1.2 Hertzian dipole, Dipole and monopole

Module:1 EM Radiation and Antenna Parameters

Course: BECE305L – Antenna and Microwave Engineering

-Dr Richards Joe Stanislaus

Assistant Professor - SENSE

Email: 51749@vitstudent.ac.in / richards.stanislaus@vit.ac.in

VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

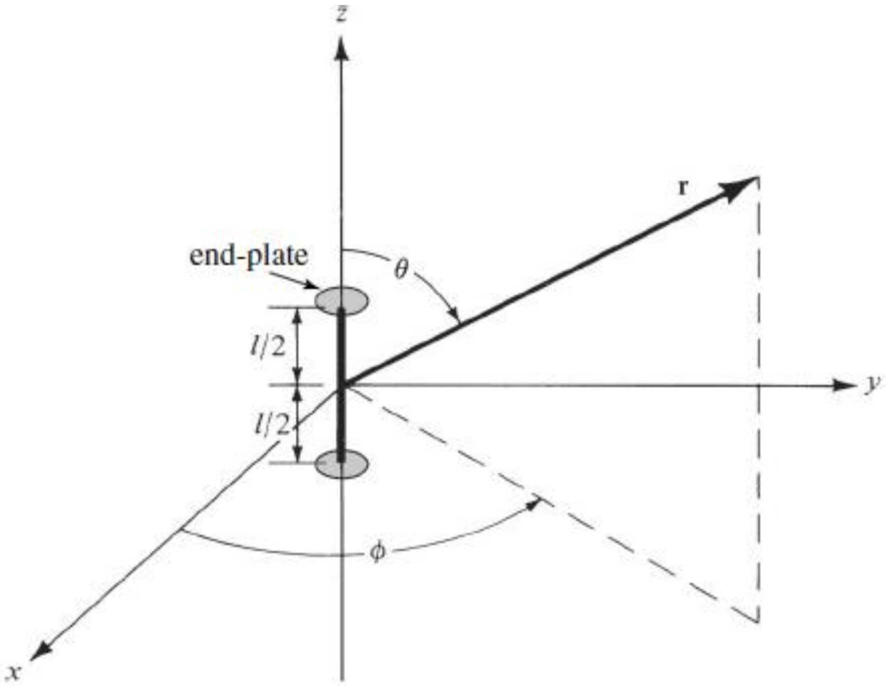
Module:1 EM Radiation and Antenna Parameters

- Radiation mechanism - single wire, two wire and current distribution, **Hertzian dipole, Dipole and monopole** - Radiation pattern, beam width, field regions, radiation power density, radiation intensity, directivity and gain, bandwidth, polarization, input impedance, efficiency, antenna effective length and area, antenna temperature. Friis transmission equation, Radar range equation

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

1. Infinitesimal dipole (Hertzian dipole)

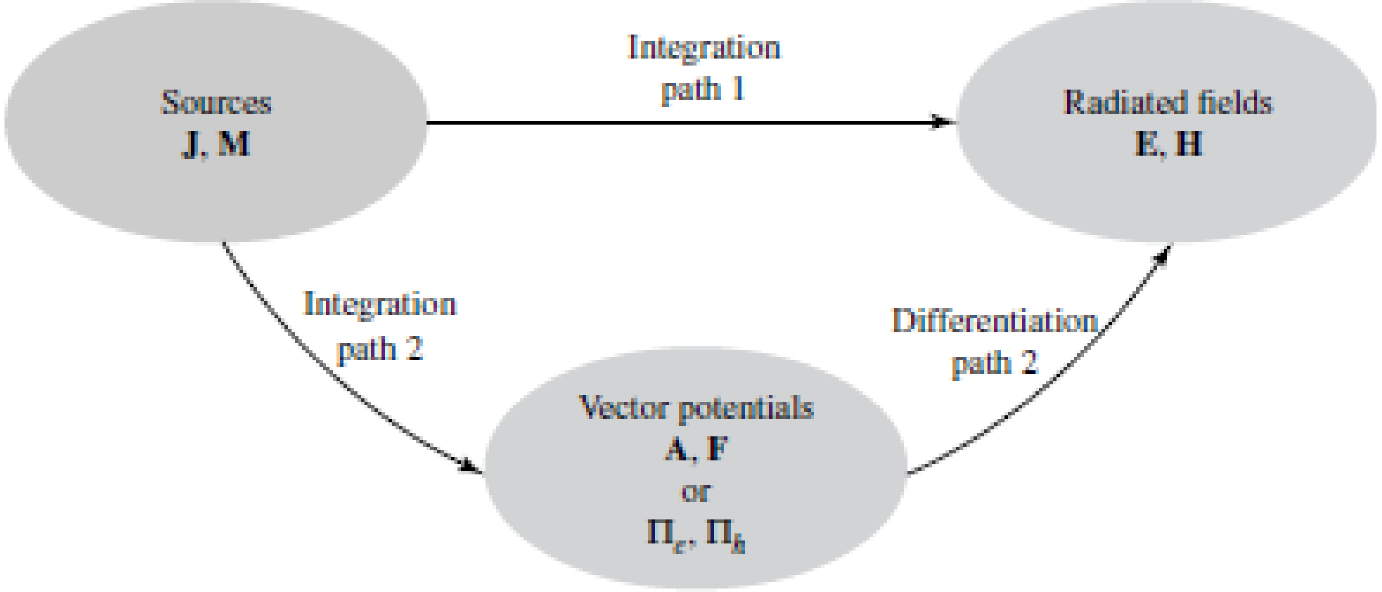
- An infinitesimal **linear wire** ($l \ll \lambda$) is positioned symmetrically at the origin of the coordinate system - **Oriented along the *z* axis**.
- Infinitesimal dipoles are **not very practical**, they are **<u>used to represent capacitor-plate</u>** (also referred to as *top-hat-loaded*) antennas - utilized as <u>building blocks of more complex geometries</u>.
- The end plates are used to provide capacitive loading in order to maintain the current on the dipole nearly uniform.
  Since the end plates are assumed to be small, their radiation is usually negligible.

