10/2/23

Stability :

To analyse stability four techniques are available:

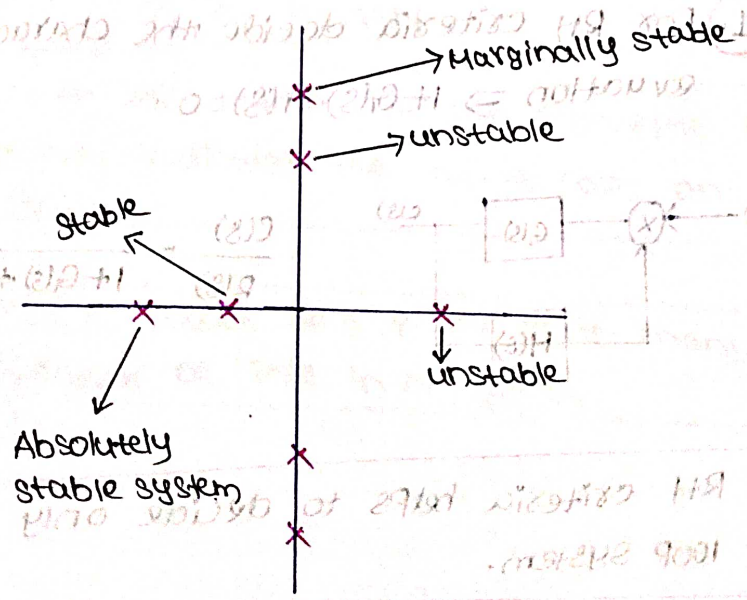
1. RH criteria
2. Root Locus
3. Nyquist Plot
4. Bode plot

A System will be stable when it satisfies:

* BIBO condition
* My signal must be absolutely summable or integrable.
* My system must have finite Maxima and Minima
* My system or response must be converging in nature.
* More than one pole on imaginary axis then unstable.

→ Bounded means Amplitude should be finite.

→ Pole will decide the stability.



Note: More than one pole on imaginary axis then unstable.

RH criteria

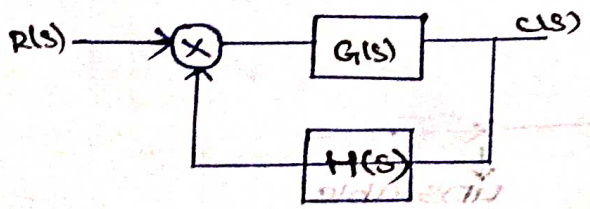
When the order of the system increases it is very difficult to calculate the roots as a result to determine the system stability will be challenging. Therefore RH criteria is an alternative solution to find out the roots of higher order Equation.

Outcomes of RH criteria: It will give information of stability of the system.

1. we can know stability of system by these method.
2. we can find out number of poles in the LHS, RHS and imaginary axis.
3. we can findout range of 'k' to decide system stability.
4. The frequency of oscillation of system can be determined.

Rules for RH criteria:

1. For RH criteria decide the characteristic equation $\Rightarrow 1 + G(s) \cdot H(s) = 0$.



$$\frac{C(s)}{R(s)} = \frac{1}{1 + G(s) H(s)}$$

RH criteria helps to decide only closed loop system.

(2.) From characteristic equation formulate RH table.

Ex:- characteristic eqn $= s^6 + 5s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1 = 0$.

1st column

s^6	1	4	2	1
s^5	5	3	1	0
s^4	$\frac{5\cdot4-1\times3}{5}$	$\frac{5\times2-1\times1}{5}$	$\frac{5\times1-1\times0}{5}$	0
s^3	A			
s^2	B			
s	C			
s^0	D			

$\boxed{\frac{(1)-(2)}{\text{dotted value}}}$

(3.) If all the system of first column are equal in sign (either '+'ve or '-'ve) then system is stable.

(4.) If any sign change in the elements of first column will be unstable.

(i) No. of sign changes of the elements in first column indicates the no. of poles on Right side.

(ii) Highest power of s gives that many number of pole in a system.

15. Find the no. of pole in RHS of s plane for the characteristic equation $s^4+2s^3+3s^2+4s+5=0$.

s^4	1	3	5
s^3	2	4	0
s^2	1	5	0
s^1	-6	0	0
s^0	5		

NSC - No sign change

Sc - Sign change.

