6.7 Richards Transformation, Stepped impedance

Module:6 Microwave Passive circuits

Course: BECE305L – Antenna and Microwave Engineering

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CHENNAI

Module:6 Microwave Passive circuits 7 hours

- T junction and resistive power divider, Wilkinson power divider, branch line coupler (equal & unequal), Rat Race Coupler, Filter design: Low pass filter (Butterworth and Chebyshev) - Richards transformation and stepped impedance methods.

- Source of the contents: Pozar

4.1 Filter implementation: Problems

- The lumped-element filter designs discussed (binomial, chebychev) generally work well at low frequencies,
- but two problems arise at higher RF and microwave frequencies.
  1) lumped-element inductors and capacitors are generally available only for a limited range of values – are difficult to implement at microwave frequencies.
  Distributed elements, such as open-circuited or short-circuited transmission line stubs, are often used to approximate ideal lumped elements.
  2) At microwave frequencies the distances between filter components is not negligible.

4.1 Filter implementation: Solution

- The first problem is treated with **Richards' transformation**, used to convert lumped elements to transmission line sections.
- **Kuroda's identities** can then be used to physically separate filter elements by using transmission line sections.
- Because such additional transmission line sections do not affect the filter response, this type of design is called **redundant filter synthesis**.
improve the filter response; such non-redundant synthesis does not have a lumped-element counterpart.

4.2 Richards' Transformation

$$\Omega = \tan \beta \ell = \tan\left(\frac{\omega \ell}{v_p}\right)$$

- Frequency ω is mapped to Ω plane
- Ω repeats itself at a period of $\frac{\omega l}{v_p} = 2\pi$
- This was introduced to synthesize LC network using open- and short- circuited transmission line stubs
- Impedance in Ω plane is obtained by replacing ω with $\Omega = \tan \beta l$
- Reactance of inductor: $jX_L = jL\Omega = jL \tan \beta l$
  (Replace inductor with short circuited stub of length βl and characteristic impedance L)
- Susceptance of capacitor: $jB_C = jC\Omega = jC \tan \beta l$
  (Replace capacitor with open circuit stub of length βl and characteristic impedance C)
- Unity filter impedance is assumed.

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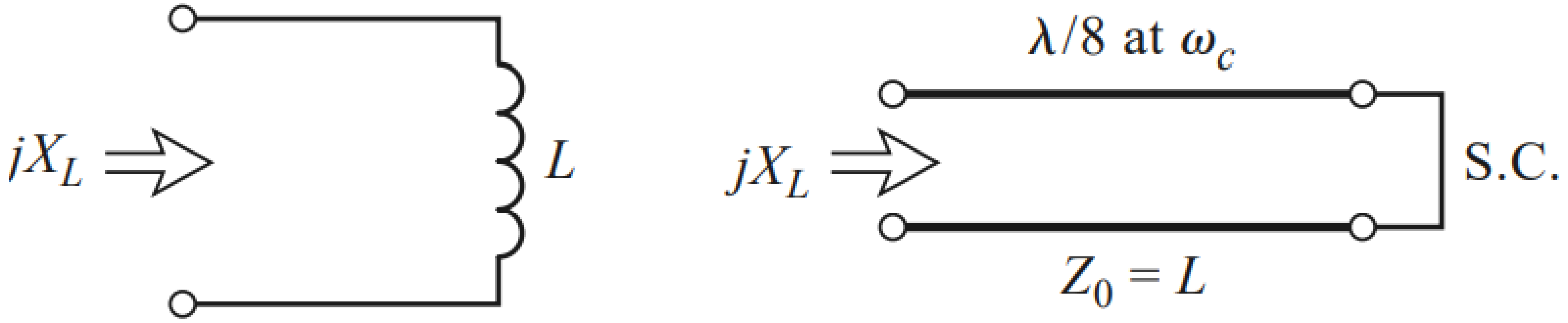

