2.1 Two element array

Module:2 Linear and Planar Arrays

Course: BECE305L – Antenna and Microwave Engineering

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Module:2 Linear and Planar Arrays

- Two element array, N-element linear array - broadside array, End fire array - Directivity, radiation pattern, pattern multiplication. Non-uniform excitation - Binomial, Chebyshev distribution, Arrays: Planar array, circular array, Phased Array antenna (Qualitative study).

- Source of the contents: Constantine A. Balanis - Antenna theory analysis and design (2016)

1. Introduction

- In many applications it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication.
- This can only be accomplished by **increasing the electrical size of the antenna**. Enlarging the dimensions of single elements often leads to more directive characteristics.
- Another way to enlarge the dimensions of the antenna, without necessarily increasing the size of the individual elements, is to form an assembly of radiating elements in an electrical and geometrical configuration. This new antenna, formed by multi-elements, is referred to as an ***array*****.**

1. Introduction

- The total field of the array is determined by the vector addition of the fields radiated by the individual elements.
- This assumes that the current in each element is the same as that of the isolated element (neglecting coupling)
  Or
  depends on the separation between the elements.
- To provide very directive patterns,
  it is necessary that the fields from the elements of the array
  * interfere constructively (add) in the desired directions and
  * interfere destructively (cancel each other) in the remaining space.

2. Factors controlling shape of radiation pattern in array

In an array of identical elements, there are at least five controls that can be used to shape the overall pattern of the antenna. These are:

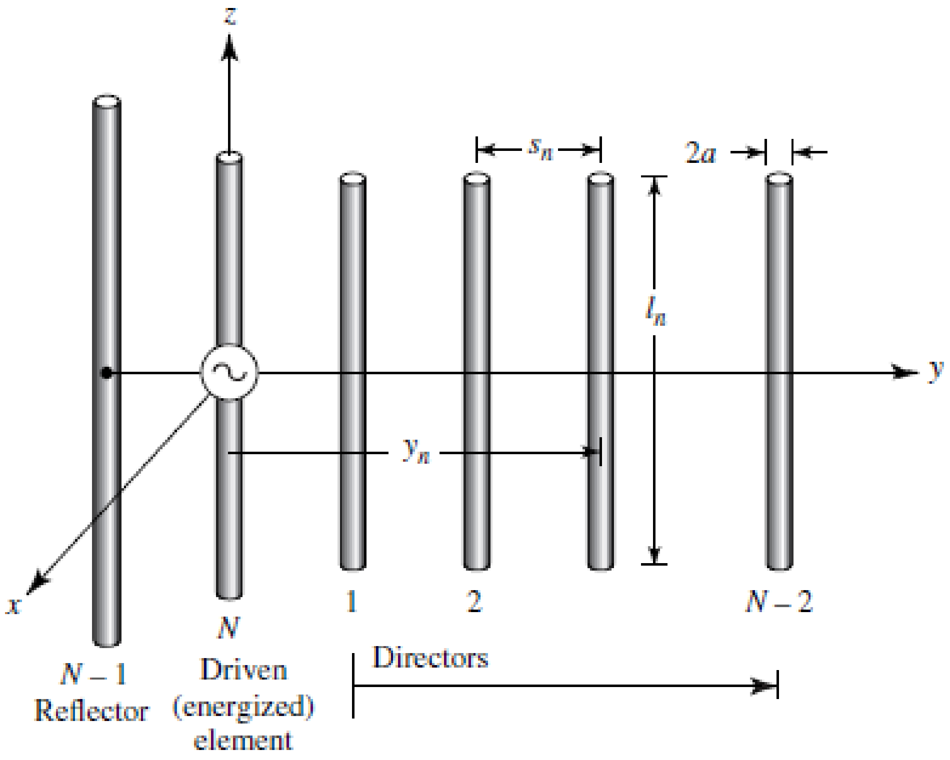
1. the geometrical configuration of the overall array (linear, circular, rectangular, spherical, etc.)
2. the relative displacement between the elements
3. the excitation amplitude of the individual elements
4. the excitation phase of the individual elements
5. the relative pattern of the individual elements

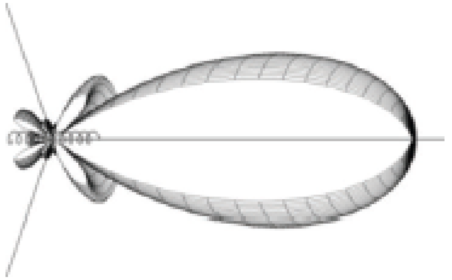
3. Examples

- 1. Sectoral Array
  widely used as a base-station antenna for mobile communication. It is a triangular array consisting of twelve dipoles, with four dipoles on each side of the triangle. Each four-element array, on each side of the triangle, is basically used to cover an angular sector of 120◦ forming what is usually referred to as a *sectoral array*.

3. Examples

- *Yagi-Uda* array, and it is primarily used for TV and amateur radio applications.





3. Examples

- an array of dipoles, which is referred to as the *log-periodic* antenna, which is primarily used for TV reception and has wider bandwidth than the Yagi-Uda array but slightly smaller directivity.

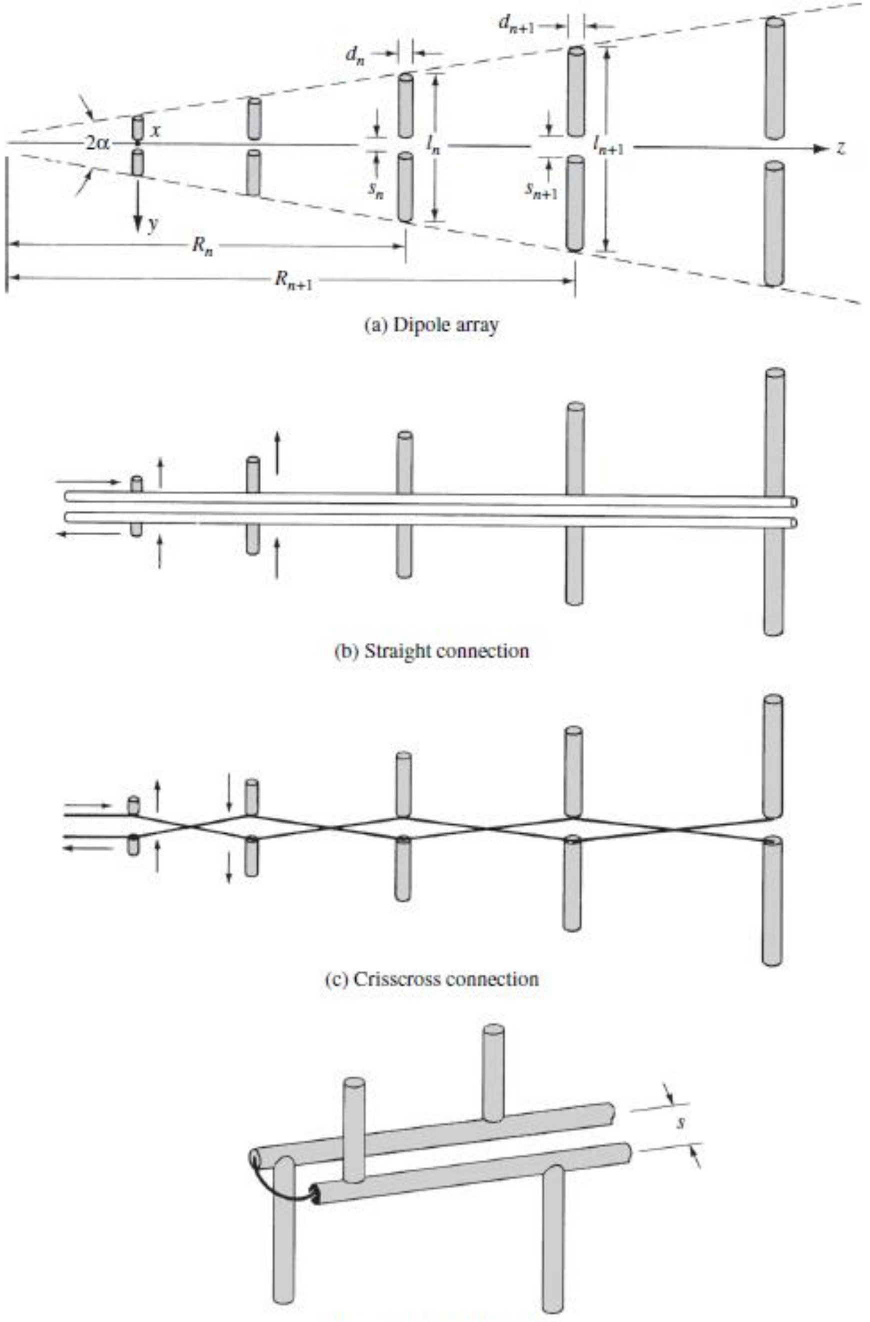
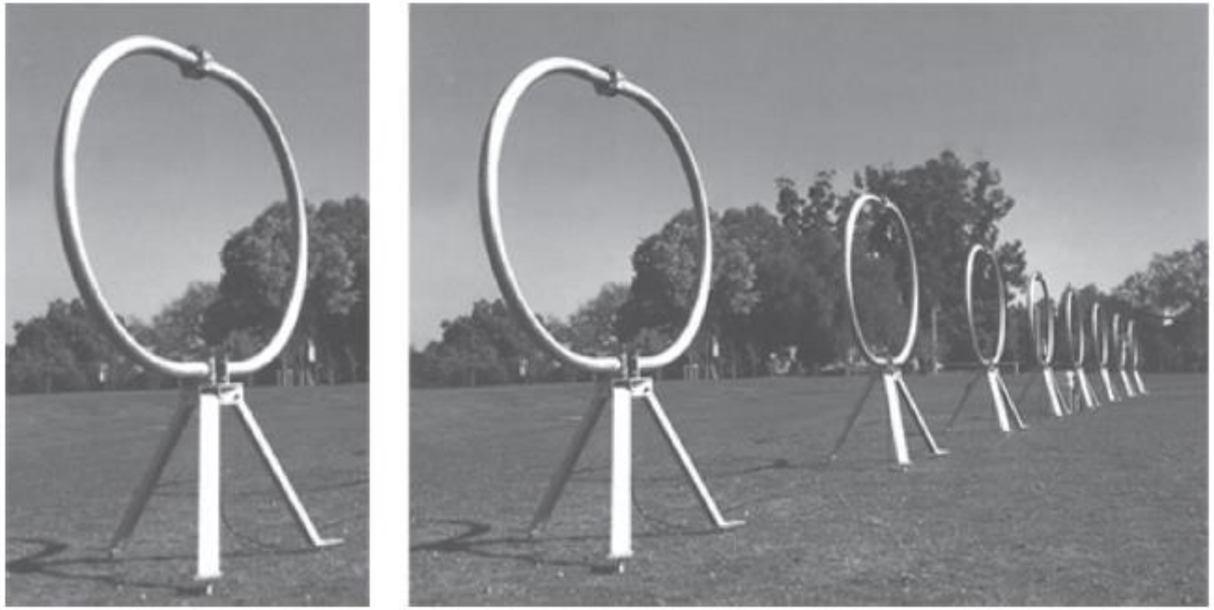


Figure 11.9 Log-periodic dipole array and associated connections.

