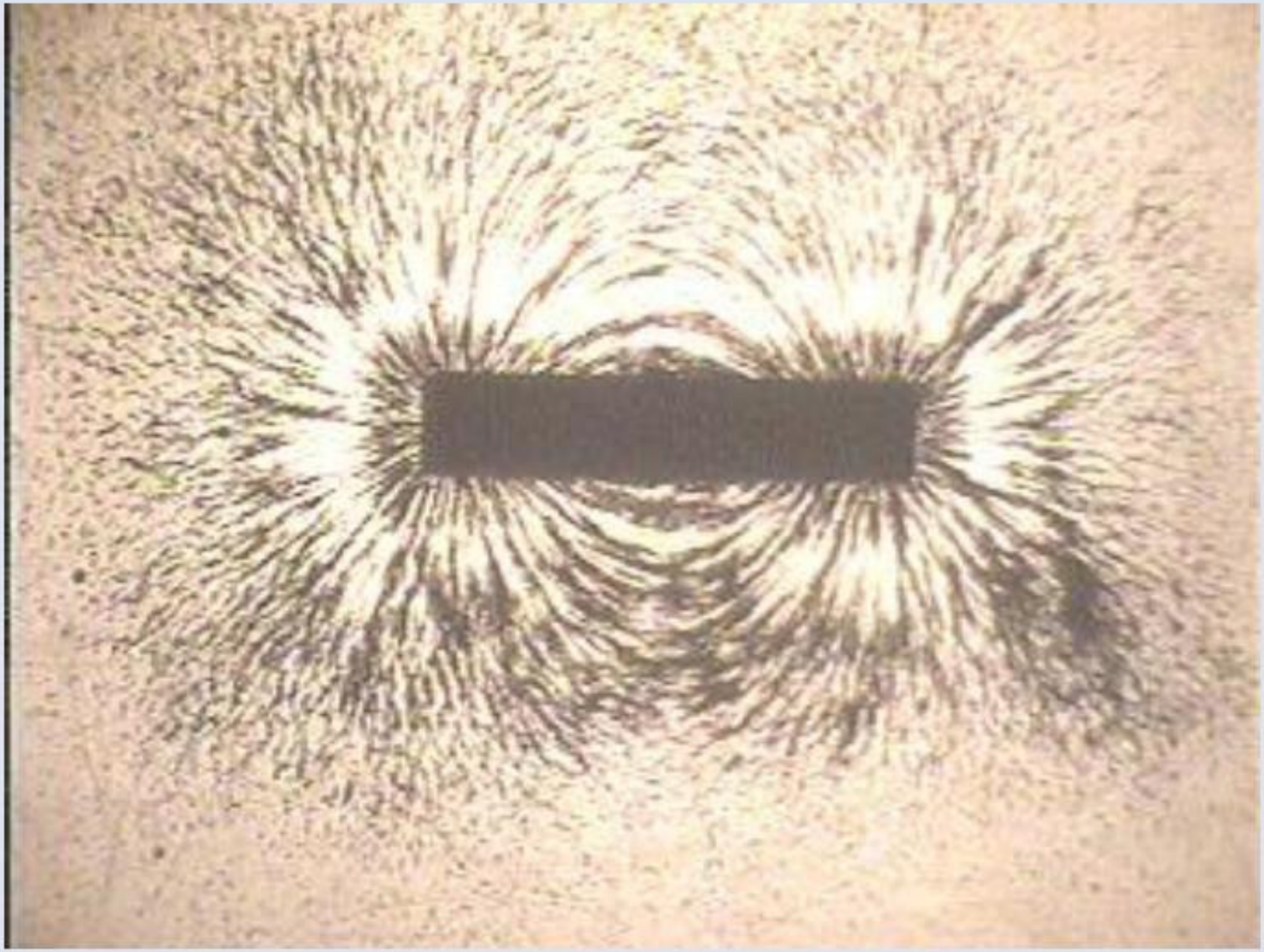
Magnetostatics:

