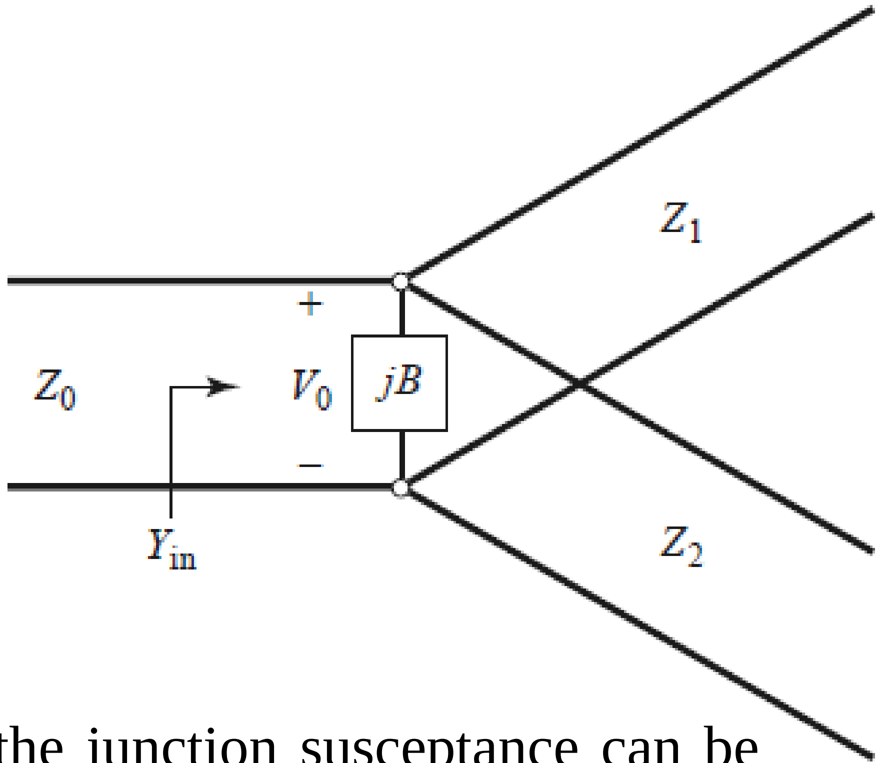
Module 5 - Microwave Passive Networks

T junction and resistive power divider, Wilkinson power divider, branch line coupler (equal & unequal), Rat Race Coupler, Filter design: Low pass filter (Butterworth and Chebyshev) - Richards transformation and stepped impedance method

Lossless T Junction power Divider

- The lossless T – Junction can be modelled as three transmission lines joined together
- At the junction of the three lines, there will be fringing fields, leading to stored energy represented as shunt susceptance



- The input admittance must equal the characteristic admittance

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = Y_0 = \frac{1}{Z_0}$$

- If we use suitable reactive matching, the junction susceptance can be neutralized giving

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

- We can have various power division ratios by selecting the corresponding branch impedances Z_1 and Z_2