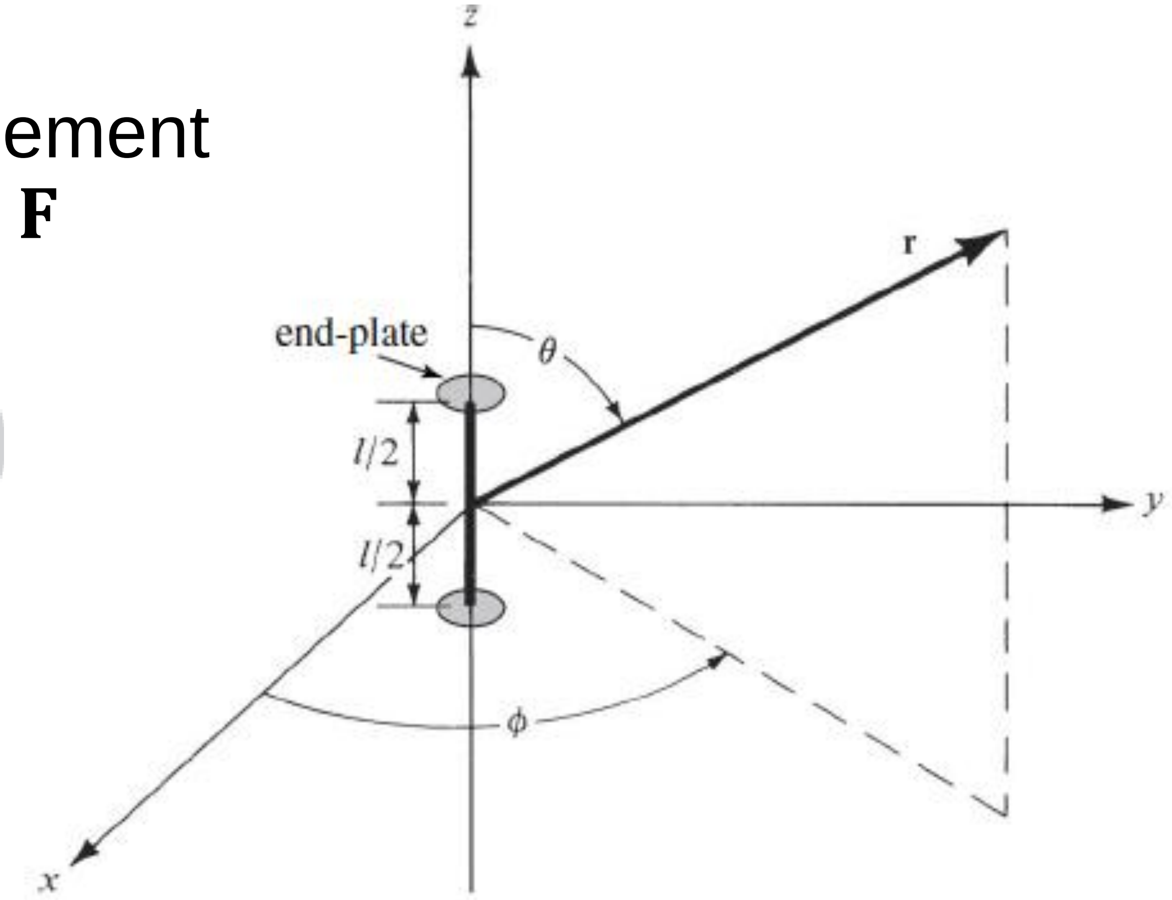


Note: an infinitesimal dipole is usually taken to have a length $l \leq \lambda/50$,

1. Infinitesimal dipole (Hertzian dipole)

- The wire, in addition to being very small ($l \ll \lambda$), is very thin ($a \ll \lambda$). The spatial variation of the current is assumed to be constant and given by
  $\mathbf{I}(z') = \hat{\mathbf{a}}_z I_0$ where $I_0 = constant$
- To find radiated fields due to current element
  Two step procedure, Determine $\mathbf{A}$ and $\mathbf{F}$
  Then find **E** and **H**



