5.1 Microwave Networks ‘S’ parameter and its properties

Module:5 Microwave Passive components

Course: BECE305L – Antenna and Microwave Engineering

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Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Module:5 Microwave Passive components 6 hours

- Microwave Networks - ABCD, ‘S’ parameter and its properties. E-Plane Tee, H-Plane Tee, Magic Tee and Multi-hole directional coupler. Principle of Faraday rotation, isolator, circulator and phase shifter.

- Source of the contents: Pozar

1. Need for Scattering matrix

- In a standing wave – **Direct measurement of voltages and currents** for non-TEM lines **are difficult**.
- Measurements involve **magnitude (obtained from Power)** and **phase of a wave in a given direction** / of a standing wave.

1. Need for Scattering matrix

- In a standing wave – Direct measurement of voltages and currents for non-TEM lines are difficult.
- Measurements involve magnitude (obtained from Power) and phase of a wave in a given direction / of a standing wave.
- Conventional impedance and admittance matrices that use the **equivalent total** voltages and currents become abstraction at high frequency networks.
- The ideas of **incident, reflected and transmitted waves** is used in Scattering matrix.

2. Scattering matrix

- A N port network : **Scattering matrix** provides complete **description of the network as seen at its N ports**.
- Scattering matrix: Relates **voltage waves incident on the port** to those **reflected from the ports**.

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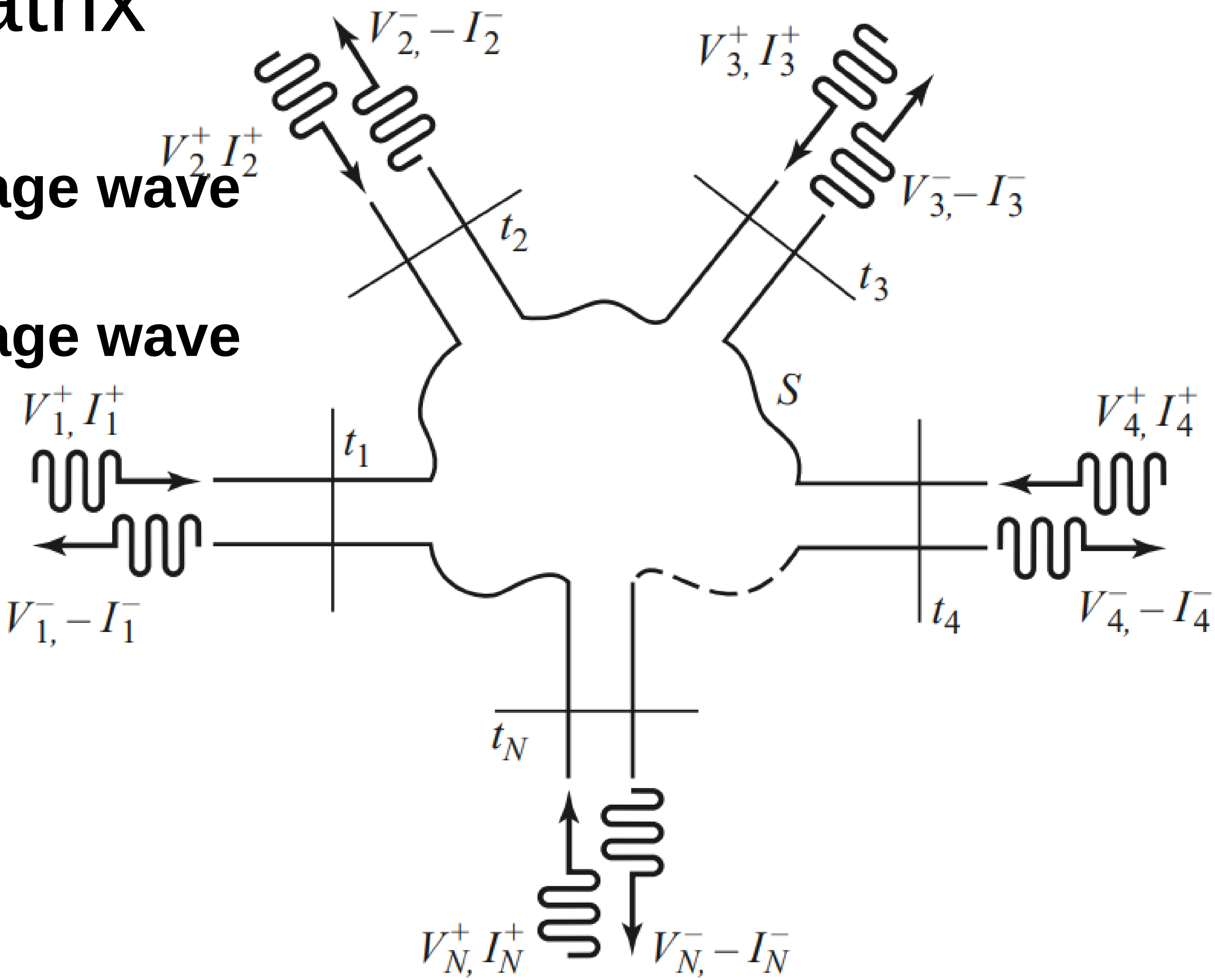
- A N port network : Scattering matrix provides complete description of the network as seen at its N ports.
- Scattering matrix: Relates voltage waves incident on the port to those reflected from the ports.
- S parameters may be computed for Some components and circuit through network analysis.
- Otherwise, **S parameters are measured using vector network analyzer**.

2. Scattering matrix

- For a network with N ports
- V_n^+ **amplitude of the voltage wave incident on port n**
- V_n^- **amplitude of the voltage wave reflected from port n**

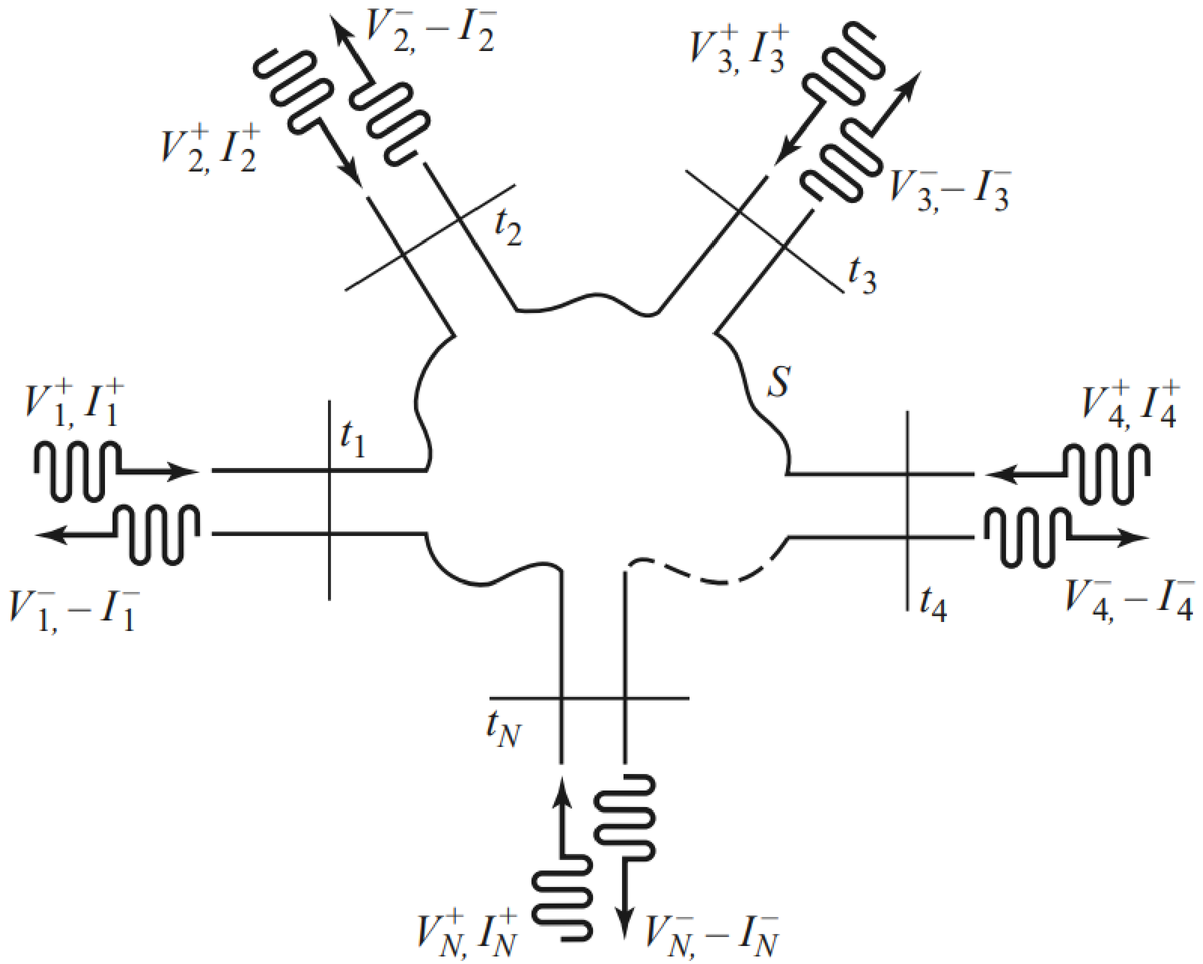


2. Scattering matrix

- For a network with N ports
- V_n^+ amplitude of the voltage wave incident on port n
- V_n^- amplitude of the voltage wave reflected from port n
- S matrix

