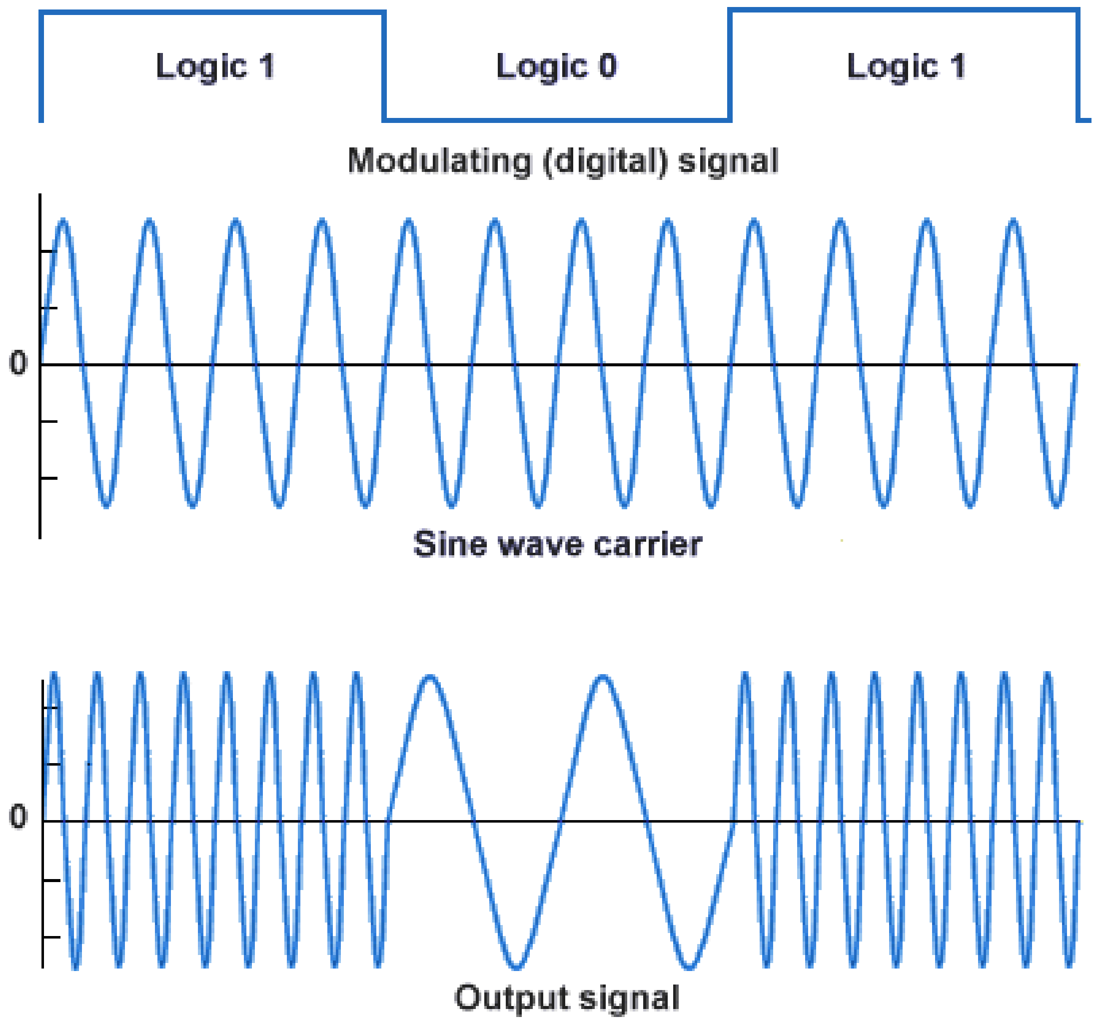
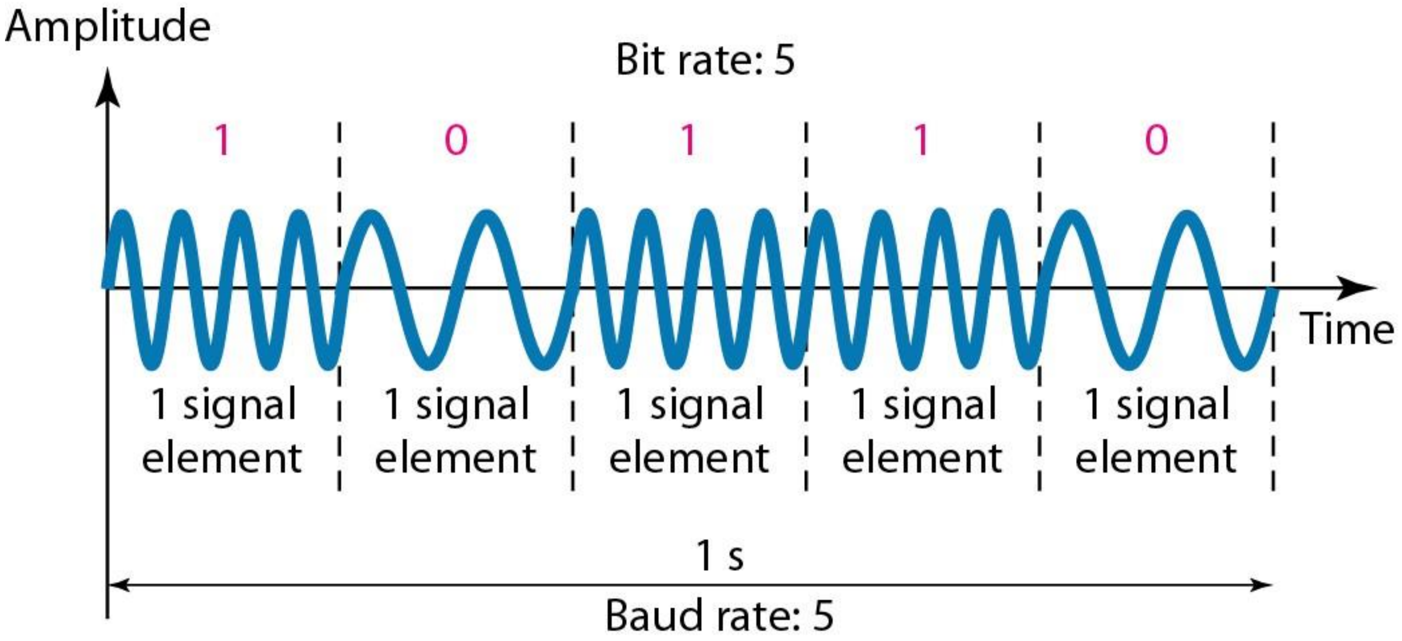
Frequency Shift Keying (FSK)