3. Examples

- Array of loop elements



(a) Single element (b) Array of eight elements

- Array of microstrip antennas

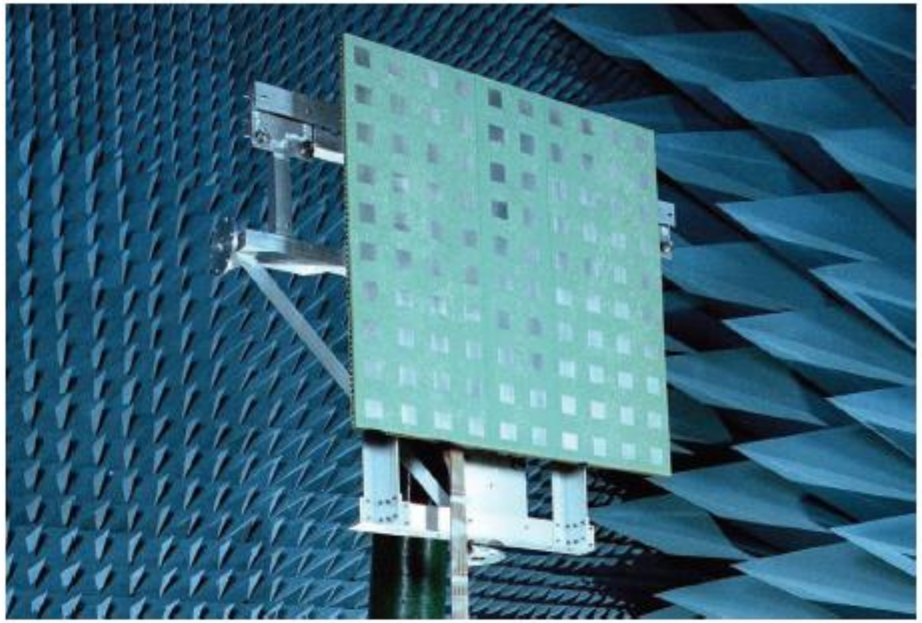


Figure 14.40 Antenna array of 10 × 10 rectangular microstrip patches, 2–2.3 GHz, for space-to-space communications. (Courtesy: Ball Aerospace & Technologies Corp.).

3. Examples

- advanced array design of slots, used in the AWACS (Airborne Warning and Control System (AWACS))

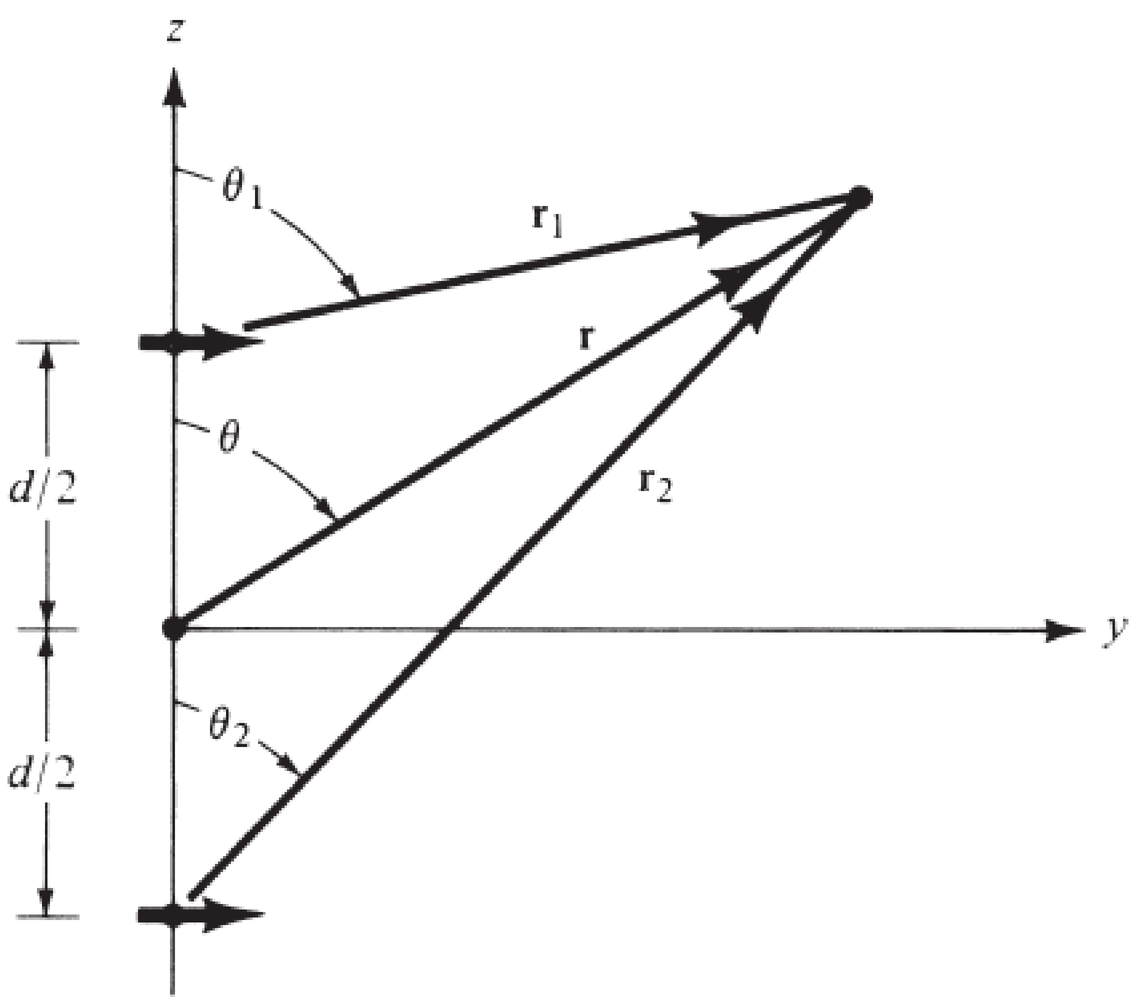
Figure 6.29 AWACS antenna array of waveguide slots. (PHOTOGRAPH COURTESY: Northrop Grumman Corporation).

4. Two element array

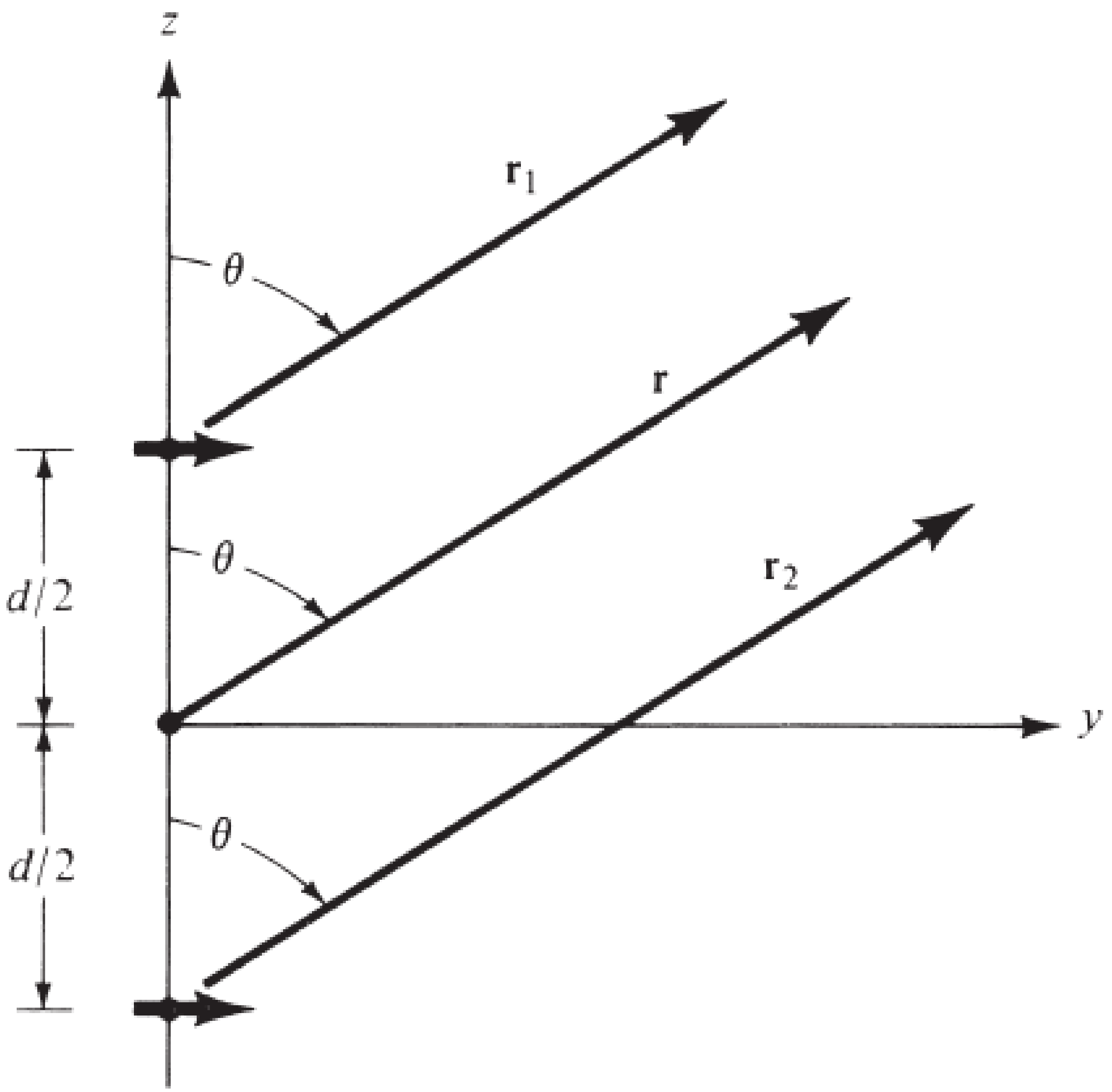
- simplest and one of the most practical arrays is formed by placing the elements along a line.
- the antenna under investigation is an array of two infinitesimal horizontal dipoles positioned along the *z*-axis.
- **total field radiated by the two elements**, *assuming no coupling between the elements*, is equal to the sum of the two and in the *y-z* plane, is

