† Relationships Between Parameters

Given the z parameters, let us obtain the y parameters.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}]^{-1} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{z}]^{-1}$$

The adjoint of the [z] matrix is

$=$

$$\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$\boxed{[y] = [z]^{-1}}$$

and its determinant is (z)

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{\Delta_z} = \begin{bmatrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{11}}{\Delta z} \end{bmatrix}$$

$$y_{11} = \frac{z_{22}}{\Delta_z}, \qquad y_{12} = -\frac{z_{12}}{\Delta_z}, \qquad y_{21} = -\frac{z_{21}}{\Delta_z}, \qquad y_{22} = \frac{z_{11}}{\Delta_z}$$

let us determine the h parameters from the z parameters.

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \checkmark$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \checkmark$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ -\dfrac{\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}\mathbf{V}_2$$

$$\mathbf{I}_2 = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{z}_{22}}\mathbf{V}_2$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{21} = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}}$$

	$\mathbf{z}$	$\mathbf{y}$	$\mathbf{h}$
$\mathbf{z}$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y} \\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$
$\mathbf{y}$	$\begin{matrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$
$\mathbf{h}$	$\begin{matrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$
$\mathbf{g}$	$\begin{matrix} \frac{1}{z_{11}} & -\frac{z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta_y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ -\frac{y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta_h} & -\frac{h_{12}}{\Delta_h} \\ -\frac{h_{21}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{matrix}$

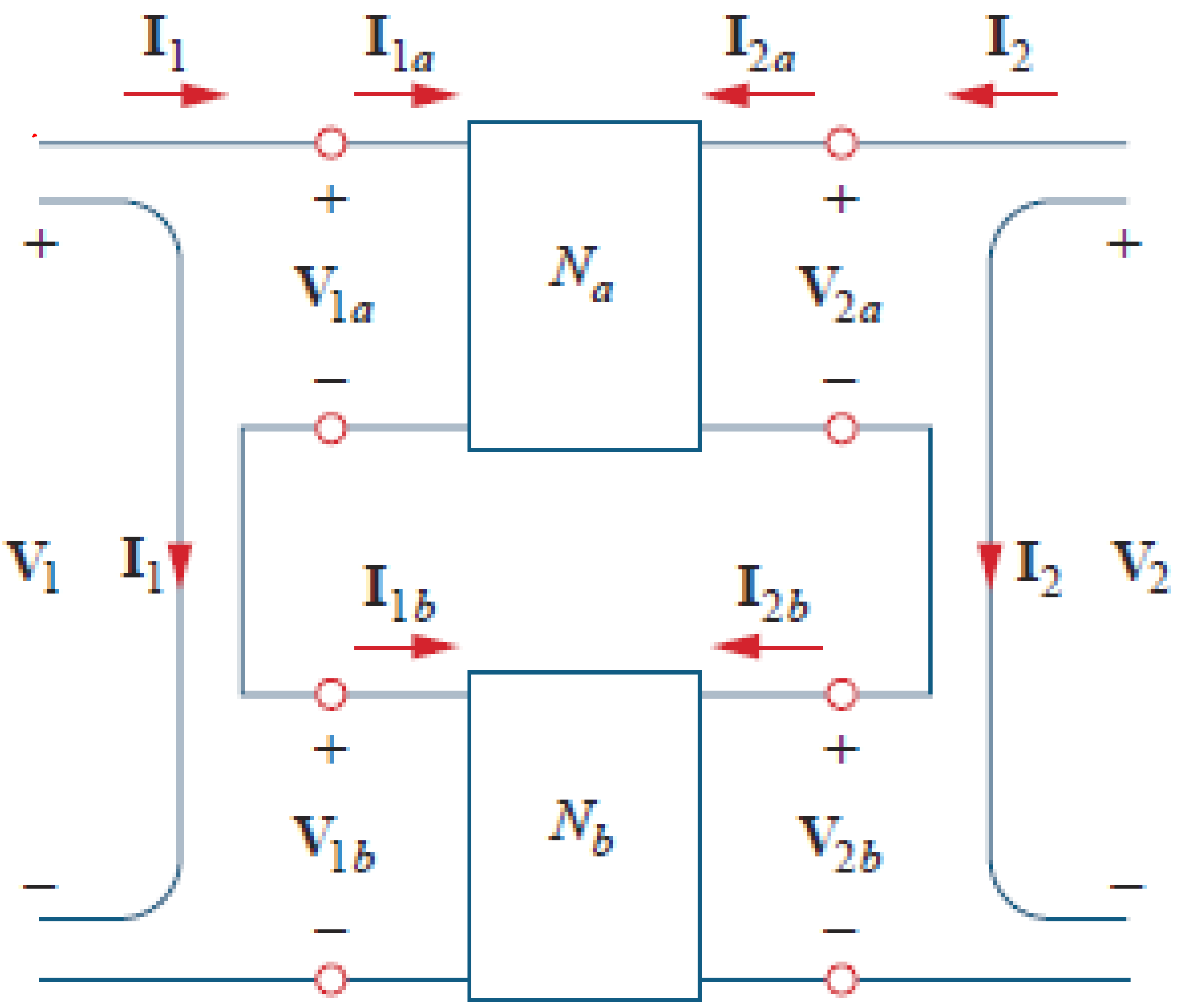
$$\Delta_z = z_{11}z_{22} - z_{12}z_{21},$$
$$\Delta_y = y_{11}y_{22} - y_{12}y_{21},$$