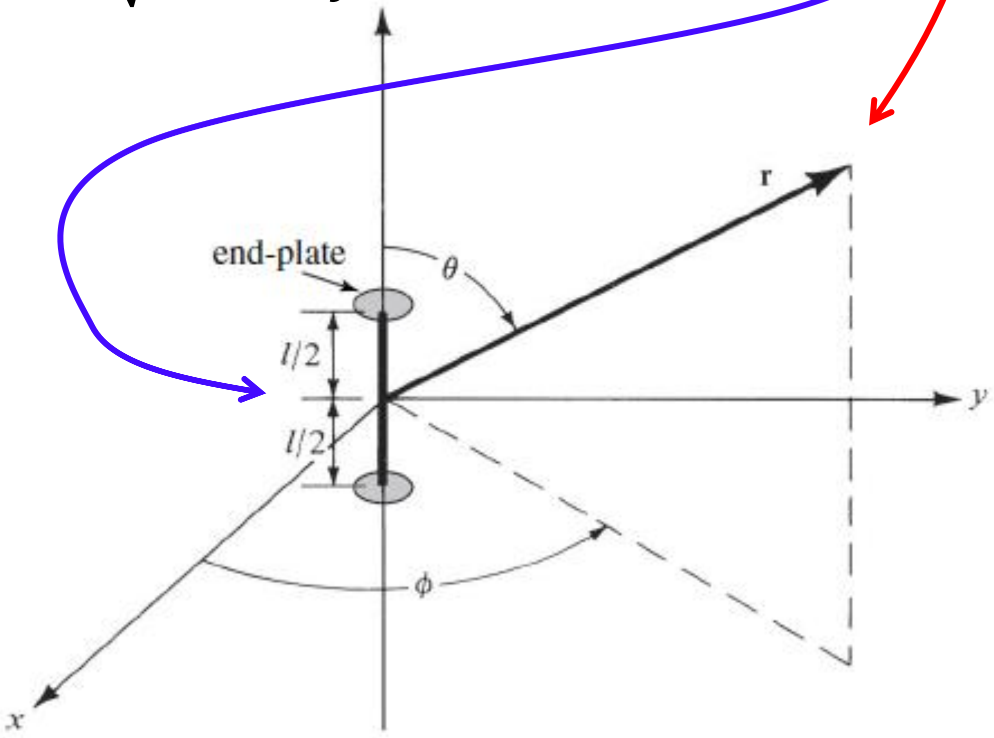


1. Infinitesimal dipole (Hertzian dipole)

- $\mathbf{I}(z') = \hat{\mathbf{a}}_z I_0$ where $I_0 = constant$
- With electric current I_e only, the potential function $F = 0$. With observation point as (x, y, z) and source coordinates as (x', y', z'), also, $x' = y' = z' = 0$ due to infinitesimal dipole, **Distance between two points** $R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} = \sqrt{x^2 + y^2 + z^2} = r$ (constant) and $dl' = dz'$
- **Electric vector potential**

$$A(x, y, z) = \frac{\mu}{4\pi} \int_C I_e(x', y', z') \frac{e^{-jkR}}{R} dl'$$

$$= \hat{\mathbf{a}}_z \frac{\mu I_0}{4\pi r} e^{-jkr} \int_{-\frac{l}{2}}^{\frac{l}{2}} dz' = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}$$



1. Infinitesimal dipole (Hertzian dipole)

- $\mathbf{I}(z') = \hat{\mathbf{a}}_z I_0$; Electric vector potential $A(x,y,z) = \hat{\mathbf{a}}_z \frac{\mu I_0 l}{4\pi r} e^{-jkr}$ $(A_x = A_y = 0)$

Convert cartesian (x, y, z) to spherical coordinate system (r, θ, ϕ) to find fields.

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

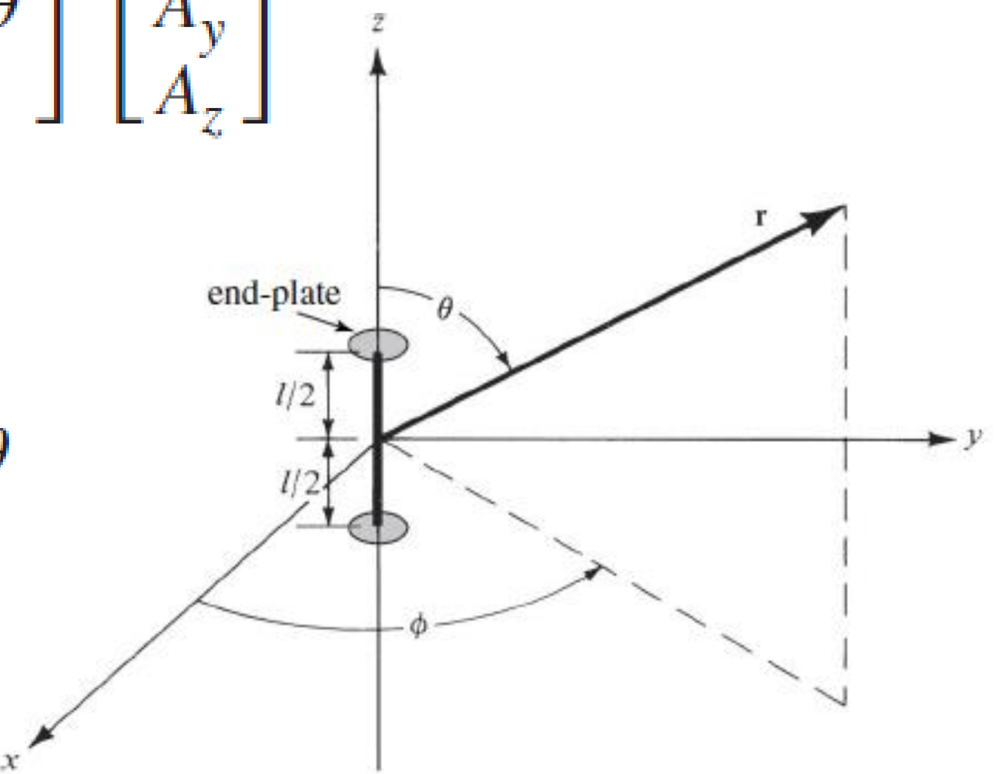
$$A_r = A_z\cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta$$

