TRIGONOMETRIC IDENTITIES

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x)$$

$$\cos(2x) = \cos^{2}(x) - \sin^{2}(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$2\cos(x) = e^{jx} + e^{-jx}$$

$$2j\sin(x) = e^{jx} - e^{-jx}$$

$$2\sin(x)\sin(y) = \cos(x - y) + \cos(x + y)$$

$$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$$

$$2\cos^{2}(x) = 1 + \cos(2x)$$

$$2\sin^{2}(x) = 3\cos(x) + \cos(3x)$$

$$4\sin^{3}(x) = 3\sin(x) - \sin(3x)$$

$$8\cos^{4}(x) = 3 + 4\cos(2x) + \cos(4x)$$
(C-1)
$$(C-2)$$

$$\cos(x) + \cos(x) + \cos(x)$$

INDEFINITE INTEGRALS

Rational Algebraic Functions

$$\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad 0 < n$$
 (C-20)

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln|a+bx|$$
 (C-21)

$$\int \frac{dx}{(a+bx)^n} = \frac{-1}{(n-1)b(a+bx)^{n-1}}$$
 1 < n (C-22)

$$\int \frac{dx}{c + bx + ax^2} = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \left(\frac{2ax + b}{\sqrt{4ac - b^2}} \right) \qquad b^2 < 4ac$$

$$= \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| \qquad b^2 > 4ac$$

$$= \frac{-2}{2ax + b} \qquad b^2 = 4ac \qquad (C-23)$$

$$\int \frac{x \, dx}{c + bx + ax^2} = \frac{1}{2a} \ln|ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{c + bx + ax^2}$$
 (C-24)

$$\int \frac{dx}{a^2 + b^2 x^2} = \frac{1}{ab} \tan^{-1} \left(\frac{bx}{a} \right)^{1/2 + 1/2} = (1)^{1/2 + 1/2}$$
(C-25)

$$\int \frac{x \, dx}{a^2 + x^2} = \frac{1}{2} \ln(a^2 + x^2) \qquad \text{(C-26)}$$

$$\int \frac{x^2 dx}{a^2 + x^2} = x - a \tan^{-1} \left(\frac{x}{a}\right)^{\frac{1}{2}} \int_{0}^{\infty} e^{-\frac{x^2}{a^2 + x^2}} dx = (x)^{\frac{1}{2}} \int_{0}^{\infty$$

$$\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \left(\frac{x}{a}\right)$$
 (C-28)

$$\int \frac{x \, dx}{(a^2 + x^2)^2} = \frac{-1}{2(a^2 + x^2)} \tag{C-29}$$

$$\int \frac{x^2 dx}{(a^2 + x^2)^2} = \frac{-x}{2(a^2 + x^2)} + \frac{1}{2a} \tan^{-1} \left(\frac{x}{a}\right)$$
 (C-30)

$$\int \frac{dx}{(a^2 + x^2)^3} = \frac{x}{4a^2(a^2 + x^2)^2} + \frac{3x}{8a^4(a^2 + x^2)} + \frac{3}{8a^5} \tan^{-1} \left(\frac{x}{a}\right)$$
 (C-31)

$$\int \frac{x^2 dx}{(a^2 + x^2)^3} = \frac{-x}{4(a^2 + x^2)^2} + \frac{x}{8a^2(a^2 + x^2)} + \frac{1}{8a^3} \tan^{-1} \left(\frac{x}{a}\right)$$
 (C-32)

$$\int \frac{x^4 dx}{(a^2 + x^2)^3} = \frac{a^2 x}{4(a^2 + x^2)^2} - \frac{5x}{8(a^2 + x^2)} + \frac{3}{8a} \tan^{-1} \left(\frac{x}{a}\right)$$
 (C-33)

$$\int \frac{dx}{(a^2 + x^2)^4} = \frac{x}{6a^2(a^2 + x^2)^5} + \frac{5x}{24a^4(a^2 + x^2)^2} + \frac{5x}{16a^6(a^2 + x^2)} + \frac{5}{16a^7} \tan^{-1} \left(\frac{x}{a}\right)$$
(C-34)

