Lecture - 1

Introduction to Control System

Control System terminology

Engineering System

1. Process 2. Plant 3. Control System

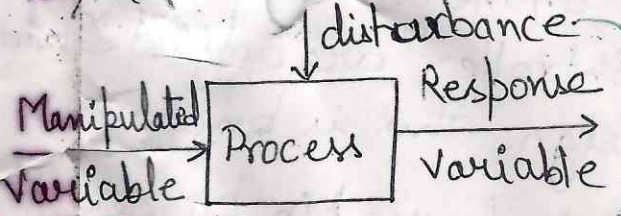
Process Industries requires Control of
→ Temp, pressure, liquid level, humidity, Compositions.

Control System techniques

① Classical techniques → 1900 - 1940 era
⇒ Automatic Airplane pilots, Gun positioning systems, Radar ~~Control~~ Antenna Control

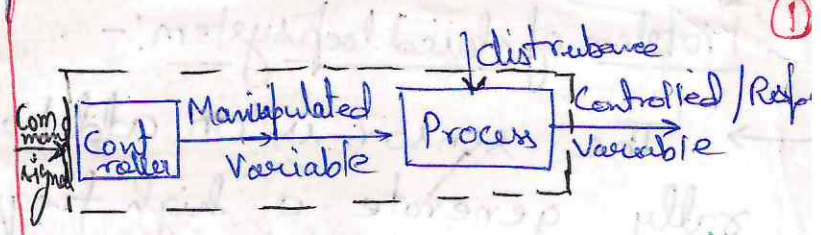
Theory of Servomechanism

Servo + mechanism
↓
Servent + system → which is slave to command.



subject to Control by Controller which we want to design

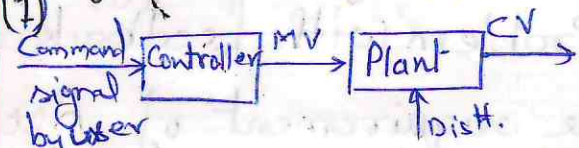
obey - and Control & Random in nature



disturbance may be present inside or outside process. Parameter change is process of inside originated disturbance

Possible scheme to achieve above objective :→ MV : Manipulated Variable
CV : Controlled Variable

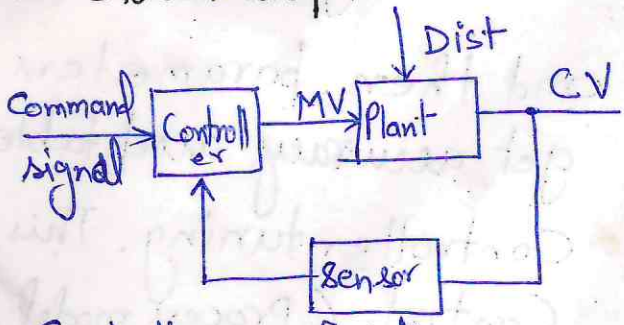
(1)



Dist. may produce unknownable changes in Command variable since Controller will not be aware of current changes as it is designed ~~for~~ with previously known parameter so CV will not follow Command which produces error.

Open loop Control

(2) Closed loop Control



Controller → function

It compares the actual Controlled Variable (Information avilable from sensor) from the Command signal and generates an error signal. That error signal will be used to manipulate variable to plant to reduce the error to zero.

Problem of closed loop system:-

→ The sensor is an additional hardware. Sensor generally generate a high frequency noise and this high frequency noise also get injected in to the loop. However, suitable noise filter can be installed in to loop to take care of noise problem.

Requirement of Controller:- ① To Make system "Robust". ② Accuracy at steady state ③ speed of Response

→ Problem with feedback structure → "as you increase the requirement of Accuracy there is a loss in stability."

Feedback Control theory accomplish a tradeoff b/w Accuracy and stability of system.

→ Basic requirement of Robustness make feed back Control a neccesity.

Design Aspects :-

How to design "Controller":→ ① Experimental Approach

Controller is installed on the basis experience and then parameters of Controller are adjusted to get accuracy acceptable to design. This is known as Controller tuning. This is majorly used in process Control. (Process model is not known due to high nonlinearity)

This is an Adhoc approach of design.

② Model based / Analytical Approach:- If requirement of Control are high or system is too complex to be tuned using adhoc approach.

In this approach the dynamics of system in a mathmatical form. (may be differential eqn, transfer fun, State space Approach).

3 Knowledge based Approach (Under stage of Active development)

→ Expert Control → fuzzy control, neural network

In this course we are primarly concern for Model based approach.

Mathemical Model → Physical laws ✓ Modelling using physical laws.
Mathemical Model → Experimentation

Physical System → Plant

System identification

→ 1940 - 1960 → frequency Domain Design Methods

Classical Methods of Design
→ ~~Ny~~ Nyquist stability Criterion
→ Bode plots
→ Root - locus plots

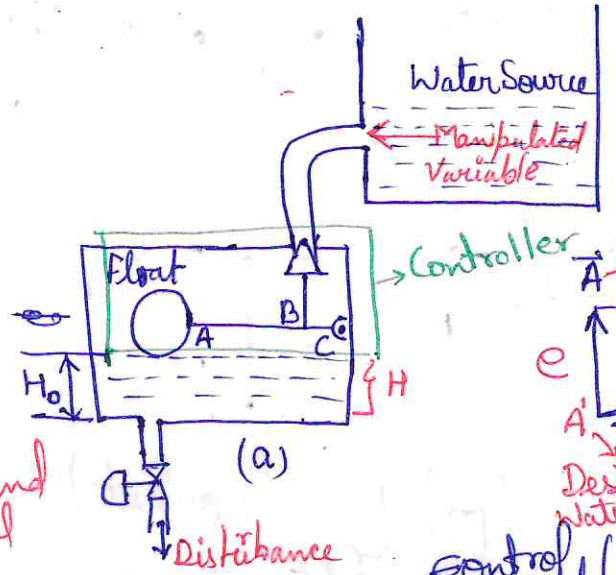
→ State space Methods of design → 1960 onwards
Modern control design (for ~~state~~ Space vehicle design)

Ex Bathroom toilet Tank, Automobile Driving, Residential Heating, Hydraulic steering Mechanism, Servo system for an antenna, speed Control system.

Control Engineering

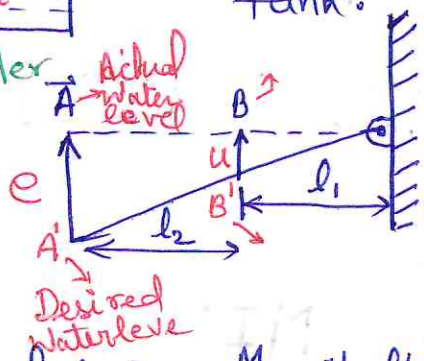
Prof. Madan Gopal

Lecture -2

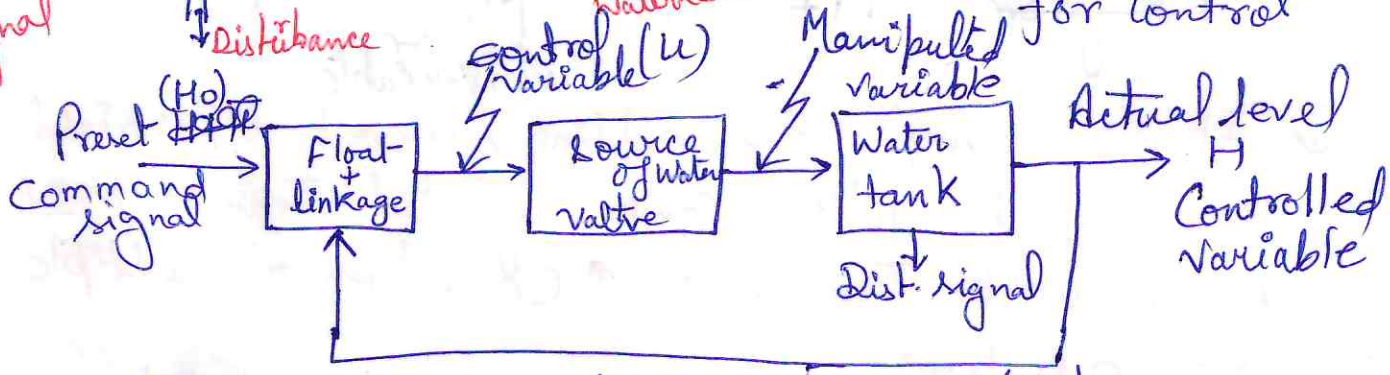


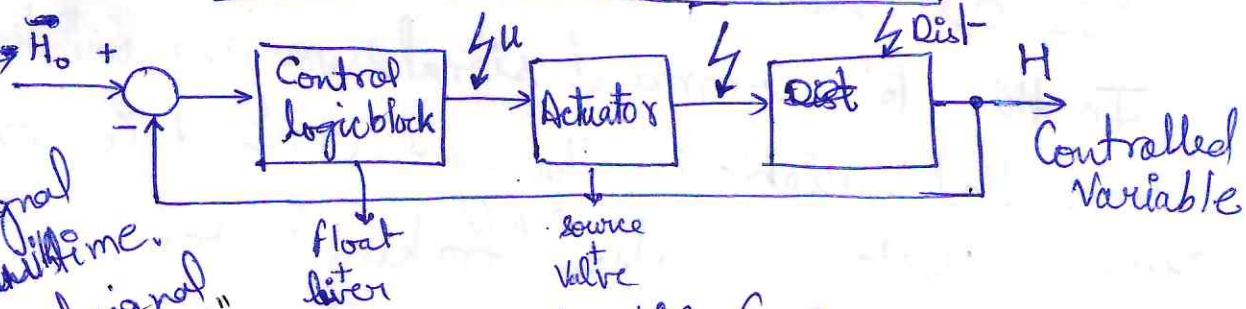
Preset hight is Command signal

Objective → To Control the hight of Water (H) in tank. so controlled Variable is H.