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_n^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_n^+ \end{bmatrix}$$

$$[V^-] = [S][V^+]$$



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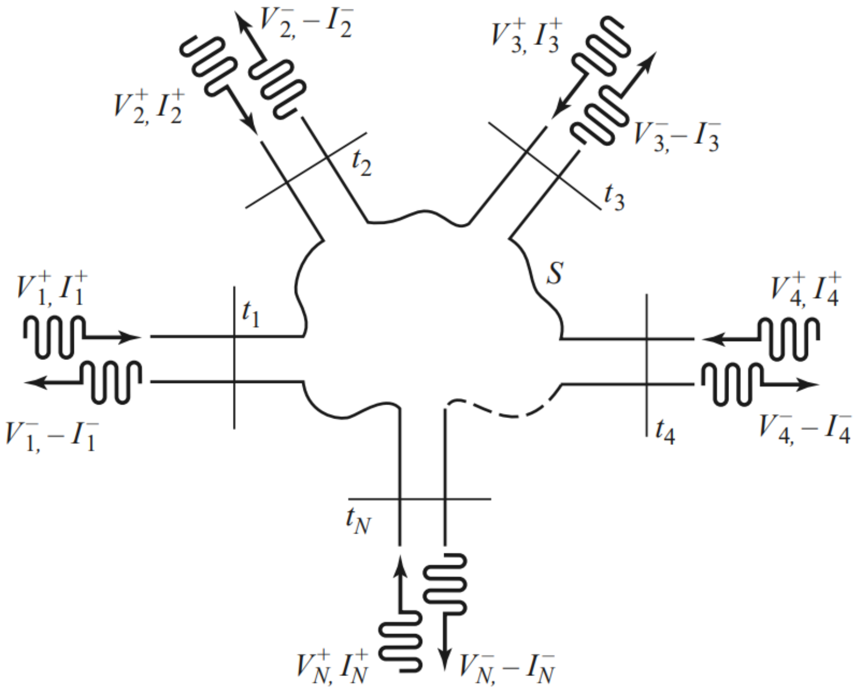

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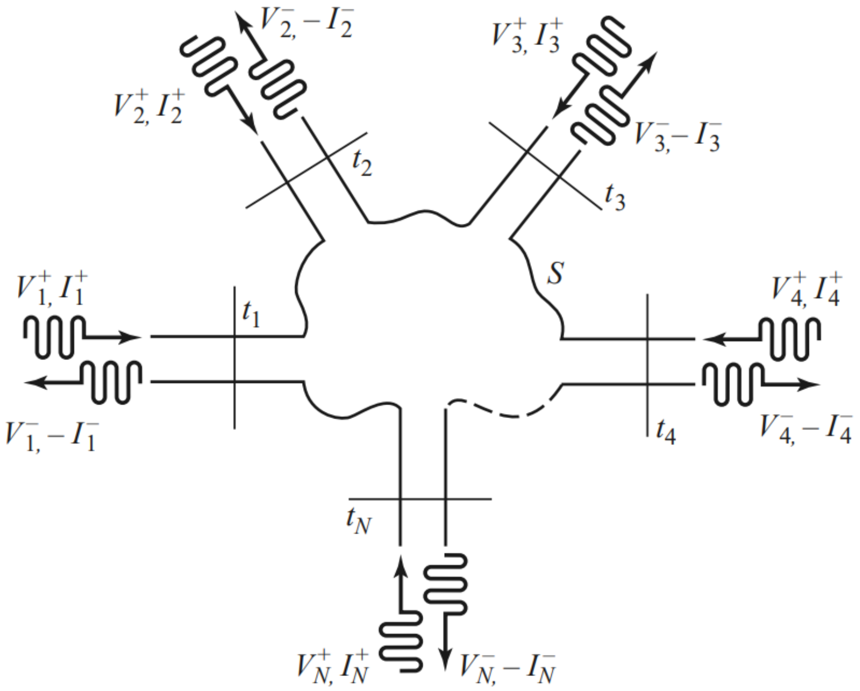

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- S_{ij} found by **driving port *j*** with **incident wave of voltage V_j^+** and **measuring the reflected wave amplitude V_i^-** coming out of **port *i***.

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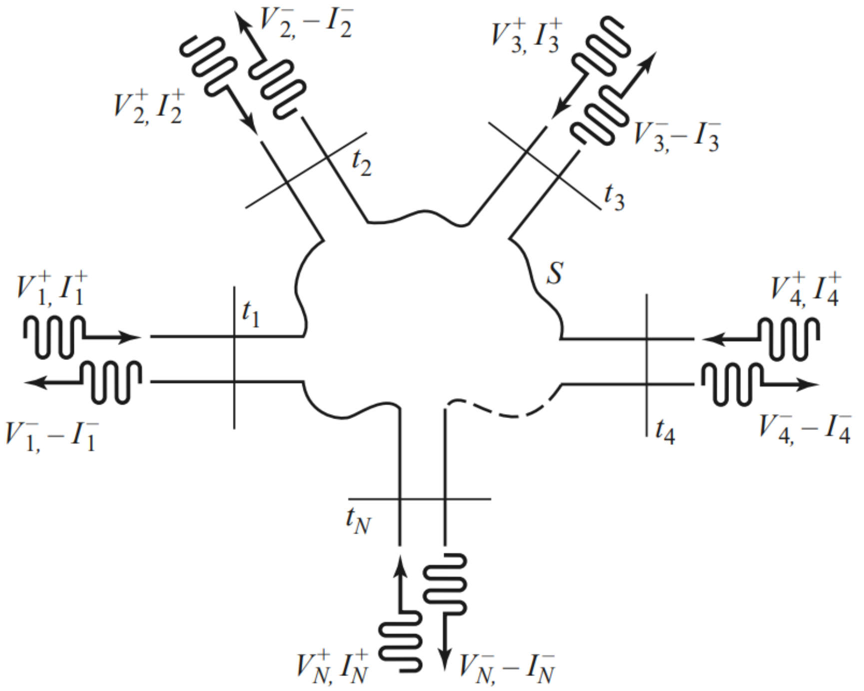

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- **Incident waves on all ports except j^{th} port are set to zero** (**All ports other than j^{th} port** is **terminated in matched loads** to avoid reflections.

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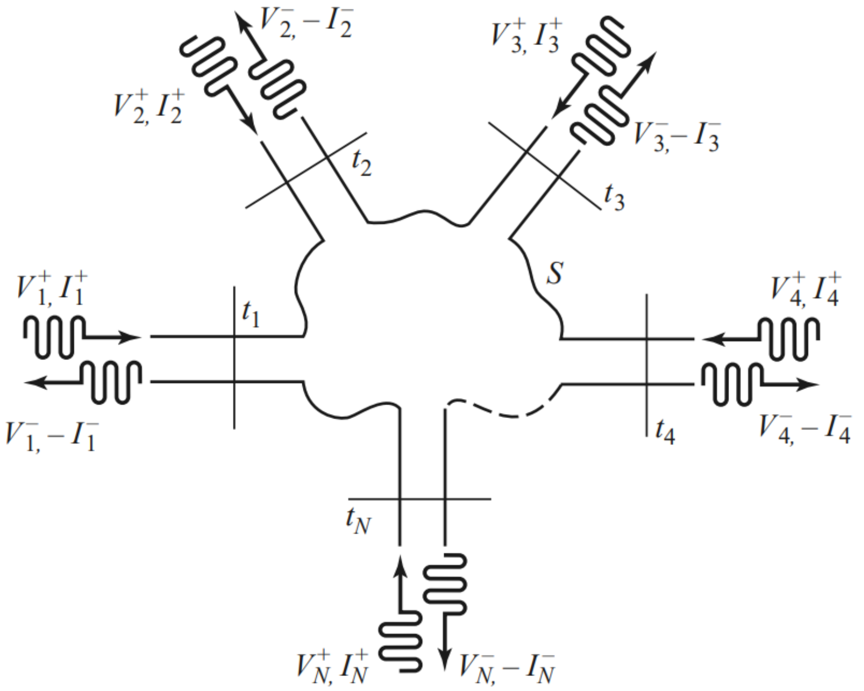

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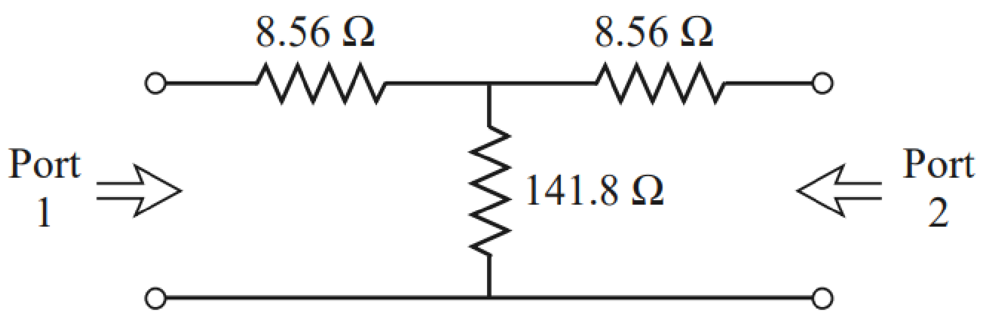
- Incident waves on all ports except j^{th} port are set to zero (All ports other than j^{th} port is terminated in matched loads to avoid reflections).
- S_{ii} is the **reflection coefficient** seen looking into port i when all ports are terminated at matched loads, and
  S_{ij}: **transmission coefficient** from port j to port i (all other ports are terminated in matched)

