

5.5 Faraday Rotation – Ferromagnetic properties

Module:5 Microwave Passive components

Course: BECE305L – Antenna and Microwave Engineering

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Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

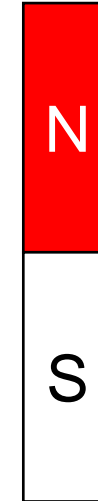
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6 hours

- Microwave Networks - ABCD, 'S' parameter and its properties. E-Plane Tee, H-Plane Tee, Magic Tee and Multi-hole directional coupler. Principle of Faraday rotation, isolator, circulator and phase shifter.
- Source of the contents: Pozar

1. Basics of Magnetic permeability

- Magnet – North and South



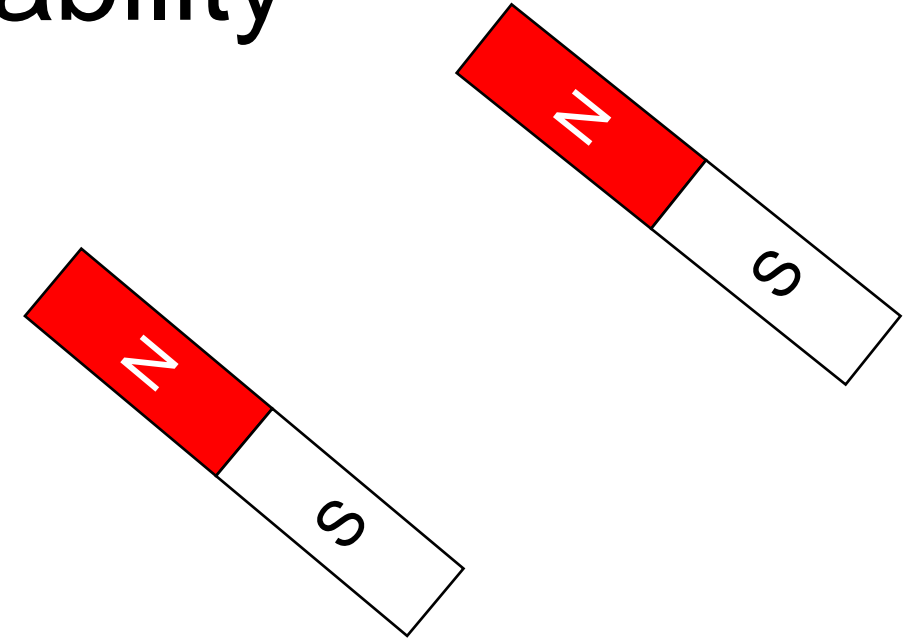
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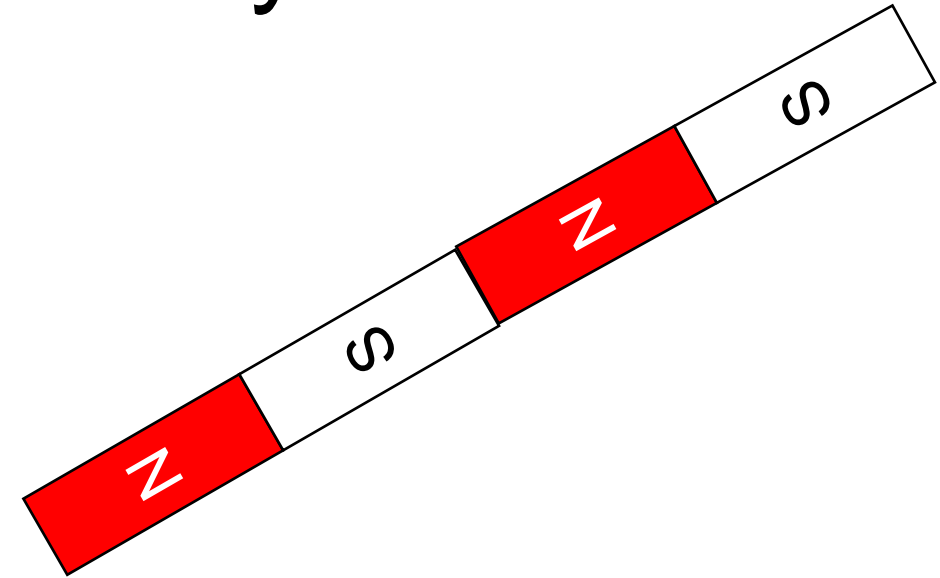
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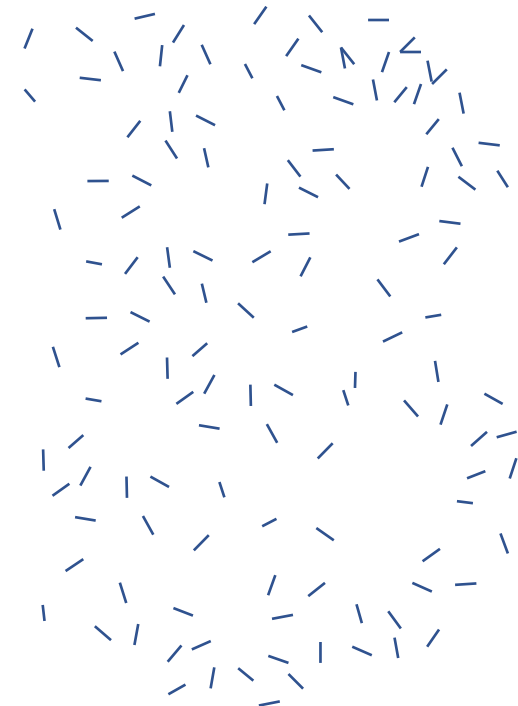
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 - Allows magnetic field in a certain pattern
- This can be measured by placing iron filings around the magnet