$$\Delta_h = h_{11}h_{22} - h_{12}h_{21},$$
$$\Delta_g = g_{11}g_{22} - g_{12}g_{21},$$

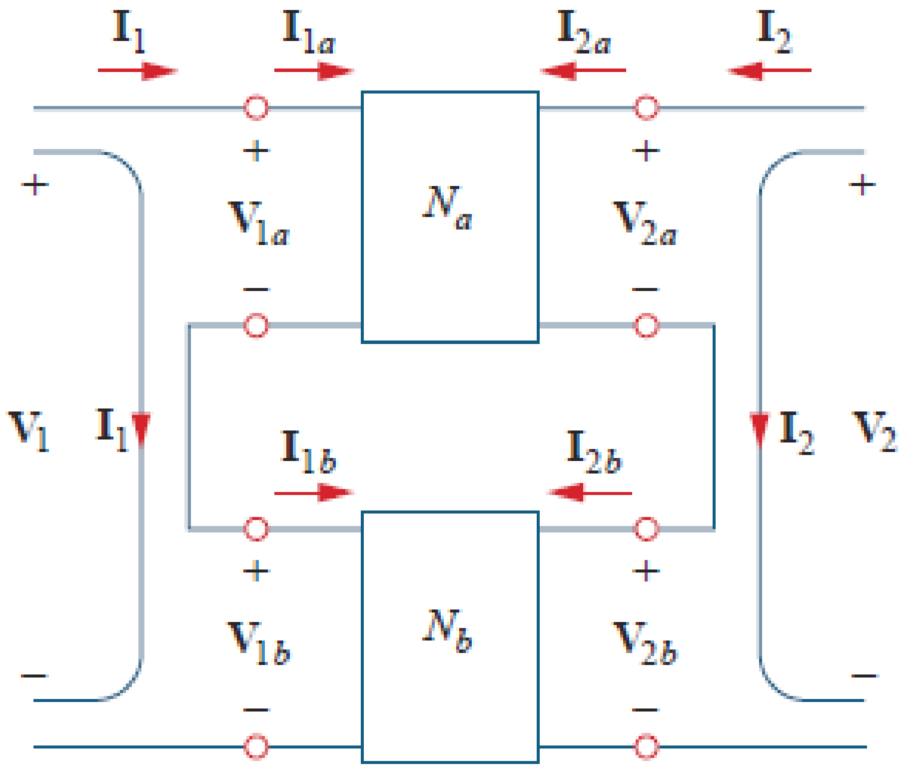
Interconnection of Networks

- A large, complex network may be divided into subnetworks for the purposes of analysis and design.
- The subnet works are modeled as two port networks, interconnected to form the original network.
- The two-port networks are building blocks a complex network.
- The interconnection can be in series, in parallel, or in cascade.

