Signal Types :

① Periodic & Aperiodic

→ Seq. repeated at particular int. of time

② Causal & Non - Causal.

→ $t \geq 0$ (Present & Future)

→ Periodic

→ Non - periodic

→ $t < 0$ Present, Past and future

→ Periodic

→ Aperiodic

Problem :

① $\sin(0.02\pi n)$

Soln,

$$\omega = 2\pi f = 0.02\pi$$

$$f = \frac{0.02}{2} = 0.01 = \frac{1}{100}$$

→ If this no. is rational, then it's periodic

$$\text{Time Period} = \frac{1}{f} = 100.$$

$\boxed{\text{Periodic}}$.

② $\sin(5\pi n)$

$$f = \frac{5}{2}$$ → Rational, So periodic

$$N = \frac{2}{5}$$

③ $\cos(4n)$

$$2\pi f = 4$$

$$f = \frac{2}{\pi}$$ → Irrational, So no period.

④ $\sin \frac{2\pi}{3} + \cos \frac{2\pi}{5}$.

$2\pi f_1 = \frac{2\pi}{3}$ $\qquad$ $2\pi f_2 = \frac{2\pi}{5}$

$\boxed{f_1 = 1/3}$ $\qquad$ $\boxed{f_2 = 1/5}$

$N_1 = 3$ $\qquad$ $N_2 = 5$

To find periodic, $\frac{N_1}{N_2}$ → Need to be rational.

$\frac{N_1}{N_2} = \frac{3}{5}$ → $\boxed{\text{Rational}}$

LCM (N_1, N_2) ⇒ Time Period

LCM $(3, 5) = 15$.

⑤ $\cos\left(\frac{n}{6}\right)$. $\cos\left(\frac{n\pi}{6}\right)$

↓ NP $\qquad$ ↓

$2\pi f_2 = \frac{\pi}{6}$

$f_2 = \pi/12$

$N_2 = 12$

$\boxed{\text{Non - Periodic}}$

⑥ $1 + e^{j\frac{2\pi}{3}} - e^{j\frac{4\pi}{3}}$

$N_1 = 1 \qquad N_2 = 3 \qquad N_3 = \frac{7}{2}$

$\frac{N_1}{N_2} = \frac{1}{3}, \quad \frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7}, \quad \frac{N_2}{N_3} = \frac{2/7}{3} = \frac{6}{7}.$

If all are rational, then it is periodic.

27.04.23

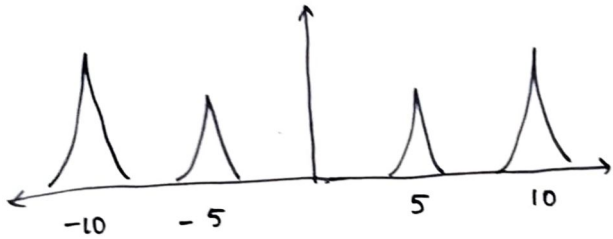
Odd or Even Signals :

Even → Symmetric

Odd → Anti-symmetric

Even :

Eg :



It is symmetric.

$x(-n) = x(n)$: Even Signal

$x(-n) = -x(n)$: Odd signal

$$x(n) = x_e(n) + x_o(n)$$

$$x(-n) = x_e(n) - x_o(n)$$

$$2x_e(n) = x(n) + x(-n)$$

$$x_e(n) = \frac{x(n) - x(-n)}{2}$$

$$x_e(n) = \frac{x(n) - x(-n)}{2}$$

$$x(n) = e^{-2n} \cos(n)$$

Soln.

$$x(-n) = e^{2n} \cos n$$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \frac{e^{-2n} \cos n + e^{2n} \cos n}{2}$$

$$= \frac{\cos n (e^{-2n} + e^{2n})}{2}$$

$$= \cos n \cdot \cosh(2n)$$

$$x_o(n) = \frac{\cos n (e^{-2n} - e^{2n})}{2}$$

$$x_o(n) = -\cos n \cdot \sinh(2n)$$

<u>Energy and Power Signal</u> : How much information carried by signal.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Energy signal has 0 time averaged power.

Power signal has ∞ energy.

Eg: Periodic and Random signal are usually power signal.

Deterministic and Non-periodic are usually energy signal.

Note: Deterministic signal can be expressed in mathematical form

$$x(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$E = \sum_{n=-\infty}^{\infty} \left| \left(\frac{1}{3}\right)^n u(n) \right|^2$$

$$E = \sum_{n=0}^{\infty} \left| \left(\frac{1}{3}\right)^n \right|^2 = \sum_{n=0}^{\infty} \left| \frac{1}{9} \right|^n = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$E = \frac{1}{1 - \frac{1}{9}}$$

$$\boxed{E = \frac{9}{8}}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = 0$$

$$x(n) = e^{2n}\, u(n)$$

$$E = \sum_{n=-\infty}^{\infty} \left| e^{2n} \cdot u(n) \right|^2$$

$$= \sum_{n=0}^{\infty} e^{4n} = \infty$$

Not a energy signal

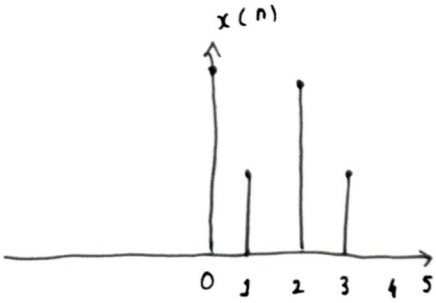
$$P = \lim_{N \to \infty} \frac{1}{2N+1} \left[\sum_{n=0}^{N} e^{4n} \right]$$

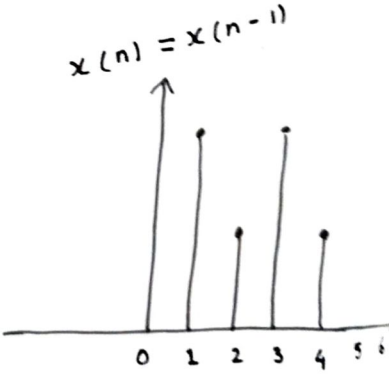
$$= \lim_{N \to \infty} \frac{1}{2N+1} \left[\frac{e^{4(N+1)} - 1}{e^{4} - 1} \right] = \infty$$

H.W. : $x(n) = \sin\left(\frac{\pi}{4} n\right)$

Signal Operation :

1. Shifting

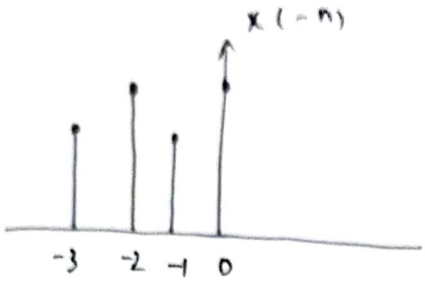




$y(n) = x(n-k)$: Right Shift

$y(n) = x(n+k)$: Left Shift

2. Time Reversal :



3. Time Reversal & Shifting :

$y(n) = x(-n-k)$: Folding and shift left

$y(n) = x(-n+k)$: Folding and left shift

3.5.23

System :

1. Linear or Non-Linear :

↓

When output varies linearly w.r.t. input.

2. Causal or Non-Causal :

→ ~~Doesn't~~ depend on past values and present

A system need to LTI system

3. ~~M~~ Static and Dynamic

↓ Memoryless ↓ Memory

4. Stable or Unstable

↳ Range is finite

⇒ BIBO Stable

E.g.: DFT System

5. Time Varient and Invariant :

↓

Output independent on time

Problem:

1. $y(n) = 2x^2(n)$: Static
2. $y(n) = x(2n)$: Dynamic
3. $y(n) = x(n)$: Static
4. $y(n) = x(n) + x(n-2)$: Dynamic

Static, Dynamic

5. $y(n) = x(n-2) + x(n-1) + x(2n)$: Non causal
6. $y(n) = x(n) + x(2n)$: NC
7. $y(n) = x^2(n) + 2x(n+2)$: NC
8. $y(n) = n(x(n))$: C

Causal, Non causal

9. $y(n) = 2x^2(n)$ → N-Linear

Soln,

to $x_1(n)$:

$$y_1(n) = 2x_1^2(n)$$

to $x_2(n)$:

$$y_2(n) = 2x_2^2(n)$$

$$a_1 y_1(n) + a_2 y_2(n) = 2a_1 x_1^2(n) + 2a_2 x_2^2(n)$$

$$= 2(a_1 x_1^2(n) + 2a_2 x_2^2(n)$$

→ ①

$$y(n) = T[x(n)]$$

$$= 2[[a_1 x_1(n) + a_2 x_2(n)]^2]$$

$$= 2[[a_1^2 x_1^2(n) + a_2^2 x_2(n) + 2a_1 a_2 x_1(n) x_2(n)$$

→ ②

① ≠ ②

∴ The system is non-linear

10. $y(n) = x(n) + x(n-1)$

Soln

$y(n-k) = x(x-k) + x(n-k-1) \rightarrow ①$

$y(n,k) = T[x(n-k)]$

$= x(n-k) + x(n-k-1) \rightarrow ②$

It is Time invarient system.

11. $y(n) = x(-n)$

$y(n-k) = x(-(n-k)) = x(-x+k) \rightarrow ①$

$y(n,k) = x[x(-n)] = x(-n-k) \rightarrow ②$

$① \neq ②$

It is time varient system.

12. $y(n) = n x^2(n)$

Stable:

$\sum |h(n)| \leq \infty$ (finite value $\rightarrow$ Stable)

13. $h(n) = 2^n u(n)$

$$\sum_{n=0}^{\infty} 2^n = \frac{1}{1-2} = \frac{1}{-1} = -1$$

$\therefore$ It is stable ~~sty~~ system.

14. $h(n) = u(n)$: Unstable

Correlation and Convolution:

↳ Mixing of 2 signals in natural phenomenon.

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l)$$

$$r_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)\, x(n-l)$$

$$\rho_{xy} = \frac{r_{xy}}{\sqrt{\sigma_x \sigma_y}}$$

5.5.23

Z - Transforms:

$Z = e^{jw}$ (Relation between Z-domain and freq. domain)

↳ **Application**: Stability Analysis

$$x(n) \longleftrightarrow X(z)$$

$$x(z) = \underbrace{\sum_{n=-\infty}^{\infty} x(n)\, z^{-n}}$$

$$\underbrace{+ \cdots + x(-1)\, z^{+1}}_{\text{Anyone is unilateral}} + \underbrace{x(0)\, z^{0} + x(1)\, z^{-1}}_{} + \cdots$$

ROC : Region where Z - transform exists.

Practice :

1. $u(n)$
2. $\delta(n)$
3. $n\,u(n)$
4. $e^{j\omega n}\,u(n)$

Problem :

1. $x(n) = a^n\,u(n)$
2. $x(n) = -a^n\,u(-n-1)$
3. $x(n) = a^n\,u(n) - b^n\,u(-n-1)$.

Solution :

1. $$x(z) = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (a z^{-1})^n$$

$$= \frac{1}{1 - \boxed{a z^{-1}}}$$

ROC :

$$|a z^{-1}| < 1$$

$$\left|\frac{a}{z}\right| < 1$$

$$\frac{|a|}{|z|} < 1$$

$$|a| < |z|$$

$$|z| > |a|$$

$$= \frac{z}{z - a}$$

2.

$$x(n) = -a^n \underbrace{u(-n-1)}$$

$$-a^n \quad n \leq -1$$

$$0 \quad n > 0$$

$$X(z) = \sum_{n=-\infty}^{-1} -a^n z^{-n}$$

$$= \sum_{n=1}^{\infty} -a^{-n} z^{n}$$

$$= -\sum_{n=1}^{\infty} (a^{-1} z)^n$$

$$= -\left[\sum_{n=0}^{\infty} (a^{-1} z)^n - 1\right]$$

$$= 1 - \sum_{n=0}^{\infty} (a^{-1} z)^n$$

$$= 1 - \frac{1}{1 - a^{-1} z}$$

$$= \frac{\cancel{1} - a^{-1} z - \cancel{1}}{1 - a^{-1} z}$$

$$X(z) = \frac{-a^{-1} z}{1 - a^{-1} z} = \frac{-z/a}{1 - z/a} = \frac{-z}{a - z}$$

$$X(z) = \frac{z}{z - a}$$

ROC : $|z| < |a|$

3. $x(n) = a^n (u(n)) - b^n u(-n-1)$

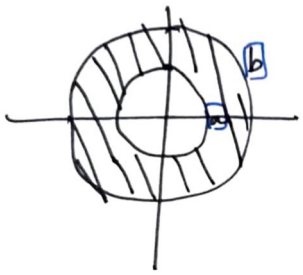
$$= \sum_{n=0}^{\infty} a^n z^{-n} - \sum_{n=-\infty}^{-1} b^n z^{-n}$$

$$= \underbrace{\frac{z}{z-a}}_{\text{From Problem ①}} + \underbrace{\frac{z}{z-b}}_{\text{From Problem ②}}$$

$|z| > |a|$ $\quad$ $|z| < |b|$

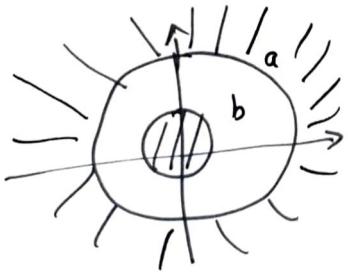
~~Let~~ If $|b| > |a|$

~~$|b| > |a|$~~



This is valid

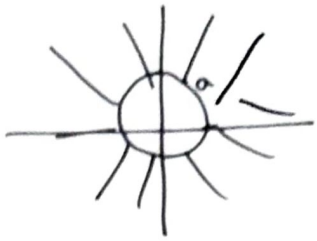
If $|a| > |b|$



X.

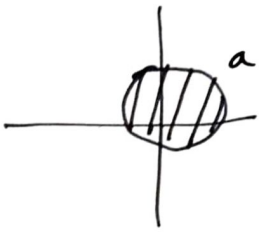
ROC of Causal Sequence:

1. ROC exists exterior of a particular circle.



ROC of Non-Causal sequence:

1. ROC exists interior of a particular circle.



Other Problems:

3.

$$x(n) = 3\left(\frac{5}{7}\right)^n u(n) + 2\left(\frac{-1}{3}\right)^n u(n)$$

4.

$$x(n) = \left(\frac{1}{2}\right)^n u(-n) - 2^n u(-n-1)$$

5.

$$x(n) = 2\left(\frac{5}{6}\right)^n u(-n-1) + 3\left(\frac{1}{2}\right)^{2n} u(n)$$

Properties :

1. ~~Lin~~ Linearity :

$$x\left[a_1 x_1(n) + a_2 x_2(n)\right]$$
$$= a_1 X_1(z) + a_2 X_2(z)$$

2. Time Shifting :

z-transform

$$\boxed{x(n \pm m) \xleftrightarrow{ZT} z^{\pm m} X(z)}$$

$$x(n-2) \xleftrightarrow{ZT} z^{-2} X(z)$$

8.6.23

3. Time Reversal :

$$x(-n) \xleftrightarrow{ZT} X(z^{-1})$$

Eg :

$$y(n) = 5y(n-1) + 6\,x(n) + 7\,x(n-1)$$
$$Y(z) = 5z^{-1} Y(z) + 6\,X(z^{-1}) + 7z^{-1} X(z)$$

4. Differentiation :

$$n\,x(n) \longleftrightarrow -z \frac{d}{dz} X(z)$$

5. Conjugation :

$$x^*(n) \longleftrightarrow X^*(z^*)$$

6. Convolution :

$$x(n) * h(n) \longleftrightarrow \underbrace{X(z) \cdot H(z)}_{\text{Multiplication}} = Y(z)$$

$$\downarrow I.Z$$

$$y(n)$$

$$y(n) = \sum [x(n)\, h(n-k)]$$

7. Correlation :

$$x_1(n) \circledast x_2(n) \xleftrightarrow{ZT} X_1(z) \cdot X_2(z^{-1})$$

$$y_1(\ell) = \sum x_1(n)\, x_2(n-\ell)$$

$y(+1)$

$y(-1)$

8. Parseval's Theorem :

$$\boxed{\sum_{n=\infty}^{\infty} x_1(n)\, x_2^*(n) = \frac{1}{2\pi j} \oint X_1(z) \cdot X_2^*(1/z^*)\, z^{-1}}$$

9. Initial Value Theorem :

$$\lim_{n \to \boxed{0}} x(n) = x(\boxed{0}) = \lim_{z \to \boxed{\infty}} X(z)$$

10. Final Value Theorem :

$$\lim_{n \to \boxed{\infty}} x(n) = x(\boxed{\infty}) = \lim_{z \to \boxed{1}} \boxed{(z-1)}\, X(z)$$

Problem :

1. $x(n) = n^2 u(n)$

$$Z[u(n)] = \cancel{\frac{z}{z-1}} \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + \dots$$

$$\boxed{x(z) = \frac{z}{z-1}}$$

$$n x(n) = -z \frac{d}{dz}\left[\frac{z}{z-1}\right]$$

$$n^2 x(n) = -z \frac{d}{dz}\left[-z \frac{d}{dz}\left[\frac{z}{z-1}\right]\right]$$

~~Contd...~~
problem not solved ....