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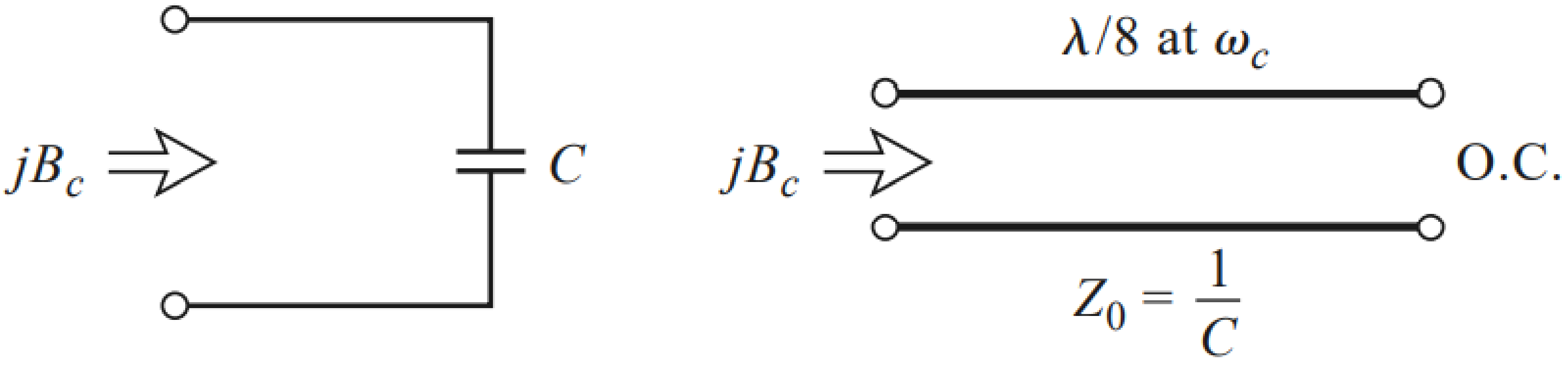


4.2 Richards' Transformation

$$\Omega = \tan \beta\ell = \tan\left(\frac{\omega\ell}{v_p}\right)$$

- At cut-off frequency ω_c in ω plane, with λ is wavelength at ω_c $\Omega = 1 = \tan \beta l$ which means $\beta l = \pi/4$ or the stub length of $\lambda/8$.

  Electrical lengths of all the stubs are same.

  Hence, they are known as ***commensurate lines.***
- At frequency $\omega_0 = 2\omega_c$, the stub length will be $\lambda/4$ and attenuation pole occurs ($\beta l = \pi/2$ and $\tan \beta l = \infty$)
- The response away from ω_c will be different and not same.
- In addition, the response will be periodic in frequency, repeating every $4\omega_c$.

4.3 Kuroda's identities

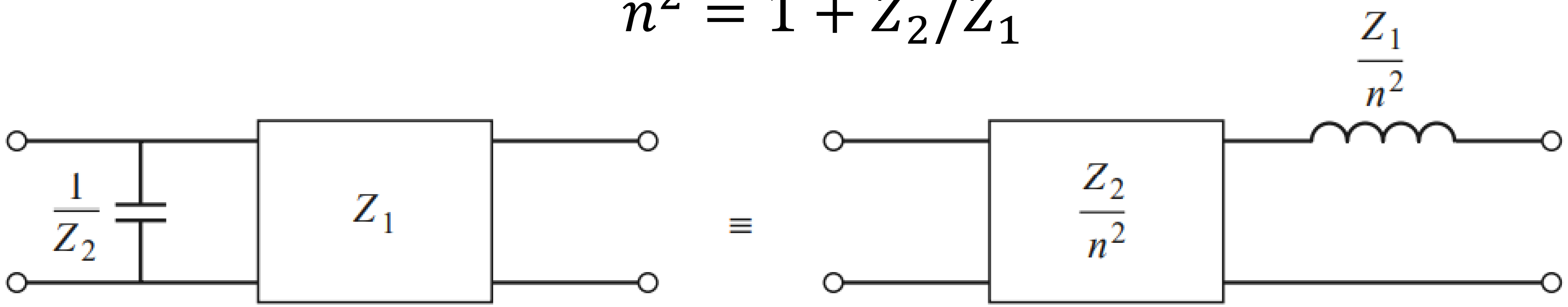
- The four Kuroda identities use redundant transmission line sections to achieve a more practical microwave filter implementation by performing any of the following operations
  - Physically separate transmission line stubs
  - Transform series stubs into shunt stubs, or Transform shunt stubs into series stubs
  - Change impractical characteristic impedances into more realizable values
  - The additional transmission line sections are called unit elements and are $\lambda/8$ long at ωc;
- The unit elements are thus commensurate with the stubs used to implement the inductors and capacitors of the prototype design

