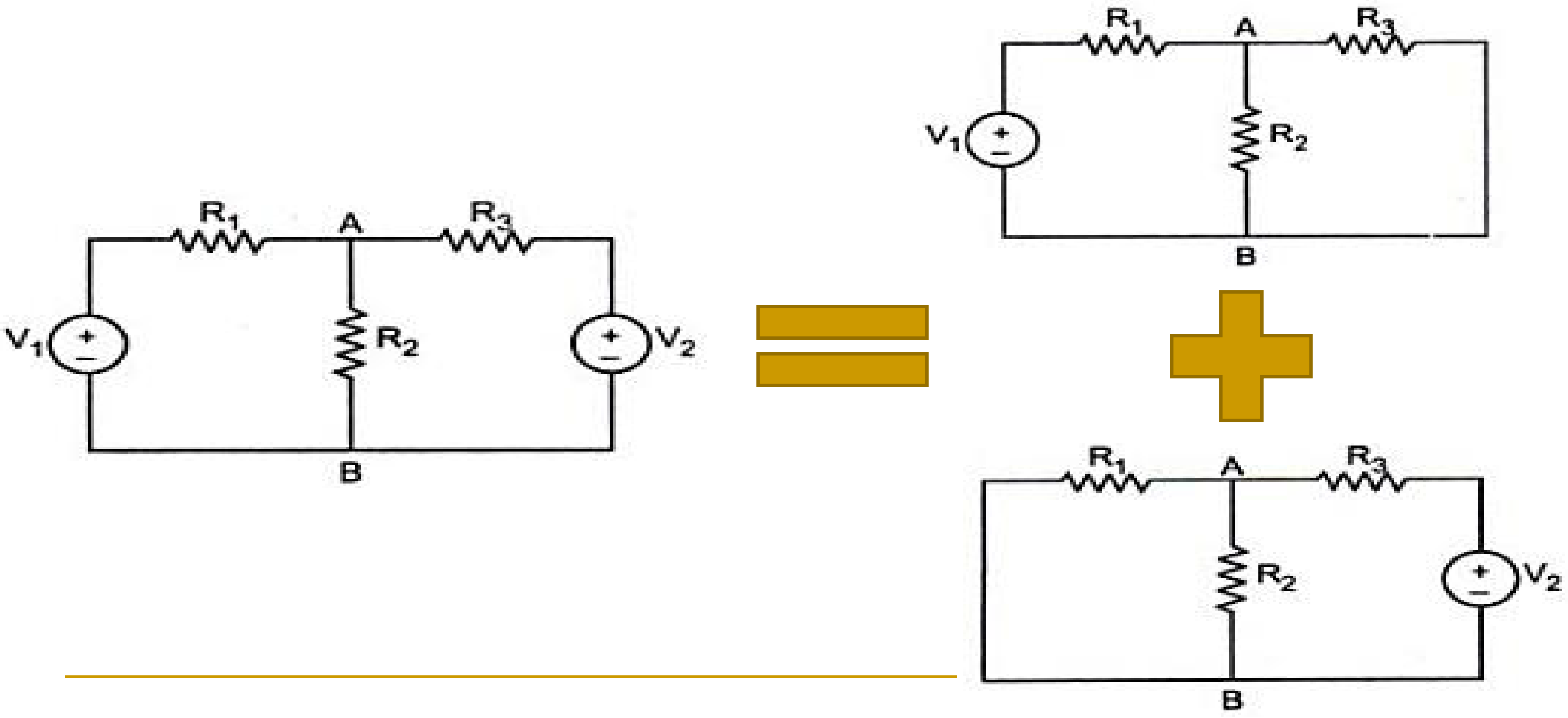
Superposition Theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

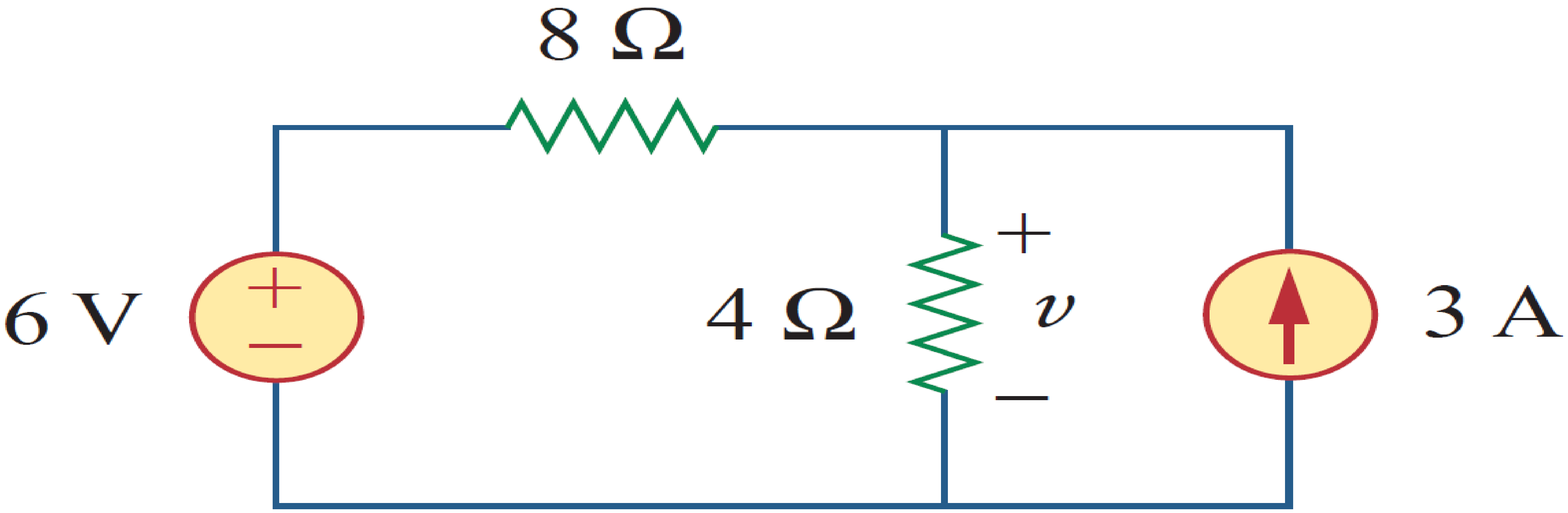
Superposition Theorem



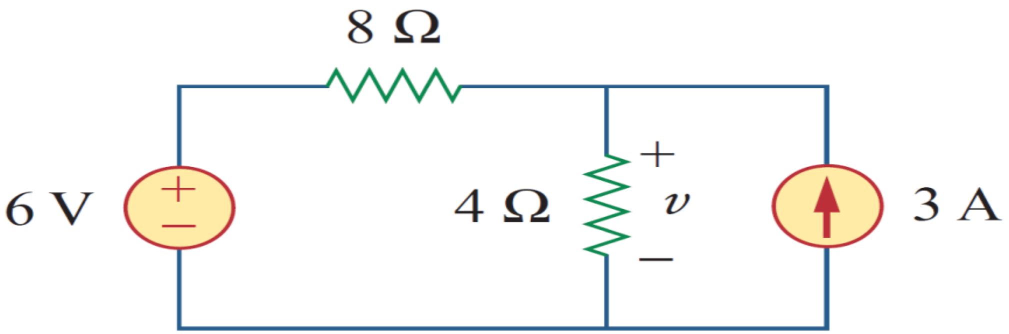
Steps to Apply Superposition theorem

1. Select one of the independent sources. Set all other independent sources to zero (This means voltage sources are replaced with short circuits and current sources are replaced with open circuits. Leave dependent sources in the circuit)
2. Analyze the simplified circuit to find the desired currents and/or voltages.
3. Repeat steps 1 & Step 2 until each independent source has been considered.
4. Add the partial currents and/or voltages obtained from the separate analyses.

