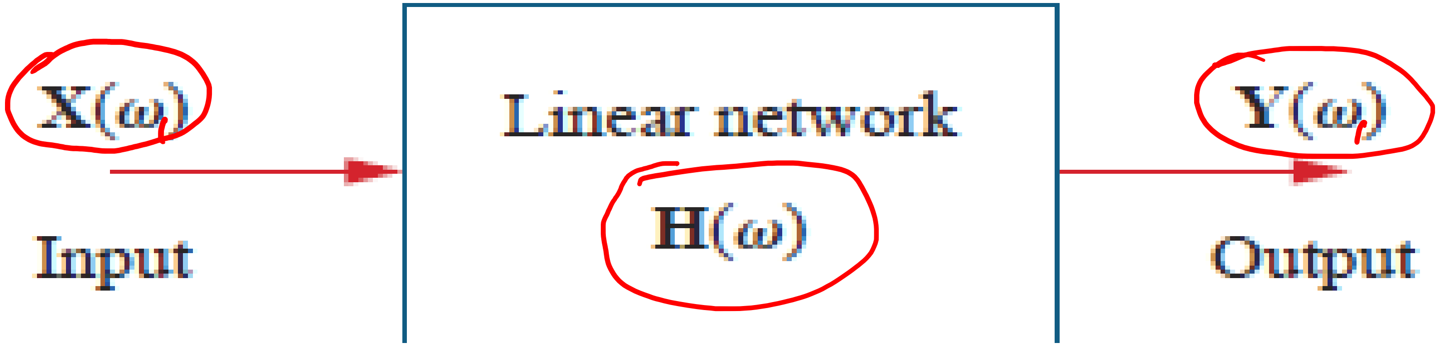
- ✓ Voltages and currents in a circuit with a constant frequency source

- ✓ let the amplitude of the sinusoidal source remain constant and vary the frequency, we obtain the circuit's *frequency response*

The **frequency response** of a circuit is the variation in its behavior with change in signal frequency.

Transfer Function

The transfer function $\mathbf{H}(\omega)$ (also called the *network function*) is a useful analytical tool for finding the frequency response of a circuit.



The **transfer function** $\mathbf{H}(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\omega)$ (an element voltage or current) to a phasor input $\mathbf{X}(\omega)$ (source voltage or current).

$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}}$$

Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions:

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

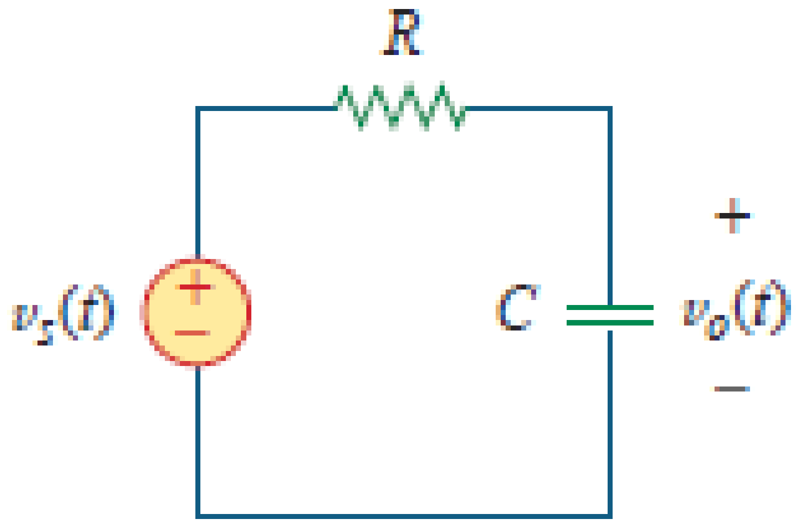
$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

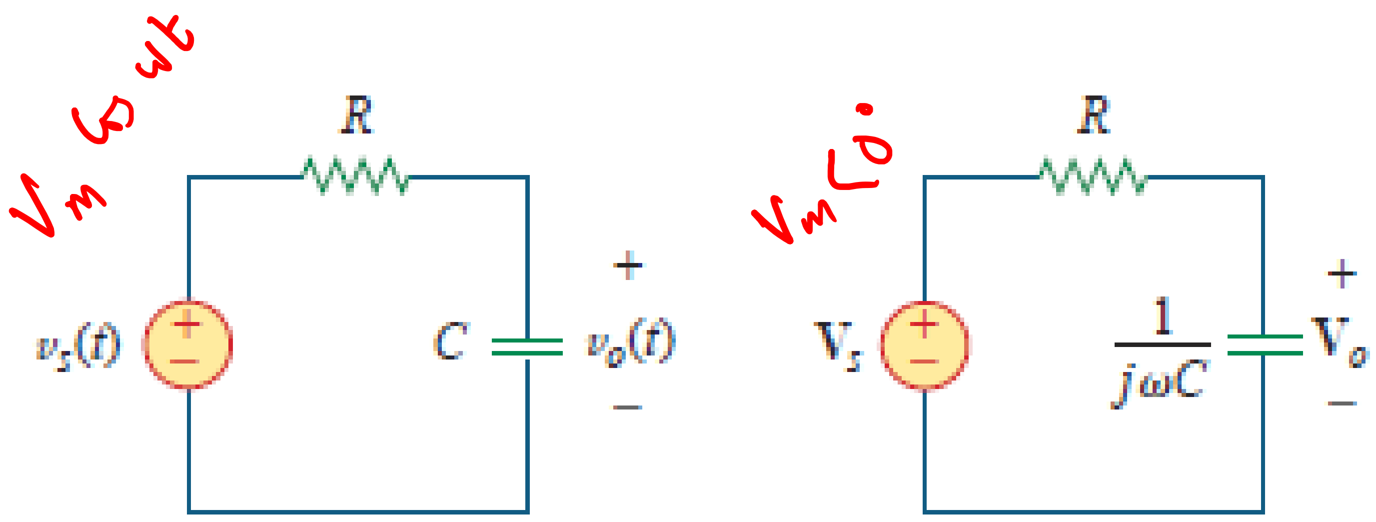
$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

where subscripts i and o denote input and output values.

Being a complex quantity, $\mathbf{H}(\omega)$ has a magnitude $H(\omega)$ and a phase ϕ; that is, $\mathbf{H}(\omega) = H(\omega)\underline{/\phi}$.

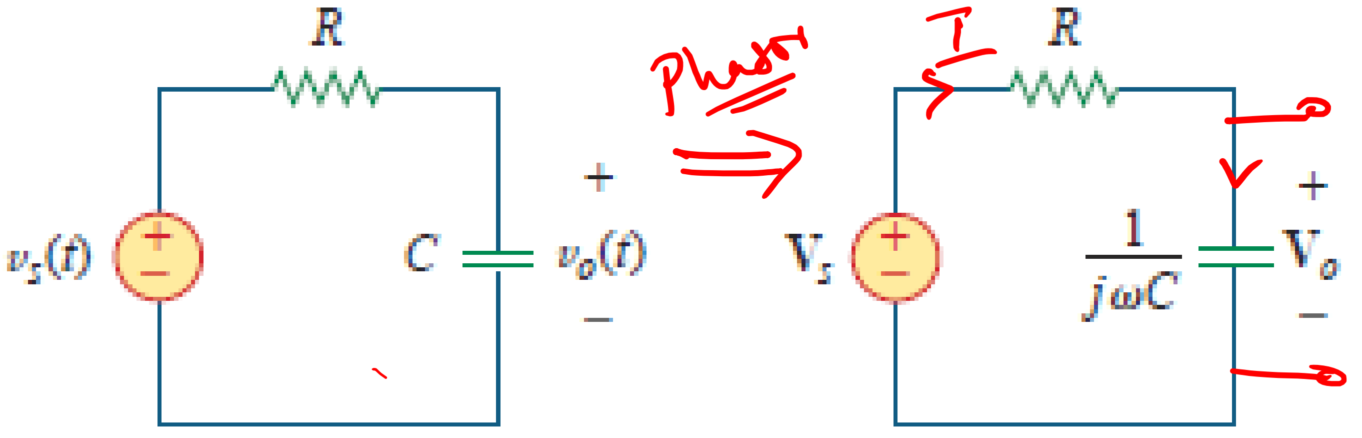
For the RC circuit in Fig. , obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and its frequency response. Let $v_s = V_m \cos \omega t$.





(a) (b)

(a) time-domain RC circuit,
(b) frequency-domain RC circuit.



(a) (b)

$$\frac{V_o}{V_s} = H(\omega) \qquad \frac{1}{Z} = \frac{1}{r} \angle -\phi$$

$$H(\omega) \angle \phi$$

$$z = x + jy$$

$$r = \sqrt{x^2 + y^2}$$

$$r \angle \phi$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

$$\phi = \tan^{-1}\left(\boxed{y}/x\right)$$

$$\phi = -\tan^{-1}(y/x)$$

the magnitude and phase of

amplitude

Phase

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0}$$

where $\omega_0 = 1/RC$. To plot H and ϕ for $0 < \omega < \infty$, we obtain their values at some critical points and then sketch.

$H(\omega)$ amplitud

$=$

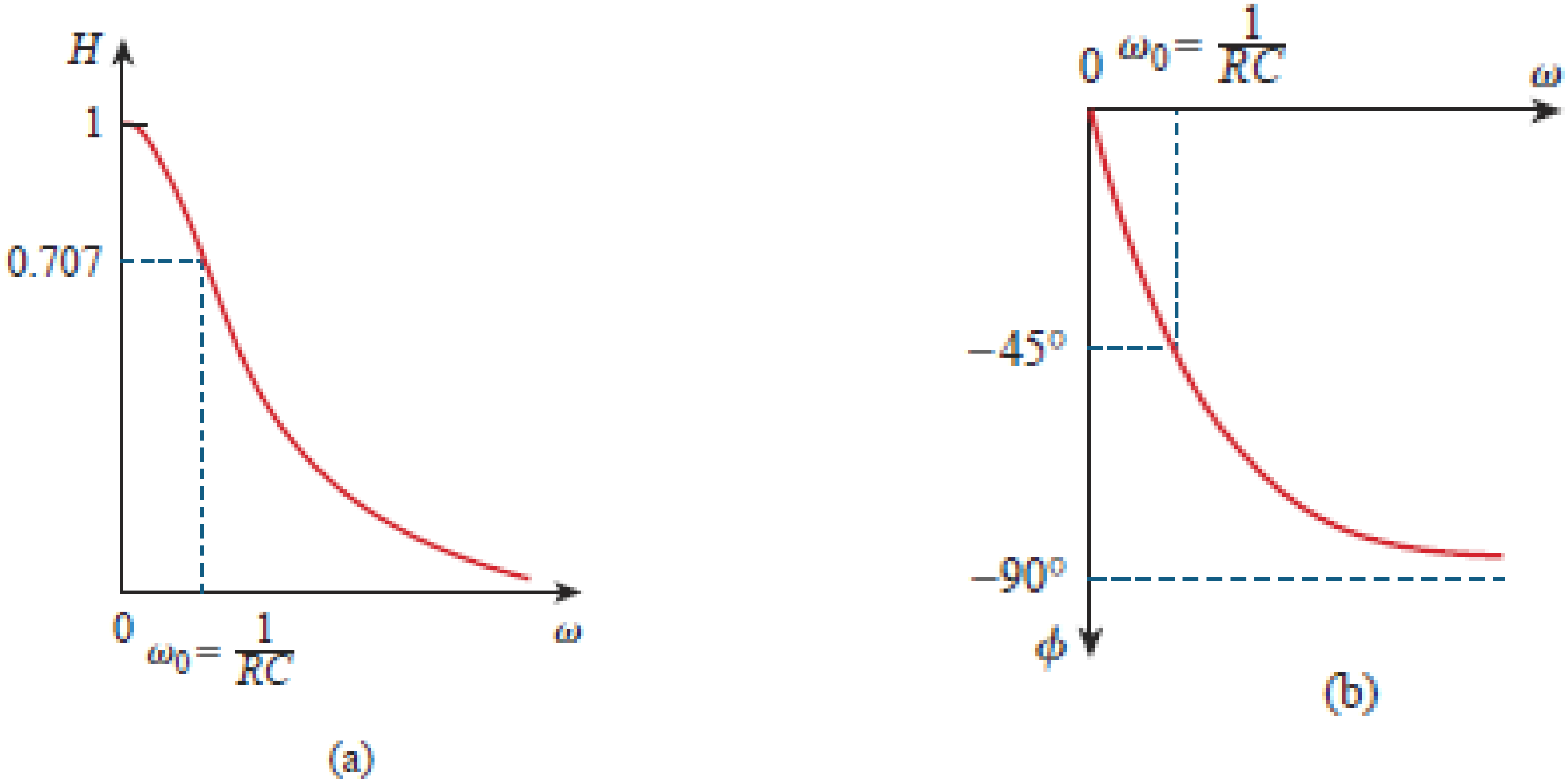
$H\angle\phi$

$1/RC = \omega_0$

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0}$$

At $\omega = 0$, $H = 1$ and $\phi = 0$. At $\omega = \infty$, $H = 0$ and $\phi = -90°$. Also, at $\omega = \omega_0$, $H = 1/\sqrt{2}$ and $\phi = -45°$.

