Module 5:
Transmission Lines: Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Expressions for Characteristic Impedance, Propagation Constant, Phase velocity, input impedance, Reflection Coefficient, VSWR. Characterization of lossless, low loss and distortionless transmission lines. Significance of short circuit and open circuit lines of length λ/8, λ/4, λ/2
Coaxial line, Planar transmission lines –Types, Microstrip Lines: field distribution, design equations, Q factor, losses in microstrip lines.

Definition

- A transmission line is the conductive connection between system elements that carry signal power

- A **transmission line** is a specialized cable designed to carry alternating current of radio frequency.

- Transmission lines are used for connecting radio transmitters and receivers with their antennas, distributing cable television signals, and computer network connections.

Types of Transmission Lines

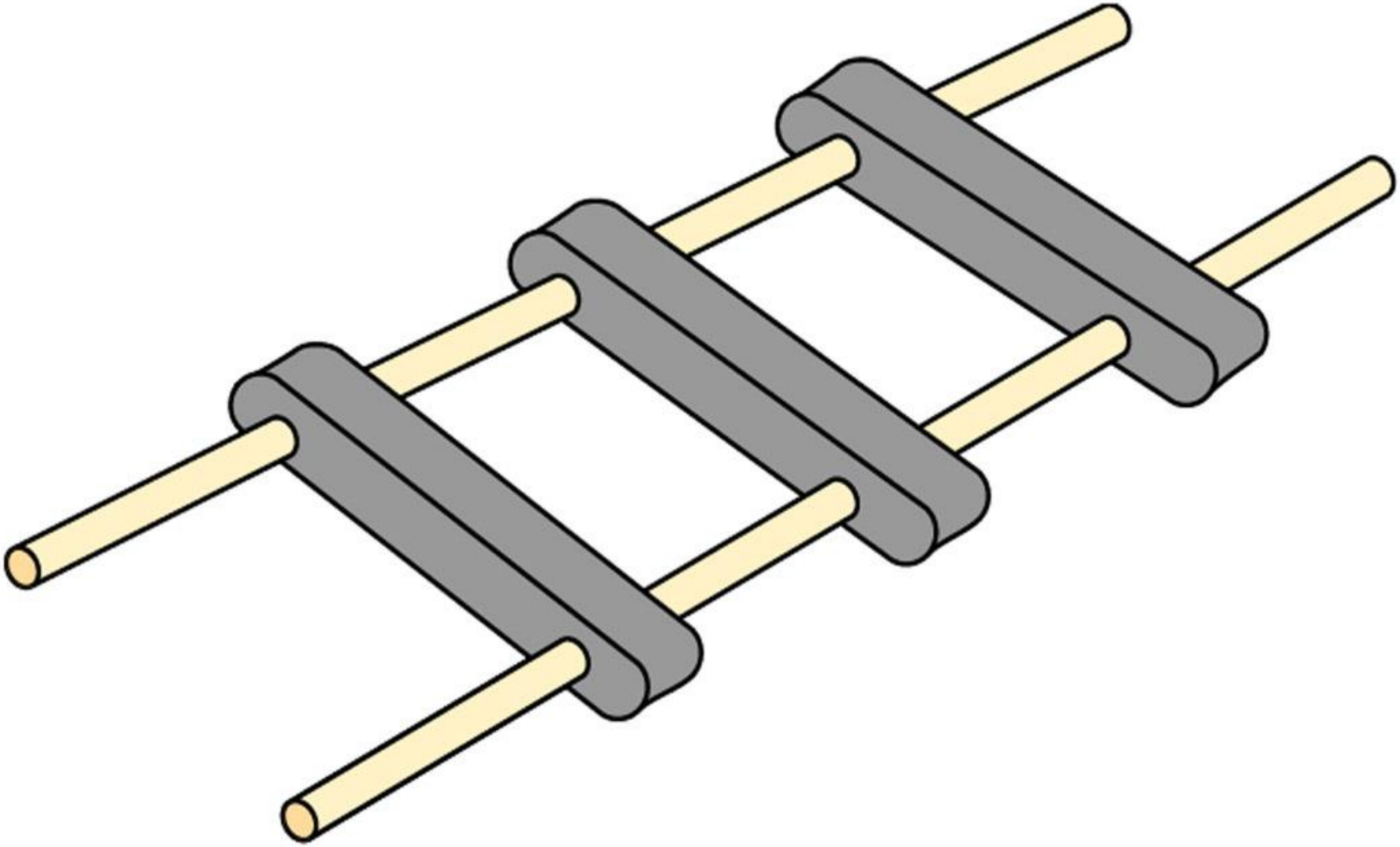
Types

- Two-wire open line
- Twisted pair
- Shielded Pair
- Coaxial Lines
- Striplines and Microstrip Lines

