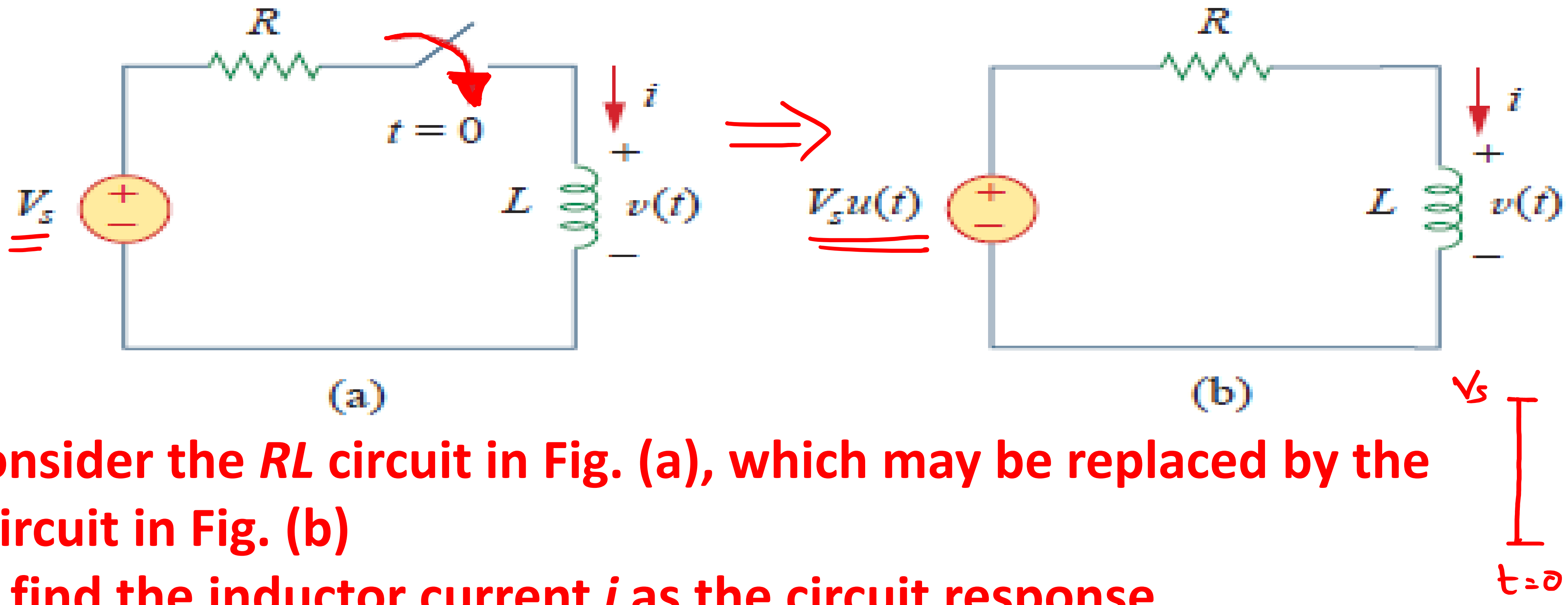
# Step Response of an *RL* Circuit

t=0




- Consider the *RL* circuit in Fig. (a), which may be replaced by the circuit in Fig. (b)
- to find the inductor current *i* as the circuit response

Let the response be the sum of the transient response and the steady-state response

$$\boxed{i = i_t + i_{ss}}$$

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**We know that the transient response is always a decaying exponential,**

$$i_t = Ae^{-t/\tau}, \qquad \tau = \frac{L}{R}$$

$t = \infty$

- **The steady-state response is the value of the current a long time after the switch is closed**
- **Once the transient response dies out, the inductor becomes a short circuit, and the voltage across it is zero. Then**

$$i_{ss} = \frac{V_s}{R}$$

$i = i_t + i_{ss}$

$$\boxed{i = Ae^{-t/\tau} + \frac{V_s}{R}}$$

**Let $I_0$ be the initial current through the inductor, which may come from a source other than $V_s$, Since the current through the inductor cannot change instantaneously,**

$$i(0^+) = i(0^-) = I_0$$

Thus, at $t = 0$,

$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$I_0 = A + \frac{V_s}{R}$$

$$\boxed{A = I_0 - \frac{V_s}{R}}$$

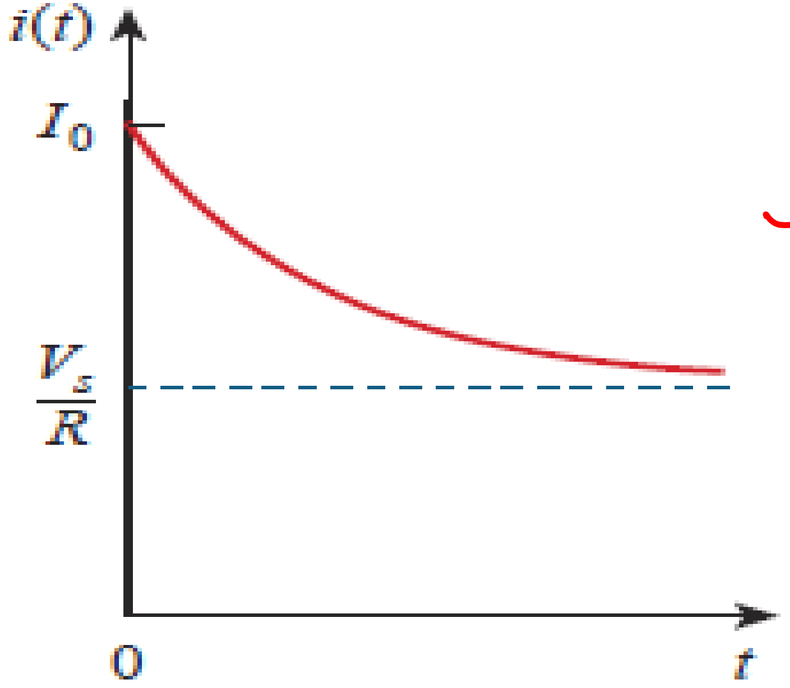
$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}}$$