ω/ω_0	H	ϕ	ω/ω_0	H	ϕ
0	1	0	10	0.1	−84°
1	0.71	−45°	20	0.05	−87°
2	0.45	−63°	100	0.01	−89°
3	0.32	−72°	∞	0	−90°



Frequency response of the *RC* circuit:
(a) amplitude response, (b) phase response.

Practice Problem 14.1

Obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the *RL* circuit in Fig. 14.4, assuming $v_s = V_m \cos \omega t$. Sketch its frequency response.

Answer: $j\omega L/(R + j\omega L)$; see Fig. 14.5 for the response.

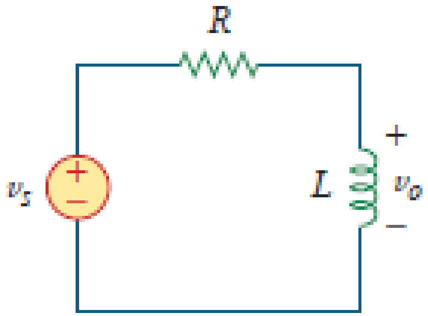
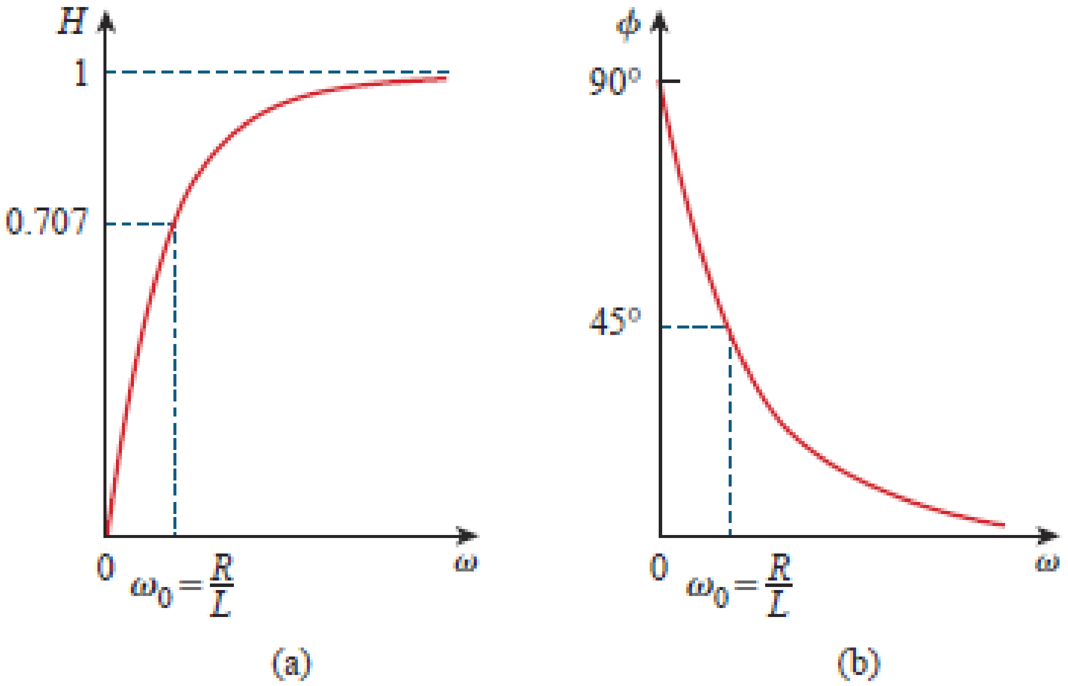


Figure 14.4
RL circuit for Practice Prob. 14.1.



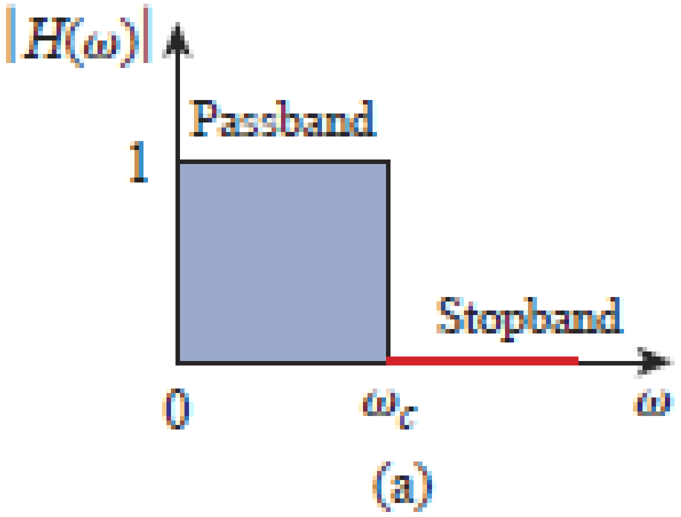
A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

- **As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies.**

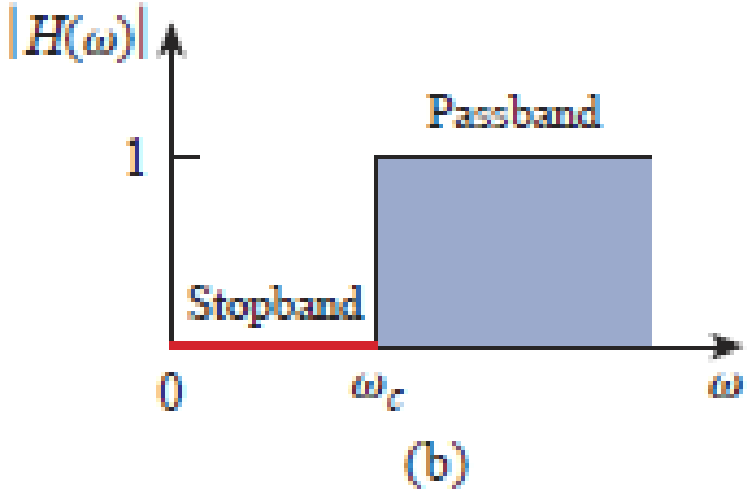
- **Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment.**

- **A filter is a passive filter if it consists of only passive elements R, L, and C.**

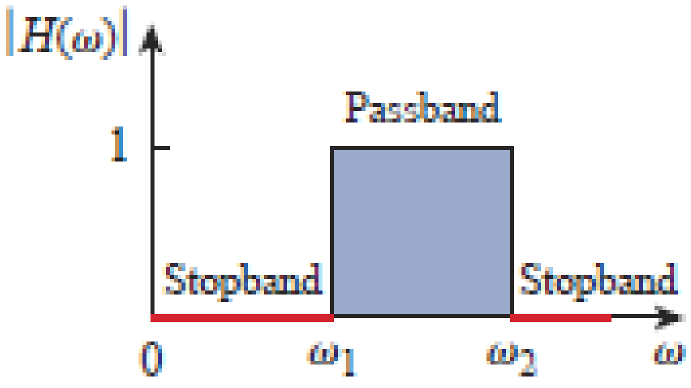
- **It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L, and C.**



A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig.

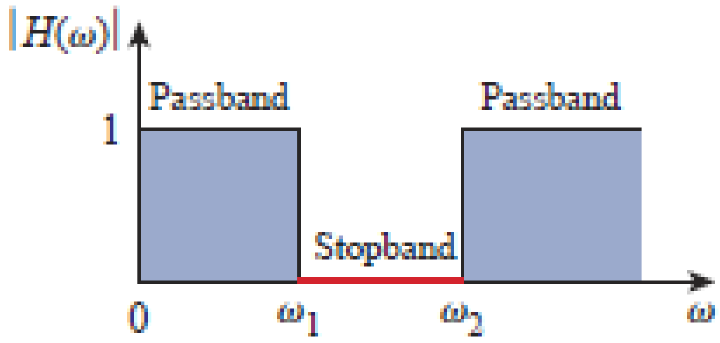


A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig.



(c)

A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig.

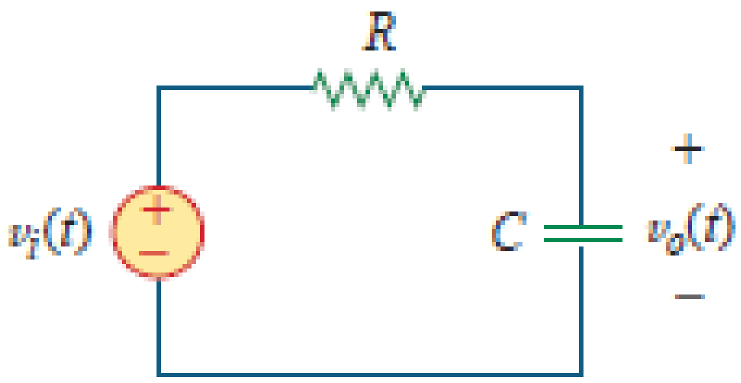


(d)

A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig.

Lowpass Filter

- A typical lowpass filter is formed when the output of an RC circuit is taken off the capacitor

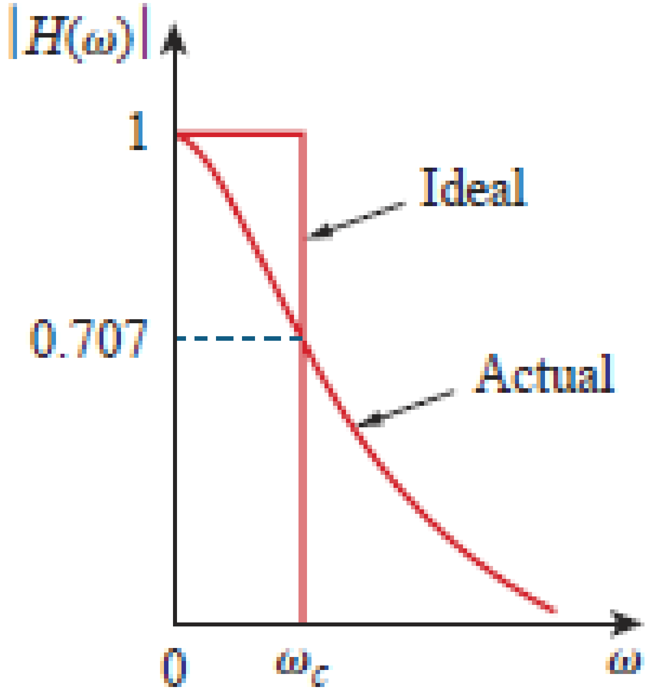


$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

Note that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 0$.

cutoff frequency ω_c,



Ideal and actual frequency response of a lowpass filter

setting the magnitude of $\mathbf{H}(\omega)$ equal to $1/\sqrt{2}$, thus,

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$

The cutoff frequency is also called the *rolloff frequency*.