Use the superposition theorem to find *v* in the circuit of Fig. **(DC)**

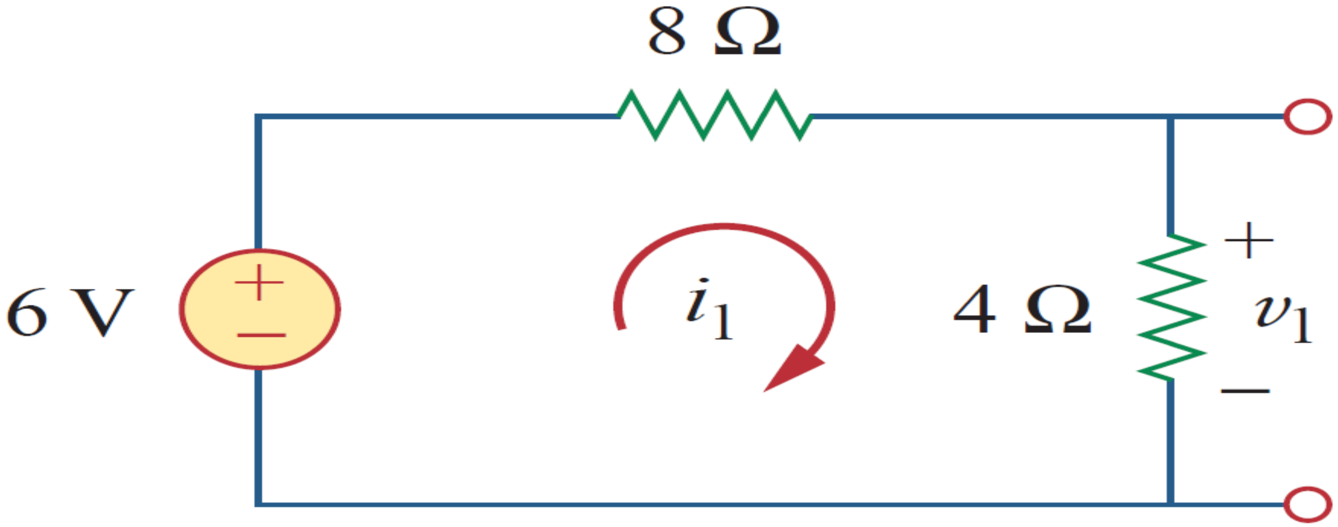


Solving with one independent source..



