Module 2 Linear Arrays and Planar Arrays

Two element array, N-element linear array - broadside array, End fire array - Directivity, radiation pattern, pattern multiplication. Non-uniform excitation - Binomial, Chebyshev distribution

Arrays: Planar array, circular array, Phased Array antenna (Qualitative study)

Array definition

Configuration of multiple radiating elements arranged to yield given radiation pattern.

Why Array?

- Long distance communication – directive antenna
- Single element - wide radiation characteristics
- Solution
- Increase electrical size

Array Design Variables

1. General Array Shape (Linear, Circular, Planar etc
2. Element Spacing
3. Element excitation Amplitude
4. Element excitation phase
5. Patterns of array elements

Array Types

Depending on

- Elements (Wire, Printed, Slot, Horn)
- Application (Communication, mobile, satellite, Radar)
- Geometry (Linear, Planar, Conformal)
- Network (Passive, Active, Adaptive)

Array Types: Elements

Depending on elements

wires – wire array antenna

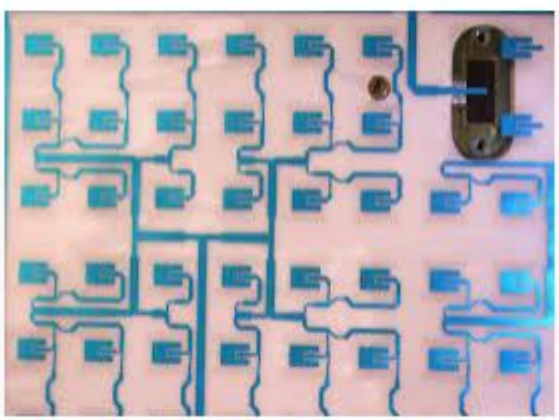
Printed elements – printed array antenna

Slot – slot array antenna

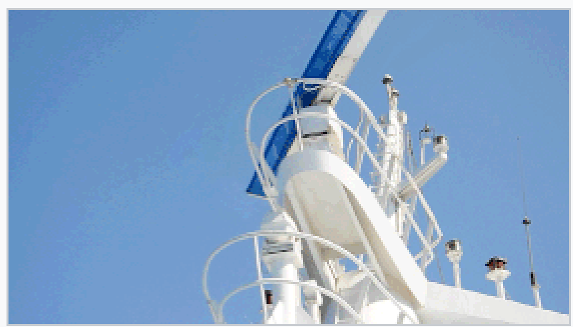
Horn – Horn array antenna

Log Periodic Dipole Array 13 elem...

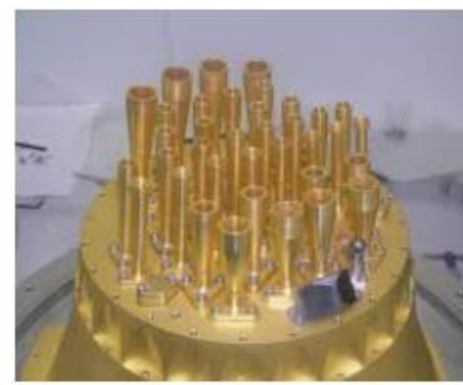
...qrznow.com



Microstrip antenna - Wikipedia
en.wikipedia.org



X-band slotted waveguide marine radar antenna on ship, 8 - 12 GHz. The antenna radiates a narrow vertical fan-shaped beam of microwaves, scanning the entire 360° water surface around the ship with each rotation.



Corrugated Horns
terahertz.co.uk

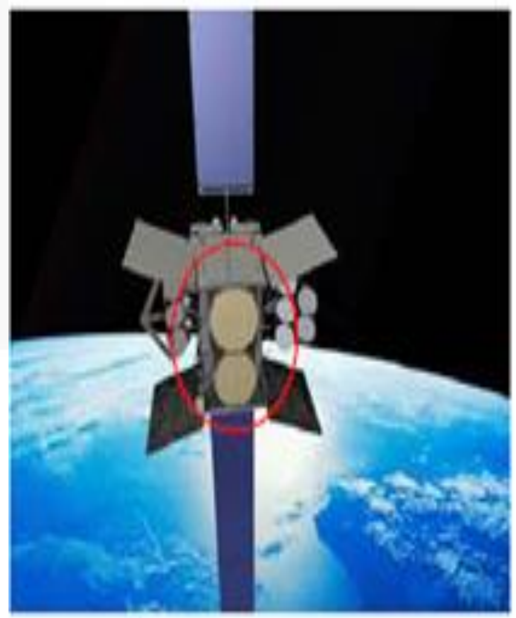
For General Information https://en.wikipedia.org/wiki/Slot_antenna

Array Types: Application

Depending on Application:
-Communication (mobile, satellite)
-Radar

Turnstile antenna array used for satellite communication

Array gain - Wikipedia
en.wikipedia.org



Air Force eyes passive radar ...
militaryaerospace.com

For General Information

Array Types: Geometry

- **Depending on it geometry**
  - Linear
  - Planar
  - Conformal
    - » Cylindrical
    - » Spherical

This classification depends on the position where the different elements are placed:

- Linear (elements in a line)
- Planar (elements in a plane): rectangular (elements in a rectangular shape), triangular (elements in a triangle shape, circular (elements on concentric circumferences)
- Conformal (elements in a 3D-surface): cylinder, sphere...

For General Information

Array Types: Geometry (Linear Arrays)

Base station antennas for mobile systems application: DECT (3.5 GHz): Vertical 65°, 90° antennas

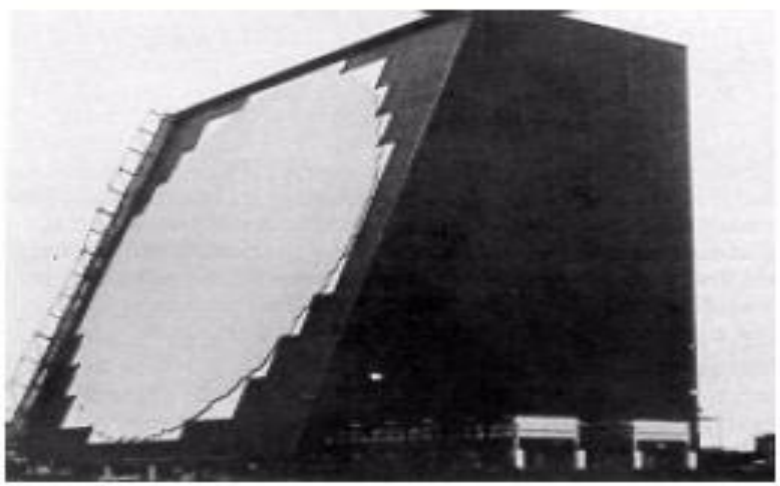
Base station antennas for mobile systems applications: GSM 1800 MHz: Vertical pol.° sectorial 65° & 90° antennas

Base station antennas for mobile systems applications: UMTS: crosspolar ± 45° sectorial 65° antennas

The printed antennas have the advantage to be easy to fabricate and low cost

For General Information Google

Array Types: Geometry (Planar Arrays)



- Satcom antenna

  -airborne radar technology for satellite communications placed on the F16

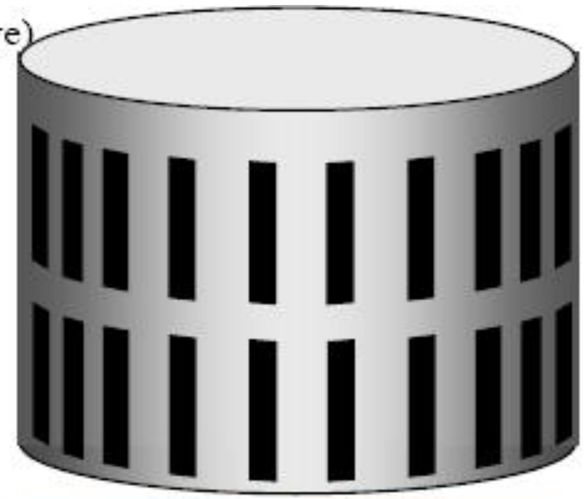
- Cobra Dane
  - A big antenna formed of 34769 radiating elements
  - works at 1200 MHz
  - part of the security radar in USA

For General Information Google

Array Types: Geometry (Conformal Arrays)

Radiating elements placed on a no planar surface (for example curve)

- Cylindrical (Elements placed over a cylinder)
- Conical (Elements placed over a cone)
- Spherical (Elements placed on a sphere)
- Different surfaces (flight wings, vehicle, etc.)



Example: Cylindrical array of slots

Omnidirectional Circularly Polarized Slot Antenna Fed by a Cylindrical Waveguide for Identification Friend or Foe System in the 37GHz band

Circularly Polarized Omnidirectional Millimeter Wave Monopole with Parasitic Strip Elements for Identification Friend or Foe System

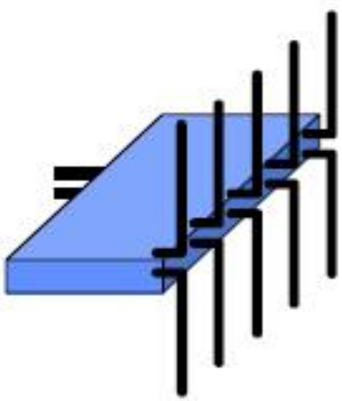
For General Information Google

Array Types: Network

- **Depending on the network**
  - Passive
    - » A single beam
    - » Multibeam
  - Active
  - Adaptative

Array Types: Network (Passive Arrays)

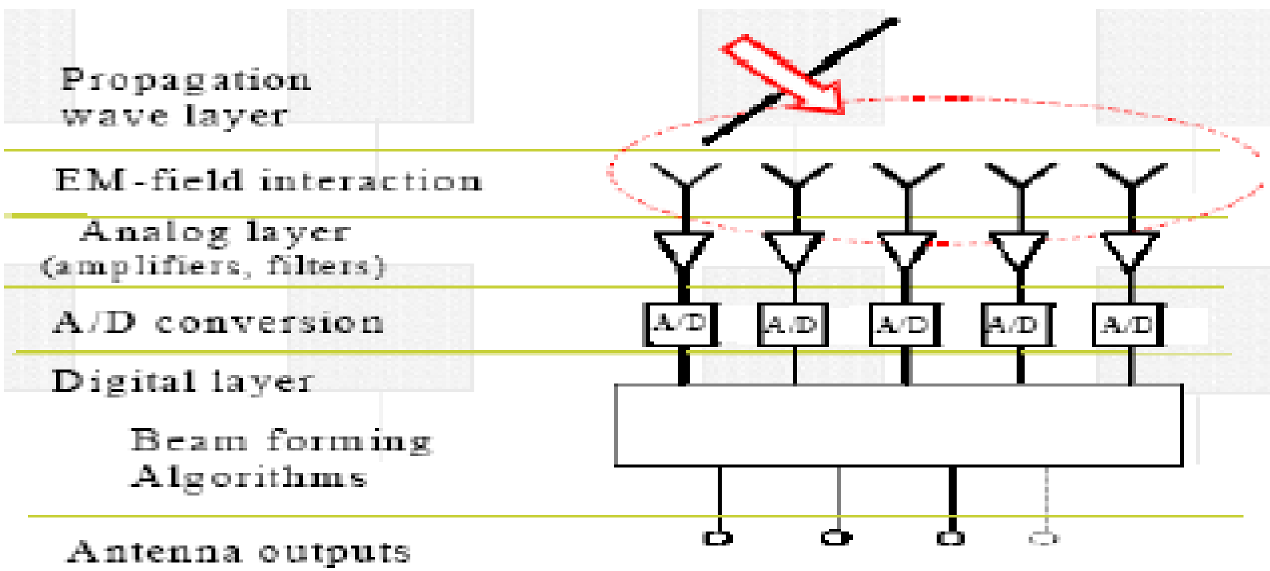
- Use a feeding network with passives elements (power divider, transmission lines, matching network etc.)
  - The radiation pattern and polarization are fixed.
  - Work as a unique antenna.
  - Depending on the network
    - » A single beam
    - » multibeam
  - Can have different input terminals in the network (multi-diagram or multibeam antenna).
  - Are reciprocal, works in transmission and reception.



For General Information

Google

Array Types: Network (Active Arrays)

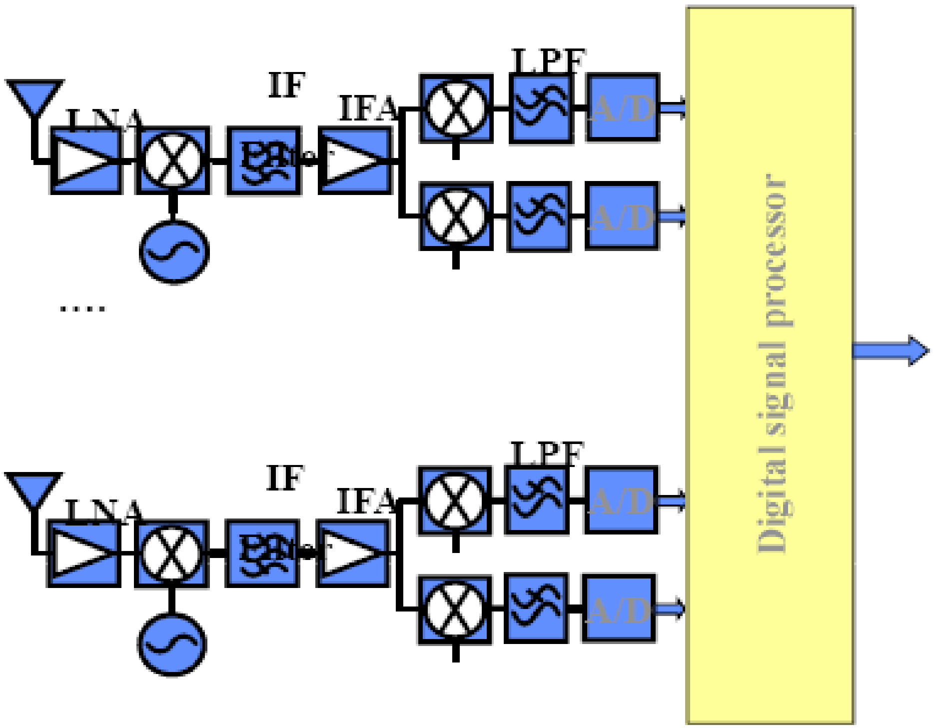


The active arrays are antennas with variable phase, that allow beam steering in a variable direction (very useful in Radar systems).

For General Information

Array Types: Network (Adaptive Arrays)

- A digital processor allow:
  - Digital control of patterns
  - Patterns dependent on
    - » frequency
    - » time
    - » code
  - Simultaneous variables patterns



The adaptative arrays are kind of antennas that works with active feeding modifying instantaneously the radiation pattern depending on the signal that it receives (These antennas are <u>very useful in communication systems</u>)

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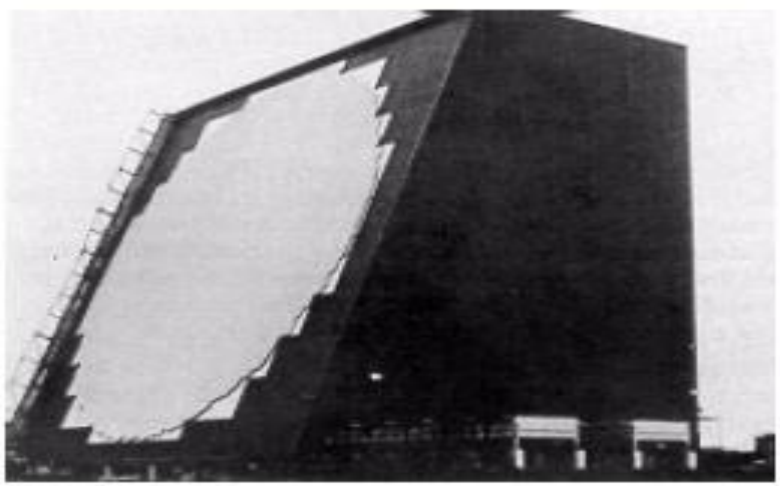

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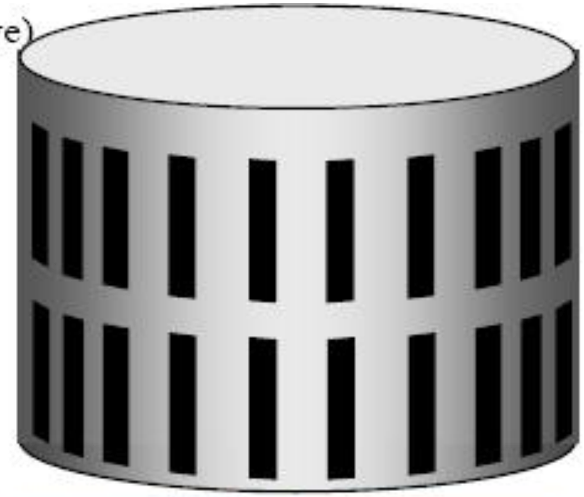

