First-Order Circuits

First order circuits are circuits that contain only one energy storage element (capacitor or inductor), and that can, therefore, be described using only a first order differential equation. The two possible types of first-order circuits are:

- ❑ RC (resistor and capacitor)
- ❑ RL (resistor and inductor)

A **first-order** circuit is characterized by a first-order differential equation.

- **Applying Kirchhoff's Laws to:**

Purely resistive circuits → Algebraic equations

RC and RL Circuits → Differential equations

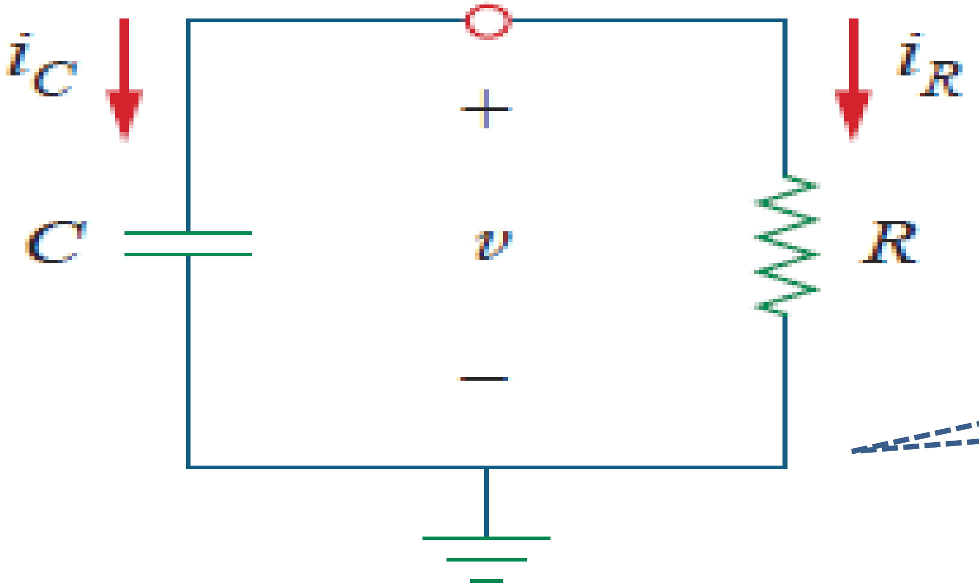
<u>Two ways of excitations of first order circuits</u>

- **Source free circuits: Assume energy is initially stored in the Inductor or Capacitor element. The energy causes current flow in the circuit and dissipates through resistors**

- **Exciting by DC sources (independent source)**

The Source-Free *RC* Circuit

A circuit response is the manner in which the circuit reacts to an excitation.



