Operation on Signals

- Basic operations on signals can be represented mathematically as:
- $Y(t) = TX(t)$
- This is also known as signal transformation where Y(t) represents the resulting signal/ Transferred signal derived from the original signal X(t) having only one independent variable t.

1. Basic Signal Operations Performed on Dependent Variables:

In this transformation, only the quadrature axis values are modified i.e magnitude of the signal changes, with no effects on the horizontal axis values or periodicity of signals like.

a. Amplitude scaling of signals
b. Addition of signals
c. Multiplication of signals
d. Subtraction of signals
e. Integration of signals and Differentiation of signals

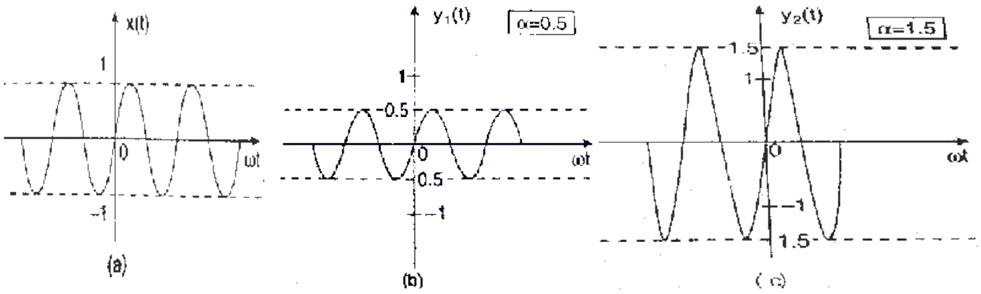
a. Amplitude scaling of signals

Amplitude scaling is a very basic operation performed on signals to vary its strength. It can be mathematically represented as Y(t) = **α** X(t).

Here, **α** is the scaling factor, where:-

α<1 → signal is attenuated.

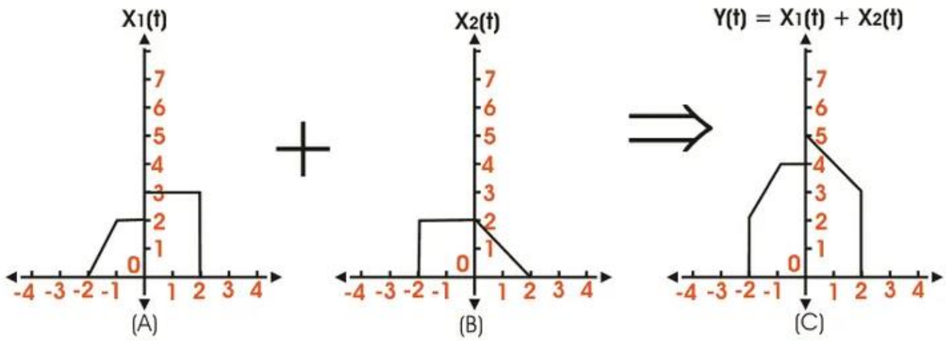
α>1 → signal is amplified.



b. **Addition of signals**

This particular operation involves the addition of amplitude of two or more signals at each instance of time or any other independent variables which are common between the signals. Addition of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, performing the additional operation on them we get,