$$u = \underbrace{\left(\frac{l_1}{l_1 + l_2}\right)}_{\text{Propotinal logic for Control}} e$$





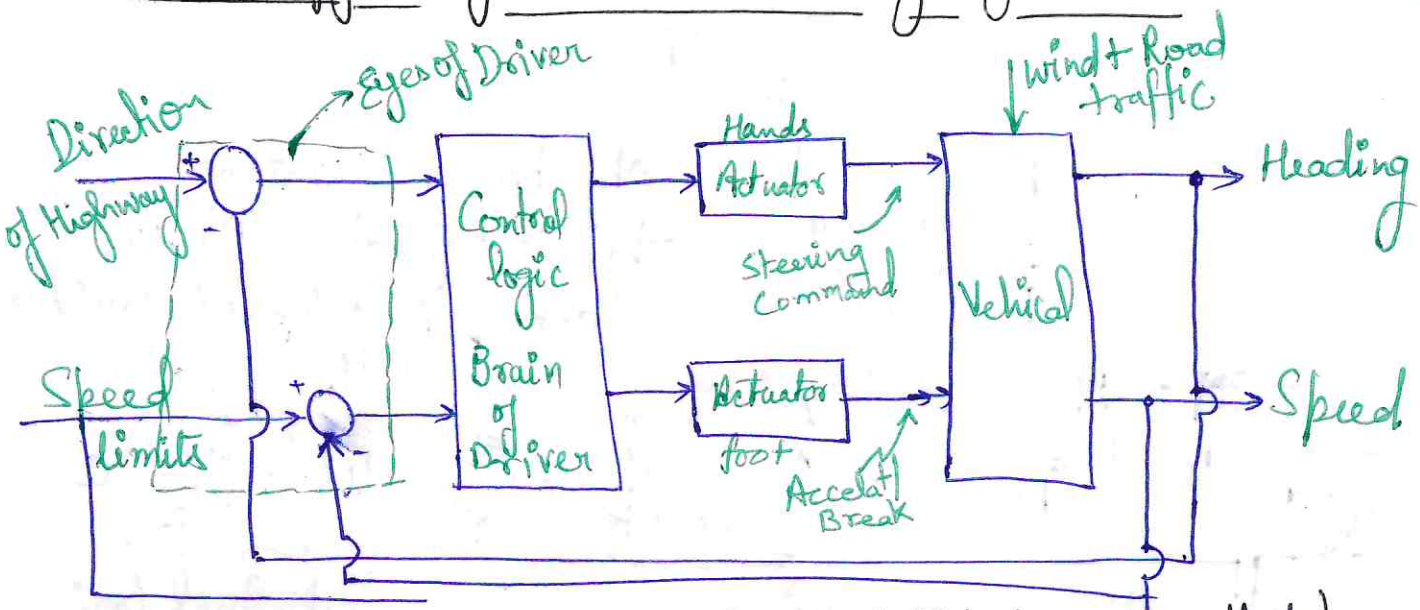
In this Case Command signal is constant in time. Such a Command signal is known as "Set point".

Requirement in this Case

$$\underbrace{\text{Controlled Variable} = \text{Set Point}}_{\text{Such a system is Called} \rightarrow \text{Regulator.}}$$

SISO (Single I/P Single O/P)
Scaler

Block Diagram of Automobile driving System :→



~~A~~ System MIMO (Multi Input Multi output) or Multivariable

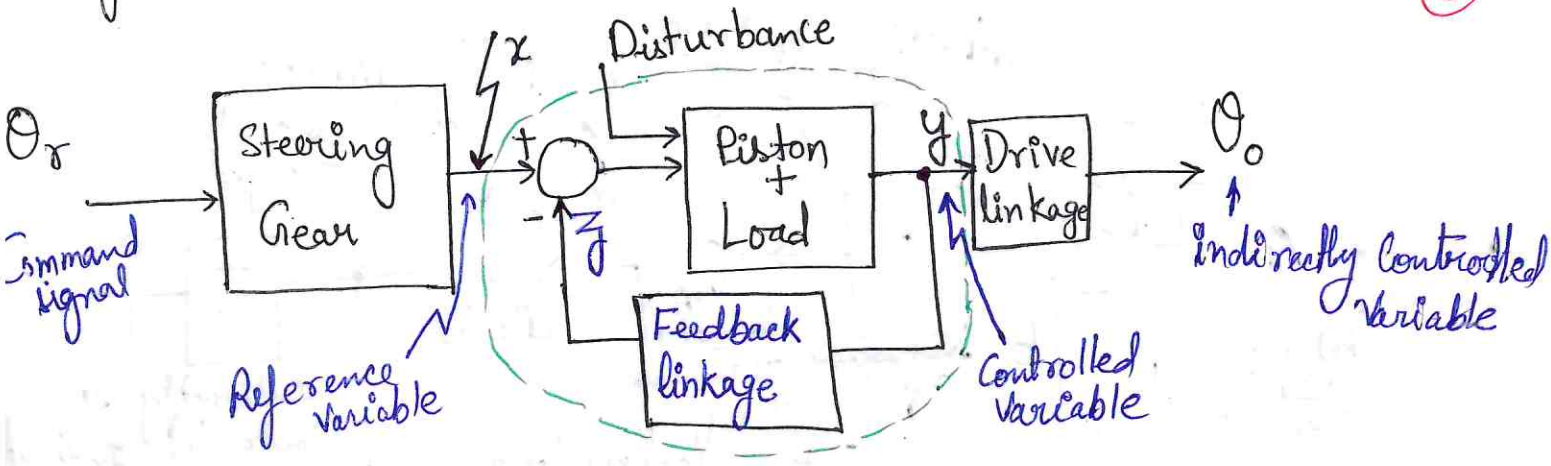
* If we can neglect the coupling different i/P variables and their interactions can be neglected. Than a MIMO system can be treated as a collection of Multiple SISO systems.

In this states command signal is varying with time. Controlled variable is following this type of Command signal. Such a system is called "tracking system".

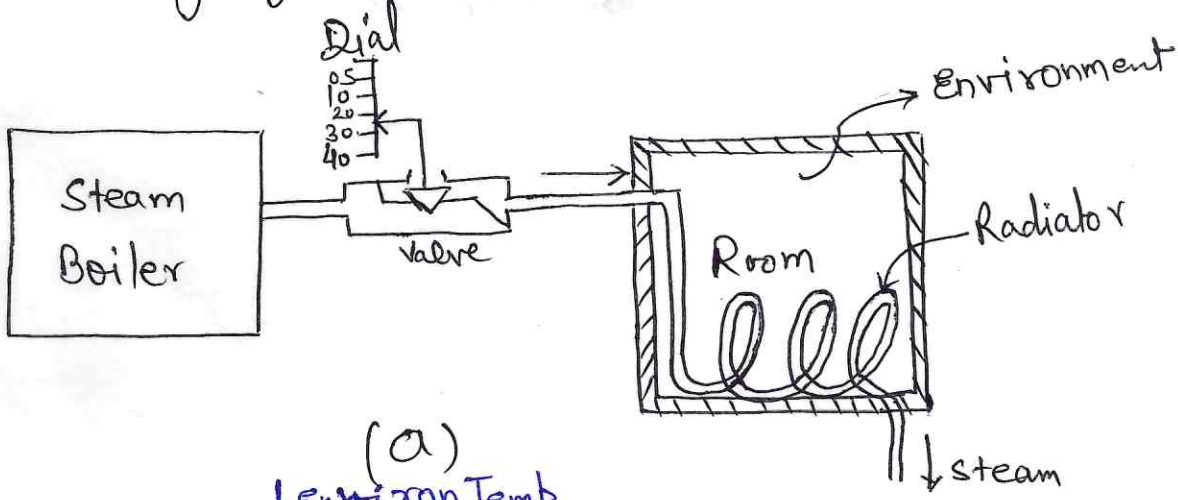
~~I~~ The systems where there is no human operator in loop (as ~~Bathroom~~ toilet-level system), called as "Automatic Control system."