Problem

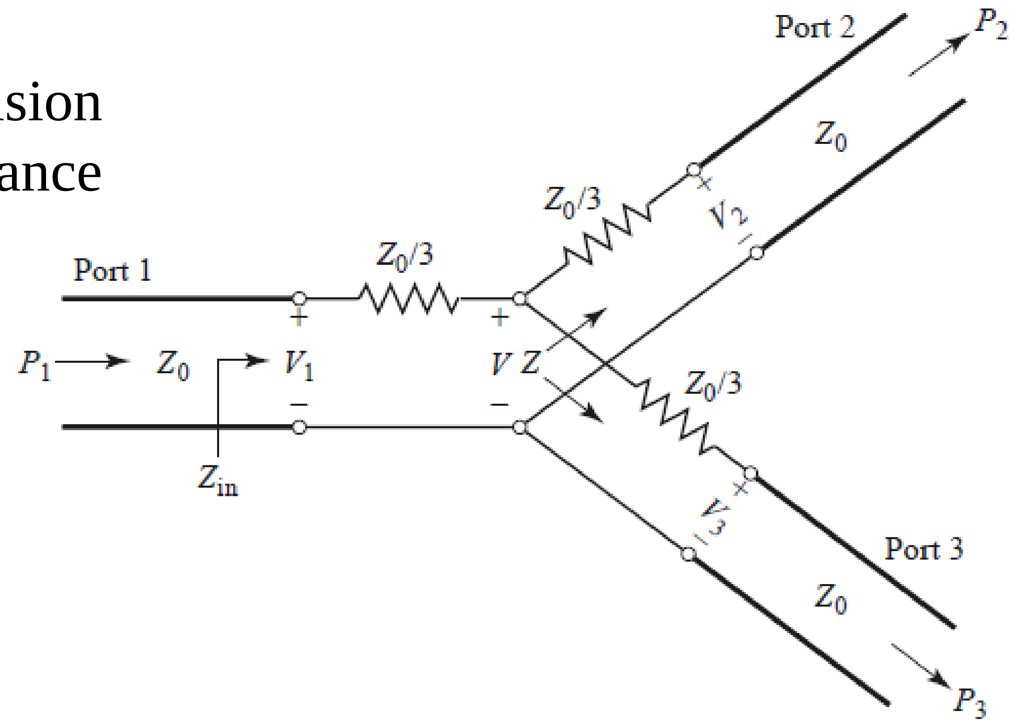
1. A lossless T junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the input power is divided in a 2:1 ratio. Compute the reflection coefficient seen looking into the output ports.

Note: power division of ratio 1:2 means the power at port 2 is twice as much as power at port 3

Resistive Power Divider

- For a lossless T – Junction there is no isolation between the output ports and a mismatch of impedance is seen looking into output ports

➢ Resistive Power Divider (Alternative) : If we consider the network to be matched at all ports and reciprocal, then we necessarily have a lossy network



- For an equal power division resistive divider, the impedance looking into output port is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4}{3} Z_0$$

- And the input impedance

$$Z_{in} = \frac{Z_0}{3} + \frac{2}{3} Z_0 = Z_0$$

Resistive Power Divider

- Since the resistive power divider is symmetric, the impedances of all branches will be equal to the characteristic impedance. Therefore, $S_{11} = S_{22} = S_{33} = 0$

- If V_1 is the voltage at the port 1, then by voltage division, the voltage V at the centre of the divider is

$$V = \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} V_1 = \frac{2}{3} V_1$$

- And the voltages across the output ports is obtained just as above

$$V_2 = V_3 = \frac{Z_0}{Z_0 + Z_0/3} V = \frac{3}{4} V = \frac{1}{2} V_1$$

- And the power across the output ports in terms of input powers is

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0}, P_2 = \frac{1}{2}\frac{V_2^2}{Z_0} = \frac{1}{2}\frac{(V_1^2/4)}{Z_0}, P_3 = \frac{1}{2}\frac{V_3^2}{Z_0} = \frac{1}{2}\frac{(V_1^2/4)}{Z_0}$$

Resistive Power Divider

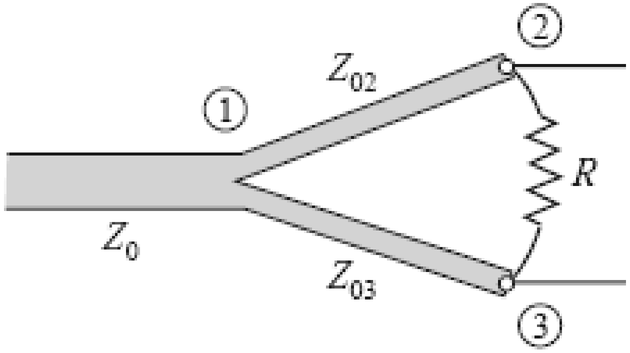
- Which means to show that the output powers are one fourth of the input power, or 6 dB below the input level
- We have seen in a lossless T – junction power divider of equal power division, that the outputs are 3 dB below the input level. Hence, we can clearly note the losses in a resistive power divider
- The S – matrix of a resistive power divider can be written as

$$S = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- It has to be noted that, just as the lossless T – junction power divider, the resistive power divider has no isolation between the output ports

