Singularity Functions

useful for circuit analysis
describe circuit switching operations

Singularity functions are functions that either are discontinuous or have discontinuous derivatives.

Step input

The three most widely used singularity functions in circuit analysis are

- **The unit step function**
- **The unit impulse function**
- **The unit ramp function**

The unit step function

The unit step function $u(t)$ is 0 for negative values of t and 1 for positive values of t.

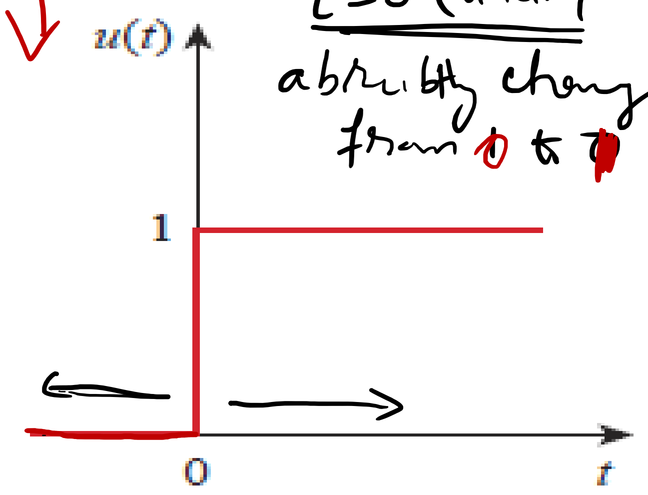
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

$t = 0$ (undefined)

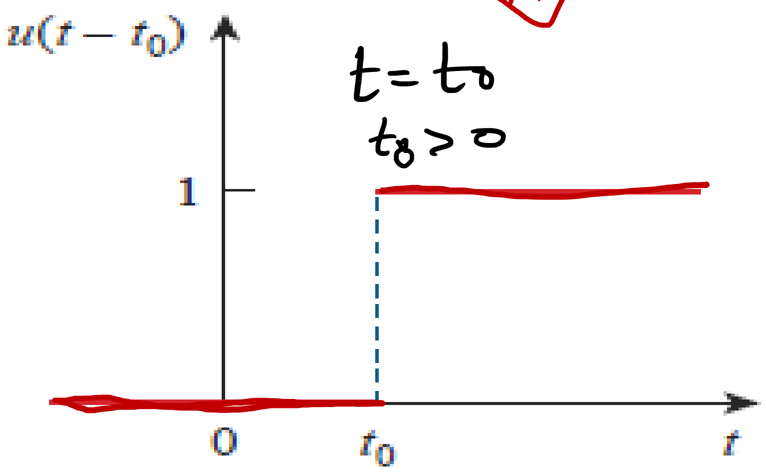
abruptly changes from 0 to 1



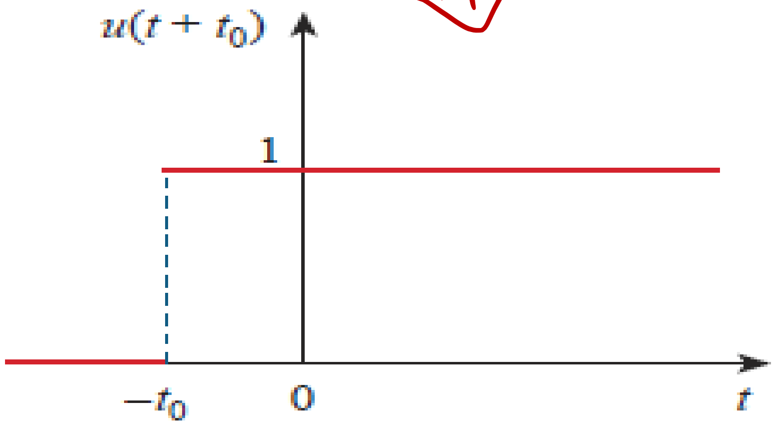
The unit step function.

$t = t_0$

$t_0 > 0$



The unit step function delayed by t_0.



the unit step advanced by t_0.

