BECE205L Engineering Electromagnetics

Dr M. Saranya Nair
School of Electronics Engineering, VIT-Chennai

Module 3: Magnetostatics

Topic1 : Biot-Savart’s law

Topic 2: Ampere’s circuital law and Applications, Magnetic flux density

Topic 3: Magnetic scalar and vector potentials

Topic 4: Forces due to Magnetic Fields

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Magnetostatics

Biot-Savart's law

- So far, we limited our discussions to static electric fields characterized by E or D.
- We now focus our attention on static magnetic fields, which are characterized by H or B.
- As E and D are related according to $D = \varepsilon E$ for linear material space,

H and B are related according to $B = \mu H$

- The analogy show that most of the equations we have derived for the electric fields may be readily used to obtain corresponding equations for magnetic fields if the equivalent analogous quantities are substituted.

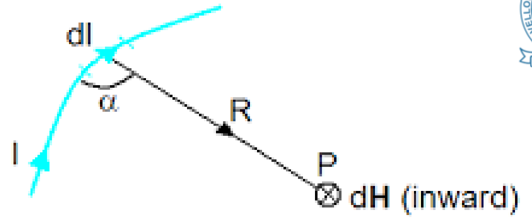
Magnetostatics

Biot-Savart’s law, Magnetic field intensity

- As we have noticed, an electrostatic field is produced by static or stationary charges.
- If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced.
- A magnetostatic field is produced by a constant current flow (or direct current).
- In this chapter, we consider magnetic fields due to direct current.

Magnetostatics

Biot-Savart's law, Magnetic field intensity



Biot-Savart's law states that the magnetic field intensity dH produced at a point P, as shown in Figure, by the differential current element $I\,dl$ is proportional to the product $I\,dl$ and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.

$$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

$$dH = \frac{kI\,dl \sin\alpha}{R^2}$$

$$\boxed{d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}}$$

Magnetostatics

Biot-Savart's law, Magnetic field intensity

Just as we can have different charge configurations, we can have different current distributions: line current, surface current, and volume current.

If we define K as the surface current density (in amperes/meter) and J as the volume current density (in amperes/meter square), the source elements are related as

$$\mathbf{H} = \int_L \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(line current)}$$

$$\mathbf{H} = \int_S \frac{\mathbf{K}\,dS \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(surface current)}$$

$$\mathbf{H} = \int_v \frac{\mathbf{J}\,dv \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(volume current)}$$

Magnetostatics

Biot-Savart's law, Magnetic field intensity

The magnetic field due to a *straight* current carrying filamentary conductor of finite length

$$\mathbf{H}_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

As a special case - when the conductor is *infinite* in length.

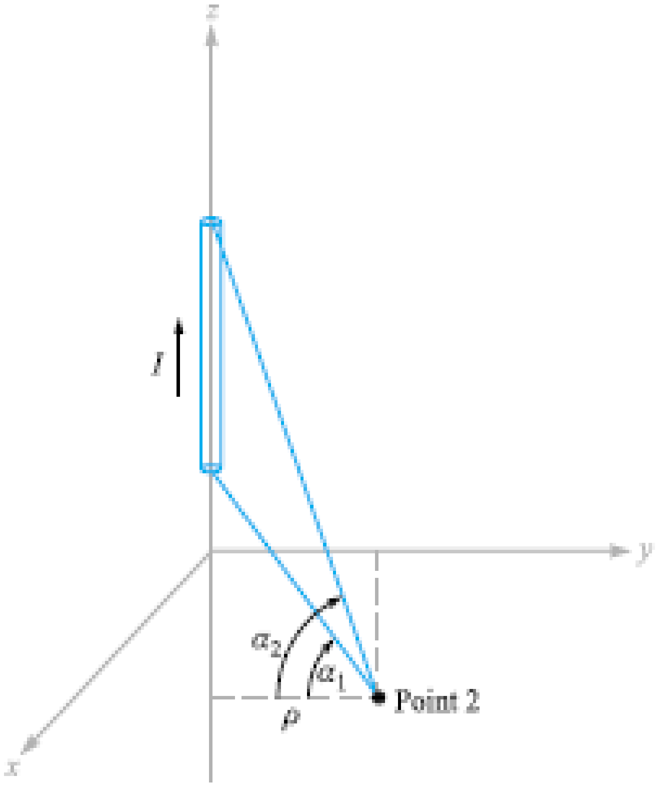
$$\boxed{\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi}$$

When the conductor is *semiinfinite*

$$\mathbf{H} = \frac{I}{4\pi\rho}\mathbf{a}_\phi$$

$$\mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho$$

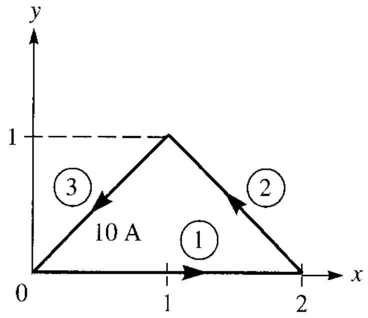
where a_l is a unit vector along the line current and a_ρ is a unit vector along the perpendicular line from the line current to the field point.



