1.3 Antenna parameters: Radiation pattern, beam width, field regions, radiation power density, radiation intensity

Module:1 EM Radiation and Antenna Parameters

Course: BECE305L – Antenna and Microwave Engineering

-Dr Richards Joe Stanislaus

Assistant Professor - SENSE

Email: 51749@vitstudent.ac.in / richards.stanislaus@vit.ac.in

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CHENNAI

Module:1 EM Radiation and Antenna Parameters

- Radiation mechanism - single wire, two wire and current distribution, **Hertzian dipole, Dipole and monopole** - Radiation pattern, beam width, field regions, radiation power density, radiation intensity, directivity and gain, bandwidth, polarization, input impedance, efficiency, antenna effective length and area, antenna temperature. Friis transmission equation, Radar range equation

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

Antenna fundamental parameters and figures of merit

- To describe the performance of antenna
- Some are inter-related

1. Radiation pattern

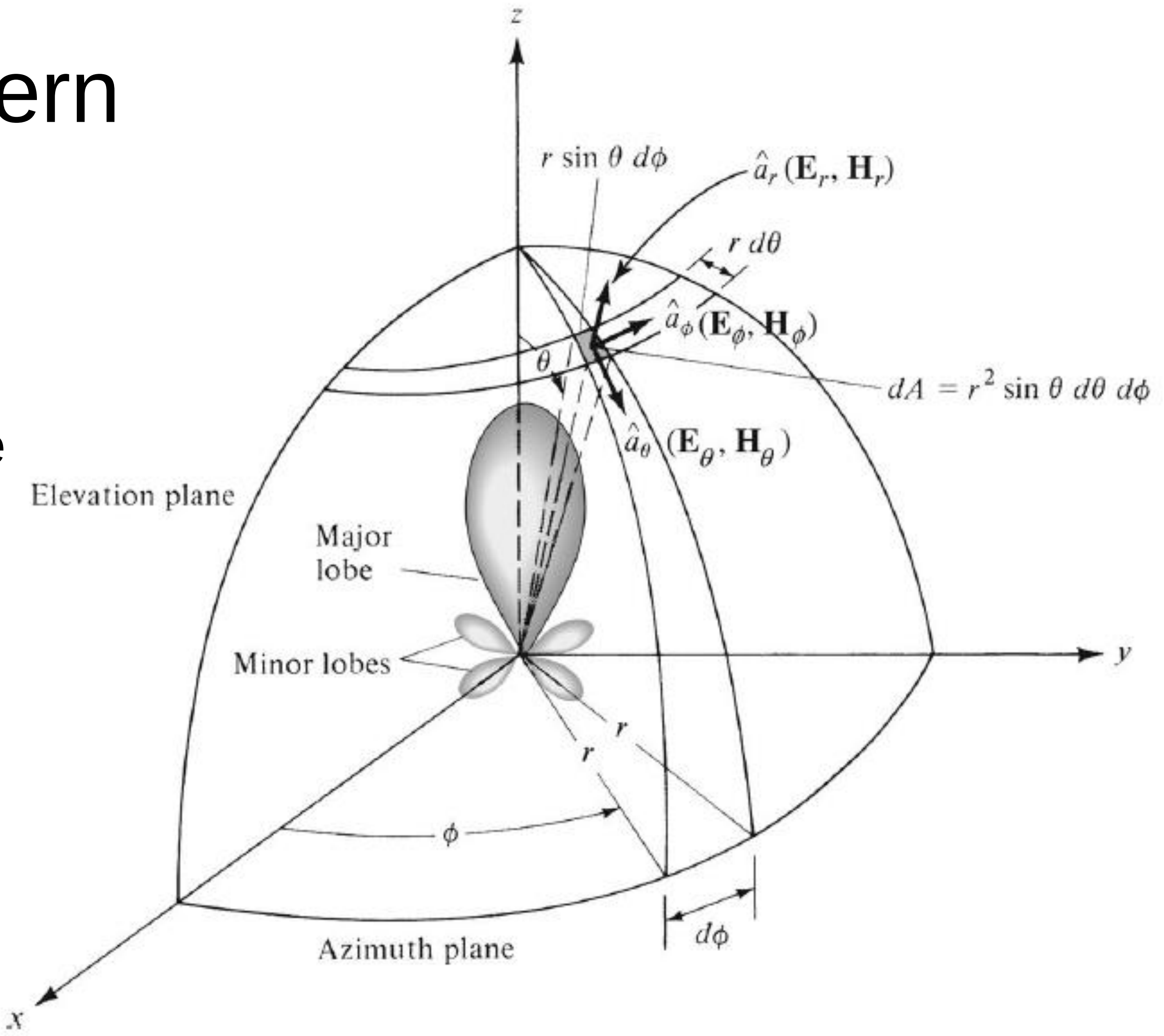
- An antenna radiation pattern or antenna pattern is defined as "**a mathematical function or a graphical representation of the radiation properties** of the antenna as **a function of space coordinates**.
- In most cases, the radiation pattern is **determined in the far-field region** and is represented as a **function of the directional coordinates**.
- Radiation properties include power flux density, radiation intensity, field strength, directivity, phase or polarization."
- The radiation property of most concern is the **two or three-dimensional spatial distribution of radiated energy** as a **function of the observer's position along a path or surface of constant radius**.

1. Radiation pattern

- A trace of the received electric (magnetic) field at a constant radius is called the **amplitude field *pattern***.
- A graph of the spatial variation of the power density along a constant radius is called an **amplitude *power pattern***.
- Often the *field* and *power* patterns are **normalized with respect to their maximum value**, yielding ***normalized field*** and ***power patterns***.
- usually plotted on a logarithmic scale or more commonly in decibels (dB). usually desirable because a **logarithmic scale can provide more details those parts of the pattern that have very low values**, which later we will refer to as minor lobes.

1. Radiation pattern

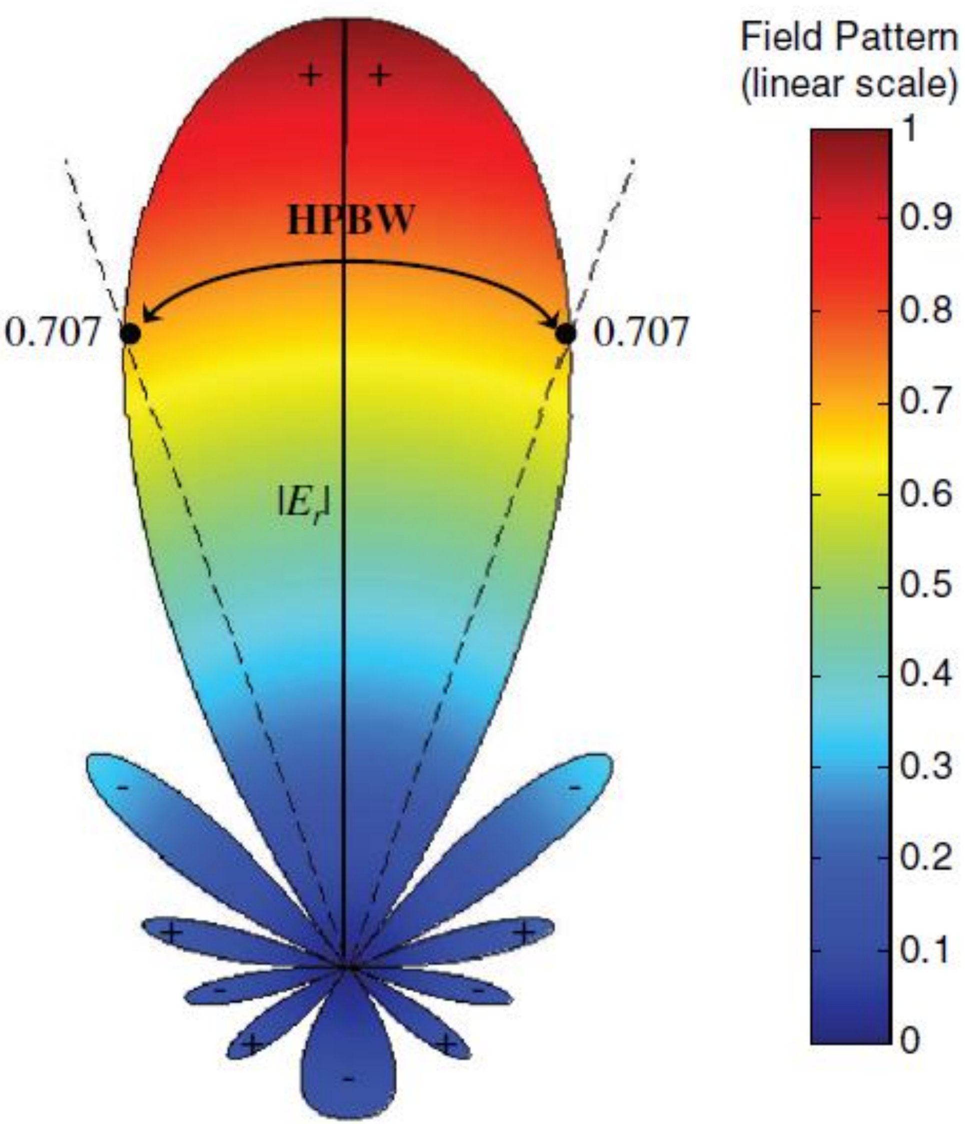
- For an amplitude pattern of an antenna, there would be, in general, three electric-field components (E_r, E_θ, E_ϕ) at each observation point on the surface of a sphere of constant radius $r = r_c$,
- In general, the magnitude of the total electric field would be
  $|\mathbf{E}| = \sqrt{(|E_r|^2 + |E_\theta|^2 + |E_\phi|^2)}$



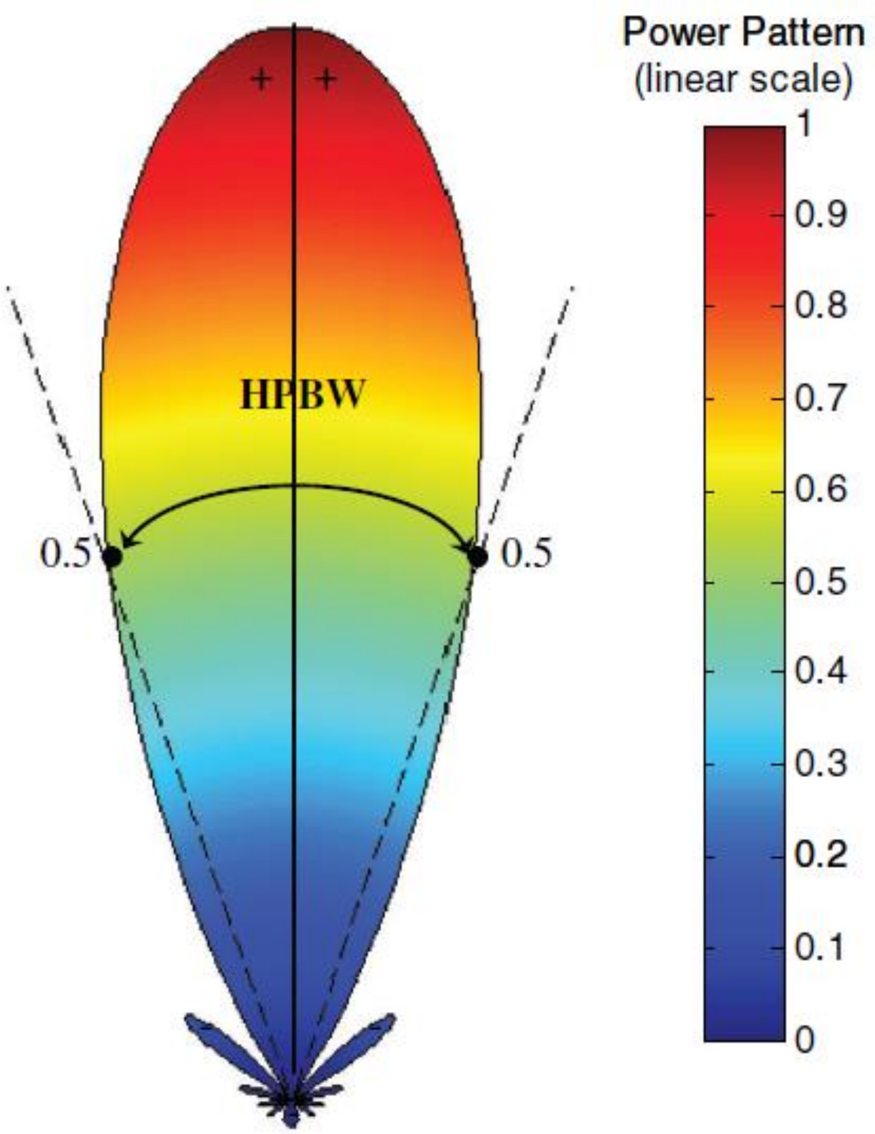
1. Radiation pattern

- For an antenna, the
  a. ***field* pattern (*in linear scale*)** typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.
  b. ***power* pattern (*in linear scale*)** typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.
  c. ***power* pattern (*in dB*)** represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

