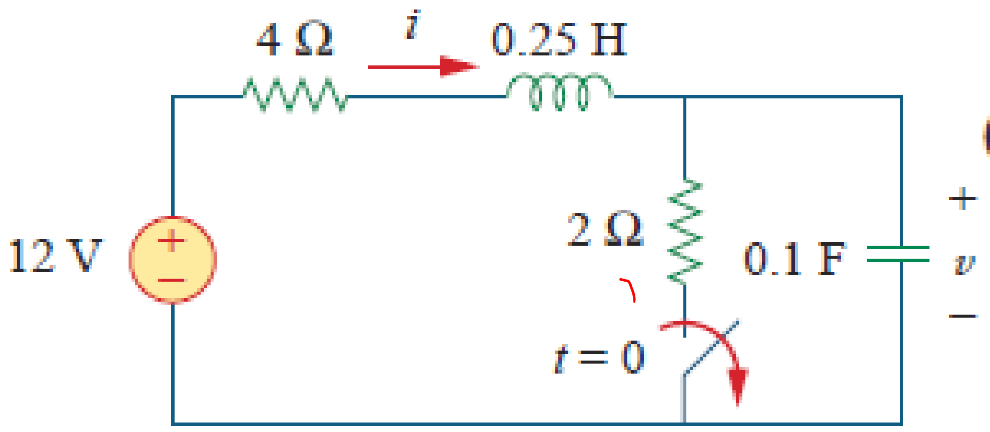
Example 8.1

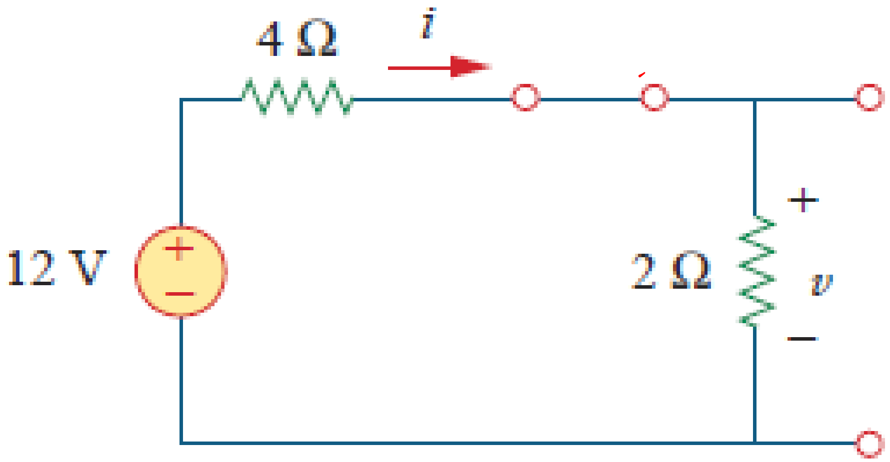
The switch in Fig. has been closed for a long time. It is open at $t = 0$. Find: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)/dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.



(a) If the switch is closed a long time before $t = 0$,

it means that the circuit has reached dc steady state at t=0

At dc steady state, the inductor acts like a short circuit, while the capacitor acts like an open circuit



