Control System

A control system is a combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired o/p. This cause and effect relationship is governed by a mathematical relation.

If the aforesaid mathematical relation is linear the control system is termed as linear control system.

OR

Control system is an interconnection of components forming a system configuration that will provide a desired response.

Based on control action

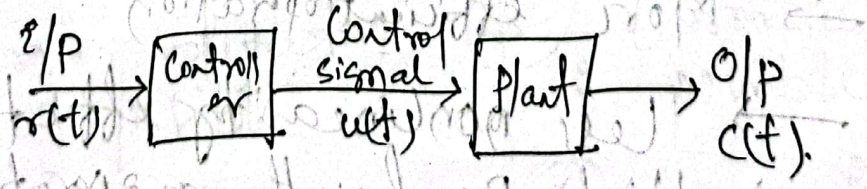
Control system is classified into two types

(i) Open loop control system

(ii) Close loop control system

1 Open Loop Control System

Its control action only depends on i/p signal and doesnot depend on its o/p response.



Ex traffic signal, washing machine Bread toaster etc

Adv → Simple design & easy to construct
→ Economical
→ Easy for maintenance
→ Highly stable operation.

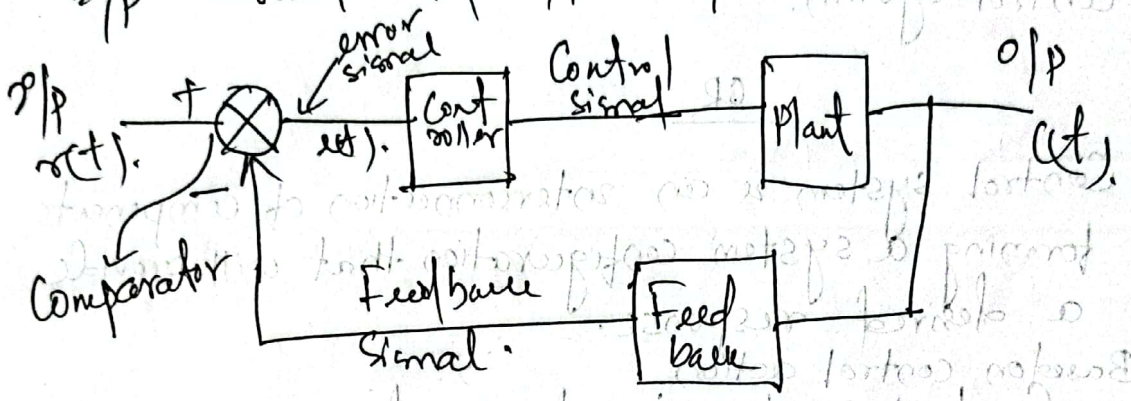
Dis Adv

→ Not accurate and reliable

→ Recalibration of the parameters are required.

Close loop control system

Its control action depends on both of its i/p signal and o/p response



Ex → automatic electric iron

→ Automatic washing machine

→ Automatic air conditioner

→ Missile launcher.

Adv → More accurate

→ More efficient operation

→ Less nonlinearity effect

→ High Bandwidth of operation

→ Automation facility

→ Time to time recalibration of the parameters are not required.

Dis Adv

→ complex design & difficult to construct
→ Expensive
→ Complicate for maintenance
→ less stable operation than O.

Comparision

O Control system	C: Loop control system
(1) No feedback	(1) Feedback is present
(2) Not intelligent	(2) Intelligent
(3) No possiblity of undesirable system oscillation	(3) Introduce the undesirred system oscillation.
(4) O/p not ~~depend~~ vary with i/p.	(4) O/p may vary with constant i/p.
(5) Error detection not possible	(5) Error detection is possible
(6) Small BW.	(6) Large BW.
(7) More stable	(7) less stable
(8) Less accurate	(8) More accurate o/p.
(9) Affected by non-linearity	(9) Not affected by nonlinearity
(10) Very sensitive to disturbance	(10) Less sensitive to disturbance
(11) Simple design	

Laplace Transform

It helps to transform a time domain differential eq into a frequy domain algebric equation.

$$L f(t) = F(s).$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$$

$\delta(t) \longrightarrow 1$

$u(t) \rightarrow \frac{1}{s}$

$t \rightarrow 1/s^2$

$\frac{t^2}{2} \rightarrow 1/s^3$

$t^n \rightarrow \frac{n!}{s^{n+1}}$

$e^{-at} \rightarrow \frac{1}{s+a}$

$e^{at} = \frac{1}{s-a}$

$t e^{-at} \rightarrow \frac{1}{(s+a)^2}$

$t e^{at} \rightarrow \frac{1}{(s-a)^2}$

$t^n e^{-at} \rightarrow \frac{n!}{(s+a)^{n+1}}$

$\sin\omega t \rightarrow \frac{\omega}{s^2+\omega^2}$

$\cos\omega t \rightarrow \frac{s}{s^2+\omega^2}$

$e^{-\alpha t} \sin\omega t \rightarrow \frac{\omega}{(s+\alpha)^2+\omega^2}$

$e^{-\alpha t} \cos\omega t \rightarrow \frac{s+a}{(s+a)^2+\omega^2}$

$\frac{d}{dt} \rightarrow s$

$\frac{d^2}{dt^2} \rightarrow s^2$

$\frac{d^n}{dt^n} \rightarrow s^n$

$\int \rightarrow \frac{1}{s}$

$\delta(t-a) = e^{-as}$

$u(t-1) = \frac{e^{-as}}{s}$

Initial value theorm :-

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s \cdot F(s)$$

Final value theorm

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s \cdot F(s).$$

* Initial value theorm is defined for both stable & unstable system.
Final value theorm is defined only for absolutely stable system.

Ex Inverse laplace transform

$$F(s) = \frac{1}{s(s+1)}$$

$$= \frac{A}{s} + \frac{B}{s+1}$$

$$= \frac{1}{s} - \frac{1}{s+1}.$$

$$L^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$f(t) = 1 - e^{-t}$$

$$1 = A(s+1) + Bs$$
$$= As + A + Bs$$
$$1 = (A+B)s + A$$
$$\Rightarrow A = 1.$$
$$B = -1.$$

Ex $$\frac{s+6}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = 2,\ B = -2.5,\ C = 1/2$$

$$F(s) = \frac{2}{s} - \frac{2.5}{s+1} + \frac{1}{2} \cdot \frac{1}{s+3}$$

$$f(t) = L^{-1} F(s) = \left(2 - 2.5 e^{-t} + \frac{1}{2} e^{-3t}\right)$$

CamScanner

Ex $\dfrac{1}{s^2+4s+8}$

as^2+bs+c

$s = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$-4 \pm \sqrt{16}$

$$s^2+4s+8 = \frac{s^2+4s+4+4}{(s+2)^2+(2)^2}$$

$$= \frac{1}{(s+2)^2+2^2}$$

$$f(t) = L^{-1}\left\{\frac{1}{2}\cdot\frac{2}{(s+2)^2+2^2}\right\}$$

$$= \frac{1}{2}e^{-2t}\sin(2t) \quad \text{Ans}$$

Ex $\dfrac{5}{s(s^2+4s+5)}$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+4s+5}$$

$A = 1,\ B = -1,\ C = -4.$

$$F(s) = \frac{1}{s} - \frac{s+4}{s^2+4s+5}$$

$$= \frac{1}{s} - \frac{s+4}{(s+2)^2+1^2}$$

$$= \frac{1}{s} - \frac{(s+2)}{(s+2)^2+1^2} - \frac{2}{(s+2)^2+1^2}$$

$$= \frac{1}{s} - \frac{s+2}{(s+2)^2+1^2} - 2\cdot\frac{1}{(s+2)^2+1}$$

$$= 1 - e^{-2t}\cos t - 2e^{-2t}\sin t \quad \text{Ans}$$

Q A system is represented by a relation below $X(s) = R(s) \cdot \frac{100}{s^2+2s+50}$

If $r(t) = 1$ unit, find the value of $x(t)$ when $t \to \infty$.

$r(t) = 1$, $R(s) = \frac{1}{s}$.

$$X(s) = \frac{1}{s} \cdot \frac{100}{s^2+2s+50}$$

According to the final value theorem

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s \cdot X(s).$$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{100}{s^2+2s+50}.$$

$$= 2 \text{ unit}$$ Ans

Q The L.T of the error signal in a control system is expressed by a relation

$$E(s) = R(s) \cdot \frac{s}{s^2+6s+25}$$

Calculate the steady state value of error $r(t) = t$

$r(t) = t$, $R(s) = \frac{1}{s^2}$

$$E(s) = \frac{1}{s^2} \cdot \frac{s}{s^2+6s+25}$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{s}{s^2+6s+25}$$

$$= \frac{1}{25}$$ Ans

Transfer function

Mathematical model of a physical system.

$$T/F = \frac{L(o/p)}{L(i/p)} \Bigg|_{\text{initial condition}=0}$$

Why initial condition is zero.

* T.F. is only valid for LTI system

$$\text{Transfer function} = G(s) = \frac{C(s)}{R(s)}$$

Pole and Zero of a Transfer function

$$G(s) = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_m}$$

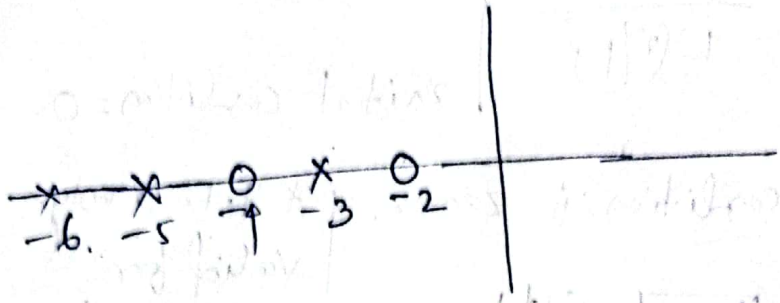
$$= \frac{K(s-s_1)(s-s_2)\cdots(s-s_n)}{(s-s_a)(s-s_b)\cdots(s-s_m)}$$

Pole If denominator will be equal to zero the roots of s is known as pole.

* Pole of the T/F is the freq for which the value of the denominator of a T/F become zero. $s_a, s_b, \ldots s_m$

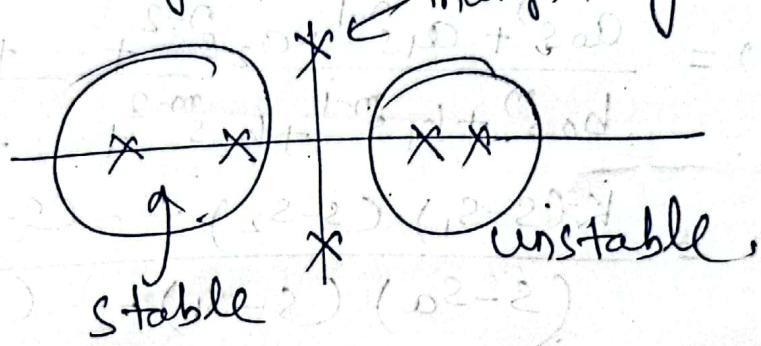
Zero If Numarator is made equal to zero. Then root of s is known as zero.

Ex $\dfrac{(s+2)(s+4)}{(s+3)(s+5)(s+6)}$.

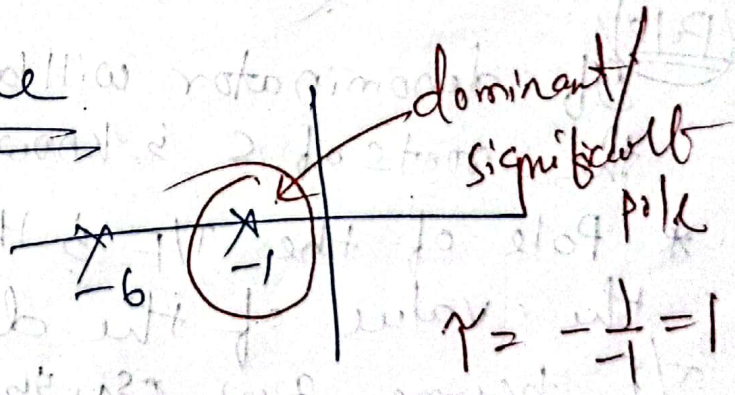


* Time constant $= -\dfrac{1}{\text{Real part of pole}}$

* Time constant is only defined for Stable system.



* Dominant pole



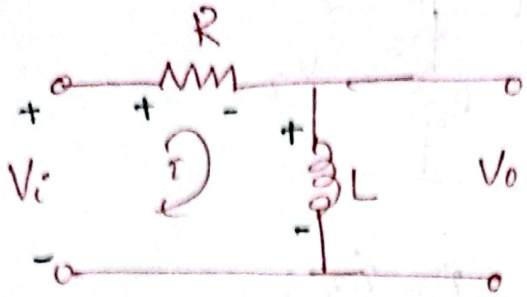
$\tau = -\dfrac{1}{-1} = 1$

* $P \geq Z \rightarrow$ Proper T/F

$P < Z \rightarrow$ Improper T/F.

How to calculate T/F

(1)



i/p loop

$$-V_i + Ri + L\frac{di}{dt} = 0$$

$$V_i = Ri + L\frac{di}{dt}$$

$$V_i(s) = RI(s) + sLI(s) \quad \text{--- (1)}$$

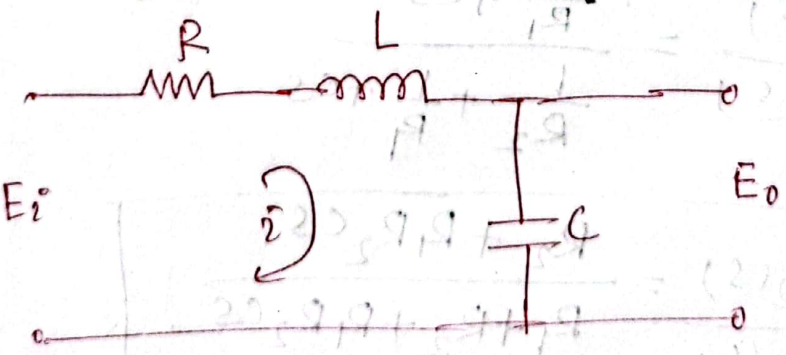
o/p loop

$$V_o(s) = Ls\,I(s). \quad \text{--- (2)}$$

$$T/F = \frac{V_o(s)}{V_i(s)} = \frac{Ls\,I(s)}{RI(s) + sLI(s)}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{sL}{R+sL}}$$

(2)

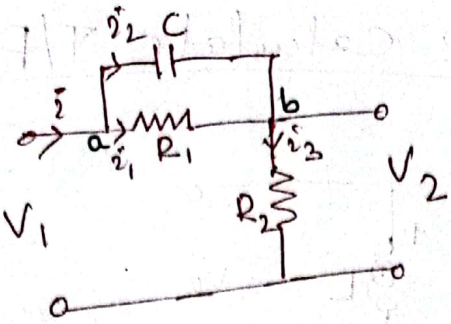


$$E_i(s) = RI(s) + sLI(s) + \frac{1}{Cs}I(s).$$

$$E_o(s) = \frac{1}{Cs}I(s).$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1/Cs}{R + sL + \frac{1}{Cs}} = \frac{1}{1 + sRC + s^2LC}$$

(3)



KCL at 'a'

$i = i_1 + i_2$ $\qquad i = i_3$.

