6.6 MaximallyFlat(Binomial or Butterworth) and Chebychev(EqualRipple) filters

Module:6 Microwave Passive circuits

Course: BECE305L – Antenna and Microwave Engineering

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Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Module:6 Microwave Passive circuits 7 hours

- T junction and resistive power divider, Wilkinson power divider, branch line coupler (equal & unequal), Rat Race Coupler, Filter design: Low pass filter (Butterworth and Chebyshev) - Richards transformation and stepped impedance methods.

- Source of the contents: Pozar

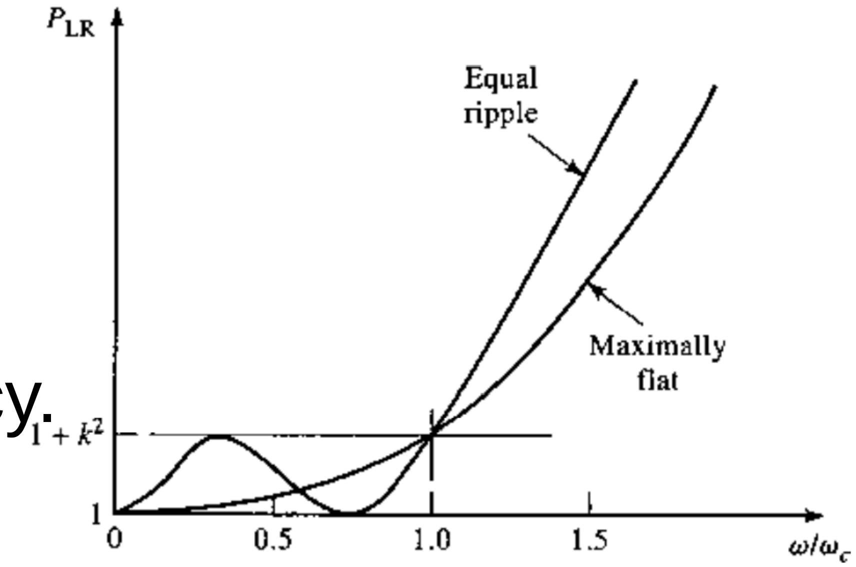
2. FILTER DESIGN BY THE INSERTION LOSS METHOD: Characterization by Power loss ratio

- **Insertion loss method**:
  a filter response is defined by its insertion loss, or power loss ratio, P_{LR}:
- $P_{LR} = \frac{Power\ available\ from\ source}{Power\ delivered\ to\ the\ load} = \frac{P_{inc}}{P_{load}} = \frac{1}{1-|\Gamma(\omega)|^2}$
- This quantity is reciprocal of $|S_{12}|^2$ if both load and source are matched.
- Insertion loss (IL)=$10\log_{10} P_{LR}$
- $|\Gamma(\omega)|^2$ is an even function of ω. It can be expressed as polynomials M and N in ω^2: $|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)}$
- $P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$ Reflection ratio is also constrained.

2.1 Characterization by Power loss ratio:
a) Maximally flat **(Binomial or Butterworth response)**

- Provides flattest possible passband response for given filter complexity.
- Low pass filter, it is specified by

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$



- N is order of the filter and ω_c is cutoff frequency.
- Pass band extends from $\omega = 0$ to $\omega = \omega_c$
- At the edge, the power loss ratio is $1 + k^2$ (If -3dB point, $k = 1$)

2.1 Characterization by Power loss ratio:
a) Maximally flat **(Binomial or Butterworth response)**

- For $\omega > \omega_c$, attenuation increases monotonically with frequency.
- For $\omega \gg \omega_c$, $P_{LR} \approx k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$ (Insertion loss increases at rate of 20dB/decade
- Like binomial response, for multisection quarter-wave matching transformer, the first $(2N-1)$ derivatives are zero at $\omega = 0$.

2.1 Characterization by Power loss ratio:
b) Equal ripple (Chebyshev)

- Chebyshev polynomial is used to specify insertion loss $\boldsymbol{Nth}$ **order low pass filter** as

  $$P_{LR} = 1 + k^2 T_N^2\left(\frac{\omega}{\omega_c}\right)$$

2.1 Characterization by Power loss ratio: b) Equal ripple (Chebyshev)

- Chebyshev polynomial is used to specify insertion loss Nth order low pass filter as
  $P_{LR} = 1 + k^2 T_N^2 \left(\frac{\omega}{\omega_c}\right)$
  Results in a **sharper cutoff** but with **ripples in passband**
  Ripple amplitude $1 + k^2$, with $T_N(x)$ oscillates between ± 1 for $|x|^2 < 1$
- Thus Passband ripple level are dependent on k^2

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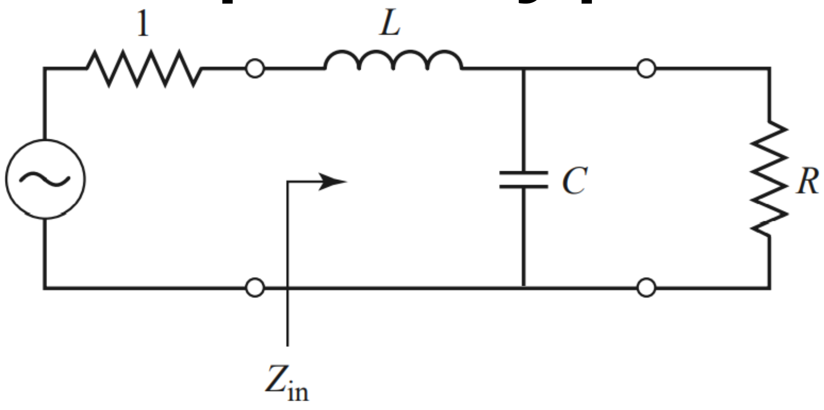
  Ripple amplitude $1 + k^2$, with $T_N(x)$ oscillates between ± 1 for $|x|^2 < 1$
- Thus Passband ripple level are dependent on k^2
- For **large** $\boldsymbol{x}$, $\mathrm{T_n(x)} \approx \frac{1}{2}(2x)^N$
  For $\boldsymbol{\omega} \gg \boldsymbol{\omega_c}$, insertion loss $P_{LR} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$
  Chebyshev case>$\frac{2^{2N}}{4}$ times binomial casd
- This also increases at $20N\ dB/decade$