Systems, where Human operator is in loop referred as "Man-Machine" Control systems.

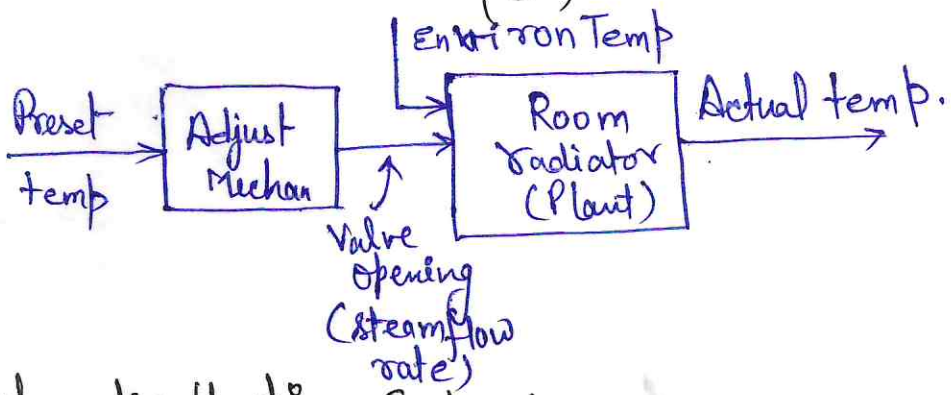
Hydralic Power steering



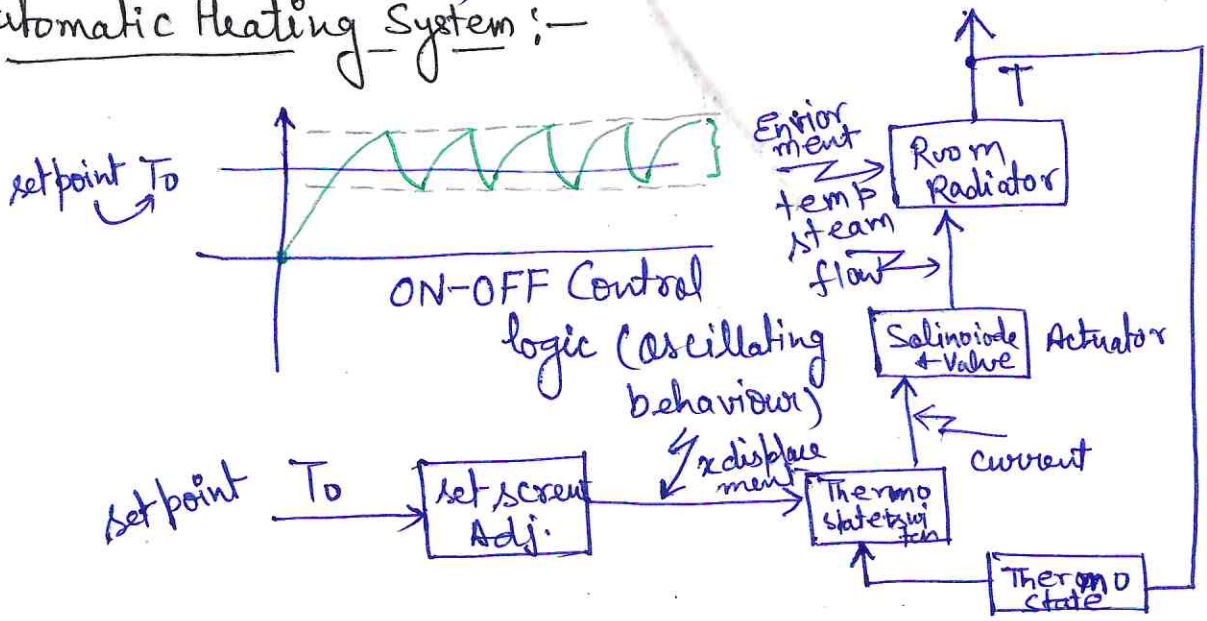
Residential Heating System :→



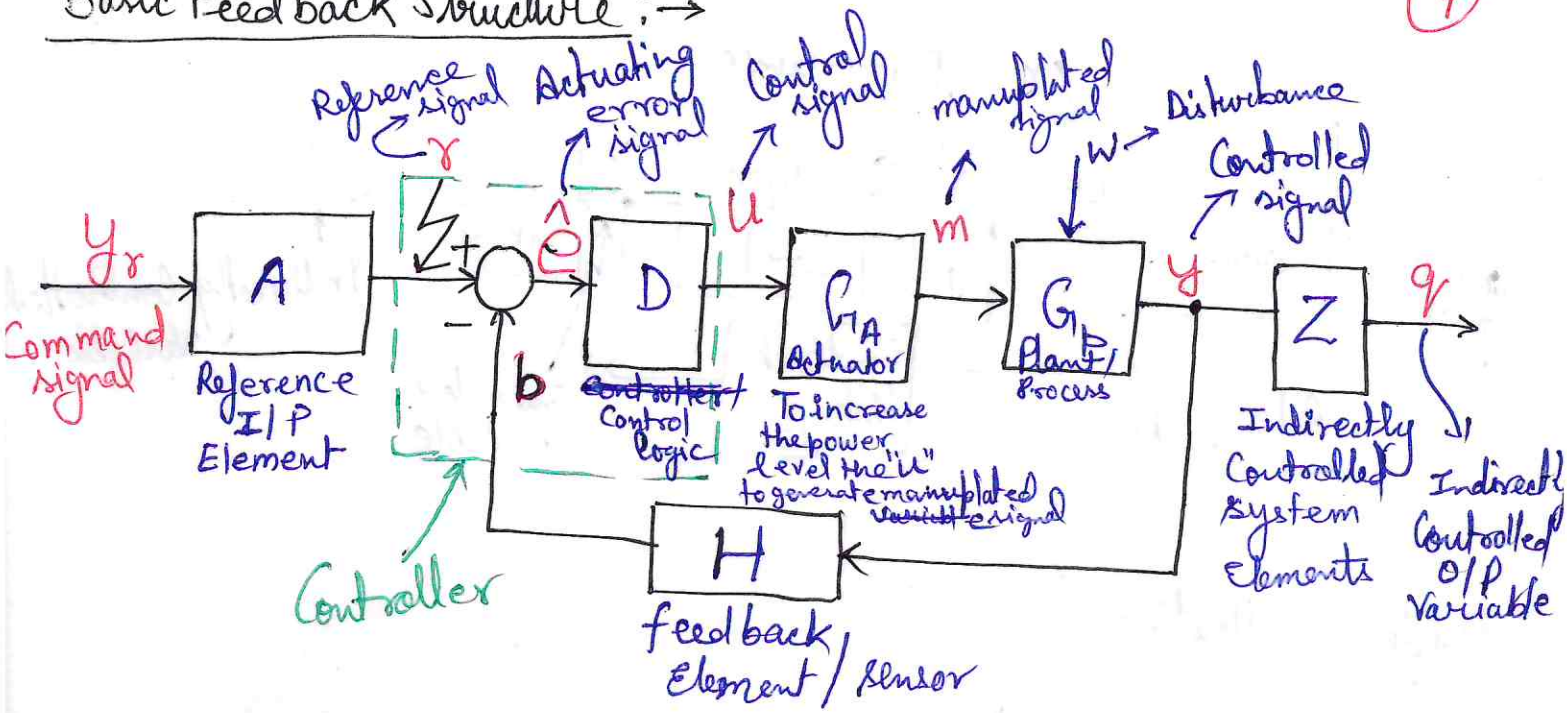
(a)



