Module 5 - Microwave Networks

ABCD, 'S' parameter and its properties. E-Plane Tee, H-Plane Tee, Magic Tee and Multi-hole directional coupler. Principle of Faraday rotation, isolator, circulator and phase shifter

Microwaves

- Microwaves as the word spells are extremely short waves.
- Microwaves are a part of electromagnetic spectrum from 3 GHz to 30 GHz. But more often the spectrum between 300 MHz (100 cm) to 300 GHz (1 mm) covering 'RF Microwave Millimeter Waves' is may be broadly referred as microwave region

Why Microwave?

- 1. Antenna gain is proportional to the electric size of the antenna.
- f \uparrow , gain \uparrow miniature microwave system possible f \uparrow \Rightarrow available bandwidth \uparrow
- e.g., TV BW=6MHz

10% BW of VHF @60MHz for 1channel

1% BW of U-band @60GHz for 100 channels

(Wide bandwidth and smaller component size)

Why Microwave?

- 2. Line of sight propagation and not affected by cloud, fog ⇒frequency reuse in satellite and terrestrial communications (frequency division duplexing, FDD)
- 3. Radar cross section (RCS) is proportional to the target electrical size. \Rightarrow frequency \uparrow , RCS \uparrow
- 4. Molecular, atomic and nuclear resonances occur at microwave frequency
- ⇒astronomy, medical diagnostics and treatment, remote sensing and industrial heating applications
- 5. Biological effects and safety

non-ionized radiation \Rightarrow thermal effect

Excessive radiation may be dangerous to brain, eye, genital, stomach organs ⇒cataract, sterility, cancer,.....

Demerits

- Problem in generation, transmission and circuit design.
- More expensive components

Component of Microwave System

- Device to generate MW signal
- A medium to guide the signal
- Some components to perform tasks such as adding, coupling, amplifying, filtering
- Some equipments and methods to assess the performance characteristics
- Antenna

Microwave Applications

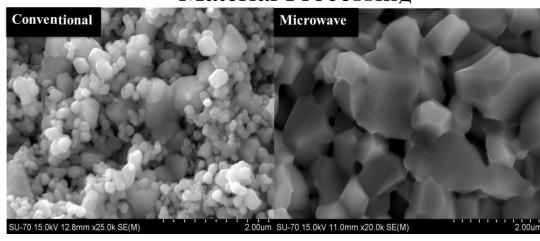
Microwaves applications:

- Scientific
- Medical
- Military and Civilian War
- Industrial Applications
 - Food Chemical -Rubber -Textiles —Plastics Paper
- Ceramic Cosmetics
- It is used in different industry for;
 - Cooking Baking Puffing Drying
 Curing Evaporating Sterilizing Molding ... etc.