$$\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi} \left\{ \frac{e^{-j[kr_1 - (\beta/2)]}}{r_1} \cos\theta_1 + \frac{e^{-j[kr_2 + (\beta/2)]}}{r_2} \cos\theta_2 \right\}$$

- β is the **difference in phase excitation between the elements**.
- The **magnitude** excitation of the radiators **is identical**



(a) Two infinitesimal dipoles



(b) Far-field observations

4. Two element array

- Assuming far-field observations $\mathbf{E}_t = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l}{4\pi} \left\{ \frac{e^{-j[kr_1-(\beta/2)]}}{r_1} \cos\theta_1 + \frac{e^{-j[kr_2+(\beta/2)]}}{r_2} \cos\theta_2 \right\}$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta [e^{+j(kd\cos\theta+\beta)/2} + e^{-j(kd\cos\theta+\beta)/2}]$$

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta \left\{ 2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right\}$$

$$\left.\begin{aligned} &\theta_1 \simeq \theta_2 \simeq \theta \\ &r_1 \simeq r - \frac{d}{2}\cos\theta \\ &r_2 \simeq r + \frac{d}{2}\cos\theta \end{aligned}\right\} \text{ for phase variations}$$

$$r_1 \simeq r_2 \simeq r \quad \text{for amplitude variations}$$

- for the two-element array of constant amplitude, the array factor

$$\mathrm{AF} = 2\cos[\tfrac{1}{2}(kd\cos\theta+\beta)]$$

- When normalized, array factor is:

$$(\mathrm{AF})_n = \cos[\tfrac{1}{2}(kd\cos\theta+\beta)]$$

4. Two element array

- **array factor** is a function of the **geometry of the array** and the **excitation phase**.
- By varying the separation *d* and/or the phase β between the elements, the characteristics of the array factor and of the total field of the array can be controlled.
- The **far-zone field of a uniform two-element array of identical elements** is equal to the ***product of the field of a single element***, *at a selected reference point (usually the origin), and* ***the array factor of that array***.

$$\boxed{\mathbf{E}(\text{total}) = [\mathbf{E}(\text{single element at reference point})] \times [\text{array factor}]}$$

pattern multiplication for arrays of identical elements

4. Two element array

- The array factor, in general, is a function of
  a) the number of elements,
  b) their geometrical arrangement,
  c) their relative magnitudes,
  d) their relative phases, and
  e) their spacings.
- The array factor will be of simpler form if the elements have identical amplitudes, phases, and spacings.

4. Two element array

- The array factor does not depend on the directional characteristics of the radiating elements themselves.
- To obtain **field pattern of array** of directive radiating elements
  a) **Array factor can be formulated by replacing the actual elements with isotropic (point) sources**. (Note: Each point-source is assumed to have the amplitude, phase, and location of the corresponding element it is replacing)
  B) Once the array factor has been derived using the point-source array, **the total field of the actual array is obtained by the use of the equation**

$$\boxed{\mathbf{E}(\text{total}) = [\mathbf{E}(\text{single element at reference point})] \times [\text{array factor}]}$$

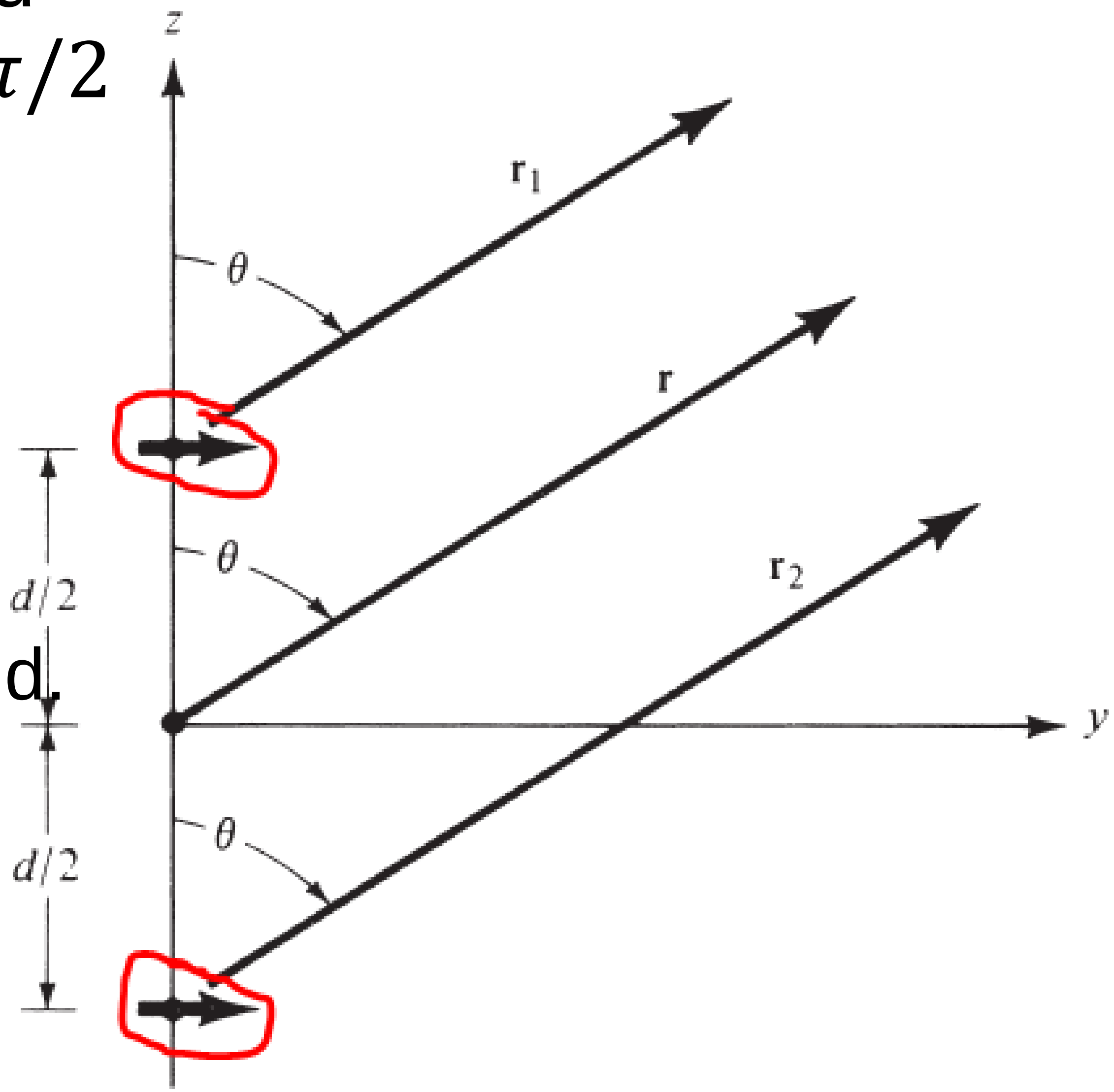
Problem 1: For the two element infinitesimal array, find the nulls of the total field when $d = \lambda/4$ and a) $\beta = 0$, b) $\beta = +\pi/2$ and c) $\beta = -\pi/2$

- Solution: Two element array:

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta \left\{ 2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]\right\}$$

In this problem, $\mathrm{kd} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$

And magnitude will be 1 for normalised field.