No. of Poles at RHS = 2

No. of Poles at LHS = 2.

16. Find the no. of pole in the LHS of s plane if the characteristic equation $s^4+2s^3+3s^2+s+2=0$.

s^4	1	3	2
s^3	2	1	0
s^2	5/2	2	0
s^1	-3/5	0	0
s^0	2	0	0

No. of Poles at LHS = 2.

5. If any coefficient of characteristic equation is negative then my system is unstable.

Ex:- $s^4+2s^3+3s^2-2s+2=0$.

6. If any power of s term is missing in the characteristic system then system is unstable.

Ex:- $s^4+2s^3+3s^2+2=0$.

17. Determine the stability of system if characteristic equation $3s^3+4s^2+5s+8=0$.

s^3	3	5	0
s^2	4	8	0
s	-1	0	0
s^0	8	0	0

The characteristic equation $3s^3+4s^2+5s+8=0$ is unstable.

7. $a_3s^3+a_2s^2+a_1s+a_0=0$.

$a_2a_1 > a_3a_0 \longrightarrow$ stable

$a_2a_1 = a_3a_0 \longrightarrow$ Marginally stable

$a_2a_1 < a_3a_0 \longrightarrow$ unstable.

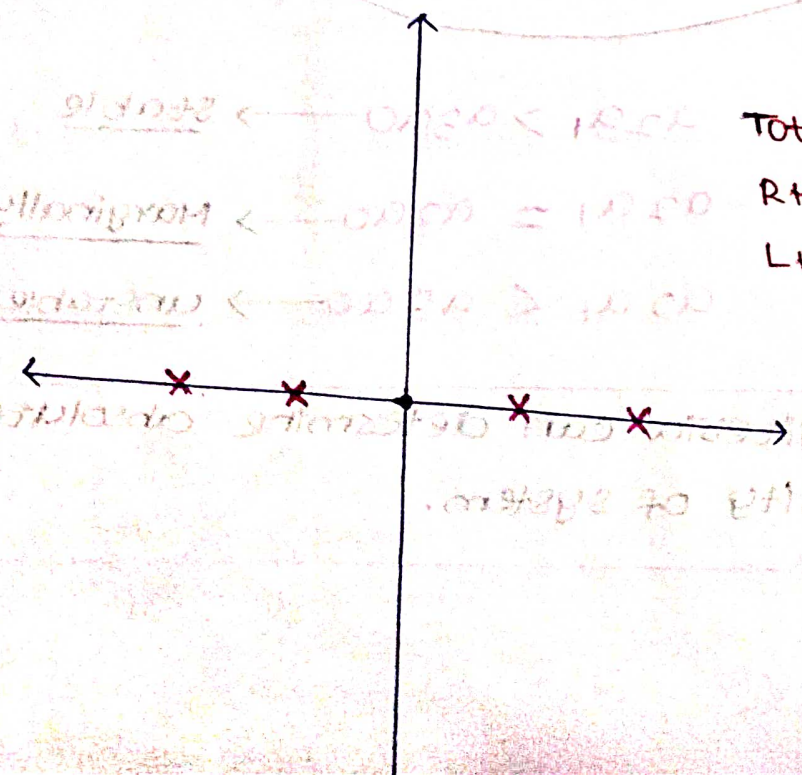
RH criteria can determine absolute stability of system.

8. If any element in the first column is zero, then replace zero with a small positive number 'ϵ' and proceed. Finally ϵ=0 and check the number of sign changes.

s^4	1	2	8
s^3	2	4	0
s^2	ϵ	8	0
s^1	$\frac{4\epsilon-16}{\epsilon}$ = −∞	0	
s^0	8		

NSC, NSC, SC, SC

High power of s in the characteristic equation will decide that many number of poles.