Microwave Oven



Material Processing



Wood Drying



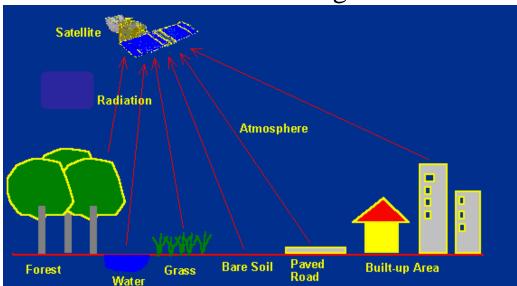
Agriculture



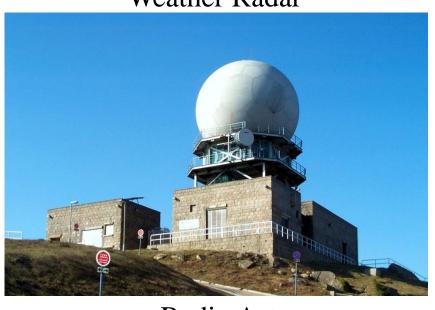
Military Radar



Remote Sensing



Weather Radar



Radio Astronomy



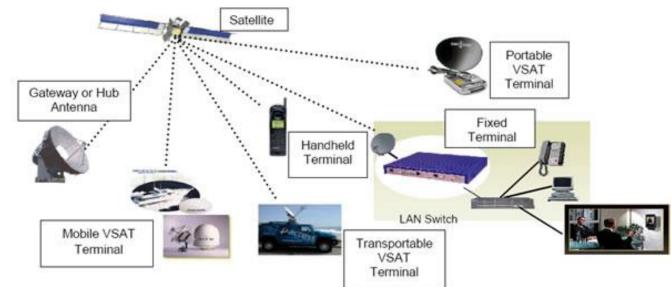
Imaging

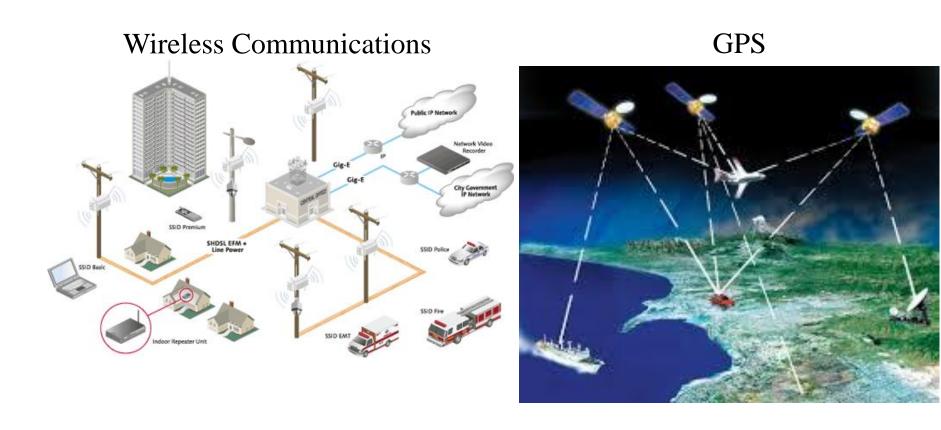


Medicine



Satellite Comm.





• Apart from the above applications microwaves find applications in spectroscopy, cancer treatment, collision avoidance systems, and in basic science (particle and nuclear physics)

Microwave Network Analysis

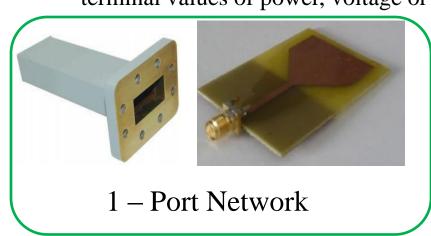
Microwave Network

• A *Microwave Network* is single or group of components or devices (active or passive) connected together using transmission lines (eg: waveguides, coaxial cables or microstrip lines etc)

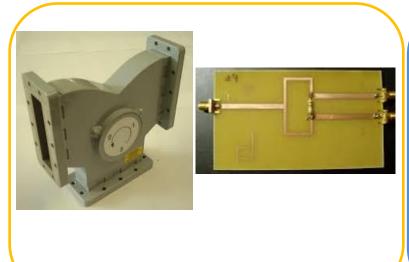
Low Frequency Analysis	High Frequency Analysis
Theorems are applied to determine V and I	Maxwell eqn to determine E and H
The physical length of the Interconnecting elements are very small compared to wavelength of signal transmitted.	Interconnecting elements are several wavelengths and propagation delay cannot be ignored
Measurable V and I are related in terms of Z, Y, h and ABCD parameters	S parameter
Cascading and analysis is easier	Difficult

Microwave Network

Microwave networks are basically characterized by the number of ports or terminals they have. Since in designing a network we are interested only in the terminal values of power, voltage or current.







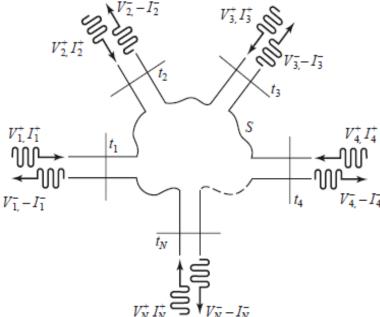
3 – Port Network



Impedance Matrix

- If voltages and vurrents of either TEM or non-TEM modes is given then we can use impedance or admittance information of that network to find voltages and currents at other locations
 - We consider an N-port network, with either a two terminal transmission line or a waveguide port for a given mode

• If more than one mode needs to be analysed then we define more ports for each of these modes



• The Impedance Matrix [Z] relates the voltages and currents as

$$[V] = [Z][I]$$

Or a more concise matrix representation is
$$[V] = [Z][I]$$

$$Z_{ij} = \frac{V_i}{I_j}|_{I_k = 0, k \neq j}$$

- This equation tells us that Z_{ij} can be found if we drive port j with the current I_i , while keeping all ports open, and measuring open circuit voltage V_i at port i
- Now Z_{II} is the input impedance of port 1 looking into the network and Z_{12} is the transfer impedance from port 1 to port 2, when all other ports are kept open

Use of Z matrix

Easy to combine series network

Admittance Matrix

The Admittance Matrix [Y] relates the voltages and currents as

or a more concise matrix representation is

$$[I] = [Y][V]$$
 From which we can note $[Y] = [Z]^{-1}$

where you can write
$$Y_{ij} = \frac{I_i}{V_j}|_{V_k=0,k\neq j}$$

- This equation tells us that Y_{ij} can be found if we drive port j with the Voltage V_i , while shorting all other ports, and measuring short circuit current I_i at port i
- Now Y_{11} is the input admittance of port 1 looking into the network and Y_{12} is the transfer admittance from port 1 to port 2, when all other ports are short circuited

S- Parameter

Scattering parameter

- Scattering parameters or S Parameters are defined as ratio of reflected to incident wave voltage or power
- The other network parameters require short or open circuit conditions. At high frequencies the shorting stub can give rise to an inductive load, similarly an open circuit will act as a capacitive load
- A sudden discontinuity in the form of open or short circuits can also result in reflected waves that can destroy the source or result in undesirable oscillations
- Since the *S* parameters does not require the above conditions, the Device-Under-Test (DUT) is safe
- Let the voltage and current at the terminals of a N port network be

$$V_n = V_n^+ + V_n^ I_n = I_n^+ - I_n^-$$
 at $z = 0$
 V_1^+
 V_2^+
 V_2^+
 V_2^+
 $V_2^ V_2^+$
 $V_2^ V_2^ V_2^ V_2^ V_2^ V_2^ V_2^ V_2^ V_2^ V_2^-$

