5.1a S Parameter Problem

Module:5 Microwave Passive components

Course: BECE305L – Antenna and Microwave Engineering

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Assistant Professor - SENSE

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Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Module:5 Microwave Passive components 6 hours

- Microwave Networks - ABCD, ‘S’ parameter and its properties. E-Plane Tee, H-Plane Tee, Magic Tee and Multi-hole directional coupler. Principle of Faraday rotation, isolator, circulator and phase shifter.

- Source of the contents: Book by Pozar
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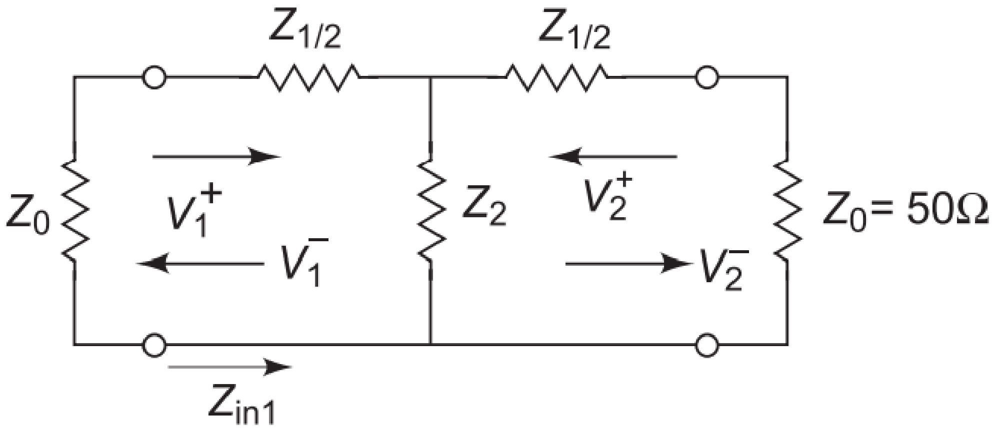
Determine the S matrix of 3dB T network attenuator terminated in 50Ω matched load with $Z_1 = 17.12\ \Omega$ abd $Z_2 = 141.78\Omega$



- $\frac{Z_1}{2} = \frac{17.12}{2} = 8.56$
- $S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \frac{b_1}{a_1} = \frac{Z_{in1}-Z_0}{Z_{in1}+Z_0}$

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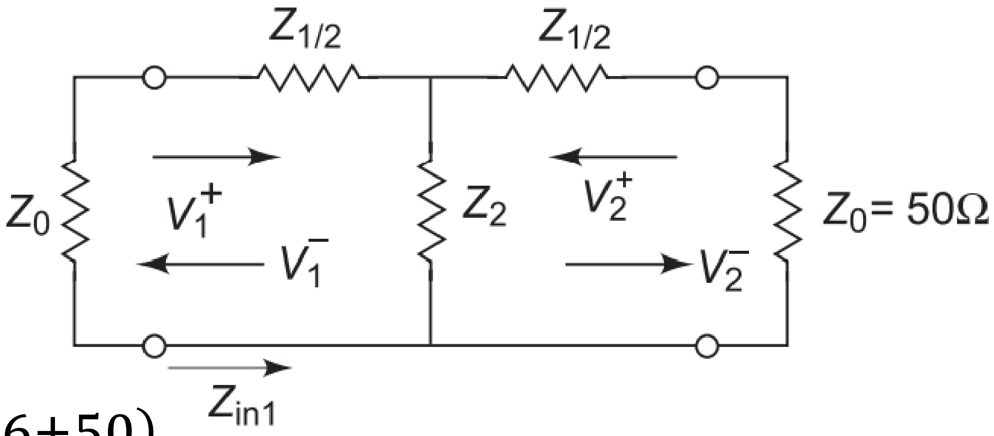