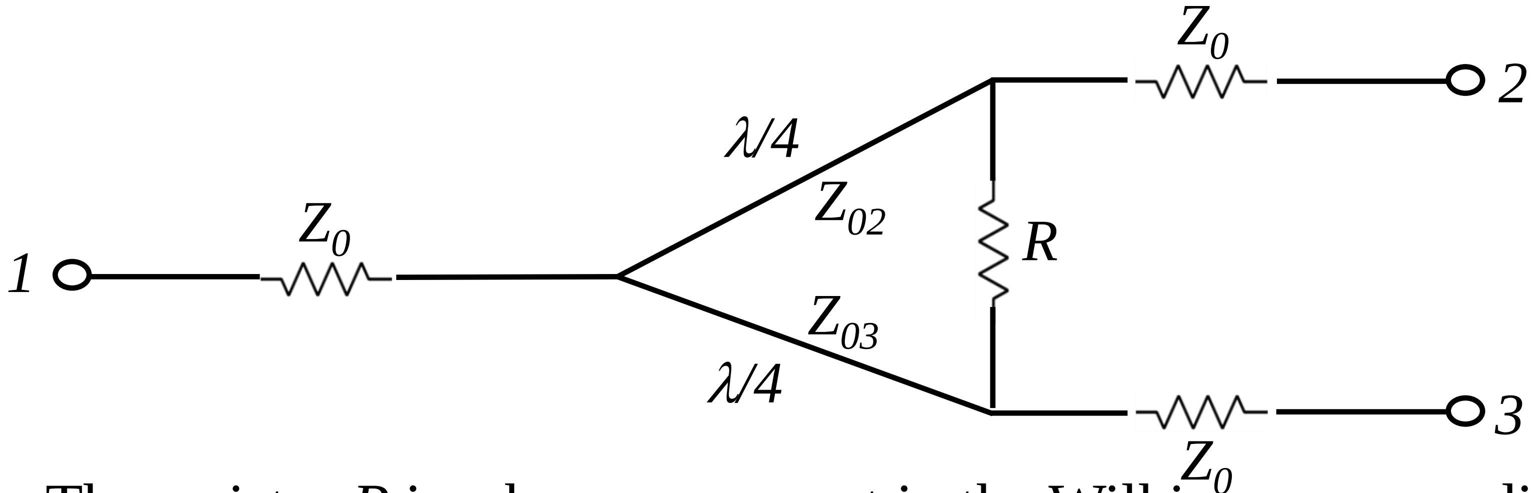
Wilkinson Power Divider

- T-Junction (lossless) and resistive power dividers have no isolation between the output ports (Ports 2 & 3),
- The above limitation Wilkinson had designed a three port network that not only matches all the ports and is reciprocal, but also induces much needed isolation between output ports
- Consider the case of equal power division, which means that the power fed to port 1 will divide equally, half of which will go to port 2 and remaining half to port 3



