7.2 Microwave Transistor Amplifiers and Stability

Module:7 Microwave Active Circuits

Course: BECE305L – Antenna and Microwave Engineering

-Dr Richards Joe Stanislaus

Assistant Professor - SENSE

Email: richards.stanislaus@vit.ac.in

VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Module:7 Microwave Active Circuits

- Microwave transistors, Microwave amplifiers: Two port power gains, stability of the amplifier, Microwave oscillators

- Source of the contents: Pozar

4.1 Microwave Transistor Amplifiers

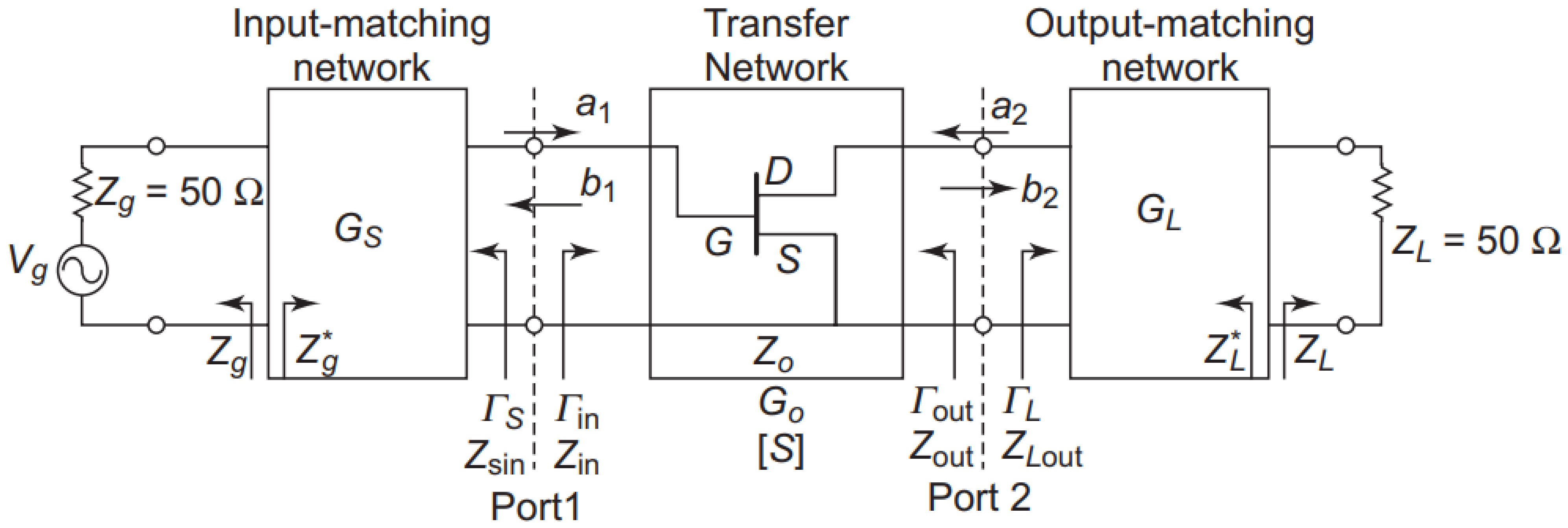
- Analyzed and designed based on the S-parameters of the device and certain performance requirements such as
  stability,
  power gain,
  bandwidth,
  noise, and
  dc biasing.
- classified as
  (i) narrow band amplifiers
  (ii) low-noise amplifiers,
  (ii) broadband amplifiers, and
  (iv) power amplifiers.

4.1 Microwave Transistor Amplifiers

- In a system, **a low-noise transistor** is used in the first stage since this is the major determinant of the overall system noise.
- **Driver stage transistors** are used for power amplification and power transistors are used for final power output.
- **Impedance-matching circuits**: designed at the input and output ports of the transistor
  for maximum power transfer from the source to the transmitter input and from the transistor output to the load.