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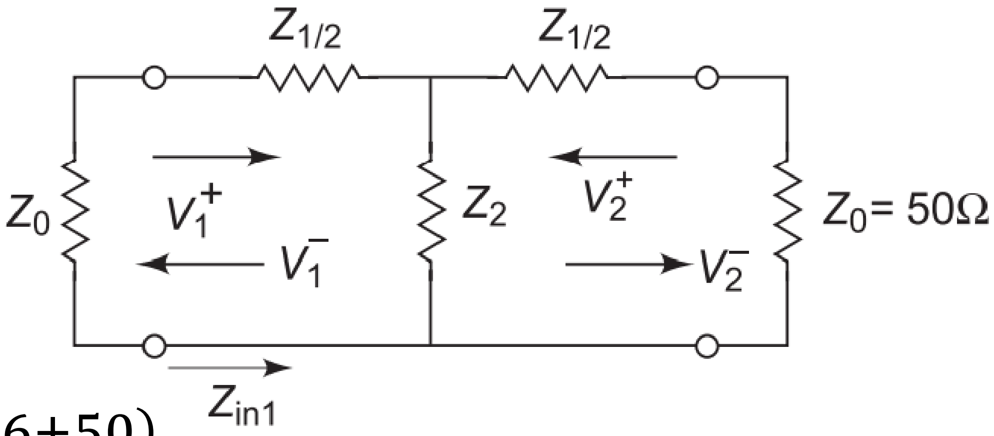

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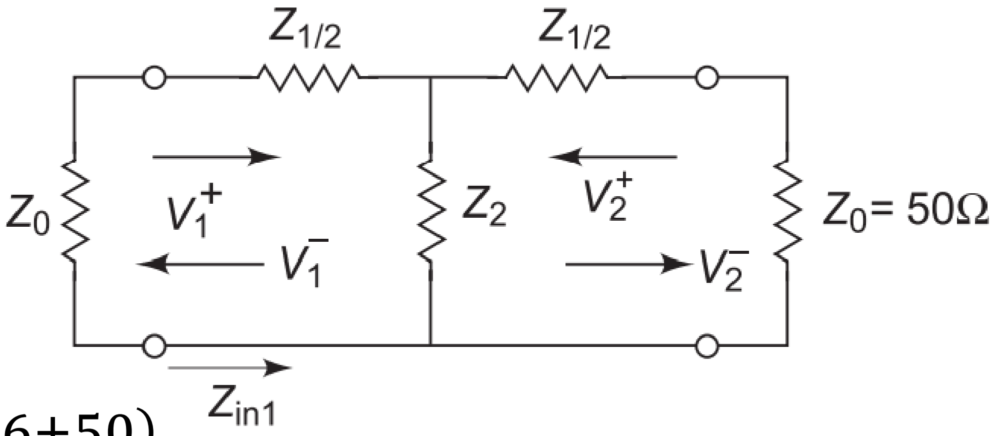

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For symmetry, matched network, $S_{22} = S_{11} = 0, V_1^- = 0, \mathrm{V}_2^+ = 0$

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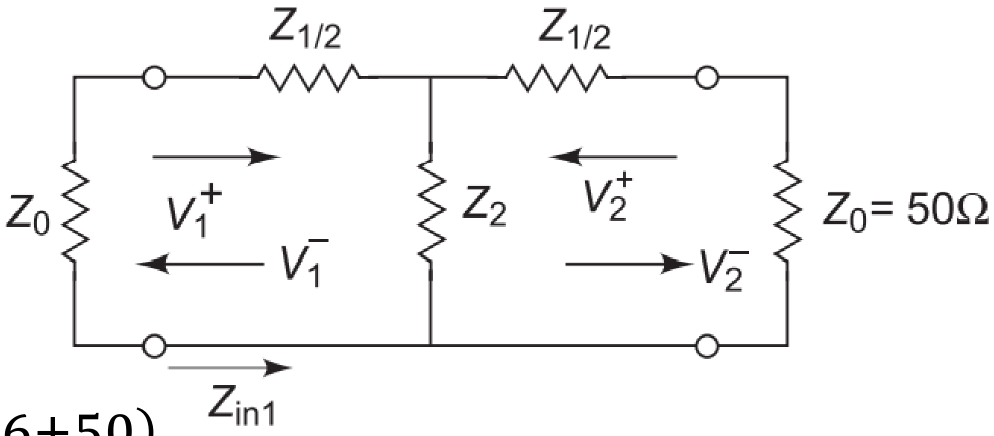