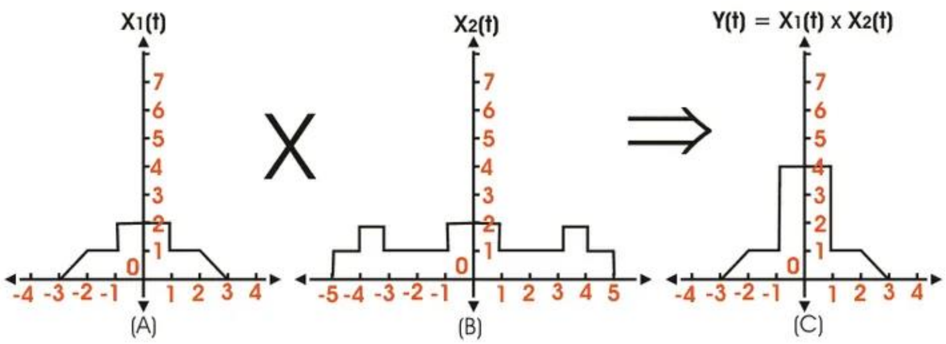
$$Y(t) = X_1(t) + X_2(t)$$



c. **Multiplication of signals**

Like addition multiplication of signals also falls under the category of basic signal operations. Here multiplication of amplitude of two or more signals at each instance of time or any other independent variables is done which are common between the signals. The resultant signal we get has values equal to the product of amplitude of the parent signals for each instance of time. Multiplication of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, on whom after performing the multiplication operation we get,

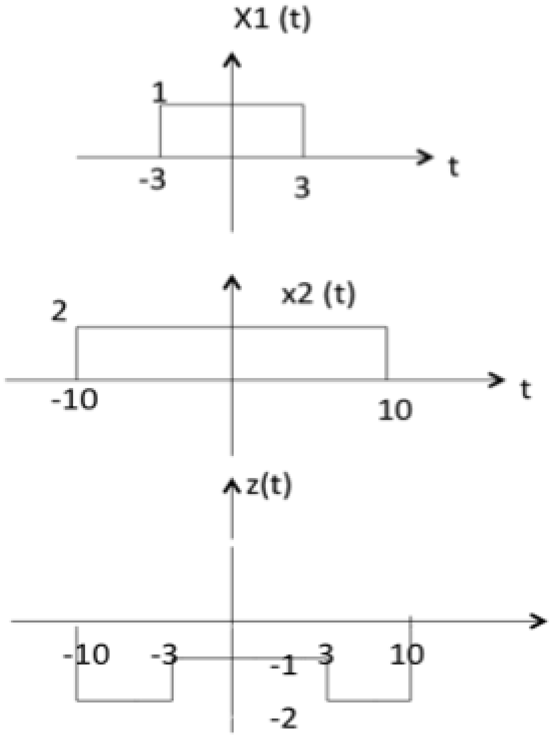
$$Y(t) = X_1(t)X_2(t)$$



d. Subtraction of signals

Subtraction of two signals is nothing but subtraction of their corresponding amplitudes. This can be best explained by the following example:

$$Y(t) = X_1(t) - X_2(t)$$

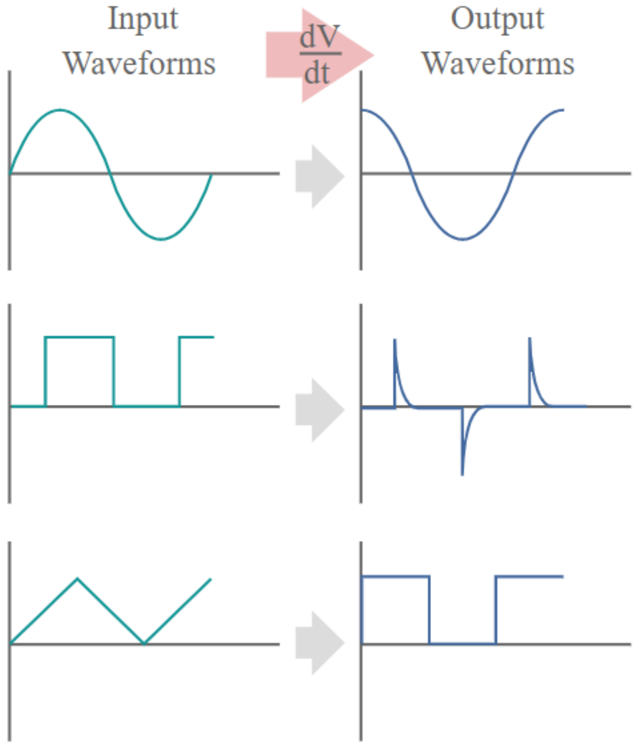


e. Integration of signals and Differentiation of signals

These operations are only applicable for only continuous signals, as a discrete function cannot be differentiated or integrated.