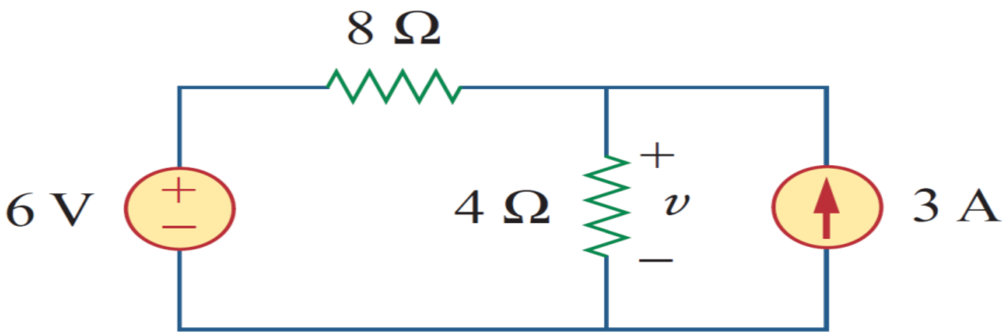
$$12i_1 - 6 = 0$$

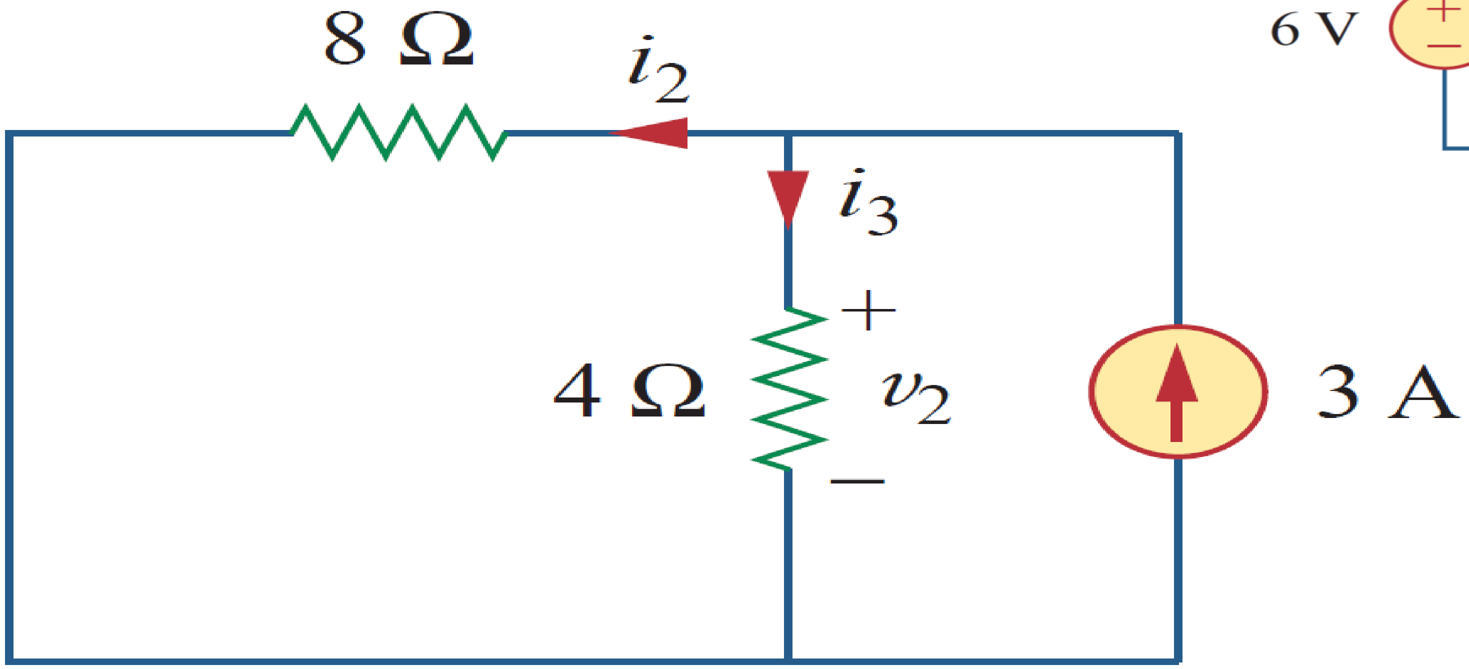
$$i_1 = 0.5 \text{ A}$$



$$v_1 = 4i_1 = 2 \text{ V}$$

Solving with another independent source…





$$i_3 = \frac{8}{4 + 8}(3) = 2 \text{ A}$$

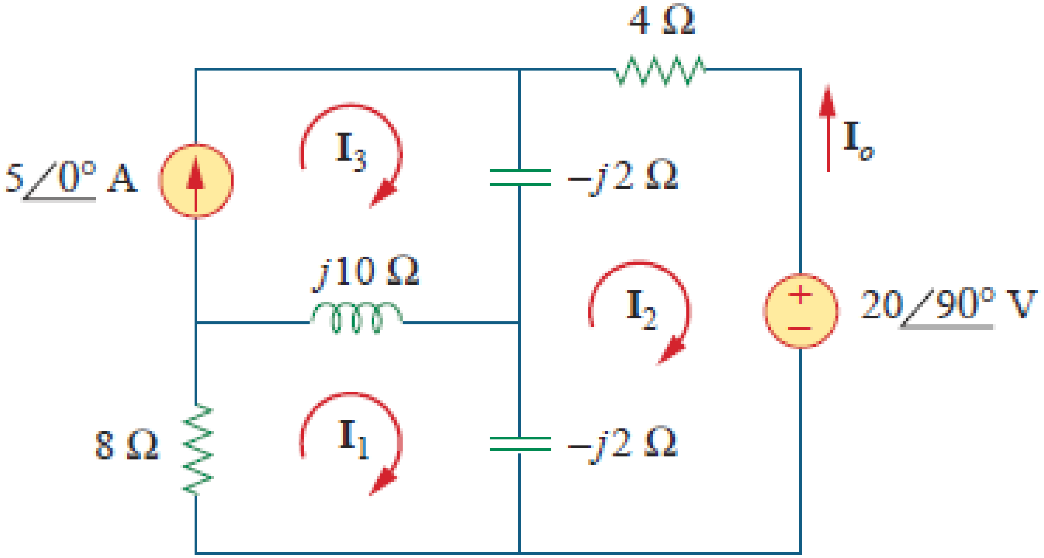
$$v_2 = 4i_3 = 8 \text{ V}$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

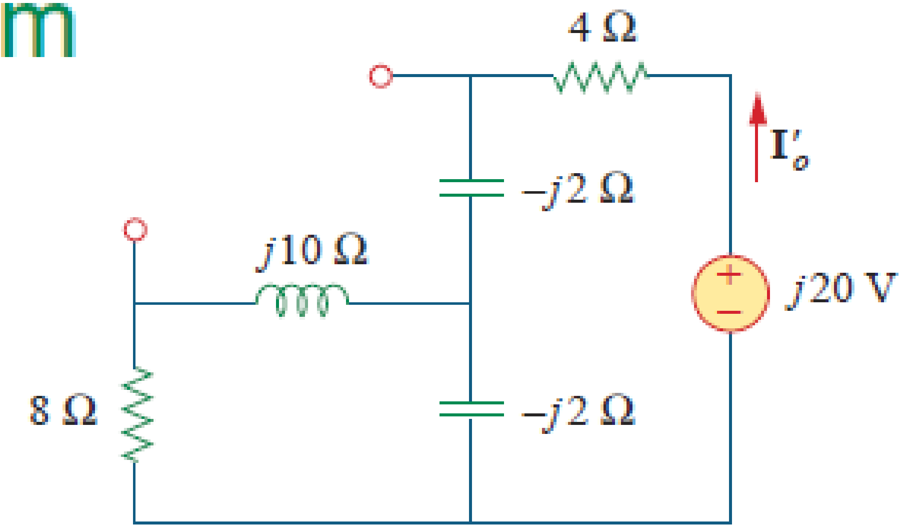
10.4 Superposition Theorem

(AC-sinusoidal)

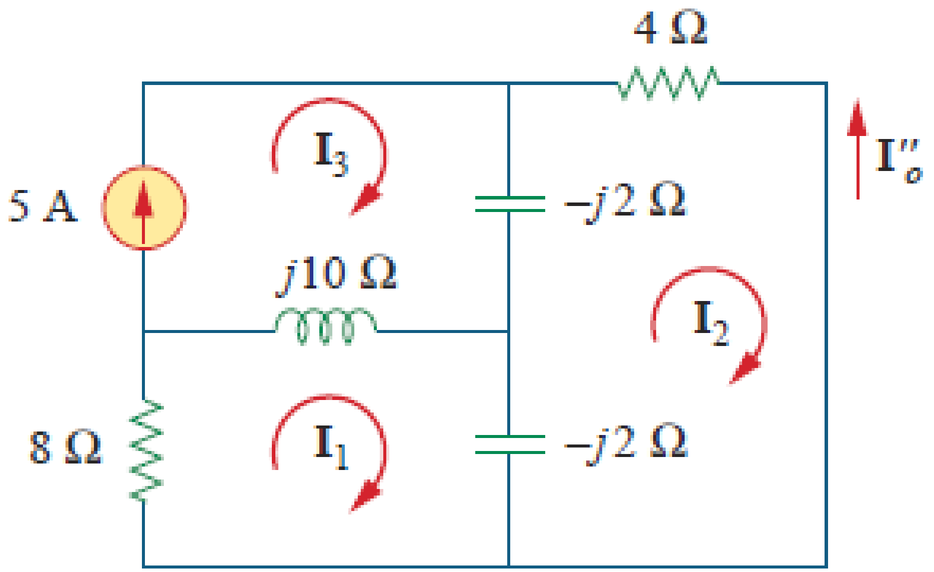
Use the superposition theorem to find $\mathbf{I}_o$ in the circuit in Fig.



$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o''$$



(a)

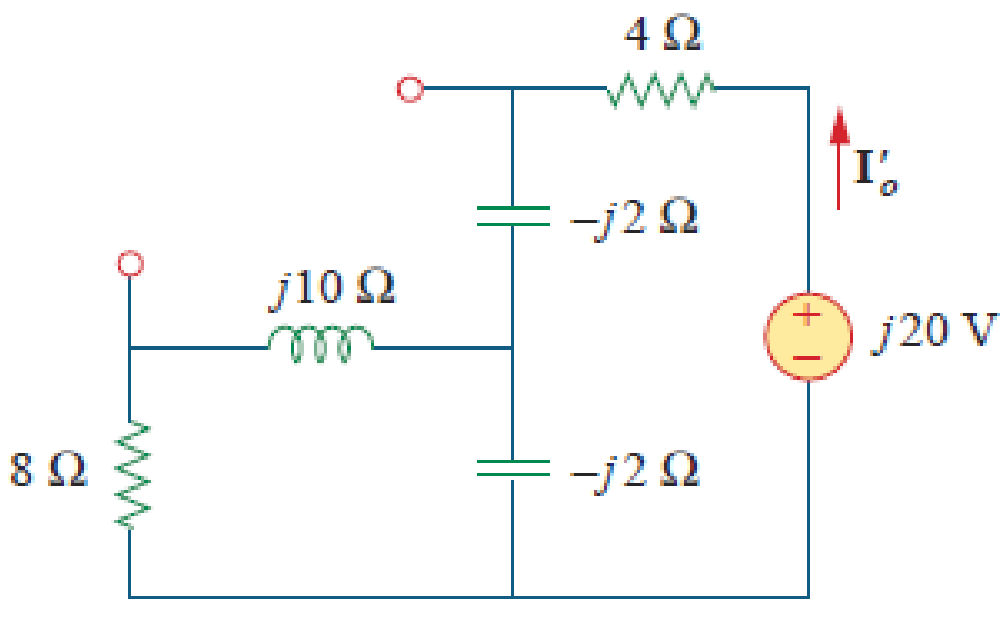


(b)