$\cancel{i_1 = \frac{V_1 - V_2}{R_1}}$,

$i_3 = i_1 + i_2$

$$\frac{V_2(s)}{R_2} = \frac{V_1(s) - V_2(s)}{R_1} + \frac{V_1(s) - V_2(s)}{1/cs}$$

$$\frac{V_2(s)}{R_2} = \frac{V_1(s)}{R_1} - \frac{V_2(s)}{R_1} + cs\,V_1(s) - cs\,V_2(s)$$

$$\Rightarrow \frac{V_2(s)}{R_2} + \frac{V_2(s)}{R_1} + cs\,V_2(s) = \frac{V_1(s)}{R_1} + cs\,V_1(s)$$

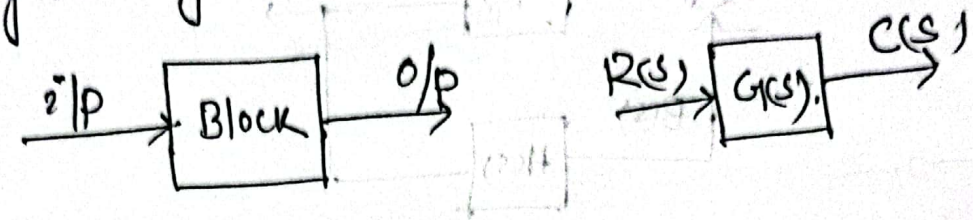
$$\Rightarrow V_2(s)\left(\frac{1}{R_2} + \frac{1}{R_1} + cs\right) = V_1(s)\left(\frac{1}{R_1} + cs\right)$$

$$\Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{R_1} + cs}{\frac{1}{R_2} + \frac{1}{R_1} + cs}$$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{R_2 + R_1R_2cs}{R_1 + R_2 + R_1R_2cs}.}$$

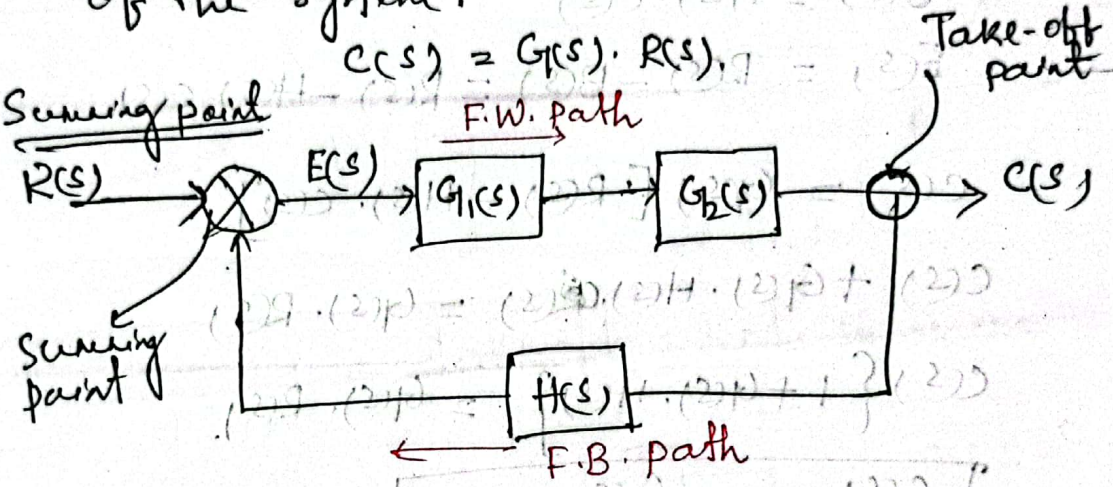
(4) Block diagram Representation

It is the pictorial representation of the cause-and-response relationship bet i/p and o/p of a physical system.



o/p - The value of i/p multiplied by the gain of the system.

$$C(s) = G(s) \cdot R(s).$$



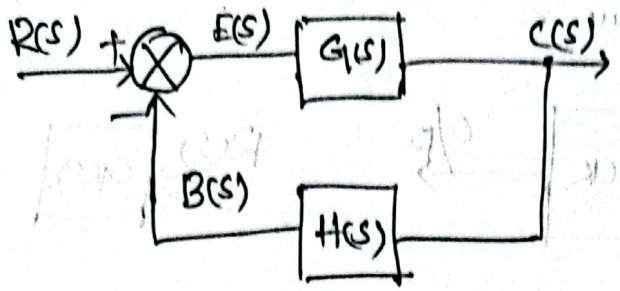
Take-off point - It is the component of a blockdiagram model at which a signal can be taken directly and supplied to one or more points as shown in fig above

Forward path - It is the direction of signal flow from i/p towards o/p.

Feedback path - It is the direction of signal flow from output towards i/p.

~~Rules for Reduction of Block diagram~~

Transfer Function of CL system



$$C(s) = G(s) \cdot E(s)$$

$$B(s) = H(s) \cdot C(s)$$

$$E(s) = R(s) - B(s) = R(s) - H(s) \cdot C(s)$$

$$C(s) = G(s)\left[R(s) - H(s) \cdot C(s)\right]$$

$$C(s) + G(s) \cdot H(s) \cdot C(s) = G(s) \cdot R(s)$$

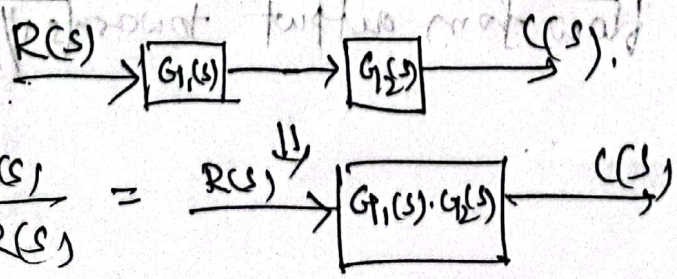
$$C(s)\{1 + G(s) \cdot H(s)\} = G(s) \cdot R(s)$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}}$$

Block diagram Reduction :-

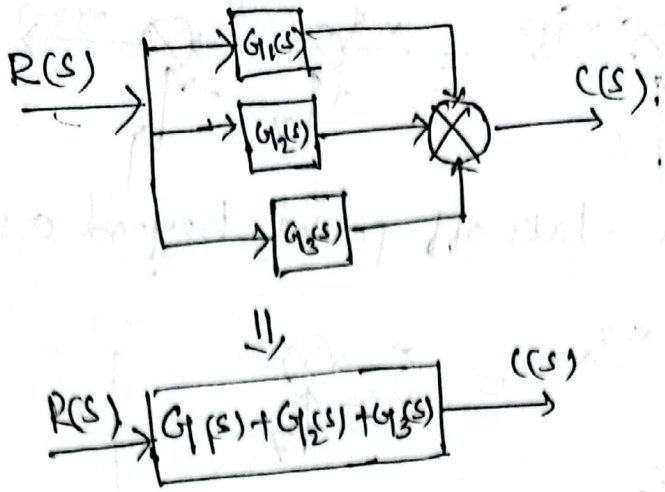
When a number of blocks are connected, the overal transfer function can be obtained by block diagram reduction technique

Rule-1 Block in cascade

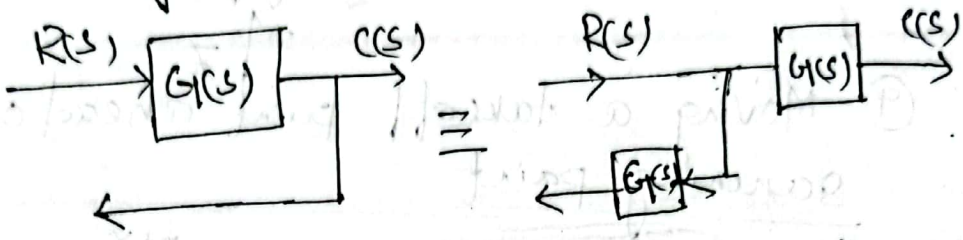


$$\frac{C(s)}{R(s)} =$$

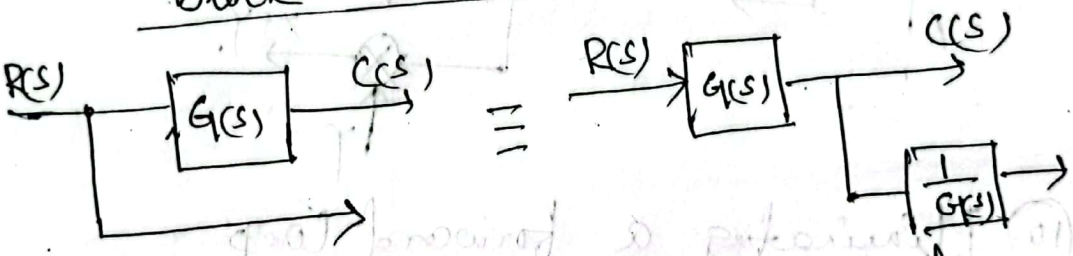
② Blocks in parallel



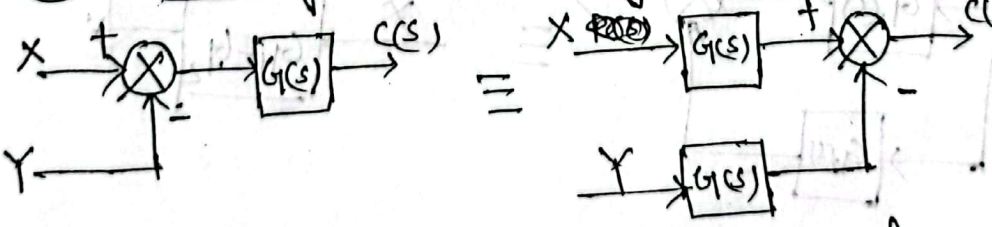
③ Moving a takeoff point ahead of block



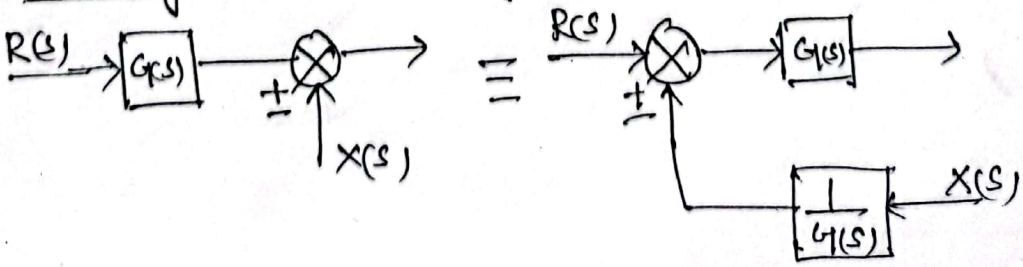
④ Moving a takeoff point after the block



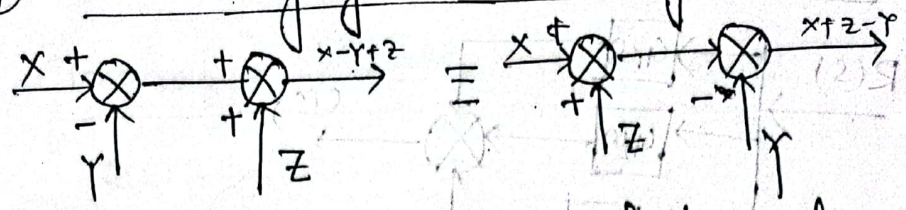
⑤ Moving a summing point beyond a block



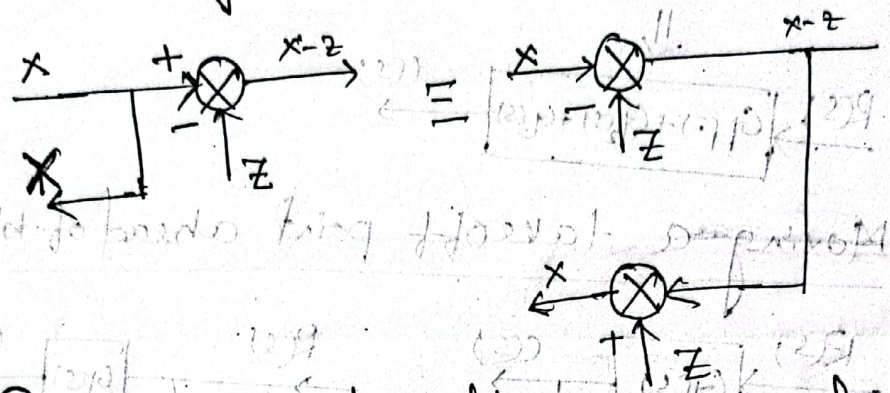
⑥ Moving a summing point ahead of a block



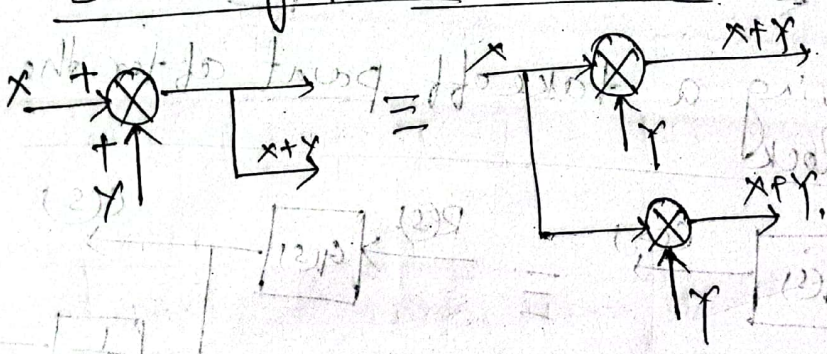
⑦ Interchanging two summing points



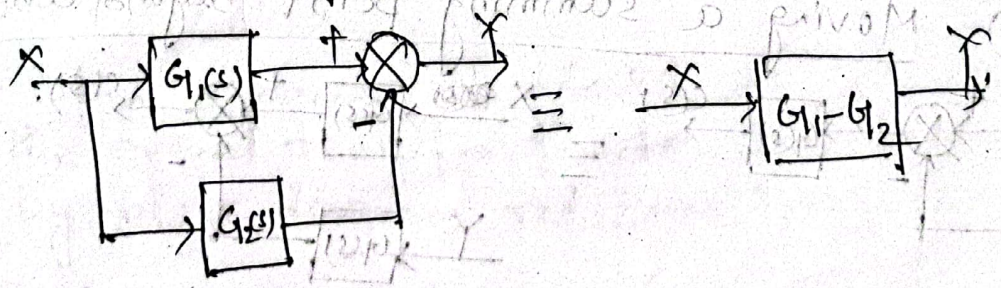
⑧ Moving a take off point beyond a summing point



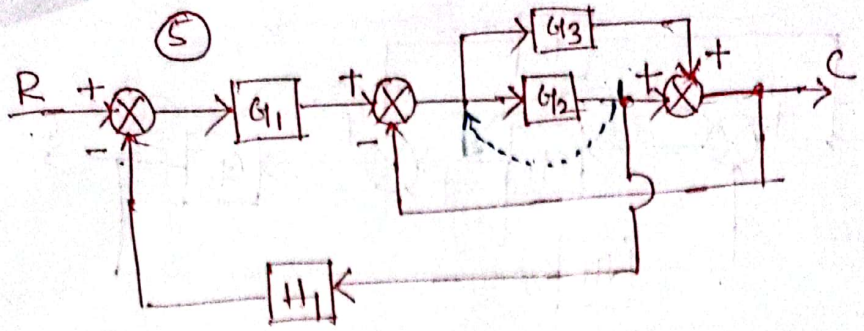
⑨ Moving a takeoff point ahead of a summing point



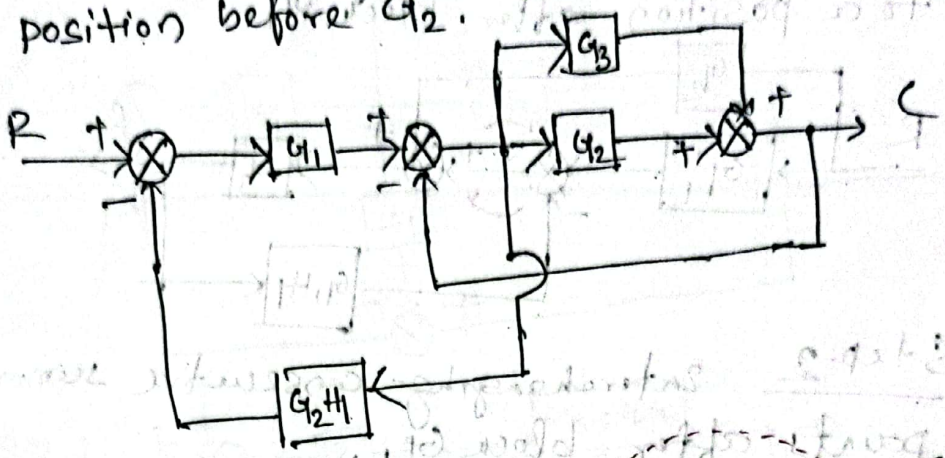
⑩ Eliminating a forward loop



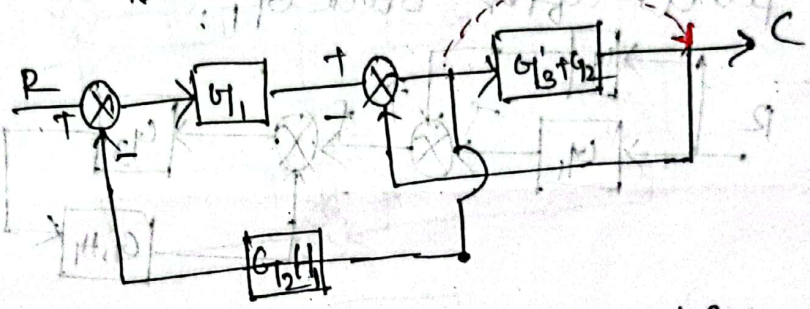
Q ⑤



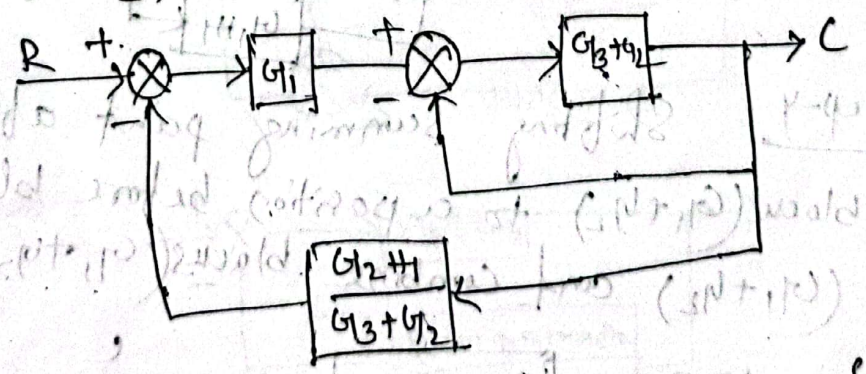
Step-1 shift the take off point after G_2 to a position before G_2.



