Fundamental Cut-set Matrix

- Fundamental cut set or **f-cut set** is the minimum number of branches that are removed from a graph in such a way that the original graph will become two isolated subgraphs.

- **The f-cut set contains only one twig and one or more links.**

- **The number of f-cut sets will be equal to the number of twigs.**

- Fundamental cut set matrix is represented with letter C. This matrix gives the relation between branch voltages and twig voltages.

- If there are ‘n’ nodes and ‘b’ branches are present in a directed graph, then the number of twigs present in a selected Tree of given graph will be n-1.

- The fundamental cut set matrix will have ‘n-1’ rows and ‘b’ columns.

- Rows and columns are corresponding to the twigs of selected tree and branches of given graph.

- The order of fundamental cut set matrix will be $(n-1) \times b$.

The elements of fundamental cut set matrix will be having one of these three values, +1, -1 and 0.

- **The value of element will be +1 for the twig of selected f-cutset.**
- The value of elements will be 0 for the remaining twigs and links, which are not part of the selected f-cut set.
- If the direction of link current of selected f-cut set is same as that of the cut set twig current, then the value of element will be +1.
- If the direction of link current of selected f-cut set is opposite to that off-cut set twig current, then the value of element will be -1.

Procedure to find Fundamental Cut-set Matrix

- To find the fundamental cut set matrix of given directed graph.
- Select a Tree of given directed graph and represent the links with the dotted lines.
- By removing one twig and necessary links at a time, we get one f-cut set. Fill the values of elements corresponding to this f-cut set in a row of fundamental cut set matrix.
- Repeat the above step for all twigs.

Example

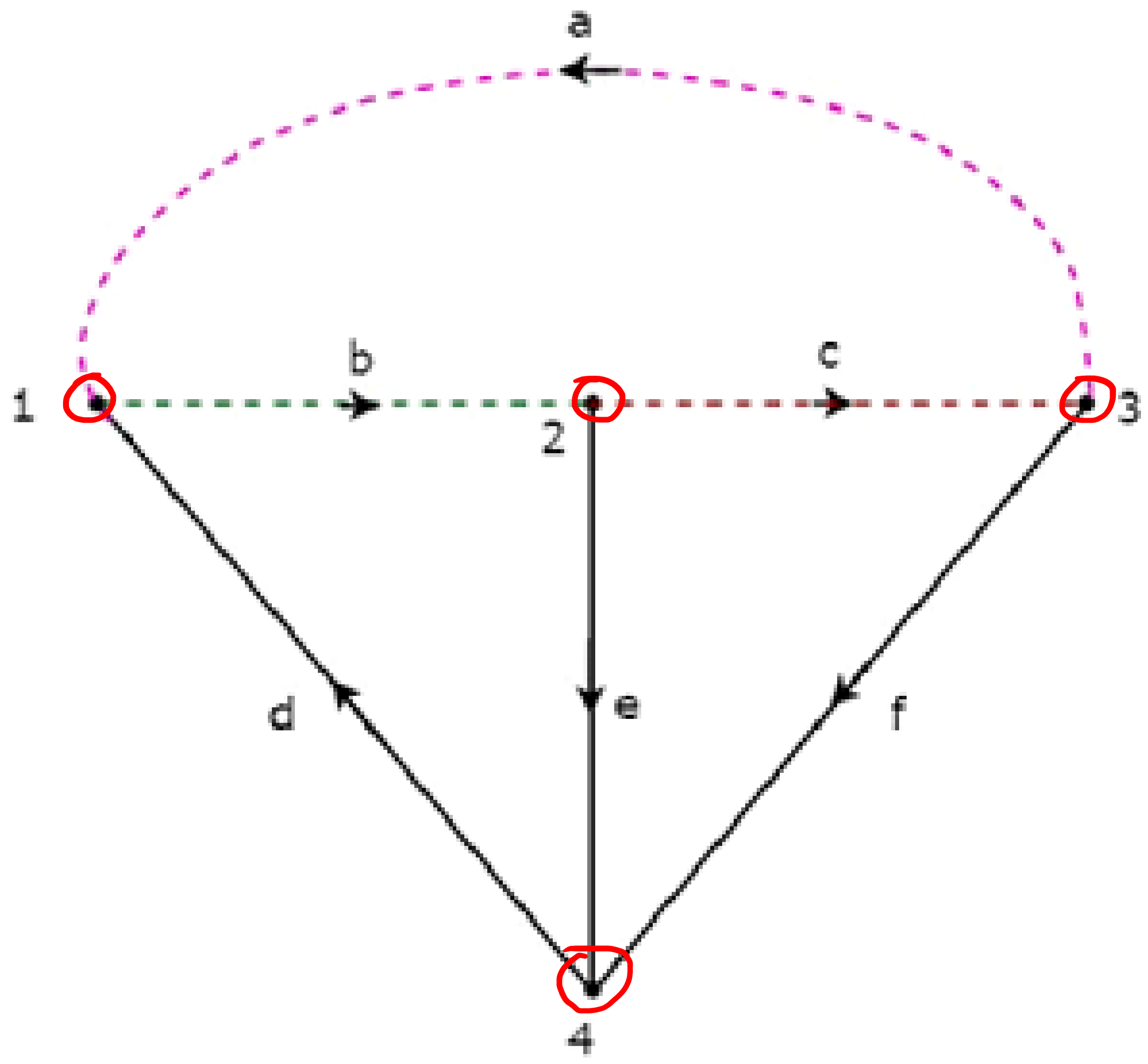
- Consider the same **directed graph**

- Select the **branches d, e & f** of this directed graph as **twigs.**

- The remaining **branches a, b & c** of this directed graph will be the **links**.

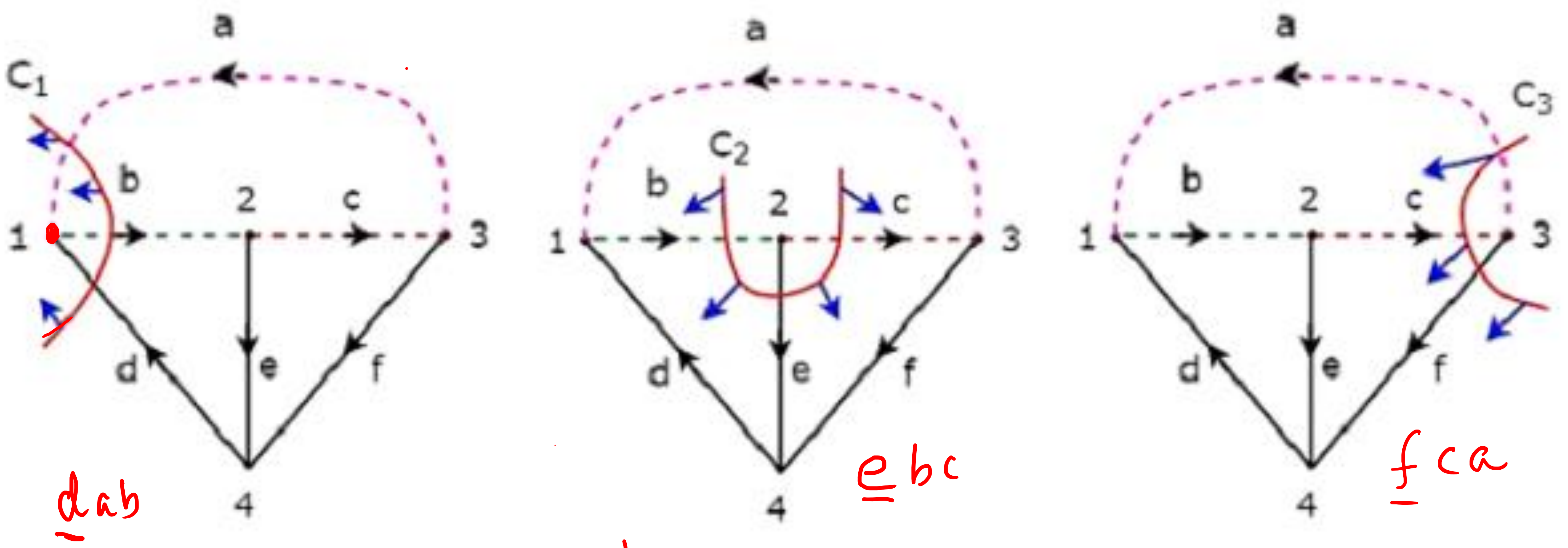
- The **twigs** **d, e & f** are represented with solid lines

- The **links** **a, b & c** are represented with dotted lines in the figure.

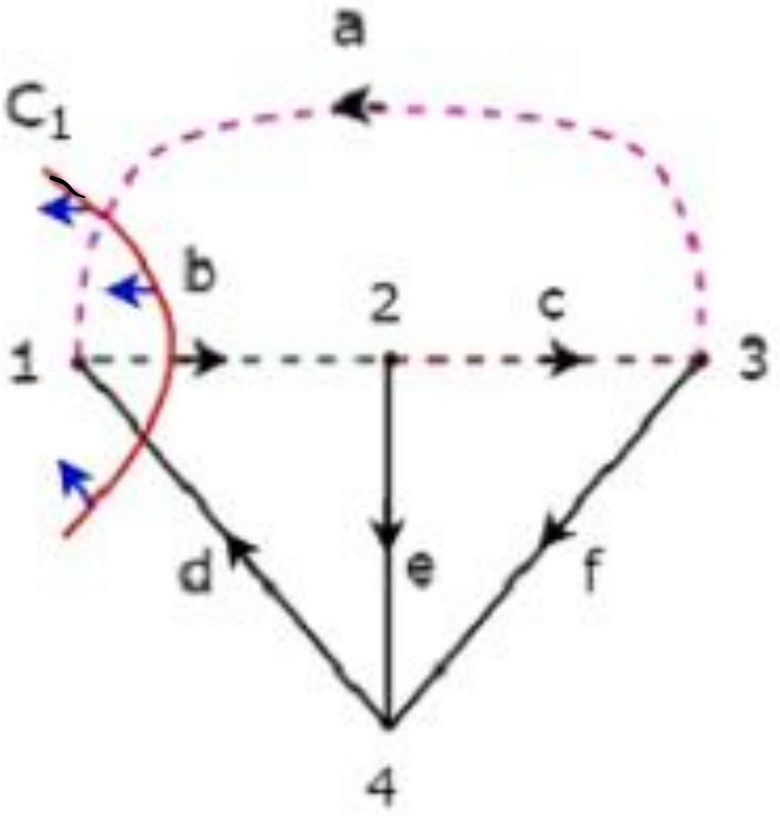


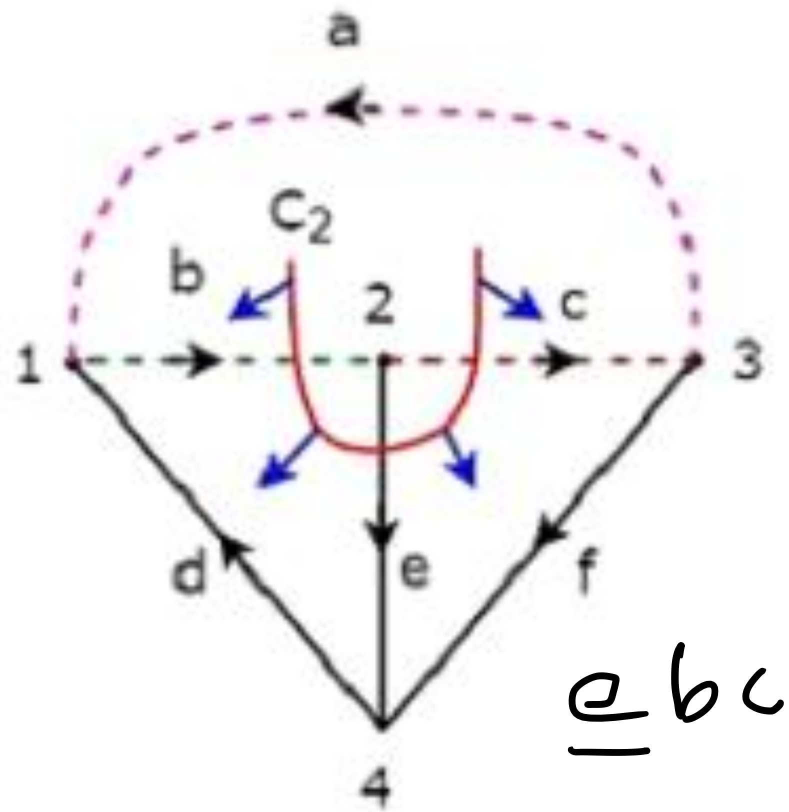
- By removing one twig and necessary links at a time, we will get one f-cut set.