Total = 4

RHS = 2

LHS = 2

(9.) If we get complete row is zero it means poles are symmetrical about the origin. These create Auxulary equation from the coefficient of row which is about the zero line then differentiate the Auxulary equation and Replace the row of zero by Differential equation. (ROZ)

A)

s^5	1	3	2
s^4	1	3	2
ROZ → s^3	0 → 4	0 → 6	0 → 2
s^2	$\frac{3}{2}$	2	
s^1	$2/3$	0	
s^0	2		

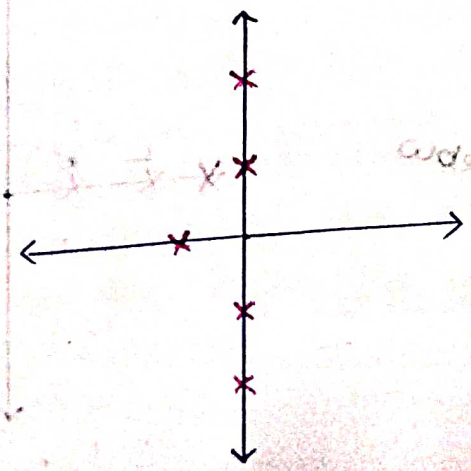
→ Auxulary Equation.

AE = Auxulary Eqn = $s^4 + 3s^2 + 2 = 0$.

$$\frac{dAE}{ds} = 4s^3 + 6s = 0.$$

Auxulary Equation always can be formulated by even power of s.

Highest power of s in the Auxulary Eqn can create that many number of pole as symmetric pole.



Total = 5

Marginally stable!

The imaginary root of s in the auxuliary equation will give the frequency of oscillation.

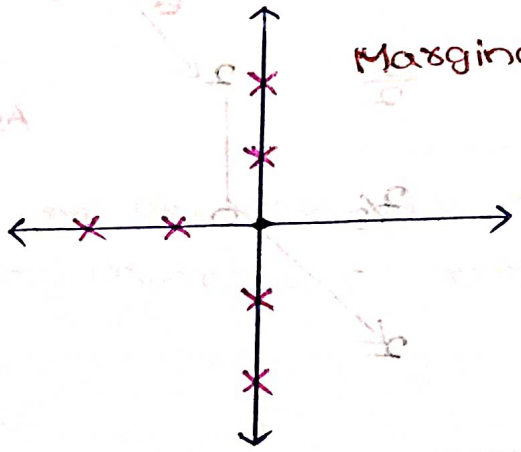
20.

s^6	+				
s^5	+				
s^4	+				
ROZ → s^3	0	0	0	0	
s^2	+				
s^1	+				
s^0	+				

Total Poles = 6

Symmetric Pole = 4

RHS = 0.



Marginally Stable

21.

s^5	+				
s^4	+				
ROZ → s^3	0	0	0	0	
s^2	−				
s^1	+				
s^0	+				

AE

SC

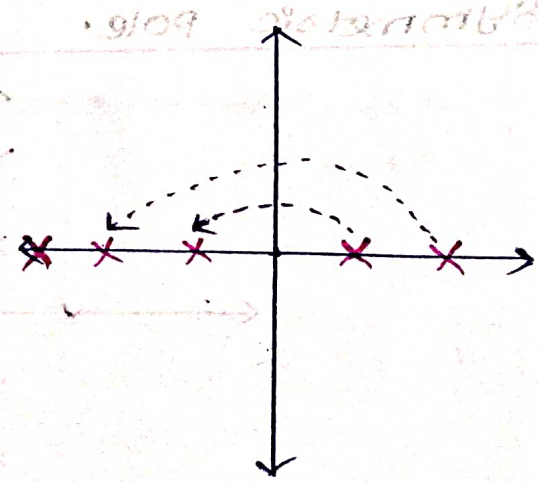
SC

Below

Total Poles = 5

RHS = 2

Symmetrical = 4



22.

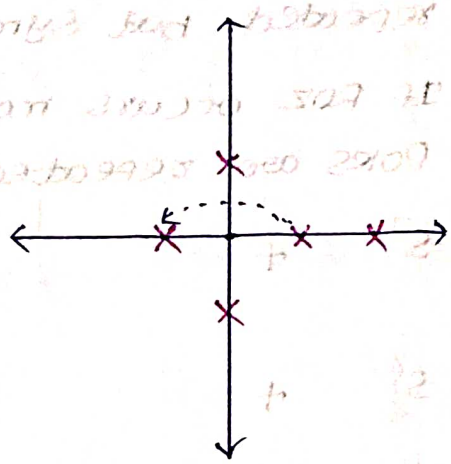
s^5	−				
s^4	+				
ROZ → s^3	0	0	0	0	
s^2	+				
s^1	+				
s^0	−				

SC, Above, AE, SC, Below

Total poles = 5

Symmetric = 4

RHS = 2.



