Design of IIR Digital Filters part II

Dr K. Mohanaprasad
Associate Professor
School of Electronics Engineering (SENSE)
VIT Chennai

Problem 1

- A digital lowpass filter is required to meet the following specifications.

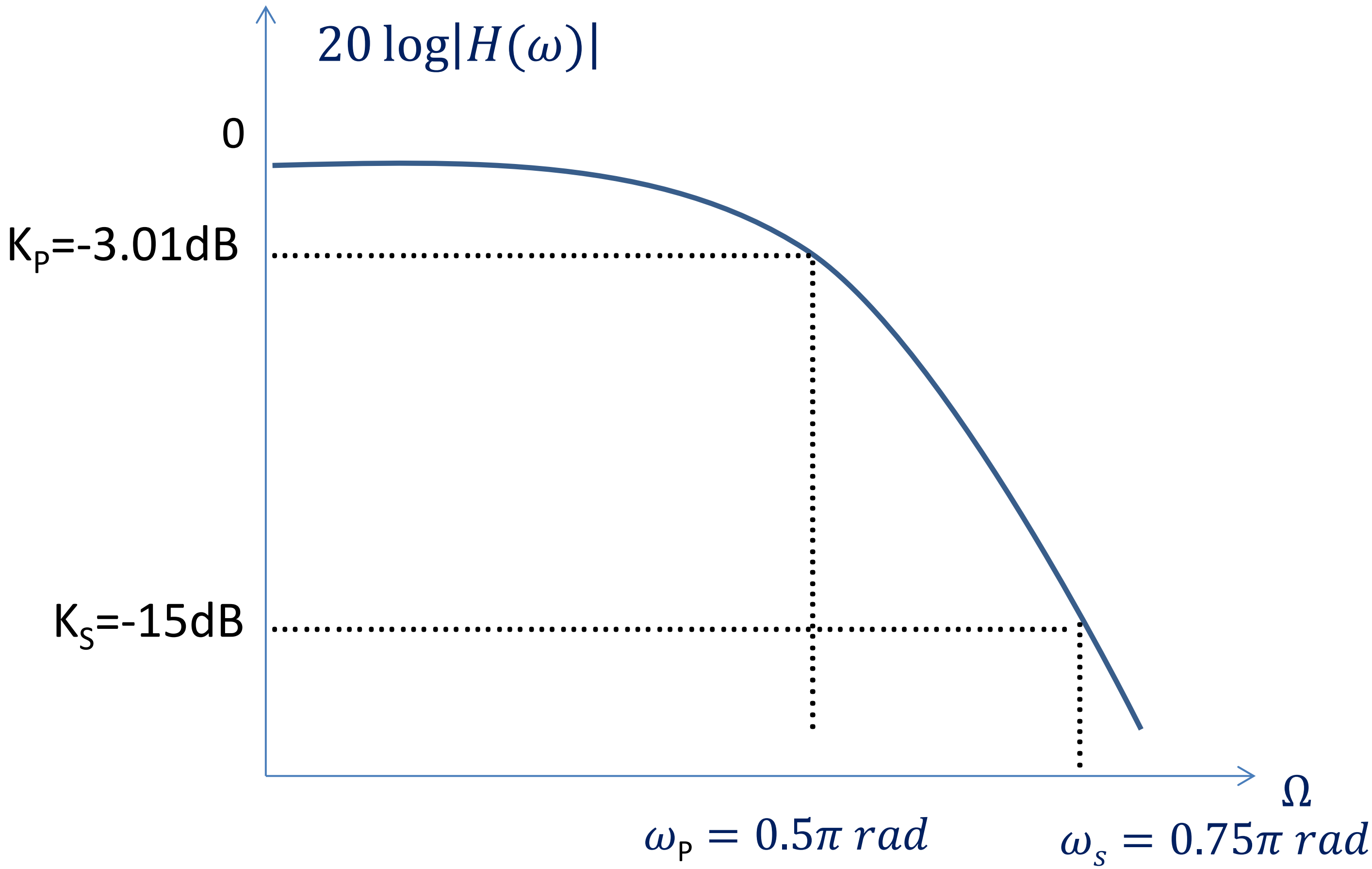
a. Monotonic passband and stop band

b. $-3.01\text{dB}\ cuto\ off\ frequency\ of\ 0.5\pi\ rad.$

c. Stopband attenuation atleast 15dB at $0.75\,\pi\ rad.$