Wilkinson Power Divider

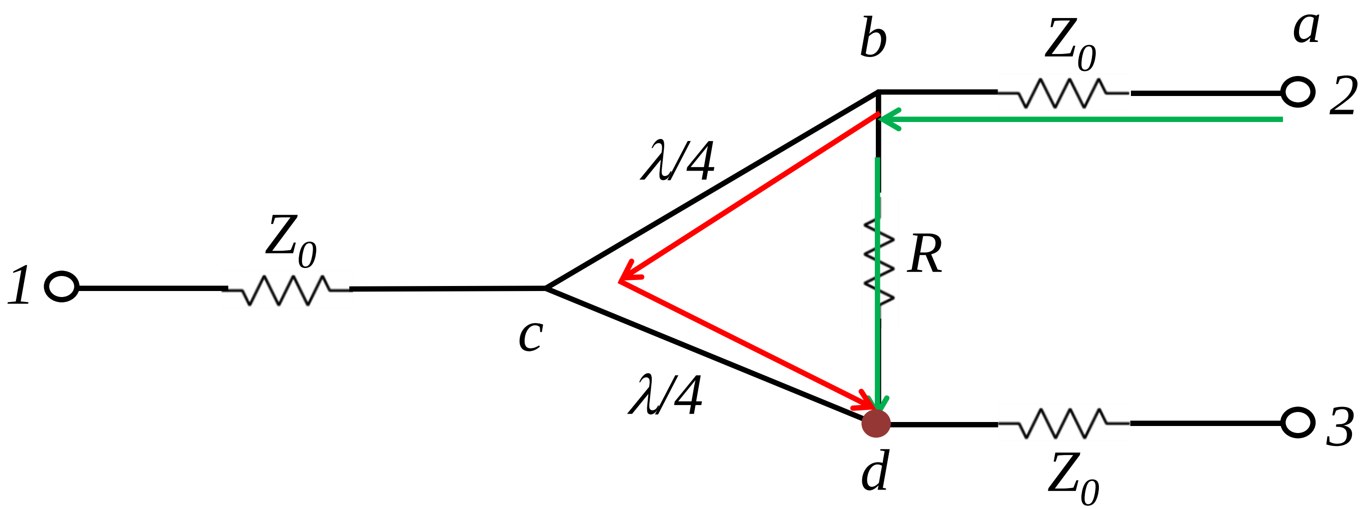
- ports impedance is Z_0 and port 1 is connected to ports 2 & 3 with two quarter wave transformers of impedances Z_{02} and Z_{03} respectively.
- By choosing appropriate values for Z_{02} and Z_{03}, various power division ratios can be achieved



- The resistor R is a key component in the Wilkinson power divider. It acts as a matching element for arms with impedances Z_{02}, Z_{03} with their corresponding port impedances. Also it helps in creating isolation between ports 2 & 3 along with the quarter wave sections

Wilkinson Power Divider

- But how can we visualize isolation in Wilkinson power divider ?



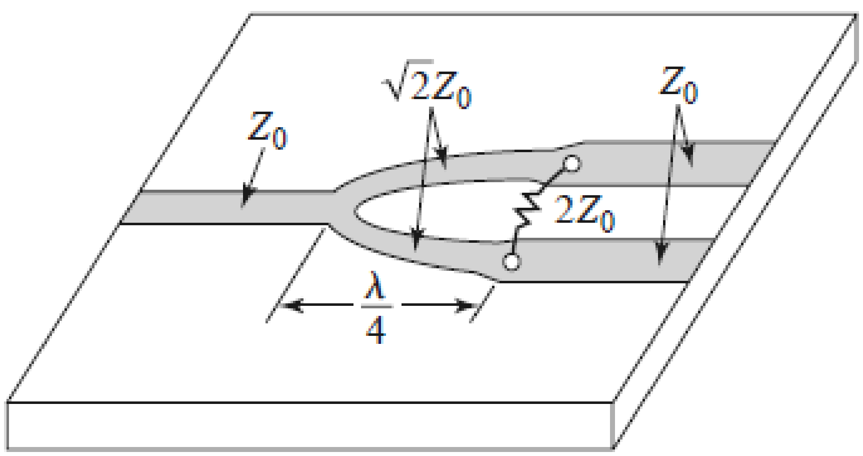
- Let us imagine a signal coming from port 2. This will reach point '*d*' without any phase delay by taking the path *abd*
- Another part of the same signal from port 2 will travel the path *abcd* to reach point '*d*'. In this process, it crosses two $\lambda/4$ lines which means a total length of $\lambda/2$ ($\lambda/4+\lambda/4$). This corresponds to a phase lag of 180^0
- Signal taking path *abd* and signal taking path *abcd*, meet at point '*d*' with a phase difference of 180^0 and cancel each other, and nothing flows into port 3. Same happens from port 3 to port 2 direction