4.3 Kuroda's identities

- Each box represents a unit element, or transmission line, of the indicated characteristic impedance and length (λ/8 at ω_c)
- The inductors and capacitors represent short-circuit stub and open-circuit stubs respectively.

$$n^2 = 1 + Z_2/Z_1$$

1



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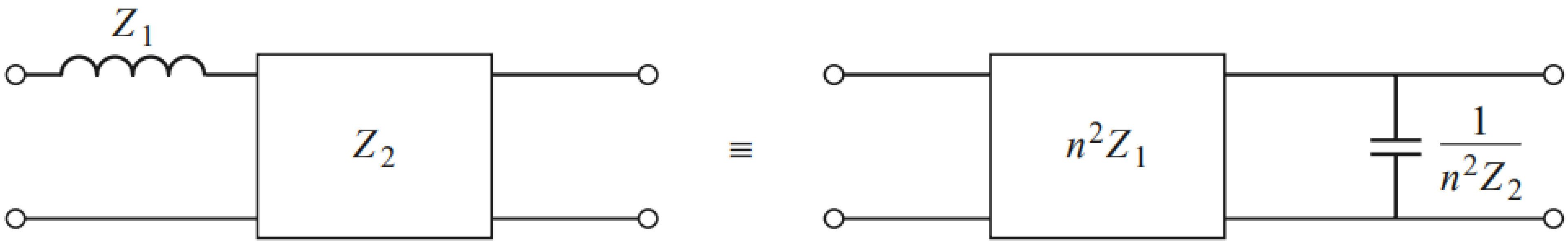

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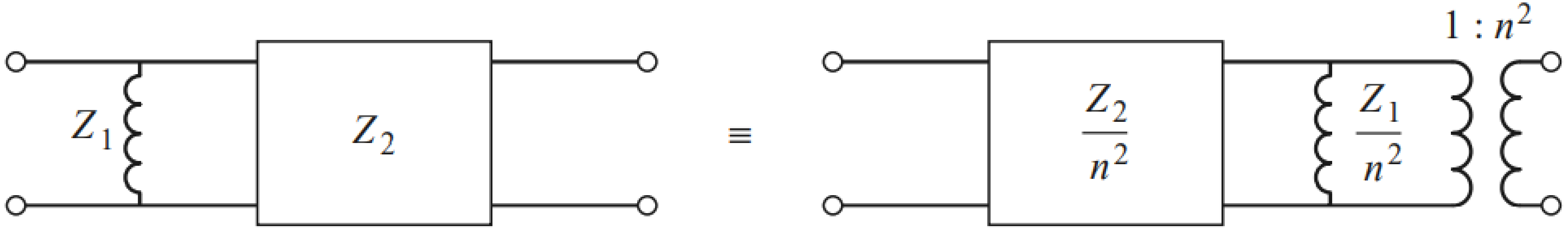

