Digital Assignment - 2

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Q1) A unity feedback system has

$$G(S) = \frac{K}{S(S+1)(0.1S+1)}$$ and $r(t) = 10t$

Solution:

a) Input $r(t) = 10t$

$\therefore R(S) = \frac{10}{S^2}$

The steady-state error for unity feedback system is given by,

$$e_{ss} = \underset{S \to 0}{Lt} \; S\,E(S) = \underset{S \to 0}{Lt} \; S \frac{R(S)}{1 + G(S)}$$

$$= \underset{S \to 0}{Lt} \frac{S\left(10/S^2\right)}{1 + \frac{K}{S(S+1)(0.1S+1)}}$$

$$= \underset{S \to 0}{Lt} \frac{\cancel{S}\left[\frac{10}{\cancel{S^2}}\right] \cancel{S}(S+1)(0.1S+1)}{S(S+1)(0.1S+1) + K}$$

If $K = 2$

$$e_{ss} = \underset{S \to 0}{Lt} \frac{10(S+1)(0.1S+1)}{S(S+1)(0.1S+1) + 2} = \frac{10}{2} = 5$$

(b) Input $r(t) = t$

$$R(s) = \frac{1}{s^2}$$

It is desired to have $e_{ss}(t) < 0.1$

$$\therefore \quad e_{ss}(t) = \underset{s \to 0}{Lt} \frac{s\left[\frac{1}{s^2}\right]}{1 + \frac{K}{s(s+1)(0.1s+1)}} < 0.1$$

$$\Rightarrow \underset{s \to 0}{Lt} \frac{(s+1)(0.1s+1)}{s(s+1)(0.1s+1)+K} < 0.1$$

$$\Rightarrow \frac{1}{K} < 0.1$$

$$K > 10$$

Therefore, minimum value of K need to be 10.

Q2) The open-loop transfer function of a servo system with unity feedback is

$$G(s) = \frac{10}{s(0.1s+1)}$$

Evaluate the static error coefficients for a system. Obtain the steady-state error of the system when subjected to an input given by the polynomial,

$$r(t) = a_0 + a_1 t + \frac{a_2}{2} t^2$$

Evaluate the dynamic error using the dynamic error co-efficients.

(a) The position error constant,

$$K_p = \underset{s \to 0}{Lt} \; G(s)$$

$$= \underset{s \to 0}{Lt} \; \frac{10}{s(0.1s+1)} = \infty$$

Therefore, the steady-state error for unit-step input is,

$$e_{ss}(t) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

The velocity error constant

$$K_v = \underset{s \to 0}{Lt} \; s\,G(s)$$

$$K_v = \underset{s \to 0}{Lt} \; s\left(\frac{10}{s(0.1s+1)}\right) = 10$$

∴ The steady state error for a unit-ramp input is,

$$e_{ss}(t) = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

The acceleration error constant,

$$K_a = \underset{s \to 0}{Lt} \; s^2 G(s)$$

$$= \underset{s \to 0}{Lt} \; s^2\left(\frac{10}{s(0.1s+1)}\right) = 0$$

The steady state error for unit-accelerati[on] input is,

$$e_{ss}(t) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

For linear system, we can apply the principle of superposition. Therefore, the steady state error for an input,

$$r(t) = a_0 + a_1 t + \frac{a_2 t^2}{2}$$

$= a_0 \times$ (Steady-state error for unit-step input)

$+ a_1 \times$ (Steady-state error for unit ramp input)

$+ a_2 \times$ (steady-state error for unit-parabolic input)

$$= a_0 \times 0 + a_1 \times 0.1 + a_2 \times \infty$$

$$= 0 + 0.1 a_1 + \infty = \infty$$

(b)

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1 + \frac{10}{s(0.1s+1)}}$$

$$= \frac{0.1s^2 + s}{0.1s^2 + s + 10}$$

Writing Numerator and Denominator polynomial in ascending power of s, for expanding it into infinite series,

