TRIGONOMETRIC IDENTITIES

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y) \tag{C-1}$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \tag{C-2}$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x) \tag{C-3}$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x) \tag{C-4}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) \tag{C-5}$$

$$\sin(2x) = 2\sin(x)\cos(x) \tag{C-6}$$

$$2\cos(x) = e^{jx} + e^{-jx} \tag{C-7}$$

$$2j\sin(x) = e^{jx} - e^{-jx} \tag{C-8}$$

$$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y) \tag{C-9}$$

$$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y) \tag{C-10}$$

$$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y) \tag{C-11}$$

$$2\cos^2(x) = 1 + \cos(2x) \tag{C-12}$$

$$2\sin^2(x) = 1 - \cos(2x) \tag{C-13}$$

$$4\cos^3(x) = 3\cos(x) + \cos(3x) \tag{C-14}$$

$$4\sin^3(x) = 3\sin(x) - \sin(3x) \tag{C-15}$$

$$8\cos^4(x) = 3 + 4\cos(2x) + \cos(4x) \tag{C-16}$$

INDEFINITE INTEGRALS

Rational Algebraic Functions

$$\int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad 0 < n \tag{C-20}$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a+bx| \tag{C-21}$$

$$\int \frac{dx}{(a+bx)^n} = \frac{-1}{(n-1)b(a+bx)^{n-1}} \qquad 1 < n \tag{C-22}$$

$$\begin{aligned} \int \frac{dx}{c+bx+ax^2} &= \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) && b^2 < 4ac \\ &= \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| && b^2 > 4ac \\ &= \frac{-2}{2ax+b} && b^2 = 4ac \end{aligned} \tag{C-23}$$

$$\int \frac{x\,dx}{c+bx+ax^2} = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{dx}{c+bx+ax^2} \tag{C-24}$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right) \tag{C-25}$$

$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2) \tag{C-26}$$

$$\int \frac{x^2\,dx}{a^2+x^2} = x - a\tan^{-1}\left(\frac{x}{a}\right) \tag{C-27}$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-28}$$

$$\int \frac{x\,dx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)} \tag{C-29}$$

$$\int \frac{x^2\,dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-30}$$

$$\int \frac{dx}{(a^2+x^2)^3} = \frac{x}{4a^2(a^2+x^2)^2} + \frac{3x}{8a^4(a^2+x^2)} + \frac{3}{8a^5}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-31}$$

$$\int \frac{x^2\,dx}{(a^2+x^2)^3} = \frac{-x}{4(a^2+x^2)^2} + \frac{x}{8a^2(a^2+x^2)} + \frac{1}{8a^3}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-32}$$

$$\int \frac{x^4\,dx}{(a^2+x^2)^3} = \frac{a^2x}{4(a^2+x^2)^2} - \frac{5x}{8(a^2+x^2)} + \frac{3}{8a}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-33}$$

$$\int \frac{dx}{(a^2+x^2)^4} = \frac{x}{6a^2(a^2+x^2)^3} + \frac{5x}{24a^4(a^2+x^2)^2} + \frac{5x}{16a^6(a^2+x^2)} + \frac{5}{16a^7}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-34}$$

$$\int \frac{x^2\,dx}{(a^2+x^2)^4} = \frac{-x}{6(a^2+x^2)^3} + \frac{x}{24a^2(a^2+x^2)^2} + \frac{x}{16a^4(a^2+x^2)} + \frac{1}{16a^5}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-35}$$

$$\int \frac{x^4\,dx}{(a^2+x^2)^4} = \frac{a^2x}{6(a^2+x^2)^3} - \frac{7x}{24(a^2+x^2)^2} + \frac{x}{16a^2(a^2+x^2)} + \frac{1}{16a^3}\tan^{-1}\left(\frac{x}{a}\right) \tag{C-36}$$

$$\int \frac{dx}{a^4+x^4} = \frac{1}{4a^3\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) + \frac{1}{2a^3\sqrt{2}}\tan^{-1}\left(\frac{ax\sqrt{2}}{a^2-x^2}\right) \tag{C-37}$$

$$\int \frac{x^2\,dx}{a^4+x^4} = -\frac{1}{4a\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) + \frac{1}{2a\sqrt{2}}\tan^{-1}\left(\frac{ax\sqrt{2}}{a^2-x^2}\right) \tag{C-38}$$

