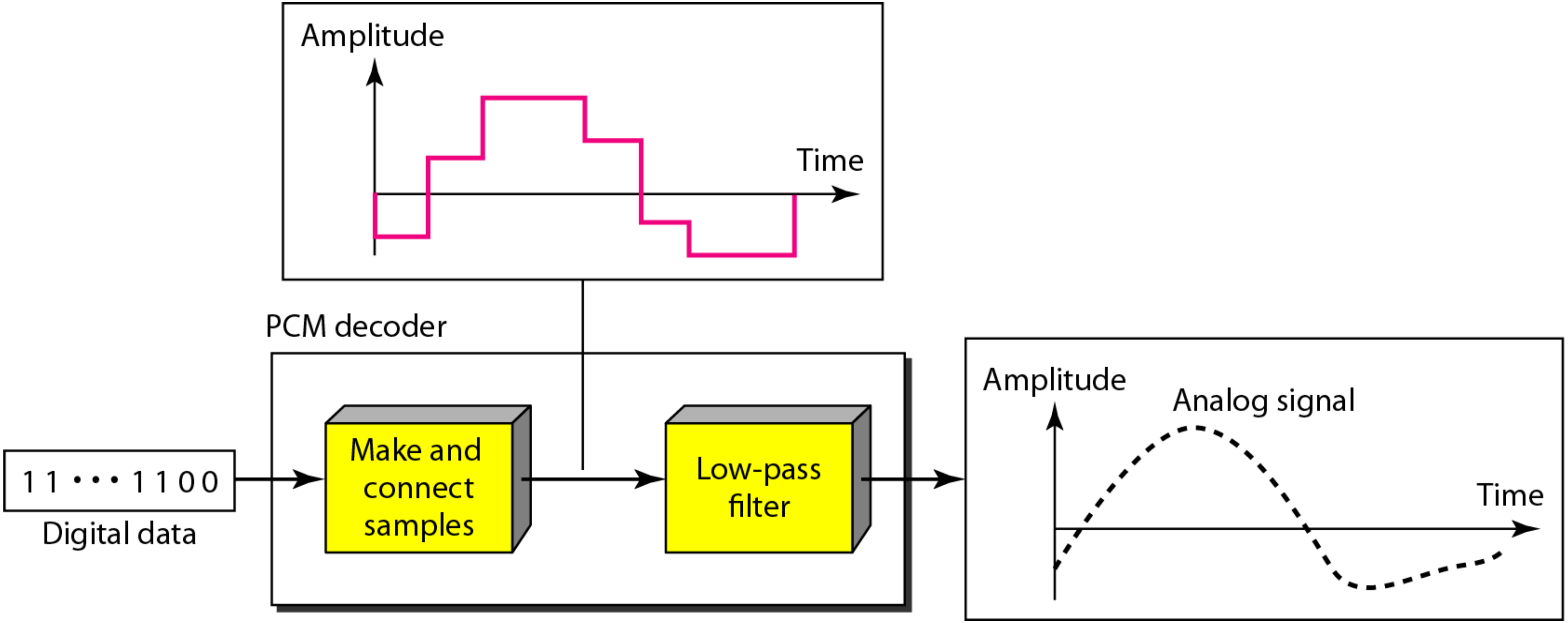
Pulse code modulation (PCM)

PCM Decoder

- To recover an analog signal from a digitized signal we follow the following steps:

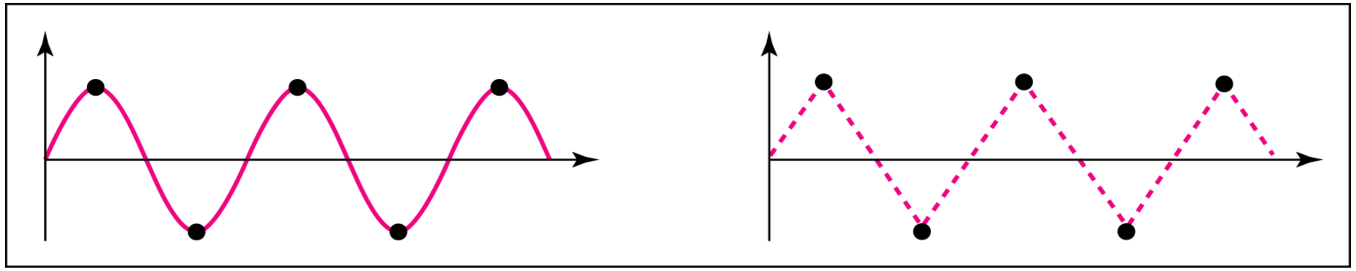
1. We use a hold circuit that holds the amplitude value of a pulse till the next pulse arrives.
2. We pass this signal through a low pass filter with a cutoff frequency that is equal to the highest frequency in the pre-sampled signal.
3. The higher the value of L, the less distorted a signal is recovered.

Pulse code modulation (PCM)

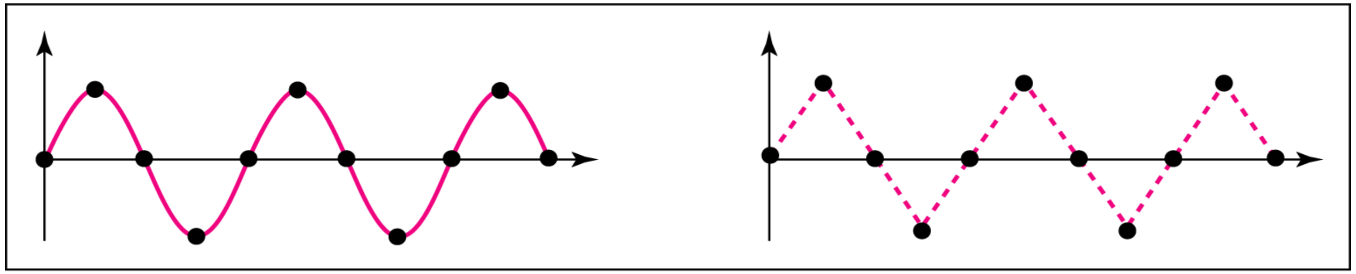


Pulse code modulation (PCM)

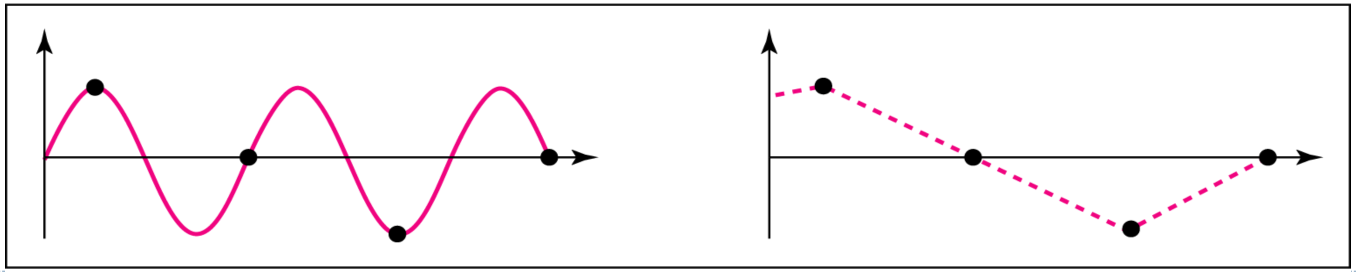
- Recovery of a sampled sine wave for different sampling rates



