Quantization (rounding off)

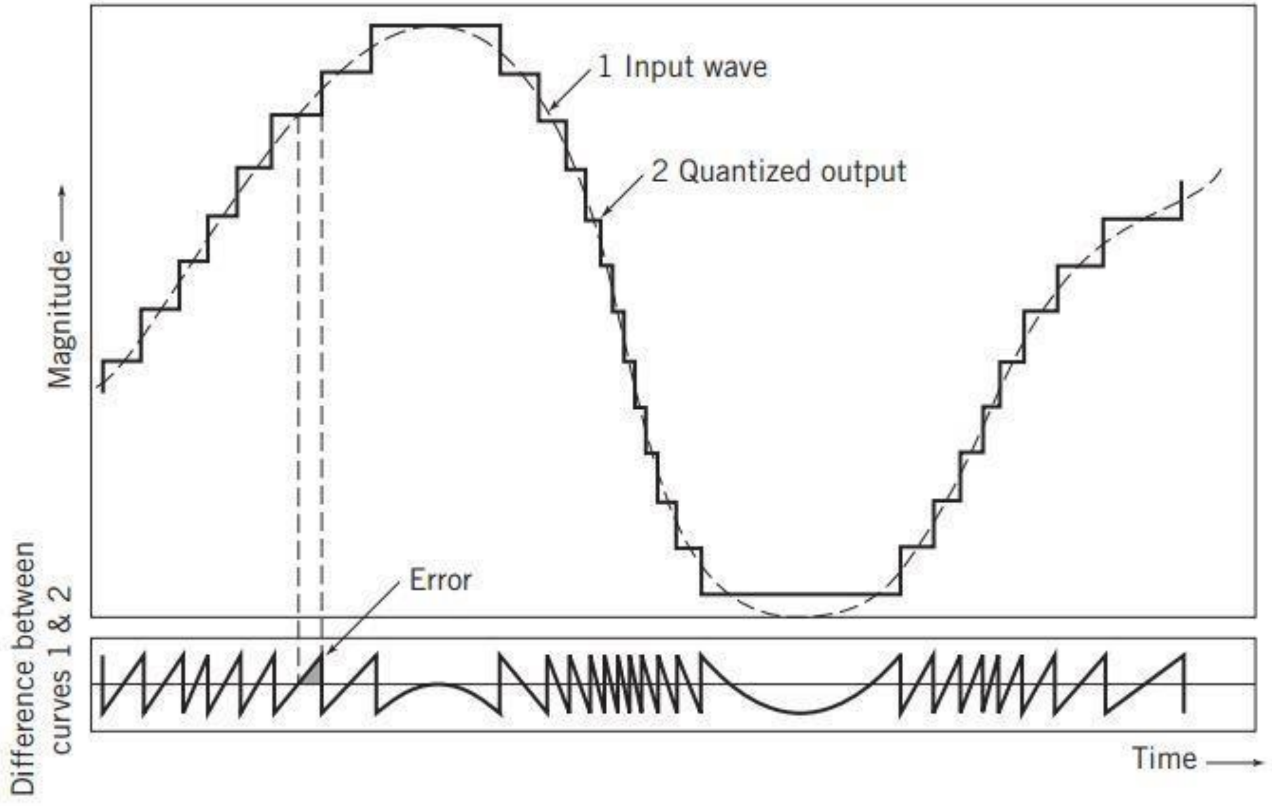
- The sampled signal is **still analog** through though discretised in time.
- Therefore it has to be passed through **Quantizer** to be discretised in amplitude.

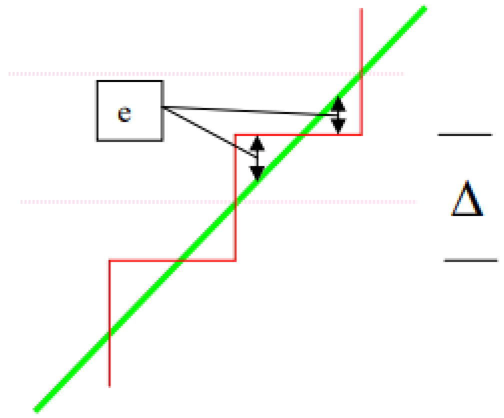
"Quantization is the process of transforming the sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $v\ (nT_s)$ taken from a finite set of possibilities"

- The definition assumes that $quantizer$ is $memoryless$ and $instantaneous$
- It means that the transformation at time $t = nT_s$ is not affected by earlier or later samples of the message signal $m(t)$

Quantization

- Amplitude quantization is shown below as the process of transforming the sample amplitude of a message signal into discrete amplitudes taken from a finite set of possible amplitudes.





Quantization Levels

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$Step\ Size\ (\Delta) = \frac{(max - min)}{L}$$

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Quantization Zones

- Assume we have a voltage signal with amplitudes

$$V_{min} = -20V \text{ } and \text{ } V_{max} = +20V$$

- We want to use $L = 8$ quantization levels.
- Zone width or Step Size:

$$Step\ Size\ (\Delta) = \frac{(20-(-20))}{8} = 5$$

- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

Quantization: Assigning Codes to Zones

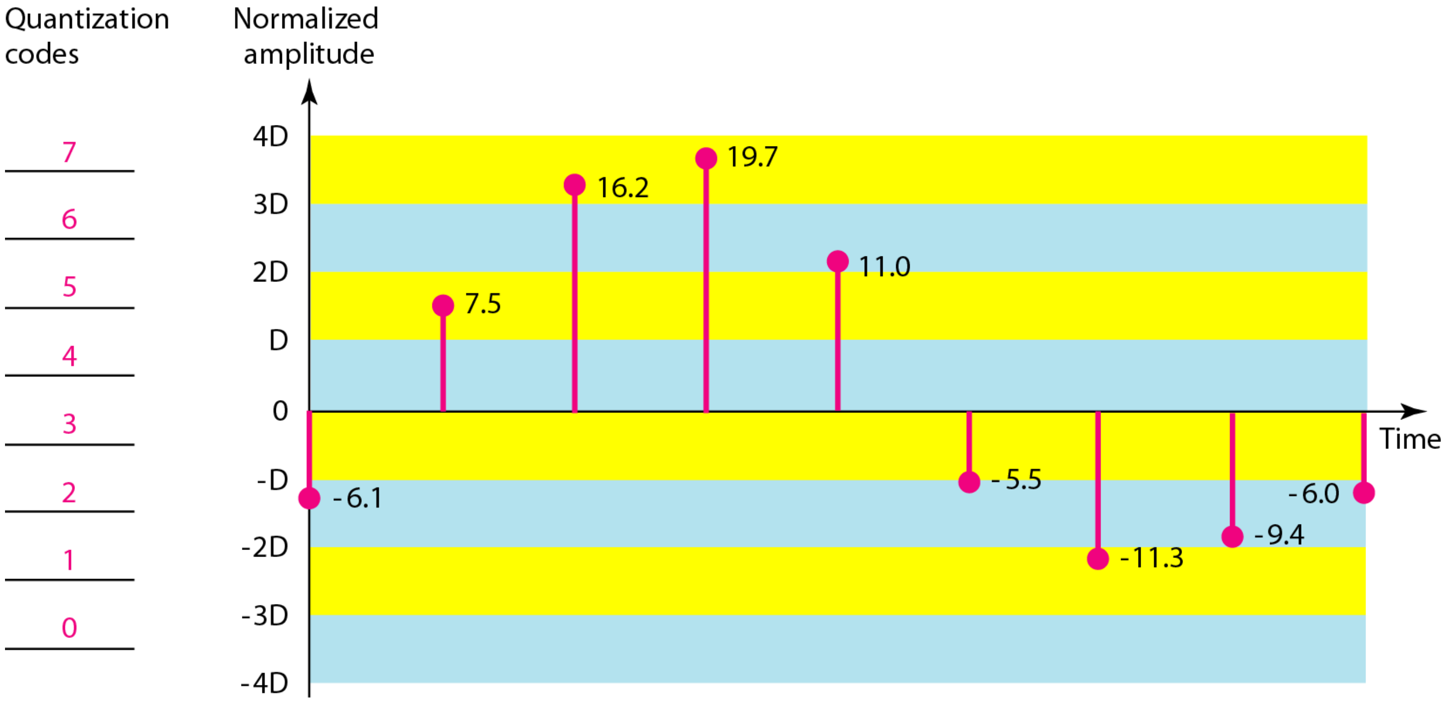
- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to,is obtained as follows:

$$n_b = log_2 L = log_2 8 = 3$$

$$n_b = 3$$

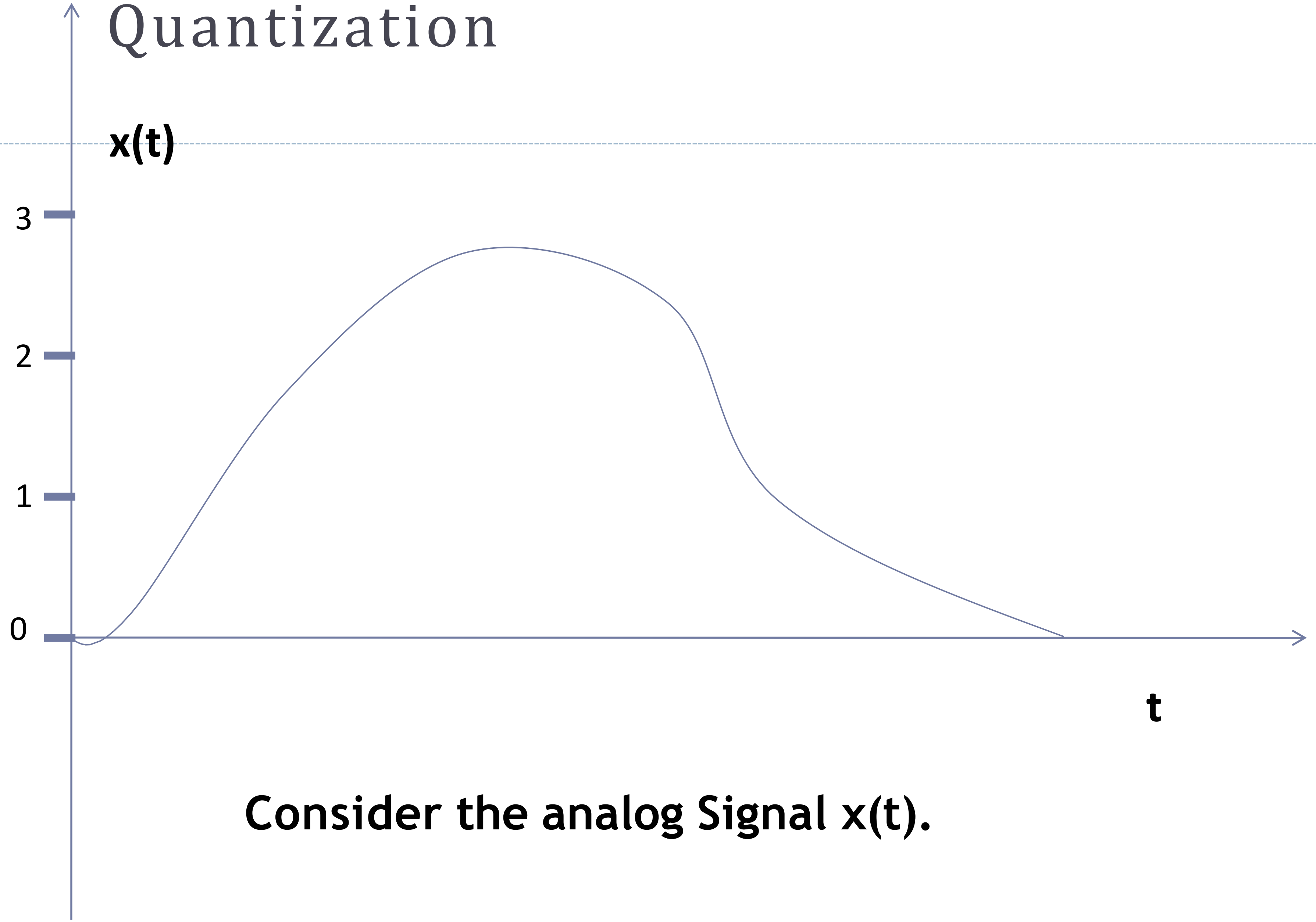
- The 8 zone (or level) codes are therefore: 000,001,010,011,100,101,110,and 111
- Assigning codes to zones:
- 000 will refer to zone -20 to -15
- 001 to zone -15 to -10,etc.