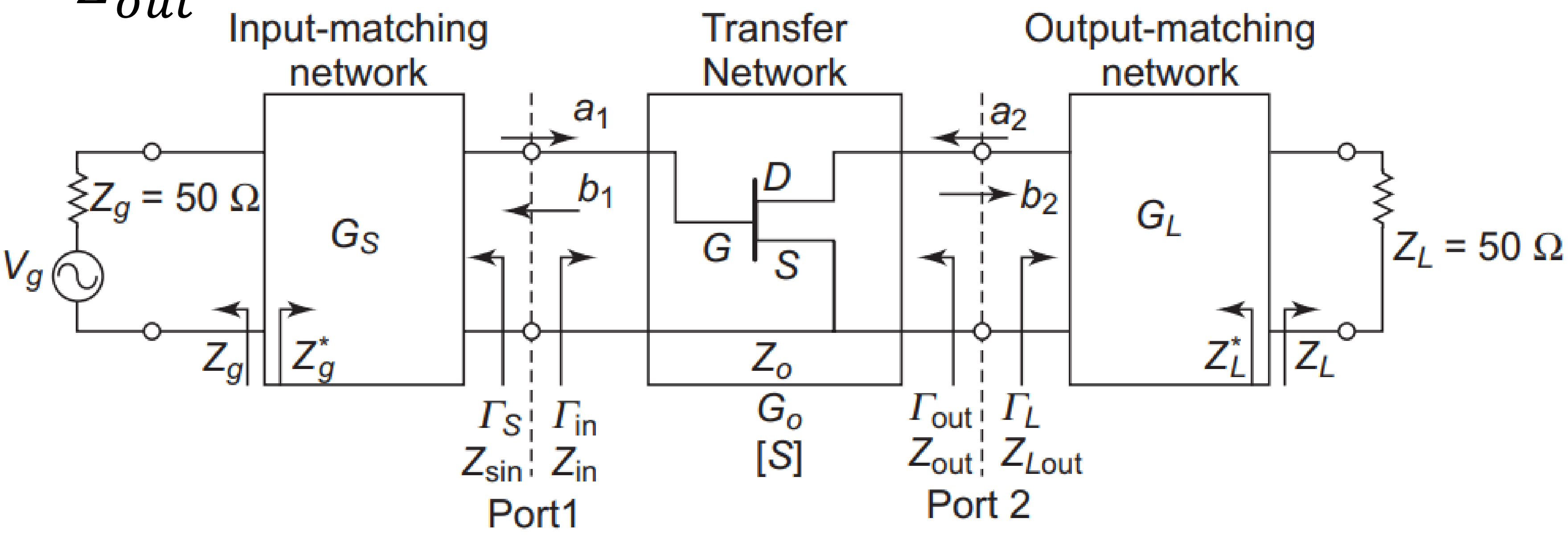
4.2 Impedance matching condition

- Power is transferred from input to outside port efficiently when there is **impedance match between the ports**.
- Therefore, a single-stage microwave transistor amplifier can be modeled by the circuit :.