Two-Wire Open

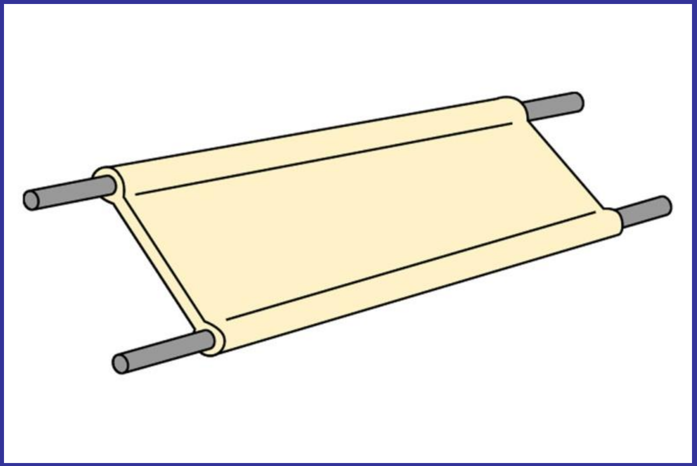
- Also called twin lead or two wire ribbon cable
- Usually spaced from ¼ to 6 inches apart
- Used as the transmission line from antenna to receiver or antenna to transmitter
- The end connected to the source is called the generator or input end
- The end connected to the load is the load or output end.

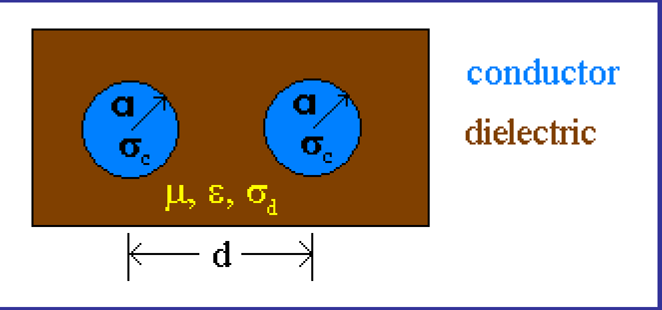
FIGURE -1 Parallel two-wire line.



Two-wire ribbon-type lines.

- Basic Construction
  - Two parallel circular conductors of equal radius and conductivity enclosed in a plastic insulating material