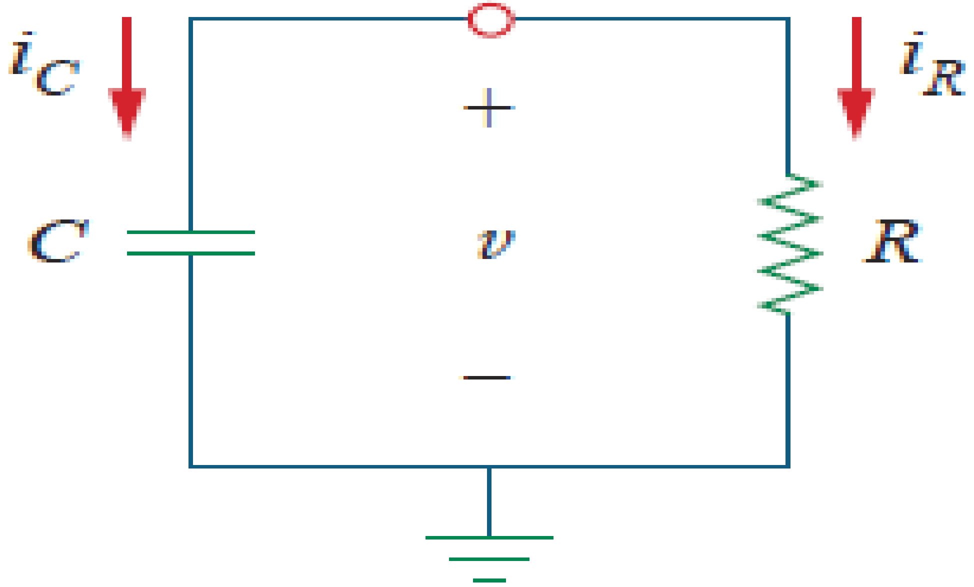
DC source is suddenly disconnected

at time $t = 0$, the initial voltage is

$$v(0) = V_0$$

with the corresponding value of the energy stored as

$$w(0) = \frac{1}{2} C V_0^2$$



Applying KCL at the top node

$$i_C + i_R = 0$$

By definition, $i_C = C\,dv/dt$ and $i_R = v/R$. Thus,

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

This is a *first-order differential equation*.

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

To solve it, we rearrange the terms as

$$\frac{dv}{v} = -\frac{1}{RC}dt$$

Integrating both sides,

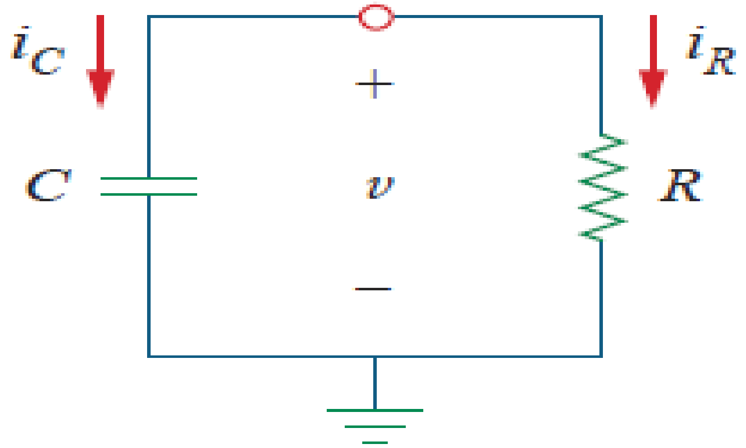
$$\ln v = -\frac{t}{RC} + \ln A$$

$$\ln \frac{v}{A} = -\frac{t}{RC}$$

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0 e^{-t/RC}$$

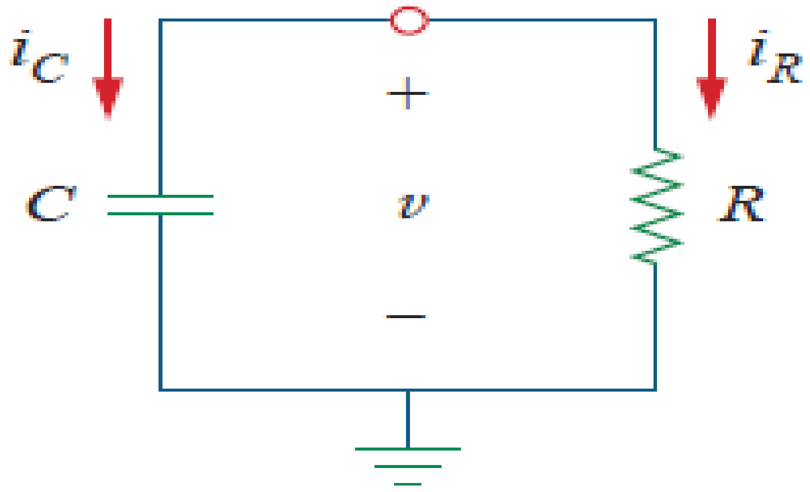


A circuit response is the manner in which the circuit reacts to an excitation.