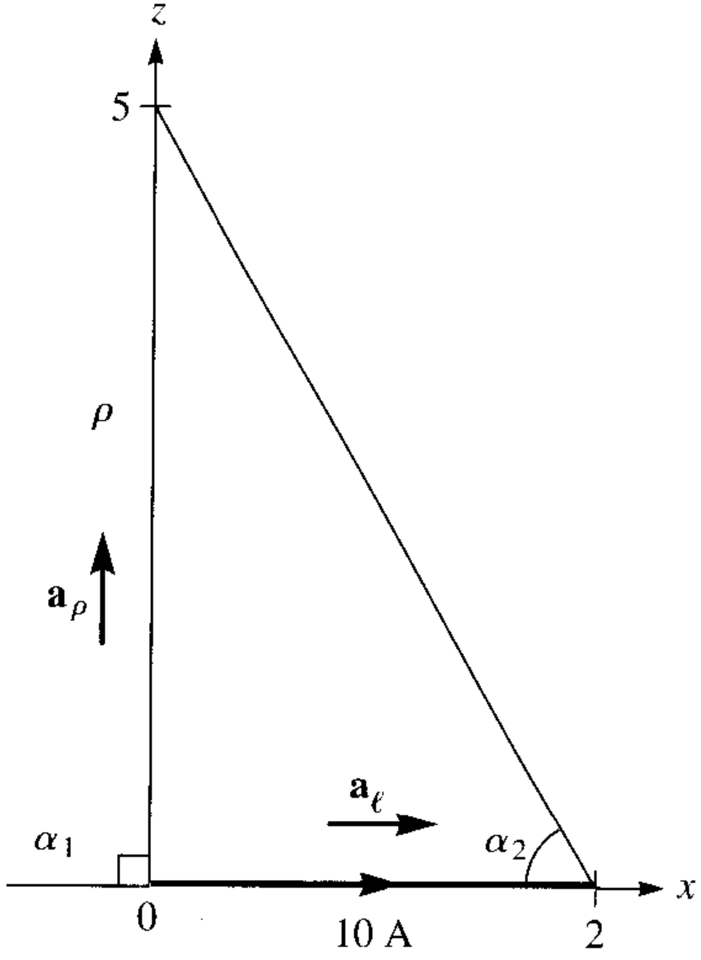
(a) Find $\mathbf{H}$ in rectangular components at $P(2, 3, 4)$ if there is a current filament on the z axis carrying 8 mA in the $\mathbf{a}_z$ direction.
(b) Repeat if the filament is located at $x = -1$, $y = 2$.
(c) Find $\mathbf{H}$ if both filaments are present.

Problem

A current filament carrying 15 A in the $\mathbf{a}_z$ direction lies along the entire z axis. Find $\mathbf{H}$ in rectangular coordinates at: (a) $P_A(\sqrt{20}, 0, 4)$; (b) $P_B(2, -4, 4)$.



The conducting triangular loop in Figure carries a current of 10 A. Find H at (0, 0, 5) due to side 1 of the loop.



$$\mathbf{H}_1 = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

$$= \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right)(-\mathbf{a}_y)$$

$$= -59.1\mathbf{a}_y \text{ mA/m}$$

Magnetostatics

Ampere’s circuital law & Applications - Magnetic flux and flux density

Ampere's circuit law states that the line integral of H around a closed path is the same as the net current enclosed by the path.

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}}$$

Ampere's law is similar to Gauss's law and it is easily applied to determine H when the current distribution is symmetrical.

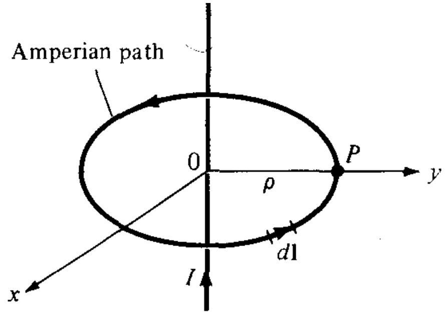
$$I_{enc} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$I_{enc} = \oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$

Magnetostatics

Ampere's circuital law, Magnetic flux and flux density



Infinite Line Current

Consider an infinitely long filamentary current along the z-axis as in Figure. To determine H at an observation point *P,* we allow a closed path pass through *P.* This path, on which Ampere's law is to be applied, is known as an *Amperian path* (analogous to the term Gaussian surface).

