Module:6 Active filters and Data Converters

Course: BECE206L – Analog Circuits

DA3 Quiz

11.11.2022, 8:30 pm – 8:50 pm

Module 4,5,6 (Filters)

Talk Title: Automotive MIMO RADAR

Time: 6:30 pm-7:30 pm

Resources Person from Valeo:

Mr. Aabilash BALACHANDRAN

Electronics Design Engineer - Research

and Development

Valeo India Private Limited,

Module:6 Active filters and Data Converters

Module:6 Active filters and Data Converters

• Filter classifications: First and second order Low-pass and High pass filter designs, Band-pass filter, Notch filter. Sample-and-hold circuits, DAC characteristics, D/A conversion techniques, A/D characteristics, A/D conversion techniques.

1. Introduction

- Electric Filters In circuits requiring separation of signals according to frequency of operations
- Almost used in all sophisticated electronic instruments
- Consists of : a) Passive RLC components
 - b) Crystals
 - c) Resistors, capacitors, Op-amps(active filters)

- Electric filter: A frequency selective electric circuit
 Passes electric signals of specified band of frequencies
 Attenuates signals of frequencies outside this band
- Filters can be analog or digital.
- Passive filters use R, L, C components.
 High frequencies, works well
 Low frequencies, Inductors require more turns, which add series resistance(coil resistance) which results in high power dissipation
- Active filters overcome these problems.
 Op-amp will be active element (Can provide gain if necessary)
 RC will be passive elements
 Inductors are avoided by enclosing capacitors in feedback loop

- Active filters
 Op-amp will be active element (Can provide gain if necessary)
 Op-amp Non inverting configuration offers high input impedance and low output impedance
 Load is isolated from filter circuit
- Limitations of Active filters
 High frequency response is limited by
 - a) gain bandwidth product (GBW)
 - b) slew rate (response time)

High frequency active filters are expensive than passive filters

- Common filters
- a) Low pass filter(LPF)
- b) High pass filter (HPF)
- c) Band pass filter (BPF)
- d) Band reject filter or Band stop filter (BSF)

Frequency response Dashed curve – ideal Solid curve – practical

Impossible for ideal Can achieve close to ideal with special design

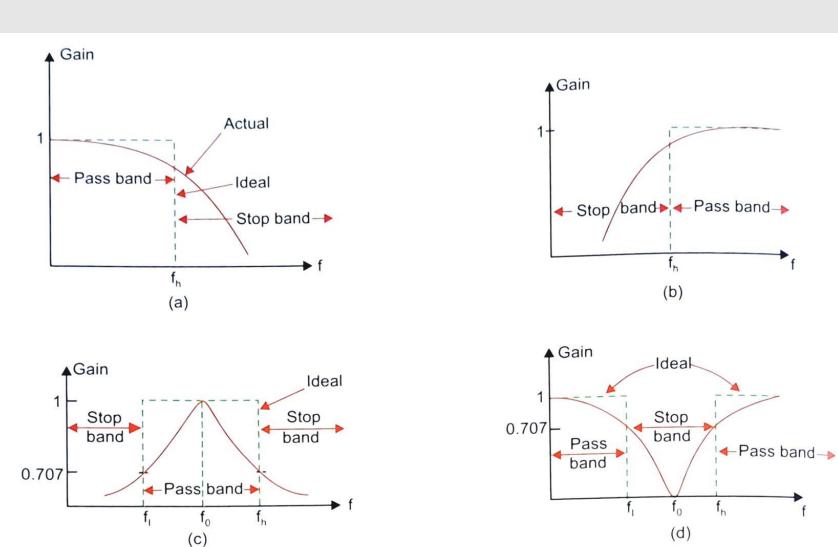


Fig. 7.1 Frequency response of filters, (a) Low-pass, (b) High-pass, (c) Band-pass, (d) Band-reject

Active filters are specified by voltage transfer function

$$H(s) = \frac{V_0(s)}{V_i(s)}$$

- Under steady state conditions $s = j\omega$ $H(j\omega) = |H(j\omega)|e^{j\phi(\omega)}$
- Where $|H(j\omega)|$ is magnitude or gain function and $\phi(\omega)$ is the phase function
- Usually, magnitude response is given in dB as $20 \log_{10} |H(j\omega)|$

And phase response in degrees as

$$-\phi(\omega) \times \frac{180}{\pi}$$
 degrees

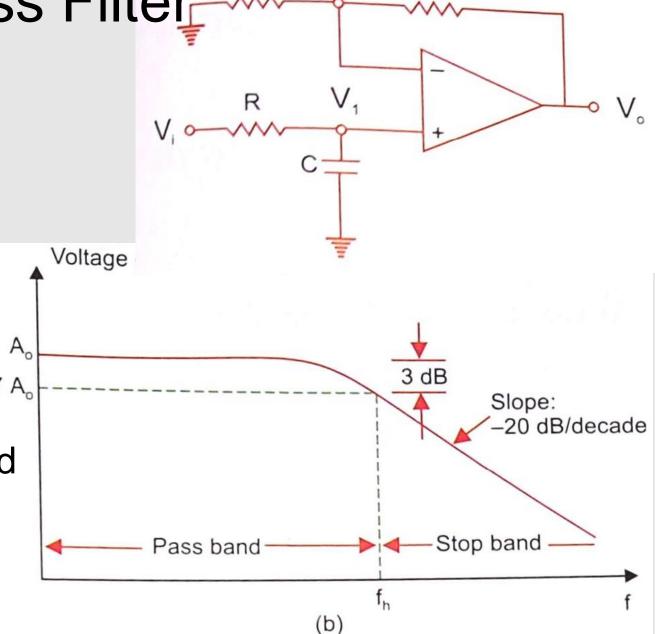
First order LPF:
 Single RC network connected to
 (+) input terminal of non-inverting
 op-amp.

 Pass band: Gain: from max gain until -3dB from max gain.

• Stop band:

Gain below -3dB from max gain

Slope: -20dB / decade in stop band

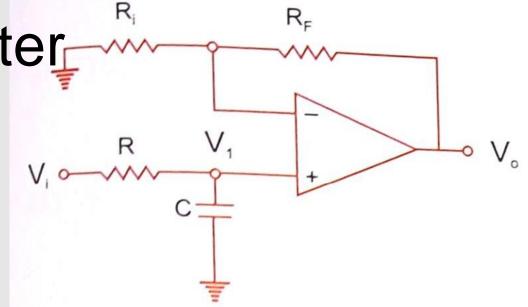


- If V is in time domain, V(s) is Laplace transform in freq domain.
- Voltage V_1 across capacitor C in s-domain

$$V_1(s) = \frac{\frac{1}{sc}}{R + \frac{1}{sc}} V_i(s)$$

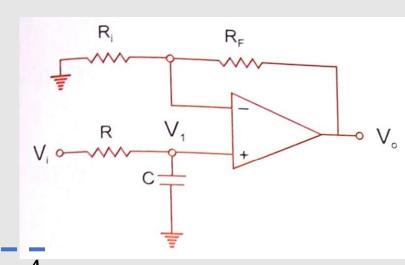
$$\bullet \frac{V_1(s)}{V_i(s)} = \frac{1}{RCs+1}$$