Quantization and encoding of a sampled signal



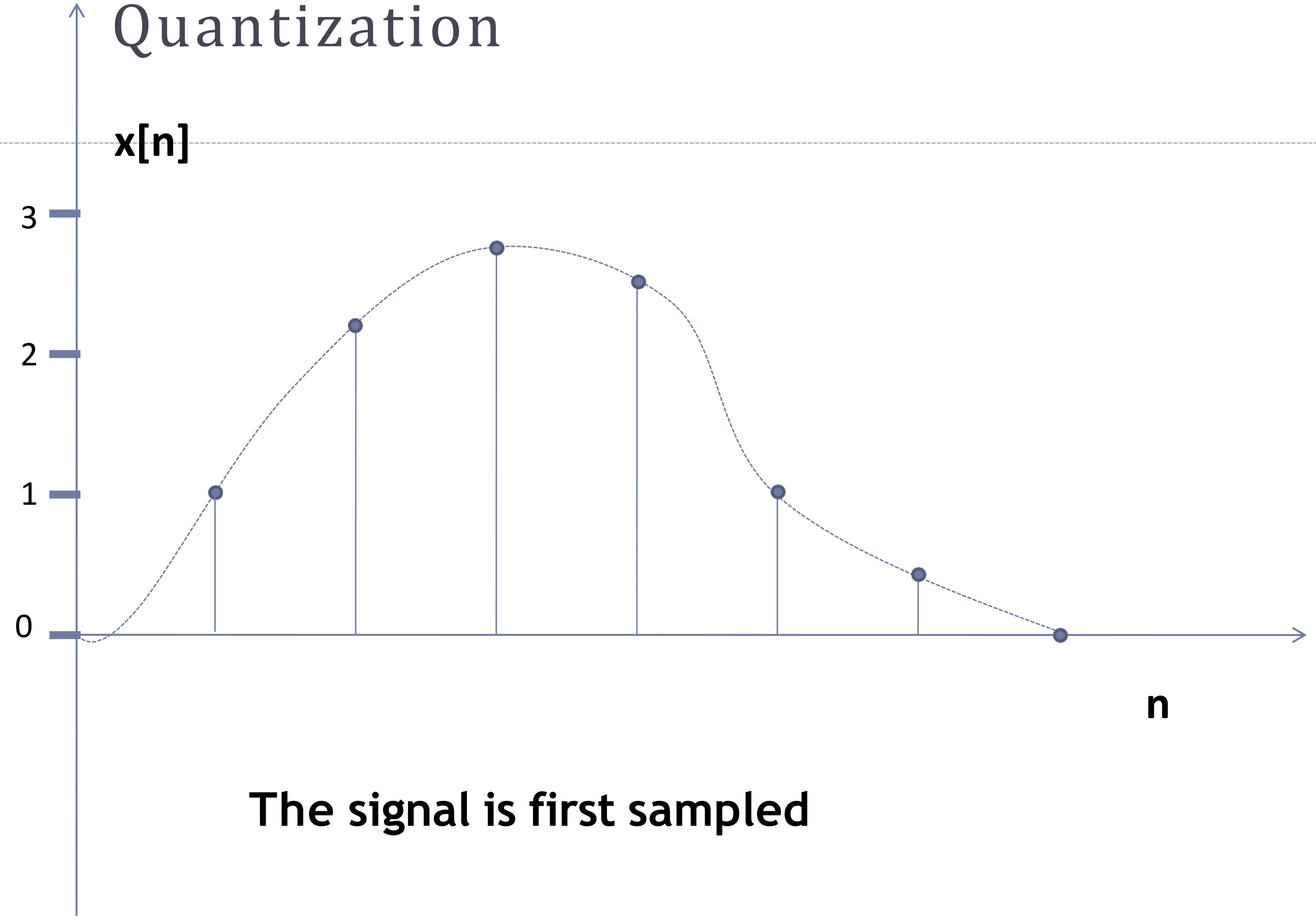
Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Quantization



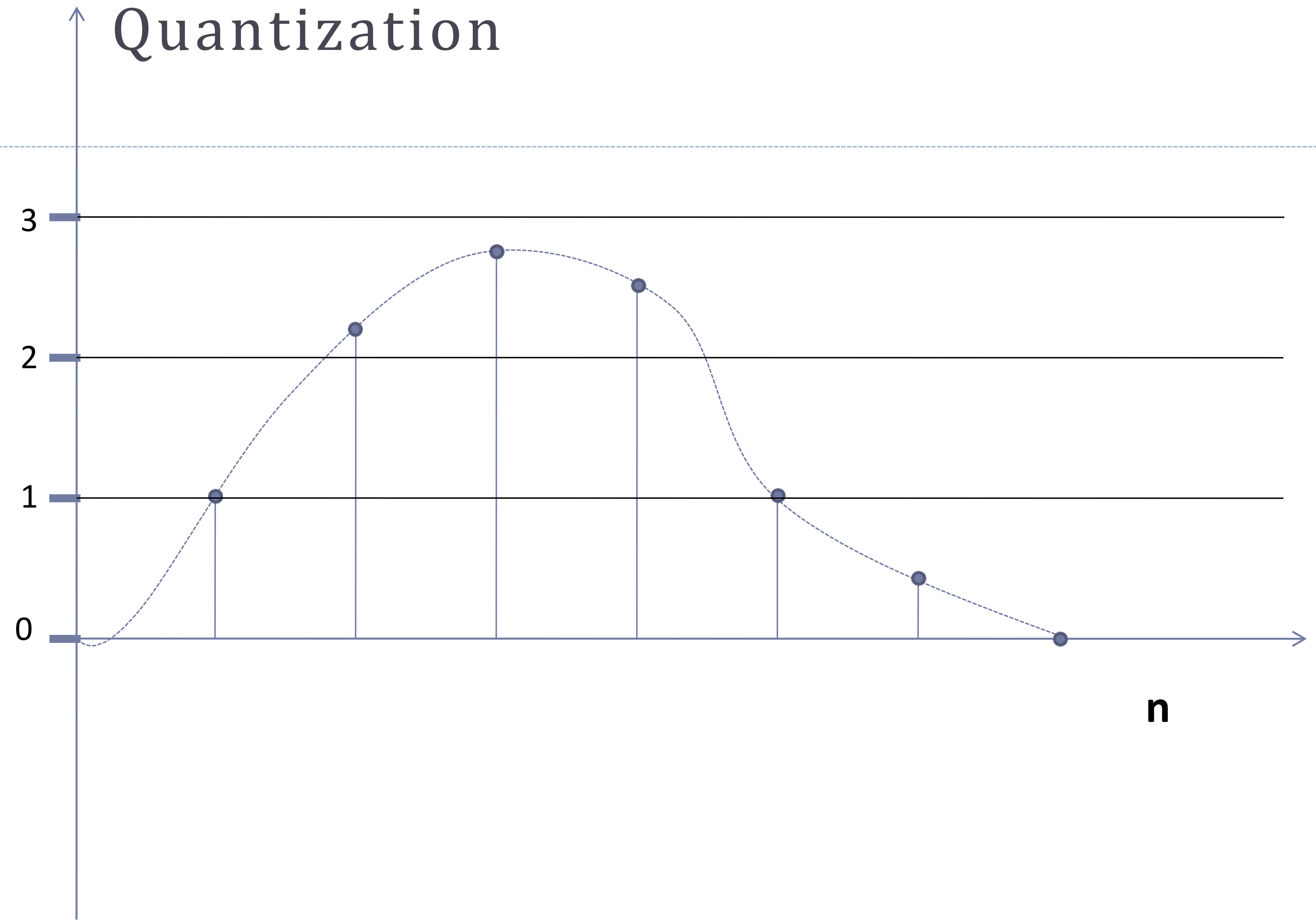
Consider the analog Signal x(t).