10-element linear antenna array of isotropic sources, with a spacing of $d = 0.25\lambda$ between the elements

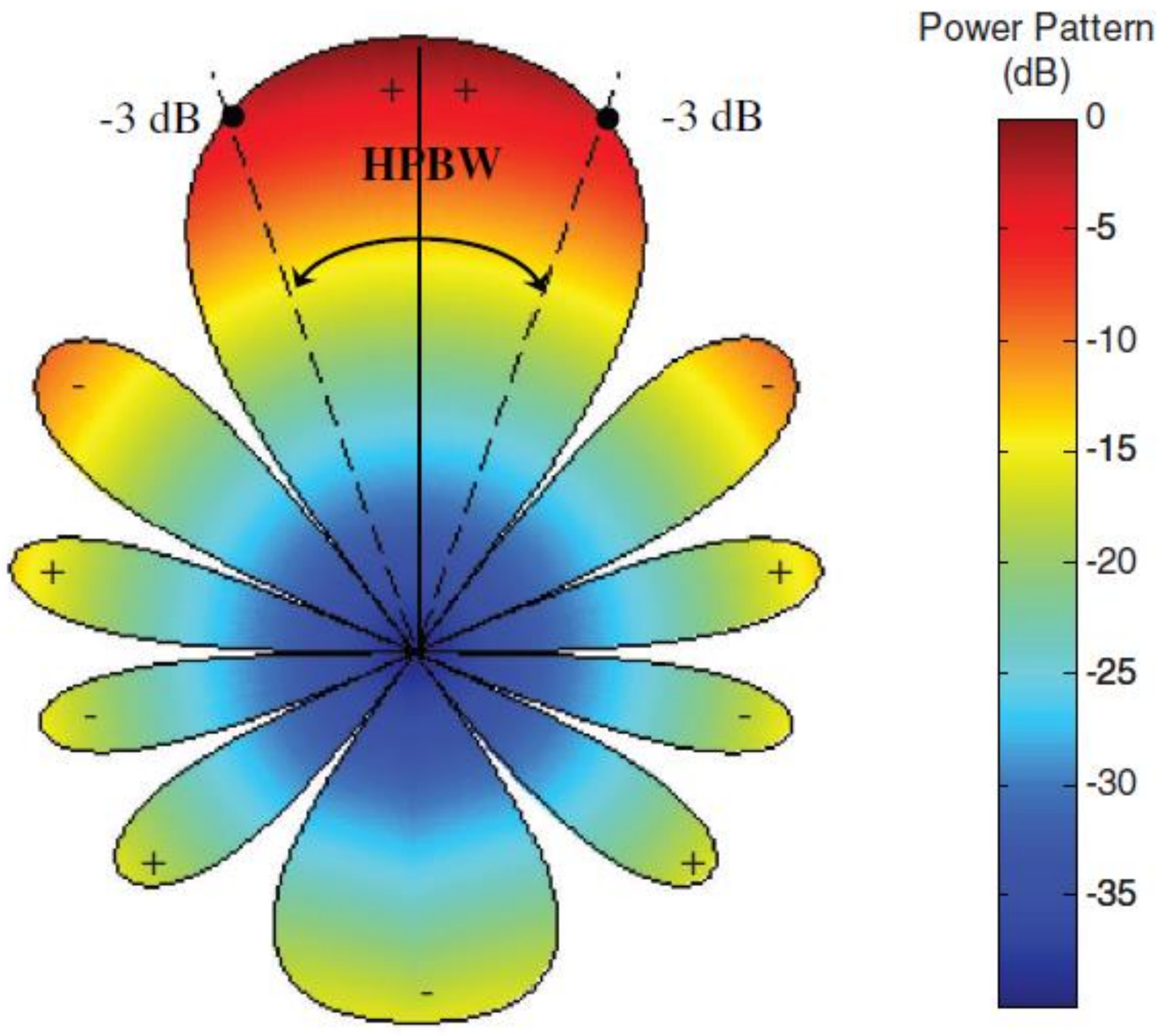


(a) Field pattern (in linear scale)



(b) Power pattern (in linear scale)

10-element linear antenna array of isotropic sources, with a spacing of $d = 0.25\lambda$ between the elements



(c) Power pattern (in dB)

1. Radiation pattern

- *In this and subsequent patterns, the **plus (+) and minus (−) signs in the lobes indicate the relative polarization (positive or negative) of the amplitude between the various lobes**, which changes (alternates) as the nulls are crossed.*
- To find the points where the **pattern achieves its half-power (−3 dB points), relative to the maximum value of the pattern**, you set the value of the

 a. ***field pattern at 0.707 value of its maximum**, as shown in Figure (a)*

 b. ***power pattern (in a linear scale) at its 0.5 value of its maximum**, as shown in Figure (b)*

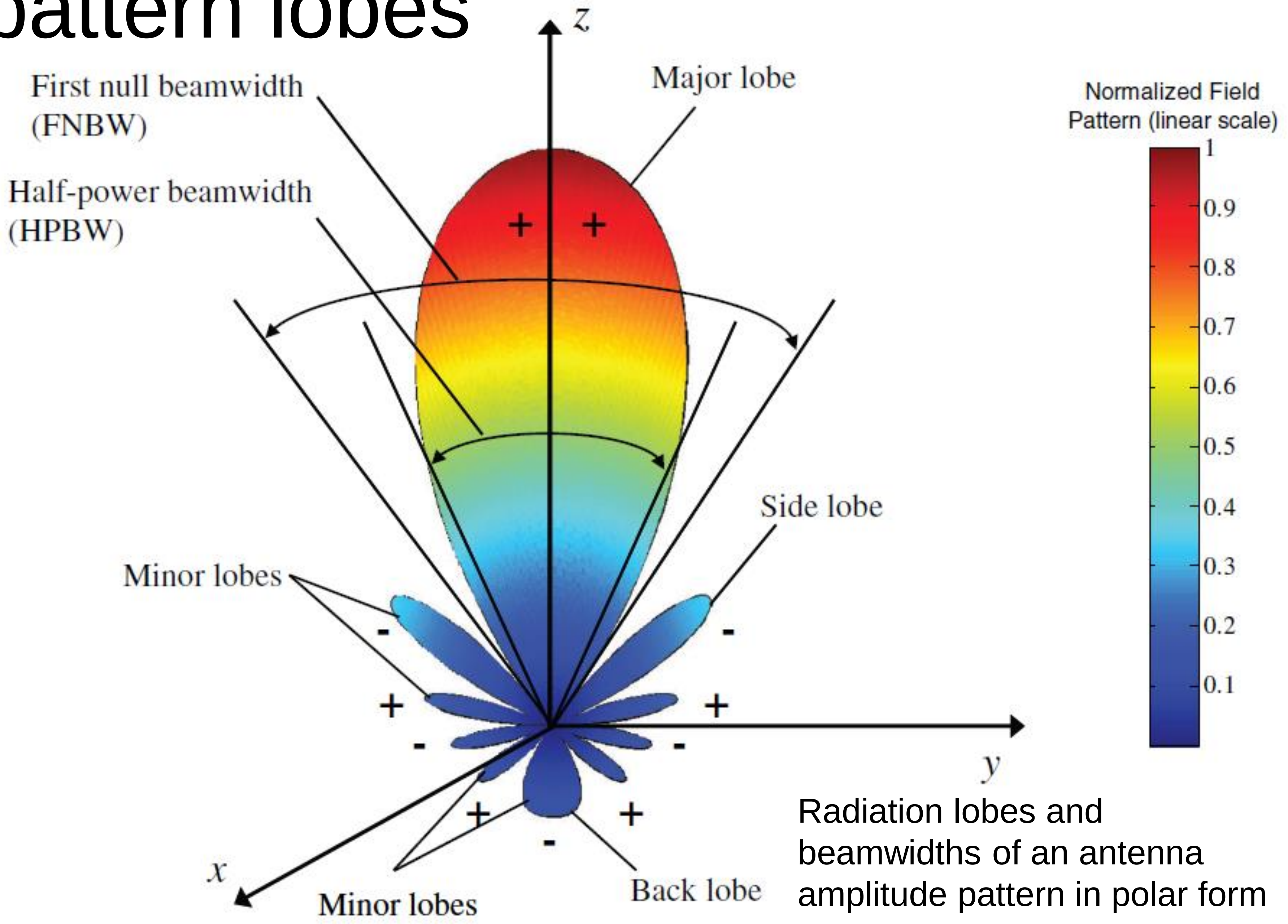
 c. ***power pattern (in dB) at −3 dB value of its maximum** Figure (c)*

1. Radiation pattern

- All three patterns: **same angular separation between the two half-power points, 38.64◦**, on their respective patterns, *referred to as **HPBW*** and illustrated in Figure.
- In practice, the three-dimensional pattern is measured and recorded in a series of two-dimensional patterns.
  However, for most practical applications, a few plots of the pattern as a function of θ for some particular values of ϕ,
  plus
  a few plots as a function of ϕ for some particular values of θ, give most of the useful and needed information.

1.1 Radiation pattern lobes

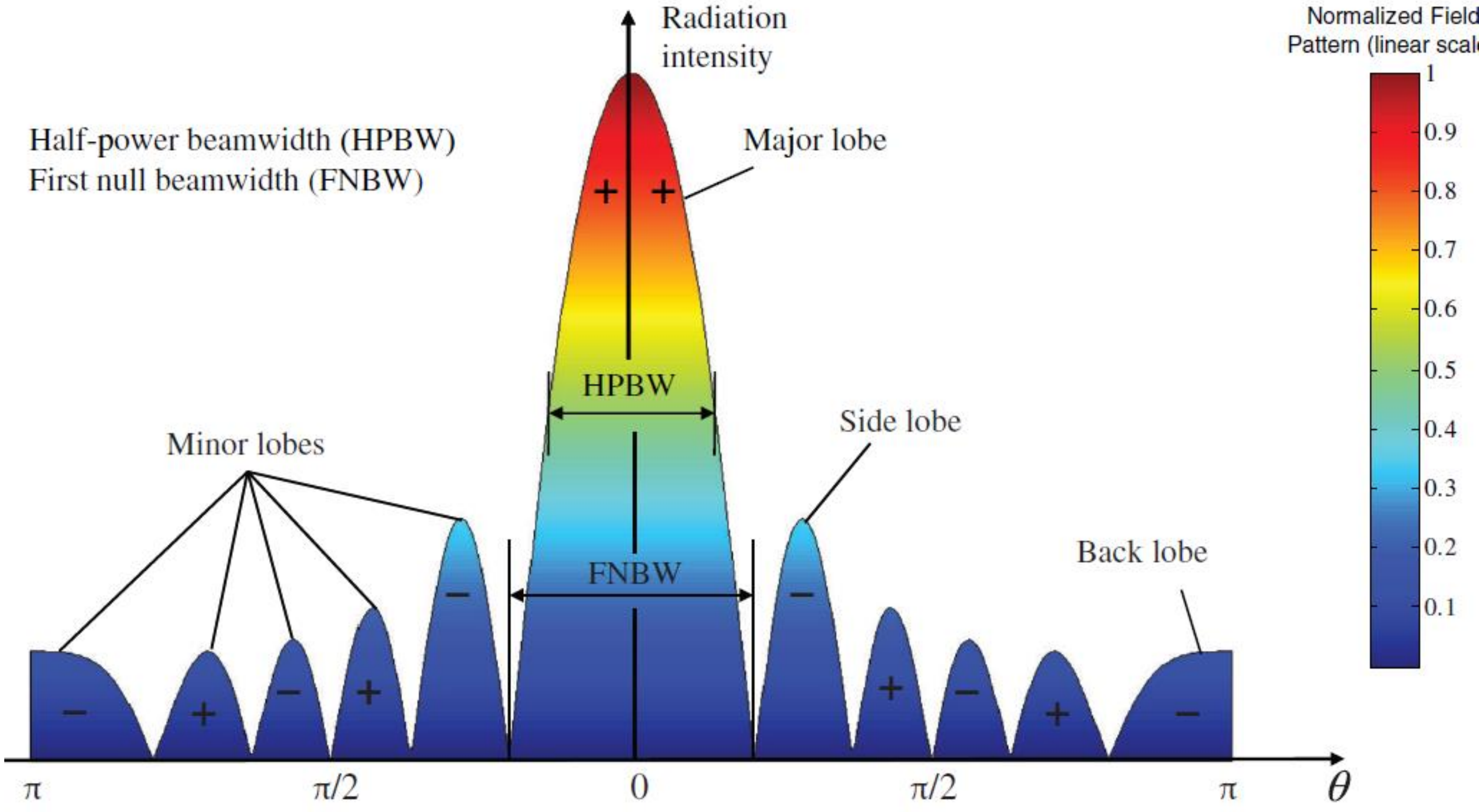
- Various parts of a radiation pattern: *lobes*
  major or *main, minor, side*, and *back* lobes
- A *radiation lobe* is a "**portion of the radiation pattern bounded by regions of relatively weak radiation intensity**".



