4.4 Magnetron Oscillator

Module:4 Microwave Sources

Course: BECE305L – Antenna and Microwave Engineering

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Vellore Institute of Technology

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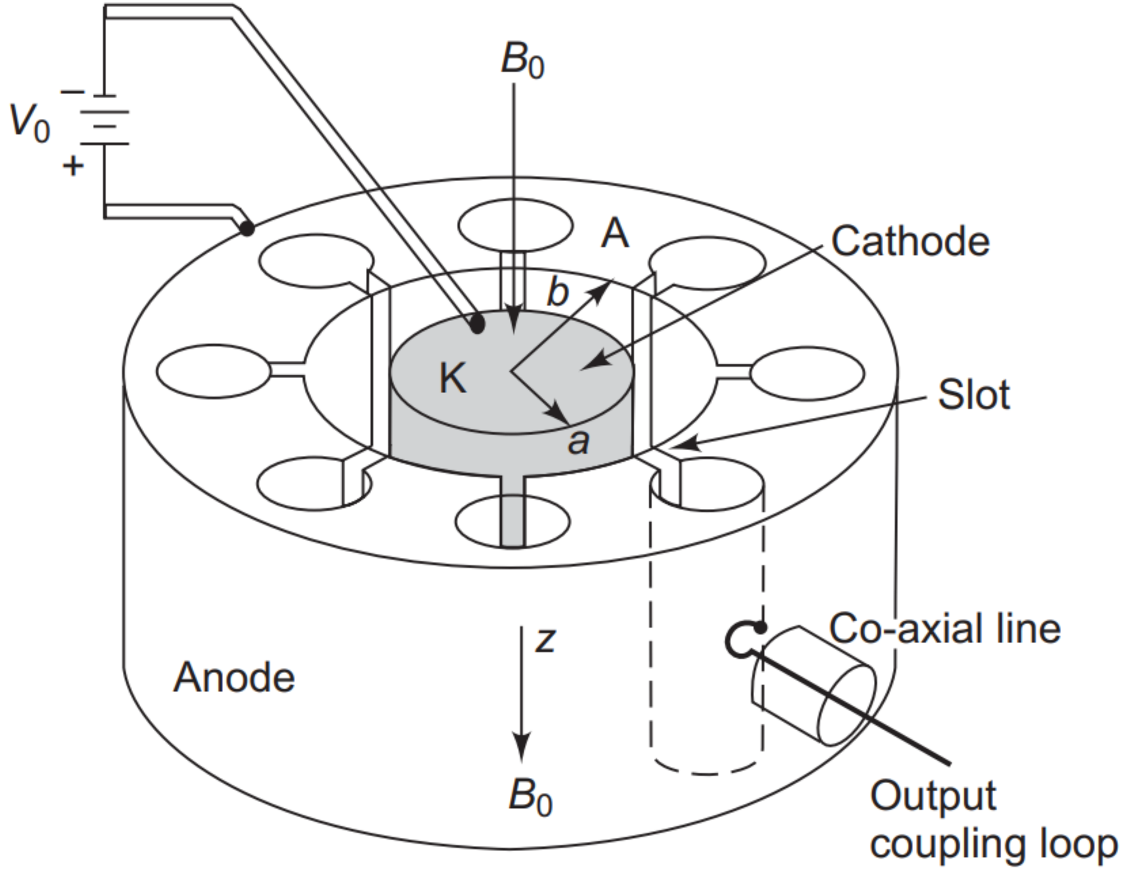
CHENNAI

Module:4 Microwave Sources 5 hours

- Microwave frequencies and applications, Microwave Tubes: TWT, Klystron amplifier, Reflex, Klystron & Magnetron. Semiconductor Devices: Gunn diode, Tunnel diode, IMPATT – TRAPATT - BARITT diodes, PIN Diode.

7.1 Magnetron Oscillator

- High output power (kilowatts)
- Noise exists – Frequency instability at higher output power
- Multiple re-entrant microwave cavities as a resonators.



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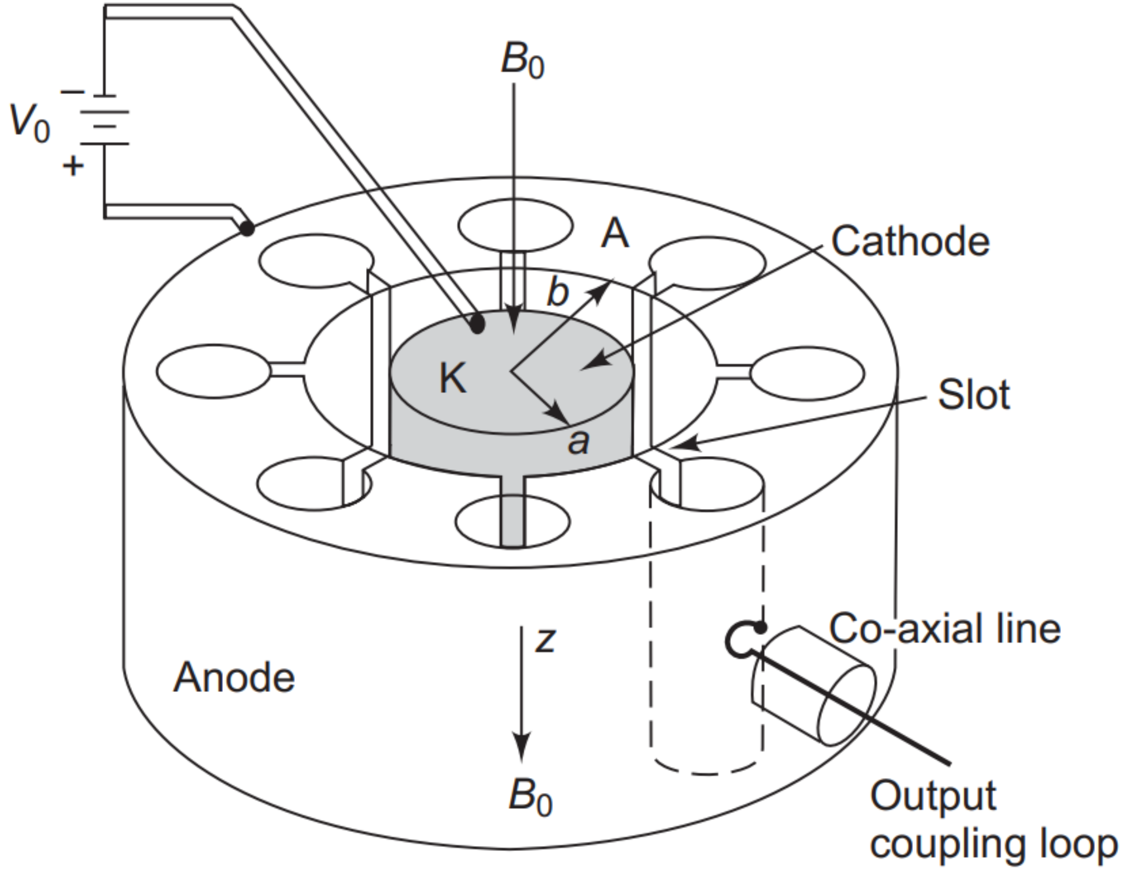
7.1 Magnetron Oscillator

- High output power (kilowatts)
- Noise exists – Frequency instability at higher output power
- Multiple re-entrant microwave cavities as a resonators.
- M-type cross field tube
- DC magnetic field and DC electric field are perpendicular to each other



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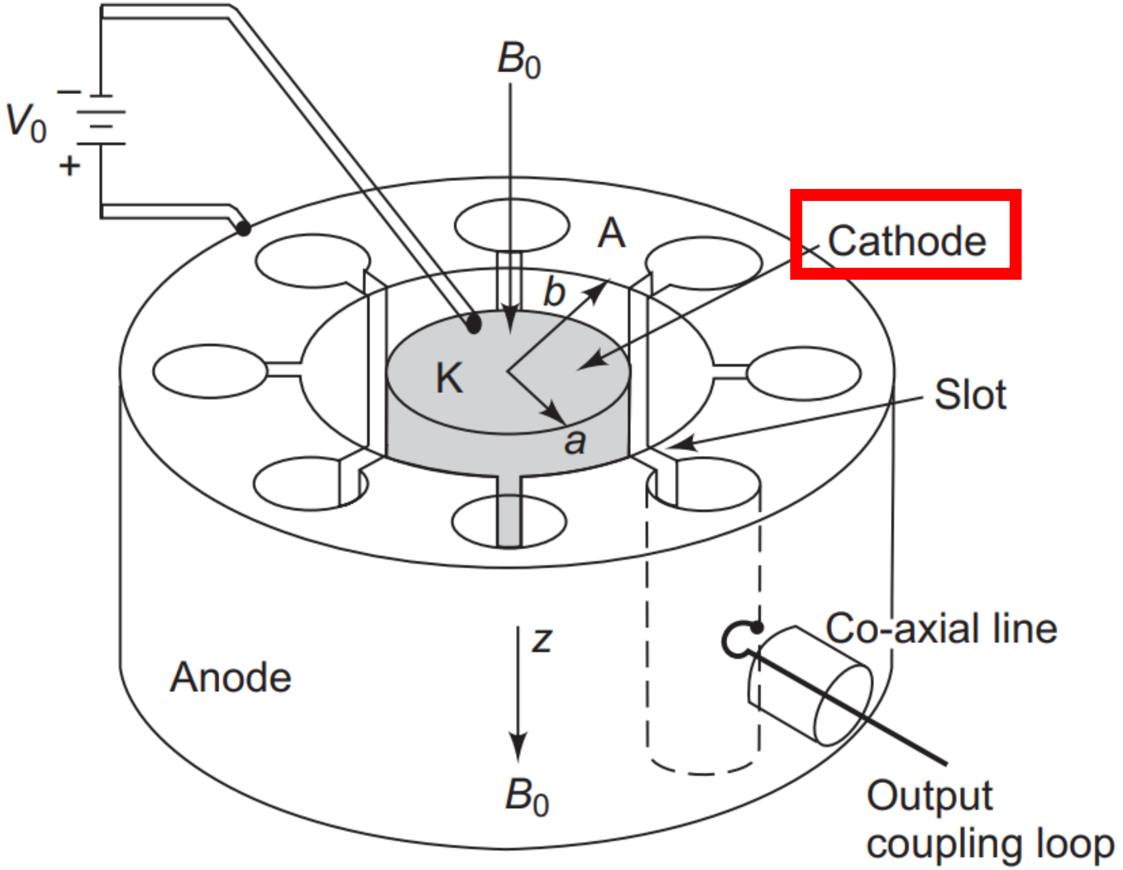
7.2 Magnetron - Construction

1) Cylindrical cathode K: radius a



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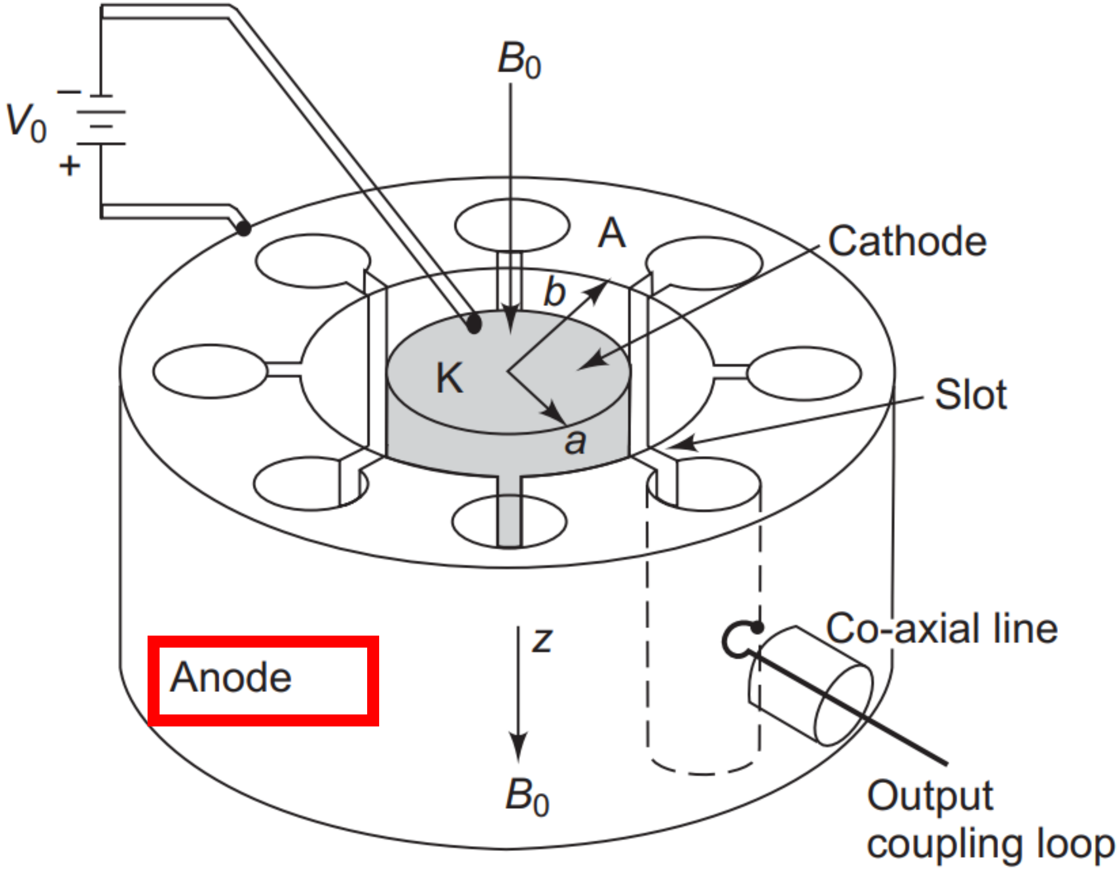
7.2 Magnetron - Construction

1) Cylindrical cathode K: radius a
2) Cylindrical anode A: radius b

Anode is slow wave structure with several re-entrant cavities equispaced around the circumference

Re-entrant cavities are coupled through a slot to the anode-cathode space.