The magnetic and Electric fields are:

$$A_\theta = -A_z\sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$\mathbf{H}_A = \frac{1}{\mu}\nabla\times\mathbf{A}$$

$$A_\phi = 0$$



1. Infinitesimal dipole (Hertzian dipole)

$$A_r = A_z \cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta$$

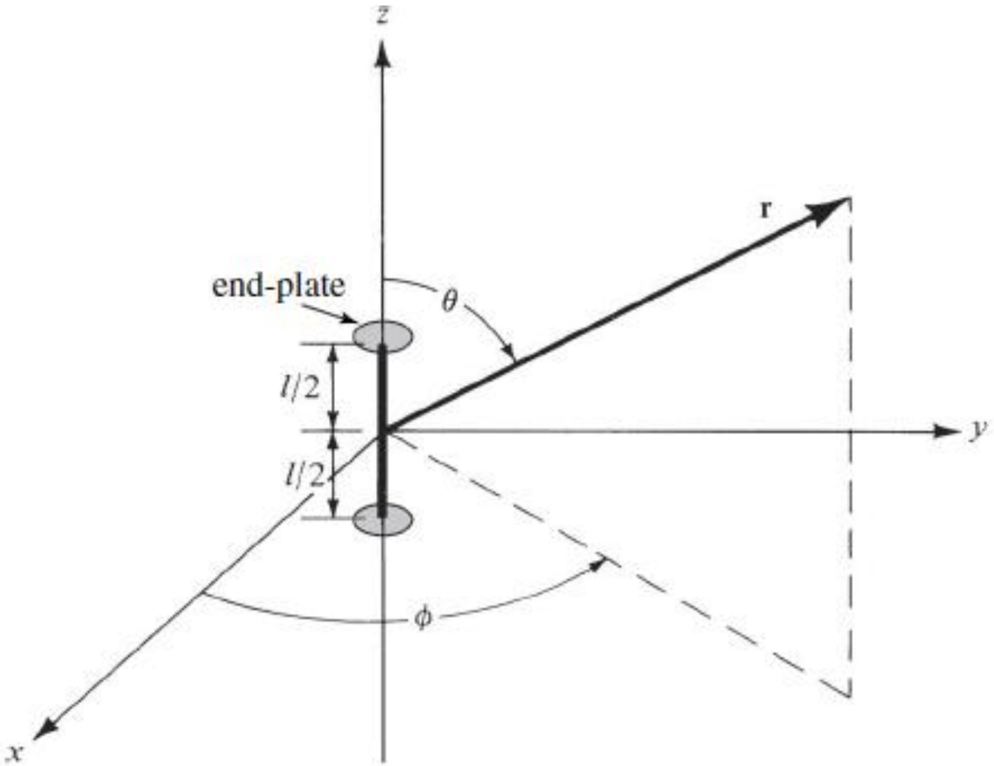
$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$A_\phi = 0$$

$$\nabla \times \mathbf{A} = \frac{\hat{\mathbf{a}}_r}{r\sin\theta}\left[\cancel{\frac{\partial}{\partial\theta}(A_\phi \sin\theta)} - \cancel{\frac{\partial A_\theta}{\partial\phi}}\right] + \frac{\hat{\mathbf{a}}_\theta}{r}\left[\frac{1}{\sin\theta}\cancel{\frac{\partial A_r}{\partial\phi}} - \cancel{\frac{\partial}{\partial r}(rA_\phi)}\right] + \frac{\hat{\mathbf{a}}_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

with no dependence on ϕ, $\frac{\partial}{d\phi} = 0$,

- The magnetic field $\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}$



1. Infinitesimal dipole (Hertzian dipole)

$$A_r = A_z \cos\theta = \frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta$$

$$A_\phi = 0$$

$$\nabla \times \mathbf{A} = \frac{\hat{\mathbf{a}}_r}{r\sin\theta}\left[\cancel{\frac{\partial}{\partial\theta}(A_\phi \sin\theta)} - \cancel{\frac{\partial A_\theta}{\partial\phi}}\right] + \frac{\hat{\mathbf{a}}_\theta}{r}\left[\frac{1}{\sin\theta}\cancel{\frac{\partial A_r}{\partial\phi}} - \cancel{\frac{\partial}{\partial r}(rA_\phi)}\right] + \frac{\hat{\mathbf{a}}_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

