1.4 Antenna parameters 2: Directivity and gain, bandwidth, polarization, input impedance

Module:1 EM Radiation and Antenna Parameters

Course: BECE305L – Antenna and Microwave Engineering

-Dr Richards Joe Stanislaus

Assistant Professor - SENSE

Email: 51749@vitstudent.ac.in / richards.stanislaus@vit.ac.in

VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Module:1 EM Radiation and Antenna Parameters

- Radiation mechanism - single wire, two wire and current distribution, **Hertzian dipole, Dipole and monopole** - Radiation pattern, beam width, field regions, radiation power density, radiation intensity, directivity and gain, bandwidth, polarization, input impedance, efficiency, antenna effective length and area, antenna temperature. Friis transmission equation, Radar range equation

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

1. Directivity

- ***Directivity of an antenna***: "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions"
- Note: The average radiation intensity = the total power radiated by the antenna divided by 4π
- *Also, if direction is not specified, directivity is calculated at direction of maximum radiation.*
- **Directivity of a nonisotropic source** is equal to the ratio of its radiation intensity in a given direction over that of an isotropic source.

1. Directivity

- Directivity $D = \dfrac{U}{U_0} = \dfrac{4\pi U}{P_{\text{rad}}}$

- If direction is not specified, directivity is w.r.t direction of max radiation intensity,

$$D_{\max} = D_0 = \frac{U|_{\max}}{U_0} = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{\text{rad}}}$$

$$\begin{aligned}
D &= \text{directivity} \quad (\text{dimensionless}) \\
D_0 &= \text{maximum directivity} \quad (\text{dimensionless}) \\
U &= \text{radiation intensity} \quad (\text{W/unit solid angle}) \\
U_{\max} &= \text{maximum radiation intensity} \quad (\text{W/unit solid angle}) \\
U_0 &= \text{radiation intensity of isotropic source} \quad (\text{W/unit solid angle}) \\
P_{\text{rad}} &= \text{total radiated power} \quad (\text{W})
\end{aligned}$$

1. Directivity $D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}$

- For isotropic source, directivity =1
- For antenna with orthogonal polarization components, *partial directivity of an antenna for a given polarization in a given direction* as “that part of the radiation intensity corresponding to a given polarization divided by the total radiation intensity averaged over all directions.
- Total maximum directivity: $D_0 = D_\theta + D_\phi$

$$D_\theta = \frac{4\pi U_\theta}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi}$$

U_θ = radiation intensity in a given direction contained in θ field component

U_ϕ = radiation intensity in a given direction contained in ϕ field component

$(P_{\text{rad}})_\theta$ = radiated power in all directions contained in θ field component

$(P_{\text{rad}})_\phi$ = radiated power in all directions contained in ϕ field component

Problem: find the maximum directivity of the antenna whose radiation intensity is $U = r^2 W_{rad} = A_0 \sin\theta$. Write an expression for the directivity as a function of the directional angles θ and ϕ.

Problem: find the maximum directivity of the antenna whose radiation intensity is $U = r^2 W_{rad} = A_0 \sin\theta$. Write an expression for the directivity as a function of the directional angles θ and ϕ.

- Radiation intensity is $U = r^2 W_{rad} = A_0 \sin\theta$

The maximum radiation is directed along $\theta = \pi/2$. Thus

$$U_{\max} = A_0$$

- Total Power radiated:

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 A_0$$

- Maximum Directivity:

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4}{\pi} = 1.27$$

Problem: find the maximum directivity of the antenna whose radiation intensity is $U = r^2 W_{rad} = A_0 \sin\theta$. Write an expression for the directivity as a function of the directional angles θ and ϕ.

- Since the radiation intensity is only a function of θ, the directivity as a function of the directional angles is represented by

$$D = D_0 \sin\theta = 1.27 \sin\theta$$

Problem 2:

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{\text{av}} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

Problem 2:

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

- Radiation intensity: $U = r^2 W_r = A_0 \sin^2\theta$
- Maximum radiation is directed along $\theta = \frac{\pi}{2}$, $U_{max} = A_0$
- Total radiated power: $P_{rad} = \oiint_{\Omega} U \, d\Omega = A_0 \int_0^{2\pi} \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi = A_0 \left(\frac{8\pi}{3}\right)$

Problem 2:

The radial component of the radiated power density of an infinitesimal linear dipole of length $l \ll \lambda$ is given by

$$\mathbf{W}_{av} = \hat{\mathbf{a}}_r W_r = \hat{\mathbf{a}}_r A_0 \frac{\sin^2\theta}{r^2} \quad (\text{W/m}^2)$$

where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{\mathbf{a}}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and ϕ.

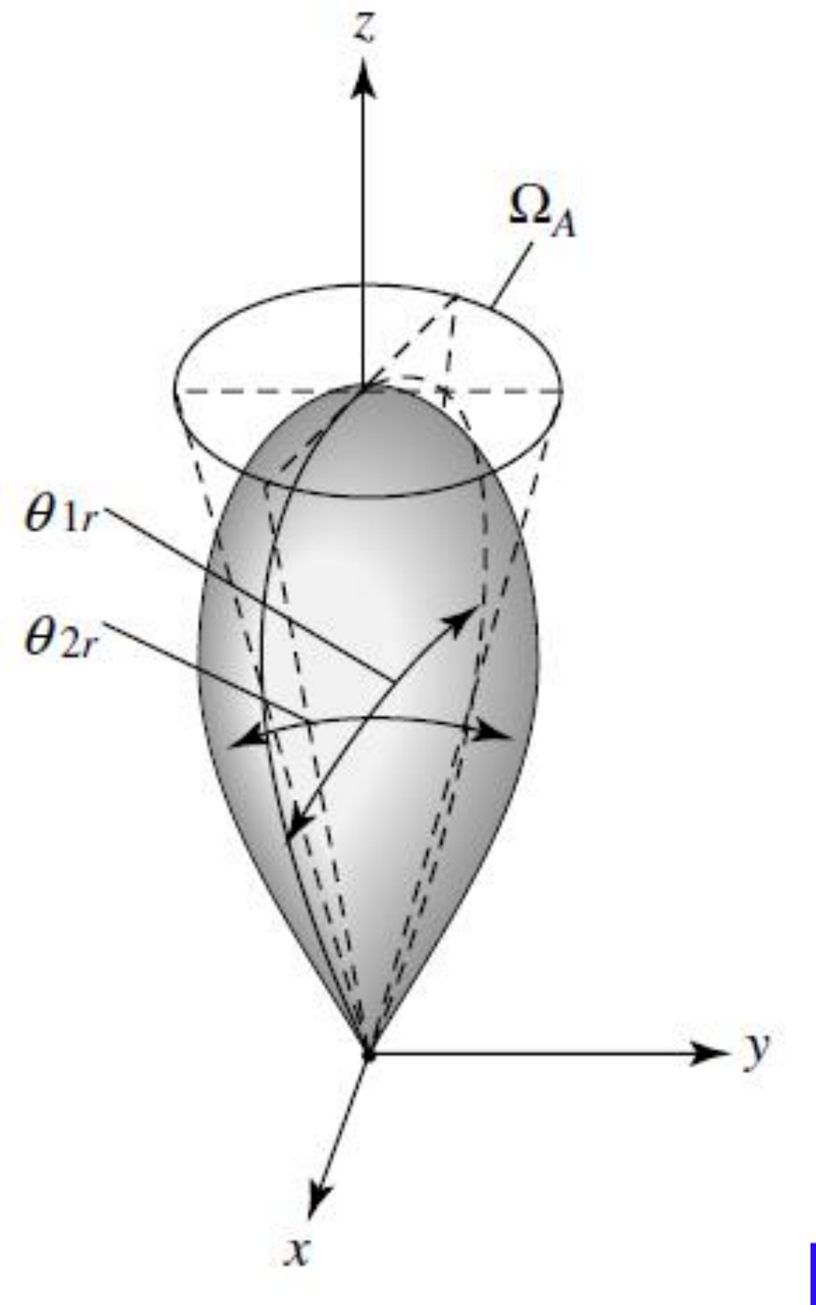
- maximum directivity is equal to $D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{\frac{8\pi}{3}(A_0)} = \frac{3}{2}$
- Directivity: $D = D_0 \sin^2\theta = 1.5 \sin^2\theta$

1. Directivity

- The directivity of an isotropic source is unity since its power is radiated equally well in all directions.
- *For all other sources, the maximum directivity will always be greater than unity, and it is a relative "figure of merit" which gives an indication of the directional properties of the antenna as compared with those of an isotropic source*.
- *The values of directivity will be equal to or greater than zero and equal to or less than the maximum directivity*
- ($0 \leq D \leq D0$).

