ANALOG COMMUNICATION SYSTEMS

BECE304L
Module-4

COURSE- MODULE-4

Module:4	Angle Modulation	10 hours
Principles of Frequency Modulation (FM) and Phase Modulation (PM) – Relation between FM and PM, Frequency deviation and bandwidth of FM, Narrow band and Wide band FM, Bessel functions and Carson's rule. FM generation and detection. Comparison of amplitude and angle modulation.		

ANGLE MODULATION

Angle modulation is a class of analog modulation. These techniques are based on altering the angle (or phase) of a carrier signal to transmit data. This as opposed to varying the amplitude of the carrier, such as in amplitude modulation transmission.

Angle Modulation is modulation in which the angle of a sine-wave carrier is varied by a modulating wave.
Frequency modulation (FM) and phase modulation (PM) are the two main types of angle modulation.
In frequency modulation, the modulating signal causes the carrier frequency to vary. These variations are controlled by both the frequency and the amplitude of the modulating wave.
In phase modulation, the phase of the carrier is controlled by the modulating waveform.

PHASE MODULATION

Phase modulation (PM) *is that form of angle modulation in which the angle* $\theta_i(t)$ *is varied linearly with the message signal* $m(t)$, as shown by

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

The term $2\pi f_c t$ represents the angle of the *unmodulated* carrier; and the constant k_p represents the *phase sensitivity* of the modulator, expressed in radians per volt on the assumption that $m(t)$ is a voltage waveform. For convenience, we have assumed in Equation (2.22) that the angle of the unmodulated carrier is zero at $t = 0$. The phase-modulated signal $s(t)$ is thus described in the time domain by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

FREQUENCY MODULATION

Frequency modulation (FM) *is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$*, as shown by

$$f_i(t) = f_c + k_f m(t)$$

The term f_c represents the frequency of the unmodulated carrier, and the constant k_f represents the *frequency sensitivity* of the modulator, expressed in Hertz per volt

on the assumption that $m(t)$ is a voltage waveform. Integrating Equation with respect to time and multiplying the result by 2π, we get

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

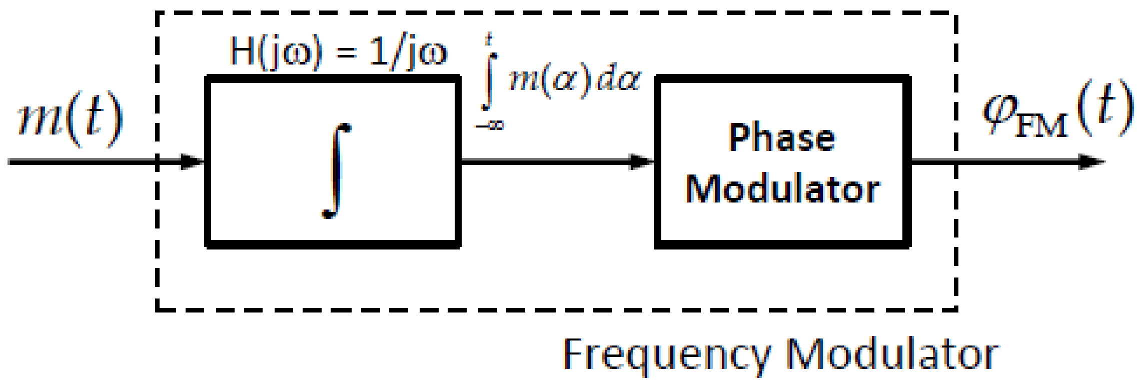
where, for convenience, we have assumed that the angle of the unmodulated carrier wave is zero at $t = 0$. The frequency-modulated signal is therefore described in the time domain by