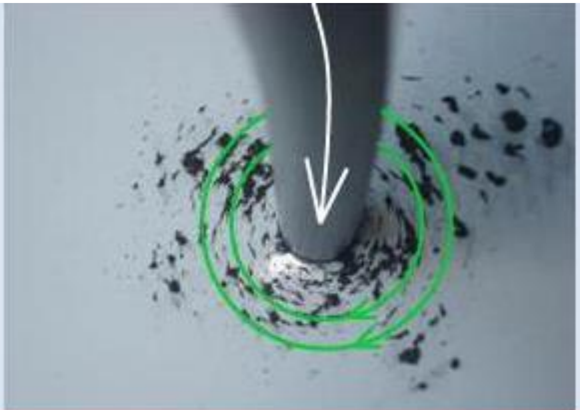
Biot-Savart's Law, Ampere's Circuit Law and Applications, Magnetic Flux Density, Magnetic Scalar and Vector Potentials, Forces due to Magnetic Fields, Magnetic Dipole, Magnetization in materials, Boundary conditions, Inductances and Magnetic Energy.

Introduction

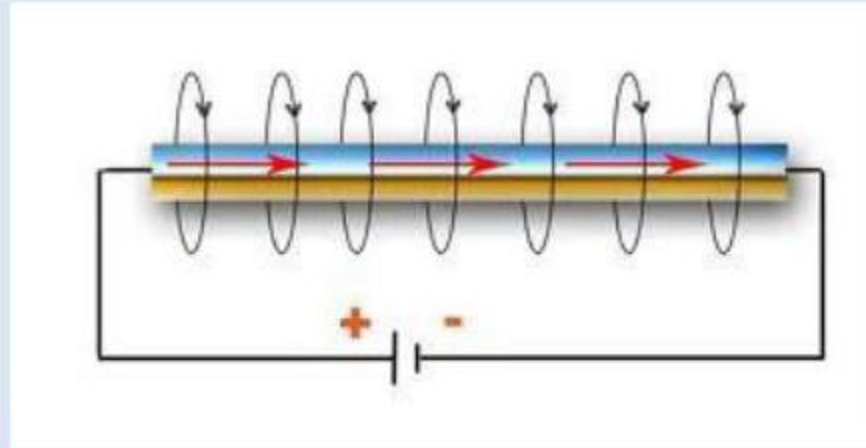
- static electric fields characterized by **E** or **D** (**D**=ε**E**) were discussed.
- considers static magnetic fields, characterised by **H** or **B** (**B**=μ**H**).
- As we have noticed, a distribution of ***static*** or ***stationary*** **charges** produces static electric field.
- If the charges move at a constant rate (**direct current- DC**), a static magnetic field is produced (**magnetostatic field**).
- Static magnetic field are also produced by **stationary permanent magnets**.

Magnet and Magnetic Field





The iron filings form circles around the **wire** along the **magnetic field**



Magnetic Field Around **Current** Carrying **Wires**

A compass needle is deflected by the **direct current** flowing in a conductor

Electrostatic Fields have dual equations for magnetostatic fields

Term	Electric	Magnetic
Basic laws	$\mathbf{F} = \dfrac{Q_1 Q_2}{4\pi\varepsilon_r^2}\mathbf{a}_r$	$d\mathbf{B} = \dfrac{\mu_o I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2}$
	$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}$
Force law	$\mathbf{F} = Q\mathbf{E}$	$\mathbf{F} = Q\mathbf{u} \times \mathbf{B}$
Source element	dQ	$Q\mathbf{u} = I\, d\mathbf{l}$
Field intensity	$E = \dfrac{V}{\ell}$ (V/m)	$H = \dfrac{I}{\ell}$ (A/m)
Flux density	$\mathbf{D} = \dfrac{\Psi}{S}$ (C/m^2)	$\mathbf{B} = \dfrac{\Psi}{S}$ (Wb/m^2)
Relationship between fields	$\mathbf{D} = \varepsilon\mathbf{E}$	$\mathbf{B} = \mu\mathbf{H}$
Potentials	$\mathbf{E} = -\nabla V$	$\mathbf{H} = -\nabla V_m\ (\mathbf{J} = 0)$
	$V = \displaystyle\int \dfrac{\rho_L dl}{4\pi\varepsilon r}$	$\mathbf{A} = \displaystyle\int \dfrac{\mu\, I\, d\mathbf{l}}{4\pi R}$
Flux	$\Psi = \int \mathbf{D} \cdot d\mathbf{S}$	$\Psi = \int \mathbf{B} \cdot d\mathbf{S}$
	$\Psi = Q = CV$	$\Psi = LI$
	$I = C\dfrac{dV}{dt}$	$V = L\dfrac{dI}{dt}$
Energy density	$w_E = \dfrac{1}{2}\mathbf{D} \cdot \mathbf{E}$	$w_m = \dfrac{1}{2}\mathbf{B} \cdot \mathbf{H}$
Poisson's equation	$\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$	$\nabla^2 \mathbf{A} = -\mu\mathbf{J}$

