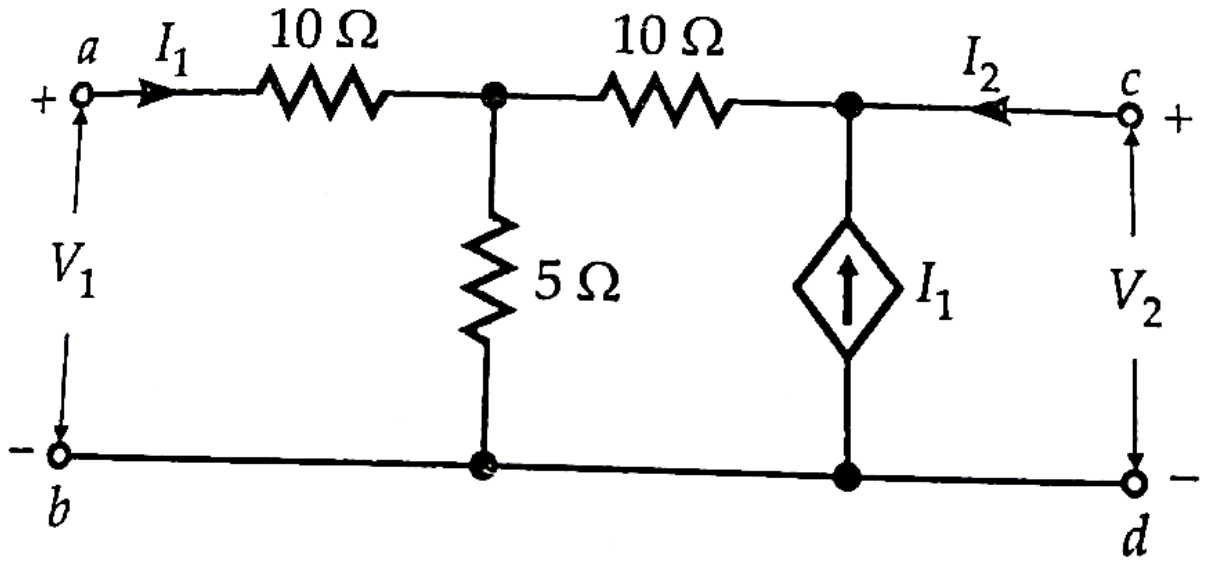
Determine the Z parameters of the network

PROBLEM 1

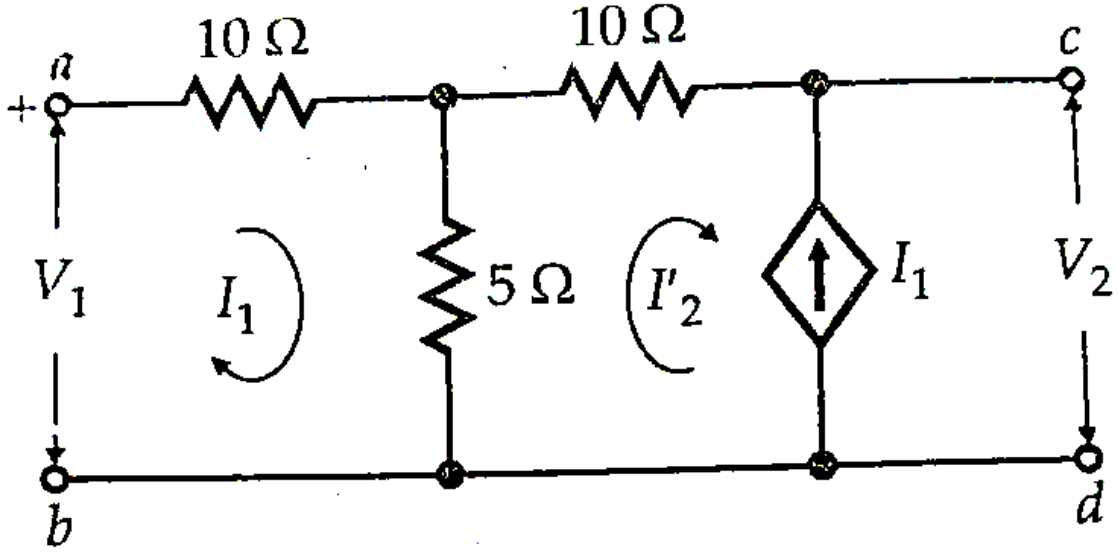


(a b) → Input port (i/p)