$t-$ $t=0$ $t+$

$i(0^-) = i(0^+) = I_0$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}}$$




Total response of the $RL$ circuit with initial inductor current $I_0$.

$$\boxed{i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}}$$

Again, if the switching takes place at time $t = t_0$ instead of $t = 0$,

$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau}$$

If $I_0 = 0$, then

$$i(t) = \begin{cases} 0, & t < 0 \\ \frac{V_s}{R}(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

$I_0 = 0$

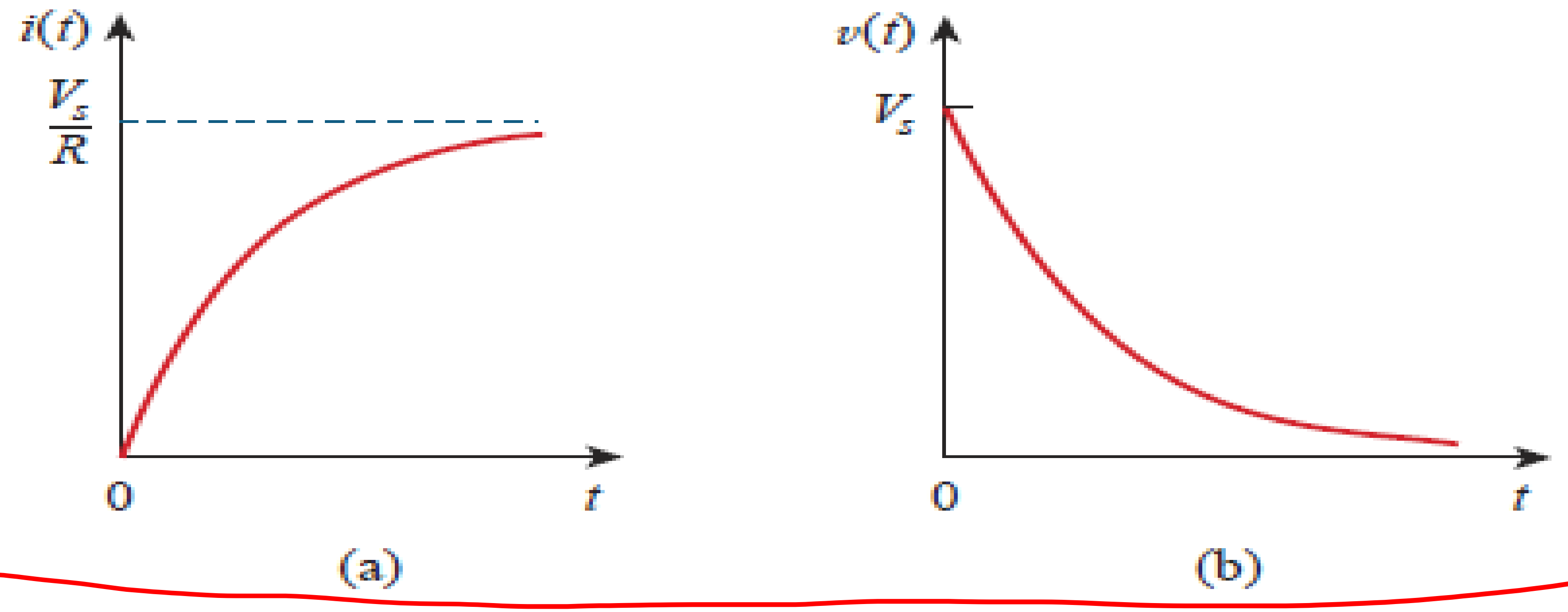
$$\boxed{i(t) = \frac{V_s}{R}(1 - e^{-t/\tau})u(t)}$$

This is the step response of the *RL* circuit with no initial inductor current.

$$v = L\,di/dt.$$

$$v(t) = L\frac{di}{dt} = V_s\frac{L}{\tau R}e^{-t/\tau}, \qquad \tau = \frac{L}{R}, \qquad t > 0$$

$$v(t) = V_s e^{-t/\tau}u(t)$$




**Step responses of an *RL* circuit with no initial inductor current: (a) current response, (b) voltage response**

In a series RL circuit, the application of a direct voltage results a steady state current of 0.632 I in one second, I being the final steady state value of the current. However the current has reached its final value, a sudden short circuit is applied against the source. What would be the value of the current after 1 sec?

$$i = I\left(1 - e^{-t/\tau}\right)$$

$t = \underline{\underline{1 \text{ sec}}}$

$$i = 0.632\, I$$

decay of steadystate current

$$i = I_0 e^{-t/\tau}$$

# Chapter 7 (First order circuit)

1. Source free RC circuit

   Examples 7.1, 7.2

   Practice problem 7.1

2. The source free RL circuit.

   Example 7.3, 7.4

   Practice problem 7.3

3. Singularity functions

   Ex. 7.6, 7.7

4. Step response of RC circuit

   Ex. 7.10 ; 7.11 (Practice problem)

5. Step response of RL circuit

   Ex. 7.12 and 7.13

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√ Second order circuit. (RLC)

→ Second order differential eqn

series & parallel