23.

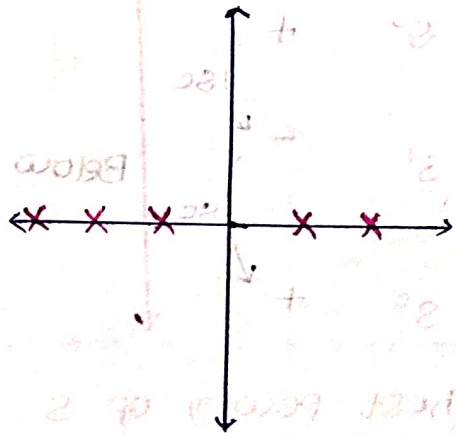
s^5	+				
s^4	+				
ROZ → s^3	0	0	0	0	
s^2	+				
s^1	−				
s^0	+				

AE, SC, SC

Total poles = 5

RHS = 2.

Symmetric = 4



If sign change occurs below ROZ then there must be a symmetric pole in the left to the pole placed in the right side of s-plane.

If sign change occurs above ROZ then there is no symmetric pole present left to the pole placed in the right side of S plane.

(10.) If ROZ occurs once it means poles are not repeated but symmetric.

If ROZ occurs more than once it means Poles are repeated and symmetric.

24.

S^7	+				
S^6	+				
S^5	0	0	0	0	
S^4	+				
S^3	0	0	0	0	
S^2	+				
S^1	−				
S^0	+				

SC

SC

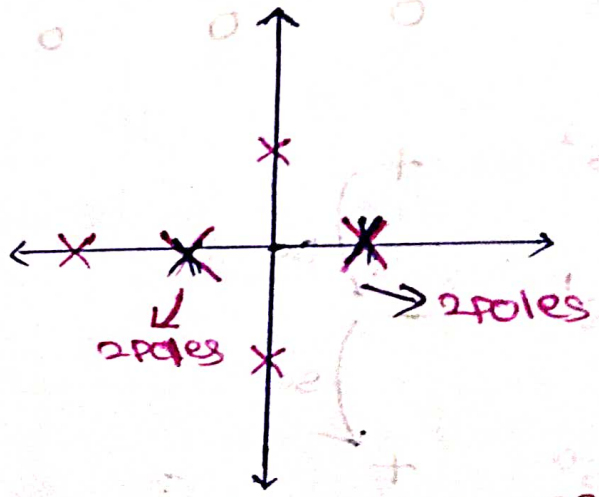
Below

Highest S power final AE

Total = 7

RHS = 2

Symmetric = 6.



Highest power of S Auxuloary Equation we should consider finally.

How many zeros that many repeated poles. X → 2poles

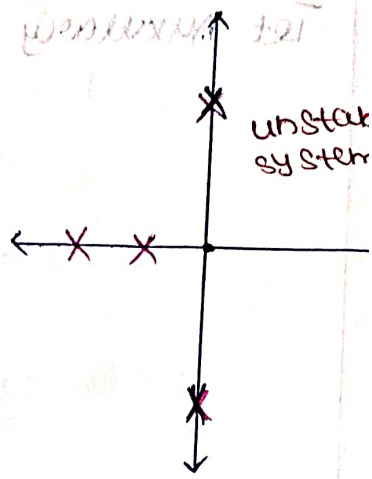
25.

s^6	+			
s^5	+			
s^4	+			
s^3	0	0	0	0
s^2	+			
s^1	0	0	0	0
s^0	+			

Total = 6

RHS = 0.

