MODULATION BY SEVERAL SINE WAVES (Multitone)

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Let a modulating signal contains two frequencies fm1 and fm2, that is

$$v_m(t) = V_{m1} \sin(2\pi f_{m1}t) + V_{m2} \sin(2\pi f_{m2}t)$$

This signal modulates a single-frequency carrier signal represented by

$$v_{c(t)} = V_c \sin(2\pi f_c t)$$

As per the basic definition of amplitude modulation, we have

$$v_{AM}(t) = [V_c + v_m(t)] \sin(2\pi f_c t)$$

Substituting $v_m(t) = V_{m1} \sin(2\pi f_{m1} t) + V_{m2} \sin(2\pi f_{m2} t)$, we have

$$v_{AM}(t) = [V_c + V_{m1} \sin(2\pi f_{m1} t) + V_{m2} \sin(2\pi f_{m2} t)] \sin(2\pi f_c t)$$

On expanding various terms, we get

$$v_{AM}(t) = V_c \sin(2\pi f_c t) + V_{m1} \sin(2\pi f_c t) \sin(2\pi f_{m1} t) + V_{m2} \sin(2\pi f_c t) \sin(2\pi f_{m2} t)$$

$$v_{AM}(t) = V_c \left[\sin(2\pi f_c t) + \frac{V_{m1}}{V_c} \sin(2\pi f_c t) \sin(2\pi f_{m1} t) + \frac{\mathrm{V}m_2}{V_c} \sin(2\pi f_c t) \sin(2\pi f_{m2} t) \right]$$

Using the expression of modulation index as $m_a = \frac{V_m}{V_c}$, we obtain

$$v_{AM}(t) = V_c\,[\sin(2\pi f_c t) + m_{a1}\sin(2\pi f_c t)\sin(2\pi f_{m1} t) + m_{a2}\sin(2\pi f_c t)\sin(2\pi f_{m2} t)]$$

Expanding the terms, we get

$$v_{AM}(t) = V_c\sin(2\pi f_c t) + m_{a1}V_c\sin(2\pi f_c t)\sin(2\pi f_{m1} t) + m_{a2}V_c\sin(2\pi f_c t)\sin(2\pi f_{m2} t)$$

Using trigonometric identity, $\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$, we obtain

$$v_{AM}(t) = V_c\sin(2\pi f_c t) + m_{a1}\frac{V_c}{2}[\cos\{2\pi(f_c - f_{m1})t\} - \cos\{2\pi(f_c + f_{m1})t\}]$$
$$+ m_{a2}\frac{V_c}{2}[\cos\{2\pi(f_c - f_{m2})t\} - \cos\{2\pi(f_c + f_{m2})t\}]$$

$$v_{AM}(t) = V_c \sin(2\pi f_c t) + m_{a1}\frac{V_c}{2}\left[\cos\{2\pi(f_c - f_{m1})t\} - \cos\{2\pi(f_c + f_{m1})t\}\right]$$
$$+ m_{a2}\frac{V_c}{2}\left[\cos\{2\pi(f_c - f_{m2})t\} - \cos\{2\pi(f_c + f_{m2})t\}\right]$$

