BECE201L

Electronic Materials & Devices

Course Overview

Total Hours: 45

- **Module 1: Electrical and Thermal conduction in Solids**
- **Module 2: Semiconductor Fundamentals**
- **Module 3: Carrier Transport Mechanism**
- **Module 4: Junction diodes**
- **Module 5: Bipolar Junction Transistor**
- **Module 6: Field Effect Transistor**
- **Module 7: Other Electronic Materials**

Course Objectives

1. To introduce the students with concepts of electronic materials and their properties

2. To demystify semiconductor device physics and electronics.

3. To equip the students with the tools for solving problems of semiconductor devices and circuits.

4. To familiarize the students with various electronic devices and their circuit applications.

Course Outcomes

Students will be able to

1. Comprehend the basics of electronic materials, crystal structure, electrical and thermal conduction in solids.
2. Draw and analyze the band diagrams of semiconductor devices.
3. Understand and model the carrier transport mechanisms in semiconductors.
4. Design and model the PN- junctions for given specifications.
5. Develop small signal models for BJT and also design BJT amplifiers under different configurations
6. Model MOS capacitors, MOSFETs; learn and mitigate the short channel effects and design future technology nodes.

Classification of Materials

Energy required by electrons to move from valence to conduction band

Every atom wants stability, so it gains, loses or shares to complete its outermost shell so that it becomes stable

Insulator

Do not conduct under normal Conditions
E.g. Wood

Complete Valence shell, 5-7 Valence Electrons

Semiconductor

Conductivity lies between Conductor and insulator
E.g. Wet Wood
Silicon, germanium

4 Valence Electrons

Conductor

Conducts easily
E.g. Water

1-3 Valence Electrons

What are Semiconductors?

Ref: https://youtu.be/Gxn1bXlNGws

Semiconductors In Everyday Life

Solid state transistors and light emitting devices are perhaps the greatest inventions in last 75 years ... Modern life is inconceivable with out these inventions

Autonomous Navigation

Airplane Cockpit

5 MB in 1956

1 TB today

Evolution of IC's

- 1945, ENIAC
- 17,468 vacuum tubes, 10K R, 70K Caps
- Add 5000 numbers/sec
- 15kW power
- 1800sq.ft

- 1971, Intel 4004
- 10um process
- 2300 transistors
- 400-800KHz
- 4-bit word size
- 16-pin DIP
- 12mm^2

- 1974, Intel 8080
- 6um process
- 4500 transistors
- 2MHz
- 8-bit word size
- 40-pin DIP
- 20mm2

- 1978, Intel 8086
- 3um process
- 29K transistors
- 5-10MHz
- 16-bit word size
- 40-pin DIP
- 33mm2

- 1989, Intel 80486
- 0.6um process
- 1.2M transistors
- 25-100MHz
- 32-bit word size
- 168-pin
- 173mm2

- 2001, Intel P4
- 180nm process
- 125M transistors
- 3.4GHz
- 32-bit word size
- 478-pin
- 217mm2

- 2020, Nvidia GPU GA100Ampere
- 7nm FINFET
- 54 billion transistors
- 826mm2

MORE MOORE !!!

image source: www.wikipedia

Moore's Law

Dr. Gordon Moore

Why the Devices are scaled down?

- **To increase the speed**
- **To increase the number of components per chip**
- **To reduce the cost**

Gordon Moore, a physical chemist working in electronics, made a prediction in 1965, the number of transistors on an integrated chip, would double every 18 months.

Electronics – Major Classification

Components & Devices

Passive

Resistors,
Capacitors,
Diode,
Inductors,
etc.

Active

Transistors,
Op.Amps,
etc.

Measuring Instruments

- Digital Multimeters
- Power supplies
- Voltage & current source
- Oscilloscopes
- Function generator

Circuits

- Rectifiers
- Amplifiers
- Oscillators
- Filters
- etc.

Ref: https://quick-learn.in/introduction-to-basics-electronics/

Ref: http://www.electronicsandyou.com/basic-electronic-components-types-functions-symbols.html

Assessment Pattern

Assessment Type	Date	Max. Marks	Weightage	Remarks	Course Outcomes
Digital Assignment 1	30/08/2022	20	10	Question and Rubrics given separately.	**CO1 CO2**
Digital Assignment 2	30/09/2022	20	10	Question and Rubrics given separately.	**CO3 CO4**
Online Quiz	15/10/2022	30	10	Separate quiz will be created and conducted.	**CO1 to CO6**
CAT-1	**As per the announcement by the University**		15	**Schedule will be announced by the University**	**CO1, CO2, CO3**
CAT-2			15		**CO3, CO4, CO5**
FAT			**40**		**All**

My YouTube Channel – Electronics Insight

https://www.youtube.com/channel/UCwx4ggqs96Q_ufsvKZd3T9Q

Books & Weblinks

❑ TEXT BOOK

1. S. O. Kasap, Principles of Electronic Materials and Devices , 2018, 4th Edition, McGraw Hill Education.

❑ BOOK REFERENCES

1. Simon Sze, Ming-Kwei Lee, "Semiconductor Devices, Physics and Technology",2012, 3rd Edition, Wiley International Student Version.
2. Ben G Streetman and Sanjay Kumar Banerjee, "Solid State Electronic Devices", 2015, 7th Edition, Pearson.
3. Adel S. Sedra, Kenneth C. Smith & Arun N. Chandorkar, "Microelectronic Circuits: Theory and Applications", 2014, 7th Edition, Oxford University Press, New York.
4. Donald A. Neamen, "Semiconductor Physics and Devices", 2017, 4th Edition, McGraw Hill.

❑ WEBSITE LINKS

- https://nptel.ac.in/courses/108/108/108108112/
- https://nptel.ac.in/courses/113/106/113106065/

Module - 1 – Electrical & Thermal conduction in Solids

Hours: 6

S. No.	Topic	Hours Required	Week	Book to be referred
1.	Crystalline state – Crystalline defects	1	Week -1	S. O. Kasap
2.	Single Crystal Growth -Czochralski Growth – Amorphous Semiconductor	1		S. O. Kasap
3.	Classical Theory: Drude Model – Temperature dependence of resistivity	1		S. O. Kasap
4.	The Hall Effect and Hall Devices	1	Week -2	S. O. Kasap
5.	Thermal conduction – Electrical conductivity of non-metals, Skin Effect	1		S. O. Kasap
6.	Thin metal films, Problems	1		S. O. Kasap

Solid State

Characteristics:

- Definite volume and shape.
- Regular order of arrangement of particles.
- Strong intermolecular forces.
- Low kinetic energy.

Classification of Solids

- Solids can be classified as crystalline or amorphous.

(on the basis of arrangement of constituent atoms/molecules)

- A **crystalline solid** is composed of one or more crystals; each crystal has a well-defined, ordered structure in three dimensions. They have sharp melting point
  - Examples include sodium chloride and sucrose.
- An **amorphous solid** has a disordered structure. It lacks the well-defined arrangement of basic units found in a crystal. They melt over a wide range of temperature.
  - Glass is an example of amorphous solid.

Sodium chloride

Classification of Solids

Crystalline

Amorphous

A - - - B

A - - - A

A - - - B

A - - - B

Anisotropic → P.P. → change direction of measurement

2-Dimensional Unit Cells

Unit Cell ≡ The Smallest Component

of the crystal (group of atoms, ions or molecules), which, when stacked together with pure translational repetition, reproduces the whole crystal.

Unit Cell

2D-Crystal

Unit Cell ≡ The Smallest Component

of the crystal (group of atoms, ions or molecules), which, when stacked together with pure translational repetition, reproduces the whole crystal.

Note that the choice of unit cell is not unique!

2D-Crystal

2-Dimensional Unit Cells

Artificial Example: "NaCl"

Lattice points are points with ***identical environments***.

