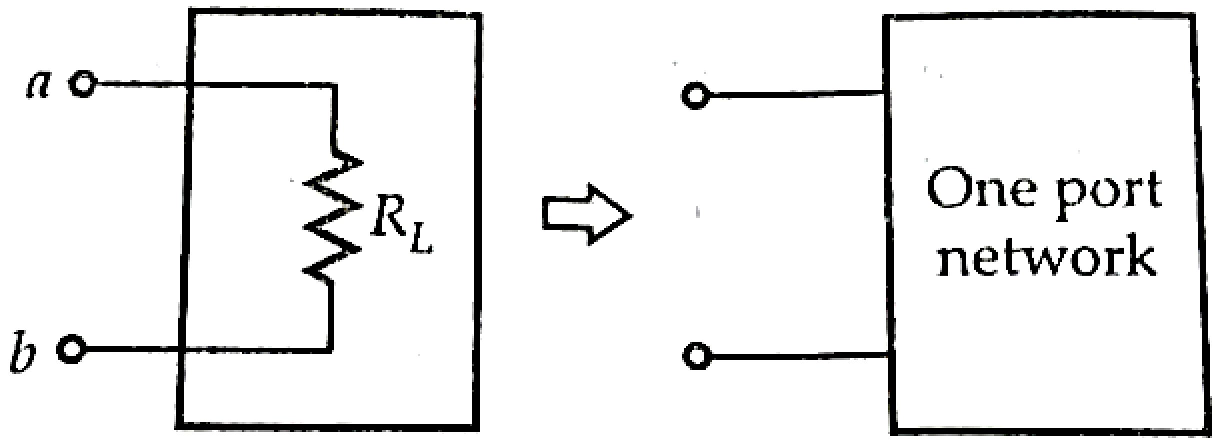
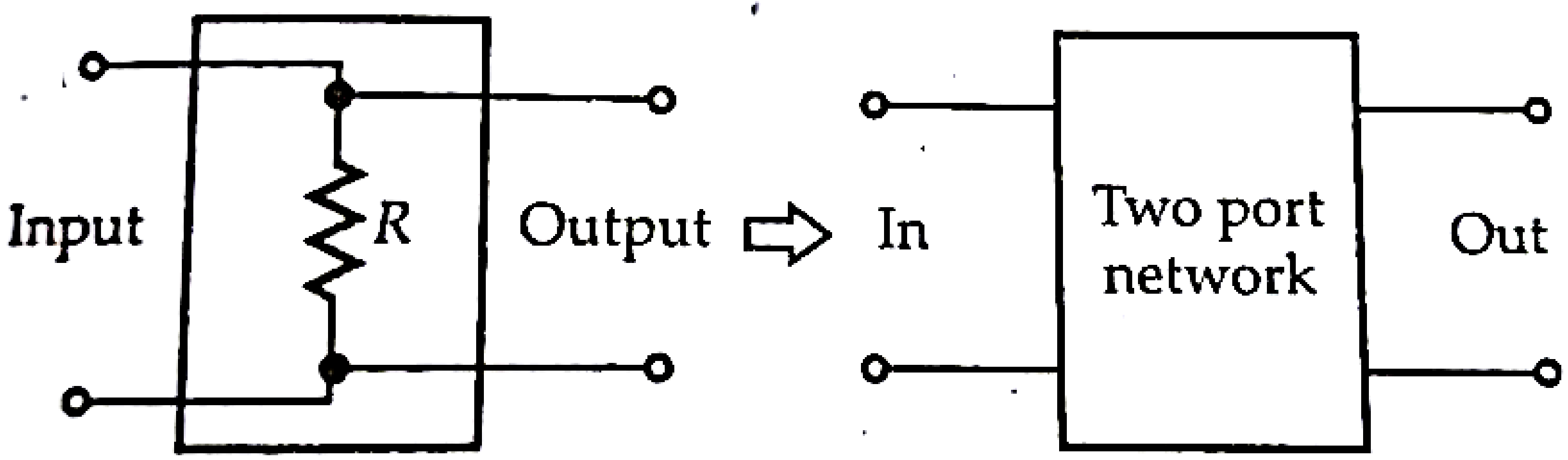
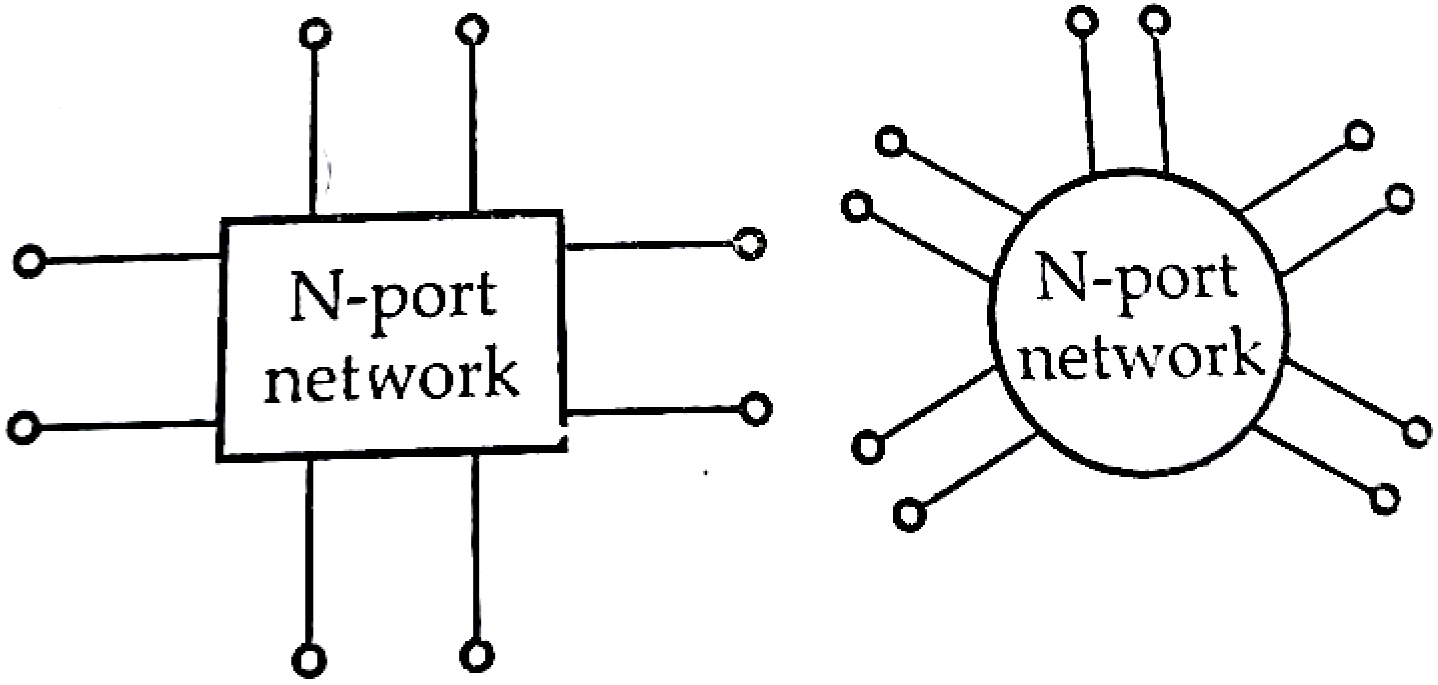


Schematic representation of one port network.



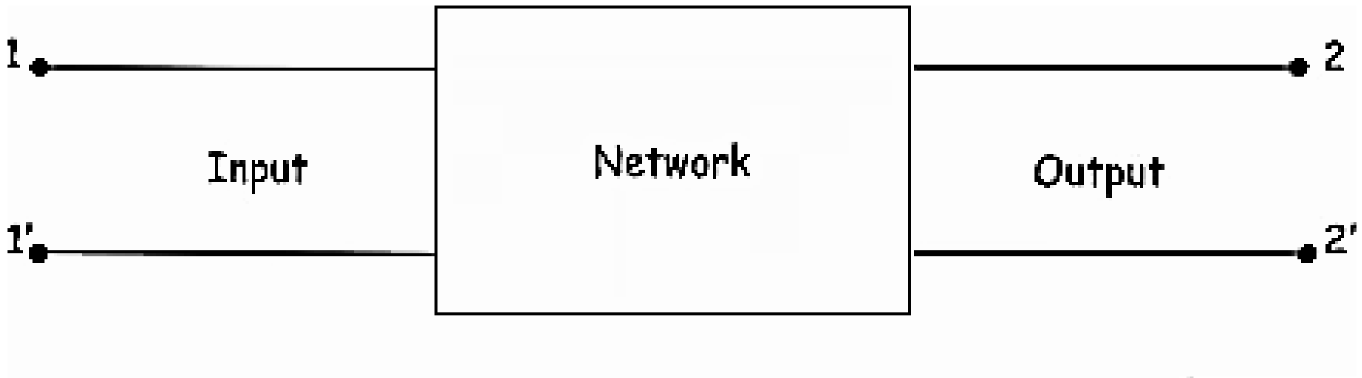
Diagrammatic representation of
two port network.