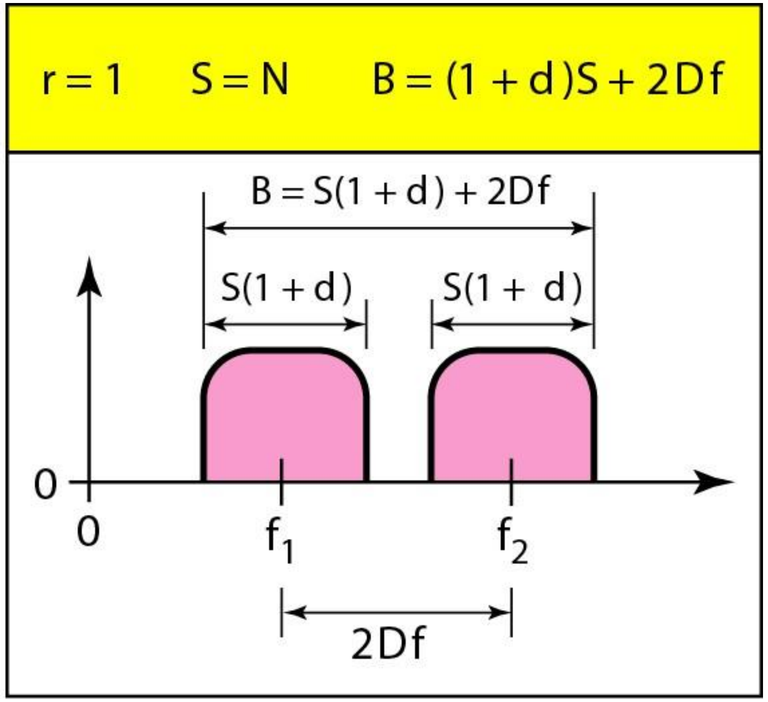


Frequency Shift Keying (FSK)

- If the difference between the two frequencies (f_1 and f_2) is $2\Delta f$, then the required BW B will be:

$$\boldsymbol{B = (1 + d)S + 2\Delta f}$$





Frequency Shift Keying (FSK)

