Causal and Non-Causal System

- A system is said to be causal if it’s response is dependent upon present and past inputs and doesn’t depends upon Future inputs.
- For non-causal system, the output depends upon future inputs too.

Ex1: Find if $y(n) = x(n) + \frac{1}{x(n-1)}$ is causal or non-causal.

Solution: If $n = 1$,

$$y(1) = x(1) + \frac{1}{x(1-1)} = x(1) + \frac{1}{x(0)}$$

System is dependent upon present and past input so it’s causal.

Ex2: $y(n) = 2x(n) + \frac{1}{x^2(n)}$ is causal

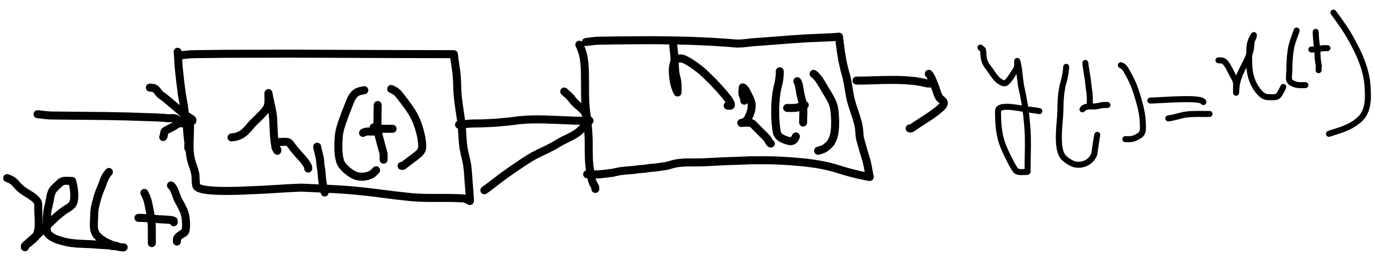
Ex3: $y(n) = x(n) + \frac{1}{2x(n+1)}$ is non causal

Ex4: $y(t) = x(t) + x(t-1) + \frac{1}{x(t+1)}$ is non causal

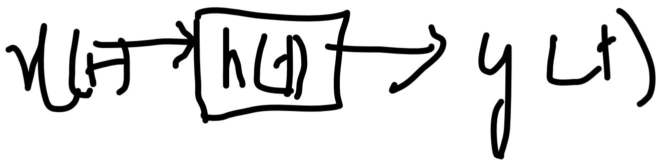
- All non-causal systems are dynamic systems, but all dynamic systems are noncausal.
- All static systems are causal but all causal systems are not static.

Invertible and Non-invertible System

- A system is said to be non-invertable if the input of the system appears at the output.



- If $y(t) \neq x(t)$, then the system is invertible.



- $y(t) = x(t) * h(t)$, where $h(t)$ is the impulse response of the system and * represents convolution.
- Convolution in time domain means multiplication in frequency domain.
- In frequency domain,
- $Y(s) = X(s)H_1(s)H_2(s)$, to make the system non-inertable, we need to make $H_2(s) = \frac{1}{H_1(s)}$, so that $Y(s) = X(s)$.
- Apply ILT,
- $L^{-1}[Y(s)] = L^{-1}[X(s)]$

Ex1: $y(t) = 2x(t)$

Solution:

$x(t) = \frac{1}{2}y(t) = \frac{1}{2}2x(t) = x(t)$, invertible

Ex2. $y(t) = x(2t)$

Solution:

$x(t) = y\left(\frac{1}{2}t\right) = y\left(\frac{1}{2}.2t\right) = x(t)$ invertible

Ex2. $y(t) = x^2(t)$ is non-invertible

<u>Stable and Non-Stable System</u>

- A system is said to be stable when it produces bounded output for bounded input.
- A system is said to be unstable when it produces unbounded output for bounded input.

  Ex: $y[n] = x[n]$, $y[n] = x^2[n]$ are bounded systems

Ex: $y[n] = \frac{1}{x[n]}$ is unstable system.