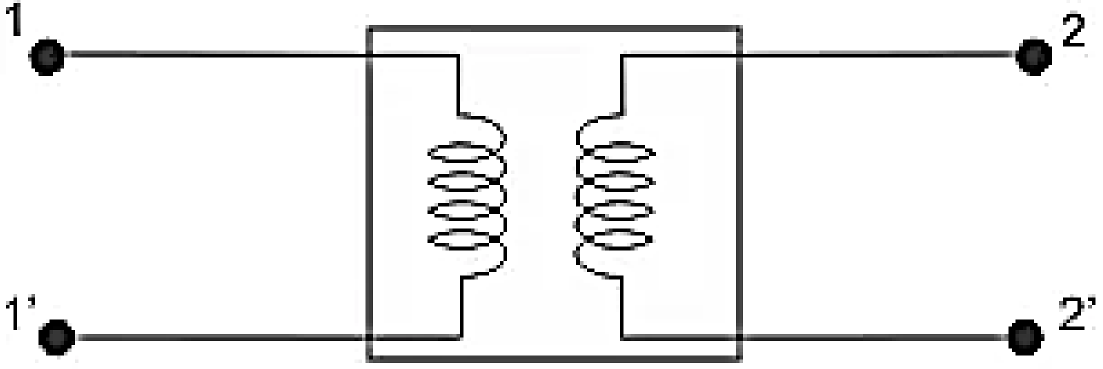
Diagrammatic representation of a n-port network.

Two Port Network

A **two port network** is an electrical network model with one pair of input terminals and one pair of output terminals. It is commonly used to model the voltage and current characteristics of complex electrical networks.



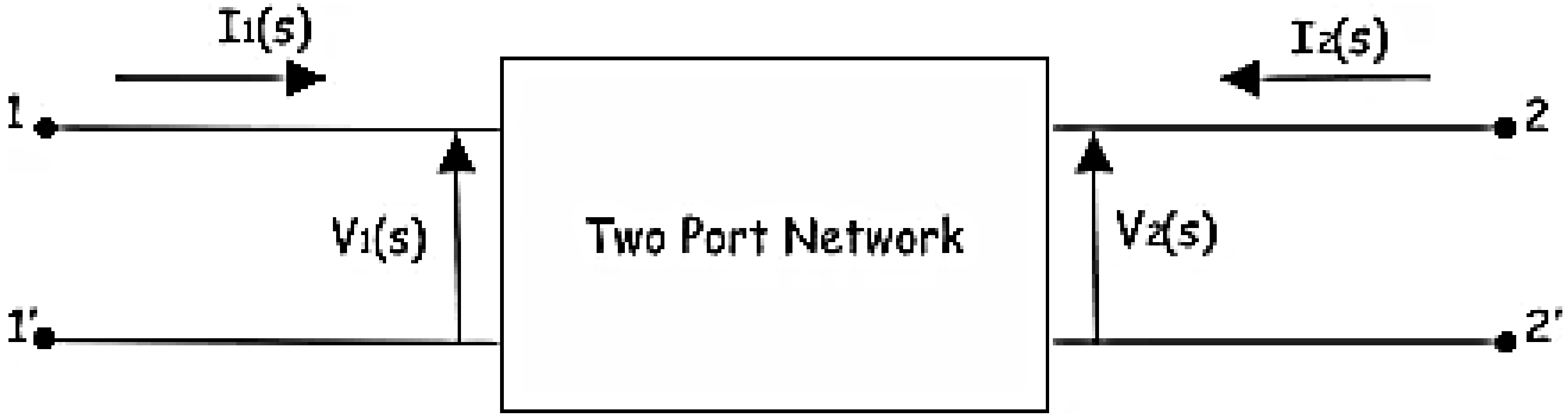
Two Port Network

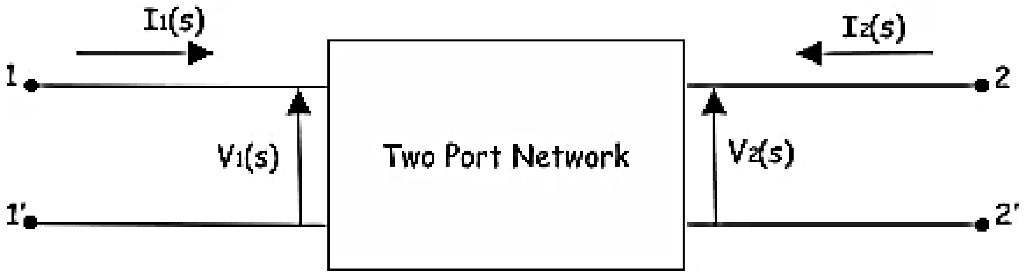


Single Phase Transformer

The relation between input and output signals of the network can be determined by transferring various network parameters, such as,

- Impedance
- Admittance
- Voltage ratio
- Current ratio.





The transfer voltage ratio function is,

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{Transform\ function\ of\ output\ voltage}{Transform\ function\ of\ input\ voltage}$$

The transfer current ratio function is,

$$\alpha = \frac{I_2(s)}{I_1(s)} = \frac{Transform\ function\ of\ output\ current}{Transform\ function\ of\ input\ current}$$

The transfer impedance function is,

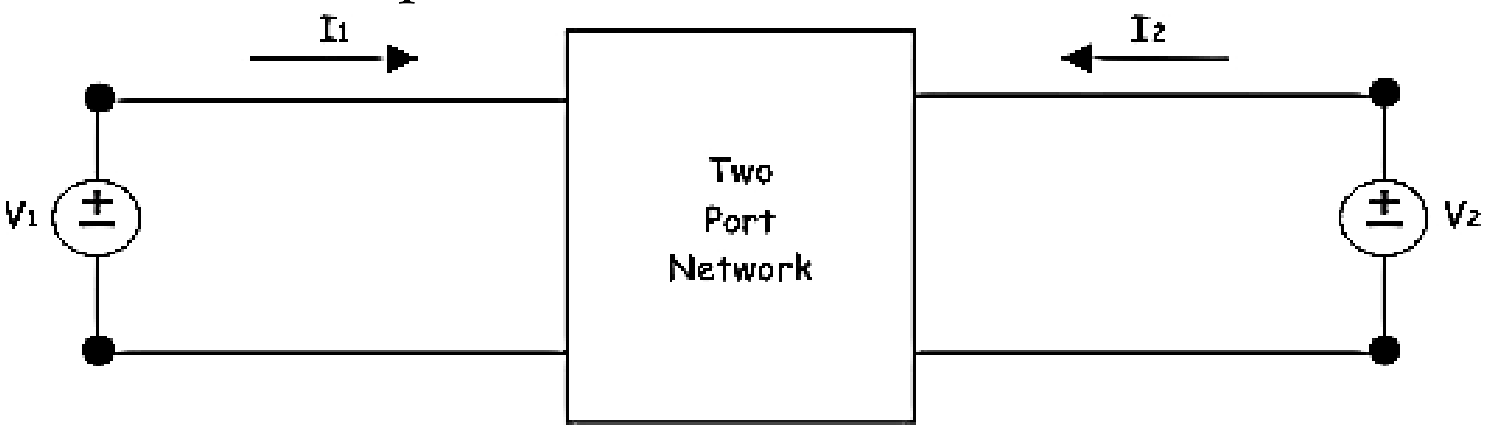
$$Z = \frac{V_1(s)}{I_2(s)} = \frac{Transform\ function\ of\ input\ voltage}{Transform\ function\ of\ output\ current}$$

The transfer admittance function is,

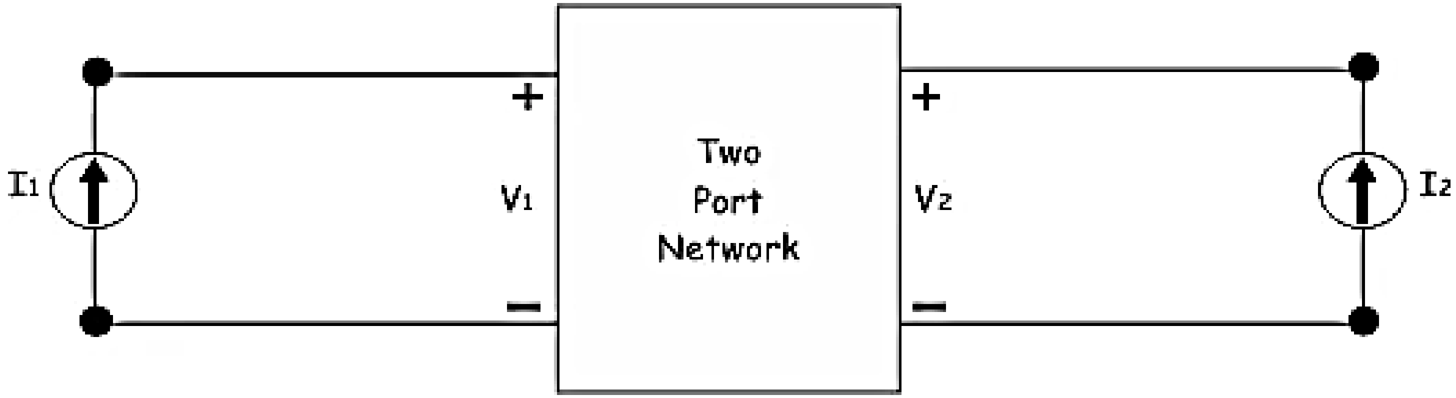
$$Y = \frac{I_1(s)}{V_2(s)} = \frac{Transform\ function\ of\ input\ current}{Transform\ function\ of\ output\ voltage}$$

Z- PARAMETERS (Open circuit impedance parameters)

- If the network is voltage driven, that can be represented as below



- If the network is driven by current, that can be represented as shown below.