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

$$\mathbf{I}_o' = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

$$\mathbf{I}_o' = -2.353 + j2.353$$



(a)

For mesh 1,

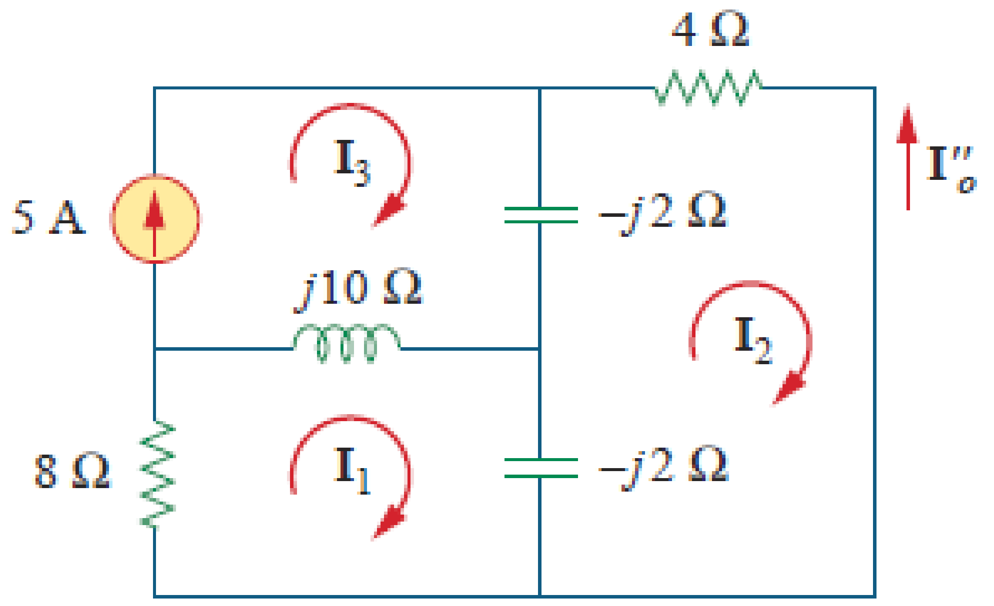
$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0$$

For mesh 3,

$$\mathbf{I}_3 = 5$$



$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \qquad \textbf{(10.5.6)}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}_o''$ is obtained as

Current $\mathbf{I}_o''$ is obtained as

$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176$$

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529 = 6.12\underline{/144.78^\circ}\text{ A}$$

Practice Problem 10.5

Find current $\mathbf{I}_o$ in the circuit of Fig. 10.8 using the superposition theorem.

Answer: $3.582\angle 65.45^\circ$ A.

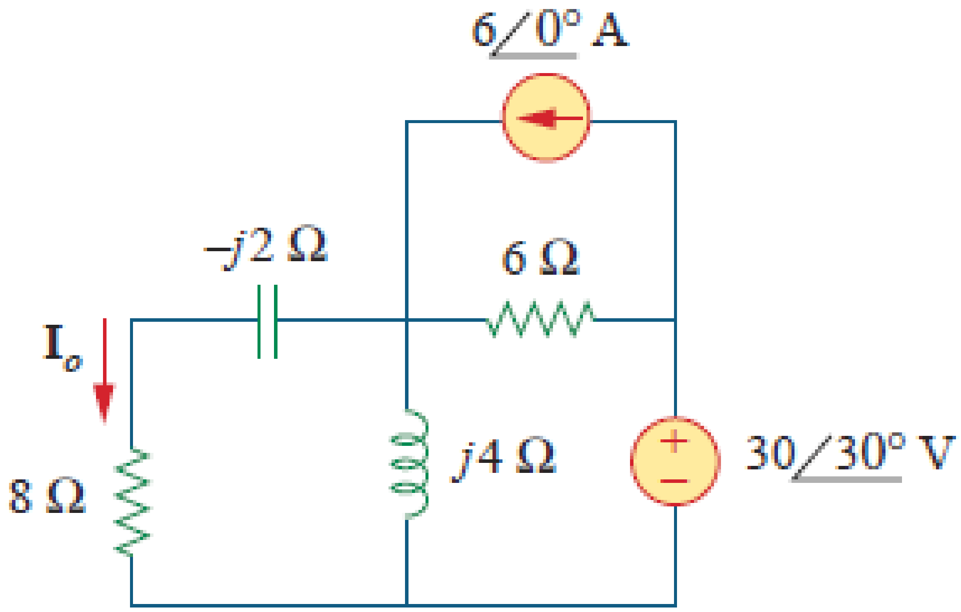


Figure 10.8

Source Transformation

- **Delta-Wye Transform** simplifies circuits
- **Source transformation** is another tool for simplifying circuits.
- It is based on the concept of ***equivalence***

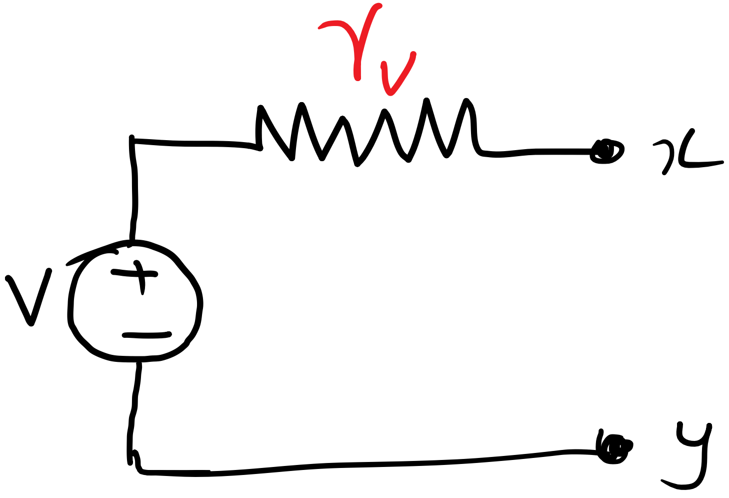
- **Practical voltage source consists of an ideal voltage source in series with a internal impedance (Resistance)**
  - **For ideal source this internal impedance is zero**

- **Practical current source is an ideal current source in parallel with internal impedance**
  - **For an ideal current source this parallel impedance is infinity (thus source current does not branch**

