FM&PM (Bessel function)

FM&PM (Bessel function)

- Thus, for general equation:

$$v_{FM}(t) = V_C \sin(\omega_C t + m_f \sin \omega_m t)$$

$$= V_C \sin \omega_C t \cos(m_f \sin \omega_m t) + \cos \omega_c t \sin(m_f \sin \omega_m t)$$

Bessel function

$$v(t)_{FM} = V_C[J_0(m_f)\sin\omega_C t + J_1(m_f)[\sin(\omega_C + \omega_m)t - \sin(\omega_C - \omega_m)t]$$

$$+V_C[J_2(m_f)\sin(\omega_C + 2\omega_m)t - \sin(\omega_C - 2\omega_m)t$$

$$+V_C[J_3(m_f)\sin(\omega_C + 3\omega_m)t - \sin(\omega_C - 3\omega_m)t] + ..$$

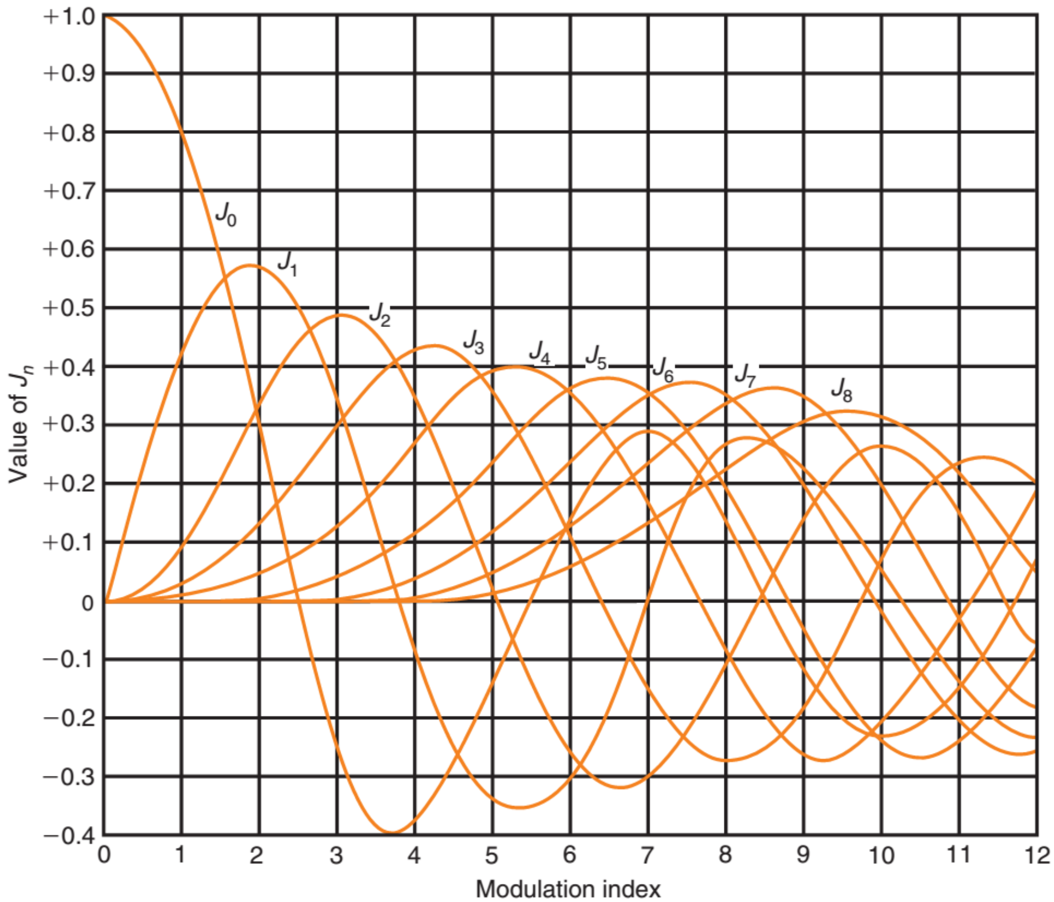
B.F. (cont'd)

- It is seen that each pair of side band is preceded by **J** coefficients. The order of the coefficient is denoted by subscript m. The Bessel function can be written as

$$J_m(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n!} - \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} - \ldots\right]$$