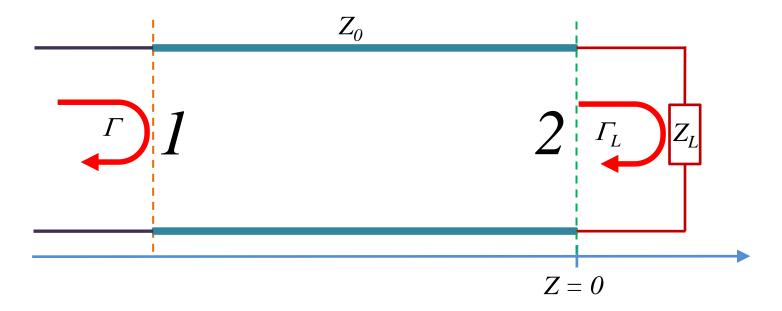
$$egin{bmatrix} V_1^- \ V_2^- \ . \ V_n^- \end{bmatrix} = egin{bmatrix} S_{11} & . & . & . & S_{1n} \ S_{21} & S_{22} & . & S_{2n} \ . & . & . & . \ . & . & . & . \ S_{n1} & . & . & . & S_{nn} \end{bmatrix} imes egin{bmatrix} V_1^+ \ V_2^+ \ . \ . \ . \ . \ . \ . \ V_n^+ \end{bmatrix}$$

Scattering Matrix

$$S_{ij} = \frac{V_i^-}{V_j^+} \big|_{V_k^+ = 0, k \neq j}$$

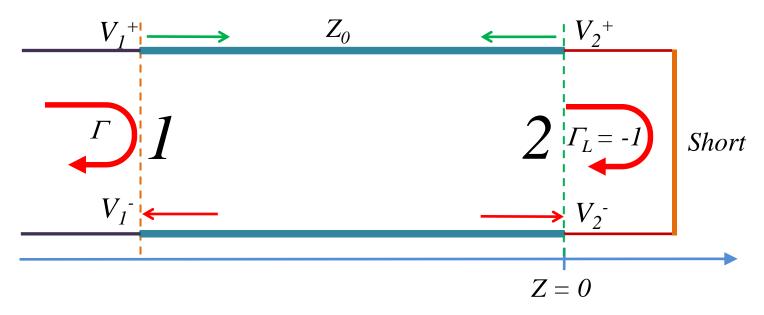
- This equation tells us that S_{ij} can be found by driving port j with an incident wave of voltage V_j^+ and finding reflected wave amplitude V_i^- at port i, while terminating all other ports with matched loads
- Now S_{II} is the Reflection Coefficient of port 1 looking into the network and S_{2I} is the Transmission Coefficient from port 1 to port 2

• The definition of the return loss Γ , given by S_{II} , holds good when the network is terminated with a matched load



- When $Z_0 = Z_L \Rightarrow \Gamma_L = 0$ Return loss (dB) is $20 \log \frac{1}{\Gamma} = 20 \log \frac{1}{|S_{11}|}$
- When there is a reflection at the load end due to mis-match, or a short or open circuit, the return loss at port 1 is not simply S_{11} anymore

• When the port 2 is short circuited $Z_L = 0 \Rightarrow \Gamma_L = -1$



- For $\Gamma_L = -1$ the port 2 voltages can be written as $V_2^+ = -V_2^-$
- We can re-write the scattering matrix in terms of the incident and reflected voltages as

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \Rightarrow V_1^- = S_{11}V_1^+ + S_{12}V_2^+ \\ V_2^- = S_{21}V_1^+ + S_{22}V_2^+ \end{bmatrix}$$

Applying the condition of short circuit $(V_2^+ = -V_2^-)$ into previous algebraic equations, we get

$$V_1^- = S_{11}V_1^+ - S_{12}V_2^- - \dots (1)$$

$$V_2^- = S_{21}V_1^+ - S_{22}V_2^- \qquad (2)$$

By rearranging eq (2) we can write

$$V_2^- + S_{22}V_2^- = S_{21}V_1^+ \Longrightarrow V_2^- = \frac{S_{21}}{1 + S_{22}}V_1^+$$

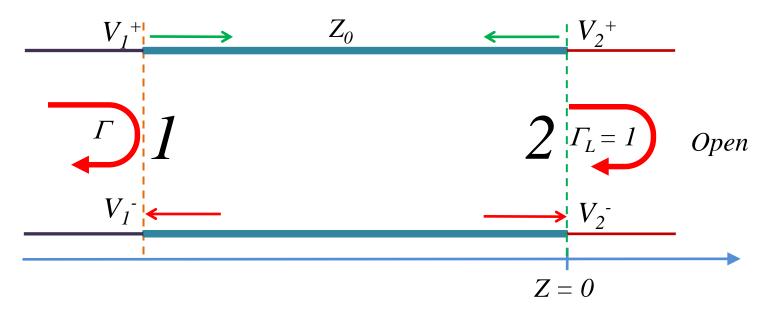
Dividing the eq (1) with V_I^+ and using the above equation yields

