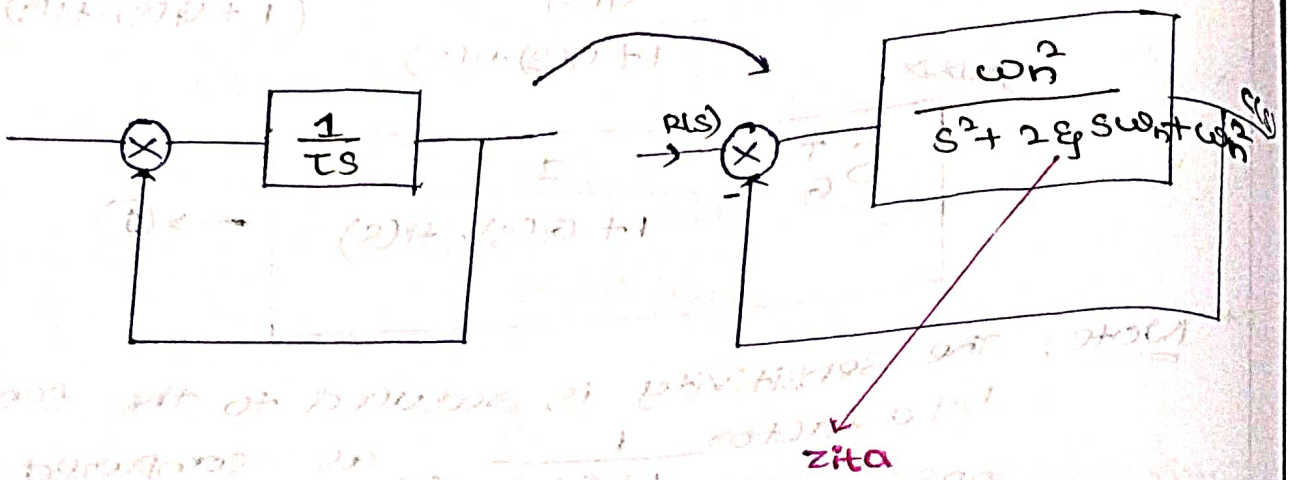
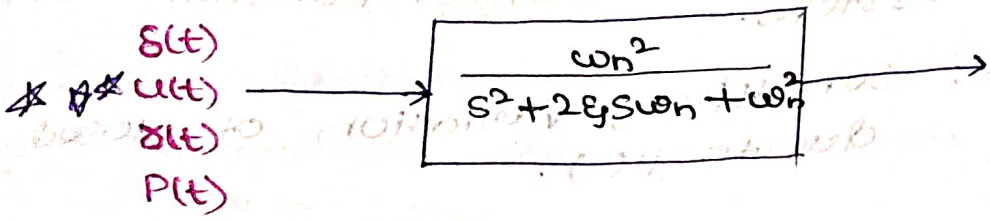
30/01/23

Time Response analysis of 2nd order system:





- ω_n = Natural frequency
- ξ = Damping quotient
  - Damping → Reducing

$\xi \rightarrow$ will decide the system behaviour.

$\xi = 0$ ✓

$0 < \xi < 1$ ✓

$\xi = 1$

$\xi > 1$

In pole,
Imaginary axis gives frequency of oscillation
Real Part system time constant.

Impulse response of second order equation

$$\frac{G(s)}{1+H(s)\cdot G(s)} \Rightarrow 1+G(s)\cdot H(s)=0 \longrightarrow \text{Characteristic Equation}$$

Characteristic equation of second order:

$$s^2+2\xi s\omega_n+\omega_n^2=0$$

$$\delta(t) \longrightarrow \boxed{\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} \longrightarrow C(s)$$

$\delta(s) = 1.$

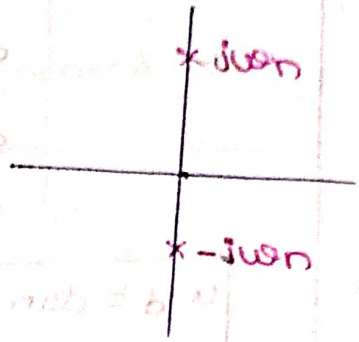
✓ $\xi = 0$

$$C(s) = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$s = \sqrt{-\omega_n^2}$

$s_1 = j\omega_n$

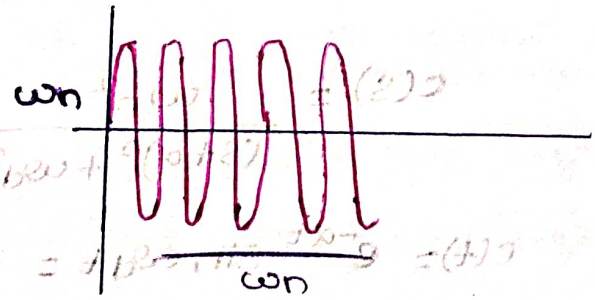
$s_2 = -j\omega_n$



$\rightarrow c(t) = \omega_n \sin\omega_n t$

when $\xi=0$ the poles lies only at imaginary axis of s plane there will complex conjugate. The oscillation of the system will be undamped oscillation | constant oscillation | sustained oscillation.