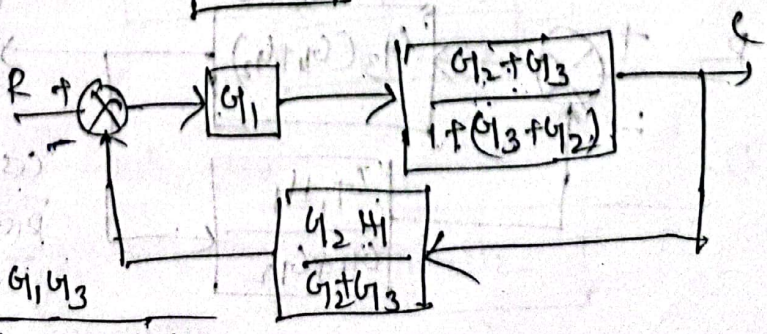
Step-2



Step-3 shift take off point before block (G_3+G_2) to a position after block (G_2+G_3)

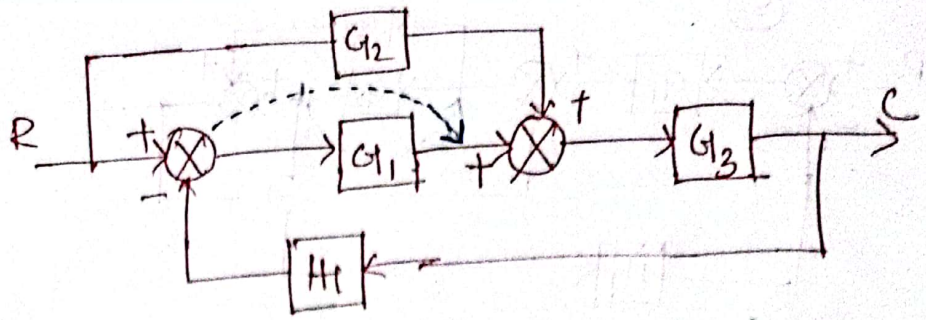


Step-4

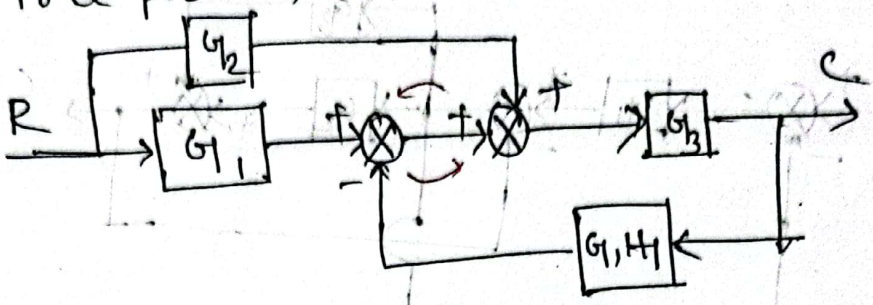


$$\frac{C(s)}{R(s)} = \frac{G_1G_2+G_1G_3}{1+G_2+G_3+G_1G_2H_1}$$

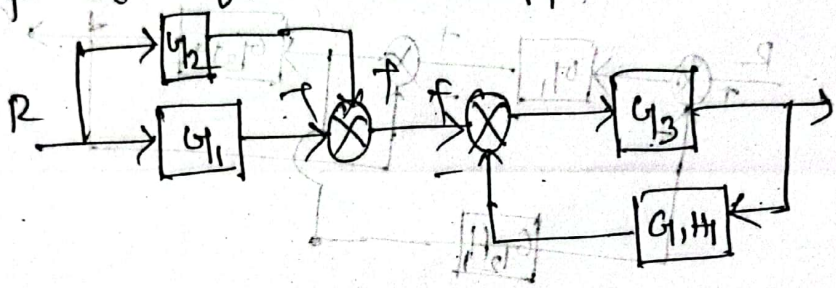
Q



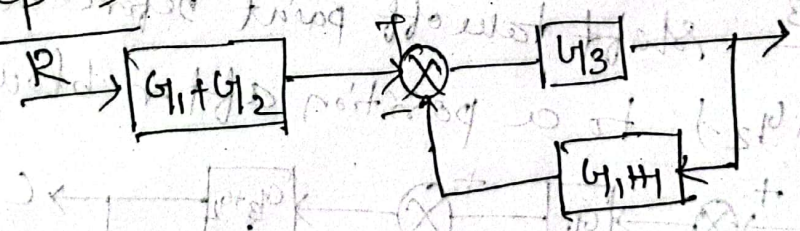
Step-1 shift the summing point before block G_1 to a position after block G_1



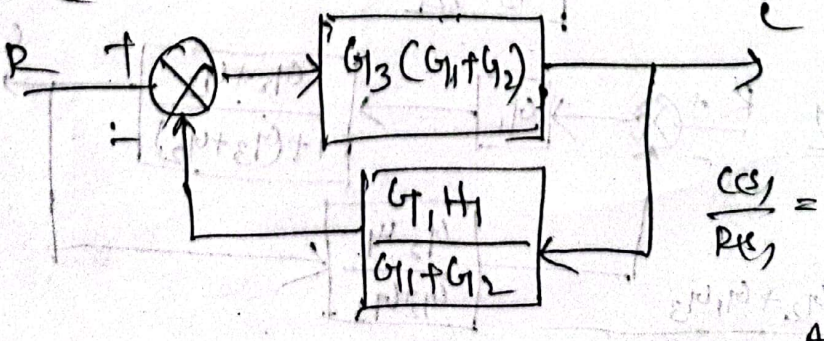
Step-2 interchanging consecutive summing points after block G_1.



Step-3



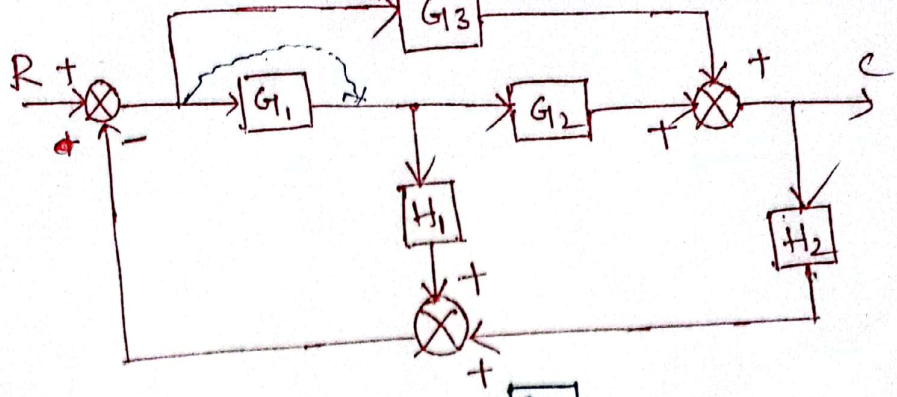
Step-4 Shifting summing point after block (G_1+G_2) to a position before block (G_1+G_2) and combine blocks (G_1+G_2) and G_3



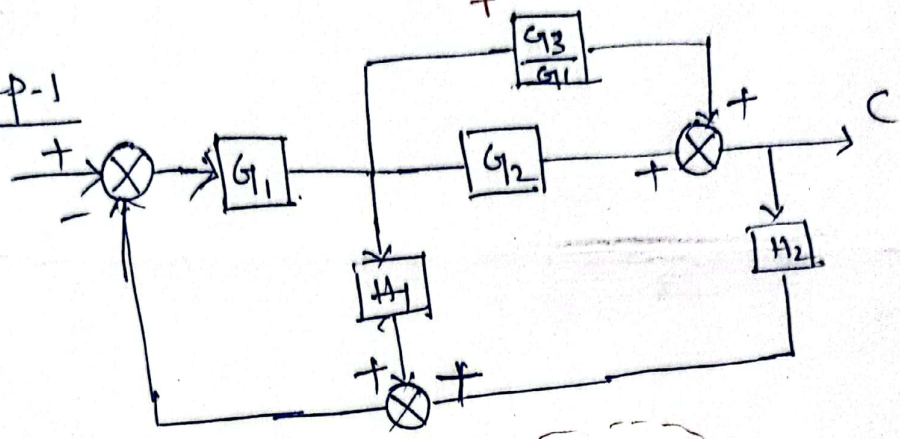
$$\frac{C(s)}{R(s)} = \frac{G_1G_3+G_2G_3}{1+G_1G_3H_1}$$

Ans

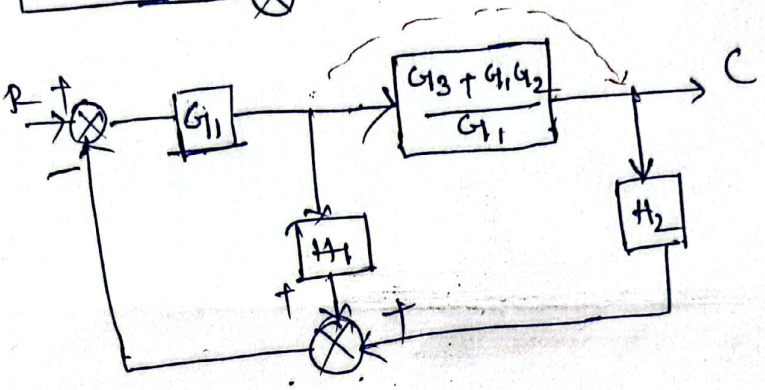
Q-



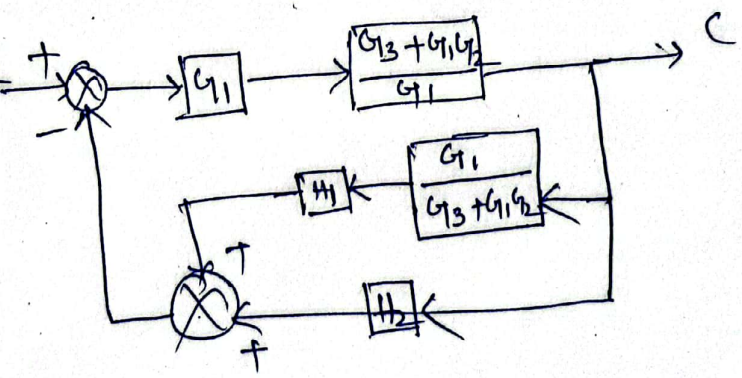
Step-1



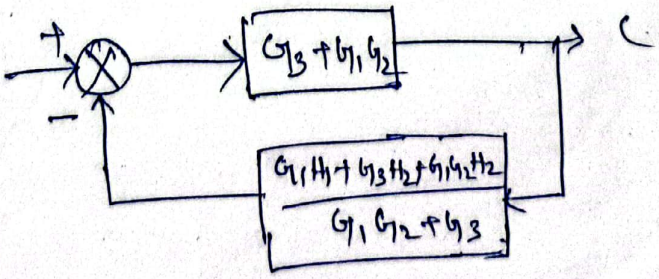
Step-2

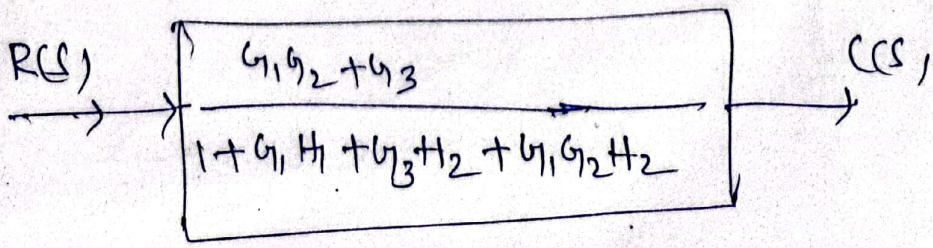


Step-3



Step-4





Effect of Feedback on Sensitivity

The parameters of any control system changes with the change in environmental conditions. These parameter variation affects the performance of the system.

So a control system should be insensitive to the parameter variations.

Let P is a gain parameter that may vary due to the variations in parameters 'R' of the system.

Then sensitivity 'S' of the system

$$S = \frac{\%\text{ change in } R}{\%\text{ change in } P} = \frac{\partial R/R}{\partial P/P} = S_P^R$$

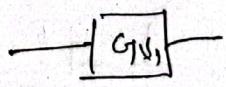
Sensitivity of OLTF

If $T(s)$ = overall transfer function

$G(s)$ = Forward path T/F is varying

Then

$$S_G^T = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)}$$



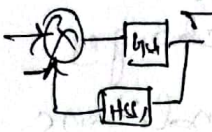
For openloop system $T(s) = G(s)$

$$S_G^T = \frac{\partial G(s)/G(s)}{\partial G(s)/G(s)} = 1.$$

Thus the sensitivity of openloop system is unity.

due to variation of G(s)

Sensitivity of close loop system



$$T(s) = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$\frac{u}{v} = \frac{v\cdot u' - u\cdot v'}{v^2}$$

$$\frac{\partial T(s)}{\partial G(s)} = \frac{[1+G(s)\cdot H(s)]\cdot 1 - G(s)\cdot H(s)}{[1+G(s)\cdot H(s)]^2}$$

$$= \frac{1}{[1+G(s)\cdot H(s)]^2}$$

then $$S_G^T = \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)}$$

$$= \frac{G(s)}{T(s)}\cdot\frac{\partial T(s)}{\partial G(s)}$$

$$= \frac{G(s)}{T(s)}\cdot\frac{1}{[1+G(s)\cdot H(s)]^2}$$

$$= \frac{G(s)}{\frac{G(s)}{1+G(s)\cdot H(s)}}\cdot\frac{1}{[1+G(s)\cdot H(s)]^2}$$

$$S_G^T = \frac{1}{1+G(s)\cdot H(s)} \quad \text{--- (1)}$$

The sensitivity is reduced due to the feedback by a factor $\frac{1}{1+G(s)\cdot H(s)}$ as compared to open loop system.

Sensitivity due to the variation of H(s)

$$\frac{\partial T(s)}{\partial H(s)} = -\frac{[G(s)]^2}{[1+G(s).H(s)]^2}$$

$$S_H^T = \frac{H(s)}{T(s)} \cdot \frac{\partial T(s)}{\partial H(s)}$$

$$= \frac{H(s)}{\frac{G(s)}{1+G(s).H(s)}} \left\{ \frac{-[G(s)]^2}{[1+G(s)H(s)]^2} \right\}$$

$$S_H^T = -\frac{G(s).H(s)}{1+G(s).H(s)} \quad \text{--- (2)}$$

From eq (1) & (2) it is clear that the close loop system is more sensitive to variations in feedback path parameters than variations in forward path variations.

Signal Flow Graph (SFG)

A signal flow graph is a diagram which represents a set of simultaneous equations. SFG consists of nodes and these nodes are connected by a directed lines called branches. Every branch of signal flow graph having an arrow, which represents the flow of signal.

