2.4 Phased Array, Planar Array

Module:2 Linear and Planar Arrays

Course: BECE305L – Antenna and Microwave Engineering

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CHENNAI

Module:2 Linear and Planar Arrays

- Two element array, N-element linear array - broadside array, End fire array - Directivity, radiation pattern, pattern multiplication. Non-uniform excitation - Binomial, Chebyshev distribution, Arrays: Planar array, circular array, Phased Array antenna (Qualitative study).

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

1. Phased array (scanning array)

- In 2.2 slides: how to direct the major radiation from an array, by controlling the phase excitation between the elements, in directions normal (broadside) and along the axis (end fire) of the array.
- maximum radiation can be oriented in any direction to form a scanning array
- maximum radiation of the array is required to be oriented at an angle $\theta_0 (0^\circ \leq \theta_0 \leq 180^\circ)$.
  phase excitation β between the elements must be adjusted

$$\psi = kd\cos\theta + \beta|_{\theta=\theta_0} = kd\cos\theta_0 + \beta = 0 \Rightarrow \beta = -kd\cos\theta_0$$

1. Phased array (scanning array)

- basic principle of electronic scanning phased array operation: .by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any desired direction to form a scanning array
- phased array technology the scanning must be continuous, the system should be capable of continuously varying the progressive phase between the elements
- by the use of
  ferrite phase shifter
  or
  diode phase shifters.

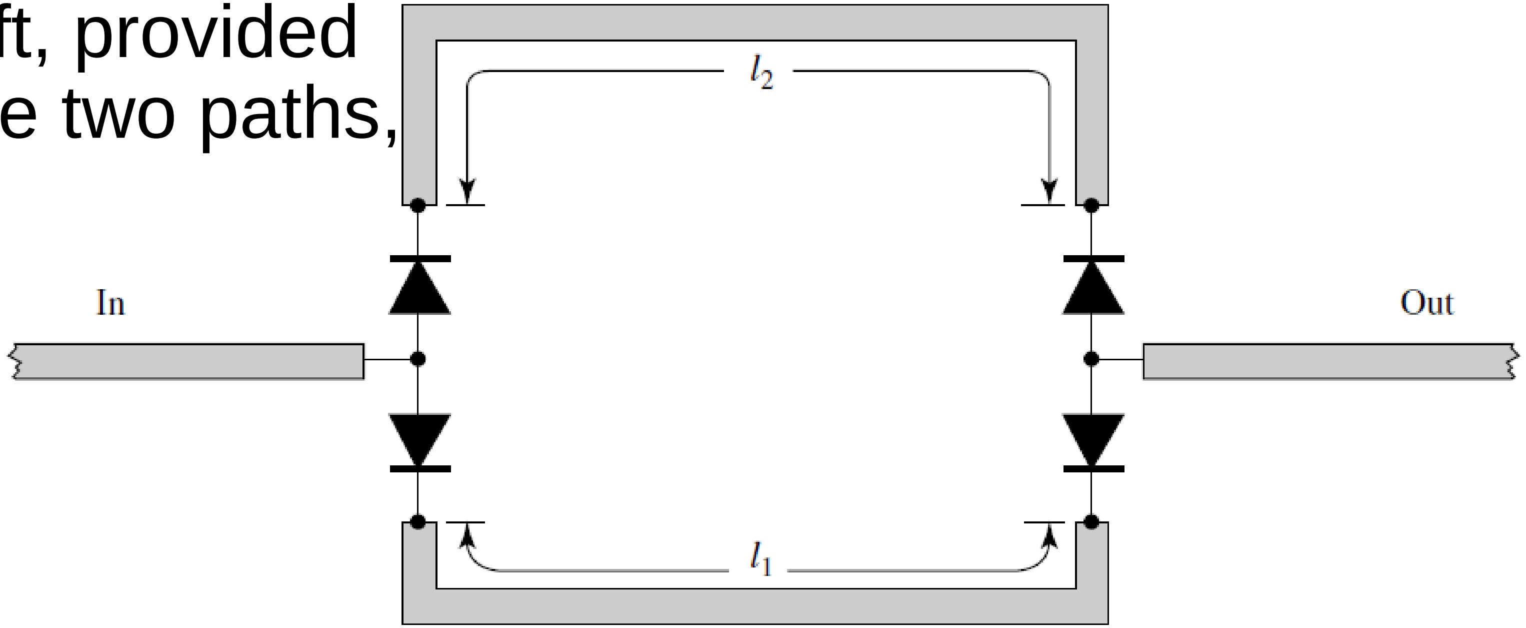
1. Phased array (scanning array)

- Ferrite phase shifter:
  the phase shift is controlled by the magnetic field within the ferrite, which in turn is controlled by the amount of current flowing through the wires wrapped around the phase shifter.
- Diode phase shifter:
  diode phase shifter using balanced, hybrid-coupled varactors, the actual phase shift is controlled
  either by varying the analog bias dc voltage (typically 0–30 volts) or by a digital command through a digital-to-analog (D/A) converter.

1. Phased array (scanning array)

- Diode phase shifter:Incremental switched-line phase shifter using PIN diodes (simple, straightforward, lightweight, and high speed.)
- The lines of lengths $l1$ and $l2$ are switched on and off by controlling the bias of the PIN diodes, using two single-pole double-throw switches
- The differential phase shift, provided by switching on and off the two paths,