* frequency of oscillation $= \omega_n$.
* The system is marginally stable.
* Time constant $\tau = \infty$
* The response c(t) will be $c(t) = \omega_n \sin\omega_n t$.



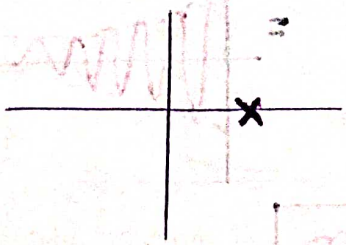
1/01/23

$\xi < 0$

$\xi = -1$

$$\frac{\omega_n^2}{s^2 - 2\omega_n s + \omega_n^2} = 0$$

$$(s-\omega_n)^2 = 0$$



Root or Pole will lie on right side so it is completely unstable.

$0 < \xi < 1$

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

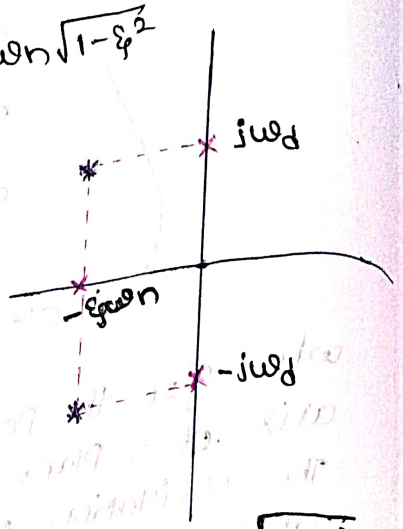
$$s_1 = -\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$$

$$s_2 = -\xi\omega_n - j\omega_n\sqrt{1-\xi^2}$$

ω_d = damping Frequency

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\xi = \cos\theta$



Frequency of oscillation $= \omega_d = \omega_n\sqrt{1-\xi^2}$

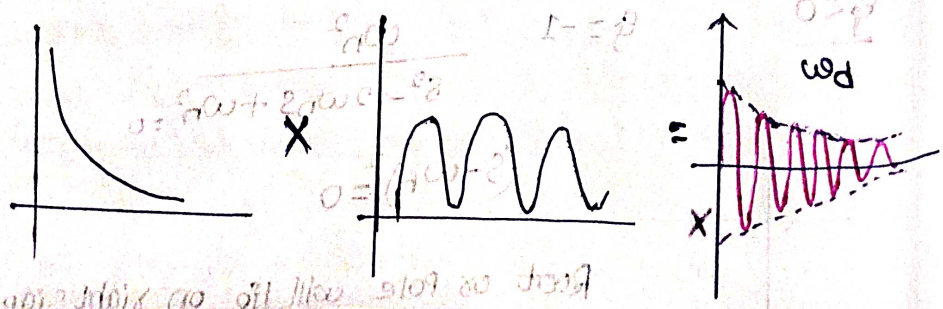
Time constant $(\tau) = \frac{-1}{-\xi\omega_n} = \frac{1}{\xi\omega_n}$

Under damped system

$$(s + \xi\omega_n - j\omega_n\sqrt{1-\xi^2})(s + \xi\omega_n + j\omega_n\sqrt{1-\xi^2})$$

$$C(s) = \frac{\omega_n^2}{(s+\alpha)^2 + \omega_d^2}$$

$$c(t) = e^{-\alpha t}\sin\omega_d t = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t$$



Stable system

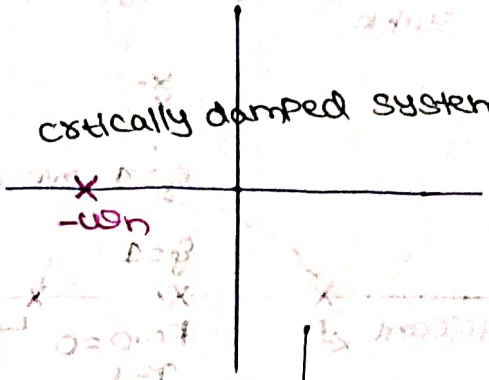
$\xi = 1$.