1. Directivity

- Radiation patterns that may be functions of both spherical coordinate angles θ and ϕ.
- *The beam solid angle ΩA is defined as the solid angle through which all the power of the antenna would flow if its radiation intensity is constant (and equal to the maximum value of U) for all angles within ΩA.*
- For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beamwidths in two perpendicular planes



1. Directivity

- For directive antennas, maximum directivity is

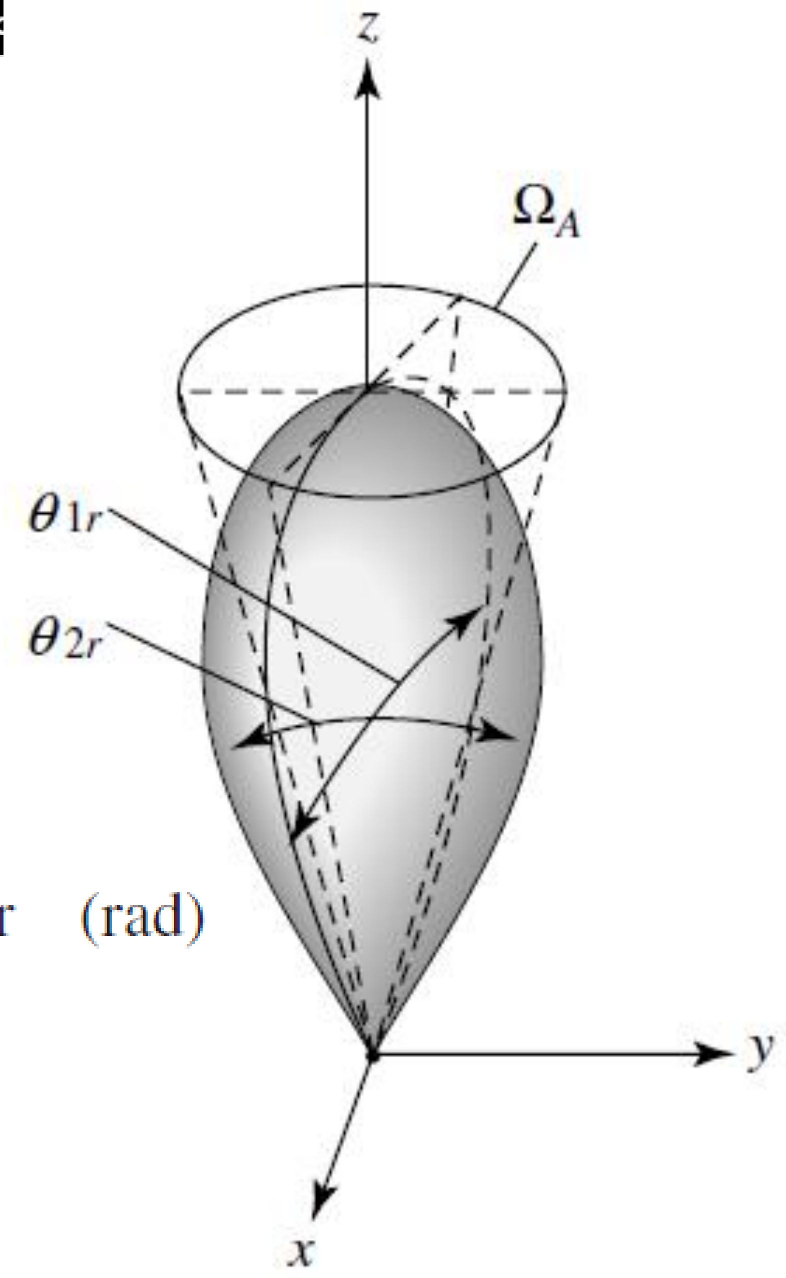
$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}}$$

- Beam solid angle:

$$\Omega_A \simeq \Theta_{1r}\Theta_{2r}$$

Θ_{1r} = half-power beamwidth in one plane (rad)

Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)



1. Directivity

- For directive antennas, maximum directivity is

$$\boxed{D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}}}$$

- Beam solid angle:

$$\Omega_A \simeq \Theta_{1r}\Theta_{2r}$$

Θ_{1r} = half-power beamwidth in one plane (rad)

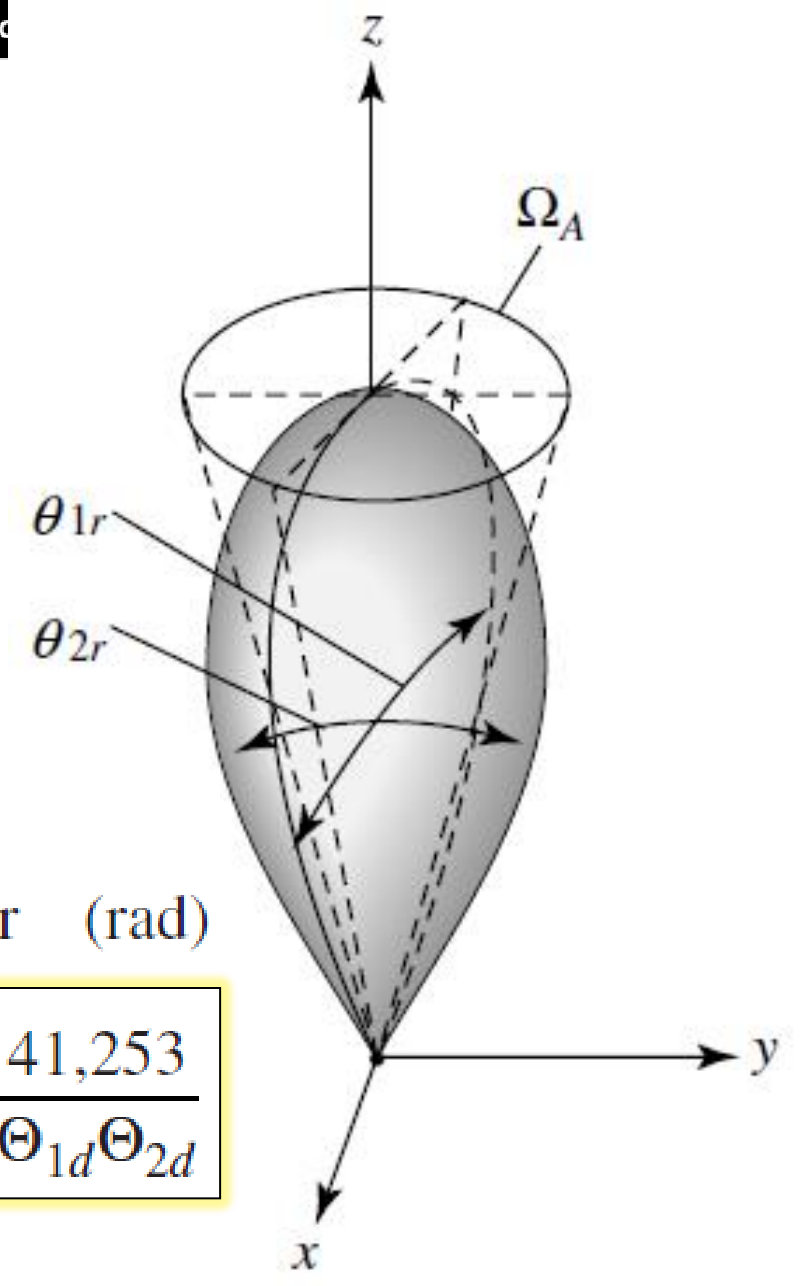
Θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

- If beam widths are in degrees, Maximum directivity is:

$$\boxed{D_0 \simeq \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}}}$$

Θ_{1d} = half-power beamwidth in one plane (degrees)

Θ_{2d} = half-power beamwidth in a plane at a right angle to the other (degrees)



1. Directivity

- For planar arrays:

$$D_0 \simeq \frac{32{,}400}{\Omega_A(\text{degrees})^2} = \frac{32{,}400}{\Theta_{1d}\Theta_{2d}}$$

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos^4 \theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.
b. maximum directivity; exact using (2-23) and approximate using (2-26).