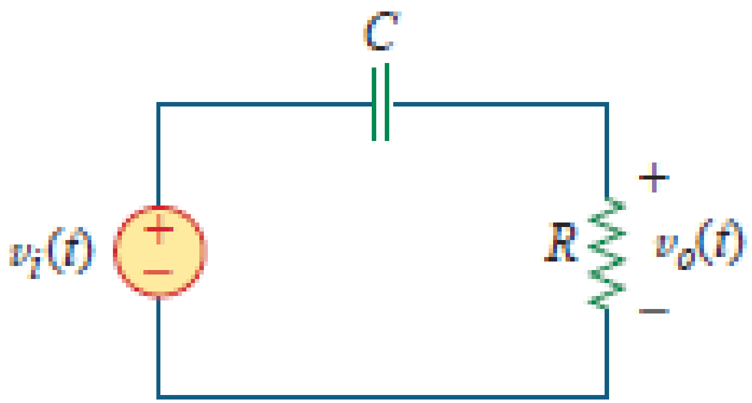
A lowpass filter is designed to pass only frequencies from dc up to the cutoff frequency ω_c.

A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor

The cutoff frequency is the frequency at which the transfer function **H** drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

Highpass Filter

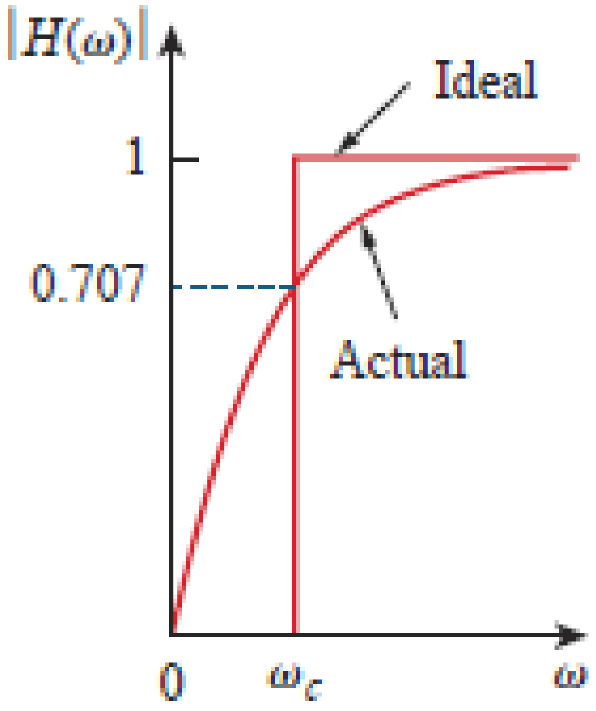
- A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

Note that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 1$.



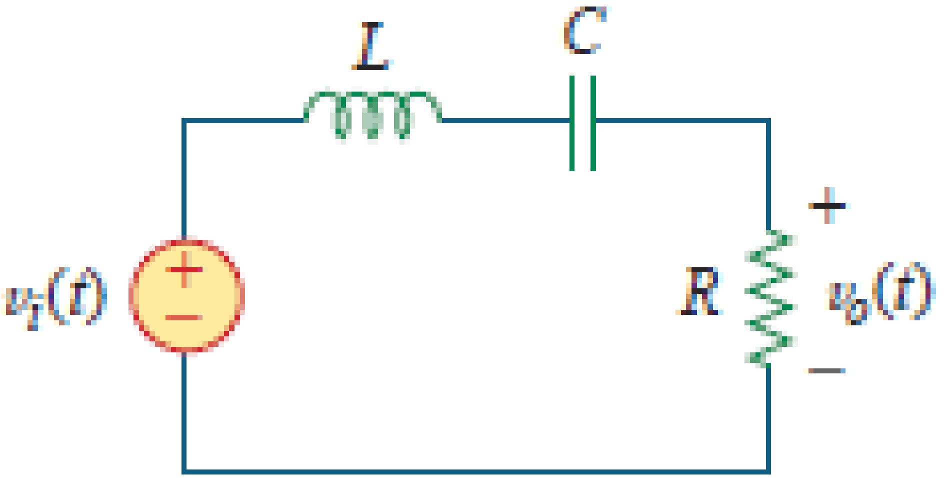
Ideal and actual frequency response of a highpass filter

A **highpass filter** is designed to pass all frequencies above its cutoff frequency ω_c.

A highpass filter can also be formed when the output of an *RL* circuit is taken off the inductor.

Bandpass Filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig.

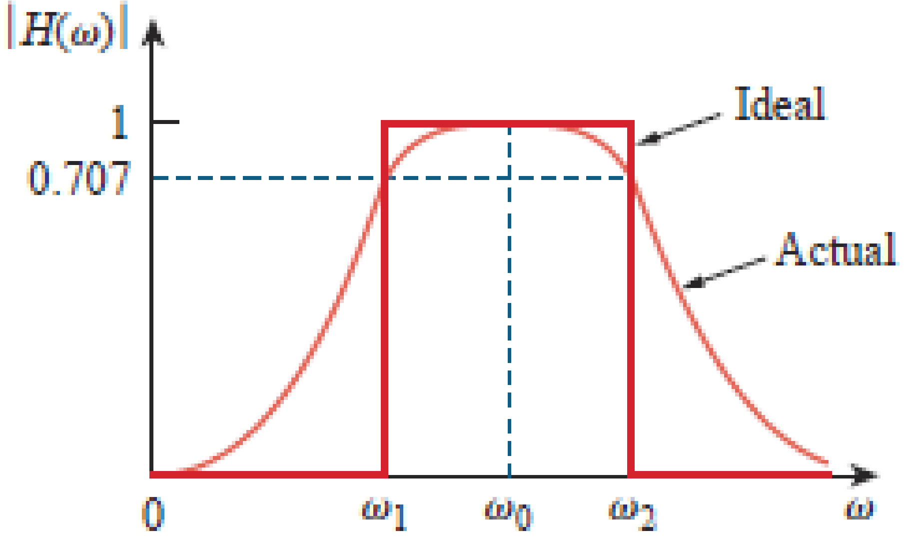


$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

$$\mathbf{H}(0) = 0, \ \mathbf{H}(\infty) = 0.$$

The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_0, the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

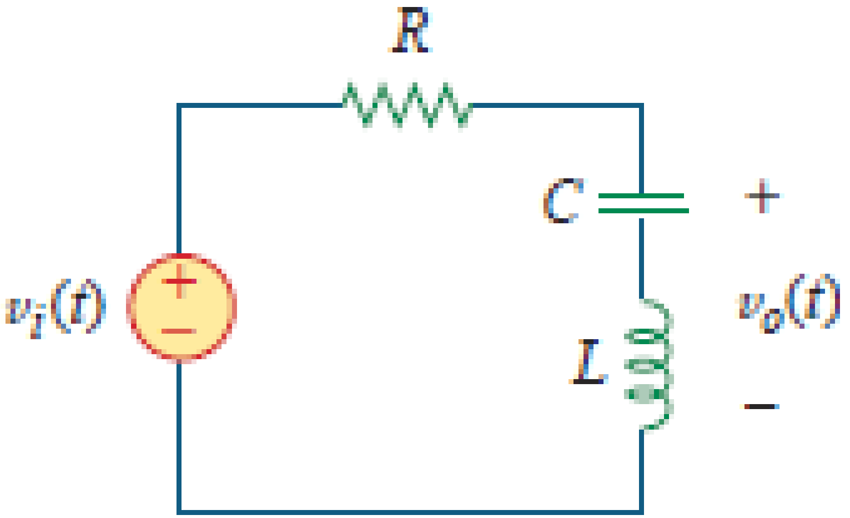


A bandpass filter is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

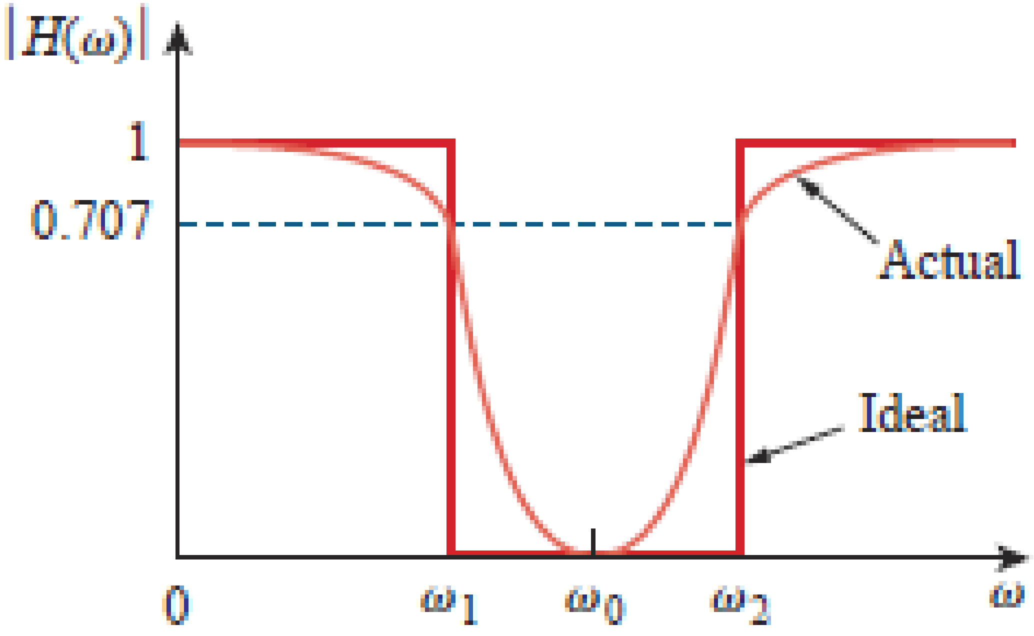
Bandstop Filter

- **A filter that prevents a band of frequencies between two designated values ω_1 and ω_2 from passing is variably known as a *bandstop*, *bandreject*, or *notch* filter.**

- **A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig**



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$



$$\mathbf{H}(0) = 1, \; \mathbf{H}(\infty) = 1.$$

the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

$(B = \omega_2 - \omega_1)$ is known as the *bandwidth of rejection.*

The Decibel Scale

In communications systems, gain is measured in *bels*. Historically, the bel is used to measure the ratio of two levels of power or power gain *G*;

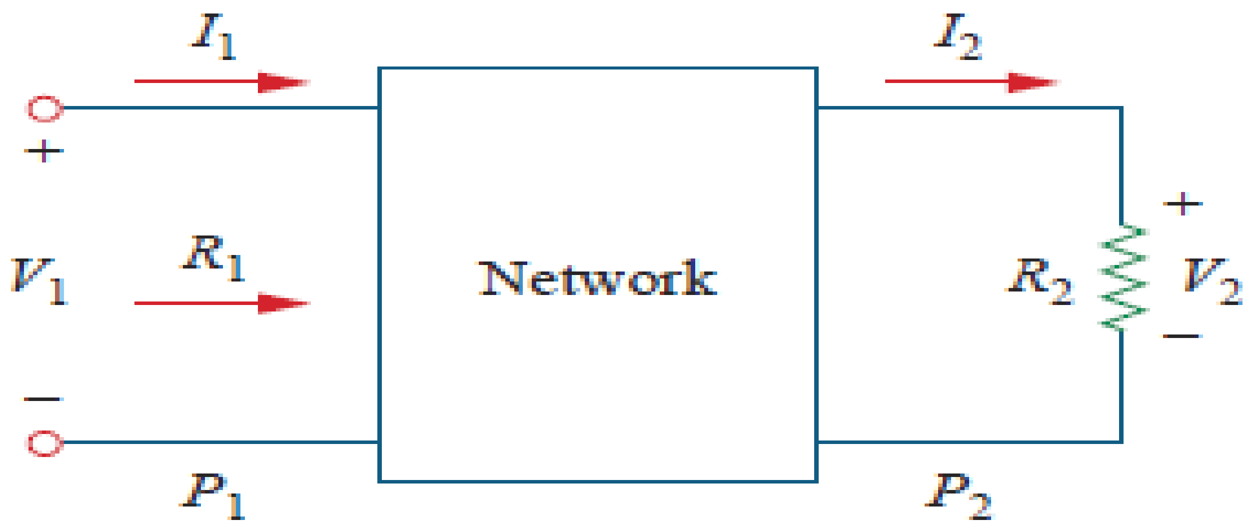
$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1}$$

When $P_1 = P_2$, there is no change in power and the gain is 0 dB. If $P_2 = 2P_1$, the gain is

$$G_{dB} = 10 \log_{10} 2 \simeq 3 \text{ dB}$$

and when $P_2 = 0.5P_1$, the gain is

$$G_{dB} = 10 \log_{10} 0.5 \simeq -3 \text{ dB}$$



$$G_{\mathrm{dB}} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2/R_2}{V_1^2/R_1}$$

$$= 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 + 10\log_{10}\frac{R_1}{R_2}$$

$$G_{\mathrm{dB}} = 20\log_{10}\frac{V_2}{V_1} - 10\log_{10}\frac{R_2}{R_1}$$

$R_2 = R_1$:

$$\boxed{G_{\mathrm{dB}} = 20\log_{10}\frac{V_2}{V_1}}$$

Instead, if $P_1 = I_1^2 R_1$ and $P_2 = I_2^2 R_2$, for $R_1 = R_2$, we obtain

$$G_{\mathrm{dB}} = 20 \log_{10} \frac{I_2}{I_1}$$

Bode Plots

Bode plots are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

Example 14.10

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2\ \text{k}\Omega$, $L = 2$ H, and $C = 2\ \mu$F.