2-Dimensional Unit Cells: "NaCl"

Note that ***the choice of origin is arbitrary!***

the lattice points need not be atoms, but

The unit cell size must always be the same.

2-Dimensional Unit Cells: "NaCl"

These are also unit cells!

***It doesn't matter if the origin is at* Na *or* Cl!**

2-Dimensional Unit Cells: "NaCl"

These are also unit cells.

The origin does not have to be on an atom!

2-Dimensional Unit Cells: "NaCl"

These are ***NOT*** unit cells!

Empty space is not allowed!

2-Dimensional Unit Cells: "NaCl"

In **2 dimensions**, **these *are* unit cells**.

In **3 dimensions**, ***they would not be*.**

3-Dimensional

Unit Cells

Cubic Unit Cell

- A **simple cubic unit cell** is a cubic cell in which the lattice points are situated only at the corners.

  - **body-centered cubic unit cell**
    lattice point in the center of the cell as well as at the corners.
  - **face-centered cubic unit cell**
    lattice points at the center of each face of the cell as well as at the corners.

Primitive (or) simple cubic unit cell (SC)

$$\therefore \text{no of atoms in a SC unit cell} = \left(\frac{Nc}{8}\right) = \left(\frac{8}{8}\right) = 1$$

Atomic Packing Factor (APF)

$$\text{APF} = \frac{\text{Volume of atoms in unit cell*}}{\text{Volume of unit cell}}$$

*assume hard spheres

- APF for a simple cubic structure = 0.52

close-packed directions contains 8 x 1/8 = 1 atom/unit cell

$$\text{APF} = \frac{\overset{\text{atoms}}{\text{unit cell}} \; 1 \; \overset{\text{volume}}{\text{atom}} \; \frac{4}{3}\pi(0.5a)^3}{a^3 \; \frac{\text{volume}}{\text{unit cell}}}$$

N_c is the number of atoms at the corners.

Primitive (or) simple cubic unit cell (SC)

Atomic Packing Factor for SC (Simple cubic)

Let a = lattice constant

r = atomic radius

The center of each atom coincides with the different corners of the cube and the atoms touch each other at their periphery.

Let us take a =2r

Each one of the atoms at the corners of the cube has effectively only 1/8th of its volume present inside the cubic cell,

$$8 \, X \frac{1}{8} = 1 \; atoms$$

$$\frac{\text{Volume of atoms}}{\text{Volume of unit cell}} = \frac{\text{No. of atoms} \times \text{vol. of each atom}}{\text{Vol. of unit cell}}$$

$$\text{APF} = \frac{1 \times \frac{4\pi r^3}{3}}{a^3} = \frac{4\pi r^3}{3(2r)^3}$$

$$\left[\begin{array}{l}\text{Since Vol. of sphere} = \frac{4\pi r^3}{3} \\ \text{and } a = 2r\end{array}\right]$$

APF = 0.52

Body – Centred cubic unit cell (BCC)

$$\therefore \text{ Number of atoms in a bccunitcell} = \left(\frac{Nc}{8}\right) + \left(\frac{N_b}{1}\right)$$
$$= \left(\frac{8}{8} + \frac{1}{1}\right)$$
$$= (1+1)$$
$$= 2$$

Atomic Packing Factor: BCC

- APF for a body-centered cubic structure = 0.68

Close-packed directions:
length = $4R = \sqrt{3}\,a$

$$\text{APF} = \frac{\dfrac{\text{atoms}}{\text{unit cell}} \cdot \dfrac{\text{volume}}{\text{atom}}}{\dfrac{\text{volume}}{\text{unit cell}}} = \frac{2 \cdot \frac{4}{3}\pi\left(\sqrt{3}a/4\right)^3}{a^3}$$

Body – Centred cubic unit cell (BCC)

Atomic Packing Factor for BCC

Solution

There is one full atom at the center and one atom at each of the eight corners.

Therefore effective number of atoms inside each unit cell is,

$$1 + 8X\frac{1}{8} = 2\ atoms$$

To find 'a' in terms of 'r'

$$xy = \sqrt{a^2 + a^2} = \sqrt{2}a\ (Solid\ diagonal)$$

$$(xz)^2 = (4r)^2 = \left(\sqrt{2}a\right)^2 + a^2$$

$$4r = \sqrt{3a^2}$$

$$a = \frac{4r}{\sqrt{3}}$$

$$APF = \frac{\text{No. of atoms} \times \text{vol. of each atom}}{\text{vol. of unit cell}}$$

$$APF = \frac{2 \times \frac{4\pi r^3}{3}}{a^3} = \frac{2 \times 4\pi r^3}{3\left(\frac{4r}{\sqrt{3}}\right)^3}$$

$$\mathbf{APF = 0.68}$$

Face – Centred cubic unit cell (FCC)

$$\therefore \text{ Number of atoms in a fcc unitcell } = \left(\frac{N_c}{8}\right) + \left(\frac{N_f}{2}\right) = \left(\frac{8}{8} + \frac{6}{2}\right) = (1+3) = 4$$

Atomic Packing Factor: FCC

- APF for a face-centered cubic structure = 0.74
  maximum achievable APF

Close-packed directions:
length = $4R = \sqrt{2}\,a$

Unit cell contains:
6 x 1/2 + 8 x 1/8
= 4 atoms/unit cell

$$\text{APF} = \frac{\dfrac{\text{atoms}}{\text{unit cell}} \cdot \dfrac{\text{volume}}{\text{atom}}}{\dfrac{\text{volume}}{\text{unit cell}}} = \frac{4 \cdot \frac{4}{3}\pi\left(\sqrt{2}a/4\right)^3}{a^3}$$

Face – Centred cubic unit cell (FCC)

Determination of FCC Unit Cell Volume

Calculate the volume of an FCC unit cell in terms of the atomic radius R.

- the atoms touch one another across a face-diagonal the length of which is $4R$.
- Since the unit cell is a cube, its volume is a^3 Where 'a' cell edge length.
- From the right triangle on the face,

$$a^2 + a^2 = (4R)^2$$

or, solving for a,

$$a = 2R\sqrt{2}$$

The FCC unit cell volume V_C may be computed from

$$V_C = a^3 = (2R\sqrt{2})^3 = 16R^3\sqrt{2}$$

Atomic Packing Factor for FCC

Solution

$$\text{APF} = \frac{\text{volume of atoms in a unit cell}}{\text{total unit cell volume}} = \frac{V_S}{V_C}$$

The total atom and unit cell volumes may be calculated in terms of the atomic radius R.

The volume for a sphere is - $\frac{4}{3}\pi R^3$

since there are 4 atoms per FCC unit cell, the total FCC atom (or sphere) volume is,

$$V_S = (4)\tfrac{4}{3}\pi R^3 = \tfrac{16}{3}\pi R^3$$

total unit cell volume is $V_C = 16R^3\sqrt{2}$

Therefore, the atomic packing factor is

$$\text{APF} = \frac{V_S}{V_C} = \frac{(\frac{16}{3})\pi R^3}{16R^3\sqrt{2}} = 0.74$$

Density of unit cell

$$\text{Density of the unit cell } \rho = \frac{\text{mass of the unit cell}}{\text{volume of the unit cell}} \quad ...(1)$$

$$\text{mass of the unit cell} = \left\{\begin{matrix}\text{total number of}\\ \text{atoms belongs to}\\ \text{that unit cell}\end{matrix}\right\} \times \left\{\begin{matrix}\text{mass of}\\ \text{one atom}\end{matrix}\right\} \quad ...(2)$$

$$\text{mass of one atom} = \frac{\text{molar mass } (gmol^{-1})}{\text{Avagadro number } (mol^{-1})}$$

$$m = \frac{M}{N_A} \quad ...(3)$$

Substitute (3) in (2)

$$\text{mass of the unit cell} = n \times \frac{M}{N_A} \quad ...(4)$$

For a cubic unit cell, all the edge lengths are equal i.e , a=b=c

$$\text{volume of the unit cell} = a \times a \times a = a^3 \quad ...(5)$$

$$\therefore \textbf{Density of the unit cell } \rho = \frac{n\,M}{a^3 N_A} \quad ...(6)$$

THE COPPER (FCC) CRYSTAL Consider the FCC unit cell of the copper crystal

a. How many atoms are there per unit cell?

b. If R is the radius of the Cu atom, show that the lattice parameter a is given by $a = R2\sqrt{2}$.

c. Calculate the **atomic packing factor** (APF) defined by

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}}{\text{Volume of unit cell}}$$

d. Calculate the **atomic concentration** (number of atoms per unit volume) in Cu and the density of the crystal given that the atomic mass of Cu is 63.55 g mol^{-1} and the radius of the Cu atom is 0.128 nm.