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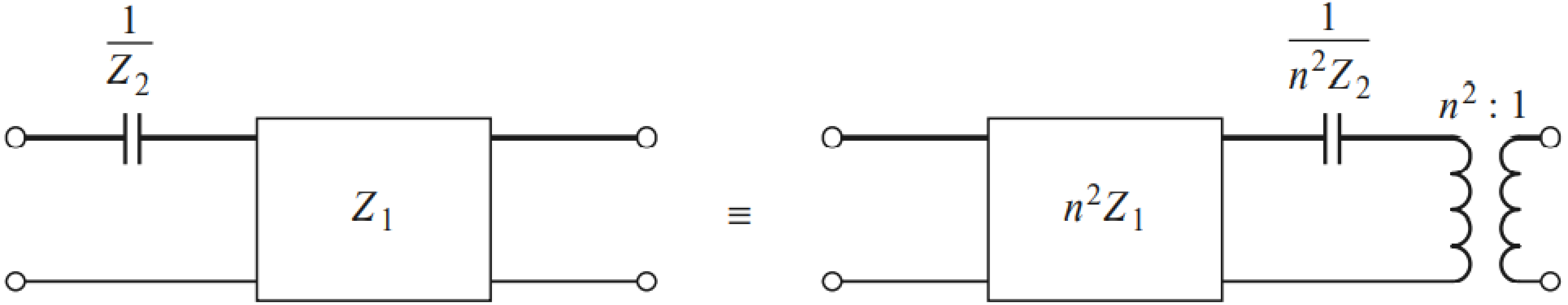
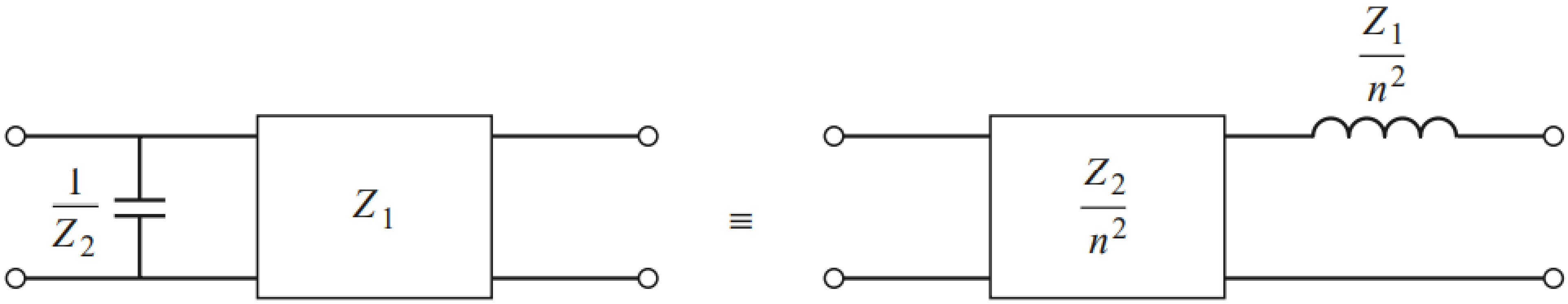
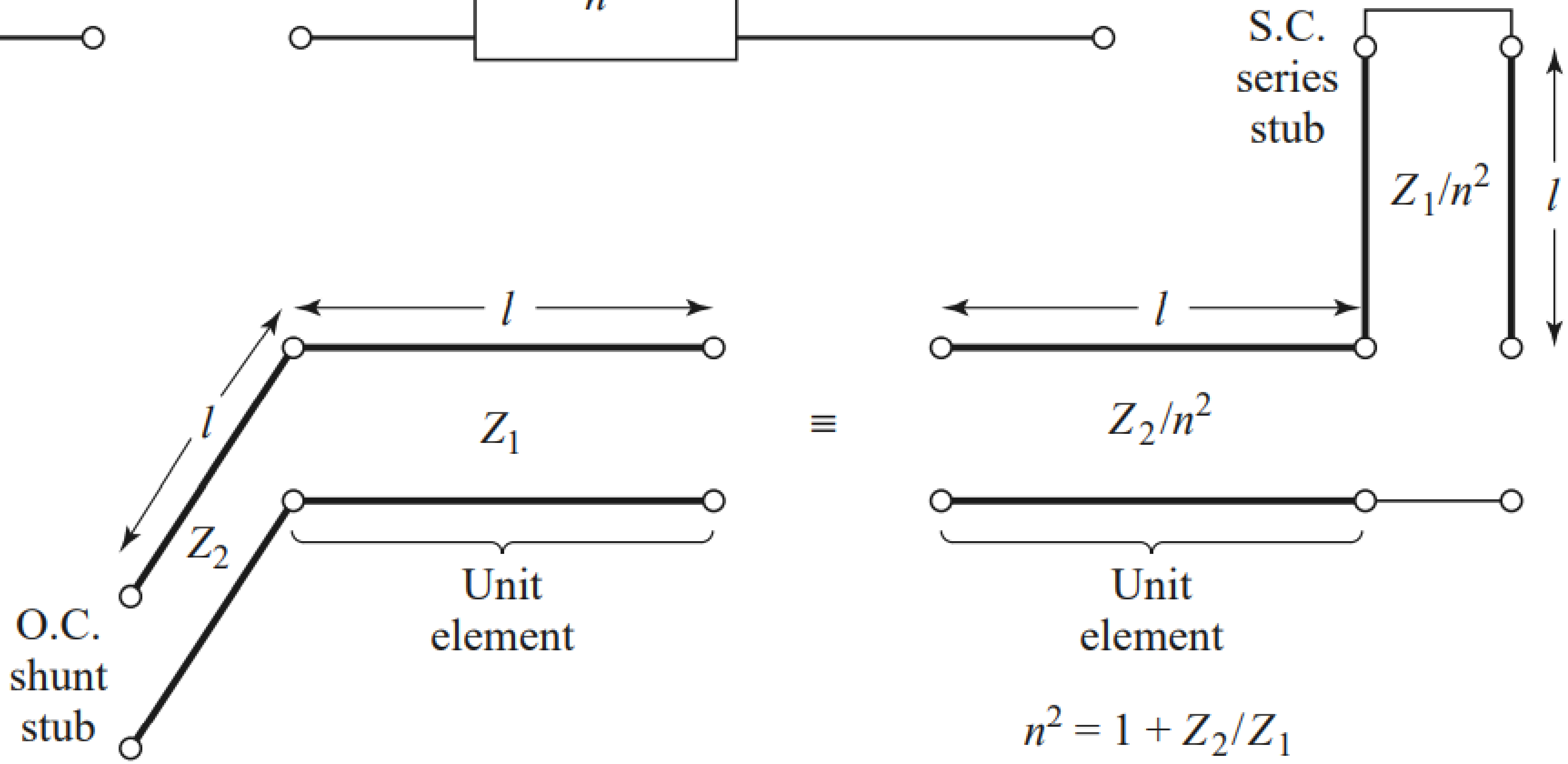