2. Scattering matrix

Find the scattering parameters of $3dB$ Attenuator (Matched load $Z_0 = 50\Omega$)



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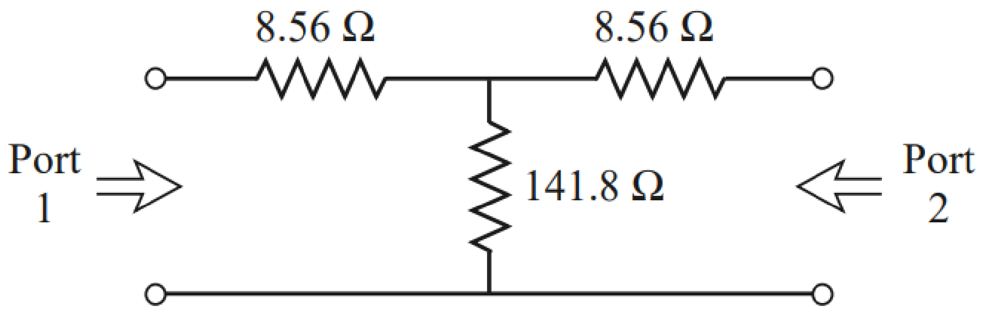

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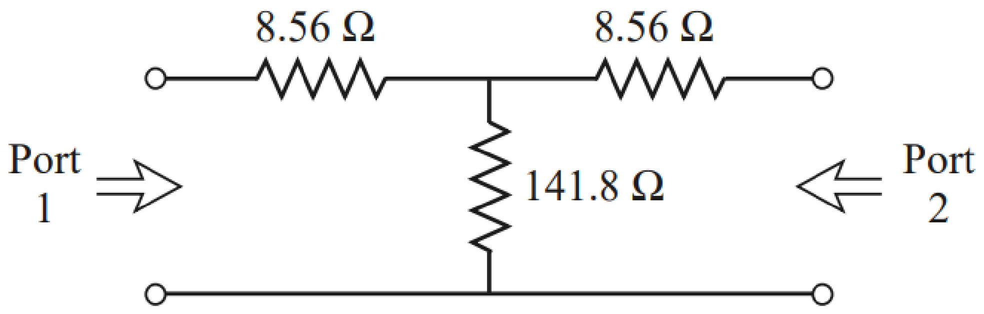

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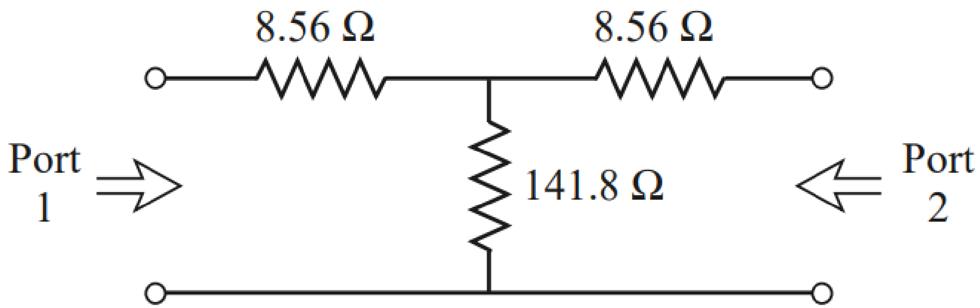

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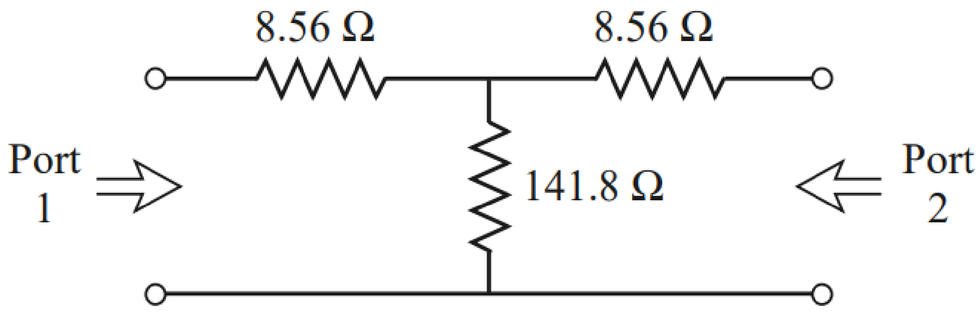

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Symmetric circuits : $S_{22} = 0$

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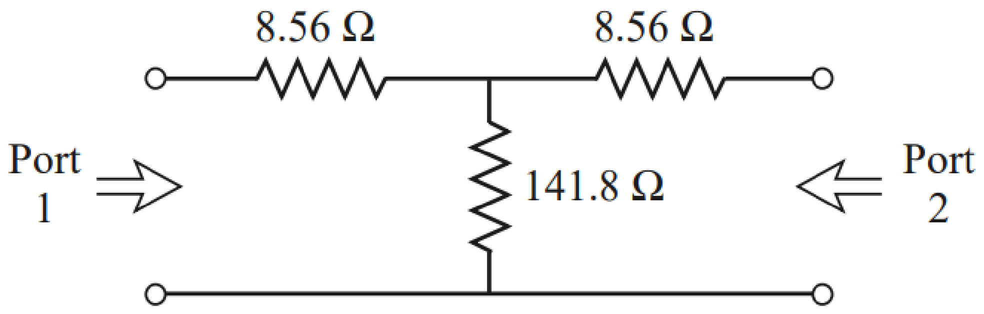

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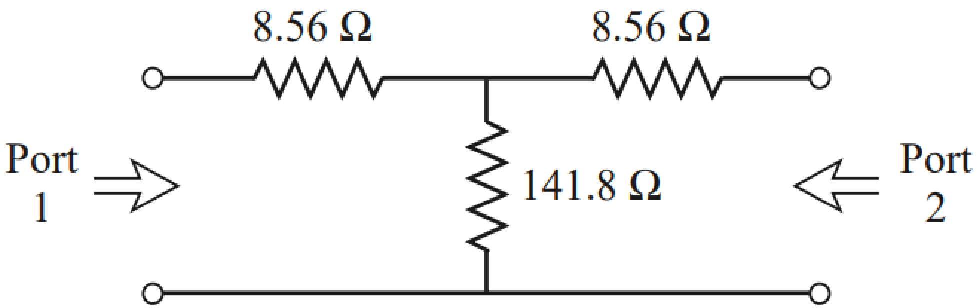

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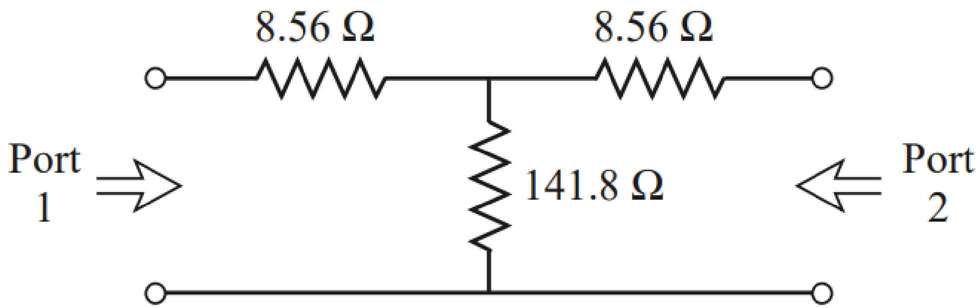

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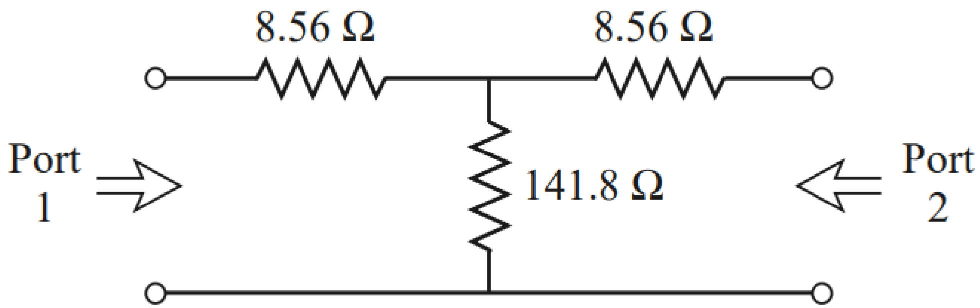

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When voltage V_1 at port 1, using division twice -> $V_2^- = V_2$ as voltage across 50Ω at port 2:

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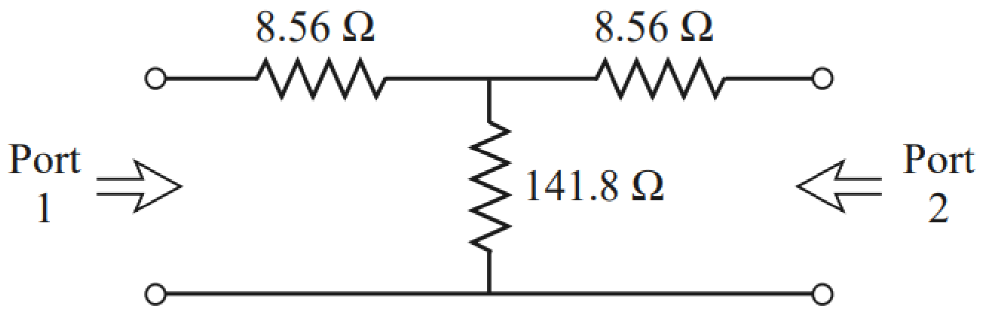

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$$V_2^- = V_2 = V_1\left(\frac{(141.18 \parallel 58.56)}{(141.18 \parallel 58.56) + 8.56}\right)\left(\frac{50}{50 + 8.56}\right) = 0.707V_1 = 0.707V_1^+$$

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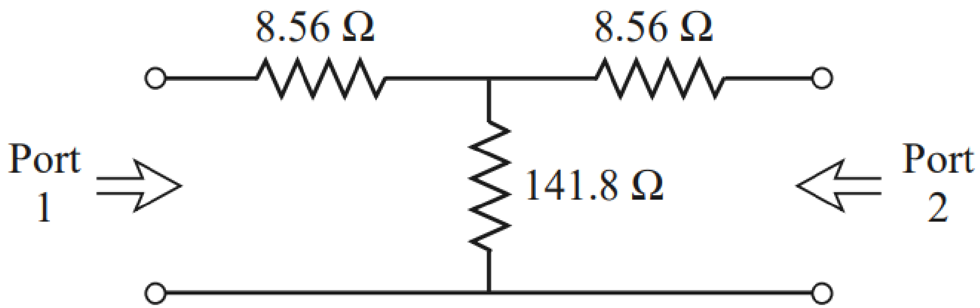

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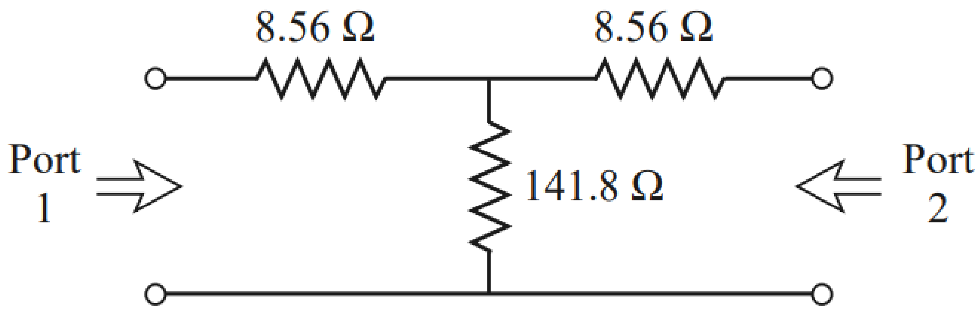

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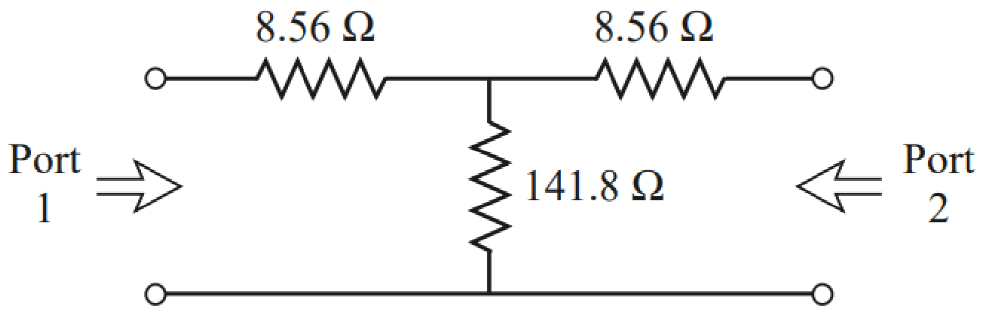

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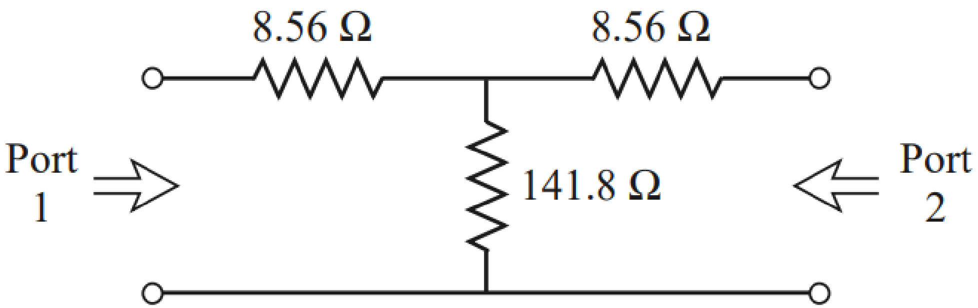
2. Scattering matrix



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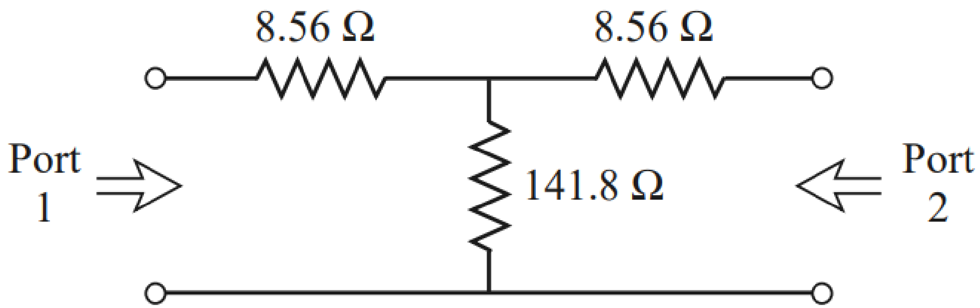

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5. Reciprocal networks and Lossless networks

- **Reciprocal**: Not containing active devices or non reciprocal media (ferrites, plasmas, etc)

$$Z_{ij} = Z_{ji} \text{ and } Y_{ij} = Y_{ji} \text{ (symmetric matrices)}$$

- **Lossless**: Z_{ij} or Y_{ij} are **purely imaginary**

5. Reciprocal networks and Lossless networks

- Total voltage and current at nth port of a N port network:
  $V_n = V_n^+ + V_n^-$ and $I_n = I_n^+ - I_n^- = V_n^+ - V_n^-$

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- $(AB)^T = B^T A^T$
- $[U]^t = [U]$ and in reciprocal network $[Z]^t = [Z]$

- Which means: **For reciprocal networks**, $[S] = [S]^t$ **the scattering matrix is symmetric**

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- If characteristic impedances of all ports are identical and 1

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- $$= \frac{1}{2}Re\{[V^+]^t[V^+]^* - [V^-]^t[V^-]^*\}$$

$$NOTE: -[V^+]^t[V^-]^* + [V^-]^t[V^+]^* = A - A^* = purely\ imnaginary$$

$$= Power\ delivered\ is\ zero$$

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- $= \frac{1}{2} Re\{[V^+]^t [V^+]^* - [V^-]^t [V^-]^*\}$ $\quad -[V^+]^t [V^-]^* + [V^-]^t [V^+]^* = A - A^* =$ $purely\ imnaginary = Power\ delivered\ is\ zero$
- $\frac{1}{2} Re\{[V^+]^t [V^+]^*\}$: $Power\ incident$
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$$[V^+]^t [V^+]^* = \{[S][V^+]\}^t \{[S][V^+]\}^* \qquad (AB)^T = B^T A^T$$
$$= [V^+]^t [S]^t [S]^* [V^+]^*$$

For non zero $[V^+]$,

$[S]^t [S]^* = [U]$ unitary matrix or $[S]^* = \{[S]^t\}^{-1}$

5. Reciprocal networks and **Lossless networks**

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- The **matrix can be written in summation form**

$$\sum_{k=1}^{N} S_{ki}S_{kj}^* = \delta_{ij} \ for\ all\ i,j$$

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Dot product of **any column** of $[S]$ with **the conjugate of the same column** is **unity.**

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Dot product of any column of $[S]$ with the conjugate of the same column is unity.

Dot product of any column of $[S]$ with **the conjugate of a different column** is **zero** (columns are orthonormal).

