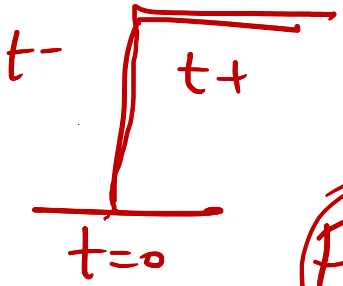
Step Response of an *RC* Circuit

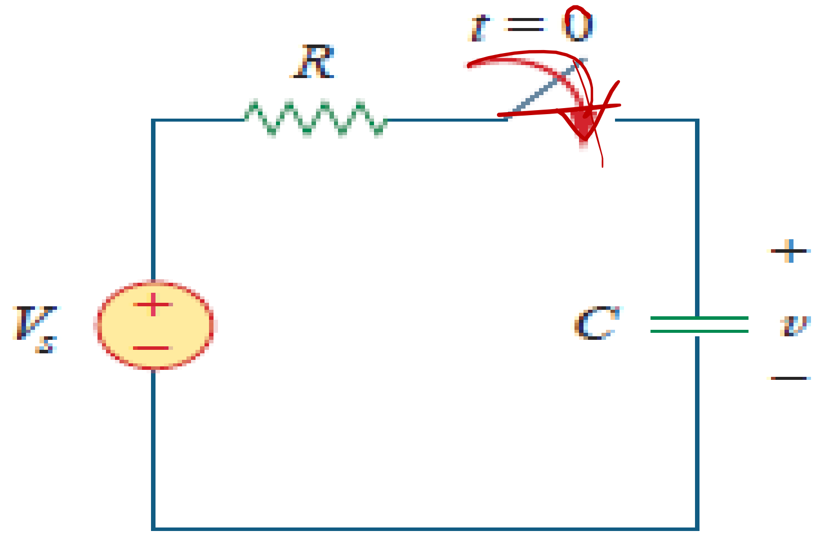
- DC source is suddenly introduced
- Voltage /current will be a step function
- The response is Step response

