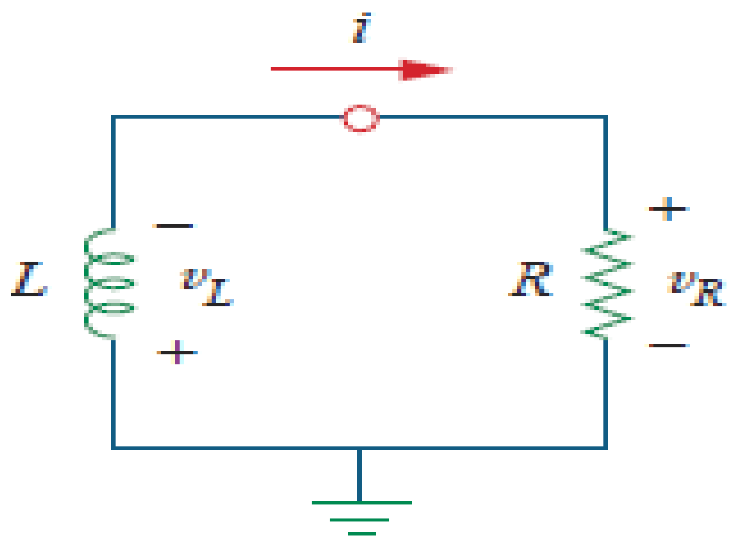
The Source-Free *RL* Circuit



At $t = 0$, assume that the inductor has an initial current I_0,

$$i(0) = I_0$$

with the corresponding energy stored in the inductor as

$$w(0) = \frac{1}{2} L I_0^2$$

Applying KVL around the loop

$$v_L + v_R = 0$$

But $v_L = L\, di/dt$ and $v_R = iR$. Thus,

$$L\frac{di}{dt} + Ri = 0$$

or

$$L\frac{di}{dt} + Ri = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt$$

$$\ln i\Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \quad \Rightarrow \quad \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

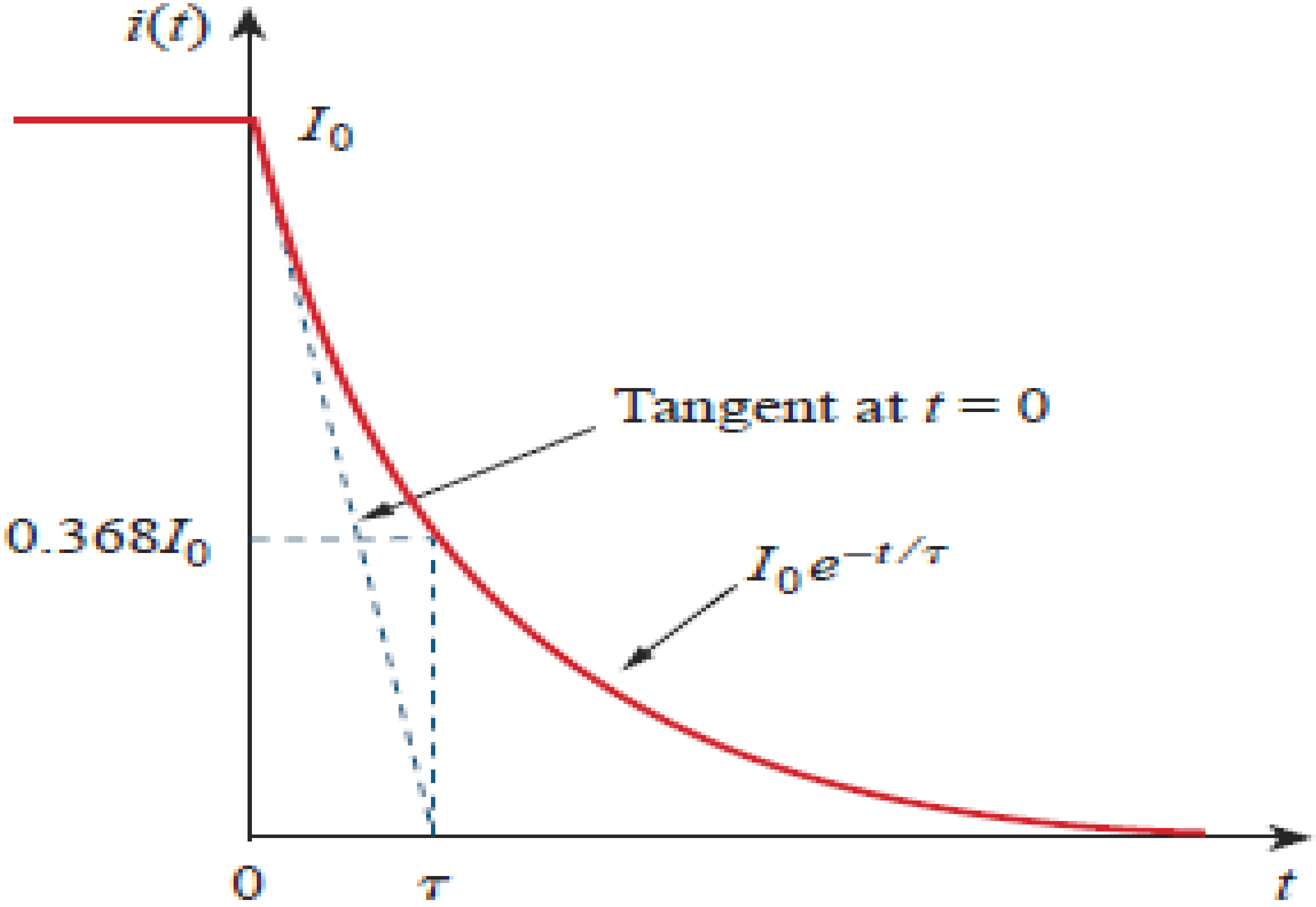
$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking the powers of e, we have

$$i(t) = I_0 e^{-Rt/L}$$

$$\tau = \frac{L}{R}$$

$$i(t) = I_0 e^{-t/\tau}$$



The current response of the RL circuit.

