1.5 Antenna Parameters 3: Effective length, area, Friis transmission equation, Radar range equation

Module:1 EM Radiation and Antenna Parameters

Course: BECE305L – Antenna and Microwave Engineering

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CHENNAI

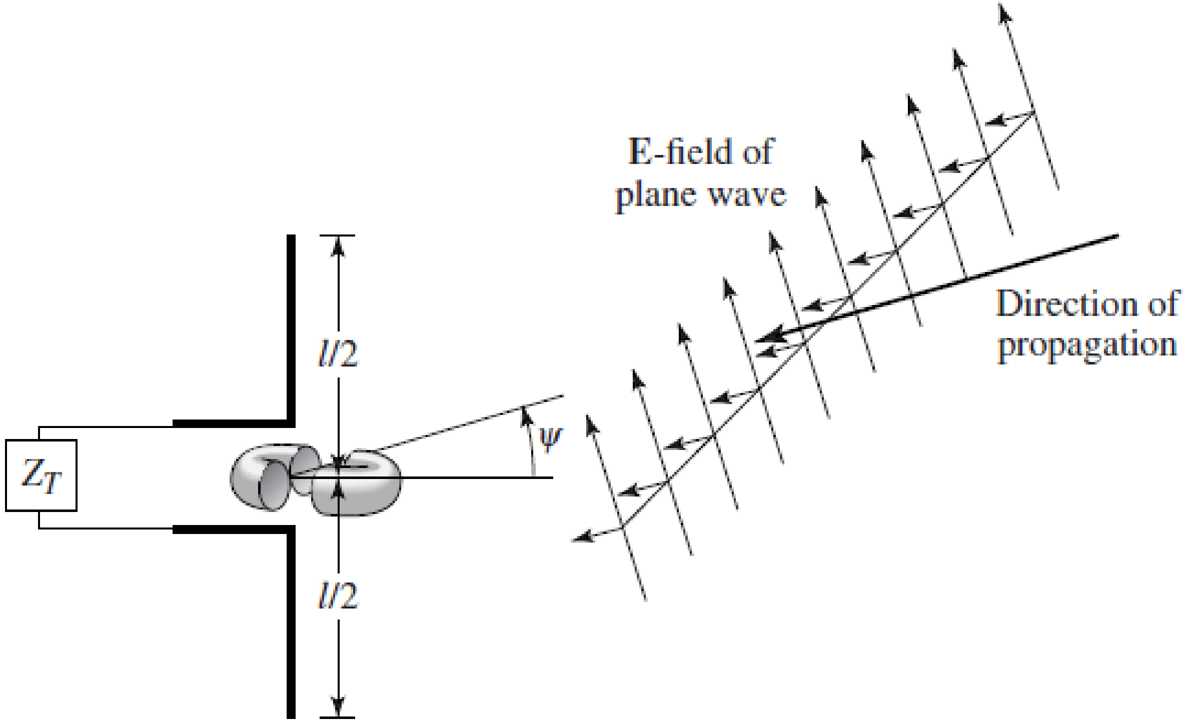
Module:1 EM Radiation and Antenna Parameters

- Radiation mechanism - single wire, two wire and current distribution, Hertzian dipole, Dipole and monopole - Radiation pattern, beam width, field regions, radiation power density, radiation intensity, directivity and gain, bandwidth, polarization, input impedance, efficiency, **antenna effective length and area, antenna temperature. Friis transmission equation, Radar range equation**

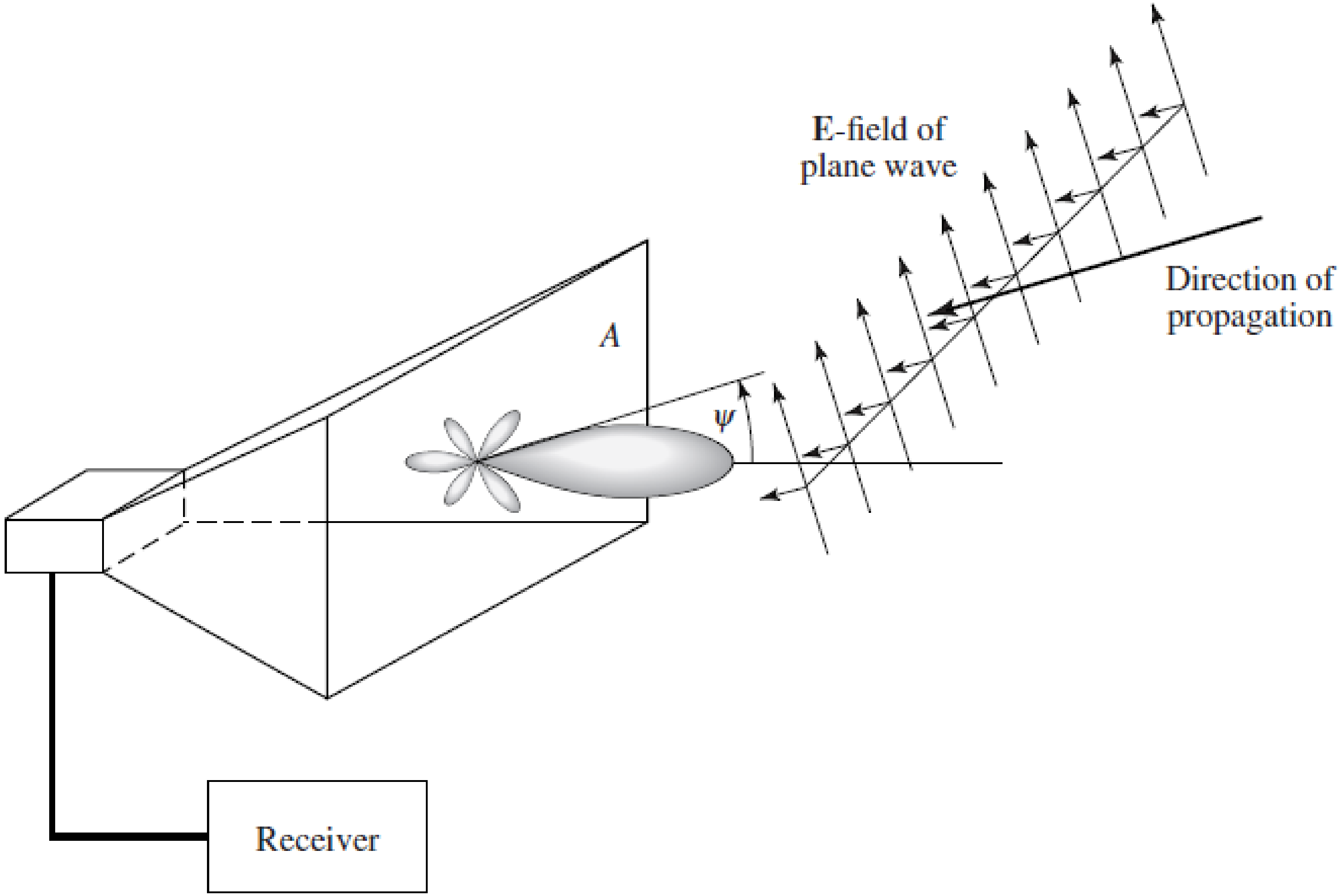
- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

