Topology

The study of geometrical properties and spatial relations unaffected by the continuous change of shape or size of figures.

Network topology is a graphical representation of electric circuits. It is useful for analyzing complex electric circuits by converting them into network graphs. Network topology is also called as **Graph theory**.

Procedure for solving circuit problem

- Selecting a number of independent branch currents as (known as loop currents or mesh currents) variables, and then to express all branch currents as functions of the chosen set of branch currents.
- A number of independent node pair voltages may be selected as variables and then express all existing node pair voltages in terms of these selected variables.
- This procedure is easier for circuit contains a few elements
- However for large scale networks particularly modern electronic circuits of integrated circuits and microcircuits with a larger number of interconnected branches, it is almost impossible to write a set of linearly independent equations by inspection or by mere intuition.
- **Network topology (graph theory approach) is used for this purpose.** By this method, a set of linearly independent loop or node equations can be written in a form that is suitable for a computer solution
- The adequacy of a set of equations for analyzing a network is more easily determined topologically than algebraically

Graph (or linear graph): A network graph is a network in which all nodes and loops are retained but its branches are represented by lines. The voltage sources are replaced by short circuits and current sources are replaced by open circuits.

(Sources without internal impedances or admittances can also be treated in the same way because they can be shifted to other branches by E-shift and/or I-shift operations.)

Branch: A line segment replacing one or more network elements that are connected in series or parallel.

Node: Interconnection of two or more branches. It is a terminal of a branch. Usually interconnections of three or more branches are nodes.

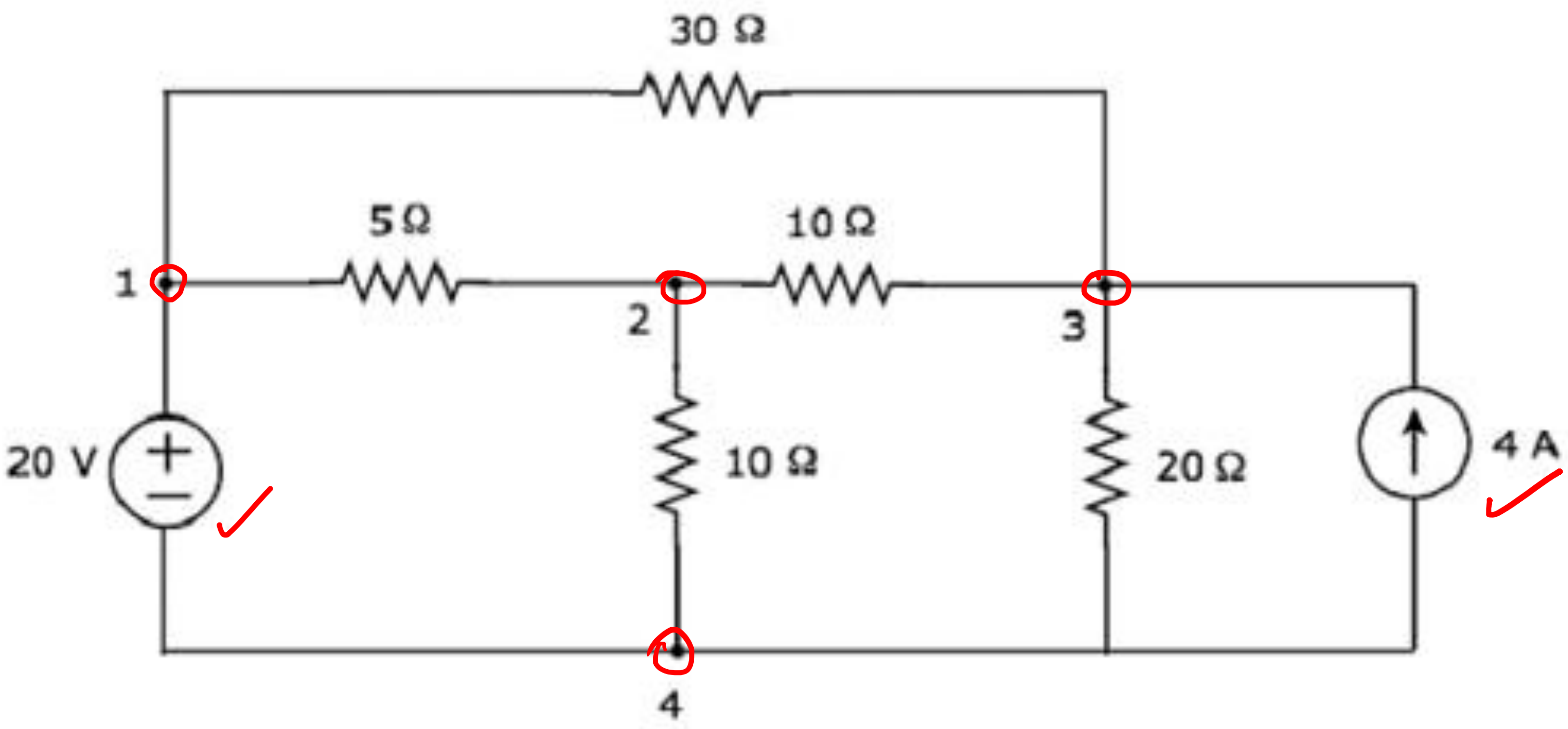
Path: A set of branches that may be traversed in an order without passing through the same node more than once.

Loop: Any closed contour selected in a graph.

Mesh: A loop which does not contain any other loop within it.

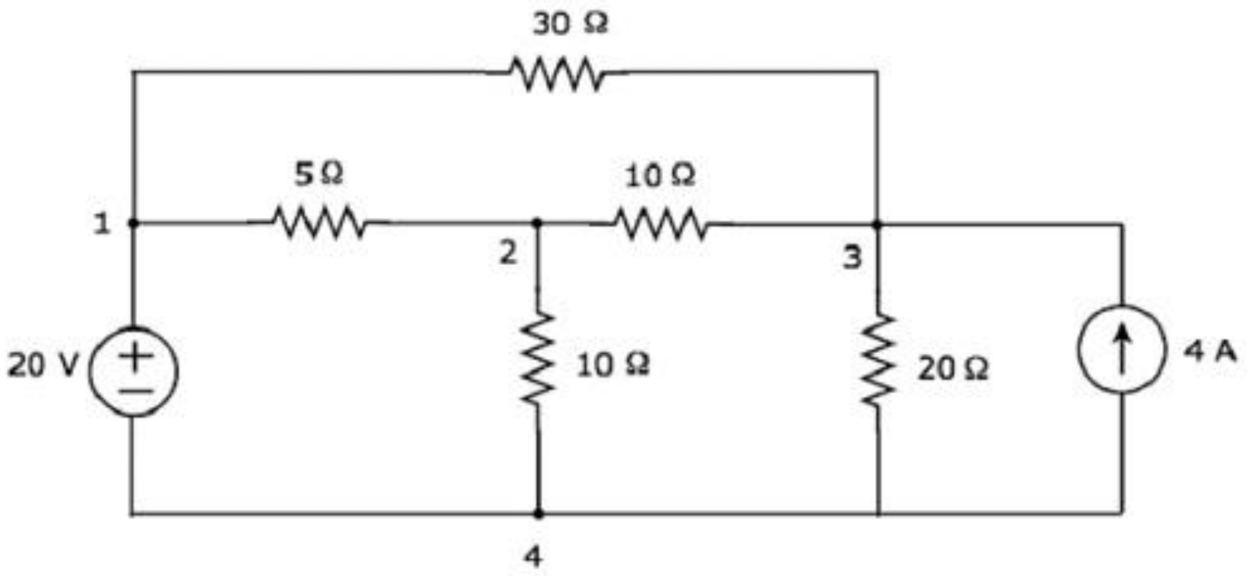
Planar graph: A graph which may be drawn on a plane surface in such a way that no branch passes over any other branch.

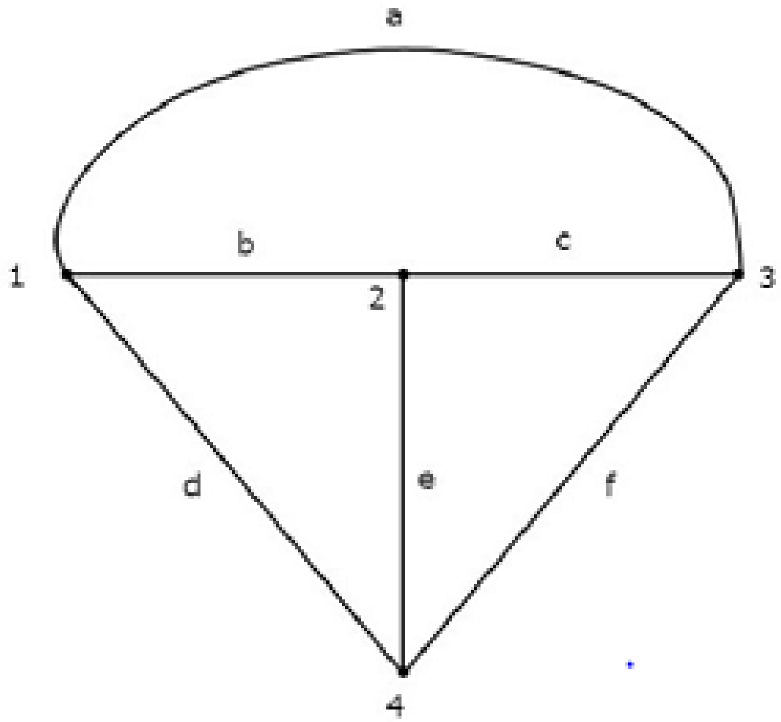
Non-planar graph: Any graph which is not planar



- There are four principal nodes and those are labelled with 1, 2, 3, and 4.
- There are seven branches in the above circuit,
- among which one branch contains a 20 V voltage source,
- another branch contains a 4 A current source and the remaining five branches contain resistors having resistances of 30 Ω, 5 Ω, 10 Ω, 10 Ω and 20 Ω respectively.