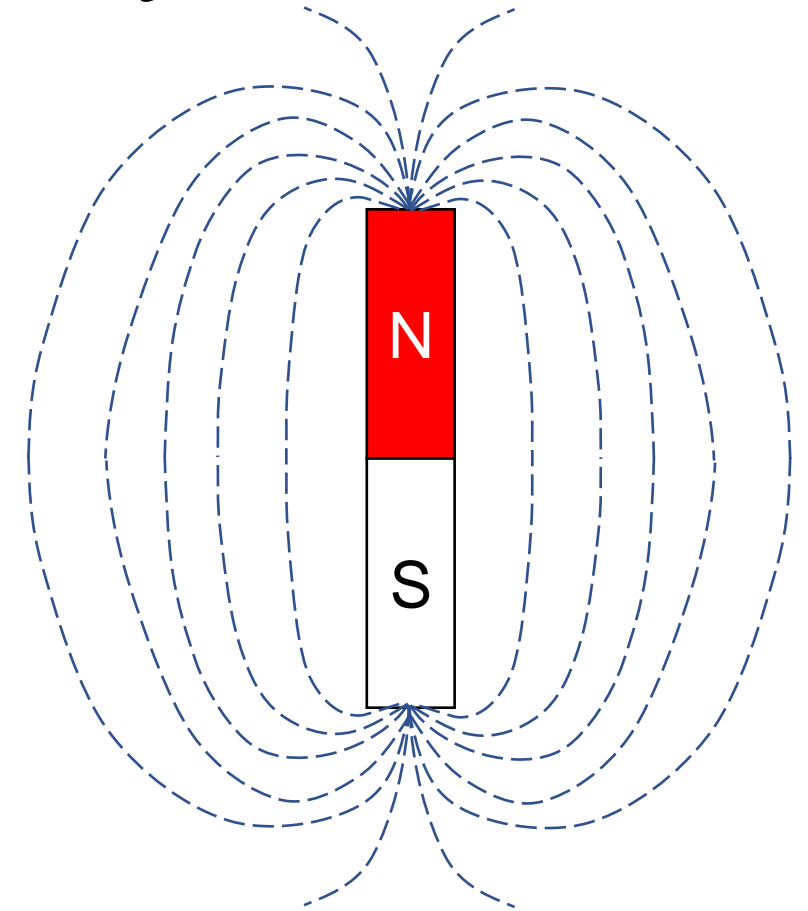
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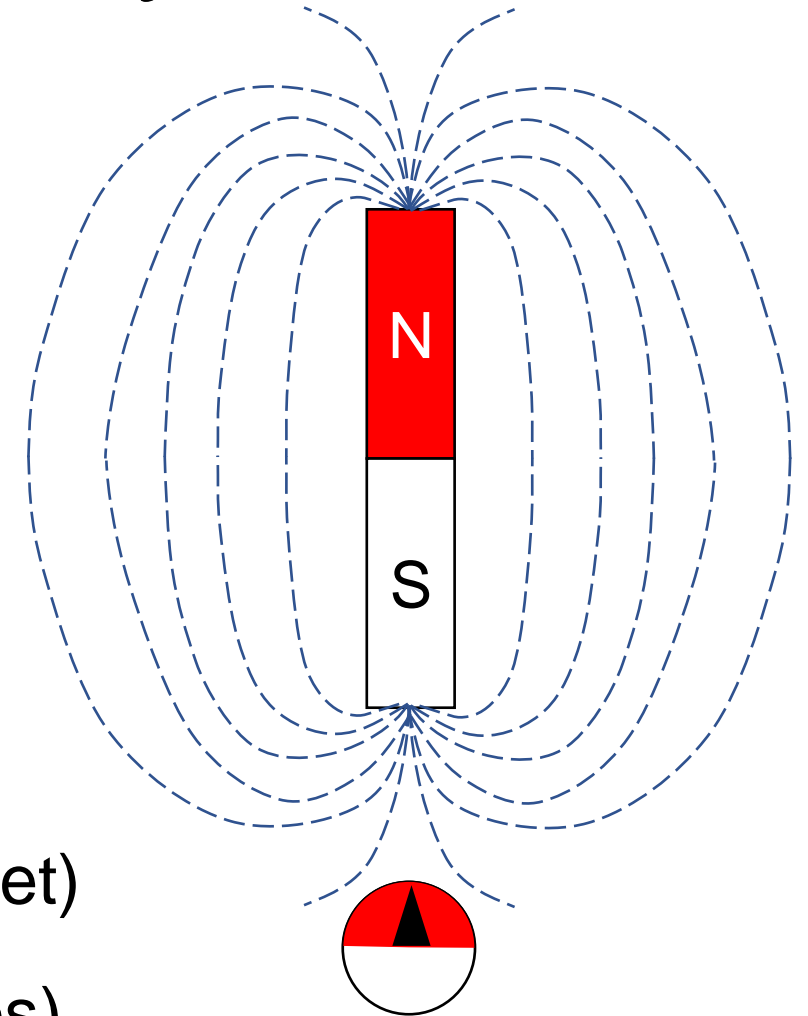
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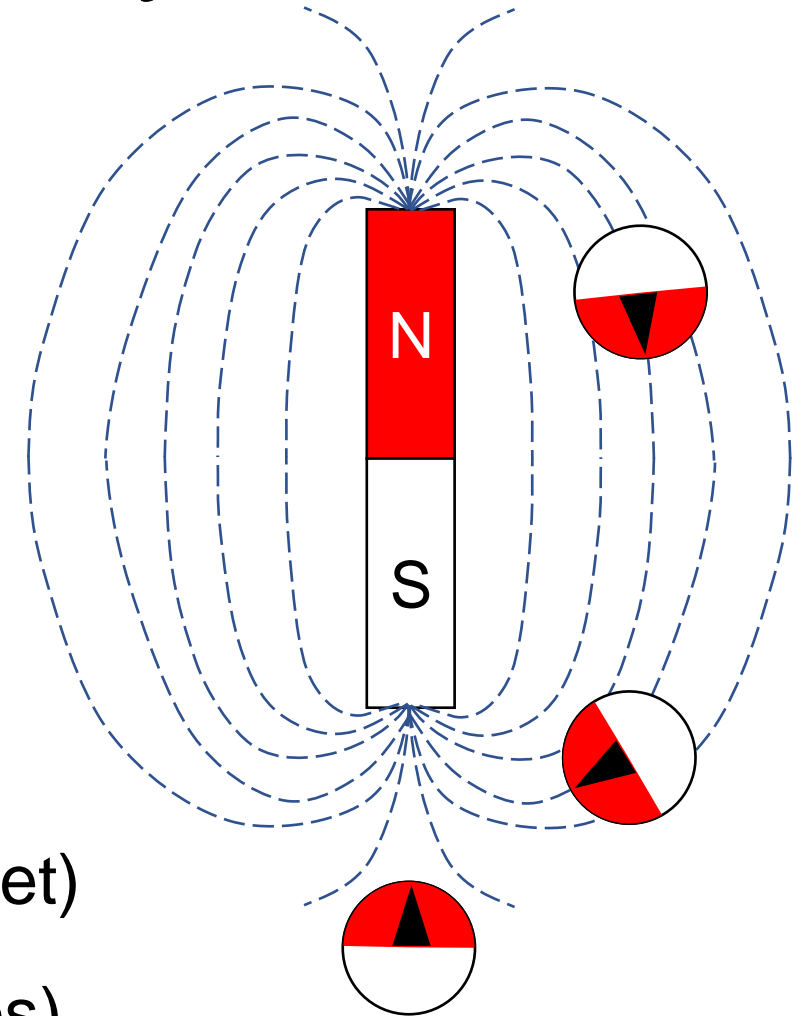
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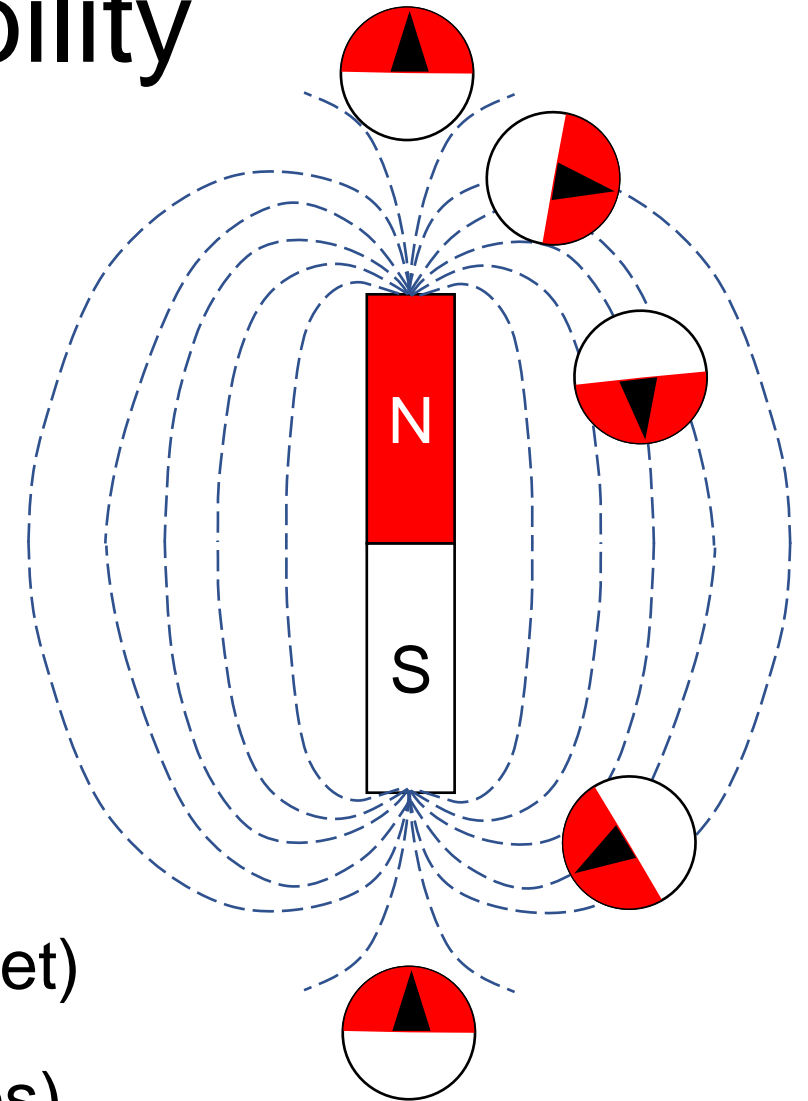
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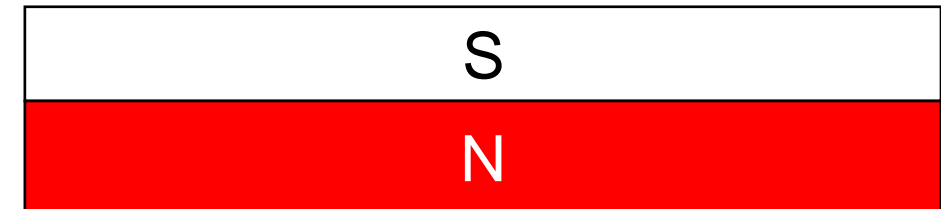
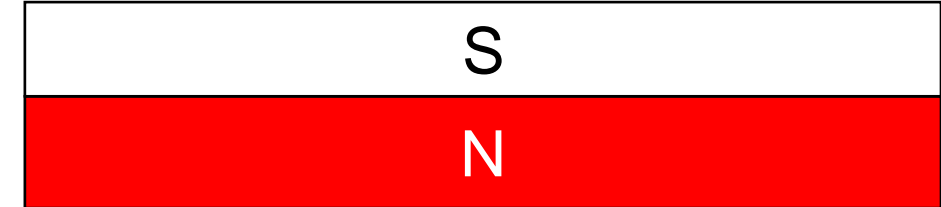
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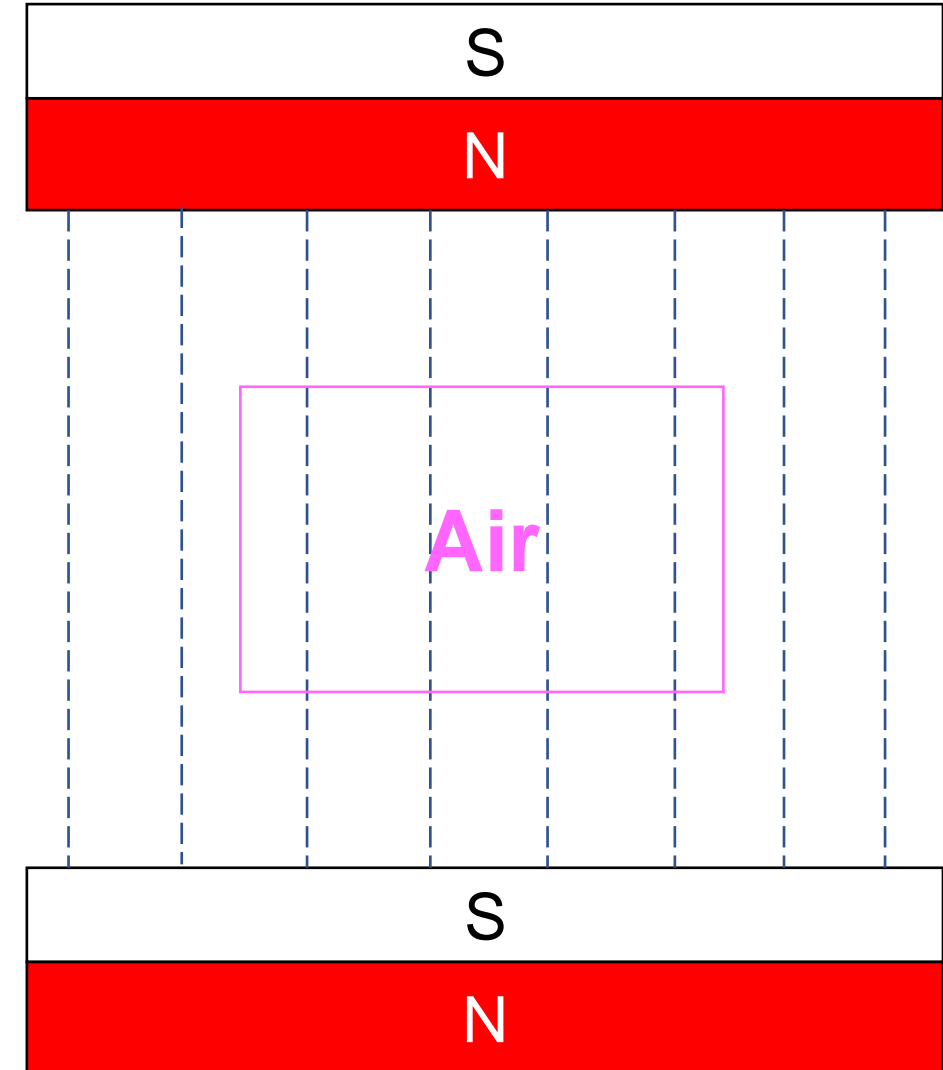
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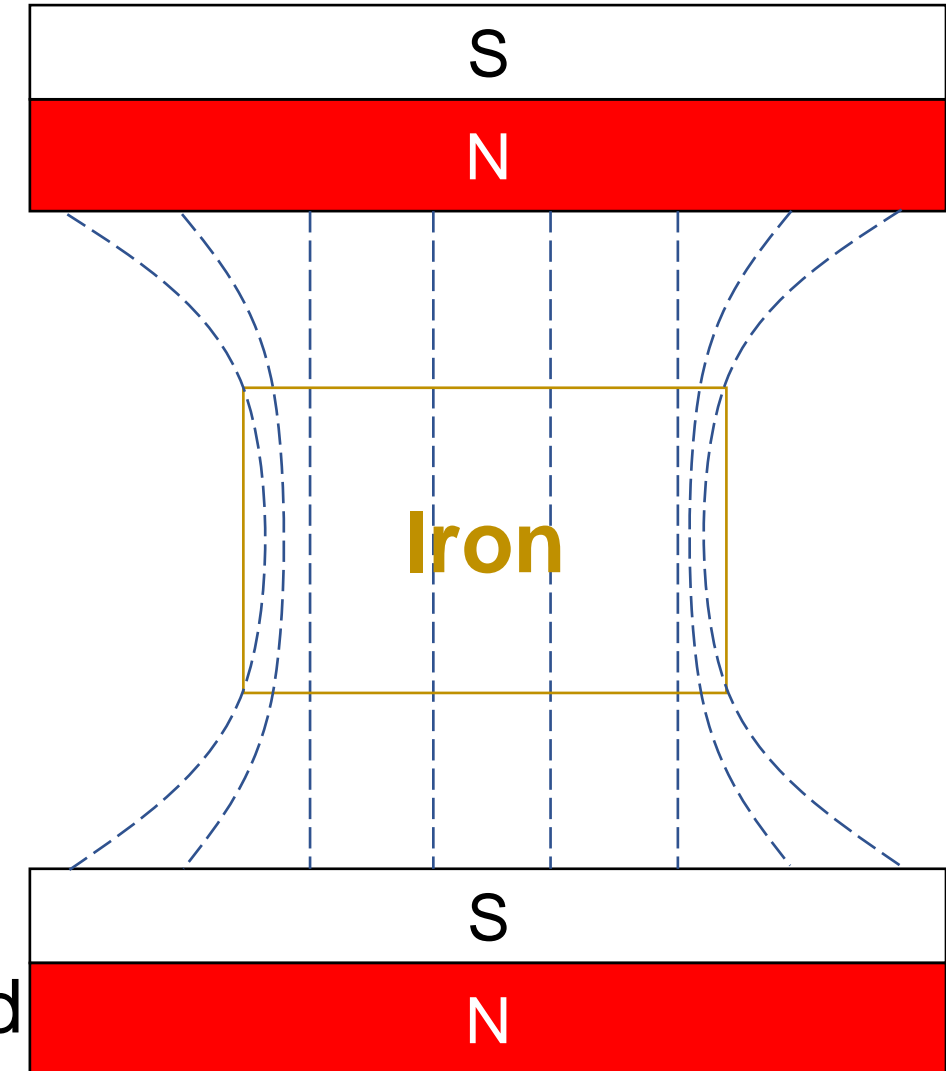
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Air/vacuum allows/permits the magnetic fields to pass through normally.
And so does **many other materials**.
 μ_0 : Ability of vacuum to create internal magnetic fields



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 μ_0 : Ability of vacuum to create internal magnetic fields
- For **iron**, the magnetic fields are warped into the medium.
This is because, fields are formed inside the material.
 $\mu = \mu_0 \mu_r$: Ability of Iron to create internal H field



1.1 Introduction - Until now

- Reciprocal networks (Response of network was independent of direction of signal flow - $S_{ij} = S_{ji}$)
- Component was passive, and materials were isotropic.

1.2 Introduction – Non-reciprocity

- Anisotropic materials and active devices produce non-reciprocal behaviour.
- Nonreciprocity - a useful property in devices like circulators and isolators.
- Nonreciprocity – an ancillary property in devices like transistor amplifiers, ferrite phase shifters.

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- Most practical materials for some microwave applications – Ferrimagnetic compounds known as ferrites.
Example: Yttrium iron garnet (YIG) and materials composed of iron oxides, various elements like aluminium, cobalt, manganese and nickel.

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Example: Yttrium iron garnet (YIG) and materials composed of iron oxides, various elements like aluminium, cobalt, manganese and nickel.
- Ferromagnetic materials (iron, steel)
- Ferrimagnetic compounds have high resistivity and significant anisotropy at microwave frequencies.

1.2 Introduction – Non-reciprocity

- **Magnetic anisotropy** is induced by applying a DC magnetic bias field.
- **Field aligns the magnetic dipoles** in ferrite material to produce a net(zero) **magnetic dipole moment**
Causing the magnetic dipoles to **precess(change in orientation of rotation axis)** at **frequency controlled by strength of the bias field**.

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- **For a given direction of rotation, sense of polarization changes with direction of propagation,**
Microwave signal will propagate through magnetically biased ferrite differently in different directions. (Used in circulators, isolators, gyrators)

1.2 Introduction – Non-reciprocity

- **Interaction** with applied microwave signal – **controlled by adjusting the strength of the bias field.**
Leads to control devices: **phase shifters, switches, tunable resonators** and filters.

1.3 Para-electric materials

- Almost dual of ferrimagnetic materials.
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- Paraelectric materials – variable phase shifters and other control components.
- Isotropic and reciprocal.
- **High dielectric constants and loss tangents** when used in **bulk form**, modern **generally use thin films of paraelectric material layered on a substrate**.
- Advantage: Need for large and heavy magnet or biasing coil is eliminated.

2. Propagation of Microwaves in Ferrite

- To develop many microwave non-reciprocal devices.
- Ferrites: Complex solids represented by $M^{+2}O \cdot Fe_2O_3$.
 M^{+2} : ion of a divalent metal (Cobalt, nickel, zinc, magnesium, cadmium, iron, manganese, chromium, copper, etc).