THE COPPER (FCC) CRYSTAL Consider the FCC unit cell of the copper crystal shown in Figure

a. How many atoms are there per unit cell?

b. If R is the radius of the Cu atom, show that the lattice parameter a is given by $a = R2\sqrt{2}$.

c. Calculate the **atomic packing factor** (APF) defined by

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}}{\text{Volume of unit cell}}$$

d. Calculate the **atomic concentration** (number of atoms per unit volume) in Cu and the density of the crystal given that the atomic mass of Cu is 63.55 g mol^{-1} and the radius of the Cu atom is 0.128 nm.

Figure The FCC unit cell.
The atomic radius is R and the lattice parameter is a.

SOLUTION

a. There are four atoms per unit cell. The Cu atom at each corner is shared with eight other adjoining unit cells. Each Cu atom at the face center is shared with the neighboring unit cell. Thus, the number of atoms in the unit cell = 8 corners ($\frac{1}{8}$ atom) + 6 faces ($\frac{1}{2}$ atom) = 4 atoms.

b. Consider the unit cell shown in Figure and one of the cubic faces. The face is a square of side a and the diagonal is $\sqrt{a^2 + a^2}$ or $a\sqrt{2}$. The diagonal has one atom at the center of diameter $2R$, which touches two atoms centered at the corners. The diagonal, going from corner to corner, is therefore $R + 2R + R$. Thus, $4R = a\sqrt{2}$ and $a = 4R/\sqrt{2} = R2\sqrt{2}$. Therefore, $a = 0.3620$ nm.

c.

$$\text{APF} = \frac{(\text{Number of atoms in unit cell}) \times (\text{Volume of atom})}{\text{Volume of unit cell}}$$

$$= \frac{4 \times \frac{4}{3}\pi R^3}{a^3} = \frac{\frac{4^2}{3}\pi R^3}{(R2\sqrt{2})^3} = \frac{4^2\pi}{3(2\sqrt{2})^3} = 0.74$$

Figure The FCC unit cell. The atomic radius is R and the lattice parameter is a.

d. In general, if there are x atoms in the unit cell, the atomic concentration is

$$n_{at} = \frac{\text{Number of atoms in unit cell}}{\text{Volume of unit cell}} = \frac{x}{a^3}$$

Thus, for Cu

$$n_{at} = \frac{4}{(0.3620 \times 10^{-7}\ \text{cm})^3} = 8.43 \times 10^{22}\ \text{cm}^{-3}$$

There are x atoms in the unit cell, and each atom has a mass of M_{at}/N_A grams. The density ρ is

$$\rho = \frac{\text{Mass of all atoms in unit cell}}{\text{Volume of unit cell}} = \frac{x\left(\dfrac{M_{at}}{N_A}\right)}{a^3}$$

that is,

$$\rho = \frac{n_{at}M_{at}}{N_A} = \frac{(8.43 \times 10^{22}\ \text{cm}^{-3})(63.55\ \text{g mol}^{-1})}{6.022 \times 10^{23}\ \text{mol}^{-1}} = 8.9\ \text{g cm}^{-3}$$

Single Crystal - CZOCHRALSKI GROWTH

State-of-the-art **200 mm Si** crystal

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https://youtu.be/xftnhfa-Dmo

Solidification and Crystallisation

- Hardening of materials from the casting of molten liquid/gels (metals and alloys).
- Size and shape of the materials structure depend on its cooling rate.
- Solidification process involves the formation of nuclei and the growth of nuclei to form grain and crystal structures.

https://www.youtube.com/watch?v=1Ou1bIDn1xE

Crystalline Defects

Defects in Solids

Imperfection or faulty or deficiencies in crystalline solid materials.

Crystal defect

Discontinuity in the lattice of a crystal caused by missing or introduction of extra atoms or ions, or by dislocations.

Defect types and amounts depend on composition, temperature, pressure, and processing history.

Classification of defects

Dimensionality

Point defects (0D)

- Vacancy atom
- Interstitial atom
- Impurities defect

Line defects (1D)

- Edge dislocation
- Screw dislocation
- Mixed dislocation

Surface defects (2D)

- External surfaces
- Grain boundaries
- Twin boundaries
- Stacking fault
- Anti phase boundaries

Volume defects (3D)

- Pores
- Cracks
- Foreign inclusions
- Voids
- Precipitates

Point Defects

1 **Vacancy** : Empty lattice space due to missing of atom that cause plane distortion

2 **Self interstitial** : Squeezing of extra atom within interstitial lattice site that cause plane distortion

↑ Higher temperature ↑ Higher distortion ↑ Greater imperfection

Point Defects

Impurities defects : Interstitial impurity and substitution impurity

3 **Interstitial impurity** : Squeezing of extra impurity atom within interstitial lattice site that cause plane distortion

(e.g., C in Fe)

4 **Substitution impurity** : Substitution of impurity atom into crystal lattice that cause plane distortion

(e.g., Cu in Ni)

Point Defects in Ionic Crystals

(a) Schottky and Frenkel defects in an ionic crystal.

(b) Two possible imperfections caused by ionized substitutional impurity atoms in an ionic crystal.

CLASSICAL THEORY: THE DRUDE MODEL

The electric current density J is defined as the net amount of charge flowing across a unit area per unit time, that is,

$$J = \frac{\Delta q}{A\ \Delta t}$$

where Δq is the net quantity of charge flowing through an area A in time Δt.

the net flow of electrons in a conductor section of cross-sectional area A in the presence of an applied field E_x. Notice that the direction of electron motion is opposite to that of the electric field E_x and of conventional current, because the electrons experience a Coulombic force eE_x in the x direction, due to their negative charge.

Figure 2.1 Drift of electrons in a conductor in the presence of an applied electric field.

Electrons drift with an average velocity v_{dx} in the x direction.

The conduction electrons are actually moving around randomly in the metal but as a result of the application of the electric field $\mathbf{E_x}$, they all acquire a net velocity in the x-direction

The average velocity of the electrons in the x-direction at time 't' is called **drift velocity**

It is denoted by $\mathbf{V_{dx}(t)}$

It is the instantaneous velocity $\mathbf{v_x}$ in the x-direction averaged over many electrons (i.e.)

$$V_{dx} = \frac{1}{N}\left[V_{x1} + V_{x2} + V_{x3} + \ldots\ldots V_{xN}\right]$$

where, V_{xi} is the x-direction velocity of the ith electron.

N : Number of conduction electrons in the metal

Number of electrons crossing A in Δt

- Suppose n is the number of electrons per unit volume in the conductor (n=N/V)
- In time Δt, electrons move a distance $\Delta x = v_{dx} \Delta t$
- The electrons within distance Δx pass through area A
- Thus, $n(A \Delta x)$ is the total number of electrons crossing A in time Δt

The current density in the x-direction is :

$$J_x = \frac{\Delta x}{A\,\Delta t} = \frac{e\,n\,A\Delta x}{A\,\Delta t}$$

$$= \frac{e\,n\,A V_{dx}\Delta t}{A\,\Delta t}$$

$$J_x = e\,n\,V_{dx}$$

This general equation relates J_x to the average velocity V_{dx} of the electrons.