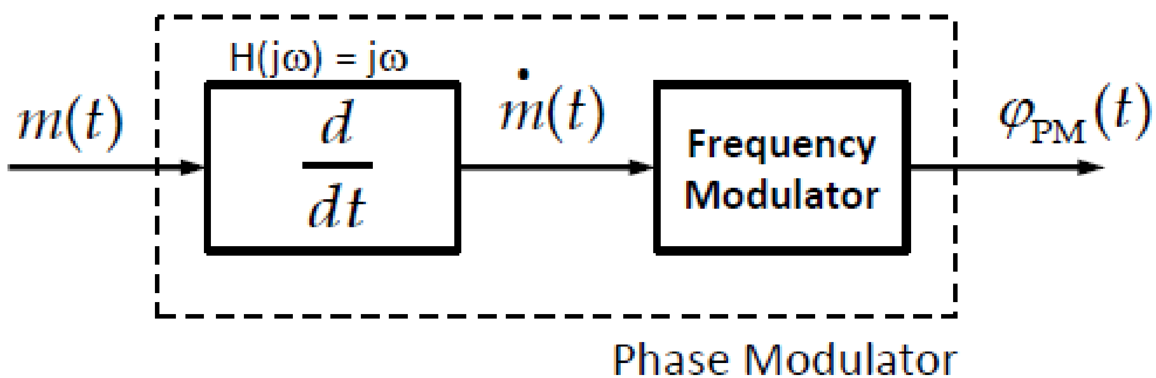
$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

FREQUENCY MODULATION

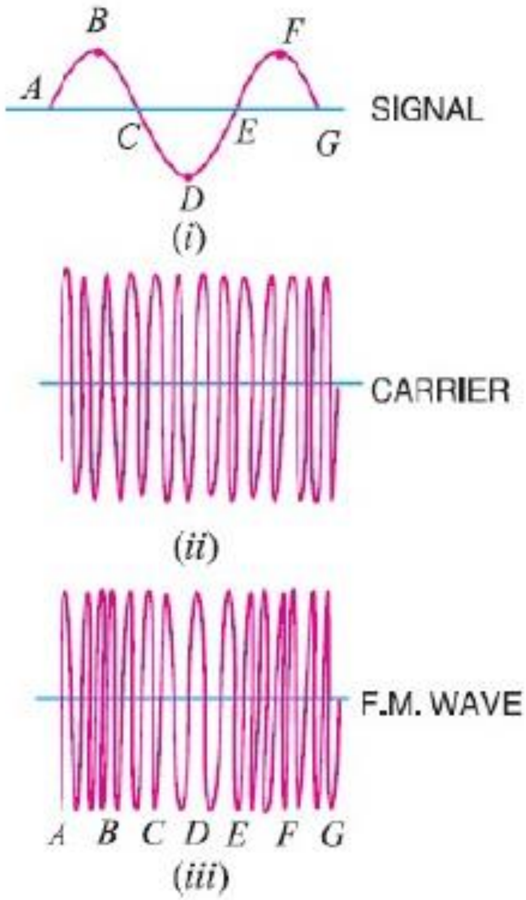
- When the frequency of carrier wave is changed in accordance with the intensity of the signal, it is called frequency modulation (FM).
- In frequency modulation, only the frequency of the carrier wave is changed in accordance with the signal. However, the amplitude of the modulated wave remains the same i.e. carrier wave amplitude.

