Delta Modulation (DM)

Delta Modulation (DM)

- In PCM, the signaling rate and transmission channel bandwidth are quite large since it transmits all the bits which are used to code a sample.
- To overcome this problem, **Delta modulation** is used.
- In DM only one bit is used to represent per sample.
- In DM the message signal is oversampled purposely to increase the correlation between adjacent samples.
- The DM provides a staircase approximation to the message signal.

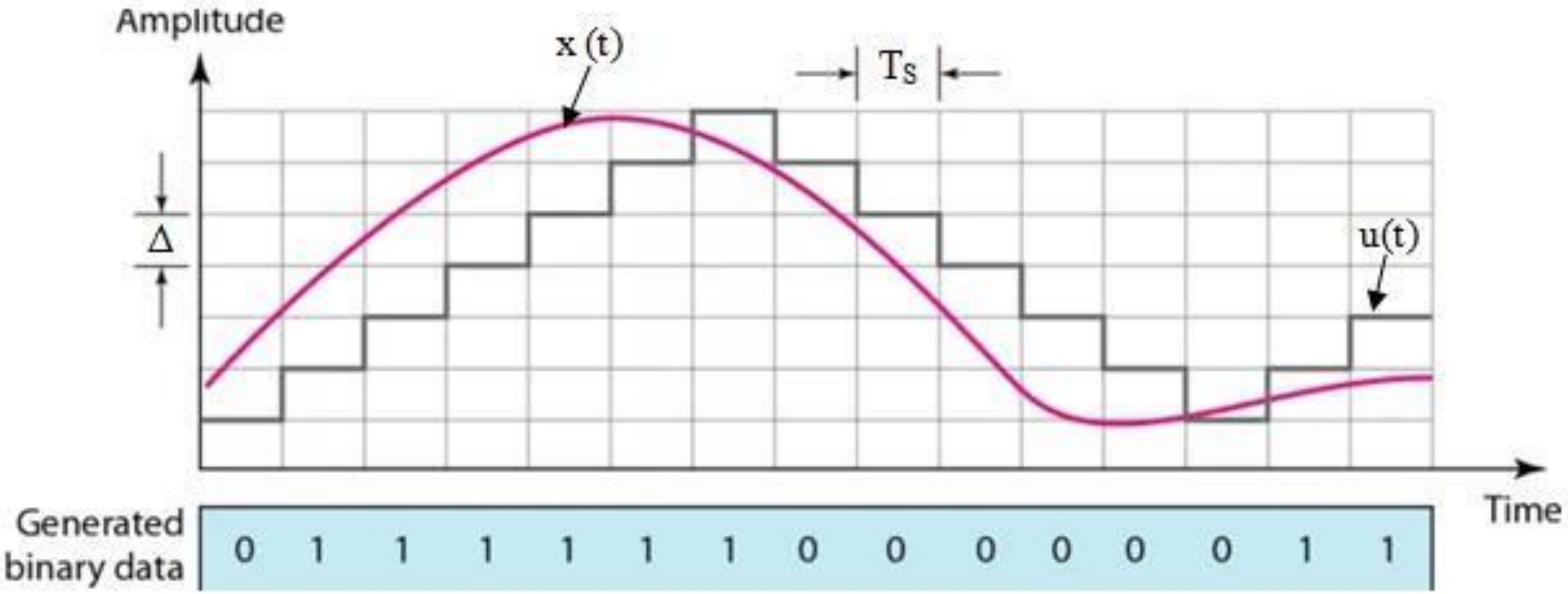
Delta Modulation (DM)

Working Principle

- Delta modulation transmits only one bit per sample.
- Here, the **present sample value is compared with the previous sample** value and this result whether the amplitude is increased or decreased is transmitted.
- Input signal **x(t)** is approximated to step signal by the delta modulator.
- This step size is kept fixed.
- The difference between the input signal x(t) and staircase approximated signal is confined to two levels, i.e., +Δ and -Δ.