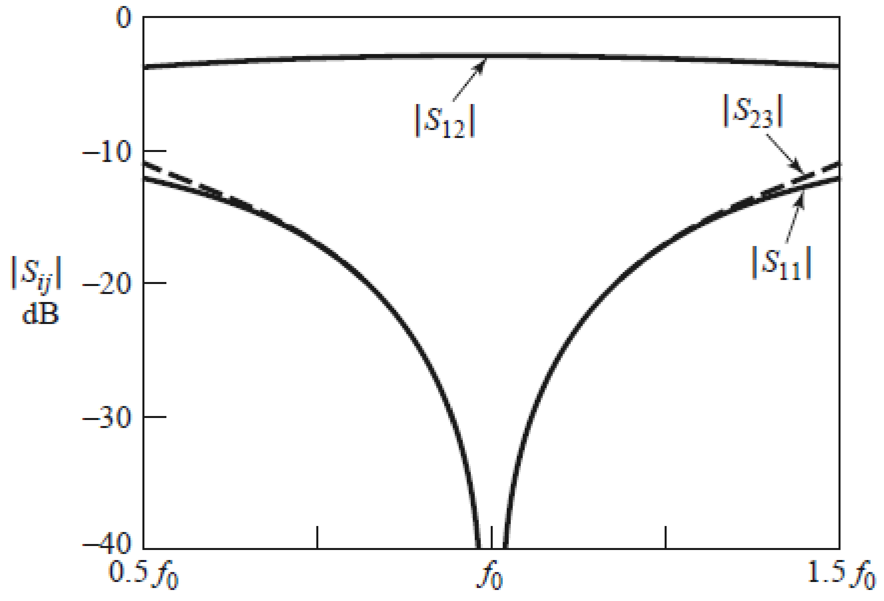
Wilkinson Power Divider – Equal

- It has to be noted that when power is fed at port 1 and output ports are matched, then no power is dissipated in the shunt resistor. Thus, the Wilkinson power divider is lossless when the outputs are matched

- Example – Wilkinson - Equal power divider $Z_0 = 50\,\Omega$

$$Z_{02} = Z_{03} = \sqrt{2}Z_0 = \sqrt{2} \times 50 = 70.7\Omega \qquad R = 2Z_0 = 100\,\Omega$$





Wilkinson Power Divider – Unequal

- Wilkinson power divider can also be designed to get unequal power divisions between the output ports (ports 2 & 3) given by a factor

$$\boxed{K^2 = \frac{P_3}{P_2}}$$

- A Wilkinson power divider of ratio 1:2 means the power at port 2 is twice as much as power at port 3

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}}$$

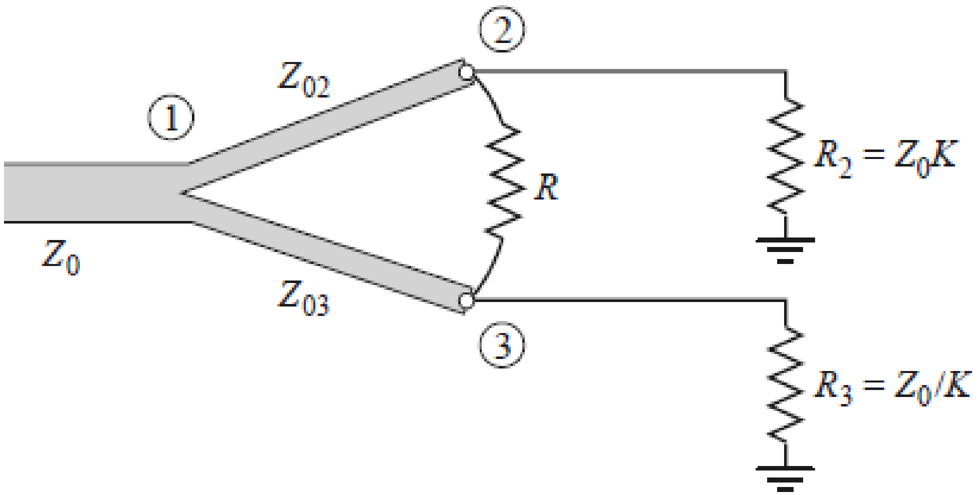
$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K\left(1+K^2\right)}$$

