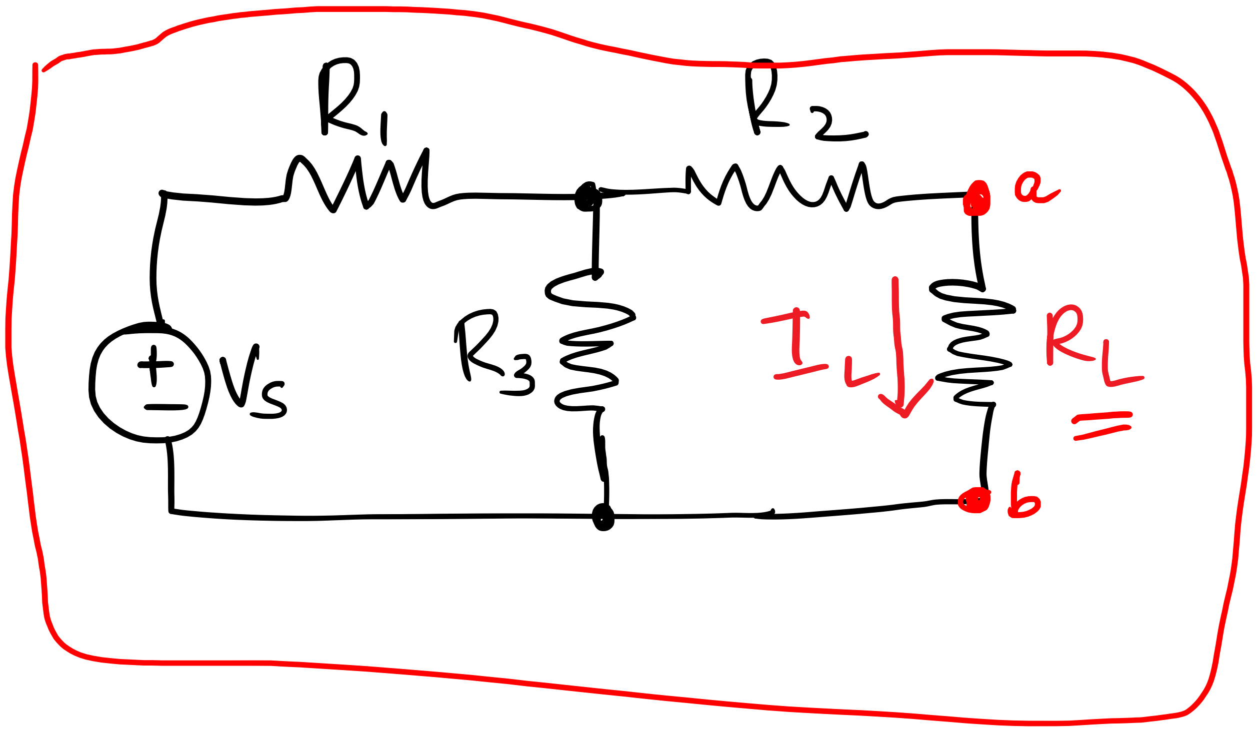
Thevenin's Theorem

the *Thevenin equivalent circuit*, was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

$\boxed{DC}$ ✓

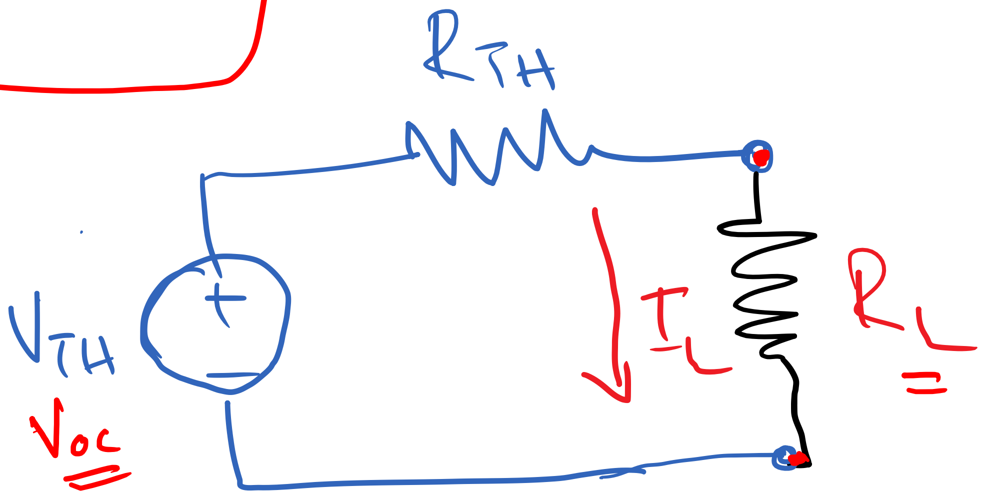
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

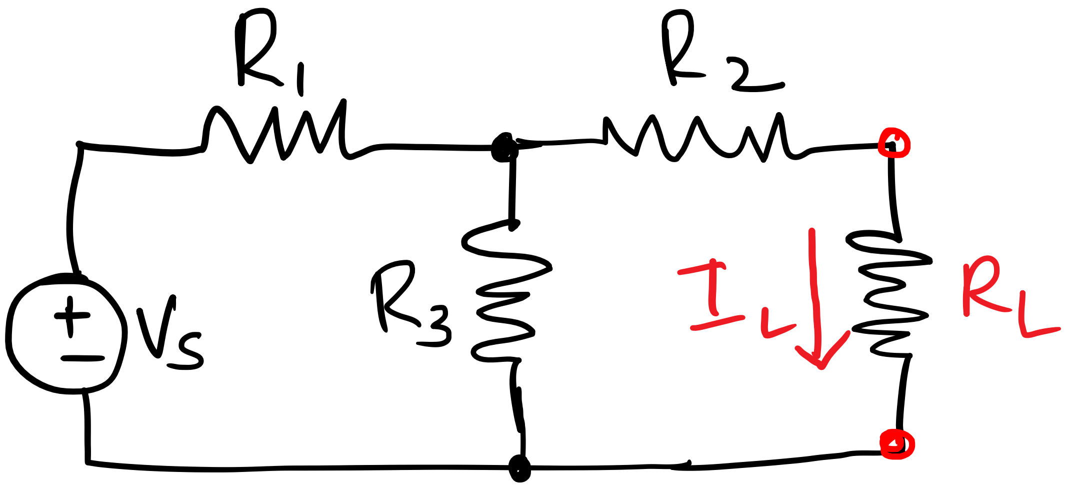


$R_L \rightarrow$ Load resistance

$I_L \rightarrow$ load current

$\boxed{DC}$





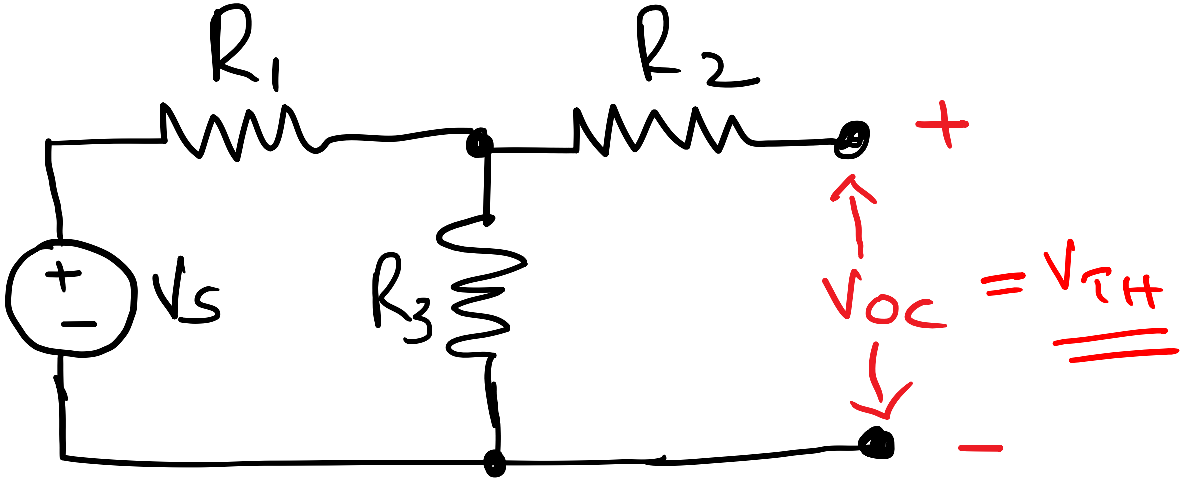
$R_L \rightarrow$ load resistance

$I_L \rightarrow$ load current

To find equivalent voltage source, R_L is removed

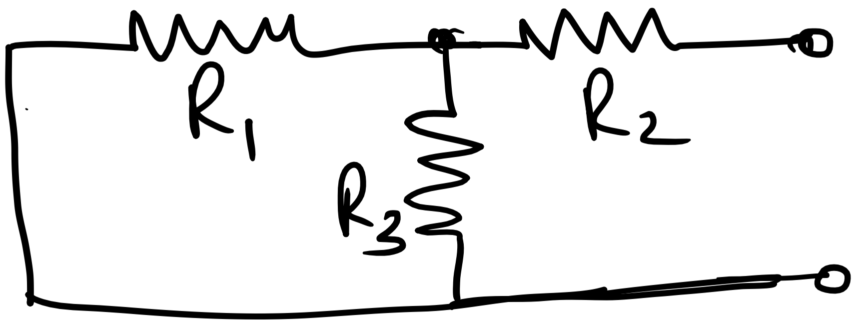
$$V_{oc} = I R_3$$

$$= \frac{V_s}{R_1 + R_3} \times R_3$$



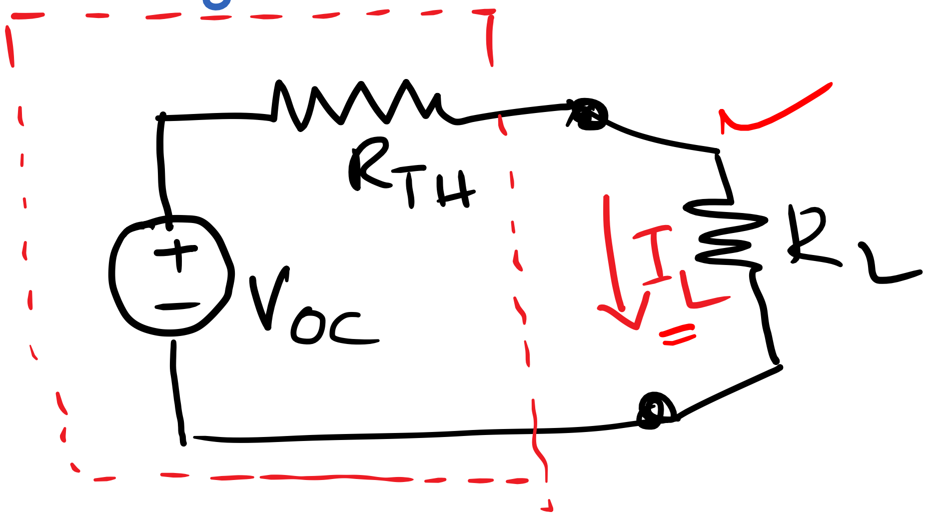
To find internal resistance of the network

→ Thevenin resistance or equivalent resistance



$$\boxed{R_{TH} = R_2 + \frac{R_1 R_3}{R_1 + R_3}}$$

According to Thevenin theorem



$$V_{OC} = V_{TH}$$

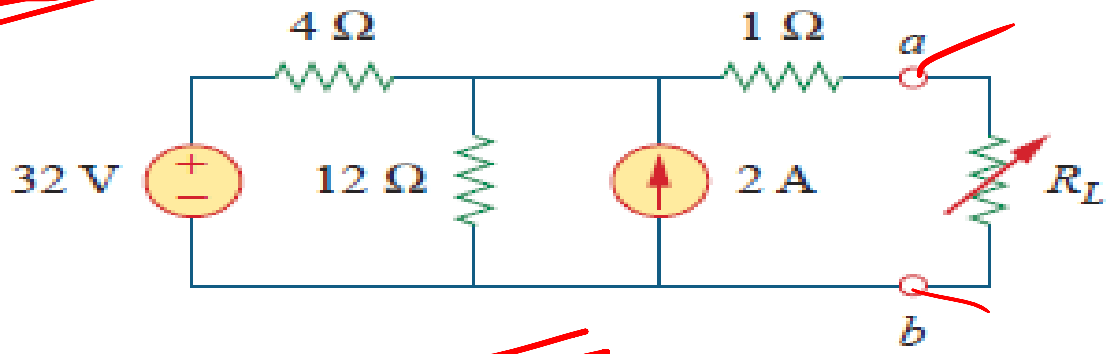
$$I_L = \frac{V_{OC}}{R_{TH} + R_L}$$

Steps to find Thevenin’s equivalent circuit

1. Identify the load terminals.
2. Remove the load resistance from the circuit (if its is connected)
3. Find the equivalent resistance (R_{Th}) across the load terminal by replacing independent voltage source by short circuit and independent current source by open circuit
4. Find the open circuit voltage (V_{Th}) using any of the circuit analysis techniques (mostly mesh/node methods)
5. Draw the Thevenin's equivalent circuit

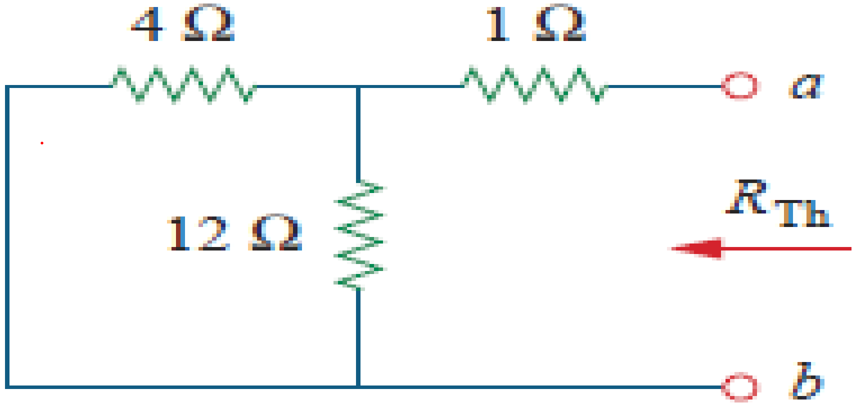
Example

DC



Find the Thevenin equivalent circuit of the circuit shown in Fig. to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