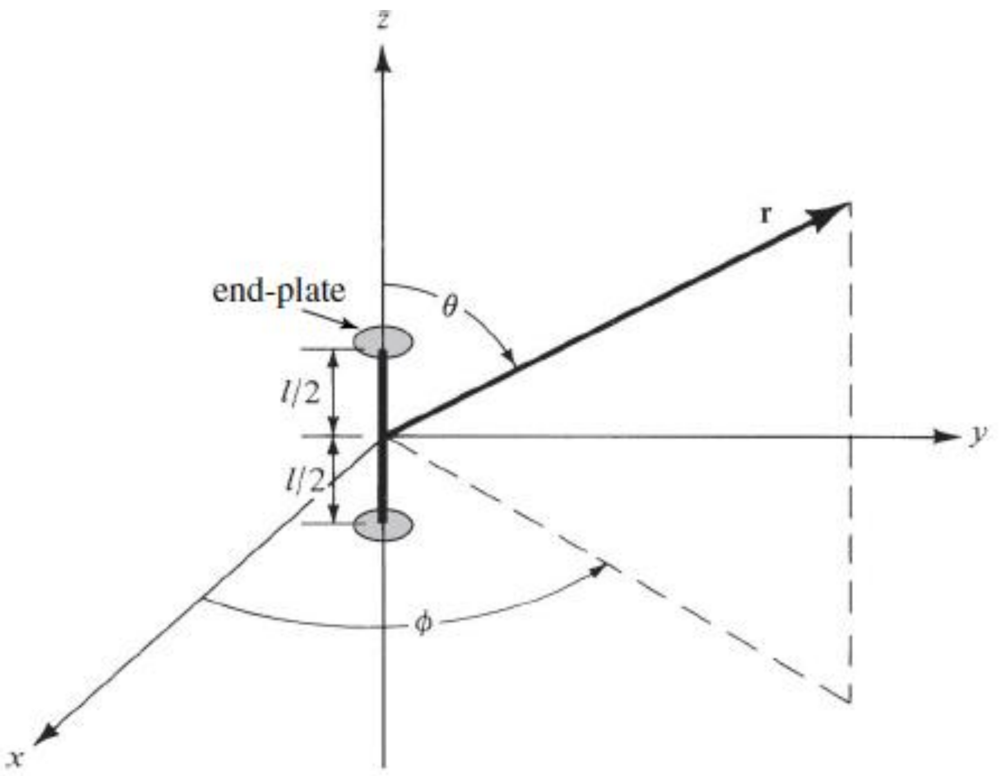
with no dependence on ϕ, $\frac{d}{d\phi} = 0$,

- The magnetic field $\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}$

$$\mathbf{H} = \hat{\mathbf{a}}_\phi \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$, on solving

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$



1. Infinitesimal dipole (Hertzian dipole)

$$\boxed{\begin{aligned} H_r &= H_\theta = 0 \\ H_\phi &= j\frac{kI_0 l \sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr} \end{aligned}}$$

Curl expression, replace A with H for $\nabla \times H$ and $H_r = H_\theta = 0$

$$\nabla \times \mathbf{A} = \frac{\hat{\mathbf{a}}_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial \cancel{A_\theta}}{\partial\phi}\right] + \frac{\hat{\mathbf{a}}_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial \cancel{A_r}}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\mathbf{a}}_\phi}{r}\left[\frac{\partial}{\partial r}(r\cancel{A_\theta}) - \frac{\partial \cancel{A_r}}{\partial\theta}\right]$$

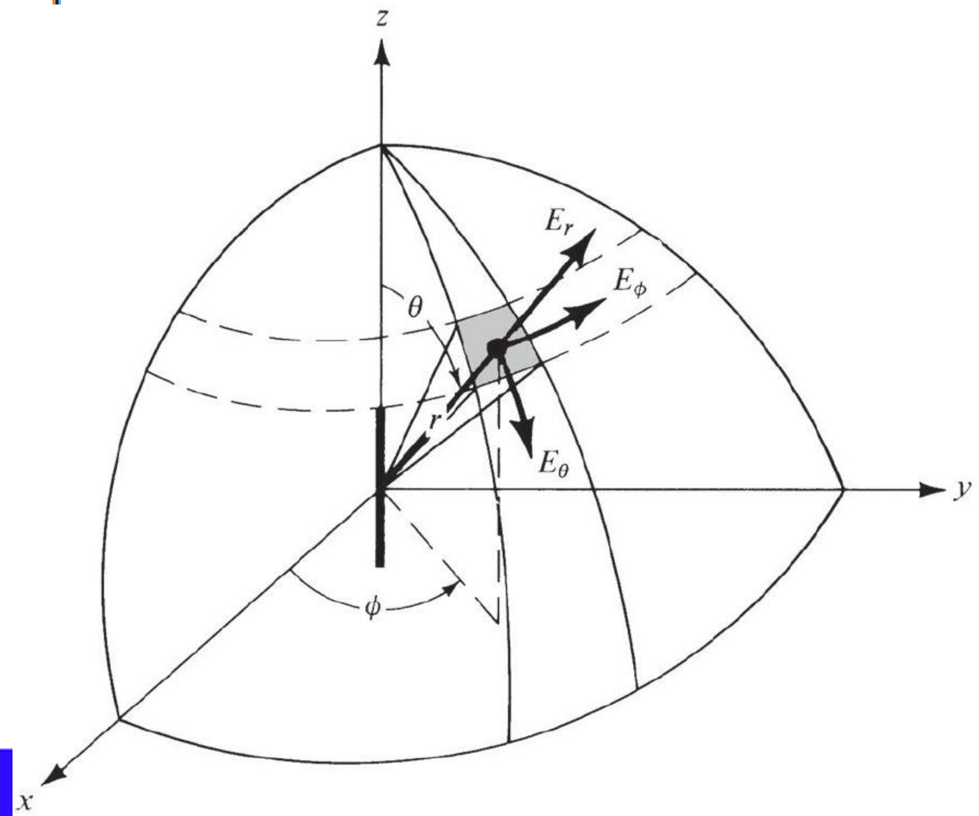
- The electric field

$$\mathbf{E} = \mathbf{E}_A = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A}) = \frac{1}{j\omega\varepsilon}\nabla\times\mathbf{H}$$

