Sampling

① Find the Nyquist rate for the signal $x(t) = 1 + \cos 10\pi t$ in Hz.

Solution

$$x(t) = 1 + \cos 10\pi t$$

1 → dc, $\cos 10\pi t$ → Ac

eq. $2\pi f_m = 10\pi$

$$f_m = 5 \text{ Hz}$$

$f_s = 2f_m$ (Nyquist Rate)

$= 2 \times 5$

$$\boxed{f_s = 10 \text{ Hz}}$$

② Find the Nyquist rate of the signal $x(t) = \sin 200\pi t - 2\cos 100\pi t$

Solution

$x(t) = x_1(t) + x_2(t)$

from $x_1(t)$

$200\pi = 2\pi f_m$

$$\boxed{f_{m_1} = 100 \text{ Hz}}$$

from $x_2(t)$

$100\pi = 2\pi f_m$

$$\boxed{f_{m_2} = 50 \text{ Hz}}$$

$f_m = \max(f_{m_1}, f_{m_2})$

$f_m = 100 \text{ Hz}$

Nyquist rate $= 2f_m$

$= 200 \text{ Hz}$

(3) Consider an analog signal $x(t) = 5\cos 200\pi t$
a) Determine the minimum sampling rate to avoid aliasing.
b) If Sampling rate $f_s = 400Hz$. what is DT Signal after sampling.

Solution

$$2\pi f_m = 200\pi$$

$$\boxed{f_m = 100\,Hz}$$

a) Minimum Sampling rate $f_s = 2f_m = 200\,Hz$

b) If $f_s = 400Hz$, $x(n) = ?$ replace $t = n/f_s$ in $x(t)$

$$x(n) = 5\cos\left(200\pi \frac{n}{400}\right)$$

$$x(n) = 5\cos(0.5\pi n)$$

(4) Consider the Signal $x(t) = \cos 2000\pi t + 10\sin(10,000\pi t) + 20\cos 5000\pi t$. Determine 1) Nyquist rate for this signal and 2) if the Sampling rate is 5000 samples/sec, then what is the discrete time signal obtained after Sampling?

Solution

From $x(t) = x_1(t) + x_2(t) + x_3(t)$

From $x_1(t)$

$$f_{m_1} \Rightarrow 2000\pi = 2\pi f_{m_1}$$

$$\boxed{f_{m_1} = 1000\,Hz}$$

From $x_2(t)$

$$10,000\pi = 2\pi f_{m_2}$$

$$\boxed{f_{m_2} = 5000\,Hz}$$

From $x_3(t)$

$5000\pi = 2\pi f_{m_3}$

$f_{m_3} = 2500$ Hz

$f_m = \max(f_{m_1}, f_{m_2}, f_{m_3})$

$\boxed{f_m = 5000 \text{ Hz}}$

$f_s = 2 \times f_m = 2 \times 5000 = 10{,}000$ Hz

2) 2) if Sampling rate is 5000 samples/sec.

replace $t = (n/f_s) = (n/5000)$ in $x(t)$

$x(t) = \cos 2000\pi t + 10 \sin 10{,}000\pi t + 20 \cos 5000\pi t$

$= \cos(2000\pi \, n/f_s) + 10 \sin(10.000\pi \, n/f_s) + 20 \cos(5000\pi \, n/f_s)$

$= \cos\left(2000\pi \, n/5000\right) + 10 \sin\left(10000\pi \, n/5000\right) + 20 \cos\left(5000\pi \, n/5000\right)$

$x(n) = \cos 0.4\pi n + 10 \sin 2\pi n + 20 \cos \pi n$