

Design of IIR Digital Filters



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- The design of digital IIR is largely based on analog filter design techniques.
- To simulate an analog filter, the digital filter is issued in A/D-Digital filter-D/A structure as shown in figure below



- The A/D-Digital-D/A structure behaves like an equivalent analog filter having a transfer function, $H_{eq}(s)$.
- A digital filter takes a discrete-time input sequence $x(n)$ and produce a discrete time output sequence $y(n)$.

Steps for typical Design procedure of a digital IIR filter

1. Selecting a method of transformation of a given analog filter to a digital filter having roughly the same frequency response.
2. Mapping the specification of the digital IIR filter to equivalent specifications of an analog IIR filter such that, after mapping from analog to digital is carried out, the digital IIR filter will meet the given specifications.
3. Designing the analog IIR filter according to the mapped specifications.
4. Transforming the analog filter to an equivalent digital filter.

- The different transformation that map a given analog filter to digital filter having roughly the same frequency response. They are
 - The backward difference method
 - The bilinear transformation
 - The impulse invariant transformation, and
 - The matched-Z transform

Bilinear Transformation

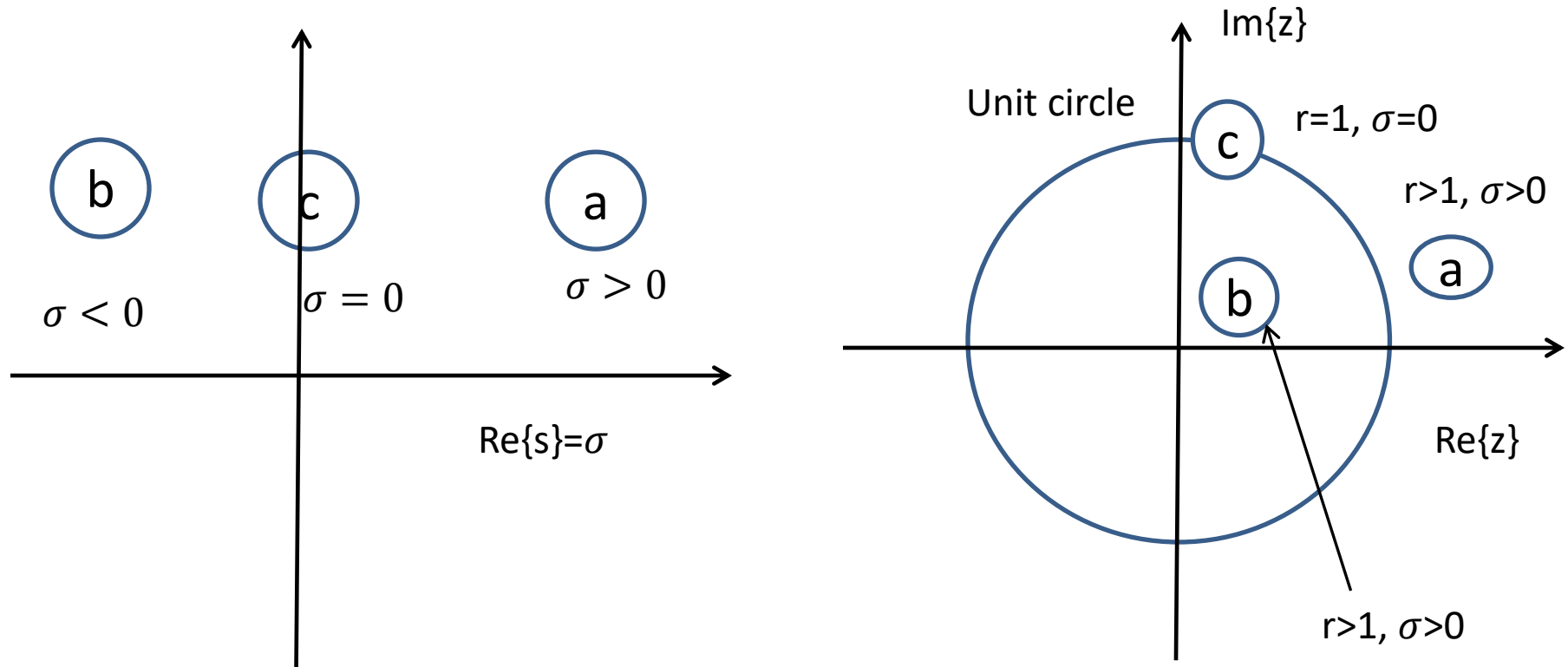
- The bilinear transformation is used for transforming an analog filter to digital filter.
- Bilinear transformation uses trapezoidal rule for integrating a continuous-time function.
- It is defined by the substitution