the voltage across the resistor as

$$v_R(t) = iR = I_0 Re^{-t/\tau}$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 Re^{-2t/\tau}$$

The energy absorbed by the resistor is

$$w_R(t) = \int_0^t p\,dt = \int_0^t I_0^2 Re^{-2t/\tau}\,dt = -\frac{1}{2}\tau I_0^2 Re^{-2t/\tau}\Big|_0^t, \qquad \tau = \frac{L}{R}$$

$$w_R(t) = \frac{1}{2}L I_0^2(1 - e^{-2t/\tau})$$

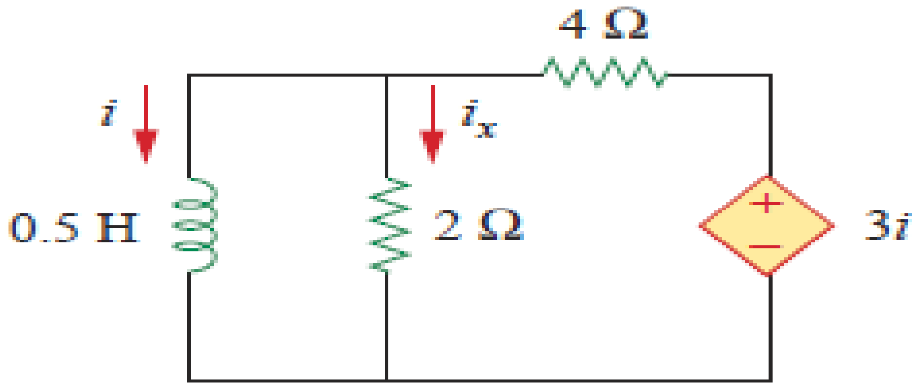
Note that as $t \longrightarrow \infty$, $w_R(\infty) \longrightarrow \frac{1}{2}L I_0^2$, which is the same as $w_L(0)$, energy initially stored in the inductor is eventually dissipated in the resistor.

The Key to Working with a Source-free *RL* Circuit Is to Find:

1. The initial current $i(0) = I_0$ through the inductor.
2. The time constant τ of the circuit.

Example

Assuming that $i(0) = 10$ A, calculate $i(t)$ and $i_x(t)$ in the circuit of

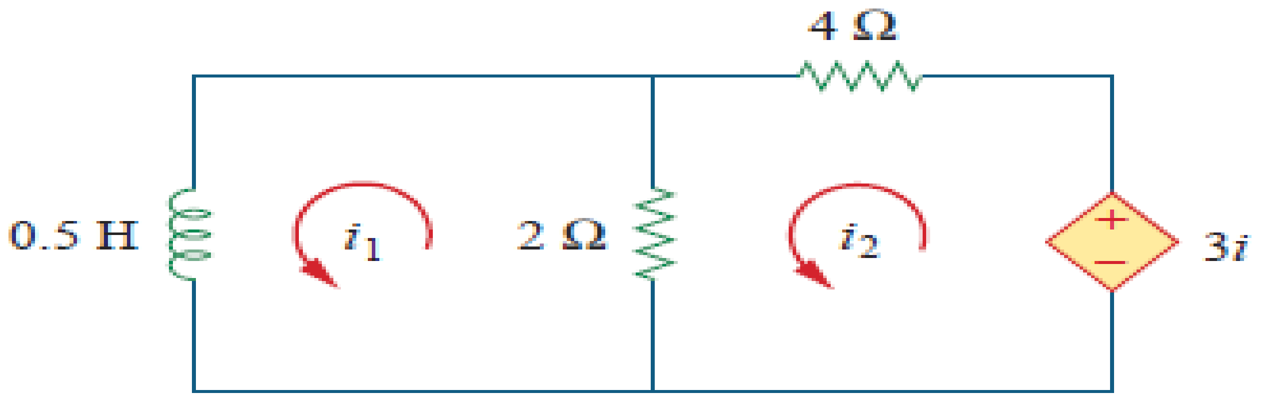


$$\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0$$

$$\frac{di_1}{dt} + 4i_1 - 4i_2 = 0$$

■ **METHOD 1**

$$6i_2 - 2i_1 - 3i_1 = 0 \quad \Longrightarrow \quad i_2 = \frac{5}{6}i_1$$



$$\frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

$$\frac{di_1}{i_1} = -\frac{2}{3}dt$$

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$$\ln i \bigg|_{i(0)}^{i(t)} = -\frac{2}{3}t \bigg|_0^t$$

$$\ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t} \text{ A}, \qquad t > 0$$

$$v = L\frac{di}{dt} = 0.5(10)\left(-\frac{2}{3}\right)e^{-(2/3)t} = -\frac{10}{3}e^{-(2/3)t} \text{ V}$$

Since the inductor and the 2-Ω resistor are in parallel,

$$i_x(t) = \frac{v}{2} = -1.6667e^{-(2/3)t} \text{ A}, \qquad t > 0$$