1.6 Additional Problems

Module:1 EM Radiation and Antenna Parameters

Course: BECE305L – Antenna and Microwave Engineering

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Module:1 EM Radiation and Antenna Parameters

- Radiation mechanism - single wire, two wire and current distribution, Hertzian dipole, Dipole and monopole - Radiation pattern, beam width, field regions, radiation power density, radiation intensity, directivity and gain, bandwidth, polarization, input impedance, efficiency, antenna effective length and area, antenna temperature. Friis transmission equation, Radar range equation

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the

(a) power density (W_{rad})

(b) power radiated (P_{rad})

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(a) power density (W_{rad}) (b) power radiated (P_{rad})

(a) $\underline{W}_{rad} = \frac{1}{2}[\underline{E} \times \underline{H}^*] = \frac{E^2}{2\eta}\hat{a}_r = \frac{5^2 \hat{a}_r}{2(120\pi)} = 0.03315\ \hat{a}_r$ watts/m^2

(b) $P_{rad} = \oint_S W_{rad}\, dS = \int_0^{2\pi}\int_0^{\pi} (0.03315)(r^2 \sin\theta\, d\theta\, d\phi)$

$= \int_0^{2\pi}\int_0^{\pi} (0.03315)(100)^2 \sin\theta\, d\theta\, d\phi$

$= 2\pi\,(0.03315)(100)^2 \int_0^{\pi} \sin\theta\, d\theta = 2\pi(0.03315)(100)^2 \cdot 2$

$= 4165.75$ watts

The maximum radiation intensity of a 90% efficiency antenna is 200 mW/unit solid angle. Find the directivity and gain (dimensionless and in dB) when the

(a) input power is 125.66 mW

(b) radiated power is 125.66 mW

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(a) $$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{0.9(125.66\times10^{-3})} = 22.22 = 13.47\text{dB}$$

$$G_o = e_t \cdot D_o = 0.9(22.22) = 20 = 13.01\text{dB}$$

(b) $$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(200\times10^{-3})}{(125.66\times10^{-3})} = 20 = 13.01\text{dB}$$

$$G_o = e_t \cdot D_o = 0.9\cdot(20) = 18 = 12.55\text{dB}$$

The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$$U = B_o \cos^3 \theta \qquad \text{(watts/unit solid angle)}$$
$$(0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi)$$

(a) maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.

(b) exact and approximate beam solid angle Ω_A.

(c) directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) gain, exact and approximate, of the antenna (dimensionless and in dB).

$$U = B_0 \cos^3\theta$$

(a) $$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \sin\theta \, d\theta \, d\phi = B_0 \int_0^{2\pi}\int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta \, d\phi$$

$$= 2\pi B_0 \int_0^{\pi/2} \cos^3\theta \sin\theta \, d\theta$$

$$P_{rad} = 2\pi B_0 \left(-\frac{\cos^4\theta}{4}\right)\Bigg|_0^{\pi/2} = \frac{\pi}{2} B_0 = 10 \Rightarrow B_0 = \frac{20}{\pi} = 6.3662$$

$$U = 6.3662 \cos^3\theta$$

$$W = \frac{U}{r^2} = \frac{6.3662}{r^2}\cos^3\theta = \frac{6.3662}{(10^3)^2} \cdot \cos^3\theta = 6.3662 \times 10^{-6} \cdot \cos^3\theta$$

$$W|_{max} = 6.3662 \times 10^{-6} \cdot \cos^3\theta\Big|_{max} = 6.3662 \times 10^{-6} \text{ Watts}/m^2$$

(b) $$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(6.3662)}{10} = 8 = 9\,dB$$

(c) $$G_0 = e_t D_0 = 8 = 9\,dB$$

2.10. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta,\phi) = \left\{ \begin{array}{cc} 1 & 0^\circ \leq \theta < 20^\circ \\ 0.342\csc(\theta) & 20^\circ \leq \theta < 60^\circ \\ 0 & 60^\circ \leq \theta \leq 180^\circ \end{array} \right\} 0^\circ \leq \phi \leq 360^\circ$$

