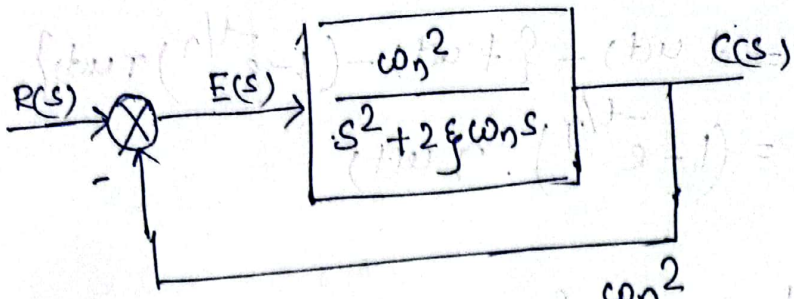
Second Order System



$$CLTF = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

✓ Order = 2 / CLTF / OLTF

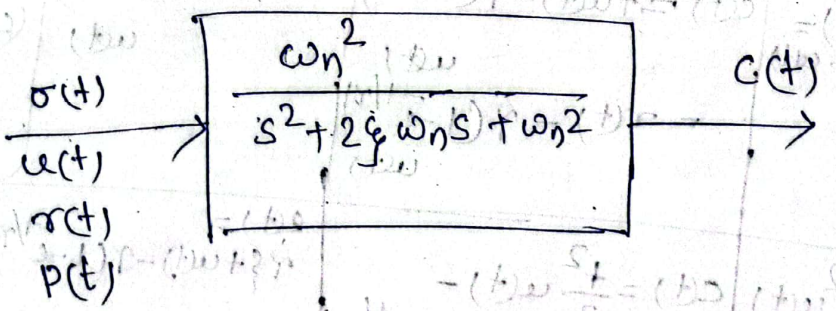
✓ Type → OLTF

$$OLTF = G(s)\cdot H(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s}$$

$$= \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

✓ Type = 1.

✓ System response analysis with standard i/p signals i.e. $\delta(t)$, $u(t)$, $r(t)$, $P(t)$.



ω_n → Natural freqy of oscillation

ξ → Damping ratio / Damping coefficient

(1) Impulse Response $\delta(t) \xrightarrow{s} 1$ $\quad C(s) = 1 \times \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

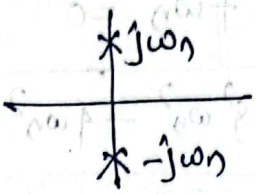
$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\boxed{1+G(s)\cdot H(s)=0}$

Characteristic eq $s^2+2\xi\omega_n s+\omega_n^2=0$

*→ ξ decides the behaviour of the system

$\xi=0$ $\quad s^2+\omega_n^2=0 \Rightarrow s=\pm j\omega_n$



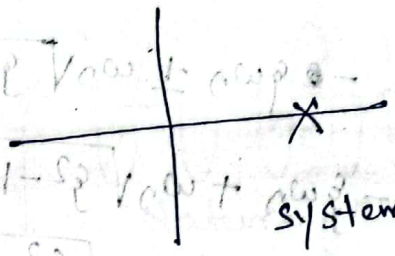
* Marginal stable system
* Poles are conjugate and completely imaginary

$\xi<0$, let $\xi=-1$

$s^2-2\omega_n s+\omega_n^2=0$

$(s-\omega_n)^2=0$

$s=\omega_n$



system unstable

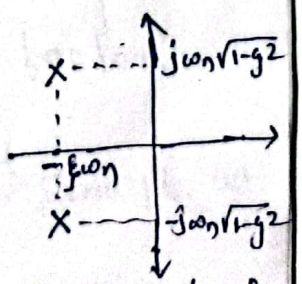
$0<\xi<1$ $\quad s^2+2\xi\omega_n s+\omega_n^2=0$

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2-4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



* Complex conjugate poles
* Stable system

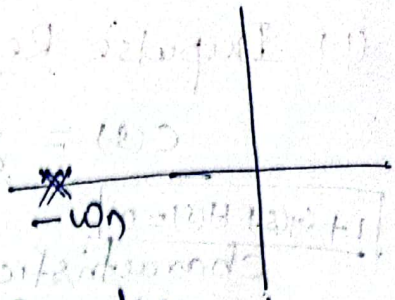
$\xi = 1$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 2\omega_n s + \omega_n^2 = 0.$$

$$(s + \omega_n)^2 = 0$$

$$s = -\omega_n, -\omega_n$$



* Repeating roots on -ve real axis.

$\xi > 1$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

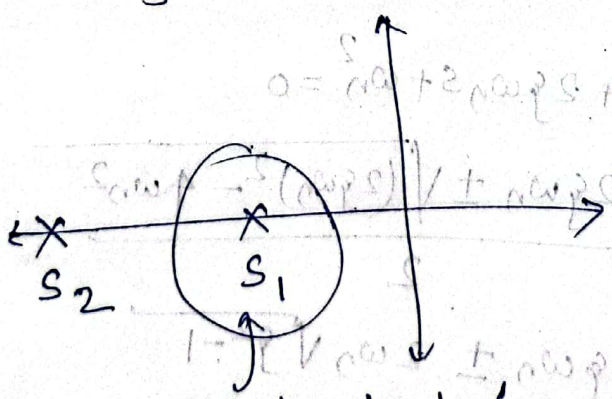
$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2 - 1}}{2}$$

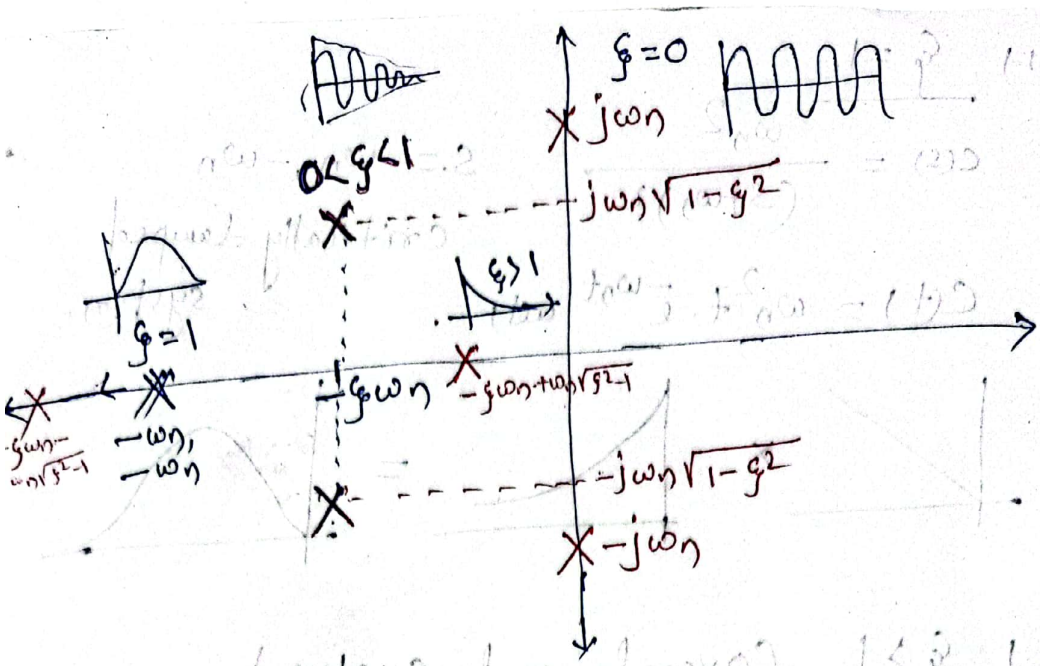
$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$s_1 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$s_2 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$



Significant pole/Dominant pole.