(b) Far-field observations

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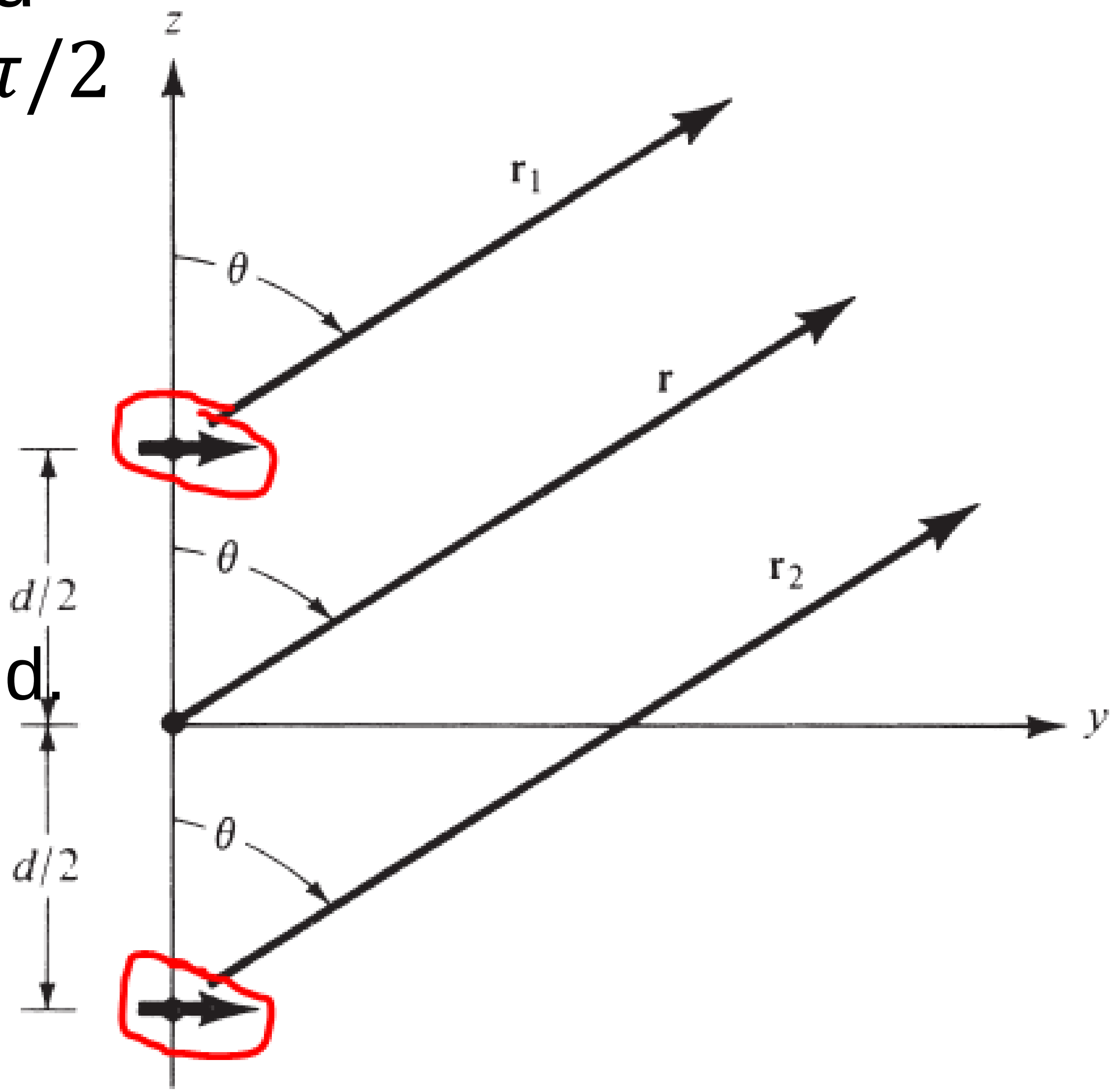

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- Solution: Two element array:

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta \left\{ 2\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]\right\}$$

In this problem, $\mathrm{kd} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$

And magnitude will be 1 for normalised field.

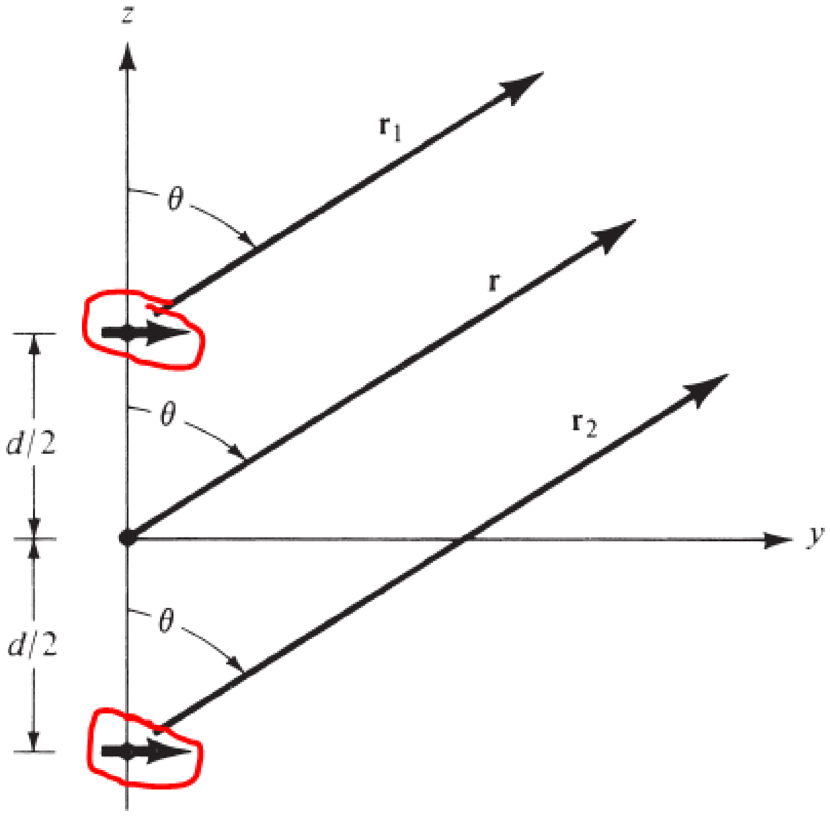
- **Case 1: $\beta = 0$**

$$E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + 0\right)\right]$$

$$= \cos\theta \cos\left[\frac{\pi}{4}\cos\theta\right]$$

Problem 1: For the two element infinitesimal array, find the nulls of the total field when $d = \lambda/4$ and

a) $\boldsymbol{\beta = 0}$, b) $\beta = +\pi/2$ and c) $\beta = -\pi/2$



(b) Far-field observations

- Normalized field: $E_{tn} = \cos\theta \cos\left(\frac{\pi}{4}\cos\theta\right)$
- At nulls, $E_{tn}|_{\theta=\theta_n} = 0$

  $\cos\theta_n \cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0$
- $\cos\theta_n = 0$ $\quad\theta_n = \pi/2$ (90°) Only solution
- Or $\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0$ **can have two soln:**

  $\frac{\pi}{4}\cos\theta_n = \frac{\pi}{2}$

or $\frac{\pi}{4}\cos\theta_n = -\frac{\pi}{2}$

-

Problem 1: For the two element infinitesimal array, find the nulls of the total field when $d = \lambda/4$ and

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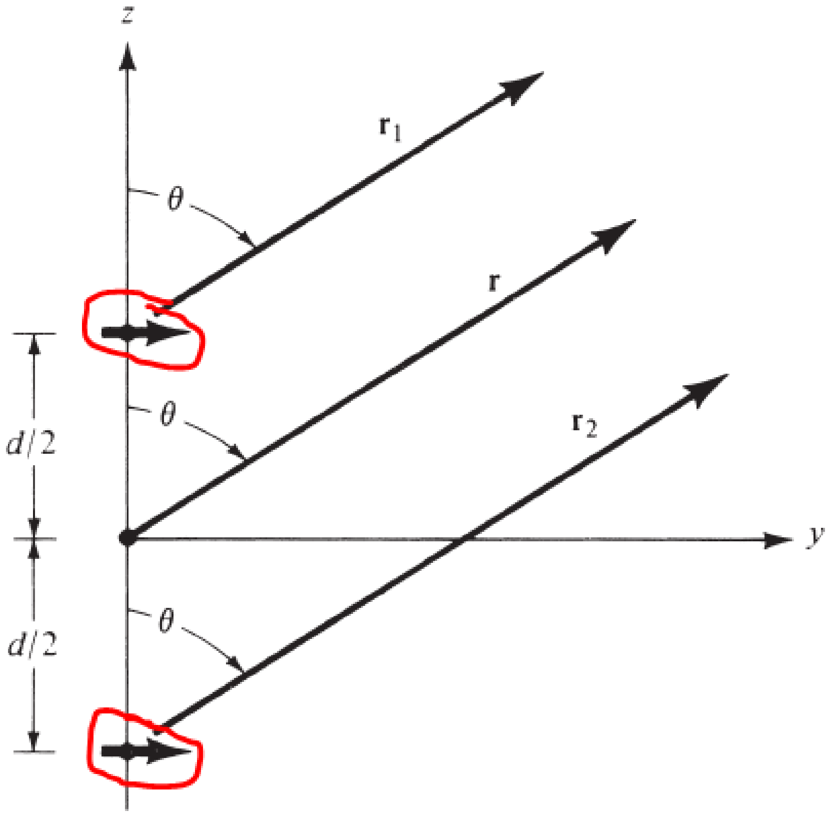

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- Or $\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0$ **can have two soln:**

  $\frac{\pi}{4}\cos\theta_n = \frac{\pi}{2}$ $\cos\theta_n = 2$ cannot be possible