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}}$$

Now the *return loss* can be written as

$$RL = 20\log\frac{1}{|\Gamma|} = -20\log|\Gamma| = -20\log\left|S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}}\right|$$

When the port 2 is open circuited $Z_L = \infty \Rightarrow \Gamma_L = 1$



For $\Gamma_L = 1$ the port 2 voltages can be written as $V_2^+ = V_2^-$

Following the previous procedure of finding the reflection coefficient, the *return loss* for open circuit condition can be written as

$$RL = 20\log\frac{1}{|\Gamma|} = -20\log|\Gamma| = -20\log\left[S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}}\right]$$

Properties of S - Parameter

- Diagonal elements zero for a perfect matched network. Which means S_{11} , S_{22} $S_{nn} = 0$ as there are no reflections back to the port
 - *S* is symmetric for a reciprocal network, which means $S_{ij} = S_{ji}$, where $i \neq j$
 - Unitary Property of S matrix helps to check power balance or power losses in network, which is not possible with Z or Y matrix
 - S is lossless, if it satisfies unitary property (both unity and zero property:

Unity property

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1,$$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0, i \neq j$$

Phase shift property

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} = \begin{bmatrix} S_{11}e^{-j(\theta_1 + \theta_2)} \\ S_{21}e^{-j(\theta_1 + \theta_2)} \\ S_{22}e^{-j(\theta_1 + \theta_2)} \end{bmatrix}$$

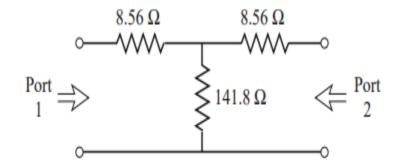
- If the reference plane is shifted, S matrix only varies in the phase whereas Z & Y matrices change both in magnitude and phase
- Phase of S_{ii} is shifted by twice the electrical length because incident wave travels twice this length upon reflection.
- Phase of Sij (i≠j) is shifted by sum of electrical length because the incident wave must pass through both length in order to travel from one shifted port to another

Problems

1. A two port network is known to have the following Scattering matrix:

$$[S] = \begin{bmatrix} 0.15 \angle 0^{\circ} & 0.85 \angle -45^{\circ} \\ 0.85 \angle 45^{\circ} & 0.2 \angle 0^{\circ} \end{bmatrix}$$

- a. Determine if the network is reciprocal and lossless.
- b. If port two is terminated with matched load, what is the return loss seen at port 1?
- c. If port 2 is terminated with a short circuit, what is the return loss seen at port 1
- 2. Find the S parameter of 3 dB attenuator shown in Fig below.



Problems

- 3. A 4 port network has the scattering matrix as given below:
- a. Determine if the network is lossless.
- b. Determine if the network reciprocal
- c. what is the return loss seen at port 1 when all other ports are matched?
- d. What is the insertion loss and phase between port 2 and port 4 when all other ports are matched?
- e. What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are matched?

Lossless power divider
Resistive power divider
Wilkinson Power divider
E-plane T
H –plane T
Circulator

 \square For a three – port network we have the S - Matrix

$$S = egin{bmatrix} S_{11} & S_{12} & S_{13} \ S_{21} & S_{22} & S_{23} \ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- ☐ If the component is passive and isotropic, then it must be reciprocal i,e, $S_{ij} = S_{ji}$
- \square For this 3-port network to be matched at all ports i,e, $S_{ii} = 0$, then we have the S Matrix as

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

☐ To be lossless it should satisfy the unitary condition, which gives

$$|S_{12}|^{2} + |S_{13}|^{2} = 1$$

$$|S_{12}|^{2} + |S_{23}|^{2} = 1$$

$$|S_{12}|^{2} + |S_{23}|^{2} = 1$$

$$|S_{13}|^{2} + |S_{23}|^{2} = 1$$

$$(1) \qquad S_{12}^{*}S_{13} = 0$$

$$|S_{23}^{*}S_{12} = 0$$

- \square From eq.'s (2), at least two of the three *S* parameters must be zero. This condition will never satisfy eq.'s (1)
- ☐ Therefore simultaneous conditions of, lossless, reciprocal and matched network do not give a solution
- Alternative 1: If the 3-port network was non-reciprocal, then the conditions of lossless and matched ports can be satisfied by the above equations. Such a device is Circulator where the non-reciprocal nature is obtained by some ferrite material

➤ <u>T - Junction (Alternative 2)</u>: If we consider the network to be lossless and reciprocal and not matched at one of the port (let us say port 3) then the S matrix is

