***Module 3***

**Find the Fourier Coefficient and series for the given signal (Train of impulse).**



**Solution:**

,

signal exist at t = 0.

, is zero as signal is even.



is a square wave signal and is the difference of two shifted version of an impulse signal given below.



Using properties of Fourier Series, we can get

1. Time shifting and linearity

Where,

1. As is the derivative of , we can give the differentiation property

Where, is the Fourier Series coefficients of .

, average value of over one period. As at k=0, we cannot find the value of .

***Frequency Response of a RLC circuit***



**Steps:**

* **Transformation of circuit analysis from time to frequency domain**
* **General format :**
* **(Multiplier in frequency domain and convolution in time domain)**
* **Step 1: Convert input into frequency domain (including and any circuit components like R, L and C)**
* **Step 2: Solve for transfer function by using voltage divider for the component you would like to analyse.**
* **Step 3: Use and functions into general format of Frequency domain shown in above equation.**
* **Plug in circuit component values into the final equation**

**Solution:**

1. **The impedance across different components are,**
2. **Transfer function by using voltage divider for the component (here its across capacitor)**

**In frequency domain,**

**=**

1. **Find the Fourier series of and**

**For the following circuit find the output across capacitor if input is and L = 2H, R = 2 and C = .**



**Solution:**

**Step 1: Find the Fourier series of .**

**We can rewrite the equation as:**

**We can relate the equation with**

**, and**

**Step 2: Transfer function by using voltage divider for the component (here its across capacitor)**

**In frequency domain,**

**=**

**Step 3:**

**, replace**

**For the following circuit find the output across capacitor if input is sin t and L = 2H, R = 2 and C = .**



**Solution:**

**Step 1: Find the Fourier series of Sin t**

**=, replace from sin t**

**Output across Capacitor is:**

**Consider a Causal LTI System whose i/p and O/P are related by following differential equation.**



**and**

**Find the o/p coefficient if i/p is**

**Solution:**

=

Given

As discussed earlier,

,

,

All other coefficients are zero.

**If x(t)= is a periodic signal given as input to the system with impulse response . Find the Output of the system and .**

**Solution:**

**From x(t) we can find the Fourier series coefficients as**

, So

**As**  and are complex and even conjugate of each other, is real.

***A signal is defined as***



***Solution:***



Fourier series is used for periodic signal.

Prove if the signal is periodic or not.

Ratio of Time periods is rational.



The fundamental frequency is, GCD[ = GCD[6,12] = 6



= 6 rad/ sec



= 1st harmonic + 2nd harmonic

***A signal is defined as***



Solution:

Fourier series is used for periodic signal.

Prove if the signal is periodic or not.

Ratio of Time periods is rational.

The fundamental frequency is, GCD[ = GCD[3,9,12] = 3

= 1st harmonic + 3rd harmonic + 4th harmonic



***A signal is defined as***



***Is it a possible Fourier series expansion? If yes, what are the harmonics present in this Fourier Series expansion?***

***Solution:***

Fourier series is used for periodic signal.

Prove if the signal is periodic or not.

, ,

Ratio of the time periods T1, T2 and T3 is rational so Fourier series expansion is possible.

The fundamental frequency is,

=> =>

=> = >

=>=>=10

The harmonics present in the Fourier series are 10th , 84th and 175th.

***Practice Problems:***

***Point to note:***

***If a signal is real and odd, then is purely imaginary and odd.***

***If a signal is real and even then is real and even.***

***If a signal is real, then is complex and even conjugate in nature.***

***If a signal is real and odd signal, then , .***

***If a signal is real and even signal, then***

**Find exponential Fourier series coefficient for .**

**Solution:**

We can rewrite it as

Where,

and ,

Comparing this equation with complex exponential Fourier series, we will get

and

The signal is real and odd, so the  ***is purely imaginary***

, ***Satisfies Odd Symmetry***

**Find exponential Fourier Series coefficient for .**

**Solution:**

and ,

and

The signal is real and even.

