Tutorial-I

(All small boldfaced letters represent Random Variables ex: **z)**

1.

The random variables $\mathbf{x}$ and $\mathbf{y}$ are jointly distributed over the region $0 < x < y < 1$ as

$$f_{xy}(x, y) = \begin{cases} kx & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

for some k. Determine k. Find the variances of $\mathbf{x}$ and $\mathbf{y}$. What is the covariance between $\mathbf{x}$ and $\mathbf{y}$?

(Hint: COV(X, Y)=E[XY]-E[X].E[Y]}

2.

Let $\mathbf{x}$ and $\mathbf{y}$ be independent random variables with variances σ_1^2 and σ_2^2, respectively. Consider the sum

$$\mathbf{z} = a\mathbf{x} + (1 - a)\mathbf{y} \qquad 0 \le a \le 1$$

Find a that minimizes the variance of $\mathbf{z}$.

(Hint: Find the variance of Z and differentiate with respect to ‘a’ and equate it to zero.

Also, the find the minimum variance by substituting the vales of ‘a’ obtained in the earlier step to get the minimum variance of Z.

3.

▶ Given

$$f_{xy}(x, y) = \begin{cases} k & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

determine $f_{x|y}(x \mid y)$ and $f_{y|x}(y \mid x)$.

(Hint: First find ‘k’ and use point conditioning formula.

4.

$\mathbf{x}$ and $\mathbf{y}$ are zero mean independent random variables with variances σ_1^2 and σ_2^2, respectively, that is, $\mathbf{x} \sim N(0, \sigma_1^2)$, $\mathbf{y} \sim N(0, \sigma_2^2)$. Let

$$\mathbf{z} = a\mathbf{x} + b\mathbf{y} + c \qquad c \neq 0$$

(*a*) Find the characteristic function $\Phi_z(u)$ of $\mathbf{z}$. (*b*) Using $\Phi_z(u)$ conclude that $\mathbf{z}$ is also a normal random variable. (*c*) Find the mean and variance of $\mathbf{z}$.

5.

x and y are independent uniform random variables in the common interval (0, 1). Determine $f_z(z)$, where $\mathbf{z} = \mathbf{x} + \mathbf{y}$. Clearly,

$$\mathbf{z} = \mathbf{x} + \mathbf{y} \Rightarrow 0 < z < 2$$

and as Fig. 6-11 shows there are two cases for which the shaded areas are quite different in shape, and they should be considered separately.

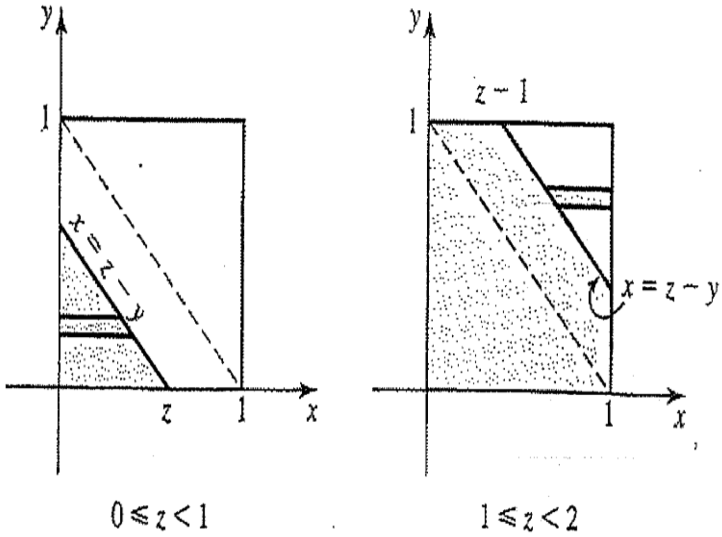

Fig. 6.11