Series connection of two two-port networks



- The networks are regarded as being in series because their input currents are the same and their voltages add.
- In addition, each network has a common reference, and when the circuits are placed in series, the common reference points of each circuit are connected together.



$$\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$$

For network N_a,

$$V_{1a} = z_{11a}I_{1a} + z_{12a}I_{2a}$$
$$V_{2a} = z_{21a}I_{1a} + z_{22a}I_{2a}$$

for network N_b,

$$V_{1b} = z_{11b}I_{1b} + z_{12b}I_{2b}$$
$$V_{2b} = z_{21b}I_{1b} + z_{22b}I_{2b}$$

$$I_1 = I_{1a} = I_{1b}, \qquad I_2 = I_{2a} = I_{2b}$$

$$V_1 = V_{1a} + V_{1b} = (z_{11a} + z_{11b})I_1 + (z_{12a} + z_{12b})I_2 \checkmark$$
$$V_2 = V_{2a} + V_{2b} = (z_{21a} + z_{21b})I_1 + (z_{22a} + z_{22b})I_2 \checkmark$$

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2$$

Thus, the z parameters for the overall network are

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix}$$

or

$$\boxed{[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]}$$

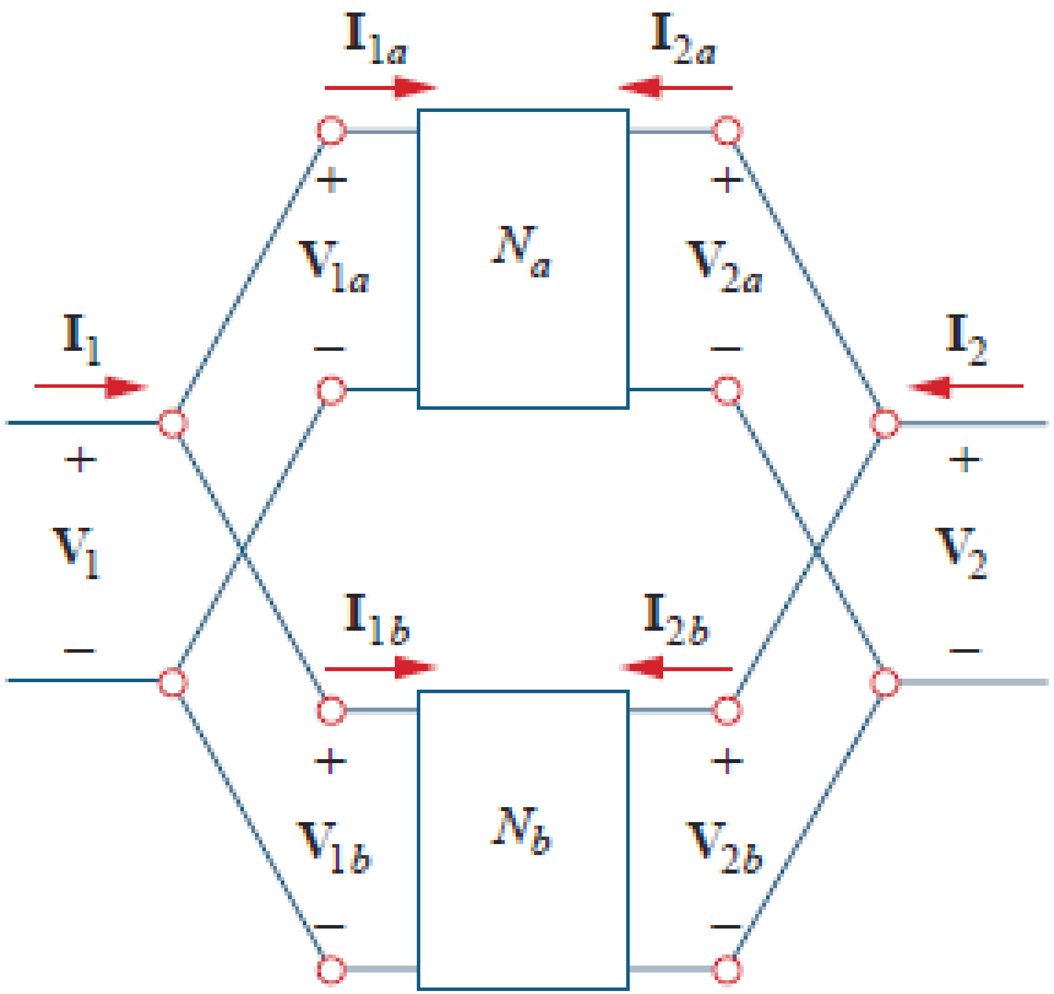
- **The z parameters for the overall network are the sum of the z parameters for the individual networks.**