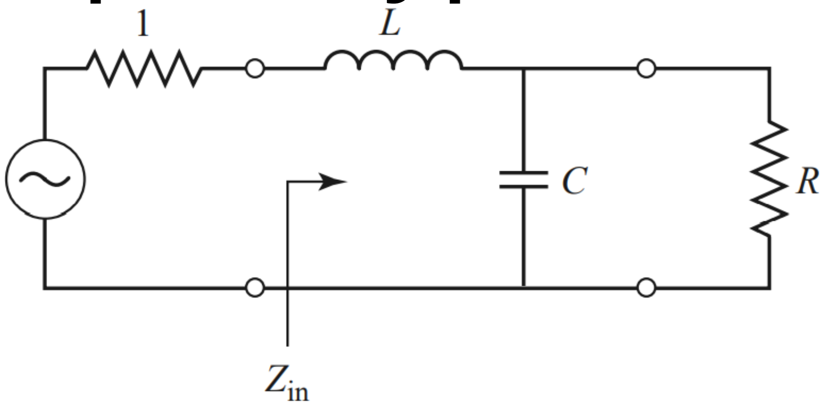
3.1 Maximally flat low pass filter prototype



- Consider **2 element Low pass filter prototype**
- Normalized element values L and C for maximally flat response.

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

3.1 Maximally flat low pass filter prototype



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$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c}\right)^{2N}$$

- Source impedance of 1Ω and cutoff frequency $\omega_c = 1$ rad/sec
- **Desired loss ratio for $N = 4$**,

$$\boldsymbol{P_{LR} = 1 + \omega^4}$$

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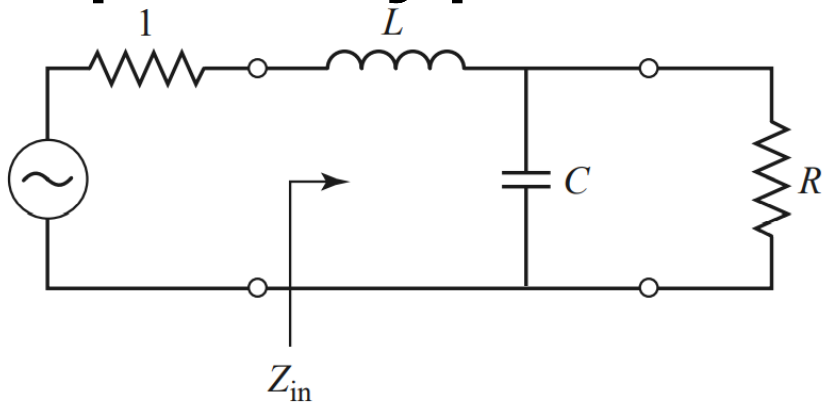

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- Source impedance of 1Ω and cutoff frequency $\omega_c = 1$ rad/sec
- Desired loss ratio for $N = 4$,

$$P_{LR} = 1 + \omega^4$$

- Input impedance of this filter $Z_{in} = j\omega L + \frac{R(1-j\omega RC)}{1+\omega^2 R^2 C^2}$
- $P_{LR} = 1 + \frac{1}{4R^2}[(1-R)^2 + (R^2C^2 + L^2R^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4]$

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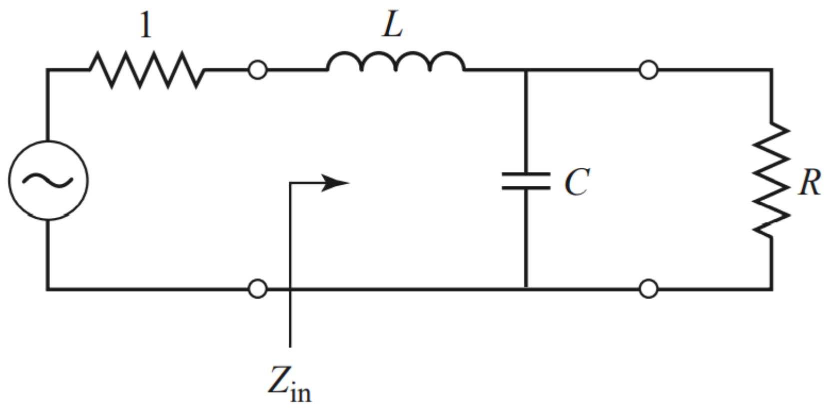

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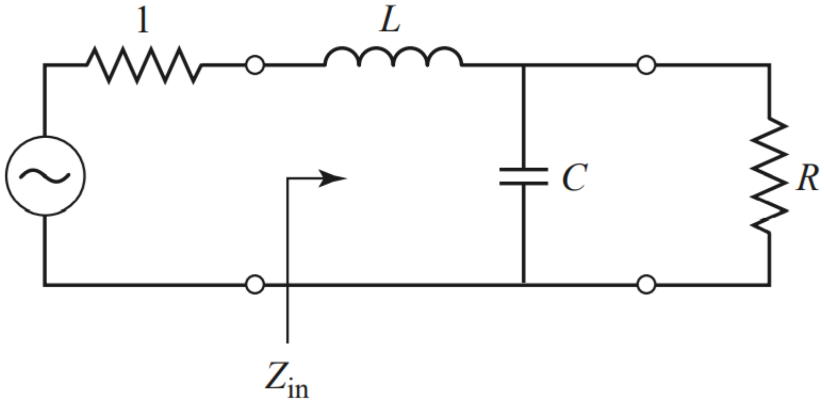
- This is expression of polynomial in ω^2
- $R = 1$ since $P_{LR} = 1$ for $\omega = 0$