1. ANTENNA VECTOR EFFECTIVE LENGTH AND EQUIVALENT AREAS

- For each antenna, an equivalent length and a number of equivalent areas can then be defined
- equivalent quantities are used to **describe the receiving characteristics of an antenna**, whether it be **a linear or an aperture type**, when a wave is incident upon the antenna.



(a) Dipole antenna in receiving mode



(b) Aperture antenna in receiving mode

2. Vector Effective Length

$$\boldsymbol{\ell}_e(\theta, \phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta, \phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta, \phi)$$

- **<u>effective length of an antenna (or *effective height*)</u>**
  (whether it be a linear or an aperture antenna)
  is a quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it.
- It is a far-field quantity and it is related to the *far-zone* field **E***a* radiated by the antenna, with current I_{in} in its terminals

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi = -j\eta \frac{kI_{in}}{4\pi r} \boldsymbol{\ell}_e e^{-jkr}$$

- useful in relating the open-circuit voltage *Voc* of receiving antennas.

$$V_{oc} = \mathbf{E}^i \cdot \boldsymbol{\ell}_e$$

V_{oc} = open-circuit voltage at antenna terminals

$\mathbf{E}^i$ = incident electric field

$\boldsymbol{\ell}_e$ = vector effective length

2. **Vector Effective Length**

- the *effective length of a linearly polarized antenna receiving a plane wave in a given direction* is defined as
“the ratio of the magnitude of the
open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization.
- the effective length is
the length of a thin straight conductor oriented perpendicular to the given direction and parallel to the antenna polarization, having a uniform current equal to that at the antenna terminals and producing the same far-field strength as the antenna in that direction

3. Antenna Equivalent Areas

- used to describe the power capturing characteristics of the antenna when a wave impinges on it.
- *effective area (aperture)*, which in a given direction is defined as "the ratio of
  the available power at the terminals of a receiving antenna to
  the power flux density of a plane wave incident on the antenna from that direction,
  the wave being polarization-matched to the antenna.
- If the direction is not specified, the direction of maximum radiation intensity is implied.

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

A_e = effective area (effective aperture) (m^2)

P_T = power delivered to the load (W)

W_i = power density of incident wave $(\mathrm{W/m}^2)$

3. Antenna Equivalent Areas

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

- maximum power transfer (*conjugate matching*), $R_r + R_L = R_T \text{ and } X_A = -X_T$,

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right]$$

- under conjugate matching
  only half of the captured power is delivered to the load; the other half is scattered and dissipated as heat.
- to account for the scattered and dissipated power we need to define, in addition to the effective area, the *scattering, loss* and *capture* equivalent areas

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

3. Antenna Equivalent Areas

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L+R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r+R_L}\right]$$

- The *scattering area* is defined as the equivalent area when multiplied by the incident power density is equal to the scattered or reradiated power

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L+R_r)^2}\right]$$

- which when multiplied by the incident power density gives the scattering power
- *loss area* is defined as the equivalent area, which when multiplied by the incident power density leads to the power dissipated as heat through *RL*.

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_L+R_r)^2}\right]$$

- which when multiplied by the incident power density gives the dissipated power

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_T/2}{W_i}$$

3. Antenna Equivalent Areas

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_L+R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r+R_L}\right]$$

- *capture area* is defined as the equivalent area, which when multiplied by the incident power density leads to the total power captured, collected, or intercepted by the antenna. Under conjugate matching

$$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T+R_r+R_L}{(R_L+R_r)^2}\right] \quad \textit{Capture Area} = \textit{Effective Area} + \textit{Scattering Area} + \textit{Loss Area}$$

- total capture area is equal to the sum of the other three

$$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}} \qquad (A_{em} \le A_p \text{ or } 0 \le \varepsilon_{ap} \le 1)$$

- *aperture efficiency* ε_{ap} of an antenna, which is defined as the ratio of the maximum effective area A_{em} of the antenna to its physical area A_p

3. MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

- Antenna 1 is used as a transmitter and 2 as a receiver
- The effective areas and directivities of each are designated A_t, A_r and D_t, D_r.
- If antenna 1 were isotropic, its radiated power density at a distance *R*

$$W_0 = \frac{P_t}{4\pi R^2}$$

P_t is the total radiated power.