(c d) → output port (o/p)

Apply V_1 in the (i/p) port

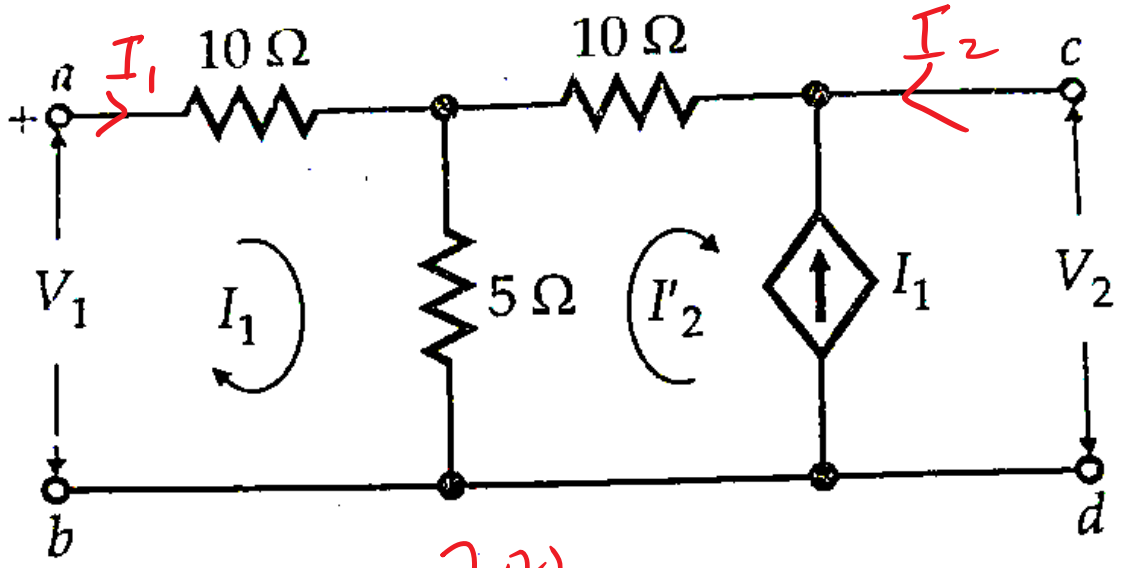
o/p port is open



Assume loop current as shown

$\therefore V_1 = 10 I_1 + 5 I_1 - 5 I'_2$

$V_1 = 15 I_1 - 5 I'_2 \longrightarrow (1)$



To find Z_{11} & Z_{21}

$V_1 = 15 I_1 - 5 I_2' \longrightarrow (1)$

Since (c d) is open, $I_2 = 0$

See the dependent source in the figure, I_1 is opposite to $-I_2'$

$\therefore -I_2' = I_1$ (Note $I_2 = 0$)

$\longrightarrow (2)$

Using (2) in (1) $\Rightarrow$ $V_1 = 15 I_1 + 5 I_1 = 20 I_1$

$V_1 = 20 I_1 \longrightarrow (3)$

From (3) $\Rightarrow \left(\frac{V_1}{I_1}\right)_{I_2=0} = Z_{11} = 10\ \Omega$