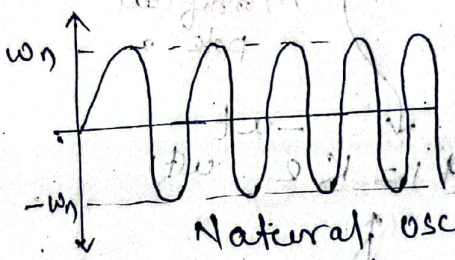


ξ = 0
0 < ξ < 1
ξ = 1
ξ > 1

(I) $\xi = 0$

$$C(s) = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$C(t) = \omega_n \sin \omega_n t \cdot u(t)$$



Natural oscillation
Undamped oscillation
sustained oscillation

(II) $0 < \xi < 1$

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n+j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n-j\omega_n\sqrt{1-\xi^2})}$$

$$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

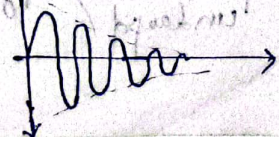
$\omega_d = \omega_n\sqrt{1-\xi^2}$ → damping frequy.

$\xi\omega_n \rightarrow$ damping factor. $\rightarrow \alpha$

$$C(s) = \frac{\omega_n^2}{(s+\alpha)^2+\omega_d^2} \xrightarrow{ILT} K e^{-\alpha t} \cdot \sin\omega_d t \; u(t)$$

$\boxed{\xi = \cos\theta}$

* Damped oscillation



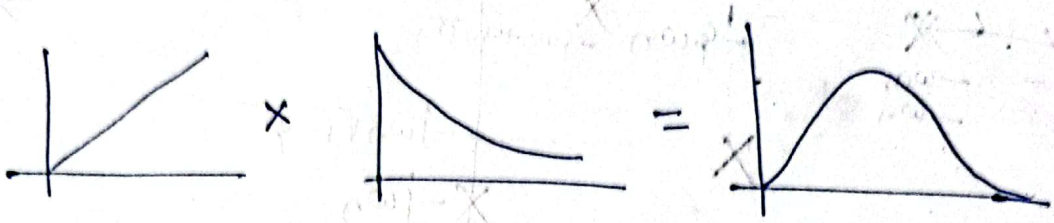
(iii) $\xi = 1$

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2}$$

$s = -\omega_n, -\omega_n$

Critically damped system.

$$c(t) = \omega_n^2 \cdot t \cdot e^{-\omega_n t} u(t)$$



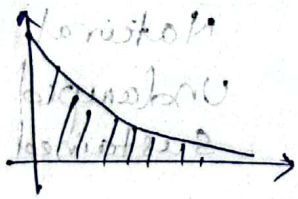
(iv) $\xi > 1$ (Overdamped system)

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1})(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1})}$$

insignificant pole

significant pole

$$c(t) = K e^{-xt} u(t)$$



Time constant $\tau = -\frac{1}{\text{Real part of pole}}$

$\xi = 0 \rightarrow \tau = \infty$

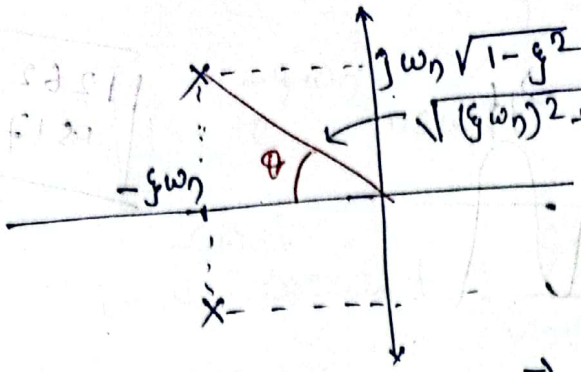
$0 < \xi < 1 \rightarrow \tau = \frac{1}{\xi\omega_n}$

$\xi = 1 \rightarrow \tau = \frac{1}{\omega_n}$

$\xi > 1 \rightarrow \tau = \frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}$

$\tau_{undamped} > \tau_{OD} > \tau_{critdamp} > \tau_{CD}$.

Under damped System



$$\sqrt{(\xi\omega_n)^2+\omega_n^2(1-\xi^2)} = \omega_n$$

$$\cos\theta = \frac{\xi\omega_n}{\omega_n}$$

$$\Rightarrow \boxed{\cos\theta = \xi}$$

$$\boxed{\theta = \cos^{-1}\xi}$$

$$\boxed{\theta \propto \frac{1}{\xi}}$$

(2) Second order system with unit step i/p signal

step Response of 2nd order system :-

$$\frac{1}{s} \longrightarrow \boxed{\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} \longrightarrow C(s).$$

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\xi = 0$

$\boxed{0<\xi<1}$ → Underdamped very important

$\xi = 1$

$\xi > 1.$

case-I $\xi = 0$

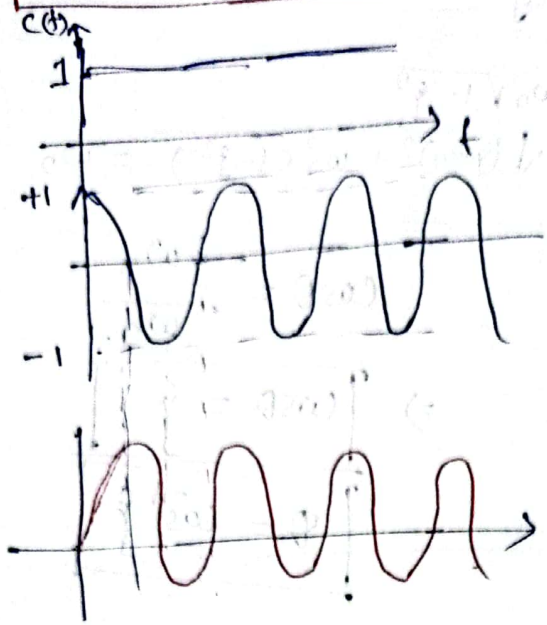
$$C(s) = \frac{1}{s}\cdot\frac{\omega_n^2}{s^2+\omega_n^2}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2}$$

$A = 1;\ B = -1,\ C = 0$

$$C(s) = \frac{1}{s} - \frac{s}{s^2+\omega_n^2}$$

$\boxed{c(t) = 1 - \cos\omega_n t}$ ← step response $\xi = 0$



1262
1517

$0 < \xi < 1$ in this condition the roots are

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$C(s) = \frac{1}{s}\,\frac{\omega_n^2}{(s+\xi\omega_n + j\omega_n\sqrt{1-\xi^2})(s+\xi\omega_n - j\omega_n\sqrt{1-\xi^2})}$$

A ⊕ B A ⊖ B

$$= \frac{1}{s}\,\frac{\omega_n^2}{(s+\xi\omega_n)^2 - (j\omega_n\sqrt{1-\xi^2})^2}$$

$$= \frac{\omega_n^2}{s\{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)\}}$$

$$= \frac{A}{s} + \frac{Bs+C}{(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)}$$