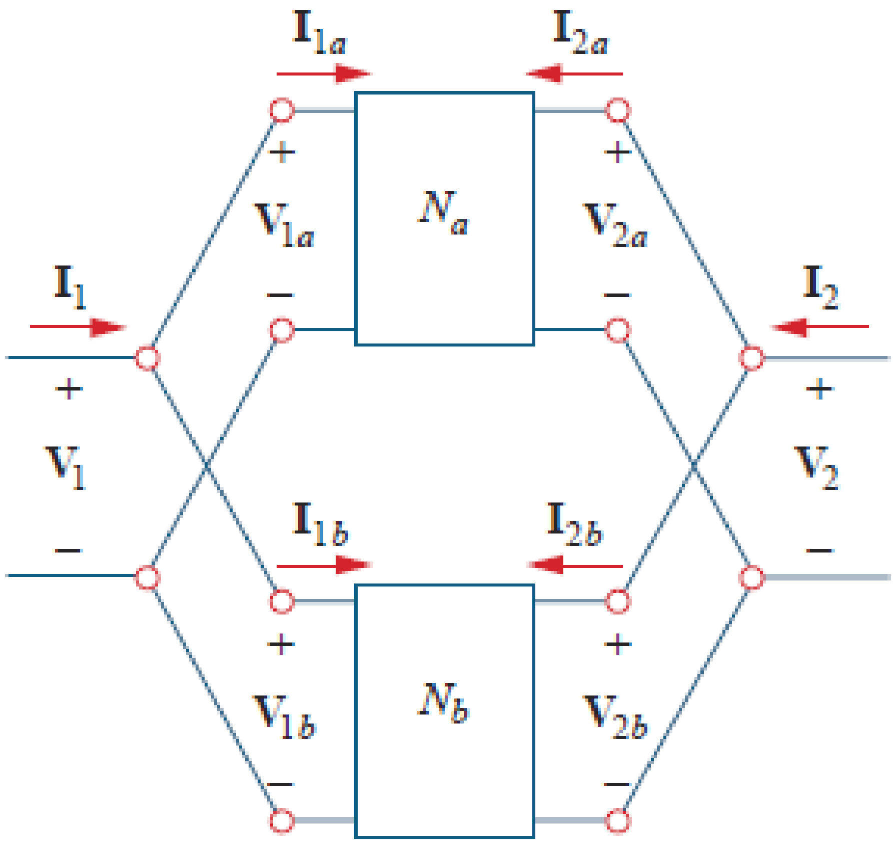
- **This can be extended to n networks in series.**

- **If two two-port networks in the h model, for example, are connected in series, we use Table given before to convert the [h] to [z] and then apply above equation. Finally convert the results back to [h] using the table**

Two-port networks in parallel.

- Two two-port networks are in parallel when their port voltages are equal
- The port currents of the larger network are the sums of the individual port currents.





$$\boxed{\begin{aligned}\mathbf{I}_1 &= \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2\\ \mathbf{I}_2 &= \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2\end{aligned}}$$

$$\mathbf{I}_{1a} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a}$$
$$\mathbf{I}_{2a} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{1b} = \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b}$$
$$\mathbf{I}_{2a} = \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b}$$

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}, \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b}$$
$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

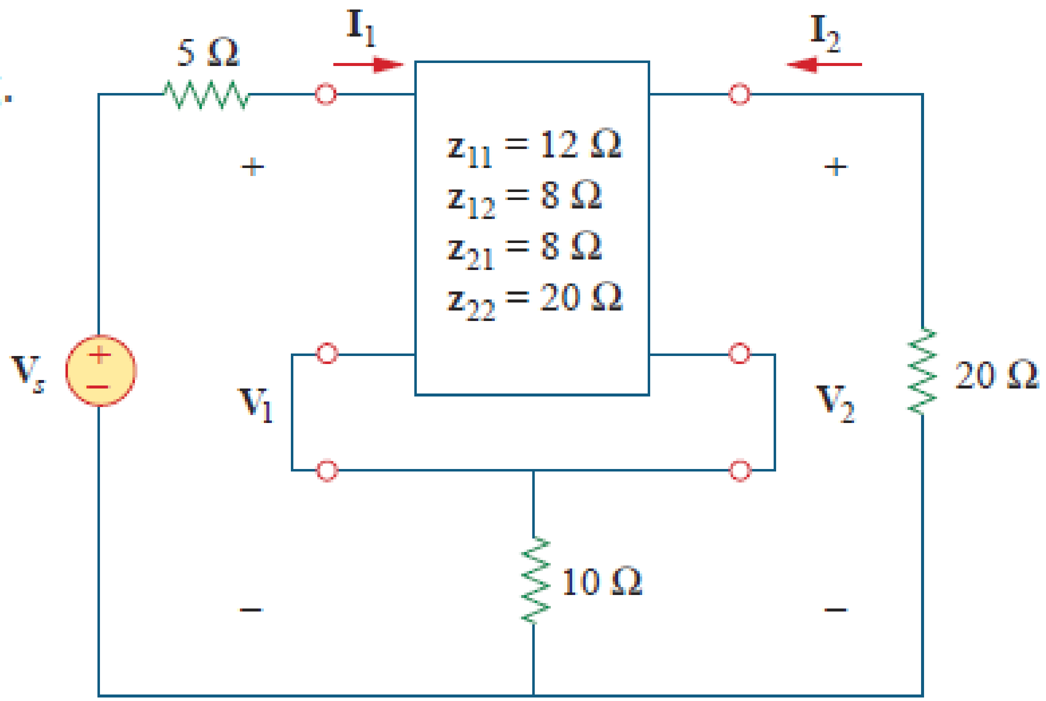
$$\mathbf{I}_1 = (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2$$
$$\mathbf{I}_2 = (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2$$

