = D_E$

Then the emitter injection efficiency is

$$\gamma = \frac{1}{1+\dfrac{p_{EO}}{n_{BO}}} = \frac{1}{1+\dfrac{n_{iE}^2}{N_E}\cdot\dfrac{N_B}{n_{iB}^2}}$$

where $n_{iB}^2 = n_i^2$.

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing,

$$n_{iE}^2 = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$$

Then

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}\exp\left(\dfrac{\Delta E_g}{kT}\right)}$$

(a) No bandgap narrowing, so $\Delta E_g = 0$

$\alpha = \gamma\alpha_T\delta = \gamma(0.995)^2$. We find

N_E	γ	α	β
10^{17}	0.5	0.495	0.980
10^{18}	0.909	0.8999	8.99
10^{19}	0.990	0.980	49.3
10^{20}	0.9990	0.989	90.2

(b) Taking into account bandgap narrowing, we find

N_E	ΔE_g (meV)	γ	α	β
10^{17}	0	0.5	0.495	0.98
10^{18}	25	0.792	0.784	3.63
10^{19}	80	0.820	0.812	4.32
10^{20}	230	0.122	0.121	0.14

12.41

(a) We have

$$\gamma = \frac{1}{1+\dfrac{p_{EO}D_E L_B}{n_{BO}D_B L_E}\cdot\dfrac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For $x_B = x_E$, $L_B = L_E$, $D_B = D_E$, we obtain

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}\exp\left(\dfrac{\Delta E_g}{kT}\right)}$$

For $N_E = 10^{19}\ \mathrm{cm}^{-3}$, we have $\Delta E_g = 80$ meV

Then

$$0.996 = \frac{1}{1+\dfrac{N_B}{10^{19}}\exp\left(\dfrac{0.080}{0.0259}\right)}$$

which yields

$N_B = 1.83\times10^{15}\ \mathrm{cm}^{-3}$

(b) Neglecting bandgap narrowing, we would have

$$\gamma = \frac{1}{1+\dfrac{N_B}{N_E}} \Rightarrow 0.996 = \frac{1}{1+\dfrac{N_B}{10^{19}}}$$

which yields

$N_B = 4.02\times10^{16}\ \mathrm{cm}^{-3}$

12.42

(a)

(i) $R = \dfrac{(\rho)(Length)}{Area} = \dfrac{(S/2)}{\sigma(x_B L)}$

$$= \frac{(S/2)}{(e\mu_p N_B)(x_B L)}$$

$$= \frac{1}{(1.6\times10^{-19})(250)(2\times10^{16})}$$

$$\times\frac{5\times10^{-4}}{(0.65\times10^{-4})(25\times10^{-4})}$$

$R = 3.846\times10^3\ \Omega = 3.846\ \mathrm{k}\Omega$

(ii) $\Delta V = (I_B/2)R = (5\times10^{-6})(3.846\times10^3)$

$= 0.01923$ V

(iii) $$\frac{\delta n_B(x=S/2)}{\delta n_B(x=0)}=\frac{\exp\left[\frac{V_{BE}(x=S/2)}{0.0259}\right]}{\exp\left[\frac{V_{BE}(x=0)}{0.0259}\right]}$$

$$=\frac{\exp\left[\frac{0.60}{0.0259}\right]}{\exp\left[\frac{0.60+\Delta V}{0.0259}\right]}$$

$$=\exp\left[\frac{-\Delta V}{0.0259}\right]$$

$$=\exp\left[\frac{-0.01923}{0.0259}\right]$$

Then

$$\frac{\delta n_B(x=S/2)}{\delta n_B(x=0)}=0.476$$

(b)

(i) $$R=\frac{1}{(1.6\times10^{-19})(250)(2\times10^{16})}\times\frac{1.5\times10^{-4}}{(0.65\times10^{-4})(25\times10^{-4})}$$

$$=1.154\times10^{3}\ \Omega=1.154\ \mathrm{k}\Omega$$

(ii) $$\Delta V=(I_B/2)R=(5\times10^{-6})(1.154\times10^{3})$$

$$=0.005769\ \mathrm{V}$$

(iii) $$\frac{\delta n_B(x=S/2)}{\delta n_B(x=0)}=\exp\left[\frac{-\Delta V}{V_t}\right]$$

$$=\exp\left[\frac{-0.005769}{0.0259}\right]$$

Then

$$\frac{\delta n_B(x=S/2)}{\delta n_B(x=0)}=0.80$$

12.43

$$\frac{\delta n_B(x=S/2)}{\delta n_B(x=0)}=0.90=\exp\left[\frac{-\Delta V}{V_t}\right]$$

Then

$$\Delta V=V_t\ln\left(\frac{1}{0.90}\right)=(0.0259)\ln\left(\frac{1}{0.90}\right)$$

$$\Delta V=0.002729\ \mathrm{V}=(I_B/2)R$$

$$=(5\times10^{-6})R$$

$$R=545.8\,\Omega$$

Now

$$R=\frac{1}{e\mu_pN_B}\cdot\frac{(S/2)}{x_BL}$$

$$545.8=\frac{1}{(1.6\times10^{-19})(250)(2\times10^{16})}\times\frac{(S/2)}{(0.65\times10^{-4})(25\times10^{-4})}$$

$$\Rightarrow S=1.42\times10^{-4}\ \mathrm{cm}=1.42\ \mu\mathrm{m}$$

12.44

(a)

$$N_B=N_B(0)\exp\left(\frac{-ax}{x_B}\right)$$

where

$$a=\ln\left[\frac{N_B(0)}{N_B(x_B)}\right]>0$$

and is a constant. In thermal equilibrium

$$J_p=e\mu_pN_B\mathrm{E}-eD_p\frac{dN_B}{dx}=0$$

so that

$$\mathrm{E}=\frac{D_p}{\mu_p}\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx}=\left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot\frac{dN_B}{dx}$$

which becomes

$$\mathrm{E}=\left(\frac{kT}{e}\right)\cdot\frac{1}{N_B}\cdot N_B(0)\cdot\left(\frac{-a}{x_B}\right)\cdot\exp\left(\frac{-ax}{x_B}\right)$$

$$=\left(\frac{kT}{e}\right)\cdot\left(\frac{-a}{x_B}\right)\cdot\frac{1}{N_B}\cdot N_B$$

or

$$\mathrm{E}=-\left(\frac{a}{x_B}\right)\left(\frac{kT}{e}\right)$$

which is a constant.

(b) The electric field is in the negative x-direction which will aid the flow of minority carrier electrons across the base.

(c)

$$J_n=e\mu_nn\mathrm{E}+eD_n\frac{dn}{dx}$$

Assuming no recombination in the base, J_n will be a constant across the base. Then

$$\frac{dn}{dx}+\left(\frac{\mu_n}{D_n}\right)\cdot n\mathrm{E}=\frac{J_n}{eD_n}=\frac{dn}{dx}+n\left(\frac{\mathrm{E}}{V_t}\right)$$

where $V_t = \left(\frac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \frac{\mathrm{E}}{V_t}$

The solution is of the form

$$n_H = n_H(0)\exp(-Ax)$$

The particular solution is found from

$$n_P \cdot A = B$$

where $B = \frac{J_n}{eD_n}$

The particular solution is then

$$n_P = \frac{B}{A} = \frac{\left(\frac{J_n}{eD_n}\right)}{\left(\frac{\mathrm{E}}{V_t}\right)} = \frac{J_n V_t}{eD_n \mathrm{E}} = \frac{J_n}{e\mu_n \mathrm{E}}$$

The total solution is then

$$n = \frac{J_n}{e\mu_n \mathrm{E}} + n_H(0)\exp(-Ax)$$

and

$$n(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right)$$

Then

$$n_H(0) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n \mathrm{E}}$$

12.45

(a) For $N_C = 2\times10^{15}\text{ cm}^{-3}$,

$$BV_{BC0} \cong 180\text{ V}$$

(b) $\beta = \frac{\alpha}{1-\alpha} = \frac{0.9930}{1-0.9930} = 141.86$

$$BV_{EC0} = \frac{BV_{BC0}}{\sqrt[n]{\beta}} = \frac{180}{\sqrt[3]{141.86}} = 34.5\text{ V}$$

(c) For $N_B = 5\times10^{16}\text{ cm}^{-3}$,

$$BV_{EB} \cong 19\text{ V}$$

12.46

We want $BV_{CEO} = 60$ V

Then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221\text{ V}$$

For this breakdown voltage, we need

$$N_C \cong 1.5\times10^{15}\text{ cm}^{-3}$$

The depletion width into the collector at this voltage is

$$x_C = x_n$$

$$= \left[\frac{2\in_s (V_{bi}+V_{BC})}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right]^{1/2}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(1.5\times10^{15})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.646\text{ V}$$

and

$$V_{BC} \cong BV_{CEO} = 60\text{ V}$$

so that

$$x_C = \left[\frac{2(11.7)(8.85\times10^{-14})(0.646+60)}{1.6\times10^{-19}} \times\left(\frac{10^{16}}{1.5\times10^{15}}\right)\left(\frac{1}{10^{16}+1.5\times10^{15}}\right)\right]^{1/2}$$

or

$$x_C = 6.75\times10^{-4}\text{ cm} = 6.75\ \mu\text{m}$$

12.47

(a) For $N_C = 8\times10^{15}\text{ cm}^{-3}$,

$$BV_{CB0} \cong 64\text{ V}$$

(b) $V_{pt} = \frac{ex_{B0}^2}{2\in_s}\cdot\frac{N_B(N_C+N_B)}{N_C}$

$$= \frac{(1.6\times10^{-19})(0.50\times10^{-4})^2}{2(11.7)(8.85\times10^{-14})} \times\frac{(5\times10^{16})(8\times10^{15}+5\times10^{16})}{8\times10^{15}}$$

$$V_{pt} = 70.0\text{ V}$$

12.48

(a) $$V_{pt}=\frac{ex_{B0}^2}{2\in_s}\cdot\frac{N_B\left(N_C+N_B\right)}{N_C}$$

$$=\frac{\left(1.6\times10^{-19}\right)\left(0.65\times10^{-4}\right)^2}{2(11.7)\left(8.85\times10^{-14}\right)}\times\frac{\left(2\times10^{16}\right)\left(5\times10^{15}+2\times10^{16}\right)}{5\times15}$$

$$V_{pt}=32.6\text{ V}$$

(b) From Chapter 7,

$$\left|\mathrm{E}_{\max}\right|\cong\left\{\frac{2eV_{pt}}{\in_s}\left(\frac{N_BN_C}{N_B+N_C}\right)\right\}^{1/2}$$

$$=\left\{\frac{2\left(1.6\times10^{-19}\right)(32.6)}{(11.7)\left(8.85\times10^{-14}\right)}\times\left[\frac{\left(5\times10^{15}\right)\left(2\times10^{16}\right)}{5\times10^{15}+2\times10^{16}}\right]\right\}^{1/2}$$

$$\left|\mathrm{E}_{\max}\right|=2.01\times10^5\text{ V/cm}$$

12.49

$$V_{pt}=\frac{ex_{B0}^2}{2\in_s}\cdot\frac{N_B\left(N_C+N_B\right)}{N_C}$$

$$15=\frac{\left(1.6\times10^{-19}\right)\left(x_{B0}^2\right)}{2(11.7)\left(8.85\times10^{-14}\right)}\times\frac{\left(5\times10^{16}\right)\left(3\times10^{15}+5\times10^{16}\right)}{3\times10^{15}}$$

$$\Rightarrow x_{B0}=1.483\times10^{-5}\text{ cm}=0.1483\,\mu\text{m}$$

12.50

We have

$$V_{CE}(sat)=V_t\cdot\ln\left[\frac{I_C\left(1-\alpha_R\right)+I_B}{\alpha_FI_B-I_C\left(1-\alpha_F\right)}\cdot\frac{\alpha_F}{\alpha_R}\right]$$

We can write

$$\exp\left[\frac{V_{CE}(sat)}{0.0259}\right]=\frac{(1)(1-0.2)+I_B}{(0.99)I_B-(1)(1-0.99)}\left(\frac{0.99}{0.20}\right)$$

or

$$\exp\left[\frac{V_{CE}(sat)}{0.0259}\right]=\left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

(a) For $V_{CE}(sat)=0.30$ V, we find

$$\exp\left[\frac{0.30}{0.0259}\right]=1.0726\times10^5$$

$$=\left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

We find

$I_B=0.01014\text{ mA}=10.14\,\mu\text{ A}$

(b) For $V_{CE}(sat)=0.20$ V, we find

$I_B=0.0119\text{ mA}=11.9\,\mu\text{ A}$

(c) For $V_{CE}(sat)=0.10$ V, we find

$I_B=0.105\text{ mA}=105\,\mu\text{ A}$

12.51

For an npn transistor biased in the active mode, we have $V_{BC}<0$, so that

$\exp\left(\frac{V_{BC}}{V_t}\right)\cong0$. Now

$$I_E+I_B+I_C=0\Rightarrow I_B=-\left(I_C+I_E\right)$$

Then we have

$$I_B=-\left\{\alpha_FI_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]+I_{CS}\right\}-\left\{-\alpha_RI_{CS}-I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

or

$$I_B=\left(1-\alpha_F\right)I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]-\left(1-\alpha_R\right)I_{CS}$$

12.52

We can write

$$I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]=\alpha_RI_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E$$

Substituting, we find

$$I_C=\alpha_F\left\{\alpha_RI_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E\right\}-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

From the definition of currents, we have $I_E = -I_C$ for the case of $I_B = 0$. Then

$$I_C = \alpha_F \alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]+\alpha_F I_C$$

$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

When a C-E voltage is applied, then the B-C becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right)\cong 0$.