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- Specific resistance of ferrites: order of $10^7 - 10^8 \Omega m$ (10^{14} times than that of metal)
- Relative permittivity $\epsilon_r \sim 10 - 15$
- Loss tangent $\tan \delta \approx 10^{-4}$ (low loss at microwave frequencies)

2. Propagation of Microwaves in Ferrite

- For magnetically saturated ferrite, magnetized in z direction with RF field propagating in z – *direction*

Permeability of ferrite: (It is asymmetric tensor) $\tilde{\mu} = \mu_0 \begin{bmatrix} \mu & -jK & 0 \\ jK & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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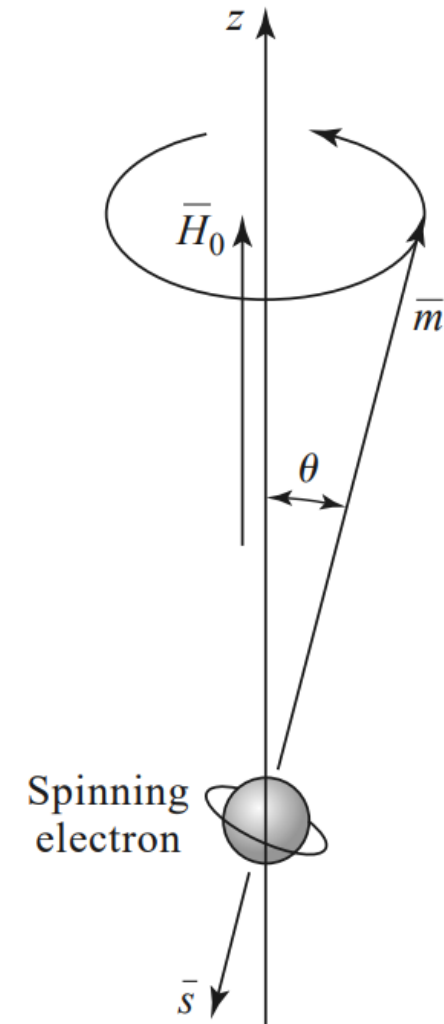
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Ferrites are good dielectrics but exhibit magnetic anisotropy.
- **Non-reciprocal electrical properties** (for microwave propagation)
 - 1) The **transmission coefficient** is **not same for different direction**
 - 2) **Non-reciprocal rotation of the plane of polarization.**

2.1 Magnetic dipole moment of an atom

- **Magnetic properties of ferrites** results from magnetic dipole moment \vec{m} **due to the electron spin**.
- Permeability tensor:
Existence of magnetic dipole moments (due to primarily from electron spin)



2.1 Magnetic dipole moment of an atom

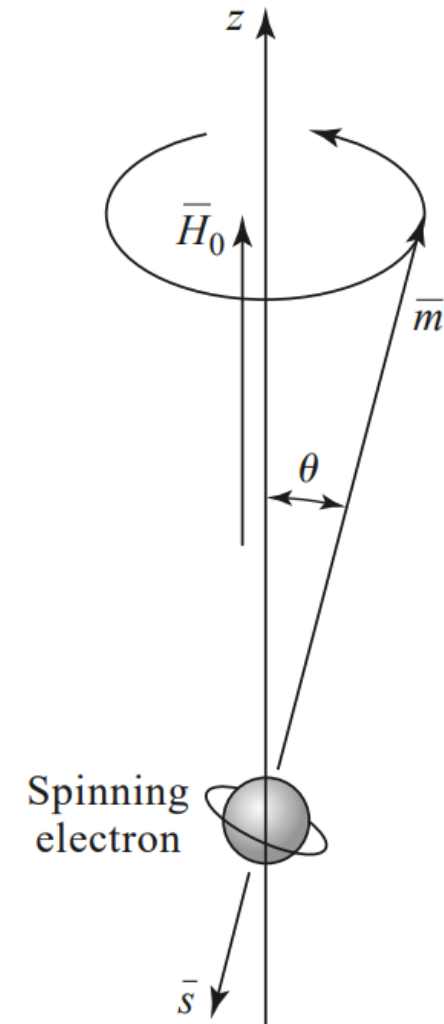
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- **Magnetic dipole moment of an electron due to its spin**

$$m = \frac{qh}{2m_e} = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

h : Plank's constant by 2π .

q : Electron charge

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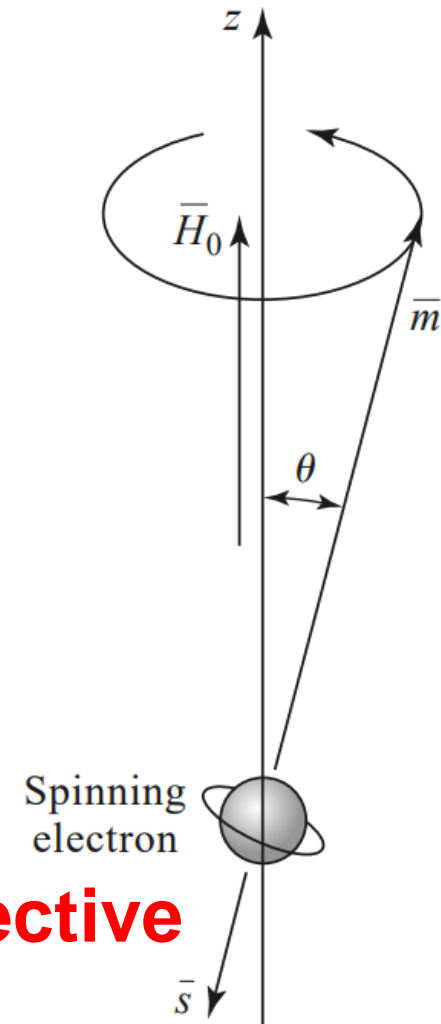
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- An electron in orbit around a nucleus gives rise to an **effective current loop**, thus an **additional magnetic moment**.



2.1 Magnetic dipole moment of an atom

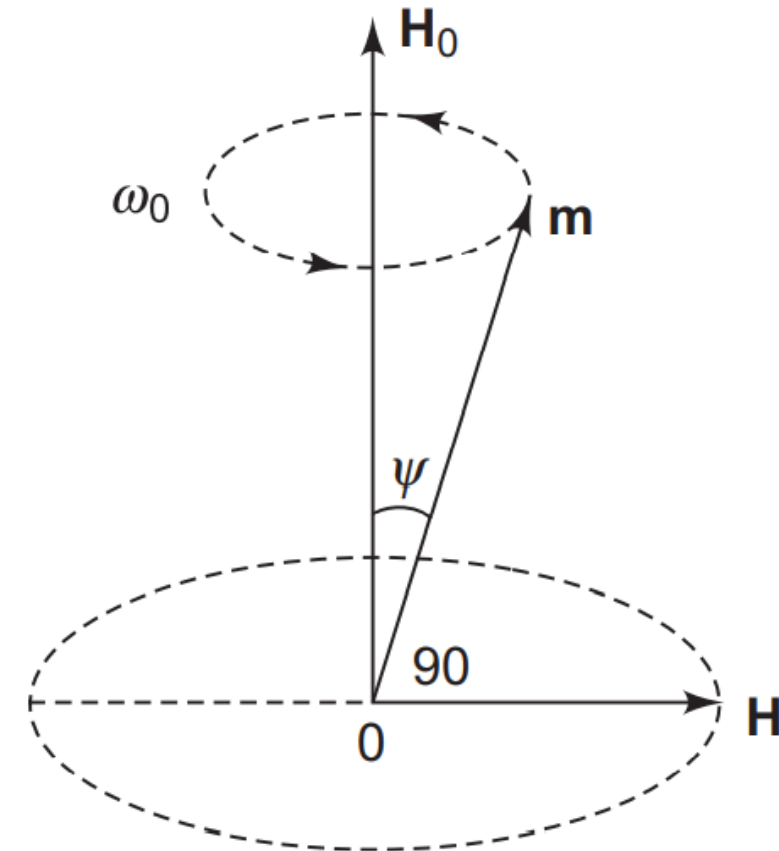
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- Ferrite: **Can be considered as collection of N effective spinning electrons per unit volume.**
- Total magnetic dipole moment $\mathbf{M} = N\mathbf{m}$.

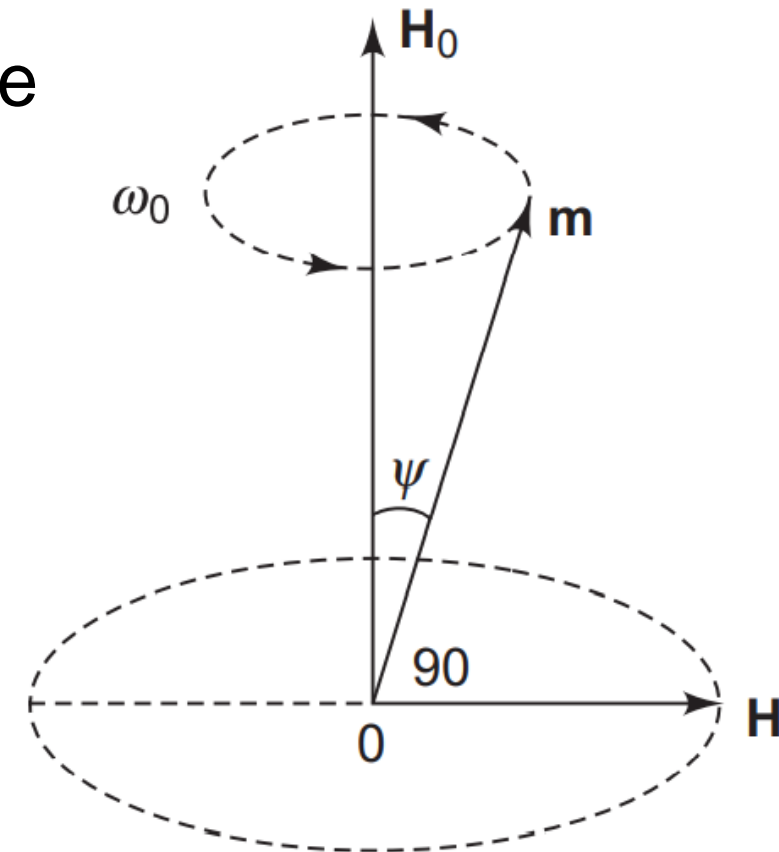
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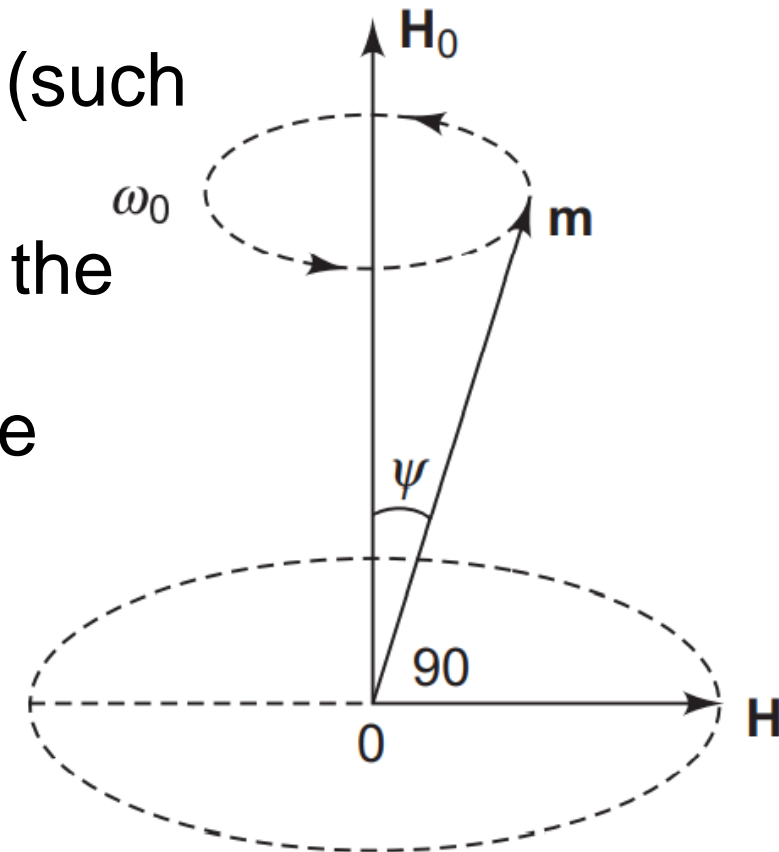


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- If **Steady external magnetic field** H_0 is present in the **direction other than that of \mathbf{m} of an electron**, \mathbf{m} precesses gyromagnetically around H_0 due to the torque

$$\boldsymbol{\tau} = \mathbf{m} \times \mu \mathbf{H}_0$$

And trends to align the electron spin with \mathbf{H}_0 .



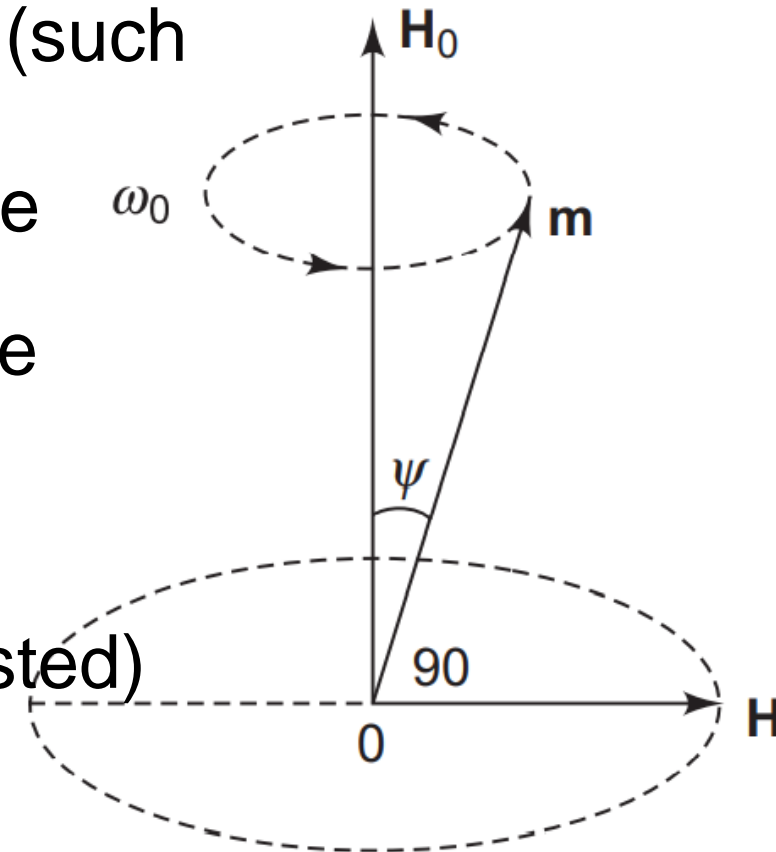
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- **Frequency of precessions** $f_0(\text{MHz}) \approx 2.8H_0$ (oersted)
Gyromagnetic ratio = $\gamma = 2.8 \text{ MHz/oersted}$



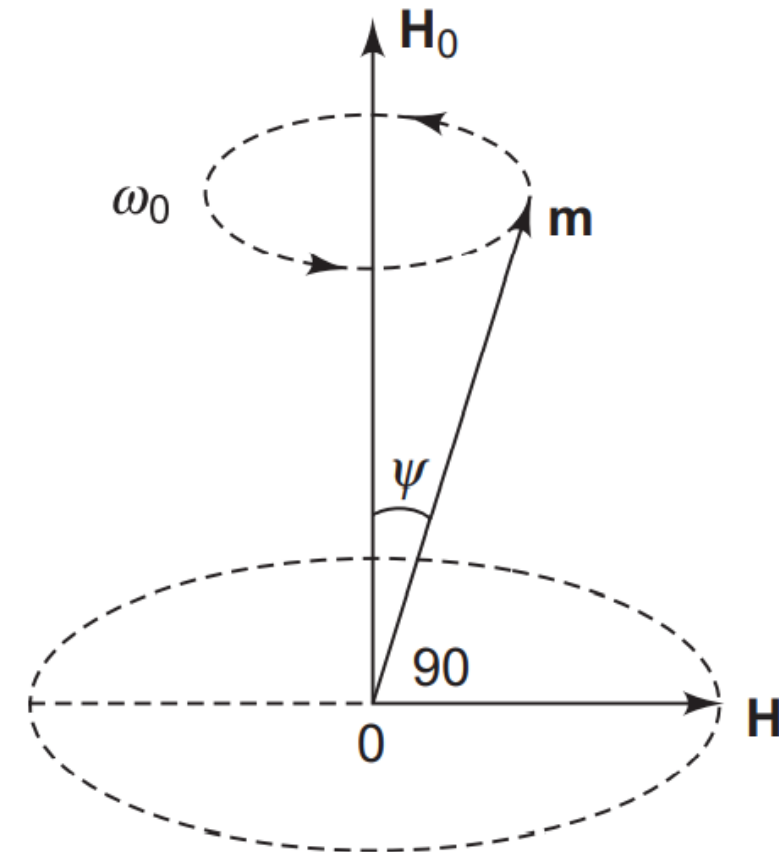
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- Direction of precession – determined by \mathbf{H}_0 and is clockwise looking along \mathbf{H}_0 .
- Angle of precession, $\psi = \angle \mathbf{H}_0 \mathbf{m}$ decreases due to friction.