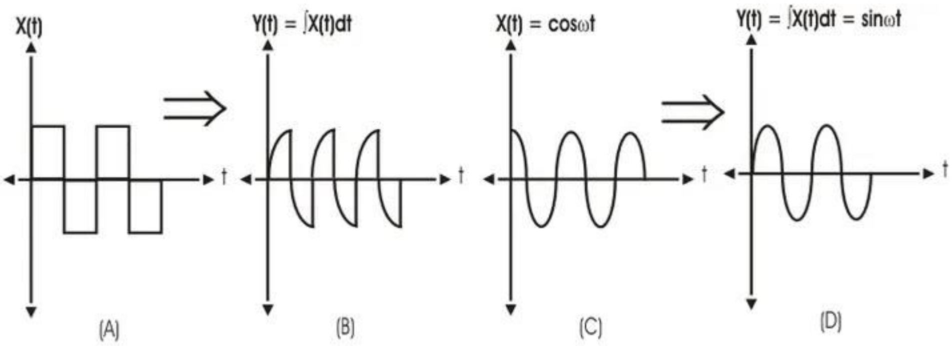
The modified signal we get on differentiation has tangential values of the original signal at all instance of time. Mathematically it can be expressed as:-

$$Y(t) = \frac{d}{dt}X(t)$$



Integration of signals is also applicable to only continuous time signals. The limits of integration will be from – ∞ to present instance of time t. It is mathematically expressed as,

$$Y(t) = \int_{-\infty}^{t} X(t)dt$$



2. Basic Signal Operations Performed on Independent Variables:

Here the periodicity of the signal is varied by modifying the horizontal axis values, while the amplitude or the strength remains constant. These are:-

a. Time scaling of signals
b. Reflection of signals/ Time Reversal
c. Time-shifting of signals

a. Time scaling of signals

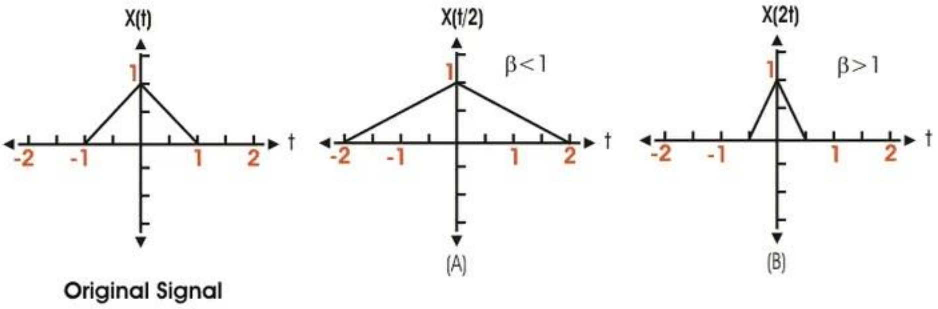
Time scaling of signals of signals involves the modification of a periodicity of the signal, keeping its amplitude constant. It's mathematically expressed as,

$$\boldsymbol{Y(t) = X(\beta t)}$$

Where, $\mathrm{X(t)}$ is the original signal and β is the scaling factor.

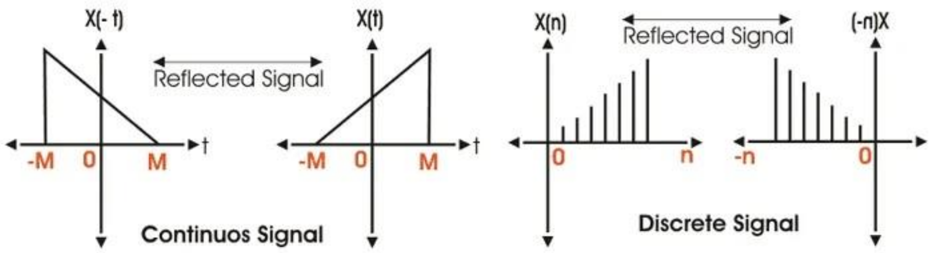
If $\beta > 1$, then the signal is compressed.