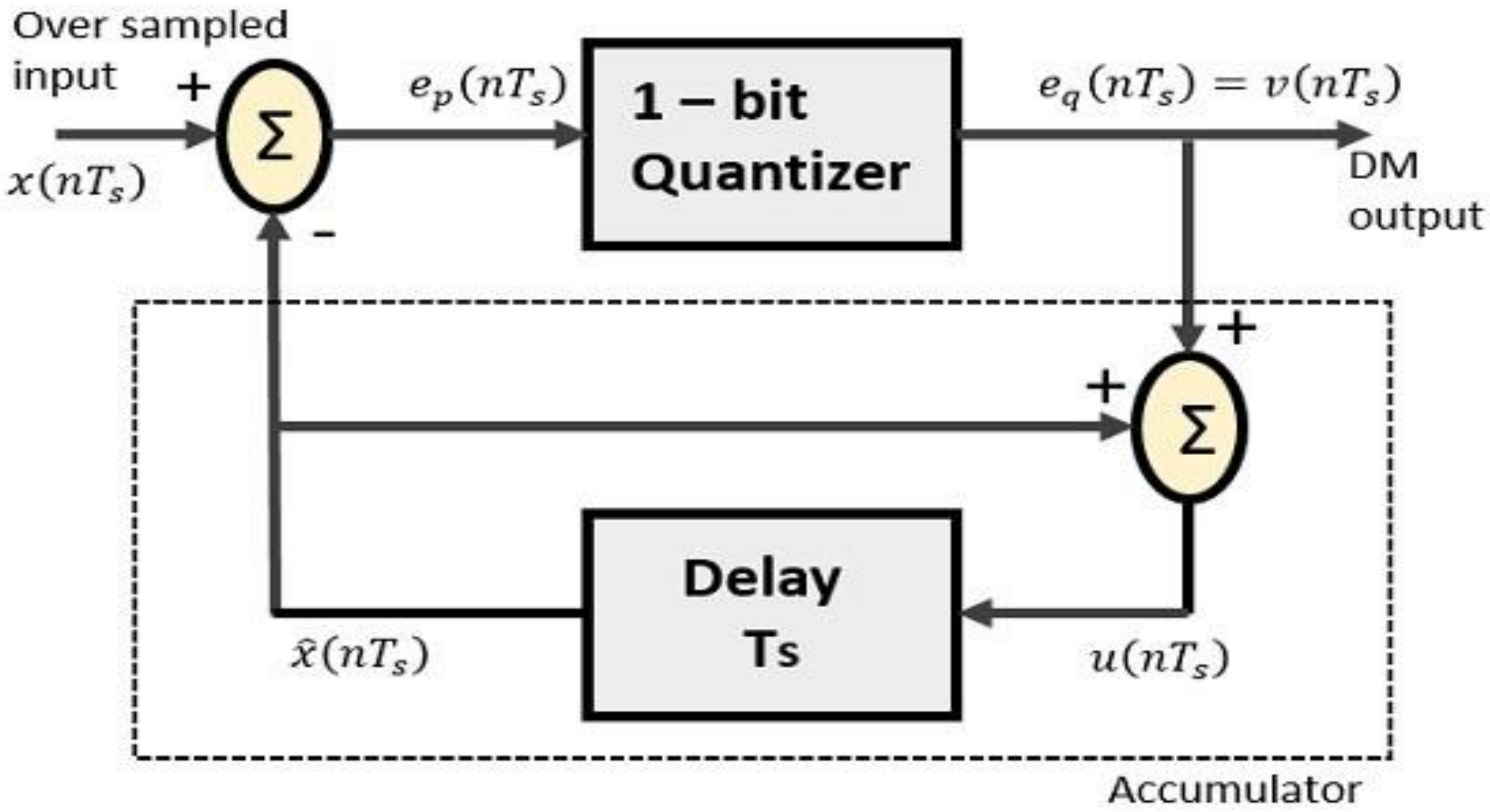
Delta Modulation (DM)

- Now, if the **difference is positive**, then approximated signal is increased by one step, i.e.'Δ'
- If the **difference is negative**, then approximated signal is reduced by 'Δ' .
- When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted.



