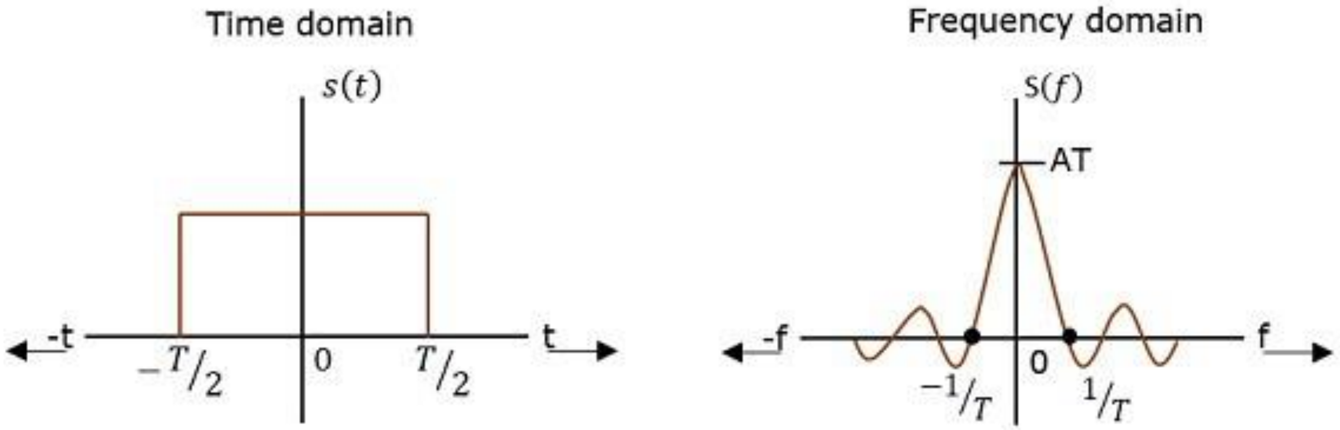
Power Spectral Density of Line codes

- The function which describes how the power of a signal got distributed at various frequencies, in the frequency domain is called as **Power Spectral Density.**
- PSD is the Fourier Transform of Auto-Correlation i.e similarity between observations



$$s(t) = A \quad |t| < T/2$$
$$= 0 \quad |t| > T/2$$

$$s(f) = AT\frac{\sin(\pi T f)}{\pi T f}$$

PSD of Polar NRZ line coding scheme

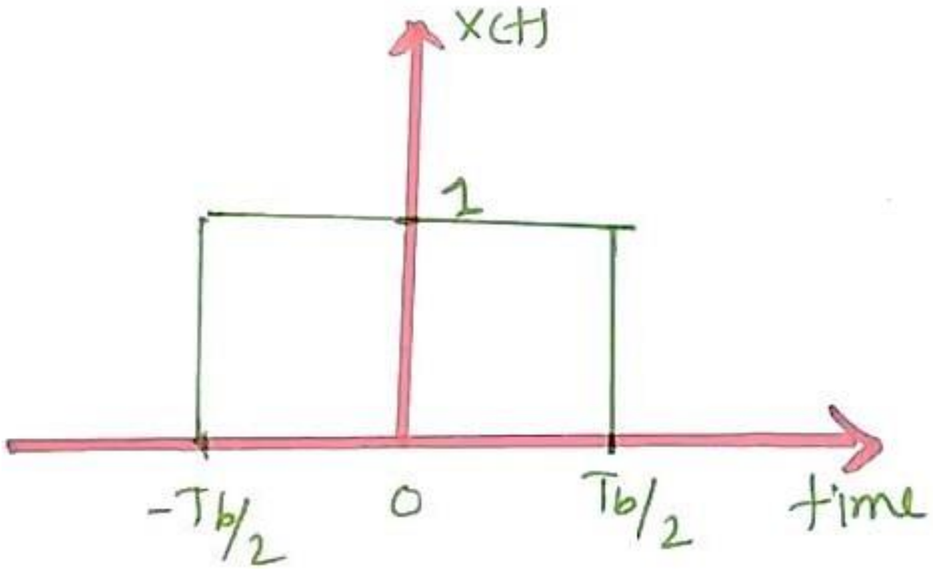
Steps to obtain the PSD

- Find the Fourier transform of NRZ pulse $\boldsymbol{X(f)}$
- Find the autocorrelation of the Polar signal $\boldsymbol{R_A(n)}$
- Calculate the PSD based on $X(f)$ and $R_A(n)$