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

☐ Again applying the unitary condition to the above matrix (Lossless), we have

$$|S_{12}|^{2} + |S_{13}|^{2} = 1$$

$$|S_{12}|^{2} + |S_{23}|^{2} = 1$$

$$|S_{12}|^{2} + |S_{23}|^{2} = 1$$

$$|S_{13}|^{2} + |S_{23}|^{2} + |S_{33}|^{2} = 1$$

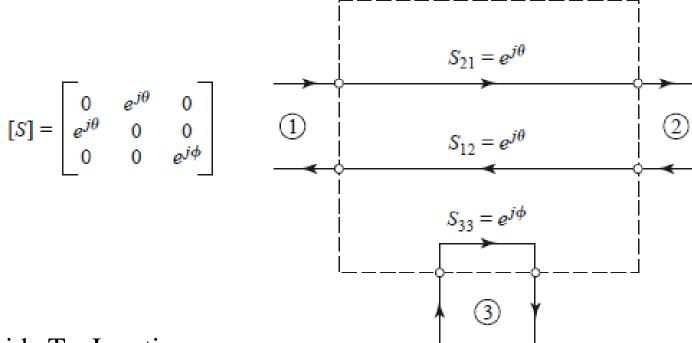
$$|S_{13}|^{2} + |S_{23}|^{2} + |S_{33}|^{2} = 1$$

$$|S_{23}|^{2} + |S_{33}|^{2} = 0$$

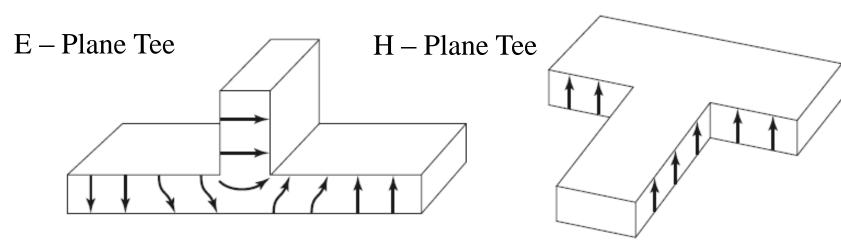
☐ For the above eq.'s to hold, we have the conditions

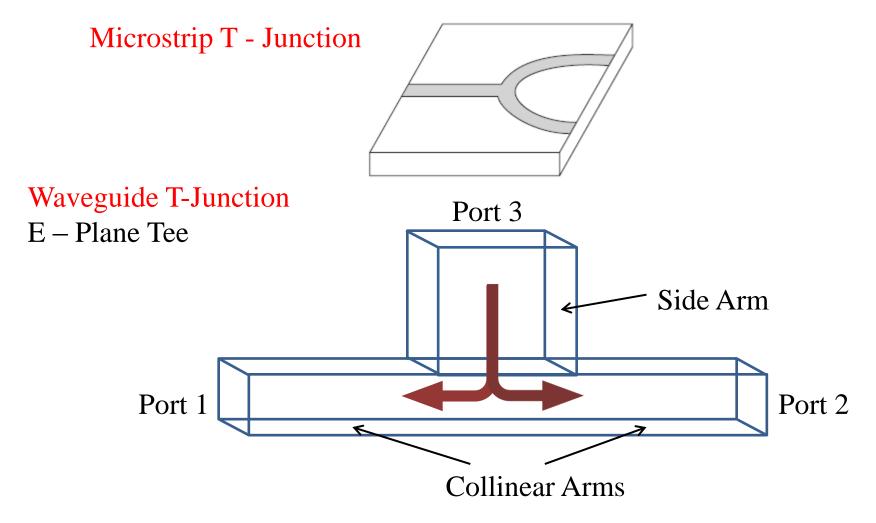
$$|S_{13}| = |S_{23}| = 0$$

 $|S_{12}| = |S_{33}| = 1$



Waveguide T - Junctions

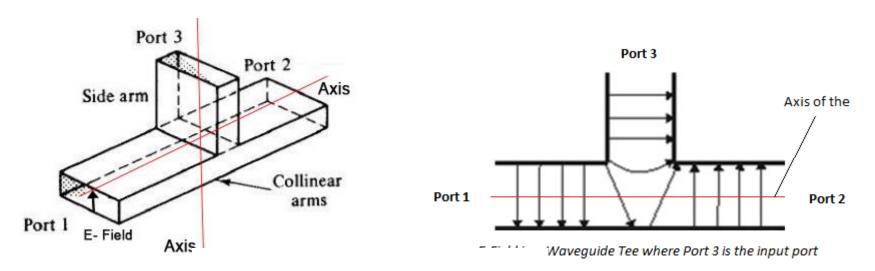


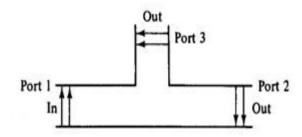


☐ The wave incident at port 3 will result in equal division of magnitude to ports 1 & 2, whereas the phase between them is opposite.