$$\frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s+\omega_n)^2}$$

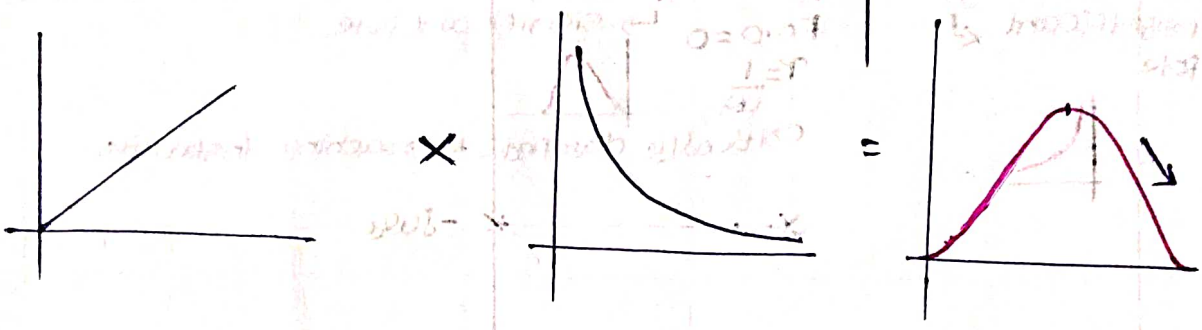
Frequency of oscillation = 0

Time constant $(\tau) = \frac{1}{\omega_n}$

critically damped system



$$C(t) = t \cdot e^{-\omega_n t} \cdot \omega_n^2$$



Note: Multiple pole at left side then system has some instability. Stability approaching instability.

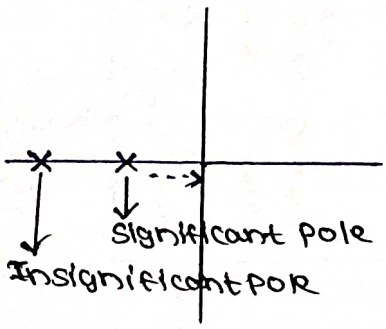
$\boxed{\text{critically damped}}$

$\xi > 1$

$$\frac{\omega_n^2}{s^2 + 2\xi s\omega_n + \omega_n^2} = \frac{\omega_n^2}{(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})\,(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})}$$

Higher Negative is insignificant pole

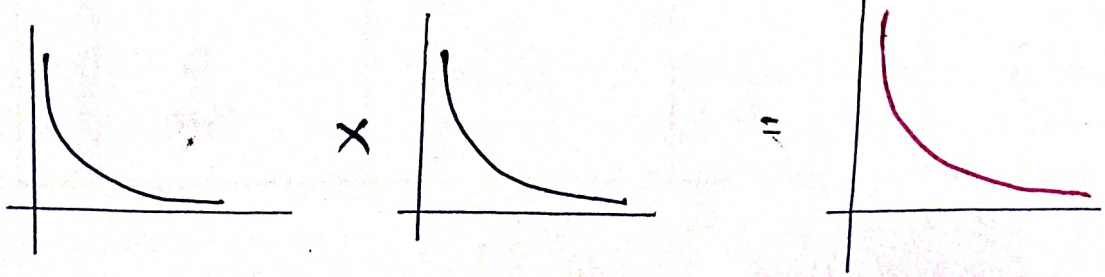
Less Negative is significant pole.



$$s_1 = -\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$$

$$s_2 = -\xi\omega_n - j\omega_n\sqrt{1-\xi^2}$$

$$c(t) = \left(\xi\omega_n - \omega_n\sqrt{\xi^2-1}\right) e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})\,t}$$



$\xi = 0$

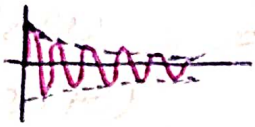
$\tau = \infty$

$F.O.O = \omega_n$

Marginally stable - Undamped

$0 < \xi < 1$

$\tau = \frac{1}{\xi \omega_n}$



$F.O.O = \omega_d$

Under damped system - stable

$j\omega_n$

$j\omega_n\sqrt{1-\xi^2} = \omega_d j$

$\xi > 1$ overdamped - stable

$\xi = 1$

Insignificant pole

$F.O.O = 0$

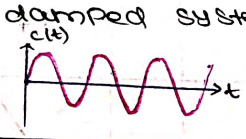
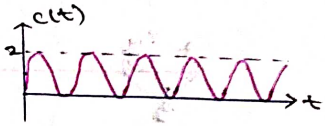
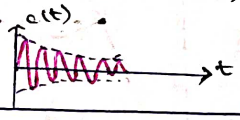
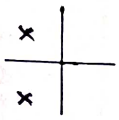
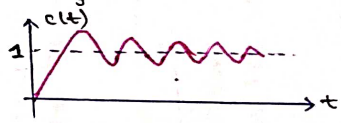
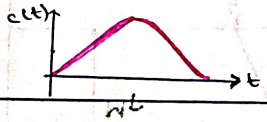
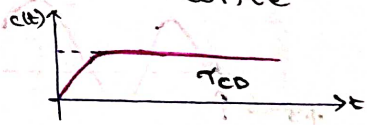
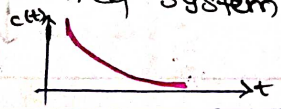
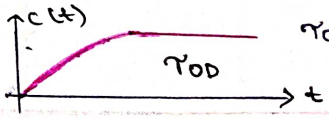
$\tau = \frac{1}{\omega_n}$