Radiation lobes and beamwidths of an antenna amplitude pattern in polar form

1.1 Radiation pattern lobes

- symmetrical three-dimensional polar pattern with a number of radiation lobes. Some are of greater radiation intensity than others, but all are classified as lobes



Linear plot of power pattern and its associated lobes and beamwidths.

1.1 Radiation pattern lobes

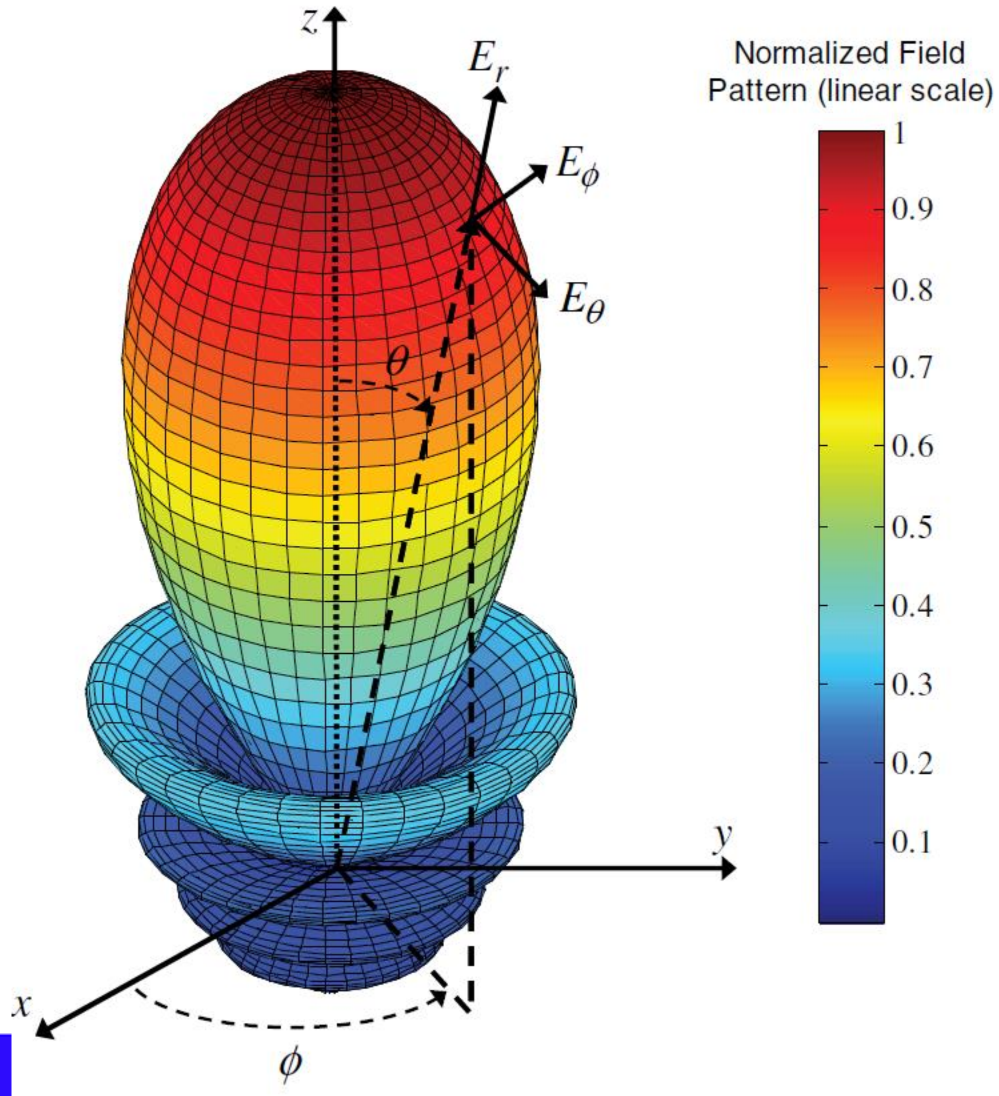
- A ***major lobe* (also called main beam)** is defined as "the radiation lobe containing the direction of maximum radiation." In Figure the major lobe is pointing in the $\theta = 0$ direction
- Some antennas, such as split-beam antennas, there may exist more than one major lobe.
- A ***minor lobe*** is any lobe except a major lobe. **Minor lobes usually represent radiation in undesired directions**, and they should be minimized.
- The **level of minor lobes** is usually **expressed as a ratio of the power density in the lobe in question to that of the major lobe**.

1.1 Radiation pattern lobes

- A ***side lobe*** is "a radiation lobe in any direction other than the intended lobe." (Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main beam.)
- **Side lobes are normally the largest of the minor lobes**
- A ***back lobe*** is "a radiation lobe whose axis makes an angle of approximately 180◦ with respect to the beam of an antenna." Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.
- Side lobe levels of −20 dB or smaller are usually not desirable in most applications. Attainment of a side lobe level smaller than −30 dB usually requires very careful design and construction. In most radar systems, low side lobe ratios are very important to minimize false target indications through the side lobes.

1.1 Radiation pattern lobes

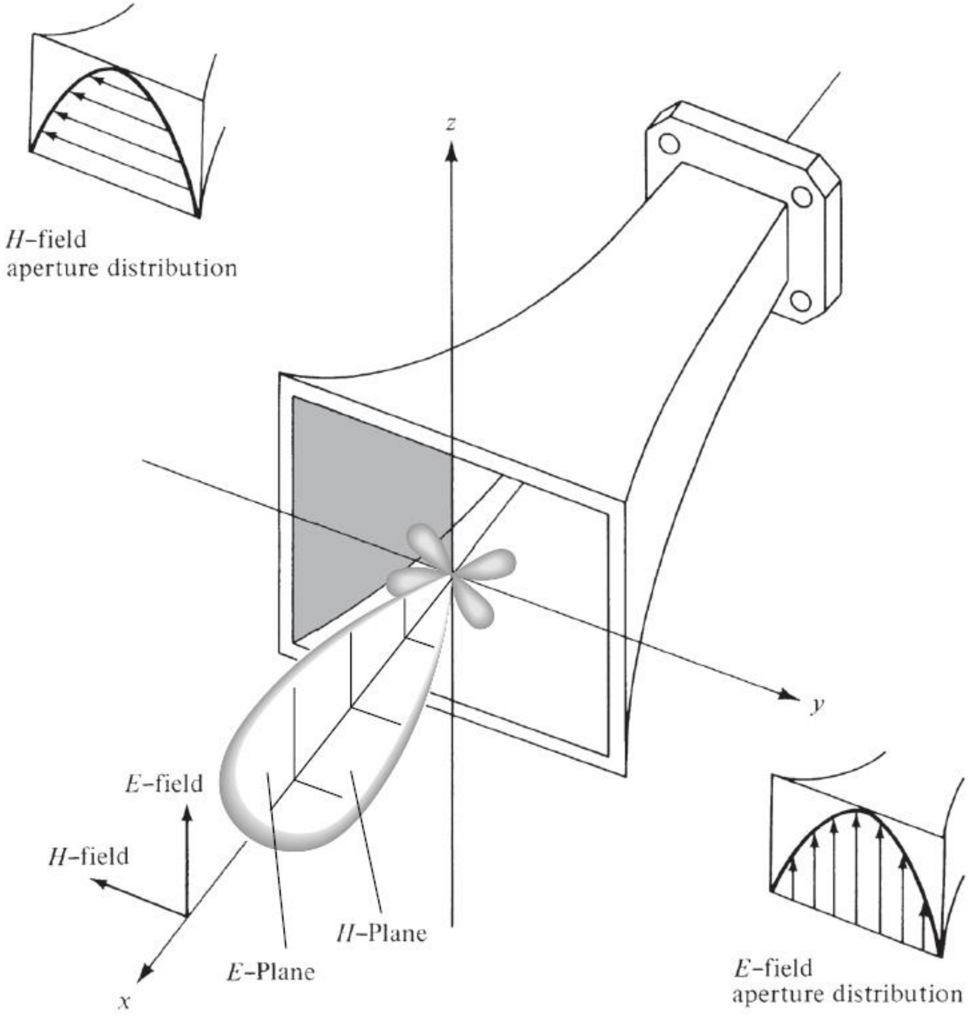
- normalized three-dimensional far-field amplitude pattern, plotted on a linear scale, of a 10- element linear antenna array of isotropic sources with a spacing of $d = 0.25\lambda$ and progressive phase shift $\beta = -0.6\pi$, between the elements.
- One major lobe, five minor lobes and one back lobe.



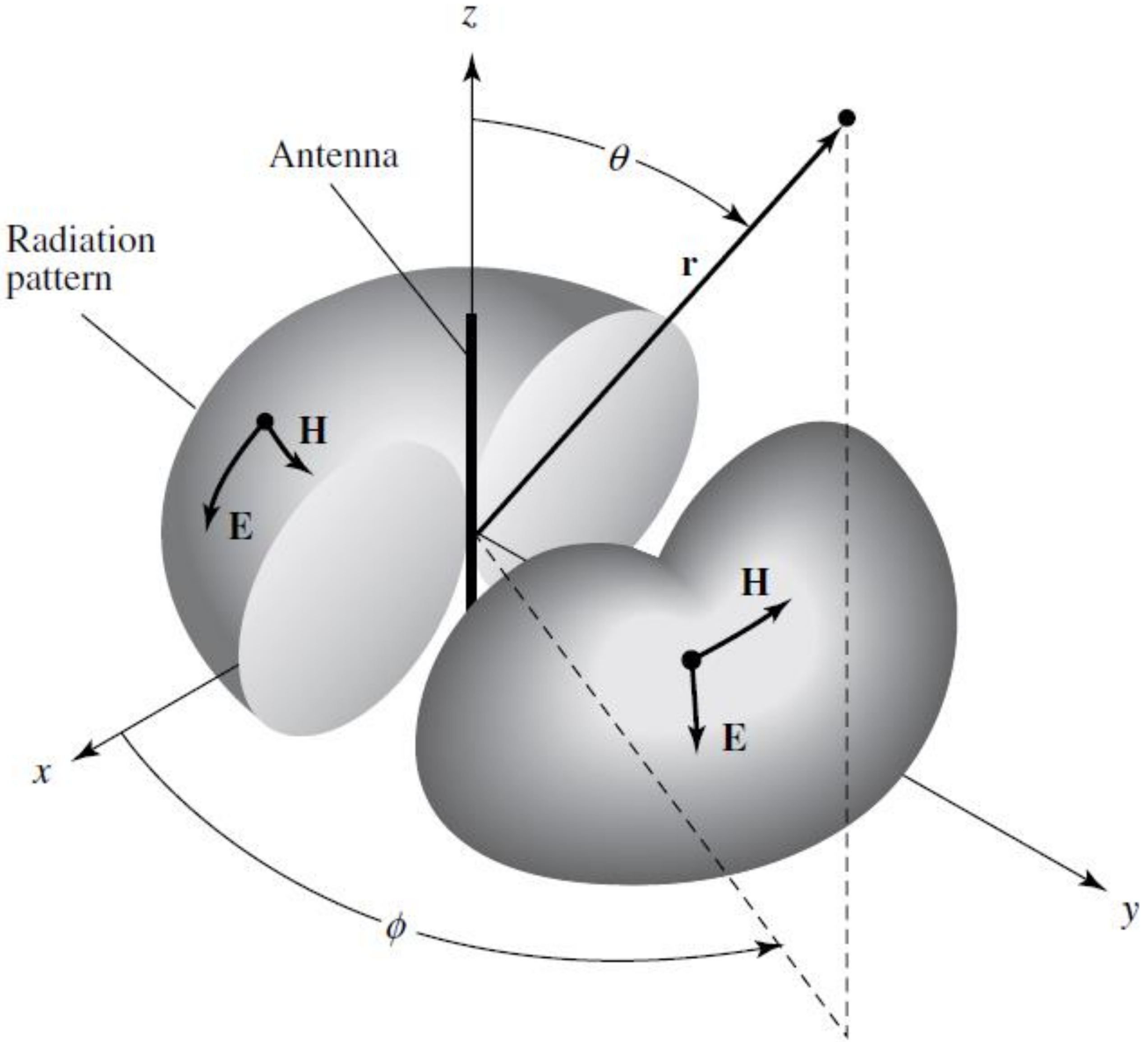
1.2 Isotropic, Directional, and Omnidirectional Patterns

- ***isotropic* radiator** is defined as “a hypothetical lossless antenna having equal radiation in all directions.”
- ***directional* antenna** is one “having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others.
  applied to an antenna whose maximum directivity is significantly greater than that of a half-wave dipole.
- ***omnidirectional*** is defined as one “having an essentially nondirectional pattern in a given plane (in this case in azimuth) and a directional pattern in any orthogonal plane (in this case in elevation).”
  An *omnidirectional* pattern is then a special type of a *directional* pattern.