Find the system function *H(z) and the difference equation realization.*

- # Solution: Step 1



Specified magnitude frequency response of the lowpass digital Butterworth filter

Step: 1: Prewarp

- Prewarp the band-edge frequencies $\omega_P = 0.5\pi\ rad$ and $\omega_S = 0.75\pi\ rad$ using T=1 sec to get

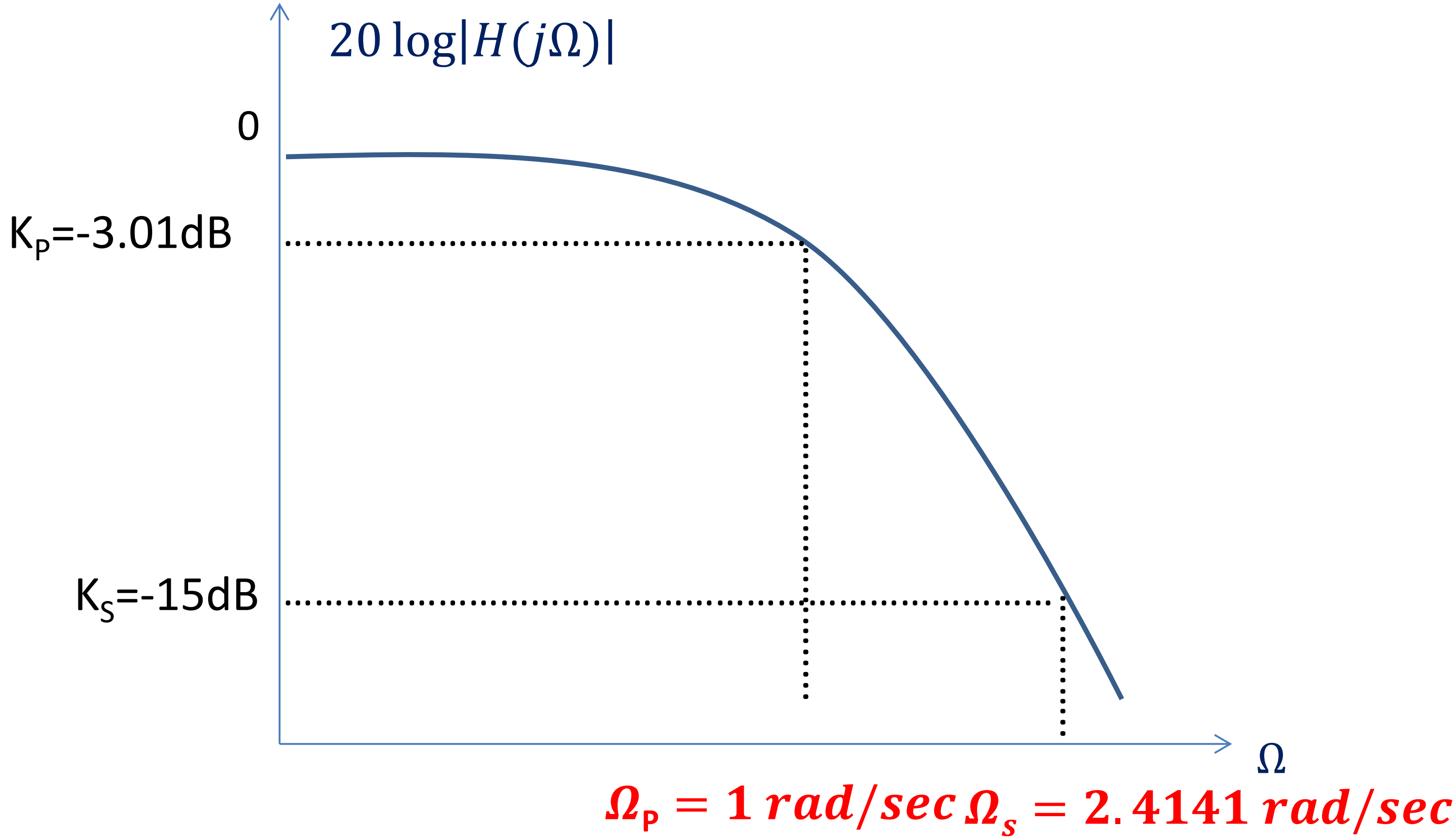
$$\Omega_P{}' = \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) = \frac{2}{1}tan\left(\frac{0.5\pi}{2}\right) = 2$$

