6.2 Wilkinson Power divider

Module:6 Microwave Passive circuits

Course: BECE305L – Antenna and Microwave Engineering

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Assistant Professor - SENSE

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Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Module:6 Microwave Passive circuits 7 hours

- T junction and resistive power divider, Wilkinson power divider, branch line coupler (equal & unequal), Rat Race Coupler, Filter design: Low pass filter (Butterworth and Chebyshev) - Richards transformation and stepped impedance methods.
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- Source of the contents: Pozar

6.4 Need for Wilkinson Power divider

- Lossless T junction divider : Not matched at all ports
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- Wilkinson power divider:
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  - Lossless when output ports are matched
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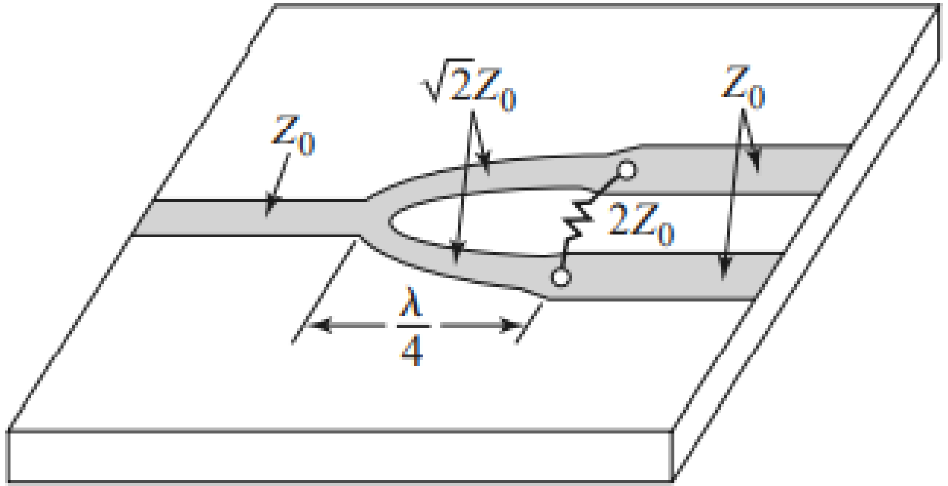
  - All ports matched
  - Isolation between output ports
  - Arbitrary power division is possible

6.4 Need for Wilkinson Power divider

- Wilkinson power divider: (Initially 3dB case – equal split)
  <u>Microstrip transmission line/stripline</u>



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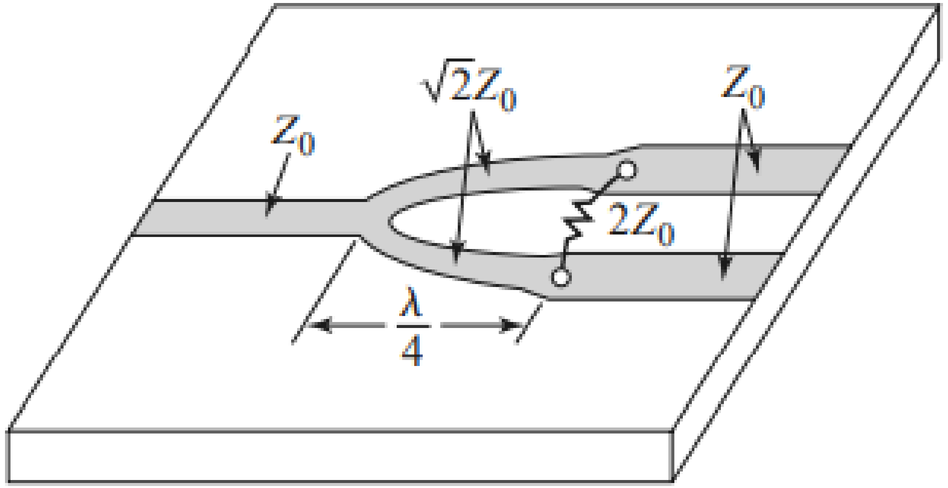
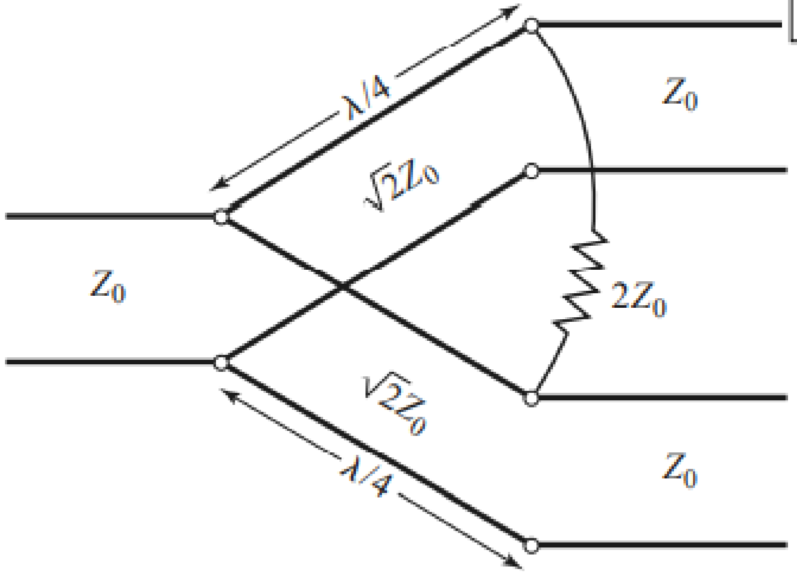
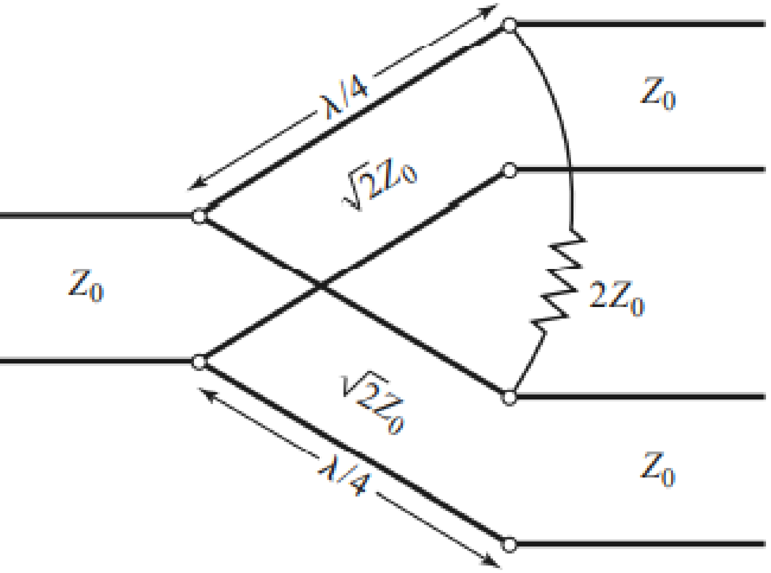


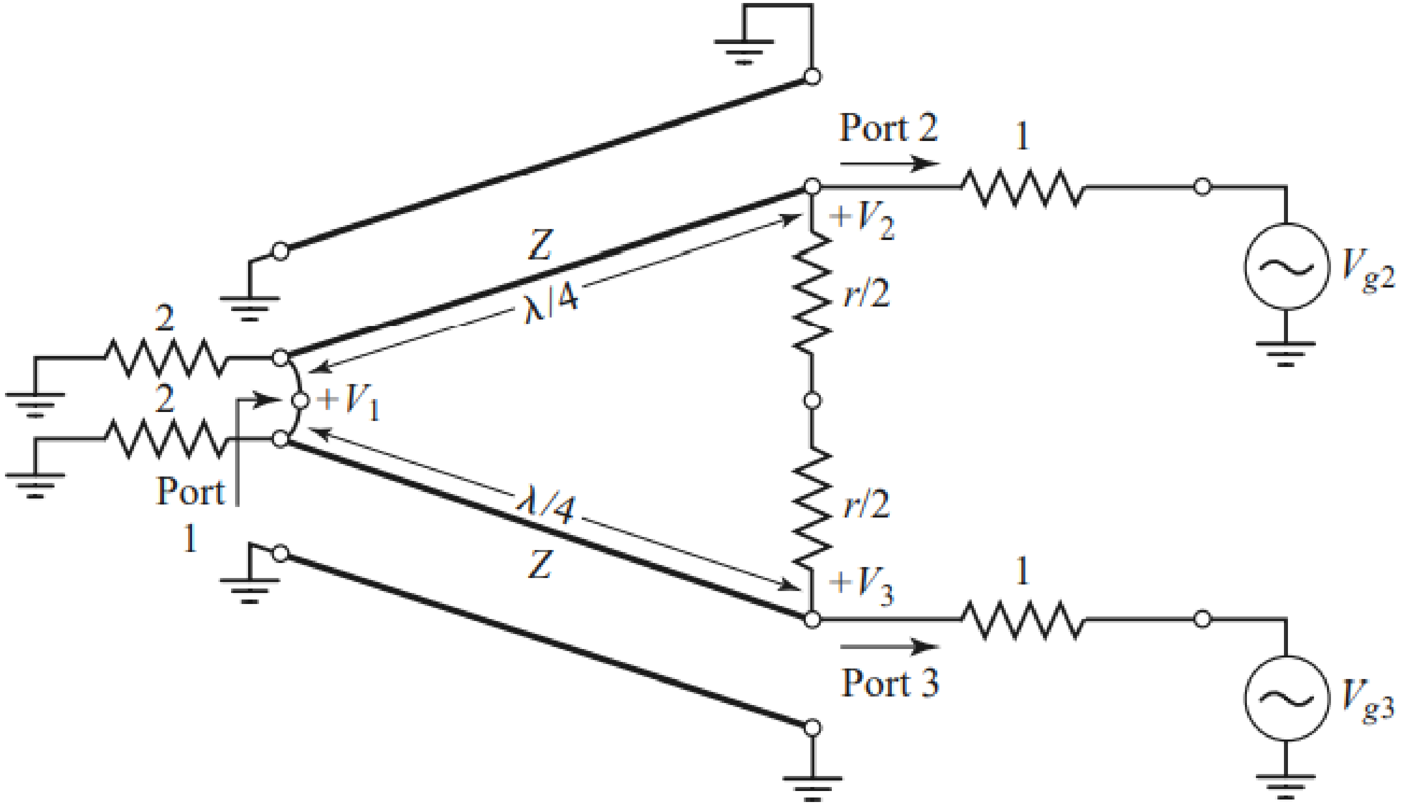
- Transmission line equivalent circuit:



6.4a Construction of Wilkinson Power divider

- (Initially 3dB case – equal split) **Symmetric across the midplane**
  Two source resistors of $2Z_0$ combine in parallel to give resistor value: $\mathbf{1Z_0}$
  <u>Quarter wave lines</u> have **<u>normalized characteristic impedance Z</u>**.
-





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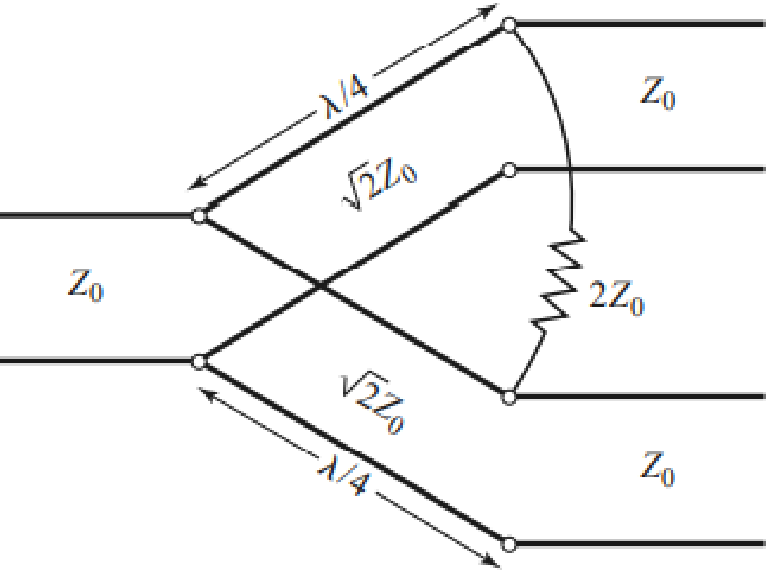