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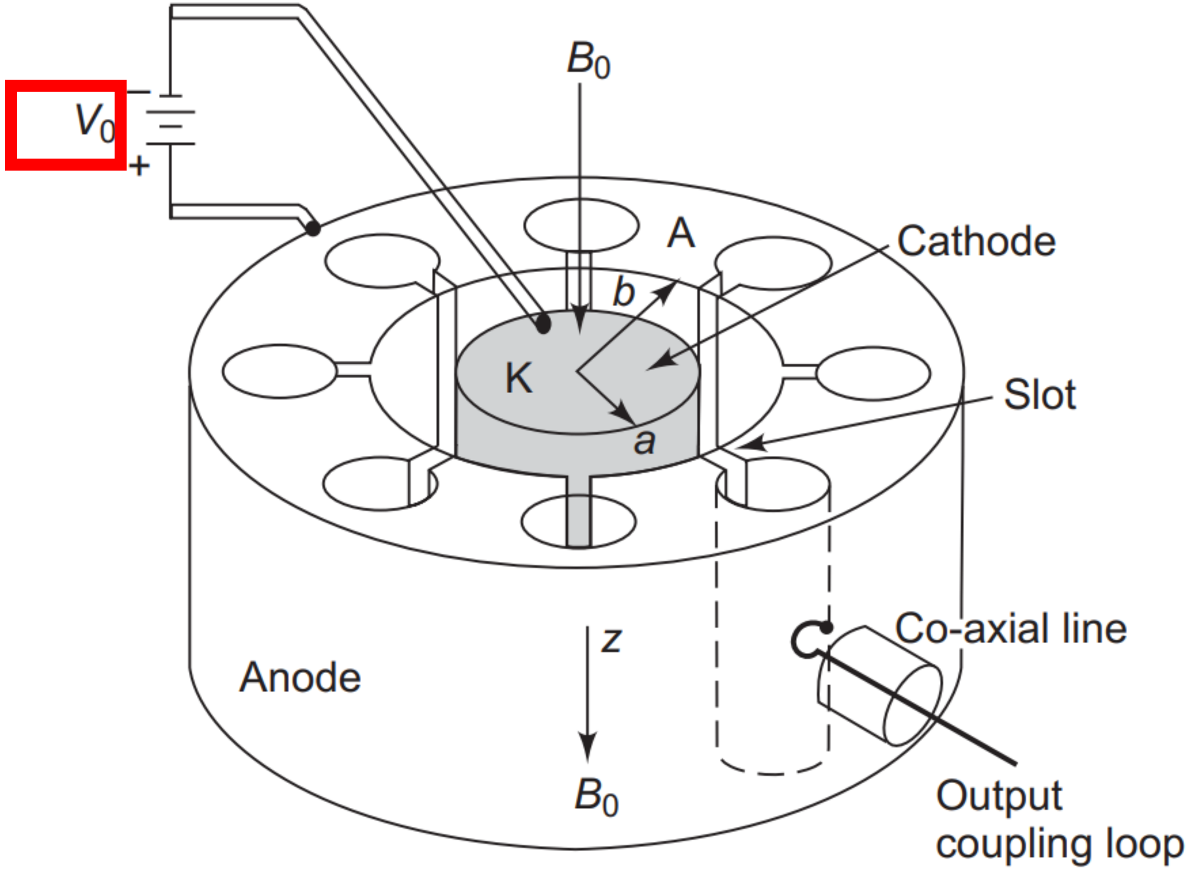
7.3. DC input sources to magnetron

1) Radial electric field dc voltage V_0 in between cathode and anode



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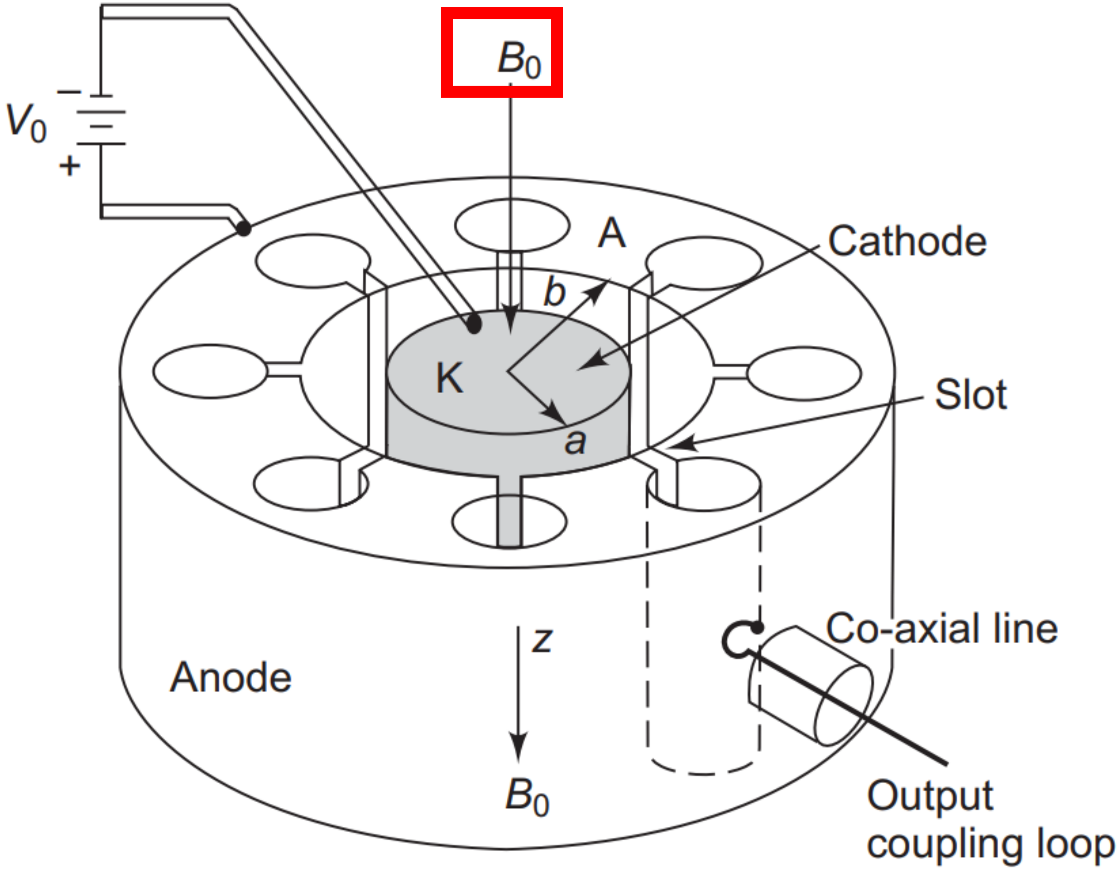
7.3. DC input sources to magnetron

1) Radial electric field dc voltage V_0 in between cathode and anode
2) Axial dc magnetic flux through permanent magnets or electromagnets B_0



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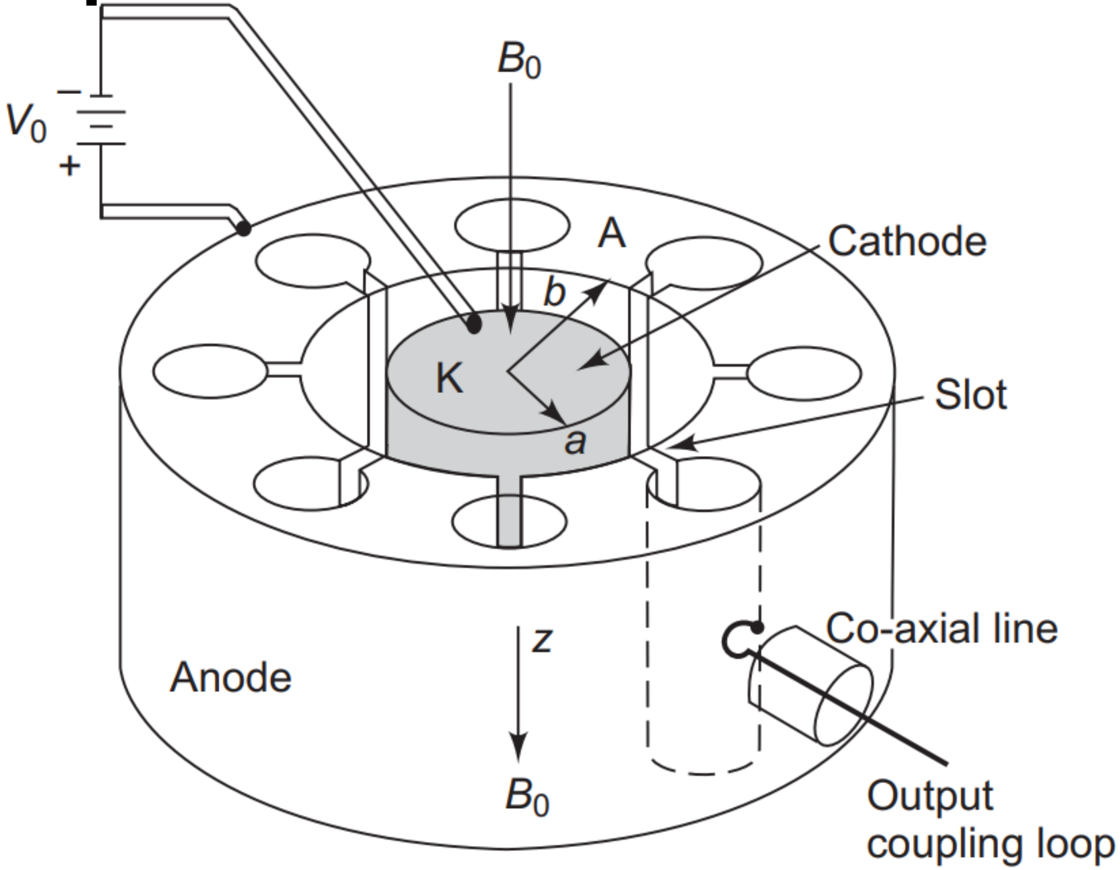
7.4. Magnetron theory of operation

1) Motion of electrons under the influence of combined electric and magnetic fields
2) Electrons emitted from cathode try to travel to anode.