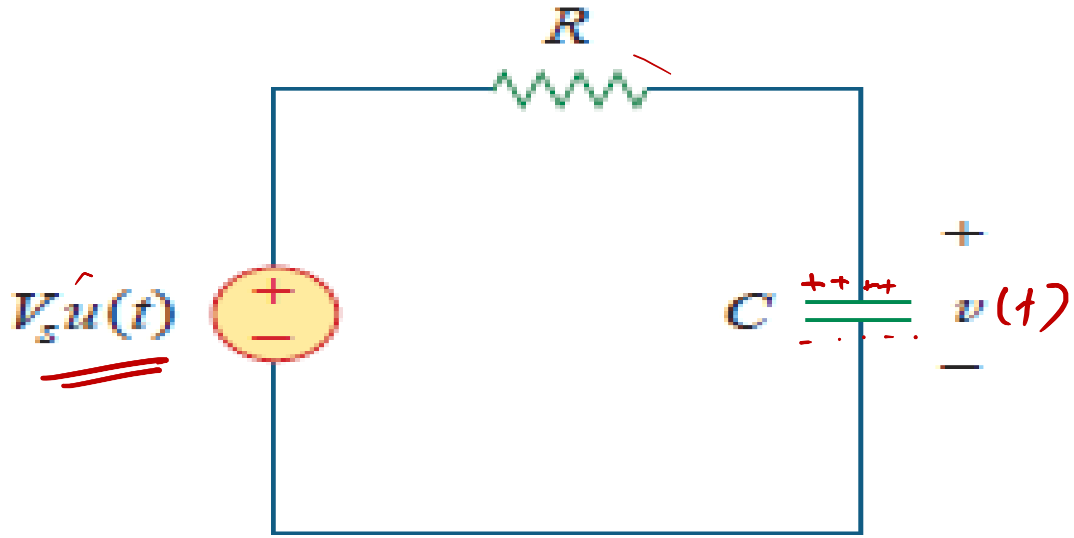


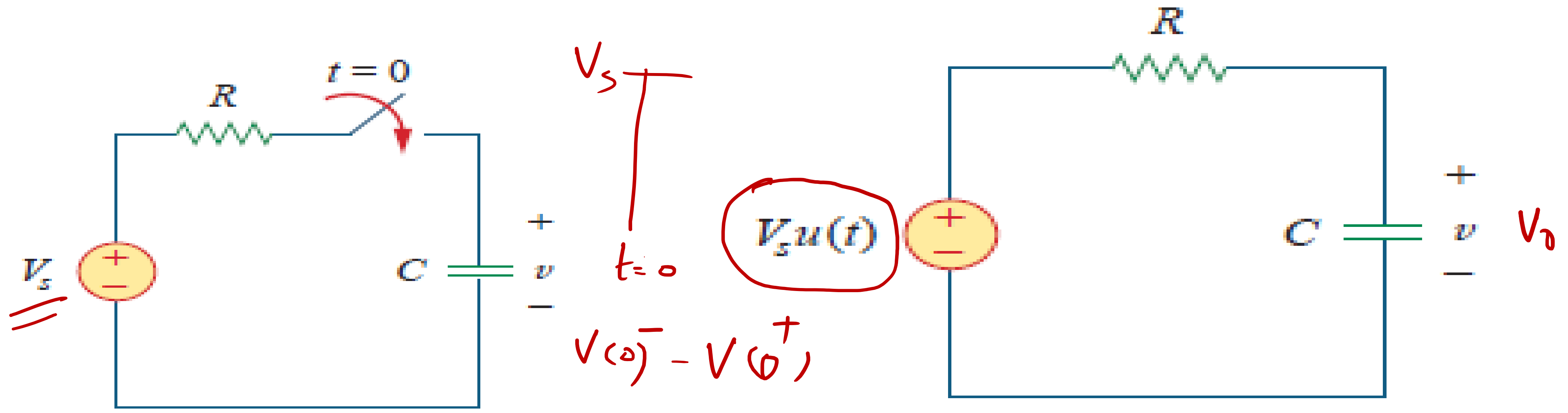
Forced + Natural

The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.

10







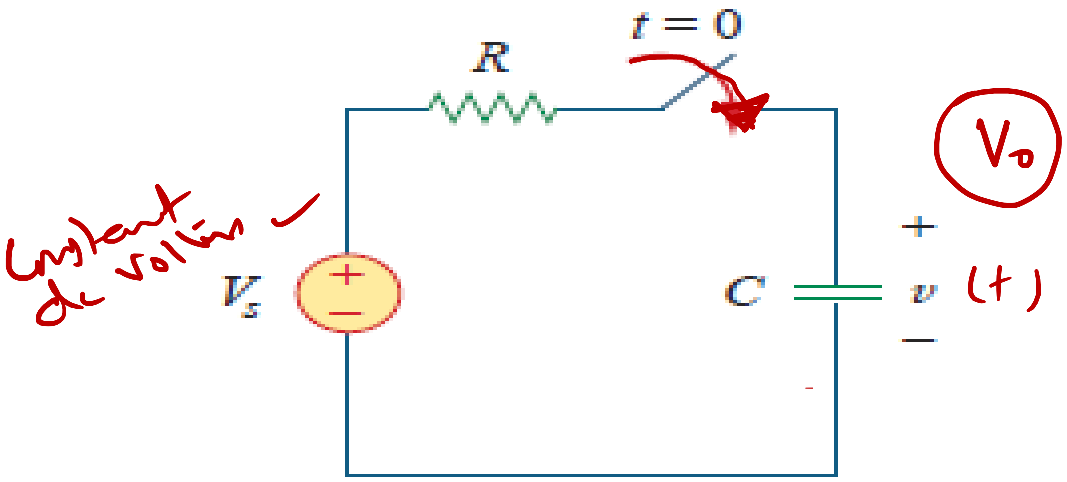
- Let initial voltage on the capacitor, V_0 (although this is not necessary for the step response.
- Since the voltage of a capacitor cannot change instantaneously