Trigonometric Functions

$$\int \cos(x)\,dx = \sin(x) \tag{C-39}$$

$$\int x\cos(x)\,dx = \cos(x) + x\sin(x) \tag{C-40}$$

$$\int x^2\cos(x)\,dx = 2x\cos(x) + (x^2-2)\sin(x) \tag{C-41}$$

$$\int \sin(x)\, dx = -\cos(x) \tag{C-42}$$

$$\int x \sin(x)\, dx = \sin(x) - x\cos(x) \tag{C-43}$$

$$\int x^2 \sin(x)\, dx = 2x\sin(x) - (x^2 - 2)\cos(x) \tag{C-44}$$

Exponential Functions

$$\int e^{ax}\, dx = \frac{e^{ax}}{a} \qquad a \text{ real or complex} \tag{C-45}$$

$$\int xe^{ax}\, dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right] \qquad a \text{ real or complex} \tag{C-46}$$

$$\int x^2 e^{ax}\, dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right] \qquad a \text{ real or complex} \tag{C-47}$$

$$\int x^3 e^{ax}\, dx = e^{ax}\left[\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right] \qquad a \text{ real or complex} \tag{C-48}$$

$$\int e^{ax} \sin(x)\, dx = \frac{e^{ax}}{a^2+1}[a\sin(x) - \cos(x)] \tag{C-49}$$

$$\int e^{ax} \cos(x)\, dx = \frac{e^{ax}}{a^2+1}[a\cos(x) + \sin(x)] \tag{C-50}$$

DEFINITE INTEGRALS

$$\int_{-\infty}^{\infty} e^{-a^2x^2+bx}\, dx = \frac{\sqrt{\pi}}{a} e^{b^2/(4a^2)} \qquad a > 0 \tag{C-51}$$

$$\int_0^{\infty} x^2 e^{-x^2}\, dx = \sqrt{\pi}/4 \tag{C-52}$$

$$\int_0^{\infty} \mathrm{Sa}(x)\, dx = \int_0^{\infty} \frac{\sin(x)}{x}\, dx = \frac{\pi}{2} \tag{C-53}$$

$$\int_0^{\infty} \mathrm{Sa}^2(x)\, dx = \pi/2 \tag{C-54}$$

FINITE SERIES

$$\sum_{n=1}^{N} n = \frac{N(N+1)}{2} \tag{C-55}$$

$$\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6} \tag{C-56}$$

$$\sum_{n=1}^{N} n^3 = \frac{N^2(N+1)^2}{4} \tag{C-57}$$

$$\sum_{n=0}^{N} x^n = \frac{x^{N+1}-1}{x-1} \tag{C-58}$$

$$\sum_{n=0}^{N} \frac{N!}{n!(N-n)!} x^n y^{N-n} = (x+y)^N \tag{C-59}$$

$$\sum_{n=0}^{N} e^{j(\theta+n\phi)} = \frac{\sin[(N+1)\phi/2]}{\sin(\phi/2)} e^{j[\theta+(N\phi/2)]} \tag{C-60}$$

$$\sum_{n=0}^{N} \binom{N}{n} = \sum_{n=0}^{N} \frac{N!}{n!(N-n)!} = 2^N \tag{C-61}$$

$$\sum_{n=N_1}^{N_2} w^n = \frac{w^{N_1} + w^{N_2+1}}{1-w} \qquad \left\{ \begin{array}{l} N_2 > N_1 \text{ and } w \\ \text{real or complex} \end{array} \right. \tag{C-62}$$

INFINITE SERIES

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{C-63}$$

TABLE E-1

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$	Notes
1	$\alpha\delta(t)$	α	
2	$\alpha/2\pi$	$\alpha\delta(\omega)$	
3	$u(t)$	$\pi\delta(\omega)+(1/j\omega)$	
4	$\frac{1}{2}\delta(t)-\frac{1}{j2\pi t}$	$u(\omega)$	
5	$\mathrm{rect}(t/\tau)$	$\tau\,\mathrm{Sa}(\omega\tau/2)$	$\tau>0$
6	$(W/\pi)\mathrm{Sa}(Wt)$	$\mathrm{rect}\,(\omega/2W)$	$W>0$
7	$\mathrm{tri}\,(t/\tau)$	$\tau\,\mathrm{Sa}^2(\omega\tau/2)$	$\tau>0$
8	$(W/\pi)\mathrm{Sa}^2(Wt)$	$\mathrm{tri}\,(\omega/2W)$	$W>0$
9	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
10	$\delta(t-\tau)$	$e^{-j\omega\tau}$	
11	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
12	$\sin(\omega_0 t)$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	
13	$u(t)\cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$u(t)\sin(\omega_0 t)$	$-j\frac{\pi}{2}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$u(t)e^{-\alpha t}$	$\frac{1}{\alpha+j\omega}$	$\alpha>0$
16	$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha+j\omega)^2}$	$\alpha>0$
17	$u(t)t^2e^{-\alpha t}$	$\frac{2}{(\alpha+j\omega)^3}$	$\alpha>0$
18	$u(t)t^3e^{-\alpha t}$	$\frac{6}{(\alpha+j\omega)^4}$	$\alpha>0$
19	$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\alpha>0$
20	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	$\sigma>0$