$$E_r = \eta\frac{I_0 l\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

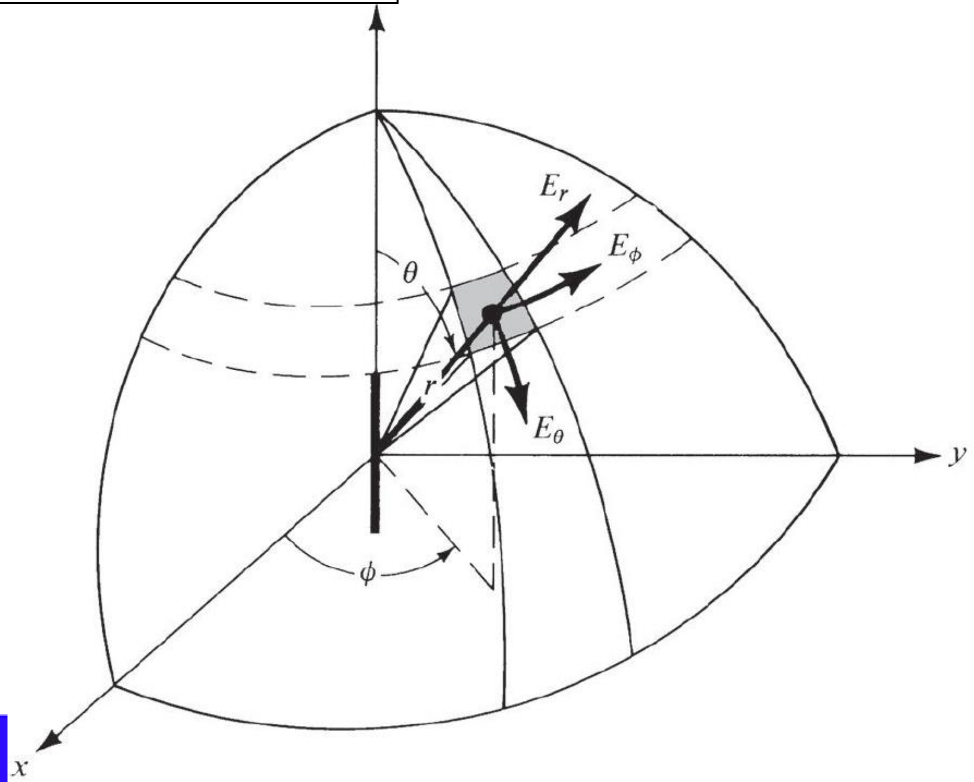


1. Infinitesimal dipole (Hertzian dipole)

$$H_r = H_\theta = 0$$
$$H_\phi = j\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

$$E_r = \eta\frac{I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr}$$
$$E_\theta = j\eta\frac{kI_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right] e^{-jkr}$$
$$E_\phi = 0$$

- The E and H fields are valid everywhere outside the infinitesimal dipole.



2. Small Dipole

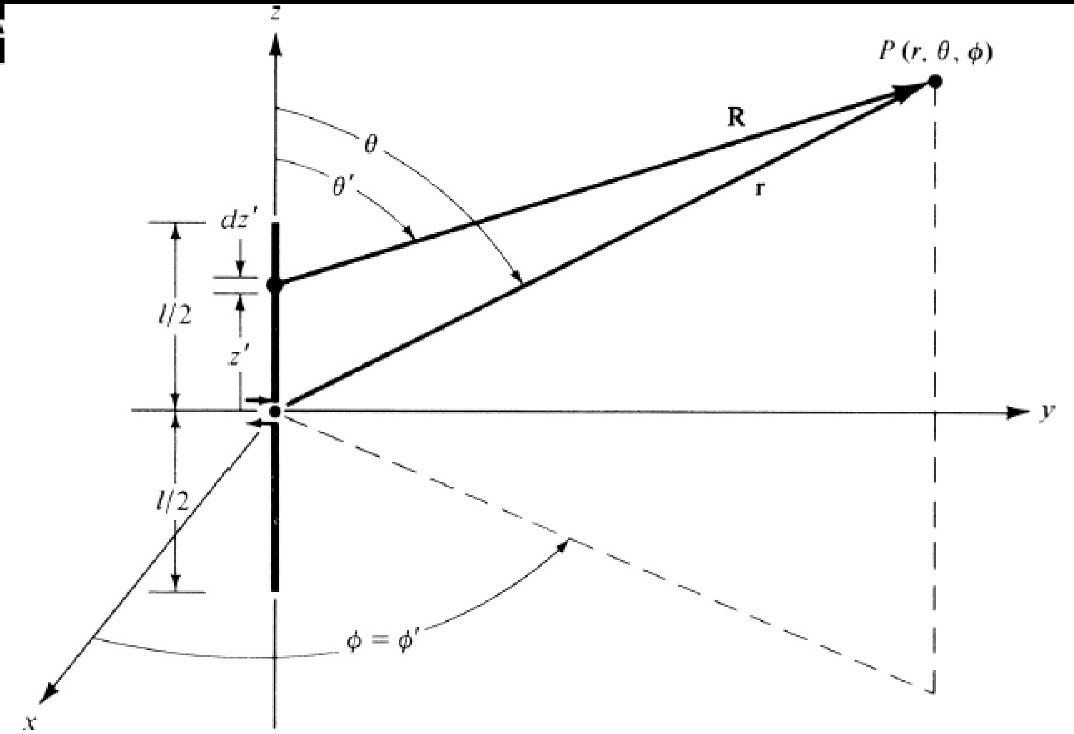
- A better approximation of the current distribution of wire antennas, whose lengths are usually λ/50 < *l* ≤ λ/10, is the triangular variation