From the figures, there are only four variables.

- **One pair of voltage variables V1 and V2 and one pair of current variables I1 and I2**
- **Thus, there are only four ratios of voltage to current, and those are,**

$$\frac{V_1}{I_1}, \frac{V_1}{I_2}, \frac{V_2}{I_1} \; and \; \frac{V_2}{I_2}$$

These four ratios are considered as impedance parameters or Z parameters of the network

$$\frac{V_1}{I_1}, \frac{V_1}{I_2}, \frac{V_2}{I_1} \; and \; \frac{V_2}{I_2}$$

$$Impedance(Z) = \frac{Voltage(V)}{Current(I)}$$

The values of these Z parameters of a two port network, can be evaluated by making two following cases

$$I_1 = 0$$

$$I_2 = 0$$

When the output is open, there will be no current in the output port. i.e. $I_2 = 0$

$$\left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

This known as the input impedance of the network, while the output port is open. This is denoted by Z_{11}

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = \textit{open circuit input impedance}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} = open\ circuit\ input\ impedance$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} = open\ circuit\ transfer\ impedance\ from\ output\ port\ to\ input\ port$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$ open circuit o/p impeden.

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} = open\ circuit\ output\ impedance\ from\ input\ port\ to\ output\ port$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2 = 0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1 = 0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2 = 0} \qquad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1 = 0}$$

- **Since all these above-shown Z parameters have been obtained by open circuiting output port or input port, the parameters are also referred to as open circuit impedance parameters.**

- **Now, we can relate all voltage and current variables of a two port network by these Z parameters.**

Now, we can relate all voltage and current variables of a two port network by these Z parameters.

$[V] = [Z][I]$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad \cdots\cdots\cdots \quad (i)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad \cdots\cdots\cdots \quad (ii)$$

These two equations can be represented in matrix form, as shown below,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{12} I_1 + Z_{12} I_2 \quad \cdots\cdots\cdots \quad (i)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \cdots\cdots\cdots \quad (ii)$$

V'

$[V] = [Z][I]$

In the equation (i), if we put I_2 = 0, We get,

$$V_1 = Z_{11} I_1 + Z_{12}.0$$

$$\Rightarrow Z_{11} = \left(\frac{V_1}{I_1}\right)_{I_2=0}$$

In the same way, by putting $I_2 = 0$ and $I_1 = 0$ alternatively in equation (ii) We can prove,

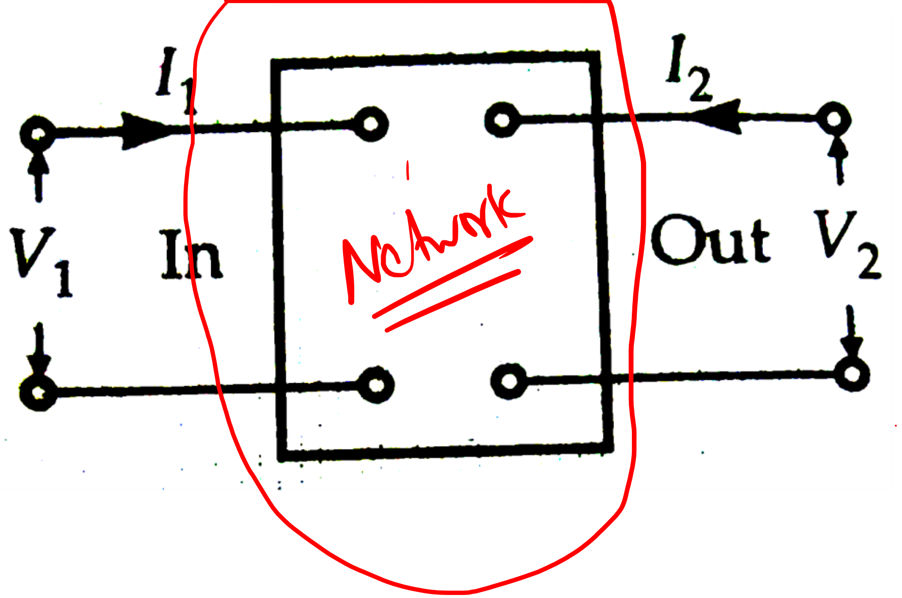
$$Z_{21} = \frac{V_2}{I_1} \; and \; Z_{22} = \frac{V_2}{I_2}$$

Similarly, if we put I_1=0, in the same equation, we get,

$$V_1 = Z_{11}.0 + Z_{12} I_2$$

$$\Rightarrow Z_{12} = \frac{V_1}{I_2}$$

- **Z_{11} and Z_{22} are also referred to as driving point impedance**
- **Z_{21} and Z_{12} are also referred to as transfer impedance**



$$[V]=[Z][I]$$

$$\boxed{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}$$

$$\boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}}$$

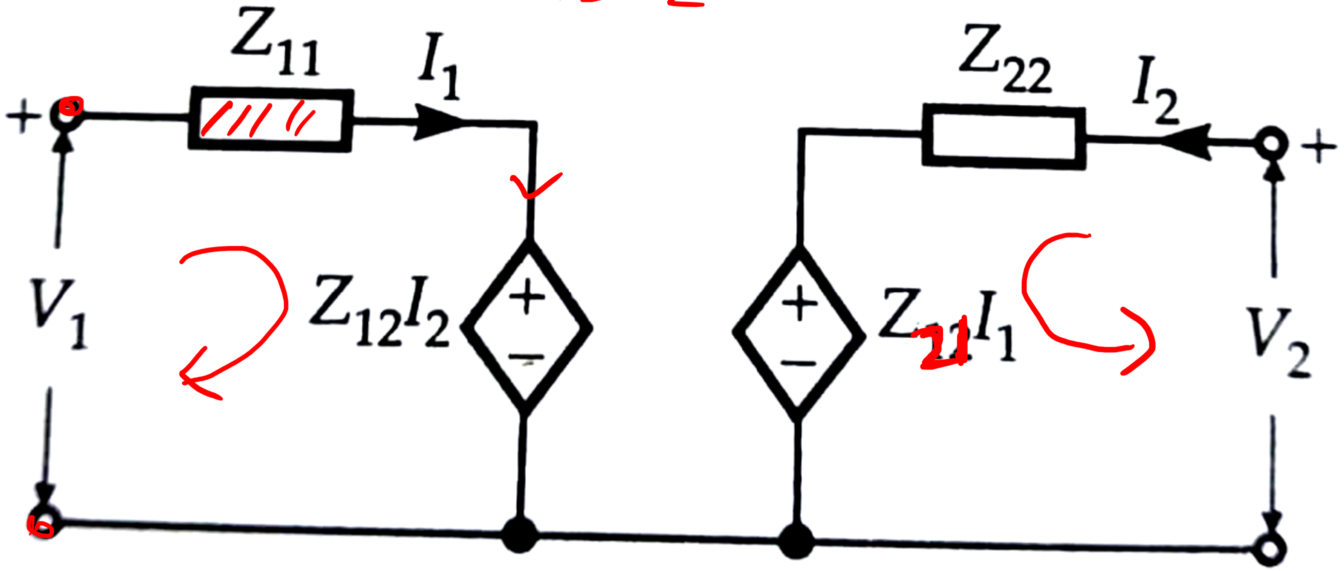
$$\boxed{\begin{aligned} V_1 &=(Z_{11}-Z_{12}) I_1 + Z_{12}(I_1+I_2) \\ V_2 &=(Z_{21}-Z_{12}) I_1 +(Z_{22}-Z_{12}) I_2 + Z_{12}(I_1+I_2) \end{aligned}}$$

KVL —

$$V_1 = Z_{11} I_1 + \underline{Z_{12} I_2}$$
$$V_2 = Z_{21} I_1 + \underline{Z_{22} I_2}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

