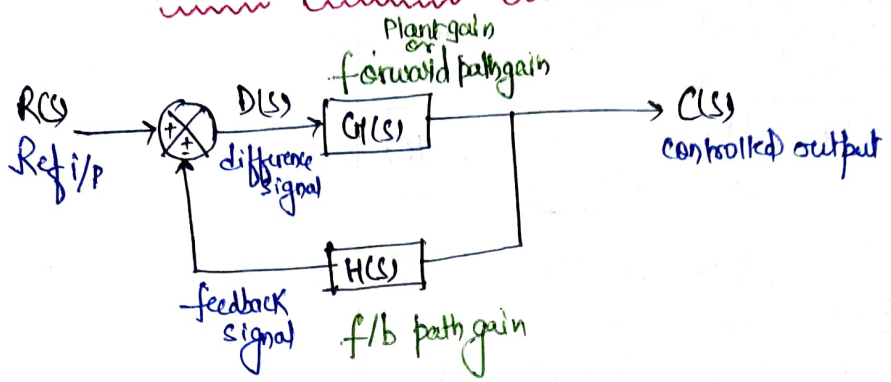
Control System

BASIC CONCEPT OF CONTROL SYSTEM



Negative f/b	Positive f/b
↑s stability	↓s stability
Degenerative f/b	Regenerative f/b
for designing of Amplifier	for designing of osscilator & multivibrator

(i) OLTF $= G(s)H(s)$

- for unity f/b system, OLTF $= G(s)\cdot(1) = G(s)$

(ii) CLTF $= \dfrac{G(s)}{1+G(s)H(s)}$

- for unity f/b system, CLTF $= \dfrac{G(s)}{1+G(s)} = \dfrac{\text{OLTF}}{1+\text{OLTF}}$

(iii) DC gain of CLTF $= T(s)\big|_{s=0}$

DC gain of OLTF $= G(s)\big|_{s=0}$

$$\boxed{\text{DC gain of } \left(G(s) = \frac{10}{s(s+2)}\right) = G(s)\big|_{s=0} = \infty}$$ (WRONG)

$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$

$G(s) = \frac{C(s)}{R(s)}$ = T.F. of open system

$\boxed{OLTF = G(s)}$ OLTF is defined only for closed system.

Sensitivity

If $A = f(B)$ $\quad S_B^A = \frac{\partial A/A}{\partial B/B} = \frac{B}{A}\frac{\partial A}{\partial B}$

by default sensitivity

$\boxed{S_G^T = \frac{G}{T}\frac{\partial T}{\partial G}}$ $\quad \boxed{S_H^T = \frac{H}{T}\frac{\partial T}{\partial H}}$ where $T(s) = \frac{G(s)}{1+G(s)H(s)}$

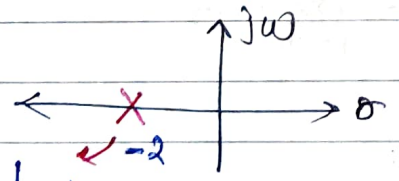
$S_G^T = \frac{1}{1+GH}$ $\quad S_H^T = \frac{-GH}{1+GH}$

Desensitivity, $\boxed{D = 1+GH}$

Significance of Poles

- If All poles on LHS of s-plane ⇒ STABLE SYSTEM
- If one of the pole on RHS of s plane ⇒ UNSTABLE
- If complex conjugate pole on Im axis ⇒ MARGINALLY STABLE
- If repeated pole on Im axis ⇒ UNSTABLE

⇒



$\tau = \left|\frac{1}{P}\right| = \frac{1}{2}$ sec

Reciprocal of magnitude of -ve real root is equal to time constant of system

$X(s) = \frac{10}{s+2} \leftrightarrow 10e^{-2t} = 10e^{-t/(1/2)}$

* Time constant doesn't exist for unstable system or Marginally stable sys.

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* Time constant is only valid for absolutely stable system

⊙ ~~tett~~ $\propto \tau$

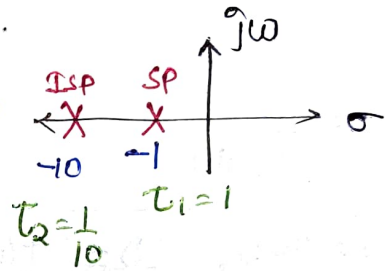
Initial & final value theorem

IVT: $\boxed{x(0) = \lim_{s\to\infty} sX(s)}$ Valid for both stable & unstable system.

FVT: $\boxed{x(\infty) = \lim_{s\to 0} sX(s)}$ FVT is valid only for absolutely stable system.

⊙ +ve f/b ↓s the stability of system by giving a right shift to poles, But not necessary to +ve real axis. So, it is not necessary ~~stable~~ +ve f/b. sys ~~are~~ unstable.

Concept of dominant pole



(i) Significant Pole is referred as dominant pole.

(ii) $\tau_{SP} > \tau_{ISP}$

(iii) Time response due to significant pole is slower than that of ISP.

(iv) In dominant pole concept we neglect the ISP for approx^n of T(s).

(v) DC gain should be same a/4 approx

(vi) for existence of dominant pole concept, $\left|\frac{ISP}{SP}\right| \geq 4$ {condition}

(vii) Overall time constant is defined as $\left|\frac{1}{SP}\right|$

(viii) Speed of dominant pole is least.

eg: $X(s) = \frac{100}{(s+1)(s+10)}$ ⇒ DC gain $= \frac{100}{1\times 10} = 10$

$\boxed{X(s) \approx \frac{100}{(s+1)}}$ ✗ T(s) gain must be same. Here DC gain = 100 ≠ 10

$\boxed{X(s) \approx \frac{10}{s+1}}$ ✓

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BLOCK DIAGRAM AND SFG

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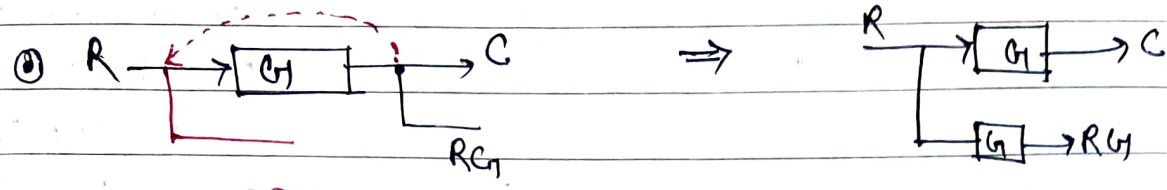

