<u>Complex Exponential Signals</u>

The complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

Using Euler's formula, this signal can be defined as

$$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \, \sin \omega_0 t$$

So, $x(t)$ is a complex signal whose real part is $\cos \omega_0 t$ and imaginary part is $\sin \omega_0 t$.

An important property of complex exponential signal $x(t)$ is, it is periodic and its fundamental period $T_0 = \frac{2\pi}{\omega_0}$

$x(t)$ is periodic with any value of ω_0.

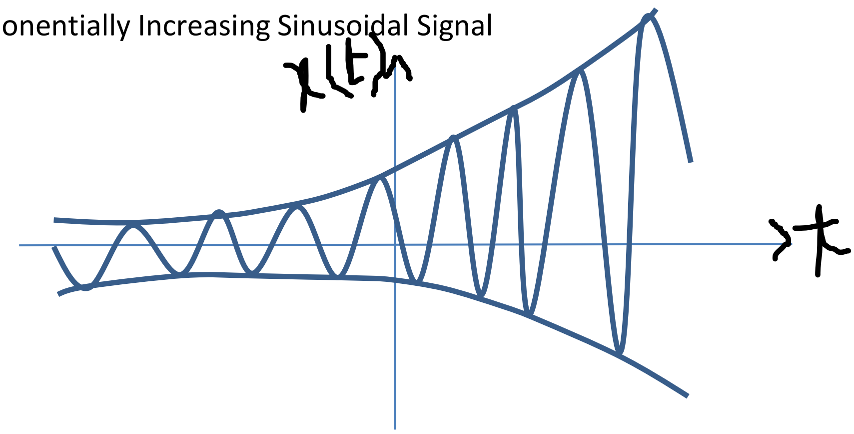
General complex exponential Signal

Let $s = \sigma + j\omega$ be a complex number.

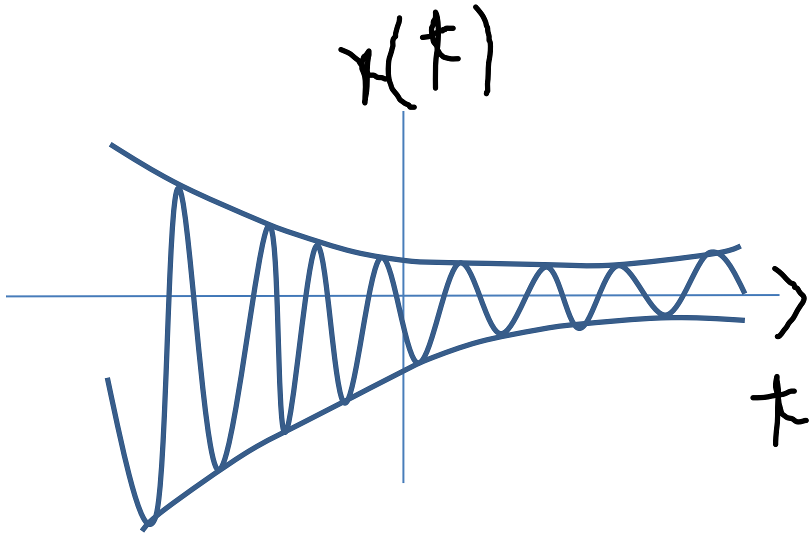
$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t}(\cos \omega_0 t + j \, \sin \omega_0 t)$$

Here, $x(t)$ is known as a general complex exponential signal whose real part $e^{\sigma t} \cos \omega_0 t$ and imaginary part $e^{\sigma t} \sin \omega_0 t$ are exponentially increasing ($\sigma > 0$) or decreasing ($\sigma > 0$) sinusoidal signal.

a. Exponentially Increasing Sinusoidal Signal

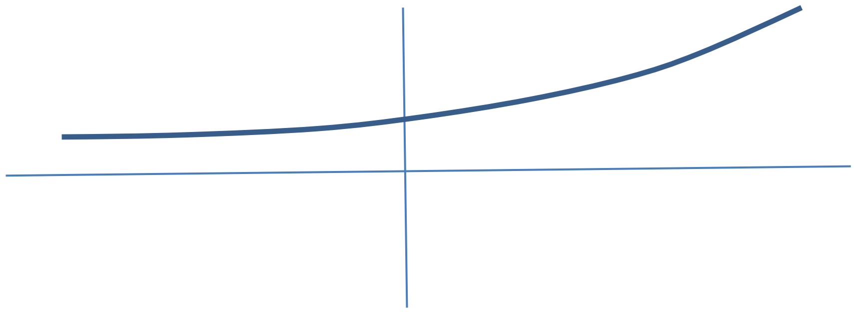


b. Exponentially Decreasing Sinusoidal Signal

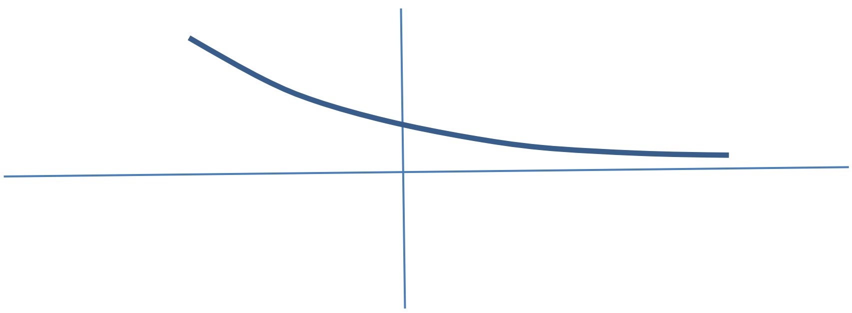


Real exponential Signal

a. Continuous time real exponential signal ($\sigma > 0$)



b. Continuous time real exponential signal ($\sigma < 0$)



Time domain to frequency domain

The analysis of signal is much easier in frequency domain. There are various transformation techniques are available to convert time domain signal into frequency domain.

- Fourier series : Used for analysis of **periodic signals**
- Fourier transform : Used for analysis of **aperiodic signals**
- Laplace transform

Fourier Series

- Fourier series and Fourier transform converts time domain signals into frequency domain (or spectral) representation.
- Apart from providing spectral representations of signals, Fourier analysis is also essential for describing certain types of system and their properties.
- The Fourier series is further divided into
  - Trigonometric Fourier series
  - Exponential Fourier Series
  - Cosine Fourier series representation

Fourier series representation of Continuous Time Periodic Signals

A continuous time signal is periodic if there is a positive nonzero value of T for which

$$x(t+T) = x(t)$$

Fundamental period T_o is the smallest value of T for which the above equation is satisfied and $\frac{1}{T_0} = f_0$ is referred to as the fundamental frequency.

Complex Exponential Fourier Series Representation

The complex exponential Fourier series representation of a periodic signal $x(t)$ with fundamental period T_0 is given by,

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$, $\omega_0 = \frac{2\pi}{T_0}$

Where, c_k are known as the complex Fourier coefficients and are given by,

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Where, $\int_{T_0}$ denotes the integral over any one period and 0 to T_0 or $-\frac{T_0}{2}$ to $\frac{T_0}{2}$.

Setting k = 0,

$$c_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

Which indicates that c_0 equals the average value of $x(t)$ over a period.

When $x(t)$ is real, then

$$c_{-k} = {c_k}^*$$