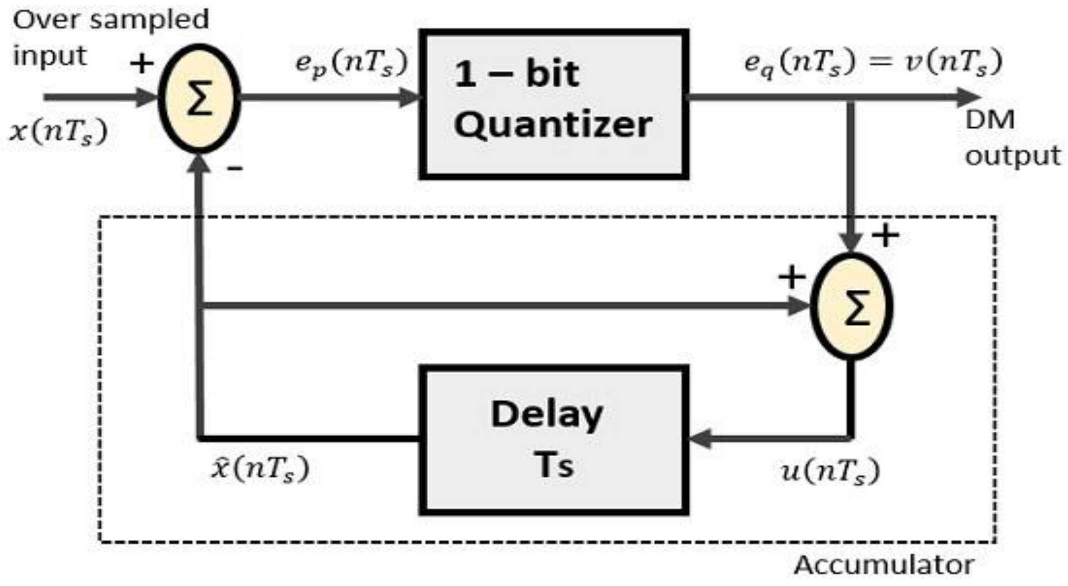
Delta Modulation (DM)

Transmitter (encoder/modulator)



Delta Modulation (DM)

- The Delta Modulator comprises of a 1-bit quantizer and a delay circuit along with two summer circuits.
- The predictor circuit in DPCM is replaced by a simple delay circuit in DM.
- From the above diagram, we have the notations as:
- $x(nT_s)$ - over sampled input
- $e_p(nT_s)$ - summer output and quantizer input
- $e_q(nT_s)$ - quantizer output $v(nT_s)$
- $\hat{x}(nT_s)$ - output of delay circuit
- $u(nT_s)$ - input of delay circuit



Delta Modulation (DM)

- Using these notations, now we shall try to figure out the process of delta modulation.

$$e_p(nT_s) = x(nT_s) - x\hat{}(nT_s)$$

$$e_p(nT_s) = x(nT_s) - u([n-1]T_s)$$

$$e_p(nT_s) = x(nT_s) - \{x\hat{}([n-1]T_s) + v([n-1]T_s)\}$$

- Further,

$$v(nT_s) = e_q(nT_s) = \Delta sgn[e_p(nT_s)]$$

- This means that depending on the sign of error $e_p(nT_s)$, the sign of step size Δ is decided.

$$v(nT_s) = \begin{cases} +\Delta & if\ x(nT_s) \geq x\hat{}(nT_s) \\ -\Delta & if\ x(nT_s) < x\hat{}(nT_s) \end{cases}$$

Delta Modulation (DM)