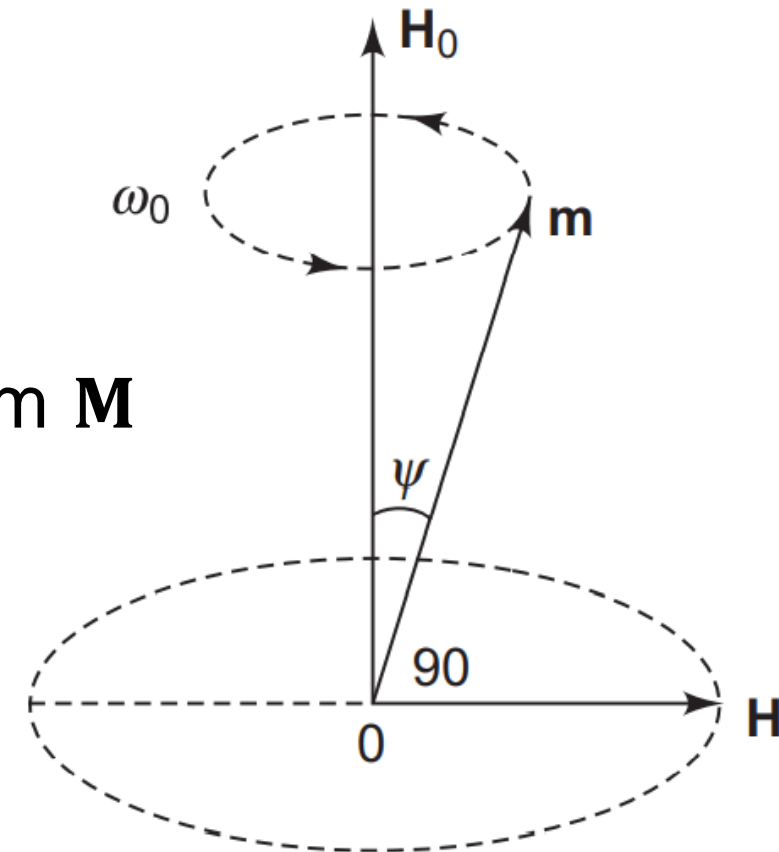
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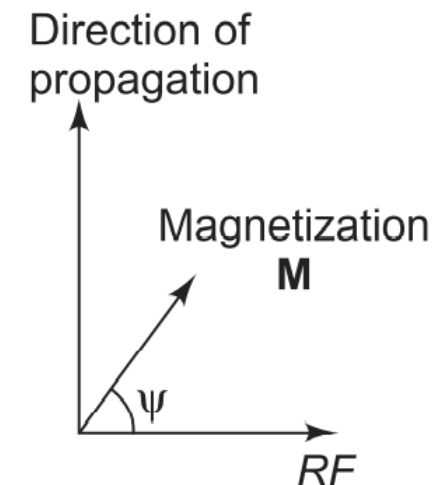
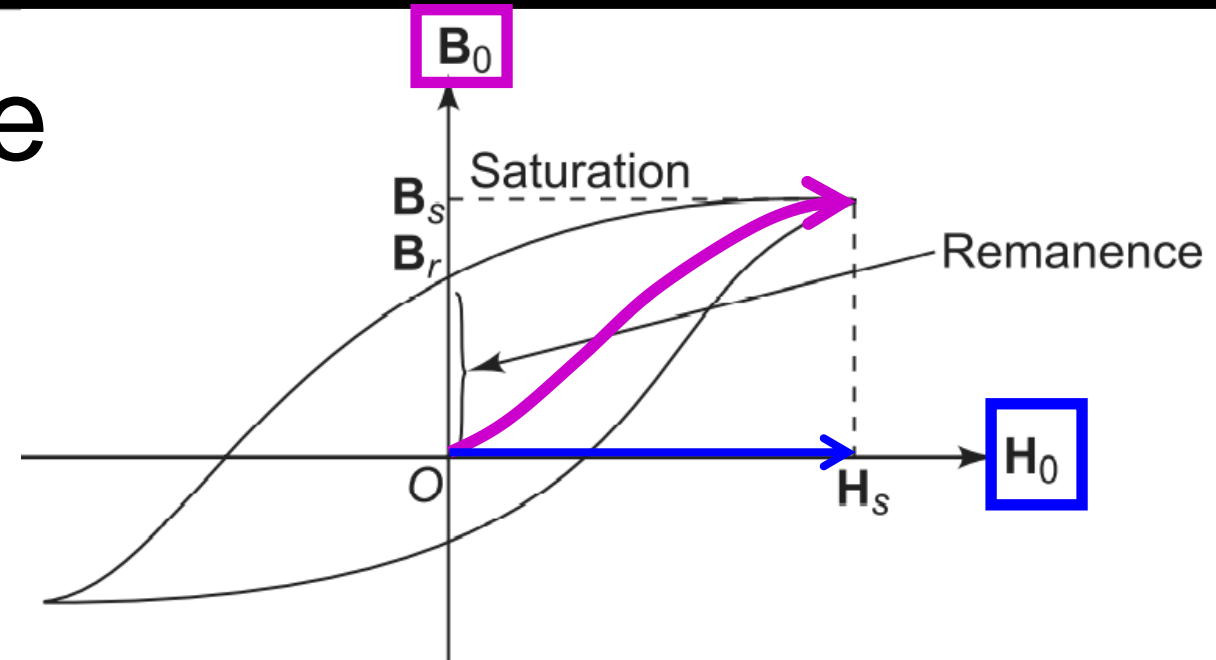
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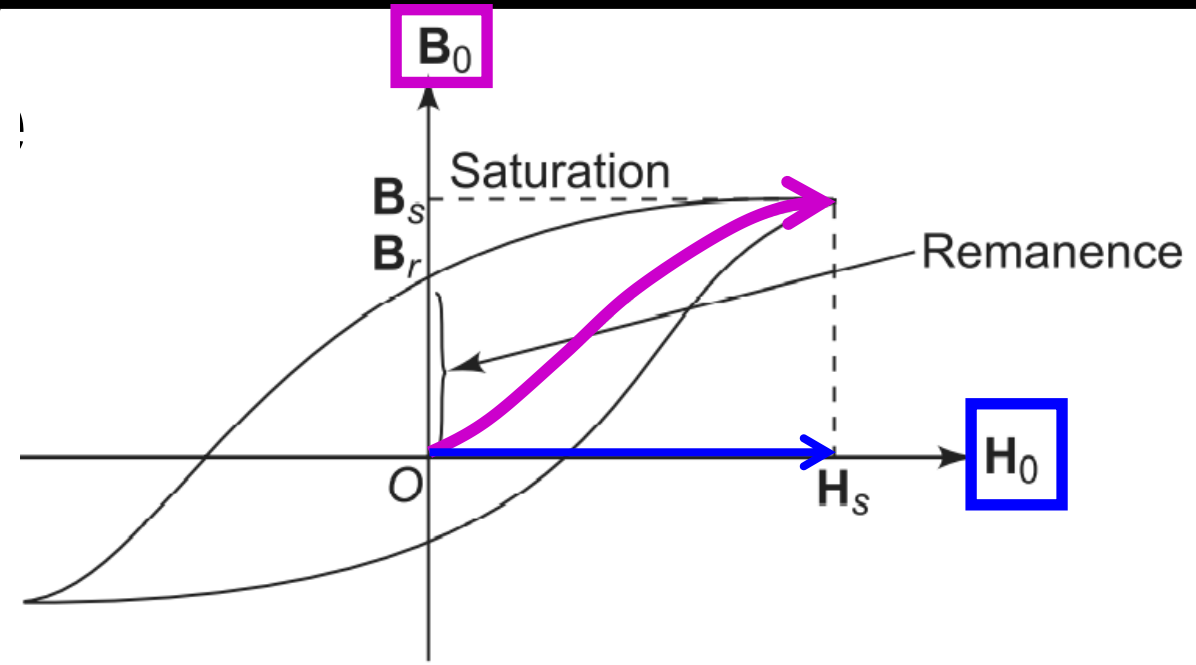
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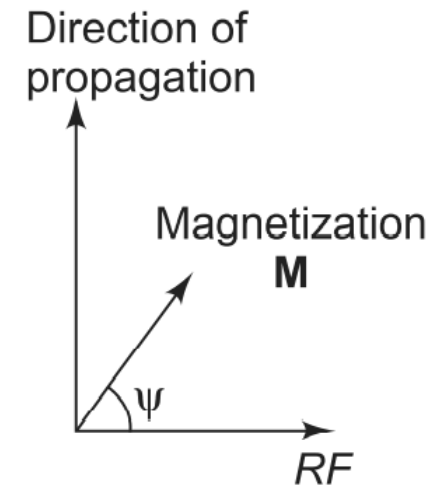
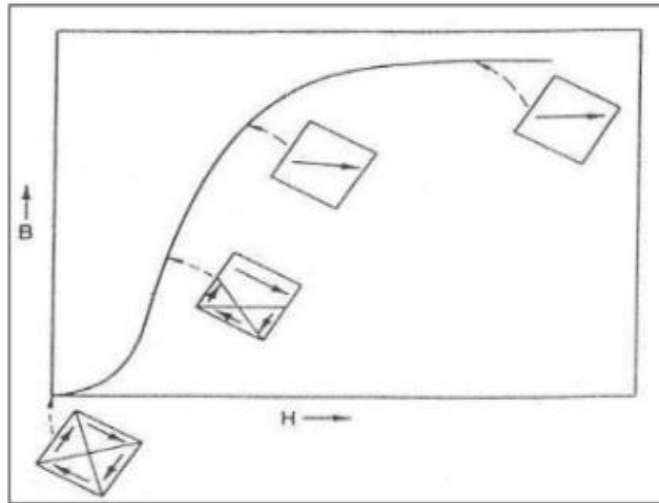
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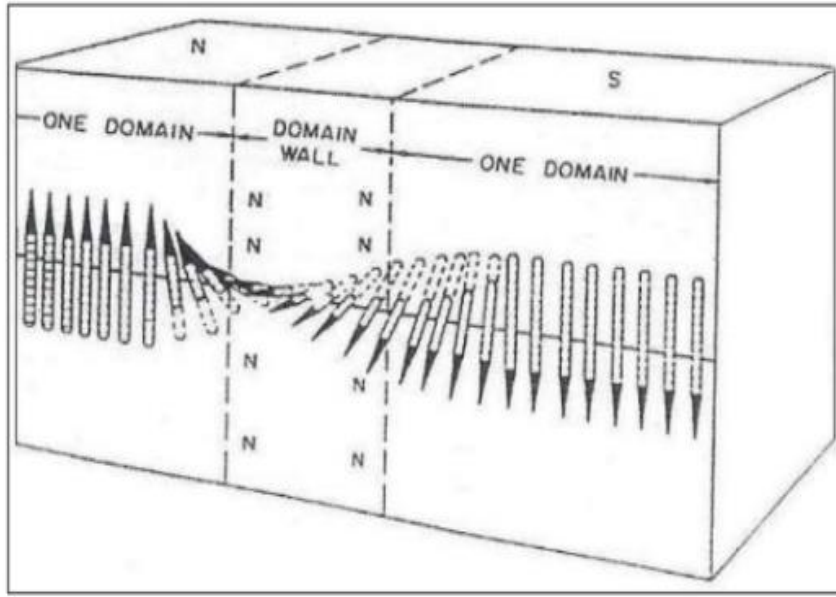
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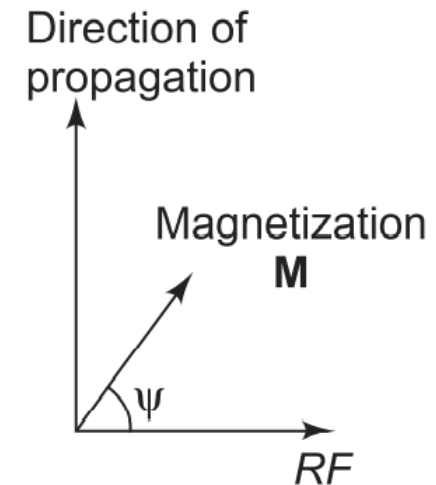
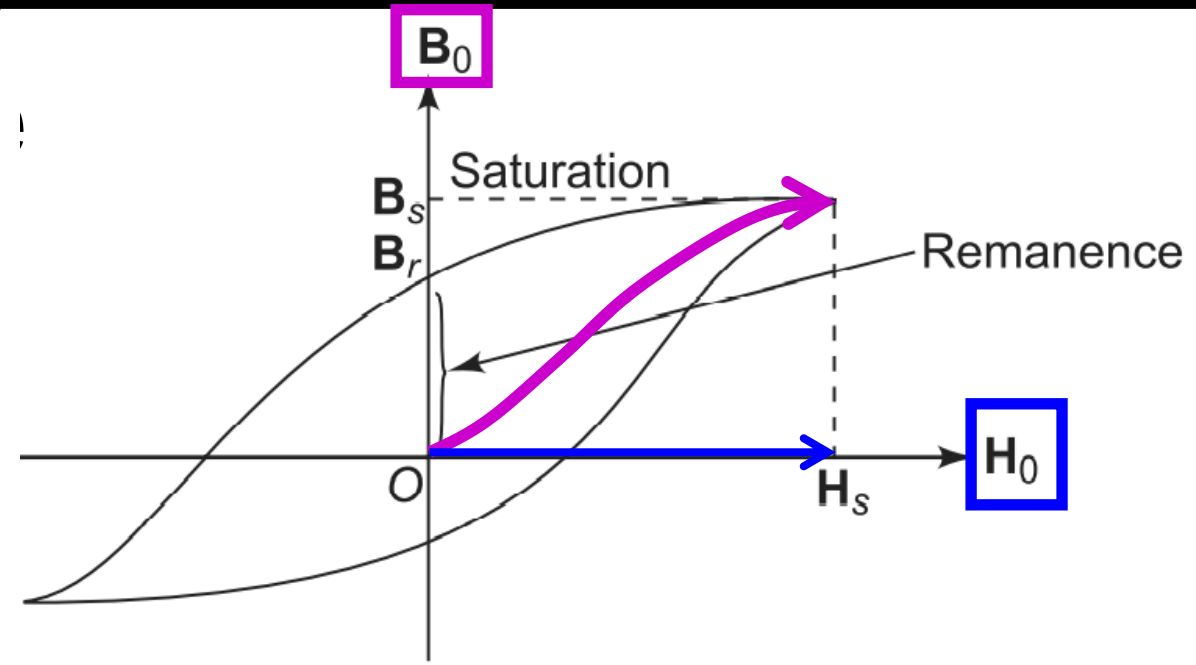
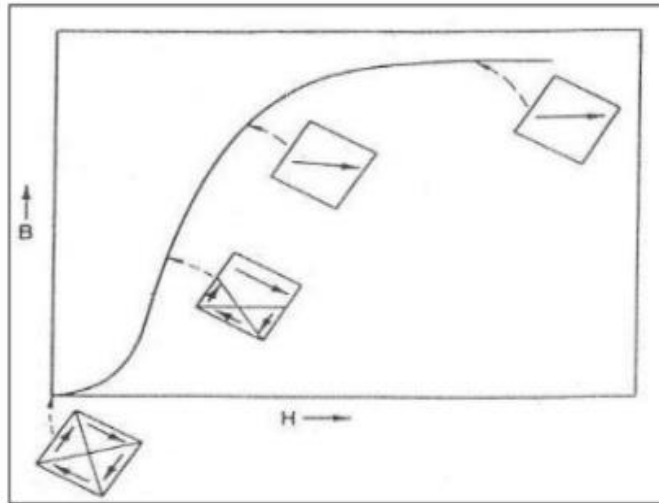