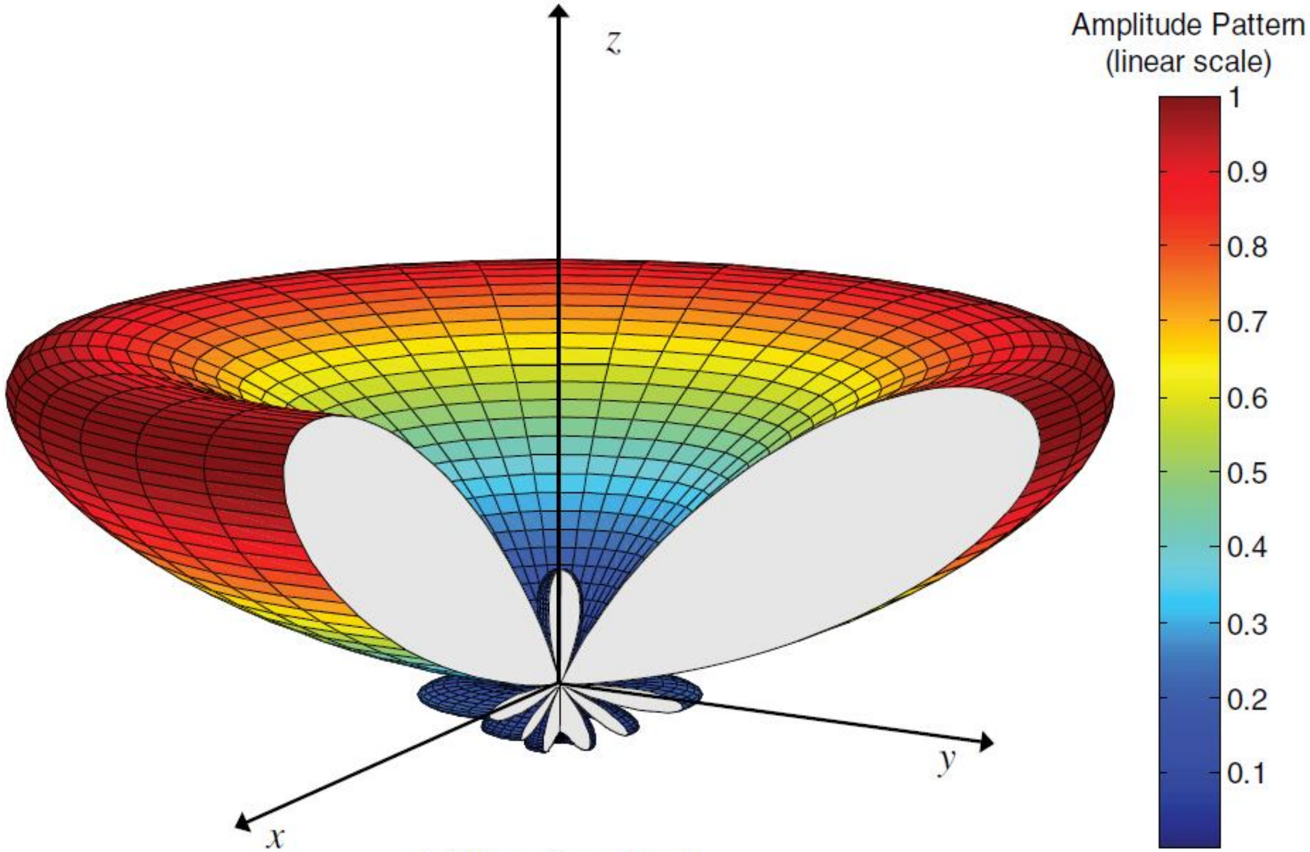
$$\Delta\phi = k(l_2 - l_1)$$



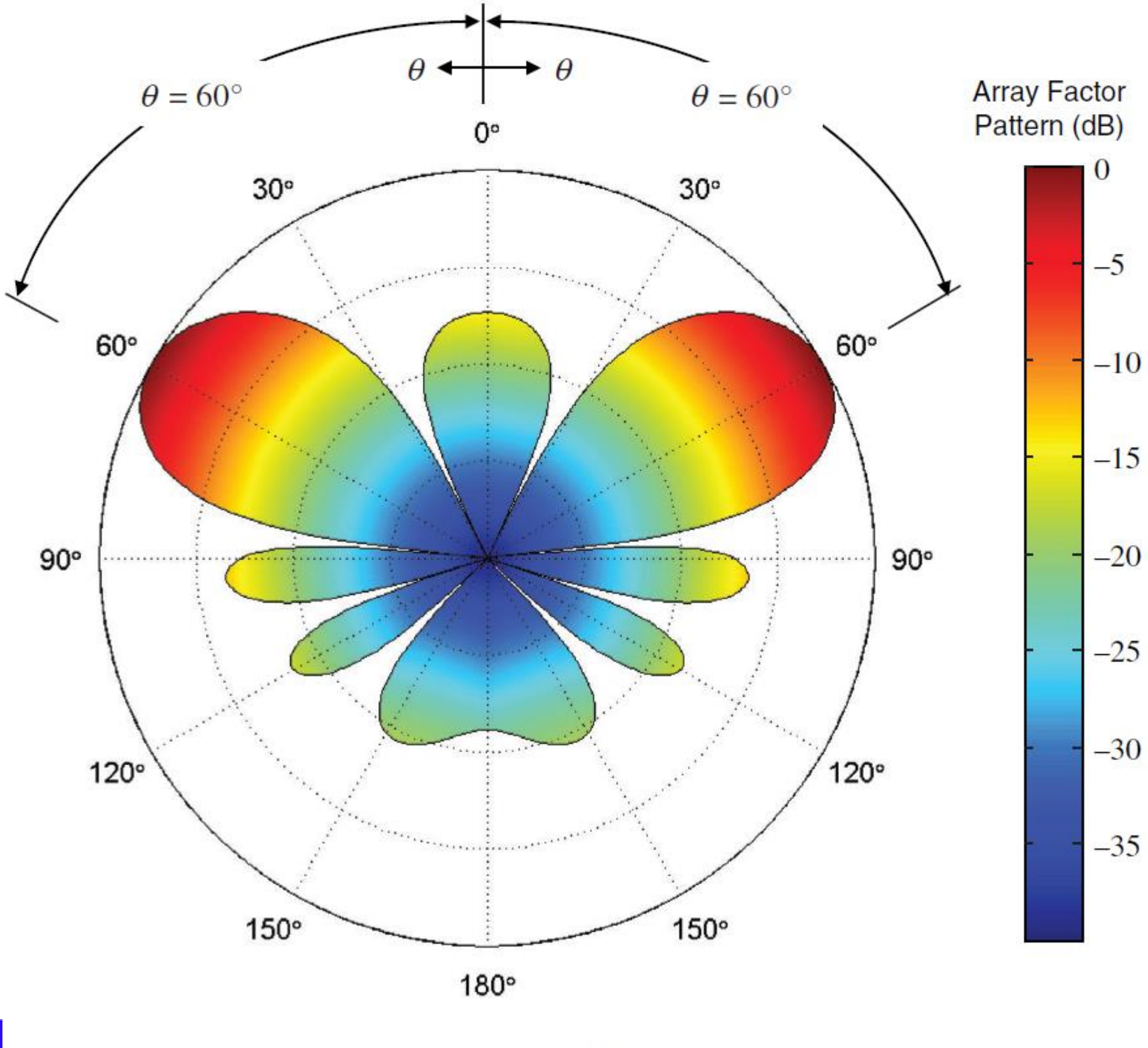
1. Phased array (scanning array)

- By properly choosing $l1$ and $l2$, and the operating frequency, the differential phase shift (in degrees) provided by each incremental line phase shifter can be as small as desired.
- it determines the resolution of the phase shifter.
- entire phase shifter typically utilizes several such incremental phase shifters to cover the entire range $(0 - 180^\circ)$ of phase
- The basic designs of a phase shifter utilizing PIN diodes are typically classified into three categories: *switched line, loaded line*, and *reflection type*.
- The *loaded-line* phase shifter can be used for phase shifts generally 45° or smaller.
- Phase shifters that utilize PIN diodes are not ideal switches since the PIN diodes usually possess finite series resistance and reactance that can contribute significant insertion loss if several of them are used

To demonstrate the principle of scanning, the three-dimensional radiation pattern of a 10-element array, with a separation of $\lambda/4$ between the elements and with the maximum squinted in the $\theta_0 = 60^\circ$ direction,



(a) Three-dimensional



(b) Two-dimensional

$$(N = 10, \beta = -kd\cos\theta_0, \theta_0 = 60^\circ, d = \lambda/4)$$

1. Phased array (scanning array)

- With $\beta = -kd\cos\theta_0$ in earlier obtained for N element array

$$\theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$$

Minus for one half, and plus for one half

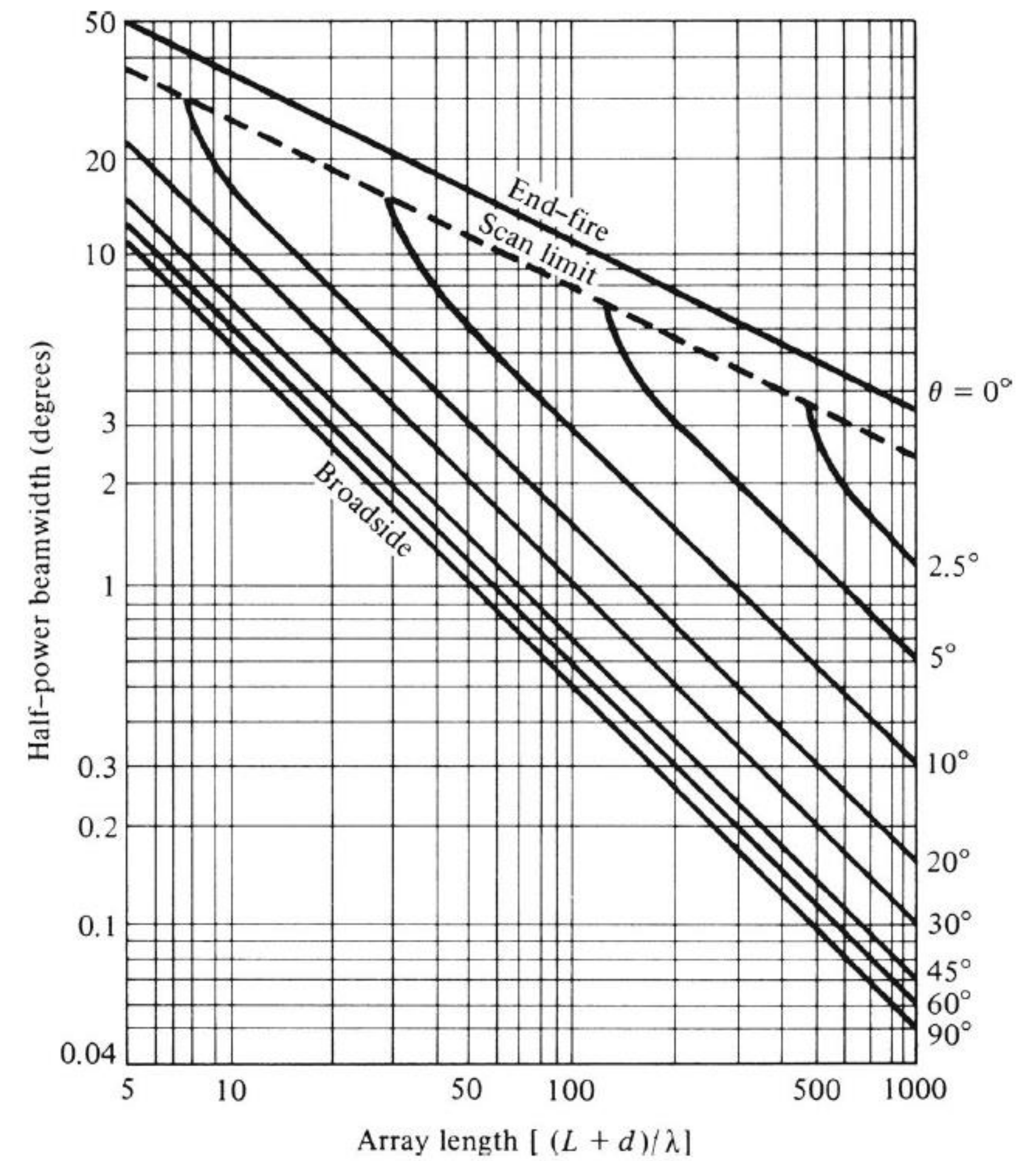
$$\Theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(kd\cos\theta_0 - \frac{2.782}{N}\right)\right] - \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(kd\cos\theta_0 + \frac{2.782}{N}\right)\right]$$

$$= \cos^{-1}\left(\cos\theta_0 - \frac{2.782}{Nkd}\right) - \cos^{-1}\left(\cos\theta_0 + \frac{2.782}{Nkd}\right)$$

- With $N = \frac{L+d}{d}$, the HPBW:

$$\Theta_h = \cos^{-1}\left[\cos\theta_0 - 0.443\frac{\lambda}{(L+d)}\right] - \cos^{-1}\left[\cos\theta_0 + 0.443\frac{\lambda}{(L+d)}\right]$$