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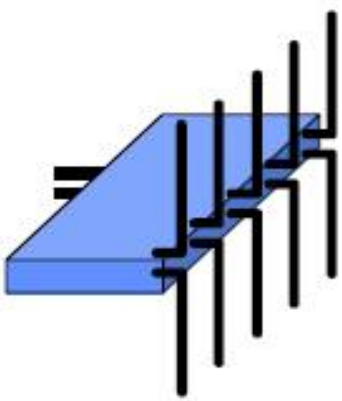
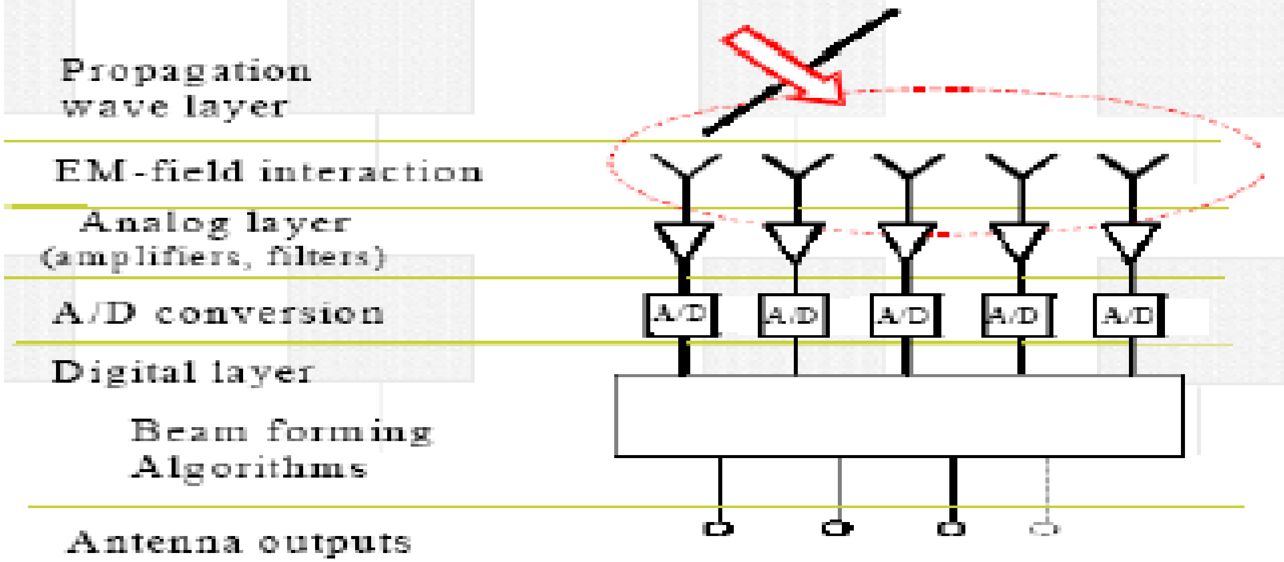


For General Information

Array Types: Network (Active Arrays)



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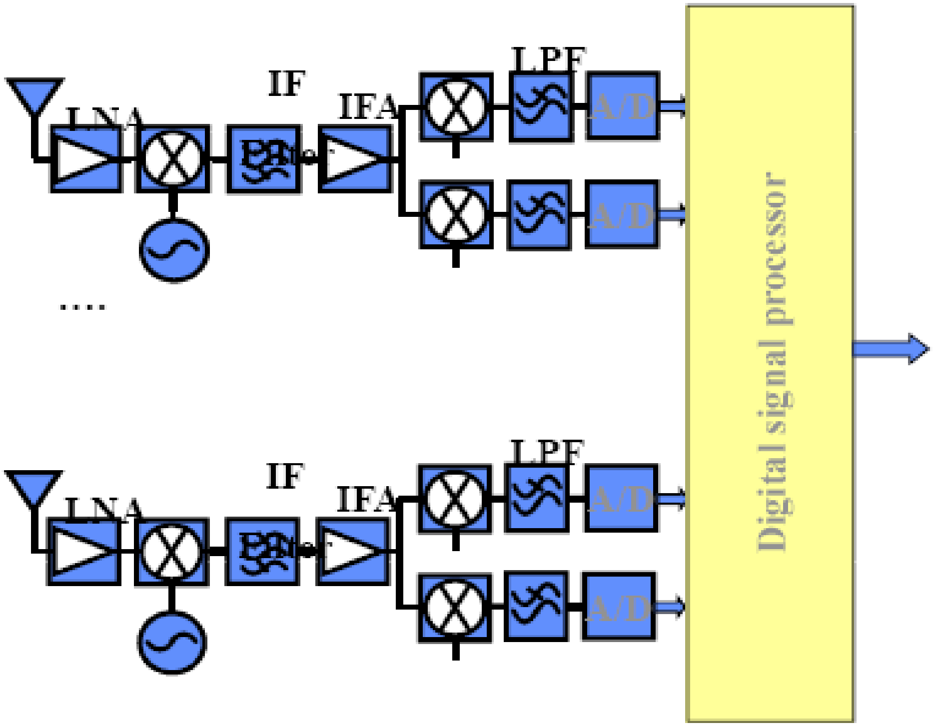

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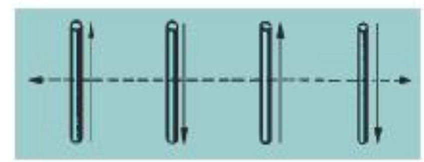
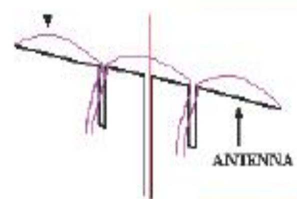
For General Information

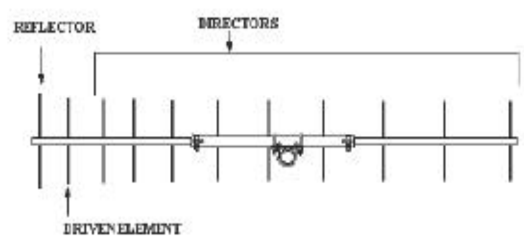
Antenna Arrays

- In general, by grouping the similar antennas in a particular way – we can increase the radiated power, gain and directivity. However, the specific phase shift and spacing should be calculated to optimize the antenna array.
- Based on the phase between each element of array, we can group the antenna in to two types:
  - Broadside array (zero phase difference)
  - Endfire array (180 degree phase difference)
- On the other hand, based on the spacing between elements – we can divide as:
  - Equal spacing
  - Unequal spacing



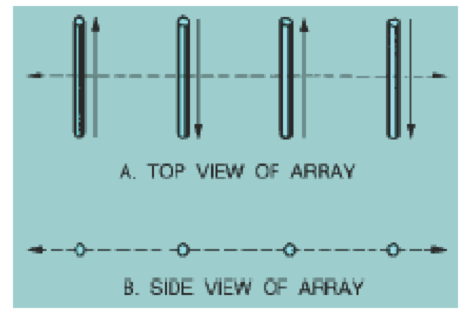
- Other than this, there are few other types
  - Collinear arrays
  - Parasitic arrays
  - Phased scanning arrays



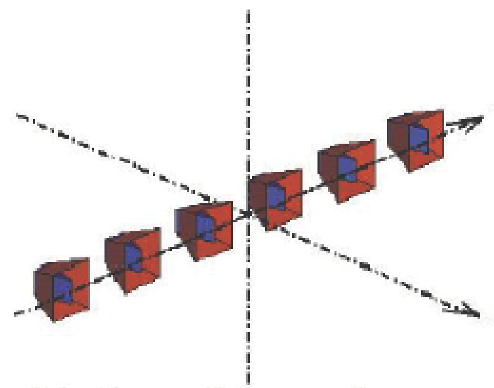


Linear antenna arrays

- Linear arrays are defined as “equal amplitude of power will be given to all elements with equal spacing between each”
- In a particular line, all the elements will be arrange, that line is called array axis



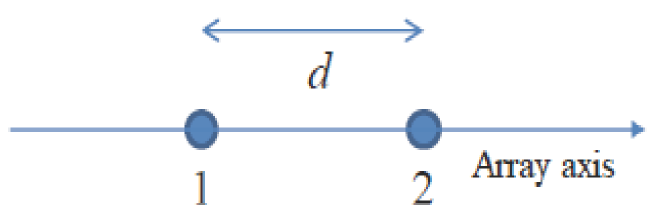
- Broadside:
  - Equal amplitude
  - Equal phase
  - Maximum radiation at perpendicular to array axis
- Endfire
  - Equal amplitude
  - 180 degree phase
  - Maximum radiation along the array axis



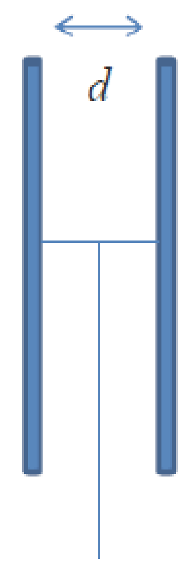
Uniform Linera Array

Two element array

- This is the least possible array and simplest
- Consider two identical isotropic (equal radiation in all direction) antennas are place at a distance "*d*"
- The radiation pattern can be obtained for three possible situations
  - Equal amplitude and phase
  - Equal amplitude with opposite phase
  - Unequal amplitude and any phase (Non-linear)



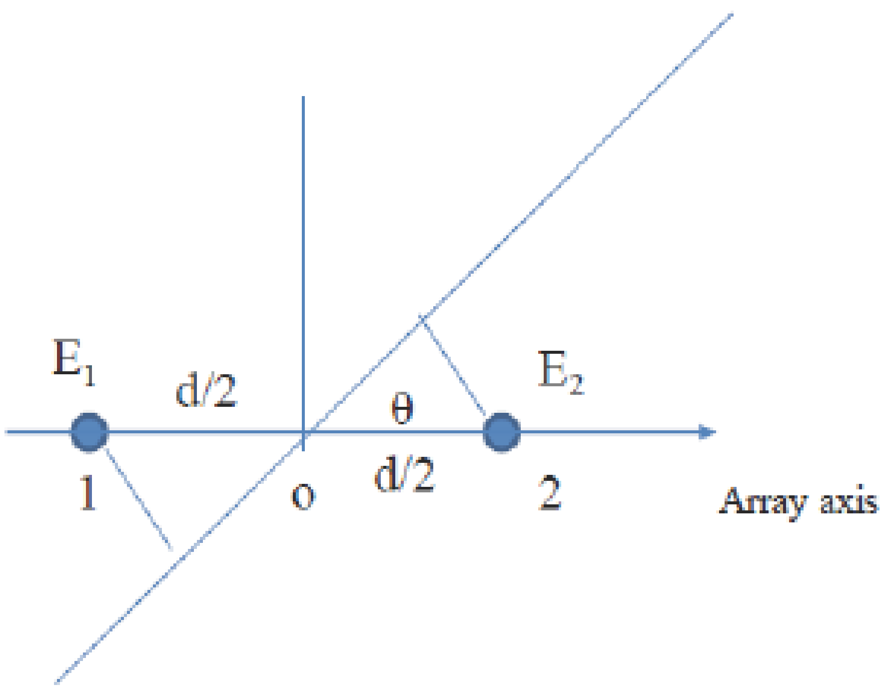
Cross section view



Side view

General case

- The same problem can be considered as "the two isotropic radiated are symmetric with respect to the origin "*o*" and located *d/2* distance from origin"
- If we expect the electric field at a point "P" located a distance "r", then with respect to the point, the phase difference between isotropic sources is



$$path\ \ \text{difference} = \frac{1}{\lambda}\left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right)$$

and

$$\text{phase difference} = 2\pi \times \text{path difference}$$

$$\Psi = 2\pi\left(\frac{d}{\lambda}\cos\theta\right)$$

$$= \beta\, d\cos\theta \ \text{ radians}$$

- Now we can easily derive the expression for total electric field due to two isotropic sources
- Be clear that with respect to the path, one is phase lag and another one is phase lead, thus

$$E_{tot} = E_1.e^{-j\Psi/2} + E_2.e^{+j\Psi/2}$$

- Now case-1, if both E1 and E2 are equal amplitude and phase, then

$$E_1 = E_2 = E_0$$

$$then$$

$$E_{tot} = E_0\left(e^{-j\Psi/2} + e^{+j\Psi/2}\right)$$

$$= 2E_0\left(\frac{e^{-j\Psi/2} + e^{+j\Psi/2}}{2}\right)$$

$$\left(\frac{e^{-j\Psi/2} + e^{+j\Psi/2}}{2}\right) = \cos(\Psi/2)$$

Resulting Radiation pattern

$$\mathrm{E}_{\mathrm{tot}} = 2E_0\cos(\Psi/2)$$

$$= 2E_0\cos\left(\frac{\beta d\cos\theta}{2}\right)$$

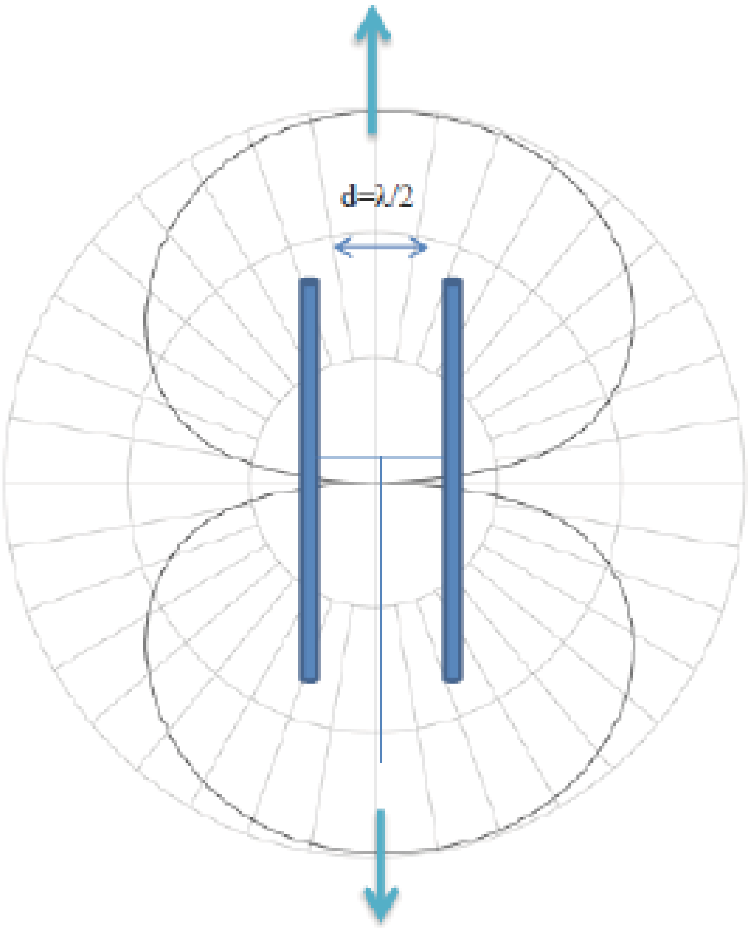
Finding maxima and minima direction

- Maxima: condition - - > $2E_0=1$
- That's $\cos(\Psi/2)=1$
- If $d=\lambda/2$
- Where $\Psi=\beta d \cos(\theta) = (2\pi/\lambda).(\lambda/2).\cos\theta$

- $\cos(0.5\,\pi \cos\theta_{max})=1$
- $\theta_{max}=90^o$ and 270^o

- Similarly, minima, $\theta_{min}=0^o$ and 180^o

So called
"Broadside Radiation" / "Broadside Array"



- Case-2: Equal amplitude and opposite phase
- From the general case,

$$E_1 = E_0, \quad E_2 = -E_0$$

$$then$$

$$E_{tot} = E_0\left(e^{-j\Psi/2} - e^{+j\Psi/2}\right)$$

$$= 2E_0\left(\frac{e^{-j\Psi/2} - e^{+j\Psi/2}}{2}\right)$$

$$\mathrm{E}_{tot} = 2jE_0 \sin(\Psi/2)$$

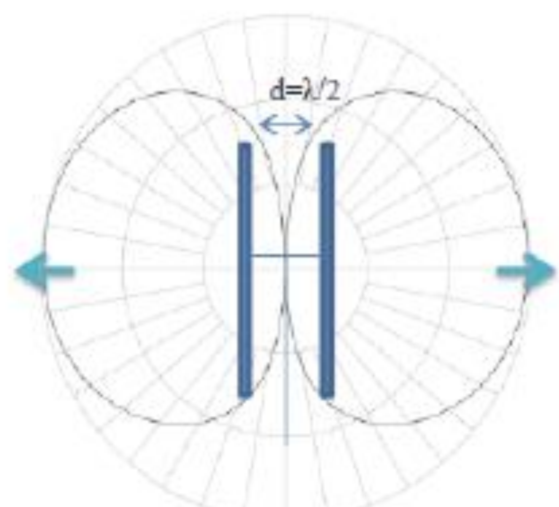
$$= 2jE_0 \sin\left(\frac{\beta d \cos\theta}{2}\right)$$

- Maxima: condition --> $2E_0=1$
- That's $\sin(\Psi/2)=1$
- If $d=\lambda/2$
- Where $\Psi=\beta d \cos(\theta) = (2\pi/\lambda).(\lambda/2).\cos\theta_{max}$

- $\sin(0.5\,\pi \cos\theta_{max})=1$
- $\theta_{max}=90^\circ$ and 270°

- Similarly, minima, $\theta_{min}=0^\circ$ and 180°

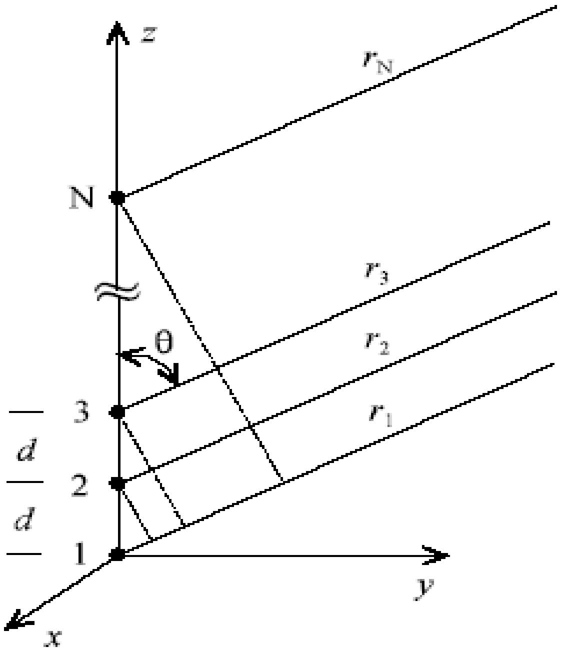
So called
"Endfire Radiation" / "Endfire Array"



N -Element Linear Array

- E field at the origin is

$$E_\theta = I_o \frac{e^{-jkr}}{4\pi r}$$



In the far field of the array

$$r_1 = r$$
$$r_2 \approx r - d\cos\theta$$
$$r_3 \approx r - 2d\cos\theta$$
$$\vdots$$
$$r_N \approx r - (N-1)d\cos\theta$$

Contd…

Uniform N-Element Linear Array

A *uniform array* is defined by uniformly-spaced identical elements of equal magnitude with a linearly progressive phase from element to element.

$$\phi_1 = 0 \qquad \phi_2 = \alpha \qquad \phi_3 = 2\alpha \qquad \cdots \qquad \phi_N = (N-1)\alpha$$

Inserting this linear phase progression into the formula for the general *N*-element array gives

$$AF = \left[1 + e^{j(\alpha + kd\cos\theta)} + e^{j2(\alpha + kd\cos\theta)} + \dots + e^{j(N-1)[\alpha + kd\cos\theta]}\right]$$

$$= \left[1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(N-1)\psi}\right] \qquad (\psi = \alpha + kd\cos\theta)$$

$$= \sum_{n=1}^{N} e^{j(n-1)\psi}$$

The function ψ is defined as the *array phase function* and is a function of the element spacing, phase shift, frequency and elevation angle. If the array factor is multiplied by $e^{j\psi}$, the result is

$$(AF)e^{j\psi} = \left[e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{jN\psi}\right]$$

Subtracting the array factor from the equation above gives

$$AF(e^{j\psi} - 1) = (e^{jN\psi} - 1)$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{e^{jN\frac{\psi}{2}}}{e^{j\frac{\psi}{2}}} \frac{e^{jN\frac{\psi}{2}} - e^{-jN\frac{\psi}{2}}}{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}} = e^{j(N-1)\psi/2} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

The complex exponential term in the last expression of the above equation represents the phase shift of the array phase center relative to the origin. If the position of the array is shifted so that the center of the array is located at the origin, this phase term goes away.