Quantization

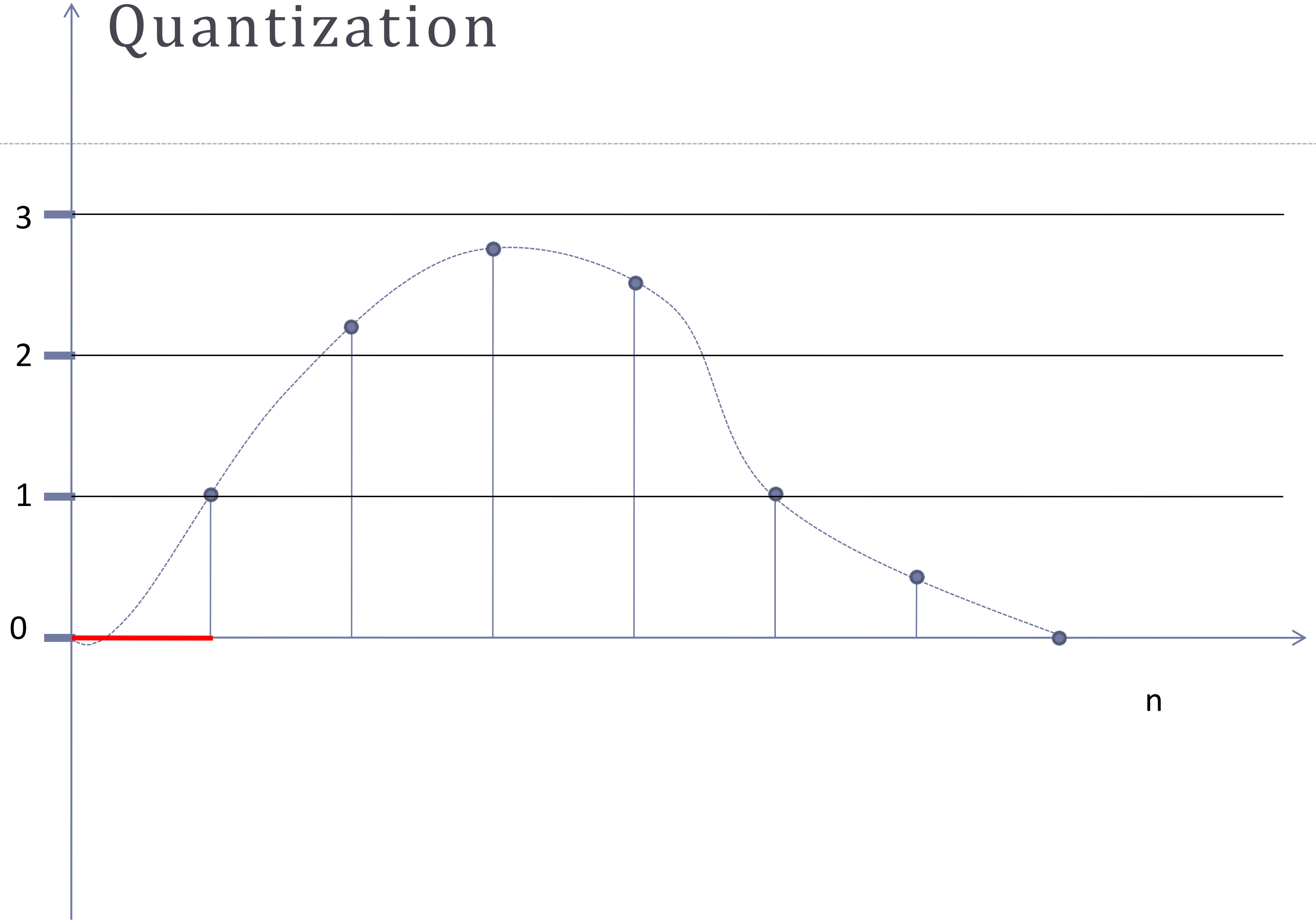


The signal is first sampled

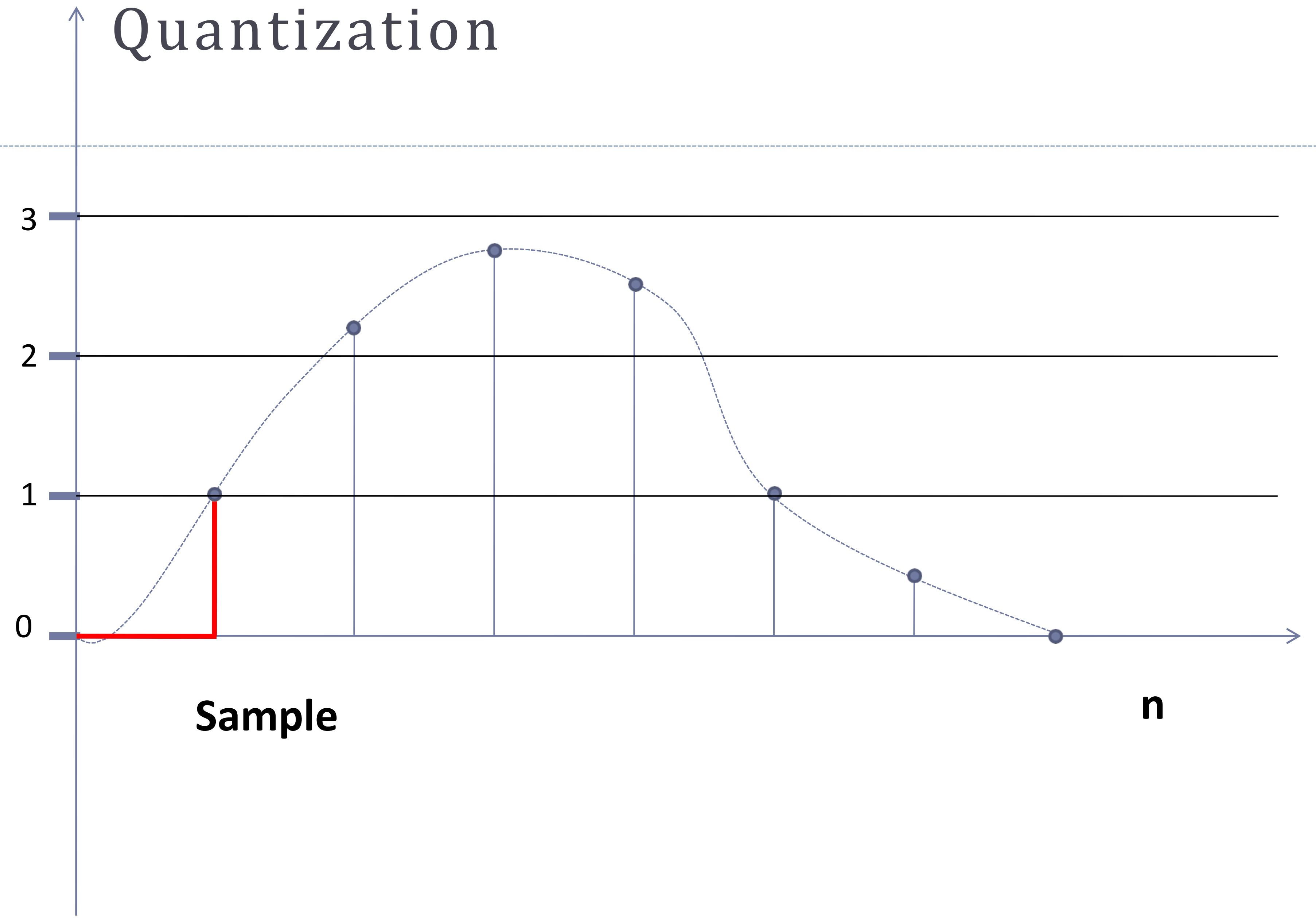
Quantization



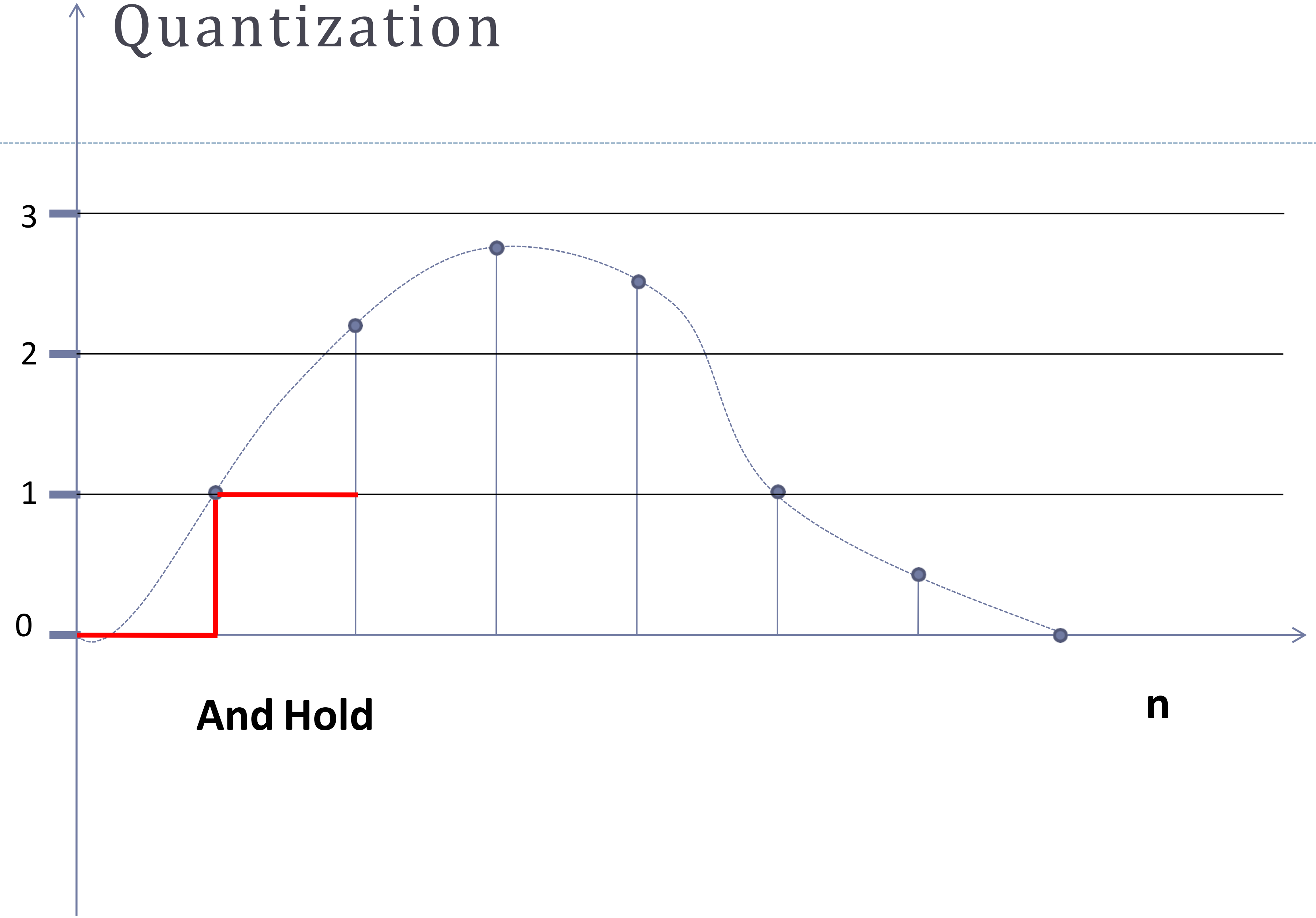
Quantization



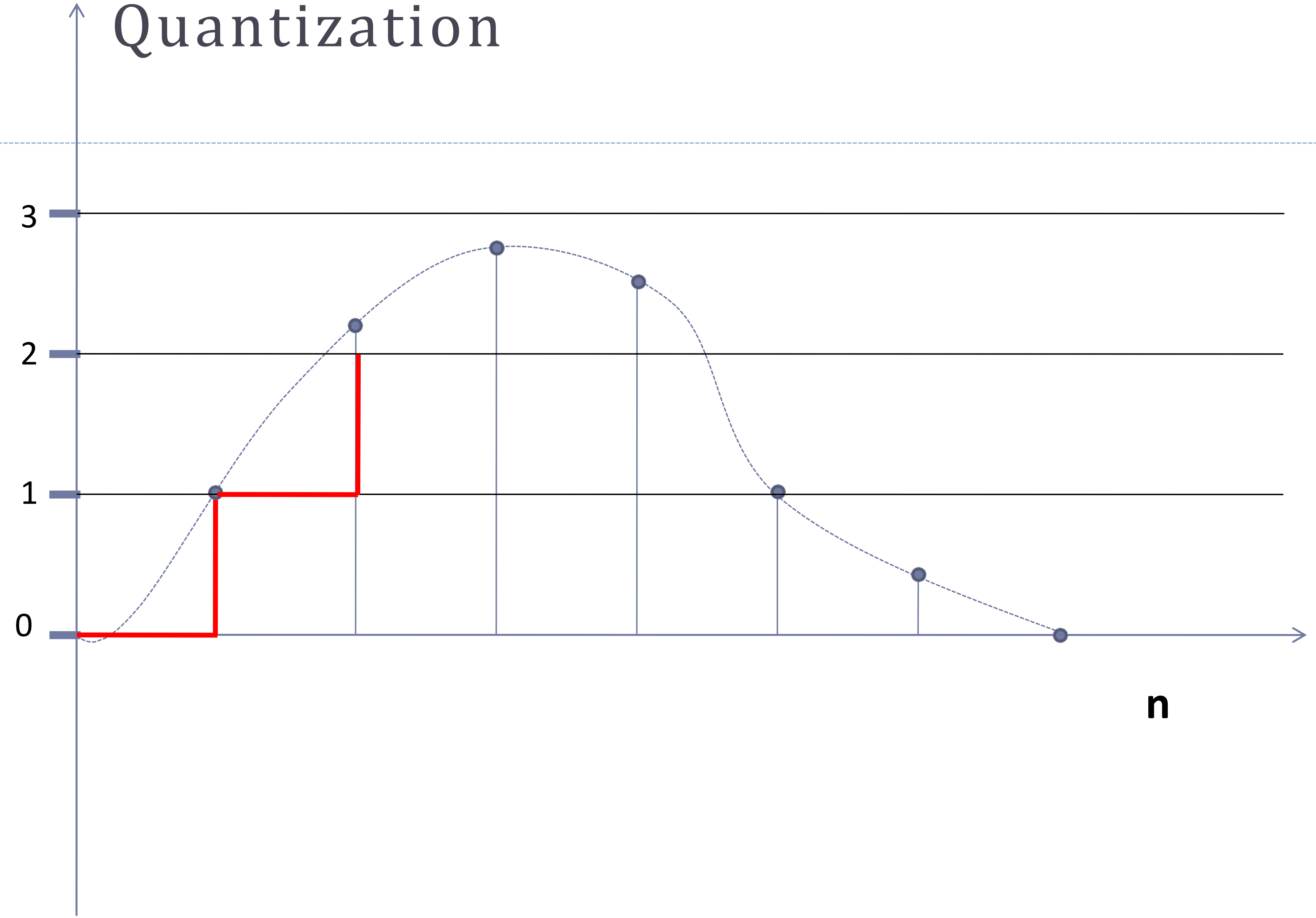
Quantization



Quantization