Biot- Savart Law

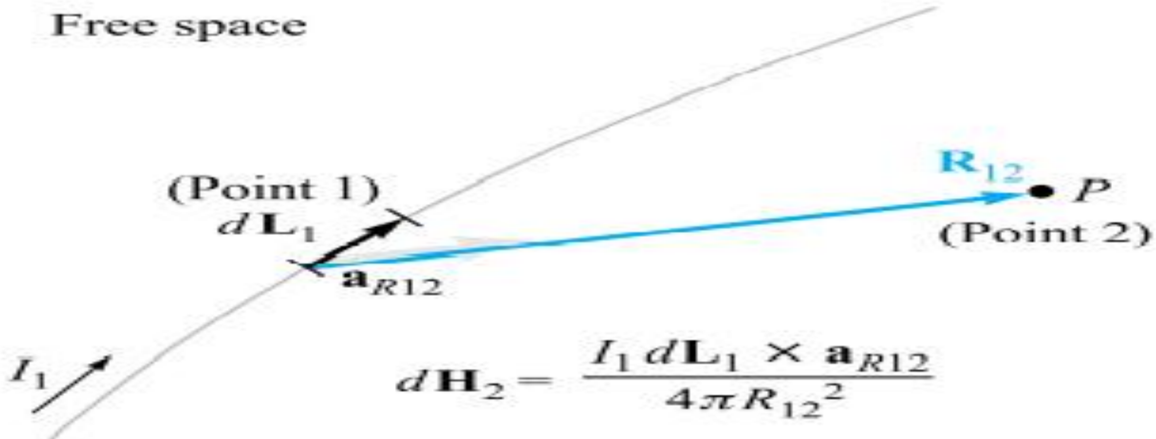
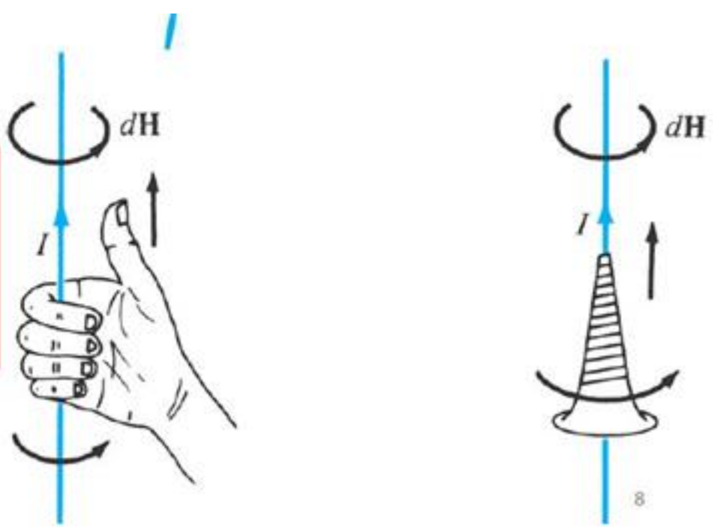
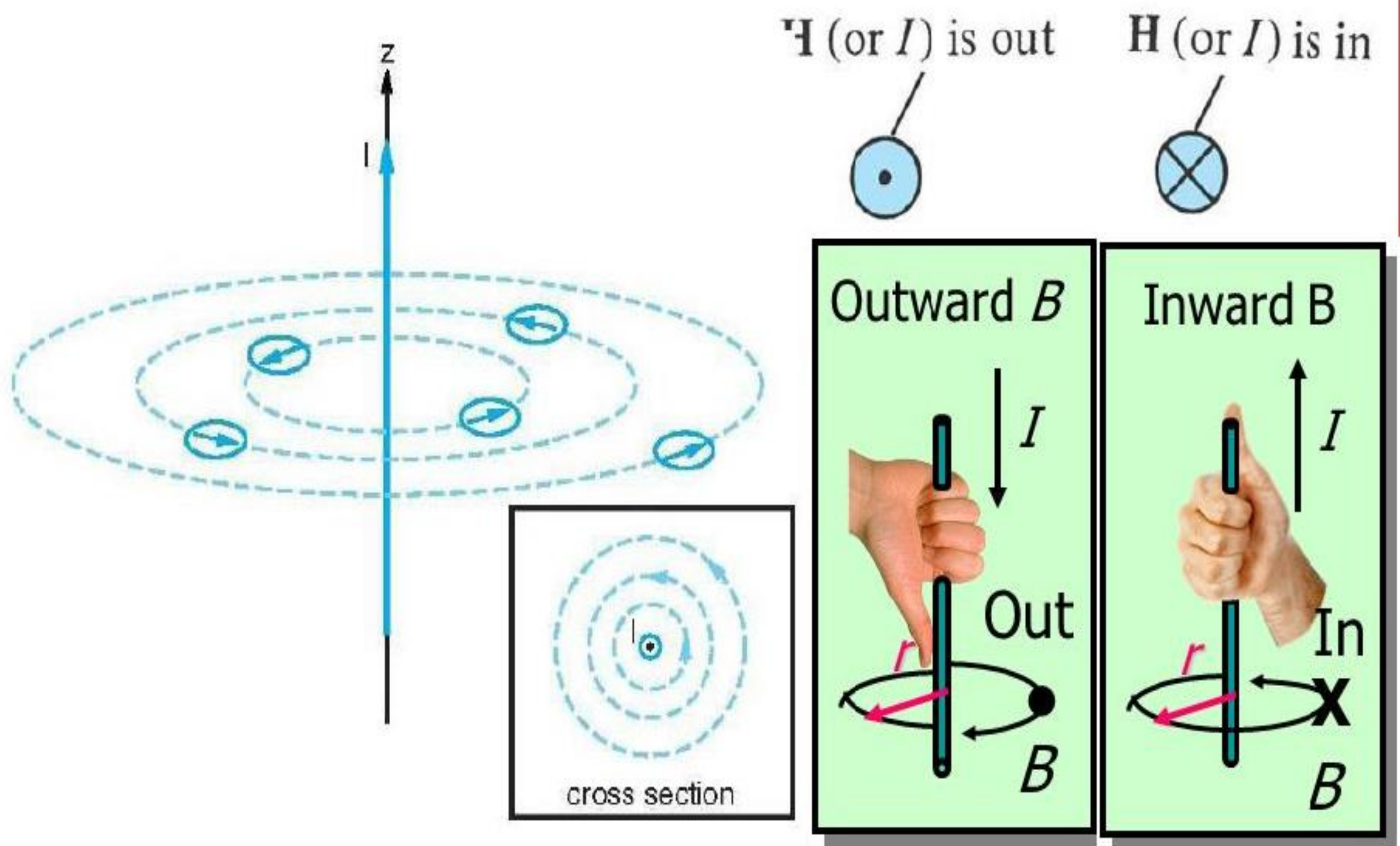


Figure 7.1 The law of Biot-Savart expresses the magnetic field intensity dH_2 produced by a differential current element $I_1 dL_1$. The direction of dH_2 is into the page.

The direction of *dH* can be determined by the right-hand rule or right-handed screw rule.



The direction of magnetic field intensity **H** (or current **I**) can be represented by a small circle with a dot or cross sign depending on whether **H** (or **I**) is out of, or into the page.