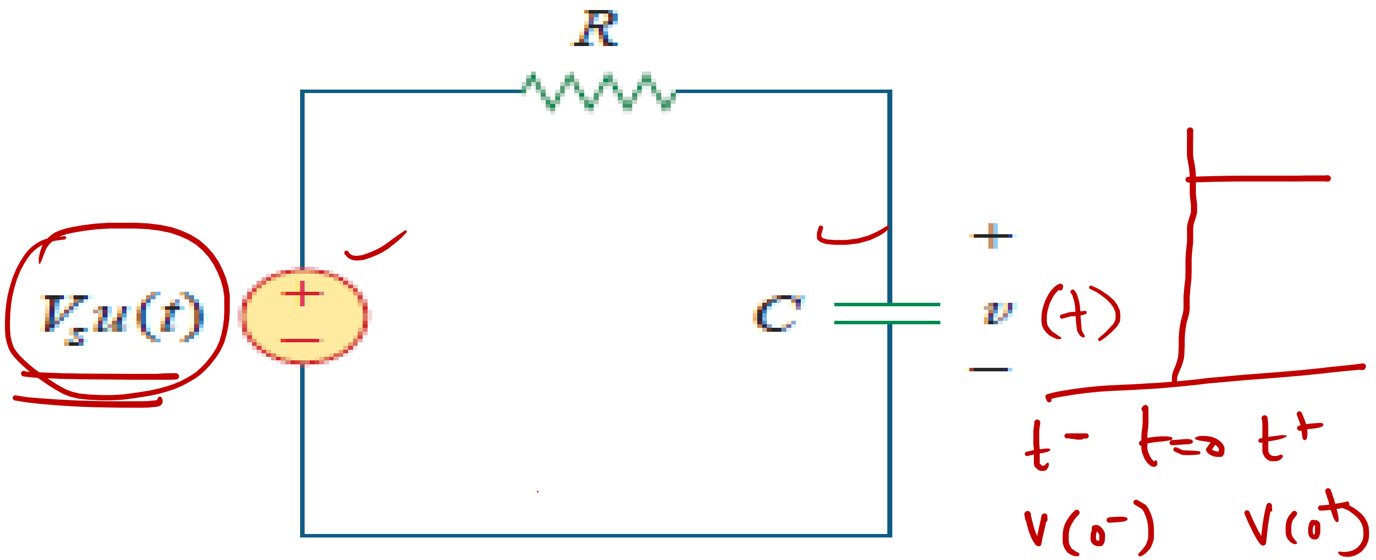
$$\boxed{v(0^-) = v(0^+) = V_0}$$

Initial voltage

$v(0^-)$ is the voltage across the capacitor just before switching

$v(0^+)$ is its voltage immediately after switching





$$C\frac{dv}{dt} + \frac{v - V_s u(t)}{R} = 0 \quad \text{or} \quad \frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}u(t)$$

where v is the voltage across the capacitor. For $t > 0$,

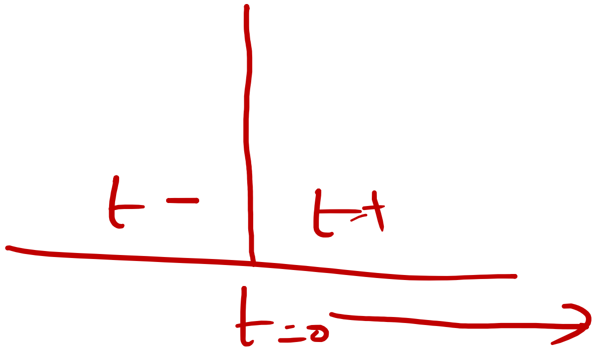
$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}$$

$$\frac{dv}{dt} = -\frac{v - V_s}{RC} \quad \text{or} \quad \boxed{\frac{dv}{v - V_s} = -\frac{dt}{RC}}$$

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0$$

V(s) u(t)



$$\ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

$$\frac{v - V_s}{V_0 - V_s} = e^{-t/\tau}, \qquad \tau = RC$$

V_0

$t > 0$

$$v - V_s = (V_0 - V_s)e^{-t/\tau}$$

$t+$

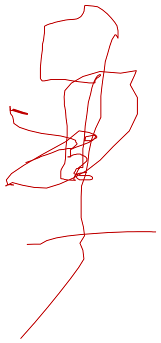
$$\boxed{v(t) = V_s + (V_0 - V_s)e^{-t/\tau},} \qquad t > 0$$