Solution:

The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega \tag{14.10.1}$$

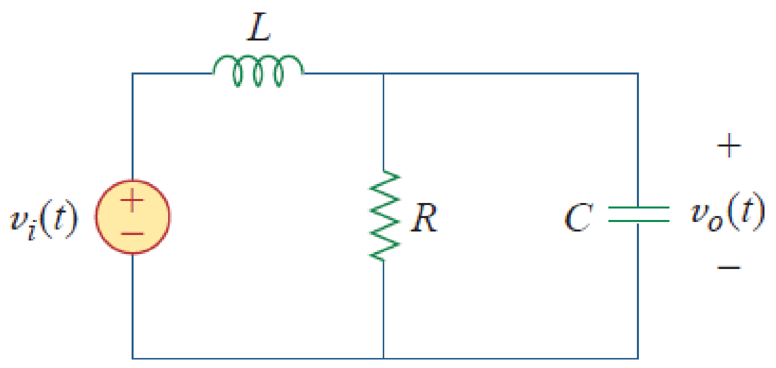


Figure 14.39
For Example 14.10.

But

$$R \left\| \frac{1}{sC} \right. = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

Substituting this into Eq. (14.10.1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \qquad s = j\omega$$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \tag{14.10.2}$$

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}} \tag{14.10.3}$$

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c\, 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

Practice Problem 14.10

For the circuit in Fig. 14.40, obtain the transfer function $\mathbf{V}_o(\omega)/\mathbf{V}_i(\omega)$. Identify the type of filter the circuit represents and determine the corner frequency. Take $R_1 = 100\ \Omega = R_2$, $L = 2$ mH.

Answer: $\dfrac{R_2}{R_1 + R_2}\left(\dfrac{j\omega}{j\omega + \omega_c}\right)$, highpass filter

$$\omega_c = \frac{R_1 R_2}{(R_1 + R_2)L} = 25 \text{ krad/s}.$$

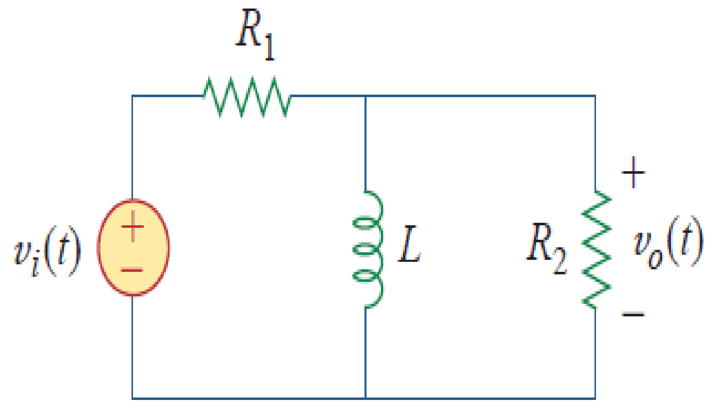


Figure 14.40

For Practice Prob. 14.10.

Example 14.11

If the bandstop filter in Fig. 14.37 is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C. Take $R = 150\ \Omega$ and the bandwidth as 100 Hz.

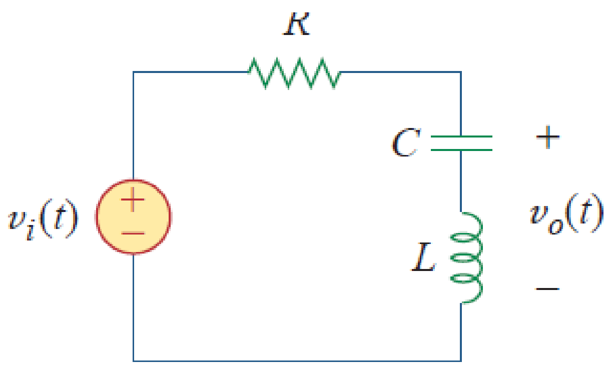


Figure 14.37
A bandstop filter.

Solution:

We use the formulas for a series resonant circuit in Section 14.5.

$$B = 2\pi(100) = 200\pi \text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387 \text{ H}$$

Rejection of the 200-Hz sinusoid means that f_0 is 200 Hz, so that ω_0 in Fig. 14.38 is

$$\omega_0 = 2\pi f_0 = 2\pi(200) = 400\pi$$

Since $\omega_0 = 1/\sqrt{LC}$,

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.653\ \mu\text{F}$$

Practice Problem 14.11

Design a bandpass filter of the form in Fig. 14.35 with a lower cutoff frequency of 20.1 kHz and an upper cutoff frequency of 20.3 kHz. Take $R = 20\ \text{k}\Omega$. Calculate L, C, and Q.

Answer: 15.92 H, 3.9 pF, 101.

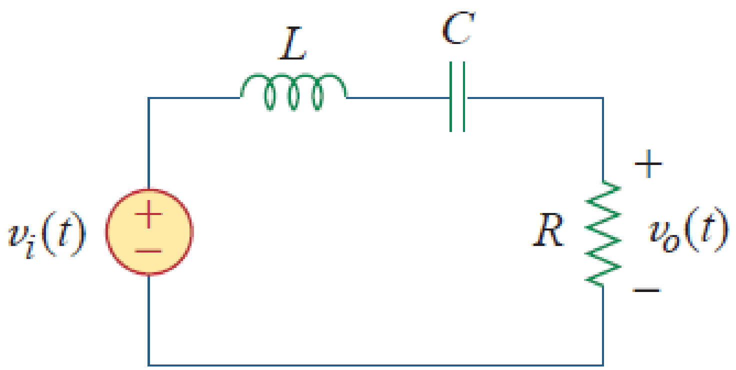


Figure 14.35
A bandpass filter.

Passive attenuator

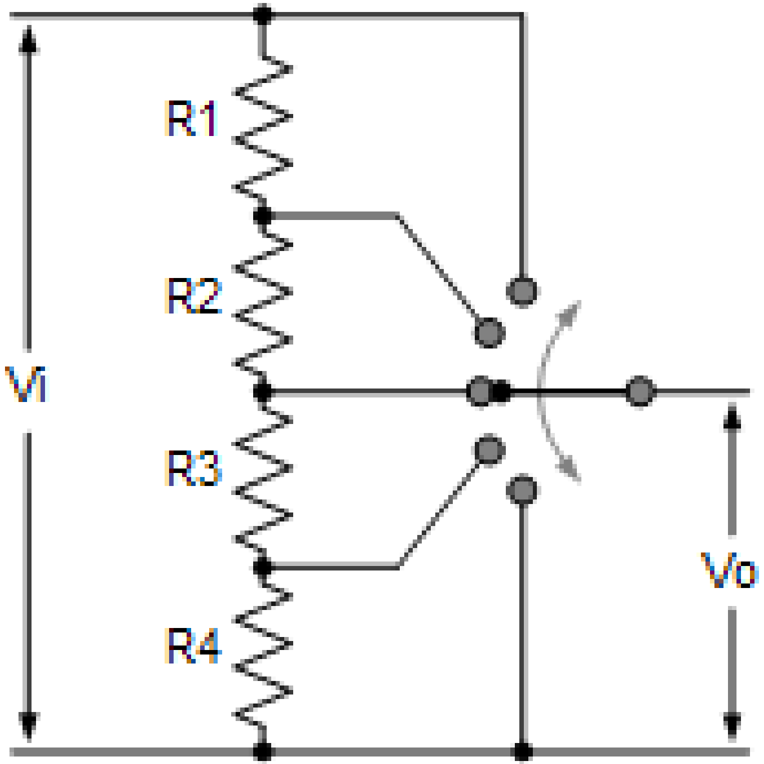
An attenuator is a two-port resistive network and is used to reduce the signal level by a given amount

- A *passive attenuator* reduces the amount of power being delivered to the connected load by either a single fixed amount, a variable amount or in a series of known switchable steps.

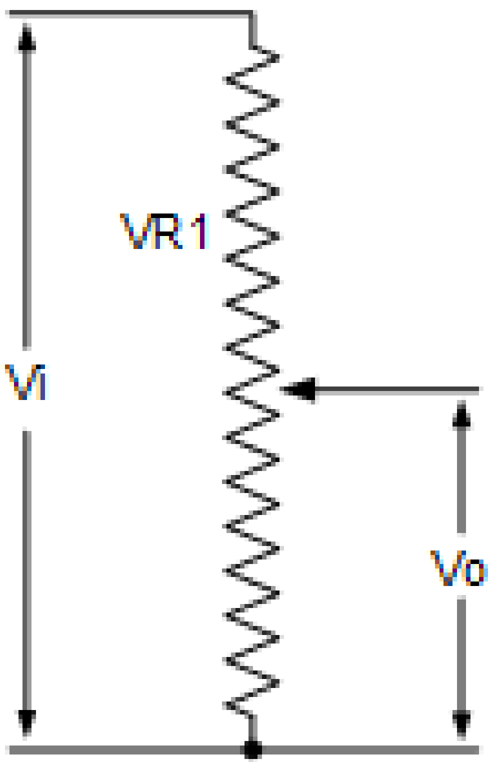
- Attenuators are generally used in radio, communication and transmission line applications to weaken a stronger signal.

