3.4 Aperture antennas – Horn antenna, Parabolic reflector antenna

Module:3 HF, UHF and Microwave Antennas

Course: BECE305L – Antenna and Microwave Engineering

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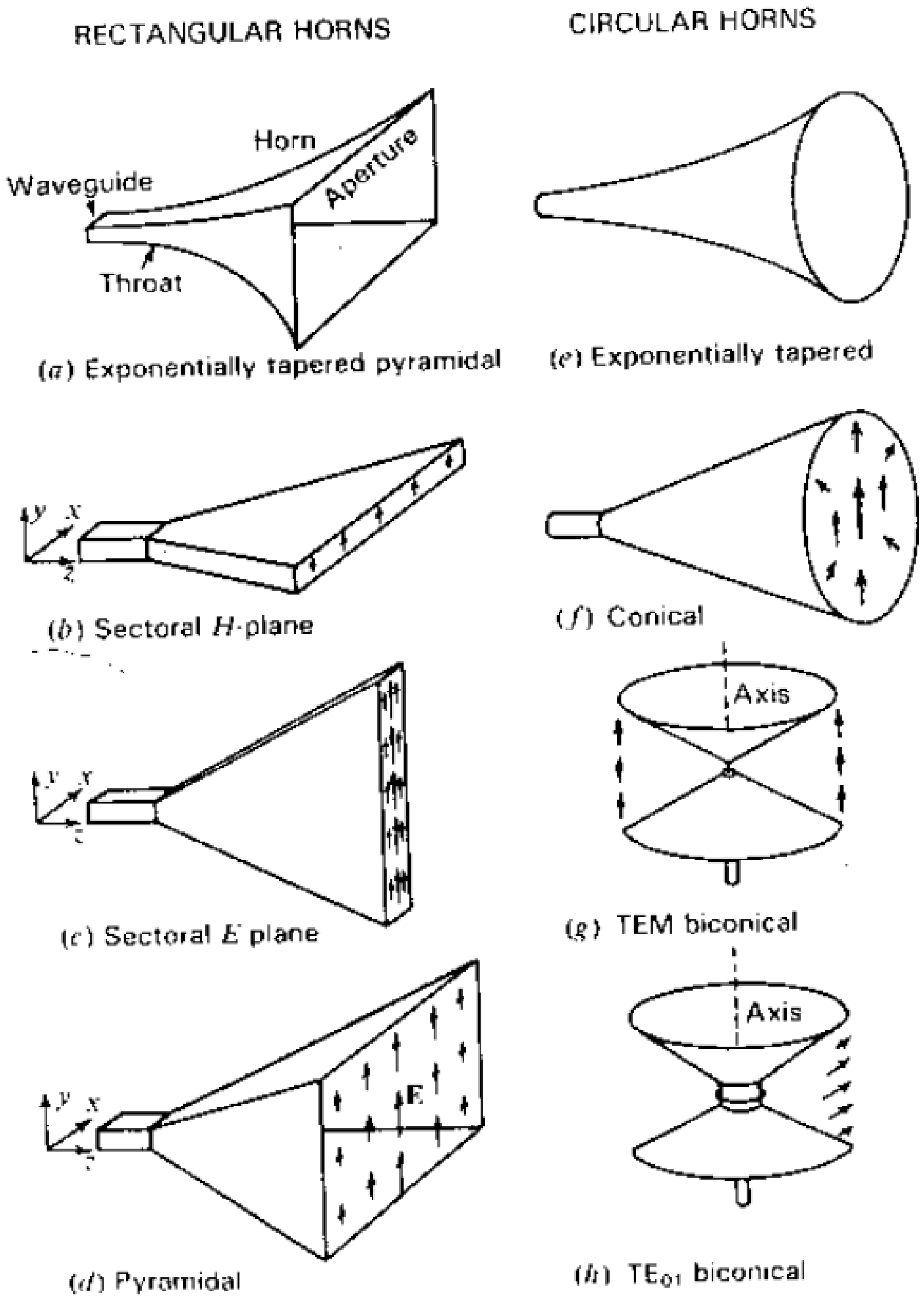
Module:3 HF, UHF and Microwave Antennas 7 hours

Wire Antennas - long wire, loop antenna - helical antenna. Yagi-Uda antenna, Frequency independent antennas - spiral and log periodic antenna - Aperture antennas – Horn antenna, Parabolic reflector antenna - Microstrip antenna

- Source of the contents: Balanais Antenna Theory and Antennas from theory to practice (Yi Huang)

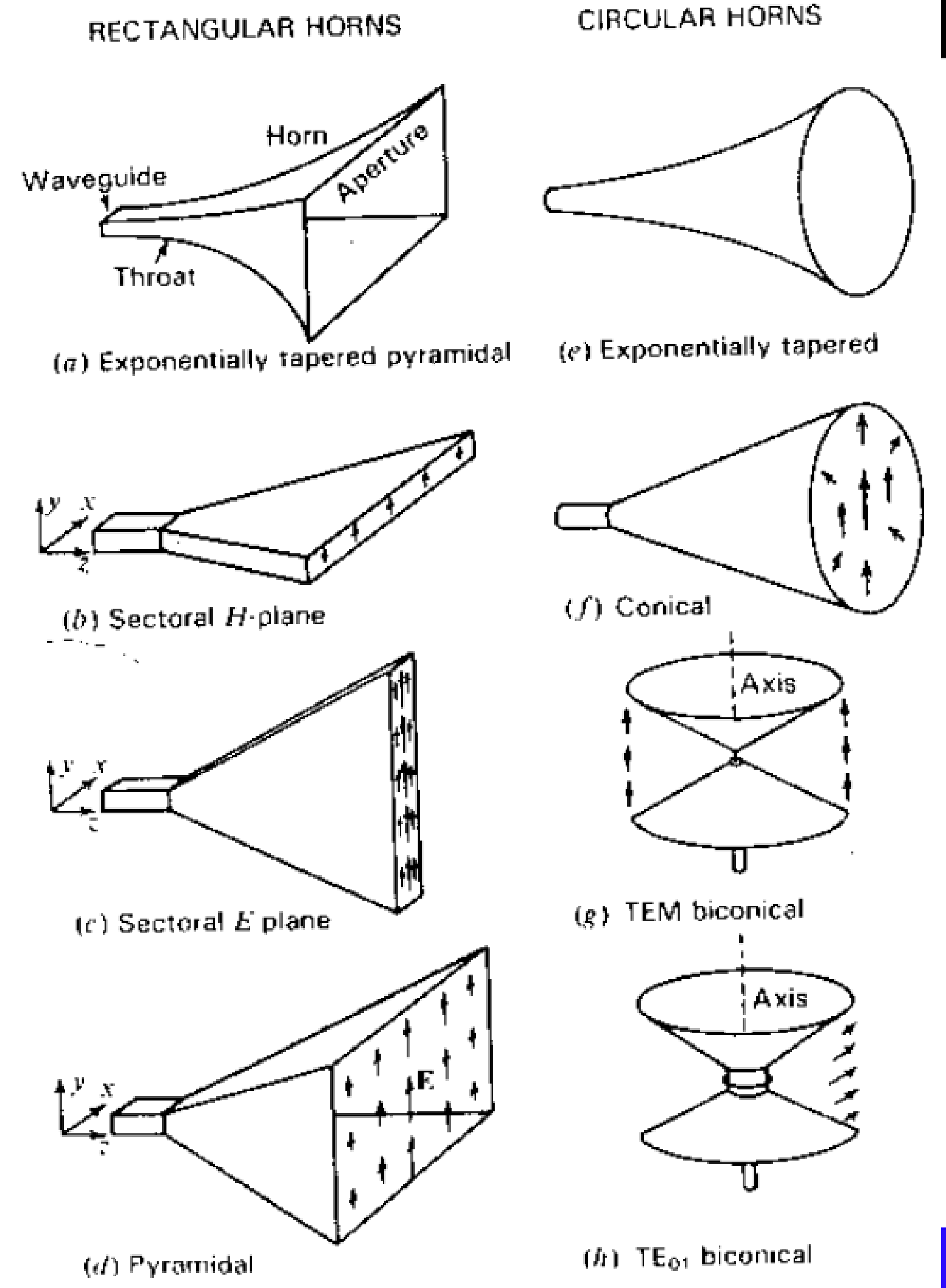
Horn Antenna

- Flared out (opened out) waveguide
- **Produces uniform phase front with large aperture than that of a waveguide** – Hence **greater directivity**
- Energized with rectangular waveguide and circular waveguide (Arrow- E field)
- Rectangular: **TE10 mode**
- **Gradual exponential taper**
- **Sectoral E plane**
  Sectoral H Plane
- **Pyramidal horn** (flare both planes)