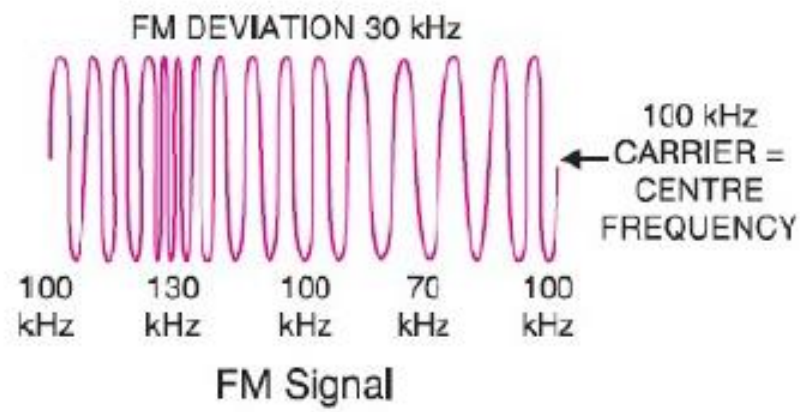
RELATIONSHIP BETWEEN FREQUENCY- PHASE MODULATION

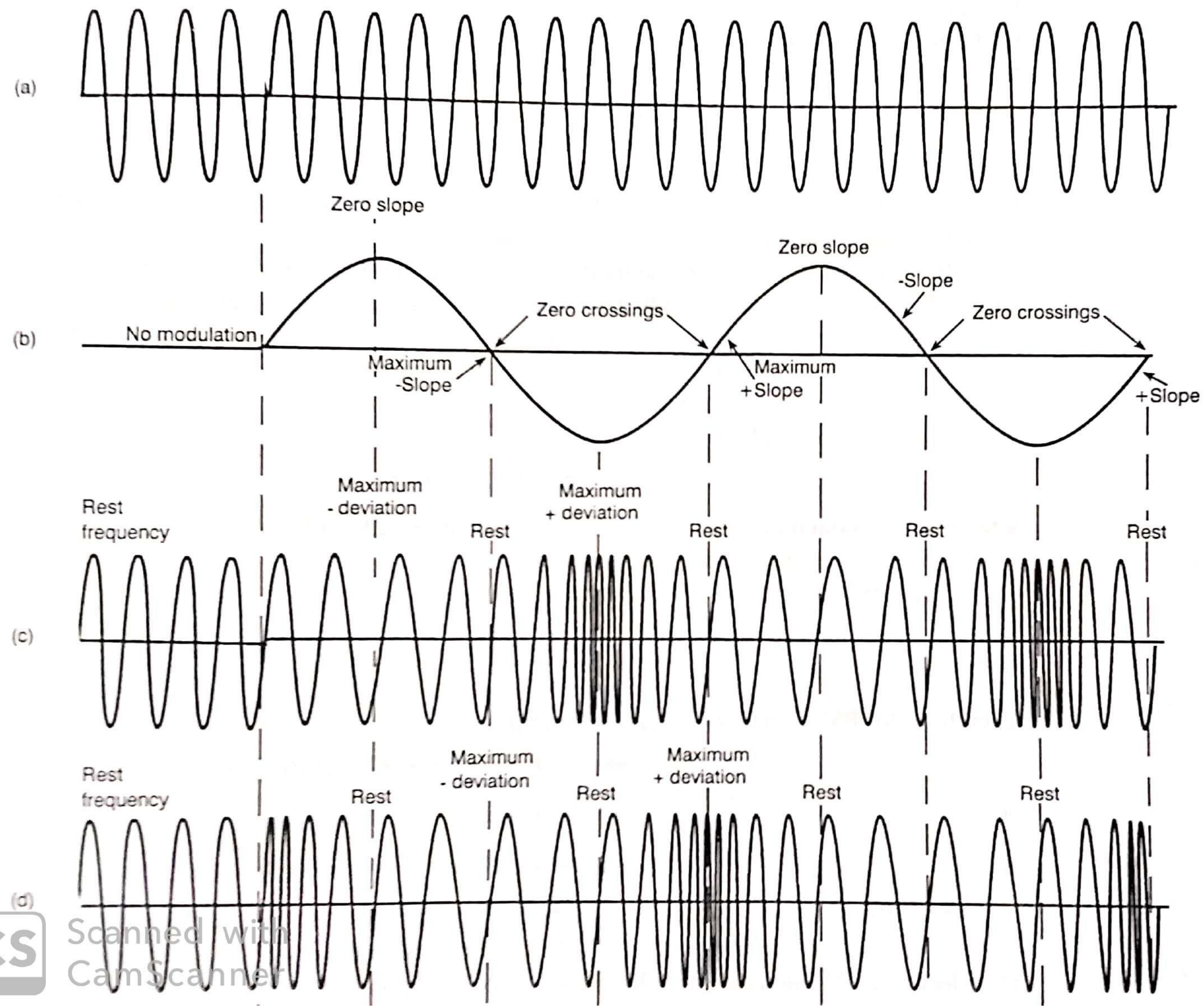




FREQUENCY MODULATION






(a)
Zero slope
Zero slope
-Slope
Zero crossings
Zero crossings
(b)
No modulation
Maximum
-Slope
Maximum
+Slope
+Slope
Maximum
- deviation
Maximum
+ deviation
Rest
frequency
Rest
Rest
Rest
Rest
(c)
Maximum
- deviation
Maximum
+ deviation
Rest
frequency
Rest
Rest
Rest
Rest
(d)

FREQUENCY MODULATION

The following points about frequency modulation (FM) may be noted carefully :
(a) The frequency deviation of FM signal depends on the amplitude of the modulating signal.
(b) The centre frequency is the frequency without modulation or when the modulating voltage is zero.
(c) The audio frequency (i.e. frequency of modulating signal) does not determine frequency deviation.

