Q. 4.6.3-

Random variables X and Y have respective density function

$$f_X(X) = \frac{1}{a}[u(x) - u(x-a)].$$

$$f_Y(Y) = b u(y) e^{-by}.$$

where a>0 & b>0. Find & sketch the density fxn W = X + Y if X and Y are statistically indep.

Sol^n

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y) f_X(w-y)\, dy.$$

$$= \int_0^{\infty} b e^{-by} \frac{1}{a} [u(w-y) - u(w-y-a)] \, dy.$$

$$= \int_0^{w} \frac{b}{a} e^{-by} dy - \int_0^{w-a} \frac{b}{a} e^{-by} dy.$$

$= 0$ for $w \leq a$.

20BEC0338

Case (i): $0 < w \le a$.

$$f_W(w) = \int_0^w \frac{b}{a} e^{-by}\, dy = \frac{1}{a}\left[1 - e^{-bw}\right].$$

Case (ii) $a < w$

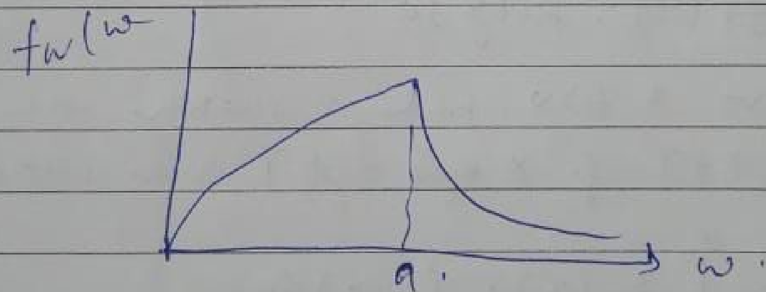
$$f_W(w) = \frac{1}{a}\left[1 - e^{-bw}\right] - \frac{b}{a}\int_0^{w-a} e^{-by}\, dy.$$

$$= \frac{e^{-bw}}{a}\left[e^{ba} - 1\right].$$

$$f_W(w) = 0 \;; \quad w \le 0.$$

$$= \frac{1}{a} e^{-bw}\left(e^{ba} - 1\right), \; a < w$$

$$= \frac{1}{a} e^{-bw}\left(e^{ba} - 1\right), \; a < w$$



Q. 4. 6. 4. Random variables X and Y have respectively density functions

$$f_X(x) = 0.1\delta(x-1) + 0.2\delta(x-2) + 0.4\delta(x-3) + 0.3\delta(x-4).$$

$$f_Y(y) = 0.4\delta(y-5) + 0.5\delta(y-6) + 0.1\delta(y-7).$$

Find and sketch the density function $W = X + Y$ if X and Y are independent.

Sol^n $f_W(w) = \int_{-\infty}^{\infty} f_Y(y)\, f_X(w-y)\, dy$

$= \int_{-\infty}^{\infty} [0.4\delta(y-5) + 0.5\delta(y-6) + 0.1\delta(y-7)]$

$[0.1\delta(w-y-1) + 0.2\delta(w-y-2) + 0.4\delta(w-y-3)$

$+ 0.3\delta(w-y-4)]\, dy.$

each integral is of form

$K\int_{-\infty}^{\infty} \delta(y-y_K)\,\delta(w-y-x_m)\, dy$ where

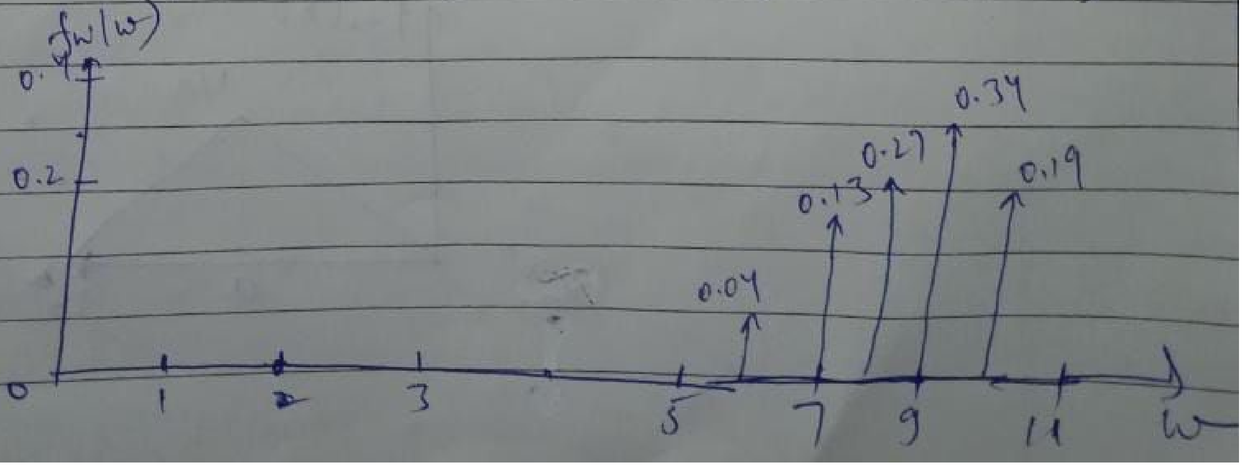
y_K and x_m are numbers & K is a constant

