Nodal Analysis

Kirchhoff's current law (KCL)

Choosing node voltages instead of element voltages as circuit variable is convenient and reduce the number of equations must solve simultaneously

Mesh Analysis

Kirchhoff's voltage law (KVL)

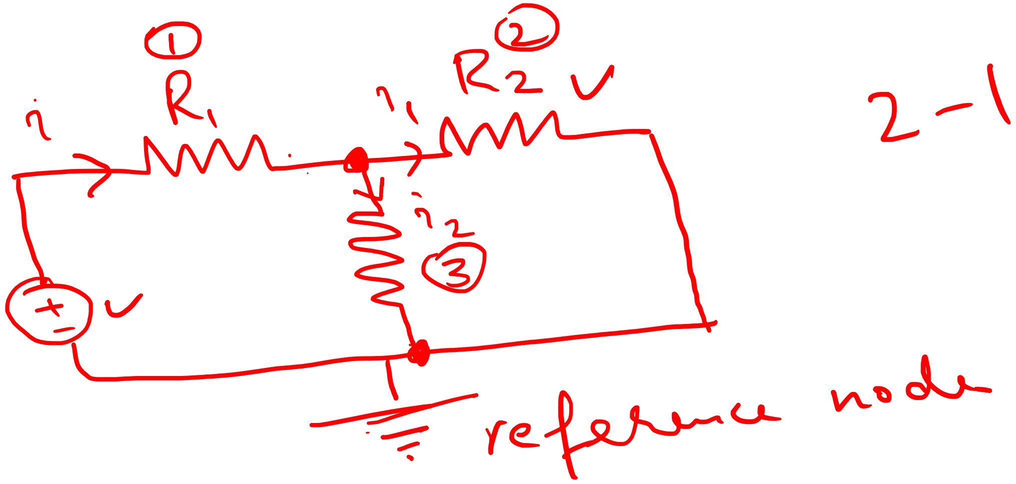
A node is a common point in the circuit where two or more elements are connected

- Simple node-two element- No current division
- Principal node- more than two elements- current division takes place
- Nodal analysis applicable for both planar and non-planar networks
- No of equations required to solve an electrical circuit using nodal analysis

$\boxed{E=N-1}$

N → node

N − 1 eqns



2 − 1 = 1

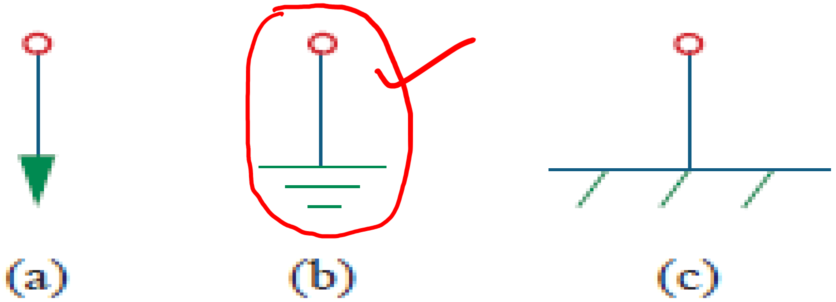
In *nodal analysis*, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

The first step in nodal analysis is selecting a node as the *reference* or *datum node*.

The reference node is commonly called the *ground* since it is assumed to have zero potential.

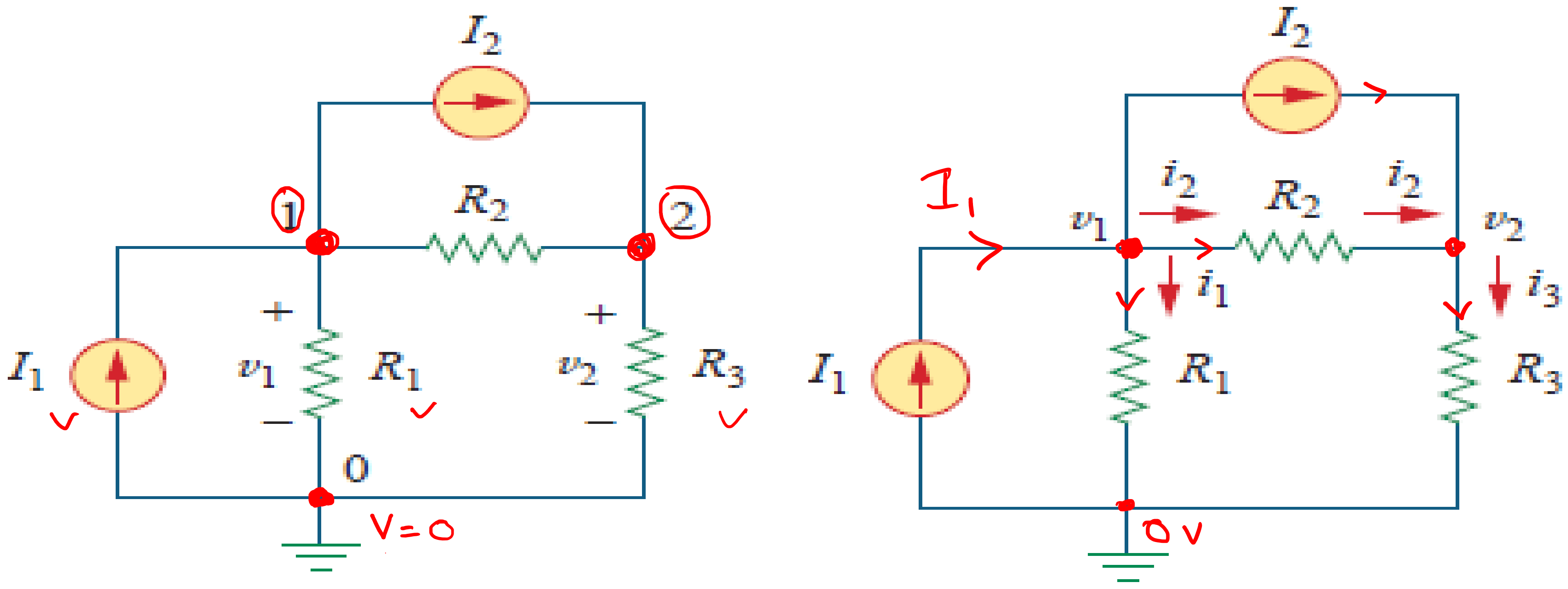


eqn = N − 1

Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis ground.