The following are the advantages of FM over AM :

(i) It gives noiseless reception. As discussed before, noise is a form of amplitude variations and a FM receiver will reject such signals.
(ii) The operating range is quite large.
(iii) It gives high-fidelity reception.
(iv) The efficiency of transmission is very high.

FREQUENCY MODULATION

The instantaneous angular frequency of FM is given by

$$\omega_i = \omega_c + \Delta\omega_c \cos \omega_s t$$

Total phase angle $\theta = \omega t$, so that if ω is variable then

$$\theta = \int_0^t \omega_i dt$$
$$\theta = \int_0^t (\omega_c + \Delta\omega_c \cos \omega_s t) dt$$

$$\theta = \omega_c t + \frac{\Delta\omega_c}{\omega_s} \sin \omega_s t$$

The term $\frac{\Delta\omega_c}{\omega_s}$ is called the modulation index β

$$\theta = \omega_c t + \beta \sin \omega_s t$$

The instantaneous value of FM voltage wave is given by

$$e = E_c \cos \theta$$

$$e = E_c \cos(\omega_c t + \beta \sin \omega_s t)$$

FREQUENCY DEVIATION AND PERCENT MODULATION

7-7-1 Frequency Deviation

Frequency deviation is the change in frequency that occurs in the carrier when it is acted on by a modulating-signal frequency. Frequency deviation is typically given as a peak frequency shift in hertz (Δf). The peak-to-peak frequency deviation ($2\Delta f$) is sometimes called *carrier swing*.

For an FM, the deviation sensitivity is often given in hertz per volt. Therefore, the peak frequency deviation is simply the product of the deviation sensitivity and the peak modulating-signal voltage and is expressed mathematically as

$$\Delta f = K_1 V_m \text{ (Hz)} \tag{7-21}$$

Equation 7-21 can be substituted into Equation 7-20, and the expression for the modulation index in FM can be rewritten as

$$m = \frac{\Delta f\text{(Hz)}}{f_m\text{(Hz)}}\text{(unitless)} \tag{7-22}$$

Therefore, for FM, Equation 7-1 can be rewritten as

$$m(t) = V_c \cos\left[\omega_c t + \frac{K_1 V_m}{f_m}\sin(\omega_m t)\right] \tag{7-23}$$

or

$$m(t) = V_c \cos\left[\omega_c t + \frac{\Delta f}{f_m}\sin(\omega_m t)\right] \tag{7-24}$$

or

$$m(t) = V_c \cos[\omega_c t + m \sin(\omega_m t)] \tag{7-25}$$

From examination of Equations 7-19 and 7-20, it can be seen that the modulation indices for FM and PM relate to the modulating signal differently. With PM, both the modulation index and the peak phase deviation are directly proportional to the amplitude of the modulating signal and unaffected by its frequency. With FM, however, both the modulation index and the frequency deviation are directly proportional to the amplitude of the modulating signal, and the modulation index is inversely proportional to its frequency. Figure 7-4 graphically shows the relationship among modulation index and peak phase deviation for PM and the modulation index and peak frequency deviation for FM in respect to the modulating-signal amplitude and frequency.

NUMERICAL-FREQUENCY DEVIATION

Example 7-1

a. Determine the peak frequency deviation (Δf) and modulation index (m) for an FM modulator with a deviation sensitivity $K_1 = 5$ kHz/V and a modulating signal $v_m(t) = 2\cos(2\pi 2000t)$.

b. Determine the peak phase deviation (m) for a PM modulator with a deviation sensitivity $K = 2.5$ rad/V and a modulating signal $v_m(t) = 2\cos(2\pi 2000t)$.