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos^4 \theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.
b. maximum directivity; exact using (2-23) and approximate using (2-26).

Solution: The half-power point of the pattern occurs at $\theta = 32.765°$. Thus the beamwidth in the θ direction is 65.53° or

$$\Theta_{1r} = 1.1437 \text{ rads}$$

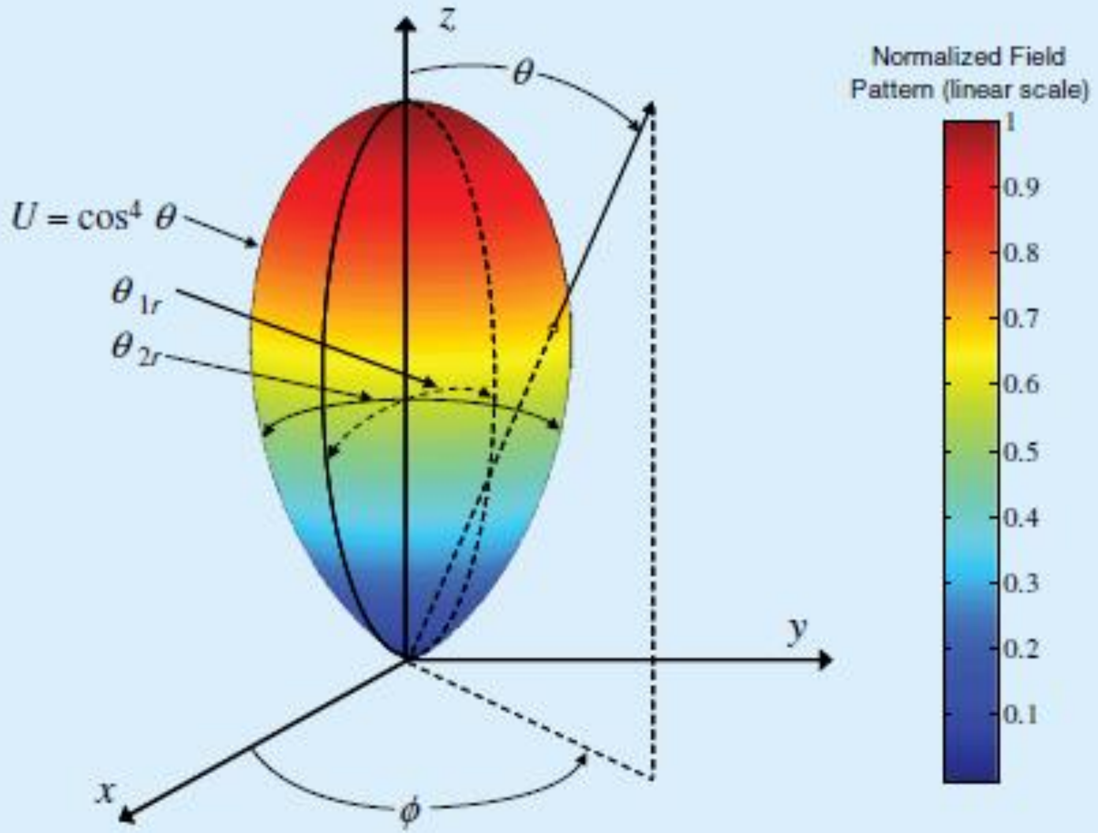


Figure 2.15 Radiation intensity pattern of the form $U = \cos^4 \theta$ in the upper hemisphere.

Since the pattern is independent of the ϕ coordinate, the beamwidth in the other plane is also equal to

$$\Theta_{2r} = 1.1437 \text{ rads}$$

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos^4\theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.

b. maximum directivity; exact using (2-23) and approximate using (2-26).

a. *Beam solid angle* Ω_A:

Exact: Using (2-24), (2-25)

$$\Omega_A = \int_0^{360^\circ}\int_0^{90^\circ} \cos^4\theta \, d\Omega = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta$$

$$= 2\pi \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta = \frac{2\pi}{5} \text{ steradians}$$

Approximate: Using (2-26a)

$$\Omega_A \approx \Theta_{1r}\Theta_{2r} = 1.1437(1.1437) = (1.1437)^2 = 1.308 \text{ steradians}$$

Solution: The half-power point of the pattern occurs at $\theta = 32.765^\circ$. Thus the beamwidth in the θ direction is 65.53° or

$$\Theta_{1r} = 1.1437 \text{ rads}$$

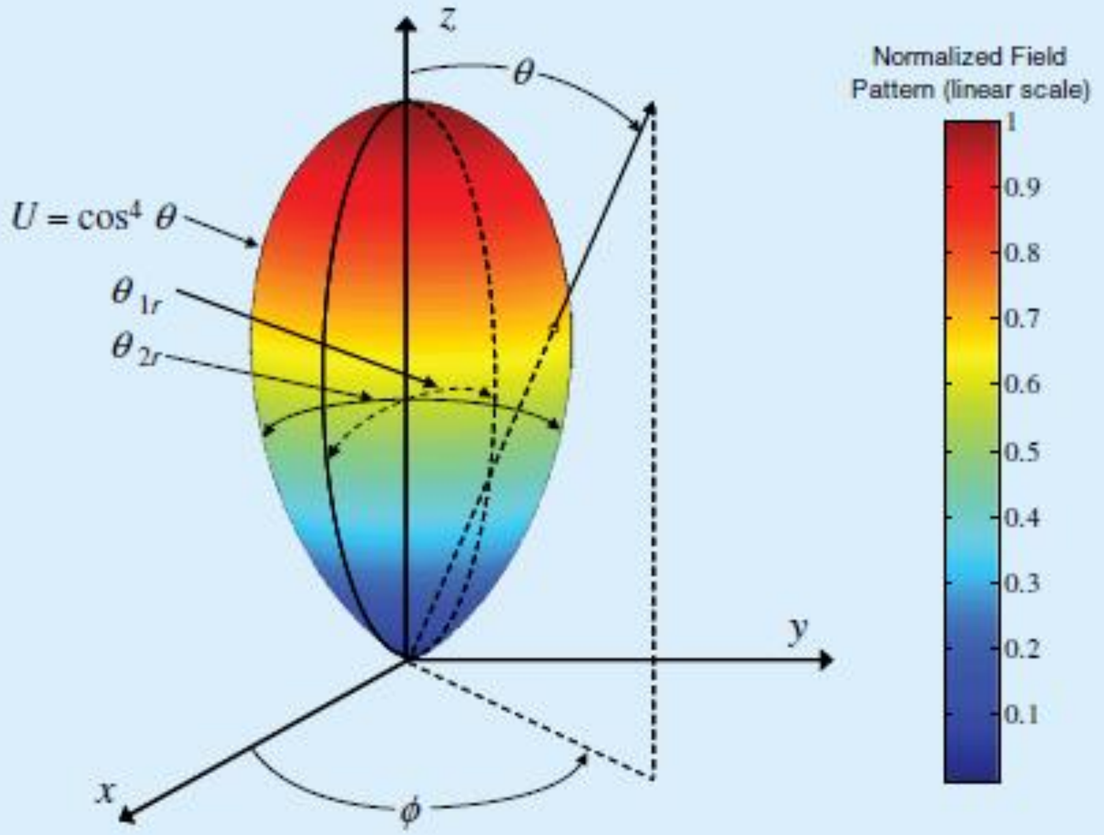


Figure 2.15 Radiation intensity pattern of the form $U = \cos^4\theta$ in the upper hemisphere.

Since the pattern is independent of the ϕ coordinate, the beamwidth in the other plane is also equal to

$$\Theta_{2r} = 1.1437 \text{ rads}$$

The radiation intensity of the major lobe of many antennas can be adequately represented by

$$U = B_0 \cos^4 \theta$$

where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$), and it is shown in Figure 2.15.

Find the

a. beam solid angle; exact and approximate.

b. maximum directivity; exact using (2-23) and approximate using (2-26).