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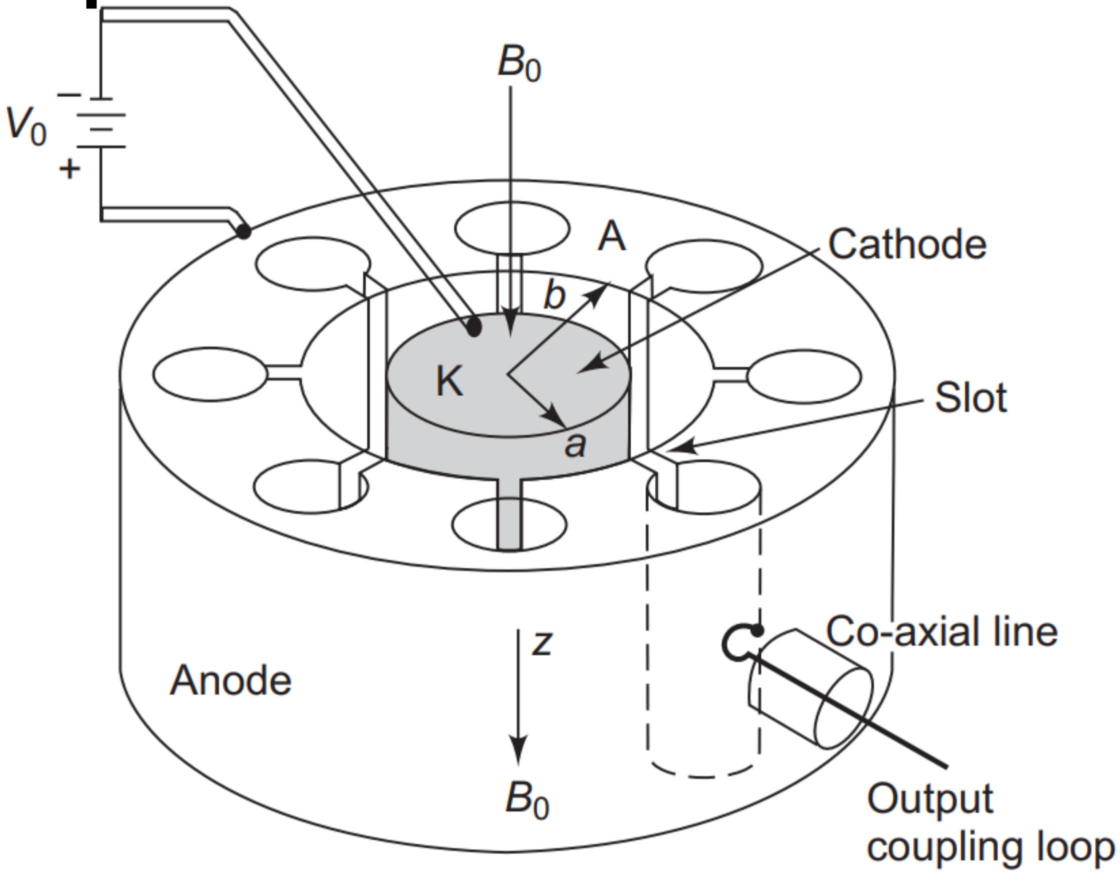
3) Under the influence of cross $\bar{E}$ and $\bar{H}$ in the space between the anode and cathode, a resultant force $\bar{F} = -e\bar{E} - e(\bar{v} \times \bar{B})$



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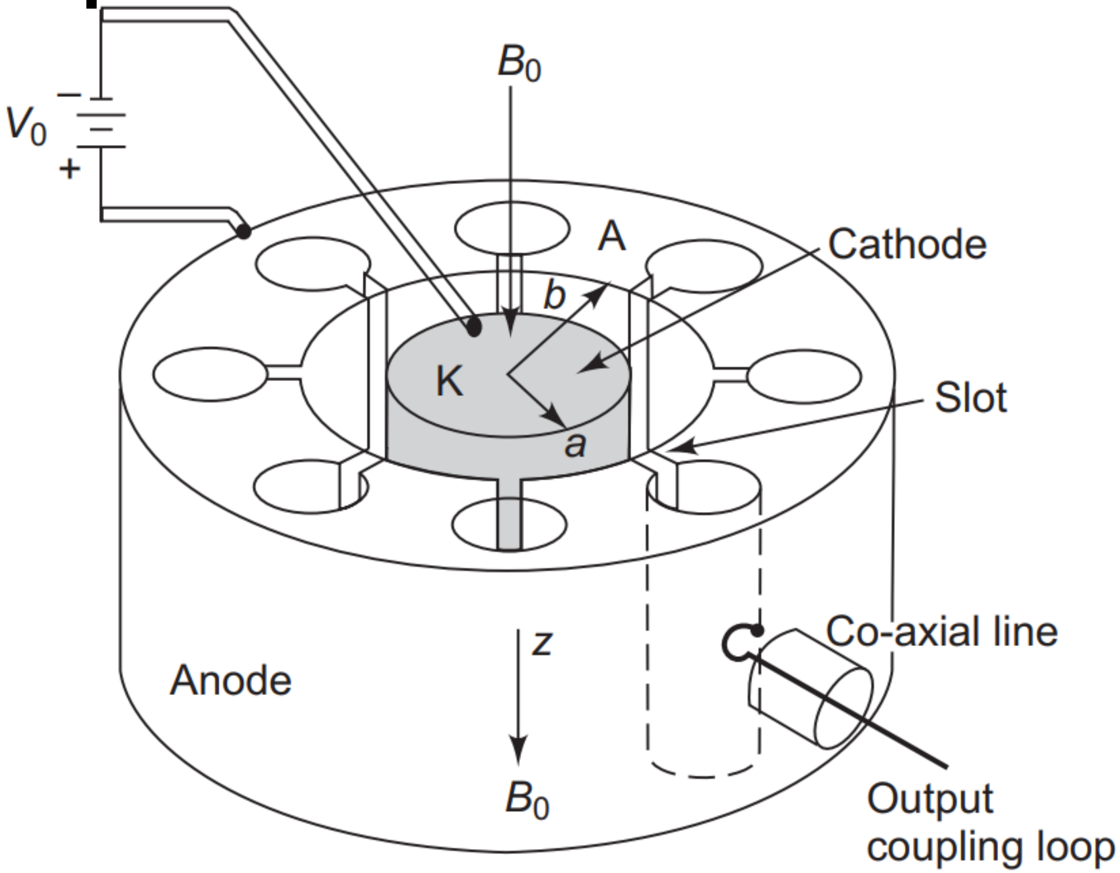
$\bar{v}$: velocity vector of the electron considered and takes curved tragectory



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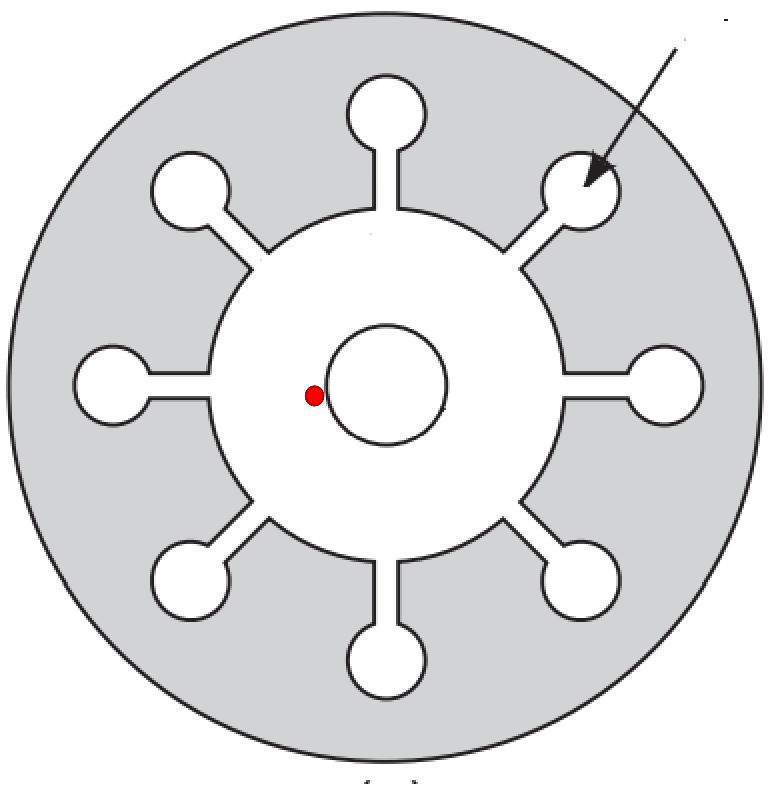

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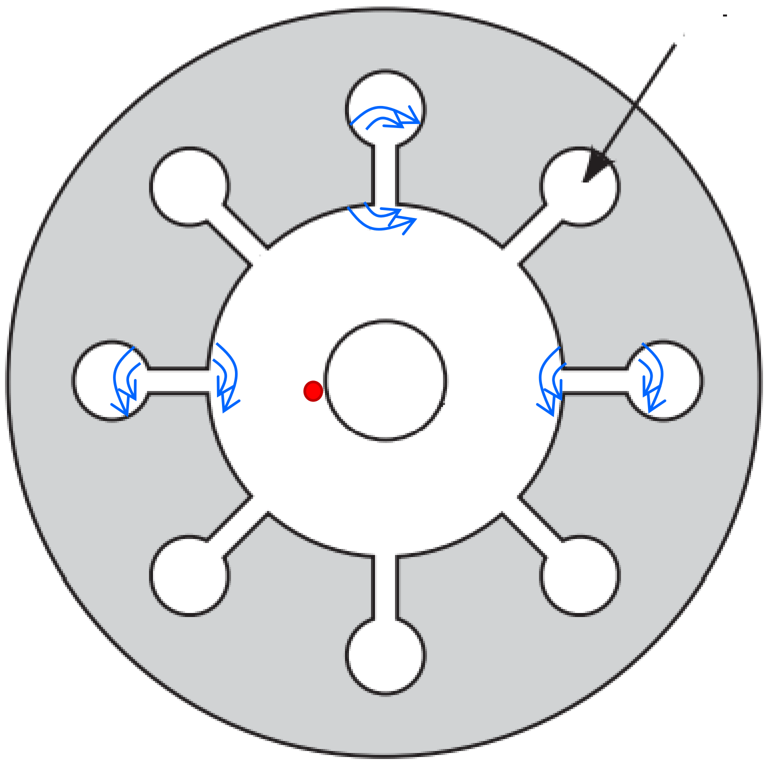
4) Noise in the anode cavities in biasing circuit, RF fields are fringed (extended) out of the cavity slot to space between anode

5) Accelerated electrons in trajectory when retarded by RF, the fields transfer energy from electrons to cavities to grow RF oscillations.

6)



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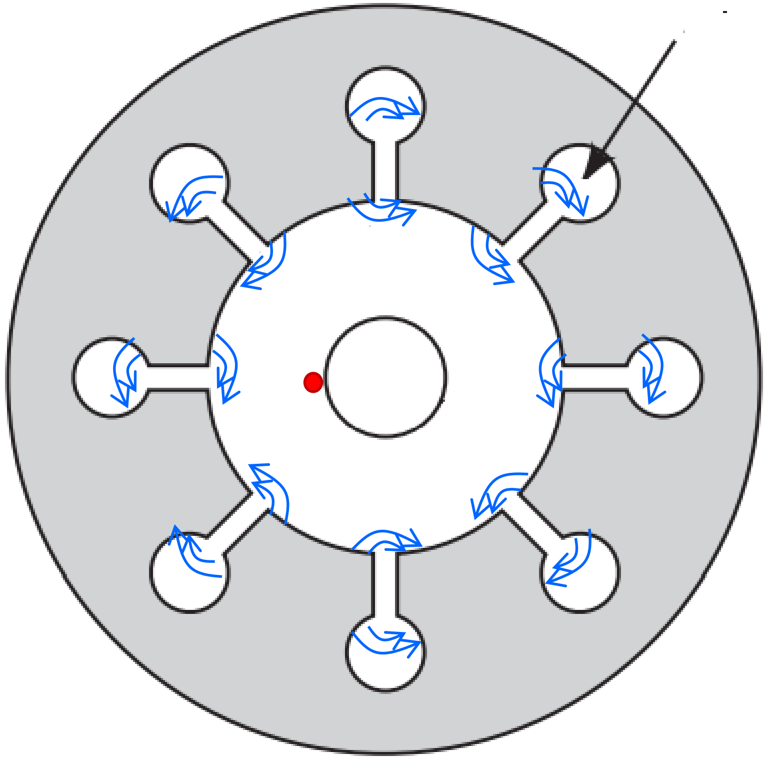
5) Accelerated electrons in trajectory when retarded by RF, the fields transfer energy from electrons to cavities to grow RF oscillations.

6) When system RF losses balance RF oscillation energy, stable oscillation is achieved.