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{s+0.1s^2}{10+s+0.1s^2}$$

$$\begin{array}{r|l} & \frac{s}{10} - \frac{s^3}{1000} + \frac{s^4}{10^4} \\ \hline 10+s+0.1s^2 & s+0.1s^2 \\ & s+0.1s^2+0.01s^3 \\ \hline & -0.01s^3 \\ & -0.01s^3 - 0.001s^4 - 0.0001s^5 \\ \hline & 0.001s^4 + 0.0001s^5 \\ \hline \end{array}$$

$$\frac{E(s)}{R(s)} = \frac{s}{10} - \frac{s^3}{1000} + \frac{s^4}{10000}$$

By equating it with,

$$\frac{E(s)}{R(s)} = C_0 + C_1 s + \frac{C_2}{2!}s^2 + \frac{C_3}{3!}s^3 + \ldots$$

The dynamic error is,

$$C_0 = 0 \qquad C_1 = \frac{1}{10} = 0.1 \qquad C_2 = 0$$

Therefore, the dynamic error is,

$$e(t) = C_0\, r(t) + C_1 \cdot \frac{d\,r(t)}{dt} + \frac{C_2}{2!} \frac{d^2 r(t)}{dt^2} + \dots$$

$$= \frac{t}{\infty}\; c_1 \left[\frac{d}{dt} \left[a_0 + a_1 t + \frac{a_2 t^2}{2} \right] \right]$$

$$= 1/10\,(a_1 + a_2 t)$$

$$= 0.1\,(a_1 + a_2 t)$$

$\therefore$ The steady-state error,

$$e_{ss}(t) = \underset{t \to \infty}{Lt}\; e(t) = \underset{t \to \infty}{Lt}\; 0.1\,(a_1 + a_2 t)$$

$$e_{ss}(t) = \infty$$

Q3) Using the Routh criterion, check whether the system represented by the following characteristic equation is stable or not. Comment on the location of the roots. Determine the frequency of sustained oscillations if any.

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

Solution:

Routh's Table :

s^4	1	6	8
s^3	2	8	0
s^2	$\frac{2\times6 - 1\times8}{2} = 2$	$\frac{2\times8 - 1\times0}{2} = 8$	0
s^1	$\frac{2\times8 - 2\times8}{2} = 0$	0	0
s^0			

All elements in s^1 are zeros. So, there are symmetrically located roots of characteristic equation with respect to the origin of the s-plane. So, the system is unstable.

To determine the location of roots, form $A(s)$ using the co-efficients of row just above ~~a~~ the zero row.

$$A(s) = 2s^2 + 8 = 0$$

$$\frac{dA(s)}{ds} = 4s = 0$$

Replace zero row with first derivative co-efficients.

s^4	1	6	8
s^3	2	8	
s^2	2	8	
s^1	4	0	
s^0	8		

No sign change in first column, so no - roots in right half of s-plane. Since, system is unstable, roots must be on imaginary axis of s-plane. We can find them by solving.

$$2s^2 + 8 = 0$$

$$s^2 + 4 = 0$$

$$s = \pm j2$$

This shows system oscillates and frequency of sustained oscillations is $\omega = 2$ rad/s.

Other roots are,

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = 0$$

$$\Rightarrow (s^2+4)(s^2+2s+2) = 0$$

$$(s^2+4)(s+1+j1)(s+1-j1) = 0$$

The other two roots a pair of complex conjugate roots in left-half of the s-plane.

Q 4) A unity feedback control system is characterized by the open-loop transfer function.

Using the Routh criterion.

(a) Calculate the range of values of K for the system to be stable.

Sol:

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+13)}{s(s+3)(s+7)} = 0$$

$$s(s+3)(s+7) + K(s+13) = 0$$

$$s^3 + 10s^2 + (21+K)s + 13K = 0$$

Routh's Table:

$$\begin{array}{lll} s^3 & 1 & 21+K \\ s^2 & 10 & 13K \\ s^1 & \dfrac{10\times(21+K) - 1\times 13K}{10} & \\ & = \dfrac{210-3K}{10} & \\ s^0 & 13K & \end{array}$$

First column should be positive, for stable system.

s^0 Row:

$$13K > 0$$

$$K > 0$$

~~21~~

s^1 Row:

$$210 - 3K \overset{>}{\cancel{=}} 0$$

$$3K < 210$$

$$K < 70$$

Hence, K range is $0 < K < 70$.

(b) What is the value of K for Marginal Stability?