Thus, the y parameters for the overall network are

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix}$$

$$\boxed{[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]}$$

Example Evaluate $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig.



$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 = 22\mathbf{I}_1 + 18\mathbf{I}_2 \longrightarrow (1)$$

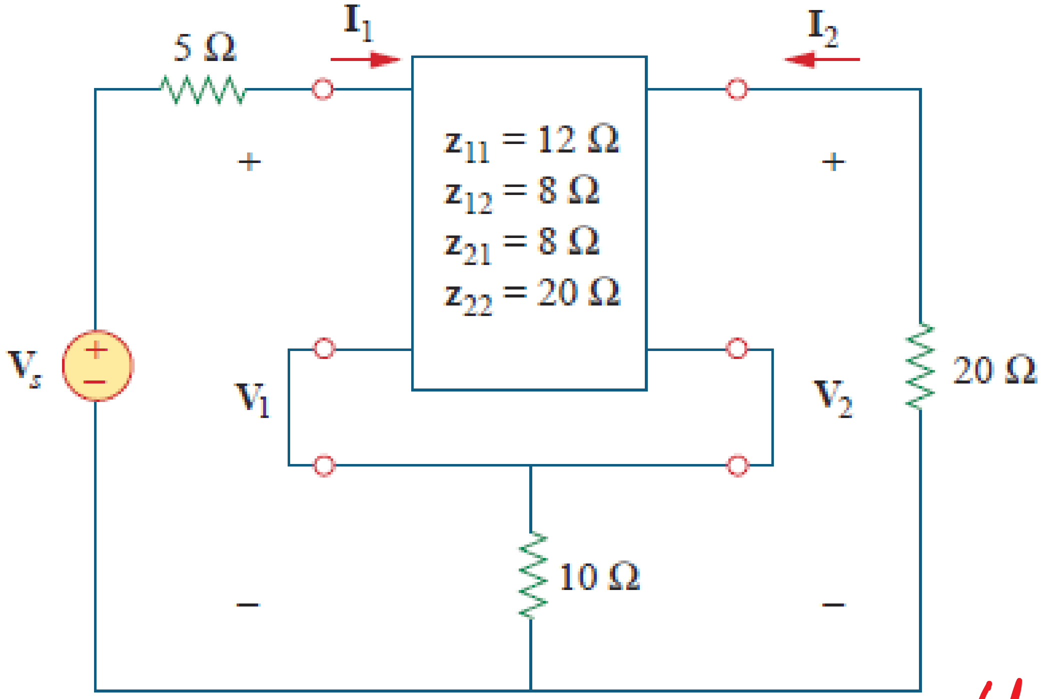
$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 = 18\mathbf{I}_1 + 30\mathbf{I}_2 \longrightarrow (2)$$

This may be regarded as two two-ports in series. For N_b,

$$\mathbf{z}_{12b} = \mathbf{z}_{21b} = 10 = \mathbf{z}_{11b} = \mathbf{z}_{22b}$$

Thus,

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$



Also, at the input port

$$V_1 = V_s - 5I_1 \longrightarrow (3)$$

and at the output port

$$V_2 = -20I_2 \quad \Rightarrow \quad I_2 = -\frac{V_2}{20} \longrightarrow (4)$$

