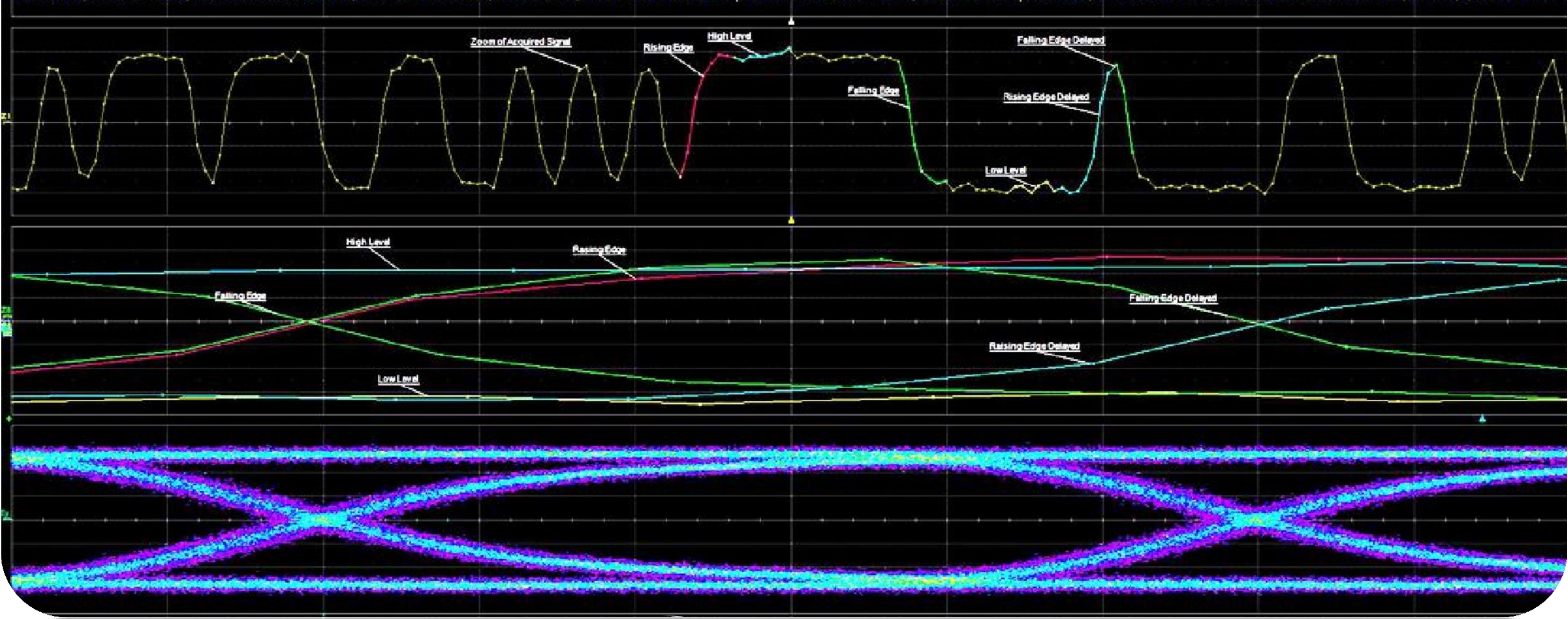
Baseband System

Module-4



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Topics to be discussed

- Introduction
- Inter Symbol Interference (ISI)
- Role of Matched Filter
- Nyquist criterion for distortion less transmission
- Raised cosine spectrum
- Correlative coding
- Eye pattern
- Equalization