- N=number of the side frequency
- M=modulation index

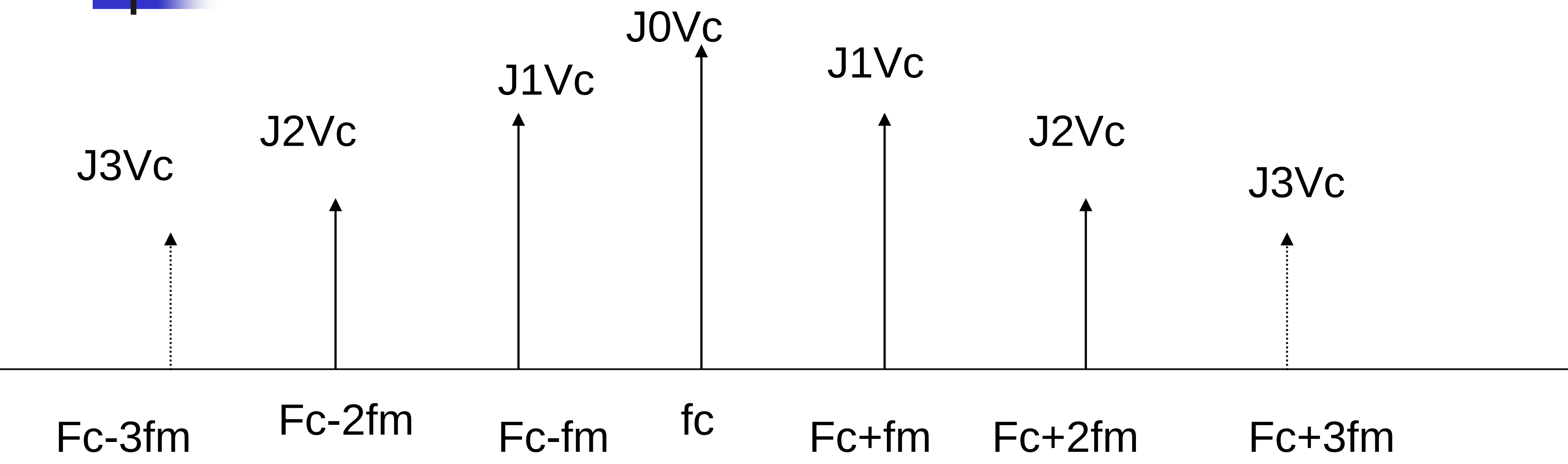
Bandwidth & Bessel's Function



x	Bessel-function order, n																
m_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.41	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—	—	—
2.5	–.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—	—	—
3.0	–.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	–.40	–.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	–.18	–.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
5.53	0	–.34	–.13	0.25	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—	—	—
6.0	0.15	–.28	–.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	–.30	–.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	–.11	–.29	–.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
8.65	0	0.27	0.06	–.24	–.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—	—	—
9.0	–.09	0.25	0.14	–.18	–.27	–.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	–.25	0.04	0.25	0.06	–.22	–.23	–.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01	—	—
12.0	0.05	–.22	–.08	0.20	0.18	–.07	–.24	–.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01

TABLE 4-1 A table of Bessel Functions of the first kind

Frequency spectrum of FM Wave –



- Observations made with Bessel functions:

1. Unlike AM where there are only three frequencies , FM has infinite number of sidebands, as well as the carrier. The sidebands are separated from carrier by fm, 2fm, 3fm……..etc

2. The J coefficient eventually decreases in value as n increases, but not in simple manner. The value fluctuates on either side of zero , gradually diminishing.

3. The sidebands at equal distance from fc have equal amplitudes , so the sideband distribution is symmetrical about the carrier.

4. In AM increased depth of modulation increases the sideband power and thereby the total transmitted power. In FM the total transmitted power always remain constant but with increased depth of modulation the required BW is increased.

5. The theoretical BW required in FM is infinite. In practice the BW used is one that has been calculated to allow for the significant amplitude of sideband components under the most exacting conditions .