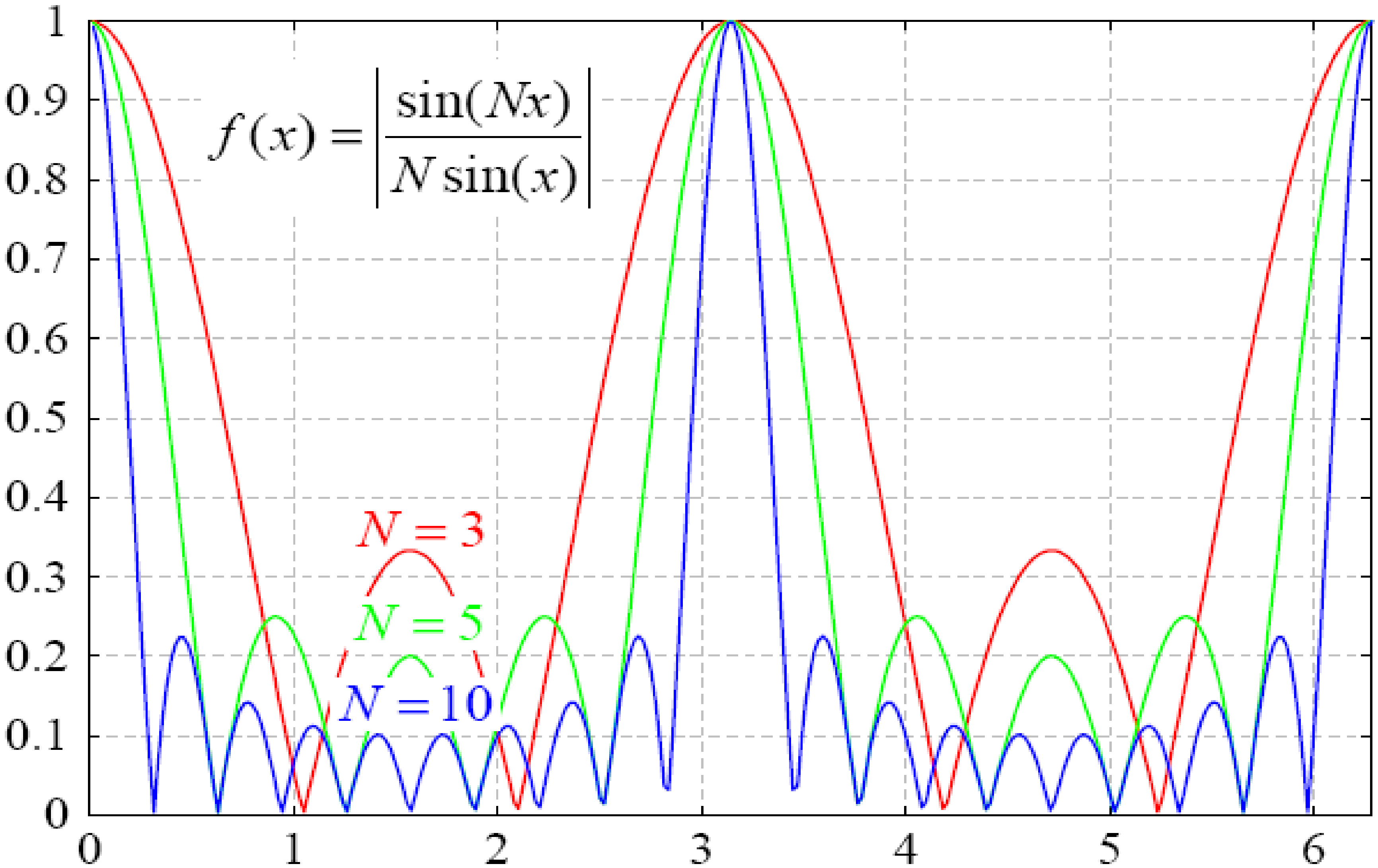
Contd…

The array factor then becomes

$$AF = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

Below are plots of the array factor AF vs. the array phase function ψ as the number of elements in the array is increased. **Note that these are not plots of AF vs. the elevation angle θ.**

Contd…



$f(x) = \left| \frac{\sin(Nx)}{N\sin(x)} \right|$
$N = 3$
$N = 5$
$N = 10$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6

Contd…

Some general characteristics of the array factor AF with respect to ψ:

(1) $[AF]_{max} = N$ at $\psi = 0$ (main lobe).

(2) Total number of lobes = N−1 (one main lobe, N−2 sidelobes).

(3) Main lobe width = $4\pi/N$, minor lobe width = $2\pi/N$

The array factor may be normalized so that the maximum value for any value of N is unity. The normalized array factor is

$$(AF)_n = \frac{1}{N}\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

Contd…

Broadside

FIRST-NULL BEAMWIDTH (FNBW) $\Theta_n = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$

- Directivity

$$\boxed{D_0 = \frac{U_{\max}}{U_0} \approx \frac{N\beta d}{\pi} = 2N\left(\frac{d}{\lambda}\right)}$$

FIRST SIDE LOBE BEAMWIDTH (FSLBW) $\Theta_s \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2dN}\right)\right]$

$\pi d/\lambda \ll 1$

- Maxima points

$$(\theta_{\max}) = \cos^{-1}\left\{\left[\pm\frac{m\lambda}{d}\right]\right\}$$

$m = 0, 1, 2, 3, \ldots.$

- Null points

$$(\theta_{null}) = \cos^{-1}\left\{\left[\pm\frac{n}{N}\frac{\lambda}{d}\right]\right\}$$

$n = 1, 2, 3\ldots.$

$\mathrm{n} \neq N, 2N, \ldots$

- Approximate Half power points

$$(\theta_h) = \cos^{-1}\left[\frac{1.391\lambda}{\pi Nd}\right]$$

Length of the Array L=(n-1)d

Endfire

FIRST-NULL BEAMWIDTH (FNBW) $\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$

- Directivity

$$\boxed{D_0 = \frac{U_{\max}}{U_0} \approx \frac{2N\beta d}{\pi} = 4N\left(\frac{d}{\lambda}\right)}$$

FIRST SIDE LOBE BEAMWIDTH (FSLBW) $\Theta_s \simeq 2\cos^{-1}\left(1 - \frac{3\lambda}{2Nd}\right)$

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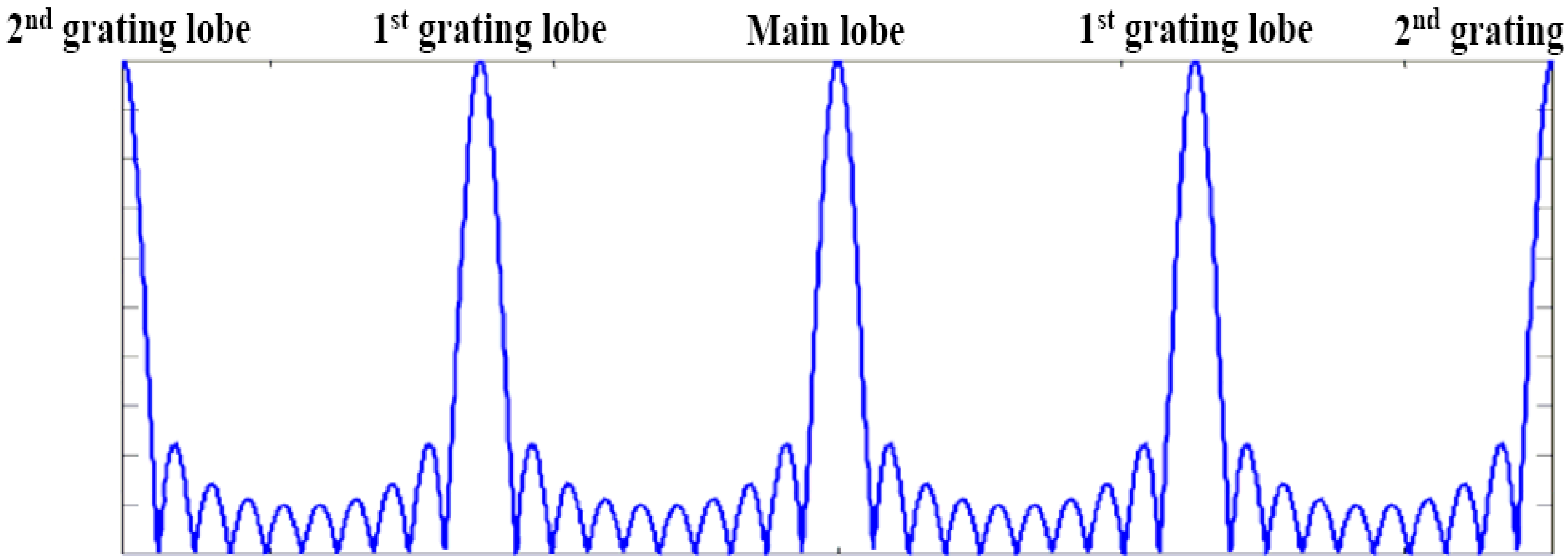
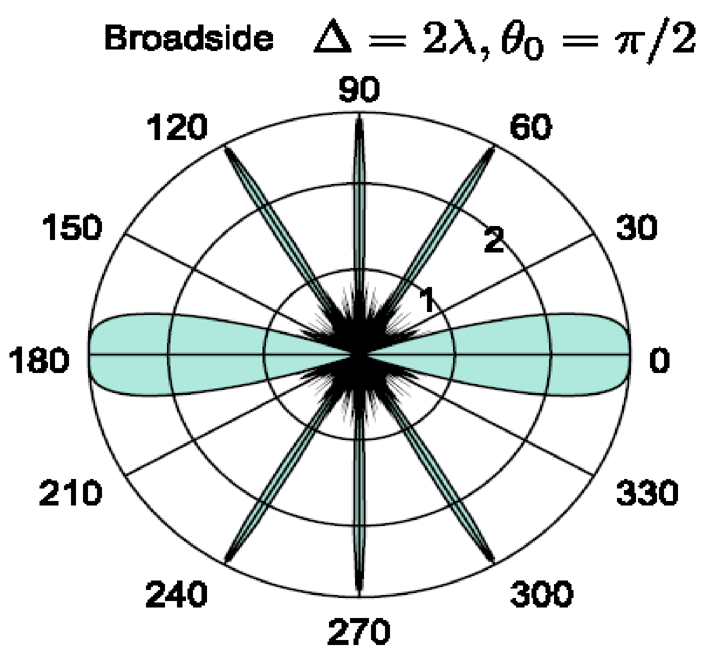




The array factor may be normalized so that the maximum value for any value of N is unity. The normalized array factor is

$$(AF)_n = \frac{1}{N}\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

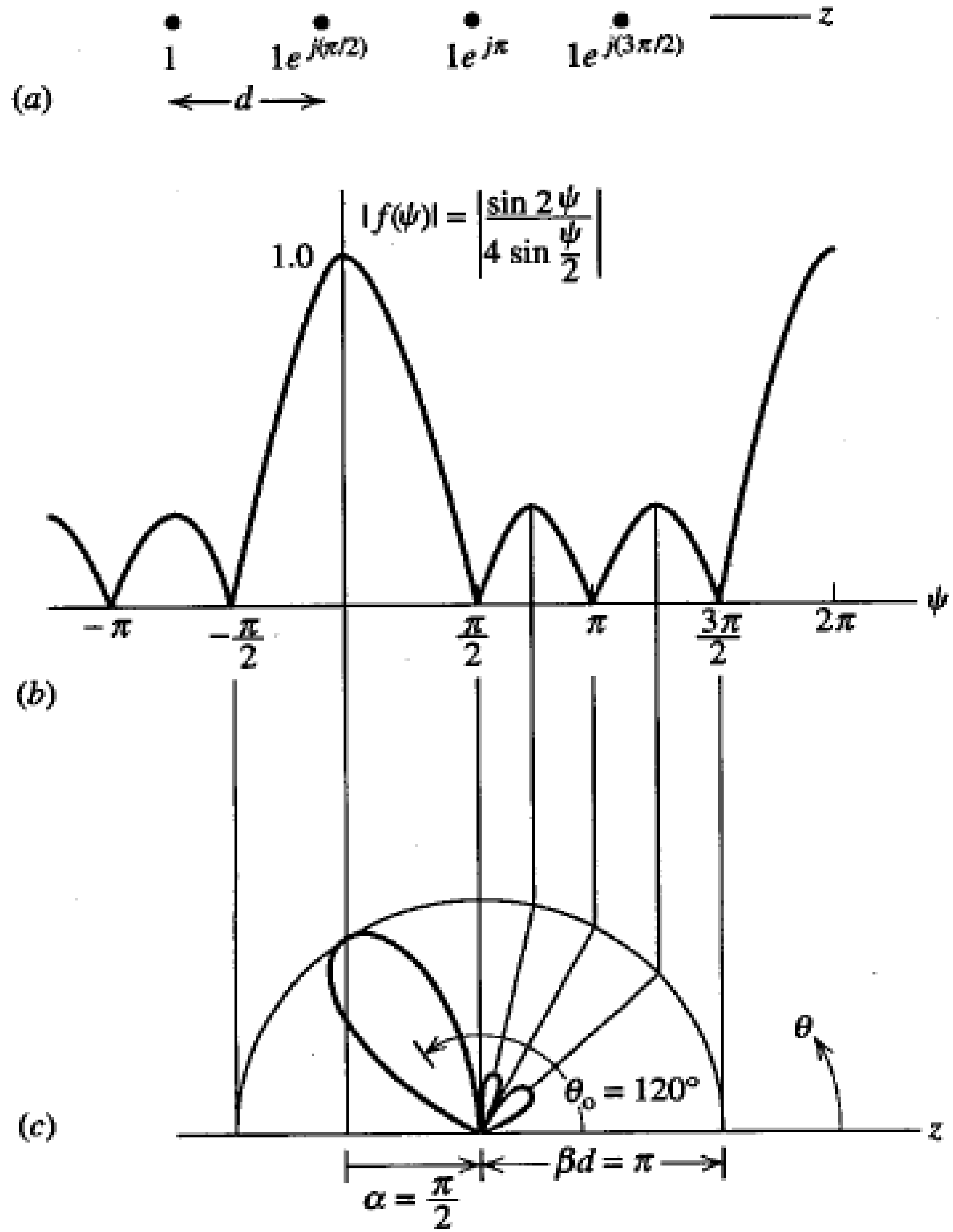


Figure 3-12 Array factor for a four-element, uniformly excited, equally spaced phased array (Example 3-5). (*a*) The array excitations. (*b*) Universal pattern for $N = 4$. (*c*) Polar plot for $d = \lambda/2$ and $\alpha = \pi/2$.