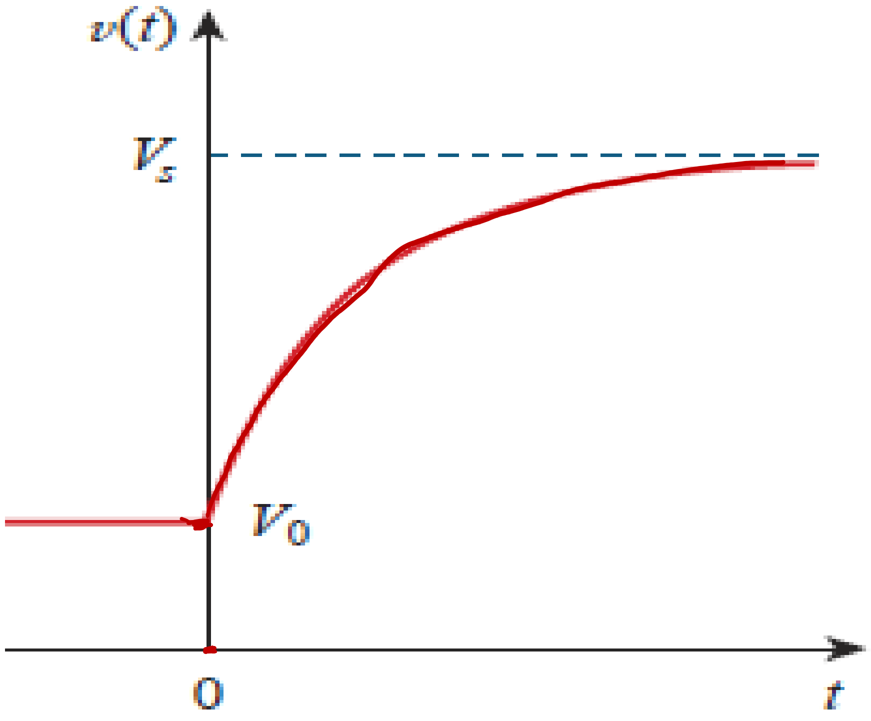
$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

t_1

t_2

t_{-1}





Response of an *RC* circuit with initially charged capacitor.

This is known as the *complete response* (or total response) of the *RC* circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged.

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

$C V(t) = C V_s (1 - e^{-t/\tau})$

$Q(t) = Q (1 - e^{-t/\tau})$

$0.8 Q = Q (1 - e^{-t/\tau})$

$0.8 = 1 - e^{-t/\tau}$

$\tau = RC$

10×10^6

If we assume that the capacitor is uncharged initially, we set $V_0 = 0$

$$v(t) = \begin{cases} 0, & t < 0 \\ V_s(1 - e^{-t/\tau}), & t > 0 \end{cases}$$

which can be written alternatively as

$$v(t) = V_s(1 - e^{-t/\tau})u(t)$$

This is the complete step response of the *RC* circuit when the capacitor is initially uncharged