Then

$$I_C = -\alpha_F\alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$

Finally, we find

$$I_C = I_{CEO} = \frac{I_{CS}(1-\alpha_F\alpha_R)}{1-\alpha_F}$$

12.53

(a) $$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$

$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

For $V_{BE} = 0.2$ V,

$$I_C = (0.992)(5\times10^{-14})\left[\exp\left(\frac{0.20}{0.0259}\right)-1\right]$$

$$-(10^{-13})\left[\exp\left(\frac{V_{BC}}{0.0259}\right)-1\right]$$

$$=1.1197\times10^{-10}$$

$$-(10^{-13})\left[\exp\left(\frac{V_{BC}}{0.0259}\right)-1\right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 1.1197\times10^{-10} - 2.4214\times10^{-5}$$

$$= -2.421\times10^{-5}\text{ A} = -24.21\,\mu\text{A}$$

For $V_{CB} = -V_{BC} = -0.25$ V

$$I_C = 1.1197\times10^{-10} - 1.5561\times10^{-9}$$

$$= -1.44\times10^{-9}\text{ A}$$

For $V_{CB} = -V_{BC} \ge 0$ V

$$I_C = 1.1197\times10^{-10}\text{ A}$$

(b) For $V_{BE} = 0.4$ V,

$$I_C = 2.5277\times10^{-7}$$

$$-(10^{-13})\left[\exp\left(\frac{V_{BC}}{0.0259}\right)-1\right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 2.5277\times10^{-7} - 2.4214\times10^{-5}$$

$$= -2.396\times10^{-5}\text{ A} \cong -24\,\mu\text{A}$$

For $V_{CB} = -V_{BC} = -0.25$ V

$$I_C = 2.5277\times10^{-7} - 1.5561\times10^{-9}$$

$$= 2.51\times10^{-7}\text{ A} = +0.251\,\mu\text{A}$$

For $V_{CB} = -V_{BC} \ge 0$ V

$$I_C = 2.5277\times10^{-7}\text{ A} = 0.2528\,\mu\text{A}$$

(c) For $V_{BE} = 0.6$ V,

$$I_C = 5.7063\times10^{-4}$$

$$-(10^{-13})\left[\exp\left(\frac{V_{BC}}{0.0259}\right)-1\right]$$

For $V_{CB} = -V_{BC} = -0.5$ V

$$I_C = 5.7063\times10^{-4} - 2.4214\times10^{-5}$$

$$= 5.464\times10^{-4}\text{ A} = 0.5464\text{ mA}$$

For $V_{CB} = -V_{BC} \ge -0.25$ V

$$I_C = 5.7063\times10^{-4}\text{ A} = 0.5706\text{ mA}$$

12.54

$$V_{CE}(sat) = V_t \ln\left[\frac{I_C(1-\alpha_R)+I_B}{\alpha_F I_B - (1-\alpha_F)I_C}\cdot\frac{\alpha_F}{\alpha_R}\right]$$

$$=(0.0259)\ln\left[\frac{(5)(1-0.15)+I_B}{(0.975)I_B-(1-0.975)(5)}\left(\frac{0.975}{0.150}\right)\right]$$

$$=(0.0259)\ln\left[\frac{4.25+I_B}{(0.975)I_B-0.125}(6.5)\right]$$

$I_B = 0.15$ A, $V_{CE}(sat) = 0.187$ V
$I_B = 0.25$ A, $V_{CE}(sat) = 0.143$ V
$I_B = 0.50$ A, $V_{CE}(sat) = 0.115$ V
$I_B = 1.0$ A, $V_{CE}(sat) = 0.0956$ V

12.55

(a) (i) $$r_e' = \frac{V_t}{I_E} = \frac{0.0259}{0.25} = 0.1036\text{ k}\Omega$$

$$\tau_e = r_e' C_{je} = (103.6)(0.35\times10^{-12})$$

$$= 3.626\times10^{-11}\text{ s} = 36.26\text{ ps}$$

(ii) $$\tau_b = \frac{x_B^2}{2D_n} = \frac{(0.65\times10^{-4})^2}{2(25)}$$

$$= 8.45\times10^{-11}\text{ s} = 84.5\text{ ps}$$

(iii) $\tau_d = \dfrac{x_{dc}}{\upsilon_s} = \dfrac{2.2\times10^{-4}}{10^7}$

$= 2.2\times10^{-11}\text{ s} = 22\text{ ps}$

(iv) $\tau_c = r_c\left(C_\mu + C_s\right)$

$= (18)(0.020+0.020)\times10^{-12}$

$= 7.2\times10^{-13}\text{ s} = 0.72\text{ ps}$

(b) $\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$

$= 36.26 + 84.5 + 22 + 0.72 = 143.48\text{ ps}$

(c) $f_T = \dfrac{1}{2\pi\tau_{ec}} = \dfrac{1}{2\pi\left(143.48\times10^{-12}\right)}$

$= 1.109\times10^9\text{ Hz} = 1.109\text{ GHz}$

(d) $f_\beta = \dfrac{f_T}{\beta} = \dfrac{1.109\times10^9}{125}$

$= 8.87\times10^6\text{ Hz} = 8.87\text{ MHz}$

12.56

$$\tau_b = \frac{x_B^2}{2D_B} = \frac{\left(0.5\times10^{-4}\right)^2}{2(20)} = 6.25\times10^{-11}\text{ s}$$

We have

$\tau_b = 0.2\tau_{ec}$

so that

$\tau_{ec} = 3.125\times10^{-10}\text{ s}$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(3.125\times10^{-10}\right)}$$

or

$f_T = 5.09\times10^8\text{ Hz} = 509\text{ MHz}$

12.57

We have

$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$

We are given

$\tau_b = 100\text{ ps}$ and $\tau_e = 25\text{ ps}$

We find

$$\tau_d = \frac{x_d}{\upsilon_s} = \frac{1.2\times10^{-4}}{10^7} = 1.2\times10^{-11}\text{ s}$$

or

$\tau_d = 12\text{ ps}$

Also

$\tau_c = r_c C_c = (10)\left(0.1\times10^{-12}\right) = 10^{-12}\text{ s}$

or

$\tau_c = 1\text{ ps}$

Then

$\tau_{ec} = 25 + 100 + 12 + 1 = 138\text{ ps}$

We obtain

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(138\times10^{-12}\right)}$$

$= 1.15\times10^9\text{ Hz}$

or

$f_T = 1.15\text{ GHz}$

Chapter 13

13.1
Sketch

13.2
Sketch

13.3

(a) $V_{pO} = \dfrac{ea^2 N_d}{2\in_s}$

(i) $V_{pO} = \dfrac{(1.6\times10^{-19})(0.40\times10^{-4})^2(3\times10^{16})}{2(13.1)(8.85\times10^{-14})}$

$= 3.312$ V

(ii) $V_{bi} = (0.0259)\ln\left[\dfrac{(3\times10^{16})(2\times10^{18})}{(1.8\times10^{6})^2}\right]$

$= 1.328$ V

$V_p = V_{bi} - V_{pO} = 1.328 - 3.312$

$= -1.984$ V

(b) $h_2 = \left[\dfrac{2\in_s(V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$

(i) h_2

$= \left[\dfrac{2(13.1)(8.85\times10^{-14})(1.328+0-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 2.97\times10^{-5}$ cm $= 0.297\ \mu$m

$a - h_2 = 0.40 - 0.297 = 0.103\ \mu$m

(ii) h_2

$= \left[\dfrac{2(13.1)(8.85\times10^{-14})(1.328+0.5-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 3.35\times10^{-5}$ cm $= 0.335\ \mu$m

$a - h_2 = 0.40 - 0.335 = 0.065\ \mu$m

(iii) h_2

$= \left[\dfrac{2(13.1)(8.85\times10^{-14})(1.328+2.5-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 4.57\times10^{-5}$ cm $= 0.457\ \mu$m

$h_2 > a \Rightarrow a - h_2 = 0$

(c) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

(i) $V_{DS}(sat) = 3.312 - (1.328 - 0)$

$= 1.984$ V

(ii) $V_{DS}(sat) = 3.312 - (1.328 - (-1.0))$

$= 0.984$ V

13.4

(a) $V_{pO} = \dfrac{ea^2 N_d}{2\in_s}$

(i) $V_{pO} = \dfrac{(1.6\times10^{-19})(0.40\times10^{-4})^2(3\times10^{16})}{2(11.7)(8.85\times10^{-14})}$

$= 3.709$ V

(ii) $V_{bi} = (0.0259)\ln\left[\dfrac{(3\times10^{16})(2\times10^{18})}{(1.5\times10^{10})^2}\right]$

$= 0.860$ V

$V_p = V_{bi} - V_{pO} = 0.860 - 3.709$

$= -2.849$ V

(b) $h_2 = \left[\dfrac{2\in_s(V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$

(i) h_2

$= \left[\dfrac{2(11.7)(8.85\times10^{-14})(0.860+0-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 2.42\times10^{-5}$ cm $= 0.242\ \mu$m

$a - h_2 = 0.40 - 0.242 = 0.158\ \mu$m

(ii) h_2

$= \left[\dfrac{2(11.7)(8.85\times10^{-14})(0.860+0.5-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 2.83\times10^{-5}$ cm $= 0.283\ \mu$m

$a - h_2 = 0.40 - 0.283 = 0.117\ \mu$m

(iii) h_2

$= \left[\dfrac{2(11.7)(8.85\times10^{-14})(0.860+2.5-(-0.5))}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$

$h_2 = 4.08\times10^{-5}$ cm $= 0.408\ \mu$m

$h_2 > a \Rightarrow a - h_2 = 0$

(c) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

(i) $V_{DS}(sat) = 3.705 - (0.860 - 0)$

$= 2.845\text{ V}$

(ii) $V_{DS}(sat) = 3.705 - (0.860 - (-1.0))$

$= 1.845\text{ V}$

13.5

(a) $V_{pO} = \dfrac{ea^2 N_a}{2\in_s} \Rightarrow N_a = \dfrac{2\in_s V_{pO}}{ea^2}$

$$N_a = \frac{2(13.1)(8.85\times10^{-14})(2.75)}{(1.6\times10^{-19})(0.65\times10^{-4})^2}$$

$= 9.433\times10^{15}\text{ cm}^{-3}$

(b) $V_{bi} = (0.0259)\ln\left[\dfrac{(9.433\times10^{15})(10^{18})}{(1.8\times10^{6})^2}\right]$

$= 1.280\text{ V}$

$V_p = V_{pO} - V_{bi} = 2.75 - 1.280$

$= 1.47\text{ V}$

(c) $a - h_2 = 0.15 = 0.65 - h_2$

$h_2 = 0.50\ \mu\text{m}$

$$h_2 = \left[\frac{2\in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a}\right]^{1/2}$$

$$(0.50\times10^{-4})^2 = \left[\frac{2(13.1)(8.85\times10^{-14})(1.28 + 0 + V_{GS})}{(1.6\times10^{-19})(9.433\times10^{15})}\right]$$

$2.5\times10^{-9} = (1.5363\times10^{-9})(1.28 + V_{GS})$

$\Rightarrow V_{GS} = 0.347\text{ V}$

(d) $h_2 = \left[\dfrac{2\in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a}\right]^{1/2}$

$$(0.65\times10^{-4})^2 = \left[\frac{2(13.1)(8.85\times10^{-14})(1.28 + V_{SD})}{(1.6\times10^{-19})(9.433\times10^{15})}\right]$$

$(0.65\times10^{-4})^2 = (1.5363\times10^{-9})(1.28 + V_{SD})$

$\Rightarrow V_{SD} = 1.47\text{ V}$

13.6

(a) $N_a = \dfrac{2\in_s V_{pO}}{ea^2}$

$$= \frac{2(11.7)(8.85\times10^{-14})(2.75)}{(1.6\times10^{-19})(0.65\times10^{-4})^2}$$

or

$N_a = 8.425\times10^{15}\text{ cm}^{-3}$

(b) $V_{bi} = (0.0259)\ln\left[\dfrac{(8.425\times10^{15})(10^{18})}{(1.5\times10^{10})^2}\right]$

$= 0.8095\text{ V}$

$V_p = V_{pO} - V_{bi} = 2.75 - 0.8095$

$= 1.9405\text{ V}$

(c) $a - h_2 = 0.15 = 0.65 - h_2$

$\Rightarrow h_2 = 0.50\ \mu\text{m}$

$$h_2 = \left[\frac{2\in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a}\right]^{1/2}$$

$$(0.50\times10^{-4})^2 = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8095 + 0 + V_{GS})}{(1.6\times10^{-19})(8.425\times10^{15})}\right]$$

$2.5\times10^{-9} = (1.536\times10^{-9})(0.8095 + V_{GS})$

$\Rightarrow V_{GS} = 0.8178\text{ V}$

(d) $h_2 = \left[\dfrac{2\in_s (V_{bi} + V_{SD} + V_{GS})}{eN_a}\right]^{1/2}$

$$(0.65\times10^{-4})^2 = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8095 + V_{SD})}{(1.6\times10^{-19})(8.425\times10^{15})}\right]$$

$(0.65\times10^{-4})^2 = (1.536\times10^{-9})(0.8095 + V_{SD})$

$\Rightarrow V_{SD} = 1.94\text{ V}$

13.7

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2\times10^{16})(3\times10^{18})}{(1.5\times10^{10})^2}\right]$$

$= 0.860\text{ V}$

$V_p = V_{pO} - V_{bi}$

$3.0 = V_{pO} - 0.860 \Rightarrow V_{pO} = 3.86\text{ V}$

Now $a = \left[\dfrac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(3.86)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$

$\cong 5.0\times10^{-5}\text{ cm} = 0.50\ \mu\text{m}$

(b) $V_{pO} = 3.86\text{ V}$

(c) $V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

(i) $V_{SD}(sat) = 3.86 - 0.86 = 3.0\text{ V}$

(ii) $V_{SD}(sat) = 3.86 - (0.86 + 1.5) = 1.5\text{ V}$

13.8

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2\times10^{16})(3\times10^{18})}{(1.8\times10^{6})^2}\right]$$

$= 1.328\text{ V}$

$V_p = V_{pO} - V_{bi}$

$3.0 = V_{pO} - 1.328 \Rightarrow V_{pO} = 4.328\text{ V}$

$$a = \left[\frac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(4.328)}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$

$= 5.60\times10^{-5}\text{ cm} = 0.560\ \mu\text{m}$

(b) $V_{pO} = 4.328\text{ V}$

(c) $V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

(i) $V_{SD}(sat) = 4.328 - (1.328 + 0) = 3.0\text{ V}$

(ii) $V_{SD}(sat) = 4.328 - (1.328 + 1.5) = 1.5\text{ V}$

13.9

(a) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(4\times10^{16})(4\times10^{18})}{(1.5\times10^{10})^2}\right]$$

$= 0.886\text{ V}$

We find

$5 = V_{pO} - 0.886 \Rightarrow V_{pO} = 5.886\text{ V}$

$$a = \left[\frac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(5.886)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

$= 4.36\times10^{-5}\text{ cm} = 0.436\ \mu\text{m}$

(b) (i) $V_{pO} = 5.886\text{ V}$

(ii) $V_p = V_{bi} - V_{pO} = 0.886 - 5.886 = -5.0\text{ V}$

13.10

(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{15})(10^{18})}{(1.8\times10^{6})^2}\right]$$

