Numerical Correlation

Convolution: $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Auto Correlation: $R_{11}(\tau) = x(t) * x(-t)$

Cross Correlation: $R_{12}(\tau) = x(t) * h(-t)$

Q1. Find the auto correlation of $x[n] = [\mathbf{0}, 1, 2, 3]$

Solution: $x[n] * x[-n]$=[0, 3,8,14,8,3,0]

	3	**2**	**1**	**0**
0	0	0	0	0
1	3	2	1	0
2	6	4	2	0
3	9	6	3	0

Q2. Find cross-correlation of $x[n] = [\mathbf{0}, 1, 2, 3]$, $h[n] = [\mathbf{1}, 1, 2, 1]$

Solution: $x[n] * h[-n]$ = [0, 1, 4, 8, 9, 5,3]

	1	**2**	**1**	**1**
0	0	0	0	0
1	1	2	1	1
2	2	4	2	2
3	3	6	3	3

Systems defined by differential & difference equations

Differential Equation: Represent Continuous time LTI systems

Difference Equation: Represent discrete time LTI systems

The standard differential equation is represented by:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \cdots + b_0 x(t)$$

Where $a_n, a_{n-1}, \ldots, a_0$ and $b_m, b_{m-1}, \ldots, b_0$ are constant coefficients of system.

Conditions:

1. Time invariance: $a_n, a_{n-1}, \ldots, a_0$ and $b_m, b_{m-1}, \ldots, b_0$ must be constant
2. Linearity : All initial conditions should be zero

The generalized form:

$$\sum_{k=0}^{n} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{m} b_k \frac{d^k}{dt^k} x(t)$$

Q1. Find whether the following differential equations are representing the equations for LTI systems or not:

i. $4\frac{d^3 y(t)}{dt^3} + 2\frac{d^2 y(t)}{dt^2} + 3\frac{dy}{dt} + y(t) = 2\frac{dx(t)}{dt} + 5x(t)$

$a_3 = 4, a_2 = 2, a_1 = 3, a_0 = 1$ and $b_1 = 2, b_0 = 5$

The coefficients are constant so TI.

The given system is LTI, if the initial conditions are zero

ii. $L\frac{d^2 y(t)}{dt^2} + R\frac{dy(t)}{dt} + \frac{1}{c} y(t) = x(t)$

Here, $a_2 = L, a_1 = R, a_0 = 1/c$ and $b_0 = 1$

The coefficients are constant so TI.

The given system is LTI, if the initial conditions are zero

iii. $4\left[\frac{dy}{dt}\right]^2 + 2y(t) = x(t)$

$Not\ matching\ the\ format\ so\ its\ not\ LTI.$

iv. $3\frac{d^2y}{dt^2} + y(t) = 2tx(t)$

As one of the coefficients is t which is not constant, its not a LTI system.

Difference Equation: Represent discrete time LTI systems

The standard difference equation is represented by:

$$a_N y[n-N] + \cdots + a_1 y[n-1] + a_0 y[n] = b_M x[n-M] + \cdots + b_1 x[n-1] + b_0 x[n]$$

The generalized form:

$$\sum_{k=0}^{N} a_k y[n-k] = \sum\nolimits_{k=0}^{m} b_k x(n-k)$$

Q1. $\boldsymbol{y[n-1] + y[n] = x[n]}$**, coefficients are constant and it's a linear equation. So its LTI.**

Q2. $\boldsymbol{y[n] + y[n-1] - 2y[n-2] = x[n-1] + 2x[n-2]}$**, constant coefficient and linear equation so LTI.**