It must be appreciated that the average velocity at one time may not be the same as at another time, because the applied field, for example, may be changing:

$$E_x = E_x(t)$$

Therefore a time-dependent current is given by

$$J_x(t) = e\,n\,V_{dx}(t)$$

- To relate the current density $\mathbf{J_x}$ to the electric field E_x, the effect of the electric field on the motion of the electrons in the conductor must be examined.
- To do so, let us consider the **copper crystal**:

- The copper atom has a single valence electron in its 4*s* subshell, and this electron is loosely bound
- The solid metal consists of positive ion cores, Cu^+, at regular sites, in the Face-Centered Cubic (FCC) crystal structure
- The valence electrons detach themselves from their parents and wander around freely in the solid, forming a kind of electron cloud or gas

The mobile electrons are free to respond to an applied field, creating a current density $\mathbf{J_x}$. The valence electrons in the electron gas are therefore **conduction electrons.**

Metallic bonding

The attractive forces between the negative electron cloud (valence electrons) and the Cu+ ions are responsible for **metallic bonding** and the existence of the solid metal.

The **electrostatic attraction** between the conduction electrons and the positive metal ions, results in the conduction electron having both potential energy *PE* and kinetic energy *KE*.

The mean *KE* of the conduction electrons in a metal is primarily determined by the electrostatic interaction of these electrons with the positive metal ions and also with each other

In Drude's model, the temperature dependence of the mean *KE is* Neglected

Absence of Electric Field

- In general, the copper crystal will not be perfect and the atoms will not be stationary. There will be crystal defects, vacancies, dislocations, impurities, etc., which will scatter the conduction electrons
- More importantly, due to their thermal energy, the atoms will vibrate about their lattice sites (equilibrium positions)
- Over a long time, the electrons therefore show no net displacement in any one direction.

(a) A conduction electron in the electron gas moves about randomly in a metal (with a mean speed *u*) being frequently and randomly scattered by thermal vibrations of the atoms. In the absence of an applied field there is no net drift in any direction.

Figure 2.2 Motion of a conduction electron in a metal.

(a) A conduction electron in the electron gas moves about randomly in a metal (with a mean speed u) being frequently and randomly scattered by thermal vibrations of the atoms. In the absence of an applied field there is no net drift in any direction.

(b) In the presence of an applied field, E_x, there is a net drift along the x direction. This net drift along the force of the field is superimposed on the random motion of the electron. After many scattering events the electron has been displaced by a net distance, Δx, from its initial position toward the positive terminal.

Figure 2.2 Motion of a conduction electron in a metal.

Presence of Electric Field

- When the conductor is connected to a battery and an electric field is applied to the crystal
- The electron experiences an acceleration in the x direction in addition to its random motion, so after some time, it will drift a finite distance in the x direction

- The electron accelerates along the x direction under the action of the force $e\mathbf{E}_x$, and then it suddenly collides with a vibrating atom and loses the gained velocity. Therefore, there is an average velocity in the x direction, which, if calculated, determines the current as

$$J_x(t) = env_{dx}(t)$$

Calculation of Drift Velocity

- To calculate the drift velocity $\mathbf{V}_{dx}$ of the electrons due to applied field $\mathbf{E}_x$, let us first consider the velocity $\mathbf{V}_{xi}$ of the $\mathbf{i}^{th}$ electron in the x-direction at time 't'
- Suppose its last collision was at time $\mathbf{t}_i$; therefore, for time $(\mathbf{t} - \mathbf{t}_i)$, it accelerated free of collisions

Figure 2.3 Velocity gained in the x direction at time t from the electric field (E_x) for three electrons. There will be N electrons to consider in the metal.

- Let u_{xi} be the velocity of electron **i** in the x-direction just after the collision, called as the initial velocity
- Since eE_x/m_e is the acceleration of the electron, the velocity v_{xi} in the x-direction at time **t** will be

$$v_{xi} = u_{xi} + \frac{eE_x}{m_e}(t - t_i)$$

- However, this is only for the ith electron. The average velocity V_{dx} for all such electrons along x. The average expression for $i = 1$ to N electrons is given by:

$$v_{dx} = \frac{1}{N}[v_{x1} + v_{x2} + \cdots + v_{xN}] = \frac{eE_x}{m_e}\overline{(t - t_i)}$$

where $\overline{(t - t_i)}$ is the **average free time** for N electrons between collisions.

- Suppose that τ is the mean free time, or the **mean time between collisions** (also known as the **mean scattering time**)
- For some electrons, $(t - t_i)$ will be greater than τ, and for others, it will be shorter Averaging $(t - t_i)$ for N electrons will be the same as τ
- Thus, we can substitute τ for $(t - t_i)$ in the previous expression to obtain

$$v_{dx} = \frac{e\tau}{m_e} E_x$$

The drift velocity increases linearly with the applied field. The constant of proportionality $e\tau/m_e$ has been given a special name and symbol. It is called the **drift mobility μ_d**, which is defined as

$$V_{dx} = \mu_d E_x$$

where,

$$\mu_d = e\tau / m_e$$

From the expression for the drift velocity $\mathbf{v}_{dx}$, the current density Jx follows immediately by substituting

$$\mathbf{v}_{dx} = \boldsymbol{\mu}_d \mathbf{E}_x$$

into

$$\mathbf{J}_x(t) = e\, n\, \mathbf{v}_{dx}(t)$$

$$\mathbf{J}_x = e\, n\, \mu_d E_x$$

Therefore, the current density is proportional to the electric field and the conductivity σ is the term multiplying $\mathbf{E}_x$, that is,

$$\boldsymbol{\sigma} = \boldsymbol{e\, n\, \mu_d}$$

Problem #1

Calculate the drift mobility and the mean scattering time of conduction electrons in copper at room temperature, given that the conductivity of copper is $5.9 \times 10^5\ \Omega^{-1}\ cm^{-1}$. [$n = 8.5 \times 10^{22}$ electrons cm^{-3}.]

Solution :

The electron drift mobility is

$$\mu_d = \frac{\sigma}{en} = \frac{5.9 \times 10^5\ \Omega^{-1}\ \text{cm}^{-1}}{[(1.6 \times 10^{-19}\ \text{C})(8.5 \times 10^{22}\ \text{cm}^{-3})]}$$
$$= 43.4\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1}$$

From the drift mobility we can calculate the mean free time τ between collisions by using Equation

$$\tau = \frac{\mu_d m_e}{e} = \frac{(43.4 \times 10^{-4}\ \text{m}^2\ \text{V}^{-1}\ \text{s}^{-1})(9.1 \times 10^{-31}\ \text{kg})}{1.6 \times 10^{-19}\ \text{C}} = 2.5 \times 10^{-14}\ \text{s}$$

Problem #2

What is the applied electric field that will impose a drift velocity equal to 0.1 percent of the mean speed 'u'($\sim 10^6$ m s^{-1})of conduction electrons in copper? What is the corresponding current density and current through a Cu wire of diameter 1 mm? [μ_d is the drift mobility, which for copper is 43.4 cm^2 V^{-1} s^{-1} and $\sigma = 5.9 \times 10^7\ \Omega^{-1}$ m^{-1}]

Solution :

The drift velocity of the conduction electrons is

$$V_{dx} = \mu_d E_x$$

where μ_d is the drift mobility, which for copper is 43.4 cm2 V^{-1} s^{-1}

With $V_{dx} = 0.001u = 10^3$ m s^{-1},

$$E_x = \frac{v_{dx}}{\mu_d} = \frac{10^3 \text{ m s}^{-1}}{43.4 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}} = 2.3 \times 10^5 \text{ V m}^{-1} \quad \text{or} \quad 2.3 \text{ kV cm}^{-1}$$

$$
\begin{aligned}
J_x &= \sigma E_x \\
&= \left(5.9 \times 10^{7}\ \Omega^{-1}\, m^{-1}\right)\left(2.3 \times 105\ V\, m^{-1}\right) \\
&= 1.4 \times 10^{13}\ A\, m^{-2} \text{ or } 1.4 \times 10^{7}\ A\, mm^{-2}
\end{aligned}
$$

TEMPERATURE DEPENDENCE OF RESISTIVITY: IDEAL PURE METALS

- When the conduction electrons are only scattered by thermal vibrations of the metal ions, then τ in the mobility expression $\mu_d = e\tau / m_e$ refers to the **mean time** between scattering events by this process.