$A = 1,\ B = -1,\ C = -2\xi\omega_n$

$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

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$$= \frac{1}{s} - \left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} + \frac{\xi\omega_n \cdot \omega_d/\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \right]$$

$$= \left[1 - e^{-\xi\omega_n t} \cdot \cos\omega_d t + \frac{\xi\omega_n}{\omega_d} \cdot e^{-\xi\omega_n} \sin\omega_d t \right] u(t)$$

$$= 1 - e^{-\xi\omega_n t} \cdot \cos\omega_d t + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\omega_d t \Big]$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sqrt{1-\xi^2} \cdot \cos\omega_d t + \xi \sin\omega_d t \right]$$

✓ Put $\xi = \cos\theta$, $\sqrt{1-\xi^2} = \sin\theta$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin\theta \cdot \cos\omega_d t + \cos\theta \cdot \sin\omega_d t \right]$$

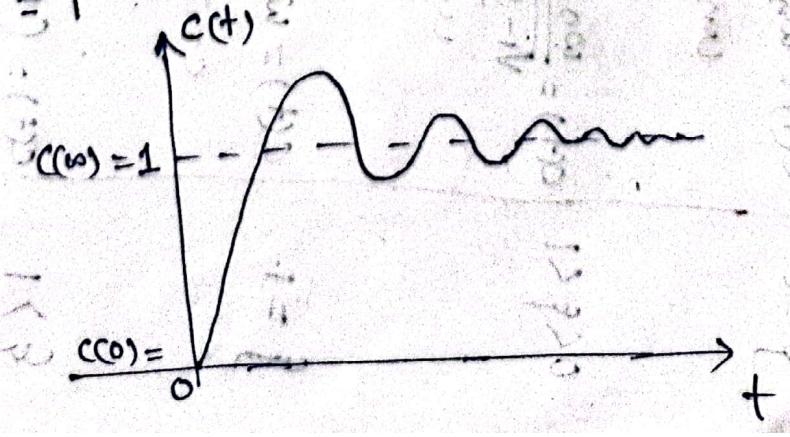
*

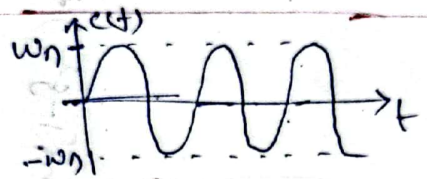
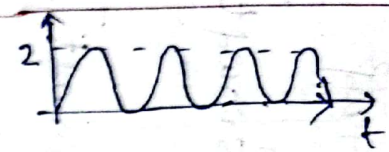
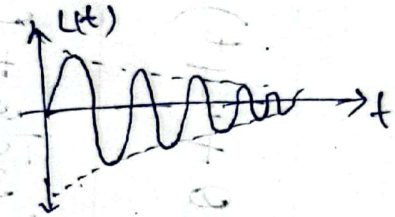
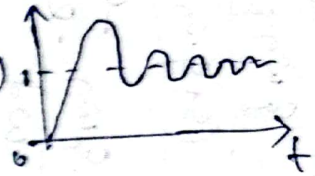
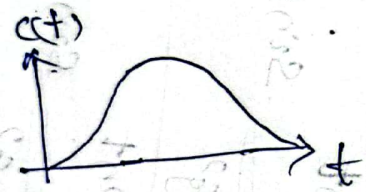
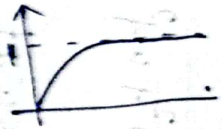
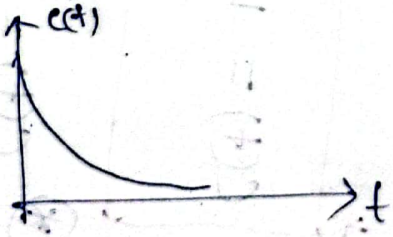
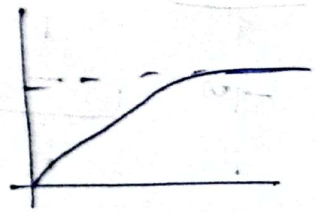
$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\theta + \omega_d t).}$$

(*) $$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\cos^{-1}\xi + (\omega_n\sqrt{1-\xi^2})t\right]$$

$$c(0) = 1 - \frac{1}{\cancel{\sqrt{1-\xi^2}}} \cancel{\sin\theta} = 0$$

$$c(\infty) = 1$$

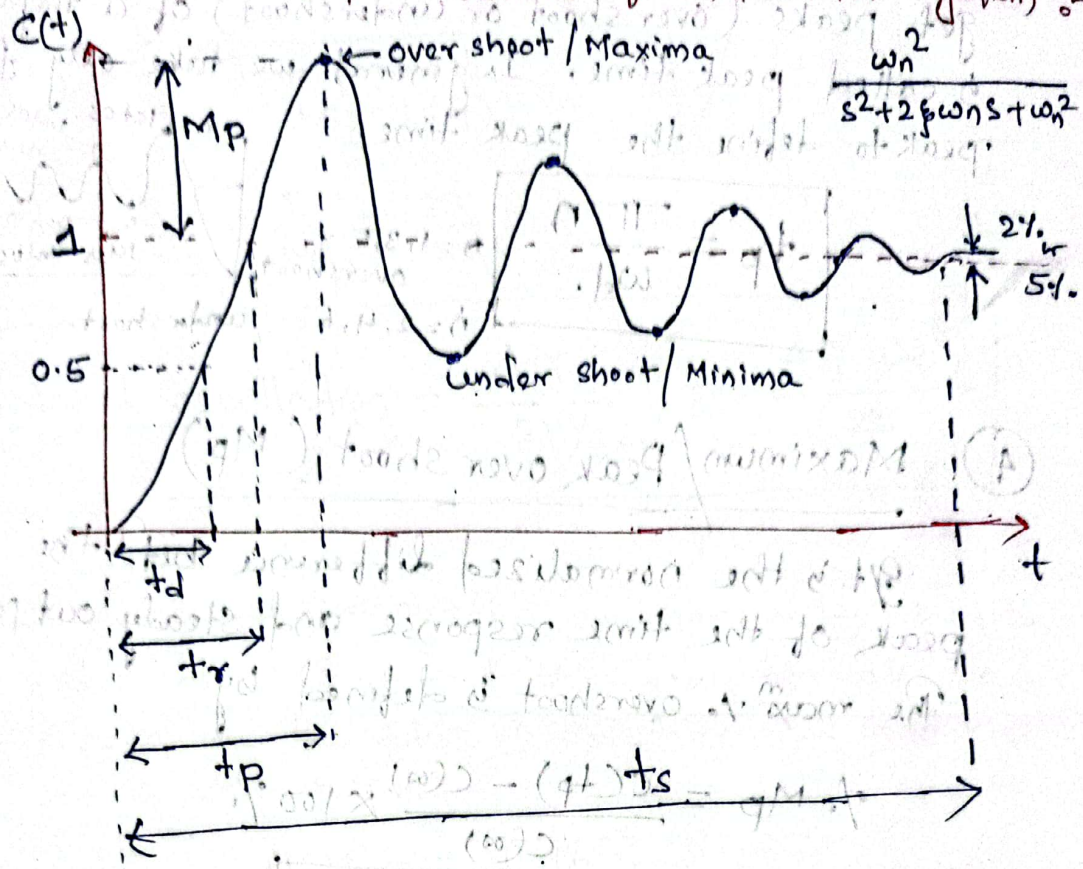


ξ	Impulse Response	Step Response
$\xi = 0$	$c(t) = \omega_n \sin \omega_n t$ undamped 	$c(t) = 1 - \cos \omega_n t$ 
$0 < \xi < 1$	$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \cdot \sin \omega_d t$ underdamped 	$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$ 
$\xi = 1$	$c(t) = \omega_n^2 t e^{-\omega_n t}$ critically damped 	$c(t) = 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$ 
$\xi > 1$	$c(t) = (\xi \omega_n - \omega_n \sqrt{\xi^2 - 1}) e^{-(\xi \omega_n - \omega_n \sqrt{\xi^2-1})t}$ 	$c(t) = 1 - e^{-[\xi \omega_n - \omega_n \sqrt{\xi^2-1}]t}$ 

