Optimum Linear Receiver

- For a binary baseband data transmission system, we consider the following channel conditions separately:

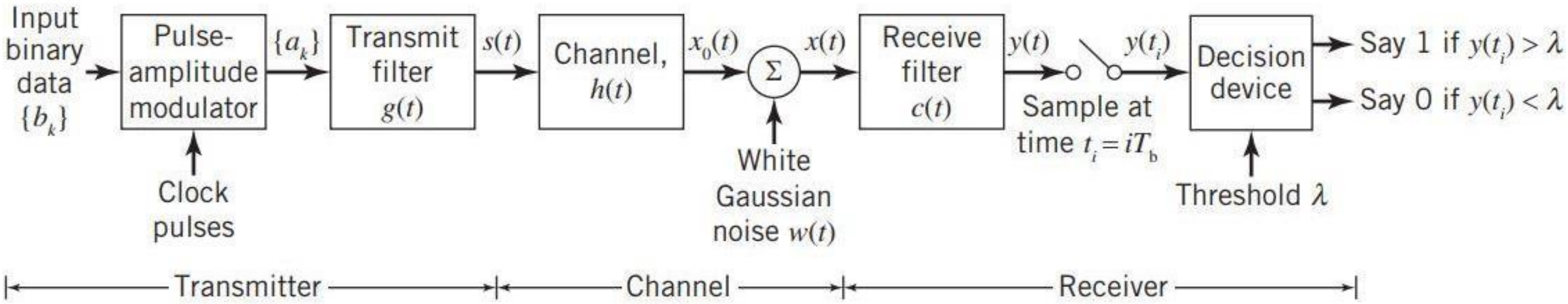
i. *Channel noise acting alone*, which led to formation of matched filter response

ii. *Intersymbol interference acting alone*, which led to formulation of pulse shaping transmit filter

- However, in **real-life situation, channel noise and intersymbol interference act together**
- It affects the transmission system in a combined manner

Optimum Linear Receiver

- Baseband binary data transmission system



Optimum Linear Receiver

- Here, we formulate the basis for designing a linear receiver optimized for both disruptive and noisy linear channel

- A more refined approach is the use of *mean-square error* criterion

- Mathematically, the frequency response $C(f)$ of the optimum linear receiver is expressed as:

$$C(f) = \frac{Q^*(f)}{S_q(f) + \frac{N_0}{2}}$$

- The receiver is periodic with period $1/T_b$

Equalization

- The expression for the frequency response $C(f)$ suggests two basic interpretation:

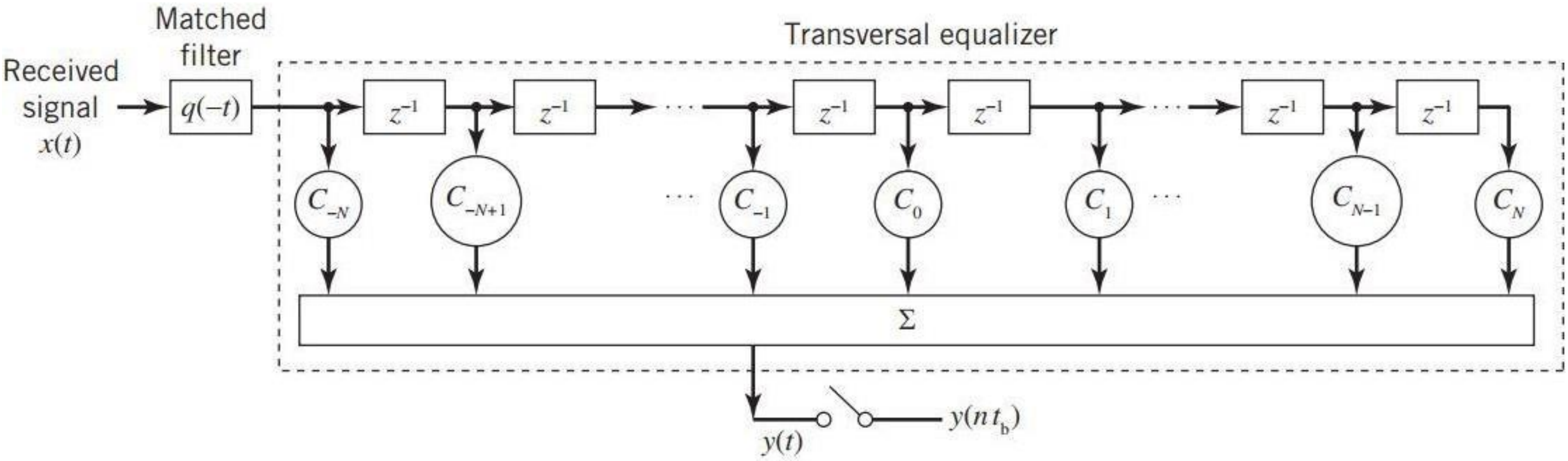
i. A *matched filter* whose impulse response is $q(-t)$, where $q(t) = g(t) * h(t)$

ii. A *transversal (tapped-delay-line)* equalizer whose frequency response is the inverse of the periodic function $S_q(f) + (N_0/2)$

- To implement this, we exactly we need an **equalizer of infinite length**
- In practice, we approximate the optimum solution by using an equalizer with a set of coefficients $\{c_k\}_k^N = -N$, provided N is large enough