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- $\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0\ if\ i \neq j \end{cases}$ is the Kronecker delta symbol
- At $i = j$, $\sum_{k=1}^{N} S_{ki}S_{ki}^* = 1$
- At $i \neq j$, $\sum_{k=1}^{N} S_{ki}S_{kj}^* = 0$
- Also: $[S][S]^{*t} = [U]$

Dot product of any column of $[S]$ with the **conjugate of the same column is unity.**

Dot product of any column of $[S]$ with the **conjugate of a different column is zero (columns are orthonormal).**

6.1 Application of Scattering matrix

- A two port network

$$[S] = \begin{bmatrix} 0.15\angle 0° & 0.85\angle -45° \\ 0.85\angle 45° & 0.2\angle 0° \end{bmatrix}$$

To find: a) If network is reciprocal and lossless

b) If port 2 is terminated with matched load, find return loss at port 1.

c) If port 2 is terminated with short ckt, find the return loss at port 1

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Condition for Lossless network: $|S_{11}|^2 + |S_{21}|^2 = 1$

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$[S]\ matrix$ is not symmetric. Hence, it is not reciprocal

Condition for Lossless network: $|S_{11}|^2 + |S_{21}|^2 = 1$

$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2 = 0.745 \neq 0$

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This means that the network is lossy (NOT Lossless)

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$\Gamma = S_{11} = (0.15)$

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b) When port 2 is terminated to matched load, Reflection coefficient seen at port 1
$\Gamma = S_{11} = (0.15)$

Return loss=$-20\log_{10}|\Gamma| = -20\log_{10} 0.15 = 16.5\text{dB}$

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based on the definition of S matrix

$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ - S_{12}V_2^-$

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$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ - S_{22}V_2^-$$

$$V_2^-(1 + S_{22}) = S_{21}V_1^+ \qquad : V_2^- = \frac{S_{21}}{1+S_{22}}V_1^+$$

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Dividing first equation by V_1^+: $\frac{V_1^-}{V_1^+} = S_{11} - S_{12}\left(\frac{V_2^-}{V_1^+}\right) =$

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For a short circuit termination at port 1, $\Gamma_{\text{port 2}} = -1 = \frac{V_2^-}{V_2^+}$: $V_2^+ = -V_2^-$

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Dividing first equation by V_1^+: $\frac{V_1^-}{V_1^+} = S_{11} - S_{12}\left(\frac{V_2^-}{V_1^+}\right) = S_{11} - S_{12}\frac{S_{21}}{1+S_{22}} = \Gamma$

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$$\Gamma = S_{11} - S_{12}\frac{S_{21}}{1+S_{22}} = 0.15 - \frac{0.85\angle 45° * 0.85\angle -45°}{0.2\angle 0°} = -0.452$$

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Return loss = $-20\ \log_{10}|\Gamma| = 6.9dB$

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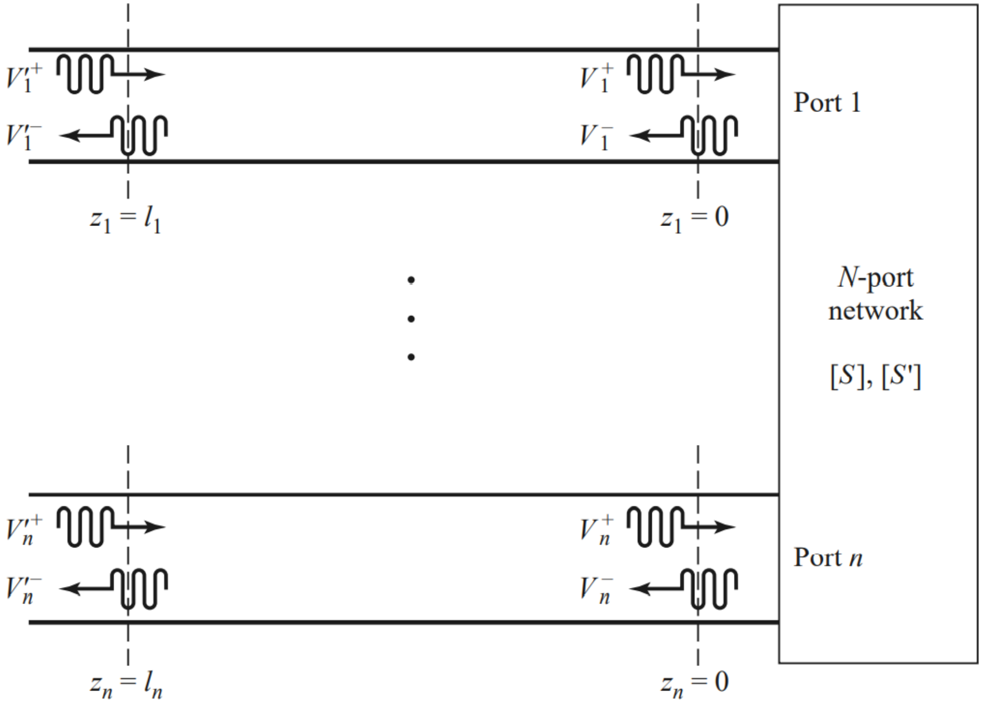
Return loss = $-20\ \log_{10}|\Gamma| = 6.9dB$

Reflection coefficient at port n is not equal to S_{nn} unless all ports are matched.

Similarly transmission coefficient from port m to port n is not equal to S_{nm} unless all other ports are matched.

7. Shift in reference plane

- S parameters – Relate incident and reflected amplitudes from network(magnitude and phase) of traveling waves : Phase reference planes must be specified for each network



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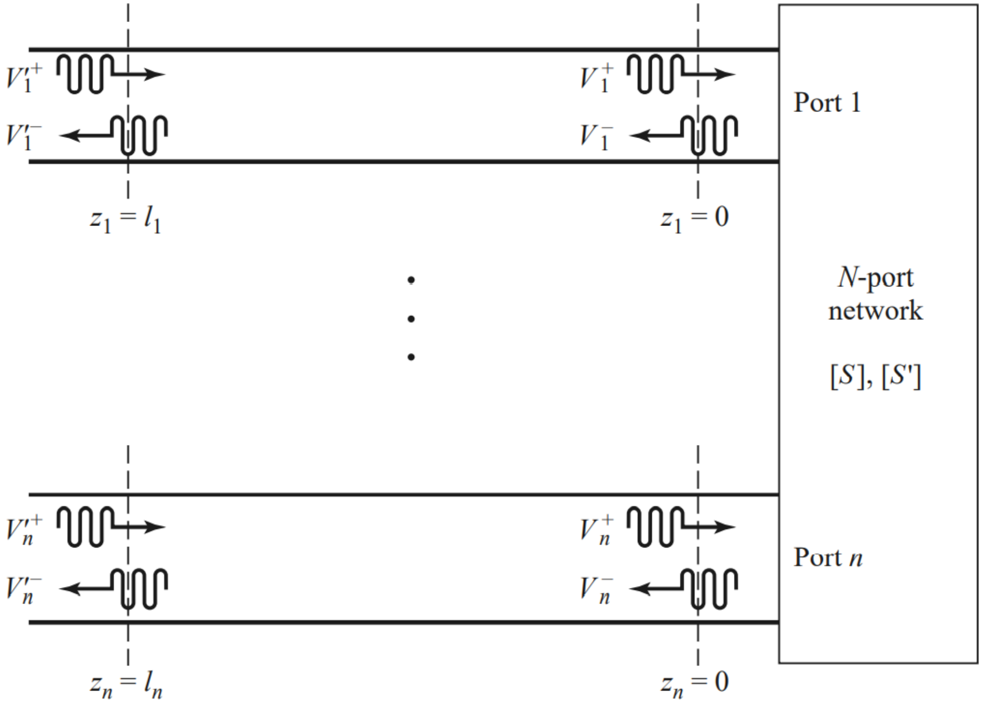

- S parameters – Relate incident and reflected amplitudes from network(magnitude and phase) of traveling waves : Phase reference planes must be specified for each network
- To show: Effect of moving reference planes from original locations on scattering parameters

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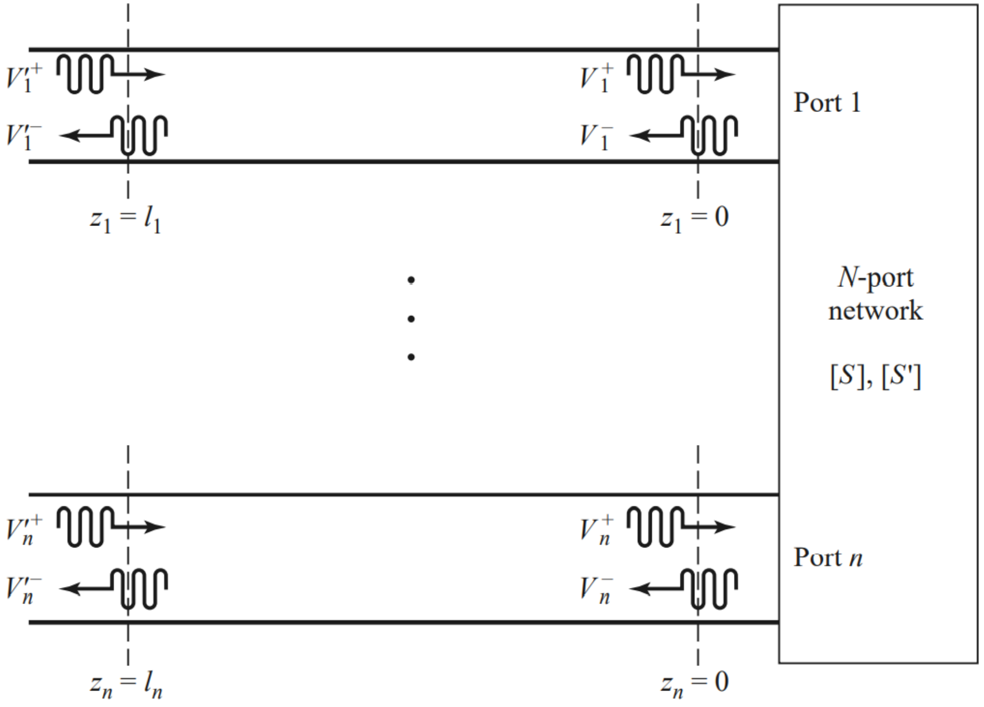
- N port network – Original termination planes at $z_n = 0$ for n th port.
- S matrix $[S]$