- Closed loop gain $A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$
- Overall transfer function: $H_{LP}(s) = \frac{V_0(s)}{V_i(s)} = \frac{V_0(s)}{V_1(s)} \cdot \frac{V_1(s)}{V_i(s)} = \frac{A_0}{RCs+1}$



- Closed loop gain $A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$
- Overall transfer function: $H_{LP}(s) = \frac{V_0(s)}{V_i(s)} = \frac{A_0}{RCs+1}$ Let $\omega_h = 1/RC$: $H_{LP}(s) = \frac{V_0(s)}{V_0(s)} = \frac{A_0}{RCs+1}$ • Let $\omega_h = 1/RC$: $H_{LP}(s) = \frac{V_0(s)}{V_i(s)} = \frac{A_0}{\frac{s}{s+1}} = \frac{A_0\omega_h}{s+\omega_h}$
- This is standard form of transfer function of first order low pass system
- To determine, put $s = j\omega$ $H_{LP}(j\omega) = \frac{A_0}{RCs + 1} = \frac{A_0}{1 + j\omega RC} = \frac{A_0}{1 + j\omega/\omega_h} = \frac{A_0}{1 + j(f/f_h)}$
- Where $f_h = \frac{1}{2\pi RC}$ and $f = \frac{\omega}{2\pi}$

•
$$A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$$
 $H_{LP}(s) = \frac{A_0}{RCs + 1}$ $H_{LP}(j\omega) = \frac{A_0}{1 + j(f/f_h)}$ $f_h = \frac{1}{2\pi RC}$ and $f = \frac{\omega}{2\pi}$



- At very low frequencies: $f \ll f_h$: $|H_{LP}(j\omega)| \approx \frac{A_0}{1+j(0)} \approx A_0$
- At $f = f_h$: $|H_{LP}(j\omega)| = \left| \frac{A_0}{1+j(f_h/f_h)} \right| = \left| \frac{A_0}{1+j} \right| = \frac{A_0}{\sqrt{2}} = 0.707A_0$

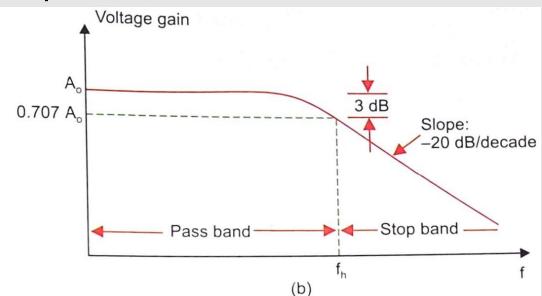
(Log scale: -3dB below max gain)

Pass band: Max gain A_0 to $0.707A_0$

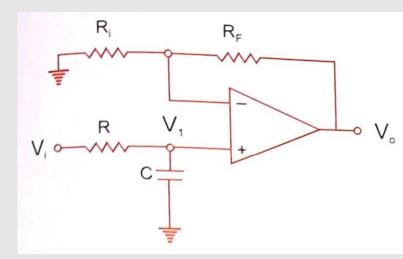
• At $f \gg f_h: |H_{LP}(j\omega)| \to \frac{A_0}{1+j(\infty)} \to 0 \ (\ll A_0)$

Log scale: -∞

Stop band: $0.707 A_0$ to 0 gain region



•
$$A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$$
 $H_{LP}(s) = \frac{A_0}{RCs + 1}$ $H_{LP}(j\omega) = \frac{A_0}{1 + j(f/f_h)}$ $f_h = \frac{1}{2\pi RC}$ and $f = \frac{\omega}{2\pi}$

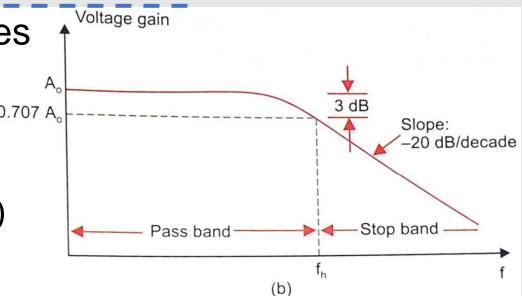


- At very low frequencies: $f \ll f_h \qquad |H_{LP}(j\omega)| \approx A_0$
- At $f = f_h$: $|H_{LP}(j\omega)| = 0.707A_0$ (Log scale: -3dB below max gain)
- At $f \gg f_h$: $|H_{LP}(j\omega)| \to 0 \ (\ll A_0)$
- When f increases above f_h , gain decreases constantly at $-\frac{20dB}{decade}$ (Roll off rate)

 When frequency is increased 10 times

 Gain drops 10 times

 or -20dB (in dB scale $20\log(1/10)$



4. Second order active LPF

- Two RC pairs
- Roll off: -40dB/decade

•
$$A_0 = (1 + R_F/R_1)$$
 (Gain)

•
$$H(s) = \frac{A_0}{(sCR)^2 + sCR(3 - A_0) + 1}$$

•
$$H_{LP}(s) = \frac{A_0 \omega_h^2}{s^2 + \alpha \omega_h s + \omega_h^2}$$

 ω_h : upper cutoff frequency
(radians/second)
 α = damping coefficient= $3 - A_0$

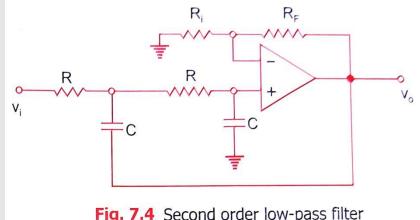


Fig. 7.4 Second order low-pass filter

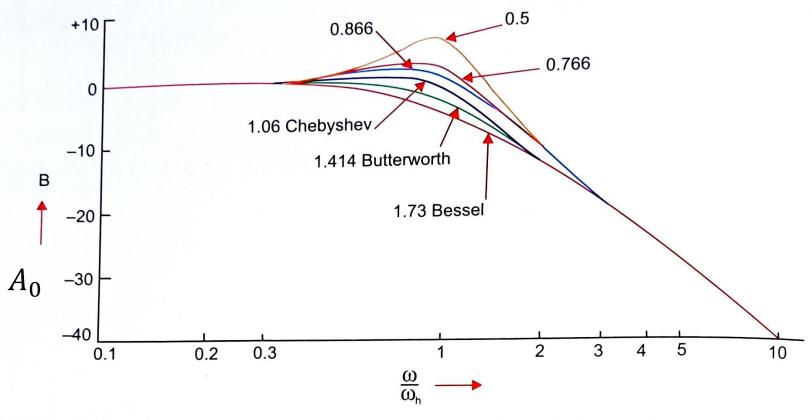


Fig. 7.5 Second order low-pass active filter response for different damping (unity gain $A_0 = 1$)

5. Higher order LPF

- With $s_n = j \omega / \omega_h$
- Normalized transfer function of nth order generalized low pass (maximally flat)

$$\left| \frac{H_{LP}(j\omega)}{A_0} \right| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_h}\right)^{2n}}}$$