$K = 70$

$$10s^2 + 13K = 0$$

$$10s^2 + 13(70) = 0$$

$$s = \pm j\sqrt{\frac{70 \times 13}{10}} = \pm j9.53$$

$$\omega = 9.53 \text{ rad/s}$$

(c) Check if for $K = 1$, all roots of characteristic equation of the above system have the damping factor greater than 0.5.

Sol:

$$s^3 + 10s^2 + 22s + 13 = 0$$

Substiute $K = 1$

$$s^3 + 10s^2 + 22s + 13 = 0$$

$$(s+1)(s^2 + 9s + 13) = 0$$

$$(s+1)(s + 4.5 + j5.38)(s + 4.5 - j5.38) = 0$$

$$s = -1, \; -4.5 \pm j5.38$$

For $s = -1$, $\theta = 0$

Damping Factor $\xi = \cos\theta = \cos 0 = 1$

For $s = -4.5 \pm j5.38$

$$\theta = \tan^{-1}\frac{5.38}{4.5} = 50.08^\circ$$

Damping Factor $\xi = \cos 50.08 = 0.64$

All the roots have damping Factor greater than 0.5.

The damping factor of $s^2 + 9s + 13$ can be found by comparing with std. equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n^2 = 13$$

$$\omega_n = \sqrt{13} = 3.6$$

$$2\xi\omega_n = 9$$

$$\xi = \frac{9}{2 \times 3.6} = 1.25$$

Q5) Draw the root locus for

$$G(s)\,H(s) = \frac{K}{s(s+2)(s+4)}$$

From the root locus, find the range of value of K for which the system will have damped oscillatory response. Also, determine the value of K for a damping ratio of 0.5. With the value of K, find closed-loop transfer function.

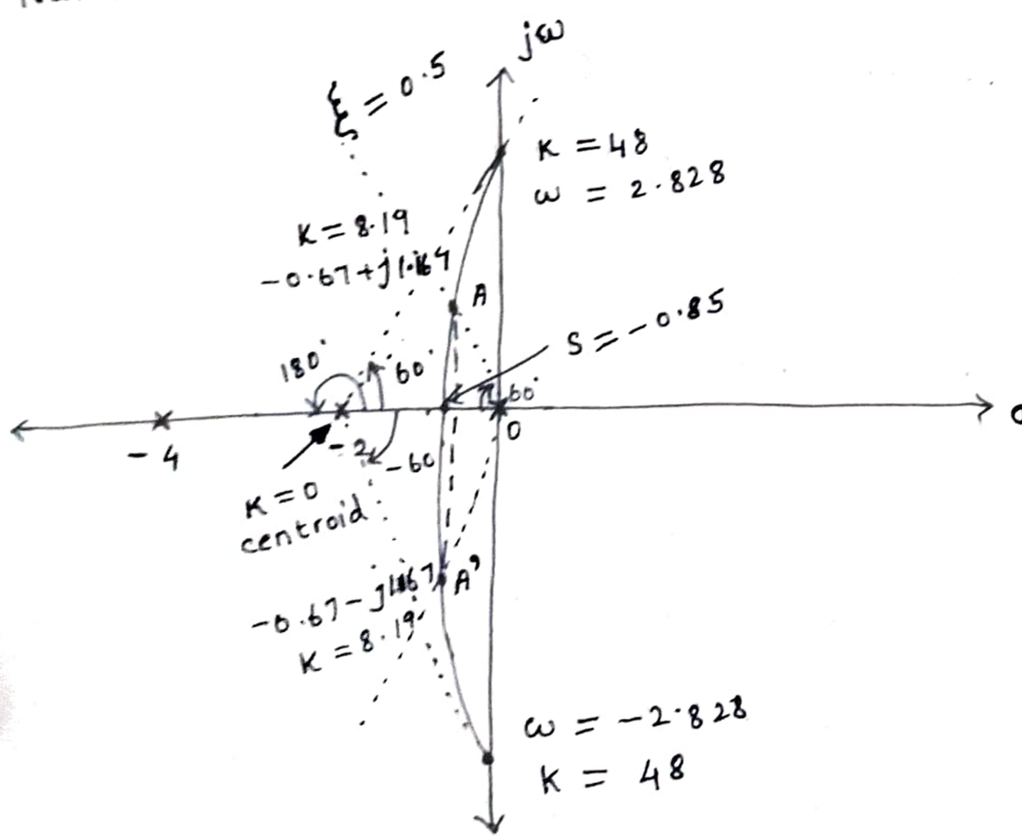
Soln:

The open Loop poles are at $s=0$, $s=-2$, $s=-4$.