Find the directivity (in dB) using the exact formula.

$$U(\theta,\phi)=\begin{cases}1 & 0^\circ\leq\theta\leq 20^\circ\\ 0.342\csc(\theta) & 20^\circ\leq\theta\leq 60^\circ\\ 0 & 60^\circ\leq\theta\leq 180^\circ\end{cases}\qquad 0^\circ\leq\phi\leq 360^\circ$$

$$P_{rad}=\int_0^{2\pi}\int_0^{\pi}U(\theta,\phi)\sin\theta\,d\theta\,d\phi=2\pi\left[\int_0^{20^\circ}\sin\theta\,d\theta+\int_{20^\circ}^{60^\circ}0.342\csc(\theta)\times\sin\theta\,d\theta\right]=2\pi\left\{-\cos\theta\Big|_0^{\pi/9}+0.342\cdot\theta\Big|_{\pi/9}^{\pi/3}\right\}$$

$$=2\pi\left\{\left[-\cos\left(\frac{\pi}{9}\right)+1\right]+0.342\left(\frac{\pi}{3}-\frac{\pi}{9}\right)\right\}$$

$$=2\pi\left\{\left[-0.93969+1\right]+0.342\pi\left(\frac{2}{9}\right)\right\}$$

$$=2\pi\left\{0.06031+0.23876\right\}=1.87912$$

$$D_0=\frac{4\pi U_{max}}{P_{rad}}=\frac{4\pi(1)}{1.87912}=6.68737=8.25255\text{ dB}.$$

The normalized radiation intensity of a given antenna is given by

(a) $U = \sin\theta \sin\phi$ (b) $U = \sin\theta \sin^2\phi$

(c) $U = \sin\theta \sin^3\phi$ (d) $U = \sin^2\theta \sin\phi$

(e) $U = \sin^2\theta \sin^2\phi$ (f) $U = \sin^2\theta \sin^3\phi$

The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the

(a) exact directivity (*dimensionless* and *in dB*).

(b) azimuthal and elevation plane half-power beamwidths (in degrees).

$$D_0 = \frac{4\pi U_{max}}{P_{rad}}$$

(a) $U = \sin\theta \sin\phi$ for $0 \leq \theta \leq \pi$, $0 \leq \phi \leq \pi$

$U|_{max} = 1$ and it occurs when $\theta = \phi = \pi/2$.

$$P_{rad} = \int_0^{\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = \int_0^{\pi} \sin\phi \, d\phi \int_0^{\pi} \sin^2\theta \, d\theta = 2\left(\frac{\pi}{2}\right) = \pi.$$

Thus $D_0 = \frac{4\pi(1)}{\pi} = 4 = 6.02 \text{ dB}$

The half-power beamwidths are equal to

$$HPBW(az.) = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

$$HPBW(el.) = 2[90^\circ - \sin^{-1}(1/2)] = 2(90^\circ - 30^\circ) = 120^\circ$$

In a similar manner, it can be shown that for

(b) $U = \sin\theta \sin^2\phi \Rightarrow D_0 = 5.09 = 7.07\text{ dB}$

$\text{HPBW (el.)} = 120^\circ$, $\text{HPBW (az.)} = 90^\circ$

(c) $U = \sin\theta \sin^3\phi \Rightarrow D_0 = 6 = 7.78\text{ dB}$

$\text{HPBW (el.)} = 120^\circ$, $\text{HPBW (az.)} = 74.93^\circ$

(d) $U = \sin^2\theta \sin\phi \Rightarrow D_0 = 12\pi/8 = 4.71 = 6.73\text{ dB}$

$\text{HPBW (el.)} = 90^\circ$, $\text{HPBW (az.)} = 120^\circ$

(e) $U = \sin^2\theta \sin^2\phi \Rightarrow D_0 = 6 = 7.78\text{ dB}$, $\text{HPBW (az.)} = \text{HPBW (el.)} = 90^\circ$