Quantization



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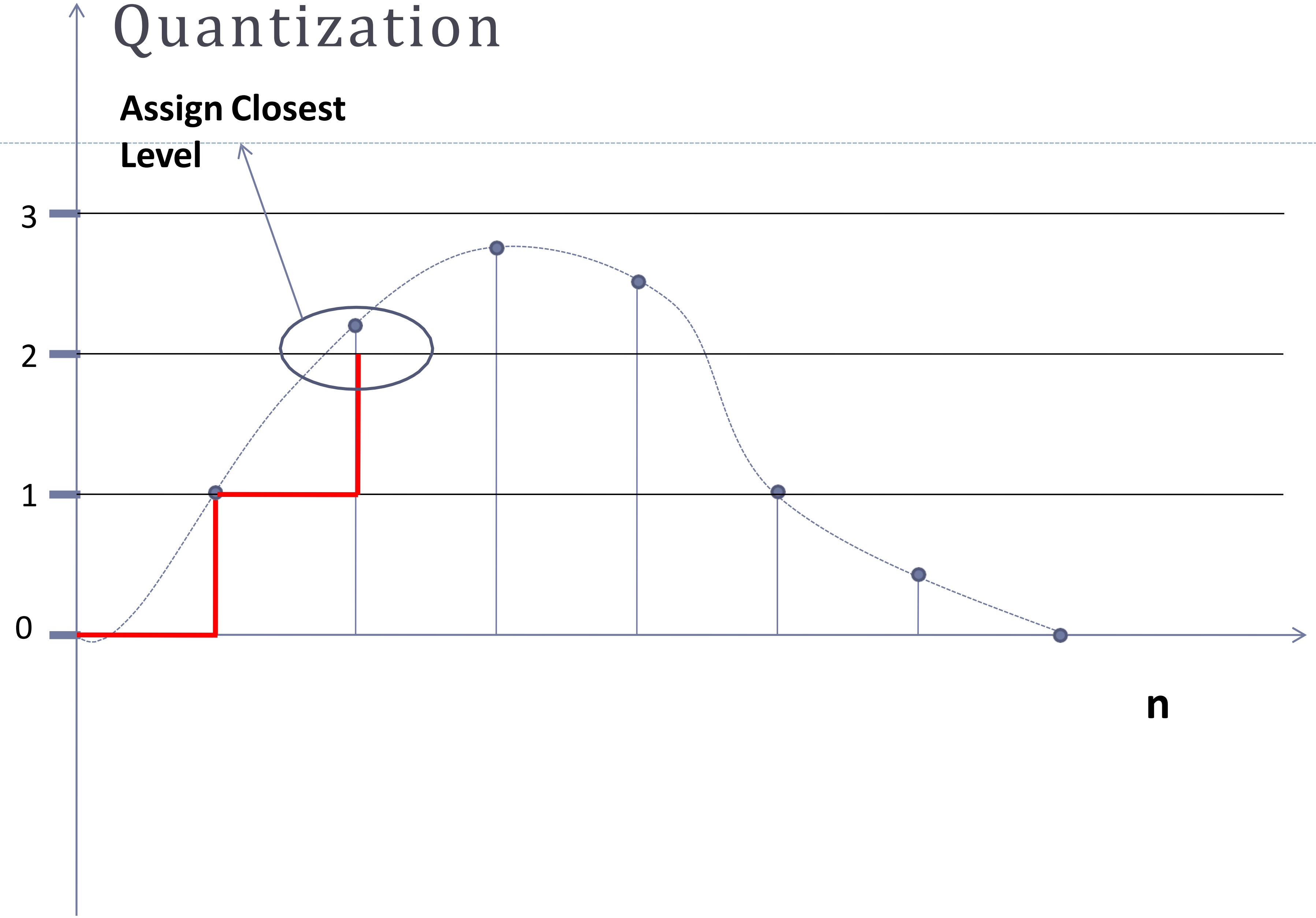

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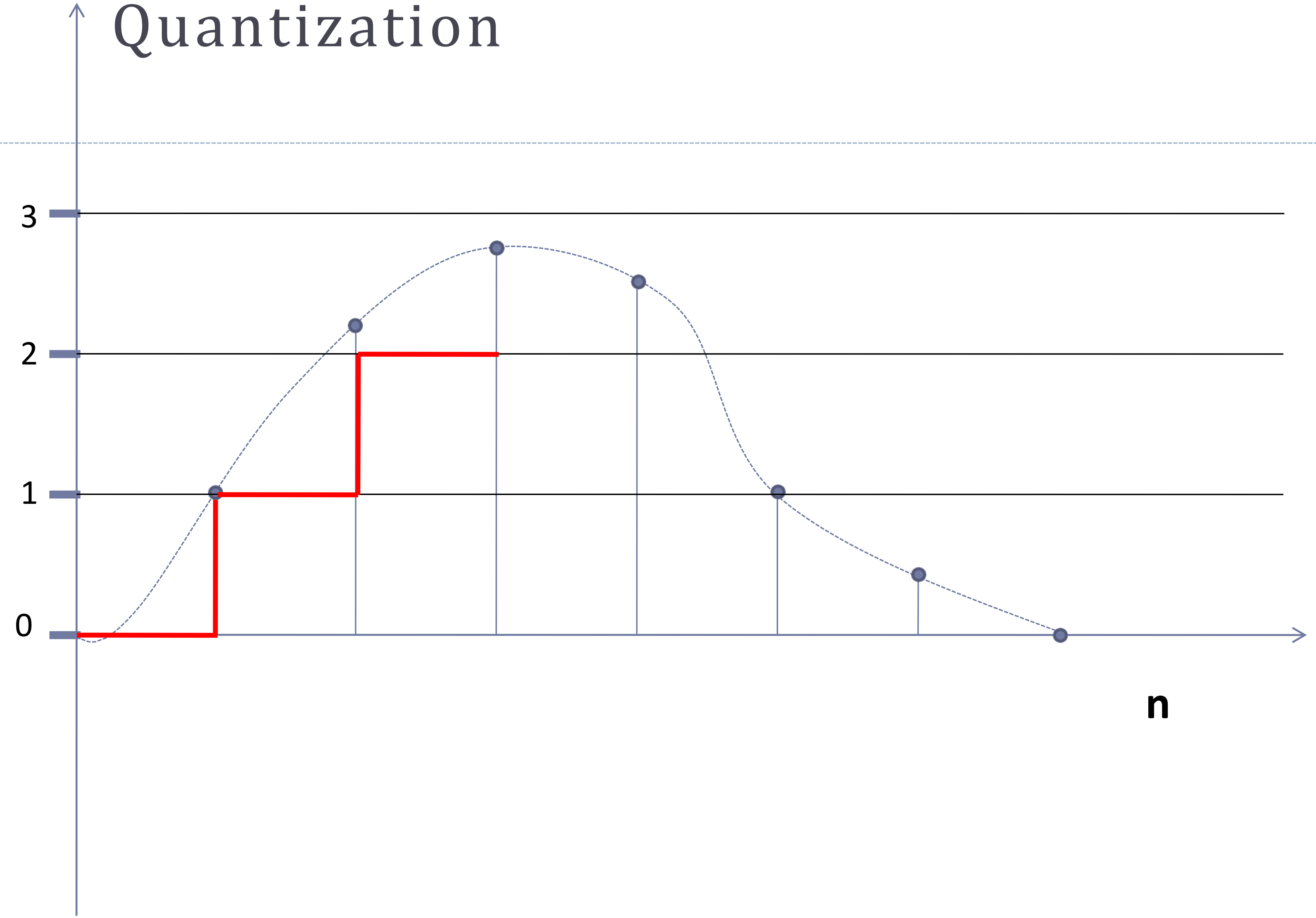
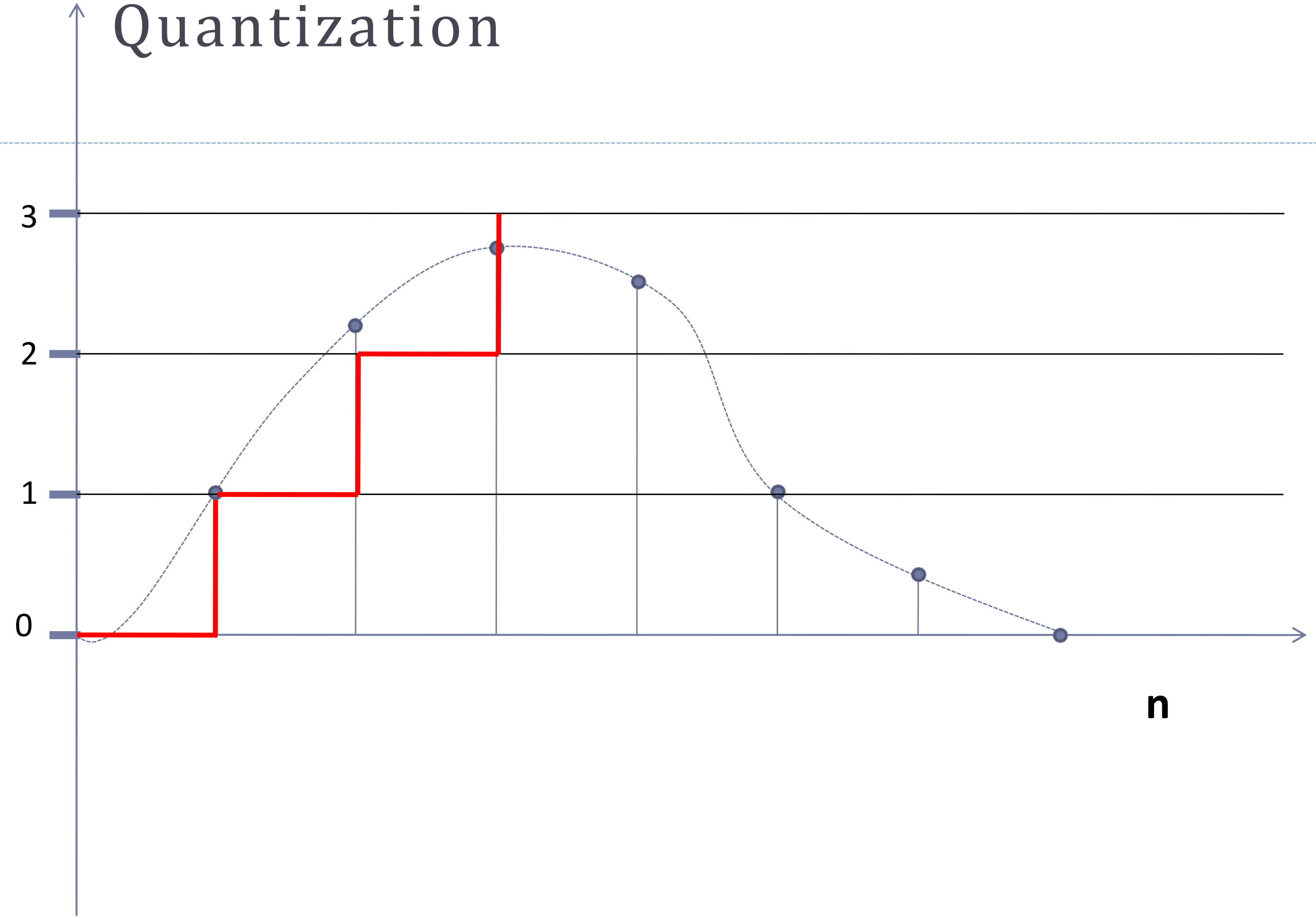
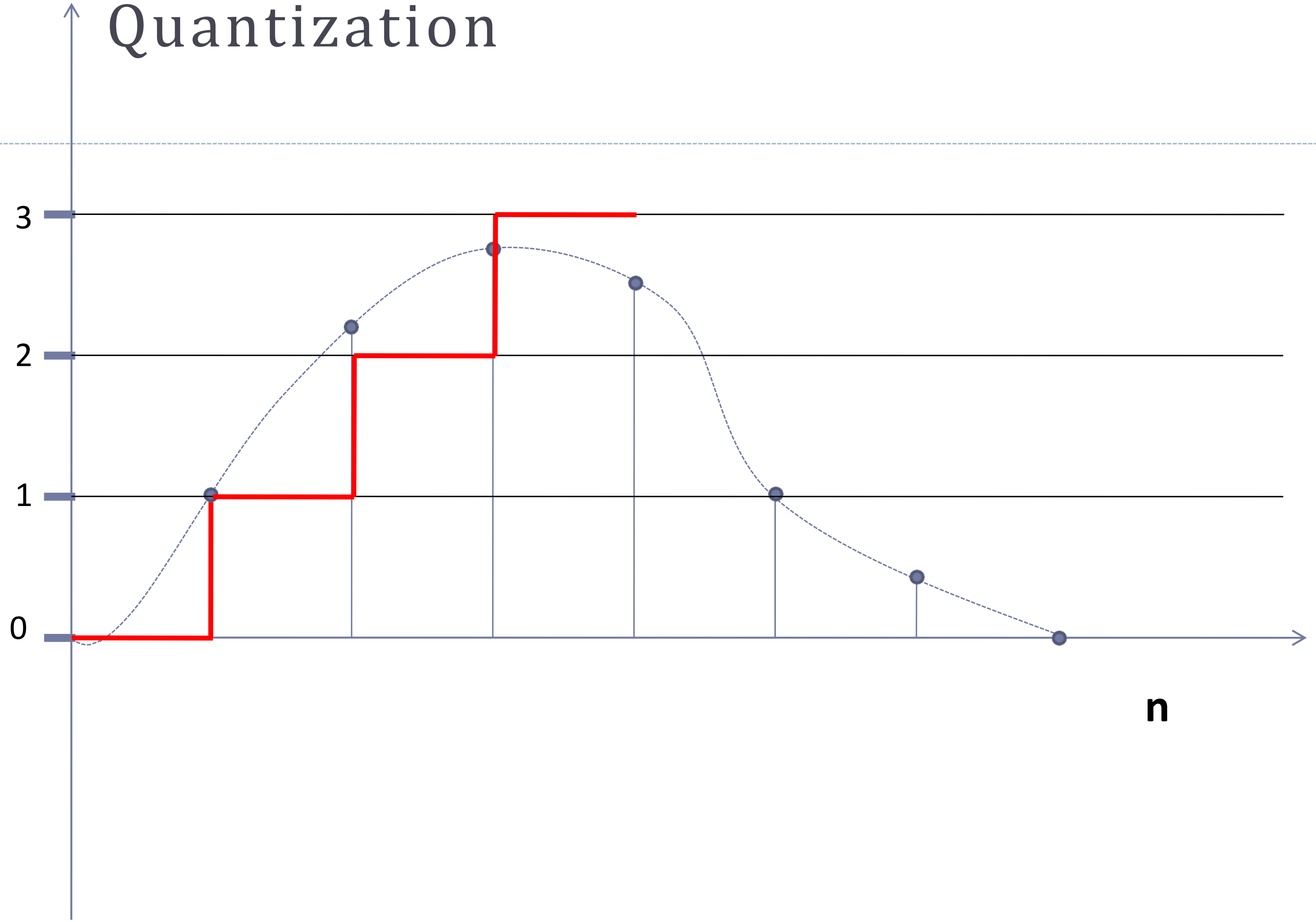