FM Bandwidth

- Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.
- The value of modulation index determine the number of sidebands that have the significant relative amplitudes

Bandwidth of FM signal

Carson's rule – Estimation of Practical Bandwidth

$$B_{FM} = 2(m_f + 1)f_m = 2(\Delta f + f_m)$$

$$B_{FM}\,(\text{NBFM}) \approx 2f_m$$

POWER IN ANGLE-MODULATED SIGNAL

- The power in an angle-modulated signal is easily computed as, (R = 1 Ohm)

$$P = \frac{1}{2} V_c^2 \sum_{n=-\infty}^{n=\infty} J_n^2(\beta)$$

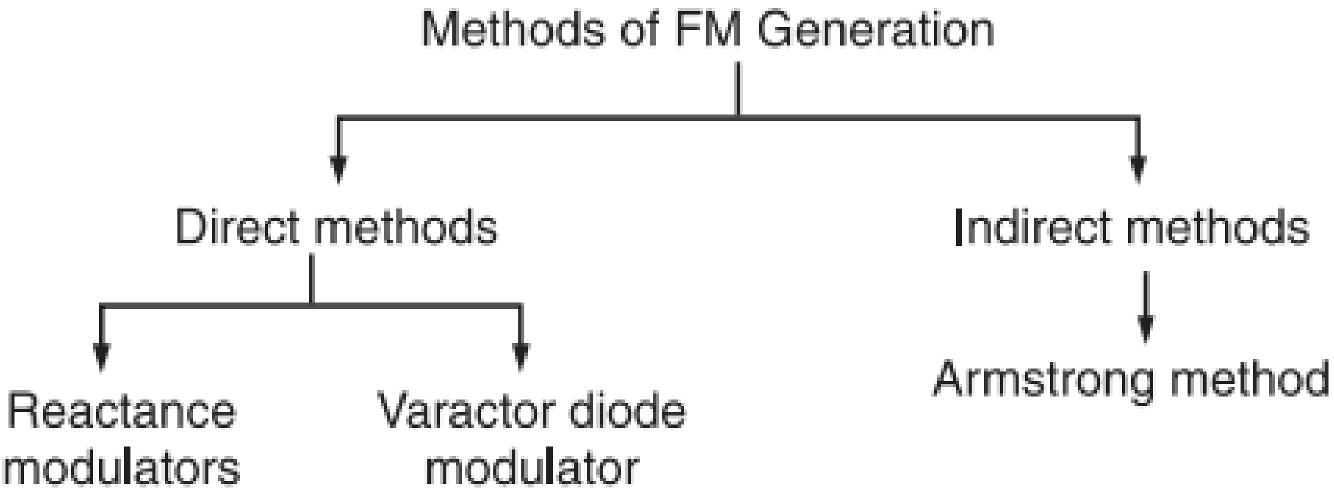
$$= \frac{V_c^2}{2}$$

- Thus the power contained in the FM signal is independent of the message signal. This is an important difference between FM and AM.

1. An FM modulator has a frequency deviation sensitivity of 4 kHz/V and a modulating signal of 10sin (2π 2000t). Determine (a) the peak frequency deviation; (b) the carrier swing; and (c) frequency modulation index. If the amplitude of the modulating signal is doubled, what is the peak frequency deviation produced?

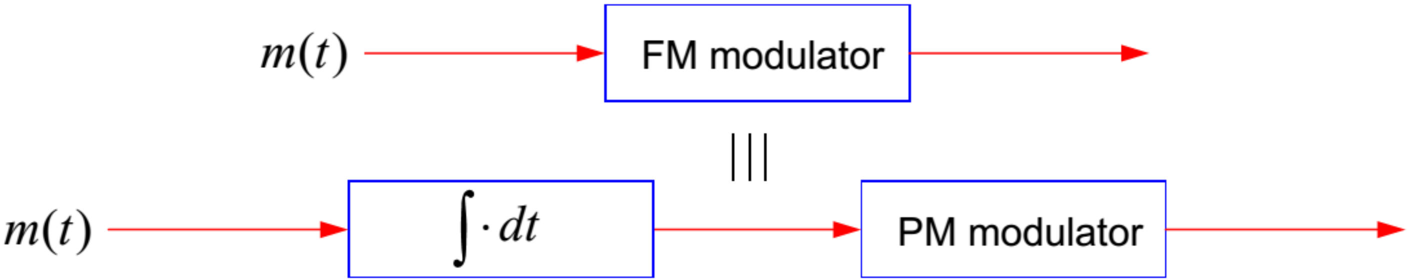
2. A PM modulator has a phase deviation sensitivity of 2.5 radians/V, and a modulating signal of 2 cos (2 π 2000t). Determine the peak phase deviation and phase-modulation index