- *For a directive antenna*

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2}$$

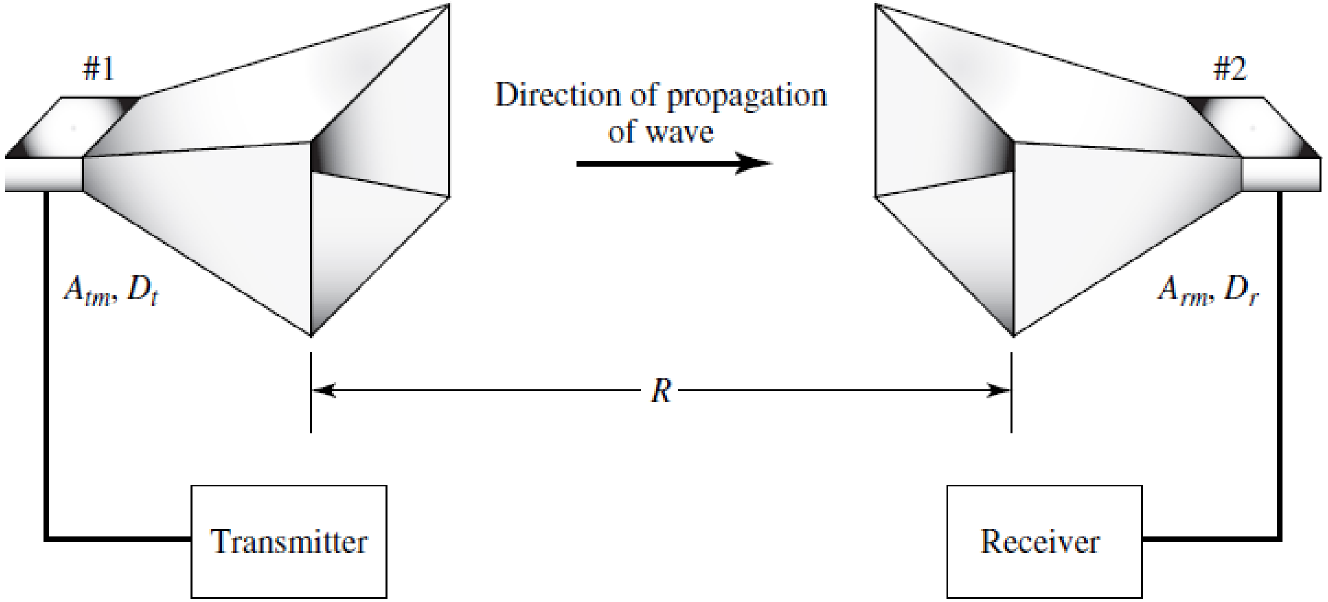


Figure 2.30 Two antennas separated by a distance *R*.

3. MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

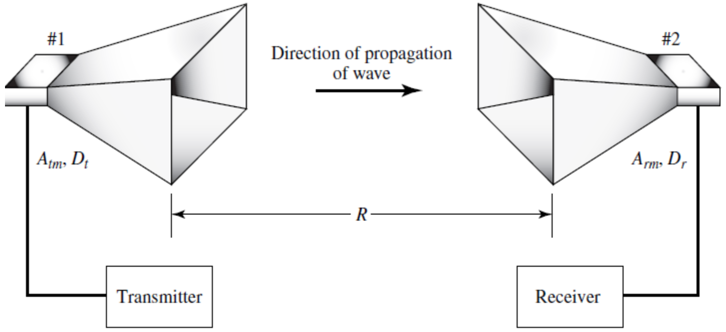


Figure 2.30 Two antennas separated by a distance R.

- power collected (received) by the antenna and transferred to the loac

$$P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2}$$

$$D_t A_r = \frac{P_r}{P_t}(4\pi R^2)$$

- antenna 2 is used as a transmitter, 1 as a receiver, and th intervening medium is linear, passive, and isotropic,

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2)$$

$$\frac{D_t}{A_t} = \frac{D_r}{A_r}$$

- Increasing the directivity of an antenna increases its effective area in direct proportion

$$\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}}$$

3. MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

- If antenna 1 is isotropic, then *D0t* = 1 and its maximum effective area

$$A_{tm} = \frac{A_{rm}}{D_{0r}}$$

- maximum effective area of the isotropic source is then equal to

$$A_{tm} = \frac{A_{rm}}{D_{0r}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}$$

$$A_{rm} = D_{0r}A_{tm} = D_{0r}\left(\frac{\lambda^2}{4\pi}\right)$$

- *maximum effective aperture* (*Aem*) *of any antenna is related to its maximum directivity* (*D0*)

$$\boxed{A_{em} = \frac{\lambda^2}{4\pi}D_0}$$

This assumes that there are no conduction-dielectric losses (radiation efficiency *ecd* is unity), the antenna is matched to the load (reflection efficiency *er* is unity), and the polarization of the impinging wave matches that of the antenna

3. MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

- If losses are accounted:

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right)D_0$$

- The maximum value assumes that the antenna is matched to the load and the incoming wave is polarization-matched to the antenna.
- If reflection and polarization efficiencies are included:

$$A_{em} = e_0\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

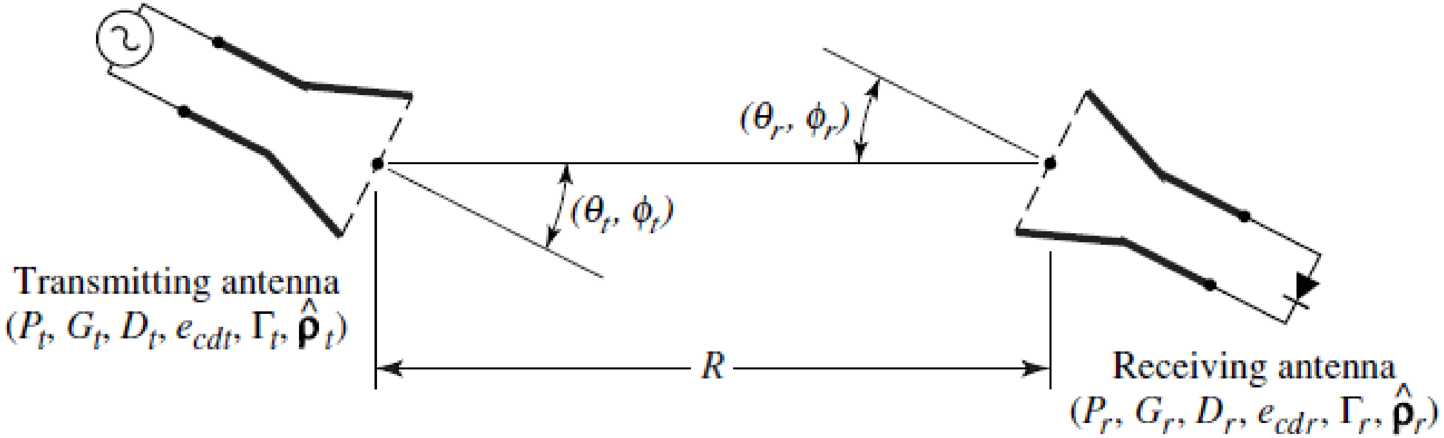
$$= e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right)D_0|\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

4. Friis Transmission Equation

- Friis Transmission Equation relates the power received to the power transmitted between two antennas separated by a distance $R > 2D^2/\lambda$
- D is largest dimension of antenna
- If the input power at the terminals of the transmitting antenna is *Pt*, then its **isotropic power density** *W*0 at distance *R* from the antenna is

$$W_0 = e_t \frac{P_t}{4\pi R^2}$$

with e_t: radiation efficiency of the transmitting antenna



4. Friis Transmission Equation

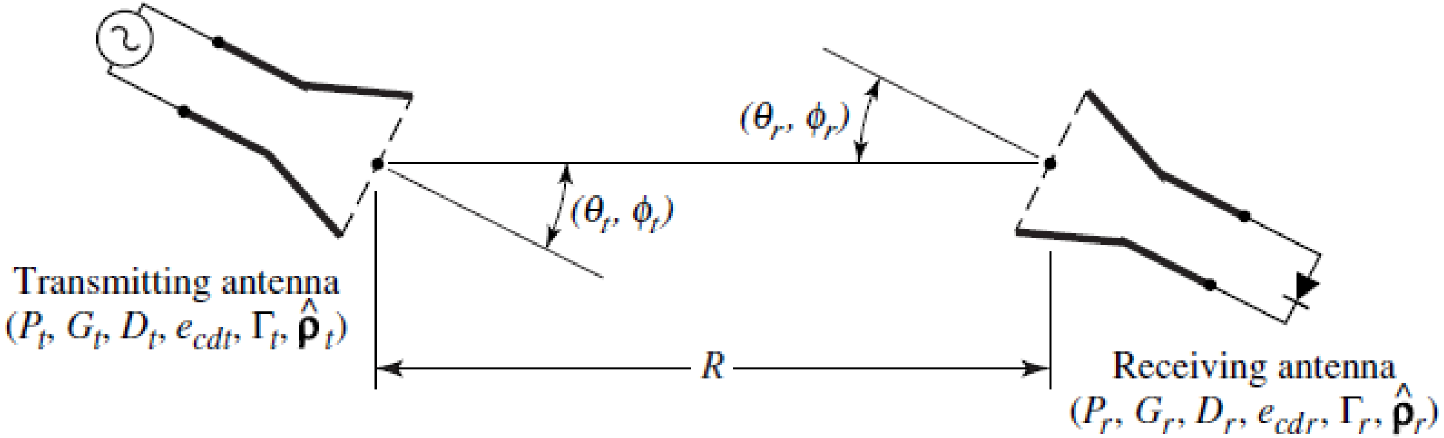
- **isotropic power density** $W_0 = e_t \dfrac{P_t}{4\pi R^2}$
- a non-isotropic transmitting antenna, the power density

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$

$G_t(\theta_t, \phi_t)$ is the gain and $D_t(\theta_t, \phi_t)$ is the directivity of the transmitting antenna

- effective area *Ar* of the receiving antenna is related to its efficiency *er* and directivity *Dr*

$$A_r = e_r D_r(\theta_r, \phi_r) \left(\frac{\lambda^2}{4\pi}\right)$$



4. Friis Transmission Equation

- the amount of power *Pr* collected by the receiving antenna

$$P_r = e_r D_r(\theta_r, \phi_r)\frac{\lambda^2}{4\pi} W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$

$$A_r = e_r D_r(\theta_r, \phi_r)\left(\frac{\lambda^2}{4\pi}\right)$$

- ratio of the received to the input power as

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2}$$

assumes that the transmitting and receiving antennas are matched to their respective lines or loads (reflection efficiencies are unity) and the polarization of the receiving antenna is polarization-matched to the impinging wave

- If these two factors are also included,

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

Two *lossless* X-band (8.2–12.4 GHz) horn antennas are separated by a distance of 100λ. The reflection coefficients at the terminals of the transmitting and receiving antennas are 0.1 and 0.2, respectively. The maximum directivities of the transmitting and receiving antennas (over isotropic) are 16 dB and 20 dB, respectively. Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2 W, and the antennas are aligned for maximum radiation between them and are polarization-matched, find the power delivered to the load of the receiver.