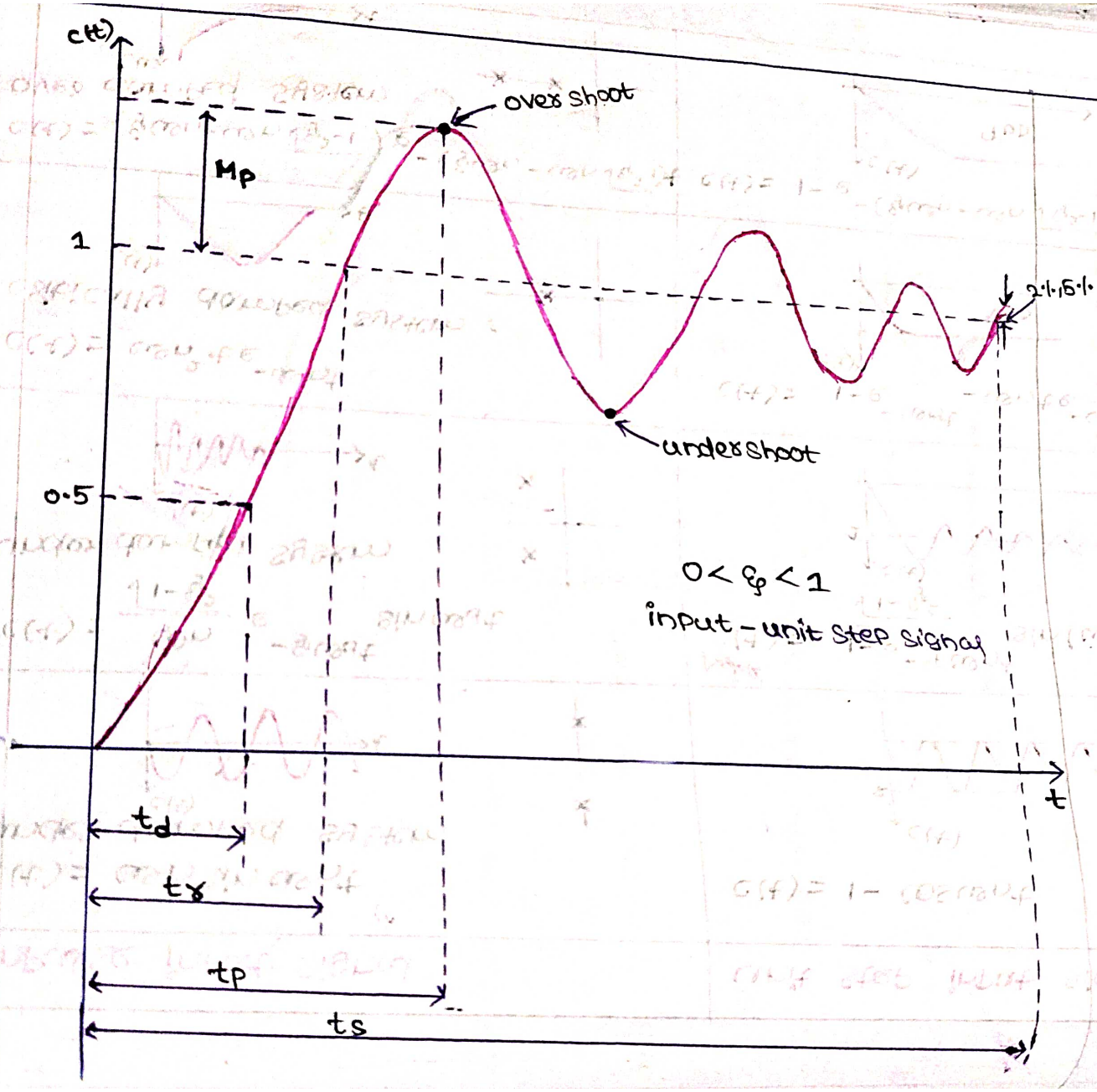
significant pole

Critically damped - Approaching instability

$-j\omega_d$

$-j\omega_n$

Values of ξ	Impulse input signal	Unit step input signal
$\xi = 0$	$c(t) = \omega_n \sin \omega_n t$ under damped system  	$c(t) = 1 - \cos \omega_n t$ 
$0 < \xi < 1$	$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot e^{-\xi \omega_n t} \sin \omega_d t$ under damped system  	$c(t) = \frac{1 - e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$ 
$\xi = 1$	$c(t) = \omega_n^2 t e^{-\omega_n t}$ critically damped system  	$c(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$ 
$\xi > 1$	$c(t) = (\xi \omega_n - \omega_n \sqrt{\xi^2 - 1})\, e^{-(\xi \omega_n - \omega_n \sqrt{\xi^2-1})t}$ over damped system  	$c(t) = 1 - e^{-(\xi \omega_n - \omega_n \sqrt{\xi^2-1})t}$  $\tau_{OD} > \tau_{CD}$


c(t)
over shoot
Mp
1
2%,5%
undershoot
0.5
0 < ξ < 1
input - unit step signal
t
td
tr
tp
ts

Delay time: The time taken by the system to reach 50% of the final value.