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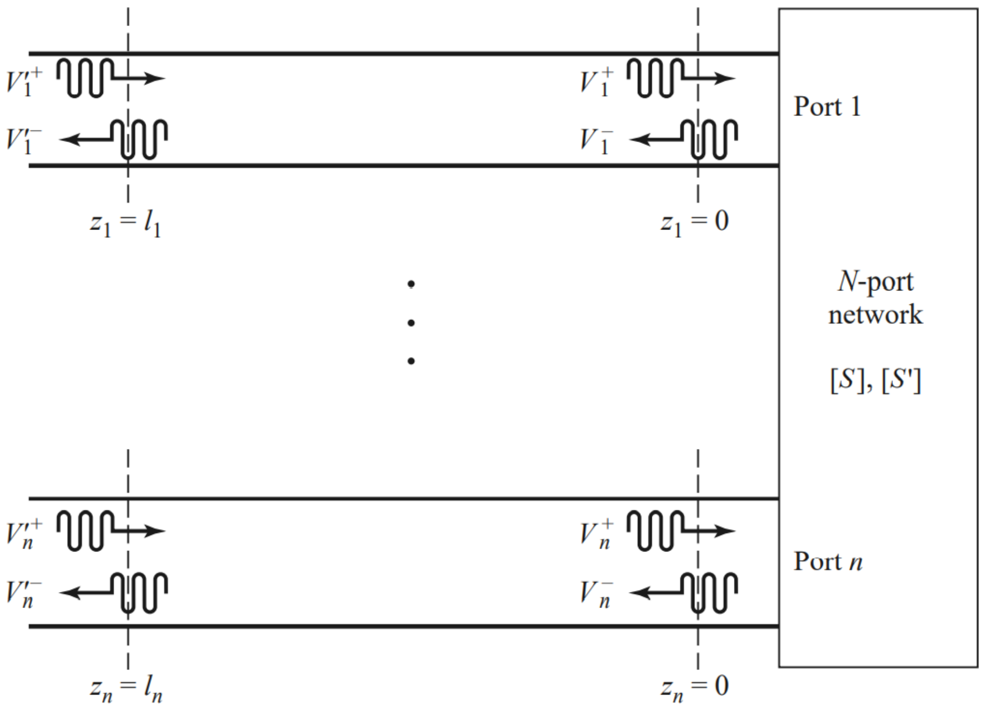
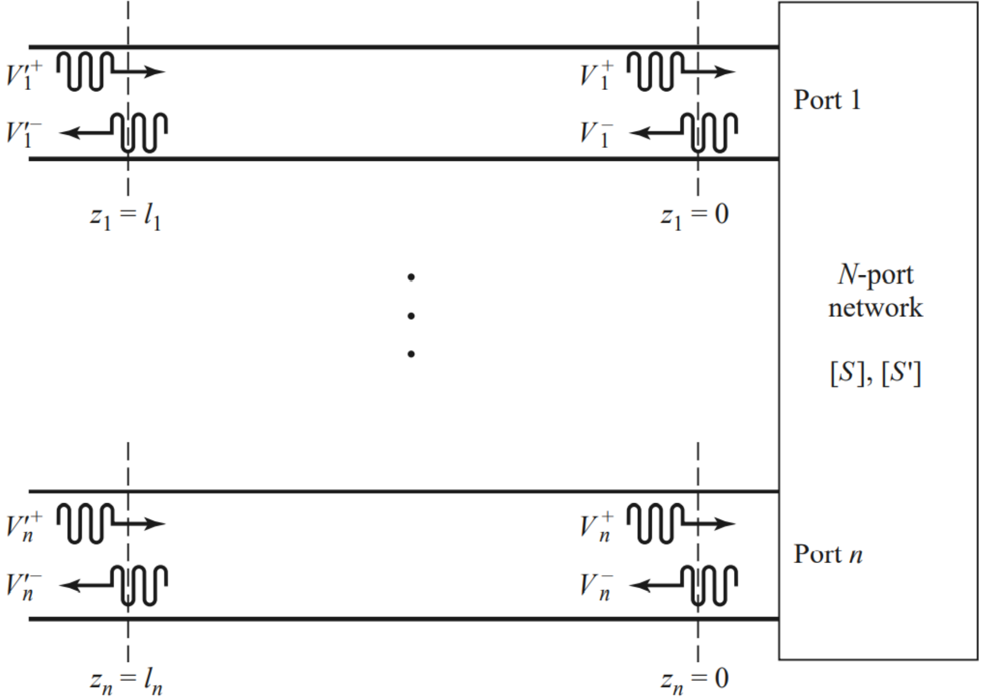


- S parameters – Relate incident and reflected amplitudes from network(magnitude and phase) of traveling waves : Phase reference planes mus be specified for each network
- To show: Effect of moving reference planes from original locations on scattering parameters
- N port network – Original termination planes at $z_n = 0$ for n th port.
- S matrix $[S]$
- Consider reference planes at $z_n = l_n$ for nth port.
  - > New scattering matrix is formed $[S']$

