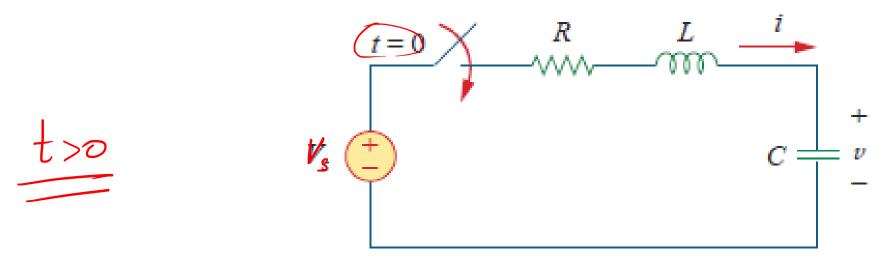
## Step Response of a Series RLC Circuit



Applying KVL around the loop for t > 0,

$$L\frac{di}{dt} + Ri + v = V_s$$
$$i = C\frac{dv}{dt}$$

## Substituting for i in Eq.

 $\int \frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$ 

$$v(t) = v_{\underline{t}(t)} + v_{\underline{ss}(t)}$$

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
 (Overdamped)  $v_t(t) = (A_1 + A_2 t) e^{-\alpha t}$  (Critically damped)  $v_t(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t) e^{-\alpha t}$  (Underdamped)

The steady-state response is the final value of v(t).

the final value of the capacitor voltage is the same as the source voltage Vs . Hence,  $\frac{1}{12}$ 

$$v_{ss}(t) = v(\infty) = V_s$$

Thus, the complete solutions for the overdamped, underdamped, and critically damped cases are:

$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(Overdamped)}$$

$$v(t) = V_s + (A_1 + A_2 t) e^{-\alpha t} \quad \text{(Critically damped)}$$

$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t) e^{-\alpha t} \quad \text{(Underdamped)}$$

The values of the constants  $A_1$  and  $A_2$  are obtained from the initial conditions: v(0) and dv(0)/dt. Keep in mind that v and i are, respectively, the voltage across the capacitor and the current through the inductor.

Work out all the Cary Ex-8.5