at $t = 0^-$

$$i(0^-) = \frac{12}{4 + 2} = 2 \text{ A},$$

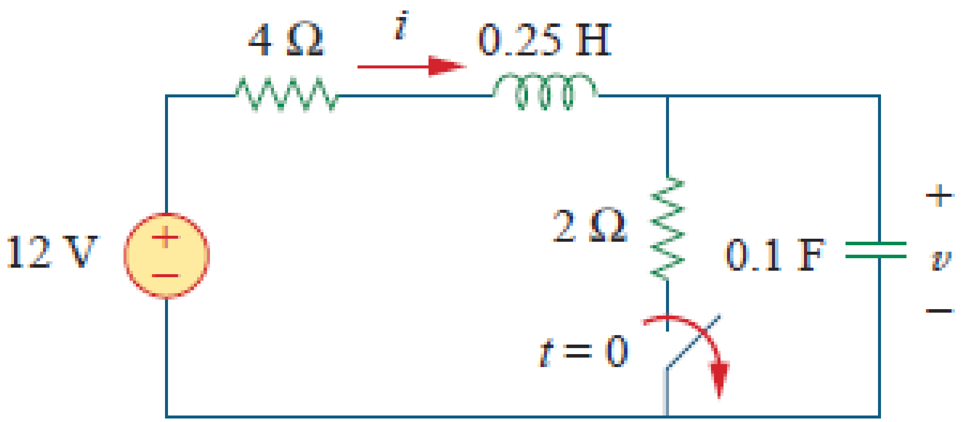
$$v(0^-) = 2i(0^-) = 4 \text{ V}$$

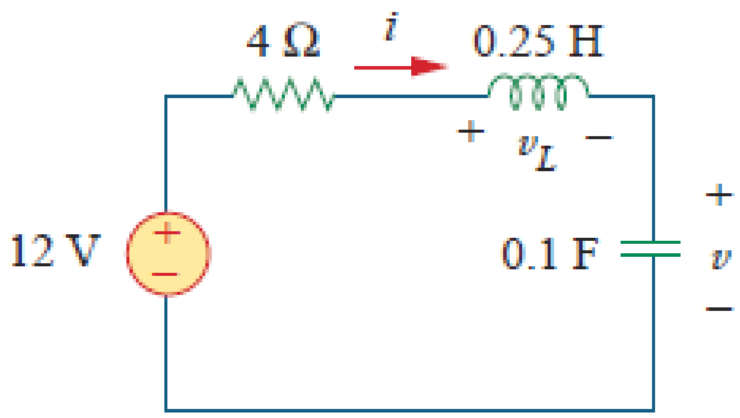
$i(0^-) = i(0) = i(0^+)$
$v(0^-) = v(0) = v(0^+)$

As the inductor current and the capacitor voltage cannot change abruptly,

$$i(0^+) = i(0^-) = 2\text{ A}, \qquad v(0^+) = v(0^-) = 4\text{ V}$$

(b) At $t = 0^+$, the switch is open;





(b)

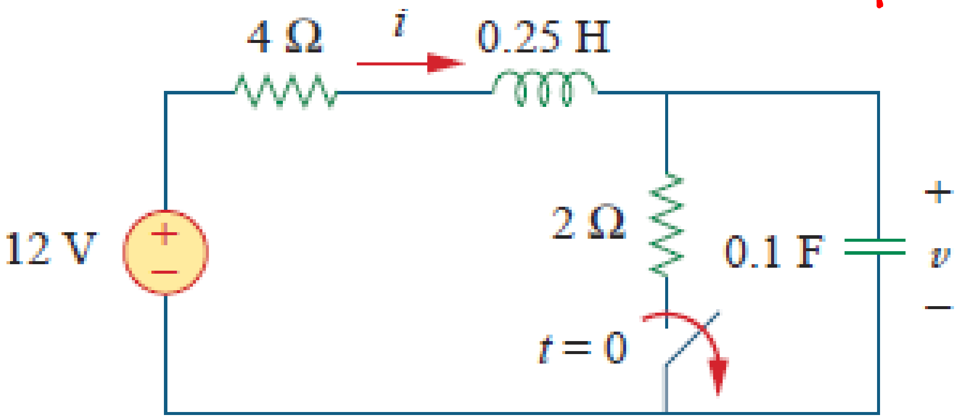
$$i_C(0^+) = i(0^+) = 2\text{ A}$$

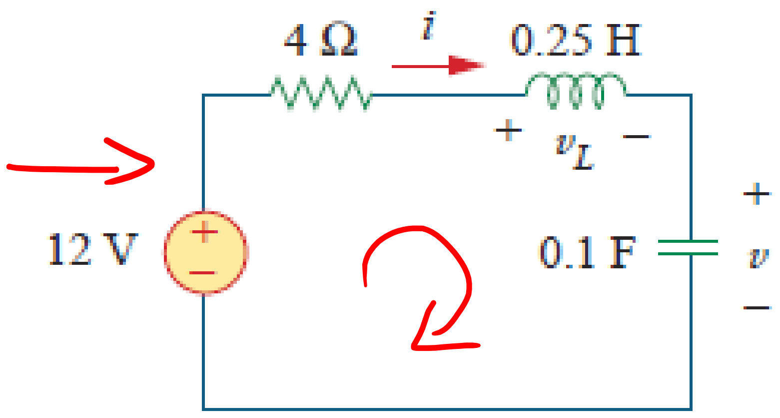
Since $C\,dv/dt = i_C$, $dv/dt = i_C/C$, and

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{2}{0.1} = 20\text{ V/s}$$

(b) At $t = 0^+$, the switch is open;

$i(0^+)$





(b)