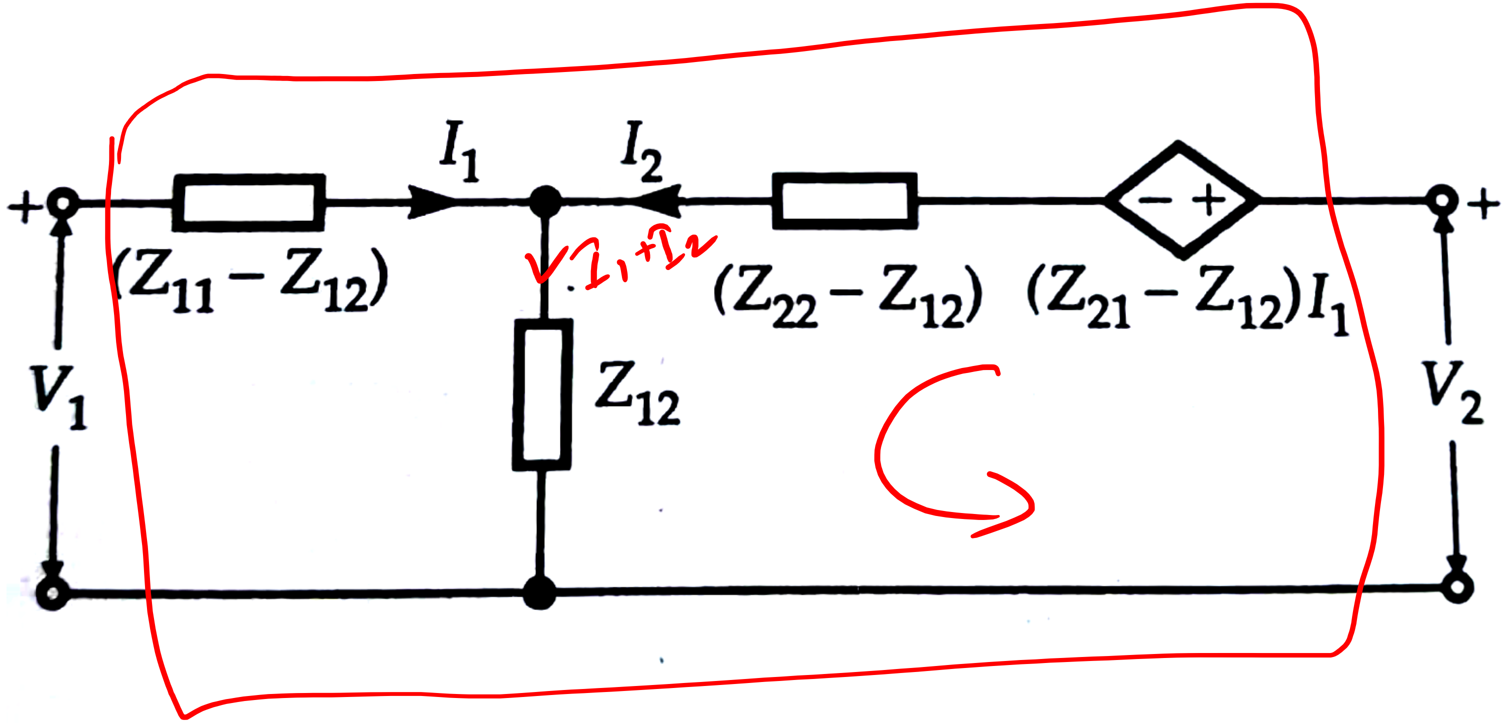
$Z_{11} I_1 + Z_{12} I_2$



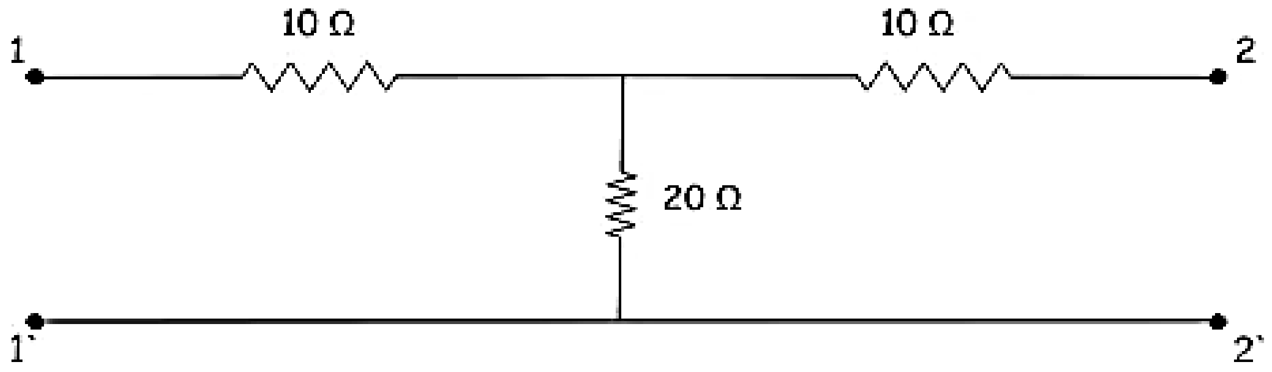
Basic Z parameter equivalent circuit

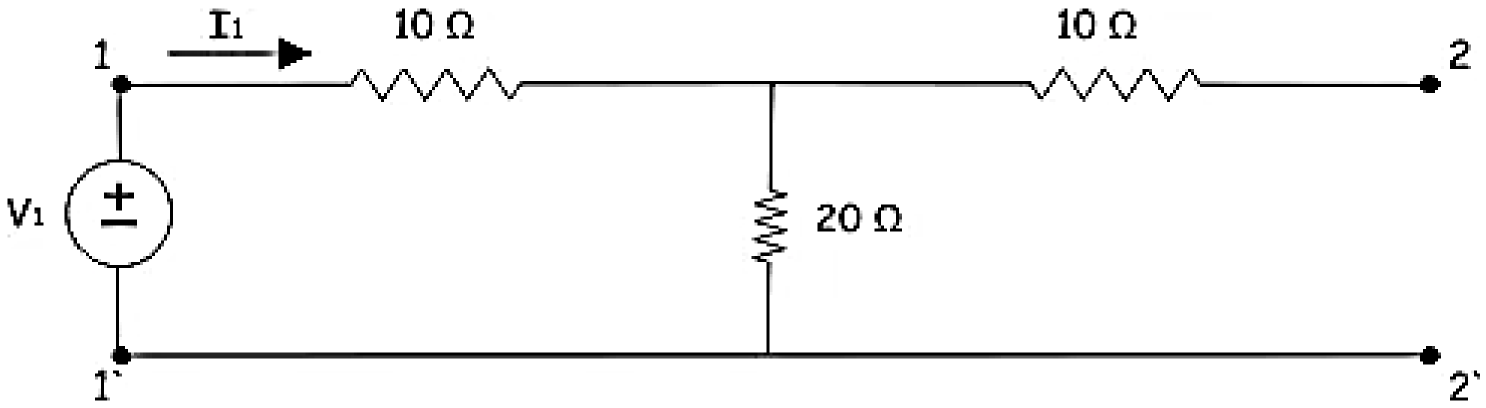
$$V_1 = (Z_{11} - Z_{12}) I_1 + Z_{12} (I_1 + I_2)$$

$$V_2 = (Z_{21} - Z_{12}) I_1 + (Z_{22} - Z_{12}) I_2 + Z_{12}(I_1 + I_2)$$



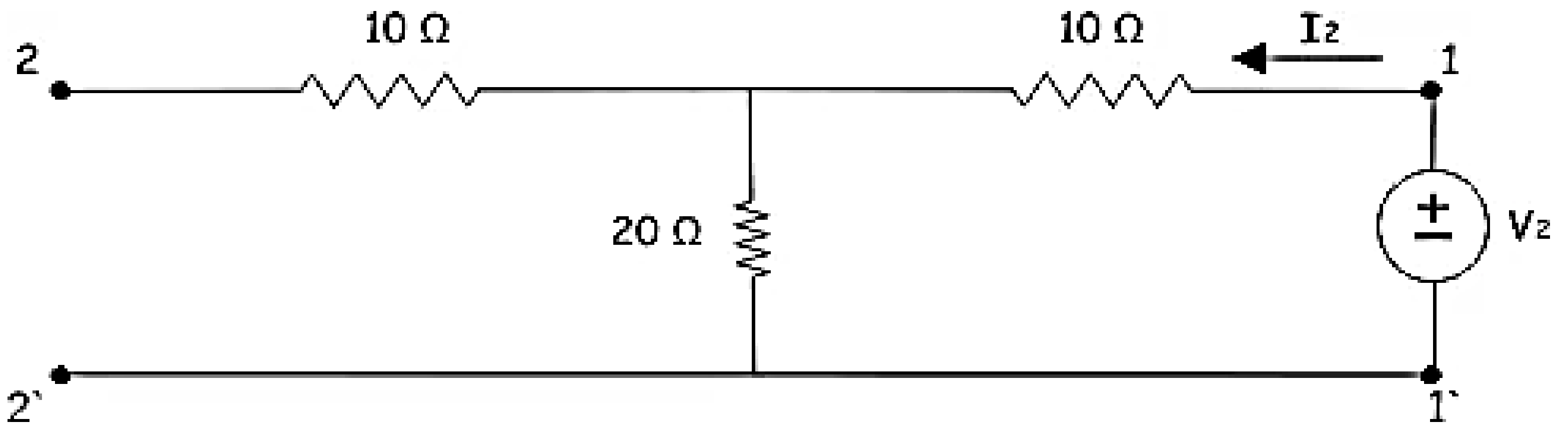
Another form of equivalent circuit with open circuit (Z) parameters





$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2 = 0} = \frac{(10 + 20)I_1}{I_1} = 30\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2 = 0} = \frac{20I_1}{I_1} = 20\ \Omega$$



$$Z_{11} = Z_{22}$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(10+20)I_2}{I_2} = 30\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20I_2}{I_2} = 20\,\Omega$$