Solution: The half-power point of the pattern occurs at $\theta = 32.765°$. Thus the beamwidth in the θ direction is 65.53° or

$$\Theta_{1r} = 1.1437 \text{ rads}$$

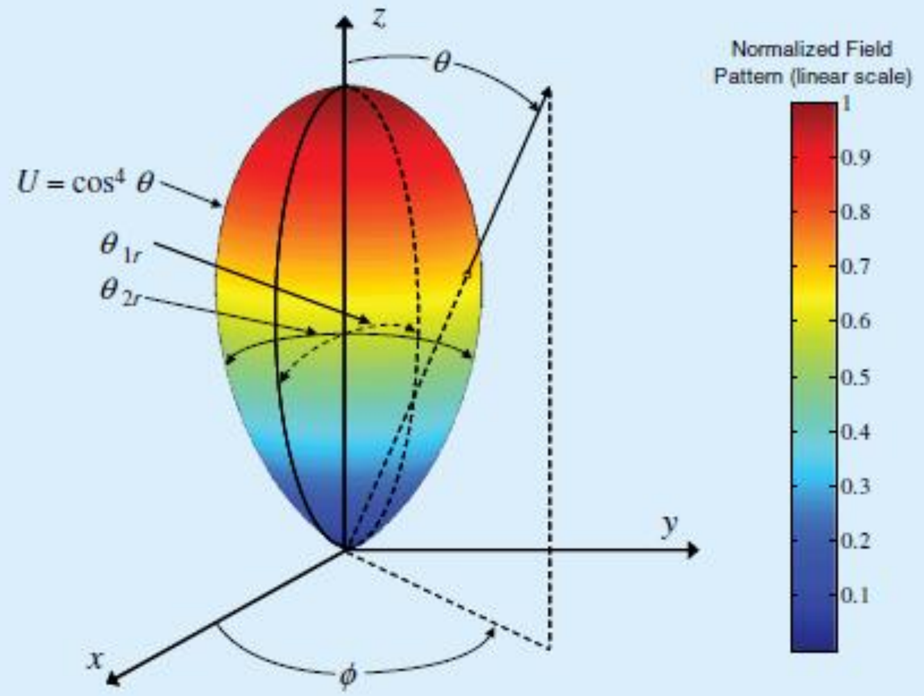


Figure 2.15 Radiation intensity pattern of the form $U = \cos^4 \theta$ in the upper hemisphere.

Since the pattern is independent of the ϕ coordinate, the beamwidth in the other plane is also equal to

$$\Theta_{2r} = 1.1437 \text{ rads}$$

a. *Beam solid angle* Ω_A:

Exact: Using (2-24), (2-25)

$$\Omega_A = \int_0^{360°} \int_0^{90°} \cos^4 \theta \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta$$

$$= 2\pi \int_0^{\pi/2} \cos^4 \theta \sin \theta \, d\theta = \frac{2\pi}{5} \text{ steradians}$$

Approximate: Using (2-26a)

$$\Omega_A \approx \Theta_{1r}\Theta_{2r} = 1.1437(1.1437) = (1.1437)^2 = 1.308 \text{ steradians}$$

b. *Directivity* D_0:

Exact: $D_0 = \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi(5)}{2\pi} = 10$ (dimensionless) $= 10$ dB

The same exact answer is obtained using (2-16a).

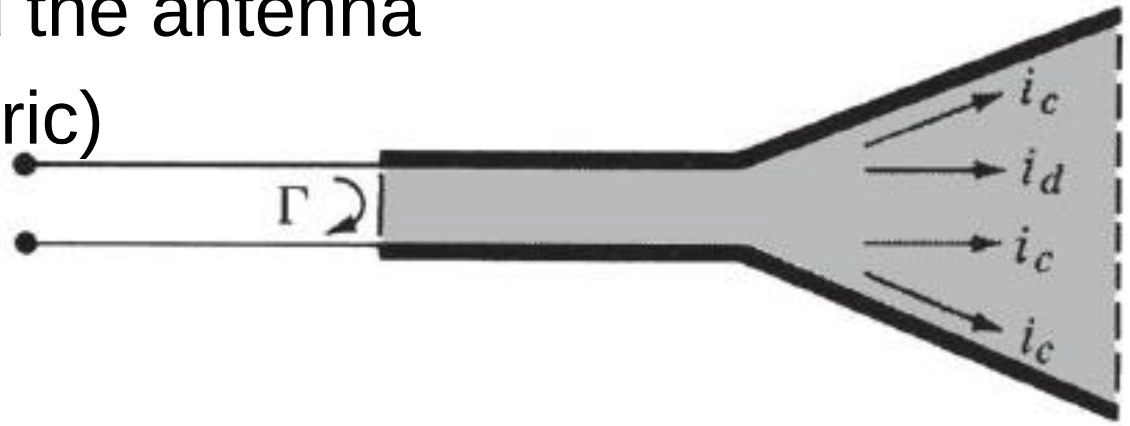
Approximate: $D_0 \approx \dfrac{4\pi}{\Omega_A} = \dfrac{4\pi}{1.308} = 9.61$ (dimensionless) $= 9.83$ dB

2. Antenna efficiency (a number of efficiencies)

- total antenna efficiency *e*0 takes into account losses at the input terminals and within the structure of the antenna.

$$e_0 = e_r e_c e_d$$

- Losses: reflections because of the mismatch between the transmission line and the antenna
- I^2R losses (conduction and dielectric)



(b) Reflection, conduction, and dielectric losses

e_c = conduction efficiency (dimensionless)

e_d = dielectric efficiency (dimensionless)

e_0 = total efficiency (dimensionless)

e_r = reflection (mismatch) efficiency $= (1 - |\Gamma|^2)$ (dimensionless)

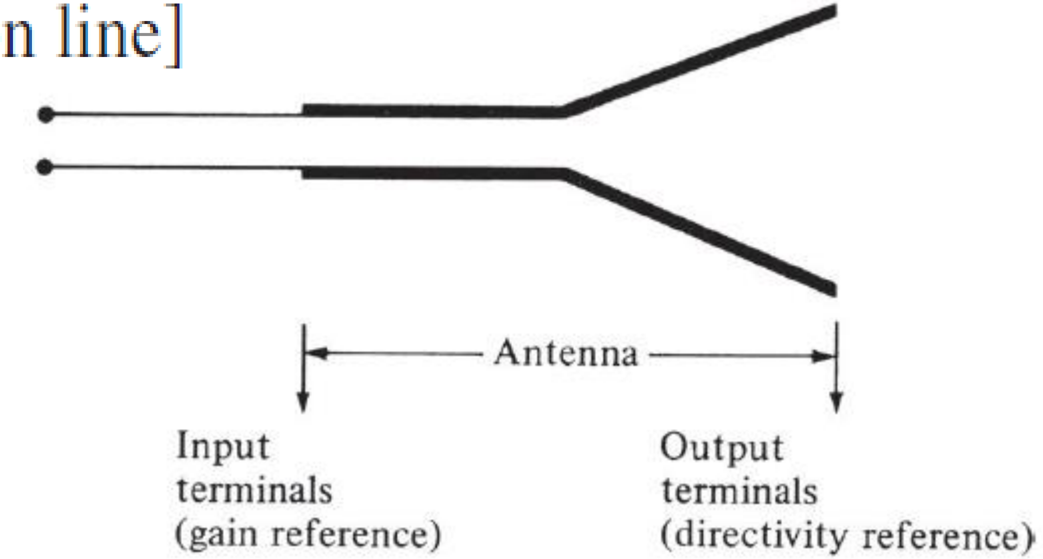
2. Antenna efficiency (a number of efficiencies)