An equivalent **graph** corresponding to the electric circuit

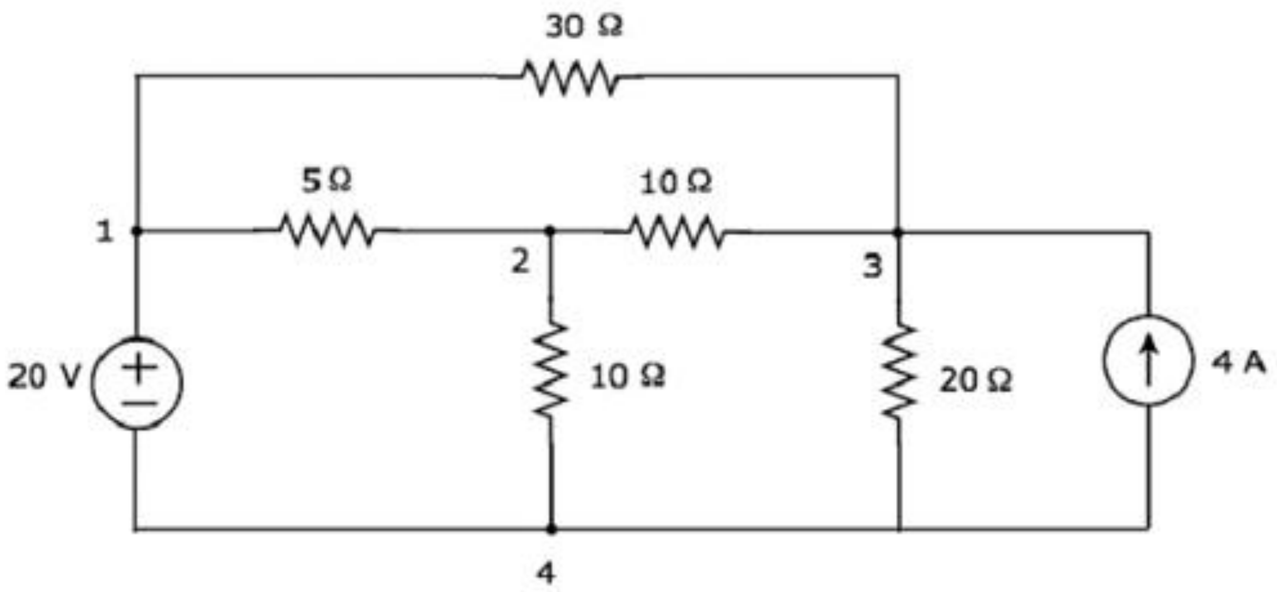


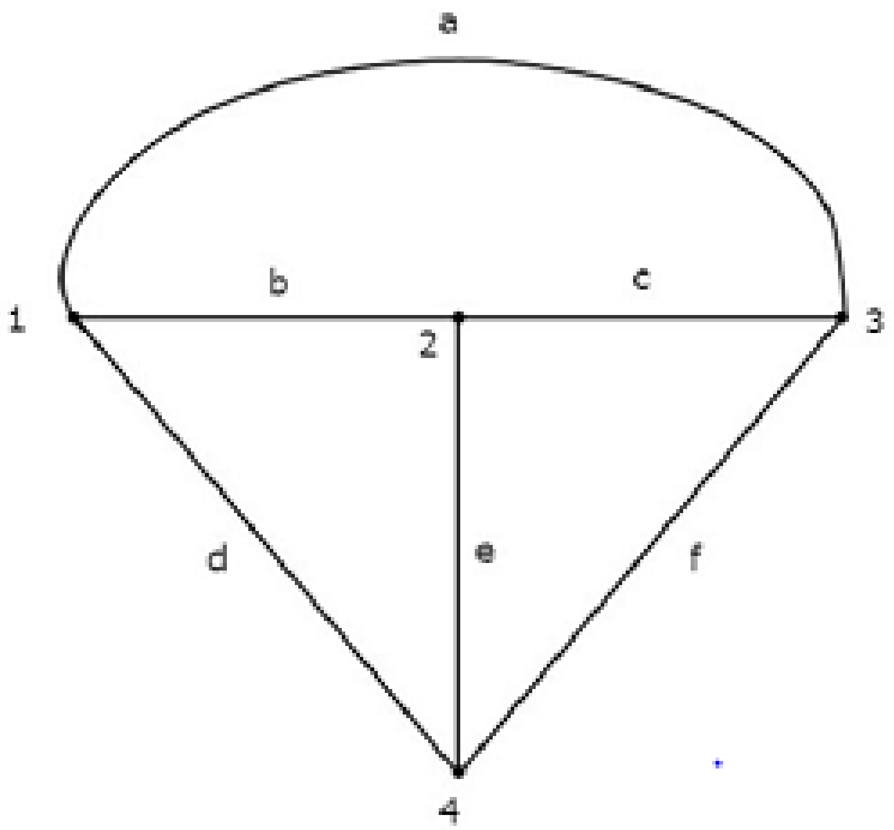


- In the above graph, there are **four nodes** and those are labelled with 1, 2, 3 & 4 respectively. These are same as that of principal nodes in the electric circuit.

- There are **six branches** in the above graph and those are labelled with a, b, c, d, e & f respectively.

- In this case, we got **one branch less** in the graph because the 4 A current source is made as open circuit, while converting the electric circuit into its equivalent graph.





we can conclude the following points:

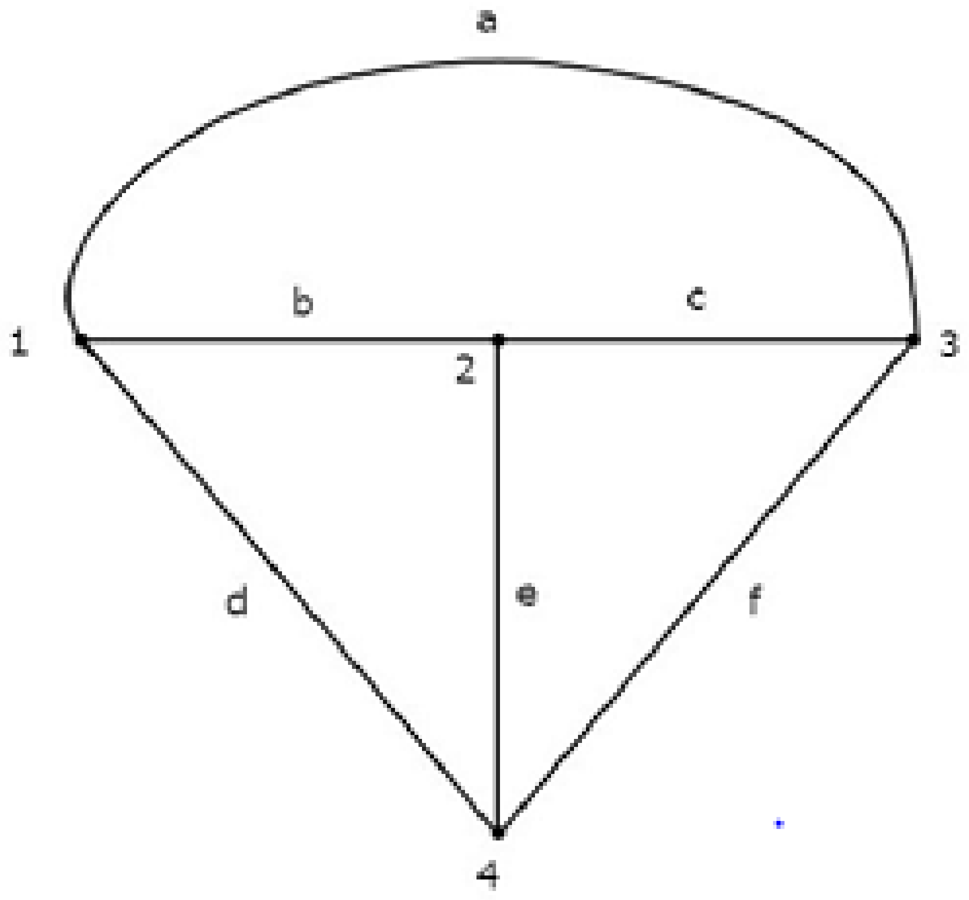
- The **number of nodes** present in a graph will be equal to the number of principal nodes present in an electric circuit.

- The **number of branches** present in a graph will be less than or equal to the number of branches present in an electric circuit.