The number of nonreference nodes is equal to the number of independent equations that we will derive.

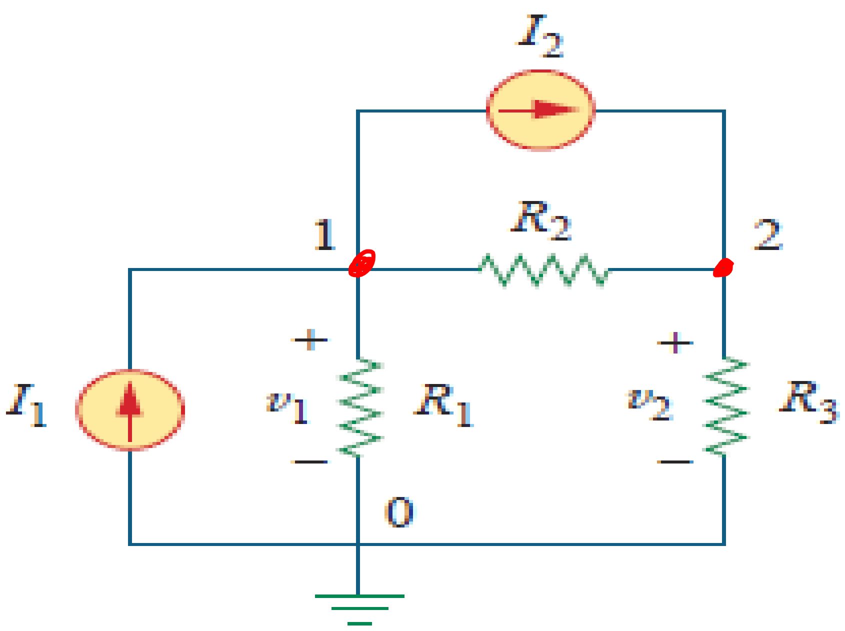
1. Select a node as the reference node. Assign voltages v_1, $v_2, \ldots, v_{n-1}$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.

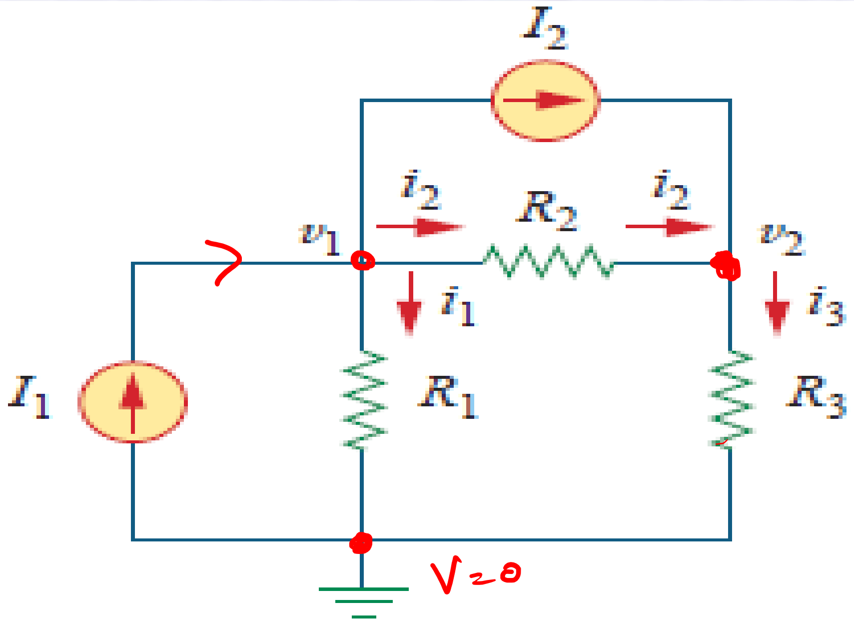


(a) (b)

Node 0 is the reference node ($v = 0$), while nodes 1 and 2 are assigned voltages v_1 and v_2, respectively. Keep in mind that the node voltages are defined with respect to the reference node.

2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.