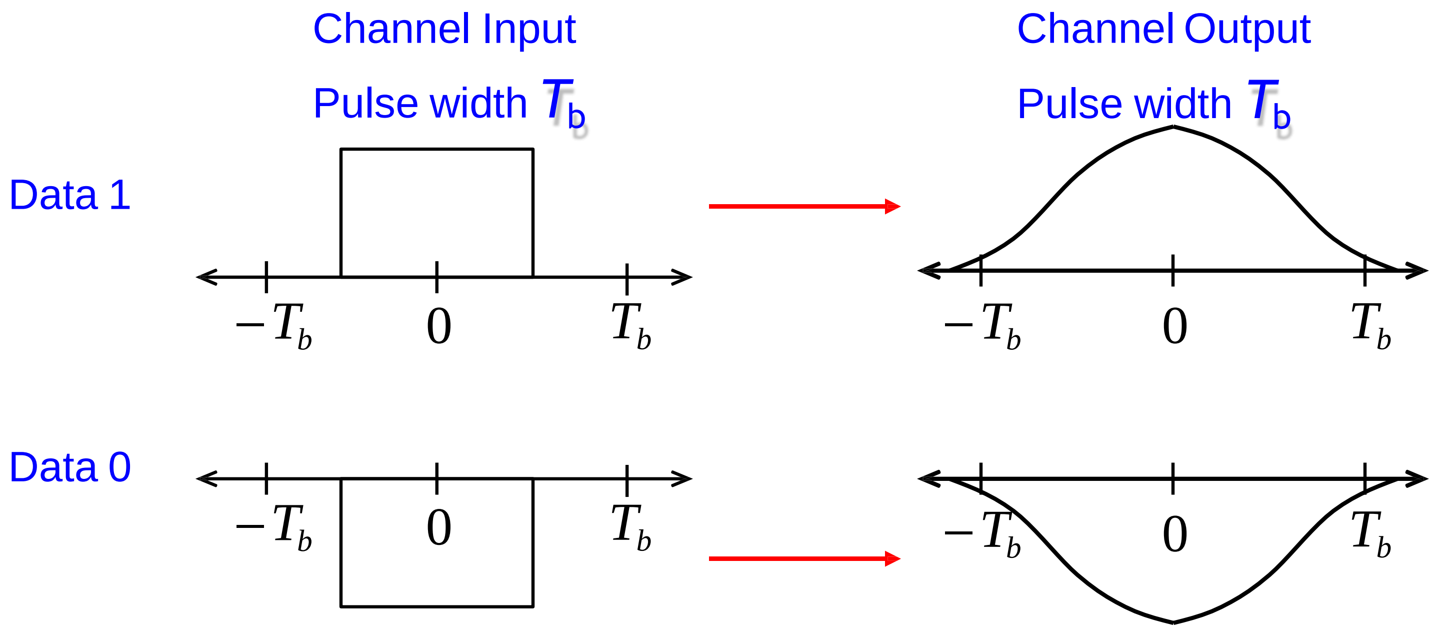
Introduction

- If any rectangular pulse representing one bit of information is applied to the channel input, the shape of the pulse will be distorted at the channel output
- The distorted pulse may consist of a main lobe representing the original bit of information along with side lobes
- These **side lobes** represent channel distortion also referred as *intersymbol interference*
- Our main objective is that of a *signal design,* whereby the effect of symbol interference is reduced to zero

InterSymbol Interference (ISI)

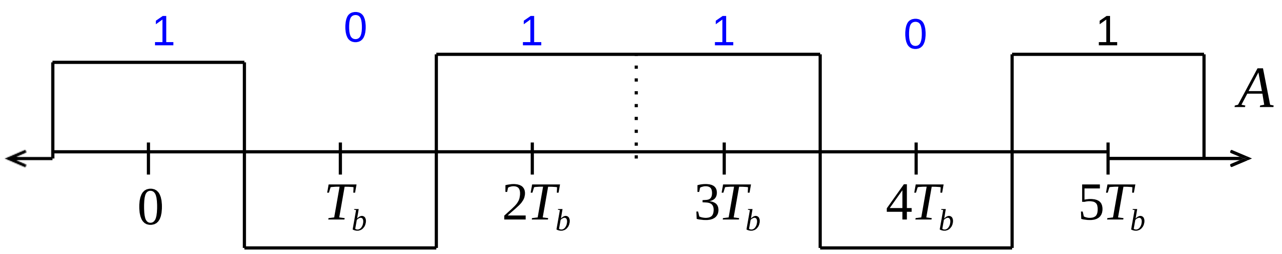
- **Intersymbol interference (ISI)** occurs when a pulse spreads out in such a way that it interferes with adjacent pulses *at the sample instant.*

- Example: assume polar NRZ line code. The channel outputs are shown as spreaded (width T_b becomes $2T_b$) pulses shown (Spreading due to bandlimited channel characteristics).