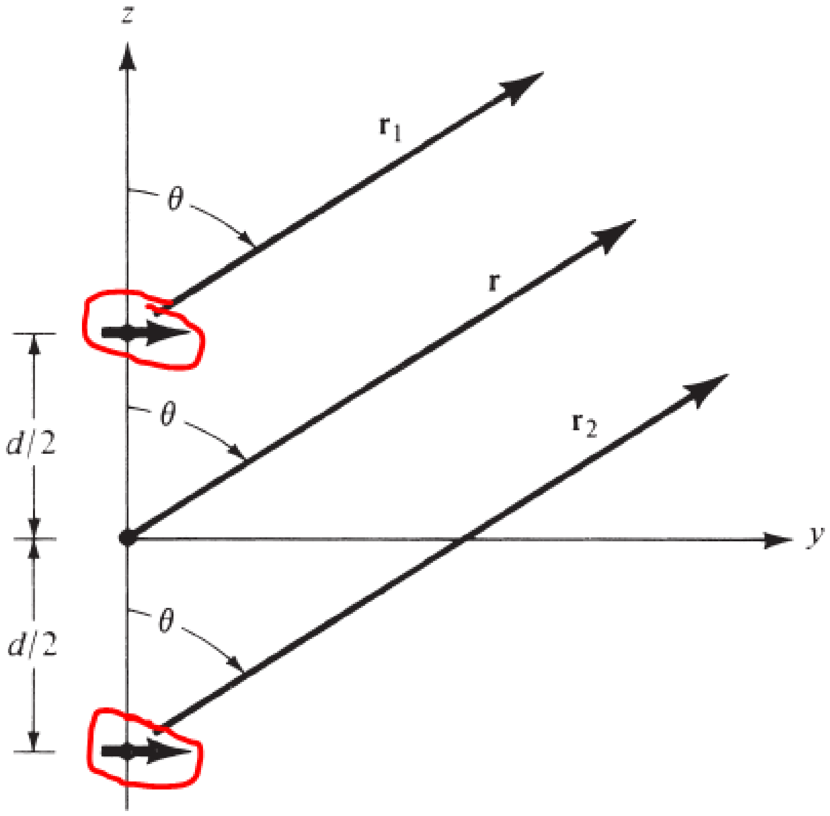
or $\frac{\pi}{4}\cos\theta_n = -\frac{\pi}{2}$ $\cos\theta_n = -2$ cannot be possible

- only null occurs at $\theta = 90^\circ$ and is due to the pattern of the individual elements. The array factor does not contribute any additional nulls because there is not enough separation between the elements to introduce a phase difference of 180° between the elements, for any observation angle.

Problem 1: For the two element infinitesimal array, find the nulls of the total field when *d* = λ/4 and
a) $\beta = 0$, **b)** $\boldsymbol{\beta = +\pi/2}$ and c) $\beta = -\pi/2$

- **Normalized field** $E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)\right]$
- At nulls, $E_{tn}|_{\theta=\theta_n} = 0$
  $\cos\theta_n \cos\left(\frac{\pi}{4}(\cos\theta_n + 1)\right) = 0$
- $\cos\theta_n = 0 \quad \theta_n = \pi/2$ (90°)
- Or $\cos\left(\frac{\pi}{4}(\cos\theta_n + 1)\right) = 0$ **can have two soln:**
  $\frac{\pi}{4}(\cos\theta_n + 1) = \frac{\pi}{2}$

Or $\frac{\pi}{4}(\cos\theta_n + 1) = -\frac{\pi}{2}$



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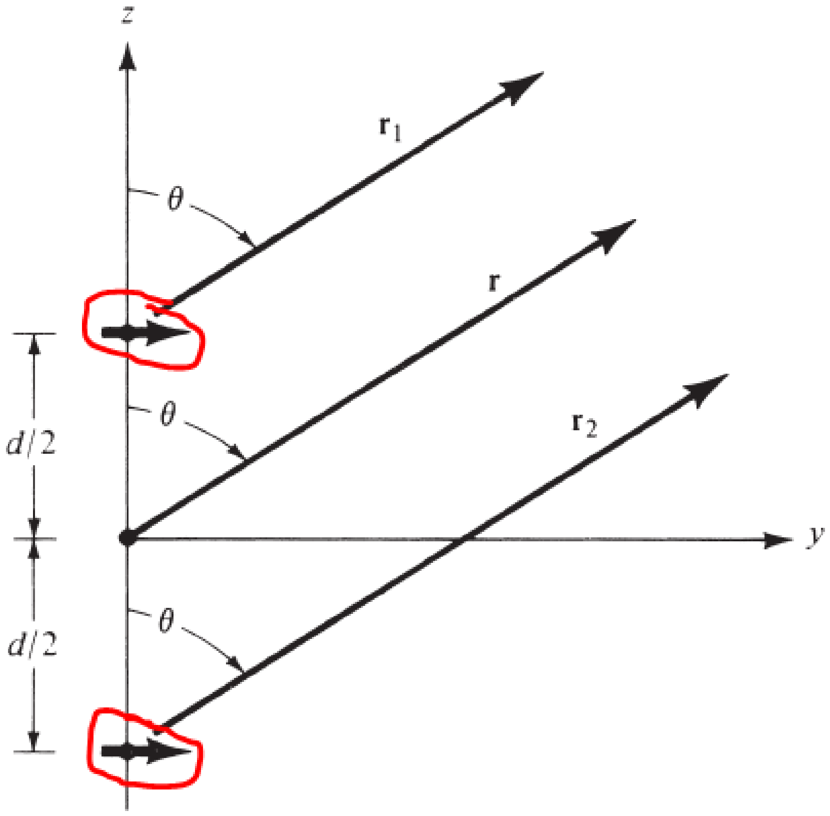


(b) Far-field observations

- **Normalized field** $E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)\right]$
- At nulls, $E_{tn}|_{\theta=\theta_n} = 0$

$$\cos\theta_n \cos\left(\frac{\pi}{4}(\cos\theta_n + 1)\right) = 0$$

- $\cos\theta_n = 0$ $\quad \theta_n = \pi/2\ (90°)$
- Or $\cos\left(\frac{\pi}{4}(\cos\theta_n + 1)\right) = 0$ **can have two soln:**

$$\frac{\pi}{4}(\cos\theta_n + 1) = \frac{\pi}{2} \qquad \cos\theta_n = 1 \quad \theta_n = 0$$

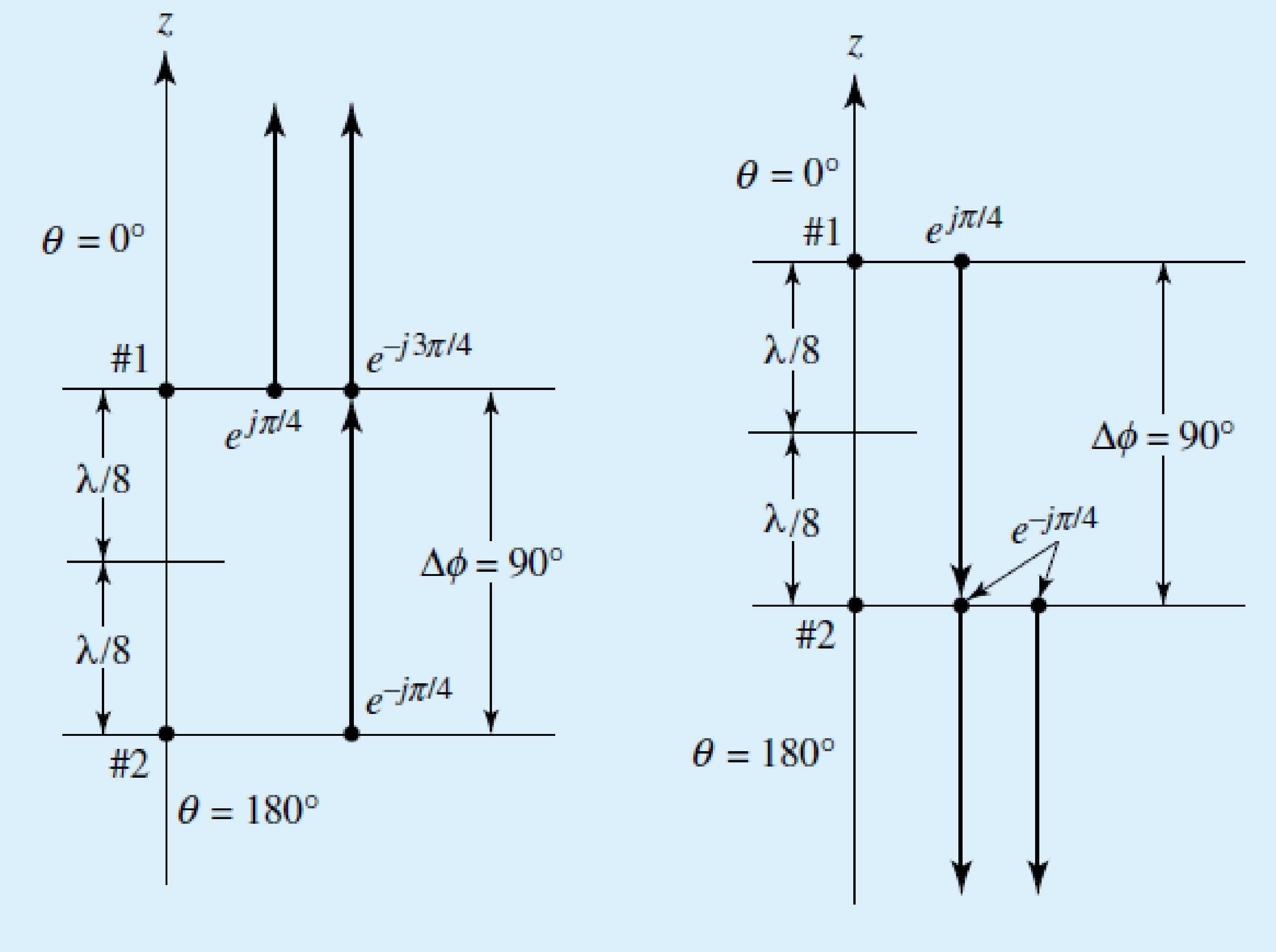
Or $\frac{\pi}{4}(\cos\theta_n + 1) = -\frac{\pi}{2}$ $\qquad \cos\theta_n = -3$ (not possible)

- The nulls of the array occur at $\theta = 90°$ and $0°$. The null at $0°$ is introduced by the arrangement of the elements (array factor).

Problem 1: For the two element infinitesimal array, find the nulls of the total field when $d = \lambda/4$ and a) $\beta = 0$, **b) $\beta = +\pi/2$** and c) $\beta = -\pi/2$

- Diagramatic explanation

- The element in the negative *z*-axis has an initial phase lag of 90◦ relative to the other element.
  As the wave from that element travels toward the positive *z*-axis ($\theta = 0$◦ direction), it undergoes an additional 90◦ phase retardation when it arrives at the other element on the positive *z*-axis.