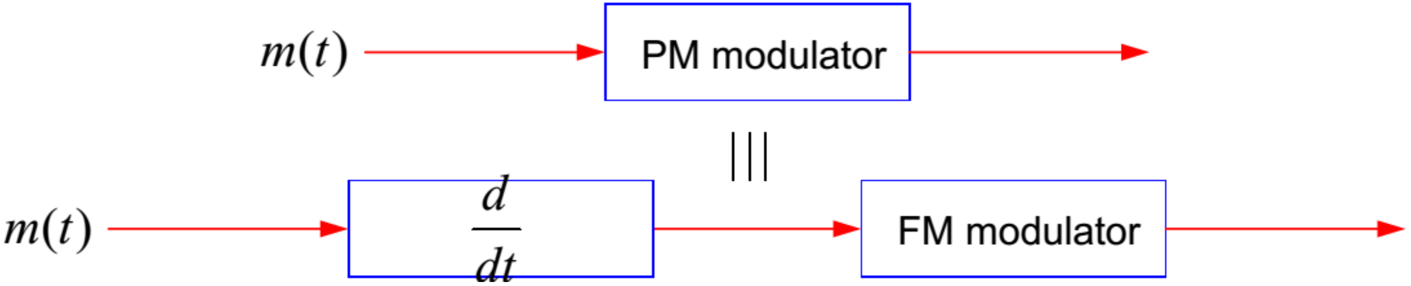
FM Generation



Angle Modulation

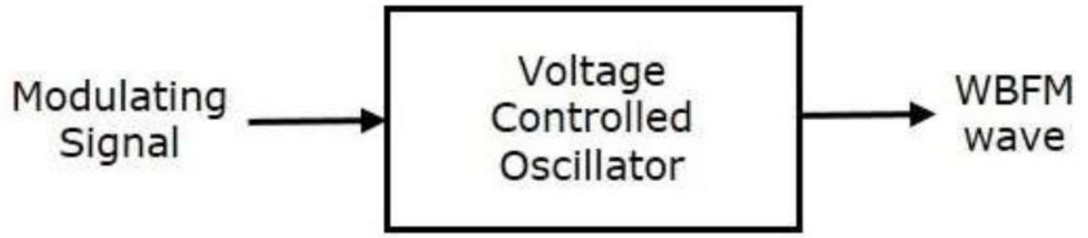
FM signal can be obtained by using phase modulator and PM wave can be obtained by using frequency modulator.





Direct method

In this method, Voltage Controlled Oscillator (VCO) is used to generate WBFM. VCO produces an output signal, whose frequency is proportional to the input signal voltage. This is similar to the definition of FM wave.