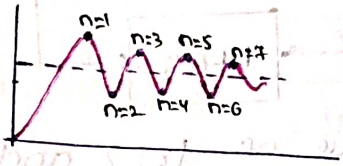
$$t_d = \frac{1 + 0.7\xi}{\omega_n} \text{ sec}$$

Rising time: The time taken by the system to reach 100% of the output.

$$t_r = \frac{\pi - \theta}{\omega_d} \text{ sec}$$

Peak time: The time taken, by the system to reach first maxima of the output signal.

$$t_p = \frac{\pi}{\omega_d} \times n$$



$n=1 \rightarrow$ first over shoot
$n=2 \rightarrow$ first under shoot
$n=3 \rightarrow$ second over shoot
$n=4 \rightarrow$ second under shoot
$n=5 \rightarrow$ Third over shoot
$n=6 \rightarrow$ Third under shoot
$n=7 \rightarrow$ fourth over shoot

Peak over shoot (%Mp): It is normalized difference between Peak value and steady state value of the output signal.

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

Settling time: The time required for the system to reach at the steady state within the specified range (2% or 5%) of its final value.

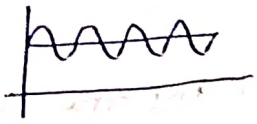
$$t_s = 4\tau = \frac{4}{\xi\omega_n} \text{ for } 2\%$$

$$t_s = 3\tau = \frac{3}{\xi\omega_n} \text{ for } 5\%$$

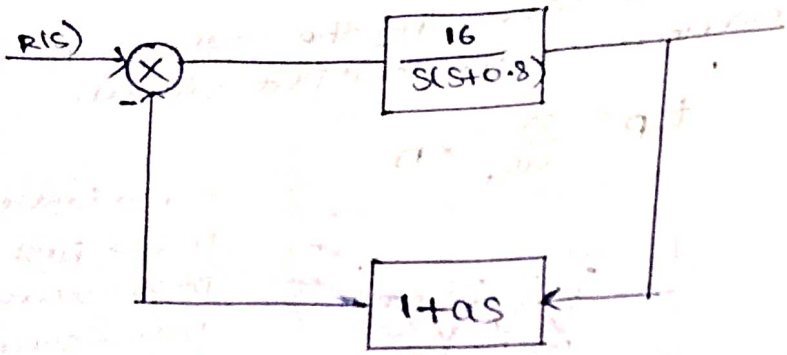
$$\text{No. of oscillations} = \frac{4\tau\omega_d}{2\pi} = \frac{\frac{4\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}}{2\pi} = \frac{2\sqrt{1-\xi^2}}{\xi\pi}$$

Note:

* $\xi = 1$ $\% MP = 0$

* $\xi = 0$  $\% MP = 100\%$

7. Consider a system as shown in the figure. Determine the value of 'a' such that the damping ratio $(\xi) = 0.5$



Also obtain the value of $t_r = ?$ and $\% M_p = ?$ for unit step input signal?

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8 + 16a)s + 16 = 0} \longrightarrow (1)$$

Normally,

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \longrightarrow (2)$$

Comparing both (1) & (2),

$$\omega_n = 4$$

$$0.8 + 16a = 2\xi\omega_n$$

$$\xi = 0.5, \quad \omega_n = 4$$

$$\boxed{a = 0.2}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\theta = \cos^{-1}(0.5)$$

$$= 60^\circ \times \frac{\pi}{180}$$

$$= 1.047 \text{ Radian}$$

$$\omega_d = 4\sqrt{1 - (0.5)^2}$$

$$= 0.866 \times 4$$

$$= 3.464$$

$$t_r = \frac{3.14 - 1.04}{3.46}$$

$$= 0.606 \text{ sec}$$

03/02/23

8.