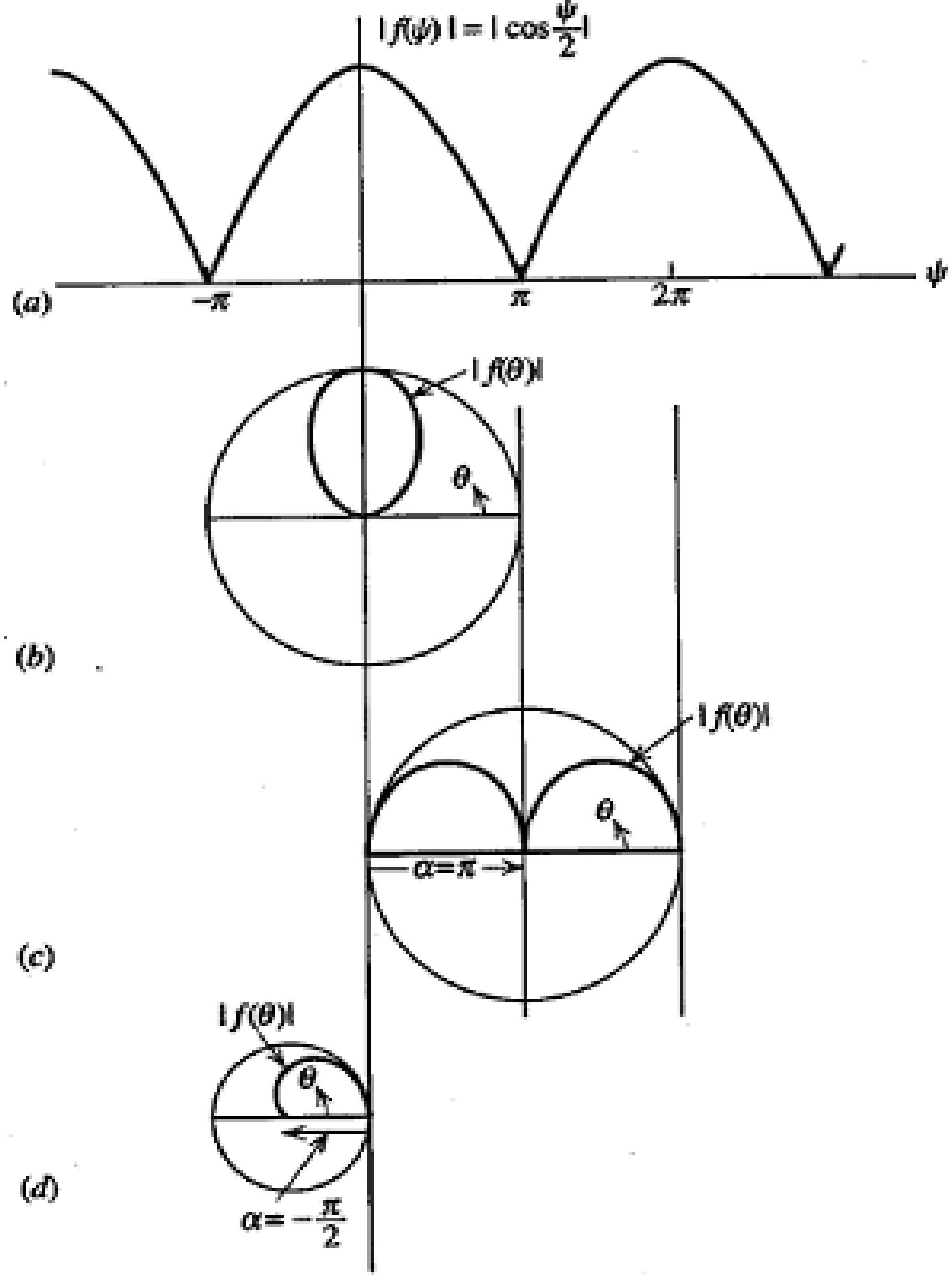
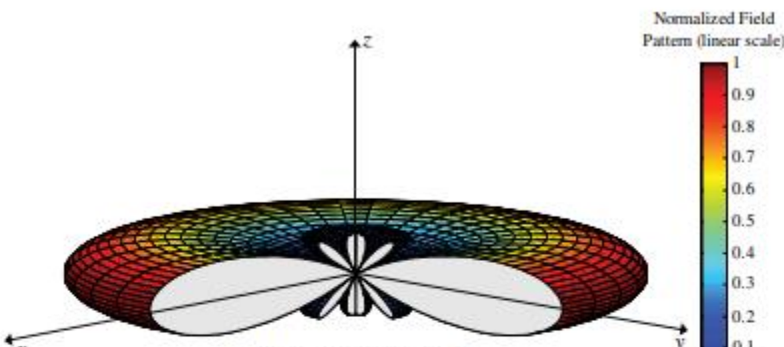
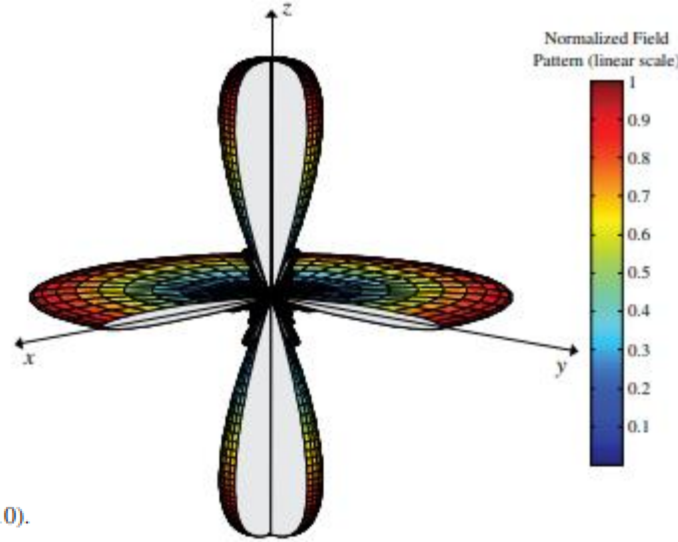


Figure 3-10 Array factors for two-element arrays with equal amplitude currents.
(*a*) Universal array factor.
(*b*) Polar plot for $d = \lambda/2$, $\beta d = \pi$, $\alpha = 0$ (Example 3-1).
(*c*) Polar plot for $d = \lambda/2$, $\beta d = \pi$, $\alpha = \pi$ (Example 3-2).
(*d*) Polar plot $d = \lambda/4$, $\beta d = \pi/2$, $\alpha = -\pi/2$ (Example 3-3).



(a) Broadside ($\beta = 0$, $d = \lambda/4$)

Three-dimensional amplitude patterns for broadside, and broadside/end-fire arrays ($N = 10$).



(b) Broadside/end-fire ($\beta = 0$, $d = \lambda$)

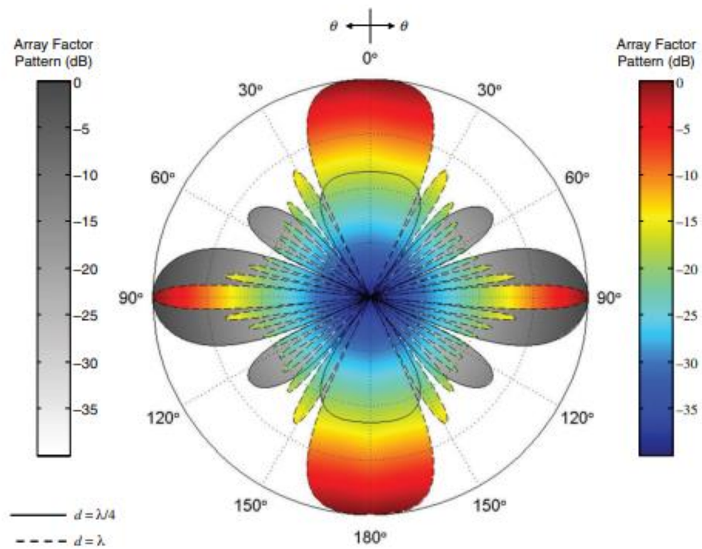


Figure 6.7 Array factor patterns of a 10-element uniform amplitude broadside array ($N = 10$, $\beta = 0$).

To avoid grating lobes and to have only one maximum, spacing between the elements should be $d_{max} < \lambda$ for broadside radiation.

Problems

1. A uniform array consists of 18 isotropic point sources each separated by a distance of $\lambda/4$. If the phase difference is -90°, calculate (i)HPBW

(ii)Directivity

(iii)Effective aperture

2. Design an ordinary endfire uniform linear array with only one maximum so that its directivity is 20dBi. The spacing between the elements is d= $\lambda/4$ and its length is much greater than spacing. Determine the

(i) Number of elements

(ii) Overall length of the array (in wavelength)

(iii) Approximate half power beamwidth (in degrees)

(iv) Amplitude level of first minor lobe maxima(in dB)

(v) Progressive phase shift between the elements(in degrees)

Problems

3. Design a two element array of isotropic sources positioned along the z axis a distance $d = \lambda/4$ apart, so that its only maximum occurs along $\theta_0 = 0°$. Assuming ordinary endfire conditions, find the

i) Relative phase excitation of each element

ii) Array factor of the array

iii) directivity

Problems

4.Design an 4 element ordinary endfire array with elements placed along the z axis a distance d apart. For spacing of d= $\lambda/2$ between the elements find the

(i) Progressive phase shift between the elements

(ii) Determine the array factor

(iii) Angle (in degree) where the nulls of the AF occur

(iv) Angle (in degree) where the maximum of the AF occur

(v) beamwidth (in degrees) between the first nulls of AF

(vi) Directivity of AF

Pattern multiplication

Pattern multiplication- The radiation pattern of an array is the product of the pattern of the individual antenna with the array pattern of isotropic point sources located at the phase center of the individual sources

$$\boxed{\text{total field} = (\text{element factor}) \times (\text{space factor})}$$

4. **Advantage:** It helps to sketch the radiation pattern of array antennas rapidly from the simple product of element pattern and array pattern.

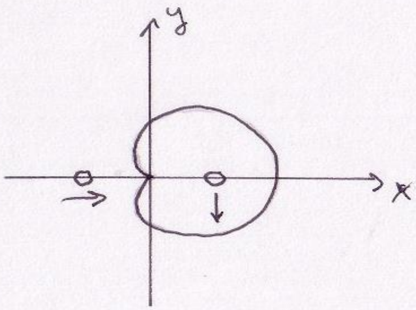
5. **Disadvantage:** This principle is only applicable for arrays containing identical elements.

6. The principle of pattern multiplication is true for any number of similar sources.

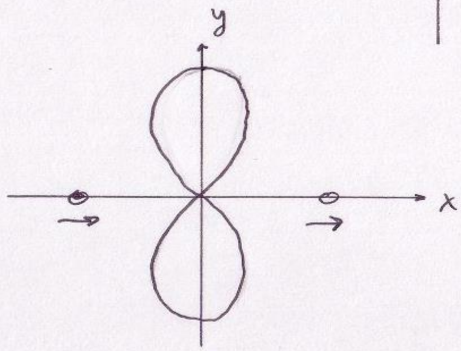
7. Total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources.

Pattern Multiplication

- **Useful 2-element Array Patterns**
- Consider the previous 2 examples and represent the currents as phasors with phase delay α of current on antenna 2. We can sketch these patterns as
- (a) $d=\lambda/4$, α=90degrees.

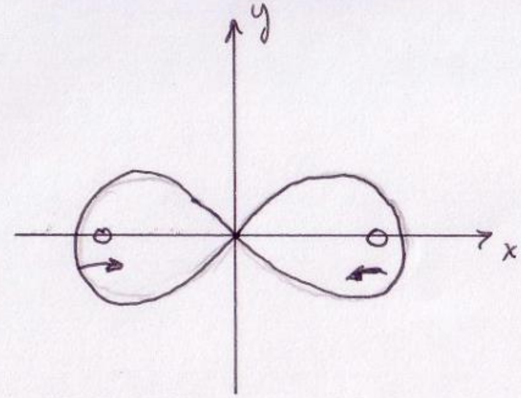


(b) $d=\lambda/2$, $\alpha=0$

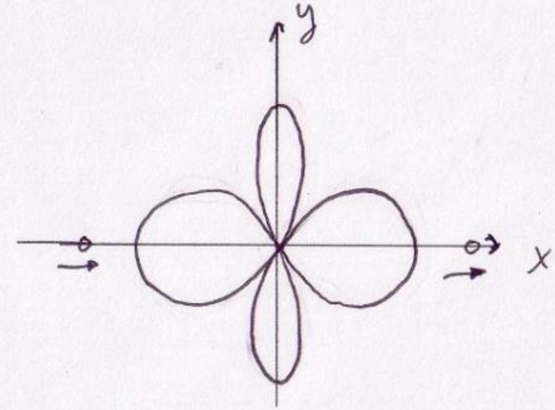


We can follow the same procedure to get more useful cases

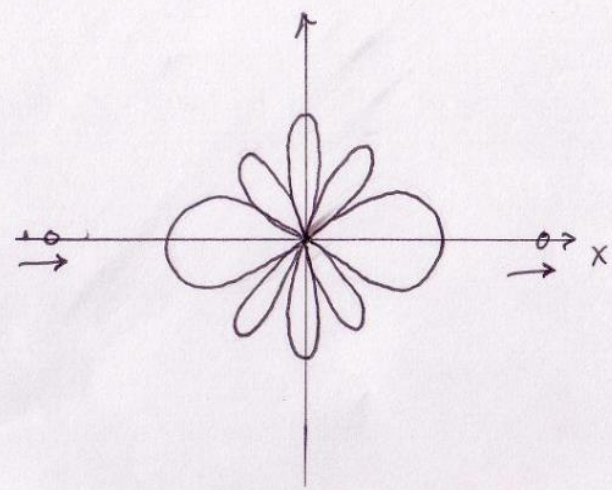
- (c) d=λ/2, α=180degrees



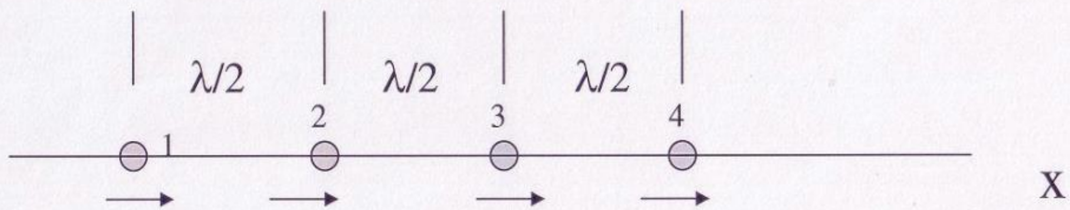
- (d) d=λ, α=0degrees



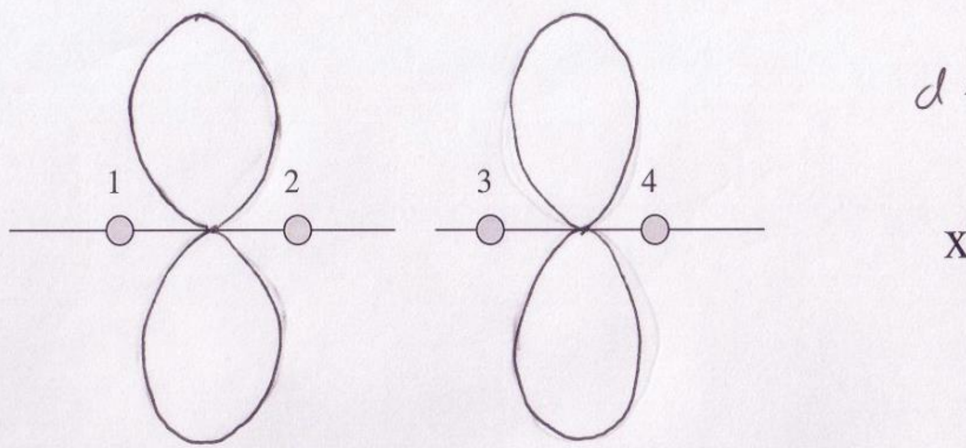
- (e) d=2λ, α=0degrees



- **Multiplication of Patterns for Larger Arrays**
- For a larger array we can sometimes break it down into smaller 2-element sub-arrays for which we know the pattern and then multiply them.
- For example, consider four isotropic elements with spacing d=λ/2 and all currents in phase so that α=0.

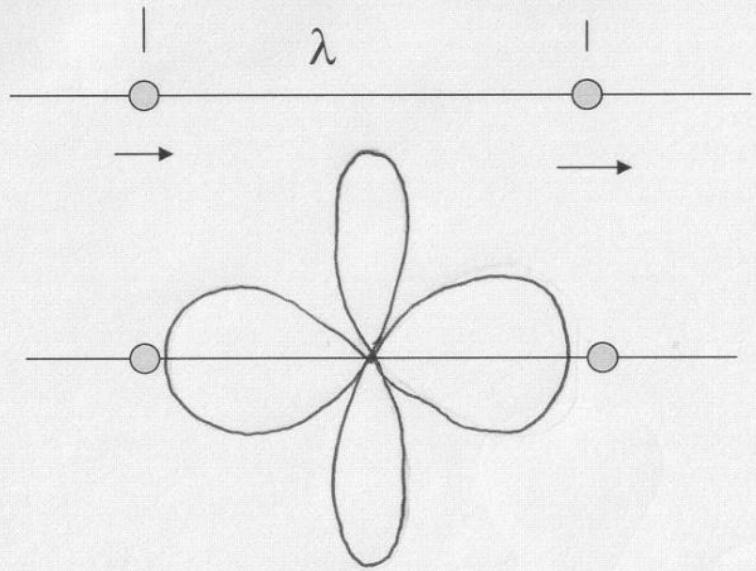


Elements 1 and 2 can be treated as one sub array, and so can elements 3 and 4.



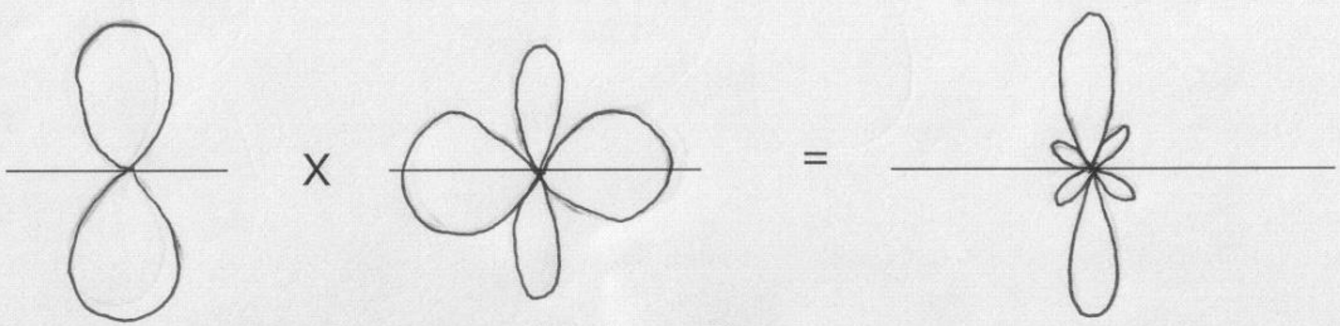
$d = \lambda/2, \ \alpha = 0$

- The two subarrays are an array of 2 elements with spacing λ and in-phase currents.