$$\mathbf{I}_e(x',y',z') = \begin{cases} \hat{\mathbf{a}}_z I_0\left(1-\frac{2}{l}z'\right), & 0 \le z' \le l/2 \\ \hat{\mathbf{a}}_z I_0\left(1+\frac{2}{l}z'\right), & -l/2 \le z' \le 0 \end{cases}$$

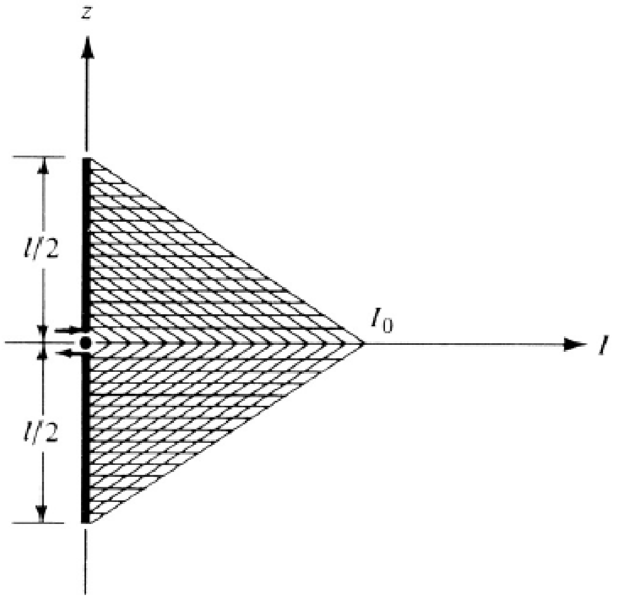
- Electric vector potential

$$\mathbf{A}(x,y,z) = \frac{\mu}{4\pi}\left[\hat{\mathbf{a}}_z \int_{-l/2}^{0} I_0\left(1+\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz' + \hat{\mathbf{a}}_z \int_{0}^{l/2} I_0\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\,dz'\right]$$

$$\boxed{\mathbf{A} = \hat{\mathbf{a}}_z A_z = \hat{\mathbf{a}}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]}$$