2. $x(n) = \left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{4}\right)^n u(n)$

$$y(z) = x_1(z) \cdot x_2(z)$$

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) = \frac{z}{z - \frac{1}{2}} \quad |z| > \frac{1}{2}$$

$$x_2(n) = \frac{z}{z - \frac{1}{4}} \quad |z| > \frac{1}{4}.$$

$$Y(z) = \frac{A}{(\quad)} + \frac{B}{(\quad)}$$

HW: 1. $x(n) = \alpha^{n-2} u(n-2)$

2. $x(n) = n\left(\frac{1}{2}\right)^n u(n) * \left[\delta(n) - \frac{1}{2}\delta(n\right.$

Inverse Z-Transform:

$$x(z) = \frac{3z^{-1}}{(1-z^{-1})(1-2z^{-1})}$$

$\downarrow$

$x(n)$

if ROC:

in (i) $|z| > 2$

(ii) $|z| < 1$

(iii) $1 < |z| < 2$.

$$\frac{x(z)}{z} = \frac{3}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$A = (z-1)\cdot\frac{x(z)}{z}$$

$$= (z-1)\boxed{\left(\frac{x(z)}{z}\right)}$$

$$A = \cancel{z-1}\ \frac{3}{\cancel{(z-1)}(z-2)}\Bigg|_{z=1}$$

$$Y(z) = \frac{A}{(\quad)} + \frac{B}{(\quad)}$$

HW: 1. $x(n) = \alpha^{n-2} u(n-2)$

2. $x(n) = n\left(\frac{1}{2}\right)^n u(n) * \left[\delta(n) - \frac{1}{2}\delta(n\right.$

<u>Inverse Z-Transform</u>:

$$x(z) = \frac{3z^{-1}}{(1-z^{-1})(1-2z^{-1})}$$

$\downarrow$

$x(n)$

if ROC:

in (i) $|z| > 2$

(ii) $|z| < 1$

(iii) $1 < |z| < 2$.

$$\frac{x(z)}{z} = \frac{3}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$A = (z-1)\frac{x(z)}{z}$$

$$= (z-1)\boxed{\left(\frac{x(z)}{z}\right)}$$

$$A = \cancel{z-1}\ \frac{3}{\cancel{(z-1)}(z-2)}\Bigg|_{z=1}$$

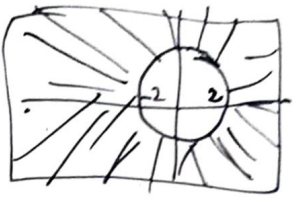
$\boxed{A = -3}$ $\boxed{B = 3}$

$$X(z) = 3 \cdot \frac{z}{z-2} - 3 \cdot \frac{z}{z-1}$$

ROC:

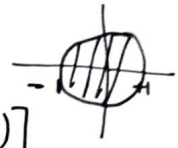
(i) $|z| > 2$.

$$x(n) = 3 \cdot 2^n u(n) - 3(1)^n u(n)$$



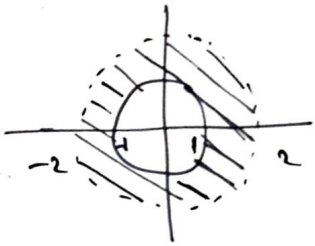
(ii) $|z| < 1$

$$x(n) = [-3 \cdot 2^n u(-n-1) + 3 \cdot 1^n u(-n-1)]$$



(iii) $1 < |z| < 2$

$$x(n) = 3[-2^n u(-n-1) - 1^n u(n)]$$



Problem 2)

$$x(n) = \left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{4}\right)^n u(n)$$

$X(z)$

Convolution :

$$y(n) = \sum_{n=-\infty}^{\infty} x(n)\, h(n-k)$$

$x(n) = \{3, 2, 1, 2\}$; $N_1 = \boxed{4}$

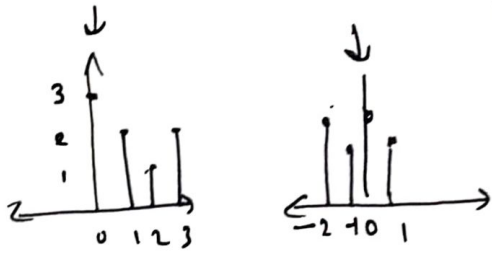
$h(n) = \{1, 2, 1, 2\}$; $N_2 = \boxed{4}$

$\rightarrow N_1 + N_2 - 1 = 8 - 1 = 7$

$y(n) = \{3, 8, 8, 12, 9, 4, 4\}$

	1	2	1	2
3	3	6	3	6
2	2	4	2	4
1	1	2	1	2
2	2	4	2	4

$$y(0) = \sum_{k=-\infty}^{+\infty} x(k)\, h(-k)$$



$$\underbrace{3 \times 2 + 2 \times 1}_{} = 8$$

Property :

1. Commutative Property

$$x(n) * h(n) = h(n) * x(n)$$

2. Associative Law.

$$x(n) * [h_1(n) * h_2(n)] = [x(n) * h_1(n)] * \cancel{x(n) *} [h_2(n)]$$

3. Distributive law :

k	-3	-2	-1	0	1	2	3	4	5	6	7
x(k)				1	2	0.5	1				
h(k)			1	2	1	-1					
h(-k)		-1	+1	2	1						
h(-1-k) -1		1	2	1							
h(1-k)			-1	1	2	1					
h(2-k)				-1	1	2	1				
h(3-k)					-1	1	2	1			
h(4-k)						-1	1	2	1		
h(5-k)							-1	1	2	1	

10.5.23

Stability Analysis

$$y(n) = y(n-1) - 0.8\, y(n-2) + x(n) + n(n)$$

noise is removed by modified coefficients

$$\underset{a_0}{1}\, y(n+1) = \underset{a_1}{0.2\, y(n-1)} - \underset{a_2}{1.2\, y(n-2)} + \underset{b_0}{0.9}\, x(n) + \underset{b_1}{0.94}$$

always $= 1$

eqn ①:

$$y(z) = z^{-1} y(z) - 0.8\, z^{-2} y(z) + x(z) + z^{-2} x(z)$$

$$y(z) - z^{-1} y(z) + 0.8\, z^{-2} y(z) = x(z) + z^{-2} x(z)$$

$$y(z)\left[1 - z^{-1} + 0.8\, z^{-2}\right] = x(z)\left[1 + z^{-2}\right]$$

Transfer function

$$\boxed{\frac{y(z)}{x(z)}} = \frac{1 + z^{-2}}{1 - z^{-1} + 0.8\, z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 - z^{-1} + 0.8\, z^{-2}}$$

$$= \frac{z^2 + 1}{z^2 - z + 0.8}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+1 \pm \sqrt{1 - 4(1)(0.8)}}{2}$$

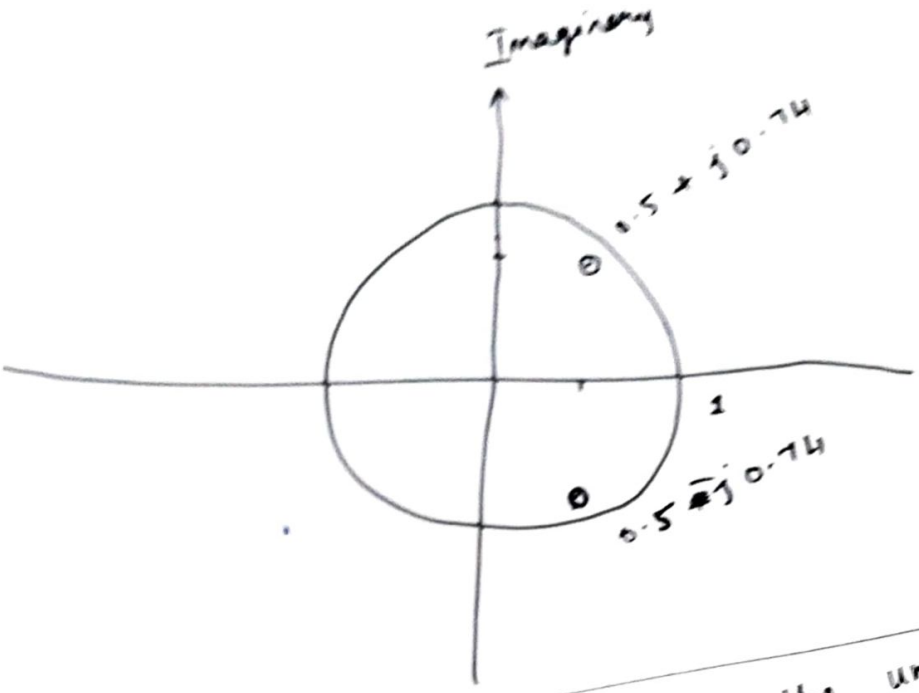
$$= \frac{1 \pm \sqrt{1 - 3.2}}{2}$$

$$= \frac{1 \pm \sqrt{-2.2}}{2}$$

$$= \frac{1 \pm 1.48i}{2}$$

$$H(z) = \frac{1 + 1.48i}{2} = 0.5 + 0.7j$$

$$\frac{1 - 1.48i}{2} = 0.5 - 0.7j$$



Since, both are inside the unit circle, both are stable.

if ___ outside - unstable

1 - mar. stable.

Problem 2)

$$y(n) = 2y(n-1) - 0.8\,y(n-2) + x(n) + 0.8\,x(n-1)$$

$$y(n) - \frac{9}{4}\,y(n-1) + \frac{1}{2}\,y(n-2) = x(n) - 3\,x(n-1)$$

- Specify the ROC of $H(z)$
- Check stable & causal

Answer

(a)

$$y(z) = 2z^{-1}y(z) - 0.8z^{-2}y(z)$$

$$\Rightarrow x(n) + 0.8z^{-1}y(z)$$

$$y(z) - 2z^{-1}y(z) + 0.8z^{-2}y(z)$$

$$= x(z) + 0.8z^{-1}y(z)$$

$$y(z)\left[1 - 2z^{-1} + 0.8z^{-2}\right]$$

$$= x(z)\left[1 + 0.8z^{-1}\right]$$

$$\frac{y(z)}{x(z)} = \frac{1 + 0.8z^{-1}}{1 - 2z^{-1} + 0.8z^{-2}}$$

$$H(z) = \cancel{\frac{0.8\frac{1}{z} + 1}{1 - \frac{2}{z} + \frac{0.8}{z^2}}} \quad \frac{z^2 + 0.8z}{\underbrace{z^2 - 2z + 0.8}}$$

$$\Rightarrow \cancel{0.8z +}$$

$$\rightarrow \boxed{1.44,\ 0.56}$$

$\downarrow$

Beyond 1.

So, unstable.

$$y(n) - \frac{9}{4} y(n-1) + \frac{1}{2} y(n-2) = x(n) - 3x(n-1)$$

By solving,

<u>Poles</u>: $z = \frac{1}{4}, 2$

$\downarrow \quad \downarrow$ greater than 1

So, unstable.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z(z-3)}{\left(z-\frac{1}{4}\right)(z-2)}$$

$$\cancel{z(z-3) = A}$$

$$\frac{H(z)}{z} = \frac{z-3}{\left(z-\frac{1}{4}\right)(z-2)} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z-2}$$

$$A = \left(z - \frac{1}{4}\right) \frac{H(z)}{z} \Bigg|_{z=\frac{1}{4}}$$

$$= \frac{z-3}{z-2} \Bigg|_{z=1/4}$$

$$= 11/7.$$

$$\boxed{B = \frac{-4}{7}}$$

$$\frac{H(z)}{z} = \frac{11}{7}\,\frac{1}{z-\frac{1}{4}} - \frac{4}{7}\,\frac{1}{z-2}.$$

$$H(z) = \frac{11z}{7\left(z-\frac{1}{4}\right)} - \frac{4}{7(z-2)}$$

$$|z| > \frac{1}{4} \qquad |z| < 2.$$

$$\frac{1}{4} < |z| < 2$$

$$h(n) = \frac{11}{7}\left(\frac{1}{4}\right)^n u(n)$$

$$+ \frac{4}{7}\left(+2^n\right) u(-n-1)$$

12.5.23

Fourier:

Time Domain → Freq. Domain

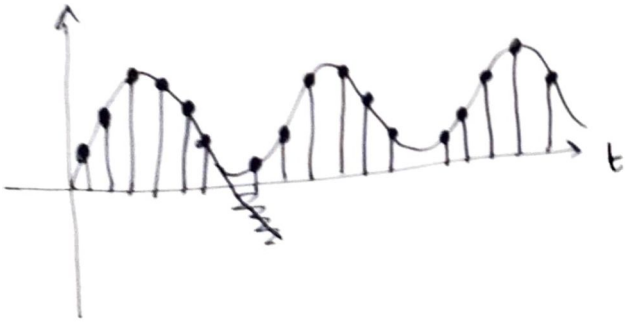
Fourier Series:

	Periodic	Aperiodic
Continuous	Fourier series	Continuous Fourier Transform
Discrete	Discrete Time Fourier Series (DTFT)	Discrete Time Fourier Transform

Sampled version of DTFT

→ DFT, FFT } Same but calc. computing methods

↳ Computations is fast



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n]\, e^{-j\boxed{\omega} n}$$

Order of DFT

2ω

N - point DFT

Now ~~Much~~ many input → That many O/P.

$$\omega = \frac{2\pi}{N}$$

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\omega_k n}$$

$$= \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N} nk}.$$

Inverse DFT:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, e^{j\frac{2\pi}{N}(n)(k)}$$

Twidle Factor:

$$W_N = e^{-\frac{j2\pi}{N}}$$

DFT:

$$x(k) = \sum_{n=0}^{N-1} x[n]\, W_N^{n.k}$$

IDFT:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk}.$$

Problem:

Compute the DFT of $x[n] = \{1, 1, 0, 0\}$ using formula method.

Solution

DFT is $X(k) = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi kn}{N}}$

$$X(k) = \sum_{n=0}^{3} x[n] e^{-\frac{j2\pi kn}{4}}$$

$$X(k) = \underbrace{x[0]}_{1} \cdot e^{-\frac{j2k(0)}{2}} + \underbrace{x[1]}_{1} \cdot e^{-\frac{j2k(1)}{2}} + \underbrace{\left\{ x[2] \cdot e^{-\frac{j2k(2)}{2}} + x[3] \cdot e^{-\frac{j2k(3)}{2}} \right\}}_{0}$$

$-j$

$$X(k) = \boxed{1 + 1 \cdot e^{-j\pi k/2}}$$

Substitute $k = 0$,

$X(0) = 1 + 1 = 2$.

$X(1) = 1 - j$

$X(2) = 1 + (-1) = 0$

$X(3) = 1 + j =$

$x[k] = [2, 1-j, 0, 1+j$

13.5.23

DFT

$\mapsto$ Twidle Factor : Sqr. Matrix

$$X = W_N \cdot x$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(n-1) \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(n-1) \end{bmatrix}$$

$n \times n$ matrix where n – no. of elements.

$$x = W_N^{-1} X : \cancel{DFT}$$

IDFT

$N : 8$ $\quad \boxed{W_N = \frac{2\pi}{N}}$ ☆

$$W_8 = \frac{2\pi}{8}$$

$$\boxed{W_8 = \frac{\pi}{4}}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$\boxed{W_4 = e^{-j \cdot 2\pi/4}}$$

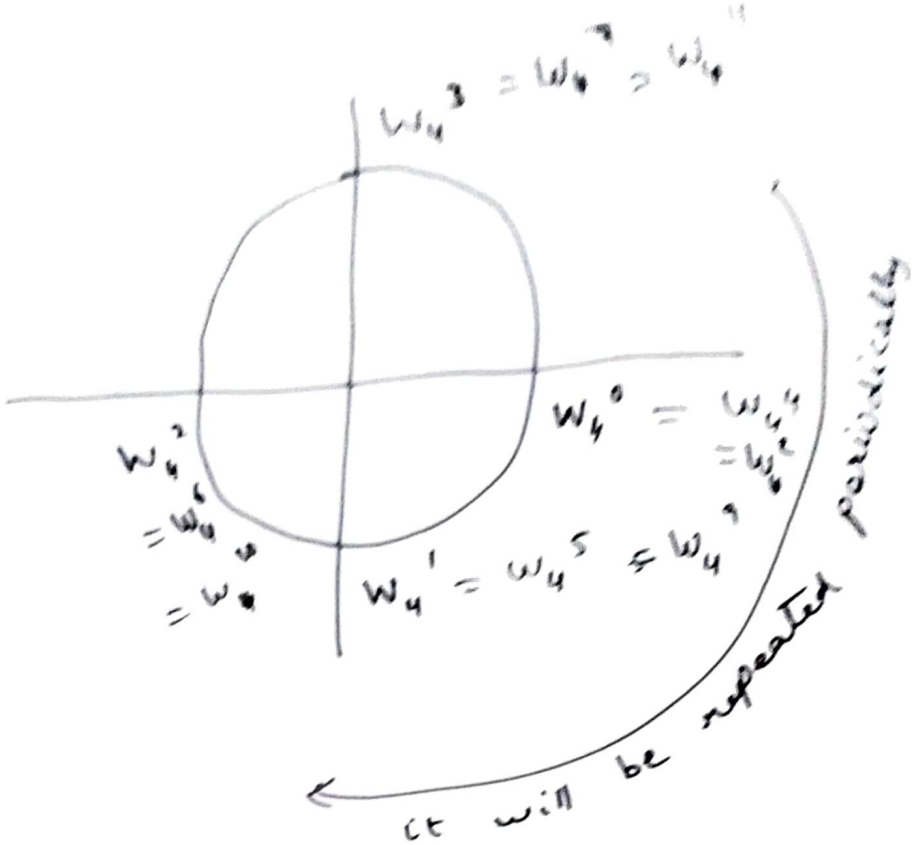
$$W_4^0 = \left(e^{-j\frac{2\pi}{4}}\right)^0$$

$N = 4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$W_4^4 = W_4^0$, $W_4^6 = W_4^2$, $W_4^9 = W_4^1$

Rewriting using that circle in next page



$$W_4^1 = \left(e^{-j\frac{2\pi}{4}}\right)^1 = e^{-j\pi/2}$$

$$= \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= 0 - j(1) = -j$$

$$W_4^2 = \left(e^{-j\frac{2\pi}{4}}\right)^2 = e^{-j\pi}$$

$$= \cos\pi - j\sin\pi$$

$$= -1$$

$$W_4^3 = \left(e^{-j\frac{3\pi}{2}}\right)^3 = +j$$

Sub. value in W_N matrix

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$x[n] = \{0, 1, 2, 3\}$

substitute

~~$x(k) =$~~

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2+3 \\ \cancel{0+1+2+3} \\ +0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$\boxed{\text{It will come upto } W_8}$

In terms of Radix-2

General Case :

$$W_N^n = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & & & & \\ 1 & W_N^3 & & & & \\ \vdots & \vdots & & & & \\ 1 & W_N^{(N-1)} & & & & W_N^{(N-1)(N-1)} \end{bmatrix}$$

1) DFT of $\delta(n)$, $u(n)$, $\cos(4\pi n)$

$$x(n) = \left[\cos \frac{2\pi r n}{N} \right] \quad 0 \leq r \leq N-1$$

1. $\delta(n)$:

Properties :

1. Periodicity :

$$x(n+N) = x(n)$$

For DFT : $X(k+N) = X(k)$

2. Linearity :

$$\text{DFT}\left[a_1 x_1(n) + a_2 x_2(n) \right] \longleftrightarrow a_1 X_1(k) + a_2 (X_2(k))$$

→ Exists only for periodic signal.

3. <u>Circular Convolution:</u>

$x_1(n) = \{2, 1, 3, 1\}$

$x_2(n) = \underbrace{\{1, 2, 3, 4\}}_{n \rightarrow}$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$

$n \times n$ matrix

n – no. of input

$$\begin{bmatrix} 2+4+9+2 \\ 4+1+12+3 \\ 6+2+3+4 \\ 8+3+6+1 \end{bmatrix}$$

$$= \begin{bmatrix} 17 \\ 20 \\ 15 \\ 18 \end{bmatrix}$$

Linear : Different length calculation is possible

Circular : length should be same

~~1 2 3 4~~

2
3
1
2

Linear Conv:

	2	1	3	1
1	2	1	3	1
2	4	2	6	2
3	6	3	9	3
4	8	4	12	4

$\{2, 5, 11, 18, 15, 15, 4\}$

Circular Conv ~~and~~ is the overlapped form of Linear Conv.

$x_1(n) = \{1, 2, 3, 4\} \rightarrow l_1 = 4$

$x_2(n) = \{1, 2\} \longrightarrow l_2 = 2$

Padding:

$l_2 - 1 \rightarrow$ add that much zeros in $x_1(n)$

now new $l_1 - 1$ no. of zeros in $x_2(n)$

Padded Become:

$\{1, 2, 3, 4, 0\}$

$\{1, 2, 0, 0, 0\}$

$$DFT\left[x_1(n) \circledast x_2(n)\right]$$

Here it is the symbol for circular conv.

15.5.23

(iv) Circular Time Shift:

$$x(n-m) = Z^{-m} \times Z(z) \longrightarrow \text{For Z-Transform}$$

For DFT:

$$DFT[x(n-m)] = W_N^{km} X(k)$$

↳ Twidle Factor : $\left(e^{-\frac{j2\pi}{N}}\right)$

km ↳ shift

↳ value of k in ques.

For. eg:

$$x(n-2) = W_N^{2k} X(k)$$

$x(n) = \{1, 1, 0, 0\}$

$x(k) = \{2, 1-j, 0, 1+j\}$ Ans

$$= W_4^{2k} \{2, 1-j, 0, 1+j\}$$

$$= 2W_4^{2(0)}, (1-j)\, W_4^{2}, 0, (1+j)\, W_4^{6}$$

$$= 2, (1-j)(-1), 0, (1+j)(-1)$$

$$= \{2, -1+j, 0, -1-j\}$$

Given $\rightarrow$ $x(n)$ $\rightarrow$ $\boxed{\text{DFT}\{x(n)\} \rightarrow x(k)}$ $\rightarrow$ do for time shift.