$0 < \xi < 1 \rightarrow$ Underdamped

Time Domain parameters of 2nd Order system :-

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$



① Delay Time (t_d) :-

The delay time is the time required for the response to reach 50% of the final value in the first time.

$$\boxed{t_d = \frac{1+0.7\xi}{\omega_n}} \text{ second}$$

② Rise time (t_r) :-

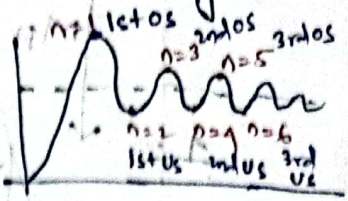
It is the time required for the response to rise from 0% to 100% of its final value in underdamped system.

$$\boxed{t_r = \frac{\pi-\theta}{\omega_d}}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\theta = \cos^{-1}\xi$$

③ **Peak time (t_p)** :— It is the time required to get peaks (over shoot or undershoot) of a response is called peak time. In general we take only first peak to define the peak time.



$$\boxed{t_p = \frac{\pi n}{\omega_d}}$$

$n = 1, 3, 5 \ldots$ overshoot

$n = 2, 4, 6 \ldots$ undershoot.

④ **Maximum/Peak over shoot (M_p)**

It is the normalized difference bet^n the peak of the time response and steady out put.

The max^m % overshoot is defined by

$$\% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

$$\boxed{\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$$

* $\xi = 1$ $\% M_p = 0$

$\xi = 0$ $\% M_p = 100\%$

⑤ **Settling time (t_s)**

It is the time required for the response to reach and stay within the specified range (2% to 5%) of its final value.

$$\boxed{\begin{aligned} t_s &= 4\tau = \frac{4}{\xi\omega_n} \quad \text{for } 2\% \\ t_s &= 3\tau = \frac{3}{\xi\omega_n} \quad \text{for } 5\% \end{aligned}}$$

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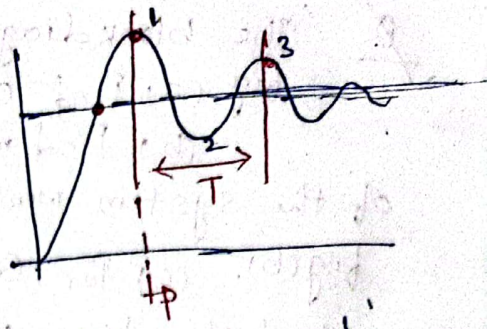
(6) No. of oscillations :-



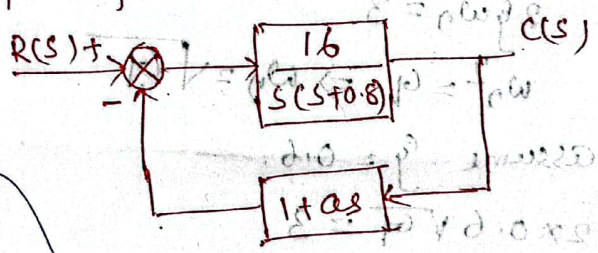
$$\text{Cycle} = \frac{t_s}{\text{Time period of 1 cycle } (t_p)}$$

$$t_p = \frac{3\pi}{\omega_d} - \frac{\pi}{\omega_d} = \frac{2\pi}{\omega_d}$$

$$\tau = \frac{1}{\xi\omega_n}$$

$$\boxed{\text{No. of oscillation} = \frac{4\tau}{2\pi/\omega_d} = \frac{4\tau\,\omega_d}{2\pi}}$$

Q Consider the system as shown in fig. Det. the value of 'a' such that the damping ratio is 0.5. Also obtain the values of rise time, and max overshoot in its step response.



$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8+16a)s + 16}$$

$$\text{Ch. eq}^n = s^2 + (0.8+16a)s + 16 = 0$$

Comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n^2 = 16, \quad \omega_n = 4$$

$$2\xi\omega_n = 0.8 + 16a$$

$$2 \times 0.5 \times 4 = 0.8 + 16a$$

$$\Rightarrow a = 0.2$$

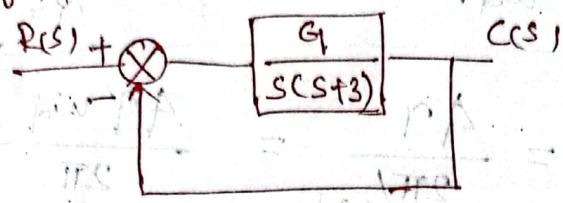
$$t_r = \frac{\pi - \theta}{\omega_d} = 0.605\,\text{sec}$$

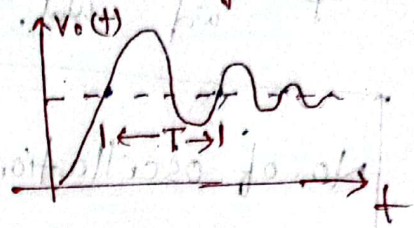
$$\%M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$= 16.31\%$$

Q. The blockdiagram of a feedback system is shown fig (a) Find CLTF

(b) Find min value of G for which the step response of the system would exhibit an overshoot as shown in fig(b). (c) for G equal to twice this min value find the time period 'T' indicated in fig(b).





(a) $$\frac{C(s)}{R(s)} = \frac{G/s(s+3)}{1+G/s(s+3)} = \frac{G}{s^2+3s+G}$$

$$\frac{G(s)}{1+G(s)\cdot H(s)}$$

(b) Ch eq $s^2+3s+G=0$

Comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$

$$2\xi\omega_n = 3$$

$$\omega_n^2 = G \Rightarrow \omega_n = \sqrt{G}$$

Lets assume $\xi = 0.6$.

$$2\times 0.6\sqrt{G} = 3$$

$$\Rightarrow G = 6.23$$

(c). $G' = 2G = 2\times 6.23 = 12.5$

$$\omega_n = \sqrt{12.5} = 3.53 \text{ rad/sec}$$

$$2\xi\omega_n = 3$$

$$\Rightarrow \xi = \frac{3}{2\omega_n} = \frac{3}{2\times 3.53} = 0.424$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.53\sqrt{1-(0.424)^2} = 3.197$$

$$\frac{2\pi}{T} = 3.197$$

$$T = 1.96 \text{ sec}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega_d}$$

Q The OLTF of a servo system with unity feedback is given by $G(s) = \frac{10}{(s+2)(s+5)}$

Det. the damping ratio, undamped natural freq of oscillation. What is % Mp of the response to a unit step input.

$$G(s) = \frac{10}{(s+2)(s+5)}$$

$$H(s) = 1$$

Ch. eq

$$1 + G(s).H(s) = 0$$

$$1 + \frac{10}{(s+2)(s+5)} = 0$$

$$s^2 + 7s + 20 = 0$$

$$\omega_n^2 = 20, \ \omega_n = 4.472$$

$$2\xi\omega_n = 7$$

$$\xi = 0.782$$

$$\% Mp = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$= 1.92\%$$

Q A feedback system is described by the following T/F

$$G(s) = \frac{12}{s^2+4s+16}, \quad H(s) = Ks$$

The damping factor of the system is 0.8. Det. the overshoot of the system and the value of K.