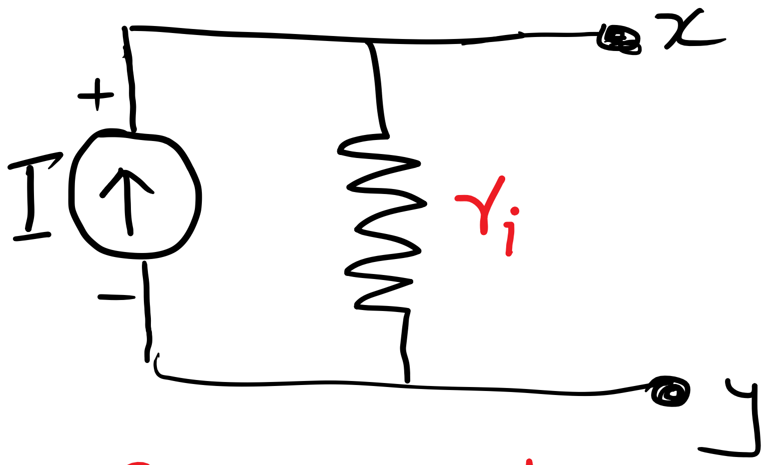
- **A voltage source is a two-terminal device which can maintain a fixed voltage. An ideal voltage source can maintain the fixed voltage independent of the load resistance or the output current. However, a real-world voltage source cannot supply unlimited current. (Eg: battery, generators etc)**

- **A current source is an electronic circuit that delivers or absorbs an electric current which is independent of the voltage across it. Eg: Most of the semiconductor devices transistor, Diodes etc**

The voltage source and current sources are mutually transferable (one is the dual of the other)



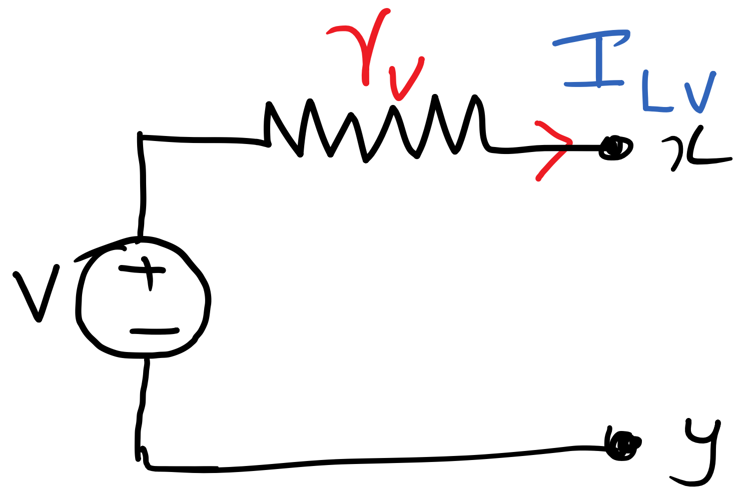
Practical voltage source

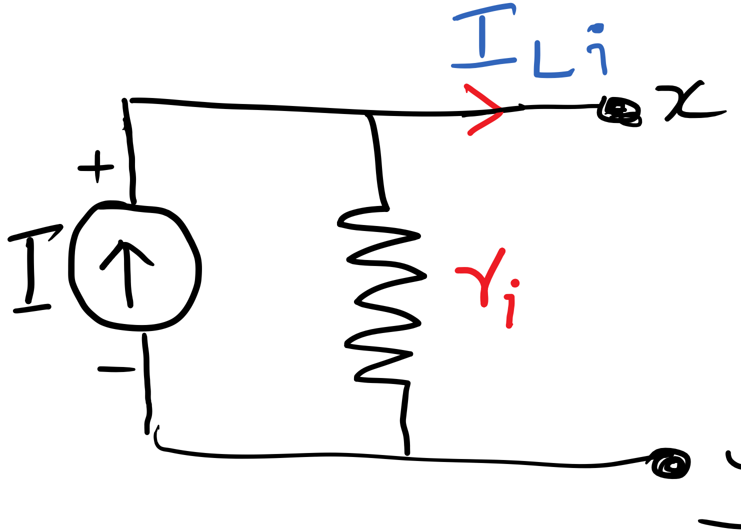


Practical current source

$r_v \rightarrow$ internal resistance of voltage source

$r_i \rightarrow$,, ,, current source.





LOAD CURRENTS (Let R_L be the load resistance)

$$\boxed{I_{Lv} = \frac{V}{r_v + R_L}}$$

$$\boxed{I_{Li} = I \, \frac{r_i}{r_i + R_L}}$$

Two sources become identical

$\Downarrow$

EQUIVALANCE

$$I_{Lv} = I_{Li}$$

$$\frac{V}{r_v + R_L} = I \frac{r_i}{r_i + R_L}$$

$$V = I r_i$$

(For current source terminal voltage at x-y would be $\underline{\underline{I r_i}}$)