Automatic Heating System :-

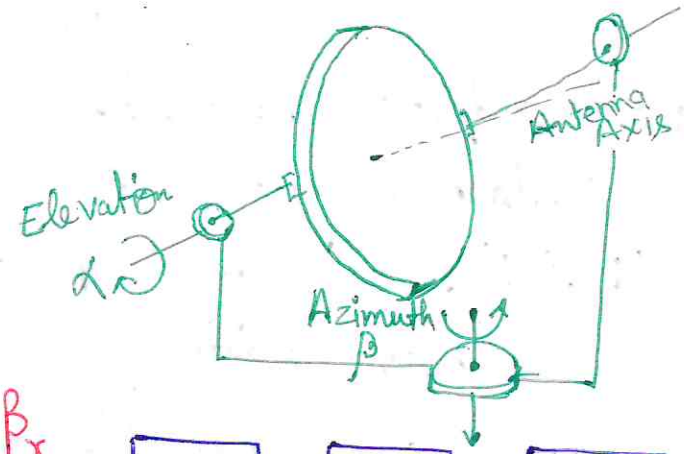


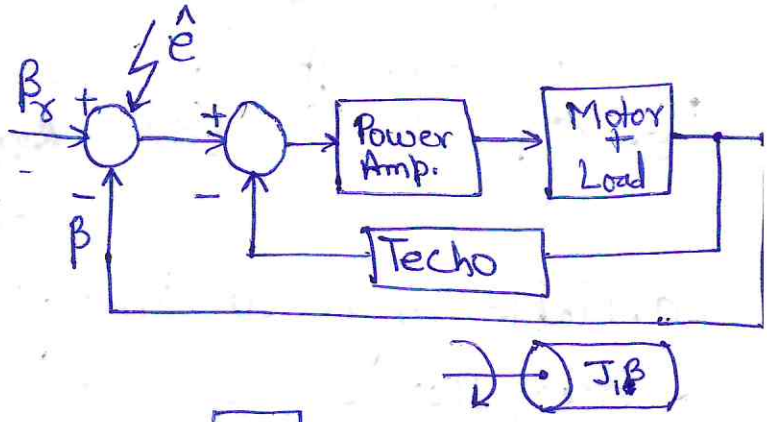
Basic Feedback Structure :→

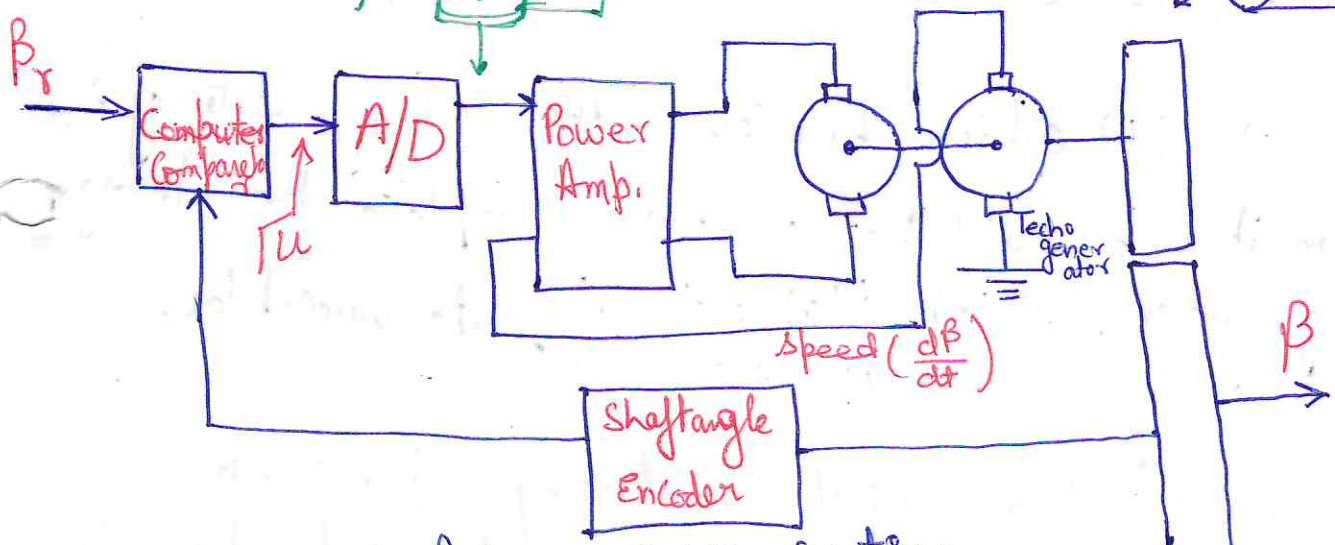


Lecture-3

Introduction to Control Problem (Contd.)

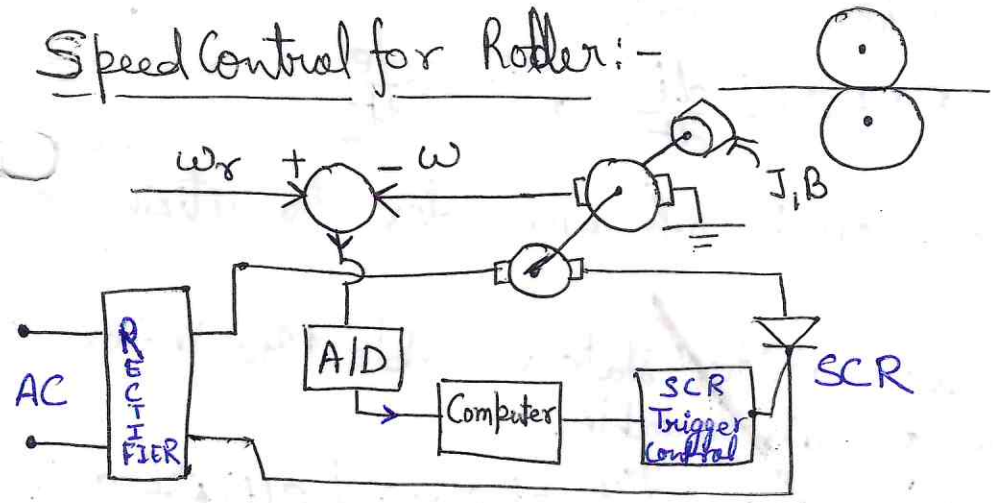






Propotional + Derivative Control

Speed Control for Roller:-



Basic terms of Control System :-

Process, Plant, Actuator,

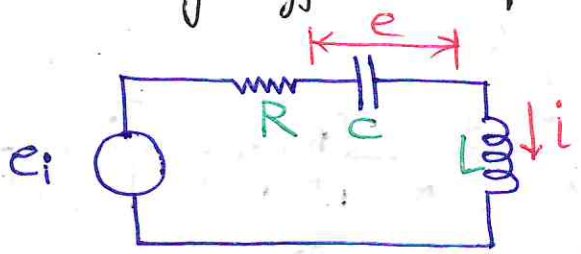
Controller → Error Detector
Controller → Control logic

Sensor

All Lectures will be limited to "Linear time invariant Sy

Basic Laws of Physics

↳ Set of Differential Equation → State variable Models
→ Transfer functions



$e(t), i(t)$ → Characterizing Variables.

Every output variable which is defined Algebracally, can be avilable to you in terms of $e(t), i(t)$.

Energy in Capacitor $\frac{1}{2}Ce^2$, inductor $\frac{1}{2}Li^2$.

→ $e(t), i(t)$ are characterizing variables.

→ $e(t), i(t)$ which characterise the energy state of system then it can be said that we have complete knowledge of system. $\left\{ \begin{matrix} e(0), i(0) ; t=0 \\ e_i(t) \quad t \geq 0 \end{matrix} \right\}$ state variables.

→ State variable are set of characterizing variable which gives complete information about a system provided initial state and external input is known to you.

$$e_i = Ri + e + L\frac{di}{dt} \quad , \quad i = C\frac{de}{dt}$$

→ State Variable Model :- which provides charactizing variable.

→ Std. Practice is to write state variable in term of first differential of state variable.

$$\frac{de(t)}{dt} = \frac{1}{C} i(t), \quad \frac{di(t)}{dt} = -\frac{R}{L} i(t) - \frac{1}{L} e(t) + \frac{1}{L} e_i(t)$$

$$e(t) = x_1 \quad , \quad i(t) = x_2 \quad , \quad e_i(t) = \gamma$$

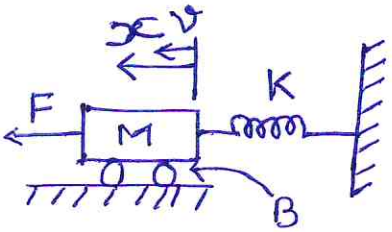
$$\frac{dx_1(t)}{dt} = \frac{1}{C} x_2 , \quad \frac{dx_2(t)}{dt} = \frac{R}{L} x_2 + \frac{1}{L} x_1 + \frac{1}{L}\gamma$$

if $y(t)$ is o/p (voltage across inductor) $\Rightarrow y(t) = -Ri - e + e_i$
$= -Rx_2 - x_1 + \gamma$

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{b}\,r$$
$$y = \underline{c}\,\underline{x} + d\,r$$

where $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ statevector, r: scaler i/p, y: scaler o/p

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad \underline{c} = [c_1 \; c_2 \; \cdots \; c_n]$$



$$F(t) = M\dot{v}(t) + Bv(t) + Kx$$

$x(t)$, $v(t) \longrightarrow$ characterizing variable of system

$x_1 = x, \quad x_2 = v$

$$\frac{dx}{dt} = v \; ; \quad \dot{x}_1 = x_2 \rightarrow (1)$$

$$\frac{dv(t)}{dt} = -\frac{K}{M}x - \frac{B}{M}v + \frac{1}{M}F(t)$$

$$\dot{x}_2 = -\frac{K}{M}x_1 - \frac{B}{M}x_2 + \frac{1}{M}r$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix}$$

displacement of Mass output variable

$$y(t) = x = \underline{c}\,\underline{x}(t) + d\,r(t)$$

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{B}{M} \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 0 \\ 1/M \end{bmatrix}, \quad \underline{c} = [1 \quad 0]$$

$d =$

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{b}\,r \quad : \text{State Equation}$$
$$y = \underline{c}\,x \quad : \text{output Equation}$$

→ State Variable is defined as Mathematical Convient.

$$x_1 = x(t) + v(t)$$
$$x_2 = v(t)$$

|| We can also this type of state variable.