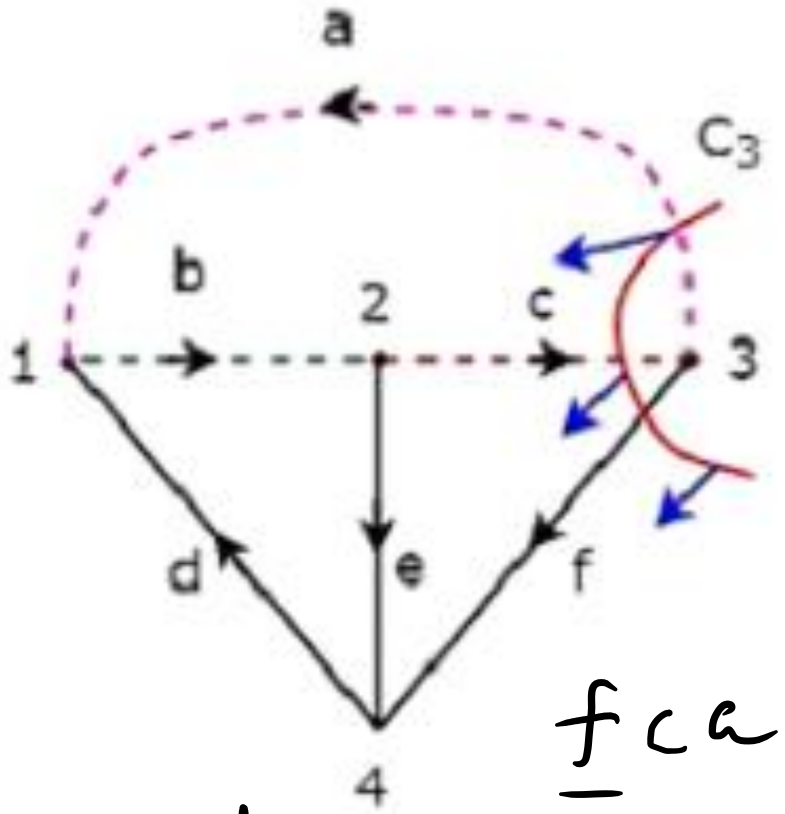
- So, there will be **three f-cut sets**, since there are three twigs. These three f-cut sets are shown in the following figure.



d







dab

ebc

fca

$$C = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

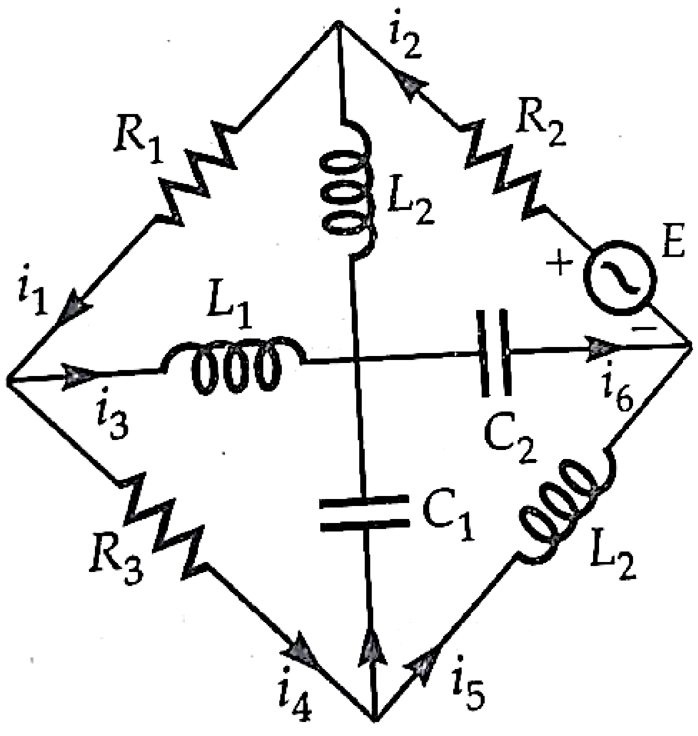
$$C = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

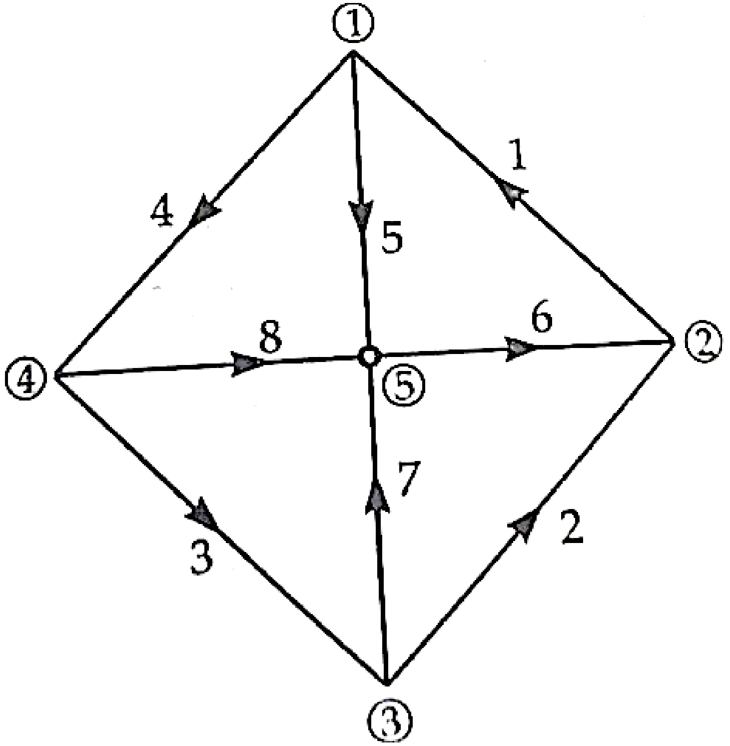
- The rows and columns of the above matrix represents the twigs and branches of given directed graph.

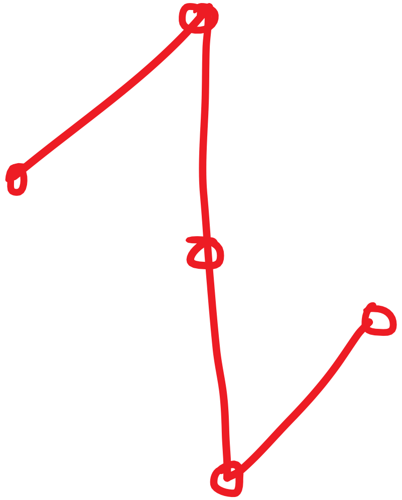
- The order of this fundamental cut set matrix is 3 × 6.

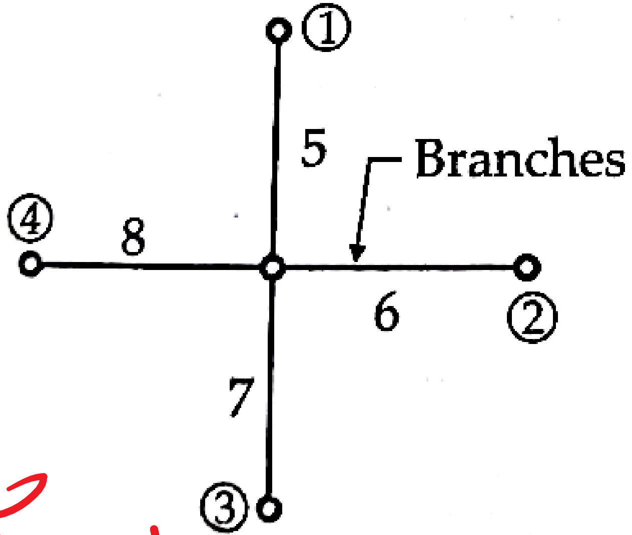
- The number of Fundamental cut set matrices of a directed graph will be equal to the number of Trees of that directed graph. Because, every Tree will be having one Fundamental cut set matrix.

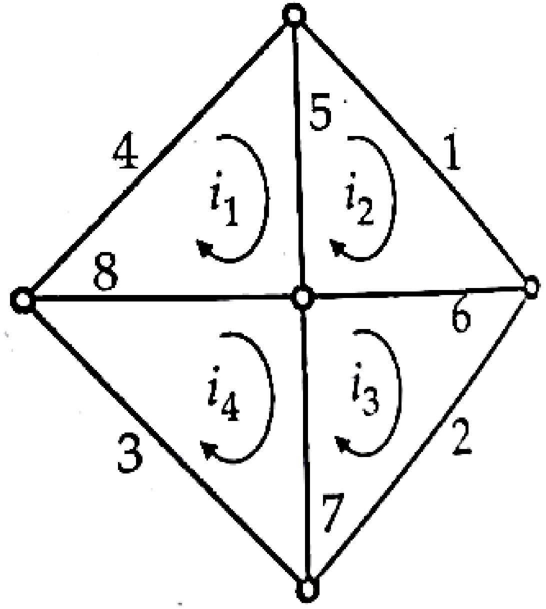
Example: Draw the directed graph of the network shown in f







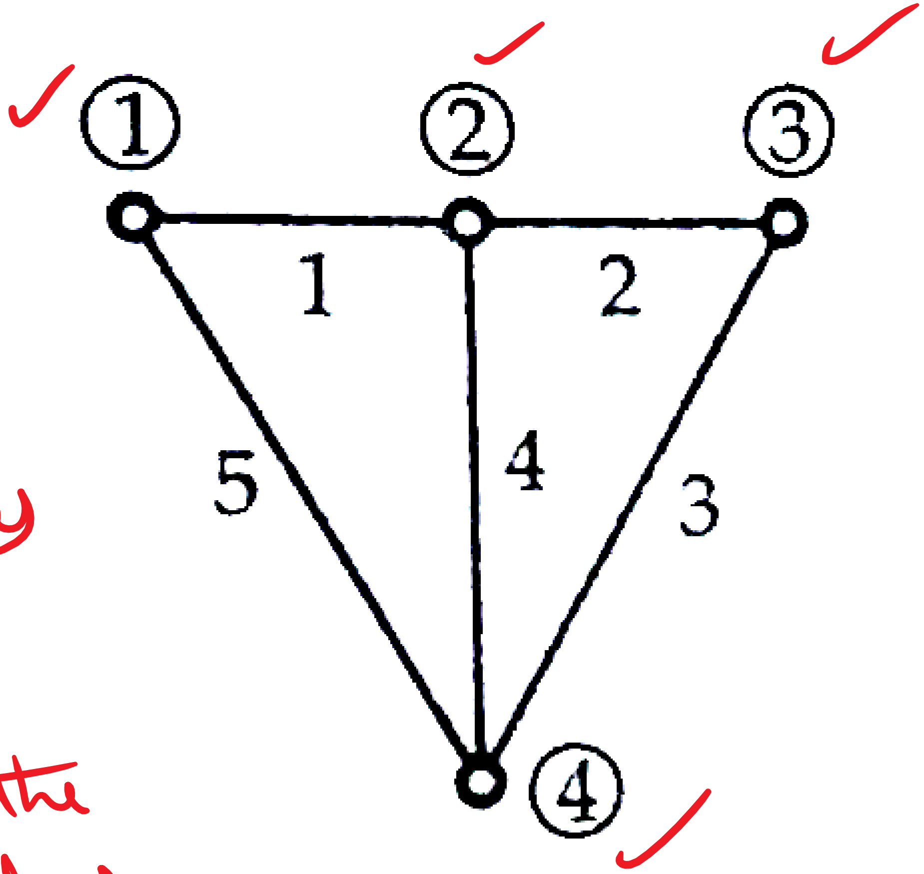




i_1, i_2, i_3 & i_4 are the arbitrary loop currents

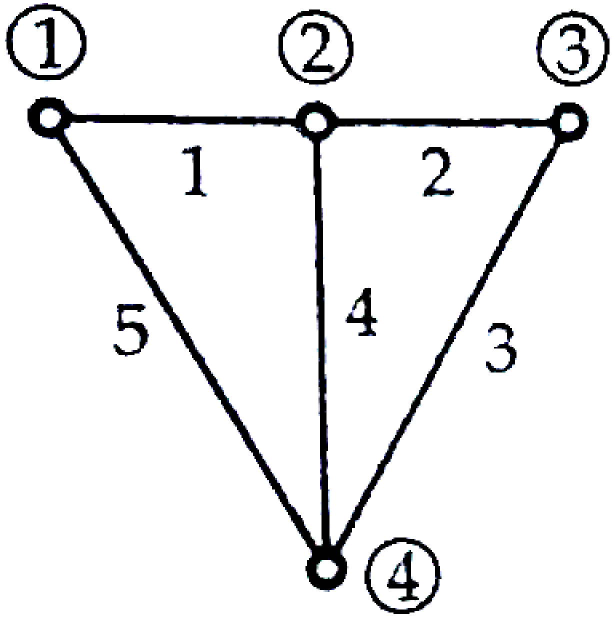
Example

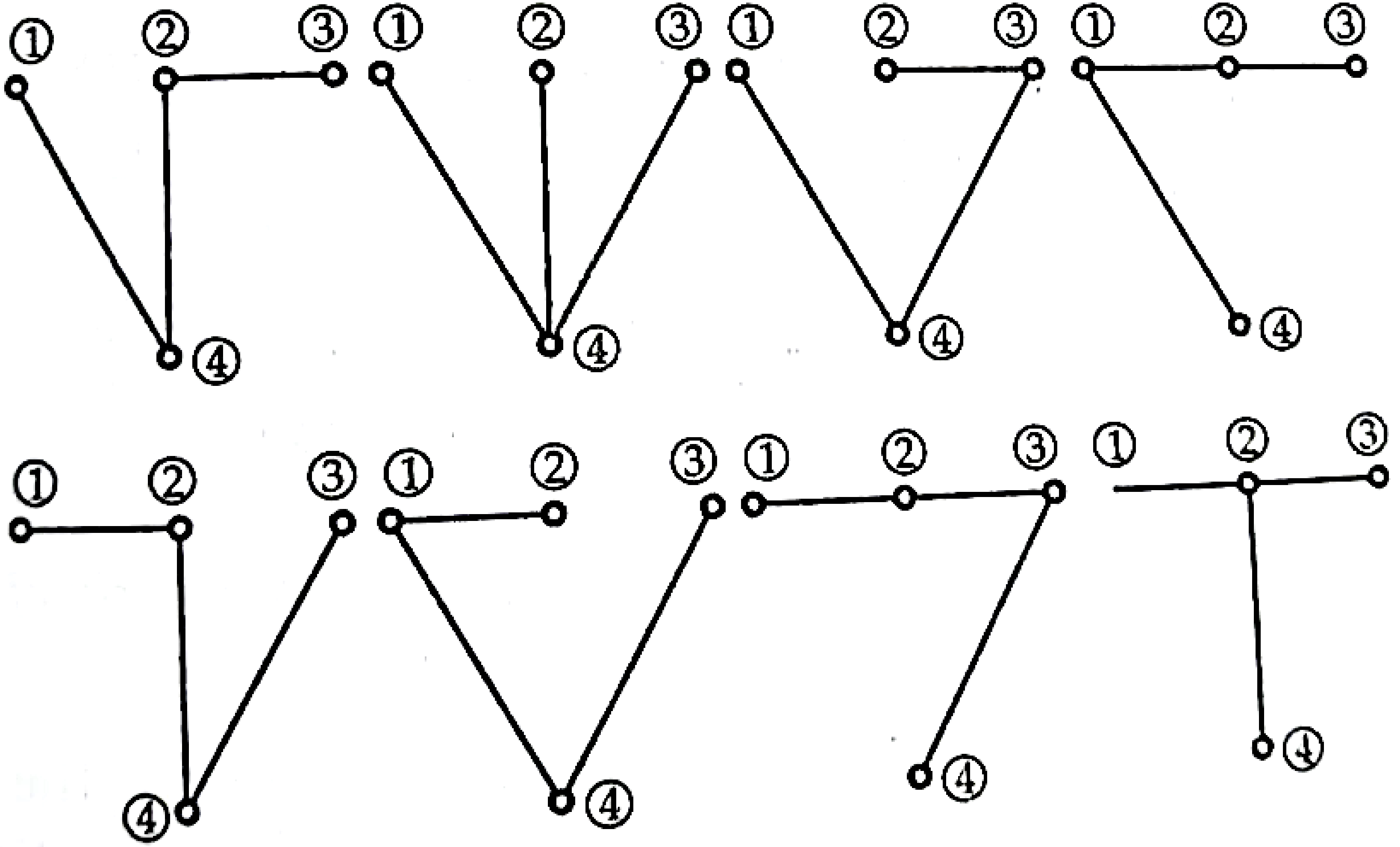
Give the graph of a network. Show the possible TREES, TWIGS and LINKS



