5.3 Directional Coupler

Module:5 Microwave Passive components

Course: BECE305L – Antenna and Microwave Engineering

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Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Module:5 Microwave Passive components 6 hours

- Microwave Networks - ABCD, 'S' parameter and its properties. E-Plane Tee, H-Plane Tee, Magic Tee and Multi-hole directional coupler. Principle of Faraday rotation, isolator, circulator and phase shifter.

- Source of the contents: Pozar

3.1 Directional couplers

- S matrix of a **reciprocal** **four port** network **matched at all ports**:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- For lossless network

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \qquad (C1,C2)$$

$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \qquad (C3,C4)$$

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- For lossless network

$$S_{13}^{*}S_{23} + S_{14}^{*}S_{24} = 0$$

$$S_{14}^{*}S_{13} + S_{24}^{*}S_{23} = 0$$

- Multiply the above two by S_{24}^{*} and S_{13}^{*} respectively and subtract,

$$S_{14}^{*}(|S_{13}|^2 - |S_{24}|^2) = 0$$

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- $S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0$
- **Multiplying Col 1 and Col 3** and **multiplication of Col 4 and Col 2** gives:

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \quad \text{and} \quad S_{14}^* S_{12} + S_{34}^* S_{23} = 0$$

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Multiply the above two by S_{12} and S_{34} and subtract:

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- $S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0$
- $S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0$

- One way the above two may be satisfied: if $S_{14} = S_{23} = 0$, which **results in a directional coupler**

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- The **self product of the rows of the unitary scattering matrix**:

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
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$$|S_{24}|^2 + |S_{34}|^2 = 1$$

This results in $|S_{13}| = |S_{24}|$ and $|S_{12}| = |S_{34}|$

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And $|S_{13}| = |S_{24}|$ and $|S_{12}| = |S_{34}|$

- Further simplification: **choose phase reference on three of the four ports**: $S_{12} = S_{34} = \alpha$ $S_{13} = \beta e^{j\theta}$ and $S_{24} = \beta e^{j\phi}$ with α, β: real and θ and ϕ are yet to be obtained.

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- Dot product of Columns 2 and 3: $S_{12}^* S_{13} + S_{24}^* S_{34} = 0$ which gives a **relation between phase constants $\boldsymbol{\theta + \phi = \pi \pm 2n\pi}$**

3.2 Directional couplers: Practical case

- S matrix of a reciprocal four port network matched at all ports:

With $S_{14} = S_{23} = 0$ $\quad [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$

And $|S_{13}| = |S_{24}|$ and $|S_{12}| = |S_{34}|$; $S_{12} = S_{34} = \alpha$ $\quad S_{13} = \beta e^{j\theta}$ and $S_{24} = \beta e^{j\phi}$; **relation between phase constants $\boldsymbol{\theta + \phi = \pi \pm 2n\pi}$**

Case 1: **Symmetric coupler**: $\theta = \phi = \pi/2$, with similar β,

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

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Case 1: Symmetric coupler: $\theta = \phi = \pi/2$;

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Case 2:Antisymmetric coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Amplitudes β are chosen 180° apart

3.2 Directional couplers: Practical case

<u>Case 1: Symmetric coupler:</u> $\theta = \phi = \pi/2$;<u>Case 2:Antisymmetric coupler</u>$\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \qquad [S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Note: The two couplers – **differ in choice of reference planes**

Amplitudes are not independent, $\alpha^2 + \beta^2 = 1$

Phase references, ideal four port directional coupler **has only one degree of freedom** – With two configurations.