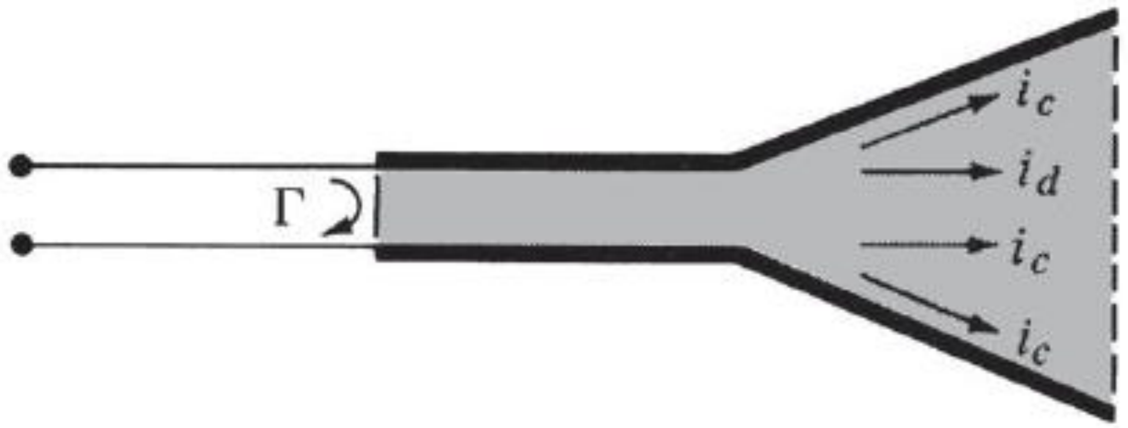
Γ = voltage reflection coefficient at the input terminals of the antenna

[$\Gamma = (Z_{in} - Z_0)/(Z_{in} + Z_0)$ where Z_{in} = antenna input impedance,

Z_0 = characteristic impedance of the transmission line]

VSWR = voltage standing wave ratio $= \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$





(b) Reflection, conduction, and dielectric losses

2. Antenna efficiency (a number of efficiencies)

- Total efficiency $e_0 = e_r e_{cd} = e_{cd}(1 - |\Gamma|^2)$
- Where $e_{cd} = antenna\, radiation\, efficiency$

$$e_{cd} = e_c e_d$$

This is used to relate the gain and directivity.

3. Gain of antenna

- ***Gain* of an antenna** (in a given direction) is
  "the ratio of the intensity, in a given direction, to
  the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$\text{Gain} = 4\pi \frac{\text{radiation intensity}}{\text{total input (accepted) power}} = 4\pi \frac{U(\theta, \phi)}{P_{in}} \quad \text{(dimensionless)}$$

- radiation intensity corresponding to the isotropically radiated power is equal to the power accepted (input) by the antenna divided by 4π."

3. Gain of antenna

- ***relative gain***, is "the ratio of the power gain in a given direction to the power gain of a reference antenna in its referenced direction." The power input must be the same for both antennas.
- reference antenna is usually a dipole, horn, or any other antenna
- In most cases, however, the reference antenna is a *lossless isotropic source*.

$$G = \frac{4\pi U(\theta, \phi)}{P_{in}(\text{lossless isotropic source})} \quad \text{(dimensionless)}$$

- Note: *When the direction is not stated, the power gain is usually taken in the direction of maximum radiation*.

Where e_{cd} =antenna radiation efficiency
- gain (G), does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).
- realized gain (Gre), takes into account the reflection/mismatch losses
- Gain is given as
- Hence, gain dependent on directivity is
- max gain is related to the max directivity

3. Gain of antenna

- total radiated power (P_{rad}) is related to the total input power (P_{in}) by

$$P_{rad} = e_{cd} P_{in}$$

Where e_{cd} =antenna radiation efficiency

- *gain* (*G*), does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).
- *realized gain* (*Gre*), takes into account the reflection/mismatch losses
- Gain is given as $$G(\theta, \phi) = e_{cd}\left[4\pi \frac{U(\theta, \phi)}{P_{\text{rad}}}\right]$$
- Hence, gain dependent on directivity is $G(\theta, \phi) = e_{cd} D(\theta, \phi)$
- max gain is related to the max directivity

$$G_0 = G(\theta, \phi)|_{\max} = e_{cd} D(\theta, \phi)|_{\max} = e_{cd} D_0$$

Where e_{cd} =antenna radiation efficiency
- gain (G), does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).
- realized gain (Gre). takes into account the reflection/mismatch losses
- Gain is given as
- Hence, gain dependent on directivity is
- max gain is related to the max directivity

3. Gain of antenna

- total radiated power (P_{rad}) is related to the total input power (P_{in}) by

$$P_{rad} = e_{cd} P_{in}$$

Where e_{cd} =antenna radiation efficiency

- *gain* (*G*), does not include losses arising from impedance mismatches (reflection losses) and polarization mismatches (losses).
- *realized gain* (*Gre*). takes into account the reflection/mismatch losses
- Gain is given as $$G(\theta,\phi) = e_{cd}\left[4\pi \frac{U(\theta,\phi)}{P_{\text{rad}}}\right]$$
- Hence, gain dependent on directivity is $G(\theta,\phi) = e_{cd} D(\theta,\phi)$
- max gain is related to the max directivity

$$G_0 = G(\theta,\phi)|_{\max} = e_{cd} D(\theta,\phi)|_{\max} = e_{cd} D_0$$

3. Gain of antenna

- When orthogonal components are present, *partial gain of an antenna for a given polarization in a given direction* as "that **part of the radiation intensity corresponding to a given polarization** divided by the **total radiation intensity** that would be obtained if the power accepted by the antenna were **radiated isotropically**."
- "the total gain is the sum of the partial gains for any two orthogonal polarizations

$$G_0 = G_\theta + G_\phi$$

partial gains G_θ and G_ϕ are expressed as $G_\theta = \dfrac{4\pi U_\theta}{P_{in}}$ $G_\phi = \dfrac{4\pi U_\phi}{P_{in}}$

U_θ = radiation intensity in a given direction contained in E_θ field component

U_ϕ = radiation intensity in a given direction contained in E_ϕ field component

P_{in} = total input (accepted) power

3. Gain of antenna

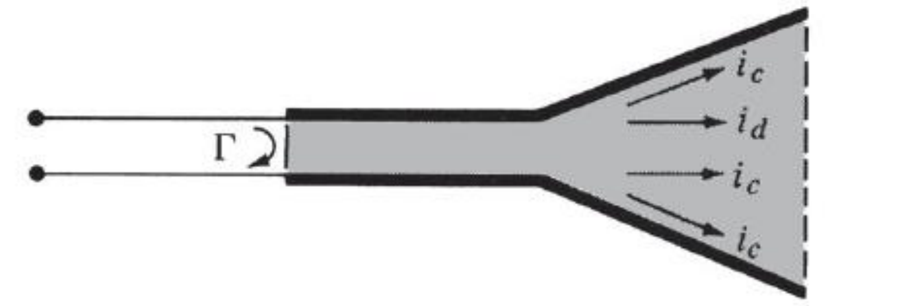
- Many practical applications,

$$G_0 \simeq \frac{30{,}000}{\Theta_{1d}\Theta_{2d}}$$

- whenever the term “gain” is used, it usually refers to the *maximum gain*
- Recollect $G(\theta, \phi) = e_{cd} D(\theta, \phi)$
- Hence gain in dB

$$G_0(\text{dB}) = 10 \log_{10}[e_{cd} D_0 \text{ (dimensionless)}]$$

3.1 Realised Gain



(b) Reflection, conduction, and dielectric losses

- connection losses are usually referred to as *reflections (mismatch) losses*, and they are taken into account by introducing a reflection (mismatch) efficiency e_r

$$e_r = (1 - |\Gamma|^2)$$

- *Realized gain* G_{re} that takes into account the reflection/mismatch losses (due to the connection of the antenna element to the transmission line),

$$G_{re}(\theta,\phi) = e_r G(\theta,\phi) = (1 - |\Gamma|^2) G(\theta,\phi)$$
$$= e_r e_{cd} D(\theta,\phi) = e_o D(\theta,\phi)$$

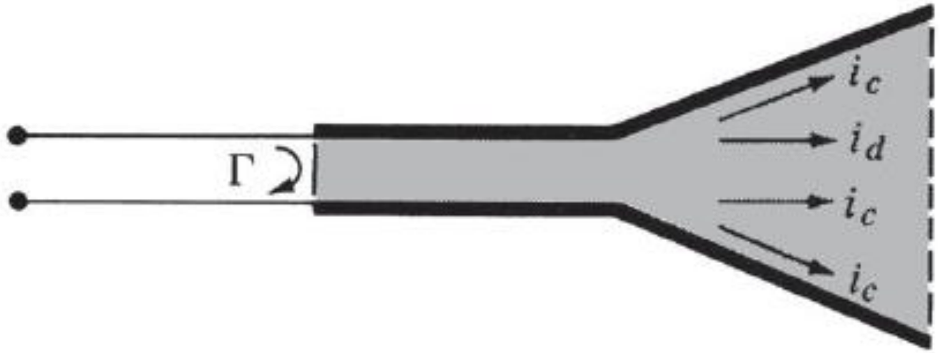
- Max realized gain

$$G_{re0} = G_{re}(\theta,\phi)|_{\max} = e_r G(\theta,\phi)|_{\max} = (1 - |\Gamma|^2) G(\theta,\phi)|_{\max}$$
$$= e_r e_{cd} D(\theta,\phi)|_{\max} = e_o D(\theta,\phi)|_{\max} = e_o D_0$$

3.1 Realised Gain

If the antenna is matched to the transmission line, that is, the antenna input impedance Z_{in} is equal to the characteristic impedance Z_c of the line ($|\Gamma| = 0$), then the two gains are equal ($G_{re} = G$).

