# Module2: Amplitude Modulation

Amplitude modulation – Single- tone and Multi-tone, Mathematical representation of AM signal, Bandwidth, current, power and transmission efficiency of AM. Generation of AM signal – Square law modulator, Switching modulator. AM demodulation – Envelope detector and Square law demodulator.

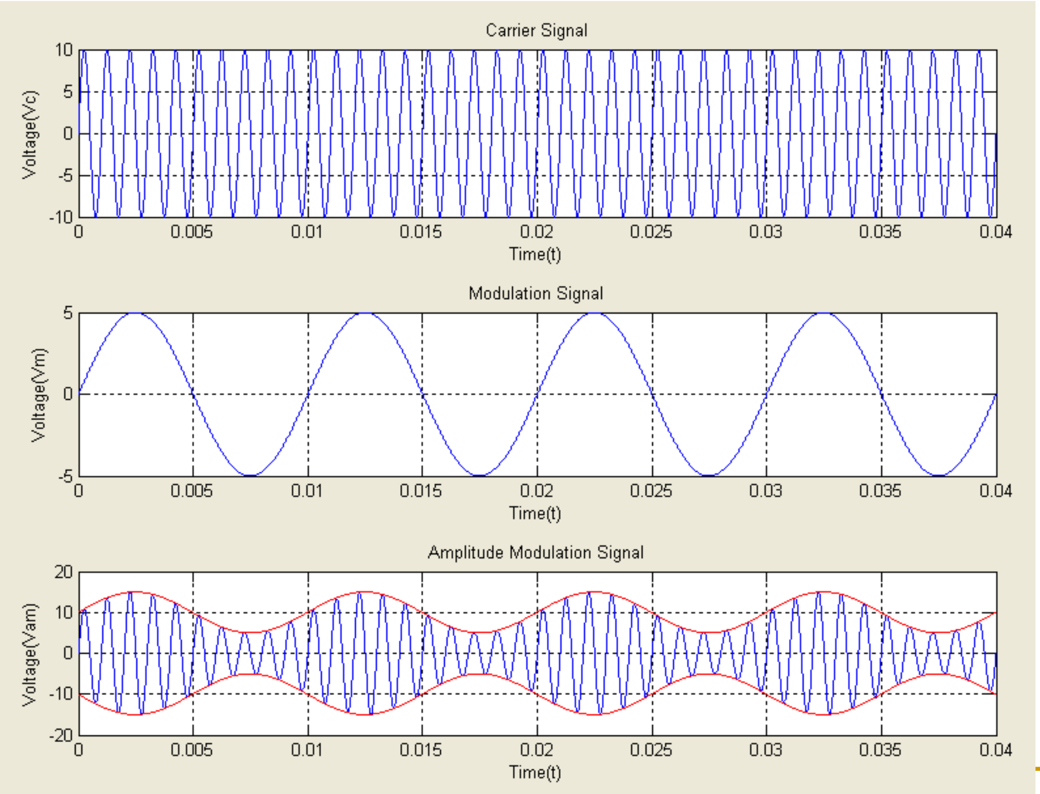
# AMPLITUDE MODULATION

- Amplitude modulation is the process of changing the amplitude of the carrier frequency in proportion with the instantaneous value of the modulation signal.

- Modulating signal will modulate the carrier signal so that the information contains in the modulating signal can be efficiently radiated by an antenna through free space at radio frequency (RF).

- Modulating signal $v_m(t)$, may contains a single frequency or multiple frequency such as human voice.

# AM SIGNAL WITH THE ENVELOPE



# MATHEMATICAL REPRESENTATION OF AM

Let the carrier signal, $v_c(t)$, and the modulating signal, $v_m(t)$ be

$$v_c(t) = V_c \sin 2\pi f_c t$$
$$v_m(t) = V_m \sin 2\pi f_m t$$

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Modulating signal uses the peak value of the carrier as a reference point, i.e. modulating signal value adds to or subtract from the peak value of the carrier.

$$V_{env}(t) = V_c + v_m(t)$$
$$V_{env}(t) = V_c + V_m \sin 2\pi f_m t$$

This signal is known as the envelope of the modulated wave, $V_{env}(t)$.