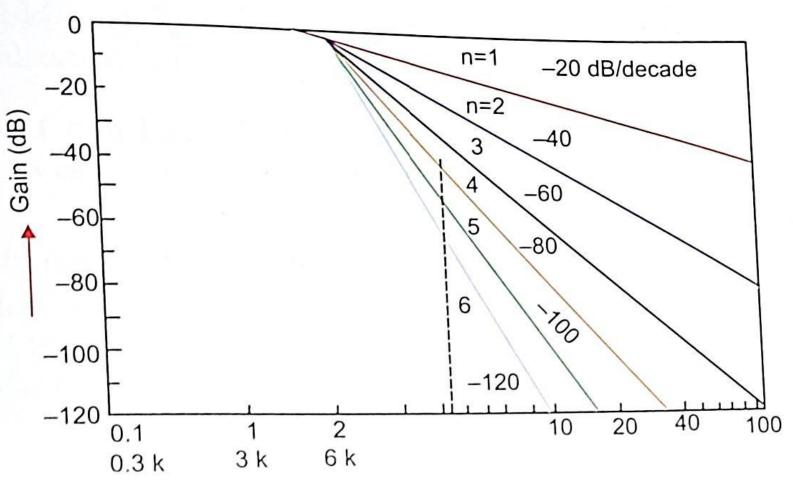


Fig. 7.6 Roll-off rate for different values of n

6. First order High pass active filter

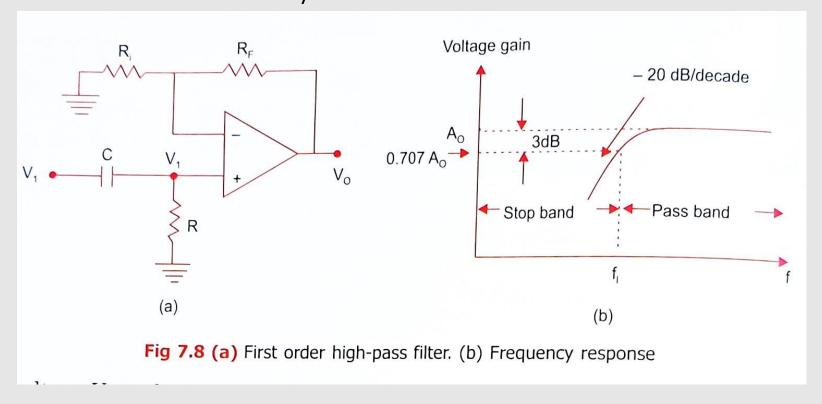
- Complement of Low pass filter (Obtained by interchanging R and C)
- Single RC network to non-inverting terminal Gain is controlled by R_iR_F in inverting configuration
- V_1 at non-inverting terminal is $V_1(s) = \frac{R}{R+1/sc} V_i(s)$

$$\frac{V_1(s)}{V_i(s)} = \frac{RCs}{1 + RCs}$$

•
$$V_0(s) = \left(1 + \frac{R_F}{R_1}\right) V_1(s)$$

CL gain:

$$A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$$



6. First order High pass active filter $\frac{V_1(s)}{V_i(s)} = \frac{RCs}{1+RCs} \qquad A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$

$$\frac{V_1(s)}{V_i(s)} = \frac{RCs}{1 + RCs} \qquad A_0 = \frac{V_0(s)}{V_1(s)} = \left(1 + \frac{R_F}{R_1}\right)$$

Overall Transfer function $H_{HP}(s) = \frac{V_0(s)}{V_i(s)} = \frac{V_0(s)}{V_1(s)} \cdot \frac{V_1(s)}{V_i(s)} = \frac{A_0 RCs}{1 + RCs}$

With
$$s = j\omega$$
 and $\omega_l = 1/RC$

$$H_{HP}(j\omega) = A_0 \frac{\left(j\frac{f}{f_l}\right)}{1+j\frac{f}{f_l}}$$

Frequency response:

$$|H_{HP}(j\omega)| = \left| \frac{V_0}{V_i} \right|$$

= $A_0 \frac{f/f_l}{\sqrt{1 + (f/f_l)^2}} = \frac{A_0}{\sqrt{1 + (f_l/f)^2}}$

With
$$s = j\omega$$
 and $\omega_l = 1/RC$ $RCs = \frac{j\omega}{\omega_l} = j\frac{2\pi f}{2\pi f_l} = j\frac{f}{f_l}$

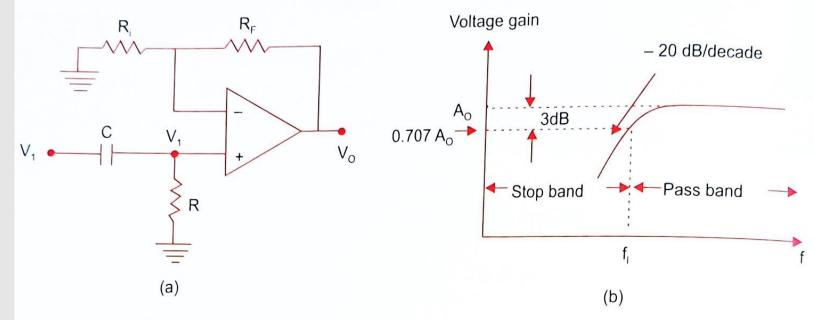
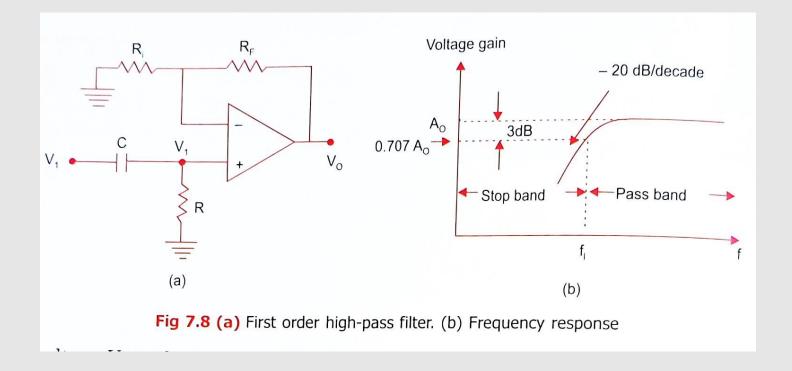


Fig 7.8 (a) First order high-pass filter. (b) Frequency response

6. First order High pass active filter Frequency response: $|H_{HP}(j\omega)| = \frac{V_0}{V_i} = \frac{A_0}{\sqrt{1+(f_1/f)^2}}$

Pass band: very high frequency $f > f_l$: Gain is constant = A_0 (as $(f_l/f)^2 <<1$)

Stop band: At $f < f_l$, gain rolls off at -20dB/decade



Problem 2: Design a first-order high pass filter at a cut-off frequency of 2 kHz with a pass band gain of 2