Problem:

$x(n) = \{1, -1, 1, -1\}$, Find $y(n) = x[(n-2)]$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^{1} & W_4^{2} & W_4^{3} \\ 1 & W_4^{2} & W_4^{4} & W_4^{6} \\ 1 & W_4^{3} & W_4^{6} & W_4^{9} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^{1} & W_4^{2} & W_4^{3} \\ 1 & W_4^{2} & W_4^{4} & W_4^{6} \\ 1 & W_4^{3} & W_4^{2} & W_4^{1} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$w_4^0 = \left(e^{-j\frac{2\pi}{4}}\right)^0 = 1$$

$$w_4^1 = \left(e^{-j\frac{2\pi}{4}}\right)^1 = e^{-j\pi/2} = -j$$

$$w_4^2 = -1$$

$$w_4^3 = +j$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} \\ 1 + j - 1 - j \\ 1 + 1 + 1 + 1 \\ 1 - j - 1 + j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$W_4^{km} \cdot X(k) = W_4^{2m} \{0, 0, 4, 0\}$$

$$= \{0, 0, 1(4), 0\}$$

$$\boxed{W_4^{km} = \{0, 0, 4, 0\}}$$

(v) Circular Freq. Shift

$$DFT\left[x(n)\cdot W_N^{-l}\right] = x((k-l))_N$$

Example:

$x(n) = \{1, 0, 1, 0\}$

$x(k) = \{2, 0, 2, 0\}$

$y(k) = x((k-2))_4$ done using property

$y(n) = W_4^{-2n}\{1, 0, 1, 0\}$

$y(n) = \{1, 0, 1, 0\}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

(vi) Parseval's Eqn:

$$DFT\left[\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |x(k)|^2 = E\right.$$

This is given → Energy will be asked

use this

(vi) Complex Conjugate: $\longrightarrow$ Symmetrical

$x(n) = a + ib$

$x^*(n) = a - ib$

$$DFT[x^*(n)] = X^*(N-k)$$: $\boxed{X(1) = X^*(N-1)}$

order

Example:

$x(k) = \{1-j, 1+j, 2+j, 0\}$

$Y(k) = X^*(N-k)$

$Y(0) = X^*(4-0) = X^*(4) = X^*(0)$

$= 1+j$

$Y(1) = X^*(4-1) = X^*(3) = 0$

$Y(2) = \ldots$

$Y(3) = \ldots$

(vii) Circular Time Reversal:

$$DFT[x(-n)] = X(N-k)$$

Similar to (vii) but conjugate is not necessary.

HW: $X(k) \longrightarrow x(n) \longrightarrow x(-n) \longrightarrow DFT$.

Problem:

$$X(k) = \{8, -1+j2, 2+j3, 1-j4, -2+j, 3+j, -1-j3, 3+j, \cancel{1-j3}\}$$

$X(k)$ is 12 point DFT:

$$X(7) = X^*(12-7)$$
$$X(8) = X^*(12-8)$$
$$X(9) = X^*(3)$$
$$X(10) = X^*(2)$$
$$X(11) = X^*(1)$$

Found using conjugate property

like that

$$X(k) = \{1, A, -1, B, -7, -j2, C, -1+j$$

8 data

Find Signal Energy:

8 Point DFT

M1 contd: Correlation:

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l)$$

$$\gamma_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)\, y(n)$$

$$\gamma_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n)\, x(n-l)$$

$$\gamma_{yx}(l) = \sum_{n=-\infty}^{\infty} y(n+l)\, x(n)$$

$$\gamma_{xy}(l) = \gamma_{yx}(-l).$$

Properties:

1. Fourier Transform of auto-correlation is ~~Given by~~ power spectrum.

$$S_{xy}(\omega) = FT\left[\gamma_{xy}(l)\right].$$

$$\text{DFT}\{x(n)\} = \{4, -j2, 0, j2\}$$

Find, :

1. $\text{DFT}\{x(n-2)\}$

2. $\text{DFT}\{x(-n)\}$

3. $\text{DFT}\{x(n) \circledast x(n)\}$

4. Energy.

<u>Solution</u>

1.

$$Y(k) = W_N^{nk} X(k) = \left(e^{-\frac{j2\pi}{4}}\right)^{k \cdot m} X(k)$$

$$= \left(e^{-\frac{j\pi}{2}}\right)^{2k} X(k)$$

$$\rightarrow \boxed{0, 1, 2, 3}$$

2.

$$Y(k) = X^*(k) = X(N-k) = X(4-0) = X($$

$$X(4-1) = X($$

$$X(4-2) = X($$

$$X(4-3) = X($$

3.

$$Y(k) = X(k) \cdot X(k) = Y(+)$$

$$= \{(4, -j2, 0, j2)\} = (4, -j2, 0, j2)\}$$

$$= \{16, -4, 0, -4\} \xrightarrow{IDFT} = \frac{4}{4}$$

4. $\frac{1}{N} \sum |x(k)|^2$

$= \frac{1}{4} \sum |4, -j2, 0, j2|^2$

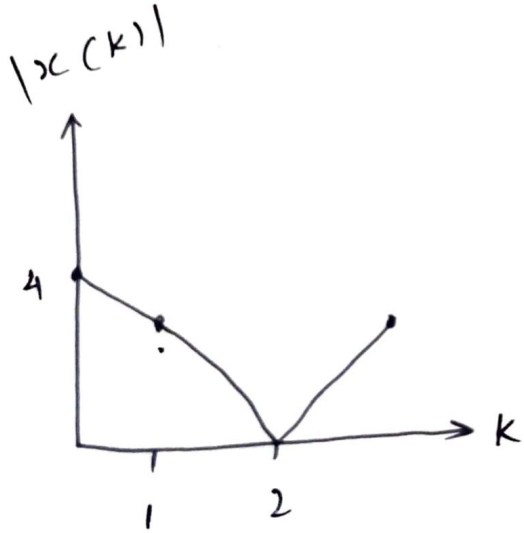
$= \frac{16 + 4 + 0 + 4}{4} = \frac{24}{4} = 6.$

$x(n) = \{2, 0, 2, 0\}$

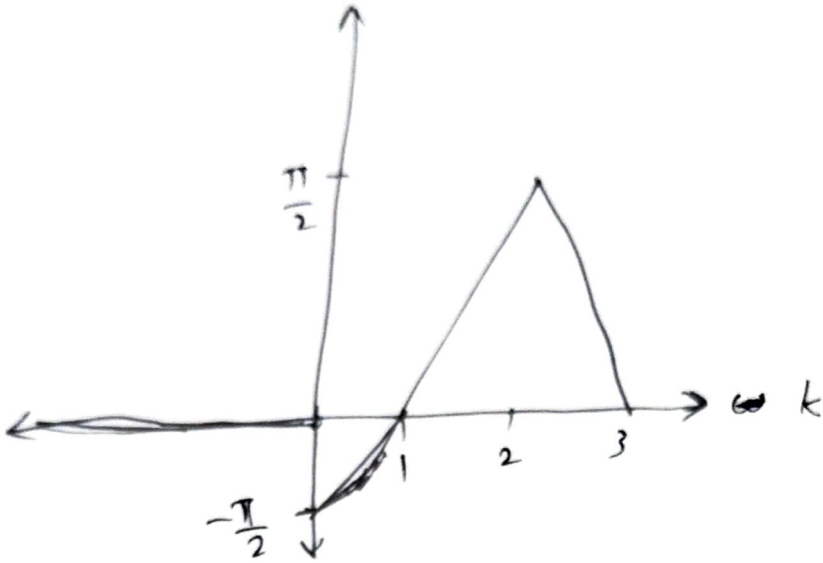
$x(k) = \{4, -j2, 0, j2\}$

$\downarrow$

$|x(k)| = \{4, 2, 0, 2\}$



$\angle X(k) = \{0, -\pi/2, 0, \pi/2\}$



Fast Fourier Transform:

$x(n) \rightarrow X(t)$

	Direct computation → DFT		Radix = 2 FFT	
	Comp. Multi $N(N-1)$	Comp. Mult. N^2	Comp. Add $N \log_2 N$	Comp Mul $\frac{N}{2} \log_2 N$
4	12	16	8	4
8	56	64	24	12

N - point $\quad N = r^m$

$4 = 2^m = 2^2$

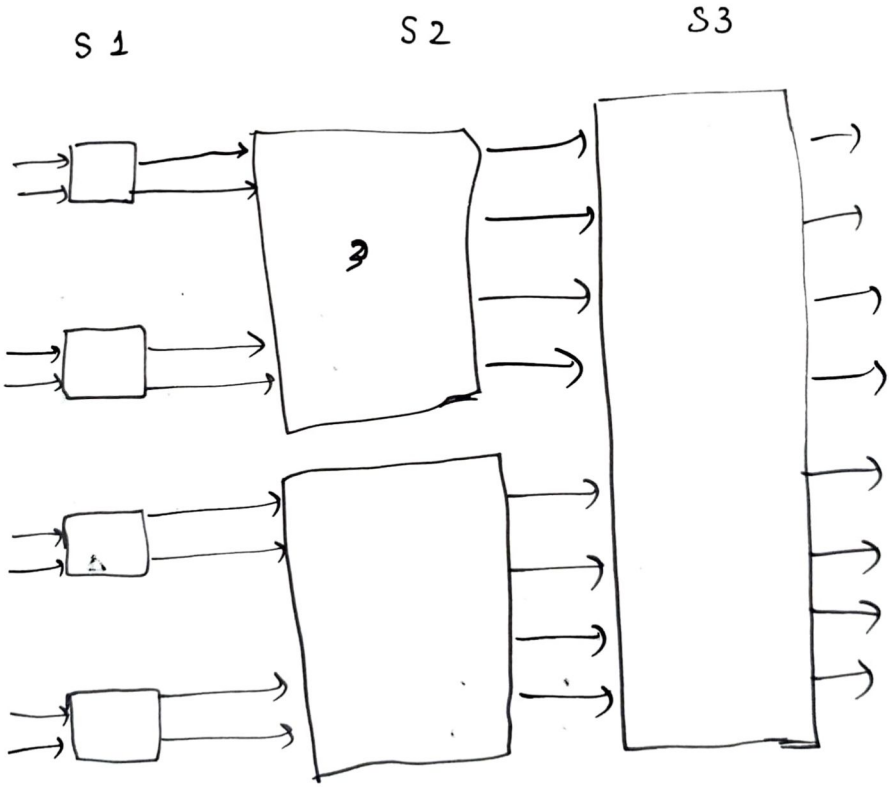
$\boxed{m = 2}$

$8 = 2^3$

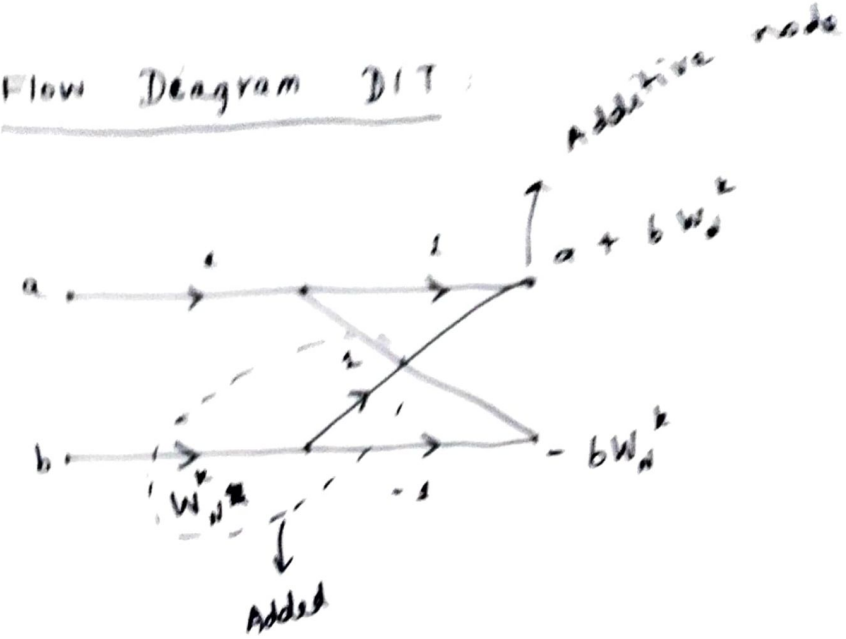
$\boxed{m = 3}$

DIT → ~~Designation~~ Destination in Time

DIF → Designation in Frequency

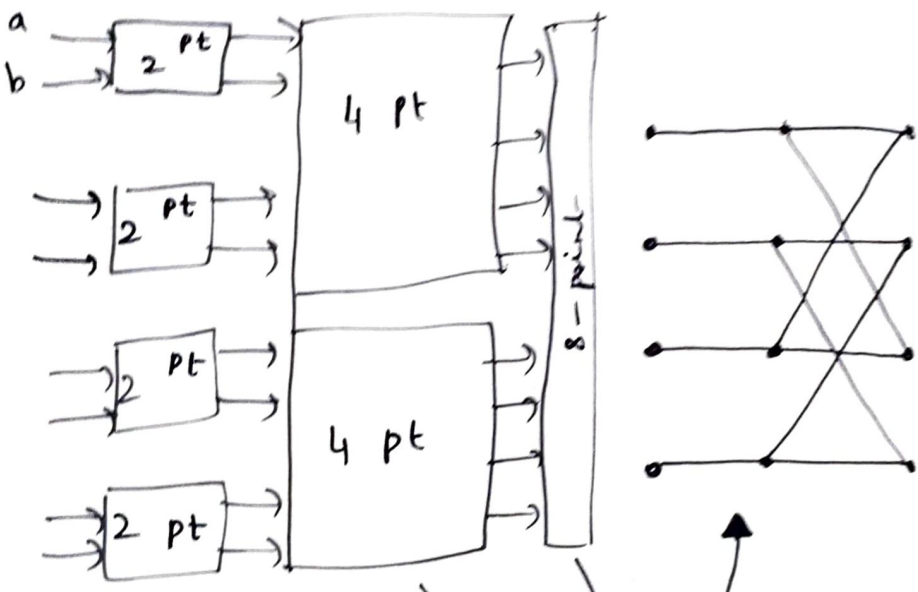


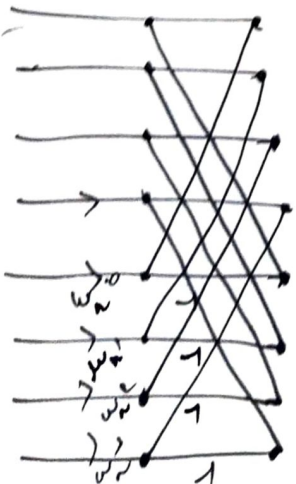
Flow Diagram DIT :



8- Point FFT :

$$\boxed{8 = 2^3 \quad , \quad 3}$$





19.5.23 CAT - 1 : 15 Marks

An 8-point sequence $x[n] = \{\overset{x(0)}{2}, \overset{x(1)}{2}, 2, 2, \boxed{1}, 1, 1, \overset{x(7)}{1}\}$

Compute 8-pt seq. DFT $[x[n]]$ by radix 2 DIT FFT method.

↓ powers of 2.

Soln

$N = 8$.

$x(0) = 000$

$x(1) = 001$

$x(2) = 010$

$x(3) = 011$

$x(4) = 100$

$x(5) = 101$

$x(6) = 110$

$x(7) = 111$

Binary

Bit Rev.

$000 = x(0) = 2$

$\boxed{100} = x(4) = 1$

$010 = x(2) = 2$

$\boxed{110} = x(6) = 1$

$\boxed{001} = x(1) = 2$

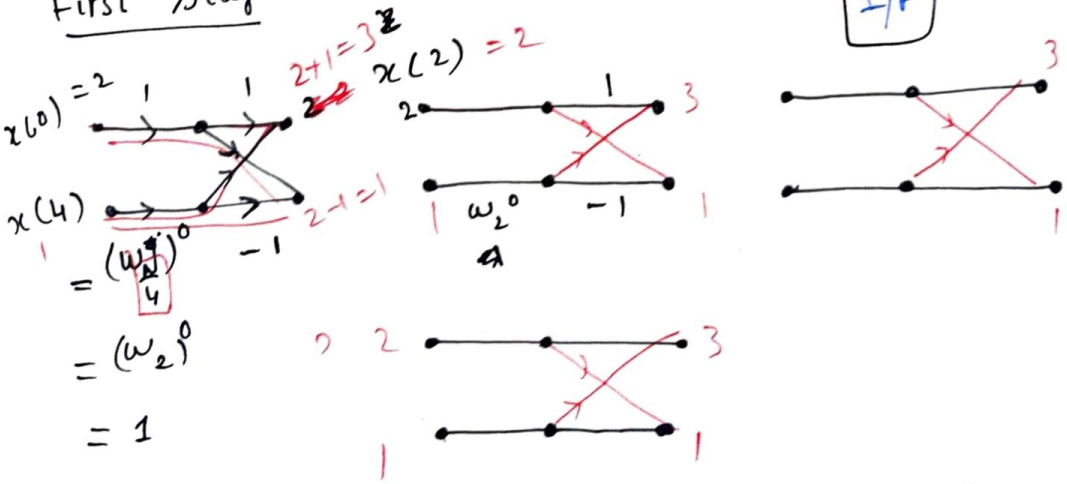
$101 = x(5) = 1$

$\boxed{011} = x(3) = 2$

$111 = x(7) = 1$

as DIT I/P

First Stage:

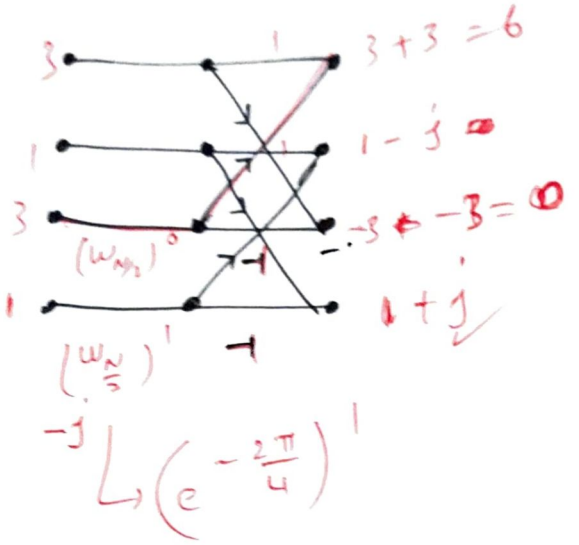


$= (\omega_{\boxed{4}})^0$

$= (\omega_2)^0$

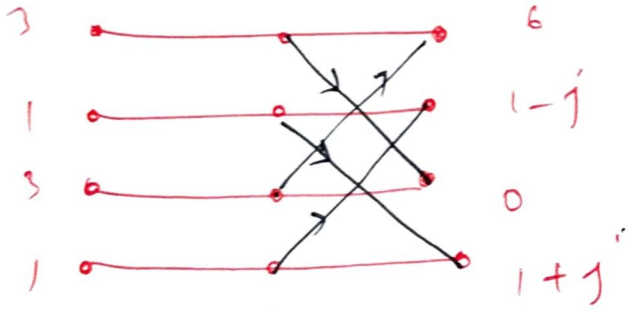
$= 1$

II - Stage :