From eqn (2) $\Rightarrow I_2' = -I_1$

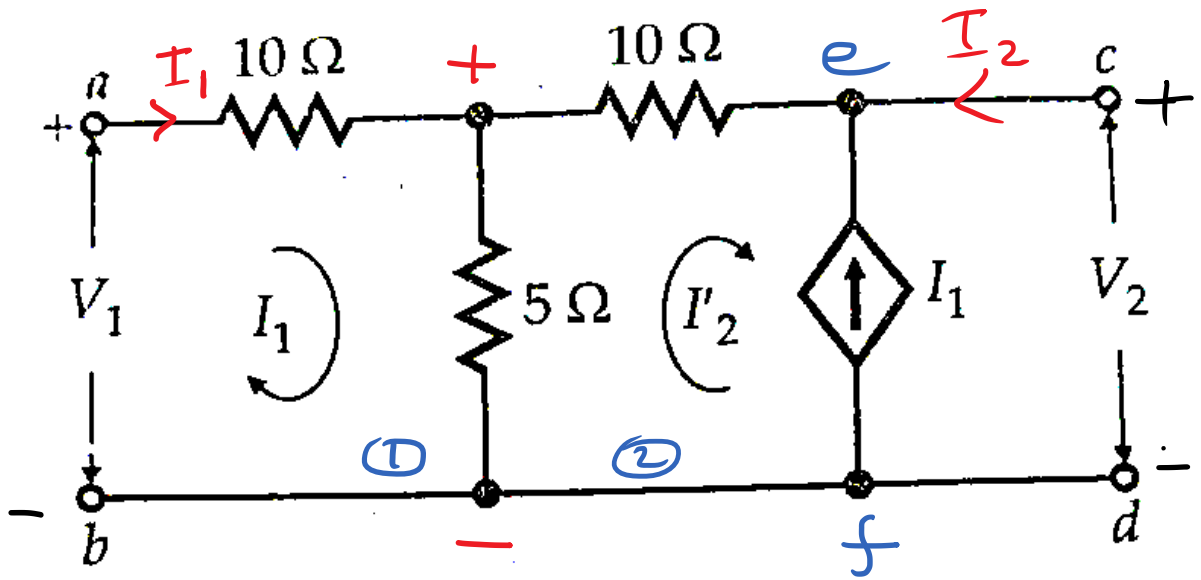
From eqn (3) $\Rightarrow I_1 = \frac{V_1}{20}$ (A)

$\therefore$ $\boxed{I_2' = -\frac{V_1}{20}\ (A)}$

A → Ampere.