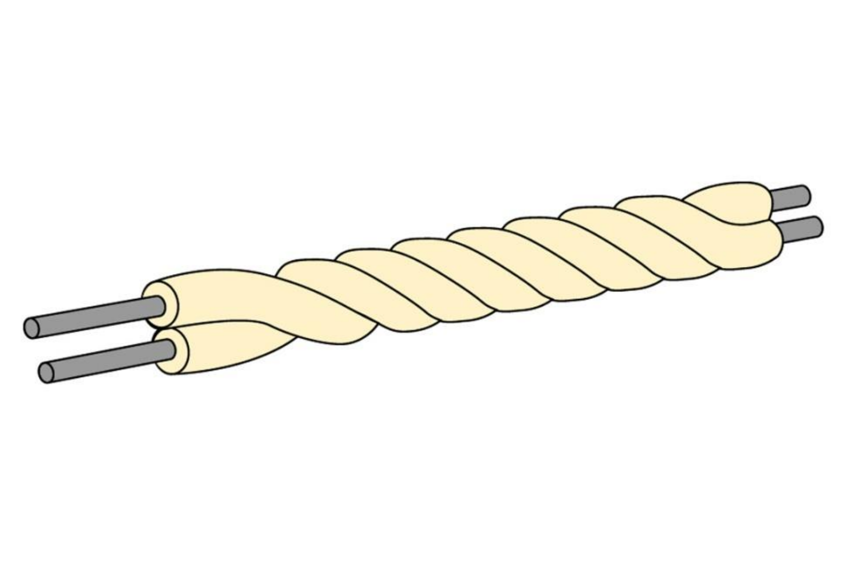
Applications of Two-wire Line

- Carry low level signals from antenna over a short run to a TV or FM Rx

- Connections in regular telephone networks

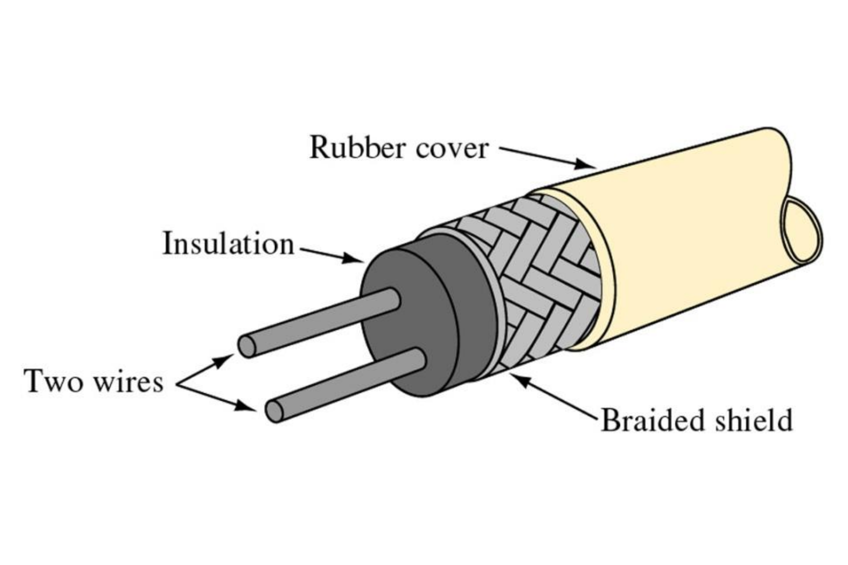
What is Twisted Pair?

- Two insulated wires twisted to form a flexible line without the use of spacers.
- It is not used for high frequencies because of the high losses that occur in the insulation
- Wet lines increase losses dramatically



What is Shielded Pair?

- Consists of parallel conductors separated from each other and surrounded by a solid dielectric.
- Conductors are contained within a copper braid tubing that acts like a shield
- Assembly is covered by a rubber coating for protection from elements and mechanical damage



Advantage of Shielded Pair

- Principal advantage is that the conductors are balanced to ground
  - The capacitance between the cables is uniform through out the line
  - Copper braid shield isolates the conductors from external noise and prevents the signal on the shielded pair cable from radiating to and interfering with other systems

