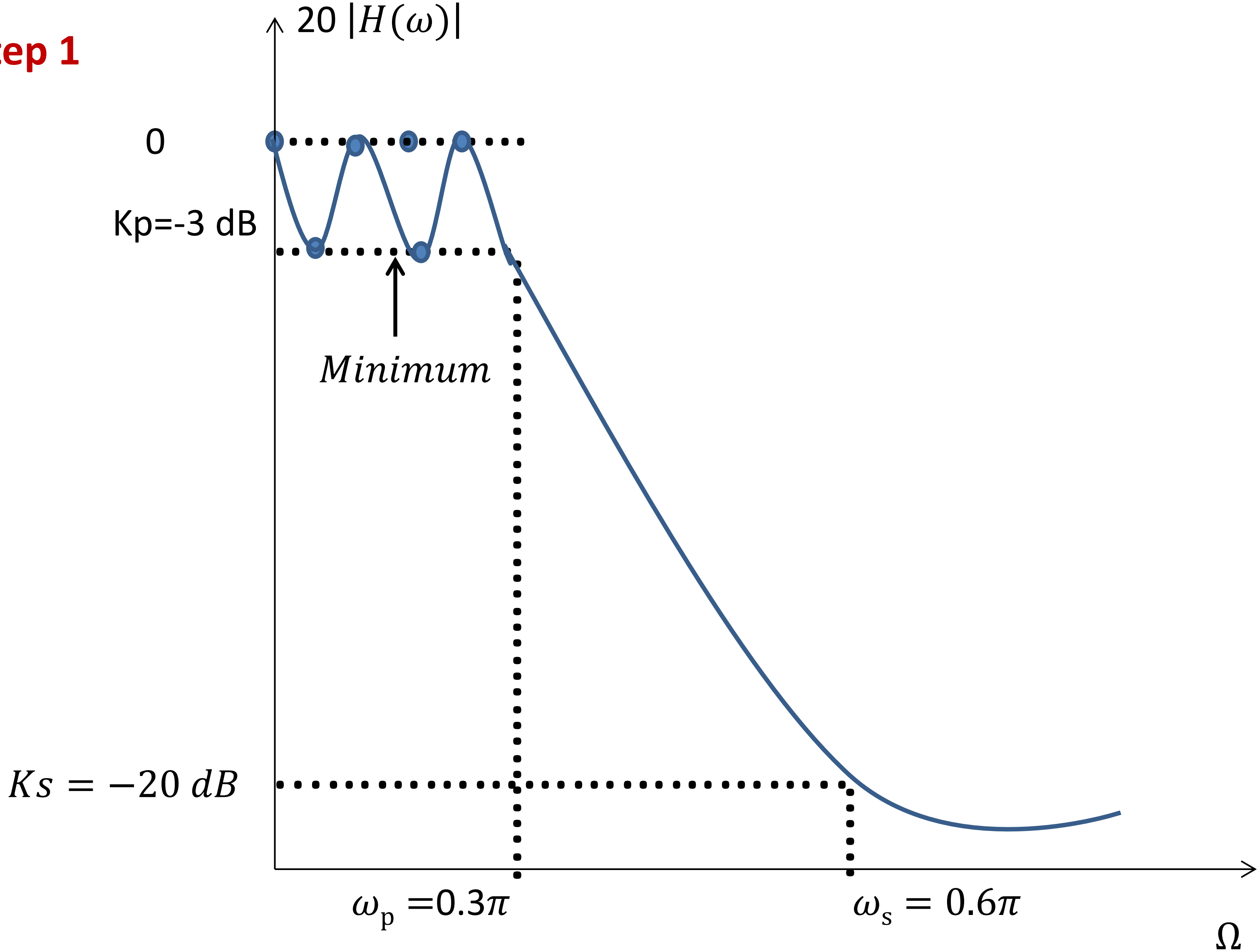
Design of IIR Digital Filters part III

Dr K. Mohanaprasad
Associate Professor
School of Electronics Engineering (SENSE)
VIT Chennai

Problem 2

- Determine the system function H(z) of the lowest-order Chebyshev I analog low pass filter that meets the following specifications:
  - a. 3 dB ripple in Passband $0 \leq |\omega| \leq 0.3\pi$
  - b. Atleast 20 dB attenuation in the Stopband $0.6\pi \leq |\omega| \geq \pi$
  - c. Use the bilinear transformation