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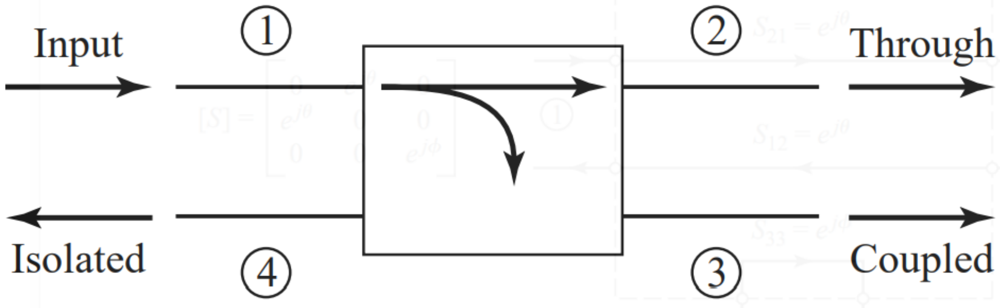
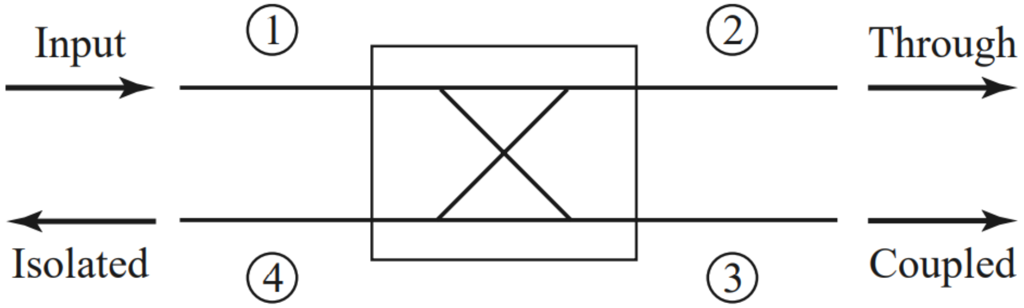
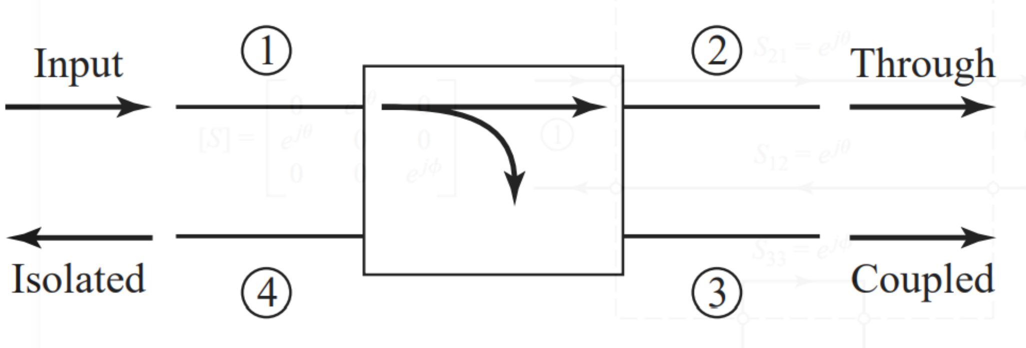
3.3 Directional couplers characteristics

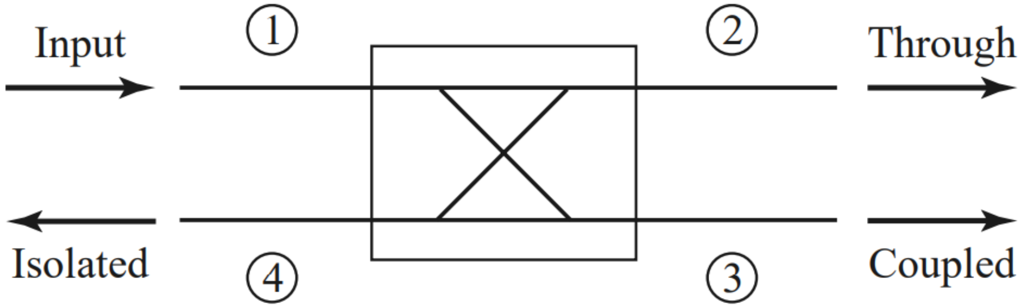




3.3 Directional couplers characteristics

- For the reciprocal network: $S_{31} = S_{13}$ and $S_{21} = S_{12}$





3.3 Directional couplers characteristics

- For the reciprocal network: $S_{31} = S_{13}$ and $S_{21} = S_{12}$
- Power supplied is coupled to port 3 with coupling factors $|S_{13}|^2 = \beta^2$
- Remainder of power to port 2 (through port) with coefficient $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$

