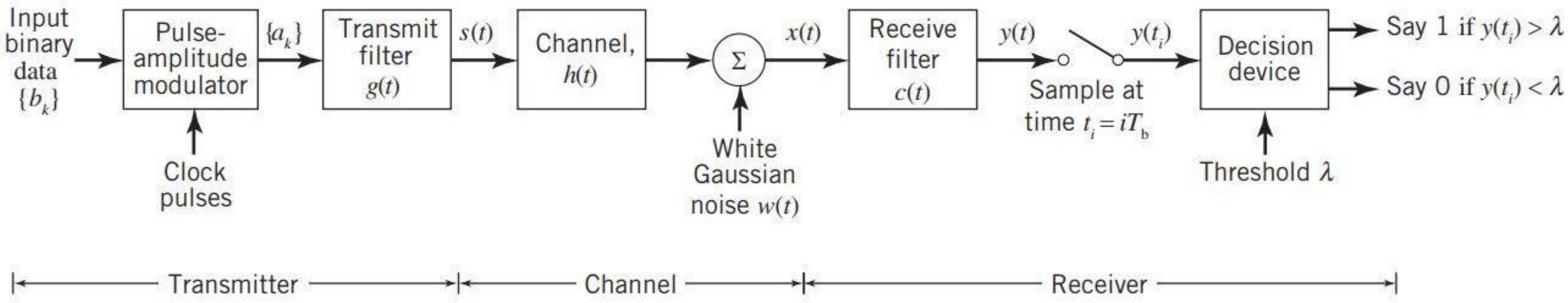
InterSymbol Interference (ISI)- Analysis

- Consider a *baseband binary PAM* system as depicted in the figure below:



InterSymbol Interference (ISI)- Analysis

- The term baseband refers to an information-bearing signal whose spectrum extends from (or near) zero up to some finite value for positive frequencies

- In this discussion, we consider the simple case of binary PAM
- In the transmission system, the pulse-amplitude modulator changes the input binary data stream $\{b_k\}$ into a new sequence of short pulses
- More specifically, the pulse amplitude a_k is represented in the polar form:

$$a_k = \begin{cases} +1 \; if \; b_k \; is \; symbol \; 1 \\ -1 \; if \; b_k \; is \; symbol \; 0 \end{cases}$$

InterSymbol Interference (ISI)- Analysis

- The sequence of short pulses so produced is applied to a *transmit filter* whose impulse response is denoted by $g(t)$
- The transmitted signal is thus defined by the sequence:

$$s(t) = \sum_{k} a_k\, g(t - kT_b)$$

- Except for the scaling factor, the received signal is further expressed as the received filter output given by:

$$y(t) = \sum_{k} a_k p(t - kT_b)$$

- The *scaled pulse* $p(t)$ is given by $p(t) = g(t) * h(t) * c(t)$

InterSymbol Interference (ISI)- Analysis

- Here, $p(t)$ is the double convolution of:

i. impulse response $g(t)$ of the transmitter,

ii. impulse response $h(t)$ of the channel

iii. impulse response $c(t)$ of the receiver

- The received filter output $y(t)$ is sampled at time $t_i = iT_b$ where i takes on integer values; thus we have:

$$y(t_i) = \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] = a_i + \sum_{k=-\infty, k\neq i}^{\infty} a_k p[(i-k)T_b]$$

InterSymbol Interference (ISI)- Analysis

- The first term a_i represents the contribution of ith transmitted bit
- The second term $\sum_{k=-\infty,k\neq i}^{\infty} a_k p[(i-k)T_b]$ represents the *residual effect* of all other transmitted bits on the decoding of the ith bit
- This residual effect due to occurrence of pulses before and after the sampling instant t_i is called the *intersymbol interference or ISI*
- In absence of ISI, the resulting equation reduces to: $y(t_i) = a_i$
- It shows that under ideal condition, the ith transmitted bit is decoded correctly

InterSymbol Interference (ISI)- Analysis

Three strategies for eliminating ISI:

- Use a line code that is absolutely bandlimited.
  - Would require Sinc pulse shape.
  - Can't actually do this (but can approximate).
- Use a line code that is zero during adjacent sample instants.
  - It's okay for pulses to overlap somewhat,as long as there is no overlap at the sample instants.
  - Can come up with pulse shapes that don't overlap during adjacent sample instants.
    - Raised-Cosine Rolloff pulse shaping
- Use a filter at the receiver to "undo" the distortion introduced by the channel.
  - Equalizer.