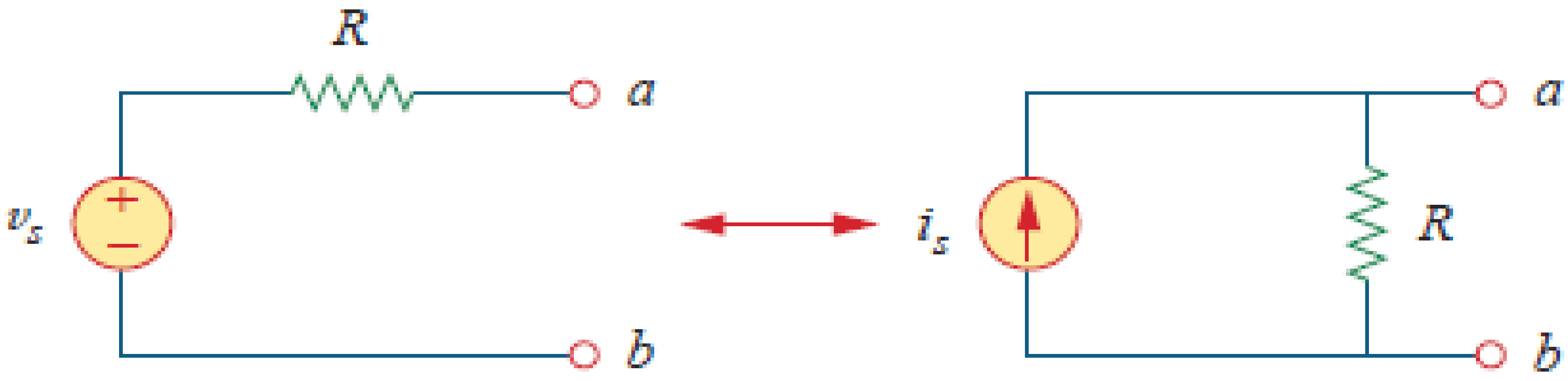
$$r_v + R_L = r_i + R_L$$

$$\boxed{r_v = r_i}$$

$$r_v = r_i$$

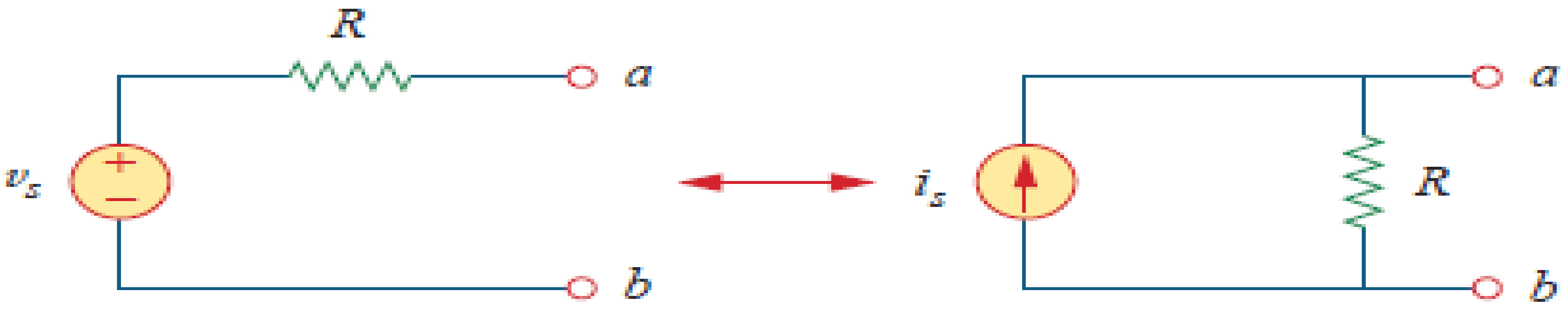
For any practical voltage source, if the ideal voltage be V and internal voltage be rv, the voltage source can be replaced by a current source I with the internal resistance in parallel to the current source

In circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa



A **source transformation** is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.

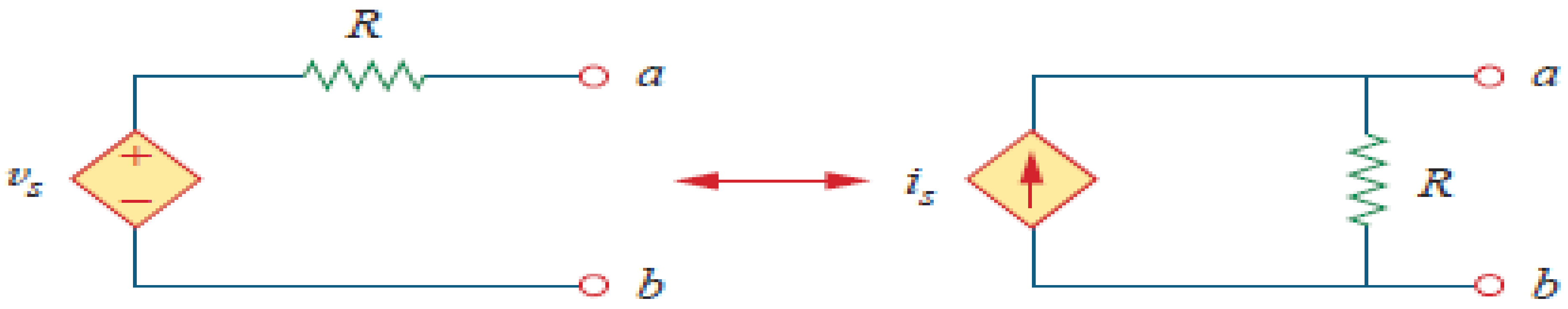
Transformation of independent sources



- **They should have same voltage-current relationship at terminals a-b.**
- **If the sources are turned off, the equivalent resistance at terminals a-b is R.**
- **Also when the terminals are short-circuited, the short circuit current flowing from a to b is same in both the circuits.**

$$i_{sc} = v_s/R \qquad i_{sc} = i_s \qquad v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Transformation of dependent sources



$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

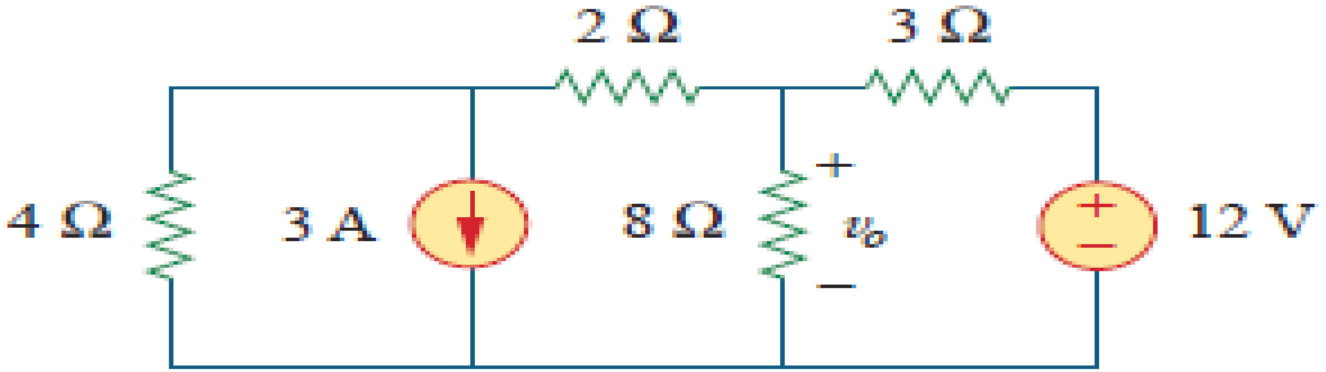
Source Transformation –Points to remember

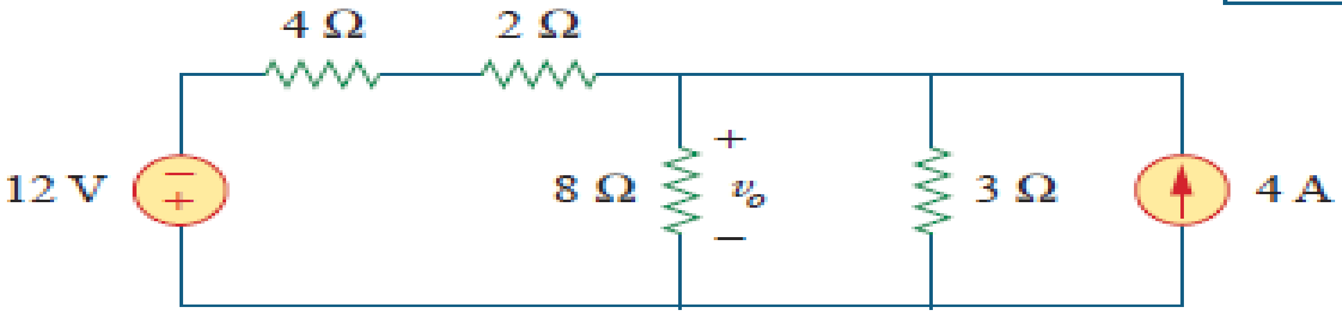
- Note from the figures that the arrow of the current source is directed towards the positive terminal of the voltage source.
- Note from the equations that source transformation is not possible when $R=0$, which is the case with an ideal voltage source.
- Similarly, an ideal current source with $R=\infty$, cannot be replaced by a finite voltage source.

Example

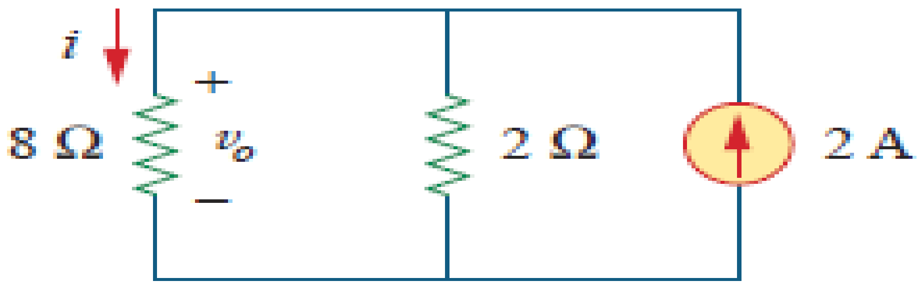
(DC)

Use source transformation to find v_o in the circuit of Fig.