$$S_{13} = S_{31} = -S_{23} = -S_{32} \& S_{12} = S_{21}$$

E-Plane Tee





E-Field in a Waveguide Tee where Port 1 is the input port

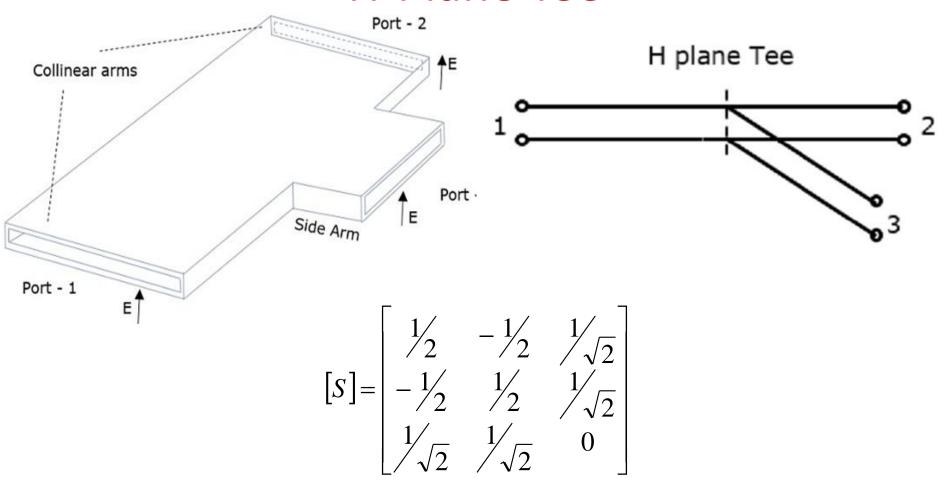
☐ The wave incident at port 3 will result in equal division of magnitude to ports 1 & 2, whereas the phase between them is opposite.

$$S_{13} = S_{31} = -S_{23} = -S_{32} \& S_{12} = S_{21}$$

□ The S – matrix for the given E - plane Tee junction can be written as

$$[S] = \begin{vmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{vmatrix}$$

H-Plane Tee

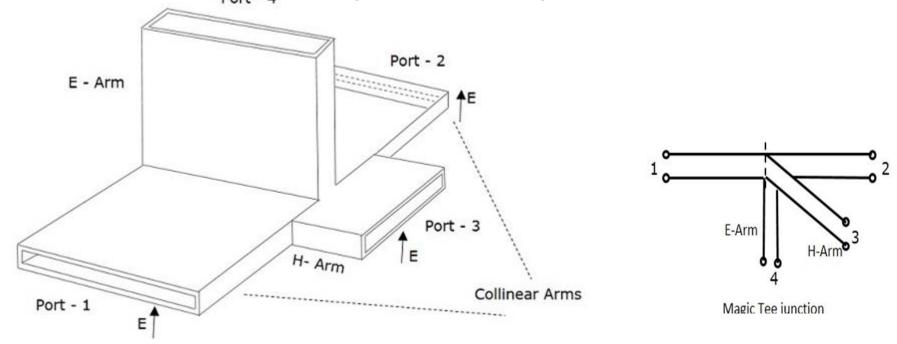


• The wave incident at port 3 will result in equal division of magnitude to ports 1 & 2, and the phase remains same

$$S_{13} = S_{31} = S_{23} = S_{32} \& S_{12} = S_{21}$$

E-H Plane Tee

- An E-H Plane Tee junction is formed by attaching two simple waveguides one parallel and the other series, to a rectangular waveguide which already has two ports. This is also called as Magic Tee, or Hybrid or 3dB coupler.
- The arms of rectangular waveguides make two ports called collinear ports i.e., Port 1 and Port 2, while the Port 3 is called as H-Arm or Sum port or Parallel port. Port 4 is called as E-Arm or Difference port or Series port.



Properties of E-H Plane Tee

- If ports 1 and 2 are perfectly matched to the junction, the other two ports are also perfectly matched to the junction.
- The junction where all the four ports are perfectly matched is called as Magic Tee Junction.

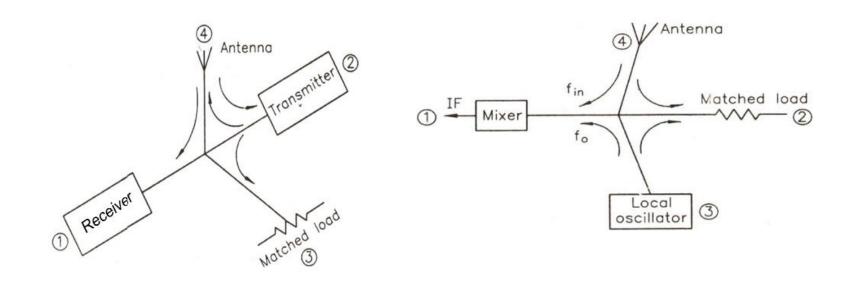
$$S = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

Characteristics of E-H Plane Tee

- If a signal of equal phase and magnitude is sent to port 1 and port 2, then the output at port 4 is zero and the output at port 3 will be the additive of both the ports 1 and 2.
- If a signal is sent to port 4, E-arm then the power is divided between port 1 and 2 equally but in opposite phase, while there would be no output at port 3. Hence, $S_{34} = 0$.
- If a signal is fed at port 3, then the power is divided between port 1 and 2 equally, while there would be no output at port 4. Hence, $S_{43} = 0$.
- If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port, as the E-arm produces a phase delay and the H-arm produces a phase advance. So, $S_{12} = S_{21} = 0$.