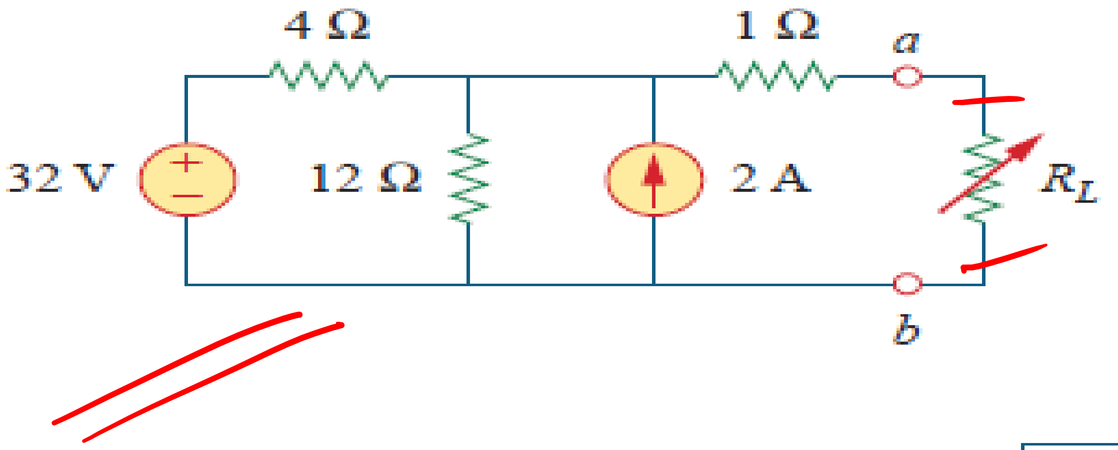
$R_{TH} = ?$



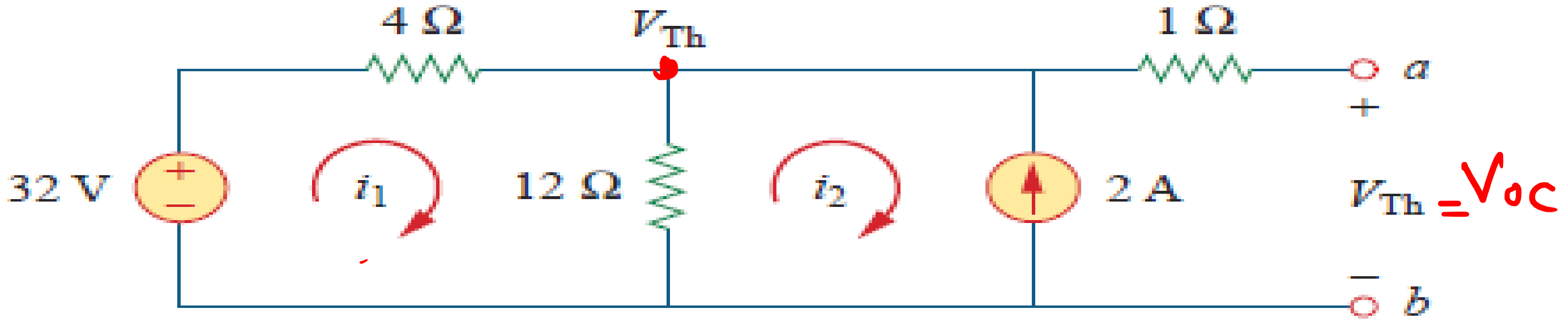
(a)

(a) finding R_{Th}.

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$



Applying mesh analysis to the two loops, we obtain



(b)

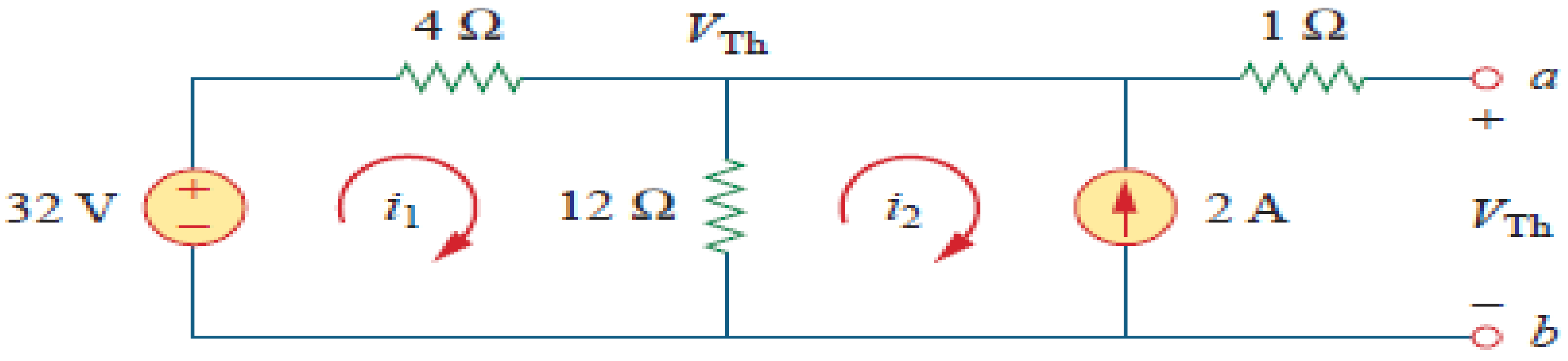
(b) finding V_{Th}.

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad \boxed{i_2 = -2 \text{ A}}$$

$V_{oc} = V_{TH} = 30\text{V}$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$



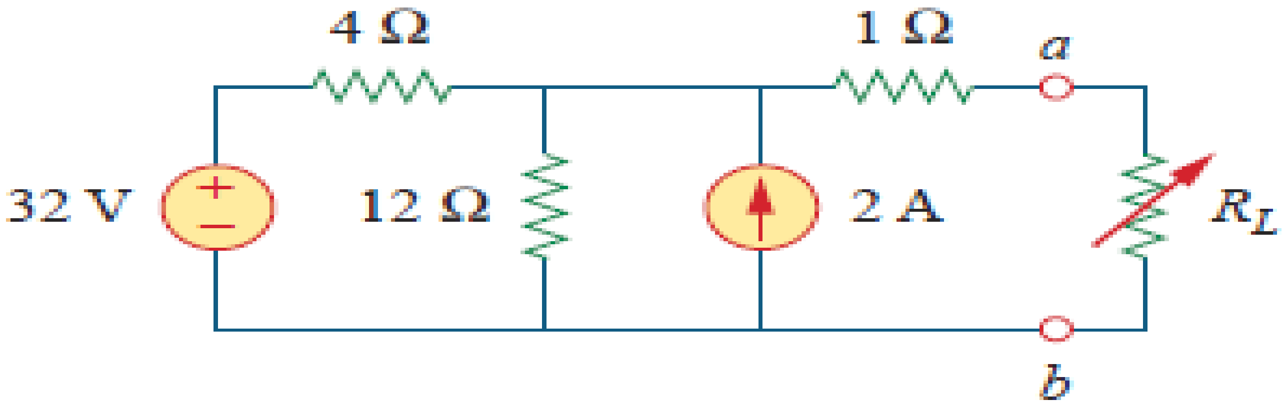
(b)

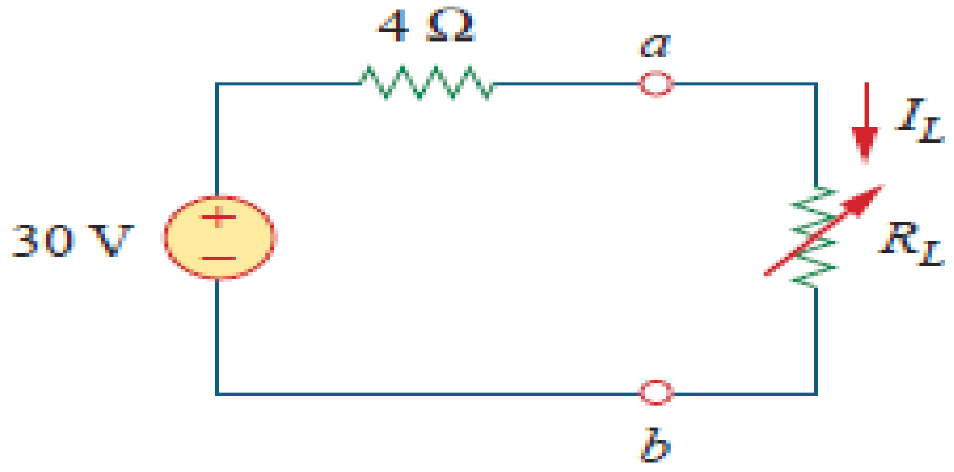
At the top node, KCL

We ignore the 1-Ω resistor since no current flows through it.

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$96 - 3V_{Th} + 24 = V_{Th} \qquad \Rightarrow \qquad V_{Th} = 30 \text{ V}$$





The Thevenin equivalent circuit

The current through R_L is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When $R_L = 6, \quad I_L = \frac{30}{10} = 3 \text{ A}$

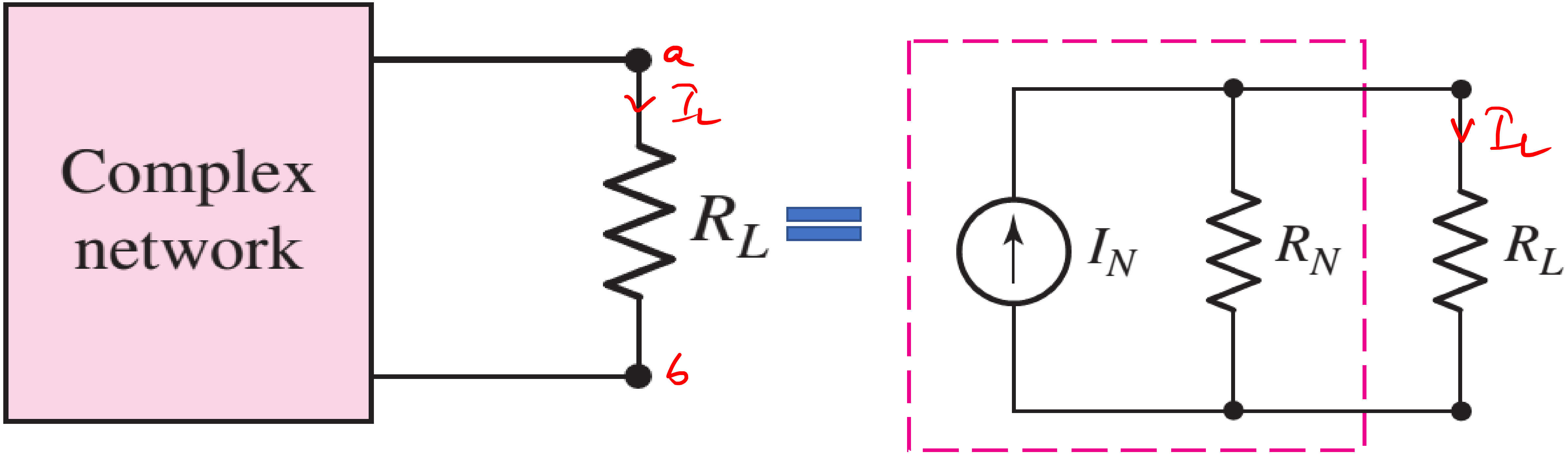
When $R_L = 16, \quad I_L = \frac{30}{20} = 1.5 \text{ A}$

When $R_L = 36, \quad I_L = \frac{30}{40} = 0.75 \text{ A}$

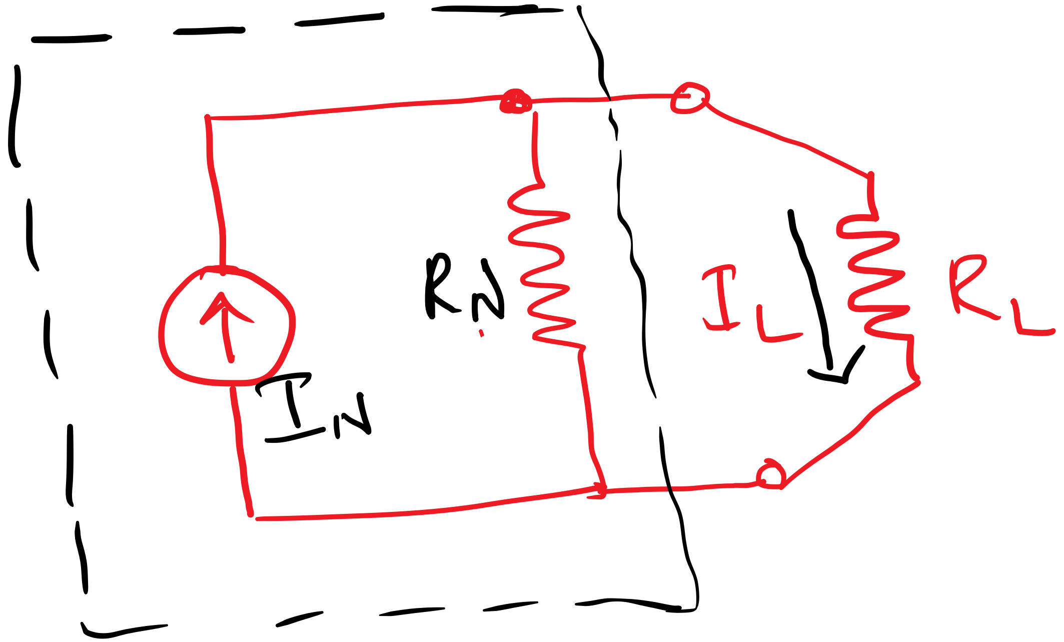
Norton's Theorem

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.