$= 1.264\text{ V}$

$V_{SD}(sat) = V_{pO} - (V_{bi} + V_{GS})$

$3.5 = V_{pO} - (1.264 + 1.0) \Rightarrow V_{pO} = 5.764\text{ V}$

$$a = \left[\frac{2\in_s V_{pO}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(5.764)}{(1.6\times10^{-19})(5\times10^{15})}\right]^{1/2}$$

$= 1.293\times10^{-4}\text{ cm} = 1.293\ \mu\text{m}$

(b)

(i) $V_{pO} = 5.764\text{ V}$

(ii) $V_p = V_{pO} - V_{bi} = 5.764 - 1.264$

$= 4.5\text{ V}$

13.11

(a)

$$I_{P1} = \frac{\mu_n (eN_d)^2 W a^3}{6\in_s L}$$

$$= \frac{(1000)\left[(1.6\times10^{-19})(10^{16})\right]^2}{6(11.7)(8.85\times10^{-14})} \times \frac{(400\times10^{-4})(0.5\times10^{-4})^3}{20\times10^{-4}}$$

or

$I_{P1} = 1.03\text{ mA}$

(b)

$$V_{PO} = \frac{ea^2 N_d}{2\in_s}$$

$$= \left[\frac{(1.6\times10^{-19})(0.5\times10^{-4})^2(10^{16})}{2(11.7)(8.85\times10^{-14})}\right]$$

or

$V_{PO} = 1.93\text{ V}$

Also

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(10^{16})}{(1.5\times10^{10})^2}\right] = 0.874\text{ V}$$

Now

$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$

$= 1.93 - 0.874 + V_{GS}$

or

$V_{DS}(sat) = 1.056 + V_{GS}$

We have

$V_P = V_{bi} - V_{PO} = 0.874 - 1.93 = -1.056$ V

Then

(i) For $V_{GS} = 0$, $V_{DS}(sat) = 1.06$ V

(ii) For $V_{GS} = \frac{1}{4}V_P = -0.264$ V,

$V_{DS}(sat) = 0.792$ V

(iii) For $V_{GS} = \frac{1}{2}V_P = -0.528$ V,

$V_{DS}(sat) = 0.528$ V

(iv) For $V_P = \frac{3}{4}V_P = -0.792$ V,

$V_{DS}(sat) = 0.264$ V

(c)

$$I_{D1}(sat) = I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

$$= (1.03)\left[1 - 3\left(\frac{0.874 - V_{GS}}{1.93}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{0.874 - V_{GS}}{1.93}}\right)\right]$$

(i) For $V_{GS} = 0$, $I_{D1}(sat) = 0.258$ mA

(ii) For $V_{GS} = -0.264$ V,

$I_{D1}(sat) = 0.141$ mA

(iii) For $V_{GS} = -0.528$ V,

$I_{D1}(sat) = 0.0608$ mA

(iv) For $V_{GS} = -0.792$ V,

$I_{D1}(sat) = 0.0148$ mA

13.12

$$g_d = G_{O1}\left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)^{1/2}\right]$$

where

$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03\times10^{-3})}{1.93}$$

or

$G_{O1} = 1.60\times10^{-3}\text{ S} = 1.60$ mS

Then

V_{GS}	$(V_{bi} - V_{GS})/V_{PO}$	g_d (mS)
0	0.453	0.523
-0.264	0.590	0.371
-0.528	0.726	0.237
-0.792	0.863	0.114
-1.056	1.0	0

13.13

n-channel JFET - GaAs

(a)

$$G_{O1} = \frac{e\mu_n N_d W a}{L}$$

$$= \frac{(1.6\times10^{-19})(8000)(2\times10^{16})}{10\times10^{-4}} \times (30\times10^{-4})(0.35\times10^{-4})$$

or

$G_{O1} = 2.69\times10^{-3}$ S

(b)

$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in_s}$$

$$= \frac{(1.6\times10^{-19})(0.35\times10^{-4})^2(2\times10^{16})}{2(13.1)(8.85\times10^{-14})}$$

or

$V_{PO} = 1.69$ V

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{18})(2\times10^{16})}{(1.8\times10^6)^2}\right]$$

or

$V_{bi} = 1.34$ V

Then

$V_P = V_{bi} - V_{PO} = 1.34 - 1.69 = -0.35$ V

We then obtain

$V_{DS}(sat) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$

For $V_{GS} = 0$, $V_{DS}(sat) = 0.35$ V

For $V_{GS} = \frac{1}{2}V_P = -0.175$ V,

$V_{DS}(sat) = 0.175$ V

(c)

$$I_{D1}(sat) = I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

where

$$I_{P1} = \frac{\mu_n (eN_d)^2 Wa^3}{6 \in_s L}$$

$$= \frac{(8000)\left[(1.6\times10^{-19})(2\times10^{16})\right]^2}{6(13.1)(8.85\times10^{-14})} \times \frac{(30\times10^{-4})(0.35\times10^{-4})^3}{10\times10^{-4}}$$

or

$$I_{P1} = 1.515\,\text{mA}$$

Then

$$I_{D1}(sat) = (1.515)\left[1 - 3\left(\frac{1.34 - V_{GS}}{1.69}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{1.34 - V_{GS}}{1.69}}\right)\right] \text{ (mA)}$$

For $V_{GS} = 0$, $I_{D1}(sat) = 0.0506$ mA

and

For $V_{GS} = -0.175$ V,

$$I_{D1}(sat) = 0.0124 \text{ mA}$$

13.14

$$g_{mS} = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)$$

We have

$I_{P1} = 1.03\,\text{mA}$, $V_{PO} = 1.93$ V, $V_{bi} = 0.874$ V

The maximum transconductance occurs when $V_{GS} = 0$. Then

$$g_{mS}(\max) = \frac{3(1.03)}{1.93}\left(1 - \sqrt{\frac{0.874}{1.93}}\right)$$

or

$$g_{mS}(\max) = 0.524\,\text{mS}$$

For $W = 400\,\mu$m, we have

$$g_{mS}(\max) = \frac{0.524}{400\times10^{-4}}$$

or

$$g_{mS}(\max) = 13.1\,\text{mS/cm} = 1.31 \text{ mS/mm}$$

13.15

The maximum transconductance occurs for $V_{GS} = 0$, so we have

(a) $$g_{mS}(\max) = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

which can be written as

$$g_{mS}(\max) = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

We found

$G_{O1} = 2.69\,\text{mS}$, $V_{bi} = 1.34$ V, $V_{PO} = 1.69$ V

Then

$$g_{mS}(\max) = (2.69)\left(1 - \sqrt{\frac{1.34}{1.69}}\right)$$

or

$$g_{mS}(\max) = 0.295 \text{ mS}$$

This is for a channel length of $L = 10\,\mu$ m.

(b) If the channel length is reduced to $L = 2\,\mu$ m, then

$$g_{mS}(\max) = (0.2947)\left(\frac{10}{2}\right) = 1.47\,\text{mS}$$

13.16

n-channel MESFET - GaAs

(a)

$$V_{PO} = \frac{ea^2 N_d}{2 \in_s}$$

$$= \frac{(1.6\times10^{-19})(0.5\times10^{-4})^2(1.5\times10^{16})}{2(13.1)(8.85\times10^{-14})}$$

or

$$V_{PO} = 2.59\text{ V}$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right) = (0.0259)\ln\left(\frac{4.7\times10^{17}}{1.5\times10^{16}}\right)$$

or

$$\phi_n = 0.0892\text{ V}$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811\text{ V}$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$V_T = -1.78\text{ V}$$

(b)If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

13.17

n-channel MESFET - GaAs

(a) We want $V_T = +0.10\text{ V}$

Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_c}{N_d}\right) - \frac{ea^2 N_d}{2\in_s}$$

which can be written as

$$(0.0259)\ln\left(\frac{4.7\times10^{17}}{N_d}\right)$$

$$+\frac{(1.6\times10^{-19})(0.35\times10^{-4})^2 N_d}{2(13.1)(8.85\times10^{-14})} = 0.89 - 0.10$$

or

$$(0.0259)\ln\left(\frac{4.7\times10^{17}}{N_d}\right)$$

$$+(8.453\times10^{-17})N_d = 0.79$$

By trial and error,

$$N_d = 8.1\times10^{15}\text{ cm}^{-3}$$

(b) At $T = 400\text{ K}$

$$\frac{N_c(400)}{N_c(300)} = \left(\frac{400}{300}\right)^{3/2} = 1.54$$

Then

$$N_c(400) = (4.7\times10^{17})(1.54)$$

$$= 7.24\times10^{17}\text{ cm}^{-3}$$

Also

$$V_t = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453)\ln\left(\frac{7.24\times10^{17}}{8.1\times10^{15}}\right)$$

$$-(8.453\times10^{-17})(8.1\times10^{15})$$

which becomes

$$V_T = +0.050\text{ V}$$

13.18

(a) $$a = \left[\frac{2\in_s V_{pO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(1.5)}{(1.6\times10^{19})(2\times10^{16})}\right]^{1/2}$$

$$= 3.30\times10^{-5}\text{ cm} = 0.330\ \mu\text{m}$$

(b) $V_T = V_{bi} - V_{pO}$

We find

$$\phi_n = V_t \ln\left(\frac{N_c}{N_d}\right) = (0.0259)\ln\left(\frac{4.7\times10^{17}}{2\times10^{16}}\right)$$

$$= 0.0818\text{ V}$$

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.0818 = 0.788\text{ V}$$

Then

$$V_T = 0.788 - 1.5 = -0.712\text{ V}$$

(c) $$h_2 = \left[\frac{2\in_s (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85\times10^{-14})(V_{bi} + V_{DS} - V_{GS})}{(1.6\times10^{-19})(2\times10^{16})}\right]^{1/2}$$

(i) $$h_2 = [(7.246\times10^{-10})(0.788 + 0 - 0.4)]^{1/2}$$

$$= 1.677\times10^{-5}\text{ cm} = 0.1677\ \mu\text{m}$$

$$a - h_2 = 0.330 - 0.1677 = 0.1623\ \mu\text{m}$$

(ii)

$$h_2 = [(7.246\times10^{-10})(0.788 + 1.0 - 0.4)]^{1/2}$$

$$= 3.171\times10^{-5}\text{ cm} = 0.3171\ \mu\text{m}$$

$$a - h_2 = 0.330 - 0.3171 = 0.0129\ \mu\text{m}$$

(iii)

$$h_2 = [(7.246\times10^{-10})(0.788 + 4.0 - 0.4)]^{1/2}$$

$$= 5.64\times10^{-5}\text{ cm} = 0.564\ \mu\text{m}$$

$$h_2 > a \Rightarrow a - h_2 = 0$$

13.19

(a) $$V_{pO} = \frac{ea^2 N_d}{2\in_s}$$

$$= \frac{(1.6\times10^{-19})(0.50\times10^{-4})^2(5\times10^{15})}{2(13.1)(8.85\times10^{-14})}$$

$$= 0.8626\text{ V}$$

We find

$$\phi_n = (0.0259)\ln\left(\frac{4.7\times10^{17}}{5\times10^{15}}\right)$$

$$= 0.1177\text{ V}$$

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.1177$$

$$= 0.7523\text{ V}$$

$$V_T = V_{bi} - V_{pO} = 0.7523 - 0.8626$$

$$= -0.1103\text{ V}$$

(b) $\phi_n = (0.0259)\ln\left(\frac{4.7\times 10^{17}}{3\times 10^{16}}\right) = 0.0713\text{ V}$

$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.0713$
$= 0.7987\text{ V}$

$V_T = V_{bi} - V_{pO}$

or $V_{pO} = V_{bi} - V_T = 0.7987 - (-0.1103)$
$= 0.909\text{ V}$

Then

$$a = \left[\frac{2\in_s V_{pO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85\times 10^{-14})(0.909)}{(1.6\times 10^{-19})(3\times 10^{16})}\right]^{1/2}$$

$$= 2.095\times 10^{-5}\text{ cm} = 0.2095\ \mu\text{m}$$

13.20

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want $V_T = 0.5\text{ V}$, so

$0.5 = 0.85 - \phi_n - V_{PO}$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7\times 10^{17}}{N_d}\right)$$

and

$$V_{PO} = \frac{ea^2 N_d}{2\in_s}$$

$$= \frac{(1.6\times 10^{-19})(0.25\times 10^{-4})^2 N_d}{2(13.1)(8.85\times 10^{-14})}$$

or

$V_{PO} = (4.31\times 10^{-17})N_d$

Then

$$0.5 = 0.85 - (0.0259)\ln\left(\frac{4.7\times 10^{17}}{N_d}\right) - (4.31\times 10^{-17})N_d$$

By trial and error

$N_d = 5.45\times 10^{15}\text{ cm}^{-3}$

13.21

n-channel MESFET - silicon

(a) For a gold contact, $\phi_{Bn} = 0.82\text{ V}$.

