<u>Revision</u>

Q1. Determine whether the following signals are energy signal or power signal and calculate their energy and power.

a. $\boldsymbol{x(t) = sin^2\, \omega_0 t}$

b. $\boldsymbol{x(t) = rect(\frac{t}{\tau})}$

c. $\boldsymbol{x(t) = u(t)}$

d. $\boldsymbol{x(t) = t\, u(t)}$

Solution:

a. $E = \int_{-\infty}^{\infty} x^2(t)dt = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt$ (Hint: When sin^2 or cos^2 are in equation)

$\lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt = \lim_{T\to\infty} \int_{-T}^{T} \boldsymbol{sin^4\, \omega_0 t} dt$

$$= \lim_{T\to\infty} \int_{-T}^{T} \left[\frac{\boldsymbol{1 - cos\, 2\;\; \omega_0 t}}{\boldsymbol{2}}\right]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{4} \int_{-T}^{T} (1 + cos^2 2\boldsymbol{\omega_0 t} - 2\cos \boldsymbol{2}\;\; \boldsymbol{\omega_0 t})dt$$

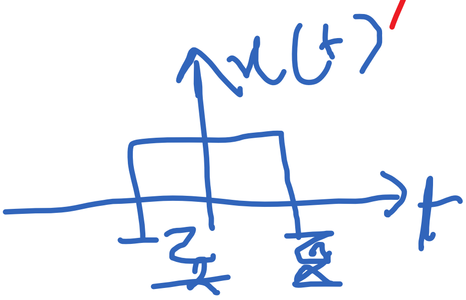
$$= \frac{1}{4} \lim_{T\to\infty} \int_{-T}^{T} (1 + \frac{\boldsymbol{1 + cos\, 4\omega_0 t}}{\boldsymbol{2}} - 2\cos \boldsymbol{2}\;\; \boldsymbol{\omega_0 t})dt$$

$$= \frac{1}{4} \lim_{T\to\infty} \int_{-T}^{T} (\frac{3}{2} + \frac{1}{2} \boldsymbol{cos\, 4\omega_0 t} - 2\cos \boldsymbol{2}\;\; \boldsymbol{\omega_0 t})dt$$

$$= \frac{1}{4} \lim_{T\to\infty} [\frac{3}{2}\;\; 2T + \frac{1}{2}\, \frac{1}{4\omega_0} \left\{\sin\left(4\, \frac{2\pi}{T} t\right)\right\}_{-T}^{T} - \frac{2}{2\omega_0} \left\{\sin\left(2\, \frac{2\pi}{T} t\right)\right\}_{-T}^{T}]$$

$$= \frac{1}{4} \lim_{T\to\infty} \frac{3}{2}\;\; 2T = \infty$$

b. $\boldsymbol{x(t) = rect(\frac{t}{\tau})}$



$$E = \int_{-\infty}^{\infty} x^2(t)dt = \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} dt = \tau\, Joules$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} dt = \lim_{T\to\infty} \frac{1}{2T}\,\tau = 0$$

This is an energy signal.

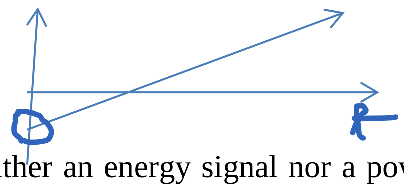
c. $\boldsymbol{x(t) = u(t)}$

$$E = \int_{-\infty}^{\infty} x^2(t)dt = \int_{0}^{\infty} dt = \infty$$

$$P = \lim\nolimits_{T\to\infty} \tfrac{1}{2T}\int_{-T}^{T} x^2(t)dt = \lim_{T\to\infty} \tfrac{1}{2T}\int_0^T dt = \tfrac{1}{2}W$$

It is a power signal.

d. $\boldsymbol{x(t) = t\,u(t)}$



It is neither an energy signal nor a power signal as at t=∞, the amplitude will be ∞. It's not a time bounded signal.

$$E = \int_{-\infty}^{\infty} x^2(t)dt$$

$$= \int_0^{\infty} t^2 dt = \left[\frac{t^3}{3}\right]_0^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_0^{T} t^2 dt = \lim_{T\to\infty} \frac{1}{2T}\,\frac{T^3}{3} = \infty$$

Classification of Signal Continue…

i. **Deterministic and Random signal:**

A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal. A deterministic signal can be completely represented by mathematical equation at any time and its nature and amplitude at any time can be predicted.

Examples: Sinusoidal sequence $x(n)$ = cos n, Exponential sequence $x(n) = e^{n}$, ramp sequence $x(n) = n$.

A signal characterized by uncertainty about its occurrence is called a non-deterministic or random signal. A random signal cannot be represented by any mathematical equation. The behaviour of such a signal is probabilistic in nature and can be analysed only stochastically. The pattern of such a signal is quite irregular. Its amplitude and phase at any time instant cannot be predicted in advance. A typical example of a non-deterministic signal is thermal noise.

iv. Causal or Non-causal signal:

A discrete-time signal $x(n)$ is said to be causal if $x(n) = 0$ for $n < 0$, otherwise the signal is non-causal. A discrete-time signal $x(n)$ is said to be anti-causal if $x(n) = 0$ for $n > 0$.

A causal signal does not exist for negative time and an anti-causal signal does not exist for positive time. A signal which exists in positive as well as negative time is called a non-casual signal.

$u(n)$ is a causal signal and $u(-n)$ an anti-causal signal, whereas $x(n) = 1$ for $-2 \leq n \leq 3$ is a non-causal signal.

***v*. Even or Odd signals:**

Any signal $x(n)$ can be expressed as sum of even and odd components. That is

$$x(n) = xe(n) + xo(n)$$

where $x_e(n)$ is even components and $x_0(n)$ is odd components of the signal.

Even (Symmetric) signal:

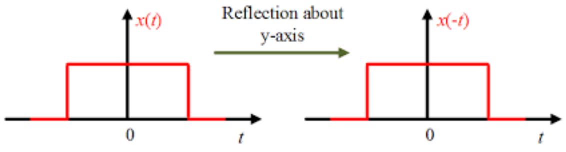
A discrete-time signal $x(n)$ is said to be an even (symmetric) signal if it satisfies the condition:

$$X[n] = x[-n] \text{ for all } n$$

Even signals are symmetrical about the vertical axis or time origin. Hence they are also called symmetric signals: cosine sequence is an example of an even signal. An even signal is identical to its reflection about the origin. For an even signal $xo(n)= 0$.

Similarly,

X(t) = x(t) for all t



Odd (anti-symmetric) signal

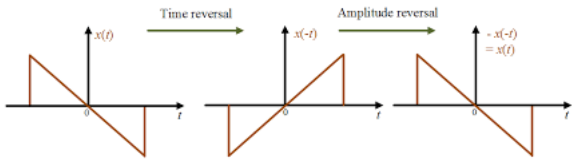
A discrete-time signal *x*(*n*) is said to be an odd (anti-symmetric) signal if it satisfies the condition:

x(–n) = –x(n) for all n

Odd signals are anti-symmetrical about the vertical axis. Hence they are called anti-symmetric signals. Sinusoidal sequence is an example of an odd signal. For an odd signal *xe*(*n*) = 0.

Similarly,

X(-t) = -x(t) for all t



Any signal can be written as a combination of even and odd signal.

$$f(t) = \frac{1}{2}[f(t) + f(-t)] + \frac{1}{2}[f(t) - f(-t)]$$

Even component:

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)]$$

$$f_o(t) = \frac{1}{2}[f(t) - f(-t)]$$

$$f(t) = f_e(t) + f_o(t)$$