Complex network
a
I_L
R_L
b
I_N
R_N
I_L
R_L

$$\boxed{R_N = R_{TH}}$$



$$\boxed{I_L = I_N \times \frac{R_N}{R_N + R_L}}$$

Steps to find Norton's equivalent circuit

1. Identify the load terminals.
2. Remove the load resistance from the circuit (if its is connected)
3. Find the equivalent resistance (R_N) across the load terminal by replacing independent voltage source by short circuit and independent current source by open circuit
4. Create a short circuit across the load terminals
5. Find the short circuit current (I_N) using any of the circuit analysis techniques (mostly mesh/node methods)
6. Draw the Norton's equivalent circuit

$I_N = \underline{\underline{I_{sc}}}$

$R_{TH} = \underline{\underline{R_N}}$

Relationship between Thevenin's & Norton's Theorem

$V_{oc} = V_{TH}$

$I_N = I_{sc}$

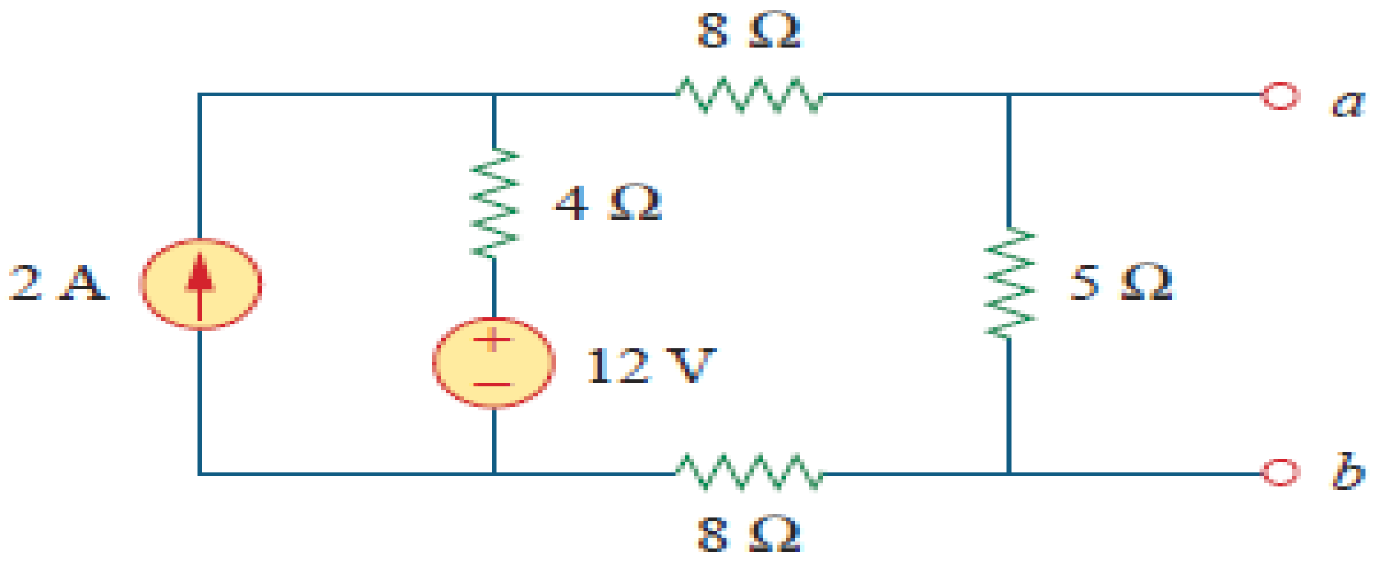
$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

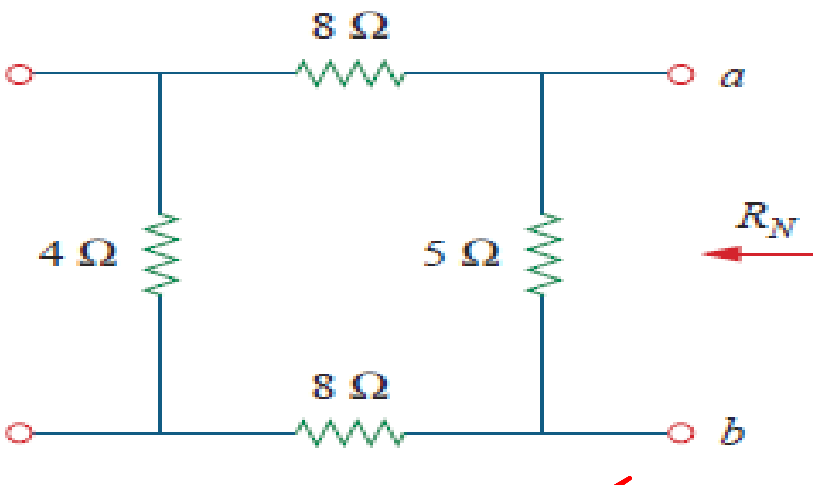
$$R_N = R_{Th}$$

Example Find the Norton equivalent circuit of the circuit in Fig. terminals *a*-*b*.

DC Source

$R_N = R_{TH}$

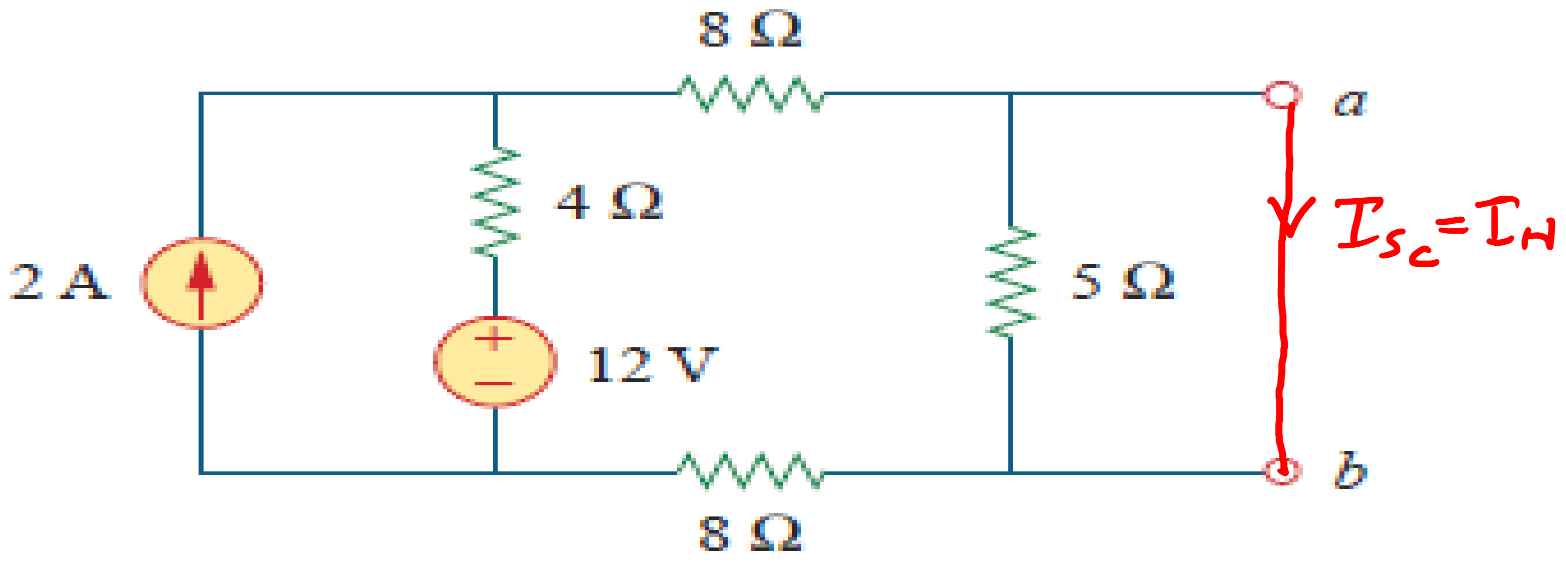


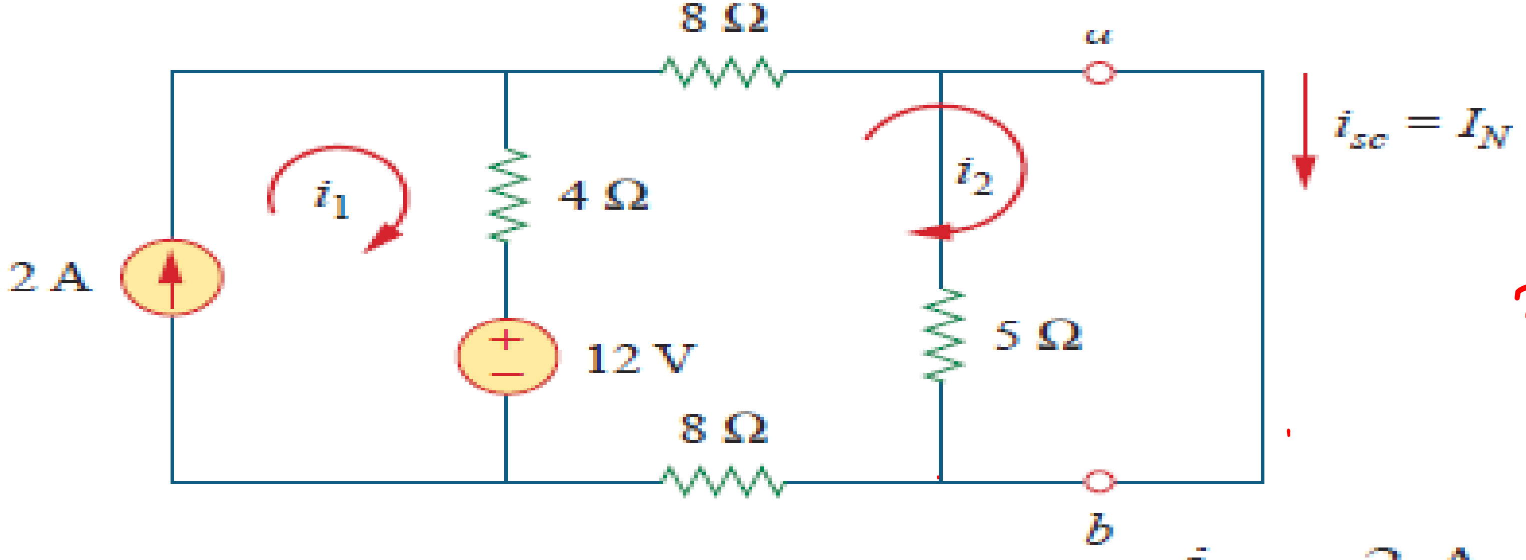


(a)

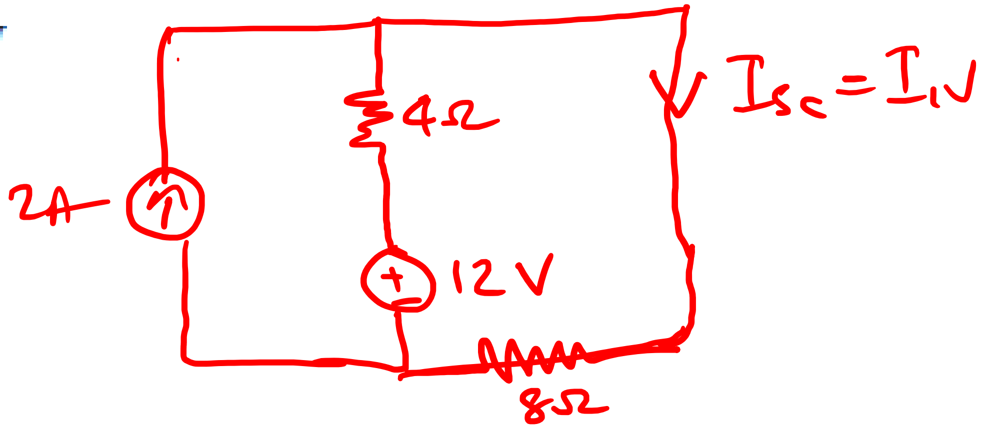
$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain





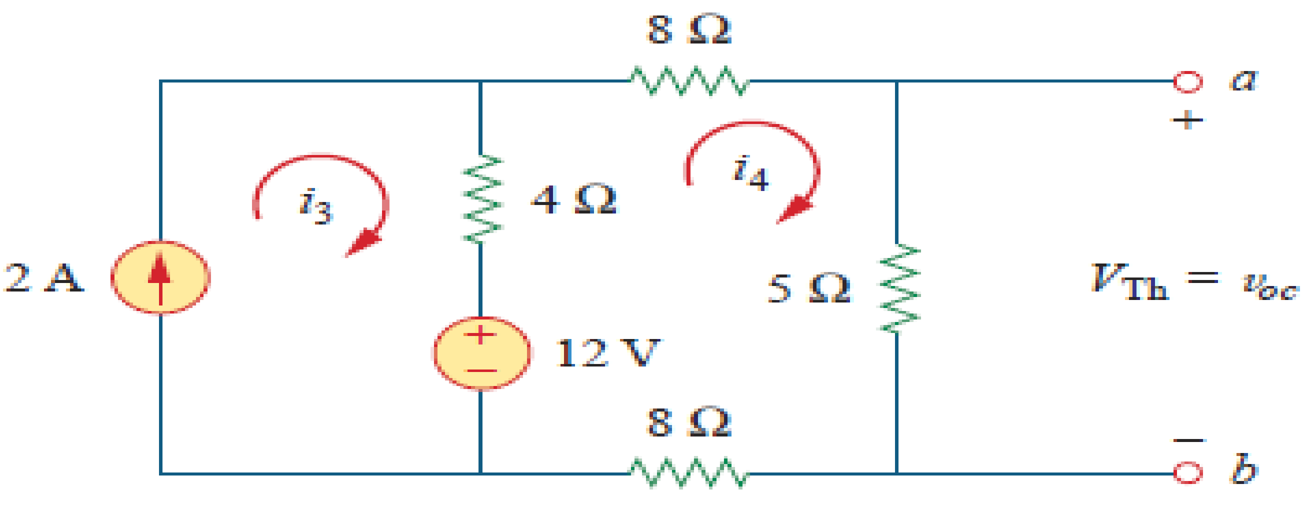
(b)



