VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Digital Assignment - 02

1. An experiment has four equally likely outcomes 0, 1, 2, and 3, i. e., $S = \{0, 1, 2, 3\}$. If a random process X_t is defined as $X_t = \cos(2\pi st)$ for all $s \in S$, then

   (a) Sketch all the possible sample functions.

   (b) Sketch the marginal CDF's of the random variables X_0, $X_{0.25}$ and $X_{0.5}$.

   (c) Determine the conditional pmf of $X_{0.25}$ given that $X_{0.5} = -1$.

   (d) Determine the conditional pmf of $X_{0.25}$ given that $X_{0.5} = 1$.

2. a) Determine the auto-correlation function of the random process $X_t = A\cos(2\pi f_c t + \Theta)$, where A and f_c are constants, and Θ is uniformly distributed in $[0, 2\pi]$. b) Can you come up with a different pdf for Θ such that X_t remains wide-sense stationary (W.S.S)?

3. A random process Y_t is defined as $Y_t = X_t \cos(2\pi f_c t + \Theta)$ where X_t is a W.S.S random process, f_c is a constant, and Θ is a random variable independent of X_t and uniform in $[0, 2\pi]$. a) Is Y_t W.S.S? b) If Y_t is defined as $Y_t = X_t \cos(2\pi f_c t)$, is Y_t W.S.S?

4. A random process X_t is defined in terms of random variables X_1 and X_2 as follows:

$$X_t = X_1 \cos 2\pi f_c t + X_2 \sin 2\pi f_c t$$

   where f_c is a constant. Determine the necessary and sufficient conditions on X_1 and X_2 such that X_t is wide-sense stationary.

5. Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates $\lambda_1=1$ and $\lambda_2=2$, respectively. Let $N(t)$ be the merged process $N(t)=N_1(t)+N_2(t)$.
   i. Find the probability that $N(1)=2$ and $N(2)=5$.
   ii. Given that $N(1)=2$, find the probability that $N_1(1)=1$.

6. Let X be a zero-mean stationary Gaussian process with auto-correlation function $R_X(\tau)$. This process is applied to a square-law device defined by $Y_t = X_t^2$. a) Show that $E[Y_t] = R_X(0)$. b) Show that the auto-covariance function of Y_t, $C_Y(\tau) = 2R_X^2(\tau)$.

7. A stationary Gaussian process X with zero-mean and power spectral density S (f) is applied to a linear filter with impulse response as shown in Figure 1. A sample Y is taken of the random process at the filter output at time T. a) Determine the mean and variance of Y. b) What is the probability density function of Y?.

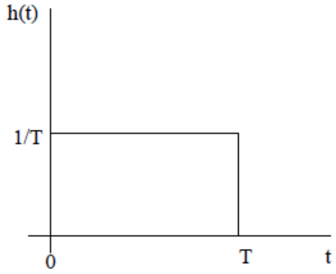

Figure 1:

8. Consider the cascaded filters shown in Figure 2 with a W. S. S. random process X_t as input. (a) Calculate the power spectral densities $S_Y(f)$ and $S_Z(f)$. (b) Calculate $S_{XY}(f)$, $S_{XZ}(f)$, and $S_{ZY}(f)$. (c) Evaluate your answers when the input is zero-mean, white Gaussian noise and $h(t) = e(t) = e^{-t}$ for $t \geq 0$ and $h(t) = e(t) = 0$ for $t < 0$.

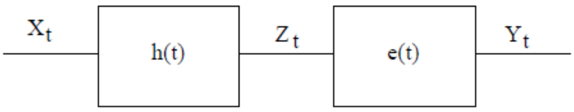

Figure 2:

9. A stationary Gaussian random process X with mean m and auto-correlation function $R_X(\tau)$ is filtered by $h_1(t)$ to obtain Y_t and $h_2(t)$ to obtain Z_t, respectively. Determine the cross-correlation function $R_{YZ}(t_1, t_2)$ and the cross-spectral density $S_{YZ}(f)$. Assuming $m_X = 0$, under what conditions on the filters are Y_t and Z_t independent? See Figure 3.

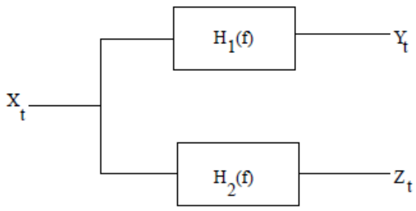

Figure 3:

Q10. Consider two independent normal random variables X ~$N(m_1, \sigma^2)$ and Y ~ $N(m_2, \sigma^2)$. Let $R = \sqrt{X^2 + Y^2}$, $\Theta = \tan^{-1}(Y/X)$. Find the pdf of R and Θ.

Q11. X= $[X1\ \ X2\ \ X3]^T$ is a three dimensional zero mean Gaussian random vector with

covariance matrix C_X given by $C_X = \begin{bmatrix} 3 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$. Determine $f_z(z)$ for the following

transform $Z = \begin{bmatrix} 5 & -3 & -1 \\ -1 & 3 & -1 \\ 1 & 0 & 1 \end{bmatrix} X$