Construction of SFG from equations

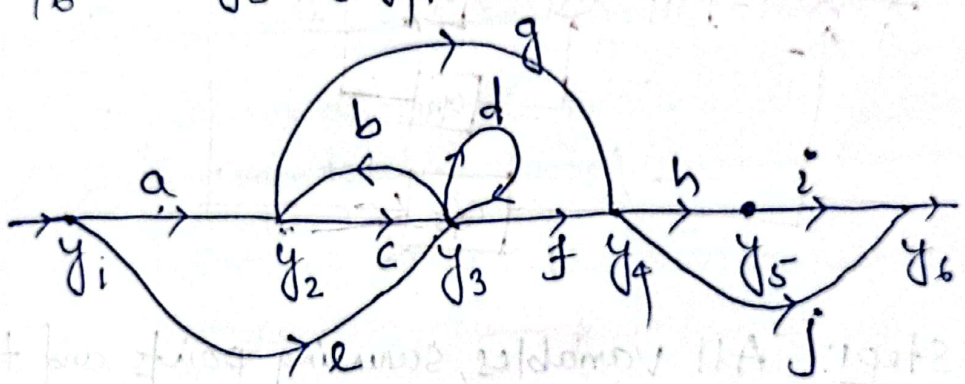
$Y_2 = a y_1 + b y_3$

$Y_3 = c y_2 + d y_3 + e y_1$

$Y_4 = f y_3 + g y_2$

$Y_5 = h y_4$

$Y_6 = i y_5 + j y_4.$



Forward path

Path which originates from i/p node and terminates at the o/p node and along which no node is traversed more than one.

loop - It is the path which originates and terminates on the same node and along which no other node is traversed more than once.

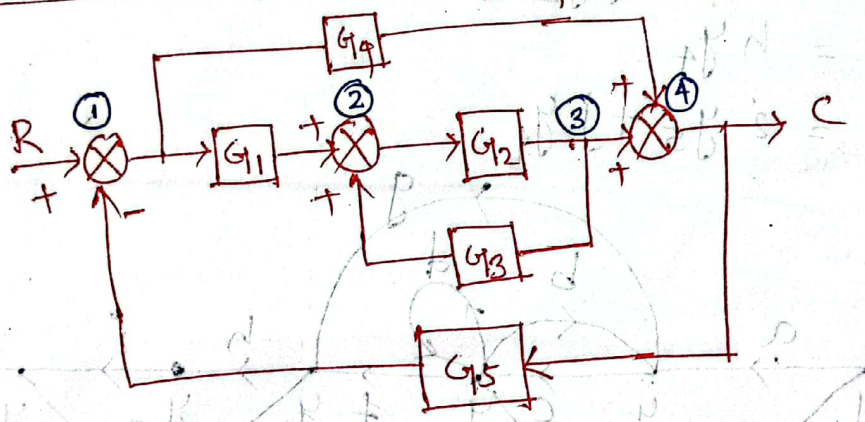
Self loop - It is the path which originates and terminates on the same node.

Path gain - The product of the branch gains along the path.

Loop gain - The gain of the loop is called loop gain.

Nontouching loop - Non-touching loops having no common nodes branch and paths.

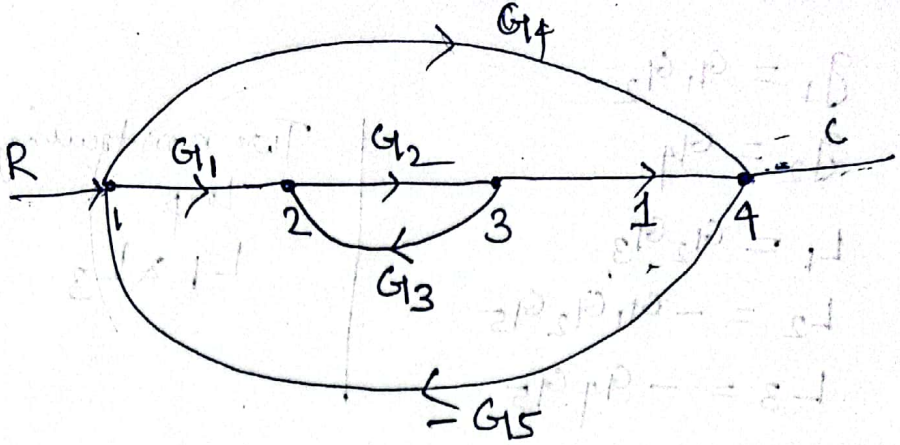
Block to signal flow graph conversion :-



Step1: All variables, summing points and takeoff points are represented by nodes.

Step-2 - If a summing point is placed before a takeoff point in the direction of signal flow, in such case represents the summing point and takeoff point by a single node.

Step-3 - If summing point is after takeoff point then represent the summing point and takeoff point by separate nodes connected by a branch having gain 1.



Mason's gain Formula

$$T/F = \frac{\sum_{K=1}^{N} g_K \Delta_K}{\Delta} = \frac{C(s)}{R(s)}$$

N = total number of forward paths

g_K = path gain of Kth F. Path.

$\Delta = 1 - \{$ sum of all individual loop gain $\}$

$+ \{$ sum of all possible gain products of two non-touching loops $\} -$

$\{$ sum of all possible gain products of three non-touching loops $\} + \cdots$

Δ_K = the part of Δ not touching the Kth forward path.

—x—

Ex $N = 2$ $\quad \frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{2} g_K \Delta_K}{\Delta}$

$$= \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}.$$

$g_1 = G_1 G_2$

$g_2 = G_4$

$L_1 = G_2 G_3$

$L_2 = -G_1 G_2 G_5$

$L_3 = -G_4 G_5$

Two nontouching loop

L_1 & L_3

$$\Delta = 1 - \{G_2 G_3 - G_4 G_5 + G_1 G_2 G_5\} + \{G_2 G_3 \cdot (-G_4 G_5)\}$$

$$\Delta = 1 - G_2 G_3 + G_4 G_5 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5$$

$$\Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - G_2 G_3$$

$$\frac{C(s)}{R(s)} = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 \cdot 1 + G_4 \cdot (1 - G_2 G_3)}{1 - G_2 G_3 + G_4 G_5 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 + G_4 - G_4 G_3 G_2}{1 - G_2 G_3 + G_4 G_5 + G_1 G_2 G_5 - G_2 G_3 G_4 G_5}$$

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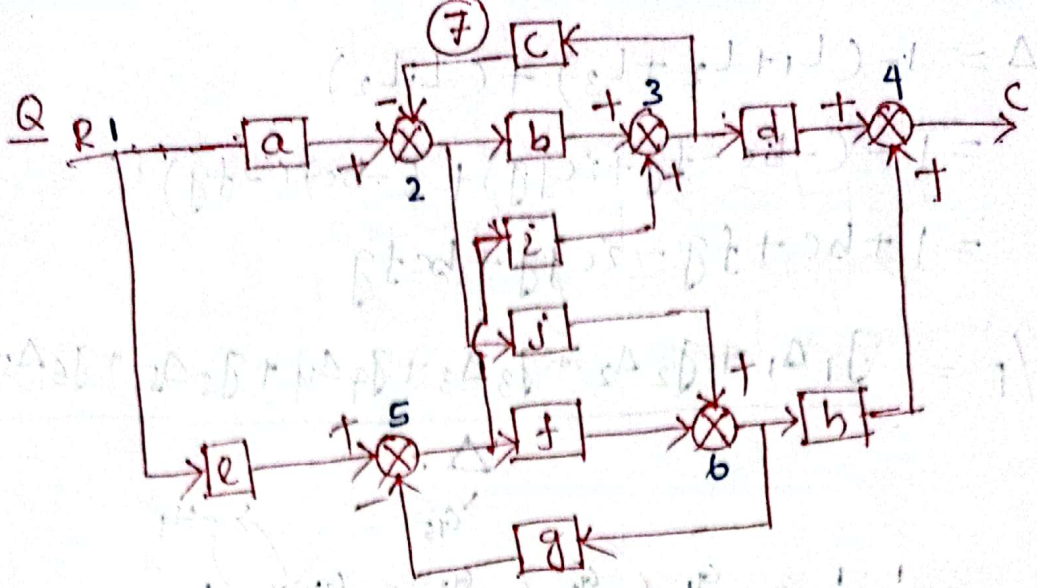
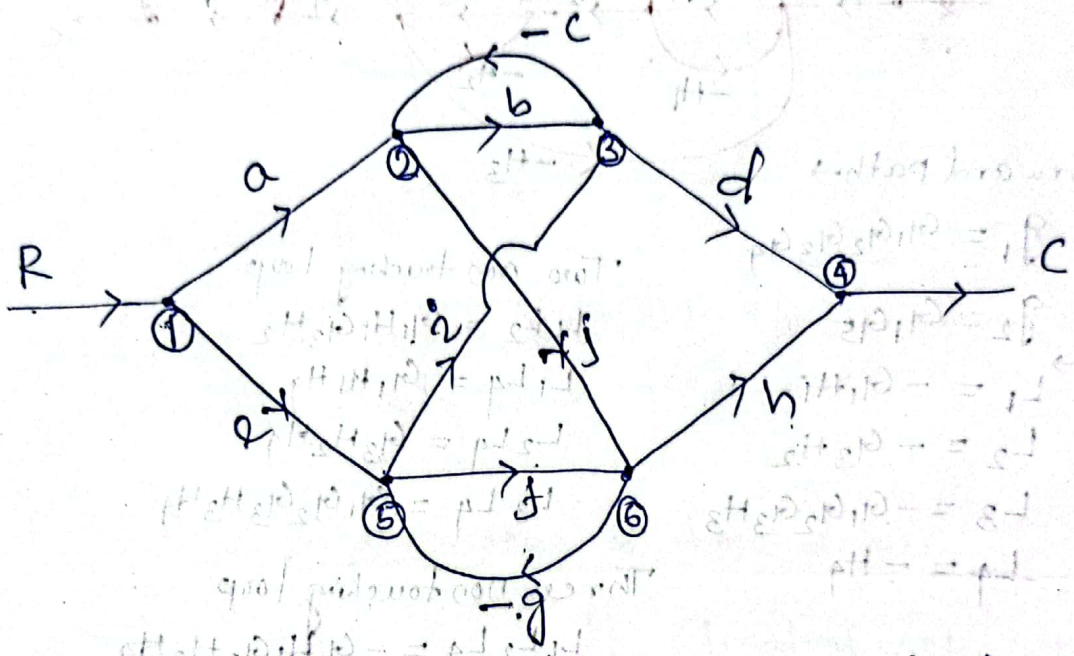




(I) 6 Forward path

$g_1 = abd$

$g_2 = efh$

$g_3 = ajh$

$g_4 = eid$

$g_5 = -eicjh$

$g_6 = -ajgid$

(II) 3 individual loops

$L_1 = -bc$

$L_2 = -fg$

$L_3 = icjg$

(III) Two nontouching loop

L_1 & L_2

$\Delta_1 = 1-(-fg) = 1+fg$

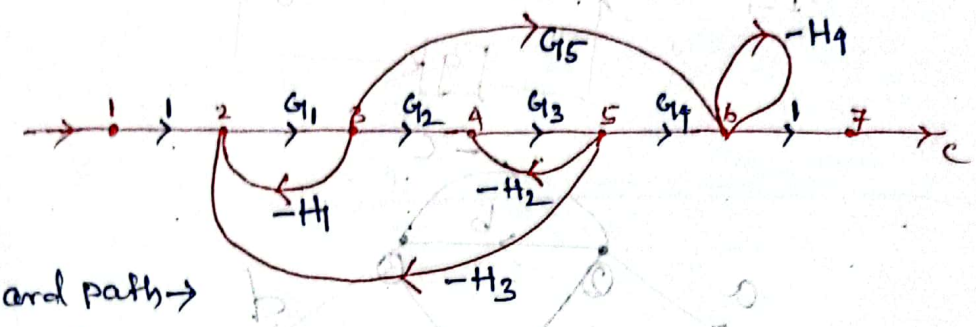
$\Delta_2 = 1-(-bc) = 1+bc$

$\Delta_3 = 1,\ \Delta_4 = 1,\ \Delta_5 = 1,\ \Delta_6 = 1.$

$$\Delta = 1-(L_1+L_2+L_3)+(L_1L_2)$$
$$= 1-(-bc-fg+icjg)+(-bc)(-fg)$$
$$= 1+bc+fg-icjg+bcfg$$

$$T/F = \frac{g_1\Delta_1+g_2\Delta_2+g_3\Delta_3+g_4\Delta_4+g_5\Delta_5+g_6\Delta_5}{\Delta}$$

Q



Forward path →

$g_1 = G_1G_2G_3G_4$

$g_2 = G_1G_5$

Loop →

$L_1 = -G_1H_1$

$L_2 = -G_3H_2$

$L_3 = -G_1G_2G_3H_3$

$L_4 = -H_4$

Two non touching loop

$L_1L_2 = G_1H_1G_3H_2$

$L_1L_4 = G_1H_1H_4$

$L_2L_4 = G_3H_2H_4$

$L_3L_4 = G_1G_2G_3H_3H_4$

Three non touching loop

$L_1L_2L_4 = -G_1H_1G_3H_2H_4$

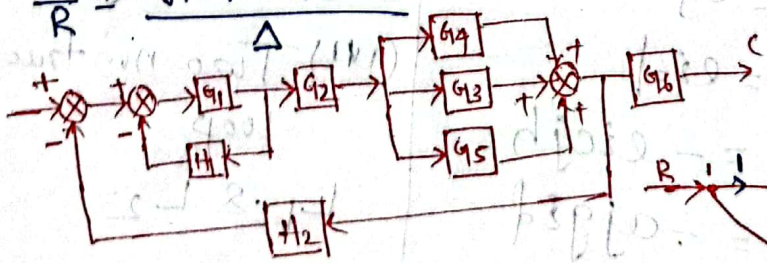
$\Delta_1 = 1-0$

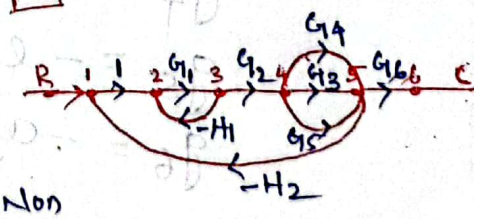
$\Delta_2 = 1+G_3H_2$

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_2+L_1L_4+L_2L_4+L_3L_4)-(L_1L_2L_4)$$

$$\frac{C}{R} = \frac{g_1\Delta_1+g_2\Delta_2}{\Delta}$$

Q





$g_1 = G_1G_2G_3G_6$

$g_2 = G_1G_2G_4G_6$

$g_3 = G_1G_2G_5G_6$

$L_1 = -G_1H_1$

$L_2 = -G_1G_2G_3H_2$

$L_3 = -G_1G_2G_4H_2$

$L_4 = -G_1G_2G_5H_2$

Two nontouching loop = Non

$\Delta_1 = 1-0 = 1$

$\Delta_2 = 1-0 = 1$

$\Delta_3 = 1-0 = 1$

$$\Delta = 1-(L_1+L_2+L_3+L_4)$$

$$\frac{C}{R} = \frac{g_1\Delta_1+g_2\Delta_2}{\Delta}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_6+G_1G_2G_4G_6+G_1G_2G_5G_6}{1+G_1H_1+G_1G_2G_3H_2+G_1G_2G_4H_2+G_1G_2G_5H_2}$$

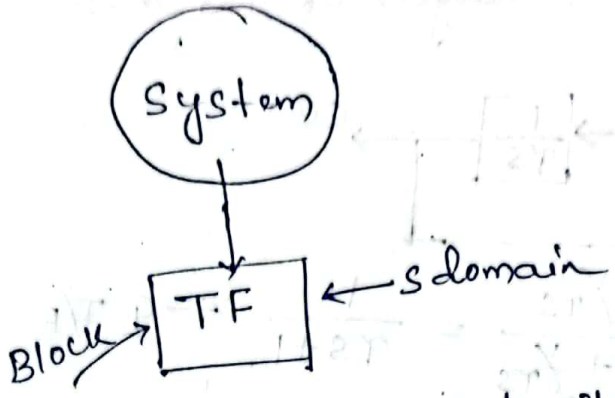
CamScanner

Time Response

(i) It describes the performance of system w.r.t time.