Signal representation of FSK

- Binary FASK is represented by

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0 & elsewhere \end{cases}$$

- Where, $i = 1,2$ and

E_b = transmitted signal energy per bit

Frequency Shift Keying (FSK)

- Transmitted frequency is $f_i = \frac{n_c + i}{T_b}$ for some fixed integer n_c and $i = 1,2$
- Symbol "1" is represented by mark frequency,

$$s_1(t) = \sqrt{2\frac{E_b}{T_b}}\, Cos\; 2\pi f_1 t \qquad \text{Where, } f_1 = f_c + \frac{\Omega}{2\pi}$$

- Symbol "0" is represented by space frequency,

$$s_2(t) = \sqrt{2\frac{E_b}{T_b}}\, Cos\; 2\pi f_2 t \qquad \text{Where, } f_2 = f_c - \frac{\Omega}{2\pi}$$

Frequency Shift Keying (FSK)

- This FSK signal is also called as **Sunde's FSK**
- The two carriers are represented in terms of orthonormal basis functions as

$$\varphi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}}\ Cos\ 2\pi f_i t, & 0 \leq t \leq T_b \\ 0, & elsewhere \end{cases}$$

Frequency Shift Keying (FSK)

Signal Space diagram of FSK

- The coefficients $s_{i\,j}$

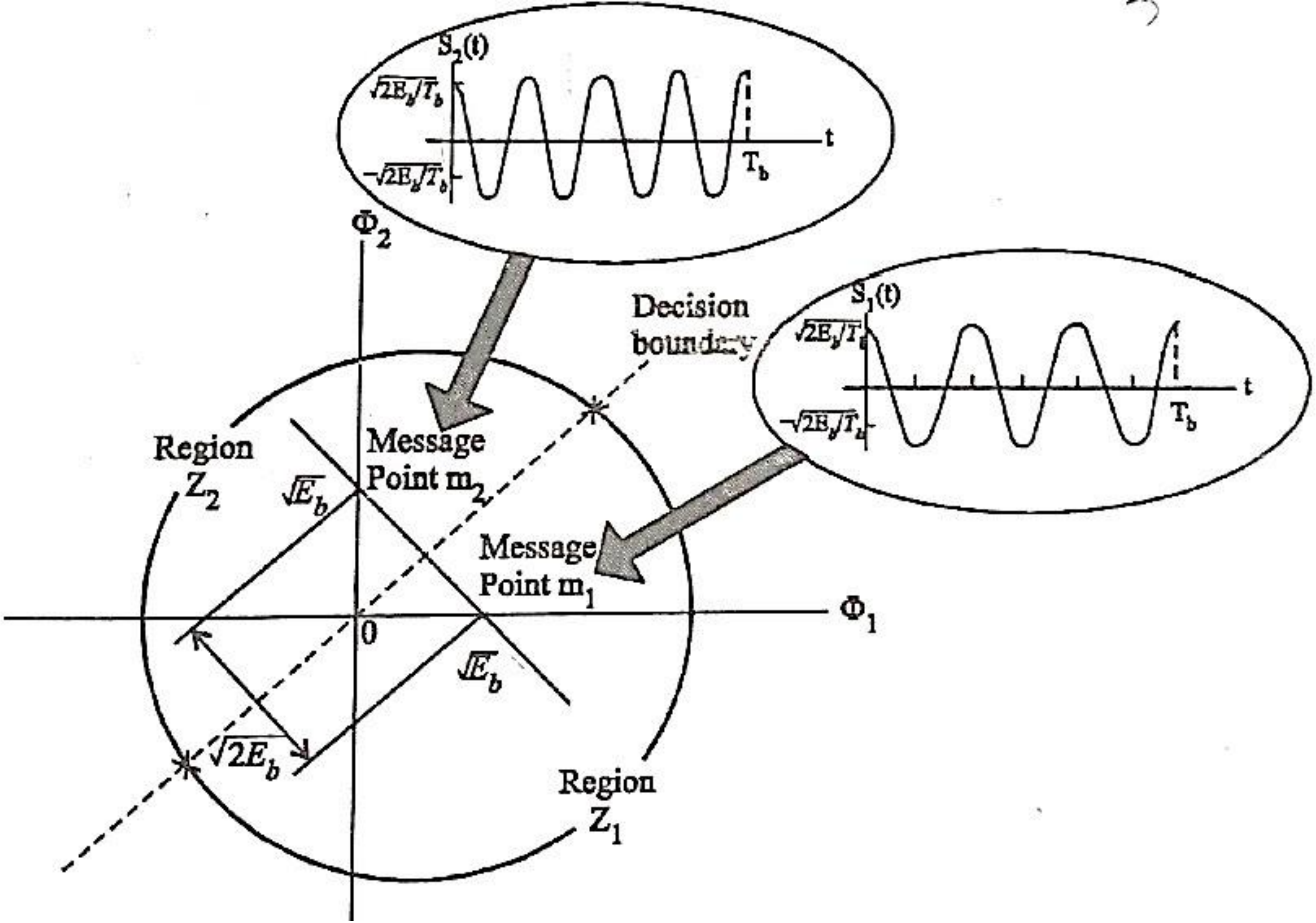
for $i = 1{,}2$ and j $= 1{,}2$ is given by,