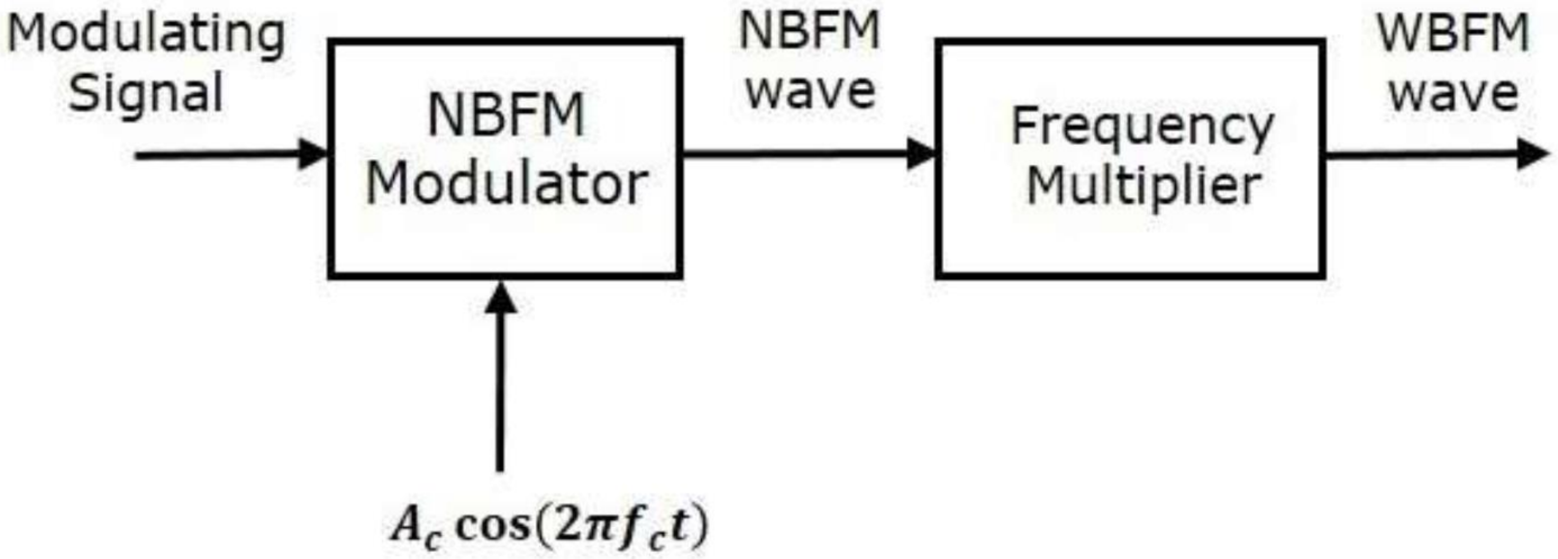


$$f_i \propto m(t)$$

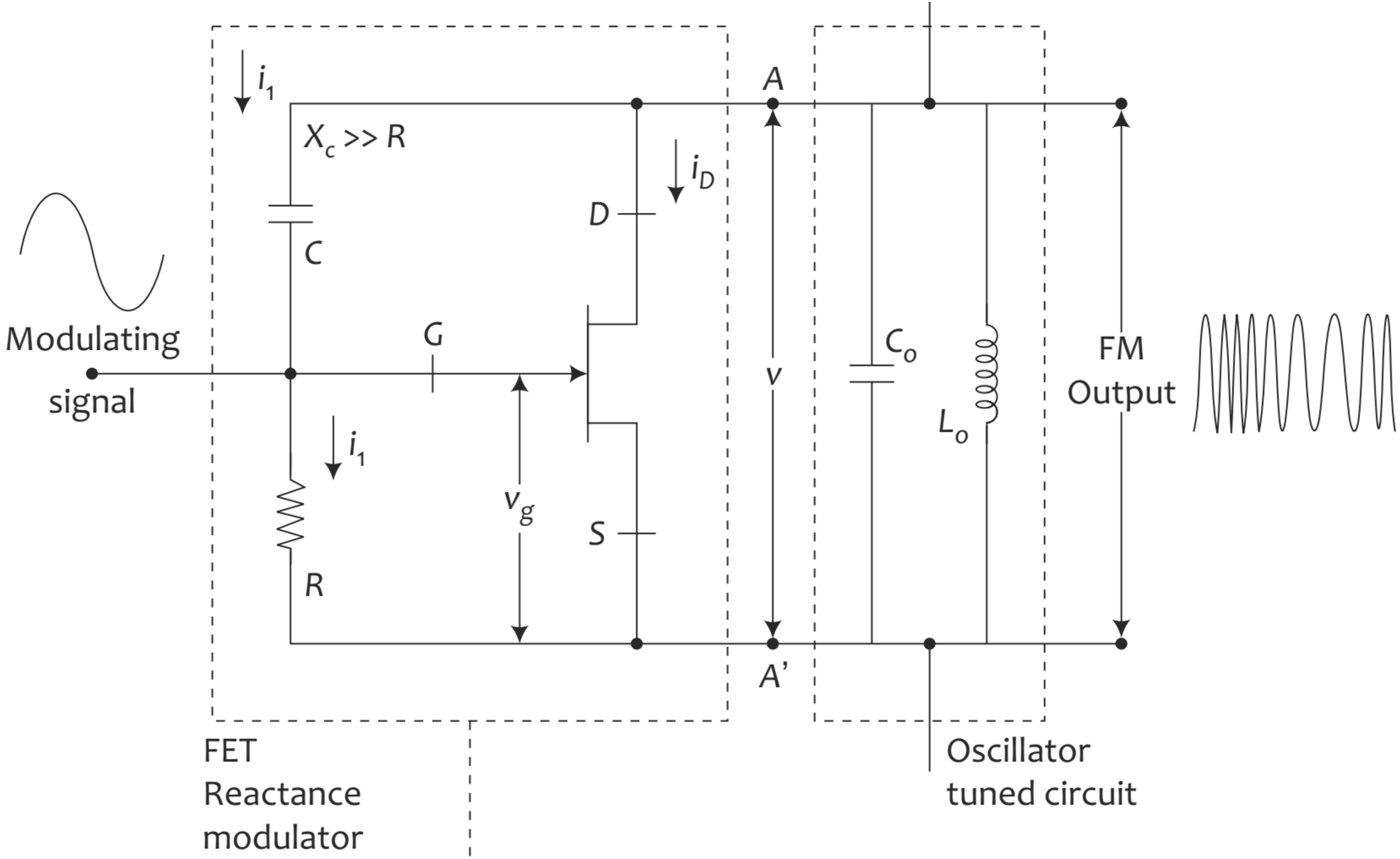
$$=> f_i = f_c + k_f m(t)$$

fi is the instantaneous frequency of WBFM wave.

Indirect method



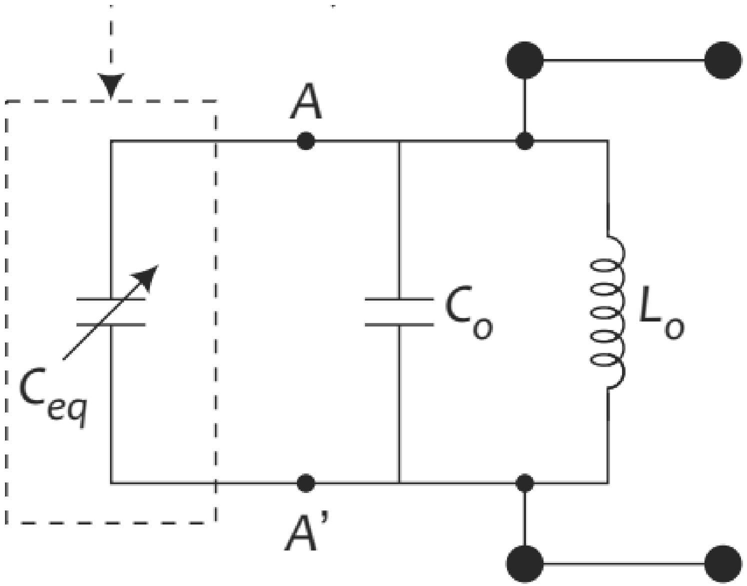
Direct method – Reactance Modulator



Direct method – Reactance Modulator

In the FM reactance modulator, an FET is operated as a variable reactance (capacitive). FET device is connected across the tuned circuit of an LC oscillator tuned circuit. As the instantaneous value of the modulating signal changes, the reactance offered by JFET will change proportionally. This will change the frequency of oscillator to produce FM signal.

