1-05

Analysis of Energy & Power Signals

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Unit I: Classification of Signals and Systems

Electrical Energy & Power

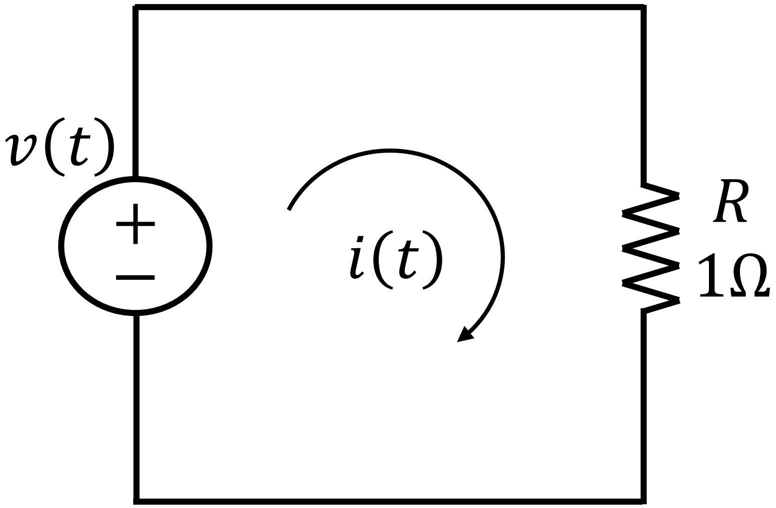
Let's consider a simple circuit shown here.

Instantaneous power dissipated in resistance is given as

$$P_i = v(t) \times i(t) = \frac{v^2(t)}{R} = i^2(t)R$$

The power generated by the source at any point of time can also be written as

$$P_i = |v^2(t)| = |i^2(t)|$$



Total Energy of the source is given as

$$E_t = \int_{-\infty}^{\infty} |v^2(t)| dt$$

Total source energy will be infinity for periodical signal.

In such case, the average power is calculated as

$$P_a = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |v^2(t)| dt$$

Energy & Power Signals – Continuous-Time Signals

The energy of a **Continuous-Time** signal $x(t)$ is determined as

$$E_x = \int_{-\infty}^{\infty} |x^2(t)|dt$$

If $0 < E_x < \infty$, the signal is termed as an Energy Signal.

There are signals which do not satisfied this condition.

For such signals we consider determination of their power given as

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x^2(t)|dt$$

If $0 < P_x < \infty$, the signal is termed as a Power Signal.

There is another category of signals which do not satisfy above both the conditions and are termed as neither an Energy Signal nor a Power Signal

Energy & Power Signals – Discrete-Time Signals

For **Discrete-Time** signal $x(n)$, the energy is determined as

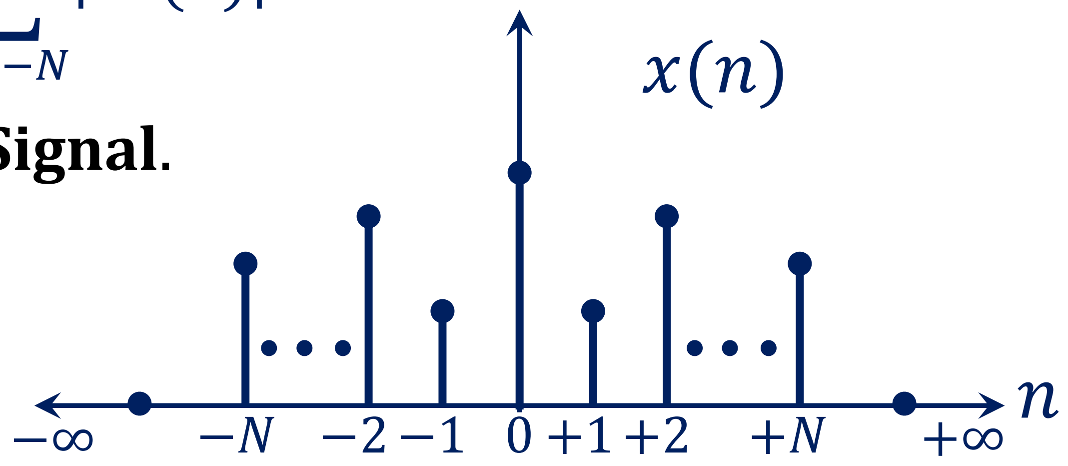
$$E_x = \sum_{n=-\infty}^{\infty} |x^2(n)|$$

If $0 < E_x < \infty$, the signal is termed as an Energy Signal.