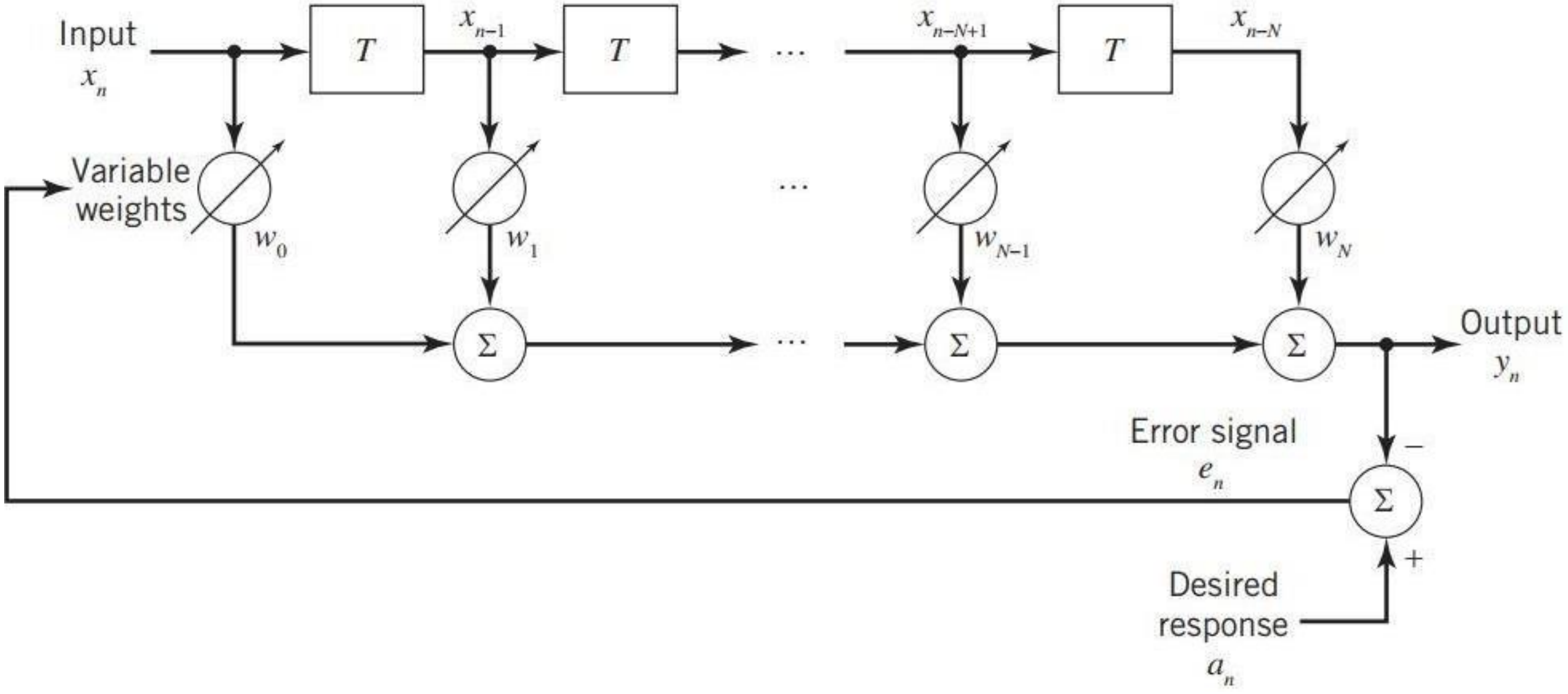
Equalization

- Optimum linear receiver consisting of the cascade connection of matched filter and transversal equalizer



Adaptive Equalization

- Block diagram of adaptive equalizer using an adjustable TDL filter

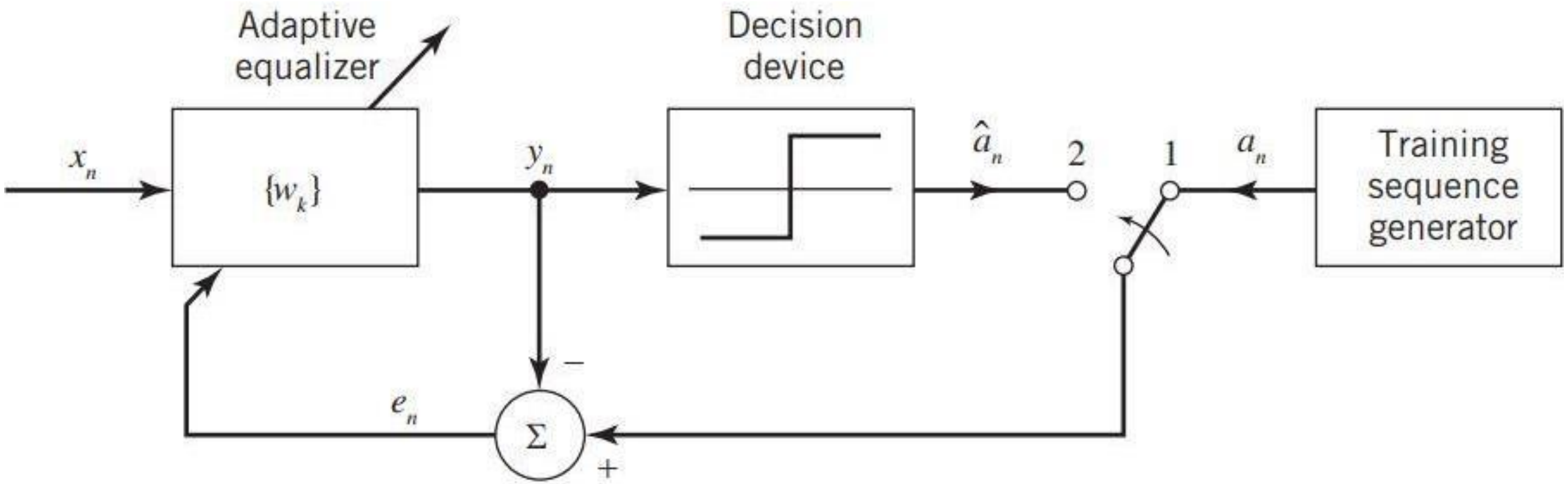


Adaptive Equalization

Operation of the Equalizer

- The two modes of operation for an adaptive equalizer are:

  a) training mode, b) tracking mode



Adaptive Equalization

Least-Mean-Square Algorithm

- In the LMS algorithm for adaptive equalization, the error signal e_n actuates the adjustments
- It is then applied to the individual tap weights of the equalizer as the algorithm proceeds from one iteration to the next
- The LMS algorithm may be restated in words as follows:

$$\begin{pmatrix}\text{Updated value}\\ \text{of } k\text{th tap weight}\end{pmatrix} = \begin{pmatrix}\text{Old value of}\\ k\text{th tap weight}\end{pmatrix} + \begin{pmatrix}\text{Step-size}\\ \text{parameter}\end{pmatrix}\begin{pmatrix}\text{Input signal applied}\\ \text{to } k\text{th tap weight}\end{pmatrix}\begin{pmatrix}\text{Error}\\ \text{signal}\end{pmatrix}$$

Adaptive Equalization

- *LMS algorithm for adaptive equalization* constitute two equations:

$$\hat{w}_{k,n+1} = \hat{w}_{k,n} + \mu x_{n-k} e_n, \qquad k = 0,1,\ldots,N$$

$$e_n = a_n - \sum_{k=0}^{N} \hat{w}_{k,n} x_{n-k}$$

- Here, e_n is the error signal,

  a_n is the desired response,

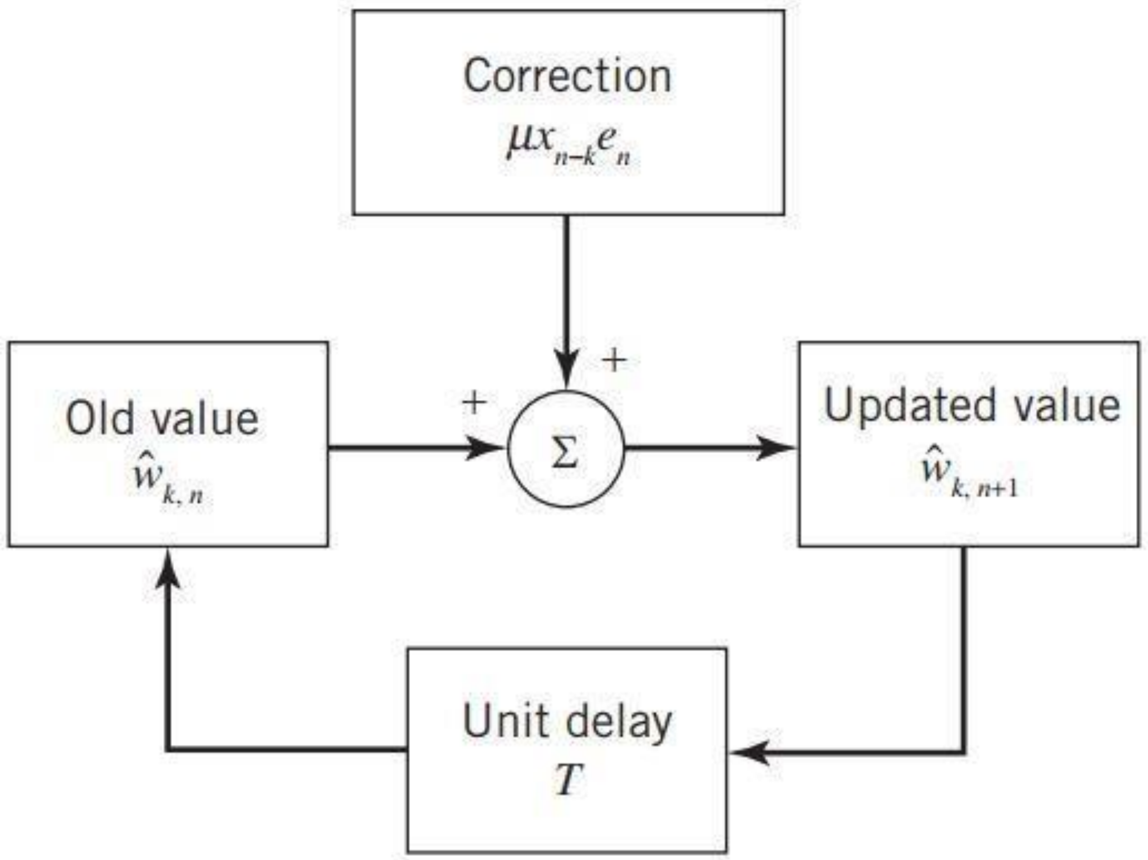
  μ denote the step-size parameter,

  x_n is the input sequence

  and $\hat{w}_k(n)$ as the old value of the kth tap weight at time step n

Adaptive Equalization

- Signal-flow graph representation of the LMS algorithm involving the *k*th tap weight



Adaptive Equalization

Decision-Feedback Equalization

- Consider a baseband channel with impulse response denoted in its sampled form by the sequence $\{h_n\}$ where $h_n = h(nT)$
- The response of this channel to an input sequence $\{x_n\}$, in the absence of noise will be:

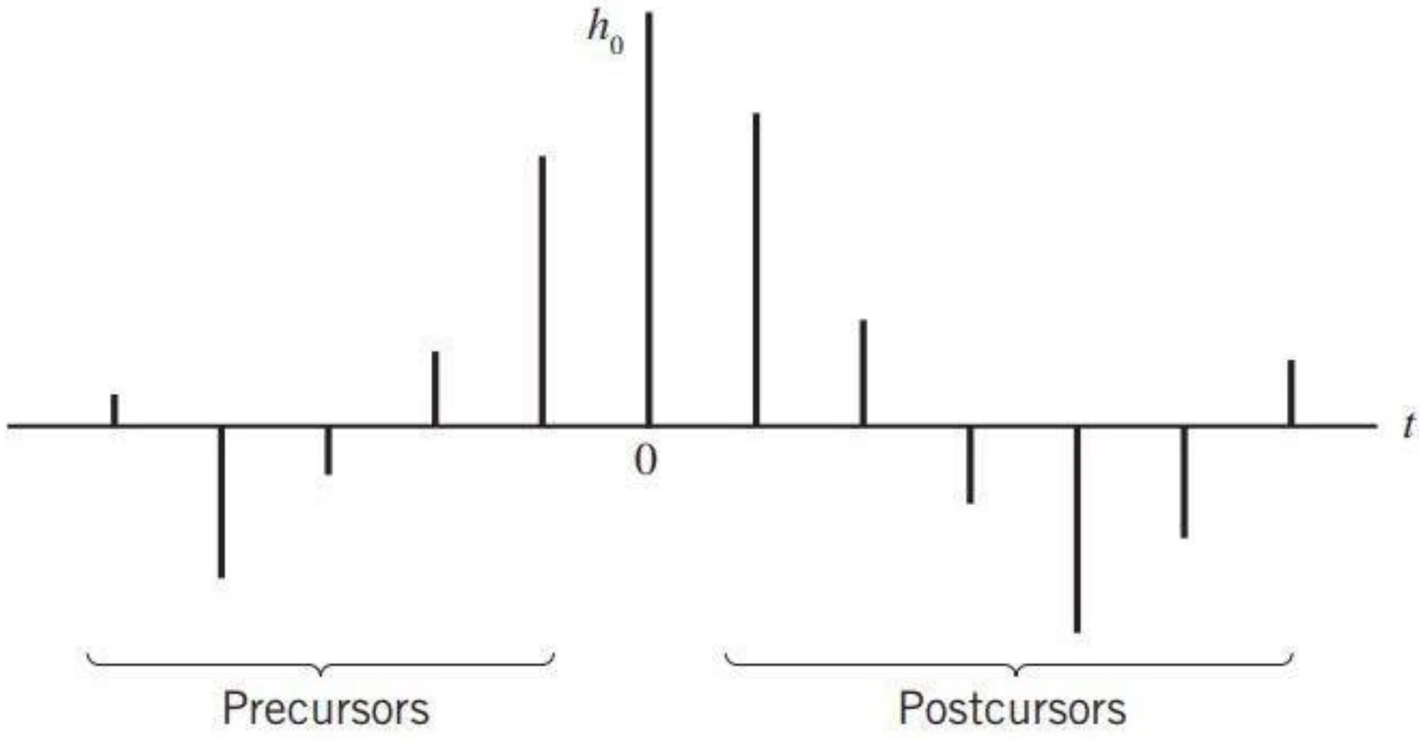
$$y_n = \sum_{k} h_k x_{n-k}$$

$$= h_0 x_n + \sum_{k<0} h_k x_{n-k} + \sum_{k>0} h_k x_{n-k}$$

- The first term here represents the *desired data symbol*, second term represents *precursors* and the third term represents the *postcursors* of the channel impulse response

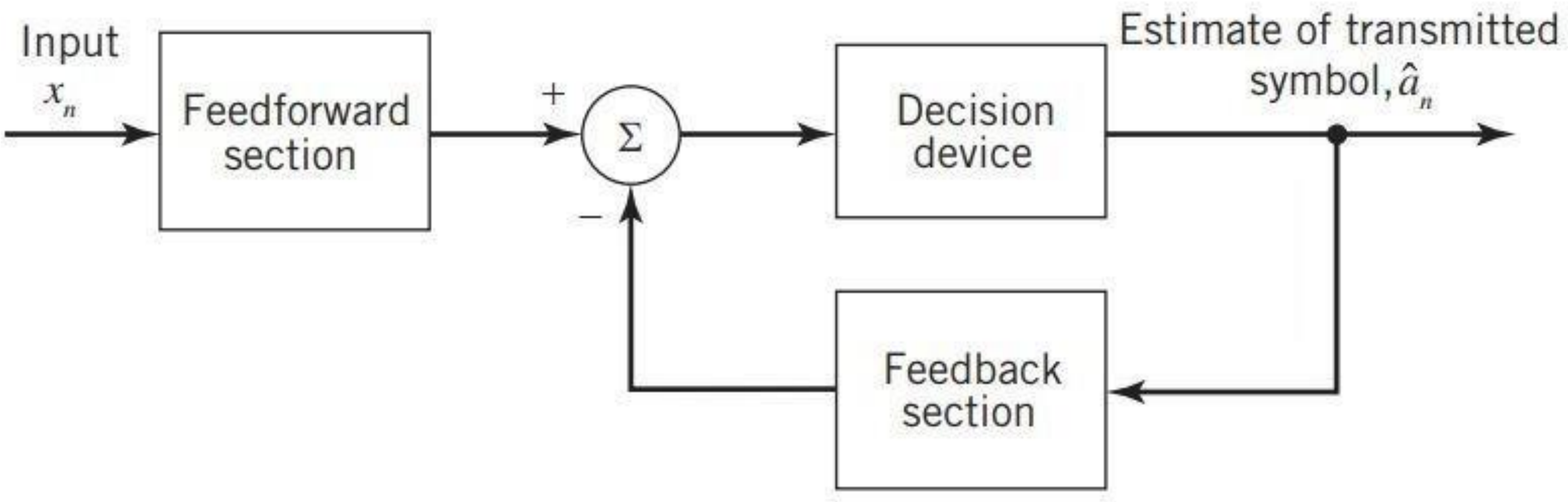
Adaptive Equalization

- Impulse response of a discrete-time channel, depicting the precursors and postcursors