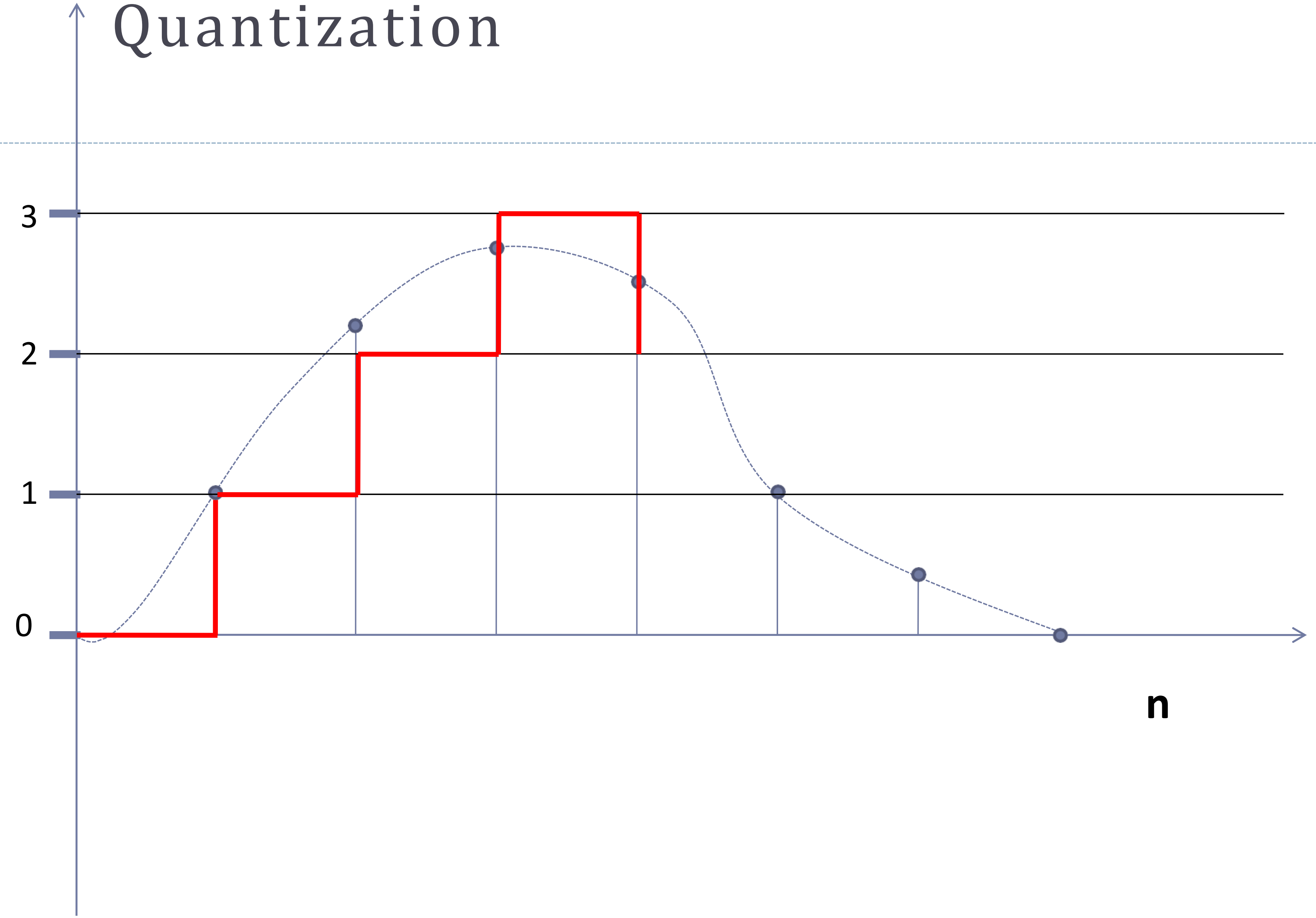
Quantization



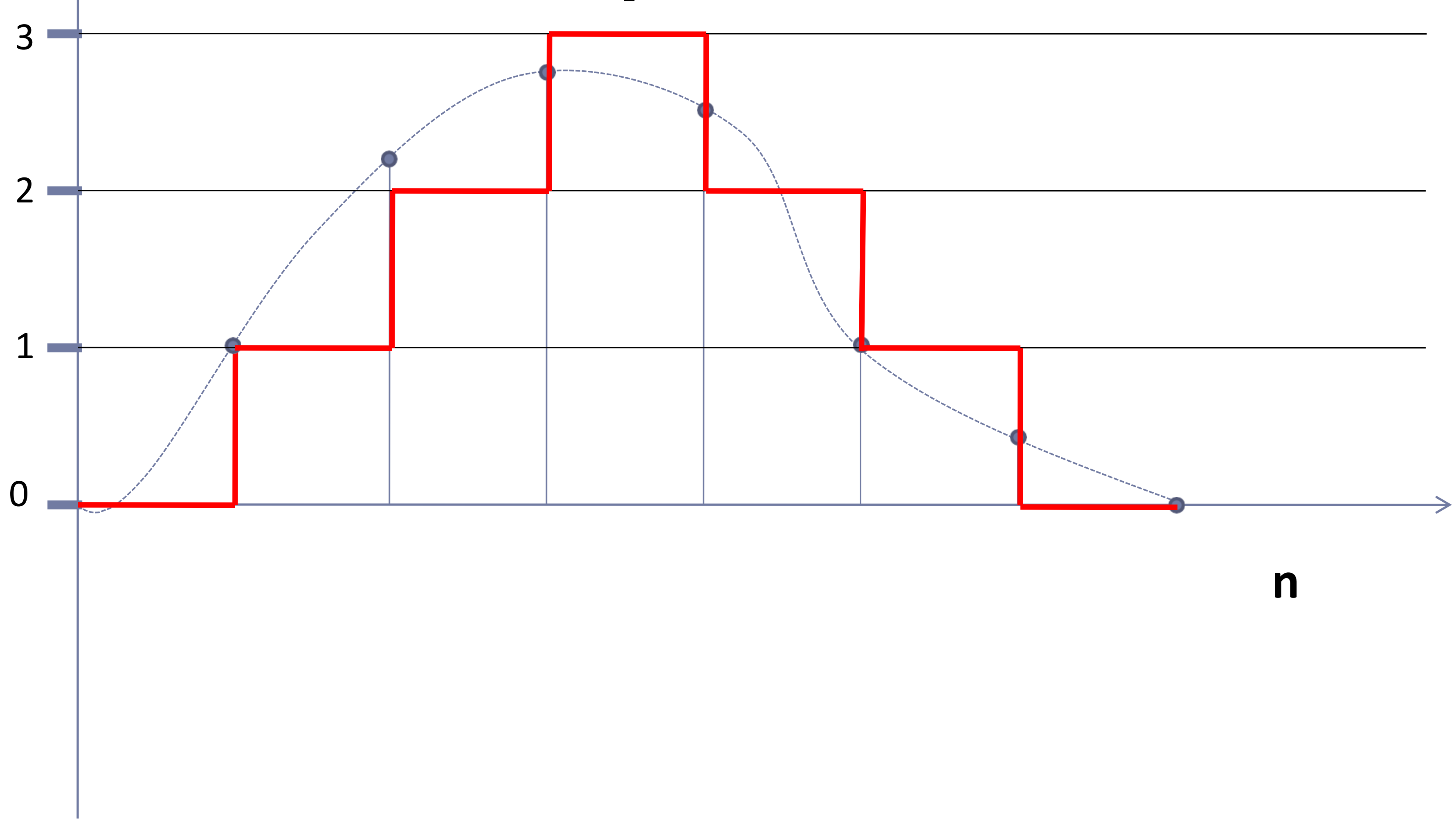
Quantization



Quantization



Each quantization level corresponds to a unique combination of bits. The analog signal is transmitted/ stored as a stream of bits and reconstructed when required.