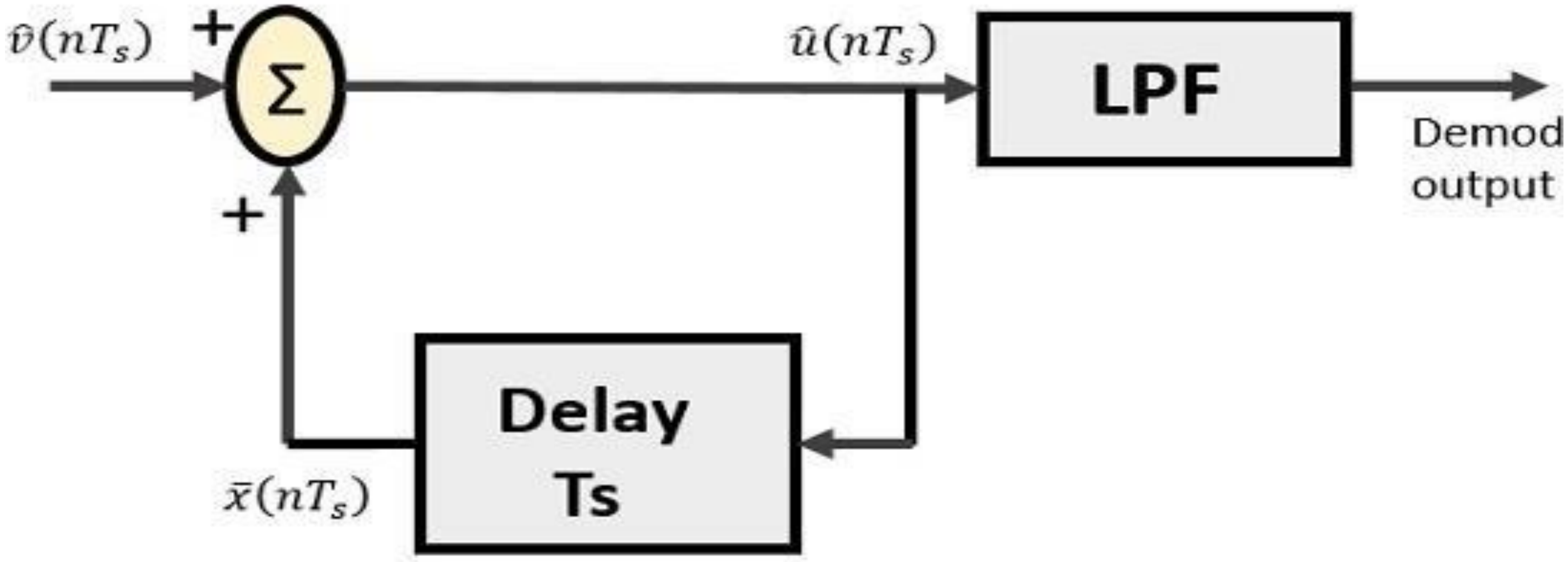
$$u(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

$$u(nT_s) = u([n-1]T_s) + v(nT_s)$$

- Present input of the delay unit = The previous output of the delay unit +The present quantizer output
- Assuming zero condition of Accumulation, $u(nT_s) = \sum_{j=1}^{n} \Delta sgn[e_{p(}jT_{s)}]$
- **Accumulated version of DM output**

$$\sum_{j=1}^{n} v(jT_s)]$$

Receiver (decoder/demodulator)



Delta Modulation (DM)

- we have the notations as :

$v\hat{}(nT_s)$ – input sample

$u\hat{}(nT_s)$ – summer output

$\bar{x}(nT_s)$ – is the delayed output

- A binary sequence will be given as an input to the demodulator.
- The stair-case approximated output is given to the LPF.
- Low pass filter is used for many reasons, but the prominent reason is noise elimination for out-of-band signals.

