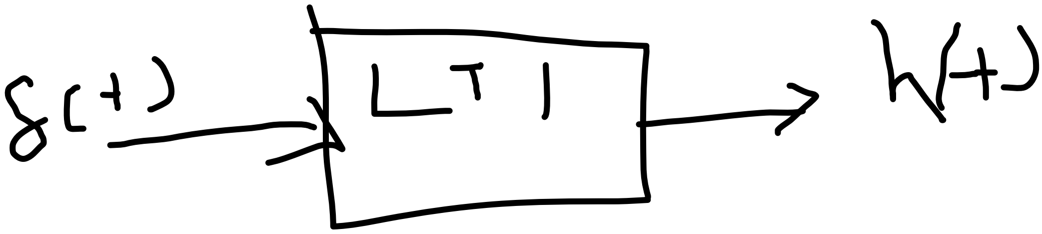
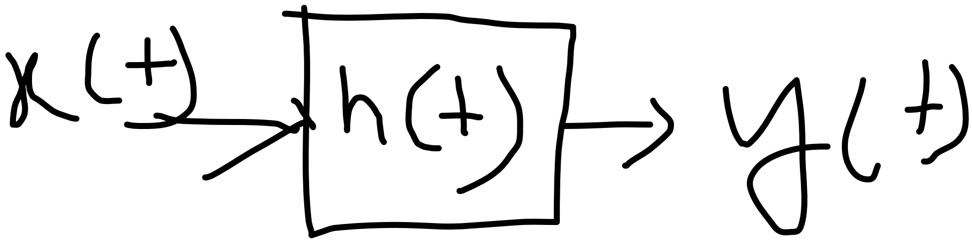
<u>**Impulse and step response of the systems**</u>



- Impulse response is calculated for LTI systems.



In time domain: $y(t) = x(t) * h(t)$

In Laplace domain : $Y(s) = X(s)H(s)$

So, $H(s) = \frac{Y(s)}{X(s)}$ is transfer function.

We can get the impulse response by taking the inverse Laplace transform of H(s),

$$L^{-1}[H(s)] = h(t)$$

Impulse response of a system is when $x(n) = \delta(n)$ or $x(t) = \delta(t)$

Step Response of a System:

Step response of a system is when $x(n) = u(n), x(t) = u(t)$.

Impulse response is represented by $s(t) or\ s(n)$.

$s(n) = x(n) * h(n)$

$s(n) = u(n) * h(n)$= $h(n)$* $u(n) = \sum_{k=-\infty}^{\infty} h(k)u(n-k)$

$$u(n-k) = 1, n-k \geq 0, n \geq k, k \leq n$$
$$u(n-k) = 0, n-k < 0, n < k, k > n$$

So step response is,

$$s(n) = \sum_{k=-\infty}^{n} h(k)$$

For continuous time:

$$s(t) = \int_{\tau=-\infty}^{t} h(\tau)d\tau$$

Q. Evaluate step response of a system

$$h(n) = \frac{1}{2}^{n} u(n)$$

Solution:

$$s(n) = \sum_{k=-\infty}^{n} h(k) = \sum_{k=-\infty}^{n} \frac{1}{2}^{k} u(k) = \sum_{k=0}^{n} \frac{1}{2}^{k} .1$$

$$s(n) = \frac{\frac{1}{2}^{n+1} - 1}{\frac{1}{2} - 1} = 2 - \frac{1}{2}^{n} \quad \text{(Formula: } \sum_{k=0}^{n} a^{n} = \frac{a^{n+1}-1}{a-1} = \frac{1-a^{n+1}}{1-a}\text{)}$$

Convolution

Q. Find convolution of two signals.

$x(t) = t^2 + 2t + 1$

$y(t) = t^2 + 3t + 4$

$z(t) = x(t) * y(t)$ (Sliding window method)