$-I_1 = -\frac{V_1}{20}$

$I_1 = V_1/20$



Since (c-d) is open $I_2 = 0$

Here $I_2' = -I_1$ and Voltage between e & d = V_2

V_2 = Voltage drop across 5 Ω + Voltage drop across 10 Ω in loop ②

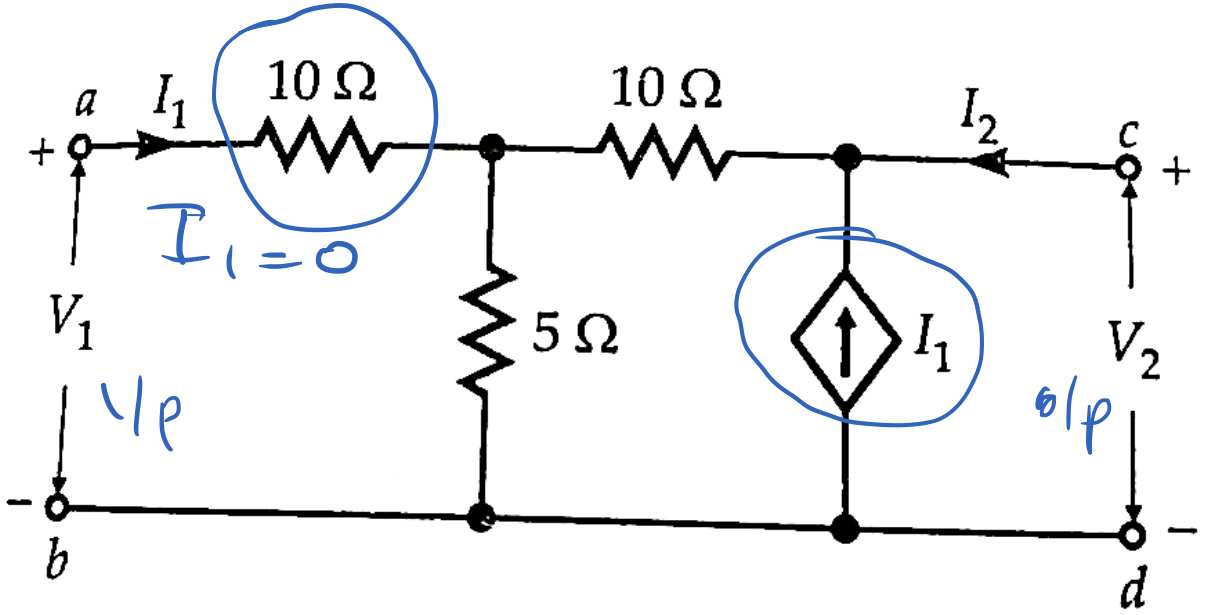
$$V_2 = 5I_1 - 5I_2' - 10I_2'$$

$$= 5I_1 + 5I_1 + 10I_1 = 20I_1$$

$$V_2 = 20 I_1 \quad \text{and} \quad I_1 = \frac{V_2}{20} \quad (A)$$

$$\left(\frac{V_2}{I_1}\right)_{I_1=0} = Z_{21}$$

$$= \frac{V_2}{V_2/20} = 20\ \Omega$$