Directional

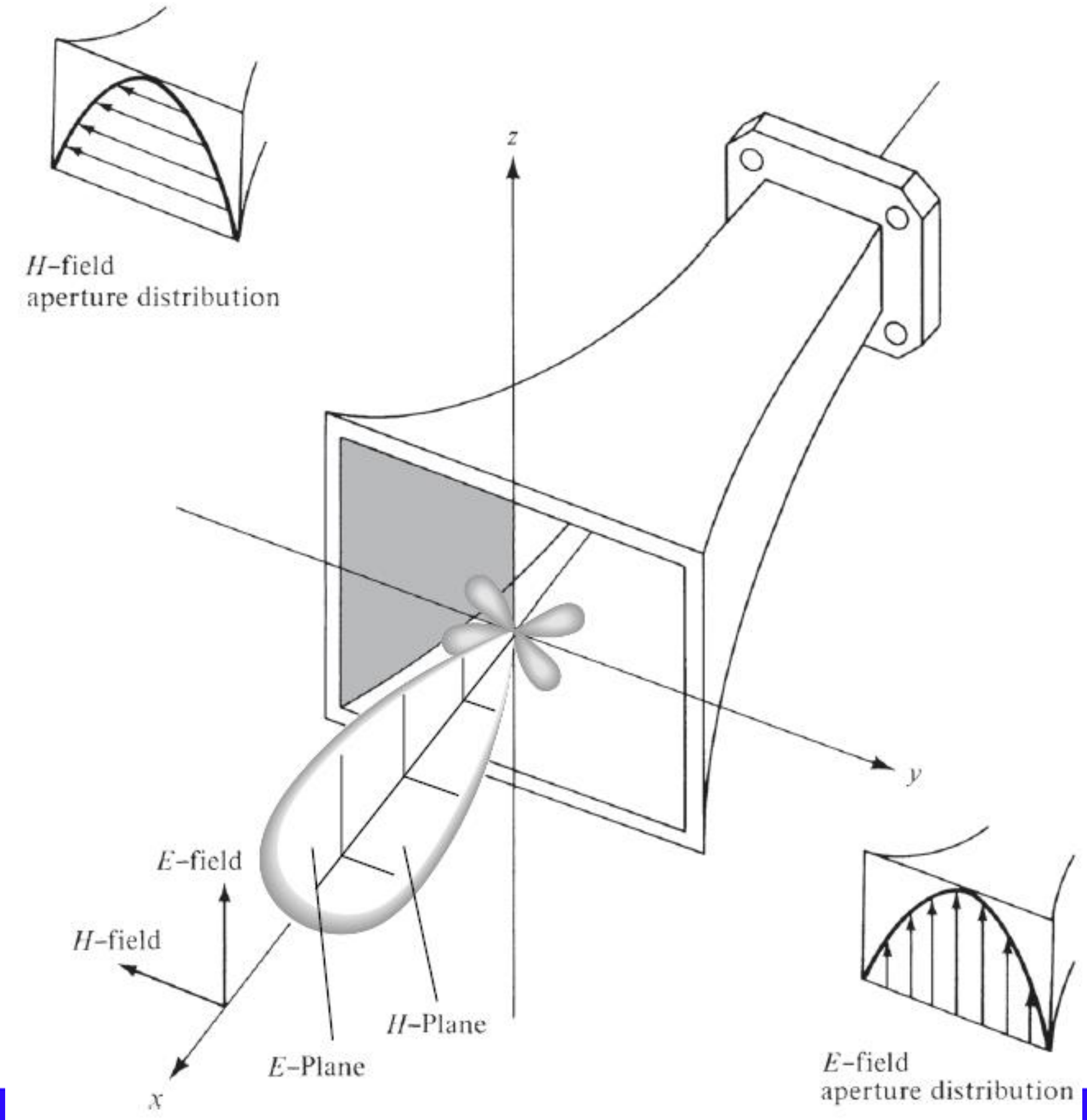


Omnidirectional



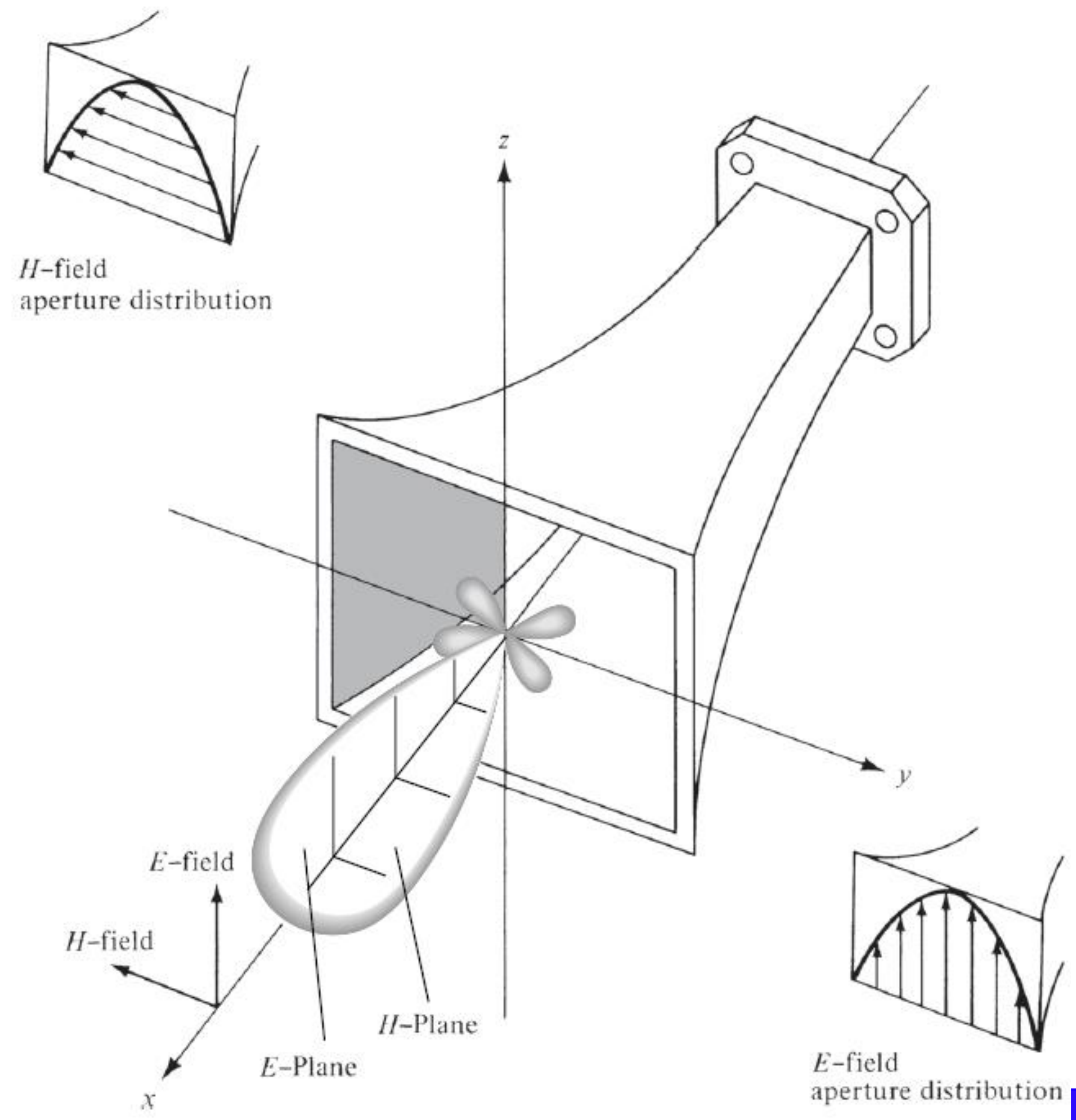
1.3 Principal Patterns

- linearly polarized antenna: performance is often described in terms of its principal *E*-plane pattern and *H*-plane pattern.
- *E-plane:* “the plane containing the electric-field vector and the direction of maximum radiation”
- H-plane: “the plane containing the magnetic-field vector and the direction of maximum radiation.”



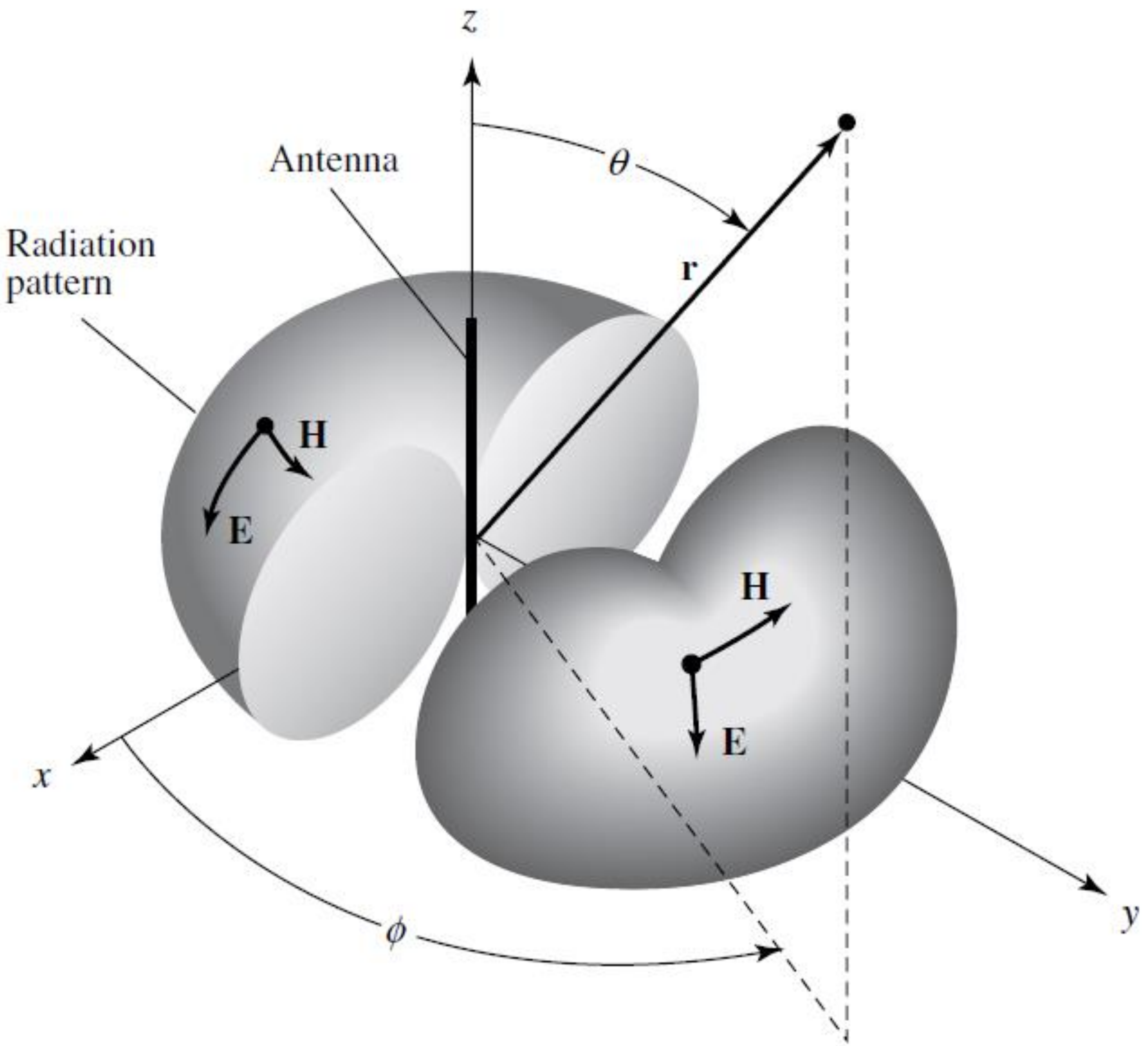
1.3 Principal Patterns

- *x*-*z* plane (elevation plane; $\phi = 0$) is the principal *E*-plane and
- *x*-*y* plane (azimuthal plane; $\theta = \pi/2$) is the principal *H*-plane