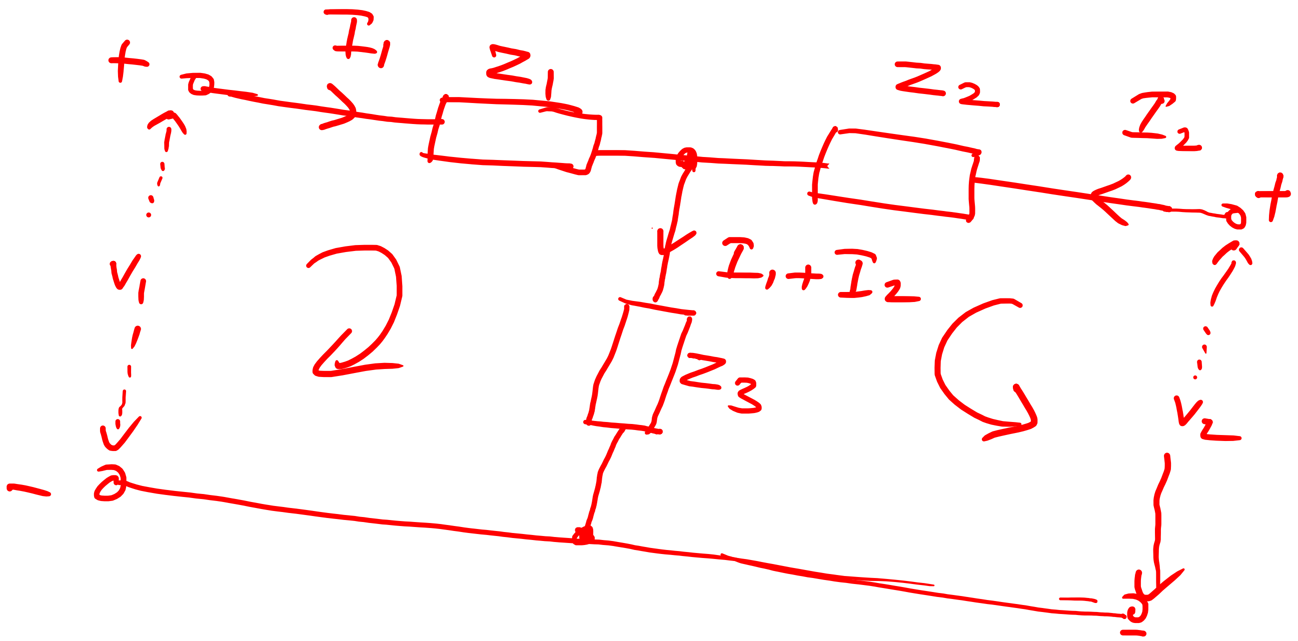
$$Z_{21} = Z_{12}$$

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{11} = Z_{22} \; and \; Z_{12} = Z_{21}$$

if input excitation and output response of the network are interchanged, the transfer impedance remains the same

Find the Z parameters!



$$V_1 = Z_1 I_1 + Z_3 (I_1 + I_2)$$

$$V_2 = Z_2 I_2 + Z_3 (I_1 + I_2)$$

Y parameters (also known as admittance parameters or short-circuit parameters) are properties used in electrical engineering to describe the electrical behavior of linear electrical networks.

These Y-parameters are used in Y-matrixes (admittance matrixes) to calculate the incoming and outgoing voltages and currents of a network.

When analyzing Z parameters (also known as impedance parameters), we express voltage in the term of current by the following equations.

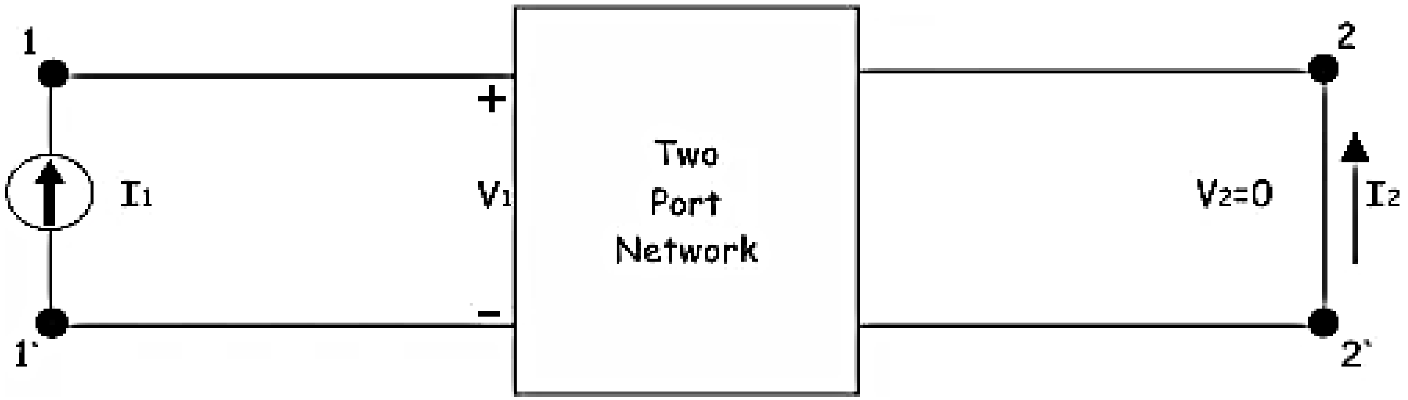
$$V_1 = Z_{11} I_1 + Z_{12} I_2 \cdots\cdots\cdots\cdots (i)$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \cdots\cdots\cdots\cdots (ii)$$

Similarly, we can represent current in terms of voltage by admittance parameters of a two port network. Then we will represent the current-voltage relations as,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \cdots\cdots\cdots\cdots (iii)$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \cdots\cdots\cdots\cdots (iv)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Apply a current source of I1 at the input port keeping the output port short-circuited as shown below



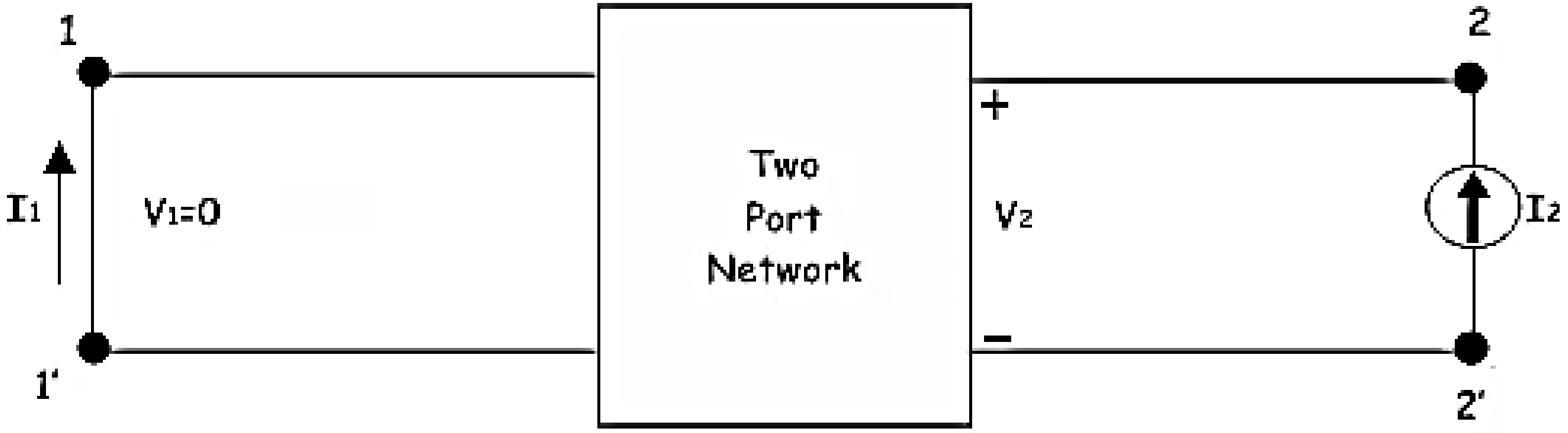
$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_{11}$$

Short circuit input admittance.