(a) $\theta = 0°$ direction (b) $\theta = 180°$ direction

Problem 1: For the two element infinitesimal array, find the nulls of the total field when $d = \lambda/4$ and a) $\beta = 0$, b) $\beta = +\pi/2$ and **c) $\boldsymbol{\beta = -\pi/2}$**

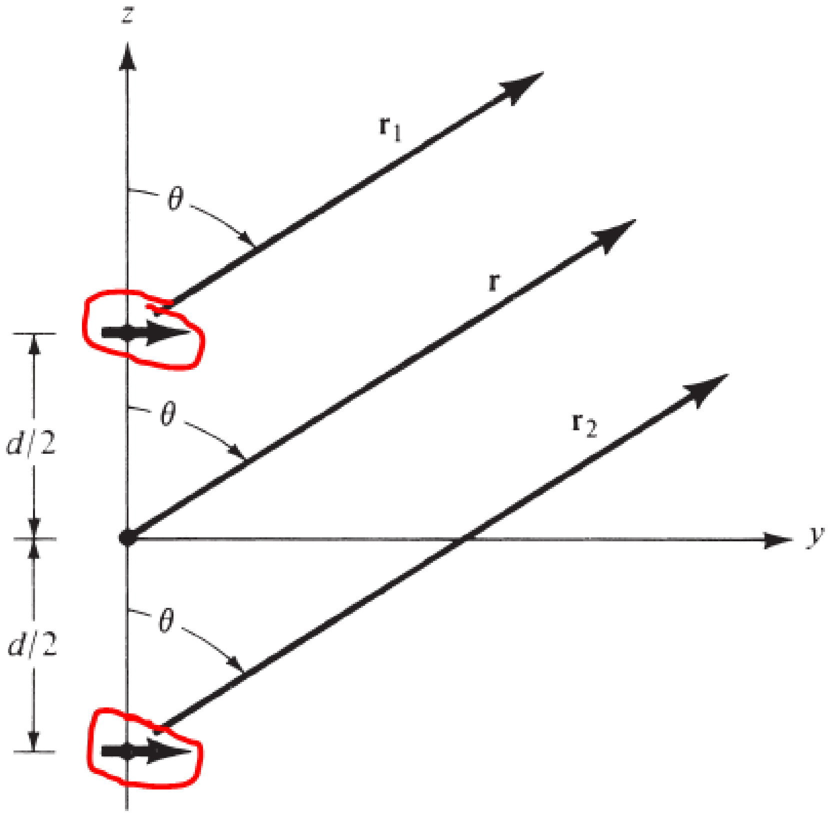
- **Normalized field** $E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta - \frac{\pi}{2}\right)\right]$
- At nulls, $E_{tn}|_{\theta=\theta_n} = 0$

$$\cos\theta_n \cos\left(\frac{\pi}{4}(\cos\theta_n - 1)\right) = 0$$

- $\cos\theta_n = 0 \quad \theta_n = \pi/2$ (90°)
- Or $\cos\left(\frac{\pi}{4}(\cos\theta_n - 1)\right) = 0$ **can have two soln:**

$$\frac{\pi}{4}(\cos\theta_n - 1) = \frac{\pi}{2}$$

or $\frac{\pi}{4}(\cos\theta_n - 1) = -\frac{\pi}{2}$



(b) Far-field observations

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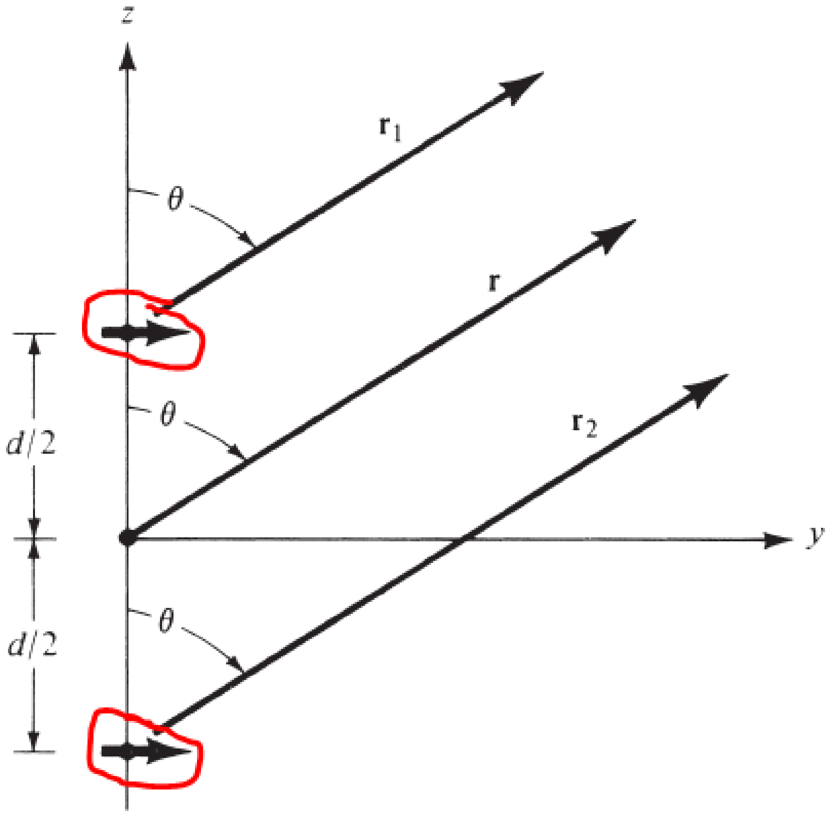

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- At nulls, $E_{tn}|_{\theta=\theta_n} = 0$

  $\cos\theta_n \cos\left(\frac{\pi}{4}(\cos\theta_n - 1)\right) = 0$
- $\cos\theta_n = 0 \quad \theta_n = \pi/2$ (90°)
- Or $\cos\left(\frac{\pi}{4}(\cos\theta_n - 1)\right) = 0$ **can have two soln:**

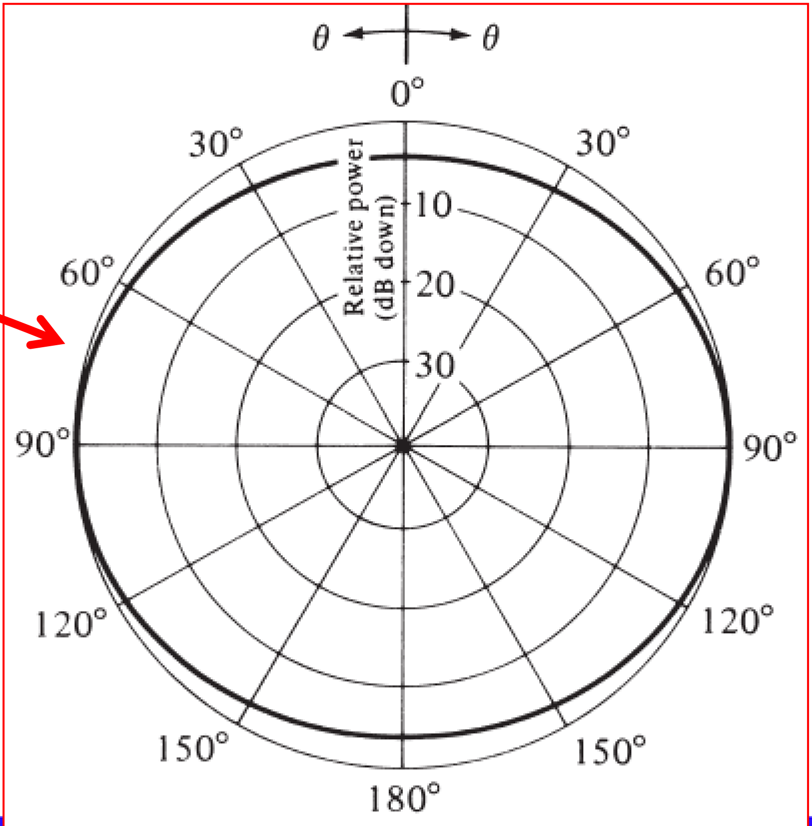
  $\frac{\pi}{4}(\cos\theta_n - 1) = \frac{\pi}{2} \qquad \cos\theta_n = 3$ (Not possible)

or $\frac{\pi}{4}(\cos\theta_n - 1) = -\frac{\pi}{2} \qquad \cos\theta_n = -1 \quad \boldsymbol{\theta_n = \pi\ (or\ 180°)}$

- The nulls of the array occur at $\theta = 90^\circ$ and 180°. The null at 180° is introduced by the arrangement of the elements (array factor).