$$S_{ij} = \int_0^{T_b} s_i(t)\,\phi_j(t)\,dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t)\,\sqrt{2/T_b}\,\cos(2\pi f_j\,t)\,dt$$

$$= \sqrt{E_b} \int_0^{T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t)\,\sqrt{2/T_b}\,\cos(2\pi f_i t)\,dt$$

$$= \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \neq j \end{cases}$$


$S_2(t)$
$\sqrt{2E_b/T_b}$
$-\sqrt{2E_b/T_b}$
t
T_b
Φ_2
Decision boundary
$S_1(t)$
$\sqrt{2E_b/T_b}$
$-\sqrt{2E_b/T_b}$
t
T_b
Region Z_2
Message Point m_2
$\sqrt{E_b}$
Message Point m_1
Φ_1
0
$\sqrt{E_b}$
$\sqrt{2E_b}$
Region Z_1

Frequency Shift Keying (FSK)

- A coherent BFSK system is characterised by two dimensional (N=2) signal space with two message points, M=2 and it is defined as,

$$s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

And

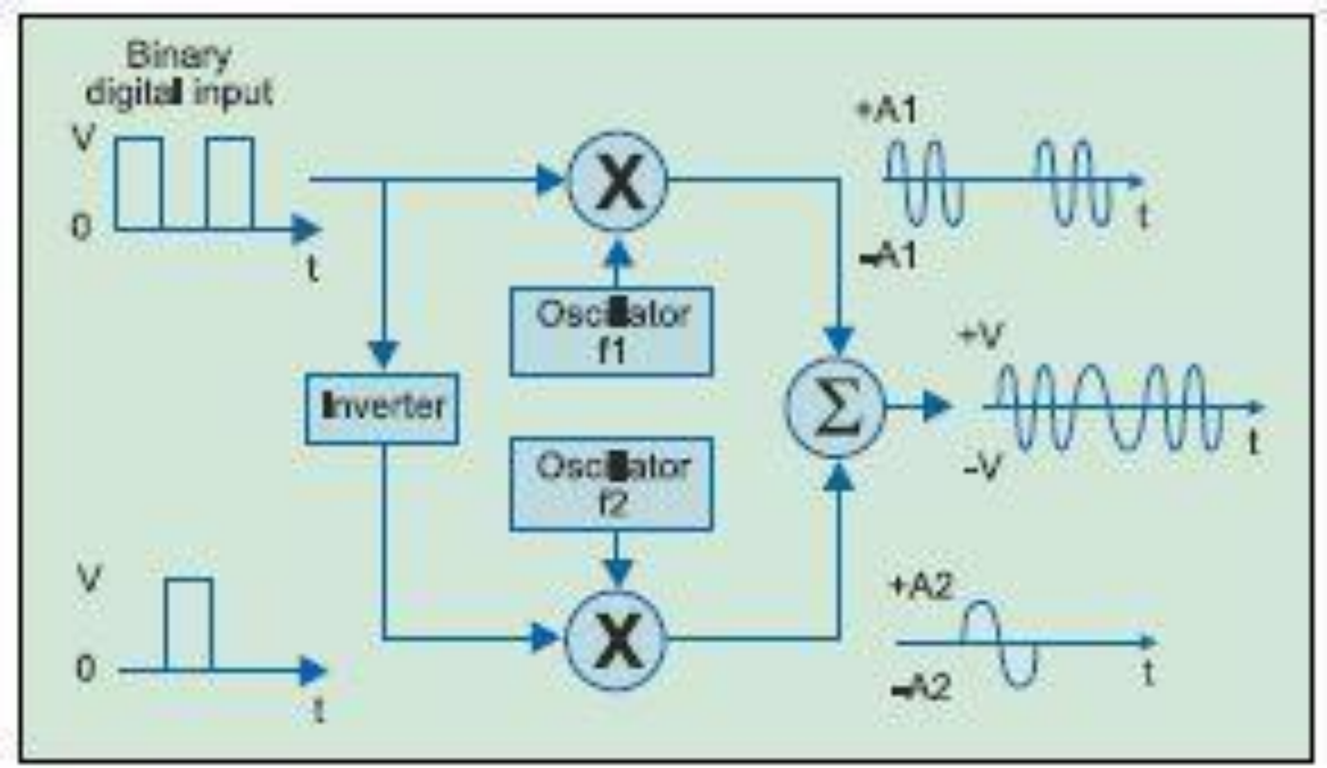
$$s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