If $\beta < 1$, then the signal is expanded.



b. Reflection of signals/ Time Reversal

Reflection of signal is a very interesting operation applicable on both continuous and discrete signals. Here in this case the vertical axis acts as the mirror, and the transformed image obtained is exactly the mirror image of the parent signal. It can be defined as Y(t) = X(- t) Where, X(t) is the original signal. But if the reflected signal X(- t) = X(t); then it's called an even signal. Whereas when X(- t) = − X(t); then its known as an odd signal.



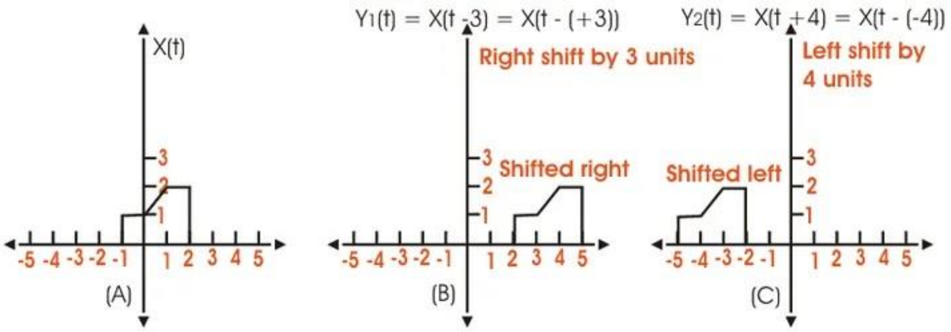
c. **Time-shifting of signals**

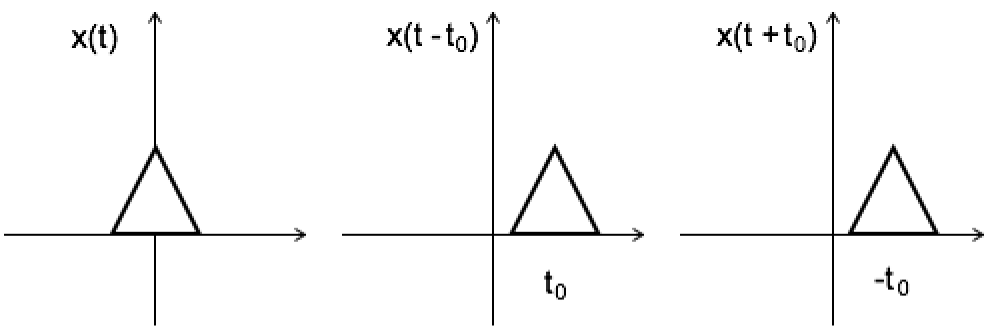
Time shifting of signals is probably the most important one, and most widely used amongst all **basic signal operations**. It's generally used to fast-forward or delay a signal, as is necessary in most practical circumstances. Time shifting is mathematically expressed as,

$$\boldsymbol{Y(t) = X(t - t_0)}$$

Where, X(t) is the original signal, and t_0 represents the shift in time. For a signal X(t) if the position shift $t_0 > 0$. Then the signal is said to be right shifted or delayed.

In the same manner, if $t_0 < 0$, implies the signal is left shifted or delayed. This has been explained diagrammatically in the figure below. Where the original signal figure (a) is right shifted and also left shifted in figure (b) and (c) respectively.





Solve:

Question 1.