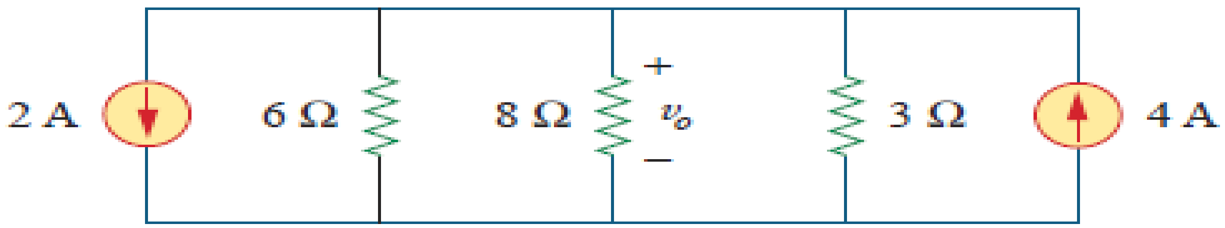




(a)



(c)



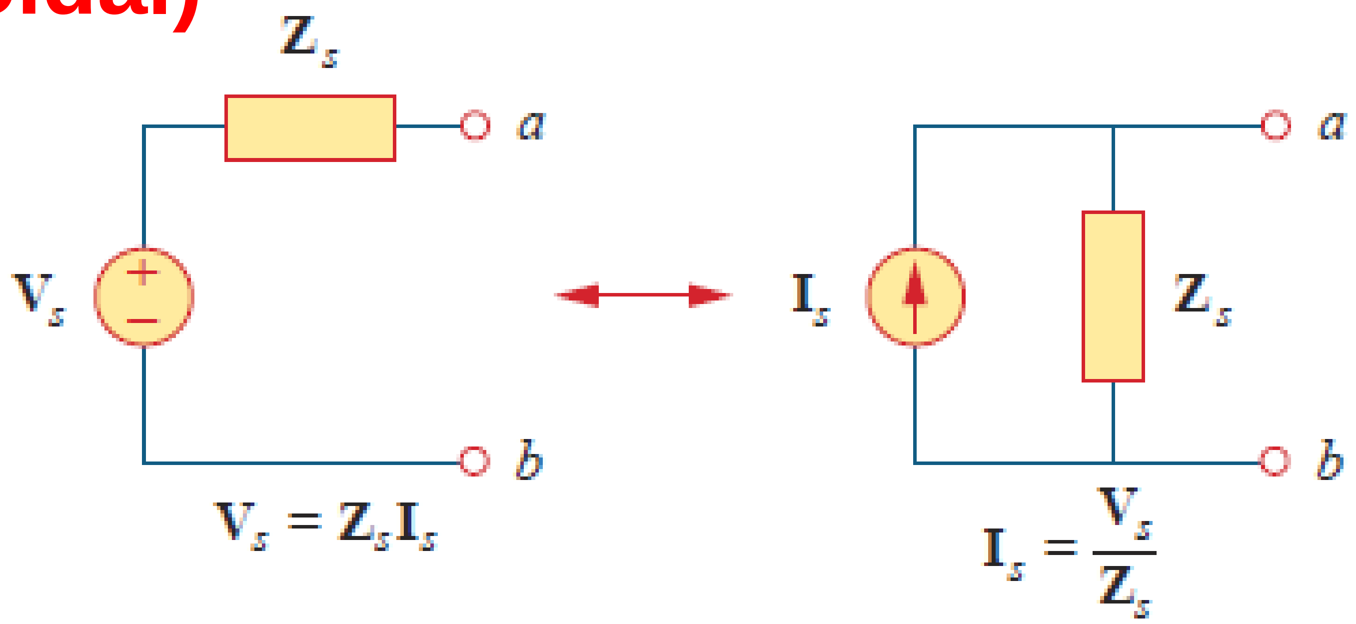
(b)

$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

10.5 Source Transformation

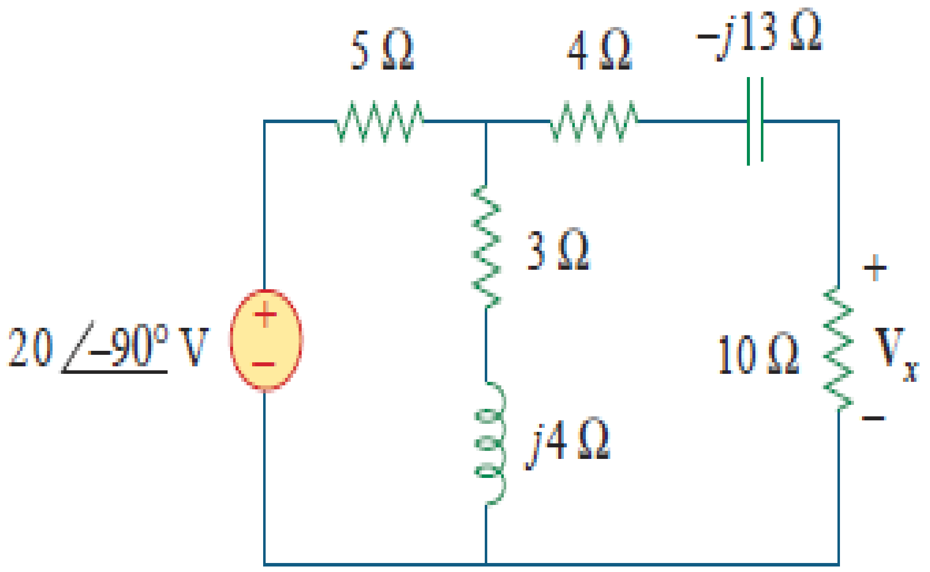
(AC-sinusoidal)