Step 1



The specified Magnitude frequency response of a Chebyshev I filter is shown above

Prewarping

- Prewarp the band-edge frequencies $\omega_P = 0.3\pi\ rad$ and $\omega_S = 0.6\pi\ rad$ using T=1 sec to get

$$\Omega_P{}' = \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) = \frac{2}{1}tan\left(\frac{0.3\pi}{2}\right) = 1.019$$

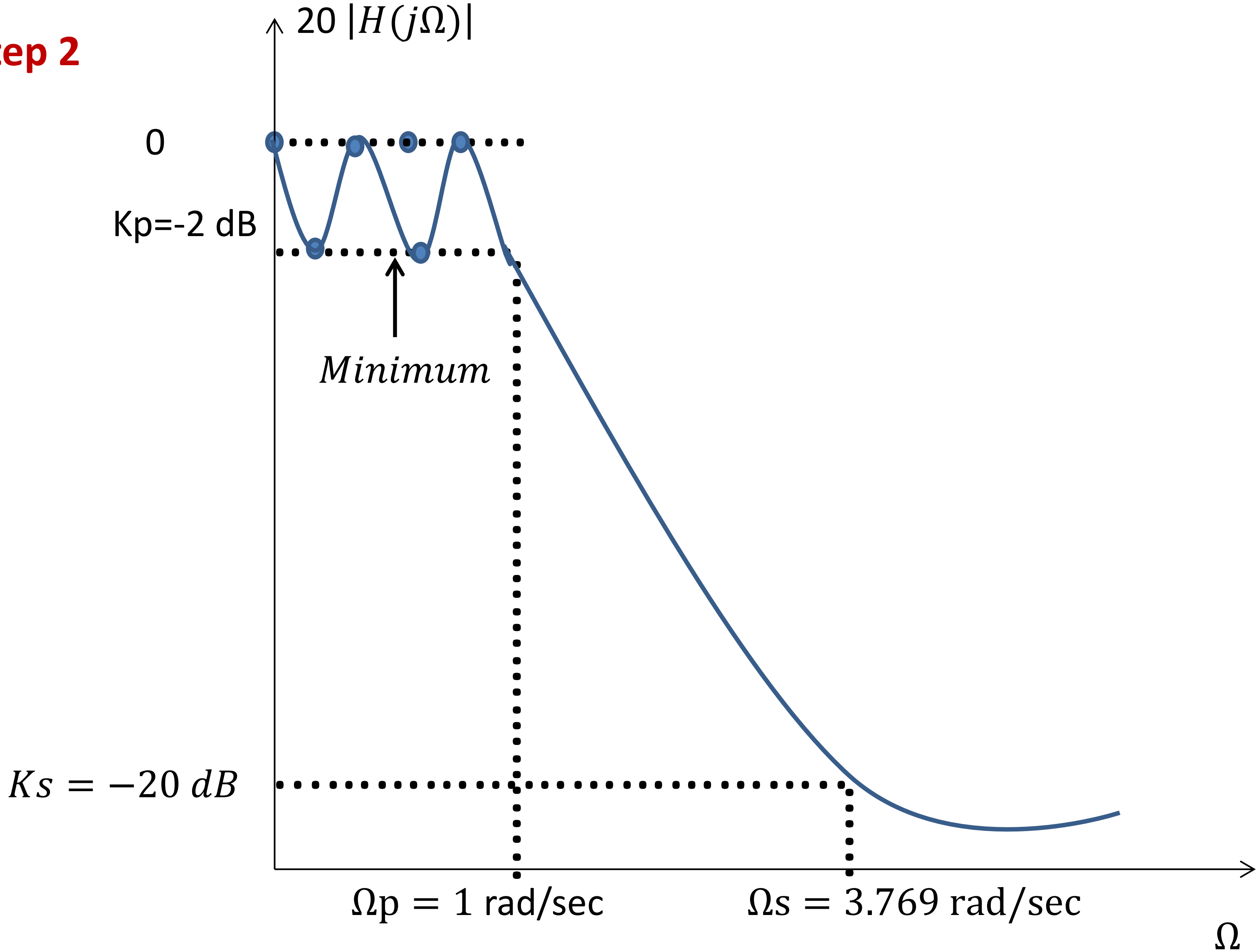
$$\Omega_S{}' = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) = \frac{2}{1}tan\left(\frac{0.6\pi}{2}\right) = 2.75$$

Choose a lowest-order Chebyshev I analog low pass filter .