4.3 Kuroda's identity - 1

1





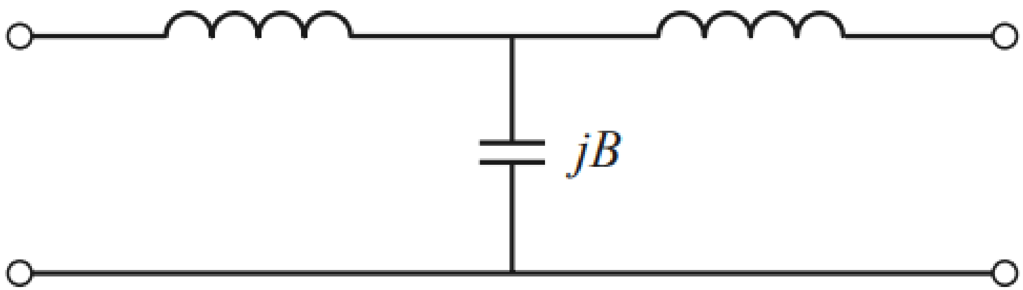
5. Stepped impedance LPF

- A relatively easy way to implement low-pass filters in microstrip or stripline: use alternating sections of very high and very low characteristic impedance lines. (stepped-impedance, or hi-Z, low-Z filters)
- Popular because they are **easier to design** and **take up less space** than a similar low-pass filter using stubs.
- Because of the approximations involved, however, their electrical performance is not as good, so the use of such filters is usually limited to applications where a sharp cutoff is not required (for instance, in rejecting out-of-band mixer products).

5. Stepped impedance LPF

- Approximate equivalent circuits for short sections of transmission lines.

Used for $\beta l \ll \pi/2$:

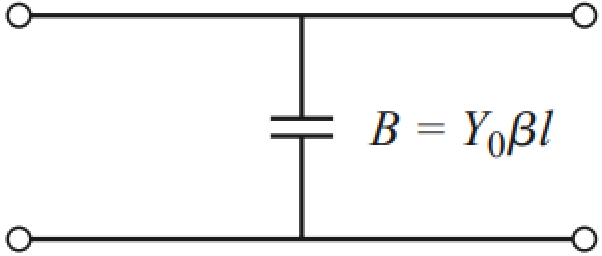


Equivalent circuit for small βl and large Z0

$$\beta\ell = \frac{LR_0}{Z_h} \quad \text{(inductor)}$$

Equivalent circuit for small βl and small Z0

$$\beta\ell = \frac{CZ_\ell}{R_0} \quad \text{(capacitor)},$$



Design a stepped-impedance low-pass filter having a maximally flat response and a cutoff frequency of 2.5 GHz. It is desired to have more than 20 dB insertion loss at 4 GHz. The filter impedance is 50 Ω; the highest practical line impedance is 120 Ω, and the lowest is 20 Ω. Consider the effect of losses when this filter is implemented with a microstrip substrate having $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.02$, and copper conductors of 0.5 mil thickness.