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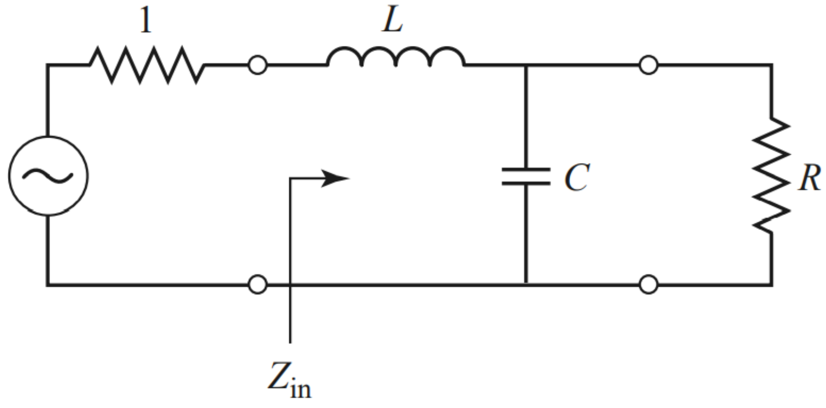
- Coefficient of ω^2 must vanish

$$C^2 + L^2 - 2LC = (C - L)^2 = 0$$



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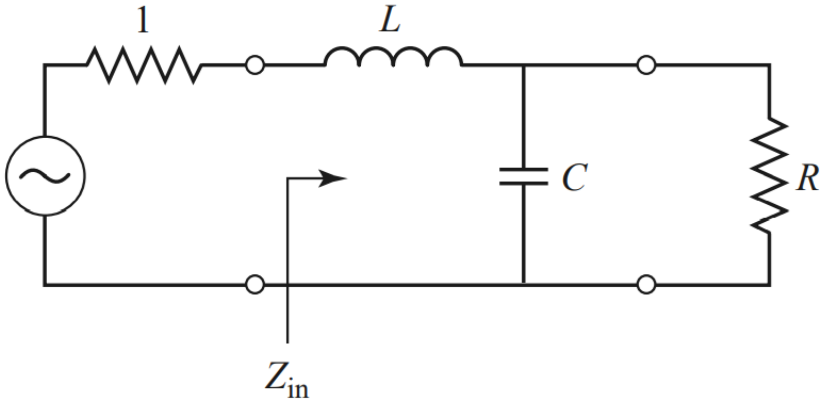
$$C^2 + L^2 - 2LC = (C - L)^2 = 0$$

- The $L = C$. For coefficient of ω^4 to be unity,

$$\frac{1}{4}L^2C^2 = \frac{1}{4}L^4 = 1 \qquad L = C = \sqrt{2}$$



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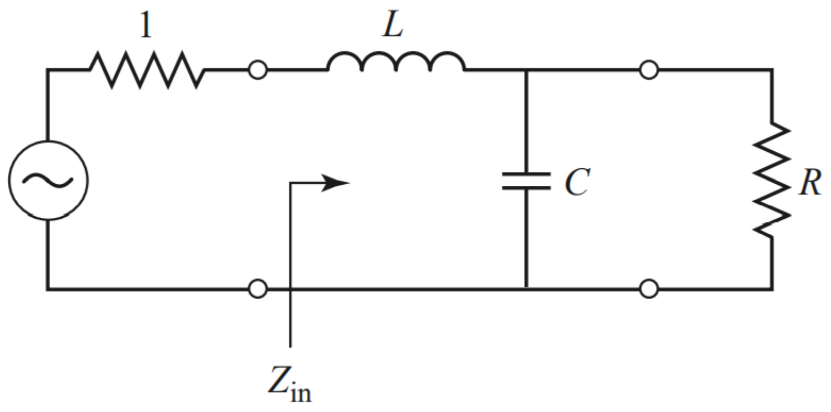
- $\mathrm{L} = \mathrm{C} = \sqrt{2}$
- **Procedure can be extended to find the element values for filters with arbitrary N elements.**



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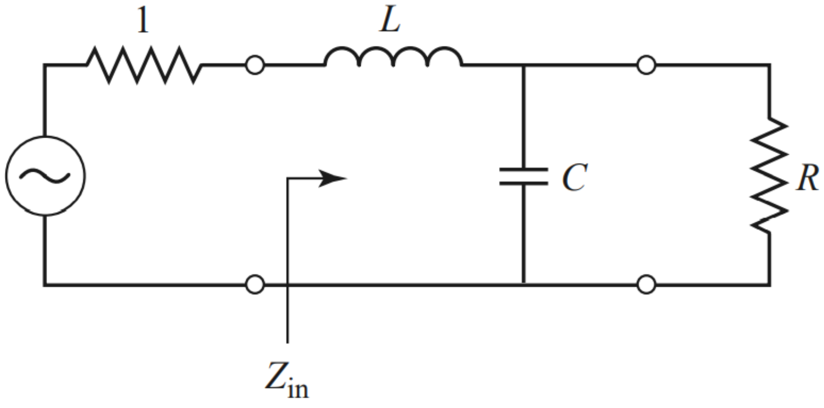
- $\mathrm{L} = \mathrm{C} = \sqrt{2}$
- Procedure can be extended to find the element values for filters with arbitrary N elements.
- **For normalized low pass design, source impedance is 1Ω** and **cutoff frequency ω_c rad/sec**
- Element values for ladder type circuit are given



3.1 Maximally flat low pass filter prototype

- Element values for ladder type circuit are given with $g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10.