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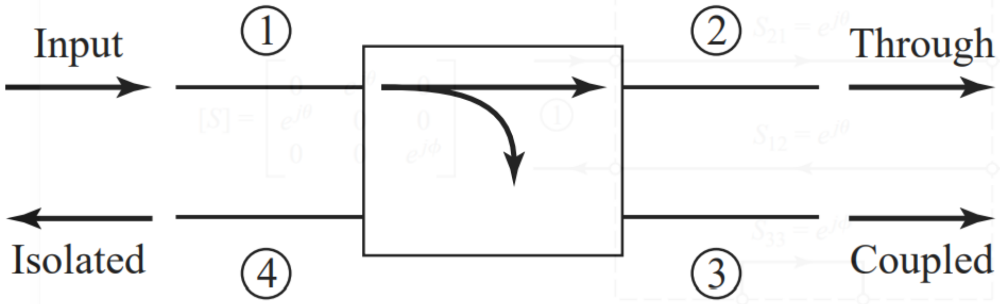

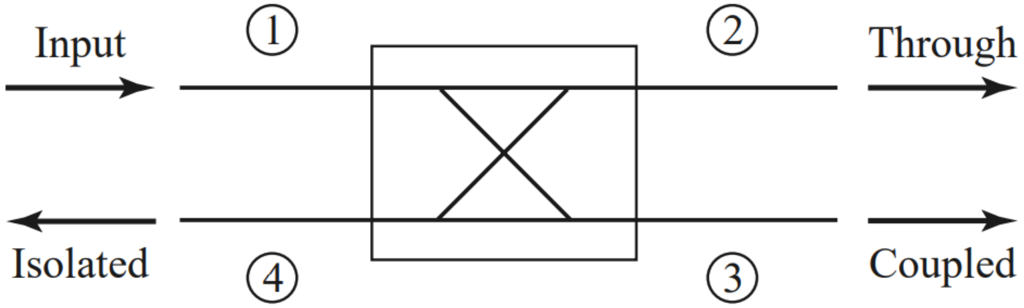

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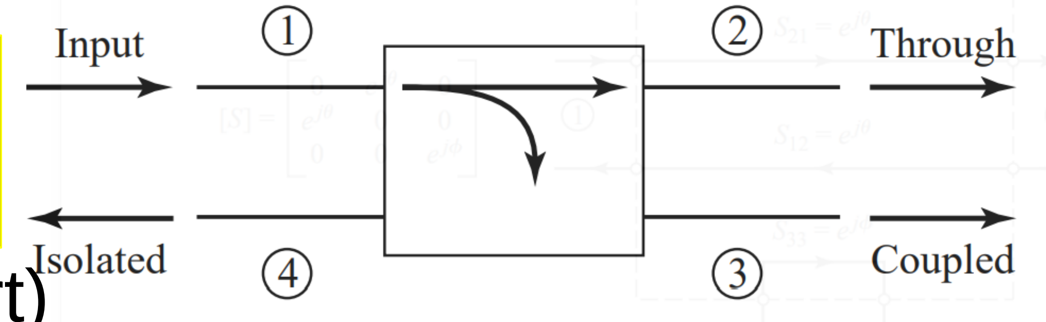
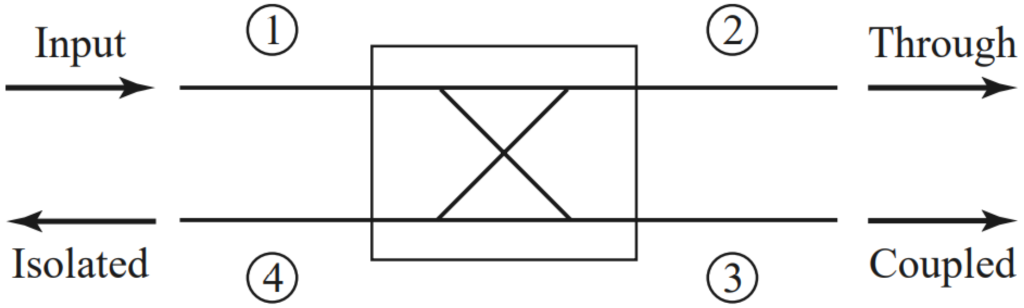
- **Ideal directional coupler:**
  Port 4 is isolated from port 1 ,
  No power is delivered to port 4 (Isolated port)