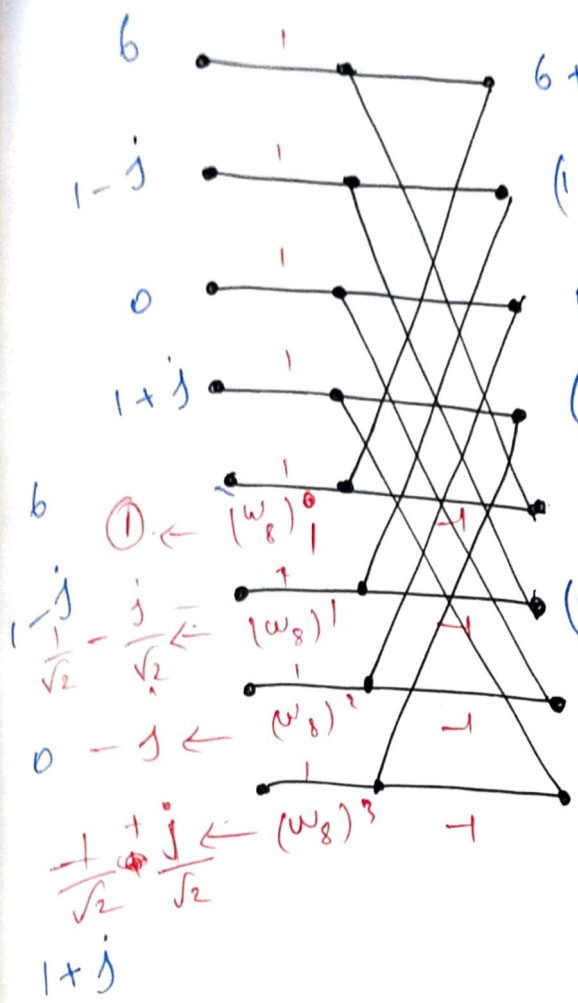
$-j \hookrightarrow \left(e^{-\frac{2\pi}{4}}\right)^1$

$= e^{-\pi/2} = -j$ ✓



III - Stage :

PTO



$6+6 = 12$

$(1-j) + \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)(1-j)$

$0+0 = 0$

$(1+j) + (1+j)\left(\frac{-1}{\sqrt{2}} + \frac{j}{2}\right)$

$6-6 = 0$

$(1-j) - \left(\frac{1}{\sqrt{2}} - \frac{j}{2}\right)(1-j)$

$0-0 = 0$

$(1+j) - (1+j)\left(\frac{-1}{\sqrt{2}} + \frac{j}{2}\right)$

$Y(0) = 12$

$Y(1) = (1-j) + \left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} - \frac{j}{\sqrt{2}} + \frac{+1}{2}\right)$

$Y(k) = \{ 12, \underline{1-j(1-\sqrt{2})}, 0, ___, 0, ___, 0, ___ \}$

$^2(n)$

Inverse DIT FFT

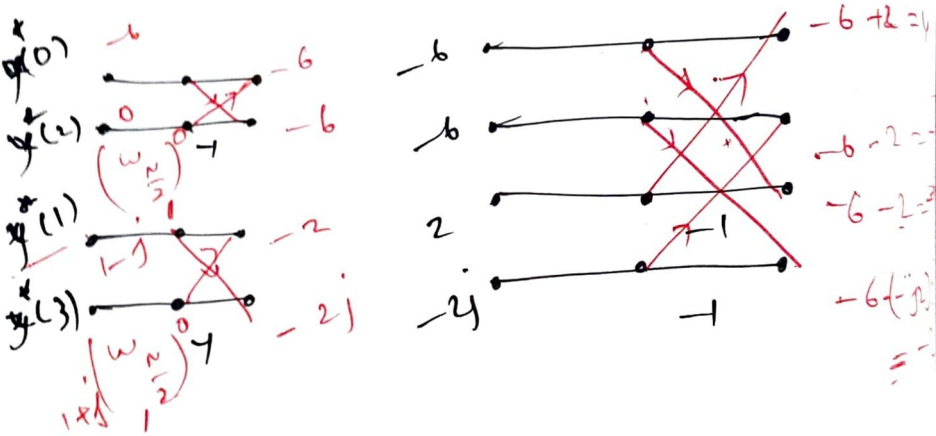
$$Y(k) = \{-6, 1+j, 0, 1-j\}$$

$$y(n) = \frac{1}{N}\left[\sum_{k=0}^{n-1} y^*(k)\, N_n^{nk}\right]$$

$$Y^*(k) = \{-6, 1-j, 0, 1+j\}$$

4 point $\quad 0, 1, 2, 3 \rightarrow$

00	00	$\rightarrow y^*(0)$
01	10	$\rightarrow y^*(2)$
10	01	$\rightarrow y^*(1)$
11	11	$\rightarrow y^*(3)$



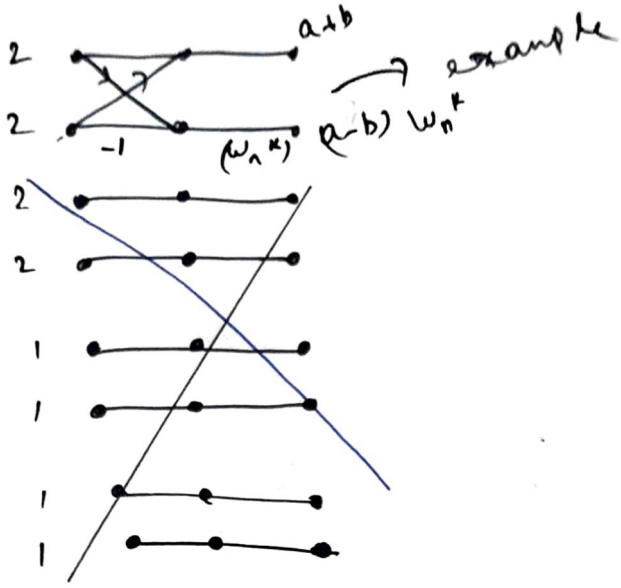
using conjugated value

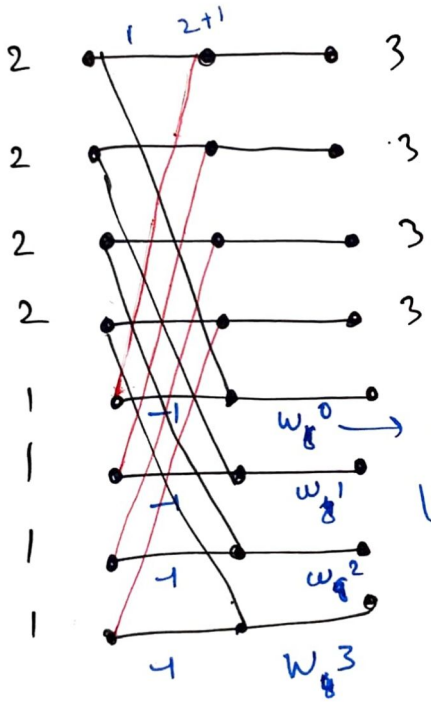
$$y^*(n) = \{-4, -8, -8, -4\}$$

$$y(n) = \{-4, -8, -8, -4\}$$

$$y(n) = \{-1, -2, -2, -1\}$$

DIF: $x(n) = [2, 2, 2, 2, 1, 1, 1, 1]$



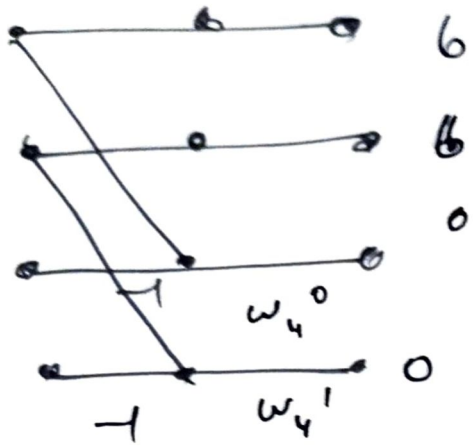


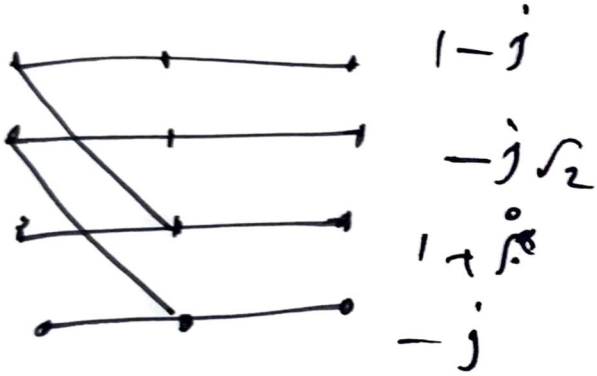
$(W_{8/2})^0$ ①

$(W_{8/2})^1$ $\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$ $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$-j$ $-j$

$\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$ $\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$


6
6
0
0
-1
W_4^0
-1
W_4^1

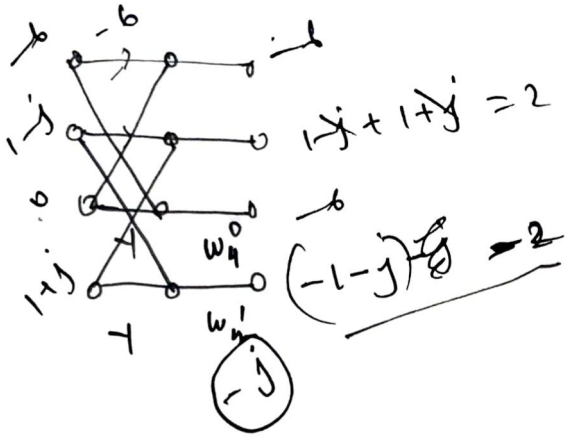

$1-j$
$-j\sqrt{2}$
$1+j$
$-j$

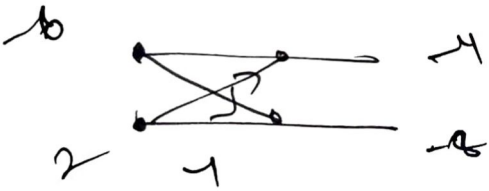
IDIF:

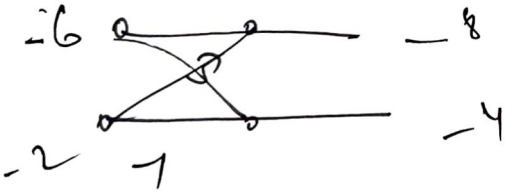
$Y(k) = \{-6, 1+j, 0, 1-j\}$

$\downarrow$

$Y^*(k) = \{-6, 1-j, 0, 1+j\}$







$Y^*(n) = \{-4, -8, -8, -4\}$

$\downarrow$ conjugate

$Y(n) = \{-4, -8, -8, -4\}$

$y(n) = \{-1, -2, -2, -1\}$

$$x(n) = \cos\left(\frac{2\pi}{N}n\right)$$

compute 8-point $\boxed{\text{DIT}}$ or DIF

$$x(n) = \cos\left(\frac{\cancel{2}^{1}\pi}{\cancel{8}_{4}}n\right)$$

$$x(n) = \left\{\cos\left(\frac{\pi}{4}0\right), \cos\left(\frac{\pi}{4}\right), \cos\left(\frac{\cancel{4}^{1}\pi}{\cancel{4}_{2}}\right), \cos(\pi), \cos\left(\frac{5\pi}{4}\right), \cos\left(\frac{\cancel{3}^{3}\pi}{\cancel{2}_{2}}\right), \cos\left(\frac{7\pi}{4}\right)\right\}$$

$$= \left\{1, \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}, -1, \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right.$$

Module 3 : Analog IIR Filter

↳ Infinite Impulse Response.

FIR → Finite " "
Filter Filter.

(μS)

$$a_0\, y(n) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \dots$$

Need for filter:

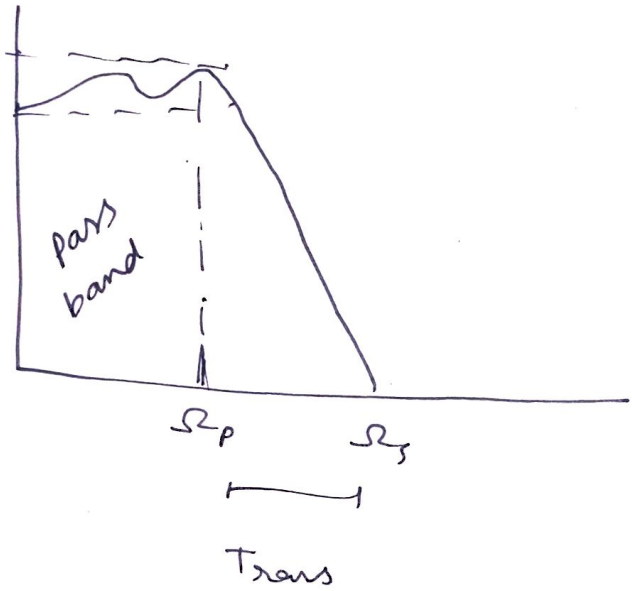
- ✱ signal Seperation
- ✱ " Restoration.

Filter Classification

- ✱ Length of impulse response.
- ✱
- ✱
- ✱

Types of Filter:

1. Butterworth Filter
2. Chebycher - I
   Chebycher - II



Steps:

① K_p K_s → gain

δ_p δ_s → Tolerance

A_p A_s → Attenuation

Ω_p Ω_s → edge freq.

② N → order

③ $H_N(s)$ → Normalised Magnitude Response of LPass Filter

Fig ④

$$H_N(s) = \frac{1}{B_N(s)} = \frac{1}{(s-s_0)(s-s_1)\cdots}$$

↳ Butterworth Polynomial.

$N = 2, 4, 8,$ like that

if $N = 4$.

$s_0 \ldots s_7$ $(2N-1)$ $e^{j\cdot c}$

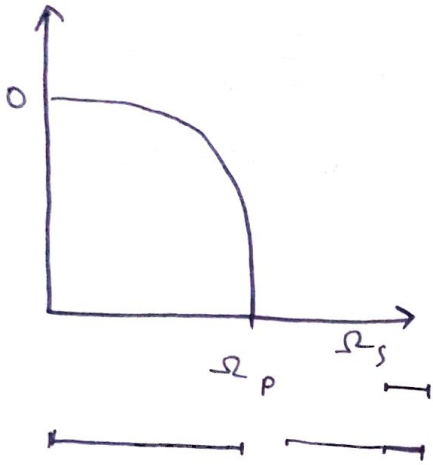
$$s_k = e^{\frac{j\pi(2k+1)}{2\pi}} \cdot e^{\frac{j\pi}{2}}$$

④ Find $H_A(s)$

For

$s \to \frac{s}{\Omega_c}$	$s \to \frac{\Omega_c}{s}$
Low pass	High pass

→ Here use the original Ω_c, Ω_p values in question



$$k_p \leq 20 \log |H(j\Omega)| \leq 0$$

$$20 \log |H(j\Omega)|$$

$$N = \frac{\log \left[\dfrac{10^{-\frac{k_p}{10}} - 1}{10^{-\frac{k_s}{10}} - 1} \right]}{2 \log \left(\dfrac{\Omega_p}{\Omega_s} \right)}$$

use normalized Ω_p and Ω_s

$$\Omega_c = \frac{\Omega_p \quad \Omega_s}{\frac{-K_p}{10} \quad -K_s}$$

$10 \qquad -1$

Either passband or stop band

5

28 M1 : Signals & Systems - 5 Marks

Z-Transform → 10

Inv-Z Transform → 5

M2 : FFT → DIT, DIF, IDIT, IDIF

Property based DFT

31.05.23

Problem :

A butterworth lowpass filter has to

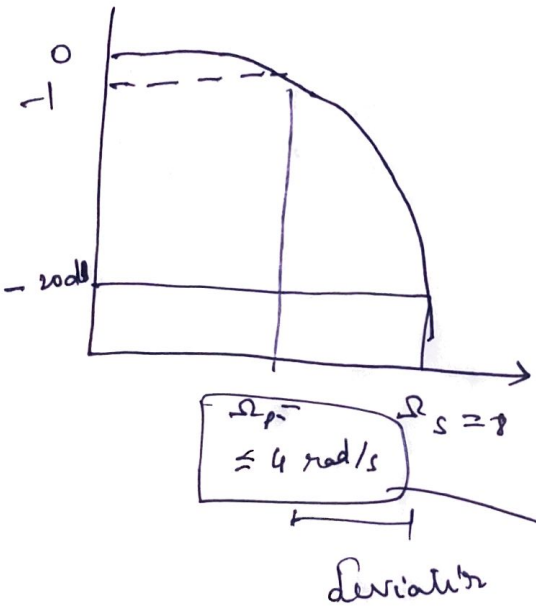
$K_p = -1$ dB

Ω_p ~~A_s~~ $= 4$ rad/s

$A_s = 20$ db

$K_s = -20$ dB

$\Omega_s = 8$ rad/s.



we need to use

$\Omega_p' = 1$ rad/s

$H_N(s) \rightarrow N$

LPF

$$\Omega_s' = \frac{\Omega_s}{\Omega_p} = \frac{8}{4} = 2 \text{ rad/s}$$

for HPF $\Rightarrow \Omega_s' = \Omega_p / \Omega_s$

Band ~~stop~~ pass Filter =

$\Omega_1, \Omega_2, \Omega_l, \Omega_u$

$\Omega_s = \text{Min}\{|A|, |B|\}$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1 (\Omega_u - \Omega_l)}$$

reciprocal

$$B = \frac{+\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2 (\Omega_u - \Omega_l)}$$

<u>Band stop Filter :</u>

$A_{BS} = 1/A_{BP}$

reciprocal.

$B_{BS} =$

$$H_5(s) = \frac{1}{(s - s_0)(s - s_1)(s - s_2)(s - s_3)(s - s_4)}$$

$$= \frac{1}{}$$

For 3

High Pass Filter:

Design a butter Analog hpf with following speech.

max passband Attenuation $A_p = 2$ dB

$\boxed{k_p = -2 \text{ dB}}$

Passband edge freq $\Omega_p = 200$ rad/s

min stopband Attenuation $A_s = +20$ dB

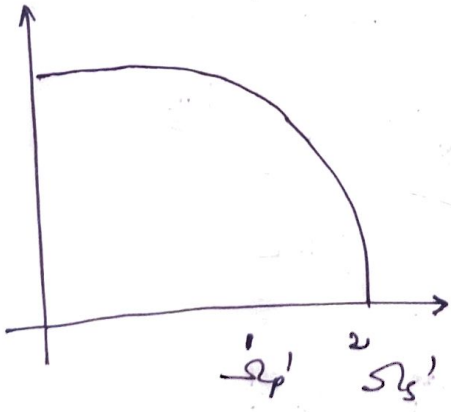
$k_s = -20$ dB

stopband edge freq $\Omega_s = 100$ rad/s

1. ~~$\Omega_p' = 1$~~

$\Omega_p' = 1$ rad/s.

$\Omega_s' = \Omega_p / \Omega_s = 200/100 = 2$ rad.



$$N = \frac{\log\left[\dfrac{10^{\frac{L_p}{10}} - 1}{10^{\frac{L_s}{10}} - 1}\right]}{2\log\left[\dfrac{\Omega_p}{\Omega_s}\right]} \begin{matrix} \to 1 \\ \to 2 \end{matrix}$$

$$= \frac{\log\left[\dfrac{10^{\frac{2}{10}} - 1}{10^{2} - 1}\right]}{2\log\left[\dfrac{1}{2}\right]}$$

$$= 3.701 \simeq 4$$

(7)

$$S_k = e^{j\pi\left(\frac{2k+1}{2N}\right)} \cdot e^{j\frac{\pi}{2}}$$

$$S_1 = e^{j\pi\left(\frac{2(1)+1}{8}\right)} \cdot e^{j\frac{\pi}{2}}$$

$$= e^{j\pi\left(\frac{3}{8}\right)} \cdot e^{j\pi/2}$$

$$= e^{j\pi\left(\frac{3}{8} + \frac{1\times 4}{2\times 4}\right)} = e^{j\pi\left(\frac{7}{8}\right)}$$

$$= -0.9239 + j\,0.3826$$

$$s_0 = -0.382 + j\,0.9238$$

$$e^{\frac{j\pi}{8}(}$$

$$s_2 = e^{j\pi\left(\frac{5}{8}\right)} \cdot e^{j\pi\left(\frac{1}{2}\right)}$$

$$= -0.928 - i\,0.382$$

$$s_3 = -0.3826 - 0.9238\,j$$

$$\Omega_c = 213.8$$

$$H_a(s) = H_4(s)\Big|_{s \to \frac{\Omega_c}{s}}$$

3.6.23

Problem 2)

Design an analog bandpass filter to meet the following freq- domain specification:

→ a -3.0103 dB lower and upper cutoff freq. 50 Hz and 20 kHz.
Ω_l Ω_u

→ a stop band attenuation of atleast 20 dB at 20 Hz and 45 kHz.
Ω_1 Ω_2

→ a monotonic freq. response.
↳ Butterworth filter

~~$\Omega_1 = 2$~~

Soln

HZ is converted to rad/s

$\Omega_1 = 2\pi \times 20 = 125.66$ r/s

$\Omega_2 = 2\pi \times 45$ kHz $= 282743.34$ rad/s

$\Omega_l = 251.32$

$\Omega_u = 125663.71$ rad/s.