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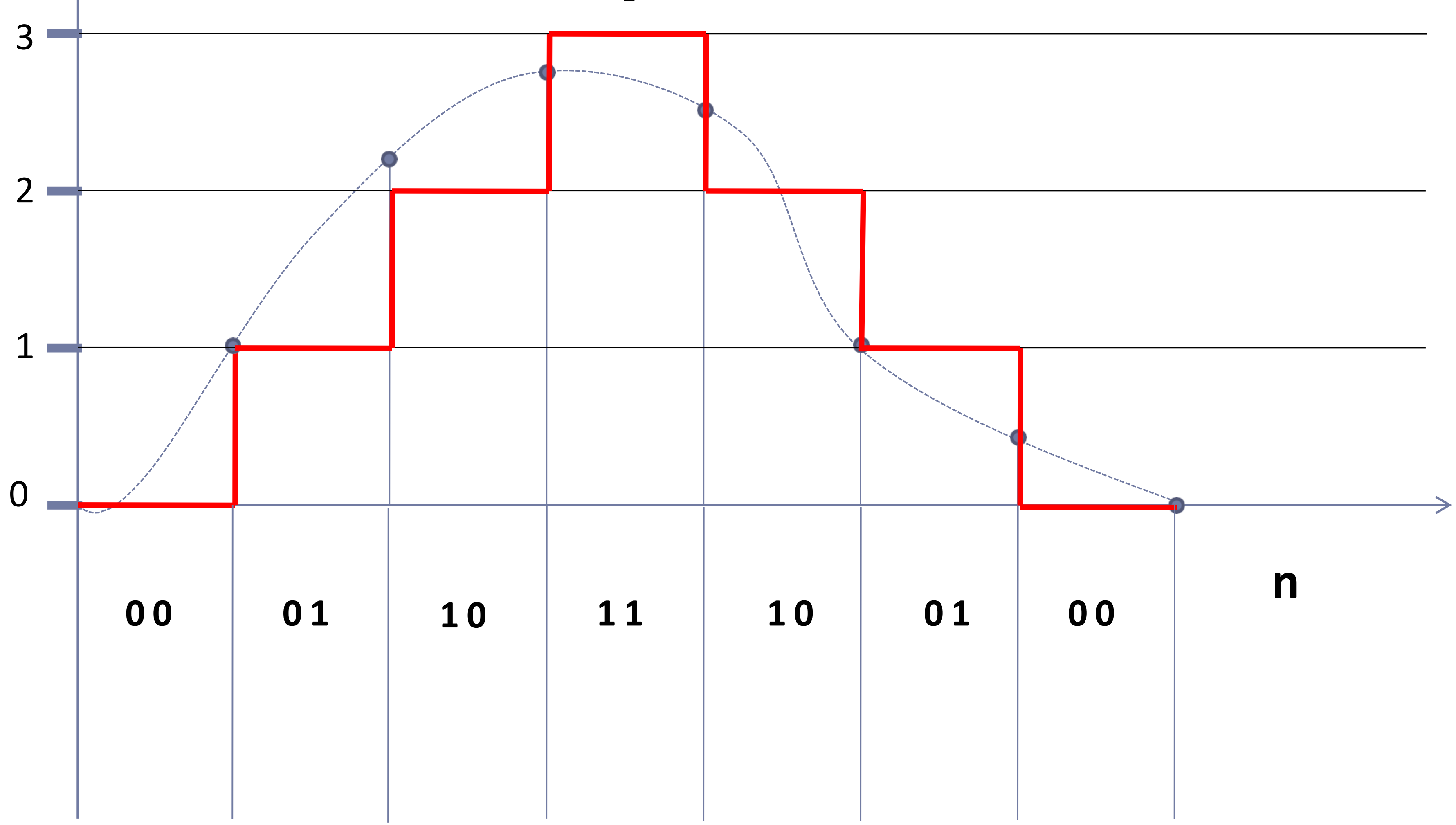
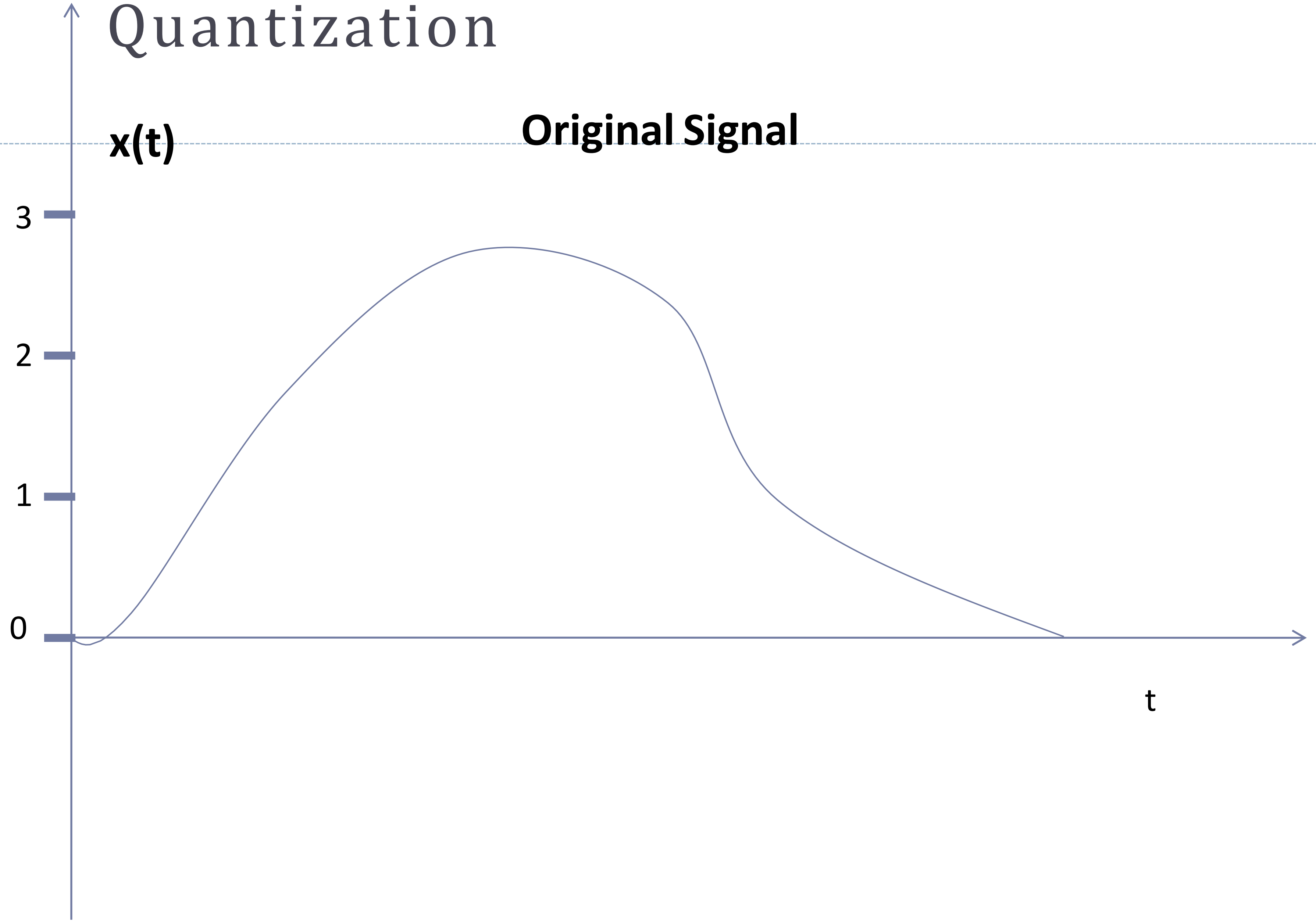
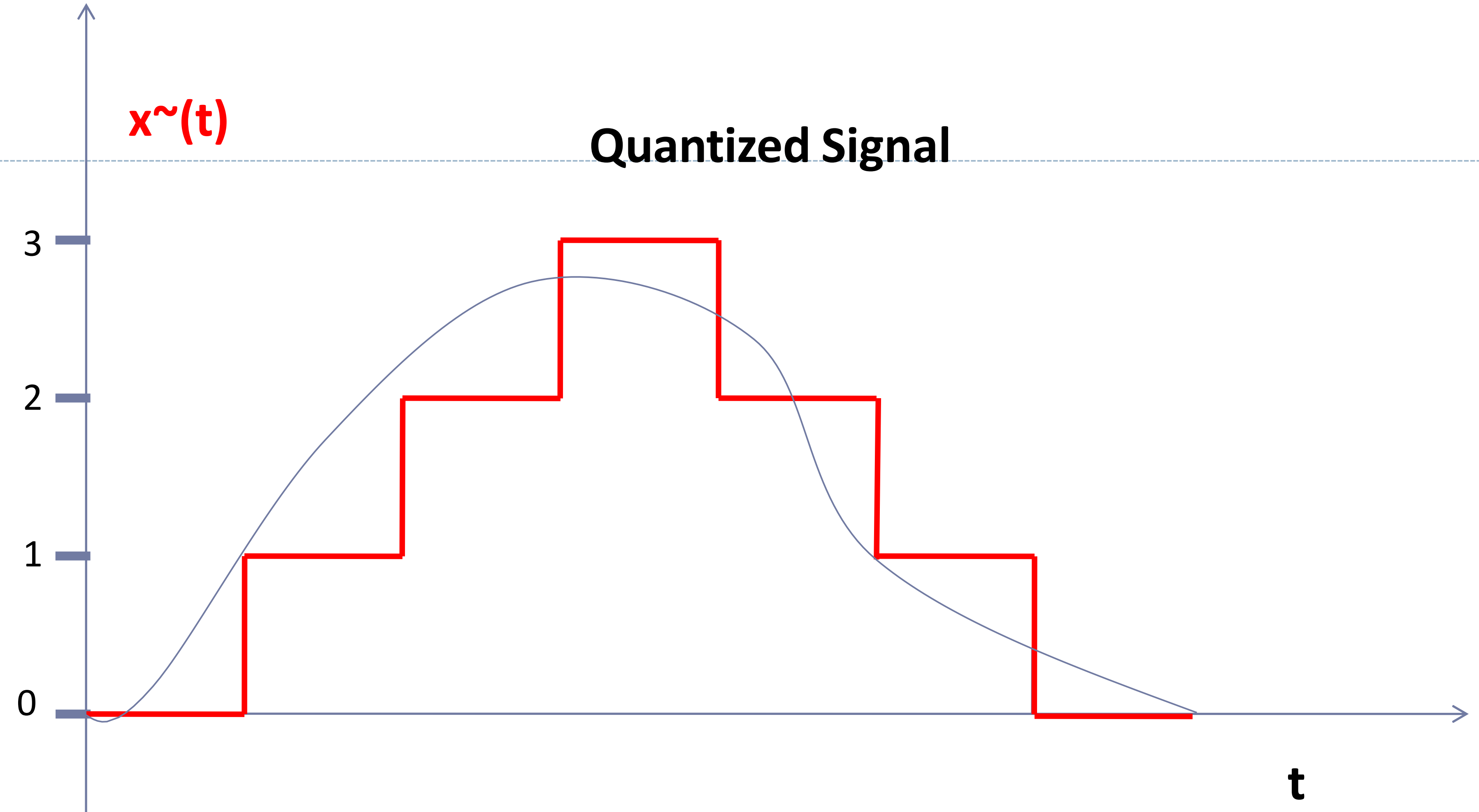


