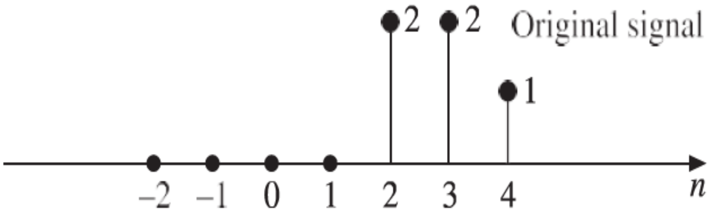
Revision

Q1. Solve : $x[n] = u(n+4) - u(n-2)$**, where**

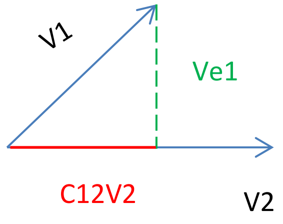
u[n] is



Analogy between vectors and signals

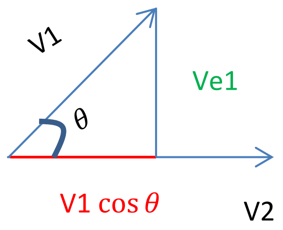
Vector: A vector contains magnitude and direction. Eg: Velocity, acceleration, force, electric field etc.

1. Consider two vectors V1 and V2.
2. Vector V1 can be represented in terms of V2 in many ways. One of them is by projecting V1 on V2.



V1 =C12V2+Ve1, where Ve1 is error vector.

The component of V1 along V2 = C12V2.



3. $V1 \cos\theta$ = C12V2
4. The dot product between two vectors V1 and V2 is

$$V1.V2 = |V1||V2|\ cos\theta$$

$$\frac{V1.V2}{|V2|} = |V1|\ cos\theta$$

$$\frac{V1.V2}{|V2|} = C12V2$$

$$C12 = \frac{V1.V2}{|V2|^2} = \frac{V1.V2}{V2.V2}$$

V1 = C12 V2

$$C12 = \frac{V1.V2}{|V2|^2}$$

If C12 = ; then V1.V2 = 0

I.e the vectors V1 and V2 are mutually perpendicular to each other. Therefore, there is no component of V1 along V2.

Ex:

V1 = 3 I +3j +3k

V2 = i+j+k

V1 = C12 V2

$$C12 = \frac{V1.V2}{V2.V2} = \frac{3+3+3}{3} = 3$$

Signals:

1. The dot product between signals V1 and V2 is equivalent to

   $V1.V2 \sim \int_{t1}^{t2} f1(t)f2(t)dt$, where f1(t) and f2(t) are two signals in the interval t1 <t<t2

2. f1(t) can be approximated using f2(t) as,

$f1(t) \approx C12f2(t)$; t1< t<t2

The error in this approximation is,

$$f_e(t) = f_1(t) - C_{12}f_2(t)$$

3. The mean square error in this approximation is,

$$\varepsilon = \frac{1}{(t_2 - t_1)} \int_{t1}^{t2} {f_e}^2(t)dt$$

$$= \frac{1}{(t_2 - t_1)} \int_{t1}^{t2} [f_1(t) - C_{12}f_2(t)]^2 dt$$

$$= \frac{1}{(t_2 - t_1)} \int_{t1}^{t2} [f_1(t)^2 + {C_{12}}^2 f_2(t)^2 - 2C_{12}f_1(t)f_2(t)]dt$$

We want to minimize mean square error,

$f(x) = x^3 - x^2 + 3x + 1$, we differentiate and equate it to zero.

Similarly, for signal we have to find the value of C12 at which mean square error is minimum by calculating

$\frac{d\epsilon}{dC12} = 0$

$\frac{d\, \frac{1}{(t_2-t_1)} \int_{t1}^{t2}[f_1(t)^2 + C_{12}{}^2 f_2(t)^2 - 2C_{12} f_1(t) f_2(t)]dt}{dC12}$ =0

$$=> \int_{t1}^{t2} [0 + 2C12 f_2(t)^2 - 2\, f_1(t) f_2(t)]dt = 0$$

$$=> C12 \int_{t1}^{t2} f_2(t)^2\, dt = \int_{t1}^{t2} f_1(t) f_2(t) dt$$

$$=> C12 = \frac{\int_{t1}^{t2} f_1(t) f_2(t) dt}{\int_{t1}^{t2} f_2(t)^2\, dt}$$

If C12 =0;

Then $\int_{t1}^{t2} f_1(t) f_2(t) dt = 0$;

That means, similar to vectors $f_1(t)$ and $f_2(t)$ and orthogonal/ perpendicular to each other.

I.e. there is no component of $f_2(t)$ in $f_1(t)$.