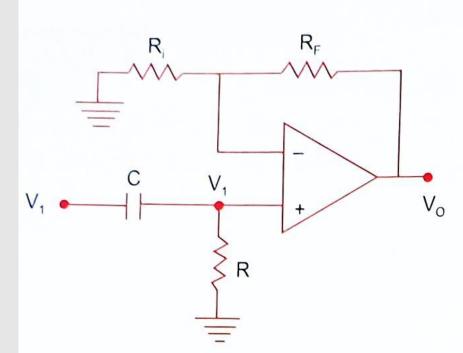
• $f_l = 2kHz$ and $C = 0.01\mu F$

•
$$\omega_l = \frac{1}{RC}$$
 $f_l = \frac{1}{2\pi RC}$ $R = \frac{1}{2\pi fC} = \frac{1}{2\pi (2k)(0.01\mu F)} = 7.95k\Omega$

To obtain pass band gain of 2

$$A_0 = 2 = 1 + \frac{R_F}{R_1}$$

 $\frac{R_F}{R_1} = 1$ $R_F = R_1 = 10k\Omega$



Problem 3: For the designed first-order high pass filter at a cut-off frequency of 2 kHz with a pass band gain of 2, plot its frequency response

• $f_l=2kHz$ and $C=0.01\mu F$ $R=7.95k\Omega$ $R_F=R_1=10k\Omega$ For frequency response: $|H_{HP}(j\omega)|=\left|\frac{V_0}{V_i}\right|=\frac{A_0}{\sqrt{1+(f_l/f)^2}}$ (Gain)

 $|H_{HP}(j\omega)|_{dB} = 20 \log_{10} \left(\frac{A_0}{1 + (f_l/f)^2} \right)$ (Gain in dB)

	\ - \- \- \- \- \- \- \- \- \- \- \- \- \-	
Frequency	Gain	Gain in dB
(Hz)	V _o V _i	20 log $rac{V_{ m o}}{V_{ m i}}$.
100	0.10	- 20.01
200	0.20	- 14.02
400	0.39	- 8.13
1000	0.89	- 0.97
3000	1.66	4.42
10,000	1.96	5.85
100 kHz	2.00	6.02

Problem 3: For the designed first-order high pass filter at a cut-off frequency of 2 kHz with a pass band gain of 2, plot its

frequency response $|H_{HP}(j\omega)|_{dB} = 20 \log_{10} \left(\frac{A_0}{1 + (f_1/f)^2} \right)$ (Gain in dB) +10 +6.023 dB Frequency Gain in dB Gain 0 dB 20 log -(Hz) /V0/V1 -20.01100 0.10 - 14.02 200 0.20 -1020 dB/decade 400 0.39 -0.971000 0.89 -14.02 4.42 3000 1.66 5.85 1.96 10,000 6.02 2.00 100 kHz -20Stop band -Pass band -30100

100 kHz

1 MHz

10 kHz

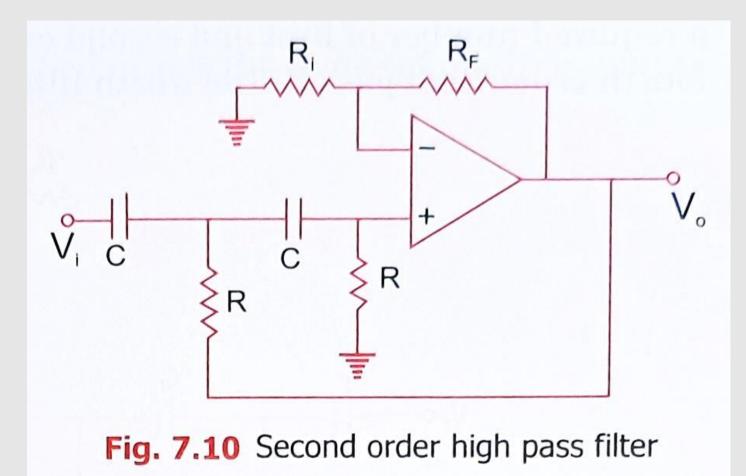
 $1000f_{i} = 2kHz$

7. Second order high pass filter

 Two pairs of RC with feedback to RC

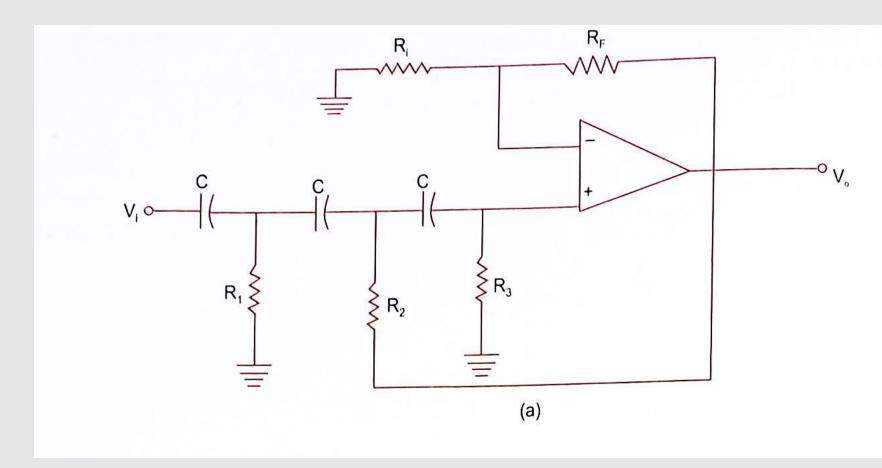
$$\bullet \ H_{HP} = \frac{A_0}{1 + \frac{\omega_l}{S} (3 - A_0) + \left(\frac{\omega_l}{S}\right)^2}$$

• Denominator has second power of $\left(\frac{\omega_l}{s}\right)$ as highest power



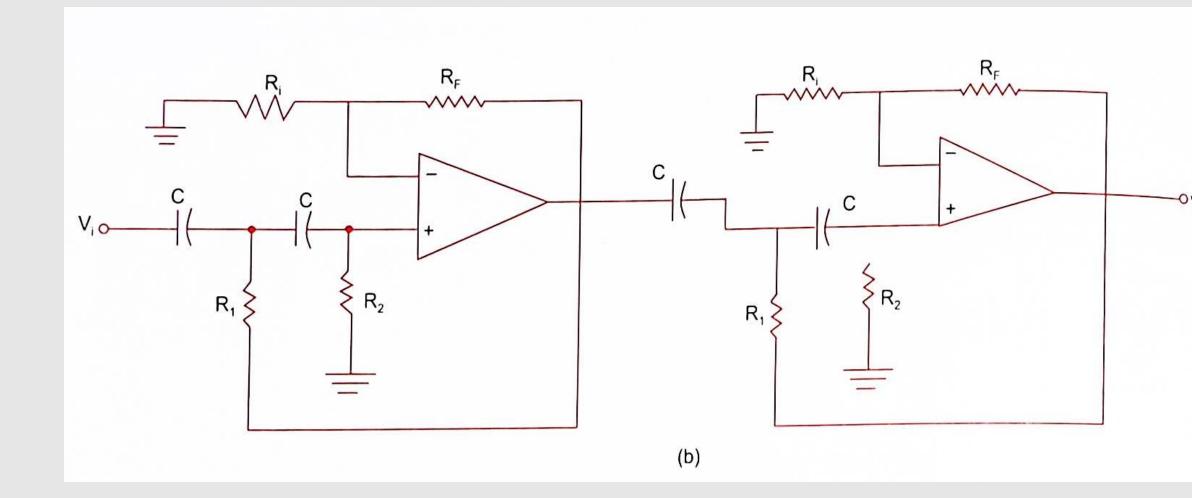
8. Higher order high pass filter

Third order HPF



8. Higher order high pass filter

Fourth Order HPF



- Most of real world quantities: Analog form (Continuous)
 Example: Voltage, current, temperature, pressure, time, etc
- Signal processing difficult to store or transmit analog: if amplitude modulation – noise gets superimposed
- For processing, transmission and storage Digital form is preferred
- Digital More accuracy and less noise

•

Analog from Sensor – Antialiasing filter limits bandwidth – Sample & hold (S/H) samples the analog signal at twice the maximum frequency and holds for A/D conversion to complete – ADC output is sequence of binary digits - Computer performs signal processing on binary digits and has output as digital binary signal.