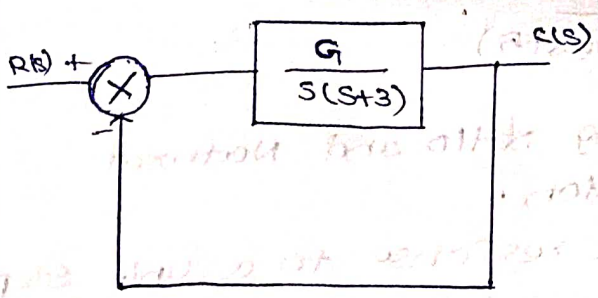


fig (a)

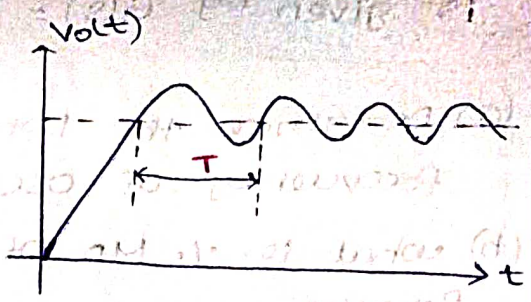


fig (b)

(a) Find CLTF

(b) Find min value of G for which the step response of the system would exhibit an overshoot as shown in fig (b).

(c) For G equal to twice of min value. Find the time period indicated in fig(b).

(a) $$\text{CLTF} = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{\frac{G}{S(S+3)}}{1+\frac{G}{S(S+3)}} = \frac{G}{S^2+3S+G} \rightarrow (1)$$

(b) $$\frac{\omega_n^2}{S^2+2\xi S\omega_n+\omega_n^2} \rightarrow (2)$$

Let us assume $\xi = 0.6$; $0<\xi<1$.

Comparing (1) & (2),

$$\omega_n = \sqrt{G}, \quad 2\xi\omega_n = 3$$

$$\xi\sqrt{G} = 3/2 \Rightarrow \sqrt{G} = \frac{3}{1.2}$$

$$G_{min} = \frac{9}{1.44} = 6.25$$

(c) $$G = 2G_{min} = 12.5$$

$$\omega_n = \sqrt{G} = \sqrt{12.5} = 3.53$$

$$\omega_d = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega_d} = \frac{2\times 3.14}{\omega_n\sqrt{1-\xi^2}} = \frac{2\times 3.14}{2.824}$$

$$T = 2.22$$

9. The OLTF of a servo system with unity feedback is given by $G(s) = \frac{10}{(s+2)(s+5)}$.

(a) Determine the Damping ratio and Natural frequency of oscillation.

(b) What is % Mp of the response to a unit step function.

(a)

$$\frac{C(s)}{R(s)} = \frac{\frac{10}{(s+2)(s+5)}}{1+\frac{10}{(s+2)(s+5)}}$$

$$= \frac{10}{s^2+7s+20}$$

$$s^2+7s+20 = s^2+2\xi s\omega_n+\omega_n^2$$

$$\omega_n = 4.47$$

$$7 = 2\times\xi\times 4.47$$

$$\xi = 0.78$$

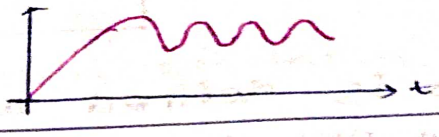
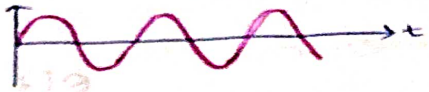
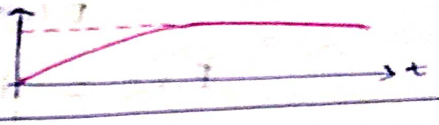
(b)

$$\% Mp = e^{\frac{-\pi\times 0.78}{\sqrt{1-(0.78)^2}}}\times 100$$

$$= e^{-\frac{2.4492}{0.625}}\times 100$$

$$= 0.0198 = 1.9\%$$

10. Match the following:

P. $\dfrac{2s}{s^2+25}$	1.
Q. $\dfrac{36}{s^2+20s+36}$	2. 
R. $\dfrac{36}{s^2+12s+36}$	3. 
S. $\dfrac{49}{s^2+7s+49}$	4. 

From P, s^2+25 comparing with $s^2+2\xi\omega_n s+\omega_n^2$ then we get $\xi=0$. (3)

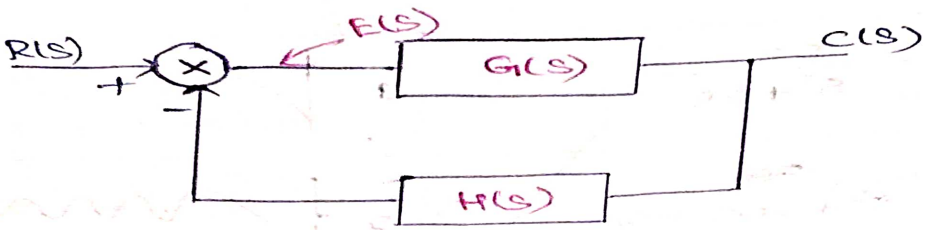
From Q, $s^2+20s+36$ comparing with $s^2+2\xi\omega_n s+\omega_n^2$ then we get $\xi=\frac{10}{6}>1$. (4)