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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3.1 Maximally flat low pass filter prototype

- g_0 is generator impedance is 1
- g_{N+1} is at the load impedance for a filter having N reactive elements
- Series
  $g_0 = generator\ resistance$ / $generator$ conductance

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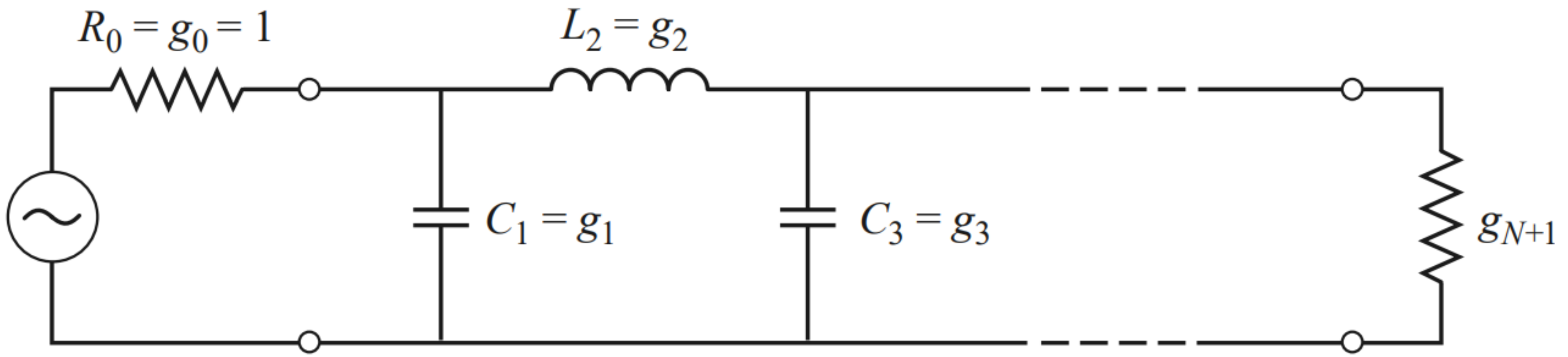
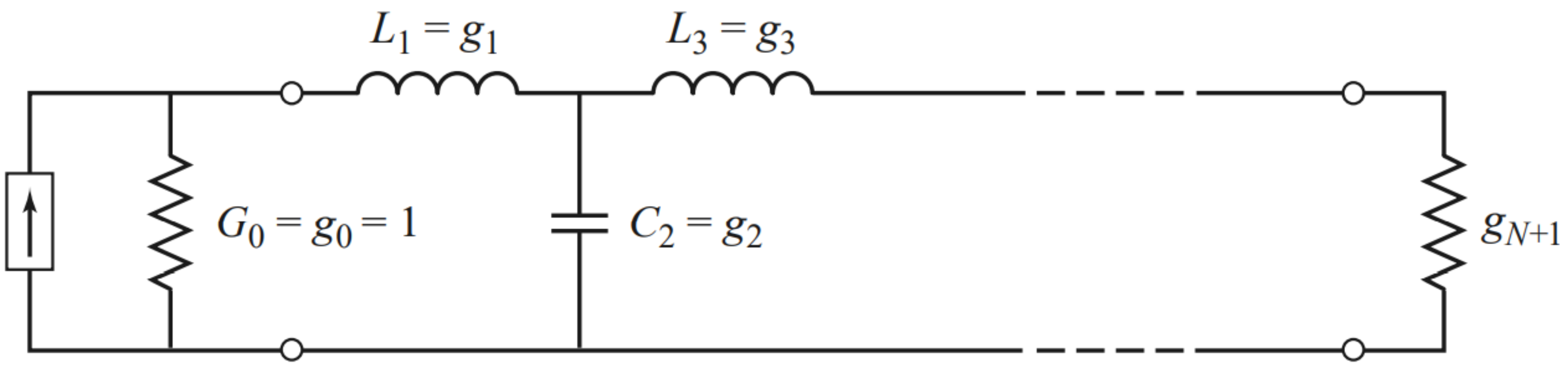
- Element values for ladder type circuit are given

Prototype beginning with shunt element



Prototype beginning with series element



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  $g_k\ (k = 1\ to\ N) = inductance\ for\ series\ inductors$

  $capacitance\ for\ shunt\ capacitors$

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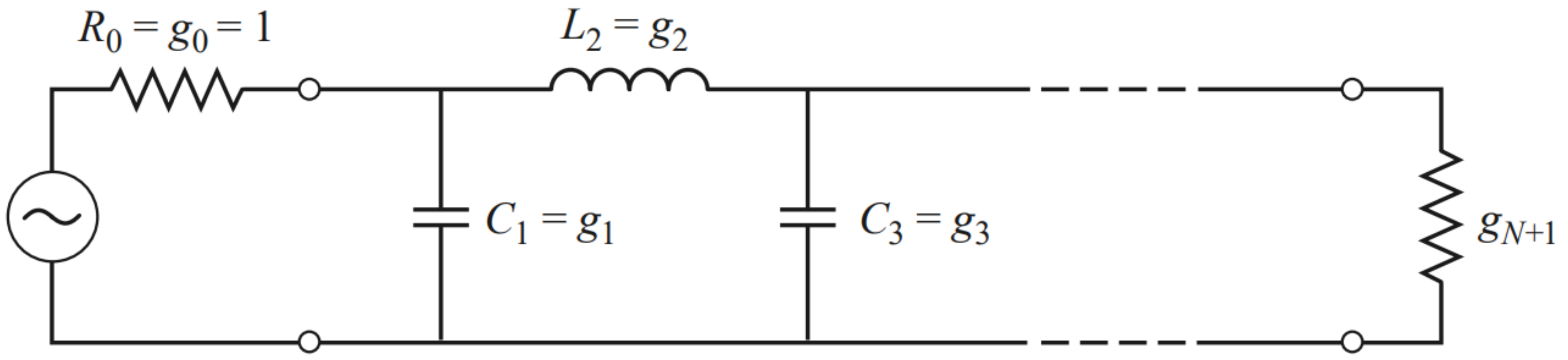