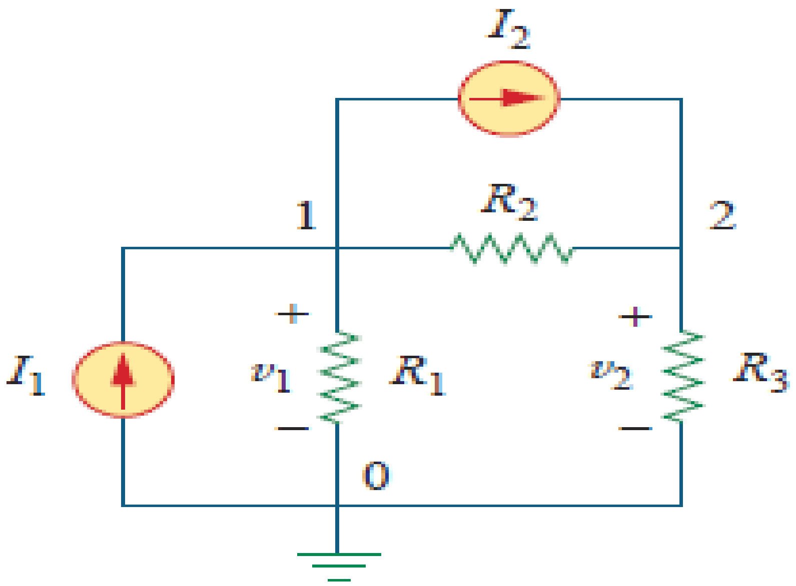


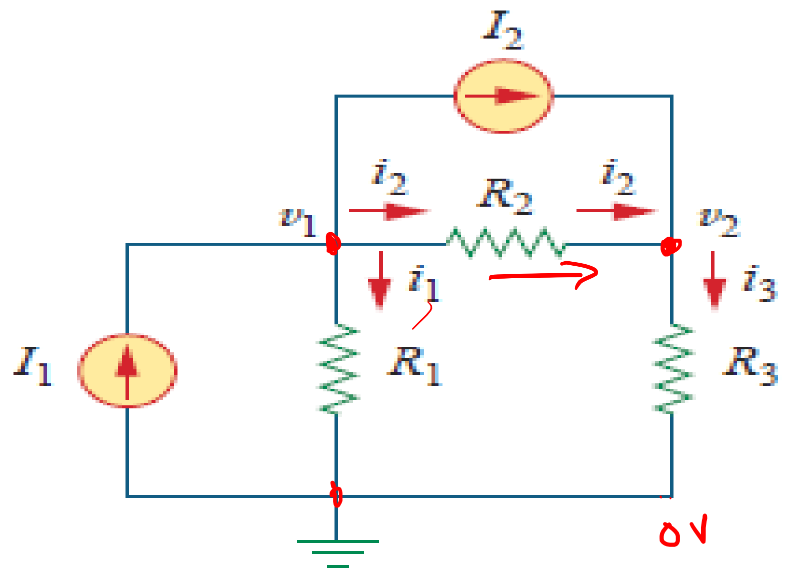
At node 1, applying KCL gives

$$\boxed{I_1 = I_2 + i_1 + i_2} \rightarrow ①$$

At node 2,

$$\boxed{I_2 + i_2 = i_3} \rightarrow ②$$





Current flows from a higher potential to a lower potential in a resistor.

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

We can express this principle as

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad \boxed{i_1 = G_1 v_1}$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad \boxed{i_2 = G_2(v_1 - v_2)}$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad \boxed{i_3 = G_3 v_2}$$

3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \longrightarrow 3$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} \longrightarrow 4$$

$$\left.\begin{aligned} I_1 &= I_2 + G_1 v_1 + G_2(v_1 - v_2) \\ I_2 + G_2(v_1 - v_2) &= G_3 v_2 \end{aligned}\right\} \begin{matrix} \longrightarrow 5 \\ 6 \end{matrix}$$