Symmetric = 4



26.

s^7	+			
s^6	−			
s^5	+			
s^4	+			
s^3	0	0	0	0
s^2	+			
s^1	−			
s^0	+			

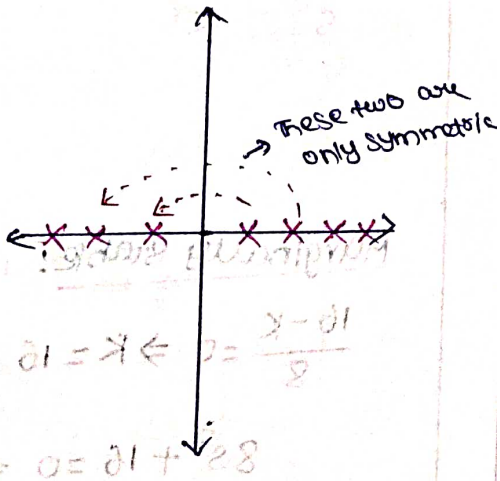
S.C. TOP

Total = 7.

Symmetric = 4

RHS = 4

SC Below



15/02/23

(11) Marginally stable system are oscillatory in nature. Then it has frequency of oscillation.
To calculate the frequency of oscillation make Auxilary equation zero and solve root of 's'. The imaginary component of root will give frequency of oscillation.

Ex:-
Let Auxilary Equation $\rightarrow 2s^2+4=0$.

$$s^2=-2$$

$$s=\pm j\sqrt{2}$$

$$\omega=\pm j\sqrt{2}$$

$$\omega=\sqrt{2}$$

27) Determine the conditional stability of system if characteristic equation is given by s^3+8s^2+2s+K.

	s^3	1	2
	s^2	8	K
ROZ →	s^1	$\frac{16-K}{8}$	0
	s^0	K	

Absolute stable:

$K>0 \rightarrow (1)$

$\frac{16-K}{8}>0$

$K<16 \rightarrow (2)$

$0<K<16$
↳ absolute stable

Marginally stable:

$\frac{16-K}{8}=0 \Rightarrow K=16$

$8s^2+16=0 \Rightarrow s^2+2=0$.

$s=j\sqrt{2}$

28 Determine the value of K and b for the system Transfer function $G(s) = \frac{K(s+1)}{s^3+bs^2+3s+1}$, $H(s)=1$. If frequency of oscillation system is $\omega = 2$ rad/sec.

Characteristic equation;

$$1+G(s)\cdot H(s)=0$$

$$1+\frac{K(s+1)}{s^3+bs^2+3s+1}=0$$

$$s^3+bs^2+3s+1+K(s+1)=0$$

$$s^3+bs^2+(3+K)s+(1+K)=0$$

$$1+K=b(3+K) \Rightarrow b=\frac{1+K}{3+K} \rightarrow (1)$$

s^3	1	3+K
s^2	b	1+K
s^1	0	0
s^0		

If a row zero means above power will be the AE equation.

$$AE = bs^2+1+K=0$$

$$\Rightarrow b(j\omega)^2+1+K=0$$

$$\Rightarrow -b\omega^2+1+K=0$$

$$-4b+1+K=0$$

$$b=\frac{1+K}{4} \rightarrow (2)$$

Now, (1) = (2)

$$\frac{\cancel{1+K}}{3+K}=\frac{\cancel{1+K}}{4}$$

$$4=3+K \Rightarrow \boxed{K=1}, \quad b=\frac{1}{2}$$

29. The open loop transfer function of a unit feedback $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$ by applying RH criterion determine value of K for stability and marginally stability and also determine the frequency of oscillation (ω)?

Characteristic equation:

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0.$$

$$(s+2)(s+4)(s^2+6s+25) + K = 0.$$

$$s^4 + 12s^3 + 69s^2 + 198s + (200+K) = 0.$$

s^4	1	69	$200+K$
s^3	12	198	0
s^2	52.5	$200+K$	
s^1	$198 - \frac{12(200+K)}{52.5}$	0	
s^0	$200+K$		

Absolute stability:

* $200+K > 0$

$K > -200$

* $198 - \dfrac{12(200+K)}{52.5} > 0$

$K < 666.25$

Marginally stable:

$K = 666.25$

Auxiliary Eqn:

$52.5s^2 + 866.25 = 0.$

$-52.5\omega^2 + 866.25 = 0$

$\omega = 4.062$

$\boxed{-200 < K < 666.25}$

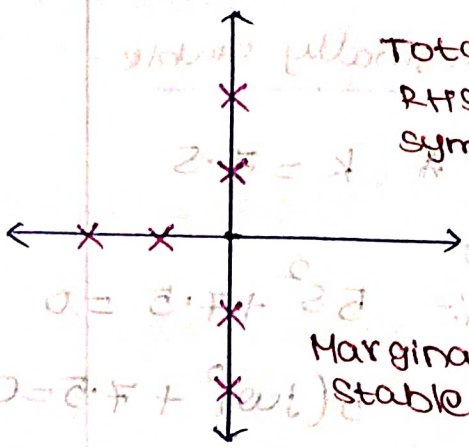
30. Determine the pole and stability of system if characteristic equation $s^6+3s^5+5s^4+9s^3+8s^2+6s+4=0$.

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	0
POZ → s^3	0 → 8	0 → 12	0 → 0	0
s^2	3	4		
s^1	4/3			
s^0	4			

AE

$AE = 2s^4+6s^2+4=0$

$\frac{dAE}{ds} = 8s^3+12s=0.$



Total=6
RHS=0
symm=4

Marginally stable.

$2s^4+6s^2+4=0$

$s^4+3s^2+2=0 \rightarrow s=\pm j1, \pm j\sqrt{2}$

31. Determine the system stability and value of range of K. $G(s)\,H(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$

$$\boxed{\begin{array}{l}\text{Replace if } e^{-s} \rightarrow 1-s \\ \qquad\qquad e^{+s} \rightarrow 1+s\end{array}}$$

$$G(s)\cdot H(s) = \frac{K(1-s)}{s(s^2+5s+9)}$$

$1+G(s)\cdot H(s)=0$

$s(s^2+5s+9)+K(1-s)=0.$

$$s^3 + 5s^2 + (9-k)s + k = 0.$$

	s^3	1	$9-k$
	s^2	5	k
ROZ →	s^1	$9-k-\frac{k}{5}$	0
	s^0	k	

Absolutely stable

* $k > 0$

* $9-k-\frac{k}{5} > 0$

$45 - 5k - k > 0$

$6k < 45$

$k < \frac{45}{6}$

$k < 7.5$

$0 < k < 7.5.$

Marginally stable.

* $k = 7.5$

AE :- $5s^2 + 7.5 = 0$

$5(j\omega)^2 + 7.5 = 0.$

$-5\omega^2 + 7.5 = 0$

$\omega^2 = \frac{7.5}{5} = 1.5$

$\omega = 1.22$ //

2. Determine the range of k such that the characteristic Equation is $s^3 + 3(k+1)s^2 + (7k+5)s + (4k+7) = 0$ has roots more negative than $s = -1$.

$\boxed{\text{Replace } s = z - 1}$

$$s^3 + 3(k+1)s^2 + (7k+5)s + (4k+7) = 0$$

$$(z-1)^3 + 3(k+1)(z-1)^2 + (7k+5)(z-1) + (4k+7) = 0.$$

$$\Rightarrow z^3 - 1 + 3z - 3z^2 + (3k+3)[z^2 + 1 - 2z] + (7k+5)(z-1) + (4k+7) = 0.$$

$$\Rightarrow z^3 - 3z^2 + 3z - 1 + 3kz^2 + 3k - 6kz + 3z^2 + 3 - 6z + 7kz - 7k + 5z - 5 + 4k + 7 = 0.$$

$$\Rightarrow z^3 + (3k - 3 + 3)z^2 + (3 - 6k - 6 + 7k + 5)z - 1 + 3k + 3 - 7k - 5 + 4k + 7 = 0.$$

$$\Rightarrow z^3 + 3kz^2 + (2+k)z + 4 = 0.$$

z^3	1	$2+k$
z^2	$3k$	4
z^1	$\frac{6k+3k^2-4}{3k}$	0
z^0	4	

$k > 0$

$$\frac{6k + 3k^2 - 4}{3k} > 0$$

$$3k^2 + 6k - 4 > 0.$$

$$k = \frac{-6 \pm \sqrt{36 + 48}}{6} > 0$$

$$k = \frac{3.46}{6} \text{ or } -2.52$$

$$= 0.526 \text{ or } -2.52$$

17/02/23

Root Locus: Path of closed loop poles when gain "K" values from "0 to ∞".