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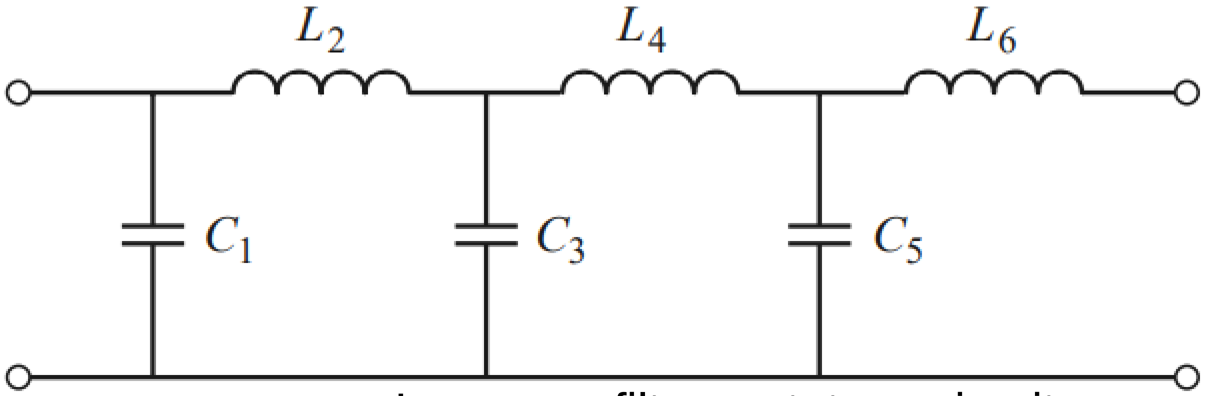
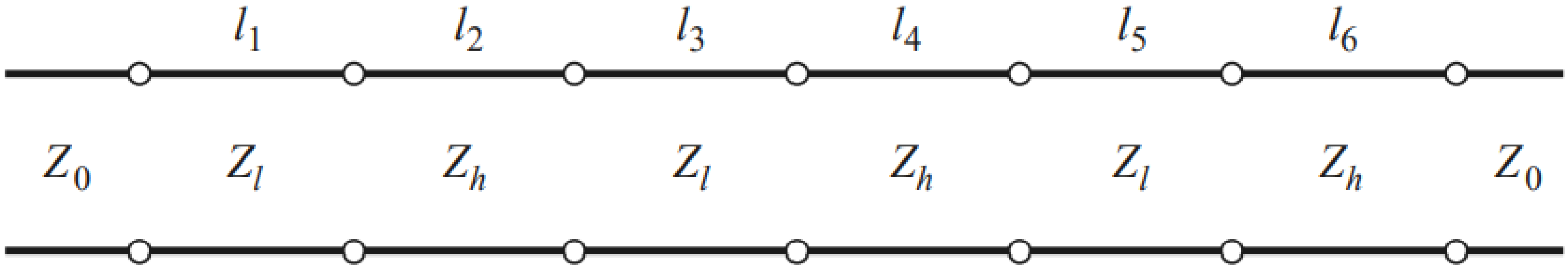
$$\frac{\omega}{\omega_c} - 1 = \frac{4.0}{2.5} - 1 = 0.6$$

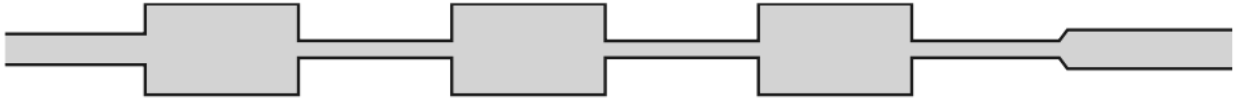


Low-pass filter prototype circuit.

Stepped impedance implementation



Microstrip layout of the final filter



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N = 6 should give the required attenuation at 4.0 GHz