$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in



(c)

$$i_3 = 2 \text{ A}$$

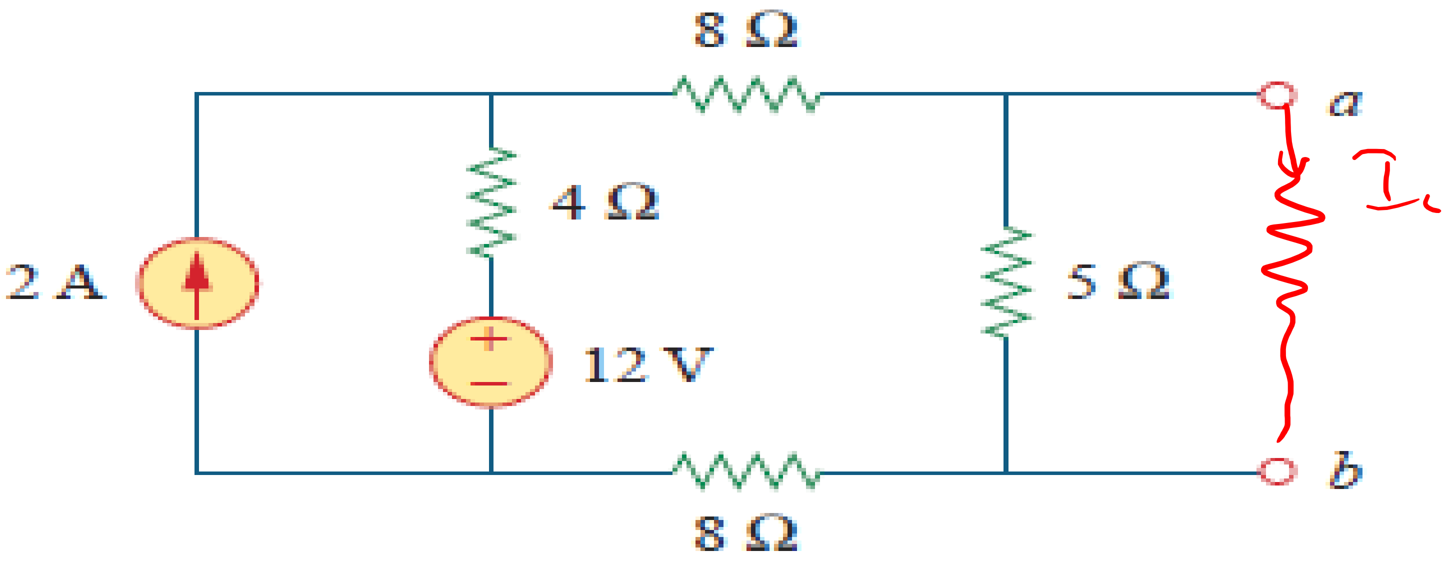
$$4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

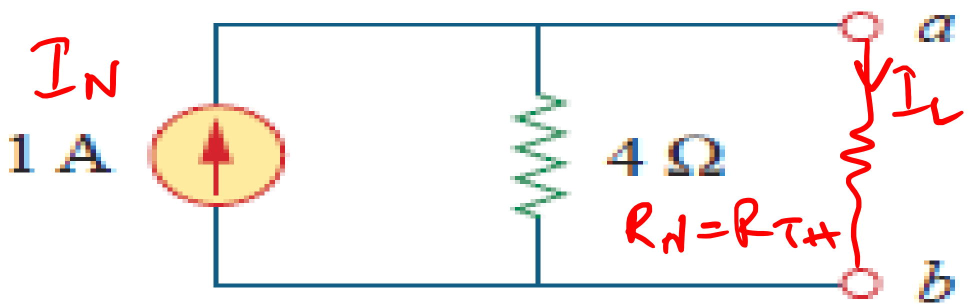
and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

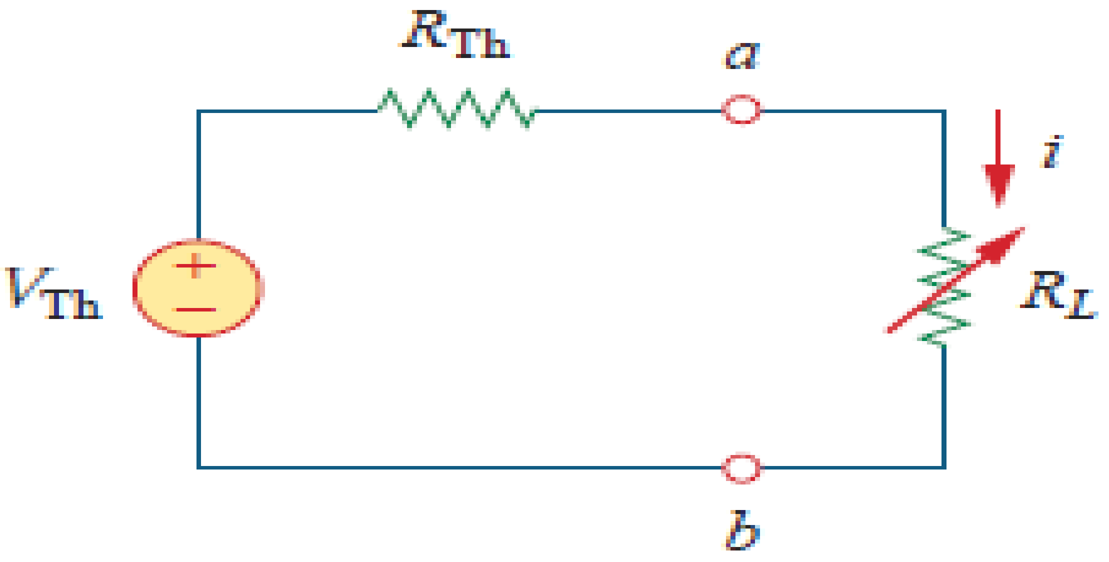
$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

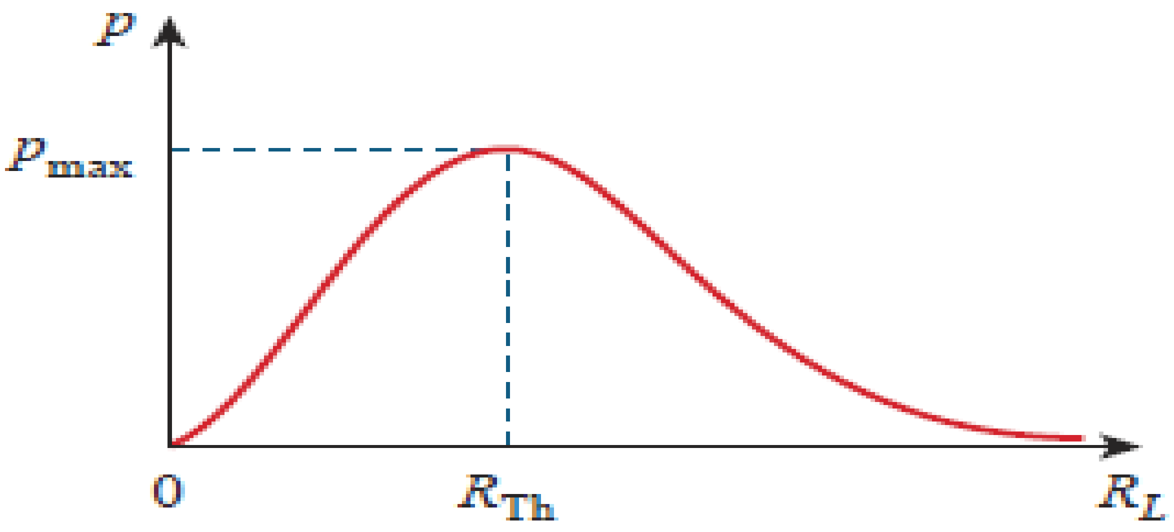




Norton equivalent of the circuit

Maximum Power Transfer





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

$$p = i^2 R_L = \left(\frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}\right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4}\right]$$

$$= V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3}\right] = 0$$

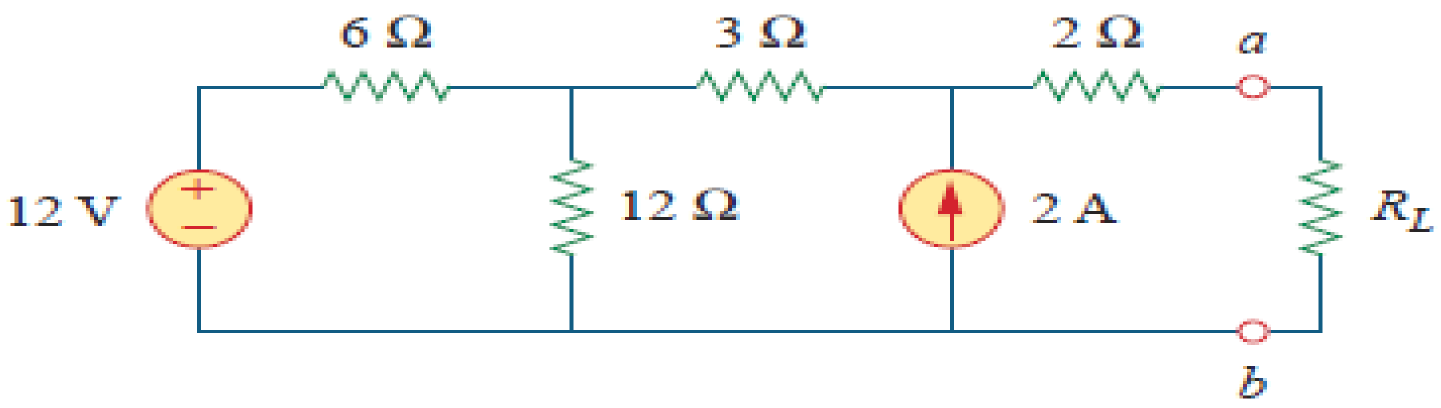
$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L)$$

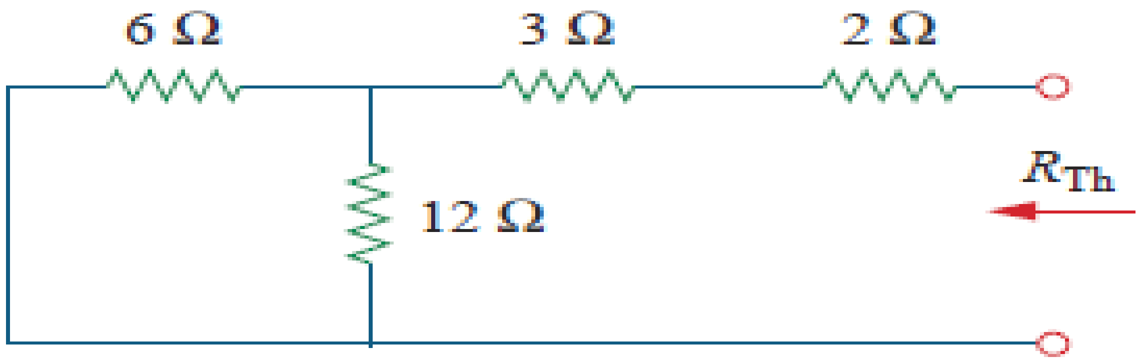
$$\boxed{R_L = R_{\text{Th}}}$$

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}}$$

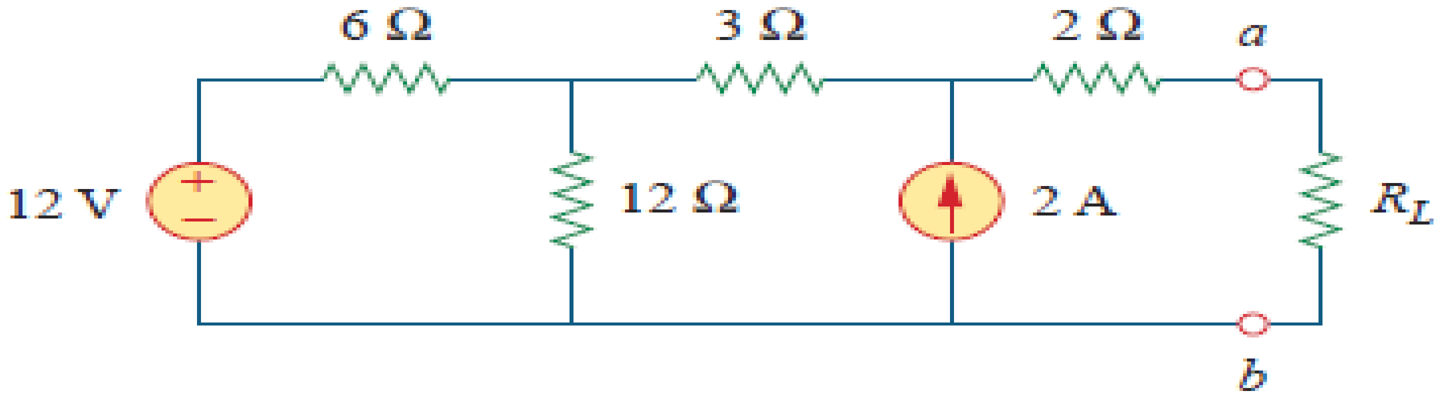
Example

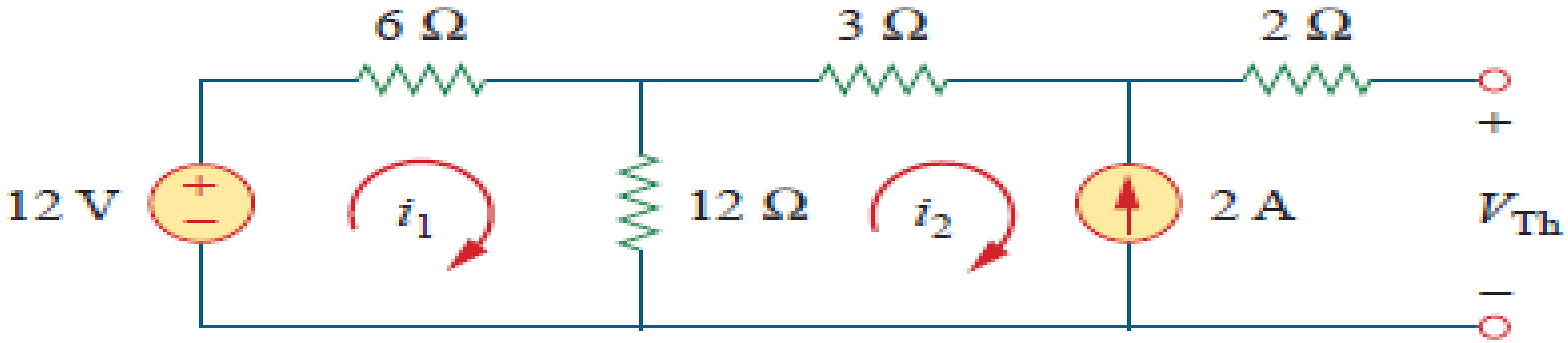
Find the value of R_L for maximum power transfer in the circuit





$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$





$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$.