→ O/P ~~will~~ Can be defined ~~for~~ above x_1, x_2 as well but for a given i/P, O/P will be Uniquely defined.

Lecture-4

Dynamic Systems & Dynamic Response

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_1 r$$

$a_{11}, a_{12}, \ldots, b_1 \rightarrow$ Constants of system (or) coefficient of $\underline{A}$ & $\underline{b}$ matrices

$x_1, x_2, \ldots x_n \rightarrow$ state variable of system, $r \rightarrow$ i/P variable

$$\text{nth order} \begin{cases} \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_2 r \\ \vdots \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_n r \end{cases}$$

$$\{ y = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n + d r$$

(SISO) system

n^{th}-order system, Order of a system is directly link to no. of state variable of system which represent energy state of SISO system.

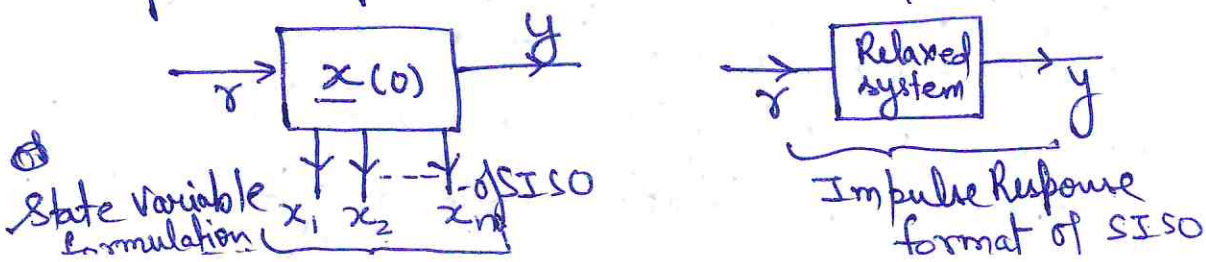
$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{A}\,\underline{x} + \underline{b}\,r \quad : \text{state Equation} \\ y &= \underline{c}\,\underline{x} + d\,r \quad : \text{output Equation} \end{aligned}\right] \quad \underline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{(n\times n)}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{(n\times 1)}, \quad \underline{c} = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}_{(1\times n)}, \quad d = \text{Const}.$$

Impulse Response Model :⇒

(i) Relaxed : $\underline{x}(t=0) = 0$, system is in Unenergied state
All intital Conditions to be zero.

(ii) Input - output relation relationship



If $r(t)$; $t \geq 0$

$$r(t) \longrightarrow \boxed{\begin{matrix}\text{Impulse Respo} \\ g(t)\end{matrix}} \longrightarrow y(t)$$

then $y(t) = \int_0^t g(t-\tau)\, r(\tau)\, d\tau$; $t \geq 0$

$= \int_0^t g(\tau)\, r(t-\tau)\, d\tau$

$g(t)$: "Impulse response" of a system is complete characterization of system.

→ Laplace transform for mathematical system

$$F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\mathcal{L}[y(t)] = Y(s) = \int_0^\infty y(t)\, e^{-st}\, dt \cancel{= \int_0^\infty (g(t-\tau)\, r(\tau))\, e^{-st}\, dt}$$

$g(t-\tau) = 0$ for $t < \tau$ Causal system

$$y(t) = \int_0^t g(t-\tau)\, r(\tau)\, d\tau = \int_0^\infty g(t-\tau)\, r(\tau)\, d\tau$$

$$Y(s) = \int_0^\infty \int_0^\infty [g(t-\tau)\, r(\tau)\, d\tau]\, e^{-st}\, dt$$

changing order of Integration

$$= \int_0^\infty \left[\int_0^\infty g(t-\tau)\, e^{-st}\, dt \right] r(\tau)\, d\tau \qquad (t-\tau) = \theta$$

$$= \int_0^\infty \left[\int_0^\infty g(\theta)\, e^{-s\theta}\, d\theta \right] e^{-s\tau}\, r(\tau)\, d\tau$$

$$= \underbrace{\int_0^\infty g(\theta)\, e^{-s\theta}\, d\theta}_{\text{Laplace transf of Impulse Respon}} \cdot \underbrace{\int_0^\infty r(\tau)\, e^{-s\tau}\, d\tau}_{\text{Laplace trans of Input signal}}$$

$$Y(s) = G(s) \cdot R(s)$$

$G(s) \triangleq \mathcal{L}[g(t)]$

$R(s) \triangleq \mathcal{L}[r(t)]$

$G(s)$ = transfer function of system

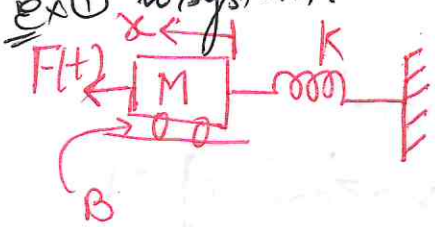
$G(s)$ = Laplace transform of Impulse response of system Under Initial condition to be zero.

However, basic physics allows me to generally "have differential Equation model of system.

For that transfer function can be defined as

$$G(s) = \frac{Y(s)}{R(s)}\Big|_{\text{system is relaxed}}$$

→ transfer function is laplace transformed of impulse response to system.

ex① 

$$M\ddot{x} + B\dot{x} + Kx = F(t)$$

Input Variable → F(t), O/P Variable : x(t)

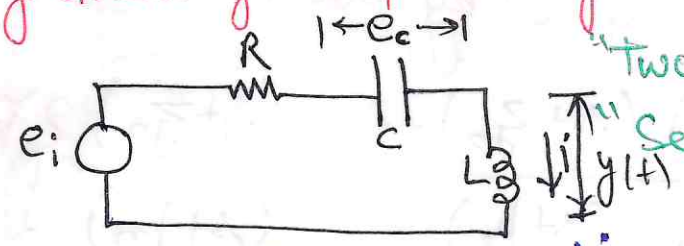
$$\mathcal{L}[\dot{x}] = sX(s) - \cancel{x(0)} = 0 \rightarrow \text{system is relaxed}$$

$$\mathcal{L}[\ddot{x}] = s^2X(s) - s\cancel{x(0)} - \cancel{\dot{x}(0)}$$

$$M s^2 X(s) + B s X(s) + K X(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} \rightarrow \text{2nd order TF mod}$$

→ In transfer fn. model the "order of system" is defin by the highest power of "s" in denominator.

ex② 

"Two energy storing" element
"Second order" system

$$Ri + e_c + L\frac{di}{dt} = e_i \quad , \quad C\frac{de_c}{dt} = i$$

$$RI(s) + E_c(s) + sLI(s) = E_i(s) \rightarrow ①$$

$$CsE_c(s) = I(s) \rightarrow ②,$$

Let i/P Variable = $E_i(s)$, O/P Variable = Voltage across Inductor = $L\frac{di(t)}{dt}$

$$Y(s) = LsI(s)$$

$$G(s) = \frac{Y(s)}{E_i(s)} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

if m=n . TF is "proper" || if $\boxed{m \leq n}$ "strictly prope TF

→ TF is physically realizable only when $m \leq n$. (or) $m > n$ TF becomes unrealizable.

→ Most physical system comes under $\boxed{m < n}$ i.e. strictly proper TF.

$$G(s) = \frac{N(s)}{\Delta(s)}$$ → Characteristics polynomial of system

$N(s) = b_0 s^m + \text{----} + b_m$ → roots → Zeros of TF

$\Delta(s) = 1 \cdot s^n + \text{-----} + a_{n-1} s + a_n$ → roots → Poles of TF

plays an imp role in dynamics Response of System

Monic Polynomial

Characteristics roots of system

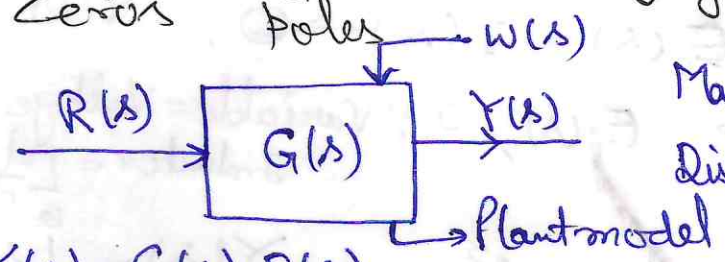
→ Zeros will play with Amplitude of Response.