Similarly, since $L\,di/dt = v_L$, $di/dt = v_L/L$. We now obtain v_L by applying KVL to the loop in Fig. 8.3(b). The result is

$$-12 + 4i(0^+) + v_L(0^+) + v(0^+) = 0$$

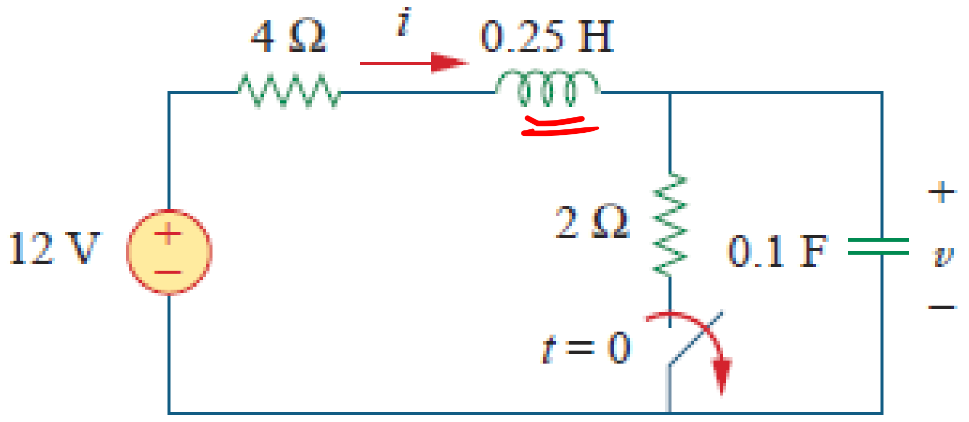
or

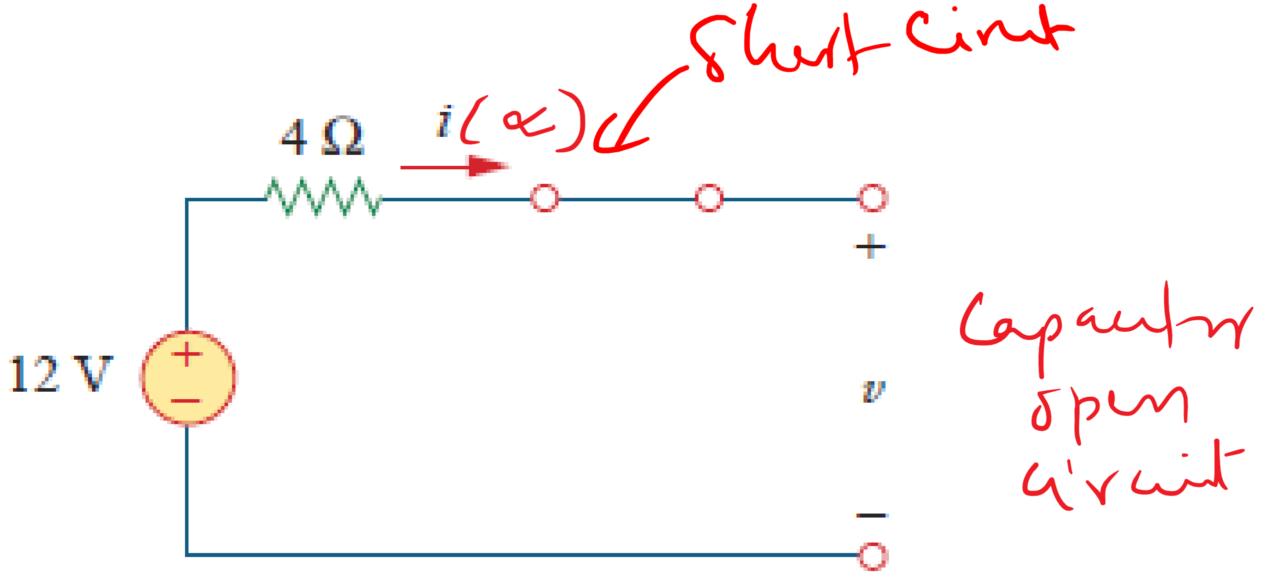
$$v_L(0^+) = 12 - 8 - 4 = 0$$

Thus,

$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{0}{0.25} = 0 \text{ A/s}$$

(c) For $t > 0$,





(c)