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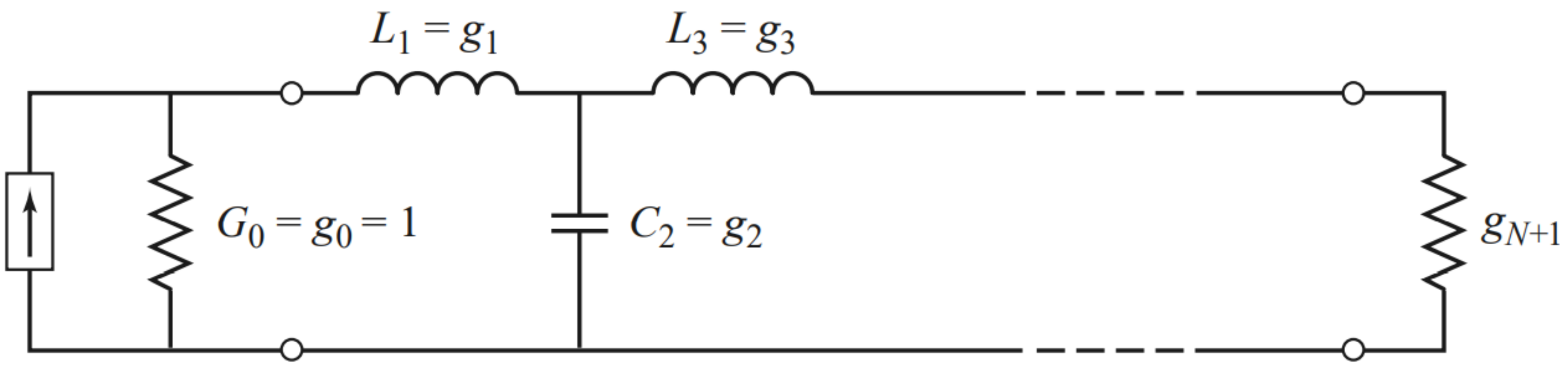

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$$g_k\ (k = 1\ to\ N) = \begin{cases} inductance\ for\ series\ inductors \\ capacitance\ for\ shunt\ capacitors \end{cases}$$

$$g_{N+1} = \begin{cases} load\ resistance\ if\ g_N\ is\ a\ shunt\ capacitor \\ load\ conductance\ if\ g_N\ is\ a\ series\ inductor \end{cases}$$

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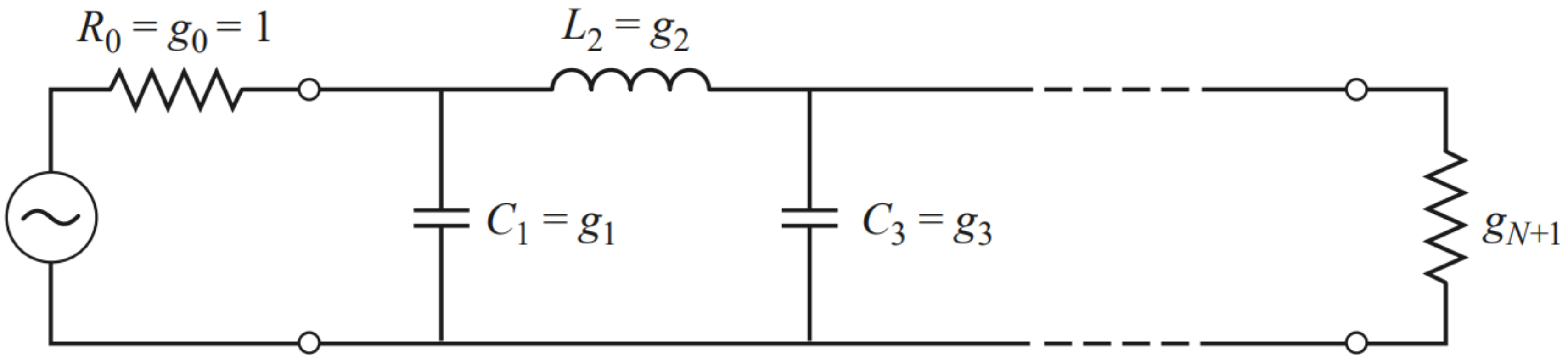