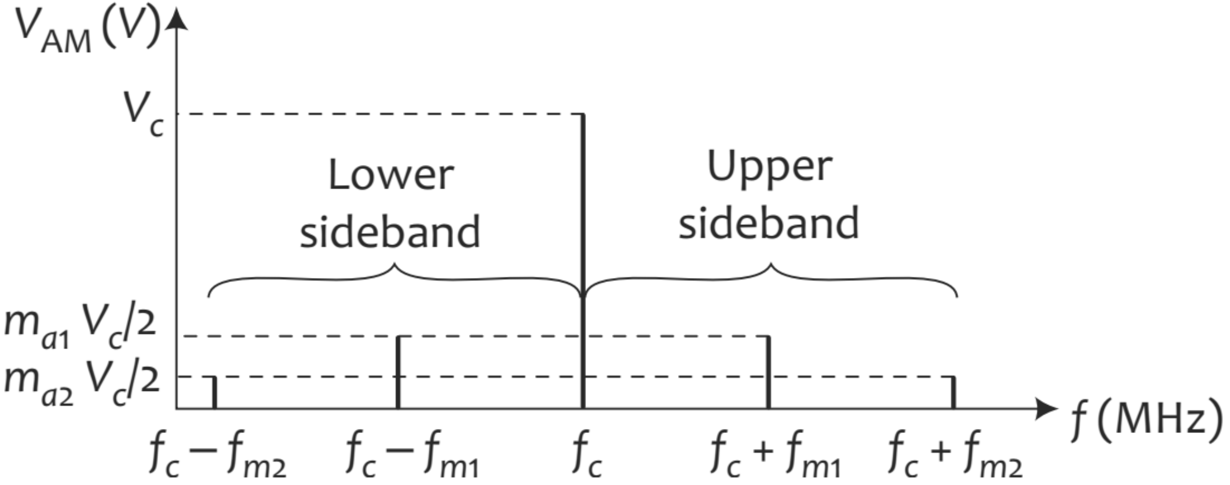
$$\boxed{\begin{aligned} v_{AM}(t) &= V_c \sin(2\pi f_c t) + \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m1})t\} - \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m1})t\} \\ &+ \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m2})t\} - \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m2})t\} \end{aligned}}$$

Frequency Spectrum

$$
\begin{aligned}
v_{AM}(t) &= V_c \sin(2\pi f_c t) + \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m1})\,t\} - \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m1})\,t\} \\
&+ \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m2})\,t\} - \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m2})\,t\}
\end{aligned}
$$

Frequency Spectrum

$$v_{AM}(t) = V_c \sin(2\pi f_c t) + \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m1})t\} - \left(m_{a1}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m1})t\}$$
$$+ \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c - f_{m2})t\} - \left(m_{a2}\frac{V_c}{2}\right)\cos\{2\pi(f_c + f_{m2})t\}$$



Modulation Index, Power & efficiency (Multitone)

Let V_1, V_2 and V_3 etc. be the peak amplitude of the information signals, the resultant voltages, V_T becomes

$$V_T = \sqrt{V_1^{\,2} + V_2^{\,2} + V_3^{\,2} + \ldots\ldots\ldots}$$

dividing both sides by V_C

$$\frac{V_T}{V_C} = \sqrt{\left(\frac{V_1}{V_C}\right)^2 + \left(\frac{V_2}{V_C}\right)^2 + \left(\frac{V_3}{V_C}\right)^2 + \ldots\ldots\ldots\ldots}$$

that is

$$\therefore m_T = \sqrt{m_1{}^2 + m_2{}^2 + m_3{}^2 + \dots\dots\dots}$$

$$P_T = P_C\left(1 + \frac{m_T{}^2}{2}\right)$$

The equation of the total current and carrier current is derived from the total power equation :-