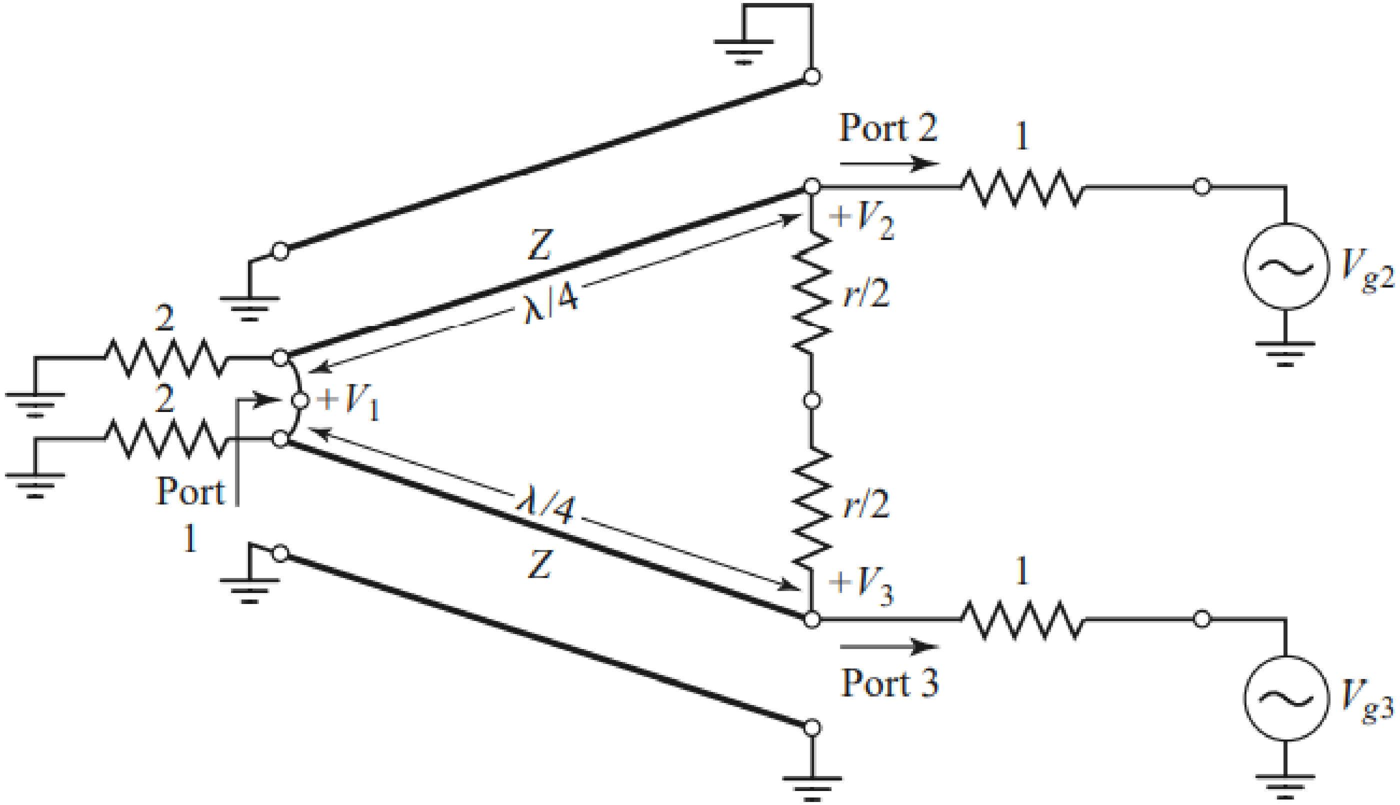

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- (Initially 3dB case – equal split) Symmetric across the midplane
  Two source resistors of $2Z_0$ combine in parallel to give resistor value: $\mathbf{1Z_0}$
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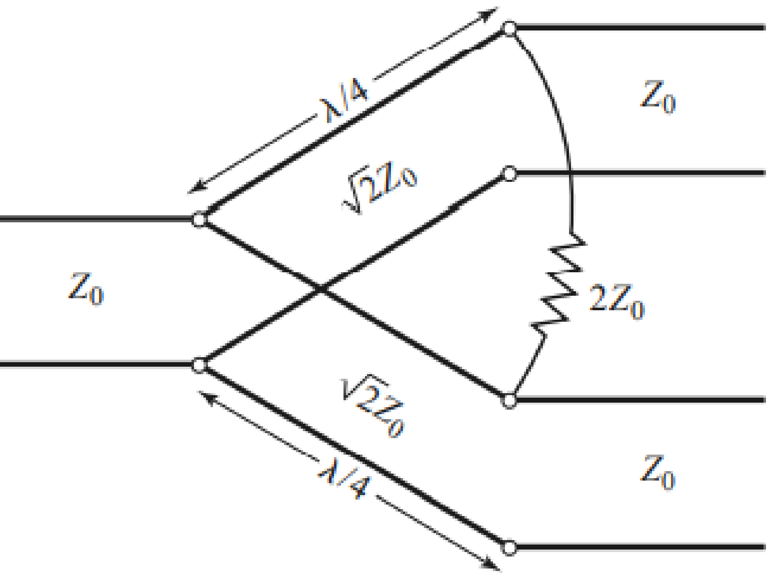
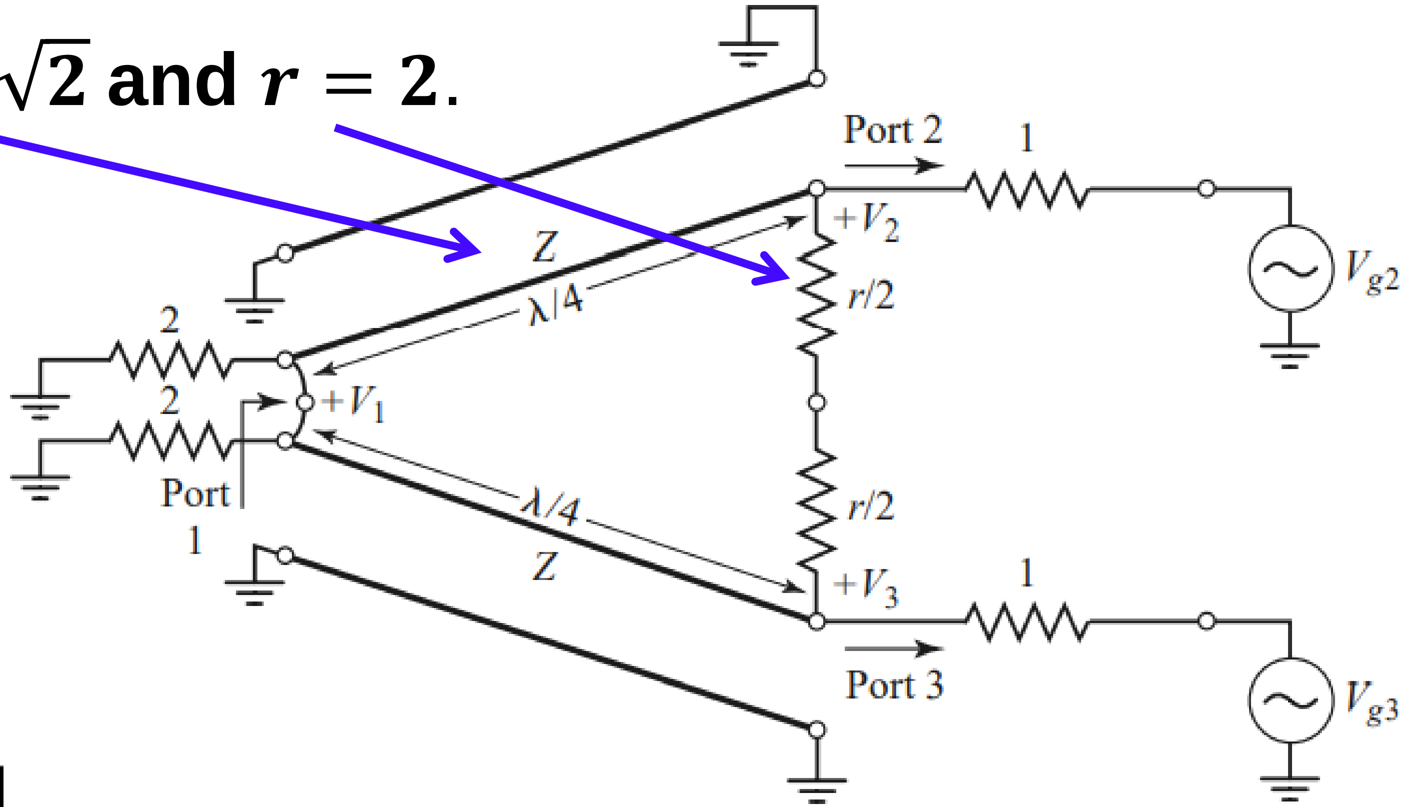
- Shunt resistor has normalized value of r.
- **For <u>equal power divider</u>, $\boldsymbol{Z = \sqrt{2}}$ and $\boldsymbol{r = 2}$.**





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6.4b Modes of operation in Wilkinson Power divider

- Two separate modes of excitation for circuit
- Even mode: $V_{g2} = V_{g3} = 2V_0$
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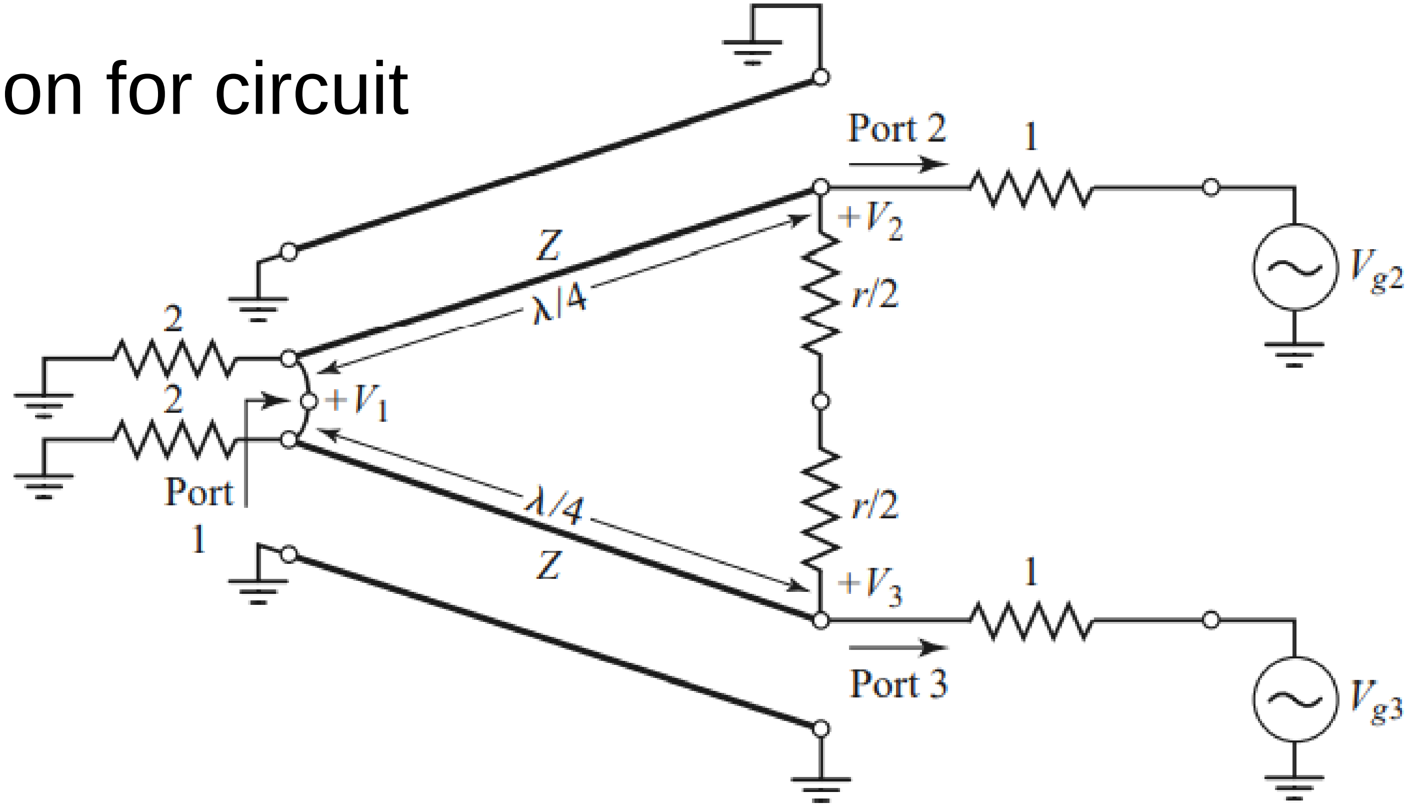

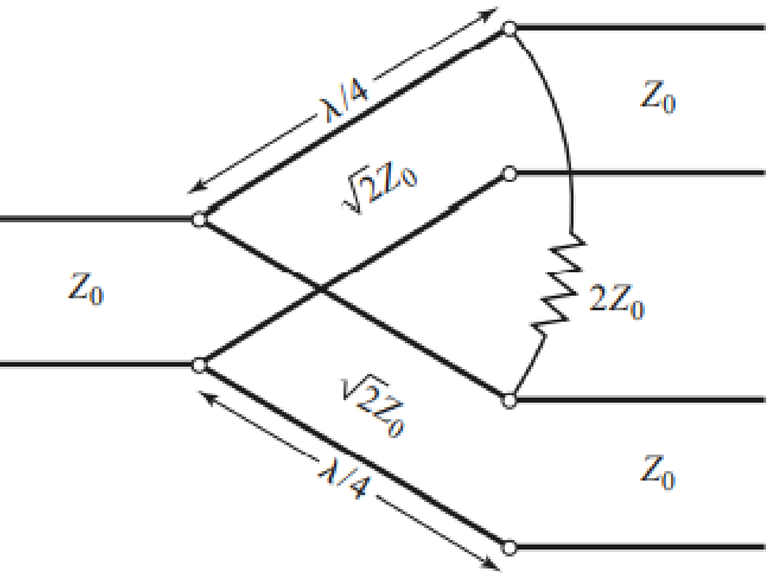

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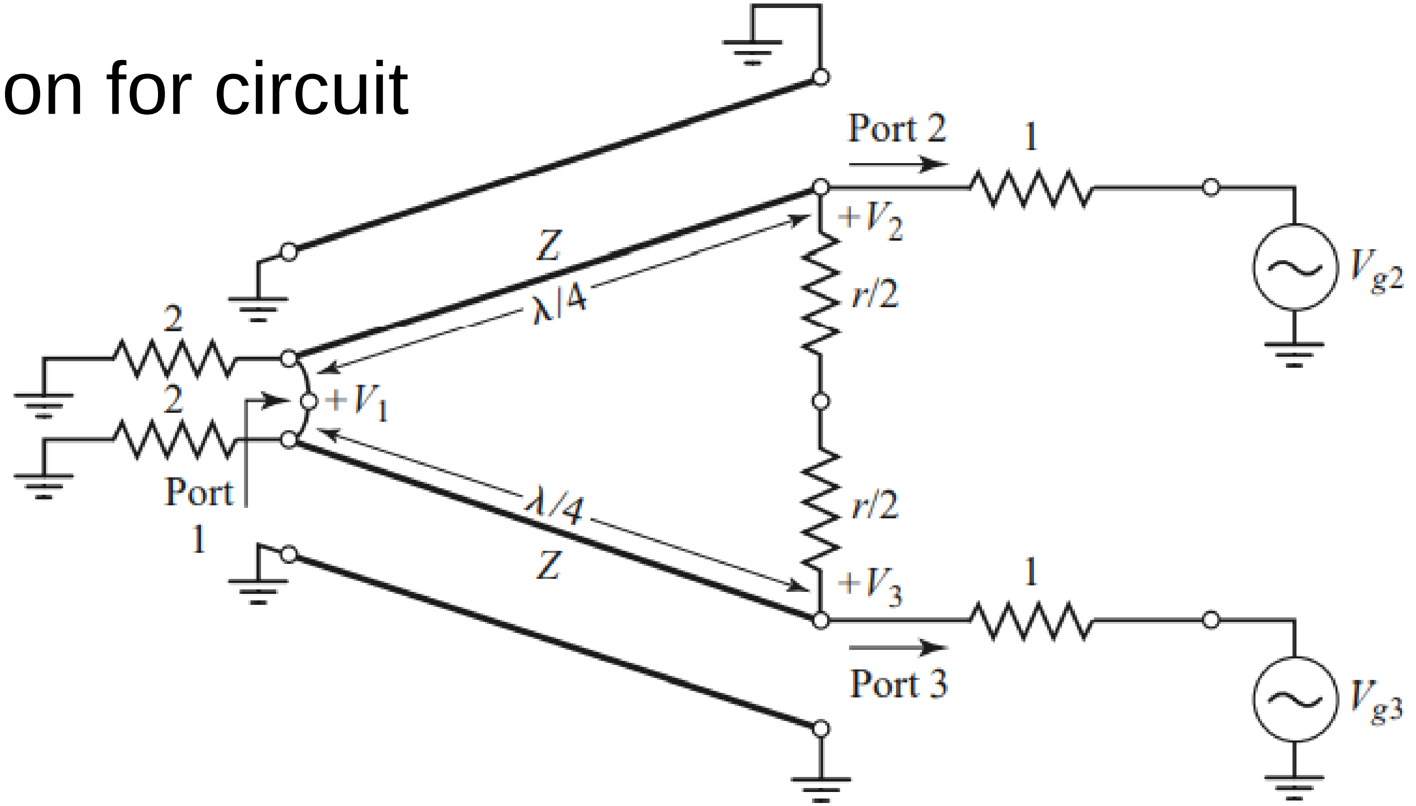