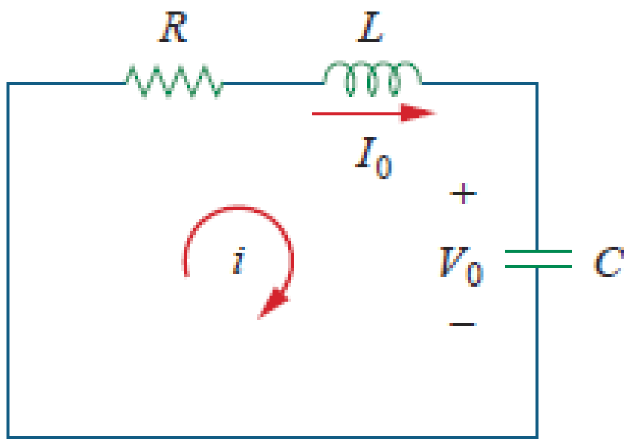
$t \to \infty$,

The circuit reaches steady state again. The inductor acts like a short circuit and the capacitor like an open circuit,

$$\boxed{i(\infty) = 0 \text{ A}, \qquad v(\infty) = 12 \text{ V}}$$

Example 8.3

In Fig. 8.8, $R = 40\ \Omega$, $L = 4$ H, and $C = 1/4$ F. Calculate the characteristic roots of the circuit. Is the natural response overdamped, underdamped, or critically damped?



Solution:

We first calculate

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5, \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times \frac{1}{4}}} = 1$$

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

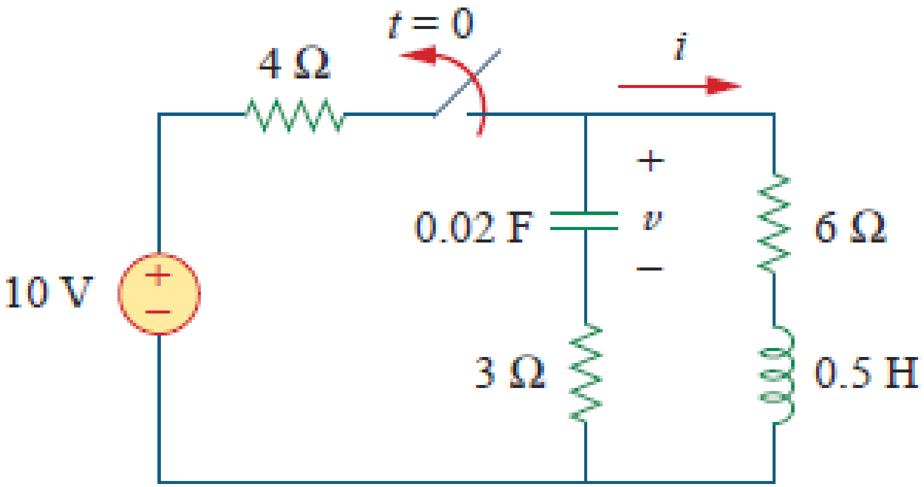
or

$$s_1 = -0.101, \qquad s_2 = -9.899$$

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

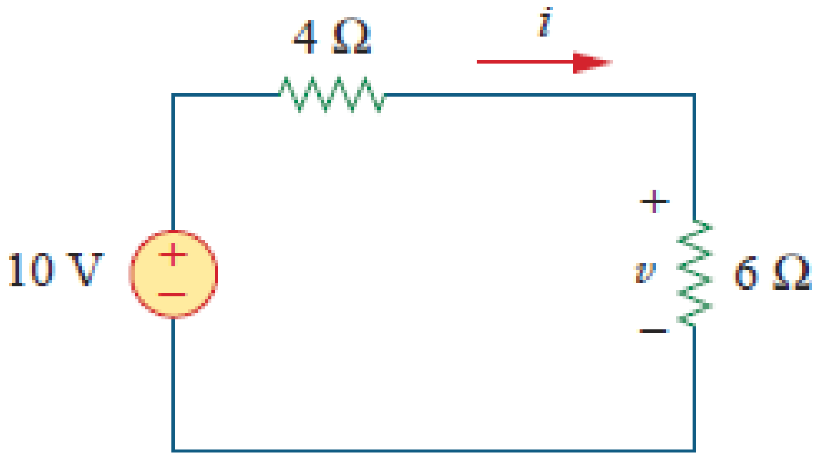
Example 8.4

Find $i(t)$ in the circuit of Fig. 8.10. Assume that the circuit has reached steady state at $t = 0^-$.



For $t < 0$, the switch is closed.

The capacitor acts like an open circuit while the inductor acts like a open circuit.