1, 2, 3, 4, 5 are the branches

1 2 3 4 are the nodes


①
②
③
④
1
2
3
4
5


①
②
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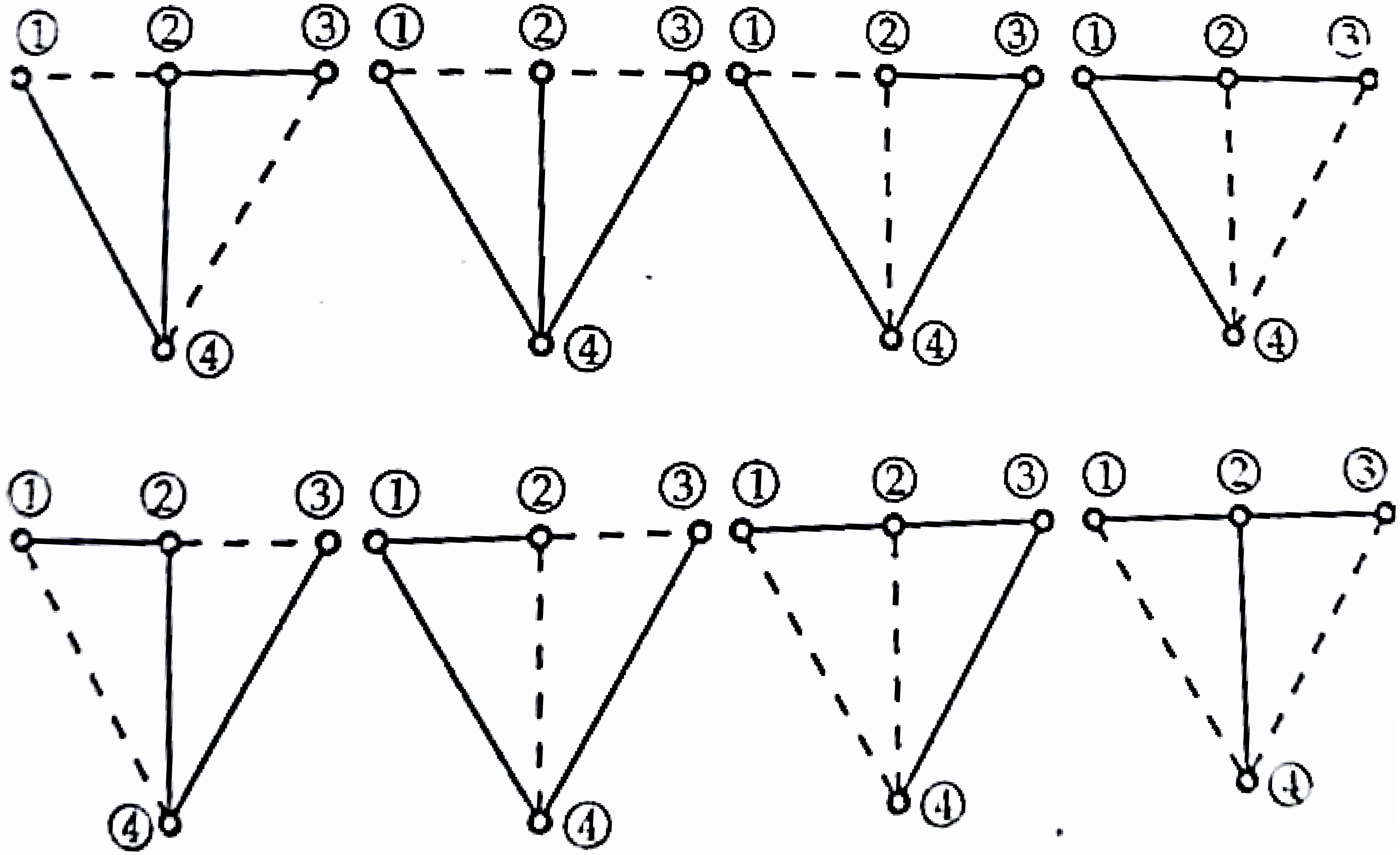
④


① ② ③ ④
① ② ③ ④
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① ② ③ ④
① ② ③ ④
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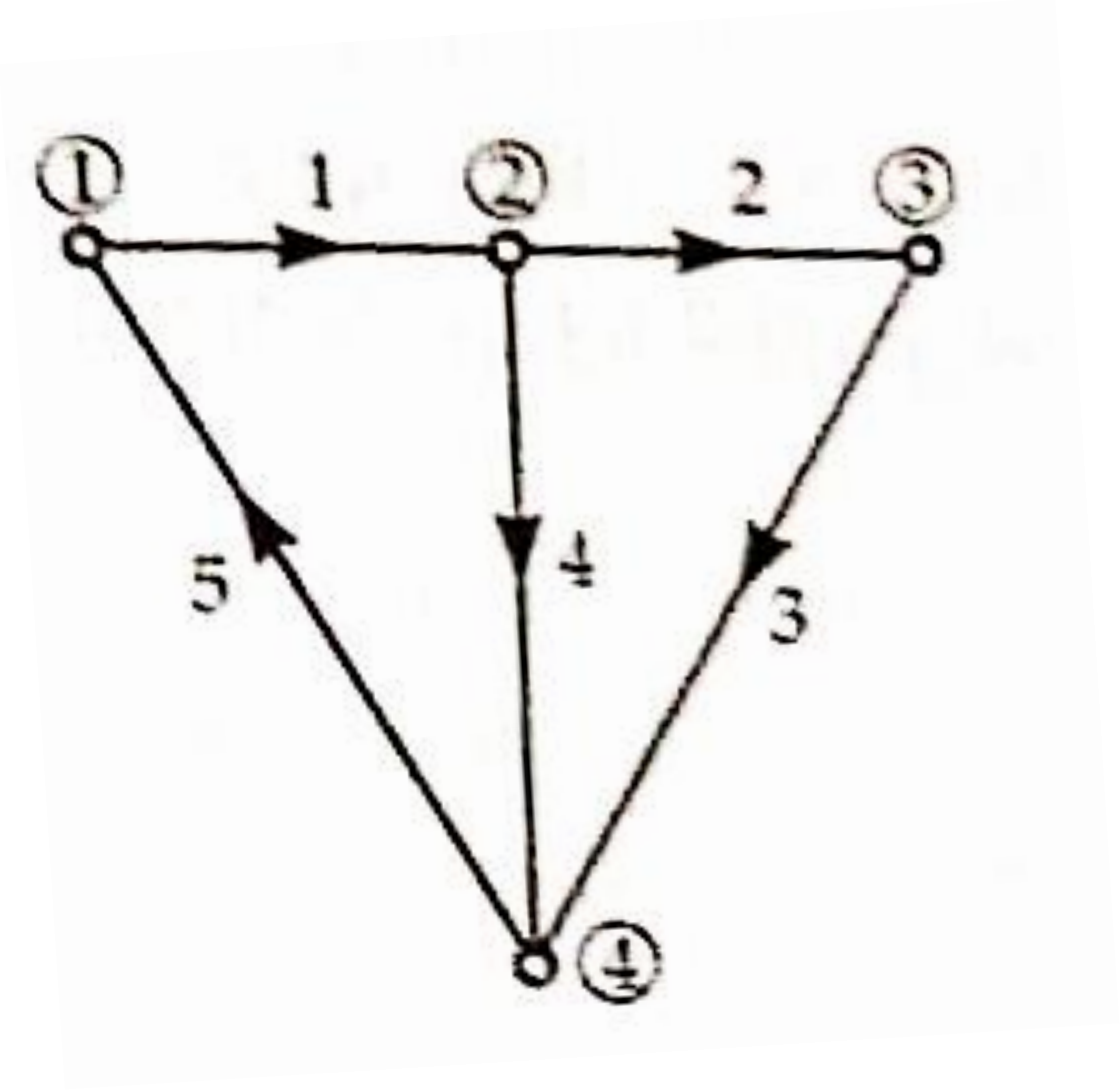
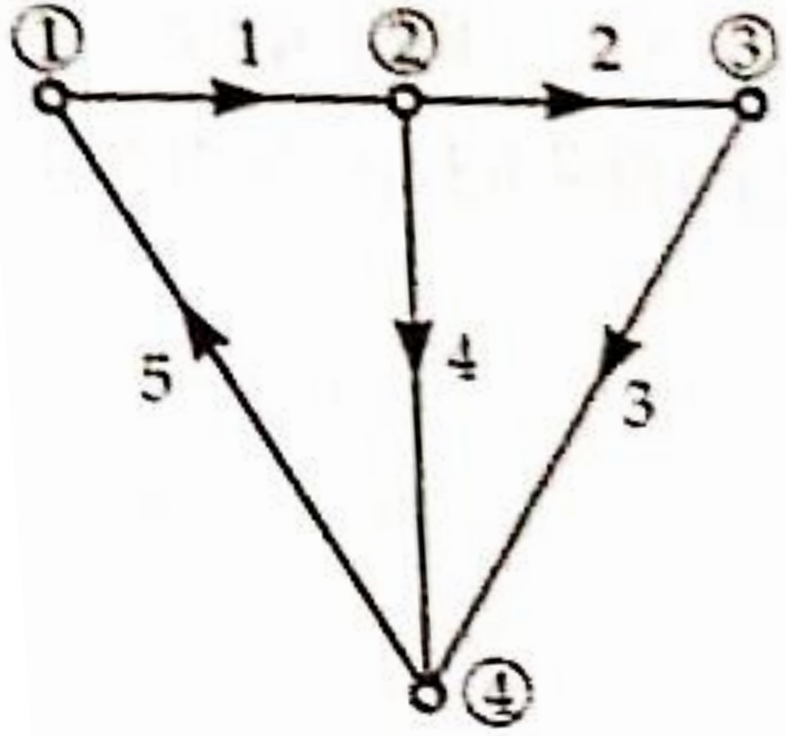
① ② ③ ④

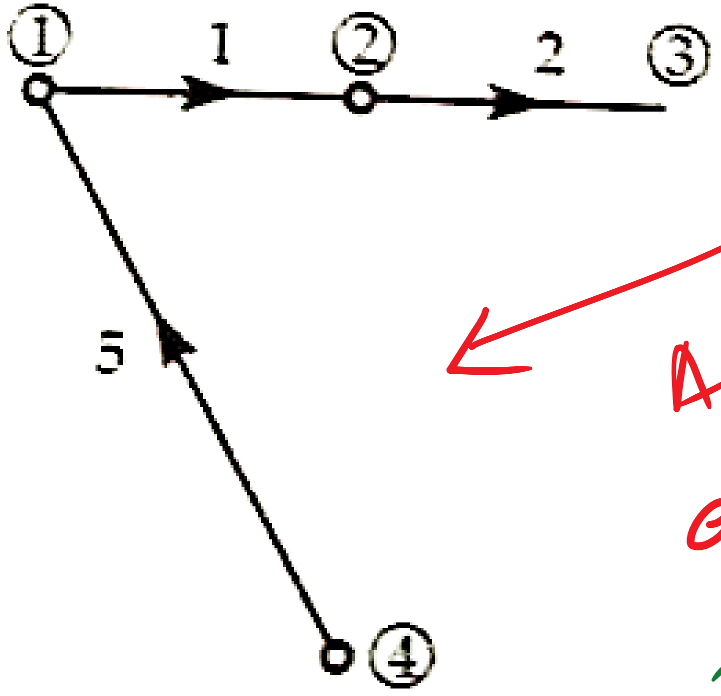
Example

Develop the fundamental TIE-SET matrix for the following directed graph



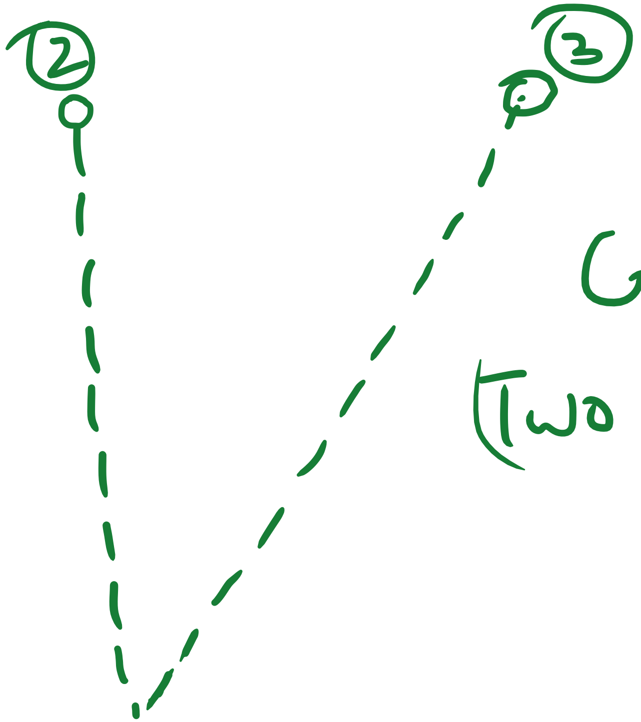


graph

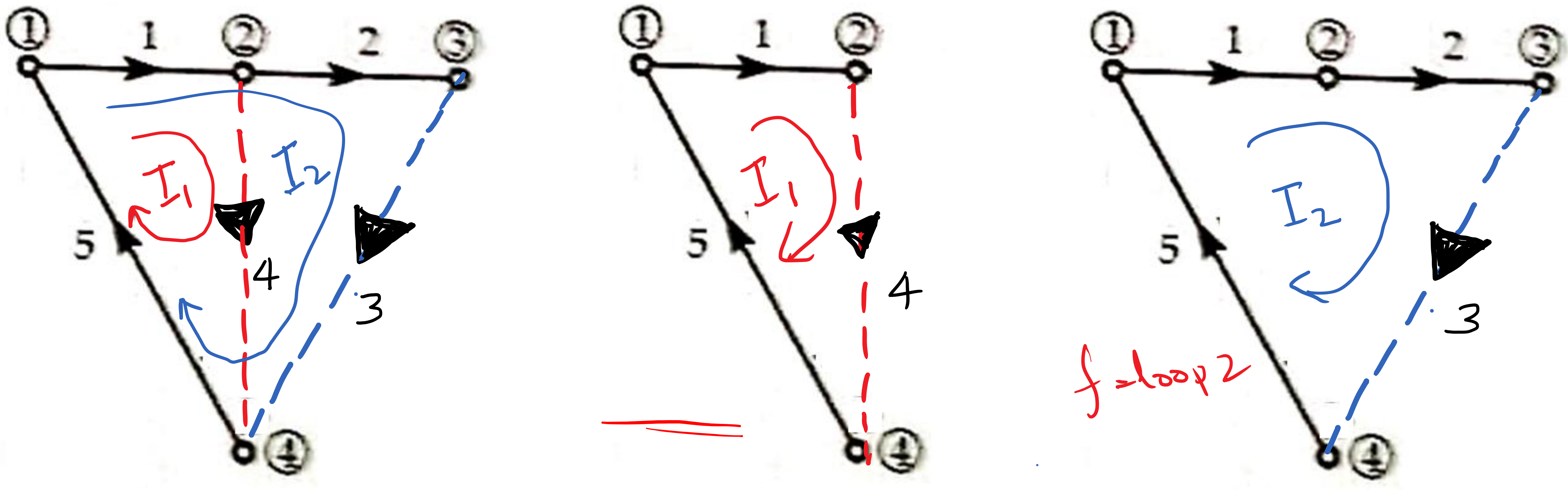


Selected Tree (3 Twigs)