a. Nyquist rate sampling: $f_s = 2\ f$



b. Oversampling: $f_s = 4\ f$

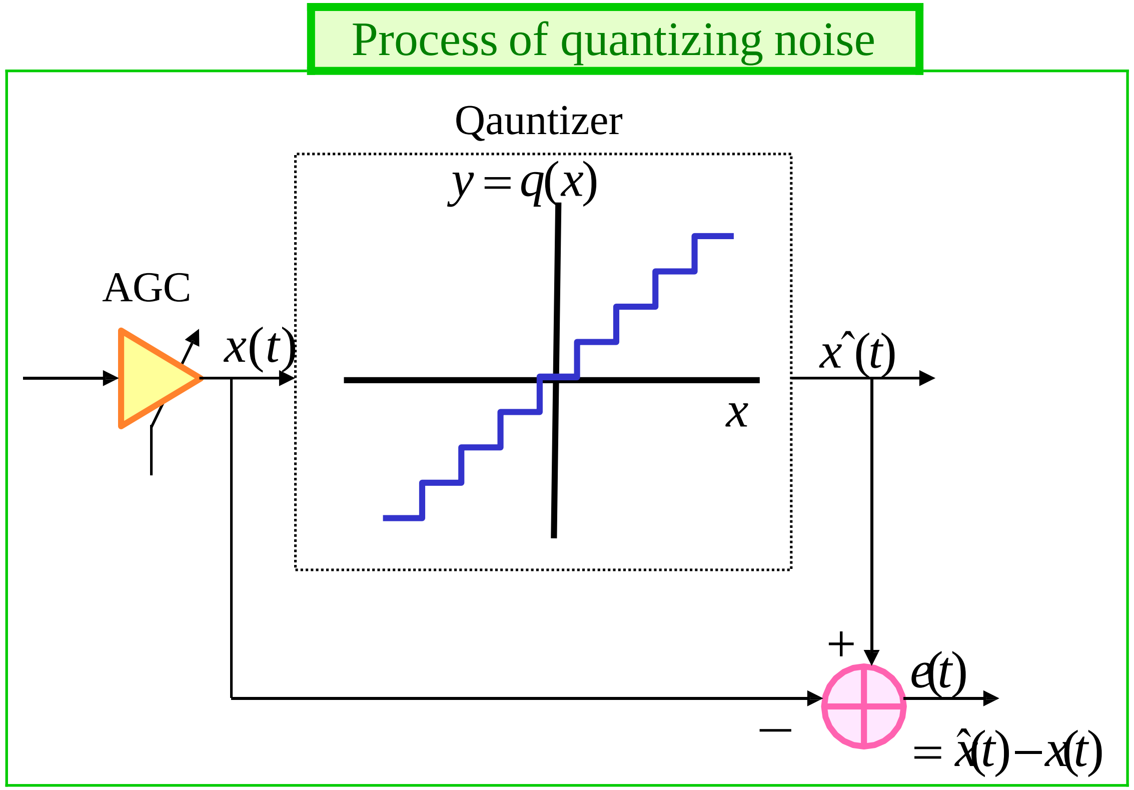


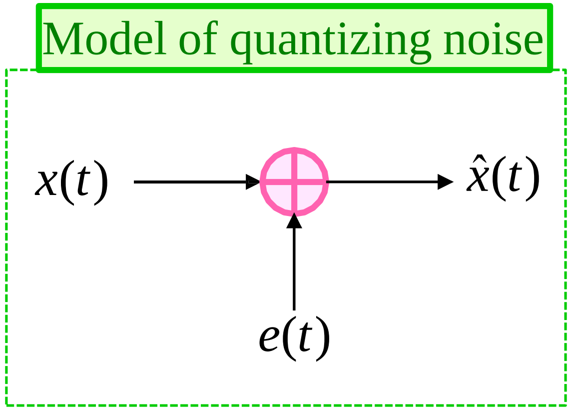
c. Undersampling: $f_s = f$

Pulse code modulation (PCM)

- **Quantizing error**:The difference between the input and output of a quantizer

$$\Rightarrow e(t) = \hat{x}(t) - x(t)$$





The Noise Model is an approximation!

Pulse code modulation (PCM)

- Quantizing error:
  - Granular or linear errors happen for inputs within the dynamic range of quantizer
  - Saturation errors happen for inputs outside the dynamic range of quantizer
    - Saturation errors are larger than linear errors (AKA as "Overflow" or "Clipping")
    - Saturation errors can be avoided by proper tuning of AGC
    - Saturation errors need to be handled by Overflow Detection!

Pulse code modulation (PCM)

Derivation of Quantization Error / Noise Power:

- The difference between the input and output signal is called Quantization error or Quantization Noise.
- Consider an input signal '**m(t)**' of continuous amplitude in the range

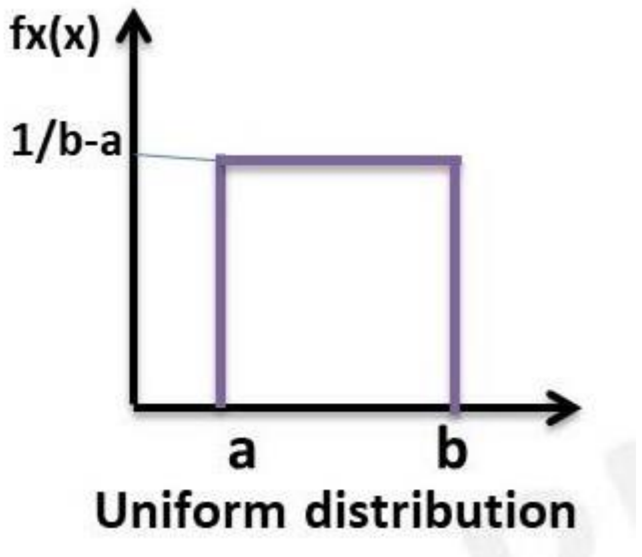
$$\left(V_p = m_{max} \,, -V_p = -\, m_{max} \right)$$

$$Step\ Size\ (\Delta) = \frac{\left(V_p - (-V_p)\right)}{L} = \frac{2V_p}{L}, where\ L\ is\ the\ number\ of\ the\ levels$$

- If **m (t)** is normalised to 1 ie, $V_p = 1 \,, -V_p = -1$
- Then the $Step\ Size\ \ (\Delta) = \frac{2}{L}$

Pulse code modulation (PCM)

- Then the quantization error **'q'** is assumed to be uniformly distributed random variable
- A continuous random variable is said to be uniformly distributed over an interval (a,b) as shown below,
- The PDF of 'x' is given by



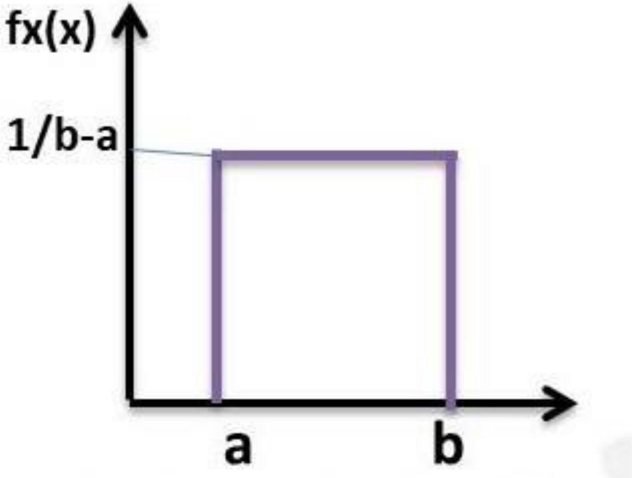
Pulse code modulation (PCM)