The coefficients are purely real.

satisfies even symmetry.

**For , the complex exponential Fourier Series coefficients are given as and . Find the trigonometric Fourier Series coefficients and explain about the type of the signal from the trigonometric Fourier Series coefficients.**

**Solution:**

**is odd, so and is zero.**

**and**

**and**

**For , the complex exponential Fourier Series coefficients are given as and . Find the trigonometric Fourier Series coefficients and explain about the type of the signal from the trigonometric Fourier Series coefficients.**

**Solution:**

**is even, so is zero.**

**and**

**and**

**A signal is defined by exponential Fourier series as,**

**Is the signal real? If real, is it even or odd.**

**Solution:**

for to 100.

Signal is not real.

**A signal is defined by exponential Fourier series as,**

**Is the signal real? If real, is it even or odd.**

**Solution:**

for to 100.

is purely real and even, so is even.

, Signal is real.

**A signal is defined by exponential Fourier series as,**

**Is the signal real? If real, is it even or odd.**

**Solution:**

for to 100

is purely imaginary and odd, so signal is odd.

, Signal is real.

**Find the Fourier Coefficient and series for the given signal (Train of impulse).**



**Solution:**

,

signal exist at t = 0.

, is zero as signal is even.

Use the Fourier series coefficient of given the period is 4.

***Solution:***



,



Ex:



Put

, if has even symmetry

=2

if has even symmetry

2

Ex:



=>

Odd Symmetry:

A signal is said to have odd symmetry or rotational symmetry, if

for all ‘t’

Ex.



Put

, if has odd symmetry

=0

If the waveform has odd symmetry,

Ex:



=>

**Find the trigonometric Fourier series of periodic signal (Even),**



Solution:

The trigonometric Fourier series of is

* ,

The given signal has even symmetry. The trigonometry Fourier series expansion consists of only “Cosine terms”.

=

**Find the trigonometric Fourier series of periodic signal (Odd),**



Solution:



(-1, -1) and (1, 2)



-1<t<1

The trigonometric Fourier series expansion of is,

* ,

Since the given has odd symmetry, its trigonometric Fourier series expansion consists of only sine terms.

So,

* ,

**Find the exponential Fourier series for the waveform shown and also draw the frequency spectrum.**



Solution:

The exponential series of periodic signal is,

Here,

If n = even,

If n = odd,

is the average value or DC value.

***Frequency spectrum:***

Plotting magnitude of exponential Fourier Series Coefficient vs frequency is called Frequency spectrum.

i.e

**Q. Plot the frequency spectrum of Exponential Fourier series coefficient**

, for odd value on n=1,3,5,7 …



The magnitude spectrum of exponential Fourier series coefficient has even symmetry.

Obtain the trigonometric Fourier series expansion of the figure shown.



Solution:

Time period of the waveform:

Find the straight line equation from point (0,0) to (.

The trigonometric Fourier series expansion of x(t) is,

* ,
* , 0
* n = 2, 4, 6, …
* n= 1, 3, 5, …
* By substituting the values, we will get

0<t<2

**Find the cosine Fourier series of the waveform shown in the figure below,**



Solution:

T = 2

Find the straight line equation from (0,0) and (2.

0<t<2

**Find the trigonometric Fourier series for the signal in the interval 0<t<1. ( ∫ uv dx = u ∫ v dx - ∫ (u' ∫ v dx) dx)**



* A straight line between (0, 0) and (1, A). So
* Straight line equation =
* =>
* Since it is periodic signal, the trigonometric Fourier series of is,
* ,
* ,
* =
* =0
* =

**Find the trigonometric Fourier series expansion of the half wave rectifier sine wave shown below.**



**Solution:**



Time period:

The formula for trigonometric Fourier series:

* ;
* ;

[Use formula , ]

For n is even

For n is odd n= 1, 3, 5, 7, . . .

If n=1; , for remaining values,

=

; 0<t<