→ Nature, type will defined by poles.

$$G(s) = \frac{K(s+z_1)(s+z_2)\text{----}(s+z_n)}{(s+p_1)(s+p_2)\text{---}(s+p_n)}$$ } Poles-Zero form of TF

z_i → Zeros, p_i → Poles

K : Gain Constant $= b_0$ of system



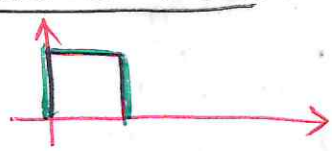
Manipulated Input R(s)
Disturbance Input W(s)

$Y(s) = G(s) R(s)$

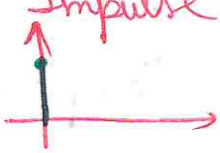
Disturbance → Erratic signals with unknown waveform (Noise) (Stochastic Models)
→ Slow-Varying Signals & Waveforms

Disturbance Models :-

① Pulse

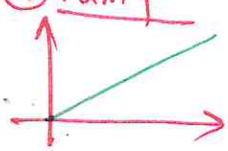


② Impulse



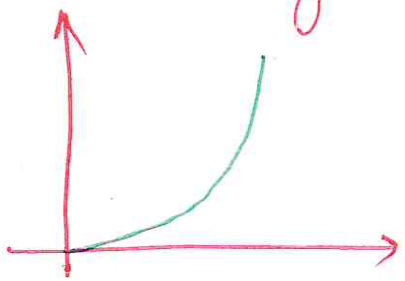
③ Constant step

④ Ramp



Disturbance &

5 Parabolic signal



$\delta(t) \xrightarrow{\mathcal{L}} \mathcal{L}[\delta(t)] = 1$

$\mathcal{L}[\mu(t)] = \frac{1}{s}$ $\quad u(t)$ control signal

$f(t) = t \; ; \; t \geq 0$

$= t\,\mu(t) \; ; \; \mathcal{L}[f(t)] = \frac{1}{s^2}$

$f(t) = \frac{t^2}{2} \; ; \; t > 0$

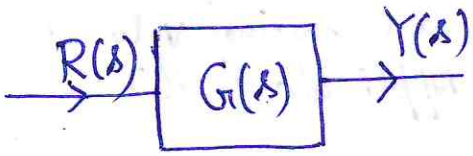
$= 0 \quad t = 0$

$f(t) = \frac{t^2}{2}\mu(t) \; ; \; \mathcal{L}[f(t)] = \frac{1}{s^3}$

Reference signal (or) Command signal

$\boxed{\text{1 Impulse} \quad \text{2 step} \quad \text{3 Ramp} \quad \text{4 Parabola}}$ → Std. test signals

Lecture-5



$Y(s) = G(s)\,R(s)$ let $G(s) = \dfrac{1}{s^2+3s+2}$

$G(s) = \dfrac{1}{(s+1)(s+2)}$ Let $r(t) = 5\mu(t)$

$R(s) = 5\dfrac{1}{s}$

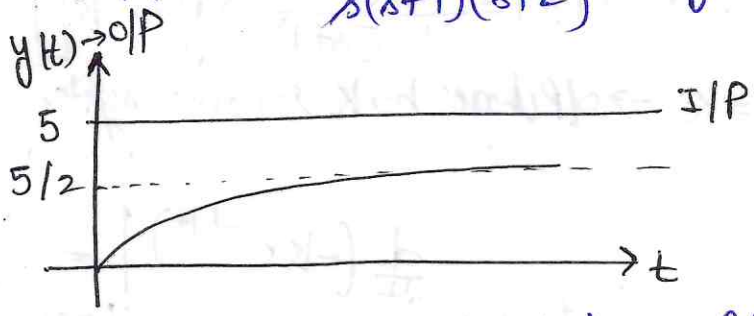
$$Y(s) = \frac{5}{s(s+1)(s+2)}$$

$$= \underbrace{\frac{5/2}{s}}_{\text{Excitation Pole}} - \underbrace{\frac{5}{s+1} + \frac{5/2}{s+2}}_{\text{system Poles}} = \overbrace{\frac{5}{2}}^{\text{Steady state Response}} \underbrace{- 5e^{-t} + \frac{5}{2}e^{-2t}}_{\text{Transient Response of System as it dies out @ } t\to\infty}$$

Final-Value Theorem to get steady-State Response

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

$$sY(s) = \frac{5\cancel{s}}{\cancel{s}(s+1)(s+2)} \quad ; \; y_{ss} = 5/2$$



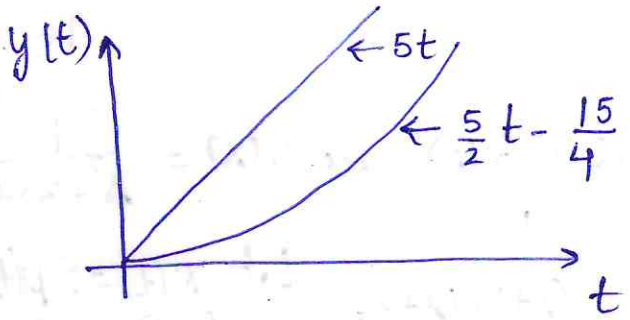
Ex ② If $r(t) = 5t\mu(t) \rightarrow R(s) = \dfrac{5}{s^2}$

$$Y(s) = \frac{5}{s^2(s+1)(s+2)} = \frac{5/2}{s^2} + \frac{K}{s} + \frac{5}{(s+1)} - \frac{5/4}{(s+2)}$$

$$K = \frac{d}{ds}\left[\frac{5}{(s+1)(s+2)}\right]\Bigg|_{s=0}$$

$$Y(s) = \frac{5/2}{s^2} - \frac{15/4}{s} + \frac{5}{(s+1)} - \frac{5/4}{(s+2)}$$

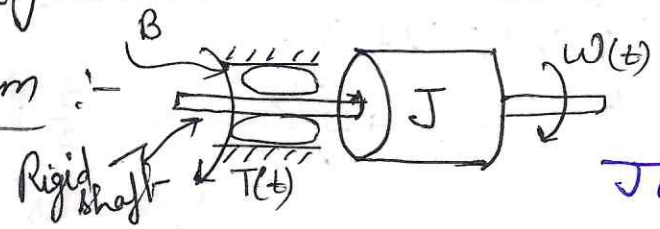
$$y(t) = \underbrace{\frac{5}{2}t - \overbrace{\frac{15/4}{}}^{y_{ss}}}_{\text{Reaction of System due to I/P. Steady State Response}} + \underbrace{5e^{-t} - 5/4\,e^{-2t}}_{\text{Reduces to zero @ } t\to\infty \text{ Transient Response}}$$



$sY(s)$ has poles on the origin in this case so final value theorem is not Applicable in this case.

Characteristics of First-Order & Second Order Models

First-Order System :-



$$J\dot{\omega} + B\omega = T$$

$$J\omega(s)s + B\omega(s) = T(s)$$

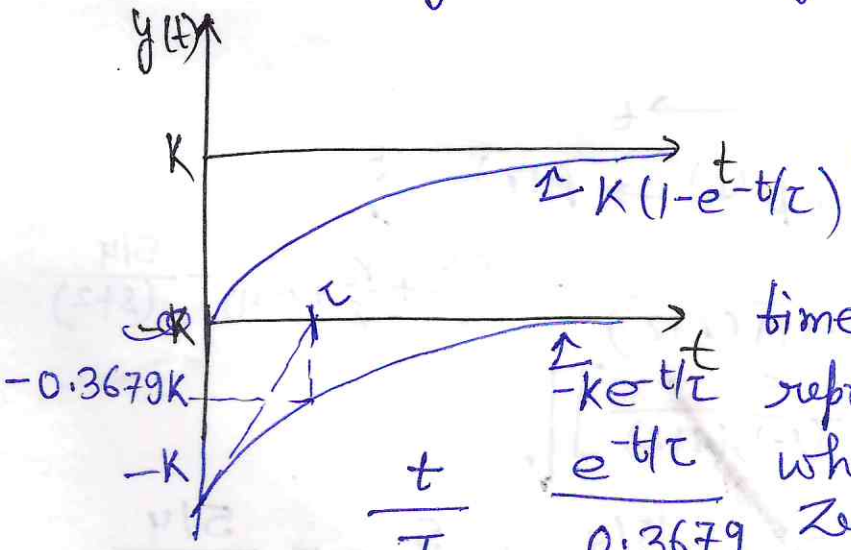
$$G(s) = \frac{\omega(s)}{T(s)} = \frac{1}{Js+B}, \quad T(s) = \frac{1}{s},$$

~~Chara~~ $$G(s) = \frac{K}{(\tau s+1)}$$ First order system

K: system gain, $\tau \rightarrow$ time Constant of system

if $T(s) = \frac{1}{s}$, $Y(s) = \frac{K}{s(\tau s+1)} \Rightarrow y(t) = K\left[1 - e^{-t/\tau}\right]$

at $t \rightarrow \infty$, $y(t) = K \rightarrow$ o/P changes by K so name system gain



$$\frac{d}{dt}\left(-Ke^{-t/\tau}\right)\Big|_{t=0} = \frac{K}{\tau}$$

time Constt of system ~~is time~~ represents the time in which transient becomes zero if it determined by intital slope of $\frac{K}{\tau}$.