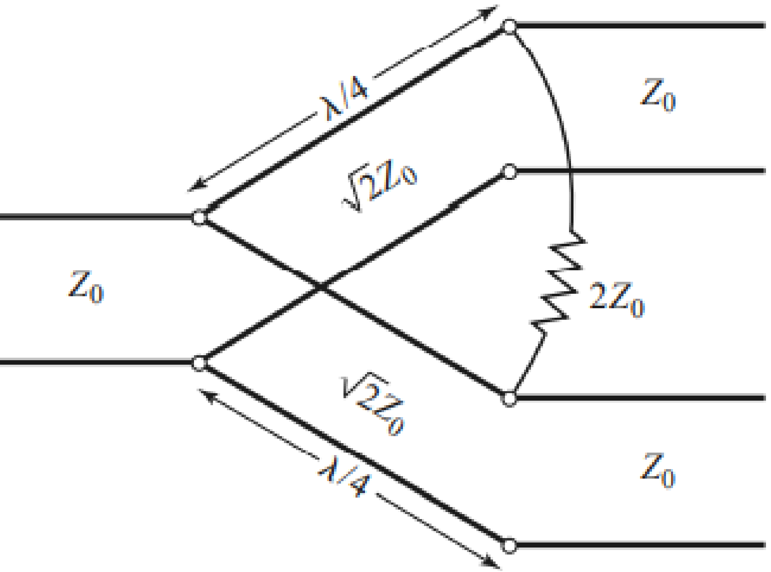

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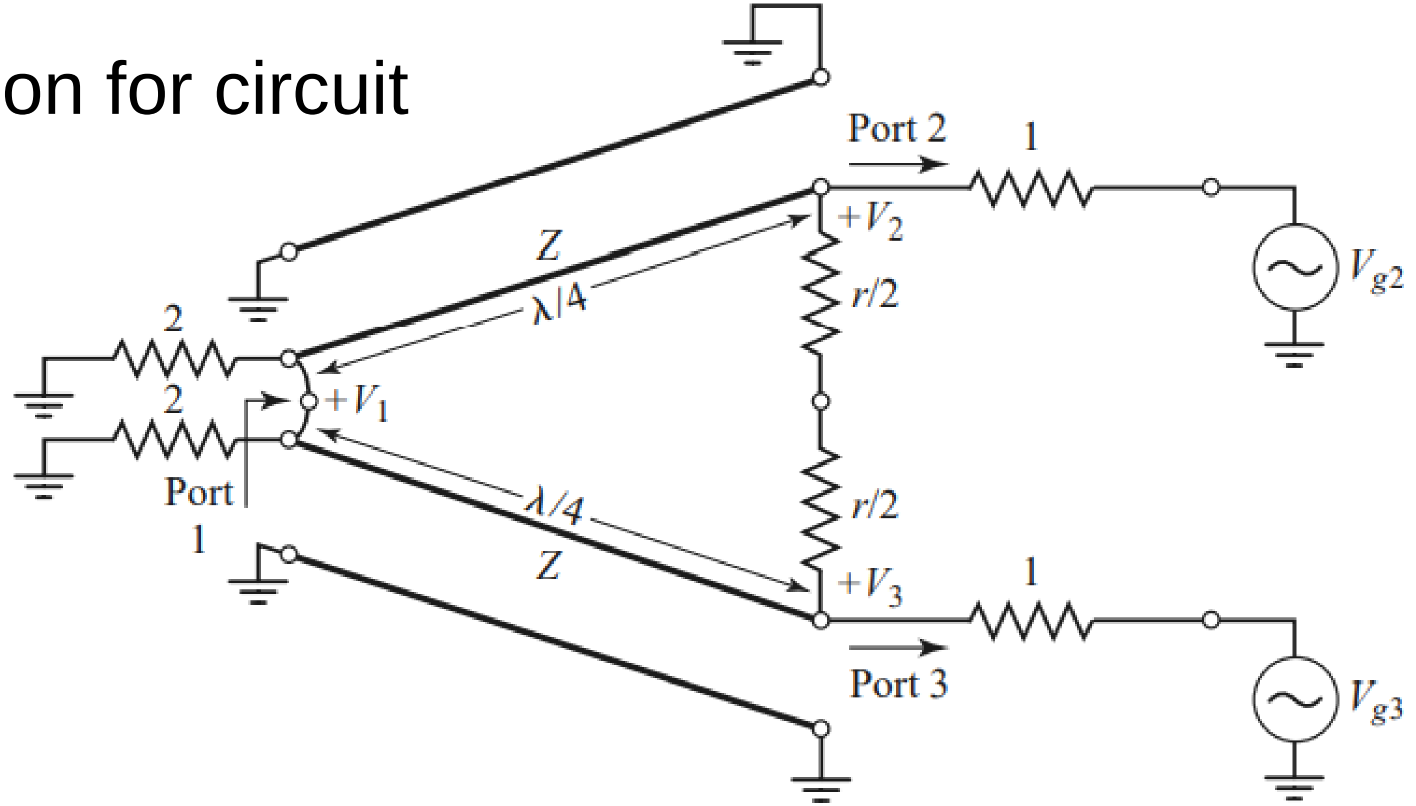
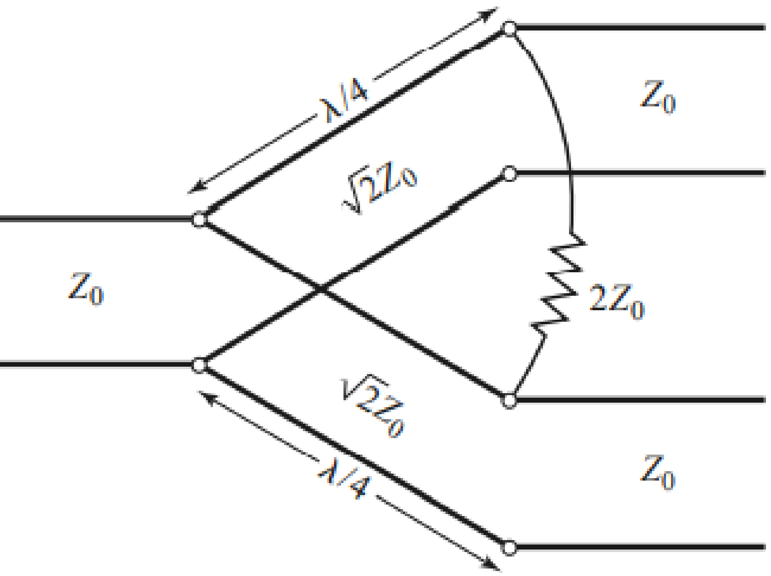
- Two separate modes of excitation for circuit
- **Even mode**: $V_{g2} = V_{g3} = 2V_0$
- **Odd mode**: $V_{g2} = -V_{g3} = 2V_0$
- **Superposition**: $V_{g2} = 4V_0$
  $V_{g3} = 0$





6.4c **Even mode** in Wilkinson Power divider

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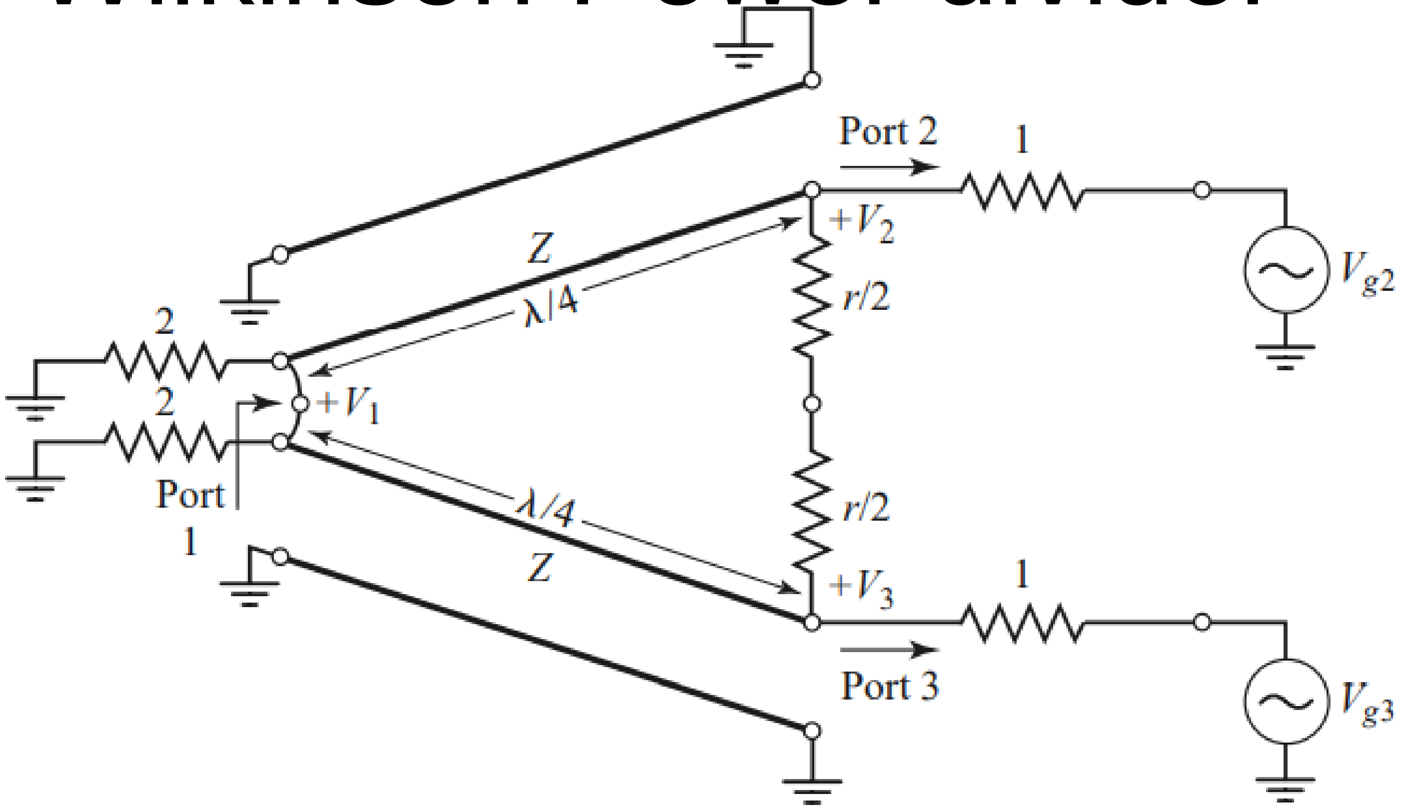

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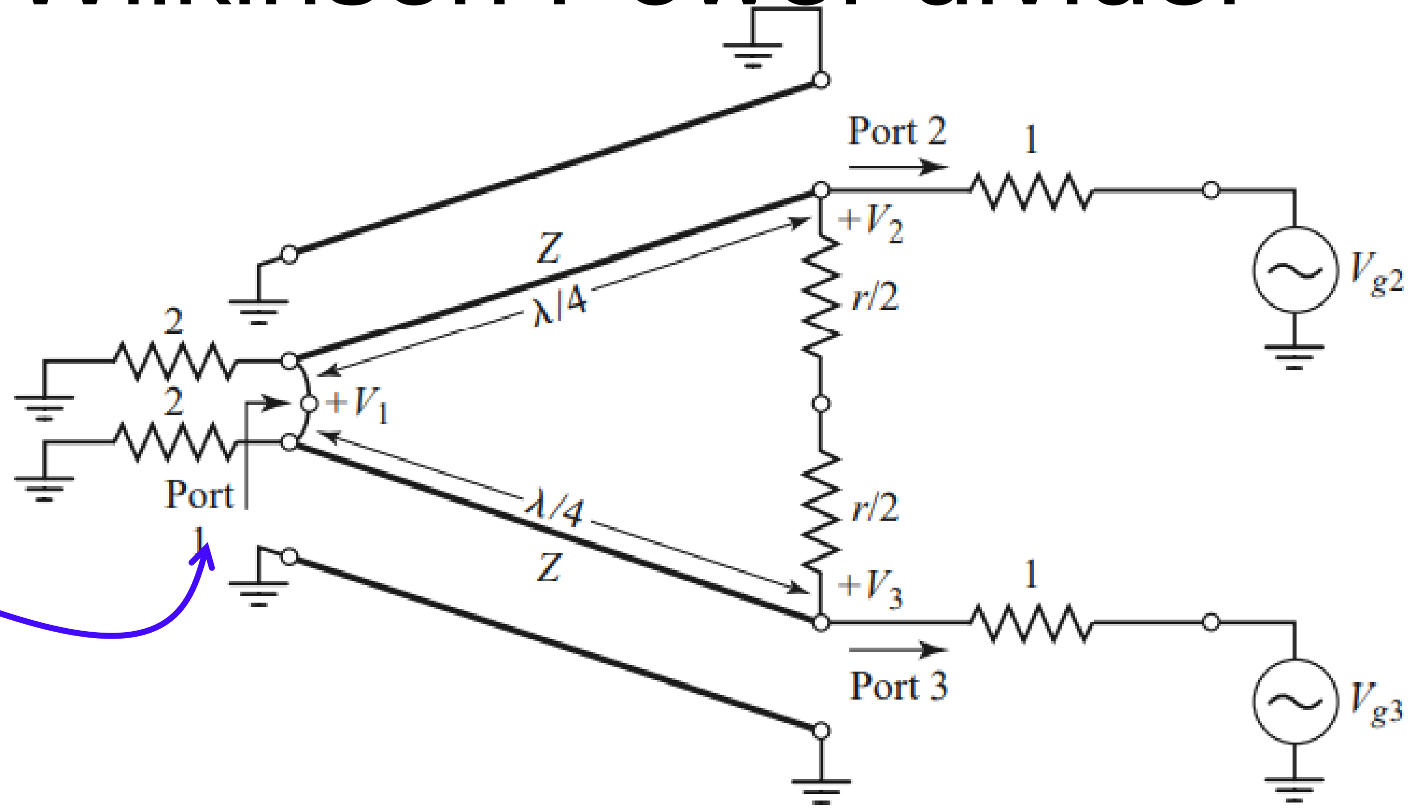
- **Voltage at port 1 is zero** as voltage will be equal after λ_4 long transmission line.
- <u>(No current in circuit)</u>