7) Output is extracted through extended line coupled to the cavity



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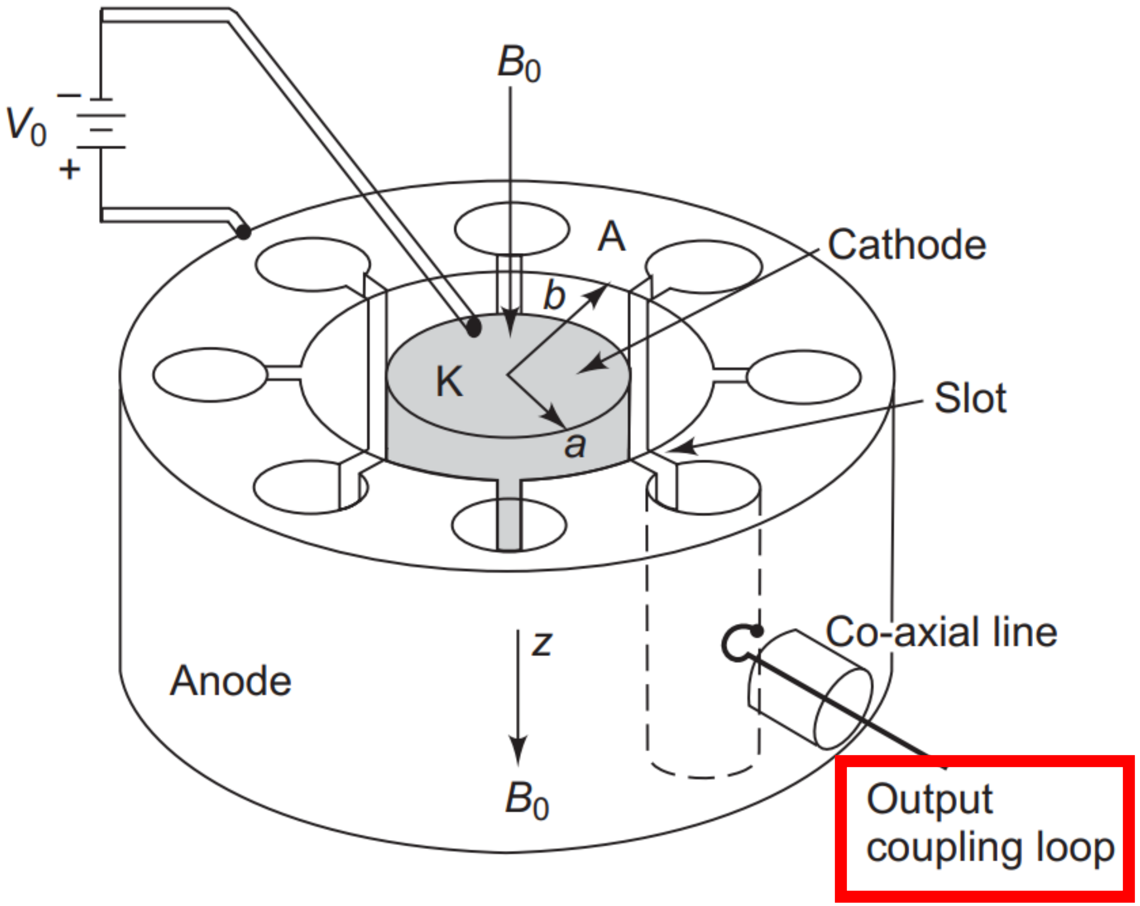

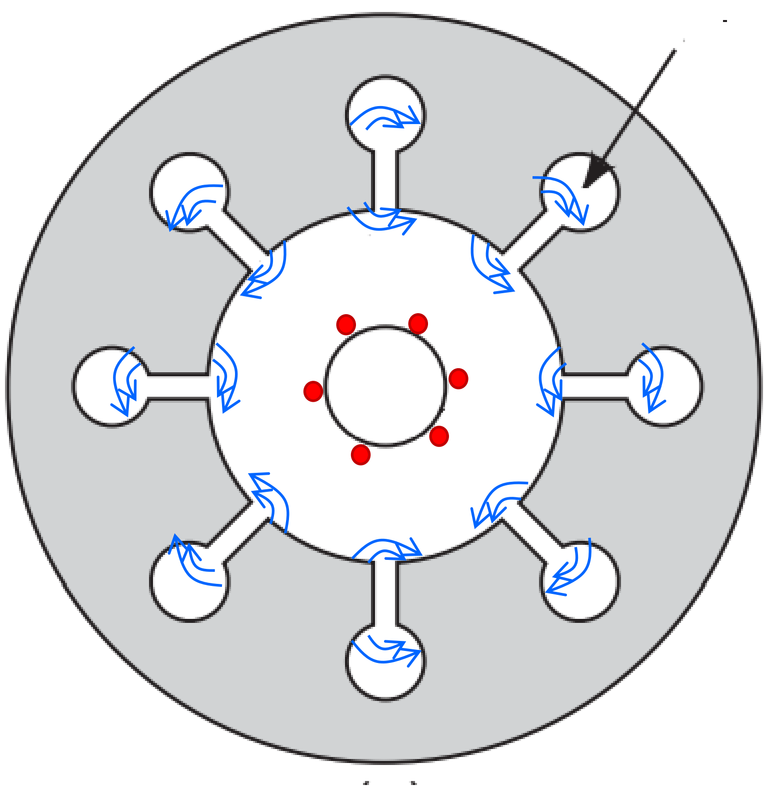

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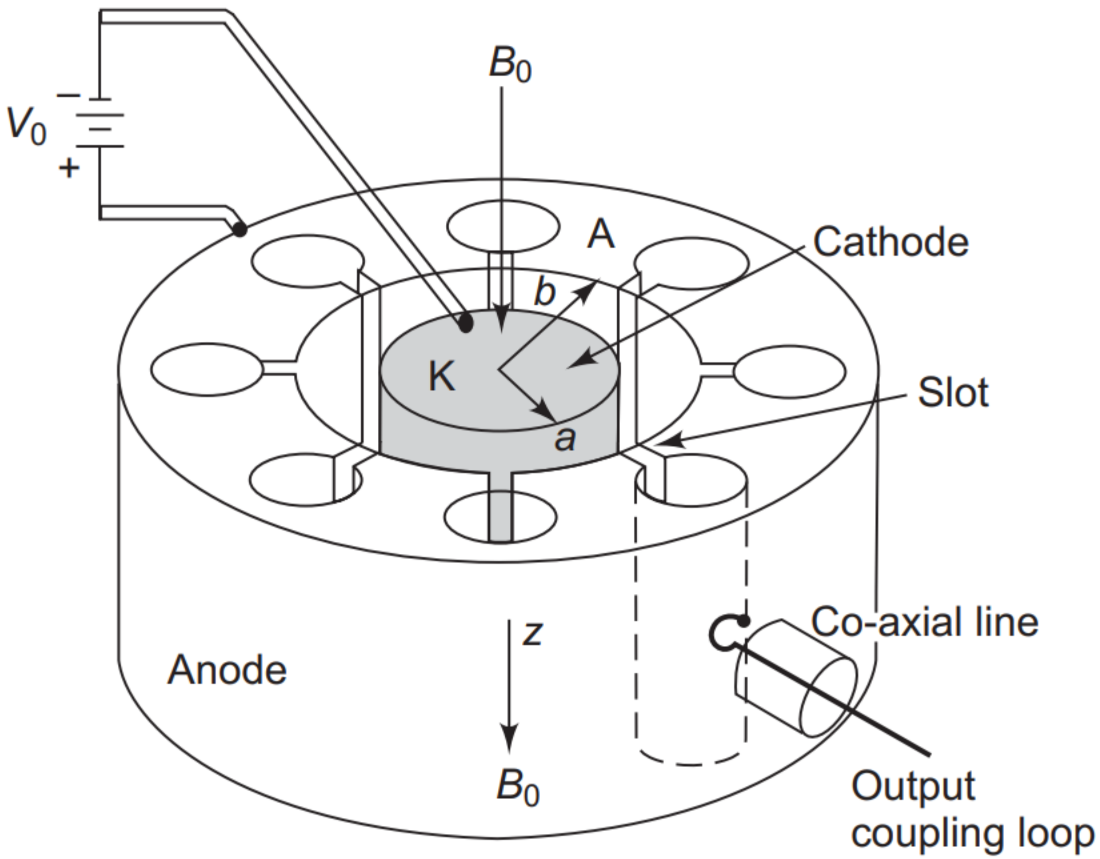
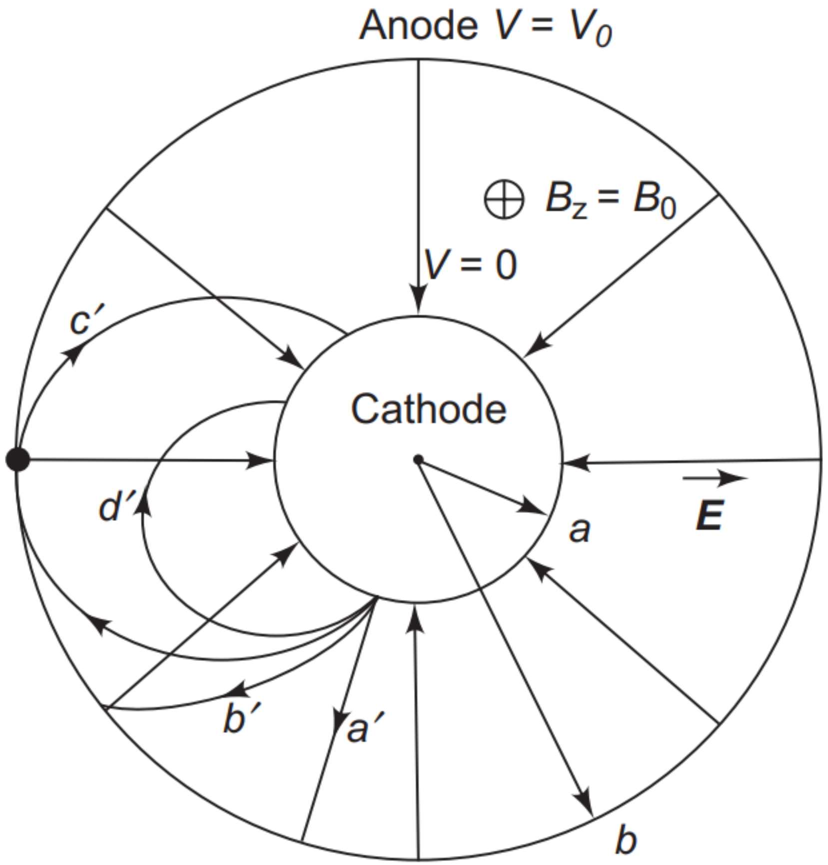
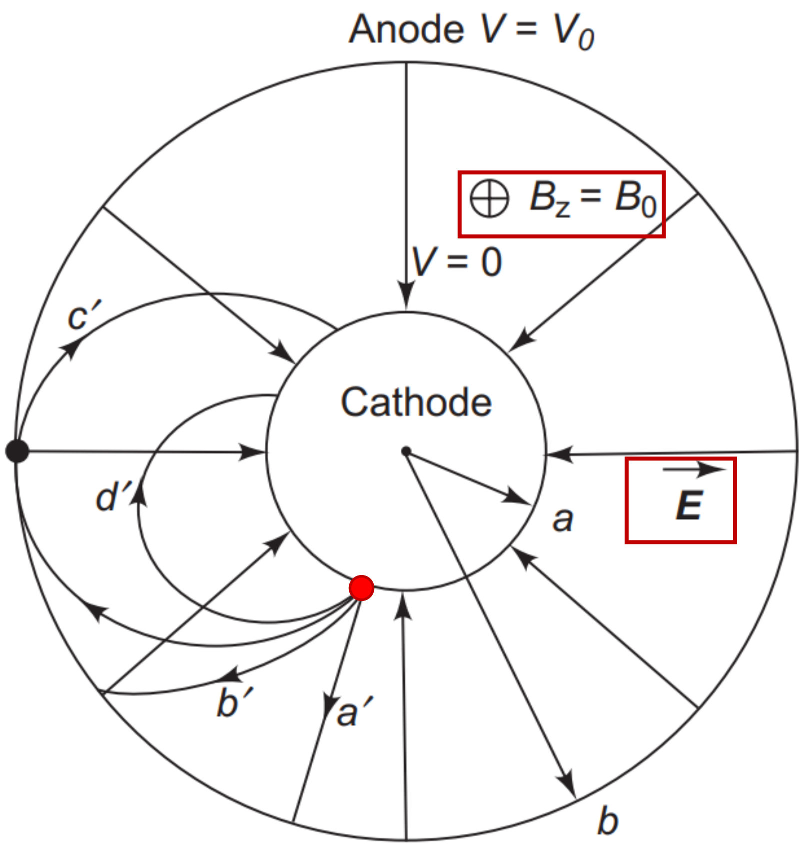
7.5 Equations of electron trajectory





7.5 Equations of electron trajectory

- After the emergence from cathode with zero velocity, electrons acquire velocity $\bar{v}$ with tangential and radial velocity components due to the force, $\bar{F} = -e\bar{E} - e(\bar{v} \times \bar{B})$