4.2 Impedance matching condition

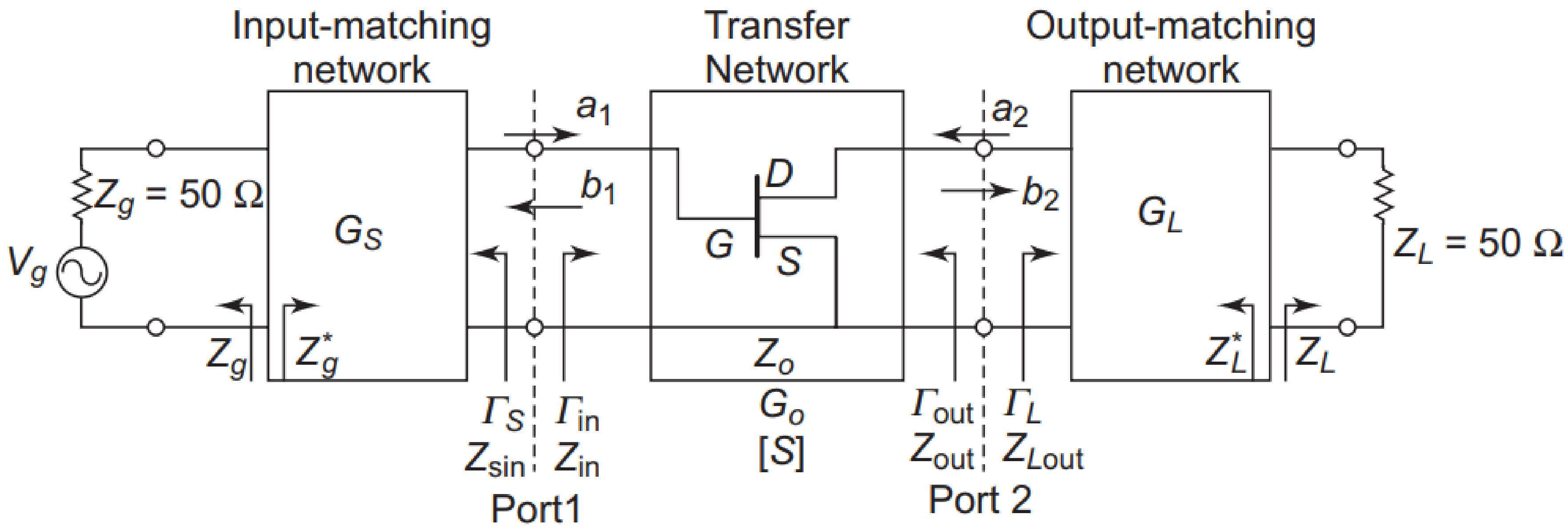
- a matching network is introduced on both sides of the transistor
  - to **transform the input impedance** of **the transistor amplifier** to **generator impedance** Z_g
  - to **transform the output impedance** to the **load impedances** Z_l
- Each of the impedance matching network input and output ports: **must have conjugate pair impedances** looking on both sides, i.e., $Z_{s_{in}} = Z_{in}^{*}$, $Z_{L_{out}} = Z_{out}^{*}$



4.2 Impedance matching condition

- **Perfect matching cannot be achieved**
- With reference to Z0 = 50 ohms, practical input and output reflection coefficients, looking towards the indicated directions at input/output ports, can be defined as follows for the given S-parameters of the transistor :

$(\Gamma_s = \Gamma_{in}{}^* = \Gamma_{sm}, \Gamma_L = \Gamma_{out}{}^* = \Gamma_{Lm}$ for conjugate match)



4.2 Impedance matching condition

- Perfect matching cannot be achieved
- With reference to Z0 = 50 ohms, practical input and output reflection coefficients, looking towards the indicated directions at input/output ports, can be defined as follows for the given S-parameters of the transistor :

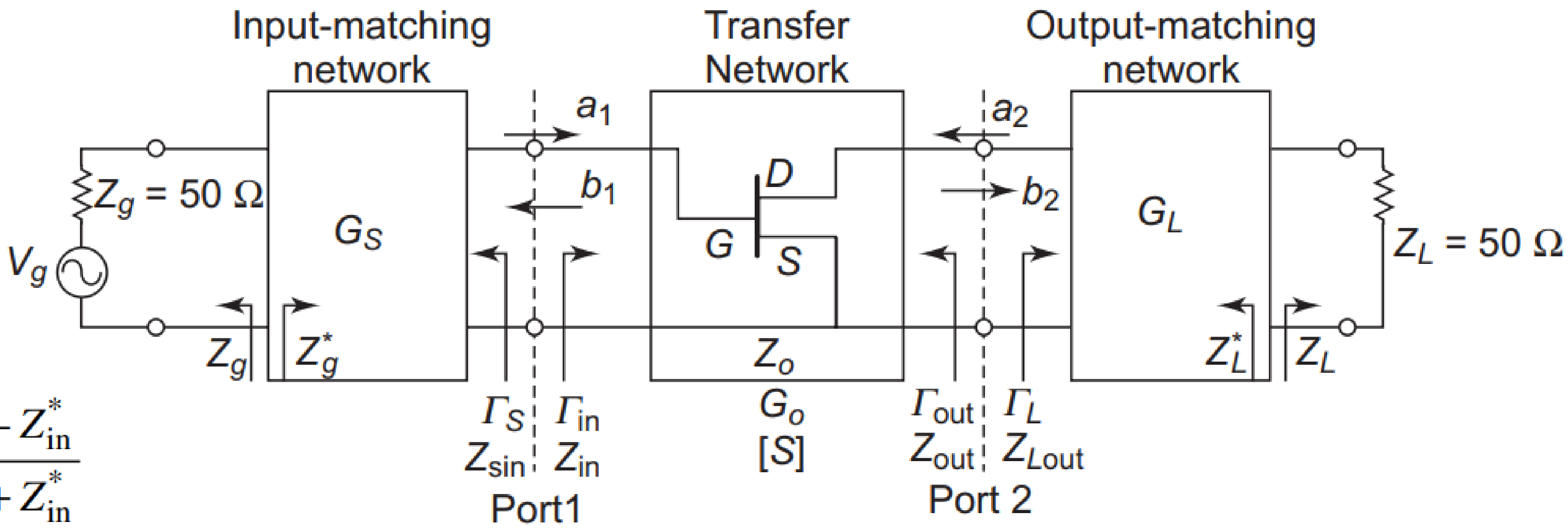
$$(\Gamma_s = \Gamma_{in}{}^* = \Gamma_{sm}, \Gamma_L = \Gamma_{out}{}^* = \Gamma_{Lm} \text{ for conjugate match})$$