7. Shift in reference plane

- $[V^-] = [S][V^+]$ Original terminal planes at $z_n = 0$

  $[V'^-] = [\boldsymbol{S'}][V'^+]$ new terminal planes at $z_n = l_n$



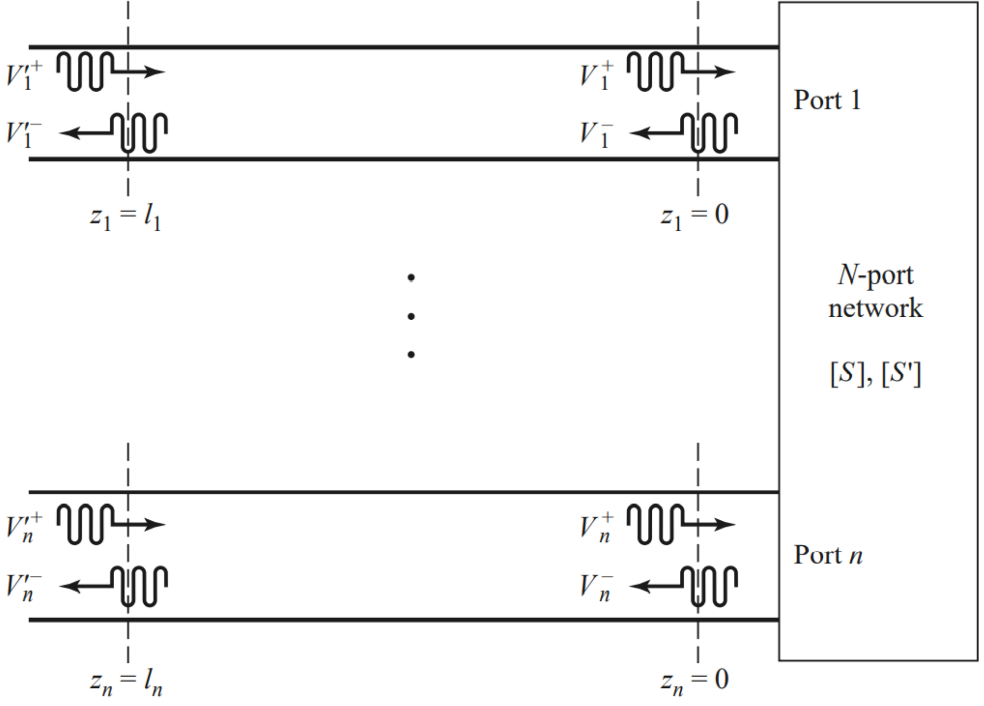
7. Shift in reference plane

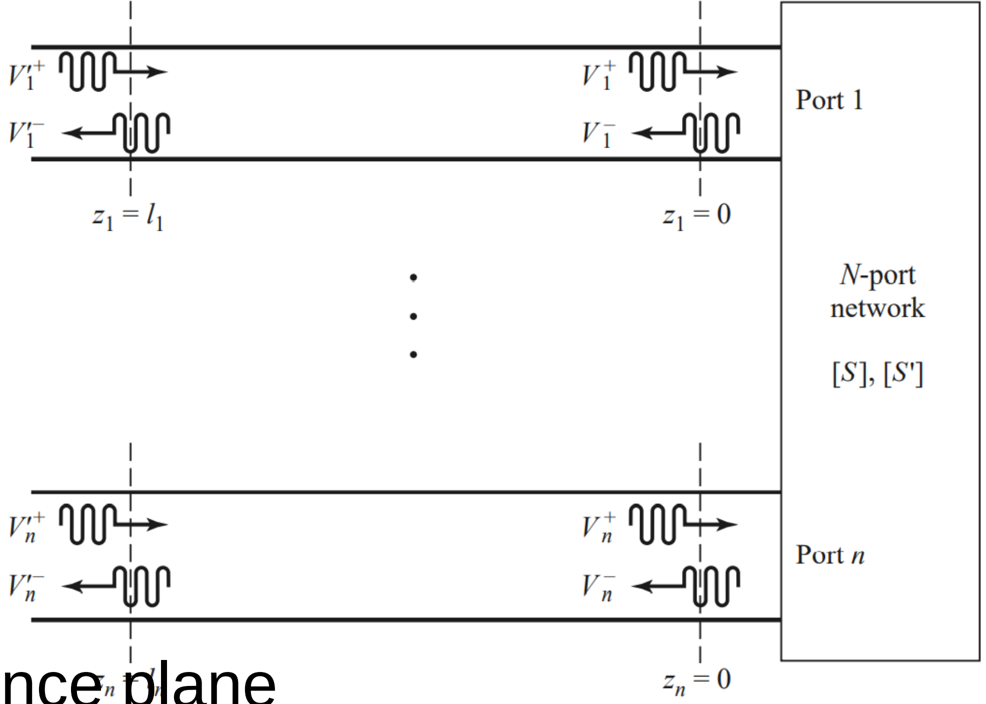
- $[V^-] = [S][V^+]$ Original terminal planes at $z_n = 0$

  $[V'^-] = [\boldsymbol{S'}][V'^+]$ new terminal planes at $z_n = l_n$

- Lossless transmission line

$$V_n'^+ = V_n^+ e^{j\theta_n} \qquad V_n'^- = V_n^- e^{-j\theta_n}$$





7. Shift in reference plane

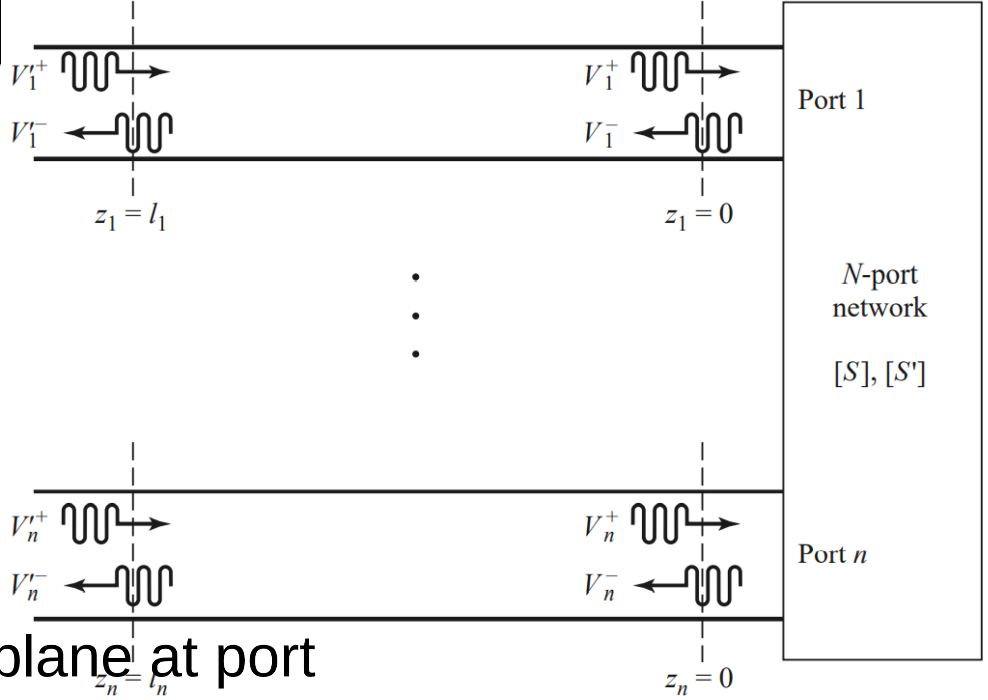
- $[V^-] = [S][V^+]$ Original terminal planes at $z_n = 0$

  $[V'^-] = [\boldsymbol{S'}][V'^+]$ new terminal planes at $z_n = l_n$

- Lossless transmission line

$$V_n'^+ = V_n^+ e^{j\theta_n} \qquad\qquad V_n'^- = V_n^- e^{-j\theta_n}$$

- $\theta_n = \beta_n l_n$ is electrical length of outward shift of reference plane at port n.



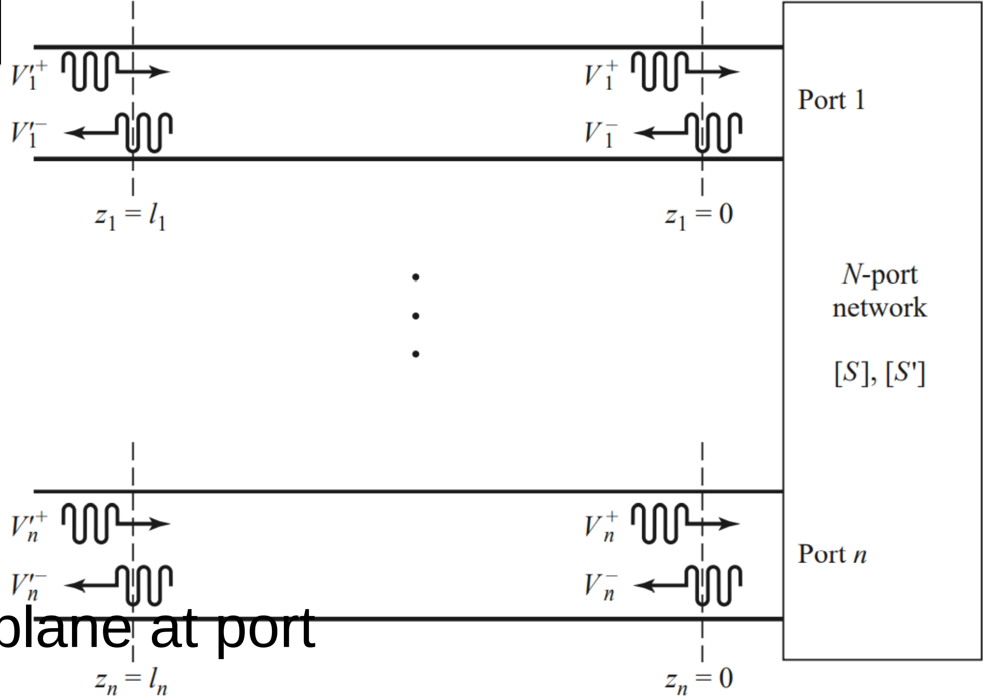
7. Shift in reference plane

- $[V^-] = [S][V^+]$ Original terminal planes at $z_n = 0$

  $[V'^-] = [S'][V'^+]$ new terminal planes at $z_n = l_n$
- Lossless transmission line

$$V_n'^+ = V_n^+ e^{j\theta_n} \qquad V_n'^- = V_n^- e^{-j\theta_n}$$

- $\theta_n = \beta_n l_n$ is electrical length of outward shift of reference plane at port n.
- $$\begin{bmatrix} e^{j\theta_1} & & & 0 \\ & e^{j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{j\theta_n} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$
- Multiplying inverse of first matrix

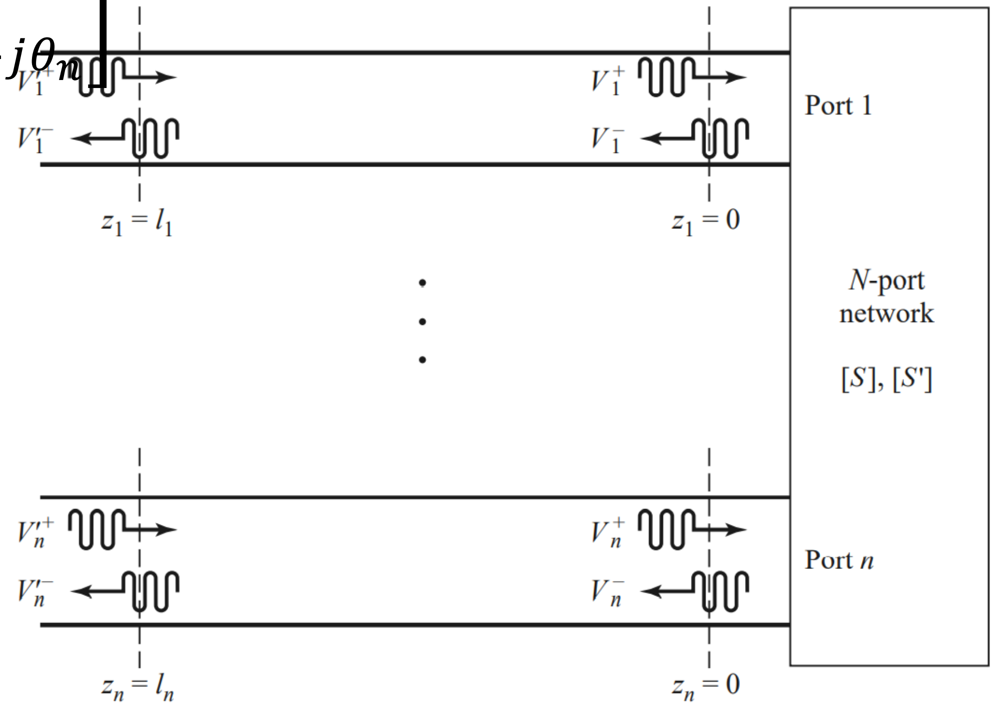


7. Shift in reference plane

- $[V^-] = [S][V^+]$ Original terminal planes at $z_n = 0$
  $[V'^-] = [S'][V'^+]$ new terminal planes at $z_n = l_n$
- Lossless transmission line
  $$V_n'^+ = V_n^+ e^{j\theta_n} \qquad V_n'^- = V_n^- e^{-j\theta_n}$$
- $\theta_n = \beta_n l_n$ is electrical length of outward shift of reference plane at port n.
- $$\begin{bmatrix} e^{j\theta_1} & & & 0 \\ & e^{j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{j\theta_n} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$$
- Multiplying inverse of first matrix
- $$[V'^-] = \underbrace{\begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix}} [V'^+]$$