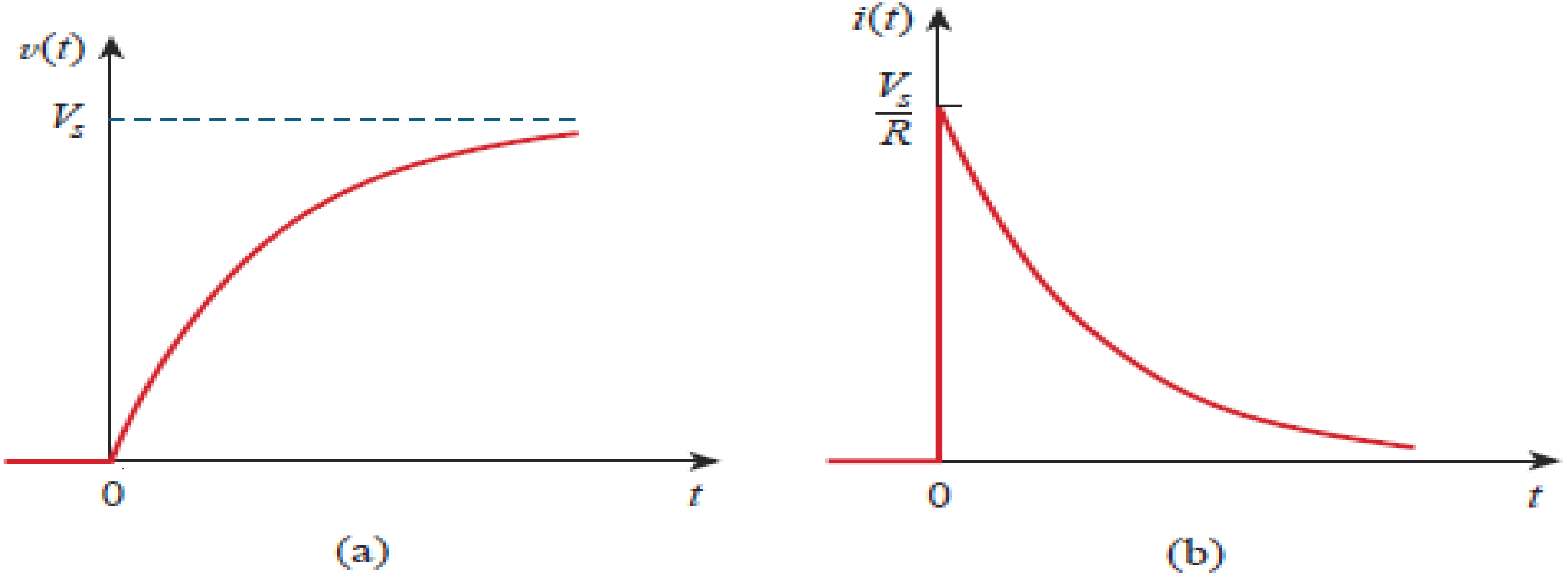
The current through the capacitor is obtained using $i(t) = C dv/dt$.

$$i(t) = C\frac{dv}{dt} = \frac{C}{\tau}V_s e^{-t/\tau}, \qquad \tau = RC, \qquad t > 0$$

$$\boxed{i(t) = \frac{V_s}{R}e^{-t/\tau}u(t)}$$

$V_0 = 0$

$$i(t) = \frac{V_s}{R} e^{-t/\tau} u(t)$$



Step response of an *RC* circuit with initially uncharged capacitor:
(a) Voltage response, (b) current response.

$$\boxed{v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \qquad t > 0}$$

- It is evident that has two components.

Classically there are two ways of decomposing this into two components.

- The first is to break it into a “natural response and a forced response”
- The second is to break it into a “transient response and a steady-state response.”

$$\boxed{\text{Complete response} = \underset{\text{stored energy}}{\text{natural response}} + \underset{\text{independent source}}{\text{forced response}}}$$

$$v = v_n + v_f$$

$$\boxed{v_n = V_o e^{-t/\tau}}$$

$$v_f = V_s(1 - e^{-t/\tau})$$

Another way of looking at the complete response is to break into two components—one temporary and the other permanent, i.e.,

$$\boxed{\text{Complete response} = \underbrace{\text{transient response}}_{\text{temporary part}} + \underbrace{\text{steady-state response}}_{\text{permanent part}}}$$

$$v = v_t + v_{ss}$$

$$v_t = (V_o - V_s)e^{-t/\tau}$$

$$v_{ss} = V_s$$

The *transient response* v_t is temporary; it is the portion of the complete response that decays to zero as time approaches infinity. Thus,

The **transient response** is the circuit's temporary response that will die out with time.

The *steady-state response* v_{ss} is the portion of the complete response that remains after the transient reponse has died out. Thus,

The **steady-state response** is the behavior of the circuit a long time after an external excitation is applied.