- The resulting conductivity and resistivity are denoted by σ_T and ρ_T, where the subscript T represents "**thermal vibration scattering**."

- To find the temperature dependence of σ, we first consider the temperature dependence of the mean free time τ, since this determines the drift mobility
- An electron moving with a mean speed u is scattered when its path crosses the cross-sectional area S of a scattering center

Figure 2.5 Scattering of an electron from the thermal vibrations of the atoms.

The electron travels a mean distance $\ell = u\tau$ between collisions. Since the scattering cross-sectional area is S, in the volume $S\ell$ there must be at least one scatterer, $N_s\,(Su\tau) = 1$.

- The scattering center may be a vibrating atom, impurity, vacancy, or some other crystal defect
- Since τ is the mean time taken for one scattering process, the **mean free path** ℓ of the electron between scattering processes is $u\tau$. If N_s is the concentration of scattering centers, then in the volume $S\ell$, there is one scattering center, that is, $(Su\tau)N_s = 1$. Thus, the mean free time is given by

$$\tau = \frac{1}{SuN_s}$$

- Because the atomic vibrations are random, the atom covers a cross-sectional area πa^2, where a is the amplitude of the vibrations
- If the electron's path crosses πa^2, it gets scattered. Therefore, the mean time between scattering events τ is inversely proportional to the area πa^2 that scatters the electron, that is, $\tau \propto 1/\pi a^2$.
- The thermal vibrations of the atom can be considered to be simple harmonic motion, much the same way as that of a mass M attached to a spring
- The average kinetic energy of the oscillations is ¼ $M a^2 \omega^2$, where ω is the oscillation frequency
- From the kinetic theory of matter, this average kinetic energy must be on the order of ½ KT. Therefore,

$$\frac{1}{4}Ma^2\omega^2 \approx \frac{1}{2}kT$$

so $a^2 \propto T$. Intuitively, this is correct because raising the temperature increases the amplitude of the atomic vibrations. Thus,

$$\tau \propto \frac{1}{\pi a^2} \propto \frac{1}{T} \qquad \text{or} \qquad \tau = \frac{C}{T}$$

where C is a temperature-independent constant. Substituting for τ in $\mu_d = e\tau/m_e$, we obtain

$$\mu_d = \frac{eC}{m_e T}$$

So, the resistivity ρ_T of a pure metal is

$$\rho_T = \frac{1}{\sigma_T} = \frac{1}{en\mu_d} = \frac{m_e T}{e^2 n C} \qquad \rho_T = AT$$

where A is a temperature-independent constant. This shows that the <u>resistivity of a pure metal wire increases linearly with the temperature.</u> We term this conductivity **lattice-scattering-limited conductivity**

Problem

TEMPERATURE DEPENDENCE OF RESISTIVITY What is the percentage change in the resistance of a pure metal wire from Saskatchewan's summer to winter, neglecting the changes in the dimensions of the wire?

SOLUTION

Assuming 20 °C for the summer and perhaps −30 °C for the winter, from $R \propto \rho = AT$, we have

$$\frac{R_{\text{summer}} - R_{\text{winter}}}{R_{\text{summer}}} = \frac{T_{\text{summer}} - T_{\text{winter}}}{T_{\text{summer}}} = \frac{(20 + 273) - (-30 + 273)}{(20 + 273)}$$

$$= 0.171 \qquad \text{or} \qquad 17\%$$

Books & Weblinks

❑ TEXT BOOK

1. S. O. Kasap, Principles of Electronic Materials and Devices , 2018, 4th Edition, McGraw Hill Education.

❑ BOOK REFERENCES

1. Simon Sze, Ming-Kwei Lee, "Semiconductor Devices, Physics and Technology",2012, 3rd Edition, Wiley International Student Version.
2. Ben G Streetman and Sanjay Kumar Banerjee, "Solid State Electronic Devices", 2015, 7th Edition, Pearson.
3. Adel S. Sedra, Kenneth C. Smith & Arun N. Chandorkar, "Microelectronic Circuits: Theory and Applications", 2014, 7th Edition, Oxford University Press, New York.
4. Donald A. Neamen, "Semiconductor Physics and Devices", 2017, 4th Edition, McGraw Hill.

❑ WEBSITE LINKS

- https://nptel.ac.in/courses/108/108/108108112/
- https://nptel.ac.in/courses/113/106/113106065/

THE HALL EFFECT AND HALL DEVICES

- An important phenomenon that we can comfortably explain using the "**electron as a particle**" concept is the Hall effect, which is illustrated in the Figure 2.16.

Figure 2.16 Illustration of the Hall effect. The z direction is out of the plane of the paper. The externally applied magnetic field is along the z direction.

- When we apply a magnetic field in a perpendicular direction to the applied field (which is driving the current), we find there is a transverse field in the sample that is perpendicular to the direction of both the applied field $\mathbf{E}_x$ and the magnetic field $\mathbf{B}_z$, that is, in the y direction
- Putting a voltmeter across the sample, gives a voltage reading V_H
- The applied field $\mathbf{E}_x$ drives a current $\mathbf{J}_x$ in the sample
- The electrons move in the $-x$ direction, with a drift velocity $\mathbf{V}_{dx}$.

Figure 2.17 A moving charge experiences a Lorentz force in a magnetic field. (a) A positive charge moving in the x direction experiences a force downward. (b) A negative charge moving in the $-x$ direction also experiences a force downward.

- Because of the magnetic field, there is a force (called the **Lorentz force**) acting on each electron and given by

$$F_y = -eV_{dx} B_z$$

- The direction of this Lorentz force is the $-y$ direction, which we can show by applying the corkscrew rule, because, in vector notation, the force **F** acting on a charge q moving with a velocity **v** in a magnetic field **B** is given through the vector product

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

- All moving charges experience the Lorentz force

- In our example of a metal this Lorentz force is the $-y$ direction, so it pushes the electrons downward, as a result of which there is a negative charge accumulation near the bottom of the sample and a positive charge near the top of the sample, due to exposed metal le of a ions (*e.g.*, Cu^+).

- The accumulation of electrons near the bottom results in an internal electric field E_H in the $-y$ direction
- This is called the **Hall field** and gives rise to a Hall voltage V_H between the top and bottom of the sample.
- Electron accumulation continues until the increase in E_H is sufficient to stop the further accumulation of electrons
- When this happens, the magnetic-field force $eV_{dx}Bz$ that pushes the electrons down just balances the force eE_H that prevents further accumulation. Therefore, in the steady state,

$$eE_H = ev_{dx}B_z$$

However, $J_x = e n V_{dx}$

Therefore, we can substitute for V_{dx} to obtain $e E_H = J_x Bz/n$ or

$$E_H = \left(\frac{1}{en}\right) J_x B_z$$

A useful parameter called the **Hall coefficient** R_H is defined as

$$R_H = \frac{E_y}{J_x B_z}$$

The quantity R_H measures the resulting Hall field, along y, per unit transverse applied current and magnetic field. The larger R_H, the greater **Ey** for a given J_x and Bz. Therefore, R_H is a gauge of the magnitude of the Hall effect.

$$R_H = -\frac{1}{en}$$

- The reason for the negative sign is that $E_H = -E_y$, which means that E_H is in the $-y$ direction.