$n = 3$

There are no finite open-loop zeros. $\therefore m = 0$

Number of Asymptotes $n - m = 3 - 0 = 3$



1. Root locus will be symmetrical about x-axis

2. $s = 0, -2, -4$ where $K = 0$ and terminate at zeros at infinity, where $K = \infty$

3. There are 3 asymptodes,

Angle of Asymptodes $\theta_q = \dfrac{(2q+1)\pi}{n-m}$, $q = 0, 1, 2$

$$\theta_0 = \frac{\pi}{3} \qquad \theta_1 = \pi \qquad \theta_2 = \frac{5\pi}{3}.$$

4. The point of intersection of the asymptodes on real axis is given by

$$-\sigma = \frac{\text{sum of real parts of poles} - \text{sum of real parts of zeros}}{\text{number of poles} - \text{number of zeros}}$$

$$-\sigma = \frac{[0-2-4] - 0}{3-0} = -2$$

5. The root locus exists on real axis from $s = 0$ to $s = -2$ and to the left of $s = -4$

6. The breakaway points are given by the solution of equation $\dfrac{dK}{ds} = 0$

$$|G(s)\,H(s)| = \left|\frac{K}{s(s+2)(s+4)}\right| = 1$$

$$K = s(s+2)(s+4)$$

$$\frac{dK}{ds} = \frac{d}{ds}\left[s(s+2)(s+4)\right] = 0$$

$$\frac{d}{ds}\left[s^3 + 6s^2 + 8s\right] = 3s^2 + 12s + 8 = 0$$

$$s = -3.15,\ -0.85$$

$s = -0.85$ is the actual ~~breaking~~ break point because the root locus exists there.

The break angle at $s = -0.85$

$$\pm\frac{\pi}{r} = \pm\frac{180^\circ}{2} = \pm 90^\circ$$

7. No Angle of departure is needed since no complex poles and zeros there.

8. Value of K is found using Routh's Criteria:

$$1 + G(s)H(s) = 1 + \frac{K}{s(s+2)(s+4)}$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0$$

The routh table is as follows:

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	

All elements in First column need to be positive for stability.

$$K > 0$$

$$48 - K > 0$$

$$K < 48$$

Therefore K range from $0 < K < 48$

The Marginal value $K_m = 48$. The freq. of sustained-oscillations is given by solution of auxiliary equation.

$$6s^2 + K = 0$$

$$6s^2 + K_m = 0$$

$$6s^2 + 48 = 0$$

$$s^2 = -8$$

$$s = \pm j\sqrt{8} = 2.828j$$

The value of K at break point,

$$K \text{ (at break point)} = 0.85 \times 1.15 \times 3.15$$
$$= 3.08$$

From the graph it is $3.08 < K < 48$

To find the value of K $\xi = 0.5$

$$\theta = \cos^{-1}(\xi) = \cos^{-1} 0.5 = 60^\circ$$

K value for $\xi = 0.5$ is

$$K = xyz = 1.3 \times 1.8 \times 3.5 = 8.19$$

The closed-loop transfer function is,

$$\frac{C(s)}{R(s)} = \frac{8.19}{(s + 0.67 + j1.16)(s + 0.67 - j1.16)(s + 4.56)}$$

Q6) Draw complete root locus for

$$G(s)H(s) = \frac{K(s+3)}{s(s+2)}$$

Find the range of K, for the system to be overdamped, critically damped, underdamped.

Soln:

For the given open-loop transfer function :

$G(S)\,H(S)$: Open loop poles $\Rightarrow S = 0, -2$

$\therefore n = 2$

The open Loop zero is at $S = -3$ $\therefore m = 1$

The number of asymptodes $n-m = 2-1 = 1$.

The angle of Asymptodes $\theta_q = \dfrac{(2q+1)\pi}{n-m}$

$q = 0$

$\theta_0 = \pi$.