All nodes included and open

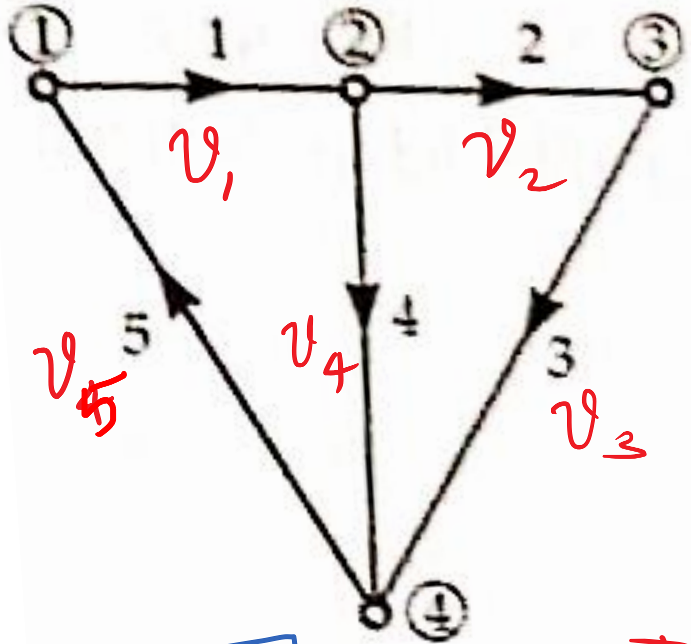


Co-tree (Two links)



Tie-sets (or loop currents)	Branches 1	2	3	4	5
I_1	1	0	0	1	1
I_2	1	1	1	0	1

Assume the direction of the link is the direction of the loop current I_1 or I_2



Tie-sets (or loop currents)	Branches 1	2	3	4	5
I_1	1	0	0	1	1
I_2	1	1	1	0	1

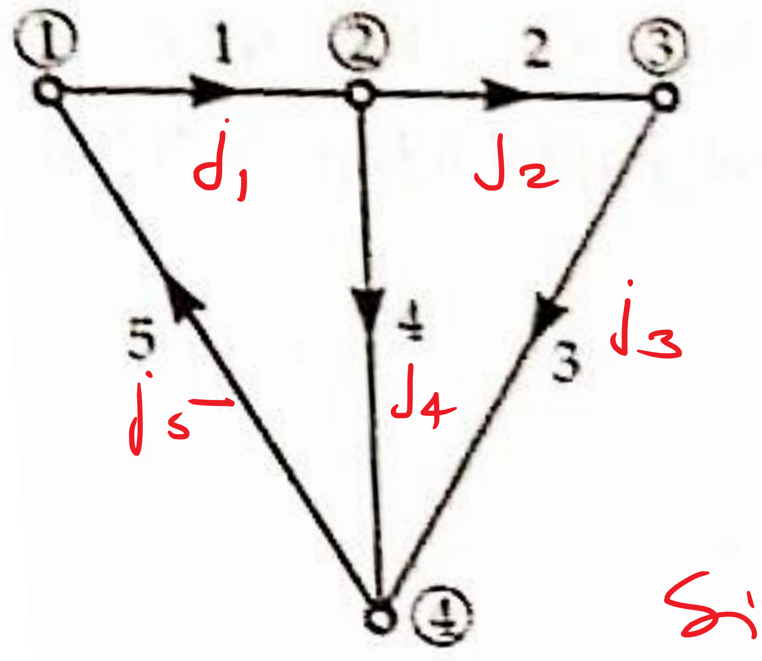
Summation of row voltages

If v_1, v_2, v_3, v_4 and v_5 are the branch Voltages

Then $v_1 + 0 + 0 + v_4 + v_5 = 0$

$\boxed{v_1 + v_4 + v_5 = 0}$ ← 1st row

Similarly, $\boxed{V_1 + V_2 + V_3 + V_5 = 0}$ ← 2nd row



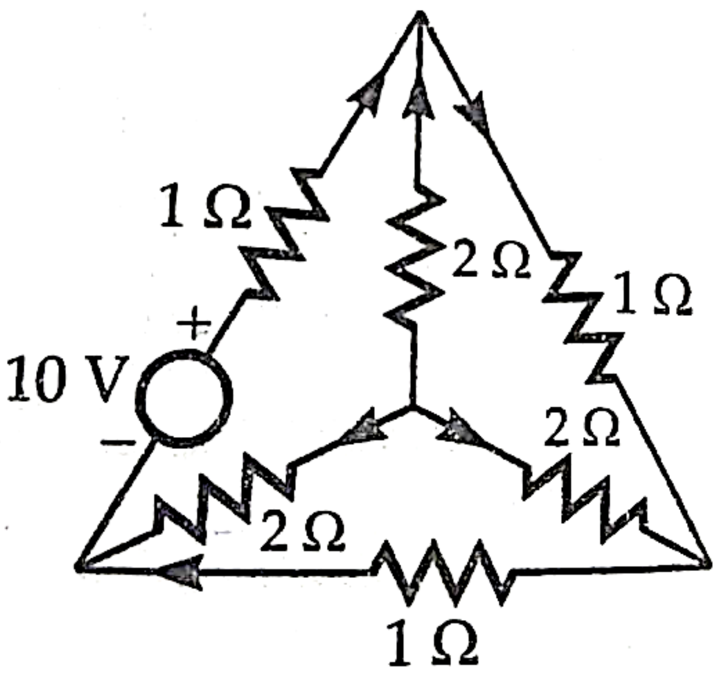
$$
\begin{array}{c|ccccc}
\text{Tie-sets (or loop currents)} & & & \text{Branches} & & \\
 & 1 & 2 & 3 & 4 & 5 \\
\hline
I_1 & 1 & 0 & 0 & 1 & 1 \\
I_2 & 1 & 1 & 1 & 0 & 1
\end{array}
$$

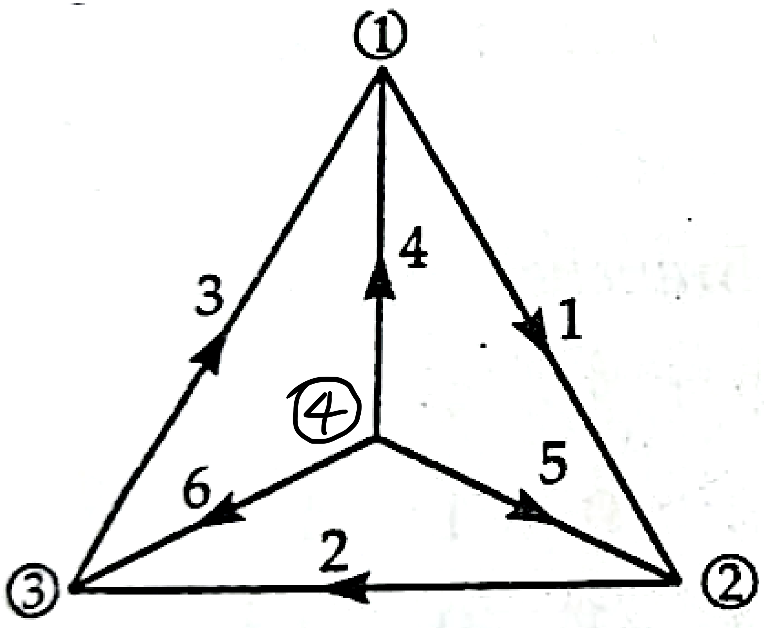
Similarly each branch currents (j) is given by the algebric sum of link currents

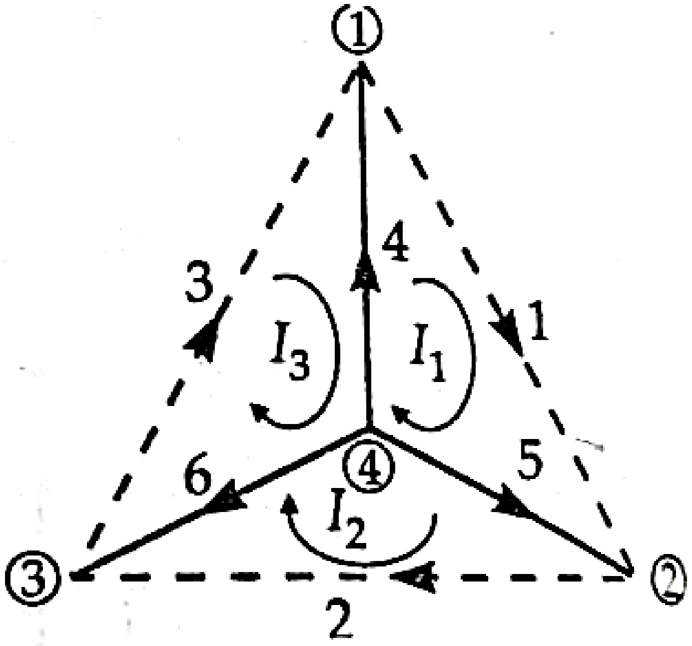
Summation of column currents

$$
\left.\begin{aligned}
I_1 + I_2 &= j_1 \\
0 + I_2 &= j_2 \\
0 + I_2 &= j_3
\end{aligned}\right]
\qquad
\left[\begin{aligned}
I_1 + 0 &= j_4 \\
I_1 + I_2 &= j_5
\end{aligned}\right]
$$