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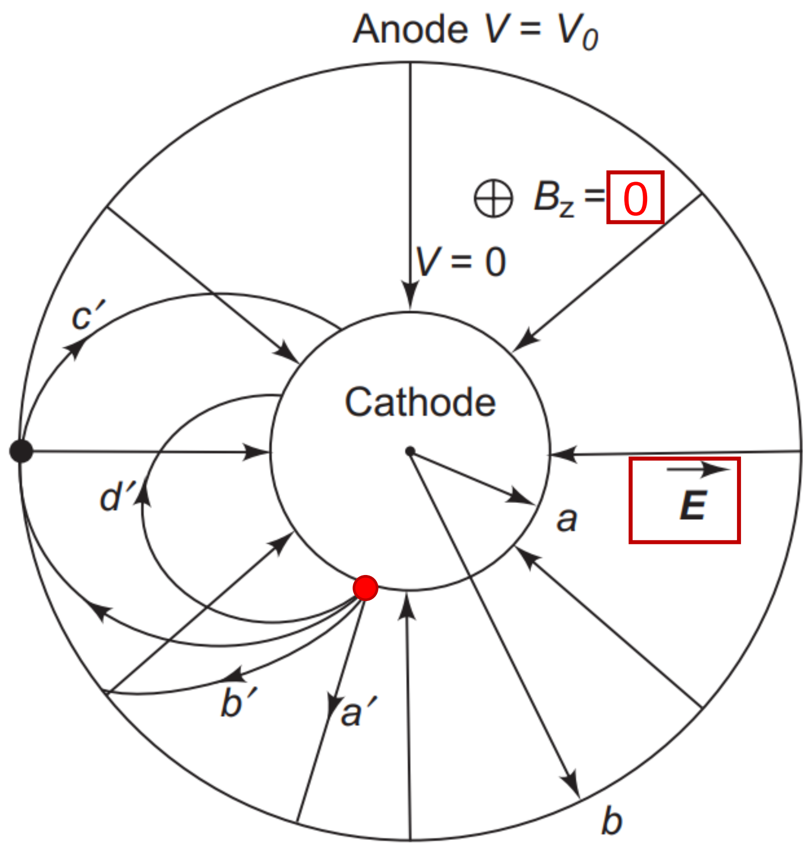
- b':



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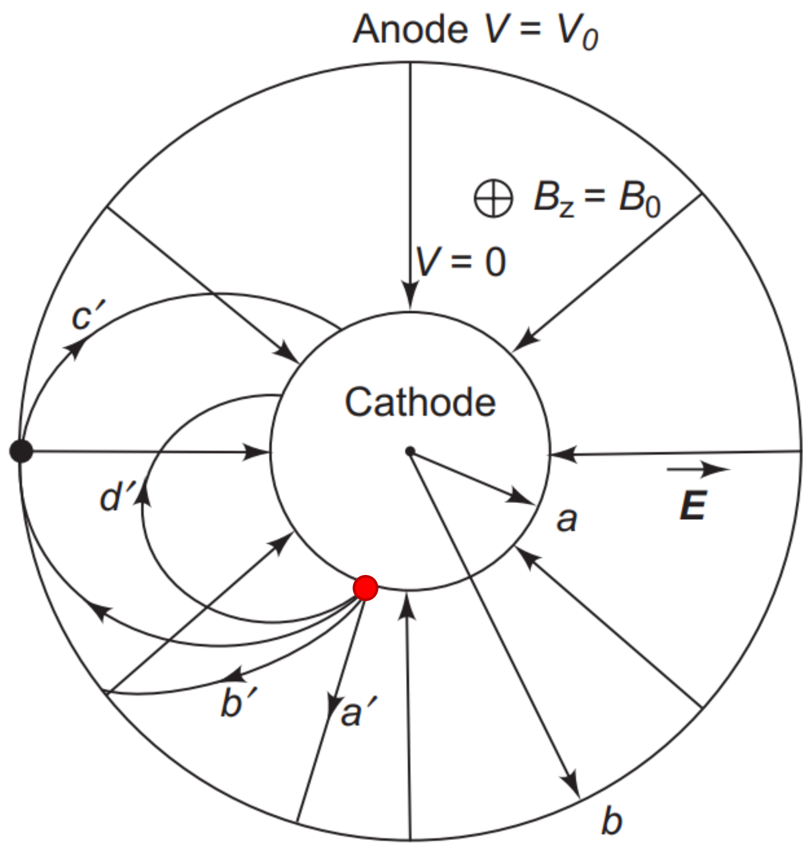
- b': by increasing magnetic field slightly, electrons reach anode in the path b' due to the force



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- c': At a critical magnetic field, B_c, electrons graze the surface of anode at radius b and take the path c' to cathode for a given voltage V_0.

  This value of magnetic field : B_C is cutoff magnetic flux density.

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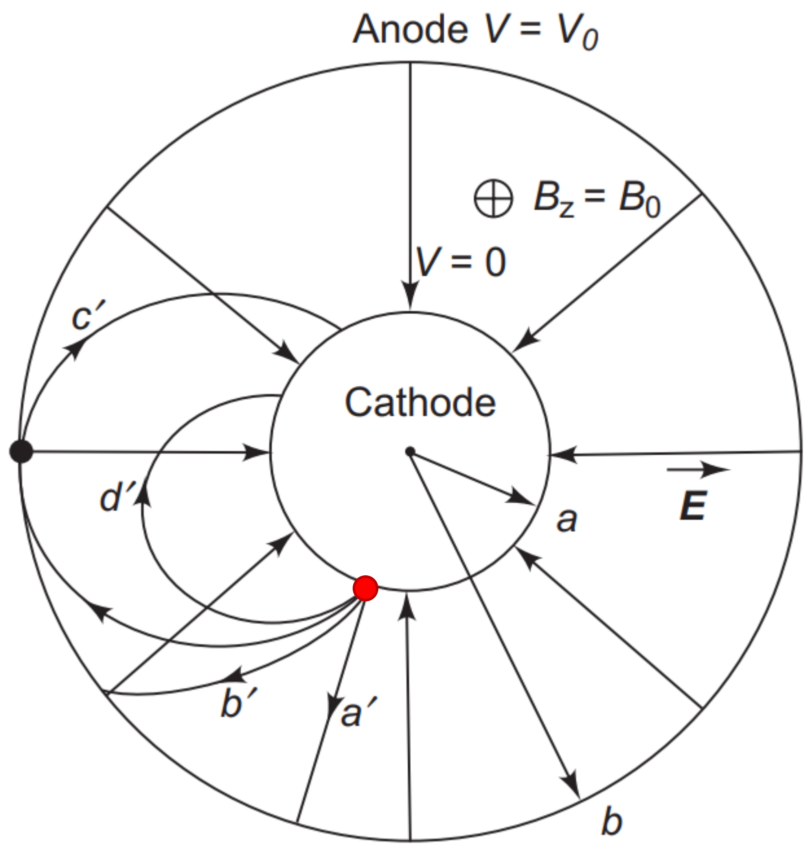

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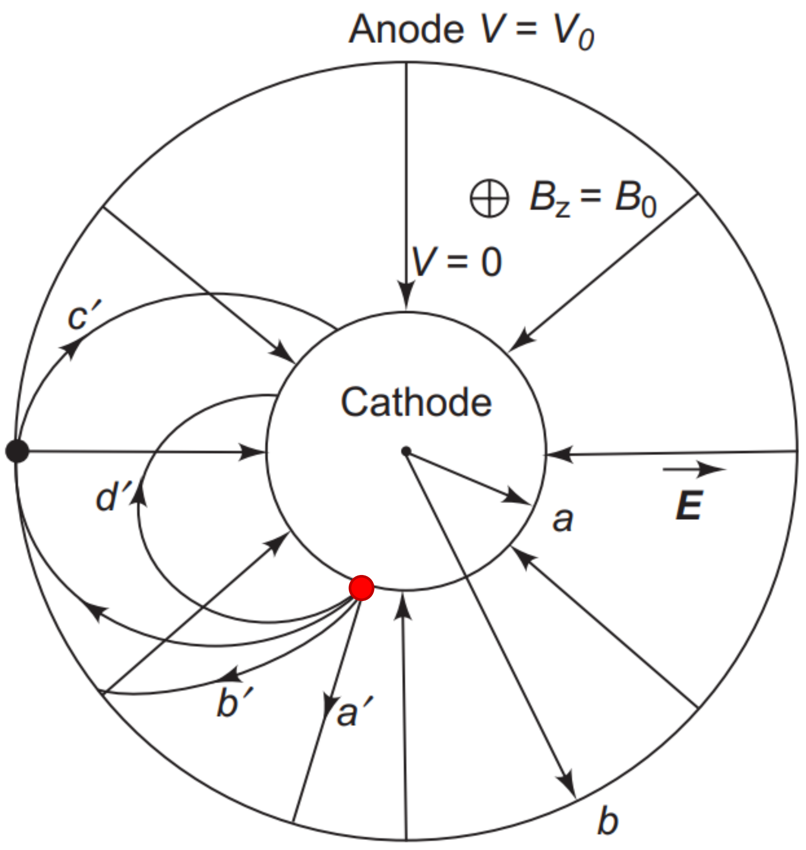
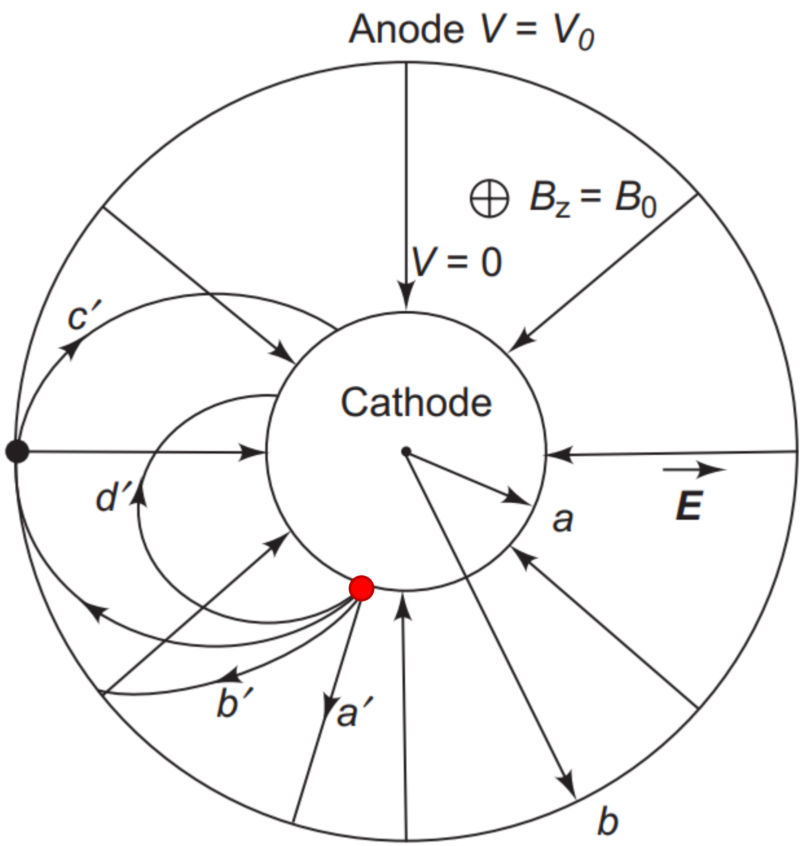
- d': When magnetic field is increased further beyond B_c, the electrons end up in a typical path as shown in d'



7.5 Equations of electron trajectory

- When magnetic field in increased from zero to maximum, Anode current(electrons hitting anode) decreases from maximum to zero.