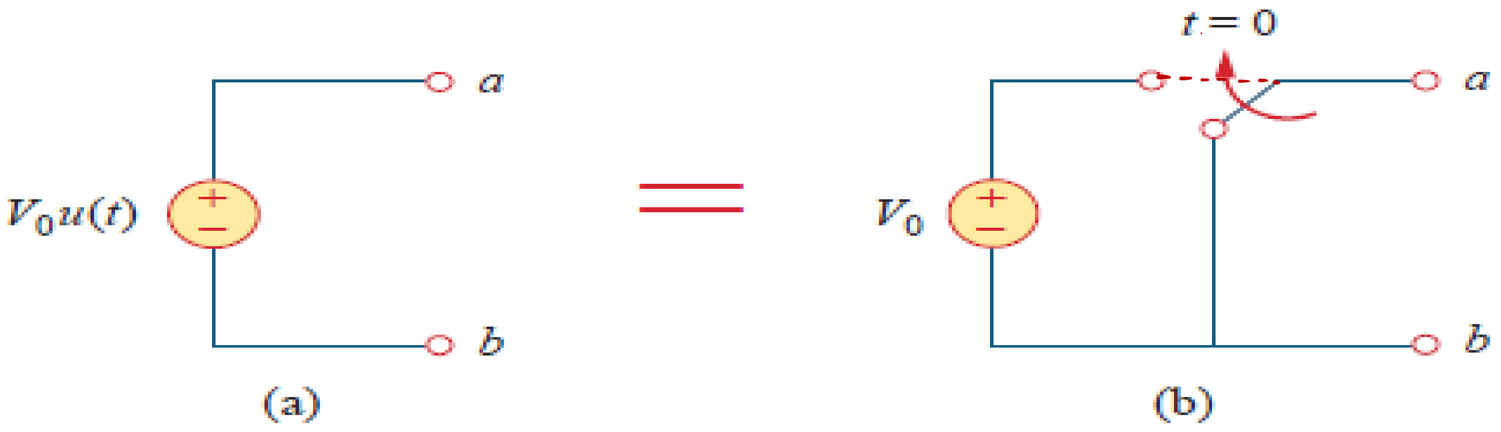
We use the step function to represent an abrupt change in voltage or current, like the changes that occur in the circuits of control systems and digital computers. For example, the voltage

$$\boxed{v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases}}$$

may be expressed in terms of the unit step function as

$$v(t) = V_0 u(t - t_0)$$

If we let $t_0 = 0$, then $v(t)$ is simply the step voltage $V_0 u(t)$

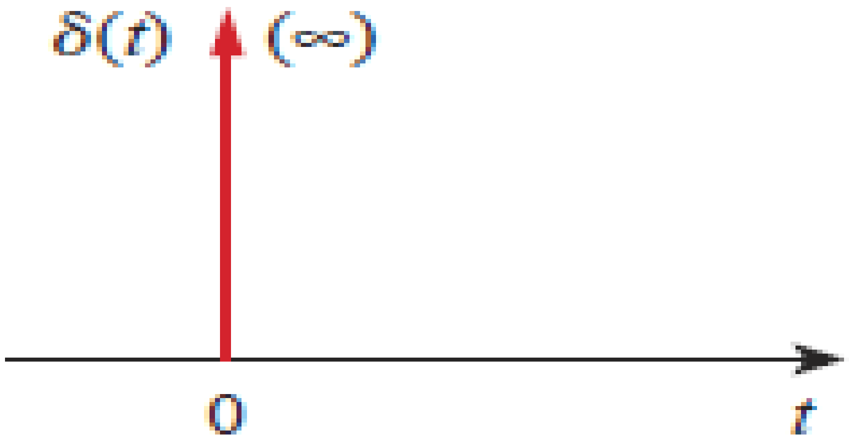


(a) Voltage source of $V_0u(t)$, (b) its equivalent circuit

The unit impulse function

The derivative of the unit step function $u(t)$ is the *unit impulse function* $\delta(t)$, which we write as