- Refection coefficient looking towards load at the output port

$$\Gamma_L = \frac{Z_{Lout} - Z_{out}^*}{Z_{Lout} + Z_{out}^*}$$

- Reflection coefficient looking towards source at the input port

$$\Gamma_s = \frac{Z_{sin} - Z_{in}^*}{Z_{sin} + Z_{in}^*}$$



4.2 Impedance matching condition

$$\Gamma_L = \frac{Z_{L\,\text{out}} - Z^*_{\text{out}}}{Z_{L\,\text{out}} + Z^*_{\text{out}}}$$

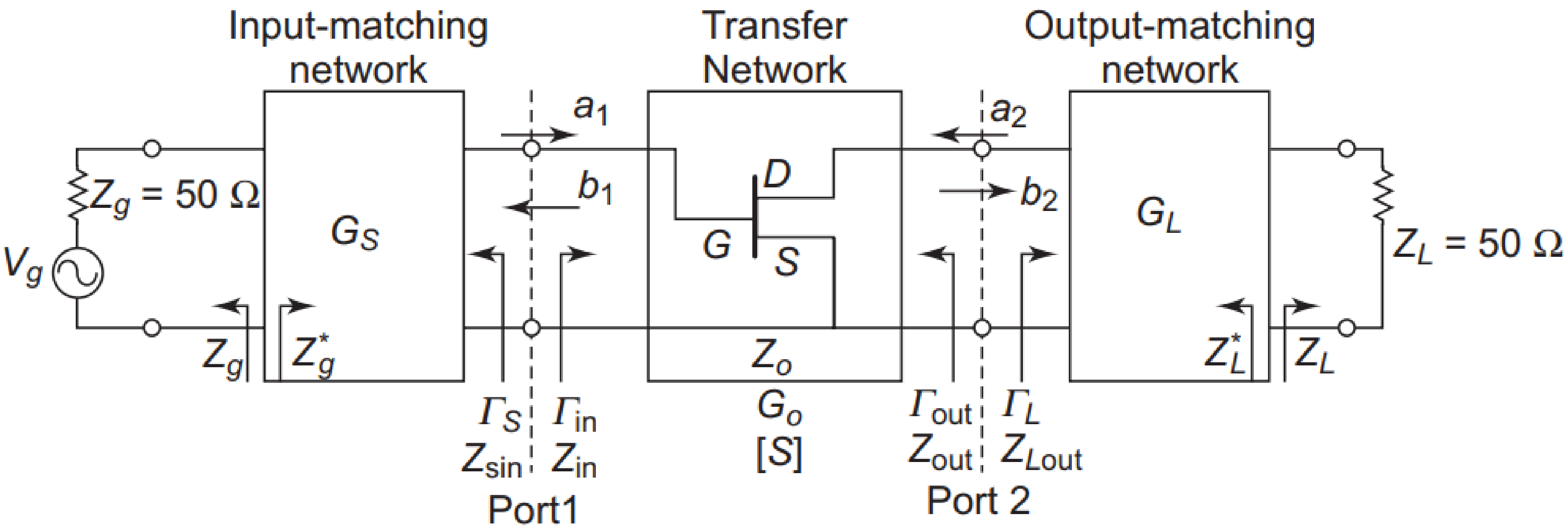
($\Gamma_s = \Gamma_{in}^* = \Gamma_{sm}$, $\Gamma_L = \Gamma_{out}^* = \Gamma_{Lm}$ for conjugate match)