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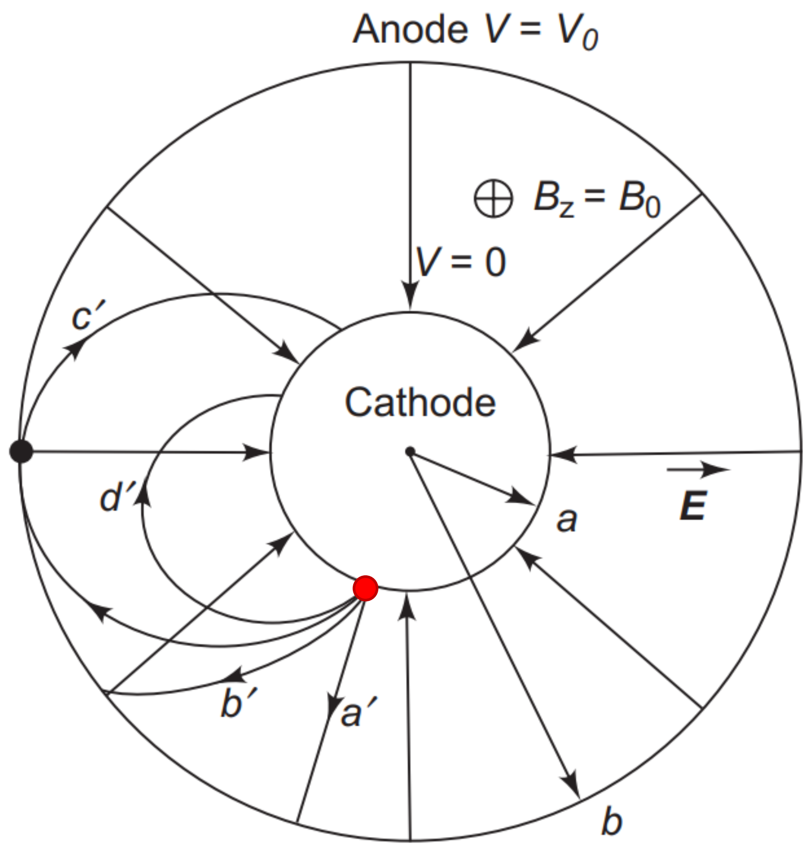
- When magnetic field in increased from zero to maximum, Anode current(electrons hitting anode) decreases from maximum to zero.
- Average velocity of electrons in z direction is constant, $v_z = E_0/B_0$ where $E_0 = \frac{V_0}{b-a}$



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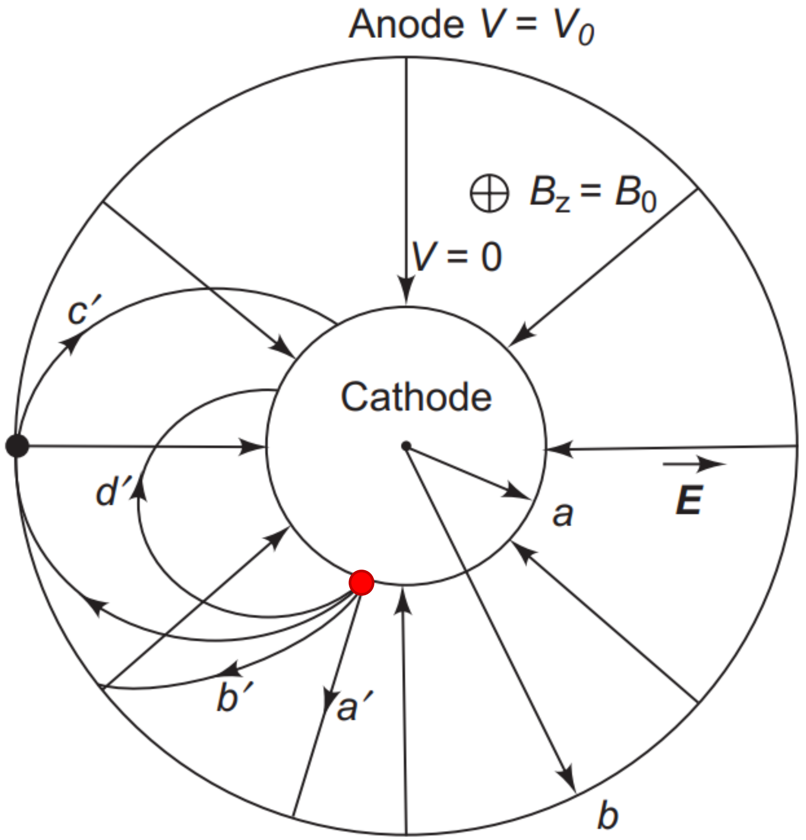
- In general, electron trajectory of radius of curvature r at velocity v experiences radial forces: $-e\bar{E}$ and $-e(\bar{v} \times \bar{B})$ and centrifugal force $\frac{mv^2}{r}$ such that, for equilibrium:

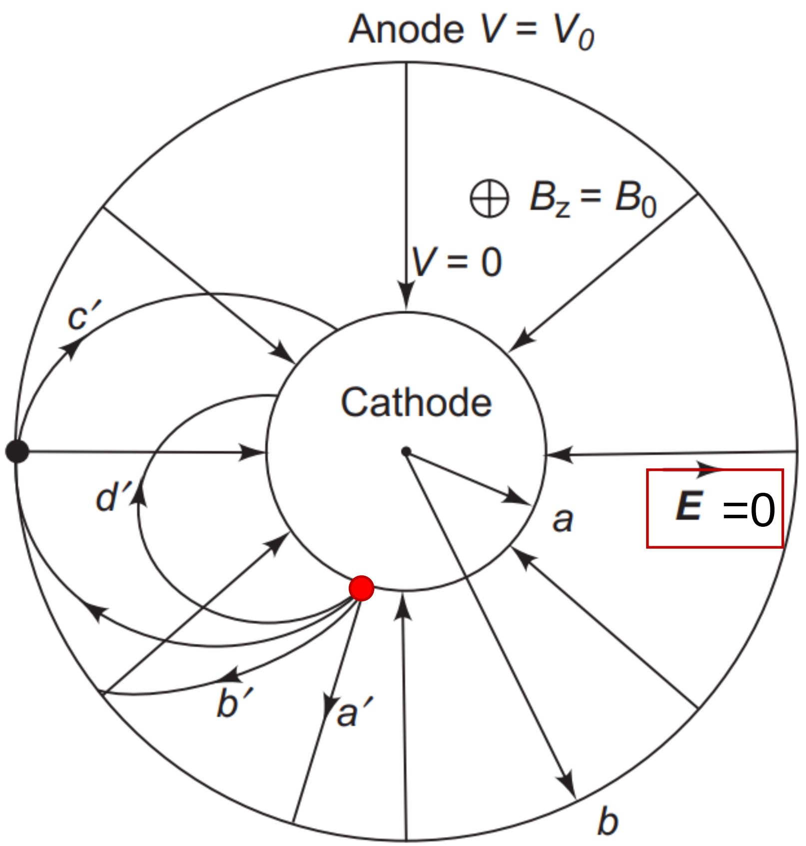


- $\frac{mv^2}{r} + eE = evB$ where $E = E(r) = -\frac{V_0}{r \ln b/a}$

7.5 Equations of electron trajectory

- $\frac{mv^2}{r} + eE = evB$;
- But when $E = 0$, electrons move in circular path to return to cathode when $\frac{mv^2}{r} = evB$

$\frac{v}{r} = \frac{eB}{m} = \omega$ the cyclotron angular frequency



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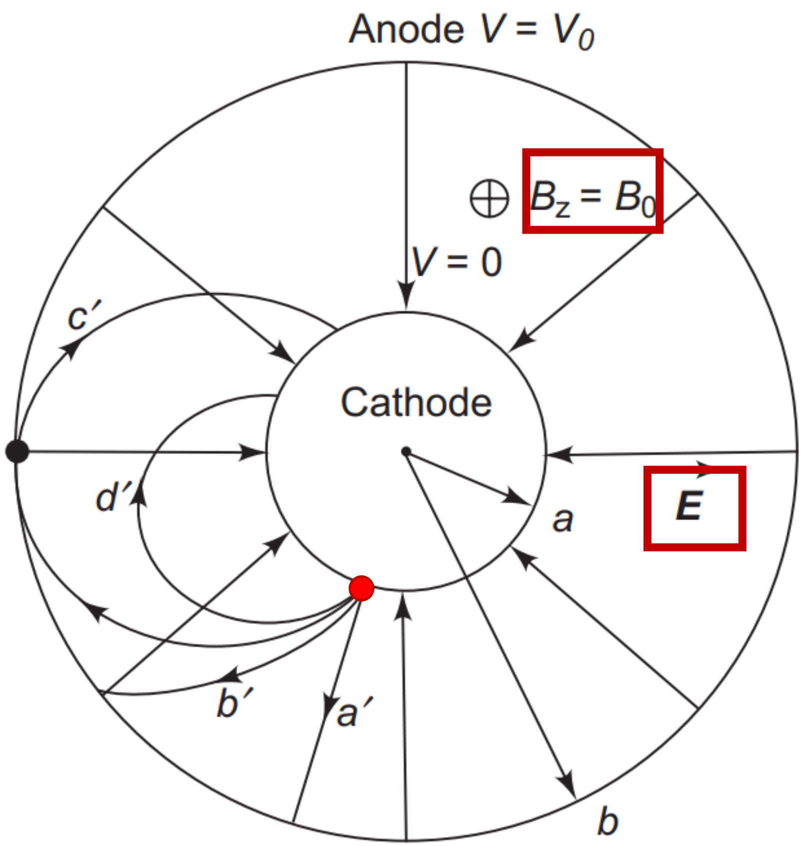
- In the presence of cross electric and magnetic fields, $m\bar{a} = -e\bar{E} - e\bar{v} \times \bar{B}$
  $$m(d\bar{v}/dt) = -e\bar{E} - e\bar{v} \times \bar{B}$$



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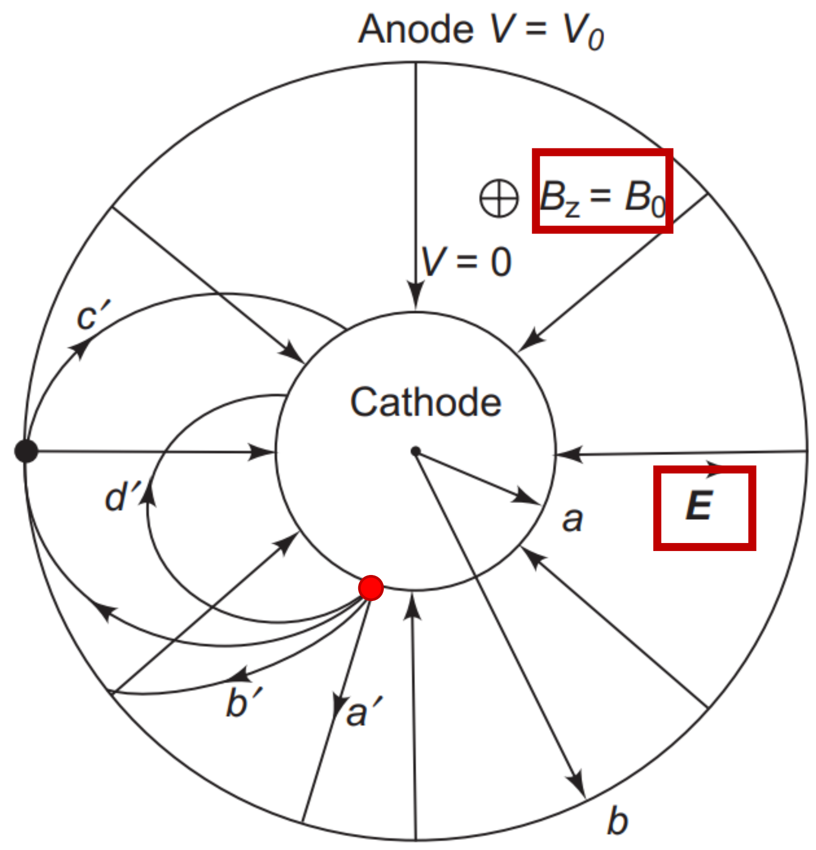
- Electron is emitted from cathode, in direction opposite to $\bar{E}$, the equation of electrons in cylindrical coordinates:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = +\frac{e}{m}\left[E_r - \frac{rB_z d\phi}{dt}\right]$$



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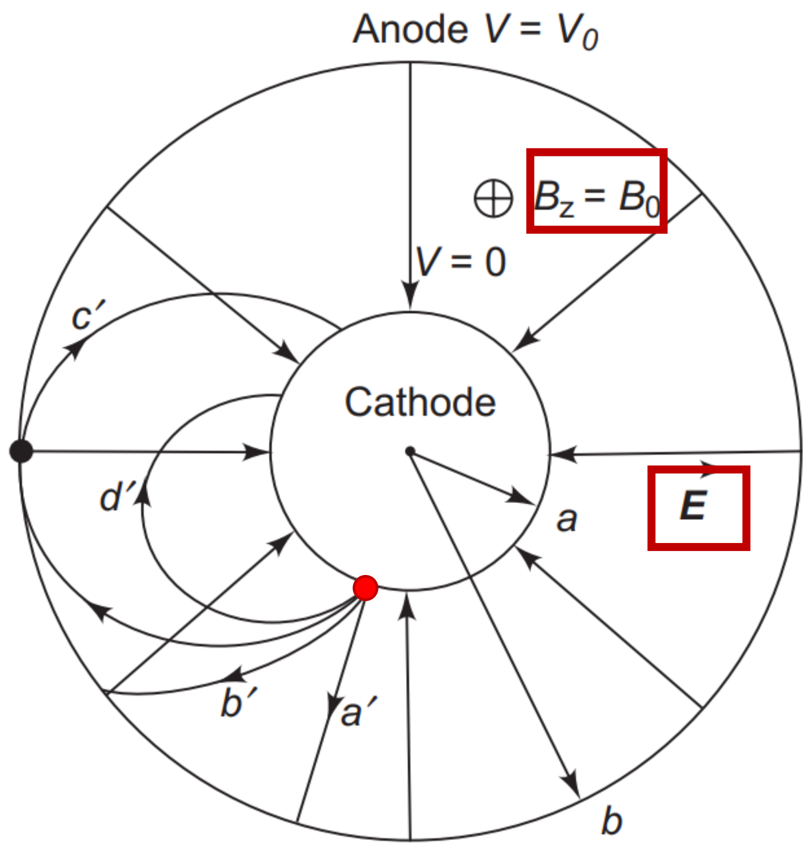