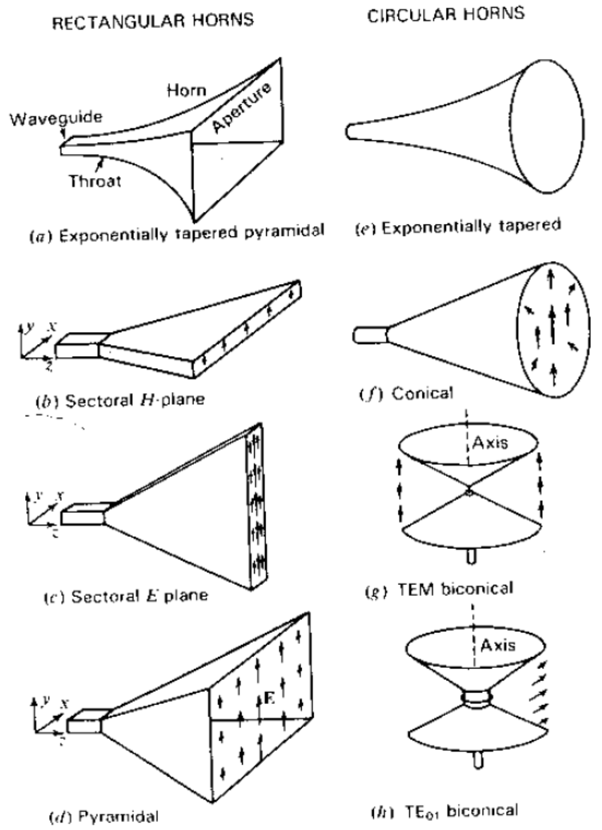


Horn Antenna

- **circular waveguide Conical (TE11 mode)**
- **Biconical (TEM and TE01 modes)**
- Biconical are non-directional in horizontal plane



Horn Antenna



- With aperture dimensions, aperture field distributions, The radiation pattern of horn can be determined
- If **uniform distribution along aperture – Maximum directivity**
- If **variations in magnitude and phase across apert Decreases directivity**
-

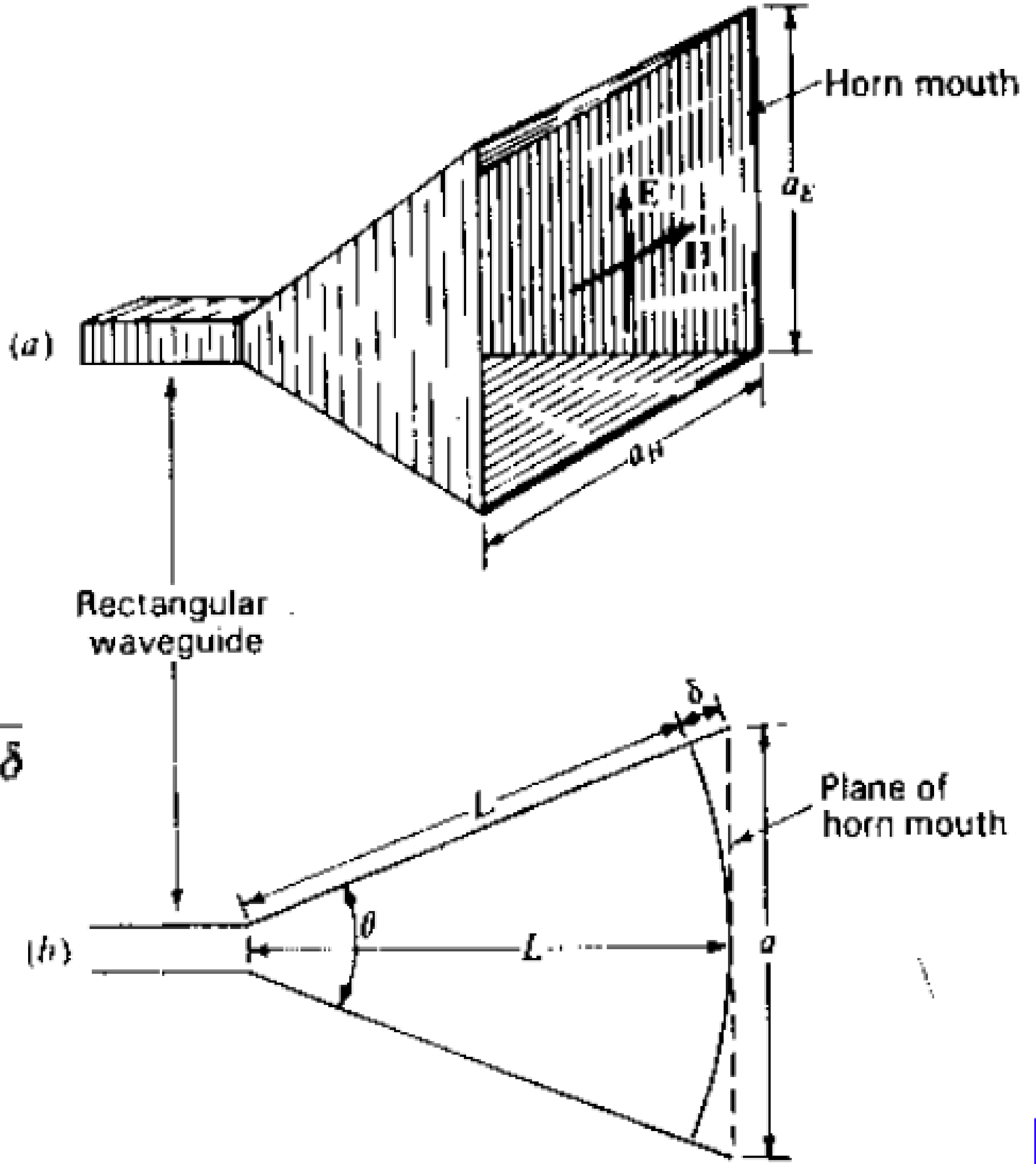
- Principle of equality of path length (Fermat's principle) is applicable
- δ: path length difference between ray along side and along axis of horn