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$$



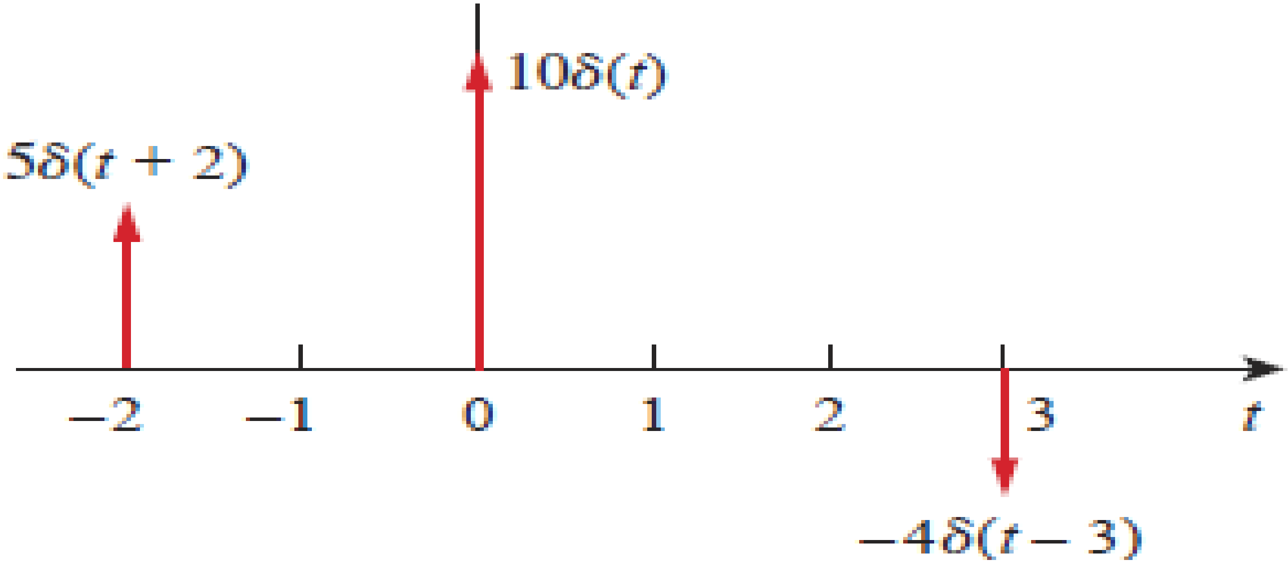
The unit impulse function.

The unit impulse function $\delta(t)$ is zero everywhere except at $t = 0$, where it is undefined.

The unit impulse function—also known as the *delta* function-

The unit impulse may be regarded as an applied or resulting shock. It may be visualized as a very short duration pulse of unit area. This may be expressed mathematically as

$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$



Three impulse functions.

$$\int_a^b f(t)\delta(t - t_0)\,dt$$

where $a < t_0 < b$. Since $\delta(t - t_0) = 0$ except at $t = t_0$, the integrand is zero except at t_0. Thus,

