Answer key

CAT-II

(i)

$$C_x = \begin{bmatrix} 3 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$y_1 = 8x_1 - 3x_2 - x_3$$

$$y_2 = -x_1 + 3x_2 - x_3$$

$$y_3 = x_1 + x_3,$$

$$C_y = \begin{bmatrix} 8 & -3 & -1 \\ -1 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 8 & -1 & 1 \\ -3 & 3 & 0 \\ -1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 99 & 18 & 9 \\ 13 & 36 & 0 \\ 9 & 0 & 9 \end{bmatrix}$$

(4) average power $P_{xx} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)\,d\omega$

$$= \frac{1}{2\pi}\left[4\omega - \frac{\omega^3}{3\times 4}\right]_{-4}^{4} = \frac{1}{2\pi}\left[16 - \frac{64}{27} + 16 - \frac{64}{27}\right]$$

$$= \frac{1}{2\pi} \times 32 = 16/\pi$$

Inverse FT $= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)\, e^{j\omega\tau}\, d\omega = \frac{1}{2\pi}\left[\int_{-4}^{4} 4 e^{j\omega\tau}\, d\omega - \int_{-4}^{4}\frac{\omega^2}{9} e^{j\omega\tau}\, d\omega\right]$

$$= \frac{1}{2\pi}\left[4e^{j\omega\tau}\left(\frac{\omega}{j\tau} + \frac{1}{\tau^2}\right) - \frac{1}{9}e^{j\omega\tau}\left(\frac{\omega^2}{j\tau} + \frac{2\omega}{\tau^2} - \frac{2}{j\tau^3}\right)\right]_{-4}^{4}$$

5(i) $R_{xx}(\tau) = 8e^{-3|\tau|}$

$$S_{xx}(\omega) = \frac{8\times 2\times 3}{\omega^2+9} = \frac{48}{\omega^2+9}$$

$$R_{xx}(0) = E[x^2(t)] = 8$$

(ii) $R_{xx}(a) = \{25^{+|a|} \quad a = 0, \pm 1, \pm 2, \pm 3, \ldots$

$$S_{xx}(\omega) = \sum_{-\infty}^{\infty} 25^{+|a|}\, e^{-j\omega a}$$

$$= \sum_{-\infty}^{-1} 25^{+a}\, e^{-j\omega a} + \sum_{0}^{\infty} 25^{+a}\, e^{-j\omega a}$$

$$= \frac{25}{25 - e^{-j\omega}} + \frac{1}{1 - 25e^{-j\omega}} = \frac{1-(25)^2}{(1+25)^2 - 2(25)\cos\omega}$$

$$\boxed{S_{xx}(\omega) = \frac{-624}{676 - 50\cos\omega}}$$

(6) $\bar{Y} = \bar{x} \int_{-\infty}^{\infty} h(t)\, dt$

$$= 6 \int t^3 e^{-8t}\, dt$$

$$= 6 \left[e^{-8t} \left[\frac{t^3}{(-8)} - \frac{3t^2}{8^2} + \frac{6t}{(-8)^3} - \frac{6}{8^4} \right] \right]$$

$$= 6 \times 6/8^4 = \frac{36}{4096}$$