- $e_r = 1$



(b) Reflection, conduction, and dielectric losses

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U = B_0 \sin^3 \theta$ find the maximum realized gain of this antenna.

Solution: Let us first compute the maximum directivity of the antenna. For this

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0 \left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697$$

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U = B_0 \sin^3 \theta$

find the maximum realized gain of this antenna.

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$. Thus, the total maximum gain is equal to

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

$$G_0(\text{dB}) = 10 \log_{10}(1.697) = 2.297$$

which is identical to the directivity because the antenna is lossless.

The gain in dB can also be obtained by converting the directivity and radiation efficiency in dB and then adding them. Thus,

$$e_{cd}(\text{dB}) = 10 \log_{10}(1.0) = 0$$

$$D_0(\text{dB}) = 10 \log_{10}(1.697) = 2.297$$

$$G_0(\text{dB}) = e_{cd}(\text{dB}) + D_0(\text{dB}) = 2.297$$

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by $U = B_0 \sin^3 \theta$

find the maximum realized gain of this antenna.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency of (2-44) or (2-45), and it is equal to

$$e_r = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73 - 50}{73 + 50}\right|^2\right) = 0.965$$

$$e_r(\text{dB}) = 10 \log_{10}(0.965) = -0.155$$

Therefore the overall efficiency is $e_0 = e_r e_{cd} = 0.965$

$$e_0(\text{dB}) = -0.155$$

Thus, the overall losses are equal to 0.155 dB. The maximum realized gain is equal to

$$G_{re0} = e_0 D_0 = 0.965(1.697) = 1.6376$$

$$G_{re0}(\text{dB}) = 10 \log_{10}(1.6376) = 2.142$$

4. Beam efficiency

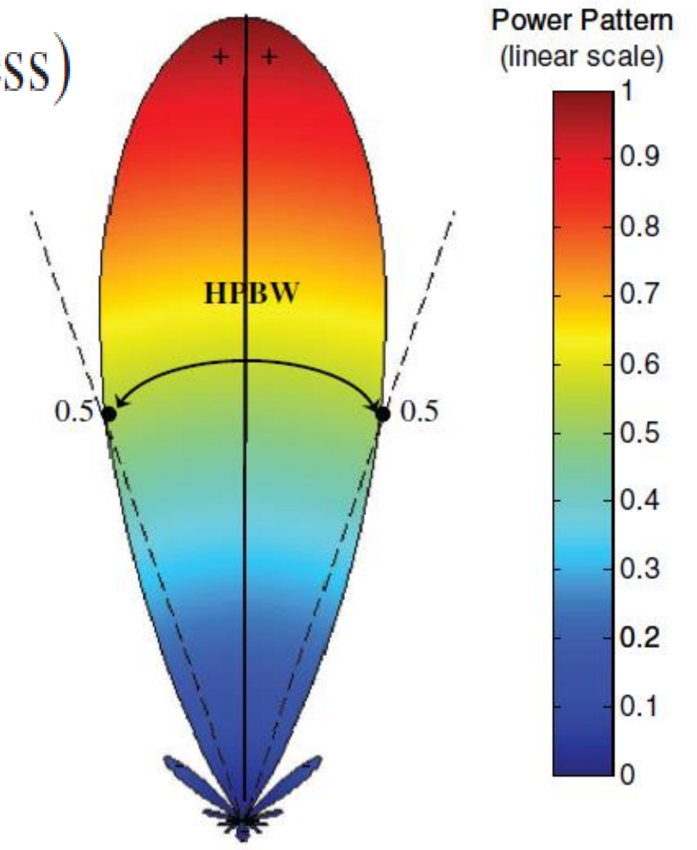
- For an antenna with its major lobe directed along the z-axis ($\theta = 0$), as shown the beam efficiency (BE) is defined by

$$\mathrm{BE} = \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}} \text{(dimensionless)}$$

-

where $\theta 1$ is the half-angle of the cone within which the percentage of the total power is to be found

$$\mathrm{BE} = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$



(b) Power pattern (in linear scale)

4. Beam efficiency

- If $\theta 1$ is chosen as the angle where the first null or minimum occurs, then the beam efficiency will indicate the amount of power in the major lobe compared to the total power.
- A very high beam efficiency (between the nulls or minima), usually in the high 90s, is necessary for antennas used in radiometry, astronomy, radar, and other applications where received signals through the minor lobes must be minimized.

5. Bandwidth

- *bandwidth* of an antenna : “the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.”
- The bandwidth can be considered to be the range of frequencies, on either side of a center frequency (usually the resonance frequency for a dipole), where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency.

5. Bandwidth

- For *broadband antennas*, the bandwidth is usually expressed as the ratio of the upper-to-lower requencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.
- For *narrowband antennas*, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency range of acceptable operation is 5% of the bandwidth center frequency.

5. Bandwidth

- *pattern bandwidth* and *impedance bandwidth* are used to emphasize this distinction.
  Associated with pattern bandwidth are gain, side lobe level, beamwidth, polarization, and beam direction

  while input impedance and radiation efficiency are related to impedance bandwidth

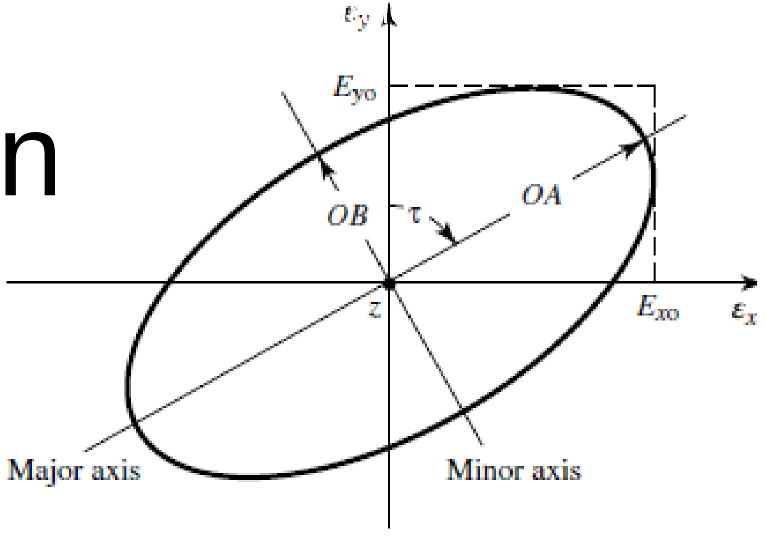
6. Polarization

- the polarization of the wave transmitted (radiated) by the antenna.
- different parts of the pattern may have different polarizations.
- When the direction is not stated, the polarization is taken to be the polarization in the direction of maximum gain
- *Polarization of a radiated wave* : "that **property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector**; specifically, the figure **traced as a function of time** by the extremity of the vector **at a fixed location in space**, and the sense in which it is traced, ***as observed along the direction of propagation***."