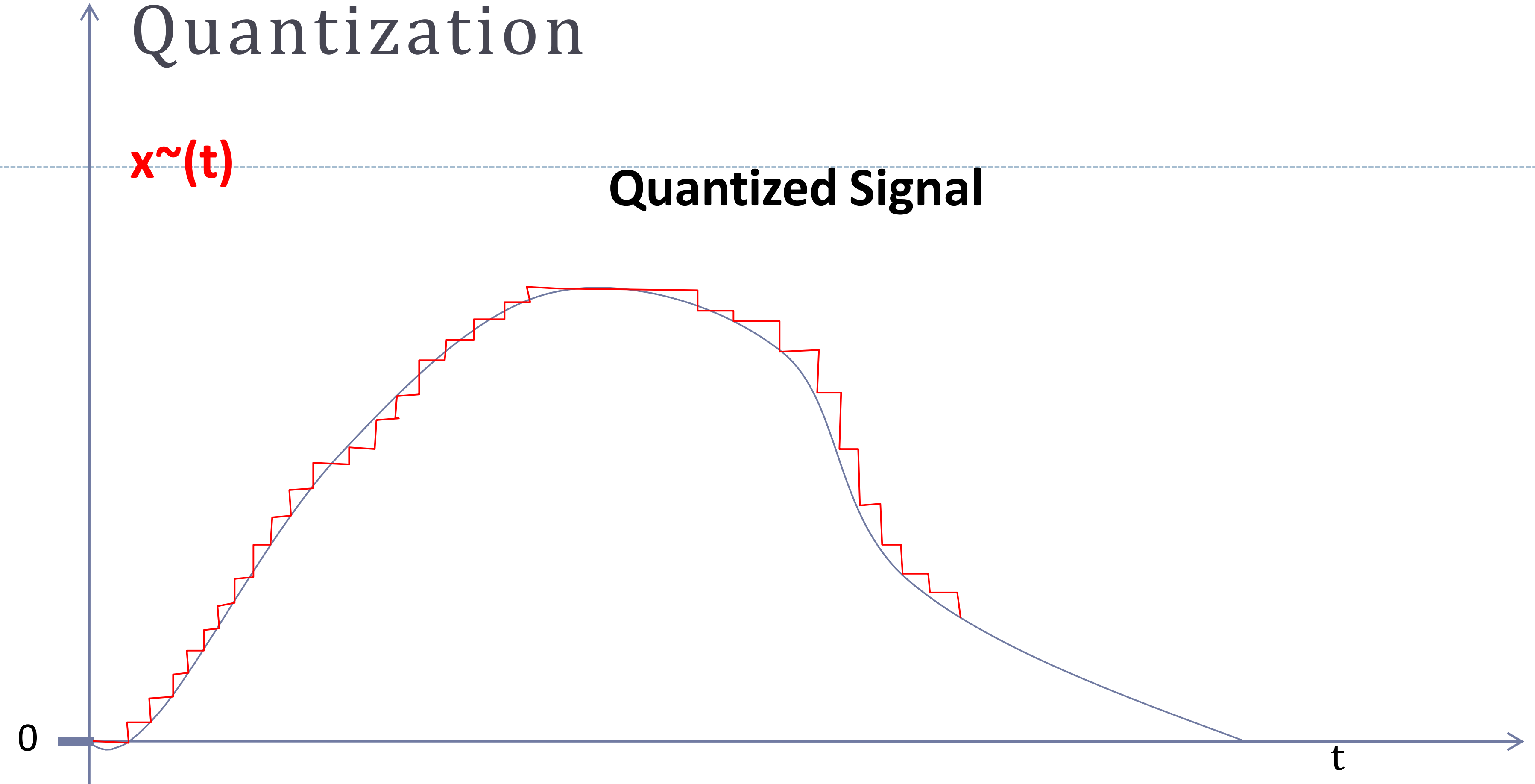
Quantization





It is quite apparent that the quantized signal is not exactly the same as the original analog signal. There is a fair degree of quantization error here. However; as the number of quantization levels is increased the quantization error is reduced and the quantized signal gets closer and closer to the original signal

Quantization



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Types of Quantization

There are two types of quantization:

Uniform Quantization:

- Step or Difference between two quantization levels remain constant over the complete amplitude range.
- So the maximum quantization error also remains same which causes problems at some amplitude levels

Non-Uniform Quantization:

- Step size or Difference between two quantization levels are different
- And mostly the relation between them is logarithmic

Quantizer Characteristics

- There are two types Quantizer Characteristics for uniform quantization.
- They are Mid-Rise type and Mid-Tread type.

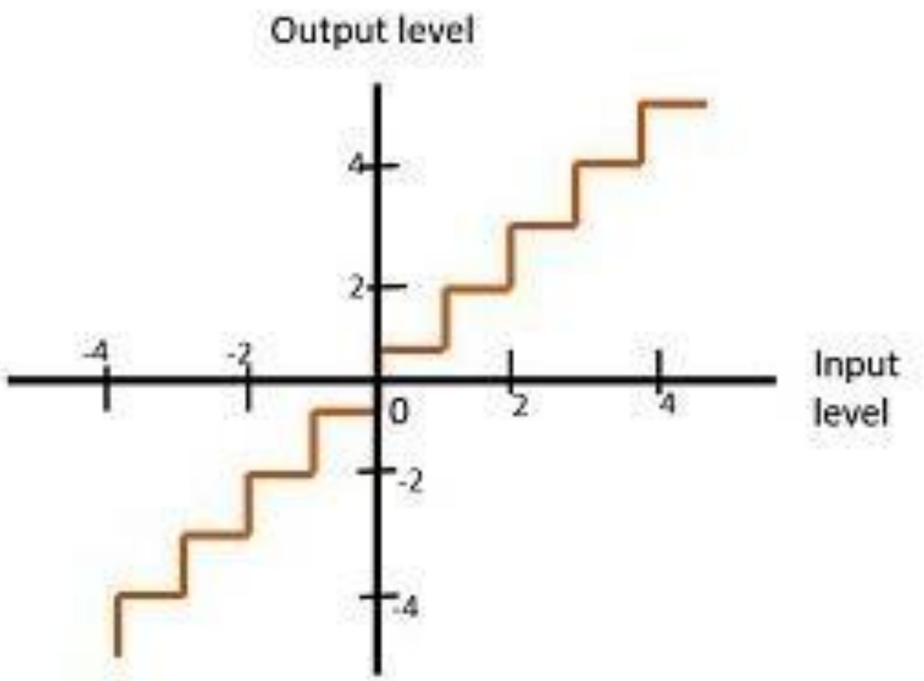


Fig 1 : Mid-Rise type Uniform Quantization

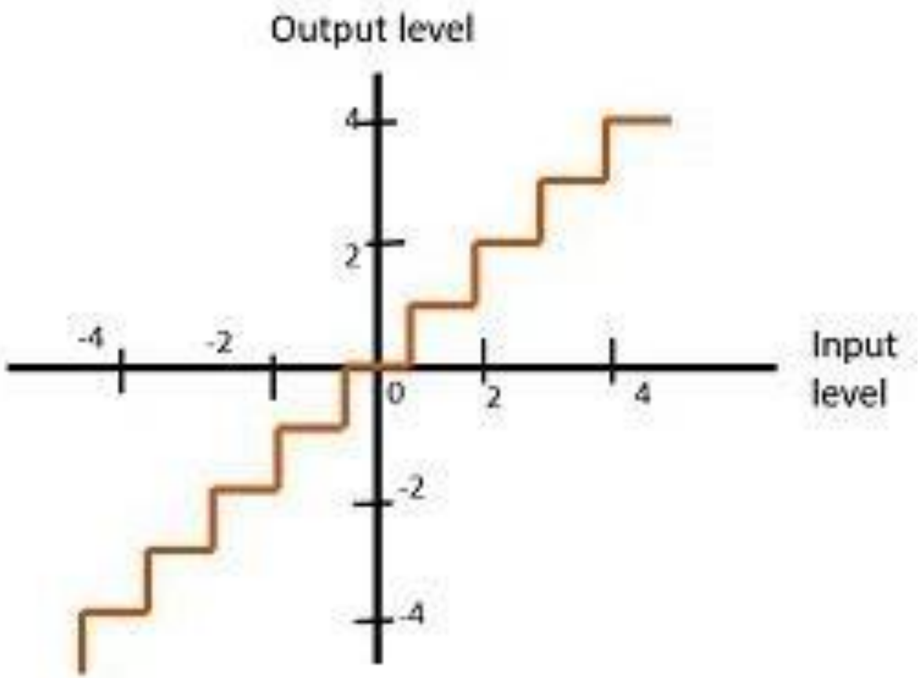


Fig 2 : Mid-Tread type Uniform Quantization

Quantizer Characteristics

- The **Mid-Rise** type is so called because the origin lies in the middle of a raising part of the stair-case like graph.
- The quantization levels in this type are even in number.

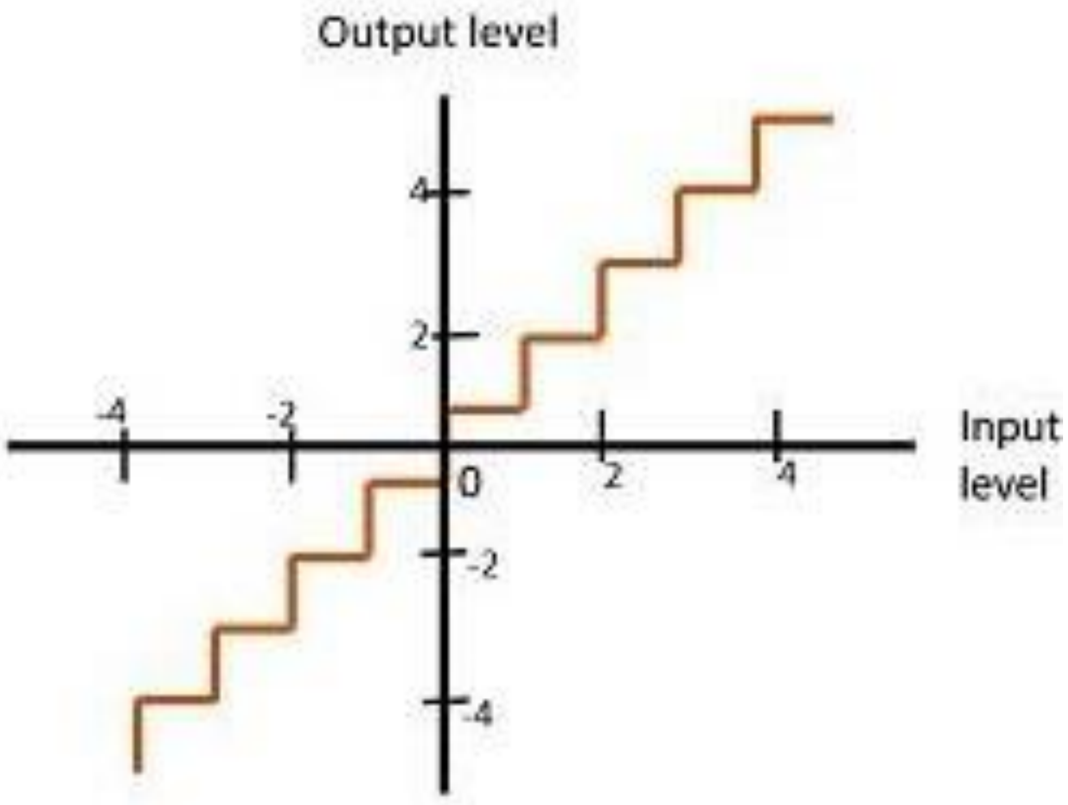


Fig 1 : Mid-Rise type Uniform Quantization

Quantizer Characteristics

- The **Mid-tread** type is so called because the origin lies in the middle of a tread of the stair-case like graph.
- The quantization levels in this type are odd in number.