- Euclidean distance between s_1 and s_2 is $\sqrt{2E_b}$

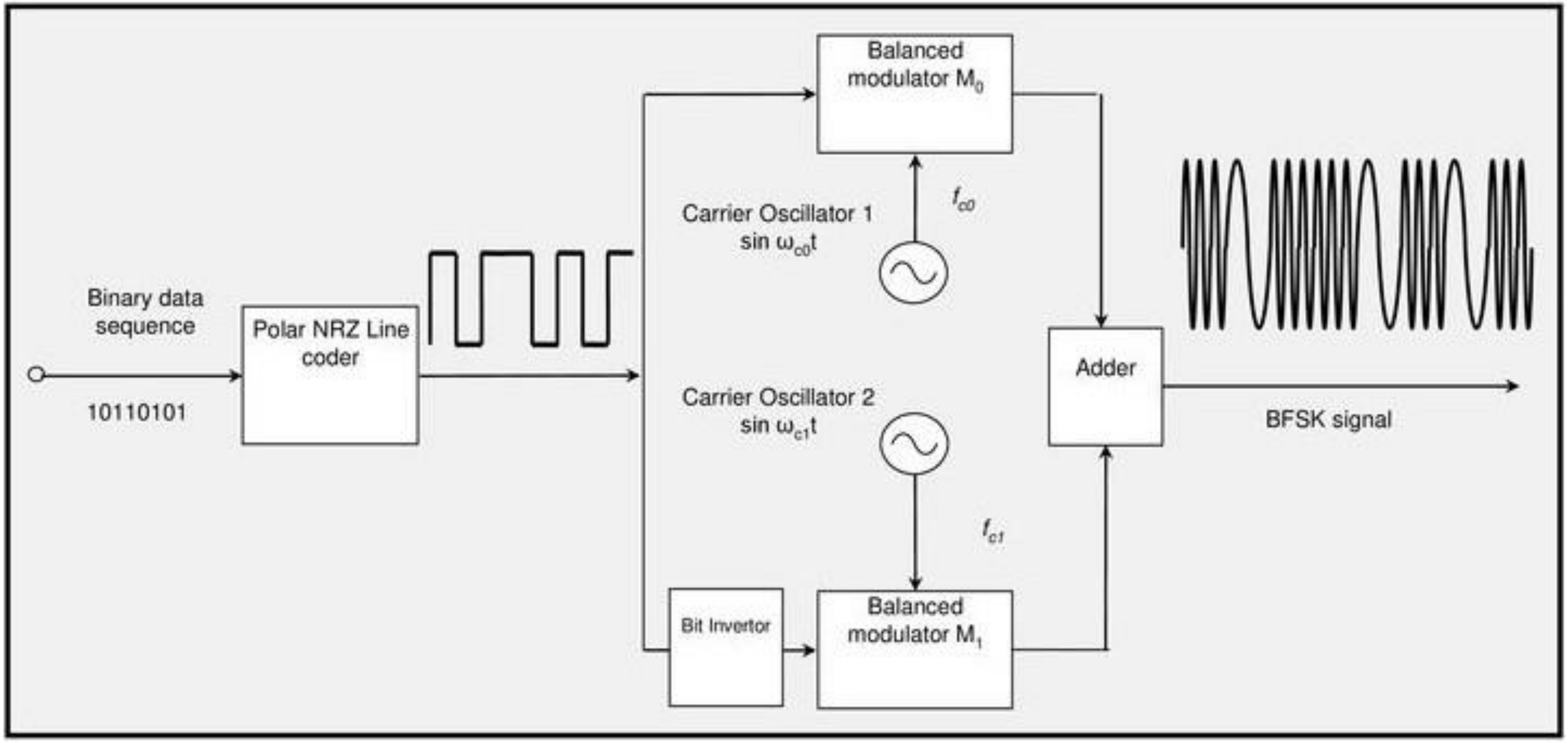
Frequency Shift Keying (FSK)