Applications of E-H Plane Tee

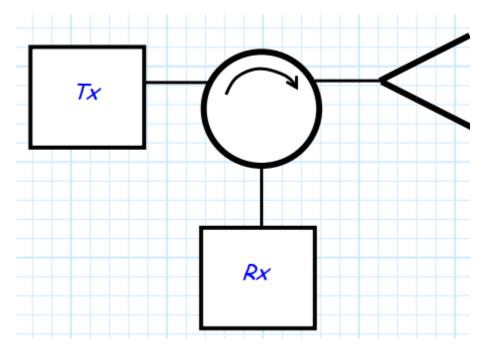
- E-H Plane junction is used to measure the impedance
- E-H Plane Tee is used as a duplexer
- E-H Plane Tee is used as a mixer



Matched Tee as duplexer

Matched Tee as mixer

 Circulators use anisotropic ferrite materials, which are often "biased" by a permanent magnet! → The result is a nonreciprocal device!



- Three port circulators: 3-port "Y-junction" circulators based on cancellation of waves propagating over two different paths near a magnetised material. Waveguide circulators may be of either type, while more compact devices based on strip-line are of the 3-port type.
- Four port circulators: 4-port waveguide circulators based on Faraday rotation of waves propagating in a magnetised material. Using this technology, they are able to route the RF signals to four ports.

Alternative 1: matched and lossless but non-reciprocal 3-port network that can be physically realizable is circulator

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
 $S_{31}^* S_{32} = 0$
 $|S_{21}|^2 + |S_{23}|^2 = 1$ $S_{21}^* S_{23} = 0$
 $|S_{31}|^2 + |S_{32}|^2 = 1$ $S_{12}^* S_{13} = 0$

The conditions to satisfy above equations are

$$S_{12} = S_{23} = S_{31} = 0, \quad |S_{21}| = |S_{32}| = |S_{13}| = 1$$
or $S_{21} = S_{32} = S_{13} = 0, \quad |S_{12}| = |S_{23}| = |S_{31}| = 1$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Two types of circulators and their scattering matrices. (a) Clockwise circulation. (b) Counterclockwise circulation. The phase references for the ports are arbitrary.

Major drawbacks with a circulator:

- 1. They're expensive.
- 2. They're heavy.
- 3. The generally produce a large, static magnetic field.
- 4. They typically exhibit a large insertion loss (e.g., $|S_{21}|^2 = |S_{32}|^2 = |S_{13}|^2 \approx 0.75$).

Isolator

 An ideal isolator is thus a two-port device with an odd looking scattering matrix:

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

- An isolator is useful for isolating a load from a source
- If the isolator was truly a matching network, then the absence of reflected power would indicate that all the incident power was absorbed by the load.
- Instead, there is no reflected power because this power is instead absorbed by the isolator—the isolator is lossy!

Problems

- 4. In a lossless H plane tee microwave signal of 100mW is fed into one of the collinear arms that is port 1. Find the power delivered to port 2 and 3 when these are terminated with the matched load.
- 5. A 40 mW signal is fed to arm 3 of E plane tee by matched terminal. Arms 1 and 2 are terminated with 40Ω and 60Ω respectively and the wave impedance of the guide is 50Ω 0. Find the power delivered to the load.
- 6. A 3 port circulator has insertion loss of 2 dB, isolation of 30 dB, VSWR=1.3. Assume phase of all coefficients are zero. Write the scattering matrix of 3 port circulator

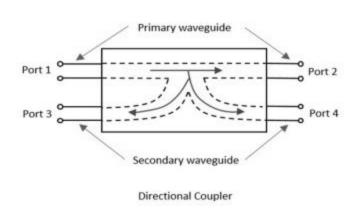
Problems

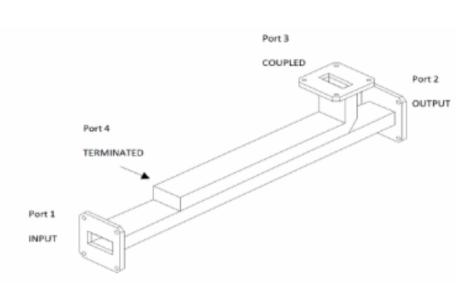
- 7. In an isolator the isolation is 30 dB and insertion loss is 0.4dB. Find its scattering parameter.
- 8. The collinear ports 1 and 2 of a magic tee are terminated by impedances of reflection coefficients Γ_1 = 0.3 and Γ_2 = 0.4 The difference port 4 is terminated by a impedance with reflection coefficient of 0.7. If 1W power is fed at sum port 3. Calculate the power reflected at port 3 and the power divisions at the other ports. Consider the reflected voltage at collinear ports are same.

A Directional coupler is a device that samples a small amount of Microwave power for measurement purposes.

Coupler is a 4-port waveguide junction consisting of a primary main waveguide and a secondary auxili

ary waveguide.

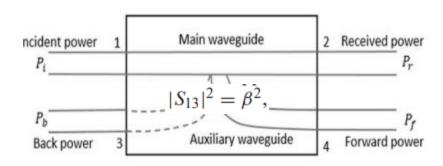




Properties of Directional Couplers:.