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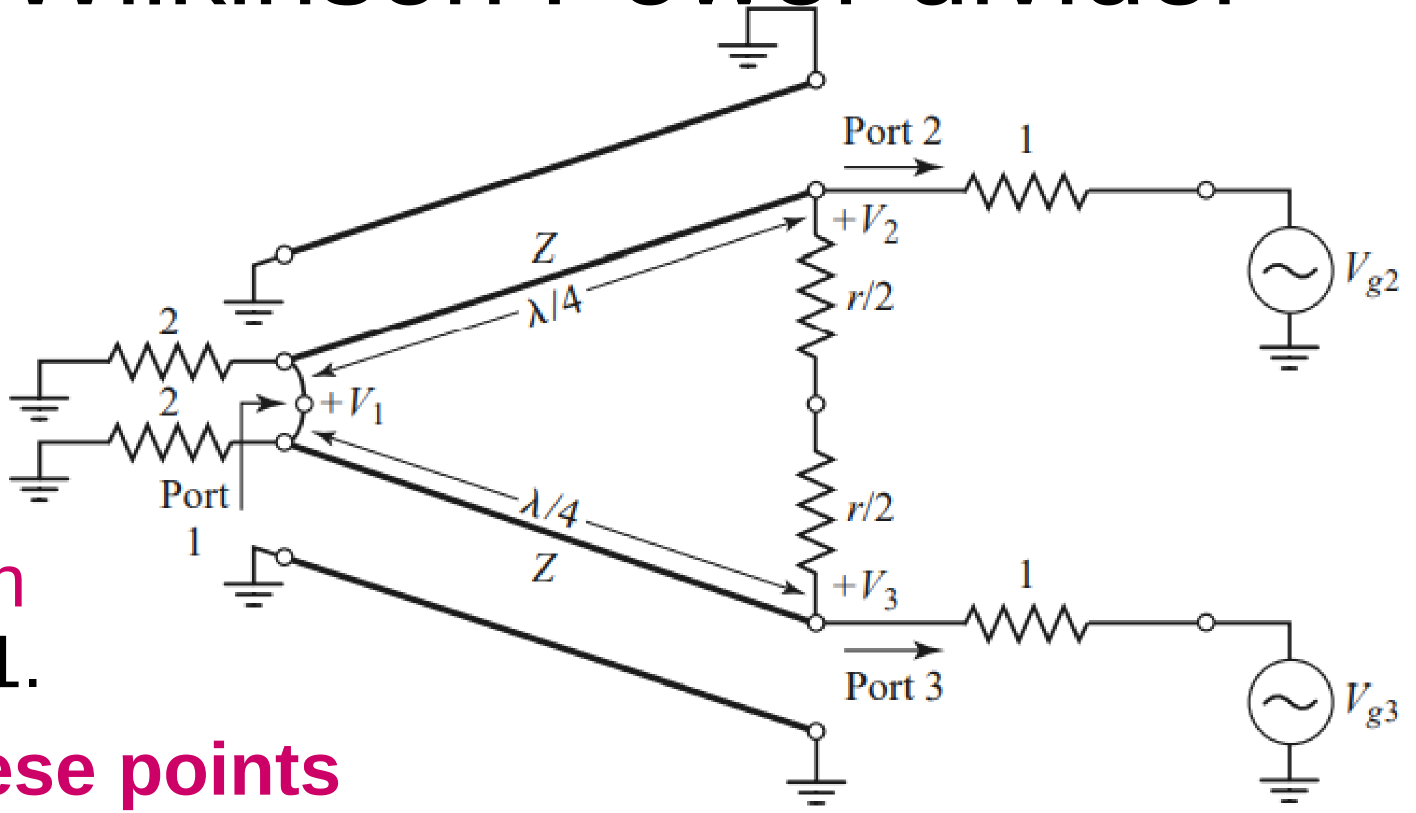
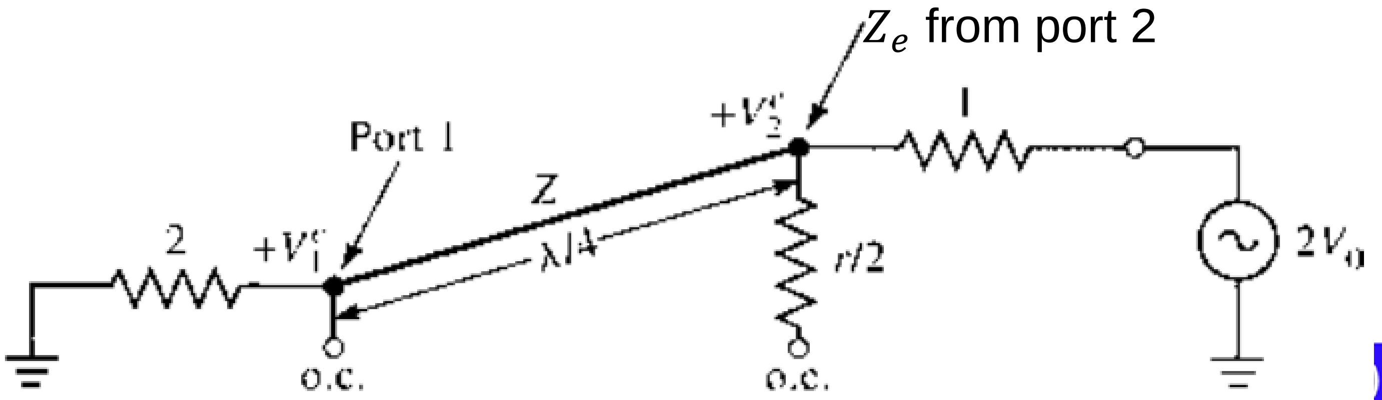
- Equivalent: **Open circuit at these points**





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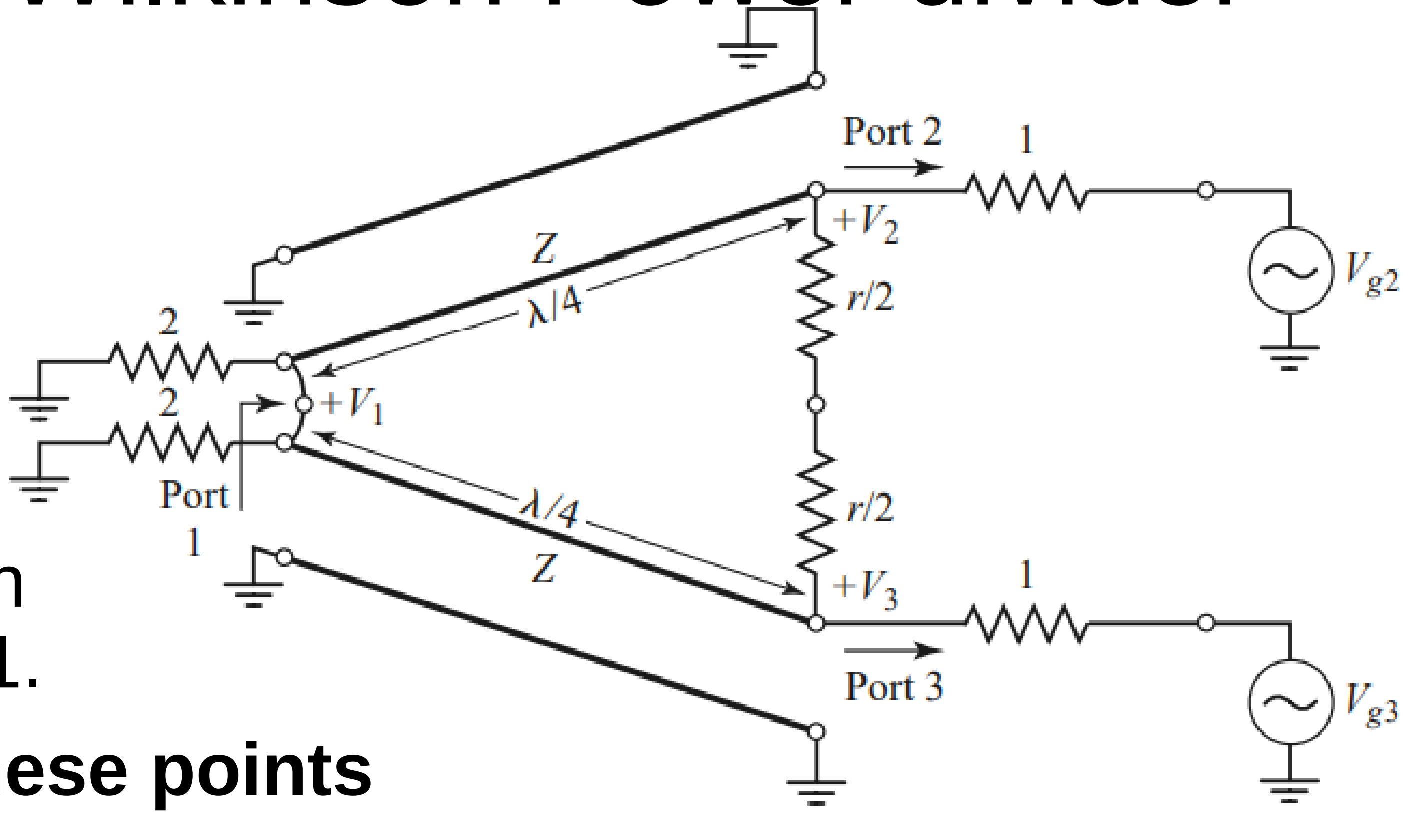

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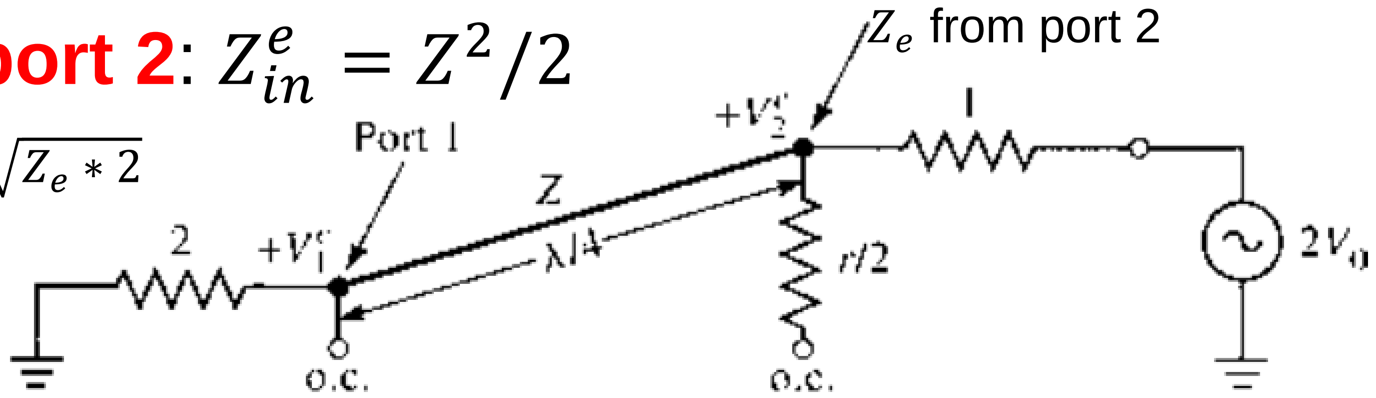
- **Impedance Z_e from port 2**: $Z_{in}^e = Z^2/2$

Quarter wave transformer $Z = \sqrt{Z_e * 2}$



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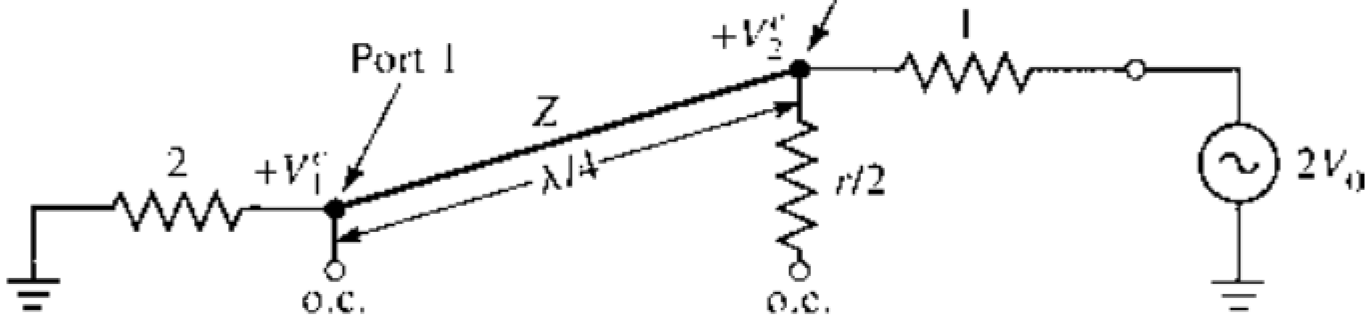

