(i) $\Phi_{xy}(\omega_1, \omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$

(i) Find correlation & mean of x & y

$$\bar{x} = m_{10} = -j \frac{\partial \Phi_{x,y}(\omega_1, \omega_2)}{\partial \omega_1} \Bigg|_{\omega_1 = 0 \ \omega_2 = 0}$$

$$= -j(-4\omega_1)\exp(-2\omega_1^2 - 8\omega_2^2)\Big|_{\omega_1 = 0,\ \omega_2 = 0}$$

$$\boxed{\bar{x} = 0}$$

$$\bar{y} = -j(-16\omega_2)\exp(-2\omega_1^2 - 8\omega_2^2)\Big|_{\omega_1 = 0 \ \omega_2 = 0}$$

$$\boxed{\bar{y} = 0}$$

$$R_{xy} = E[xy] = m_{11} = (-j)^2 \frac{\partial^2}{\partial \omega_1 \partial \omega_2} \exp(-2\omega_1^2 - 8\omega_2^2)$$

$$= -(-4\omega_1)(-16\omega_2)\exp(-2\omega_1^2 - 8\omega_2^2)\Big|_{\omega_1 = 0 \ \omega_2 = 0}$$

$$\boxed{R_{xy} = 0}$$

Scanned with OKEN Scanner

(2) $y(t) = x(t)\sin(\omega_0 t+\theta)$

(i)

$R_{yy}(t,t+\tau) = R_{xx}(\tau)\cdot\frac{1}{2}E\left[\cos(\omega_0\tau) - \cos(2\omega_0 t+\omega_0\tau+2\theta\right.$

$R_{yy}(t,t+\tau) = \frac{1}{2}R_{xx}(\tau)\cos(\omega_0\tau)$

$= R_{yy}(\tau)$

(ii) $E[y(t)] = 0$

(iii) $y(t)$ is WSS.

③ $z(t) = 0.4x(t) - 0.9y(t)$

(i) $S_{xx}(\omega) = 0.16\, S_{xx}(\omega) - 0.81\, S_{yy}(\omega) - 0.36\, S_{xy}(\omega)$
$- 0.36\, S_{yx}(\omega)$

If uncorrelated

(ii) $S_{xx}(\omega) = 0.16\, S_{xx}(\omega) - 0.81\, S_{xy} - 4\pi(0.36)\, \bar{X}\bar{Y}\, \delta($

$S_{xz}(\omega) = 0.4\, S_{xx}(\omega) - 0.9\, S_{\cancel{x}y}(\omega)$

(iii) $S_{x2}(\omega) = 0.4 S_{xx}(\omega) - 0.9 S_{xy}(\omega)$

(4) $R_{\alpha\alpha}(\tau) = A_0 \sin(2\pi\tau)$

$R_{\beta\beta}(\tau) = B_0 e^{-9(\tau/2)}$

$R_{w_1w_1}(t) = R_{\alpha\alpha}(\tau) + R_{\beta\beta}(\tau)$

$= A_0 \sin 2\pi\tau + B_0 e^{-9|\tau/2|}$

(ii) $W_2(t) = \alpha(t) - \beta(t)$

$R_{w_2w_2}(t) = R_{\alpha\alpha}(\tau) + R_{\beta\beta}(\tau)$

$= B_0 e^{-9|\tau/2|} + A_0 \sin 2\pi\tau$

(iii) $R_{w_1w_2}(t) = R_{xx}(\tau) - R_{yy}(\tau)$

$= A \sin 2\pi\tau - B_0 e^{-9|\tau/2|}$

$\cancel{y(t) = A \sin(\omega_0 t + \theta) + N(t)}$

(5)(i) $R_{xx}[k] = \delta(k) + (0.1)^{|k|}$ for $k = 0, \pm 1, \pm 2 \ldots$

ie $\boxed{|k| \leq \infty}$

$R_x(k) \leftrightarrow S_{xx}(\omega)$

$\delta(m) \leftrightarrow 1$

$\cancel{S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \theta^{|m|} e^{-j}}$

$$S_{xx}(\omega) = \sum_{k=-\infty}^{\infty} (0.1)^{|k|} e^{-jk\Omega}$$

$$= \sum_{k=-\infty}^{-1} (0.1)^{-k} e^{-jk\Omega} + \sum_{k=0}^{\infty} (0.1)^{k} e^{-jk\Omega}$$

$$= -\frac{0.1}{0.1 - e^{-j\Omega}} + \frac{1}{1 - 0.1e^{-j\Omega}}$$

$$= \frac{1-(0.1)^2}{(1+0.1)^2 - 2(0.1)\cos\Omega} = \frac{0.9999}{1.01 - 0.2\cos\Omega}$$

$$\boxed{S_{xx}(\omega) = 1 - \frac{0.9999}{1.01 - 0.2\cos\Omega}}$$

(ii) $R_x[m] = \begin{cases} 1 - 0.2|m| & |m| \le 4 \\ 0 & |m| > 4 \end{cases}$

$$= \sum_{m=-4}^{-1} (1 - 0.2(-m))\, e^{-jm\omega} + \sum_{m=0}^{4} (1-(0.2)m)\, e^{-jm\omega}$$

$$= 0.2\, e^{j4\omega} + 0.4\, e^{j3\Omega} + 0.6\, e^{j2\Omega} + 0.8\, e^{j\Omega} + 1 + 0.8\, e^{-j\Omega}$$
$$+ 0.6\, e^{-j2\Omega} + 0.4\, e^{-3j\Omega} + 0.2\, e^{-4j\Omega}$$

Cat-2

(6)

$h_1(t) = u(t)\, 4t \exp(-3t)$

$h_2(t) = u(t)\, 3 \exp(-2t)$

$$x(t) \longrightarrow \boxed{h_1(t)} \xrightarrow{y_1(t)} \boxed{h_2(t)} \longrightarrow y(t)$$

$$y_1(t) = \int_{-\infty}^{\infty} x(t-\xi_1)\, h_1(\xi_1)\, d\xi_1$$

$$y(t) \int_{-\infty}^{\infty} y_1(t-\xi_2)\, h_2(\xi_2)\, d\xi_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\xi_1-\xi_2)\, h(\xi_1)\, h(\xi_2)\, d\xi_1\, d\xi_2$$

$$= \int_0^{\infty}\int_0^{\infty} x(t-\xi_1-\xi_2)\, \cancel{h(\xi_1)}\, 3\,\xi_1 e^{-4\xi_1}\, 4 e^{-5\xi_2}\, d\xi_1\, d\xi_2$$

$$y(t) = 12 \int_0^{\infty}\int_0^{\infty} x(t-\xi_1-\xi_2)\,\xi_1\, e^{-(4\xi_1+5\xi_2)}\, d\xi_1\, d\xi_2$$

$$E[y(t)] = 12\,\overline{x} \int_0^{\infty}\int_0^{\infty} \xi_1 e^{-4\xi_1} \cdot e^{-5\xi_2}\, d\xi_2\, d\xi_2$$

$$\overline{y} = 12\times 6 \left[e^{-4\xi_1}\left(\frac{\xi_1}{-4} - \frac{1}{16}\right)\right]_0^{\infty} \times \left[\frac{e^{-5\xi_2}}{-5}\right]$$

$$\int x e^{ax} dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$= 12\times 6 \times \left[\frac{1}{16}\right] \times \frac{1}{5} = 6/10$$