Root → Pole, Locus → Path

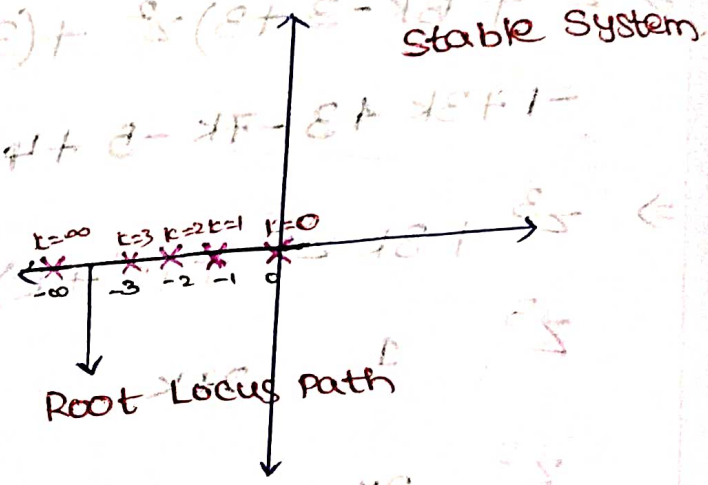
* It is a absolute and relative stability.

Ex:- $G(S)\cdot H(S) = \frac{K}{S}$

$CLTF = \frac{K}{S+K}$

$S = -K$

K	S
0	0
1	-1
2	-2
⋮	⋮
∞	-∞

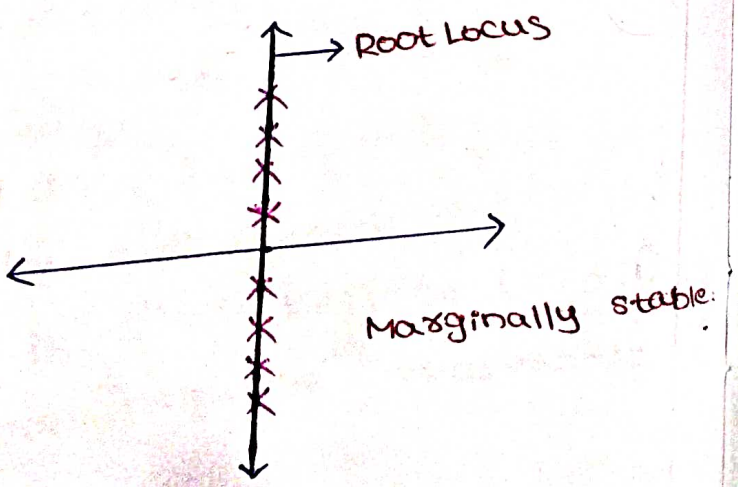


Ex:- $G(S)\cdot H(S) = \frac{K}{S^2}$

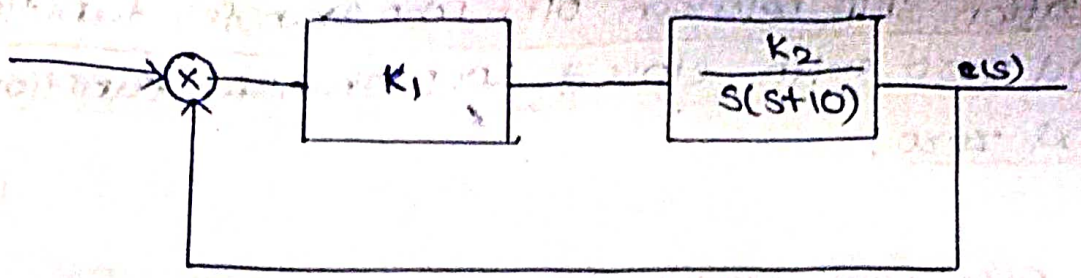
$CLTF = \frac{K}{S^2+K}$

$S = \pm j\sqrt{K}$

K	S
0	0
1	±j
2	$\pm j\sqrt{2}$
⋮	⋮
∞	±j∞



Ex 1:



$$\frac{C(S)}{R(S)} = \frac{\dfrac{K_1 K_2}{S(S+10)}}{1 + \dfrac{K_1 K_2}{S(S+10)}}$$

$$= \frac{K_1 K_2}{S^2 + 10S + K_1 \cdot K_2}$$

Let $K_1 K_2 = K$.