Delta Modulation (DM)

Advantages of DM Over DPCM

- 1-bit quantizer
- Very easy design of the modulator and the demodulator

- However, there exists some noise in DM.

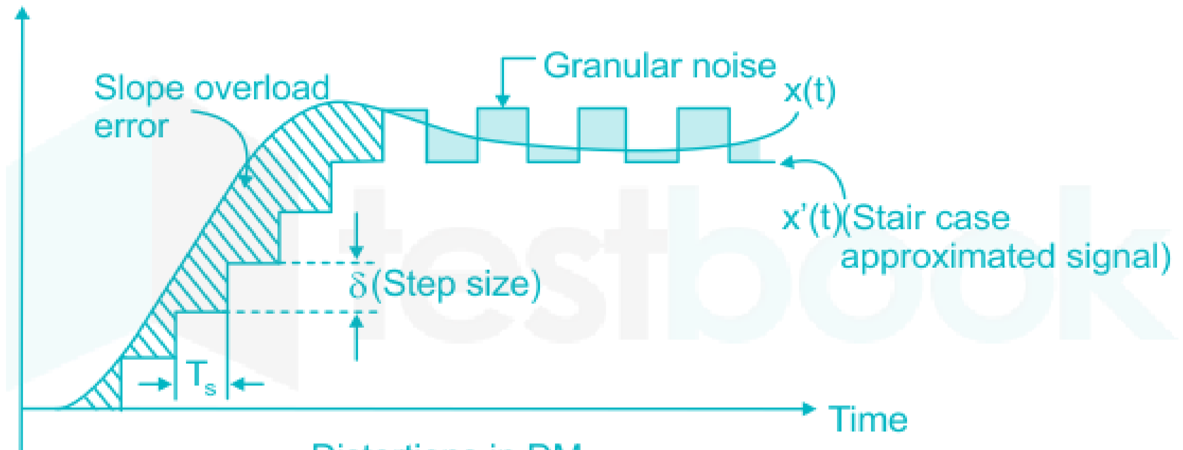
1. Slope Over load distortion (when **Δ** is small)
2. Granular noise (when **Δ** is large)

Delta Modulation (DM)

Slope Overload Noise

- It is due to the fact that the stair case approximation u(t) cant follow the actual message signal x(t) closely.
- In order for the u(t) to follow the x(t) closely, it is required that the following eqn. is met,

$$\frac{\Delta}{T_s} \geq max\left|\frac{dx(t)}{dt}\right|$$



Distortions in DM.

Delta Modulation (DM)

Granular Noise

- It occurs when Δ is too large relative to the slope characteristics of $x(t)$.
- Granular noise is similar to quantization noise in PCM.
- Large Δ is required for the rapid variation of $x(t)$ to reduce slope overload noise
- Small Δ is required to for the slowly varying $x(t)$ to reduce granular noise.
- The optimum Δ value can only be a compromise between the two cases

Delta Modulation (DM)

- If $\left|\frac{dx(t)}{dt}\right| \leq \frac{\Delta}{T_s}$,slope distortion will not occur.

$$\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| \leq \frac{\Delta}{T_s}$$

$$max|A_m 2\pi f_m\, cos(2\pi f_m t)| \leq \frac{\Delta}{T_s}$$

$$A_m 2\pi f_m \leq \frac{\Delta}{T_s} \rightarrow \boldsymbol{A_m \leq \frac{\Delta}{2\pi f_m T_s}}$$

Delta Modulation (DM)

SNR Calculation of D M

- In D M the amplitude of the modulator output changes by $\pm\Delta$ only.
- So the maximum amplitude when there is no slope overload distortion is given as,

$$A_m = \frac{\Delta f_s}{2\pi f_m}$$

- Where,
  - A_m is the peak amplitude of the sinusoidal signal
  - Δ Step size
  - f_s Sampling frequency
  - f_m Signal Frequency

Delta Modulation (DM)