(f) $U = \sin^2\theta \sin^3\phi \Rightarrow D_0 = 9\pi/4 = 7.07 = 8.49\text{ dB}$

$\text{HPBW (el.)} = 90^\circ$, $\text{HPBW (az.)} = 74.93^\circ$

The normalized radiation intensity of an antenna is rotationally symmetric in ϕ, and it is represented by

$$U = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ 0.5 & 30^\circ \le \theta < 60^\circ \\ 0.1 & 60^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?
(b) What is the directivity (above an infinitesimal dipole) of the antenna (in dB)?

(a) $D_0 = \dfrac{4\pi U_{max}}{P_{rad}} = \dfrac{U_{max}}{U_0}$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} U \sin\theta \, d\theta = 2\pi \left\{ \int_0^{30^\circ} \sin\theta \, d\theta + \right.$$

$$\left. \int_{30^\circ}^{60^\circ} (0.5)\sin\theta \, d\theta + \int_{60^\circ}^{90^\circ} (0.1)\sin\theta \, d\theta \right\} = 2\pi \left\{ (-\cos\theta)\Big|_0^{30^\circ} + \left(-\frac{\cos\theta}{2}\right)\Big|_{30^\circ}^{60^\circ} + (-0.1\cos\theta)\Big|_{60^\circ}^{90^\circ} \right\}$$

(Cont'd)

$$= 2\pi \left\{ (-0.866+1) + \left(\frac{-0.5+0.866}{2} \right) + \left(\frac{-0+0.5}{10} \right) \right\}$$

$$P_{rad} = 2\pi \{-0.866+1-0.25+0.433+0.05\} = 2\pi(0.367)$$

$$= 0.734 \cdot \pi = 2.3059$$

$$D_0 = \frac{1(4\pi)}{2.3059} = 5.4496 = 7.3636 \text{ dB}$$

(b) $D_0 \text{ (dipole)} = 1.5 = 1.761 \text{ dB}$

$D_0 \text{ (above dipole)} = (7.3636 - 1.761) \text{ dB} = 5.6026 \text{ dB}$

$D_0 \text{ (above dipole)} = \dfrac{5.45}{1.5} = 3.633 = 5.603 \text{ dB}$

A beam antenna has half-power beamwidths of 30° and 35° in perpendicular planes intersecting at the maximum of the mainbeam. Find its approximate maximum effective aperture (in λ^2) using:

(a) Kraus'

(b) Tai and Pereira's formulas.

The minor lobes are very small and can be neglected.

(a) $D_0 \simeq \dfrac{41,253}{\Theta_{1d}\,\Theta_{2d}} = \dfrac{41,253}{30(35)} = 39.29 = 15.94 \text{ dB}$

$A_{em} = \dfrac{\lambda^2}{4\pi} D_0$

(b) $D_0 \simeq \dfrac{72,815}{\Theta_{1d}^2 + \Theta_{2d}^2} = \dfrac{72,815}{(30)^2 + (35)^2} = 34.27 = 15.35 \text{ dB}$

$A_{em} = \dfrac{\lambda^2}{4\pi} D_0$

The radiation intensity of an antenna can be approximated by

$$U(\theta,\phi) = \begin{cases} \cos^4(\theta) & 0^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases} \quad \text{with } 0^\circ \le \phi \le 360^\circ$$

Determine the maximum effective aperture ($in\ \ m^2$) of the antenna if its frequency of operation is $f = 10$ GHz.