$w = x_m + y_K$ & the integral equals

$K\delta(w-y_K-x_m)$. Hence,

$f_W(w) = 0.04\delta(w-6) + 0.08\delta(w-7) + 0.16\delta(w-8)$
$+ 0.12\delta(w-9) + 0.05\delta(w-7) + 0.10\delta(w-8)$
$+ 0.20\delta(w-9) + 0.15\delta(w-10) + 0.01\delta(w-8)$
$+ 0.02\delta(w-9) + 0.04\delta(w-10) + 0.03\delta(w-11).$

$f_W(w) = 0.04\delta(w-6) + 0.13\delta(w-7) + 0.27\delta(w-8)$
$+ 0.34\delta(w-9) + 0.19\delta(w-10) + 0.03\delta(w-11)$



20BEC0338.

Date / /
Page

Q.4.6.7. Three statistically independent random variables X_1, X_2 and X_3 all have the same density fxn,

$$f_{X_i}(x_i) = \frac{1}{a}[u(x_i) - u(x_i - a)] \quad i = 1,2,3.$$

Find and sketch the density fxn of $Y = X_1 + X_2 + X_3$ if $a > 0$ is constant.

sol(Ans)

$W_1 = X_1 + X_2$.

then $f_{W_1}(w) = \int_{-\infty}^{\infty} f_{X_1}(x) \cdot f_{X_2}(w_1 - x)\,dx$

$$= \int_{-\infty}^{\infty} \frac{1}{a^2}[u(x) - u(x-a)][u(w_1 - x) - u(w_1 - x - a)]\,dx$$

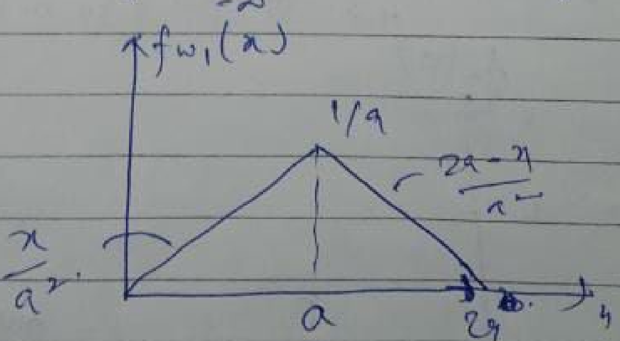
$f_{W_1}(w_1) = 0, \quad w_1 \le 0$

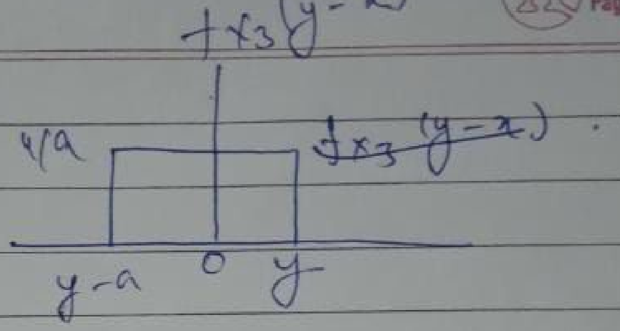
$= w_1/a^2, \quad 0 < w_1 \le a.$

$= (2a - w_1)/a^2, \quad a < w_1 \le 2a.$

$= 0, \quad 2a < w_1.$

let $Y = W_1 + X_3$, $f_Y(y) = \int_{-\infty}^{\infty} f_{W_1}(x)\, f_{X_3}(y - x)\,dx$





$f_Y(y) = 0,\ y < 0.$

$f(Y)(y) = \int_0^y \frac{x}{a^3}\,dx = y^2/2a^3,\quad 0 < y \le a.$

~~f~~ $f_Y(y) = \int_{y-a}^{a} \frac{x}{a^3}\,dx + \int_a^y \frac{(2a-x)}{a^3}\,dx.$

$$= \frac{-2y^2 + 6ay - 3a^2}{2a^3}$$

$f_Y(y) = \int_{y-a}^{2a} \frac{(2a-x)}{a^3}\,dx$

$$= \frac{(y-3a)^2}{2a^3},\quad 2a < y \le 3a.$$

$f_Y(y) = 0,\quad 3a < y$