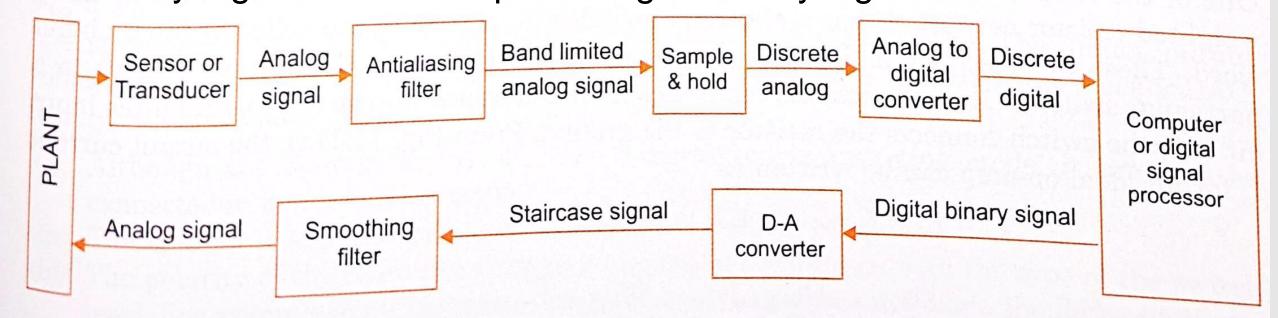


Fig. 11.1 Circuit showing application of A/D and D/A converter

Output digital binary signal of Computer(or digital signal processor)

 converted to staircase signal using D-A converter (digital to continuous) – Smoothing filter converts staircase signal to analog signal reducing quantization noise.

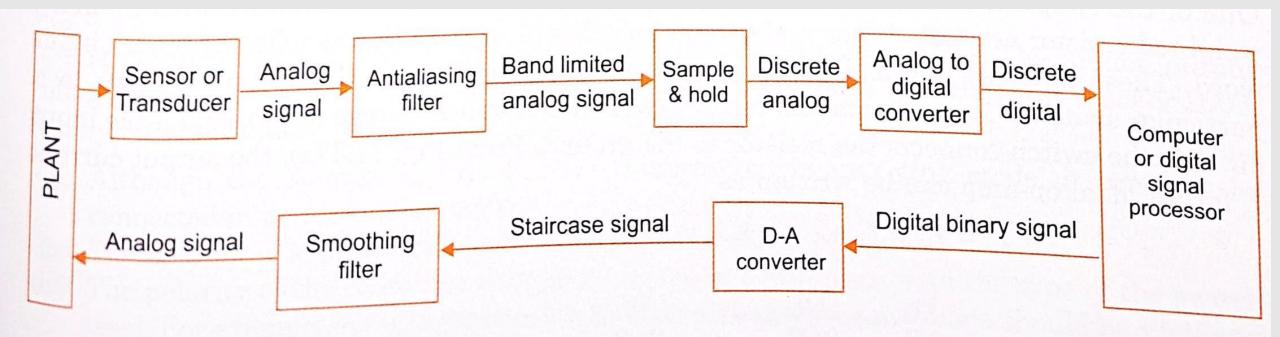


Fig. 11.1 Circuit showing application of A/D and D/A converter

 Used in either full or part in many applications: Digital audio recording and playback, computer, music synthesis, music and video synthesis, pulse code modulation transmission, data acquisition, digital multimeter, direct digital control, digital signal processing, microprocessor based instrumentation.

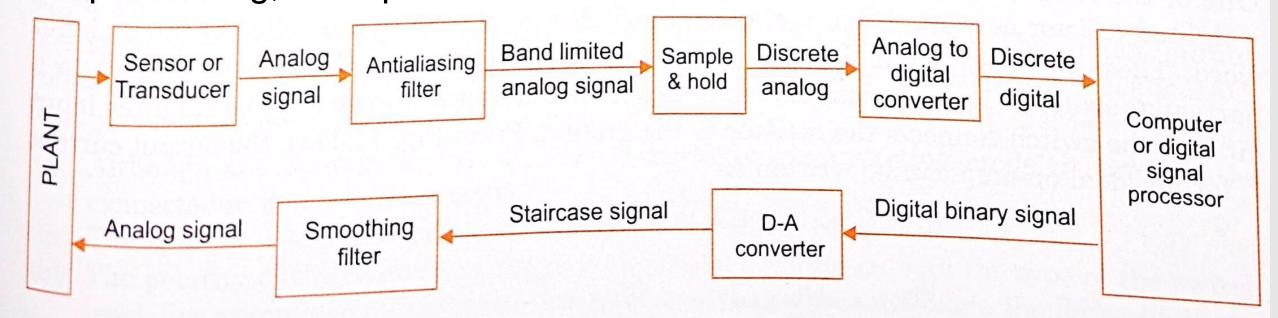


Fig. 11.1 Circuit showing application of A/D and D/A converter

- Some cases, S/H may be avoided without considerable error
- ADC(Analog to digital converter) and DAC(Digital to analog converter) are data converters and are available in IC form

2. Basic DAC techniques

- Input to DAC is n-bit binary word D and is combined with a reference voltage V_R to give analog output signal
- Output of DAC either voltage or current

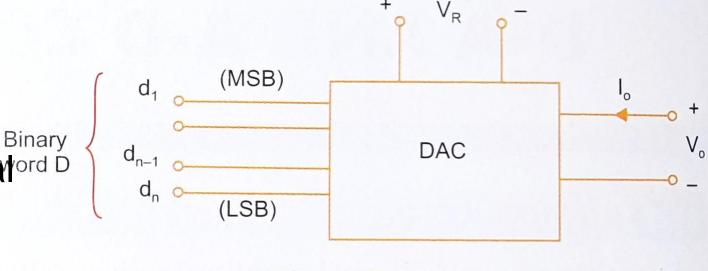
• For voltage output DAC: $V_0 = K V_{FS} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$

 V_0 : Output voltage

 V_{FS} : Full scale output voltage

K: Scaling factor (usually 1)

 d_1, d_2, \dots, d_n : n bit binary functionatord becomes word with decimal point located at left