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = Y_{21}$$

Short circuit transfer admittance from the input port to the output port

Short circuit the input port of the network and apply current I2 at the output port, as shown below.



$$\left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_{22}$$

short circuit output admittance

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = Y_{12}$$

short circuit transfer admittance from the out port to the output port

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_{11} = short\ circuit\ input\ admittance$$

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = Y_{12} = short\ circuit\ transfer\ admittance\ from\ output\ port\ to\ input\ port$$

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = Y_{21} = short\ circuit\ transfer\ admittance\ from\ intput\ port\ to\ output\ port$$

$$\left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_{22} = short\ circuit\ output\ admittance$$

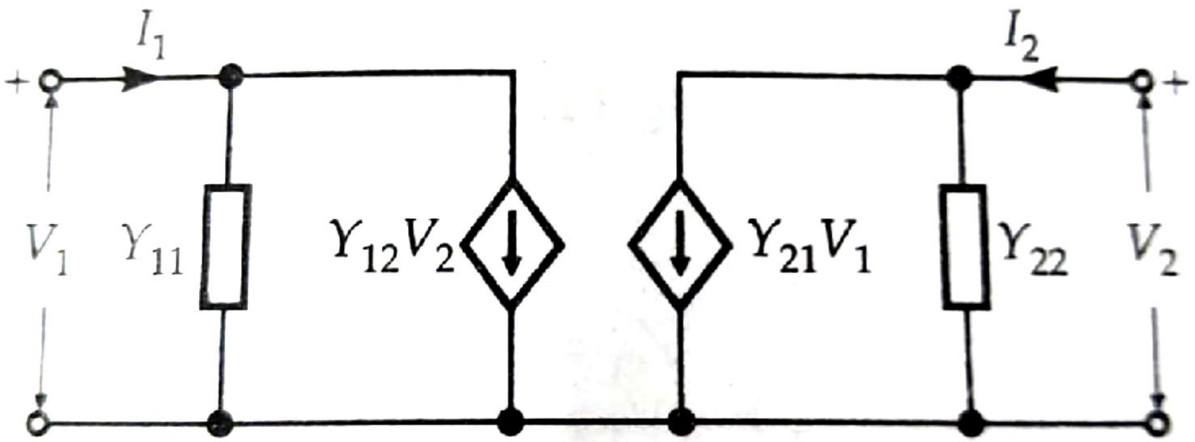
$$I_1 = Y_{11}V_1 + Y_{12}V_2 \cdots\cdots\cdots\cdots (iii)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \cdots\cdots\cdots\cdots (iv)$$

$$I_1 = (Y_{11} + Y_{12})\,V_1 - Y_{12}\,(V_1 - V_2)$$

$$I_2 = (Y_{21} - Y_{12})\,V_1 + (Y_{22} + Y_{12})\,V_2 - Y_{12}\,(V_2 - V_1)$$

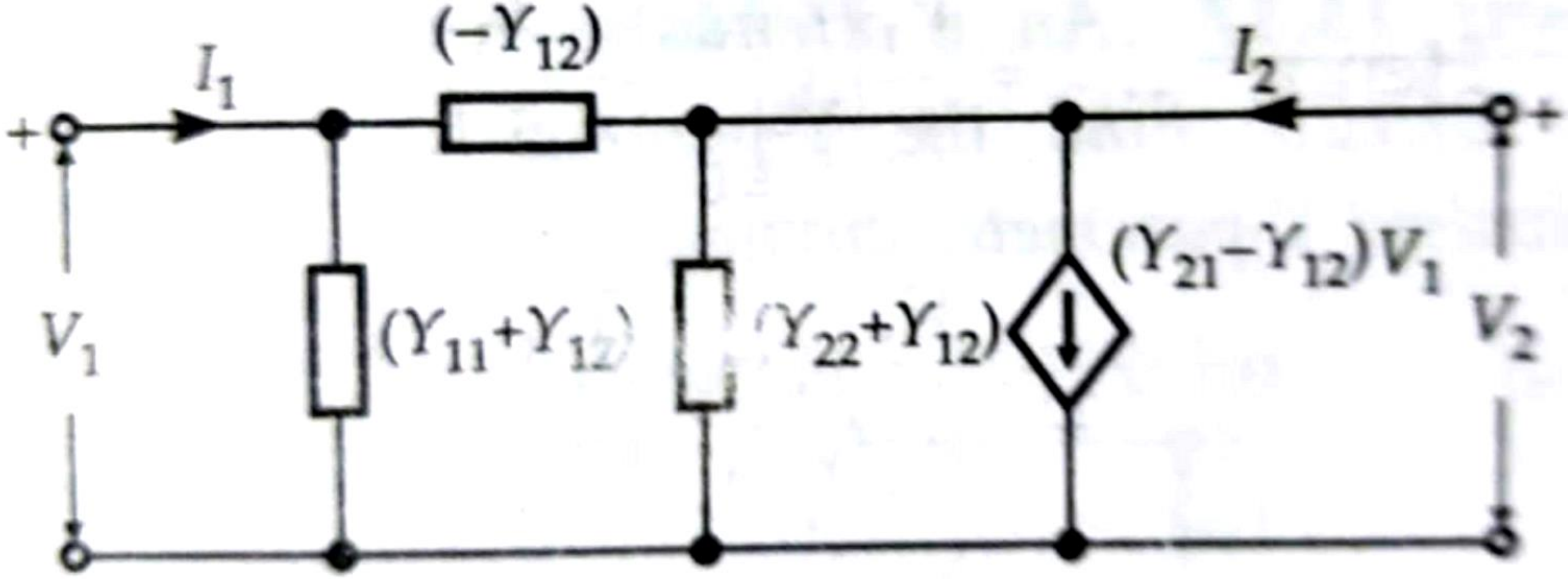
$$I_1 = Y_{11}V_1 + Y_{12}V_2 \cdots\cdots\cdots\cdots (iii)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \cdots\cdots\cdots\cdots (iv)$$



Equivalent circuit representation of short circuit (Y) parameters.

$$I_1 = (Y_{11} + Y_{12})\, V_1 - Y_{12}\,(V_1 - V_2)$$

$$I_2 = (Y_{21} - Y_{12})\, V_1 + (Y_{22} + Y_{12})\, V_2 - Y_{12}\,(V_2 - V_1)$$

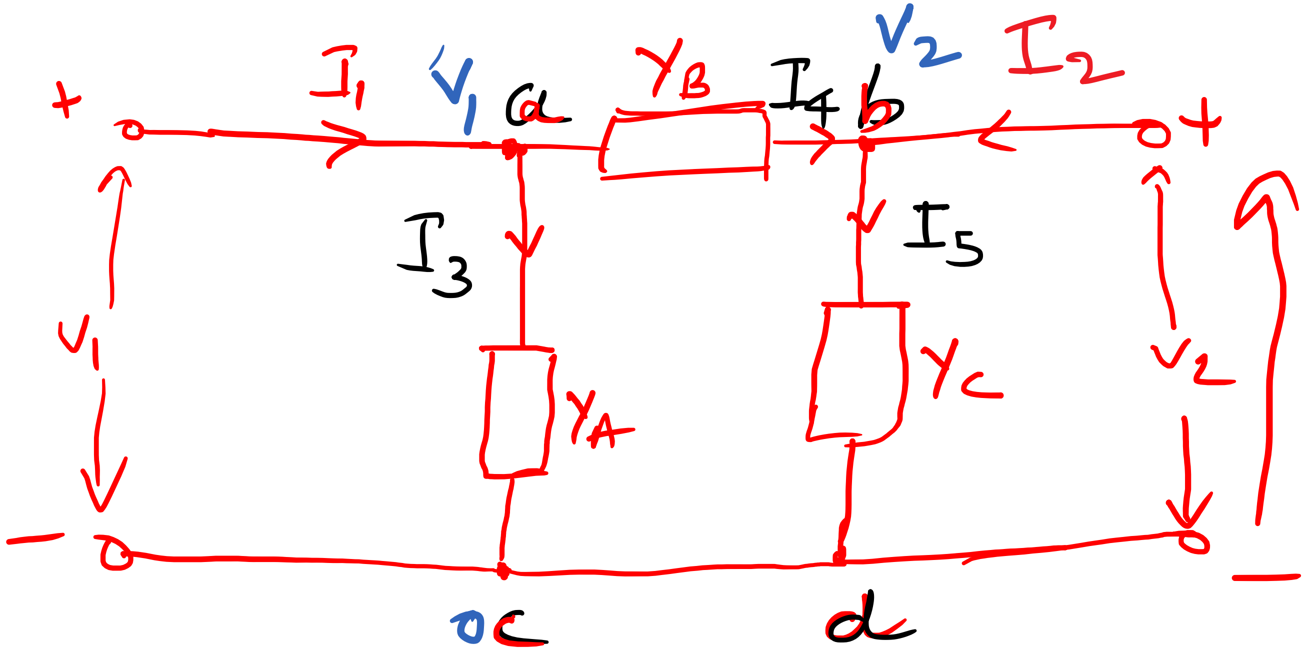


Another form of equivalent circuit with short circuit (Y) parameters.

$I_4 + I_2 = I_5$

Y parameters of the circuit?

π network.