- The **Passive Attenuator** is a purely passive resistive network (hence no supply) which is used in a wide variety of electronic equipment for extending the dynamic range of measuring

Simple Passive Attenuator

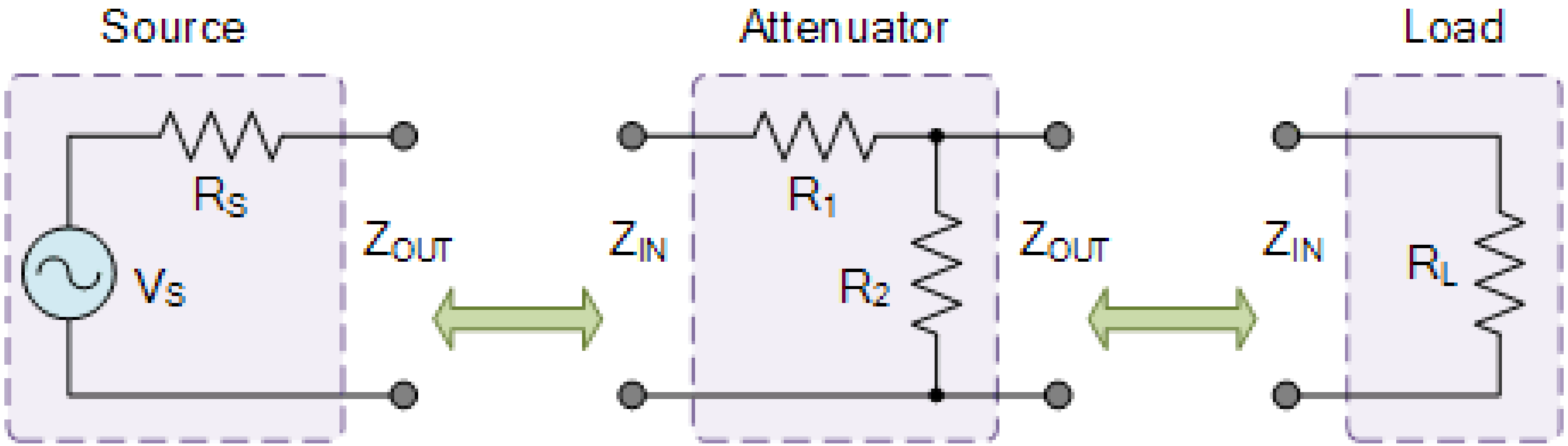


Predetermined Values of Attenuation



Variable Attenuator

Attenuator Connection



Attenuation is expressed either in decibels (dB) or in nepers

$$\text{Attenuation in dB} = 10\ \log_{10}\left(\frac{P_1}{P_2}\right)$$

where P_1 is the input power and P_2 is the output power.

For a properly matched network, both terminal pairs are matched to the characteristic resistance, *R*o of the attenuator.

$$\frac{P_1}{P_2} = \frac{I_1^2 R_0}{I_2^2 R_0} = \frac{I_1^2}{I_2^2}$$

$R_o = R_L$

where I_1 is the input current and I_2 is the output current leaving the port.

$$\frac{P_1}{P_2} = \frac{V_1^2}{V_2^2}$$

where V_1 is the voltage at port 1 and V_2 is the voltage at port 2

$$\text{Hence, attenuation in dB} = 20 \log_{10}\left(\frac{V_1}{V_2}\right) = 20 \log_{10}\left(\frac{I_1}{I_2}\right)$$

If $$\frac{V_1}{V_2} = \frac{I_1}{I_2} = N$$

then $$\frac{P_1}{P_2} = N^2$$

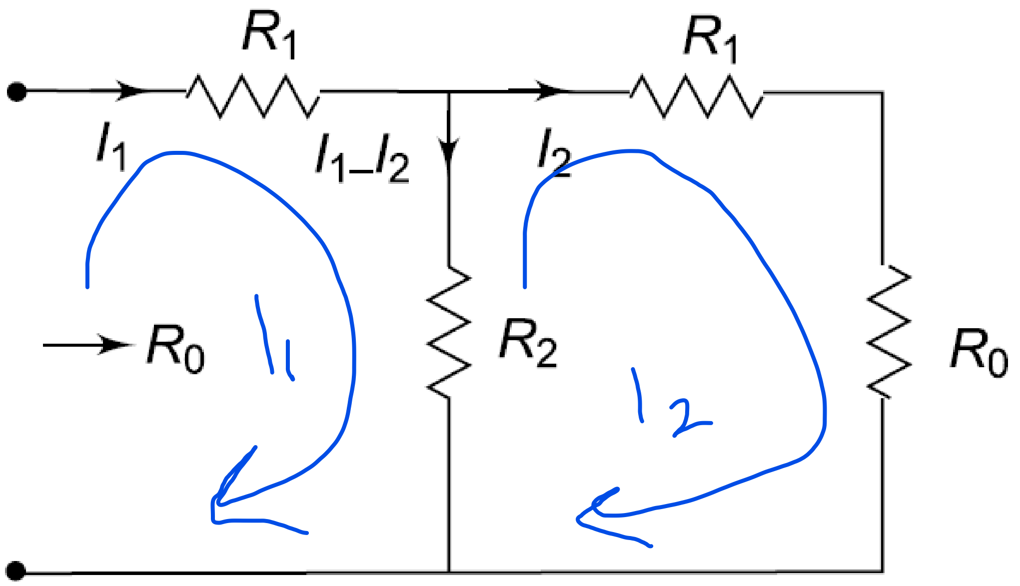
and $$\mathrm{dB} = 20 \log_{10} N$$

or $$N = \mathrm{antilog}\left(\frac{\mathrm{dB}}{20}\right)$$

Basically, there are four types of attenuators, T, π, lattice and bridged T-type.

T-TYPE ATTENUATOR

An attenuator is to be designed for desired values of characteristic resistance, *Ro* and attenuation



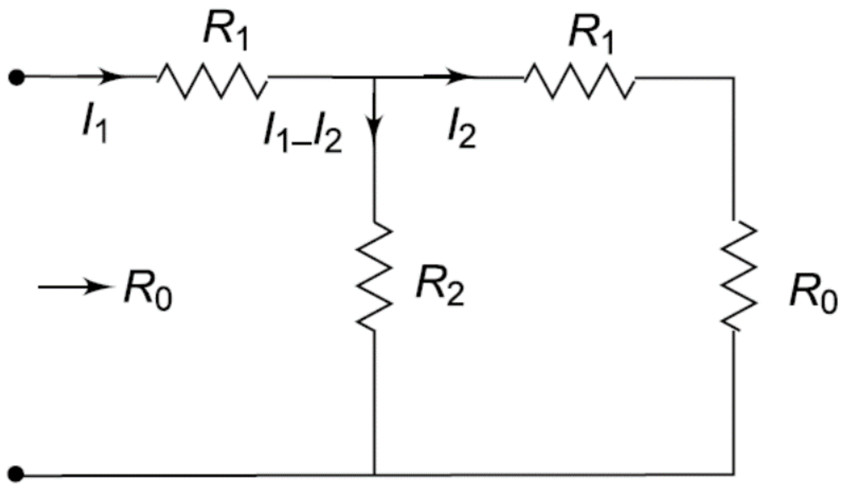
$$R_2 (I_1 - I_2) = I_2 (R_1 + R_0)$$

$$I_2 (R_2 + R_1 + R_0) = I_1 R_2$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0 + R_2}{R_2} = N \longrightarrow 1$$

The characteristic impedance of the attenuator is R_0 when it is terminated in a load of R_0

Hence,
$$R_0 = R_1 + \frac{R_2(R_1 + R_0)}{R_1 + R_0 + R_2} \longrightarrow 2$$



Substituting Eq 1 in Eq 2 ⇒

$$R_0 = R_1 + \frac{(R_1 + R_0)}{N} \longrightarrow 3$$

$$NR_0 = NR_1 + R_1 + R_0$$

$$R_0(N - 1) = R_1\,(N + 1)$$

$$R_1 = \frac{R_0(N-1)}{N+1} \longrightarrow 4$$

From Eq 1 ⇒

$$NR_2 = R_1 + R_0 + R_2$$

$$(N - 1)R_2 = (R_1 + R_0)$$

Substituting the value of *R*1 from Eq.(4)

$$(N-1)R_2 = R_0\frac{(N-1)}{N+1} + R_0$$

$$(N-1)R_2 = R_0\frac{(N-1)}{N+1} + R_0$$

$$(N-1)R_2 = \frac{2NR_0}{(N+1)}$$

$$R_2 = \frac{2NR_0}{N^2-1} \longrightarrow 5$$

Equations (4) and (5) are the design equations for the symmetrical T-attenuator

Example *Design a T-pad attenuator to give an attenuation of 60 dB and to work in a line of 500 Ω impedance.*

Solution
$$N = \frac{I_1}{I_2} = \text{antilog}\,\frac{D}{20}$$
$$= \text{antilog}\,\frac{60}{20} = 1000$$

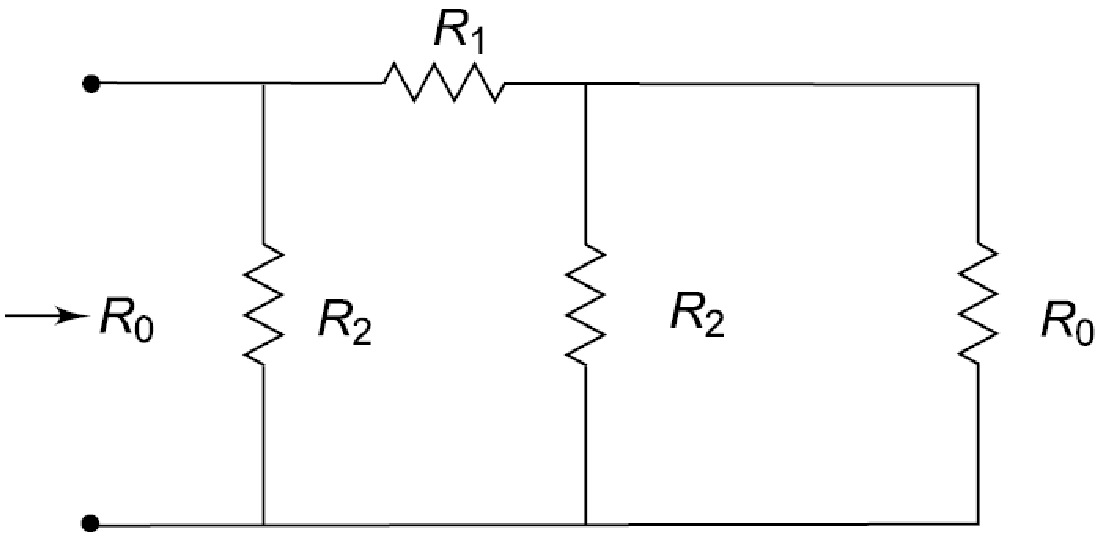
Each of the series arm is given by

$$R_1 = \frac{R_0(N-1)}{N+1} = 500\,\frac{(1000-1)}{(1000+1)} = 499\ \Omega$$

The shunt arm resistor R_2 is given by

$$R_2 = \frac{2N}{N^2-1} = R_0 = \frac{2\times 1000}{(1000)^2-1}\times 500 = 1\ \Omega$$

π-TYPE ATTENUATOR



$$R_1 = R_0 \sinh \alpha$$

$$R_2 = R_0 \coth \alpha/2$$

$$\therefore \quad R_1 = R_0 \frac{e^{\alpha} - e^{-\alpha}}{2} \longrightarrow 1$$

$$\frac{I_1}{I_2} = N \longrightarrow 2$$

$$e^{\alpha} = N \longrightarrow 3$$

$\alpha \rightarrow$ Attenuation constant

Using Eq. (3) in (1)

$$R_1 = R_0 \frac{N - \dfrac{1}{N}}{2} = R_0 \frac{N^2 - 1}{2N} \text{ — } 4$$

$$R_2 = R_0 \frac{\cosh \alpha / 2}{\sinh \alpha / 2} = R_0 \frac{e^{\alpha/2} + e^{-\alpha/2}}{e^{\alpha/2} - e^{-\alpha/2}}$$

$$R_2 = R_0 \frac{e^{\alpha} + 1}{e^{\alpha} - 1} = R_0 \frac{(N-1)}{(N+1)} \longrightarrow 5$$

Equations (4) and (5) are the design equations for the symmetrical pi-attenuator.