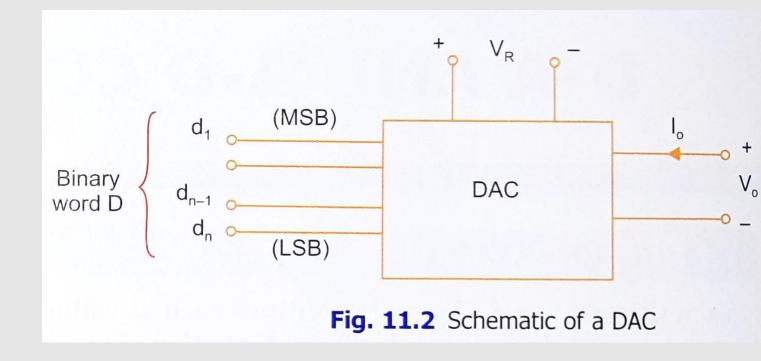
 d_1 : most significant bit (MSB) with weight of $V_{FS}/2$ Fig. 11.2 Schematic of a DAC

 d_2 : least significant bit (LSB) with weight of $V_{FS}/2^n$

2. Basic DAC techniques

- Various methods are available.
- Methods involving resistors:
 Weighted resistor DAC

R-2R ladder
Inverted R-2R ladder



3. Weighted Resistor DAC

For voltage output DAC: $V_0 = K V_{FS} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$

- Simplest: Uses summing amplifier with binary weighted resistors
- N electronic switches, d_1 , d_2 ,..., d_n controlled by binary input word
- Single pole double throw (SPDT) type switches
- If a binary input to particular switch is 1, connects resistance to reference voltage $-V_R$ If a binary input is 0, switch connects resistor to ground

3. Weighted Resistor DAC

For voltage output DAC: $V_0 = K V_{FS} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$

Output current for ideal Op-amp

$$I_0 = I_1 + I_2 + \dots + I_n$$

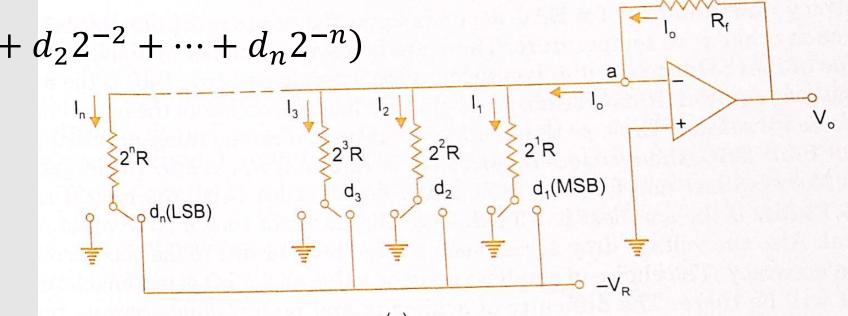
$$= \frac{V_R}{2R} d_1 + \frac{V_R}{2R^2} d_2 + \dots + \frac{V_R}{2R^n} d_n$$

$$= \frac{V_R}{R} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$$

Output voltage:

$$V_0 = I_0 R_f = V_R \frac{R_f}{R} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$$

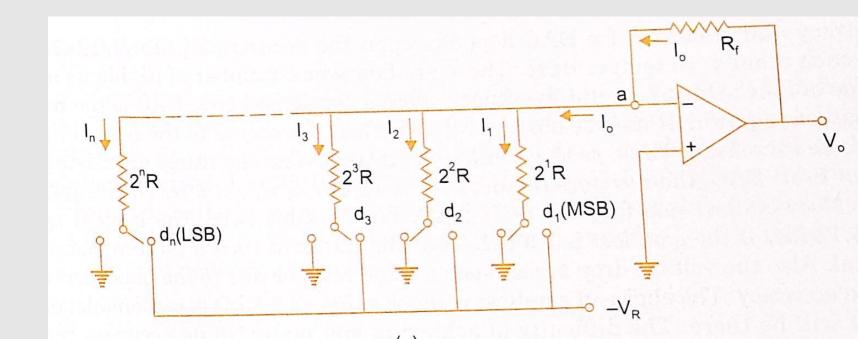
If $R_f = R$, then $K = 1$ and $V_{FS} = V_R$



3. Weighted Resistor DAC

For voltage output DAC: $V_0 = K V_{FS} (d_1 2^{-1} + d_2 2^{-2} + \dots + d_n 2^{-n})$

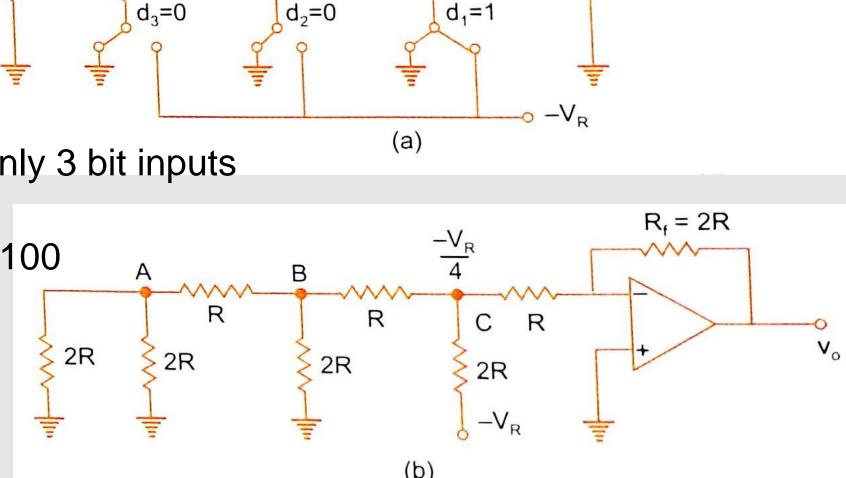
- Disadvantage:
 Resistor range increases for better resolution
 Difficult to fabricate different and large resistor values
- Accuracy is low



- Only 2 resistor values are used
- Well suitable for ICs
- Typical ranges of R:
 2.5kΩ to 10kΩ

 For example(simple) only 3 bit inputs are present

- $d_1 d_2 d_3$: binary word = 100
- Simplified circuit is:



R

2R

R

2R

(LSB)

2R

R

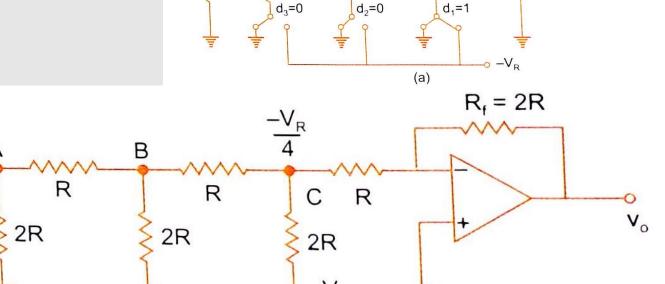
2R

(MSB)

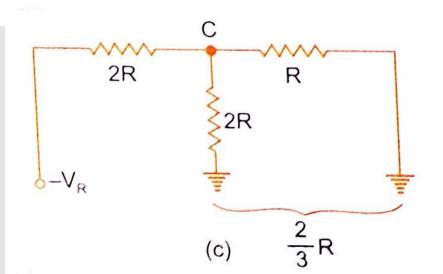
 $R_f = 2R$

- Only 2 resistor values are used
- Well suitable for ICs
- Typical ranges of R:
 2.5kΩ to 10kΩ
- For example(simple) only3 bit inputs are present
- $d_1d_2d_3$: binary word = 100
- Simplified circuit is:
 It has effective vales at C node as shown here:

2R



(LSB)

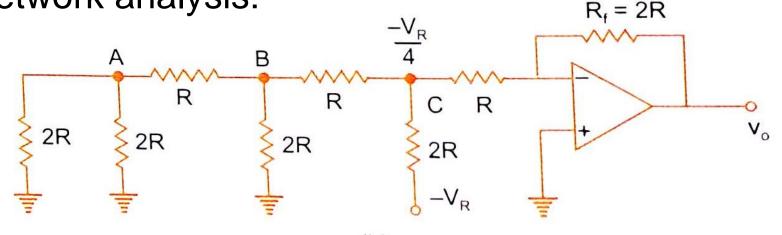


- $d_1d_2d_3$: binary word = 100
- Voltage at Node C using Network analysis:

$$= -V_R \left(\frac{2R \| R}{2R + (2R \| R)} \right)$$

$$= -V_R \left(\frac{2R/3}{2R + 2R/3} \right)$$

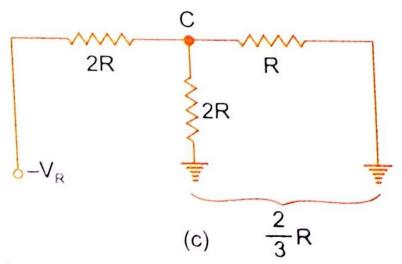
$$= -V_R/4$$



(LSB)

with input as $-V_R/4$ Output voltage of inverting configuration:

$$V_0 = -\frac{R_f}{R} \left(-\frac{V_R}{4} \right) = -\frac{2R}{R} \left(-\frac{V_R}{4} \right) = \frac{V_R}{2} = \frac{V_{FS}}{2}$$



- $d_1d_2d_3$: binary word = 100
- Similarly redraw circuit:

And apply nodal analysis

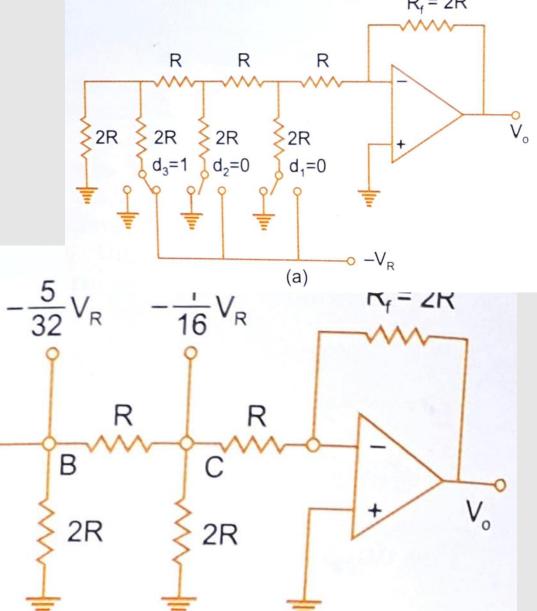
with input as $-V_R/8$ at C Output voltage of inverting opamp:

$$V_0 = -\frac{R_f}{R} \left(-\frac{V_R}{16} \right) = -\frac{2R}{R} \left(-\frac{V_R}{16} \right)^{\frac{5}{2}}$$

$$= \frac{V_R}{R} = \frac{V_{FS}}{R}$$

2R

Α



 Similarly, in the R-2R ladder type, output voltages of other 3bit binary words which are the DAC input can be calculated

1. Analog to digital converter

• Accepts analog input voltage V_a and produces output binary word $d_1d_2\dots d_n$ with functional value $D=d_12^{-1}+d_22^{-2}+\dots+d_n2^{-n}$

• MSB: *d*₁

• LSB: *d*_n

 Two control lines: START (when to start) and EOC: End of conversion)

 ADCs may give digital output to Microprocessors(interfacing) or Digital dive LCD or LED displays

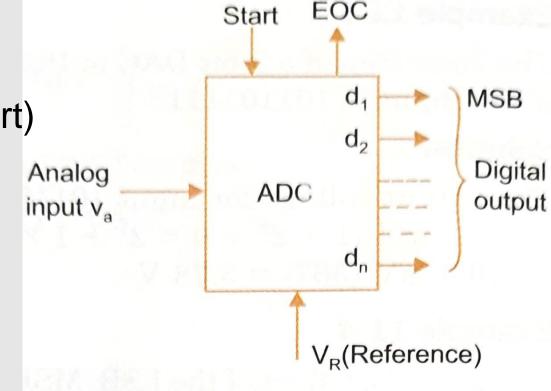


Fig. 11.9 Functional diagram of ADC

1. Analog to digital converter

ADCs classification

- Direct type ADC (Compare given analog signal with internally generated equivalent signal)
 - Flash (comparator) type converter
 - Counter type converter
 - Tracking or servo converter
 - Successive approximation type converter (Most commonly used)
- Integrating type ADC (first changes analog input to linear function of time and frequency, and then to digital code) – Most commonly used
- Charge balancing ADC
- Dual slope ADC

2.1 Direct: Parallel Comparator (Flash) A/D

input V.

Converter

- Simplest A/D converter, fastest and most expensive technique
- 3Bit A/D converter:
 Resistive divider (equal R for equal divisions)
 8 op-amp comparators
 8-line to 3 line encoder (3 bit priority encoder)

Voltage input	Logic output X
$V_{\rm a} > V_{\rm d}$	X = 1
$V_{\rm a} < V_{\rm d}$	X = 0
$V_{\rm a} = V_{\rm d}$	Previous value