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (4+16K)s + 16}$$

Ch. eq $s^2 + (4+16K)s + 16 = 0$

$\xi = 0.8$

$\omega_n = 4$

Comparing $\omega_n^2 = 16 \Rightarrow \omega_n = 4$

$$2\xi\omega_n = 16K + 4$$

$$2 \times 0.8 \times 4 = 4 + 16K \Rightarrow K = \underline{0.15}$$

$$\% Mp = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$= 1.5\% \ (\text{Ans})$$

Q A system having a forward path T/F $G(s) = \frac{16}{s(s+1)}$ and unity feedback. Det. the value of undamped natural freqy, damping ratio. If tachometer feedback is introduced, the feedback path T/F becomes $1+Ks$. What should be the value of K to obtain damping ratio of 0.6 Also calculate % Mp, first undershoot, tp and ts within 2% of final value.

$H(s) = 1$ ch. eqn is $1 + G(s) \cdot H(s) = 0$

$$1 + \frac{16}{s(s+1)} = 0 \Rightarrow s^2 + s + 16 = 0$$

$\omega_n^2 = 16$, $\omega_n = 4$,

$2\xi\omega_n = 1 \Rightarrow \xi = 0.125$

Now consider $H(s) = 1 + Ks$

$1 + G(s) \cdot H(s) = 0$

$$\Rightarrow 1 + \frac{16}{s(s+1)}(1+Ks) = 0$$

$$\Rightarrow s^2 + (1+16K)s + 16 = 0$$

$\omega_n^2 = 16$, $\omega_n = 4$

$\xi = 0.6$ $2\xi\omega_n = 1 + 16K \Rightarrow K = 0.2375$

When $\xi = 0.125$ % $M_p = 67.3\%$

$\xi = 0.6$ % $M_p = 9.48\%$

First undershoot $n = 2$

$\xi = 0.6$ % $M_p = e^{-2\pi\xi/\sqrt{1-\xi^2}} \doteq 0.009$

$$t_p = \frac{\pi}{\omega_d} = 0.79 \text{ sec } (\xi = 0.125)$$

$$t_p = 0.98 \text{ sec } (\xi = 0.6)$$

$$t_s = \frac{4}{\xi\omega_n} = 8 \text{ sec for } \xi = 0.125$$

$= 1.67$ sec for $\xi = 0.6$

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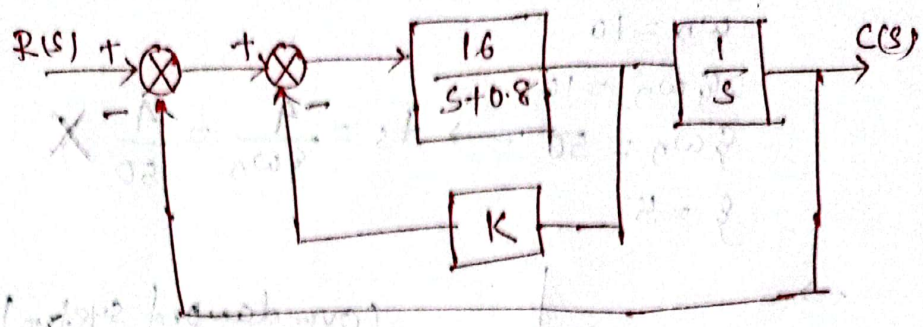
Q Consider the system shown in fig. Det. the value of K such that the damping ratio ξ is 0.5. Then obtain the t_r, t_p, % M_p and t_s in the unit step response.



$$\frac{C(s)}{R(s)} = \frac{16}{s^2+(0.8+16K)s+16}$$

Ch. eq^n $s^2+(0.8+16K)s+16=0$

Comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$

$\omega_n^2=16$, $\omega_n=4$

$2\xi\omega_n = 0.8+16K$

For $\xi=0.5$, $\boxed{K=0.2}$

$$t_r = \frac{\pi-\cos^{-1}\xi}{\omega_d} = 0.605 \text{ sec.}$$

$$t_p = \frac{\pi}{\omega_d} = 0.906 \text{ sec}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.3\%$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 4} = 2 \text{ sec.}$$

Q The T/F of the system is $G(s) = \frac{100}{(s+1)(s+100)}$ for a unit step i/p to the system the approximate settling time for 2% criterion is

(i) 100 sec (ii) 4 sec (iii) 1 sec (iv) 0.01 sec.

For 2% $\rightarrow t_s = 4\tau$ $\left(\tau = \frac{1}{\xi\omega_n}\right)$

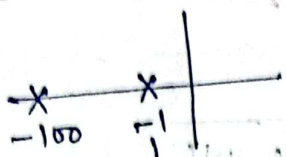
$$G(s) = \frac{100}{s^2+101s+100}$$

$\omega_n = 10$

$2\xi\omega_n = 101$

$\xi\omega_n = 50 \rightarrow t_s = \frac{4}{\xi\omega_n} = \frac{4}{50}$ ✗

$\xi = 5$

$\rightarrow$ 

(overdamped system)

$\tau = -\frac{1}{1} = 1$

So $t_s = 4\tau = 4$ sec (Ans)

Q

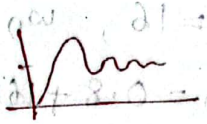
P. $\frac{25}{s^2+25}$

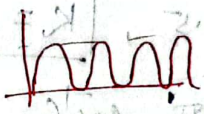
Q. $\frac{36}{s^2+20s+36}$

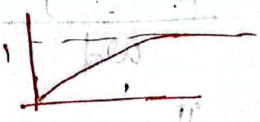
R. $\frac{36}{s^2+12s+36}$

S. $\frac{49}{s^2+7s+49}$

1.

2. 

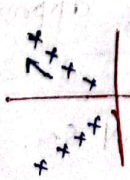
3. 

4. 

