Angle Modulation

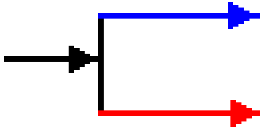
Angle modulation

- **Angle modulation is the process of *varying the total phase angle* of a carrier wave in accordance with the instantaneous value of the message signal, while keeping the amplitude of the carrier constant.**

Angle modulation *requires more complicated transmitters and receivers* as compared to amplitude modulation.

Angle Modulation

The phase angle of the carrier wave is varied in accordance with the amplitude of modulating signal is referred to as 'Angle Modulation'.



PM : Phase Modulation

FM : Frequency Modulation

PM: The phase of carrier wave is varied in accordance with the amplitude of message signal.

FM: The frequency of the carrier wave is varied in accordance with the amplitude of the modulating signal

Phase angle and Instantaneous Frequency

$$c(t) = A_C \cos(\theta(t))$$

$$= A_C \cos(\omega_C t + \theta_0)$$

$$\theta(t) = \omega_C t + \theta_0$$

Relationship between Frequency & phase angle

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

Determine the instantaneous frequency in hertz of each of the following signals:

(a) $10 \cos\left(200\pi t + \frac{\pi}{3}\right)$

(b) $10 \cos\left(20\pi t + \pi t^2\right)$

(c) $\cos 200\pi t \cos\left(5 \sin 2\pi t\right) + \sin 200\pi t \sin\left(5 \sin 2\pi t\right)$

(*a*)

$$\theta(t) = 200\pi t + \frac{\pi}{3}$$

$$\omega_i = \frac{d\theta}{dt} = 200\pi = 2\pi(100)$$

The instantaneous frequency of the signal is 100 Hz, which is constant.

(*b*)

$$\theta(t) = 20\pi t + \pi t^2$$

$$\omega_i = \frac{d\theta}{dt} = 20\pi + 2\pi t = 2\pi(10 + t)$$

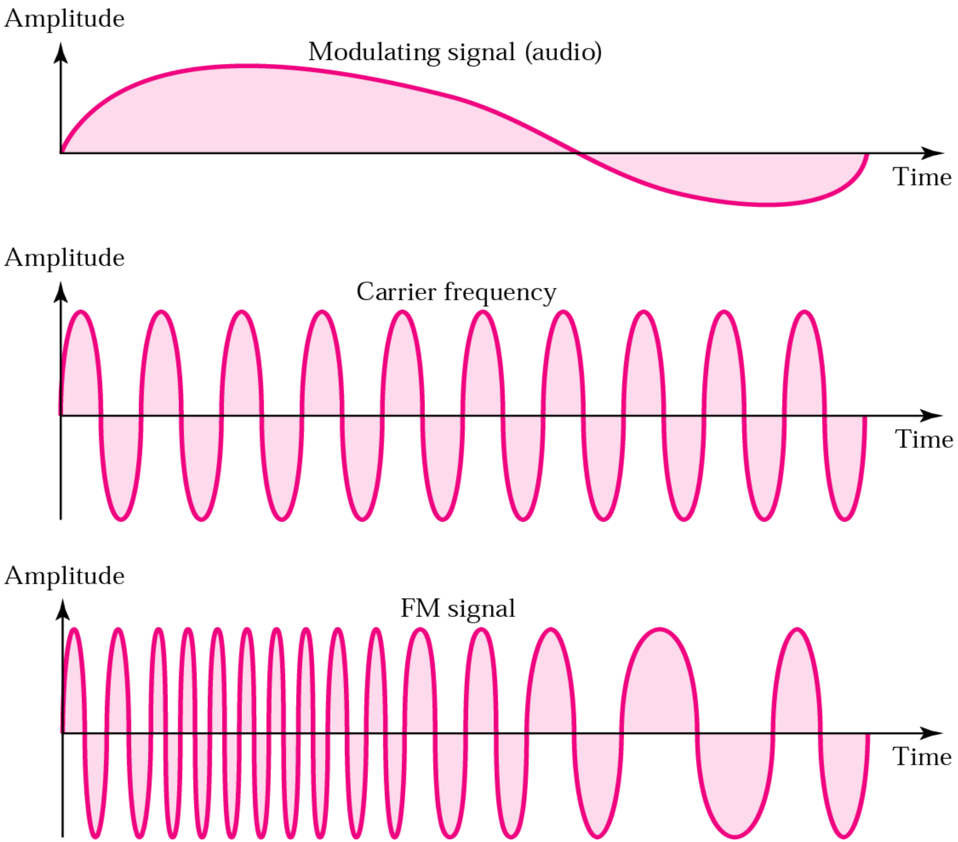
$$
\begin{aligned}
\cos(a)\cos(b) &= \frac{1}{2}\Big(\cos(a+b)+\cos(a-b)\Big)\\
\sin(a)\sin(b) &= \frac{1}{2}\Big(\cos(a-b)-\cos(a+b)\Big)\\
\sin(a)\cos(b) &= \frac{1}{2}\Big(\sin(a+b)+\sin(a-b)\Big)\\
\cos(a)\sin(b) &= \frac{1}{2}\Big(\sin(a+b)-\sin(a-b)\Big)
\end{aligned}
$$