Signal Design for Zero ISI

We design a signal-design problem as follows:

"The overlapping pulses in the binary data-transmission system are configured in such a way that they do not interfere with each other at the sampling times $t_i = iT_b$"

- Such a design procedure is rooted in the *criterion for distortionless transmission*
- It was formulated by Nyquist (1928) on telegraph transmission theory

Signal Design for Zero ISI

- For this the weighted pulse contribution $a_k p(iT_b - kT_b)$ must be zero for all k except for $k = \mathrm{i}$ for the transmission to be *ISI free*

- In other words, the overall pulse-shape $p(t)$ must be designed such that:

$$p(iT_b - kT_b) = \begin{cases} 1 \; for \; i = k \\ 0 \; for \; i \neq k \end{cases}$$

$p(t)$ satisfying this two-part condition is called a *Nyquist pulse*

- The above condition itself is referred to as "*Nyquist's criterion for distortionless binary baseband data transmission*"

Ideal Nyquist Pulse for Distortionless Baseband Data Transmission

- Consider the sequence of samples $\{p(nT_b)\}$, where $n = 0, \pm 1, \pm 2, \ldots$

- We have: $P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b)$
- Here, $R_b = 1/T_b$ is the bit rate per second
- $P_\delta(f)$ is the Fourier transform of an infinite periodic sequence of delta functions of period T_b
- The frequency domain condition for zero ISI to be satisfied is: $\sum_{n=-\infty}^{\infty} P(f - nR_b) = \mathrm{T_b}$

Ideal Nyquist Pulse

- The frequency function $p(f)$ to satisfy the zero ISI condition, it has to be in the form of a *rectangular function* given by:

$$P(f) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0, & |f| > W \end{cases}$$

$$= \frac{1}{2W} rect\left(\frac{f}{2W}\right)$$

- $rect(f)$ stands for a rectangular function of unit amplitude and unit support centered on $f = 0$

Ideal Nyquist Pulse

- The overall base system bandwidth W is defined by: $W = \frac{R_b}{2} = \frac{1}{2T_b}$

- We find that a signal waveform that produces zero ISI is defined by the *sinc function*

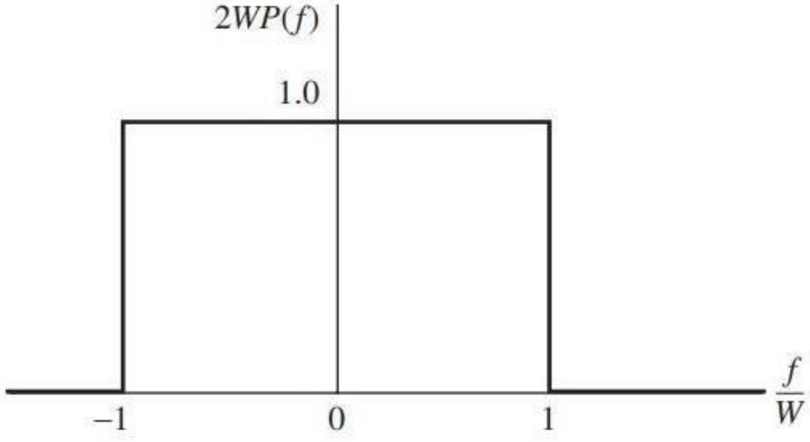
$$p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt}$$

$$= sinc(2Wt)$$

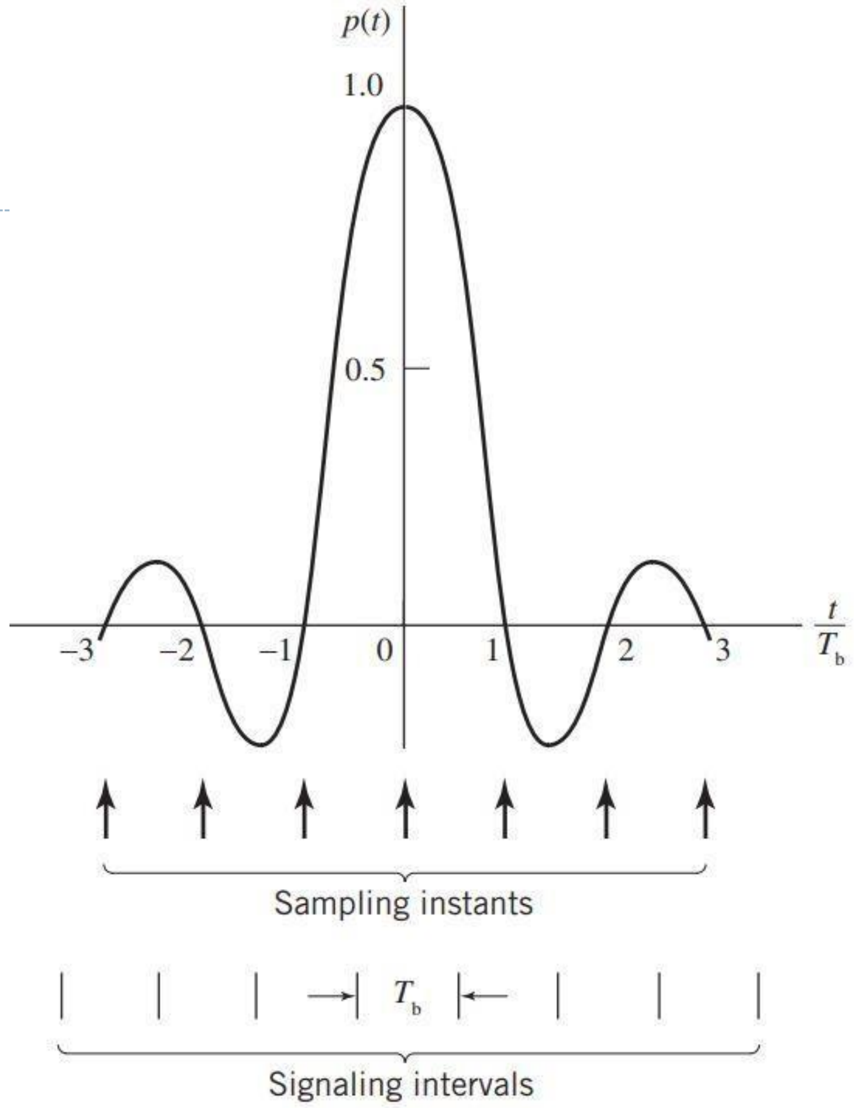
- The special value of the bit rate $R_b = 2W$ is called the *Nyquist Rate*
- W is called the *Nyquist bandwidth*