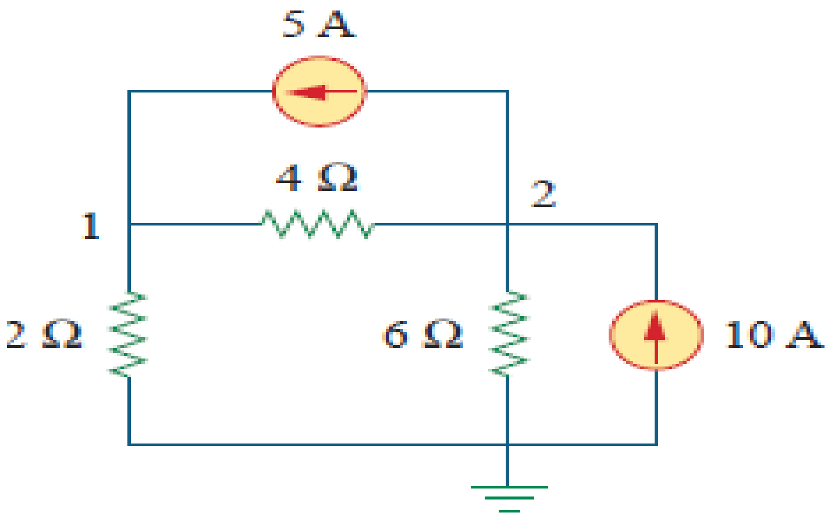
A

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

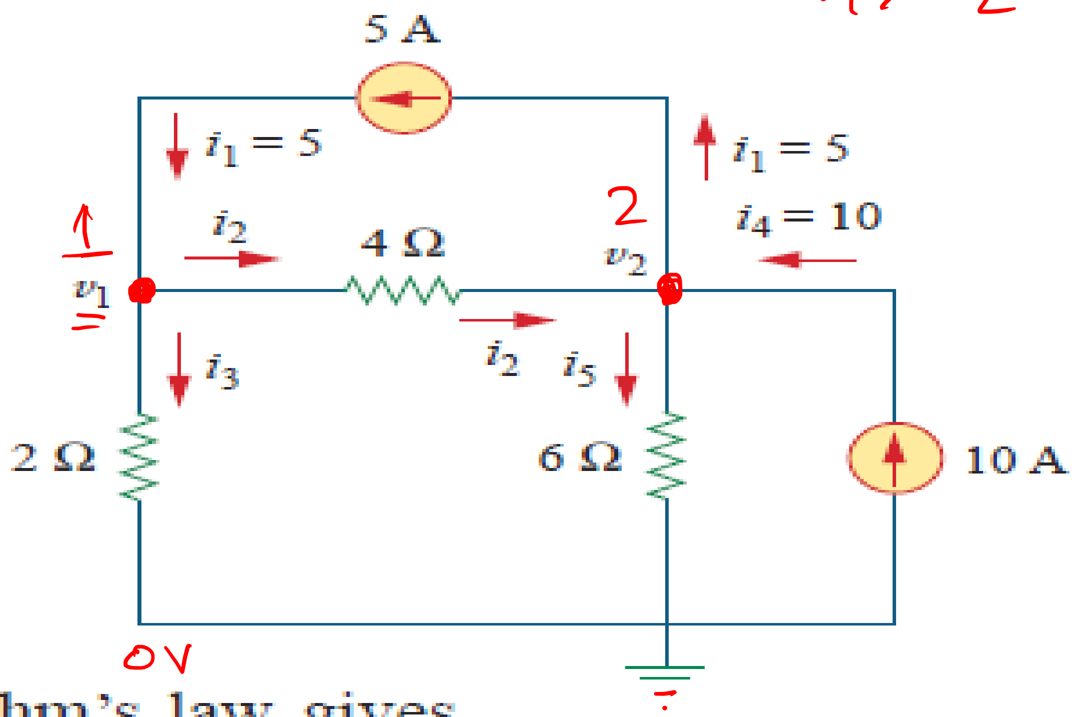
Example

Calculate the node voltages in the circuit shown in Fig.

$v_1 > v_2$



(a)



At node 1, applying KCL and Ohm's law gives

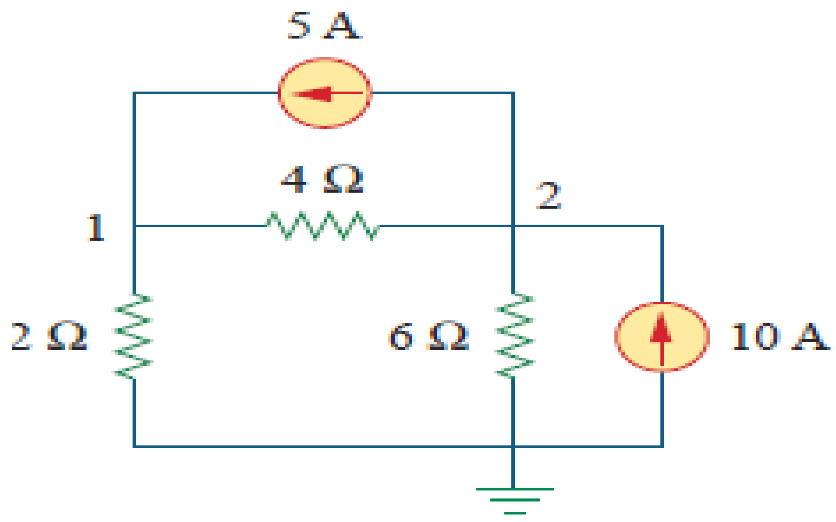
$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4, we obtain

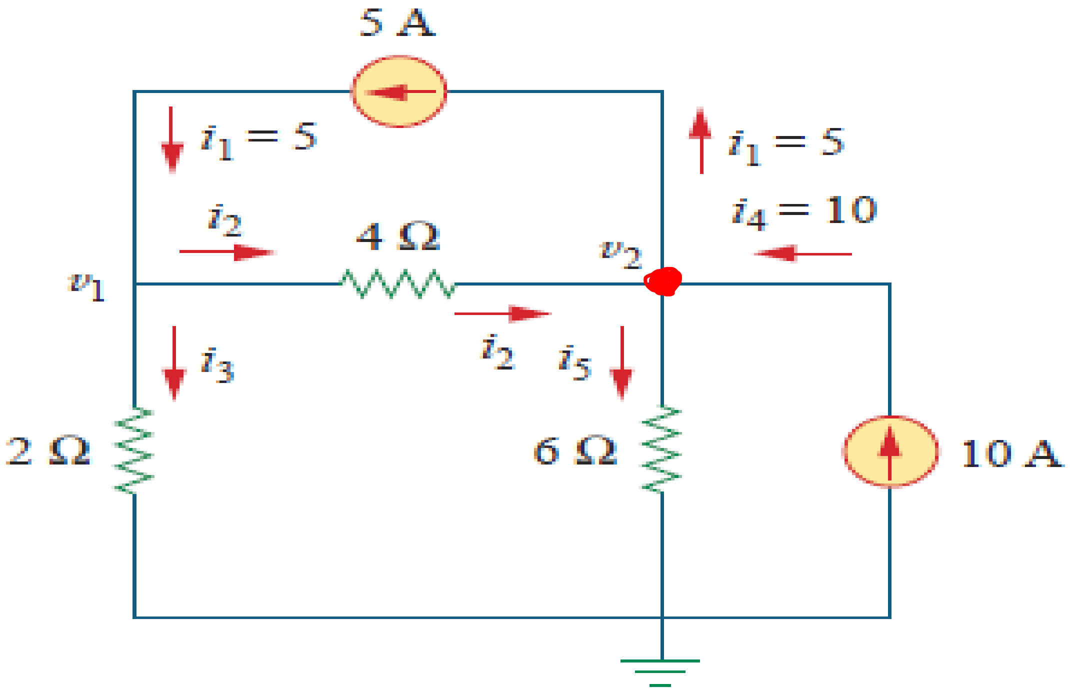
$$20 = v_1 - v_2 + 2v_1$$

or

$$\boxed{3v_1 - v_2 = 20} \rightarrow 1$$



(a)



At node 2, we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \text{ —— } 2 \qquad (3$$

METHOD 1 Using the elimination technique, we add Eqs.

$$4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V}$$

METHOD 2 To use Cramer's rule, ,

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain v_1 and v_2 as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333\text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20\text{ V}$$

giving us the same result as did the elimination method.

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5\text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6668\text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666\text{ A}$$

$$i_4 = 10\text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333\text{ A}$$

The fact that i_2 is negative shows that the current flows in the direction opposite to the one assumed.