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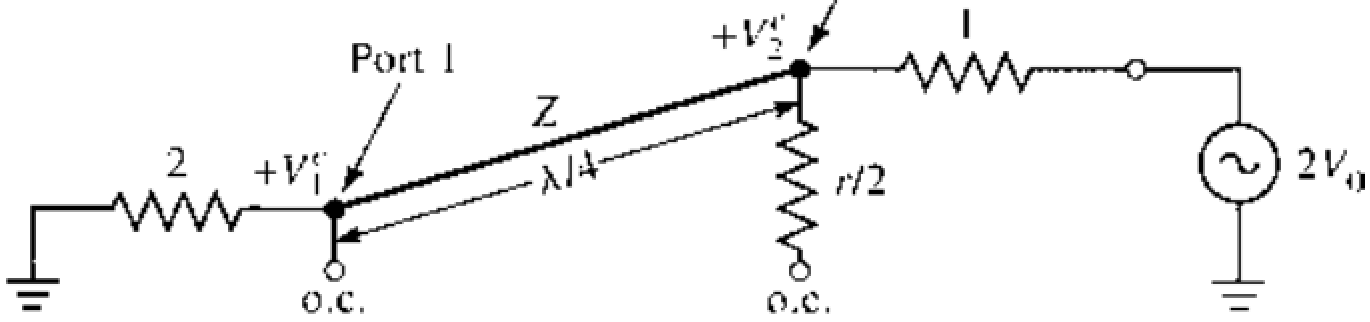

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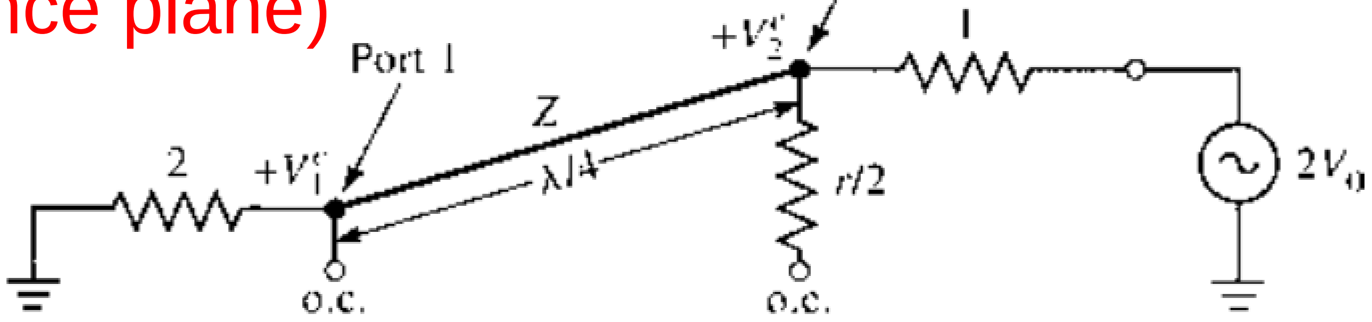
If <u>$\boldsymbol{x = 0}$ **at port 1**</u>, and <u>$x = -\lambda/4$ at port 2</u>,
$V(x) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$
(Phase shift due to shift in reference plane)



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$$V_2^e = V\left(-\frac{\lambda}{4}\right) = V^+[0 + j + \Gamma(0 - j)]$$

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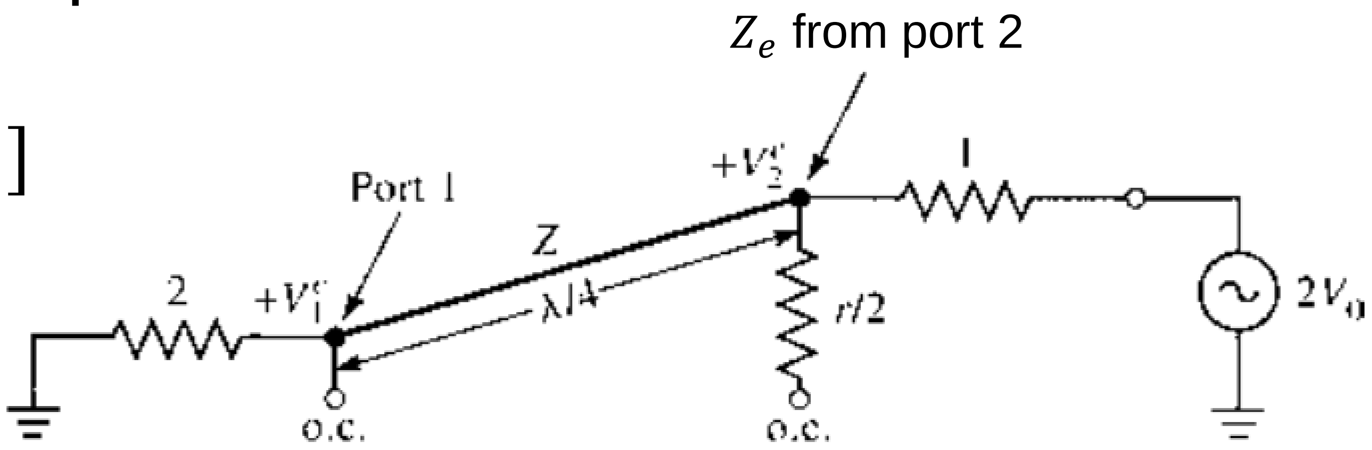

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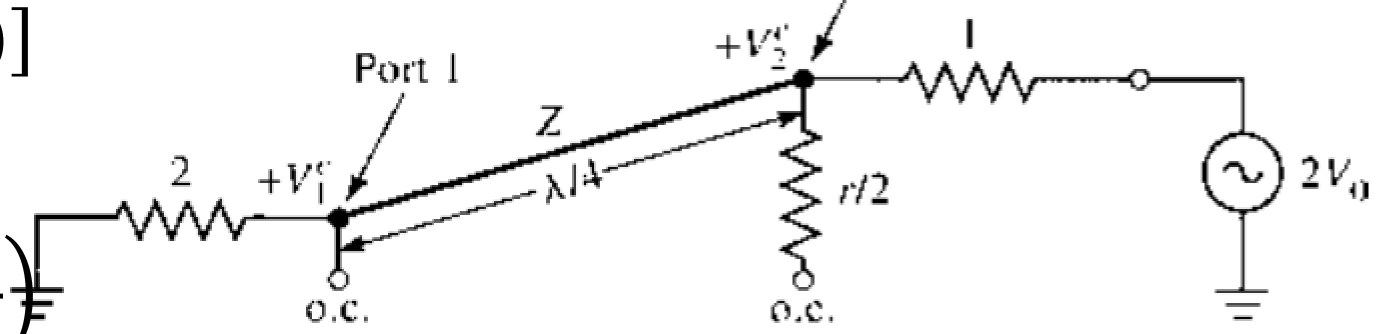
$$V_1^e = V(0) = jV^+(1 + \Gamma) = jV_0\left(\frac{\Gamma+1}{\Gamma-1}\right)$$

Z_e from port 2



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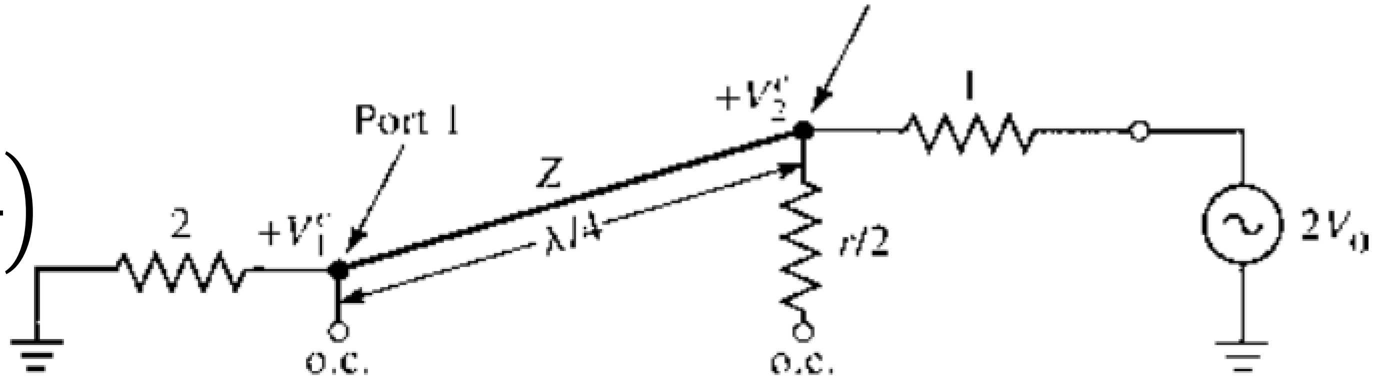


$$V_2^e = jV^+(1-\Gamma) = V_0$$

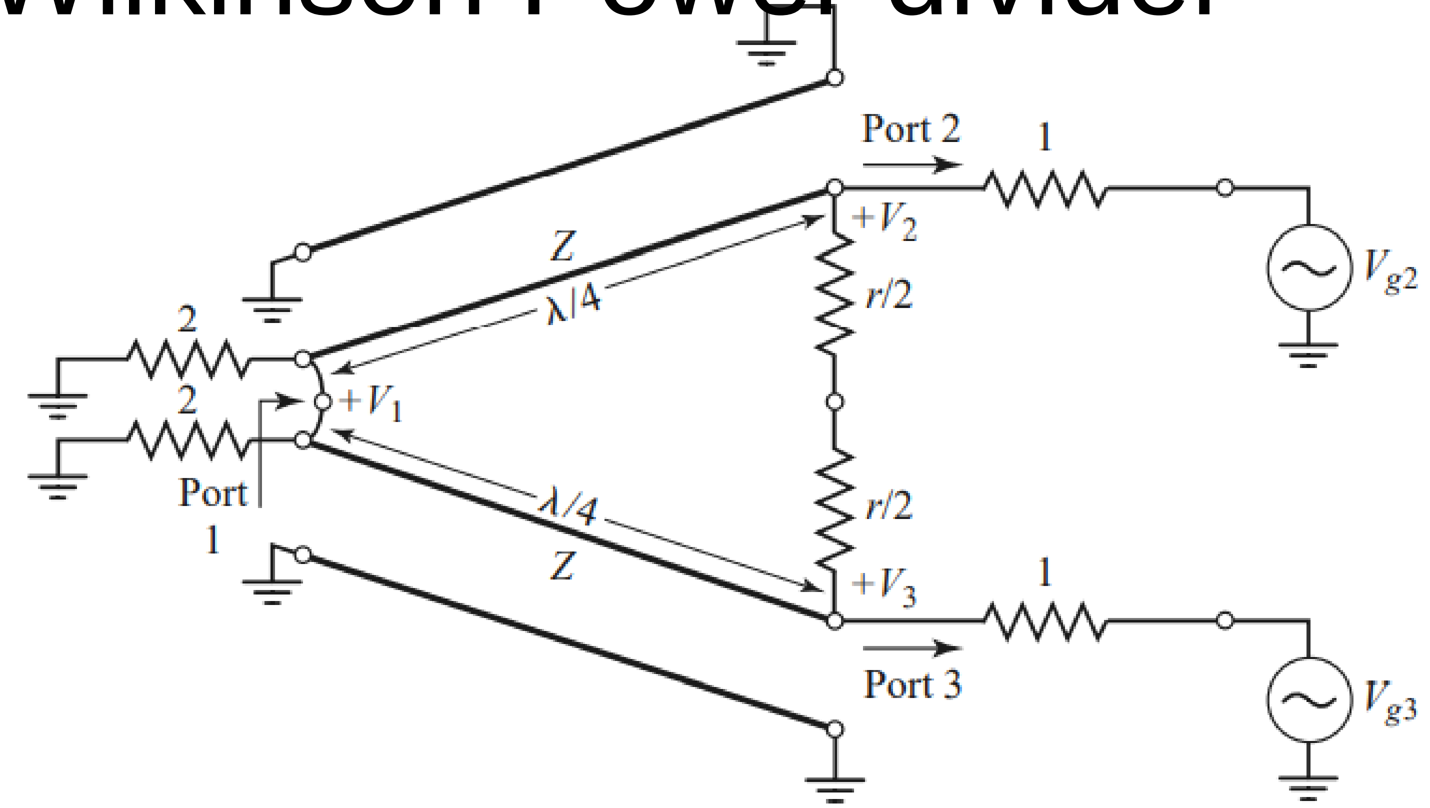
$$V_1^e = V(0) = jV^+(1+\Gamma) = jV_0\left(\frac{\Gamma+1}{\Gamma-1}\right)$$

Reflection coefficient at port 1

$\Gamma = \frac{2-Z}{2+Z} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$ and $V_1^e = -jV_0\sqrt{2}$ Note: $\Gamma = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}$

6.4d Odd mode in Wilkinson Power divider

- **Odd mode**: $V_{g2} = -V_{g3} = 2V_0$
- Port 2: $V_2^0 = -V_3^0$: Port 3



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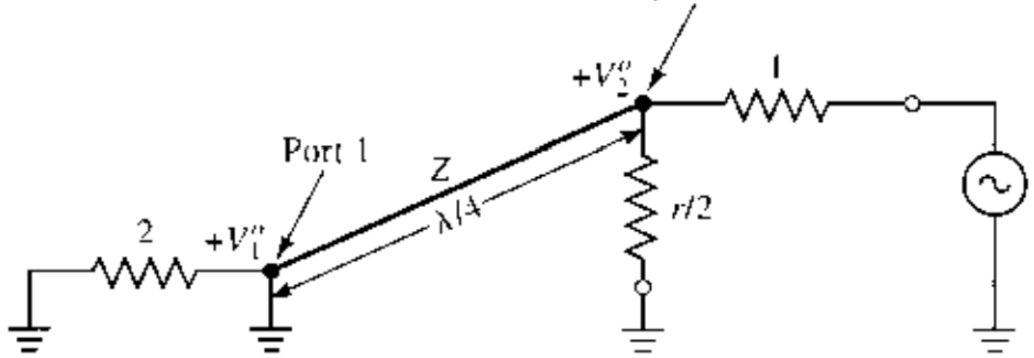