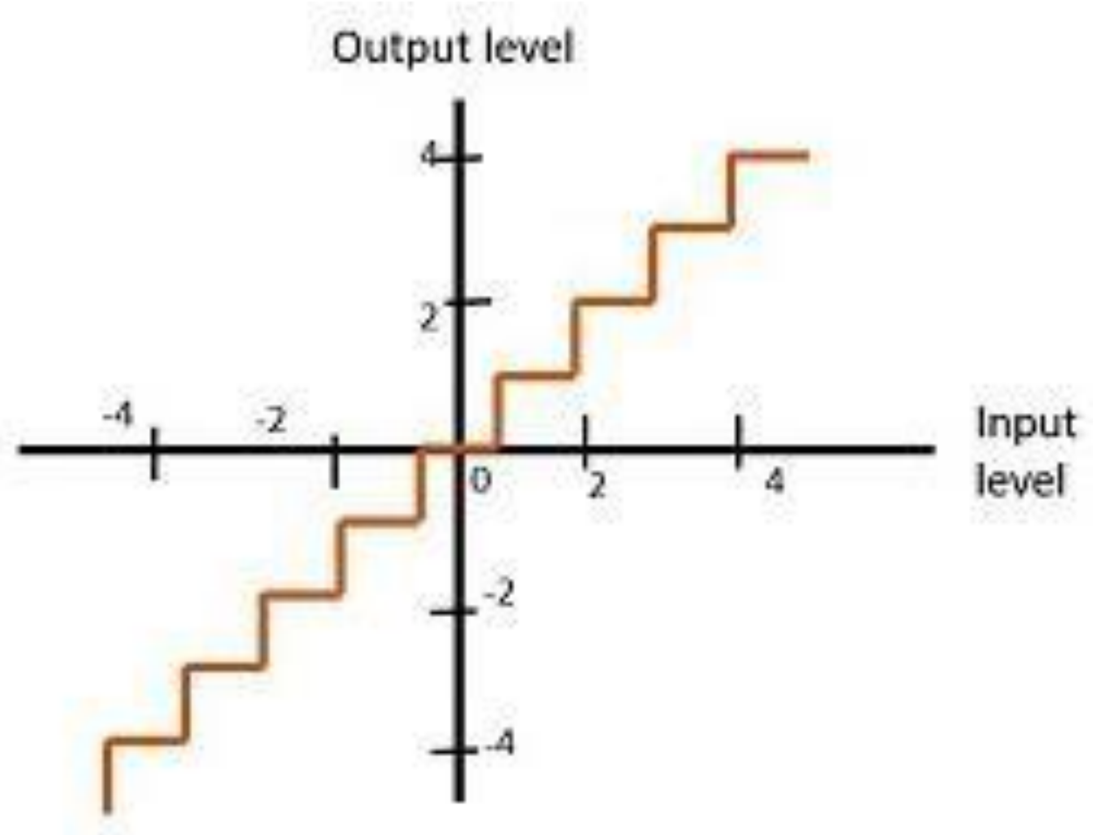


Fig 2 : Mid-Tread type Uniform Quantization

Quantization Error

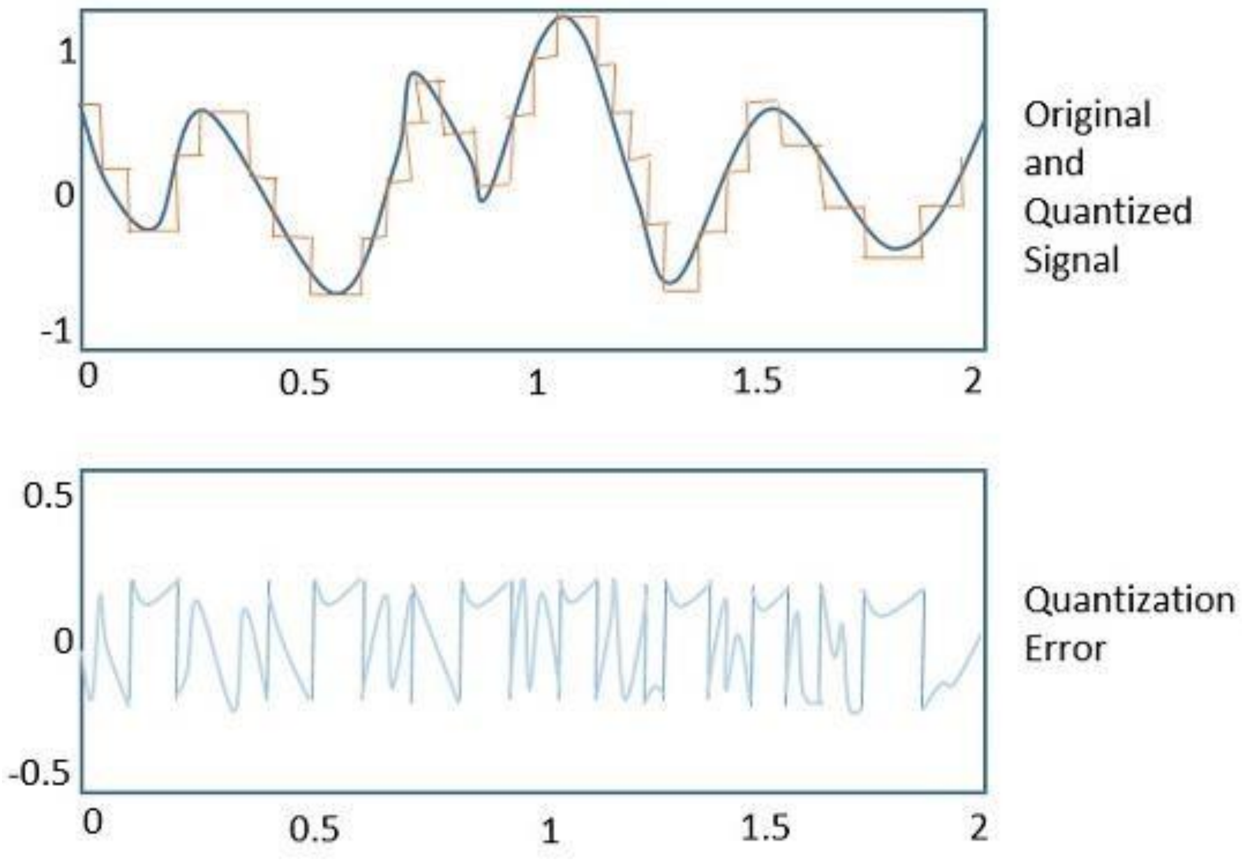
- The difference between an input value and its quantized value is called a **Quantization Error**.
- A **Quantizer** is a logarithmic function that performs Quantization.
- An analog-to-digital converter (**ADC**) works as a quantizer

Quantization Noise

- It is a type of quantization error, which usually occurs in analog audio signal, while quantizing it to digital.
- For example, in music, the signals keep changing continuously, where a regularity is not found in errors.
- Such errors create a wideband noise called as **Quantization Noise**.

Quantization Error

- The following figure illustrates an example for a quantization error, indicating the difference between the original signal and the quantized signal.



Quantization Error

- For an input m of continuous amplitude which symmetrically occupies the range $[-m_{max}, m_{max}]$ and assuming a uniform quantizer of midrise type, the *step size* will be

$$\Delta = \frac{2m_{max}}{L}$$

Here, L is the total number of levels

- The quantization error Q will have its sample value bounded by $-\frac{\Delta}{2} \leq q < \frac{\Delta}{2}$
- The probability density function of a quantization noise is given by

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} \leq q < \frac{\Delta}{2} \\ 0, & otherwise \end{cases}$$

Quantization Error

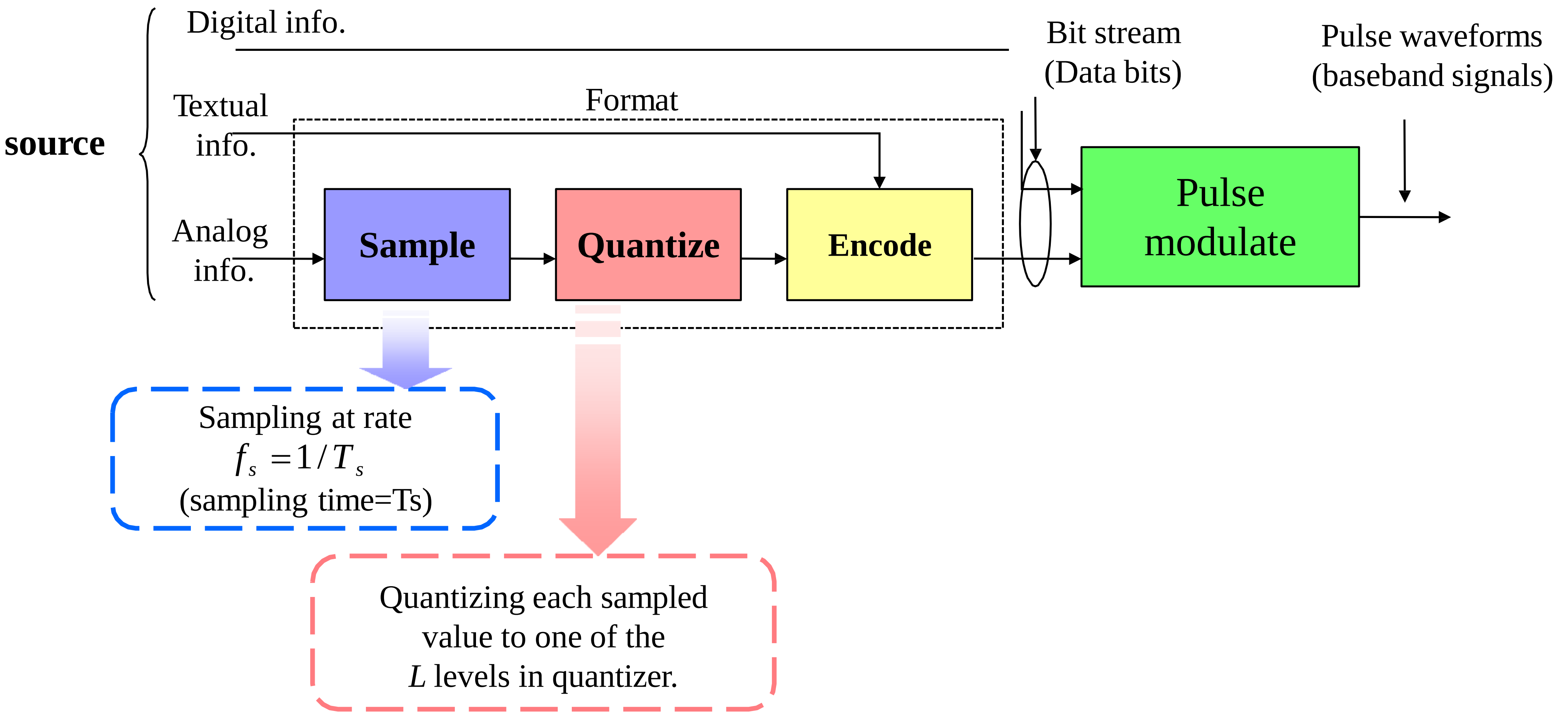
- The mean of the quantization noise is **zero**
- The variance will be: $\boldsymbol{\sigma_Q^2 = E[Q^2] = \frac{\Delta^2}{12}}$
- If R denotes the *number of bits per sample*, then $L = 2^R$ or $R = \log_2 L$
- On further substituting the above equations, we also get

$$\Delta = \frac{2m_{max}}{2^R} \text{ and } \sigma_Q^2 = \frac{1}{3} m_{max}^2 2^{-2R}$$

- If P denote the average power of the original message $m(t)$, then,

$$\boldsymbol{SQNR = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{max}^2}\right) 2^{2R}}$$

Analog to Digital Conversion Blocks in DCS



Summary

- Model of digital communication system and bandwidth of signals
- Sampling
- Types of Sampling
- Quantization
- Types and Characteristics of Quantization
- Quantization error and Quantization noise
- Reconstruction of a message from its samples