Whereas as the power of a discrete-time signal $x(n)$ given by

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x^2(n)|$$

If $0 < P_x < \infty$, the signal is termed as a Power Signal.



Periodic Signals

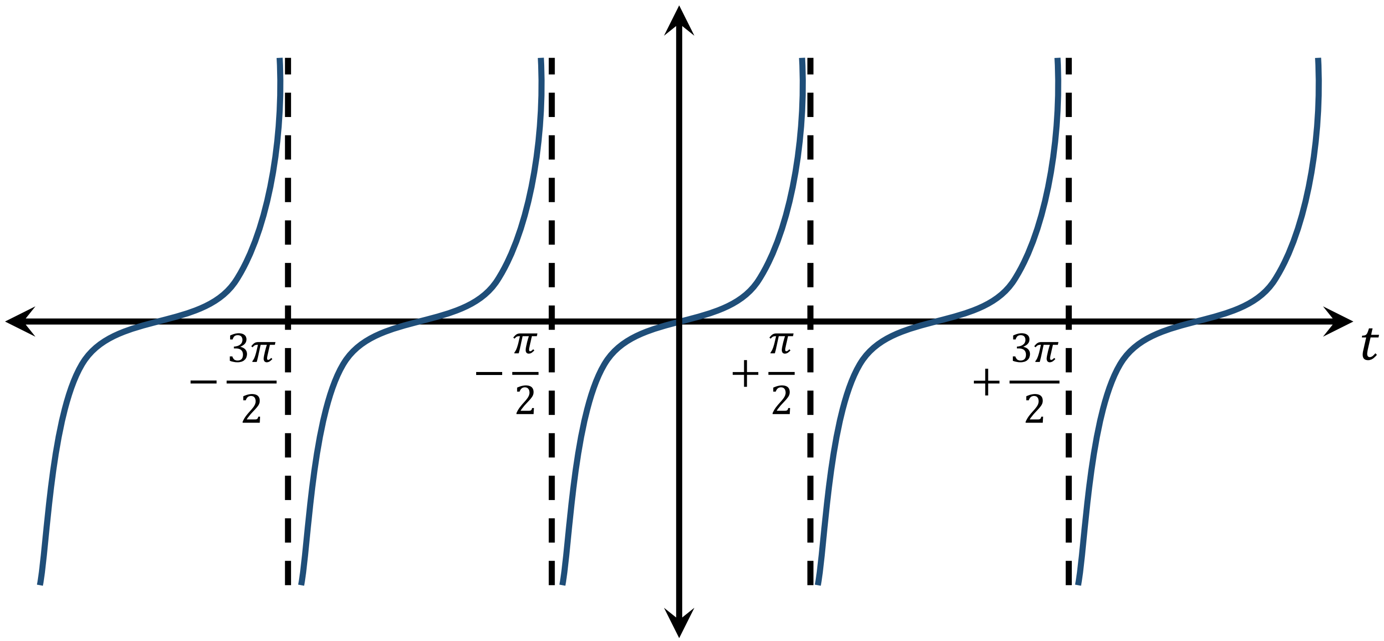
All periodic signals are power signals, if their magnitude never reach infinity at any point of time. For example,

$$x_1(t) = \sin \omega t \quad \text{and} \quad x_2(t) = \sin \omega t + \cos \omega t$$

If the magnitude of a signal reaches infinity at any point of time then signal is **neither an Energy nor a Power Signal.**

For example,

$$x(t) = \tan t$$

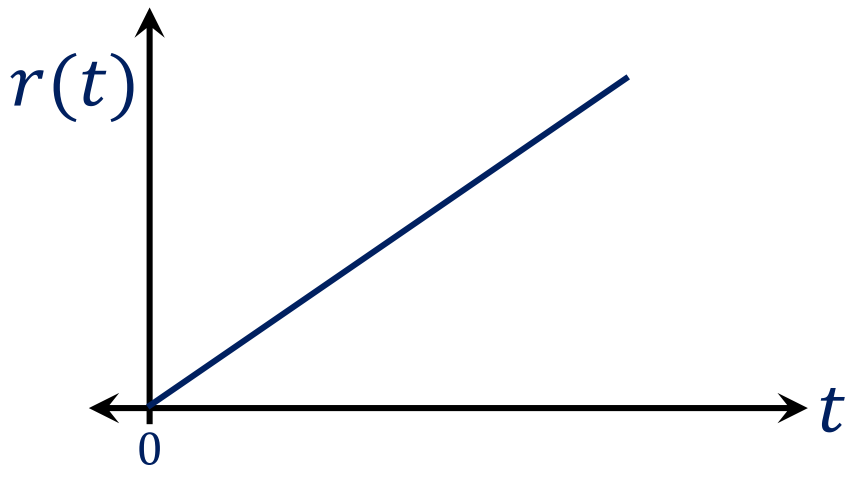


This is a periodic signal but not a power signal, since its magnitude reaches infinity at multiple instances of time.