Generation of FSK (FSK Modulator)

- The FSK transmitter (Modulator) consists of an On-Off Level encoder, product modulator and summer.



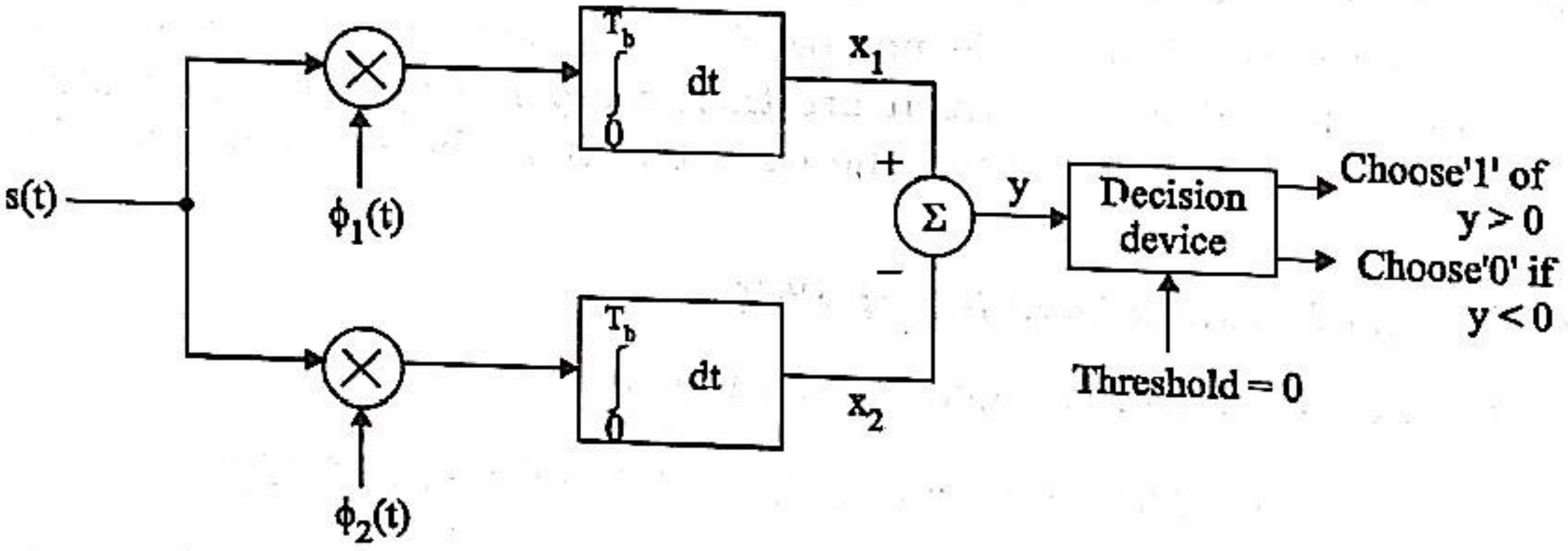
Frequency Shift Keying (FSK)



Frequency Shift Keying (FSK)