$$\cos\frac{\theta}{2} = \frac{L}{L+\delta}$$

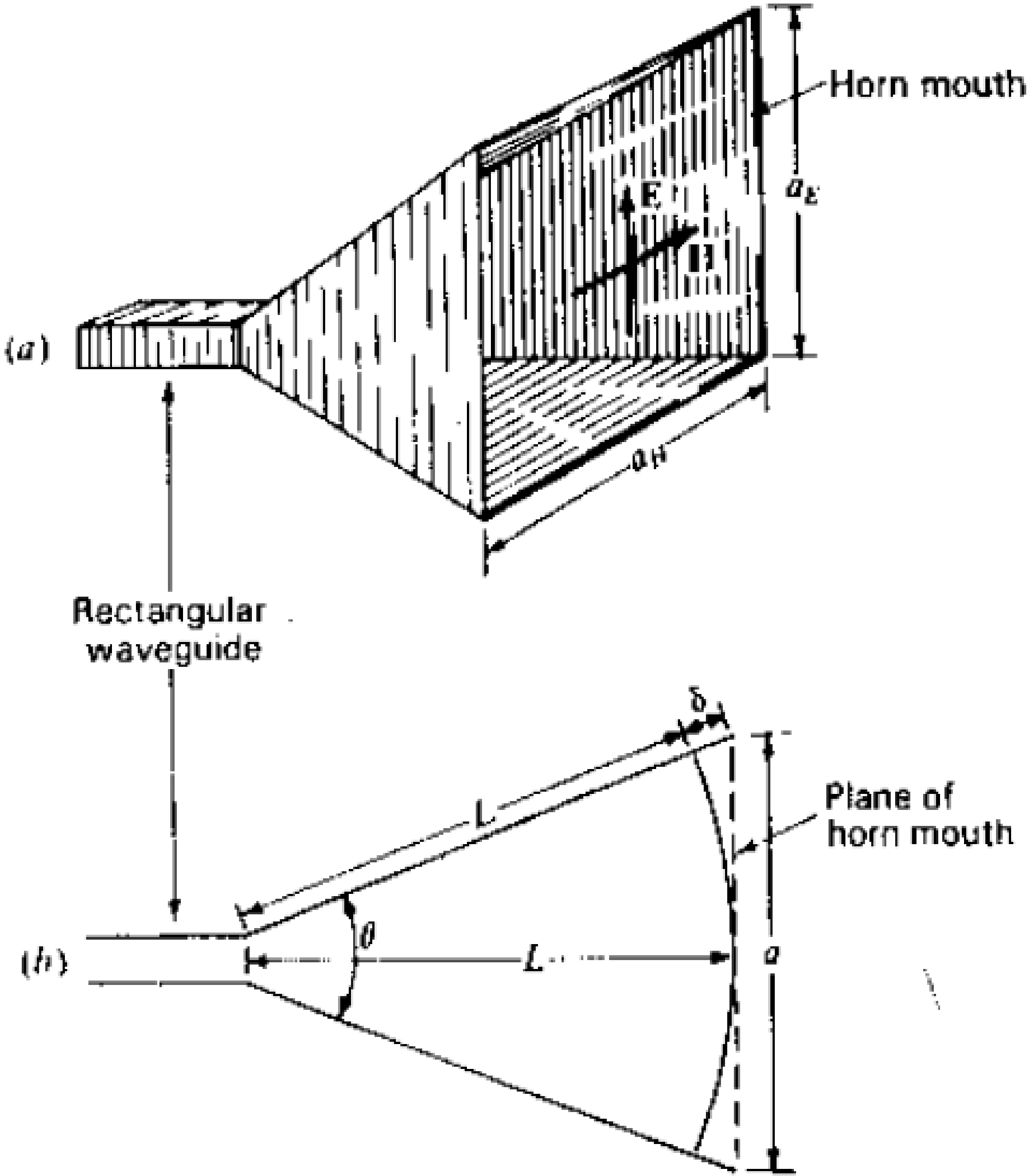
$$\sin\frac{\theta}{2} = \frac{a}{2(L+\delta)} \qquad L = \frac{a^2}{8\delta} \qquad (\delta \ll L)$$

$$\tan\frac{\theta}{2} = \frac{a}{2L} \qquad \theta = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\frac{L}{L+\delta}$$

θ = flare angle (θ_E for E plane, θ_H for H plane)
a = aperture (a_E for E plane, a_H for H plane)
L = horn length



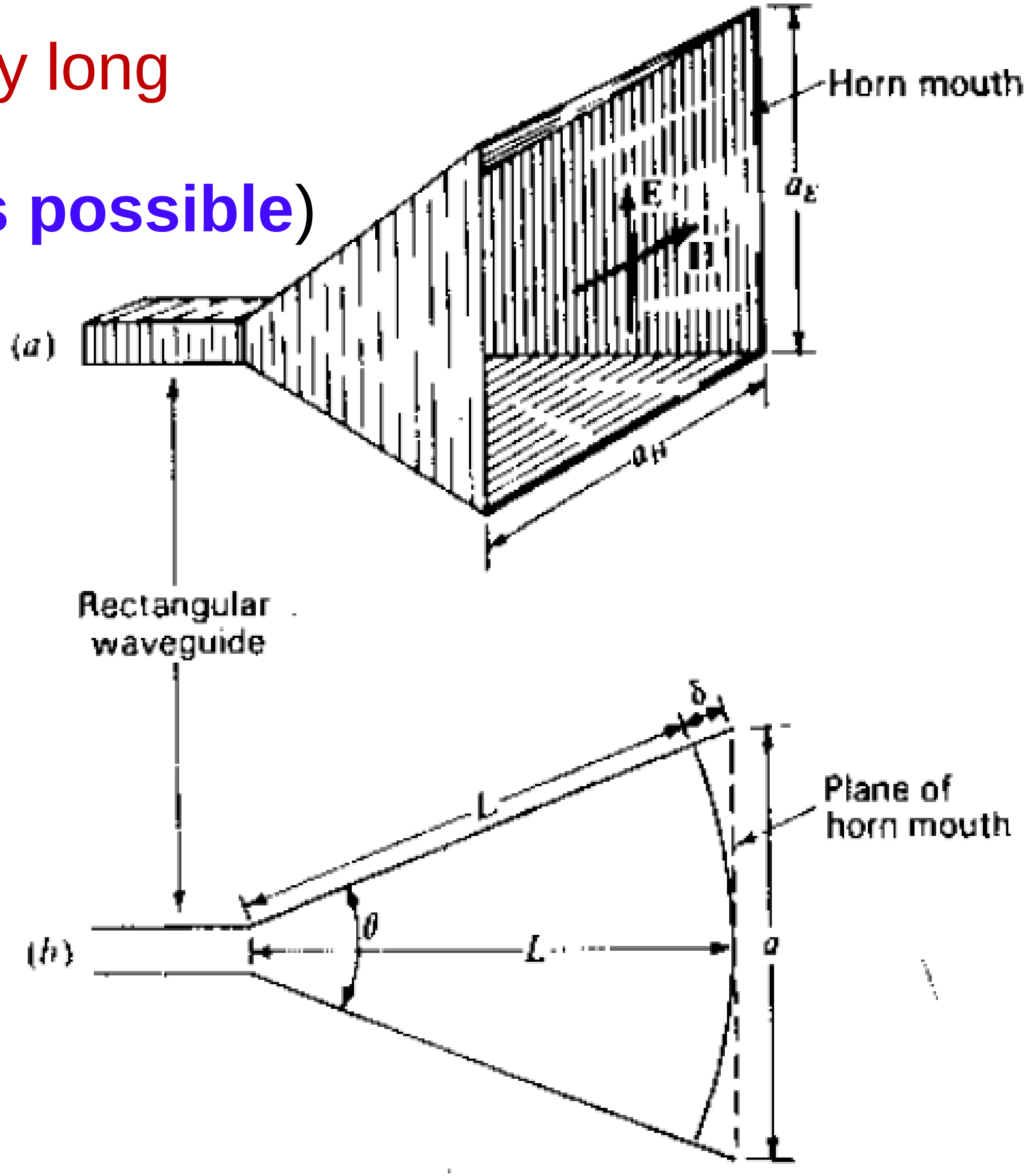
- **E plane of horn**
  δ is usually held to 0.25λ or less
- **H plane**
  δ cab be larger or about 0.4λ
  since E goes to zero at horn edges
  E_t=0 (Boundary condition)



δ: path length difference between ray along side and along axis of horn

θ = flare angle (θ_E for E plane, θ_H for H plane)
a = aperture (a_E for E plane, a_H for H plane)
L = horn length