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

$I_1 = I_3 + I_4$

$I_3 = V_1 Y_A$

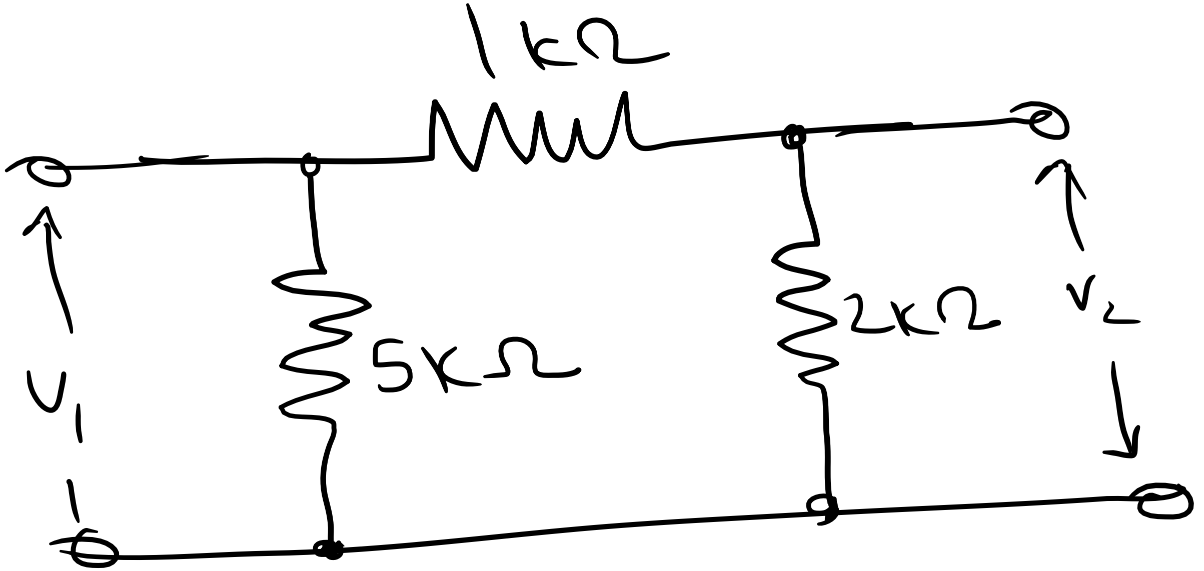
$I_2 = I_5 - I_4$

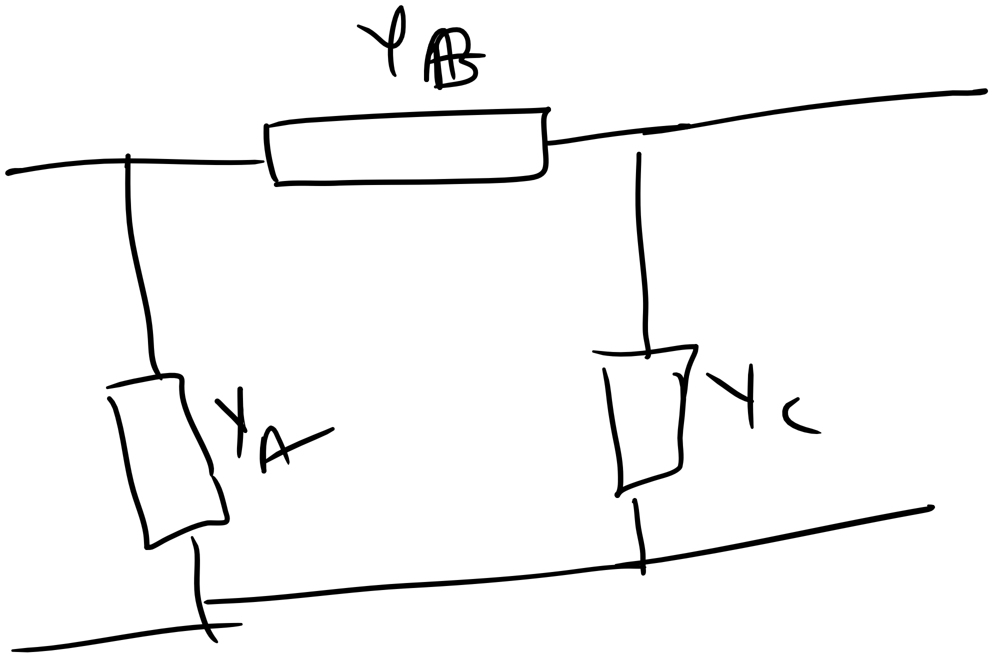
$Y_{11} = (Y_A + Y_B)$

$Y_{12} = -Y_B$

$Y_{21} = -Y_B$

$Y_{22} = Y_c + Y_B$





$1/5k\Omega$

Ω^{-1}

(mho)

H Parameters (Hybrid Parameters)

- Hybrid parameters use Z parameters, Y parameters, voltage ratio, and current ratios to represent the relationship between voltage and current in a two port linear network.

- H parameters are useful in describing the input-output characteristics of circuits where it is hard to measure Z or Y parameters (such as in a transistor).

- H parameters encapsulate all the important linear characteristics of the circuit, so they are very useful for simulation purposes.

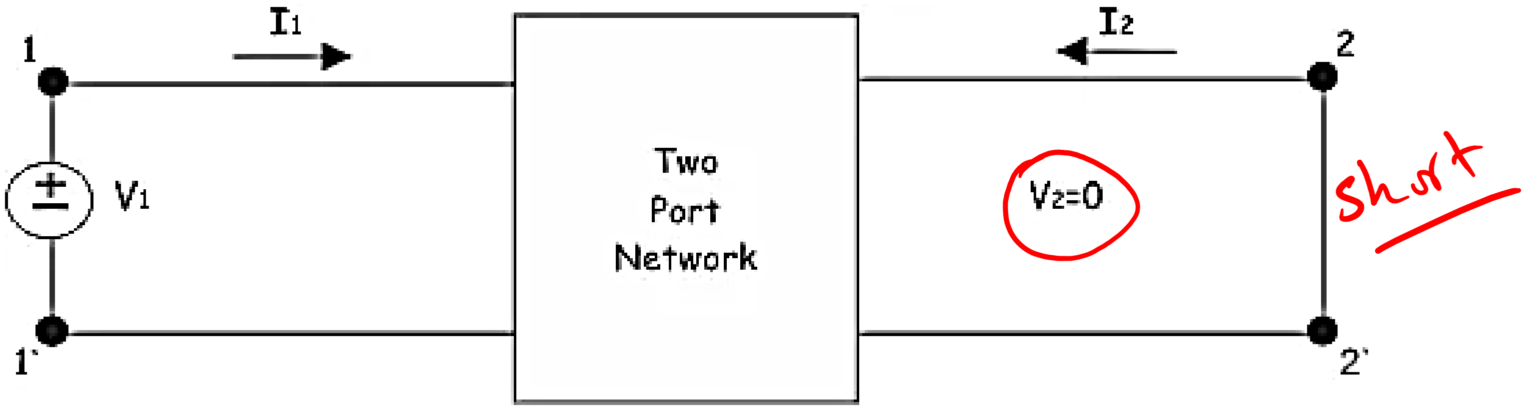
The relationship between voltages and current in h parameters can be represented as:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$V_1 = h_{11} I_1$



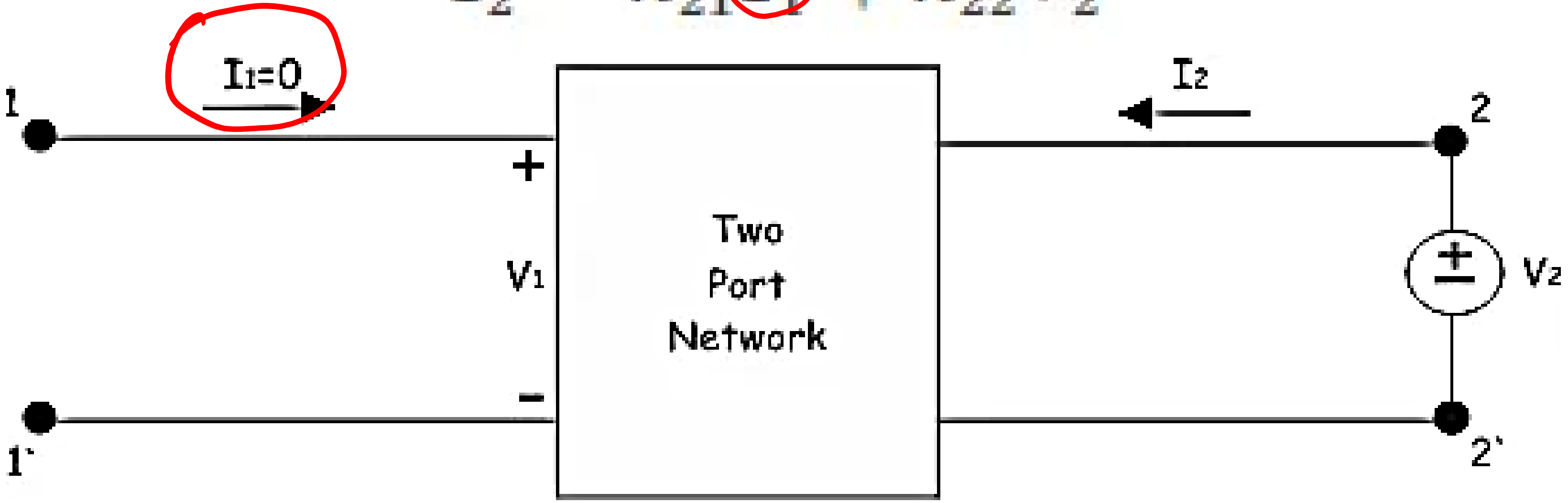
$$\left.\frac{V_1}{I_1}\right|_{V_2=0} = h_{11}$$

