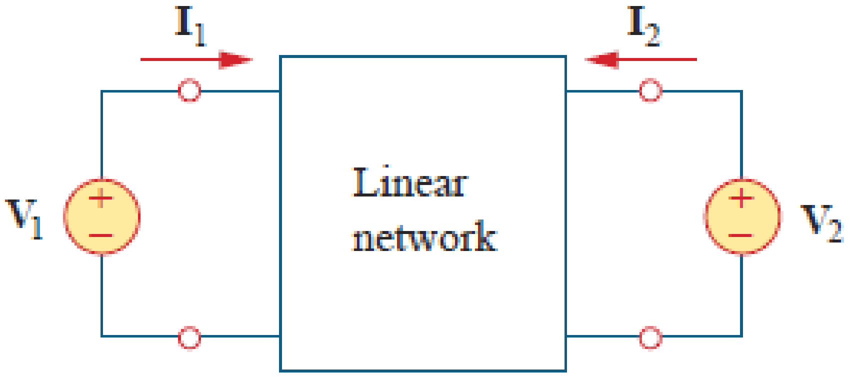
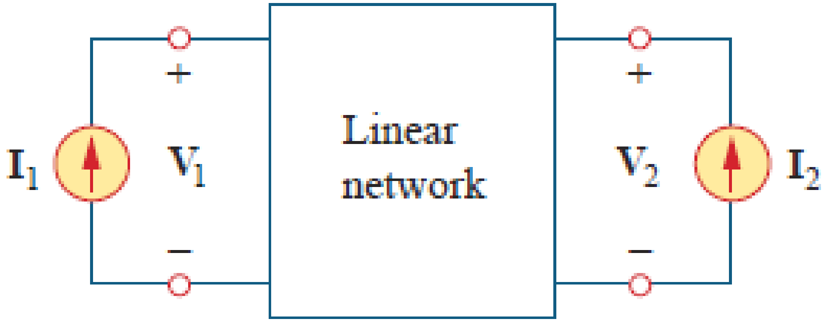
Impedance Parameters

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$



(a)

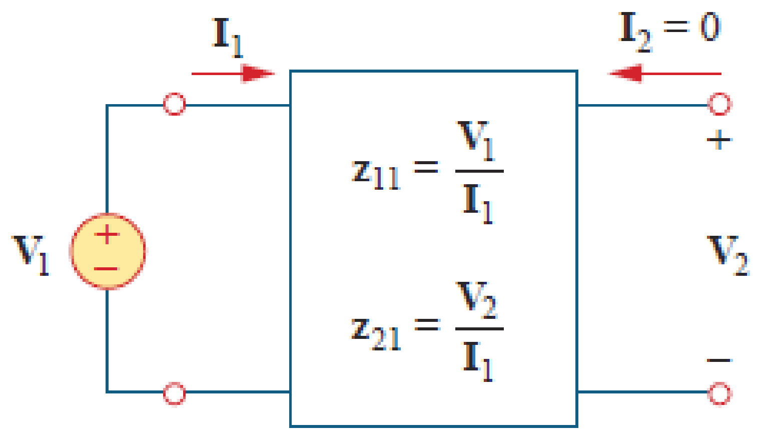


(b)

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

A two-port network may be voltage-driven as in Fig. (a) or current-driven Fig (b)

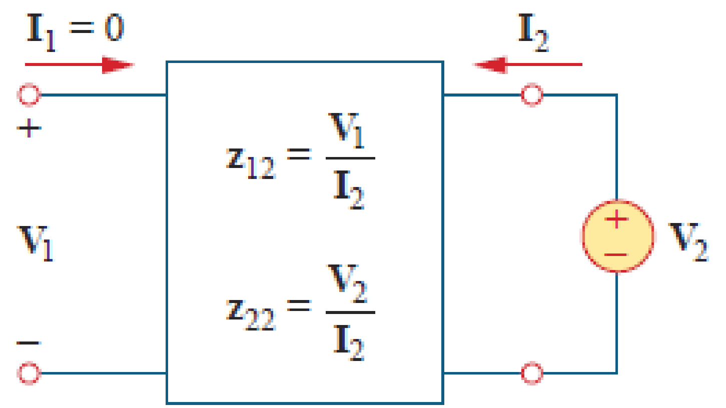
$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$
$$\mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}$$



(a)

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}, \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}, \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



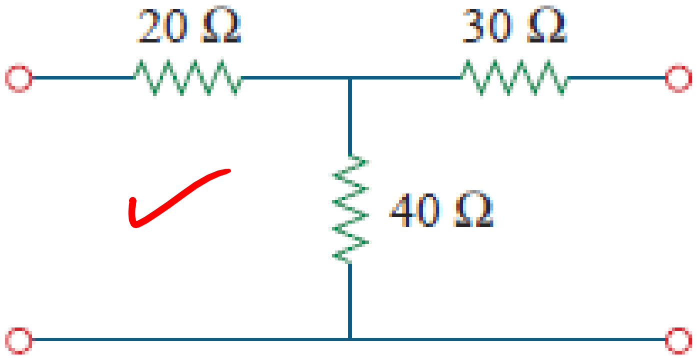
z_{11} = Open-circuit input impedance

z_{12} = Open-circuit transfer impedance from port 1 to port 2

z_{21} = Open-circuit transfer impedance from port 2 to port 1

z_{22} = Open-circuit output impedance

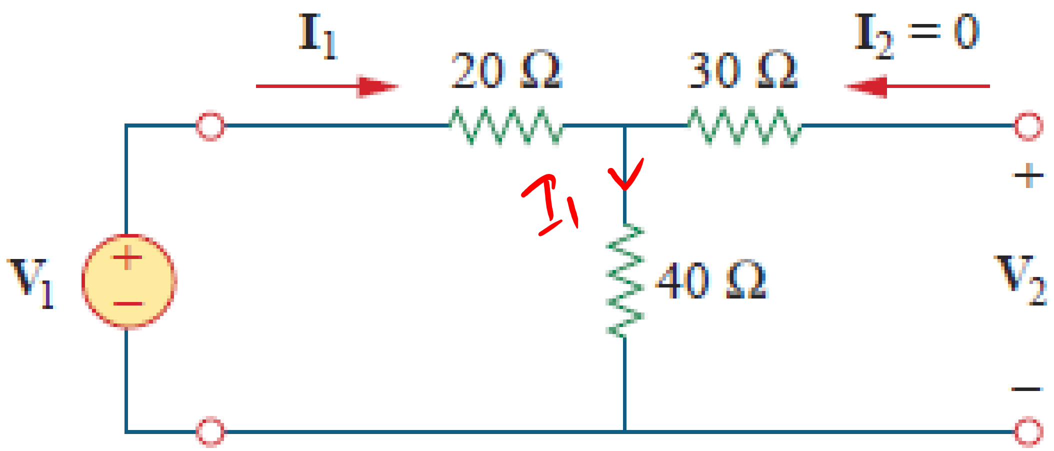
Determine the z parameters for the circuit in Fig.



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

METHOD 1

To determine $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we apply a voltage source $\mathbf{V}_1$ to the input port and leave the output port open as in Fig.

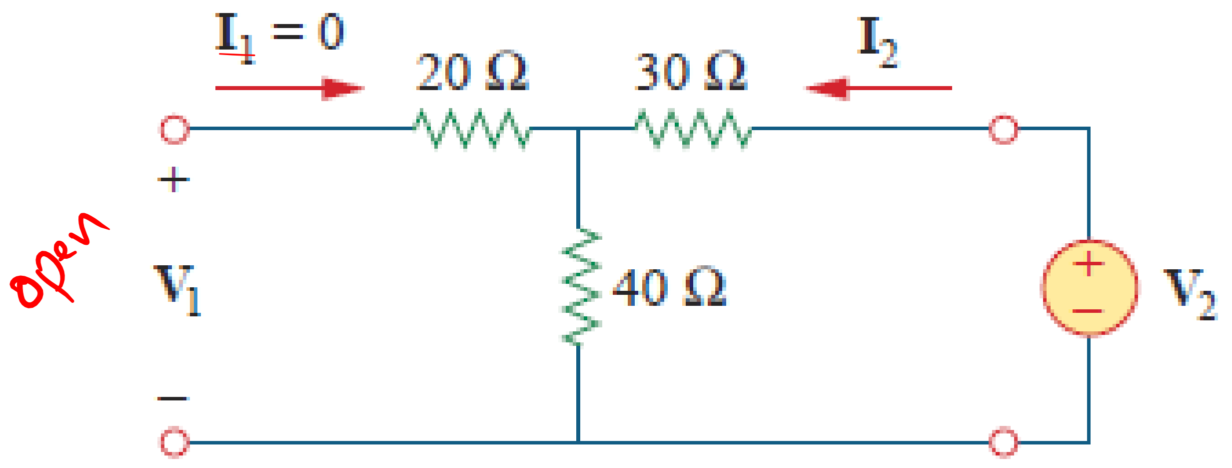


open circuit voltage

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20 + 40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$

To find $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we apply a voltage source $\mathbf{V}_2$ to the output port and leave the input port open as in Fig.



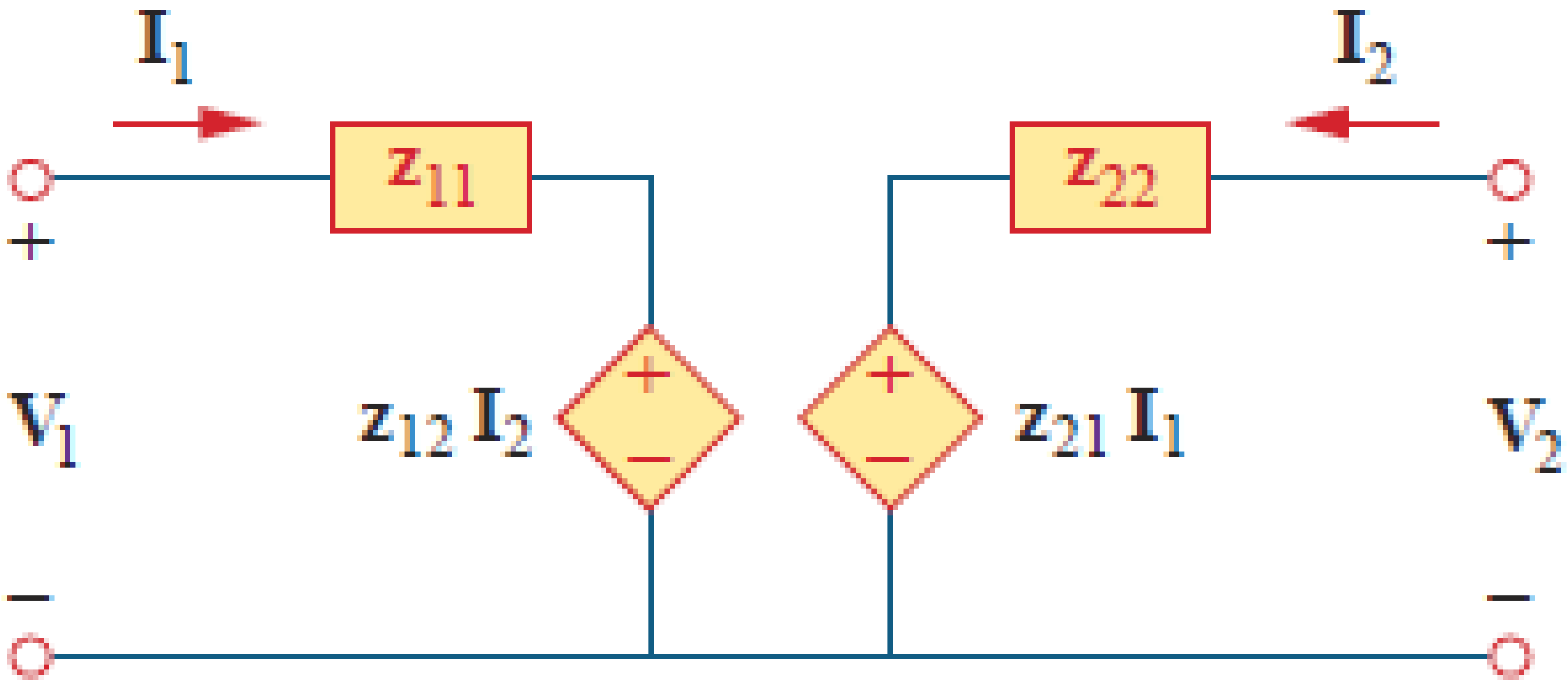
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega,$$