To Obtain Signal Power

- Signal power is given as
- $P = \frac{V^2}{R}$, where V is the rms value of the signal
- Substituting $V = \frac{A_m}{\sqrt{2}}$ we get

$$P = \frac{{A_m}^2}{2}$$

Therefore the signal power is,

$$S_q = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2}$$

Delta Modulation (DM)

To obtain Noise Power

- In Delta Modulation the maximum qunatization error is equal to step size ‘Δ’
- Let the qunatization error be uniformly distributed over an interval (-Δ,+Δ)
- The PDF of the qunatization error can be expressed as

$$f_Q(q) = \begin{cases} 0 & for \;\; q < -\Delta \\ \dfrac{1}{2\Delta} \; for & -\Delta < q < \Delta \\ 0 & for \;\; q < \Delta \end{cases}$$

Delta Modulation (DM)

- The noise power $\sigma^2 = N_q$

$$N_q = \int_{-\Delta}^{\Delta} q^2 \frac{1}{2\Delta} dq = \frac{1}{2\Delta}\left[\frac{q^3}{3}\right]_{-\Delta}^{\Delta}$$

$$\boldsymbol{N_q = \frac{\Delta^2}{3}}$$

- The low pass reconstruction filter with cut off frequency 'W', passes part of the noise power uniformly distributed over $-f_s$ to $+f_s$ range to the output.

$$Output\ Noise\ Power = \frac{W}{f_s} \times Noise\ Power = \frac{W}{f_s} \times \frac{\Delta^2}{3} = \boldsymbol{\frac{W\Delta^2}{3f_s}\ or\ \frac{WT_s\Delta^2}{3}}$$

Delta Modulation (DM)

- Therefore SNR can be expressed as,

$$SNR = \frac{Normalised\ Signal\ Power}{Normalised\ Noise\ Power} = \frac{S_q}{N_q} = \frac{\frac{\Delta^2 f_s^2}{8\pi^2 f_m^2}}{\frac{W\Delta^2}{3f_s}}$$

$$\boldsymbol{SNR = \frac{3f_s^3}{8\pi^2 W f_m^2}\ or\ \frac{3}{8\pi^2 W f_m^2 T_s^3}}$$

- Minimum Transmission Bandwidth, $\boldsymbol{B_{Tmin} = \frac{f_s}{2}}$
- Bandwidth expansion factor, $\boldsymbol{b = \frac{transmission\ bandwidth}{signal\ bandwidth} = \frac{B_T}{W} = \frac{f_s}{2W}}$

Numerical

- A delta modulation system is designed to operate at 5 times the Nyquist rate for a signal with 3 KHz bandwidth. Determine the maximum amplitude of a 2 KHz input signal for which the delta modulator does not show slope overload distortion. The qunatizing step size is 250mv.

Solution:

Numerical

- A delta modulation system is designed to operate at 5 times the Nyquist rate for a signal with 3 KHz bandwidth. Determine the maximum amplitude of a 2 KHz input signal for which the delta modulator does not show slope overload distortion. The qunatizing step size is 250mv.

Solution:

- W = 3 KHz
- Nyquist Rate = 2 X 3KHz = 6 KHz
- f_s = 5 X Nyquiat Rate = 5 X 6 KHz = 30 KHz
- Δ = 250mv
- f_m = 2KHz

- $A_m \leq \frac{\Delta f_s}{2\pi f_m} \leq \frac{250mv \times 30\ KHz}{2 \times \pi \times 2 \times 10^3} \leq 0.6\ V$

Numerical

- A Delta Modulation system is tested with a 10 KHz sinusoidal signal with I V peak to peak at the input. It is sampled at 10 times the Nyquist rate. What is the step size required to prevent slope over load?What is the corresponding SNR?

- Solution:

Numerical

- A Delta Modulation system is tested with a 10 KHz sinusoidal signal with I V peak to peak at the input. It is sampled at 10 times the Nyquist rate. What is the step size required to prevent slope over load? What is the corresponding SNR?