- Then the quantization error **‘q’** is assumed to be uniformly distributed random variable
- A continuous random variable is said to be uniformly distributed over an interval (a,b) as shown below,
- The PDF of ‘x’ is given by

$$f_x(x) = 0 \; for \; x \leq a$$

$$= \frac{1}{b-a} \; for \; a < x \leq b$$

$$= 0 \; for \; x > b$$



Uniform distribution

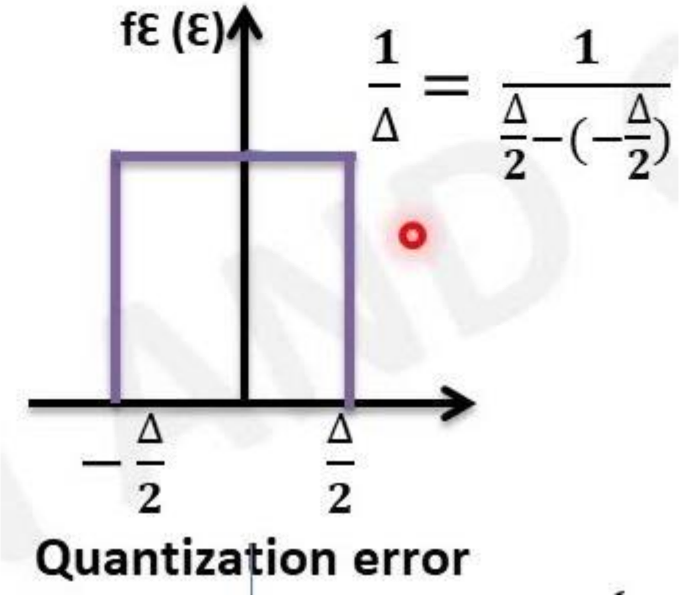
Pulse code modulation (PCM)

- Similarly, the PDF of the quantization error 'q' can be written as

$$f_Q(q) = 0 \; for \; q \leq \frac{-\Delta}{2}$$

$$= \frac{1}{\Delta} for \; \frac{-\Delta}{2} < x \leq \frac{\Delta}{2}$$

$$= 0 \; for \; x > \frac{\Delta}{2}$$



- For this to be true, the incoming signal should not overload the quantizer.

Pulse code modulation (PCM)

- Mean square value of a random variable is equivalent to its variance

$$\sigma_q{}^2 = E[q^2]$$

- Variance of $f_x\,(x)$ is given by

$$\sigma_x^2 = \int_a^b (x-\mu)^2\, f_x(x)dx$$

- Where $\mu = expected\ value\ E(x)$ or the mean value

Pulse code modulation (PCM)

- Similarly, the variance of $f_Q\,(q)$ is given by

$$\sigma_q^2 = \int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}} (q-\mu)^2\, f_Q(q)dq = \int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}} (q)^2\, f_Q(q)dq$$

- Since mean value or $E(q) = \mu = 0$

- Substituting the value of $f_Q(q)$,$\sigma_q^2 = \frac{1}{\Delta}\int_{\frac{-\Delta}{2}}^{\frac{\Delta}{2}}(q)^2\, dq$

- **Quantization Noise power $\boldsymbol{\sigma_q^2(N_q) = \frac{\Delta^2}{12}}$**

Pulse code modulation (PCM)

Derivation of maximum signal to quantization noise ratio (non-sinusoidal)

- The number of bits per sample 'R' and the quantization levels 'L' are related as

$$L = 2^R; R = log_2 L;\ \Delta = \frac{2V_p}{L}; \Delta = \frac{2V_p}{2^R}$$

$$N_q = \frac{\Delta^2}{12} = \frac{\frac{4V_p^2}{L^2}}{12} = \frac{V_p^2}{3L^2}$$

$$V_p^2 = \frac{\Delta^2 L^2}{4}$$

$$\frac{Signal\ Power}{Noise\ Power} = (SNR)_q = \frac{\Delta^2 L^2}{4} \Big/ N_q = \frac{\Delta^2 L^2}{4} \Big/ \frac{\Delta^2}{12} = 3L^2$$

Pulse code modulation (PCM)

- If ‘P’ is the average power of the message signal, then

$$S/_N = \frac{P}{N_q} = \frac{P}{\frac{V_p^2}{3L^2}} = \frac{3P}{V_p^2} \times 2^{2R}$$

- If input V_p and power ‘P’ is normalised, then $S/_N = 3 \times 2^{2R}$
- In decibels, $\left(\frac{S}{N}\right)_{dB} = 10log_{10}\left(\frac{S}{N}\right)_{dB}$

$$\leq 10log_{10}[3 \times 2^{2R}]$$

$$\leq 10log_{10}3 + 10log_{10}2^{2R}$$

$$\leq 4.8 + 2R \times 10 \times 0.3$$

$$\boldsymbol{\left(\frac{S}{N}\right)_{dB} \leq (4.8 + 6R)dB}$$

Pulse code modulation (PCM)

Derivation of maximum signal to quantization noise ratio (sinusoidal)

- For full-load Sinusoidal Signal with peak amplitude A_m
- Power $P = \frac{V^2}{R}$, where V = RMS value

$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2$$

- The normalised power P, when resistance R=1

$$P = \frac{A_m^2}{2}$$

$$S/N = \frac{3P}{V_p^2} \times 2^{2R} = \frac{3 \times \frac{A_m^2}{2}}{A_m^2} \times 2^{2R} = \frac{3}{2} \times 2^{2R} = \mathbf{1.5 \times 2^{2R}}$$

Pulse code modulation (PCM)

- Expressing in dB,

$$\left(\frac{S}{N}\right)_{dB} = 10log_{10}\left(\frac{S}{N}\right)_{dB} = 10log_{10}(1.5 \times 2^{2R})$$

$$= 10log_{10}(1.5) + 10log_{10}2^{2R}$$

$$= 1.76 + 2R \times 10 \times 0.3$$

$$= 1.8 + 6R$$

- Therefore, for Sinusoidal Signal $\boldsymbol{\left(\frac{S}{N}\right)_{dB} = 1.8 + 6R\ dB}$

Pulse code modulation (PCM)