Direct method – Reactance Modulator Equivalent Circuit



$$f = \frac{1}{2\pi\sqrt{LC}}$$

f ⟶ Resonant Frenquency

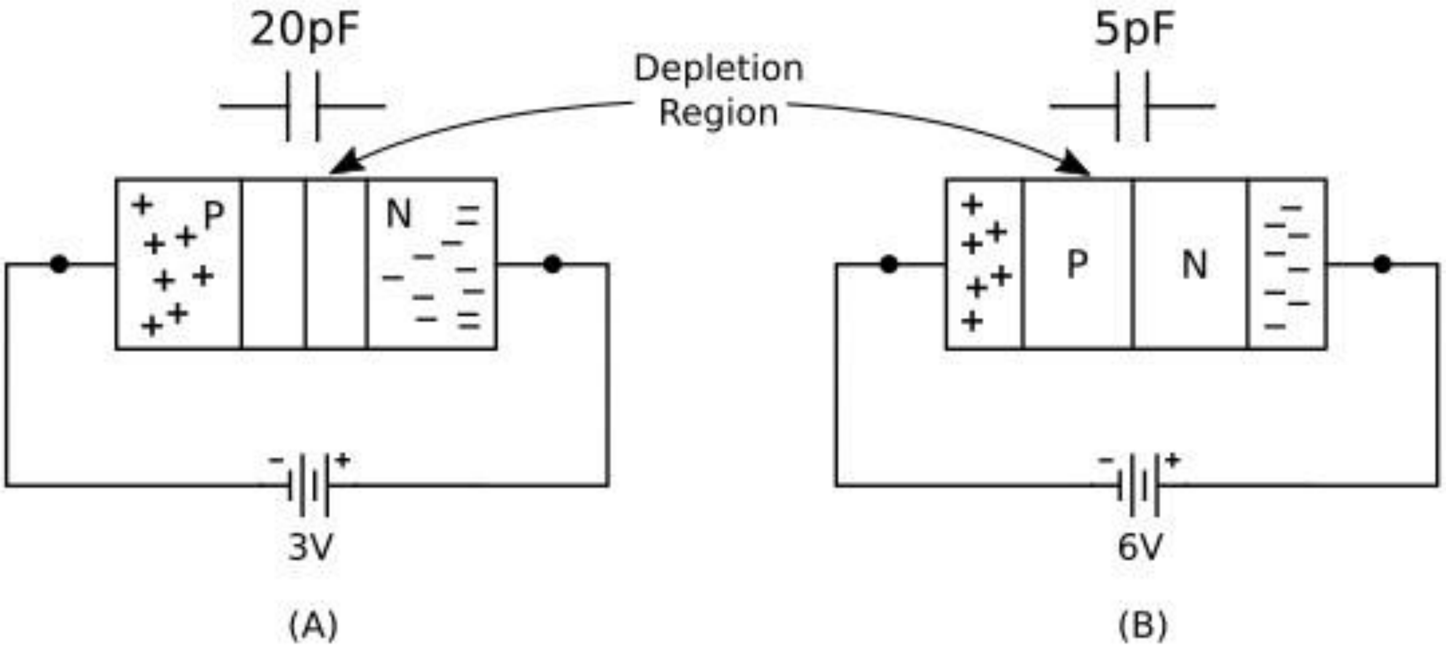
L ⟶ inductance

C ⟶ capacitance

Direct method – Varactor Diode Modulator

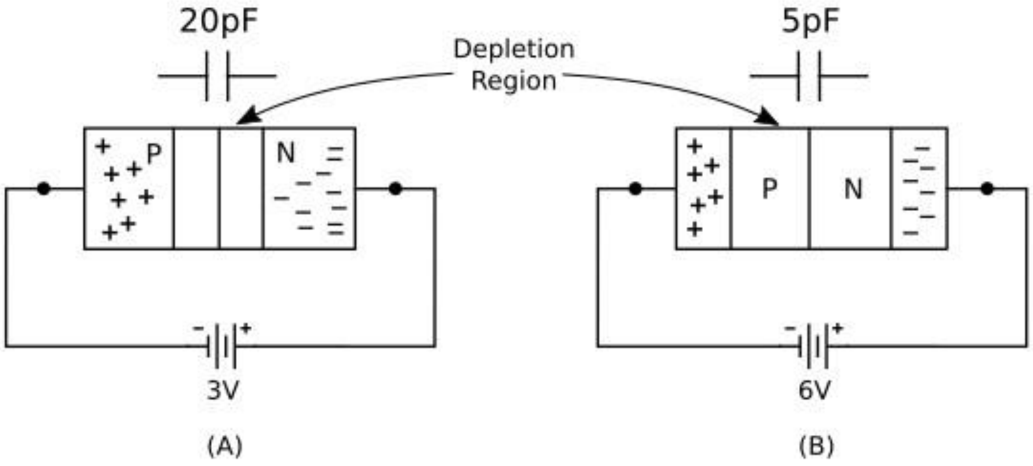
How Varactor Diode Works

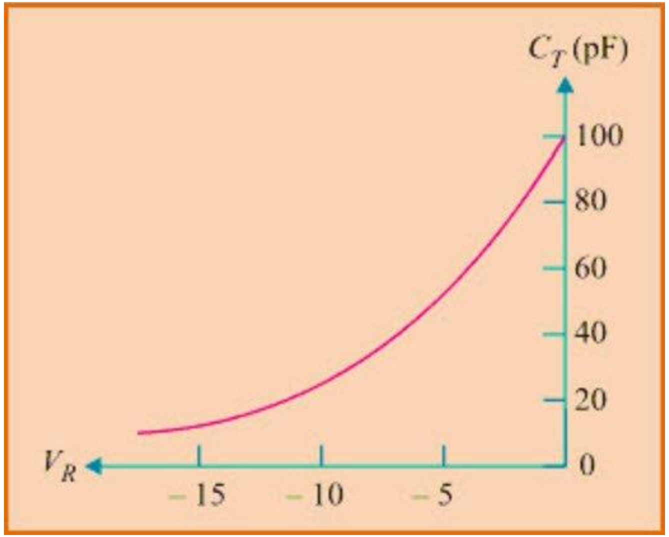
How Varactor Diode Works



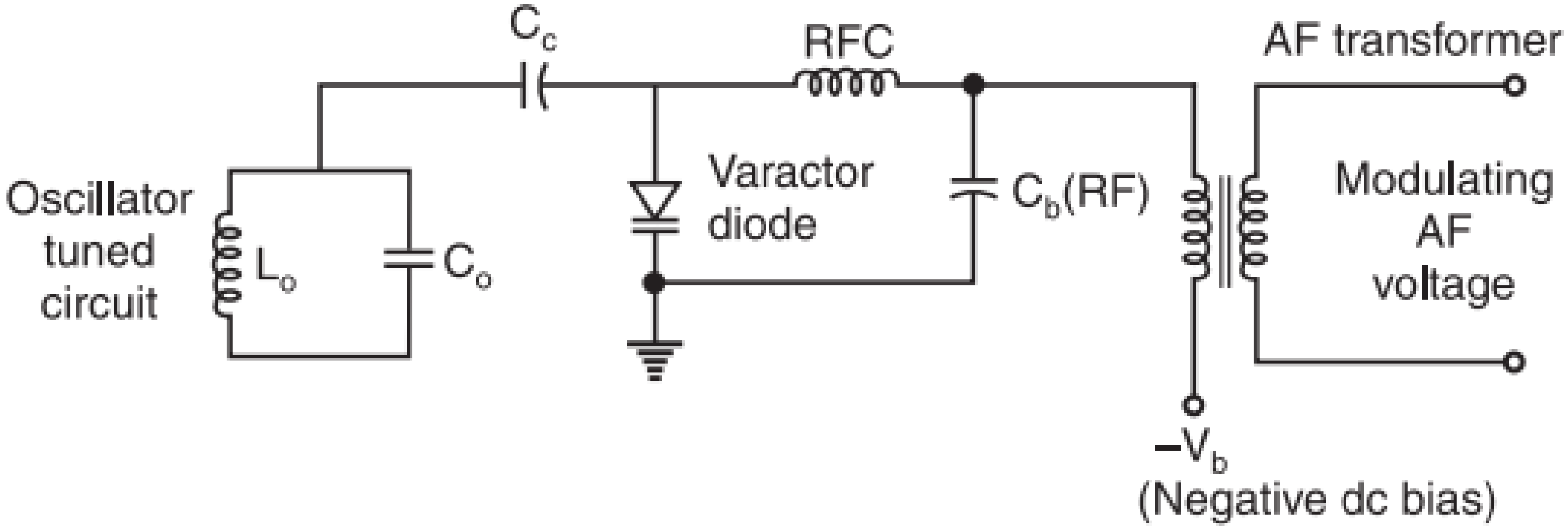
How Varactor Diode Works

$$C = \frac{\varepsilon A}{d}$$





Varactor Diode Modulator



Varactor Diode Modulator

The centre frequency for varactor-diode oscillator

$$f_c = \frac{1}{2\pi\sqrt{LC_v}} \text{ Hz}$$

Cv is the value of capacitance of varactor diode when modulating signal is not present

Varactor Diode Modulator

The centre frequency for varactor-diode oscillator

$$f_c = \frac{1}{2\pi\sqrt{LC_v}}\ \text{Hz}$$

Cv is the value of capacitance of varactor diode when modulating signal is not present

When a modulating signal is applied, then the output frequency is given by

$$f = \frac{1}{2\pi\sqrt{L(C_v + \Delta C_v)}}\ \text{Hz}$$

Varactor Diode Modulator

The varactor diode is reverse biased by the negative dc source $-V_b$.
The modulating AF voltage appears in series with the negative supply voltage. Hence, the voltage applied across the varactor diode varies in proportion with the modulating voltage.

This will vary the junction capacitance of the varactor diode. The varactor diode appears in parallel with the oscillator tuned circuit.

Varactor Diode Modulator

Hence the oscillator frequency will change with change in varactor dioide capacitance and FM wave is produced.

The RFC will connect the dc and modulating signal to the varactor diode but it offers a very high impedance at high oscillator frequency. Therefore, the oscillator circuit is isolated from the dc bias and modulating signal.