Array length, HPBW

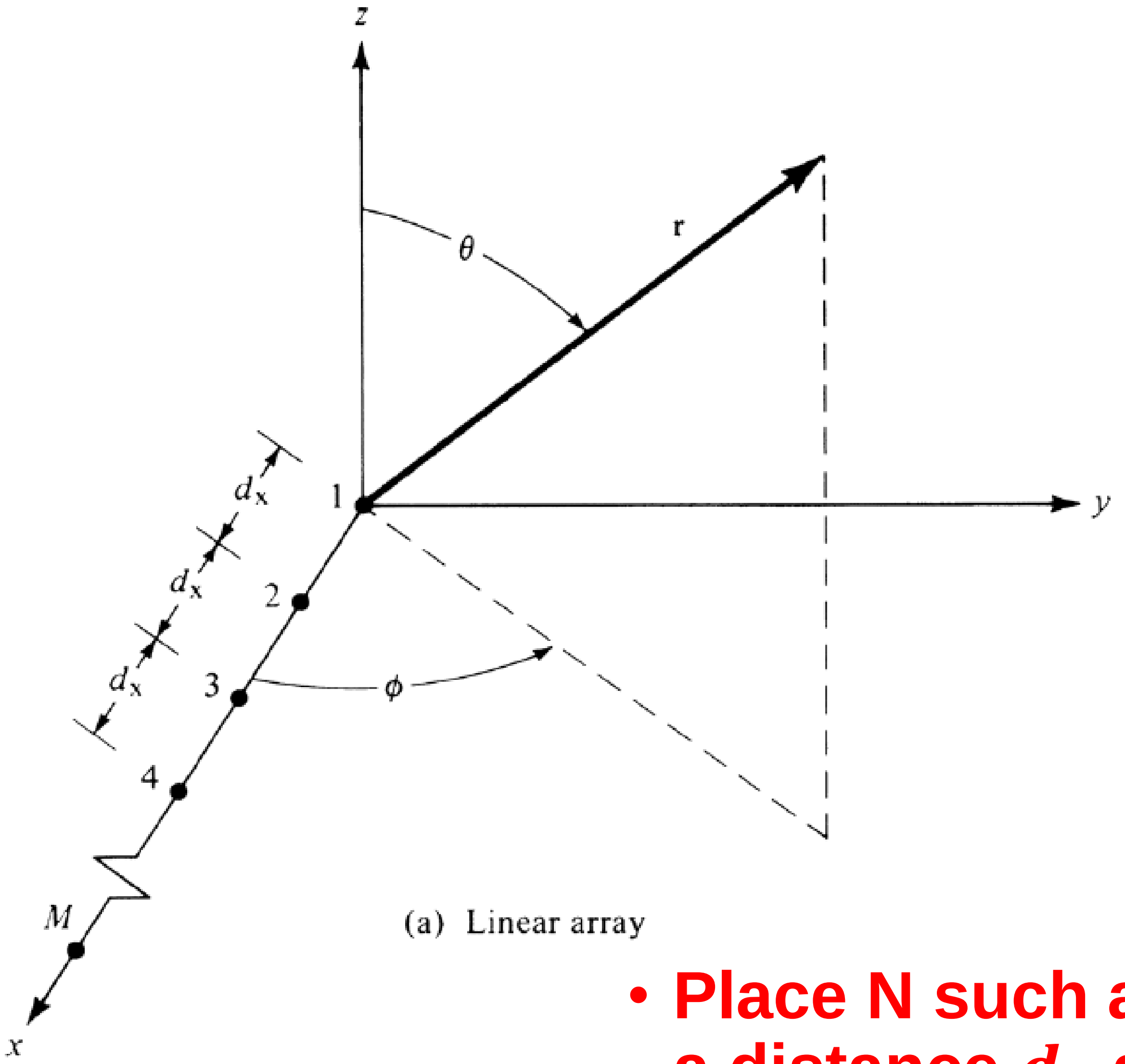


2. Planar Array:

- In addition to placing elements along a line (to form a linear array), individual radiators can be positioned along a rectangular grid to form a rectangular or planar array
- to control and shape the pattern of the array
- can provide more symmetrical patterns with lower side lobes.
- to scan the main beam of the antenna toward any point in space
- Applications: include tracking radar, search radar, remote sensing, communications, and many others (vehicular radar for autonomous vehicles)

- AWACS antenna array of waveguide slots.

2.1 Planar Array: Array Factor



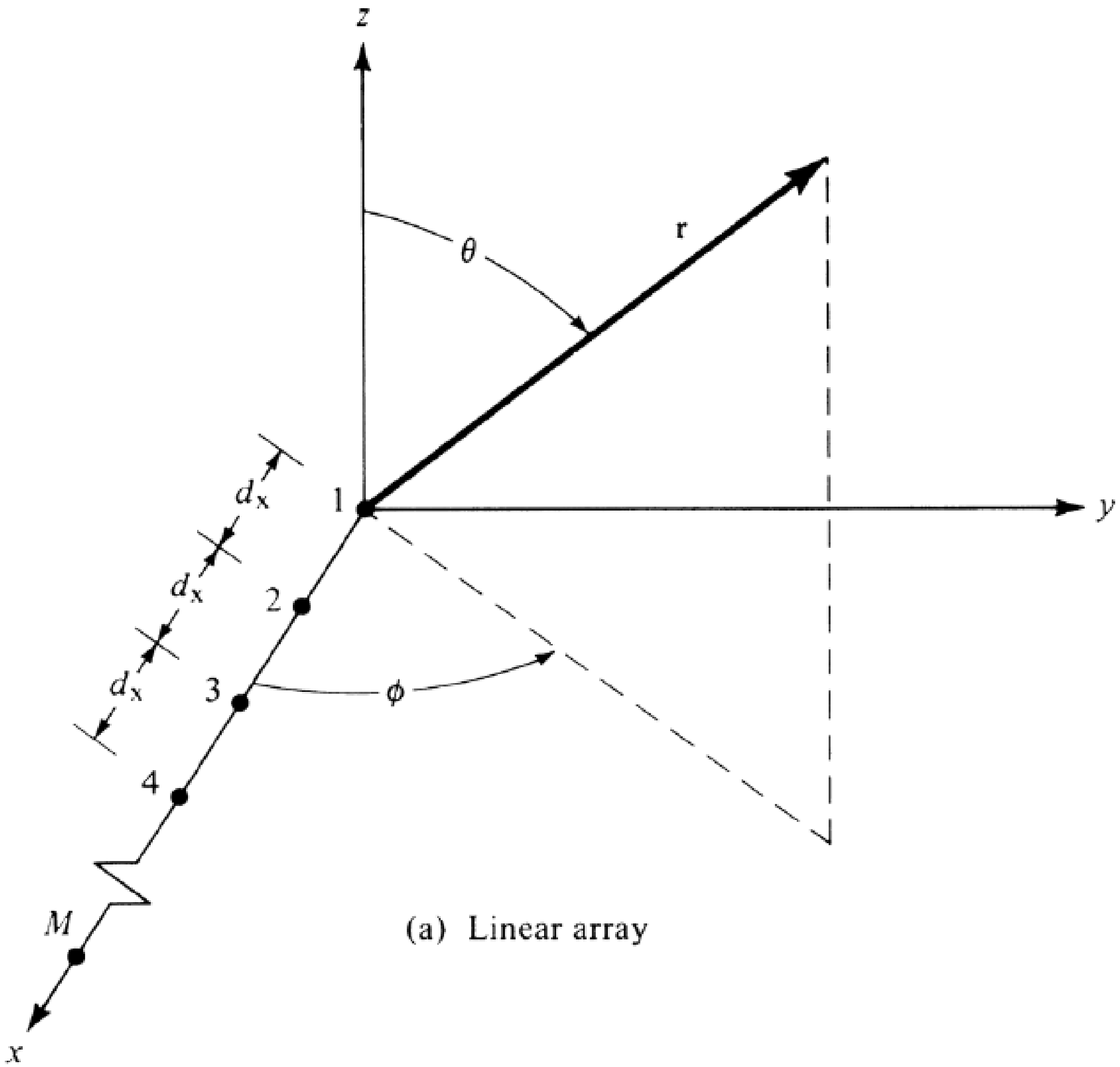
(a) Linear array

- If *M* elements are initially placed along the *x*-axis,

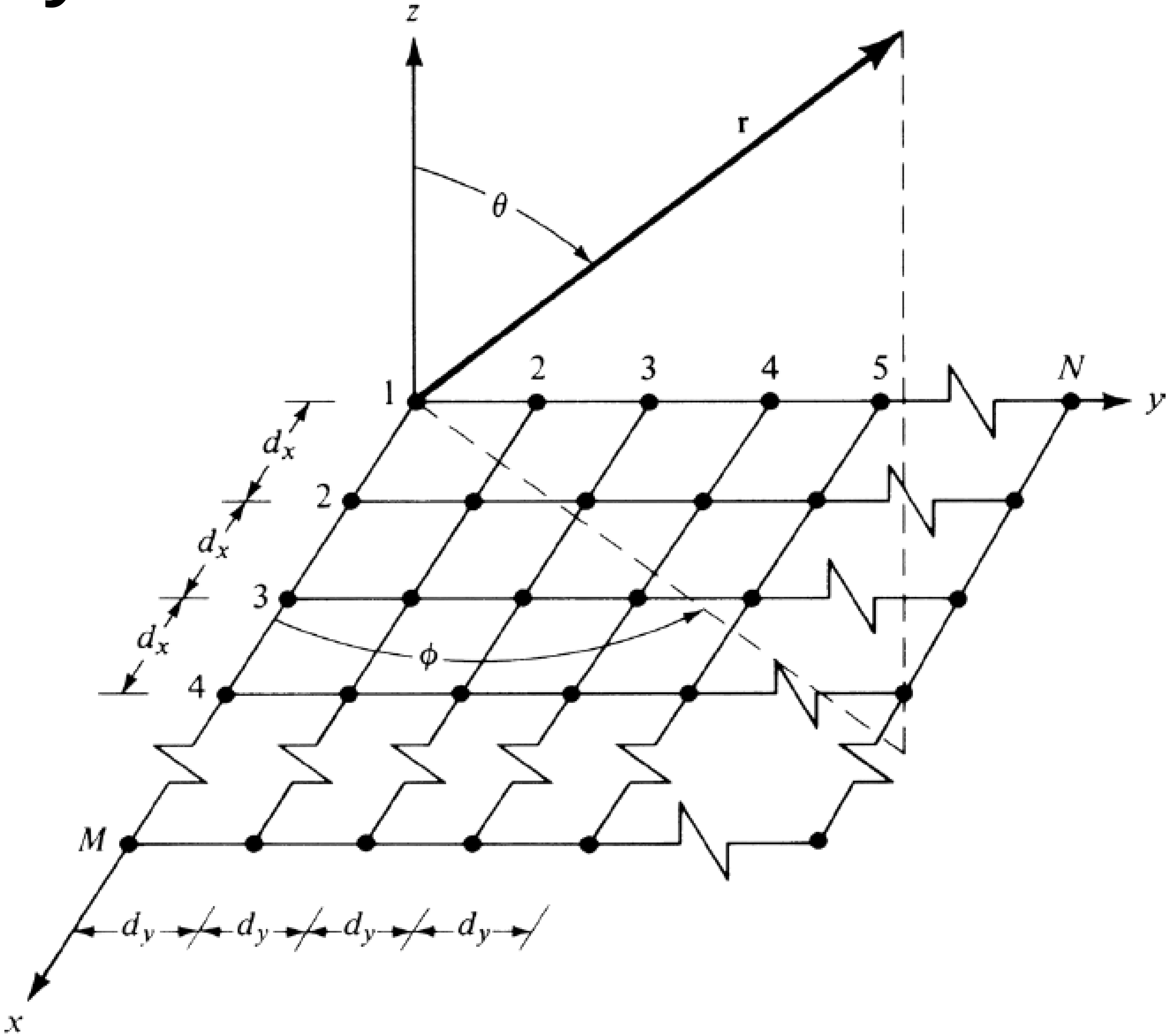
$$\mathrm{AF} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)}$$