We find

$$\phi_n = (0.0259)\ln\left(\frac{2.8\times 10^{19}}{10^{16}}\right) = 0.206\text{ V}$$

and

$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614\text{ V}$

With $V_{DS} = 0$ and $V_{GS} = 0.35\text{ V}$, we find

$$a - h = 0.075\times 10^{-4}$$

$$= a - \left[\frac{2\in_s (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so that

$$a = 0.075\times 10^{-4} + \left[\frac{2(11.7)(8.85\times 10^{-14})(0.614 - 0.35)}{(1.6\times 10^{-19})(10^{16})}\right]^{1/2}$$

or

$a = 0.26\times 10^{-4}\text{ cm} = 0.26\ \mu\text{m}$

Now

$$V_T = V_{bi} - V_{PO} = 0.614 - \frac{ea^2 N_d}{2\in_s}$$

or

$$V_T = 0.614 - \frac{(1.6\times 10^{-19})(0.26\times 10^{-4})^2(10^{16})}{2(11.7)(8.85\times 10^{-14})}$$

We obtain

$V_T = 0.092\text{ V}$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= (V_{bi} - V_T) - (V_{bi} - V_{GS})$$

or

$V_{DS}(sat) = V_{GS} - V_T = 0.35 - 0.092$

which yields

$V_{DS}(sat) = 0.258\text{ V}$

13.22

(a) $\phi_n = (0.0259)\ln\left(\frac{4.7\times 10^{17}}{2\times 10^{16}}\right)$
$= 0.0818\text{ V}$

(i) $V_{bi} = \phi_{Bn} - \phi_n = 0.90 - 0.0818$
$= 0.818\text{ V}$

(ii) $V_{pO} = \frac{ea^2 N_d}{2\in_s}$

$$= \frac{(1.6\times 10^{-19})(0.65\times 10^{-4})^2(2\times 10^{16})}{2(13.1)(8.85\times 10^{-14})}$$

$= 5.83\text{ V}$

(iii) $V_T = V_{bi} - V_{pO} = 0.818 - 5.83$
$= -5.012\text{ V}$

(b) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$

(i) $V_{DS}(sat) = 5.83 - (0.818 - (-1.0))$
$= 4.01 \text{ V}$

(ii) $V_{DS}(sat) = 5.83 - (0.818 - (-2.0))$
$= 3.01 \text{ V}$

(iii) $V_{DS}(sat) = 5.83 - (0.818 - (-3.0))$
$= 2.01 \text{ V}$

13.23

(a) $k_n = \dfrac{\mu_n \in_s W}{2aL}$

$$= \frac{(6500)(13.1)(8.85\times10^{-14})(12\times10^{-4})}{2(0.25\times10^{-4})(1.5\times10^{-4})}$$

$= 1.206\times10^{-3} \text{ A/V}^2 = 1.206 \text{ mA/V}^2$

(b) $I_{D1}(sat) = k_n (V_{GS} - V_T)^2$

(i) $I_{D1}(sat) = (1.206)(0.25 - 0.15)^2$
$= 0.01206 \text{ mA} = 12.06\ \mu\text{A}$

(ii) $I_{D1}(sat) = (1.206)(0.45 - 0.15)^2$
$= 0.1085 \text{ mA}$

(c) $V_{DS}(sat) = V_{GS} - V_T$

(i) $V_{DS}(sat) = 0.25 - 0.15 = 0.10 \text{ V}$

(ii) $V_{DS}(sat) = 0.45 - 0.15 = 0.30 \text{ V}$

13.24

(a) $g_{ms} = \dfrac{\partial I_D}{\partial V_{GS}} = \dfrac{\partial}{\partial V_{GS}}\left[k_n (V_{GS} - V_T)^2\right]$

$= 2k_n (V_{GS} - V_T)$

$1.25 = 2k_n (0.45 - 0.15)$

$\Rightarrow k_n = 2.083 \text{ mA/V}^2$

$k_n = \dfrac{\mu_n \in_s W}{2aL}$

$$2.083\times10^{-3} = \frac{(6500)(13.1)(8.85\times10^{-14})W}{2(0.25\times10^{-4})(1.5\times10^{-4})}$$

$\Rightarrow W = 2.073\times10^{-3} \text{ cm} = 20.73\ \mu\text{m}$

(b) $I_{D1}(sat) = k_n (V_{GS} - V_T)^2$

(i) $I_{D1}(sat) = (2.083)(0.25 - 0.15)^2$
$= 0.02083 \text{ mA} = 20.83\ \mu\text{A}$

(ii) $I_{D1}(sat) = (2.083)(0.45 - 0.15)^2$
$= 0.1875 \text{ mA}$

13.25
Plot

13.26
Plot

13.27

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(3\times10^{16})}{(1.5\times10^{10})^2}\right]$$

$= 0.8424 \text{ V}$

$V_{pO} = \dfrac{ea^2 N_d}{2 \in_s}$

$$= \frac{(1.6\times10^{-19})(0.50\times10^{-4})^2(3\times10^{16})}{2(11.7)(8.85\times10^{-14})}$$

$= 5.795 \text{ V}$

(a) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$
$= 5.795 - 0.8424 = 4.953 \text{ V}$

$$\Delta L = \left[\frac{2 \in_s (V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(10 - 4.953)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$\Delta L = 4.666\times10^{-5} \text{ cm}$

Now

$\dfrac{L'}{L} = 1 - \dfrac{(1/2)\Delta L}{L} = 0.90$

$L = \dfrac{\Delta L}{2(0.10)} = \dfrac{4.666\times10^{-5}}{2(0.10)}$

$L = 2.333\times10^{-4} \text{ cm} = 2.333\ \mu\text{m}$

(b) $V_{DS}(sat) = V_{pO} - (V_{bi} - V_{GS})$
$= 5.795 - (0.8424 + 3)$
$= 1.953 \text{ V}$

$$\Delta L = \left[\frac{2(11.7)(8.85\times10^{-14})(10 - 1.953)}{(1.6\times10^{-19})(3\times10^{16})}\right]^{1/2}$$

$= 5.892\times10^{-5} \text{ cm}$

Then

$L = \dfrac{\Delta L}{2(0.10)} = \dfrac{5.892\times10^{-5}}{2(0.10)}$

$= 2.946\times10^{-4} \text{ cm} = 2.946\ \mu\text{m}$

13.28

We have that

$$I'_{D1} = I_{D1}\left(\frac{L}{L-(1/2)\Delta L}\right)$$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$.

We can write

$$I'_{D1}(sat) = I_{D1}(sat)\cdot\frac{1}{1-\frac{1}{2}\cdot\frac{\Delta L}{L}}$$

If $\Delta L << L$, then

$$I'_{D1}(sat) \cong I_{D1}(sat)\left[1+\frac{1}{2}\cdot\frac{\Delta L}{L}\right]$$

We have that

$$\Delta L = \left[\frac{2\in_s (V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2\in_s V_{DS}}{eN_d}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

which can be written as

$$\Delta L = V_{DS}\left[\frac{2\in_s}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat)(1+\lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L}\left[\frac{2\in_s}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

The parameter λ is not independent of V_{DS}.

Define

$$x \equiv \frac{V_{DS}}{V_{DS}(sat)}$$

and consider the function

$$f = \frac{1}{x}\left(1-\frac{1}{x}\right)$$

which is directly proportional to λ. Then

x	$f(x)$
1.5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

13.29

(a) Saturation occurs when $\mathrm{E} = 1\times10^4$ V/cm. As a first approximation, let

$$\mathrm{E} = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = \mathrm{E}\cdot L = (10^4)(2\times10^{-4}) = 2\text{ V}$$

(b) We have that

$$h_2 = h_{sat} = \left[\frac{2\in_s (V_{bi}+V_{DS}-V_{GS})}{eN_d}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{18})(4\times10^{16})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.8915\text{ V}$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8915+2)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

or

$$h_{sat} = 0.306\times10^{-4}\text{ cm} = 0.306\ \mu\text{m}$$

(c) We then find

$$I_{D1}(sat) = eN_d\upsilon_{sat}(a-h_{sat})W$$

$$= (1.6\times10^{-19})(4\times10^{16})(10^7)$$
$$\times(0.50-0.306)(10^{-4})(30\times10^{-4})$$

or

$$I_{D1}(sat) = 3.72\text{ mA}$$

(d) For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1}\left[1-3\left(\frac{V_{bi}}{V_{PO}}\right)\left(1-\frac{2}{3}\sqrt{\frac{V_{bi}}{V_{PO}}}\right)\right]$$

Now

$$I_{P1} = \frac{\mu_n(eN_d)^2Wa^3}{6\in_s L}$$

$$= \frac{(1000)[(1.6\times10^{-19})(4\times10^{16})]^2}{6(11.7)(8.85\times10^{-14})}$$
$$\times\frac{(30\times10^{-4})(0.5\times10^{-4})^3}{(2\times10^{-4})}$$

or

$$I_{P1} = 12.36\text{ mA}$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2 \in_s}$$

$$= \frac{(1.6\times10^{-19})(0.5\times10^{-4})^2(4\times10^{16})}{2(11.7)(8.85\times10^{-14})}$$

or

$$V_{PO} = 7.726\text{ V}$$

Then

$$I_{D1}(sat) = (12.36)\left[1 - 3\left(\frac{0.8915}{7.726}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{0.8915}{7.726}}\right)\right]$$

or

$$I_{D1}(sat) = 9.05\text{ mA}$$

13.30

(a) If $L = 1\,\mu$m, then saturation will occur when

$$V_{DS} = \text{E} \cdot L = (10^4)(1\times10^{-4}) = 1\text{ V}$$

We find

$$h_2 = h_{sat} = \left[\frac{2\in_s (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$

We have $V_{bi} = 0.8915$ V and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)(8.85\times10^{-14})(0.8915+1)}{(1.6\times10^{-19})(4\times10^{16})}\right]^{1/2}$$

or

$$h_{sat} = 0.247\times10^{-4}\text{ cm} = 0.247\,\mu\text{m}$$

Then

$$I_{D1}(sat) = eN_d \upsilon_{sat}(a - h_{sat})W$$

$$= (1.6\times10^{-19})(4\times10^{16})(10^7) \times (0.50 - 0.247)(10^{-4})(30\times10^{-4})$$

or

$$I_{D1}(sat) = 4.86\text{ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(sat) = (9.05)\left(\frac{2}{1}\right) = 18.1\text{mA}$$

(b) If velocity saturation occurs, then the relation $I_{D1}(sat) \propto (1/L)$ does not apply.

13.31

(a)

$$\upsilon = \mu_n \text{E} = (8000)(5\times10^3) = 4\times10^7\text{ cm/s}$$

Then

$$t_d = \frac{L}{\upsilon} = \frac{2\times10^{-4}}{4\times10^7} = 5\times10^{-12}\text{ s}$$

or

$$t_d = 5\text{ ps}$$

(b) Assume $\upsilon = \upsilon_{sat} = 10^7$ cm/s

Then

$$t_d = \frac{L}{\upsilon_{sat}} = \frac{2\times10^{-4}}{10^7} = 2\times10^{-11}\text{ s}$$

or

$$t_d = 20\text{ ps}$$

13.32

(a)

$$\upsilon = \mu_n \text{E} = (1000)(10^4) = 10^7\text{ cm/s}$$

Then

$$t_d = \frac{L}{\upsilon} = \frac{2\times10^{-4}}{10^7} = 2\times10^{-11}\text{ s}$$

or

$$t_d = 20\text{ ps}$$

(b) For $\upsilon = \upsilon_{sat} = 10^7$ cm/s

$$t_d = \frac{L}{\upsilon_{sat}} = \frac{2\times10^{-4}}{10^7} = 2\times10^{-11}\text{ s}$$

or

$$t_d = 20\text{ ps}$$

13.33

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{18})(3\times10^{16})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.884\text{ V}$$

Also $V_{PO} = \dfrac{ea^2 N_d}{2 \in_s}$

$$= \left[\frac{(1.6\times10^{-19})(0.3\times10^{-4})^2(3\times10^{16})}{2(11.7)(8.85\times10^{-14})}\right]$$

or
$V_{PO} = 2.086\text{ V}$
Then
$V_P = 0.884 - 2.086 = -1.20\text{ V}$
Let $V_{GS} = -1.20\text{ V}$
Now
$$x_n = \left[\frac{2\in_s (V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$
$$= \left[\frac{2(11.7)(8.85\times10^{-14})}{(1.6\times10^{-19})} \times \frac{(0.884 + V_{DS} - (-1.20))}{(3\times10^{16})}\right]^{1/2}$$
or
$$x_n = \left[(4.314\times10^{-10})(2.084 + V_{DS})\right]^{1/2}$$
(a) For $V_{DS} = 0$, $x_n = 0.30\ \mu\text{m}$
(b) For $V_{DS} = 1\text{ V}$, $x_n = 0.365\ \mu\text{m}$
(c) For $V_{DS} = 5\text{ V}$, $x_n = 0.553\ \mu\text{m}$

The depletion region volume at the drain is
$$Vol = (a)\left(\frac{L}{2}\right)(W) + (x_n)(2a)(W)$$
$$= (0.3\times10^{-4})\left(\frac{2.4\times10^{-4}}{2}\right)(30\times10^{-4}) + (x_n)(0.6\times10^{-4})(30\times10^{-4})$$
or
$$Vol = 10.8\times10^{-12} + x_n(18\times10^{-8})$$
(a) For $V_{DS} = 0$, $Vol = 1.62\times10^{-11}\text{ cm}^3$
(b) For $V_{DS} = 1\text{ V}$, $Vol = 1.737\times10^{-11}\text{ cm}^3$
(c) For $V_{DS} = 5\text{ V}$, $Vol = 2.075\times10^{-11}\text{ cm}^3$

The generation current at the drain is
$$I_{DG} = e\left(\frac{n_i}{2\tau_O}\right)\cdot Vol$$
$$= (1.6\times10^{-19})\left[\frac{1.5\times10^{10}}{2(5\times10^{-8})}\right]\cdot Vol$$
or
$$I_{DG} = (2.4\times10^{-2})\cdot Vol$$
(a) For $V_{DS} = 0$, $I_{DG} = 0.39\text{ pA}$
(b) For $V_{DS} = 1\text{ V}$, $I_{DG} = 0.42\text{ pA}$
(c) For $V_{DS} = 5\text{ V}$, $I_{DG} = 0.50\text{ pA}$

13.34
(a) The ideal transconductance for $V_{GS} = 0$ is
$$g_{mS} = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$
where
$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$
$$= \frac{(1.6\times10^{-19})(4500)(7\times10^{16})}{1.5\times10^{-4}} \times (5\times10^{-4})(0.3\times10^{-4})$$
or
$G_{O1} = 5.04\text{ mS}$
We find
$$V_{PO} = \frac{ea^2 N_d}{2\in_s}$$
$$= \frac{(1.6\times10^{-19})(0.3\times10^{-4})^2(7\times10^{16})}{2(13.1)(8.85\times10^{-14})}$$
or
$V_{PO} = 4.347\text{ V}$
We have
$$\phi_n = (0.0259)\ln\left(\frac{4.7\times10^{17}}{7\times10^{16}}\right) = 0.049\text{ V}$$
so that
$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.049 = 0.841\text{ V}$
Then
$$g_{mS} = (5.04)\left(1 - \sqrt{\frac{0.841}{4.347}}\right)$$
or
$g_{mS} = 2.82\text{ mS}$
(b) With a source resistance
$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = \frac{1}{1 + g_m r_s}$$
For
$$\frac{g'_m}{g_m} = 0.80 = \frac{1}{1 + (2.823)r_s}$$
we obtain
$r_s = 88.6\ \Omega$
(c)
$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n n)A}$$

so

$$L = (88.56)(1.6\times10^{-19})(4500)(7\times10^{16}) \times (0.3\times10^{-4})(5\times10^{-4})$$

or

$$L = 0.67\times10^{-4} \text{ cm} = 0.67\ \mu\text{m}$$

13.35

Considering the capacitance charging time, we have

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\in_s WL}{a}$$

$$= \frac{(13.1)(8.85\times10^{-14})(5\times10^{-4})(1.5\times10^{-4})}{0.3\times10^{-4}}$$

or

$$C_G = 2.9\times10^{-15} \text{ F}$$

We must use g'_m, so we obtain

$$f_T = \frac{(2.82\times10^{-3})(0.80)}{2\pi(2.9\times10^{-15})} = 1.238\times10^{11} \text{ Hz}$$