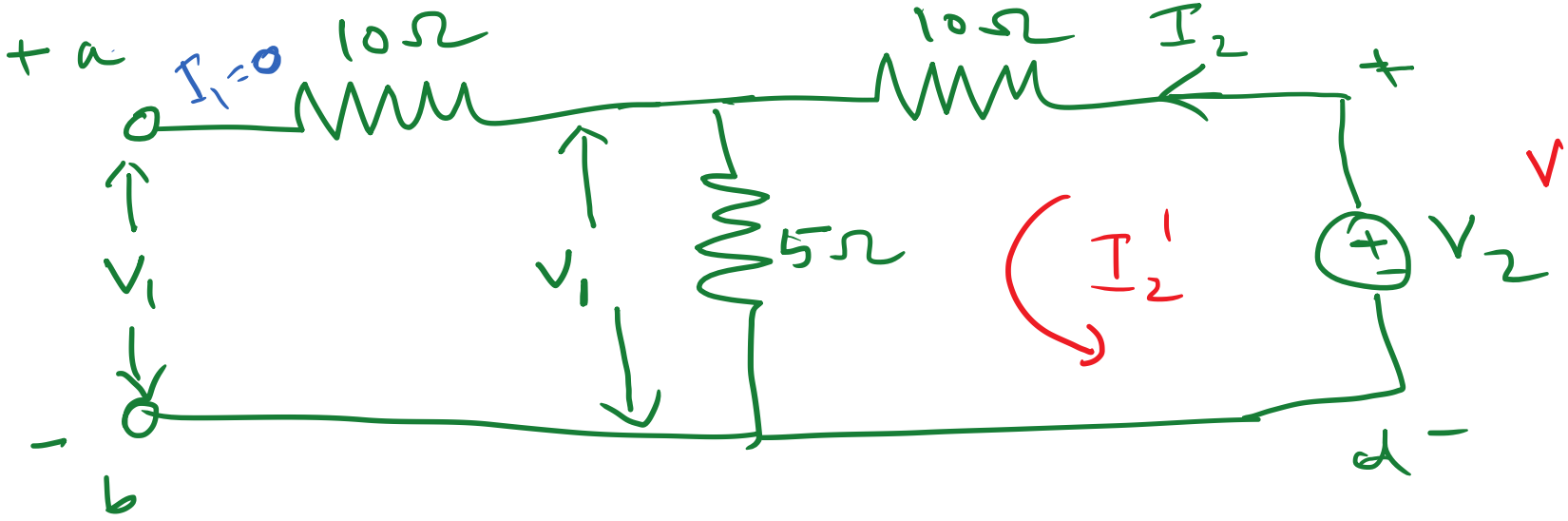
To find Z_{22} and Z_{12}

i/p is open ; $\therefore I_1 = 0$

V_2 is applied between c & d

Since $I_1 = 0$; the dependent current source is useless

Thus the circuit can be redrawn

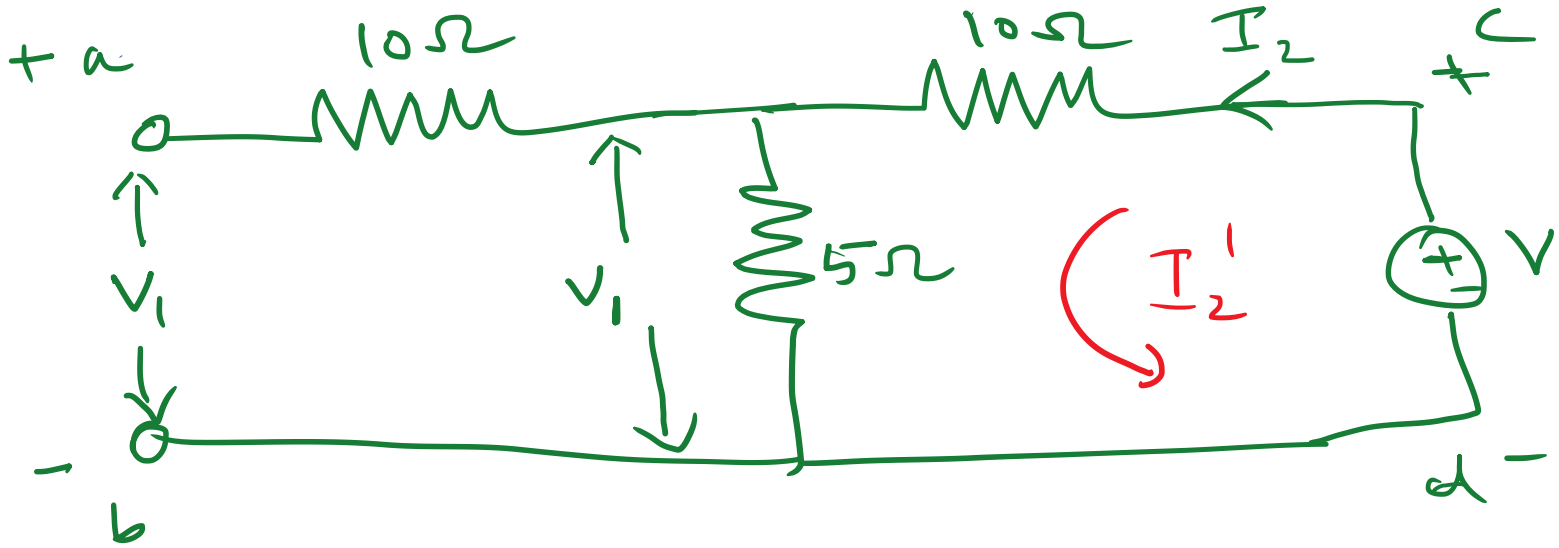


Here $I_2 = I_2'$

$V_2 = (10 + 5)\, I_2$

$= 15\, I_2$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 20\Omega & 5\Omega \\ 20\Omega & 15\Omega \end{bmatrix}$$



Here $I_2 = I_2'$

$V_2 = (10+5)\, I_2$

$= 15\, I_2$ (V)