Explicit formulas for small systems [edit]

Consider the linear system

$$\begin{cases} a_1x + b_1y & = c_1 \\ a_2x + b_2y & = c_2 \end{cases}$$

which in matrix format is

$$\boxed{\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.}$$

Assume $a_1b_2 - b_1a_2$ nonzero. Then, with help of determinants, x and y can be found with Cramer's rule as

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - b_1c_2}{a_1b_2 - b_1a_2}, \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - c_1a_2}{a_1b_2 - b_1a_2}.$$

The rules for 3 × 3 matrices are similar. Given

$$\begin{cases} a_1 x + b_1 y + c_1 z &= d_1 \\ a_2 x + b_2 y + c_2 z &= d_2 \\ a_3 x + b_3 y + c_3 z &= d_3 \end{cases}$$

which in matrix format is

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}.$$

Then the values of x, y and z can be found as follows:

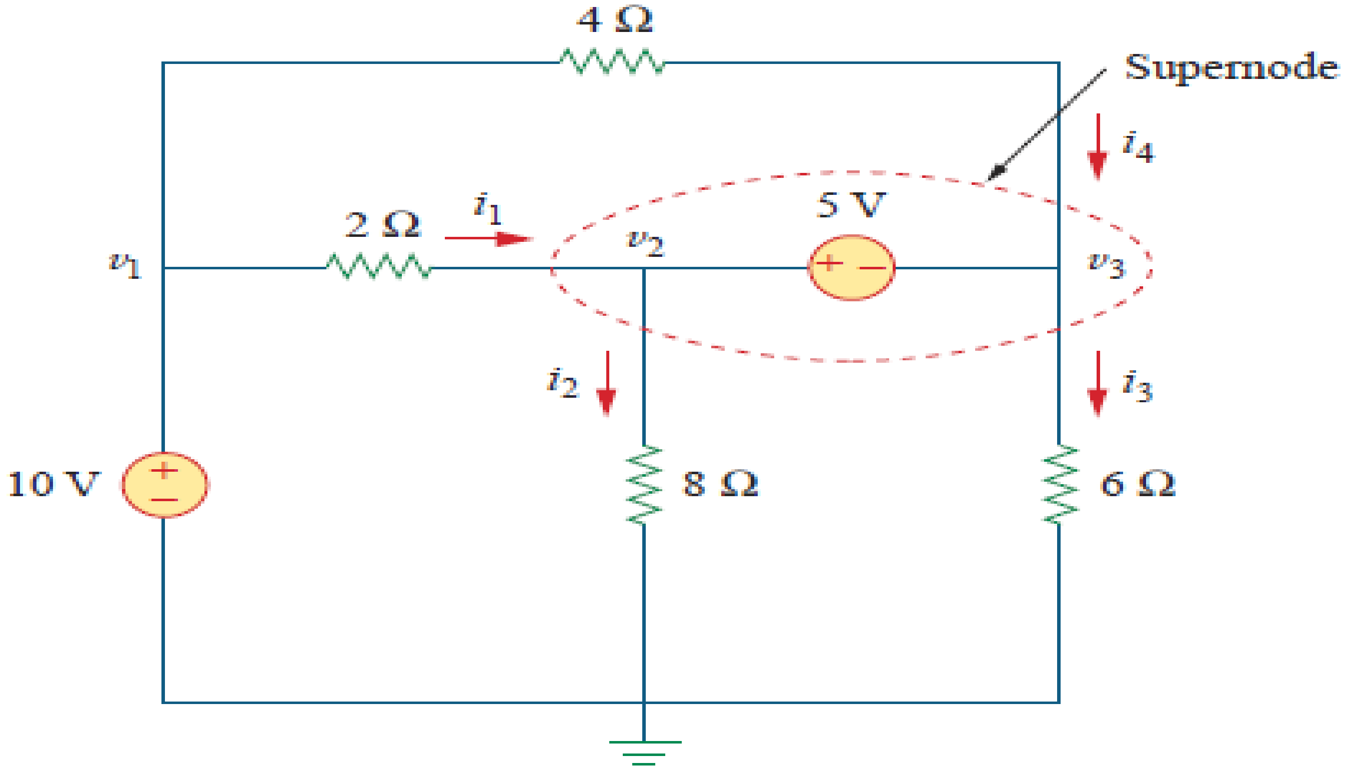
$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \text{ and } z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}.$$

METHOD 3 We now use *MATLAB* to solve the matrix. Equation (3.2.6) can be written as

$$\mathbf{AV} = \mathbf{B} \qquad \Longrightarrow \qquad \boxed{\mathbf{V} = \mathbf{A}^{-1}\mathbf{B}}$$

where $\mathbf{A}$ is the 3 by 3 square matrix, $\mathbf{B}$ is the column vector, and $\mathbf{V}$ is a column vector comprised of v_1, v_2, and v_3 that we want to determine. We use *MATLAB* to determine $\mathbf{V}$ as follows:

Nodal Analysis with Voltage Sources

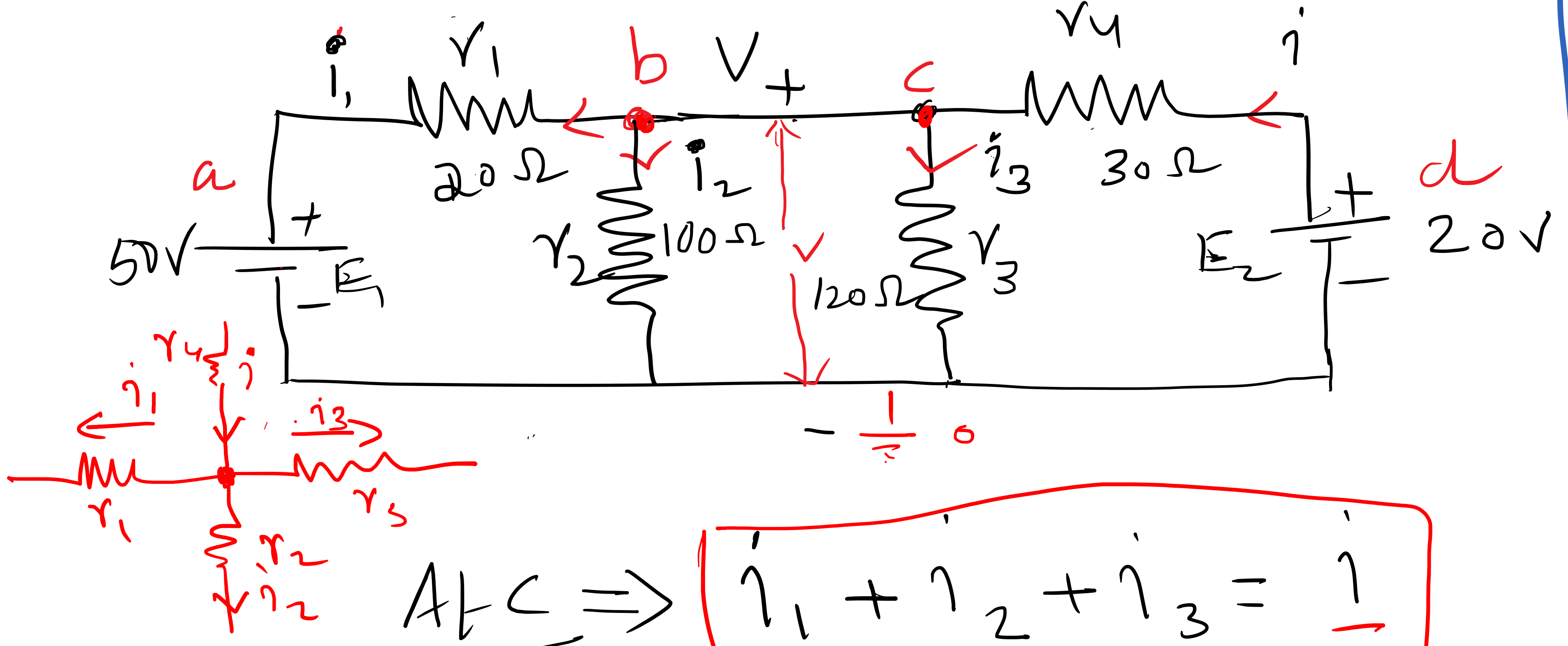


DA 1

Q 1

A **supernode** is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

Develop the nodal eqns for the circuit



At c $\Rightarrow$ $\boxed{i_1 + i_2 + i_3 = i}$

$$\frac{V-50}{r_1} + \frac{V}{r_2} + \frac{V}{r_3} = \frac{20-V}{r_4}$$