Schematic Illustration of Domain Structure at Various Stages of the Magnetization Process



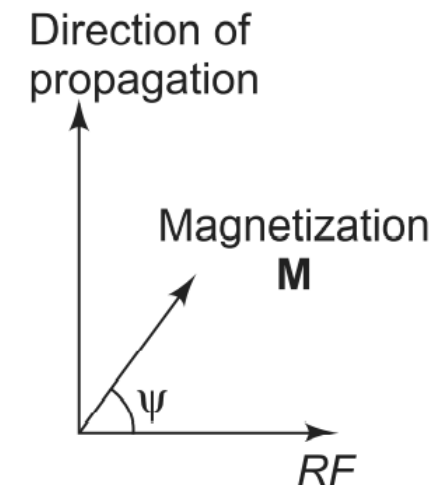
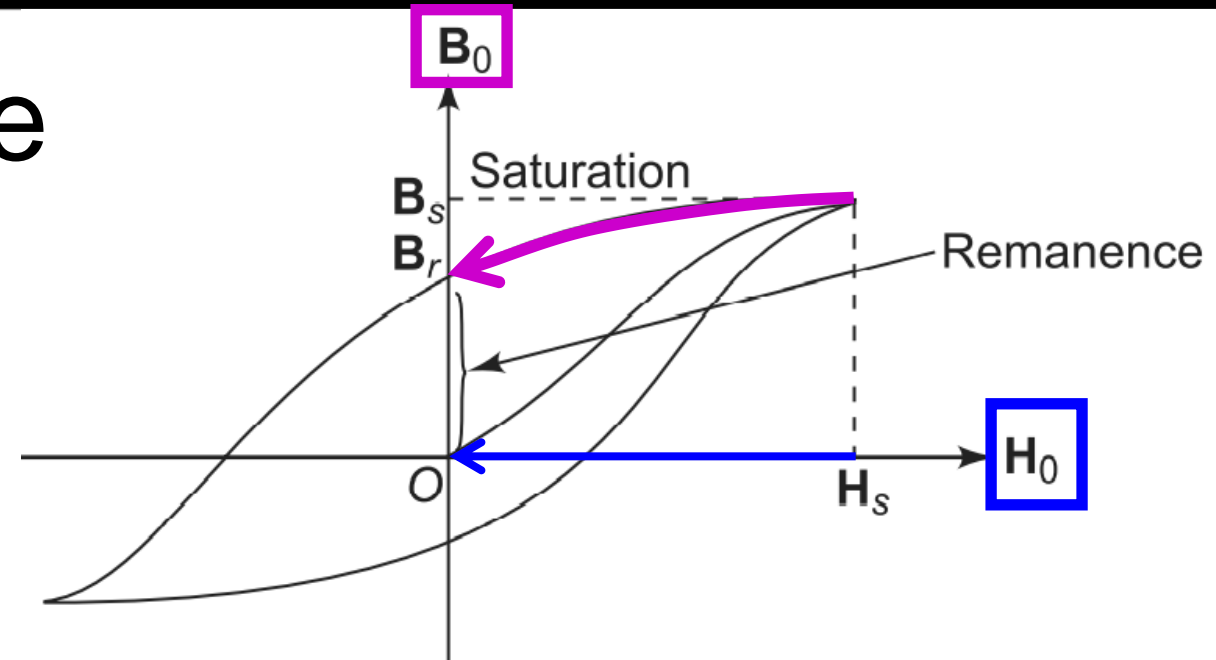


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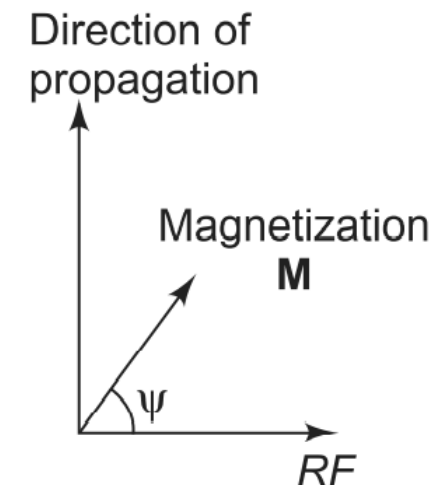
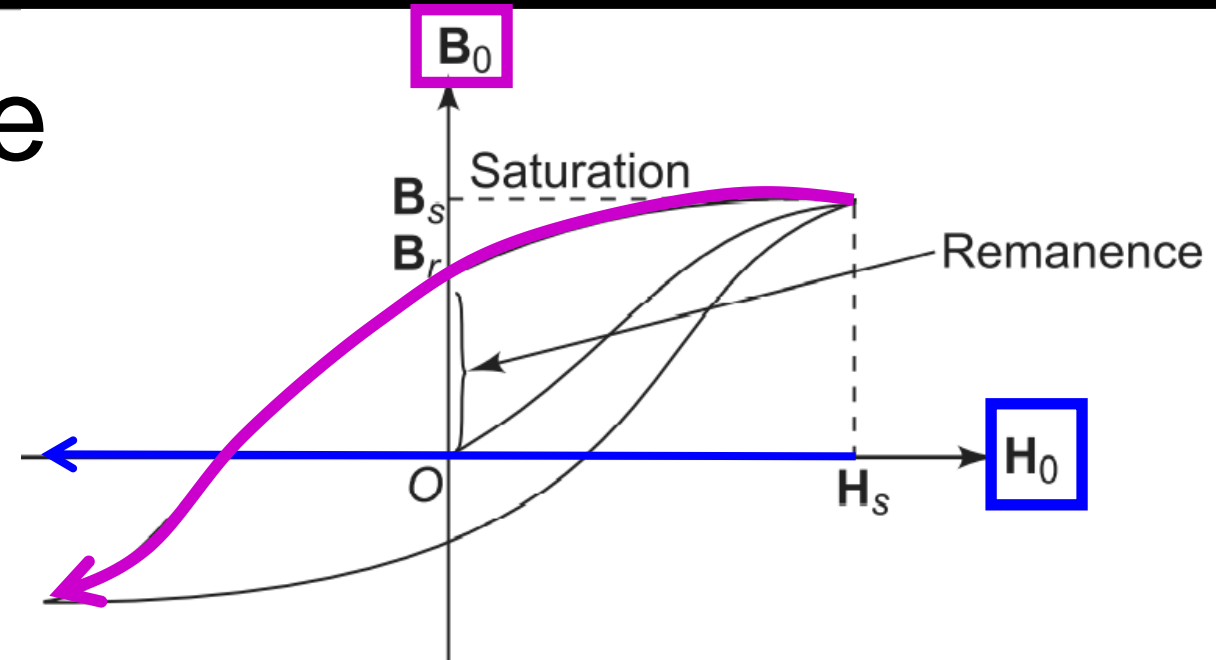
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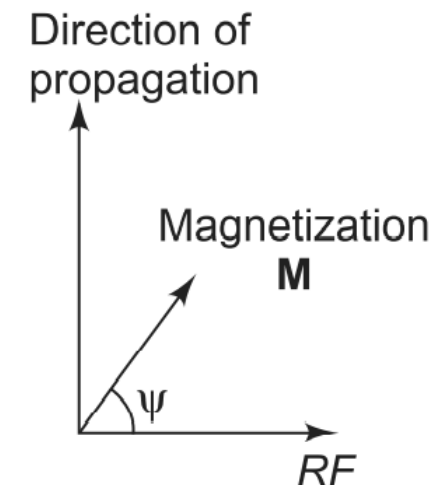
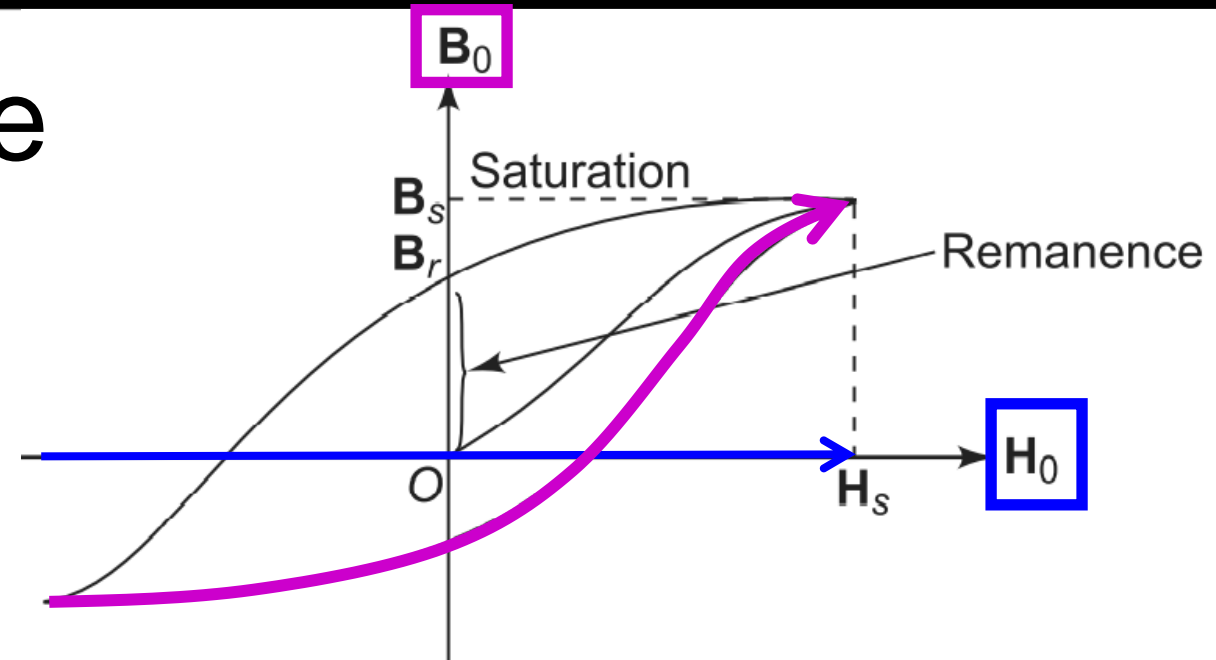
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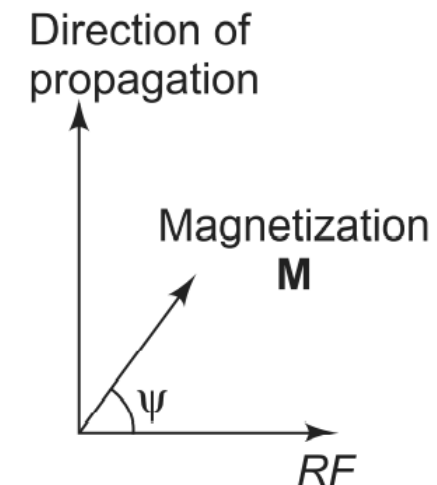
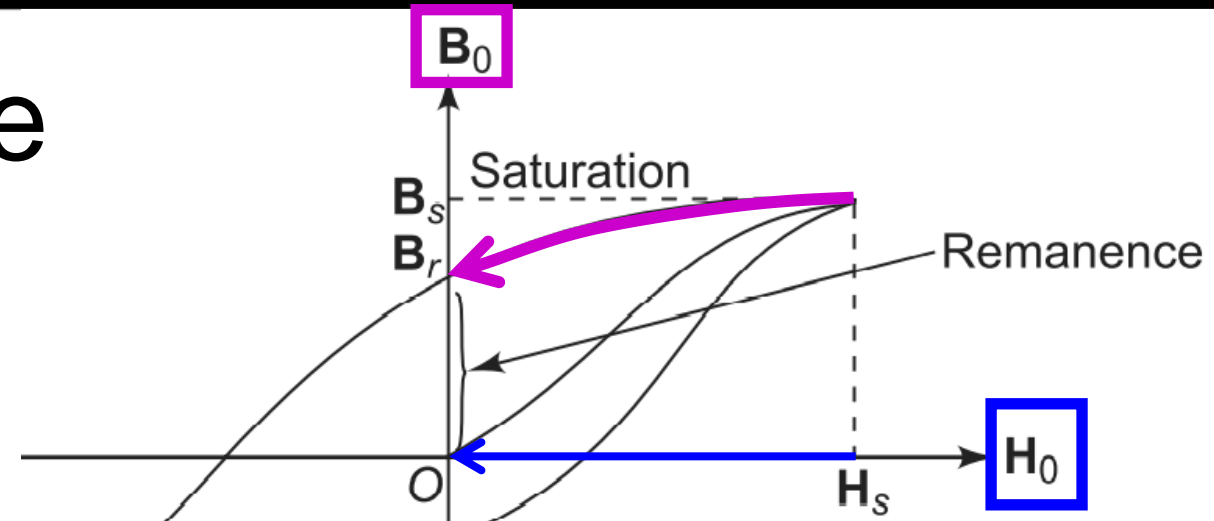
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- Hysteresis: The behaviour of magnetization and demagnetization of ferrite.
- Ferrite has high **remanence value**, B_r : sample remains magnetized even when applied external dc magnetic field H_0 is reduced to zero from its saturation magnetization H_s .