Let us design a lowpass filter to the meet the following specification.

- Step 2:
- The pass band edge frequency Ω_P of the normalized low pass filter is 1 rad/sec
- Let us the backward design equation to find the stopband edge frequency Ω_s of the normalized low pass filter $\Omega_s = \frac{\Omega'_s}{\Omega'_P} = \frac{2.75}{1.019} = 2.69872\, rad/sec$.
- This backward equation is for Low pass to Lowpass transformation

Step 2



Normalized Magnitude frequency response of a Chebyshev I Low pass filter for N being odd

- If the given filter is high pass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s}$ is $\frac{\boldsymbol{\Omega_P}}{\boldsymbol{\Omega_S}}$.

- If the given filter is Bandpass then the backward design equation to find the stopband edge frequency
$\boldsymbol{\Omega_s} = Min\{|A|, |B|\}$. Where $A = \frac{-\Omega_1{}^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)}$, $B = \frac{\Omega_2{}^2-\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)}$

- If the given filter is Bandstop then the backward design equation to find the stopband edge frequency
$\boldsymbol{\Omega_s} = Min\{|A|, |B|\}$. Where $A = \frac{\Omega_1(\Omega_u-\Omega_l)}{-\Omega_1{}^2+\Omega_l\Omega_u}$, $B = \frac{\Omega_2(\Omega_u-\Omega_l)}{\Omega_2{}^2-\Omega_l\Omega_u}$

- The normalized passband edge frequency $\boldsymbol{\Omega_P}$ is always equal to 1 rad/sec irrespective of the given filter.

- **Step 3**

$$K_P = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$-3 = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\epsilon = 0.997628337608751$$

$$\delta_P = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.2920541$$

$$K_S = -20\ dB$$

$$20 \log \delta_S = -20$$

$$\delta_S = 0.1$$

$$K = \frac{\Omega'_P}{\Omega'_S} = \frac{1}{2.69872} = 0.3705$$

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{{\delta_S}^{-2}-1}}=0.1$$

Step 4:

Minimum filter order (of normalized filter) is

$$N \geq \frac{cosh^{-1}\left(\frac{1}{d}\right)}{cosh^{-1}\left(\frac{1}{K}\right)}= 1.8$$

Rounding off to the next integer we get, N=2

Appendix-II

Polynomials $V_N(s)$ used in Chebyshev I filter design for $\frac{1}{2}$, 1, 2, and 3 dB ripples

$$\text{Chebyshev filter } H_N(s) = \frac{K_N}{V_N(s)}, \text{ where } K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N \text{ odd} \end{cases}$$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0$$

a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	2.8627752									
2	1.5162026	1.4256245								
3	0.7156938	1.5348954	1.2529130							
4	0.3790506	1.0254553	1.7168662	1.1973856						
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909					
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761				
7	0.0447309	0.2820722	0.7556511	1.6479029	1.8694079	2.4126510	1.1512176			
8	0.0236907	0.1525444	0.5735604	1.1485894	2.1840154	2.1492173	2.6567498	1.1460801		
9	0.0111827	0.0941198	0.3408193	0.9836199	1.6113880	2.7814990	2.4293297	2.9027337	1.1425705	
10	0.0059227	0.0492855	0.2372688	0.6269689	1.5274307	2.1442372	3.4409268	2.7097415	3.1498757	1.1400664

b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	1.9652267									
2	1.1025103	1.0977343								
3	0.4913067	1.2384092	0.9883412							
4	0.2756276	0.7426194	1.4539248	0.9528114						
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201					
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510				
7	0.0307066	0.2136712	0.5486192	1.3575440	1.4287930	2.1760778	0.9231228			
8	0.0172267	0.1073447	0.4478257	0.8468243	1.8369024	1.6551557	2.4230264	0.9198113		
9	0.0076767	0.0706048	0.2441864	0.7863109	1.2016071	2.3781188	1.8814798	2.6709468	0.9175476	
10	0.0043067	0.0344971	0.1824512	0.4553892	1.2444914	1.6129856	2.9815094	2.1078524	2.9194657	0.9159320