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\text{ V}$$

$$R_L = R_{\mathrm{Th}} = 9\ \Omega$$

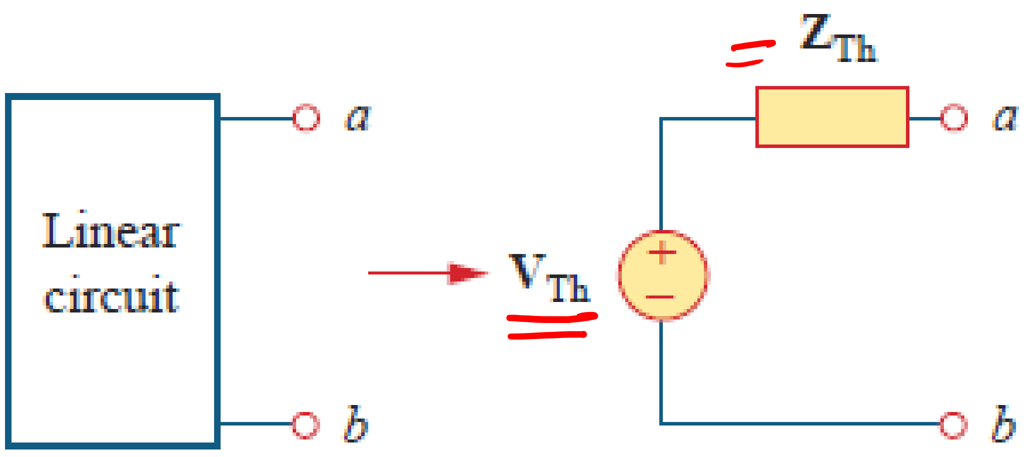
$$p_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\ \mathrm{W}$$

10.6 Thevenin and Norton Equivalent Circuits

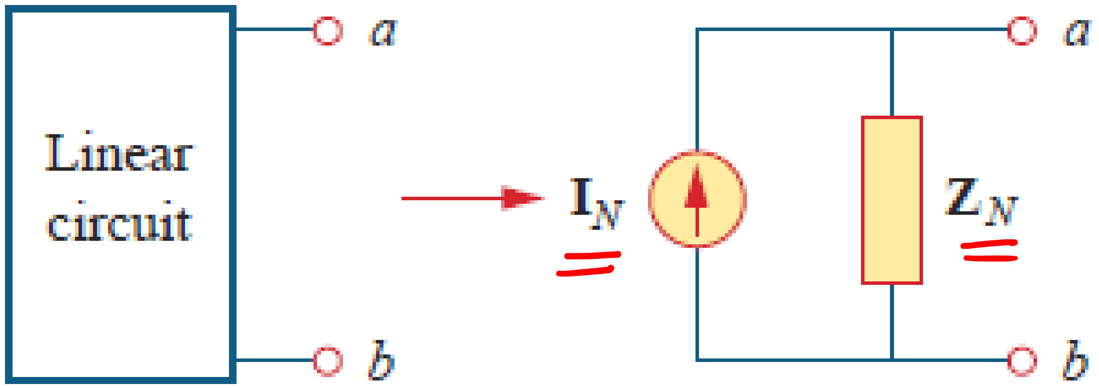
Sinusoidal voltage sources & current.

(AC with constant omega or Sinusoidal)

Phasors



Thevenin equivalent



Nortan equivalent

$$\mathbf{V}_{Th} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{Th} = \mathbf{Z}_N$$

$\mathbf{V}_{Th}$ → Thevenin voltage (open circuit voltage)

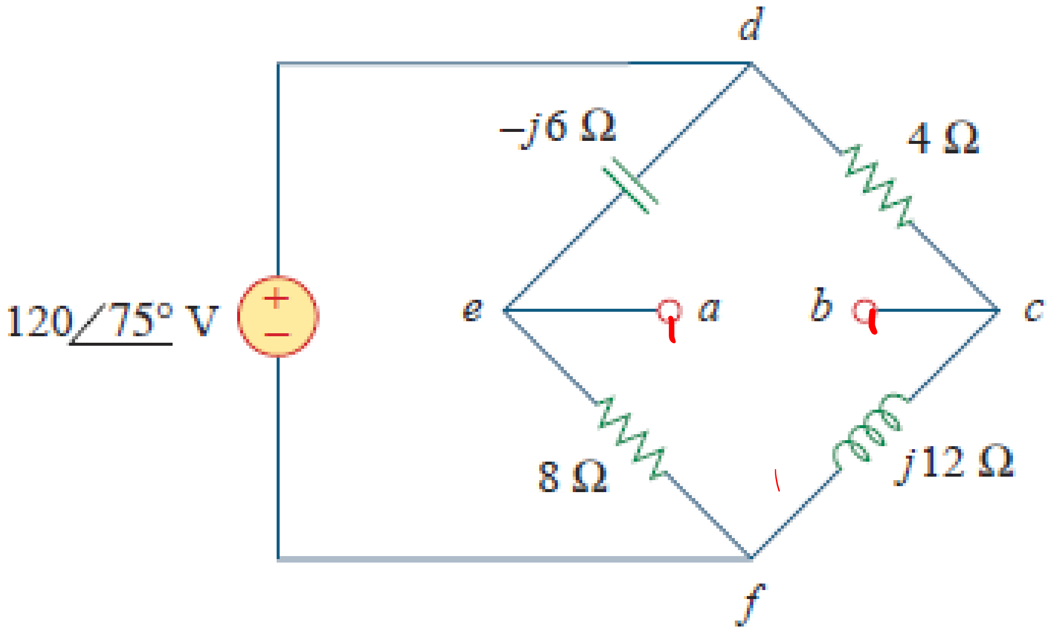
$\mathbf{I}_N$ → Nortan current (short-circuit current)

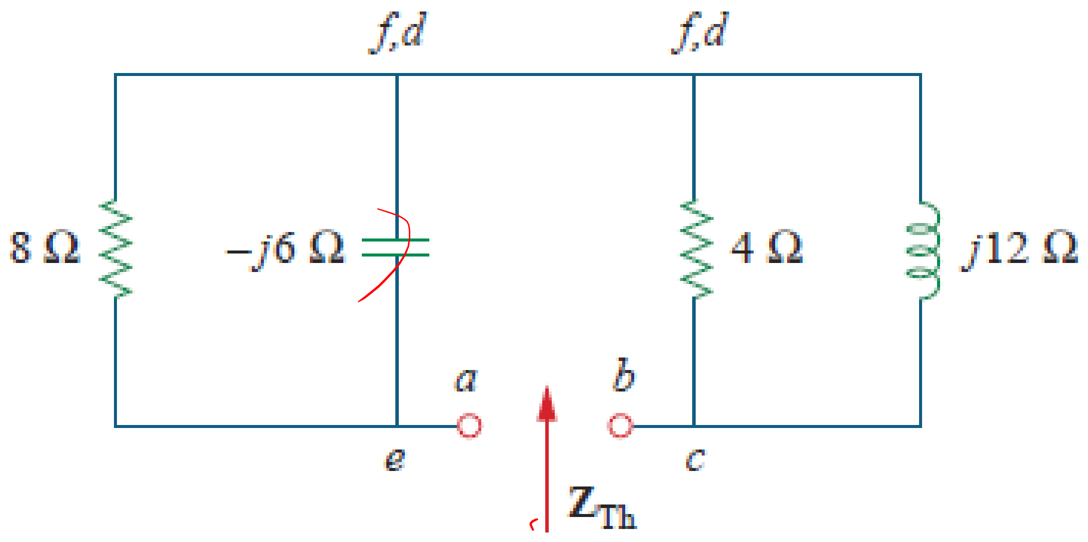
$\mathbf{Z}_{Th}$ → Thevenin equivalent impedance

$\mathbf{Z}_N$ → Nortan equivalent impedance

Example 10.8

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in Fig.



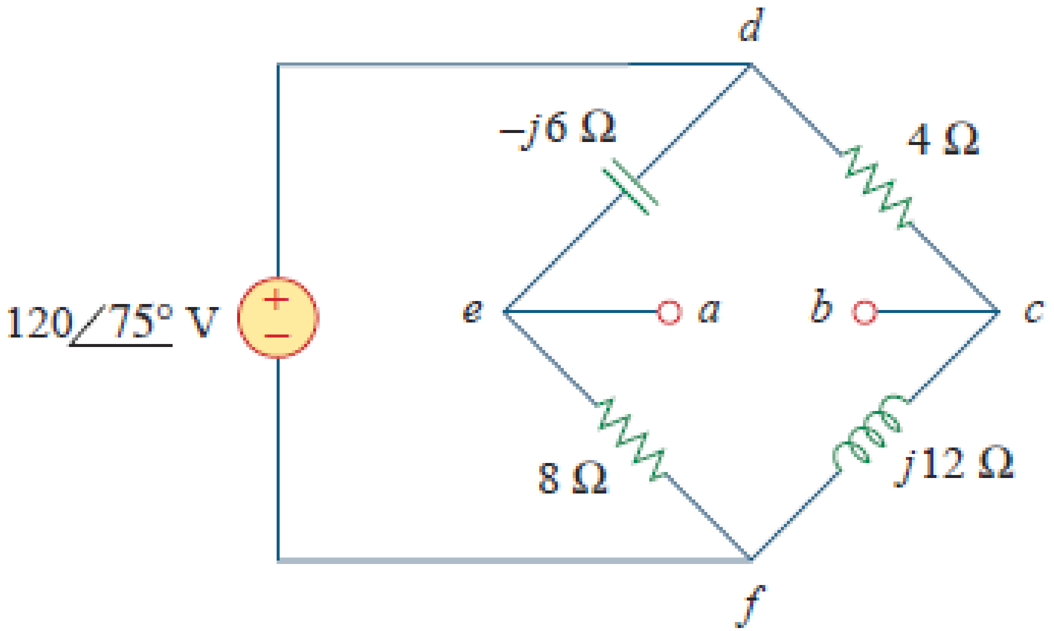


(a)

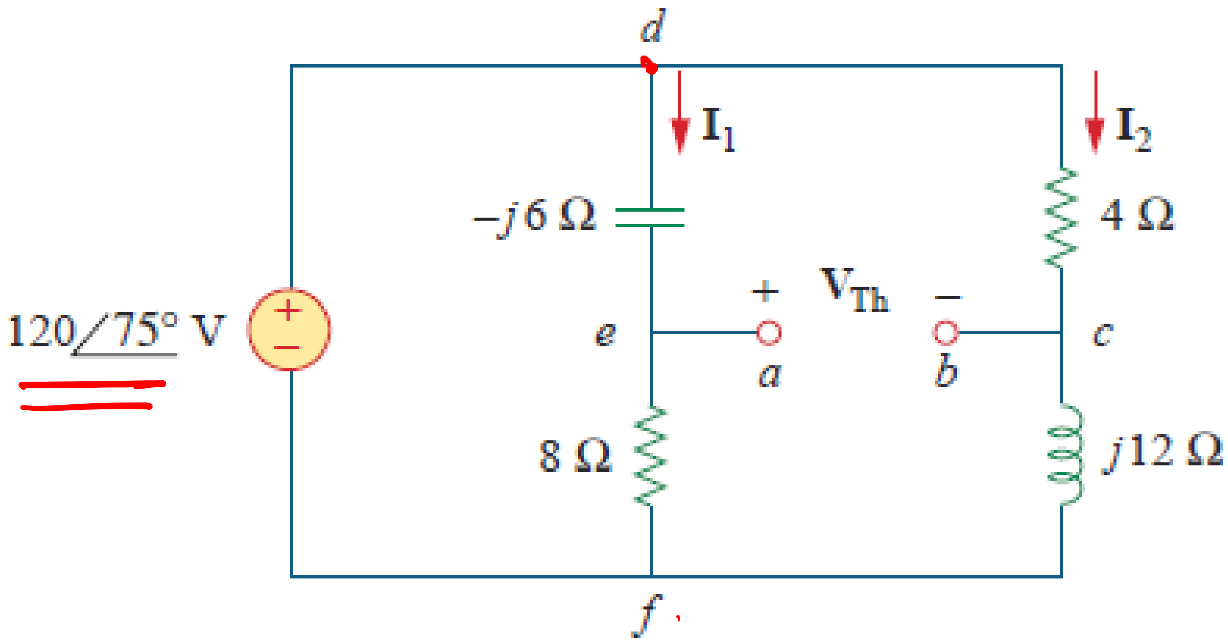
$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3$$