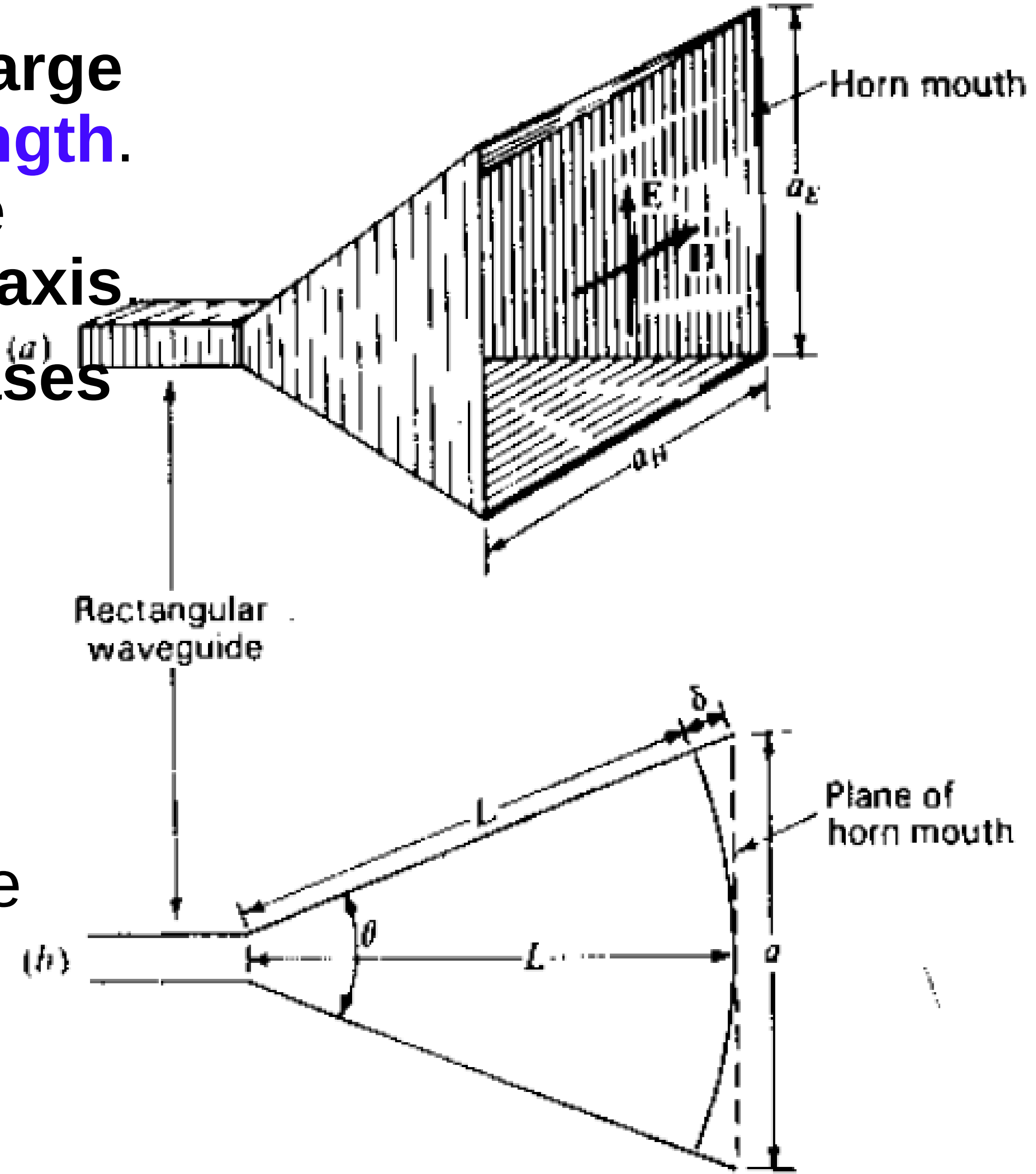
- For **uniform aperture distribution**, very long aperture with small flare is required. **Practical** (**Horn should be as small as possible**)
- **Optimum horn** is between these extremes and has **minimum beam width without excessive sidelobe level** for a given length.
- **If δ is sufficiently small fraction of wavelength**, **field has nearly uniform phase over entire aperture**.
- If aperture ***a*** and **flare angle θ** are **increased**, then **directivity increases** and **beamwidth decreases**,



- If **aperture and flare angles are very large** then **δ may result in 180° electrical length**. **Field at centre and at edge** of aperture is in **opposition phases** to **field along axis**.
- **This increases side lobes and decreases directivity.**
- For very large flare angles $\frac{L}{L+\delta} \approx 1$ (path length δ on distribution of field may be neglected.
- **Maximum directivity occurs at large flare angle** for which δ does not exclude certain value

$$\delta_0 = \frac{L}{\cos(\theta/2)} - L$$

$$L = \frac{\delta_0 \cos(\theta/2)}{1 - \cos(\theta/2)}$$



$$\delta_0 = \frac{L}{\cos(\theta/2)} - L$$

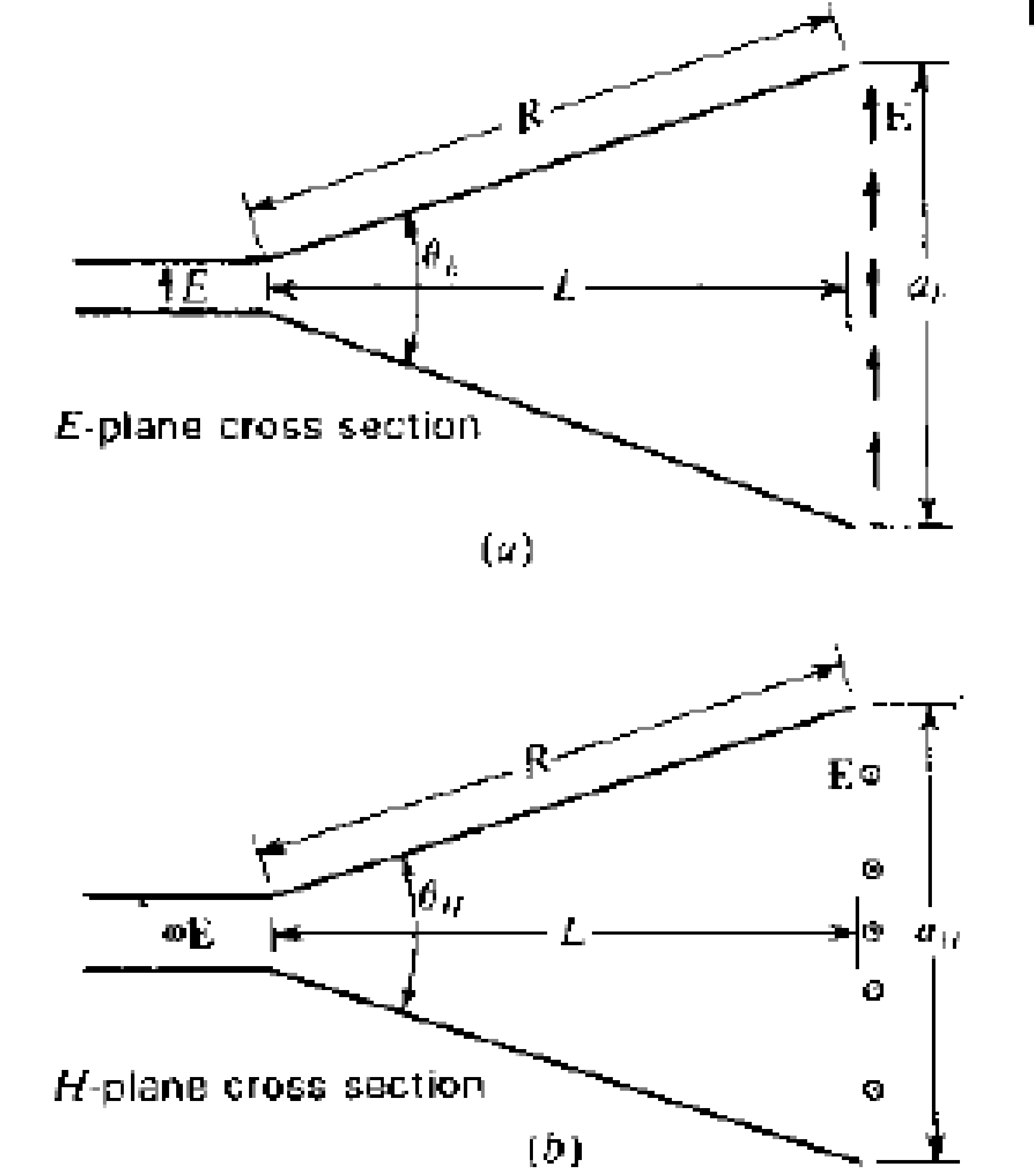
$$L = \frac{\delta_0 \cos(\theta/2)}{1 - \cos(\theta/2)}$$