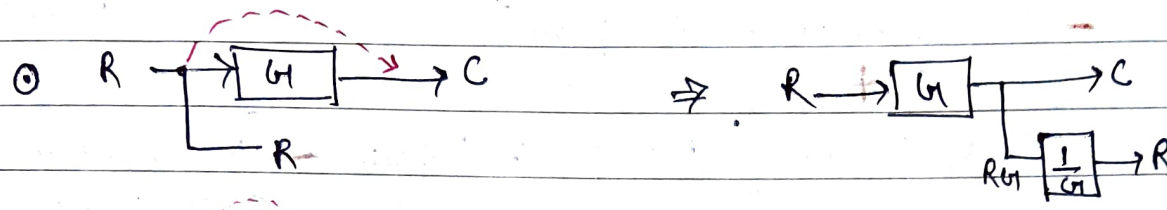

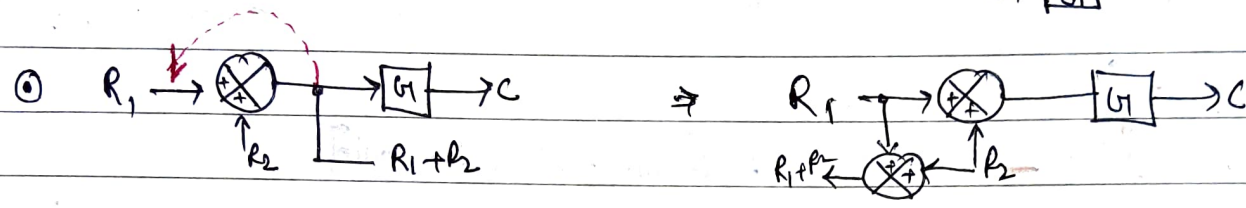

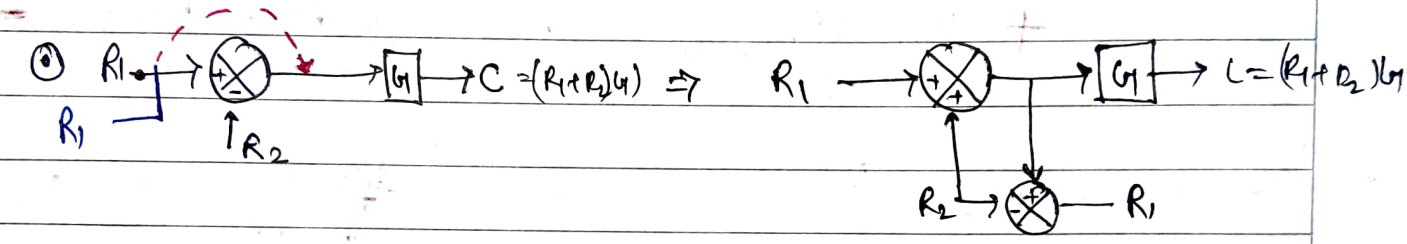

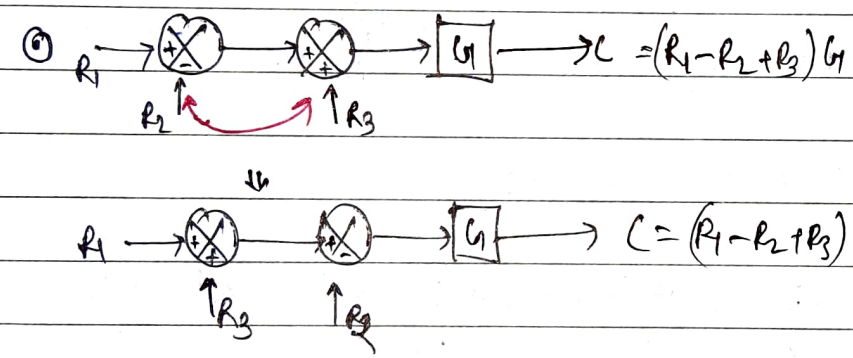

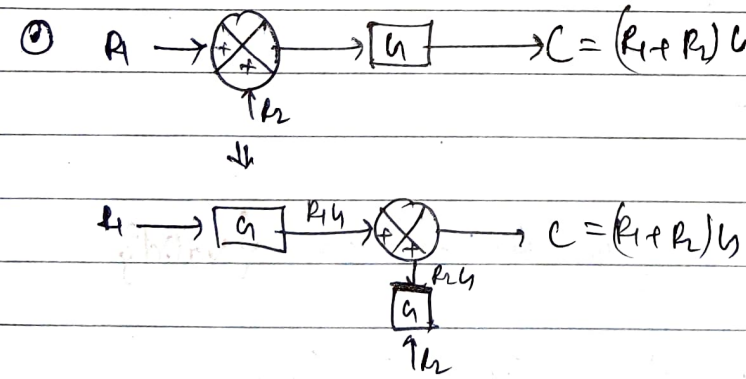

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Mason's gain formula:

$$\boxed{T(s) = \frac{\sum_{K=1}^{n} P_K \Delta_K}{\Delta}}$$

P_K : f/w path gain of Kth path

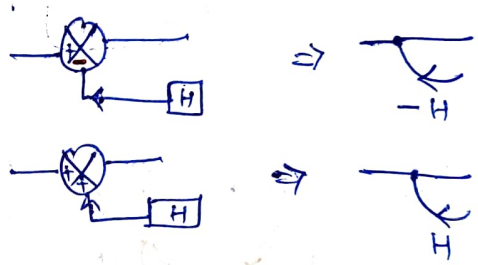
Δ = Determinant of STG

Δ_K = Associated path factor depends on f/w path & isolated loop

$\Delta = \{[1 - \text{Sum of all individual loop gain}]$
$+ [\text{Sum of product of 2 nontouching loop gain}] - [\text{Sum of product of 3 non touching loop gain}]$
$+ \ldots\ldots\}$

$\Delta_K = \{[1 - \text{Sum of all individual isolated loop gain}]$
$+ [\text{Sum of product of 2 nontouching isolated loop gain}] + \ldots\}$

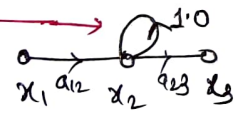
- $\{- + - + \ldots\}$
- Repetion of nodes is not allowed.



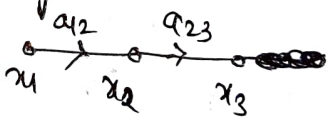
Limitation of Mason's gain formula:

(i) Not valid for self sustaining loop with unity gain

Invalid →

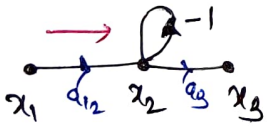


modified fig. →



$x_2 = a_{12}x_1 + x_2$ {Invalid eqn}

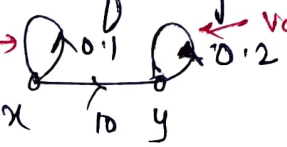
Valid →



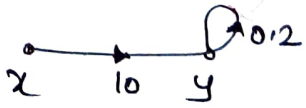
$x_2 = a_{12}x_1 - x_2 \Rightarrow x_2 = \frac{a_{12}x_1}{2}$ {Valid}

(ii) Self loop with any gain at input is invalid

Invalid → Valid



modified fig →



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Time response $\longrightarrow$

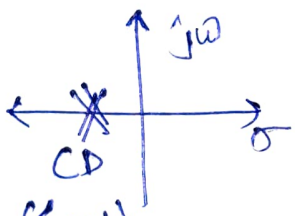
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{4\omega_n^2(\zeta^2-1)}$$

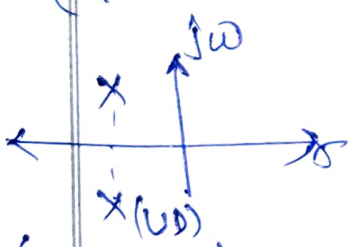
$$D = 4\zeta^2\omega_n^2 - 4\omega_n^2$$
$$= 4\omega_n^2(\zeta^2-1)$$

$\zeta > 1$	$D > 0$	Real roots & unequal
$\zeta < 1$	$D < 0$	Complex roots
$\zeta = 1$	$D = 0$	Real & equal roots
$\zeta = 0$	$\alpha = 0$	Pure Im roots

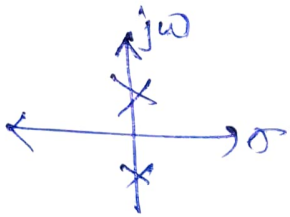
OD
$(\zeta > 1)$



CD
$(\zeta = 1)$



(UD)
$(0 < \zeta < 1)$



Undamped
$(\zeta = 0)$

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Time Response

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* response means $C(s)$ or $c(t)$.
* 1st order system $G(s) = \frac{1}{s\tau}$ $CLTF = \frac{1}{1+s\tau}$

$r(t)$	ess
$\delta(t)$	doesnt exist → as initial & final values can't be calculated
$u(t)$	0
$t\,u(t)$	τ
$\frac{t^2}{2}u(t)$	∞