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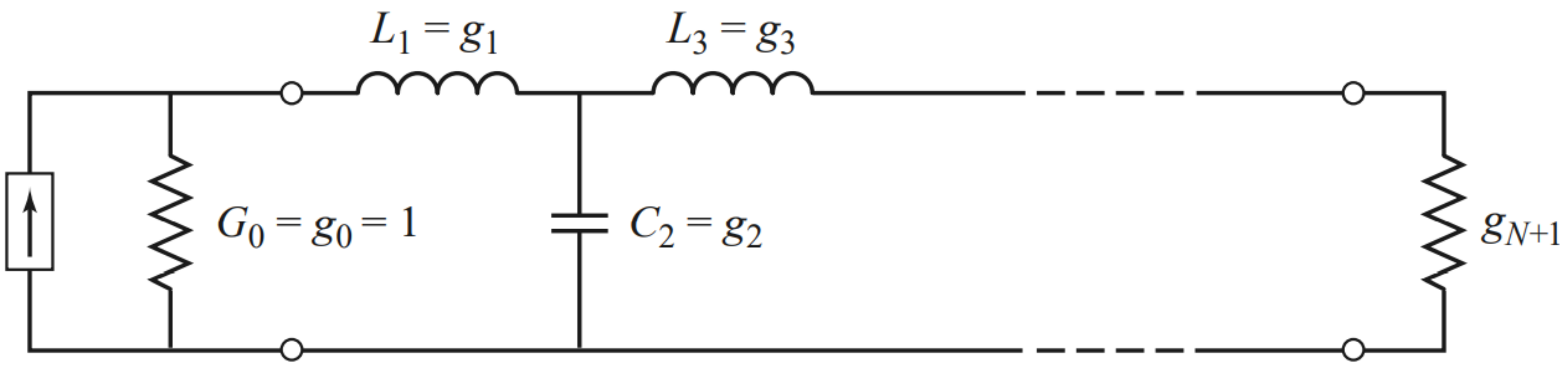
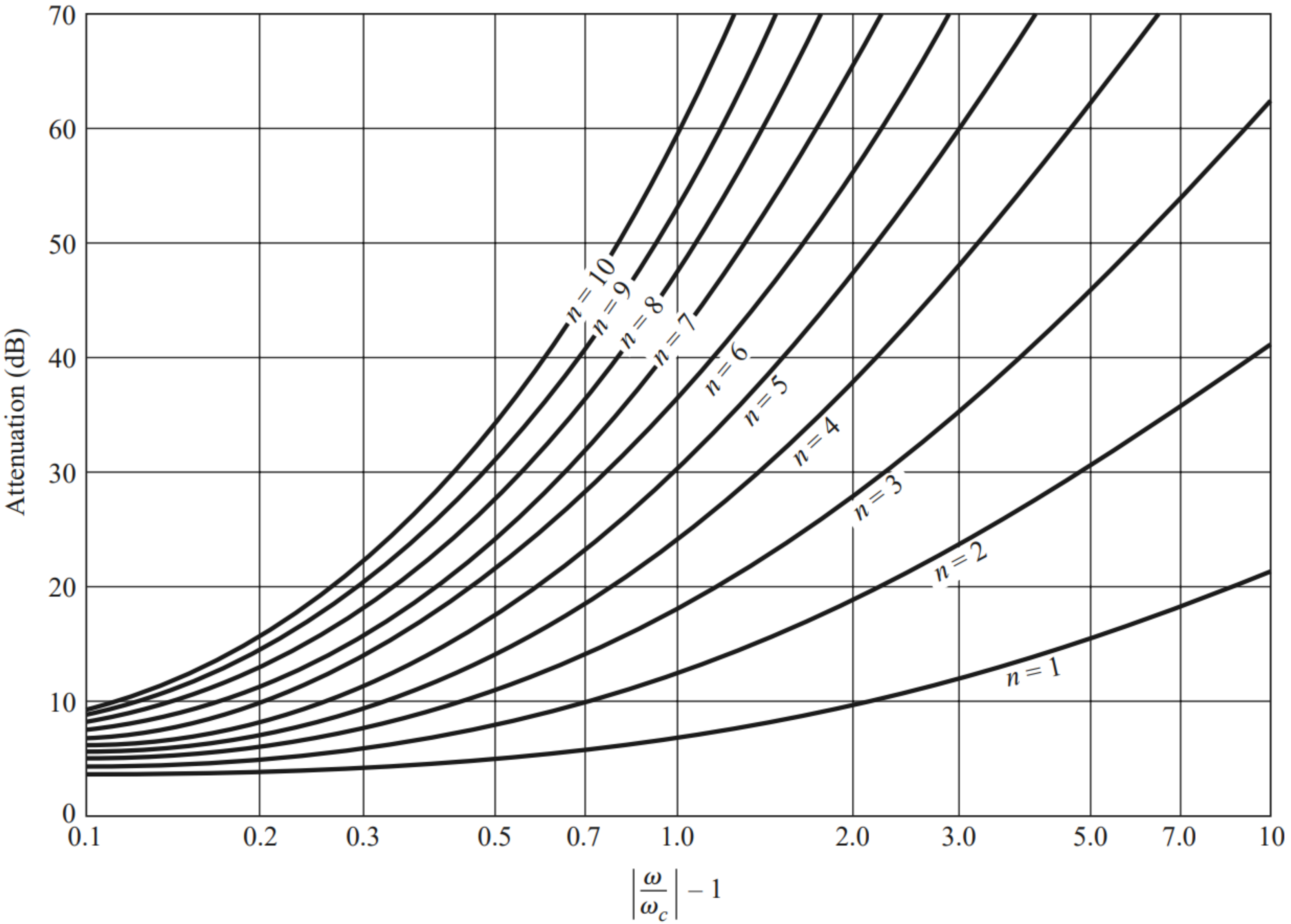


3.1 Maximally flat low pass filter prototype

- Both the circuits will give same response.

- Attenuation characteristics for various N vs Frequency
- For N>10, Two lower order filters in cascade are used.