We can also write

$$f_T = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = \frac{1}{2\pi f_T}$$

so

$$\tau_C = \frac{1}{2\pi(1.238\times10^{11})} = 1.285\times10^{-12} \text{ s}$$

The channel transit time is

$$t_t = \frac{1.5\times10^{-4}}{10^7} = 1.5\times10^{-11} \text{ s}$$

The total time constant is

$$\tau = 1.5\times10^{-11} + 1.285\times10^{-12}$$
$$= 1.629\times10^{-11} \text{ s}$$

Taking into account the channel transit time and the capacitance charging time, we find

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.629\times10^{-11})}$$

or

$$f_T = 9.77\times10^9 \text{ Hz} = 9.77 \text{ GHz}$$

13.36

(a) For constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi\in_s L^2}$$

$$= \frac{(1.6\times10^{-19})(7500)(4\times10^{16})(0.30\times10^{-4})^2}{2\pi(13.1)(8.85\times10^{-14})(1.2\times10^{-4})^2}$$

$$f_T = 4.12\times10^{11} \text{ Hz} = 412 \text{ GHz}$$

(b) For saturation velocity model

$$f_T = \frac{\upsilon_{sat}}{2\pi L} = \frac{10^7}{2\pi(1.2\times10^{-4})}$$

$$f_T = 1.33\times10^{10} \text{ Hz} = 13.3 \text{ GHz}$$

13.37

$$f_T = \frac{e\mu_n N_d a^2}{2\pi\in_s L^2}$$

$$= \frac{(1.6\times10^{-19})(1000)(2\times10^{16})(0.40\times10^{-4})^2}{2\pi(11.7)(8.85\times10^{-14})L^2}$$

$$f_T = \frac{786.975}{L^2}$$

(a) $$f_T = \frac{786.975}{(3\times10^{-4})^2}$$

$$= 8.74\times10^9 \text{ Hz} = 8.74 \text{ GHz}$$

(b) $$f_T = \frac{786.975}{(1.5\times10^{-4})^2}$$

$$= 3.50\times10^{10} \text{ Hz} = 35.0 \text{ GHz}$$

13.38

$$f_T = \frac{e\mu_p N_a a^2}{2\pi\in_s L^2}$$

or

$$L = \left[\frac{e\mu_p N_a a^2}{2\pi\in_s f_T}\right]^{1/2}$$

$$= \left[\frac{(1.6\times10^{-19})(420)(2\times10^{16})(0.40\times10^{-4})^2}{2\pi(11.7)(8.85\times10^{-14})f_T}\right]^{1/2}$$

$$L = \frac{18.18}{\sqrt{f_T}}$$

(a) $L = \dfrac{18.18}{\sqrt{5\times 10^{9}}}$

$= 2.57\times 10^{-4}\text{ cm} = 2.57\ \mu\text{m}$

(b) $L = \dfrac{18.18}{\sqrt{12\times 10^{9}}}$

$= 1.66\times 10^{-4}\text{ cm} = 1.66\ \mu\text{m}$

13.39

(a)

$$V_{off} = \phi_B - \frac{\Delta E_c}{e} - V_{P2}$$

where

$$V_{P2} = \frac{eN_d d_d^2}{2\in_N}$$

$$= \frac{(1.6\times 10^{-19})(3\times 10^{18})(350\times 10^{-8})^2}{2(12.2)(8.85\times 10^{-14})}$$

or

$$V_{P2} = 2.72\text{ V}$$

Then

$$V_{off} = 0.89 - 0.24 - 2.72$$

or

$$V_{off} = -2.07\text{ V}$$

(b)

$$n_S = \frac{\in_N}{e(d+\Delta d)}(V_g - V_{off})$$

For $V_g = 0$, we have

$$n_S = \frac{(12.2)(8.85\times 10^{-14})}{(1.6\times 10^{-19})(350+80)(10^{-8})}(2.07)$$

or

$$n_S = 3.25\times 10^{12}\text{ cm}^{-2}$$

13.40

(a) We have

$$I_D(sat) = \frac{\in_N W}{(d+\Delta d)}(V_g - V_{off} - V_O)\cdot \upsilon_s$$

We find

$$\left(\frac{g_{mS}}{W}\right) = \frac{\partial}{\partial V_g}\left[\frac{I_D(sat)}{W}\right] = \frac{\in_N \upsilon_s}{(d+\Delta d)}$$

$$= \frac{(12.2)(8.85\times 10^{-14})(2\times 10^{7})}{(350+80)(10^{-8})}$$

or

$$\left(\frac{g_{mS}}{W}\right) = 5.02\text{ S/cm} = 502\text{ mS/mm}$$

(b) At $V_g = 0$, we obtain

$$\frac{I_D(sat)}{W} = \frac{\in_N}{(d+\Delta d)}(-V_{off} - V_O)\cdot \upsilon_s$$

$$= \frac{(12.2)(8.85\times 10^{-12})}{(350+80)(10^{-8})}(2.07-1)(2\times 10^{7})$$

or

$$\frac{I_D(sat)}{W} = 5.37\text{ A/cm} = 537\text{ mA/mm}$$

13.41

$$V_{off} = \phi_B - \frac{\Delta E_c}{e} - V_{P2}$$

We want $V_{off} = -0.3$ V, so

$$-0.30 = 0.85 - 0.22 - V_{P2}$$

or

$$V_{P2} = 0.93\text{ V}$$

We have

$$V_{P2} = \frac{eN_d d_d^2}{2\in_N}$$

or

$$d_d^2 = \frac{2\in_N V_{P2}}{eN_d}$$

$$= \frac{2(12.2)(8.85\times 10^{-14})(0.93)}{(1.6\times 10^{-19})(2\times 10^{18})}$$

We then obtain

$$d_d = 2.51\times 10^{-6}\text{ cm} = 251\ \overset{o}{A}$$

Chapter 14

14.1

$$\lambda_{max} = \frac{1.24}{E_g} \mu\text{m}$$

(a) Si: $\lambda_{max} = \frac{1.24}{1.12} = 1.11 \mu\text{m}$

(b) Ge: $\lambda_{max} = \frac{1.24}{0.66} = 1.88 \mu\text{m}$

(c) GaAs: $\lambda_{max} = \frac{1.24}{1.42} = 0.873 \mu\text{m}$

(d) InP: $\lambda_{max} = \frac{1.24}{1.35} = 0.919 \mu\text{m}$

14.2

(a) For $\lambda = 480$ nm,

$$E = \frac{1.24}{\lambda} = \frac{1.24}{0.480} = 2.58 \text{ eV}$$

For $\lambda = 725$ nm,

$$E = \frac{1.24}{0.725} = 1.71 \text{ eV}$$

(b) For $E = 0.87$ eV,

$$\lambda = \frac{1.24}{E} = \frac{1.24}{0.87} = 1.43 \mu\text{m}$$

For $E = 1.32$ eV,

$$\lambda = \frac{1.24}{1.32} = 0.939 \mu\text{m}$$

For $E = 1.90$ eV,

$$\lambda = \frac{1.24}{1.90} = 0.653 \mu\text{m}$$

14.3

(a) $\lambda = \frac{1.24}{1.65} = 0.752 \mu\text{m}$

(i) From Figure 14.4, $\alpha \cong 9 \times 10^3 \text{ cm}^{-1}$

(ii) $\frac{I_\nu(d)}{I_{\nu 0}} = \exp(-\alpha d)$

$$= \exp\left[-\left(9 \times 10^3\right)\left(1.2 \times 10^{-4}\right)\right]$$
$$= 0.340$$

Fraction absorbed $= 1 - 0.34 = 0.66$

(b) $\lambda = \frac{1.24}{1.90} = 0.653 \mu\text{m}$

(i) From Figure 14.4, $\alpha \cong 2.6 \times 10^4 \text{ cm}^{-1}$

(ii) $\frac{I_\nu(d)}{I_{\nu 0}} = \exp(-\alpha d)$

$$= \exp\left[-\left(2.6 \times 10^4\right)\left(0.80 \times 10^{-4}\right)\right]$$
$$= 0.125$$

Fraction absorbed $= 1 - 0.125 = 0.875$

14.4

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3$ eV, $\lambda = \frac{1.24}{1.3} = 0.95 \mu\text{m}$

For silicon: $\alpha \cong 3 \times 10^2 \text{ cm}^{-1}$

Then for $I(x) = 10^{-2}$ W/cm^2, we obtain

$$g' = \frac{\left(3 \times 10^2\right)\left(10^{-2}\right)}{\left(1.6 \times 10^{-19}\right)(1.3)}$$

or

$$g' = 1.44 \times 10^{19} \text{ cm}^{-3}\text{ s}^{-1}$$

The excess concentration is

$$\delta n = g'\tau = \left(1.44 \times 10^{19}\right)\left(10^{-6}\right)$$

or

$$\delta n = 1.44 \times 10^{13} \text{ cm}^{-3}$$

14.5

(a) $\delta p = g'\tau_{p0} \Rightarrow g' = \frac{\delta p}{\tau_{p0}}$

$$g' = \frac{5 \times 10^{15}}{2 \times 10^{-7}} = 2.5 \times 10^{22} \text{ cm}^{-3}\text{ s}^{-1}$$

For $h\nu = 1.65$ eV,

$$\Rightarrow \lambda = \frac{1.24}{1.65} = 0.752 \mu\text{m}$$

From Figure 14.4, $\alpha \cong 9 \times 10^3 \text{ cm}^{-1}$

$$I_{\nu 0} = \frac{(g')(h\nu)}{\alpha}$$
$$= \frac{\left(2.5 \times 10^{22}\right)\left(1.6 \times 10^{-19}\right)(1.65)}{9 \times 10^3}$$
$$= 0.733 \text{ W/cm}^2$$

(b) $\dfrac{I_\nu(d)}{I_{\nu 0}} = 0.1 = \exp(-\alpha d)$

$$0.1 = \exp\left[-\left(9\times10^3\right)(d)\right]$$

$$d = \frac{1}{9\times10^3}\ln\left(\frac{1}{0.1}\right)$$

$$= 2.56\times10^{-4}\text{ cm} = 2.56\ \mu\text{m}$$

14.6

$$\lambda = \frac{1.24}{E} = \frac{1.24}{1.40} = 0.886\ \mu\text{m}$$

From Figure 14.4, $\alpha \cong 4.5\times10^2\text{ cm}^{-1}$

(a) $\dfrac{I_\nu(d)}{I_{\nu 0}} = 0.1 = \exp(-\alpha d)$

$$d = \frac{1}{\alpha}\ln\left(\frac{1}{0.1}\right) = \frac{1}{4.5\times10^2}\ln\left(\frac{1}{0.1}\right)$$

$$= 5.12\times10^{-3}\text{ cm} = 51.2\ \mu\text{m}$$

(b) $d = \dfrac{1}{4.5\times10^2}\ln\left(\dfrac{1}{0.3}\right)$

$$= 2.68\times10^{-3}\text{ cm} = 26.8\ \mu\text{m}$$

14.7

GaAs:

For $x = 1\ \mu\text{m} = 10^{-4}$ cm, we have 50% absorbed or 50% transmitted, then

$$\frac{I(x)}{I_O} = 0.50 = \exp(-\alpha x)$$

We can write

$$\alpha = \left(\frac{1}{x}\right)\cdot\ln\left(\frac{1}{0.5}\right) = \left(\frac{1}{10^{-4}}\right)\cdot\ln(2)$$

or

$$\alpha = 0.69\times10^4\text{ cm}^{-1}$$

This value corresponds to

$$\lambda = 0.75\ \mu\text{m},\quad E = 1.65\text{ eV}$$

14.8

The ambipolar transport equation for minority carrier holes in steady state is

$$D_p\frac{d^2(\delta p_n)}{dx^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p = \sqrt{D_p\tau_p}$

The photon flux in the semiconductor is

$$\Phi(x) = \Phi_O\exp(-\alpha x)$$

and the generation rate is

$$G_L = \alpha\Phi(x) = \alpha\Phi_O\exp(-\alpha x)$$

so the differential equation becomes

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{\alpha\Phi_O}{D_p}\exp(-\alpha x)$$

The general solution is of the form

$$\delta p_n(x) = A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}\cdot\exp(-\alpha x)$$

As $x\to\infty$, $\delta p_n = 0$ so that $B = 0$. Then

$$\delta p_n(x) = A\exp\left(\frac{-x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}\cdot\exp(-\alpha x)$$

At $x = 0$, we have

$$D_p\left.\frac{d(\delta p_n)}{dx}\right|_{x=0} = s\,\delta p_n\Big|_{x=0}$$

so we can write

$$\delta p_n\Big|_{x=0} = A - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}$$

and

$$\left.\frac{d(\delta p_n)}{dx}\right|_{x=0} = -\frac{A}{L_p} + \frac{\alpha^2\Phi_O\tau_p}{\alpha^2L_p^2-1}$$

Then we have

$$-\frac{AD_p}{L_p} + \frac{\alpha^2\Phi_O\tau_pD_p}{\alpha^2L_p^2-1} = sA - \frac{s\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}$$

Solving for A, we find

$$A = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}\cdot\left[\frac{s+\alpha D_p}{s+\left(D_p/L_p\right)}\right]$$

The solution can now be written as

$$\delta p_n(x) = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2-1}\times\left[\frac{s+\alpha D_p}{s+\left(D_p/L_p\right)}\cdot\exp\left(\frac{-x}{L_p}\right) - \exp(-\alpha x)\right]$$

14.9

We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n = \sqrt{D_n \tau_n}$

The general solution can be written in the form

$$\delta n_p(x) = A\cosh\left(\frac{x}{L_n}\right) + B\sinh\left(\frac{x}{L_n}\right) + G_L \tau_n$$

For $s = \infty$ at $x = 0$ means $\delta n_p(0) = 0$. Then

$$0 = A + G_L \tau_n \Rightarrow A = -G_L \tau_n$$

At $x = W$,

$$-D_n \frac{d(\delta n_p)}{dx}\bigg|_{x=W} = s_o \delta n_p\bigg|_{x=W}$$

Now

$$\delta n_p(W) = -G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n$$

and

$$\frac{d(\delta n_p)}{dx}\bigg|_{x=W} = -\frac{G_L \tau_n}{L_n}\sinh\left(\frac{W}{L_n}\right) + \frac{B}{L_n}\cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\frac{G_L \tau_n D_n}{L_n}\sinh\left(\frac{W}{L_n}\right) - \frac{BD_n}{L_n}\cosh\left(\frac{W}{L_n}\right) = s_o\left[-G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n\right]$$

Solving for B, we obtain

$$B = \frac{G_L\left\{L_n \sinh\left(\frac{W}{L_n}\right) + s_o \tau_n\left[\cosh\left(\frac{W}{L_n}\right) - 1\right]\right\}}{\frac{D_n}{L_n}\cosh\left(\frac{W}{L_n}\right) + s_o \sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p(x) = G_L \tau_n\left[1 - \cosh\left(\frac{x}{L_n}\right)\right] + B\sinh\left(\frac{x}{L_n}\right)$$

where B was just given.