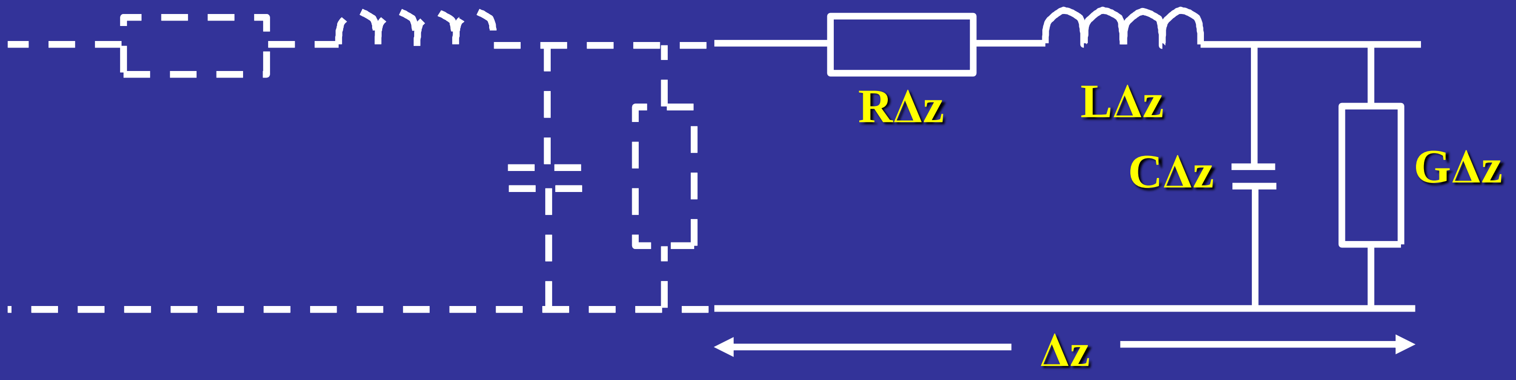
Transmission line theory

- Key difference between circuit theory and transmission line theory - Electrical size
- Circuit theory assumes that the physical dimensions of a network are much smaller than the electrical wavelength
- Transmission lines may be a considerable fraction of a wavelength, or many wavelengths, in size
- Transmission line is a distributed parameter network, where voltages and currents can vary in magnitude and phase over its length

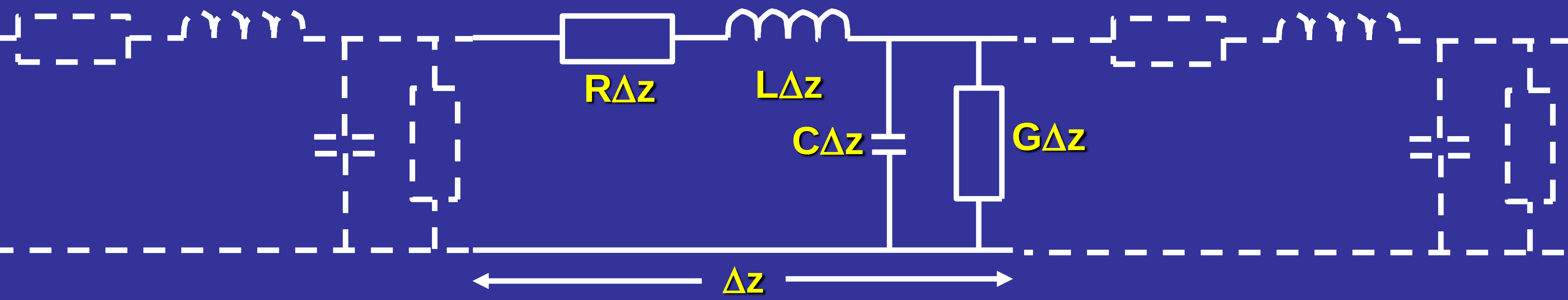
EQUIVALENT CIRCUIT FOR A TRANSMISSION LINE

- The existence of an inductance, capacitance, resistance and conductance (per unit length) allows us to represent the transmission line by an equivalent circuit

- Infinitesimal length of transmission line is represented by the same combination of 4 components:



To make up the whole line, repeat the equivalent circuit a sufficient number of times.



PRIMARY LINE CONSTANTS

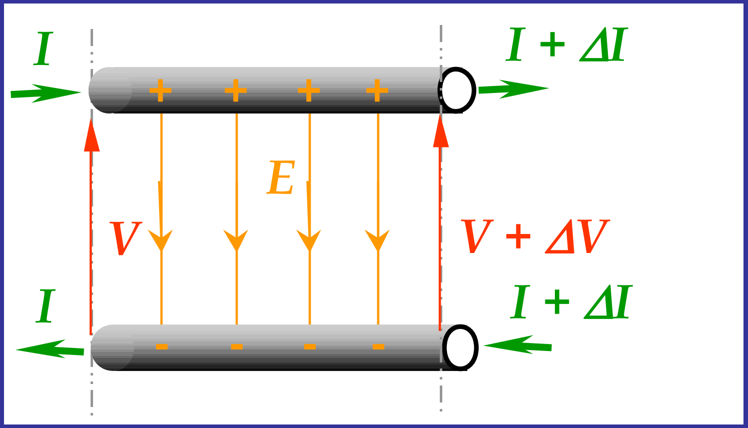
C = capacitance per unit length (F/m)
L = inductance per unit length (H/m)
R = resistance per unit length (Ω/m)
G = conductance per unit length (S/m)