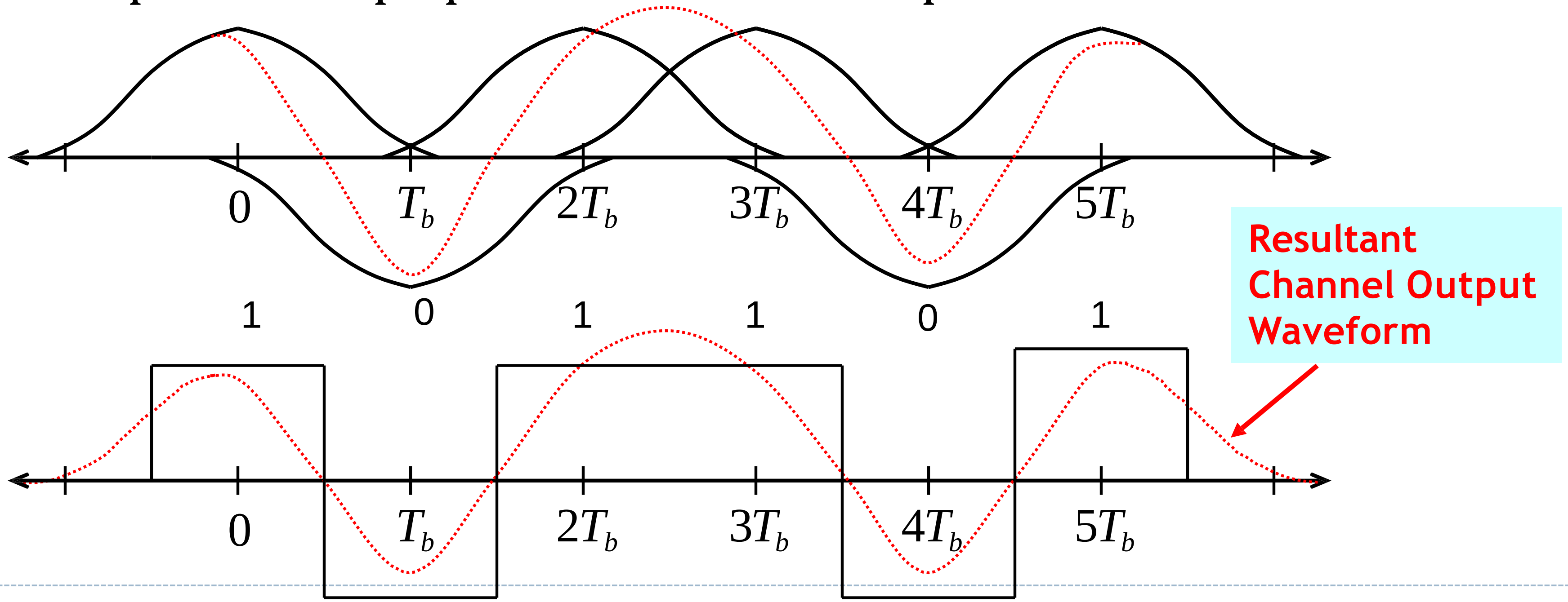


InterSymbol Interference (ISI)

- For the input data stream:

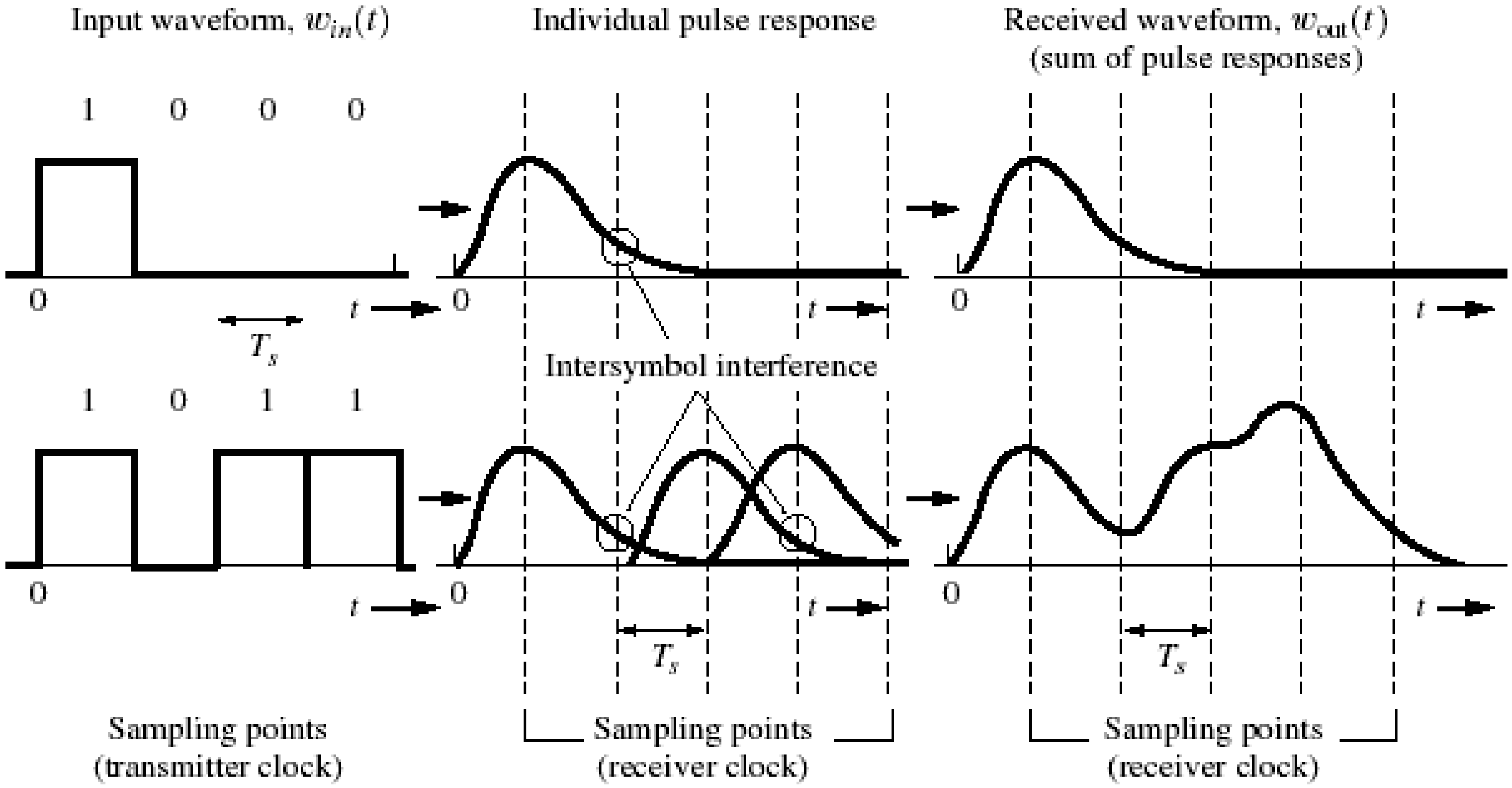


- The channel output is the superposition of each bit's output:



InterSymbol Interference (ISI)

- If the rectangular multilevel pulses are filtered improperly, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause ***ISI.***



Matched Filter

- Matched filter is a device for the optimal detection of digital pulse
- The impulse response of the filter is matched to the shape of the input signal
- Matched filter generates an output to maximize the output peak power ratio to mean noise power within its frequency response.
- In telecommunications, it is the optimal linear filter used to increase the SNR or signal-to-noise ratio in the existence of additive stochastic noise.