14.10

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{(25)(10^{-6})} = 5\times10^{-3}\text{ cm}$$

$$L_p = \sqrt{D_p \tau_{p0}} = \sqrt{(10)(5\times10^{-7})} = 2.236\times10^{-3}\text{ cm}$$

Now $J_S = en_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$

$$= (1.6\times10^{-19})(1.5\times10^{10})^2 \times\left[\frac{25}{(5\times10^{-3})(10^{16})} + \frac{10}{(2.236\times10^{-3})(10^{15})}\right]$$

$$J_S = 1.790\times10^{-10}\text{ A/cm}^2$$

$$I_S = AJ_S = (5)(1.79\times10^{-10}) = 8.950\times10^{-10}\text{ A}$$

(a) $I_L = eG_L AW$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.6350\text{ V}$$

$$W = \left\{\frac{2\in_s V_{bi}}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.635)}{1.6\times10^{-19}} \times\left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$W = 9.508\times10^{-5}\text{ cm}$$

Then

$$I_L = (1.6\times10^{-19})(5\times10^{21})(5)(9.508\times10^{-5}) = 0.380\text{ A} = 380\text{ mA}$$

(b) $V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_S}\right)$

$$= (0.0259)\ln\left(1 + \frac{0.380}{8.95\times10^{-10}}\right)$$

$$V_{oc} = 0.5145\text{ V}$$

(c) $\frac{V_{oc}}{V_{bi}} = \frac{0.5145}{0.635} = 0.810$

14.11

From Problem 14.10, $I_S = 8.95\times10^{-10}$ A

(a) $V_{oc} = V_t \ln\left(1+\frac{I_L}{I_S}\right)$

$= (0.0259)\ln\left(1+\frac{120\times10^{-3}}{8.95\times10^{-10}}\right)$

$= 0.4847$ V

(b) $I = I_L - I_S\left[\exp\left(\frac{V}{V_t}\right)-1\right]$

$100\times10^{-3} = 120\times10^{-3}$

$-8.95\times10^{-10}\left[\exp\left(\frac{V}{V_t}\right)-1\right]$

$\Rightarrow V = 0.4383$ V

(c) $\left(1+\frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1+\frac{I_L}{I_S}$

$= 1+\frac{120\times10^{-3}}{8.95\times10^{-10}}$

$= 1.341\times10^{8}$

By trial and error, $V_m \cong 0.412$ V

Now

$I_m = I_L - I_S\left[\exp\left(\frac{V_m}{V_t}\right)-1\right]$

$= 120\times10^{-3}$

$-\left(8.95\times10^{-10}\right)\left[\exp\left(\frac{0.412}{0.0259}\right)-1\right]$

$\Rightarrow I_m = 112.75\times10^{-3}$ A $= 112.75$ mA

$P_m = I_m V_m = (112.75)(0.412)$

$= 46.5$ mW

(d) $V_m = I_m R_L \Rightarrow R_L = \frac{V_m}{I_m} = \frac{0.412}{0.11275}$

$R_L = 3.65\,\Omega$

14.12

From Problem 14.10, $I_S = 8.95\times10^{-10}$ A

(a)

(i) $V_{oc} = V_t \ln\left(1+\frac{I_L}{I_S}\right)$

$= (0.0259)\ln\left(1+\frac{10\times10^{-3}}{8.95\times10^{-10}}\right)$

$= 0.420$ V

(ii) $\left(1+\frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1+\frac{I_L}{I_S}$

$= 1+\frac{10\times10^{-3}}{8.95\times10^{-10}}$

$= 1.117\times10^{7}$

By trial and error, $V_m \cong 0.351$ V

Now

$I_m = I_L - I_S\left[\exp\left(\frac{V_m}{V_t}\right)-1\right]$

$= 10\times10^{-3}$

$-\left(8.95\times10^{-10}\right)\left[\exp\left(\frac{0.351}{0.0259}\right)-1\right]$

$I_m = 9.31\times10^{-3}$ A $= 9.31$ mA

Then

$P_m = I_m V_m = (9.31)(0.351) = 3.27$ mW

(b)

(i) $V_{oc} = (0.0259)\ln\left(1+\frac{100\times10^{-3}}{8.95\times10^{-10}}\right)$

$= 0.480$ V

(ii) $\left(1+\frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1+\frac{I_L}{I_S}$

$= 1+\frac{100\times10^{-3}}{8.95\times10^{-10}}$

$= 1.117\times10^{8}$

By trial and error, $V_m \cong 0.407$ V

Now

$I_m = I_L - I_S\left[\exp\left(\frac{V_m}{V_t}\right)-1\right]$

$= 100\times10^{-3}$

$-\left(8.95\times10^{-10}\right)\left[\exp\left(\frac{0.407}{0.0259}\right)-1\right]$

$I_m = 9.40\times10^{-2}$ A $= 94.0$ mA

Then

$P_m = I_m V_m = (94.0)(0.407) = 38.3$ mW

(c) $\frac{P_{m2}}{P_{m1}} = \frac{38.3}{3.27} = 11.7$

14.13

$$V_{OC}=V_t\ln\left(1+\frac{J_L}{J_S}\right)$$
$$=(0.0259)\ln\left(1+\frac{30\times10^{-3}}{J_S}\right)$$

where

$$J_S=en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_n}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_p}}\right]$$

which becomes

$$J_S=(1.6\times10^{-19})(1.8\times10^6)^2$$
$$\times\left[\frac{1}{N_a}\sqrt{\frac{225}{5\times10^{-8}}}+\frac{1}{10^{19}}\sqrt{\frac{7}{5\times10^{-8}}}\right]$$

or

$$J_S=(5.184\times10^{-7})\left[\frac{6.708\times10^4}{N_a}+1.183\times10^{-15}\right]$$

Then

N_a (cm^{-3})	J_S (A/cm^2)	V_{OC} (V)
10^{15}	3.477×10^{-17}	0.891
10^{16}	3.478×10^{-18}	0.950
10^{17}	3.484×10^{-19}	1.01
10^{18}	3.539×10^{-20}	1.07

14.14

(a)

$$I_L=J_L\cdot A=(25\times10^{-3})(2)=50\times10^{-3}\text{ A}$$

We have

$$J_S=en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_n}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_p}}\right]$$

or

$$J_S=(1.6\times10^{-19})(1.5\times10^{10})^2$$
$$\times\left[\frac{1}{3\times10^{16}}\sqrt{\frac{18}{5\times10^{-6}}}+\frac{1}{10^{19}}\sqrt{\frac{6}{5\times10^{-7}}}\right]$$

which becomes

$$J_S=2.289\times10^{-12}\text{ A/cm}^2$$

or

$$I_S=4.579\times10^{-12}\text{ A}$$

We have

$$I=I_L-I_S\left[\exp\left(\frac{V}{V_t}\right)-1\right]$$

or

$$I=50\times10^{-3}-(4.579\times10^{-12})\left[\exp\left(\frac{V}{V_t}\right)-1\right]$$

We see that when $I=0$, $V=V_{OC}=0.599$ V. We find

V (V)	I (mA)
0	50
0.1	50
0.2	50
0.3	50
0.4	49.98
0.45	49.84
0.50	48.89
0.55	42.36
0.57	33.46
0.59	14.19

(b) The voltage at the maximum power point is found from

$$\left[1+\frac{V_m}{V_t}\right]\cdot\exp\left(\frac{V_m}{V_t}\right)=1+\frac{I_L}{I_S}$$
$$=1+\frac{50\times10^{-3}}{4.58\times10^{-12}}$$
$$=1.092\times10^{10}$$

By trial and error,

$$V_m=0.520\text{ V}$$

At this point, we find

$$I_m=47.6\text{ mA}$$

so the maximum power is

$$P_m=I_mV_m=(47.6)(0.520)$$

or

$$P_m=24.8\text{ mW}$$

(c) We have

$$V=IR\Rightarrow R=\frac{V}{I}=\frac{V_m}{I_m}=\frac{0.520}{47.6\times10^{-3}}$$

or

$$R=10.9\,\Omega$$

14.15

(a) $$V_{oc}=(0.0259)\ln\left(1+\frac{180\times10^{-3}}{2\times10^{-9}}\right)$$
$$=0.474\text{ V}$$

(b) $$\left(1+\frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right)=1+\frac{I_L}{I_S}$$
$$=1+\frac{180\times10^{-3}}{2\times10^{-9}}=9\times10^7$$

By trial and error, $V_m \cong 0.402\text{ V}$

$$I_m \cong I_L - I_S \exp\left(\frac{V_m}{V_t}\right)$$

$$= 180\times10^{-3} - \left(2\times10^{-9}\right)\exp\left(\frac{0.402}{0.0259}\right)$$

$$= 1.69\times10^{-1}\text{ A} = 169\text{ mA}$$

$$P_m = I_m V_m = (169)(0.402) = 67.9\text{ mW}$$

(c) $R_L = \dfrac{V_m}{I_m} = \dfrac{0.402}{0.169} = 2.379\,\Omega$

(d) $R_L \to (1.5)(2.379) = 3.568\,\Omega$

Now

$$I = \frac{V}{R_L} = I_L - I_S \exp\left(\frac{V}{V_t}\right)$$

$$\frac{V}{3.568} = 180\times10^{-3} - \left(2\times10^{-9}\right)\exp\left(\frac{V}{0.0259}\right)$$

By trial and error, $V \cong 0.444\text{ V}$

Then $I = \dfrac{V}{R_L} = \dfrac{0.444}{3.568} = 0.1244\text{ A}$

$$P = IV = (124.4)(0.444) = 55.2\text{ mW}$$

14.16

(a) $V_{oc} = (0.0259)\ln\left(1 + \dfrac{100\times10^{-3}}{10^{-10}}\right)$

$$= 0.5367\text{ V}$$

(b) $\left(1 + \dfrac{V_m}{V_t}\right)\exp\left(\dfrac{V_m}{V_t}\right) = 1 + \dfrac{I_L}{I_S}$

$$= 1 + \frac{100\times10^{-3}}{10^{-10}}$$

$$= 10^9$$

By trial and error, $V_m \cong 0.461\text{ V}$

Then

$$I_m = 100\times10^{-3} - \left(10^{-10}\right)\exp\left(\frac{0.461}{0.0259}\right)$$

$$= 9.463\times10^{-2}\text{ A} = 94.63\text{ mA}$$

$$P_m = I_m V_m = (94.63)(0.461) = 43.62\text{ mW}$$

(c) $n = \dfrac{10}{0.461} = 21.7 \to n = 22$ cells

(d) Now $V = (22)(0.461) = 10.14\text{ V}$

$P = IV$

$5.2 = I(10.14) \Rightarrow I = 0.5128\text{ A}$

Then $n' = \dfrac{0.5128}{0.09463} = 5.42 \to n' = 6$

(e) Then $I = (6)(0.09463) = 0.5678\text{ A}$

So $R_L = \dfrac{V}{I} = \dfrac{10.14}{0.5678} = 17.86\,\Omega$

14.17

Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then in the p-region

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha\Phi_O}{D_n}\exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p(x) = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\exp(-\alpha x)$$

As $x \to \infty$, $\delta n_p = 0$ so that $B = 0$. Then

$$\delta n_p(x) = A\exp\left(\frac{-x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\cdot\exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \dfrac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p(x) = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x)\right]$$

where Φ_O is the incident flux at $x = 0$.