$$\Gamma_s = \frac{Z_{s\,\text{in}} - Z^*_{\text{in}}}{Z_{s\,\text{in}} + Z^*_{\text{in}}}$$

Reflection coefficient looking into Port 2 $\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = \frac{Z_{\text{out}} - Z^*_{L\,\text{out}}}{Z_{\text{out}} + Z^*_{L\,\text{out}}} = \frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s}$

Reflection coefficient seen looking into Port 1 $\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{\text{in}} - Z^*_{s\,\text{in}}}{Z_{\text{in}} + Z^*_{s\,\text{in}}} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$



4.3 Amplifier Gain Equations

$P_L =$ Power absorbed in the load $Z_L = \frac{1}{2}\left(|b_2|^2 - |a_2|^2\right)$

$P_{in} =$ Power delivered to the input of the transistor $= \frac{1}{2}\left(|a_1|^2 - |b_1|^2\right)$

P_{avn} = Power available from the transistor network $= P_L|_{\Gamma_L=\Gamma^*_{out}}$

$P_{avs} =$ Power available from the source to the network $= P_{in}|_{\Gamma in = \Gamma s*}$

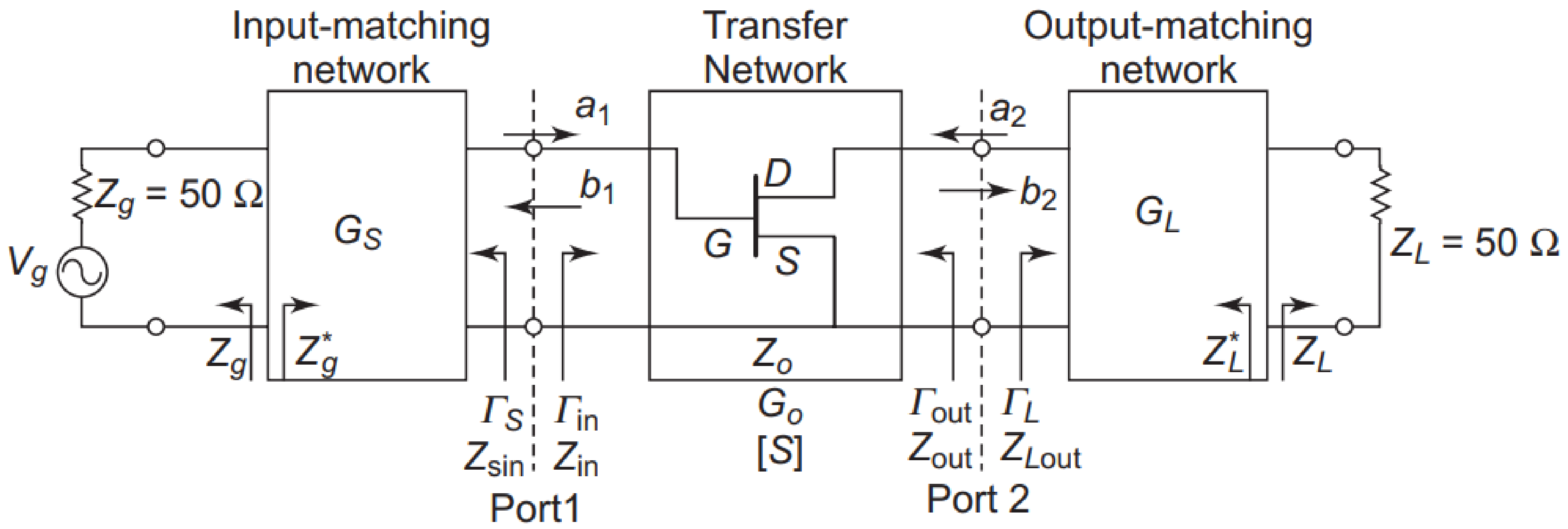
Power gain $G_p = \frac{P_L}{P_{in}}$, independent of $Z_{s_{in}}$ but depends of $Z_{L_{out}}$