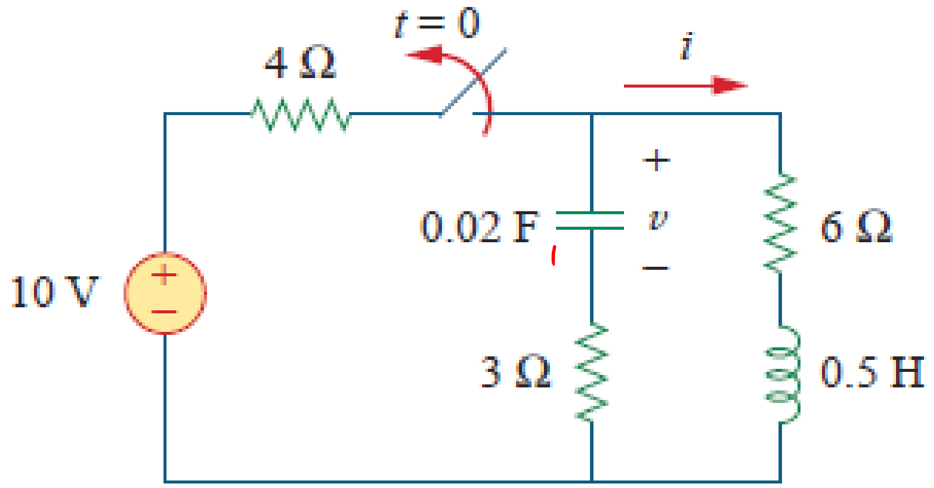


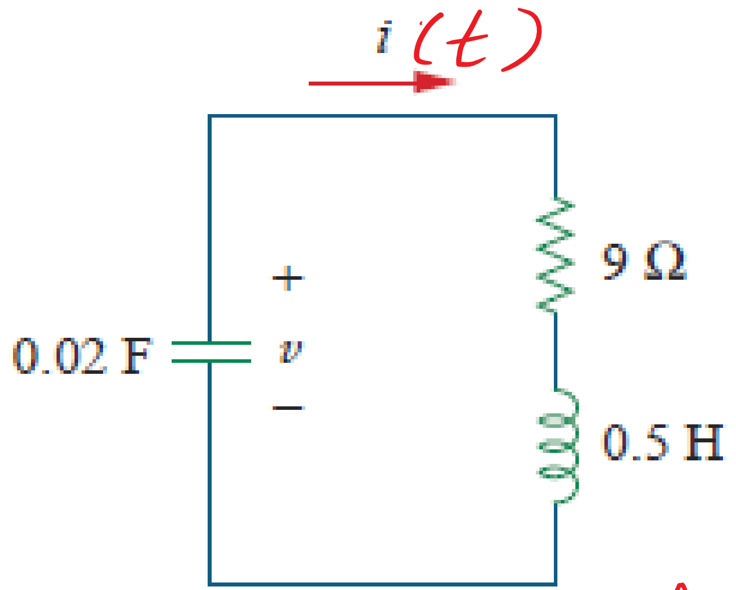
Thus, at $t = 0$,

$$i(0) = \frac{10}{4 + 6} = 1 \text{ A}, \qquad v(0) = 6i(0) = 6 \text{ V}$$

where $i(0)$ is the initial current through the inductor and $v(0)$ is the initial voltage across the capacitor.

For t > 0 the switch is opened and the voltage source is disconnected.





Source free Series RLC circuit

$$\alpha = \frac{R}{2L} = \frac{9}{2(\frac{1}{2})} = 9, \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times \frac{1}{50}}} = 10$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100}$$

or

$$s_{1,2} = -9 \pm j4.359$$

Hence, the response is underdamped ($\alpha < \omega$); that is,

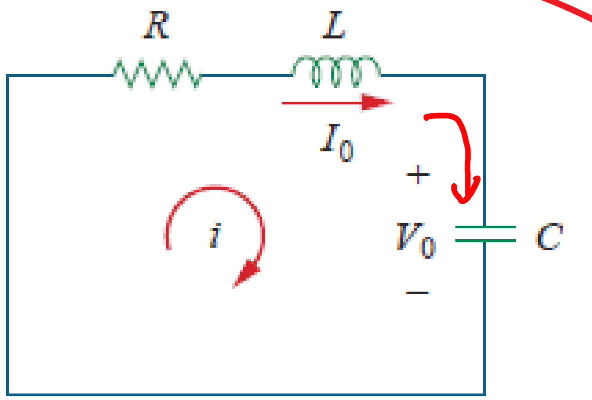
$$\boxed{i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)}$$