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = +\frac{e}{m}\left[E_r - \frac{rB_z d\phi}{dt}\right]$$

And $\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{eB_z}{m}\frac{dr}{dt}$

7.6 Cutoff magnetic field and voltage

- $\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = +\frac{e}{m}\left[E_r - \frac{rB_z d\phi}{dt}\right]$
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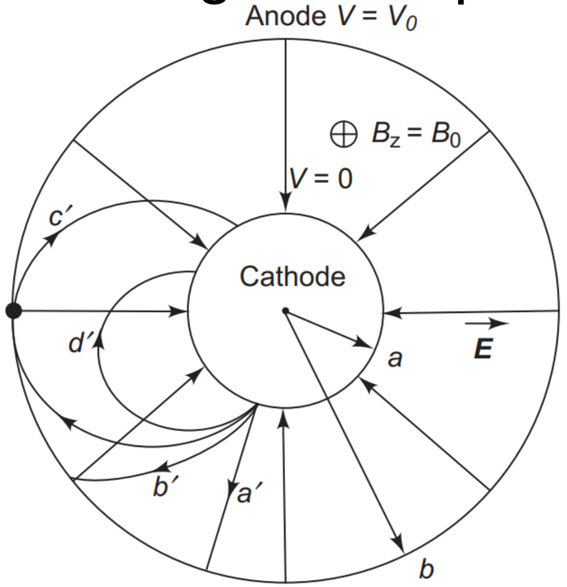

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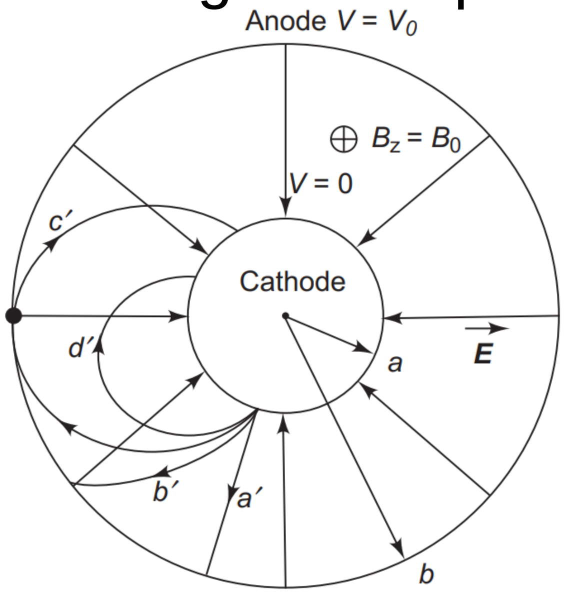
- From the second equation, $\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \omega\frac{rdr}{dt}$

$r^2\frac{d\phi}{dt} = \frac{\omega r^2}{2} + K$ with K being the integration constant



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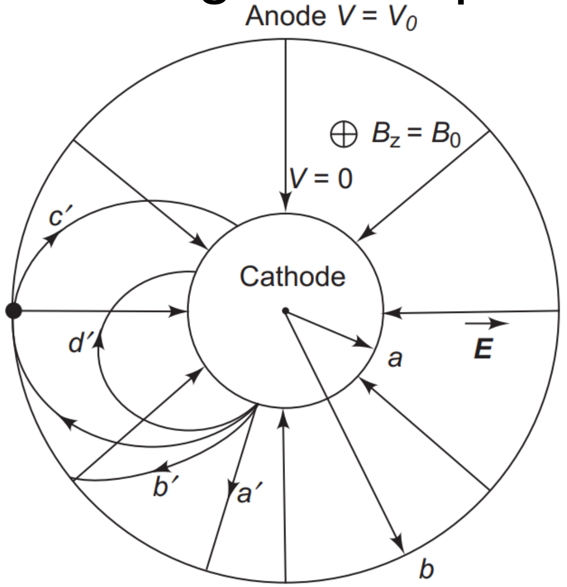

- At $r = a$, $K = -\frac{\omega a^2}{2}$
- $\frac{d\phi}{dt} = \frac{\omega}{2} - \frac{\omega a^2}{2r^2} = \frac{\omega}{2}\left(1 - \frac{a^2}{r^2}\right)$

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- $eV = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right]$

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$$\frac{2eV_0}{m} = \left(b\frac{d\phi}{dt}\right)^2 =$$

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- When the electrons graze, $B = B_c$, the cyclotron frequency $\omega = \frac{eB}{m}$

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- When the electrons graze, $B = B_c$, the cyclotron frequency $\omega = \frac{eB}{m}$
- $b^2\left[\frac{eB_c}{2m}\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2eV_0}{m}$ or

7.6 Cutoff magnetic field and voltage

- $\frac{d\phi}{dt} = \frac{\omega}{2}\left(1 - \frac{a^2}{r^2}\right)$ $\frac{v}{r} = \frac{eB}{m} = \omega$ the cyclotron angular frequency
- $eV = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right]$
- At $r = b$, $V = V_0$ $\frac{dr}{dt} = 0$ for electrons to just graze the anode

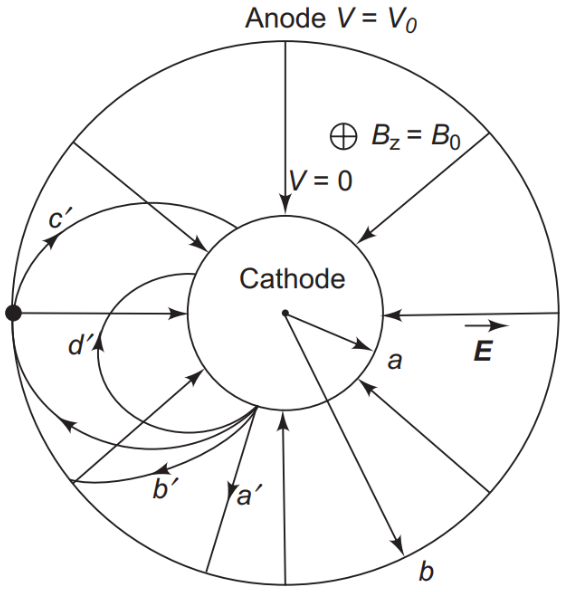
$$\frac{2eV_0}{m} = \left(b\frac{d\phi}{dt}\right)^2 = \left(b\frac{\omega}{2}\left(1 - \frac{a^2}{b^2}\right)\right)^2 = b^2\left[\frac{\omega}{2}\left(1 - \frac{a^2}{b^2}\right)\right]^2$$

- When the electrons graze, $B = B_c$, the cyclotron frequency $\omega = \frac{eB}{m}$
- $b^2\left[\frac{eB_c}{2m}\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2eV_0}{m}$ or $\frac{e^2}{m^2}\frac{B_c^2}{2^2} = \frac{2V_0\frac{e}{m}}{b^2\left(1-\frac{a^2}{b^2}\right)^2}$ or $B_c = \frac{(8V_0 m/e)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$

7.6 Cutoff magnetic field and voltage

- $\frac{d\phi}{dt} = \frac{\omega}{2}\left(1 - \frac{a^2}{r^2}\right)$
- $B_c = \frac{(8V_0 m/e)^{1/2}}{b\left(1 - \frac{a^2}{b^2}\right)}$
- $B_0 > B_c$ for given V_0, electrons will not reach the anode.

$\frac{v}{r} = \frac{eB}{m} = \omega$ the cyclotron angular frequency



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- The cutoff voltage for a given B_0 is given by

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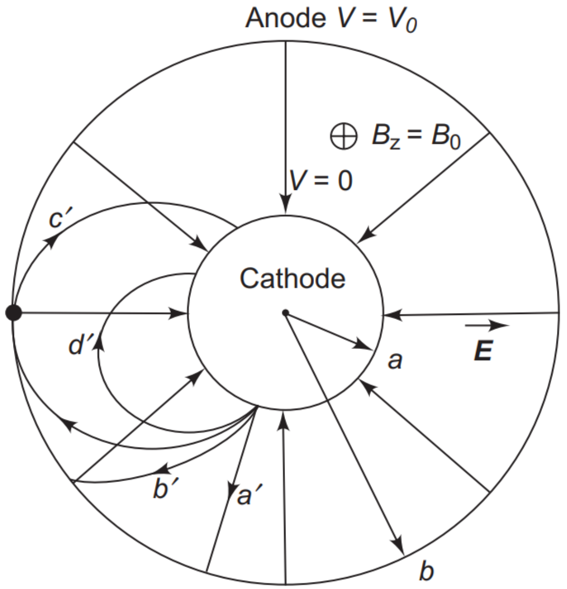

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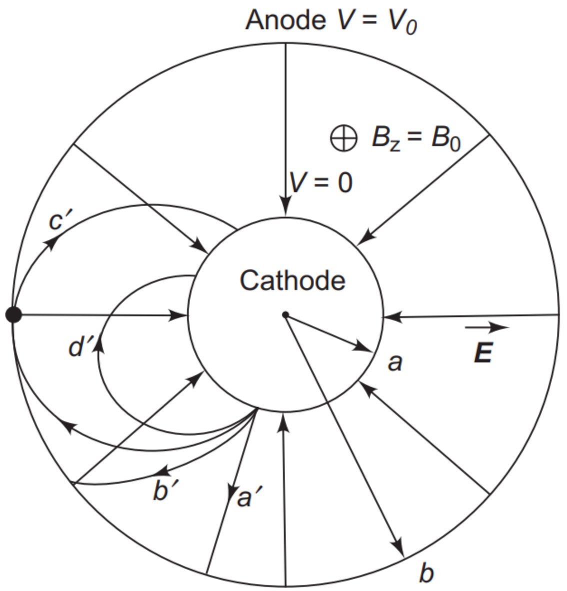
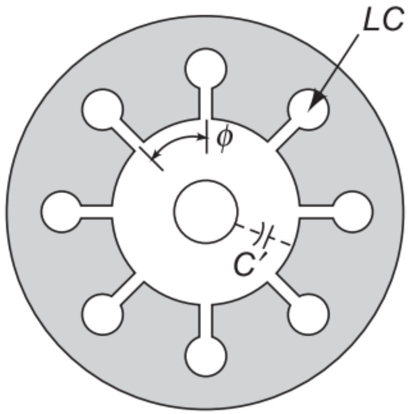


7.7 Resonant modes in magnetron

- Field distribution: Alternating RF magnetic flux lines through cavities parallel to cathode – RF Electric fields at cathode are concentrated across the slot and fringe out to the interaction space between anode and cathode (transverse direction)



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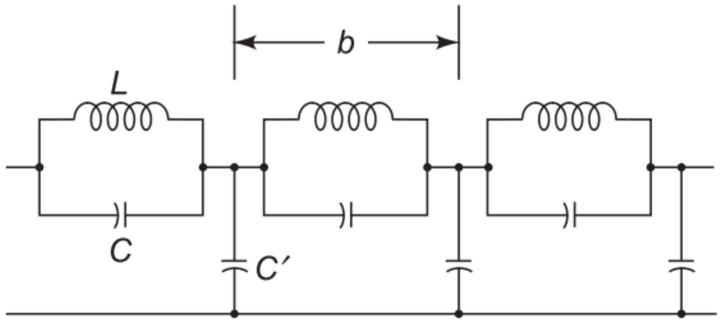

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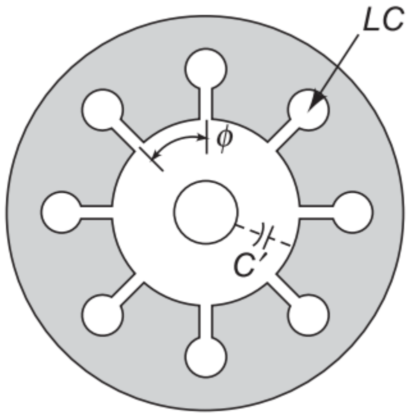

