(1) Assume that the joint sample space S_J has only three possible elements: (1, 1), (2, 1) and (3,3). The probabilities of these elements are to be $P(1,1) = 0.2$, $P(2,1) = 0.3$ and $P(3,3) = 0.5$

Find $F_{X,Y}(x,y)$ (Joint distribution function) & $f_{X,Y}(x,y)$ joint density function. Also Find ~~Also~~ $F_X(x)$, and $F_Y(y)$ for the joint sample space.

(2) Discrete random variables X and Y have a joint distribution function.

$$F_{X,Y}(x,y) = 0.10\, u(x+4)\, u(y-1) + 0.15\, u(x+3)\, u(y+5)$$
$$+ 0.17\, u(x+1)\, u(y-3) + 0.05\, u(x)\, u(y-1)$$
$$+ 0.18\, u(x-2)\, u(y+2) + 0.23\, u(x-3)\, u(y-4)$$
$$+ 0.12\, u(x-4)\, u(y+3)$$

Find (a) the marginal distribution $F_X(x)$ & $F_Y(y)$

(b) $\overline{X}$ and $\overline{Y}$

(c) the probability $P\{-1 < X \leq 4, -3 < Y \leq 3\}$

3) Find the value of the constant b so that the function

$$f_{X,Y}(x,y) = bxy^2 \exp(-2xy)\, u(x-2)\, u(y-1)$$

is a valid joint probability density

4) Random variables X and Y have the joint density

$$f_{X,Y}(x,y) = \frac{1}{12} u(x)\, u(y)\, e^{-(x/4)-(y/3)}$$

Find

(1) $P\{2 < X \leq 4, -1 < Y \leq 5\}$ and

(2) $P\{0 < X < \infty, -\infty < Y \leq -2\}$

5) Statistically independent random variables X and Y have respective densities

$$f_X(x) = 5u(x)\exp(-5x)$$

$$f_Y(y) = 2u(y)\exp(-2y)$$

Find the density of the sum $W = X+Y$

(6) Statistically independent random variables X and Y have probability densities

$$f_X(x) = \begin{cases} \frac{3}{32}(4-x^2) & -2 \le x \le 2 \\ 0 & \text{elsewhere in } x \end{cases}$$

$$f_Y(y) = \frac{1}{2}\left[u(y+1) - u(y-1)\right]$$

Find the exact probability density of the sum $W = X + Y$

(7) Let X be a random variable that has a mean value $\bar{X} = E[X] = 3$ and variance $\sigma_X^2 = 2$. Determine the second moment of X about the origin. Another random variable $Y = -6X + 22$. Also prove that X and Y are orthogonal; Is X and Y uncorrelated or not.

(8) Three Statistically independent random variables X_1, X_2 & X_3 have mean values $\overline{X_1} = 3$, $\overline{X_2} = 6$ and $\overline{X_3} = -2$. Find the mean values of the following functions

(a) $g(X_1, X_2, X_3) = X_1 + 3X_2 + 4X_3$

(b) $g(X_1, X_2, X_3) = X_1 X_2 X_3$

(c) $g(X_1, X_2, X_3) = -2X_1X_2 + 3X_1X_2 + 4X_2X_3$

(d) $g(X_1, X_2, X_3) = X_1 + X_2 + X_3$

(9) Discrete random variables X and Y have the joint density

$$f_{X,Y}(x,y) = 0.4\,\delta(x+\alpha)\,\delta(y-2) + 0.3\,\delta(x-\alpha)\,\delta(y-2) + 0.1\,\delta(x-\alpha)\,\delta(y-\alpha) + 0.2\,\delta(x-1)\,\delta(y-1)$$

Determine the values of α, if any that minimizes the correlation between X and Y

And find the minimum correlation.
Are X and Y orthogonal?

(10) Given $W = (aX + 3Y)^2$ where X and Y are zero mean random variables with variance $\sigma_X^2 = 4$ and $\sigma_Y^2 = 16$. Their correlation co-efficient is $\rho = -0.5$.

(a) Find a value for the parameter 'a' that minimizes the mean value of W.

(b) Find the minimum mean value.

(11) Let X and Y be statistically independent random variables with $\overline{X} = 3/4$, $\overline{X^2} = 4$, $\overline{Y} = 1$ and $\overline{Y^2} = 5$. For a random variable $W = X - 2Y + 1$. find (a) R_{XY} (b) R_{XW} (c) R_{YW} and (d) C_{XY} (e) Are X and Y uncorrelated?

(12) Statistically independent random variables X and Y have moments $m_{10} = 2$, $m_{20} = 14$, $m_{02} = 12$ and $m_{11} = -6$. Find the moment μ_{22}.

(13) Three random variables X_1, X_2 and X_3 represents samples of a random noise voltage taken at three times. Their covariance matrix is defined by

$$[C_X] = \begin{bmatrix} 3.0 & 1.8 & 1.1 \\ 1.8 & 3.0 & 1.8 \\ 1.1 & 1.8 & 1.1 \end{bmatrix}$$

A transformation matrix

$$[T] = \begin{bmatrix} 4 & -1 & -2 \\ 2 & 2 & 1 \\ -3 & -1 & 3 \end{bmatrix}$$

Converts the variables to new random variable Y_1, Y_2, Y_3. Find the co-variance matrix of the new random variables.

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(14) Two gaussian random variable X_1 & X_2 are defined by the mean and covariance matrices

$$[\bar{X}] = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \qquad [C_x] = \begin{bmatrix} 5 & -2/\sqrt{5} \\ -2/\sqrt{5} & 4 \end{bmatrix}$$

Two new random variables Y_1 and Y_2 are formed using the transformation

$$[T] = \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$$

Find the matrices (a) $[\bar{Y}]$, (b) $[C_y]$, and

(c) Find the correlation co-efficient of Y_1 & Y_2

(15) Determine a constant 'b' such that each of the following are valid joint density function

(a) $f_{X,Y}(x,y) = \begin{cases} 3xy & 0<x<1,\ 0<y<b \\ 0 & \text{elsewhere.} \end{cases}$

(b) $f_{X,Y}(x,y) = \begin{cases} bx(1-y) & 0<x<0.5 \ \& \ 0<y<1 \\ 0 & \text{elsewhere.} \end{cases}$

(c) $f_{X,Y}(x,y) = \begin{cases} b(x^2+4y^2) & 0\le|x|<1 \text{ and } 0\le y<2 \\ 0 & \text{elsewhere} \end{cases}$