$$\mathbf{H} = \frac{I}{2\pi\rho}\,\mathbf{a}_\phi$$

Magnetostatics

Ampere's circuital law, Magnetic flux and flux density

$$\mathbf{B} = \mu_o \mathbf{H}$$

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

D7.5. Calculate the value of the vector current density: (*a*) in rectangular coordinates at $P_A(2, 3, 4)$ if $\mathbf{H} = x^2 z\mathbf{a}_y - y^2 x\mathbf{a}_z$; (*b*) in cylindrical coordinates at $P_B(1.5, 90°, 0.5)$ if $\mathbf{H} = \frac{2}{\rho}(\cos 0.2\phi)\mathbf{a}_\rho$; (*c*) in spherical coordinates at $P_C(2, 30°, 20°)$ if $\mathbf{H} = \frac{1}{\sin\theta}\mathbf{a}_\theta$.

Ans. $-16\mathbf{a}_x + 9\mathbf{a}_y + 16\mathbf{a}_z$ A/m^2; $0.055\mathbf{a}_z$ A/m^2; $\mathbf{a}_\phi$ A/m^2

Magnetostatics

Ampere’s circuital law, Magnetic flux and flux density

The magnetic flux through a surface *S* is given by

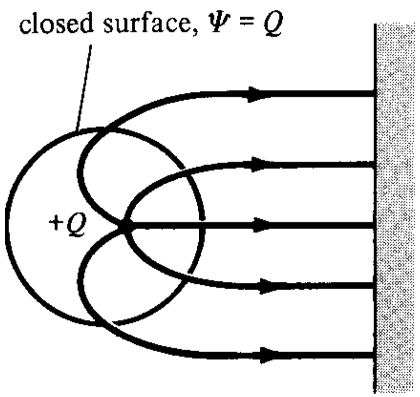
$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

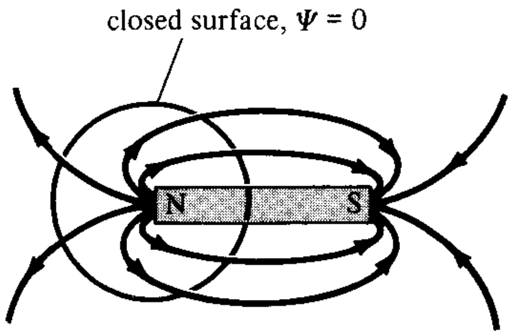
In an electrostatic field, the flux passing through a closed surface is the same as the charge enclosed, Thus it is possible to have an isolated electric charge which also reveals that electric flux lines are not necessarily closed. Unlike electric flux lines, magnetic flux lines always close upon themselves.

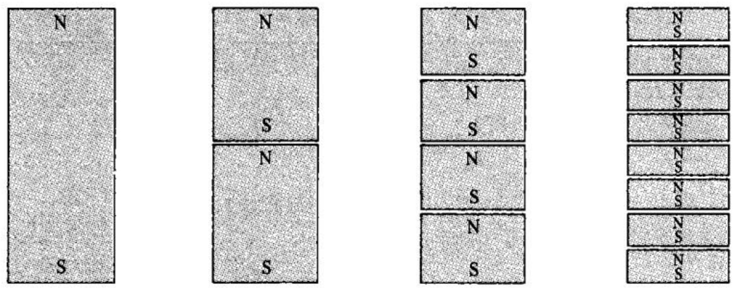
This is due to the fact that *it is not possible to have isolated magnetic poles (or magnetic charges).*

Magnetostatics

Ampere's circuital law, Magnetic flux and flux density







Thus the total flux through a closed surface in a magnetic field must be zero; that is,

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

This equation is referred to as the *law of conservation of magnetic flux* or *Gauss's law for magnetostatic fields* just as § D • *dS* = *Q* is Gauss's law for electrostatic fields.

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B} \, dv = 0$$

$$\boxed{\nabla \cdot \mathbf{B} = 0}$$

Maxwell's Equations for Static EM Fields

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \mathbf{E} = 0$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = 0$	Conservativeness of electrostatic field
$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$	Ampere's law

Magnetostatics

Ampere's circuital law, Magnetic flux and flux density

For an infinite sheet of current density K A/m

$$\mathbf{H} = \frac{1}{2}\mathbf{K} \times \mathbf{a}_n$$

where a_n is a unit normal vector directed from the current sheet to the point of interest

Planes z = 0 and *z* = 4 carry current K = -10ax A/m and K = 10ax A/m, respectively.