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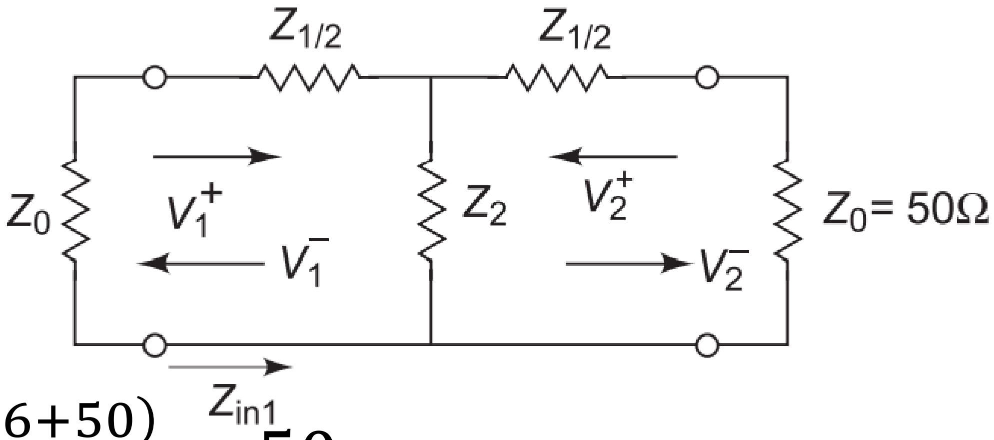

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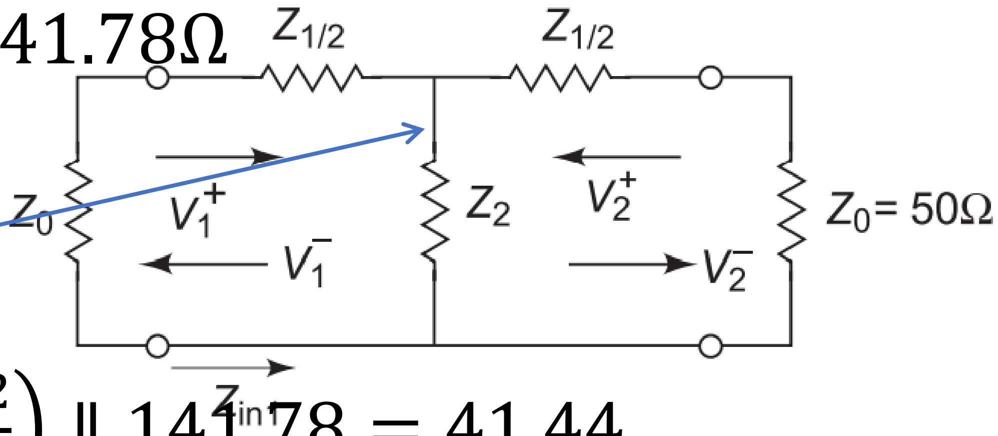

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Again, Impedance seen at center= $\left(50 + \frac{17.12}{2}\right) \parallel 141.78 = 41.44$