$$s = \frac{2}{T} \left[\frac{1 - z^{-1}}{1 + z^{-1}} \right]$$

Where T is the sampling period.

(Derivative part of the above equation is given in notes)

Mapping of the s-plane into the z-plane with bilinear transformation.



- The left-half s-plane is now mapped entirely inside the unit circle $|z| = 1$, rather than to a part of it.
- Imaginary axis is mapped on the unit circle.
- Right-half s-plane is mapped outside the unit circle

Bilinear is Frequency to Frequency transformation

$$s = \frac{2}{T} \left[\frac{1-z^{-1}}{1+z^{-1}} \right] \text{ or } s = \frac{2}{T} \left[\frac{z-1}{z+1} \right]$$

Sub $s = \sigma + j\Omega$ and $z = re^{j\omega}$ in the above equation

$$\begin{aligned} \sigma + j\Omega &= \frac{2}{T} \left[\frac{re^{j\omega} - 1}{re^{j\omega} + 1} \right] \\ &= \frac{2}{T} \left[\frac{(r \cos\omega - 1) + jr \sin\omega}{(r \cos\omega + 1) + jr \sin\omega} \right] \end{aligned}$$

Rationalizing the right-hand side of the above equation

$$\sigma + j\Omega = \frac{2}{T} \left[\frac{(r \cos \omega - 1) + jr \sin \omega}{(r \cos \omega + 1) + jr \sin \omega} \right] \times \left[\frac{(r \cos \omega + 1) - jr \sin \omega}{(r \cos \omega + 1) - jr \sin \omega} \right]$$
$$\sigma + j\Omega = \frac{2}{T} \left[\frac{r^2 - 1}{r^2 + 1 + 2r \cos \omega} + j \frac{2r \sin \omega}{r^2 + 1 + 2r \cos \omega} \right]$$

Equating real and imaginary parts on both the sides of the above equation

$$\sigma = \frac{2}{T} \left[\frac{r^2 - 1}{r^2 + 1 + 2r \cos \omega} \right]$$
$$\Omega = \frac{2}{T} \left[\frac{2r \sin \omega}{r^2 + 1 + 2r \cos \omega} \right]$$

From the above equation

1. If $r < 1$, we get , $\sigma < 0$. This means that the left-hand side of the s-plane is mapped inside the unit circle $|z| = 1$.
2. If $r < 1$, we get , $\sigma > 0$. This means that the right-hand side of the s-plane is mapped outside the unit circle $|z| = 1$.
3. If $r = 1$, we get , $\sigma = 0$. This means that the Imaginary axis of the s-plane is mapped to the circle of unit radius centered at $z = 0$ in the Z-domain.