Determine **H** at

(a) (1,1,1)

(b) (0, - 3 , 10)

Magnetostatics

Ampere's circuital law, Magnetic flux and flux density

Plane $y = 1$ carries current K = 50az mA/m. Find H at

(a) (0,0,0)

(b) (1,5,-3)

Magnetostatics

Magnetic scalar and vector potentials

We recall that some electrostatic field problems were simplified by relating the electric potential *V* to the electric field intensity E (E = -∇V). Similarly, we can define a potential associated with magnetostatic field B. In fact, the magnetic potential could be scalar *Vm* or vector A.

To define *Vm* and A involves recalling two important identities

$$\nabla \times (\nabla V) = 0$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

Magnetostatics

Magnetic scalar and vector potentials

Just as $E = -\nabla V$, we define the *magnetic scalar potential Vm* (in amperes) as related to H according to

$$\boxed{\mathbf{H} = -\nabla V_m} \qquad \text{if } \mathbf{J} = 0$$

$$J = \nabla \times H = \nabla \times (-\nabla V_m) = 0$$

Vm satisfies Laplace's equation just as *V* does for electrostatic fields; hence, $\nabla^2 Vm = 0$, $(J = 0)$

We know that for a magnetostatic field, $\nabla \bullet B = 0$, we can define the *vector magnetic potential* A (in Wb/m) such that $B = \nabla \times A$

Magnetostatics

Magnetic scalar and vector potentials

Given the magnetic vector potential $\mathbf{A} = -\rho^2/4\ \mathbf{a}_z$ Wb/m, calculate the total magnetic flux crossing the surface $\phi = \pi/2$, $1 \leq \rho \leq 2$ m, $0 \leq z \leq 5$ m.

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_L \mathbf{A} \cdot d\mathbf{l}$$

$$\boxed{\Psi = \oint_L \mathbf{A} \cdot d\mathbf{l}}$$

Magnetostatics

Magnetic scalar and vector potentials

A current distribution gives rise to the vector magnetic potential $\mathbf{A} = x^2y\mathbf{a}_x + y^2x\mathbf{a}_y - 4xyz\mathbf{a}_z$ Wb/m. Calculate

(a) $\mathbf{B}$ at $(-1, 2, 5)$

(b) The flux through the surface defined by $z = 1, 0 \leq x \leq 1, -1 \leq y \leq 4$

Magnetic Forces

The second half of the magnetic field problem, that of determining the forces and torques exerted by the magnetic field on other charges.

The electric field causes a force to be exerted on a charge that may be either stationary or in motion;

But the steady magnetic field is capable of exerting a force only on a *moving* charge. This result appears reasonable; a magnetic field may be produced by moving charges and may exert forces on moving charges; a magnetic field cannot arise from stationary charges and cannot exert any force on a stationary charge.

FORCE ON A MOVING CHARGE

In an electric field, the definition of the electric field intensity shows us that the force on a charged particle is $\mathbf{F} = Q\mathbf{E}$ **(1)**

The force is in the same direction as the electric field intensity (for a positive charge) and is directly proportional to both **E** and *Q*. If the charge is in motion, the force at any point in its trajectory is then given by (1).

A charged particle in motion in a magnetic field of flux density **B** is found experimentally to experience a force whose magnitude is proportional to the product of the magnitudes of the charge *Q*, its velocity **v**, and the flux density **B**, and to the sine of the angle between the vectors **v** and **B**. The direction of the force is perpendicular to both **v** and **B** and is given by a unit vector in the direction of $\mathbf{v} \times \mathbf{B}$. The force may therefore be expressed as

$$\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$$

The force on a moving particle arising from combined electric and magnetic fields is obtained easily by superposition,

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

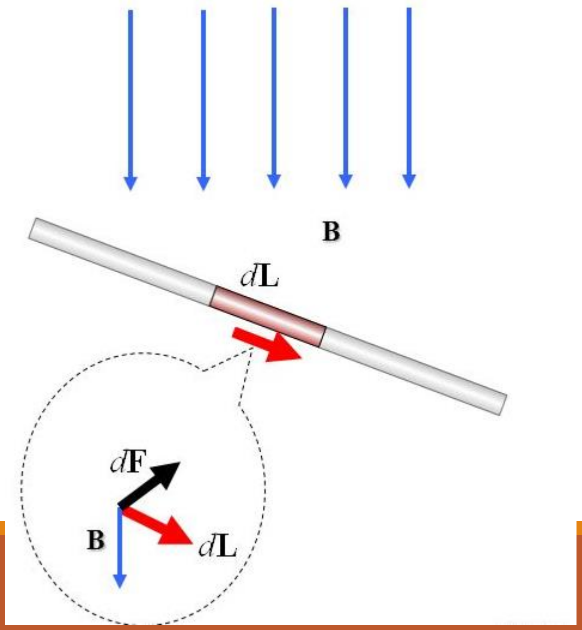
This equation is known as the *Lorentz force equation,* and its solution is required in determining electron orbits in the magnetron, proton paths in the cyclotron, plasma characteristics in a magneto hydrodynamic (MHD) generator, or, in general, charged particle motion in combined electric and magnetic fields.