Uniform quantization

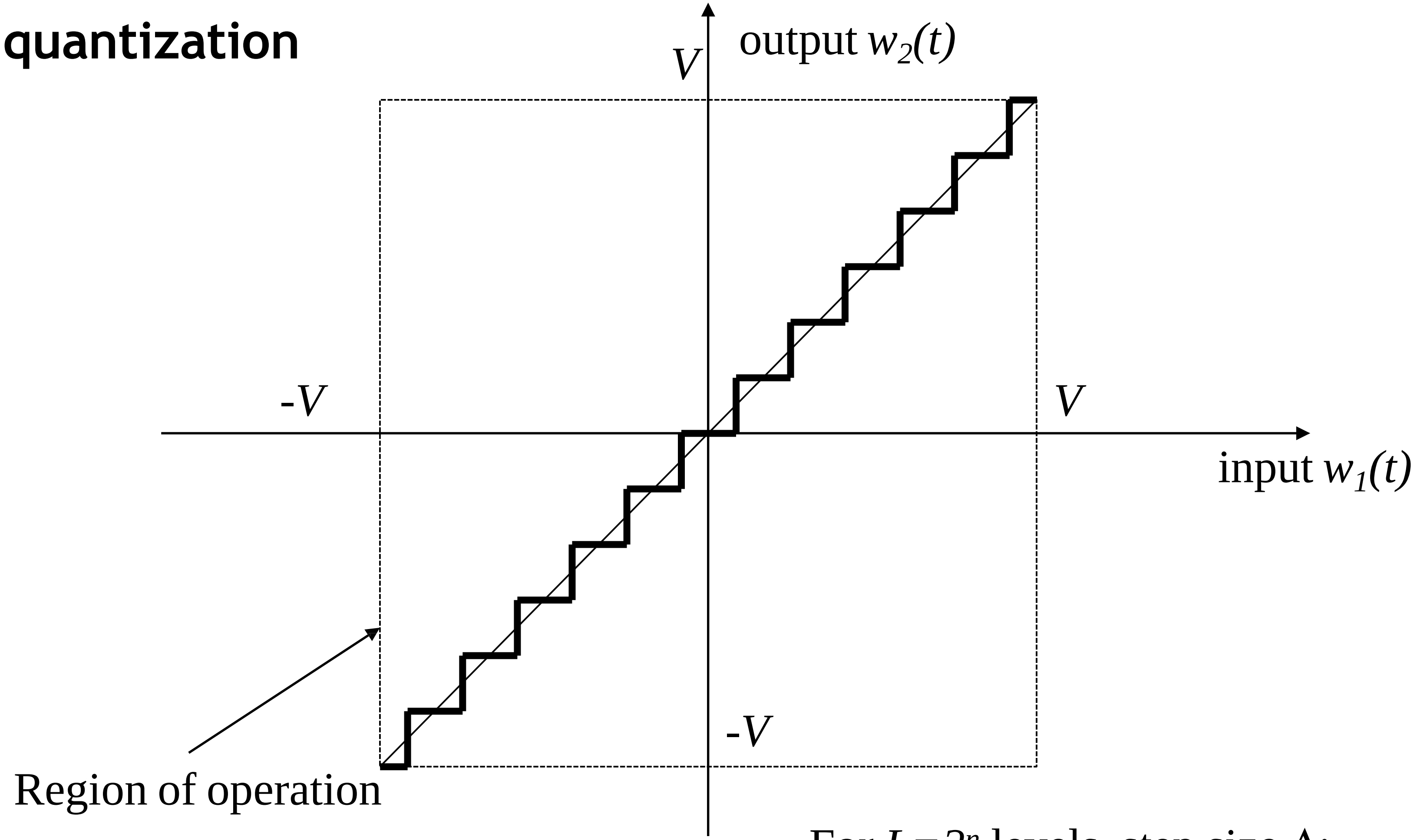
- When the quantization levels are uniformly distributed over the full amplitude range of the input signal,the quantizer is called an **uniform or linear quantizer**.
- In uniform quantization, the step size between quantization levels remains the same throughout the input range.

Non-uniform quantization

- If the quantizer characteristic is nonlinear, then the quantization is known as non-uniform quantization.
- In non-uniform quantization,the step size is not constant.
- The step size is variable,depending on the amplitude of input signal.

Pulse code modulation (PCM)

- **Uniform quantization**

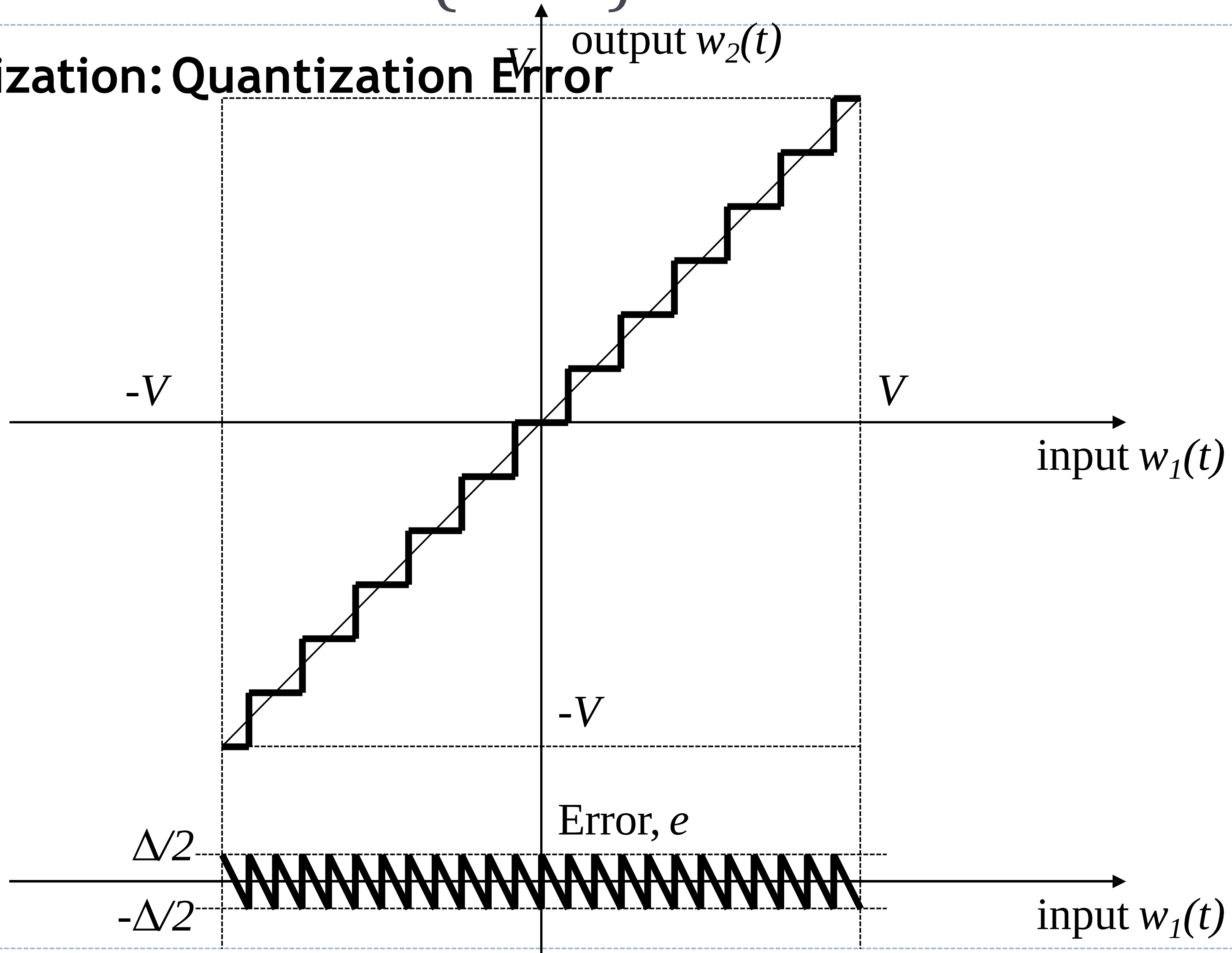


For $L=2^n$ levels, step size Δ:

$$\Delta = 2V/2^n = V(2^{-n+1})$$

Pulse code modulation (PCM)

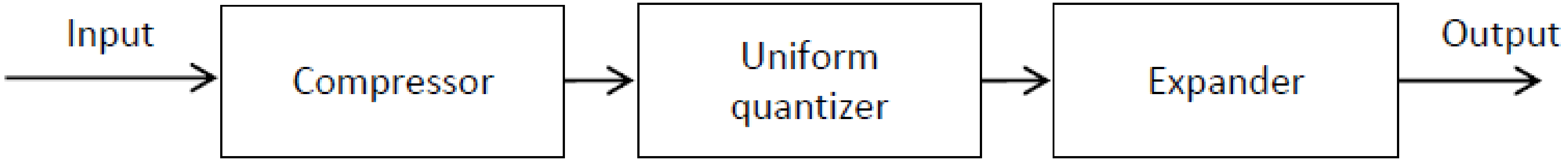
- **Uniform quantization: Quantization Error**