$$\frac{I_T}{I_C} = \sqrt{\left(1 + \frac{m_T{}^2}{2}\right)}$$

$$\%\,\eta = \frac{m_T^2}{2 + m_T^2} \times 100$$

AM Generation

AM Generation

The instantaneous value of the modulated wave is

$$V_{AM} = (V_c + V_m sin2\pi f_m t)\sin 2\pi f_c t$$

$$V_{AM} = V_c \sin 2\pi f_c t + V_m sin2\pi f_m t \sin 2\pi f_c t$$

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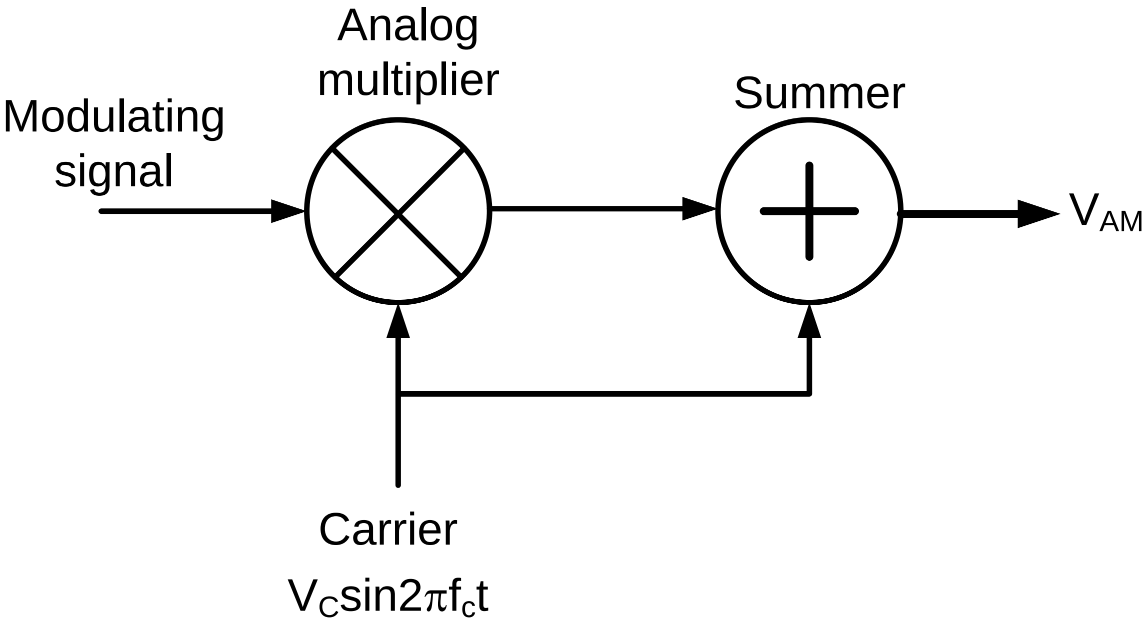


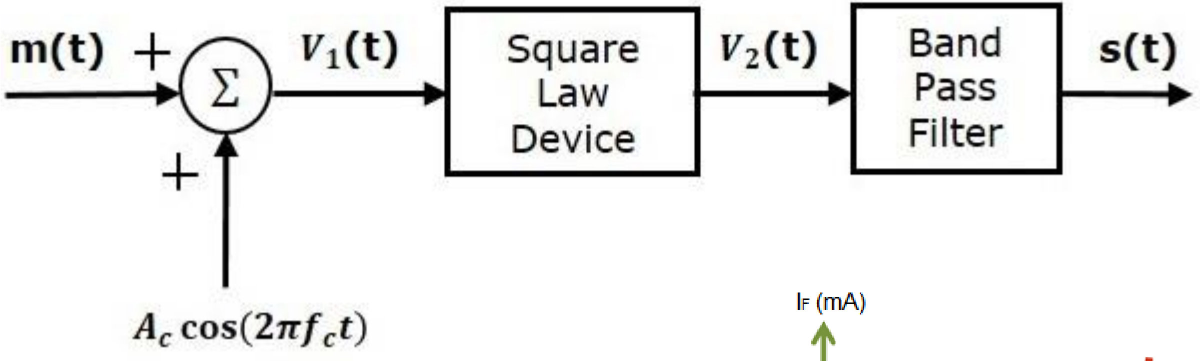
AM MODULATOR CIRCUIT

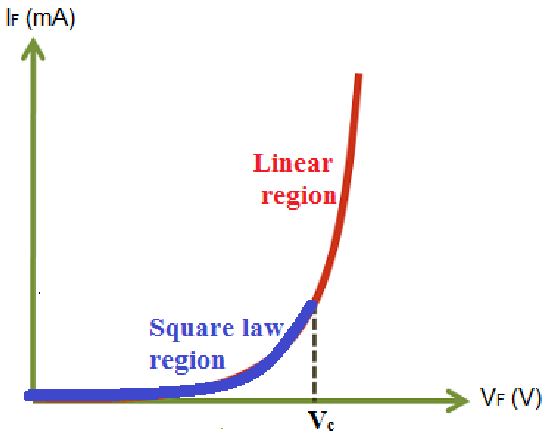
- Amplitude modulation voltage is produced by a circuit that can multiply the carrier by the modulating signal and then add the carrier.