- I_{m1}:excitation coefficient of each element
- The spacing and progressive phase shift between the elements along the *x*-axis are represented, respectively, by d_x and β_x.
- **Place N such arrays along Y axis next to each other , a distance d_y apart with progressive phase shift β_y**

2.1 Planar Array: Array Factor



(a) Linear array



(b) Planar array

2.1 Planar Array: Array Factor

- The array factor for the entire planar array

$$\text{AF} = \sum_{n=1}^{N} I_{1n} \left[\sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \right] e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

$$\text{AF} = S_{xm} S_{yn} \qquad S_{xm} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \qquad S_{yn} = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

- pattern of a rectangular array is the product of the array factors of the arrays in the *x*- and *y*-directions.
- If the amplitude excitation coefficients of the elements of the array in the *y*-direction are proportional to those along the *x*, $I_{mn} = I_m I_n = I_0$

$$\text{AF} = I_0 \sum_{m=1}^{M} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)} \sum_{n=1}^{N} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

2.1 Planar Array: Array Factor

- On simplification and normalization:

$$\mathrm{AF}_n(\theta, \phi) = \left\{ \frac{1}{M} \frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}$$

where

$$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$$

$$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$$

- To form or avoid grating lobes in a rectangular array, the same principles must be satisfied as for a linear array. *x*-*z* and *y*-*z* planes, the spacing between the elements in the *x*- and *y*-directions, respectively, must be less than $\lambda/2$ $(d_x < \lambda/2 \ and \ d_y < \lambda/2)$.

Array	Distribution	Type	Direction of Maximum	Element Spacing
Linear	Uniform	Broadside	$\theta_0 = 90°$ *only*	$d_{max} < \lambda$
			$\theta_0 = 0°, 90°, 180°$ *simultaneously*	$d = \lambda$
Linear	Uniform	Ordinary end-fire	$\theta_0 = 0°$ *only*	$d_{max} < \lambda/2$
			$\theta_0 = 180°$ *only*	$d_{max} < \lambda/2$
			$\theta_0 = 0°, 90°, 180°$ *simultaneously*	$d = \lambda$
Linear	Uniform	Hansen-Woodyard end-fire	$\theta_0 = 0°$ *only*	$d \simeq \lambda/4$
			$\theta_0 = 180°$ *only*	$d \simeq \lambda/4$
Linear	Uniform	Scanning	$\theta_0 = \theta_{max}$	$d_{max} < \lambda$
			$0 < \theta_0 < 180°$	
Linear	Nonuniform	Binomial	$\theta_0 = 90°$ *only*	$d_{max} < \lambda$
			$\theta_0 = 0°, 90°, 180°$ *simultaneously*	$d = \lambda$
Linear	Nonuniform	Dolph-Tschebyscheff	$\theta_0 = 90°$ *only*	$d_{max} \leq \frac{\lambda}{\pi} \cos^{-1}\left(-\frac{1}{z_o}\right)$
			$\theta_0 = 0°, 90°, 180°$ *simultaneously*	$d = \lambda$
Planar	Uniform	Planar	$\theta_0 = 0°$ *only*	$d_{max} < \lambda$
			$\theta_0 = 0°, 90°$ and $180°$; $\phi_0 = 0°, 90°, 180°, 270°$ *simultaneously*	$d = \lambda$

$$\mathrm{AF}_n(\theta,\phi) = \left\{\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

where

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$
$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$

2.1 Planar Array: Array Factor

- the major lobe and grating lobes of *Sxm* and *Syn* are located at

$$kd_x \sin\theta\cos\phi + \beta_x = \pm 2m\pi \qquad m = 0, 1, 2, \ldots$$
$$kd_y \sin\theta\sin\phi + \beta_y = \pm 2n\pi \qquad n = 0, 1, 2, \ldots$$

$$S_{xm} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}$$

$$S_{yn} = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)}$$

- If it is desired to have only one main beam that is directed along $\theta = \theta_0$ *and* $\phi = \phi_0$, the progressive phase shift between the elements in the *x*- and *y*-directions must be equal to

$$\beta_x = -kd_x \sin\theta_0\cos\phi_0$$
$$\beta_y = -kd_y \sin\theta_0\sin\phi_0$$

On solving gives

$$\tan\phi_0 = \frac{\beta_y d_x}{\beta_x d_y}$$

$$\sin^2\theta_0 = \left(\frac{\beta_x}{kd_x}\right)^2 + \left(\frac{\beta_y}{kd_y}\right)^2$$

- The principal maximum ($m = n = 0$) and the grating lobes can be located by

$$kd_x(\sin\theta\cos\phi - \sin\theta_0\cos\phi_0) = \pm 2m\pi, \quad m = 0, 1, 2, \ldots$$

$$kd_y(\sin\theta\sin\phi - \sin\theta_0\sin\phi_0) = \pm 2n\pi, \quad n = 0, 1, 2, \ldots$$

- On solving:

$$\sin\theta\cos\phi - \sin\theta_0\cos\phi_0 = \pm\frac{m\lambda}{d_x}, \quad m = 0, 1, 2, \ldots$$

$$\sin\theta\sin\phi - \sin\theta_0\sin\phi_0 = \pm\frac{n\lambda}{d_y}, \quad n = 0, 1, 2, \ldots$$

$$\phi = \tan^{-1}\left[\frac{\sin\theta_0\sin\phi_0 \pm n\lambda/d_y}{\sin\theta_0\cos\phi_0 \pm m\lambda/d_x}\right]$$

$$\mathrm{AF}_n(\theta,\phi) = \left\{\frac{1}{M}\frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)}\right\}\left\{\frac{1}{N}\frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)}\right\}$$

where

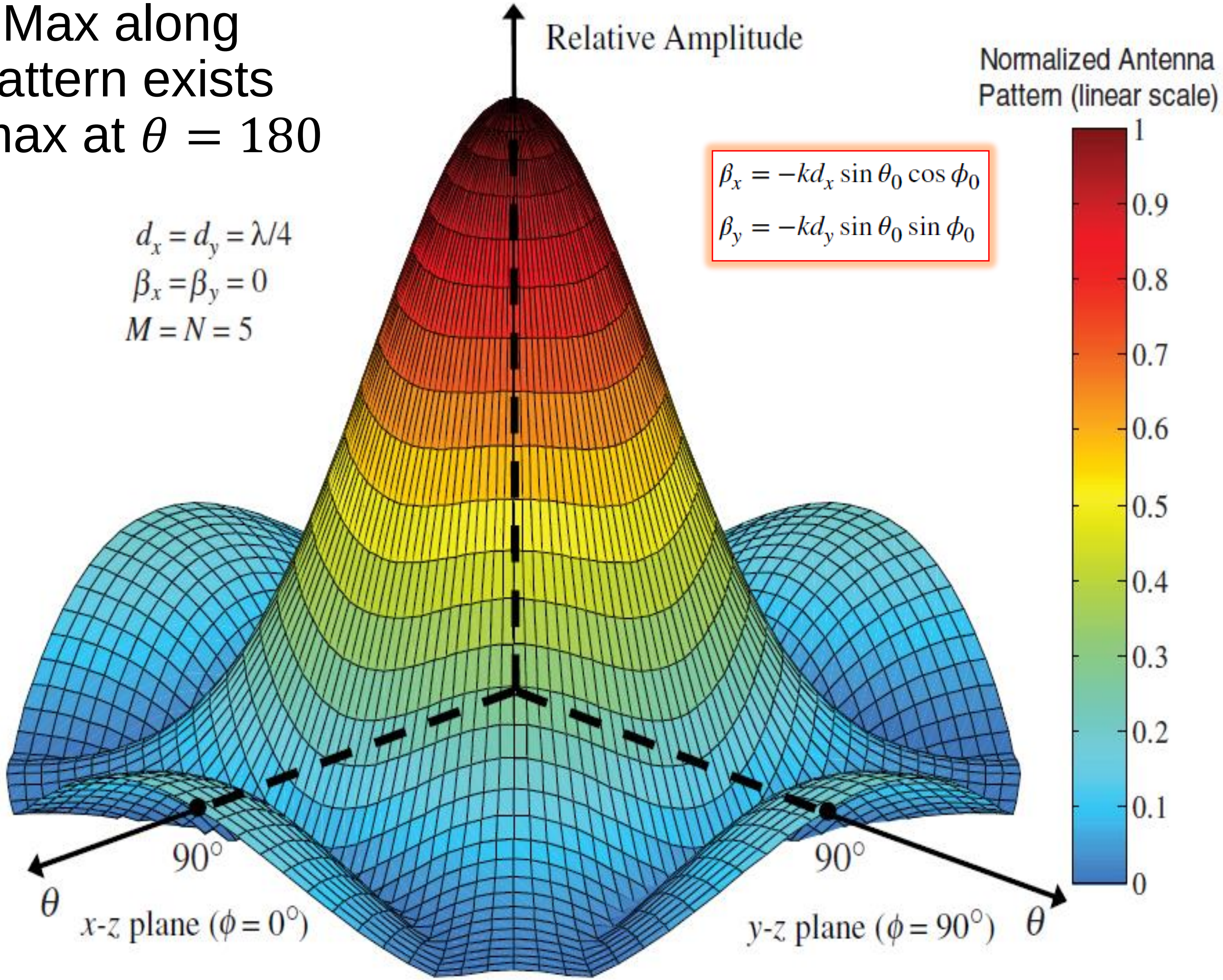
$$\psi_x = kd_x\sin\theta\cos\phi + \beta_x$$

$$\psi_y = kd_y\sin\theta\sin\phi + \beta_y$$

$$\theta = \sin^{-1}\left[\frac{\sin\theta_0\cos\phi_0 \pm m\lambda/d_x}{\cos\phi}\right] = \sin^{-1}\left[\frac{\sin\theta_0\sin\phi_0 \pm n\lambda/d_y}{\sin\phi}\right]$$