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

$$Z_{TH} = Z_1 + Z_2$$
$$= (6.48 - j2.64)\Omega$$



Voltge applied between d & f is same as voltge applied betn



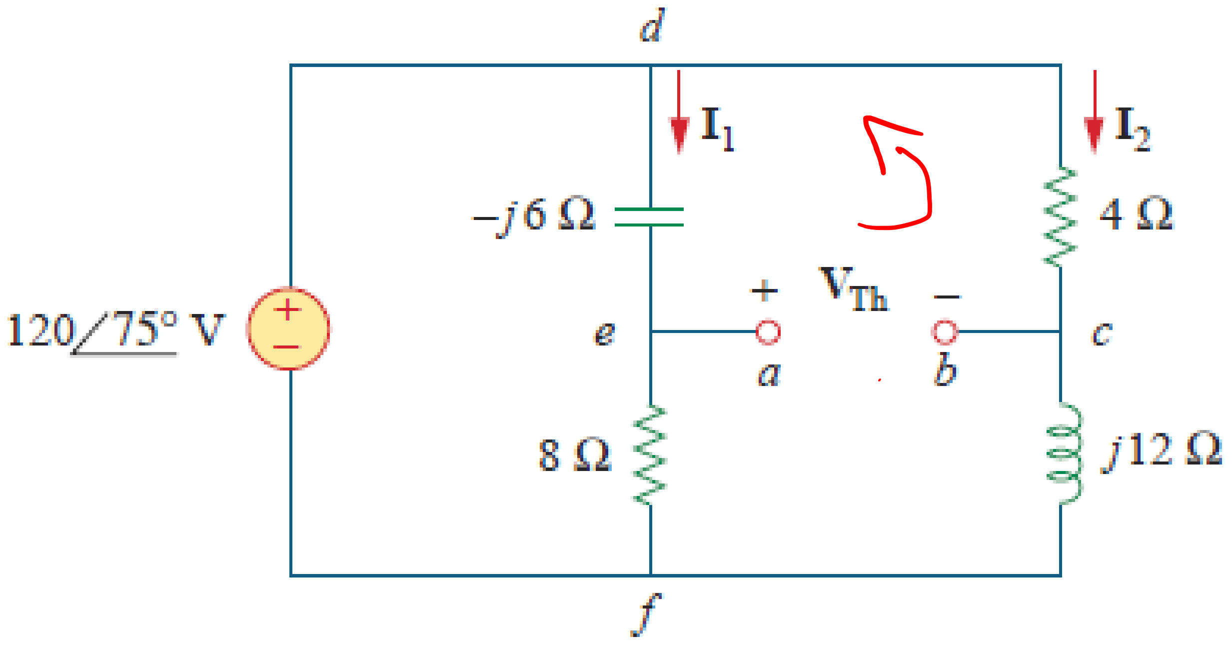
(b)

$$\underline{\underline{I_1}} = \frac{120\angle 75^\circ}{8 - j6} \, A$$

$$\underline{\underline{I_2}} = \frac{120\angle 75^\circ}{4 + j12} \, A$$

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{Th} = \underline{\underline{\mathbf{Z}_1 + \mathbf{Z}_2}} = 6.48 - j2.64\ \Omega$$



(b)

$$V_{TH} = \underline{\underline{-28.936 - j\,24.55}}$$

$$= \underline{\underline{37.95 \angle 220.31° \text{ V}}}$$

$\underline{\underline{bcdeab}} \Rightarrow \underline{\underline{KVL}}$

Using $I_1 + I_2 \Longrightarrow$

$$V_{TH} - 4I_2 + (-j6 I_1) = 0$$

$$\underline{\underline{V_{TH} = 4I_2 + j6I_1}}$$

$$\mathbf{V}_{\mathrm{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

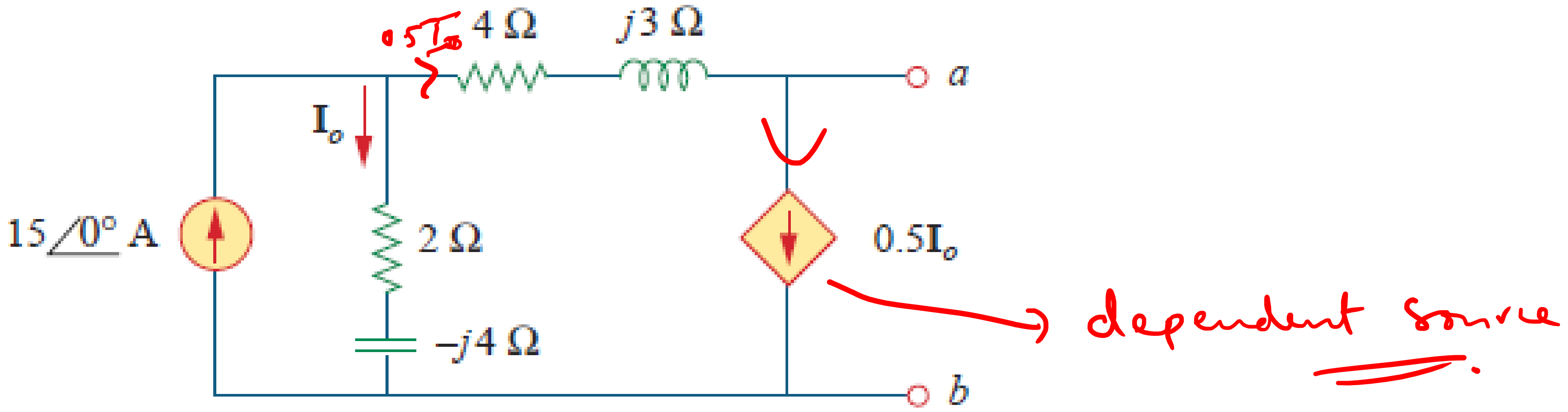
$$\mathbf{V}_{\mathrm{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\underline{/75^\circ}}{4 + j12} + \frac{720\underline{/75^\circ + 90^\circ}}{8 - j6}$$

$$= 37.95\underline{/3.43^\circ} + 72\underline{/201.87^\circ}$$

$$= -28.936 - j24.55 = 37.95\underline{/220.31^\circ}\ \mathrm{V}$$

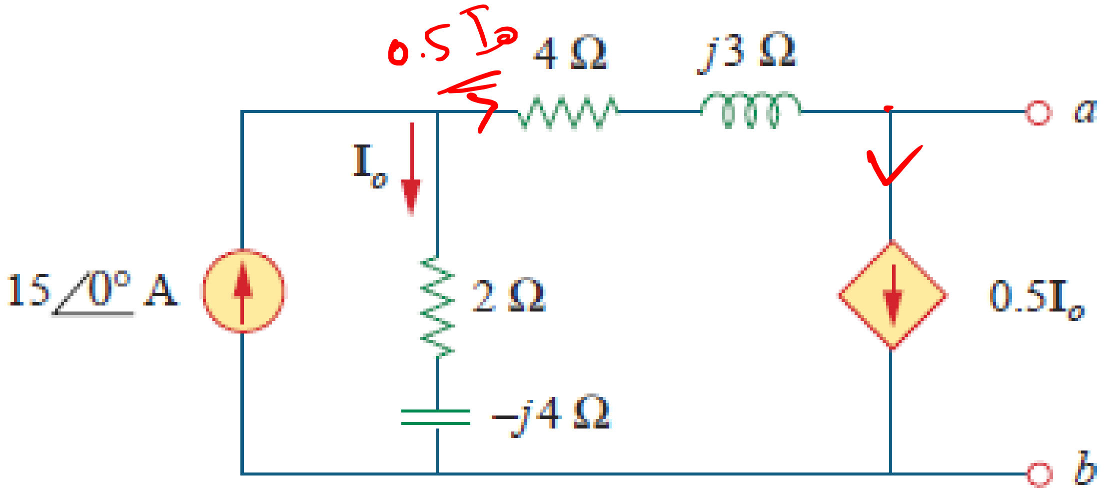
Example 10.9

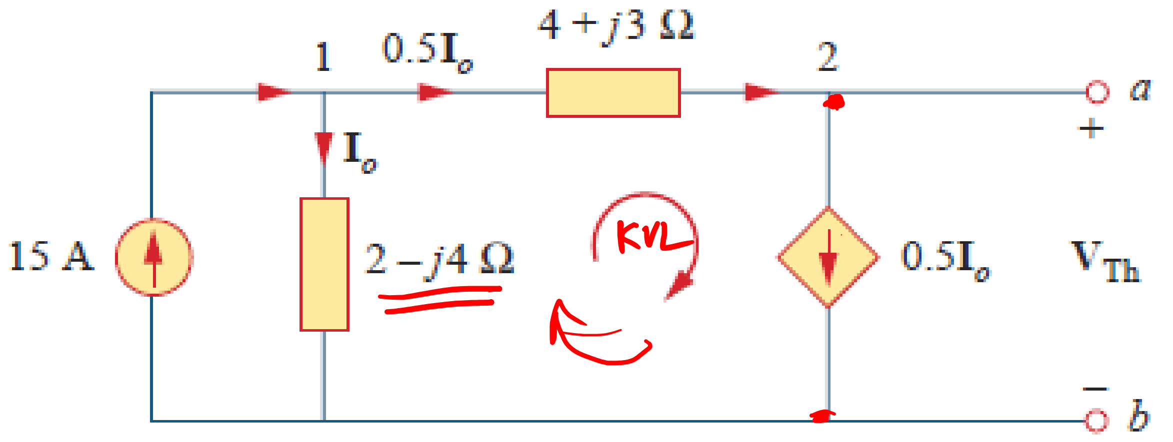
Find the Thevenin equivalent of the circuit in Fig.



To find V_{Th}, we apply KCL at node 1 in Fig.

$$15 = I_o + 0.5I_o \quad \Rightarrow \quad I_o = 10 \text{ A}$$





(a)

Applying KVL to the loop on the right-hand side in Fig.

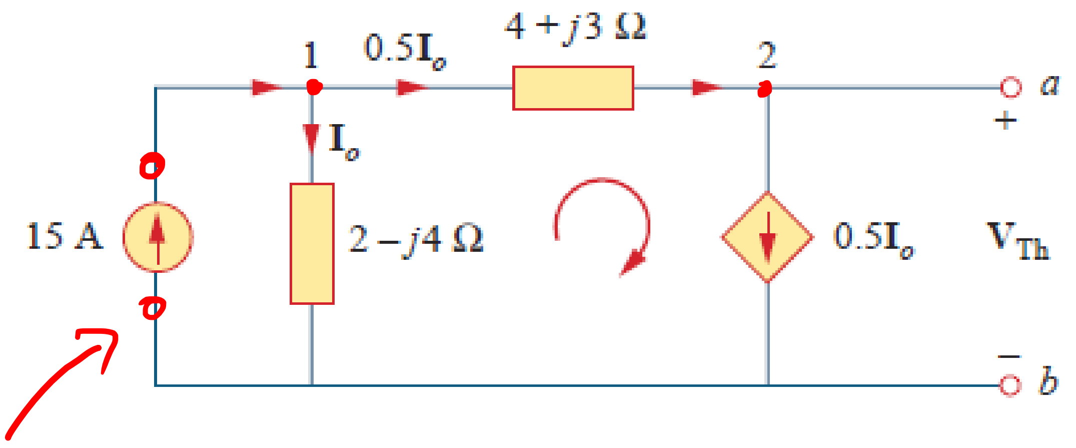
$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 10(2 - j4) - 5(4 + j3) = -j55$$

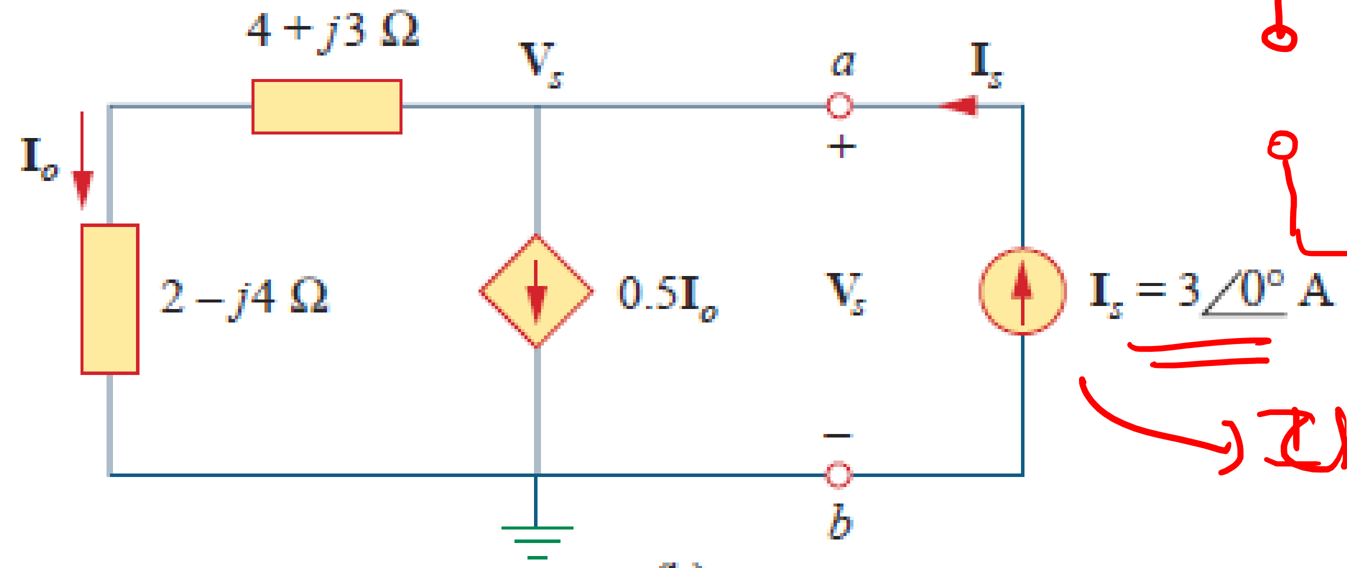
$$\boxed{\mathbf{V}_{\text{Th}} = 55\angle{-90^\circ}\ \text{V}}$$

To obtain $\mathbf{Z}_{Th}$, we remove the independent source.