$$\cos 200\pi t \cos(5 \sin 2\pi t) + \sin 200\pi t \sin(5 \sin 2\pi t) = \cos(200\pi t - 5 \sin 2\pi t)$$

$$\theta(t) = 200\pi t - 5 \sin 2\pi t$$

$$\omega_i = \frac{d\theta}{dt} = 200\pi - 10\pi \cos 2\pi t = 2\pi(100 - 5 \cos 2\pi t)$$

Frequency modulation



Frequency modulation (FM) Mathematical Representation

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The equation for instantaneous frequency $\mathbf{f}_i$ in FM modulation is

$$\mathbf{f_i} = \mathbf{f_c} + \mathbf{k_f}\, \boldsymbol{m}(\boldsymbol{t})$$

Where,

$\boldsymbol{f}_{\mathbf{c}}$ is the carrier frequency

$\boldsymbol{k}_{\mathbf{f}}$ is the frequency sensitivity

$\mathbf{m(t)}$ is the message signal

We know the relationship between angular frequency ωi and angle $\theta i(t)$

$$\omega_i = \frac{d\theta_i(t)}{dt}$$

$$=> 2\pi f_i = \frac{d\theta_i(t)}{dt}$$

$$=> \theta_i(t) = 2\pi \int f_i \, dt$$

$$\boldsymbol{\theta_i(t) = 2\pi \int f_i \, dt}$$

Substitute, $\mathbf{f}_i$ value in the above equation.

$$\boldsymbol{\theta_i(t) = 2\pi \int (\mathrm{f_c} + \mathrm{k_f}\, m(t))\, dt}$$

$$\boldsymbol{=> \theta_i(t) = 2\pi \mathrm{f_c} t + 2\pi \mathrm{k_f} \int m(t)\, dt}$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t)\, dt$$

Substitute, $\theta i(t)$ value in the standard equation of angle modulated wave.

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t) dt\right)$$

This is the **equation of FM wave**.

FM Modulation – Single Tone Signal

If the modulating signal is $\mathbf{m(t)} = \boldsymbol{A_m\, cos(2\pi f_m t)}$,

FM Equation can be obtained from

$$\mathbf{s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right)}$$

If the modulating signal is $\mathbf{m(t)} = \boldsymbol{A_m}\ \boldsymbol{cos}(\mathbf{2}\boldsymbol{\pi f_m t})$,

$$s_{FM}(t) = A_C \cos\left[\omega_C t + 2\pi k_f \int A_m \cos 2\pi f_m t \, dt\right]$$

$$= A_C \cos\left[\omega_C t + \frac{2\pi k_f A_m}{2\pi f_m} \sin 2\pi f_m t\right]$$

$$= A_C \cos\left[\omega_C t + \beta \sin 2\pi f_m t\right]$$

$$\boldsymbol{\beta} = \boldsymbol{modulation\ index} = \frac{\boldsymbol{\Delta f}}{\boldsymbol{f_m}} = \frac{\mathbf{k_f}\boldsymbol{A_m}}{\boldsymbol{f_m}}$$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as **Frequency Deviation**. It is denoted by **Δf**, which is equal to the product of $\boldsymbol{kf}$ and $\boldsymbol{Am}$.

FM can be divided into **Narrowband FM** and **Wideband FM** based on the values of modulation index $\boldsymbol{\beta}$.

Narrowband FM

This frequency modulation has a small bandwidth when compared to wideband FM.

The modulation index β is small, i.e., less than 1.

Its spectrum consists of the carrier, the upper sideband and the lower sideband.

This is used in mobile communications such as police wireless, ambulances, taxicabs, etc.

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This frequency modulation has a small bandwidth when compared to wideband FM.

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This is used in mobile communications such as police wireless, ambulances, taxicabs, etc.

Wideband FM

This frequency modulation has infinite bandwidth.

The modulation index β is large, i.e., higher than 1.

Its spectrum consists of a carrier and infinite number of sidebands, which are located around it.

This is used in entertainment, broadcasting applications such as FM radio, TV, etc.

Bandwidth of FM signal

The bandwidth of an FM signal depends on the frequency deviation. When the frequency deviation (and hence m_f) is high, the bandwidth will be large.