FORCE ON A DIFFERENTIAL CURRENT ELEMENT

The force on a charged particle moving through a steady magnetic field may be written as the differential force exerted on a differential element of charge,

$$d\mathbf{F} = dQ\,\mathbf{v} \times \mathbf{B}$$

Physically, the differential element of charge consists of a large number of very small, discrete charges occupying a volume which, although small, is much larger than the average separation between the charges.



$\mathbf{J} = \rho_v \mathbf{v}$

$dQ = \rho_v dv$

Thus $d\mathbf{F} = \rho_v dv\, \mathbf{v} \times \mathbf{B}$

$$\boxed{\mathbf{F} = \oint I\, d\mathbf{L} \times \mathbf{B} = -I \oint \mathbf{B} \times d\mathbf{L}}$$

Or $d\mathbf{F} = \mathbf{J} \times \mathbf{B}\, dv$

$\mathbf{J}\, dv = \mathbf{K} dS = I\, d\mathbf{L}$

$d\mathbf{F} = \mathbf{K} \times \mathbf{B}\, dS$

$d\mathbf{F} = I\, d\mathbf{L} \times \mathbf{B}$

$$\boxed{\mathbf{F} = I\mathbf{L} \times \mathbf{B}}$$

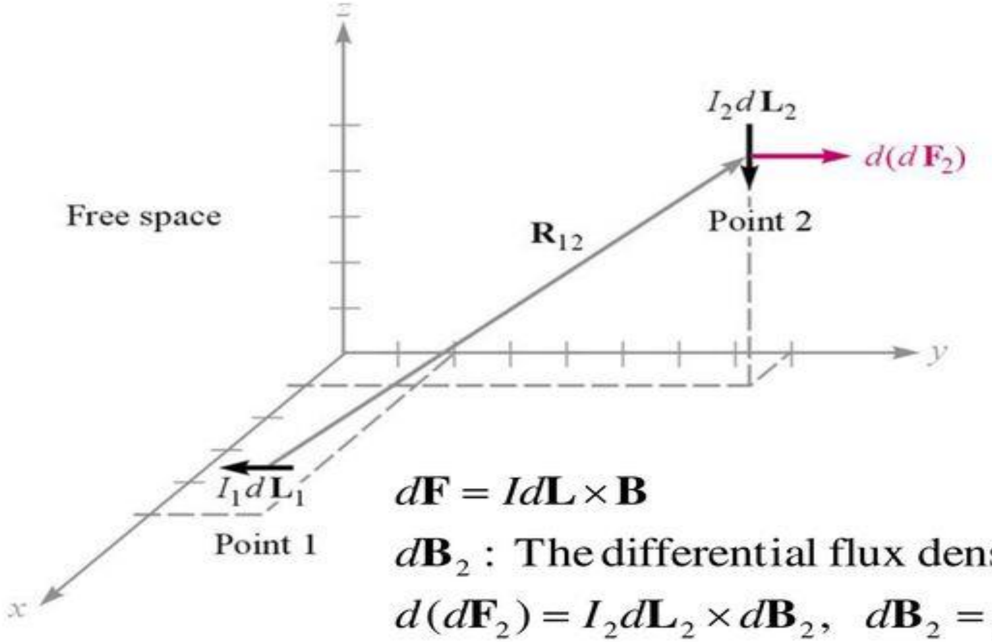
$$F = BIL \sin\theta$$

FORCE BETWEEN DIFFERENTIAL CURRENT ELEMENTS

The concept of the magnetic field was introduced to break into two parts the problem of finding the interaction of one current distribution on a second current distribution.

It is possible to express the force on one current element directly in terms of a second current element without finding the magnetic field.

Force between Differential Current Element



$$dH_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

$$d\mathbf{F} = Id\mathbf{L} \times \mathbf{B}$$

$d\mathbf{B}_2$: The differential flux density at point 2 caused by current element 1

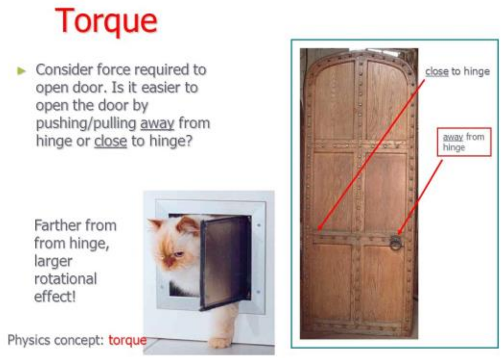
$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times d\mathbf{B}_2, \quad d\mathbf{B}_2 = \mu_o d\mathbf{H}_2$$