Available power gain $G_A = \frac{P_{avn}}{P_{avs}}$

independent of Z_L but depends of $Z_{s_{in}}$

Transducer power gain $G_T = \frac{P_L}{P_{avs}}$

depends on both $Z_{s_{in}}$ and $Z_{L_{out}}$



4.3 Amplifier Gain Equations

power gain

$$G_P = \left|\frac{b_2}{a_1}\right|^2 \frac{1-|\Gamma_L|^2}{1-|\Gamma_{in}|^2} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{in}|^2)}$$

$$\Gamma_L = \Gamma_{out}^*$$

Available power gain

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}$$

Transducer Power Gain

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}$$

When both input and output are matched

$$\Gamma_L = \Gamma_s = 0,$$

$$\therefore \quad G_T = |S_{21}|^2$$

Transducer power gain could have three cases:

(a) Unilateral transducer power gain (non-reciprocal), where $S_{12} = 0$

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_s|^2|1-S_{22}\Gamma_L|^2}$$

(b) Maximum unilateral transducer gain, where $\Gamma_s = S_{11}^*$, $\Gamma_L = S_{22}^*$:

$$G_{TU\max} = \frac{|S_{21}|^2}{(|1-|S_{11}|^2)(|1-|S_{22}|^2)}$$

(c) Matched transducer gain ($\Gamma_s = \Gamma_L = 0$)

$$G_{TM} = |S_{21}|^2$$

4.3 Amplifier Gain Equations

- In the amplifier design, the most useful gain is the transducer power gain which accounts for both source and load mismatch and is defined by

$$G_T = G_S \,.\, G_0 .\, G_L$$

$$G_s = \frac{1-|\Gamma_s|^2}{|1-\Gamma_{in}\Gamma_s|^2}$$

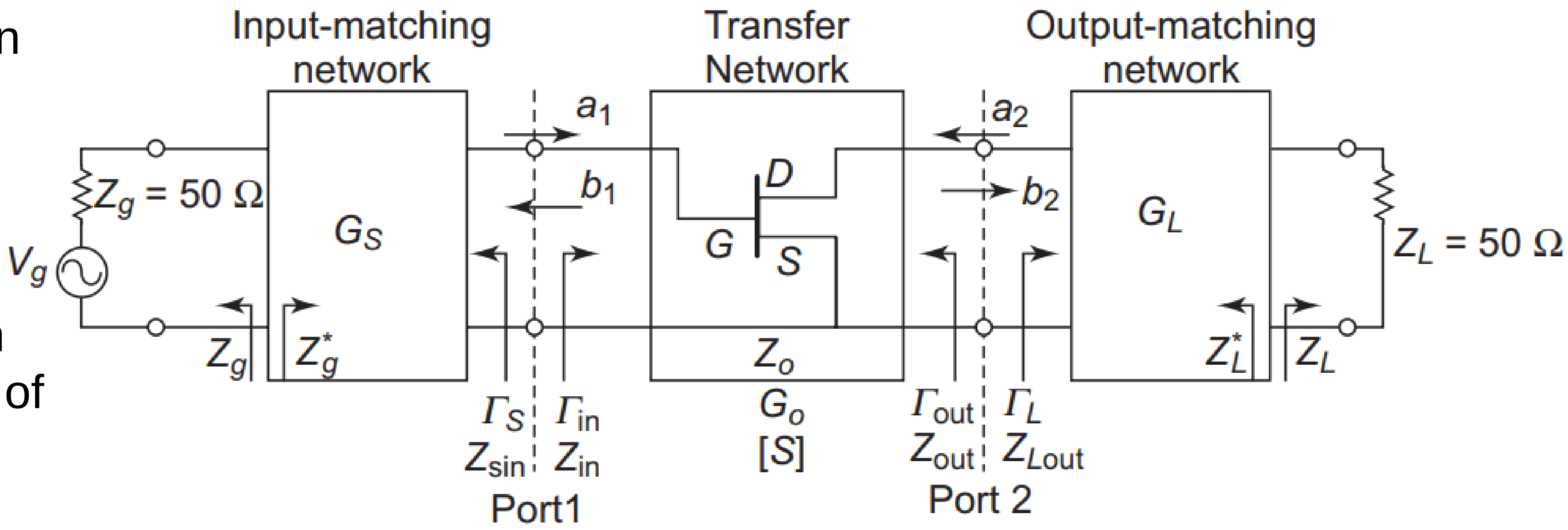
The source-matching network gain due to the impedance matching of the transistor to the impedance Z0,