- Inasmuch as R_H depends inversely on the free electron concentration, its value in metals is much less than that in semiconductors
- In fact, Hall-effect devices (such as magnetometers) always employ a semiconductor material, simply because the R_H is larger.

Applications of Hall Effect

- The Hall effect is widely used in **magnetically actuated electronic switches**. The application of a magnetic field, say from a magnet, results in a Hall voltage that is amplified to trigger an electronic switch. The switches invariably use Si and are readily available from various companies

- Hall-effect electronic switches are used as **non-contacting keyboard and panel switches** that last almost forever, as they have no mechanical contact assembly. Another advantage is that the electrical contact is "bounce" free

- There are a variety of interesting applications for Hall-effect switches, ranging from ignition systems, to speed controls, position detectors, alignment controls, brushless dc motor commutators, etc.

THERMAL CONDUCTIVITY

- Metals are both <u>good electrical and thermal conductors</u>
- Free conduction electrons in a metal must also play a role in heat conduction
- The transport of heat in a metal is accomplished by the electron gas (conduction electrons), whereas in nonmetals, the conduction is due to lattice vibrations
- When a metal piece is heated at one end, the amplitude of the atomic vibrations, and thus the average kinetic energy of the electrons, in this region increases, as depicted in Figure

Figure Thermal conduction in a metal involves transferring energy from the hot region to the cold region by conduction electrons.

More energetic electrons (shown with longer velocity vectors) from the hotter regions arrive at cooler regions, collide with lattice vibrations, and transfer their energy. Lengths of arrowed lines on atoms represent the magnitudes of atomic vibrations.

THERMAL CONDUCTIVITY (contd..)

- Electrons gain energy from energetic atomic vibrations when the two collide
- By virtue of their increased random motion, these energetic electrons
- then transfer the extra energy to the colder regions by colliding with the atomic vibrations there.
- Thus, electrons act as "energy carriers."
- **The thermal conductivity of a material, as its name implies, measures the ease with which heat, that is, thermal energy, can be transported through the medium**

- Consider the metal rod shown in Figure, which is heated at one end
- Heat will flow from the hot end to the cold end.

Figure Heat flow in a metal rod heated at one end.

Consider the rate of heat flow, dQ/dt, across a thin section δx of the rod. The rate of heat flow is proportional to the temperature gradient $\delta T/\delta x$ and the cross-sectional area A.

- Experiments show that the rate of heat flow, $Q' = dQ/dt$, through a thin section of thickness δx is proportional to the temperature gradient $\delta T/\delta x$ and the cross-sectional area A.
- By ***Fourier's law of thermal conduction***

$$Q' = -A\kappa\frac{\delta T}{\delta x}$$

where, κ is a material-dependent **constant of proportionality** that we call the **thermal conductivity.** The negative sign indicates that the heat flow direction is that of decreasing temperature.

Figure Thermal conductivity κ versus electrical conductivity σ for various metals (elements and alloys) at 20 °C.

The solid line represents the WFL law with $C_{WFL} \approx 2.44 \times 10^8$ W Ω K^{-2}.

Comparing Fourier's law of heat conduction with Ohm's Law

- **The Fourier's law** of heat conduction is a defining equation for κ.

$$Q' = -A\kappa \frac{\delta T}{\delta x}$$

- The driving force for the heat flow is the temperature gradient $\delta T / \delta x$
- If we compare the above equation with Ohm's law for the electric current I, we see that

$$I = -A\sigma \frac{\delta V}{\delta x}$$

which shows that in this case, the driving force is the potential gradient, that is, the electric field.

In metals, electrons participate in the processes of charge and heat transport, which are characterized by σ and κ, respectively

Therefore, it is not surprising to find that the two coefficients are related by the **Wiedemann–Franz–Lorenz law,** which is

$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}}$$

where,

$$C_{\mathrm{WFL}} = \pi^2 k^2 / 3e^2 = 2.44 \times 10^{-8}\ \mathrm{W\ \Omega\ K^{-2}}$$

is a constant called the **Lorenz number** (or the Wiedemann–Franz–Lorenz coefficient).

Experiments on a wide variety of metals, ranging from pure metals to various alloys, show that equation

$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}}$$

is reasonably well obeyed at close to room temperature and above

Figure Thermal conductivity versus temperature for two pure metals (Cu and Al) and two alloys (brass and Al–14% Mg).

SOURCE: Data extracted from Touloukian, Y.S., et al., "Thermal Conductivity, Metallic Elements and Alloys," Thermophysical Properties of Matter, vol.1, 1970. New York, NY: Plenum, 1970.

- Figure shows the temperature dependence of κ for copper and aluminium down to the lowest temperatures. It can be seen that for these two metals, above ~100 K, the thermal conductivity becomes temperature independent, in agreement with Equation

$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}}$$

- Qualitatively, above ~100 K, κ is constant, because heat conduction depends essentially on the rate at which the electron transfers energy from one atomic vibration to another as it collides with them

Thermal Conduction in Non-Metals

- Nonmetals do not have any free conduction electrons inside the crystal to transfer thermal energy from hot to cold regions of the material
- In nonmetals, the energy transfer involves lattice vibrations, that is, atomic vibrations of the crystal
- We know that we can view the atoms and bonds in a crystal as balls connected together through springs as shown for one chain of atoms in Figure.

Figure Conduction of heat in insulators involves the generation and propagation of atomic vibrations through the bonds that couple the atoms (an intuitive figure).

- According to the kinetic molecular theory, all the atoms would be vibrating and the average vibrational kinetic energy would be proportional to the temperature.

- When we heat one end of a crystal, we set up large-amplitude atomic vibrations at this hot end

- The springs *couple* the vibrations to neighbouring atoms and thus allow the large-amplitude vibrations to propagate, as a **vibrational wave,** to the cooler regions of the crystal.

- The efficiency of heat transfer depends not only on the efficiency of coupling between the atoms, and hence on the nature of interatomic bonding, but also on how the vibrational waves propagate in the crystal and how they are scattered by crystal imperfections and by their interactions with other vibrational waves
- The stronger the coupling, the greater will be the thermal conductivity, a trend that is intuitive but also borne out by experiments
- Diamond has an exceptionally strong covalent bond and also has a very high thermal conductivity; $\kappa \approx 1000\ W\ m^{-1}\ K^{-1}$
- On the other hand, polymers have weak secondary bonding between the polymer chains and their thermal conductivities are very poor; $\kappa < 1\ W\ m^{-1}\ K^{-1}$.

Diamond Bonding

Polymer Bonding

THERMAL RESISTANCE

Consider a component of length L that has a temperature difference ΔT between its ends as in Figure. The temperature gradient is $\Delta T/L$. Thus, the rate of heat flow Q', or the **heat current,** is

Fourier's Law:

$$Q' = A\kappa\frac{\Delta T}{L} = \frac{\Delta T}{(L/\kappa A)}$$

This should be compared with Ohm's law in electric circuits,

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{(L/\sigma A)}$$

where ΔV is the voltage difference across a conductor of resistance R, and I is the electric current.
In analogy with electrical resistance, we may define **thermal resistance θ** by

$$Q' = \frac{\Delta T}{\theta}$$

where, in terms of thermal conductivity,

Thermal Resistance :

$$\theta = \frac{L}{\kappa A}$$

The rate of heat flow Q' and the temperature difference ΔT correspond to the electric current I and potential difference ΔV, respectively. Thermal resistance is the thermal analog of electrical resistance

Figure 2.24 Conduction of heat through a component in (a) can be modeled as a thermal resistance θ shown in (b) where $Q' = \Delta T/\theta$.