Connected Graph

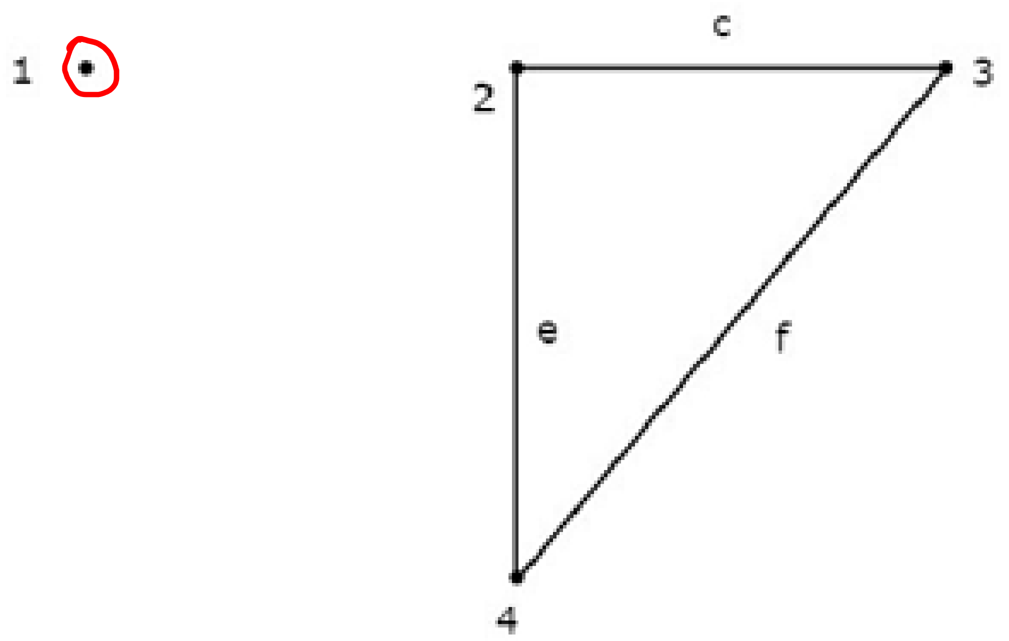
If there exists at least one branch between any of the two nodes of a graph, then it is called as a **connected graph**. That means, each node in the connected graph will be having one or more branches that are connected to it. So, no node will present as isolated or separated.

The graph shown in the previous Example is a **connected graph**. Here, all the nodes are connected by three branches.



Unconnected Graph

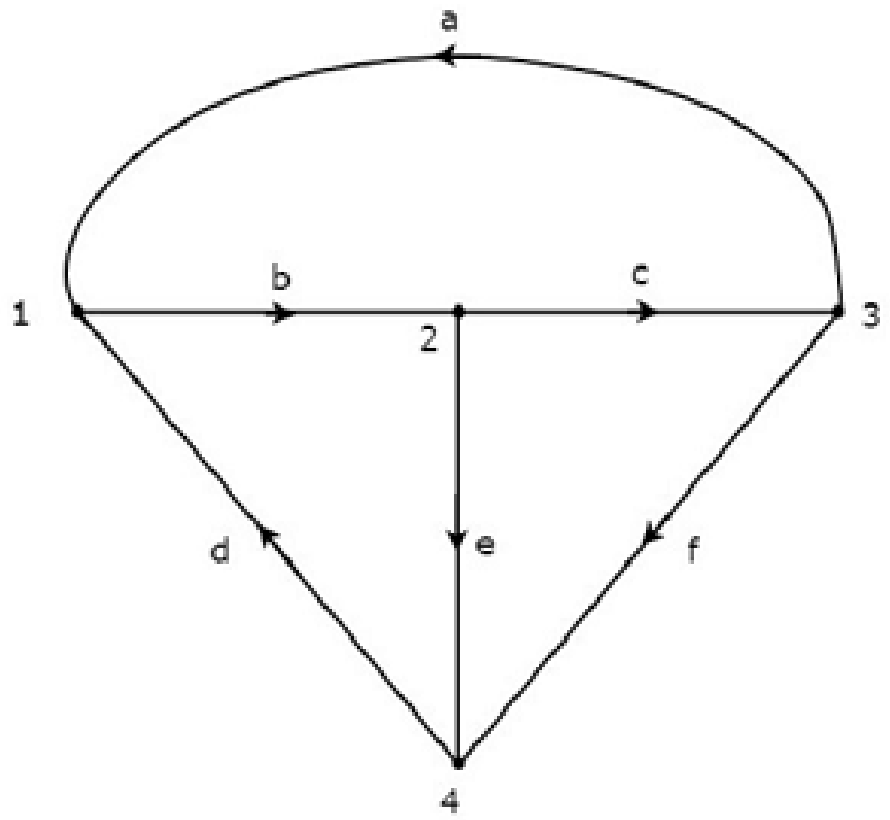
If there exists at least one node in the graph that remains unconnected by even single branch, then it is called as an **unconnected graph**. So, there will be one or more isolated nodes in an unconnected graph.



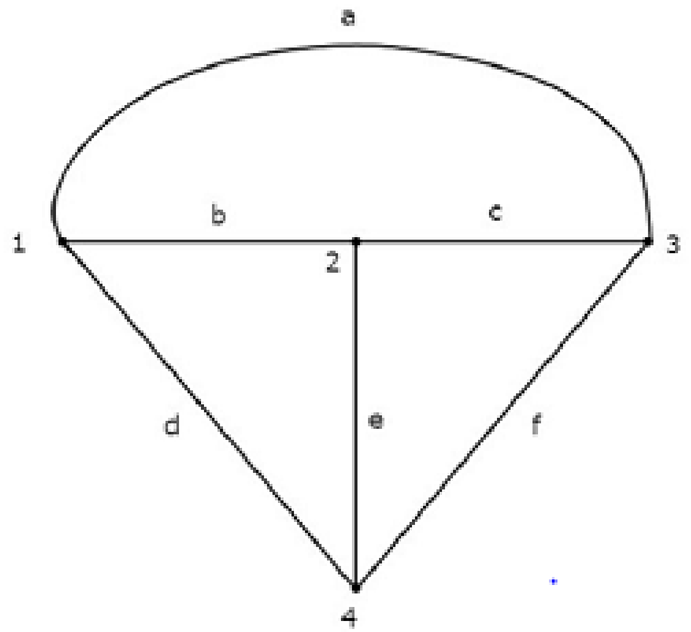
In this graph, the nodes 2, 3, and 4 are connected by two branches each. But, not even a single branch has been connected to the **node 1**. So, the node 1 becomes an **isolated node**. Hence, the above graph is an **unconnected graph**.

Directed Graph

If all the branches of a graph are represented with arrows, then that graph is called as a **directed graph**. These arrows indicate the direction of current flow in each branch. Hence, this graph is also called as **oriented graph**.



In the above graph, the direction of current flow is represented with an arrow in each branch. Hence, it is a **directed graph**.



Undirected Graph

If the branches of a graph are not represented with arrows, then that graph is called as an **undirected graph**. Since, there are no directions of current flow, this graph is also called as an **unoriented graph**.

The graph that was shown in the first Example of this chapter is an **unoriented graph**, because there are no arrows on the branches of that graph.

Subgraph and its Types

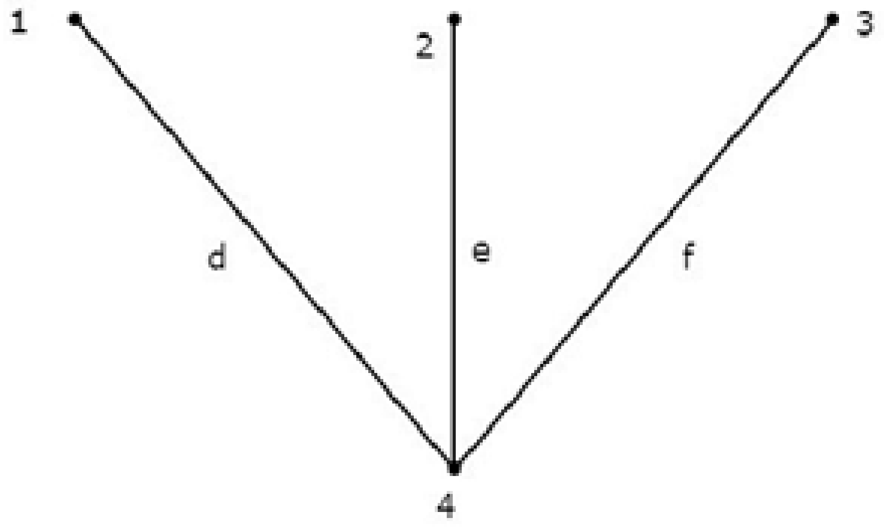
A part of the graph is called as a **subgraph**. We get subgraphs by removing some nodes and/or branches of a given graph. So, the number of branches and/or nodes of a subgraph will be less than that of the original graph. Hence, we can conclude that a subgraph is a subset of a graph.

Following are the **two types** of subgraphs.

- Tree
- Co-Tree

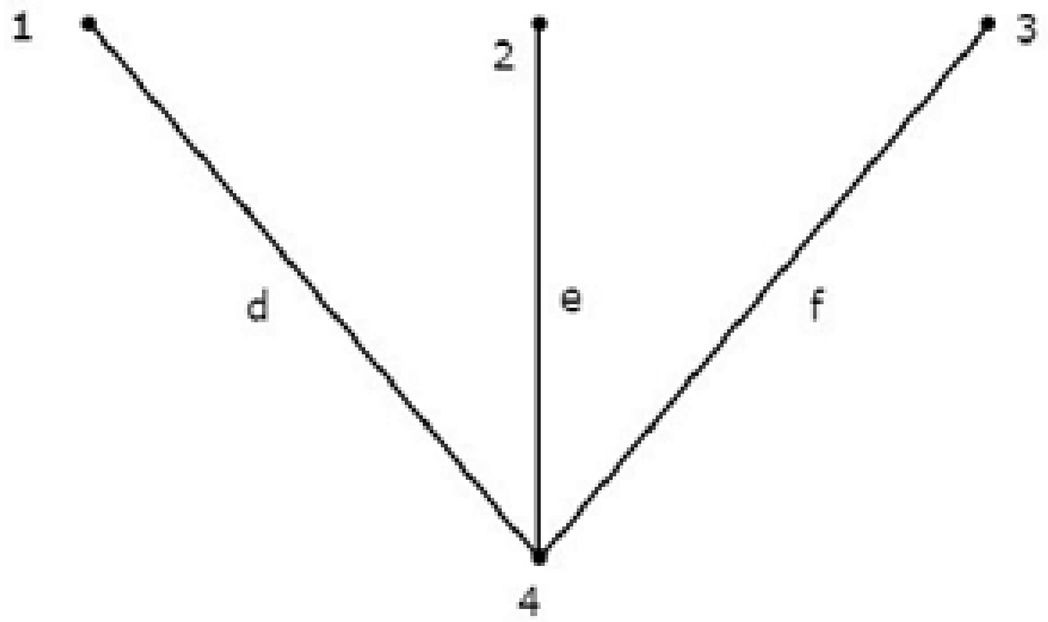
Tree

Tree is a connected subgraph of a given graph, which contains all the nodes of a graph. But, there should not be any loop in that subgraph. The branches of a tree are called as **twigs**.