The point of intersection of the asymptodes on the real axis is given by,

$$-\sigma = \frac{-2-(-3)}{2-1} = 1$$

The root Locus Lies between real axis between $S=0$ and $S=-2$ and to left ~~of~~ of $S=-3$

Break Point $\dfrac{dK}{ds} = 0$

$$|G(s)\,H(s)| = \left|\frac{K(s+3)}{s(s+2)}\right| = 1$$

$$K = \frac{s(s+2)}{s+3}$$

$$\frac{d}{ds}\left[\frac{s(s+2)}{s+3}\right] = 0$$

$$(s+3)(2s+2) - s(s+2) = 0$$

$$2s^2 + 8s + 6 - s^2 - 2s = 0$$

$$s^2 + 6s + 6 = 0$$

$$s = -1.29, \; -4.71$$

The break angles at $s = -1.29$ and $s = -4.71$ are

$$\pm \frac{\pi}{r} = \pm \frac{180^\circ}{2} = \pm 90^\circ$$

The system is stable for all positive values of K.

* There are two branches of root locus. The branches starting $s = 0$ to the left $s = -2$. Both will meet at $s = -1.29$ & breakaway enter the complex plane, opposite directions on real axis. One terminates at $s = -3$ and the other one travels along asymptodes at 180° and terminates at $s = -\infty$

To find ξ_{min}, from origin draw a tangential to the circle. Let it meet the circle at A.

$$K\ (\text{at } A) = \frac{xy}{z} = \frac{2.4 \times 1.4}{1.75} = 1.92$$

$K = 1.92$

The pole of at A for $K = 1.92$ is $-2 + j1.4$. It's complex conjugate is $s = -2 - j1.4$.

$$\frac{C(s)}{R(s)} = \frac{1.92}{(s+2+j1.4)(s+2-j1.4)}$$

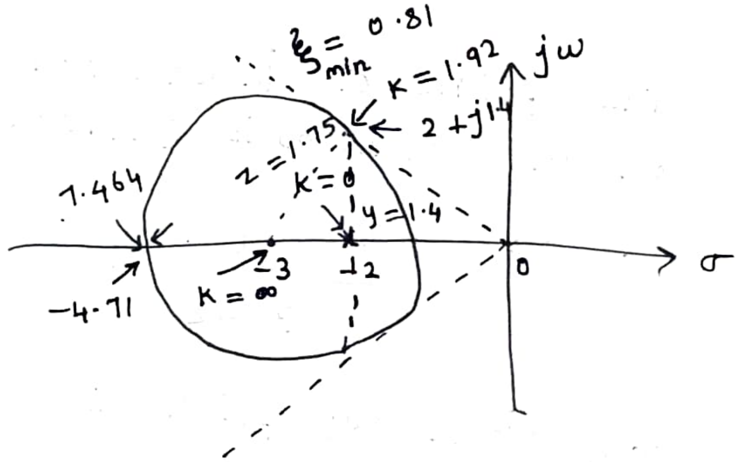
The K value at breakaway point is $K = 0.573$ and break-in point is $K = 7.464$

Therefore, the range of values of K for a system to be overdamped is,

$$0 < K < 0.573$$

The range of values for a system to be underdamped is,

$$0.573 < K < 7.464$$



Q7) Draw Complete root locus for

$$G(s)\,H(s) = \frac{K(s^2+2s+10)}{s^2(s+2)}$$

The open loop poles are at $s=0$, $s=0$, $s=-2$ $\therefore n=3$

The open loop zeros are at

$$s = \frac{-2 \pm \sqrt{4-40}}{2}$$

$$= -1 \pm j3$$

$$\therefore m = 2$$

The number of Asymptodes $= n-m$

$$= 3-2$$

$$= 1$$

The angle of asymptode $\theta_q = \frac{(2q+1)\pi}{n-m}$

$$\theta_0 = \pi$$

The points of intersection of asymptodes on the real axis (centroid) is given by

$$-\sigma = \frac{-2-(-1-1)}{3-2} = 0$$

The breakaway points is at origin

$$\pm\frac{\pi}{r} = \pm\frac{180^\circ}{2} = \pm 90^\circ$$

The root locus exists on the real axis to the left of $s = -2$

The angle of arrival,

$$\phi = \theta_3 - (\theta_1 + \theta_2 + \theta_4) = 90^{\circ} - (108.4^{\circ} + 108.4 + 71.6)$$

$$= -198.4^{\circ}$$

The angle of arrival at the complex zero is $\theta_a = -18.4^{\circ}$.