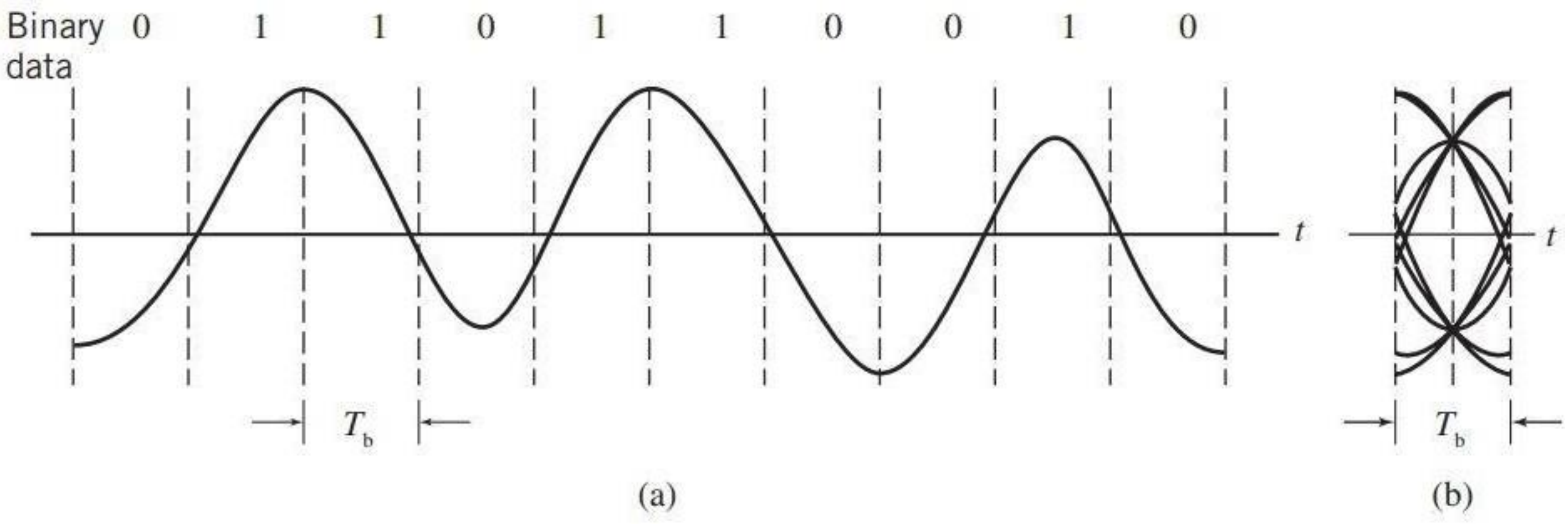
Adaptive Equalization

- Block diagram of decision-feedback equalizer



Post-Processing Techniques: The Eye Pattern

"The *eye pattern* or *eye diagram* is produced by the synchronized superposition of successive symbol intervals of the distorted waveform appearing at the output of the receive filter prior to thresholding"



a) Binary data sequence and its waveform b) Corresponding eye pattern

Post-Processing Techniques: The Eye Pattern

Timing Features

- we may infer three timing features pertaining to a binary data transmission system:

1. *Optimum sampling time*:

It is the sampling time at which the eye opening is the widest

2. *Zero-crossing jitter*:

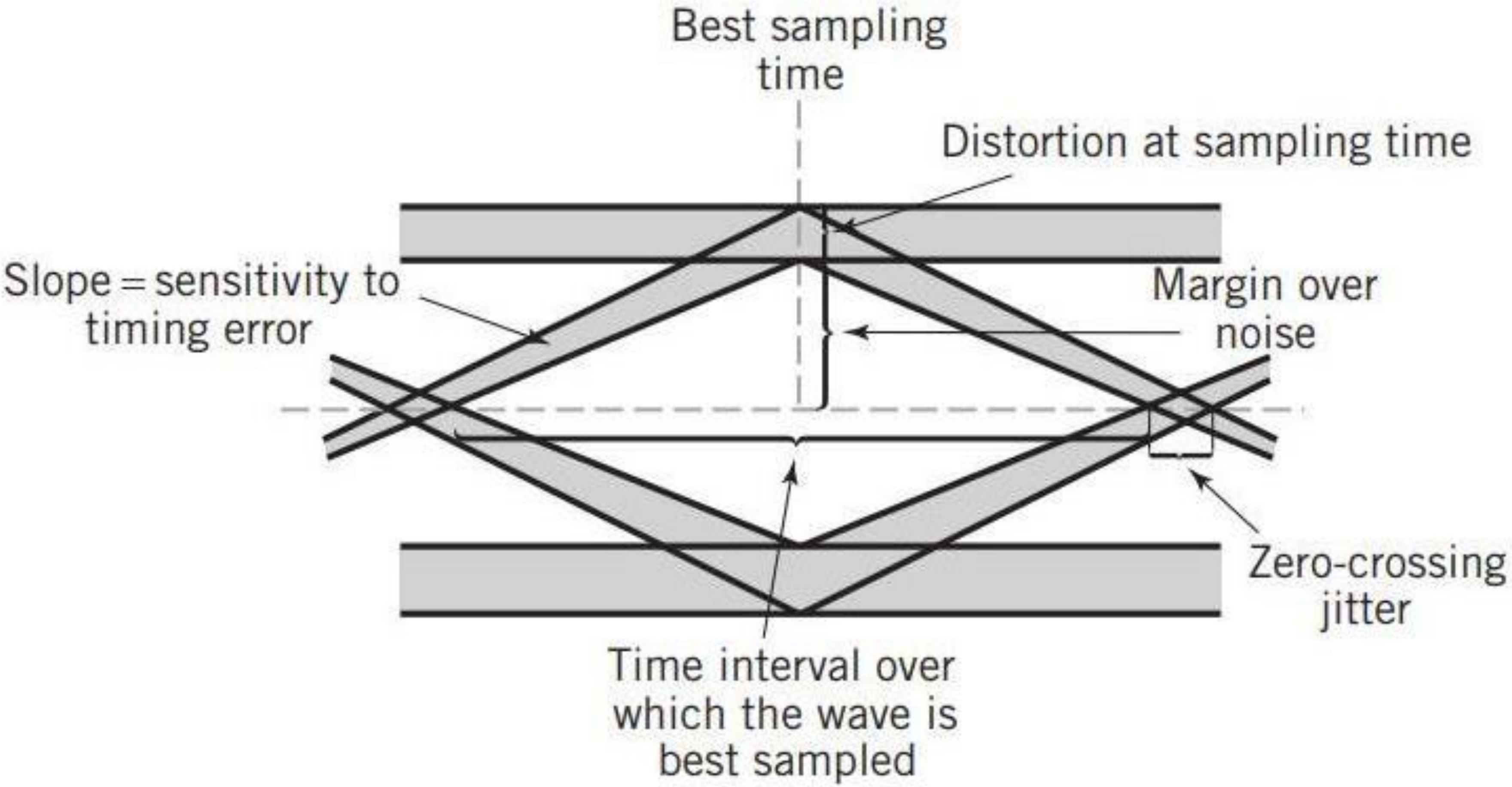
The timing signal is extracted from the zero crossings of the waveform appearing at the receive-filter output and irregularities give rise to jitter