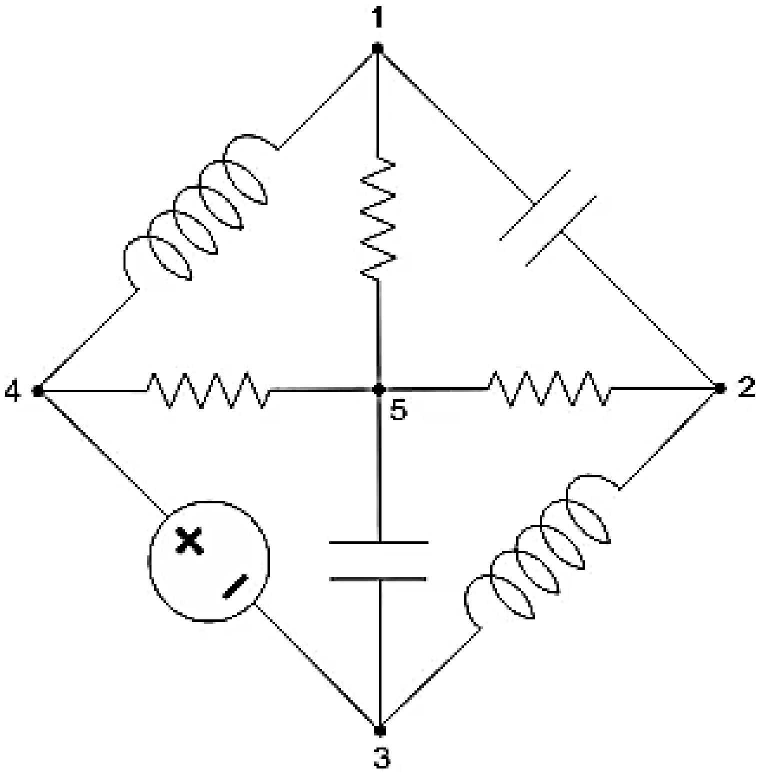


This connected subgraph contains all the four nodes of the given graph and there is no loop. Hence, it is a **Tree**.

This Tree has only three branches out of six branches of given graph. Because, if we consider even single branch of the remaining branches of the graph, then there will be a loop in the above connected subgraph. Then, the resultant connected subgraph will not be a Tree.



From the above Tree, we can conclude that the number of branches that are present in a Tree should be equal to n - 1 where 'n' is the number of nodes of the given graph.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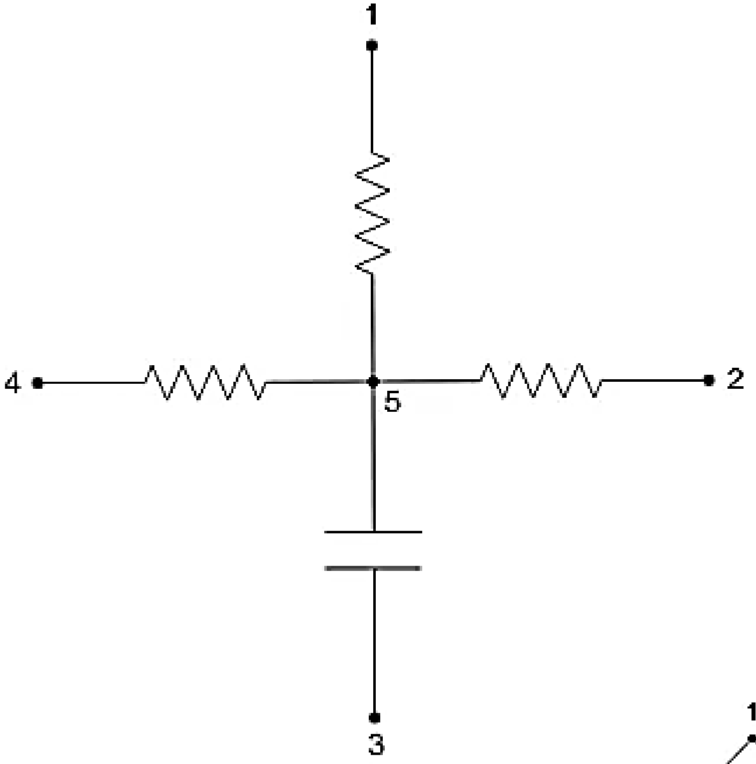
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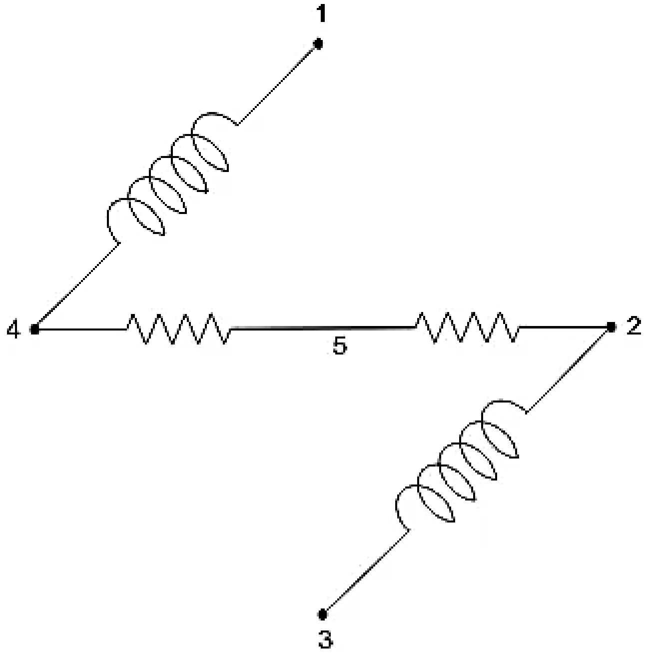
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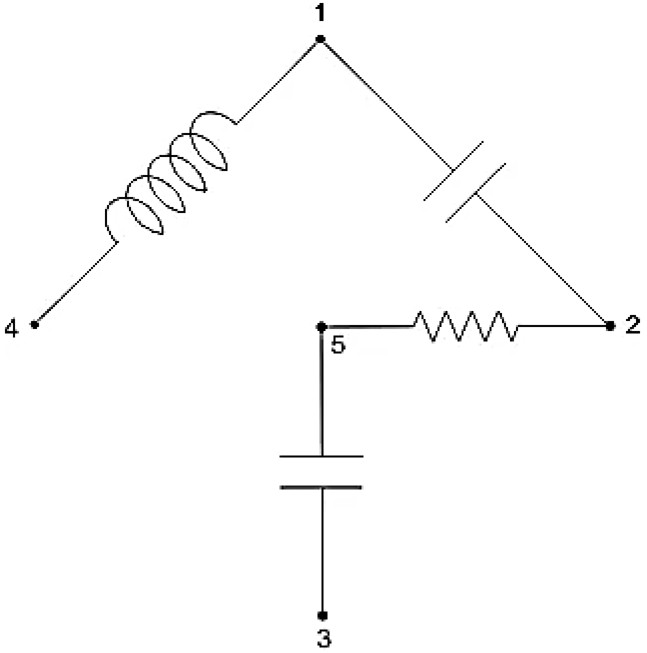
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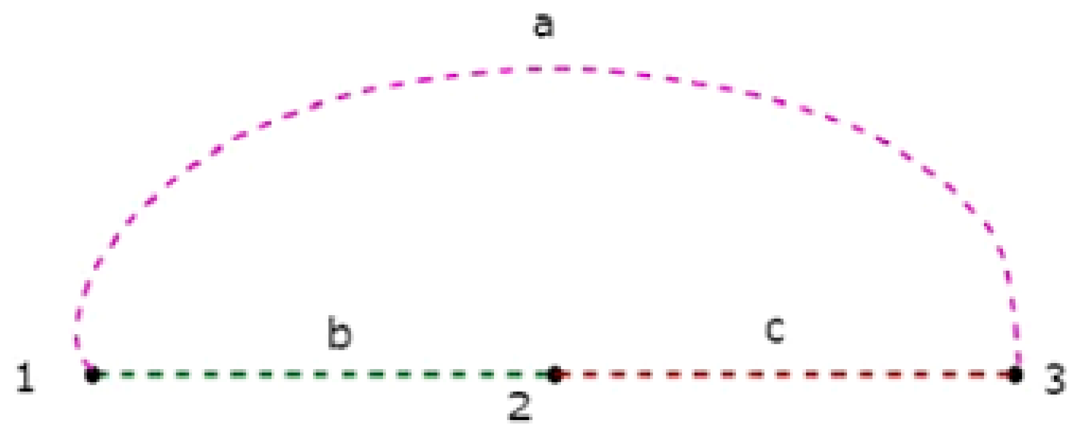
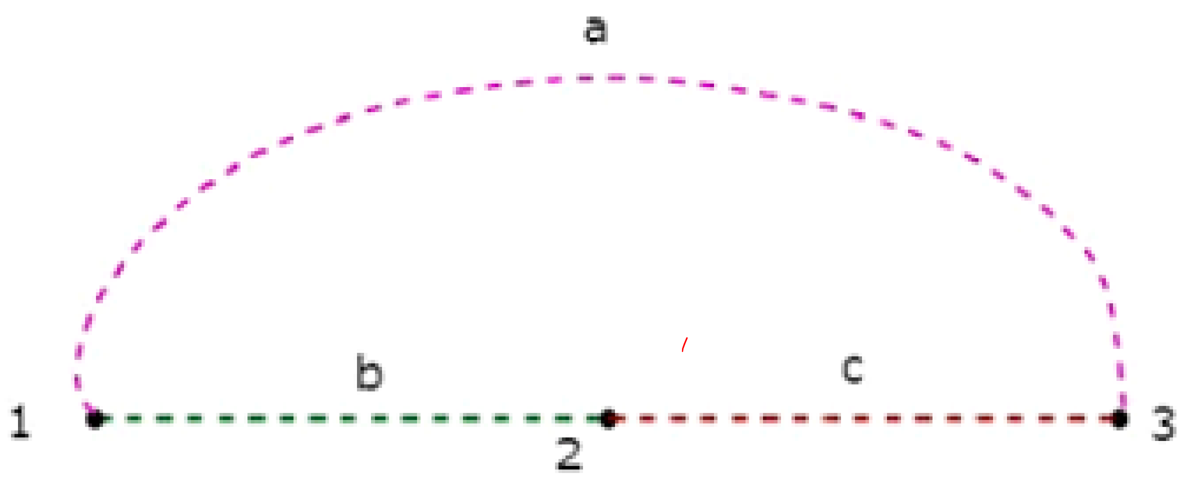
Co-Tree