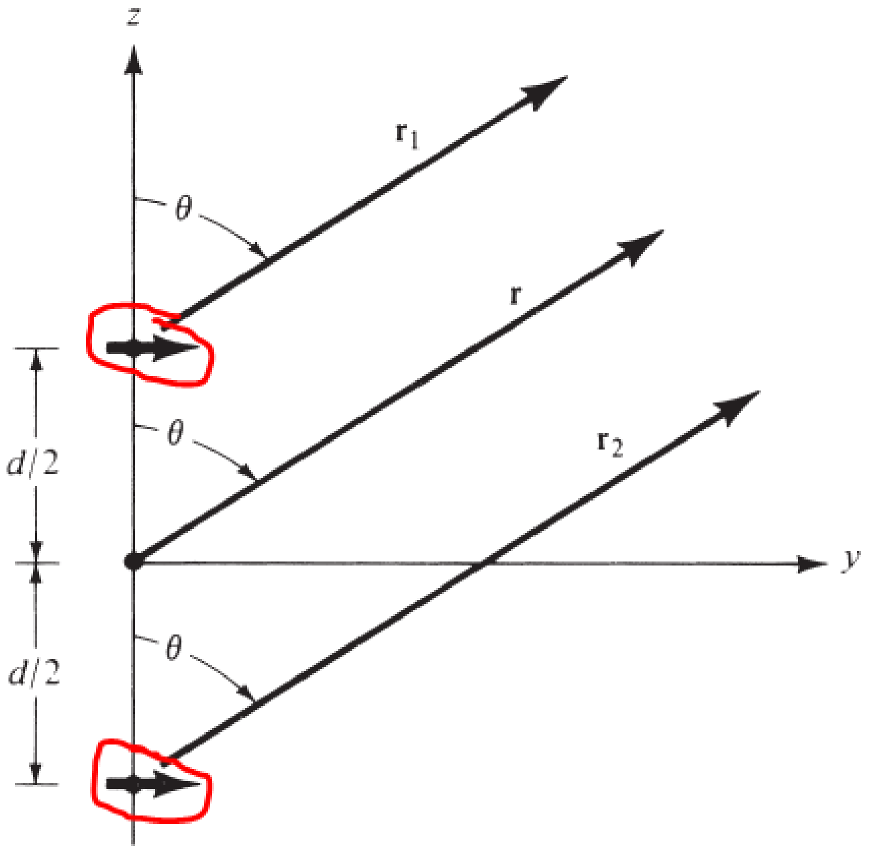
4.2 Pattern multiplication for two element antenna

- To better illustrate the pattern multiplication rule, the normalized patterns of the single element, the array factor, and the total array for each of the above array examples
- *In each case, the pattern is normalized to its own maximum*.
- Since the **array factor** for the example is nearly isotropic (within 3 dB), the element pattern and the total pattern are almost identical in shape. Sometimes, array factor can have nulls.



Problem: Consider an array of two identical infinitesimal dipoles oriented. For a separation *d* and phase excitation difference β between the elements, find the angles of observation where the nulls of the array occur. The magnitude excitation of the elements is the same.

$$\mathbf{E}_t = \hat{\mathbf{a}}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r}\cos\theta \left\{2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right\}$$



(b) Far-field observations

Problem: Consider an array of two identical infinitesimal dipoles oriented. For a separation *d* and phase excitation difference β between the elements, find the angles of observation where the nulls of the array occur. The magnitude excitation of the elements is the same.

- **Case 1: $\boldsymbol{\beta = 0}$** $\quad E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + 0\right)\right]$

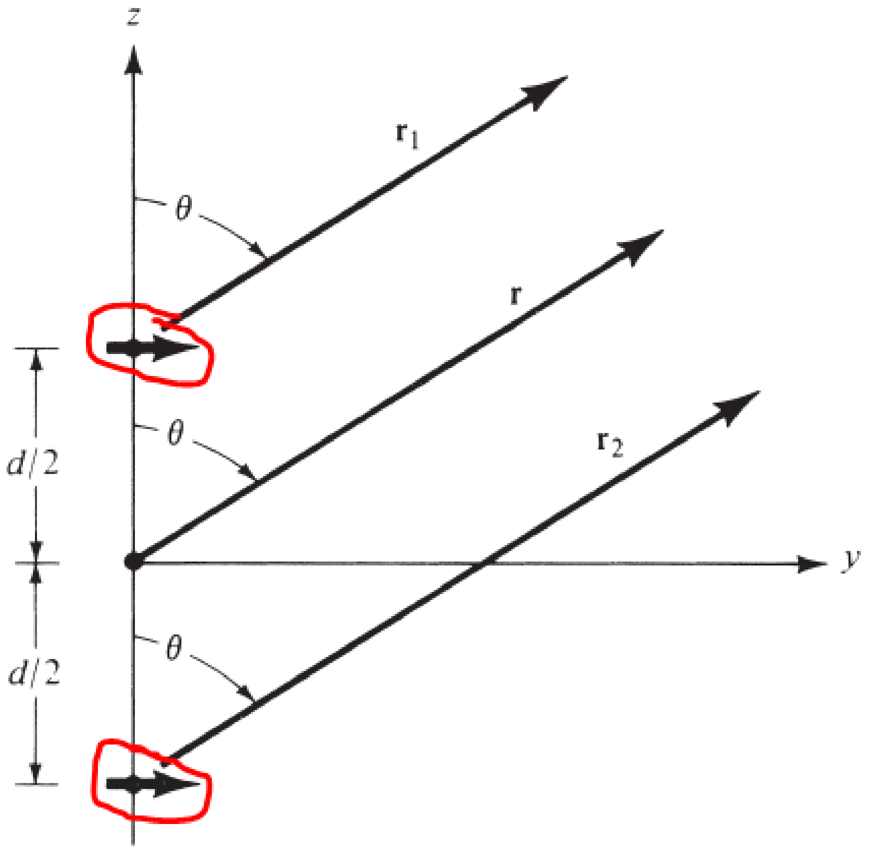
$$= \cos\theta \cos\left[\frac{\pi}{4}\cos\theta\right]$$

Case 2: $\boldsymbol{\beta = \frac{\pi}{2}}$ $\quad E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right)\right]$

$$= \cos\theta \cos\left[\frac{\pi}{2}(\cos\theta + 1)\right]$$

Case 3: $\boldsymbol{\beta = -\frac{\pi}{2}}$ $\quad E_{tn} = \cos\theta \cos\left[\frac{1}{2}\left(\frac{\pi}{2}\cos\theta - \frac{\pi}{2}\right)\right]$

$$= \cos\theta \cos\left[\frac{\pi}{2}(\cos\theta - 1)\right]$$

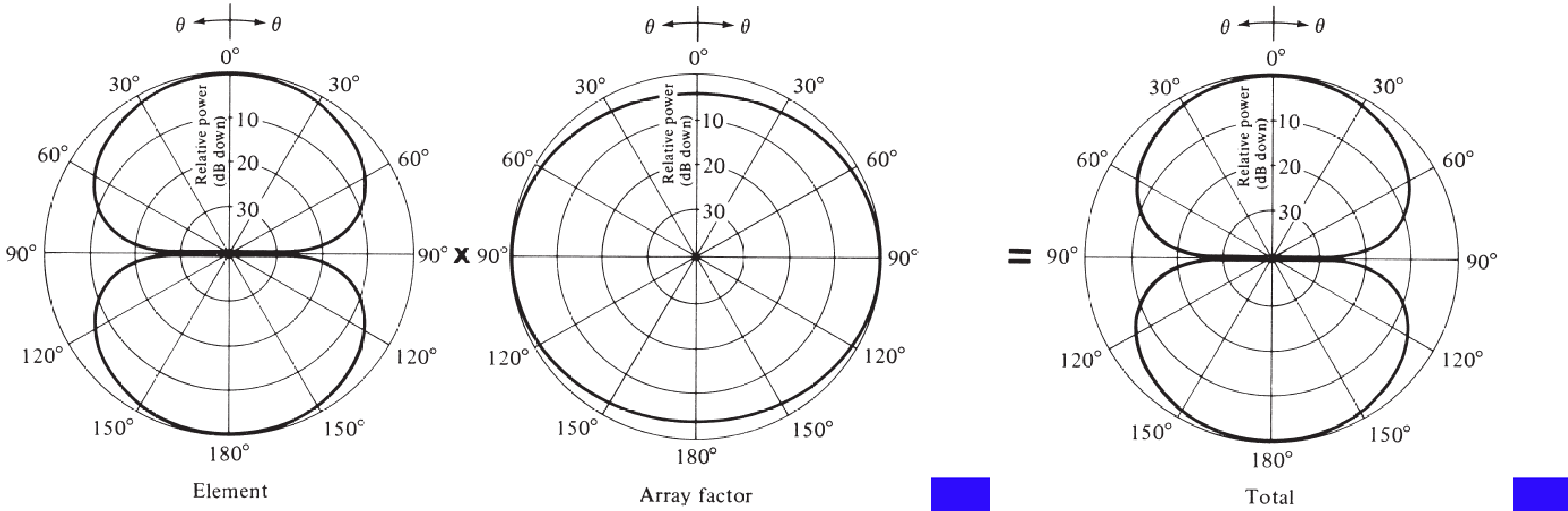


(b) Far-field observations

Problem: Consider an array of two identical infinitesimal dipoles oriented. For a separation *d* and phase excitation difference β between the elements, find the angles of observation where the nulls of the array occur. The magnitude excitation of the elements is the same.

- Pattern multiplication of Element, array factor, and total field patterns of a two-element array of infinitesimal horizontal dipoles with **identical phase excitation (β = 0°, *d* = λ/4).**

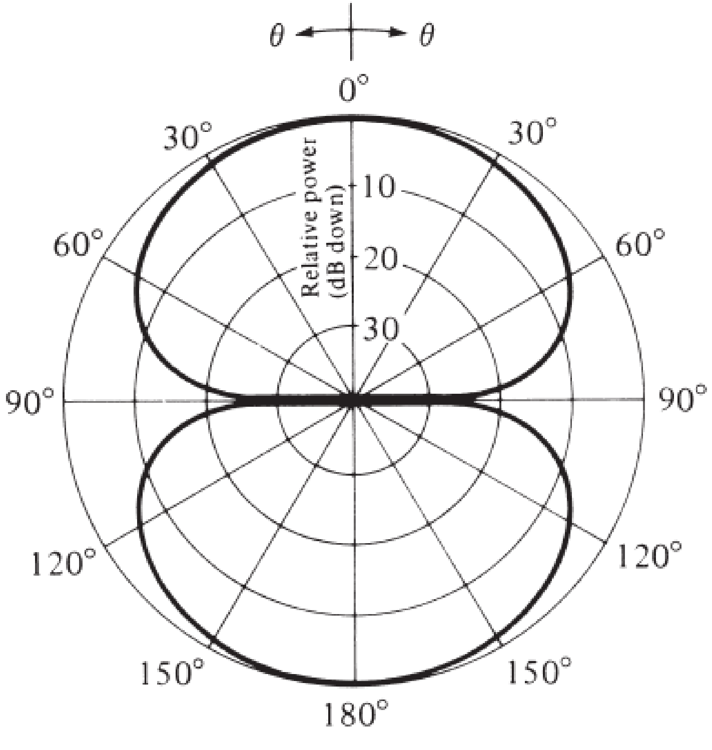
Case 1: $\beta = 0$ $E_{tn} = \cos\theta \cos\left[\frac{\pi}{4}\cos\theta\right]$



Problem: Consider an array of two identical infinitesimal dipoles oriented. For a separation *d* and phase excitation difference β between the elements, find the angles of observation where the nulls of the array occur. The magnitude excitation of the elements is the same.