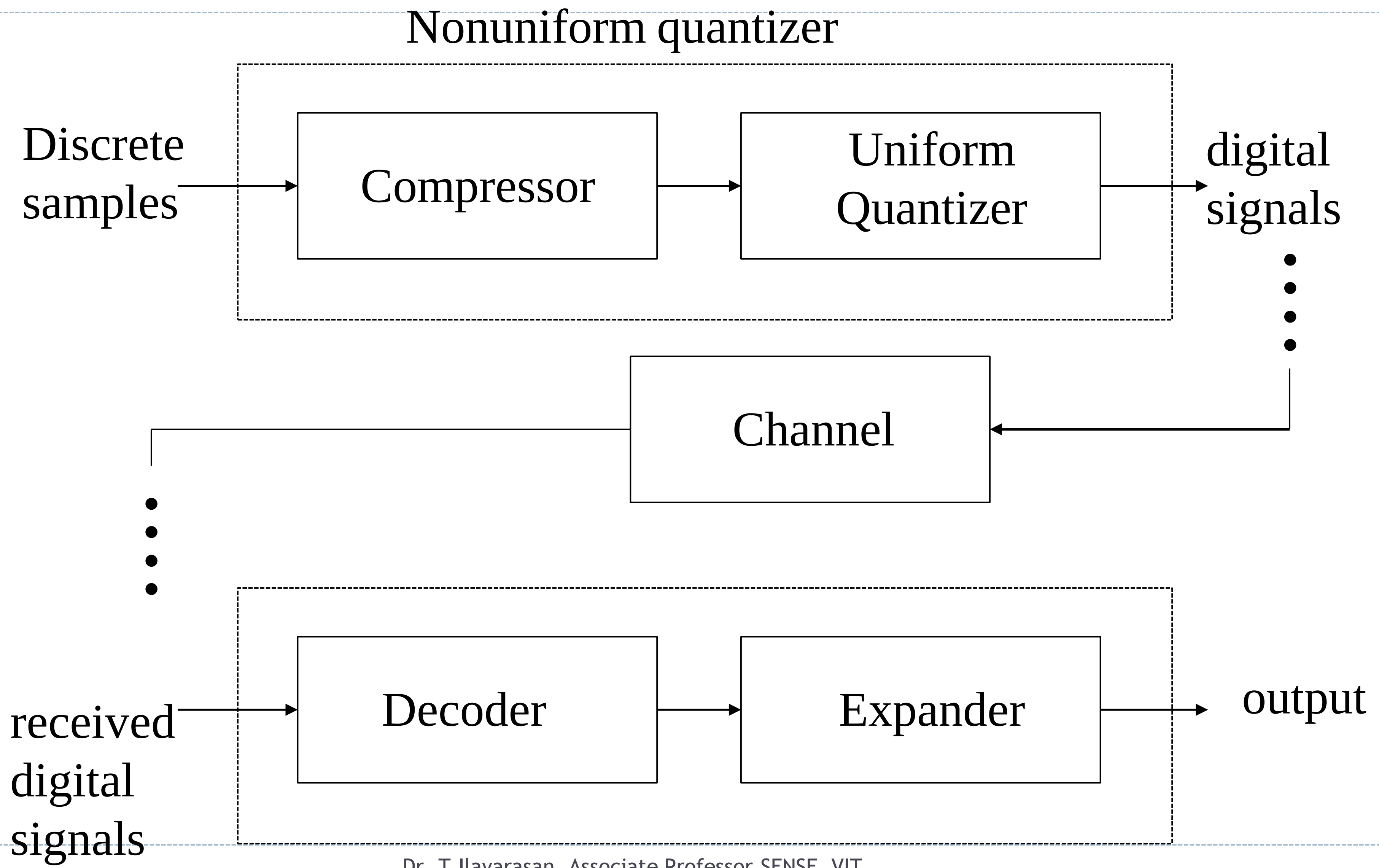
Pulse code modulation (PCM)

Companding

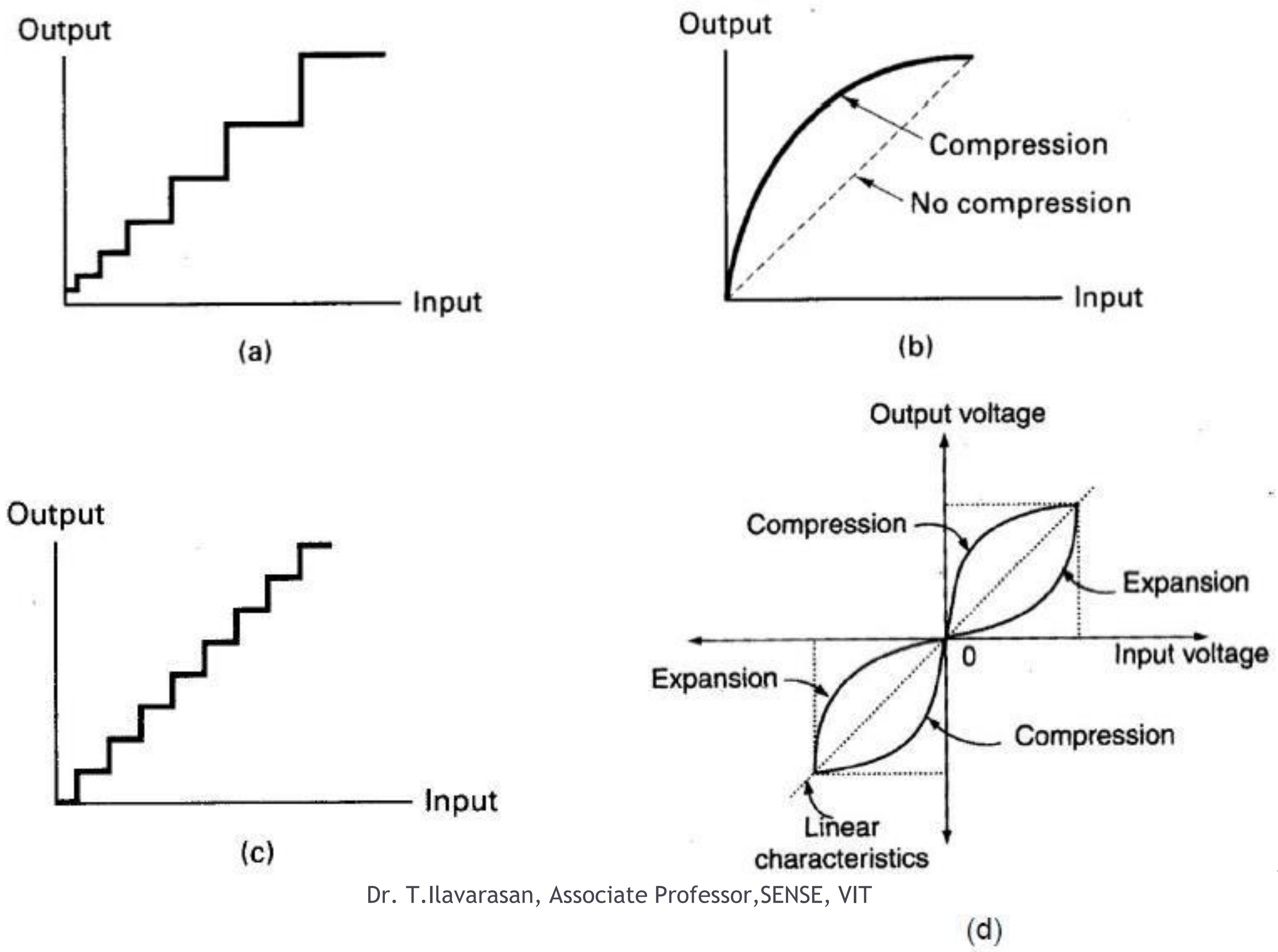
- The non-uniform quantization is practically achieved through a process called **companding**.
- The word **Companding** is a combination of Compressing and Expanding, which means that it does both.
- This is a non-linear technique used in PCM which compresses the data at the transmitter and expands the same data at the receiver.
- The effects of noise and crosstalk are reduced by using this technique.



