Norm of a vector

Norm: It is the magnitude/ length of a vector.

If x is a vector such that $\vec{x} = \{x1, x2, \ldots, xn\}$, then

The L2 norm of a vector is $||\vec{x}|| = \sqrt{x1^2 + x2^2 + \cdots + xn^2}$ (Distance from origin)

Properties of Norm:

1. $||\vec{x}|| \geq 0$
2. $||\vec{x}|| = 0$ iff $\vec{x} = \vec{0}$
3. $||t\vec{x}|| = |t|||\vec{x}||$, where t is a scalar.
4. $||x + y|| \leq ||x|| + ||y||$, triangular inequality

Unit Vector:

$\vec{V}$ is said to be an unit vector if $\|\vec{V}\| = 1$

L2 Norm:

Q. If $\vec{V} = (-5, 3, 9)$, find the norm of the vector.

Answer: $\|\vec{V}\| = \sqrt{(-5)^2 + (3)^2 + (9)^2} = \sqrt{115}$

Euclidean Distance: Distance between two vectors.

$$||\vec{x} - \vec{y}|| = \sqrt{(x1 - y1)^2 + (x2 - y2)^2 + \cdots + (xn - yn)^2}$$

Q. If $\overrightarrow{V1} = (0, 3, 9)$ and $\overrightarrow{V2} = (-5, 0, 0)$ find the Euclidean Distance between the two vectors.

$$\|\overrightarrow{V1} - \overrightarrow{V2}\| = \sqrt{(0 - (-5))^2 + (3 - 0)^2 + \cdots + (9 - 0)^2} = \sqrt{115}$$

Norm of a Signal

Properties of Norm:

1. $\|\vec{x}\| \geq \phi$
2. $\|\vec{x}\| = \phi$ iff $x(t) = \phi \quad \forall t$
3. $\|\alpha\vec{x}\| = |\alpha|\|\vec{x}\|$, where α is a scalar.
4. $\|u1 + u2\| \leq \|u1\| + \|u2\|$, triangular inequality

L1 Norm:

$$\|u\|_1 = \int_{-\infty}^{\infty} |u(t)|dt$$

L2 Norm:

$$\|u\|_2 = \int_{-\infty}^{\infty} (u(t))^2 dt$$

Used in calculation of power and energy.

Euclidean Distance between two signals:

$$\|u1(t) - u2(t)\|^2 = \int_0^T (u1(t) - u2(t))^2 dt$$

Moment of a Signal

1st order moment:

$$E[t] = \int_{-\infty}^{\infty} tu(t)dt$$

2nd order moment:

$$E[t^2] = \int_{-\infty}^{\infty} t^2 u(t)dt$$

Linearly Dependent and Linearly Independent Vectors

Linearly Independent:

The vectors v1, v2, …, vp are linearly independent, if

$a1V1 + a2V2 + \cdots + apVp = 0$, for a1=a2=…=ap=0

That is you cannot find the linear combination of vectors.

Linearly Dependent:

The vectors v1, v2, …, vp are linearly dependent, if

$a1V1 + a2V2 + \cdots + apVp = 0$, for a1=a2=…=ap≠0

That is you can find the linear combination of vectors.

Q. V1 = [1 2 3], V2= [2 -1 4], v3 = [0 5 2] is linearly independent or not?

Answer:

$a1V1 + a2V2 + \cdots + apVp = 0$

$\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} a1 \\ a2 \\ a3 \end{bmatrix}$=0

a1 = -2a3

a2=a3

2V1+V2+V3=0, so V1, V2 and V3 are linearly dependent.

Module 2:

Continuous Time and Discrete Time System

Classification of Systems:

I. Linear and Non-Linear System
II. Time Variant and Time invariant System
III. Linear Time variant (LTV) and Linear time invariant System (LTI)
IV. Static and Dynamic System
V. Causal and Non-Causal system
VI. Invertible and non-invertible System
VII. Stable or Unstable systems

Linear and Non-Linear System

A system is said to be **linear if it satisfies the superposition** principle.

Consider a system with input $x1(t) and\ x2(t)$ and output $y1(t) and\ y2(t)$.

For linearity:

$$T[a1x1(t) + a2x2(t)] = a1T[x1] + a2t[x2]$$

Q. Find a system $\boldsymbol{y(t) = x^2(t)}$ is linear or non-linear.

Ans:

$T[x1(t)] = x1^2(t)$

$T[x2(t)] = x2^2(t)$

$T[\boldsymbol{a1x1(t) + a2x2(t)}] = [\boldsymbol{a1x1(t) + a2x2(t)}]^2 = (\boldsymbol{a1x1(t)})^2 + (\boldsymbol{a2x2(t)})^2 + \boldsymbol{2a1x1(t)a2x2(t)}$

$[a1x1(t) + a2x2(t)] \neq a1T[x1] + a2t[x2]$, so its violating **superposition** principle. So it's a **Non-Linear system**.

Q. Find a system $y(t) = x(t)$ is linear or non-linear.

Ans:

$T[x1(t)] = x1(t)$

$T[x2(t)] = x2(t)$

$T[\boldsymbol{a1x1(t)} + \boldsymbol{a2x2(t)}] = \boldsymbol{a1x1(t)} + \boldsymbol{a2x2(t)}$

$T[a1x1(t) + a2x2(t)] = a1T[x1] + a2t[x2]$, so it satisfies **superposition** principle. So it's a **Linear system**.

Time Variant and Time invariant System

A system is said to be time variant if its input/ output characteristic changes with time otherwise the system is time invariant.

The condition for time invariance is:

For discrete time:

$y(n,k) = y(n-k)$, where $y(n,k) = T[x(n-k)]$

For continuous Time:

$y(t,k) = y(t-k)$, where $y(t,k) = T[x(t-k)]$

Ex 1: $y(n) = x(n) + x(n-2)$, find if the system is time variant or Time variant.

$y(n,k) = T[x(n-k)] = x(n-k) + x(n-2-k)$ (Delay the input and get the result)

$y(n-k) = x(n-k) + x(n-k-2)$

$y(n,k) = y(n-k)$, So it is a time invariant system.