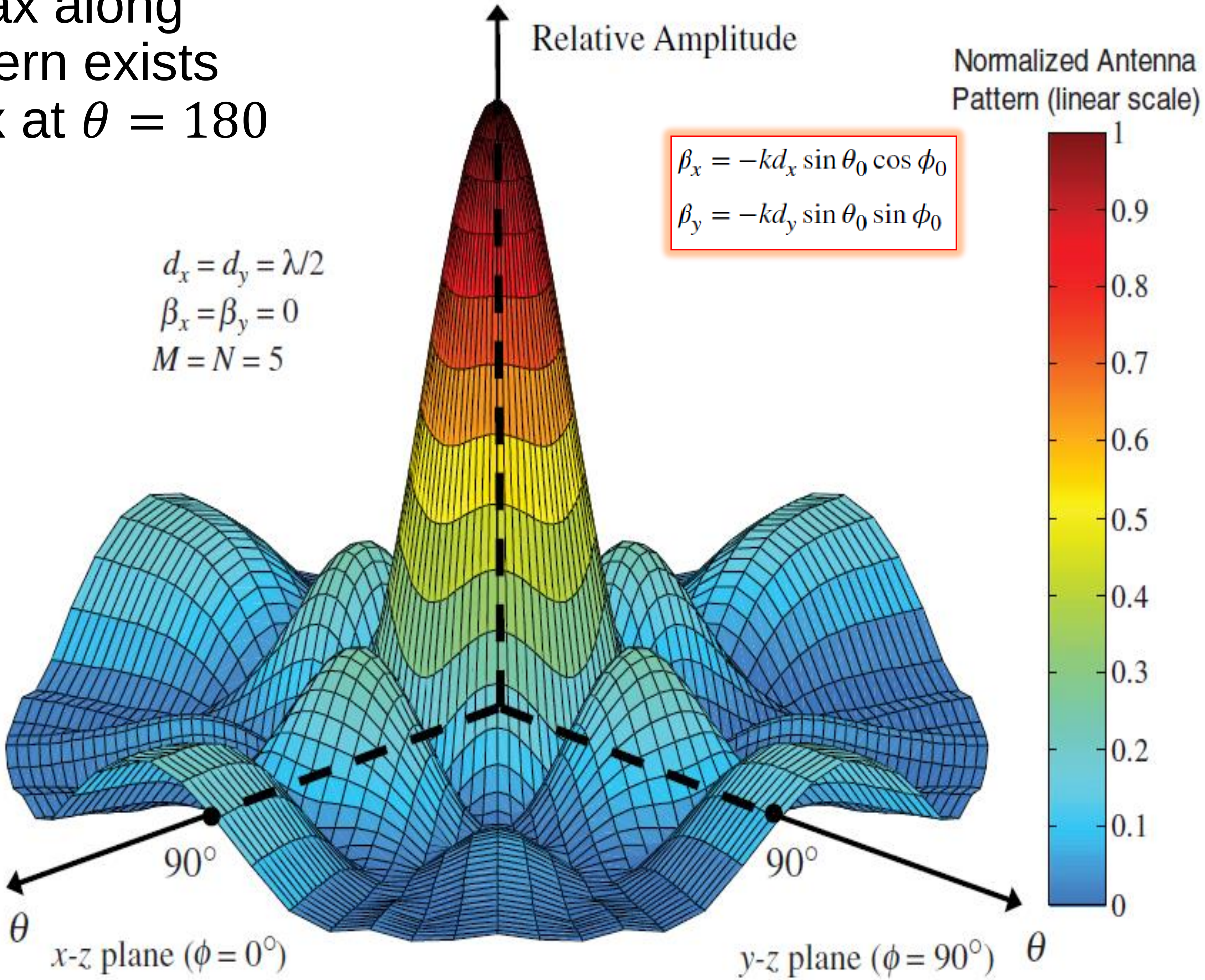
- One main beam, Max along $\theta = 0$ (Identical pattern exists below also with max at $\theta = 180$

- Three-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 4$, and equal amplitude and phase excitations



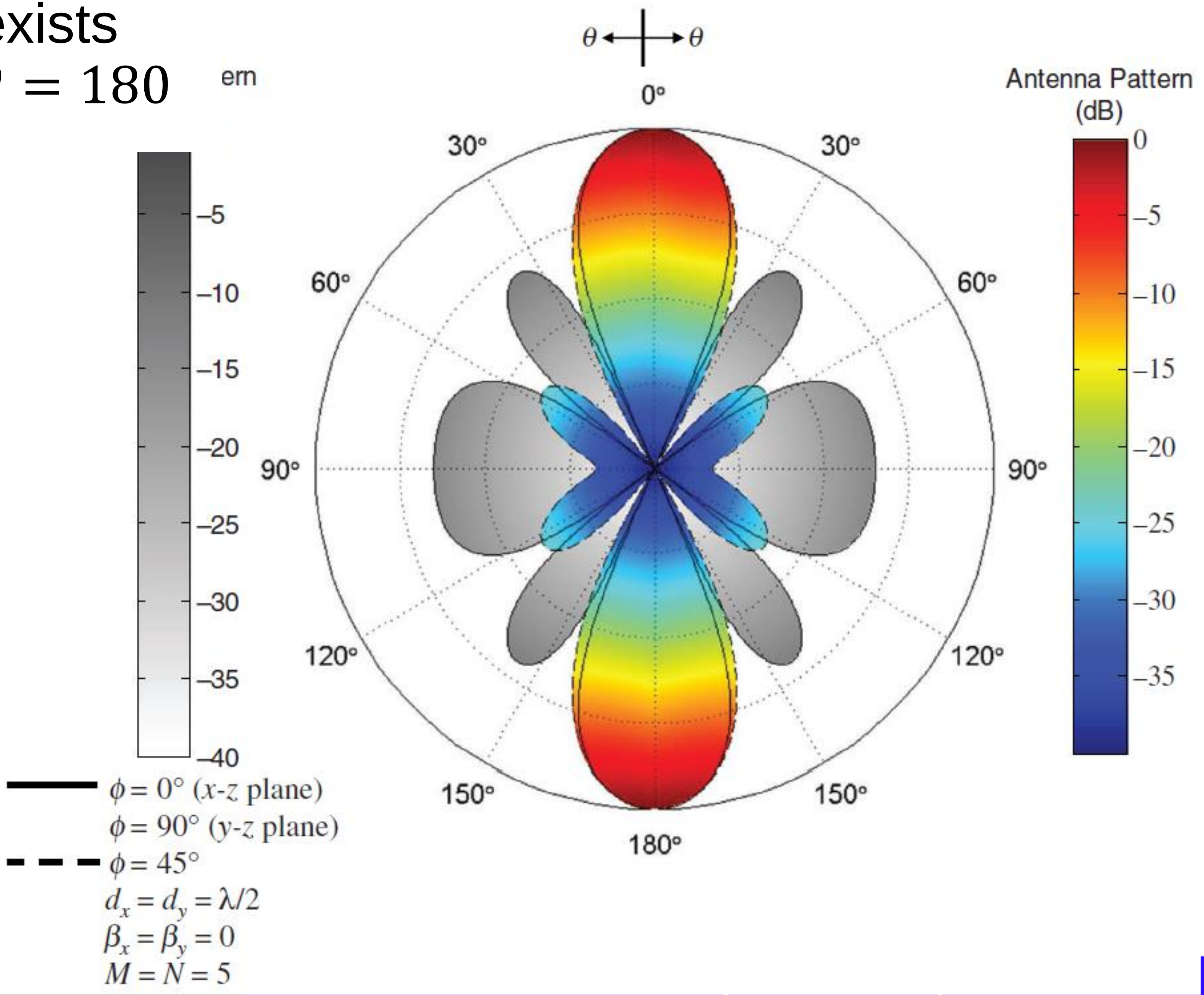
- One main beam, Max along $\theta = 0$ (Identical pattern exists below also with max at $\theta = 180$

- Three-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 2$, and equal amplitude and phase excitations