→ Polar

$0 \rightarrow -a$

$1 \rightarrow +a$



Dr. T.Ilavarasan, Associate Professor,SENSE, VIT

PSD of Unipolar NRZ line coding scheme

Fourier transform of NRZ pulse $X(f)$

- Apply Fourier transform

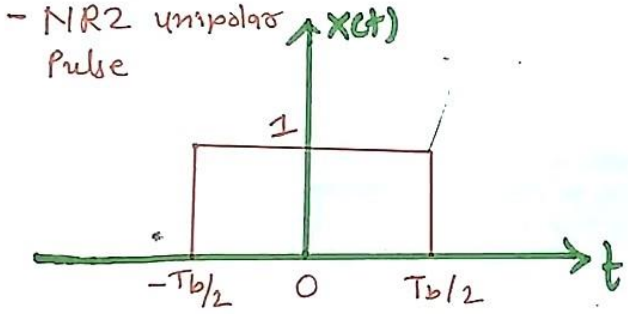
$$X(f) = \int_{-T_b/2}^{T_b/2} (1)\, e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-T_b/2}^{T_b/2}$$

$$= \frac{e^{j\pi f T_b} - e^{-j\pi f T_b}}{2j\pi f}$$

$$= \boxed{\frac{e^{j\pi f T_b} - e^{-j\pi f T_b}}{2j\pi f}} \quad \sin(\pi f T_b)$$

$$\rightarrow X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$



$$\rightarrow \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

PSD of Unipolar NRZ line coding scheme

$$= \boxed{\frac{e^{j\pi f T_b} - e^{-j\pi f T_b}}{2j}}\,\boxed{\pi f} \qquad \sin(\pi f T_b)$$

$$= \frac{\sin(\pi f T_b)}{\pi f T_b} \times T_b \qquad \rightarrow \operatorname{sinc}\theta = \frac{\sin \pi\theta}{\pi\theta}$$

$$= \boxed{\frac{\sin(\pi f T_b)}{\pi f T_b}} \times T_b \qquad \operatorname{sinc}(f T_b)$$

$$\boxed{X(f) = T_b \operatorname{sinc}(f T_b)}$$

Fourier transform of NRZ pulse $X(f)$

PSD of Polar NRZ line coding scheme

- Autocorrelation of the Polar signal $\boldsymbol{R_A(n)}$

For Polar format

$$A_l = \begin{cases} +a & , \text{Symbol } 1 \\ -a & , \text{Symbol } 0 \end{cases}$$

→ Auto correlation

$$R_A(n) = E[A_l A_{l-n}]$$

→ If $n = 0$

$$\Rightarrow R_A(0) = E[A_l A_l] = E[A_l^2]$$

	A_l	A_l	A_l^2	Prob
1	a	a	a^2	$1/2$
0	$-a$	$-a$	a^2	$1/2$

$$\Rightarrow R_A(0) = \sum A_l^2 P[X = x]$$

$$= a^2(1/2) + a^2(1/2)$$

$$= \underline{a^2}$$

PSD of Polar NRZ line coding scheme

If $n \neq 0$

$\Rightarrow R_A(n) = E[A_l A_{l-n}]$

$$R_A(n) = E[A_l A_{l-n}]$$

$$= \sum A_l A_{l-n} P[X=x]$$

$$= a^2(1/4) + (-a^2)(1/4) + (-a^2)(1/4) + a^2(1/4)$$

$$= 0$$

A_l	A_{l-n}	$A_l A_{l-n}$	Prob		
$-a$	$-a$	a^2	1/4	0	0
$-a$	a	$-a^2$	1/4	0	1
a	$-a$	$-a^2$	1/4	1	0
a	a	a^2	1/4	1	1

PSD of Polar NRZ line coding scheme

So Auto correlation (Polar)

$$R_A(n) = \begin{cases} a^2, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$X(f) = T_b \operatorname{sinc}(fT_b)$$

- PSD based on $X(f)$ and $R_A(n)$
- According to the Einstein-Wiener-Khintchine theorem,