Biot-Savart's Law

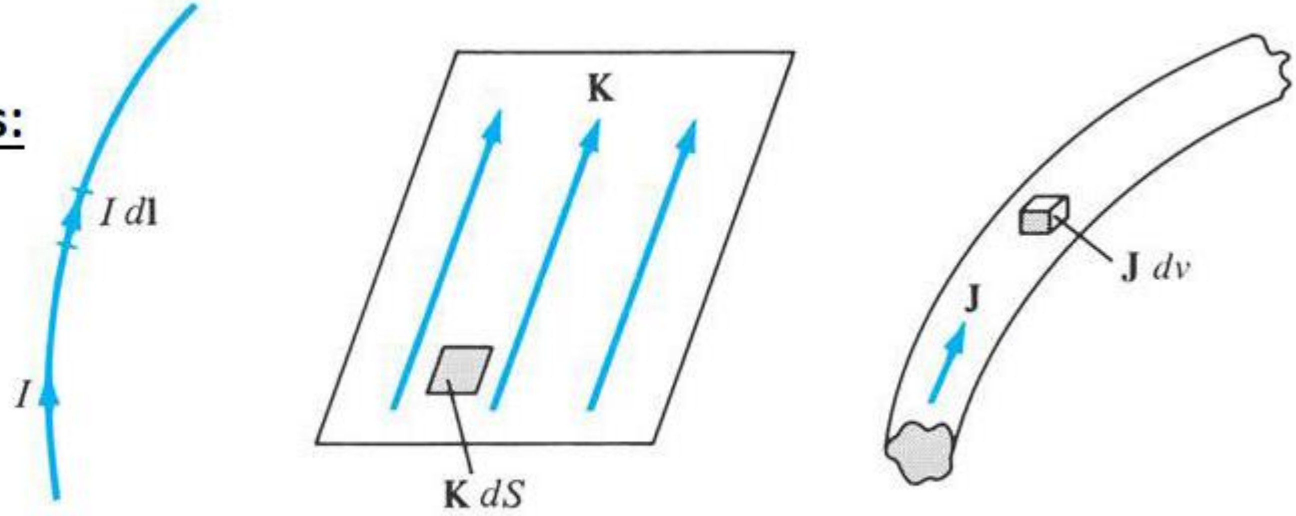
For different current distributions:

Current distributions:

(a) line current,

(b) surface current,

(c) volume current.



(a) (b) (c)

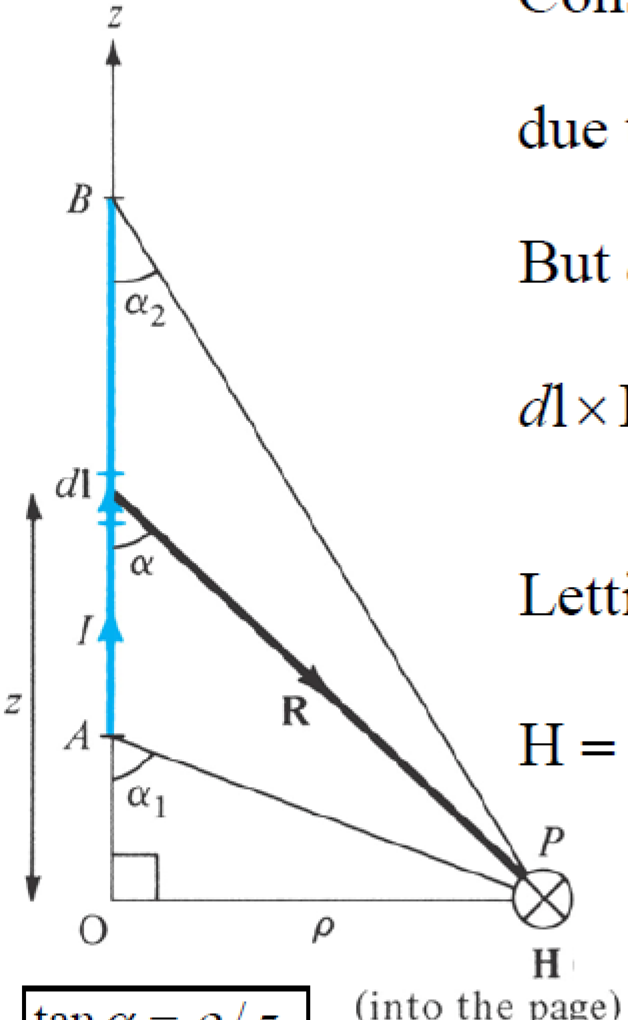
$$H = \int_L \frac{I\, d\mathrm{l} \times a_R}{4\pi R^2} \quad \text{(line current)}$$

$$H = \int_S \frac{\mathrm{K}\, d\mathrm{S} \times a_R}{4\pi R^2} \quad \text{(surface current)} \quad \text{(K: surface current density)}$$

$$H = \int_v \frac{\mathrm{J}\, dv \times a_R}{4\pi R^2} \quad \text{(volume current)} \quad \text{(J: volume current density)}$$