Two *lossless* X-band (8.2–12.4 GHz) horn antennas are separated by a distance of 100λ. The reflection coefficients at the terminals of the transmitting and receiving antennas are 0.1 and 0.2, respectively. The maximum directivities of the transmitting and receiving antennas (over isotropic) are 16 dB and 20 dB, respectively. Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2 W, and the antennas are aligned for maximum radiation between them and are polarization-matched, find the power delivered to the load of the receiver.

$$e_{cdt} = e_{cdr} = 1 \text{ because the antennas are lossless.}$$

$$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1 \text{ because the antennas are polarization-matched}$$

$$\left.\begin{array}{l} D_t = D_{0t} \\ D_r = D_{0r} \end{array}\right\} \begin{array}{l} \text{because the antennas are aligned for} \\ \text{maximum radiation between them} \end{array}$$

$$D_{0t} = 16\text{ dB} \Rightarrow 39.81 \text{ (dimensionless)}$$

$$D_{0r} = 20\text{ dB} \Rightarrow 100 \text{ (dimensionless)}$$

$$P_r = [1-(0.1)^2][1-(0.2)^2][\lambda/4\pi(100\lambda)]^2(39.81)(100)(2)$$

$$= 4.777 \text{ mW}$$

4. Friis Transmission Equation

- With mismatch in reflection and polarization:

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)|\hat{\rho}_t\cdot\hat{\rho}_r|^2$$

- For reflection and polarization-matched antennas aligned for maximum directional radiation and reception

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)}{(4\pi R)^2}$$

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t}G_{0r}$$

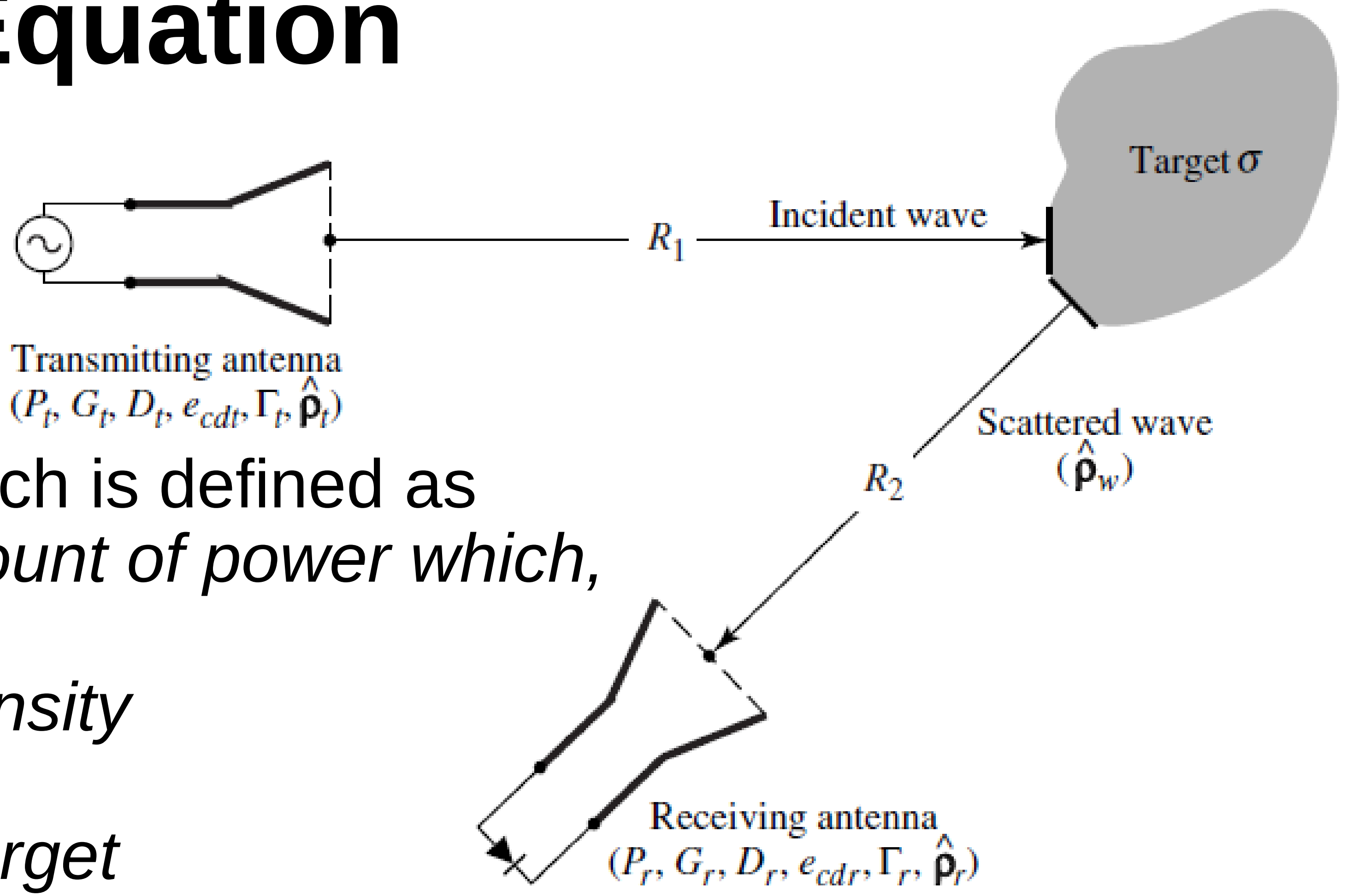
Friis Transmission Equation relates the power *Pr* (delivered to the receiver load) to the input power of the transmitting antenna *Pt*.