- **δ must be between 0.1 and 0.4 free space wavelengths**
- Optimum horn $\boldsymbol{\delta_0 = 0.25\lambda}$ **and** axial length $\boldsymbol{L = 10\lambda}$
- From $\theta = 25°$ results in max directivity for 10λ horn
- **Width of waveguide at throat** of horn must be **between $\boldsymbol{\lambda/2}$ and $\boldsymbol{1\lambda}$**
- Excitation system is symmetrical so that **even modes are not energized**, the width must be between $\frac{\lambda}{2}$ and $3\lambda/2$.

$$L = \frac{a^2}{8\delta} \qquad (\delta \ll L)$$

$$\theta = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\frac{L}{L+\delta}$$

Rectangular horn



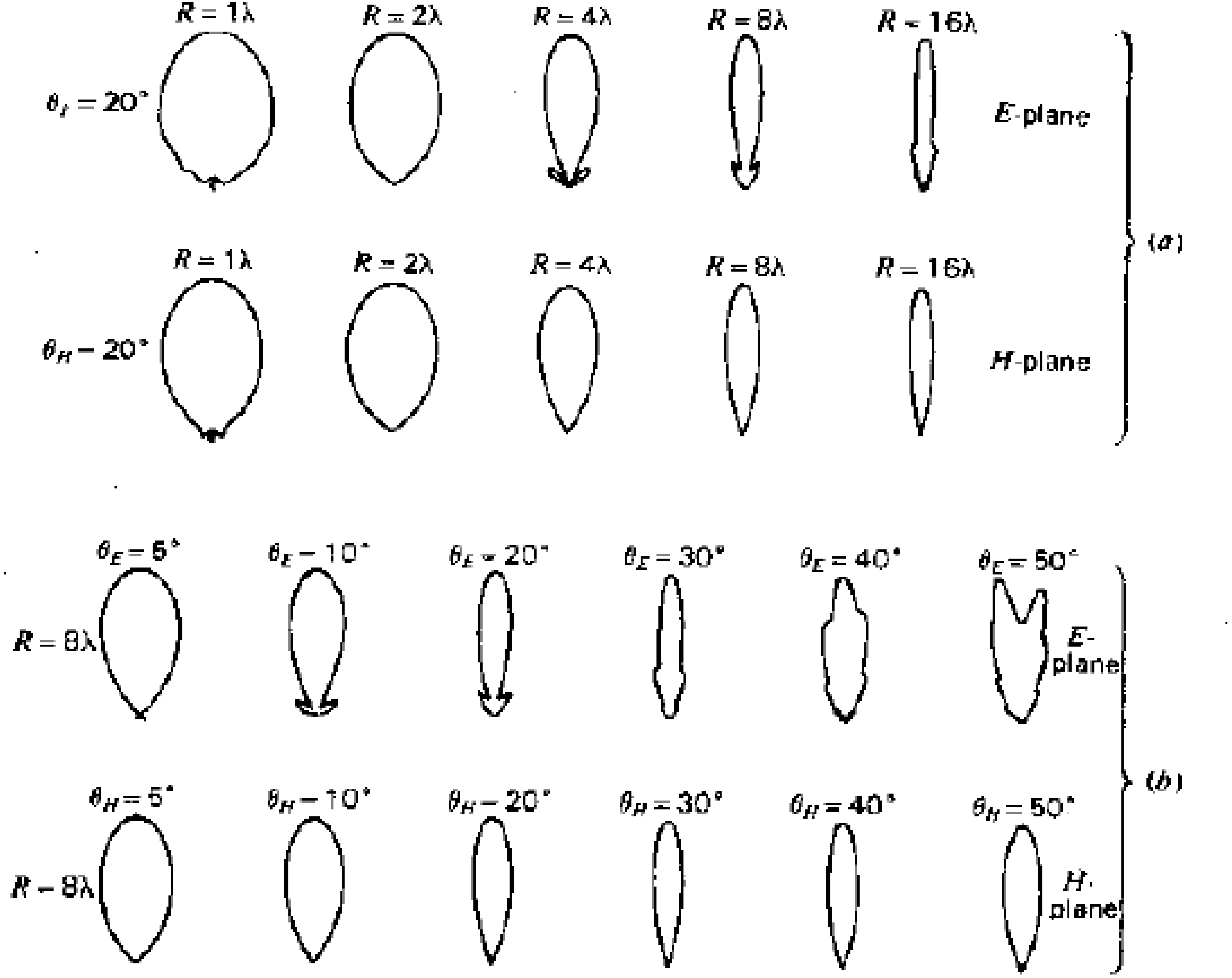


Figure 13-23 Measured *E*- and *H*-plane field patterns of rectangular horns as a function of flare angle and horn length. (*After D. R. Rhodes, "An Experimental Investigation of the Radiation Patterns of Electromagnetic Horn Antennas,"* Proc. IRE, 36, *1101–1105, September 1948.*)

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2}$$

A_e = effective aperture, m^2
A_p = physical aperture, m^2
ε_{ap} = aperture efficiency = A_e/A_p
λ = wavelength, m

- For rectangular horn
  $A_p = a_E\, a_H$
- For conical horn $A_p = \pi r^2$
- a_E or a_H or r must be atleast 1λ taking $\varepsilon_{ap} \approx 0.6$

$$D \simeq \frac{7.5 A_p}{\lambda^2}$$

$$D \simeq 10 \log\left(\frac{7.5 A_p}{\lambda^2}\right) \qquad \text{(dBi)}$$

Pyramidal (or rectangular horn)

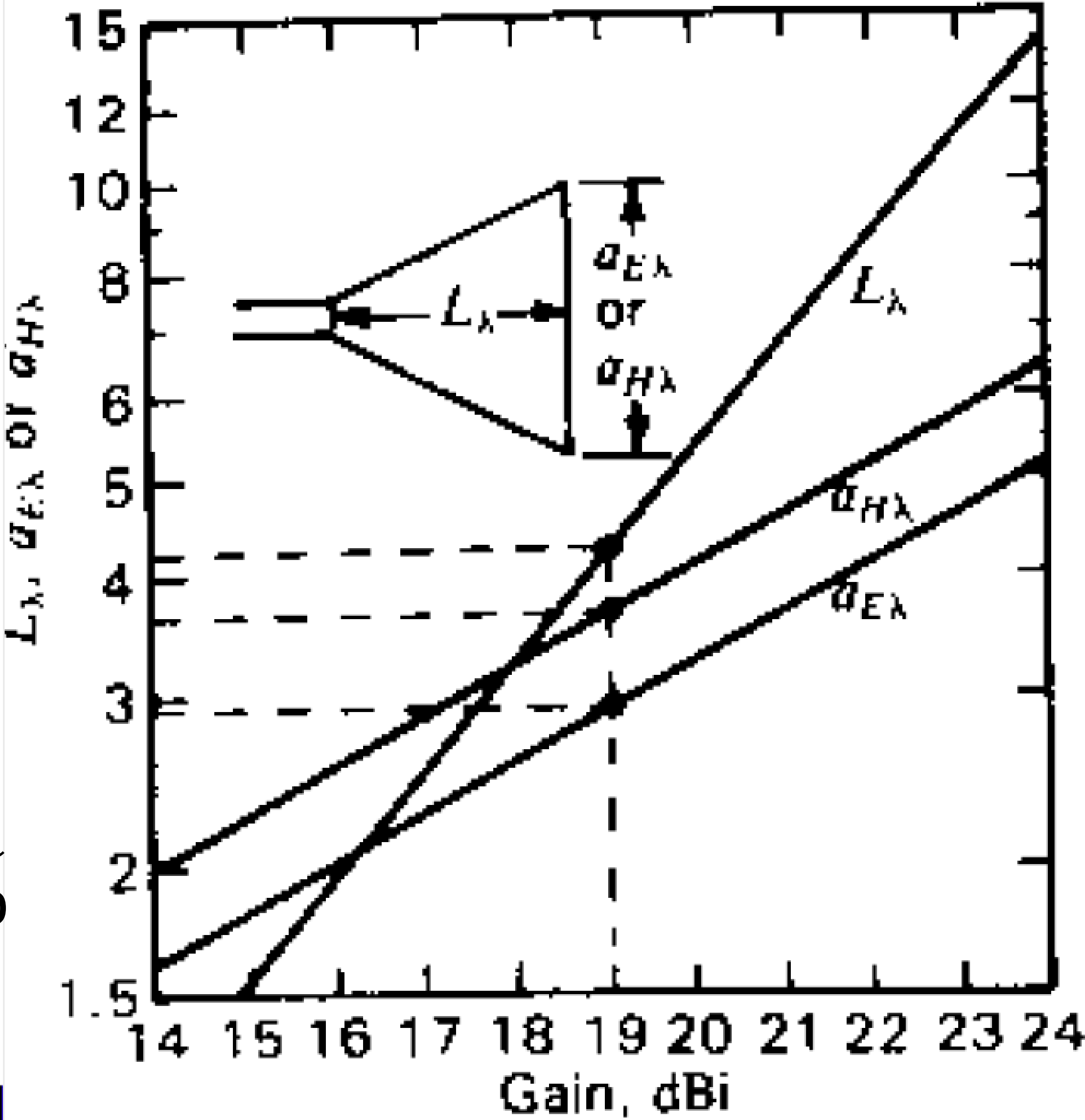
$$D \simeq 10 \log (7.5 a_{E\lambda} a_{H\lambda})$$