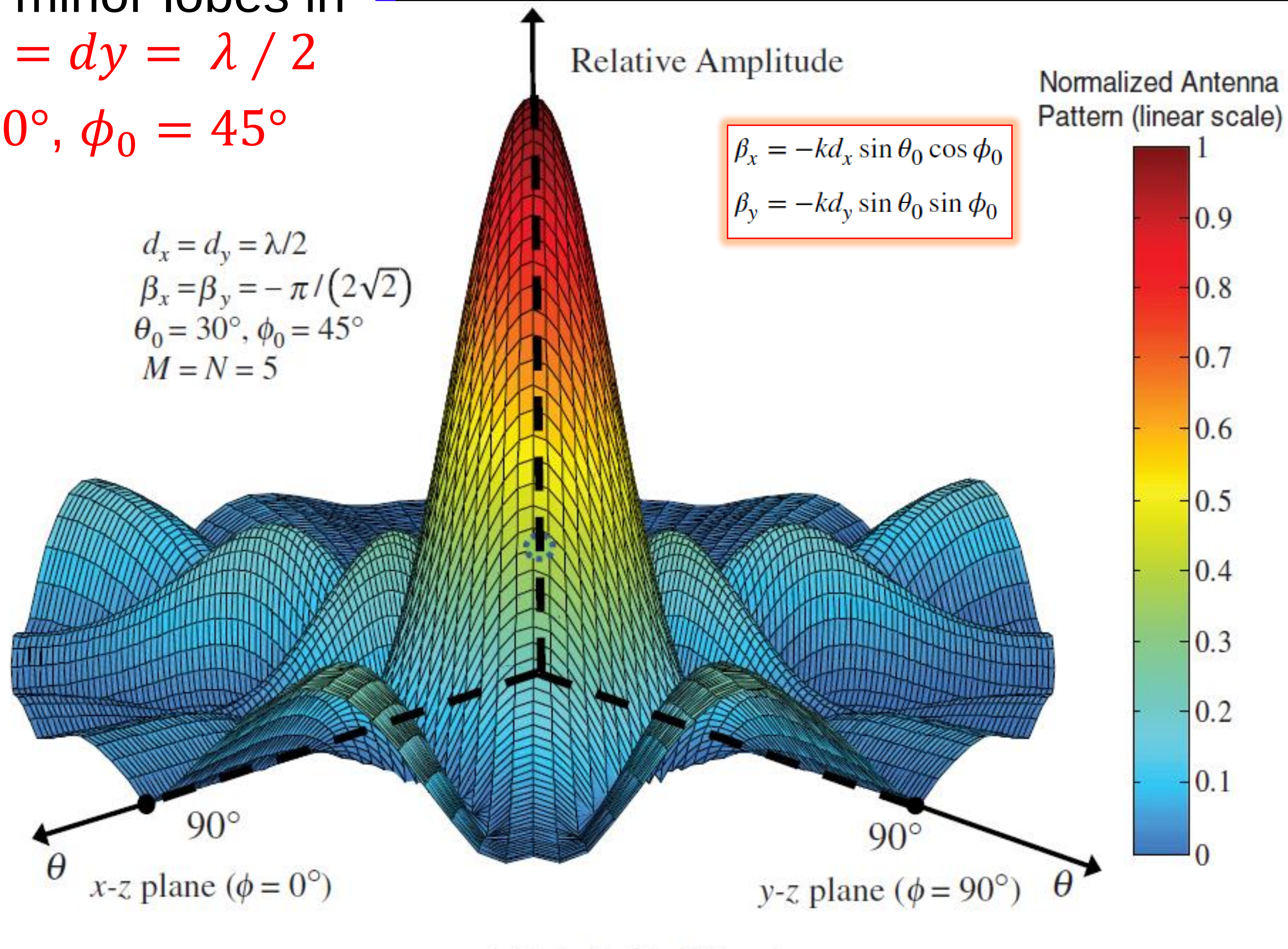
- One main beam, Max along $\theta = 0$ (Identical pattern exists below also with max at $\theta = 180$

- Two-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 2$, and equal amplitude and phase excitations



- exhibits complete minor lobes in all planes with $dx = dy = \lambda / 2$
- Max along $\theta_0 = 30°, \phi_0 = 45°$

- Three-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 2$, and equal amplitude and **Progressive phase excitations**

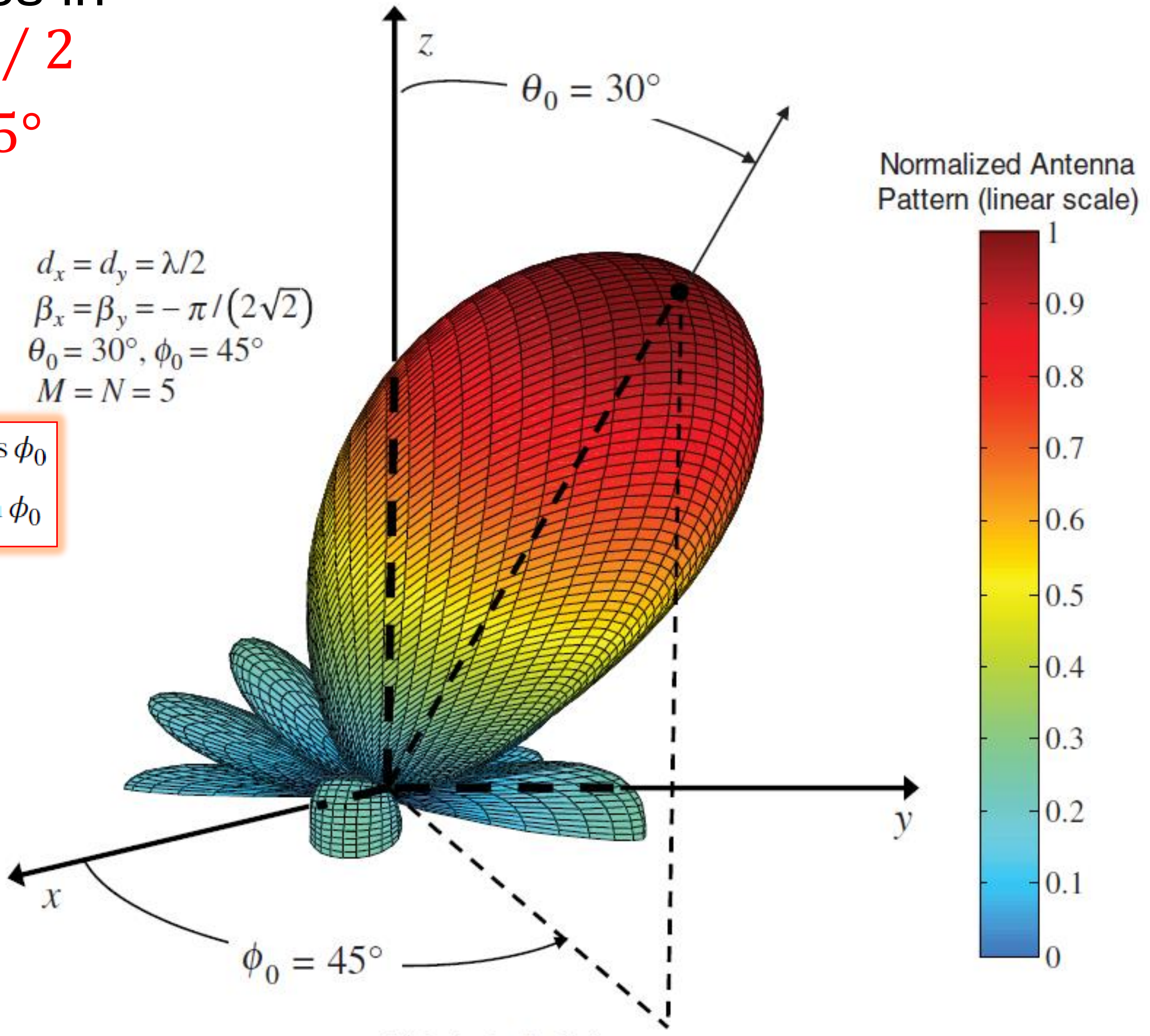


(a) in 'cylindrical' format

- exhibits complete minor lobes in all planes with $dx = dy = \lambda / 2$
- Max along $\theta_0 = 30°, \phi_0 = 45°$

- Three-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 2$, and equal amplitude and **Progressive phase excitations**

$$\boxed{\begin{aligned}\beta_x &= -kd_x \sin\theta_0 \cos\phi_0 \\ \beta_y &= -kd_y \sin\theta_0 \sin\phi_0\end{aligned}}$$