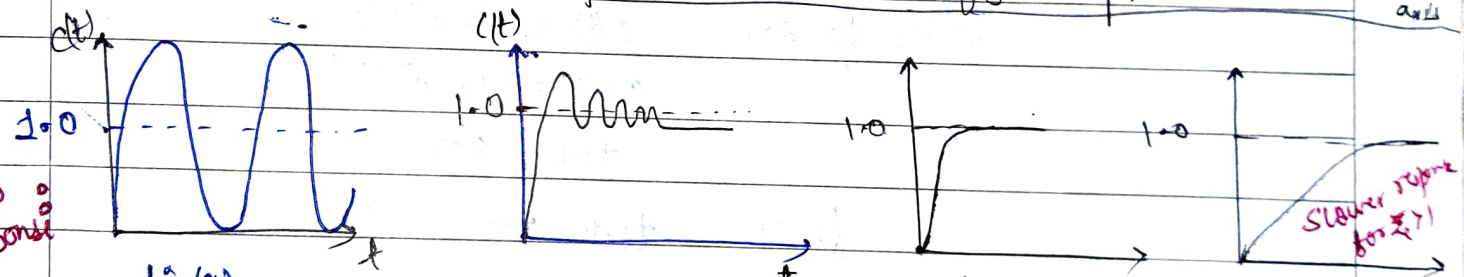
* 2nd Order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}$$

$$CLTF = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

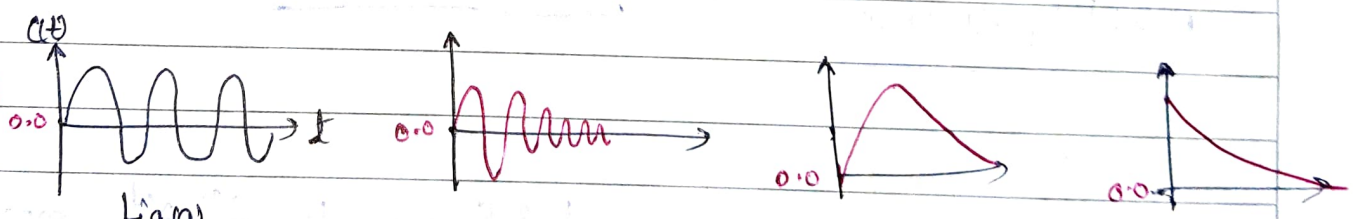
M.S.	$\xi = 0$	Undamped oscillation	fig(a)	Poles on Im axis
Stable	$0 < \xi < 1$	Underdamped oscillation	fig(b)	Poles: $-\alpha \pm j\omega d$
"	$\xi = 1$	Critical damped oscillation	fig(c)	Poles:- -ve real axis & equal
"	$\xi > 1$	Over damped oscillation	fig(d)	Poles:- unequal on -ve R axis

Step response:



fig(a) fig(b) fig(c) fig d

Impulse response:



fig(a)

** Over damped gives slower response & critical gives faster for step input although they look almost similar.

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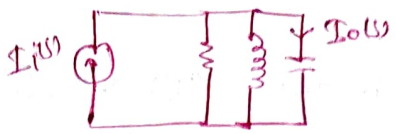
* RLC ckt. for 2nd order system



o/p is taken from 'C' if not mentioned in mall questions.

* $\alpha = \xi\omega_n$ & $\omega_d = \omega_n\sqrt{1-\xi^2}$ $\qquad \tau = \left|\frac{1}{\alpha}\right|$

* $\theta = \cos^{-1}\xi$

* C/S eqn $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$
* Don't take care of N°, always calculate ω_n & ξ from D°.
* 1st order:

delay time to reach 50% $\boxed{t_d = 0.693\tau}$

rise time 10% - 90% $\boxed{t_r = 2.2\tau}$

Settling time 2% to 5% error band

for eq: for 2%. $c(t) = 1 - e^{-t/\tau}$

$0.98 = 1 - e^{-t_{sett}/\tau}$

$t_{sett} = 4\tau$

for 5%. $t_{sett} = 3\tau$

for E%. $\boxed{t_{sett} = \tau \ln\frac{100}{E}}$

Note: $\boxed{t_d < t_r < t_{sett}}$

** 2nd Order:

$\boxed{\frac{dc}{dt} = 0}$, $t_p = \frac{n\pi}{\omega_d}$ where $n = 1, 3, 5 \ldots$ for $1^{st}, 2^{nd} \ldots$ maxima

$n = 2, 4, \ldots$ for $1^{st}, 2^{nd} \ldots$ minima

$M.P._o = e^{-\pi\cot\theta}$ or $e^{-(2m-1)\pi\cot\theta}$ // first or Generalized

$MP_4 = e^{-2\pi\cot\theta}$ or $e^{-2m\pi\cot\theta}$

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$$t_r = \frac{\pi - \theta}{\omega_d} \quad \{ \text{when } c(t) = 100\% \}$$

$$t_s = \left(\frac{4 \ (=4\tau) \text{ for } 2\% \text{ tolerance}}{\xi \omega_n} \right) = \frac{3 \ (=3\tau) \text{ for } 5\% \text{ tolerance}}{\xi \omega_n}$$

No. of oscillations, $N_o = \frac{2\sqrt{1-\xi^2}}{\pi \xi} = \frac{2}{\pi} \tan\theta$

evergreen formula for error calculation

$$\boxed{E_{s0} = \frac{R(s)}{1+G(s)H(s)}} \; ; \; \boxed{e_{ss} = \lim_{s \to 0} \frac{s E(s)}{} = \lim_{s \to 0} \frac{s R(s)}{1+G(s)H(s)}}$$

Only valid for unity f/b system

$K_p = \lim_{s \to 0} G(s)H(s)$; $K_v = \lim_{s \to 0} s\,G(s)H(s)$; $K_a = \lim_{s \to 0} s^2 G(s)H(s)$

$$\boxed{e_{ss} = \frac{1}{1+K_p} = \frac{1}{K_v} = \frac{1}{K_a}}$$

* Type of CLTF = Type of OLTF

Type of system	error coefficients			Steady state error		
	K_p	K_v	K_a	Step i/p	Ramp i/p	Parabolic i/p
0	K	∞	∞	$\frac{1}{1+K}$	∞	∞
1	∞	K	0	0	$1/K$	∞
2	∞	∞	K	0	0	$1/K$

for Non unity f/b system : $e_{ss} = \lim_{s \to 0} s\,E(s)$

$$= \lim_{s \to 0} s R(s) \left[\frac{1}{KH} - T(s) \right]$$

P.T.O

GATE 2019

$$e_{ss} = \lim_{s \to 0} \frac{s^N}{H(s)}$$

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order (finite error)

order 2 (zero error) type order 3 (∞ error)

- If order of i/p is greater than type of system the error will be infinite.
- If type of system > order of i/p ⇒ $e_{ss} = 0$
- If type of system = order of i/p ⇒ e_{ss} = finite
- Error coefficient is valid for stable CLTF

i/p	order
$u(t)$	0
$t\,u(t)$	1
$\frac{t^2}{2}u(t)$	2

In case of non-unity f/b system

(i) when f/b system consist no poles & zeroes at origin

$$K_H = \lim_{s\to 0} H(s)$$

$$e_{ss} = \lim_{s\to 0} sR(s)\left[\frac{1}{K_H} - T(s)\right]$$

(ii) when f/b system consits N number of poles at origin

$$K_H = \lim_{s\to 0} s^N H(s)$$

$$e_{ss} = \lim_{s\to 0} sR(s)\left[\frac{s^N}{K_H} - T(s)\right]$$

(iii) when f/b system consits N no. of zeroes at origin

$$K_H = \lim_{s\to 0} \frac{H(s)}{s^N}$$

$$e_{ss} = \lim_{s\to 0} sR(s)\left[\frac{1}{s^N K_H} - T(s)\right]$$

** Avoid this mistake

Doubt ⇒ why can't we use this for non unity f/b. as H(s) is already included & if H(s) is not to be used in non unity f/b & formula is valid for U.F.S. H(s) = 1 then why H(s) is included in formula ?

- Never use $e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$ in case of non-unity f/b system.

Ans: In actual $E(s) = R(s) - C(s)$ but in case of non unity f/b $K_H E(s) = R(s) - K_H C(s)$

video lecture by Shande sir, QR code scan.