: $a_{E\lambda}$ = E-plane aperture in λ

$a_{H\lambda}$ = H-plane aperture in λ

Optimum dimensions

- Dimensions of rectangular Pyramidal horn in wavelengths, versus directivity or gain (if lossless)
- 19dBi gain requires horn of length $L_\lambda = 4.25$
  H plane aperture $a_{H\lambda} = 3.7$
  E plane aperture $a_{E\lambda} = 2.9$
- These are inside dimensions
  Assumed that $\delta(E\ plane) = 0.25\lambda$
  $\delta(H\ plane) = 0.4\lambda$ (optimum) also
  ε_{ap}=0.6



Type of aperture	Beam width, deg	
	Between first nulls	Between half-power points
Uniformly illuminated rectangular aperture or linear array	$\frac{115}{L_\lambda}$	$\frac{51}{L_\lambda}$
Uniformly illuminated circular aperture	$\frac{140}{D_\lambda}$	$\frac{58}{D_\lambda}$
Optimum E-plane rectangular horn	$\frac{115}{a_{E\lambda}}$	$\frac{56}{a_{E\lambda}}$
Optimum H-plane rectangular horn	$\frac{172}{a_{H\lambda}}$	$\frac{67}{a_{H\lambda}}$

† L_λ = length of rectangular aperture or linear array in free-space wavelengths

D_λ = diameter of circular aperture in free-space wavelengths

$a_{E\lambda}$ = aperture in E plane in free-space wavelengths

$a_{H\lambda}$ = aperture in H plane in free-space wavelengths

Determine the length L, H Plane aperture and flare angles θ_E and θ_H (in E and H planes respectively) of pyramidal horn for which the E plane aperture is $a_E = 10\lambda$. The horn is fed by rectangular waveguide with TE_{10} mode. Let $\delta = 0.2\lambda$ in E plane and 0.375λ in H plane. b) what are the beam widths. c) what is the directivity

- $\delta = 0.2\lambda = \lambda/5$ in E plane,
  Horn length $L = \frac{a_E^2}{8\delta} = \frac{100\lambda^2}{8\lambda/5} = 62.5\lambda$
- Flare angle in E plane $\theta_E = 2\tan^{-1}\frac{a}{2L} = 2\tan^{-1}\frac{10}{125} = 9.1°$
- Taking $\delta = 0.375\lambda = 3\lambda/8$ in H plane,

$$\theta_H = 2\cos^{-1}\frac{L}{L+\delta} = 2\cos^{-1}\frac{62.5}{62.5+0.375} = 12.52°$$

- H plane aperture $a_H = 2L\tan\frac{\theta_H}{2} = 2 \times 62.5\lambda \tan 6.26° = 13.7\lambda$

Determine the length L, H Plane aperture and flare angles θ_E and θ_H (in E and H planes respectively) of pyramidal horn for which the E plane aperture is $a_E = 10\lambda$. The horn is fed by rectangular waveguide with TE_{10} mode. Let $\delta = 0.2\lambda$ in E plane and 0.375λ in H plane. b) what are the beam widths. c) what is the directivity

- From table,

$$HPBW\ (E\ plane) = \frac{56°}{a_{E\lambda}} = \frac{56°}{10} = 5.6°$$

- $HPBW\ (H\ plane) = \frac{67°}{a_{H\lambda}} = \frac{67°}{13.7} = 4.9°$

- Directivity

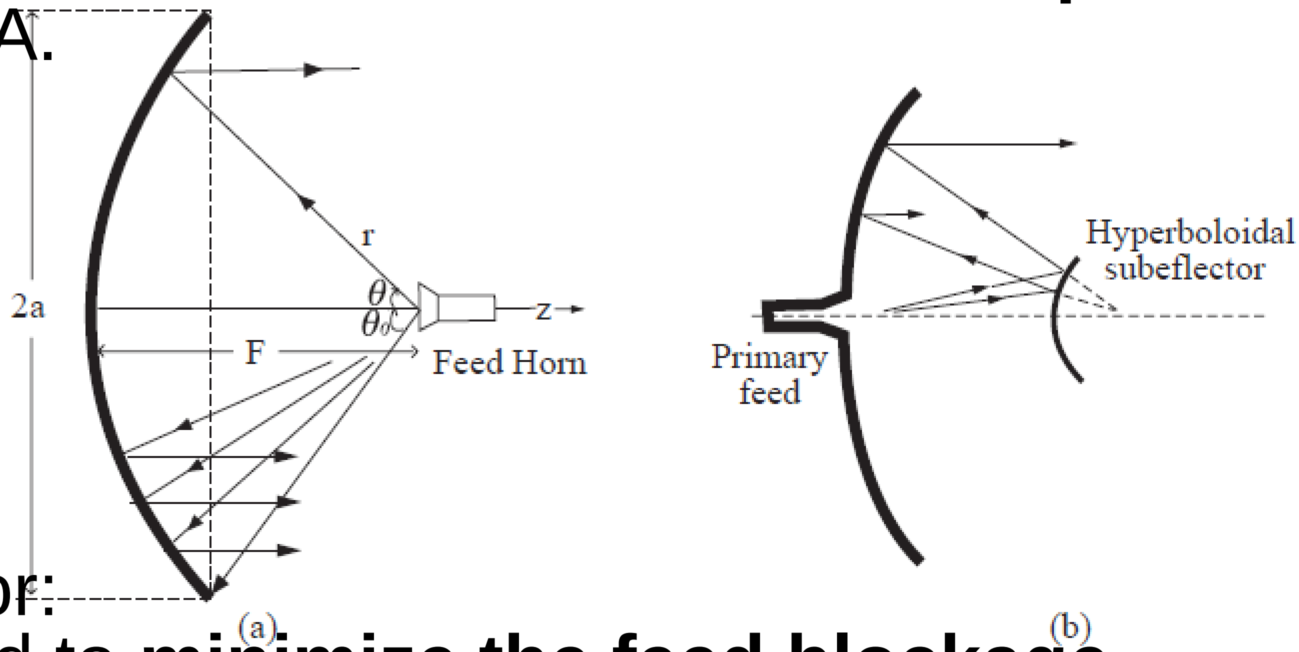
$$D = 10 \log\left(\frac{7.5\ A_p}{\lambda^2}\right) = 10 \log(7.5 \times 10 \times 13.7) = 30.1\ \text{dBi}$$

2. Parabolic Reflector antenna

- The typical gain of a **practical horn antenna** is **up to 20 dBi** or so.
- **Reflector and lens antennas** can offer **much higher gains** than horn antennas and are normally relatively easy to design and construct.
- Most widely used antennas for **high-frequency and high-gain** applications in **radio astronomy**, **radar**, **microwave and millimeter wave communications and satellite tracking and communications**
- Most popular shape is the **paraboloid** – because of its excellent ability to produce
  a pencil beam (high gain) with
  low side lobes and
  good cross-polarization characteristics in the radiation pattern.