$$\int_a^b f(t)\delta(t - t_0)\,dt = \int_a^b f(t_0)\delta(t - t_0)\,dt$$

$$= f(t_0)\int_a^b \delta(t - t_0)\,dt = f(t_0)$$

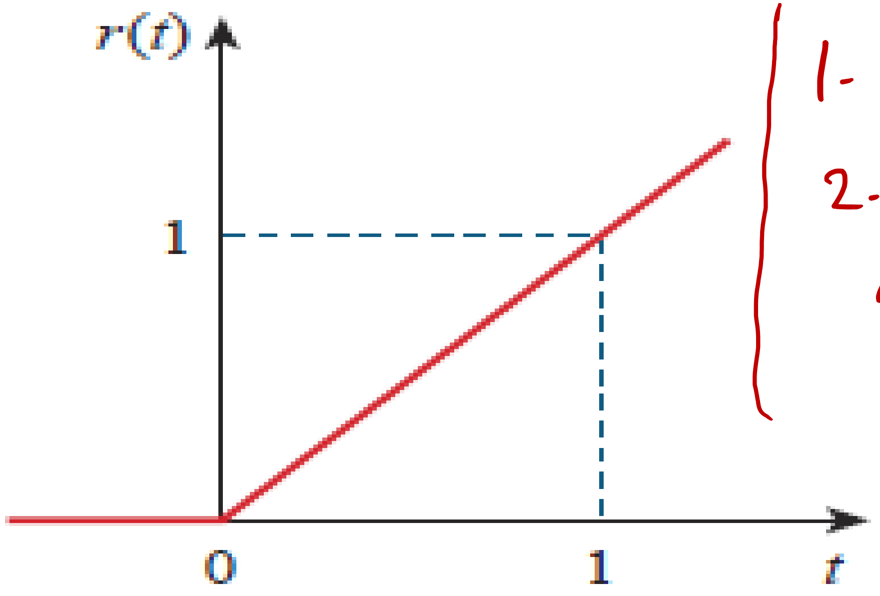
$$\boxed{\int_a^b f(t)\delta(t - t_0)\,dt = f(t_0)}$$

Integrating the unit step function $u(t)$ results in the *unit ramp function* $r(t)$; we write

$$r(t) = \int_{-\infty}^{t} u(t)dt = tu(t)$$

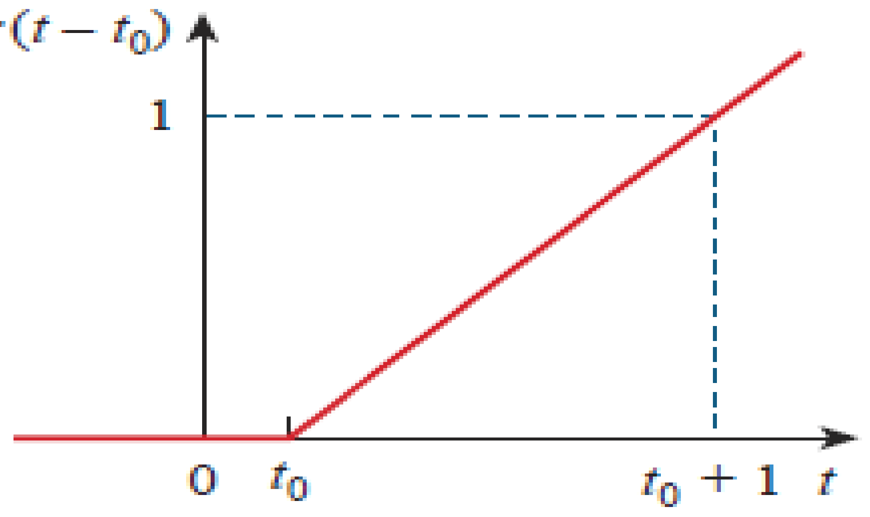
$$\boxed{r(t) = \begin{cases} 0, & t \le 0 \\ t, & t \ge 0 \end{cases}}$$

The **unit ramp function** is zero for negative values of t and has a unit slope for positive values of t.



