FREQUENCY MODULATION

The instantaneous angular frequency of FM is given by

$$\omega_i = \omega_c + \Delta\omega_c \cos \omega_s t$$

Total phase angle $\theta = \omega t$, so that if ω is variable then

$$\theta = \int_0^t \omega_i dt$$
$$\theta = \int_0^t (\omega_c + \Delta\omega_c \cos \omega_s t) dt$$

$$\theta = \omega_c t + \frac{\Delta\omega_c}{\omega_s} \sin \omega_s t$$

The term $\frac{\Delta\omega_c}{\omega_s}$ is called the modulation index β

$$\theta = \omega_c t + \beta \sin \omega_s t$$

The instantaneous value of FM voltage wave is given by

$$e = E_c \cos \theta$$

$$e = E_c \cos(\omega_c t + \beta \sin \omega_s t)$$

FREQUENCY DEVIATION AND PERCENT MODULATION

7-7-1 Frequency Deviation

Frequency deviation is the change in frequency that occurs in the carrier when it is acted on by a modulating-signal frequency. Frequency deviation is typically given as a peak frequency shift in hertz (Δf). The peak-to-peak frequency deviation ($2\Delta f$) is sometimes called *carrier swing*.

For an FM, the deviation sensitivity is often given in hertz per volt. Therefore, the peak frequency deviation is simply the product of the deviation sensitivity and the peak modulating-signal voltage and is expressed mathematically as

$$\Delta f = K_1 V_m \text{ (Hz)} \tag{7-21}$$

Equation 7-21 can be substituted into Equation 7-20, and the expression for the modulation index in FM can be rewritten as

$$m = \frac{\Delta f \text{(Hz)}}{f_m \text{(Hz)}} \text{(unitless)} \tag{7-22}$$

Therefore, for FM, Equation 7-1 can be rewritten as

$$m(t) = V_c \cos\left[\omega_c t + \frac{K_1 V_m}{f_m} \sin(\omega_m t)\right] \tag{7-23}$$

or

$$m(t) = V_c \cos\left[\omega_c t + \frac{\Delta f}{f_m} \sin(\omega_m t)\right] \tag{7-24}$$

or

$$m(t) = V_c \cos[\omega_c t + m \sin(\omega_m t)] \tag{7-25}$$

From examination of Equations 7-19 and 7-20, it can be seen that the modulation indices for FM and PM relate to the modulating signal differently. With PM, both the modulation index and the peak phase deviation are directly proportional to the amplitude of the modulating signal and unaffected by its frequency. With FM, however, both the modulation index and the frequency deviation are directly proportional to the amplitude of the modulating signal, and the modulation index is inversely proportional to its frequency. Figure 7-4 graphically shows the relationship among modulation index and peak phase deviation for PM and the modulation index and peak frequency deviation for FM in respect to the modulating-signal amplitude and frequency.

NUMERICAL-FREQUENCY DEVIATION

Example 7-1

a. Determine the peak frequency deviation (Δf) and modulation index (m) for an FM modulator with a deviation sensitivity $K_1 = 5$ kHz/V and a modulating signal $v_m(t) = 2\cos(2\pi 2000t)$.

b. Determine the peak phase deviation (m) for a PM modulator with a deviation sensitivity $K = 2.5$ rad/V and a modulating signal $v_m(t) = 2\cos(2\pi 2000t)$.

Solution

a. The peak frequency deviation is simply the product of the deviation sensitivity and the peak amplitude of the modulating signal, or

$$\Delta f = \frac{5\text{ kHz}}{\text{V}} \times 2\text{ V} = 10\text{ kHz}$$

The modulation index is determined by substituting into Equation 7-22:

$$m = \frac{10\text{ kHz}}{2\text{ kHz}} = 5$$

b. The peak phase shift for a phase-modulated wave is the modulation index and is found by substituting into Equation 7-15:

$$m = \frac{2.5\text{ rad}}{\text{V}} \times 2\text{ V} = 5\text{ rad}$$

NUMERICALS-FM

Example *A frequency modulated voltage wave is given by the equation :*

$$e = 12 \cos (6 \times 10^8 t + 5 \sin 1250\, t)$$

Find (i) carrier frequency (ii) signal frequency (iii) modulation index (iv) maximum frequency deviation (v) power dissipated by the FM wave in 10-ohm resistor.