Step 2:

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1 (\Omega_u - \Omega_l)} = 2$$

$B = -\Omega_2^2 -$

$K_p = -3.0103$

$K_s = 20$

$\Omega_p = 1$

$\Omega_s = 2.25$

$$N = \frac{\log\left[\dfrac{10^{-\frac{K_p}{10}} - 1}{10^{-\frac{K_s}{10}} - 1}\right]}{2\log\left[\dfrac{\Omega_p}{\Omega_s}\right]}$$

$$= \frac{\log\left[\dfrac{10^{+\frac{3.0103}{10}} - 1}{10^{\frac{20}{10}} - 1}\right]}{2\log\left[\dfrac{1}{2.25}\right]} = 2.83 \approx 3.$$

$$S_k = e^{j\pi\left(\frac{2k+1}{2N}\right)} \cdot e^{\frac{j\pi}{2}}.$$

S_0 to S_3:

$$\cancel{H_a(s) = 1.9695 \times 10^{15} \; s^3}$$

$S_k =$

5.6.23

Chebyshev Filter

Chebyshev-I

1. Equiripple in pass band and monotonic in stop band.

Chebyshev-II

2. Equiripple in stop band and monotonic in pass band.

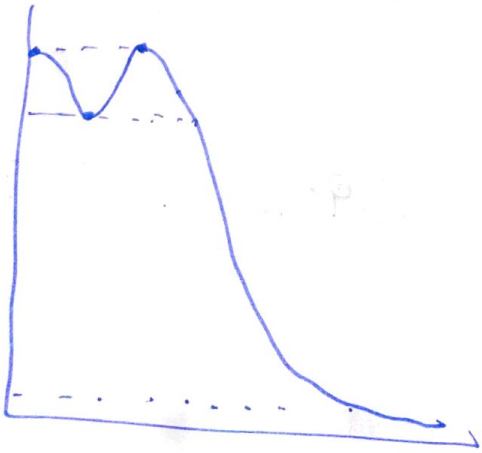
P1) Design a Chebyshev-I analog low pass filter to meet the following specification.

$k_p = -3$ dB

$\Omega_p = 100$ rad/s

$k_s = -25$ dB

$\Omega_s = 250$ rad/s.



$\Omega_p' = 1$

$\Omega_s' = \frac{\Omega_s}{\Omega_p} = \frac{250}{100}$

$= 2.5$

Step 3

$$K_p = 20 \log \left(\frac{1}{\sqrt{1+\epsilon^2}} \right)$$

$$N \geq \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)}$$

$d \rightarrow$ can be found using δ_p δ_s

$$\boxed{\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}}}$$

$$\boxed{K_s = 20 \log \delta_s}$$

$$k = \frac{\Omega_p}{\Omega_s} \rightarrow \text{Normalized}$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$$N = \frac{3.5755}{1.5667} = 2.28$$

6.6.23

B.P :

$$s \rightarrow \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$$

B.S :

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$$

$$K = \frac{\Omega_p}{\Omega_s}$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

2)

Passband ripple ≤ 2 dB

Passband edge : 1 rad/s

Stopband attenuation ≥ 20 dB

Stop band edge : 1.3 rad/s

$\boxed{\text{LPF}}$

$\Omega_p' = 1$ ✓

$\Omega_s' = \frac{\Omega_s}{\Omega_p} = \frac{1.3}{1} = 1.3$ rad/s ✓

$K_p = 20 \log \left(\frac{1}{\sqrt{1+\epsilon^2}} \right)$

$K_2 = 20 \log \left(\frac{1}{\sqrt{1+\varepsilon^2}} \right)$

$$\frac{12}{20} = \log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$10^{\frac{-1}{10}} = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$10^{\frac{-1}{10}} = \sqrt{1+\epsilon^2}$$

$$10^{-2/10} = 1+\epsilon^2$$

$$10^{-2/10} - 1 = \epsilon^2$$

$$\epsilon \cdot \epsilon$$

$$2 = 20 \log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\frac{1}{10} = \log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$10^{\frac{1}{10}} = \cancel{\log} \frac{1}{\sqrt{1+\epsilon^2}}$$

$$10^{-1/10} = \sqrt{1+\epsilon^2}$$

$$10^{-2/10} = 1+\epsilon^2$$

$$10^{-1/5} = 1+\epsilon^2$$

$$10^{-1/5} - 1 = \epsilon^2 \qquad \epsilon^2 = -0.3690$$

$$\epsilon = 0.6074\, i$$

$$\delta_p = 1 - \frac{1}{\sqrt{1-\epsilon^2}}$$

$$= 1 - \frac{1}{\sqrt{1+0.3690}}$$

$$\boxed{\delta_p = 0.145}$$

$\cancel{\delta_s =}$.

$$K_s = 20 \log \delta_s$$

$$\cancel{20} = \cancel{20} \log \delta_s$$

$$\delta_s = 10$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}}$$

$$\boxed{d = 0.6096\, i}$$

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)} =$$

$b_0 = 0.08172$

$b_1 = 0.459349$

$b_2 = 0.6934$

$b_3 = 1.4995$

$b_4 = 0.70646$

Module 4:

7.6.23

Ω_p & Ω_s also fine

$$\Omega_p' = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega_s' = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$$

Frequency Warping:

$$\cancel{\Omega = \frac{2}{T}\left[\frac{2 \sin \omega}{}\right]}$$

Flowchart for Digital Filter:

Digital values
$k_p, k_s, \omega_p, \omega_s$

↓ Ω_p, Ω_s found $\boxed{T = 1}$ if not given

Prewarped analog specification
$\Omega_p', \Omega_s', k_p, k_s$

↓

Design of analog filter.

↓

Problem 1:

Design a digital freq.

a.

b. -3.01 dB cut off 0.5π rad

K_p ω_p

c. Stop band attenuation

15 dB $\longrightarrow A_s \longrightarrow K_s \rightarrow -15$ dB

0.15π rad.

ω_s

Step 1:

$$\Omega_p' = \frac{2}{T} \tan\left(\frac{0.5\pi}{2}\right) = 2$$

$$\Omega_s' = \frac{2}{T} \tan\left(\frac{0.75\pi}{2}\right) = 4.828$$

$$\Omega_p = 1$$

$$\Omega_s = \frac{\Omega_s}{\Omega_p}$$

$$= \frac{4.828}{2} = 2.414 \text{ rad/s.}$$

$$N = \frac{\log\left[\dfrac{10^{\frac{-K_p}{10}} - 1}{10^{\frac{-K_s}{10}} - 1}\right]}{2\log\left(\dfrac{\Omega_p}{\Omega_s}\right)}$$

$$= 1.94 \simeq 2$$

$$= \cancel{\log\left[\right.}$$

$0 \rightarrow 2N-1$

$\boxed{0-3}$

$$S_k = e^{j\pi\left(\frac{2k+1}{4}\right)} \cdot e^{\frac{j\pi}{2}}$$

$$S_0 = e^{j\pi\left(\frac{2(0)+1}{4}\right)} \cdot e^{\frac{j\pi}{2}}$$

$$= e^{j\pi\left(\frac{1}{4}\right)} \cdot e^{j\pi/2}$$

$$= e^{\frac{j\pi}{4}} \cdot e^{\frac{j\pi}{2}}$$

$$= \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$$

$$S_1 = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$$

Lowest Order :

Chebyshev I LPF :

ω_p

- 3 dB ripple in Passband $0 \leq |\omega| \leq 0.3\pi$

Atleast

- 20 dB attenuation is stop band

$0.6\pi \leq |\omega| \geq \pi$

- use Bilinear Transformation

Given,

$k_p = -3$ dB

$\omega_p = 0.3\pi$

$k_p = -20$ dB

$\omega_s = 0.6\pi$.

1. Prewarping:

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{T}\tan\left(\frac{0.3}{2}\right)$$

$$= \cancel{28.20}\ 1.019$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{T}\tan\left(\frac{0.6}{2}\right)$$

$$= 2.752$$

$$\Omega_p = 1 \qquad \Omega_s = \frac{\Omega_s'}{\Omega_p} = \frac{2.752}{1.019} = 2.701$$

2. $$k_p = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$-3 = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\frac{-3}{20} = \log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$10^{\frac{-3}{20}} = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$\sqrt{1+\epsilon^2} = 10^{3/20} = 1.41253754$$

$$1+\epsilon^2 = 1.995262315$$

$$\epsilon^2 = 0.995262315$$

$$\epsilon = 0.9976283451$$

$$K_s = 20 \log \delta_s$$

$$-\cancel{20} = \cancel{20} \log \delta_s$$

$$\boxed{\delta_s = 0.1}$$

$$\delta_p = 1 - \frac{1}{1.4125} = 0.29205$$

$$k = \frac{\Omega_p'}{\Omega_s'} = 0.37$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1}} = 0.15$$

$$\cancel{N = \log_{10}}$$

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{k}\right)}$$

$$= 1.81$$

$$\boxed{N \approx 2.}$$

$b_1 = 0.6448996$

$b_0 = 0.7079478$

$H_2(s)$

$$\cancel{H_N} = \frac{b_0 / \sqrt{1+\varepsilon^2}}{s^2 + b_1 s + b_0}$$

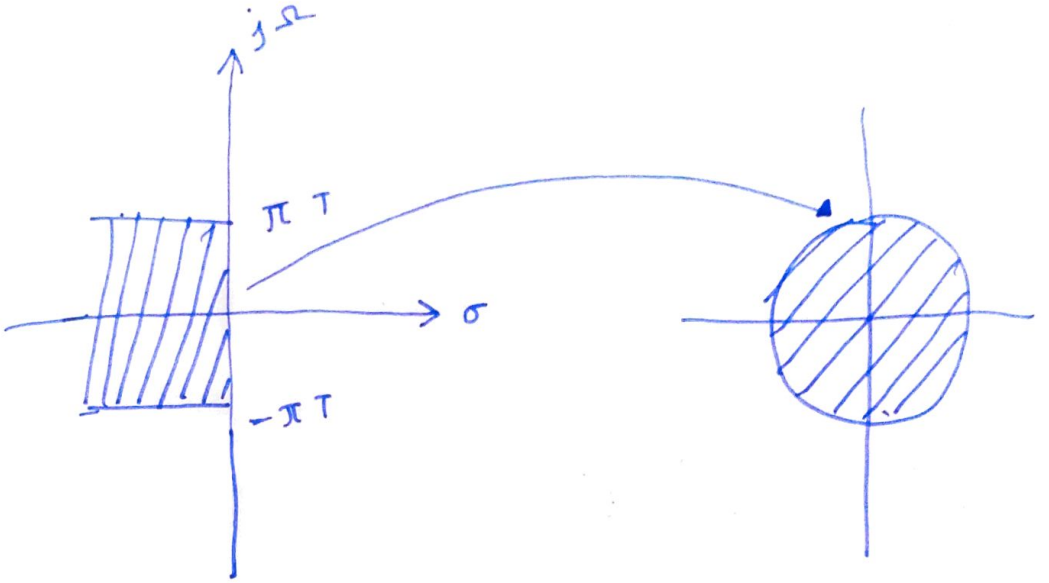
$$= \frac{\dfrac{0.7079478}{\sqrt{1.4125}}}{s^2 + ____ \cancel{b_0}\, s + ____}$$

$$= \frac{0.5011 9}{s^2 + 0. \quad +} \Bigg|_{s \to \frac{s}{1.01}}$$

Replace $H(z)$ by $H_a(s)\Big|_{s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

$$= \frac{0.52}{_____}$$

Impulse Invarient Technique – IIT :



$$H_A(s) = \frac{1}{(s-1)(s^2+2s+1)}$$

$\downarrow$ $H(z)$, For that

Convert into partial fraction.

$$\frac{2}{s-4} + \frac{3}{s-5} - \frac{5}{s-6}$$

$\downarrow$

$$\frac{2}{1-e^{4T}z^{-1}} + \frac{3}{1-e^{5T}z^{-1}}$$

$$+ \frac{5}{1-e^{6T}z^{-1}}$$

13.6.23

Impulse Invariant :

$$H_a(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$H_a(s) \Rightarrow H(z)$ using IIT method

$$\frac{1}{s-s_i} = \frac{1}{\cancel{(s+1)}\ 1-e^{sT} z^{-1}}$$

$$\frac{1}{\underbrace{(s+1)}_{s_1 = -1}\ \underbrace{(s^2+s+1)}_{s_2 = \frac{-1+\sqrt{3}j}{2},\ s_3 = \frac{-1-\sqrt{3}j}{2}}}$$

$, -1, \ -0.5+0.866j, \ -0.5-0.866j$

$$= \frac{1}{(s+1)(s-(0.5-0.866j))(s-(0.5+0.866j))}$$

$$= \frac{c_1}{s+1} + \frac{c_2}{s-(0.5-0.866j)} + \frac{c_3}{s-(0.5+0.866j)}$$

$$= c_1(s^2+s+1) + c_2(s+1)\left(\frac{-1-\sqrt{3}j}{2}\right) + c_3(s+1)\left(\frac{-1+\sqrt{3}j}{2}\right)$$

$$= \frac{1}{s+1} + \frac{0.577e^{-2.62}}{s+0.5-j0.866} + \frac{0.577e^{-2.4}}{s+0.5+j0.866}$$

$$= \frac{1}{1-e^{-T} z^{-1}} + \frac{0.577 e^{-j2.62}}{1-e^{(0.5-j0.866)T} z^{-1}}$$

$$+ \frac{0.577 e^{-j2.62}}{1-e^{(0.5+0.866j)T} z^{-1}}$$

$$\boxed{\begin{aligned} \frac{b}{(s+a)^2+b^2} &= \frac{(e^{-aT} \sin bT) z^{-1}}{1-2e^{-2T}\cos bT z^{-1} + e^{-2T} z^{-2}} \\ \frac{s+a}{(s+a)^2+b^2} &= \frac{1-e^{-2T}\cos bT z^{-1}}{1-2e^{-2T}\cos b z^{-1}} \end{aligned}}$$

0.26332085

$$H_a(s) = \frac{0.51302 \times}{(s+0.296T)^2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{z^{-2}+2z^{-1}+1}$$

$$Y(z)\, z^{-2} + 2z^{-1} Y(z) + Y(z) = X(z) + 2z^{-1} X(z)$$

$$y(n-2) + 2y(n-1) + y(n) = x(n) + 2x(n-1)$$

Module 6 : Structures and Realization

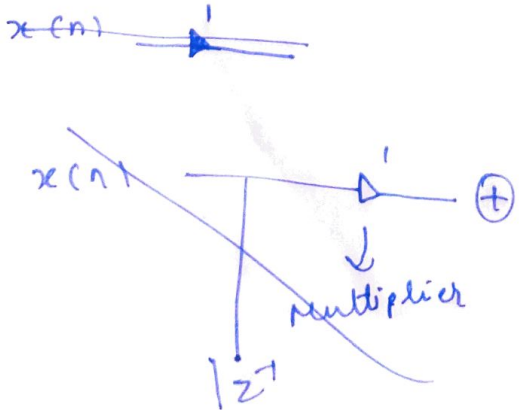
→ D – I (Direct Form I)
→ D – II (Direct Form II)
→ Cascade
→ Parallel.

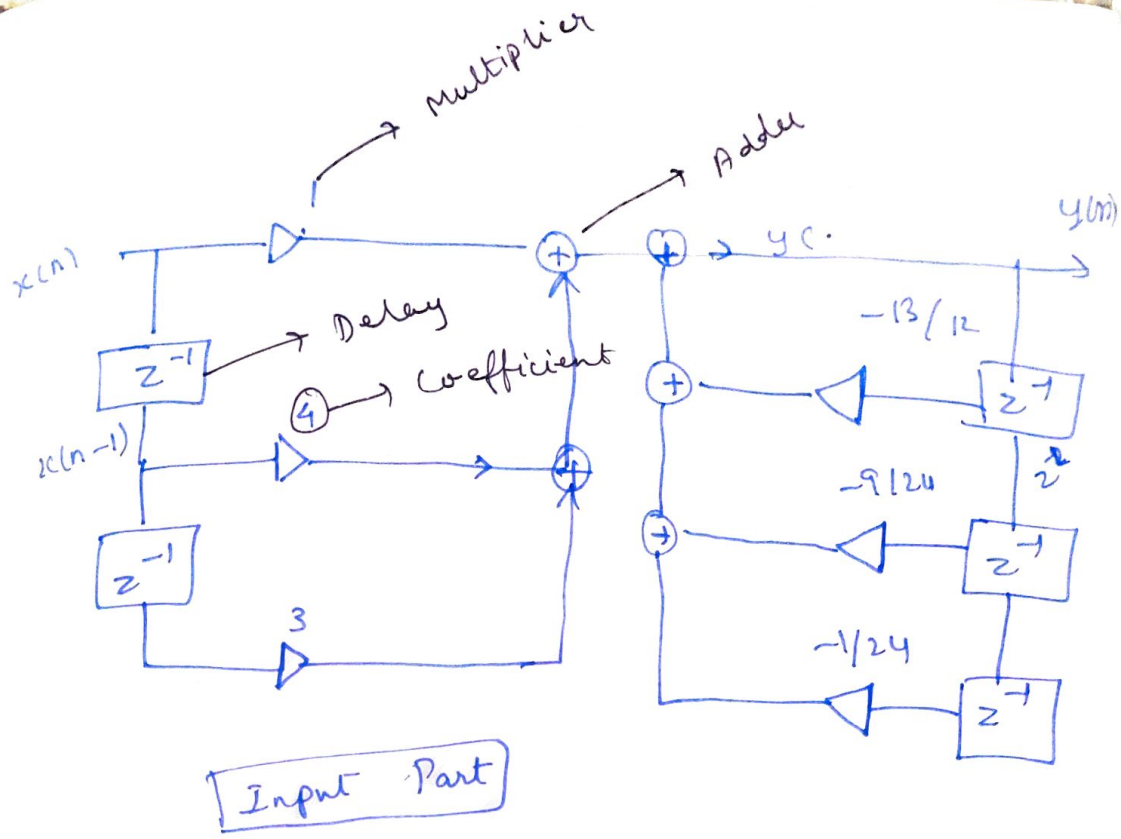
→ Lattice Structures
→ Lattice Ladder structures