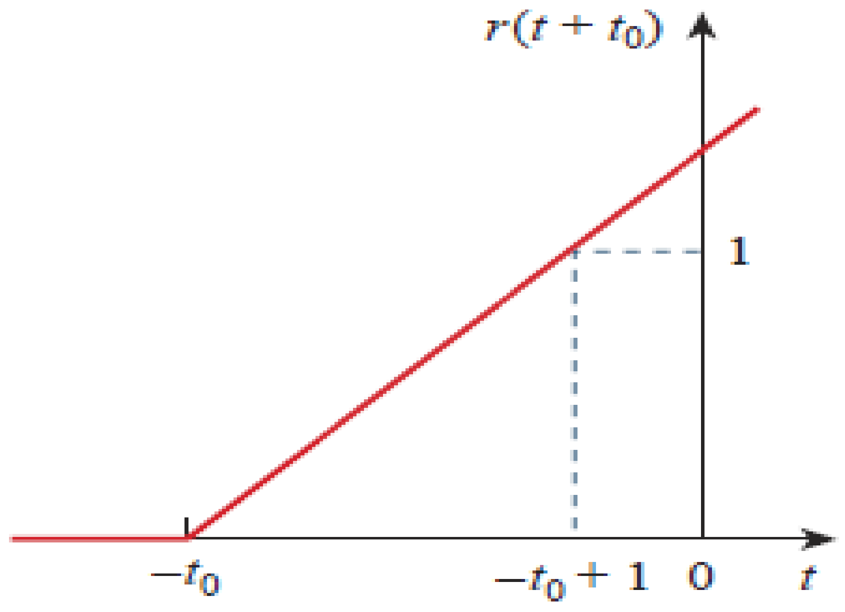
1- Unit impulse function
2. Unit ramp function
Eg. 7.6, 7.7, 7.8
7.9

The unit ramp function.



$$r(t-t_0)=\begin{cases}0, & t \le t_0\\ t-t_0, & t \ge t_0\end{cases}$$

(a) delayed by t_0,

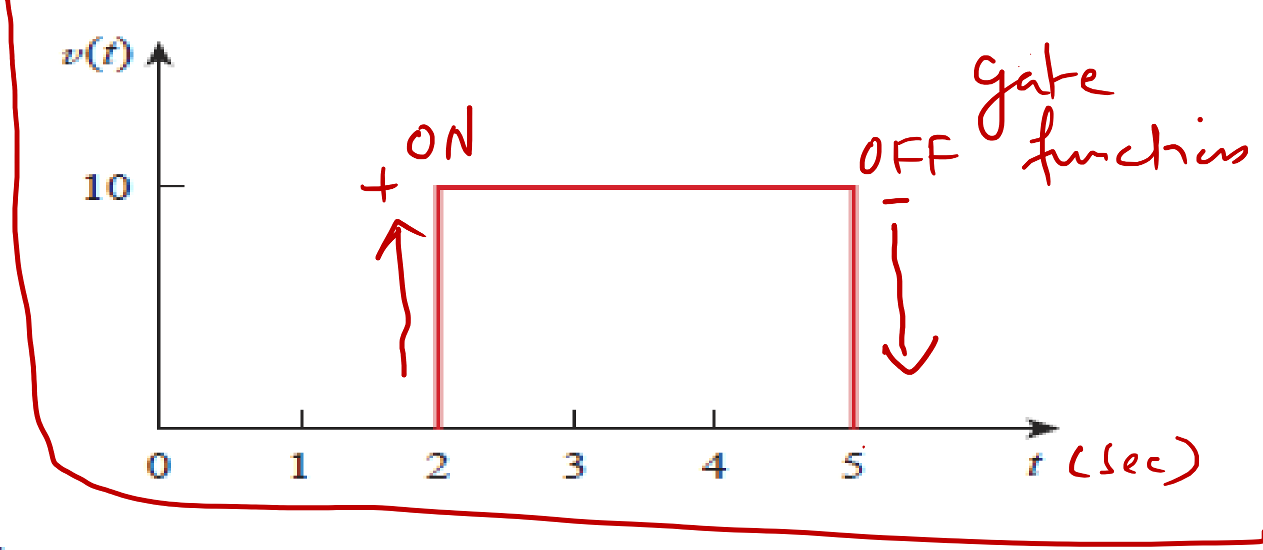


$$r(t+t_0)=\begin{cases}0, & t \le -t_0\\ t+t_0, & t \ge -t_0\end{cases}$$

(b) advanced by t_0.

Example

Express the voltage pulse in the fig in terms of the unit step. Calculate
its derivative and sketch it.