$\left(\frac{\lambda}{4\pi R}\right)^2$ is called the *free-space loss factor*

(loss due to spherical spreading of the energy by the antenna)

5. Radar Range Equation

- transmitted power is incident upon a target
- ***radar cross section*** or ***echo area*** **(σ) of a target** which is defined as *the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target*



$$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s$$

$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\mathbf{E}^s|^2}{|\mathbf{E}^i|^2}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\mathbf{H}^s|^2}{|\mathbf{H}^i|^2}\right]$$

σ = radar cross section or echo area (m^2)

R = observation distance from target (m)

W_i = incident power density (W/m^2)

W_s = scattered power density (W/m^2)

$\mathbf{E}^i$ ($\mathbf{E}^s$) = incident (scattered) electric field (V/m)

$\mathbf{H}^i$ ($\mathbf{H}^s$) = incident (scattered) magnetic field (A/m)

amount of captured power *Pc* is obtained by multiplying the incident power density of (2-114) by the radar cross section σ

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^2} = e_t \sigma \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R_1^2}$$

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^2} = e_t \sigma \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R_1^2}$$

- The **power captured by the target is reradiated isotropically**, and the **scattered power density** can be written as

$$W_s = \frac{P_c}{4\pi R_2^2} = e_{cdt} \sigma \frac{P_t D_t(\theta_t, \phi_t)}{(4\pi R_1 R_2)^2}$$

- The **amount of power delivered to the receiver load** is

$$P_r = A_r W_s = e_{cdt} e_{cdr} \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2$$

- *Ar* is the effective area of the receiving antenna

$$P_r = A_r W_s = e_{cdt} e_{cdr} \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2$$

- **ratio of the received power to the input power** (with only conduction-dielectric losses (radiation efficiency) of the transmitting and receiving antennas)

$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2}$$

- **include reflection losses (reflection efficiency)** and polarization losses (polarization loss factor or polarization efficiency)

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \times \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

$\hat{\rho}_w$ = polarization unit vector of the scattered waves

$\hat{\rho}_r$ = polarization unit vector of the receiving antenna

$$\frac{P_r}{P_t} = e_{cdt}e_{cdr}(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\sigma\frac{D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)}{4\pi} \times \left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

- For **polarization-matched antennas aligned for maximum directional radiation and reception**

$$\frac{P_r}{P_t} = \sigma\frac{G_{0t}G_{0r}}{4\pi}\left[\frac{\lambda}{4\pi R_1 R_2}\right]^2$$

- This is known as the ***Radar Range Equation***
- It relates the power *Pr* (delivered to the receiver load) to the input power *Pt* transmitted by an antenna, after it has been scattered by a target with a radar cross section (echo area) of σ.

6. Polarization Loss Factor and Efficiency

- Polarization mismatch occurs when transmitting and receiving antennas are of different polarization.
- E field of incoming wave is

$$\boldsymbol{E}_i = \hat{\boldsymbol{\rho}}_w E_i,$$

where $\hat{\boldsymbol{\rho}}_w$ is the unit vector of the wave. The polarization of the electric field of the receiving antenna can be expressed as

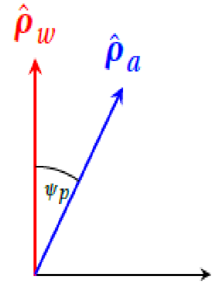
$$\boldsymbol{E}_a = \hat{\boldsymbol{\rho}}_a E_a,$$

where $\hat{\boldsymbol{\rho}}_a$ is its unit vector.

- The polarization loss can be taken into account by introducing a *polarization loss factor* (PLF). It is defined, based on the polarization of the antenna in its transmitting mode, as

$$\text{PLF} = |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 = |\cos\psi_p|^2.$$

where ψ_p is the angle between the two unit vectors.

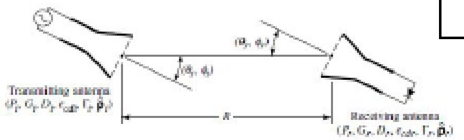


- The PLF is dimensionless and corresponds to the polarization efficiency e_p.

If reflection and polarization losses are also included, then the maximum effective area of is represented by

$$\boxed{\begin{aligned} A_{em} &= e_0 \left(\frac{\lambda^2}{4\pi}\right) D_0 |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 \\ &= e_{cd}(1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi}\right) D_0 |\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a|^2 \end{aligned}} \quad (2\text{-}112)$$

Friis Transmission Equation



$$\boxed{\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) \left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\boldsymbol{\rho}}_t \cdot \hat{\boldsymbol{\rho}}_r|^2}$$

7. ANTENNA TEMPERATURE

- Every object with a physical temperature above absolute zero (0 K = −273◦C) radiates energy
- amount of energy radiated is usually represented by an equivalent temperature *TB*, better known as brightness temperature, and it is defined as

$$T_B(\theta, \phi) = \varepsilon(\theta, \phi) T_m = (1 - |\Gamma|^2) T_m$$

T_B = brightness temperature (equivalent temperature; K)

ε = emissivity (dimensionless)

T_m = molecular (physical) temperature (K)

$\Gamma(\theta, \phi)$ = reflection coefficient of the surface for the polarization of the wave

7. ANTENNA TEMPERATURE

- Since the values of emissivity are $0 \le \varepsilon \le 1$, the maximum value the brightness temperature can achieve is equal to the molecular temperature.
- Emissivity: function of the frequency of operation, polarization of the emitted energy, and molecular structure of the object
- better natural emitters of energy at microwave frequencies are
  (a) the ground with equivalent temperature of about 300 K and
  (b) the sky with equivalent temperature of about 5 K when looking toward zenith and
  about 100–150 K toward the horizon

7. ANTENNA TEMPERATURE

- The brightness temperature emitted by the different sources is intercepted by antennas, and it appears at their terminals as an antenna temperature.

$$T_A = \frac{\int_0^{2\pi} \int_0^{\pi} T_B(\theta,\phi) G(\theta,\phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi} \int_0^{\pi} G(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

T_A = antenna temperature (effective noise temperature of the antenna radiation resistance; K)