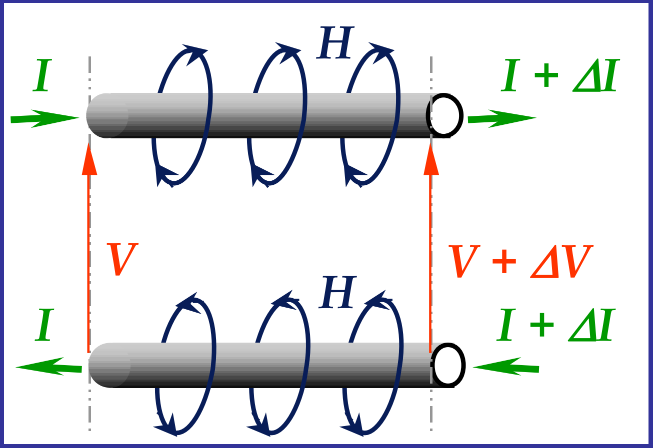
Note:-

- R, C, L and G are all expressed per unit length
- R & G should be small for a good transmission line.

If R = 0 and G = 0, the line is termed lossless.

LUMPED CIRCUIT MODEL





L represents total self inductance of the two conductors

C represents shunt capacitance due to the close proximity of the two conductors

R represents the resistance due to the finite conductivity of the conductors

G represents the shunt conductance due to dielectric loss in the material between the conductors

Lumped circuit model

The short piece of line of length Δz of figure 1a can be modeled as a lumped-circuit element as shown in figure 1b, where R, L, G and C are per unit length quantities defined as follows:

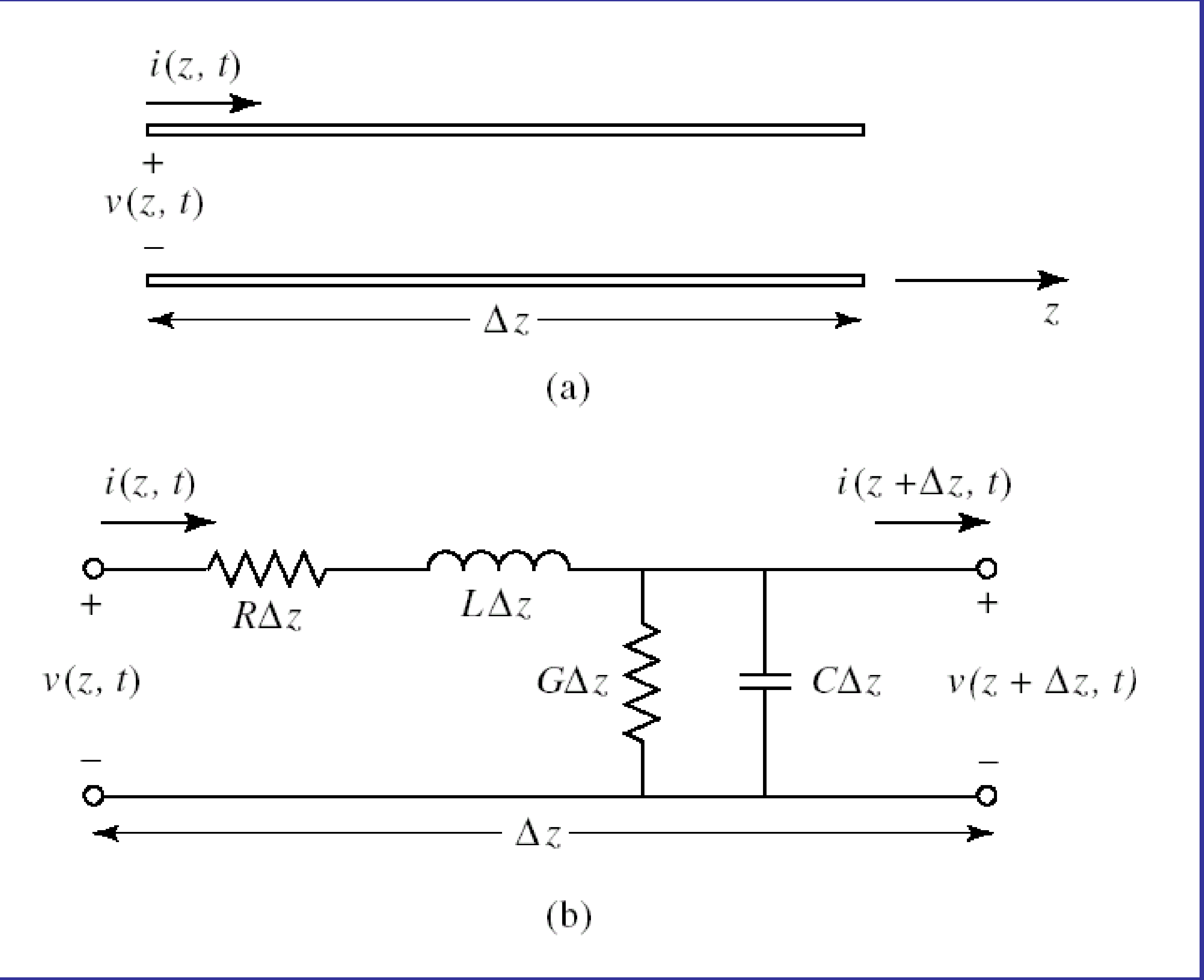
R – Series resistance per unit length, for both conductors, in Ω/m