3. Timing sensitivity:

It is the rate at which the eye pattern is closed as the sampling time is varied

Post-Processing Techniques: The Eye Pattern

- Interpretation of the eye pattern for a baseband binary data transmission system



Post-Processing Techniques: The Eye Pattern

The Peak Distortion for Intersymbol Interference

- In a noisy environment, ideal signal amplitude is scaled to occupy range from -1 to $+1$
- In the absence of channel noise, the eye opening assumes two extreme values:

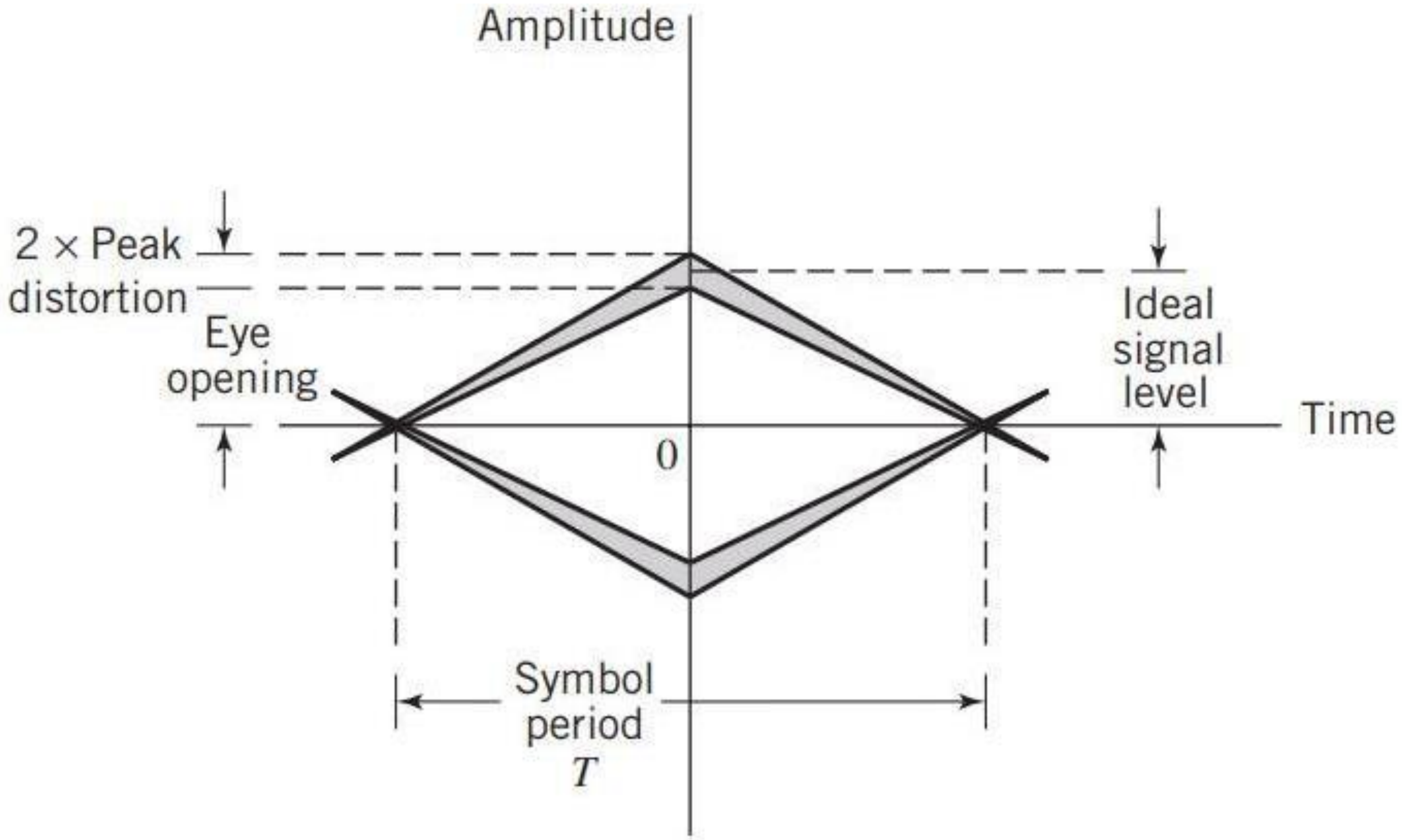
i. *An opening of unity*, which corresponds to zero ISI

ii. *An eye opening of zero*, which corresponds to maximum ISI

- Also, eye opening can be mathematically calculated as: $Eye\ opening = 1 - D_{peak}$
- D_{peak} is new criterion called the *peak distortion*