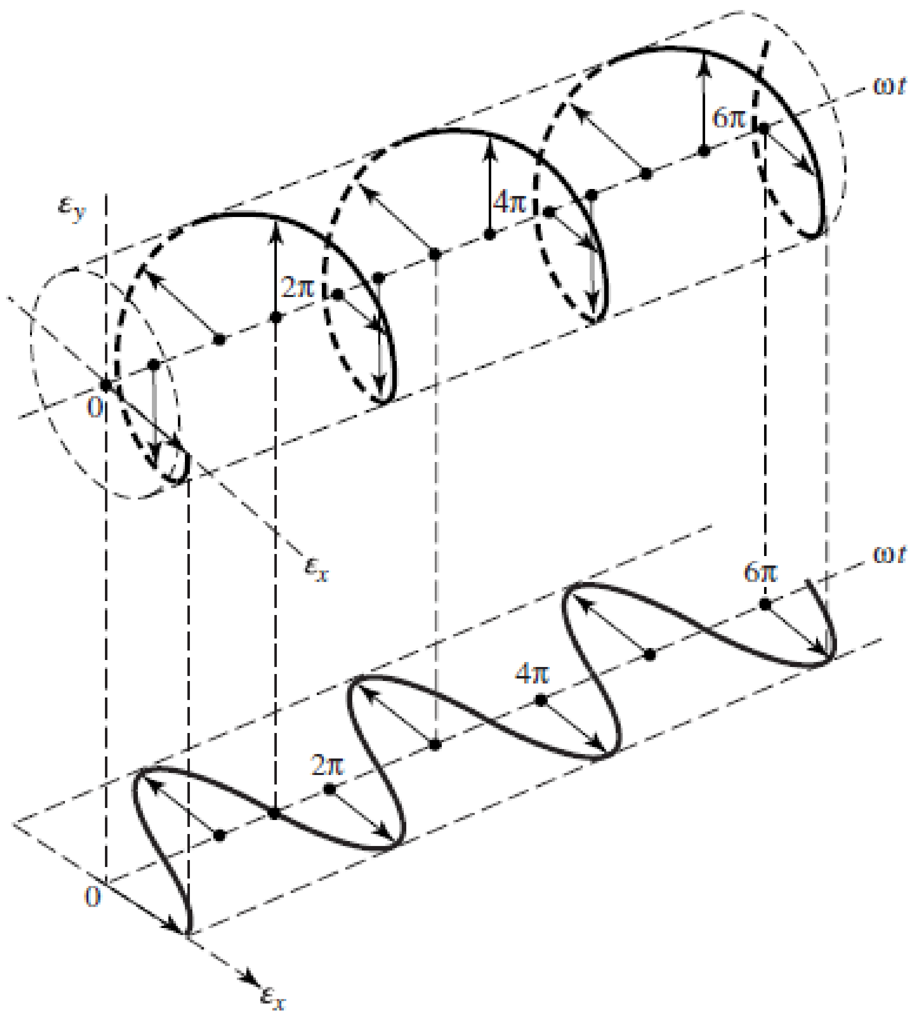
6. Polarization

- Polarization then is the curve traced by the end point of the arrow (vector) representing the instantaneous electric field.
- The polarization of a wave *radiated* by an antenna in a specified direction at a point in the far field is defined as "**the polarization of the (locally) plane wave** which is used to represent the radiated wave at that point.
- The polarization of a wave *received* by an antenna is defined as the "polarization of a plane wave, incident from a given direction and having a given power flux density, <u>which results in maximum available power at the antenna terminals</u>."

6. Polarization



(b) Polarization ellipse



(a) Rotation of wave

- The figure of the electric field is traced in a *clockwise* (CW) or *counterclockwise* (CCW) sense.
- *Clockwise* rotation of the electric-field vector is also designated as *right-hand polarization* and
- *counterclockwise* as *left-hand polarization*.

6. Polarization

- Polarization may be classified as linear, circular, or elliptical.
- If the vector that describes the electric field at a point in space as a function of time is always directed along a line, the field is said to be *linearly* polarized.
- In general, however, the figure that the electric field traces is an ellipse, and the field is said to be elliptically polarized.
- Linear and circular polarizations are special cases of elliptical, and they can be obtained when the ellipse becomes a straight line or a circle, respectively

6. Polarization

- At each point on the radiation sphere the polarization is usually resolved into a pair of orthogonal polarizations, the *co-polarization* and *cross polarization*.
- “*Co-polarization* represents the polarization the antenna is intended to radiate (receive) while *cross-polarization* represents the polarization orthogonal to a specified polarization, which is usually the co-polarization.”
- “In practice, the axis of the antenna’s main beam should be directed along the polar axis of the radiation sphere. The antenna is then appropriately oriented about this axis to align the direction of its polarization with that of the defined co-polarization at the pole.”
- “This manner of defining co-polarization can be extended to the case of elliptical polarization by defining the constant angles using the major axes of the polarization ellipses rather than the co-polar electric-field vector. The sense of polarization (rotation) must also be specified.”

6.1 Linear, Circular, and Elliptical Polarizations

- instantaneous field of a plane wave, traveling in the negative *z* direction

$$\mathscr{E}(z;t) = \hat{\mathbf{a}}_x \mathscr{E}_x(z;t) + \hat{\mathbf{a}}_y \mathscr{E}_y(z;t)$$

- instantaneous components are related to their complex counterparts by

$$\mathscr{E}_x(z;t) = \mathrm{Re}[E_x^{-} e^{j(\omega t + kz)}] = \mathrm{Re}[E_{xo} e^{j(\omega t + kz + \phi_x)}]$$
$$= E_{xo} \cos(\omega t + kz + \phi_x)$$
$$\mathscr{E}_y(z;t) = \mathrm{Re}[E_y^{-} e^{j(\omega t + kz)}] = \mathrm{Re}[E_{yo} e^{j(\omega t + kz + \phi_y)}]$$
$$= E_{yo} \cos(\omega t + kz + \phi_y)$$

- E_{x0} and E_{y0} are, respectively, the maximum magnitudes of the *x* and *y* components.

6.1 Linear, Circular, and Elliptical Polarizations

- Linear polarization: the time-phase difference between the two components must be

$$\Delta\phi = \phi_y - \phi_x = n\pi, \quad n = 0, 1, 2, 3, \ldots$$

- Circular polarization: *only* when the magnitudes of the two components are the same *and* the time-phase difference between them is odd multiples of $\pi/2$.

$$|\mathscr{E}_x| = |\mathscr{E}_y| \Rightarrow E_{xo} = E_{yo}$$

- If the direction of wave propagation is reversed (i.e., +*z* direction), the phases for CW and CCW rotation must be interchanged.

$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +\left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CW} \\ -\left(\frac{1}{2} + 2n\right)\pi, n = 0, 1, 2, \ldots & \text{for CCW} \end{cases}$$

6.1 Linear, Circular, and Elliptical Polarizations

- Elliptical polarization: *only* when the time-phase difference between the two components is odd multiples of $\pi/2$ *and* their magnitudes are not the same *or* when the time-phase difference between the two components is not equal to multiples of $\pi/2$

$$|\mathscr{E}_x| \neq |\mathscr{E}_y| \Rightarrow E_{xo} \neq E_{yo}$$

$$\text{when } \Delta\phi = \phi_y - \phi_x = \begin{cases} +(\frac{1}{2} + 2n)\pi & \text{for CW} \\ -(\frac{1}{2} + 2n)\pi & \text{for CCW} \end{cases} \quad n = 0, 1, 2, \ldots$$

$$\Delta\phi = \phi_y - \phi_x \neq \pm\frac{n}{2}\pi = \begin{cases} > 0 & \text{for CW} \\ < 0 & \text{for CCW} \end{cases} \quad n = 0, 1, 2, 3, \ldots$$

6.1 Linear, Circular, and Elliptical Polarizations

- The ratio of the major axis to the minor axis is referred to as the axial ratio (AR), and it is equal to

$$AR = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}, \quad 1 \le AR \le \infty$$

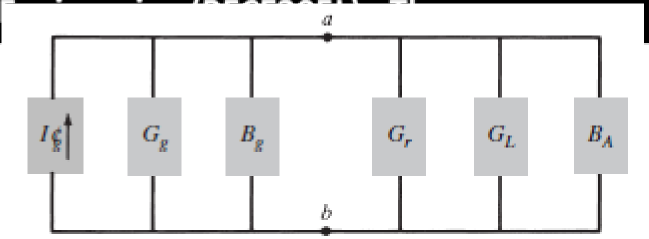
$$OA = \left[\frac{1}{2}\{E_{xo}^2 + E_{yo}^2 + [E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2 E_{yo}^2 \cos(2\Delta\phi)]^{1/2}\}\right]^{1/2}$$

$$OB = \left[\frac{1}{2}\{E_{xo}^2 + E_{yo}^2 - [E_{xo}^4 + E_{yo}^4 + 2E_{xo}^2 E_{yo}^2 \cos(2\Delta\phi)]^{1/2}\}\right]^{1/2}$$