$$U(\theta,\phi) = \begin{cases} \cos^4(\theta), & 0° \leq \theta \leq 90° \\ 0, & 90° \leq \theta \leq 180° \end{cases} \quad 0° \leq \phi \leq 360°$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}}$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi/2} \cos^4(\theta)\sin\theta \, d\theta = 2\pi\left[-\frac{\cos^5\theta}{5}\right]_0^{\pi/2}$$

$$P_{rad} = 2\pi\left(-0+\frac{1}{5}\right) = \frac{2\pi}{5},$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(1)}{2\pi/5} = 10$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o = \frac{\lambda^2}{4\pi} \cdot 10 = \frac{10\lambda^2}{4\pi}, \quad \lambda = \frac{3\times10^8}{10^{10}} = 3\times10^{-2} = 0.03\text{ m}$$

$$A_{em} = \frac{10(0.03)^2}{4\pi} = \frac{10\cdot(3\times10^{-2})^2}{4\pi} = \frac{10\cdot(9\times10^{-4})}{4\pi} = 7.16197\times10^{-4}$$

$$A_{em} = 7.16197\times10^{-4}$$

A communication satellite is in stationary (synchronous) orbit about the earth (assume altitude of 22,300 statute miles). Its transmitter generates 8.0 W. Assume the transmitting antenna is isotropic. Its signal is received by the 210-ft diameter tracking paraboloidal antenna on the earth at the NASA tracking station at Goldstone, California. Also assume no resistive losses in either antenna, perfect polarization match, and perfect impedance match at both antennas. At a frequency of 2 GHz, determine the:

(a) power density (*in watts/m^2*) incident on the receiving antenna.

(b) power received by the ground-based antenna whose gain is 60 dB.

1 status mile = 1609.3 meter, 22,300 (status miles) = 3.588739×10^{7} m

a. $P_i = \dfrac{P_{rad}}{4\pi R^2} = \dfrac{8 \times 10^{-14}}{4\pi \times (3.58874)} = 4.943 \times 10^{-16}$ Watts/m².

b. $A_{em} = \dfrac{\lambda^2}{4\pi} D_o$, ($\leftarrow D_o = 60\,dB, = 10^6$)
($\leftarrow \lambda = 0.15\,m$)

$A_{em} = \dfrac{(0.15)^2}{4\pi} \cdot 10^6 = 1790.493\ m^2$

$P_{received} = A_{em} \cdot P_i = (1790.493) \cdot (4.943 \times 10^{-16})$

$= 8.85 \times 10^{-13}$ watts.

A lossless ($e_{cd} = 1$) antenna is operating at 100 MHz and its maximum effective aperture is 0.7162 m^2 at this frequency. The input impedance of this antenna is 75 ohms, and it is attached to a 50-ohm transmission line. Find the directivity (dimensionless) of this antenna if it is polarization-matched.

$$A_{em} = 0.7162 \ m^2$$

$$A_{em} = \left(\frac{\lambda}{4\pi}\right)^2 \cdot \overset{1}{e_{cd}} (1 - |\Gamma|^2) \, \overset{1}{|\hat{\rho}_w \cdot \hat{\rho}_a|} \cdot D_o$$

$$D_o = \frac{A_{em}}{\left(\frac{\lambda}{4\pi}\right)^2 (1 - |\Gamma|^2)} , \quad \Gamma = \frac{75-50}{75+50} = 0.2 , \quad \lambda = \frac{3 \times 10^8}{100 \times 10^6} = 3m$$

$$D_o = \frac{0.7162}{\frac{3^2}{4\pi} (1 - |0.2|^2)}$$

$$D_o = 1.0417$$

A resonant, lossless ($e_{cd} = 1.0$) half-wavelength dipole antenna, having a directivity of 2.156 dB, has an input impedance of 73 ohms and is connected to a lossless, 50 ohms transmission line. A wave, having the same polarization as the antenna, is incident upon the antenna with a power density of 5 W/m^2 at a frequency of 10 MHz. Find the received power available at the end of the transmission line.

$$P_r = W_i A_{em} = W_i e_{cd} (1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi}\right) D_o |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$W_i = 5\ \text{W/m}^2$, $e_{cd} = 1$ (lossless), $\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} = \frac{73-50}{73+50} = 0.187$

$\lambda = \frac{3\times10^8}{10\times10^6} = 30\ \text{m}$, $D_o = 2.156\ \text{dB} = 1.643$, $PLF = 1$

$$P_r = (5)(1)(1-(0.187)^2)\left(\frac{30^2}{4\pi}\right)(1.643)(1) = 567.78\ \text{Watts} = P_r$$

$P_r = 567.78$ Watts.