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

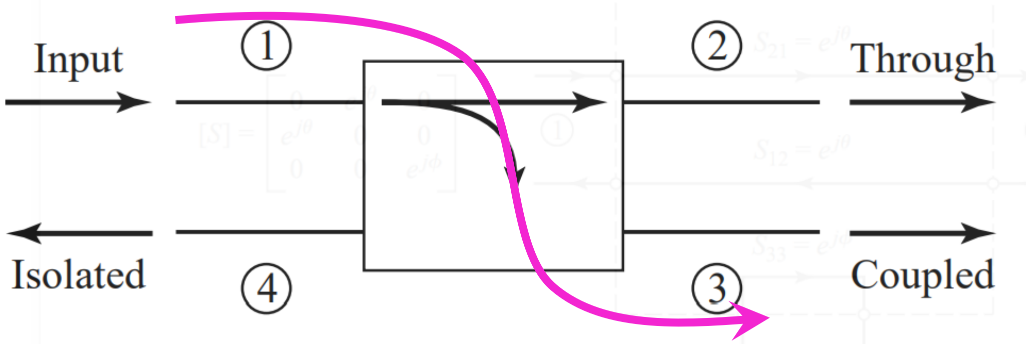




3.3 Directional couplers characteristics

- **Coupling=C** $= 10\log_{10}\frac{P_1}{P_3} = -20\log_{10}\beta$ dB

Fraction of input power coupled to output port



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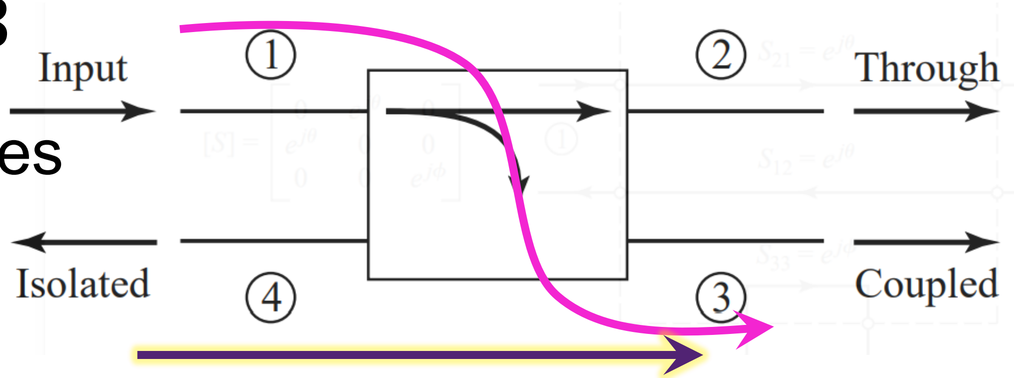
- **Coupling=C** $= 10\log_{10}\frac{P_1}{P_3} = -20\log_{10}\beta$ dB

Fraction of input power coupled to output port

- **Directivity=D** $= 10\log_{10}\frac{P_3}{P_4} = 20\log_{10}\frac{\beta}{|S_{14}|}$ dB

Coupler's ability to isolate forward and backward waves (or coupled and decoupled ports)



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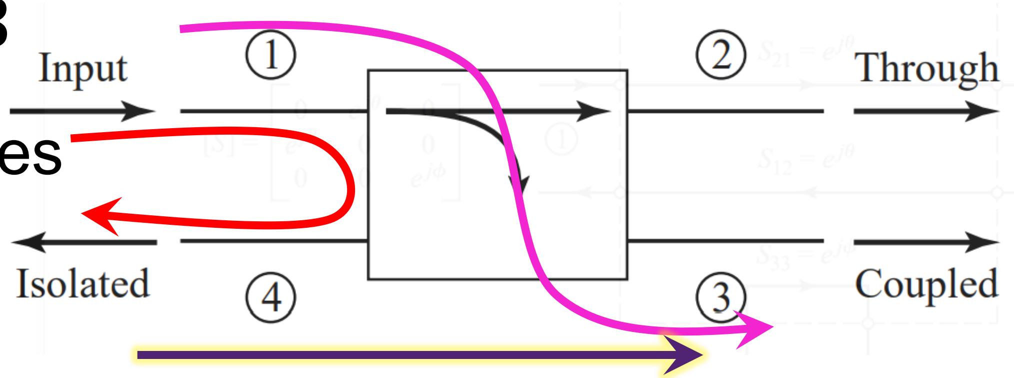
Power delivered to the uncoupled port $I = D + C\ dB$