$$R = Z_0\left(K + \frac{1}{K}\right)$$

$$R_2 = Z_0 K$$

$$R_3 = Z_0 / K$$

Design equations

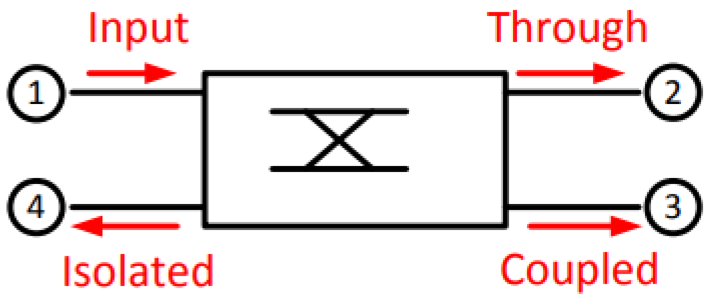


Problems

2. Design a Wilkinson power divider with a power division ratio of $P_3/P_2 = 1/3$ and a source impedance of 50Ω

Branchline Coupler

Directional coupler symbol:



- All ports are matched.
- No power flows to port 4 from in the input port 1 (i.e., port 4 is “isolated”).
- Ports 2 and 3 are isolated (power incident on port 2 does not get to port 3 and vice versa).

Branch Line Coupler

Choose

$$S_{12} = S_{34} = \alpha$$

$$S_{13} = \beta e^{j\theta}, \quad S_{24} = \beta e^{j\phi}$$

The first equation is really just a choice of reference planes on ports 2 and 3, which makes these parameters real.

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad \Rightarrow \quad \alpha^2 + \beta^2 = 1$$

$$S_{12}S_{13}^* + S_{24}S_{34}^* = 0 \quad \Rightarrow \quad \alpha\beta\, e^{-j\theta} + \alpha\beta\, e^{j\phi} = 0$$

$$\Rightarrow \quad \alpha\beta e^{-j\theta}\left(1 + \; e^{j(\phi+\theta)}\right) = 0$$

$$\Rightarrow \quad \theta + \phi = \pi + 2\pi n \qquad \text{(usually, } n = 0\text{)}$$

Branchline Coupler

Two possible choices:

1) Symmetrical coupler $(\theta = \phi = \pi / 2)$

$$\Rightarrow [S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

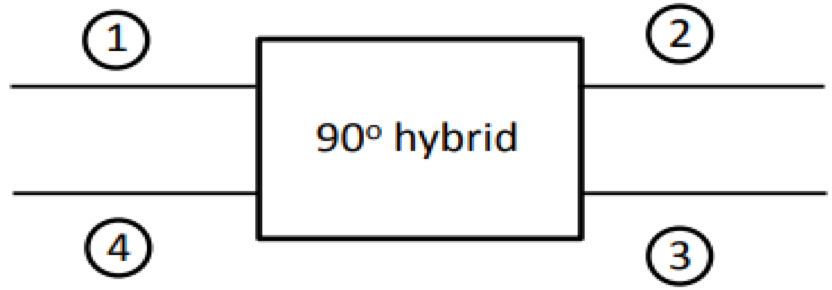
Example: 90° quadrature hybrid coupler

2) Anti-symmetrical coupler $(\theta = 0; \phi = \pi)$

$$\Rightarrow [S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Example: 180° rat-race hybrid coupler