Two X-band (8.2–12.4 GHz) rectangular horns, with aperture dimensions of 5.5 cm and 7.4 cm and each with a gain of 16.3 dB (over isotropic) at 10 GHz, are used as transmitting and receiving antennas. Assuming that the input power is 200 mW, the VSWR of each is 1.1, the conduction-dielectric efficiency is 100%, and the antennas are polarization-matched, find the maximum received power when the horns are separated in air by

(a) 5 m (b) 50 m (c) 500 m

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{or} G_{ot}\ , \quad G_{or} = G_{ot} = 16.3 \Rightarrow G_o \text{ (power ratio)} = 42.66$$

$f = 10\text{ GHz} \Rightarrow \lambda = 0.03$ meters.

$P_t = 200$ m watts $= 0.2$ Watts

a. $R = 5\text{ m}$: $P_r = \left[\frac{0.03}{4\pi(5)}\right]^2 (42.66)^2 (0.2) = 82.9\ \mu$ Watts

b. $R = 50\text{ m}$: $P_r = 0.829\ \mu$ Watts

c. $R = 500\text{ m}$: $P_r = 8.29$ n watts

Transmitting and receiving antennas operating at 1 GHz with gains (over isotropic) of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the maximum power delivered to the load when the input power is 150 W. Assume that the

(a) antennas are polarization-matched

(b) transmitting antenna is circularly polarized (either right- or left-hand) and the receiving antenna is linearly polarized.

1. $\frac{P_r}{P_t} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t}\ G_{0r}$

$G_{0t} = 20\,dB \Rightarrow G_{0t}$ (power ratio) $= 10^2 = 100$

$G_{0r} = 15\,dB \Rightarrow G_{0r}$ (power ratio) $= 10^{1.5} = 31.623$

$f = 1\,GHz \Rightarrow \lambda = 0.3$ meters

$R = 1 \times 10^3$ meters

a. For $|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1$

$$P_r = \left(\frac{0.3}{4\pi \times 10^3}\right)^2 (100)(31.623)(150 \times 10^{-3}) = 270.344\ \mu\ \text{Watts}$$

b. When transmitting antennas is circularly polarized and receiving antenna is linearly polarized, the PLF is equal to

$$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = \left|\left(\frac{\hat{a}_x \pm j\hat{a}_y}{\sqrt{2}}\right) \cdot \hat{a}_x\right|^2 = \frac{1}{2}$$

Thus $P_r = \frac{1}{2}(270.344 \times 10^{-6}) = 135.172 \times 10^{-6} = 135.172\ \mu\text{Watts}$

2.99. Two lossless, polarization-matched antennas are aligned for maximum radiation between them, and are separated by a distance of 50λ. The antennas are matched to their transmission lines and have directivities of 20 dB. Assuming that the power at the input terminals of the transmitting antenna is 10 W, find the power at the terminals of the receiving antenna.

.100. Repeat Problem 2.99 for two antennas with 30 dB directivities and separated by 100λ. The power at the input terminals is 20 W.

.101. Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the power delivered to the load when the input power is 150 W. Assume the PLF = 1.