$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times \mu_o \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2} = \mu_o \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12})$$

$$\mathbf{F}_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[d\mathbf{L}_2 \times \oint \frac{d\mathbf{L}_1 \times \mathbf{a}_{R12}}{R_{12}^2} \right]$$

$$= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[\oint \frac{\mathbf{a}_{R12} \times d\mathbf{L}_1}{R_{12}^2} \right] \times d\mathbf{L}_2$$

TORQUE ON A CLOSED CIRCUIT

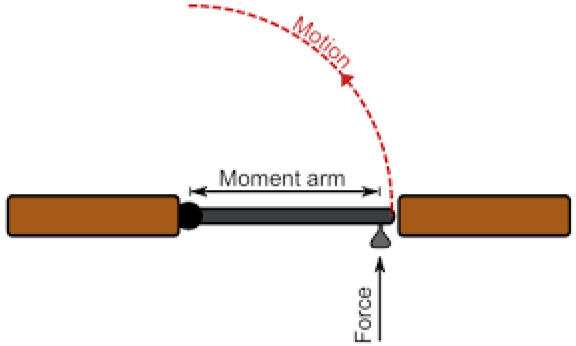


$$\mathbf{F} = -I \oint \mathbf{B} \times d\mathbf{L}$$

Assume a *uniform* magnetic flux density, then **B** may be removed from the integral:

$$\mathbf{F} = -I\mathbf{B} \times \oint d\mathbf{L} \qquad \oint d\mathbf{L} = 0$$

Although the force is zero, the torque is generally not equal to zero.
Torque is a measure of the force that can cause an object to rotate about an axis. Just as force is what causes an object to accelerate in linear kinematics, torque is what causes an object to acquire angular acceleration.
Torque is a vector quantity. The direction of the torque vector depends on the direction of the force on the axis.



In defining the *torque,* or *moment,* of a force, it is necessary to consider both an origin at or about which the torque is to be calculated, and the point at which the force is applied.

$$\boxed{d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}}$$

where $d\mathbf{S}$ is the vector area of the differential current loop

$$\boxed{d\mathbf{m} = I\, d\mathbf{S}}$$

the product of the loop current and the vector area of the loop as the differential *magnetic dipole moment* $d\mathbf{m}$, with units of Am^2.

$$\boxed{d\mathbf{T} = d\mathbf{m} \times \mathbf{B}}$$

If we extend the results for the differential *electric* dipole by determining the torque produced on it by an *electric* field

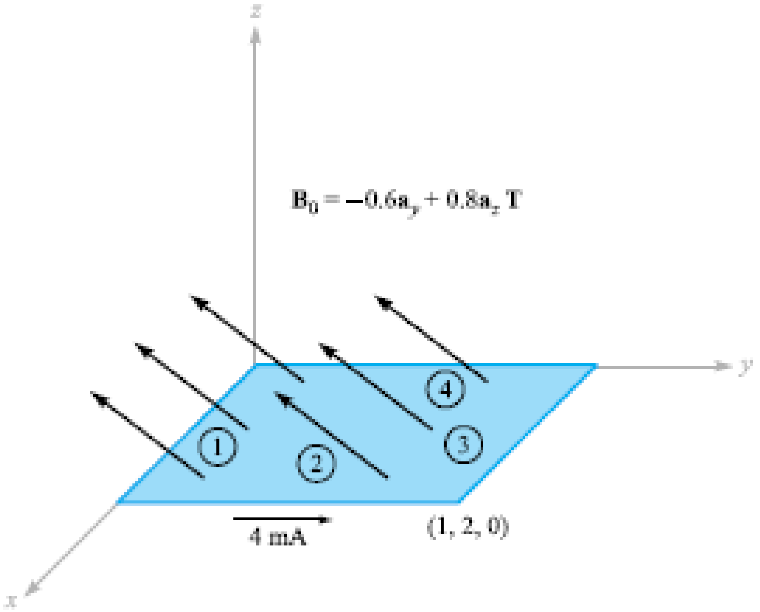
$$d\mathbf{T} = d\mathbf{p} \times \mathbf{E}$$

$$\boxed{\mathbf{T} = I\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}}$$

To illustrate some force and torque calculations, consider the rectangular loop shown in Figure. Calculate the torque

The loop has dimensions of 1 m by 2 m and lies in the uniform field

B = −0.6**a***y* +0.8**a***z*T. The loop current is 4 mA,



$$\mathbf{T} = 4 \times 10^{-3}[(1)(2)\mathbf{a}_z] \times (-0.6\mathbf{a}_y + 0.8\mathbf{a}_z) = 4.8\mathbf{a}_x \text{ mN} \cdot \text{m}$$

The point charge Q = 18nC has a velocity of 5×10^6 m/s in the direction $\mathbf{a}_v = 0.60\mathbf{a}_x + 0.75\mathbf{a}_y + 0.30\mathbf{a}_z$. Calculate the magnitude of the force exerted on the charge by the field: (*a*) $\mathbf{B} = -3\mathbf{a}x + 4\mathbf{a}y + 6\mathbf{a}z$ mT; (*b*) $\mathbf{E} = -3\mathbf{a}x + 4\mathbf{a}y + 6\mathbf{a}z$ kV/m; (*c*) B and E acting together.