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{K(s^2 + 2s + 10)}{s^2(s+2)}$$

$$s^3 + (2+K)s^2 + 2Ks + 10K = 0$$

The Routh Table is as Follows:

s^3	1	$2K$
s^2	$2+K$	$10K$
s^1	$\frac{2K^2 + 4K - 10K}{2+K}$	
s^0	$10K$	

For stability, all the elements in first column of the Routh array must be positive.

$$10K > 0$$

$$K > 0$$

$$2 + K > 0$$

$$K < -2$$

$$2K^2 - 6K > 0$$

$$K > 3$$

So the range of values of K is

$3 < K < \infty$.

The marginal stability is $K_m = 3$

The frequency of oscillations is given by solution of auxiliary equation,

$$(2 + K) s^2 + 10K = 0$$

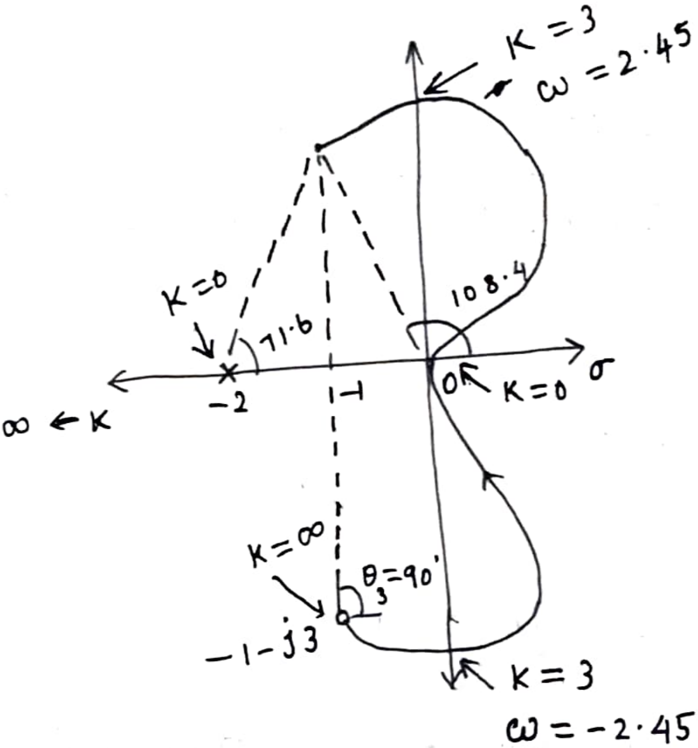
$$(2 + K_m) s^2 + 10 K_m = 0$$

$$(2 + 3) s^2 + 10(3) = 0$$

$$s^2 = \frac{-30}{5} = -6$$

$$s = \pm j2.45$$

$$\omega = 2.45 \text{ rad/s}.$$



Q8) Sketch the polar plot of Transfer Function given below. Determine whether these plots cross the real axis.

$$G(s) = \frac{1}{(2s+1)(s+1)}$$

Soln,

$$G(j\omega) = \frac{1}{(1+j\omega)(1+2j\omega)}$$

$$G(j\omega) = \frac{(1-j\omega)(1-j2\omega)}{(1+j\omega)(1+2j\omega)(1-j\omega)(1-2j\omega)}$$

$$= \frac{(1-2\omega)^2 - j3\omega}{(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{1-2\omega^2}{(1+\omega^2)(1+4\omega^2)} - j\frac{3\omega}{(1+\omega^2)(1+4\omega^2)}$$

when $\omega = 0$

$$G(j0) = 1 - j0$$

when $\omega = \infty$

$$G(j\infty) = -0 - j0$$

This plot does not cross real axis.
It crosses the imaginary axis at the frequency given by the solution of