$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30+40)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

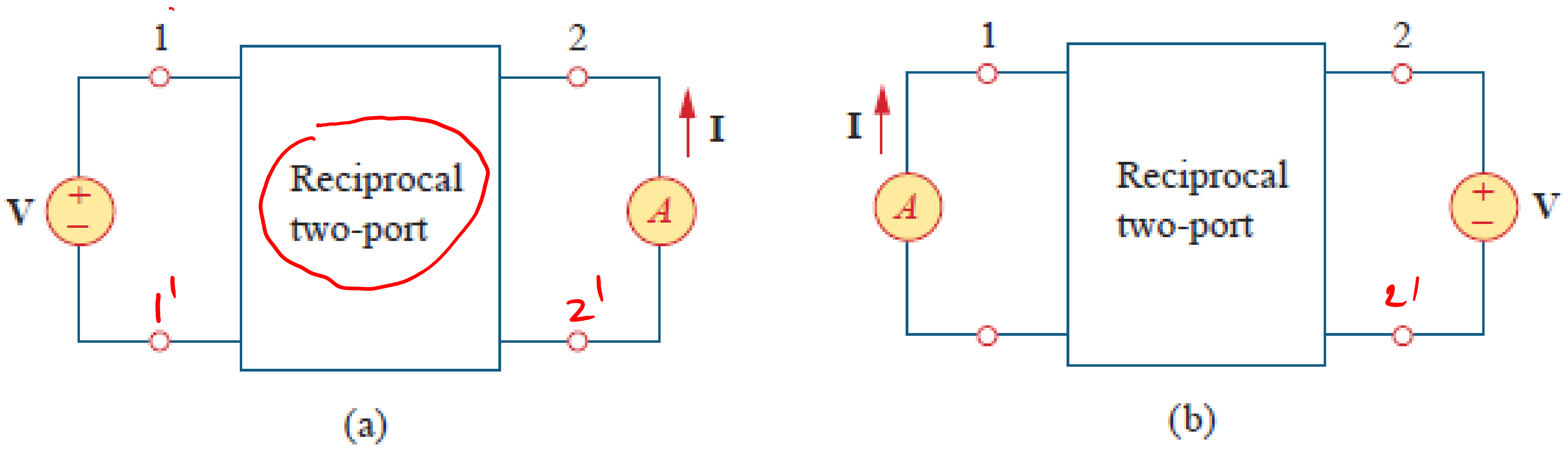
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



general equivalent network equivalent diagram

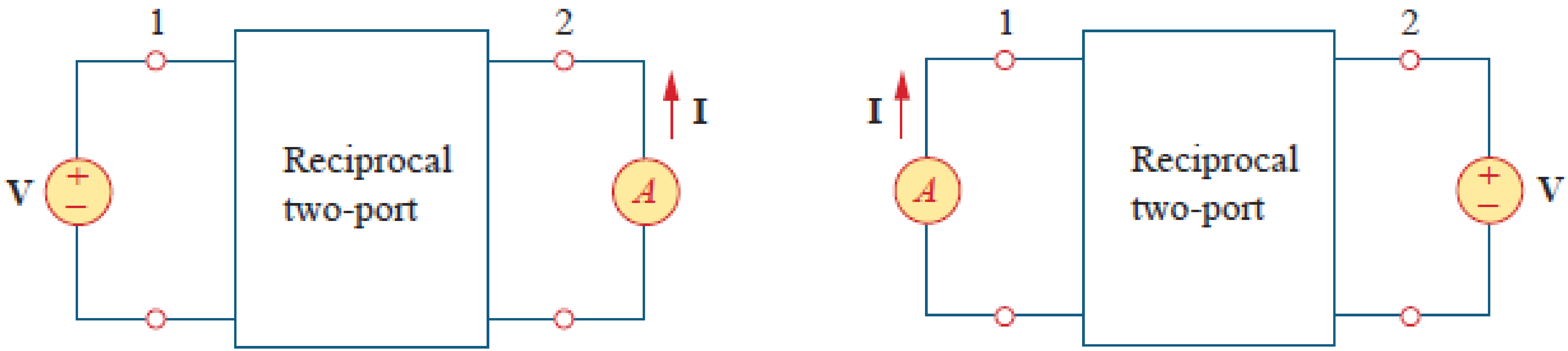
When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be *symmetrical*. This implies that the network has mirrorlike symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($\mathbf{z}_{12} = \mathbf{z}_{21}$), and the two-port is said to be *reciprocal*. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same.



Interchanging a voltage source at one port with an ideal ammeter at the other port produces the same reading in a reciprocal two-port.

$$\text{if } z_{12} = z_{21}$$



(a)

(b)

$$V = z_{12} I$$

$$V = z_{21} I$$

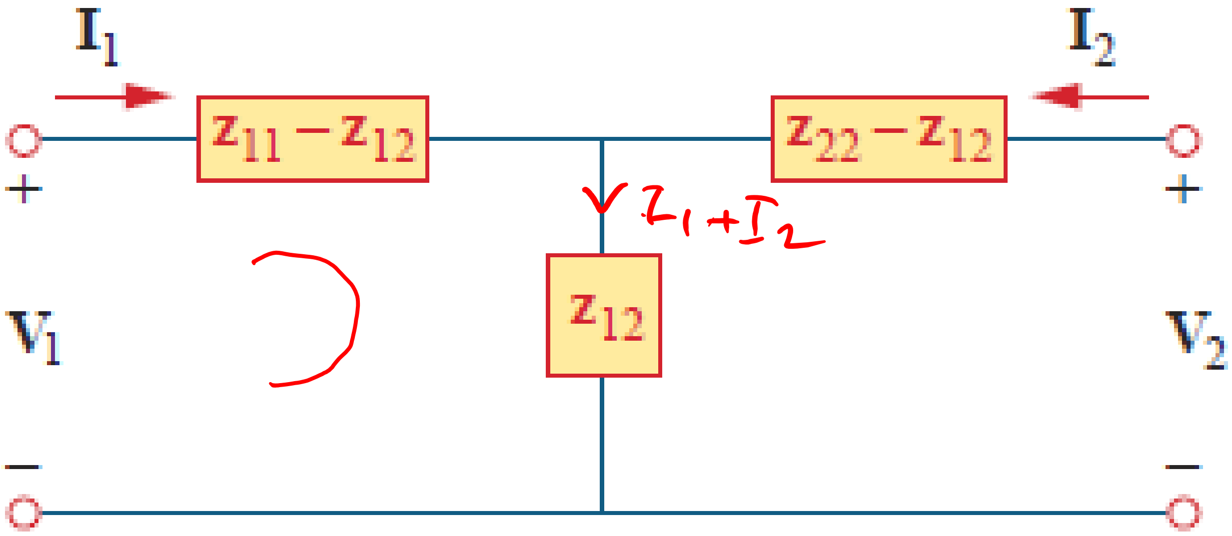
This means that if the points of excitation and response are interchanged, the transfer impedances remain the same

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$
$\rightarrow$ ①

From the diagram

$$V_1 = (Z_{11}-Z_{12})I_1 + Z_{12}(I_1+I_2)$$
$$= Z_{11}I_1 - Z_{12}I_1 + Z_{12}I_1 + Z_{12}I_2$$
$$\boxed{V_1 = Z_{11}I_1 + Z_{12}I_2}$$

Eqn 1 holds good



$$V_2 = (Z_{22}-Z_{12})I_2 + Z_{12}(I_1+I_2)$$
$$= Z_{22}I_2 - Z_{12}I_2 + Z_{12}I_1 + Z_{12}I_2$$
$$V_2 = Z_{12}I_1 + Z_{22}I_2$$

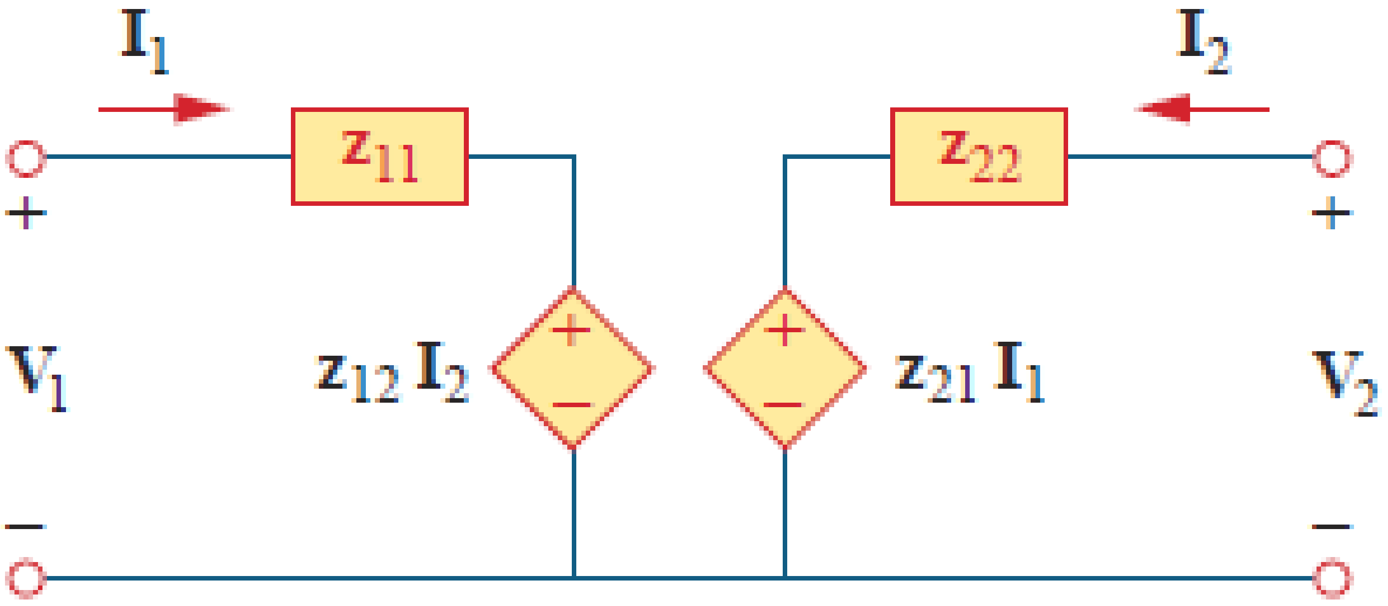
For reciprocal circuits

$$Z_{12} = Z_{21}$$

A reciprocal network can be replaced by the T-equivalent circuit

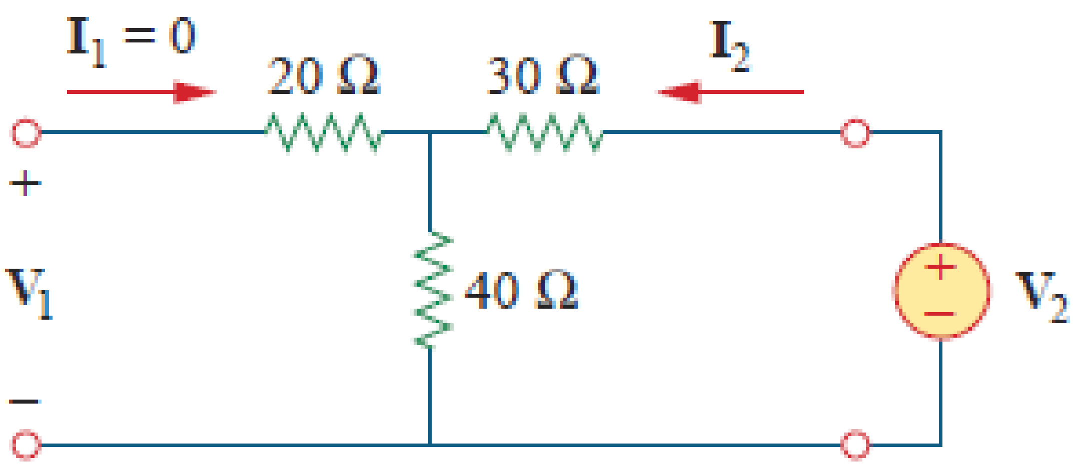
Eqn 1 holds good $\rightarrow \therefore \boxed{V_2 = Z_{21}I_1 + Z_{22}I_2}$