Low-Temperature Behavior of Resistivity

Ex) Cu

At low temperatures, the number of lattice vibrations are greatly suppressed due to the quantum nature, resulting in much more rapid decrease with temperature. (The equipartition theorem for zone-center acoustic modes does not hold.)

See Kasap pp 387 for explaining why ρ is proportional to T^5 at low temperatures.

Mathiessens's Rule

$\sigma = ne^2\tau / m^*$

Another major contribution to the resistivity is from the defects/impurities, which breaks the translational symmetry of crystal. The defects can be classified into point defects (vacancies, interstitials, substitutional impurities) and extended defects (dislocations and grain boundaries). The point defects induce local variation of potential, which cause the electron scattering.

Let's first assume there is one type of defects. The mean time between the collisions with the defects is τ_I. Most discussions on the phonon scattering equally apply to the impurity scattering if they exist separately. However, the **two scattering mechanisms** should exist simultaneously. Then how to add them? The inverse of scattering time is the scattering frequency.

$1/\tau_T$: scattering frequency from thermal vibrations - ***Phonons***

$1/\tau_I$: scattering frequency from lattice imperfections - ***Lattice Imperfections***

If the two scattering mechanisms are independent, the mean scattering rate is the sum of these two scattering frequency.

$$\frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I}$$

The total resistivity is equal to the sum of the resistivity due to the lattice scattering and due to the impurities.

This is called the Mathiessen's rule.

$1/\tau$ = mean frequency of collision

Two Scattering Mechanisms

$$\frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I}$$

For the mobility

$$\mu_d = \frac{e}{m}\tau \rightarrow \frac{1}{\mu_d} = \frac{m}{e}\frac{1}{\tau} = \frac{m}{e}\left(\frac{1}{\tau_T} + \frac{1}{\tau_I}\right) = \frac{1}{\mu_T} + \frac{1}{\mu_I}$$

That is to say, the drift mobility is the harmonic average of lattice-scattering-limited drift mobility (μ_T) and the impurity-scattering-limited drift mobility (μ_I). In terms of resistivity,

$$\rho = \frac{1}{en\mu_d} = \frac{1}{en\mu_T} + \frac{1}{en\mu_I} = \rho_T + \rho_I$$

- Note that ρ_I is (approximately) temperature independent and depends on the defect concentration.
- In general, there are several types of defects. The scattering with each defect type is independent if the density is not too high. Therefore, one can apply to Matthiesen's rule, and total resistivity is the sum of resistivity caused by each defect type. The impurity scattering remains even at low temperatures.

The concentration of equilibrium defects such as vacancies depends on the temperature. Its concentrations are typically very low so do not affect the conductivity much. Here we rather concern on the non-equilibrium defects.

$$\sigma = ne\mu = ne^2\tau / m^*$$

$$\sigma = n e \mu_e + p e \mu_h$$

Demonstration of Mathiessen's law: resistivity of Cu

Lattice imperfections

Impurities

Defects

Typical temperature dependence of the resistivity of annealed and cold-worked (deformed) copper containing various amounts of Ni in atomic percentage. The deformed samples contain extended defects such as dislocations while they disappear in the annealed samples. The alloying element substitutes Cu atoms randomly so they play as impurities and scattering centers.

Problem

A brass disk of electrical resistivity 50 nΩ m conducts heat from a heat source to a heat sink at a rate of 10 W. If its diameter is 20 mm and its thickness is 30 mm, what is the temperature drop across the disk, neglecting the heat losses from the surface?

SOLUTION

We first determine the thermal conductivity:

$$\kappa = \sigma T C_{\mathrm{WFL}} = (5 \times 10^{-8}\ \Omega\ \mathrm{m})^{-1}(300\ \mathrm{K})(2.44 \times 10^{-8}\ \mathrm{W}\ \Omega\ \mathrm{K}^{-2})$$
$$= 146\ \mathrm{W\ m^{-1}\ K^{-1}}$$

The thermal resistance is

$$\theta = \frac{L}{\kappa A} = \frac{(30 \times 10^{-3}\ \mathrm{m})}{(146\ \mathrm{W\ m^{-1}\ K^{-1}})\pi(10 \times 10^{-3}\ \mathrm{m})^2} = 0.65\ \mathrm{K\ W^{-1}}$$

Therefore, the temperature drop is

$$\Delta T = \theta Q' = (0.65\ \mathrm{K\ W^{-1}})(10\ \mathrm{W}) = 6.5\ \mathrm{K\ or\ ^\circ C}$$

ELECTRICAL CONDUCTIVITY OF NONMETALS

All metals are good conductors because they have a very large number of conduction electrons free inside the metal. We should therefore expect solids that do not have metallic bonding to be very poor conductors, indeed insulators.

Figure Range of conductivities exhibited by various materials.

It is apparent that nonmetals are not perfect insulators with zero conductivity. There is no well-defined sharp boundary between what we call insulators and semiconductors. Conductors are intimately identified with metals. It is more appropriate to view insulators as **high resistivity** (or **low conductivity**) **materials.**

SEMICONDUCTORS

- A perfect Si crystal has each Si atom bonded to four neighbors, and each covalent bond has two shared electrons
- We know from classical physics (the kinetic molecular theory and Boltzmann distribution) that all the atoms in the crystal are executing vibrations with a distribution of energies.
- As the temperature increases, the distribution spreads to higher energies. Statistically some of the atomic vibrations will be sufficiently energetic to rupture a bond as indicated in Figure

Figure 2.26 (a) Thermal vibrations of the atoms rupture a bond and release a free electron into the crystal. A hole is left in the broken bond, which has an effective positive charge. (b) An electron in a neighboring bond can jump and repair this bond and thereby create a hole in its original site; the hole has been displaced. (c) When a field is applied, both holes and electrons contribute to electrical conduction.

Rupture of a bond releases an electron from the bond which is *free* to wander inside the crystal.

The free electron can drift in the presence of an applied field; it is called a **conduction electron.**

As an electron has been removed from a region of the crystal that is otherwise neutral, the broken-bond region has a *net positive charge.*

This broken-bond region is called a **hole** (*h*+). An electron in a neighboring bond can jump and repair this bond and thereby create a hole in its

- Effectively, the hole has been displaced in the opposite direction to the electron jump by this *bond switching.*

- Holes can also wander in the crystal by the repetition of bond switching.

- When a field is applied, both holes and electrons contribute to electrical conduction

- For all practical purposes, these holes behave as if they were *free* positively charged particles (independent of the original electrons) inside the crystal
- In the presence of an applied field, holes drift along the field direction and contribute to conduction just as the free electrons released from the broken bonds drift in <u>the opposite direction</u>
- and contribute to conduction.

Doping

- It is also possible to create free electrons or holes by intentionally doping a semiconductor crystal, that is **substituting impurity atoms** for some of the Si atoms

Defects can also generate free carriers. The simplest example is nonstoichiometric **ZnO** that is which has excess Zn. The electrons from the excess Zn are free to wander in the crystal and hence contribute to conduction

- Suppose that n and p are the concentrations of electrons and holes in a semiconductor crystal. If electrons and holes have drift mobilities of μ_e and μ_h, respectively,

The overall conductivity of the crystal is given by $\sigma = ep\mu_h + en\mu_e$

- Unless a semiconductor has been heavily doped, the concentrations n and p are much smaller than the electron concentration in a metal. Even though carrier drift mobilities in most semiconductors are higher than electron drift mobilities in metals, semiconductors have much lower conductivities due to their lower concentration of free charge carriers.

SKIN EFFECT: HF RESISTANCE OF A CONDUCTOR

- Consider the cylindrical conductor which is carrying a current I into ×
- The magnetic field B of the current I is clockwise. Consider two magnetic field values B_1 and B_2, is inside the core and B_2 is just outside the conductor.

Figure 2.30 Illustration of the skin effect.