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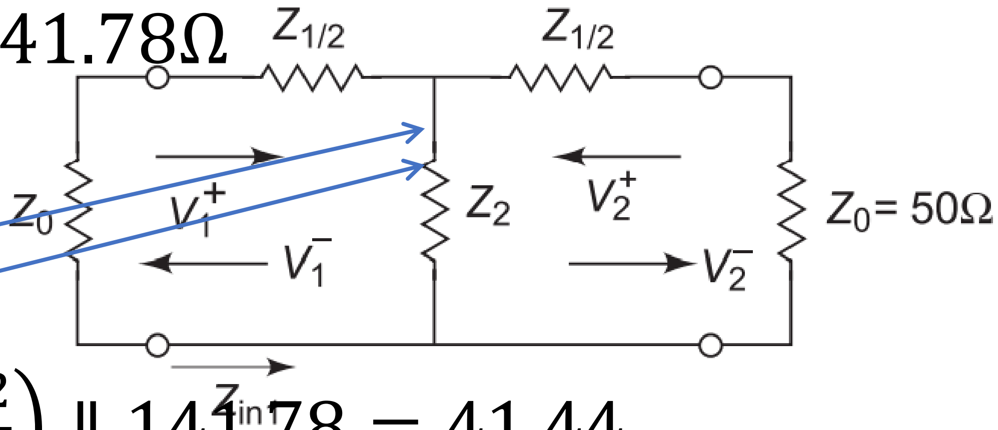

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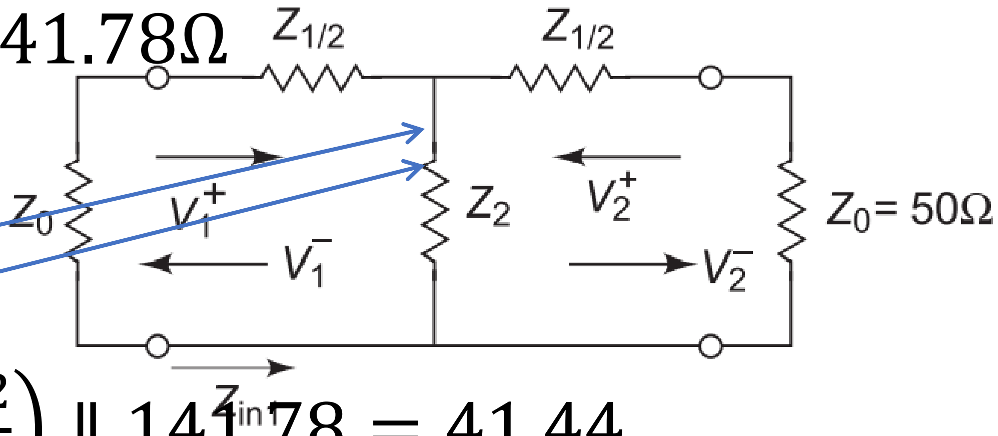

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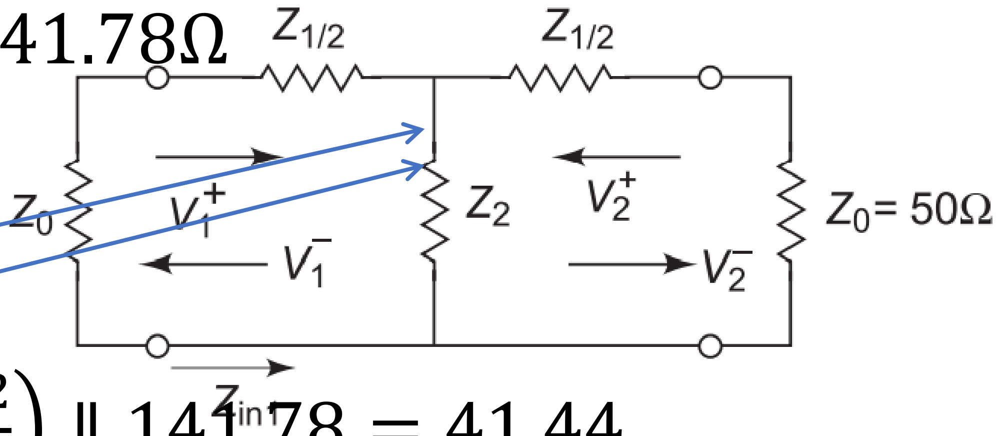