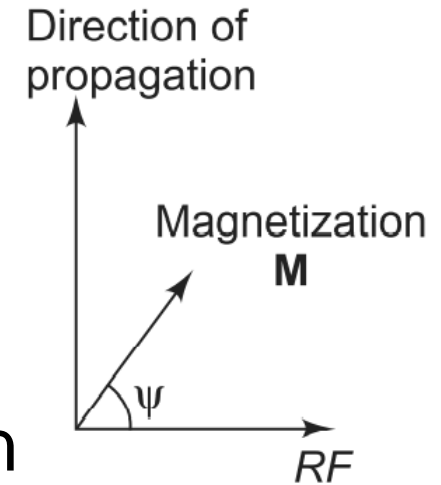


3. TEM wave in ferrite

1) If Ferrite sample is magnetized to saturation in its plane, in direction to RF magnetic field $\psi = 0^\circ$, then for small RF signal strength, minimum interaction between ferrite and RF magnetic field.

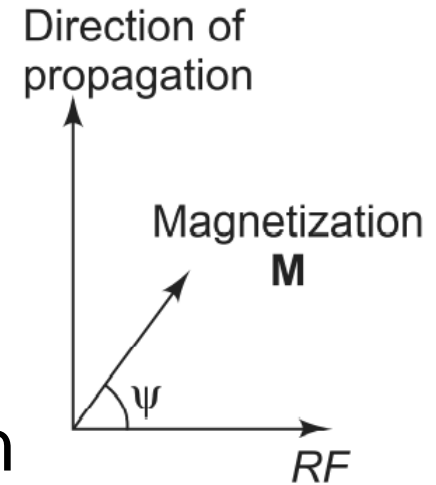
- Effective RF relative permeability $\mu_r \approx 1$

2)



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- Effective RF relative permeability $\mu_r \approx 1$

2) If Ferrite sample is magnetized to saturation in direction perpendicular to RF magnetic field $\psi = 90^\circ$, maximum interaction takes place between ferrite medium and RF magnetic field.

- Effective RF relative permeability $\mu_r = \frac{\mu^2 - K^2}{\mu^2}$ where μ and K are components of permeability tensor $K = \frac{\gamma_m 4\pi M_s}{\omega}$.

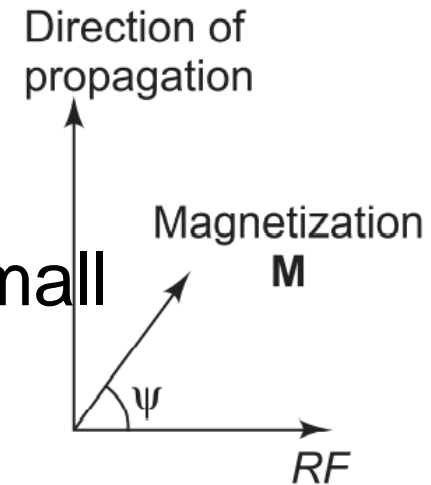
3. TEM wave in ferrite

In the frequency range of interest where $\frac{\gamma_m 4\pi M_s}{\omega} < 1$, and with small applied magnetic field, the effective RF relative permeability becomes:

$$\mu_r \approx 1 - \left(\frac{\gamma_m 4\pi M_s}{\omega} \right)^2$$

Where the ferrite's magnetization = $4\pi M_s$.

When $\omega = \gamma_m 4\pi M_s$, magnetic loss becomes significant and the μ_r equation changes.



3. TEM wave in ferrite

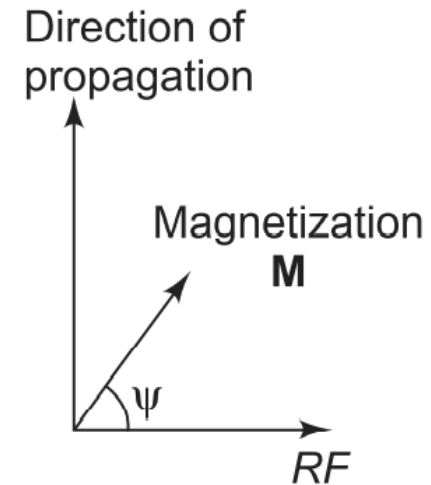
- The propagation constant of TEM wave in ferrite

$$\beta = \omega \sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}$$

- When the ferrite is magnetized with

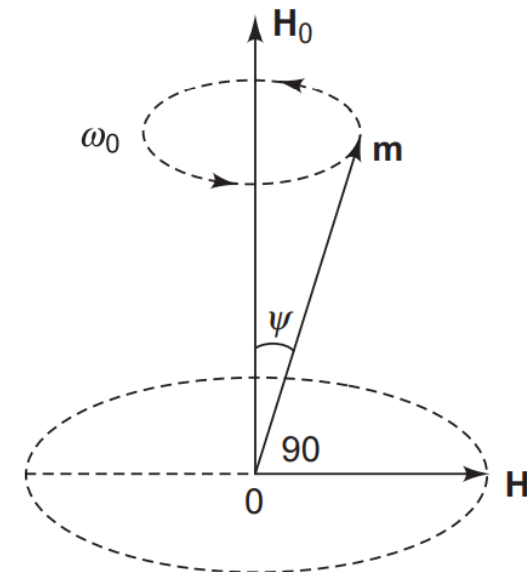
$$\mu_r = \begin{cases} 1, & \text{non interacting state} \\ \frac{\mu^2 - K^2}{\mu^2}, & \text{Interacting state} \end{cases}$$

- Propagation constant changes its state with the direction of magnetization.



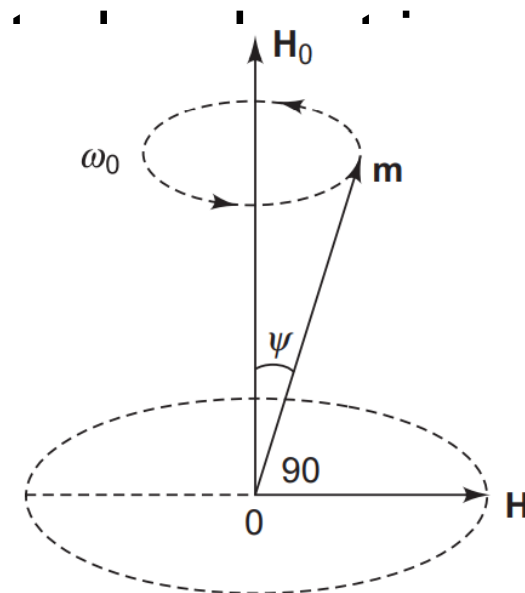
4. Gyromagnetic resonance

- In addition to external steady magnetic field, if RF or microwave circularly polarized magnetic field \mathbf{H} at frequency $f \approx f_0$ is applied perpendicular to \mathbf{H}_0 ,



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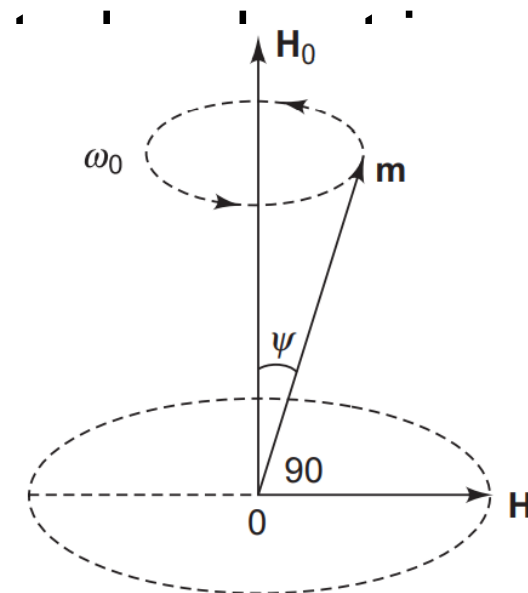
- In addition to external steady magnetic field, if RF or microwave circularly polarized magnetic field \mathbf{H} at frequency $f \approx f_0$ is applied perpendicular to \mathbf{H}_0 ,
When the direction of rotation of \mathbf{H} and \mathbf{M} coincide:
 the angle of precession ψ of \mathbf{M} and also the amplitude of induced precession will tend to increase at $f = f_0$ and reach a steady state due to magnetic frictional loss.
 The energy continuously supplied by the RF field is dissipated in the ferrite.



4. Gyromagnetic resonance

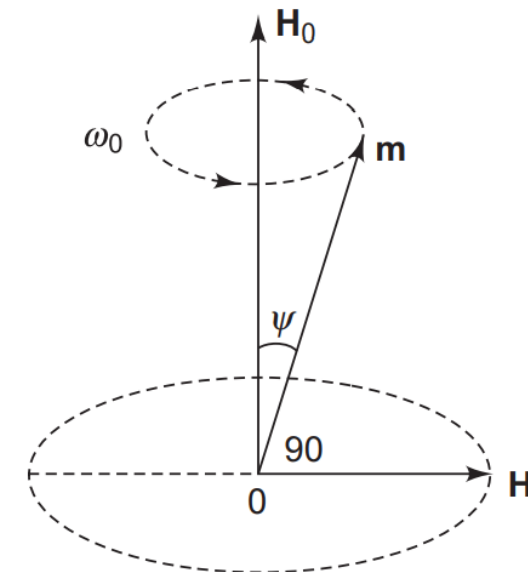
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When the interaction between RF/Microwave field and electrons is reduced: Low losses in ferrite.



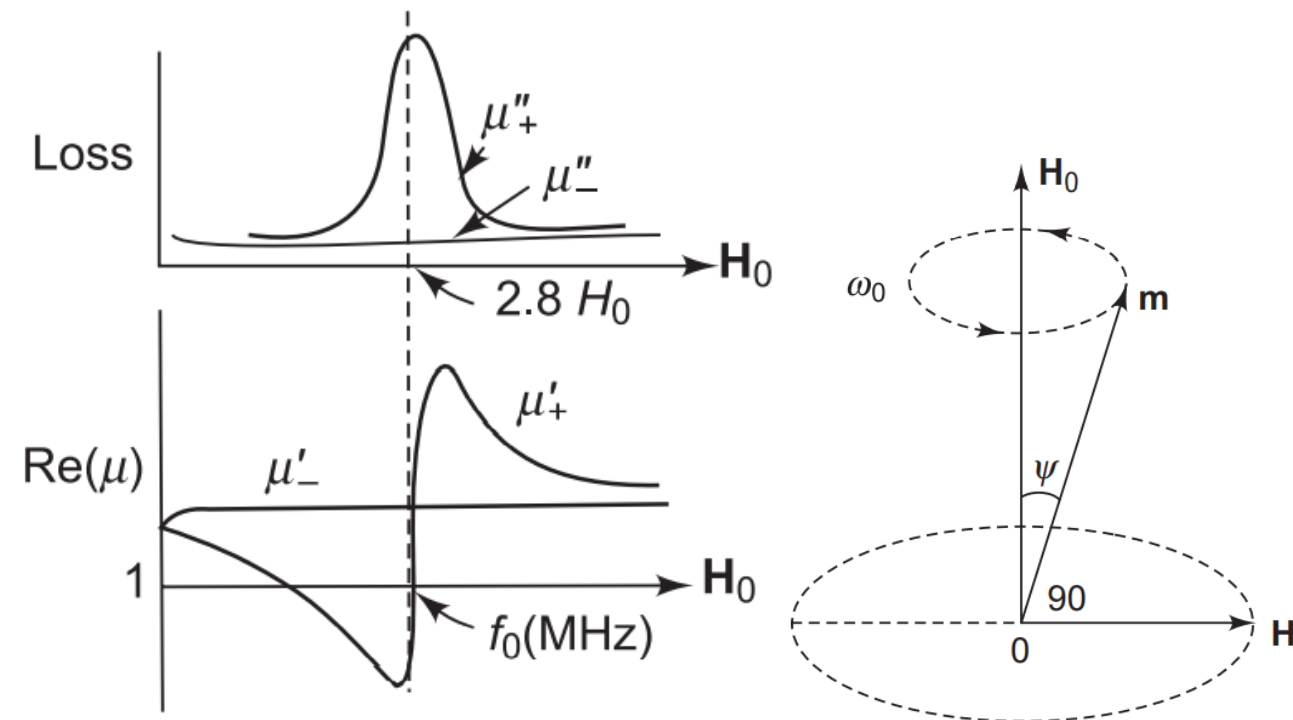
4. Gyromagnetic resonance

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When the direction of rotation of \mathbf{H} and \mathbf{M} coincide:
the angle of precession ψ of \mathbf{M} and also the amplitude of induced precession will tend to increase at $f = f_0$ and reach a steady state due to magnetic frictional loss.
The energy continuously supplied by the RF field is dissipated as heat in the ferrite.
- When the interaction between RF/Microwave field and electrons is reduced: Low losses in ferrite.
- When \mathbf{H} and \mathbf{M} rotate in opposite direction:
Ferrite dissipates no time averaged power, and hence exhibits low losses.