$V = 31.18\ V$

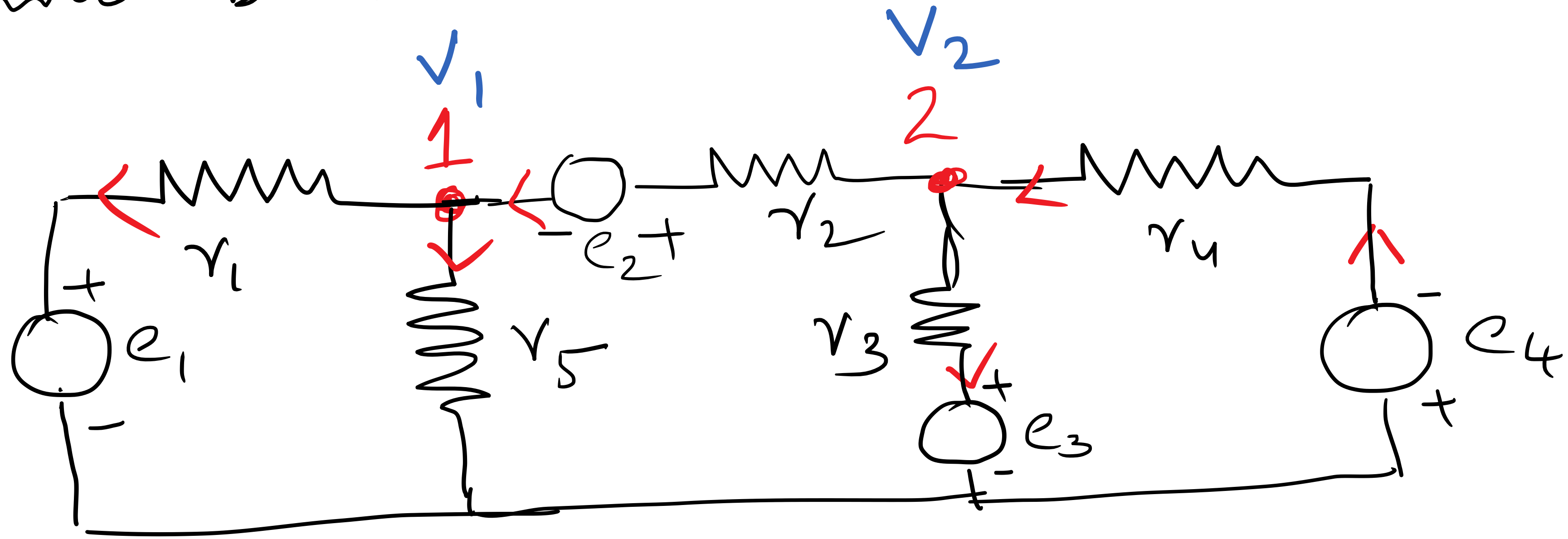
$i_1 = \frac{V-50}{r_1}$

$i_2 = \frac{V}{r_2}$

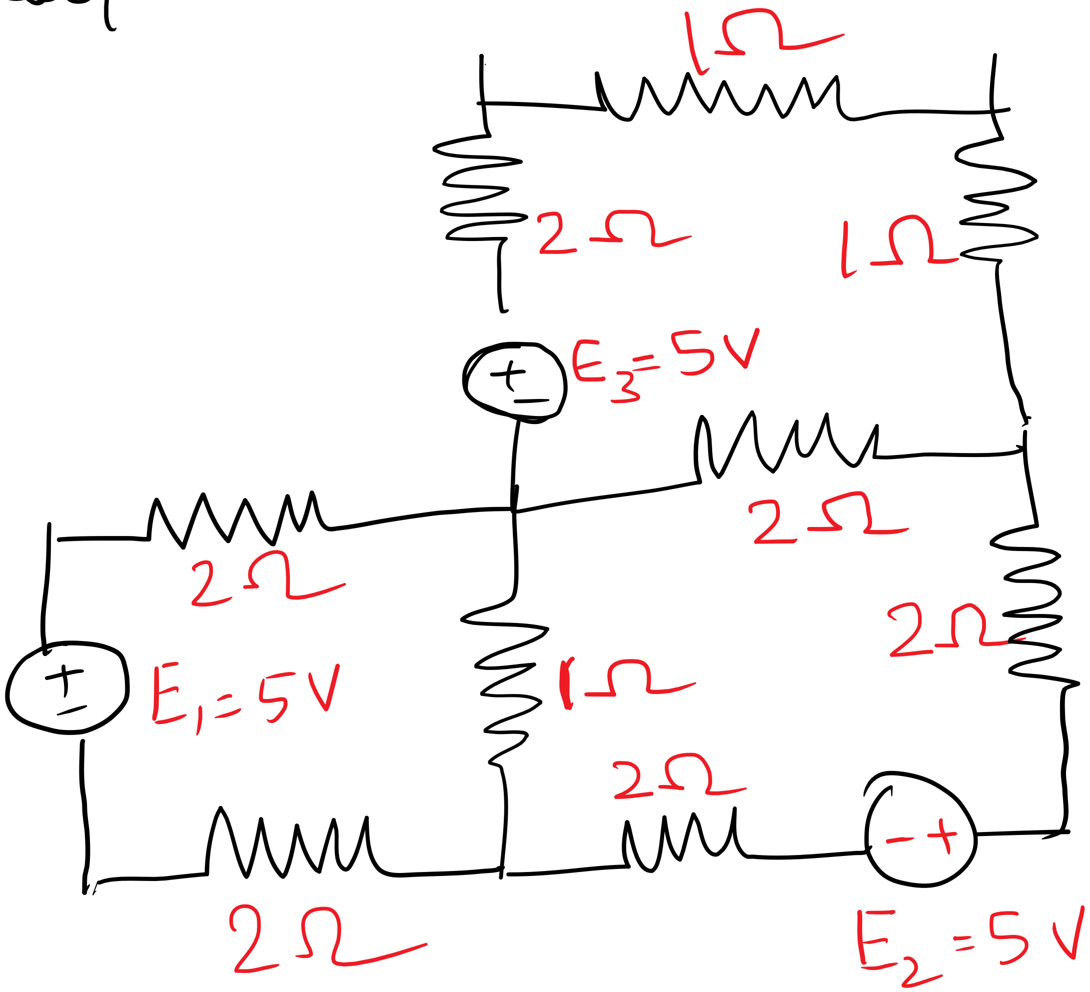
$i_3 = \frac{V}{r_3}$

$i = \frac{20-V}{r_4}$

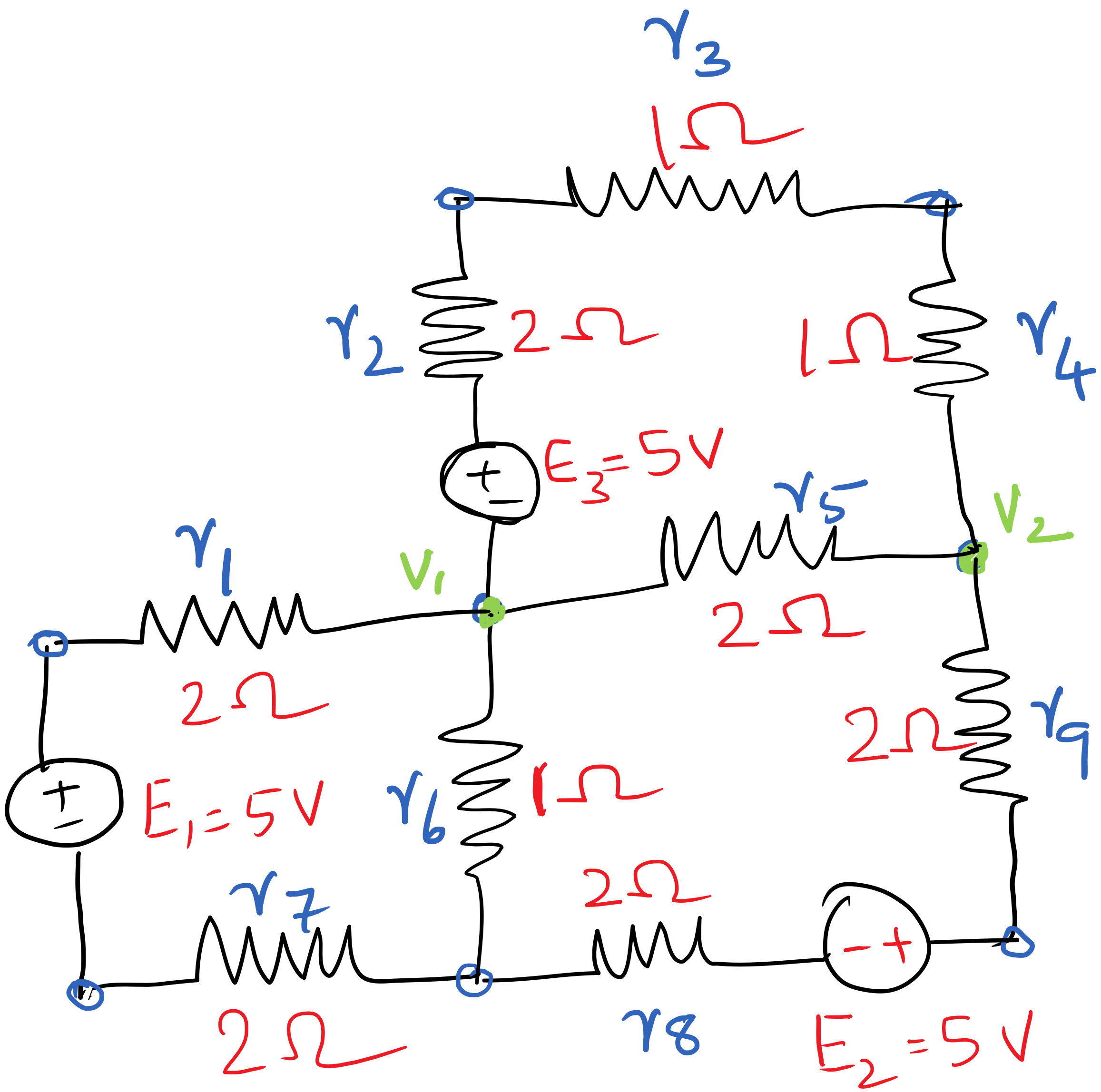
Develop nodal equations in the circuit shown below