$\frac{t}{\tau}$	$e^{-t/\tau}$
	0.3679
4τ	0.0183
5τ	0.0067

→ In this range, transient of system "almost" decays to zero.

Speed of Response :-

→ ~~Faster time~~ larger time Constant → sluggish system

Process Models

$\frac{K}{\tau s+1}$: First order lag (simple lag) is completly Characterisitics by two parameter (K, τ).

Second Order System :→ Mostly 1oth order Can be approximated by a second order system.

Flexible shaft (k) ← small "k"

O/P → $\theta(t)$, I/P → $T(t)$

$$J\ddot{\theta} + B\dot{\theta} + k\theta = T$$

$$Js^2\theta(s) + B\theta(s).s + k\,\theta(s) = T(s)$$

$$G(s) = \frac{\theta(s)}{T(s)} = \frac{1}{Js^2+Bs+k}$$

$$G(s) = \frac{1}{Js^2+Bs+k}$$

K = system gain = $1/k$

$$= \frac{K}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

$\xi = \frac{1}{2}\frac{B}{\sqrt{kJ}}$ ← damping ratio

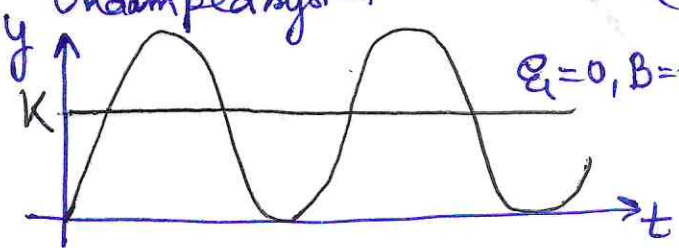
$\omega_n = \sqrt{k/J}$ ← Undamped Natural frequency

$\{K, \xi, \omega_n\}$ → describes a second order system

let $T(s) = \frac{1}{s} \Rightarrow \theta(s) = \frac{K\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$ (step Input)

(i) $\underbrace{\xi = 0 \; ; \; (B=0)}_{\text{Undamped system}}$, $\underbrace{\text{(ii) } 0<\xi<1 \text{ (iii) } \xi=1, \text{ (iv) } \xi>1}_{(B\neq 0)\text{ (damped system)}}$

(i)



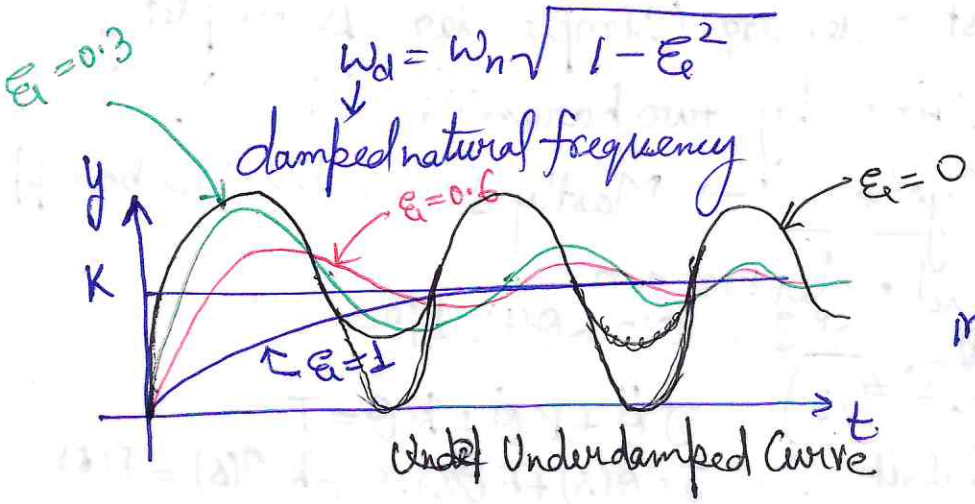
$\xi=0, B=0$ ω_n: Undamped Natural frequency

(iii) $0<\xi<1$ $(B\neq 0)$ $\quad \frac{K}{J} > \left(\frac{B}{2J}\right)^2$

$$\theta(t) = K\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

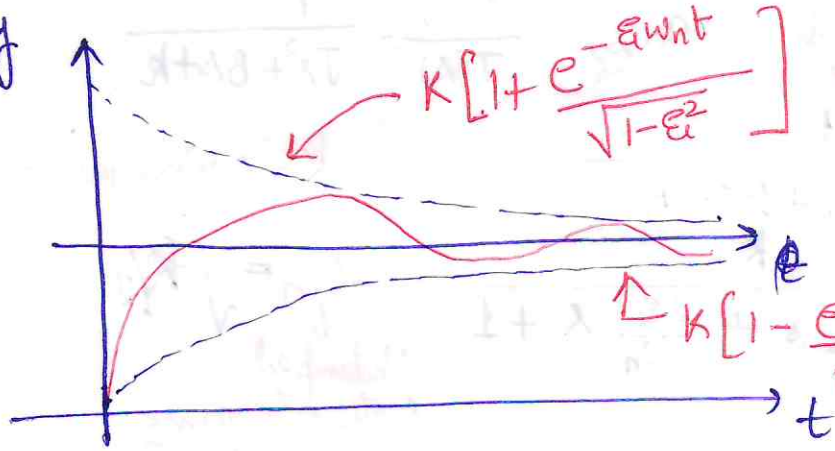
damped natural frequency



$\xi=0 \rightarrow$ oscillatory Reponse

$\xi=1 \rightarrow$ sluggish system

Most industrial systems are $0<\xi<1$



$e^{-\xi\omega_n t} = e^{-t/\tau}$

$\boxed{\tau = \frac{1}{\xi\omega_n}}$ time Constant

larger the values of $\xi\omega_n$, $\rightarrow$ smaller time Const $\rightarrow$ faster response.

(iii) $\xi=1$, $\quad \frac{K}{J} = \left(\frac{B}{2J}\right)^2 \leftarrow$ Critically damped

In borderline situation $\xi=1$, $\xi=0$ may be accepted

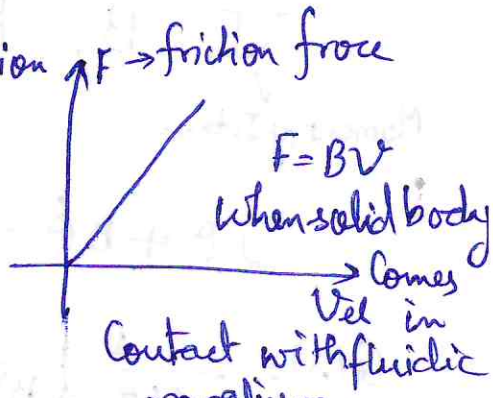
Lecture-6

Mechanics of Translation:-

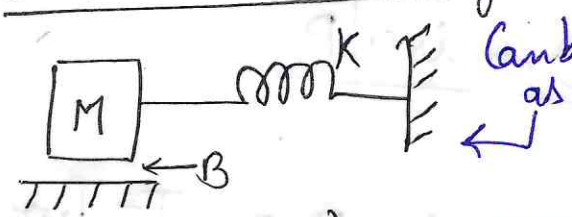
M, B, k Mechanical Component

$F = kx$
↳ spring Constant

Coulomb-friction

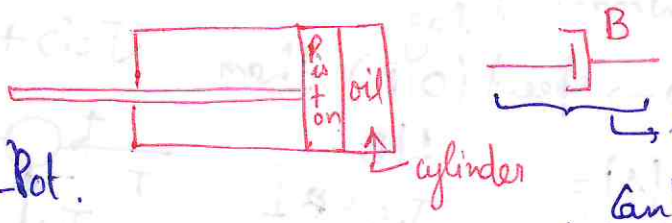


Hydraulic Power steering -

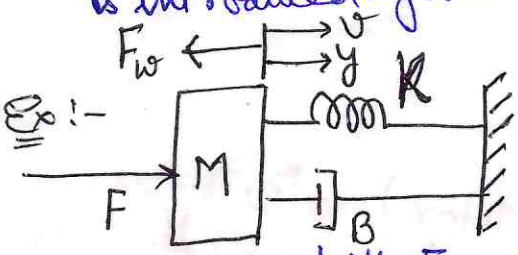


Damper (Dash-Pot)

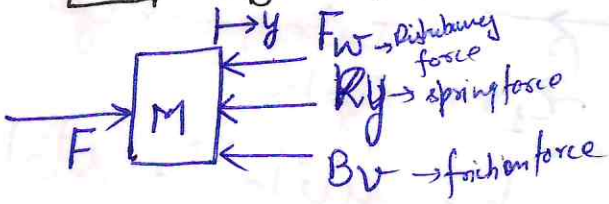
intential friction is introduced by dash-Pot.