Co-Tree is a subgraph, which is formed with the branches that are removed while forming a Tree. Hence, it is called as **Complement** of a Tree. For every Tree, there will be a corresponding Co-Tree and its branches are called as **links** or chords. In general, the links are represented with dotted lines.

The **Co-Tree** corresponding to the above Tree is shown in the following figure.



The **Co-Tree** corresponding to the above Tree is shown in the following figure.



This Co-Tree has only three nodes instead of four nodes of the given graph, because Node 4 is isolated from the above Co-Tree. Therefore, the Co-Tree need not be a connected subgraph. This Co-Tree has three branches and they form a loop.

The **number of branches** that are present in a co-tree will be equal to the difference between the number of branches of a given graph and the number of twigs. Mathematically, it can be written as

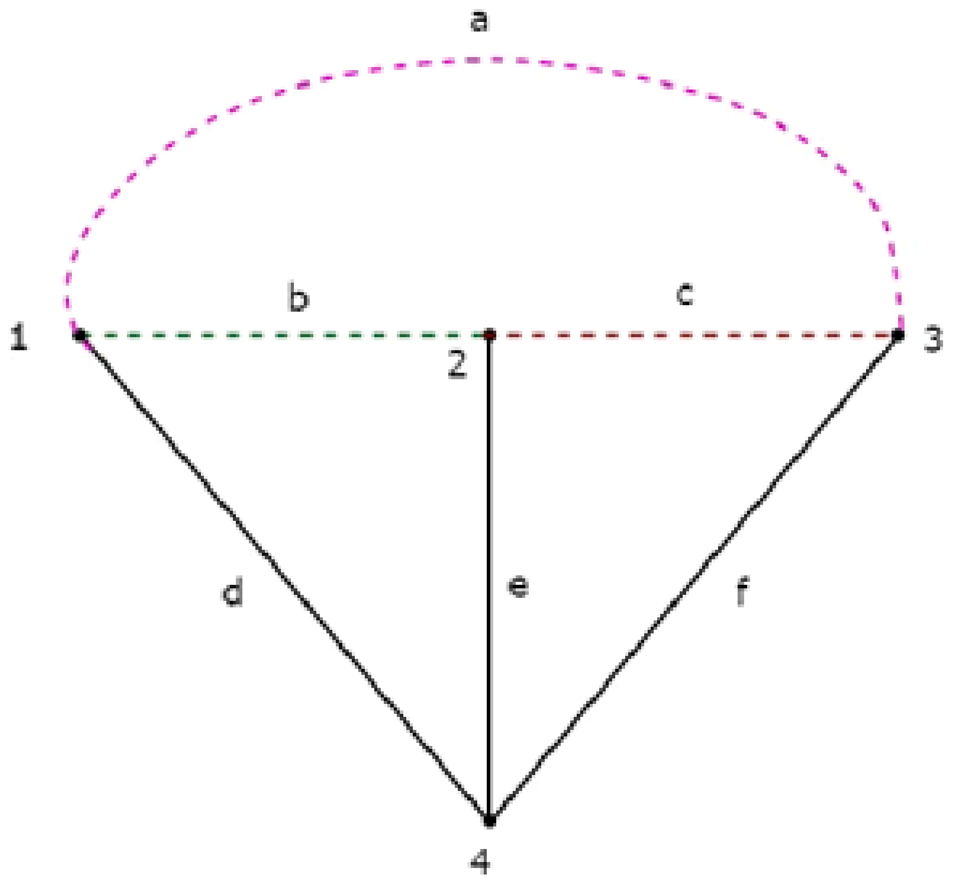
$$l = b - (n - 1)$$

$$l = b - n + 1$$

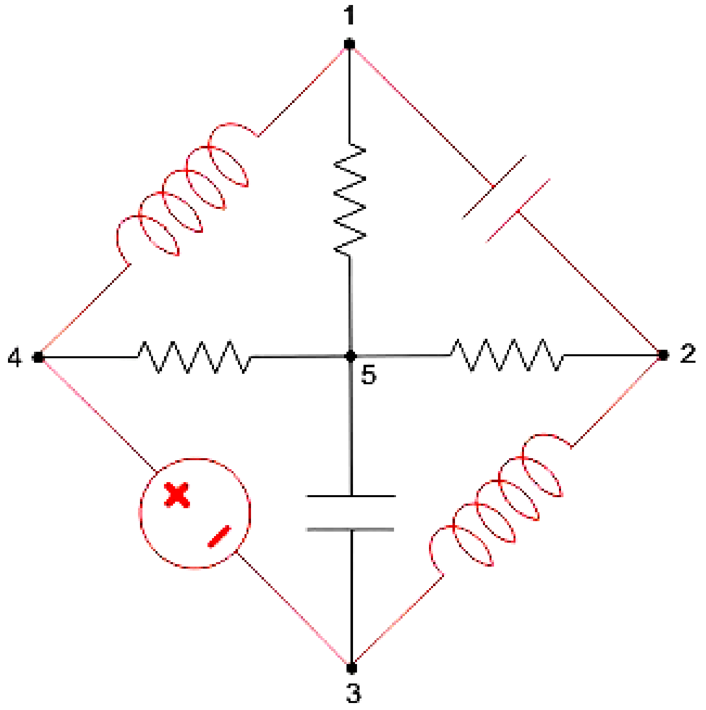
Where,

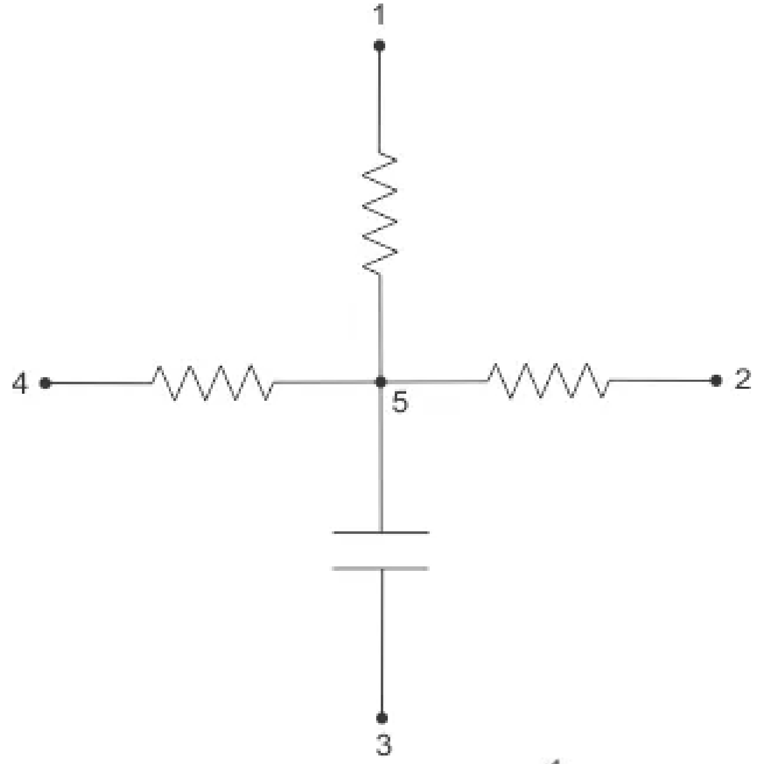
- *l* is the number of links.
- *b* is the number of branches present in a given graph.
- *n* is the number of nodes present in a given graph.

If we combine a Tree and its corresponding Co-Tree, then we will get the **original graph** as shown below.



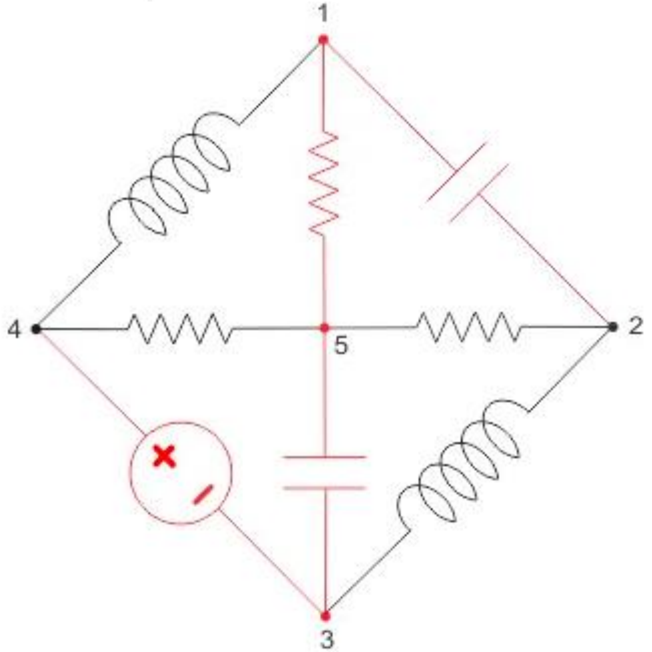
The Tree branches d, e & f are represented with solid lines. The Co-Tree branches a, b & c are represented with dashed lines.