Q. $\frac{36}{s^2+20s+36}$ $\omega_n = 6$, $\xi = 10/6 > 1$ overdamped.

R. $\frac{36}{s^2+12s+36}$ $\omega_n = 6$, $\xi = 1$ critically damped

S. $\omega_n = 7$, $\xi = \frac{1}{2}$ under damped.

Q 

$\theta_1 = \theta_2 = \theta_3$

$\xi_1 = \xi_2 = \xi_3$

$M_{p_1} = M_{p_2} = M_{p_3}$

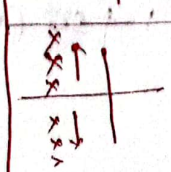
$\omega_d \uparrow$

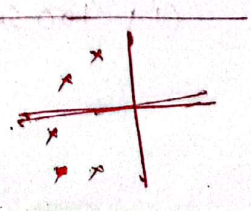
$\downarrow t_r = \frac{\pi-\theta}{\omega_d}$ $\downarrow t_p = \frac{\pi}{\omega_d}$, $t_d \downarrow$

$\downarrow t_s = \frac{4}{\xi\omega_n}$

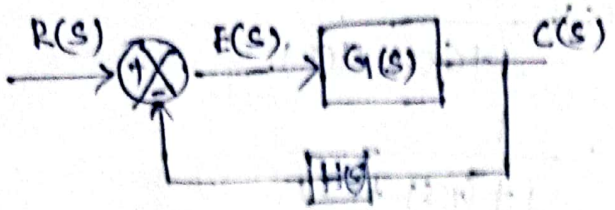
$\theta \downarrow$ $\xi \updownarrow$ ω_d-Fix

$\uparrow t_r, t_p$-Fix, $t_s \downarrow$





Steady State Error



$e(t) = r(t) - c(t)$

$E(s) = R(s) - C(s)$

steady state error $(E_{ss}) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s \cdot E(s)$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} \qquad (C(s) = E(s) \cdot G(s))$$

$$\frac{E(s) \cdot G(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$\Rightarrow \boxed{\frac{E(s)}{R(s)} = \frac{1}{1 + G(s) \cdot H(s)}}$$

$$\boxed{e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}}$$

S.S. Error with Step i/p signal

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{1 + \lim_{s \to 0} G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1 + K_p}} \qquad \boxed{K_p = \lim_{s \to 0} G(s) \cdot H(s)}$$

K_p → Position error coefficient.

S.S. error with Ramp i/p.

$$e_{ss} = \lim_{s \to 0} s \cdot R(s) \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s + s \cdot G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s \cdot G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \qquad \boxed{K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)}$$

Velocity error coefficient.

S.S. error with parabolic i/p.

$$e_{ss} = \lim_{s \to 0} s \cdot R(s) \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{1}{s^3} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2} \cdot \frac{1}{1+G(s) \cdot H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 \cdot G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}} \qquad \boxed{K_v = \lim_{s \to 0} s^2 \, G(s) \cdot H(s)}$$

acceleration ~~velocity~~ error coefficient.

① Step i/p	$E_{ss} = \frac{1}{1+K_p}$	$K_p = \lim_{s\to 0} G(s).H(s).$
② Ramp i/p	$E_{ss} = \frac{1}{K_v}$	$K_v = \lim_{s\to 0} s.G(s).H(s)$
③ Parabolic i/p	$E_{ss} = \frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2 G(s).H(s).$

$$E_{ss} = f(R(s), G(s))$$

i/p, system type

steady state error for different "Type" of system

① Type-0 System

Type is for Q T/F?

(i) step i/p type-0 system :-

$$R(s) = \frac{1}{s}$$

$$G(s).H(s) = \frac{K(s+s_a)(s+s_b)(s+s_c)\cdots}{(s+s_1)(s+s_2)(s+s_3)\cdots}$$

$$\cancel{K_p = \lim_{s\to 0} G(s).H(s) = \lim_{s\to 0}}$$

$$\cancel{= \lim_{s\to 0} \frac{K(s+s_a)(s+s_b)(s+s_c)\cdots}{(s+s_1)(s+s_2)(s+s_3)\cdots}}$$

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$K_p = \lim_{s\to 0} G(s).H(s) = \lim_{s\to 0} \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots}$$

$$= K$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} \qquad \boxed{e_{ss} = \frac{1}{1+K}}$$ constant

(ii) Ramp i/p:

$$K_v = \lim_{s\to 0} s.G(s).H(s)$$

$$= \lim_{s\to 0} s.\frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots} = 0$$

$$e_{ss} = \frac{1}{K_v} = \infty \Rightarrow \boxed{e_{ss} = \infty}$$

(iii) Parabolic i/p.

$$K_a = \lim_{s\to 0} s^2 G(s).H(s)$$

$$= \lim_{s\to 0} s^2 \frac{K(1+sT_1)(1+sT_2)\cdots}{(1+sT_a)(1+sT_b)\cdots} = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty \Rightarrow \boxed{e_{ss} = \infty}$$

② Type-1 System

$$G(s).H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s(1+sT_a)(1+sT_b)\cdots}$$

(i) Step i/p.

$$K_p = \lim_{s\to 0} G(s).H(s) = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(ii) Ramp i/p.

$$K_v = \lim_{s\to 0} s.G(s).H(s) = K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K} \qquad \boxed{e_{ss} = \frac{1}{K}} \text{ constant}$$

(iii) Parabolic i/p

$$K_a = \lim_{s\to 0} s^2 G(s).H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty \qquad \boxed{e_{ss} = \infty}$$

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3 Type-2 system

$$G(s)\cdot H(s) = \frac{K(1+sT_1)(1+sT_2)\cdots}{s^2(1+sT_a)(1+sT_b)\cdots}$$

(i) Step i/p.

$$K_p = \lim_{s\to 0} G(s)\cdot H(s) = \infty.$$

$$e_{ss} = \frac{1}{1+K_p} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(ii) Ramp i/p

$$K_v = \lim_{s\to 0} s\cdot G(s)\cdot H(s) = \infty.$$

$$e_{ss} = \frac{1}{K_v} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

(iii) Parabolic i/p

$$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = K.$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K} \Rightarrow \boxed{e_{ss} = \frac{1}{K}}$$ constant

i/p \ Type	0	1	2
t^0 Step	$\frac{1}{1+K_p}$	0	0
t^1 Ramp	∞	$\frac{1}{K_v}$	0
t^2 Parabolic	∞	∞	$\frac{1}{K_a}$

type = i/p $e_{ss} \to$ const.
type > i/p $e_{ss} \to 0$
type < i/p $e_{ss} \to \infty$

Q The OLTF of a unity feedback system is given by $G(s) = \frac{50}{(1+0.1s)(s+10)}$.

Det. the static error coefficient K_p, K_v, K_a

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$= \lim_{s \to 0} \frac{50}{(1+0.1s)(s+10)} = 5$$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s \cdot \frac{50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s^2 \frac{50}{(1+0.1s)(s+10)} = 0$$

Q Forward path T/F of a unity feedback control system is given by $G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$

Det. the step, ramp & parabolic error coefficient

Also det. type of the system.

$$K_p = \lim_{s \to 0} G(s) \cdot H(s) = \lim_{s \to 0} \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$$= \infty.$$

$$K_v = \lim_{s \to 0} s\, G(s) \cdot H(s) = \infty.$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s) = \frac{500}{50} = 10$$

Type = 2.