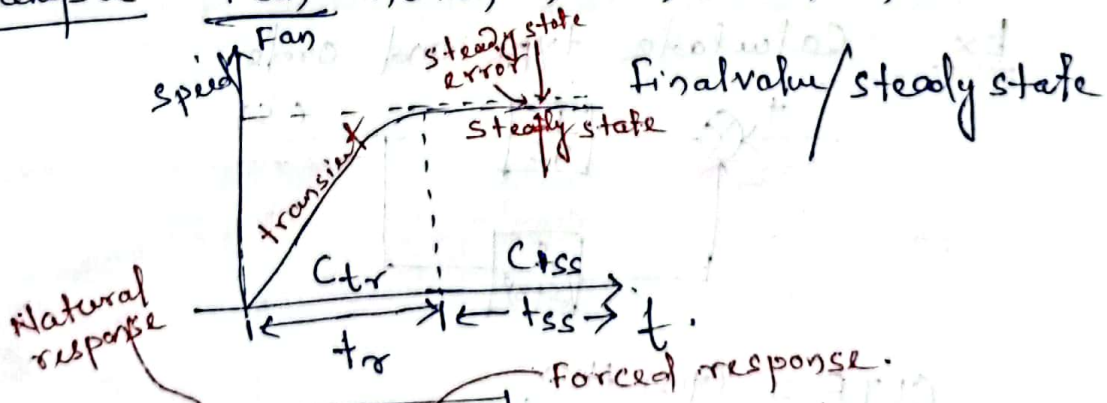
(ii) It is the combination of ~~tr & steady~~ transient & steady state response.

(iii) T.R. describes the speed & error of the system.



How system behaves w.r.t i/p signal we will do analysis in time domain.

Example Fan motion in time domain



$$\boxed{C(t) = C_{tr}(t) + C_{ss}(t)}$$

$C(t) \rightarrow$ system response.

$C_{tr}(t)$ → Change response → $\lim_{t \to \infty} C(t) = 0$ → at $t = \infty$ doesnot exist

$C_{ss}(t)$ → Fixed/constant → $t = \infty$ It exists

Ex $C(t) = 2\sin t + 3e^{-t} + 5$

($3e^{-t}$ → transient part/response)

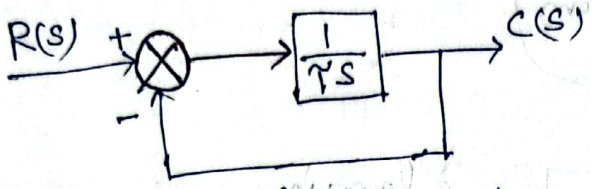
* Steady state error $e = \lim_{t \to \infty} e(t) = R(t) - C(t)$

Order :- Total number of pole is order.

Type :- No. of pole at origin. * Type is always defined for OLTF of unity feedback system.

Ex $T/F = \dfrac{K}{s^2(s+1)}$ Order = 3, Type = 2

Time domain Analysis of 1st order system :-



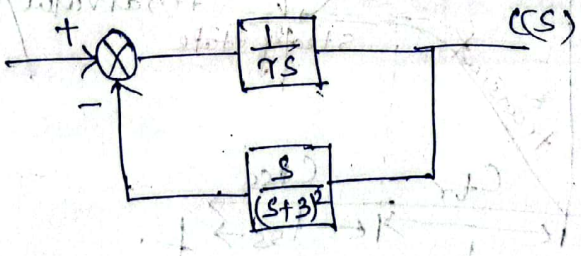
$$\frac{C(s)}{R(s)} = \frac{1/\tau s}{1 + 1/\tau s} = \frac{1}{\tau s + 1} \leftarrow \text{CL T/F}$$

Type = 0 ✗

→ OLTF = G(s). H(s).

$= \dfrac{1}{\tau s}$ ⇒ Type = 1.

Ex Calculate type and order.



OLTF = G(s). H(s)

$= \dfrac{1}{\tau \cancel{s}} \times \dfrac{\cancel{s}}{(s+3)^2}$ Type = 0 ✗

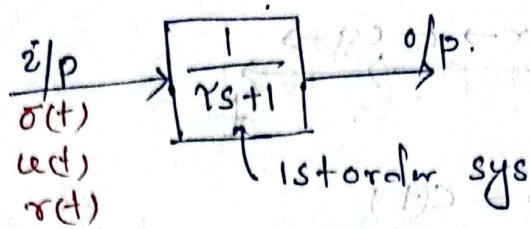
Convert it to unity feedback system. then we can conclude about Type and order.

* How to convert will explain during error analysis

Q OLTF with unity feedback $= \dfrac{(s+1)(s+2)}{s^3(s+4)(s+6)}$

Type = 3, Order = 5

Time Domain analysis of First order system.

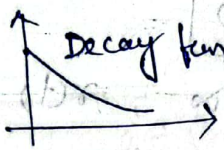


① Response of FOS for unit impulse i/p

$\delta(s) \rightarrow 1$

$$C(s) = \left(\frac{1}{\tau s+1}\right)\cdot 1 = \frac{1}{\tau s+1} = \frac{1}{\tau\left(\frac{1}{\tau}+s\right)}$$

$$\boxed{c(t) = \frac{1}{\tau} e^{-t/\tau}}$$



$$\boxed{e^{-at} \leftrightarrow \frac{1}{s+a}}$$

* Error $e(t) = r(t) - c(t)$

$$= \delta(t) - \frac{1}{\tau} e^{-t/\tau}.$$

$$\lim_{t\to\infty} e(t) = \delta(\infty) - 0$$

Not defined.

at $t=\infty$ o/p doesn't exist. If no i/p then how can you calculate $e(t)$?

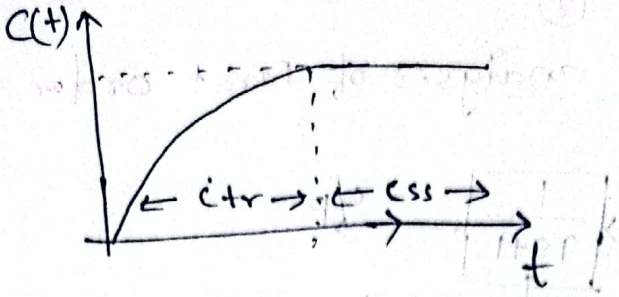
Imp For unit impulse i/p for 1st order system steady state error is not defined.

② Response of FOS for unit step i/p.

$$C(s) = \frac{1}{s}\cdot\frac{1}{\tau s+1}$$

$$C(s) = \frac{1}{s} - \frac{\tau}{1+\tau s} = \frac{1}{s} - \frac{1}{s+1/\tau}.$$

$$\Rightarrow c(t) = (1 - e^{-t/\tau})\, u(t).$$



$$e(s) = r(t) - c(t)$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s)$$

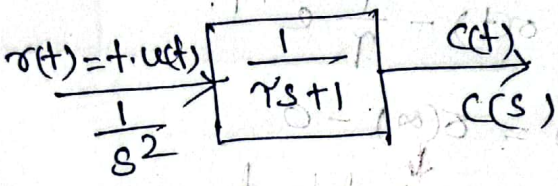
$$e_{ss} = r(\infty) - c(\infty)$$

$$= 1 - \{1 - e^{-\infty}\}$$

$$\boxed{e_{ss} = 0}$$

$\because$ at $t = \infty$, $r(t) = c(t)$.

③ <u>With ramp i/p.</u>



$$C(s) = \frac{1}{s^2} \cdot \frac{1}{\tau s+1}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{\tau s+1}$$

$$A = -\tau, \quad B = 1, \quad C = \tau^2$$

$$C(s) = -\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{\tau s+1}$$

$$C(s) = -\frac{\tau}{s} + \frac{1}{s^2} + \frac{\tau}{(s+1/\tau)}$$

ILT ↓

$$c(t) = -\tau u(t) + t\,u(t) + \tau e^{-t/\tau} u(t)$$

$$c(t) = t\,u(t) - \tau u(t)\left(1 - e^{-t/\tau}\right)$$

Error $e(t) = r(t) - c(t)$

$$= t\,u(t) - \{t\,u(t) - (1-e^{-t/\tau})\tau\,u(t)\}$$

$$= (1-e^{-t/\tau}).\,\tau\,u(t)$$

~~ESS~~

Steady state error $e_{ss} =$

$$e(\infty) = (1-e^{-\infty/\tau})\,\tau\,u(\infty)$$

$$= (1-0)\,\tau$$

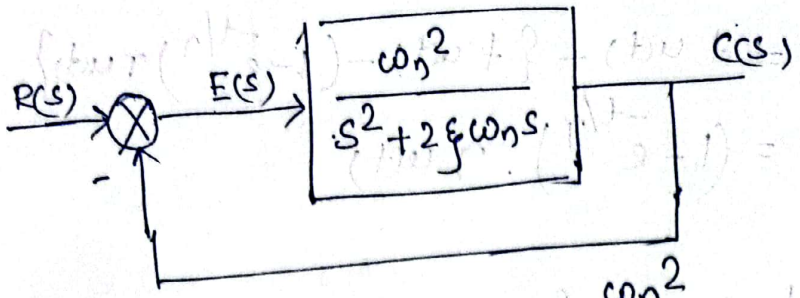
$$\boxed{e(\infty) = \tau.}$$

i/p	c(t)	e(t) = r(t) − c(t)	e(∞) e_{ss}
$\delta(t)$	$c(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$	$\delta(t) - \frac{1}{\tau} e^{-t/\tau} u(t)$	× Not defined
$u(t)$	$c(t) = (1-e^{-t/\tau})u(t)$ $= u(t) - e^{-t/\tau}u(t)$	$e(t) = e^{-t/\tau}u(t)$	Zero
$r(t) = t\,u(t)$	$c(t) = t\,u(t) - \tau(1-e^{-t/\tau})u(t)$ $= r(t) - \tau(1-e^{-t/\tau})u(t)$	$e(t) = \tau(1-e^{-t/\tau})u(t)$	τ (constant)
$P(t) = \frac{t^2}{2}u(t)$	$c(t) = \frac{t^2}{2}u(t) - \tau\{t\,u(t) - \tau(1-e^{-t/\tau})u(t)\}$	$e(t) = \tau\{t\,u(t) - \tau(1-e^{-t/\tau})u(t)\}$	∞

↓ i/p − τ (previous signal c(t))

↓ last part of c(t) retained

Second Order System



$$CLTF = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

✓ Order = 2 / CLTF / OLTF

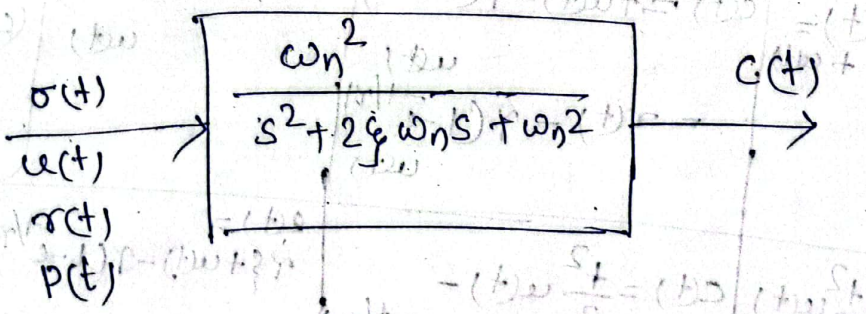
✓ Type → OLTF

$$OLTF = G(s)\cdot H(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s}$$

$$= \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

✓ Type = 1.

✓ System response analysis with standard i/p signals i.e. $\delta(t)$, $u(t)$, $r(t)$, $p(t)$.



ω_n → Natural frequency of oscillation

ξ → Damping ratio / Damping coefficient

(1) Impulse Response $\delta(t) \xrightarrow{s} 1$ $\quad C(s) = 1 \times \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

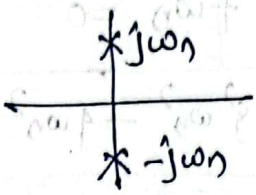
$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\boxed{1+G(s)\cdot H(s)=0}$

Characteristic eq $s^2+2\xi\omega_n s+\omega_n^2=0$

*→ ξ decides the behaviour of the system

$\xi=0$ $\quad s^2+\omega_n^2=0 \Rightarrow s=\pm j\omega_n$



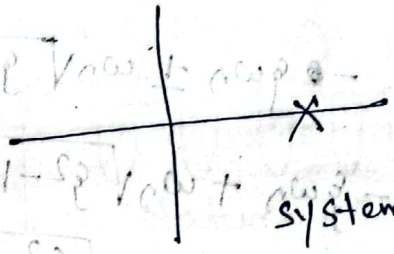
* Marginal stable system
* Poles are conjugate and completely imaginary

$\xi<0$ $\quad$ let $\xi=-1$

$s^2-2\omega_n s+\omega_n^2=0$

$(s-\omega_n)^2=0$

$s=\omega_n$



system unstable

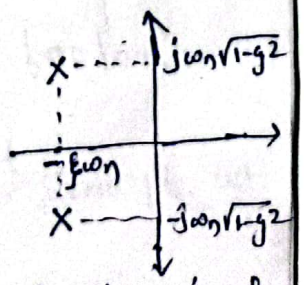
$0<\xi<1$ $\quad s^2+2\xi\omega_n s+\omega_n^2=0$

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2-4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



* Complex conjugate poles
* Stable system

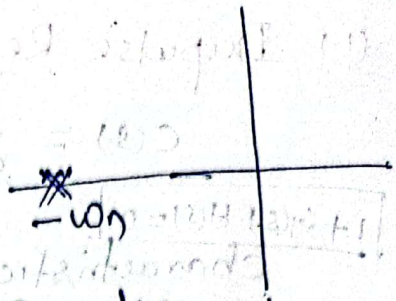
$\xi = 1$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 2\omega_n s + \omega_n^2 = 0.$$

$$(s + \omega_n)^2 = 0$$

$$s = -\omega_n, -\omega_n$$



* Repeating roots on -ve real axis.

$\xi > 1$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

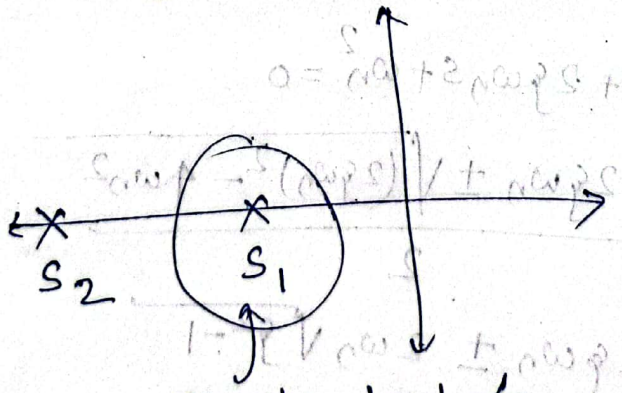
$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2 - 1}}{2}$$

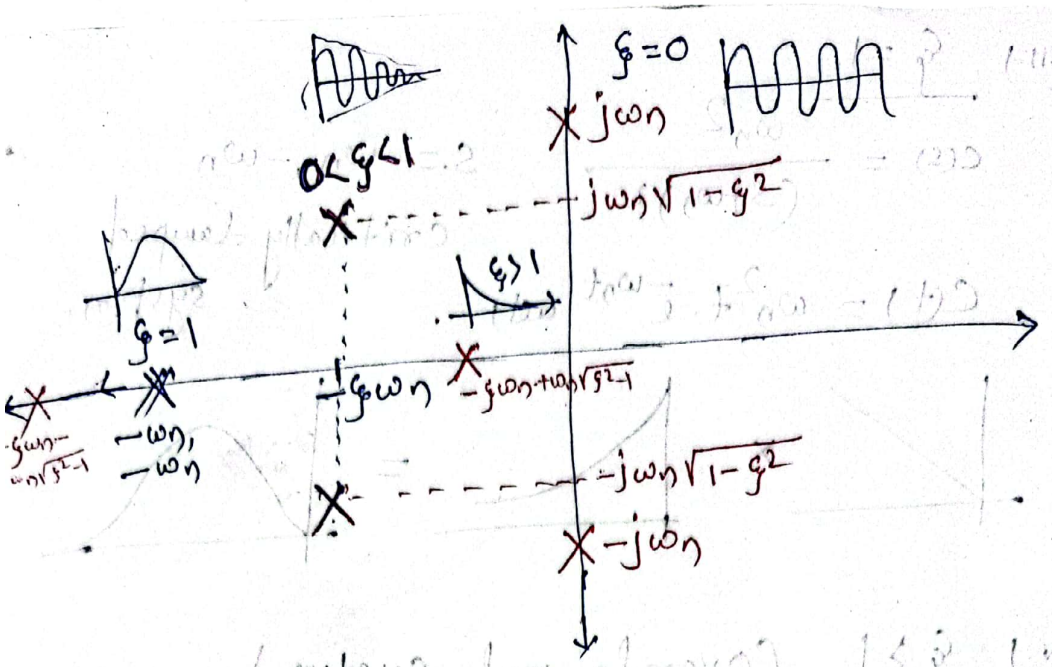
$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$



Significant pole / Dominant pole.