Lossless : $e_{cd} = 1$, polarization matched : $|\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$,
line matched : $(1 - |\Gamma|^2) = 1$

$$D_o = 20\,dB = 10^2 = 100 = D_{or} = D_{ot}$$

$$P_r = P_t \left(\frac{\lambda}{4\pi R}\right)^2 D_{ot} D_{or} = 10\left(\frac{\lambda}{4\pi \cdot 50\lambda}\right)^2 (100)(100) = 0.253 \text{ watts}$$

$$P_r = 0.253 \text{ watts}$$

Lossless : $e_{cd} = 1$, PLF = 1. line matched : $(1 - |\Gamma|^2) = 1$.

$$D_o = 30dB = 10^3 = 1000 = D_{or} = D_{ot}$$

$$P_r = P_t \left(\frac{\lambda}{4\pi \cdot 100\lambda}\right)^2 \cdot (1000)^2 = 20 \cdot \left(\frac{1}{4\pi}\right)^2 \cdot 100 = 12.665 \text{ watts}$$

$G_{or} = 20dB = 100$, $G_{ot} = 25\,dB = 316.23$. $\lambda = \dfrac{3 \times 10^8}{3 \times 10^9} = 0.1$ m.

$$P_r = P_t \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \cdot \left(\frac{\lambda}{4\pi R}\right)^2 \cdot G_{or} \cdot G_{ot} = 100 \cdot (1) \cdot \left(\frac{0.1}{4\pi \times 500}\right)^2 (100)(316.23)$$

$$= 8 \times 10^{-4} \text{ watts} = P_r$$

A series of microwave repeater links operating at 10 GHz are used to relay television signals into a valley that is surrounded by steep mountain ranges. Each repeater consists of a receiver, transmitter, antennas, and associated equipment. The transmitting and receiving antennas are identical horns, each having gain over isotropic of 15 dB. The repeaters are separated in distance by 10 km. For acceptable signal-to-noise ratio, the power received at each repeater must be greater than 10 nW. Loss due to polarization mismatch is not expected to exceed 3 dB. Assume matched loads and free-space propagation conditions. Determine the minimum transmitter power that should be used.

$f = 10\,\text{GHz}, \rightarrow \lambda = \frac{3\times10^8}{10^{10}} = 0.03\,\text{m}$

$G_{ot} = G_{or} = 15\,\text{dB} = 10^{1.5} = 31.62$

$R = 10\,\text{km} = 10^4\,\text{m}$

$P_r \geqslant 10\,\text{nW} = 10^{-8}\,\text{W}$

$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = -3\,\text{dB} = \frac{1}{2}$

Friis Transmission Equation :

$$\frac{P_r}{P_t} = G_{ot}\, G_{or} \left(\frac{\lambda}{4\pi R}\right)^2 \cdot |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

$$= (10^{1.5})^2 \cdot \left(\frac{0.03}{4\pi\times10^4}\right)^2 \cdot \left(\frac{1}{2}\right) = 2.85\times10^{-11}$$

$$P_t = \frac{P_r}{2.85\times10^{-11}}$$

$P_r \geqslant 10^{-8}\,\text{W} \rightarrow (P_t)_{min} = 351\,\text{W}$

A one-way communication system, operating at 100 MHz, uses two identical $\lambda/2$ vertical, resonant, and lossless dipole antennas as transmitting and receiving elements separated by 10 km. In order for the signal to be detected by the receiver, the power level at the receiver terminals must be at least 1 μW. Each antenna is connected to the transmitter and receiver by a lossless 50-Ω transmission line. Assuming the antennas are polarization-matched and are aligned so that the maximum intensity of one is directed toward the maximum radiation intensity of the other, determine the minimum power that must be generated by the transmitter so that the signal will be detected by the receiver. Account for the proper losses from the transmitter to the receiver.