$t^2 \quad 2t \quad 1$

$4 \quad 3t \quad t^2$

$4 \quad 3t \quad t^2 \qquad t^4$

$4 \quad 3t \quad t^2 \rightarrow 3t^3 + 2t^3 = 5t^3$

$4 \quad 3t \quad t^2 \rightarrow 4t^2 + 6t^2 + t^2 = 11t^2$

$4 \quad 3t \rightarrow 8t + 3t = 11t$

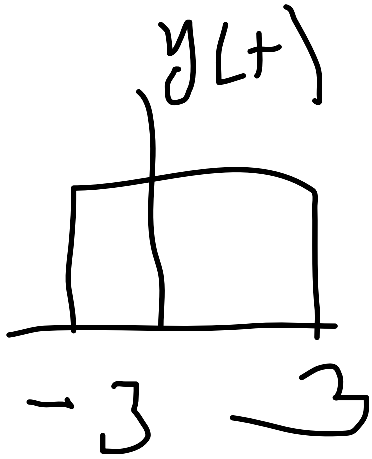
$4 \rightarrow 4$

$$z(t) = t^4 + 5t^3 + 11t^2 + 11t + 4$$

Standard method:

$z(t) = \int_{-\infty}^{\infty} x(\tau) y(t-\tau) d\tau = \int_{-\infty}^{\infty} y(\tau) x(t-\tau) d\tau$

Q2.



Convoluted signal limits are,

Sum of lower limits <t<Sum of Upper Limits

-2+(-3) <t<2+3

-5<t<5

Q3. A linear System with input $x(n)$ and output $y(n)$ related as

$y(n) = \sum_{k=-\infty}^{\infty} x(k) g(n-2k)$, where $g(n) = u(n) - u(n-4)$. Find $y(n)$ when $x(n) = \delta(n-2)$

Solution:

$y(n)$ when $x(n) = \delta(n-2)$ -> $\delta(n-k)$, here k=2

$\delta(n) = 1$ at n=0

$\delta(n-2) = 1$ at n=2

$y(n) = x(2) g(n-4)$

$y(n) = \delta(2-2)g(n-4)$= $\delta(0)g(n-4)$= $g(n-4)$

$y(n) = \ u(n-4) - u(n-8)$

Q4. A linear System with input $x(n)$ and output $y(n)$ related as

$y(n) = \sum_{k=-\infty}^{\infty} x(k)g(n-2k)$, where $g(n) = u(n) - u(n-4)$. Find $y(n)$ when $x(n) = u(n)$

Solution:

$y(n) = \sum_{k=-\infty}^{\infty} x(k)g(n-2k)$=$\sum_{k=0}^{\infty} g(n-2k)$

Discrete Convolution:

Q. x(n) = {1, 2, 3, 4}, h(n)={1,1,-1,1}

Find z(n)=x(n)* h(n)

If Length of x(n) is m and length of h(n) is n, then total length is m+n-1.

Solution:

	1	**2**	**3**	**4**
1	1	2	3	4
1	1	2	3	4
-1	-1	-2	-3	-4
1	1	2	3	4

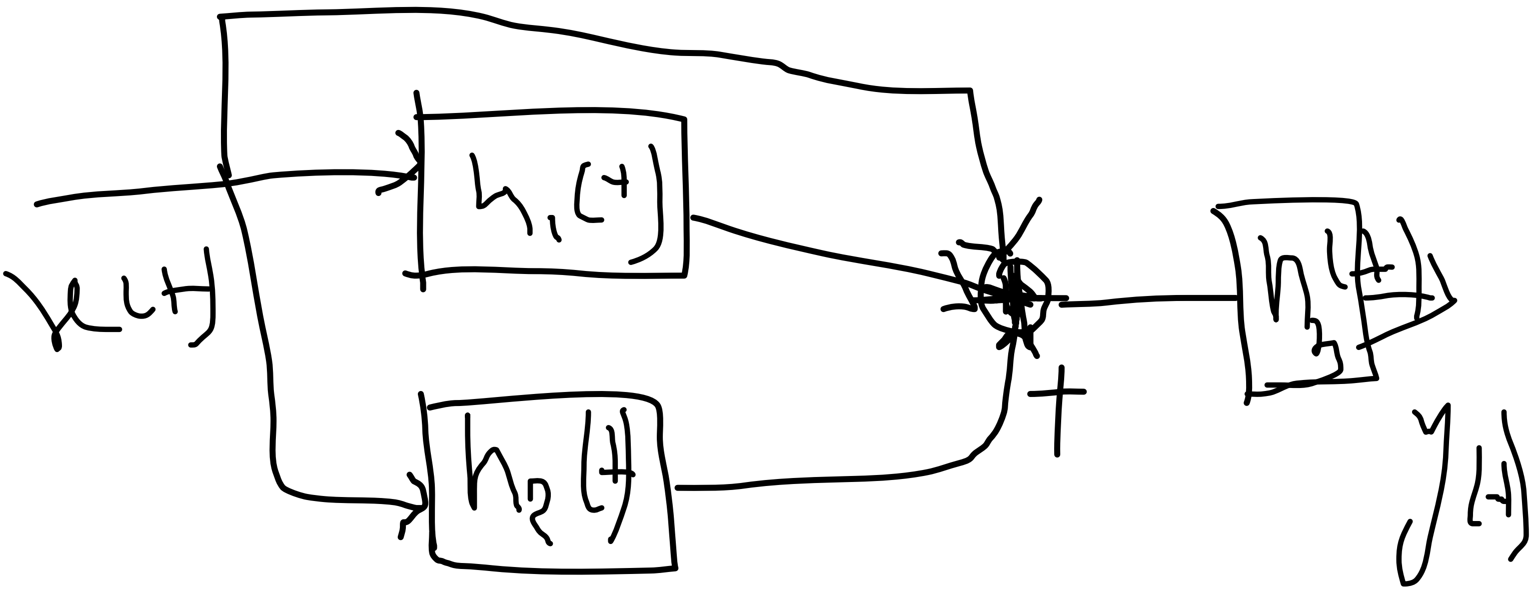
z(n)=x(n)* h(n) = {1, 3, 4, 6, 3, -1,4}

Q. x(n) = {1, 2, 3}, h(n)={1,-2}

	1	**2**	**3**
1	1	2	3
-2	-2	-4	-6

z(n)=x(n)* h(n) = {1, 0, -1,-6}

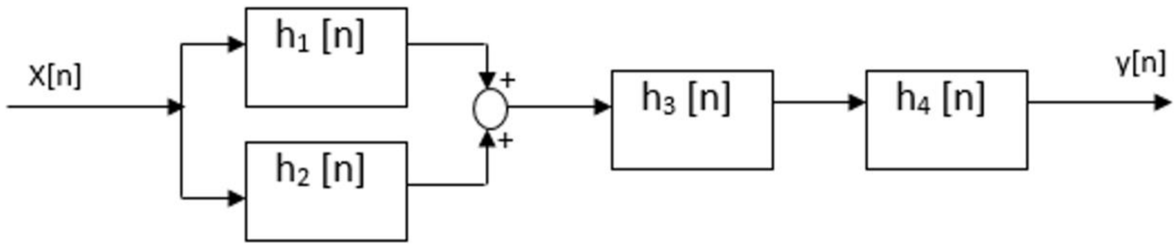
Q. Find the overall response of the given LTI System



Solution:

$y(t) = [x(t) + x(t) * h1(t) + x(t) * h2(t)] * h3(t)$

Q.



a. Find the overall System response
b. Find y(n), if x(n) = {1, **2**, 3, 4}, h(n)={1,**1**,-1,1}, where h1(n)=h(n-1), h2(n)=h(n), h3(n)=2h(n) and h3(n)= h(n+1)

Solution:

a. Overall response : $[h_1[n] + h_2[n]] * h_3[n] * h_4[n]$= [{**1**, 2, 3, 4}+{1,**1**,-1,1}]*{2,**2**,-2,2}*{1,1,**-1**,1}={1,**2**,1,4,4}*{2,**2**,-2,2}*{1,1,**-1**,1}

b. $y[n] = x[n] * Overall\ response$