$$G_0 = |S_{21}|^2$$

The transistor gain

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

The load-matching network gain due to the impedance matching of the transistor to Z0



4.4 Problem: The S-parameters of a transistor at 5 GHz for a conjugate matched transistor amplifier are given by S11 = 0.9 ∠-100°, S21 = 2.4 ∠90°, S12 = 0, S22= 0.8 ∠40°. Determine the maximum gain.

- For maximum gain: $\Gamma_s = S_{11}^*, \Gamma_L = S_{22}^*$
- The maximum gain of the conjugate matched transistor amplifier is

$$G_{TU\max} = \frac{|S_{21}|^2}{(|1-|S_{11}|^2)(|1-|S_{22}|^2)}$$

$$= \frac{(2.4)^2}{\left[\{1-(0.9)^2\}\{(1-(0.8)^2\}\right]} = 84.21 = 19.3 \text{ dB}$$

4.5 A GaAs MESFET has the following S-parameters at 5 GHz with a 50 ohm reference

$S_{11} = 0.5 \angle 160°$, $S_{12} = 0.1 \angle 10°$, $S_{21} = 2.0 \angle 10°$, $S_{22} = 0.4 \angle 150°$

The source and load impedances are $Z_{s\,in} = 30$ ohms, $Z_{L\,out} = 40\ \Omega$. Calculate G, G_A and G_T.

4.5 A GaAs MESFET has the following S-parameters at 5 GHz with a 50 ohm reference

$S_{11} = 0.5\ \angle 160°$, $S_{12} = 0.1\ \angle 10°$, $S_{21} = 2.0\ \angle 10°$, $S_{22} = 0.4\ \angle 150°$

The source and load impedances are $Z_{sin} = 30$ ohms, $Z_{L\,out} = 40\ \Omega$. Calculate G, G_A and G_T.

Since S-parameters are measured with 50 ohm reference $Z^*_{in} = Z^*_{out} = 50$ ohms

$$\Gamma_s = \frac{30-50}{30+50} = -0.25 \qquad \Gamma_L = \frac{40-50}{40+50} = -0.11$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.5\ \angle 160° + \frac{0.1\angle 10° * 2.0\angle 10° * (-0.11)}{1 - 0.4\angle 150° * (-0.11)}$$

$$= 0.5182\ \angle -161.5970°$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = 0.4\ \angle 150° + \frac{0.1\angle 10° * 2.0\angle 10° * (-0.25)}{1 - 0.5\angle 160° * (-0.25)}$$

$$= 0.4404\ \angle -155.4148°$$

4.5 A GaAs MESFET has the following S-parameters at 5 GHz with a 50 ohm reference

$S_{11} = 0.5\ \angle 160°$, $S_{12} = 0.1\ \angle 10°$, $S_{21} = 2.0\ \angle 10°$, $S_{22} = 0.4\ \angle 150°$

The source and load impedances are $Z_{sin} = 30$ ohms, $Z_{L\,out} = 40\ \Omega$. Calculate G, G_A and G_T.