$$\frac{P_r}{P_t} = \rho\, e_t e_r\, D_{ot}\, D_{or}\left(\frac{\lambda}{4\pi R}\right)^2$$

$$= (\rho)(e_{rt}\cdot e_{cdt})(e_{rr}\cdot e_{cdr})\left(\frac{\lambda}{4\pi R}\right)^2 \cdot D_{ot}\cdot D_{or}$$

$$\frac{P_r}{P_t} = (1)(e_{rt}\cdot(1))(e_{rr}\cdot(1))\left(\frac{\lambda}{4\pi R}\right)^2 \cdot D_{ot}\cdot D_{or}$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^8} = 3m, \quad R = 10\times10^3 = 10^4.$$

$$\left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{3}{4\pi\times10^4}\right)^2 = \left(\frac{3}{4\pi}\times10^{-4}\right)^2 = \left(0.2387\times10^{-4}\right)^2 = 5.699\times10^{-2}\times10^{-8}$$

$$\left(\frac{\lambda}{4\pi R}\right)^2 = 5.699\times10^{-10}$$

$$e_{rt} = e_{rr} = (1-|\Gamma|^2) = \left(1-\left|\frac{73.3-50}{73.3+50}\right|^2\right) = \left(1-\left|\frac{23.3}{123.3}\right|^2\right)$$

$$= (1-(0.18897)^2) = (1-0.0357) = 0.9643$$

$$e_{cdt} = e_{cdr} = 1$$

$$D_{ot} = D_{or} = 1.643$$

$$\frac{P_r}{P_t} = (0.9643)^2(1.643)^2(5.699\times10^{-10}) = (0.92987)(2.699)(5.699\times10^{-10})$$

$$= 2.51\,(5.699\times10^{-10}) = 14.305\times10^{-10}$$

$$P_t = \frac{P_r}{14.305\times10^{-10}} = 6.99\times10^{-2}\times10^{10}(1\times10^{-6}) = 6.99\times10^2 = 699.$$

$$P_t = 699 \text{ Watts}$$

RADAR Problems

A rectangular X-band horn, with aperture dimensions of 5.5 cm and 7.4 cm and a gain of 16.3 dB (over isotropic) at 10 GHz, is used to transmit and receive energy scattered from a perfectly conducting sphere of radius $a = 5\lambda$. Find the maximum scattered power delivered to the load when the distance between the horn and the sphere is

(a) 200λ (b) 500λ

Assume that the input power is 200 mW, and the radar cross section is equal to the geometrical cross section.

4. $\sigma = \pi a^2 = 25\pi\lambda^2$

$G_{ot} = G_{or} = 16.3\text{ dB} \Rightarrow G_{ot}(\text{power ratio}) = 10^{1.63} = 42.66$

$f = 10\text{GHz} \Rightarrow \lambda = 0.03\text{ m}$

$$\frac{P_r}{P_t} = \sigma \frac{G_{ot} \cdot G_{or}}{4\pi} \left(\frac{\lambda}{4\pi R_1 \cdot R_2} \right)^2$$

a. $R_1 = R_2 = 200\lambda = 6$ meters;

$$P_r = 25\pi\lambda^2 \frac{(42.66)^2}{4\pi} \cdot \left[\frac{\lambda}{4\pi (200\lambda)^2} \right]^2 \cdot (0.2) = 9.00\ n\text{ watts}$$

b. $R_1 = R_2 = 500\lambda = 15$ meters;

$P_r = 0.23\ n\text{ watts}$

A radar antenna, used for both transmitting and receiving, has a gain of 150 (dimensionless) at its operating frequency of 5 GHz. It transmits 100 kW, and is aligned for maximum directional radiation and reception to a target 1 km away having a radar cross section of 3 m^2. The received signal matches the polarization of the transmitted signal. Find the received power.

$$P_r = P_t\, \sigma \cdot \frac{G_{ot} \cdot G_{or}}{4\pi} \cdot \left[\frac{\lambda}{4\pi R_1 \cdot R_2}\right]^2, \quad \lambda = \frac{3\times10^8}{5\times10^9} = 0.06\ \text{m}$$

$$P_r = 10^5 \cdot (3) \cdot \frac{150^2}{4\pi} \cdot \left[\frac{0.06}{4\pi(10^6)}\right]^2$$

$$P_r = 1.22\times10^{-8}\ \text{Watts}$$

In an experiment to determine the radar cross section of a Tomahawk cruise missile, a 100 W, 10 GHz signal was transmitted toward the target, and the received power was measured to be –160 dB. The same antenna, whose gain was 80 (*dimensionless*), was used for both transmitting and receiving. The polarizations of both signals were identical (PLF = 1), and the distance between the antenna and missile was 10^4 m. What is the radar cross section of the cruise missile?