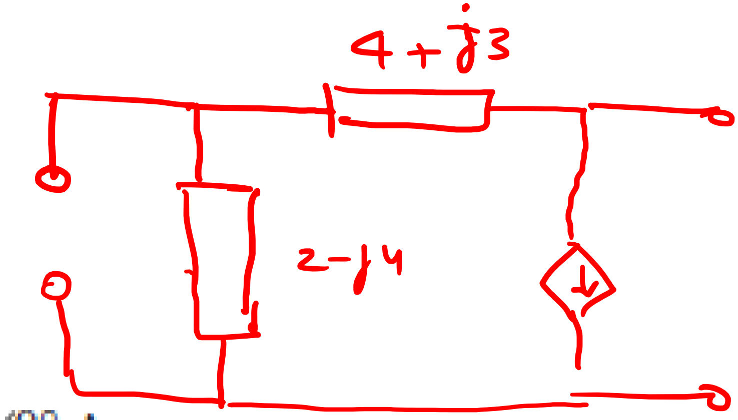
- Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a*-*b*



(a)

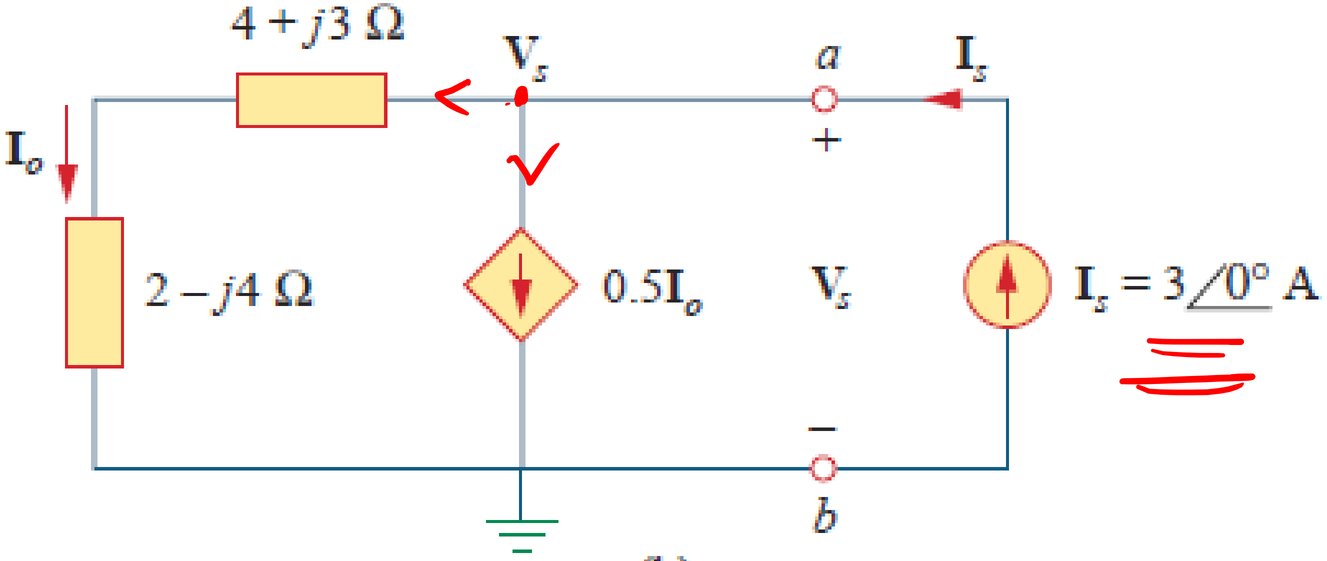


(b)



Choosing I_s

- **At the node, KCL gives**



(b)

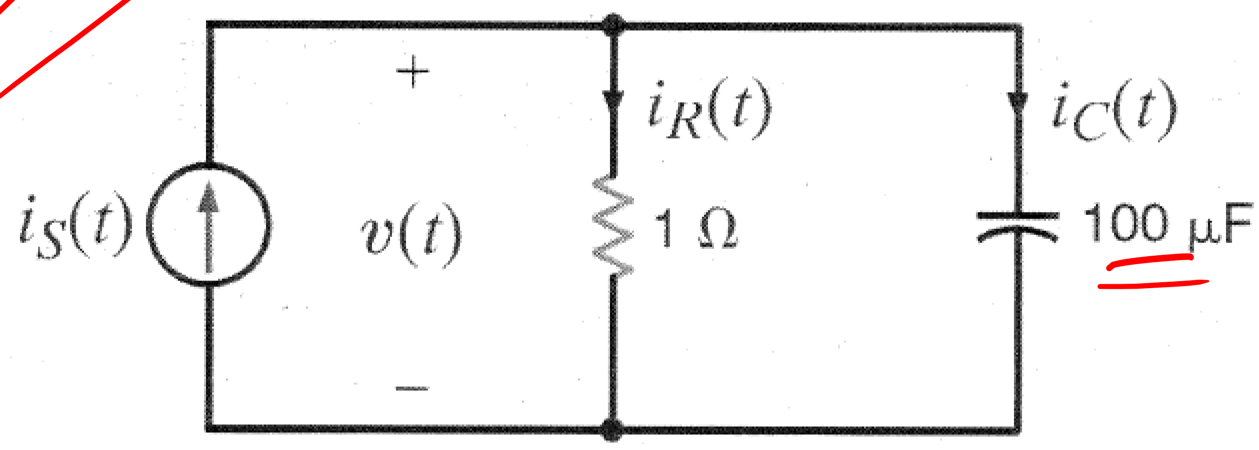
$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Rightarrow \quad \mathbf{I}_o = 2\text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$

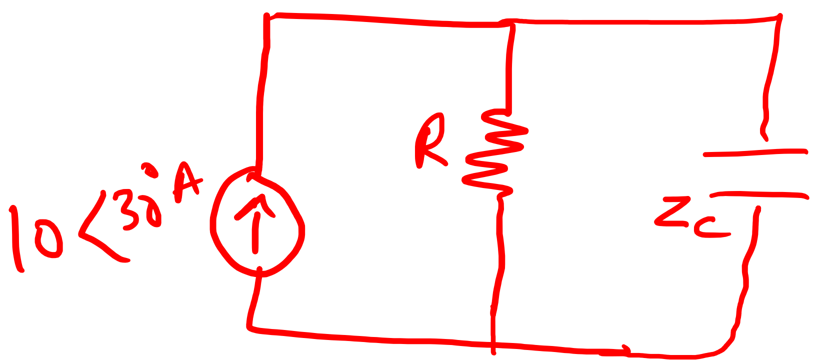


Draw the frequency domain circuit and calculate $v(t)$

$i_s(t) = 10\cos(377t + 30^\circ)A$

$\phi = 30^\circ$

$\omega = 377$ rad/sec.



$Z_c = \frac{1}{j\omega C}$

$R = 1\Omega$.

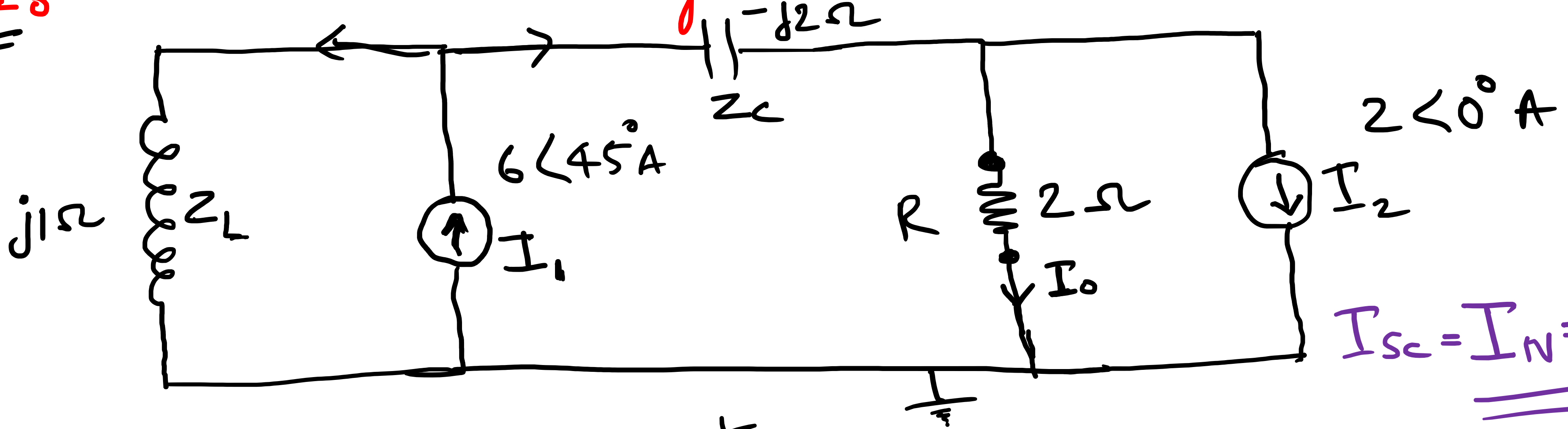
$\frac{V}{Z} = I$

$V = IZ$

$= 10\angle 30^\circ \left(\frac{RZ_c}{R+Z_c}\right)$

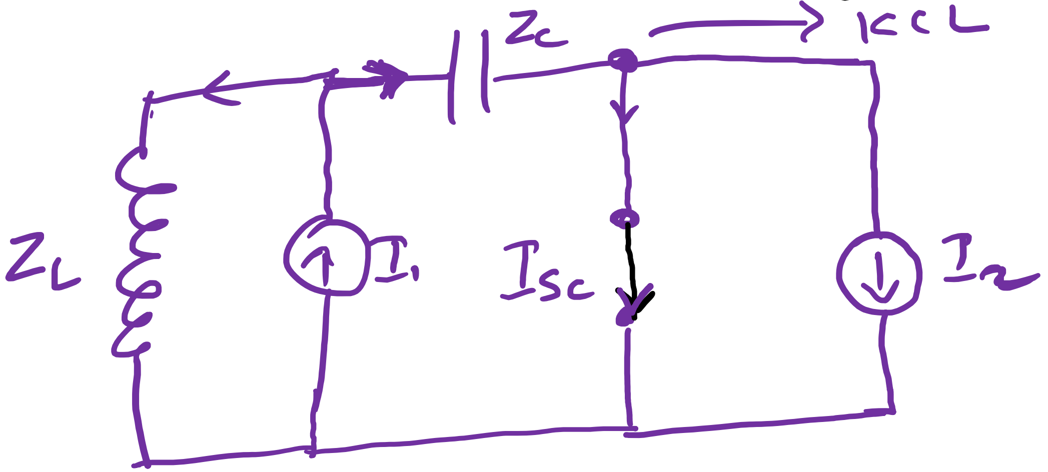
$v(t) = 9.99\cos(377t + 27.8^\circ)$

Find I_o in the network using Norton's theorem



$I_{sc} = I_N = I_o$

The current through the 2Ω resistor (Load)

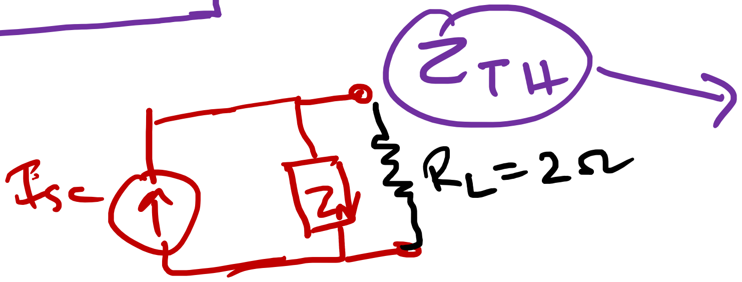


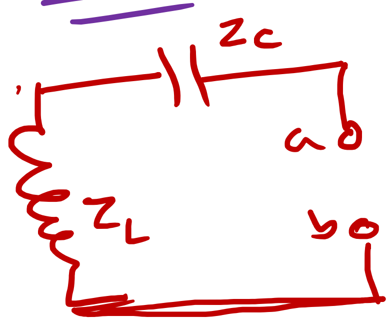
$$I_{sc} + I_2 - \frac{I_1 Z_L}{Z_L + Z_c} = 0 \quad \text{(KCL)}$$