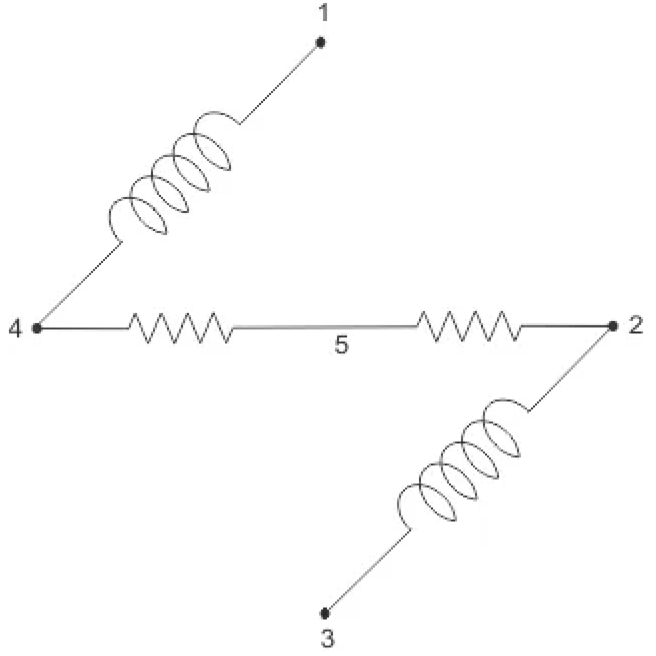


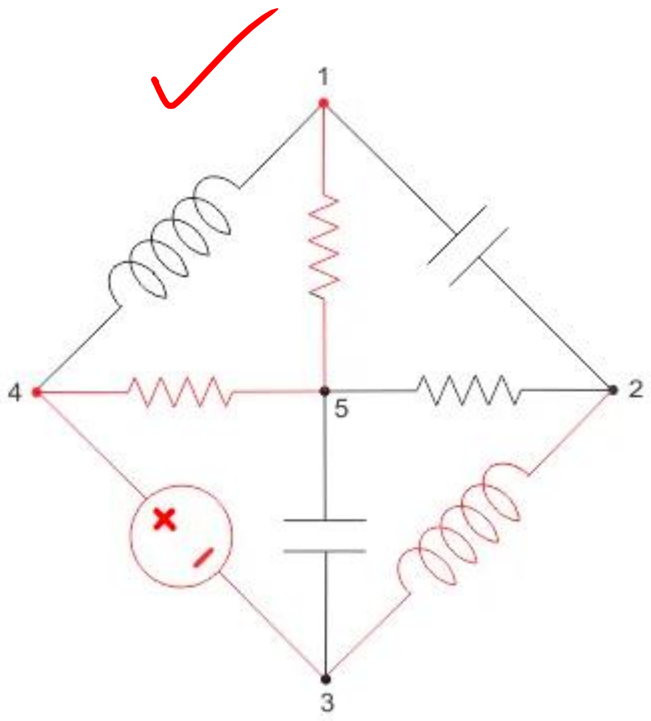


Possible trees

Co-trees



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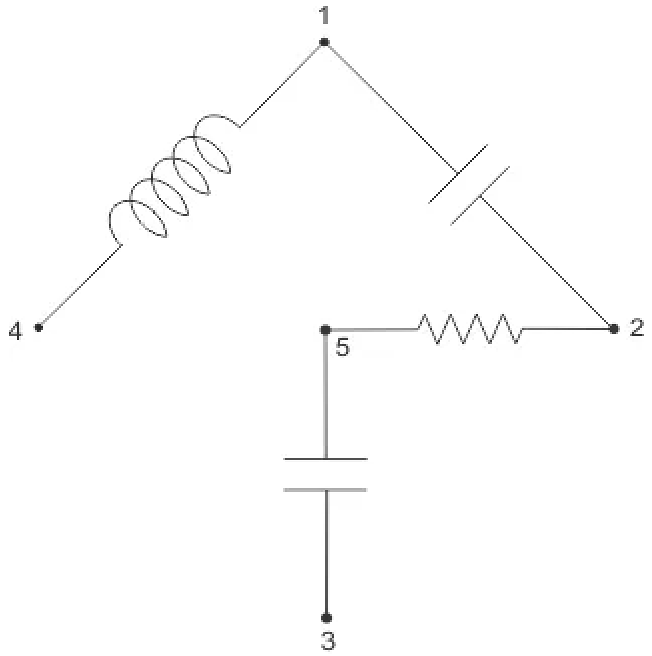
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Network Topology Matrices

Network Topology Matrices which are useful for solving any electric circuit or network problem by using their equivalent graphs.

Matrices Associated with Network Graphs

Following are the three matrices that are used in Graph theory

- Incidence Matrix
- Fundamental Loop Matrix
- Fundamental Cut set Matrix

Incidence Matrix

- An Incidence Matrix represents the graph of a given electric circuit or network. Hence, it is possible to draw the graph of that same electric circuit or network from the **incidence matrix**.

- Incidence matrix is represented with the letter A. It is also called as node to branch incidence matrix or node incidence matrix.

- If there are ‘n’ nodes and ‘b’ branches are present in a directed graph, then the incidence matrix will have ‘n’ rows and ‘b’ columns.

- The rows and columns are corresponding to the nodes and branches of a directed graph. Hence, the order of incidence matrix will be n × b.

The elements of incidence matrix will be having one of these three values, +1, -1 and 0.

- **If the branch current is leaving from a selected node, then the value of the element will be +1.**

- **If the branch current is entering towards a selected node, then the value of the element will be -1.**

- **If the branch current neither enters at a selected node nor leaves from a selected node, then the value of element will be 0.**

Procedure to find Incidence Matrix

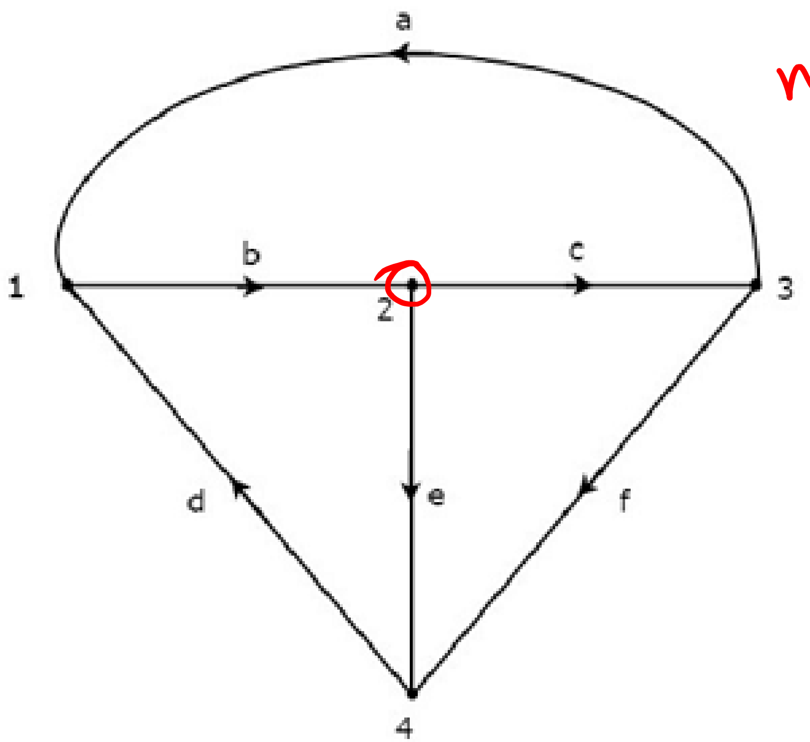
Steps to find the incidence matrix of directed graph

- Select a node at a time of the given directed graph and fill the values of the elements of incidence matrix corresponding to that node in a row.

- Repeat the above step for all the nodes of the given directed graph.

Example

Consider the following **directed graph**.



Branches

nodes

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \end{array}$$

- The rows and columns of the above matrix represents the nodes and branches of given directed graph. The order of this incidence matrix is 4 × 6.
- By observing the above incidence matrix, we can conclude that the summation of column elements of incidence matrix is equal to zero. That means, a branch current leaves from one node and enters at another single node only.
- If the given graph is an un-directed type, then convert it into a directed graph by representing the arrows on each branch of it. We can consider the arbitrary direction of current flow in each branch.