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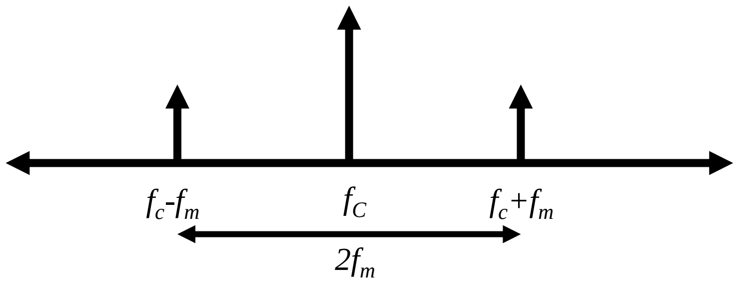
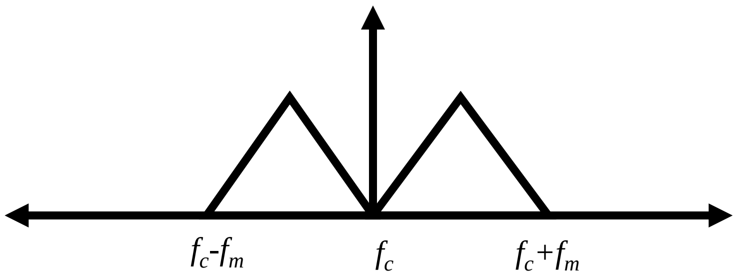
Using the formula $\sin A \sin B = \frac{\cos(A-B)}{2} - \frac{\cos(A+B)}{2}$

$$V_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi(f_c - f_m)t - \frac{V_m}{2}\cos 2\pi(f_c + f_m)t$$

# AM FREQUENCY SPECTRUM & BANDWIDTH

- Output envelope is complex wave made up of dc voltage, the carrier frequency and the sum (fm + fc) and difference (fc – fm) frequencies
- Sum and difference frequency are displaced from the carrier frequency by an amount equal to the modulating signal frequency
- Therefore, AM spectrum contains frequency component spaced fm Hz on either side of the carrier
- Figure shows the frequency spectrum of AM DSBFC wave



- Bandwidth = Maximum freq. - minimum freq.

$$
\begin{aligned}
BW &= (f_c + f_m) - (f_c - f_m) \\
&= f_c + f_m - f_c + f_m \\
&= 2f_m
\end{aligned}
$$

# MODULATION INDEX

- Modulation index, $m$ is merely defined as a parameter, which determines the amount of modulation.
- For proper AM to occur

$$V_m \leq V_c$$

- Modulation index,

$$m = \frac{V_m}{V_c}$$

- Or in percentage ,

$$M = \frac{V_m}{V_c} \times 100\%$$

- By this definition, we could distinguished three different types of amplitude modulation.
  - Under modulated AM for $m < 1$
  - Ideal AM for $m = 1$
  - Over modulated AM for $m > 1$

- $m < 1$ : under modulation
  - $V_m < V_c$
  - signal strength obtained at the receiver is not exactly the same as the signal strength at the transmitter.
  - No distortion to the signal, just reduced signal strength.

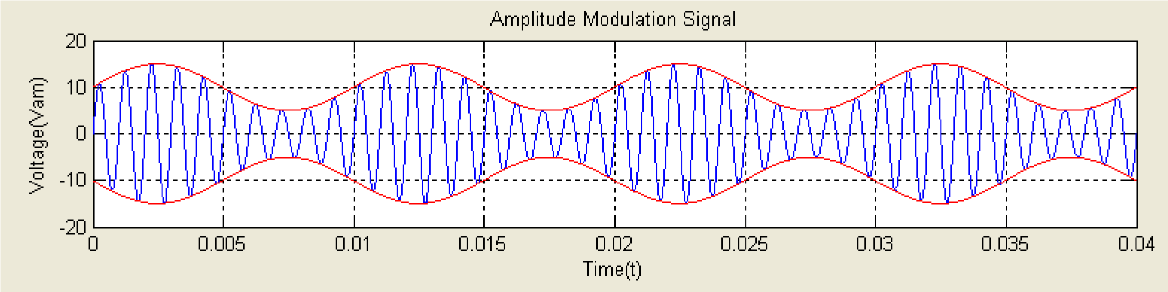



Figure 2.4 (a) $m < 1$, under modulation.

- $m = 1$ : ideal modulation
  - $V_m = V_c$
  - will produce greatest output at the receiver without distortion
  - maximum info signal amplitude is transmitted
  - more info signal power is transmitted → producing stronger, more intelligible signal
  - hard to achieve especially when the modulating signal amplitude varies randomly over a wide range – only the peak of the signal will produce 100% modulation.