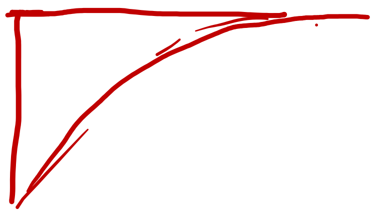
Eg = 7.10
7.10
Practice book
7.11

$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}, \qquad t > 0$$

t = 0, 1, 2, 3 . . .

∞

$$\boxed{v(t) = \underbrace{v(\infty)}_{V_s} + [v(0) - v(\infty)]e^{-t/\tau}}$$



where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an RC circuit requires three things:

t −
V(−)

t +
V(+)

t=0

→ V_0
t > 0
v(t)

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

Calculate the time taken by a capacitor $1\mu F$ and in series with a $1M\Omega$ resistance to be charged up to 80% of the final value

$t = 1.61$

$C = Q/V$; $Q = CV$

$V(t) = V_s(1 - e^{-t/RC})$

$CV(t) = CV_s[1 - e^{-t/RC}]$

$0.8 \times Q = Q[1 - e^{-t/RC}]$

$RC = 1 \times \cancel{10^6}\,\Omega \times 1 \times \cancel{10^{-6}}$

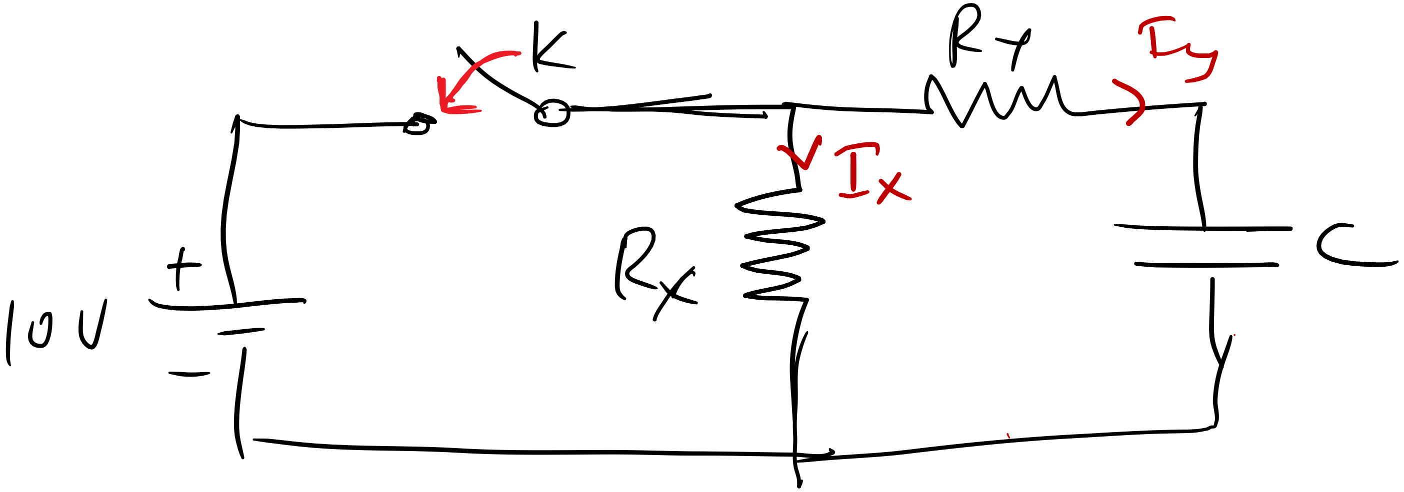
$q = CV_c$

$C = Q/V$

Charge stored in the capacitor during the charging

$0.8 = [1 - e^{-t}]$

A dc constant voltage source feeds a resistance of 2000 KΩ in series with a 5 μF capacitor. Find the time taken for the capacitor when the charge retained will be decayed to 50% initial value, the voltage source being short circuited



Switch K is closed find the time when the current from the battery reaches 500mA

$R_x = 50\,\Omega$

$R_Y = 70\,\Omega$

$C = 100\,\mu F$

$$I = I_x + I_y$$

$$500 = I_x + I_y$$

$$500 = \frac{10}{50} + I_y$$

I_y

$$I_y = 300\,mA$$

$$\boxed{I_y = \frac{V}{R_y}(1}$$