Aperiodic Signals

Let's consider now an aperiodic signal

$$r(t) = t.u(t)$$



Since magnitude of $r(t)$ reaches infinity at $t = \infty$, therefore, this is also a **neither an energy signal nor a power signal**.

Practice Problem

Problem: Determine if signal $x(t)$ is energy signals or a power signal

$$\boldsymbol{x(t) = ae^{-\alpha t}u(t) \qquad \alpha > 0}$$

Solution: The energy of a continuous-time signal $x(t)$ is given as

$$E_x = \int_{-\infty}^{\infty} |x^2(t)|dt = \int_{0}^{\infty} |ae^{-\alpha t}|dt = a\left|\frac{e^{-\alpha t}}{-\alpha}\right|_0^{\infty}$$

$$E_x = a\left[\frac{e^{-\infty}}{-\alpha} - \frac{e^0}{-\alpha}\right] = a\left[\frac{0}{-\alpha} - \frac{1}{-\alpha}\right] = \frac{a}{\alpha}$$

The signal is an Energy Signal, since $E_x < \infty$.

Practice Problem

Problem: Determine if signal $x(t)$ is energy signals or a power signal

$$\boldsymbol{x(t) = ae^{-\alpha t}u(t) \qquad \alpha > 0}$$

Solution: The energy of a continuous-time signal $x(t)$ is given as

$$E_x = \int_{-\infty}^{\infty} |x^2(t)| dt = \int_{0}^{\infty} |ae^{-\alpha t}| dt = a \left| \frac{e^{-\alpha t}}{-\alpha} \right|_0^{\infty}$$

$$E_x = a \left[\frac{e^{-\infty}}{-\alpha} - \frac{e^{0}}{-\alpha} \right] = \frac{a}{\alpha}$$

The signal is an Energy Signal, since $\boldsymbol{E_x < \infty}$.

Practice Problem

Problem: **Determine if following Discrete-Time signal $x(n)$ is an Energy signal or a Power signal**

$$\boldsymbol{x(n) = \left(\frac{1}{4}\right)^{n} u(n)}$$

Solution: Let's determine energy of the signal as

$$E_x = \sum_{n=-\infty}^{\infty} |x^2(n)| = \sum_{n=0}^{\infty} \left[\left(\frac{1}{4}\right)^{n}\right]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^{n}$$

It's a Geometric Progression Series $\left(\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}\right)$. Hence

$$E_x = \frac{1}{1 - \frac{1}{16}} = \frac{16}{15} < \infty$$

Hence $x(n)$ is a Energy Signal.

Practice Problem

Problem: **A signal is given as $\boldsymbol{x(t) = A\cos(\omega_0 t)}$.**

Determine if it is an Energy Signal or a Power signal

Solution: This a periodic signal that exists for infinite time, therefore the energy of the signal $E_x = \infty$. Let's now determine the signal power as

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x^2(t)|dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (A\cos(\omega_0 t))^2 dt = \lim_{T\to\infty} \frac{A^2}{2T}\int_{-T}^{T} \left[\frac{1}{2} + \frac{1}{2}\cos 2(\omega_0 t)\right] dt$$

$$P_x = \lim_{T\to\infty} \frac{A^2}{2T}\left|\frac{t}{2} + \frac{\sin 2(\omega_0 t)}{2\omega_0}\right|_{-T}^{T} = \lim_{T\to\infty} \frac{A^2}{2T}\left[\frac{T}{2} - \frac{-T}{2}\right] = \frac{A^2}{2} < \infty$$

Hence $\boldsymbol{x(t) = A\cos(\omega_0 t)}$ is a Power Signal.

1-05 Questions (Exam Point of View)

Lower Order Thinking Skills (LOTS)

1. What is the difference between energy and power signal?
2. Identify if signal $x(t) = A \sin t$ is an energy signal or a power signal.

Higher Order Thinking Skills (HOTS)

1. Determine whether signal $x(t) = u(t+1)$ is a power or energy signals.
2. Compute the energy of $x(n) = a^n u(n)$ when $a < 1$.
3. Determine energy of the signal $x(t) = u(t+2) - u(t-2)$?

Thank You

Next Topic: Continuous-Time and Discrete-Time Systems

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