Types of frequency modulation

If $m_f < 1$, this is known as *narrowband FM.*

If $m_f > 1$ it is *wideband FM* (also known as broadcast FM).

Bandwidth of FM signal

Bandwidth of a sinusoidally modulated FM signal (Carson's rule)

$$B_{FM} = 2(m_f + 1)f_m = 2(\Delta f + f_m)$$

$$B_{FM}\,(\text{NBFM}) \approx 2f_m$$

Bandwidth of a sinusoidally modulated FM/PM signal (Carson's rule)

$$B_{FM} = 2(m_f + 1)f_m = 2(\Delta f + f_m)$$

Similarly $B_{PM} = 2(m_p + 1)f_m$

A sinusoidal modulating waveform of amplitude 5 V and a frequency of 2 KHz is applied to FM generator, which has a frequency sensitivity of 40 Hz/volt. Calculate the frequency deviation, modulation index, and bandwidth.

Given, the amplitude of modulating signal, $\boldsymbol{A_m = 5\,V}$

Frequency of modulating signal, $\boldsymbol{f_m = 2\,KHz}$

Frequency sensitivity, $\boldsymbol{k_f = 40\,Hz\,/\,volt}$

We know the formula for Frequency deviation as

$$\boldsymbol{\Delta f = k_f\,A_m}$$

Substitute $\boldsymbol{k_f}$ and $\boldsymbol{A_m}$ values in the above formula.

$$\boldsymbol{\Delta f = 40\ \times 5 = 200\,Hz}$$

Therefore, **frequency deviation**, Δf is $\boldsymbol{200\,Hz}$.

The formula for modulation index is

$$\boldsymbol{\beta} = \frac{\Delta \boldsymbol{f}}{\boldsymbol{f_m}}$$

Substitute $\Delta \boldsymbol{f}$ and $\boldsymbol{f_m}$ values in the above formula.

$$\boldsymbol{\beta} = \frac{\mathbf{200}}{\mathbf{2 \times 1000}} = \mathbf{0.1}$$

The formula for Bandwidth of Narrow Band FM is the same as that of AM wave.

$$BW = 2f_m$$

Substitute f_m value in the above formula.

$$BW = 2 \times 2K = 4\ KHz$$

Therefore, the **bandwidth** of Narrow Band FM wave is **4 KHz**.

An FM wave is given by

$$\boldsymbol{s(t) = 20\cos(8\pi \times 106t + 9\sin(2\pi \times 10^3 t))}.$$

Calculate the frequency deviation, bandwidth, and power of FM wave.

Given, the equation of an FM wave as

$$\boldsymbol{s(t) = 20\cos(8\pi \times 10^{6}t + 9\sin(2\pi \times 10^{3}t))}$$

We know the standard equation of an FM wave as

$$\boldsymbol{s(t) = A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))}$$

Amplitude of the carrier signal, $\boldsymbol{A_c = 20\,V}$

Frequency of the carrier signal, $\boldsymbol{f_c = 4 \times 10^6\,Hz = 4\,MHz}$

Frequency of the message signal, $\boldsymbol{f_m = 1 \times 10^3\,Hz = 1\,KHz}$

Modulation index, $\boldsymbol{\beta = 9}$

We know the formula for modulation index as

$$\boldsymbol{\beta} = \frac{\Delta \boldsymbol{f}}{\boldsymbol{f_m}}$$

Rearrange the above equation as follows.

$$\Delta \boldsymbol{f} = \boldsymbol{\beta}\, \boldsymbol{f_m}$$

Substitute $\boldsymbol{\beta}$ and $\boldsymbol{f_m}$ values in the above equation.

$$\Delta \boldsymbol{f} = \mathbf{9} \times \mathbf{1}\boldsymbol{K} = \mathbf{9}\, \boldsymbol{KHz}$$

Therefore, **frequency deviation**, Δf is **9 KHz**.

The formula for Bandwidth of Wide Band FM wave is

$$\boldsymbol{BW = 2(\beta + 1)f_m}$$

Substitute $\boldsymbol{\beta}$ and $\boldsymbol{f_m}$ values in the above formula.

$$\boldsymbol{BW = 2(9 + 1)\ 1K = 20\ KHz}$$

power of FM wave is

$$P = \frac{{A_c}^2}{2R}$$

power of FM wave is

$$\boldsymbol{P = \frac{{A_c}^2}{2R}}$$

Assume, $\boldsymbol{R = 1\,\Omega}$ and substitute $\boldsymbol{A_c}$ value in the above equation.

$$\boldsymbol{P = \frac{(20)^2}{2(1)} = 200\,W}$$

Therefore, the **power** of FM wave is **200 watts**.