Superposition

$I_{sc} = ?$

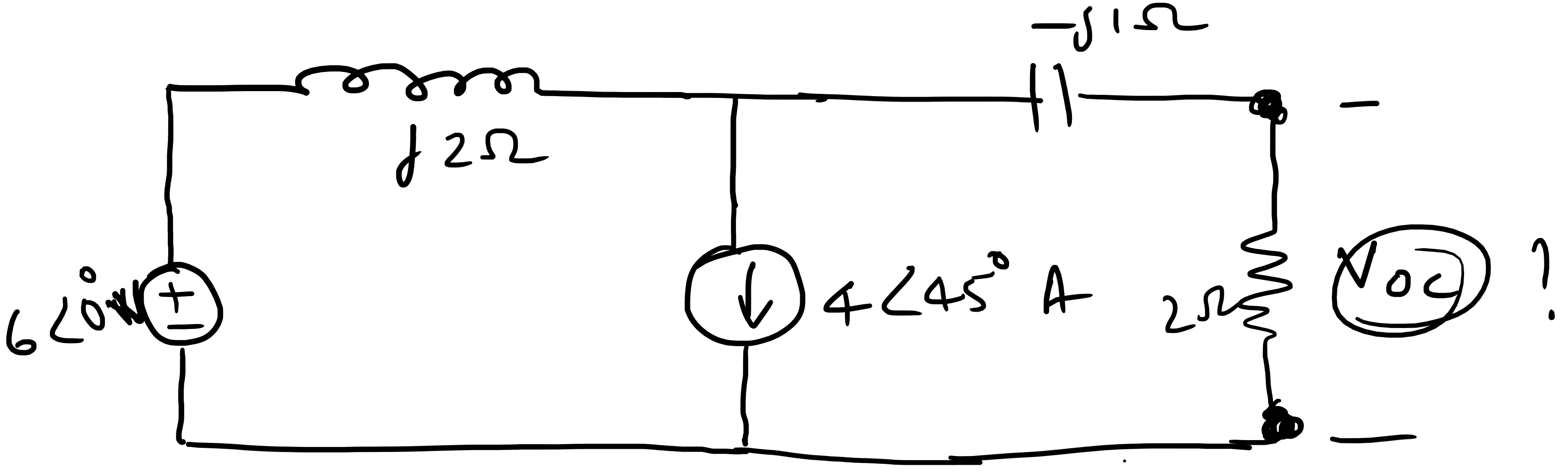
Eq Circuit





Z_N

$$Z_{TH} = Z_L + Z_c$$
$$= j1\Omega - j2\Omega$$
$$= -j\Omega$$

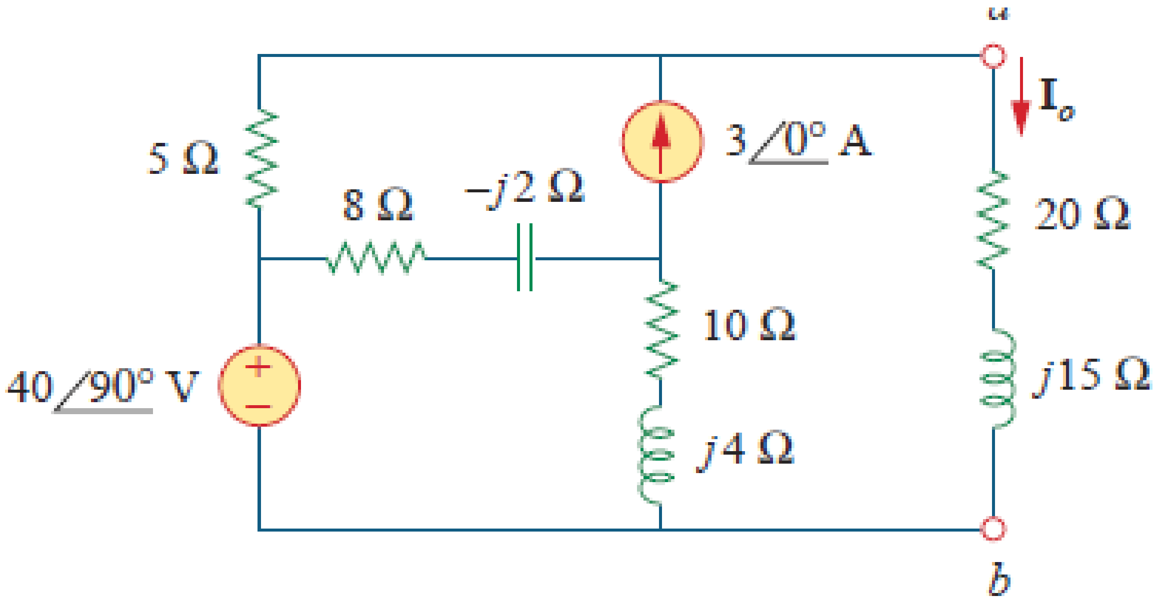


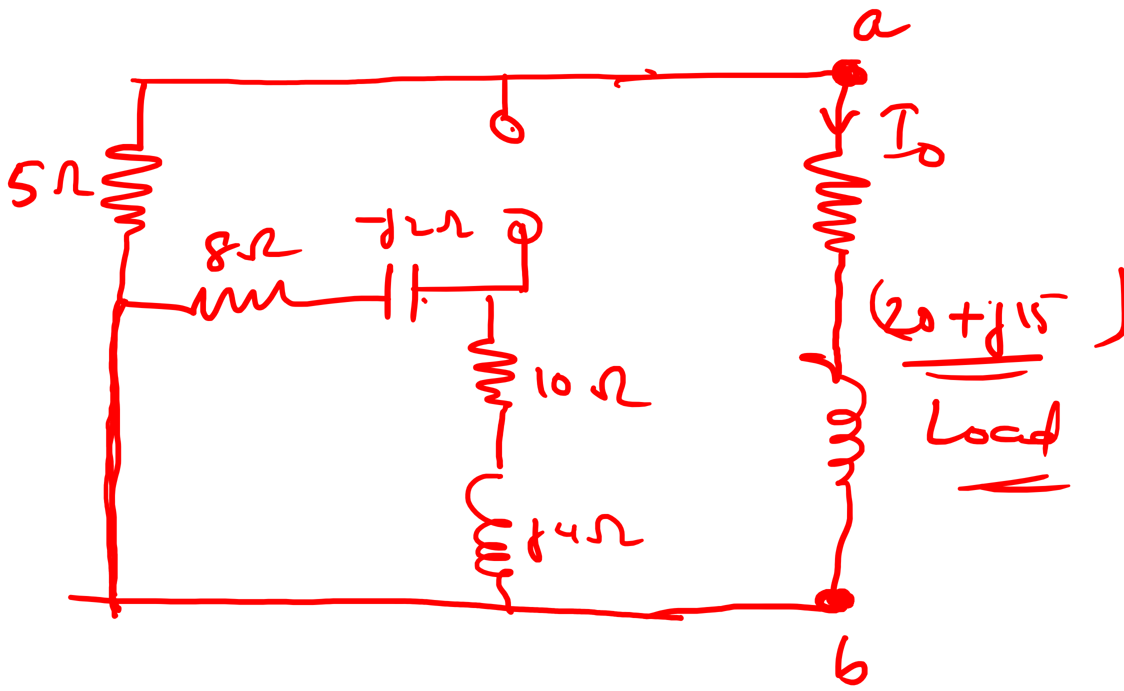
$R_L = 2\Omega$

V_{TH} and Z_{TH} ?

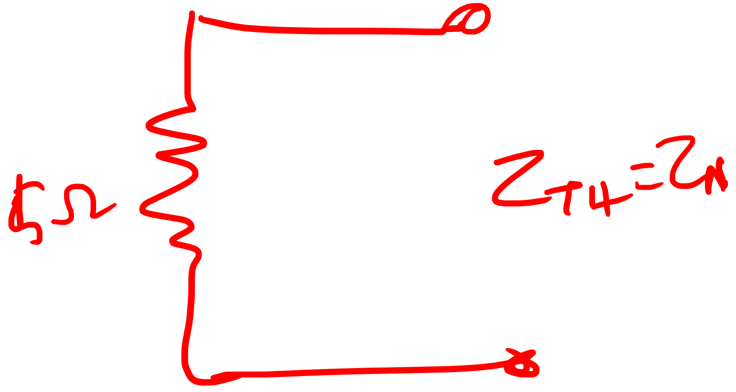
Example 10.10

Obtain current $\mathbf{I}_o$ in Fig. using Norton's theorem.

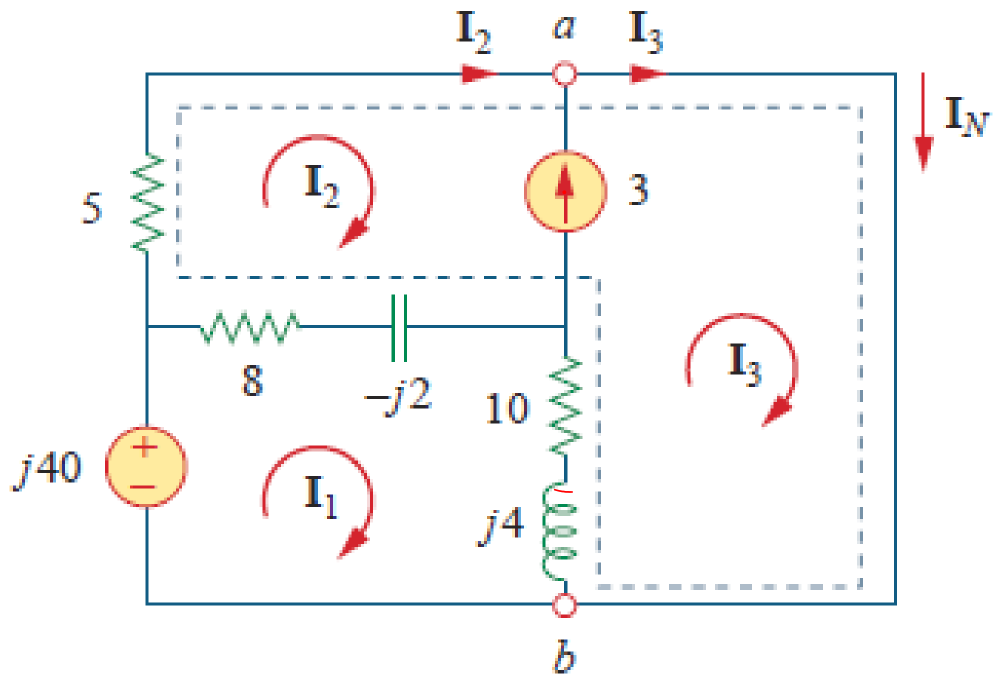




As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that



$$Z_{TH} = \mathbf{Z}_N = 5\ \Omega$$

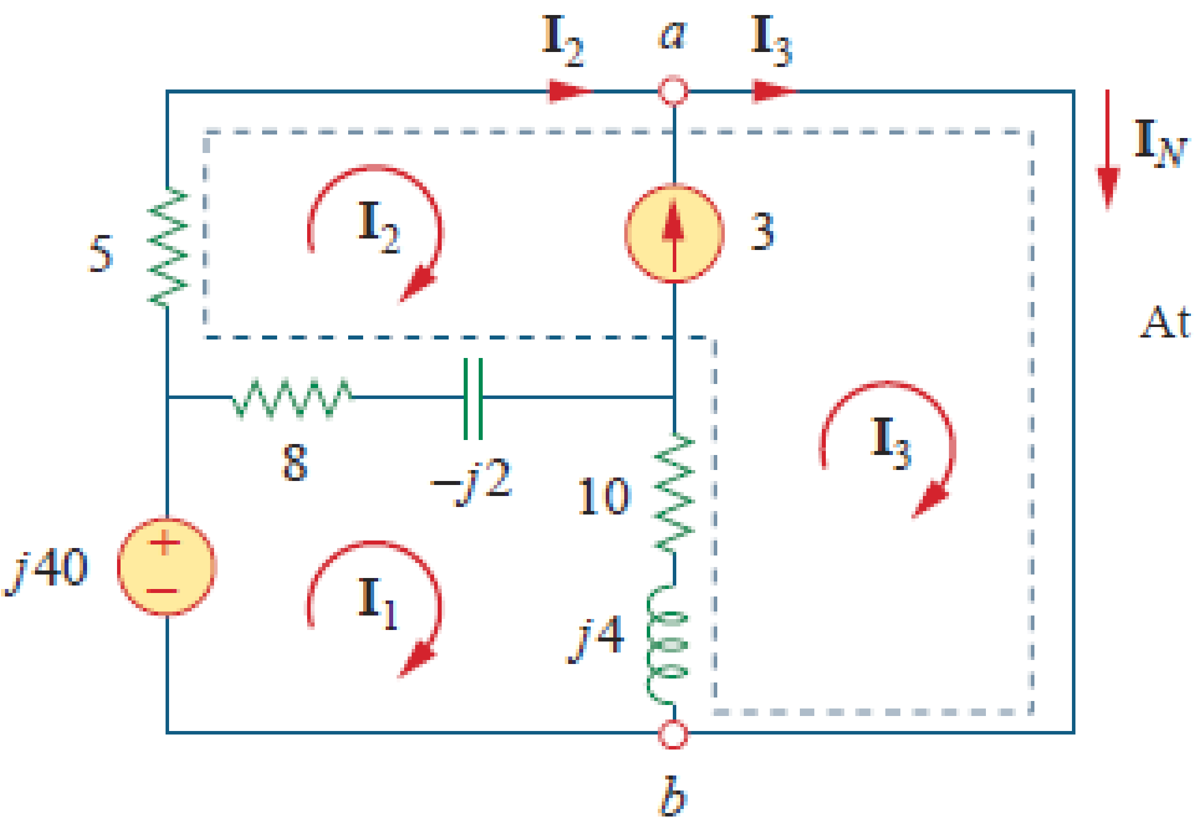


Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0$$



At node a, due to the current source between meshes 2 and 3,

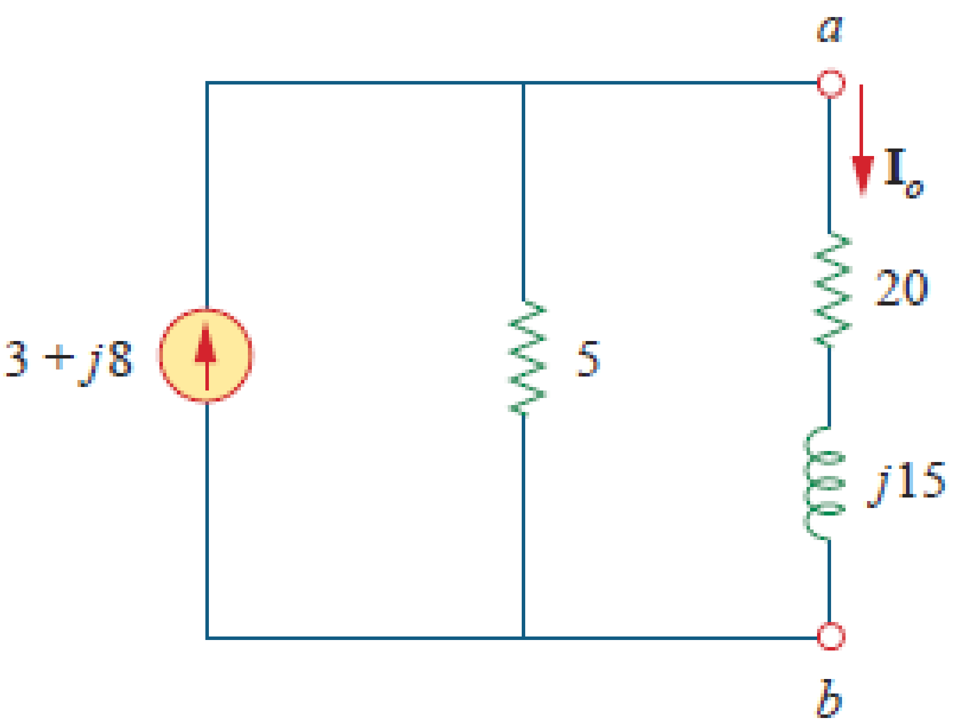
$$\mathbf{I}_3 = \mathbf{I}_2 + 3$$

$$-j40 + 5\mathbf{I}_2 = 0 \quad \Rightarrow \quad \mathbf{I}_2 = j8$$

The Norton current is

$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\text{ A}$$

The Norton equivalent circuit along with the impedance at terminals *a*-*b*.



(c)

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\underline{/38.48^\circ} \text{ A}$$