Branchline Coupler

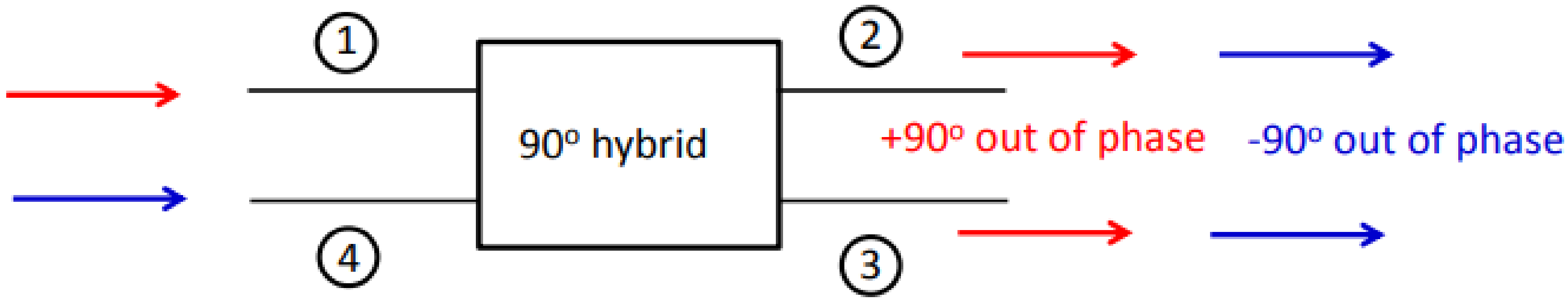


$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

- The quadrature hybrid is a lossless 4-port (the S matrix is unitary).
- All four ports are matched.
- The device is reciprocal (the S matrix is symmetric.)
- Port 4 is isolated from port 1 and ports 2 and 3 are isolated from each other.

Branchline Coupler

The quadrature hybrid is usually used as a splitter:



- The signal from port 1 splits evenly between ports 2 and 3, with a 90º phase difference.

$$S_{21} = jS_{31}$$

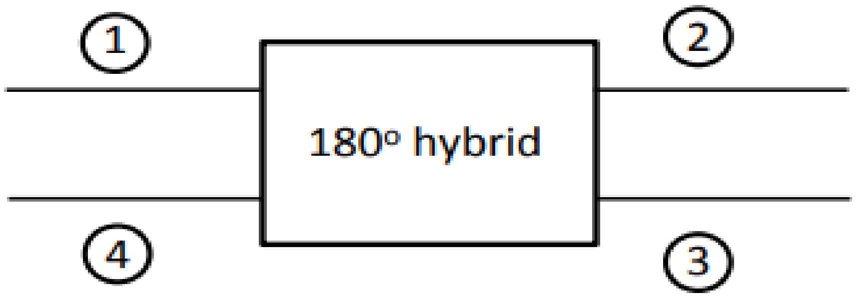
Can be used to produce right-handed circular polarization.

- The signal from port 4 splits evenly between ports 2 and 3, with a -90º phase difference.

$$S_{24} = -jS_{34}$$

Can be used to produce left-handed circular polarization.