A continuous-time signal $x(t)$ is shown in Fig. 1-17. Sketch and label each of the following signals.
(a) $x(t-2)$; (b) $x(2t)$; (c) $x(t/2)$; (d) $x(-t)$

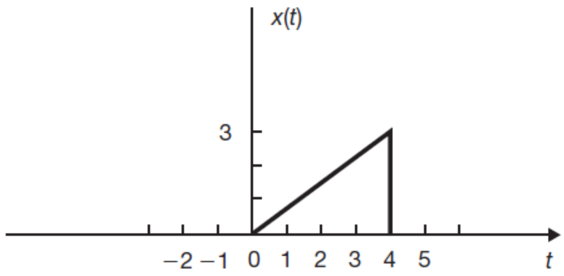
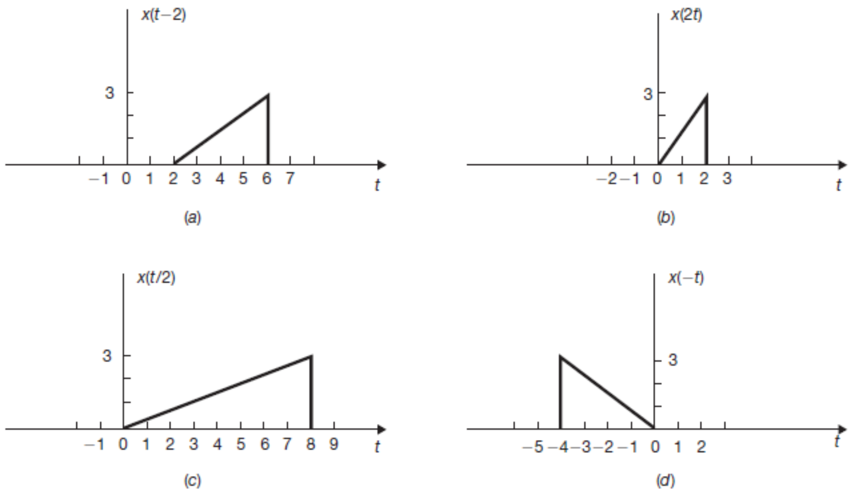


Fig. 1-17

Solution 1.



Question 2.

A discrete-time signal $x[n]$ is shown in Fig. 1-19. Sketch and label each of the following signals.
(a) $x[n-2]$; (b) $x[2n]$; (c) $x[-n]$; (d) $x[-n+2]$

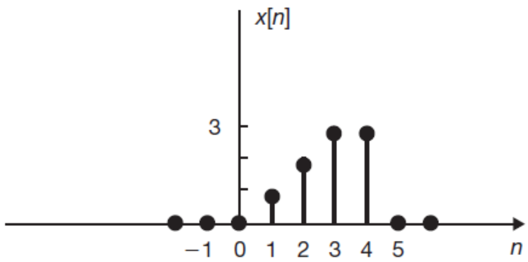
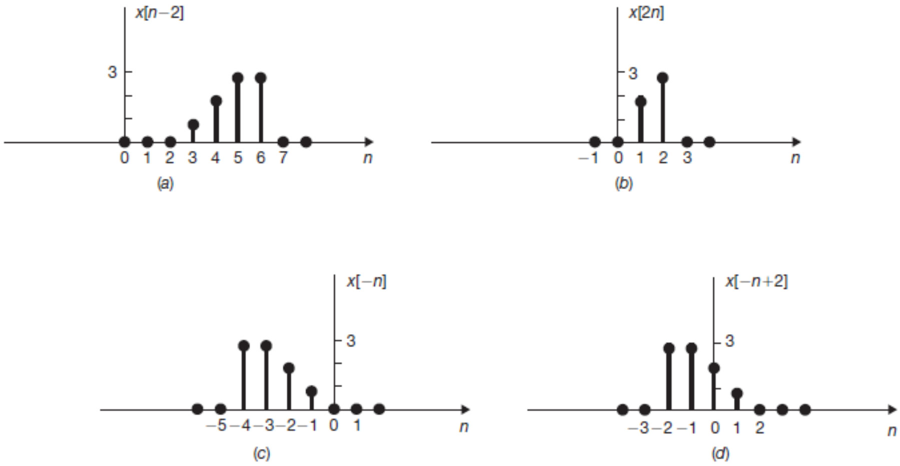


Fig. 1-19

Solution 2.