Using (3) & (4) in (1)

$$V_s - 5I_1 = 22I_1 - \frac{18}{20}V_2 \quad \Rightarrow \quad V_s = 27I_1 - 0.9V_2 \longrightarrow (5)$$

Using (4) in (2)

$$V_2 = 18I_1 - \frac{30}{20}V_2 \quad \Rightarrow \quad I_1 = \frac{2.5}{18}V_2 \longrightarrow (6)$$

Using 6 in 5

$$\mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$$

And so,

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{1}{2.85} = 0.3509$$

Practice Problem 19.12

Find $\mathbf{V}_2/\mathbf{V}_s$ in the circuit in Fig. 19.43.

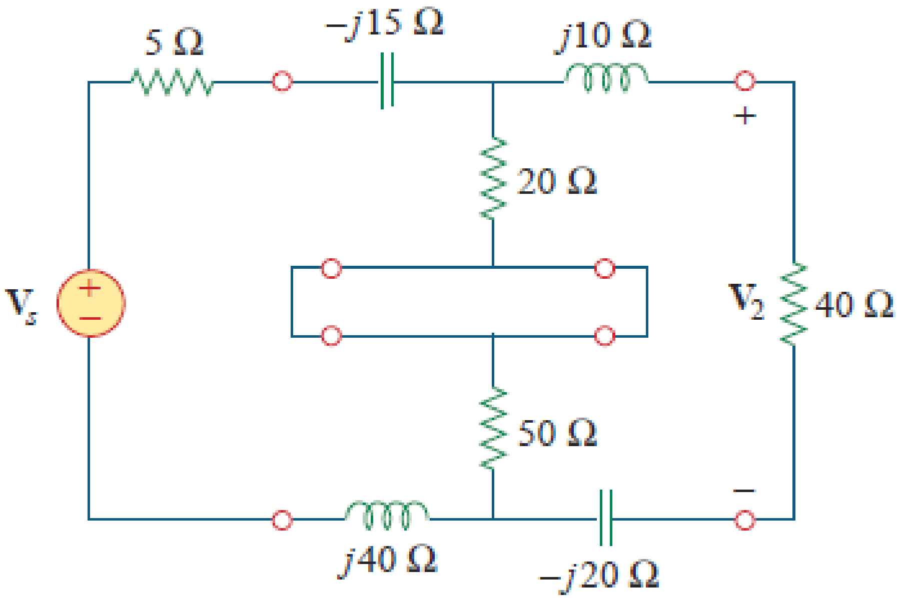
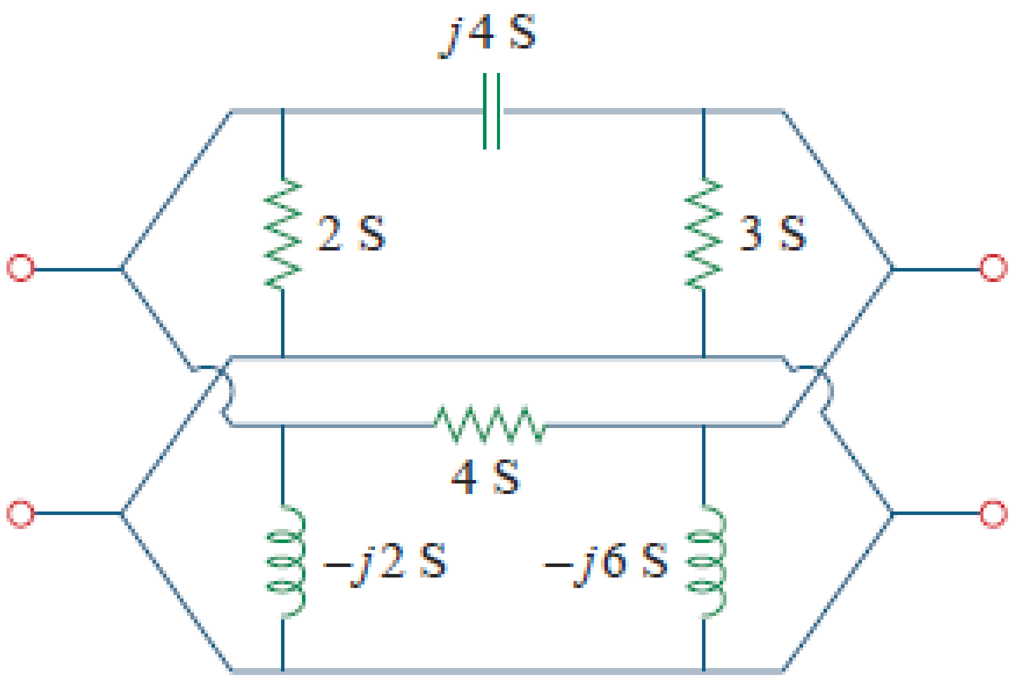


Figure 19.43
For Practice Prob. 19.12.