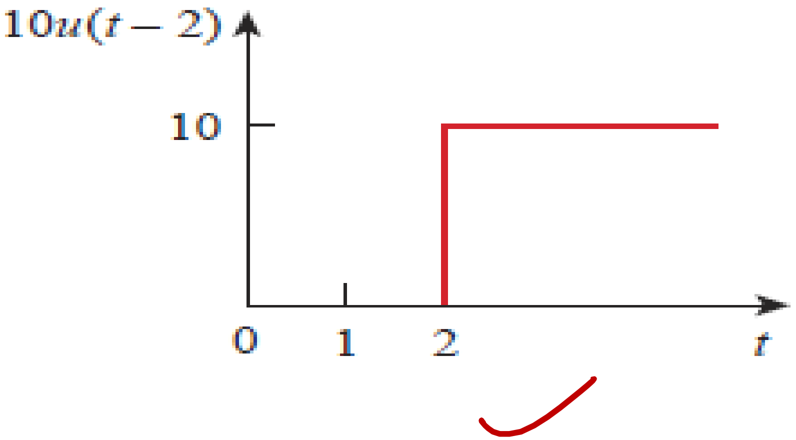


$t=2$ $t=5$

$$v(t) = 10u(t-2) - 10u(t-5)$$
$$= 10[u(t-2) - u(t-5)]$$

Taking the derivative of this gives

$$\frac{dv}{dt} = 10[\delta(t-2) - \delta(t-5)]$$



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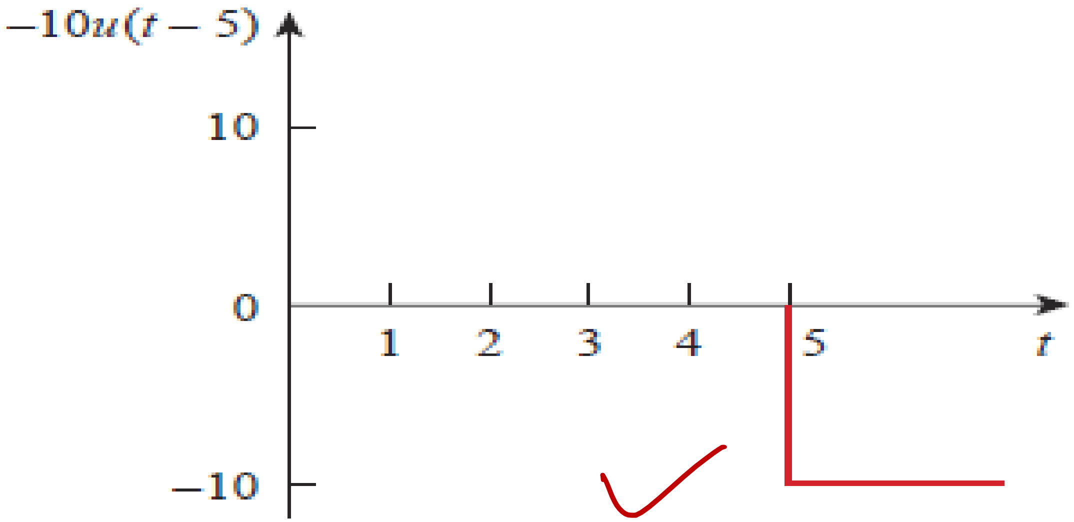