Pulse code modulation (PCM)

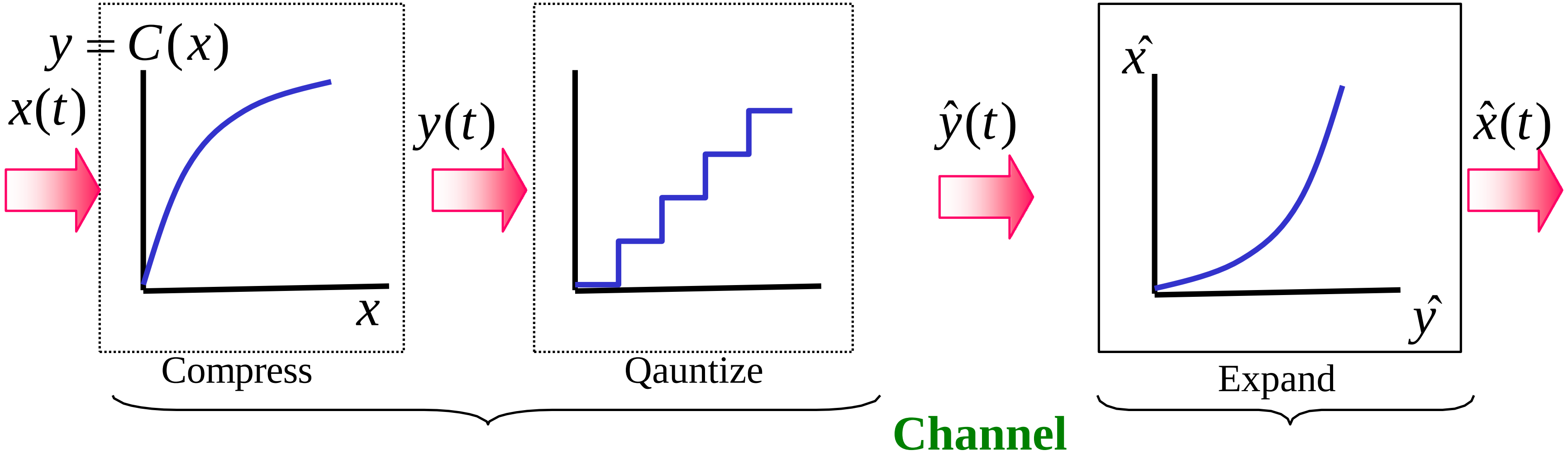


Pulse code modulation (PCM)

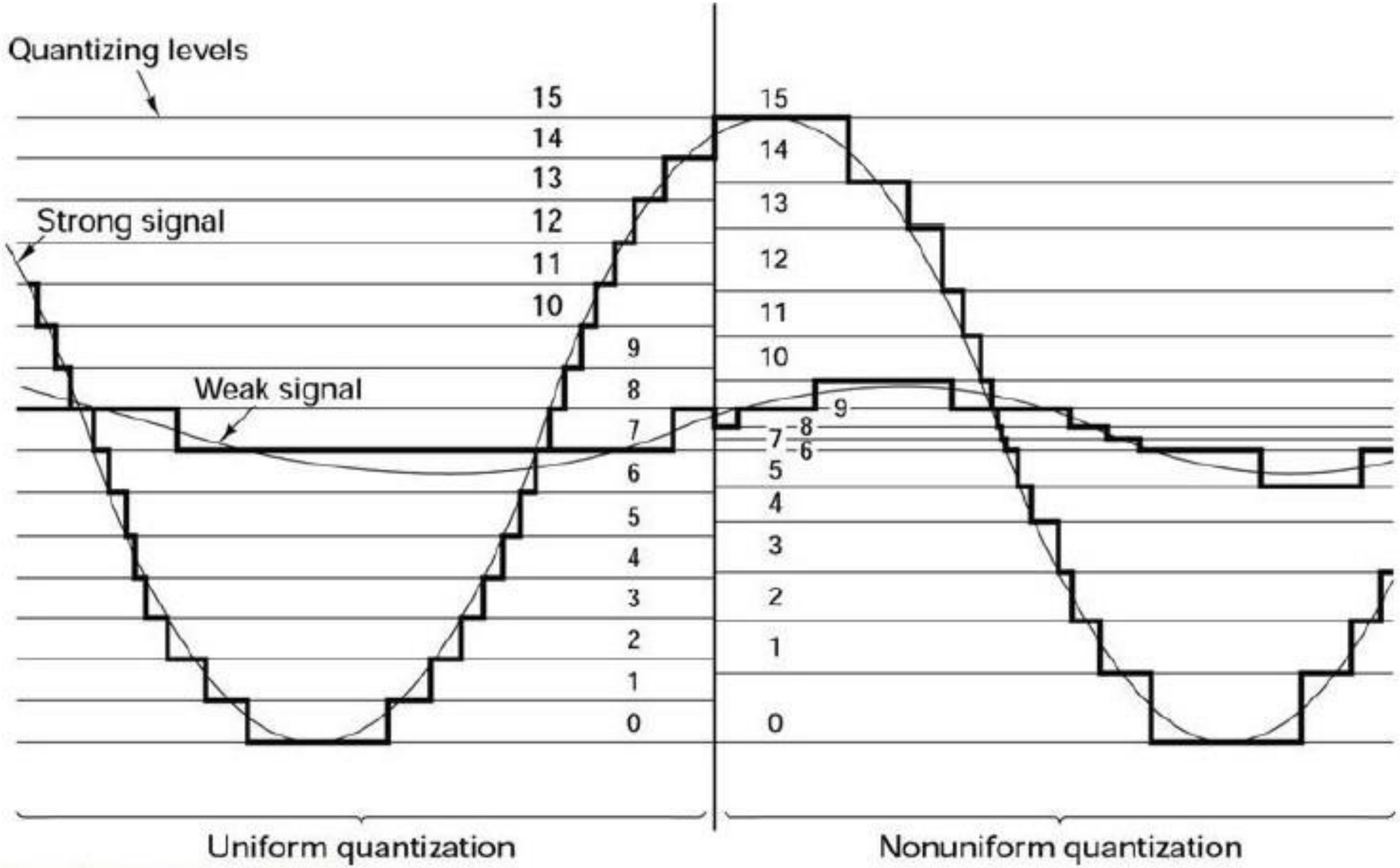


Pulse code modulation (PCM)