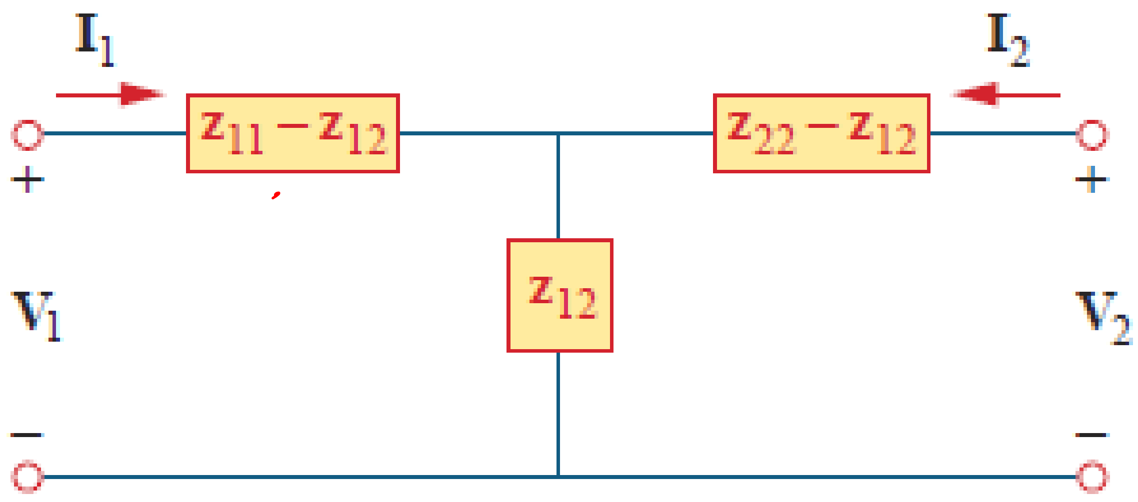
$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$



If the network is not reciprocal, the above general equivalent network is sufficent

METHOD 2 Alternatively, since there is no dependent source in the given circuit, $\mathbf{z}_{12} = \mathbf{z}_{21}$





$$\mathbf{z}_{12} = 40\ \Omega = \mathbf{z}_{21}$$

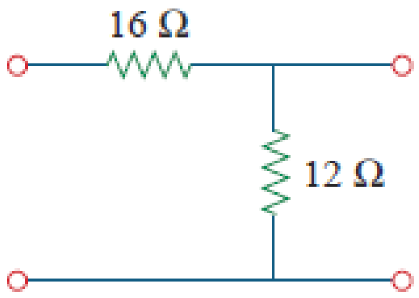
$$\mathbf{z}_{11} - \mathbf{z}_{12} = 20 \quad \Rightarrow \quad \mathbf{z}_{11} = 20 + \mathbf{z}_{12} = 60\ \Omega$$

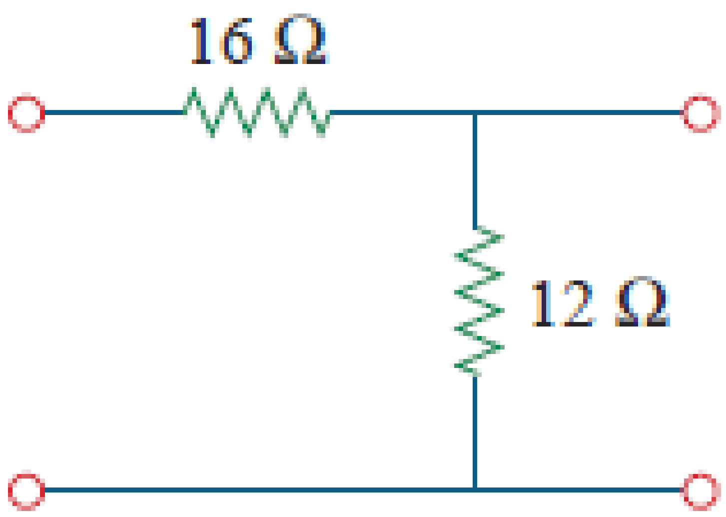
$$\mathbf{z}_{22} - \mathbf{z}_{12} = 30 \quad \Rightarrow \quad \mathbf{z}_{22} = 30 + \mathbf{z}_{12} = 70\ \Omega$$

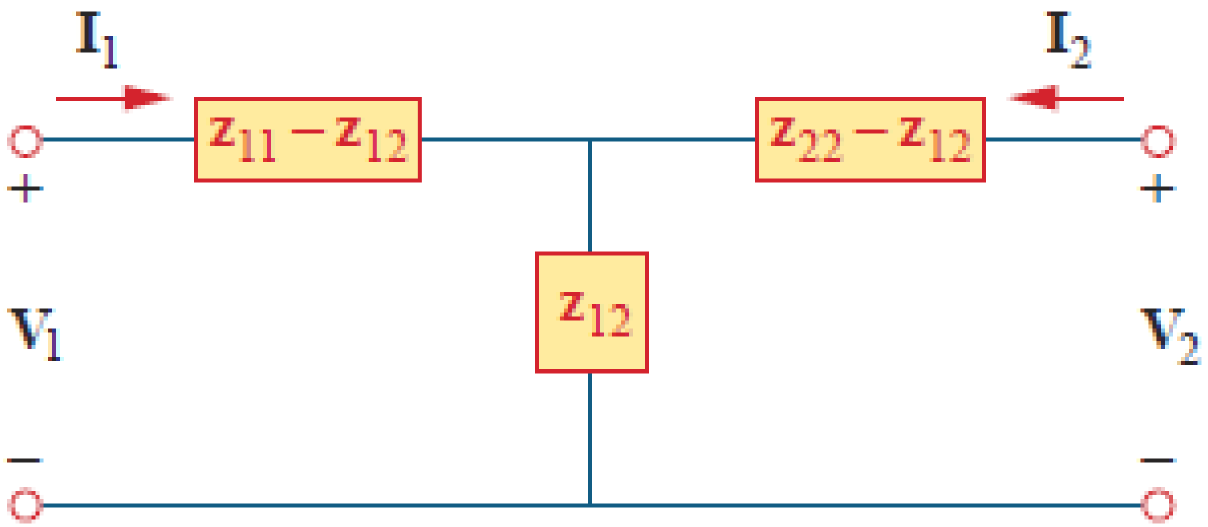
Practice Problem 19.1

Find the z parameters of the two-port network in Fig. 19.9.

Answer: $\mathbf{z}_{11} = 28\ \Omega$, $\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 12\ \Omega$.







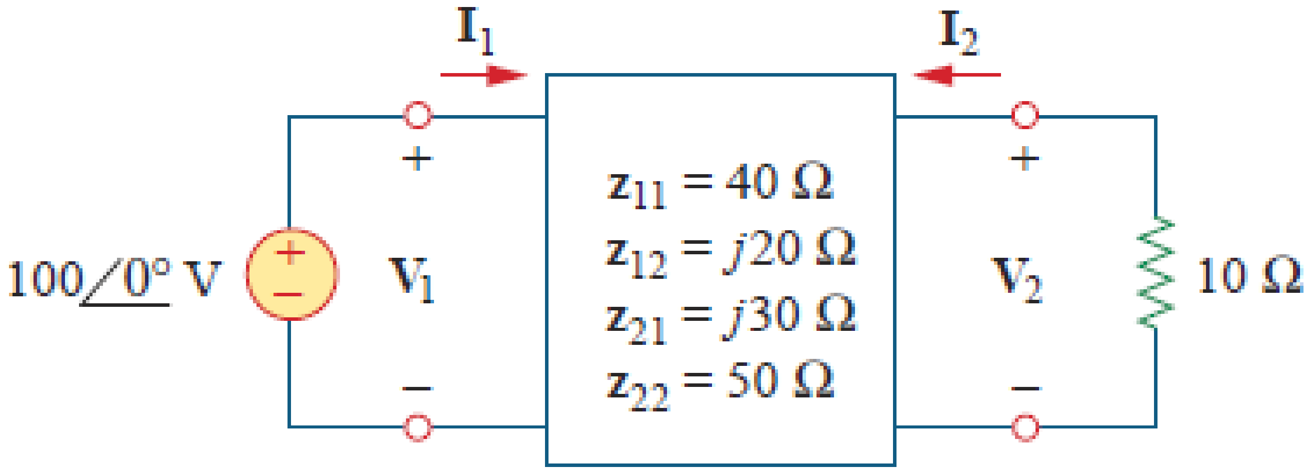
$Z_{11} - Z_{12} = 16\ \Omega$

$Z_{22} - Z_{12} = 0$

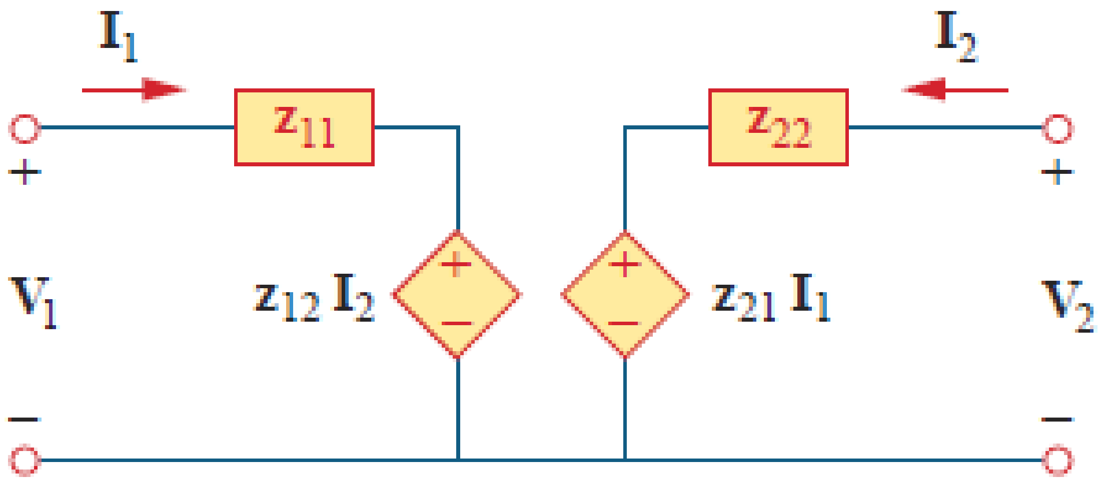
$Z_{12} = 12\ \Omega$

Example 19.2

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig.