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frequency response of 2nd Order System

① $T(s) = \dfrac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \dfrac{K}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)}$

$T(j\mu) = \dfrac{K}{1 - \mu^2 + 2j\xi\mu}$

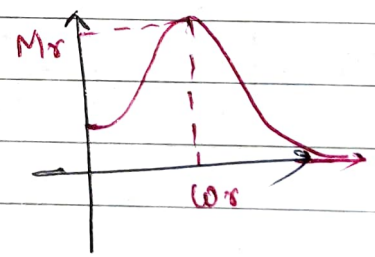
*** $\boxed{|T j\mu|_{max} = M_r = \text{resonant peak} = \dfrac{K}{2\xi\sqrt{1-\xi^2}}}$ $= K\operatorname{cosec} 2\theta$

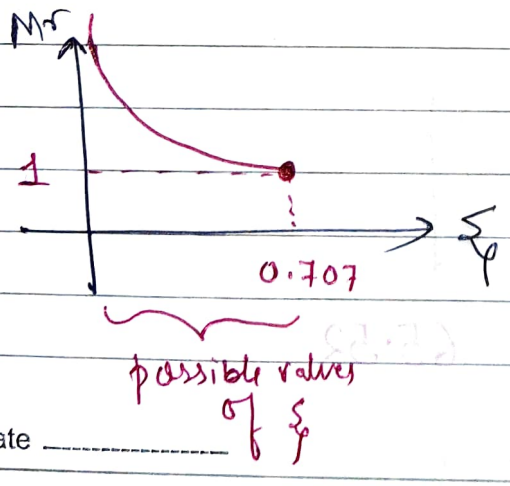
occurs at $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$ ⇐ resonant frequency

or $\boxed{\mu_r = \sqrt{1-2\xi^2} =}$ Normalised resonant frequency

② $\boxed{\xi_{max} = \dfrac{1}{\sqrt{2}} = 0.707}$ as $\xi > 0.707 \Rightarrow \omega_r =$ Imaginary

$\boxed{\omega_r \text{ min} = 0}$





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(3) ** Let $M_r = 2.5$, & $K=1$

$$2.5 = \frac{1}{\sin 2\theta} \Rightarrow 2\theta = \sin^{-1}(0.4) = (23.57^\circ) \text{ or } (180^\circ - 23.57^\circ)$$

Don't forget to calculate this

$$\Rightarrow \theta = 11.78 \text{ or } 78.215$$

$$\Rightarrow \xi = \cos\theta = \underset{\times\ (>0.707)}{0.978} \text{ or } \underset{\checkmark}{0.204}$$

$$\boxed{\xi = 0.204}$$

(4) 3-dB Bandwidth or simply Bandwidth.

$$\Rightarrow |T(j\omega)| = \frac{1}{\sqrt{2}} \text{ for } \omega_b$$

$$\boxed{\omega_b = \omega_n \sqrt{a + \sqrt{a^2+1}}} \quad \text{where } \boxed{a = (1 - 2\xi^2)}$$

** Bandwidth decreases the damping ratio.

(5) Relative stability parameters

** $$\boxed{\omega_{gc} = \omega_n \sqrt{-2\xi^2 + \sqrt{4\xi^4+1}}}$$

** $$\boxed{PM = \tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2 + \sqrt{4\xi^4+1}}}\right)}$$

$\xi_{max} = 0.707 \Rightarrow$ max P.M. $= 65.52$

** $$\boxed{\xi = 0.5\sqrt{\sin(PM) \times \tan(PM)}}$$

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Routh's stability criteria

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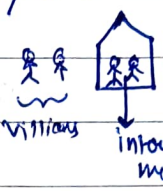
* ROZ occurs at only odd powers of s

***** IP = EP for value of Kmar. Marginally stable

IP > EP stable

IP < EP Unstable

{ Yad rakhne k liye



for safety Internal > Ext. members

Hence for stability IP > EP

* Key pt. Pg 5.58

eg: $a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0 s^0 = 0$

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = \frac{a_5 a_4 - a_3 a_6}{a_5}$	b_2	a_0	
s^3	c_1			
s^2	d_1			
s^1	e_1			
s^0	f_1			

0 0 0 0 (AE) ROZ

- No. of sign changes = No. of RH poles
- No. of Im axis poles = order(AE) − 2 × No. of sign changes below AE
- No. of LH pole = Total No. of poles − RH pole − Im axis poles

** ~~Rout~~ ~~Routh~~

[Imaginary axis poles ⇒ ROZ always
But ROZ doesn't always ⇒ Im axis poles]

Whenever ROZ occurs & if formula for Im axis poles not remembered then check roots of AE. → If roots are Imaginary then Im axis poles.

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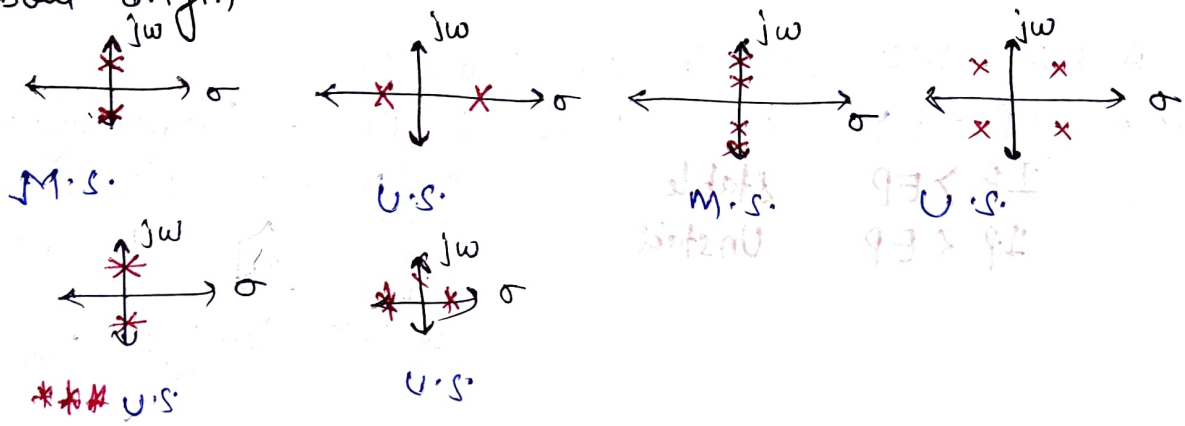
⊙ ROZ occurs only when the poles are located symmetrical about origin.



⊙ ROZ > 1 ⇒ Repeated poles (any where ~~on splane~~ Re or Im axis)

No. of repeated poles = No. of ROZ

⊙ $as^3 + bs^2 + cs + d = 0$

IP IP

IP > EP	Stable
IP < EP	Unstable
IP = EP	M. Stable

* This concept is valid only for 3rd order system.
* This concept is valid for hurwitz eqn only

$s^3 - 4s^2 + s + 6$

IP = −4

EP = 6

EP > IP

← Not valid ∵ non Hurwitz polynomial

⊙ There is no significance of ROZ in last Row

⊙ If only first element of Row = 0 & Not a ROZ condn then replace 0 by K & put K = 0 at last for checking sign change

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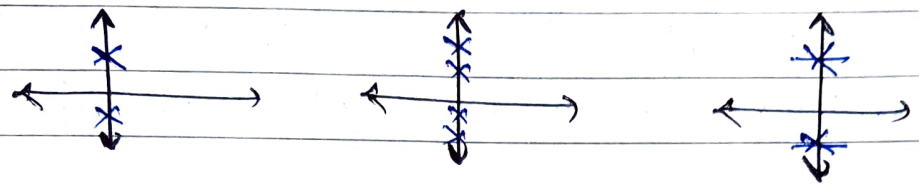
⊙ Symmetricity about origin is used ↓

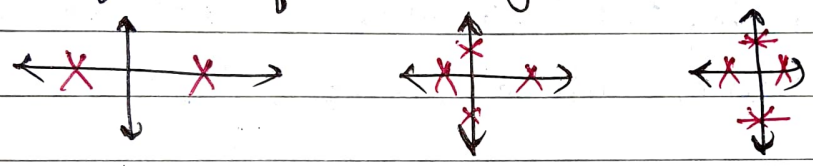
(i) If no. of sign changes below AE = 0 [⇒ NO RH pole found below AE]



O[A.E.] = 2 O[A.E.] = 4 O[A.E.] = 6

2 ROZ

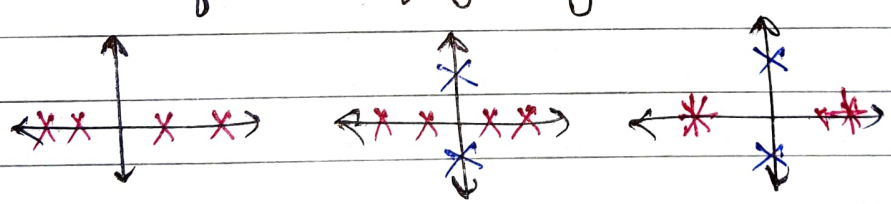
(ii) If no. of sign changes below AE = 1 [1 RH pole found below AE but symmetry ⇒ 1 pair]



⊙[A.E.] = 2 O[A.E.] = 4 O[A.E.] = 6

2 ROZ

(iii) If no. of sign changes below AE = 2 [2 RH pole found below AE but from symmetry ⇒ 2 pair]



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ROOT LOCUS

Poles of CLTF

- Valid for -ve f/b system
- Path of poles of CLTF for different values of system gain of OLTF Xfer function. $0 < K < \infty$
- In root locus diagram, arrow indicates, movement of poles with ↑ value of K

Rules for sketching root locus:

(i) Root locus is always symmetrical about real axis.