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

PSD of Polar NRZ line coding scheme

- So the PSD of Polar NRZ code is given by,

So power spectral density

$$P(f) = \frac{1}{T_b} |X(f)|^2 \boxed{\sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}} \quad a^2$$

$$= \frac{1}{T_b} T_b^2 \operatorname{sinc}^2(fT_b) \left[a^2\right]$$

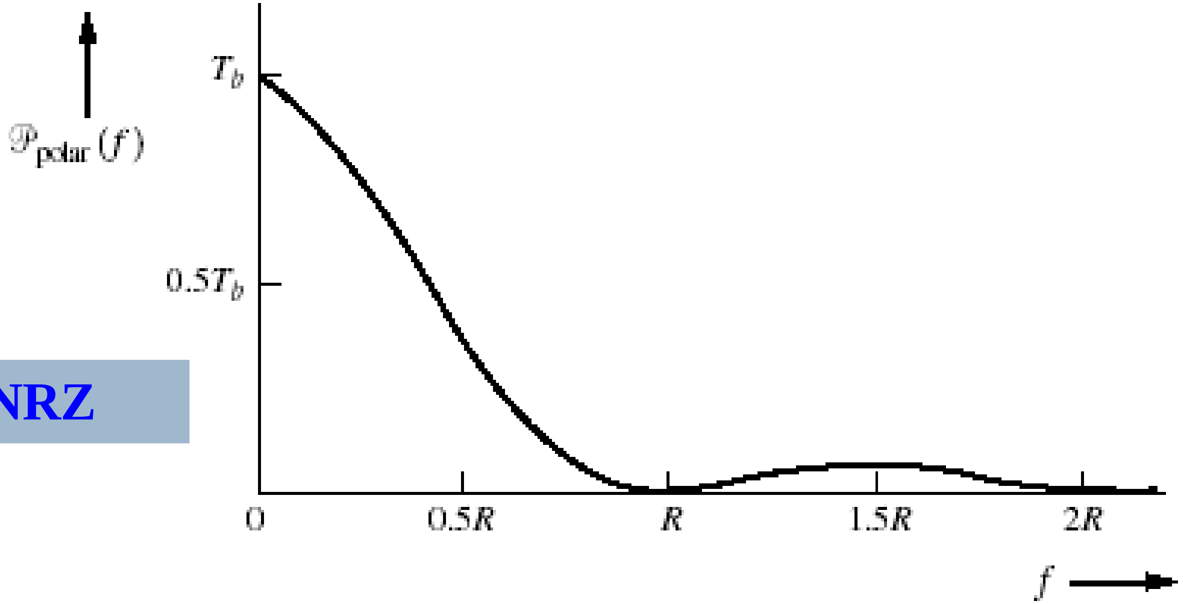
$$= a^2 T_b \operatorname{sinc}^2(fT_b)$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2(fT_b)} \longleftarrow \text{PSD of NRZ Polar form}$$

PSD of Polar NRZ line coding scheme

$$P_{\text{Polar NRZ}}(f) = A^2 T_b \left(\frac{\sin \pi f T_b}{\pi f T_b} \right)^2$$

Polar NRZ

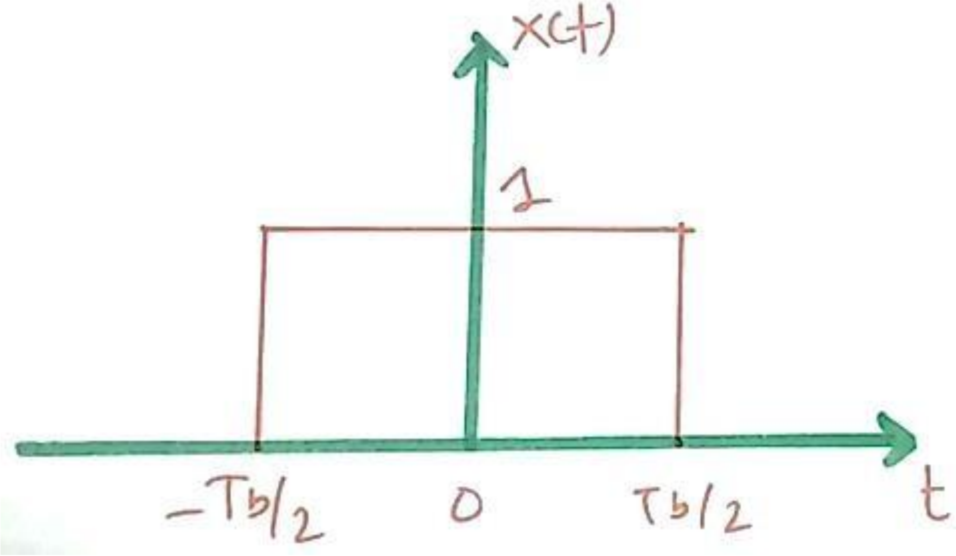


PSD of Bipolar NRZ line coding scheme

Steps to obtain the PSD

- Find the Fourier transform of NRZ pulse $\boldsymbol{X(f)}$
- Find the autocorrelation of the Bipolar signal $\boldsymbol{R_A(n)}$
- Calculate the PSD based on $X(f)$ and $R_A(n)$

$$\boxed{X(f) = T_b \sin(fT_b)}$$



For bipolar format

$$A_L = \begin{cases} 0, & \text{Symbol } 0 \\ +a \\ -a & ,\ \text{Symbol } 1 \end{cases}$$

PSD of Bipolar NRZ line coding scheme

For bipolar format

$$A_L = \begin{cases} 0, & \text{Symbol } 0 \\ +a \\ -a & , \text{Symbol } 1 \end{cases}$$

1	1	1
$+a$	$-a$	$+a$

Bipolar format is Pseudo ternary code.