This is not a reciprocal network. We may use the equivalent circuit in Fig below.



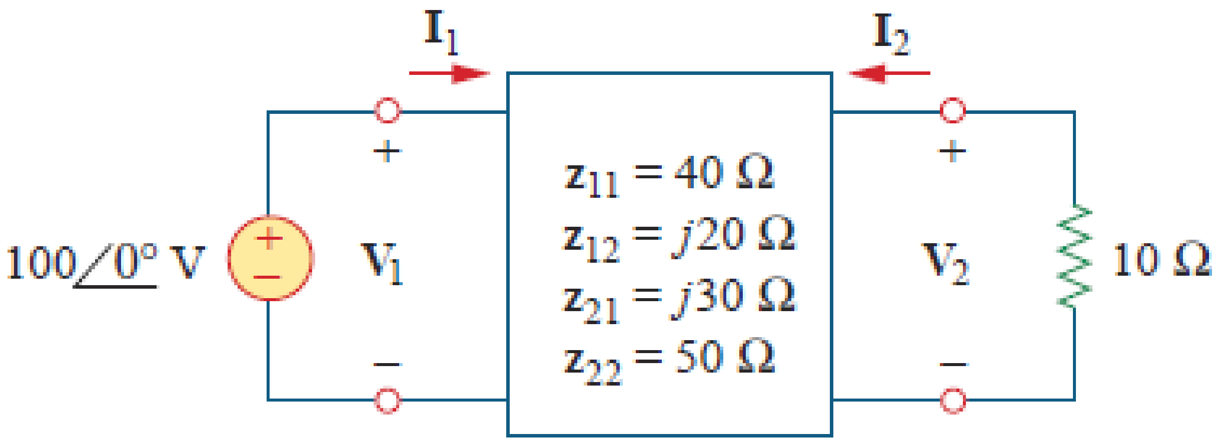
Phasors

$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned}}$$

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2$$

$$\mathbf{V}_1 = 100\underline{/0^\circ}, \qquad \mathbf{V}_2 = -10\mathbf{I}_2$$



$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2$$

$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{I}_1 = j2\mathbf{I}_2$$

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \tag{19.2.3}$$

$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \qquad \Rightarrow \qquad \mathbf{I}_1 = j2\mathbf{I}_2 \tag{19.2.4}$$

Substituting Eq. (19.2.4) into Eq. (19.2.3) gives

$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \qquad \Rightarrow \qquad \mathbf{I}_2 = \frac{100}{j100} = -j$$

From Eq. (19.2.4), $\mathbf{I}_1 = j2(-j) = 2$. Thus,

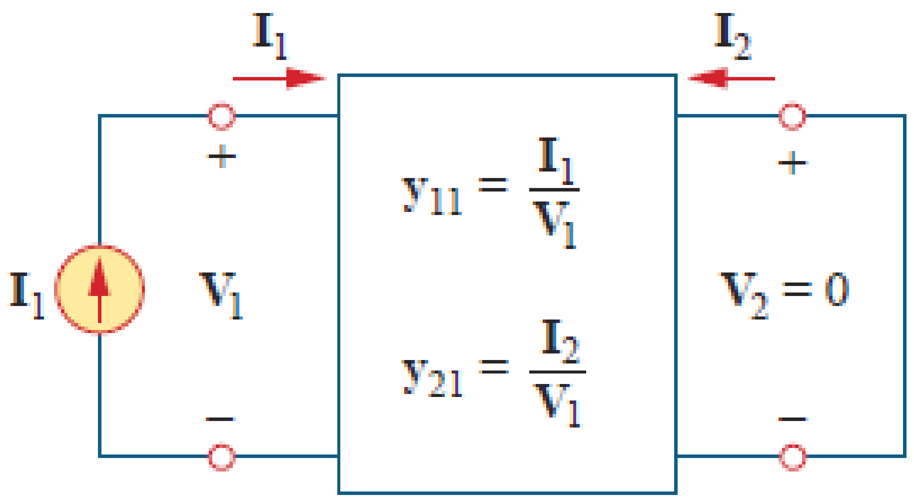
$$\mathbf{I}_1 = 2\underline{/0^\circ}\text{ A}, \qquad \mathbf{I}_2 = 1\underline{/-90^\circ}\text{ A}$$

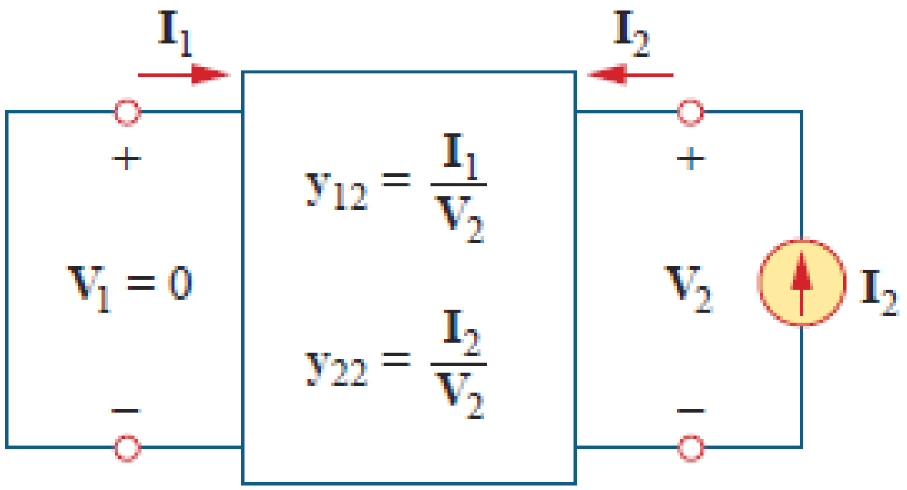
Admittance Parameters

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

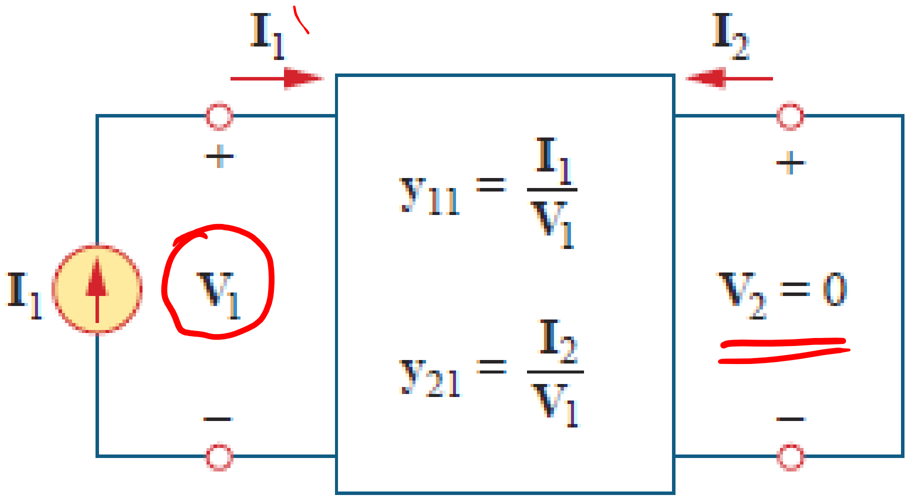
$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$
$$\mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}$$

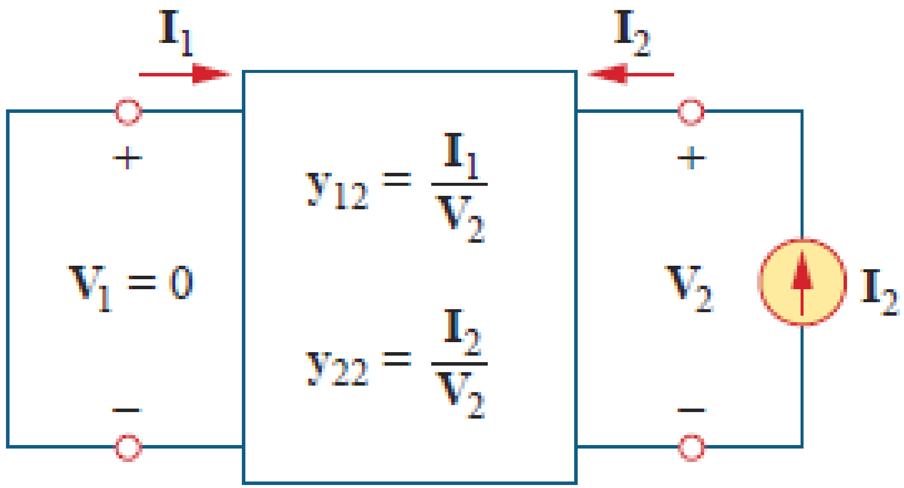




$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$





y_{11} = Short-circuit input admittance

y_{12} = Short-circuit transfer admittance from port 2 to port 1

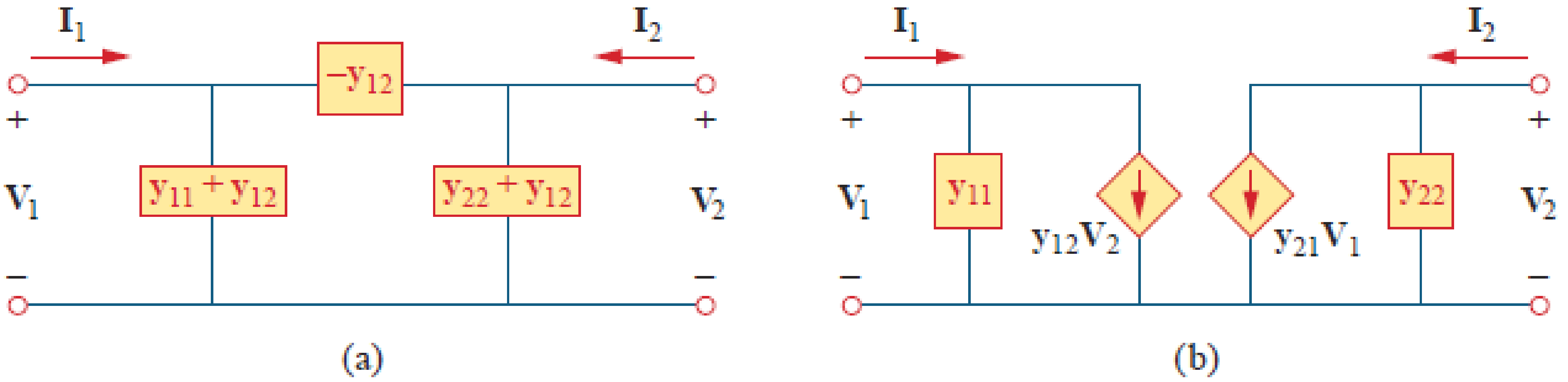
y_{21} = Short-circuit transfer admittance from port 1 to port 2

y_{22} = Short-circuit output admittance

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \end{aligned}$$

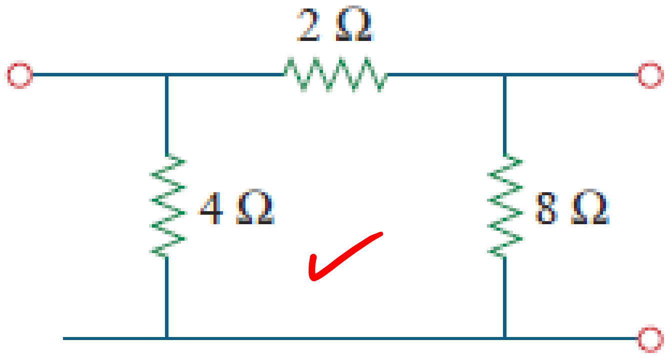
Reciprocal network is a linear two-port network that has no dependent sources, the transfer admittances are equal:

$$(\mathbf{y}_{12} = \mathbf{y}_{21})$$



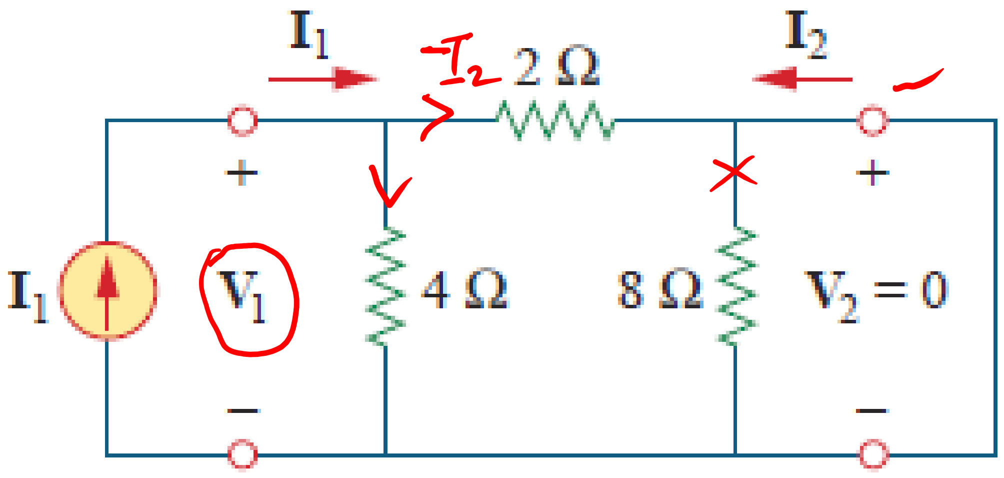
(a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

Obtain the y parameters for the Π network shown in Fig.



METHOD 1

To find y_{11} and y_{21},



$I_2 = \frac{4}{4+2} \times I_1$

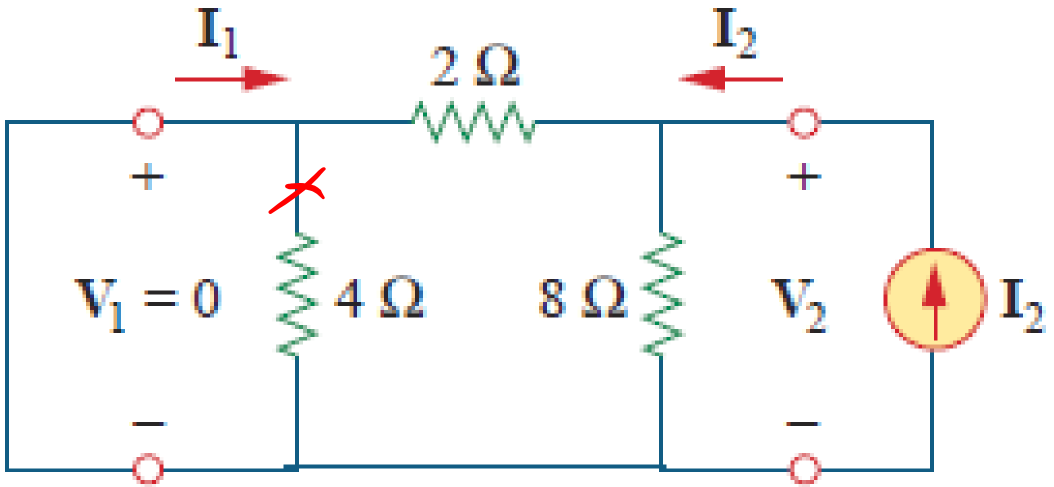
the 8-Ω resistor is short-circuited, the 2-Ω resistor is in parallel with the 4-Ω resistor. Hence,

$$\mathbf{V}_1 = \mathbf{I}_1(4 \parallel 2) = \frac{4}{3}\mathbf{I}_1, \qquad y_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = 0.75 \text{ S}$$

By current division,

$$-\mathbf{I}_2 = \frac{4}{4+2}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1, \qquad y_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-\frac{2}{3}\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = -0.5 \text{ S}$$

To get y_{12} and y_{22}



The 4-Ω resistor is short-circuited so that the 2-Ω and 8-Ω resistors are in parallel.

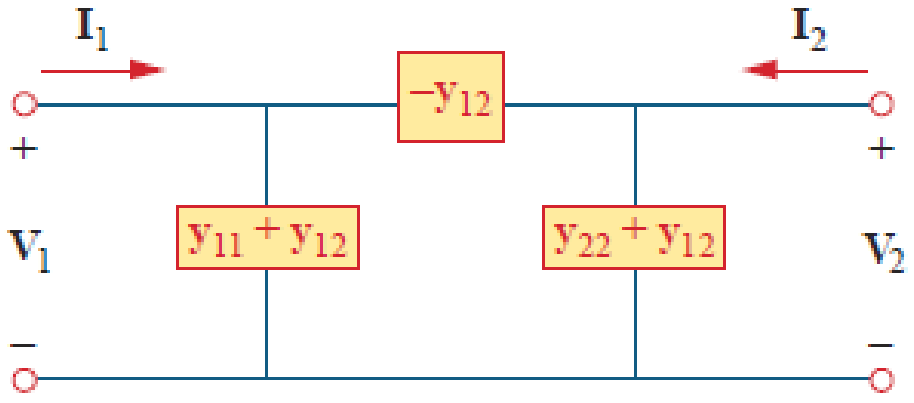
$$\mathbf{V}_2 = \mathbf{I}_2(8 \parallel 2) = \frac{8}{5}\mathbf{I}_2, \qquad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = \frac{5}{8} = 0.625\ \text{S}$$

By current division,

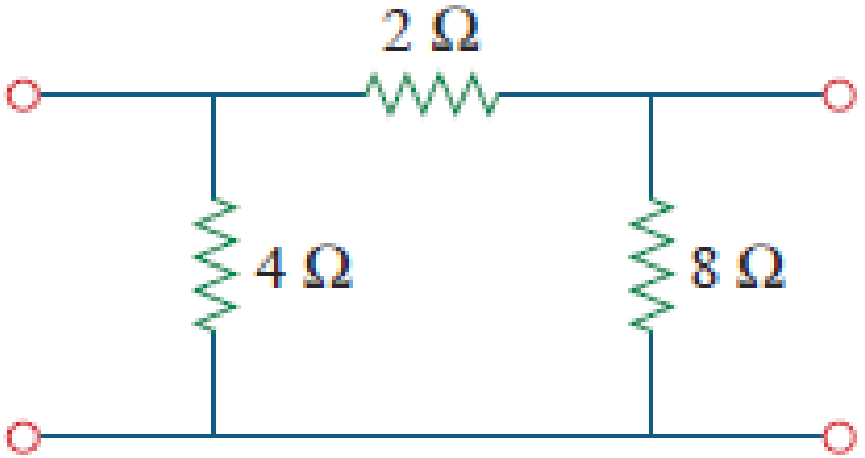
$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2, \qquad \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5\ \text{S}$$

METHOD 2 Alternatively, comparing Fig.

There are no dependent sources



Π-equivalent circuit (for reciprocal case only).



$$y_{12} = -\frac{1}{2}\,\text{S} = y_{21}$$

$$y_{11} + y_{12} = \frac{1}{4} \quad \Rightarrow \quad y_{11} = \frac{1}{4} - y_{12} = 0.75\ \text{S}$$

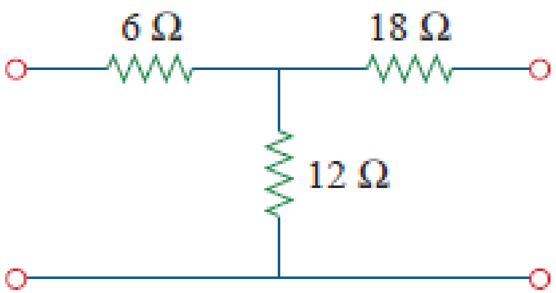
$$y_{22} + y_{12} = \frac{1}{8} \quad \Rightarrow \quad y_{22} = \frac{1}{8} - y_{12} = 0.625\ \text{S}$$

as obtained previously.

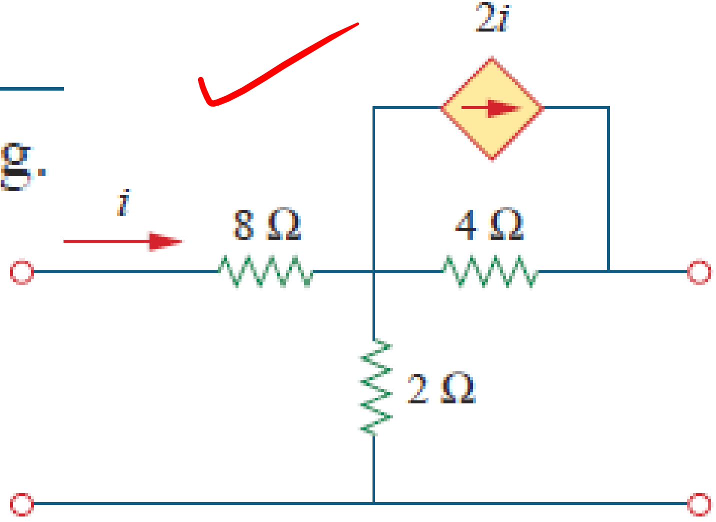
Practice Problem 19.3

Obtain the y parameters for the T network shown in Fig. 19.16.

Answer: $\mathbf{y}_{11} = 75.77$ mS, $\mathbf{y}_{12} = \mathbf{y}_{21} = -30.3$ mS, $\mathbf{y}_{22} = 45.47$ mS.