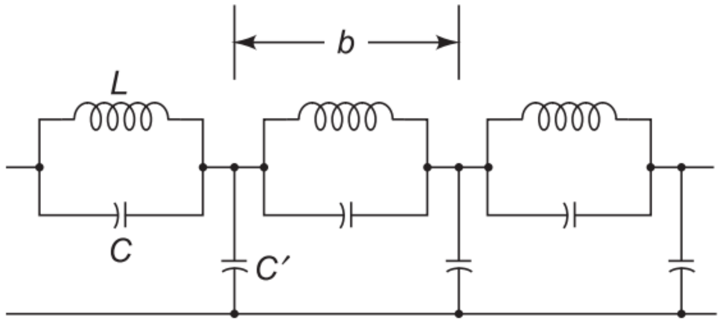

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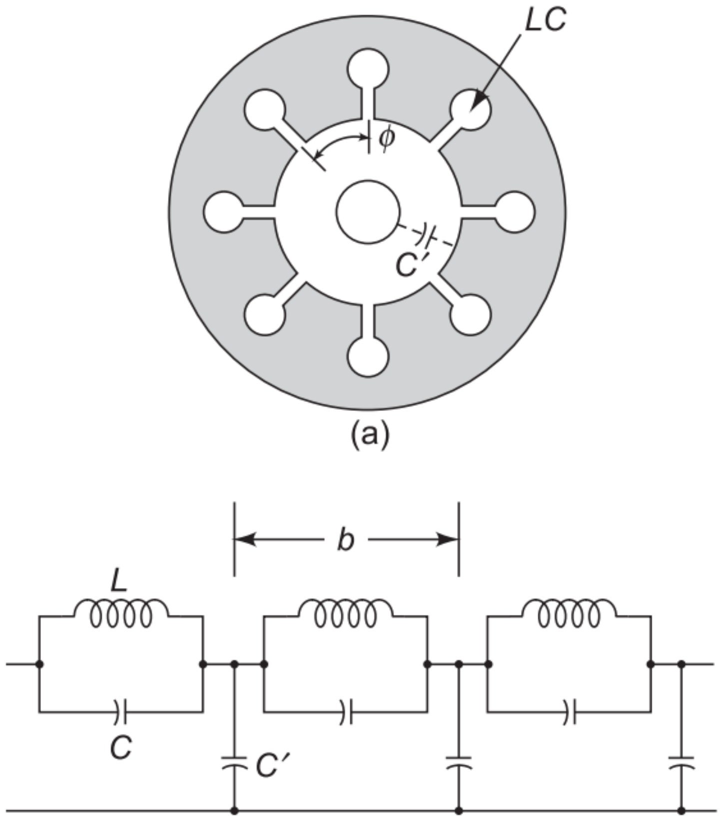
- As the Slow wave structure is closed on itself, total phase shift around internal periphery – should be multiple of 2π for possible oscillations.
- .



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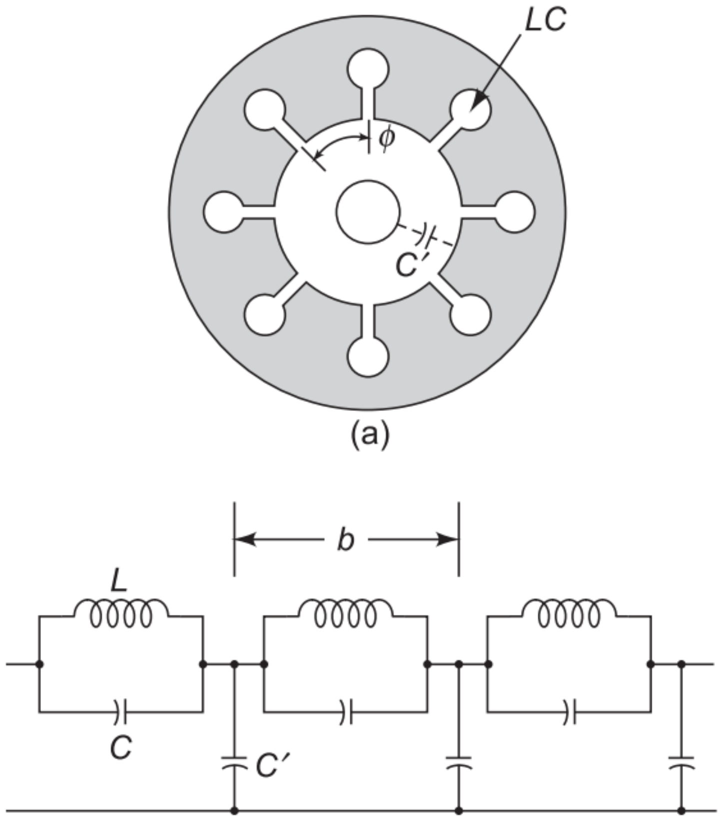
- As the Slow wave structure is closed on itself, total phase shift around internal periphery – should be multiple of 2π for possible oscillations.
- Phase-shift between adjacent cavities $\phi = \frac{2\pi n}{N}$ where $n = \pm 1, \pm 2, \ldots \pm \frac{N}{2}$ n: nth mode of oscillation and $n \neq 0$.



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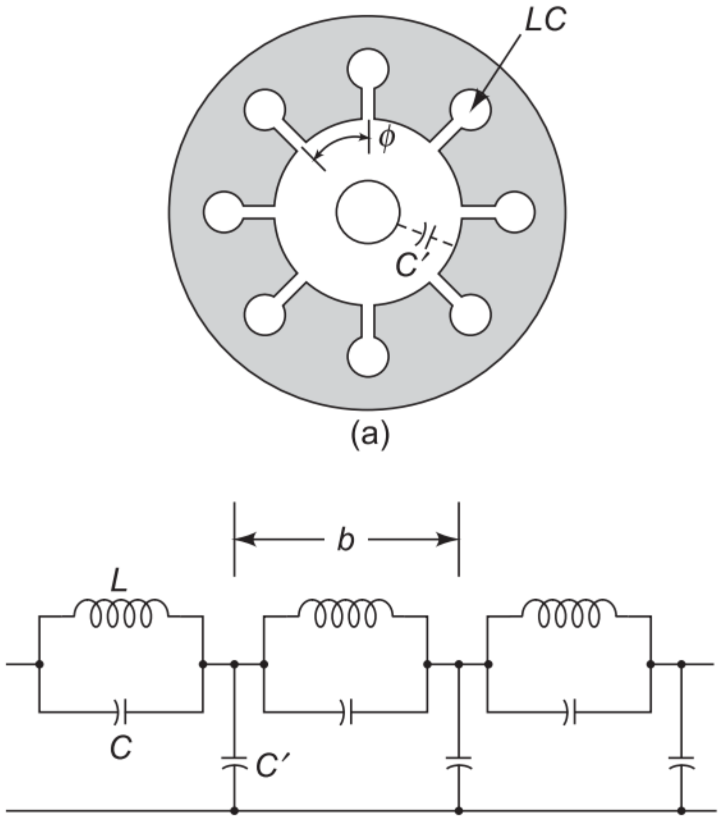

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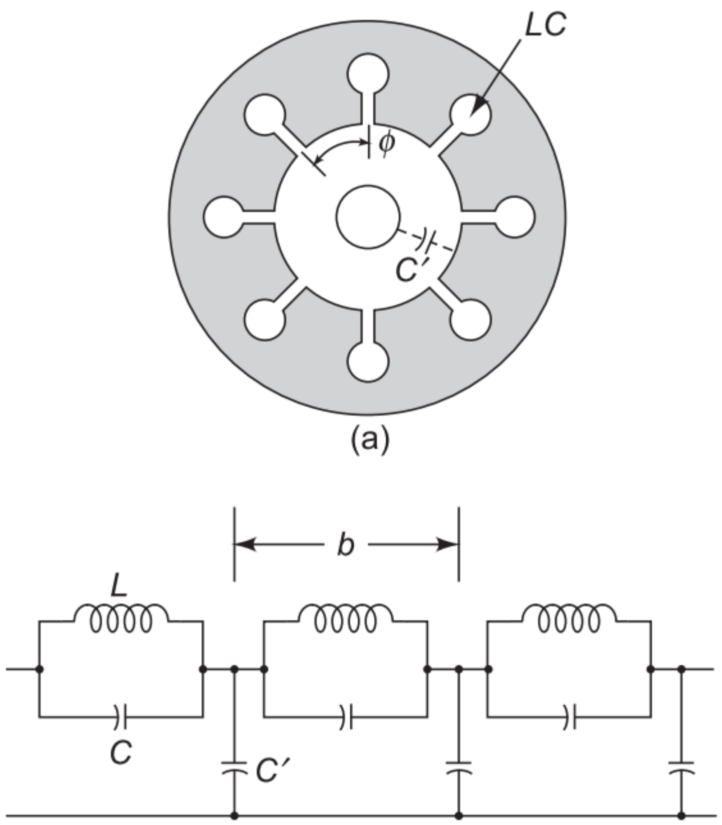

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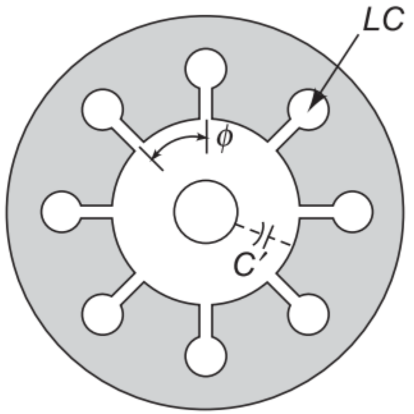
- Continuous interaction between electrons and RF fields for transfer of energy, anode dc voltage V_0 - to match the average rotational velocity of electrons with phase velocity of RF field in the interaction space
- Since opposite phases in cavities - excitation is maximum

$\Phi_n = \pi$ or π mode is preferred with $n = \frac{N}{2}$



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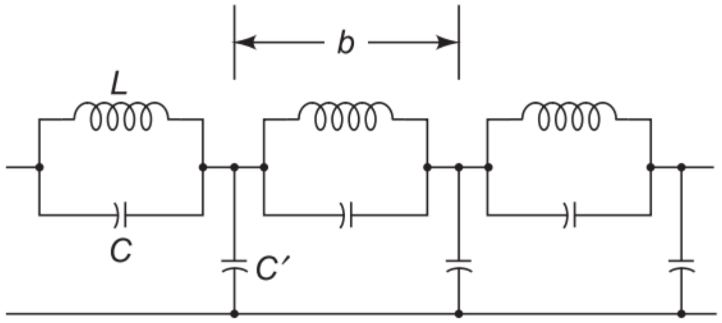

7.8 Average drift velocity

- Electrons move in both ϕ and r directions
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- $v_\phi = \frac{E_r}{B_z}$
- If $v_\phi < \frac{E_r}{B_z}$ then electrons will be deflected to anode, collected at anode
- If $v_\phi > \frac{E_r}{B_z}$ then electrons will be deflected to cathode

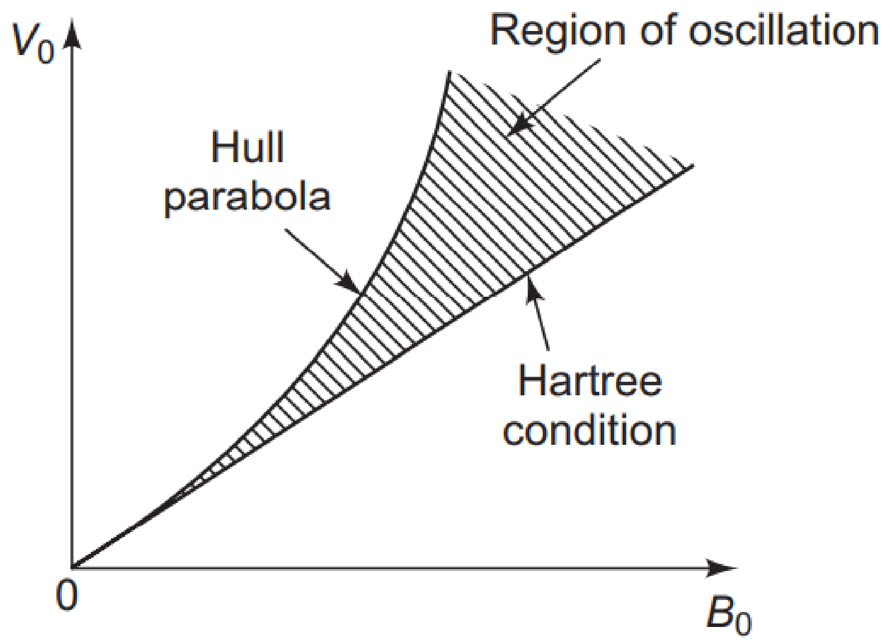
7.9 Hartree voltage

$\frac{v}{r} = \frac{eB}{m} = \omega$ the cyclotron angular frequency

- The cutoff voltage for a given B_0 is given by

$$V_c = \frac{e}{8m} b^2 \left(1 - \frac{a^2}{b^2}\right) B_0^2$$

Magnetrons usually operate in π mode where phase difference between adjacent cavities is 180°.



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Phase velocity should be near drift velocity v_ϕ and oscillations for π mode need to start at beam voltage

$$V_{oh} = \frac{2\pi f}{N} b^2 (b^2 - a^2) B_0$$

Is the Hartree voltage

f: Operating frequency with N resonators