Question 3.

Using the discrete-time signals $x_1[n]$ and $x_2[n]$ shown in Fig. 1-22, represent each of the following signals by a graph and by a sequence of numbers.

(*a*) $y_1[n] = x_1[n] + x_2[n]$; (*b*) $y_2[n] = 2x_1[n]$; (*c*) $y_3[n] = x_1[n]x_2[n]$

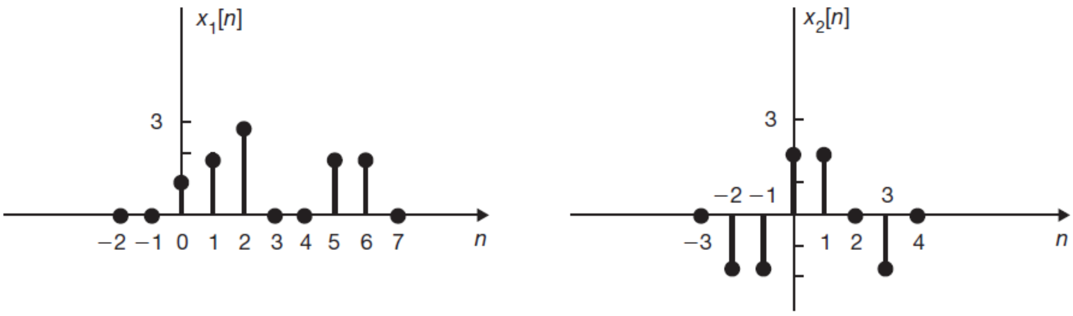


Fig. 1-22

Solution 3.

(*a*) $y_1[n]$ is sketched in Fig. 1-23(*a*). From Fig. 1-23(*a*) we obtain

$$y_1[n] = \{\ldots, 0, -2, -2, \underset{\uparrow}{3}, 4, 3, -2, 0, 2, 2, 0, \ldots\}$$

(*b*) $y_2[n]$ is sketched in Fig. 1-23(*b*). From Fig. 1-23(*b*) we obtain

$$y_2[n] = \{\ldots, 0, \underset{\uparrow}{2}, 4, 6, 0, 0, 4, 4, 0, \ldots\}$$

(*c*) $y_3[n]$ is sketched in Fig. 1-23(*c*). From Fig. 1-23(*c*) we obtain

$$y_3[n] = \{\ldots, 0, \underset{\uparrow}{2}, 4, 0, \ldots\}$$

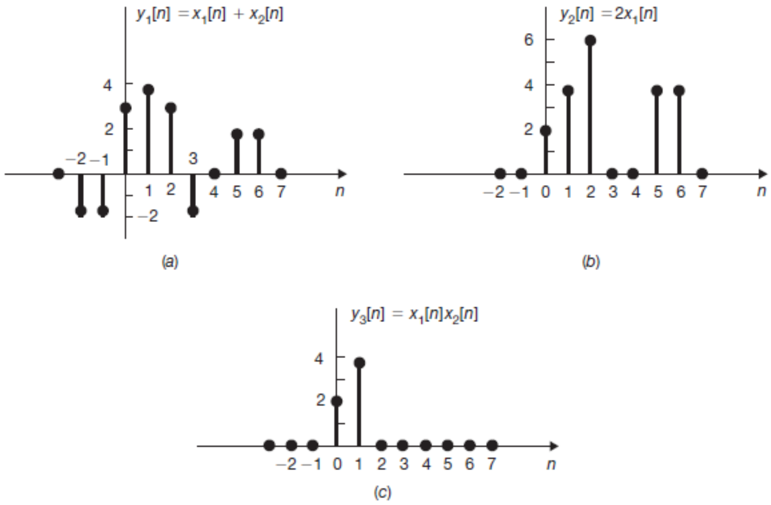


Fig. 1-23