$d = \lambda, \alpha = 0$

The complete pattern will be the multiplication of the two sub-array patterns



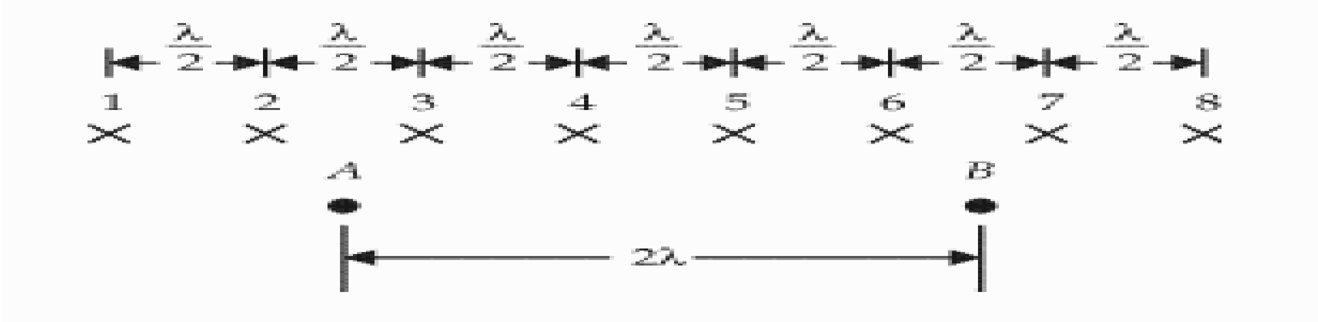


Fig: Array of eight elements

- Centre of the first four elements and last four elements are marked as A and B.
- The unit pattern is the pattern of four elements.
- The group pattern is the pattern of two elements spaced at 2λ.
- The resultant pattern is again the product of unit pattern and group pattern.

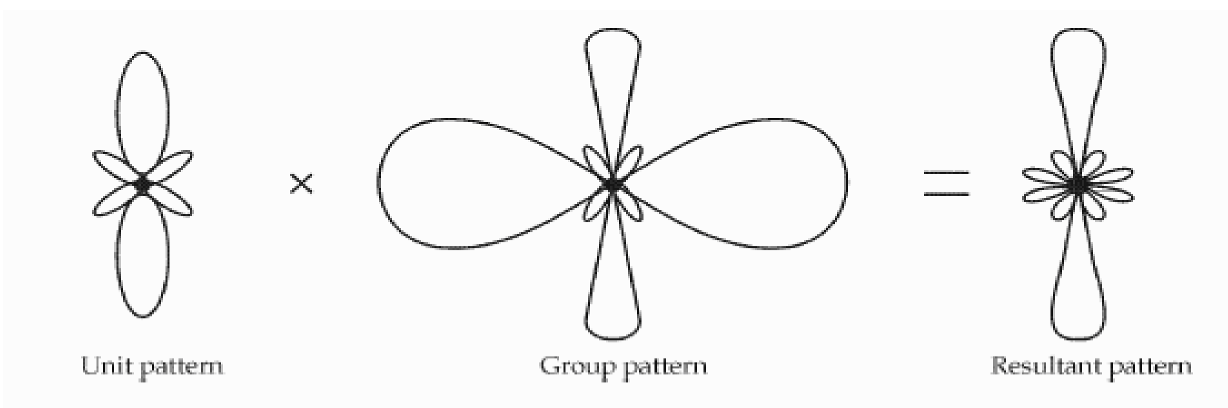


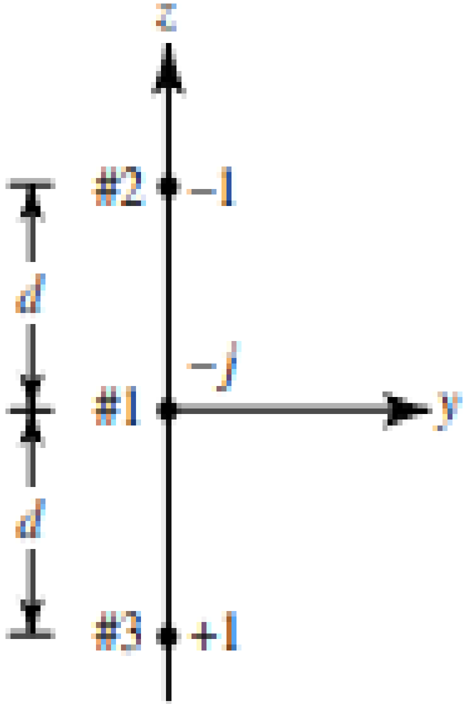
Fig: Resultant pattern of eight element array

Problems

A three element array of isotropic sources has phase and magnitude relationships as shown in Figure. The spacing between the elements is d= $\lambda/2$.

(i) Find the array facto

(ii) Find all the nulls



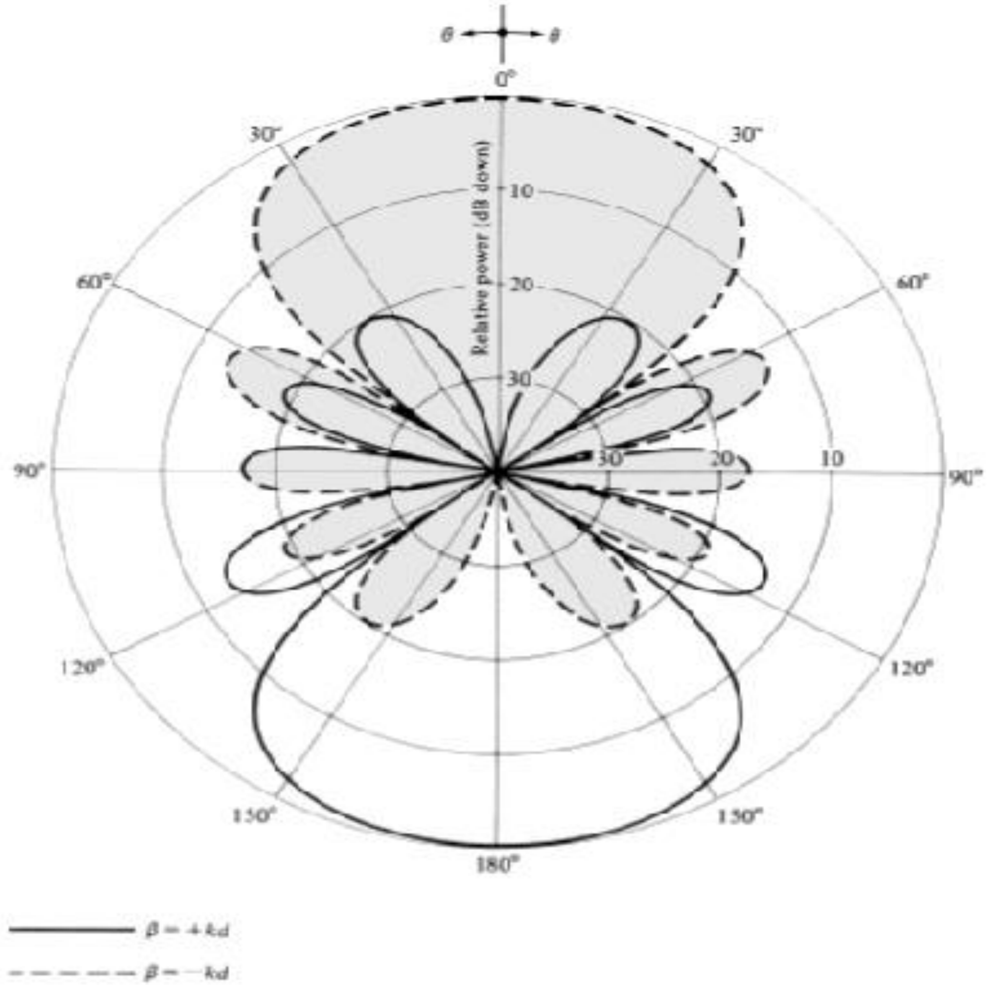


Figure 6.9 Array factor patterns of a 10-element uniform amplitude end-fire array ($N = 10$, $d = \lambda/4$).

To avoid grating lobes and to have only one maximum spacing between the elements should be $d_{max} < \lambda/2$ for endfre radiation.

Non-Uniform Excitation

Increased Flexibility

- Weights are general
- Similar to a filter synthesis problem

Example methods

Binomial Array

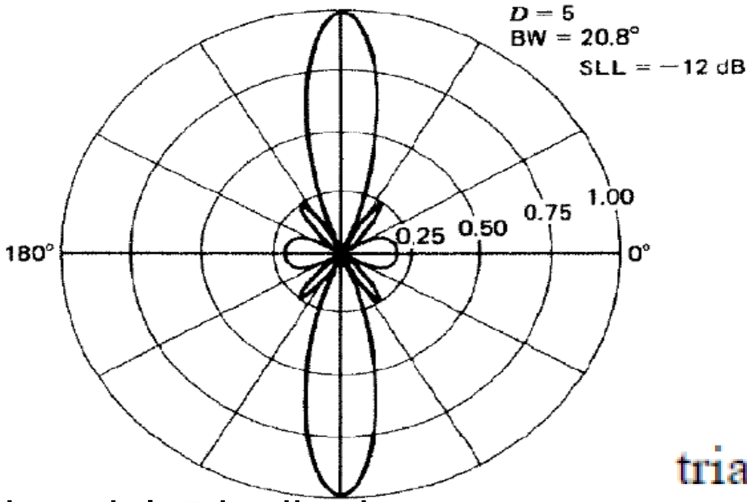
- Similar to “maximally flat” filter
- No side lobes for $\Delta < \lambda/2$

Tschebyscheff Array

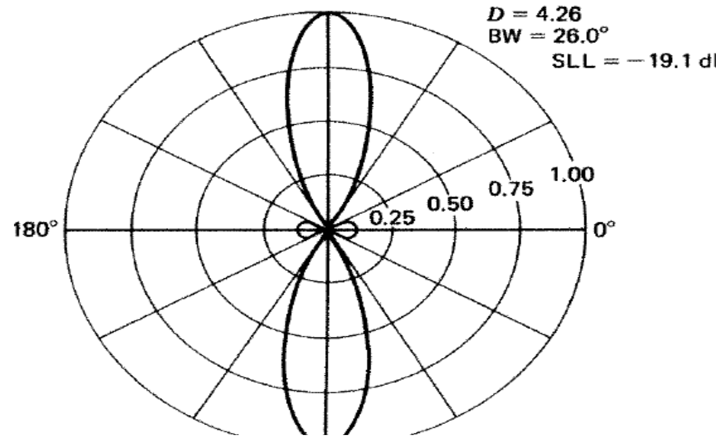
- Similar to “equiripple” filter
- Produces smallest beamwidth
  for given sidelobe level

Non- uniform amplitude excitation

Uniform Distribution

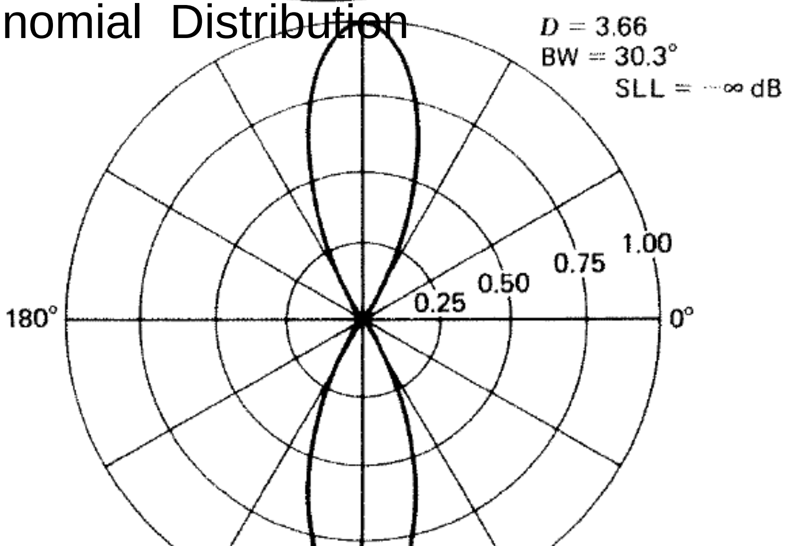


Triangular Distribution



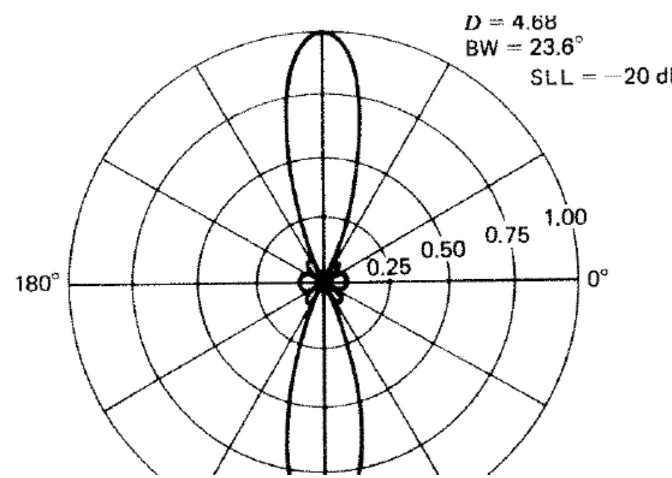
triangular (1:2:3:2:1) amplitude distribution

Binomial Distribution



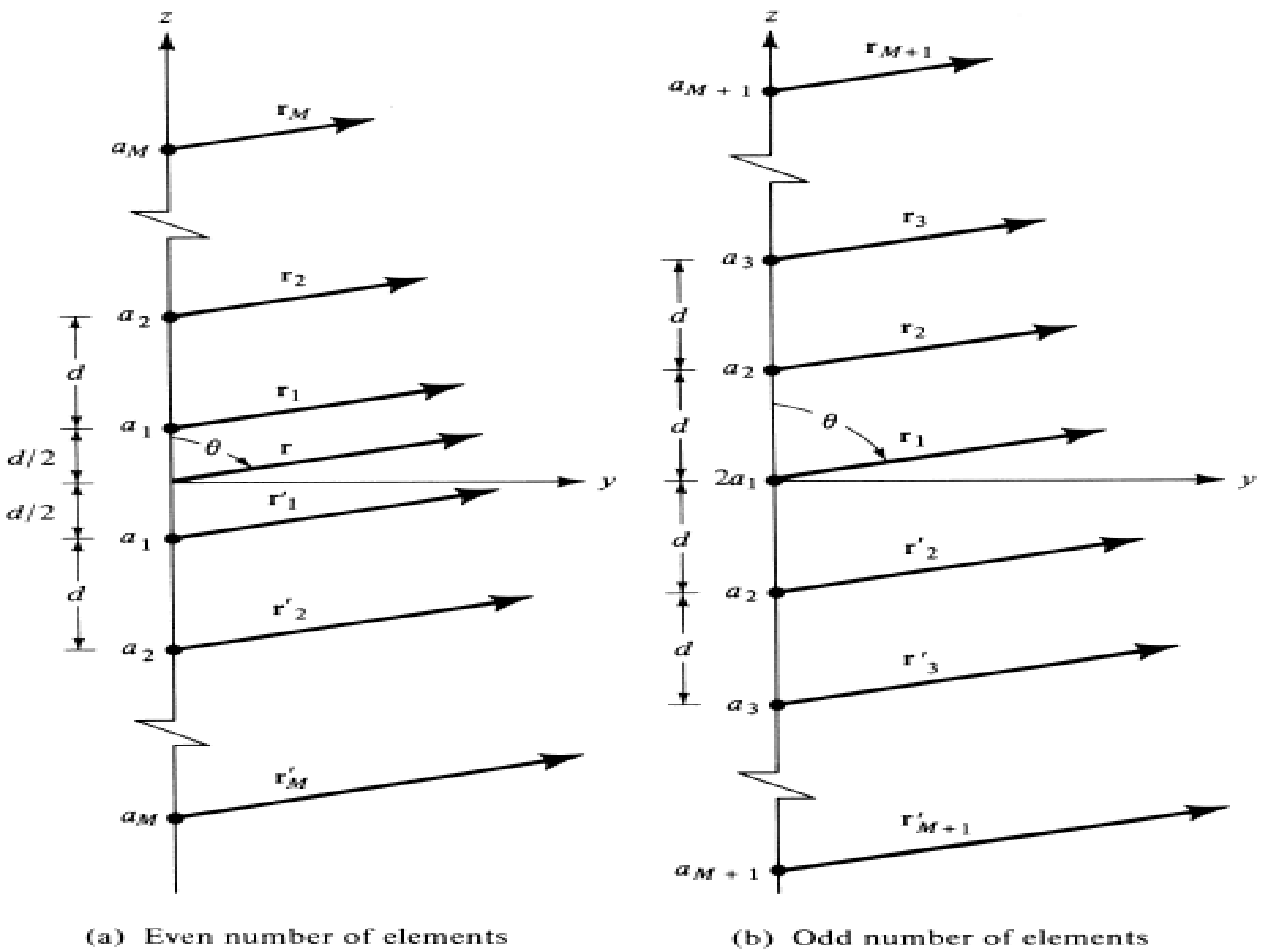
binomial (1:4:6:4:1)

Dolph Chebyschev Distribution



Dolph-Tschebyschev (1:1.61:1.94:1.61:1)

Non Uniform Array- Uniform spacing non uniform amplitude



Array factor- Non uniform Amplitude distribution (even number of elements)