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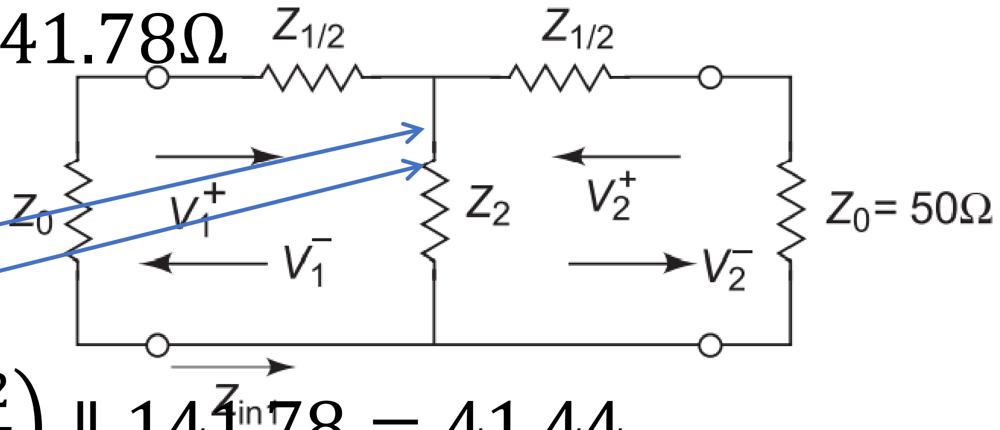

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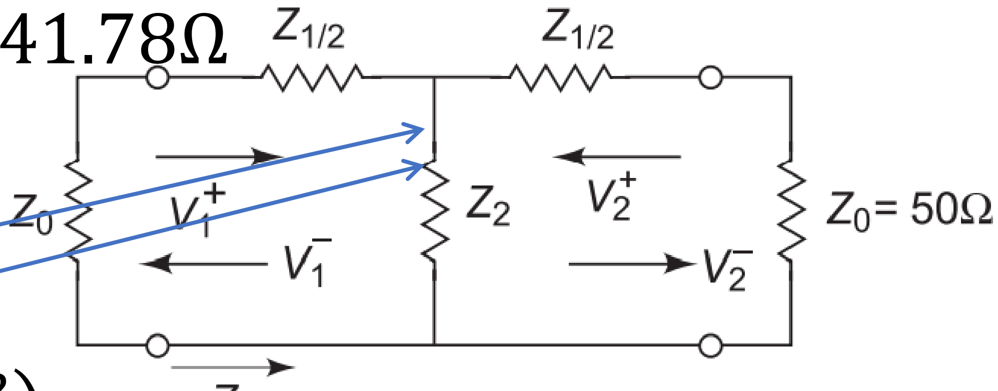

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$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

2. S matrix of a two port network is

$$[S] = \begin{bmatrix} 0.2\angle 0° & 0.6\angle 90° \\ 0.6\angle 90° & 0.1\angle 0° \end{bmatrix}$$

Prove that network is reciprocal but not lossless.

Find return loss at port 1 when port 2 is short circuited.

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$$= \begin{bmatrix} 0.4 + 0.36 & 0.12j - 0.6j \\ -0.12j + 0.6j & 0.1 \end{bmatrix} =$$

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$$= \begin{bmatrix} 0.4 + 0.36 & 0.12j - 0.6j \\ -0.12j + 0.6j & 0.1 \end{bmatrix} = \begin{bmatrix} 0.76 & 0.6j \\ -0.6j & 0.1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$[\boldsymbol{S}^*]^{\boldsymbol{T}}[\boldsymbol{S}] \neq [\boldsymbol{I}]$ **Not lossless**

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$$Input\ reflection\ coefficient\ \Gamma_1 = S_{11} + \frac{S_{12}^2\Gamma_2}{1 - S_{22}\Gamma_2}$$

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Γ_2 for short circuit is -1.

$$\Gamma_1 = S_{11} + \frac{S_{12}^2(-1)}{1-S_{22}(-1)} =$$

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Return loss $= -20\log_{10}|\Gamma_1| = -20\log_{10}(0.5273) = 5.56dB$