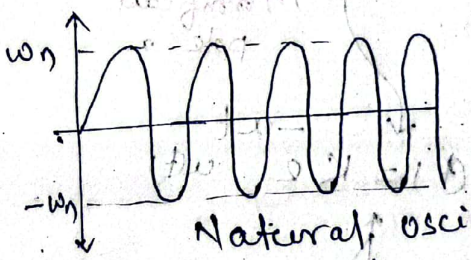


ξ = 0
0 < ξ < 1
ξ = 1
ξ > 1

(I) $\xi = 0$

$$C(s) = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$C(t) = \omega_n \sin \omega_n t \cdot u(t)$$



Natural oscillation
Undamped oscillation
sustained oscillation

(II) $0 < \xi < 1$

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n+j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n-j\omega_n\sqrt{1-\xi^2})}$$

$$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

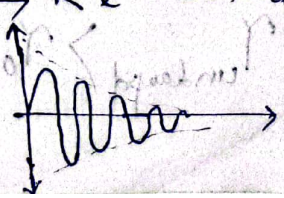
$\omega_d = \omega_n\sqrt{1-\xi^2}$ → damping frequy.

$\xi\omega_n \rightarrow$ damping factor. ($\xi\omega_n \rightarrow \alpha$)

$$C(s) = \frac{\omega_n^2}{(s+\alpha)^2+\omega_d^2} \xrightarrow{ILT} K e^{-\alpha t} \cdot \sin\omega_d t \; u(t)$$

$\boxed{\xi = \cos\theta}$

* Damped oscillation



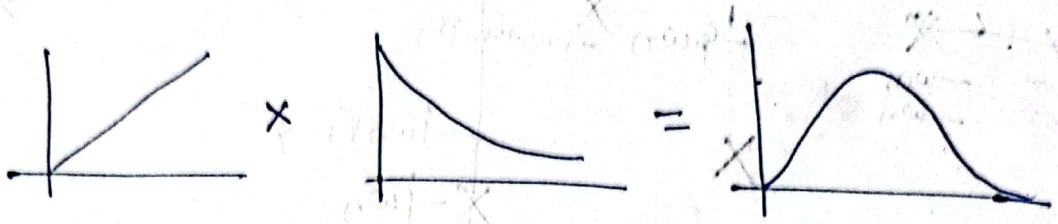
(iii) $\xi = 1$

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$s = -\omega_n, -\omega_n$

Critically damped system.

$$C(t) = \omega_n^2 \cdot t \cdot e^{-\omega_n t} u(t)$$



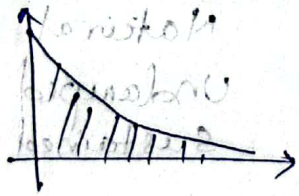
(iv) $\xi > 1$ (Overdamped system)

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1})(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1})}$$

Insignificant pole

Significant pole

$$C(t) = K e^{-xt} u(t)$$



Time constant $\tau = -\frac{1}{\text{Real part of pole}}$

$\xi = 0 \rightarrow \tau = \infty$

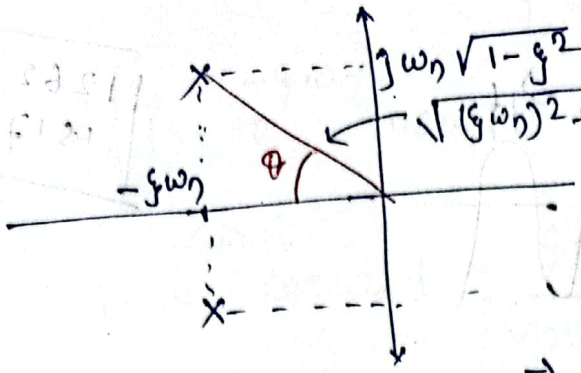
$0 < \xi < 1 \rightarrow \tau = \frac{1}{\xi\omega_n}$

$\xi = 1 \rightarrow \tau = \frac{1}{\omega_n}$

$\xi > 1 \rightarrow \tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$

$\tau_{undamped} > \tau_{OD} > \tau_{udamp} > \tau_{CD}$.

Under damped System



$$\sqrt{(\xi\omega_n)^2 + \omega_n^2(1-\xi^2)} = \omega_n$$

$$\cos\theta = \frac{\xi\omega_n}{\omega_n}$$

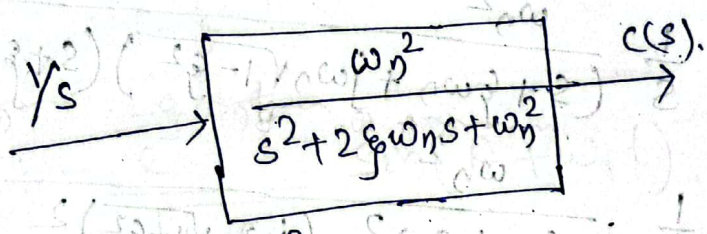
$$\Rightarrow \boxed{\cos\theta = \xi}$$

$$\boxed{\theta \propto \frac{1}{\xi}}$$

$$\boxed{\theta = \cos^{-1}\xi}$$

(2) Second order system with unit step i/p signal

step Response of 2nd order system :-



$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\xi = 0$

$\boxed{0<\xi<1}$ Under damped very important

$\xi = 1$

$\xi > 1.$

case-I $\xi = 0$

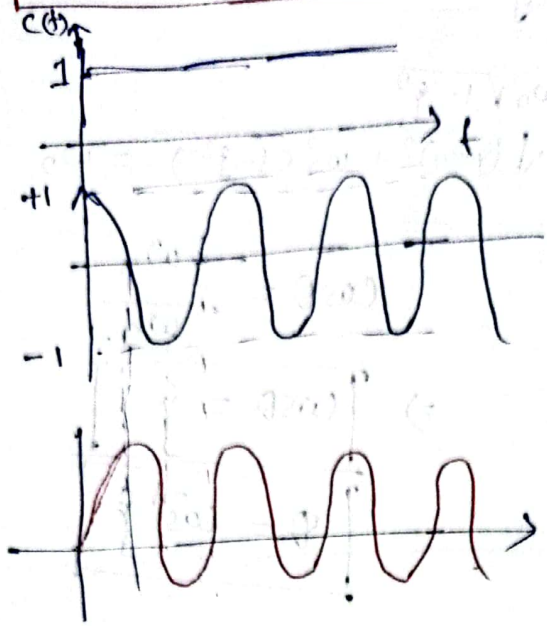
$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2}$$

$A = 1;\ B = -1,\ C = 0$

$$C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

$\boxed{c(t) = 1 - \cos\omega_n t}$ ← step response $\xi = 0$



1262
1517

$0 < \xi < 1$ in this condition the roots are

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$C(s) = \frac{1}{s}\,\frac{\omega_n^2}{(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})}$$

A ⊕ B A ⊖ B

$$= \frac{1}{s}\,\frac{\omega_n^2}{(s+\xi\omega_n)^2 - (j\omega_n\sqrt{1-\xi^2})^2}$$

$$= \frac{\omega_n^2}{s\{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)\}}$$

$$= \frac{A}{s} + \frac{Bs+C}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$A = 1,\ B = -1,\ C = -2\xi\omega_n$

$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

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$$= \frac{1}{s} - \left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n \cdot \omega_d/\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \right]$$

$$= \left[1 - e^{-\xi\omega_n t} \cdot \cos\omega_d t + \frac{\xi\omega_n}{\omega_d} \cdot e^{-\xi\omega_n} \sin\omega_d t \right] u(t)$$

$$= 1 - e^{-\xi\omega_n t} \cdot \cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\omega_d t \Big]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sqrt{1-\xi^2} \cdot \cos\omega_d t + \xi \sin\omega_d t \right]$$

✓ Put $\xi = \cos\theta$, $\sqrt{1-\xi^2} = \sin\theta$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin\theta \cdot \cos\omega_d t + \cos\theta \cdot \sin\omega_d t \right]$$

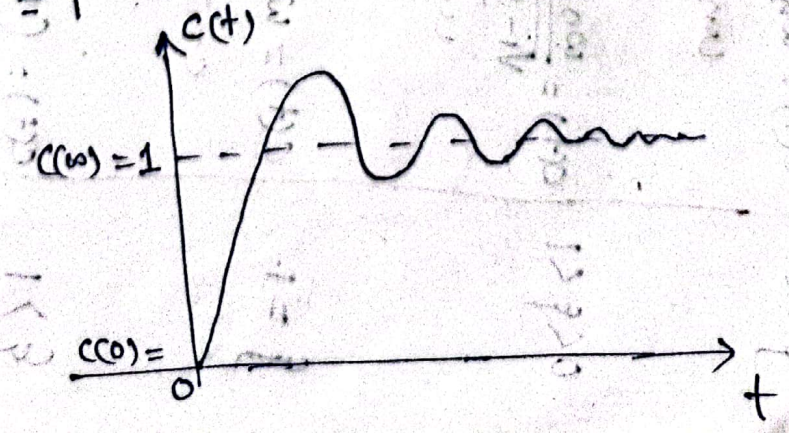
*

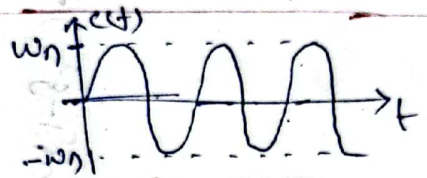
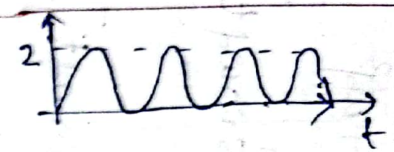
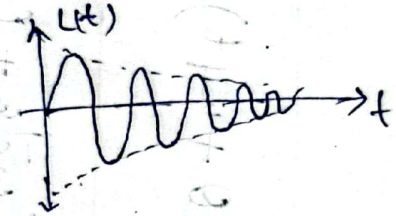
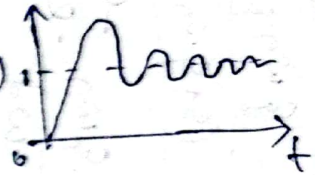
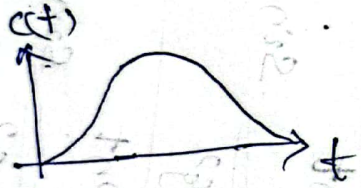
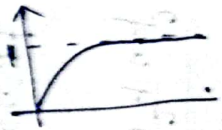
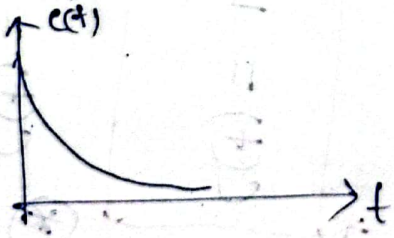
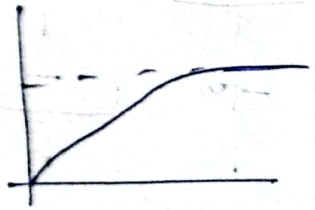
$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\theta + \omega_d t).}$$

(*) $$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\cos^{-1}\xi + (\omega_n\sqrt{1-\xi^2})t\right]$$

$$c(0) = 1 - \frac{1}{\cancel{\sqrt{1-\xi^2}}} \cancel{\sin\theta} = 0$$

$$c(\infty) = 1$$



ξ	Impulse Response	Step Response
$\xi = 0$	$c(t) = \omega_n \sin \omega_n t$ undamped 	$c(t) = 1 - \cos \omega_n t$ 
$0 < \xi < 1$	$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \cdot \sin \omega_d t$ underdamped 	$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$ 
$\xi = 1$	$c(t) = \omega_n^2 t e^{-\omega_n t}$ critically damped 	$c(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$ 
$\xi > 1$	$c(t) = (\xi \omega_n - \omega_n \sqrt{\xi^2 - 1}) e^{-(\xi \omega_n - \omega_n \sqrt{\xi^2-1})t}$ 	$c(t) = 1 - e^{-[\xi \omega_n - \omega_n \sqrt{\xi^2-1}]t}$ 

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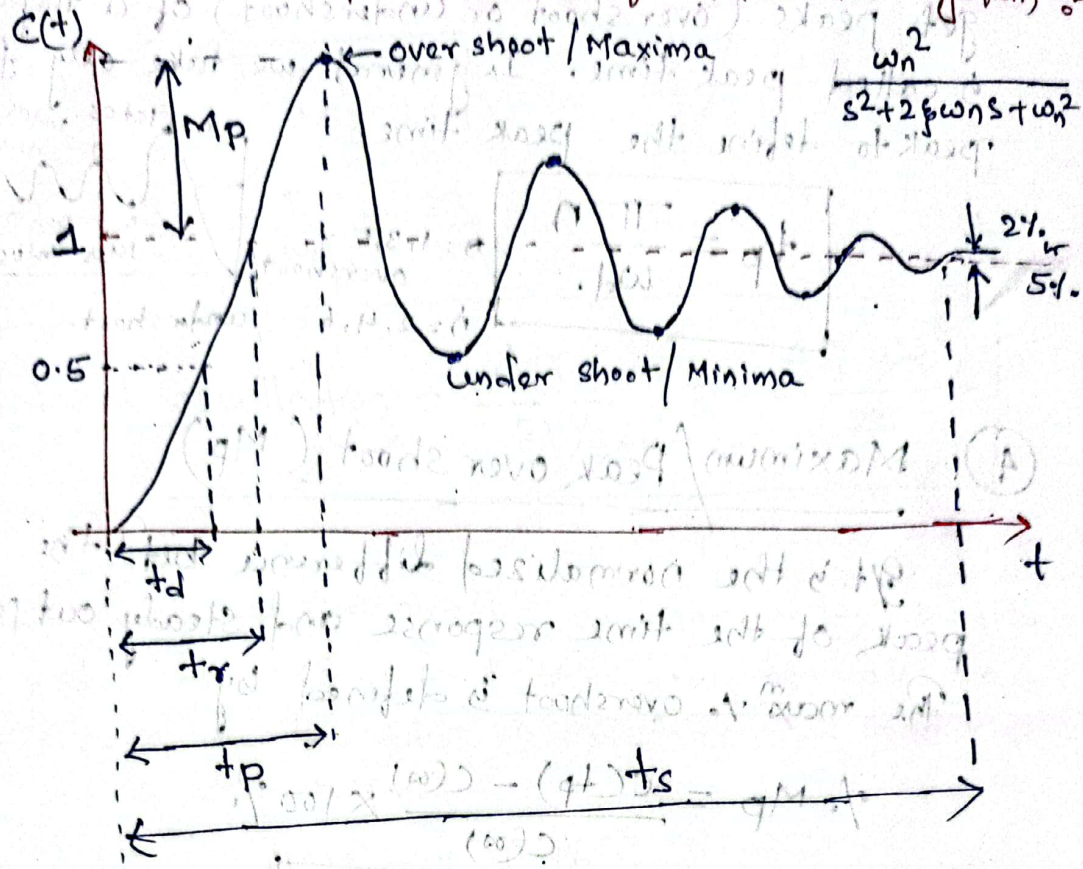
$0 < \xi < 1 \rightarrow$ Underdamped

Time Domain parameters of 2nd Order system :-

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$



① Delay Time (t_d) :-

The delay time is the time required for the response to reach 50% of the final value in the first time.

$$\boxed{t_d = \frac{1+0.7\xi}{\omega_n}} \text{ second}$$

② Rise time (t_r) :-

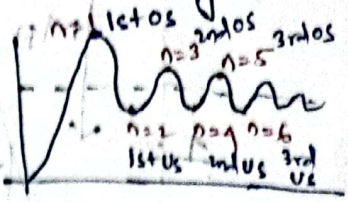
It is the time required for the response to rise from 0% to 100% of its final value in underdamped system.

$$\boxed{t_r = \frac{\pi-\theta}{\omega_d}}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\theta = \cos^{-1}\xi$$

③ **Peak time (t_p)** :— It is the time required to get peaks (over shoot or undershoot) of a response is called peak time. In general we take only first peak to define the peak time.



$$\boxed{t_p = \frac{\pi n}{\omega_d}}$$

$n = 1, 3, 5 \ldots$ overshoot

$n = 2, 4, 6 \ldots$ undershoot.

④ **Maximum/Peak over shoot (M_p)**

It is the normalized difference bet^n the peak of the time response and steady out put.