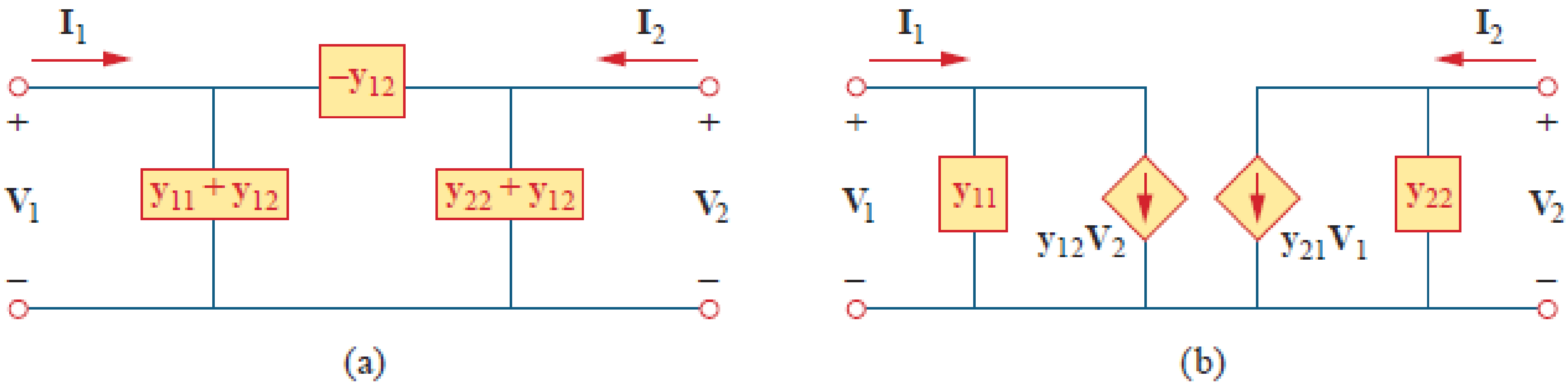
Answer: $0.6799\underline{/-29.05^\circ}$.

Example

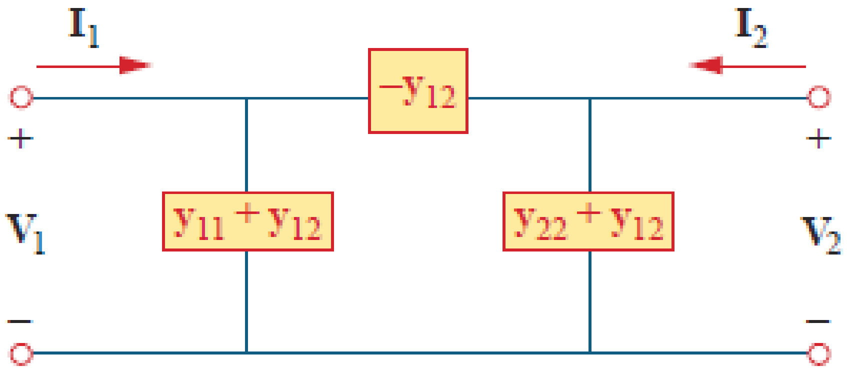
Find the y parameters of the two-port in Fig.