Example: Draw the graph and develop the fundamental TIE-SET matrix





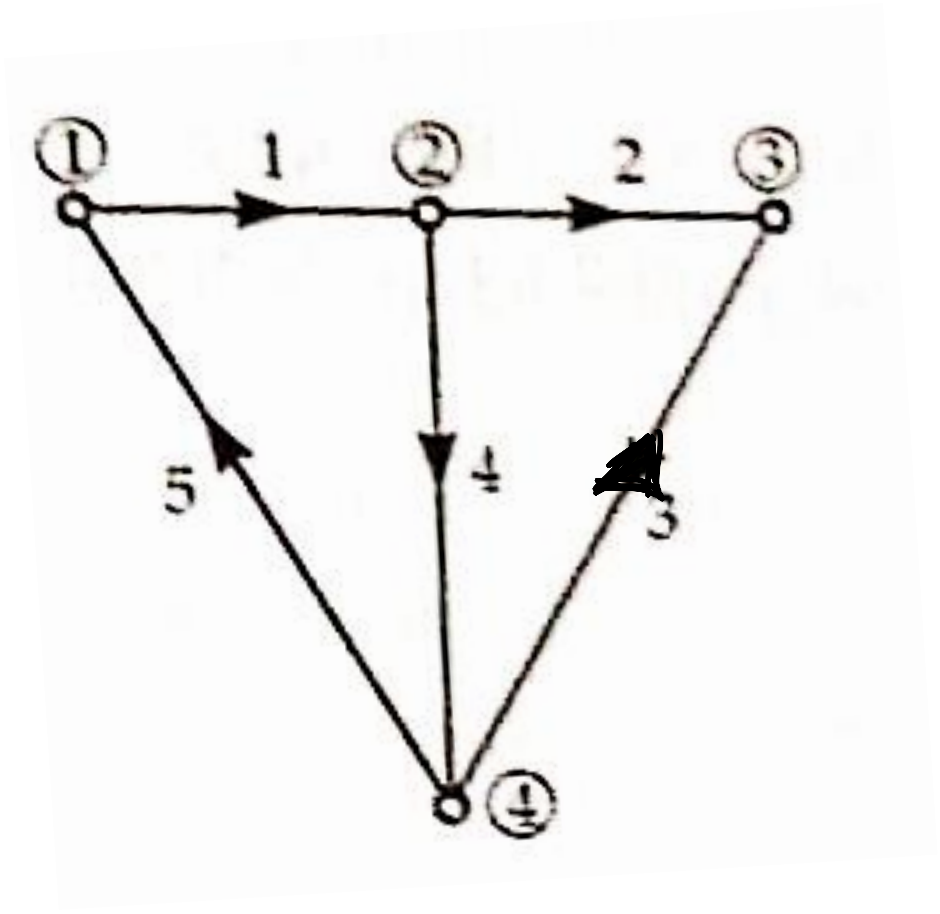


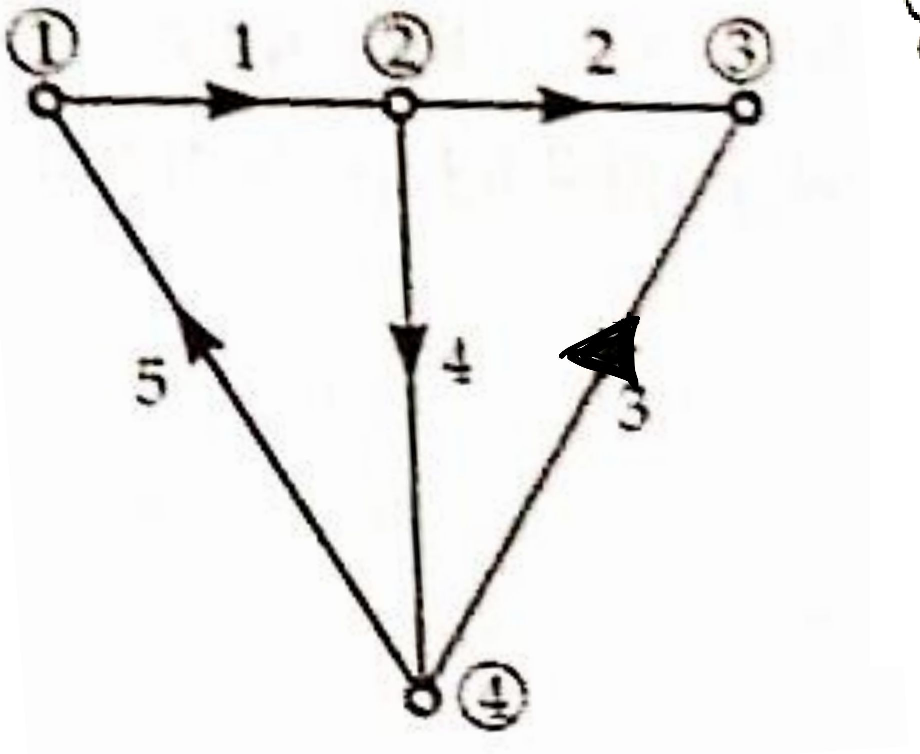
$$
\begin{array}{c|c}
\textit{Tie-set (loop)} & \textit{Branches} \\
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}
\end{array}
$$

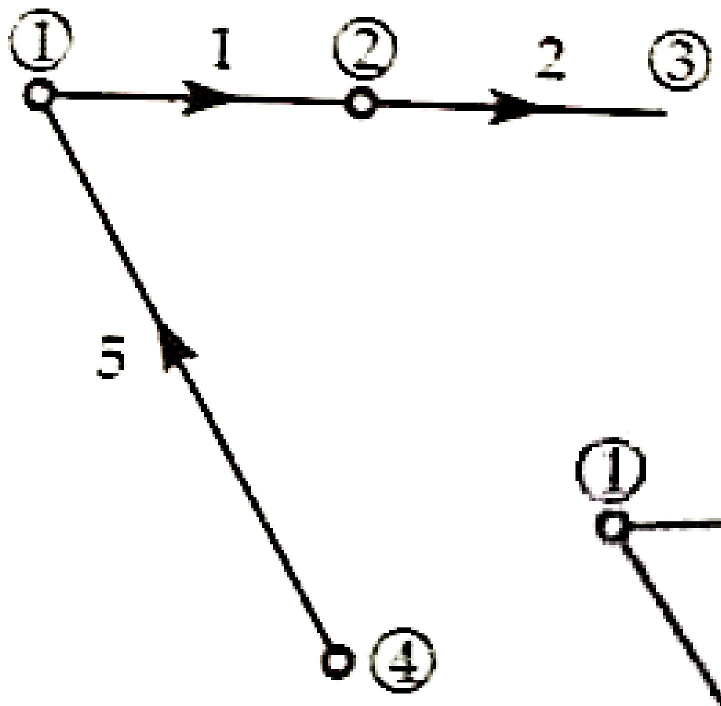
Find the branch currents?

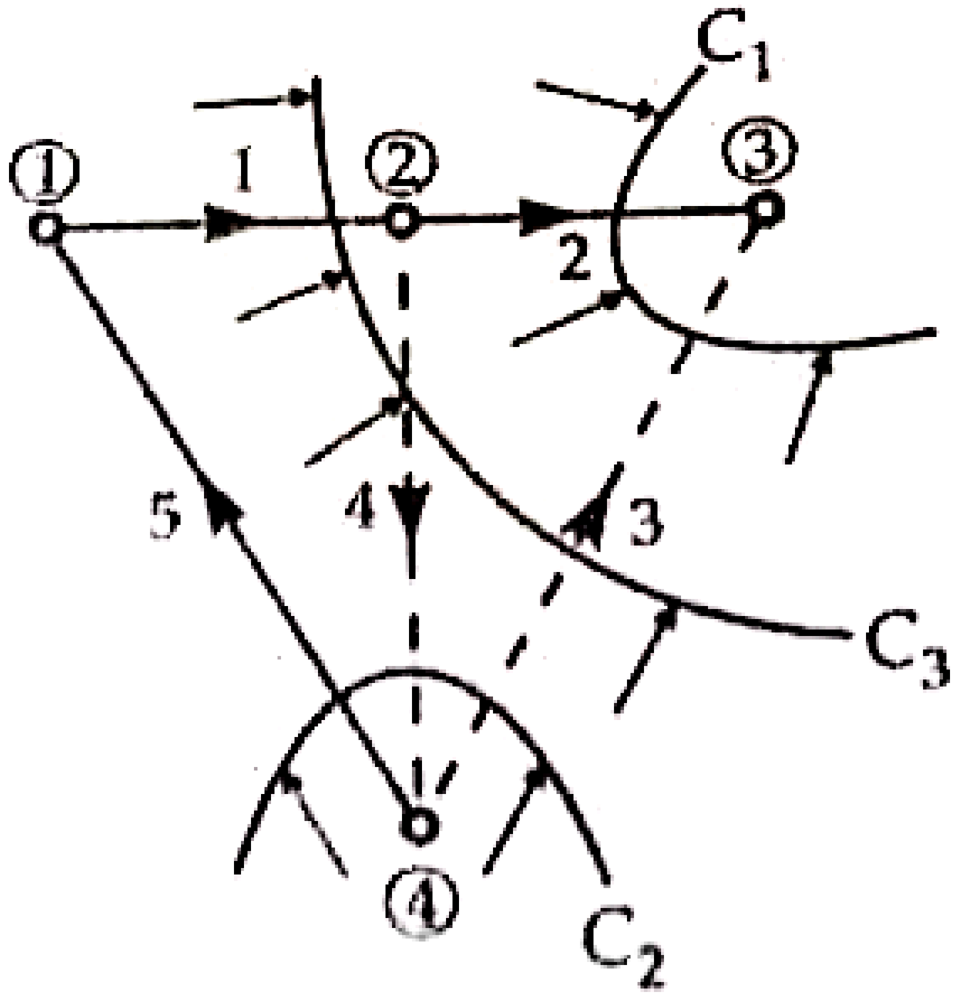
Example

Develop the fundamental CUT-SET matrix for the following directed graph

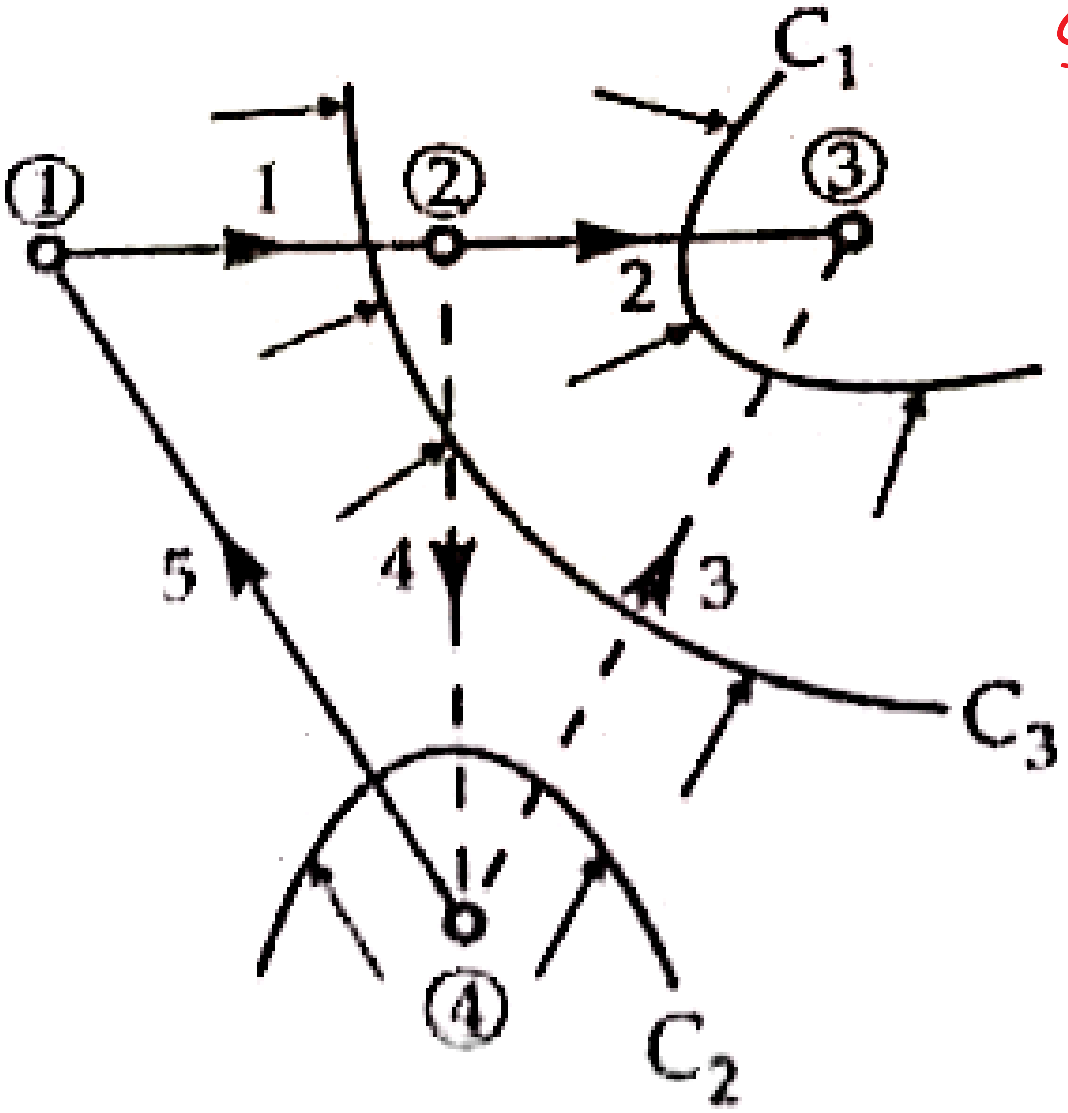








The direction of the twig current
is taken as reference



CUT SET C_1 : TWIG 2 & LINK 3

CUT SET C_2 : TWIGS 5 & LINKS 3 & 4

CUT SET C3 : TWIG 1 & LINKS 4 & 3

Cut-sets	Branches				
	1	2	3	4	5
C_1	0	+1	+1	0	0
C_2	0	0	+1	−1	+1
C_3	+1	0	+1	−1	0

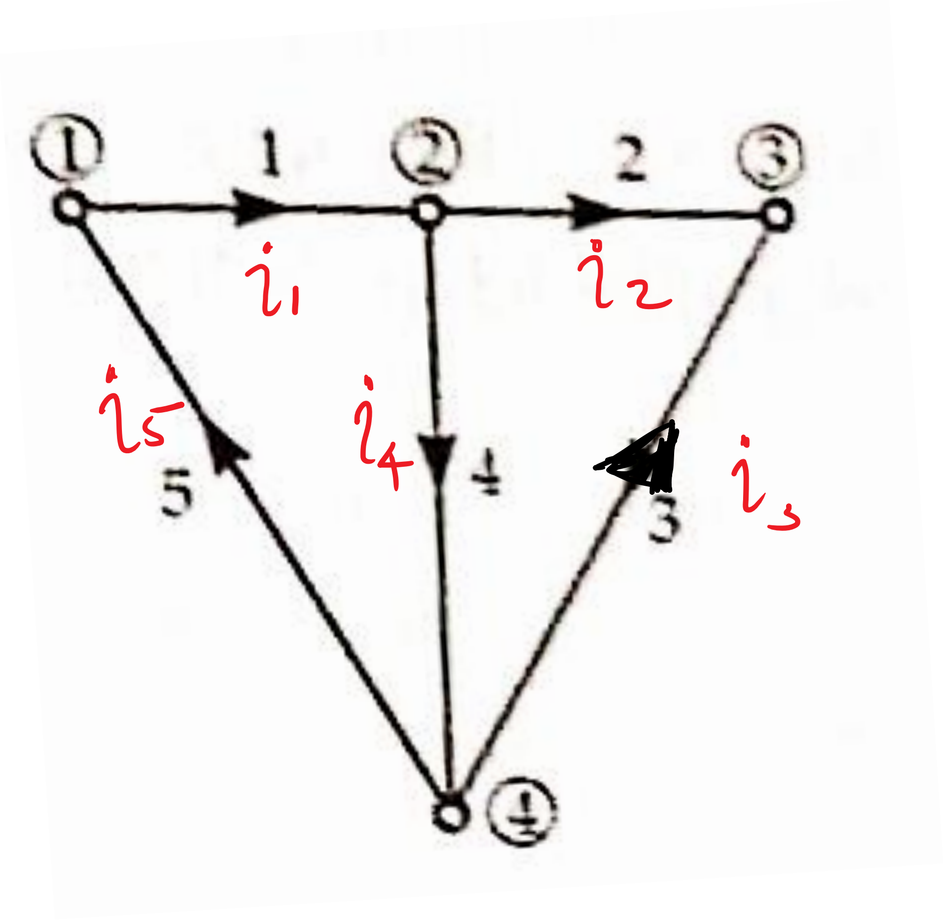
$$
\begin{array}{c|ccccc}
\text{Cut-sets} & \multicolumn{5}{c}{\textit{Branches}} \\
 & 1 & 2 & 3 & 4 & 5 \\
\hline
\begin{array}{c} C_1 \\ C_2 \\ C_3 \end{array} & \multicolumn{5}{c}{\begin{bmatrix} 0 & +1 & +1 & 0 & 0 \\ 0 & 0 & +1 & -1 & +1 \\ +1 & 0 & +1 & -1 & 0 \end{bmatrix}}
\end{array}
$$