$i(0)$

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

We now obtain A_1 and A_2 using the initial conditions. At $t = 0$,

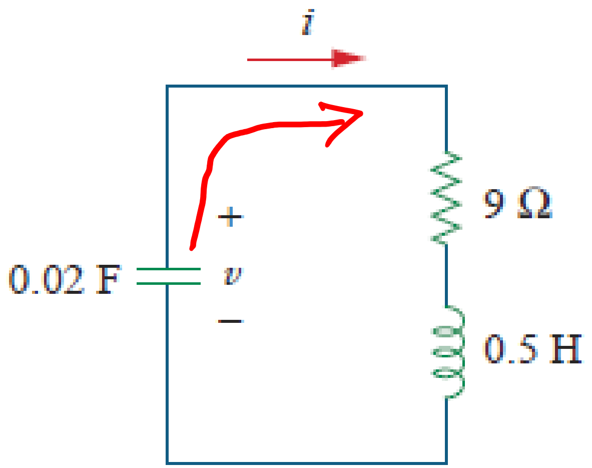
$$i(0) = 1 = A_1$$



$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$

$V_0 = v(0)$; $I_0 = i(0)$

$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9(1) - 6] = -6 \text{ A/s}$$



Polarity is opposite in this circuit

Hence Note that $v(0) = V_0 = -6$ V

$$\frac{di}{dt} = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

$$+ e^{-9t}(4.359)(-A_1 \sin 4.359t + A_2 \cos 4.359t)$$

Imposing the condition in Eq. (8.4.3) at $t = 0$ gives

$$-6 = -9(A_1 + 0) + 4.359(-0 + A_2)$$

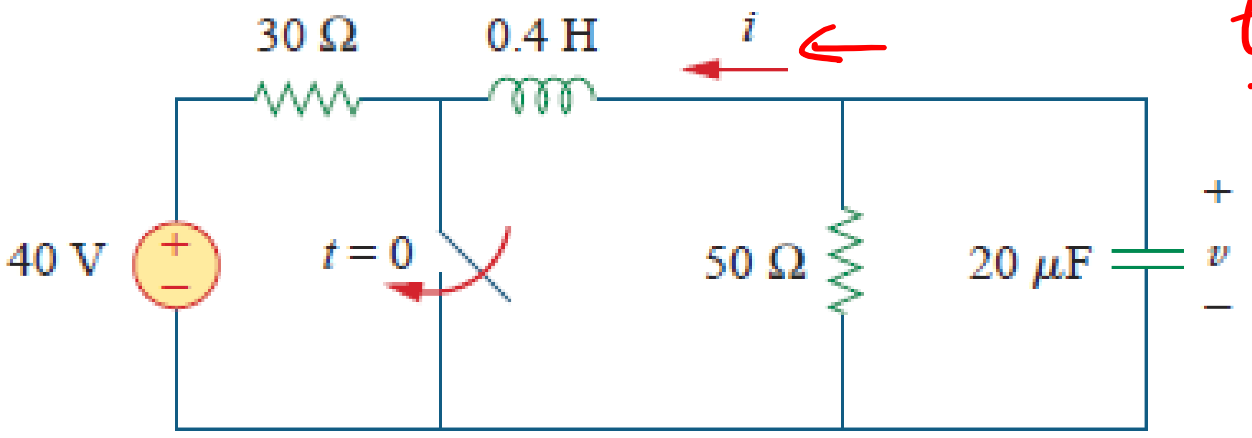
But $A_1 = 1$ from Eq. (8.4.2). Then

$$-6 = -9 + 4.359A_2 \quad \Rightarrow \quad \underline{\underline{A_2 = 0.6882}}$$

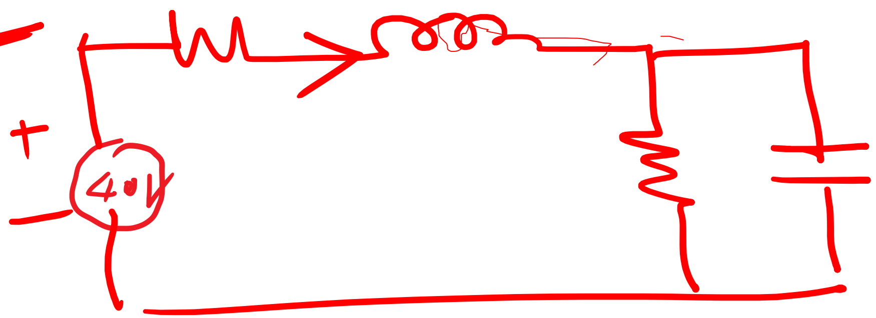
$$\boxed{i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t)\ \text{A}}$$