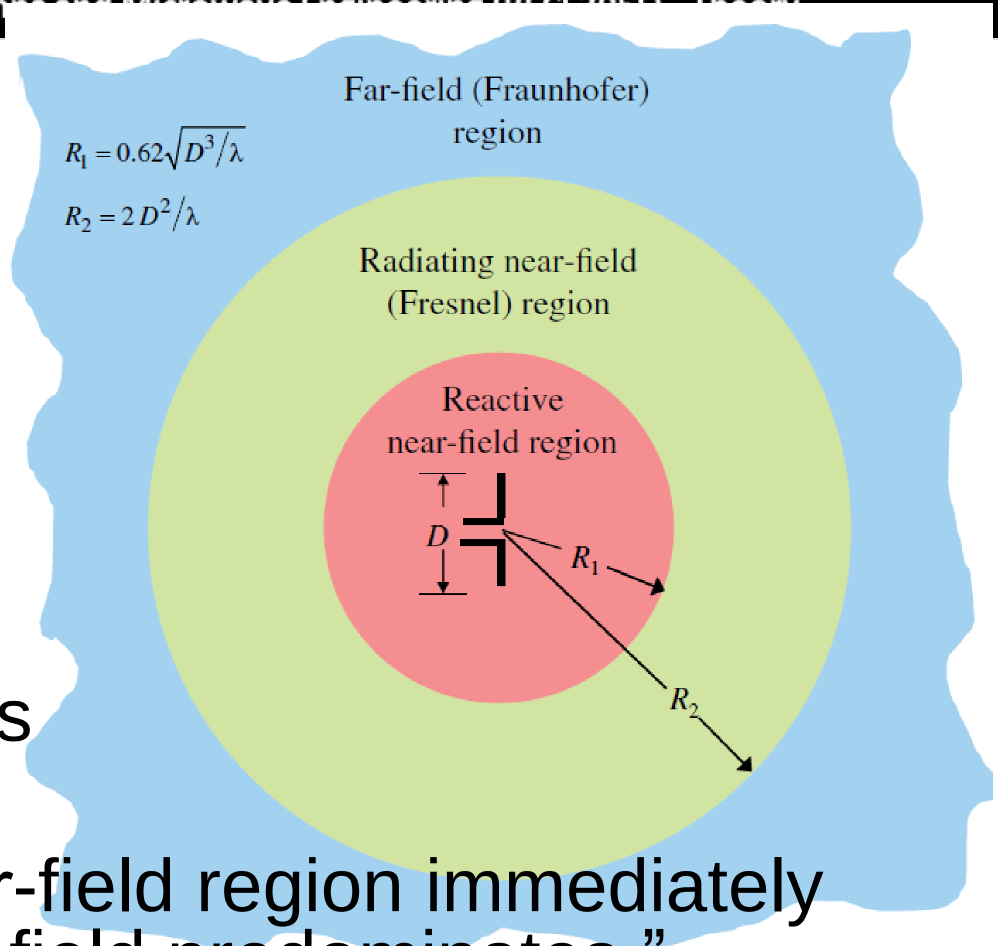


1.3 Principal Patterns

- *For* Omnidirectional antenna: Infinite E planes and H planes Exist



1.4 Field regions



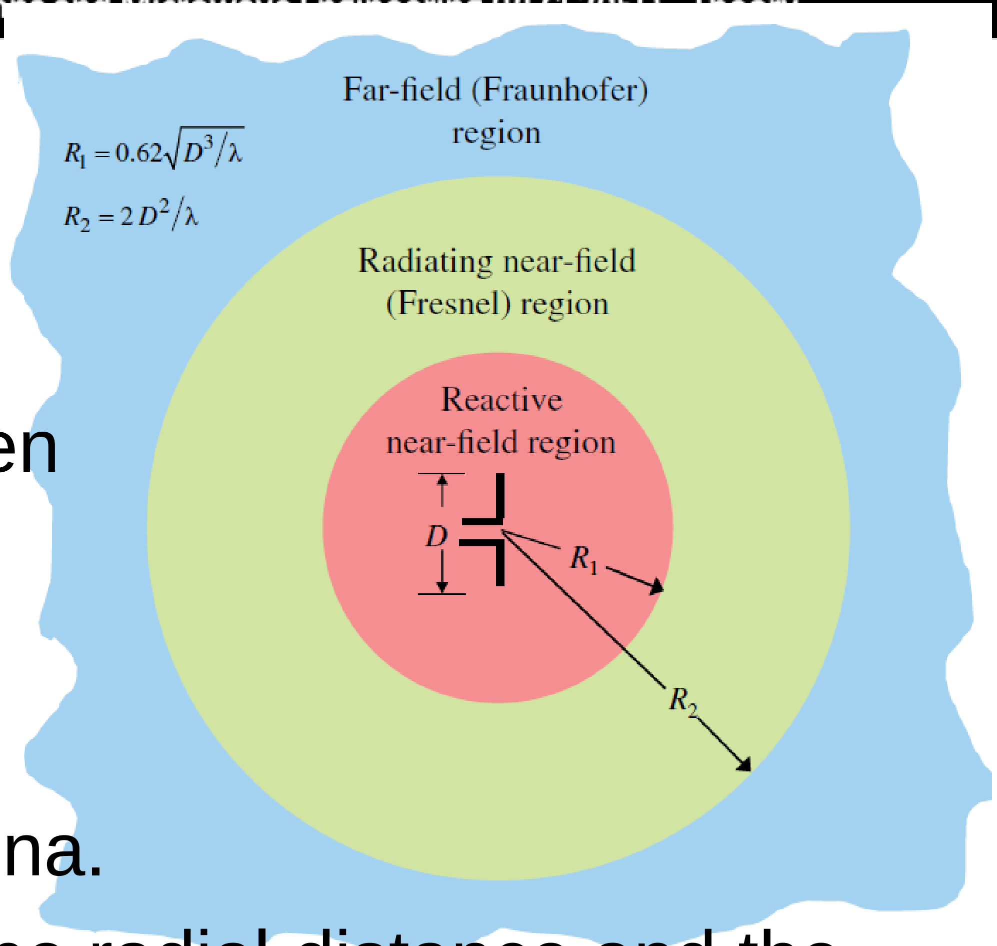
- space surrounding an antenna is usually subdivided into three regions:
  (a) reactive near-field,
  (b) radiating near-field (Fresnel) and
  (c) far-field (Fraunhofer) regions
- Distinct differences exists in field expressions in all three regions

a) Reactive near field: “that portion of the near-field region immediately surrounding the antenna wherein the reactive field predominates.”

R<$0.62\sqrt{D^3/\lambda}$: D is largest dimension of antenna, λ: wavelength

“For a very short dipole, or equivalent radiator, the outer boundary is commonly taken to exist at a distance $\lambda/2\pi$ from the antenna surface.”

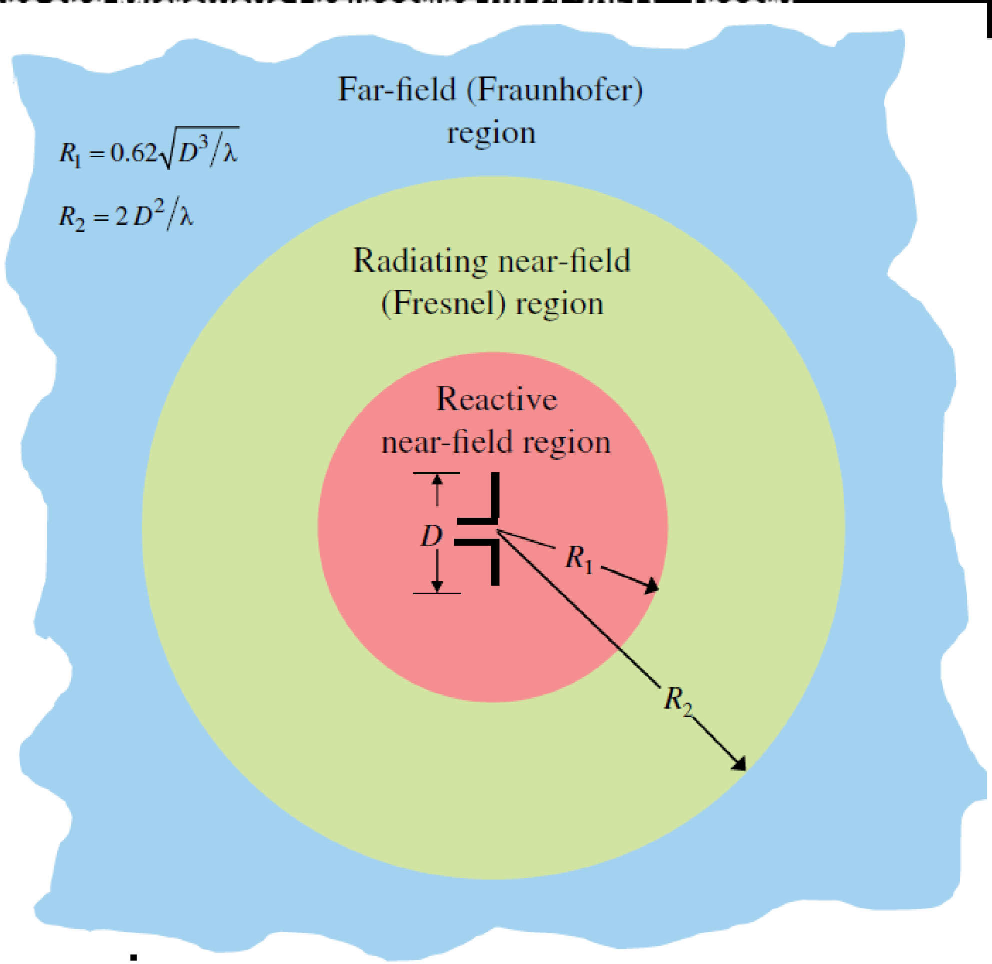
1.4 Field regions



b) radiating near-field (Fresnel)

- "that region of the field of an antenna between the reactive near-field region and the far-field region wherein radiation fields predominate and wherein the angular field distribution is dependent upon the distance from the antenna.
- the field pattern is, in general, a function of the radial distance and the radial field component may be appreciable.
- Inner boundary $0.62\sqrt{D^3/\lambda} \leq R < 2D^2/\lambda$
- Note: If $D \ll \lambda$, this region will not exist.