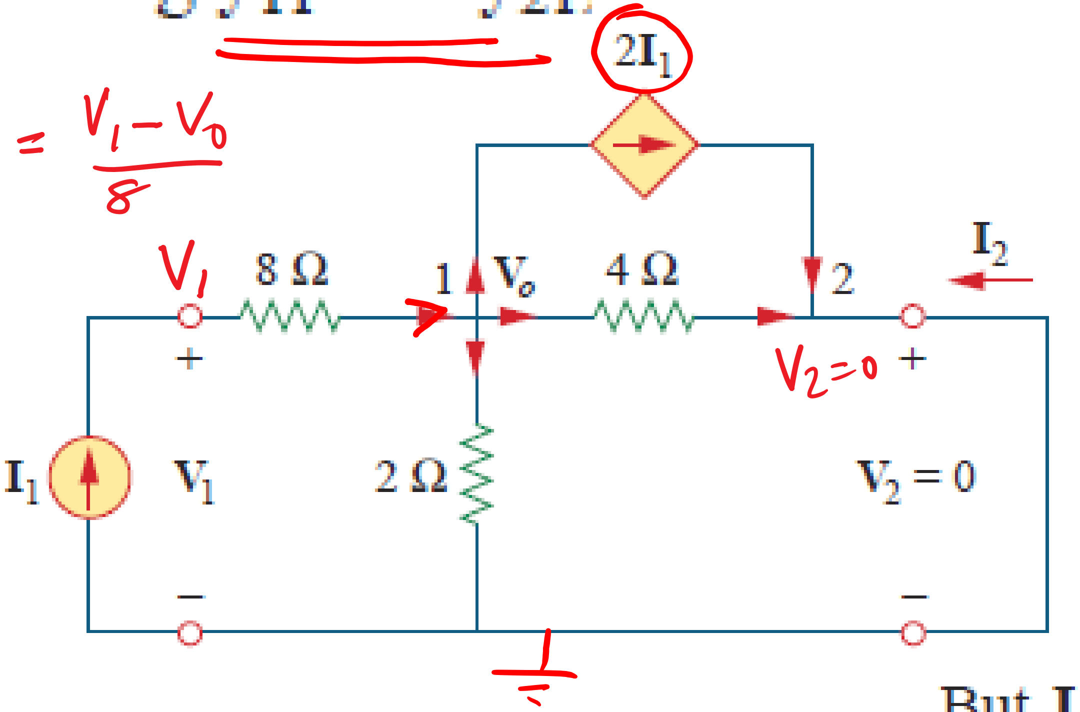


Determine the y parameters for the two-port shown in Fig.



finding y_{11} and y_{21}:

$I_1 = \frac{V_1 - V_o}{8}$



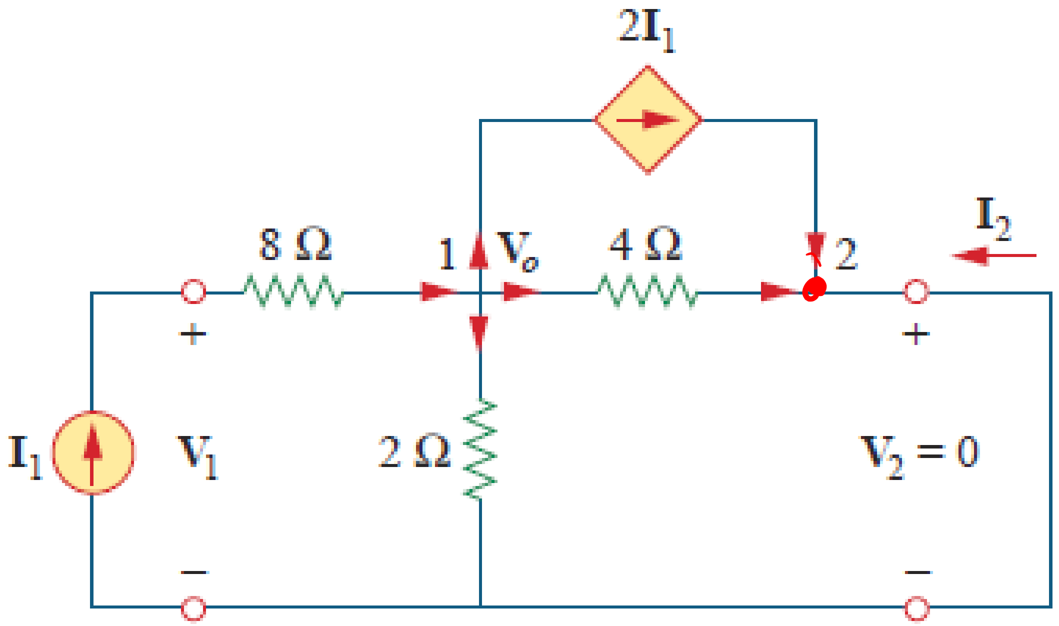
KCL

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - 0}{4}$$

But $\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8}$; therefore,

$$0 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} + \frac{3\mathbf{V}_o}{4}$$

$$0 = \mathbf{V}_1 - \mathbf{V}_o + 6\mathbf{V}_o \quad \Rightarrow \quad \mathbf{V}_1 = -5\mathbf{V}_o$$



Hence,

$$\mathbf{I}_1 = \frac{-5\mathbf{V}_o - \mathbf{V}_o}{8} = -0.75\mathbf{V}_o$$

and

$$\boxed{y_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{-0.75\mathbf{V}_o}{-5\mathbf{V}_o} = 0.15\text{ S}}$$

At node 2,

$$\frac{\mathbf{V}_o - 0}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

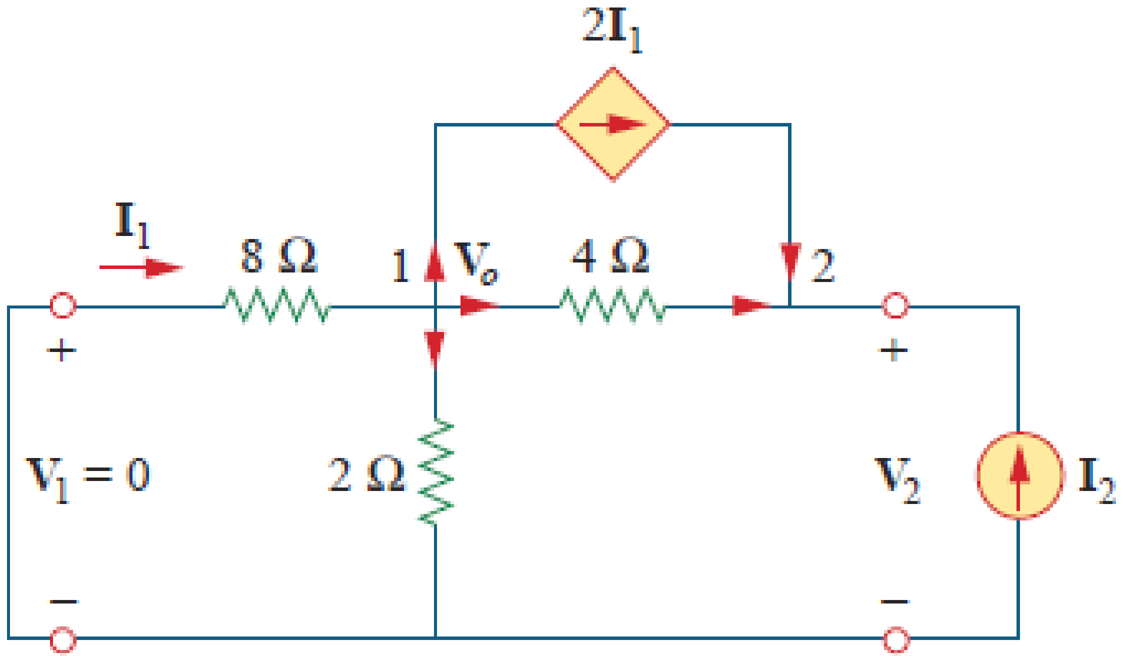
or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - 1.5\mathbf{V}_o = -1.25\mathbf{V}_o$$

Hence,

$$y_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1.25\mathbf{V}_o}{-5\mathbf{V}_o} = -0.25\text{ S}$$

Similarly, we get y_{12} and y_{22} using Fig.



At node 1,

$$\frac{0 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

But $\mathbf{I}_1 = \dfrac{0 - \mathbf{V}_o}{8}$; therefore,

$$0 = -\frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

or

$$0 = -\mathbf{V}_o + 4\mathbf{V}_o + 2\mathbf{V}_o - 2\mathbf{V}_2 \qquad \Rightarrow \qquad \mathbf{V}_2 = 2.5\mathbf{V}_o$$

Hence,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_o/8}{2.5\mathbf{V}_o} = -0.05 \text{ S}$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

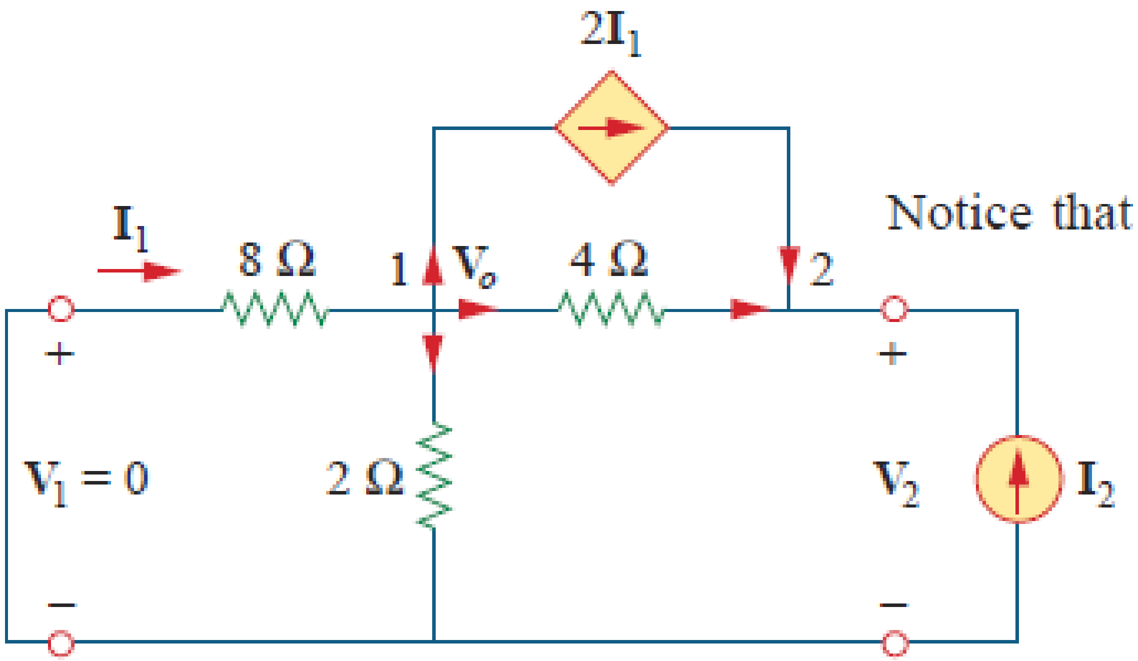
or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - \frac{1}{4}(2.5\mathbf{V}_o) - \frac{2\mathbf{V}_o}{8} = -0.625\mathbf{V}_o$$

Thus,

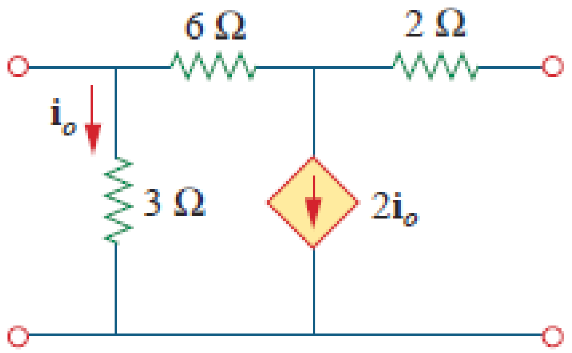
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_o}{2.5\mathbf{V}_o} = 0.25 \text{ S}$$

Notice that $\mathbf{y}_{12} \neq \mathbf{y}_{21}$ in this case, since the network is not reciprocal.



Practice Problem 19.4

Obtain the y parameters for the circuit in Fig. 19.19.



Answer: $\mathbf{y}_{11} = 0.625$ S, $\mathbf{y}_{12} = -0.125$ S, $\mathbf{y}_{21} = 0.375$ S, $\mathbf{y}_{22} = 0.125$ S.