Post-Processing Techniques: The Eye Pattern

- The *peak distortion* is the maximum value assumed by the intersymbol interference over all possible transmitted sequences



Eye Patterns for M-ary Transmission

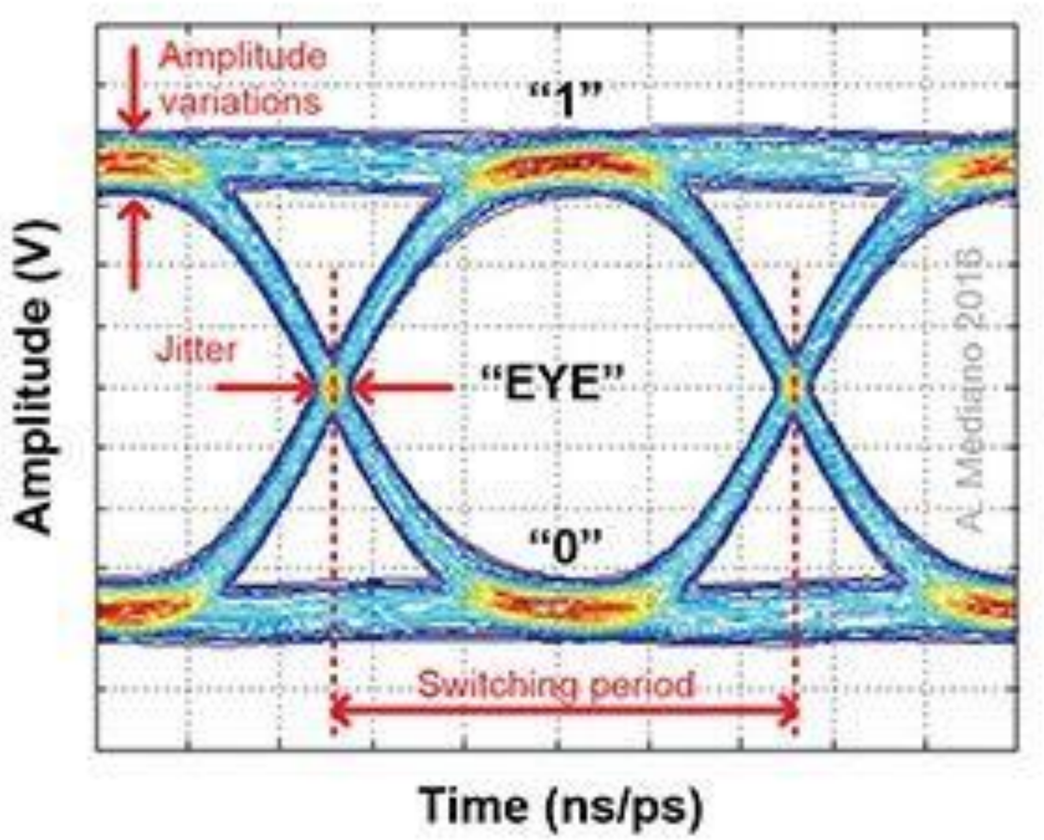
- The eye pattern for an M-ary data transmission system contains $M - 1$ eye openings stacked vertically one on top of the other

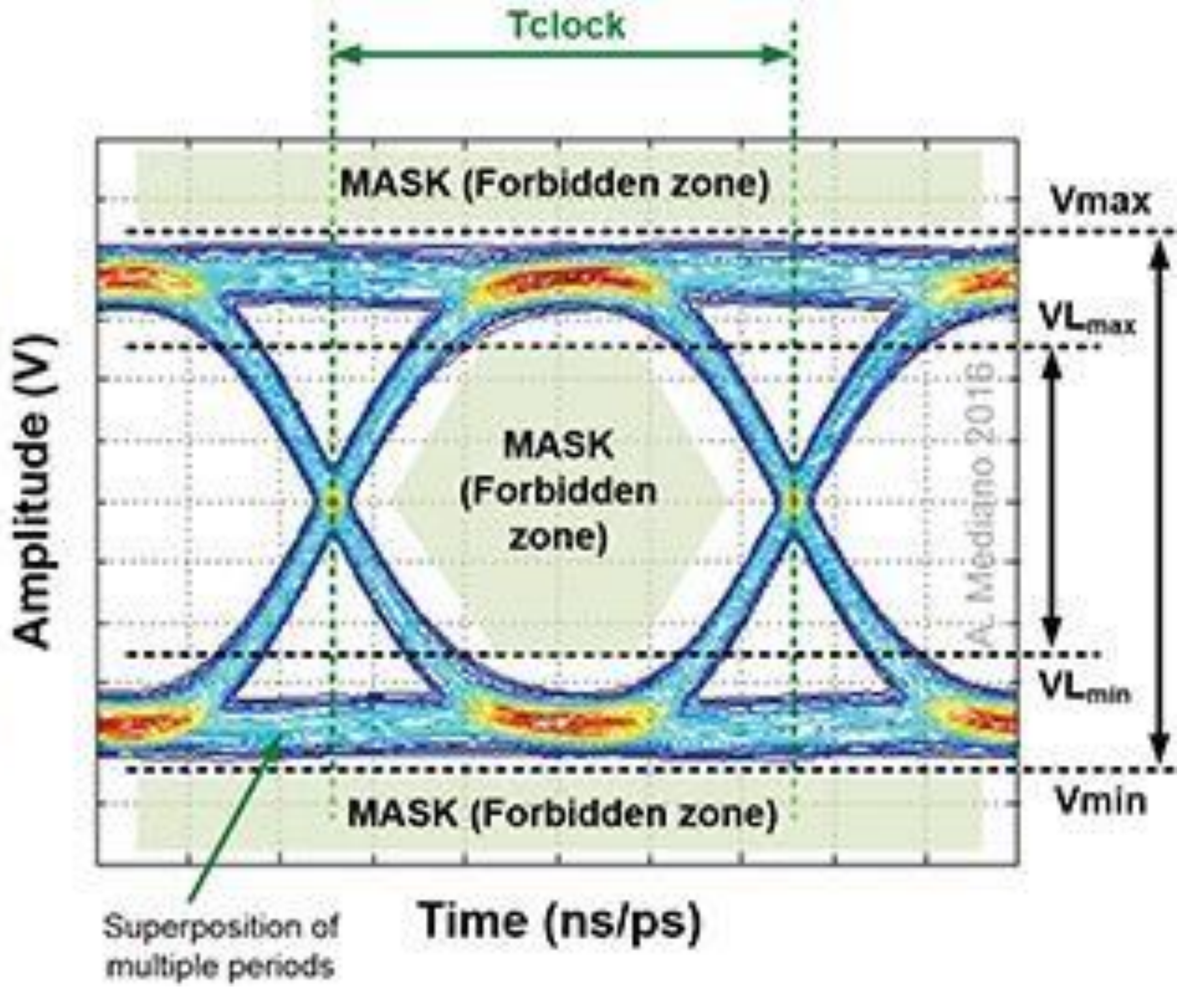
- The thresholds are defined by the amplitude-transition levels as we move up in the stack