1. A four port network has the scattering matrix shown below.
a) Is the network lossless? b) Is the network reciprocal?
c) Find return loss at port 1 when all other ports are terminated with matched loads.
d) Find insertion loss between Ports 2 and 4 when all other ports are terminated with matched load.
e) Find phase delay between Ports 2 and 4 when all other ports are terminated with matched load.
f) Find reflection coefficient seen at port 1 if a short circuit is placed at terminal plane of port 3 and all other ports are terminated with matched load.

$$[S] = \begin{bmatrix} 0.178\angle 90° & 0.6\angle 45° & 0.4\angle 45° & 0 \\ 0.6\angle 45° & 0 & 0 & 0.3\angle -45° \\ 0.4\angle 45° & 0 & 0 & 0.5\angle -45° \\ 0 & 0.3\angle -45° & 0.5\angle -45° & 0 \end{bmatrix}$$

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a) For Lossless

$[S]$ must be unitary

First row

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2$$

$$= 0.178^2 + 0.6^2 + 0.4^2 + 0$$

$$= 0.552 \neq 1$$

The network is not lossless

1. A four port network has the scattering matrix shown below.

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b) For Reciprocal

$[S]$ must be Symmetric

Given matrix is symmetric and hence it is reciprocal

$$[S] = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix}$$

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c) When ports 2,3,4 are matched, $\Gamma = S_{11}$

Return loss$= -20\log_{10}|\Gamma|$

$$= -20\log_{10}|0.178|$$

$$= 15dB$$

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d) Find insertion loss between Ports 2 and 4 when all other ports are terminated with matched load.

e) Find phase delay between Ports 2 and 4 when all other ports are terminated with matched load.

f) Find reflection coefficient seen at port 1 if a short circuit is placed at terminal plane of port 3 and all other ports are terminated with matched load.

$$[S] = \begin{bmatrix} 0.178\angle 90° & 0.6\angle 45° & 0.4\angle 45° & 0 \\ 0.6\angle 45° & 0 & 0 & 0.3\angle -45° \\ 0.4\angle 45° & 0 & 0 & 0.5\angle -45° \\ 0 & 0.3\angle -45° & 0.5\angle -45° & 0 \end{bmatrix}$$

d) When ports 1 and 3 are matched with Z_0 , $V_1^+ = 0$ $V_3^+ = 0.$ $V_4^- = S_{42}V_2^+$

Insertion loss $IL = -20\log_{10}|S_{42}|$

$$= -20\log_{10}|0.3|$$

$$= 10.45dB$$

e) Phase delay = $+45°$

1. A four port network has the scattering matrix shown below.

a) Is the network lossless? b) Is the network reciprocal?

c) Find return loss at port 1 when all other ports are terminated with matched loads.

d) Find insertion loss between Ports 2 and 4 when all other ports are terminated with matched load.

e) Find phase delay between Ports 2 and 4 when all other ports are terminated with matched load.

f) Find reflection coefficient seen at port 1 if a short circuit is placed at terminal plane of port 3 and all other ports are terminated with matched load.

$$[S] = \begin{bmatrix} 0.178\angle 90° & 0.6\angle 45° & 0.4\angle 45° & 0 \\ 0.6\angle 45° & 0 & 0 & 0.3\angle -45° \\ 0.4\angle 45° & 0 & 0 & 0.5\angle -45° \\ 0 & 0.3\angle -45° & 0.5\angle -45° & 0 \end{bmatrix}$$

f) Ports 2 and 4 are matched

$$V_2^+ = V_4^+ = 0$$

Short at port 3: $V_3^+ = -V_3^-$

$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+$$
$$= S_{11}V_1^+ - S_{13}V_3^-$$
$$V_3^- = S_{31}V_1^+$$

$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+} = S_{11} - S_{13}S_{31} = 0.178j - (0.4\angle 45°)(0.4\angle 45°)$$
$$= 0.018\angle 90°$$