$$\Omega_S{}' = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) = \frac{2}{1}tan\left(\frac{0.75\pi}{2}\right) = 4.8282$$

Choose a Butterworth filter to meet the monotonic passband and stopband requirement.

Let us design a lowpass filter to the meet the following specification.

- **Step 2**: Magnitude frequency response of the normalized lowpass filter



The pass band edge frequency $\boldsymbol{\Omega_P}$ of the normalized low pass filter is 1 rad/sec
Let us the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s}$ of the normalized low pass filter $\boldsymbol{\Omega_s = \frac{\Omega'_s}{\Omega'_P}} = \frac{4.8282}{2} = 2.4141\, rad/sec.$

This backward equation is for Low pass to Lowpass transformation

- If the given filter is high pass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s}$ is $\frac{\boldsymbol{\Omega_P}}{\boldsymbol{\Omega_S}}$.

- If the given filter is Bandpass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} = Min\{|A|, |B|\}$. Where $A = \frac{-\Omega_1{}^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)}$, $B = \frac{\Omega_2{}^2-\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)}$

- If the given filter is Bandstop then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} = Min\{|A|, |B|\}$. Where $A = \frac{\Omega_1(\Omega_u-\Omega_l)}{-\Omega_1{}^2+\Omega_l\Omega_u}$, $B = \frac{\Omega_2(\Omega_u-\Omega_l)}{\Omega_2{}^2-\Omega_l\Omega_u}$

- The normalized passband edge frequency $\boldsymbol{\Omega_P}$ is always equal to 1 rad/sec irrespective of the given filter.

Step 3: Find the order N of the filter using equation (14).

Sub K_P=-3.01 dB, K_S=-15 dB, Ω_P=1 rad/sec, Ω_S=2.4141 rad/sec.

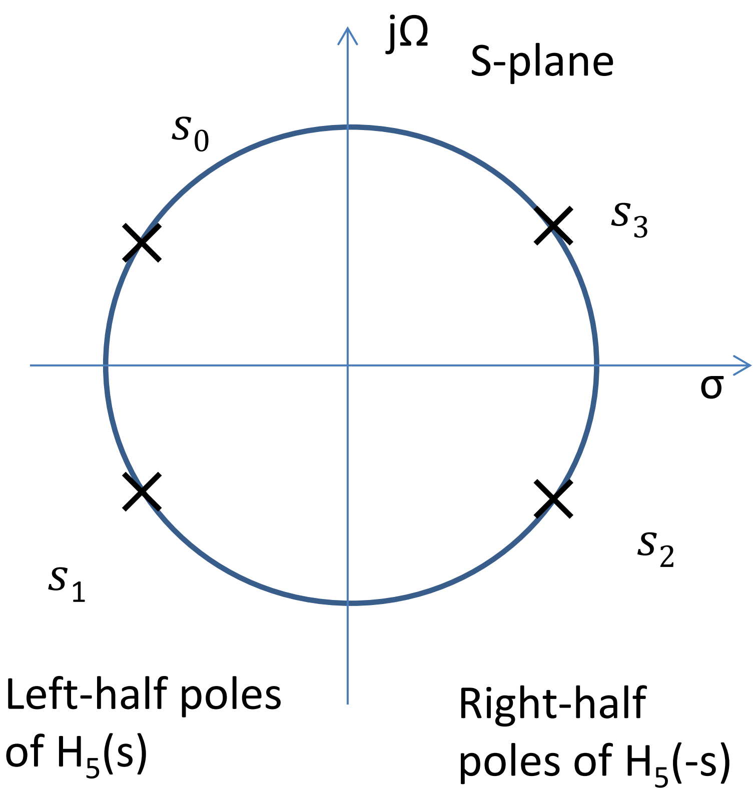
$$N = \frac{log\left[\left(10^{\frac{-K_P}{10}}-1\right)\Big/\left(10^{\frac{-K_S}{10}}-1\right)\right]}{2log\left(\frac{\Omega_P}{\Omega_S}\right)} \text{ 1.94=2}$$