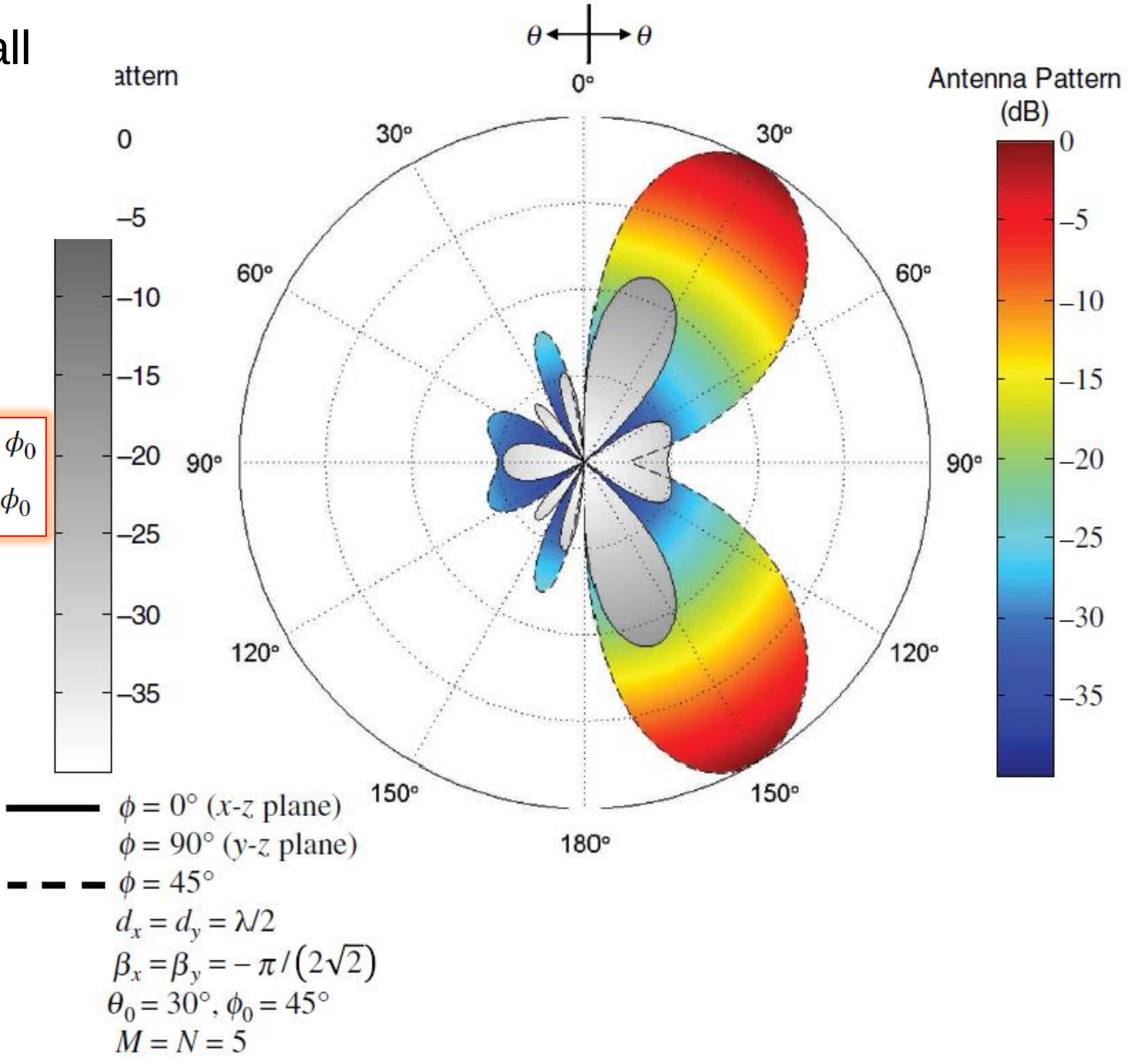


(b) in 'spherical' format

- exhibits complete minor lobes in all planes with $dx = dy = \lambda / 2$
- Max along $\theta_0 = 30°$, $\phi_0 = 45°$

- Two-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda / 2$, and equal amplitude and **Progressive phase excitations**

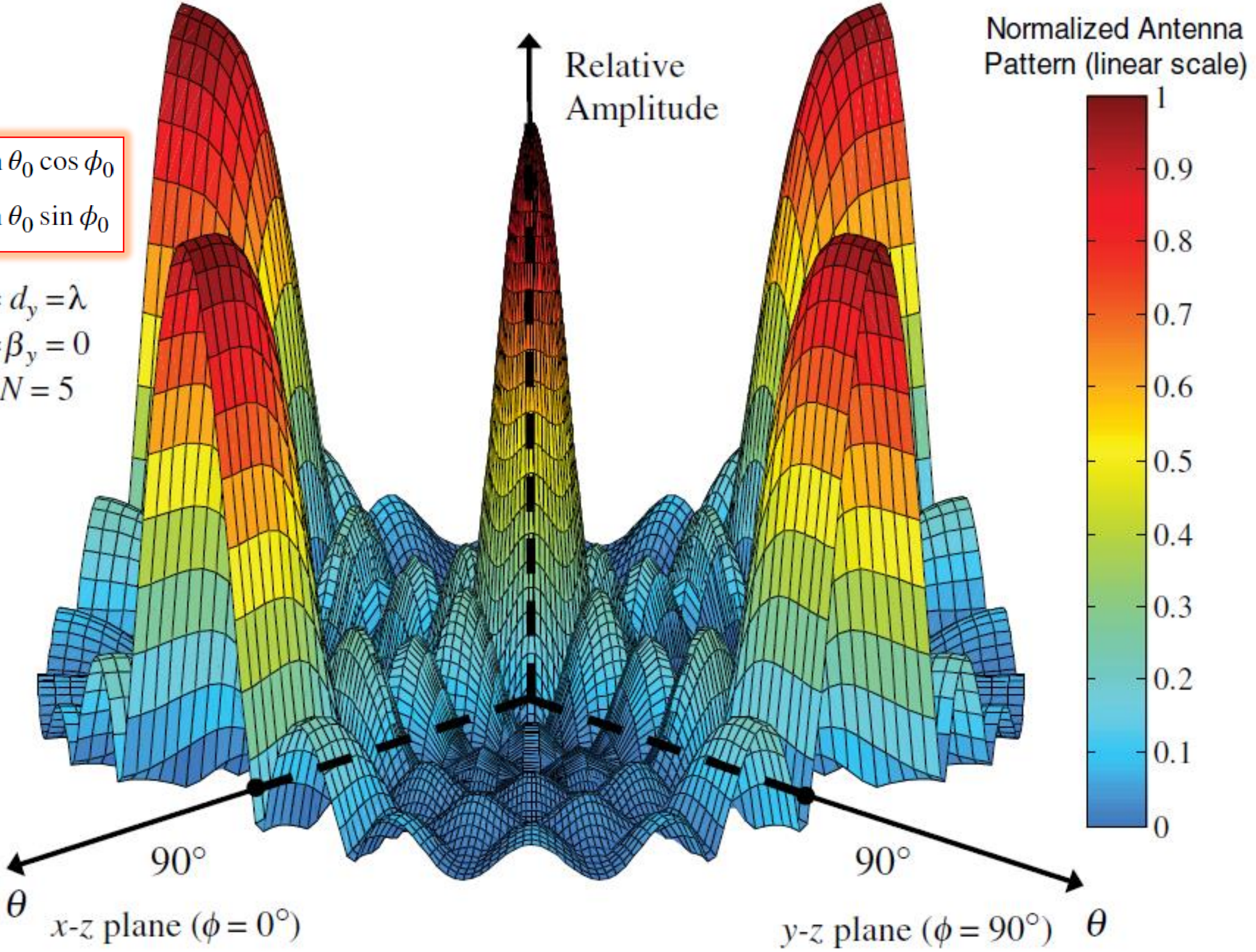
$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0$$
$$\beta_y = -kd_y \sin\theta_0 \sin\phi_0$$



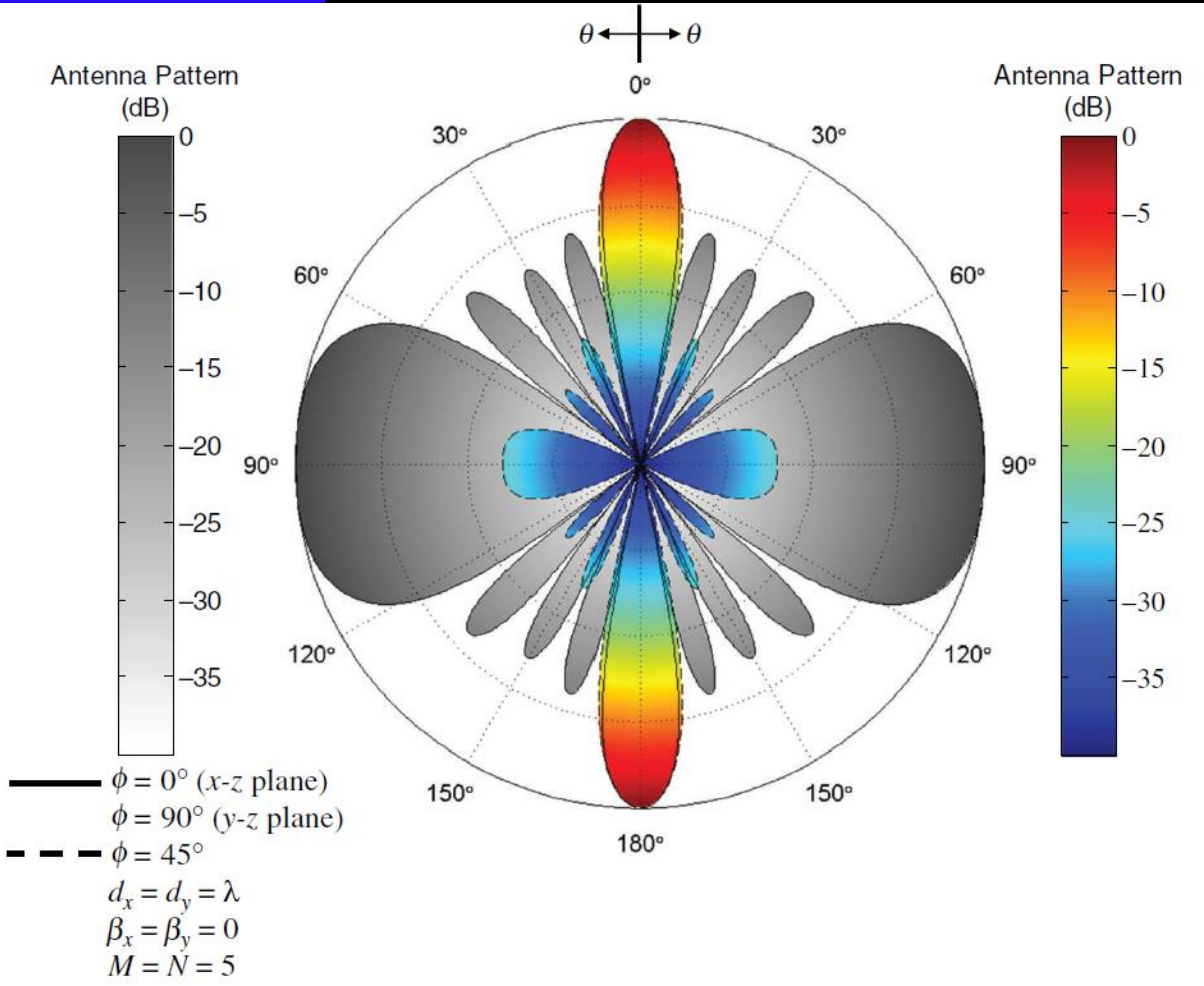
- Three-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda$, and equal amplitude and Equal phase excitations

$$\boxed{\begin{aligned}\beta_x &= -kd_x \sin\theta_0 \cos\phi_0 \\ \beta_y &= -kd_y \sin\theta_0 \sin\phi_0\end{aligned}}$$

$$d_x = d_y = \lambda$$
$$\beta_x = \beta_y = 0$$
$$M = N = 5$$



- Two-dimensional antenna pattern of a planar array of isotropic elements with a spacing of $dx = dy = \lambda$, and equal amplitude and Equal phase excitations