4. Equal ripple Low Pass filter prototype (Chebyshev)

- $\omega_c = 1\, rad/sec$
  Power loss ratio: $P_{LR} = 1 + k^2 T_N^2(\omega)$
  $1 + k^2$: ripple level in passband
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  $1 + k^2$: ripple level in passband
- Chebyshev polynomial property: $$T_N(0) = \begin{cases} 0 & \text{for } N \text{ odd,} \\ 1 & \text{for } N \text{ even,} \end{cases}$$
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  $P_{LR} = 1$ at $\omega = 0$ for N odd
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  $1 + k^2$: ripple level in passband
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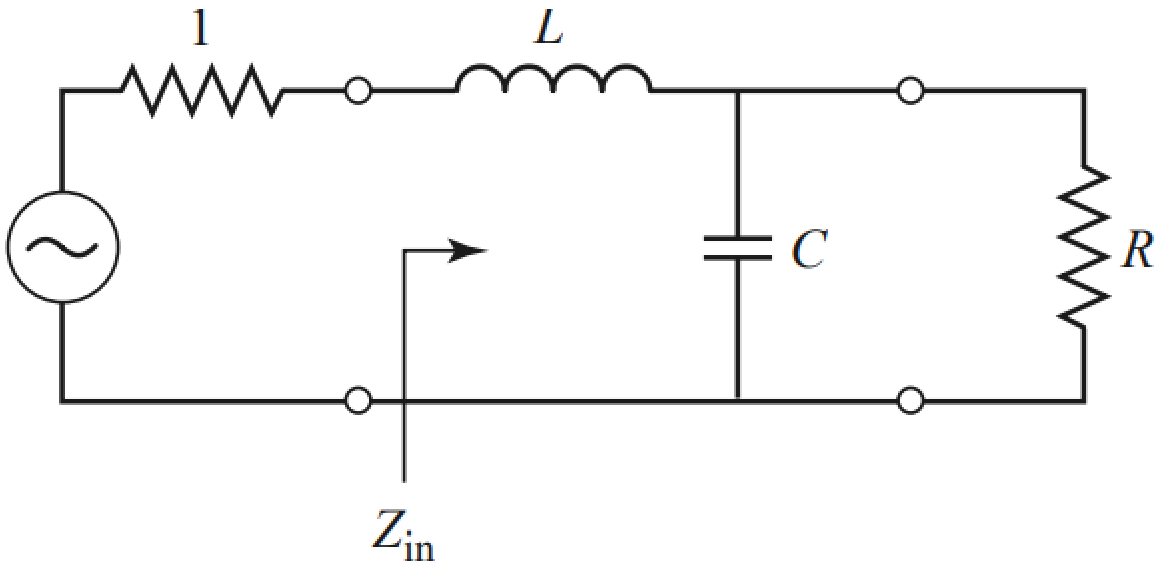
  $P_{LR} = 1$ at $\omega = 0$ for N odd
  $P_{LR} = 1 + k^2$ at $\omega = 0$ for N even

- $k^2 = \frac{(1-R)^2}{4R}$ or

- $R = 1 + 2k^2 \pm 2k\sqrt{1 + k^2}$ for N even



4. Equal ripple Low Pass filter prototype (Chebyshev): 0.5dB ripple

0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

4. Equal ripple Low Pass filter prototype (Chebyshev) : 3dB ripple

3.0 dB Ripple

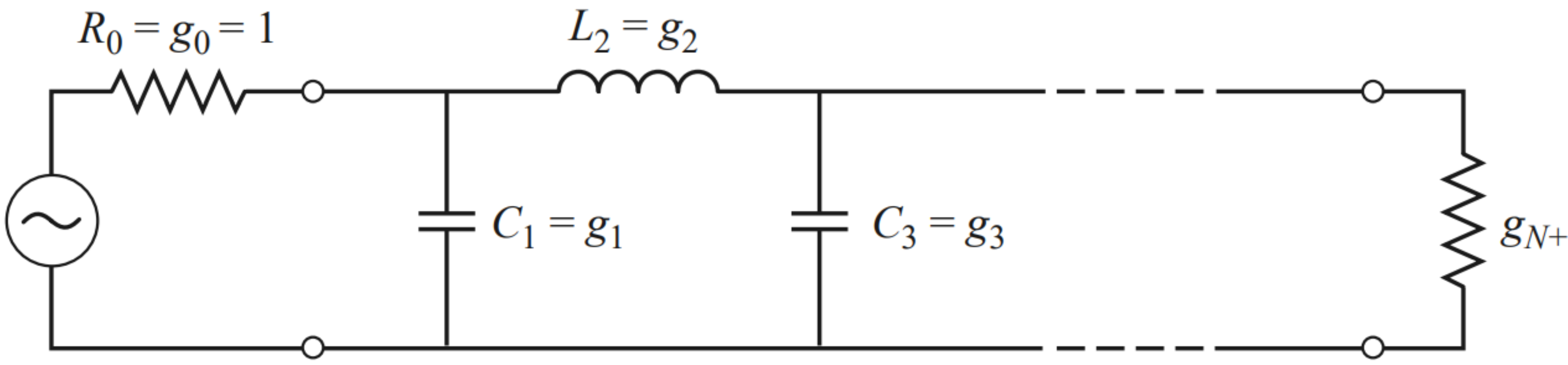
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

4. Equal ripple Low Pass filter prototype (Chebyshev)

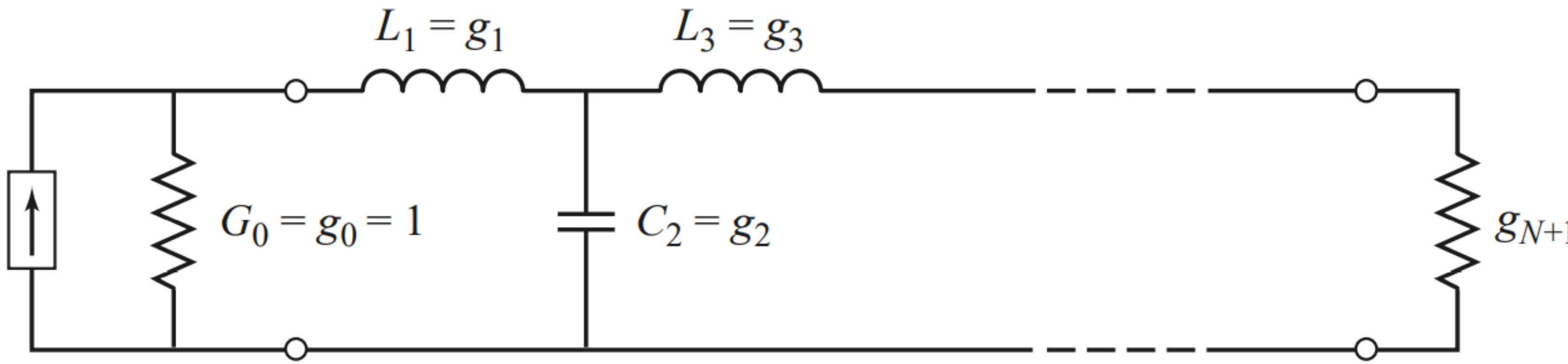
- Table for designing equal ripple low pass filters with normalized source impedance
- Cutoff frequency $\omega_c' = \frac{1rad}{}$