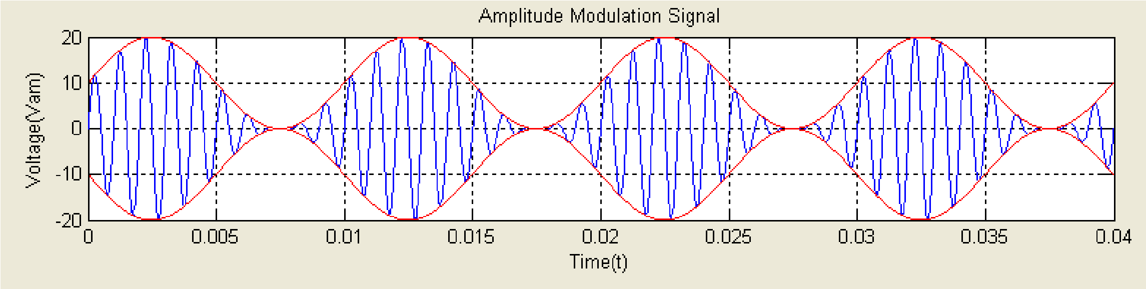



Figure 2.4(b) $m = 1$, ideal modulation.

- $m > 1$ : over modulation
  - $V_m > V_c$
  - cause distortion
  - negative peaks have been clipped off.
  - The original shape of the signal is destroyed.