The field $\mathbf{B} = -2\mathbf{a}x + 3\mathbf{a}y + 4\mathbf{a}z$ mT is present in free space. Find the vector force exerted on a straight wire carrying 12 A in the $\mathbf{a}_{AB}$ direction, given $A(1, 1, 1)$ and $B(3, 5, 6)$.

Two differential current elements, $I1\mathrm{d}\mathbf{L}1 = 3 \times 10^{-6}\mathbf{a}y$ Am at $P1(1, 0, 0)$ and $I2\mathrm{d}\mathbf{L}2 = 3\times10^{-6}(-0.5\mathbf{a}x + 0.4\mathbf{a}y + 0.3\mathbf{a}z)$ Am at $P2(2, 2, 2)$, are located in free space. Find the vector force exerted on: (*a*) $I2\mathrm{d}\mathbf{L}2$ by $I1\mathrm{d}\mathbf{L}1$; (*b*) $I1\mathrm{d}\mathbf{L}1$ by $I2\mathrm{d}\mathbf{L}2$.

Magnetic Boundary Conditions

We define magnetic boundary conditions as the conditions that H (or B) field must satisfy at the boundary between two different media.

$$\boxed{\mathbf{B}_{1n} = \mathbf{B}_{2n}} \quad \text{or} \quad \mu_1 \mathbf{H}_{1n} = \mu_2 \mathbf{H}_{2n}$$

$$H_{1t} - H_{2t} = K$$

$$\boxed{(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}}$$

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

where $\mathbf{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2.

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

$$\boxed{\mathbf{H}_{1t} = \mathbf{H}_{2t}} \quad \text{or} \quad \frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2}$$

$$\frac{B_1}{\mu_1}\sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

Problems

The xy-plane serves as the interface between two different media. Medium 1 ($z < 0$) is filled with a material whose $\mu r = 6$, and medium 2 ($z > 0$) is filled with a material whose $\mu r = 4$. If the interface carries current $(1/\mu o)\, a_y$ mA/m, and $B2 = 5ax + 8az$ mWb/m^2 find B1.

$$\mathbf{B}_{1n} = \mathbf{B}_{2n} = 8\mathbf{a}_z \rightarrow B_z = 8$$

$$\mathbf{H}_2 = \frac{\mathbf{B}_2}{\mu_2} = \frac{1}{4\mu_o}(5\mathbf{a}_x + 8\mathbf{a}_z)\mathrm{mA/m}$$

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{6\mu_o}(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z)\ \mathrm{mA/m}$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$

$$\mathbf{H}_1 \times \mathbf{a}_{n12} = \mathbf{H}_2 \times \mathbf{a}_{n12} + \mathbf{K}$$

$$\frac{1}{6\mu_o}(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z) \times \mathbf{a}_z = \frac{1}{4\mu_o}(5\mathbf{a}_x + 8\mathbf{a}_z) \times \mathbf{a}_z + \frac{1}{\mu_o}\mathbf{a}_y$$

Equating components yields

$$B_y = 0, \quad \frac{-B_x}{6} = \frac{-5}{4} + 1, \quad \text{or} \quad B_x = \frac{6}{4} = 1.5$$

$$\mathbf{B}_1 = 1.5\mathbf{a}_x + 8\mathbf{a}_z \text{ mWb/m}^2$$

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{\mu_o}(0.25\mathbf{a}_x + 1.33\mathbf{a}_z) \text{ mA/m}$$

$$\mathbf{H}_2 = \frac{1}{\mu_o}(1.25\mathbf{a}_x + 2\mathbf{a}_z) \text{ mA/m}$$

Inductance

A circuit (or closed conducting path) carrying current I produces a magnetic field B which causes a flux to pass through each turn of the circuit.

The flux linkage is proportional to the current I producing it.

Inductance L of an inductor is the ratio of the magnetic flux linkage to the current through the inductor

$$\boxed{L = \frac{\lambda}{I} = \frac{N\Psi}{I}}$$

1. Choose a suitable coordinate system.
2. Let the inductor carry current I.
3. Determine $\mathbf{B}$ from Biot–Savart's law (or from Ampère's law if symmetry exists) and calculate Ψ from $\Psi = \int_s \mathbf{B} \cdot d\mathbf{S}$.
4. Finally find L from $L = \dfrac{\lambda}{I} = \dfrac{N\Psi}{I}$.