Ideal Nyquist Pulse

$$\boxed{W = \frac{1}{2T_b} = \frac{R_b}{2}}$$



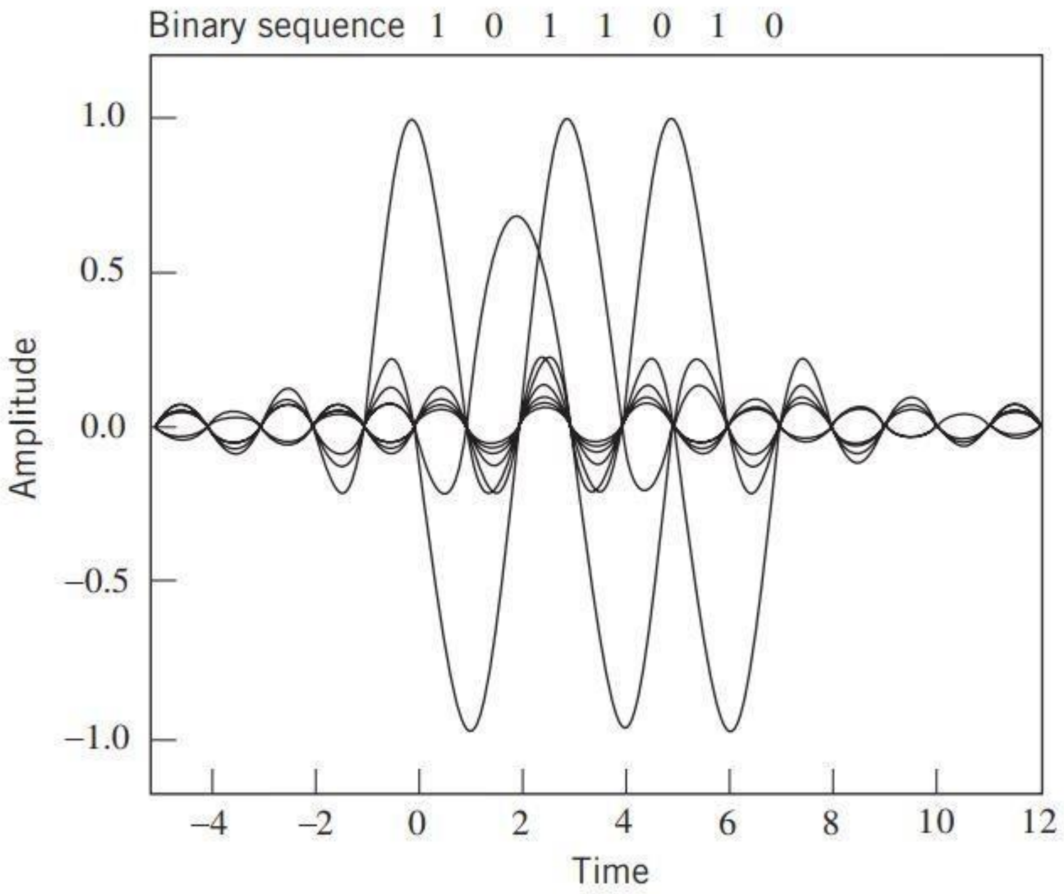
(a)



(b)

a) Ideal magnitude response b) Ideal basic pulse shape

Ideal Nyquist Pulse



A series of sinc pulses corresponding to the sequence 1011010