Properties of Incidence matrix

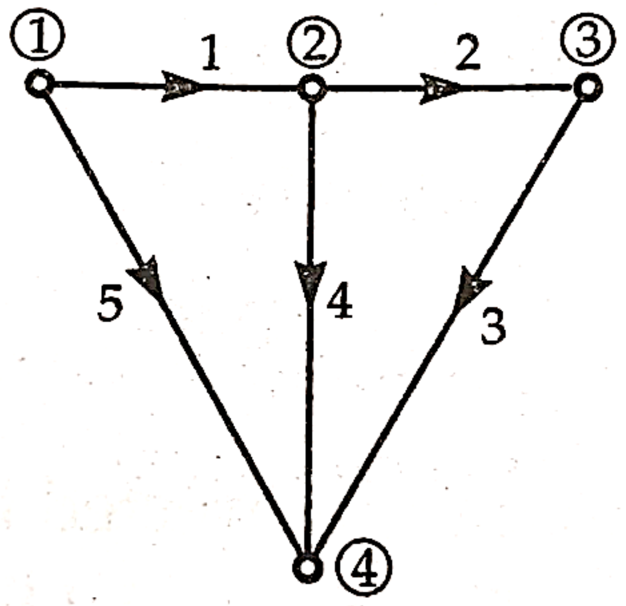
1. **Algebraic sum of column entries of an incidence matrix is zero**

2. **Determinant of the incidence matrix of a closed loop is zero**

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

Reduced Incidence matrix: It is possible to remove any one row from incidence matrix [A] with out sacrificing the information in it. When one row is deleted, the remaining matrix is called a reduced incidence matrix

Example: Develop the incidence matrix and reduced incidence matrix of the following graph



Network Graph

$$\begin{array}{cc} & \textit{Branches} \\ \textit{Nodes} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array} = [A_i]$$

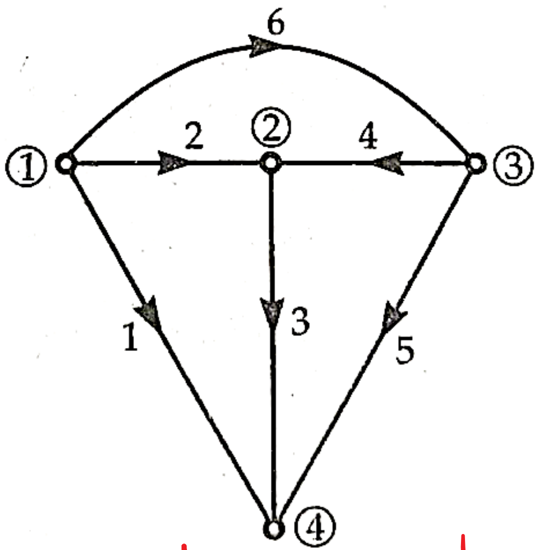
Incidence Matrix

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

Reduced incidence Matrix

Node 4 is removed so that the 4th row is removed

Example: **Develop Twig matrix and Link matrix from the following directed graph of a network:**



Directed graph

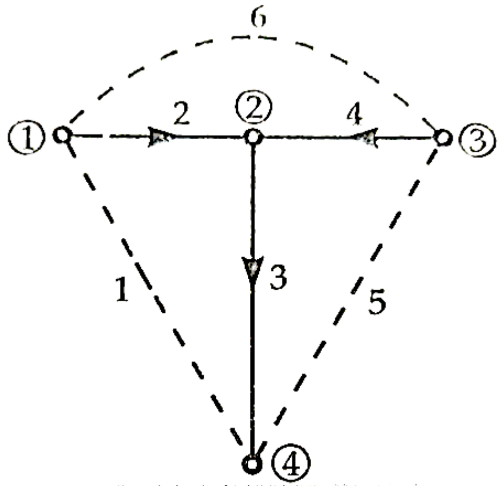
$$[A_i] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

Incidence Matrix

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Reduced incidence Matrix

Node 3 removed, 3rd row eleminated



[Twigs : (2, 3, 4)
Links : (1, 5, 6)]

Twigs

Links

column rearranged

$$\begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \qquad \begin{array}{ccc} 1 & 5 & 6 \end{array} \end{array}$$

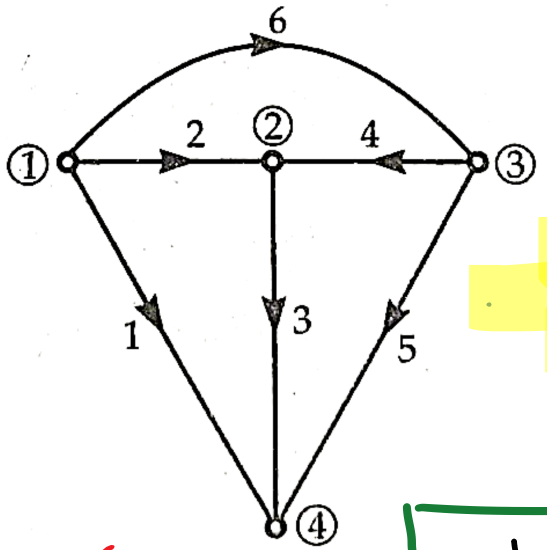
$$[A] = \left[\boxed{\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{matrix}} \quad \boxed{\begin{matrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & 0 \end{matrix}} \right]$$

Matrix of Twigs $[A_T]$ Matrix of Links $[A_L]$

$$[A] = [A_T : A_L]$$

Matrix $[A]$ subdivided to matrices $[A_T]$ & $[A_L]$

Example: **Determine the number of possible trees of the given graph**



Graph

$$[A_i] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

Incidence Matrix

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Reduced incidence Matrix (Column 3, Row 3 dropped)

$$\boxed{\text{No. of possible Trees, } T = \text{Det}\,[A]\,[A^T]}$$

$$T = \text{Det}\left\{ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \right\} = \text{Det}\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} = 16$$

$\therefore$ No. of possible trees $= 16$

Fundamental Loop Matrix (TIE-SET Matrix)

- **Fundamental loop or f-loop is a loop contains only one link and one or more twigs**
- **The number of f-loops will be equal to the number of links.**
- Fundamental loop matrix is represented with letter B.
- This matrix gives the relation between branch currents and link currents.
- If there are ‘n’ nodes and ‘b’ branches are present in a directed graph, then the number of links present in a co-tree, which is corresponding to the selected tree of given graph will be b-n+1.
- The fundamental loop matrix will have ‘b-n+1’ rows and ‘b’ columns.
- The rows and columns correspond to the links of co-tree and branches of given graph
- Hence, the order of fundamental loop matrix will be $(b - n + 1) \times b$.

The **elements of fundamental loop matrix** will be having one of these three values, +1, -1 and 0

- The value of element will be +1 for the link of selected f-loop
- The value of elements will be 0 for the remaining links and twigs, which are not part of the selected f-loop
- If the direction of twig current of selected f-loop is same as that of f-loop link current, then the value of element will be +1
- If the direction of twig current of selected f-loop is opposite to that of f-loop link current, then the value of element will be -1.

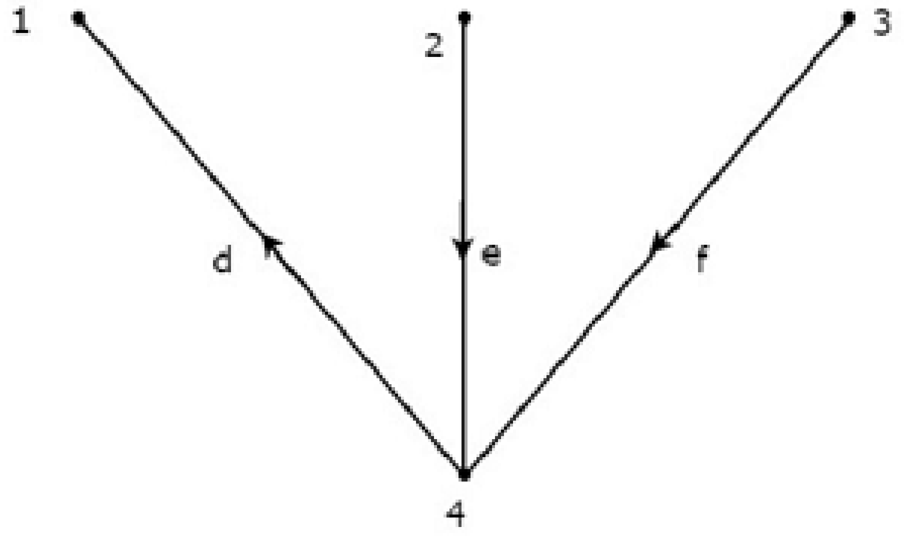
Procedure to find Fundamental Loop Matrix

Steps to find the fundamental loop matrix of given directed graph

- Select a tree of given directed graph
- By including one link at a time, we will get one f-loop. Fill the values of elements corresponding to this f-loop in a row of fundamental loop matrix
- Repeat the above step for all links.