$$v(t) = V_0 e^{-t/RC}$$

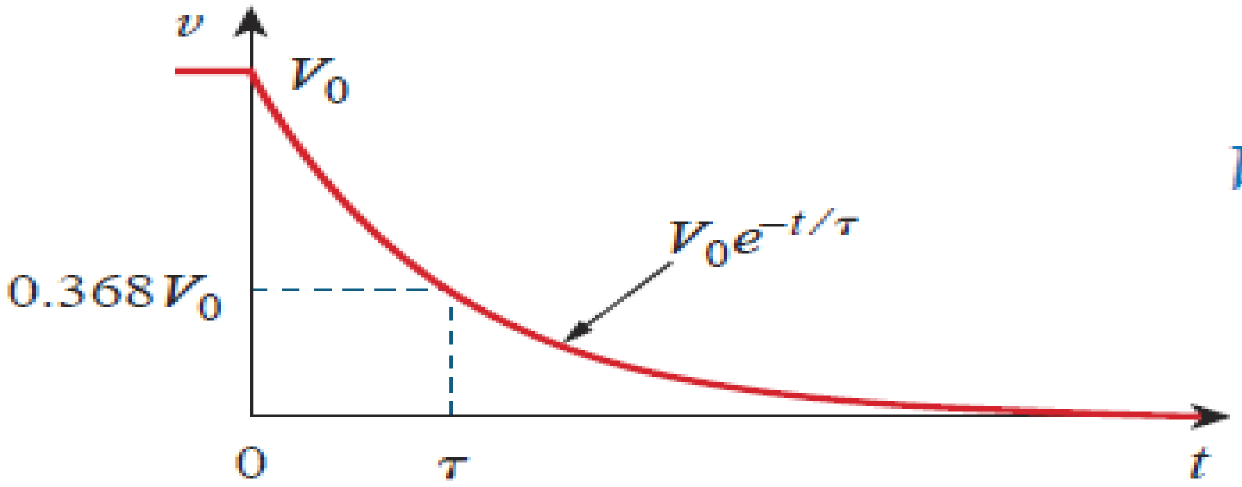
The voltage response of RC circuit is the exponential decay of initial voltage

The **natural response** of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.



$$v(t) = V_0 e^{-t/RC}$$

The **time constant** of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8 percent of its initial value.[1]