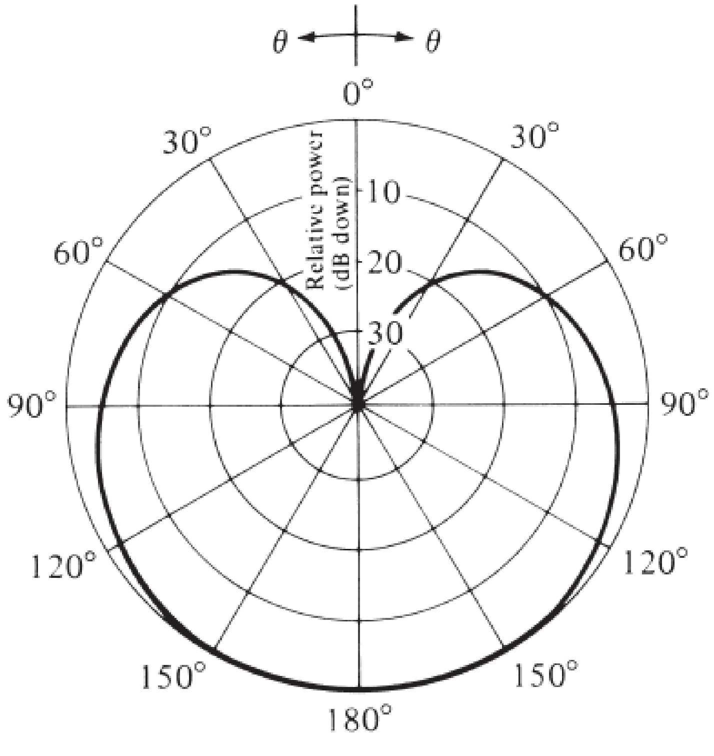
- Pattern multiplication of Element, array factor, and total field patterns of a two-element array of infinitesimal horizontal dipoles with **phase excitation (β = 90°, *d* = λ/4).**

Case 2: $\boldsymbol{\beta = \frac{\pi}{2}}$ $E_{tn} = \cos\theta \cos\left[\frac{\pi}{2}(\cos\theta + \mathbf{1})\right]$



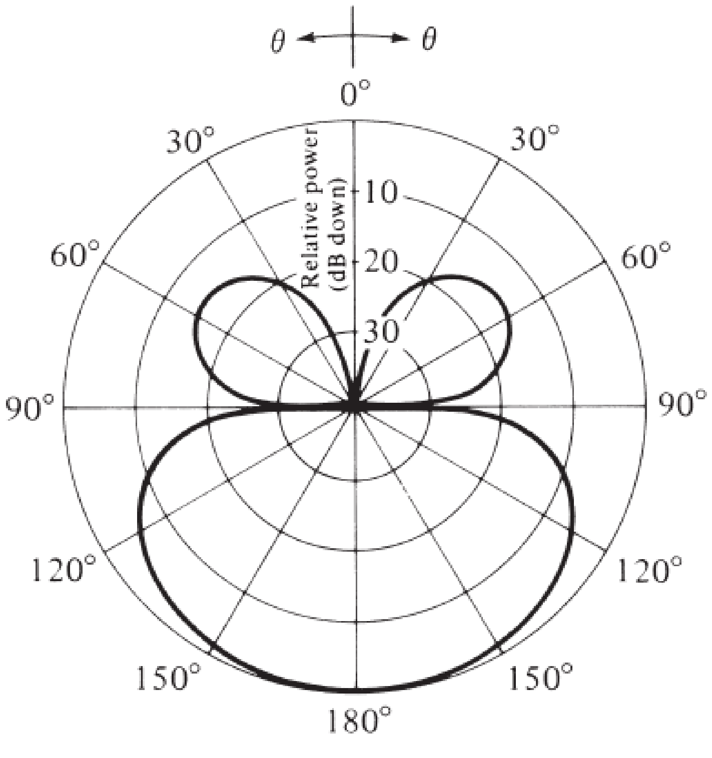
Element

x



Array factor

=

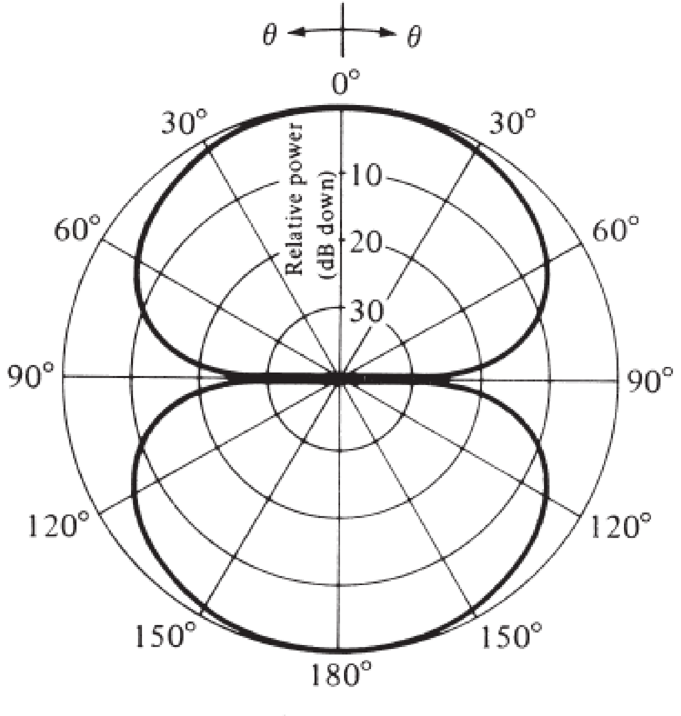


Total

Problem: Consider an array of two identical infinitesimal dipoles oriented. For a separation *d* and phase excitation difference β between the elements, find the angles of observation where the nulls of the array occur. The magnitude excitation of the elements is the same.

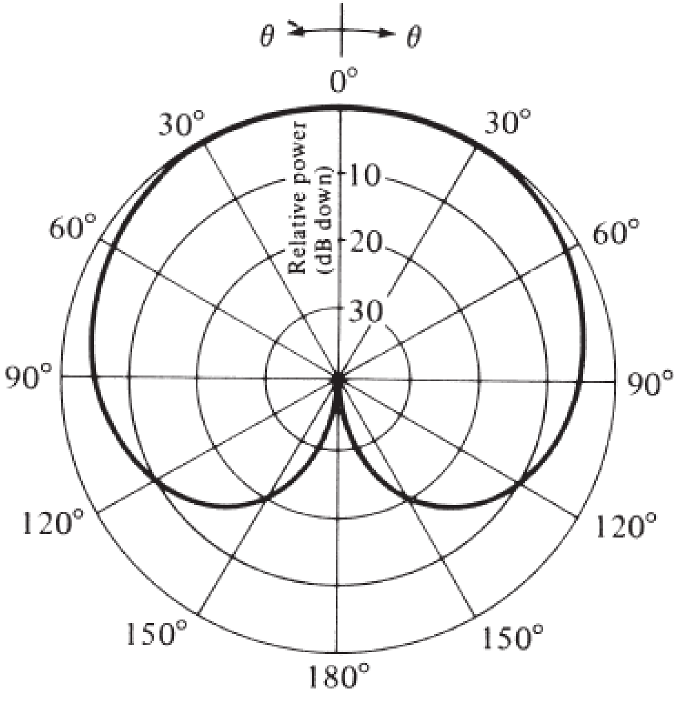
- Pattern multiplication of Element, array factor, and total field patterns of a two-element array of infinitesimal horizontal dipoles with **phase excitation (β = -90∘, *d* = λ/4).**

Case 3: $\boldsymbol{\beta = -\frac{\pi}{2}}$ $E_{tn} = \cos\theta \cos\left[\frac{\pi}{2}(\cos\theta - 1)\right]$



Element

x



Array factor

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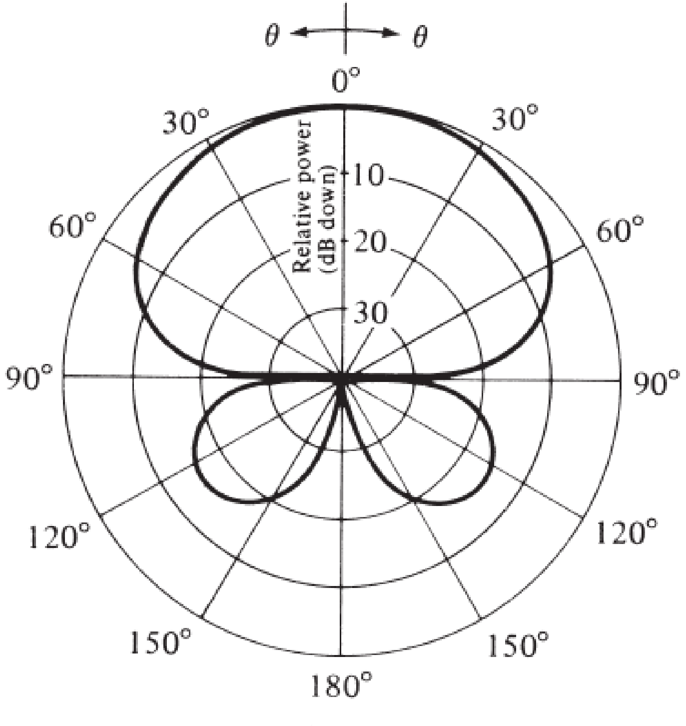

Total