- When the encoded symbols are equiprobable, the thresholds will be equidistant from each other

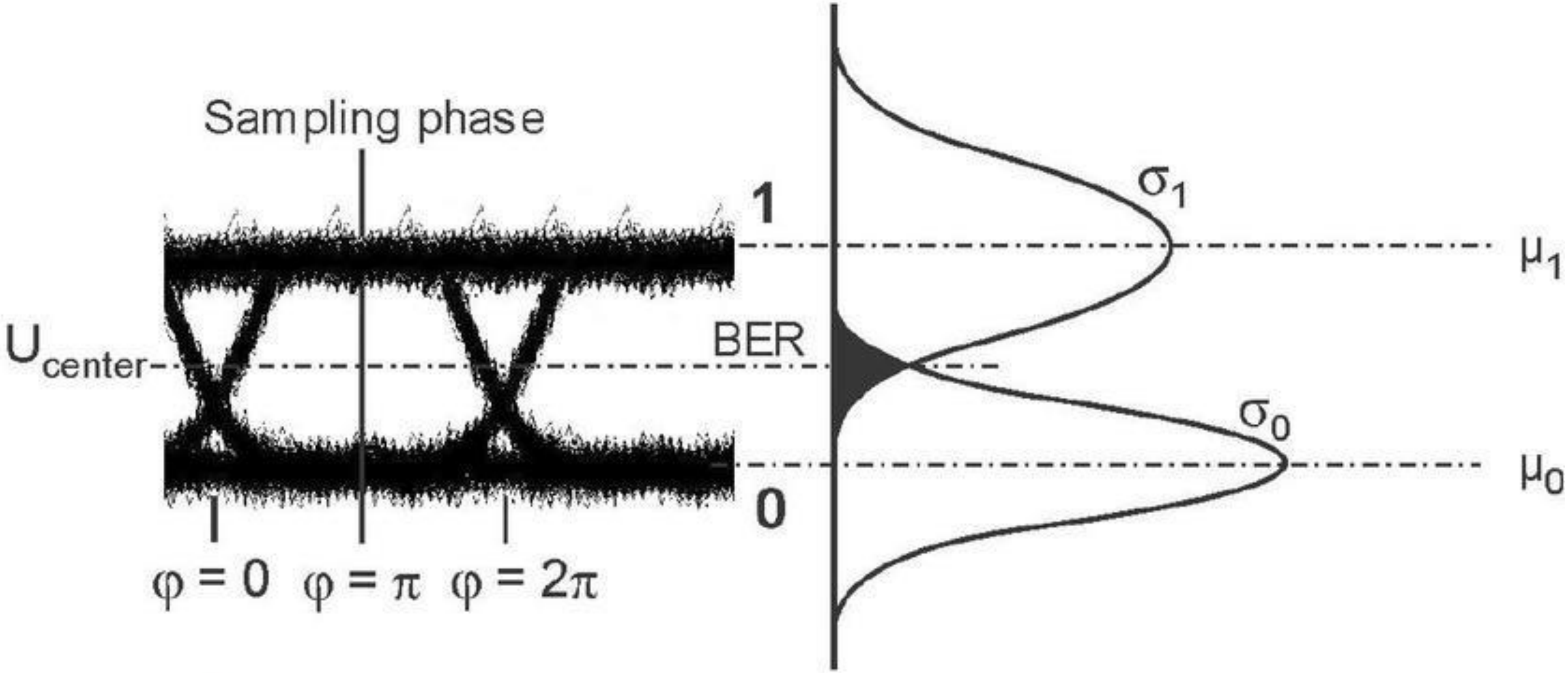
- In a strictly, linear data transmission, all the $M - 1$ openings would be identical

Eye Patterns for M-ary Transmission





Decision Circuit



$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0}$$

$$BER\,(V_{th}) = \tfrac{1}{2}\left(\mathrm{erfc}\left(\frac{|\mu_1 - V_{th}|}{\sigma_1}\right) + erfc\left(\frac{|V_{th} - \mu_0|}{\sigma_0}\right)\right)$$