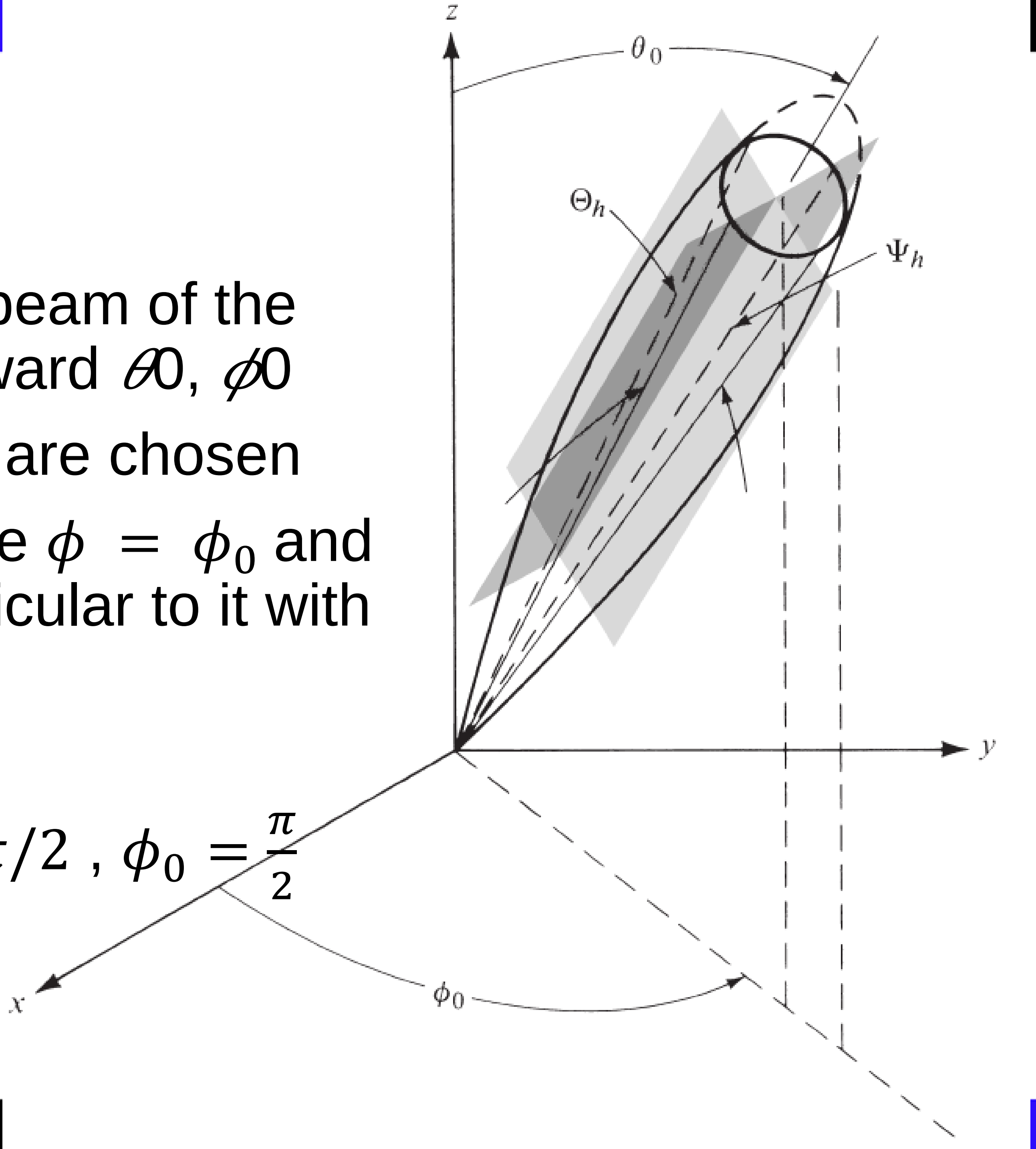


2.2 Planar Array: Beamwidth

- The maximum of the conical main beam of the array is assumed to be directed toward $\theta 0$, $\phi 0$
- To define a beamwidth, two planes are chosen
- elevation plane defined by the angle $\phi = \phi_0$ and the other is a plane that is perpendicular to it with θ_0 inclination from z axis
- Respective HPBWs are: Θ_h and Ψ_h
- Example: If maxima is along $\theta_0 = \pi/2$, $\phi_0 = \frac{\pi}{2}$

  Θ_h : Beam width along $\theta_0 = \frac{\pi}{2}$

  and Ψ_h: Beam width along $\phi_0 = \frac{\pi}{2}$



2.2 Planar Array: Beamwidth

- For large array: Θ_{x0} represents the half-power beamwidth of a *broadside* linear array of*M* elements. Similarly, Θ_{y0} represents the half-power beamwidth of a *broadside* array of *N* element

$$\Theta_h = \sqrt{\frac{1}{\cos^2\theta_0[\Theta_{x0}^{-2}\cos^2\phi_0 + \Theta_{y0}^{-2}\sin^2\phi_0]}}$$

$$\Psi_h = \sqrt{\frac{1}{\Theta_{x0}^{-2}\sin^2\phi_0 + \Theta_{y0}^{-2}\cos^2\phi_0}}$$

- For square array: $M = N, \Theta_{x0} = \Theta_{y0}$

$$\Theta_h = \Theta_{x0}\sec\theta_0 = \Theta_{y0}\sec\theta_0$$

$$\Psi_h = \Theta_{x0} = \Theta_{y0}$$

2.2 Planar Array: Beamwidth

- For a planar array

$$\Omega_A = \Theta_h \Psi_h$$

$$\Omega_A = \frac{\Theta_{x0}\Theta_{y0}\sec\theta_0}{\left[\sin^2\phi_0 + \frac{\Theta_{y0}^2}{\Theta_{x0}^2}\cos^2\phi_0\right]^{1/2}\left[\sin^2\phi_0 + \frac{\Theta_{x0}^2}{\Theta_{y0}^2}\cos^2\phi_0\right]^{1/2}}$$

2.3 Planar array: Directivity of Array factor

$$D_0 = \frac{4\pi[\mathrm{AF}(\theta_0, \phi_0)][\mathrm{AF}(\theta_0, \phi_0)]^*|_{\max}}{\int_0^{2\pi}\int_0^{\pi}[\mathrm{AF}(\theta, \phi)][\mathrm{AF}(\theta, \phi)]^* \sin\theta \, d\theta \, d\phi}$$

- the directivity of an array with bidirectional characteristics (two-sided pattern in free space) would be half the directivity of the same array with unidirectional (one-sided pattern) elements
- For large planar arrays, which are nearly broadside, the directivity reduces

$$D_0 = \pi \cos\theta_0 D_x D_y$$

where *Dx* and *Dy* are the directivities of broadside linear arrays each, respectively, of length and number of elements *Lx*, *M* and *Ly*, *N*. $\cos\theta_0$ accounts for the decrease of the directivity because of the decrease of the projected area of the array

2.3 Planar array: Directivity of Array factor

- For most practical amplitude distributions

$$D_0 \simeq \frac{\pi^2}{\Omega_A(\text{rads}^2)} = \frac{32{,}400}{\Omega_A(\text{degrees}^2)}$$

- ΩA is expressed in square radians or square degrees

Compute the half-power beamwidths, beam solid angle, and directivity of a planar square array of 100 isotropic elements (10 × 10). Assume a Tschebyscheff distribution, $\lambda/2$ spacing between the elements, -26 dB side lobe level, and the maximum oriented along $\theta_0 = 30°$, $\phi_0 = 45°$.

$$L_x + d_x = L_y + d_y = 5\lambda$$

Broadside array beam width:

$$\Theta_h = \cos^{-1}\left[\cos\theta_0 - 0.443\frac{\lambda}{(L+d)}\right] - \cos^{-1}\left[\cos\theta_0 + 0.443\frac{\lambda}{(L+d)}\right]$$

$$\Theta_h = 10.17°$$

$$\Theta_{x0} = \Theta_{y0} = 10.97°$$

$$\Theta_h = \Theta_{x0}\sec\theta_0 = 10.97° \sec(30°) = 12.67°$$

$$\Psi_h = \Theta_{x0} = 10.97°$$

$$\Omega_A = \Theta_h \Psi_h = 12.67(10.97) = 138.96\ (\text{degrees}^2)$$

Compute the half-power beamwidths, beam solid angle, and directivity of a planar square array of 100 isotropic elements (10×10). Assume a Tschebyscheff distribution, $\lambda/2$ spacing between the elements, -26 dB side lobe level, and the maximum oriented along $\theta_0 = 30°$, $\phi_0 = 45°$.

$$D_0 = \pi \cos\theta_0 D_x D_y$$

The directivity can be obtained using (6-103). Since the array is square, $D_x = D_y$, each one is equal to the directivity of Example 6.10. Thus

$$D_0 = \pi\cos(30°)(9.18)(9.18) = 229.28 \text{ (dimensionless)} = 23.60 \text{ dB}$$

$$D_0 \simeq \frac{\pi^2}{\Omega_A(\text{rads}^2)} = \frac{32{,}400}{\Omega_A(\text{degrees}^2)}$$

$$D_0 \simeq \frac{32{,}400}{\Omega_A(\text{degrees}^2)} = \frac{32{,}400}{138.96} = 233.16 \text{ (dimensionless)} = 23.67 \text{ dB}$$