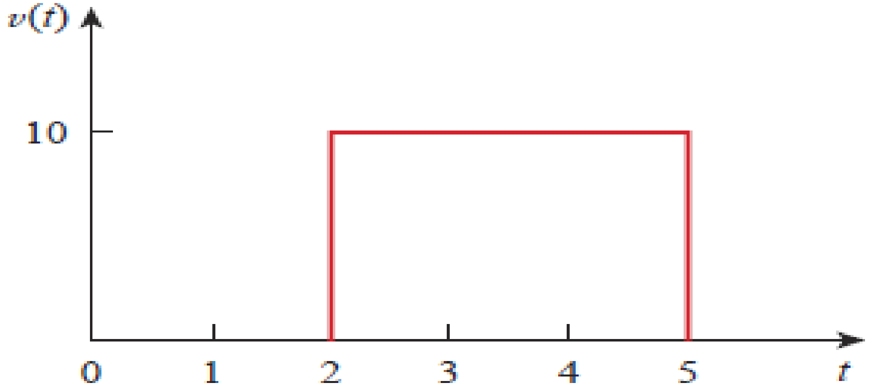
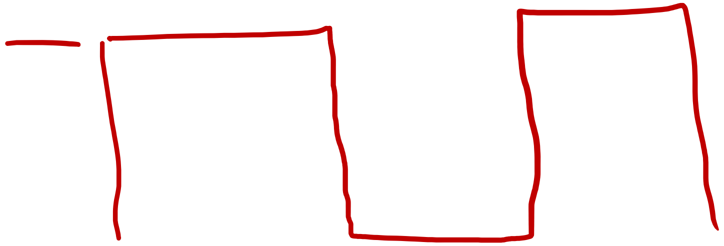
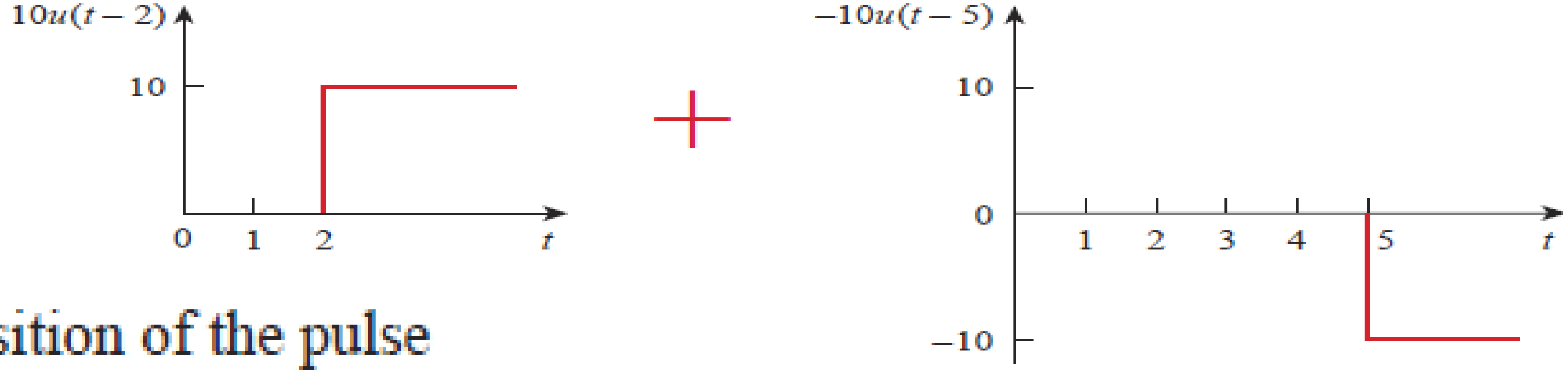
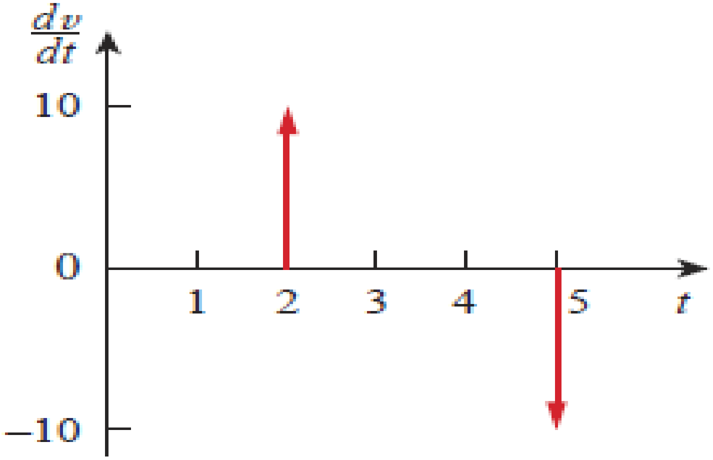








(a) Decomposition of the pulse



(b) (b) derivative of the pulse