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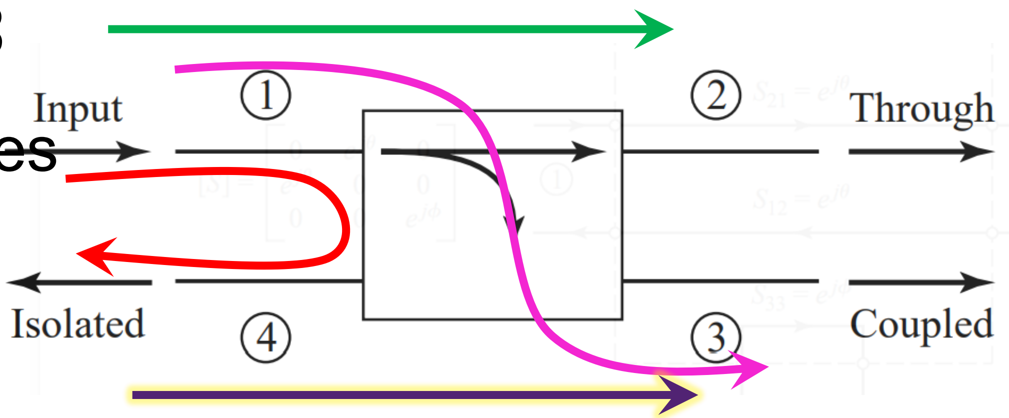
- **Insertion loss L**$=10\log\frac{P_1}{P_2} = -20\log|S_{12}|$dB

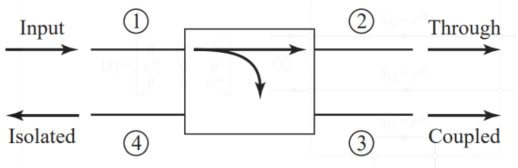
Input power delivered to through port, diminished by power delivered to coupled and isolated ports



3.3 Directional couplers characteristics

- **Ideal directional coupler:** Directivity $D = \infty = 10\log_{10}\frac{P_3}{P_4}$ and

  $S_{14} = 0 \quad I = 10\log_{10}\frac{P_1}{P_4} = -20\log|S_{14}| = -\infty$

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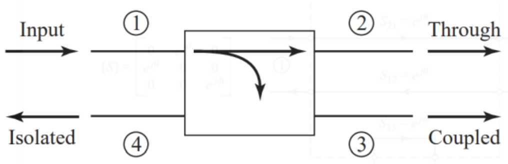

- **Hybrid couplers**

  Special case of directional couplers.

  Coupling factor $C = 3dB$

  $\alpha = \beta = 1/\sqrt{2}$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta e^{j\phi} \\ \beta e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta e^{j\phi} & \alpha & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & e^{j\theta} & 0 \\ 1 & 0 & 0 & e^{j\phi} \\ e^{j\theta} & 0 & 0 & 1 \\ 0 & e^{j\phi} & 1 & 0 \end{bmatrix}$$

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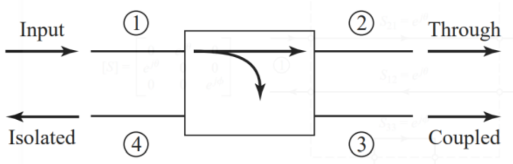

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1) Quadrature hybrid: 90° phase shift between port 2 and 3 ($\theta = \phi = \pi/2$)

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

2)

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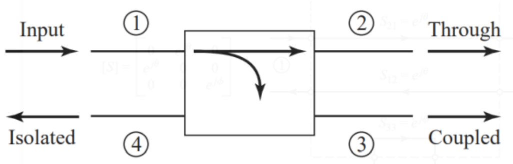
- **Ideal directional coupler:** Directivity $D = \infty$ and Isolation $S_{14} = 0$

  Both α, β can be obtained from coupling factor



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$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

2) Magic T hybrid and Rat race hybrid:
   180° phase shift between ports 2,3
   $\theta = 0, \phi = \pi$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$