L – Series inductance per unit length, for both conductors, in H/m

G – Shunt conductance per unit length, in S/m

C – Shunt capacitance per unit length, in F/m

Fig 1. (a) Voltage and Current definitions (b) lumped-element equivalent circuit



- From the circuit of Fig 1b Kirchoff's voltage law can be applied to give

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \ldots (1)$$

and Kirchoff's current law leads to

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \ldots (2)$$

Divide eqn. 1 by Δz and rearranging it gives

$$\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = \frac{-R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t}}{\Delta z}$$

Taking the limit as $\Delta z \longrightarrow 0$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \quad ...(3)$$

Similarly equation 2 becomes

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t} \quad ...(4)$$

Eqn. (3) & (4) are time domain form of transmission line or telegrapher equations

- For steady state condition, with cosine – based phasors simplify to

$$v(z,t) = V(z)e^{jwt} \;\&\; i(z,t) = I(z)e^{jwt}$$

$$\frac{dV(z)}{dz} = -(R + jwL)I(z) \ldots (5)$$

$$\frac{dI(z)}{dz} = -(G + jwC)V(z) \ldots (6)$$

Wave propagation on a transmission line

$$(5) \Rightarrow \frac{dV(z)}{dz} = -(R + jwL)I(z)$$

Differentiating with respect to z

$$\frac{d^2V(z)}{dz^2} = -(R + jwL)\frac{dI(z)}{dz}$$

Substituting $\frac{dI(z)}{dz}$ from(6) in above we get

$$\frac{d^2V(z)}{dz^2} = (R + jwL)(G + jwC)V(z)$$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \ldots (7)$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \ldots (8)$$

where $$\gamma = \alpha + j\beta = \sqrt{(R + jwL)(G + jwC)} \ldots (9)$$

Solution to eqn. (7) & (8) are

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \ldots (10) \;\; \&$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \ldots (11)$$

where $e^{-\gamma z}$ represents wave propagation in +z direction and

$e^{\gamma z}$ represents wave propagation in -z direction

Differentiation eqn. 10 w.r.t z we get

$$\frac{dV(z)}{dz} = V_0^+(-\gamma e^{-\gamma z}) + V_0^-(\gamma e^{\gamma z})$$

Substituting eqn. (5) in above we get

$$-(R + jwL)I(z) = -\gamma(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

$$I(z) = \frac{\gamma}{R + jwL}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \ldots (12)$$

Comparing eqn. (12) with eqn. (11)

$$I_0^+ = \frac{V_0^+}{(R + jwL)/\gamma}$$

$$Z_0 = \frac{R + jwL}{\gamma} = \sqrt{\frac{R + jwL}{G + jwC}} \ldots(13)$$

Equation (12) can be written as

$$I(z) = \left(\frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}\right) \ldots(14) \ \&$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} \ldots(15)$$

Converting back to time domain, the voltage waveform can be expressed as

$$v(z,t) = \left|V_0^+\right| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + \left|V_0^-\right| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}$$

Where $\Phi^{\pm}$ is the phase angle of the complex voltage $\left|V_0^{\pm}\right|$

- Phase velocity

$$wt - \beta z = cons\tan t$$

$$Let\ \Phi^{+} = 0(the\ initial\ value)\ then$$

$$wt - \beta z = cons\tan t$$

$$z = \frac{wt - cons\tan t}{\beta}$$

$$v_p = \frac{dz}{dt} = \frac{d}{dt}\left(\frac{wt - cons\tan t}{\beta}\right) = \frac{w}{\beta} \ldots (17)$$