Example *Design a π-type attenuator to give 20 dB attenuation and to have a characteristic impedance of 100 Ω.*

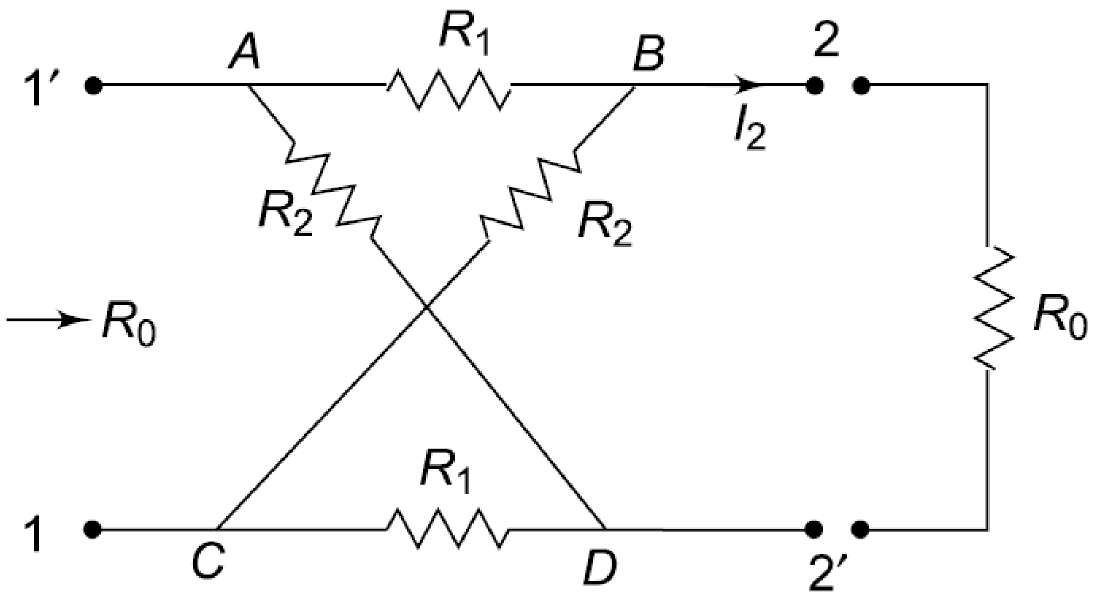
Solution Given $R_0 = 100\ \Omega$, $D = 20$ dB.

$$N = \text{Antilog}\ \frac{D}{20} = 10$$

$$R_1 = R_0 \frac{(N^2 - 1)}{2N} = 100 \frac{(10^2 - 1)}{2 + 10} = 495\ \Omega$$

$$R_2 = R_0 \frac{(N + 1)}{(N - 1)} = 100 \left(\frac{10 + 1}{10 - 1} \right) = 122.22\ \Omega$$

LATTICE ATTENUATOR



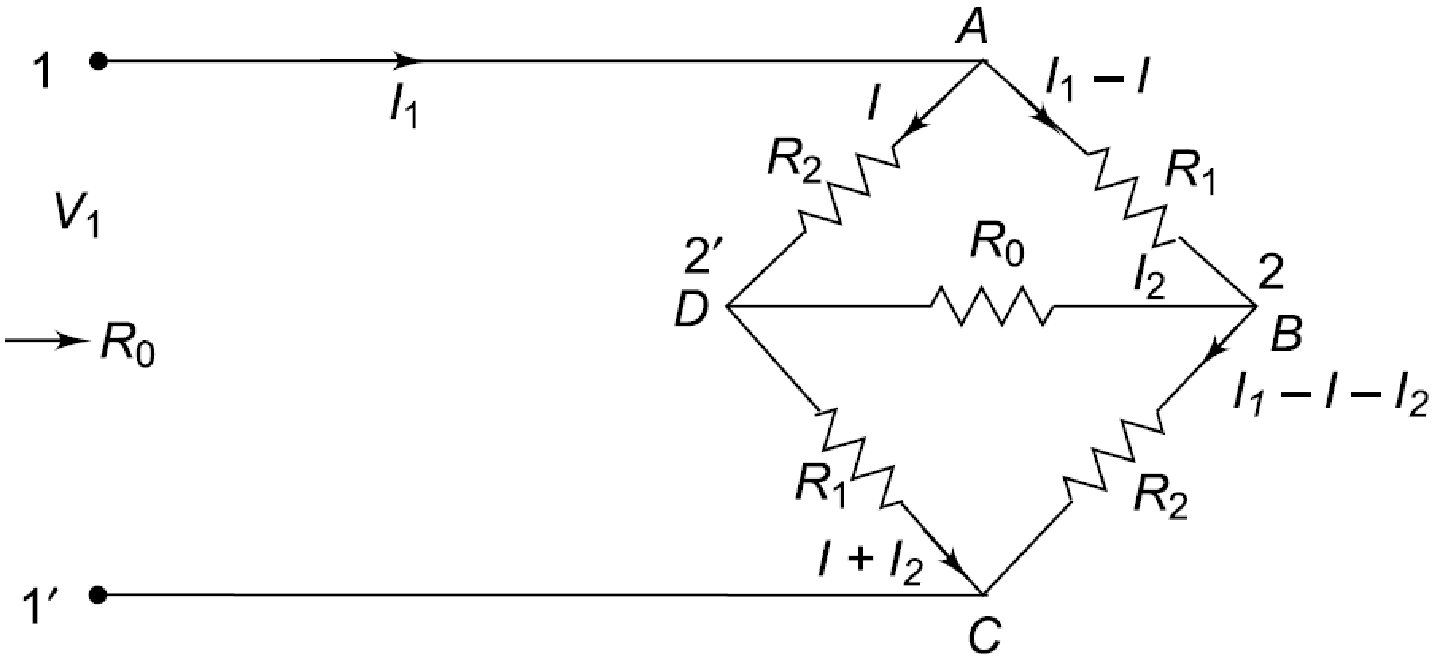
Thus, $$Z_{sc} = \frac{2R_1R_2}{R_1 + R_2}$$

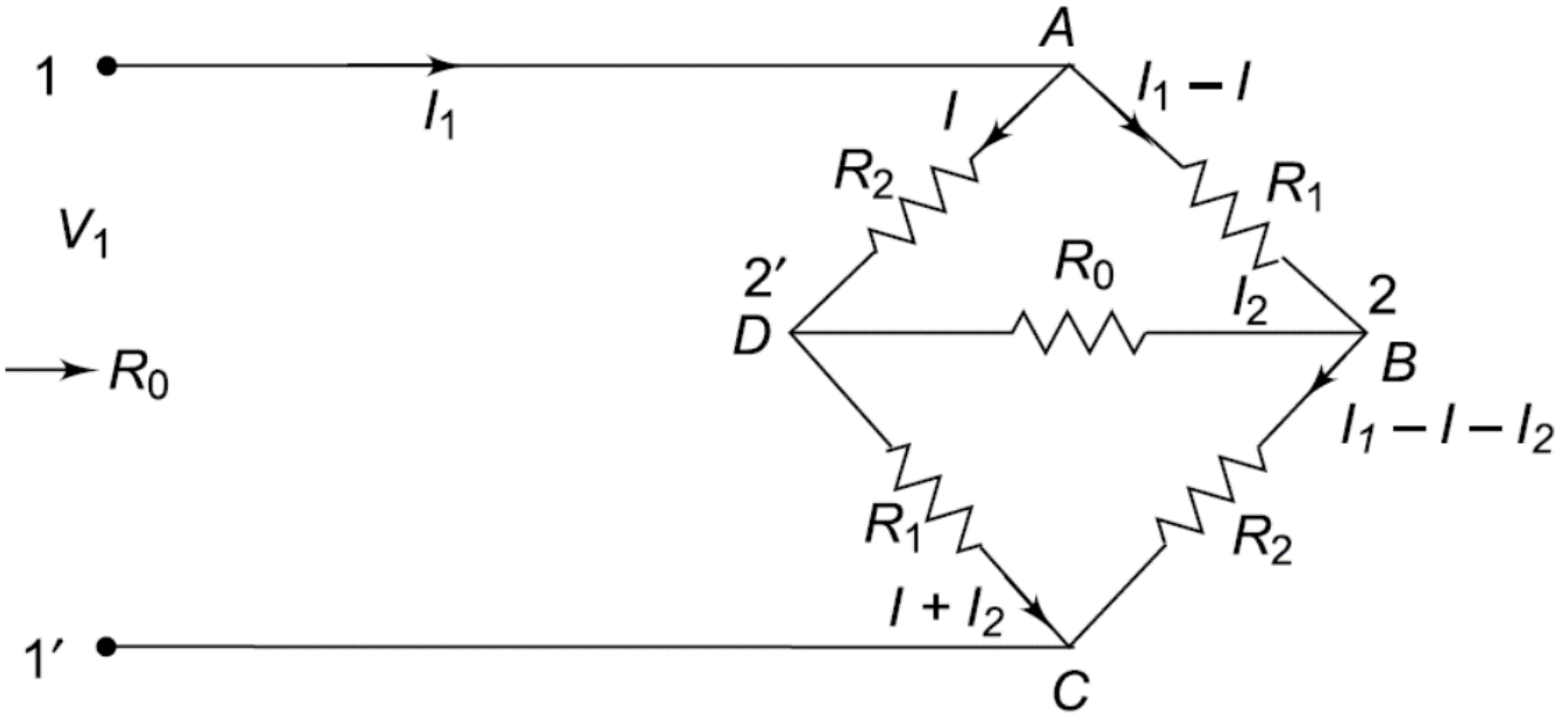
$$Z_{0c} = \frac{R_1 + R_2}{2}$$

Hence, $$Z_0 = R_0 = \sqrt{Z_{0c}Z_{sc}}$$

$$R_0 = \sqrt{R_1R_2}$$

→ 1





the input impedance at 1-1’ is *R*o when the network is terminated in *R*o at 2-2’

By applying Kirchhoff’s voltage law, we get

$$V_1 = I_1R_0 = (I_1 - I)R_1 + I_2R_0 + (1 + I_2)R_1$$

$$I_1R_0 = R_1(I_1 + I_2) + I_2R_0$$

$$I_1(R_0 - R_1) = I_2(R_1 + R_0)$$

$$\frac{I_1}{I_2} = \frac{R_1 + R_0}{R_0 - R_1} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}}$$

Using $R_0 = \sqrt{R_1 R_2}$

$$N = e^{\alpha} = \frac{I_1}{I_2} = \frac{1 + \dfrac{R_1}{R_0}}{1 - \dfrac{R_1}{R_0}} \longrightarrow ②$$

$$\alpha = \log\left[\frac{1 + \sqrt{\dfrac{R_1}{R_2}}}{1 - \sqrt{\dfrac{R_1}{R_2}}}\right] \longrightarrow ③$$

$$e^{\alpha} = \frac{1 + \sqrt{R_1 / R_2}}{1 - \sqrt{R_1 / R_2}}$$

From Eq 2 $\Rightarrow$ $$N\left(1-\frac{R_1}{R_0}\right)=\left(1+\frac{R_1}{R_0}\right)$$

$$R_1 = R_0\frac{(N-1)}{(N+1)} \longrightarrow 4$$

Similarly, we can express $$R_2 = R_0\frac{(N+1)}{(N-1)} \longrightarrow 5$$

Equations (4)and (5) are the design equations for lattice attenuator.