$$\int \frac{x^2 dx}{(a^2 + x^2)^4} = \frac{-x}{6(a^2 + x^2)^3} + \frac{x}{24a^2(a^2 + x^2)^2} + \frac{x}{16a^4(a^2 + x^2)} + \frac{x}{16a^5} \tan^{-1}\left(\frac{x}{a}\right)$$
(C-35)

$$\int \frac{x^4 dx}{(a^2 + x^2)^4} = \frac{a^2 x}{6(a^2 + x^2)^3} - \frac{7x}{24(a^2 + x^2)^2} + \frac{x}{16a^2(a^2 + x^2)} + \frac{1}{16a^3} \tan^{-1} \left(\frac{x}{a}\right)$$
(C-36)

$$\int \frac{dx}{a^4 + x^4} = \frac{1}{4a^3\sqrt{2}} \ln\left(\frac{x^2 + ax\sqrt{2} + a^2}{x^2 - ax\sqrt{2} + a^2}\right) + \frac{1}{2a^3\sqrt{2}} \tan^{-1}\left(\frac{ax\sqrt{2}}{a^2 - x^2}\right)$$
(C-37)

$$\int \frac{x^2 dx}{a^4 + x^4} = -\frac{1}{4a\sqrt{2}} \ln \left(\frac{x^2 + ax\sqrt{2} + a^2}{x^2 - ax\sqrt{2} + a^2} \right) + \frac{1}{2a\sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) \tag{C-38}$$

Trigonometric Functions

$$\int \cos(x) dx = \sin(x)$$
(C-39)

$$\int \cos(x) dx = \sin(x)$$
(C-39)
$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$
(C-40)

$$\int x^2 \cos(x) \, dx = 2x \cos(x) + (x^2 - 2) \sin(x) \tag{C-41}$$

$$\int \sin(x) \, dx = -\cos(x) \tag{C-42}$$

$$\int x \sin(x) dx = \sin(x) - x \cos(x)$$
 (C-43)

$$\int x^2 \sin(x) \, dx = 2x \sin(x) - (x^2 - 2) \cos(x) \tag{C-44}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} \qquad a \text{ real or complex}$$
 (C-45)

$$\int xe^{ax} dx = e^{ax} \left[\frac{x}{a} - \frac{1}{a^2} \right] \qquad a \text{ real or complex}$$
(C-46)

$$\int x^2 e^{ax} dx = e^{ax} \left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right] \qquad a \text{ real or complex}$$
 (C-47)

$$\int x^3 e^{ax} \, dx = e^{ax} \left[\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4} \right] \qquad a \text{ real or complex}$$
 (C-48)

$$\int e^{ax} \sin(x) \, dx = \frac{e^{ax}}{a^2 + 1} [a \sin(x) - \cos(x)] \tag{C-49}$$

$$\int e^{ax} \cos(x) \, dx = \frac{e^{ax}}{a^2 + 1} [a \cos(x) + \sin(x)] \tag{C-50}$$

 $\int \frac{dx}{dx} = \frac{1}{4\pi^2 \sqrt{3}} \ln \left(\frac{x^2 + ax\sqrt{2} + a^2}{2a^2 \sqrt{2}} + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} \tan^{-1} \left(\frac{ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} + \frac{1-ax\sqrt{2}}{a^2 - x^2} \right) + \frac{1-ax}{2a^2 \sqrt{2}} + \frac{1-ax}{2a^2 \sqrt{2}} + \frac{1-ax}{2a^2$