V_1 is the voltage drop across 5 Ω resistance

$\therefore V_1 = 5\, I_2$

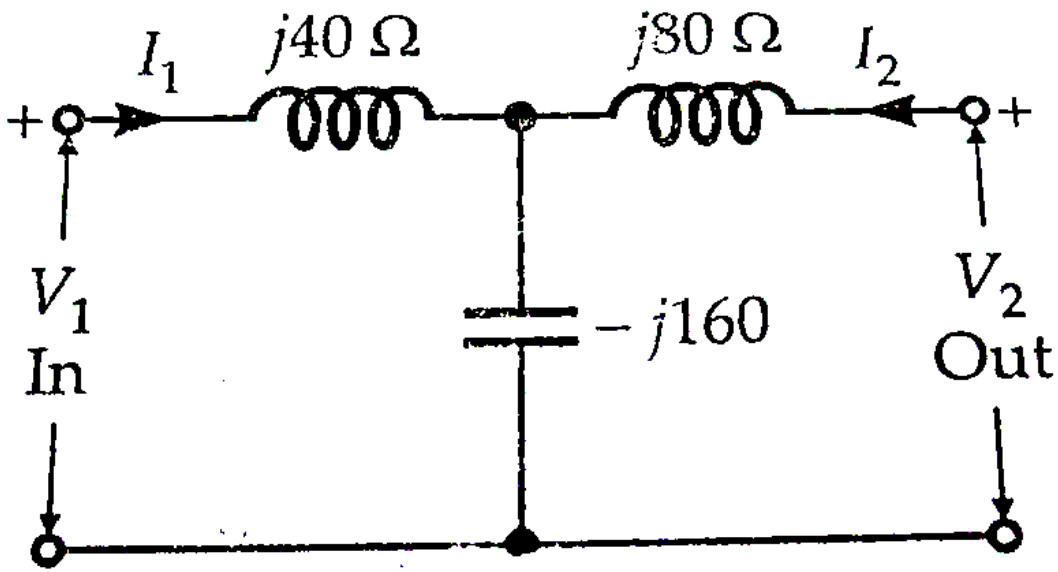
$$z_{12} = \left(\frac{V_1}{I_2}\right)_{I_1=0}$$

$$= \frac{5 I_2}{I_2} = 5\,\Omega$$

$$z_{22} = \left(\frac{V_2}{I_2}\right)_{I_1=0}$$

$$= \frac{15 I_2}{I_2} = 15\,\Omega$$

PROBLEM 2



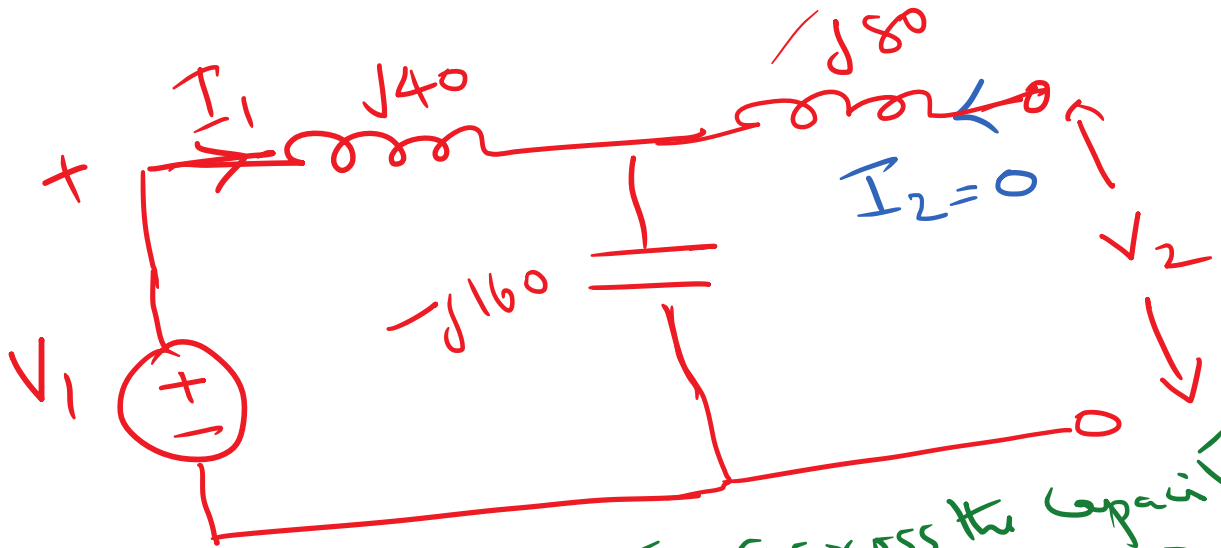
Find the open circuit parameters of the circuit (Z parameters)