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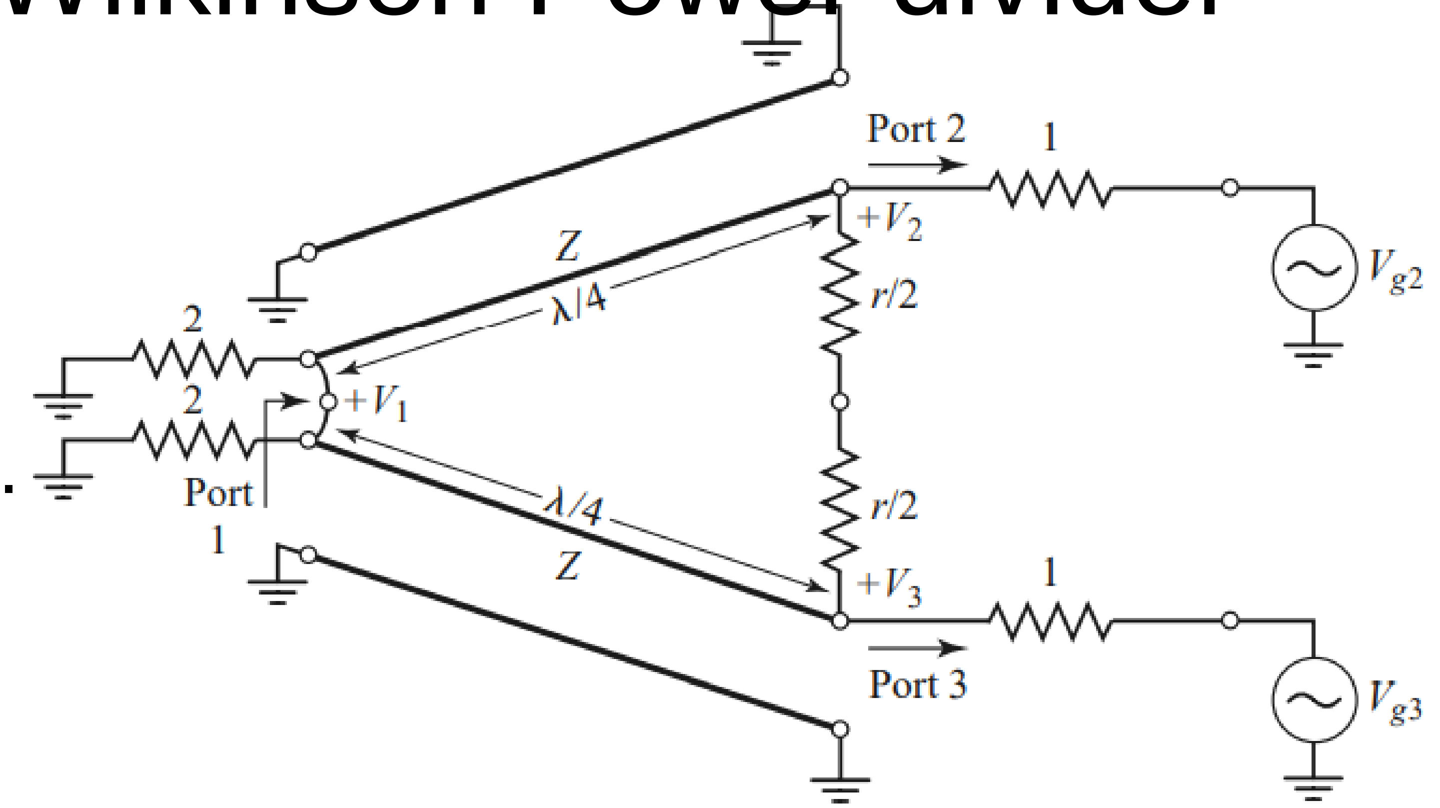

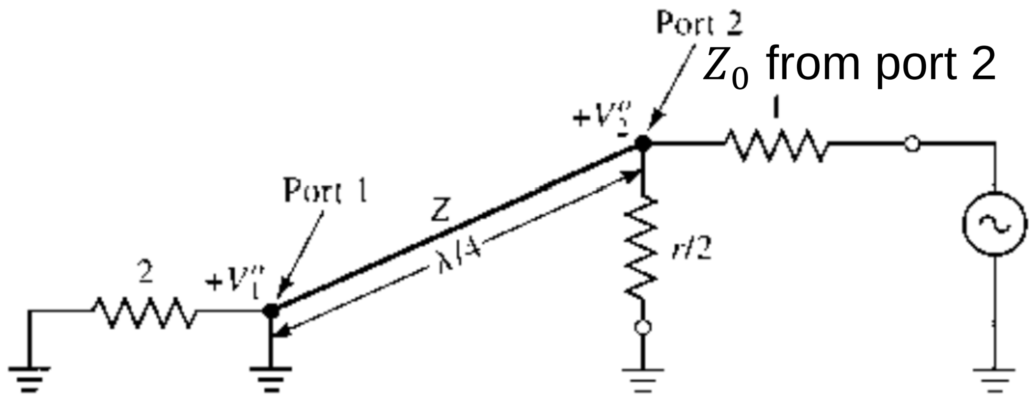

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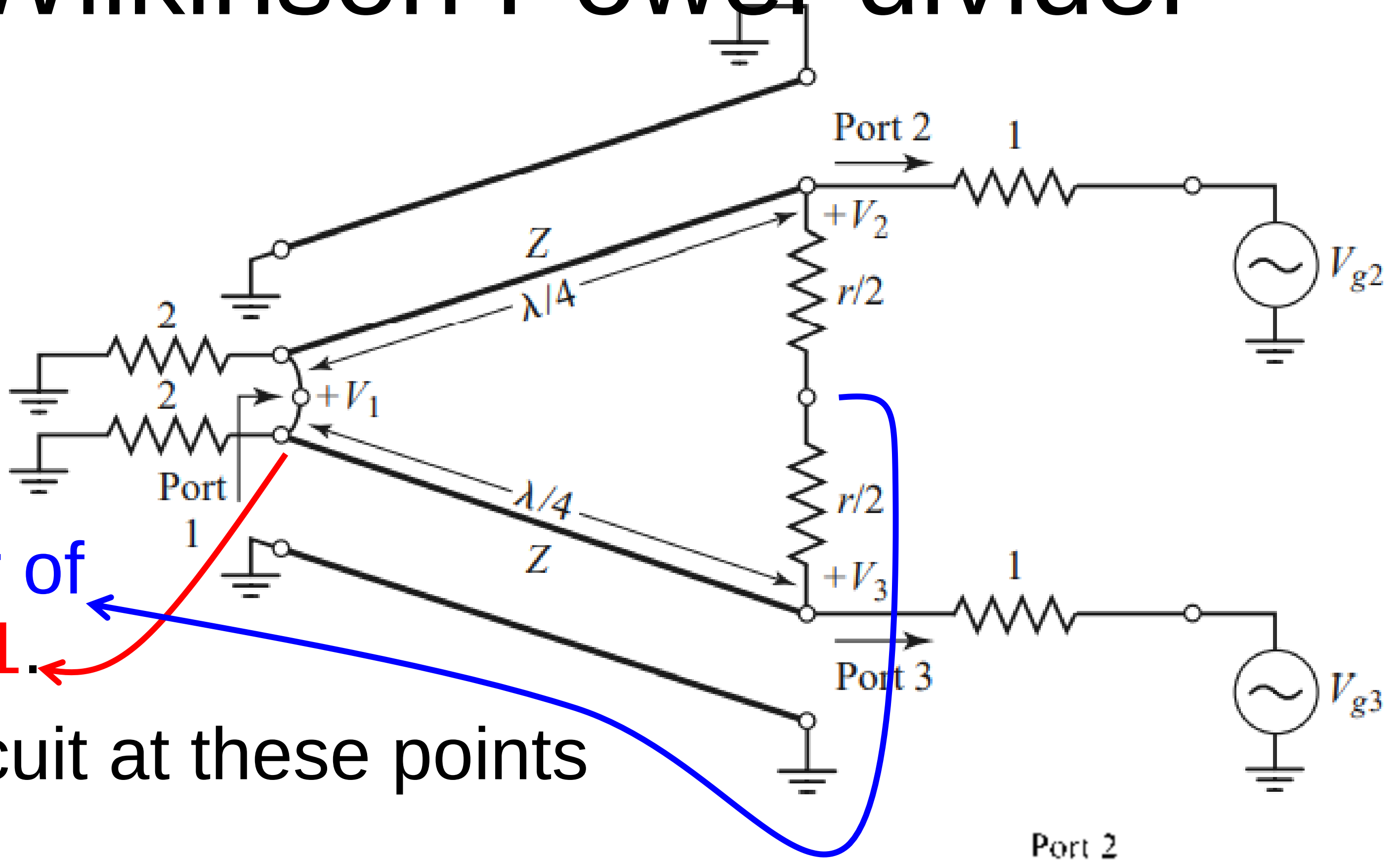

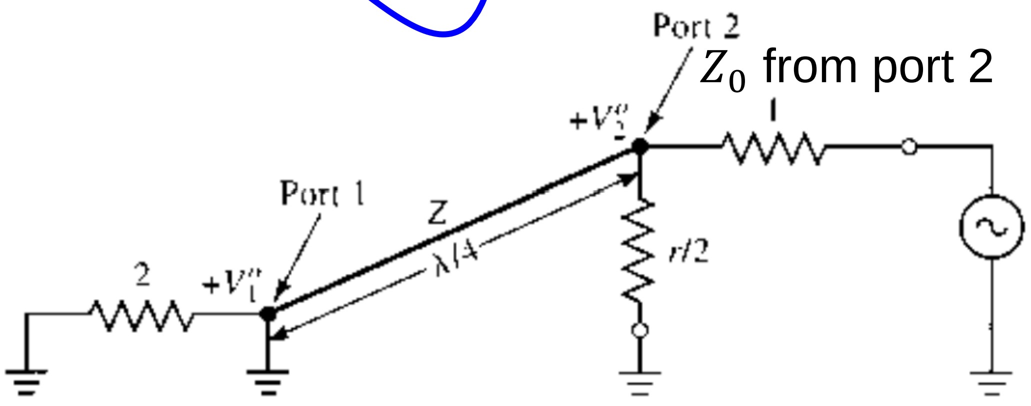

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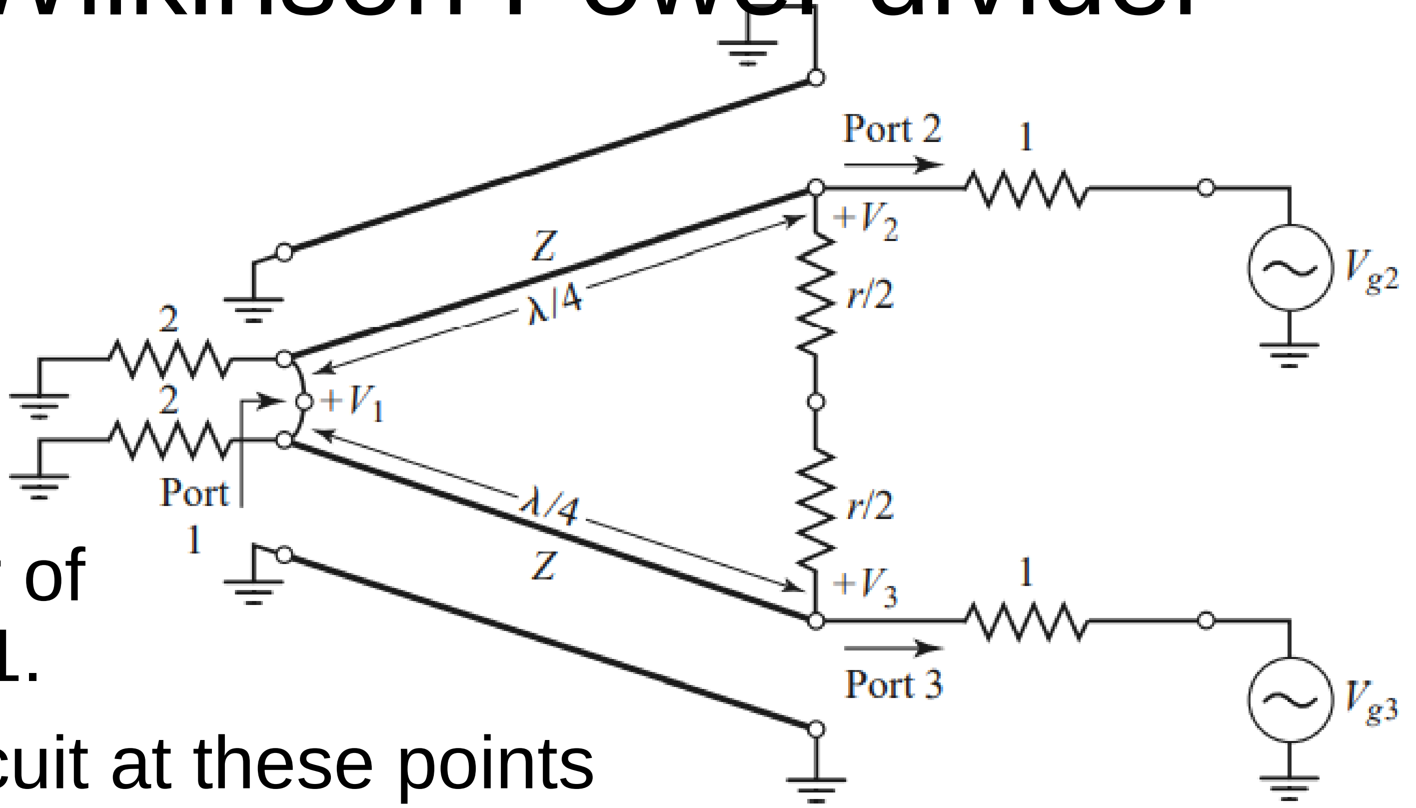
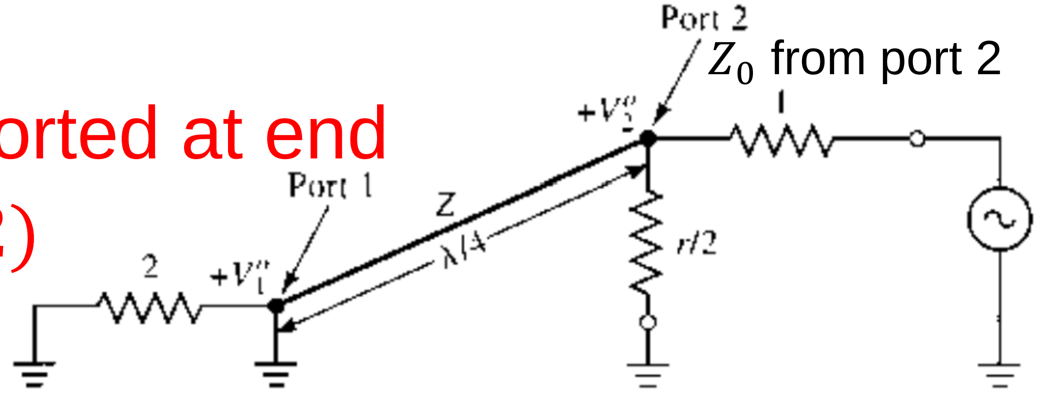
- Impedance Z_0 from port 2:

  $Z_{in}^0 = r/2$ parallel to $\frac{\lambda}{4}$ transmission line shorted at end