(ii) Root locus always start from open loop poles & terminate at either ∞ open loop zero (Virtual zero) or at finite zero of open loop

If $P > Z$ then RL ends at virtual zero

Poles of CLTF = Poles of OLTF [K=0]
= Zeroes of OLTF [K=∞]

(iii) RL exist on real axis only where where total odd no. of [Zeroes + poles] open loop to RHS of that pnt.

or

RL will exist only that section of real axis where total angle contribution by all open loop poles & zeroes towards that observation pt. is odd integral multiples of 180°.

(iv)

(v)

Shortcut

$\angle A = 90, 270$	$\angle A = 60, 180, 300$
diff $= 2\times 90$	diff: 2×60 2×60

← Just remember the starting $\angle A$

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(vi) for $P > Z$ $(B = P)$

$Z > P$ $(B = Z)$

$Z = P$ $(B = P = Z)$

(vii) for Break Pnt Calculation : $1 + G\,H(s) = 0 \Rightarrow K = f(s)$

Roots of $\frac{dK}{ds} = 0$

(viii) No. of Asymptotes : $A = \underset{(P>Z)}{P-Z} = \underset{(Z>P)}{Z-P}$

(ix) Angle of Asymptotes :

$P > Z$ $\angle A = \frac{(2\alpha+1)180^\circ}{P-Z}$ $\alpha = 0, 1, 2 \ldots, P-Z-1$

$P-Z=1 \Rightarrow \angle A = 180^\circ$

$P-Z=2 \Rightarrow \angle A = 90^\circ, 270^\circ$ $= \frac{(2\alpha+1)180^\circ}{Z-P}$ $\alpha = 0, 1, 2, \ldots Z-P-1$

$P-Z=3 \Rightarrow \angle A = 60, 180^\circ, 300$

$Z = P \Rightarrow$ No Asymptotes

(x) $\sigma = \frac{\Sigma Re(P) - \Sigma Re(Z)}{P - Z}$

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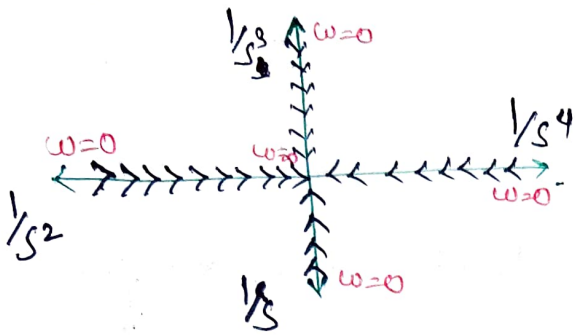
6. Polar plot ($\omega = 0$ to $\omega = \infty$)

- Relative stability of CLTF by using frequency response of OLTF



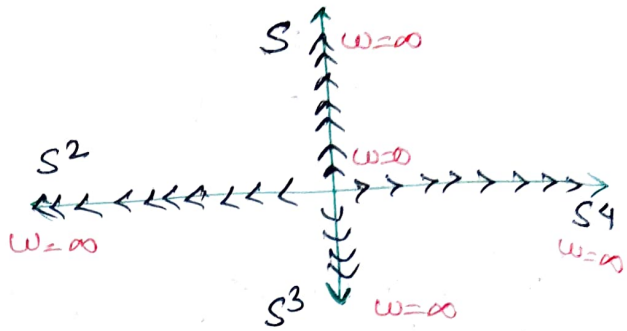
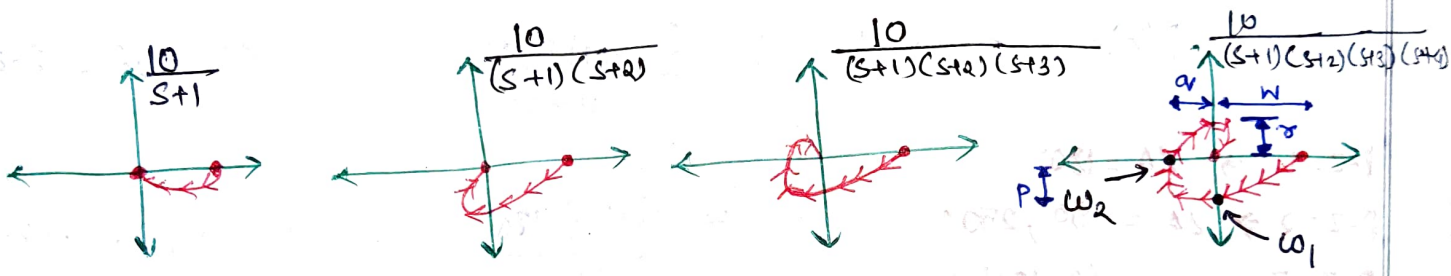


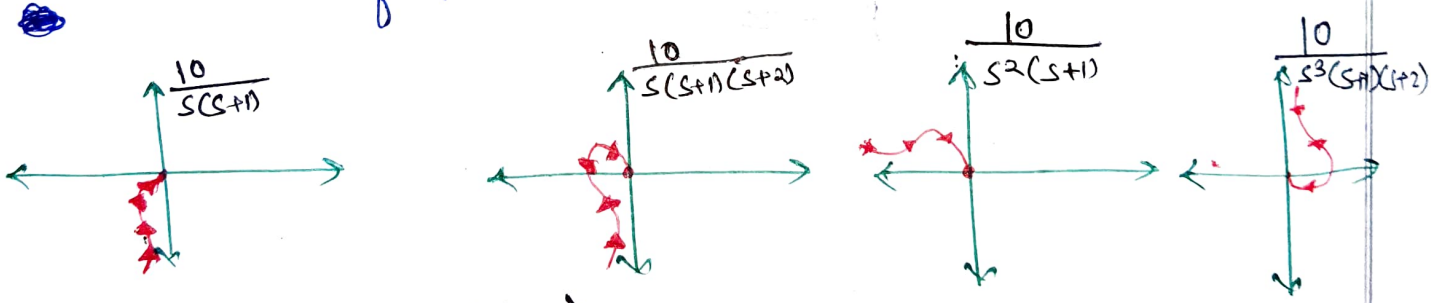
Table:

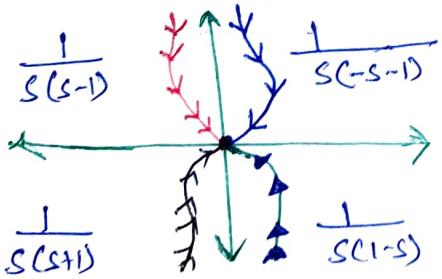
eg: $\frac{10}{s+1}$

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0	10.0	0°
1.0	7.07	−45°
∞	0.0	−90°



- Calculation of ω_1 : $Re[GH(s) = 0]$
- Calculation of ω_2 : $Im[GH(s) = 0]$
- Calculation of ϕ : $|GH(j\omega_1)|$





$\angle\left(\frac{1}{s^3(s+1)}\right)$

$-\omega^3(j\omega+1)$

$-\omega^3(-j\omega-1)$

- $s \to$ −ve hoga to +s wale Ka Im axis Ka mirror image
- Pole $\to$ −ve hoga to Real Ke about

eg: $(-s-1)$: $(s+1)$ Real, $(s-1)$ Im, $(-s-1)$

Relative stability Parameter:

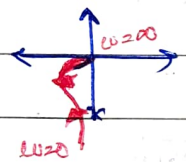
Sure shot Question.