Hybrid Parameters

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

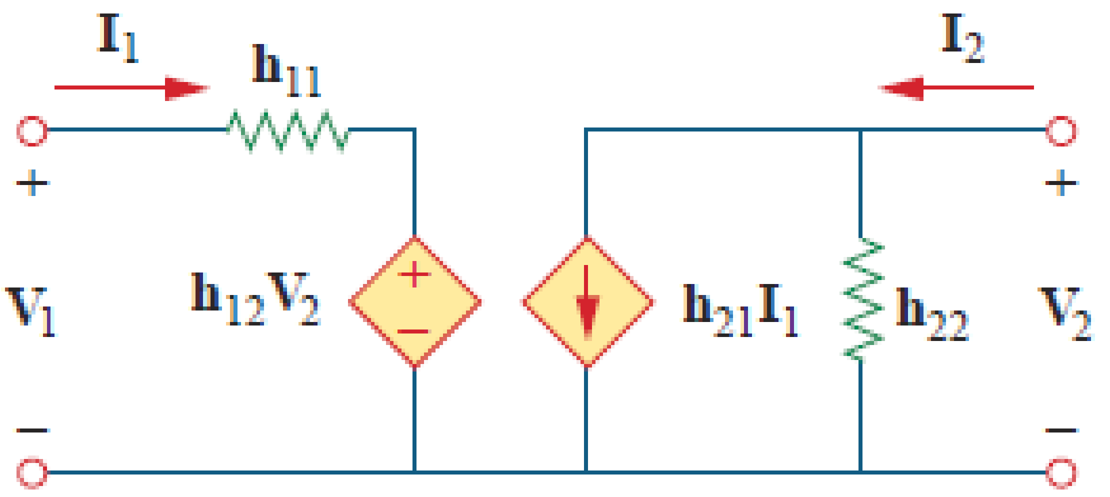
$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

$\mathbf{h}_{11}$ = Short-circuit input impedance

$\mathbf{h}_{12}$ = Open-circuit reverse voltage gain

$\mathbf{h}_{21}$ = Short-circuit forward current gain

$\mathbf{h}_{22}$ = Open-circuit output admittance



For reciprocal networks

$$\mathbf{h}_{12} = -\mathbf{h}_{21}$$

A set of parameters closely related to the h parameters are the *g parameters* or *inverse hybrid parameters*. These are used to describe the terminal currents and voltages as

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

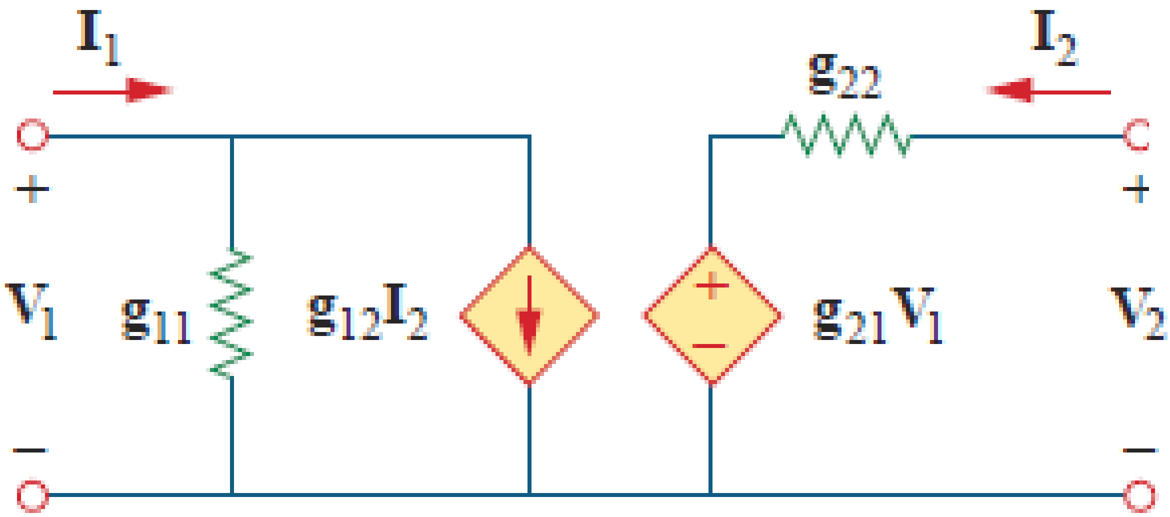
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{g}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$\mathbf{g}_{11}$ = Open-circuit input admittance

$\mathbf{g}_{12}$ = Short-circuit reverse current gain

$\mathbf{g}_{21}$ = Open-circuit forward voltage gain

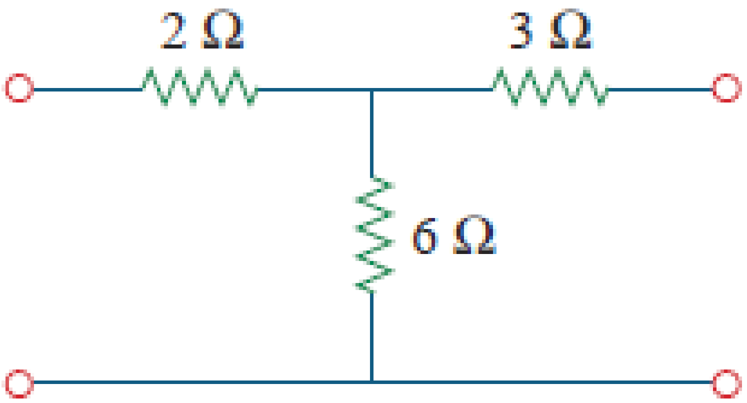
$\mathbf{g}_{22}$ = Short-circuit output impedance

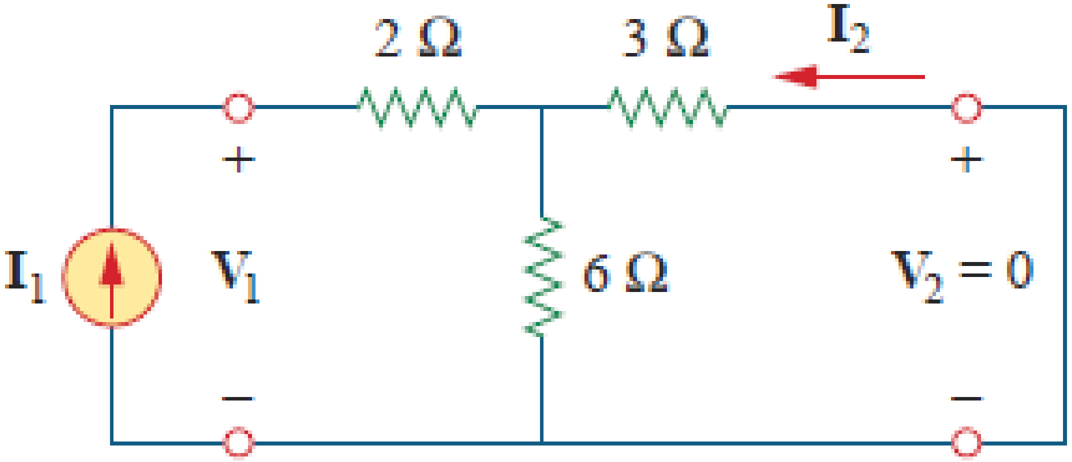


$$\mathbf{g}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{g}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}$$

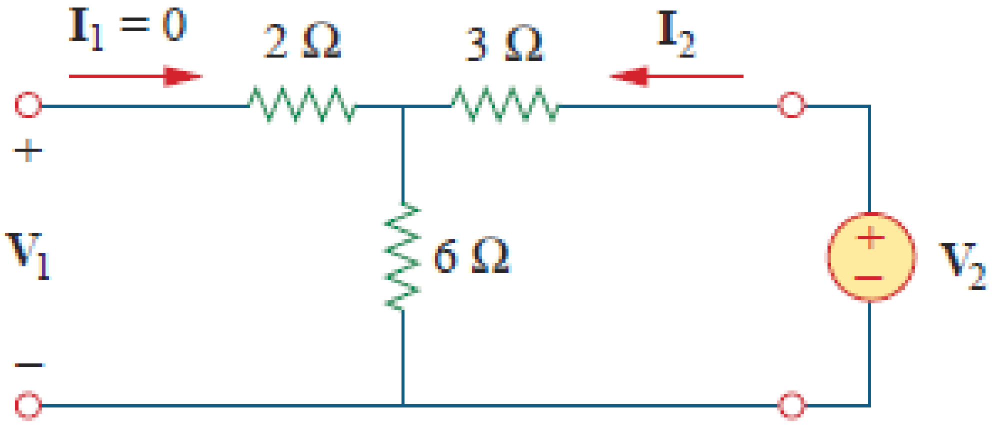
Find the hybrid parameters for the two-port network of Fig.





(a)

$$h_{11} = \frac{V_1}{I_1} \qquad h_{21} = \frac{I_2}{I_1}$$



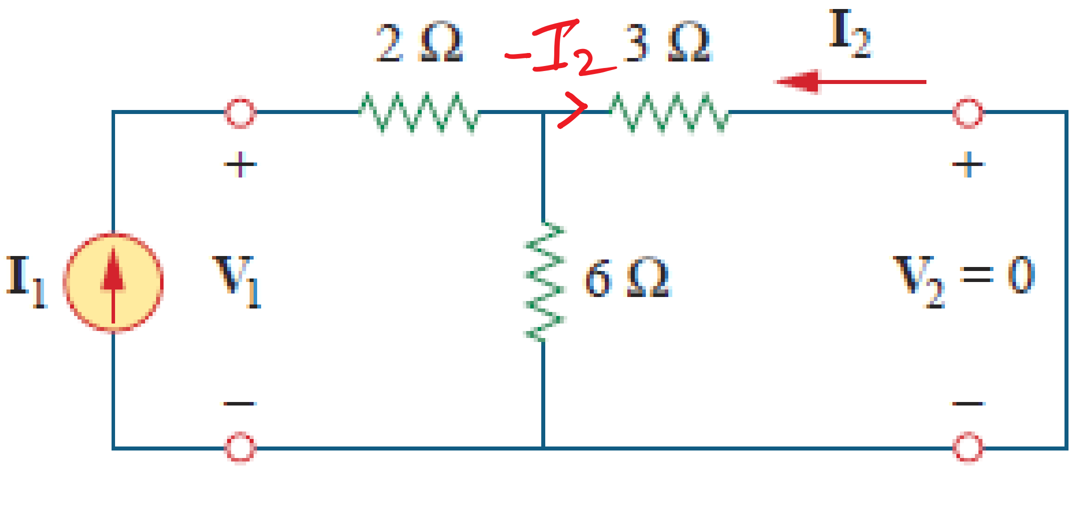
(b)

$$h_{12} = \frac{V_1}{V_2} \qquad h_{22} = \frac{I_2}{V_2}$$

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$



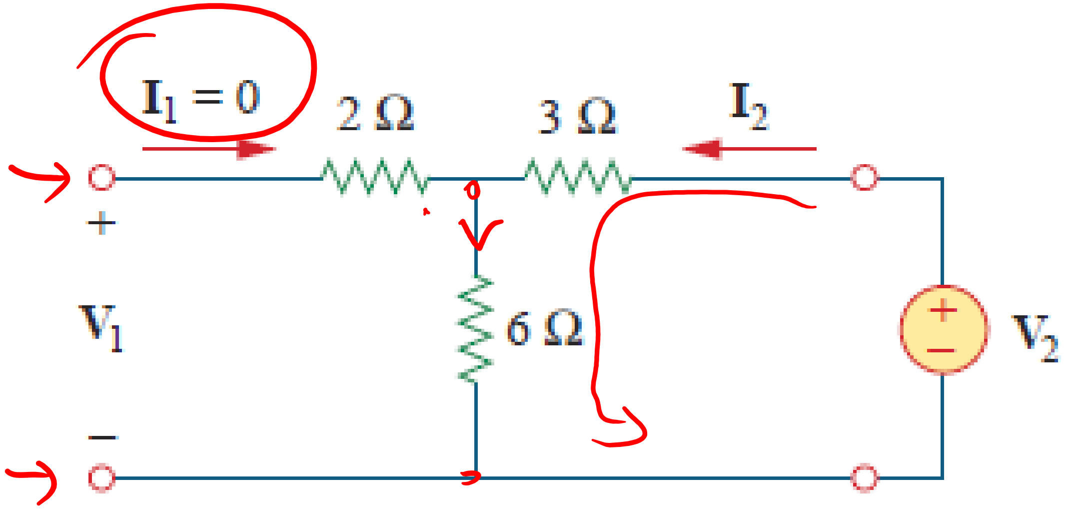
(a)