A hypothetical cut produces a hollow outer cylinder and a solid inner cylinder. Cut is placed where it would give equal current in each section. The two sections are in parallel so that the currents in (b) and (c) sum to that in (a).

Assume that the conductor is divided into two conductors

The conductor in is now cut into a hollow cylinder and a smaller solid cylinder The currents I_1 and I_2 in the solid and hollow cylinders sum to I.

We can arrange things and choose B_1 such that our cut gives $I_1 = I_2 = 1/2\ I$

Obviously, I_1 flowing in the inner conductor is threaded (or linked) by both B_1 and B_2. (Remember that B_1 is just inside the conductor. So it threads at least 99% of I_1.)

On the other hand, the outer conductor is only threaded by B_2, simply because I_2 flows in the hollow cylinder and there is no current in the hollow, which means that B_1 is not threaded by I_2

- Clearly, I_1 threads more magnetic field than I_2 and thus <u>conductor (c) has a higher inductance</u> than (b)
- Recall that **inductance** is defined as the *total magnetic flux threaded per unit current*
- Consequently, an ac current will prefer paths near the surface where the inductive impedance is smaller. <u>As the frequency increases, the current is confined more and more to the surface region.</u>

Figure 2.30 Illustration of the skin effect.
A hypothetical cut produces a hollow outer cylinder and a solid inner cylinder. Cut is placed where it would give equal current in each section. The two sections are in parallel so that the currents in (b) and (c) sum to that in (a).

- For a given conductor, we can assume that most of the current flows in a surface region of depth δ, called the **skin depth,** as indicated in Figure <u>in the central region, the current will be negligibly small</u>
- The skin depth will obviously depend on the frequency ω. To find δ, we must solve Maxwell's equations in a conductive medium

$$\delta = \frac{1}{\sqrt{\frac{1}{2}\omega\sigma\mu}}$$

where ω is the angular frequency of the current, σ is the conductivity, and μ is the magnetic permeability of the medium, which is the product of the absolute (free space) permeability μ_o and the relative permeability μ_r.

Figure At high frequencies, the core region exhibits more inductive impedance than the surface region, and the current flows in the surface region of a conductor defined approximately by the skin depth, δ.

THIN METAL FILMS

Conduction in thin metal films

In certain applications, notably microelectronics, metal films are widely used to provide electrical conduction paths to and from the semiconductor devices.

Methods used to deposit thin films:

- Physical Vapor Deposition (PVD)
- Thermal evaporation
- Electron beam deposition
- Sputtering
- Electrodeposition

Physical Vapor Deposition (PVD), that is, by **vacuum deposition,** which typically involves either evaporation or sputtering

In **thermal evaporation,** the metal is evaporated from a heated source in a vacuum chamber. As the metal atoms, evaporated from the source, impinge and adhere to the semiconductor surface, they form a metal film which is often highly polycrystalline.

Stated differently, the metal atoms in the vapor condense to form a metal film on a suitably placed substrate

Physical Vapor Deposition

Thermal evaporation

In **electron beam deposition,** an energetic electron beam is used to melt and evaporate the metal

Sputtering is a vacuum deposition process that involves bombarding a metal target material with energetic Ar ions, which dislodges the metal atoms and then condenses them onto a substrate. The use of sputtering is quite common in microelectronic fabrication.

Copper metal interconnect films used in microelectronics are usually grown by **electrodeposition** (i.e.) electroplating, an electrochemical process, to deposit the metal film onto the required chip areas

Electrodeposition

Electron beam deposition

Sputtering

- In many applications, especially in microelectronics, we are interested in the resistivity of a metal film in which the thickness of the film or the average size of the grains is comparable to the mean distance between scattering events ℓ_{bulk} (the mean free path) in the bulk material

- In such cases, the resistivity of the metal film is greater than the corresponding resistivity of the bulk crystal

- A good example is the resistivity of interconnects and various metal films used in the "shrinking" world of microelectronics, in which more and more transistors are packed into a single Si crystal, and various device dimensions are scaled down.

Polycrystalline Films and Grain Boundary Scattering

In a highly polycrystalline sample the conduction electrons are more likely to be scattered by grain boundaries than by other processes as depicted in Figure 2.33a

Consider the resistivity due to scattering from grain boundaries alone as shown in Figure 2.33b. The conduction electron is free within a grain, but becomes scattered at the grain boundary.

Its mean free path ℓgrain is therefore roughly equal to the average grain size *d*.
If $\lambda = \ell_{crystal}$ is the mean free path of the conduction electrons in the *single crystal* (no grain boundaries), then

$$\frac{1}{\ell} = \frac{1}{\ell_{\text{crystal}}} + \frac{1}{\ell_{\text{grain}}} = \frac{1}{\lambda} + \frac{1}{d}$$

Figure 2.33 (a) Grain boundaries cause scattering of the electron and therefore add to the resistivity by Matthiessen's rule. (b) For a very grainy solid, the electron is scattered from grain boundary to grain boundary and the mean free path is approximately equal to the mean grain diameter. (c) TEM (transmission electron microscope) image of an annealed polycrystalline Cu thin film of thickness 41.7 nm, encapsulated in SiO_2. The films structure is composed of grains with an average size 87.7 nm. The resistivity of this film is 30 nΩ m, higher than the bulk resistivity of Cu (17 nΩ m).

| (c) Courtesy of Tik Sun and Bo Yao.

The resistivity is inversely proportional to the mean free path which means that the resistivity of the bulk single crystal $\rho_{crystal} \propto 1/\lambda$ and the resistivity of the polycrystalline sample $\rho \propto 1/\ell$. Thus,

$$\frac{\rho}{\rho_{\text{crystal}}} = 1 + \left(\frac{\lambda}{d}\right)$$

Polycrystalline metal films with a smaller grain diameter d (*i.e.,* more grainy films) will have a higher resistivity.

In a more rigorous theory we have to consider a number of effects. It may take more than one scattering at a grain boundary to totally randomize the velocity, so we need to calculate the effective mean free path that accounts for how many collisions are needed to randomize the velocity.

There is a possibility that the electron may be totally reflected back at a grain boundary (bounce back). Suppose that the probability of reflection at a grain boundary is R. Suppose that the probability of reflection at a grain boundary is R and d is the average grain size (diameter), then the resistivity can be calculated by the **Mayadas–Shatzkes formula**

$$\frac{\rho}{\rho_{\text{crystal}}} = \left[1 - \frac{3}{2}\beta + 3\beta^2 - 3\beta^3 \ln(1 + 1/\beta)\right]^{-1}$$

in which the quantity β is defined by

$$\beta = \frac{\lambda}{d}\left(\frac{R}{1 - R}\right)$$

Figure 2.34 Conduction in thin films may be controlled by scattering from the surfaces.

Figure 2.35 The mean free path of the electron depends on the angle θ after scattering.

The β represents the λ/d ratio adjusted for the reflection to transmission ratio of the electron at the grain boundary

When the grain size is large, β is small and Equation simplifies to

$$\frac{\rho}{\rho_{\text{crystal}}} \approx 1 + \frac{3}{2}\beta$$

For highly polycrystalline films, the grain size would be small and $\beta >> 1$

Books & Weblinks

❑ TEXT BOOK

1. S. O. Kasap, Principles of Electronic Materials and Devices , 2018, 4th Edition, McGraw Hill Education.

❑ BOOK REFERENCES

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3. Adel S. Sedra, Kenneth C. Smith & Arun N. Chandorkar, "Microelectronic Circuits: Theory and Applications", 2014, 7th Edition, Oxford University Press, New York.
4. Donald A. Neamen, "Semiconductor Physics and Devices", 2017, 4th Edition, McGraw Hill.

❑ WEBSITE LINKS

- https://nptel.ac.in/courses/108/108/108108112/
- https://nptel.ac.in/courses/113/106/113106065/