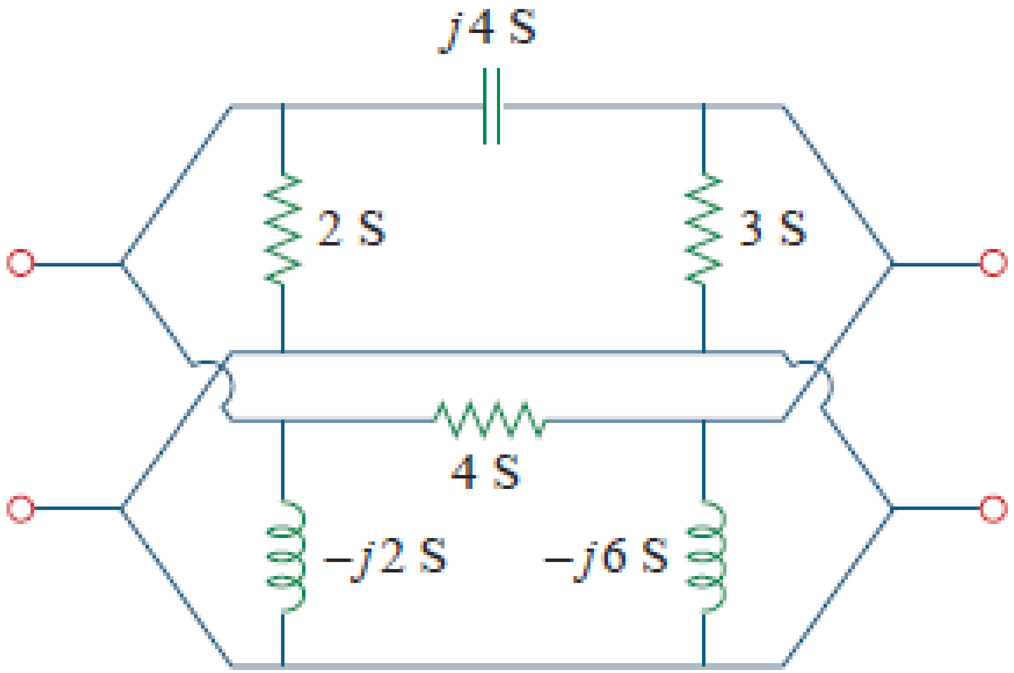


$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$



(a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.





$$\mathbf{y}_{12a} = -j4 = \mathbf{y}_{21a}, \qquad \mathbf{y}_{11a} = 2 + j4, \qquad \mathbf{y}_{22a} = 3 + j4$$

$$[\mathbf{y}_a] = \begin{bmatrix} 2 + j4 & -j4 \\ -j4 & 3 + j4 \end{bmatrix} \text{S}$$

$$\mathbf{y}_{12b} = -4 = \mathbf{y}_{21b}, \qquad \mathbf{y}_{11b} = 4 - j2, \qquad \mathbf{y}_{22b} = 4 - j6$$

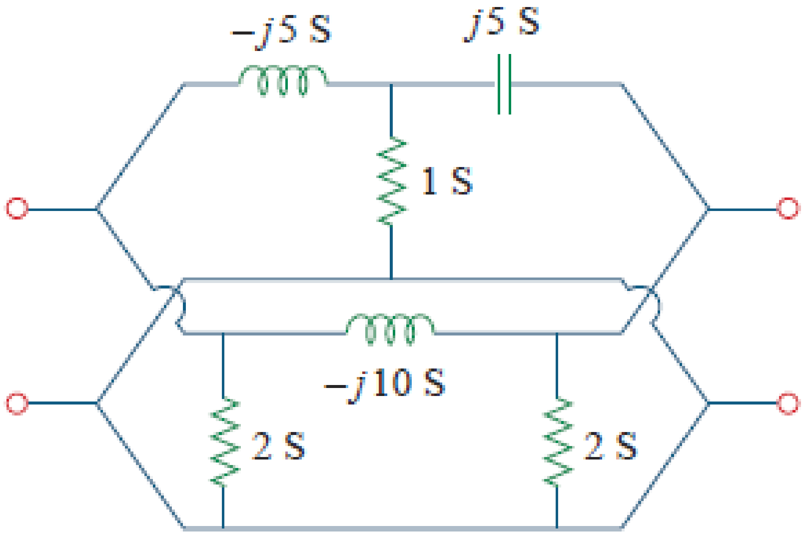
$$[\mathbf{y}_b] = \begin{bmatrix} 4 - j2 & -4 \\ -4 & 4 - j6 \end{bmatrix} \text{S}$$

The overall y parameters are

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 + j2 & -4 - j4 \\ -4 - j4 & 7 - j2 \end{bmatrix} \text{S}$$

Practice Problem 19.13

Obtain the y parameters for the network in Fig. 19.45.



Answer: $\begin{bmatrix} 27 - j15 & -25 + j10 \\ -25 + j10 & 27 - j5 \end{bmatrix}$ S.