short circuit input impedance of the network

$$\left.\frac{I_2}{I_1}\right|_{V_2=0} = h_{21}$$

short-circuit current gain of the network

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

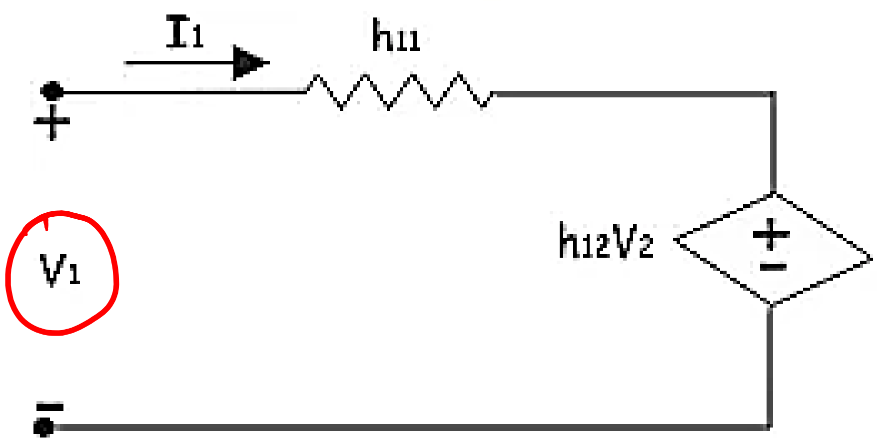


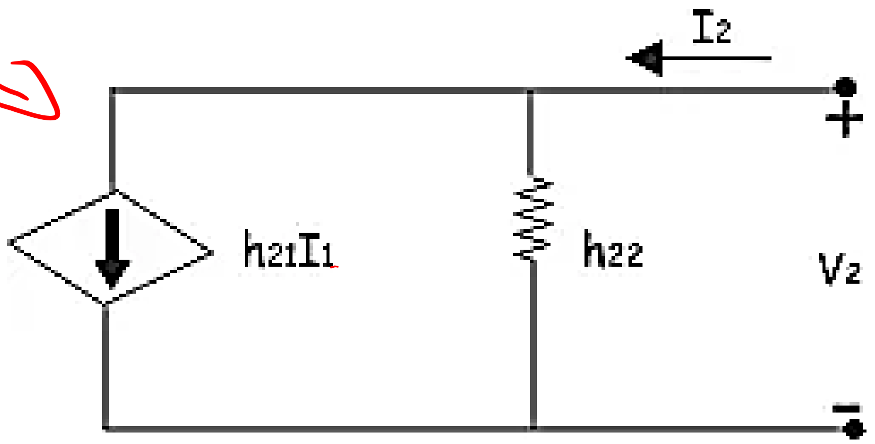
$$\left.\frac{V_1}{V_2}\right|_{I_1 = 0} = h_{12} = \textit{open circuit reverse voltage gain}$$

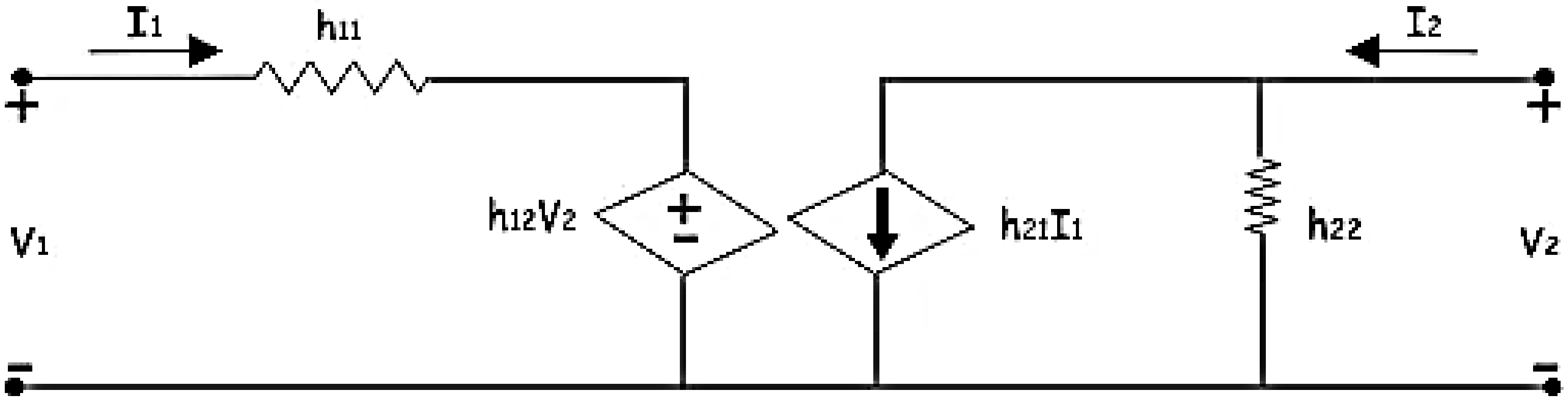
$$\left.\frac{I_2}{V_2}\right|_{I_1 = 0} = h_{22}$$ *open circuit output admittance.*

$$V_1 = h_{11} I_1 + h_{12} V_2 \cdots\cdots\cdots\cdots\cdots\cdots (i)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \cdots\cdots\cdots\cdots\cdots\cdots (ii)$$







The h parameters equivalent network of a two port network

Inverse Hybrid Parameters or g Parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = open\ circuit\ input\ admittance$$

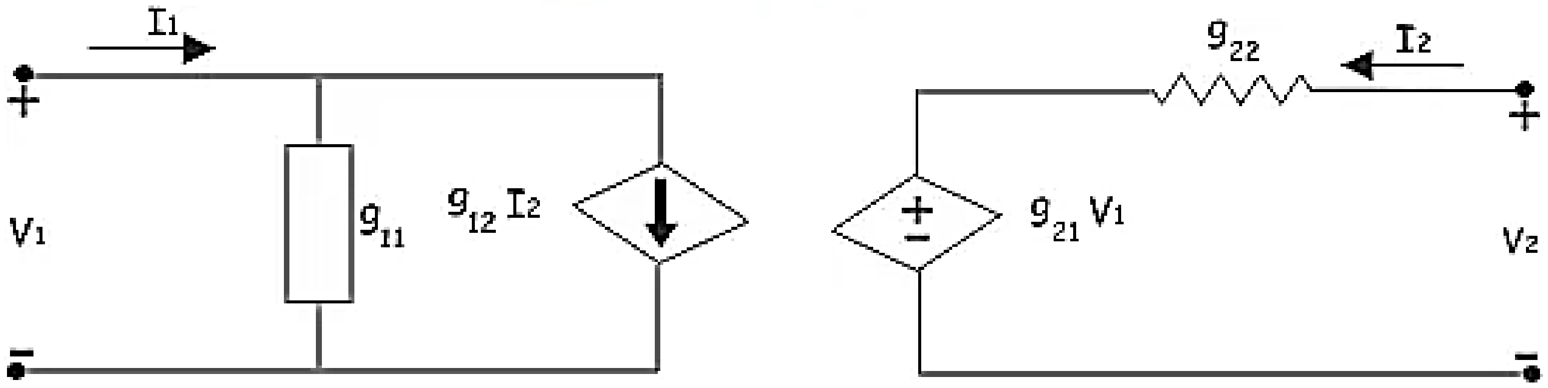
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = short\ circuit\ reverse\ current\ gain$$

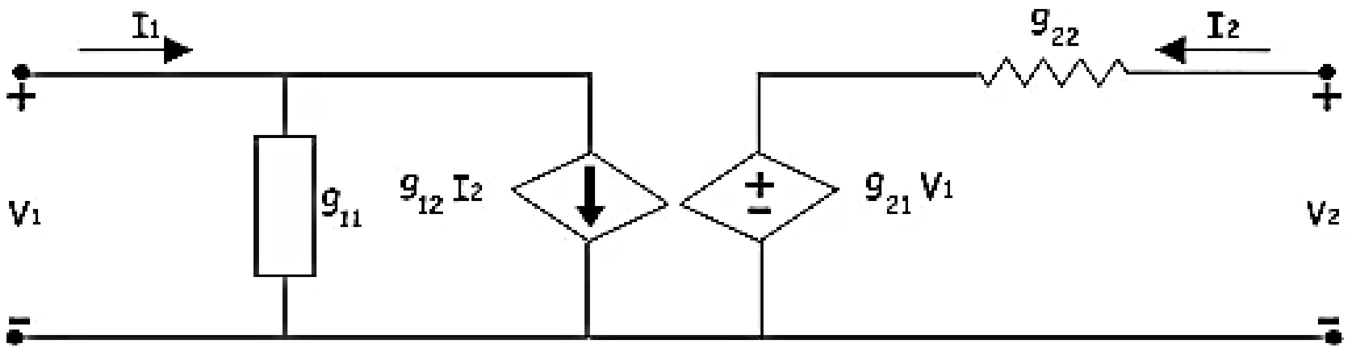
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = open\ circuit\ voltage\ gain$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = short\ circuit\ output\ impedance$$

$$I_1 = g_{11}V_1 + g_{12}I_2 \cdots\cdots\cdots\cdots\cdots (iii)$$

$$V_2 = g_{21}V_1 + g_{22}I_2 \cdots\cdots\cdots\cdots\cdots (iv)$$





The g parameters equivalent network of a two port network

- The h parameters are used to analyze Bipolar Junction Transistor or BJT.
- Whereas, g parameter are used to analyze Junction Field Effect Transistor or JFET