Example 8.6

Find $v(t)$ for $t > 0$ in the RLC circuit of Fig.



t<0



When $t < 0$, the switch is open; the inductor acts like a short circuit while the capacitor behaves like an open circuit. The initial voltage across the capacitor is the same as the voltage across the 50-Ω resistor; that is,

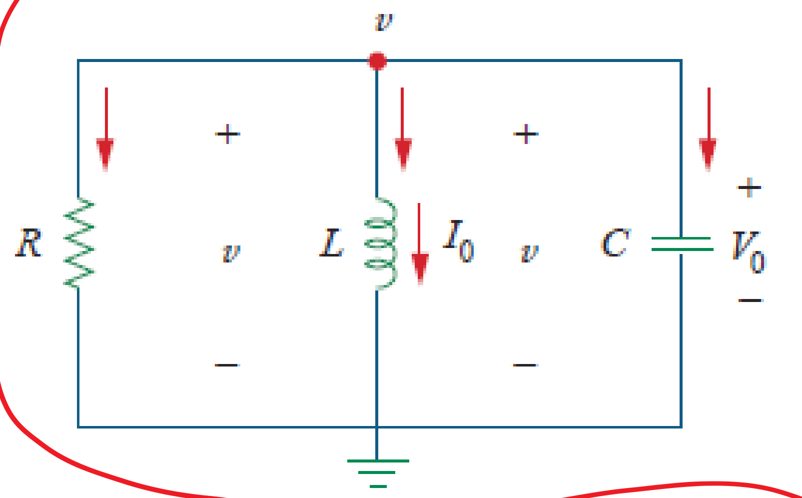
$$v(0) = \frac{50}{30 + 50}(40) = \frac{5}{8} \times 40 = 25 \text{ V}$$

The initial current through the inductor is

$$i(0) = -\frac{40}{30 + 50} = -0.5 \text{ A}$$

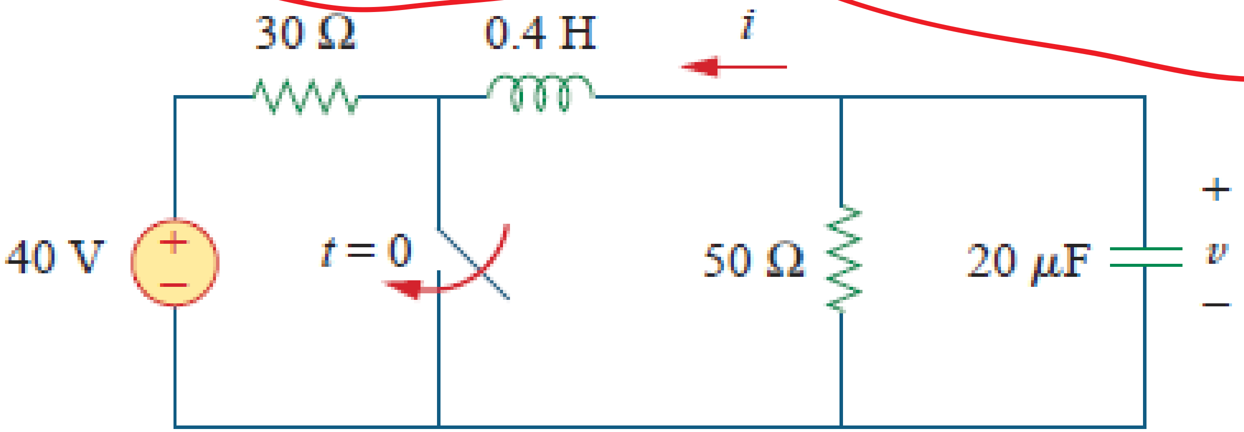
(current is opposite)

$V_0 = v(0)$; $I_0 = i(0)$



$$\frac{dv(0)}{dt} = -\frac{(V_0 + RI_0)}{RC}$$

The direction of i is as to conform with the direction of I_0 which is in agreement with the convention that current flows into the positive terminal of an inductor