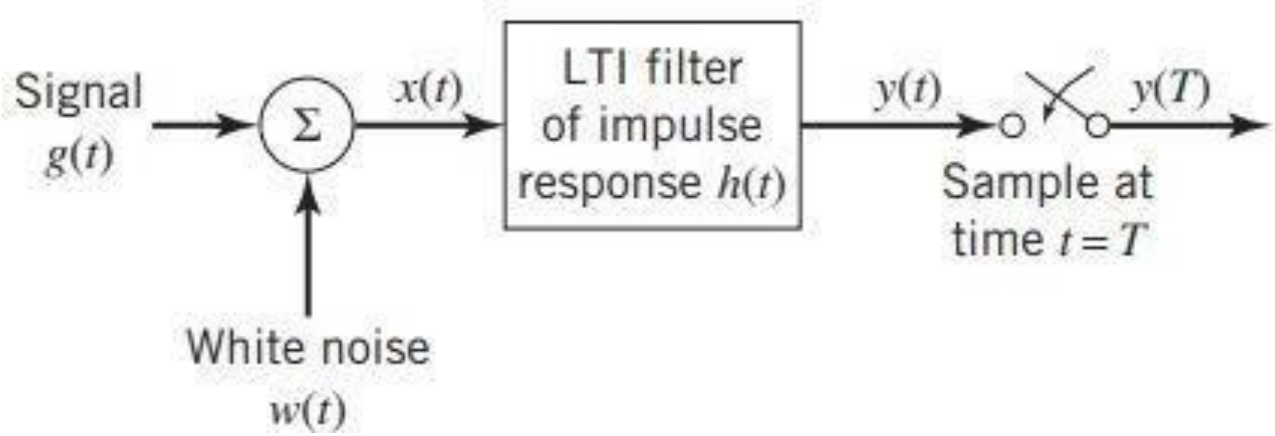
- The characteristics of matched filter include the following.

➢ The Signal to noise ratio maximization is possible even for non-Gaussian noise.
➢ The output generated by this filter is like signal energy in the nonexistence of noise.
➢ They are applicable for the detection of signals.

Matched Filter

Optimum Receiver Filter

- Here, before the transmitted signal $g(t)$ is received at the receiver, it is first passed through a band-limited linear receiver with impulse response h (t) as shown below:



Matched Filter

- We assume that the source of uncertainty lies with noise $w(t)$
- The function of receiver is to detect the signal $g(t)$ in an optimum manner from the received signal $x(t)$,given by:

$$\boldsymbol{x(t) = g(t) + w(t), \; 0 \leq t \leq T}$$

- Here, $g(t)$ is the transmitted pulse signal which is corrupted by additive white noise process $w(t)$
- T is an arbitrary observation interval
- Since, the filter is linear, the resulting output $y(t)$ may be expressed as:

$$\boldsymbol{y(t) = g_0(t) + n(t)}$$

Matched Filter

- Here, $\boldsymbol{g_0(t) = g(t) * h(t)}$ and $\boldsymbol{n(t) = w(t) * h(t)}$
- $g_0(t)$ and $n(t)$ are produced by the convolution of signal and noise components of $x(t)$ with impulse response $h(t)$ respectively
- For $g_0(t)$ to be considerably greater than $n(t)$ we maximize the *peak pulse signal-to-noise ratio* given by:

$$\eta = \frac{|g_0(T)|^2}{E[n^2(t)]}, \text{measured at } t = T$$

Matched Filter

- Here, we investigate the optimization of impulse response of a filter to maximize the peak signal-to-noise ratio at the output of the filter

- Using Fourier transform, $g_0(t)$ can be expressed in terms of $G(f)H(f)$ as:

$$g_0(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi f t)\,df$$

- At the sample time $t = T$, the signal power is given by:

$$|g_0(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi f \mathrm{T})\,df\right|^2$$

Role of Matched Filter

- The power spectral density is given by:

$$S_{NN}(f) = \frac{N_0}{2}|H(f)|^2$$

- And, the average output noise power is:

$$E[n^2(t)] = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df$$

- Therefore, the peak signal-to-noise ratio will be:

$$\eta = \frac{\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi f\mathrm{T})\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

Role of Matched Filter

- Using Schwartz's inequality, we have:

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi f\mathrm{T})\,df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} G|f|^2 df$$