  The ladder circuits:

Prototype beginning with shunt element



Prototype beginning with series element



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- Table for designing equal ripple low pass filters with normalized source impedance
- Cutoff frequency $\omega_c' = \frac{1rad}{sec}$
  The ladder circuits:
- The design data depends on specified passband ripple (0.5dB or 3dB)
- Notice: Load impedance $g_{N+1} \neq 1$ for even N

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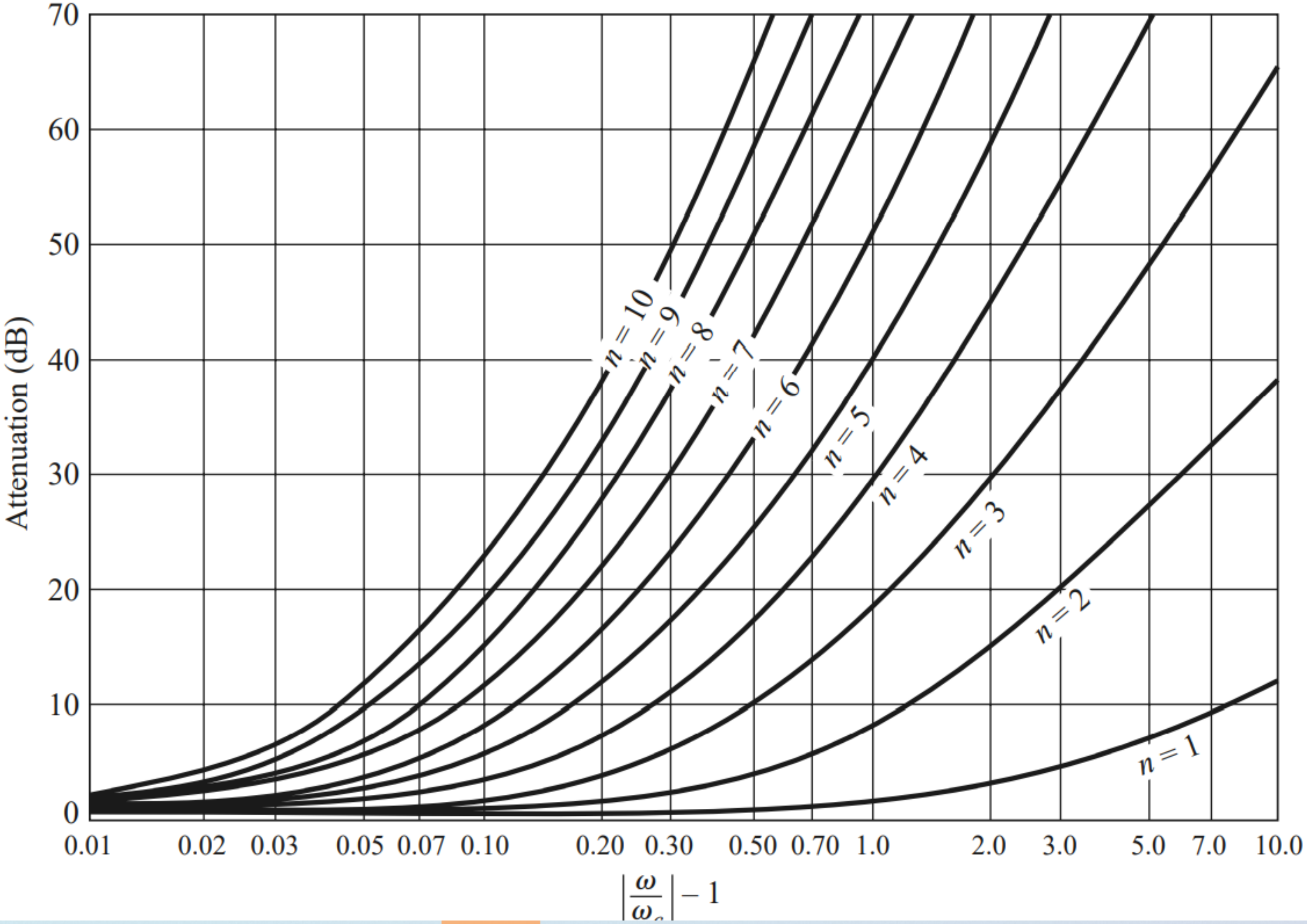
- Cutoff frequency $\omega_c' = \frac{1rad}{sec}$
  The ladder circuits:
- The design data depends on specified passband ripple (0.5dB or 3dB)
- Notice: Load impedance $g_{N+1} \neq 1$ for even N
- If stopband attenuation is specified, the curves can be used to determine the necessary value of N for the ripples.

4. Equal ripple Low Pass filter prototype (Chebyshev): 0.5dB ripple Attenuation



4. Equal ripple Low Pass filter prototype (Chebyshev) : 3dB ripple attenuation