1.4 Field regions

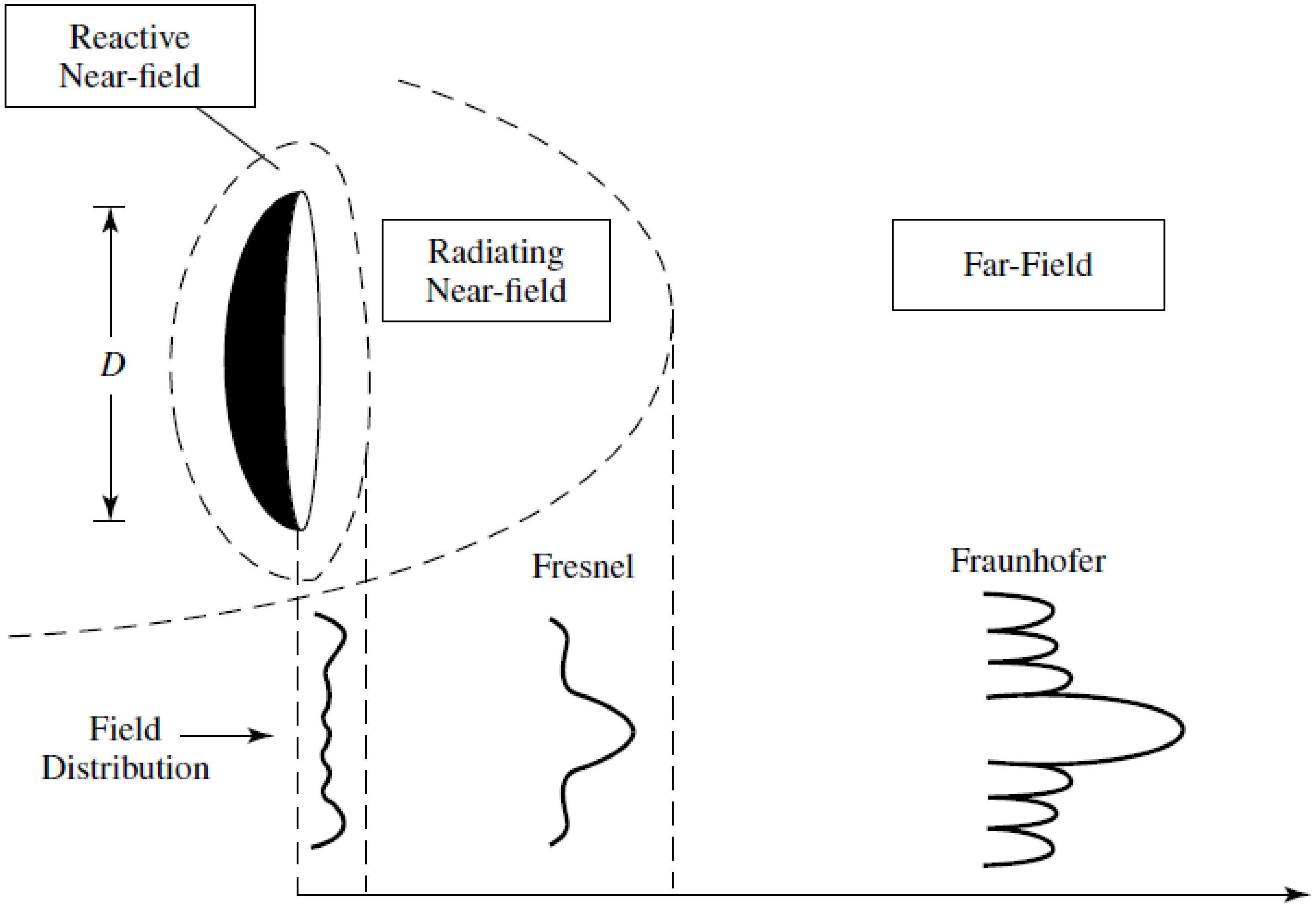


c) far-field (Fraunhofer) region

- “that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.
- $R > 2D^2/\lambda$
- Note: the propagation constant is γ, and if $D > \pi/|\gamma|$, then $R > |\gamma|D^2/\pi$ is the far field region.
- In this region, the field components are essentially transverse
- and the angular distribution is independent of the radial distance where the measurements are made.

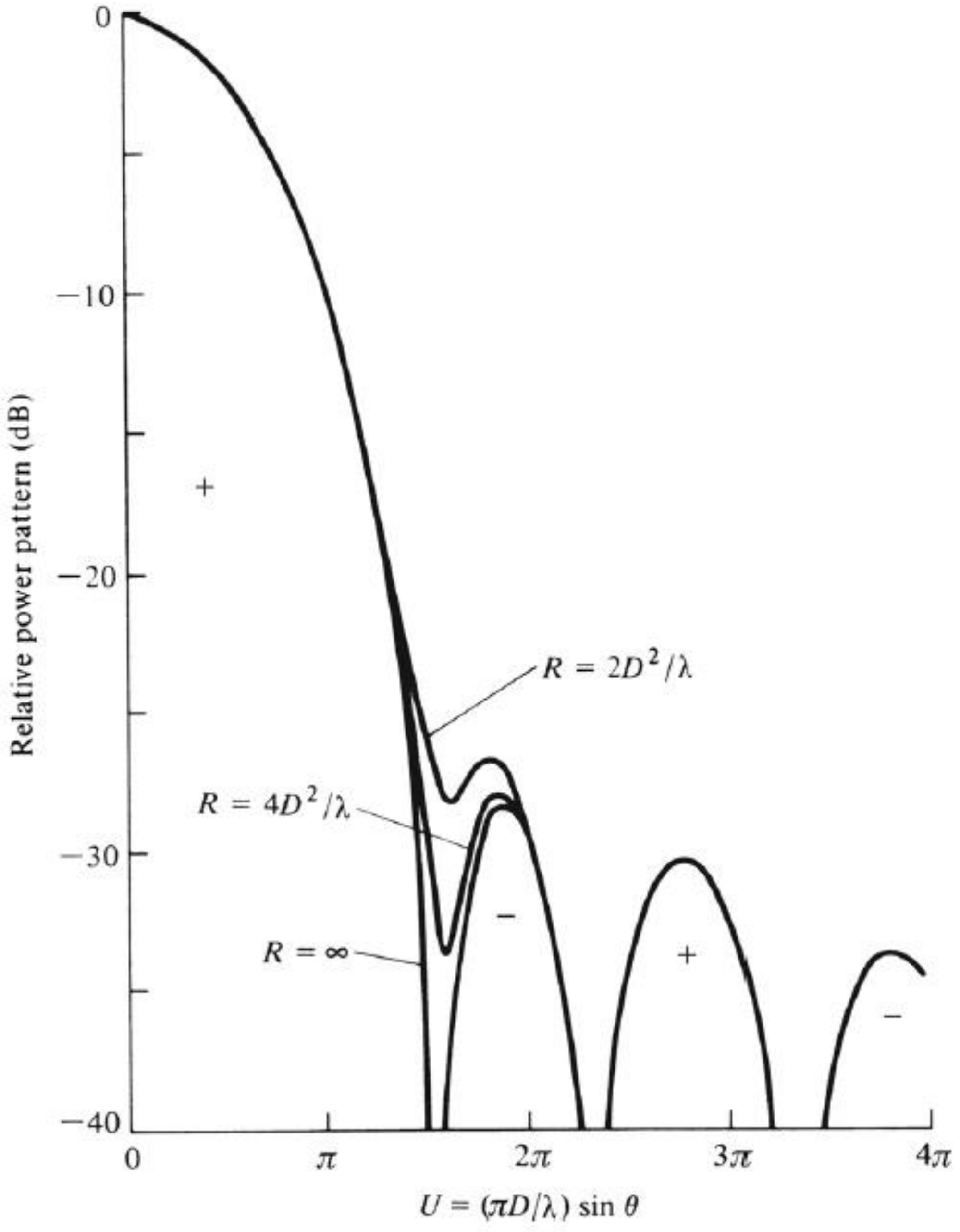
1.4 Field regions

- In example, near field is more spread out, nearly uniform with slight variation
- In radiating near field(fresnel) lobe starts to form
- In far field (Fraunhofer), pattern is well formed with many minor lobes and one (or more in some cases)majore lobe.

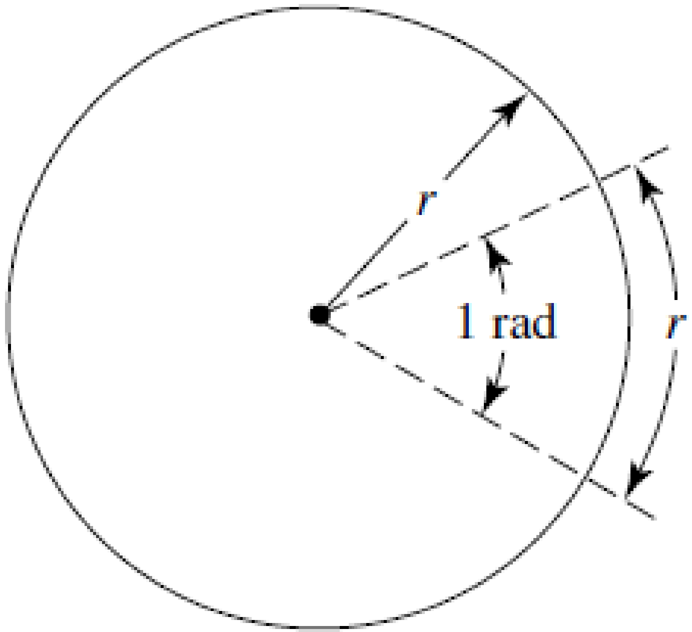


1.4 Field regions

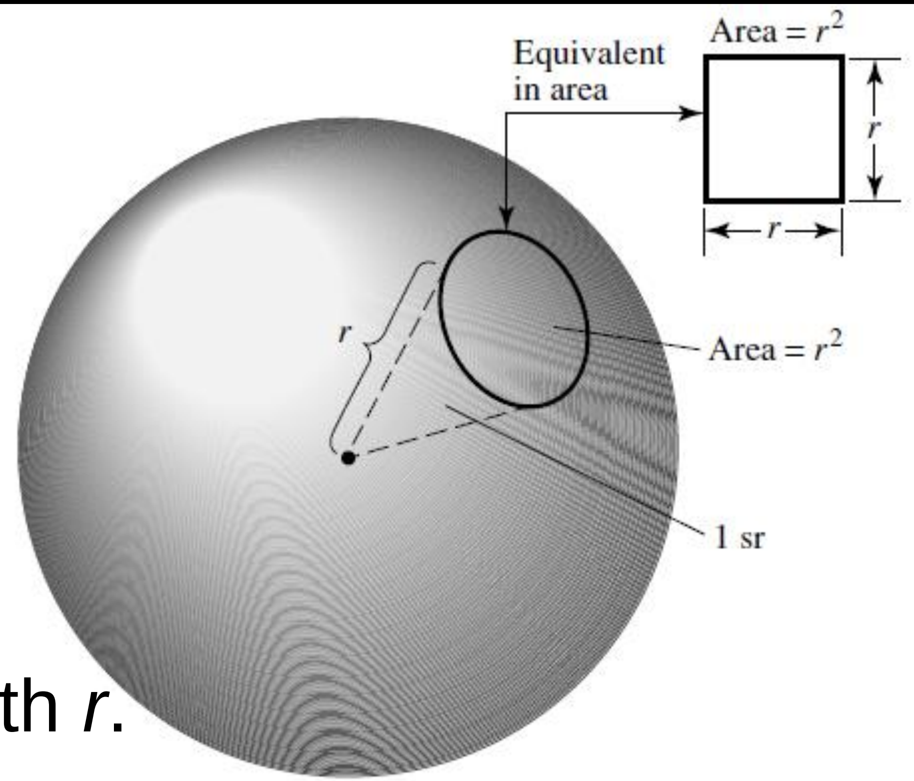
- Parabolic reflector's field pattern at
  a) $R = \frac{2D^2}{\lambda}$
  b) $R = 4D^2/\lambda$
  c) $R = \infty$
- First null, difference is observed below -25 dB



1.5 Radian and Steradian



- Measure of plane angle is a radian
- One *radian*: the plane angle with its vertex at the center of a circle of radius *r* that is subtended by an arc whose length is *r*.
- Full circle circumference is $2\pi r$: Full circle has has 2π radians



1.5 Radian and Steradian

- Measure of solid angle is a steradian
- One ste*radian*: solid angle with its vertex at the center of a sphere of radius *r* that is subtended by a spherical surface area equal to that of a square with each side of length *r*.
- Full sphere area $4\pi r^2$: Full circle has has 4π steradians
- If infinitesimal area dA on surface of sphere of radius r,
  $dA = r^2 \sin\theta \; d\theta \, d\phi \qquad (m^2)$
- Element of solid angle $d\Omega$ (in steradian) subtended by dA

$$d\Omega = \frac{dA}{r^2} = \sin\theta \; d\theta \, d\phi \quad \text{(sr)}$$

Problem: For a sphere of radius r, find the solid angle ΩA (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of $0 \leq \theta \leq 30^\circ$, $0 \leq \phi \leq 360^\circ$.Do this: a. exactly. b. using $\Omega A \approx \Delta\Theta 1 \cdot \Delta\Theta 2$, where $\Delta\Theta 1$ and $\Delta\Theta 2$ are two perpendicular angular separations of the spherical cap passing through the north pole.

