Worked Examples Using Nyquist’s Theorem

Problems involving Nyquist's Theorem generally require the use of the Fourier Transform. However, being able to compute the Fourier Transform is not required to understand the concepts of Nyquist's Theorem, and thus these computations will not be included in these example problems. For more information about Fourier Series and the Fourier Transform, click here.

1) Through the Fourier Transform, it is revealed that a signal is made up of constituent frequencies 1000 Hz, 1800 Hz, and 2000 Hz. Assuming that this information was collected accurately, what was the minimum sample rate required to find this information?

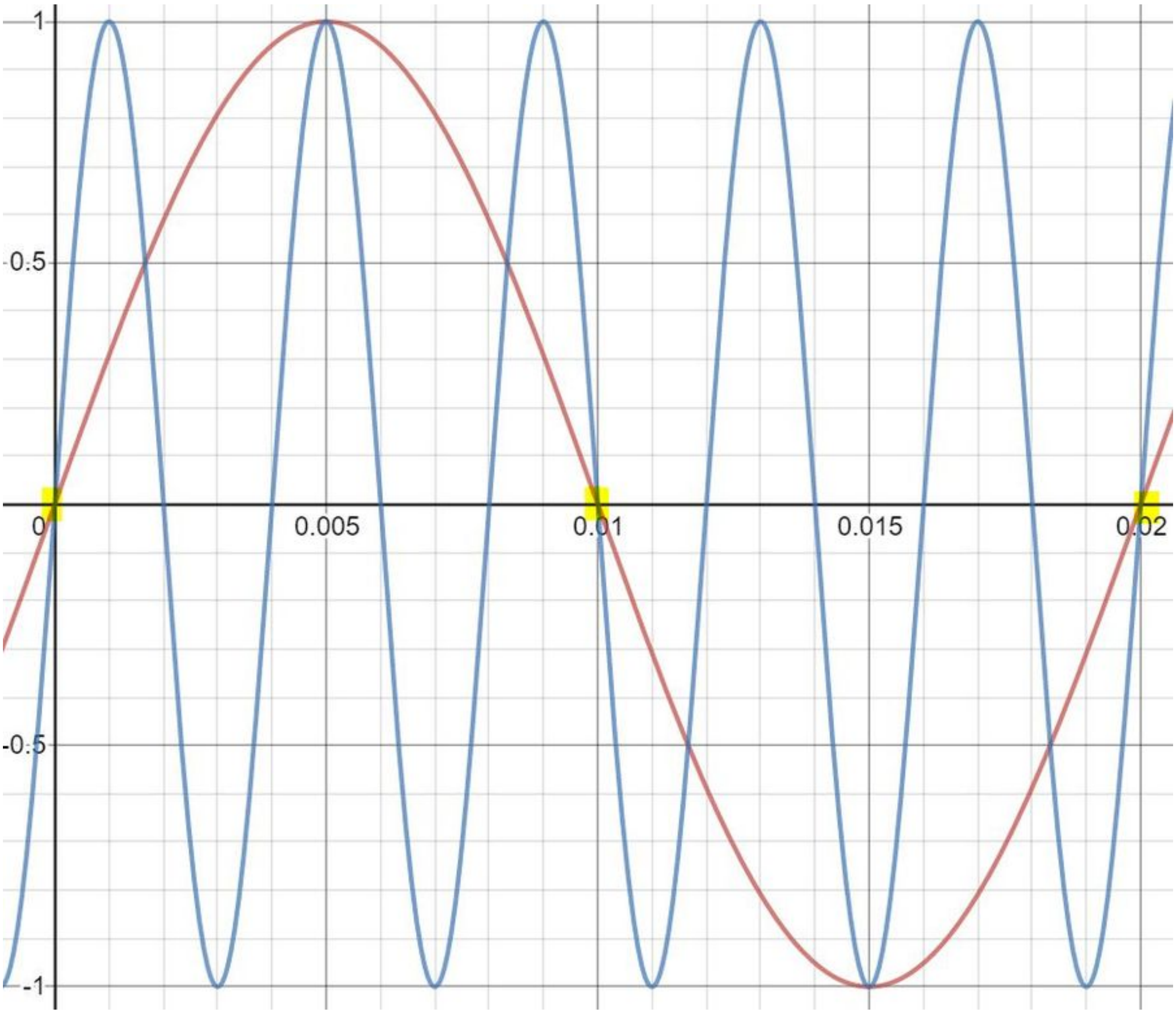
A: The minimum sample rate is the Nyquist Rate, which is two times the maximum frequency contained within the signal. Therefore, the answer is 2000 Hz * 2 = 4000 Hz.

2) If a signal is thought to have a maximum frequency between 1000 Hz and 4000 Hz, which of the following would be the most appropriate sample rate?
a. 500 Hz
b. 8000 Hz
c. 9000 Hz
d. 24000 Hz

A: The correct answer is *c*. Answer choice *c* is correct because the sample rate must be greater than two times the maximum frequency contained within the signal. Answer *a* is of course incorrect, since it is less than the estimates given for maximum frequency. Answer choice *b* is incorrect because if the signal has a frequency of 4000 Hz, the sample rate must be *greater* than 8000 Hz, not equal to 8000 Hz. Answer choice *d* is incorrect because it is unnecessarily large, leading to excessive oversampling.

3) Signals can be modeled by sine waves, such as $X = \sin(100\pi t)$. For the signal modeled by X, is it appropriate to utilize a sample rate of 100 Hz?

A: No. The sine function has a period $2\pi/c$, where c is the coefficient inside the sine function. In this case, the period is $2\pi/(100\pi)$, so it is 1/50, or 0.02 seconds. Because period is the inverse of frequency, the frequency of this sine wave is 50 Hz. The Nyquist Rate is thus 100 Hz. However, Nyquist's Theorem states that the sample rate must be *greater*, and not equal to, the Nyquist Rate. This is a great example to illustrate why this is the case. The image below shows the graph of X, in red, as well as the graph of $X_2 = \sin(500\pi t)$, in blue. The points that would be collected at a sample rate of 100 Hz would be at t = 0, 0.01, and 0.02 seconds. As evidenced by the graph, all three of these samples have the same value. Therefore, these two signals are indistinguishable at a sample rate of exactly 100 Hz. In fact, the entire family of signals $X_c = \sin((100c)\pi t)$ is indistinguishable for integer values of c that are at least 1, when sampling begins at $t = 0$ and the sampling rate is 100 Hz. This is why it is so important to recognize that the sample rate must be greater than, and not equal to the Nyquist Rate.



These two sine waves have the same sampled values with a sample rate of 100 Hz, the Nyquist Rate of *X*. [9]
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