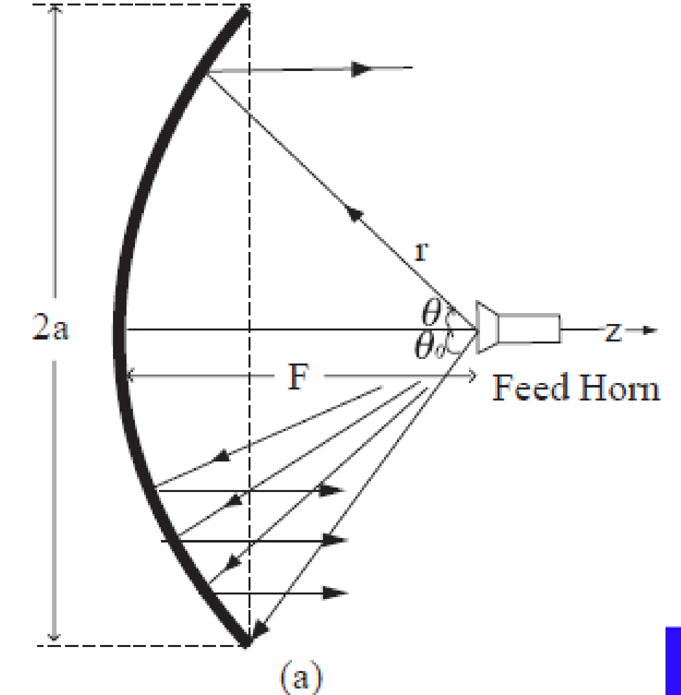
2. Parabolic Reflector antenna

- The **largest fully steerable reflector** in the world is the **100 m** diameter **radio telescope of the Max Planck Institute** for **Radioastronomy** at Effelsberg in Germany, whereas one of the **largest reflector antenna is the 305 m radio telescope** at Arecibo Observatory in the USA.
- typical feed for such an antenna is a **horn antenna**,
- **Paraboloid**: placed at the focal point in front of the reflector (front-fed)
  Cassegrain reflector antenna: at the back (vertex) of the reflector: to avoid using a long feed line and to **minimize the feed blockage** problems of a conventional paraboloidal antenna (the **side lobes can be reduced as well**).



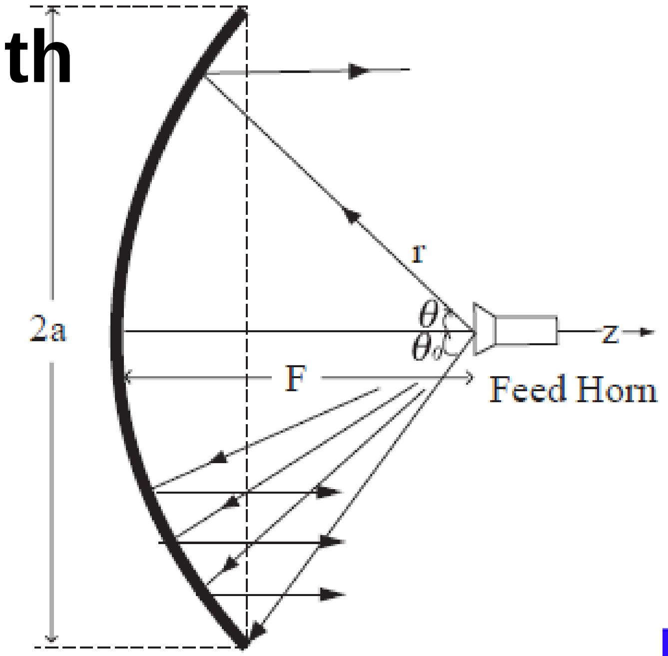
2. Parabolic Reflector antenna

- **Parabolic Reflector antenna is most popular**,:
  a front-fed paraboloidal antenna consists of
  a **reflector** and a **feed antenna**.
- surface of a paraboloidal reflector, which is illuminated by the feed antenna, is formed by **rotating a parabola about its axis**
- Its surface is therefore a paraboloid of revolution
  and **rays emanating from the focus of the reflect**
  are transformed into plane waves
  – this means that it is **highly directional**.



2. Parabolic Reflector antenna

- If the reflector was extremely large (infinite) and the feed was a point source at the focal point having radiation only towards the paraboloid, **all the radiated EM energy would be directed in one direction** (the *z*-direction) with zero beamwidth
- the **structure is much greater than the wavelength**
- **extremely suitable for high-frequency and high-gain applications**



(a)

2.1 Parabolic Reflector antenna: Analysis and design

- **not possible to make the reflector infinitely large** (actually we always try to make it as small as possible) and truncation has to take place.
- Also, **the feed antenna cannot be a point source**, which means that the actual performance of the antenna will be different from the ideal one.
- assume that the **diameter of the reflector is 2*a*** and **the focal length is *F***
- Any point on this paraboloid must satisfy the following condition:
-
$$r = \frac{2F}{1+\cos\theta} = F\sec^2(\theta/2) \qquad \theta \le \theta_0$$

subtended/angular aperture angle θ_0 (also known as the *edge angle*) is **determined** by the **reflector diameter** and **the focal length**

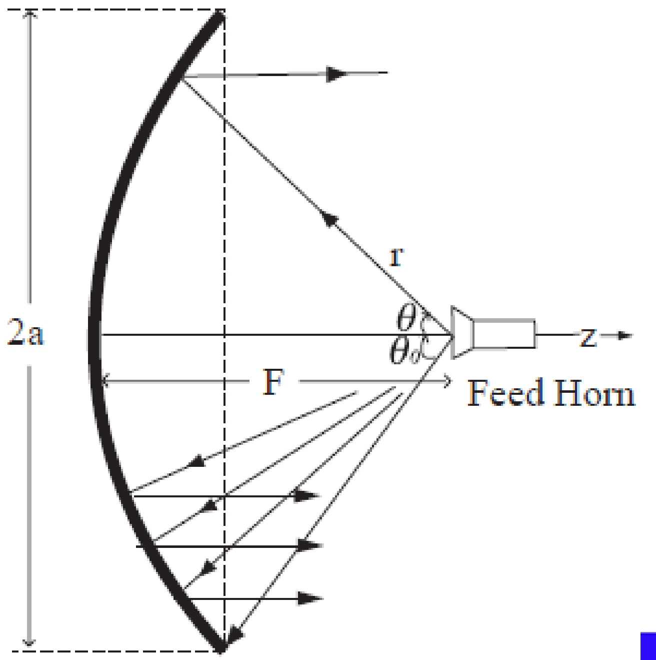
$$\theta_0 = \tan^{-1}\left|\frac{a}{F - a^2/(4F)}\right|$$

2.1 Parabolic Reflector antenna: Analysis and design

- another element of the reflector antenna is the feed, which is an antenna on its own and has many variables.
- the complete design of the reflector antenna is actually a complex task since there are many parameters which could be changed to meet (or fail) the specifications, some are independent and some are interlinked.
- *The reflector design problem consists primarily of* ***matching the feed antenna pattern to the reflector****.*
  The usual goal is to have the feed pattern about 10 dB down in the direction of the rim, that is the *edge taper =*
  (**the field at the edge**)/(**the field at the center**) ≈ **10 dB**.

2.1 Parabolic Reflector antenna: Analysis and design

- With *edge taper* = (the field at the edge)/(the field at the center) ≈ 10 dB.
- Feed antennas with this property can be constructed for the commonly used ***F/2a* value of 0.3 to 1.0**.
- **Higher values lead to better cross-polarization performance**, but require a **narrower feed pattern** and hence a **physically larger feed antenna**.