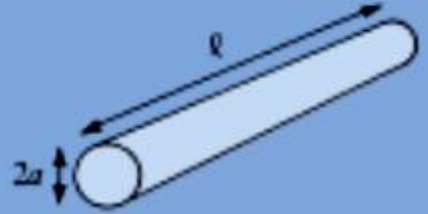
Mutual inductance M_{12} *is* the ratio of the flux linkage on circuit 1 to current *in circuit 2*

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1\Psi_{12}}{I_2}}$$

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2\Psi_{21}}{I_1}$$

1. Wire

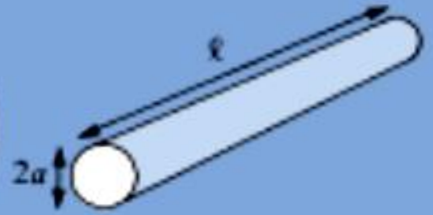
$$L = \frac{\mu_o \ell}{8\pi}$$



2. Hollow cylinder

$$L = \frac{\mu_o \ell}{2\pi}\left(\ln \frac{2\ell}{\mathbf{a}} - 1\right)$$

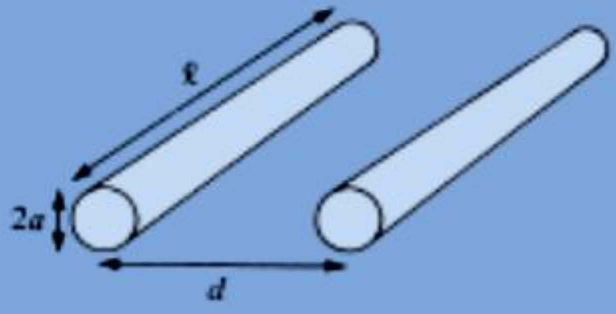
$$\ell \gg \mathbf{a}$$



3. Parallel wires

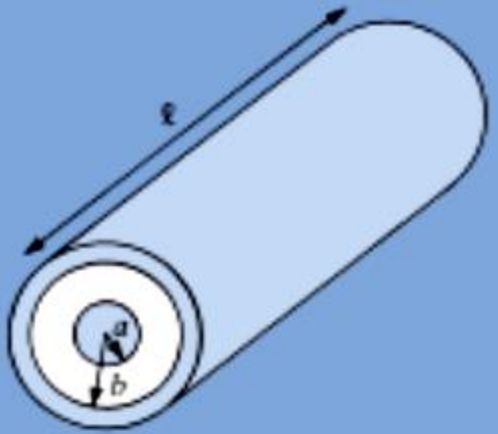
$$L = \frac{\mu_o \ell}{\pi} \ln \frac{d}{a}$$

$$\ell \gg d,\ d \gg a$$



4. Coaxial conductor

$$L = \frac{\mu_o \ell}{2\pi} \ln \frac{b}{a}$$



5. Circular loop

$$L = \frac{\mu_o \ell}{2\pi}\left(\ln \frac{4\ell}{d} - 2.45\right)$$

$$\ell = 2\pi\rho_o,\ \rho_o \gg d$$

6. Solenoid

$$L = \frac{\mu_o N^2 S}{\ell}$$

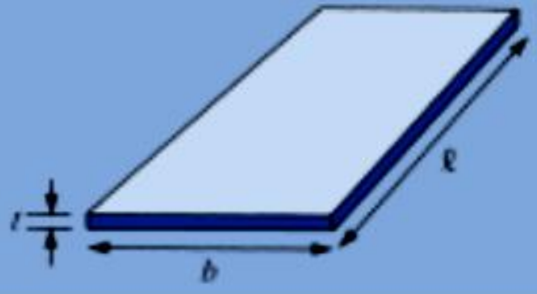
$$\ell \gg a$$

7. Torus (of circular cross section)

$$L = \mu_o N^2[\rho_o - \sqrt{\rho_o^2 - a^2}]$$

8. Sheet

$$L = \mu_o\, 2\ell\left(\ln \frac{2\ell}{b + t} + 0.5\right)$$



A solenoid is 50 cm long, 2 cm in diameter, and contains 1500 turns. The cylindrical core has a diameter of 2 cm and a relative permeability of 75. This coil is coaxial with a second solenoid, also 50 cm long, but with a 3 cm diameter and 1200 turns. Calculate: (*a*) L for the inner solenoid; (*b*) L for the outer solenoid; (*c*) M between the two solenoids.

Ans. 133.2 mH; 192 mH; 106.6 mH