From R, $s^2+12s+36$ comparing with $s^2+2\xi\omega_n s+\omega_n^2$ then we get $\xi=1$. (1)

From S, $s^2+7s+49$ comparing with $s^2+2\xi\omega_n s+\omega_n^2$ then we get $\xi=\frac{1}{2}$. $0<\xi=1/2<1$. (2)

07/02/23

Steady State Error calculation



Error is defined as deviation between input and output signal.

$$e(t) = r(t) - c(t)$$

$$E(S) = R(S) - C(S)$$

$$E_{ss} = \lim_{t \to \infty} e(t)$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(S)$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S) \cdot H(S)}$$

$$\frac{E(S) \cdot \cancel{G(S)}}{R(S)} = \frac{\cancel{G(S)}}{1 + G(S) \cdot H(S)} \Rightarrow E(S) = \frac{R(S)}{1 + G(S) \cdot H(S)}$$

$$\boxed{E(S) = \frac{R(S)}{1 + G(S) \cdot H(S)}}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(S)$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s \cdot R(S)}{1 + G(S) \cdot H(S)}}$$

for unity feedback system,

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(S)}{1 + G(S)}$$

$$e_{ss} = f(R(s), G(s))$$

↓ input

Impulse $\delta(t)$ ✗

unit step $u(t)$

Ramp $r(t)$

Parabolic $p(t)$

* Steady state error depends on input $R(s)$ and system characteristic $(G(s))$.

* steady state not valid for impulse input signal. Therefore, ess, we will consider 3 input signal that is $u(t)$, $r(t)$, $p(t)$.

* Type has significant impant on ess analysis. order has nothing to do with Ess analysis.

* Type : type 0, type 1, type 2. System parameters depends on type of the system.

* Higher order type can be generated by combing lower order type system.

<u>Steady state error for unit step i/p signal response.</u>

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{1}{1+G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{1+G(s)\cdot H(s)}$$

$$= \frac{1}{1+\lim_{s \to 0} G(s)\cdot H(s)} = \frac{1}{1+K_p}$$

$$\boxed{K_p \rightarrow \lim_{s \to 0} G(s)\cdot H(s)}$$

↓

positional error coefficient.

Steady state error for Ramp i/p signal

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s(1+G(s) \cdot H(s))}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s + s(G(s) \cdot H(s))}$$

$$= \frac{1}{\lim_{s \to 0} s \cdot G(s) \cdot H(s)} = \frac{1}{K_v}$$

$$\boxed{K_v \rightarrow \lim_{s \to 0} s \cdot G(s) \cdot H(s)}$$

$\downarrow$

velocity error coefficient.

Steady state error for Parabolic i/p signal.

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{s^{\cancel{3}2}(1+G(s) \cdot H(s))}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2(G(s) \cdot H(s))}$$

$$= \frac{1}{\lim_{s \to 0} s^2(G(s) \cdot H(s))} = \frac{1}{K_a}$$

$$\boxed{K_a \rightarrow \lim_{s \to 0} s^2 G(s) \cdot H(s)}$$

$\downarrow$

Acceleration error coefficient

Input	ess	Coefficient
$u(t)$	$e_{ss} = \frac{1}{1+K_p}$	$K_p = \lim_{s\to 0} G(s)\cdot H(s)$
$r(t)$	$e_{ss} = \frac{1}{K_v}$	$K_v = \lim_{s\to 0} s\cdot G(s)\, H(s)$
$p(t)$	$e_{ss} = \frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2\cdot G(s)\, H(s)$

position x

Velocity $\frac{dx}{dt} \to s$

Accelarsation $\frac{d^2x}{dt^2} \to s^2$

Type-0 system with different input:

Genetic Eqn.

$$G(s)\cdot H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^0(1+sT_a)(1+sT_b)\cdots}$$

(i) step signal:

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\lim_{s\to 0} G(s)\cdot H(s)}$$

$$K_p = \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$= K \to$ constant value

$$e_{ss} = \frac{1}{1+K} \to \text{constant value.}$$

(ii) Ramp signal:

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} s\, G(s)\cdot H(s) = \lim_{s\to 0} \frac{s\,K(1+sT_1)\cdots}{(1+sT_a)\cdots} = 0$$

$K_v = 0$, $e_{ss} = \infty$.

