Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	**B.Tech**	Semester	**Winter Semester 2022-23**
Course Title	**RANDOM PROCESSES**	Course Code	**BECE207L**
Faculty Name	Prof. **Chandrasekaran N**	Slot	**B1+TB1**
		Class Nbr	CH2022235000477
Time	**3 Hours**	Max. Marks	**100**

Section A (2 X 5 Marks)
Answer All questions

01. Two Random variables X and Y have the following Joint PDF. [5]

$$f_{XY}(x,y) = \begin{cases} e^{-x(2y+1)} & 0 \le x \le \infty, 0 \le y \le \infty \\ 0 & otherwise \end{cases}$$

(i) Determine the conditional PDF of X given Y

(ii) Determine the conditional PDF of Y given X

02. Suppose the discrete autocorrelation function of a random sequence is given by, [5]

$R_{XX}[a] = \{5^{|a|}$, for $a = 0, \pm 1, \pm 2, \pm 3 \ldots$

compute its power spectral density.

Section B (9 X 10 Marks)
Answer All questions

03. Let X, Y, Z be three jointly continuous random variables with joint PDF: [10]

$$f_{XYZ}(x,y,z) = \begin{cases} k(2x + y + 4z)) & 0 \le x, y, z \le 1 \\ 0 & otherwise \end{cases}$$

(i) Find the value k.

(ii) Find the marginal PDF of X and Y.

04. Random variables X_1 and X_2 have the joint characteristic function: [10]

$\phi_{X_1,X_2}(\omega_1, \omega_2) = [(1 - j2\omega_1)(1 - j2\omega_2)]^{-(\frac{N}{2})}$

where N > 0 is an integer.

(i) Find the correlation, and moments (m_{02} and m_{20}).

(ii) Determine mean of X_1 and X_2.

05. For two random variable X and Y, [10]

$$f_{XY}(x,y) = 0.2\,\delta(x)\delta(y+1) + 0.1\delta(x)\delta(y) + 0.1\delta(x-2)\delta(y)$$
$$+ 0.4\delta(x+2)\delta(y-1) + 0.3\delta(x-1)\delta(y-1) + 0.5\delta(x-3)\delta(y-1)$$

Find

(i) Correlation

(ii) Covariance

(iii) The correlation coefficient of X and Y.

(iv) Are X and Y either uncorrelated or orthogonal?

06. A random process is defined by, [10]

$X(t)\ \mathrm{Sin}(\omega_0 t + \theta)$

where X(t) is a wide sense stationary random process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase θ independent of X(t) and uniformly distributed on $(-\pi,\ \pi)$.

(i) Find $E[Y(t)]$.

(ii) Find the autocorrelation function of $Y(t)$.

(iii) Is $Y(t)$ wide sense stationary?

07. Consider two jointly W. S. S. random processes $X(t)$ and $Y(t)$, with known $R_X(\tau)$, $R_Y(\tau)$, and $R_{XY}(\tau)$. Let $P(t) = X(t) + Y(t)$ and $Q(t) = X(t) - Y(t)$. [10]

(i) Find the auto-correlation functions of $P(t)$ and $Q(t)$.

(ii) Find the cross-correlation function $R_{XP}(t, t+\tau)$.

(iii) Find the cross-correlation function $R_{PQ}(t, t+\tau)$.

08. A random process is given by,

$W(t) = e^{-\alpha}X(t) - e^{-\beta}Y(t)$ [10]

Where α and β are constants, and $X(t)$ and $Y(t)$ are jointly wide sense stationary processes.

(i) Find $S_{WW}(\omega)$ if $X(t)$ and $Y(t)$ are uncorrelated.

(ii) Find the cross power spectrum $S_{XW}(\omega)$ and $S_{YW}(\omega)$.

09. A WSS process $X(t)$ with mean value 5 and power spectrum:

$S_{XX}(\omega) = \frac{4}{16+\omega^2} + \frac{9}{25+\omega^2} + 20\pi\delta(\omega)$ [10]

is applied to a network with impulse response.

$h(t) = 4te^{-4t}u(t)$

(i) Find $H(w)$ for the network.

(ii) Determine the mean $\overline{Y}$.

(iii) Determine the power spectrum of the response $Y(t)$.

10. A random noise X(t) having power spectrum

$S_{XX}(\omega) = 6\omega^2$

is applied to the differentiator that has a transfer function $H_1(\omega) = \frac{2}{3+j\omega}$. [10]

The differentiator's output is applied to a network for which,

$h_2(t) = t^2e^{-3t}u(t)$

The network's response is a noise denoted by $Y(t)$.

(i) What is the average power of $X(t)$ over the interval (0, 2)?

(ii) Find the Power spectrum of $Y(t)$.

11. The auto correlation functions of a random signal $X(t)$ and additive uncorrelated noise $N(t)$ are: [10]

$R_{XX}(\tau) = \frac{5}{8}e^{-4|\tau|}$ and

$R_{NN}(\tau) \equiv \frac{7}{6}e^{-3|\tau|}$

(i) Find the power spectrum of $X(t)$ and $N(t)$.

(ii) Find the transfer function of the Wiener filter for the given signal and noise.

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