- Let us consider a linear array with an even number ($2M$) of elements, located symmetrically along the z-axis, with excitation, which is also symmetrical with respect to z=0. For a *broadside array* (β=0),

$$AF^e = a_1 e^{j\frac{1}{2}kd\cos\theta} + a_2 e^{j\frac{3}{2}kd\cos\theta} + \ldots + a_M e^{j\frac{2M-1}{2}kd\cos\theta} +$$

$$+ a_1 e^{-j\frac{1}{2}kd\cos\theta} + a_2 e^{-j\frac{3}{2}kd\cos\theta} + \ldots + a_M e^{-j\frac{2M-1}{2}kd\cos\theta},$$

$$\Rightarrow AF^e = 2\sum_{n=1}^{M} a_n \cos\left[\left(\frac{2n-1}{2}\right)kd\cos\theta\right].$$

Array factor- Non uniform Amplitude distribution (odd number of elements)

If the linear array consists of an odd number $(2M+1)$ of elements, located symmetrically along the z-axis, the array factor is

$$AF^{o} = 2a_1 + a_2 e^{jkd\cos\theta} + a_3 e^{j2kd\cos\theta} + \ldots + a_{M+1} e^{jMkd\cos\theta} +$$
$$+ a_2 e^{-jkd\cos\theta} + a_3 e^{-j2kd\cos\theta} + \ldots + a_{M+1} e^{-jMkd\cos\theta},$$

$$\Rightarrow AF^{o} = 2\sum_{n=1}^{M+1} a_n \cos\left[(n-1)kd\cos\theta\right].$$

The normalized AF derived can be written in the form

$$AF^{e} = \sum_{n=1}^{M} a_n \cos\left[(2n-1)u\right], \text{ for } N = 2M,$$

$$AF^{o} = \sum_{n=1}^{M+1} a_n \cos\left[2(n-1)u\right], \text{ for } N = 2M+1,$$

where $u = \dfrac{1}{2}kd\cos\theta = \dfrac{\pi d}{\lambda}\cos\theta.$

Binomial Expansion

$$(a+b)^n = \frac{a^n b^0}{0!} + n\frac{a^{n-1}b^1}{1!} + n(n-1)\frac{a^{n-2}b^2}{2!} + \cdots$$

$$(1+x)^{m-1} = (+1)^{m-1}\frac{(x)^0}{0!} + (m-1)(+1)^{m-2}\frac{x^1}{1!} + \cdots$$

$$(1+x)^{m-1} = \boxed{1} + \boxed{m-1}\,x + \boxed{\frac{(m-1)(m-2)}{2!}}\,x^2 + \cdots \quad \text{(6-62)}$$

$\underline{m=1}: (1+x)^0 = 1 \Rightarrow$ 1

$\underline{m=2}: (1+x)^1 = 1+(1)x \Rightarrow$ 1 1

$\underline{m=3}: (1+x)^2 = 1+2x+(1)x^2 \Rightarrow$ 1 2 1

$\underline{m=4}: (1+x)^3 = 1+3x+3x^2+(1)x^3 \Rightarrow$ 1 3 3 1

Example: Binomial ($N = 5$; odd)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$$2M+1=5 \Rightarrow M=2$$

$$2a_1 = 6 \Rightarrow a_1 = 3$$

$$a_2 = 4 \qquad a_2 = 4$$

$$a_3 = 1 \qquad a_3 = 1$$

Binomial Design

1. Excitation Coefficients from Pascal's triangle ($N = 10$ Elements)

$$\underbrace{1}_{a_5} \quad \underbrace{9}_{a_4} \quad \underbrace{36}_{a_3} \quad \underbrace{84}_{a_2} \quad \underbrace{126}_{a_1} \quad \underbrace{126}_{a_1} \quad \underbrace{84}_{a_2} \quad \underbrace{36}_{a_3} \quad \underbrace{9}_{a_4} \quad \underbrace{1}_{a_5}$$

$$\left.\begin{aligned} a_1 &= 126 \\ a_2 &= 84 \\ a_3 &= 36 \\ a_4 &= 9 \\ a_5 &= 1 \end{aligned}\right\} \text{Excitation Coefficients}$$

2. Spacing

A. For No Side Lobes, Select

$$d \leq \lambda / 2$$

B. For No "Grating" Lobes, Select

$$d < \lambda$$

4. Normalized $a_n^{'}s$

Tschebyscheff		Binomial
$a_1 = 1$	$a_1 = 2.798$	$a_1 = 126$
$a_2 = 0.890$	$a_2 = 2.496$	$a_2 = 84$
$a_3 = 0.706$	$a_3 = 1.974$	$a_3 = 36$
$a_4 = 0.485$	$a_4 = 1.357$	$a_4 = 9$
$a_5 = 0.357$	$a_5 = 1$	$a_5 = 1$

Array Factor Power Patterns for a 10-Element Broadside Binomial Array

$N = 10$

$d = \lambda/4,$

$\lambda/2,$

$3\lambda/4,$

λ

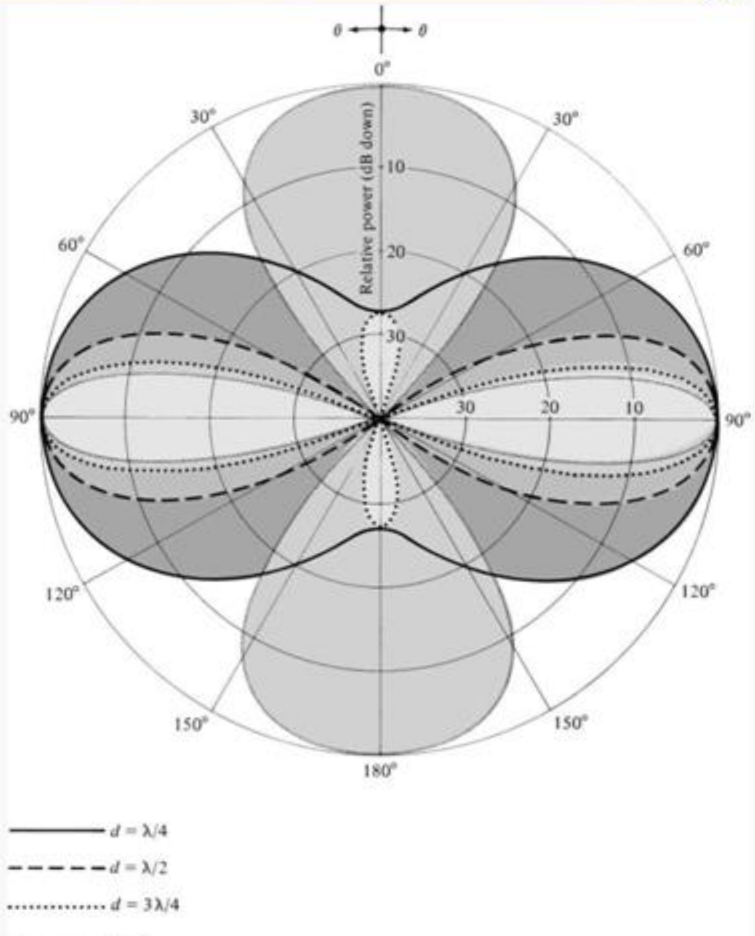


Fig. 6.20

Dolph-Tschebyscheff Design

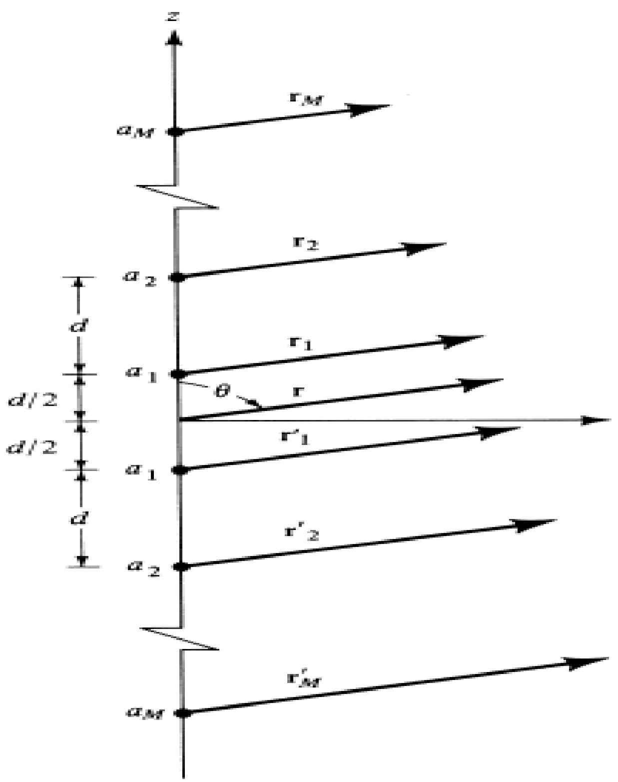
Equal Ripple; Chebyshev

The normalized AF derived can be written in the form

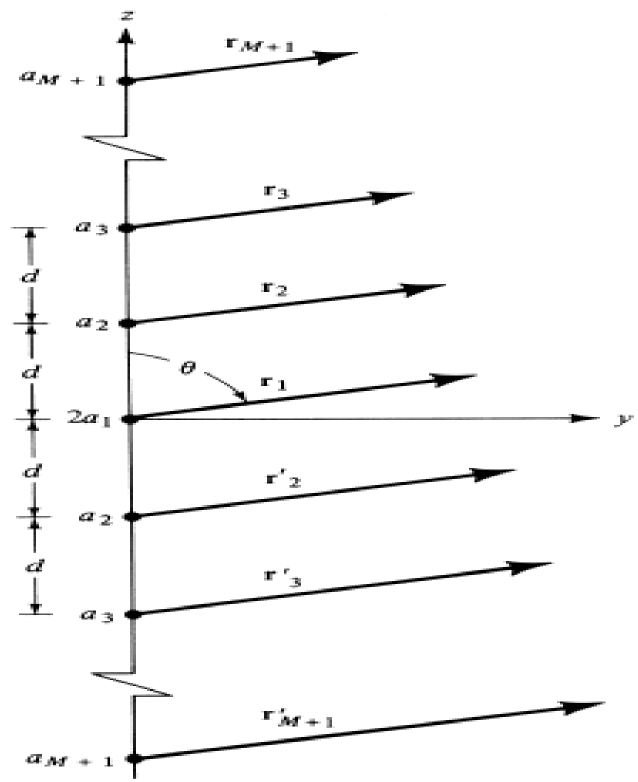
$$AF^e = \sum_{n=1}^{M} a_n \cos[(2n-1)u], \text{ for } N = 2M,$$

$$AF^o = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u], \text{ for } N = 2M+1,$$

where $u = \frac{1}{2}kd\cos\theta = \frac{\pi d}{\lambda}\cos\theta$.



(a) Even number of elements



(b) Odd number of elements

$$
\begin{aligned}
&m = 0 \quad \cos(mu) = 1 \\
&m = 1 \quad \cos(mu) = \cos u \\
&m = 2 \quad \cos(mu) = \cos(2u) = 2\cos^2 u - 1 \\
&m = 3 \quad \cos(mu) = \cos(3u) = 4\cos^3 u - 3\cos u \\
&m = 4 \quad \cos(mu) = \cos(4u) = 8\cos^4 u - 8\cos^2 u + 1 \\
&m = 5 \quad \cos(mu) = \cos(5u) = 16\cos^5 u - 20\cos^3 u + 5\cos u \\
&m = 6 \quad \cos(mu) = \cos(6u) = 32\cos^6 u - 48\cos^4 u + 18\cos^2 u - 1 \\
&m = 7 \quad \cos(mu) = \cos(7u) = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u \\
&m = 8 \quad \cos(mu) = \cos(8u) = 128\cos^8 u - 256\cos^6 u + 160\cos^4 u \\
&\qquad\qquad\qquad\qquad\qquad\qquad - 32\cos^2 u + 1 \\
&m = 9 \quad \cos(mu) = \cos(9u) = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u \\
&\qquad\qquad\qquad\qquad\qquad\qquad - 120\cos^3 u + 9\cos u
\end{aligned}
\tag{6-66}
$$

The above are obtained by the use of Euler's formula

$$
[e^{ju}]^m = (\cos u + j\sin u)^m = e^{jmu} = \cos(mu) + j\sin(mu) \tag{6-67}
$$

and the trigonometric identity $\sin^2 u = 1 - \cos^2 u$.

The recursion formula for Tschebyscheff polynomials is

$$\boxed{T_m(z) = 2zT_{m-1}(z) - T_{m-2}(z)}$$

If we let

$$z = \cos u$$

(6-66) can be written as

$$\begin{aligned}
m &= 0 \quad \cos(mu) = 1 = T_0(z)\\
m &= 1 \quad \cos(mu) = z = T_1(z)\\
m &= 2 \quad \cos(mu) = 2z^2 - 1 = T_2(z)\\
m &= 3 \quad \cos(mu) = 4z^3 - 3z = T_3(z)\\
m &= 4 \quad \cos(mu) = 8z^4 - 8z^2 + 1 = T_4(z)\\
m &= 5 \quad \cos(mu) = 16z^5 - 20z^3 + 5z = T_5(z)\\
m &= 6 \quad \cos(mu) = 32z^6 - 48z^4 + 18z^2 - 1 = T_6(z)\\
m &= 7 \quad \cos(mu) = 64z^7 - 112z^5 + 56z^3 - 7z = T_7(z)\\
m &= 8 \quad \cos(mu) = 128z^8 - 256z^6 + 160z^4 - 32z^2 + 1 = T_8(z)\\
m &= 9 \quad \cos(mu) = 256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z = T_9(z)
\end{aligned}$$

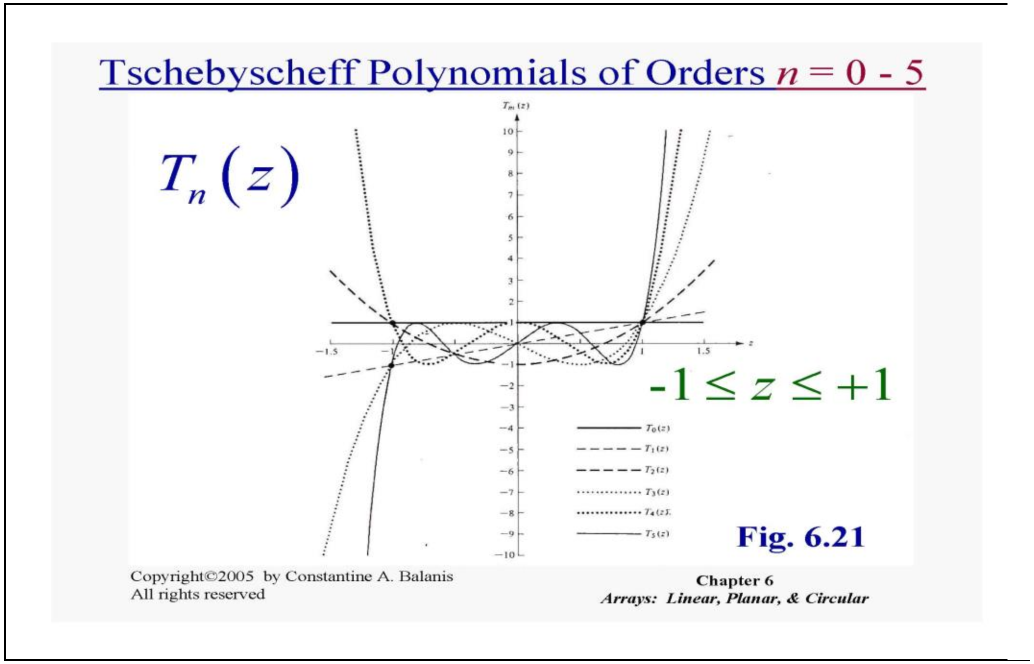
(6-69)

$$\begin{aligned}
m &= 0 \quad \cos(mu) = 1\\
m &= 1 \quad \cos(mu) = \cos u\\
m &= 2 \quad \cos(mu) = \cos(2u) = 2\cos^2 u - 1\\
m &= 3 \quad \cos(mu) = \cos(3u) = 4\cos^3 u - 3\cos u\\
m &= 4 \quad \cos(mu) = \cos(4u) = 8\cos^4 u - 8\cos^2 u + 1\\
m &= 5 \quad \cos(mu) = \cos(5u) = 16\cos^5 u - 20\cos^3 u + 5\cos u\\
m &= 6 \quad \cos(mu) = \cos(6u) = 32\cos^6 u - 48\cos^4 u + 18\cos^2 u - 1\\
m &= 7 \quad \cos(mu) = \cos(7u) = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u\\
m &= 8 \quad \cos(mu) = \cos(8u) = 128\cos^8 u - 256\cos^6 u + 160\cos^4 u\\
&\qquad - 32\cos^2 u + 1\\
m &= 9 \quad \cos(mu) = \cos(9u) = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u\\
&\qquad - 120\cos^3 u + 9\cos u
\end{aligned} \tag{6-66}$$

The above are obtained by the use of Euler's formula

$$[e^{ju}]^m = (\cos u + j\sin u)^m = e^{jmu} = \cos(mu) + j\sin(mu) \tag{6-67}$$

and the trigonometric identity $\sin^2 u = 1 - \cos^2 u$.