The current balance equatins can be obtaind from cut-set matrix

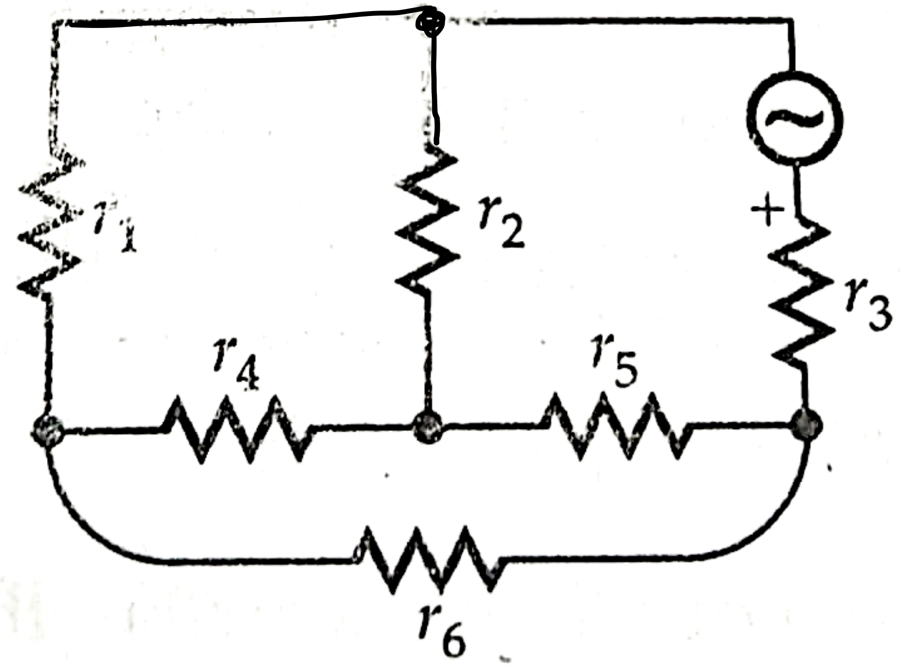
$$i_2 + i_3 = 0 \longrightarrow 1$$

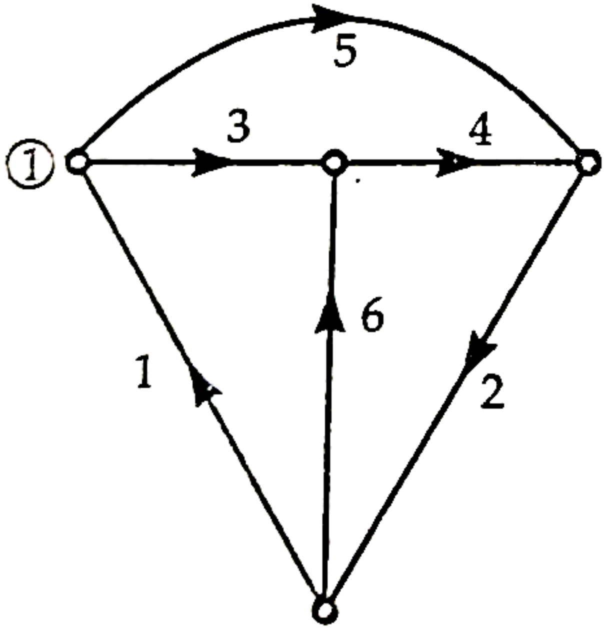
$$i_3 - i_4 + i_5 = 0 \longrightarrow 2$$

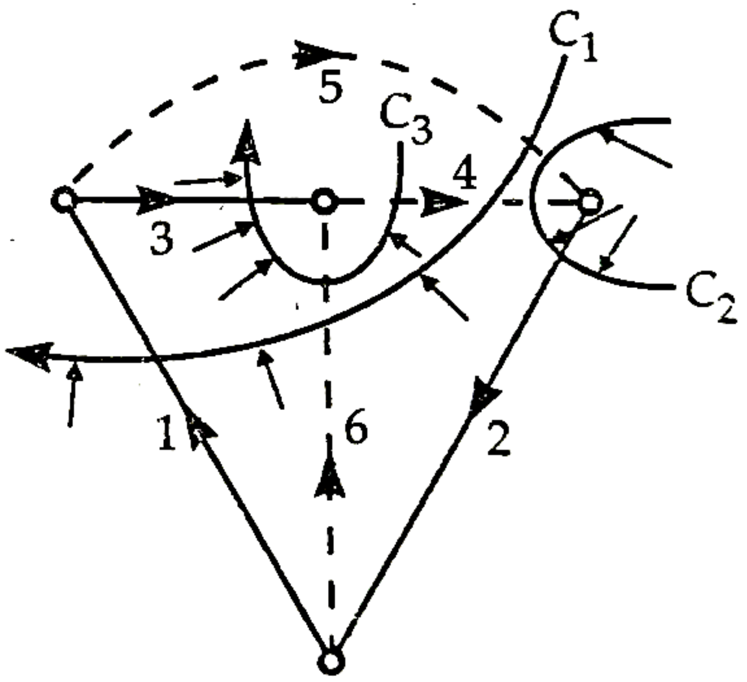
$$i_1 + i_3 - i_4 = 0 \longrightarrow 3$$



Example: Develop the fundamental CUT-SET matrix for the following circuit





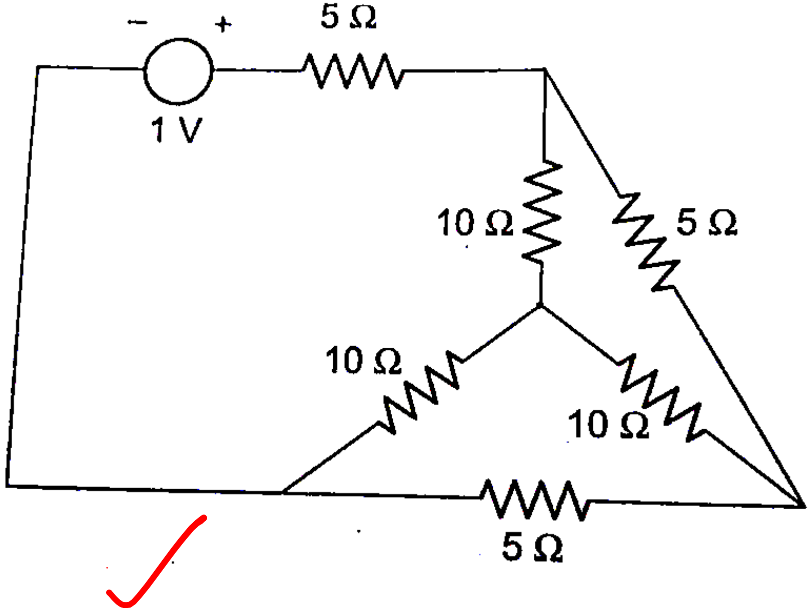


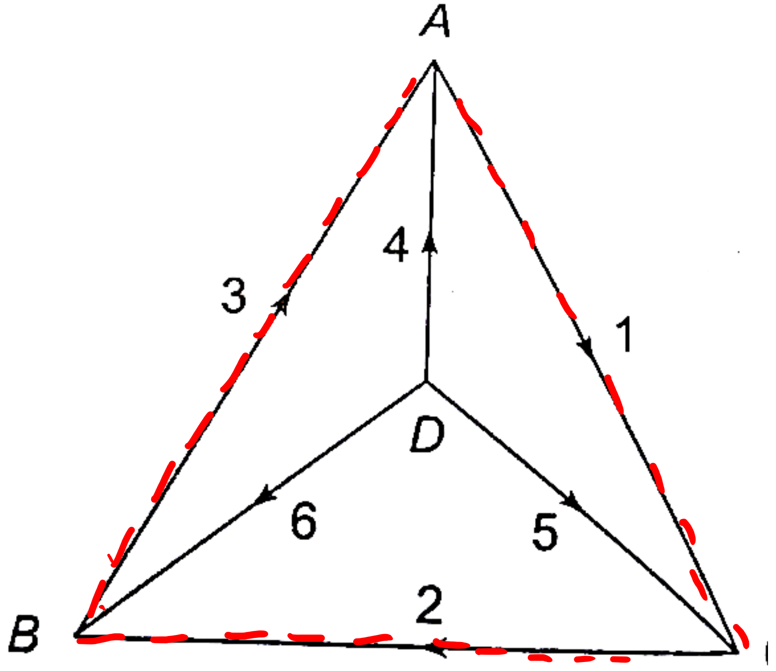
Obtain the current balance equations ?

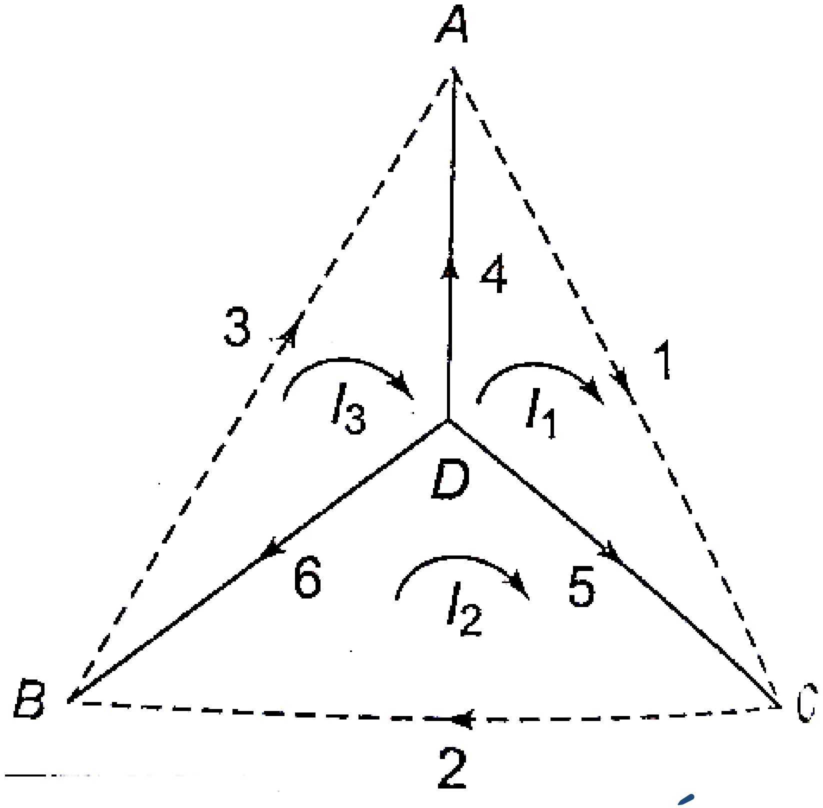
Branches

$$\begin{array}{c c} \textit{Cut-sets} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} C_1\,[1,6,4] \\ C_2\,[2,4,5] \\ C_3\,[3,6,4] \end{array} & \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & +1 & -1 & 0 & +1 \end{bmatrix} \end{array}$$

Example: For the given network, draw the network graph . Select tree branches as 4, 5,6. Write TIE-SET matrix and develop equilibrium equations and calculate the loop currents.