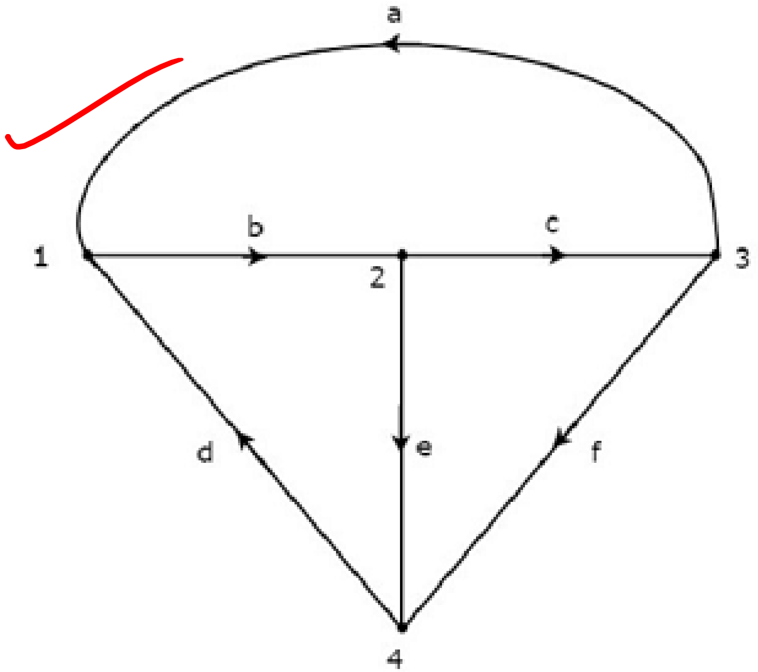
Example

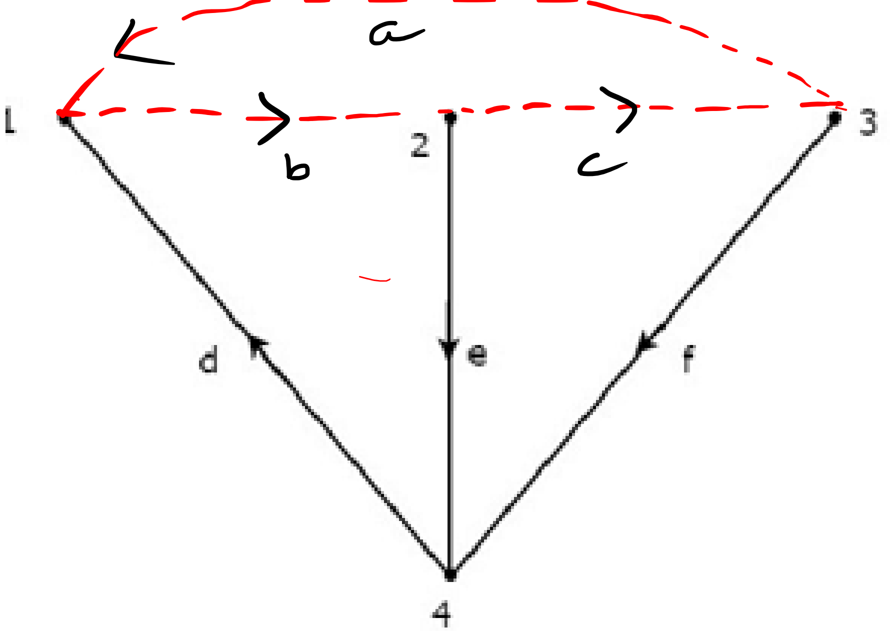
Tree of **directed graph**, which is considered for incidence matrix.

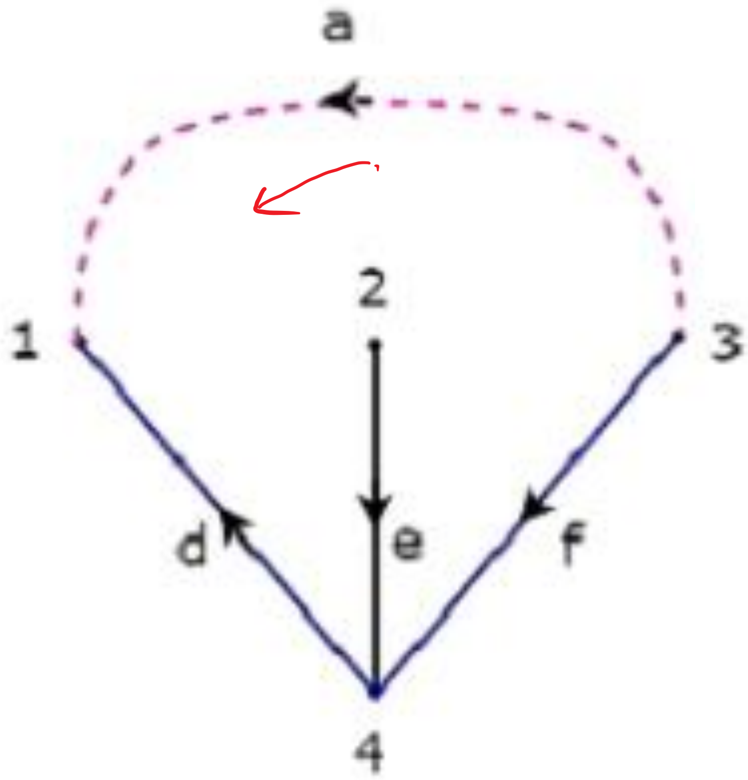


- This tree contains three branches d, e & f
- The branches a, b & c will be the links of the Co-Tree corresponding to the above Tree.
- By including one link at a time to the above Tree, we will get one **f-loop**.
- there will be three **f-loops**, since there are three links. These three f-loops are shown in the following figure.

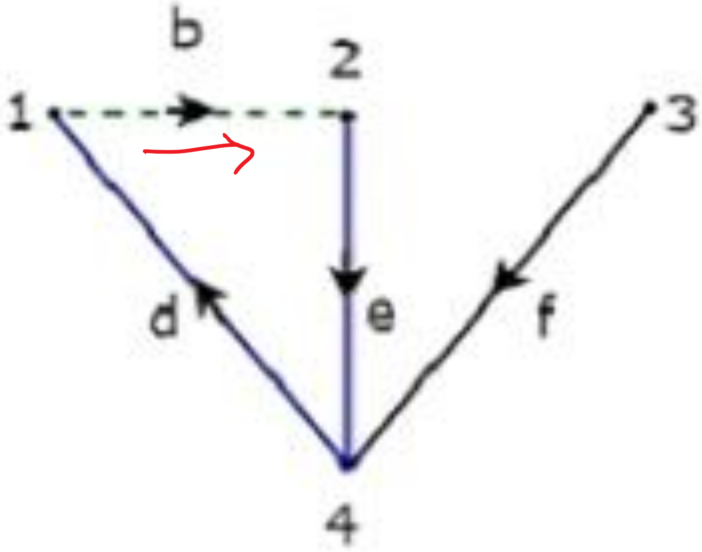
f-loops or fundamental loops



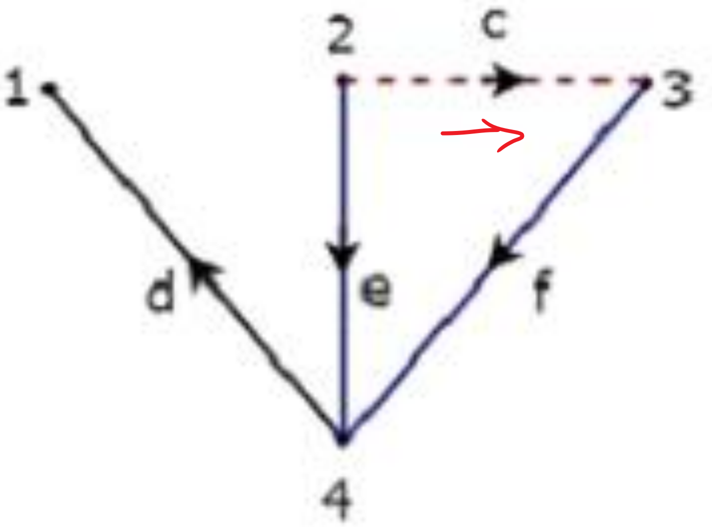




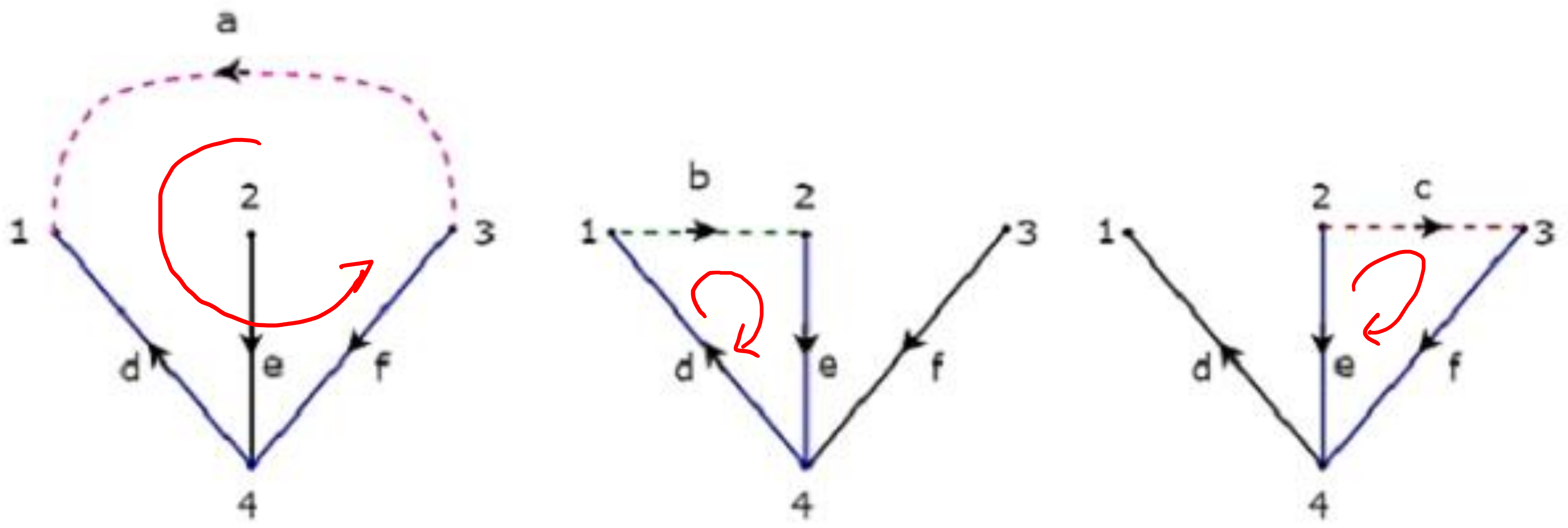
f-loop 1



f-loop 2



f-loop 3



f-loop 1 f-loop 2 f-loop 3

$$
B = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}
\end{array}
$$

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

- The rows and columns of the above matrix represents the links and branches of given directed graph.

- The order of this incidence matrix is 3 × 6.

- The **number of Fundamental loop matrices** of a directed graph will be equal to the number of Trees of that directed graph. Because, every Tree will be having one Fundamental loop matrix.