(iii) Parabolic signal: $e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$

$K_a = 0$, $e_{ss} = \infty$

08/02/23

Type-1 with different input system

$$G(s) \cdot H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s(1+sT_a)(1+sT_b)\cdots}$$

(i) Step signal:

$$e_{ss} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s \to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{s(1+sT_a)(1+sT_b)\cdots}$$

$$= \infty$$

$$e_{ss} = 0$$

(ii) Ramp signal:

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s \to 0} \frac{\cancel{s} \cdot K(1+sT_1)(1+sT_2)\cdots}{\cancel{s}(1+sT_a)(1+sT_b)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{K}$$

(iii) Parabolic signal:

$$e_{ss} = \frac{1}{K_a}$$

$$K_a = \lim_{s \to 0} \frac{s^{\cancel{2}} \cdot K(1+sT_1)(1+sT_2)\cdots}{\cancel{s}(1+sT_a)(1+sT_b)\cdots}$$

$$= 0$$

$$e_{ss} = \infty$$

Type-2 System with different inputs.

$$G(s)\cdot H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

(i) Step signal:

$$e_{ss} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

$$= \infty$$

$$e_{ss} = 0.$$

(ii) Ramp signal:

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} \frac{s\cdot K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

$$= \infty$$

$$e_{ss} = 0$$

(iii) Parabolic signal:

$$e_{ss} = \frac{1}{K_a}$$

$$K_a = \lim_{s\to 0} \frac{s^2\cdot K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{K}$$

Input	Type 0	Type 1	Type 2
t^0 $u(t)$	$\frac{1}{1+K_p}$ constant	0	0
t^1 $r(t)$	∞	$\frac{1}{K_v}$ constant	0
t^2 $p(t)$	∞	∞	$\frac{1}{K_a}$ constant

i/p (power of t) < type = 0

i/p (power of t) > type = ∞

i/p (power of t) = type = constant

11. The open loop transfer function (OLTF) of a unity feedback system is given by $G(s) = \frac{50}{(1+0.1s)(s+10)}$. Determine static error coeffecient K_p, K_v, K_a.

$$K_p = \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)} = \frac{50}{10} = 5$$

$$K_v = \lim_{s \to 0} \frac{s \cdot 50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s \to 0} \frac{s^2 \cdot 50}{(1+0.1s)(s+10)} = 0.$$

12. The forward path Transfer function of a unit feedback system $G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$.
Determine step, ramp, parabolic coefficient (K_p, K_v, K_a) and determine type of system.

Type = 2

$$K_p = \lim_{s\to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)} = \infty$$

$$K_v = \lim_{s\to 0} \frac{s \cdot 5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)} = \infty$$

$$K_a = \lim_{s\to 0} \frac{s^2 \cdot 5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$= \frac{5\times 100}{5\times 10} = 10$$

13. The unity feedback control system the forward path transfer function is given $G(s) = \frac{20}{s(s+2)(s^2+2s+20)}$. Determine the steady state error of the system where inputs are (i) 5
(ii) $5t$
(iii) $\frac{3t^2}{2}$.

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)\cdot H(s)}$$

(i) $5 \xrightarrow{L} 5/s$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot 5/s}{1+\frac{20}{s(s+2)(s^2+2s+20)}} = 0$$

(ii) $5t \xrightarrow{L} 5/s^2$

$$e_{ss} = \lim_{s\to 0} s \cdot 5/s^2 \cdot \frac{1}{1+\frac{20}{s(s+2)(s^2+2s+20)}}$$

$$= \lim_{s\to 0} \frac{5}{s} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$$= \frac{5 \times 2 \times \cancel{20}}{\cancel{20}}$$

$$= 10$$

(iii) $\frac{3t^2}{2} \longrightarrow \frac{3}{\cancel{2}} \times \frac{\cancel{2}}{s^3} = \frac{3}{s^3}$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{3}{s^{\cancel{3}2}} \cdot \frac{1}{1 + \frac{20}{s(s+2)(s^2+2s+20)}}$$

$$= \lim_{s \to 0} \frac{3}{s^{\cancel{2}}} \cdot \frac{\cancel{s}(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20) + 20}$$

$$= \infty$$

14. The unity feedback with forward path transfer function is $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$ with input $r(t) = 2 + 5t + 2t^2$. find steady state error.

$$r(t) \longrightarrow R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

$$\downarrow$$

$$e_{ss} = \frac{2}{1+K_p} + \frac{5}{K_v} + \frac{4}{K_a}$$

$$K_p = \lim_{s \to 0} \frac{108}{s^2(s+4)(s^2+3s+12)} = \infty$$

$$K_v = \lim_{s \to 0} \cancel{s} \cdot \frac{108}{s^{\cancel{2}}(s+4)(s^2+3s+12)} = \infty$$

$$K_a = \lim_{s \to 0} \cancel{s^2} \cdot \frac{108}{\cancel{s^2}(s+4)(s^2+3s+12)}$$

$$= \frac{108}{4 \times 12} = \frac{9}{4}$$

$$e_{ss} = \infty + \infty + \frac{4 \times 4}{9} = 16/9$$