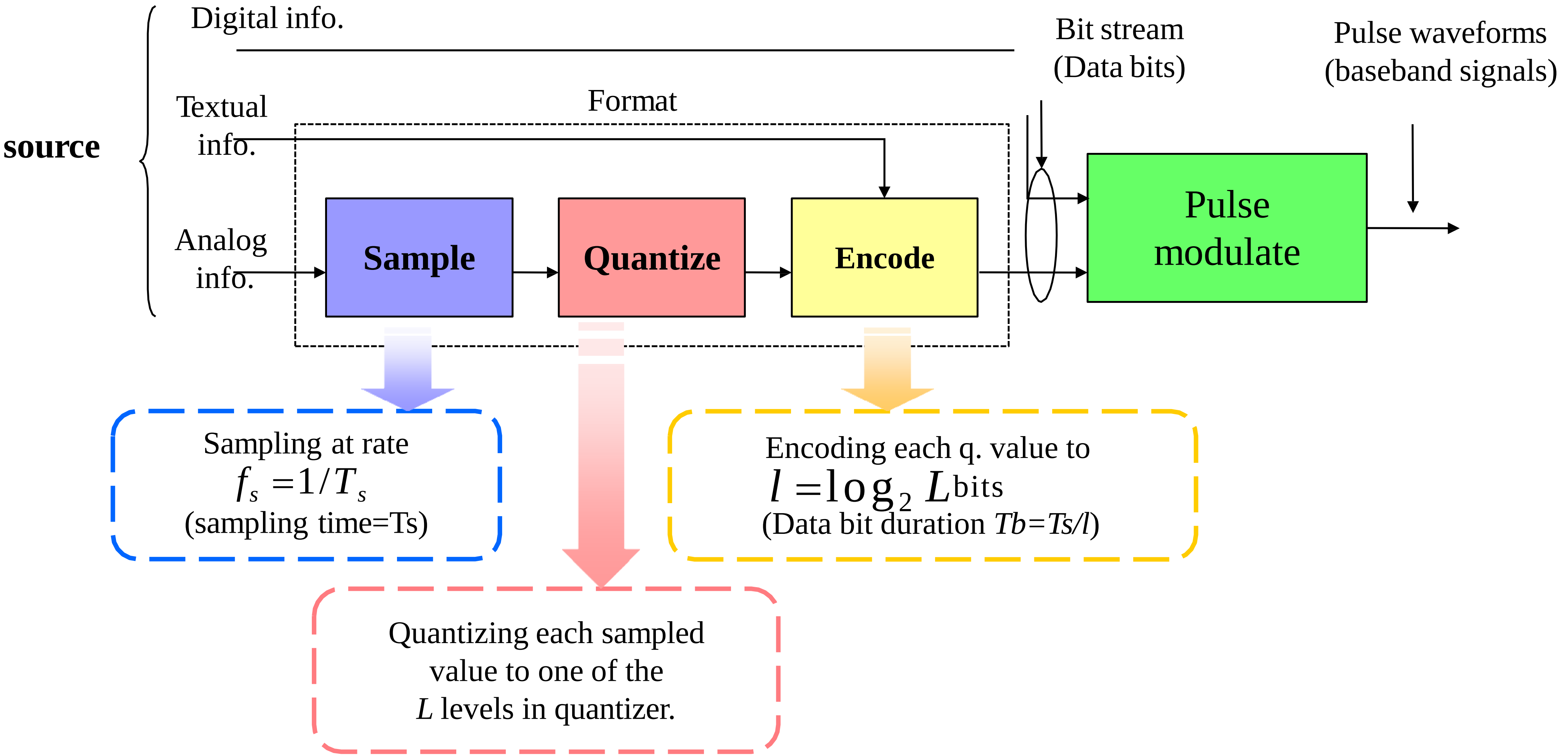
compression+expansion ⇨ companding



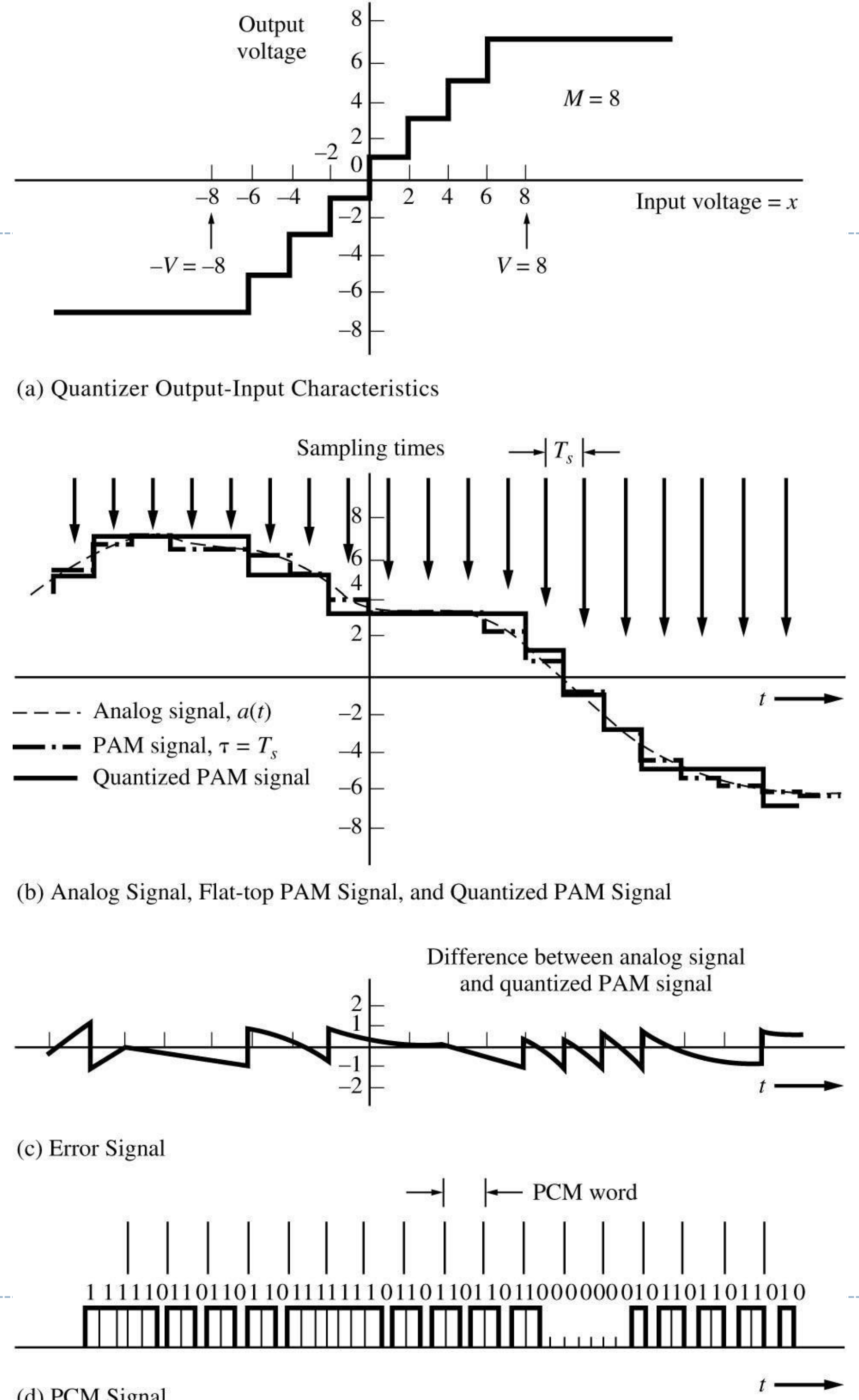
Pulse code modulation (PCM)