- Wavelength

$$(wt - \beta z) - [wt - \beta(z + \lambda)] = 2\pi$$

$$\beta\lambda = 2\pi;\ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w/v_p} = \frac{v_p}{f} \ldots (18)$$

Classification of transmission lines

- Lossless line
- Low loss line
- Distortionless line

The Lossless line

- In practice most of the times the line loss is very small and can be neglected. Thus R = G = 0.
- So Propagation constant from eqn. (9) reduces to

$$\gamma = \alpha + j\beta = jw\sqrt{LC} \ldots(19)$$

$$\beta = w\sqrt{LC}$$

$$\alpha = 0$$

- And Characteristic impedance from eqn. (13) reduces to

$$Z_0 = \sqrt{\frac{L}{C}} \ldots(20)$$

The Lossless line contd..

- The general solutions for voltage and current on a lossless transmission line can be written as

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z} \ldots(21) \;\&$$

$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{j\beta z} \ldots(22)$$

- Wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{w\sqrt{LC}} = \frac{T}{\sqrt{LC}} \ldots(23)$$

- Phase velocity

$$v_p = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} \ldots(24)$$

Low loss line

- For low-loss line, the conductor loss and dielectric loss are small.

That is, $R \ll \omega L$ and $G \ll \omega C$

Then $\quad RG \ll \omega^2 LC$

$$WKT$$

$$\gamma = \alpha + j\beta = \sqrt{(R + jwL)(G + jwC)}$$

$$\gamma = \sqrt{(jwL)(jwC)(1 + \frac{R}{jwL})(1 + \frac{G}{jwC})}$$

$$\gamma = jw\sqrt{LC}\sqrt{1 - j\left(\frac{R}{wL} + \frac{G}{wC}\right) - \frac{RG}{w^2LC}}$$

Using (1) and (2) in above, this equation reduces to

$$\gamma = jw\sqrt{LC}\sqrt{1 - j\left(\frac{R}{wL} + \frac{G}{wC}\right)}$$

$$By\ Taylor's\ series$$

$$\sqrt{1+x} \approx 1 + x/2 + \ldots.$$

$$Therefore,$$

$$\gamma \simeq jw\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{wL} + \frac{G}{wC}\right)\right]$$

$$so\ that$$

$$\alpha \approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0 \right)$$

$$\beta \approx \omega\sqrt{LC}$$

The characteristics impedance

$$Z_0 = \sqrt{\frac{R + jwL}{G + jwC}} \approx \sqrt{\frac{L}{C}}$$

Distortionless line

- General expression for complex propagation constant is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$v_p = \frac{\omega}{\beta}$$

$$\alpha = \frac{1}{\sqrt{2}}\sqrt{RG - \omega^2 LC + [(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}}$$

$$\beta = \frac{1}{\sqrt{2}}\sqrt{-RG - \omega^2 LC + [(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}}$$

- If β is not linear function of frequency then phase velocity will be different for different frequencies ω
- Various frequency components of wideband signal will travel with different phase velocities, and arrive at receiver end at slightly different times, this lead to "Dispersion"

- A lossy line that has linear phase factor as a function of frequency such line is called Distortionless line that satisfy the relation

  R/L = G/C

- Substitute above condition in general propagation constant equation

$$\gamma = R\sqrt{C/L} + j\omega\sqrt{LC} = \alpha + j\beta$$

$\beta = \omega\sqrt{LC}$ is a linear function of frequency

$\alpha = R\sqrt{C/L}$ Not a function of frequency so all frequency component attenuated by same amount

2.2 A transmission line has the following per-unit-length parameters: $L = 0.5\ \mu H/m$, $C = 200\ pF/m$, $R = 4.0\ \Omega/m$, and $G = 0.02\ S/m$. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. Recalculate these quantities in the absence of loss ($R = G = 0$).

9.2 (Cheng)

It is found that the attenuation on a 50Ω distortionless transmission line is 0.01dB/m. The line capacitance is 0.1pF/m. Find:

A. the resistance, inductance, conductance per meter of the line and velocity of propagation

Ans:0.057 Ω/m, .25H/m, 22.8μS/m, 2×10^{8}m/s