- The peak signal-to-noise ratio can thus be expressed as:

$$\eta \leq \frac{2}{N_0}\int_{-\infty}^{\infty} G|f|^2 df$$

- Correspondingly, $H(f)$ assumes its optimum value denoted by $H_{opt}(f)$
- The optimum value is given by:

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$$

- Here, k is a scaling factor of appropriate dimensions

Role of Matched Filter

- $G^*(f)$ is the complex conjugate of the Fourier transform of the input signal $g(t)$

- In terms of time domain, using inverse Fourier transform we have:

$$h_{opt}(t) = kG^*(f)\exp[-j2\pi f(T-t)]\,df$$

- For a real signal $g(t)$ we have, $G^*(f) = G(-f)$, we may rewrite:

$$h_{opt} = k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]\,df = kg(T-t)$$

Role of Matched Filter

- Thus, the impulse response of the optimum filter will be given by:

$$h_{opt}(t) = kg(T - t)$$

- The above equation shows that h_{opt} is the delayed version of the input signal $g(t)$

- It means that $h_{opt}(t)$ is "*matched*" to the input signal

- *A linear time-invariant filter defined in this way is called a matched filter*

Error Rate Due to Channel Noise in a Matched Filter Receiver

- Consider a binary transmission system based on polar *nonreturn-to-zero (NRZ) signaling*
- Here, 0 and 1 are represented by positive and negative rectangular pulses of equal amplitude and duration
- The noise is modeled as additive white Gaussian noise $w(t)$ of zero mean and power spectral density $N_0/2$
- In the signaling interval $0 \leq t \leq T_b$, the received signal $x(t)$ is given by:

$$x(t) = \begin{cases} +A + w(t), & symbol\ 1\ was\ sent \\ -A + w(t), & symbol\ 0\ was\ sent \end{cases}$$

- T_b is the *bit duration* and A is the *transmitted pulse amplitude*

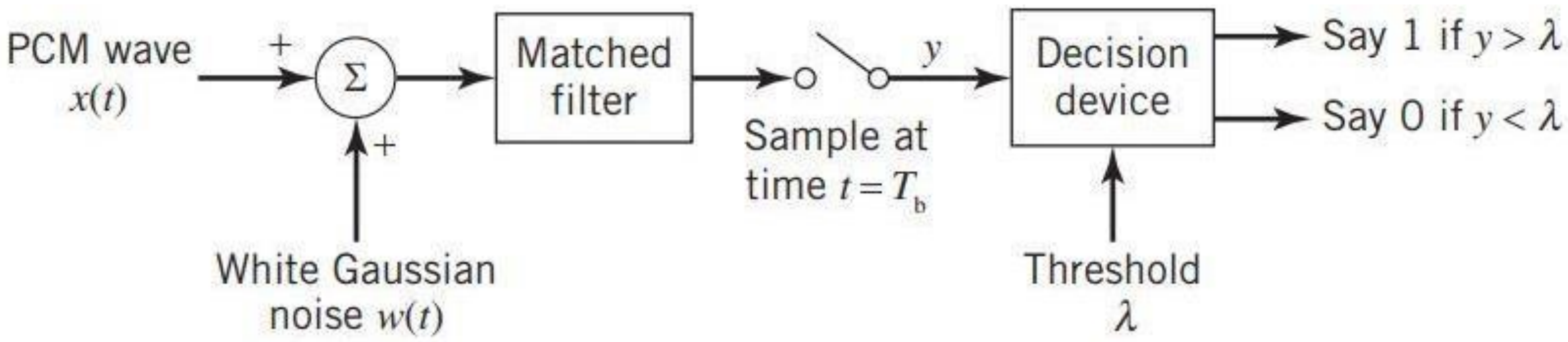
Error Rate Due to Channel Noise in a Matched Filter Receiver

- Given the noisy signal $x(t)$, the receiver is required to make a decision in each signalling interval

- Two possible kinds of errors are considered:

i. *Error of the first kind*: Symbol 1 is chosen when 0 was actually transmitted

ii. *Error of the second kind*: Symbol 0 is chosen when 1 was actually transmitted