The voltage response of the RC circuit

This implies that at $t = \tau$,

$$V_0e^{-\tau/RC} = V_0e^{-1} = 0.368V_0$$

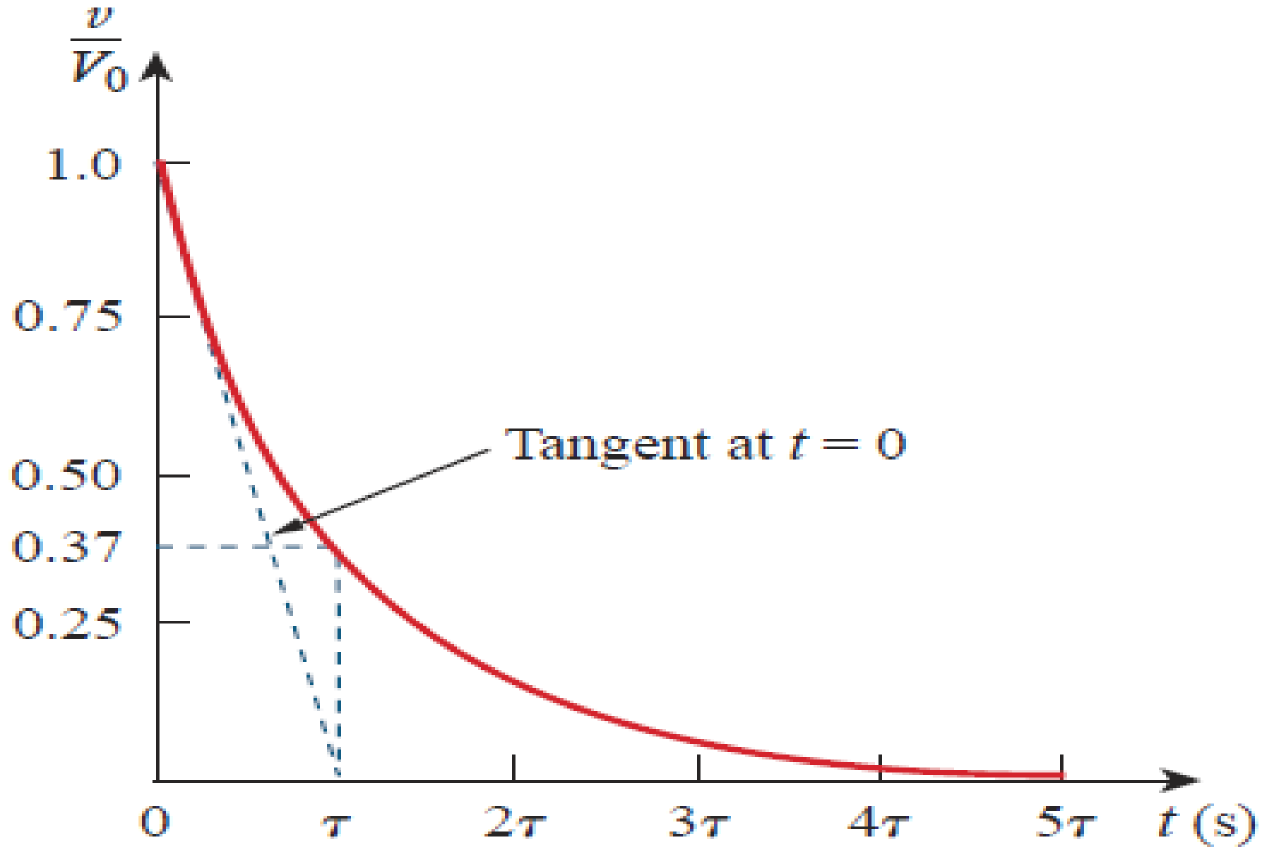
$$\boxed{\tau = RC}$$

$$\boxed{v(t) = V_0e^{-t/\tau}}$$

TABLE

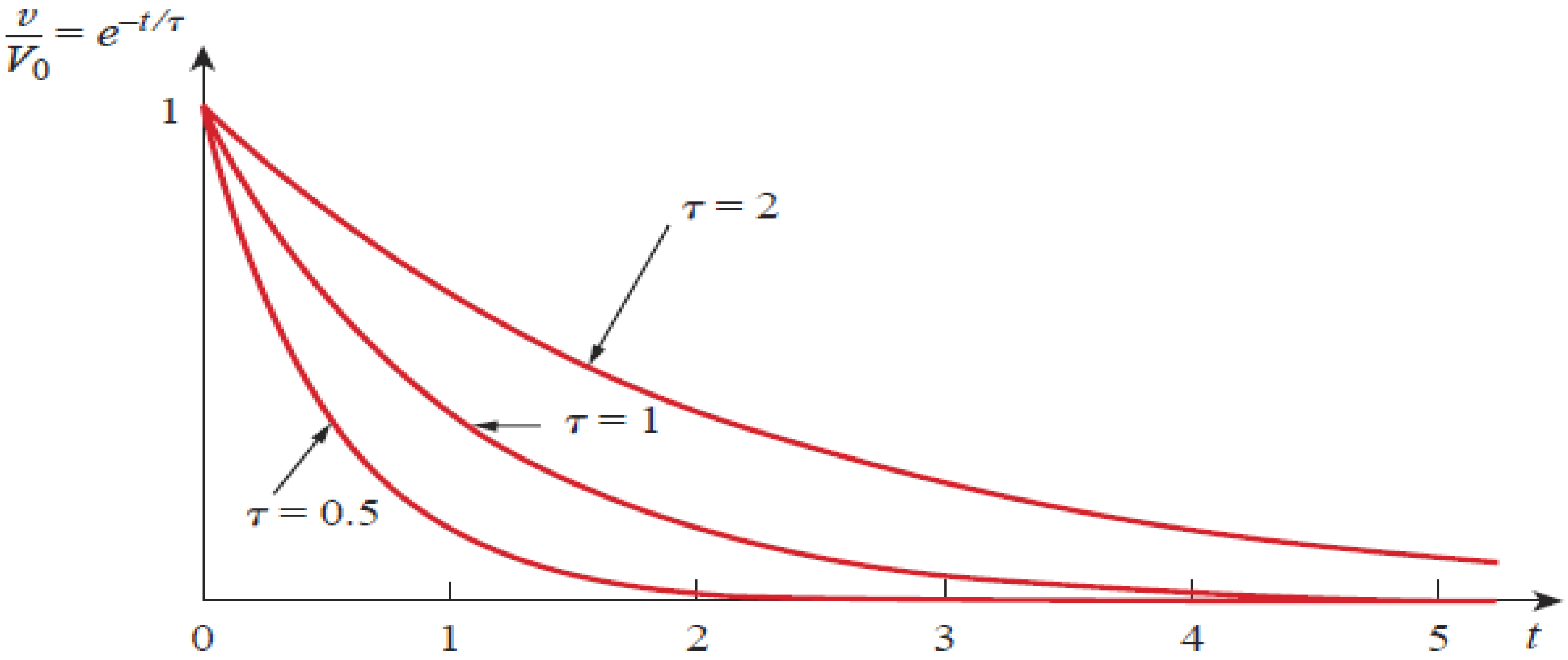
Values of $v(t)/V_0 = e^{-t/\tau}$.