Since S-parameters are measured with 50 ohm reference $Z^*_{in} = Z^*_{out} = 50$ ohms

$$\Gamma_s = \frac{30-50}{30+50} = -0.25 \qquad \Gamma_L = \frac{40-50}{40+50} = -0.11$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = 0.5\ \angle 160° + \frac{0.1\angle 10° * 2.0\angle 10° * (-0.11)}{1-0.4\angle 150° * (-0.11)}$$

$$= 0.5182\ \angle -161.5970°$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s} = 0.4\ \angle 150° + \frac{0.1\angle 10° * 2.0\angle 10° * (-0.25)}{1-0.5\angle 160° * (-0.25)}$$

$$= 0.4404\ \angle -155.4148°$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = 5.8392$$

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)} = 5.9591$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_2|^2} = 5.1398$$

4.6 Amplifier Stability

- The passive matching networks at the input and output produces $Re\ (Zs)\ >\ 0$ and $Re\ (ZL) > 0$ such that $|\Gamma_s|\ <\ 1$ and $|\Gamma_L|\ <\ 1$.
- From expressions of Γ_{in} and Γ_{out} it can be seen that for certain values of S-parameters, even if $|\Gamma_s|\ <\ 1$ and $|\Gamma_L|\ <\ 1$ and/or $|\,\Gamma_{out}\,| > 1$ representing presence of negative resistance at the input and/or output and oscillations can occur.
- Stability of an amplifier, or its resistance to oscillate is a very important design consideration.
- The stability is determined from the S-parameters of the transistor, the matching network and the terminations.

4.6 Amplifier Stability

- There are two types of stability defined:
- **<u>Unconditional Stability</u>**: Achieved when $Re(Zin) > 0$ and $Re(Zout) > 0$

$$\left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = |\Gamma_{in}| < 1 \text{ and } |\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| < 1 \qquad |\Gamma_s| < 1 \text{ and } |\Gamma_L| < 1$$

for all passive source and load impedances, i.e.,

- **<u>Conditional Stability</u>** Achieved when $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$

only for a certain range of passive source and load impedances. This is potentially unstable. If the device is unilateral (S_{12} = 0), the condition for unconditional stability becomes

$$|S_{11}| < 1 \text{ and } |S_{22}| < 1.$$

4.6 Amplifier Stability

- Stability can be expressed by a factor

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12} S_{21}|^2} > 1$$

$$|\Delta| = |S_{11} S_{22} - S_{12} S_{21}| < 1$$

- If $K > 1$ and $|\Delta| > 1$ or,

  $K < 1$ and $|\Delta| < 1$

the network becomes unstable.

- For unilateral device (S12 = 0) $K = \infty$, $|\Delta| < 1$ for stability.

4.7 Amplifier Stability Problem:

A GaAs FET has the following S-parameters at 6 GHz with $V_{ds} = 4\text{V}$ and $I_{ds} = 30$ mA and reference impedance $Z_0 = 50$ ohms.

$$S_{11} = 0.9\ \angle{-60^\circ}$$

$$S_{21} = 3.1\ \angle 124^\circ$$

$$S_{12} = 0.02\ \angle 62^\circ$$

$$S_{22} = 0.8\ \angle{-28^\circ}$$

Determine the stability of this transistor.

4.7 Amplifier Stability Problem:

A GaAs FET has the following S-parameters at 6 GHz with $V_{ds} = 4$V and $I_{ds} = 30$ mA and reference impedance $Z_0 = 50$ ohms.

Determine the stability of this transistor.

$S_{11} = 0.9 \angle -60°$

$S_{21} = 3.1 \angle 124°$

$S_{12} = 0.02 \angle 62°$

$S_{22} = 0.8 \angle -28°$

Delta factor $|\Delta| = S_{11} S_{22} - S_{12} S_{21} = 0.7183$

$$\text{Stability factor } K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2 |S_{12} S_{21}|^2} = 0.5324$$

Since $\Delta = 0.72 < 1$ and $K = 0.53 < 1$, the device is potentially unstable.