$G(\theta,\phi)$ = gain (power) pattern of the antenna

7. ANTENNA TEMPERATURE

- Assuming no losses or other contributions between the antenna and the receiver, the noise power transferred to the receiver is given by

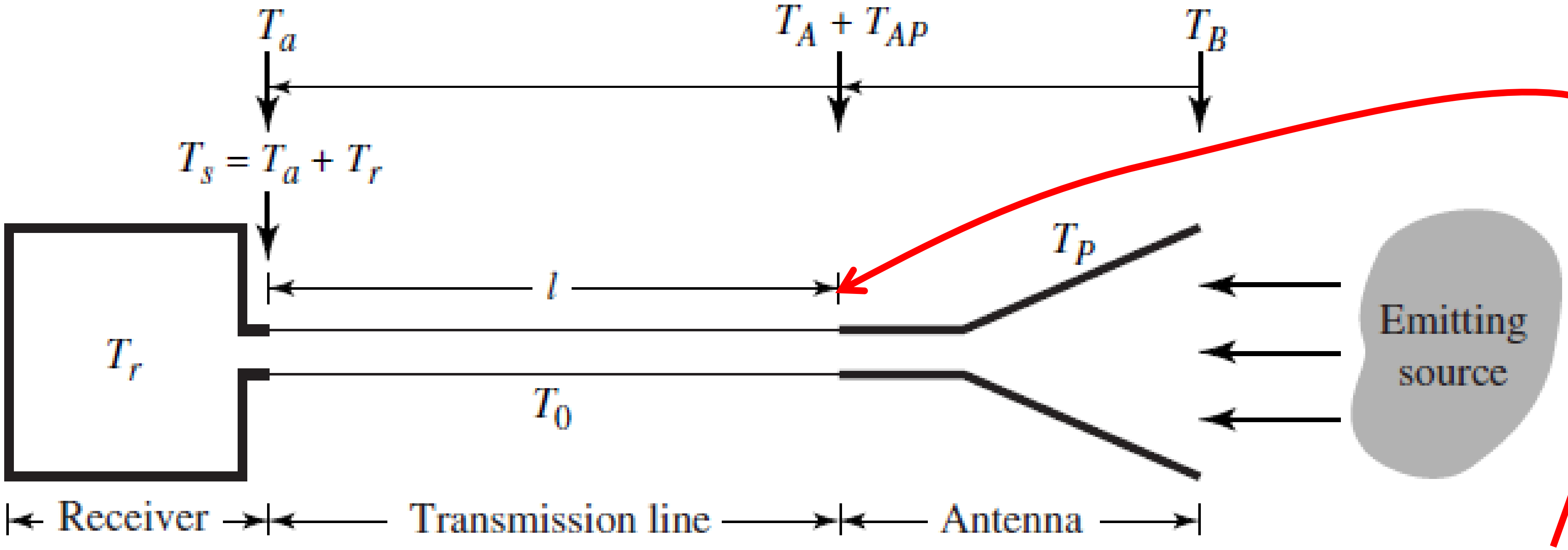
$$P_r = kT_A\Delta f$$

P_r = antenna noise power (W)

k = Boltzmann's constant (1.38×10^{-23} J/K)

T_A = antenna temperature (K)

Δf = bandwidth (Hz)



$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_p$$

T_a = antenna temperature at the receiver terminals (K)

T_A = antenna noise temperature at the antenna terminals (2-138) (K)

T_{AP} = antenna temperature at the antenna terminals due to physical temperature (2-140a) (K)

T_p = antenna physical temperature (K)

α = attenuation coefficient of transmission line (Np/m)

e_A = thermal efficiency of antenna (dimensionless)

l = length of transmission line (m)

T_0 = physical temperature of the transmission line (K)

7. ANTENNA TEMPERATURE

- The antenna noise power: $P_r = kT_a \Delta f$

T_a is the antenna temperature at the receiver input

- If receiver itself has a certain noise temperature *Tr* (due to thermal noise in the receiver components), the *system noise power at the receiver terminals* is given by

$$P_s = k(T_a + T_r)\Delta f = kT_s \Delta f$$

P_s = system noise power (at receiver terminals)

T_a = antenna noise temperature (at receiver terminals)

T_r = receiver noise temperature (at receiver terminals)

$T_s = T_a + T_r =$ effective system noise temperature (at receiver terminals)

The effective antenna temperature of a target at the input terminals of the antenna is 150 K. Assuming that the antenna is maintained at a thermal temperature of 300 K and has a thermal efficiency of 99% and it is connected to a receiver through an X-band (8.2–12.4 GHz) rectangular waveguide of 10 m (loss of waveguide = 0.13 dB/m) and at a temperature of 300 K, find the effective antenna temperature at the receiver terminals.

Solution: We first convert the attenuation coefficient from dB to Np by $\alpha(\text{dB/m})$ $20(\log_{10} e)\alpha(\text{Np/m}) = 20(0.434)\alpha(\text{Np/m}) = 8.68\alpha(\text{Np/m})$. Thus $\alpha(\text{Np/m}) = \alpha(\text{dB/m})/8.68$ $0.13/8.68 = 0.0149$. The effective antenna temperature at the receiver terminals can be writte using (2-140a) and (2-140), as

$$T_{AP} = 300\left(\frac{1}{0.99} - 1\right) = 3.03$$

$$T_a = 150e^{-0.149(2)} + 3.03e^{-0.149(2)} + 300[1 - e^{-0.149(2)}]$$

$$= 111.345 + 2.249 + 77.31 = 190.904 \text{ K}$$

$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_p$$

SUMMARY

Parameter	Formula
Infinitesimal area of sphere	$dA = r^2 \sin\theta \, d\theta \, d\phi$
Elemental solid angle of sphere	$d\Omega = \sin\theta \, d\theta \, d\phi$
Average power density	$\mathbf{W}_{av} = \frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*]$
Radiated power/average radiated power	$P_{rad} = P_{av} = \oint\!\!\oint_S \mathbf{W}_{av} \cdot d\mathbf{s} = \frac{1}{2} \oint\!\!\oint_S \mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot d\mathbf{s}$
Radiation density of isotropic radiator	$W_0 = \frac{P_{rad}}{4\pi r^2}$
Radiation intensity (far field)	$U = r^2 W_{rad} = B_0 F(\theta, \phi) \simeq \frac{r^2}{2\eta} \times \left[\lvert E_\theta(r, \theta, \phi) \rvert^2 + \lvert E_\phi(r, \theta, \phi) \rvert^2 \right]$