Step 4: Now proceed to find the transfer function of the 5^th order normalized lowpass filter. Find the poles of the 5^th order normalized low pass filter using equation (5)

$$s_k = e^{j\pi\frac{(2k+1)}{2N}}e^{j\frac{\pi}{2}} \qquad k = 0,1,2,\ldots\ldots 2N-1$$

N=2, so, $K = 0, 1, 2,3$

Total 4 poles from S_0 to S_3



Poles	σ+jΩ
s_0	$\frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$
s_1	$\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$
s_2	$\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$
s_3	$\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

Step 5:

Hence, the transfer function of the 2nd order normalized lowpass Butterworth filter is

$$H_{N(s)} = \frac{1}{\prod_{LHP}(s - s_k)}$$

- $H_2(s) = \frac{1}{(s-s_0)(s-s_1)}$

$$H_2(s) = \frac{1}{(s^2 + \sqrt{2}s + 1)}$$

STEP 6: Find the Cutoff frequency Ω_c to exactly meet the pass band requirement.

$$\Omega_c = \frac{\Omega'_P}{\left(10^{\frac{-K_P}{10}} - 1\right)^{\frac{1}{2N}}} = 2\, rad/sec$$

Sub $\Omega'_P = 2\, rad/\sec$ $N = 2$ K_P=-3.01dB

- Step 7: The specified Lowpass filter is obtained by applying lowpass to lowpass transformation on the normalized low pass filter.

- $H_a(s) = H_2(s)|s \rightarrow \frac{s}{\Omega_c=2}$

$$=\frac{1}{s^2+\sqrt{2}s+1}\bigg|\; s \rightarrow \frac{s}{2}$$

$$H_a(s) = \frac{4}{s^2 + 2\sqrt{2}s + 4}$$

- If the specified filter is high pass then apply lowpass to highpass transformation on the normalized low pass filter by replacing $s \rightarrow \frac{\Omega_c}{s}$.

- If the specified filter is Bandpass then apply lowpass to Bandpass transformation on the normalized low pass filter by replacing

$$s \rightarrow \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}.$$

- If the specified filter is Bandstop then apply lowpass to Bandstop transformation on the normalized low pass filter by replacing

$$s \rightarrow \frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_u\Omega_l}.$$

- Step 8: $H(z) = H_a(s)|s \rightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$= \frac{4}{\left[\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right]^2 + 2\sqrt{2}\left[\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right] + 4}$$

$$= \frac{\left(1+z^{-1}\right)^2}{3.4142+0.5858z^{-2}}$$

Step 9: Difference equation realization

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+z^{-2}}{0.5858z^{-2}+3.4142}$$

Cross multiplying, we get

$$0.5858z^{-2}Y(z) + 3.4142Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}X(z)$$

- Step 9: Take inverse Z-transform on both the sides and rearranging
- $y(n) = -0.1715y(n-2) + 0.2928x(n) + 0.5857x(n-1) + 0.2928x(n-2)$