- All the terminations are matched to the ports.
- When the power travels from Port 1 to Port 2, some portion of it gets coupled to Port 4 but not to Port 3.
- As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.
- If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.
- If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2.

Coupling =
$$C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \, dB$$
,
Directivity = $D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \, dB$,
Isolation = $I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \, dB$,
Insertion loss = $L = 10 \log \frac{P_1}{P_2} = -20 \log |S_{12}| \, dB$.



Directional Coupler indicating powers

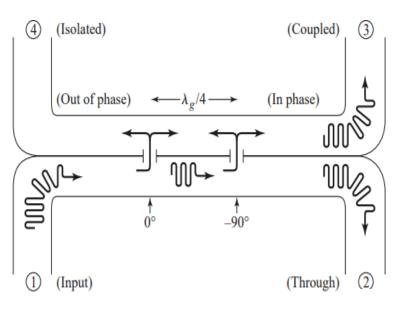
The coupling factor indicates the fraction of the input power that is coupled to the output port

The directivity is a measure of the coupler's ability to isolate forward and backward waves (or the coupled and uncoupled ports) delivered to the uncoupled port.

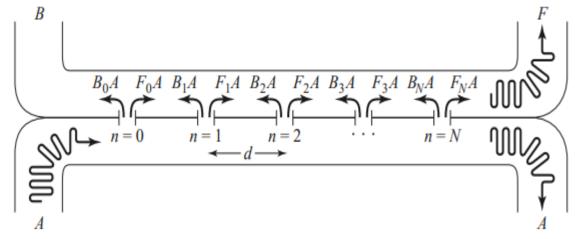
The isolation is a measure of the power delivered to the uncoupled port.

These quantities are related as I = D + C dB.

The insertion loss accounts for the input power delivered to the through port, d



Basic operation of a two-hole directional coupler.



7.19 Geometry of an (N + 1)-hole waveguide directional coupler.

Problems

8. A 2W power source is connected to the input of a directional coupler with C=20 dB, D=25 dB and an insertion loss of 0.7 dB. Find the output powers in watts and dBm at the through, coupled and isolated ports. Assume all ports are matched.

Ferrites

- Ferrites are non metallic high resistivity material >10¹⁴ for metal, ϵ_r =10-15, μ_r =1000
- Ferrite atoms have large number of electrons spinning resulting in strong magnetic properties, which is due to the dipole moment.
- It is used in microwave devices to reduce reflected power up to 100GHz.
- Ferrite is nonreciprocal, behaves differently for clockwise and anticlockwise rotation and present different effective permeability's.

Principle of Faraday rotation- isolator,

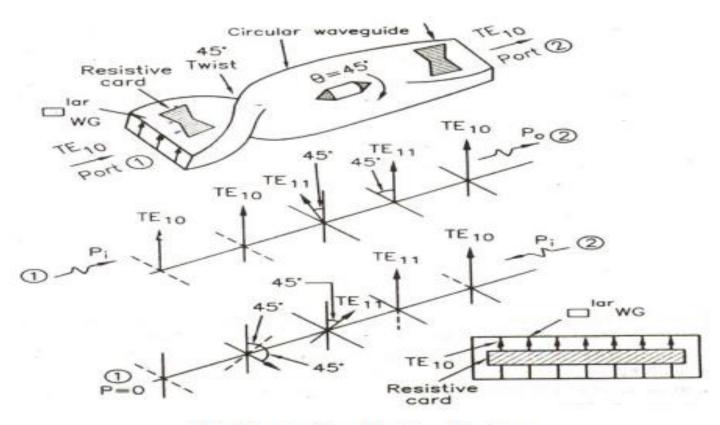


Fig. Constructional details of isolator

Principle of Faraday rotation-circulator

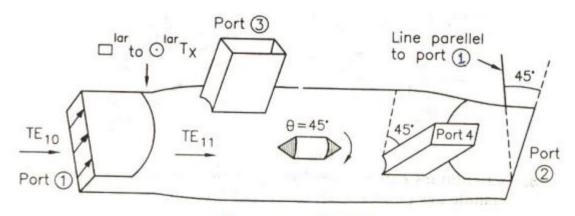


Fig. Four port circulator

S – Parameters

• The ratio of transmitted to the power received at the output terminals is transmission loss. The difference of transmitted and received power gives us the attenuation on the network

Transmission loss or Attenuation (dB) =
$$10 \log \frac{P_i - P_r}{P_o} = 10 \log \frac{1 - |S_{11}|^2}{|S_{21}|^2}$$

• The ratio of incident to the transmitted power $(P_t = P_i - P_r)$ is the reflection loss. This signifies the power lost due to reflection by measuring the actual power transmitted

Reflection loss (dB) =
$$10 \log \frac{P_i}{P_i - P_r} = 10 \log \frac{1}{1 - |S_{11}|^2}$$

• While reflection loss gives what fraction of incident power is transmitted and return loss gives the fraction of incident power that is reflected back

Return loss (dB) =
$$10 \log \frac{P_i}{P_r} = 20 \log \frac{1}{\Gamma} = 20 \log \frac{1}{|S_{11}|}$$