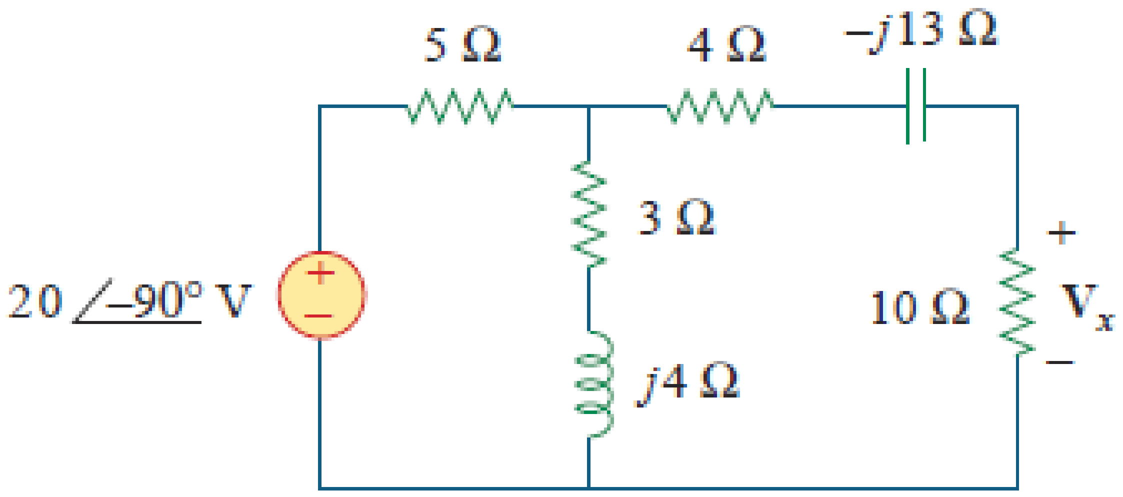


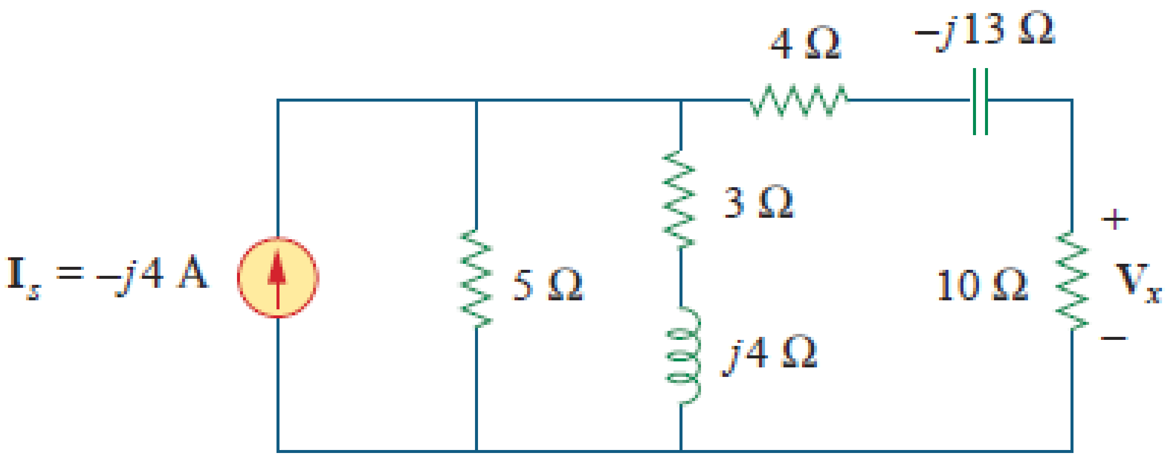
$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

Example 10.7

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.





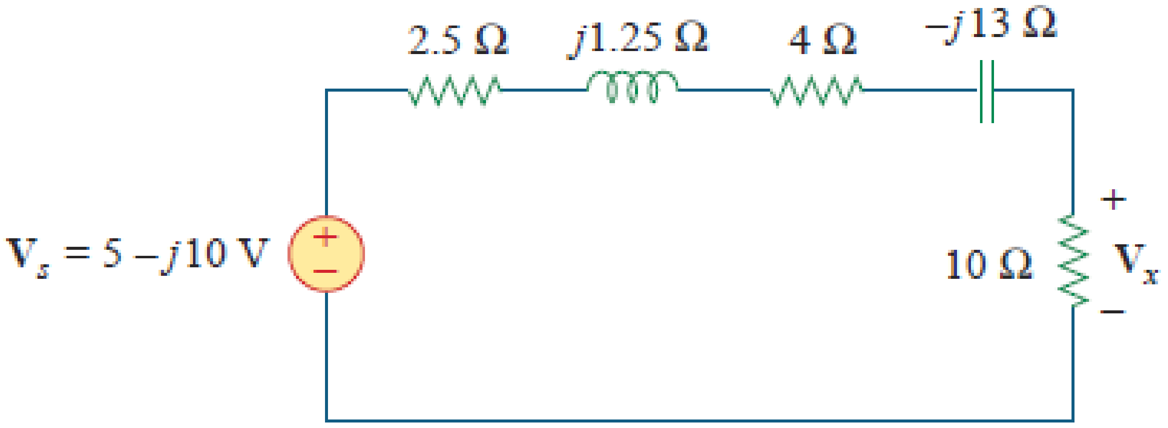


(a)

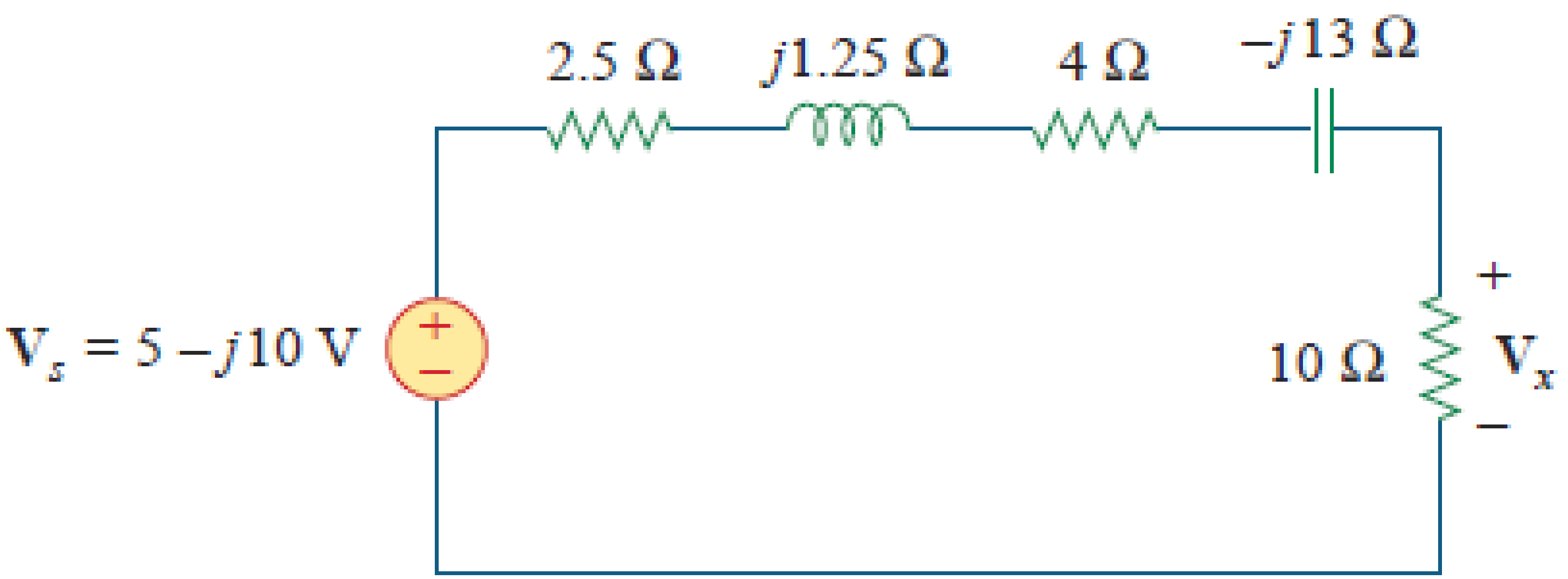
$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$



(b)



By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\underline{/-28^\circ} \text{ V}$$