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
c. 2 dB Ripple ($\epsilon = 0.7647831,\ \epsilon^2 = 0.5848932$)									
1.3075603									
0.6367681	0.8038164								
0.3268901	1.0221903	0.7378216							
0.2057651	0.5167981	1.2564819	0.7162150						
0.0817225	0.4593491	0.6934770	1.4995433	0.7064606					
0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257				
0.0204228	0.1660920	0.3825056	1.1444390	1.0392203	1.9935272	0.6978929			
0.0128603	0.0729373	0.3587043	0.5982214	1.5795807	1.2117121	2.2422529	0.6960646		
0.0051076	0.0543756	0.1684473	0.6444677	0.8568648	2.0767479	1.3837464	2.4912897	0.6946793	
0.0032151	0.0233347	0.1440057	0.3177560	1.0389104	1.1585287	2.6362507	1.5557424	2.7406032	0.6936904
d. 3 dB Ripple ($\epsilon = 0.9976283,\ \epsilon^2 = 0.9952623$)									
1.0023773									
0.7079478	0.6448996								
0.2505943	0.9283480	0.5972404							
0.1769869	0.4047679	1.1691176	0.5815799						
0.0626391	0.4079421	0.5488626	1.4149847	0.5744296					
0.0442467	0.1634299	0.6990977	0.6906098	1.6628481	0.5706979				
0.0156621	0.1461530	0.3000167	1.0518448	0.8314411	1.9115507	0.5684201			
0.0110617	0.0564813	0.3207646	0.4718990	1.4666990	0.9719473	2.1607148	0.5669476		
0.0039154	0.0475900	0.1313851	0.5834984	0.6789075	1.9438443	1.1122863	2.4101346	0.5659234	
0.0027654	0.0180313	0.1277560	0.2492043	0.9499208	0.9210659	2.4834205	1.2526467	2.6597378	0.5652218

- Referring to the normalized 3 dB ripple Chebyshev I filter tables, we get for N=2 the following filter coefficients
- $b_0 = 0.7079478$
- $b_1 = 0.6448996$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + b_2 s^{N-2} + \cdots + b_0$$

$$= (s^2 + b_1 s + b_0)$$

Step 5: $H_2 = \frac{K_N}{\prod_{LHP}(s-s_k)} = \frac{K_N}{(s-s_1)(s-s_2)} = \frac{K_N}{s^2+b_1 s+b_0}$

$$H_2(s) = \frac{K_N = \frac{b_0}{\sqrt{1+\epsilon^2}}}{s^2 + b_1 s + b_0}$$

$$= \frac{\frac{0.7079478}{\sqrt{1 + 0.995263^2}} = 0.50119}{s^2 + 0.6448996s + 0.7079478}$$

Where $K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}}, & N = even \\ b_0 & N = odd \end{cases}$

Since N is odd, $K_N = \frac{b_0}{\sqrt{1+\epsilon^2}} = 0.50119$

Step 6

- If the specified filter is Low pass then apply lowpass to Lowpass transformation on the normalized lowpass filter by replacing $s \rightarrow \frac{s}{\Omega_P}$.

- If the specified filter is high pass then apply lowpass to highpass transformation on the normalized low pass filter by replacing

  $s \rightarrow \frac{\Omega_P}{s}$.

- If the specified filter is Bandpass then apply lowpass to Bandpass transformation on the normalized low pass filter by replacing

$$\mathrm{s} \rightarrow \frac{s^2+\Omega_u\Omega_l}{s(\Omega_{u-}\Omega_l)}.$$

- If the specified filter is Bandstop then apply lowpass to Bandstop transformation on the normalized low pass filter by replacing

$$\mathrm{s} \rightarrow \frac{s(\Omega_{u-}\Omega_l)}{s^2+\Omega_u\Omega_l}.$$

- The required lowpass filter $H_a(s)$ is obtained by applying a lowpass-to lowpass transformation to $H_3(s)$

$$H_a(s) = H_2(s)|\mathrm{s} \to \frac{s}{\Omega'_P}$$

$$= \frac{0.50119}{s^2+0.6448996s+0.7079478} \Bigg| \mathrm{s} \to \frac{s}{1.019}$$

$$H_a(s) = \frac{0.52}{s^2+0.657152s+0.7351053}$$

- Step 7: Finally the transfer function of the digital filter is obtained by applying bilinear transformation to $H_a(s)$ with T=1 sec.
- $H(z) = H_a(s)|s \rightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$$= \frac{0.52}{\left[\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right]^2 + 0.65715269\left[\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\right] + 0.73510538}$$

$$= \frac{0.52\left(1+z^{-1}\right)^2}{6.0494-6.53z^{-1}+3.420805z^{-2}}$$