Since $I_2 = 0$; $j80$ is excluded

$V_1 = I_1(j40 + -j160)$

$= I_1 j(40 - 160)$ V

Z_{11} and Z_{21}

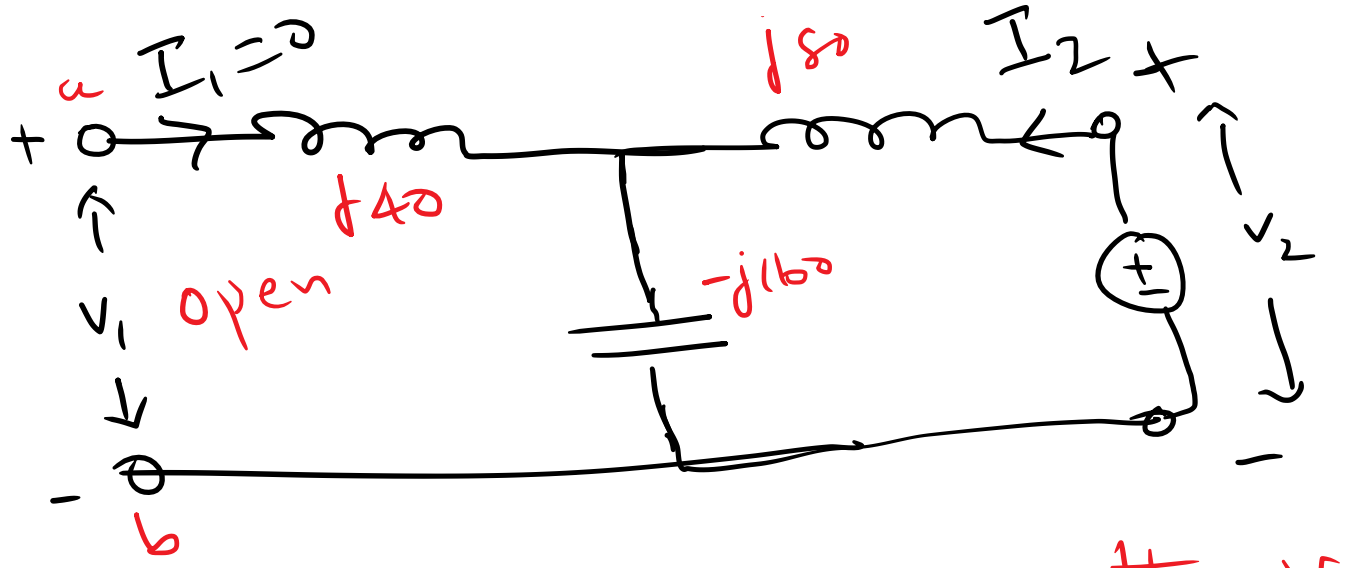


Since $I_2 = 0$; V_2 is the voltage across the capacitor

$V_2 = I_1(-j160)$ V

$Z_{11} = \left(\frac{V_1}{I_1}\right)_{I_2=0}$

$Z_{22} = \left(\frac{V_2}{I_1}\right)_{I_2=0}$



To find Z_{22} & Z_{12}

Since (ab) is open, $I_1 = 0$

$\therefore$ j40 is excluded.

Also the voltage V_1 is the voltage across the capacitor.

$$V_2 = I_2(j80 + -j160)$$

$$= I_2(j80 - j160)\ V$$

$$V_2 = I_2\, j(80 - 160)$$

$$V_1 = I_2 \angle -j160$$

$$Z_{22} = \left(\frac{V_2}{I_2}\right)_{I_1 = 0}$$

$$Z_{12} = \left(\frac{V_1}{I_2}\right)_{I_1 = 0}$$

Finally $Z = \begin{bmatrix} -j120\,\Omega & -j160\,\Omega \\ -j160\,\Omega & -j80\,\Omega \end{bmatrix}$

$\Longrightarrow$