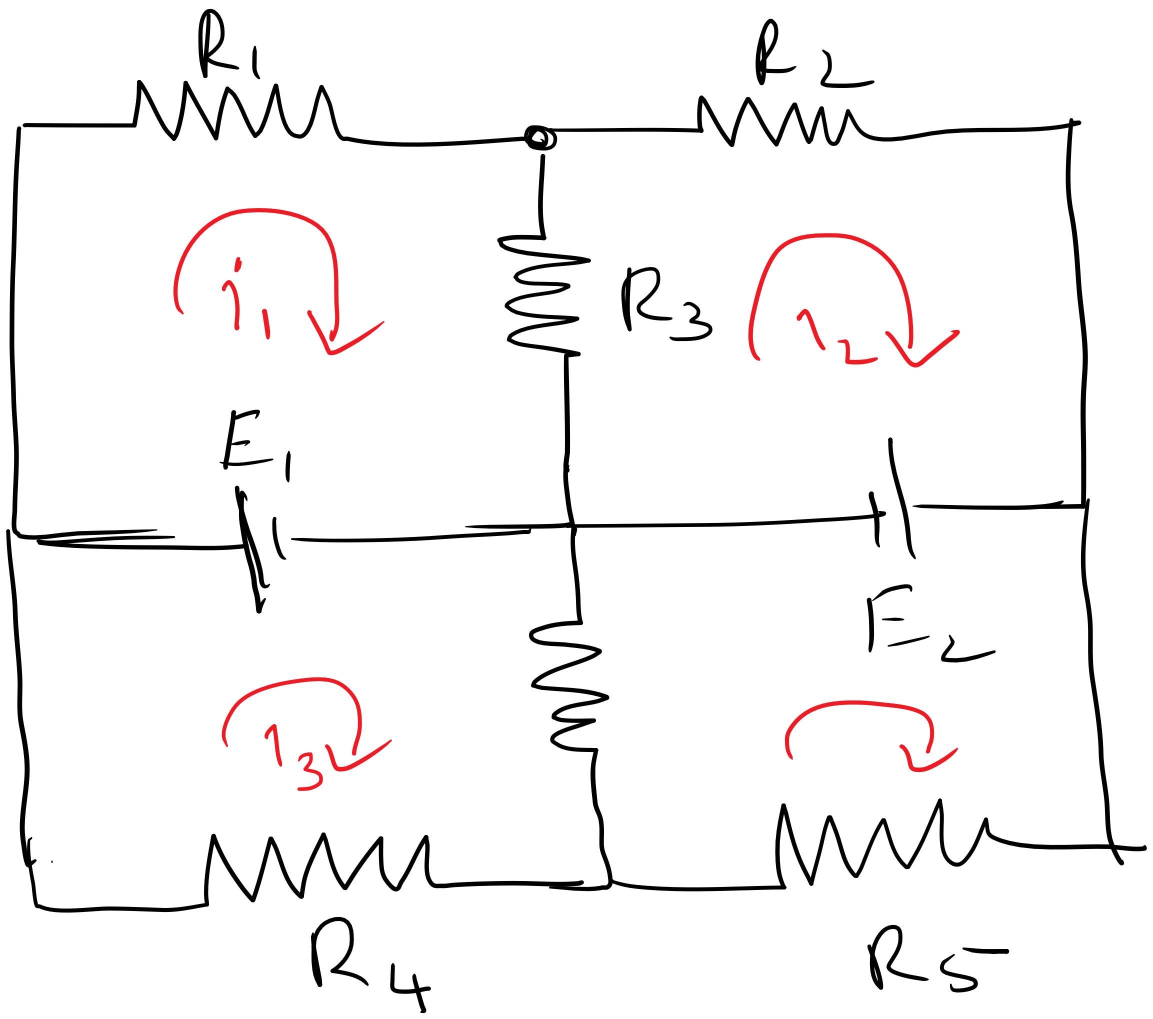
Using nodal method, find the battery currents in the circuit



Node 1



r3
1Ω
r2
2Ω
1Ω
r4
E3=5V
r5
V2
r1
V1
2Ω
2Ω
2Ω
r9
E1=5V
r6
1Ω
r7
2Ω
2Ω
r8
E2=5V


R1
R2
R3
i1
i2
E1
E2
i3
R4
R5