- Product of the carrier and modulating signal can be generated by applying both signals to a nonlinear device

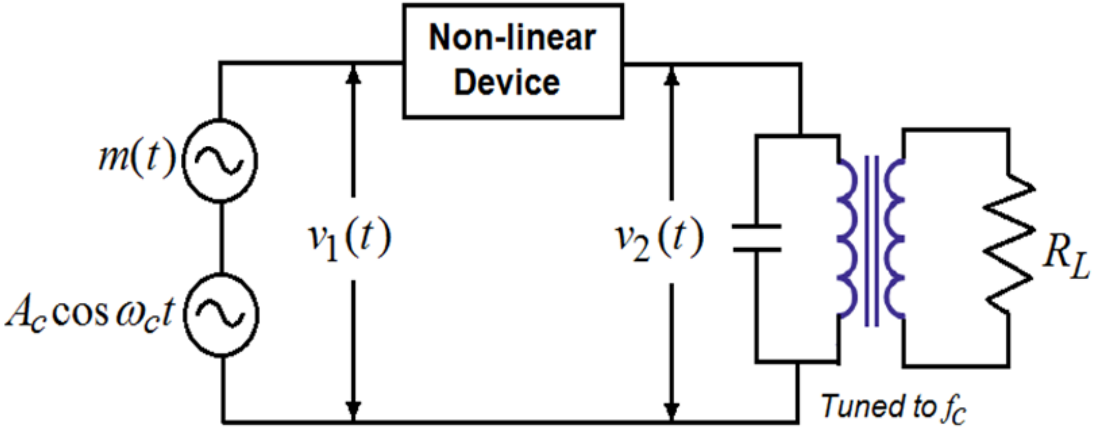
- Tuned circuits **filter out the modulating signal and carrier harmonics**, leaving only carrier and sidebands.

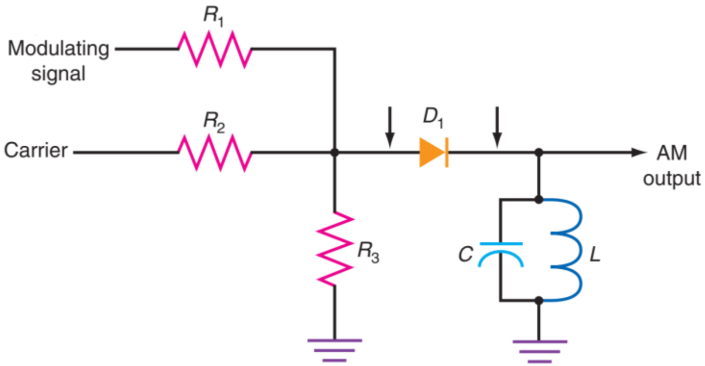
Square Law Modulator





Diode - Square-Law Modulator





The diode-resistor equation is given by

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

Where a_1 and a_2 are constants & m(t) is the message signal

The output of the non-linear device (diode)

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \qquad \text{where} \qquad v_1(t) = m(t) + A_c \cos\omega_c t$$

$$v_2(t) = a_1\left[m(t) + A_c \cos\omega_c t\right] + a_2\left[m(t) + A_c \cos\omega_c t\right]^2$$

$$= a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right]\cos\omega_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c \cos^2\omega_c t$$

The first term is the desired AM wave with amplitude sensitivity

$$k_a = \frac{2a_2}{a_1}$$

Remaining unwanted three terms are removed by appropriate filtering.