DEFINITE INTEGRALS

(14-1)

$$\int_{-\infty}^{\infty} e^{-a^2 x^2 + bx} dx = \frac{\sqrt{\pi}}{a} e^{b^2/(4a^2)} \qquad a > 0$$
 (C-51)

$$\int_0^\infty x^2 e^{-x^2} \, dx = \sqrt{\pi}/4 \tag{C-52}$$

$$\int_0^\infty \operatorname{Sa}(x) \, dx = \int_0^\infty \frac{\sin(x)}{x} \, dx = \frac{\pi}{2} \tag{C-53}$$

$$\int_{0}^{\infty} \text{Sa}^{2}(x) \, dx = \pi/2 \tag{C-54}$$

 $v = cos(x) dx = 2v \cos(x) + (x^2 - 2) \sin(x)$

FINITE SERIES

$$\sum_{n=1}^{N} n = \frac{N(N+1)}{2} \tag{C-55}$$

$$\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$
 (C-56)

$$\sum_{n=1}^{N} n^3 = \frac{N^2(N+1)^2}{4} \tag{C-57}$$

$$\sum_{n=0}^{N} x^n = \frac{x^{N+1} - 1}{x - 1} \tag{C-58}$$

$$\sum_{n=0}^{N} \frac{N!}{n!(N-n)!} x^n y^{N-n} = (x+y)^N$$
 (C-59)

$$\sum_{n=0}^{N} e^{j(\theta + n\phi)} = \frac{\sin[(N+1)\phi/2]}{\sin(\phi/2)} e^{j[\theta + (N\phi/2)]}$$
(C-60)

$$\sum_{n=0}^{N} {N \choose n} = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} = 2^{N}$$
(C-61)

$$\sum_{n=N_1}^{N_2} w^n = \frac{w^{N_1} + w^{N_2+1}}{1 - w} \qquad \begin{cases} N_2 > N_1 \text{ and } w \\ \text{real or complex} \end{cases}$$
(C-62)

INFINITE SERIES in parties transform puriSeries (D-1) and (D-2) form a fourier transform puriSeries (D-1) and (D-1) an

general, is a complex finicion of a even for real signals x(a). Yind describes the relative complex contages (amplicates and phases) as a timetion of that are present in a waveform will from (D-1), be see that the unit of X(a) is volts per force if a(t) is a voltage-time wavelerm. Time, A(cc) can be consider sed as the discrete of voltage in vertice increase of sugular frequency as

TABLE E-1
Fourier Transform Pairs

Pair	x(t)	$X(\omega)$	Notes
1	$\alpha\delta(t)$	α	properties also and the second
2	u(t)	$\alpha\delta(\omega)$ $\pi\delta(\omega) + (1/j\omega)$	able o
4	$\frac{1}{2}\delta(t) - \frac{1}{j2\pi t}$	$u(\omega)$	
5	$rect(t/\tau)$	$\tau \operatorname{Sa}(\omega \tau/2)$	$\tau > 0$
6	(W/π) Sa (Wt)	$rect (\omega/2W)$	W > 0
7	tri (t/τ)	$\tau \operatorname{Sa}^2(\omega \tau/2)$	$\tau > 0$
8	(W/π) Sa ² (Wt)	$\operatorname{tri}(\omega/2W)$	W > 0
9	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
10	$\delta(t- au)$	$e^{-j\omega \tau}$	
11	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
12	$\sin(\omega_0 t)$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	
13	$u(t)\cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	Table E-1 o
[14]	$u(t)\sin(\omega_0 t)$	$-j\frac{\pi}{2}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$u(t)e^{-\alpha t}$	$\overline{\alpha+j\omega}$	$\alpha > 0$
(5-8)		$\frac{\overline{\alpha + j\omega}}{1}$ $\frac{1}{(\alpha + j\omega)^2}$	
16	$u(t)te^{-\alpha t}$		$\alpha > 0$
(17-1)	$u(t)t^2e^{-\alpha t}$	$\frac{2}{(\alpha+j\omega)^3} = (3)62$	$\alpha > 0$
(18-()	$u(t)t^3e^{-\alpha t}$	$(\alpha + j\omega)^{3}$ $\frac{6}{(\alpha + j\omega)^{4}}$ $\frac{2\alpha}{\alpha^{2} + \omega^{2}}$ $\sigma\sqrt{2\pi}e^{-\sigma^{2}\omega^{2}/2}$	$\alpha > 0$
19	$e^{-\alpha I }$	$\alpha^2 + \omega^2$	$\alpha > 0$
20	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	$\sigma > 0$