14.18

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

Then

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right) \cdot \ln(10)$$

For $h\nu = 1.7$ eV , $\alpha \cong 10^4 \text{ cm}^{-1}$
Then

$$x = \left(\frac{1}{10^4}\right) \cdot \ln(10) = 2.3 \times 10^{-4} \text{ cm}$$

or

$$x = 2.3 \,\mu\text{m}$$

and for $h\nu = 2.0$ eV, $\alpha \cong 10^5 \text{ cm}^{-1}$.
Then

$$x = \left(\frac{1}{10^5}\right) \cdot \ln(10) = 0.23 \times 10^{-4} \text{ cm}$$

or

$$x = 0.23 \,\mu\text{m}$$

14.19

(a) $n_o = N_d = 5 \times 10^{15} \text{ cm}^{-3}$

$$I = e\mu_n n_o A\mathrm{E}$$
$$= (1.6 \times 10^{-19})(1200)(5 \times 10^{15}) \times (5 \times 10^{-4})\left(\frac{3}{120 \times 10^{-4}}\right)$$
$$I = 0.12 \text{ A} = 120 \text{ mA}$$

(b) $\delta p = G_L \tau_{p0} = (10^{21})(10^{-7}) = 10^{14} \text{ cm}^{-3}$

(c) $\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$

$$= (1.6 \times 10^{-19})(10^{14})(1200 + 400)$$
$$= 2.56 \times 10^{-2} \ (\Omega\text{-cm})^{-1}$$

(d) $I_L = (\Delta\sigma) A\mathrm{E}$

$$= (2.56 \times 10^{-2})(5 \times 10^{-4})\left(\frac{3}{120 \times 10^{-4}}\right)$$
$$= 3.2 \times 10^{-3} \text{ A} = 3.2 \text{ mA}$$

(e) $\Gamma_{ph} = \dfrac{I_L}{eG_L AL}$

$$= \frac{3.2 \times 10^{-3}}{(1.6 \times 10^{-19})(10^{21})(5 \times 10^{-4})(120 \times 10^{-4})}$$
$$\Gamma_{ph} = 3.33$$

14.20

n-type, so holes are the minority carrier

(a)

$$\delta p = G_L \tau_p = (10^{21})(10^{-8})$$

or

$$\delta p = \delta n = 10^{13} \text{ cm}^{-3}$$

(b)

$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$
$$= (1.6 \times 10^{-19})(10^{13})(8000 + 250)$$

or

$$\Delta\sigma = 1.32 \times 10^{-2} \ (\Omega\text{-cm})^{-1}$$

(c)

$$I_L = J_L \cdot A = (\Delta\sigma) A\mathrm{E} = \frac{(\Delta\sigma)AV}{L}$$
$$= \frac{(1.32 \times 10^{-2})(10^{-4})(5)}{100 \times 10^{-4}}$$

or

$$I_L = 0.66 \text{ mA}$$

(d)

$$\Gamma_{ph} = \frac{I_L}{eG_L AL}$$
$$= \frac{0.66 \times 10^{-3}}{(1.6 \times 10^{-19})(10^{21})(10^{-4})(100 \times 10^{-4})}$$

or

$$\Gamma_{ph} = 4.125$$

14.21

$$\Phi(x) = \Phi_O \exp(-\alpha x)$$

The electron-hole generation rate is

$$g' = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

and the excess carrier concentration is

$$\delta p = \tau_p \alpha \Phi(x)$$

Now

$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$

and

$$J_L = (\Delta\sigma)\mathrm{E}$$

The photocurrent is now found from

$$I_L = \iint (\Delta\sigma)\mathrm{E} \cdot dA = \int_0^W dy \int_0^{x_O} (\Delta\sigma)\mathrm{E} \cdot dx$$
$$= We(\mu_n + \mu_p)\mathrm{E} \int_0^{x_O} \delta p \cdot dx$$

Then

$$I_L = We(\mu_n + \mu_p)\text{E}\alpha\Phi_O\tau_p \int_0^{x_O} \exp(-\alpha x)dx$$

$$= We(\mu_n + \mu_p)\text{E}\alpha\Phi_O\tau_p \times\left[-\frac{1}{\alpha}\exp(-\alpha x)\right]_0^{x_O}$$

which becomes

$$I_L = We(\mu_n + \mu_p)\text{E}\Phi_O\tau_p[1-\exp(-\alpha x_O)]$$

Now

$$I_L = (50\times10^{-4})(1.6\times10^{-19})(1200+450)(50) \times(10^{16})(2\times10^{-7})[1-\exp(-(5\times10^4)(10^{-4}))]$$

or

$$I_L = 0.131\,\mu\text{A}$$

14.22

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(2\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.6530\text{ V}$$

$$L_n = \sqrt{D_n\tau_{n0}} = \sqrt{(25)(5\times10^{-7})}$$

$$= 3.536\times10^{-3}\text{ cm}$$

$$L_p = \sqrt{D_p\tau_{p0}} = \sqrt{(10)(10^{-7})}$$

$$= 10^{-3}\text{ cm}$$

$$W = \left\{\frac{2\in_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.653+5)}{(1.6\times10^{-19})} \times\left[\frac{10^{16}+2\times10^{15}}{(10^{16})(2\times10^{15})}\right]\right\}^{1/2}$$

Then

$$W = 2.095\times10^{-4}\text{ cm}$$

(a) $I_{L1} = eWG_LA$

$$= (1.6\times10^{-19})(2.095\times10^{-4})(10^{21})(10^{-3})$$

$$= 3.352\times10^{-5}\text{ A} = 33.52\,\mu\text{A}$$

(b) In n-region,

$$\delta p = G_L\tau_{p0} = (10^{21})(10^{-7}) = 10^{14}\text{ cm}^{-3}$$

In p-region,

$$\delta n = G_L\tau_{n0} = (10^{21})(5\times10^{-7}) = 5\times10^{14}\text{ cm}^{-3}$$

(c) $I_L = eG_LA(W+L_n+L_p)$

$$= (1.6\times10^{-19})(10^{21})(10^{-3}) \times(2.095+35.36+10.0)\times10^{-4}$$

$$I_L = 7.593\times10^{-4}\text{ A} = 0.7593\text{ mA}$$

14.23

In the n-region under steady state and for $\text{E}=0$, we have

$$D_p\frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p = \sqrt{D_p\tau_p}$ and where x' is positive in the negative *x* direction. The homogenerous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh}(x') = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$-\frac{\delta p_{np}}{L_p^2} = -\frac{G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_LL_p^2}{D_p} = G_L\tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n(x') = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{x'}{L_p}\right) + G_L\tau_p$$

One boundary condition is that δp_n remains finite as $x'\to\infty$ which means that $B=0$. Then at $x'=0$, $p_n(0) = 0 = \delta p_n(0) + p_{nO}$, so that $\delta p_n(0) = -p_{nO}$.

We find that

$$A = -(p_{nO} + G_L\tau_p)$$

The solution is then written as

$$\delta p_n(x') = G_L\tau_p - (G_L\tau_p + p_{nO})\exp\left(\frac{-x'}{L_p}\right)$$

The diffusion current density is found as

$$J_p = -eD_p \left. \frac{d\left(\delta p_n(x')\right)}{dx} \right|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = -\frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions.
So

$$J_p = +eD_p \left. \frac{d(\delta p_n)}{dx'} \right|_{x'=0}$$

$$= eD_p\left[-\left(G_L\tau_p + p_{nO}\right)\right]$$

$$\times \left.\left(\frac{-1}{L_p}\right)\exp\left(\frac{-x'}{L_p}\right)\right|_{x'=0}$$

Finally

$$J_p = eG_L L_p + \frac{eD_p p_{nO}}{L_p}$$

14.24

(a) $J_L = e\Phi_o\left[1-\exp(-\alpha W)\right]$

Diode A: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(10^4\right)\left(2\times10^{-4}\right)\right]\right\}$$

$J_L = 6.92\times10^{-2}$ A/cm^2

Diode B: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(10^4\right)\left(10\times10^{-4}\right)\right]\right\}$$

$J_L \cong 8.0\times10^{-2}$ A/cm^2

Diode C: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(10^4\right)\left(80\times10^{-4}\right)\right]\right\}$$

$J_L = 8.0\times10^{-2}$ A/cm^2

(b) $J_L = e\Phi_o\left[1-\exp(-\alpha W)\right]$

Diode A: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(5\times10^2\right)\left(2\times10^{-4}\right)\right]\right\}$$

$J_L = 7.613\times10^{-3}$ A/cm^2

Diode B: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(5\times10^2\right)\left(10\times10^{-4}\right)\right]\right\}$$

$J_L = 3.148\times10^{-2}$ A/cm^2

Diode C: $J_L = \left(1.6\times10^{-19}\right)\left(5\times10^{17}\right)$

$$\times\left\{1-\exp\left[-\left(5\times10^2\right)\left(80\times10^{-4}\right)\right]\right\}$$

$J_L = 7.853\times10^{-2}$ A/cm^2

14.25

(a) $G_{L0} = \dfrac{\alpha I_{\nu 0}}{h\nu} = \dfrac{\left(10^3\right)(0.080)}{\left(1.6\times10^{-19}\right)(1.5)}$

$= 3.33\times10^{20}$ cm^{-3} s^{-1}

Then

$$G_L(x) = G_{L0}\exp(-\alpha x)$$

$$= \left(3.33\times10^{20}\right)\exp\left[-\left(10^3\right)(x)\right]$$

(b) $J_L = e\Phi_o\left[1-\exp(-\alpha W)\right]$

$$= \frac{eG_{L0}}{\alpha}\left[1-\exp(-\alpha W)\right]$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(3.333\times10^{20}\right)}{\left(10^3\right)}$$

$$\times\left\{1-\exp\left[-\left(10^3\right)\left(100\times10^{-4}\right)\right]\right\}$$

$J_L = 5.33\times10^{-2}$ A/cm^2 $= 53.3$ mA/cm^2

14.26

(a) $J_L = eWG_L$

$= \left(1.6\times10^{-19}\right)\left(20\times10^{-4}\right)\left(10^{21}\right)$

$= 0.32$ A/cm^2

(b) $J_L = e\Phi_o\left[1-\exp(-\alpha W)\right]$

$$= \frac{eG_{L0}}{\alpha}\left[1-\exp(-\alpha W)\right]$$

$$= \frac{\left(1.6\times10^{-19}\right)\left(10^{21}\right)}{\left(10^3\right)}$$

$$\times\left\{1-\exp\left[-\left(10^3\right)\left(20\times10^{-4}\right)\right]\right\}$$

$J_L = 0.138$ A/cm^2

14.27

The minimum α occurs when $\lambda = 1\,\mu$m which gives $\alpha = 10^2$ cm^{-1}. We want

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

which can be written as

$$\exp(+\alpha x) = \frac{1}{0.10} = 10$$

Then

$$x = \frac{1}{\alpha}\ln(10) = \frac{1}{10^2}\ln(10) = 2.30\times10^{-2}\ \text{cm}$$

or

$x = 230\,\mu$m

14.28

For $Al_xGa_{1-x}As$ system, a direct bandgap for $0 \le x \le 0.45$, we have

$$E_g = 1.424 + 1.247x$$

At $x = 0.45$, $E_g = 1.985$ eV, so for the direct bandgap

$$1.424 \le E_g \le 1.985 \text{ eV}$$

which yields

$$0.625 \le \lambda \le 0.871\,\mu\text{m}$$

14.29

(a) From Figure 14.24, $E_g \cong 1.64$ eV

$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.64} = 0.756\,\mu\text{ m}$$

(b) From Figure 14.24, $E_g \cong 1.78$ eV

$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.78} = 0.697\,\mu\text{ m}$$

14.30

$$E_g = \frac{1.24}{\lambda} = \frac{1.24}{0.670} = 1.85 \text{ eV}$$

From Figure 14.23, $x \cong 0.35$

14.31

$$E_g = \frac{1.24}{\lambda} = \frac{1.24}{0.670} = 1.85 \text{ eV}$$

From Figure 14.24, $x \cong 0.38$

14.32

(a) For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.

The critical angle is

$$\theta_C = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.86°$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\theta_C}{360} = \frac{2(15.86)}{360} \Rightarrow 8.81\%$$

(b)Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2 = \left(\frac{3.66 - 1}{3.66 + 1}\right)^2 = 0.3258$$

The fraction of photons emitted is then

$$(0.0881)(1 - 0.3258) = 0.0594 \Rightarrow 5.94\%$$

14.33

We can write the external quantum efficiency as

$$\eta_{ext} = T_1 T_2$$

where $T_1 = 1 - R_1$ and where R_1 is the reflection coefficient (Fresnel loss), and the factor T_2 is the fraction of photons that do not experience total internal reflection. We have

$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

so that

$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

which reduces to

$$T_1 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}$$

Now consider the solid angle from the source point. The surface area described by the solid angle is πp^2. The factor T_1 is given by

$$T_1 = \frac{\pi p^2}{4\pi R^2}$$

From the geometry, we have

$$\sin\left(\frac{\theta_C}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R\sin\left(\frac{\theta_C}{2}\right)$$

Then the area is

$$A = \pi p^2 = 4R^2\pi\sin^2\left(\frac{\theta_C}{2}\right)$$

Now

$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_C}{2}\right)$$

From a trig identity, we have

$$\sin^2\left(\frac{\theta_C}{2}\right) = \frac{1}{2}(1 - \cos\theta_C)$$

Then

$$T_1 = \frac{1}{2}(1 - \cos\theta_C)$$

The external quantum efficiency is now

$$\eta_{ext} = T_1 T_2 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2} \cdot \frac{1}{2}(1 - \cos\theta_C)$$

or

$$\eta_{ext} = \frac{2\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}(1 - \cos\theta_C)$$

14.34

For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1\lambda_1}{2} = \frac{(N_1+1)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1\lambda_1}{2} - \frac{(N_1+1)\lambda_2}{2} = \frac{N_1\lambda_1}{2} - \frac{N_1\lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2}\Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1\lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2}\cdot\frac{2L}{\lambda}\Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\Delta\lambda = \frac{\lambda^2}{2L}$$

14.35

For GaAs:

$$h\nu = 1.42 \text{ eV} \Rightarrow \lambda = \frac{1.24}{1.42} = 0.873\,\mu\text{ m}$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{\left(0.873\times10^{-4}\right)^2}{2\left(75\times10^{-4}\right)} = 5.08\times10^{-7}\text{ cm}$$

or

$$\Delta\lambda = 5.08\times10^{-3}\,\mu\text{ m}$$

Chapter 15

15.1 See diagrams in Figure 8.29

15.2

$$R = \frac{\Delta V}{\Delta I} = \frac{0.60 - 0.15}{(2-20)\times 10^{-3}} = -25\,\Omega$$

15.3

$$f_r = \frac{1}{2\pi R_{\min} C_j}\sqrt{\frac{R_{\min}}{R_p} - 1}$$

$$= \frac{1}{2\pi (10)(2\times 10^{-9})}\sqrt{\frac{10}{1} - 1}$$

$$= 2.39\times 10^7\ \text{Hz} = 23.9\ \text{MHz}$$

15.4

(a) $n_o L = 10^{12}\ \text{cm}^{-2}$

(i) $L = \dfrac{10^{12}}{10^{15}} = 10^{-3}\ \text{cm} = 10\ \mu\text{m}$

(ii) $\tau = \dfrac{L}{\upsilon_d} = \dfrac{10^{-3}}{1.5\times 10^7} = 6.667\times 10^{-11}\ \text{s}$

(iii) $f = \dfrac{1}{\tau} = \dfrac{1}{6.667\times 10^{-11}}$

$= 1.5\times 10^{10}\ \text{Hz} = 15\ \text{GHz}$

(b)

(i) $L = \dfrac{10^{12}}{10^{16}} = 10^{-4}\ \text{cm} = 1\ \mu\text{m}$

(ii) $\tau = \dfrac{L}{\upsilon_d} = \dfrac{10^{-4}}{1.5\times 10^7} = 6.667\times 10^{-12}\ \text{s}$

(iii) $f = \dfrac{1}{\tau} = \dfrac{1}{6.667\times 10^{-12}}$

$= 1.5\times 10^{11}\ \text{Hz} = 150\ \text{GHz}$

15.5

(a) $\text{E} = \dfrac{V}{L} = \dfrac{9}{15\times 10^{-4}} = 6\times 10^3\ \text{V/cm}$