Coherent BFSK Receiver

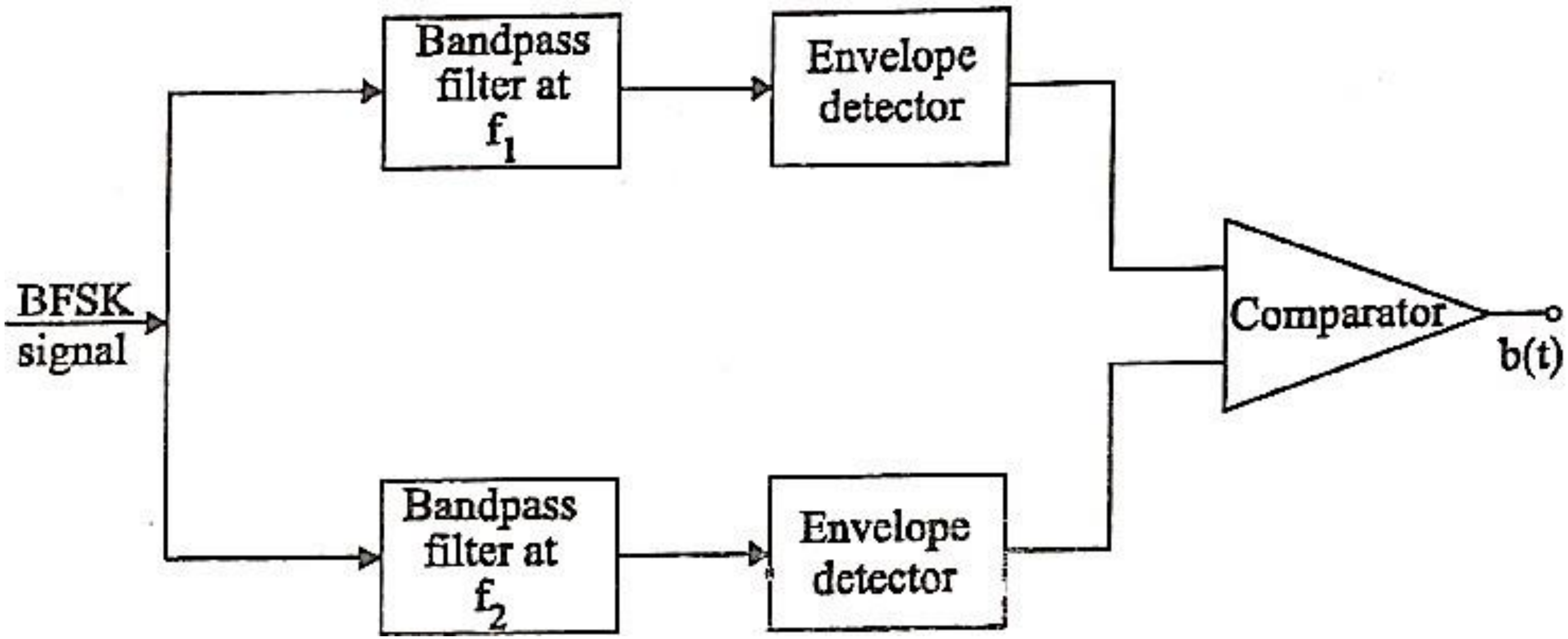
- The block diagram of (coherent) Synchronous FSK detector consists of two mixers with local oscillator circuits, two band pass filters and a decision circuit.



Frequency Shift Keying (FSK)

Non-Coherent BFSK receiver

- The block diagram of (non-coherent) Asynchronous FSK detector consists of two band pass filters, two envelope detectors, and a decision circuit.



Frequency Shift Keying (FSK)

- **Probability of error in BFSK**

$$P_e = \frac{1}{2} erfc \left(\frac{s_1(t) - s_2(t)}{2\sqrt{2}\,\sigma} \right)$$

$$\left(\frac{s_1(t) - s_2(t)}{\sigma} \right)^2 = \int_0^{T_b} \frac{s^2(t)\, dt}{N_o/2}$$

$$s(t) = s_1(t) - s_2(t)$$

$$s_1(t) = \sqrt{2P} \cos(2\pi f_c + \Omega)\, t$$

$$s_2(t) = \sqrt{2P} \cos(2\pi f_c - \Omega)\, t$$

$$s_1(t) - s_2(t) = \sqrt{2P}\, [\cos(2\pi f_c + \Omega)\, t - \cos(2\pi f_c - \Omega)\, t]$$

By formula,

$$\cos(A+B) - \cos(A-B) = 2 \sin A \sin B$$

we get,

$$s_1(t) - s_2(t) = \sqrt{2P}\, [2 \sin 2\pi f_c t \sin \Omega t]$$

Frequency Shift Keying (FSK)

$$s^2(t) = [s_1(t) - s_2(t)]^2 = \int_0^{T_b} (\sqrt{2P}\, 2 \sin 2\pi f_c t \sin \Omega t)^2 \, dt$$

$$= \int_0^{T_b} 2P \times 4\sin^2 2\pi f_c t \sin^2 \Omega t \, dt$$

$$\therefore \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$= \int_0^{T_b} 2P \times 4 \left(\frac{1 - \cos 4\pi f_c t}{2} \right) \left(\frac{1 - \cos 2\Omega t}{2} \right) dt$$

$$= 2P \int_0^{T_b} (1 - \cos 2\Omega t - \cos 4\pi f_c t + \cos 4\pi f_c t \cos 2\Omega t) \, dt$$