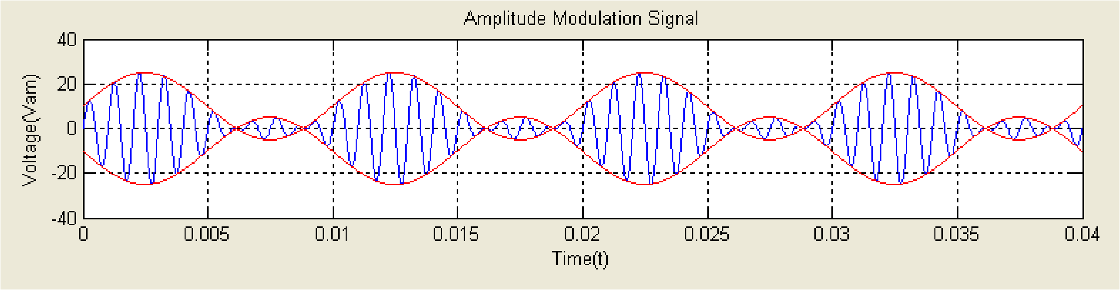
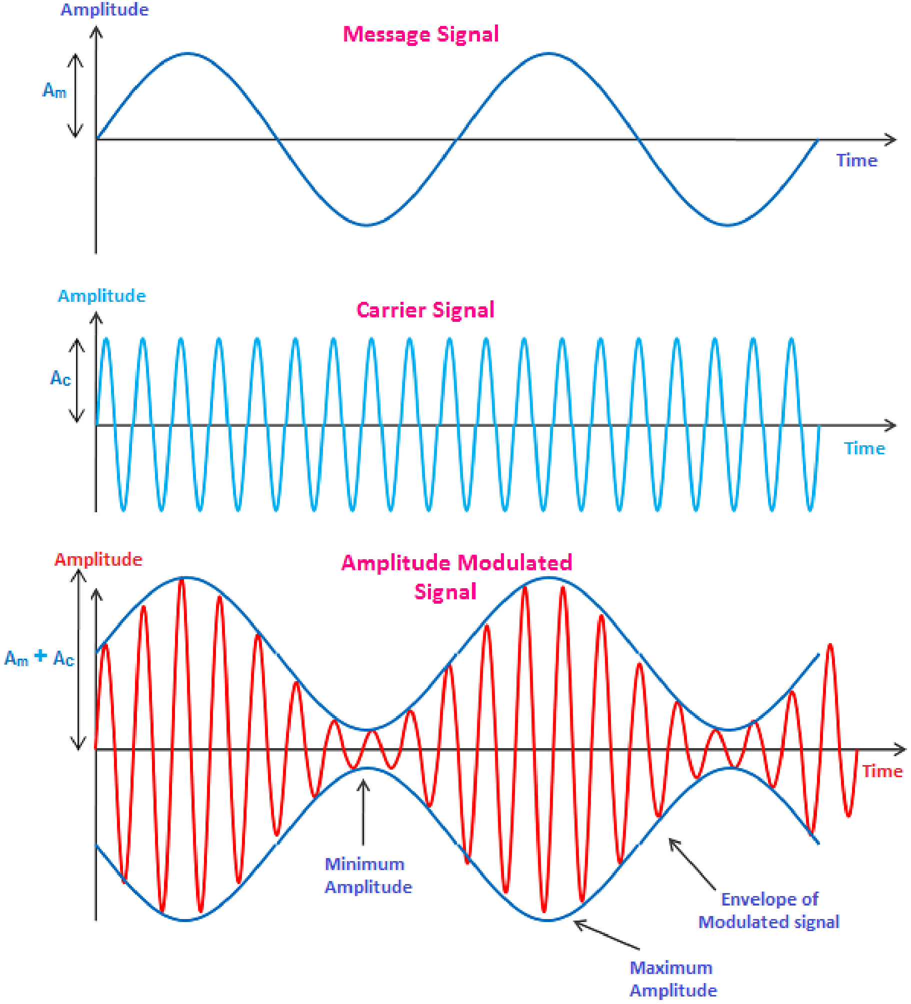
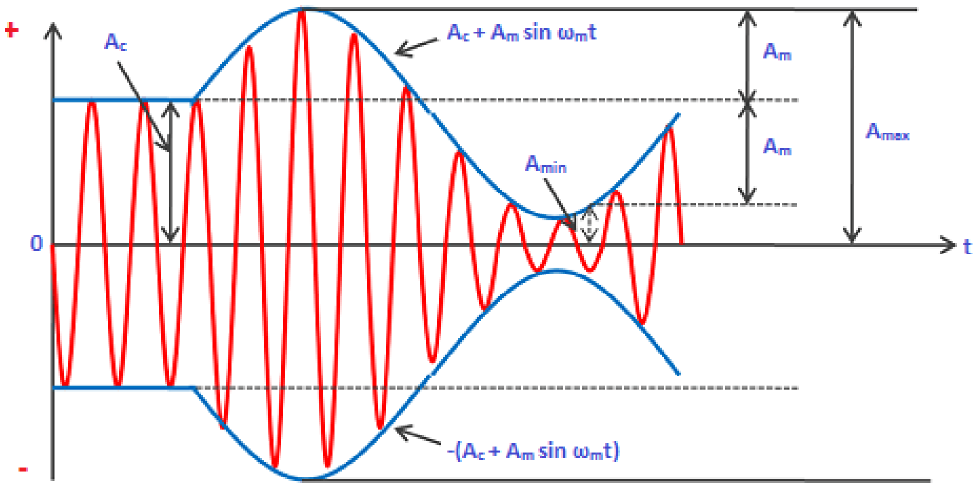
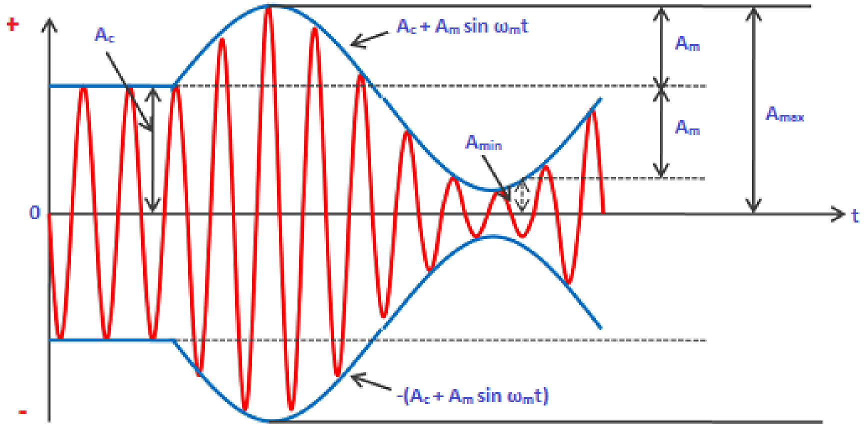