4. Gyromagnetic resonance

- Hence a resonance (gyromagnetic) with
 - a peak of loss exists for microwave propagation in ferrites, for clockwise polarization of \mathbf{H} coinciding with \mathbf{M}
 - a flat low loss for opposite polarization



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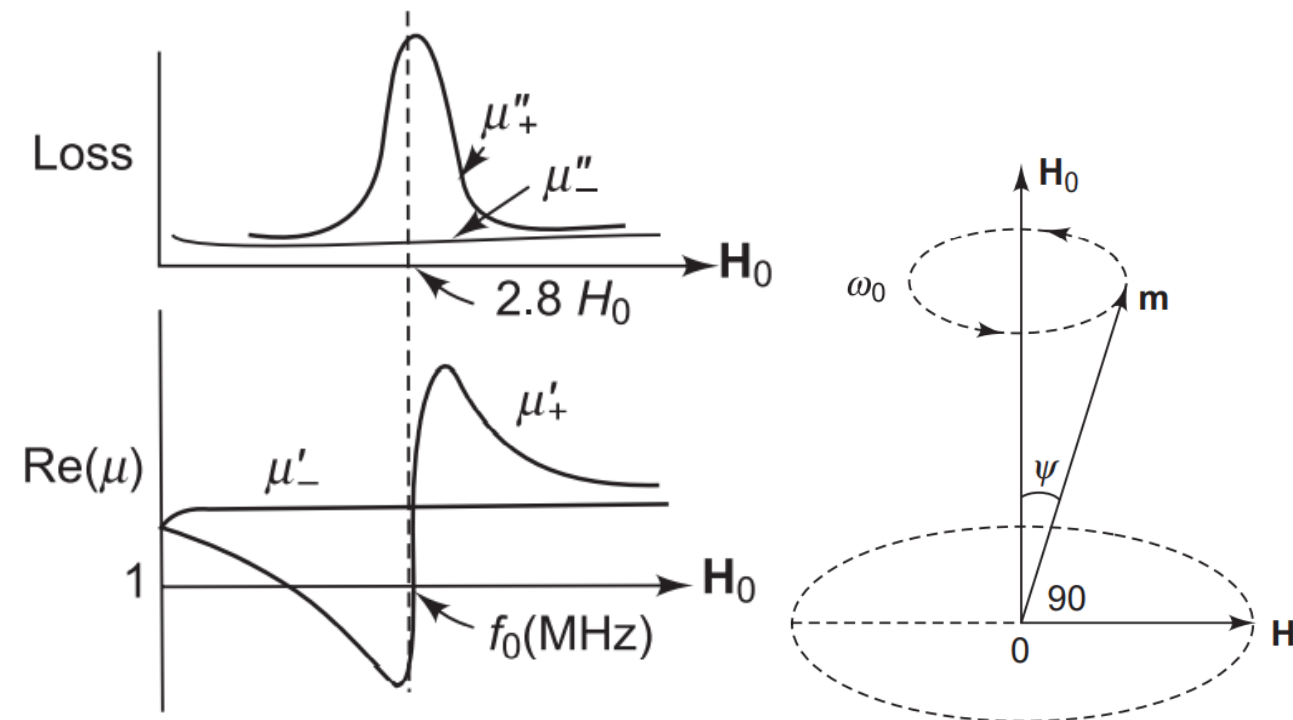
Clockwise polarization:

$$B_t^+ = \mu_0 M^+ + B^+ = \mu_+ H^+$$

Anticlockwise polarization

$$B_t^- = \mu_0 M^- + B^- = \mu_- H^-$$

Two complex permeabilities μ_+ , μ_-



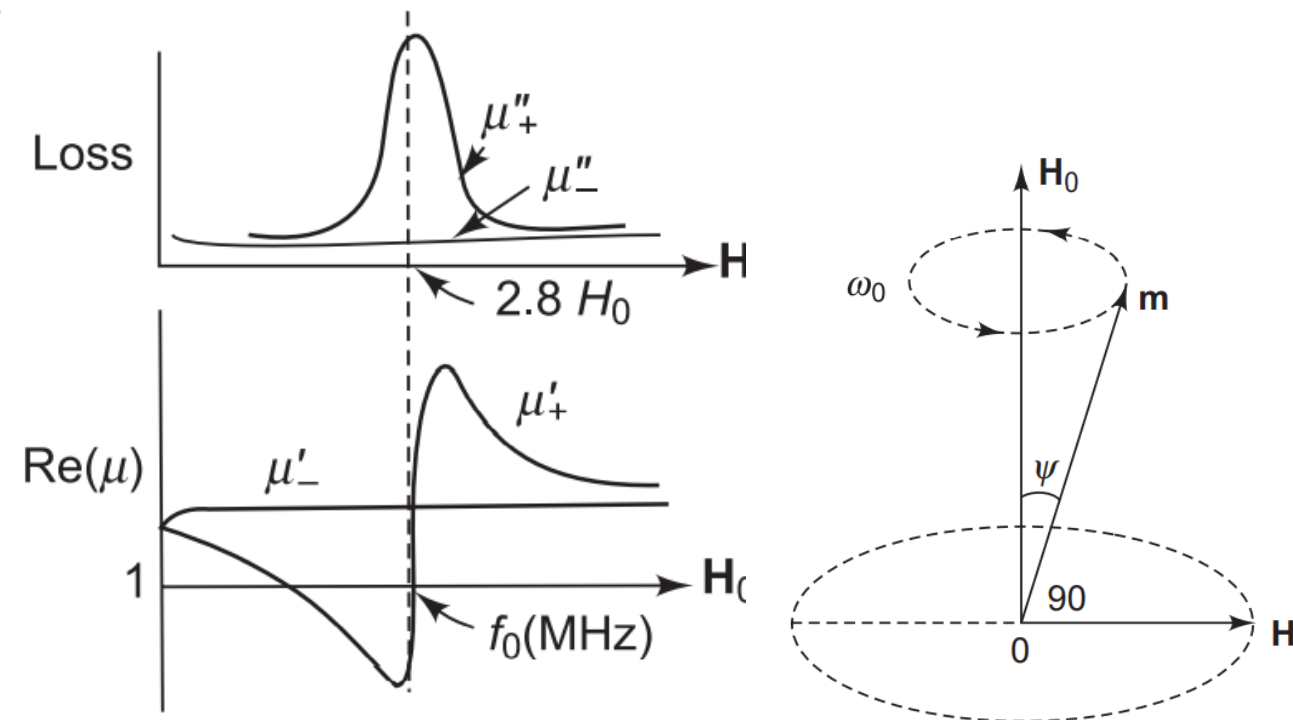
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$$\mu_+ = \mu'_+ - j\mu''_+$$

$$\mu_- = \mu'_- - j\mu''_-$$

Real and imaginary parts of μ_- : are independent of applied steady magnetic field.

μ_+ : Resonant behaviour at H_0 value



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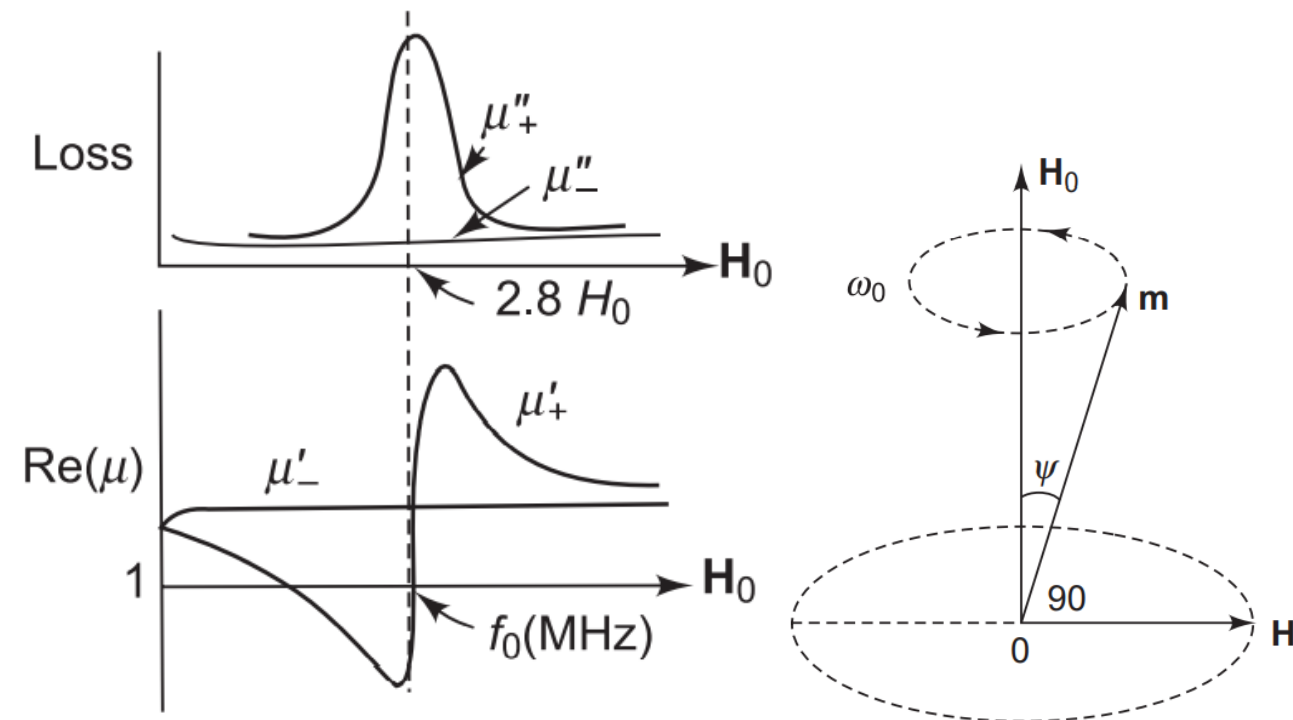
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Linearly polarized: Combination of Two circularly polarized. Therefore Non-reciprocal behaviour of ferrite materials to microwave propagation.



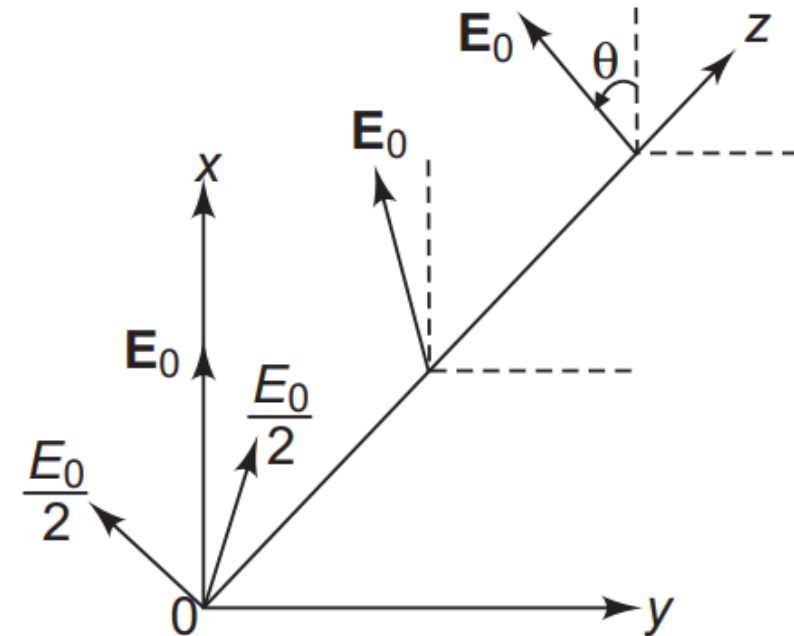
5. Faraday Rotation

$$\mu_+ = \mu'_+ - j\mu''_+ \quad \mu_- = \mu'_- - j\mu''_-$$

Plane circularly polarized wave propagating in H_0 direction will have two propagation constants

$$\beta^+ =$$

$$\beta^- =$$



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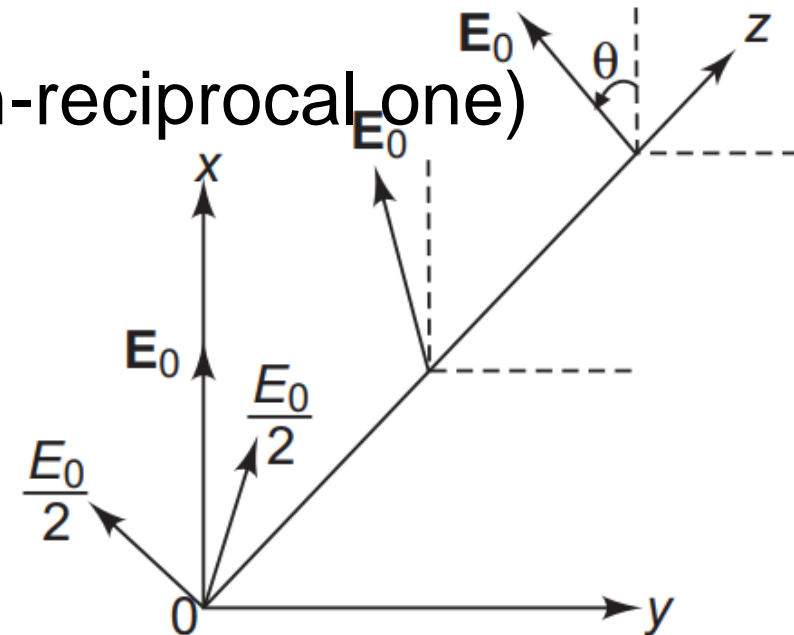
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$$\beta^+ = \omega\sqrt{\epsilon\mu_+} = \frac{2\pi}{\lambda^+} \text{ Clockwise}$$

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Which results in plane of polarization to rotate. (Non-reciprocal one)



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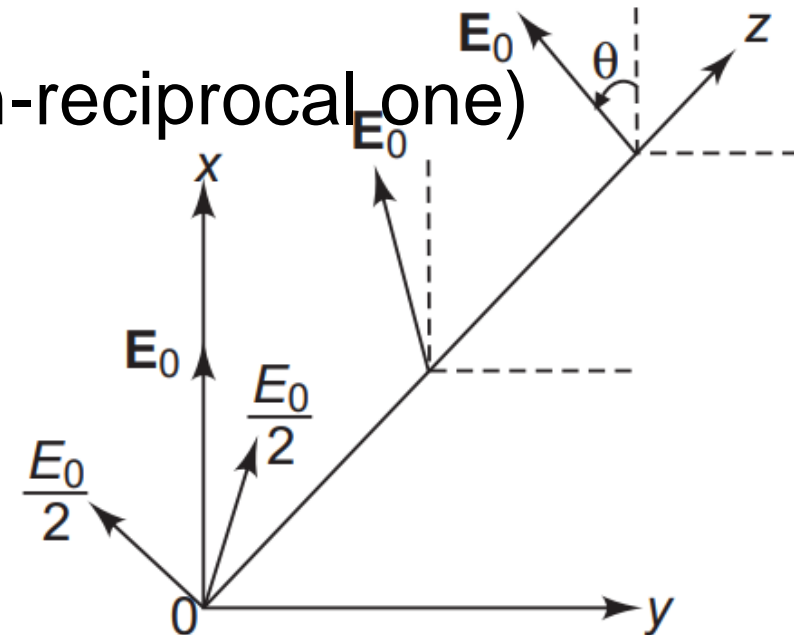
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Rotation of Electric field of linearly polarized wave passing through a magnetized ferrite is

Faraday rotation in ferrites.