Links ⟶ 1, 2, 3 Branches

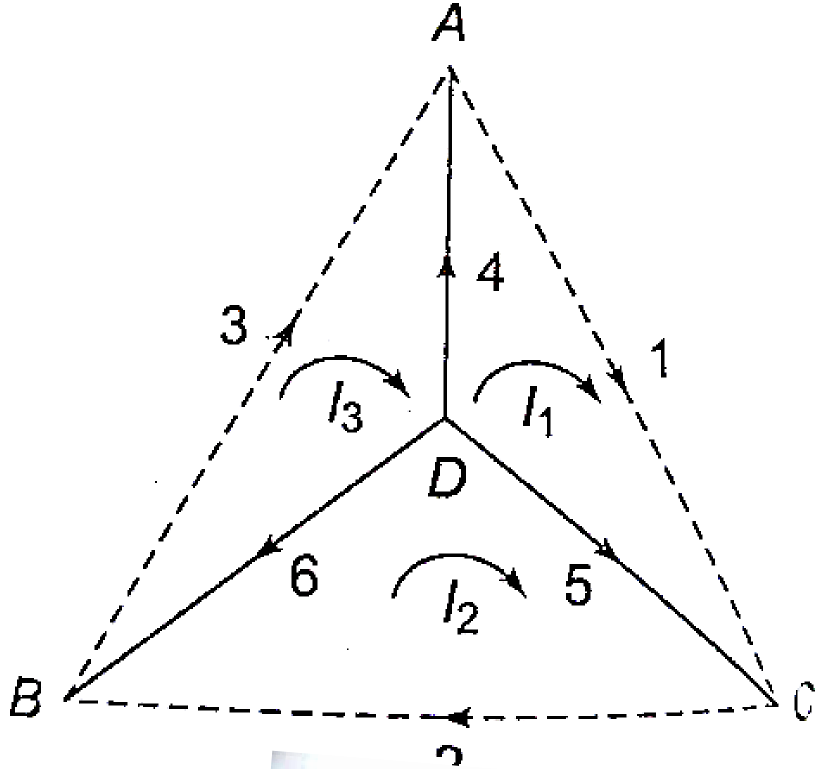
Twigs ⟶ 4, 5, 6 Branches

The directions of I_1 ⟶ direction of link 1

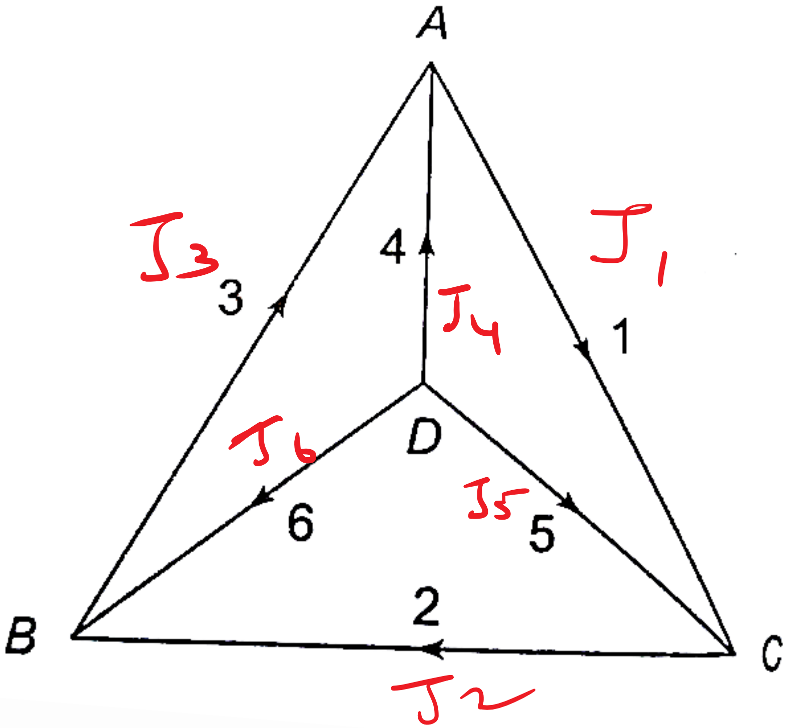
Loop 1 ⟶ Link 1, Twigs 4 & 5

2 ⟶ Link 2, Twigs 5, 6

3 ⟶ Link 3, Twi 6, 4



Branch Currents



$J_1 = I_1$

$J_2 = I_2$

$J_3 = I_3$

$J_4 = I_1 - I_2$

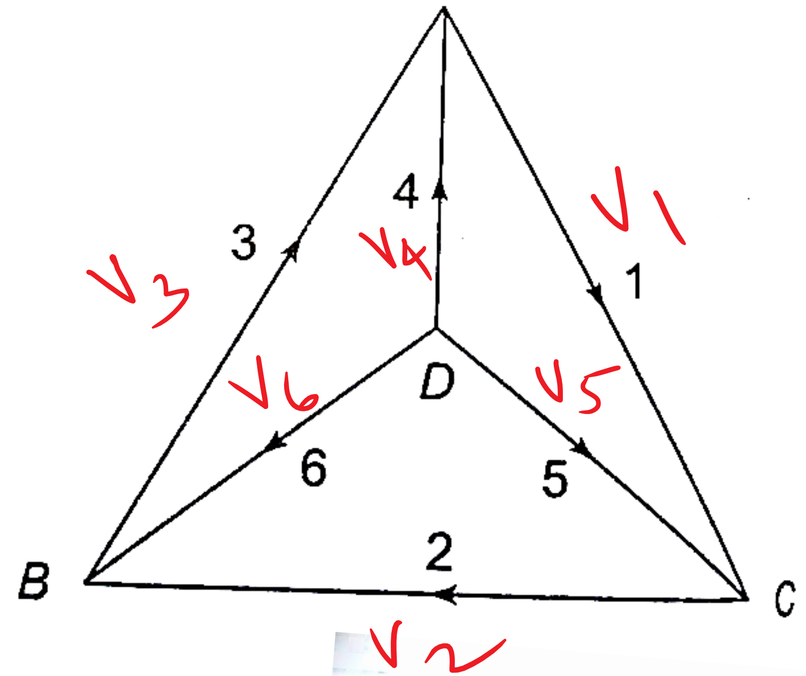
$J_5 = I_2 - I_1$

$J_6 = I_3 - I_2$

Add columns

Link No.	Branch No.					
	1	2	3	4	5	6
$I_1(1, 5, 4)$	+1	0	0	+1	−1	0
$I_2(2, 6, 5)$	0	+1	0	0	+1	−1
$I_3(3, 4, 6)$	0	0	+1	−1	0	+1

Branch Voltage



Add rows

$$V_1 + V_4 - V_5 = 0$$

$$V_2 + V_5 - V_6 = 0$$

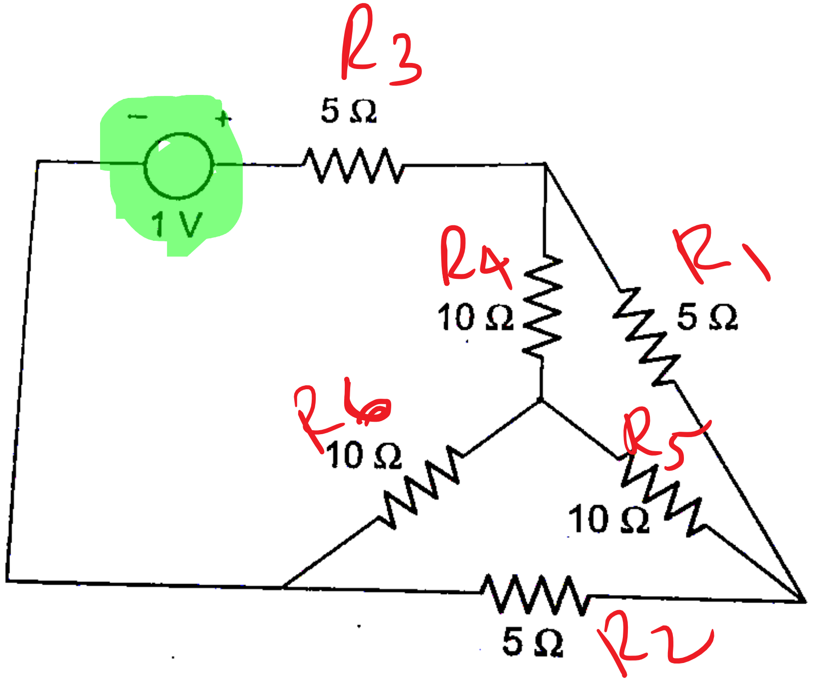
$$V_3 - V_4 + V_6 = 0$$

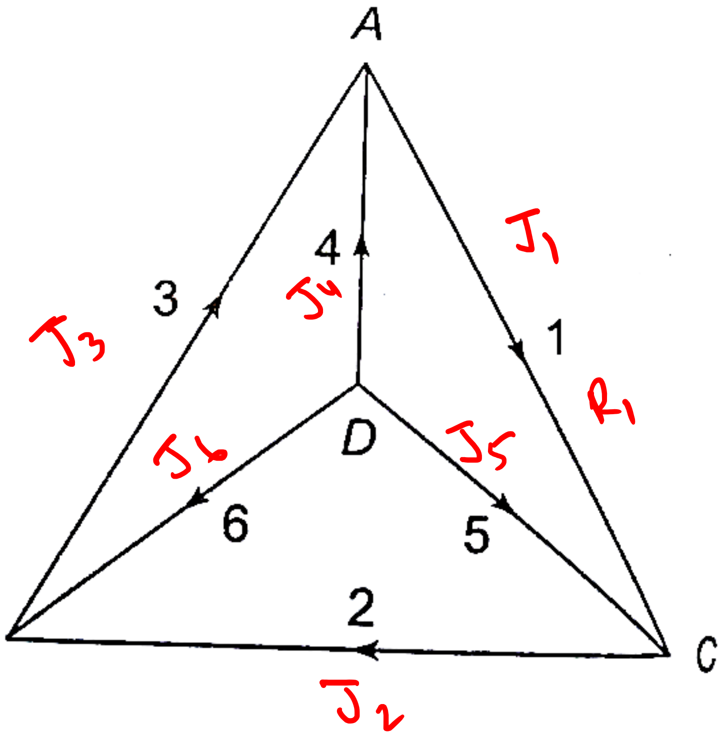
Link No.	Branch No.					
	1	2	3	4	5	6
$I_1(1, 5, 4)$	+1	0	0	+1	−1	0
$I_2(2, 6, 1)$	0	+1	0	0	+1	−1
$I_3(3, 4, 5)$	0	0	+1	−1	0	+1

$J_1, J_2, \ldots J_6$ are the branch currents

Substitute values of Js & Rs

Link No.	Branch No.					
	1	2	3	4	5	6
$I_1(1, 5, 4)$	+1	0	0	+1	−1	0
$I_2(2, 6, \cancel{1}\,5)$	0	+1	0	0	+1	−1
$I_3(3, 4, \cancel{5}\,6)$	0	0	+1	−1	0	+1





$$V_1 = J_1R_1 = R_1I_1 = 5I_1$$

$$V_2 = J_2R_2 = R_2I_2 = 5I_2$$

$$V_3 = J_3R_3 \underline{-1} = 5I_3 - 1$$

$$V_4 = J_4R_4 = (I_1 - I_3)10$$

$$V_5 = J_5R_5 = (I_2 - I_1)10$$

$$V_6 = J_6R_6 = (I_3 - I_2)10$$

$$5I_1 - 10(I_2 - I_1) + 10(I_1 - I_3) = 0$$

$$25I_1 - 10I_2 - 10I_3 = 0$$

$$10(I_2 - I_1) + 5I_2 - 10(I_3 - I_2) = 0$$

$$-10I_1 + 25I_2 - 10I_3 = 0$$

$$(5I_3 - 1) - 10(I_2 - I_3) + 10(I_3 - I_2) = 0$$

$$-10I_1 + 10I_2 - 25I_3 = 1$$

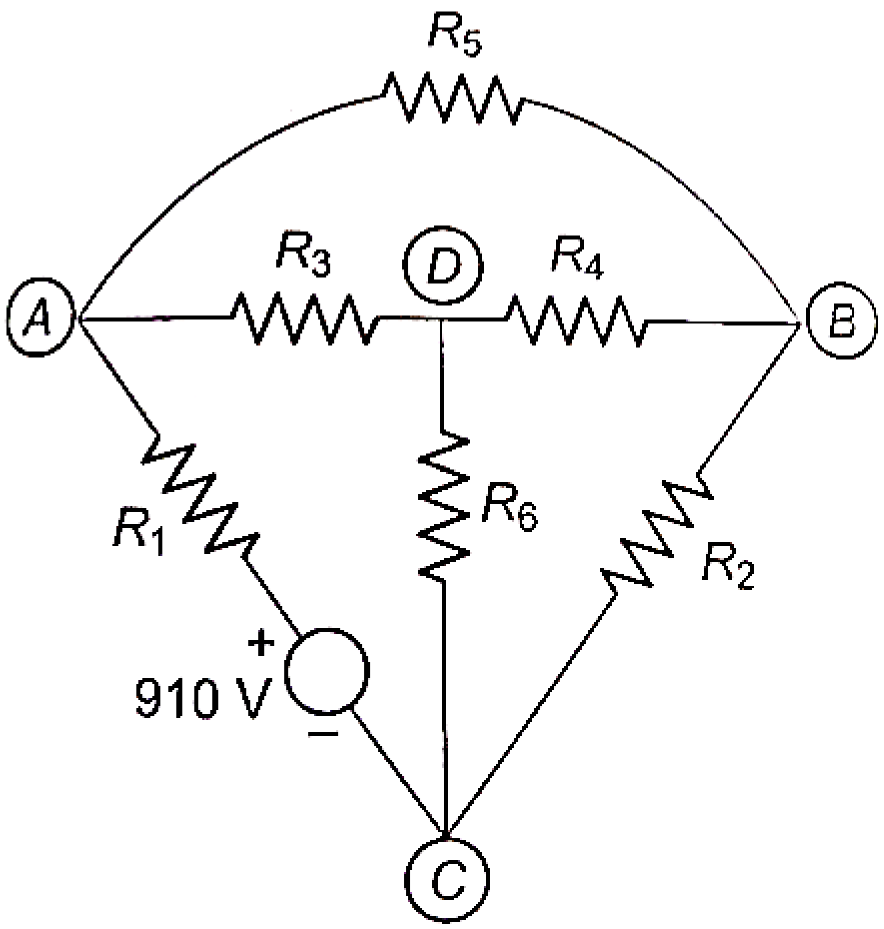
$$\begin{bmatrix} 25 & -10 & -10 \\ -10 & 25 & -10 \\ -10 & -10 & 25 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

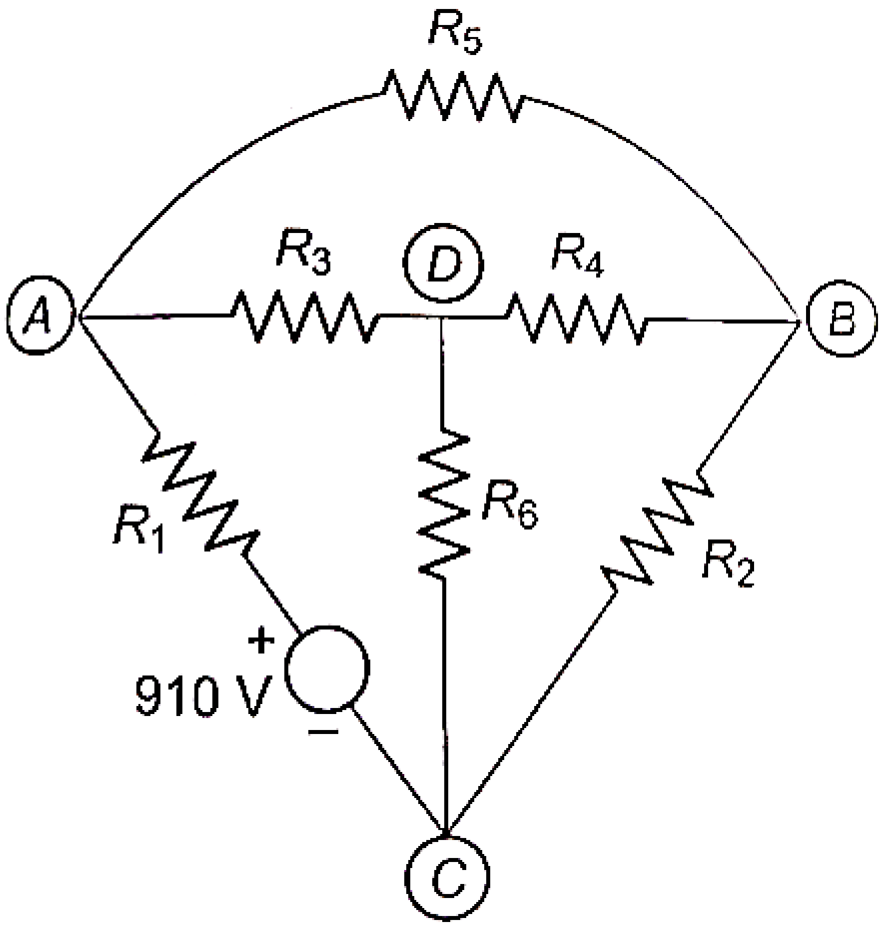
$$I_1 = \frac{1}{17.5} = 0.057 \text{ A}$$