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$

$V_2 = 3I_2 + 6I_2$; $9I_2 = V_2$



(b)

$$V_1 = \frac{6}{6+3}V_2 = \frac{2}{3}V_2$$

$$h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

$$V_2 = (3+6)I_2 = 9I_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{9}\,S$$

V_1 = Voltage across 6Ω resistance

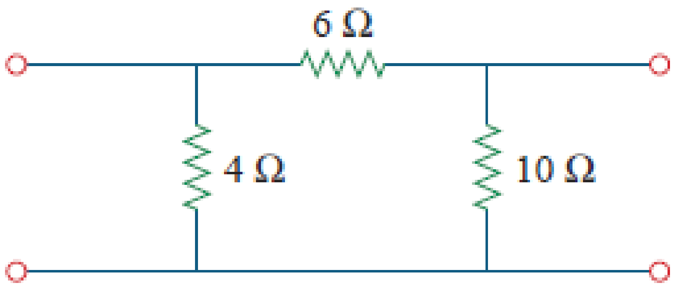
$$I_2 = \frac{V_2}{9}$$

$$V_1 = \frac{6V_2}{9} = \frac{2}{3}V_2$$

Practice Problem 19.5

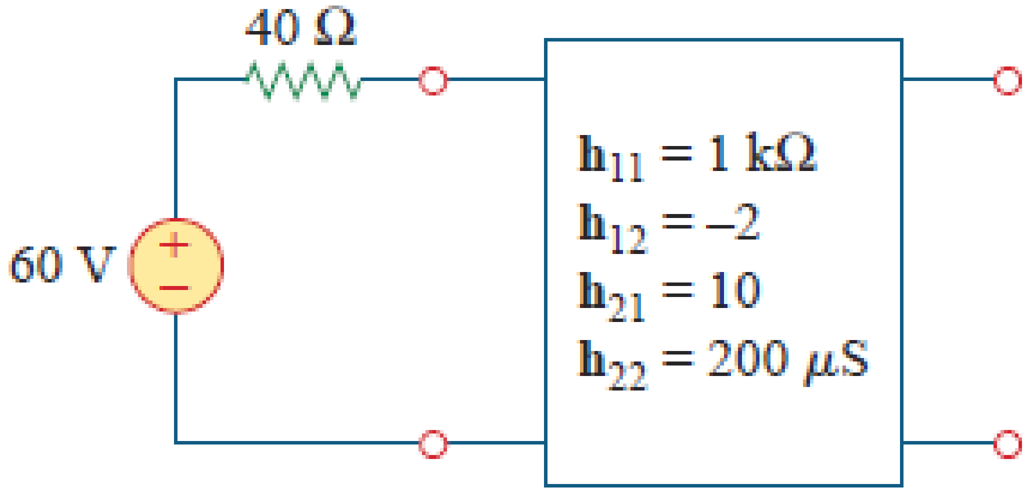
Determine the h parameters for the circuit in Fig. 19.24.

Answer: $\mathbf{h}_{11} = 2.4\ \Omega$, $\mathbf{h}_{12} = 0.4$, $\mathbf{h}_{21} = -0.4$, $\mathbf{h}_{22} = 200$ mS.



Determine the Thevenin equivalent at the output port of the circuit in Fig.

To Find Z_{TH} + V_{TH}

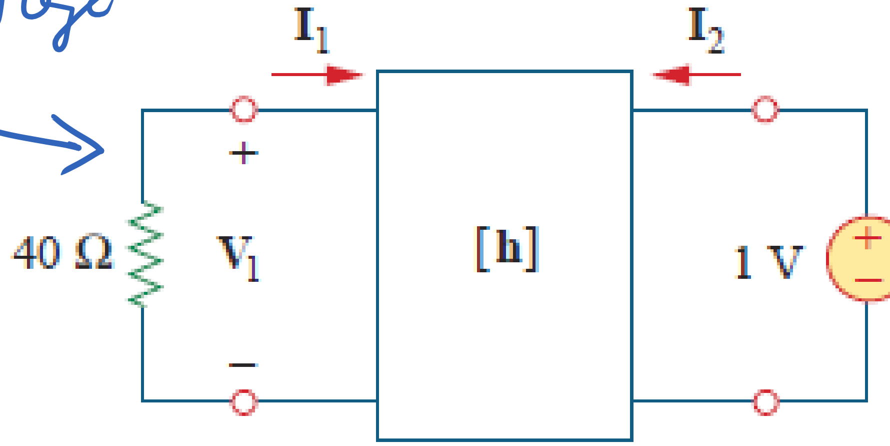


$$V_1 = h_{11}I_1 + h_{12}V_2 \longrightarrow (1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \longrightarrow (2)$$

Z_{TH}

Short the voltage source



(a)

Connect test voltage source of 1V

Using (3) in (1)

$$\longrightarrow -40I_1 = h_{11}I_1 + h_{12}$$

But $V_2 = 1$, and $V_1 = -40I_1 \longrightarrow (3)$

$$I_1 = -\frac{h_{12}}{40 + h_{11}} \longrightarrow (4)$$

Since $V_2 = 1V$; Eqn 2 becomes

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22} \longrightarrow 5$$

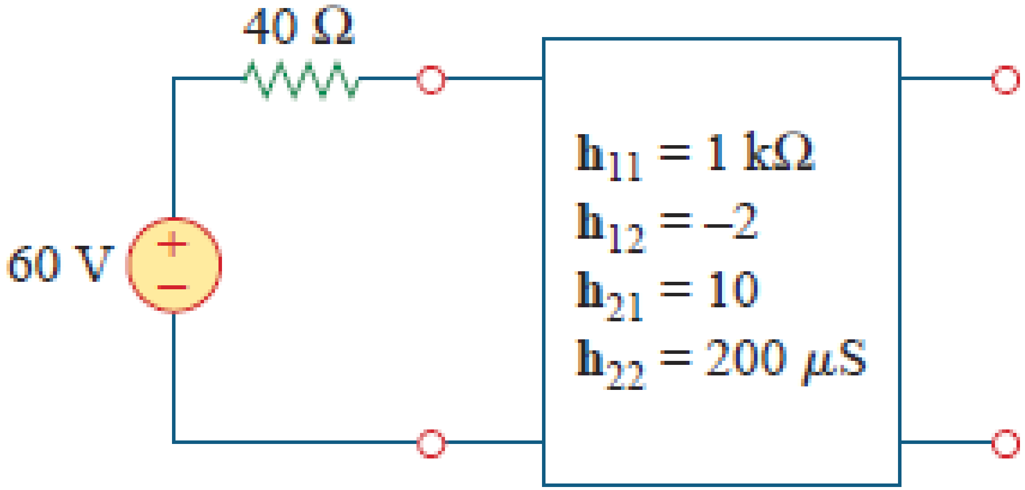
Using Eqn 4 in Eqn 5

$$\mathbf{I}_2 = \mathbf{h}_{22} - \frac{\mathbf{h}_{21}\mathbf{h}_{12}}{\mathbf{h}_{11} + 40} = \frac{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{21}\mathbf{h}_{12} + \mathbf{h}_{22}40}{\mathbf{h}_{11} + 40} \longrightarrow 6$$

Therefore,

$$\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \frac{\mathbf{h}_{11} + 40}{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{21}\mathbf{h}_{12} + \mathbf{h}_{22}40} \longrightarrow 7$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \frac{\mathbf{h}_{11} + 40}{\mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{21}\mathbf{h}_{12} + \mathbf{h}_{22}40}$$

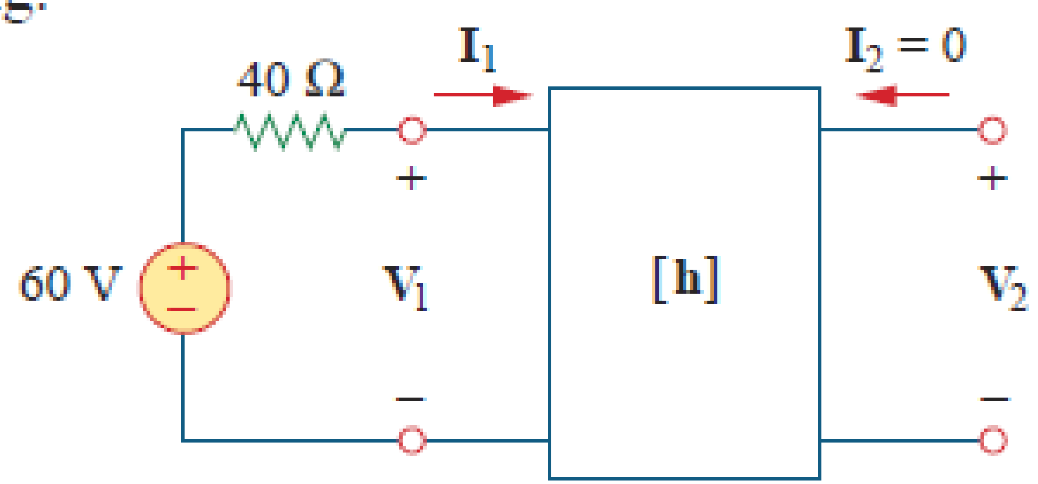


Substituting the values of the *h* parameters,

$$\mathbf{Z}_{Th} = \frac{1000 + 40}{10^3 \times 200 \times 10^{-6} + 20 + 40 \times 200 \times 10^{-6}}$$

$$= \frac{1040}{20.21} = 51.46\ \Omega$$

To get $\mathbf{V}_{Th}$, we find the open-circuit voltage $\mathbf{V}_2$ in Fig.



Apply KVL in the input port

$$-60 + 40\mathbf{I}_1 + \mathbf{V}_1 = 0 \quad \Rightarrow \quad \boxed{\mathbf{V}_1 = 60 - 40\mathbf{I}_1 \longrightarrow (8)}$$

At the output,

$$\boxed{\mathbf{I}_2 = 0} \longrightarrow (9)$$

Using Eqn 8 & 9 in Eqns. 1 and 2

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \rightarrow (1)$$

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \rightarrow (2)$$

$$60 - 40\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

$$\boxed{60 = (\mathbf{h}_{11} + 40)\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2} \rightarrow (10)$$

$$0 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \quad \Rightarrow \quad \boxed{\mathbf{I}_1 = -\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2} \rightarrow (11)$$

Using Eqn 11 in Eqn 10

$$60 = \left[-(\mathbf{h}_{11} + 40)\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}} + \mathbf{h}_{12}\right]\mathbf{V}_2$$

or

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_2 = \frac{60}{-(\mathbf{h}_{11} + 40)\mathbf{h}_{22}/\mathbf{h}_{21} + \mathbf{h}_{12}} = \frac{60\mathbf{h}_{21}}{\mathbf{h}_{12}\mathbf{h}_{21} - \mathbf{h}_{11}\mathbf{h}_{22} - 40\mathbf{h}_{22}}$$

Substituting the values of the h parameters,

$$\mathbf{V}_{\mathrm{Th}} = \frac{60 \times 10}{-20.21} = \underline{-29.69\ \mathrm{V}}$$

V_{TH} is the open circuit voltage