(a) Dipole and geometry



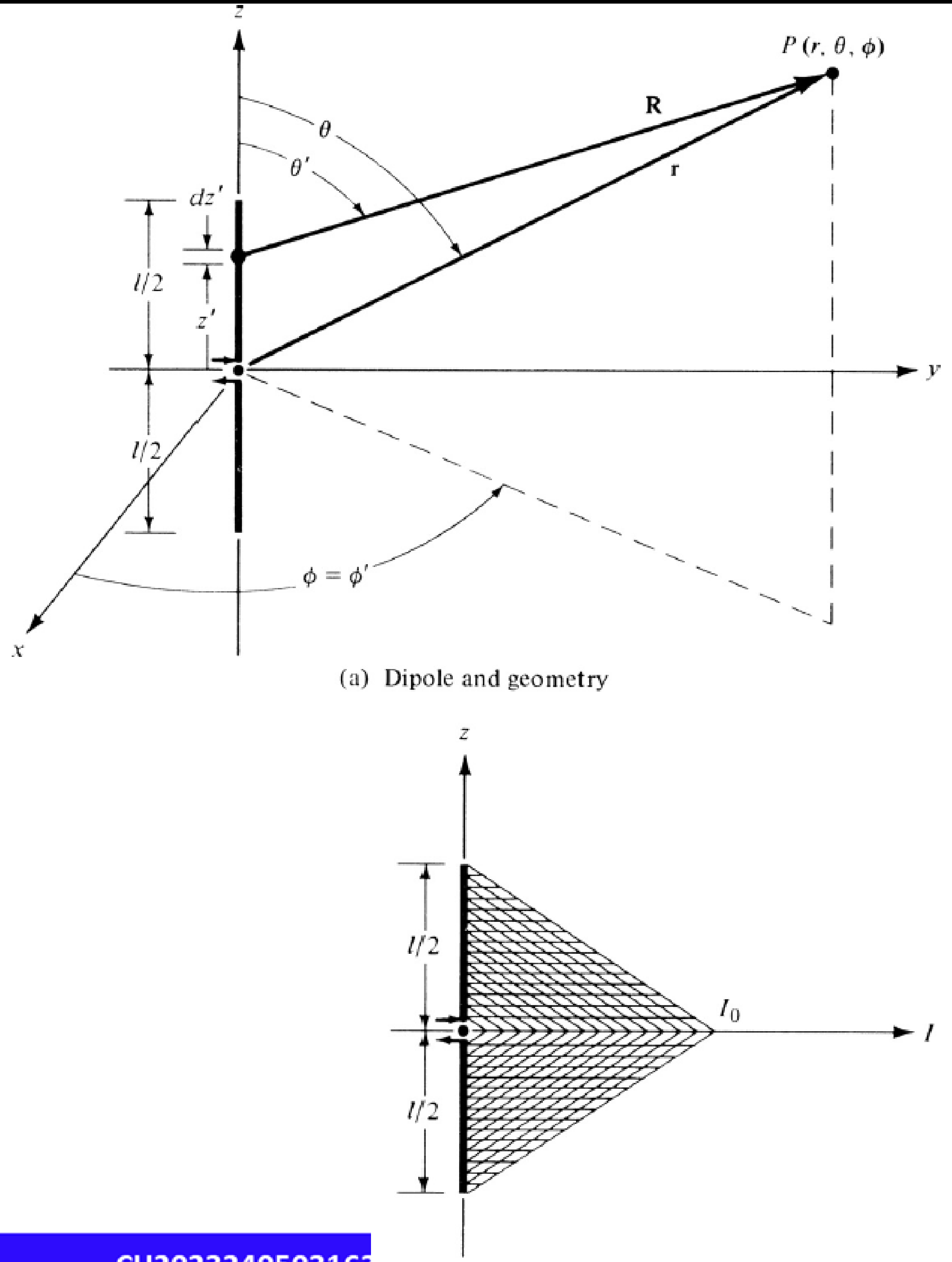
(b) Current distribution

2. Small Dipole

$$\mathbf{I}_e(x', y', z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \left(1 - \frac{2}{l} z'\right), & 0 \le z' \le l/2 \\ \hat{\mathbf{a}}_z I_0 \left(1 + \frac{2}{l} z'\right), & -l/2 \le z' \le 0 \end{cases}$$

$$\boxed{\mathbf{A} = \hat{\mathbf{a}}_z A_z = \hat{\mathbf{a}}_z \frac{1}{2}\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\right]}$$

$$\left.\begin{aligned} E_\theta &\simeq j\eta \frac{k I_0 l e^{-jkr}}{8\pi r} \sin\theta \\ E_r &\simeq E_\phi = H_r = H_\theta = 0 \\ H_\phi &\simeq j \frac{k I_0 l e^{-jkr}}{8\pi r} \sin\theta \end{aligned}\right\} \quad kr \gg 1$$



(a) Dipole and geometry

(b) Current distribution

Field regions

reactive near-field $[0.62\sqrt{D^3/\lambda} > r > 0]$

radiating near-field (Fresnel) $[2D^2/\lambda > r \geq 0.62\sqrt{D^3/\lambda}]$

far-field (Fraunhofer) $[\infty \geq r \geq 2D^2/\lambda]$

3. Finite length dipole

- very thin dipole (ideally zero diameter), the current distribution can be written, to a good approximation, as

$$\mathbf{I}_e(x'=0, y'=0, z') = \begin{cases} \hat{\mathbf{a}}_z I_0 \sin\left[k\left(\frac{l}{2} - z'\right)\right], & 0 \le z' \le l/2 \\ \hat{\mathbf{a}}_z I_0 \sin\left[k\left(\frac{l}{2} + z'\right)\right], & -l/2 \le z' \le 0 \end{cases}$$

$$dE_\theta \simeq j\eta \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

$$dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$$

$$dH_\phi \simeq j \frac{kI_e(x', y', z')e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

Far-field Approximations

$$r \ge 2\frac{D^2}{\lambda}$$

$R \simeq r - z'\cos\theta$ for phase terms

$R \simeq r$ for amplitude terms

$$dE_\theta \simeq j\eta \frac{kI_e(x', y', z')e^{-jkr}}{4\pi r} \sin\theta e^{+jkz'\cos\theta} \, dz'$$

3. Finite length dipole

- Outside brackets:
  Element factor
  Inside brackets: Space factor

$$E_\theta = \int_{-l/2}^{+l/2} dE_\theta = j\eta \frac{ke^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{+l/2} I_e(x', y', z') e^{jkz'\cos\theta}\, dz' \right]$$

- For this antenna, element factor =field of infinitesimal antenna

$$E_\theta \simeq j\eta \frac{kI_0 e^{-jkr}}{4\pi r} \sin\theta \left\{ \int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2} + z'\right)\right] e^{+jkz'\cos\theta}\, dz' + \int_{0}^{+l/2} \sin\left[k\left(\frac{l}{2} - z'\right)\right] e^{+jkz'\cos\theta}\, dz' \right\}$$

$$\int e^{\alpha x} \sin(\beta x + \gamma)\, dx = \frac{e^{\alpha x}}{\alpha^2 + \beta^2}[\alpha \sin(\beta x + \gamma) - \beta \cos(\beta x + \gamma)]$$

$$\alpha = \pm jk\cos\theta$$

$$\beta = \pm k$$

$$\gamma = kl/2$$

3. Finite length dipole

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]$$

$$H_\phi \simeq \frac{E_\theta}{\eta} \simeq j\frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]$$

4. Half wavelength Dipole

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$

$$H_\phi \simeq \frac{E_\theta}{\eta} \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$

- When length of dipole is $l = \frac{\lambda}{2}$ and phase constant $k = \frac{2\pi}{\lambda}$, with far field approximations

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

$$H_\phi \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$