<i> $\angle GH(j\omega_{pc}) = -180^\circ$ or $Im[GH(j\omega_{pc})] = 0$

<ii> $|GH(j\omega_{gc})| = 1$
$= 0\,dB$

<iii> $GM = \dfrac{1}{|GH(j\omega_{pc})|}$ (= +ve value) ** { GM dB can be negative }

<iv> $PM = 180^\circ + \angle GH(j\omega_{gc})$



- $\omega_{pc} = \infty$
- B/4 Calculating $Im[GH(j\omega_{pc})] = 0$. first check weather polar plot crosses the 180° line or not, otherwise it will give wrong answer.
- eg: $\dfrac{5}{s(s+1)} = \dfrac{5}{-\omega^2 + j\omega}$ $\boxed{\omega_{pc} = 0}$ (WRONG)

So, Calculate angle at $\omega = 0$ & $\omega = \infty$ b/4 solving for ω_{pc}.

- If ω_{gc} = real, if no real values comes then ω_{gc} doesn't exist.

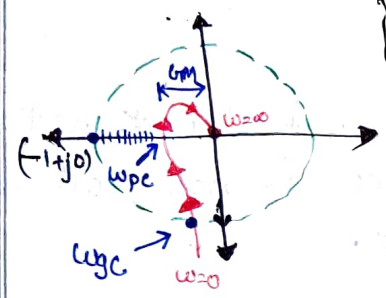
$GH(s)$	$\lvert GH(j\omega)\rvert_{\omega=0}$		$\lvert GH(j\omega=\infty)\rvert$	$\lvert GH(j\omega)\rvert$	ω_{gc}
$\dfrac{10}{(s+1)(s+2)}$	5.0	→	0.0	$0 < \lvert GH(j\omega)\rvert < 5$	exist
$\dfrac{1}{(s+1)(s+2)}$	0.5	→	0.0	$0 < \lvert GH(j\omega)\rvert < 0.5$ *	doesn't exist

* for concept of ω_{gc} we will calculate magnitude at $\omega = 0$ & $\omega = \infty$. and we will decide the calculation of ω_{gc}.

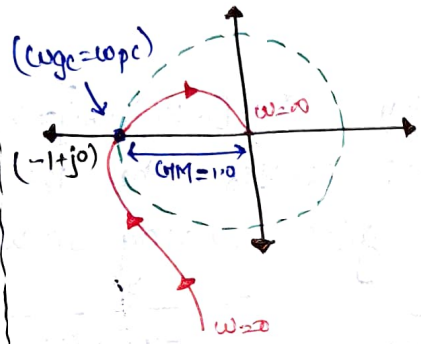
Stability concept

(i) GM > 0 dB & PM > 0°	Stable
(ii) GM = 0 dB & PM = 0°	Marginally stable
(iii) GM < 0 dB & PM < 0°	Unstable

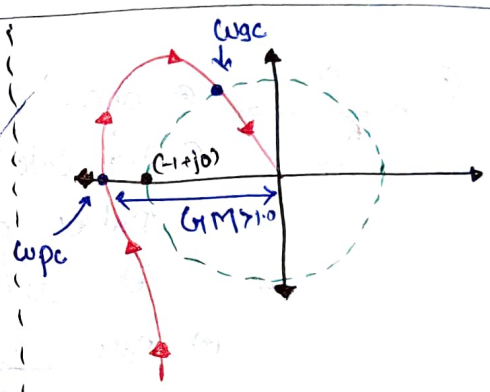
- $\omega_{gc} < \omega_{pc}$ → CLTF (or system) will be stable
- $\omega_{gc} > \omega_{pc}$ → System will be unstable
- $\omega_{gc} = \omega_{pc}$ → System will be marginally unstable



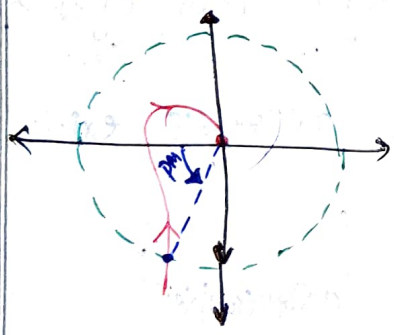
Stable



Marginally stable

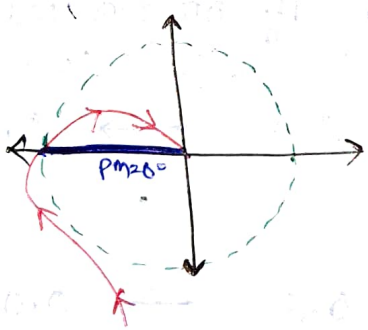


Unstable



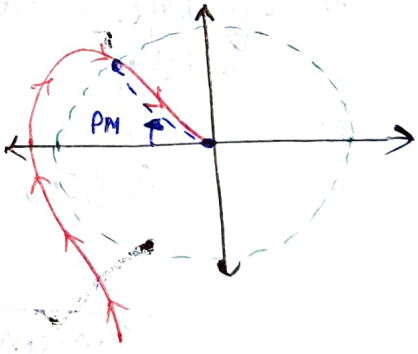
PM > 0

Stable



PM = 0

Marginally stable



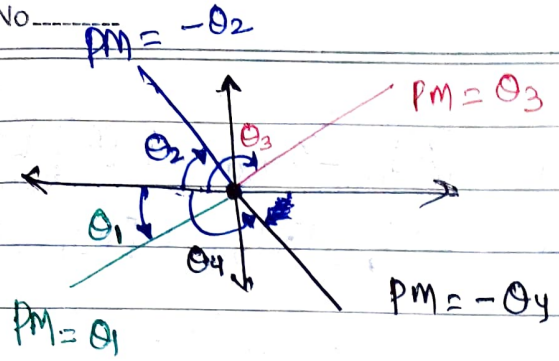
PM < 0

Unstable

from fig.

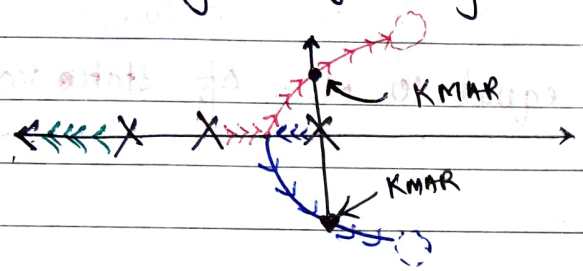
$\angle GH(j\omega) = -180° + PM$

$$\boxed{PM = 180° + \angle GH(j\omega)}$$



- ACW → θ = +ve PM = θ
  CW → θ = −ve

Gain Margin by using root locus :



- KMAR from root locus
- for order − 3 GH(s)
  IP = EP for Kmar
- Kdesired = K ← $GH(s) = \frac{K}{(s+P_1)(s+P_2)}$

$$\boxed{GM = \frac{K_{MAR}}{K_{desired}}}$$

- $|GH(s)| = |GH(s)| \times GM$
  $K_{MAR} = K \times GM$

State space Analysis

$\dot{X} = AX + BU$ $\qquad [\dot{X}]_{n\times1} = [A]_{n\times n}[X]_{n\times1} + [B]_{n\times m}[U]_{m\times1}$

$Y = CX + DU$ $\qquad [Y]_{p\times1} = [C]_{p\times n}[X]_{n\times1} + [D]_{p\times m}[U]_{m\times1}$

* n: No. of state variables = No. of effective storing elements
= No. of initial values
= order of T.F. = order of D.E.
= No. of time constants.

* No. of Ys terms in SFG is equal to No. of states variables.