Tschebyscheff Polynomials of Orders n = 0 - 5
T_n(z)
-1 ≤ z ≤ +1
Fig. 6.21
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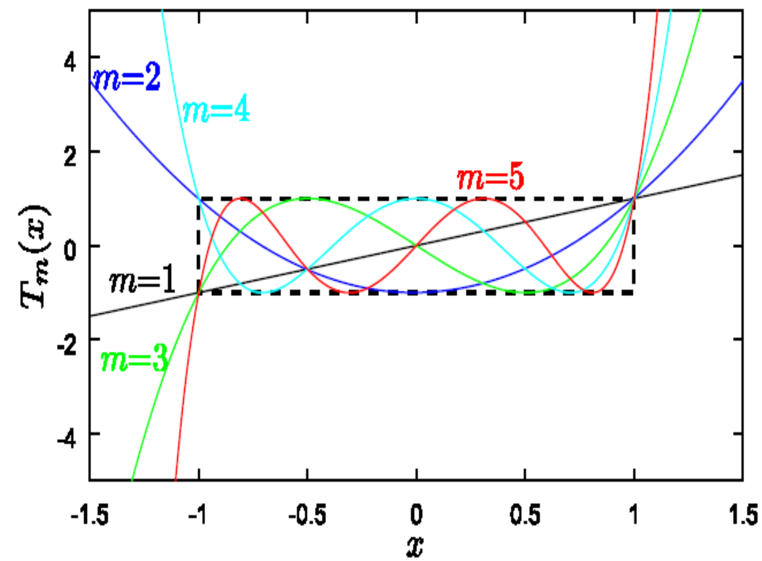
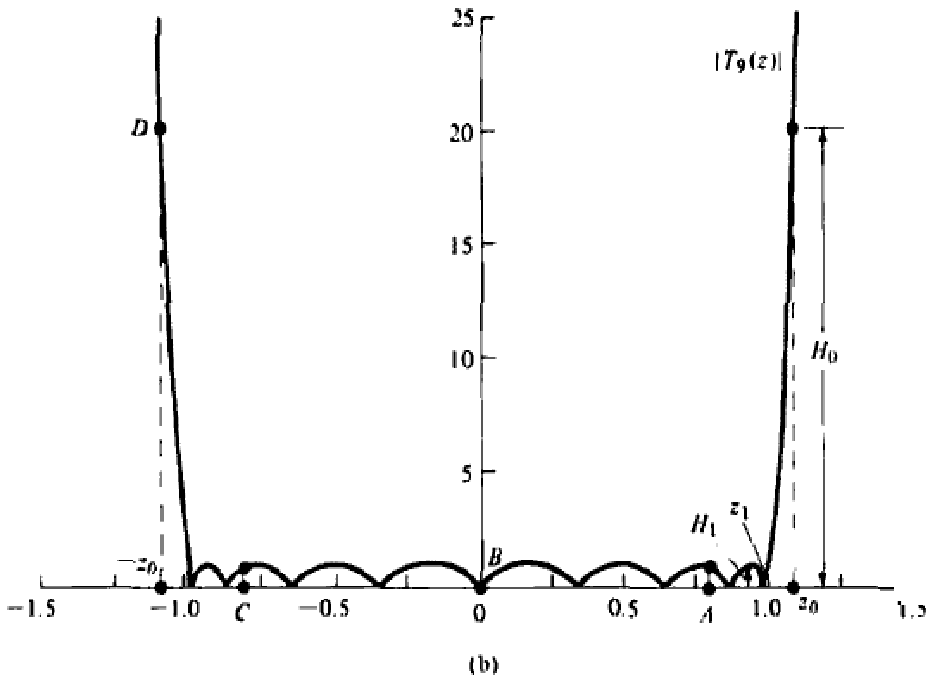
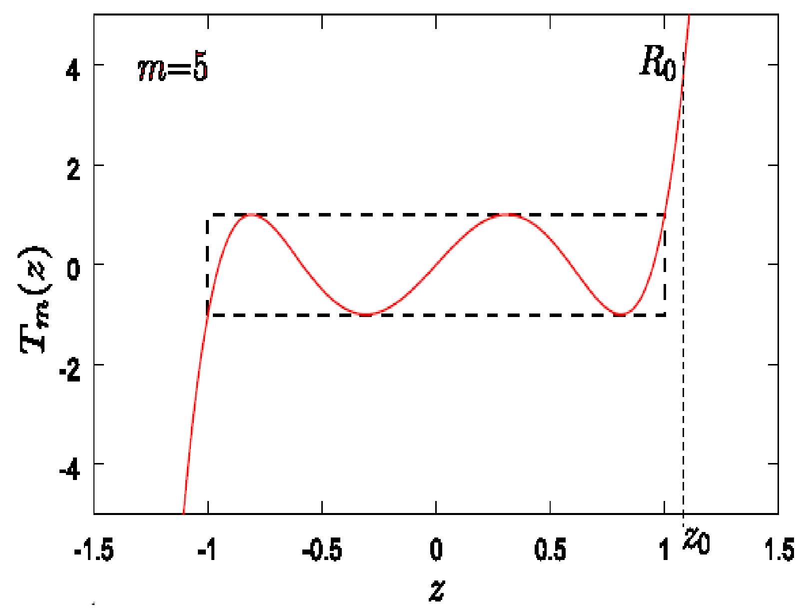
Chapter 6
Arrays: Linear, Planar, & Circular


m=1
m=2
m=3
m=4
m=5
T_m(x)
x


|T_9(z)|
D
C
B
A
H_0
H_1
z_1
z_0
-z_0
(b)


m=5
R_0
T_m(z)
z
z_0

Design of Dolph-Tchybysheff Array

1. Determine z_o such that $T_m\ (z_o)=R_0$ (voltage ratio) i.e, $\cosh[m \cosh^{-1}(z_o)] = R_{0,}$ m=N-1

2. Formulate Array Factor for given N element

3. Expand the AF by replacing cos (mu) with power series of cosu and substitute cosu $=z/\ z_o$

4. Equate the AF of step 3 to $T_m(z)$ and determine the coefficients for each power of z.

Problems

Design a Dolph Tchbysheff array for a sidelobe level of 20 dB with N=5 elements

Example: Design a DCA (broadside) of N=10 elements with a major-to-minor lobe ratio of $R_0 = 26$ dB. Find the excitation coefficients and form the AF.

Solution:

The order of the Chebyshev polynomial is $m = N - 1 = 9$. The AF for an even-number array is:

$$AF_{2M} = \sum_{n=1}^{5} a_n \cos\left[(2n-1)u\right], \quad u = \frac{\pi d}{\lambda}\cos\theta, \; M = 5.$$

Step 1: Write AF_{10} explicitly:

$$AF_{10} = a_1 \cos u + a_2 \cos 3u + a_3 \cos 5u + a_4 \cos 7u + a_5 \cos 9u .$$

Expand the $\cos(mu)$ terms of powers of $\cos u$:

$$\cos 3u = 4\cos^3 u - 3\cos u ,$$

$$\cos 5u = 16\cos^5 u - 20\cos^3 u + 5\cos u ,$$

$$\cos 7u = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u ,$$

$$\cos 9u = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u - 120\cos^3 u + 9\cos u .$$

Step 2: Determine z_0:

$$R_0 = 26 \text{ dB} \Rightarrow R_0 = 10^{26/20} \simeq 20 \Rightarrow T_9(z_0) = 20,$$

$$\cosh\left[9\operatorname{arccosh}(z_0)\right] = 20,$$

$$9\operatorname{arccosh}(z_0) = \operatorname{arccosh}20 = 3.69,$$

$$\operatorname{arccosh}(z_0) = 0.41,$$

$$z_0 = \cosh 0.41 \Rightarrow z_0 = 1.08515.$$

Step 3: Express the AF from Step 1 in terms of $\cos u = z / z_0$:

$$\begin{aligned}
AF_{10} = {} & \frac{z}{z_0}(a_1 - 3a_2 + 5a_3 - 7a_4 + 9a_5) \\
& + \frac{z^3}{z_0^3}(4a_2 - 20a_3 + 56a_4 - 120a_5) \\
& + \frac{z^5}{z_0^5}(16a_3 - 112a_4 + 432a_5) \\
& + \frac{z^7}{z_0^7}(64a_4 - 576a_5) \\
& + \frac{z^9}{z_0^9}(256a_5) = \\
& = \underbrace{9z - 120z^3 + 432z^5 - 576z^7 + 256z^9}_{T_9(z)}
\end{aligned}$$

Step 4: Finding the coefficients by matching the power terms:

$$256a_5 = 256z_0^9 \Rightarrow a_5 = 2.0860$$

$$64a_4 - 576a_5 = -576z_0^7 \Rightarrow a_4 = 2.8308$$

$$16a_3 - 112a_4 + 432a_5 = 432z_0^5 \Rightarrow a_3 = 4.1184$$

$$4a_2 - 20a_3 + 56a_4 - 120a_5 = -120z_0^7 \Rightarrow a_2 = 5.2073$$

$$a_1 - 3a_2 + 5a_3 - 7a_4 + 9a_5 = 9z_0^9 \Rightarrow a_1 = 5.8377$$

Normalize coefficients with respect to edge element (N=5):

$$a_5 = 1;\ a_4 = 1.357;\ a_3 = 1.974;\ a_2 = 2.496;\ a_1 = 2.789$$

$$\Rightarrow AF_{10} = 2.789\cos(u) + 2.496\cos(3u) + 1.974\cos(5u) + 1.357\cos(7u) + \cos(9u)$$

where $u = \dfrac{\pi d}{\lambda}\cos\theta$.

Planar Array, Circular Array and Phased Array

Antenna Arrays

Advantages of Arrays

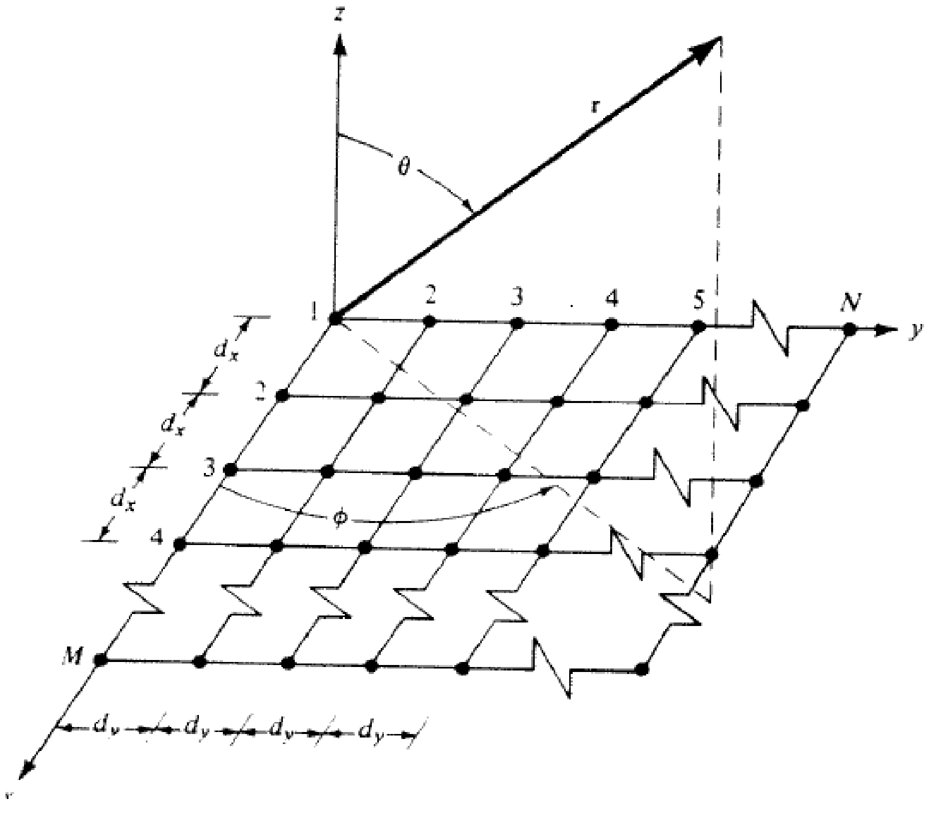
- Provide beam steering
- Provide high gain using simple antenna elements
- Provide diversity gain in multipath signal reception
- Enable array signal processing

Planar Arrays

Planar Array Geometry

- Directional Beams
- Symmetrical Patterns
- Low Side Lobes
- Higher Directivity
- To point the beam

toward any direction



• Array Factor of a Planar Array

• The pattern of a rectangular array is the product of the array factors of the linear arrays in the *x* and *y* directions

$$AF = S_{x_M} \cdot S_{y_N}$$

$$S_{x_M} = AF_{x1} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta \cos\phi + \beta_x)}$$

$$S_{y_N} = AF_{1y} = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta \sin\phi + \beta_y)}$$

- Array Factor of a Planar Array:
- In the case of a uniform planar (rectangular) array, $I_{m1} = I_{1n} = I_0$ for all m and n

$$AF_n(\theta,\phi) = \left[\frac{1}{M} \frac{\sin\left(M\frac{\psi_x}{2}\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right] \cdot \left[\frac{1}{N} \frac{\sin\left(N\frac{\psi_y}{2}\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right]$$

$$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$$
$$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$$

- Major and grating lobes occur at:

$$kd_x \sin\theta_m \cos\phi_m + \beta_x = \pm 2m\pi, \quad m = 0, 1, \ldots$$

$$kd_y \sin\theta_n \sin\phi_n + \beta_y = \pm 2n\pi, \quad n = 0, 1, \ldots$$

and the principal maximum occurs at m=n=0

•To get the main beam in specified direction progressive phases in x and y directions must satisfy

$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0$$

$$\beta_y = -kd_y \sin\theta_0 \sin\phi_0$$

Circular Arrays

- Circular arrays can provide 2D angular scan in both horizontal ϕ and vertical θ scan.
- Unlike 2D planar arrays, circular arrays are basically 1D linear array in circular form.
- Unlike linear arrays a circular array can scan horizontally for 360° with no distortions near the end fire direction.
- Unlike linear arrays, distortions in the array pattern of a circular array due to mutual coupling effect are same for each element and this makes it easier to deal with mutual coupling effect

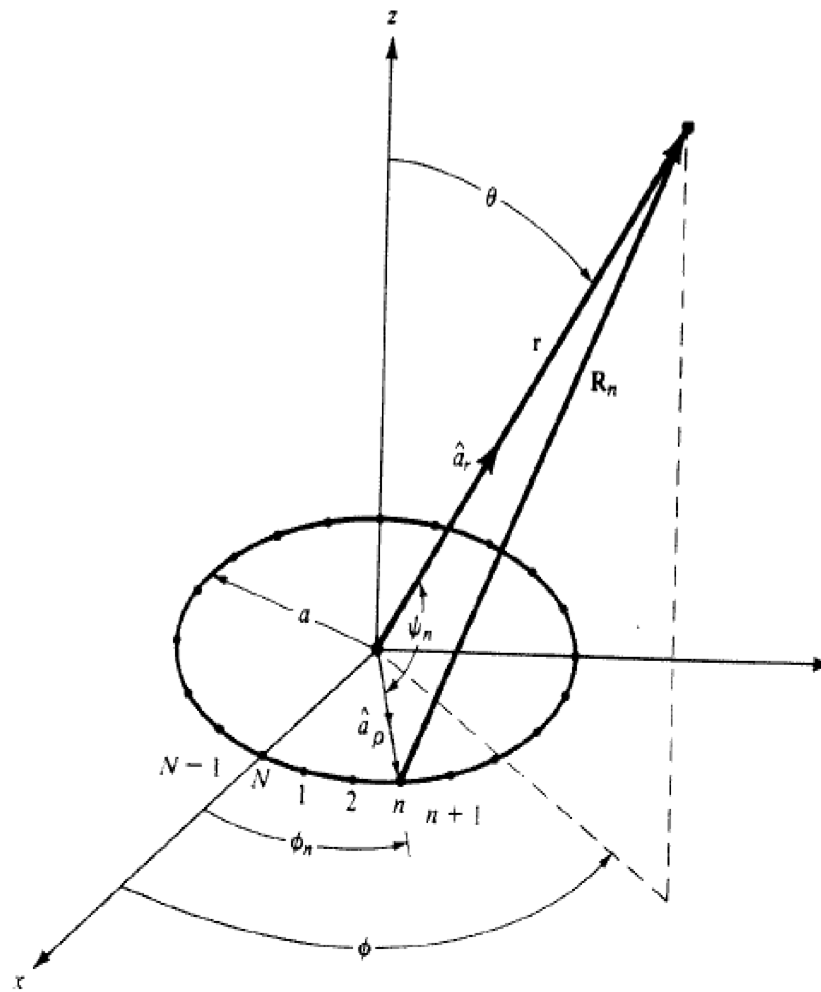
Circular Arrays

- Normalized E field is given

$$E(r,\theta,\phi) = \sum_{n=1}^{N} a_n \frac{e^{-jkR_n}}{R_n}$$

- Where

$$R_n = \sqrt{r^2 + a^2 - 2ar\cos\psi_n}\ .$$



Circular Arrays

And the array factor is obtained as:

$$AF(\theta,\phi)=\sum_{n=1}^{N} I_n e^{j\left[ka\sin\theta\cos(\phi-\phi_n)+\alpha_n\right]}$$

- The maximum of the AF occurs when all the phase terms equals unity

$$ka\sin\theta\cos(\phi-\phi_n)+\alpha_n=2m\pi,\quad m=0,\pm1,\pm2,\ \text{all } n$$