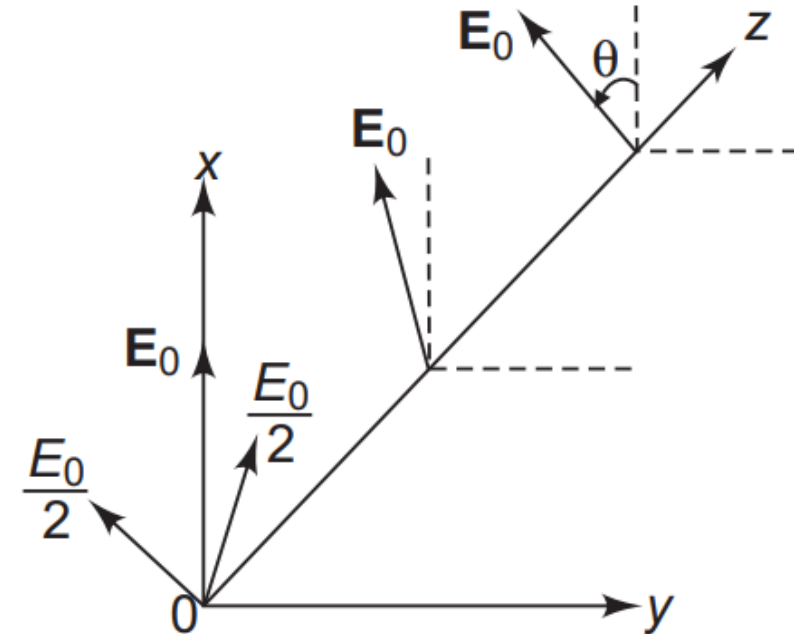


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Linearly polarized TEM wave propagates in ferrite along +z axis.

$$\mathbf{E} = \bar{a}_x E_x \text{ at } z = 0$$



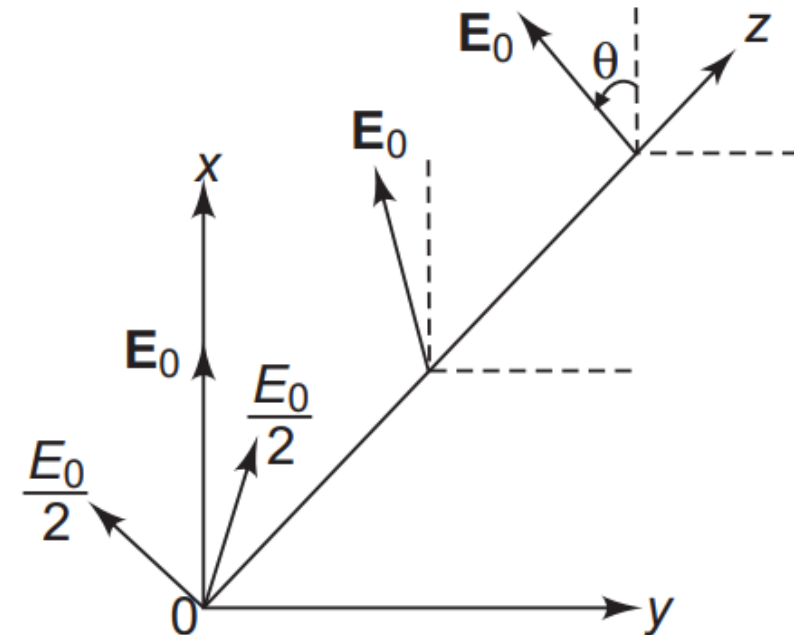
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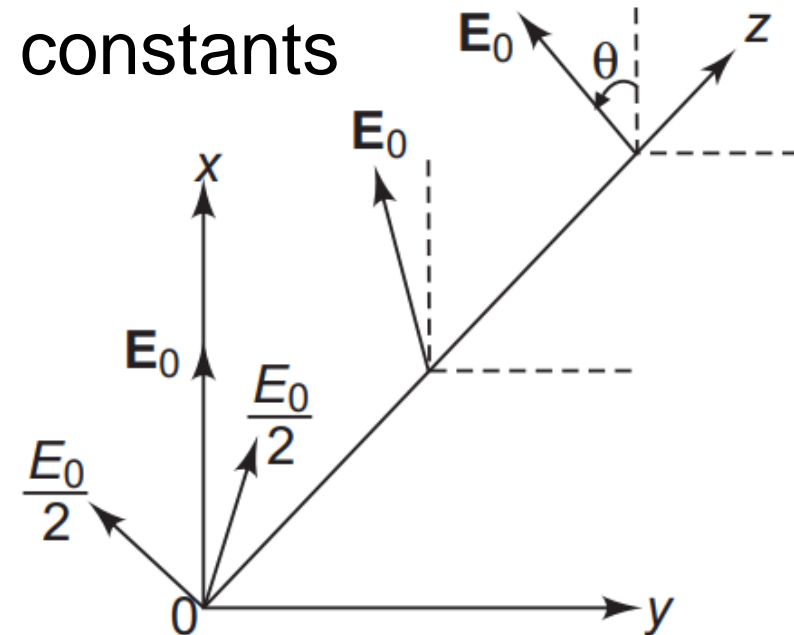
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The component waves propagate at different phase constants β^+ , β^- .

Two electric field vectors rotate at different rates.



5. Faraday Rotation

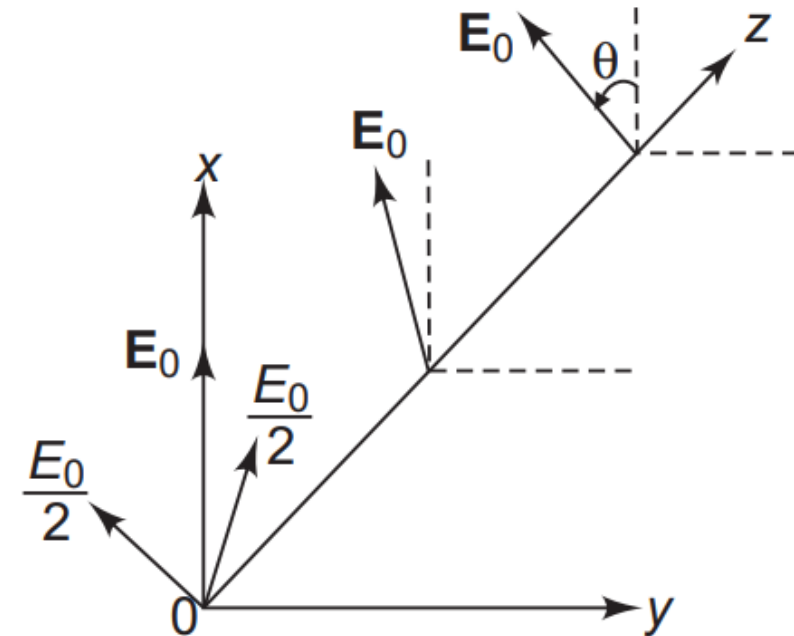
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Over a distance λ , resultant polarized wave undergoes phase shift of $(\beta^+ + \beta^-)\lambda = 4\pi$

$$\lambda = \frac{4\pi}{\beta^+ + \beta^-} = \frac{2\lambda^+\lambda^-}{\lambda^+ + \lambda^-} \quad \text{with angle } \frac{(\beta^+ - \beta^-)\lambda}{2} = \frac{(\beta^+ - \beta^-)}{(\beta^+ + \beta^-)} 2\pi$$

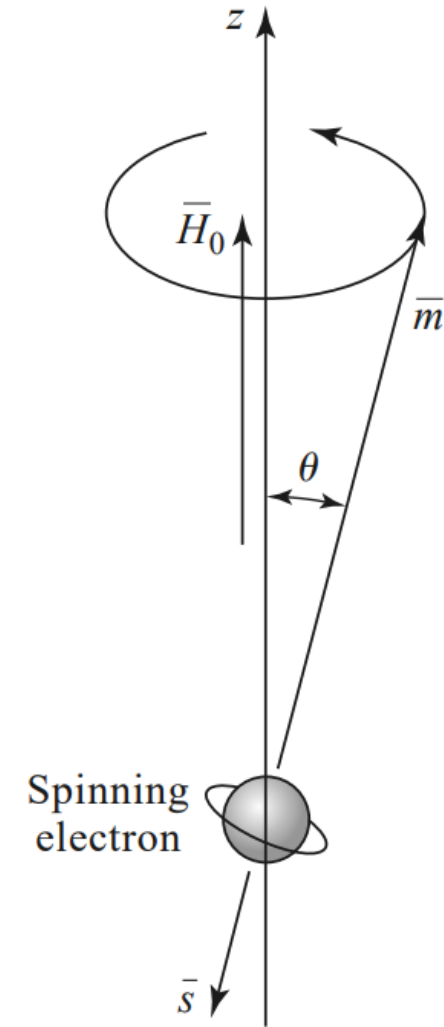
$$\text{Rotation per unit distance } \theta = \frac{(\beta^+ - \beta^-)\lambda}{2}$$

Application in ferrite isolators



2.1 Basic properties of Ferrimagnetic materials

- Permeability tensor:
Existence of magnetic dipole moments (due to primarily from electron spin)



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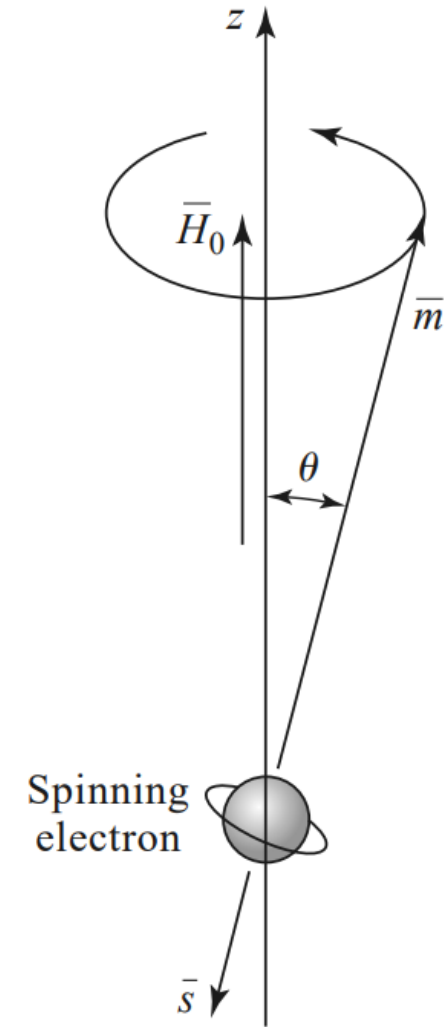
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$$m = \frac{qh}{2m_e} = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

h : Plank's constant by 2π .

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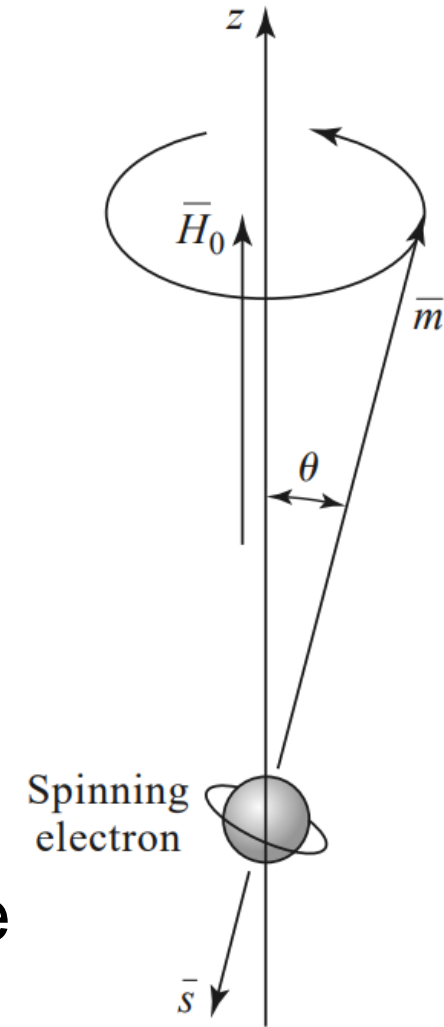
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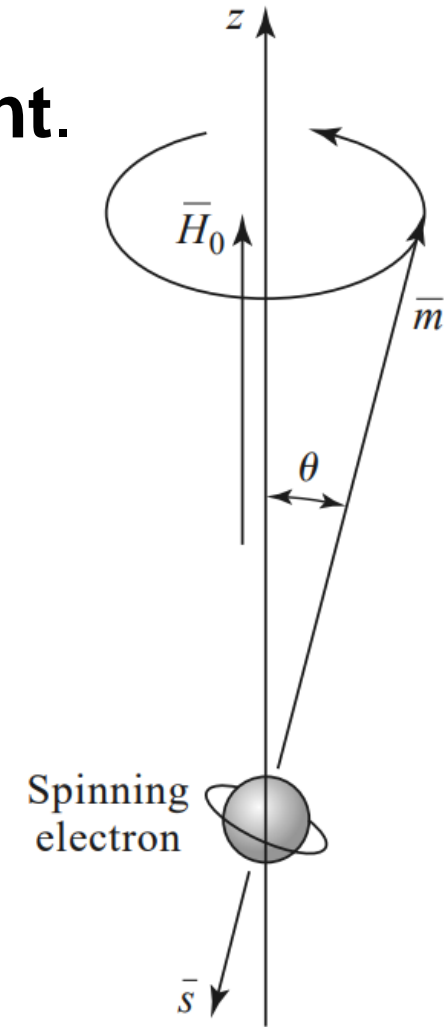
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- This is **generally insignificant** when **compared to magnetic moment due to spin**.
- *Lande' g factor*: measure of relative contributions of orbital moment and the spin moment to the total magnetic moment
 - $g = 1$ when moment is due only to orbital motion.
 - $g = 2$ moment is due to spin only.
 - Most ferrite materials: g : in the range of 1.98 – 2.01 ($g=2$)