Ex:-



Free body Diagram shows Masses as nodes and forces by suitable by arrows.

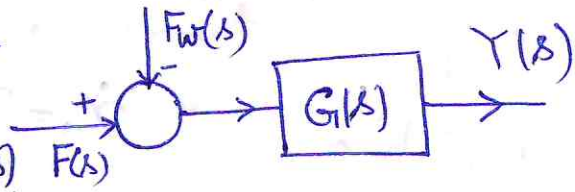


$$M\dot{v} + Bv + ky = F - F_w$$

→ It is an algebraic quantity. It can have any sign

$$x_1 = y, \quad x_2 = v = \dot{y}$$

$$M\ddot{y} + B\dot{y} + ky = F - F_w$$



$$T.F = \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k} = G(s)$$

$$= \frac{1/k}{\frac{M}{k}s^2 + \frac{B}{k}s + 1} = \frac{K_s}{\frac{1}{\omega_n^2}s^2 + \frac{2\xi}{\omega_n}s + 1}$$

Quadratic Lag (or) second order lag

$F = M\ddot{y} \Rightarrow M = F/\ddot{y}$
N/m/sec²

Kg ← M F → Newton 'N'
N/(m/sec) ← B y → m
N/m ← k v → m/s
a → m/s²

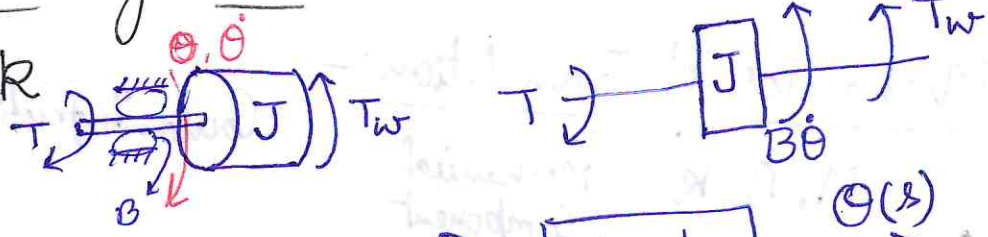
$F = Bv$, $F = kx$

$F/v = B$, $\frac{F}{x} = k$
N/m/sec N/m

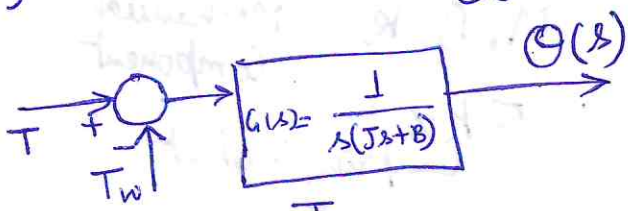
Fixed-Rotation System

J, B, K

Moment of Interia



$$J\ddot{\theta} + B\dot{\theta} = T - T_w$$



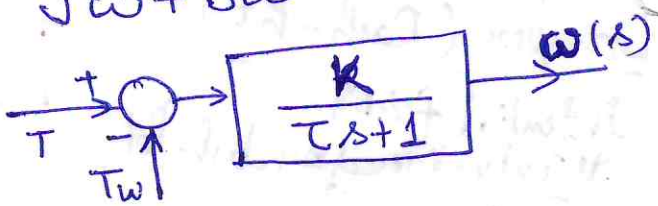
$$G(s) = \frac{1}{s(Js+B)} = \frac{1/B}{s(\tau s+1)} \; ; \; \tau = \frac{J}{B}$$

$$= \underbrace{\frac{1}{s}}_{\text{Integrator}} \cdot \underbrace{\frac{1/B}{(\tau s+1)}}_{\text{first order lag}}$$

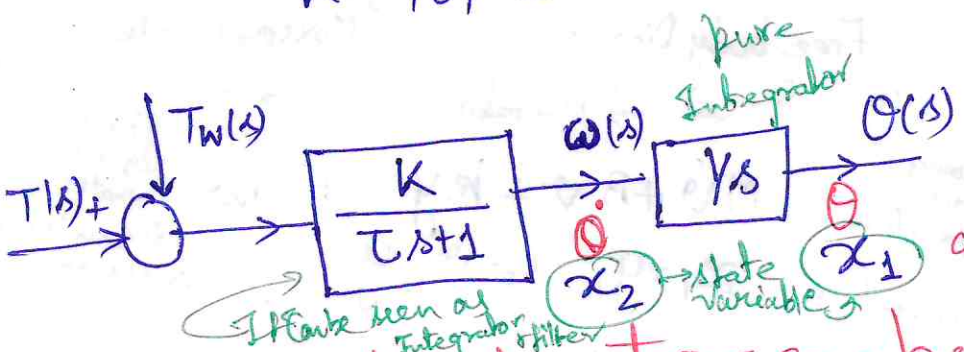
$\theta \rightarrow$ is O/P

If $\omega \rightarrow$ speed (O/P) than $J\dot{\omega} + B\omega = T - T_w$

$$G(s) = \frac{1/B}{J/B\, s + 1}$$



$K = 1/B$, $\tau = J/B$



Motor produces an Integrator effect in a system.

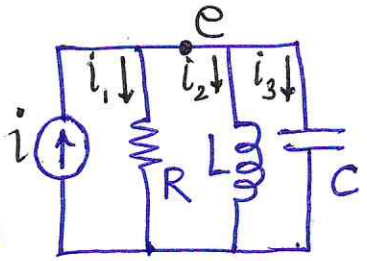
→ O/P of Integrator can be suitable state variable.

$$\dot{x}_1 = x_2, \quad \tau\dot{x}_2 + x_2 = \cancel{K}B(T - T_w)$$

T	Newton-m
θ	rad
$\dot{\theta}$	rad/sec
$\ddot{\theta}$	rad/sec^2
B	Newton-m/(rad/sec) $T = B\dot{\theta}$

J	Newton-m/(rad/sec^2) or ~~Kg-m²~~ Kg-m^2

Analogus System :-



$$\frac{e}{R} + i_2 + C\frac{de}{dt} = i \rightarrow (1)$$

$$L\frac{di_2}{dt} = e \rightarrow (2)$$

By (1) & (2)

Current through Inductor
Voltage across Capacitor } state variable

$$\frac{e}{R} + C\frac{de}{dt} + \frac{1}{L}\int_{-\infty}^{t} e(\tau)\,d\tau = i \rightarrow (3)$$

Mechanical system

$$M\ddot{y} + B\dot{y} + Ky = F$$

rearrange

$$M\dot{v} + Bv + K\int_{-\infty}^{t} v(\tau)\,d\tau = F \rightarrow (4)$$

Compare (3) & (4)

T ↔ force F ⟷ Current i

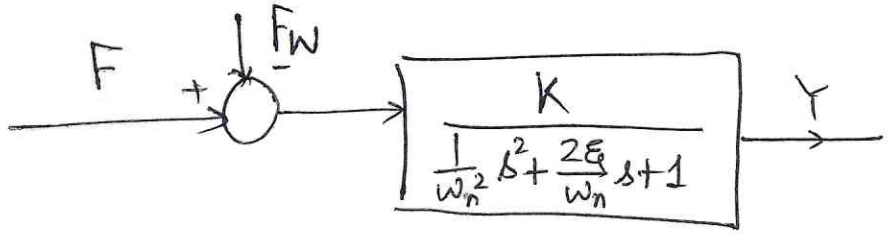
$\omega \longleftrightarrow v \longleftrightarrow e$

$J \longleftrightarrow \underline{M} \longleftrightarrow \underline{C}$

$B \longleftrightarrow B \longleftrightarrow 1/R$

$K \longleftrightarrow K \longleftrightarrow 1/L$

Force-Current Analogy



$x_1 = y$
$x_2 = \dot{y} = \dot{x}_1$ | $\dot{x}_1 = x_2$

$$\frac{1}{\omega_n^2}\ddot{y} + \frac{2\xi}{\omega_n}\dot{y} + y = K(F - F_W)$$

$$\dot{x}_2 = \left\{-x_1 - \frac{2\xi}{\omega_n}x_2 + K(F - F_W)\right\}\omega_n^2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix} & \\ & \end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{B_1 \rightarrow \text{Vector}} F(t) + \underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{B_2 \rightarrow \text{Vector}} F_W(t)}_{\text{It is a two i/P system.}}$$