The max^m % overshoot is defined by

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

$$\boxed{\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$$

* $\xi = 1$ $\% M_p = 0$

$\xi = 0$ $\% M_p = 100\%$

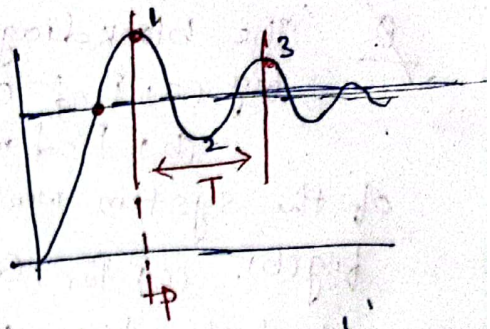
⑤ **Settling time (t_s)**

It is the time required for the response to reach and stay within the specified range (2% to 5%) of its final value.

$$t_s = 4\tau = \frac{4}{\xi\omega_n} \quad \text{for } 2\%$$

$$t_s = 3\tau = \frac{3}{\xi\omega_n} \quad \text{for } 5\%$$

(6) No. of oscillations :-



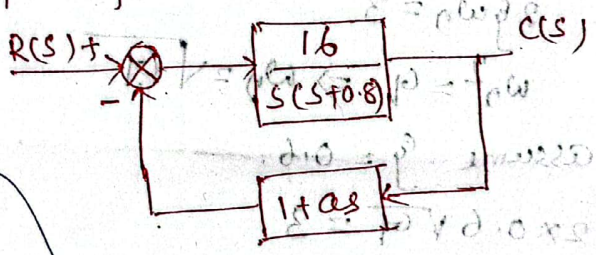
$$\text{Cycle} = \frac{t_s}{\text{Time period of 1 cycle } (t_p)}$$

$$t_p = \frac{3\pi}{\omega_d} - \frac{\pi}{\omega_d} = \frac{2\pi}{\omega_d}$$

$$\tau = \frac{1}{\xi\omega_n}$$

$$\boxed{\text{No. of oscillation} = \frac{4\tau}{2\pi/\omega_d} = \frac{4\tau\,\omega_d}{2\pi}}$$

Q Consider the system as shown in fig. Det. the value of 'a' such that the damping ratio is 0.5. Also obtain the values of rise time, and max overshoot in its step response.



$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8+16a)s + 16}$$

Ch. eqn $= s^2 + (0.8+16a)s + 16 = 0$

Comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n^2 = 16$, $\omega_n = 4$

$2\xi\omega_n = 0.8 + 16a$

$2 \times 0.5 \times 4 = 0.8 + 16a$

$\Rightarrow a = 0.2$

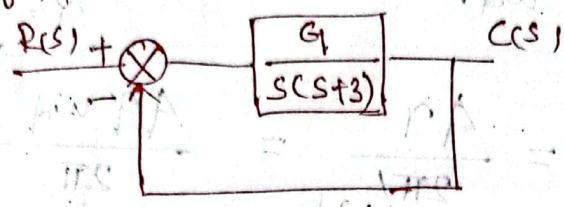
$$t_r = \frac{\pi - \theta}{\omega_d} = 0.605\,\text{sec.}$$

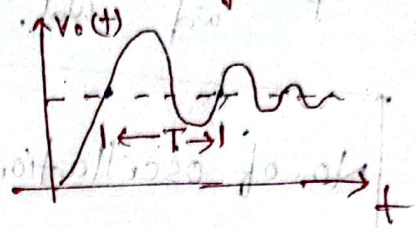
$$\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$= 16.31\%$.

Q. The block diagram of a feedback system is shown fig (a) Find CLTF

(b) Find min value of G for which the step response of the system would exhibit an overshoot as shown in fig(b). (c) for G equal to twice this min value, find the time period 'T' indicated in fig(b).





(a) $$\frac{C(s)}{R(s)} = \frac{G/s(s+3)}{1+G/s(s+3)} = \frac{G}{s^2+3s+G}$$

$$\frac{G(s)}{1+G(s)\cdot H(s)}$$

(b) Ch eq^n $s^2+3s+G=0$

Comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$

$$2\xi\omega_n = 3$$

$$\omega_n^2 = G \Rightarrow \omega_n = \sqrt{G}$$

Lets assume $\xi = 0.6$.

$$2\times 0.6\sqrt{G} = 3$$

$$\Rightarrow G = 6.23$$

(c). $G' = 2G = 2\times 6.23 = 12.5$

$$\omega_n = \sqrt{12.5} = 3.53 \text{ rad/sec}$$

$$2\xi\omega_n = 3$$

$$\Rightarrow \xi = \frac{3}{2\omega_n} = \frac{3}{2\times 3.53} = 0.424$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.53\sqrt{1-(0.424)^2} = 3.197$$

$$\frac{2\pi}{T} = 3.197$$

$$T = 1.96 \text{ sec}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega_d}$$

Q The OLTF of a servo system with unity feedback is given by $G(s) = \frac{10}{(s+2)(s+5)}$

Det. the damping ratio, undamped natural freq of oscillation. What is % Mp of the response to a unit step input.

$$G(s) = \frac{10}{(s+2)(s+5)}$$

$$H(s) = 1$$

Ch. eq

$$1 + G(s).H(s) = 0$$

$$1 + \frac{10}{(s+2)(s+5)} = 0$$

$$s^2 + 7s + 20 = 0$$

$$\omega_n^2 = 20, \ \omega_n = 4.472$$

$$2\xi\omega_n = 7$$

$$\xi = 0.782$$

$$\% Mp = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$= 1.92\%$$

Q A feedback system is described by the following T/F

$$G(s) = \frac{12}{s^2+4s+16}, \quad H(s) = Ks$$

The damping factor of the system is 0.8. Det. the overshoot of the system and the value of K.

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (4+16K)s + 16}$$

Ch. eq $s^2 + (4+16K)s + 16 = 0$

$\xi = 0.8$

$\omega_n = 4$

Comparing $\omega_n^2 = 16 \Rightarrow \omega_n = 4$

$$2\xi\omega_n = 16K + 4$$

$$2 \times 0.8 \times 4 = 4 + 16K \Rightarrow K = \underline{0.15}$$

$$\% Mp = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$= 1.5\% \ (\text{Ans})$$

Q A system having a forward path T/F $G(s) = \frac{16}{s(s+1)}$ and unity feedback. Det. the value of undamped natural freqy, damping ratio. If tachometer feedback is introduced, the feedback path T/F becomes $1+Ks$. What should be the value of K to obtain damping ratio of 0.6 Also calculate % Mp, first undershoot, tp and ts within 2% of final value.

$H(s) = 1$ Ch. eqn is $1 + G(s).H(s) = 0$

$$1 + \frac{16}{s(s+1)} = 0 \Rightarrow s^2 + s + 16 = 0$$

$\omega_n^2 = 16$, $\omega_n = 4$,

$2\xi\omega_n = 1 \Rightarrow \xi = 0.125$

Now consider $H(s) = 1 + Ks$

$1 + G(s).H(s) = 0$

$$\Rightarrow 1 + \frac{16}{s(s+1)}(1+Ks) = 0$$

$$\Rightarrow s^2 + (1+16K)s + 16 = 0$$

$\omega_n^2 = 16$, $\omega_n = 4$

$\xi = 0.6$ $2\xi\omega_n = 1 + 16K \Rightarrow K = 0.2375$

When $\xi = 0.125$ % $M_p = 67.3\%$

$\xi = 0.6$ % $M_p = 9.48\%$

First undershoot $n = 2$

$\xi = 0.6$ % $M_p = e^{-2\pi\xi/\sqrt{1-\xi^2}} = 0.009$

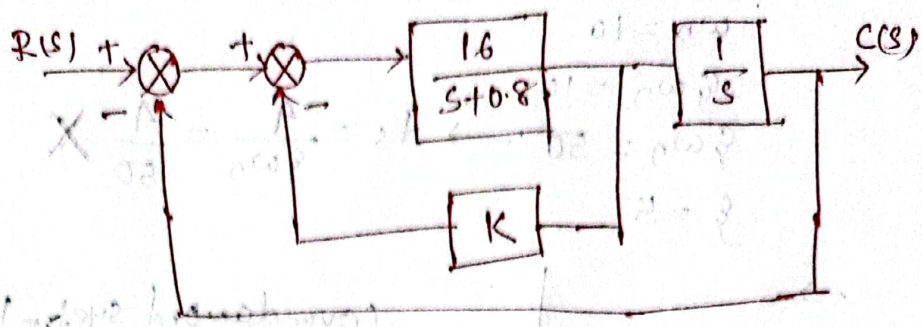
$$t_p = \frac{\pi}{\omega_d} = 0.79 \text{ sec } (\xi = 0.125)$$

$t_p = 0.98$ sec $(\xi = 0.6)$

$$t_s = \frac{4}{\xi\omega_n} = 8 \text{ sec for } \xi = 0.125$$

$= 1.67$ sec for $\xi = 0.6$

Q Consider the system shown in fig. Det. the value of K such that the damping ratio ξ is 0.5. Then obtain the t_r, t_p, % M_p and t_s in the unit step response.



$$\frac{C(s)}{R(s)} = \frac{16}{s^2+(0.8+16K)s+16}$$

Ch. eqn $s^2+(0.8+16K)s+16=0$

Comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$

$\omega_n^2=16$, $\omega_n=4$

$2\xi\omega_n = 0.8+16K$

For $\xi=0.5$, $\boxed{K=0.2}$

$$t_r = \frac{\pi-\cos^{-1}\xi}{\omega_d} = 0.605 \text{ sec.}$$

$$t_p = \frac{\pi}{\omega_d} = 0.906 \text{ sec}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.3\%$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 4} = 2 \text{ sec. (A)}$$

Q The T/F of the system is $G(s) = \frac{100}{(s+1)(s+100)}$ for a unit step i/p to the system the approximate settling time for 2% criterion is

(i) 100 sec (ii) 4 sec (iii) 1 sec (iv) 0.01 sec.

For 2% $\rightarrow t_s = 4\tau$ $\left(\tau = \frac{1}{\xi\omega_n}\right)$

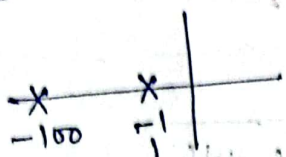
$$G(s) = \frac{100}{s^2+101s+100}$$

$\omega_n = 10$

$2\xi\omega_n = 101$

$\xi\omega_n = 50 \rightarrow t_s = \frac{4}{\xi\omega_n} = \frac{4}{50}$ ✗

$\xi = 5$

$\rightarrow$ 

(overdamped system)

$\tau = -\frac{1}{1} = 1$

So $t_s = 4\tau = 4$ sec (Ans)

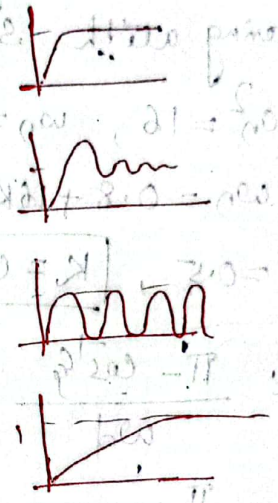
Q

P. $\frac{25}{s^2+25}$

Q. $\frac{36}{s^2+20s+36}$

R. $\frac{36}{s^2+12s+36}$

S. $\frac{49}{s^2+7s+49}$



Q. $\frac{36}{s^2+20s+36}$ $\omega_n = 6$, $\xi = 10/6 > 1$ overdamped.

R. $\frac{36}{s^2+12s+36}$ $\omega_n = 6$, $\xi = 1$ critically damped

S. $\omega_n = 7$, $\xi = \frac{1}{2}$ under damped.

Q

$\theta_1 = \theta_2 = \theta_3$

$\xi_1 = \xi_2 = \xi_3$

$M_{P_1} = M_{P_2} = M_{P_3}$

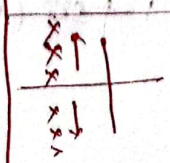
$\omega_d \uparrow$

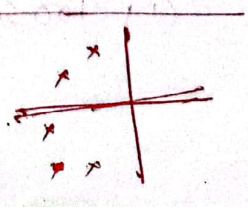
$\downarrow t_r = \frac{\pi-\theta}{\omega_d}$ $\downarrow t_p = \frac{\pi}{\omega_d}$, $t_d \downarrow$

$\downarrow t_s = \frac{4}{\xi\omega_n}$

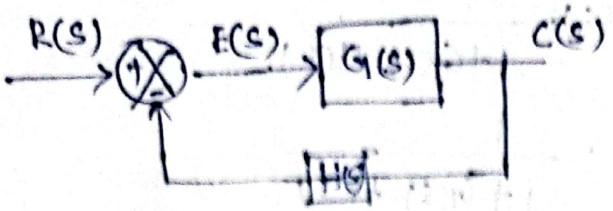
$\theta \downarrow$ $\xi \uparrow$ ω_d-Fix

$\uparrow t_r, t_p$-Fix, $t_s \downarrow$





Steady State Error



$e(t) = r(t) - c(t)$

$E(s) = R(s) - C(s)$

steady state error $(E_{ss}) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s)$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} \qquad (C(s) = E(s) \cdot G(s))$$

$$\frac{E(s) \cdot G(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\Rightarrow \boxed{\frac{E(s)}{R(s)} = \frac{1}{1 + G(s) \cdot H(s)}}$$

$$\boxed{e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}}$$

S.S. Error with step i/p signal

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$R(s) = \frac{1}{s}$.

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \lim_{s \to 0} G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1 + K_p}} \qquad \boxed{K_p = \lim_{s \to 0} G(s) \cdot H(s)}$$

K_p → Position error coefficient.

S.S. error with Ramp i/p.

$$e_{ss} = \lim_{s \to 0} s \cdot R(s) \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s + s \cdot G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s \cdot G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \qquad \boxed{K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)}$$

Velocity error coefficient.

S.S. error with parabolic i/p.

$$e_{ss} = \lim_{s \to 0} s \cdot R(s) \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{1}{s^3} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 \cdot G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \qquad \boxed{K_v = \lim_{s \to 0} s^2 \, G(s) \cdot H(s)}$$

accelarating ~~velocity~~ error coefficient.

① Step i/p	$E_{ss} = \frac{1}{1+K_p}$	$K_p = \lim_{s\to 0} G(s).H(s).$
② Ramp i/p	$E_{ss} = \frac{1}{K_v}$	$K_v = \lim_{s\to 0} s.G(s).H(s)$
③ Parabolic i/p	$E_{ss} = \frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2 G(s).H(s).$

$$E_{ss} = f(R(s), G(s))$$

i/p, system type

steady state error for different "Type" of system

① Type-0 system

Type is for Q T/F?

(i) step i/p type-0 system :—

$$R(s) = \frac{1}{s}$$

$$G(s).H(s) = \frac{K(s+s_a)(s+s_b)(s+s_c)\cdots}{(s+s_1)(s+s_2)(s+s_3)\cdots}$$

$$\cancel{K_p = \lim_{s\to 0} G(s).H(s) = \lim_{s\to 0}}$$

$$\cancel{= \lim_{s\to 0} \frac{K(s+s_a)(s+s_b)(s+s_c)\cdots}{(s+s_1)(s+s_2)(s+s_3)\cdots}}$$

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$K_p = \lim_{s\to 0} G(s).H(s) = \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} \qquad \boxed{e_{ss} = \frac{1}{1+K}}$$ constant

(ii) Ramp i/p:

$$K_v = \lim_{s\to 0} s.G(s).H(s)$$

$$= \lim_{s\to 0} s.\frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots} = 0$$

$$e_{ss} = \frac{1}{K_v} = \infty \Rightarrow \boxed{e_{ss} = \infty}$$

(iii) Parabolic i/p.

$$K_a = \lim_{s\to 0} s^2 G(s).H(s)$$

$$= \lim_{s\to 0} s^2 \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots} = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty \Rightarrow \boxed{e_{ss} = \infty}$$

② Type-1 System

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s(1+sT_a)(1+sT_b)\cdots}$$

(i) Step i/p.

$$K_p = \lim_{s\to 0} G(s).H(s) = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(ii) Ramp i/p.

$$K_v = \lim_{s\to 0} s.G(s).H(s) = K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K} \qquad \boxed{e_{ss} = \frac{1}{K}} \text{ constant}$$

(iii) Parabolic i/p

$$K_a = \lim_{s\to 0} s^2 G(s).H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty \qquad \boxed{e_{ss} = \infty}$$

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3 Type-2 system

$$G(s)\cdot H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

(i) **Step i/p.**

$$K_p = \lim_{s\to 0} G(s)\cdot H(s) = \infty.$$

$$e_{ss} = \frac{1}{1+K_p} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(ii) **Ramp i/p**

$$K_v = \lim_{s\to 0} s\cdot G(s)\cdot H(s) = \infty.$$

$$e_{ss} = \frac{1}{K_v} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(iii) **Parabolic i/p**

$$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = K.$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K} \Rightarrow \boxed{e_{ss} = \frac{1}{K}}$$ constant

i/p	Type 0	Type 1	Type 2
t^0 Step	$\frac{1}{1+K_p}$	0	0
t^1 Ramp	∞	$\frac{1}{K_v}$	0
t^2 Parabolic	∞	∞	$\frac{1}{K_a}$

type = i/p $e_{ss} \to$ const.
type > i/p $e_{ss} \to 0$
type < i/p $e_{ss} \to \infty$

Q The OLTF of a unity feedback system is given by $G(s) = \frac{50}{(1+0.1s)(s+10)}$

Det. the static error coefficient K_p, K_v, K_a

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)} = 5$$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s \cdot \frac{50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s^2 \frac{50}{(1+0.1s)(s+10)} = 0$$

Q Forward path T/F of a unity feedback control system is given by $G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$

Det. the step, ramp & parabolic error coefficient

Also det. type of the system.

$$K_p = \lim_{s \to 0} G(s) \cdot H(s) = \lim_{s \to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$= \infty.$$

$$K_v = \lim_{s \to 0} s\, G(s) \cdot H(s) = \infty.$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s) = \frac{500}{50} = 10$$

Type = 2.