Problem: For a sphere of radius *r*, find the solid angle Ω*A* (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of $0 \leq \theta \leq 30^\circ$, $0 \leq \phi \leq 360^\circ$. Do this: a. exactly. b. using $\Omega A \approx \Delta\Theta 1 \cdot \Delta\Theta 2$, where ΔΘ1 and ΔΘ2 are two perpendicular angular separations of the spherical cap passing through the north pole.

$$\Omega_A = \int_0^{360^\circ} \int_0^{30^\circ} d\Omega = \int_0^{2\pi} \int_0^{\pi/6} \sin\theta \, d\theta \, d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta \, d\theta$$

$$= 2\pi[-\cos\theta]\Big|_0^{\pi/6} = 2\pi[-0.867 + 1] = 2\pi(0.133) = 0.83566$$

b. $$\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2 \overset{\Delta\Theta_1 = \Delta\Theta_2}{\underbrace{\qquad}_{=}} (\Delta\Theta_1)^2 = \frac{\pi}{3}\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} = 1.09662$$

It is apparent that the approximate beam solid angle is about 31.23% in error.

1.6 Radiation power density

- power and energy are associated with electromagnetic fields.
- Power associated with an electromagnetic wave is the **instantaneous Poynting vector** $\mathscr{W} = \mathscr{E} \times \mathscr{H}$

$$\mathscr{W} = \text{instantaneous Poynting vector} \quad (\mathrm{W/m^2})$$

$$\mathscr{E} = \text{instantaneous electric-field intensity} \quad (\mathrm{V/m})$$

$$\mathscr{H} = \text{instantaneous magnetic-field intensity} \quad (\mathrm{A/m})$$

- Script letters denote instantaneous fields and quantities
  Roman letters describe complex fields and quantities

$\mathscr{W}$ = instantaneous Poynting vector (W/m^2)

$\mathscr{E}$ = instantaneous electric-field intensity (V/m)

$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m)

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}$$

1.6 Radiation power density

- Total power crossing a closed surface: obtained by integrating the normal component of Poynting vector over entire surface

$$\mathscr{P} = \oiint_S \mathscr{W} \cdot d\mathbf{s} = \oiint_S \mathscr{W} \cdot \hat{\mathbf{n}}\, da$$

$\mathscr{P}$ = instantaneous total power (W)

$\hat{\mathbf{n}}$ = unit vector normal to the surface

da = infinitesimal area of the closed surface (m^2)

1.6 Radiation power density

$\mathscr{W}$ = instantaneous Poynting vector (W/m^2)

$\mathscr{E}$ = instantaneous electric-field intensity (V/m)

$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m)

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}$$

$$\mathscr{P} = \oiint_S \mathscr{W} \cdot d\mathbf{s} = \oiint_S \mathscr{W} \cdot \hat{\mathbf{n}}\, da$$

$\mathscr{P}$ = instantaneous total power (W)

$\hat{\mathbf{n}}$ = unit vector normal to the surface

da = infinitesimal area of the closed surface (m^2)

- For time varying fields, average power density is obtained by integrating instantaneous Poynting vector over one period and dividing by period.
- For time harmonic variations ($e^{j\omega t}$) ,

$$\mathscr{E}(x, y, z; t) = \mathrm{Re}[\mathbf{E}(x, y, z)e^{j\omega t}]$$

$$\mathscr{H}(x, y, z; t) = \mathrm{Re}[\mathbf{H}(x, y, z)e^{j\omega t}]$$

$$\mathrm{Re}[\mathbf{E}e^{j\omega t}] = \frac{1}{2}[\mathbf{E}e^{j\omega t} + \mathbf{E}^* e^{-j\omega t}],$$

- Hence
first : not function of time
second: time variation is twice the given frequency

$$\mathscr{W} = \mathscr{E} \times \mathscr{H} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] + \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}e^{j2\omega t}]$$

- Time averaged Poynting vector (average power density)

$$\boxed{\mathbf{W}_{\mathrm{av}}(x, y, z) = [\mathscr{W}(x, y, z; t)]_{\mathrm{av}} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]} \quad (\mathrm{W/m^2})$$

1.6 Radiation power density

$\mathscr{W}$ = instantaneous Poynting vector (W/m^2)

$\mathscr{E}$ = instantaneous electric-field intensity (V/m)

$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m)

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}$$

$$\mathscr{P} = \oiint_S \mathscr{W} \cdot d\mathbf{s} = \oiint_S \mathscr{W} \cdot \hat{\mathbf{n}}\, da$$

$\mathscr{P}$ = instantaneous total power (W)

$\hat{\mathbf{n}}$ = unit vector normal to the surface

da = infinitesimal area of the closed surface (m^2)

- Note: ½ is used as we have considered E and H to have peak values.
- If rms values are considered, then ½ has to be omitted

$$\boxed{\mathbf{W}_{av}(x, y, z) = [\mathscr{W}(x, y, z; t)]_{av} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]} \quad (\mathrm{W/m^2})$$

- Average power radiated by antenna (radiated power)

$$P_{rad} = P_{av} = \oiint_S \mathbf{W}_{rad} \cdot d\mathbf{s} = \oiint_S \mathbf{W}_{av} \cdot \hat{\mathbf{n}}\, da$$

$$= \frac{1}{2}\oiint_S \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s}$$

1.6 Radiation power density

$\mathscr{W}$ = instantaneous Poynting vector (W/m^2)

$\mathscr{E}$ = instantaneous electric-field intensity (V/m)

$\mathscr{H}$ = instantaneous magnetic-field intensity (A/m)

$$\mathscr{W} = \mathscr{E} \times \mathscr{H}$$

- Power pattern of antenna is a function of
  a) direction
  b) average power density radiated by antenna.
- Observations are made on large sphere of constant radius (far field)
- Absolute power pattern are generally replaced with gain (will be discussed later) and in relative power terms (normalized)
- 3D pattern is generally not measured
  Only 2D cuts of 3D space are measured

$$\mathscr{P} = \oiint_S \mathscr{W} \cdot d\mathbf{s} = \oiint_S \mathscr{W} \cdot \hat{\mathbf{n}}\, da$$

$\mathscr{P}$ = instantaneous total power (W)

$\hat{\mathbf{n}}$ = unit vector normal to the surface

da = infinitesimal area of the closed surface (m^2)

$$\boxed{\mathbf{W}_{av}(x, y, z) = [\mathscr{W}(x, y, z; t)]_{av} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]} \quad (\mathrm{W/m^2})$$

$$P_{rad} = P_{av} = \oiint_S \mathbf{W}_{rad} \cdot d\mathbf{s} = \oiint_S \mathbf{W}_{av} \cdot \hat{\mathbf{n}}\, da$$

$$= \frac{1}{2} \oiint_S \mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s}$$

The radial component of the radiated power density of an antenna is given by

Problem

$$\mathbf{W}_{\text{rad}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the total radiated power.

The radial component of the radiated power density of an antenna is given by

$$\mathbf{W}_{\text{rad}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the total radiated power.

Solution: For a closed surface, a sphere of radius r is chosen. To find the total radiated power, the radial component of the power density is integrated over its surface. Thus

$$P_{\text{rad}} = \oiint_S \mathbf{W}_{\text{rad}} \cdot \hat{\mathbf{n}}\, da$$

$$= \int_0^{2\pi} \int_0^{\pi} \left(\hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \right) \cdot (\hat{\mathbf{a}}_r r^2 \sin\theta\, d\theta\, d\phi) = \pi^2 A_0 \quad (\text{W})$$

1.6 Radiation power density

- Isotropic radiator (ideal source which radiates equally in all directions) does not exist in practical, but is used for reference.
- Poynting vector for isotropic radiator: function of r only, independent of θ, ϕ.
- Total power radiated:

$$P_{\rm rad} = \oiint_S \mathbf{W}_0 \cdot d\mathbf{s} = \int_0^{2\pi} \int_0^{\pi} [\hat{\mathbf{a}}_r W_0(r)] \cdot [\hat{\mathbf{a}}_r r^2 \sin\theta \, d\theta \, d\phi] = 4\pi r^2 W_0$$

- Power density:
  Uniformly distributed over sphere of radius r

$$\mathbf{W}_0 = \hat{\mathbf{a}}_r W_0 = \hat{\mathbf{a}}_r \left(\frac{P_{\rm rad}}{4\pi r^2} \right) \quad ({\rm W/m^2})$$

1.7 Radiation intensity (U)

- Radiation intensity: power radiated from an antenna per unit solid angle.
- Multiply radiation density (W_{rad}) by square of distance to get radiation intensity (U)

$$\boxed{U = r^2 W_{\text{rad}}}$$

$U =$ radiation intensity (W/unit solid angle)

$W_{\text{rad}} =$ radiation density (W/m^2)

- Radiation intensity is also related to far zone electric field of antenna:

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\mathbf{E}(r,\theta,\phi)|^2 \simeq \frac{r^2}{2\eta}\left[|E_\theta(r,\theta,\phi)|^2 + |E_\phi(r,\theta,\phi)|^2\right]$$
$$\simeq \frac{1}{2\eta}\left[|E_\theta^\circ(\theta,\phi)|^2 + |E_\phi^\circ(\theta,\phi)|^2\right]$$

$\mathbf{E}(r,\theta,\phi) =$ far-zone electric-field intensity of the antenna $= \mathbf{E}^\circ(\theta,\phi)\dfrac{e^{-jkr}}{r}$

$E_\theta, E_\phi =$ far-zone electric-field components of the antenna

$\eta =$ intrinsic impedance of the medium

1.7 Radiation intensity (U)

$$\boxed{U = r^2 W_{\text{rad}}}$$

U = radiation intensity (W/unit solid angle)
W_{rad} = radiation density (W/m^2)

- Total power : Obtained by integrating radiation intensity over entire solid angle:

$$P_{\text{rad}} = \oiint_{\Omega} U \, d\Omega = \int_0^{2\pi} \int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

- $d\Omega$ = element of solid angle = $\sin\theta \, d\theta \, d\phi$.
-

1.7 Radiation intensity

The radial component of the radiated power density of an antenna is given by

$$\mathbf{W}_{rad} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2} \quad (\mathrm{W/m^2})$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the total radiated power.