Figure 2.4(c) $m > 1$, over modulated AM

## Amplitude Modulation








$$A_m = \frac{A_{max} - A_{min}}{2} \quad \ldots\ldots\ldots\ldots (1)$$

$$A_c = A_{max} - A_m \quad \ldots\ldots\ldots\ldots (2)$$

Put $A_m$ value from eq (1) into eq (2), then we get

$$A_c = A_{max} - \frac{A_{max} - A_{min}}{2} \quad \ldots\ldots\ldots\ldots (3)$$

$$A_c = \frac{A_{max} + A_{min}}{2} \quad \ldots\ldots\ldots\ldots (4)$$

Taking the ratio of equation (1)and (4),

$$M_i = \frac{A_m}{A_c}$$

$$M_i = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}}$$

$$M_i = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \quad \ldots\ldots\ldots\ldots (5)$$

- From equation

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi(f_c - f_m)t - \frac{V_m}{2}\cos 2\pi(f_c + f_m)t$$

- We know that $m = V_m/V_c$ , $V_m = mV_c$
- Thus

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{mV_c}{2}\cos 2\pi(f_c - f_m)t - \frac{mV_c}{2}\cos 2\pi(f_c + f_m)t$$

- Its Fourier transform then
- $V_{AM}(f) = \frac{V_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{mV_c}{4}[\delta\big(f - (f_c - f_m)\big) + \delta\big(f + (f_c - f_m)\big)] + \frac{mV_c}{4}[\delta\big(f - (f_c + f_m)\big) + \delta\big(f + (f_c + f_m)\big)]$

*A 100 V, 100 kHz carrier is modulated with the help of a 10V, 1 kHz signal to the extent of 50%. Write down the equation for the A.M. wave*

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$$m = 50\% = 0.5$$

$$E_c = 100\text{ V};\ E_m = 10\text{ V}$$

$$f_c = 100\text{ kHz}$$

$$f_m = 1\text{ kHZ}$$

$$\omega_c = 2\pi f_c = 2 \times 3.14 \times 100 \times 10^3 = 628000$$

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*A 100 V, 100 kHz carrier is modulated with the help of a 10V, 1 kHz signal to the extent of 50%. Write down the equation for the A.M. wave*

Putting these values in the standard equation for the modulated voltage wave ;

$$e = E_c \sin \omega_c t - \frac{mE_c}{2} \cos (\omega_c - \omega_m)\, t - \frac{mE_c}{2} \cos (\omega_c + \omega_m)\, t$$

$$e = 100 \sin 628000\, t + \frac{0.5 \times 100}{2} \cos (628000 - 6280)\, t$$

$$- \frac{0.5 \times 100}{2} \cos (628000 + 6280)\, t$$

$$e = 100 \sin 628000\, t + 25 \cos 621720\, t - 25 \cos 634280\, t$$

*An amplitude modulated wave is represented by the equation*

$e = 20\,(1 + 0.7 \sin 6280\, t) \sin 628000\, t$

Determine:

(*i*) Modulation factor

(*ii*) Carrier amplitude

(*iii*) Signal frequency

(*iv*) Carrier frequency

(*v*) Maximum amplitude of A.M. wave

(*vi*) Minimum amplitude of A.M. wave

(*vii*) Bandwidth.

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Comparing with the given equation

$$e = 20\,(1 + 0.7 \times \sin 6280\,t).\ \sin 628000\,t$$

We get

(*i*) Modulation factor $m = 0.7$

(*ii*) Carrier amplitude $E_c = 20\text{V}$

(*iii*) $\omega_m = 6280$ $[\omega_m = 2\pi f_m]$

$\therefore$ Signal frequency, $$f_m = \frac{\omega_m}{2\pi} = \frac{6280}{2\pi} = 1\text{kHz}\ \textbf{Ans.}$$

(*iv*) $\omega_c = 628{,}000$

Carrier frequency, $$f_m = \frac{\omega_c}{2\pi} = \frac{628000}{2\pi} = 100\ \text{kHz}\ \textbf{Ans.}$$

(*v*) Maximum amplitude of AM wave

$$E_{\text{max}} = E + mE_c = 20 + (0.7 \times 20) = 34\text{V}\ \textbf{Ans.}$$

(*vi*) Minimum amplitude of AM wave

$$E_{\text{min}} = E_c - mE_c$$

$$E_{\text{min}} = 20 - mE_c = 20 - (0.7 \times 20) = 6\text{V} \quad \textbf{Ans.}$$

(*vii*) Bandwidth = $2f_m = 2 \times 1 = 2$ kHz.