Repeat Problem 2.108 for a radar system with 100 W, 3 GHz transmitted signal, –160 dB received signal, an antenna with a gain of 80 (*dimensionless*), and separation between the antenna and target of 10^4 m.

$$\frac{P_r}{P_t} = \sigma \frac{G_{or} \cdot G_{ot}}{4\pi} \cdot \left[\frac{\lambda}{4\pi R_1 R_2}\right]^2 \Rightarrow \sigma = \frac{P_r \cdot 4\pi}{P_t \cdot G_{or} \cdot G_{ot}} \left[\frac{4\pi R_1 \cdot R_2}{\lambda}\right]^2$$

$$\lambda = \frac{3\times 10^8}{3\times 10^8} = 1\,m$$

$$\therefore \sigma = \frac{0.1425\times 10^{-3}(4\pi)}{1000\,(75)(75)} \left[\frac{4\pi(500)(500)}{1}\right]^2 = 3142\,m^2 = \sigma$$

$$\sigma = \frac{P_r \cdot 4\pi}{P_t \cdot G_{or} \cdot G_{ot}} \left[\frac{4\pi R_1 R_2}{\lambda}\right]^2$$

$$\lambda = \frac{3\times 10^8}{1\times 10^8} = 3\,m$$

$$\sigma = \frac{0.01 \cdot (4\pi)}{1000\,(75)(75)} \left[\frac{4\pi(700)(700)}{3}\right]^2 = 94,114.5\,m^2$$

$$\sigma = 94,114.5\,m^2$$

The maximum radar cross section of a resonant linear $\lambda/2$ dipole is approximately $0.86\lambda^2$. For a monostatic system (i.e., transmitter and receiver at the same location), find the received power (in W) if the transmitted power is 100 W, the distance of the dipole from the transmitting and receiving antennas is 100 m, the gain of the transmitting and receiving antennas is 15 dB each, and the frequency of operation is 3 GHz. Assume a polarization loss factor of -1 dB.

$$\sigma = 0.85\lambda^2$$

$$\frac{P_r}{P_t} = \sigma \cdot \frac{G_{ot} \cdot G_{or}}{4\pi} \left(\frac{\lambda}{4\pi R_1 \cdot R_2} \right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

$\sigma = 0.85\lambda^2$, $G_{ot} = G_{or} = 15\text{dB} \Rightarrow G_{ot} = G_{or} = 31.6228$ (dimensionless)

$R_1 = R_2 = 100$ meter $\Rightarrow R_1 = R_2 = 1{,}000\,\lambda$

$f = 3\text{GHz} \Rightarrow \lambda = \dfrac{3\times10^8}{3\times10^9} = 0.1$ meters

$|\hat{\rho}_w \cdot \hat{\rho}_r|^2 = 1\text{ dB} \Rightarrow |\hat{\rho}_w \cdot \hat{\rho}_r|^2 = 0.7943$ ((linear)

$$\frac{P_r}{P_t} = 0.85\lambda^2 \cdot \frac{(31.6228)^2}{4\pi} \cdot \left(\frac{\lambda}{4\pi \times 10^6 \lambda^2} \right)^2 \cdot (0.7943)$$

$$= \frac{0.85\,(31.6228)^2\,(0.7943)}{(4\pi)^3\,(10^{12})} = 0.3402 \times 10^{-12}$$

$$P_r = 0.3402\times10^{-12}(10^2) = 0.3402\times10^{-10} = 34.02\times10^{-12} \text{ watts}$$

$$P_r = 34.02 \text{ pWatts}$$