Direct Form I Structure : 5 Marks

$$y(n) = -\frac{13}{12}\, y(n-1) - \frac{9}{24}\, y(n-2) - \frac{1}{24}\, y(n-3)$$

$$+ x(n) + 4\, x(n-1) + 3\, x(n-2)$$





Input Part

Direct Form 2:

$$y(n) = \frac{-13}{12} y(n-1)$$

$$y(z) = \frac{-13}{12} z^{-1} y(z)$$

$$y(z) \longrightarrow H(z)$$

$$Y(z)\left[1 + \frac{13}{12} z^{-1} + \frac{9}{24} z^{-2} + \frac{1}{24} z^{-3}\right]$$

$$= X(z)\left[1 + 4z^{-1} + 3z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1 + 4z^{-1} + 3z^{-2}}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}$$

$$H(z) = \frac{Y(z)}{W(z)} \cdot \boxed{\frac{W(z)}{X(z)}}$$

$W(z)$ → dummy variable

$$\frac{Y(z)}{W(z)} = \frac{1 + 4z^{-1} + 3z^{-2}}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}$$

$\frac{Y(z)}{W(z)}$ → Eq ①

$$\boxed{\frac{W(z)}{X(z)} = \frac{1}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}}$$

Eq 2

$$\frac{Y(z)}{W(z)} = 1 + 4z^{-1} + 3z^{-2}$$

o/p i/p

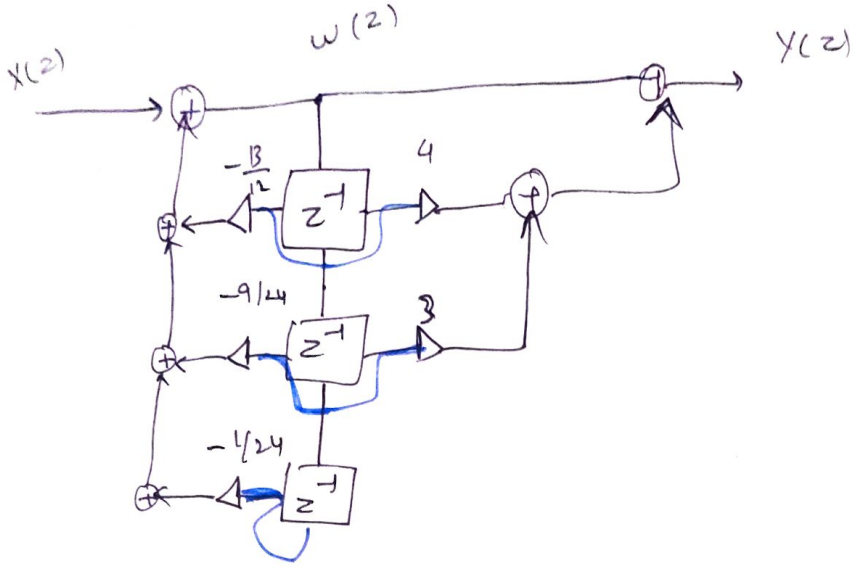
$$\boxed{Y(z) = W(z) + 4z^{-1}W(z) + 3z^{-2}W(z)} \rightarrow (3)$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}$$

$$\boxed{\begin{array}{l} \cancel{X(z)}\ W(z) = X(z) - \frac{13}{12}z^{-1}W(z) - \frac{9}{24}z^{-2}W(z) \\ \qquad - \frac{1}{24}z^{-3}W(z) \end{array}} \rightarrow (4)$$

o/p i/p

$\cancel{X(z)}$



$$H(z) = \frac{3 - 5z^{-1} + 9z^{-2} - 3z^{-3}}{1 - \frac{3}{2}z^{-1} + \frac{5}{6}z^{-2} - \left(\frac{1}{16}\right)z}$$

$$\frac{Y(z)}{X(z)} = \frac{3 - 5z^{-1} + 9z^{-2} - 3z^{-3}}{1 - \frac{3}{2}z^{-1} + \frac{5}{6}z^{-2} - \left(\frac{-1}{16}\right)z^{-3}}$$

$$\frac{Y(z)}{W(z)} = 3 - 5z^{-1} + 9z^{-2} - 3z^{-3}$$

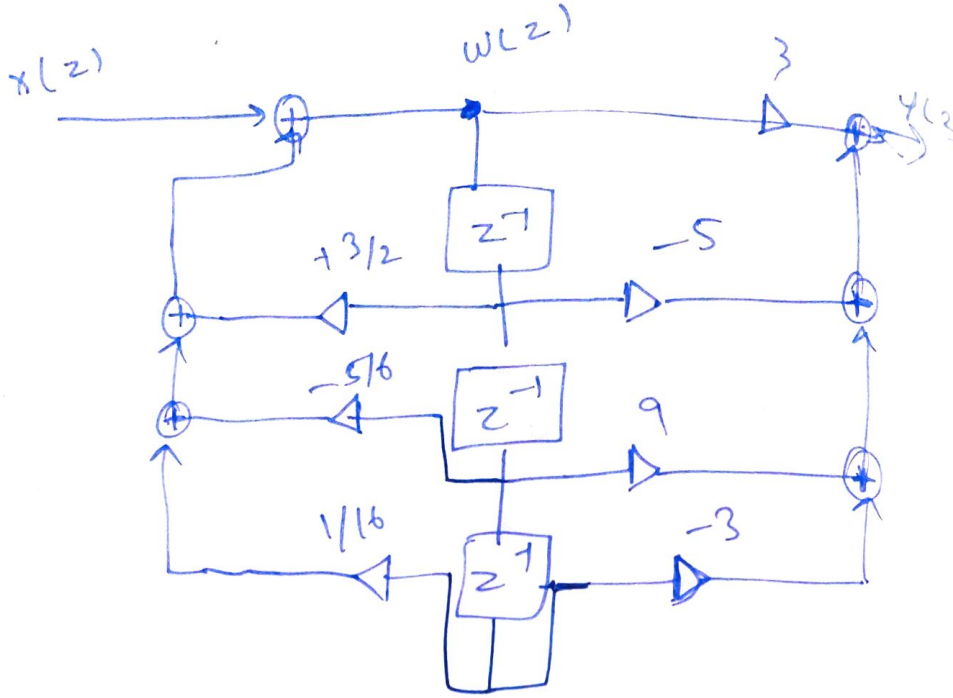
$$\boxed{Y(z) = 3W(z) - 5z^{-1}W(z) + 9z^{-2}W(z) - 3z^{-3}W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - \frac{3}{2} z^{-1} + \frac{5}{6} z^{-2} - \left(\frac{1}{16}\right) z^{-3}}$$

$$\cancel{W(z)} \quad \frac{1}{X(z)} = \frac{1}{W(z) - \frac{3}{2} z^{-1} W(z) + \frac{5}{6} z^{-2} W(z) - \frac{1}{16} z^{-3} W(z)}$$

$$X(z) = W(z) - \frac{3}{2} z^{-1} W(z) + \frac{5}{6} z^{-2} W(z) - \frac{1}{16} z^{-3} W(z)$$

$$W(z) = X(z) + \frac{3}{2} z^{-1} W(z) - \frac{5}{6} z^{-2} W(z) + \frac{1}{16} z^{-3} W(z)$$



16.6.23

Cascade:

$$H(z) \longrightarrow H_1(z) \cdot H_2(z)$$

$$\boxed{\frac{z+1}{z^2-1}} \Longrightarrow \frac{z+1}{(z-1)} \cdot \frac{1}{(z+1)}$$

$(z-1)(z+1)$

$H_1(z)$ $H_2(z)$

Direct form 2 separately

Direct form 2 separately

Parallel

$$\frac{z+1}{z^2-1} = \frac{A}{(z-1)} + \frac{B}{(z+1)}$$

Problem:

$$H(z) = \frac{1+4z^{-1}+3z^{-2}}{1+\frac{13}{12}z^{-1}+\frac{9}{24}z^{-2}+\frac{1}{24}z^{-3}}$$

$$\cancel{\frac{Y(z)}{X(z)}} = \left(\frac{1+z^{-1}}{1+\frac{z^{-1}}{2}}\right)\left(\frac{1+3z^{-1}}{1+\frac{z^{-1}}{3}}\right)$$

$$H(z) = \frac{(1+z^{-1})(1+3z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)} \left(\frac{1}{1+\frac{z^{-1}}{4}}\right)$$

$$\frac{H_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)}.$$

$$\frac{Y_1(z)}{W_1(z)} = 1 + z^{-1}$$

$$Y_1(z) = W_1(z) + W_1(z)\, z^{-1}$$

$$= \frac{1}{\left(1 + \frac{1}{2} z^{-1}\right)}$$

$$H(z) = \frac{(1+z^{-1})(1+3z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\cdot\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z) = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} + \frac{C}{\left(1+\frac{1}{4}z^{-1}\right)}$$

$\cancel{A} =$

17.6.23

$$H(z) = \frac{30}{1+\frac{1}{2}z^{-1}} - \frac{128}{1+\frac{1}{3}z^{-1}} + \frac{99}{1+\frac{1}{4}z^{-1}}$$

$$H_1(z) + H_2(z) + H_3(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)}$$

$$\boxed{Y_1 = 30 \cdot W_1(z)}$$

$$\boxed{W_1(z) = X(z) - \frac{1}{2} W_1(z) z^{-1}}$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{Y_2(z)}{W_2(z)} \cdot \frac{W_2(z)}{X(z)}$$

$$\frac{Y_2(z)}{W_2(z)} = -128$$

$$\frac{W_2(z)}{X(z)} = \frac{1}{1+\frac{1}{3}z^{-1}}$$

$$W_2(z) = X(z) - \frac{1}{3} z^{-1} W_2(z)$$

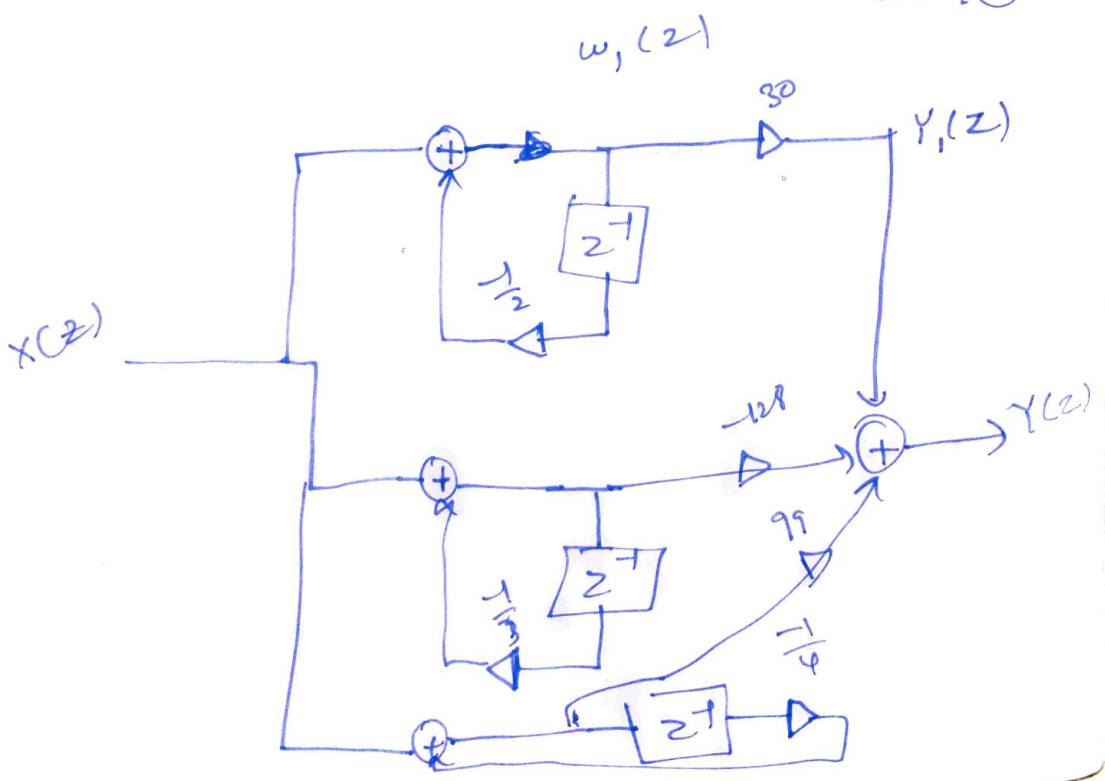
$$\frac{Y_3(z)}{X(z)} = \frac{Y_3(z)}{W_3(z)} \cdot \frac{W_3(z)}{X(z)}$$

$$= \frac{99}{1+\frac{1}{4}z^{-1}}$$

$$\frac{Y_3(z)}{\cancel{X(z)}\ W_3(z)} = 99$$

$$\boxed{Y_3(z) = 99\, W_3(z)}$$

$$W_3(z) = X(z) - \frac{1}{4} z^{-1} W_3(z) \rightarrow (6)$$



$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)}$$

$$\cancel{H(z) = \frac{1+z^{-1}}{1+1}}$$

$$H(z) = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{4}z^{-1}} + \frac{C}{1+\frac{1}{8}z^{-1}}$$

$$A = \left.\frac{(1+z^{-1})(1+2z^{-1})}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{8}z^{-1}\right)}\right|_{z^{-1}=-2}$$

$$= \frac{\cancel{1+2}\ (1-2)(1-4)}{\left(1+\frac{1}{2}\right)\left(1-\frac{1}{4}\right)}$$

$$= \frac{(-1)\cdot(\cancel{+}3)}{\frac{3}{2}\cdot\frac{3}{4}} = \frac{1}{\frac{3}{8}}$$

$$\boxed{A = \frac{8}{3}}$$

$$B = \left.\frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{8}z^{-1})}\right|_{z^{-1}=+4}$$

$$= \frac{(+4)(1+8)}{(1+2)(1+1/2)}$$

$$= \frac{5\cdot\cancel{9}}{\cancel{3}\cdot 3/2} = \frac{\cancel{45}^{\,5}}{\frac{\cancel{9}}{2}} = 10$$

$$\boxed{B = 10}$$

$$C = \left.\frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}\right|_{z^{-1}=-8}$$

$$= \frac{(1-8)(1-16)}{(1-4)(1-2)} = \frac{\cancel{-7}\cdot\cancel{-15}}{\cancel{-3}\cdot\cancel{-1}}$$

$$\boxed{C = 35}$$

$$C = \left.\frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}\right|_{z^{-1}=-2}$$

$$= \frac{-35}{3}$$

$$H(z) = \frac{8/3}{1+\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{4}z^{-1}} - \frac{\frac{35}{3}}{1+\frac{1}{8}z^{-1}}$$

$$H_1(z) = \frac{Y_1(z)}{X(z)}$$

$$= \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)}$$

$$\boxed{Y_1(z) = \frac{8}{3} W_1(z)} \longrightarrow (1)$$

$$W_1(z) = X(z) - \frac{1}{2} z^{-1} W(z)$$

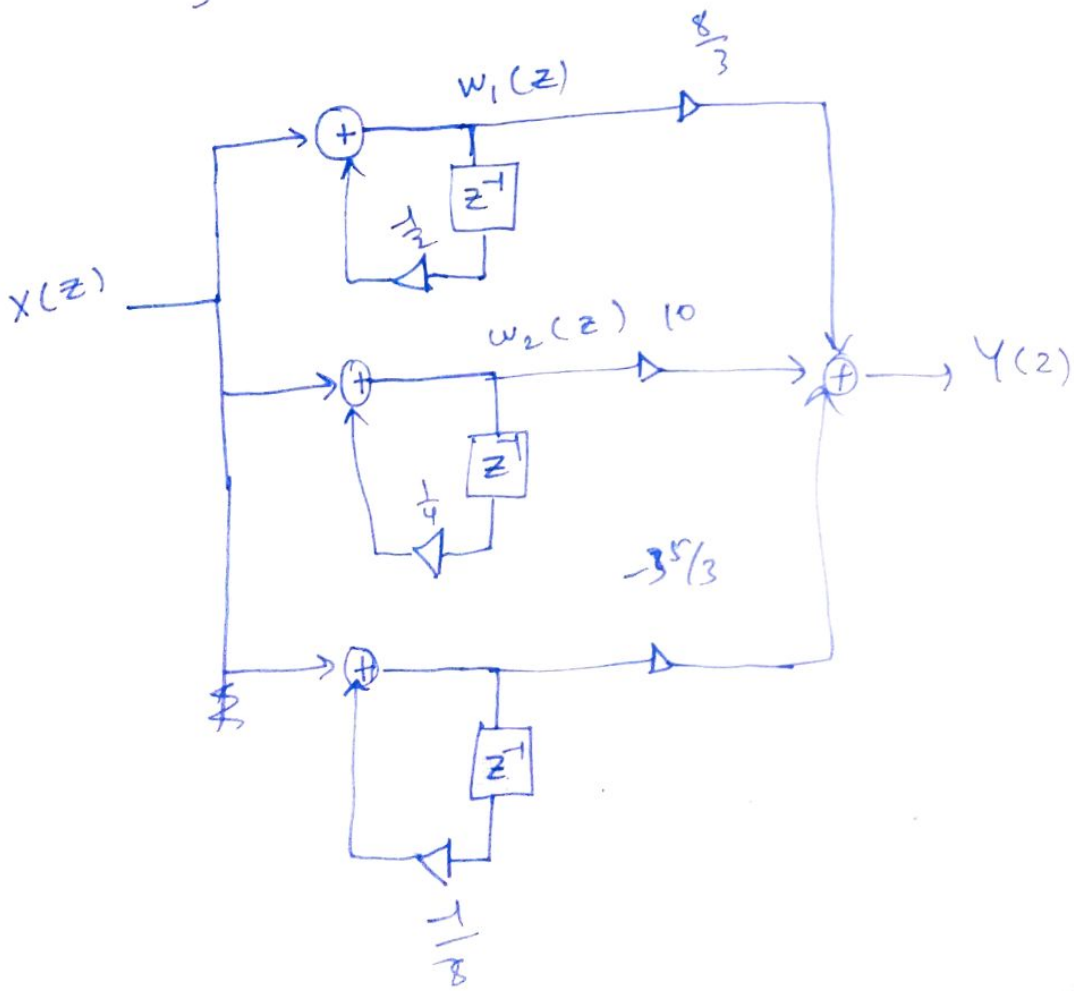
$$\boxed{Y_2(z) = 10 \cdot W_2(z)} \longrightarrow (8)$$

$$\boxed{W_2(z) = X(z) + \frac{1}{4} z^{-1} W_2(z)} \longrightarrow (4)$$

$$H_3(z) = \frac{Y_3(z)}{X(z)}$$

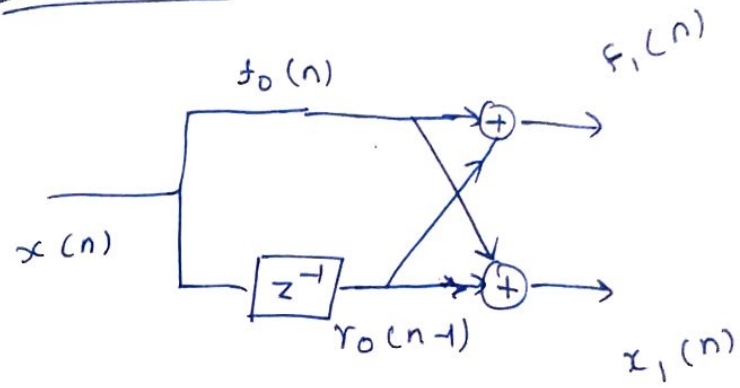
$$Y_3(z) = \frac{-35}{3} W_3(z)$$

$$W_3(z) = X(z) - \frac{1}{8} z^{-1} W_3(z)$$



19.6.23

FIR Lattice:



$$f_1(n) = f_0(n) + k_1 r_0(n-1)$$

$$r_1(n) = k_1 f_0(n) + r_0(n-1)$$

$m = M, M-1, \cdots, 2, 1$

$k_m = a_m(m)$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-1)}{1 - k_m^2}$$

$1 \leq i \leq m-1$

Lattice to direct form 1:

$a_m(0) = 1$

$a_m(m) = k_m$

(4)

$$a_m(i) = a_{m-1}(i) + a_m(m)\, a_{m-1}(m-i)$$

$$\boxed{1 \leq i \leq m-1}$$

$$H(z) = \underset{a_3(0)}{1} + \underset{a_3(1)}{2}z^{-1} + \underset{a_3(2)}{3}z^{-2} + \underset{a_3(3)}{4}z^{-3}$$

Problem:

Determine the co-efficients K_m of the lattice ~~vector~~ filter corresponding to FIR filter described by system function

$$\boxed{H(z) = \underset{a_2(0)}{1} + \underset{a_2(1)}{2}z^{-1} + \underset{a_2(2)}{\frac{1}{3}}z^{-2}}$$

Also draw the corresponding second order lattice structure.

$$k_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - k_m^2}$$

$m = 2,$

$$\boxed{k_2 = a_2(2) = \frac{1}{3}}$$

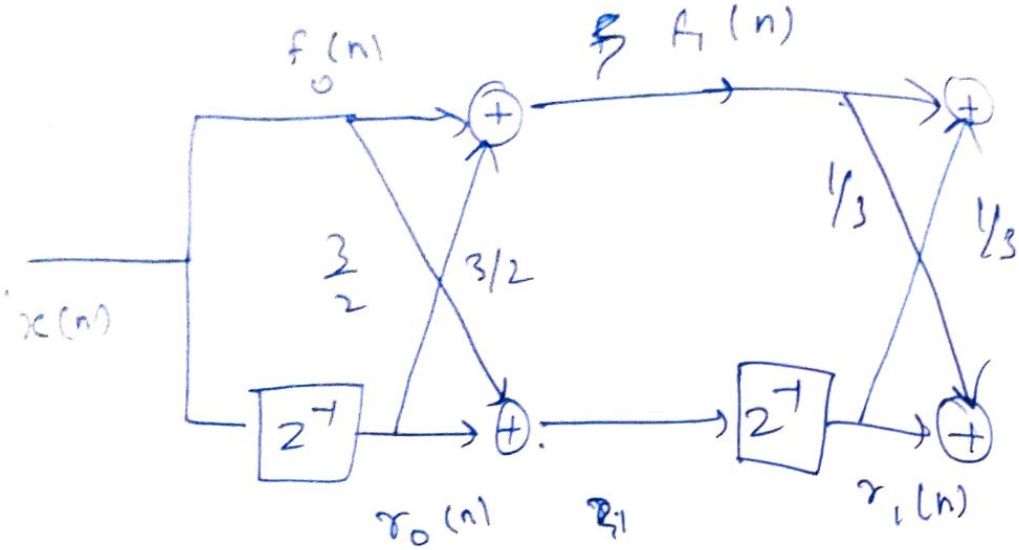
$m = 1,$

$$a_1(i) = \frac{a_2(i) - a_2(2)\, a_2(2-i)}{1 - k_2^2}$$

$$= \frac{2 - \frac{1}{3} \cdot 2}{1 - \frac{1}{9}}$$

$$= \frac{4/3}{8/9} = \frac{3}{2}$$

$$\boxed{k_1 = 3/2}$$



2) Let the coefficients of a 3 state FIR Lattice structure be $k_1 = 0.1$, $k_2 = 0.2$, $k_3 = 0.3$. Find the co-efficient of direct form 1 FIR filter and draw its block diagram.

<u>Solution,</u>

$k_1 = 0.1$, $k_2 = 0.2$, $k_3 = 0.3$

$\boxed{m = 1, 2, 3}$

$a_m(0) = 1$

$a_m(m) = k_m$

$a_m(i) = a_{m-1}(i) + a_m(m)\, a_{m-1}(m-$

$1 \leq i \leq m-1$

recursion Formula

$m = 1$,

$a_1(0) = 1$

$a_1(1) = k_1 = 0.1$

Recursion Formula is not needed

$m = 2$,

$a_2(0) = 1$

$a_2(2) = k_2 = 0.2$

~~$a_2(i$~~

$a_2(i) = a_1(i) + a_2(2)\, a_1(2-i)$

For $i = 1$:

$a_2(1) = a_1(1) + a_2(2)\, a_1(1)$

$= 0.1 + 0.2 \times 0.1$

$\boxed{a_2(1) = 0.12}$

For $m = 3$:

$a_3(0) = 1$

$a_3(3) = k_3 = 0.3$

$i = 1$:

$a_3(1) = a_2(1) + a_3(3) \cdot a_2(2)$

$= 0.12 + 0.3 \times 0.2$

$= 0.18$

$a_3(2) = a_2(2) + a_3(3) \cdot a_2(1)$

$= 0.2 + (0.3)(0.12)$

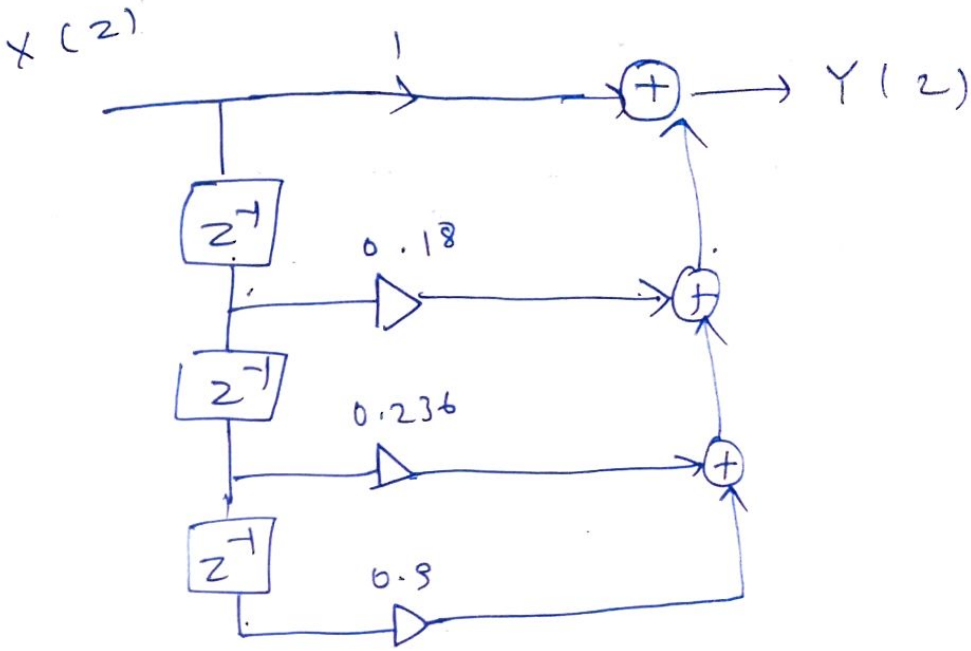
$= 0.236.$

$$\boxed{H(z) = 1 + 0.18 z^{-1} + 0.236 z^{-2} + 0.3 z^{-3}}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)}}$$

Direct Form I :

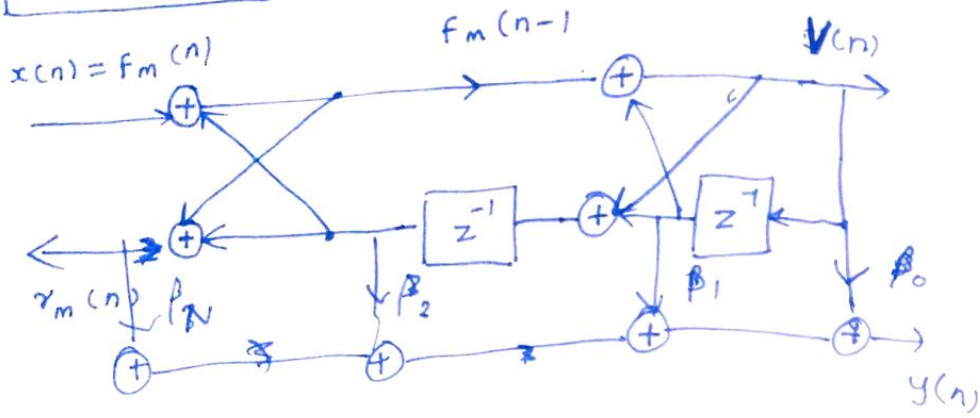
$$Y(z) = X(z) + 0.18\, z^{-1} X(z) + 0.236\, z^{-2} X(z) + 0.3\, z^{-3} X(z)$$



21.06.23

IIR Lattice Structure :

$$\boxed{H(z) = \frac{N}{D}}$$



Question

A LTI system described by,

$$H(z) = \frac{0.129 + 0.3867\, z^{-1} + 0.3869\, z^{-2} + 0.129\, z^{-3}}{1 - 0.2991\, z^{-1} + 0.3564\, z^{-2} - 0.0276\, z^{-3}}$$

Realize by using Lattice Ladder.

Solution

$$H(z) = \frac{1}{A(z)} \cdot B(z)$$

$A(z)$

$$= 1 - 0.2991 z^{-1} + 0.3564 z^{-2} - 0.0276 z^{-3}$$

$B(z)$

$$= \underset{b_0}{0.129} + \underset{b_1}{0.3867 z^{-1}} + \underset{b_2}{0.3869 z^{-2}} + \underset{b_3}{0.129 z^{-3}}.$$

$\boxed{\text{order } 3, \text{ so } 3 \text{ stages}}$

$$a_m(m) = \cancel{a_m} \; k_m$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - k_m^2}$$

$$a_3(3) = k_3 = -0.0276$$

$i = 1$

$$a_2(1) = \frac{a_3(1) - a_3(3)\, a_3(2)}{1 - k_3^2}$$

$$= \frac{-0.2991 - (-0.0276 \times 0.3564)}{1 - (-0.0276)^2}$$

$$= -0.2875$$

$i = 2$

$$a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - k_3^2} = \frac{0.3564 - (-0.0276 \times -0.2975)}{1 - (-0.0276)^2}$$

$$a_2(2) = 0.3485$$

$$k_2 = a_2(2) = 0.3485$$

$i = 1$

$$a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - k_2^2}$$

$$= \frac{-0.2875 - (0.3485 \times (-0.2875))}{1 - (0.3485)^2}$$

$$\boxed{a_1(1) = -0.2132} \rightarrow k_1$$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m (m-i)$$

$$\beta_3 = b_3 - \cancel{\sum_{m=4}^{3} \beta_m a_m (m-i)} \rightarrow \boxed{\text{Not possible}}$$

$$\boxed{\beta_3 = b_3 = 0.129}$$

$i = 2$:

$$\beta_2 = b_2 - \sum_{m=3}^{3} \beta_m a_m (m-2)$$

$$= b_2 - \beta_3 a_3 (1)$$

$$= 0.3869 - 0.129 \times (-0.2971)$$

$$\boxed{\beta_2 = 0.4252}$$

$i = 1$:

$$\beta_1 = b_1 - \sum_{m=2}^{3} \beta_m a_m (m-1)$$

$$= 0.3867 - [\beta_2 a_2 (1) + \beta_3 a_3 (2)]$$

$$= 0.3867 - [0.4252 \times (-0.2875)$$

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + 1.2 z^{-1} + 0.47 z^{-2} + 0.06 z^{-3}}$$

$$= \frac{B(z)}{A(z)}$$

$m = 3, 2, 1$

$a_m(m) = K_m$

$a_3(3) = K_3 = 0.06$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

$i = 1, 2$:

$$a_2(1) = \frac{a_3(1) - a_3(3)\, a_3(2)}{1 - 0.06^2} = 1.176$$

$$a_2(2) = \frac{0.47 - (0.06)(1.2)}{1 - K_3^2} = 0.4$$

$\cancel{\beta_i = b_i}$

$m = 2$:

$$a_1(1) = \frac{1.176 - \cancel{a_2} \cdot 0.4\, \overset{1.17}{a_2(1)}}{1 - \underbrace{(K_2)^2}_{0.4^2}}$$

$$\boxed{a_1(1) = 0.84}$$

$\underline{m=1}$:

$$K_m = a_m(m)$$

$$K_1 = a_1(1) = 0.84$$

$$K_1 = 0.84$$

$$K_2 = 0.4$$

$$K_3 = 0.06$$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$$

$M = 2$:

$i = 2$:

$$\beta_2 = b_2 = 1$$

$$\beta_0 = 1 - [-0.176 \times 0.84 + 0.1 \times 0.4]$$

$$= 0.748$$

$$\beta_1 = b_1 - \beta_2 a_2(2-1)$$

$$= 1 - 1.176$$

$$= -0.176$$

$$\beta_2 = b_0 - [\beta_1 a_1(1) + \beta_2 a_2(2)]$$

$$= 1 - _____ = _____$$

26.6.23

Design of FIR Filter :

LP

Linear Phase

Case

i. Symmetric $h(n)$ N = odd

ii. Symmetric $h(n)$ N = even

iii. Anti - Symmetric $h(n)$, N = odd

iv. " " $h(n)$, N = even

Ques :

Design LPF using rectangular window using 9 samples with 1.2 rad/s as cutoff frequency.

1.

For LPF, desired freq is :

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & -\omega_c \leq \omega \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

2.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) \cdot e^{j\omega n} \, d\omega$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha} e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)} \cdot d\omega$$

$$h_d(n) = \frac{1}{2\pi} \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi(n-\alpha)} \left[\frac{e^{j\omega(n-\alpha)}}{2j} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi(n-\alpha)} \left[\frac{e^{j\omega_c(n-\alpha)}}{2j} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi(n-\alpha)} \left[\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j} \right]$$

$$= \frac{\sin(\cdot \quad)}{2\pi(n-\alpha)}$$

$$=$$

$$h(n) =$$

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$H(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s+1)$$

Let $s = -2$

$$2 = -B$$

$$\boxed{B = -2}$$

Let $s = -1$

$$2 = A$$

$$\boxed{A = 2}$$

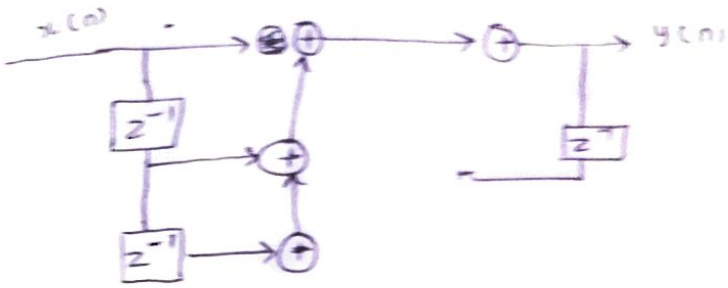
$$H(s) = \frac{+2}{s+1} + \frac{-2}{s+2}$$

$$\frac{1}{s - P_i} \xrightarrow{\text{transformed to}} \frac{1}{1 - e^{P_i T} z^{-1}}$$

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} + \frac{-2}{1 - e^{-2T} z^{-1}}$$

$$y(n) = y(n-1) - \frac{1}{2} y(n-2) - x(n) - x(n-1) - x(n-2)$$

Direct Form - I



Direct Form - II:

$$Y(z) = z^{-1} Y(z) - \frac{1}{2} z^{-2} Y(z) + X(z) - z^{-1} X(z) + z^{-2} X(z)$$

$$Y(z) \left(1 + z^{-1} - \frac{z^{-2}}{2}\right) = X(z)\left(1 - z^{-1} + z^{-2}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + z^{-2}}{1 + z^{-1} - \frac{z^{-2}}{2}}$$

$$\frac{Y(z)}{W(z)} = 1 - z^{-1} + z^{-2}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + z^{-1} - z}$$

$$\frac{Y(z)}{W(z)} = 1 - z^{-1} - z^{-2} \qquad 1 - z^{-1} + \frac{1}{2} z^{-2}$$

$$= z^{-2}\left(z^{2} - z - \tfrac{1}{2}\right)$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - z^{-1} + \frac{z^{-2}}{2}}$$

$$Y(z) = W(z) - W(z) z^{-1} - z\, W(z)\, z^{-2}$$

$$X(z) = W(z) - W(z) z^{-1} + \frac{z^{-2} W(z)}{2}.$$

$$W(z) = X(z) + W(z) z^{-1} - \frac{1}{2} z^{-2} W(z)$$

$$H(z) = \frac{1 + 4z^{-1} + 3z^{-2}}{1 + \frac{13}{12} z^{-1} + \frac{9}{24} z^{-2} + \frac{1}{24} z^{-3}}$$

$$H(z) = \frac{(1 + z^{-1})(1 + 3z^{-1})}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{3} z^{-1}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z)$$

$$\frac{Y_2(z)}{Y_1(z)} = \boxed{\frac{(1 - z^{-1})}{\left(1 + \frac{1}{2} z^{-1}\right)} \cdot \frac{1 + 3z^{-1}}{1 + \frac{1}{3} z^{-1}} \cdot \frac{1}{1 + \frac{1}{4} z^{-1}}}$$

$\cancel{Y_2(z) =}$

$\cancel{W}$

5.7.23

$$\cancel{H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; |\omega| \\ \end{cases}}$$

Design a HPF with a freq. response given $H_d(e^{j\omega})$ as

$$= \begin{cases} 1 & ; \pi/4 \leq |\omega| \leq \pi \\ 0 & ; |\omega| < \pi/4 \end{cases}$$

$M = 11$, using hamming window.