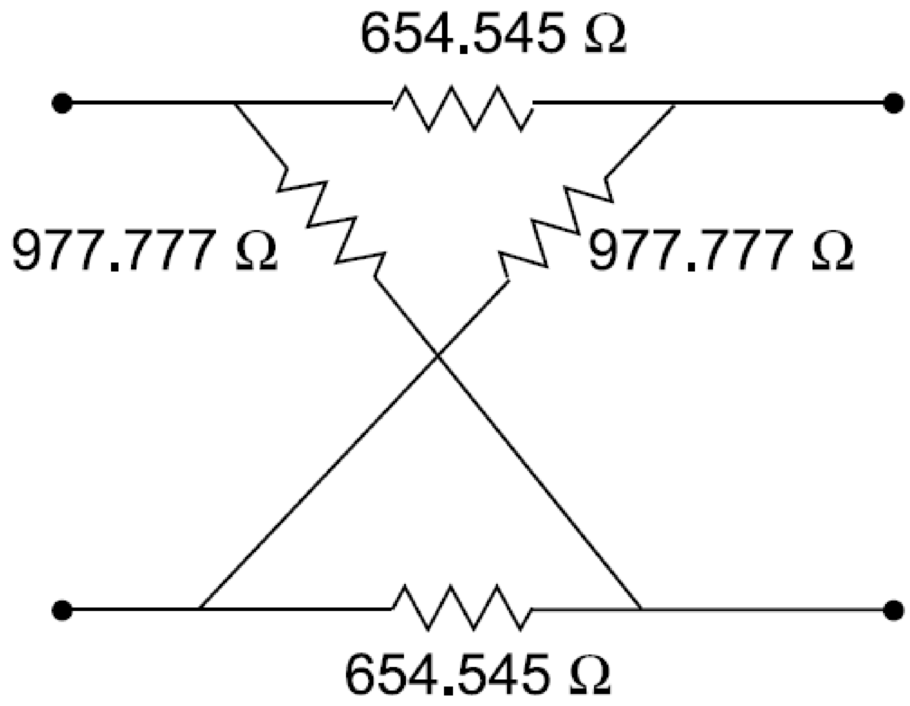
Example *Design a symmetrical lattice attenuator to have characteristic impedance of 800 Ω and attenuation of 20 dB.*

Solution Given $R_0 = 800\ \Omega$ and $D = 20$ dB

$$N = \text{antilog}\,\frac{D}{20} = \text{antilog}\,\frac{20}{20} = 10$$

From the design equations of lattice attenuator

Series arm resistance $R_1 = R_0 \dfrac{(N-1)}{(N+1)}$



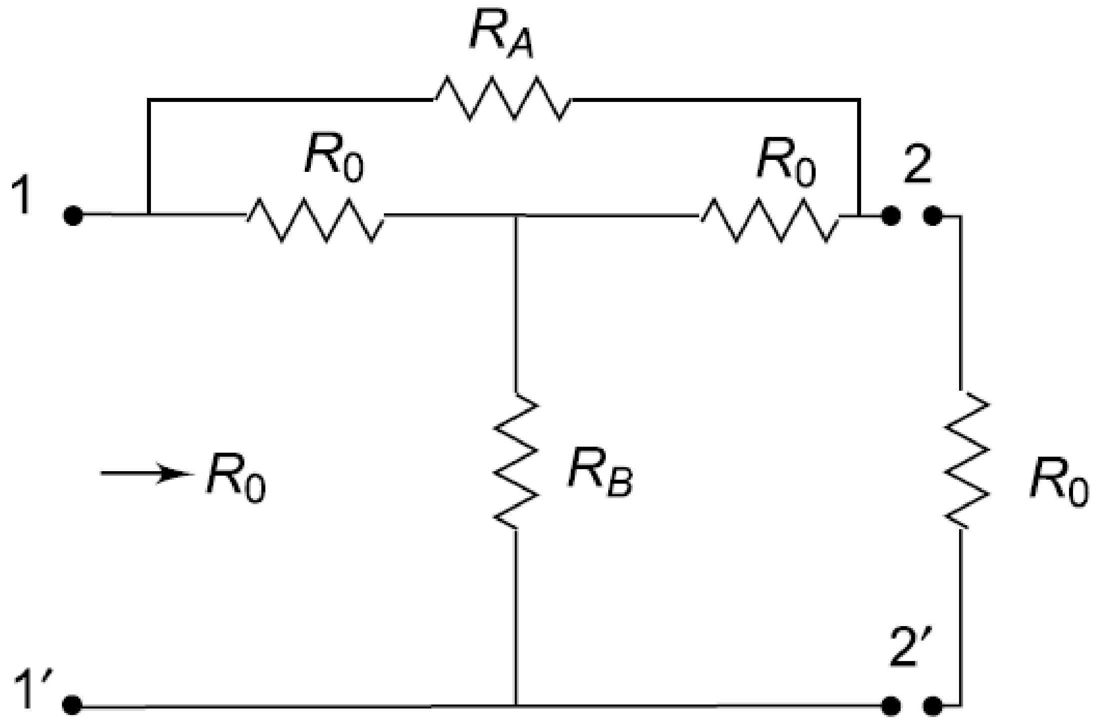
$$= 800\frac{(10-1)}{(10+1)} = 654.545\ \Omega$$

Diagonal arm resistance $R_2 = R_0 \dfrac{(N+1)}{(N-1)}$

$$= 800\frac{(10+1)}{(10-1)} = 977.777\ \Omega$$

The resulting lattice attenuator is shown in Fig.

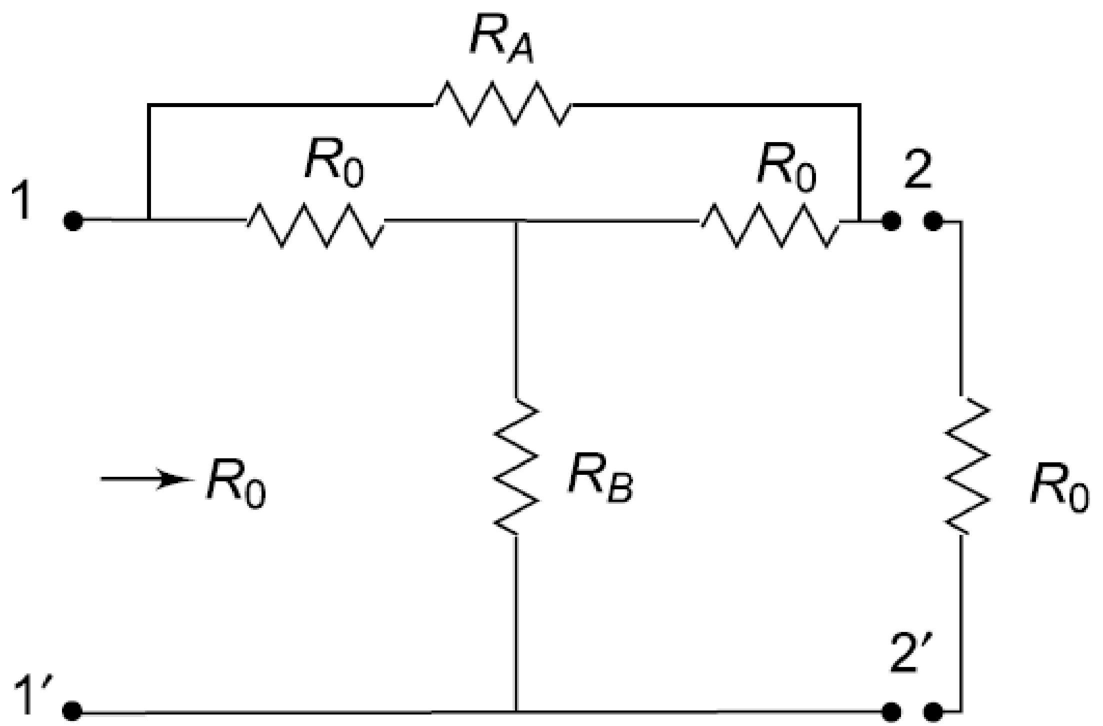
BRIDGED-*T* ATTENUATOR



- The bridged-*T* network may be designed to have any characteristic resistance $R0$ and desired attenuation by making $R_A\, R_B = R_0^2$
- Here R_A and R_B are variable resistances and all other resistances are equal to the characteristic resistance R_0 of the network

- According to the bisection theorem, a network having mirror image symmetry can be reduced to an equivalent lattice structure.
- The series arm of the equivalent lattice is found by bisecting the given network into two parts, short circuiting all the cut wires and equating the series impedance of the lattice to the input impedance of the bisected network; the diagonal arm is equal to the input impedance of the bisected network when cut wires are open circuited

BRIDGED-*T* ATTENUATOR



When the cut wires *A*, *B*, *C* are shorted, the input resistance of the network is given by

$$R_{sc} = \frac{R_0 \times R_{A/2}}{R_0 + R_{A/2}} = \frac{R_0 R_A}{2R_0 + R_A}$$

$$\therefore \qquad \frac{R_0 R_A}{2R_0 + R_A} = R_1$$

$$R_1 = R_0 \frac{(N-1)}{(N+1)} \qquad R_2 = R_0 \frac{(N+1)}{(N-1)}$$

$$R_A = R_0 (N - 1) \qquad R_B = \frac{R_0}{N-1}$$

Example *Design a symmetrical bridged T - attenuator with an attenuation of 20 dB and terminated into a load of 500 Ω.*

Solution $D = 20\ dB;\ R_0 = 500\,\Omega$

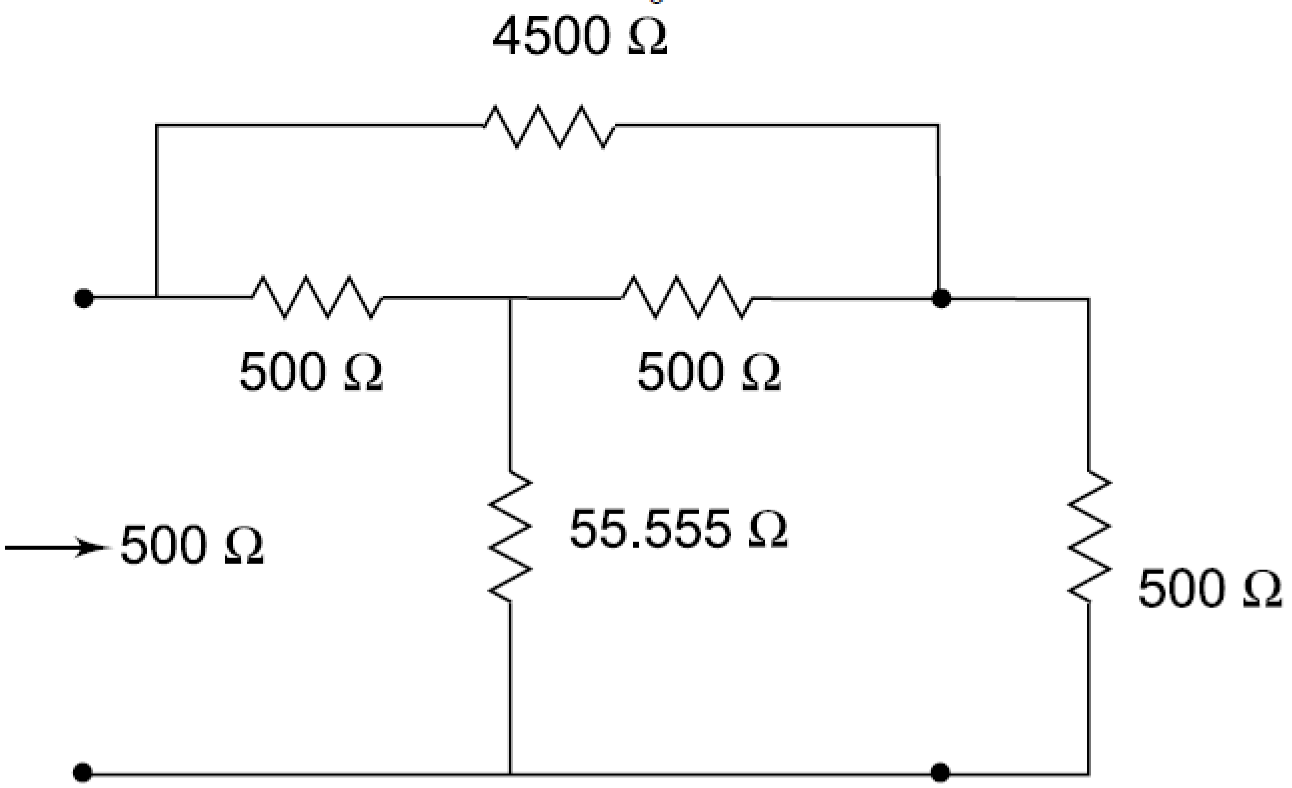


Fig.