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Q For a unity F.B control system the F. path transfer function is given by $G(s) = \frac{20}{s(s+2)(s^2+2s+20)}$ Det steady state error of the system. When the i/ps are (i) 5 (ii) 5t (iii) $\frac{3t^2}{2}$

(i) $r(t) = 5$ $R(s) = 5/s$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s).H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{5/s}{1+\frac{20}{s(s+2)(s^2+2s+20)}} = 0.$$

(ii) $r(t) = 5t$, $R(s) = \frac{5}{s^2}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s).H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{5/s^2}{1+\frac{20}{s(s+2)(s^2+2s+20)}}$$

$$= \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$= 10.$

(iii) $r(t) = \frac{3t^2}{2}$ $R(s) = \frac{3}{s^3}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{3}{s^2} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$= \infty.$

Q The OLTF of a unity feedback system is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$

For the static error coefficients and steady state error of the system when subjected to a i/p given by $r(t) = 2+5t+2t^2$

$$K_p = \lim_{s\to 0} G(s)\cdot H(s) = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \infty$$

$$K_v = \lim_{s\to 0} s\, G(s)\cdot H(s) = \lim_{s\to 0} s\cdot \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = \lim_{s\to 0} s^2 \cdot \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \frac{108}{4\times 12} = \frac{108}{48}$$

$$r(t) = 2+5t+2t^2$$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

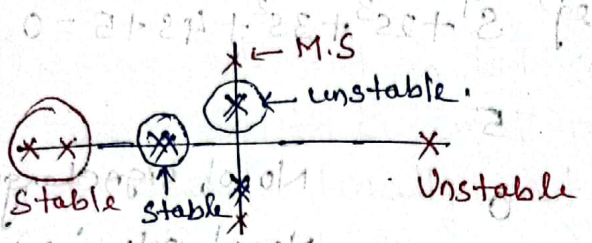
$$e_{ss} = \frac{2}{1+K_p} + \frac{5}{K_v} + \frac{4}{K_a}$$

$$= \frac{2}{1+\infty} + \frac{5}{\infty} + \frac{4}{108/48}$$

$$e_{ss} = 1.77 \text{ Ans}$$

(15) Stability Analysis. → BIBO
→ Finite no. of minima & maxima
→ Absolutly integrables

From higher order system how to get poles. position. For that we need a techique, that is Routh hurwitz criteria.



→ What can we conclude from RH criteria :-

1. We can conclude the stability of system
2. We can findout number of poles in RHS, LHS, on imaginary axis of s-plane.
3. We can find out range of 'K' for stability & marginally stability of a system.
4. We can findout frequy of oscillation.

By watching close loop poles we can define system stability. For that we need Close loop T/F.

→ i.e. characteristics eq we need.

$1 + G(s) \cdot H(s) = 0$ ← Poles of CTF.

1. ch. equation
2. Form the RH table
3. If all elements of first column are either -ve or +ve (same sign) then the system is stable.
4. Number of sign change of elements in 1st column equal to number of poles in RHS.

Ex $s^6 + 5s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1 = 0$

s^6	1	4	2	1
s^5	5	3	1	0
s^4	$\frac{5\times4 - 1\times3}{5}$	$\frac{5\times2 - 1\times1}{5}$	0	
s^3	A			
s^2	B			
s^1	C			
s^0	D.			

(5) If sign change occurs then system will be unstable.

(6)

<u>Example</u> Find the number of poles in RHS of s-plane for ch. eq $s^4+2s^3+3s^2+4s+5=0$

s^4	1	3	5
s^3	2	4	0
s^2	1	5	0
s^1	-6	0	
s^0	5		

No. of sign change = 2

No. of pole in RHS = 2

<u>Ex</u> Find the poles in LHS of s-plane for $s^4+2s^3+3s^2+s+2=0$

s^4	1	3	2
s^3	2	1	0
s^2	$5/2$	2	0
s^1	$-35/5$	0	0
s^0	2	0	0

Total pole = 4.

R.H.S = 2 poles

L.H.S = 2 poles.

(6) If any coefficient of ch. eq is -ve then system would be unstable.

(7) If any power of s term is missing then system would be unstable.

* $a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$ For stable system

all coefficients must be +ve

$a_2 a_1 > a_3 a_0$.

(8) If any element in 1st column is zero then replace 0 with a small positive number 'ε'. and proceed. Finally put ε = 0 and check the number of sign change.

s^4	1	2	8
s^3	2	4	0
s^2	ε	8	
s^1	$\frac{4\varepsilon - 16}{\varepsilon}$	0	
s^0	8		

$\to -\infty$ system is unstable.

(9) If we get complete row is zero, it means poles are symmetrical about origin. Then make the auxulary eqn from the coefficient of row which is above the zero line then differentiate the auxulary eqn and replace the ROZ by coefficient of differ eqn.

Case-1

s^5	1	3	2
s^4	1	3	2
s^3	0	0	0
s^2	3/2	2	
s^1	2/3		
s^0	2		

Auxulary eqn $= s^4 + 3s^2 + 2$

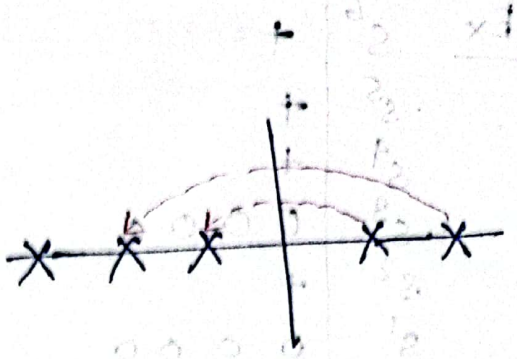
$\frac{d}{ds}(AE) = 4s^3 + 6s$

No sign change so no pole at RHS.

* System is MS.

Case-4

	s^5	+			
	s^4	+			
ROZ	s^3	0	0	0	
	s^2	+			below
	s^1	−			
	s^0	+			



* If the sign change occurs below ROZ then there must be a symmetric pole in the left to the pole placed in the right side.

* If the sign change occurs above ROZ then there is no symmetric poles left to the poles placed in the right side.

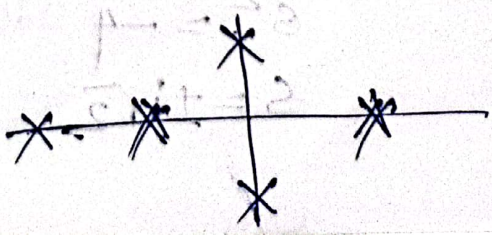
(10) If ROZ occurs once, it means poles are not repeated but symmetric.

→ If ROZ ocurs more than once, it means poles are repeated and symmetric. (Unstable system.)

The number of symmetric poles = Highest order of s from two different Auxulary eq.

	s^7	+			
	s^6	+			
	s^5	0	0	0	0
	s^4	+			
	s^3	0	0	0	0
	s^2	+			
	s^1	−			below
	s^0	+			

ROZ = 2
Order of AE = 6
Symmetric pole = 6
Sign change = 2
R.H.S pole = 2



Ex

s^6	+
s^5	+
s^4	+
s^3	0 0 0 0
s^2	+
s^1	0 0 0 0
s^0	+

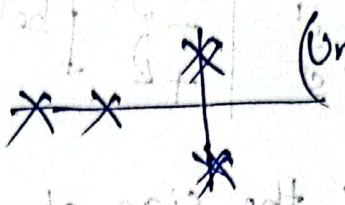
ROZ = 2

order of AE = 4

Symmetric pole = 4

Sign change = 0

RHS = 0



(Unstable system)

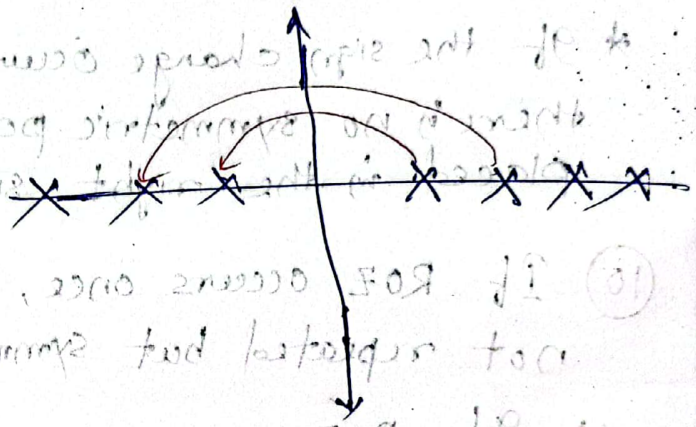
→ repeated pole

Ex

s^7	+
s^6	−
s^5	+
s^4	+
s^3	0 0 0 0
s^2	+
s^1	−
s^0	+

↑ Above

↓ below



* If ROZ is present then the system is Marginally stable system. (if no sign change).

(11)

Marginaly stable system is the oscillatory system. Then it has frequency of oscillation. So the F.O.O will be calculated from A.E = 0.

Ex

$$2s^2+4=0.$$

$$s^2=-4$$

$$s=\pm j\sqrt{2}$$

$\omega=\sqrt{2}$ rad/sec

Conditionally stable system :-

Ex $s^3+8s^2+2s+K=0$

s^3	1	2	0
s^2	8	K	0
s^1	$\frac{16-K}{8}$	0	0
s^0	K		

To make stable $\frac{16-K}{8} > 0$

$16-K>0$.

$K<16$,

$K>0$

$\boxed{0<K<16}$

Q $G(s)\cdot H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$

$1+G(s)\cdot H(s)=0$.

$s(s+2)(s+4)(s+6)+K=0$

$s^4+12s^3+44s^2+48s+K=0$.

s^4	1	44	K
s^3	12	48	0
s^2	40	K	0
s^1	$\frac{40\times48-12K}{40}$	K	
s^0	K		

$\frac{40\times48-12K}{40}>0$

$40\times48>12K$

$160>K$

$K>0$

$\boxed{0<K<160}$

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Q Det. K & b such that the system

TF $G(s) = \dfrac{K(s+1)}{s^3+bs^2+3s+1}$; $H(s)=1$

oscillation with freq of 2 rad/sec.

$s^3+bs^2+3s+1+K(s+1)=0.$

$s^3+bs^2+(3+K)s+(1+K)=0$

$1+K = b(3+K) \longrightarrow \boxed{b = \dfrac{1+K}{3+K}}$

$\omega = 2$

AE $\Rightarrow bs^2+K+1=0.$

$b(j\omega)^2+K+1=0.$

$b(2j)^2+K+1=0.$

$-b\times 4+K+1=0$

$b = \dfrac{K+1}{4}$ —— (2).

$b=0.5,\ K=1.$

Q. The OLTF of a unity F.B control system is given by $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$

by applying RH criteria, discuss the stability of the CLS as a fun of K. Det. the value of K which will cause sustained oscillation in the close loop system. What is the corresponding oscillation frequency.

Sol: the ch. eq is given by

$$1+G(s)\cdot H(s)=0$$

$$1+\frac{K}{(s+2)(s+4)(s^2+6s+25)}=0$$

$$s^4+12s^3+69s^2+198s+(200+K)=0$$

s^4	1	69	$200+K$
s^3	12	198	
s^2	52.5	$200+K$	
s^1	$198-\frac{12(200+K)}{52.5}$		
s^0	$200+K$		

System will be stable when

$200+K>0$ or $K>-200$

$$198-\frac{12(200+K)}{52.5}>0 \Rightarrow K<\boxed{666.25}$$

Oscillation will occure when $K=666.25$

Auxiliary eq $52.5s^2+(200+K)=0$

$$52.5s^2+(200+666.25)=0$$

$$52.5s^2=-866.25$$

$$s^2=-16.5,\quad s=\pm j4.06$$

$$j\omega=\pm j4.06 \Rightarrow \omega=4.06 \text{ rad/sec}$$

Q A feedback system has an OLT/F

$$G(s)H(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$$

Det. the max^m value of 'K' for the close loop system to be stable.

Sol^n For low frequency $e^{-s} = (1-s)$.

$$G(s)\cdot H(s) = \frac{K(1-s)}{s(s^2+5s+9)}$$

ch. eq^n $1+G(s)\cdot H(s)=0$.

$$1+\frac{K(1-s)}{s(s^2+5s+9)} = 0$$

$$s(s^2+5s+9)+K(1-s)=0$$

$$s^3+5s^2+(9-K)s+K=0.$$

s^3	1	$9-K$
s^2	5	K
s^1	$9-\frac{6K}{5}$	
s^0	K	

For stability $K>0$.

$$9-\frac{6K}{5}>0 \Rightarrow K<7.5$$

$\boxed{0<K<7.5}$ Range of K.

For oscillation.

$$9-\frac{6K}{5}=0.$$

$$45-6K=0. \Rightarrow K=\frac{45}{6}.$$

AE = $5s^2+K$.

Q Det the pole & stability

$$s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$$

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	
s^3	~~08~~	~~012~~	0	
s^2	3	4		
s^1	4/3			
s^0	4			

$AE = 2s^4 + 6s^2 + 4$

$\frac{dA}{ds} = 8s^3 + 12s$

No sign change so R.H.S pole = 0.

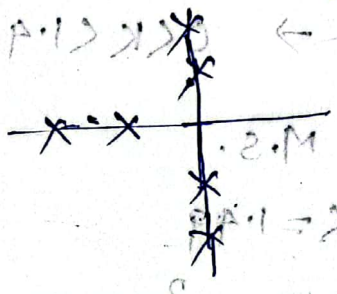
symmetric pole = 4.

i.e. 4 poles on the imaginary axis.

$2s^4 + 6s^2 + 4 = 0$

$s = \pm j1$

$s = \pm j\sqrt{2}$



Q Det. the range of values of K such that the characteristic eq

$$s^3 + 3(K+1)s^2 + (7K+5)s + (4K+7) = 0$$ has roots more negative than $s = -1$.

Sol Put $s = z - 1$ in the given ch. eq

$$(z-1)^3 + 3(K+1)(z-1)^2 + (7K+5)(z-1) + (4K+7) = 0$$

$$\Rightarrow z^3 + 3Kz^2 + (K+2)z + 4 = 0.$$

z^3	1	$K+2$
z^2	$3K$	4
z	$\frac{3K(K+2)-4}{3K}$	
z^0	4	

$\frac{3K(K+2)-4}{3K} > 0$

$3K > 0 \Rightarrow K > 0$

$3K^2 + 6K - 4 > 0$

$K > -2.527, 0.527$

Q $s^4 + 20s^3 + 15s^2 + 2s + K = 0$

Det range of K for the system to be stable

Can the system M.S. if so determine K & F.O.O

s^4	1	15	K
s^3	20	2	
s^2	14.9	K	
s^1	$\frac{29.8 - 20K}{14.9}$		
s^0	K		

(a) For stability $K > 0$

$29.8 - 20K > 0$

$K < 1.49$

Range → $0 < K < 1.49$

(b) For M.S.

$K = 1.49$

$AE = 14.9s^2 + 1.49 = 0$

$s = \pm j\, 0.316$

$\omega = 0.316$ rad/sec.

F.O.O. = 0.316 rad/sec.