(a)

2.2 Parabolic Reflector antenna: **Aperture efficiency** and Directivity

- Since the feed antenna is closely linked to the reflector, the aperture efficiency should surely reflect this linkage. Let ***g*(*θ*) be the power radiation pattern of the feed located at the focus** – it is circularly symmetrical (not a function of *φ*).
- It has been shown that the **aperture efficiency** is given

$$\eta_{ap} = \cot^2\left(\frac{\theta_0}{2}\right)\left|\int_0^{\theta_0} g(\theta)\tan\left(\frac{\theta}{2}\right)\,d\theta\right|^2$$

- this is determined by both the reflector and the feed. The **maximum aperture efficiency is around 82%**, which is higher than that of a pyramidal horn

2.2 Parabolic Reflector antenna: Aperture efficiency and Directivity

- Aperture efficiency is generally a product of the following:

1. **Spillover efficiency: the fraction** of the **total power intercepted** and **collimated by the reflector**; it reduces the gain and increases the side-lobe levels.

2. **Taper efficiency**: the **uniformity of the amplitude distribution of the feed pattern** over the surface of the reflector.

3. **Phase efficiency**: the **phase uniformity of the field** over the aperture plane; it affects the gain and side lobes.

4. **Polarization efficiency**: the **polarization uniformity** of the field over the aperture plane.

5. **Blockage efficiency**: by the feed; it reduces gain and increases side-lobe levels. The **support structure can also contribute to the blockage**.

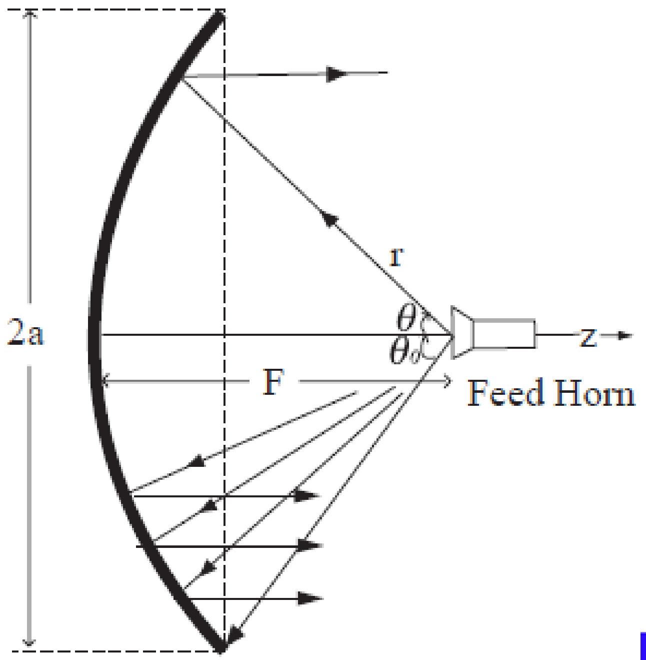
6. **Random error efficiency**: over the reflector surface.

2.2 Parabolic Reflector antenna: Aperture efficiency and Directivity

- The **larger the reflector angular aperture angle θ_0**, the larger the spillover efficiency, but the smaller the taper efficiency
- The **optimum aperture efficiency is** the **best trade-off** from all aspects.
- Once the aperture efficiency is found, the directivity can be readily obtained using

$$D = \frac{4\pi}{\lambda^2}\eta_{ap}(\pi a^2)$$

- The analysis here has not taken the feed antenna efficiency into account, which is about 70– 80% if it is a horn antenna.



(a)

2.2 Parabolic Reflector antenna: Aperture efficiency and Directivity

- the **overall reflector efficiency factor** is in the region of **50–70%**.
- It is very common to make a **reflector antenna with a gain of over 30 dBi**
- Some **world's largest antennas have a gain over 70 or even 80 dBi**,
- remember the typical gain for a**Yagi–Uda** or **log-periodic** antenna is about **10–15 dBi.**
- HPBW:

$$HPBW \approx 70^\circ \frac{\lambda}{2a}$$

- The **beamwidth also depends on the edge illumination**. Typically, as the edge attenuation increases, the beamwidth widens and the side lobes decrease.

2.3 *Design Considerations and Procedures*

- In addition to the aperture efficiency and directivity/gain, **cross-polarization**, due to **phase errors** (the **maximum fractional reduction in directivity** is $\delta^2(1 - \delta^2/4)$, where δ is the phase error), HPBW (which can be estimated the directivity is known) and side lobes must also be considered in design.
- The priority is really down to the specific application.

2.3 ***Design Considerations and Procedures***

- **selection of the feed antenna, which determines** the **antenna polarization** and the **reflector *F* and 2*a***
- let the **feed pattern be about 10 dB down in the direction of the reflector rim** (The feed and reflector are interlinked; an iterative process may be required to ensure that the feed antenna pattern is well matched with the reflector)
- Once the feed pattern and the reflector are known, the radiated field can be calculated

Example 5.6: Edge taper and spillover efficiency. A circular parabolic reflector has $F/2a = 0.5$. The field pattern of the feed antenna is $E(\theta) = \cos\theta,\ \theta < \pi/2$. Find the edge taper, spillover efficiency and aperture efficiency. **For reference only**

$$\theta_0 = \tan^{-1}\left|\frac{a}{F - a^2/(4F)}\right| = \tan^{-1}\left|\frac{(F/2a)/2}{(F/2a)^2 - 1/16}\right| = \tan^{-1}\left|\frac{0.25}{(0.5)^2 - 1/16}\right| = 53.13^\circ$$

- aperture illumination function

$$A(\theta) = E(\theta)\frac{1}{r}e^{-j\beta r} = \cos(\theta)\frac{1}{r}e^{-j\beta r}$$

$$|A(\theta)| = \frac{\cos(\theta)(1+\cos\theta)}{2F}$$

- Edge taper: $$\frac{A(\theta_0)}{A(0)} = \frac{\cos(\theta_0)(1+\cos(\theta_0))}{(1+1)} = 0.4800 = -6.3752\,\text{dB}$$

Example 5.6: Edge taper and spillover efficiency. A circular parabolic reflector has $F/2a = 0.5$. The field pattern of the feed antenna is $E(\theta) = \cos\theta$, $\theta < \pi/2$. Find the edge taper, spillover efficiency and aperture efficiency. For reference only

$$\theta_0 = \tan^{-1}\left|\frac{a}{F - a^2/(4F)}\right| = \tan^{-1}\left|\frac{(F/2a)/2}{(F/2a)^2 - 1/16}\right| = \tan^{-1}\left|\frac{0.25}{(0.5)^2 - 1/16}\right| = 53.13^\circ$$

- aperture illumination function

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- Edge taper: $\dfrac{A(\theta_0)}{A(0)} = \dfrac{\cos(\theta_0)(1+\cos(\theta_0))}{(1+1)} = 0.4800 = -6.3752\,\text{dB}$

- Spillover efficiency: the spillover loss is about −1.08 dB from below:

$$\eta_s = \frac{P_{intercepted}}{P_{radiated}} = \frac{\int_0^{\theta_0} g(\theta)\sin\theta d\theta}{\int_0^{\pi} g(\theta)\sin\theta d\theta} = \frac{\int_0^{\theta_0} E(\theta)^2\sin\theta d\theta}{\int_0^{\pi} E(\theta)^2\sin\theta d\theta} = \frac{\int_0^{\theta_0} \cos^2(\theta)\sin\theta d\theta}{\int_0^{\pi} \cos^2(\theta)\sin\theta d\theta} \approx 0.78$$

Example 5.6: Edge taper and spillover efficiency. A circular parabolic reflector has $F/2a = 0.5$. The field pattern of the feed antenna is $E(\theta) = \cos\theta,\ \theta < \pi/2$. Find the edge taper, spillover efficiency and aperture efficiency. For reference only

- Aperture efficiency is about 0.59

$$\eta_{ap} = \cot^2\left(\frac{\theta_0}{2}\right)\left|\int_0^{\theta_0} g(\theta)\tan\left(\frac{\theta}{2}\right)\, d\theta\right|^2$$

- If a uniform aperture illumination is needed, we have:

$$\frac{A(\theta)}{A(0)} = \frac{E(\theta)(1+\cos(\theta))}{E(0)(1+1)} \equiv 1$$

- Feed pattern should be $E(\theta) = \dfrac{2}{1+\cos\theta}$

- a trough at the center (θ = 0), which is hard to achieve in practice

Other parabolic reflectors:

- Offset parabolic reflector