Numerical

- Consider the four signals given below and apply GSOP
- a) Determine the basis functions.
- b) Represent all the signals by using basis functions.

$$S1(t) = 1 \qquad (0 \le t \le 2)$$

$$S2(t) = \begin{cases} 1 & (0 \le t \le 1) \\ -1 & (1 \le t \le 2) \end{cases}$$

$$S3(t) = \begin{cases} 1 & (0 \le t \le 2) \\ -1 & (2 \le t \le 3) \end{cases}$$

$$S4(t) = -1 \qquad (0 \le t \le 3)$$

Numerical

Soln

$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$, $E_1 = \int_0^2 1^2 dt = 2$

$\boxed{\phi_1(t) = \frac{1}{\sqrt{2}}}$ $0 \le t \le 2$.

$\phi_2 = \frac{s_2(t)}{\sqrt{E_2 - s_{21}^2}}$ $E_2 = \int_0^1 1 + \int_1^2 1 \, dt = 2$

$s_{21} = \int_0^1 s_2(t)\, \phi_1(t)\, dt + \int_1^2 s_2(t)\, \phi_1(t)\, dt$

$s_2(t) \begin{cases} = \frac{1}{\sqrt{2}} & 0 \le t < 1 \\ -\frac{1}{\sqrt{2}} & 1 \le t \le 2 \end{cases}$ $= 1 \cdot \frac{1}{\sqrt{2}} dt + (-) \frac{1}{\sqrt{2}}$

$= 0$

Numerical

(3) $\phi_2(t) = \dfrac{s_2(t)}{\sqrt{E_2 - s_{21}^2 - s_{22}^2}}$

$s_{21} = \int s_2(t)\,\phi_1(t)\,dt$

$= \int_0^2 (-1)\frac{1}{\sqrt{2}}$

$\boxed{s_{21} = -\sqrt{2}}$

$s_{22} = \int s_2(t)\,\phi_2(t)\,dt$

$= \int_0^1 (-1)\frac{1}{\sqrt{2}} + \int_1^2 (-1)\left(-\frac{1}{\sqrt{2}}\right)$

$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$\boxed{s_{22} = 0}$

$E_2 = \int_0^3 (-1)^2 = 3$

$\phi_3(t) = \dfrac{-1}{\sqrt{3 - (\sqrt{2})^2}} = \dfrac{-1}{\sqrt{3-2}} = -1$

$\boxed{\phi_3(t) = -1}$ $\quad 2 < t \le 3$

Numerical

Consider that the tap weights of an equalizing transversal filter are to be determined by transmitting a single impulse as a training signal. Let the equalizer circuit in Figure 3.26 be made up of just three taps. Given a received distorted set of pulse samples $\{x(k)\}$, with voltage values 0.0, 0.2, 0.9, –0.3, 0.1, as shown in Figure 3.25, use a zero-forcing solution to find the weights $\{c_{-1}, c_0, c_1\}$ that reduce the ISI so that the equalized pulse samples $\{z(k)\}$ have the values, $\{z(-1) = 0,\ z(0) = 1,\ z(1) = 0\}$.