Magnetic Field of straight Conductor

Consider conductor of finite length AB, carrying current from point A to point B.



$\tan\alpha = \rho / z$
$\rightarrow z = \rho\cot\alpha$

$1+\cot^2 x = \mathrm{cosec}^2 x$

Consider the contribution dH at P

due to dl at (0,0,z): $d\mathbf{H} = \dfrac{I\; d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$,

But $d\mathbf{l} = dz\; a_z$ and $\mathbf{R} = \rho \mathbf{a}_\rho - z\mathbf{a}_z$, *so*,

$d\mathbf{l} \times \mathbf{R} = \rho dz\; \mathbf{a}_\phi$, Hence $\mathbf{H} = \int \dfrac{I \rho dz}{4\pi\left[\rho^2 + z^2\right]^{3/2}} \mathbf{a}_\phi$

Letting $z = \rho\cot\alpha$, $dz = -\rho\,\mathrm{cosec}^2\alpha\, d\alpha$,

$$\mathbf{H} = -\frac{I}{4\pi}\int_{\alpha 1}^{\alpha 2} \frac{\rho^2 \mathrm{cosec}^2\alpha\, d\alpha}{\rho^3 \mathrm{cosec}^3\alpha} a_\phi = -\frac{I}{4\pi\rho} a_\phi \int_{\alpha 1}^{\alpha 2} \sin\alpha\, d\alpha$$

or $$\boxed{\mathbf{H} = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right) a_\phi}$$

$$\boxed{H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a_\phi}$$

When the conductor is semi-infinite, so that point A is now at O(0,0,0) while B is at $(0,0,\infty)$, $\alpha_1 = 90^\circ$, $\alpha_2 = 0^\circ$:

$$\boxed{H = \frac{I}{4\pi\rho} a_\phi}$$

When the conductor is of infinite length, point A is at $(0,0,-\infty)$ while B is at $(0,0,\infty)$, $\alpha_1 = 180^\circ$, $\alpha_2 = 0^\circ$:

$$\boxed{H = \frac{I}{2\pi\rho} a_\phi}$$

A simple approach to determine a_ϕ :

$$\boxed{a_\phi = a_l \times a_\rho}$$

where a_l is is a unit vector along the line current, and a_ρ is a unit vector along the perpendicular line from the line current to the field point.

D7.1. Given the following values for P_1, P_2, and $I_1 \Delta L_1$, calculate ΔH_2:

(a) $P_1(0, 0, 2)$, $P_2(4, 2, 0)$, $2\pi \mathbf{a}_z \mu A \cdot m$;

(b) $P_1(0, 2, 0)$, $P_2(4, 2, 3)$, $2\pi \mathbf{a}_z \mu A \cdot m$;

(c) $P_1(1, 2, 3)$, $P_2(-3, -1, 2)$, $2\pi(-\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)\ \mu A \cdot m$.

Ans. $-8.51\mathbf{a}_x + 17.01\mathbf{a}_y$ nA/m; $16\mathbf{a}_y$ nA/m; $18.9\mathbf{a}_x - 33.9\mathbf{a}_y + 26.4\mathbf{a}_z$ nA/m

D7.2. A current filament carrying 15 A in the $\mathbf{a}_z$ direction lies along the entire z axis. Find **H** in rectangular coordinates at:

(a) $P_A(\sqrt{20}, 0, 4)$;

(b) $P_B(2, -4, 4)$.

Ans. $0.534\mathbf{a}_y$ A/m; $0.477\mathbf{a}_x + 0.239\mathbf{a}_y$ A/m