Auto Correlation

$$R_A(n) = E[A_l A_{l-n}]$$

If $n = 0$

$$R_A(0) = E[A_L A_L] = E[A_L^2]$$

A_l	A_l	A_l^2	Prob		bit
0	0	0	1/2	←	0
a	a	a^2	1/4	} ←	1
$-a$	$-a$	a^2	1/4		

PSD of Bipolar NRZ line coding scheme

- For $n = 0$

A_k	A_k	A_k^2	Prob	bit
0	0	0	1/2	0
a	a	a^2	1/4	1
$-a$	$-a$	a^2	1/4	

$$R_A(0) = E[A_k^2]$$

$$= \sum A_k^2 \, P[X=x]$$

$$= 0 \times 1/2 + a^2(1/4) + a^2(1/4)$$

$$= a^2/2$$

PSD of Bipolar NRZ line coding scheme

For $n=1$

$$R_A(1) = \sum A_L A_{L-1}\, P$$

A_L	A_{L-1}	$A_L A_{L-1}$	Prob		
0	0	0	1/4	0	0
0	$\pm a$	0	1/4	0	1
$\pm a$	0	0	1/4	1	0
$+a$	$-a$	$-a^2$	1/8	1	1
$-a$	$+a$	$-a^2$	1/8		

$$R_A(1) = E[A_L A_{L-1}]$$

$$= \sum A_L A_{L-1}\, P[X=x]$$

$$= -a^2/4$$

$$\rightarrow R_A(1) = R_A(-1) = -a^2/4$$

PSD of Bipolar NRZ line coding scheme

For $n > 1$

A_L	A_{L-n}	$A_L A_{L-n}$	Prob		
0	0	0	1/4	0	0
0	$\pm a$	0	1/4	0	1
$\pm a$	0	0	1/4	1	0
$+a$	$+a$	a^2	1/16	1	1
$-a$	$+a$	$-a^2$	1/16		
$+a$	$-a$	$-a^2$	1/16		
$-a$	$-a$	a^2	1/16		

PSD of Bipolar NRZ line coding scheme

$$R_A(n) = \sum A_L A_{L-n} P = 0 \times 1/4 + 0 \times 1/4 + 0 \times 1/4 + a^2(1/16) + (-a^2)(1/16) + (-a^2)(1/16) + a^2(1/16)$$

$$= 0$$

so Autocorrelation

$$R_A(n) = \begin{cases} a^2/2 & , \quad n = 0 \\ -a^2/4 & , \quad n = \pm 1 \\ 0 & , \quad |n| \geq 2 \end{cases}$$

$$X(f) = T_b \operatorname{Sinc}(f T_b)$$

PSD of Bipolar NRZ line coding scheme

- Now the PSD can be derived as,

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} \left(T_b^2\, \text{sinc}^2(fT_b) \right) \left[R_A(0) + R_A(1)\, e^{-j2\pi f T_b} + R_A(-1)\, e^{j2\pi f T_b} \right]$$

$$T_b\, \text{sinc}^2(fT_b) \left[\frac{a^2}{2} - \frac{a^2}{4} \left[e^{-j2\pi f T_b} + e^{j2\pi f T_b} \right] \right]$$

$$= T_b\, \text{sinc}^2(fT_b) \left[\frac{a^2}{2} - \frac{a^2}{2} \cos(2\pi f T_b) \right]$$

PSD of Bipolar NRZ line coding scheme

$$= T_b \operatorname{sinc}^2(fT_b)\left[\frac{a^2}{2} - \frac{a^2}{2}\cos(2\pi f T_b)\right]$$

$$= \frac{a^2 T_b}{2}\operatorname{sinc}^2(fT_b)\left[1 - \cos(2\pi f T_b)\right]$$

$$1 - \cos 2\theta = 2\sin^2\theta$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2(fT_b)\sin^2(\pi f T_b)}$$

PSD of Bipolar NRZ line coding scheme