  ($Open\ tx\ line\ at\ port\ 2$)





Z_0 from port 2

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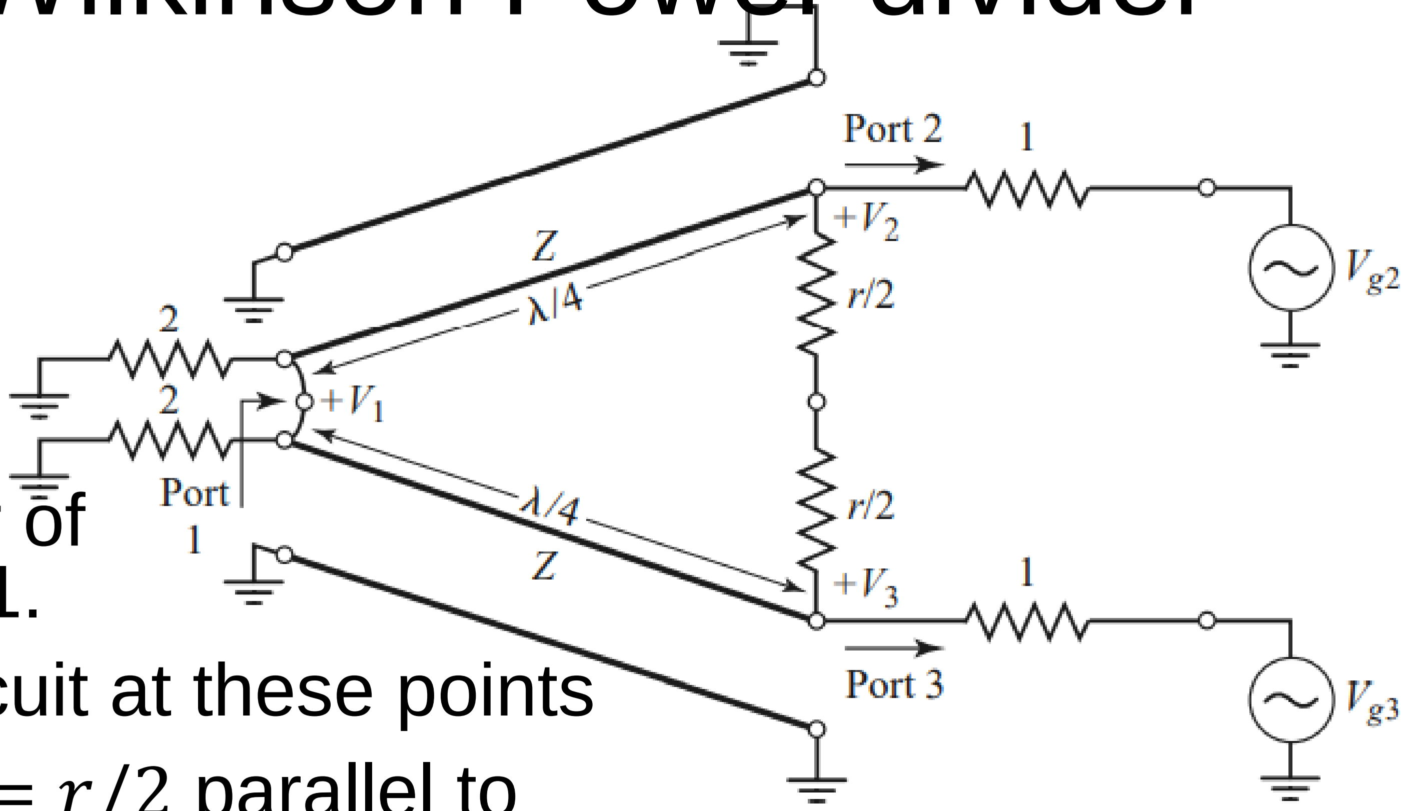
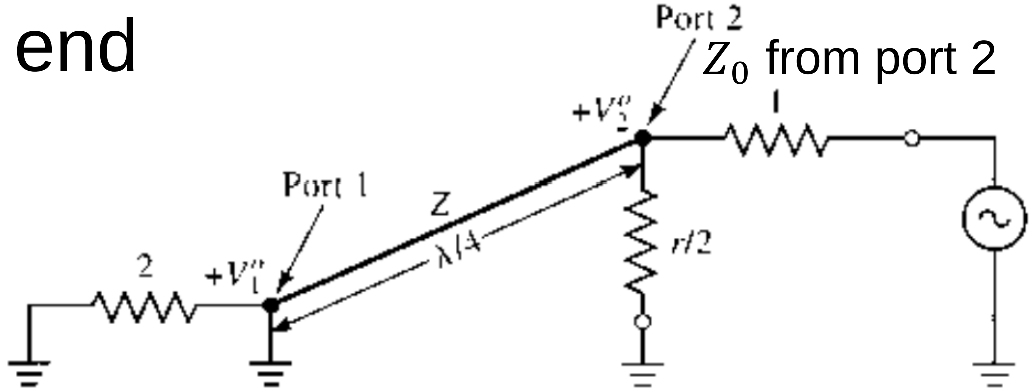
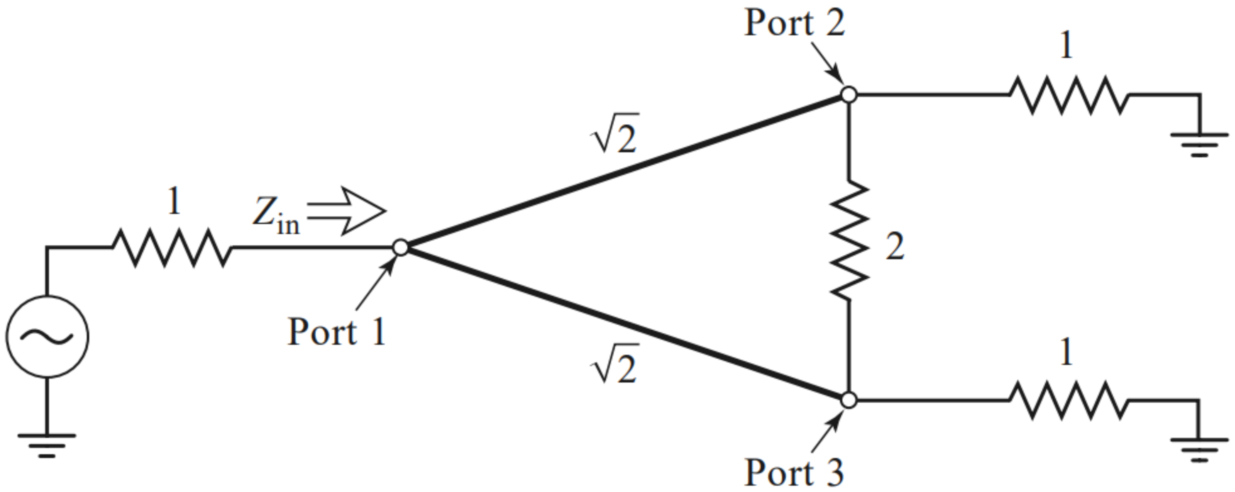
- Impedance Z_o from port 2: $Z_{in}^0 = r/2$ parallel to $\frac{\lambda}{4}$ transmission line shorted at end $(Open\ tx\ line\ at\ port\ 2)$
- $Z_{in} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$

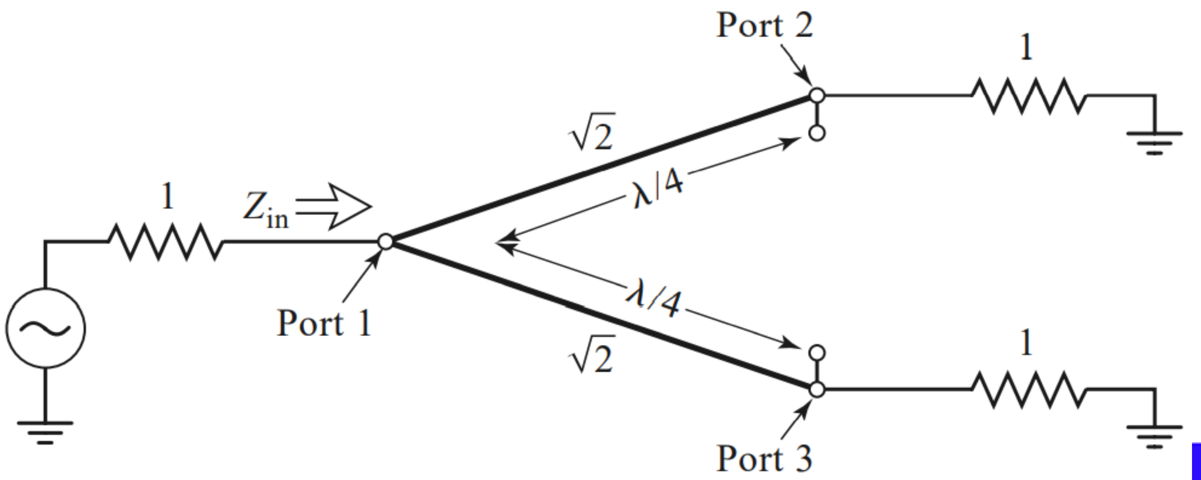




6.5 Summary Wilkinson Power divider

- $S_{11} = 0$ $\quad Z_{in} = 1$ at port 1
- $S_{22} = S_{33} = 0$ $\quad (port$ 2 and port 3 matched for even and odd modes as $Z_{in}^{e} = 1 = Z_{in}^{o})$





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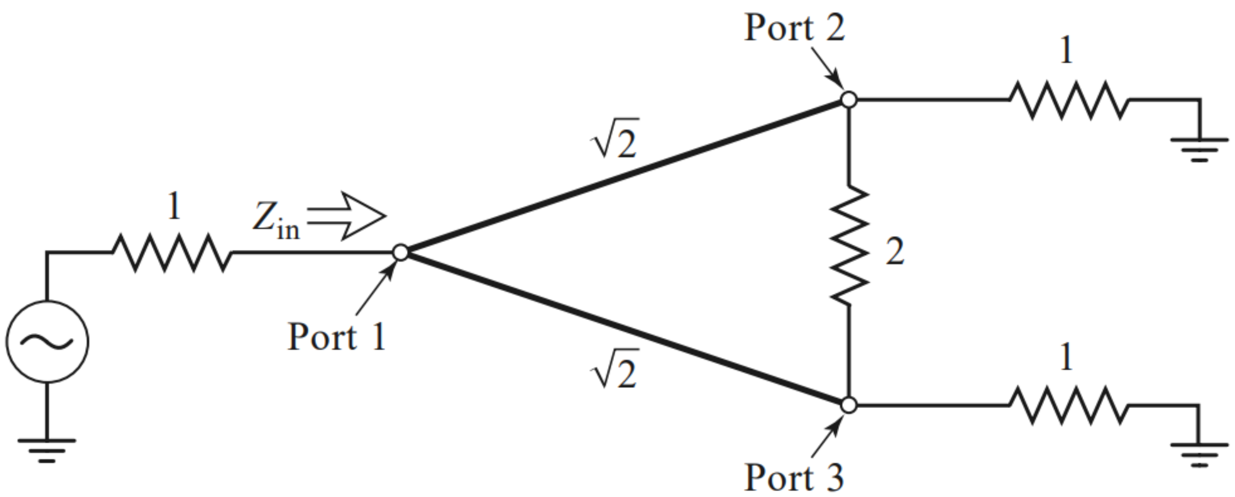
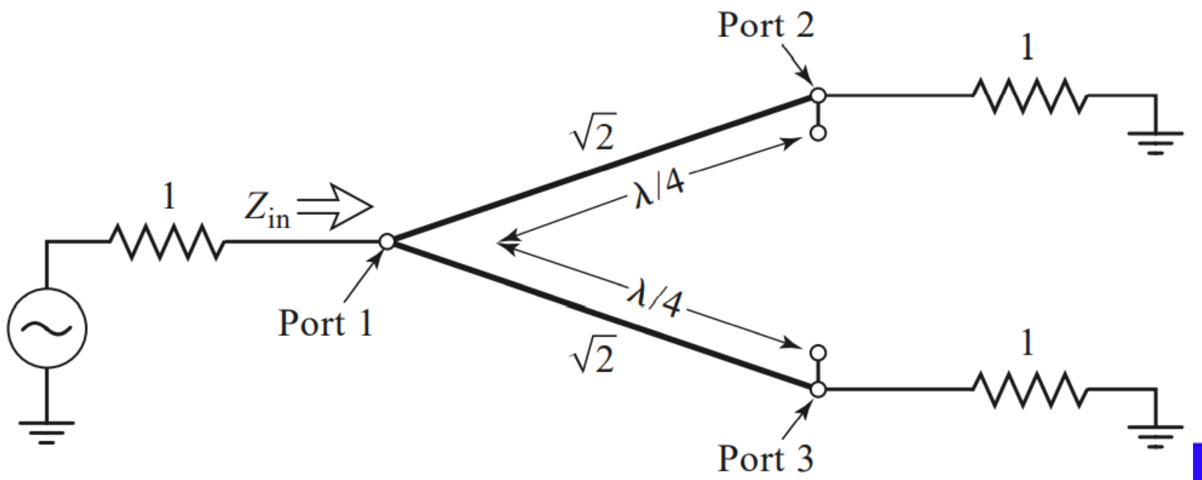
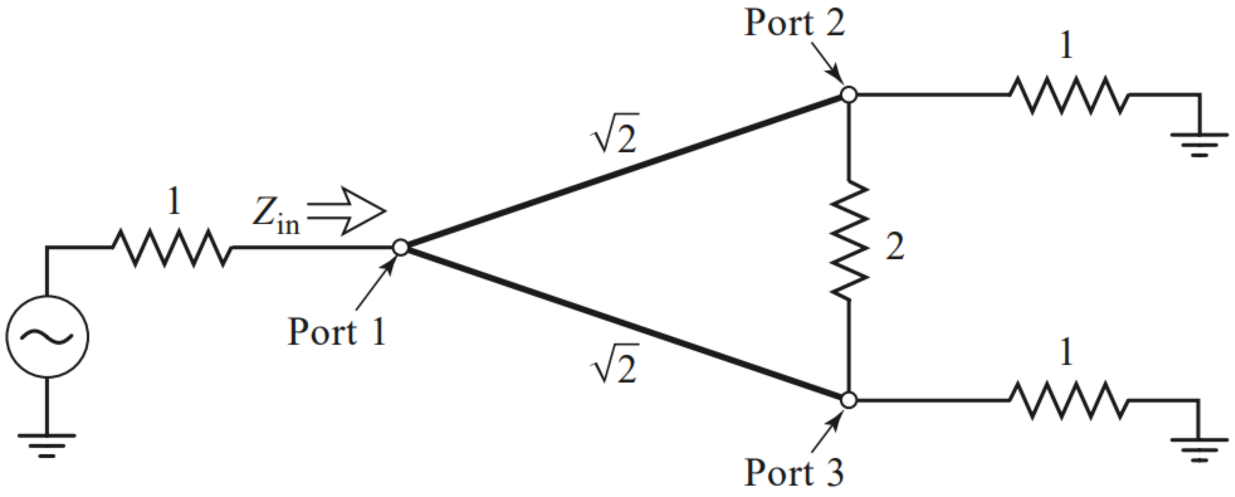
- $S_{11} = 0$ $Z_{in} = 1$ at port 1
- $S_{22} = S_{33} = 0$ $(port$ 2 and port 3 matched for even and odd modes)
- $S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$ (symmetry due to reciprocity - all matched ports)