Solution: For a closed surface, a sphere of radius r is chosen. To find the total radiated power, the radial component of the power density is integrated over its surface. Thus

$$P_{rad} = \oiint_S \mathbf{W}_{rad} \cdot \hat{\mathbf{n}}\, da$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\hat{\mathbf{a}}_r A_0 \frac{\sin\theta}{r^2}\right) \cdot (\hat{\mathbf{a}}_r r^2 \sin\theta\, d\theta\, d\phi) = \pi^2 A_0 \quad (\mathrm{W})$$

- For the previous problem:

$$U = r^2 W_{rad} = A_0 \sin\theta$$

- Radiated power can also be found using:

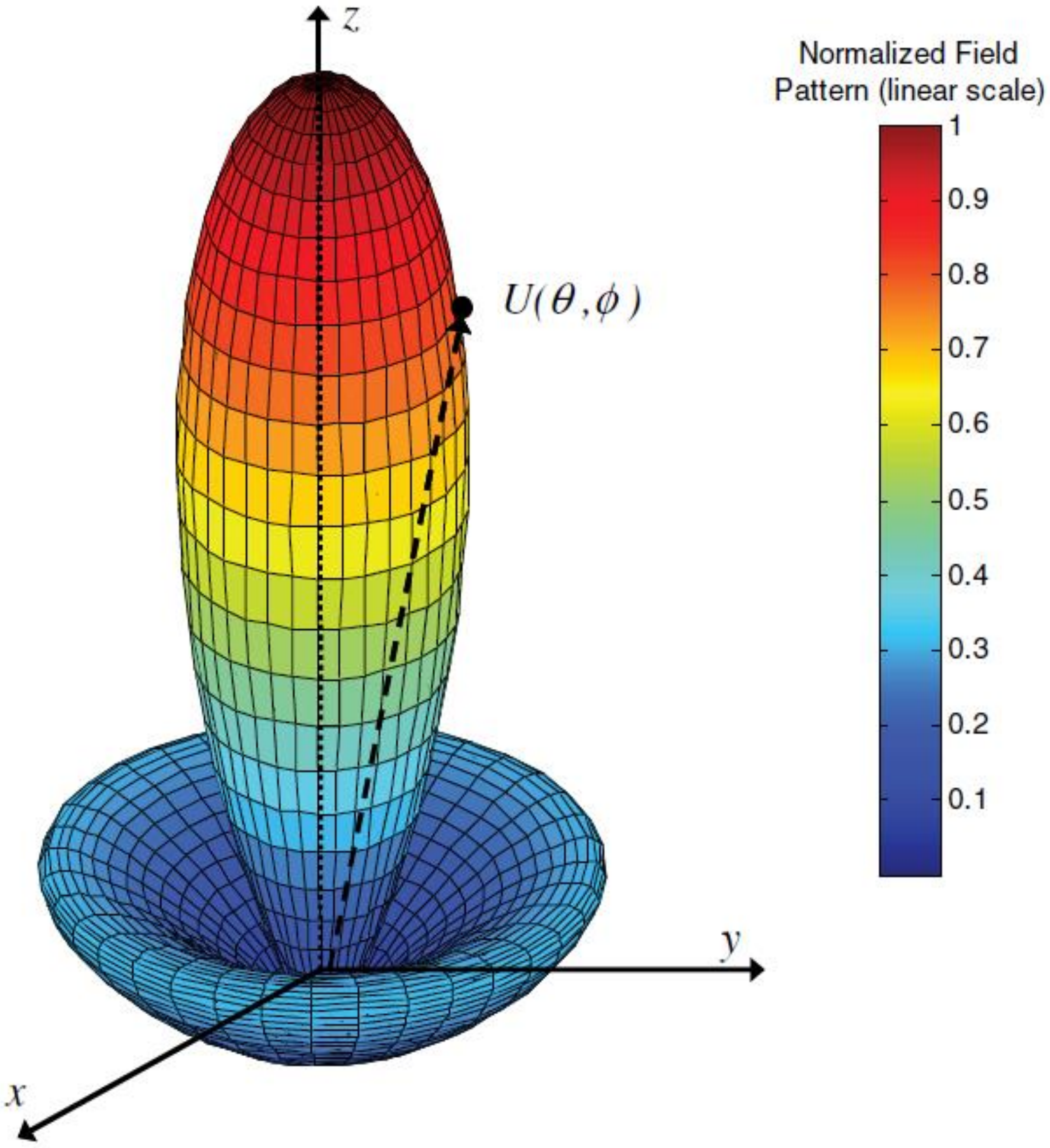
$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\, d\phi = \pi^2 A_0$$

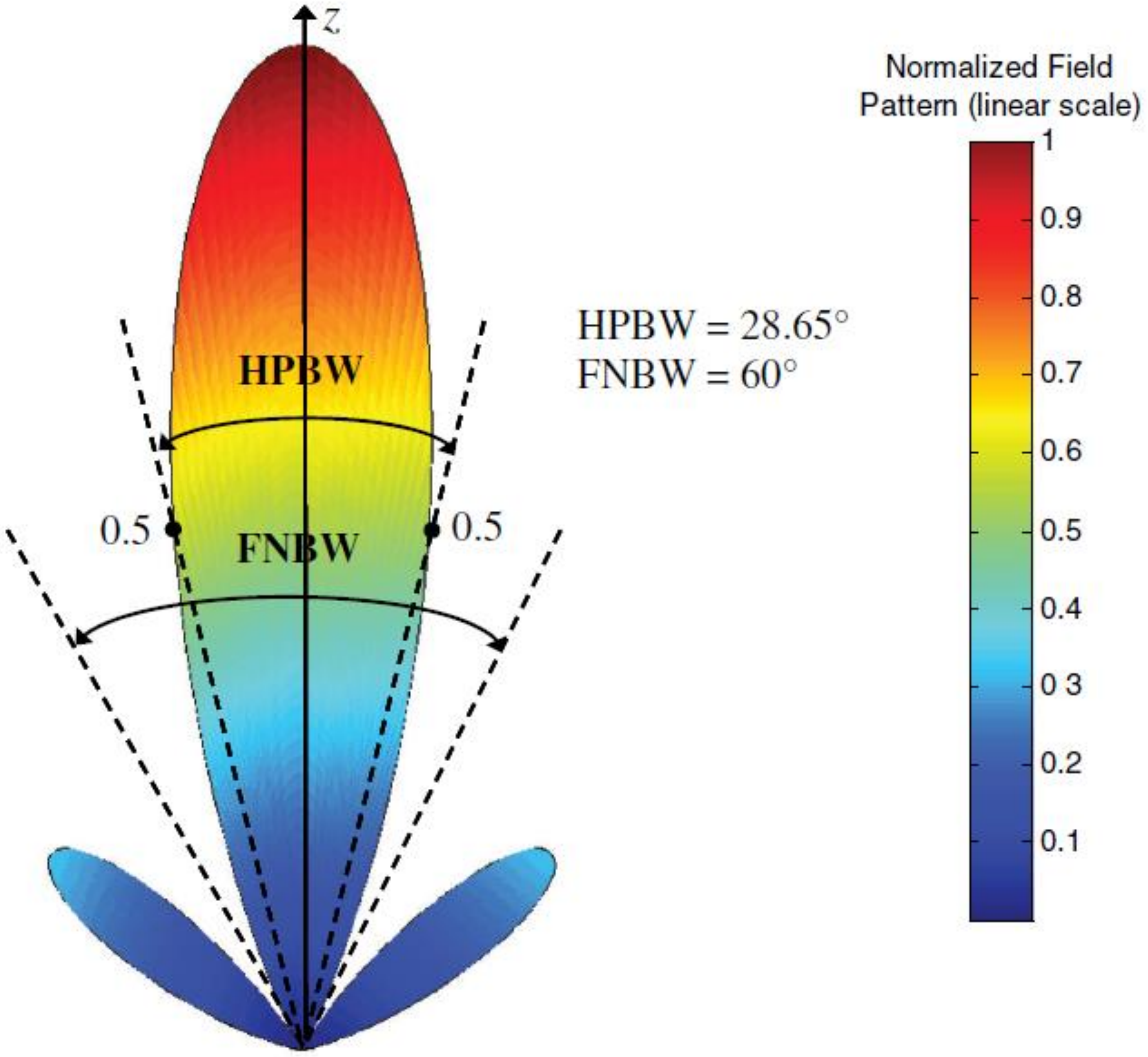
1.7 Radiation intensity (U)

- For isotropic source, U is independent of angles θ, ϕ

$$P_{\text{rad}} = \oiint_{\Omega} U_0 \, d\Omega = U_0 \oiint_{\Omega} d\Omega = 4\pi U_0$$

- Radiation intensity of isotropic source: $U_0 = \dfrac{P_{\text{rad}}}{4\pi}$





(b) Two-dimensional

Three- and two-dimensional power patterns (in linear scale) of $U(\theta) = \cos^2(\theta) \cos^2(3\theta)$.

1.8 Beam width

- *Beamwidth* of a pattern: the angular separation between two identical points on opposite side of the pattern maximum.
- *Half-Power Beamwidth* (*HPBW*): “In a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam.” (IEEE)
- *First-Null Beamwidth* (*FNBW*): the angular separation between the first nulls of the pattern,
- Generally, beamwidth corresponds to HPBW.
- Beamwidth is very important figure of merit and often is used as a trade-off between it and the side lobe level; as the beamwidth decreases, the side lobe increases and vice versa

1.8 Beam width

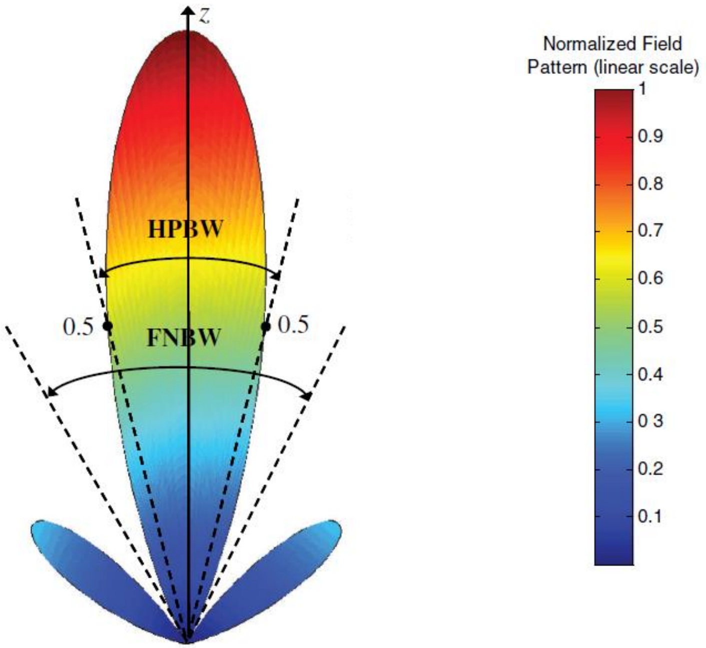
- Beamwidth: describe the resolution capabilities of the antenna to distinguish between two adjacent radiating sources or radar targets.
- most common resolution criterion states that *the resolution capability of an antenna to distinguish between two sources is equal to half the first-null beamwidth (FNBW/2), which is usually <u>used to</u> <u>approximate the half-power beamwidth (HPBW)</u>*

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), \quad (0 \le \theta \le 90^\circ, \quad 0^\circ \le \phi \le 360^\circ)$$

The three- and two-dimensional plots of this, plotted in a linear scale, are shown
Find the

a. half-power beamwidth HPBW (*in radians and degrees*)
b. first-null beamwidth FNBW (*in radians and degrees*)

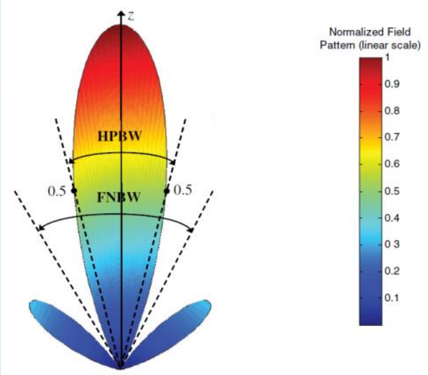


(b) Two-dimensional

a. Since the $U(\theta)$ represents the *power* pattern, to find the half-power beamwidth you set the function equal to half of its maximum, or

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5 \Rightarrow \cos\theta_h \cos 3\theta_h = 0.707$$

$$\theta_h = \cos^{-1}\left(\frac{0.707}{\cos 3\theta_h}\right)$$



(b) Two-dimensional

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25\ radians = 14.325°$$

Since the function $U(\theta)$ is symmetrical about the maximum at $\theta = 0$, then the HPBW is

$$\text{HPBW} = 2\theta_h \approx 0.50\ radians = 28.65°$$

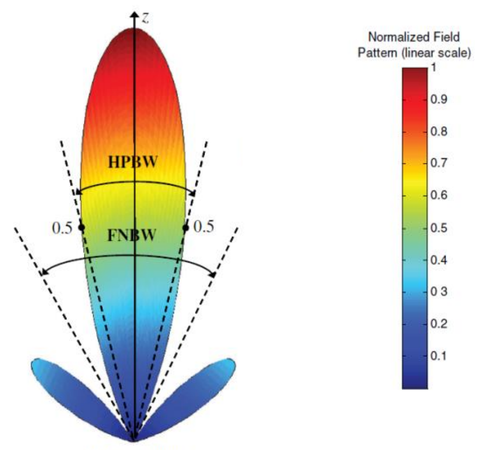
b. To find the first-null beamwidth (FNBW), you set the $U(\theta)$ equal to zero, or

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This leads to two solutions for θ_n.

$$\cos\theta_n = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2}\ radians = 90°$$

$$\cos 3\theta_n = 0 \Rightarrow \theta_n = \frac{1}{3}\cos^{-1}(0) = \frac{\pi}{6}\ radians = 30°$$



(b) Two-dimensional

The one with the smallest value leads to the FNBW. Again, because of the symmetry of the pattern, the FNBW is

$$\text{FNBW} = 2\theta_n = \frac{\pi}{3}\ radians = 60°$$