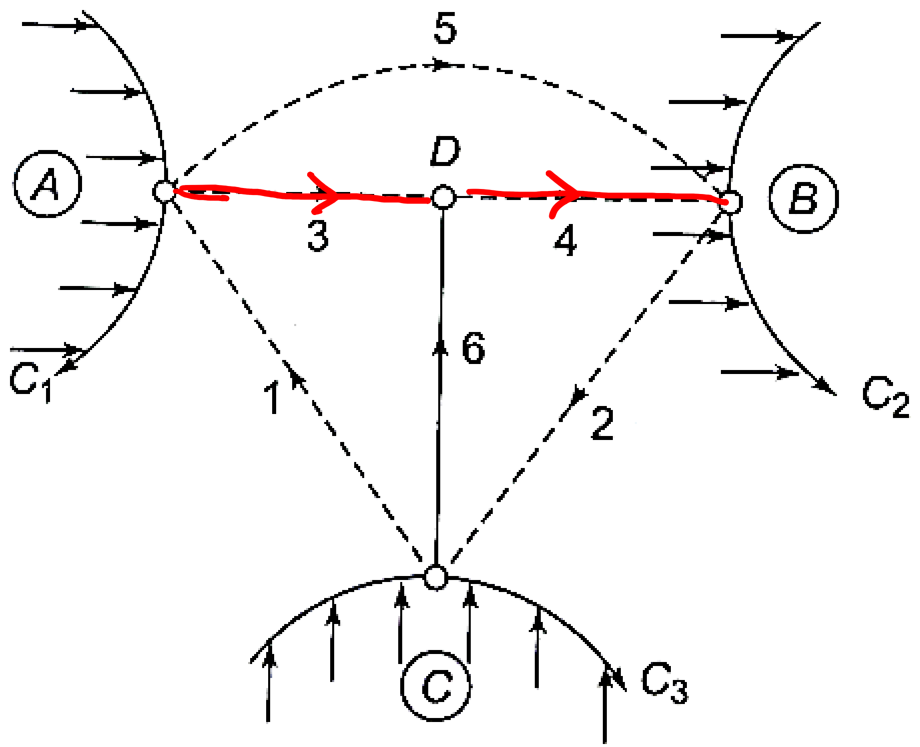
$$I_2 = \frac{1}{17.5} = 0.057 \text{ A}$$

$$I_3 = \frac{3}{35} = 0.0857 \text{ A}$$

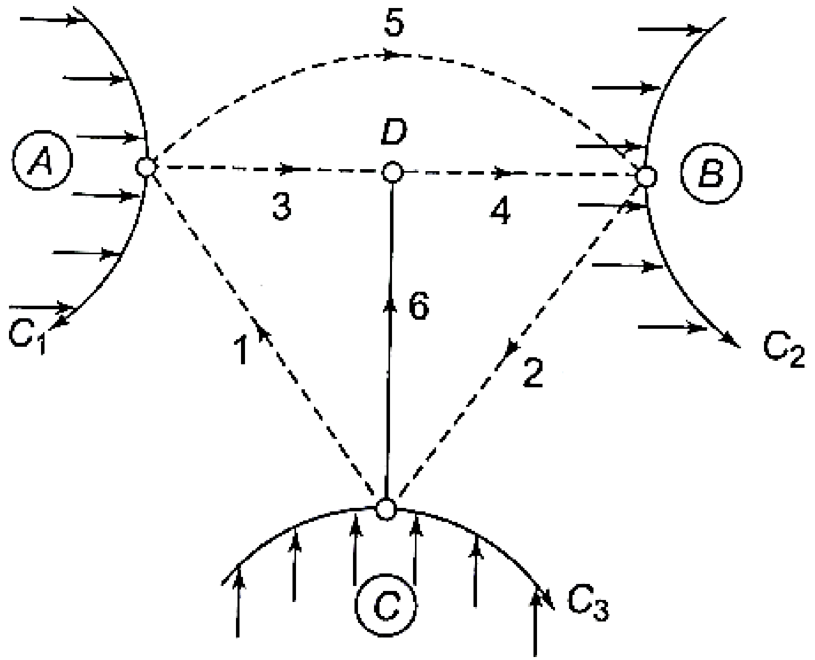
Example: Write the cut-set matrix and equilibrium equation on voltage basis. Hence obtain the values of branch voltages and branch currents. Given, R1=R2= 5 Ohm and R3=R4=R5= 10 Ohm



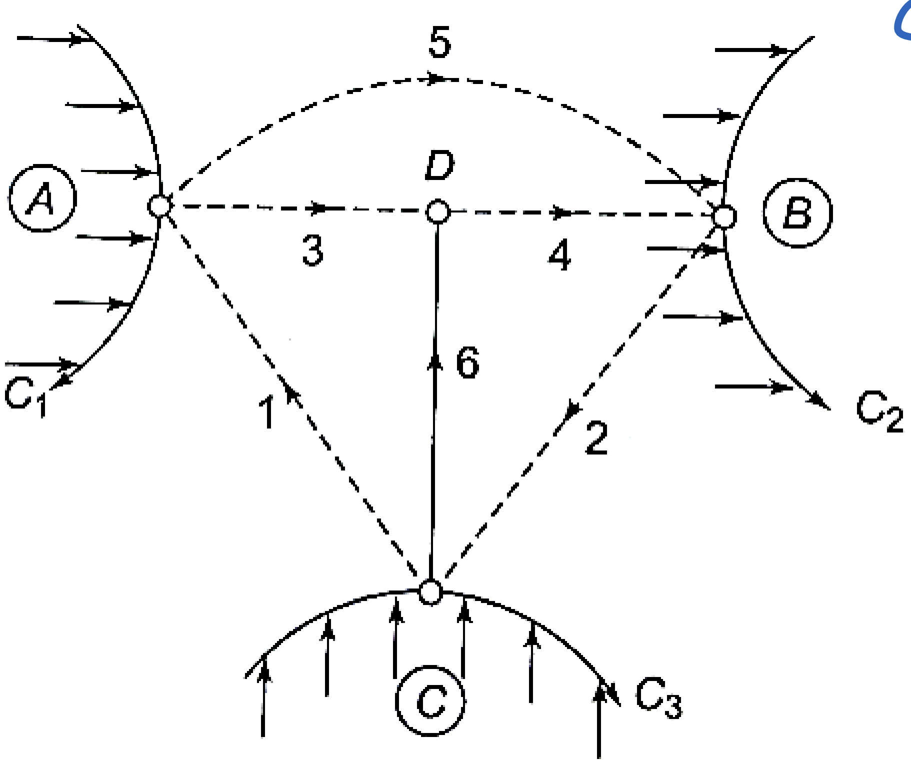




Tree branch	Basic cut-sets
3	1, 3, 5
4	2, 4, 5
6	1, 2, 6



Basic cut-set tree branch	Elements					
	1	2	3	4	5	6
3 (1, 3, 5)	–1	0	+1	0	+1	0
4 (2, 4, 5)	0	–1	0	+1	+1	0
6 (1, 2, 6)	+1	–1	0	0	0	+1



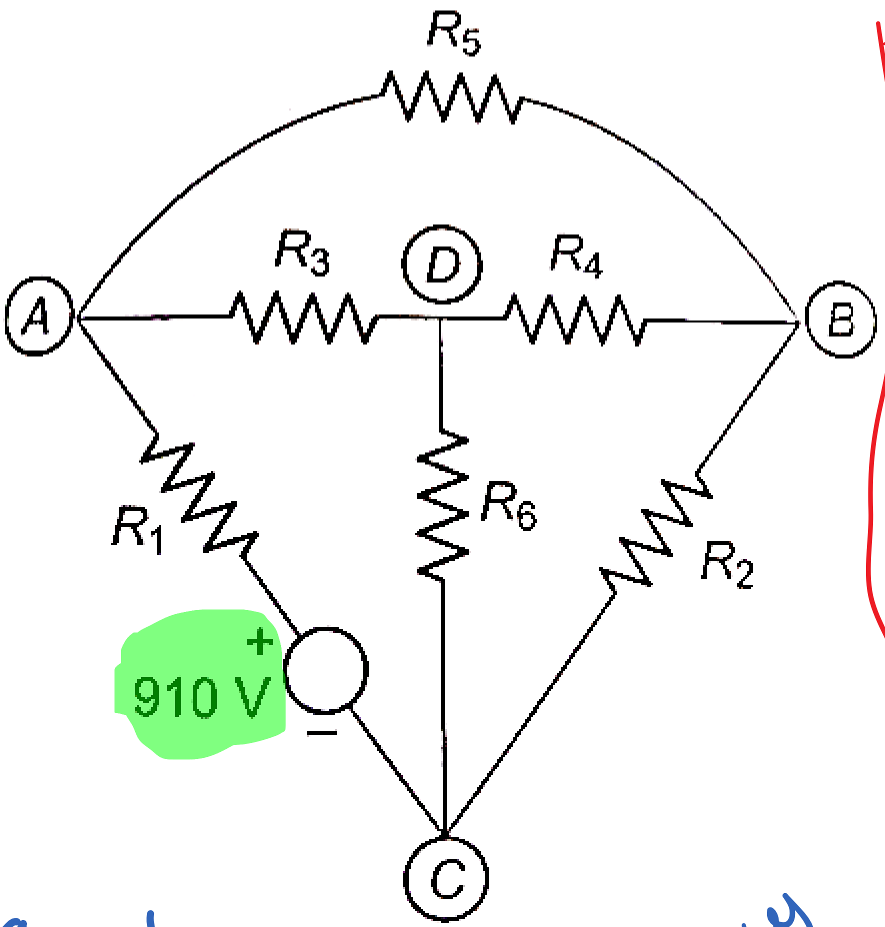
columns give branch voltages

$$
\boxed{
\begin{aligned}
e_1 &= e_6 - e_3 \\
e_2 &= -e_4 - e_6 \\
e_3 &= e_3 \\
e_4 &= e_4 \\
e_5 &= e_3 + e_4 \\
e_6 &= e_6
\end{aligned}
}
$$

rows give nodal Eqns

$$
\boxed{
\left.
\begin{aligned}
-j_1 + j_3 + j_5 &= 0 \\
-j_2 + j_4 + j_5 &= 0 \\
+j_1 - j_2 + j_6 &= 0
\end{aligned}
\right\}
}
$$

Basic cut-set tree branch	Elements					
	e_1 1	e_2 2	e_3 3	e_4 4	e_5 5	e_6 6
3 (1, 3, 5) e_3	j_1 –1	j_2 0	j_3 +1	j_4 0	j_5 +1	j_6 0
4 (2, 4, 5) e_4	0	–1	0	+1	+1	0
6 (1, 2, 6) e_6	+1	–1	0	0	0	+1



$$e_1 = e_6 - e_3$$
$$e_2 = -e_4 - e_6$$
$$e_3 = e_3$$
$$e_4 = e_4$$
$$e_5 = e_3 + e_4$$
$$e_6 = e_6$$

Branch current (J) = $\frac{\text{Branch voltage}}{\text{Branch resistanc}}$

$$j_1 = g_1\,(e_1 - 910) = 0.2\,(e_6 - e_3 - 910)$$
$$j_2 = g_2\,(e_2) = 0.2\,(-e_4 - e_6)$$
$$j_3 = g_3\,(e_3) = 0.1e_3$$
$$j_4 = g_4\,(e_4) = 0.1e_4$$
$$j5 = g_5(e_5) = 0.5\,(e_3 + e_4)$$
$$j_6 = g_6\,(e_6) = 0.1e_6$$

Include the voltage source in branch 1 to obtain branch voltage of the graph

$g_1 = 1/R_1$
$g_2 = 1/R_2$
$\vdots$
$g_6 = 1/R_3$
Conductancy

$$\left.\begin{aligned} -j_1 + j_3 + j_5 &= 0 \\ -j_2 + j_4 + j_5 &= 0 \\ +j_1 - j_2 + j_6 &= 0 \end{aligned}\right\}$$

Substitute branch currents ($j_1, j_2, j_3 \ldots$) in Nodal eqns

Check the steps

$$-0.2\,(e_6 - e_3 - 910) + 0.1e_3 + 0.5\,(e_3 + e_4) = 0$$

$$0.8e_3 + 0.5e_4 - 0.2e_6 = -182$$

$$0.2\,(e_4 + e_6) + 0.1e_4 + 0.5\,(e_3 + e_4) = 0$$

$$0.5e_3 + 0.8e_4 + 0.2e_6 = 0$$

$$0.2\,(e_6 - e_3 - 910) + 0.2\,(e_4 + e_6) + 0.1e_6 = 0$$

$$-0.2e_3 + 0.2e_4 + 0.5e_6 = 182$$

$$\begin{bmatrix} 0.8 & 0.5 & -0.2 \\ 0.5 & 0.8 & 0.2 \\ -0.2 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \\ e_6 \end{bmatrix} = \begin{bmatrix} -182 \\ 0 \\ 182 \end{bmatrix}$$

Use Cramer's rule

Solving above matrix for e_3, e_4, and e_6:

$$e_3 = -200 \text{ V}, \; e_4 = 60 \text{ V}, \; e_6 = 260 \text{ V}$$

The branch voltages and branch currents are:

Solve for branch voltages

$$e_1 = e_6 - e_3 = 460 \text{ V}$$

$$e_2 = -e_4 - e_6 = -320 \text{ V}$$

$$e_3 = e_3 = -200 \text{ V}$$

$$e_4 = 60 \text{ V}$$

$$e_5 = e_3 + e_4 = -140 \text{ V}$$

$$e_6 = 260 \text{ V}$$

Solve for branch currents

Branch currents are:

$$j_1 = 0.2\,(460 - 910) = -90\ \text{A}$$

$$j_2 = 0.2\,(-320) = -64\ \text{A}$$

$$j_3 = 0.1\,(-200) = -20\ \text{A}$$

$$j_4 = 0.1\,(60) = 6\ \text{A}$$

$$j_5 = 0.5\,(-140) = -70\ \text{A}$$

$$j_6 = 0.1\,(260) = 26\ \text{A}$$