Solution

a. The peak frequency deviation is simply the product of the deviation sensitivity and the peak amplitude of the modulating signal, or

$$\Delta f = \frac{5\text{ kHz}}{\text{V}} \times 2\text{ V} = 10\text{ kHz}$$

The modulation index is determined by substituting into Equation 7-22:

$$m = \frac{10\text{ kHz}}{2\text{ kHz}} = 5$$

b. The peak phase shift for a phase-modulated wave is the modulation index and is found by substituting into Equation 7-15:

$$m = \frac{2.5\text{ rad}}{\text{V}} \times 2\text{ V} = 5\text{ rad}$$

NUMERICALS-FM

Example *A frequency modulated voltage wave is given by the equation :*

$$e = 12 \cos (6 \times 10^8 t + 5 \sin 1250\, t)$$

Find (i) carrier frequency (ii) signal frequency (iii) modulation index (iv) maximum frequency deviation (v) power dissipated by the FM wave in 10-ohm resistor.

NUMERICALS-FM

Solution. The given FM voltage wave is

$$e = 12\cos(6\times 10^8 t + 5\sin 1250\,t)$$

The equation of standard FM voltage wave is

$$e = E_c\cos(\omega_c t + m_f \sin\omega_s t)$$

Comparing eqs. (*i*) and (*ii*), we have,

(*i*) Carrier frequency, $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{6\times 10^8}{2\pi} = \mathbf{95.5\times 10^6\ Hz}$

(*ii*) Signal frequency, $f_s = \dfrac{\omega_s}{2\pi} = \dfrac{1250}{2\pi} = \mathbf{199\ Hz}$

(*iii*) Modulation index, $m_f = \mathbf{5}$

(*iv*) Max. frequency deviation, $\Delta f = m_f \times f_s = 5\times 199 = \mathbf{995\ Hz}$

(*v*) Power dissipated, $P = \dfrac{E_{r.m.s.}^2}{R} = \dfrac{(12/\sqrt{2})^2}{10} = \mathbf{7.2W}$

NUMERICALS-FM

Example *A 25 MHz carrier is modulated by a 400 Hz audio sine wave. If the carrier voltage is 4V and the maximum frequency deviation is 10 kHz, write down the voltage equation of the FM wave.*

NUMERICALS-FM

Solution. The voltage equation of the FM wave is

$$e = E_c \cos(\omega_c t + m_f \sin \omega_s t)$$

$$\omega_c = 2\pi f_c = 2\pi \times 25 \times 10^6 = 1.57 \times 10^8 \text{ rad/s}$$

$$\omega_s = 2\pi f_s = 2\pi \times 400 = 2513 \text{ rad/s}$$

$$m_f = \frac{\Delta f}{f_s} = \frac{10 \text{ kHz}}{400 \text{ Hz}} = \frac{10 \times 10^3 \text{ Hz}}{400\text{Hz}} = 25$$

$$e = 4 \cos(1.57 \times 10^8 t + 25 \sin 2513t)$$ **Ans.**

NUMERICALS-FM

Example: Calculate the modulation index for an FM wave where the maximum frequency deviation is 50 kHz and the modulating frequency is 5 kHz.

NUMERICALS-FM

Solution.

$$\text{Max. frequency deviation, } \Delta f = 50 \text{ kHz}$$

$$\text{Modulating frequency, } f_s = 5 \text{ kHz}$$

$$\therefore \quad \text{Modulation index, } m_f = \frac{\Delta f}{f_s} = \frac{50 \text{ kHz}}{5 \text{ kHz}} = \mathbf{10}$$

NUMERICALS-FM

Solution.

Carrier frequency, f_c = 1000 kHz

Modulating frequency, f_s = 15 kHz

Upper sideband frequencies

$f_c + f_s$	$f_c + 2f_s$	$f_c + 3f_s$
$1000 + 15$	$1000 + 2 \times 15$	$1000 + 3 \times 15$
1015 kHz	**1030 kHz**	**1045 kHz**

Lower sideband frequencies

$f_c - f_s$	$f_c - 2f_s$	$f_c - 3f_s$
$1000 - 15$	$1000 - 2 \times 15$	$1000 - 3 \times 15$
985 kHz	**970 kHz**	**955 kHz**

NUMERICALS-FM

Example : The carrier frequency in an FM modulator is 1000 kHz. If the modulating frequency is 15 kHz, what are the first three upper sideband and lower sideband frequencies?