Revision

Q1. Show that the functions $\sin n\omega_0 t$ and $\cos m\omega_0 t$ are orthogonal over interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$ for any integers value of m, n.

Solution:

If two vectors are Orthogonal, then

V1. V2 = 0

If two signals f1(t) and f2(t) in interval t1<t<t2 are orthogonal, then

$$\int_{t1}^{t2} f1(t)f2(t)dt = 0$$

$$I = \int_{t0}^{t0+\frac{2\pi}{\omega_0}} \sin n\omega_0 t \cos m\omega_0 t \, dt$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

So,

$$I = \frac{1}{2}\int_{t0}^{t0+\frac{2\pi}{\omega_0}} 2\ \sin n\omega_0 t \cos m\omega_0 t \, dt$$

$$=> I = \frac{1}{2}\int_{t0}^{t0+\frac{2\pi}{\omega_0}} [\sin(n+m)\omega_0 t + \sin(n-m)\omega_0 t]dt$$

$$=> \ I = \frac{1}{2}\int_{t0}^{t0+\frac{2\pi}{\omega_0}} \sin(n+m)\omega_0 t\, dt + \frac{1}{2}\int_{t0}^{t0+\frac{2\pi}{\omega_0}} \sin(n-m)\omega_0 t\, dt$$

$$=> \ -\frac{1}{2(n+m)\omega_0}\{\cos(n+m)\,\omega_0 t\}_{t0}^{t0+\frac{2\pi}{\omega_0}} - \frac{1}{2(n-m)\omega_0}\{\cos(n-m)\,\omega_0 t\}_{t0}^{t0+\frac{2\pi}{\omega_0}}$$

$$=> -\frac{1}{2(n+m)\omega_0}\left\{\cos\left[(n+m)\omega_0\left(t_0+\frac{2\pi}{\omega_0}\right)\right] - \cos(n+m)\,\omega_0 t_0\right\}$$

$$-\frac{1}{2(n-m)\omega_0}\left\{\cos\left[(n-m)\omega_0\left(t_0+\frac{2\pi}{\omega_0}\right)\right] - \cos(n-m)\,\omega_0 t_0\right\}$$

$$= -\frac{1}{2(n+m)\omega_0}\{\cos[2\pi(n+m) + (n+m)\omega_0 t_0] - \cos(n+m)\,\omega_0 t_0\}$$

$$-\frac{1}{2(n-m)\omega_0}\{\cos[2\pi(n-m) + (n-m)\omega_0 t_0] - \cos(n-m)\,\omega_0 t_0\}$$

For any integer value of m and n,

$2\pi(n+m)$ *will always be an even multiple of* π, *so* $\cos[2\pi(n+m)$

$+ (n+m)\omega_0 t_0]$ *will be* $\cos(n+m)\omega_0 t_0$

$$=> I = -\frac{1}{2(n+m)\omega_0}\{\cos(n+m)\omega_0 t_0 - \cos(n+m)\,\omega_0 t_0\}$$

$$-\frac{1}{2(n-m)\omega_0}\{\cos(n-m)\omega_0 t_0 - \cos(n-m)\,\omega_0 t_0\}$$

$$=> I = 0$$

Hence, $\sin n\omega_0 t$ and $\cos m\omega_0 t$ are orthogonal over interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$ for any integers value of m, n.

$\omega_0 = \frac{2\pi}{T}$, => $T = \frac{2\pi}{\omega_0}$

So the interval can be written as, $(t_0, t_0 + T)$.