Directivity $D(\theta, \phi)$

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} = \frac{4\pi}{\Omega_A}$$

Beam solid angle Ω_A

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} F_n(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

$$F_n(\theta, \phi) = \frac{F(\theta, \phi)}{|F(\theta, \phi)|_{\max}}$$

Maximum directivity D_0

$$D_{\max} = D_0 = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}}$$

Partial directivities D_θ, D_ϕ

$$D_0 = D_\theta + D_\phi$$

$$D_\theta = \frac{4\pi U_\theta}{P_{rad}} = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi}$$

$$D_\phi = \frac{4\pi U_\phi}{P_{rad}} = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$

Approximate maximum directivity
(*one main lobe pattern*)

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41{,}253}{\Theta_{1d}\Theta_{2d}}$$

(Kraus)

$$D_0 \simeq \frac{32\ln 2}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2 + \Theta_{2r}^2} = \frac{72{,}815}{\Theta_{1d}^2 + \Theta_{2d}^2}$$

(Tai-Pereira)

Approximate maximum directivity
(*omnidirectional pattern*)

$$D_0 \simeq \frac{101}{\text{HPBW(degrees)} - 0.0027[\text{HPBW(degrees)}]^2}$$

(McDonald)

$$D_0 \simeq -172.4 + 191\sqrt{0.818 + \frac{1}{\text{HPBW(degrees)}}}$$

(Pozar)

Parameter	Formula
Gain $G(\theta, \phi)$	$G = \dfrac{4\pi U(\theta,\phi)}{P_{in}} = e_{cd}\left[\dfrac{4\pi U(\theta,\phi)}{P_{rad}}\right] = e_{cd}D(\theta,\phi)$ $P_{rad} = e_{cd}P_{in}$
Antenna radiation efficiency e_{cd}	$e_{cd} = \dfrac{R_r}{R_r + R_L}$
Loss resistance R_L (*straight wire/uniform current*)	$R_L = R_{hf} = \dfrac{l}{P}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$
Loss resistance R_L (*straight wire/ $\lambda/2$ dipole*)	$R_L = \dfrac{l}{2P}\sqrt{\dfrac{\omega\mu_0}{2\sigma}}$
Maximum gain G_0	$G_0 = e_{cd}D_{max} = e_{cd}D_0$

Partial gains G_θ, G_ϕ	$G_0 = G_\theta + G_\phi$
	$G_\theta = \dfrac{4\pi U_\theta}{P_{in}}, \quad G_\phi = \dfrac{4\pi U_\phi}{P_{in}}$
Realized gain G_{re}	$G_{re} = e_r G(\theta,\phi) = e_r e_{cd} D(\theta,\phi) = (1-\lvert\Gamma\rvert^2) e_{cd} D(\theta,\phi) = e_0 D(\theta,\phi)$
Total antenna efficiency e_0	$e_0 = e_r e_c e_d = e_r e_{cd} = (1-\lvert\Gamma\rvert^2) e_{cd}$
Reflection efficiency e_r	$e_r = (1-\lvert\Gamma\rvert^2)$
Beam efficiency BE	$\text{BE} = \dfrac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}$
Polarization loss factor (PLF)	$\text{PLF} = \lvert\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_a\rvert^2$
Vector effective length $\boldsymbol{\ell}_e(\theta,\phi)$	$\boldsymbol{\ell}_e(\theta,\phi) = \hat{\mathbf{a}}_\theta l_\theta(\theta,\phi) + \hat{\mathbf{a}}_\phi l_\phi(\theta,\phi)$
Polarization efficiency p_e	$p_e = \dfrac{\lvert\boldsymbol{\ell}_e \cdot \mathbf{E}^{inc}\rvert^2}{\lvert\boldsymbol{\ell}_e\rvert^2 \lvert\mathbf{E}^{inc}\rvert^2}$

Antenna impedance Z_A

$$Z_A = R_A + jX_A = (R_r + R_L) + jX_A$$

Maximum effective area A_{em}

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r + R_L}\right] = e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$
$$= \left(\frac{\lambda^2}{4\pi}\right) G_0 |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

Aperture efficiency ε_{ap}

$$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$$

Friis transmission equation

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

Radar range equation

$$\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi}\left[\frac{\lambda}{4\pi R_1 R_2}\right]^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

Parameter	Formula
Radar cross section (RCS) (m^2)	$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{\lvert\mathbf{E}^s\rvert^2}{\lvert\mathbf{E}^i\rvert^2}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{\lvert\mathbf{H}^s\rvert^2}{\lvert\mathbf{H}^i\rvert^2}\right]$
Brightness temperature $T_B(\theta,\phi)$ (K)	$T_B(\theta,\phi) = \varepsilon(\theta,\phi)T_m = (1-\lvert\Gamma\rvert^2)T_m$
Antenna temperature T_A (K)	$T_A = \dfrac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)G(\theta,\phi)\sin\theta\, d\theta\, d\phi}{\int_{0\pi}^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta\, d\theta\, d\phi}$