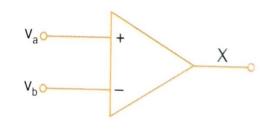
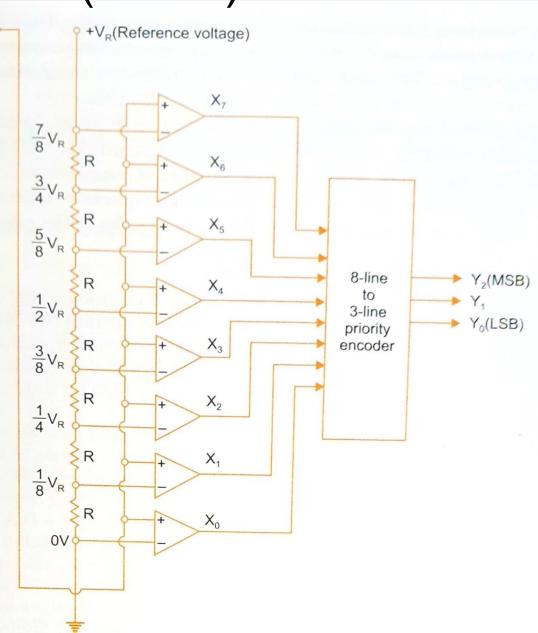
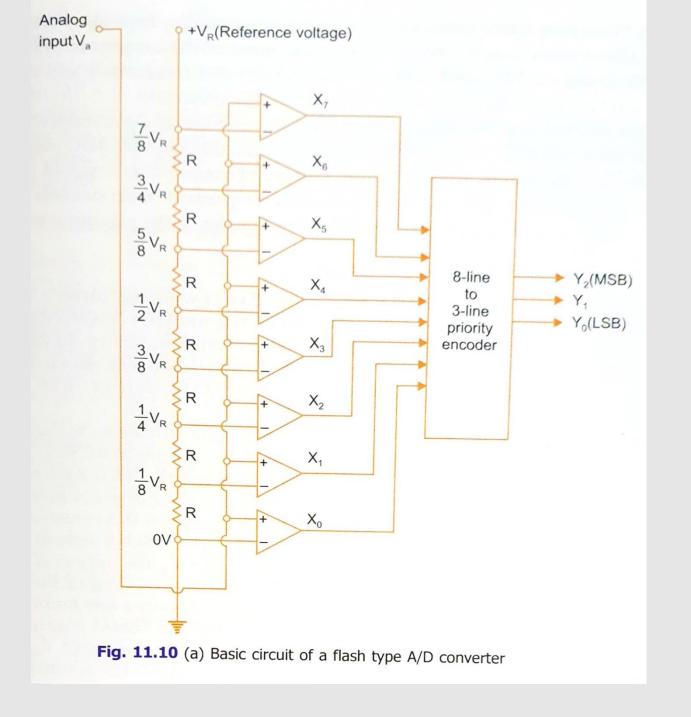


Fig. 11.10 (b) Comparator and its truth table





2.1 Direct: Parallel Comparator (Flash) A/D

input V.

Converter

- Circuit compares: Voltage levels are available at non-inverting terminals and divided between VR (reference voltage) and ground
- Circuit compares analog input V_a with each node voltages
- High speed conversion with conversion time less than 100ns

Voltage input	Logic output X				
$V_{\rm a} > V_{\rm d}$	X = 1				
$V_{\rm a} < V_{\rm d}$	X = 0				
$V_a = V_d$	Previous value				

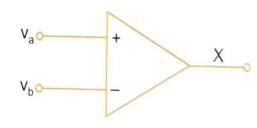
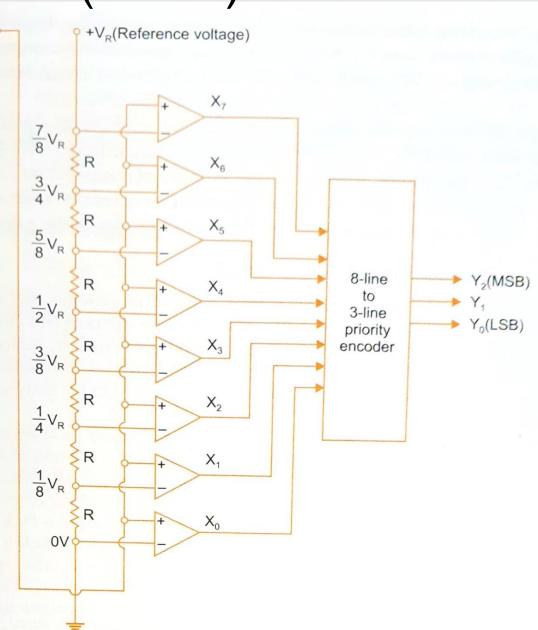


Fig. 11.10 (b) Comparator and its truth table



2.1 Direct: Parallel Comparator (Flash) A/D Converter

			-								
Input voltage V _a	X7	X_6	X_5	X_4	X_3	X_2	X_1	X_{O}	Y2	Y_1	Yo
0 to V _R /8	0	0	0	0	0	0	0	1	0	0	0
$V_R/8$ to $V_R/4$	0	0	0	0	0	0	1	1	0	0	1
$V_R/4$ to 3 $V_R/8$	0	0	0	0	0	1	1	1	0	1	0
$3 V_R/8$ to $V_R/2$	0	0	0	0	1	1	1	1	0	1	1
$V_R/2$ to 5 $V_R/8$	0	0	0	1	1	1	1	1	1	0	0
$5 V_R/8 \text{ to } 3 V_R/4$	0	0	1	1	1	1	1	1	1	0	1
$3 V_R/4 \text{ to } 7 V_R/8$	0	1	1	1	1	1	1	1	1	1	0
$7 V_R/8$ to V_R	1	1	1	1	1	1	1	1	1	1	1

Logic output X
X = 1
X = 0
Previous value

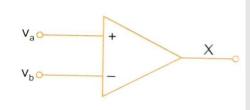
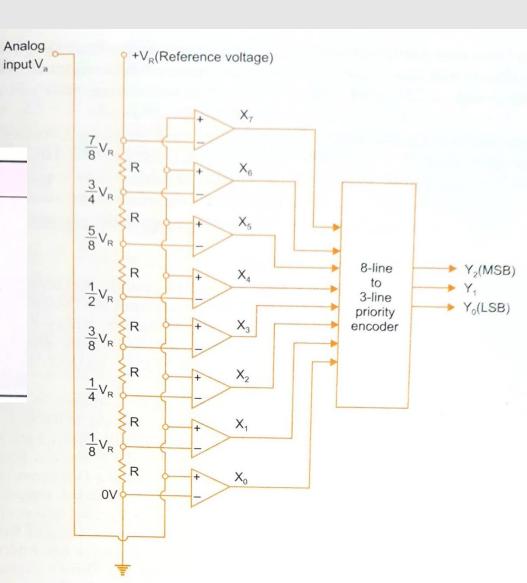


Fig. 11.10 (b) Comparator and its truth table



2.1 Direct: Parallel Comparator (Flash) A/D Converter

• Disadvantage: Number of comparators double for each bit increase $(2^n - 1$ comparators for n bit)

Input voltage V _a	X7	X_6	<i>X</i> ₅	X_4	X_3	X_2	X_1	X ₀	Y2	Y ₁	Yo
0 to V _R /8	0	0	0	0	0	0	0	1	0	0	0
$V_R/8$ to $V_R/4$	0	0	0	0	0	0	1	1	0	0	1
$V_R/4$ to 3 $V_R/8$	0	0	0	0	0	1	1	1	0	1	0
$3 V_R/8$ to $V_R/2$	0	0	0	0	1	1	1	1	0	1	1
$V_R/2$ to 5 $V_R/8$	0	0	0	1	1	1	1	1	1	0	0
5 V _R /8 to 3 V _R /4	0	0	1	1	1	1	1	1	1	0	1
3 V _R /4 to 7 V _R /8	0	1	1	1	1	1	1	1	1	1	0
7 $V_R/8$ to V_R	1	1	1	1	1	1	1	1	1	1	1

Voltage input	Logic output X				
$V_{\rm a} > V_{\rm d}$	X = 1				
$V_{\rm a} < V_{\rm d}$	X = 0				
$V_{\rm a} = V_{\rm d}$	Previous value				

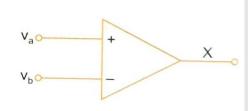


Fig. 11.10 (b) Comparator and its truth table