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Q For a unity F.B control system the F. path transfer function is given by $G(s) = \frac{20}{s(s+2)(s^2+2s+20)}$ Det steady state error of the system. When the i/ps are (i) 5 (ii) 5t (iii) $\frac{3t^2}{2}$

(i) $r(t) = 5$ $R(s) = 5/s$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{5/s}{1+\frac{20}{s(s+2)(s^2+2s+20)}} = 0.$$

(ii) $r(t) = 5t$, $R(s) = \frac{5}{s^2}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1+G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} s \cdot \frac{5/s^2}{1+\frac{20}{s(s+2)(s^2+2s+20)}}$$

$$= \lim_{s \to 0} s \cdot \frac{5}{s^2} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$= 10.$

(iii) $r(t) = \frac{3t^2}{2}$ $R(s) = \frac{3}{s^3}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{3}{s^2} \cdot \frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20)+20}$$

$= \infty.$

Q The OLTF of a unity feedback system is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$

For the static error coefficients and steady state error of the system when subjected to a i/p given by $r(t) = 2+5t+2t^2$

$$K_p = \lim_{s\to 0} G(s)\cdot H(s) = \lim_{s\to 0} \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \infty$$

$$K_v = \lim_{s\to 0} s\, G(s)\cdot H(s) = \lim_{s\to 0} s\cdot \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s)\cdot H(s) = \lim_{s\to 0} s^2\cdot \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \frac{108}{4\times 12} = \frac{108}{48}$$

$$r(t) = 2+5t+2t^2$$

$$R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

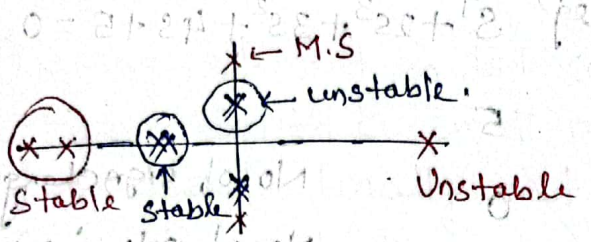
$$e_{ss} = \frac{2}{1+K_p} + \frac{5}{K_v} + \frac{4}{K_a}$$

$$= \frac{2}{1+\infty} + \frac{5}{\infty} + \frac{4}{108/48}$$

$$e_{ss} = 1.77 \text{ Ans}$$

(15) Stability Analysis. → BIBO
→ Finite no. of minima & maxima
→ Absolutly integrables

From higher order system how to get poles. position. For that we need a techique, that is Routh hurwitz criteria.



→ What can we conclude from RH criteria :-

1. We can conclude the stability of system
2. We can findout number of poles in RHS, LHS, on imaginary axis of s-plane.
3. We can find out range of 'K' for stability & marginally stability of a system.
4. We can findout frequy of oscillation.

By watching close loop poles we can define system stability. For that we need Close loop T/F.

→ i.e. characteristics eq we need.

$1 + G(s) \cdot H(s) = 0$ ← Poles of CTF.

1. ch. equation

Ex $s^6 + 5s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1 = 0$

2. Form the RH table

s^6	1	4	2	1
s^5	5	3	1	0
s^4	$\frac{5\times4-1\times3}{5}$	$\frac{5\times2-1\times1}{5}$	0	
s^3	A			
s^2	B			
s^1	C			
s^0	D.			

3. If all elements of first column are either -ve or +ve (same sign) then the system is stable.

4. Number of sign change of elements in 1st column equal to number of poles in RHS.

(5) If sign change occurs then system will be unstable.

(6)

<u>Example</u> Find the number of poles in RHS of s-plane for ch. eq $s^4+2s^3+3s^2+4s+5=0$

s^4	1	3	5
s^3	2	4	0
s^2	1	5	0
s^1	-6	0	
s^0	5		

No. of sign change = 2

No. of pole in RHS = 2

<u>Ex</u> Find the poles in LHS of s-plane for $s^4+2s^3+3s^2+s+2=0$

s^4	1	3	2
s^3	2	1	0
s^2	5/2	2	0
s^1	$-35/5$	0	0
s^0	2	0	0

Total pole = 4.

R.H.S = 2 poles

L.H.S = 2 poles.

(6) If any coefficient of ch. eq is -ve then system would be unstable.

(7) If any power of s term is missing then system would be unstable.

* $a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$ For stable system

all coefficients must be +ve

$a_2 a_1 > a_3 a_0$.

(8) If any element in 1st column is zero then replace 0 with a small positive number 'ε'. and proceed. Finally put ε = 0 and check the number of sign change.

s^4	1	2	8
s^3	2	4	0
s^2	0^{ϵ}	8	
s^1	$\frac{4\epsilon - 16}{\epsilon}$	0	
s^0	8		

$\to -\infty$ system is unstable.

(9) If we get complete row is zero, it means poles are symmetrical about origin. Then make the auxulary eqn from the coefficient of row which is above the zero line then differentiate the auxulary eqn and replace the ROZ by coefficient of differce eqn.

Case-1

s^5	1	3	2
s^4	1	3	2
s^3	0	0	0
s^2	3/2	2	
s^1	2/3		
s^0	2		

Auxulary eqn $= s^4 + 3s^2 + 2$

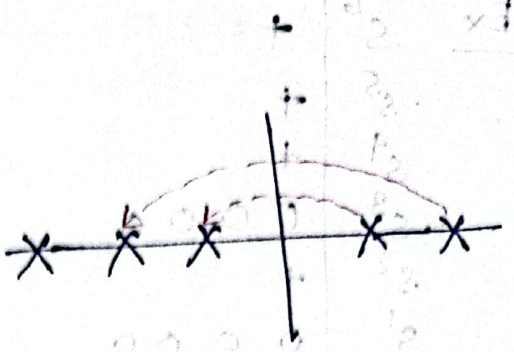
$\frac{d}{ds}(AE) = 4s^3 + 6s$

No sign change so no pole at RHS.

* System is MS.

Case-4

	s^5	+		
	s^4	+		
ROZ	s^3	0	0	0
	s^2	+		
	s^1	−		below
	s^0	+		



* If the sign change occurs below ROZ then there must be a symmetric pole in the left to the pole placed in the right side.

* If the sign change occurs above ROZ then there is no symmetric poles left to the poles placed in the right side.

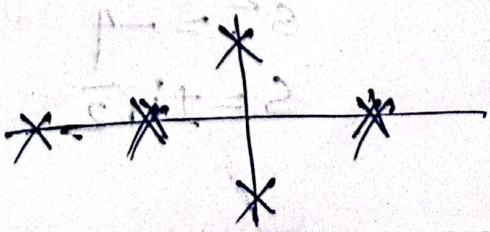
(10) If ROZ occurs once, it means poles are not repeated but symmetric.

→ If ROZ occurs more than once, it means poles are repeated and symmetric. (Unstable system.)

The number of symmetric poles = Highest order of s from two different auxiliary eq.

	s^7	+			
	s^6	+			
	s^5	0	0	0	0
	s^4	+			
	s^3	0	0	0	0
	s^2	+			
	s^1	−			below
	s^0	+			

ROZ = 2
Order of AE = 6
Symmetric pole = 6
Sign change = 2
R.H.S pole = 2



Ex

s^6	+
s^5	+
s^4	+
s^3	0 0 0 0
s^2	+
s^1	0 0 0 0
s^0	+

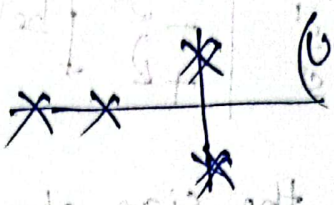
ROZ = 2

order of AE = 4

Symmetric pole = 4

Sign change = 0

RHS = 0



(Unstable system)

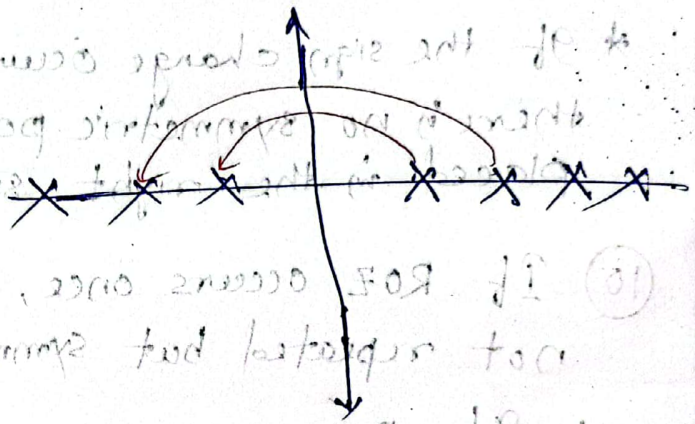
→ repeated pole

Ex

s^7	+
s^6	−
s^5	+
s^4	+
s^3	0 0 0 0
s^2	+
s^1	−
s^0	+

↑ Above

↓ below



* If ROZ is present then the system is Marginally stable system. (if no sign change).