$$N = \text{antilog}\frac{D}{20} = \text{antilog}\frac{20}{20} = 10$$

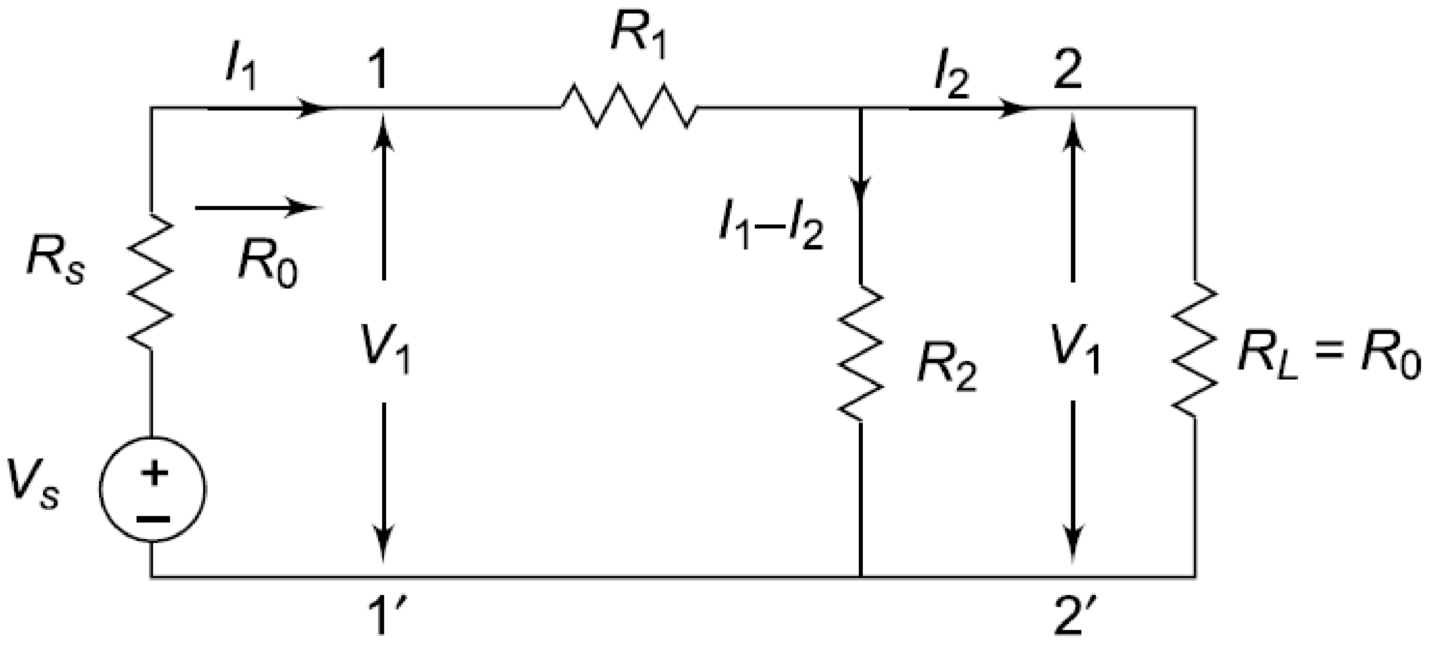
$$R_A = R_0(N-1) = 500(10-1) = 4500\,\Omega$$

$$R_B = \frac{R_0}{(N-1)} = \frac{500}{(10-1)} = 55.555\,\Omega$$

The desired configuration of the attenuator is shown in Fig.

L-TYPE ATTENUATOR

The L type attenuator is connected between a source with source resistance $R_S = R_0$ and load resistance $R_L = R_0$



$$V_2 = (I_1 - I_2)R_2 = I_2 R_L$$

$$\text{or} \quad I_1 R_2 = I_2(R_2 + R_L)$$

$$\frac{I_1}{I_2} = \frac{R_2 + R_L}{R_2} = N \longrightarrow ①$$

$$1 + \frac{R_L}{R_2} = N \longrightarrow ②$$

$$R_2 = \frac{R_L}{N-1} \longrightarrow 3$$

As $R_L = R_0$, Eq. 3 can be written as

$$\boxed{R_2 = \frac{R_0}{N-1}} \longrightarrow 4$$

The resistance of the network as viewed from 1-1′ into the network is

$$R_0 = R_1 + \frac{R_2 R_0}{R_2 + R_0}$$

$$R_1 = \frac{R_0^2}{R_2 + R_0} \longrightarrow 5$$

Substituting the value of R_2 from Eq. 4

$$R_1 = \frac{R_0^2}{\dfrac{R_0}{N-1} + R_0} = \frac{R_0^2(N-1)}{R_0 + R_0(N-1)}$$

$$\boxed{R_1 = R_0 \frac{(N-1)}{N}} \longrightarrow 6$$

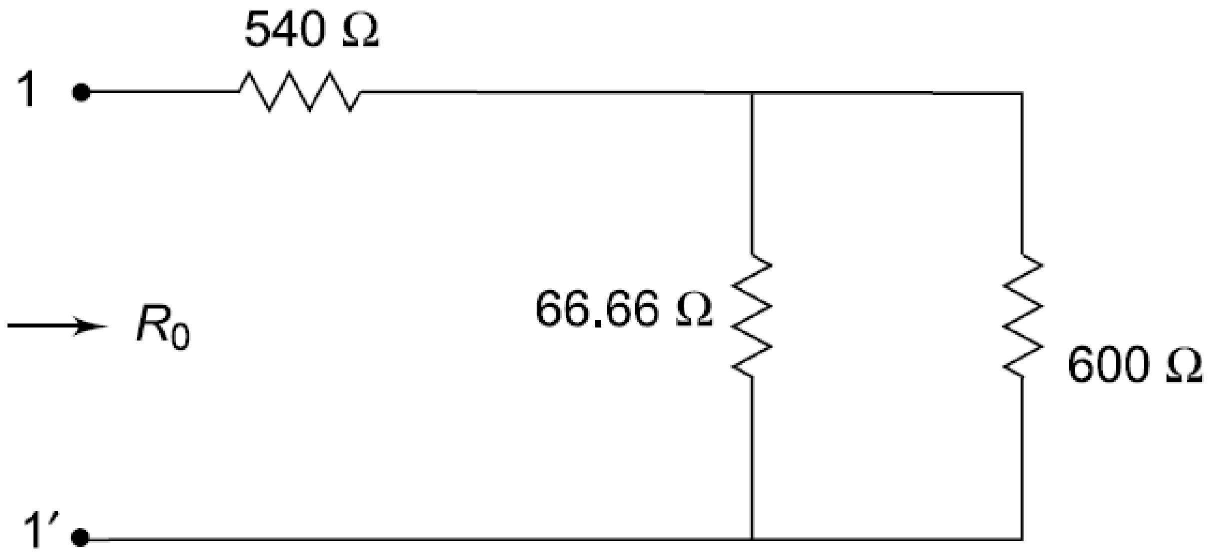
Equations 4 and 6 are the design equations. Attenuation N of the network can be varied by varying the values of R_1 and R_2.

Example *Design a L-type attenuator to operate into a load resistance of 600 Ω with an attenuation of 20 dB.*

Solution $N = \text{antilog}\,\dfrac{\text{dB}}{20} = \text{antilog}\,\dfrac{20}{20} = 10$

The series arm of the attenuator is given by

$$R_1 = R_0\left(\frac{N-1}{N}\right) = 600\left(\frac{10-1}{10}\right) = 540\,\Omega$$



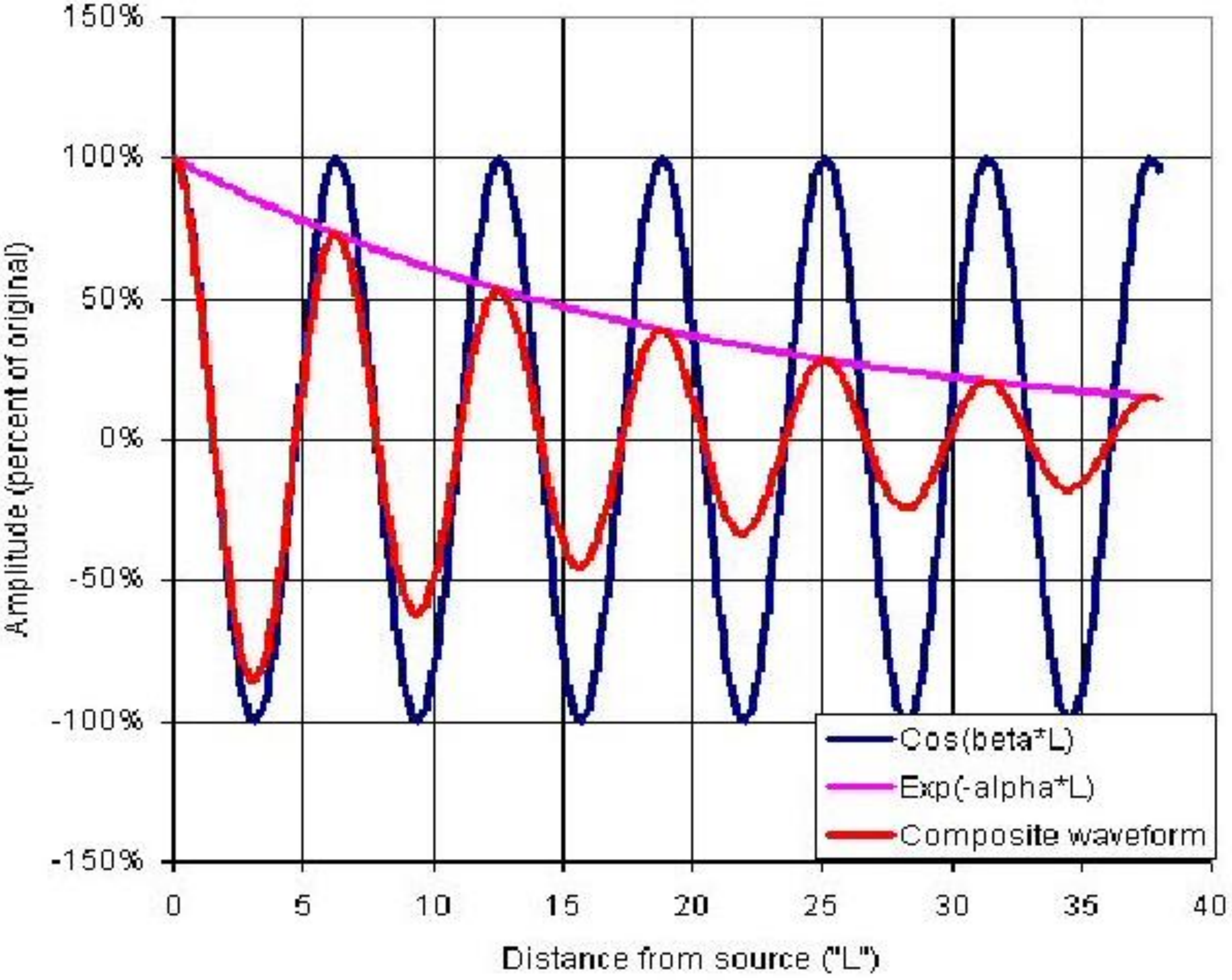
The shunt arm of the attenuator is given by

$$R_2 = \frac{R_0}{N-1} = \frac{600}{9} = 66.66\,\Omega$$

The desired configuration of the network is shown in Fig.

EQUALIZERS

Equalizers are networks designed to provide compensation against distortions that occur in a signal while passing through an electrical network. In general, any electrical network has attenuation distortion and phase distortion. Attenuation distortion occurs due to non-uniform attenuation against frequency characteristics. Phase distortion occurs due to phase delay against frequency characteristics. An attenuation equalizer is used to compensate attenuation distortion in any network. These equalizers are used in medium to high frequency carrier telephone systems, amplifiers, transmission lines and speech reproduction, etc. A phase equalizer is used to compensate phase distortion in any network. These equalizers are used in TV signal transmission lines and in facsimile.



$\gamma = \alpha + j\beta$

$\gamma =$ *propagation constant*

$\alpha =$ *attenuation constant*

$\beta =$ *phase constant*