7. Shift in reference plane

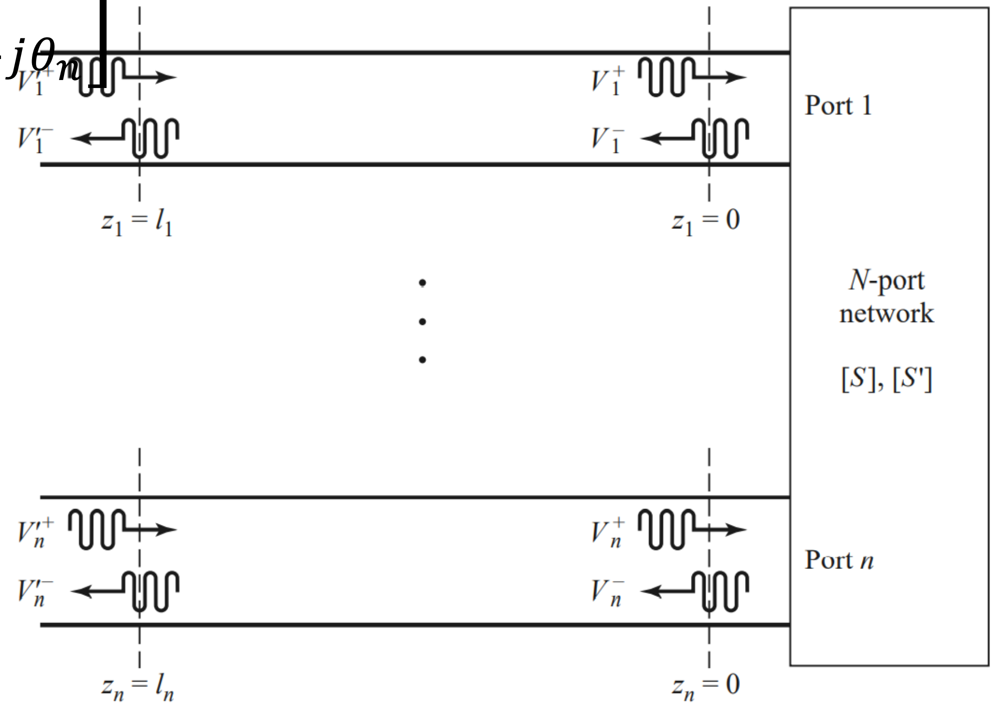
- $[S'] = \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^{+}]$



7. Shift in reference plane

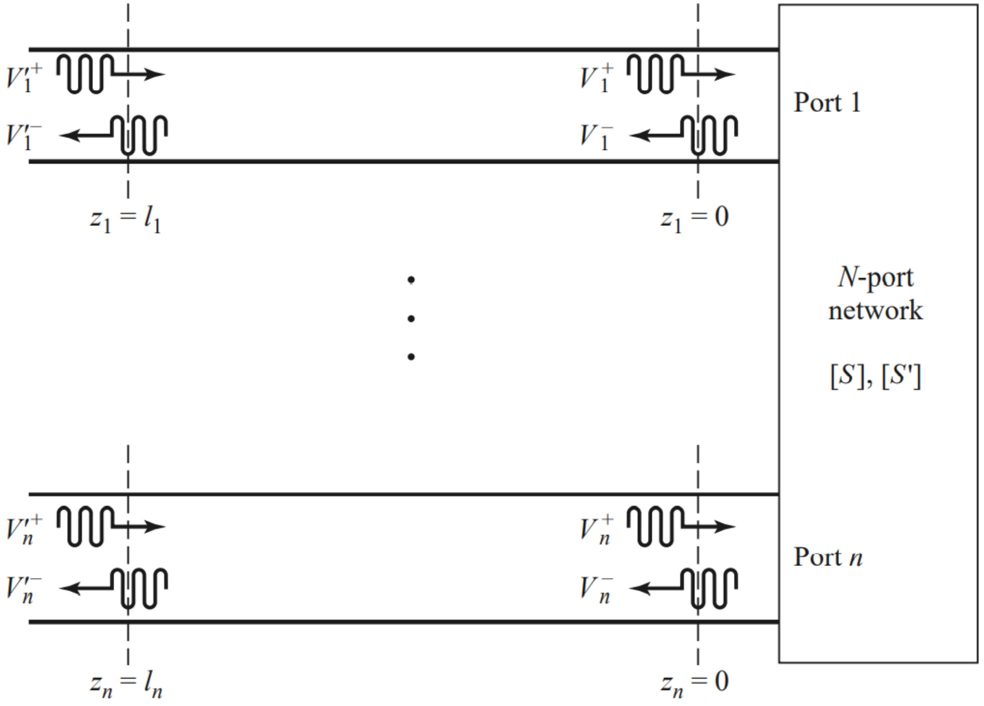
- $[S'] = \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^+]$

- $S'_{nn} = e^{-2j\theta_n} S_{nn}$



7. Shift in reference plane

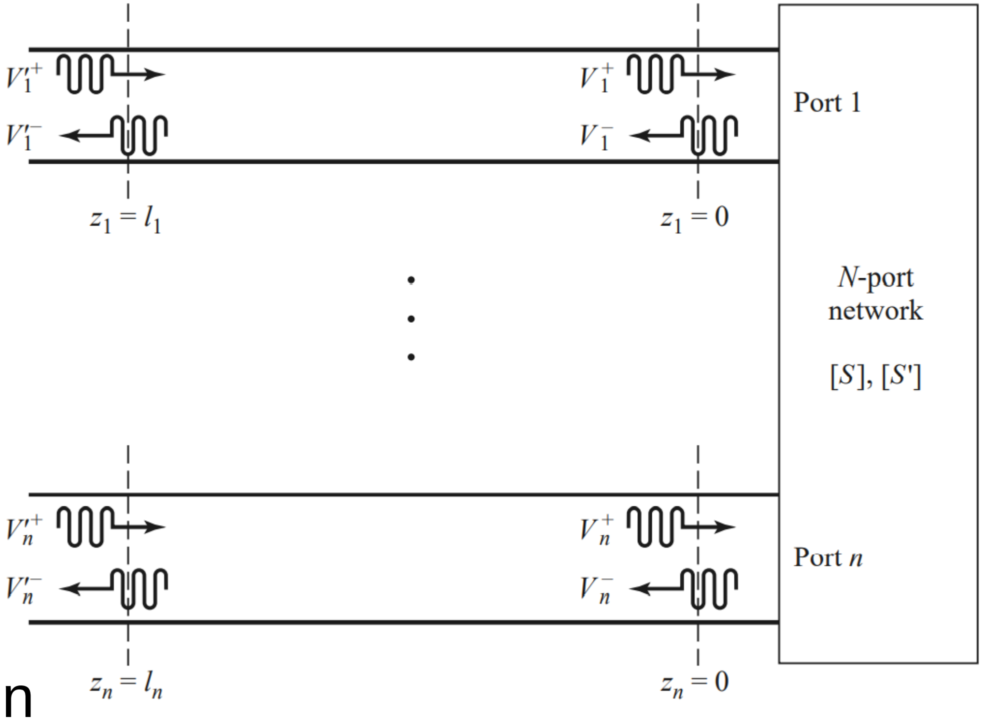
- $[S'] = \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^{+}]$



- $S'_{nn} = e^{-2j\theta_n} S_{nn}$
- Phase of S_{nn} is shifted by twice the electrical length of the shift in terminal plane
- The wave travels twice over the length upon incident and reflected.

7. Shift in reference plane

- $[S'] = \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \cdots & \\ 0 & & & e^{-j\theta_n} \end{bmatrix} [V'^{+}]$



- $S'_{nn} = e^{-2j\theta_n} S_{nn}$
- Phase of S_{nn} is shifted by twice the electrical length of the shift in terminal plane
- The wave travels twice over the length upon incident and reflected.

This is similar to change in reflection coefficient due to shift in reference plane