(11)

Marginaly stable system is the oscillatory system. Then it has frequency of oscillation. So the F.O.O will be calculated from A.E = 0.

Ex

$2s^2+4=0.$

$s^2=-4$

$s=\pm j\sqrt{2}$

$\omega=\sqrt{2}$ rad/sec

Conditionally stable system :-

Ex $s^3+8s^2+2s+K=0$

s^3	1	2	0
s^2	8	K	0
s^1	$\frac{16-K}{8}$	0	0
s^0	K		

To make stable $\frac{16-K}{8} > 0$

$16-K>0$.

$K<16$,

$K>0$

$\boxed{0<K<16}$

Q $G(s)\cdot H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$

$1+G(s)\cdot H(s)=0$.

$s(s+2)(s+4)(s+6)+K=0$

$s^4+12s^3+44s^2+48s+K=0$.

s^4	1	44	K
s^3	12	48	0
s^2	40	K	0
s^1	$\frac{40\times48-12K}{40}$	K	
s^0	K		

$\frac{40\times48-12K}{40}>0$

$40\times48>12K$

$160>K$

$K>0$

$\boxed{0<K<160}$

Q Det. K & b such that the system

TF $G(s) = \dfrac{K(s+1)}{s^3+bs^2+3s+1}$, $H(s)=1$

oscillation with freq of 2 rad/sec.

$s^3+bs^2+3s+1+K(s+1)=0.$

$s^3+bs^2+(3+K)s+(1+K)=0$

$1+K = b(3+K) \longrightarrow \boxed{b = \dfrac{1+K}{3+K}}$

$\omega = 2$

AE $\Rightarrow bs^2+K+1=0.$

$b(j\omega)^2+K+1=0.$

$b(2j)^2+K+1=0.$

$-b\times 4+K+1=0$

$b = \dfrac{K+1}{4}$ —— (2).

$b=0.5,\ K=1.$

Q. The OLTF of a unity F.B control system is given by $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$

by applying RH criteria, discuss the stability of the CLS as a fun of K. Det. the value of K which will cause sustained oscillation in the close loop system. What is the corresponding oscillation frequency.

Sol: the ch. eq is given by

$$1+G(s)\cdot H(s)=0$$

$$1+\frac{K}{(s+2)(s+4)(s^2+6s+25)}=0$$

$$s^4+12s^3+69s^2+198s+(200+K)=0$$

s^4	1	69	$200+K$
s^3	12	198	
s^2	52.5	$200+K$	
s^1	$198-\frac{12(200+K)}{52.5}$		
s^0	$200+K$		

System will be stable when

$200+K>0$ or $K>-200$

$$198-\frac{12(200+K)}{52.5}>0 \Rightarrow K<\boxed{666.25}$$

Oscillation will occure when $K=666.25$

Auxiliary eq $52.5s^2+(200+K)=0$

$$52.5s^2+(200+666.25)=0$$

$$52.5s^2=-866.25$$

$$s^2=-16.5,\quad s=\pm j4.06$$

$$j\omega=\pm j4.06 \Rightarrow \omega=4.06 \text{ rad/sec}$$

Q A feedback system has an OLT/F

$$G(s)H(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$$

Det. the max^m value of 'K' for the close loop system to be stable.

Sol^n For low frequency $e^{-s} = (1-s)$.

$$G(s)\cdot H(s) = \frac{K(1-s)}{s(s^2+5s+9)}$$

ch. eq^n $1+G(s)\cdot H(s)=0$.

$$1+\frac{K(1-s)}{s(s^2+5s+9)} = 0$$

$$s(s^2+5s+9)+K(1-s)=0$$

$$s^3+5s^2+(9-K)s+K=0.$$

s^3	1	$9-K$
s^2	5	K
s^1	$9-\frac{6K}{5}$	
s^0	K	

For stability $K>0$.

$$9-\frac{6K}{5}>0 \Rightarrow K<7.5$$

$\boxed{0<K<7.5}$ Range of K.

~~For oscillation.~~

~~$9-\frac{6K}{5}=0$.~~

~~$45-6K=0$. $\Rightarrow K=\frac{45}{6}$.~~

AE = $5s^2+K$.

Q Det the pole & stability

$$s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$$

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	
s^3	~~08~~	~~012~~	0	
s^2	3	4		
s^1	4/3			
s^0	4			

$AE = 2s^4 + 6s^2 + 4$

$\frac{dA}{ds} = 8s^3 + 12s$

No sign change so R.H.S pole = 0.

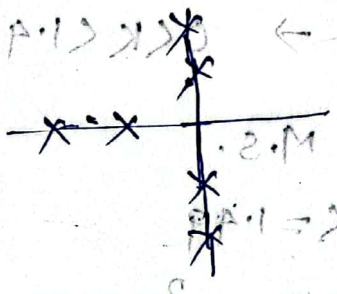
symmetric pole = 4.

i.e. 4 poles on the imaginary axis.

$2s^4 + 6s^2 + 4 = 0$

$s = \pm j1$

$s = \pm j\sqrt{2}$



Q Det. the range of values of K such that the characteristic eq

$$s^3 + 3(K+1)s^2 + (7K+5)s + (4K+7) = 0$$ has roots more negative than $s = -1$.

Sol Put $s = z - 1$ in the given ch. eq

$$(z-1)^3 + 3(K+1)(z-1)^2 + (7K+5)(z-1) + (4K+7) = 0$$

$$\Rightarrow z^3 + 3Kz^2 + (K+2)z + 4 = 0.$$

z^3	1	$K+2$
z^2	$3K$	4
z	$\frac{3K(K+2)-4}{3K}$	
z^0	4	

$\frac{3K(K+2)-4}{3K} > 0$

$3K > 0 \Rightarrow K > 0$

$3K^2 + 6K - 4 > 0$

$K > -2.527, 0.527$

Q $s^4 + 20s^3 + 15s^2 + 2s + K = 0$

Det range of K for the system to be stable

Can the system M.S. if so determine K & F.O.O

s^4	1	15	K
s^3	20	2	
s^2	14.9	K	
s^1	$\frac{29.8 - 20K}{14.9}$		
s^0	K		

(a) For stability $K > 0$

$29.8 - 20K > 0$

$K < 1.49$

Range → $0 < K < 1.49$

(b) For M.S.

$K = 1.49$

$AE = 14.9s^2 + 1.49 = 0$

$s = \pm j\, 0.316$

$\omega = 0.316$ rad/sec

F.O.O. = 0.316 rad/sec