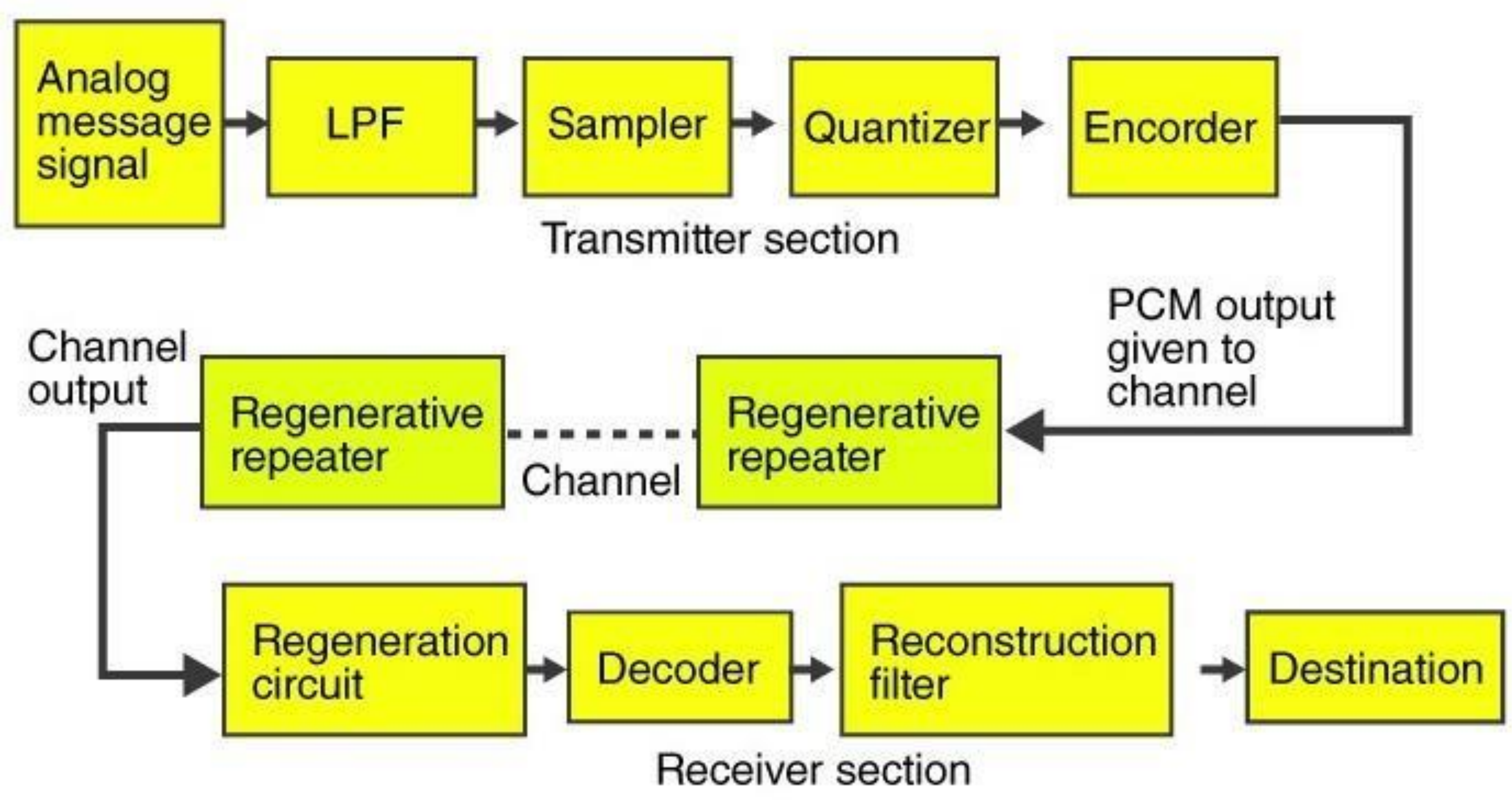
Pulse code modulation (PCM) or ADC



Pulse code modulation (PCM)



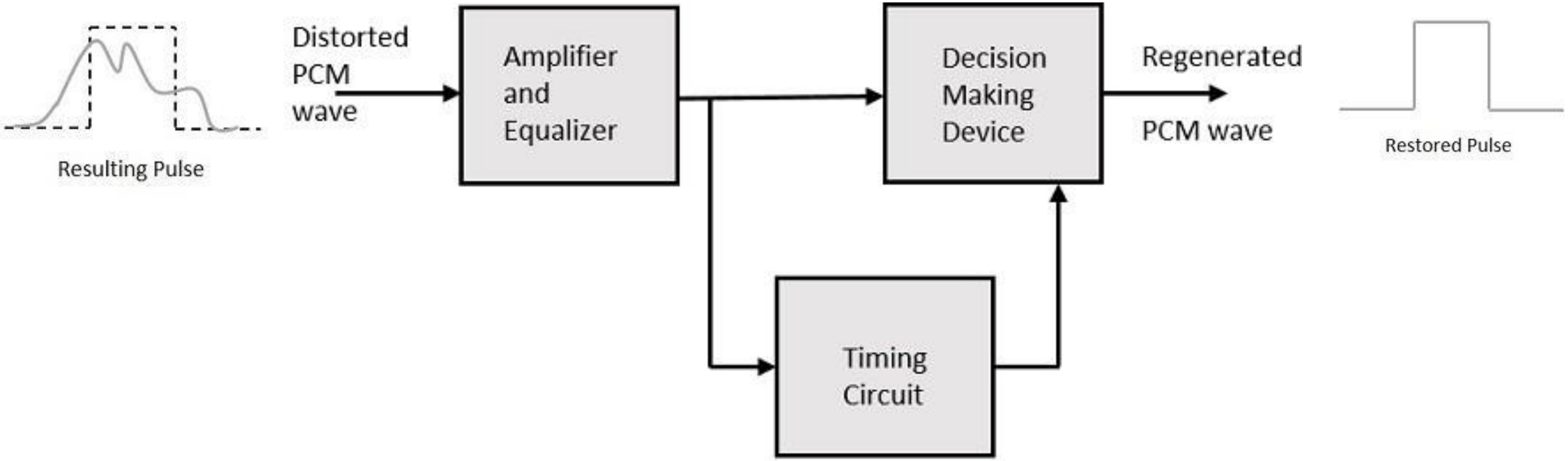
Pulse code modulation (PCM)



Regenerative Repeater (PCM)

- A regenerative repeater consists of an **equalizer**, **a timing circuit and a decision making device**.

- Regenerative repeaters are used to reconstruct the PCM signal.



Regenerative Repeater (PCM)

Equalizer

- The channel produces amplitude and phase distortions to the signals.
- This is due to the transmission characteristics of the channel.
- The Equalizer circuit compensates these losses by shaping the received pulses.

Timing Circuit

- To obtain a quality output, the sampling of the pulses should be done where the signal to noise ratio is maximum
- To achieve this perfect sampling, a periodic pulse train has to be derived from the received pulses, which is done by the timing circuit.
- Hence, the timing circuit, allots the timing interval for sampling at high SNR, through the received pulses.

Regenerative Repeater (PCM)

Decision Device

- The timing circuit determines the sampling times.
- The decision device is enabled at these sampling times.
- The decision device decides its output based on whether the amplitude of the quantized pulse and the noise, exceeds a pre-determined value or not.

Pulse code modulation (PCM)

Advantages:

- It is robust against noise and interference.
- Uniform transmission quality.
- Efficient SNR and bandwidth trade off.
- It provides secure data transmission.
- It offers efficient regeneration.
- It is easy to add or drop channels.

Pulse code modulation (PCM)

Disadvantages:

- Overload appears when modulating signal changes between samplings, by an amount greater than the size of the step.
- Large bandwidth is required for transmission.
- Noise and crosstalk leaves low but rises attenuation.
- An IDN (Integrated Digital Network) can only be realized by gradual extension of noise.