Rat Race Ring Coupler (Hybrid)

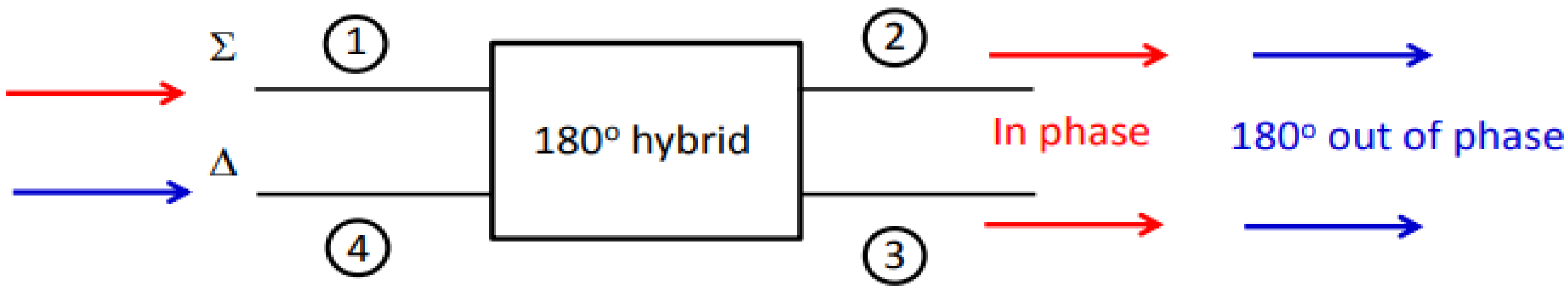


$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

- The rat race is a lossless 4-port (the S matrix is unitary).
- All four ports are matched.
- The device is reciprocal (the S matrix is symmetric).
- Port 4 is isolated from port 1 and ports 2 and 3 are isolated from each other.

Rat Race Ring Coupler (Hybrid)

The rat race can be used as a splitter:



- The signal from the "sum port" Σ (port 1) splits evenly between ports 2 and 3, in phase.

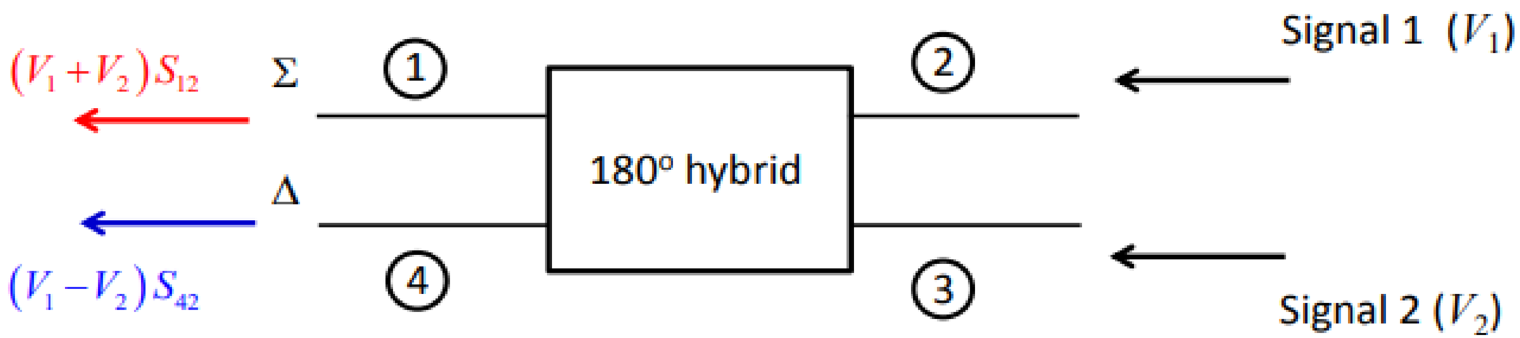
$$S_{21} = S_{31}$$

- The signal from the "difference port" Δ (port 4) splits evenly between ports 1 and 2, 180° out of phase.

$$S_{24} = -S_{34}$$

Rat Race Ring Coupler (Hybrid)

The rat race can be used as a combiner:



- The signal from the sum port Σ (port 1) is the sum of the input signals 1 and 2.

$$S_{12} = S_{13}$$

- The signal from the difference port Δ (port 4) is the difference of the input signals 1 and 2.

$$S_{42} = -S_{43}$$