Frequency Shift Keying (FSK)

$$= 2P \int_0^{T_b} (1 - \cos 2\,\Omega\, t - \cos 4\,\pi\, f_c t + \cos 4\,\pi\, f_c t \cos 2\,\Omega\, t)\, dt$$

By Formula,

$$\frac{\cos (A+B) + \cos (A-B)}{2} = \cos A \cos B$$

$$= 2P \int_0^{T_b} \left\{ 1 - \cos 2\,\Omega\, t - \cos 4\,\pi\, f_c t + \frac{1}{2} \cos (4\,\pi\, f_c t + 2\,\Omega\, t) + \frac{1}{2} \cos (4\,\pi\, f_c t - 2\,\Omega\, t) \right\} dt$$

Frequency Shift Keying (FSK)

$$= 2P \int_0^{T_b} 1\, dt - \int_0^{T_b} \cos 2\,\Omega\, t\, dt - \int_0^{T_b} \cos 4\,\pi\, f_c t\, dt$$

$$+ \frac{1}{2} \int_0^{T_b} \cos\,(4\pi\, f_c t + 2\,\Omega\, t)\, dt + \frac{1}{2} \int_0^{T_b} \cos\,(4\pi\, f_c t - 2\,\Omega\, t)\, dt$$

$$= 2P \left[[\,t\,]_0^{T_b} - \left[\frac{\sin 2\,\Omega\, t}{2\,\Omega} \right]_0^{T_b} \right]$$

Frequency Shift Keying (FSK)

$$= 2P\left[T_b - \frac{\sin 2\,\Omega\,T_b}{2\,\Omega}\right]$$

$$= 2PT_b\left[1 - \frac{\sin 2\,\Omega\,T_b}{2\,\Omega\,T_b}\right]$$

$$\int_0^{T_b} s^2(t)\,dt = 2PT_b\left[1 - \frac{\sin 2\,\Omega\,T_b}{2\,\Omega\,T_b}\right]$$

The ratio $\frac{\sin 2\,\Omega\,T_b}{2\,\Omega\,T_b}$ attains the largest value when

Frequency Shift Keying (FSK)

$$2\,\Omega\,T_b \Rightarrow \frac{3\,\pi}{2}$$

$$\Rightarrow 2PT_b\left[\,1-\frac{\sin(3\,\pi/2)}{3\,\pi/2}\,\right]$$

$$= 2PT_b\,(1.2121)$$

$$\left(\frac{s_1(t)-s_2(t)}{\sigma}\right)^2_{\max} = \frac{\int_0^{T_b} s^2(t)\,dt}{\frac{N_o}{2}}$$

$$= \frac{2.424PT_b \times 2}{N_o} = \frac{4.848\,PT_b}{N_o}$$

Frequency Shift Keying (FSK)

$$s_1(t) - s_2(t) = 2.20\sqrt{\frac{PT_b}{N_o}}$$

$$P_e = \frac{1}{2}\,erfc\left(\frac{s_1(t) - s_2(t)}{2\sqrt{2}\,\sigma}\right)$$

$$= \frac{1}{2}\,erfc\left(\frac{2.20}{2\sqrt{2}}\sqrt{\frac{PT_b}{N_o}}\right)$$

$$P_e = \frac{1}{2}\,erfc\sqrt{\frac{0.6E_b}{N_o}}$$

$$= \frac{1}{2}\,erfc\sqrt{\frac{0.6E_b}{N_o}}$$

P_e for Non-coherent BFSK depends on signal energy

$$\therefore P_e = \frac{1}{2}e^{-\frac{E}{2N_o}}$$

Frequency Shift Keying (FSK)

Advantages

- Simple circuit is required
- Generation is easier
- Has constant modulated signal envelope

Disadvantages

- Hardware is more complicated when compared to ASK
- Requires large Bandwidth

Frequency Shift Keying (FSK)

Limitations

- Bandwidth is greater than $4f_b$, which is almost double the bandwidth of BPSK.
- Error rate of BFSK is more compared to BPSK
- Only half of the transmitted energy carries the information signal