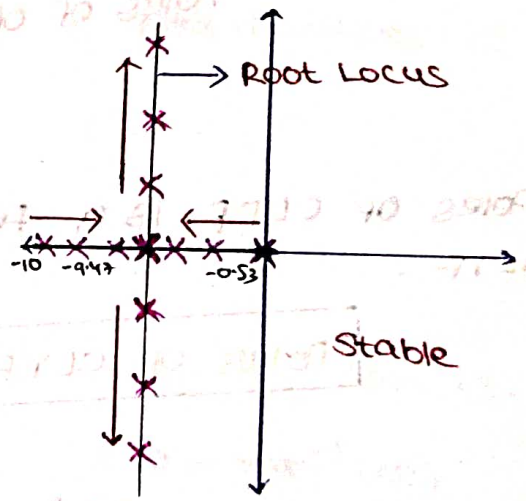
$$= \frac{K}{S^2 + 10S + K}$$

$$S = \frac{-10 \pm \sqrt{100 - 4K}}{2}$$

$$S = -5 \pm \sqrt{25 - K}$$

S_1, S_2

K	S_1	S_2
0	-10	0
5	-9.47	-0.53
10	-8.87	-1.13
25	-5	-5
30	-5+j2.24	-5-j2.24
35	-5+j3.16	-5-j3.16
40	-5+j4.47	-5-j4.47



Root Locus helps to determine closed Loop System stability by considering open Loop Transfer function.

Relation Ship between open Loop Transfer function pole and zero with closed Loop Transfer function pole and zero:

Open Loop Transfer Function:

$$OLTF = K \cdot \frac{N(S)}{D(S)} = G(S) \cdot H(S)$$

$D(S) = 0 \longrightarrow$ Poles of OLTF

$N(S) = 0 \longrightarrow$ zeros of OLTF

Closed Loop Transfer Function:

$$CLTF = 1 + G(S) \cdot H(S) = 0$$

$$1 + K \cdot \frac{N(S)}{D(S)} = 0.$$

$$\underbrace{D(S)}_{\text{Poles of OLTF}} + K \cdot \underbrace{N(S)}_{\text{zeros of OLTF}} = 0 \longrightarrow \text{Poles of CLTF}$$

Poles of CLTF is a function of poles and zeros of OLTF.

$$\boxed{\text{Poles of CLTF} = f\,(\text{Poles and zeros of OLTF})}$$

$$D(S) + K\,N(S) = 0$$

Put $K = 0 \Rightarrow D(S) = 0 \rightarrow$ Poles of OLTF

$K = \infty \Rightarrow N(S) = 0 \rightarrow$ zeros of OLTF

OLTF		CLTF
Pole	$K = 0_{start}$	Pole
zero	$K = \infty_{stop}$	Pole

All the Root Locus Branch it will originates at pole ($k=0$) and terminates at zero ($k=\infty$).

case-1

Start pole — terminates at zero.

case-2

All the Root Locus must satisfy Magnitude criteria and Angle criteria.

$$1 + G(s) \cdot H(s) = 0.$$

$$\boxed{G(s) \cdot H(s) = -1 + j0} \rightarrow \text{critical point}$$

Magnitude criteria

$|M| = 1$

Angle criteria

$\phi = \pi - \tan^{-1}(0)$

$\angle G(s) \cdot H(s) = \pm 180^\circ (2q+1)$

$q = 0, 1, 2, 3 \ldots$

odd multiples of 180°.

Ex:- $\frac{1}{s+1}$, $s = -1 + j3$

$$\rightarrow \frac{1}{-1+j3+1} = \frac{1+j0}{0+3j}$$

$M = \frac{1}{3}$ $\qquad \angle G(s) \cdot H(s) = \tan^{-1}(0) - \tan^{-1}(3/0)$

$= 0 - \tan^{-1}(\infty)$

$= -\pi/2$

Both criterias are not satisfying so it is not root locus.