$$\frac{1-2\omega^2}{(1+\omega^2)(1+4\omega^2)} = 0$$

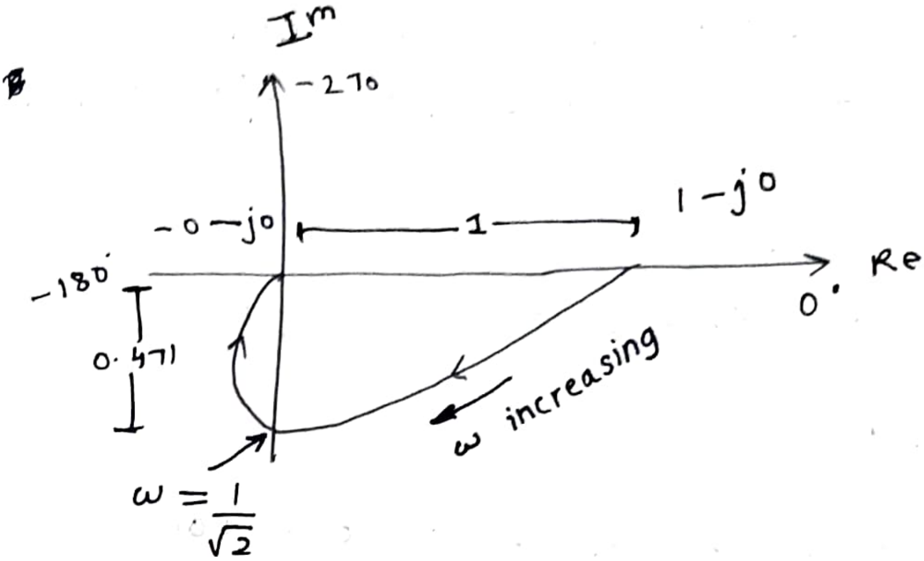
$$1 - 2\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{2}}$$

The value of $G(j\omega)$ at this frequency is,

$$-j\left.\frac{3\omega}{(1+\omega^2)(1+4\omega)^2}\right|_{\omega=\frac{1}{\sqrt{2}}} = -j\frac{\sqrt{2}}{3}$$

$$= -j0.471$$



Q9) Sketch the Nyquist plot and there from assess the stability of closed-loop system whose open-loop transfer function is,

$$G(s)\,H(s) = \frac{K\,(s+4)}{s^2\,(s+1)}$$

Soln,

$$G(j\omega)\,H(j\omega) = \frac{K\,(j\omega+4)}{(j\omega)^2\,(j\omega+1)}$$

$$= \frac{K\,(4+j\omega)\,(1-j\omega)}{-\omega^2\,(1+j\omega)\,.\,(1-j\omega)}$$

$$= \frac{K\,[\,(4+\omega^2) - j\,3\omega\,]}{-\omega^2\,(1+\omega^2)}$$

$$= \frac{-K\,(4+\omega^2)}{\omega^2\,(1+\omega^2)} + j\,\frac{3}{\omega\,(1+\omega^2)}$$

Along the segment of Nyquist contour on $j\omega$ – axis, s varies from $-j\infty$ to $+j\infty$

At $\omega = -\infty$,

$$G(j\omega)\, H(j\omega) = -0 - j0$$

At $\omega = 0^-$

$$G(j\omega)\, H(j\omega) = -\infty - j\infty$$

At $\omega = 0^+$

$$G(j\omega)\, H(j\omega) = -\infty + j\infty$$

At $\omega = +\infty$

$$G(j\omega)\, H(j\omega) = -0 + j0$$

The pole at origin of Nyquist Contour C_1 represented by $s = \epsilon e^{j\theta}$
$\epsilon \to 0$

$$\lim_{\epsilon \to 0} \frac{K(\epsilon e^{j\theta} + 4)}{(\epsilon e^{j\theta})^2 (\epsilon e^{j\theta} + 1)}$$

$$= \lim_{\epsilon \to 0} \frac{4K}{\epsilon^2 e^{j2\theta}} = \infty e^{-j2\theta}$$

$$= \infty \angle +180^\circ \longrightarrow \angle 0^\circ \longrightarrow \angle -180^\circ$$

It is infinite circular arc in clock wise direction.