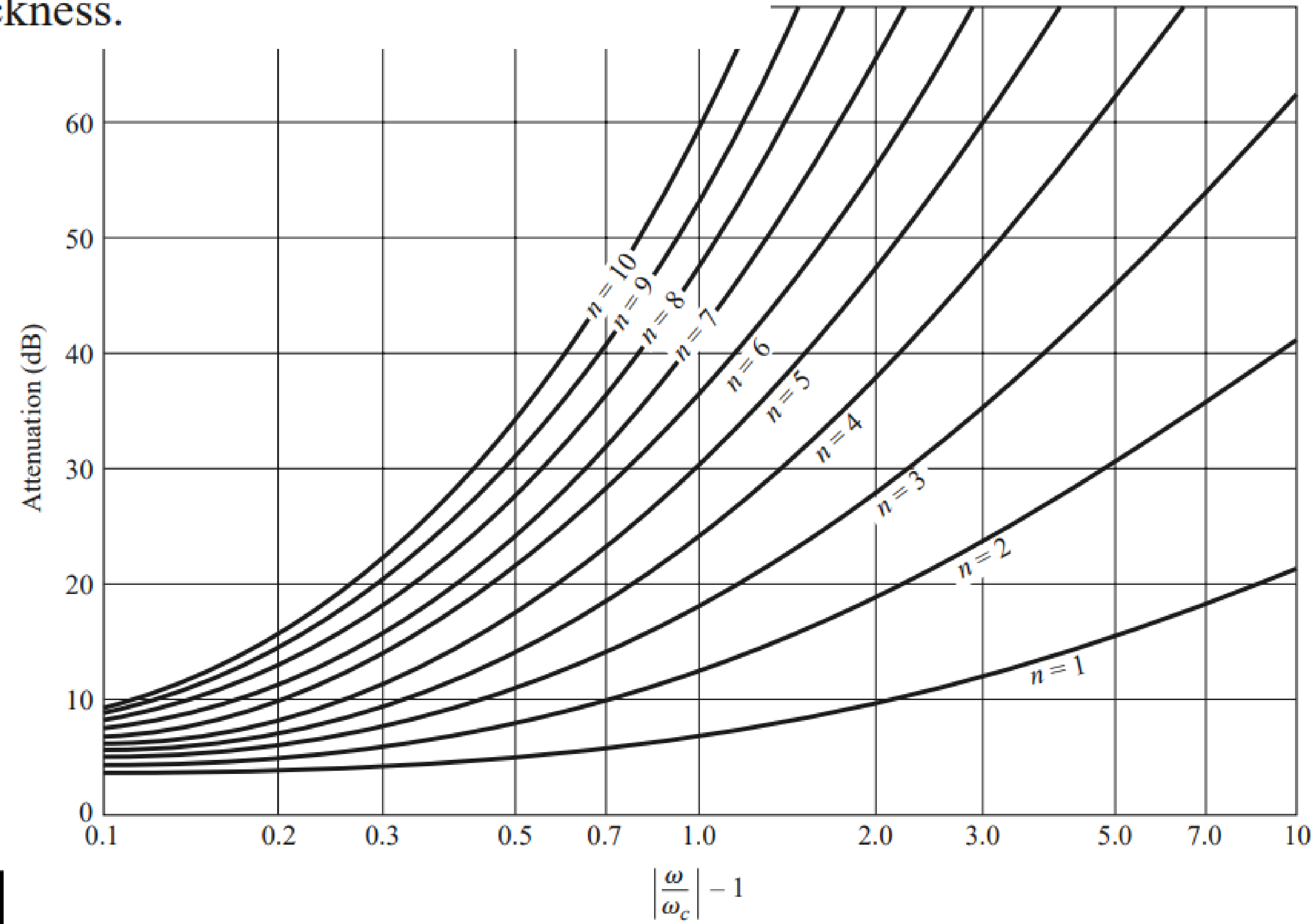

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TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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$$\boxed{\begin{aligned} g_1 &= 0.517 = C_1, \\ g_2 &= 1.414 = L_2, \\ g_3 &= 1.932 = C_3, \\ g_4 &= 1.932 = L_4, \\ g_5 &= 1.414 = C_5, \\ g_6 &= 0.517 = L_6. \end{aligned}}$$

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$$g_4 = 1.932 = L_4,$$
$$g_5 = 1.414 = C_5,$$
$$g_6 = 0.517 = L_6.$$

Section	$Z_i = Z_\ell$ or $Z_h (\Omega)$	$\beta\ell_i$ (deg)	W_i (mm)	ℓ_i (mm)
1	20	11.8	11.3	2.05
2	120	33.8	0.428	6.63
3	20	44.3	11.3	7.69
4	120	46.1	0.428	9.04
5	20	32.4	11.3	5.63
6	120	12.3	0.428	2.41

$$\beta\ell = \frac{LR_0}{Z_h} \quad \text{(inductor)} \qquad \beta\ell = \frac{CZ_\ell}{R_0} \quad \text{(capacitor)},$$

• Very important