t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674



$$\boxed{\tau = RC}$$

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$



Plot of $v/V_0 = e^{-t/\tau}$ for various values of the time constant.

- Smaller the time constant, more rapid the voltage decreases
- Small time constant gives fast response and reaches the steady state quickly
- Large time constant give slow response

With the voltage $v(t)$, we can find the current $i_R(t)$

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

The power dissipated in the resistor is

$$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$

The energy absorbed by the resistor up to time t is

$$w_R(t) = \int_0^t p\,dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau}\,dt$$

$$= -\frac{\tau V_0^2}{2R} e^{-2t/\tau} \Big|_0^t = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}), \qquad \tau = RC$$

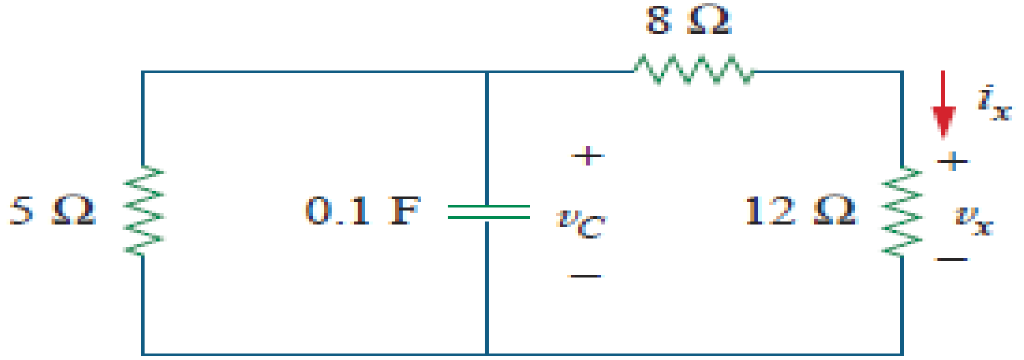
Notice that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2} C V_0^2$, which is the same as $w_C(0)$,

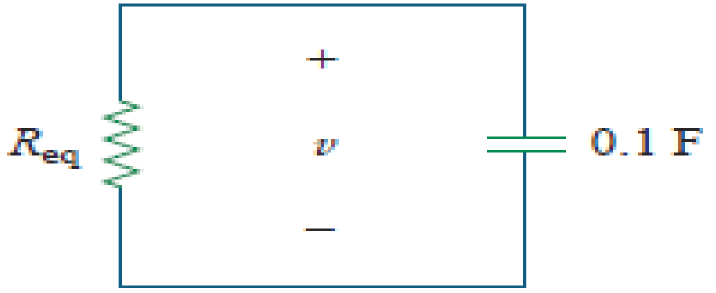
The Key to Working with a Source-free *RC* Circuit Is Finding:

1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant τ.

Example

In Fig. let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.





$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$