- The main beam occurs at m=0 where

$$\alpha_n=-ka\sin\theta_0\cos(\phi_0-\phi_n),\quad n=1,2,\ldots,N$$

Linear Array and Phased Array

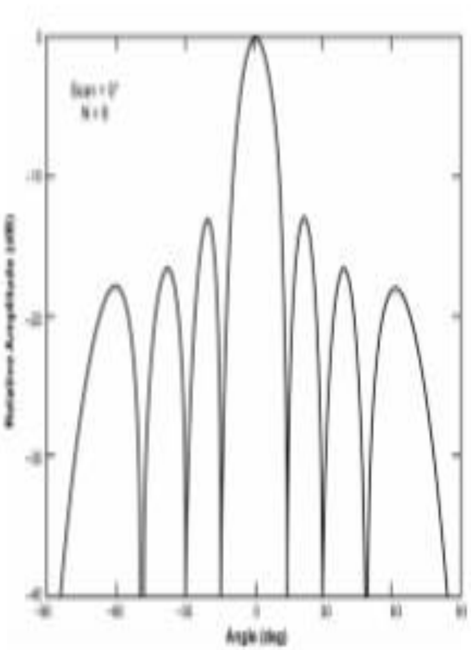
Array Factor

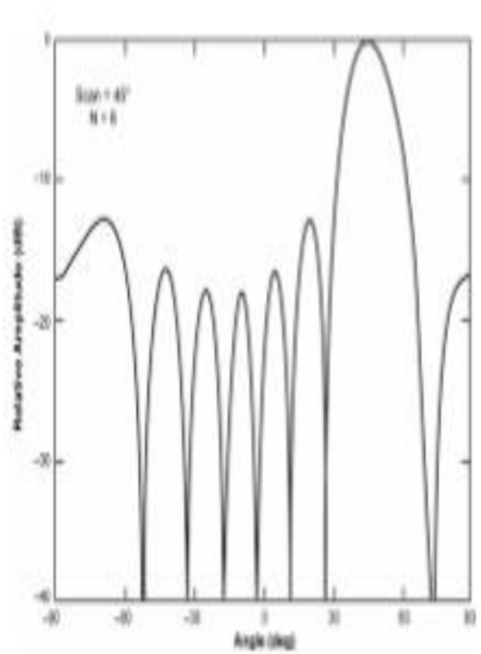
$$AF(\theta,\phi) = \sum_{n=1}^{N} i_n e^{j\Psi_n} \qquad i_n = A_n e^{j\Psi_{ns}} \qquad AF(\theta,\phi) = \sum_{n=1}^{N} A_n e^{j(\Psi_n+\Psi_{ns})}$$

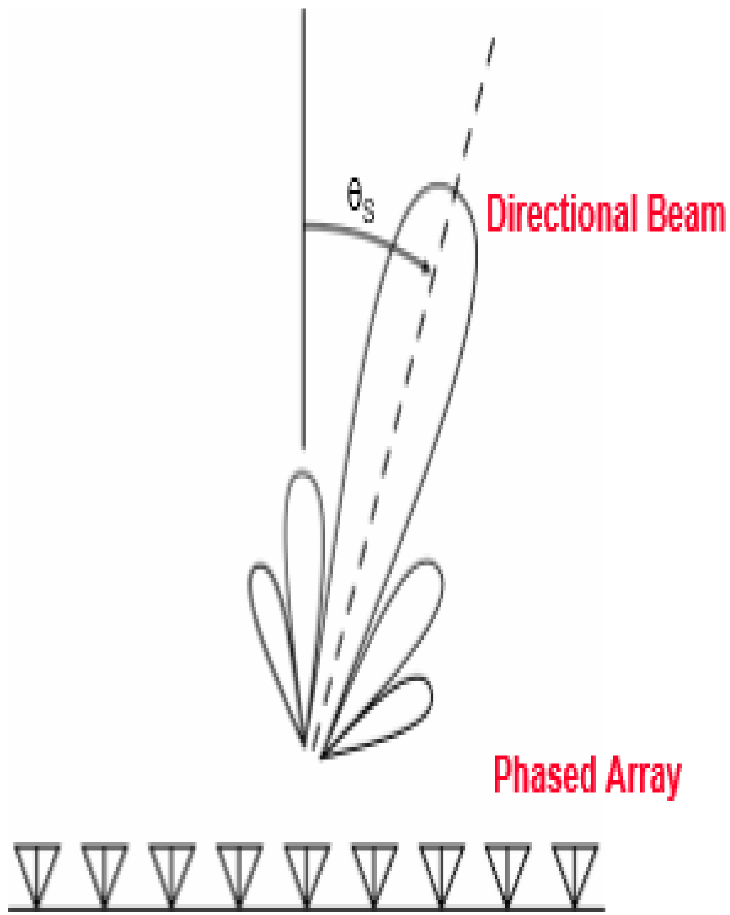
$$\Psi_n = \beta \sin\theta (x_n \cos\phi + y_n \sin\phi)$$

$$\Psi_{ns} = -\beta \sin\theta_s (x_n \cos\phi_s + y_n \sin\phi_s)$$

$$\boxed{AF(\theta) = \sum_{n=1}^{N} A_n e^{j\beta x_n(\sin\theta - \sin\theta_s)}}$$

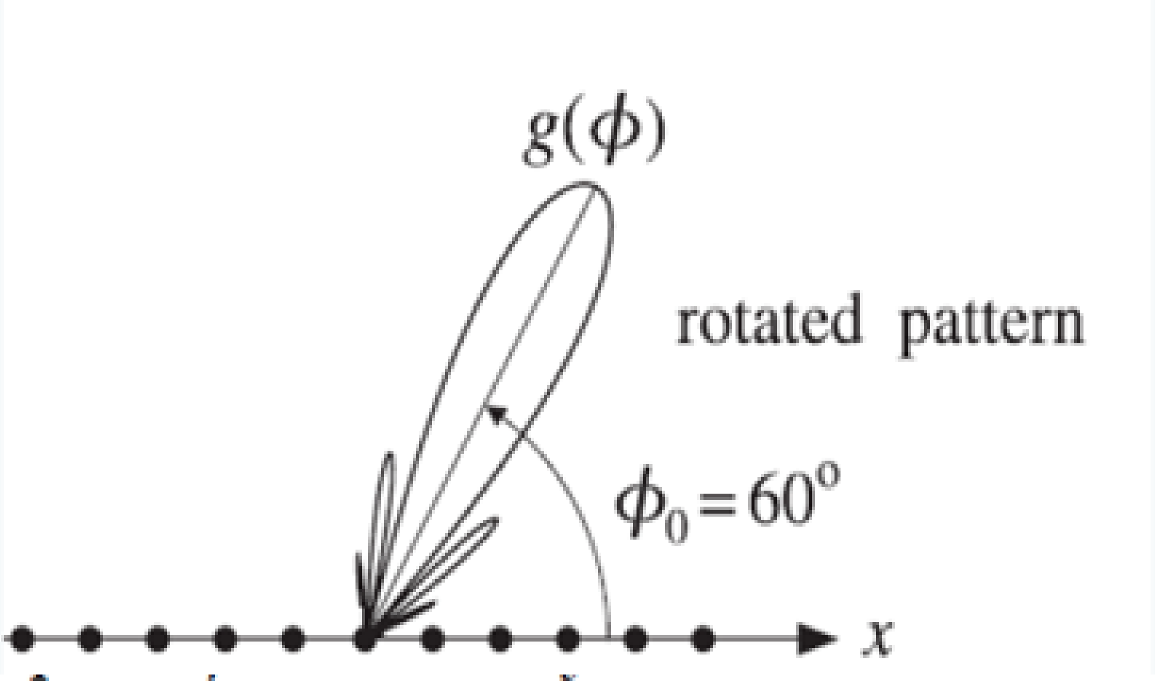






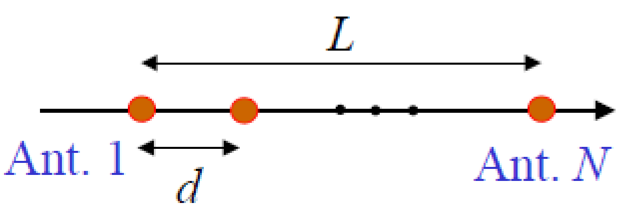
Phased Array

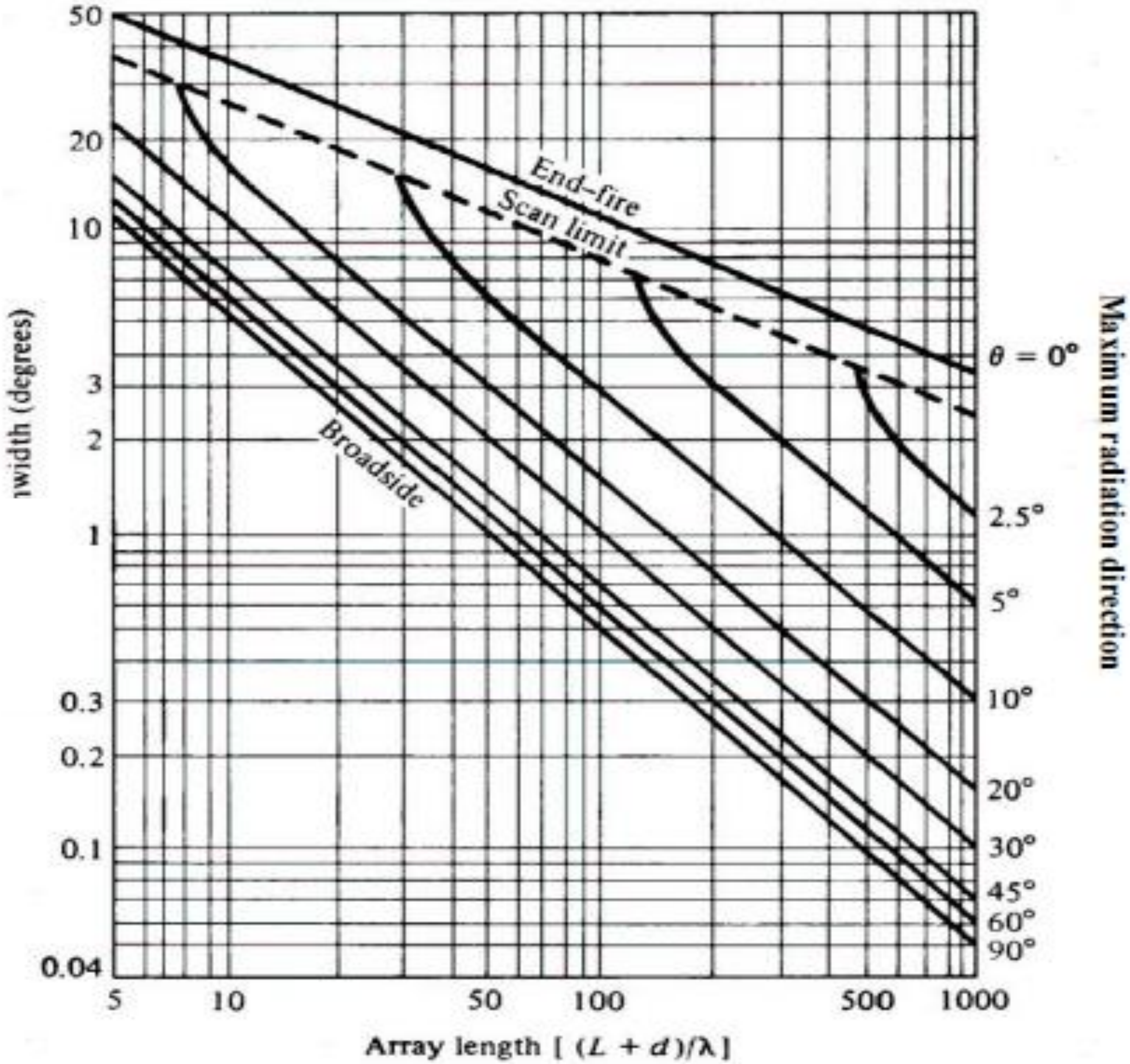
The phased arrays are the linear arrays with capability of beam steering or controlling the beam direction. Here the phase shift factor α varies between 0 to 180°



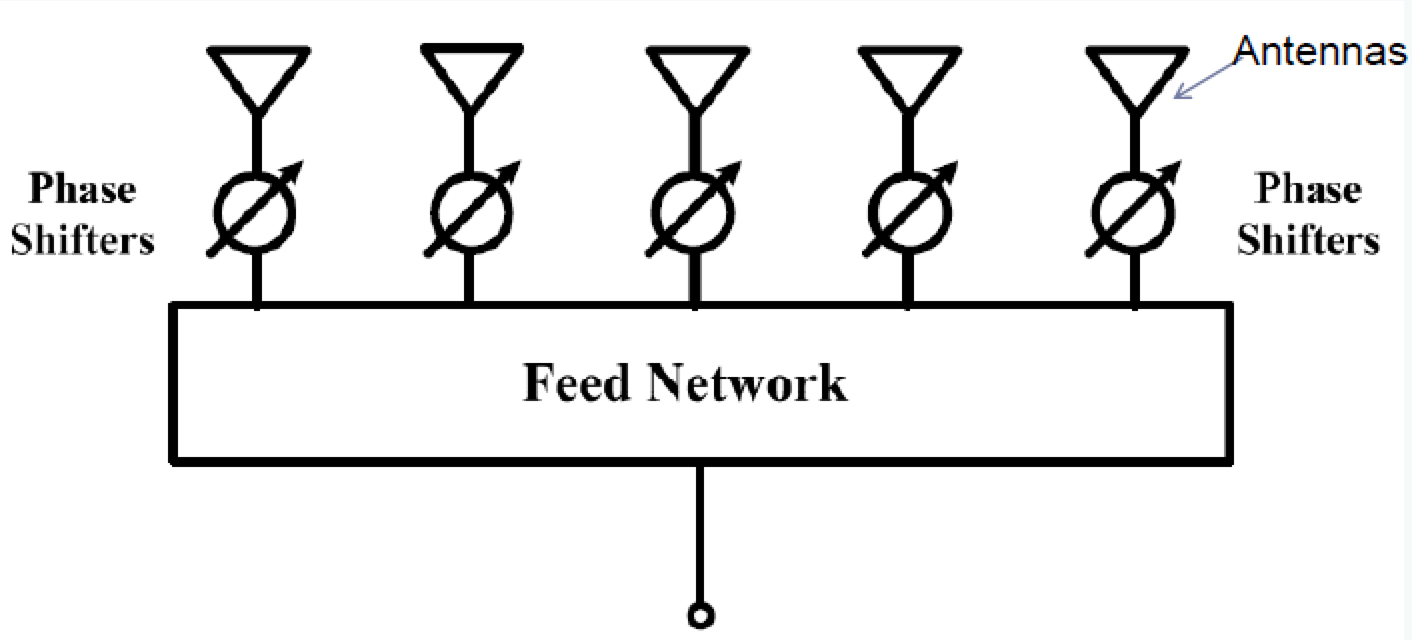
$$\boxed{\psi = kd\cos\phi + \beta\Big|_{\phi=\phi_0} = 0, \qquad \Rightarrow \qquad \beta = -kd\cos\phi_0}$$

- For a phased scanning array Length L = (N-1)d of the array can be determined from the graph





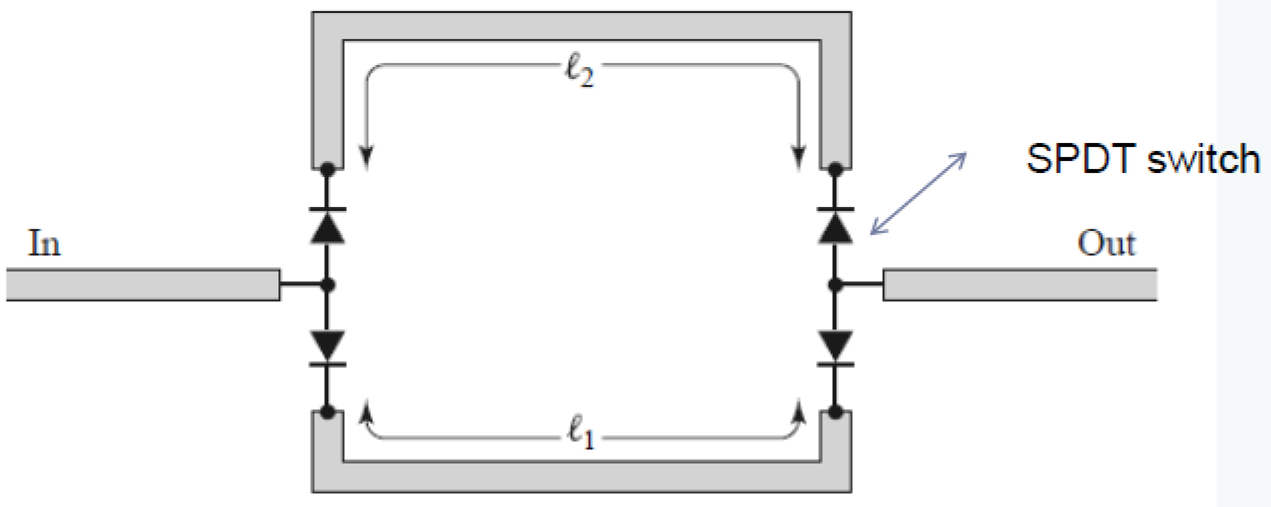
Basic Phased Array



Commonly phased arrays use phase shifters which work on time delay principle in order to cause various phase changes near the antenna input.

Delay line Phase shifter

$$\Delta\phi = \beta(\ell_2 - \ell_1),$$



A switched-line phase shifter.

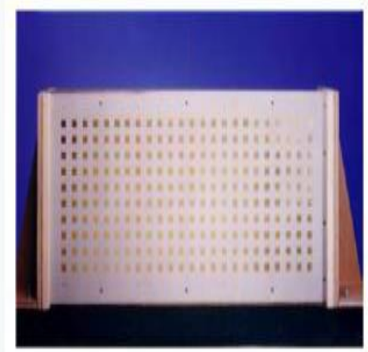
Applications of Phased Array Antenna

Ground based multi-function radar for military use

Airborne radar for surveillance (RBE2

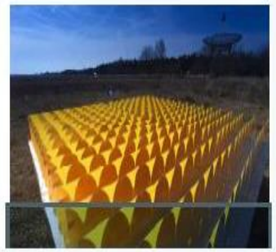
ASTRON

Spaceborne SAR and communications for remote sensing



Applications of Phased Array Antenna

Recently for radio astronomy



Navigation : tracking of ships and aircrafts

Mobile base station : smart operation i.e that is concentration of energy in the desired direction