The infinite semicircular arc of Nyquist contour (C_2) represented by $s = \underset{R \to \infty}{R e^{j\phi}}$

$$= \lim_{R \to \infty} \frac{K(Re^{j\phi} + 4)}{(Re^{j\phi})^2 (Re^{j\phi} + 1)}$$

$$= \lim_{R \to \infty} \frac{K}{R^2 e^{+j2\phi}} = 0 e^{-j2\phi} = 0 \angle -180^{\circ}$$

$$\to \angle 0^{\circ} \to \angle +180^{\circ}$$

The map turns around the origin from $\angle -180^{\circ} \to \angle 0^{\circ} \to \angle \pm 180^{\circ}$ as sketched. ~~in~~ Looking at imaginary axis of $G(j\omega) H(j\omega)$, we observe that the Nyquist plot intersects the real axis at $\omega = \infty$.

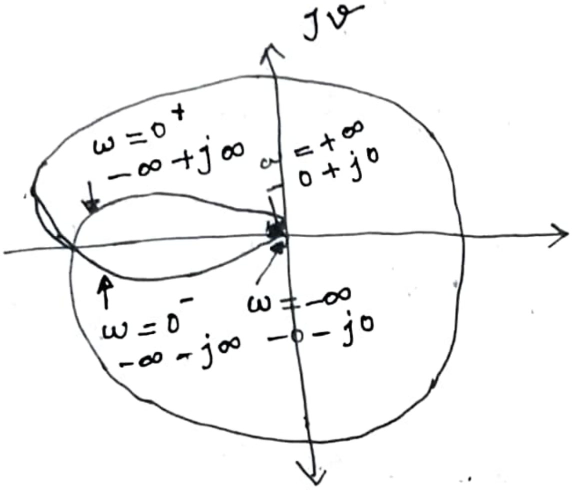
The point of intersection of Nyquist plot on real axis is obtained by setting the imag part to 0. i.e.

$$\frac{3\omega K}{\omega^2 (1+\omega^2)} = 0$$

$$\omega = 0$$

The value of real part at this frequency is obtained by substituting this value of ω in the real part $G(j\omega)H(j\omega)$ i.e.

$$\frac{-K(4+\omega^2)}{\omega^2(1+\omega^2)} = \infty$$



Q10) Sketch the Nyquist plot and comment on the stability of closed-loop system ~~whie~~ whose open-loop transfer function is,

$$G(s)H(s) = \frac{K(s-4)}{(s+1)^2}$$

Soln,

The given open loop system has one zero in right-half of the s-plane. The sinusoidal transfer function is,

$$G(j\omega)H(j\omega) = \frac{K(j\omega-4)}{(j\omega+1)^2}$$

$$= \frac{K(j\omega-4)(1-j\omega)^2}{(1+j\omega)^2(1-j\omega)^2}$$

$$= \cancel{K (j\omega - 4)} \; \frac{K(6\omega^2 - 4)}{(1+\omega^2)^2} + j\,\frac{K\omega(9-\omega^2)}{(1+\omega^2)^2}$$

Along the segment (C_1) of Nyquist contour on the $j\omega$ axis, s values from $-j\infty$ to $+j\infty$.

At $\omega = -\infty$

$G(j\omega)\,H(j\omega) = 0 + j0$

At $\omega = 0^-$

$G(j\omega)\,H(j\omega) = -4K - j0$

At $\omega = 0^+$

$G(j\omega)\,H(j\omega) = -4K + j0$

At $\omega = +\infty$

$G(j\omega)\,H(j\omega) = 0 - j0$

$$s = \lim_{R \to \infty} Re^{j\phi}$$

$$\lim_{R\to\infty} \frac{K(Re^{j\phi} - 4)}{(Re^{j\phi}+1)^2} = \lim_{R\to\infty} \frac{K}{Re^{j\phi}} = 0e^{-j\phi}$$

$$= 0\angle -90 \rightarrow \angle 0^\circ \rightarrow \angle 90^\circ$$

$$\frac{K\omega(9-\omega^2)}{(1+\omega^2)^2} = 0$$

$$\omega^2 = 9$$

$$\omega = \pm 3 \text{ rad/s}$$

$$\frac{K(6\omega^2-4)}{(1+\omega^2)^2} = \frac{K(6\times 9-4)}{(1+9)^2} = \frac{K}{2}$$

$\frac{K}{2} < -1$ or $K > \frac{1}{4}$ for $K < -2$ or $K > \frac{1}{4}$

$$N = -1$$

$$P = 0$$

$$N = P - Z$$

$$-1 = 0 - Z$$

$$Z = 1$$