* eg: $\dfrac{Y(s)}{U(s)} = \dfrac{10}{s^3 - 4s^2 + 2s - 6} = T(s)$

No. of state variables = 3

$s^3 Y(s) - 4s^2 Y(s) + 2sY(s) - 6Y(s) = 10U(s)$

** L^{-1}: $\underbrace{\dfrac{d^3[y(t)]}{dt^3}}_{\dot{x}_3} - 4\underbrace{\dfrac{d^2[y(t)]}{dt^2}}_{x_3 = \dot{x}_2} + 2\underbrace{\dfrac{d[y(t)]}{dt}}_{x_2 = \dot{x}_1} - 6\underbrace{y(t)}_{x_1} = 10u(t)$

$\dot{x}_3 = 4\dot{x}_3 - 2x_2 + 6x_1 + 10u$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$y = x_1$

$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -2 & 4 \end{bmatrix}}_{[A]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}}_{[B]} [u]$

** If N^r contains only constant term, then shortcut

$\dfrac{Y(s)}{U(s)} = \dfrac{10}{s^3 - 4s^2 + 2s - 6}$ $\longrightarrow$ Last row with sign change & order change (asc./des.)

$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -2 & 4 \end{bmatrix}$ element above diagonal is always 1 except this last row is non zero & rest all terms are zero.

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~~Recovery~~ Recovery of T.F. from state variable representation

** $\dot{X} = AX + BU$ $\qquad Y = CX + DU$

$sX(s) = AX(s) + BU(s)$

$X(s) = (sI - A)^{-1} B\, U(s) \Rightarrow \boxed{\dfrac{Y(s)}{U(s)} = [C][(sI-A)^{-1} B] + D}$

for continous:

for discrete:

** $\boxed{\dfrac{Y(z)}{U(z)} = T(z) = C\left[(zI - A)^{-1} B\right] + D}$

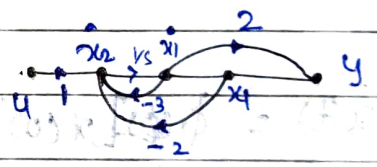
** Shortcut 2: {only valid for proper fraction}

$T(s) = \dfrac{2s+7}{s^2+3s+2}$ $\qquad Y = 2s+7$ $\qquad y = 7x_1 + 2x_2$

$U = s^2+3s+2$ $\qquad u = 2x_1 + 3x_2 + \dot{x}_2$

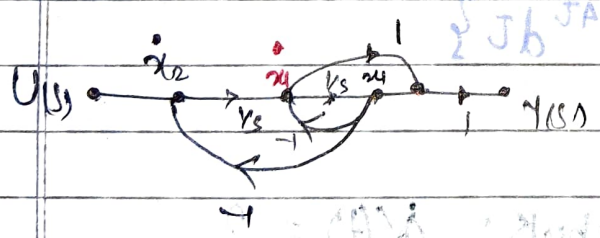
$\dot{x}_1 = x_2$

** $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$



** $[y] = [7 \quad 2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

** Note:



$\dot{x}_1 \neq x_2$

$\dot{x}_1 = x_2 - x_1$

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Note :- $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad e^{At} \neq \begin{bmatrix} 1 & e^t \\ e^t & 1 \end{bmatrix}$ $\quad e^{At} = L^{-1}\left\{\begin{bmatrix} s & -1 \\ -1 & s \end{bmatrix}^{-1}\right\}$

Solution of state space equation

$$\dot{X} = AX + BU \qquad Y = CX + DU$$

Case (i) $U(i/p) = 0$

$$\underset{n\times 1}{[X(s)]} = \underset{n\times n}{[sI - A]^{-1}} \underset{n\times 1}{[x(0)]}$$

$[\phi(s)]_{n\times n} = \underset{n\times n}{[sI - A]^{-1}}$ = Resolvent Matrix

State transition matrix

- ZIR $= f[x(0)]$
- $\phi(s) = [sI - A]^{-1} = \dfrac{1}{s-A} \Rightarrow [\phi(t) = e^{At}] = STM$

** Apply this to eliminate options for $\phi(t) = ?$

- Properties of STM $\{\phi(t)\}$

**(i) $\phi(0) = I$ **(ii) $\phi'(0) = A$ (iii) $\phi^K(t) = \phi(Kt)$ (iv) $\phi^{-1}(t) = \phi(-t)$

(v) $\phi(t_1 - t_0) \cdot \phi(t_2 - t_2) = \phi(t_1 - t_2)$

Case (ii) $U \neq 0$ {Non Homogenous equation}

$$\dot{X} = AX + BU \Rightarrow sX(s) - x(0) = AX(s) + BU(s)$$

$$\boxed{\underset{\text{State eq}^n}{X(s)} = \underset{\text{ZIR}}{\phi(s)\, x(0)} + \underset{\text{ZSR}}{\phi(s)\, B\, U(s)}}$$

$$x(t) = \phi(t)[x(0)] + \phi(t) \otimes B\,U(t)$$

$$x(t) = e^{At}\, x(0) + \int_0^{\infty} B\,U(\tau)\, e^{A(t-\tau)}\, d\tau$$

or, $$x(t) = e^{At}\left\{x(0) + \int_0^{t} B\,U(\tau)\, e^{-A\tau}\, d\tau\right\}$$

* Calculation of ZIR :

$$x(t)_{ZIR} = \underset{2\times 2}{[\phi(t)]}\, \underset{2\times 1}{[x(0)]}$$

$$\phi(t) = L^{-1}\left\{(sI - A)^{-1}\right\}$$

To check : $\phi'(t) = ?$

$\phi'(0) = ?$

$\phi'(0)$ must be equal to A

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** Calculation of ZSR

$$[X(s)]_{2\times1} = [\phi(s)]_{2\times2} [B] [U(s)]_{1\times1}$$

$$x(t)|_{ZSR} = L^{-1}[X(s)]$$

** $x(t) = [x(t)]_{ZIR} + [x(t)]_{ZSR}$

Calculate ZSR & put $t=0$

$x(t)|_{t=0} = ZIR + ZSR = ZIR$ { satisfy initial value.

Controllability: { relationship b/w i/p & SV }

SFG: ⊙ If there is path from i/p to all S.V. the controllable

Kalman's test: $Q_c = [B : AB : A^2B \dots A^{n-1}B]_{n\times n}$

If $|Q_c| \neq 0 \Rightarrow$ Controllable

$|Q_c| = 0 \Rightarrow$ uncontrollable

Observability:

Kalman's test: $Q_o = [C^T : A^TC^T : A^{2T}C^T : \dots A^{(n-1)T}C^T]_{n\times n}$

$|Q_o| \neq 0 \Rightarrow$ system is observable

$|Q_o| = 0 \Rightarrow$ system is unobservable

** If T.F. doesn't involve any pole zero cancellation then system is both controllable & observable

In case of PZ cancelation ⇒ may be uncontrollable / unobservable / both.

eg: $T(s) = \dfrac{s+2}{s^3+9s^2+26s+24} = \dfrac{\cancel{(s+2)}}{\cancel{(s+2)}(s^2+7s+12)}$

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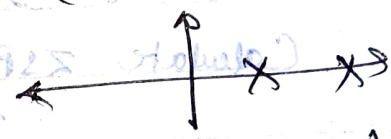
Types of decomposition : (later)

Note : $|SI - A| = 0$ is C/S eqn which decides stability of system.

Foreg: $A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$ $\quad \begin{vmatrix} s-2 & -3 \\ 0 & s-5 \end{vmatrix} = 0$ $\quad \cancel{s^2 - 7s + 7 = 0}$

$\Rightarrow (s-2)(s-5) = 0$



unstable

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Nyquist Plot

- Stability of CLTF from openloop frequency response & open loop poles.
- Zeroes [CE] = Poles [CLTF]
  Poles [CE] = Zeroes [OLTF]

- $N = P - Z$
  P: RH poles of CE = RH poles of OLTF
  Z: RH ~~pot~~ zeroes of CE = RH poles of CLTF
  N: No. of encirclement about $(-1+j0)$ critical pnt. in CCW direction

- By default, Nyquist contour = CW direction, N in CCW dir^n
  If Nyquist plot is CCW dir^n, $N = P - Z$ in CW direction.