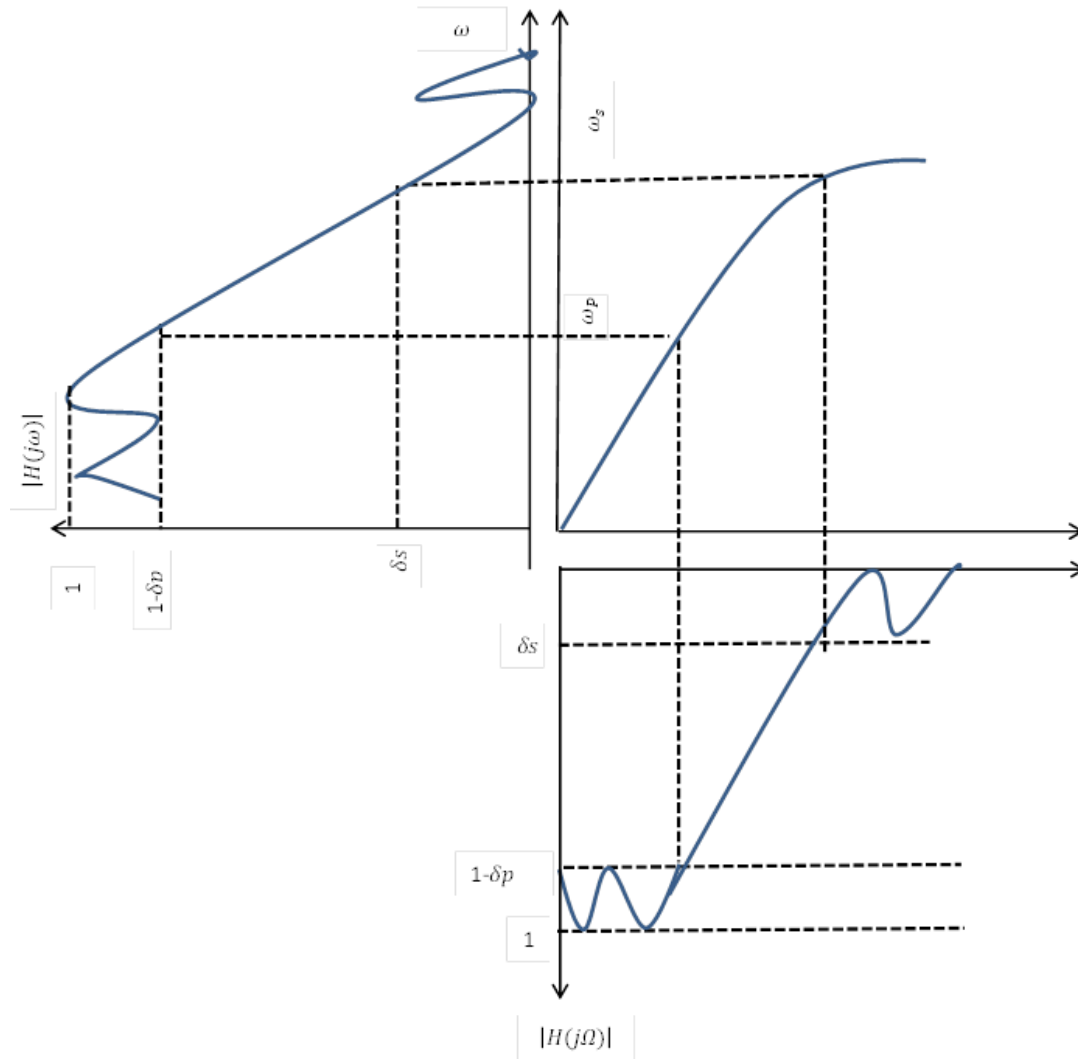
Frequency warping

- When $r=1$, in the previous equation, we get $\sigma = 0$ and

$$\Omega = \frac{2}{T} \left[\frac{2 \sin \omega}{1^2 + 1 + 2 \cos \omega} \right]$$
$$\Omega = \frac{2}{T} \tan \left(\frac{\omega}{2} \right)$$
$$\omega = \frac{2}{T} \tan^{-1} \left(\frac{\Omega T}{2} \right)$$

- The digital-domain frequency ω is therefore warped with respect to the analog frequency Ω , using the above the warping function.
- The analog frequencies, $\Omega = \pm\infty$ are mapped to digital frequencies, $\omega = \pm\pi$.
- The frequency mapping is not aliased that is the relationship between ω and Ω is one to one.