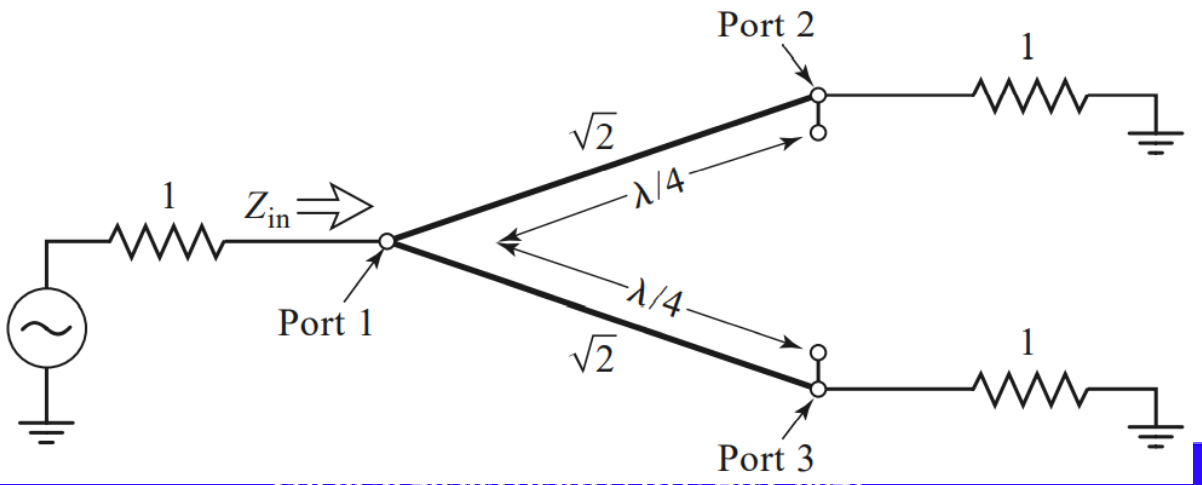




6.5 Summary Wilkinson Power divider

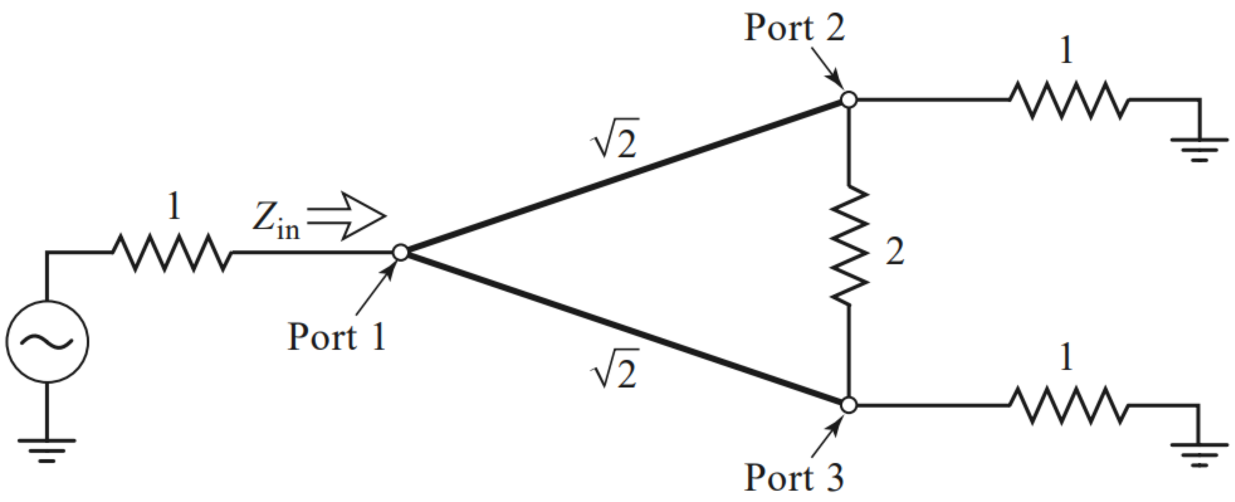
- $S_{11} = 0$ $Z_{in} = 1$ at port 1
- $S_{22} = S_{33} = 0$ $(port$ 2 and port 3 matched for even and odd modes)
- $S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$ (symmetry due to reciprocity – all matched ports)
- $S_{13} = S_{31} = -j/\sqrt{2}$ (symmetry of ports 2 and 3)
- $S_{23} = S_{32} = 0$ (due to short or open at bisection)

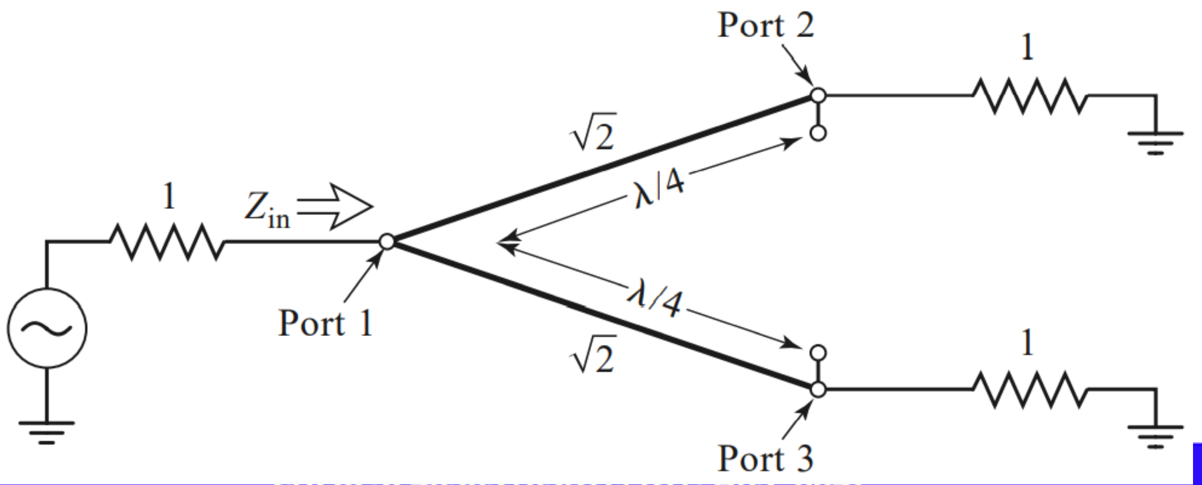




6.5 Summary Wilkinson Power divider

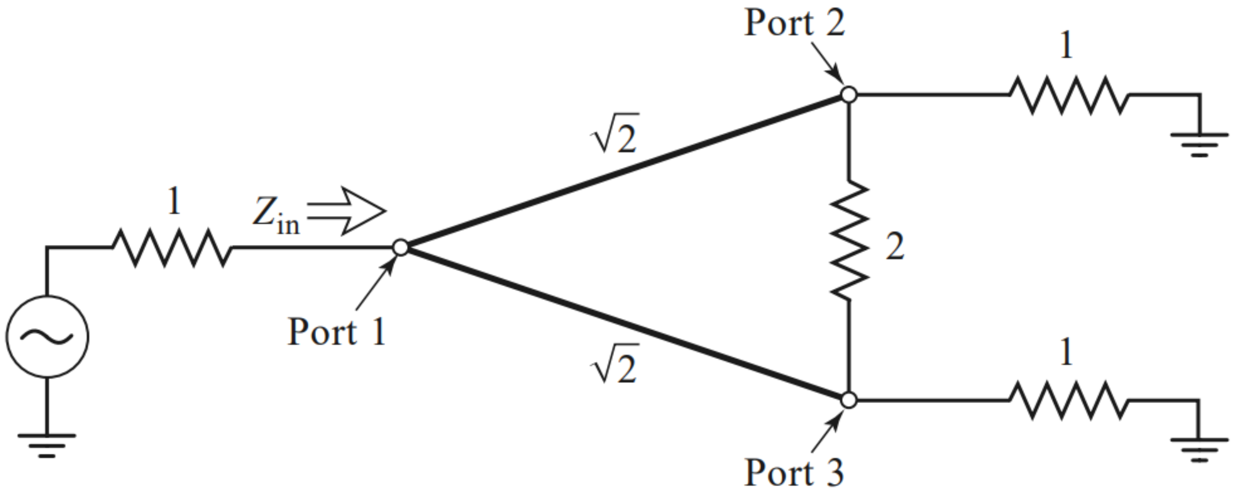
- Divider driven at port 1: **Outputs are matched**:
  <u>No power is dissipated in resistor</u>.
  Divider is lossless when outputs are matched.

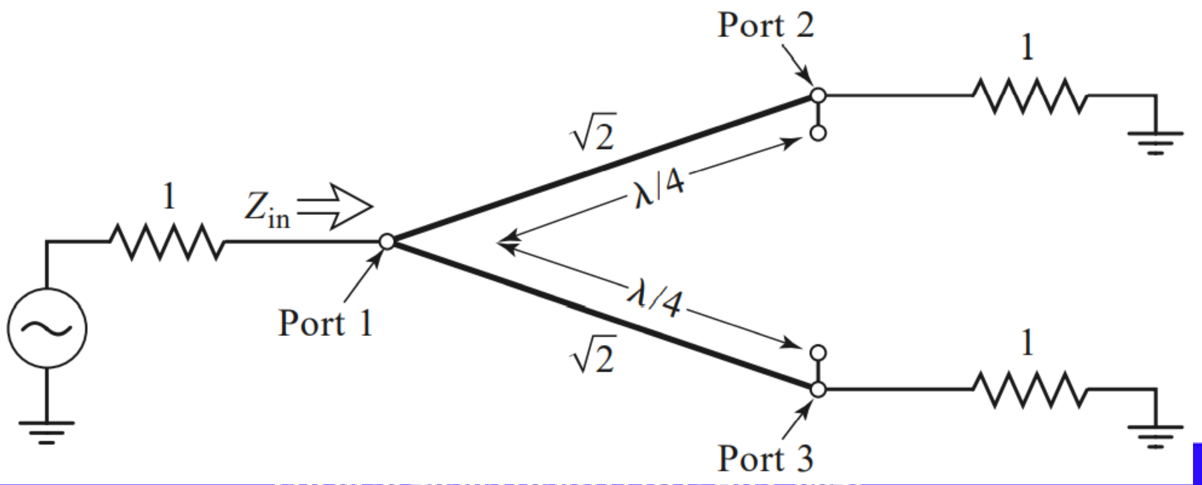




6.5 Summary Wilkinson Power divider

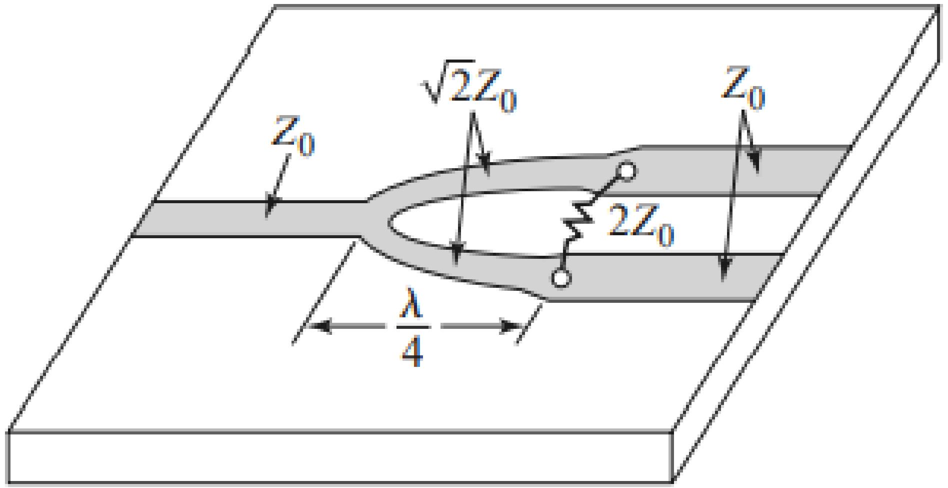
- Divider driven at port 1: Outputs are matched:
  No power is dissipated in resistor.
  Divider is lossless when outputs are matched.
- **Reflected power from ports 2 or 3 is dissipated in resistor**
- $S_{23} = S_{32} = 0$: Ports 2 and 3 are isolated

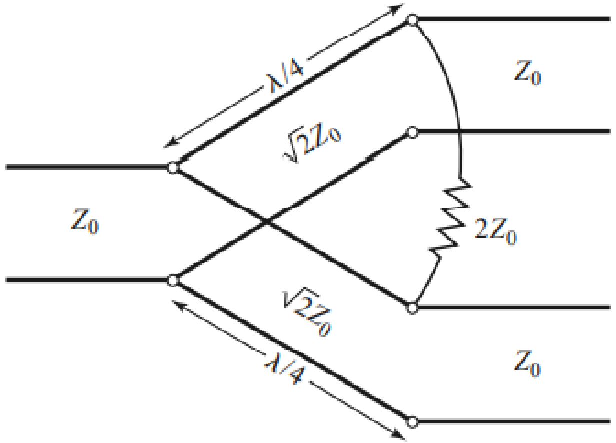




For splitting the power equally at Ports 2 and 3 in the ratio of 1:1, using Wilkinson power divider when characteristic impedance is 70 ohms, Design the Wilkinson power divider network.

- **Find the values**
  as shown in the figure:
  with the given
  characteristic impedance $Z_0 = 70$





- The S matrix : $[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$