- Tilt angle w.r.t y axis:

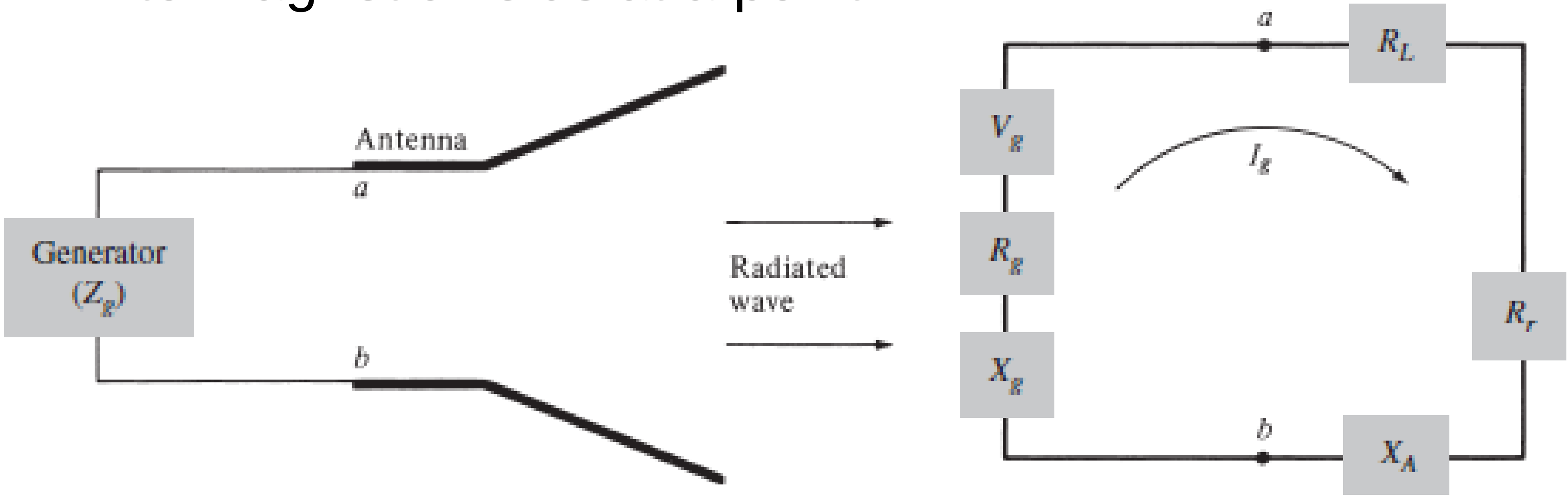
$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left[\frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2}\cos(\Delta\phi)\right]$$

7. Input Impedance



(c) Norton equivalent

- *Input impedance* is defined as “the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point.”



(b) Thevenin equivalent

7. Input Impedance

- Antenna impedance

$$\boxed{Z_A = R_A + jX_A}$$

Z_A = antenna impedance at terminals a–b (ohms)
R_A = antenna resistance at terminals a–b (ohms)
X_A = antenna reactance at terminals a–b (ohms)

$$\boxed{R_A = R_r + R_L}$$

R_r = radiation resistance of the antenna
R_L = loss resistance of the antenna

- Source(generator) impedance

$$Z_g = R_g + jX_g$$

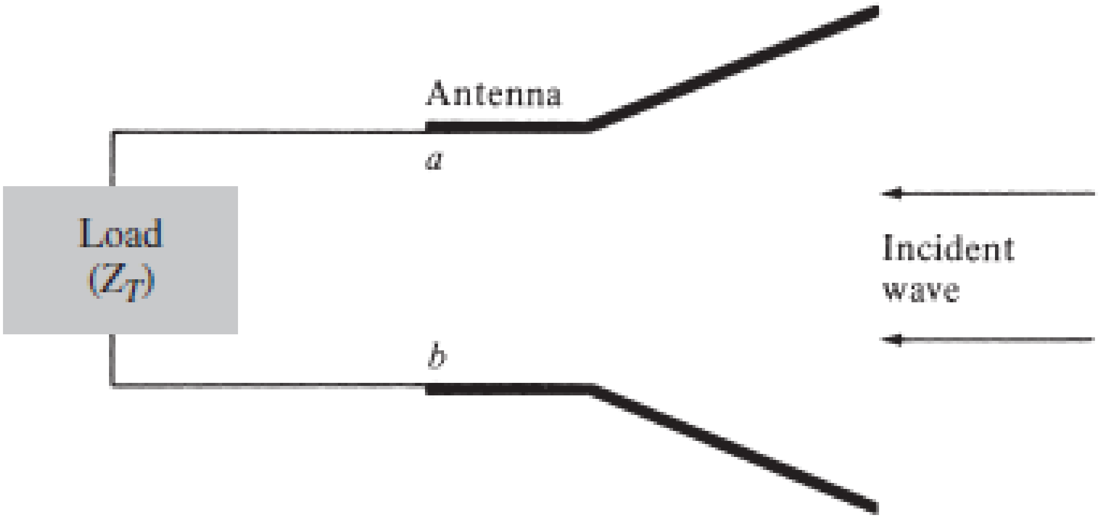
- When antenna impedance and generator impedances are complex conjugates, maximum power will be delivered.
  Source power:

$$P_s = \frac{1}{2}V_g I_g^* = \frac{1}{2}V_g\left[\frac{V_g^*}{2(R_r+R_L)}\right] = \frac{|V_g|^2}{4}\left[\frac{1}{R_r+R_L}\right] \quad (W)$$

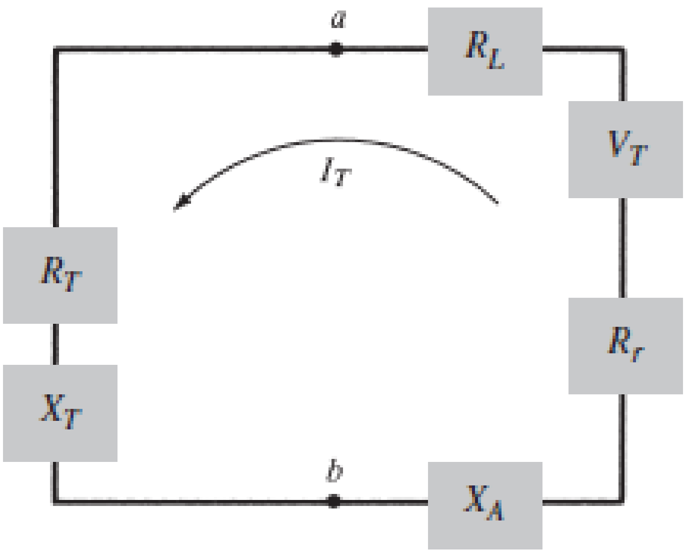
7.1 Antenna impedance Receive mode

- Power collected by antenna is dependent on Voltage induced V_T, and Power collected is

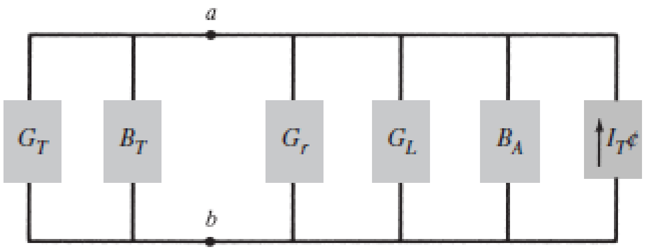
$$P_c = \frac{1}{2}V_T I_T^* = \frac{1}{2}V_T\left[\frac{V_T^*}{2(R_r+R_L)}\right] = \frac{|V_T|^2}{4}\left(\frac{1}{R_r+R_L}\right)$$



(a) Antenna in receiving mode



(b) Thevenin equivalent



(c) Norton equivalent