(b) $\upsilon_d \cong 1.5\times 10^7\ \text{cm/s}$

(c) $f = \dfrac{\upsilon_d}{L} = \dfrac{1.5\times 10^7}{15\times 10^{-4}} = 1\times 10^{10}\ \text{Hz}$

$= 10\ \text{GHz}$

15.6

$$f = \frac{\upsilon_s}{2L} = \frac{10^7}{2(10\times 10^{-4})} = 5\times 10^9\ \text{Hz}$$

$$= 5\ \text{GHz}$$

15.7

(a) $n_{po} = \dfrac{n_i^2}{N_B} = \dfrac{(1.5\times 10^{10})^2}{8\times 10^{15}}$

$= 2.8125\times 10^4\ \text{cm}^{-3}$

(i) $\delta n_p(0) \cong n_{po}\exp\left(\dfrac{V_{BE}}{V_t}\right)$

$\Rightarrow V_{BE} = V_t \ln\left(\dfrac{\delta n_p(0)}{n_{po}}\right)$

$= (0.0259)\ln\left(\dfrac{10^{14}}{2.8125\times 10^4}\right)$

$= 0.5696\ \text{V}$

(ii) Neglecting any recombination in the base

$I_C \cong \dfrac{eD_B n_{po} A}{x_B}\exp\left(\dfrac{V_{BE}}{V_t}\right)$

$= \dfrac{(1.6\times 10^{-19})(20)(2.8125\times 10^4)(0.4)}{2\times 10^{-4}} \times \exp\left(\dfrac{0.5696}{0.0259}\right)$

$I_C = 0.640\ \text{A}$

(b) $\delta n_p(0) = (0.1)N_B = 8\times 10^{14}\ \text{cm}^{-3}$

(i) $V_{BE} \cong (0.0259)\ln\left(\dfrac{8\times 10^{14}}{2.8125\times 10^4}\right)$

$= 0.6234\ \text{V}$

(ii) $I_C = \dfrac{(1.6\times 10^{-19})(20)(2.8125\times 10^4)(0.4)}{2\times 10^{-4}} \times \exp\left(\dfrac{0.6234}{0.0259}\right)$

$I_C = 5.12\ \text{A}$

15.8

(a) From Figure 7.15, $BV_{BC} \cong 450$ V

(b) $V_{pt} = \dfrac{ex_B^2}{2\in_s} \cdot \dfrac{N_B(N_C+N_B)}{N_C}$

$$= \frac{(1.6\times10^{-19})(2\times10^{-4})^2}{2(11.7)(8.85\times10^{-14})} \times \frac{(8\times10^{15})(6\times10^{14}+8\times10^{15})}{6\times10^{14}}$$

$V_{pt} = 354.4$ V

(c) From Figure 7.15, $BV_{BE} \cong 65$ V

15.9

From the junction breakdown curve, for $BV_{CBO} = 1000$ V, we need the collector doping concentration to be $N_C \cong 2\times10^{14}$ cm^{-3}.

Depletion width into the base (neglect V_{bi}).

$$x_p = \left[\frac{2\in_s V_{BC}}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_B+N_C}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(1000)}{(1.6\times10^{-19})} \times \left(\frac{2\times10^{14}}{5\times10^{15}}\right)\left(\frac{1}{5\times10^{15}+2\times10^{14}}\right)\right]^{1/2}$$

or

$x_p = 3.16\times10^{-4}$ cm $= 3.16\ \mu$m

(Minimum base width)

Depletion width into the collector

$$x_n = \left[\frac{2\in_s V_{BC}}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(1000)}{(1.6\times10^{-19})} \times \left(\frac{5\times10^{15}}{2\times10^{14}}\right)\left(\frac{1}{5\times10^{15}+2\times10^{14}}\right)\right]^{1/2}$$

or

$x_n = 78.9\times10^{-4}$ cm $= 78.9\ \mu$m

(Minimum collector width)

15.10

(a) $BV_{CEO} = \dfrac{BV_{CBO}}{\sqrt[n]{\beta}}$

(i) $BV_{CEO} = \dfrac{300}{\sqrt[3]{10}} = 139$ V

(ii) $BV_{CEO} = \dfrac{300}{\sqrt[3]{50}} = 81.4$ V

(b)

(i) $BV_{CEO} = \dfrac{125}{\sqrt[3]{10}} = 58.0$ V

(ii) $BV_{CEO} = \dfrac{125}{\sqrt[3]{50}} = 33.9$ V

15.11

(a) We have

$\beta_{eff} = \beta_A\beta_B + \beta_A + \beta_B$

so

$180 = 25\beta_B + 25 + \beta_B$

or

$155 = 26\beta_B$

which yields

$\beta_B = 5.96$

(b) We have

$\beta_B i_{EA} = i_{CB}$

or

$\beta_B\left(\dfrac{1+\beta_A}{\beta_A}\right)\cdot i_{CA} = i_{CB}$

so

$(5.96)\left(\dfrac{1+25}{25}\right)\cdot i_{CA} = 20$

which yields

$i_{CA} = 3.23$ A

15.12

(b) $P_T = \left(\dfrac{1}{2}\cdot I_{C,\max}\right)(V_{CEQ})$

$30 = \left(\dfrac{1}{2}\cdot I_{C,\max}\right)(60)$

$\Rightarrow I_{C,\max} = 1.0$ A

$R_L = \dfrac{V_{CEQ}}{I_{CQ}} = \dfrac{60}{0.5} = 120\,\Omega$

$V_{CE,\max} = 120$ V

(c) $P_T = \left(\frac{1}{2} \cdot I_{C,\max}\right)(V_{CEQ})$

$$30 = \left(\frac{2}{2}\right)V_{CEQ}$$

$$\Rightarrow V_{CEQ} = 30\text{ V}$$

$$R_L = \frac{V_{CEQ}}{I_{CQ}} = \frac{30}{1} = 30\,\Omega$$

$$V_{CE,\max} = 2V_{CEQ} = 2(30) = 60\text{ V}$$

(d) Same as part (b)

15.13

(a) $P_T = V_{CEQ} I_{CQ} = \left(\frac{V_{CC}}{2}\right) \cdot I_{CQ}$

$$10 = 6I_{CQ} \Rightarrow I_{CQ} = 1.667\text{ A}$$

$$R_L = \frac{V_{CEQ}}{I_{CQ}} = \frac{6}{1.667} = 3.60\,\Omega$$

(b) $I_{C,\max} = 2I_{CQ} = 2(1.667) = 3.333\text{ A}$

15.14

If $V_{CC} = 25$ V, then

$$I_C(\max) = \frac{V_{CC}}{R_L} = \frac{25}{100} = 0.25\text{ A} < I_{C,rated}$$

The power

$$P = I_C V_{CE} = I_C(V_{CC} - I_C R_L)$$

Now, to find the maximum power point

$$\frac{dP}{dI_C} = 0 = V_{CC} - 2I_C R_L = 25 - I_C(2)(100)$$

which yields

$$I_C = 0.125\text{ A}$$

So

$$P(\max) = (0.125)[25 - (0.125)(100)]$$

or

$$P(\max) = 1.56\text{ W} < P_T$$

So maximum V_{CC} is $V_{CC} = 25$ V

15.15

Now $R_{on} = \frac{V_{DS}}{I_D}$

Power dissipated in the transistor

$$P = I_D V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

We have

$$I_D = \frac{200 - V_{DS}}{100}$$

so we can write

$$P = \left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{R_{on}}$$

For $T = 25°$ C, $R_{on} = 2\,\Omega$.

Then

$$\left(\frac{200 - V_{DS}}{100}\right) \cdot V_{DS} = \frac{V_{DS}^2}{2}$$

which yields

$$V_{DS} = 3.92\text{ V}$$

The power is

$$P = \left(\frac{200 - 3.92}{100}\right)(3.92) = 7.69\text{ W}$$

We then have

T (C)	R_{on} (Ω)	V_{DS} (V)	P (W)
25	2.0	3.92	7.69
50	2.33	4.56	8.91
75	2.67	5.19	10.1
100	3.0	5.83	11.3

15.16

(a) We have, for three devices in parallel,

$$\frac{V}{1.8} + \frac{V}{2} + \frac{V}{2.2} = 5 \Rightarrow V(1.51) = 5$$

or

$$V = 3.311\text{ V}$$

Then, $I = \frac{V}{R}$, so that

$$I_1 = 1.839\text{ A}$$

$$I_2 = 1.656\text{ A}$$

$$I_3 = 1.505\text{ A}$$

Now, $P = IV$, so

$$P_1 = 6.09\text{ W}$$

$$P_2 = 5.48\text{ W}$$

$$P_3 = 4.98\text{ W}$$

(b) Now

$$V\left(\frac{1}{1.8} + \frac{1}{3.6} + \frac{1}{2.2}\right) = 5 \Rightarrow V = 3.882\text{ V}$$

Then

$$I_1 = 2.157\text{ A},\quad P_1 = 8.37\text{ W}$$

$$I_2 = 1.078\text{ A},\quad P_2 = 4.19\text{ W}$$

$$I_3 = 1.765\text{ A},\quad P_3 = 6.85\text{ W}$$

15.17

(a) Let the n-drift region doping concentration be $N_d = 10^{14}\ \text{cm}^{-3}$.

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{14})(10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.516\ \text{V}$$

For the base region,

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+200)}{1.6\times10^{-19}} \times\left(\frac{10^{14}}{10^{15}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_p = 4.86\times10^{-4}\ \text{cm} = 4.86\ \mu\text{m}$$

= channel length

$$x_n = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+200)}{1.6\times10^{-19}} \times\left(\frac{10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_n = 4.86\times10^{-3}\ \text{cm} = 48.6\ \mu\text{m}$$

= drift region width

(b) Assume $N_d = 10^{14}\ \text{cm}^{-3}$

$V_{bi} = 0.516$ V

$$x_p = \left\{\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+80)}{1.6\times10^{-19}} \times\left(\frac{10^{14}}{10^{15}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_p = 3.08\times10^{-4}\ \text{cm} = 3.08\ \mu\text{m}$$

= channel length

$$x_n = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.516+80)}{1.6\times10^{-19}} \times\left(\frac{10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+10^{15}}\right)\right\}^{1/2}$$

$$x_n = 3.08\times10^{-3}\ \text{cm} = 30.8\ \mu\text{m}$$

= drift region width

15.18

(b) In the saturation region,

$$I_D = K_n(V_{GS}-V_T)^2 = (0.20)(V_{GS}-2)^2$$

$$V_{DS} = V_{DD} - I_D R_L = 60 - I_D(10)$$

For $V_{GS} = 4$ V, $I_D = 0.8$ A, $V_{DS} = 52$ V

$$P = I_D V_{DS} = (0.8)(52) = 41.6\ \text{W}$$

For $V_{GS} = 6$ V, $I_D = 3.2$ A, $V_{DS} = 28$ V

$$P = (3.2)(28) = 89.6\ \text{W}$$

For $V_{GS} = 8$ V, transistor biased in the nonsaturation region.

$$\frac{60-V_{DS}}{10} = (0.20)\left[2(8-2)V_{DS} - V_{DS}^2\right]$$

We obtain $2.0V_{DS}^2 - 25V_{DS} + 60 = 0$

$\Rightarrow V_{DS} = 3.24$ V, $I_D = 5.676$ A

$\Rightarrow P = (3.24)(5.676) = 18.39$ W

For $V_{GS} = 6$ V, $P > P_T$ so transistor may be damaged.

15.19

(a) $P = \left(\frac{1}{2}\cdot V_{DD}\right)\left(\frac{1}{2}\cdot I_{D,\max}\right)$

$$45 = \left(\frac{60}{2}\right)\left(\frac{I_D}{2}\right) \Rightarrow I_{D,\max} = 3\ \text{A}$$

$$R_L = \frac{V_{DD}}{I_{D,\max}} = \frac{60}{3} = 20\ \Omega$$

(b) $P = \left(\frac{V_{DD}}{2}\right)\cdot\left(\frac{I_{D,\max}}{2}\right)$

$$R_L = \frac{V_{DD}}{I_{D,\max}} = 10 \Rightarrow I_{D,\max} = \frac{V_{DD}}{10}$$

Then

$$P = \left(\frac{V_{DD}}{2}\right)\cdot\left(\frac{V_{DD}}{20}\right)$$

Or

$$45 = \frac{V_{DD}^2}{40}$$

$$\Rightarrow V_{DD} = 42.4\ \text{V}$$

15.20

We have $\alpha_1 + \alpha_2 = 1$. Now

$$\alpha_1 = \frac{\beta_1}{1+\beta_1} \quad \text{and} \quad \alpha_2 = \frac{\beta_2}{1+\beta_2}$$

so

$$\alpha_1 + \alpha_2 = \frac{\beta_1}{1+\beta_1} + \frac{\beta_2}{1+\beta_2} = 1$$

which can be written as

$$1 = \frac{\beta_1(1+\beta_2) + \beta_2(1+\beta_1)}{(1+\beta_1)(1+\beta_2)}$$

or

$$(1+\beta_1)(1+\beta_2) = \beta_1(1+\beta_2) + \beta_2(1+\beta_1)$$

Expanding, we find

$$1 + \beta_1 + \beta_2 + \beta_1\beta_2$$
$$= \beta_1 + \beta_1\beta_2 + \beta_2 + \beta_1\beta_2$$

which yields

$$\beta_1\beta_2 = 1$$

15.21

The reverse-biased p-well to substrate junction corresponds to the J_2 junction in an SCR. The photocurrent generated in this junction will be similar to the avalanche generated current in an SCR, which can trigger the device.

15.22

Case 1: Terminal 1(+), terminal 2(-), and I_G negative: this triggering was discussed in the text.

Case 2: Terminal 1(+), terminal 2(-), and I_G positive: the gate current enters the P2 region directly so that J3 becomes forward biased. Electrons are injected from N2 and diffuse into N1, lowering the potential of N1. The junction J2 becomes more forward biased, and the increased current triggers the SCR so that P2N1P1N4 turns on.

Case 3: Terminal 1(-), terminal 2(+), and I_G positive: the gate current enters the P2 region directly so that the J3 junction becomes more forward biased. More electrons are injected from N2 into N1 so that J1 also becomes more forward biased. The increased current triggers the P1N1P2N2 device into its conducting state.

Case 4: Terminal 1(-), terminal 2(+), and I_G negative: in this case, the J4 junction becomes forward biased. Electrons are injected from N3 and diffuse into N1. The potential of N1 is lowered which increases the forward biased potential of J1. This increased current then triggers the P1N1P2N2 device into its conducting state.