Frequency warping introduced by the bilinear transformation



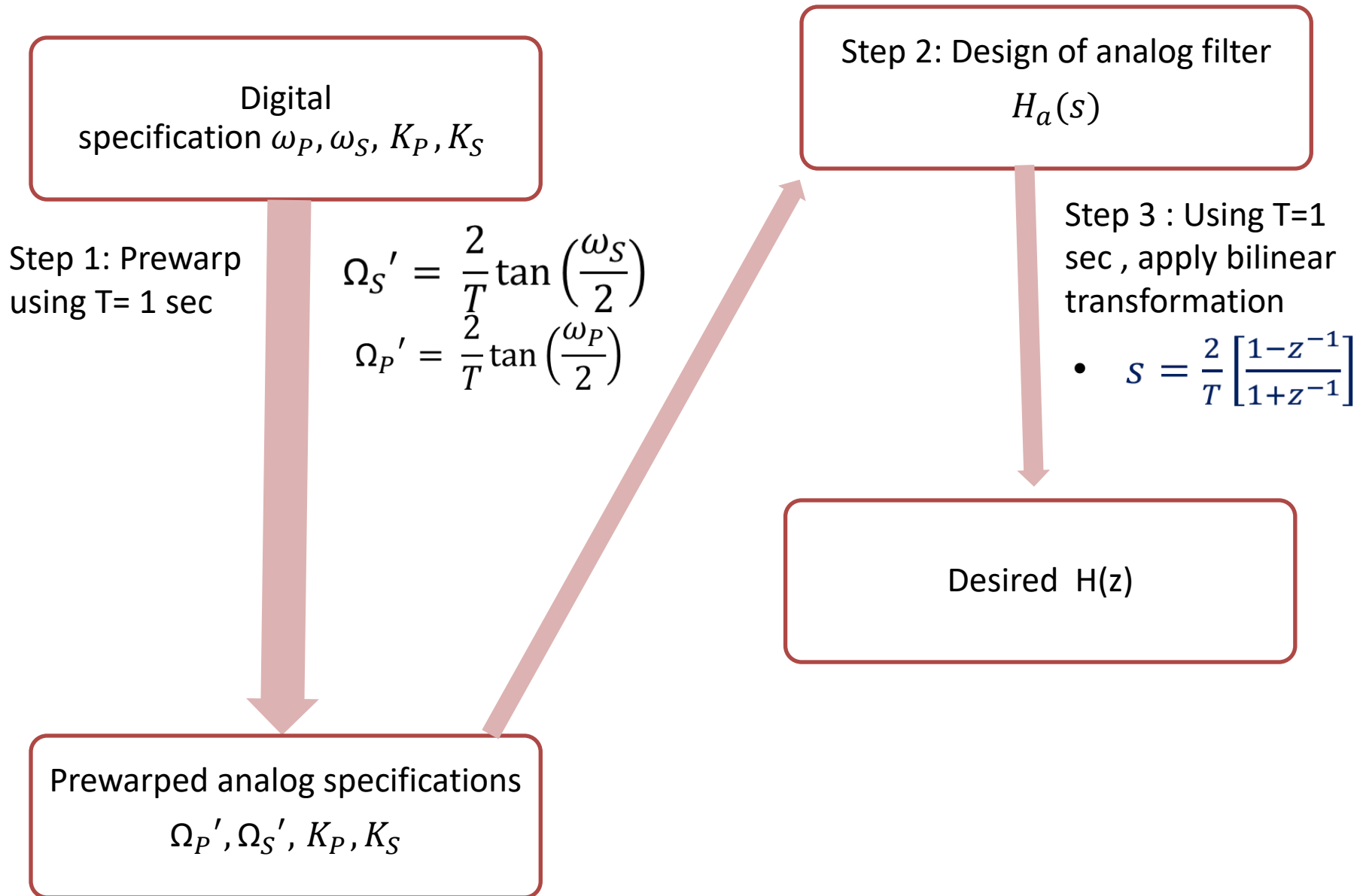
- To design a lowpass filter with passband and stopband edge frequencies ω_P and ω_S .
- Convert these frequencies to corresponding analog-domain band edge frequencies Ω_P' and Ω_S' using the following equation.

$$\Omega_P' = \frac{2}{T} \tan\left(\frac{\omega_P}{2}\right)$$

$$\Omega_S' = \frac{2}{T} \tan\left(\frac{\omega_S}{2}\right)$$

Using above equation we design analog filter. Finally, the required digital low pass filter is obtained by using the bilinear transformation on the analog filter.

Flowchart for digital filter design



Thank you