NUMERICALS-FM

Solution. The given FM voltage wave is

$$e = 12\cos(6\times10^{8}t + 5\sin 1250\,t)$$

The equation of standard FM voltage wave is

$$e = E_c\cos(\omega_c t + m_f\sin\omega_s t)$$

Comparing eqs. (*i*) and (*ii*), we have,

(*i*) Carrier frequency, $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{6\times10^{8}}{2\pi} = \mathbf{95.5\times10^{6}\ Hz}$

(*ii*) Signal frequency, $f_s = \dfrac{\omega_s}{2\pi} = \dfrac{1250}{2\pi} = \mathbf{199\ Hz}$

(*iii*) Modulation index, $m_f = \mathbf{5}$

(*iv*) Max. frequency deviation, $\Delta f = m_f\times f_s = 5\times199 = \mathbf{995\ Hz}$

(*v*) Power dissipated, $P = \dfrac{E_{r.m.s.}^2}{R} = \dfrac{(12/\sqrt{2})^2}{10} = \mathbf{7.2W}$

NUMERICALS-FM

Example *A 25 MHz carrier is modulated by a 400 Hz audio sine wave. If the carrier voltage is 4V and the maximum frequency deviation is 10 kHz, write down the voltage equation of the FM wave.*

NUMERICALS-FM

Solution. The voltage equation of the FM wave is

$$e = E_c \cos(\omega_c t + m_f \sin \omega_s t)$$

$$\omega_c = 2\pi f_c = 2\pi \times 25 \times 10^6 = 1.57 \times 10^8 \text{ rad/s}$$

$$\omega_s = 2\pi f_s = 2\pi \times 400 = 2513 \text{ rad/s}$$

$$m_f = \frac{\Delta f}{f_s} = \frac{10 \text{ kHz}}{400 \text{ Hz}} = \frac{10 \times 10^3 \text{ Hz}}{400\text{Hz}} = 25$$

$$e = 4 \cos(1.57 \times 10^8 t + 25 \sin 2513t)$$ **Ans.**

NUMERICALS-FM

Example: Calculate the modulation index for an FM wave where the maximum frequency deviation is 50 kHz and the modulating frequency is 5 kHz.

NUMERICALS-FM

Solution.

$$\text{Max. frequency deviation, } \Delta f = 50 \text{ kHz}$$

$$\text{Modulating frequency, } f_s = 5 \text{ kHz}$$

$$\therefore \quad \text{Modulation index, } m_f = \frac{\Delta f}{f_s} = \frac{50 \text{ kHz}}{5 \text{ kHz}} = \mathbf{10}$$

NUMERICALS-FM

Solution.

Carrier frequency, f_c = 1000 kHz

Modulating frequency, f_s = 15 kHz

Upper sideband frequencies

$f_c + f_s$	;	$f_c + 2f_s$	;	$f_c + 3f_s$
$1000 + 15$	;	$1000 + 2 \times 15$	;	$1000 + 3 \times 15$
1015 kHz	;	**1030 kHz**	;	**1045 kHz**

Lower sideband frequencies

$f_c - f_s$	;	$f_c - 2f_s$	;	$f_c - 3f_s$
$1000 - 15$	;	$1000 - 2 \times 15$	;	$1000 - 3 \times 15$
985 kHz	;	**970 kHz**	;	**955 kHz**

NUMERICALS-FM

Example : The carrier frequency in an FM modulator is 1000 kHz. If the modulating frequency is 15 kHz, what are the first three upper sideband and lower sideband frequencies?