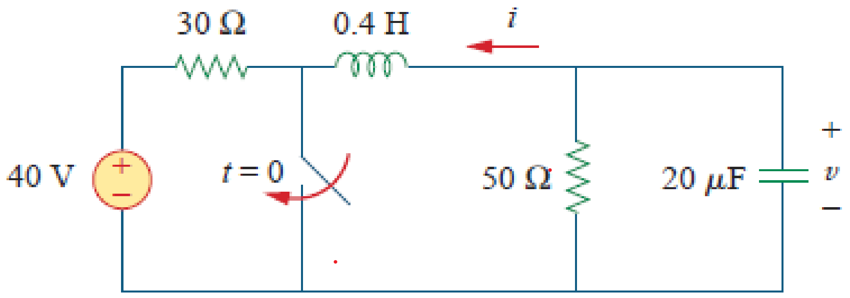


Here direction of current are same

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{25 - 50 \times 0.5}{50 \times 20 \times 10^{-6}} = 0$$

When $t > 0$, the switch is closed. The voltage source along with the 30-Ω resistor is separated from the rest of the circuit. The parallel *RLC* circuit acts independently of the voltage source, as illustrated in Fig. 8.16.

Source free parallel RLC ckt



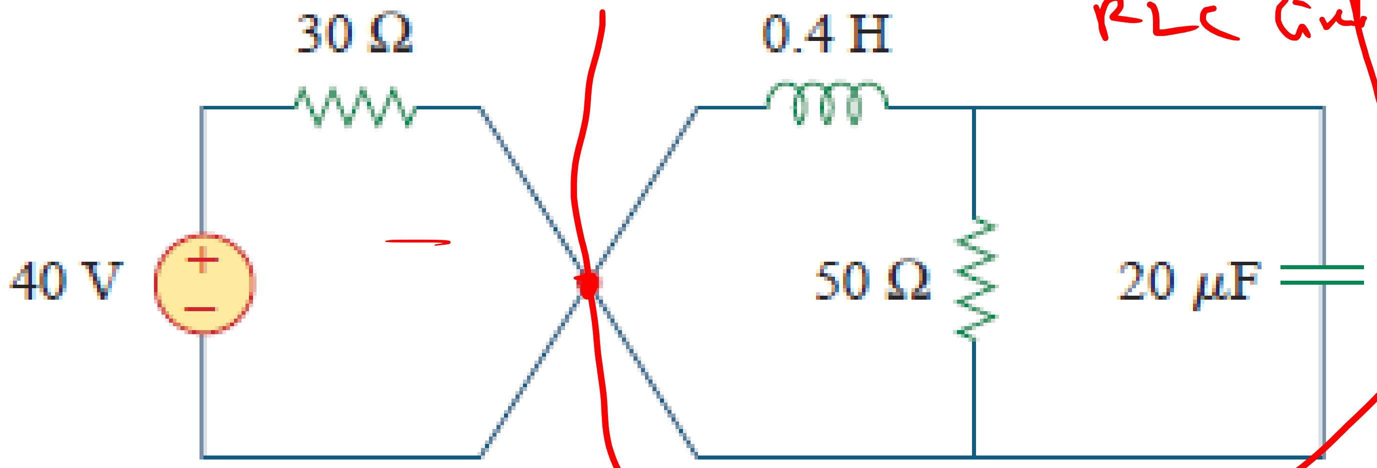


Figure 8.16
The circuit in Fig. 8.15 when $t > 0$. The parallel *RLC* circuit on the right-hand side acts independently of the circuit on the left-hand side of the junction.

source free parallel RLC circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 50 \times 20 \times 10^{-6}} = 500$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4 \times 20 \times 10^{-6}}} = 354$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -500 \pm \sqrt{250{,}000 - 124{,}997.6} = -500 \pm 354$$

$$s_1 = -854, \qquad s_2 = -146$$

Since $\alpha > \omega_0$, we have the overdamped response

$$v(t) = A_1 e^{-854t} + A_2 e^{-146t}$$

At $t = 0$, we impose the condition in Eq. (8.6.1),

$$v(0) = 25 = A_1 + A_2 \quad \Rightarrow \quad A_2 = 25 - A_1$$

Taking the derivative of $v(t)$ in Eq. (8.6.3),

$$\frac{dv}{dt} = -854A_1 e^{-854t} - 146A_2 e^{-146t}$$

Imposing the condition in Eq. (8.6.2),

$$\frac{dv(0)}{dt} = 0 = -854A_1 - 146A_2$$

$$\frac{dv(0)}{dt} = 0 = -854A_1 - 146A_2$$

or

$$0 = 854A_1 + 146A_2$$

Solving Eqs. (8.6.4) and (8.6.5) gives

$$A_1 = -5.156, \qquad A_2 = 30.16$$

Thus, the complete solution in Eq. (8.6.3) becomes

$$v(t) = -5.156e^{-854t} + 30.16e^{-146t}\ \text{V}$$