Reference: Digital communications by B.Sklar

Numerical

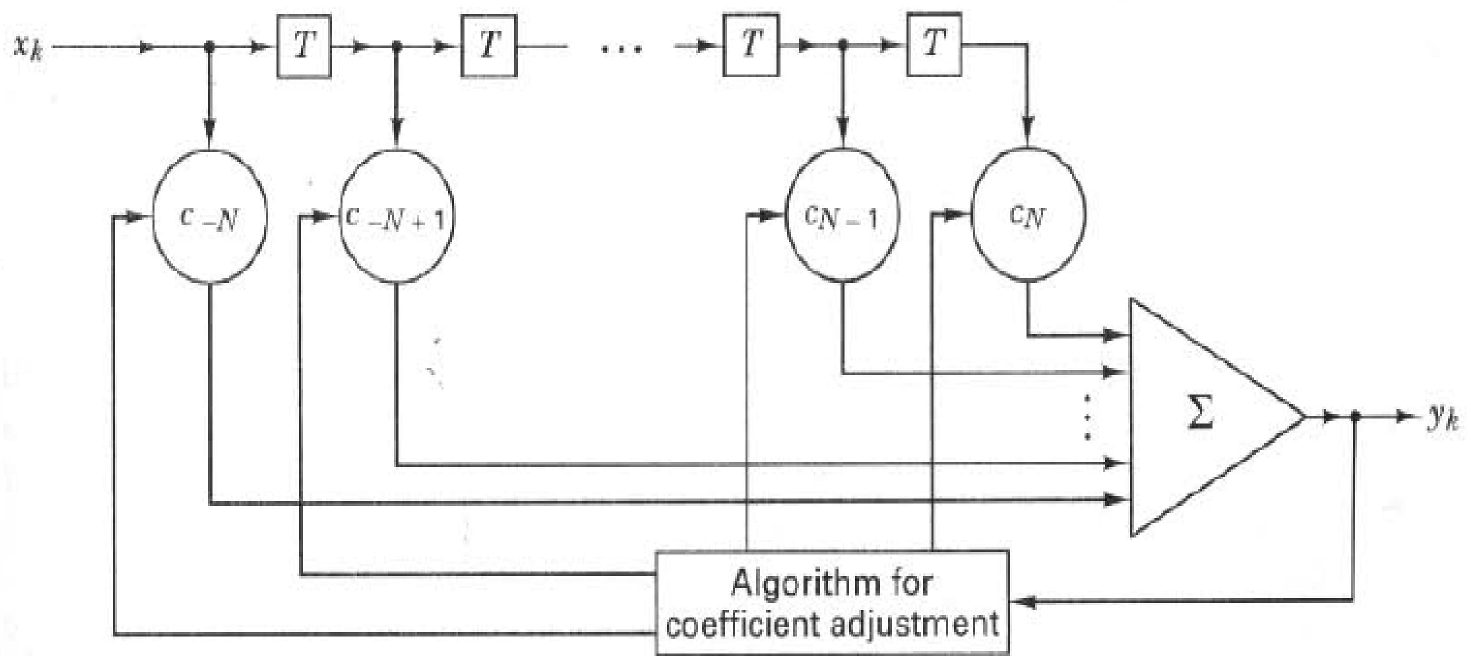
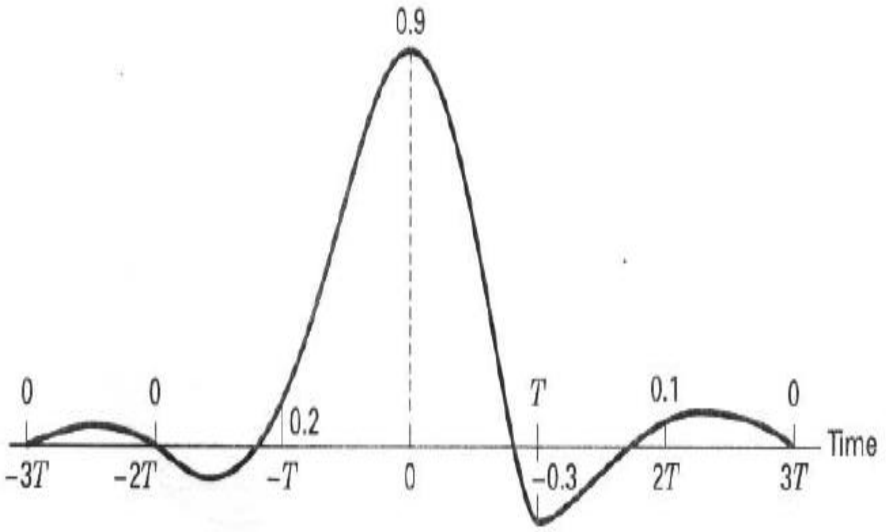


Figure 3.26 Transversal filter.



Numerical

$$\mathbf{z} = \mathbf{x}\,\mathbf{c}$$

or

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

Solving these three simultaneous equations results in the following weights:

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -0.2140 \\ 0.9631 \\ 0.3448 \end{bmatrix}$$