- <u>Objective Approach:</u>

$G(s) = \frac{10}{s(s+1)}$

(i) $P = 0$ {No. of RH pole of OLTF}

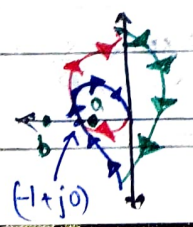
(ii) $1 + G(s) = 0 \Rightarrow s^2 + s + 10 = 0$

s^2	1	10
s	1	0
s^0	10	0

} RH pole of CLTF, $Z = 0$

(iii) $N = P - Z = 0$

- Nyquist plot is always symmetrical about real axis.
- $N > 0 \Rightarrow$ CCW encirclement } for CW contour
  $N < 0 \Rightarrow$ CW encirclement

- 

$\omega_{pc} = ?$
$\Rightarrow |GH(j\omega_{pc})| = \frac{2K}{3}$ (say)

Pnt. (a) $N = -2$, $P = 0$, $Z = 2 \Rightarrow$ } unstable for $\left[\frac{2K}{3}\right] > 1$ or $K > \frac{3}{2}$

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Bode Plot

- Only for minimum phase system {i.e. All P & Z on LHS}
- Corner frequency represent the frequency at which error will be maximum.

- $T(s) = \frac{(1.0)(s+10)}{(s+100)^2}$ $\qquad K \neq 1.0$

$= \frac{10}{10^4}\frac{(1+s/10)}{(1+\frac{s}{100})^2}$ $\qquad K = 10^{-3}$

- $K=1$, zero at $s=10$, 2 poles at $s=100$

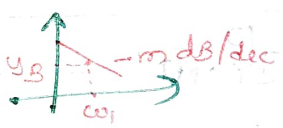
$\boxed{T(s) = \frac{1.0(s+10)}{(s+100)^2}}$ (WRONG)

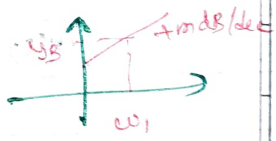
$\boxed{T(s) = 1 \cdot \frac{(1+s/10)}{(1+s/100)^2}}$

- If low frequency region or initial region consists of slope then $\boxed{20\log K = |GH(j\omega)|_{\omega=1\,rad/s}}$ *

- For initial slope {Low freq. region}

$\boxed{y_B = 20\log K \pm m\log\omega_1}$





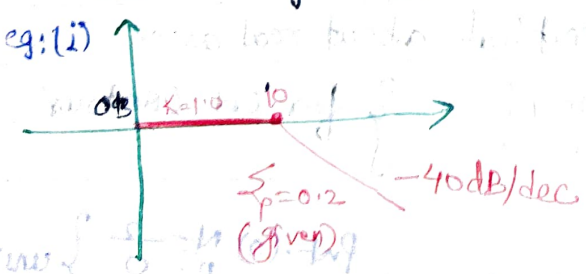
- 'K' is always calculated from low frequency region.

Bode plot for 2nd order System

$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

(i) Underdamped ($\zeta < 1$)

eg: (i)



$T(s) = \frac{1}{1+2\zeta\left(\frac{s}{\omega_n}\right)+\left(\frac{s}{\omega_n}\right)^2}$

$T(s) = \frac{1}{1+2\times0.2\times\frac{s}{10}+\frac{s^2}{100}}$

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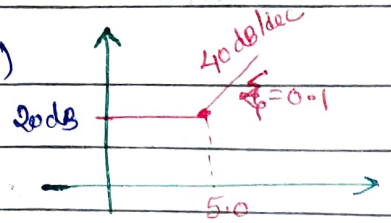
(ii)



$$G(s) = 10\left[1 + 2\times 0.1\times\left(\frac{s}{5}\right) + \left(\frac{s}{5}\right)^2\right]$$

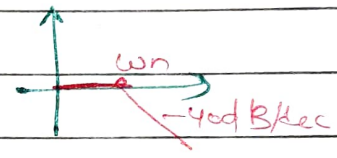
- Slope must be multiples of 40 dB/dec in 2nd order system. {except over damped}

My point → • initial slope = 0dB/dec for Under damped, always.

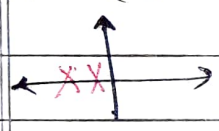
- $\omega_n = \omega_c$ = corner frequency.

(ii) ($\xi_p = 1$) C.D.

$$T(s) = \frac{1}{1 + \frac{2s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2} = \frac{1}{\left(1 + \frac{s}{\omega_n}\right)^2}$$



(iii) ($\xi_p > 1$) O.D.



$$T(s) = \frac{\omega_n^2}{\left(s + \xi_p\omega_n + \omega_n\sqrt{\xi_p^2 - 1}\right)\left(s + \xi_p\omega_n - \omega_n\sqrt{\xi_p^2 - 1}\right)}$$

$$T(s) = \frac{1}{\left(1 + \frac{s}{\xi_p\omega_n - \omega_n\sqrt{\xi_p^2 - 1}}\right)\left(1 + \frac{s}{\xi_p\omega_n + \omega_n\sqrt{\xi_p^2 - 1}}\right)}$$

* In bode plot when repeated poles ⇒ C.D. $\xi_p = 1$

" " " " unequal poles ⇒ O.D. $\xi_p > 1$

we only define ξ_p for U.D. ⇒ $\xi_p < 1$

** Bode plot is not defined for undamped ($\xi_p = 0$) since High frequency region must have constant slope & undamped system has varying slope always.

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$$\text{Error (dB)} = \text{Exact}\Big|_{u=1}^{\text{dB}} - \text{Approximate}\Big|_{u=1}^{\text{dB}}$$

$$= -20 \log 2\zeta_p$$

⊙ To remember, lag me pole origin ke pas hota h, so α is small
i.e. $(\xi\omega_n)$ is small ⇒ $\xi\downarrow$ $\omega_n\downarrow$ } ⇒ $t_{sett} = 4/\xi\omega_n$ ⇒ $t_r\uparrow$
$\omega_{gc}, BW\downarrow$; $\uparrow MPO = e^{-\pi\cot\theta}$ ⇒ stability $\downarrow$, $e_{ss}\downarrow$

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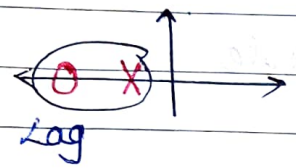
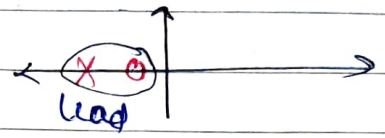
Compensator

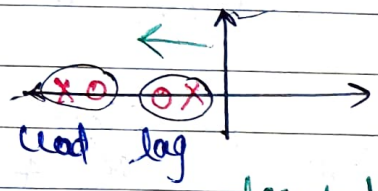
(i)



(ii)

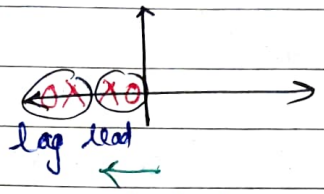


(iii)



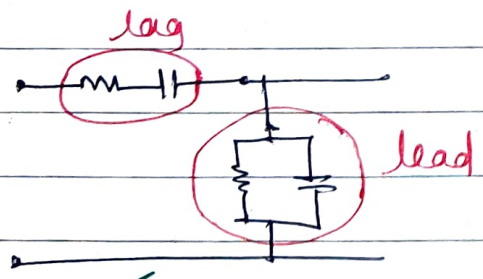
lag lead comp^r

(iv)

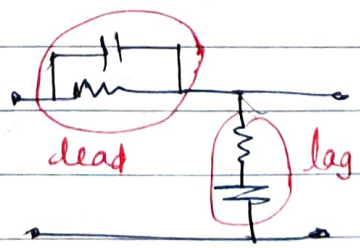


lead lag comp^r

lag compensator	lead compensator
⊙ Damping ratio $\downarrow$	⊙ Damping ratio $\uparrow$
⊙ ω_n decreases	⊙ ω_n increases
⊙ ω_{gc} decreases	⊙ ω_{gc} increases
⊙ BW decreases	⊙ BW increases
⊙ $t_{sett}\uparrow = \frac{4}{\xi\omega_n}$	⊙ $\downarrow t_{sett} = \frac{4}{\xi\omega_n}$
⊙ slow time response	⊙ fast time response
⊙ $t_r\uparrow$	⊙ $t_r\downarrow$
⊙ MPO $\uparrow$	⊙ MPO $\downarrow$
⊙ stability $\downarrow$	⊙ stability $\uparrow$
⊙ Includes steady state performance (decreases e_{ss})	⊙ Include transient state performance (decreases time domain para)



lead lag comp^r



lag lead comp^r

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