Error Rate Due to Channel Noise in a Matched Filter Receiver

- The probability density function of the random variable Y, given that symbol 0 was sent

$$f_Y(y|0) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp(-\frac{(y+A)^2}{N_0/T_b})$$

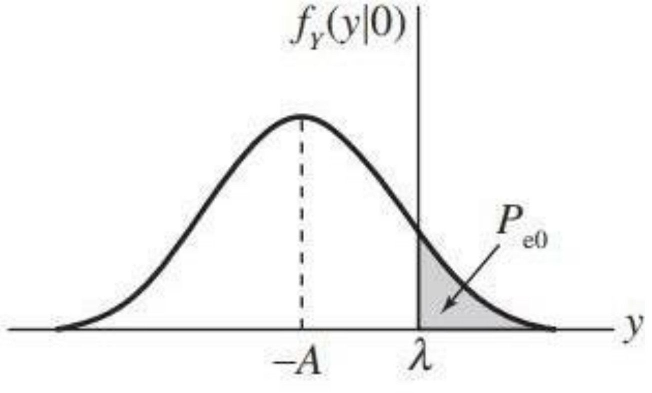
- Let P_{e0} denote the *conditional probability* of error, given that symbol 0 was sent
- This probability is defined by the shaded area under the curve of $f_Y(y|0)$ from the threshold λ to infinity
- When noise is present, y occasionally assumes a value greater than λ, in which case an error is made

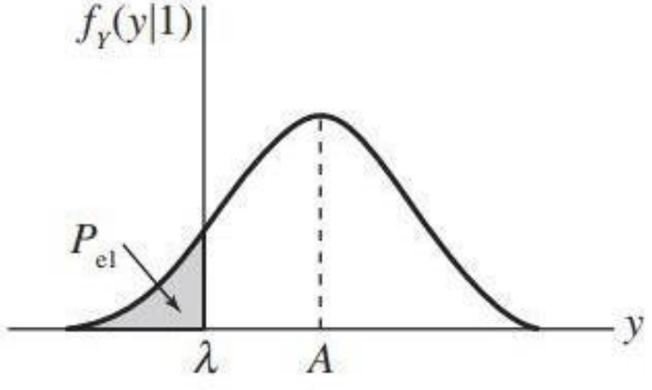
Error Rate Due to Channel Noise in a Matched Filter Receiver

- Analysis of the effect of channel noise on a binary system

- (a) Probability density function of random variable Y at matched filter output when a 0 is transmitted,

- (b) Probability density function of Y when a 1 is transmitted





Error Rate Due to Channel Noise in a Matched Filter Receiver

- The probability of this error, conditional on sending 0, is defined by:

$$P_{e0} = P(y > \lambda | symbol\ 0\ was\ sent) = \int_{\lambda}^{\infty} f_Y(y|0)dy = \frac{1}{\sqrt{\pi N_0/T_b}} \int_{\lambda}^{\infty} \exp\left(-\frac{(y+A)^2}{\frac{N_0}{T_b}}\right) dy$$

- Define a new variable: $z = \frac{y+A}{\sqrt{N_0/2T_b}}$

- We may thus reformulate: $P_{e0} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_0}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz$

- E_b is the transmitted signal energy per bit defined by $E_b = A^2 T_b$

Error Rate Due to Channel Noise in a Matched Filter Receiver

- We now introduce the definition of the so-called Q-function

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz$$

- The conditional probability of error P_{e0} in terms of the Q-function as follows:

$$P_{e0} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- We also find that $P_{e0} = P_{e1}$
- A channel for which the conditional error probabilities P_{e1} and P_{e0} are equal is said to be binary symmetric

Error Rate Due to Channel Noise in a Matched Filter Receiver

- The average probability of symbol error P_e in the receiver is given by

$$P_e = p_0 P_{e0} + p_1 P_{e1}$$

- Where, p_0 and p_1 are the *a priori* probabilities of binary symbols 0 and 1, respectively
- Since, $P_{e1} = P_{e0}$ and $p_0 = p_1 = \frac{1}{2}$, therefore we finally obtain:

$$P_e = P_{e1} = P_{e0}$$

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Error Rate Due to Channel Noise in a Matched Filter Receiver