$-\pi \qquad \pi$

Soln,

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} \boxed{e^{-j\omega\alpha}\, e^{j\omega n}}\, d\omega + \int_{\pi/4}^{\pi} \boxed{e^{-j\omega\alpha}\, e^{j\omega n}}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\pi/4} e^{j\omega(n-\alpha)}\, d\omega\right.$$

$$+ \int_{\pi/4}^{\pi} e^{j\omega(n-\alpha)} \, d\omega \Bigg]$$

$$= \frac{1}{\pi(n-\alpha)} \left[\left[\frac{e^{j\omega(n-\alpha)}}{2j} \right]_{-\pi}^{-\pi/4} + \left[\frac{e^{j\omega(n-\alpha)}}{2j} \right]_{-\pi/4}^{\pi} \right]$$

$$= \frac{1}{\pi(n-\alpha)} \left[\sin \pi(n-\alpha) + \sin \frac{\pi}{4}(n-\alpha) \right]$$

<u>If $n = \alpha$:</u>

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} d\omega + \int_{\pi/4}^{\pi} d\omega \right]$$

$$= \frac{1}{2\pi} \left[[\omega]_{-\pi}^{-\pi/4} + [\omega]_{\pi/4}^{\pi} \right]$$

$$= \frac{2\pi}{2\pi} \left[1 - 1/4 \right]$$

$$= 3/4 \, ,$$

$$W_{Han}(n) = 0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right)$$

$$0 \leq n \leq N-1$$

$N = 11$

$\alpha = \frac{N-1}{2} = 5$

$$h_d(0) = \frac{1}{\pi n}\left[\sin \pi n + \sin \frac{\pi n}{4}\right]$$

$$h_d(1) = \frac{1}{\pi(n-1)}\left[\sin \pi(n-1) + \sin \frac{\pi(n}{4}\right]$$

$$h_d(2) = \frac{1}{\pi(n-2)}\left[\sin \pi(n-2) + \sin \frac{\pi(n-2}{4}\right]$$

We Table like this :

n	$h_d(n)$ → ①	$W_{han}(n)$ → ②	$h(n) = ① \times ②$
0			

Desired Freq. response of LPF is given by

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; \quad |\omega| \leq 3\pi/4 \\ 0 & ; \quad \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$

Design FIR Bandpass filter, duration $N = 7$ using hamming window and $H_d(\omega)$

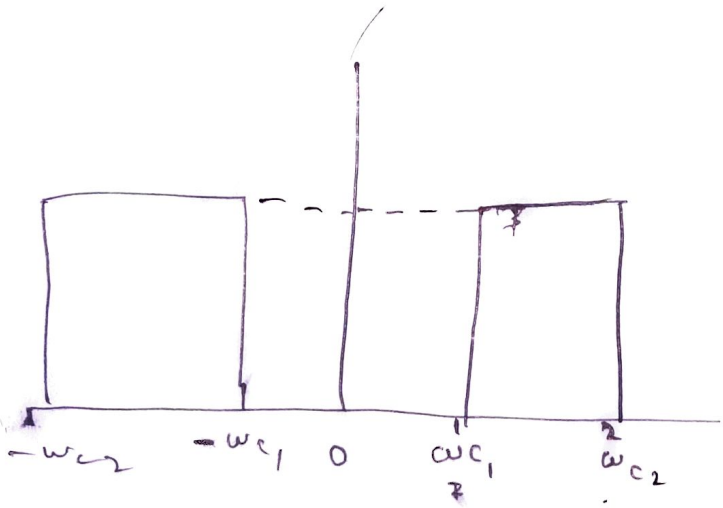
$$= \begin{cases} e^{-j\omega\alpha} ; & \omega c_1 < |\omega| < \omega c_2 \\ 0 ; & \text{otherwise} \end{cases}$$

$N = 7$, $\boxed{\begin{array}{l} \omega c_1 = 1 \text{ rad} \\ \omega c_2 = 2 \text{ rad.} \end{array}}$

For band pass and band stop

$$W_{Ham}(n) = \begin{cases} 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) & 0 \leq n \leq N-1 \end{cases}$$

This is for non causal hamming



$\cancel{h_d(n) = \frac{1}{}}$

$$\boxed{\alpha = \frac{N-1}{2} = 3}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-w_{c_2}}^{-w_{c_1}} H_d(e^{jw})\, e^{jwn}\, dw$$

$$+ \int_{w_{c_1}}^{w_{c_2}} H_d(e^{jw})\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\left[\int_{-w_{c_2}}^{-w_{c_1}} e^{-jw\alpha}\, e^{jwn}\, dw + \int_{w_{c_1}}^{w_{c_2}} e^{-jw\alpha}\, e^{jwn}\, dw\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-jw_{c_1}(n-\alpha)}}{2j} - \frac{e^{-jw_{c_2}(n-\alpha)}}{2j} + \frac{e^{jw_{c_2}(n-\alpha)}}{2j} - \frac{e^{jw_{c_1}(n-\alpha)}}{2j}\right]$$

$$h_d(n) = \left[\frac{\sin[\omega_{c_2}(n-\alpha)] - \sin[\omega_{c_1}(n-\alpha)]}{\pi(n-\alpha)} \right]$$

$n = \alpha$:

$$h_d(n) = \frac{1}{2\pi}\left[[\omega]_{-\omega_{c_2}}^{-\omega_{c_1}} + [\omega]_{+\omega_{c_1}}^{\omega_{c_2}} \right]$$

$\omega_{c_1} = 1$

$\omega_{c_2} = 2$

$$= \frac{1}{2\pi}\left[-\omega_{c_1} + \omega_{c_2} + \omega_{c_2} - \omega_{c_1} \right]$$

$$= \frac{2\omega_{c_2} - 2\omega_{c_1}}{2\pi}$$

$$= \frac{\omega_{c_2} - \omega_{c_1}}{\pi}$$

n	$h_2(n)$	$H_{Han}(n)$	$h_d(n)$
0	-0.0446	0.08	
1	0.0080	0.31	
2	0.2916	0.77	
3	0.31831	1	
4	0.2916	0.77	
5	0.0080	0.31	
6	-0.0446	0.08	

for $n=0$:

$$\frac{\sin(2(0-3)) - \sin(1(-3))}{\pi(0-3)}$$

Wavelet Transform — ⊗ — Theory (Refer PPT)

12.7.23 Multirate DSP :

↳ Multiple sampling rate

Audio Rate : 48 KHz

But CD players : 44.1 KHz

Broadcasting : 32 KHz

So, rate conversion needed. For that Multirate DSP Needed.

↳ Possible by Interpolation + Decimation

14.7.23

Decimation :

Interpolation :

$T \rightarrow$ sampling rate

$f_s \rightarrow$ sampling freq.

$T = \frac{1}{f_s}$ $\quad D = \frac{T'}{T} \Rightarrow T' = DT$

$2T = T'$; $1000 = T'$; $f_s' = \frac{1}{T'} = \frac{1}{DT}$

17.7.23

Revision

Mod 1 – 6 : 15 marks each

Mod 7 : 10 marks

① ✓ What is multirate S.P ? with 1 or 2 applications ?

(or)

✓ Explain Wavelet Transform ? with 1 or 2 example ?

Module 7

① (i) Multirate SP

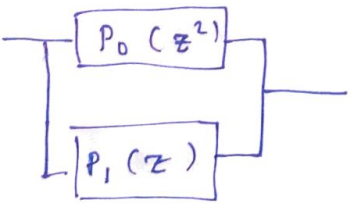
(ii) Down - sampler & Up - sampler : 2 - 3 lines

(iii) Decimation → Anti - aliasing (Helps to avoid aliasing) → Overlapping of signals

Interpolation → With Example.

(iv) Sampling Rate Conversion with Example.

(v) Polyphase Structure → Type 1 and Type 2.



(vi) Few Applications.

2) (i) WT.

(ii) CWT

(iii) DWT

(iv) Types

(v)

} Any 5 points from PPT.

$\boxed{\text{Module 6}}$

Lattice Ladder :

$\underbrace{\text{Pole - 0 Filter}}_{H(Z) \text{ with Num \& Den}}$

Example :

$$H(Z) = \frac{0.129 + 0.3867 Z^{-1} + 0.3869 Z^{-2} + 0.129 Z^{-3}}{1 - 0.2971 Z^{-1} + 0.3564 Z^{-2} - 0.027 Z^{-3}}$$

$$H(Z) = \frac{1}{\underbrace{A(Z)}_{\text{Denominator}}} \cdot \underbrace{B(Z)}_{\text{Numerator}}$$

$$A(z) = \boxed{1} \underbrace{\boxed{-0.2971}}_{a_3(1)} z^{-1} + \underbrace{\boxed{0.3564}}_{a_3(2)} z^{-2} \underbrace{\boxed{-0.027}}_{a_3(3)} z^{-3}$$

$\downarrow$ $a_3(0)$

~~$a_3(0)$~~

Order is $\boxed{3 \rightarrow N}$

$m = N, N-1, \ldots, 2, 1$

$m = 3, 2, 1$ ☆

$\boxed{a_m(m) = k_m}$ $\boxed{K_3 \cancel{(3)} = a_3(3) = -0.027}$

$$\boxed{a_{m-1}(i) = \frac{a_m(i) - a_m(m) \cdot a_m(m-i)}{1 - k_m^2} \qquad i \in [1, m-1]}$$

a_2 $m = 3$

$i = 1, 2$

$$a_2(1) = \frac{a_3(1) - a_3(3) \cdot a_3(2)}{1 - (0.027)^2}$$

$\downarrow$ $i = 1$

$a_2(1) = -0.2876$

$$a_2(2) = \frac{a_3(2) - a_3(3) \cdot a_3(1)}{1 - (0.027)^2} \rightarrow \boxed{k_2}$$

$\downarrow$ $i = 2$

$= 0.3486$

$a_3(3) = -0.027$

$m = 2$ $\boxed{i = 1}$

$$a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(1)}{1 - K_2^2}$$

$$= -0.2132$$

$m = 1$

$$\boxed{a_1(1) = K_1 = -0.2132} \quad ☆$$

$$B(z) = \underset{b_0}{\boxed{0.129}} + \underset{b_1}{\boxed{0.3867}}\, z^{-1} + \underset{b_2}{\boxed{0.3869}}\, z^{-2} + \underset{b_3}{\boxed{0.129}}\, z^{-3}$$

$$\boxed{\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)}$$

$M \rightarrow$ order

$i = M, M-1, \ldots, 0$

$i = 3, 2, 1, 0$

When $i = 3$:

$$\beta_3 = b_3 = -\boxed{\sum_{m=4}^{3} \beta_m a_m(m-1)}$$

This is not possible

$$\beta_3 = b_3 = 0.129$$

$\underline{i = 2}$:

$$\beta_2 = b_2 - \sum_{3}^{3} \beta_m \, a_m \, (m - i)$$

$$\beta_2 = b_2 - \beta_3 \, a_3 \, (1)$$

$$= 0.3869 - 0.129 \, (\qquad)$$

$$\beta_2 =$$

$\underline{i = 1}$:

$$\beta_1 = b_1 - \sum_{2}^{3} \beta_m \, a_m \, (m - i)$$

$$= b_1 - \left[\beta_2 \, a_2 \, (1) + \beta_3 \cdot a_3 \, (2)\right]$$

$$=$$

$$\beta_1 =$$

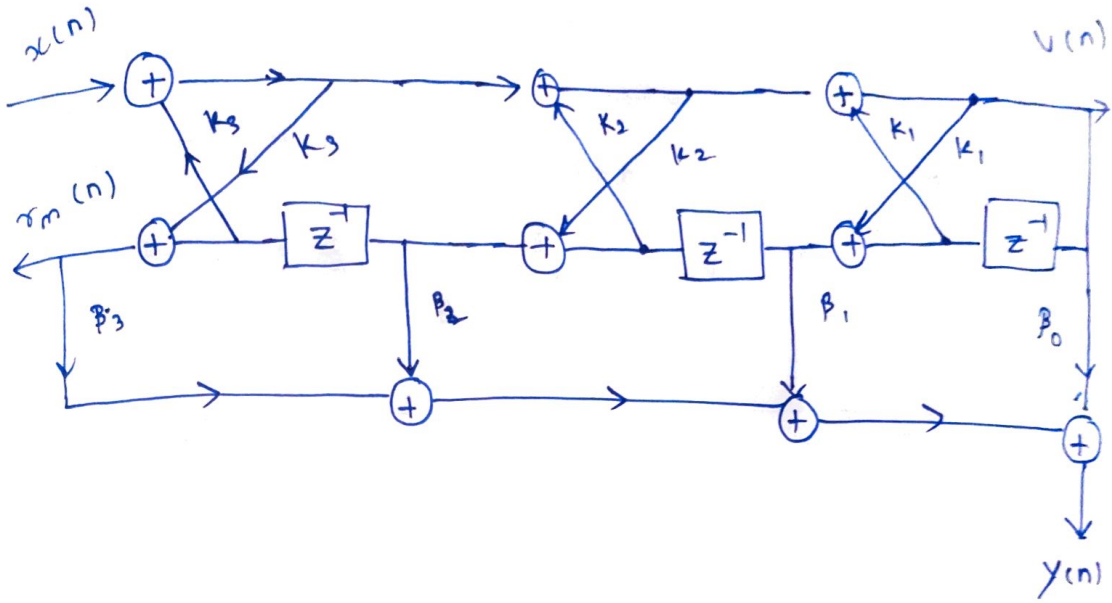
$\underline{i = 0}$:

$$\beta_0 = b_0 - \sum_{m=1}^{3} \beta_m \, a_m \, (m - i)$$

$$= b_0 - \left[\beta_1 \, a_1 \, (1) - \beta_2 \, a_2 \, (2) - \beta_3 \, a_3 \, (\right.$$

$\beta_0 =$

Structure :



18.7.23

Module 27

① FFT → DIT or DIF → 10 Marks

② $y(n) = x(n-2)$: DFT Properties

$= e^{j\pi/2} x(n)$ → find $Y(K)$

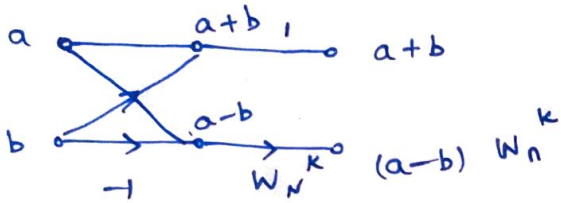
③ FFT computational complexity.

N^2 $N(N-1)$,

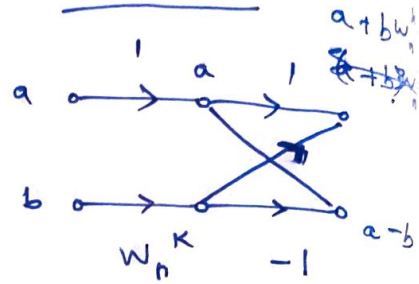
DIF :

$$x(n) = \{1, 2, 2, 2, 1, 0, 0, 0\}$$

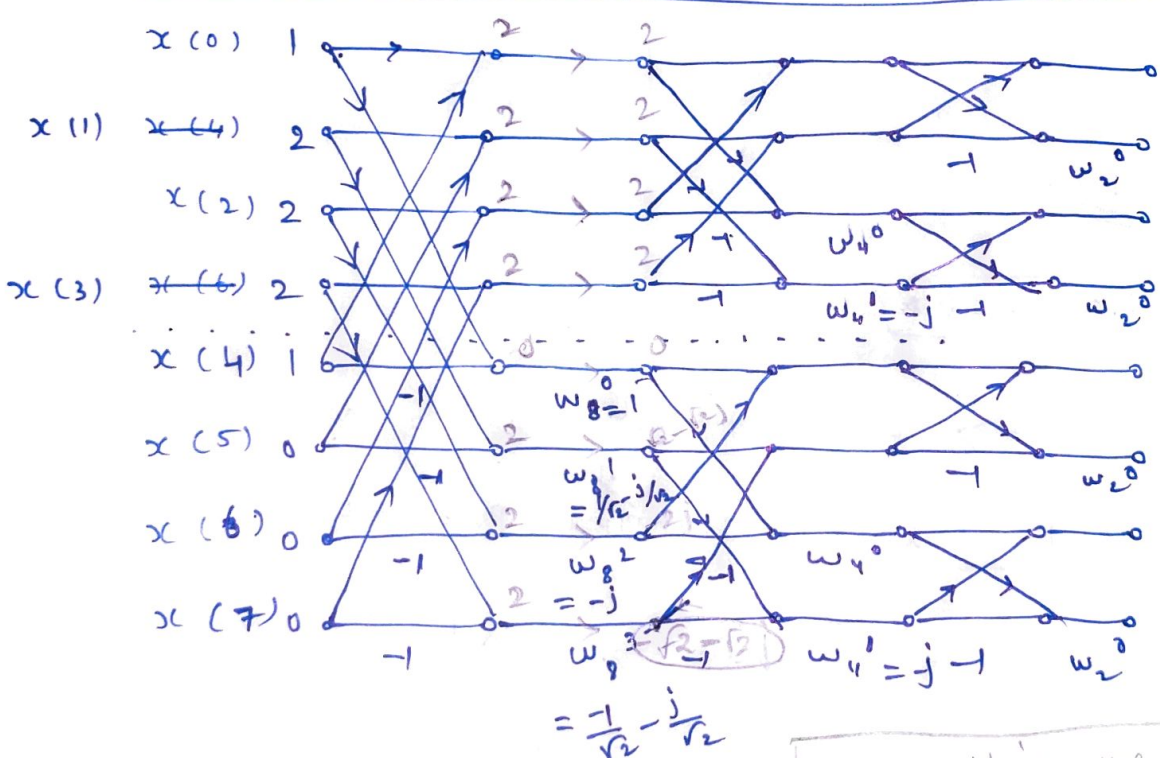
Structure of DIF :



St. of DIT



DIF, give normal order in 8-pt DFT



After this we need to reverse it

Bit reversed

$$\frac{\sqrt{2}}{\sqrt{2}} - \frac{j\sqrt{2}}{\sqrt{2}}$$

$$\{8, 0, 0, 0, -j2(1+\sqrt{2}), -j2(1-\sqrt{2}$$

$$w_2^{\,0} = 1 \qquad\qquad w_N = \left[e^{-(j\,2\pi)/N}\right]$$

$$w_4^{\,0} = 1$$

$$w_4^{\,1} = \left(e^{-j\,2\pi/4}\right) = -j$$

$$w_8^{\,0} = 1$$

$$w_8^{\,1} = \left(e^{-j\,2\pi/8}\right) = e^{-j\frac{\pi}{4}}$$

$$= \cos\frac{\pi}{4} - j\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$w_8^{\,2} = \left(e^{-j\frac{\pi}{4}}\right)^2 = e^{-j\pi/2} = -j$$

$$w_8^{\,3} = \left(e^{-j\frac{\pi}{4}}\right)^3 = \cos\frac{3\pi}{4} - j\sin\frac{\pi}{4}$$

$$w_8^{\,3} = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}.$$

$, j2(1-\sqrt{2}), +j2(1+\sqrt{2})\}$

$\boxed{\text{Module } 5}$

$$\boxed{\int_{-2}^{-1} + \int_{1}^{2}}$$

FIR :

- ☆ Low Pass
- ☆ High Pass
- ✓ ☆ $\boxed{\text{Band Pass}}$

For Example :

$\boxed{\omega_{c_1} \le |\omega| \le \omega_{c_2}}$ also can be given

$$\boxed{\int_{-\omega c_2}^{-\omega c_1} + \int_{\omega c_1}^{\omega c_2}}$$

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; \underbrace{\omega c_1 < |\omega| < \omega c_2}_{} < \pi \\ 0 & ; \text{otherwise} \end{cases}$$

$$h_d(n) = \left[\frac{1}{2\pi}\int_{-\omega c_2}^{-\omega c_1} \underbrace{H_d(e^{j\omega})}_{e^{-j\omega\alpha}} e^{j\omega n}\, d\omega\right] + \left[\frac{1}{2\pi}\int_{\omega c_1}^{\omega c_1} \underbrace{H_d(e^{j\omega})}_{e^{-j\omega\alpha}} e^{j\omega n}\, d\omega\right].$$

$$= \frac{1}{2\pi}\left[\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega c_2}^{-\omega c_1} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega c_1}^{\omega c_2}\right]$$

$$= \frac{1}{\pi(n-\alpha)} \left[\frac{e^{j\omega_1(n-\alpha)} - e^{-j\omega c_2 (n-\alpha)} + e^{j\omega c_2 (n-\alpha)} - e^{-j\omega c_1 (n-\alpha)}}{2j} \right]$$

$$= \frac{1}{\pi(n-\alpha)} \left[\sin(n-\alpha)\,\omega c_2 - \sin(n-\alpha)\,\omega c_1 \right]$$

for $n \neq \alpha$

$$\boxed{H_d(\omega) = \frac{1}{\pi}\left(\omega c_2 - \omega c_1\right) \quad n = \alpha}$$

~~H(Z)~~.

$$h(n) = h_\alpha(n) \times \underbrace{W_{Han}(n)}_{1}$$

$n = \alpha$ ——— $\rightarrow$ 1

Then find the H(Z)

H

Fourier Transform using

Formula given.

Module 3

15 Marks

(or)

Butter worth	Chebyshew
① $\Omega_p, \Omega_s, \underbrace{k_p, k_s}_{\text{gain in +ive}}$	$\delta_p \quad \delta_s$
↓	
② $\Omega_p' = 1$	$K \quad \alpha$
$\Omega_s' = \frac{\Omega_p}{\Omega_s}$	N
Mod 4	↓
③ Marks $\Omega_s' = \frac{\Omega_s}{\Omega_p}$ etc...	$H_N(s)$
③ Marks ← N	↓
④ ⑤ Marks ← $H_N(s)$	$H_a(s)$
⑤ Marks ← ⑤ $H_A(s)$ ✓	

Module 4

Butterworth	Cheby
ω_p, ω_s	ω
↓ Prewarping	
Ω_s, Ω_s → analog	→ same
↓	↓
Ω_s', Ω_p'	N
↓	↓ → use chart
N → $H_N(s)$	$H_N(s)$
↓	↓
$H(z)$ ← $H_A(s)$	$H_c(s)$ → $H(z)$