- Solution:

$$f_s = 200 \text{ KHz}$$

$$\text{(i)} \quad A_m \leq \frac{\Delta}{T_s \cdot 2\pi f_m}$$

$$\Delta \geq A_m \cdot T_s \cdot 2\pi f_m = 0.5 \times \frac{1}{200\,K} \times 2 \times \pi \times 10 \text{ KHz}$$

$$= 0.5 \times (5 \times 10^{-6}) \times 2 \times \pi \times 10 \text{ KHz}$$

$$\boxed{\Delta \geq 0.157 \text{ V}}$$

Numerical

(ii) $(SNR)_{\text{at filter o/p}} \Rightarrow \dfrac{3\, f_s^3}{8\pi^2\, W \cdot f_m^2}$

Since 'W' is not given, SNR is calculated using the formula

$$\frac{S}{N} = \frac{\dfrac{\Delta^2}{8\pi^2 f_m^2\, T_s^2}}{\dfrac{\Delta^2}{3}} = \frac{3 f_s^2}{8\pi^2 f_m^2}$$

where $\dfrac{\Delta^2}{3}$ is the noise power of delta modulation.

Numerical

$$SNR = \frac{3 \times f_s^2}{8\pi^2 \cdot f_m^2}$$

$$= \frac{3 \times (200K)^2}{8 \times \pi^2 \times (10K)^2} = 15.19$$

$$(SNR)_{dB} = 10 \log SNR$$

$$\boxed{(SNR)_{dB} = 11.8 \text{ dB}}$$

Numerical

- In a single integration delta modulation, the voice signal is sampled at the rate of 64 KHz. The maximum signal amplitude is 2V, the voice signal bandwidth is 3.5 KHz. Determine the minimum value of step size to avoid overload distortion and also calculate the noise power.
- Solution:

Numerical

- In a single integration delta modulation, the voice signal is sampled at the rate of 64 KHz. The maximum signal amplitude is 2V, the voice signal bandwidth is 3.5 KHz. Determine the minimum value of step size to avoid overload distortion and also calculate the noise power.

- Solution:

$$A_m = 2V$$

$$f_m = 3.5 \text{ KHz}$$

$$\Delta \geq \frac{2\pi f_m A_m}{f_s}$$

$$\geq \frac{2 \times \pi \times 3.5 \times 10^3 \times 2}{64 \times 10^3}$$

$$\boxed{\Delta \geq 0.68 \text{ V}}$$

$$P_{\text{Noise}} = \frac{\Delta^2}{3} = \frac{(0.68)^2}{3}$$

$$= 0.1541$$

$$\boxed{P_{\text{Noise}} \Rightarrow \text{noise power} = 0.154 \text{ W}}$$

Numerical

***Problem: 7. Consider a DM system designed to accommodate analog message signals limited to bandwidth* W = 5 KHz. *A sinusoidal test signal of amplitude* A = 1 volt *and frequency* f_m = 1 kHz *is applied to the system. The sampling rate of the system is* 50 kHz.**

(a) Calculate the step size Δ ***required to minimize slope overload.***

(b) Calculate the signal-to-(quantization) noise ratio of the system for the specified sinusoidal test signal.

Numerical

(a) $A \le \dfrac{\Delta f_s}{2\pi f_m}$

$$\Delta \ge \frac{2\pi f_m A}{f_s}$$

$$\Delta \ge \frac{2 \times \pi \times 10^3 \times 1}{50 \times 10^3}$$

$$= 0.126 \text{ V}$$

(b) $(SNR)_{out} = \dfrac{3}{8\pi^2}\dfrac{f_s^3}{f_m^2 W}$

$$= \frac{3}{16\pi^2} \times \frac{(50 \times 10^3)^3}{10^6 \times 5 \times 10^3}$$

$$= 475$$

In decibels,

$$(SNR)_{out} = 10 \log_{10} 475$$

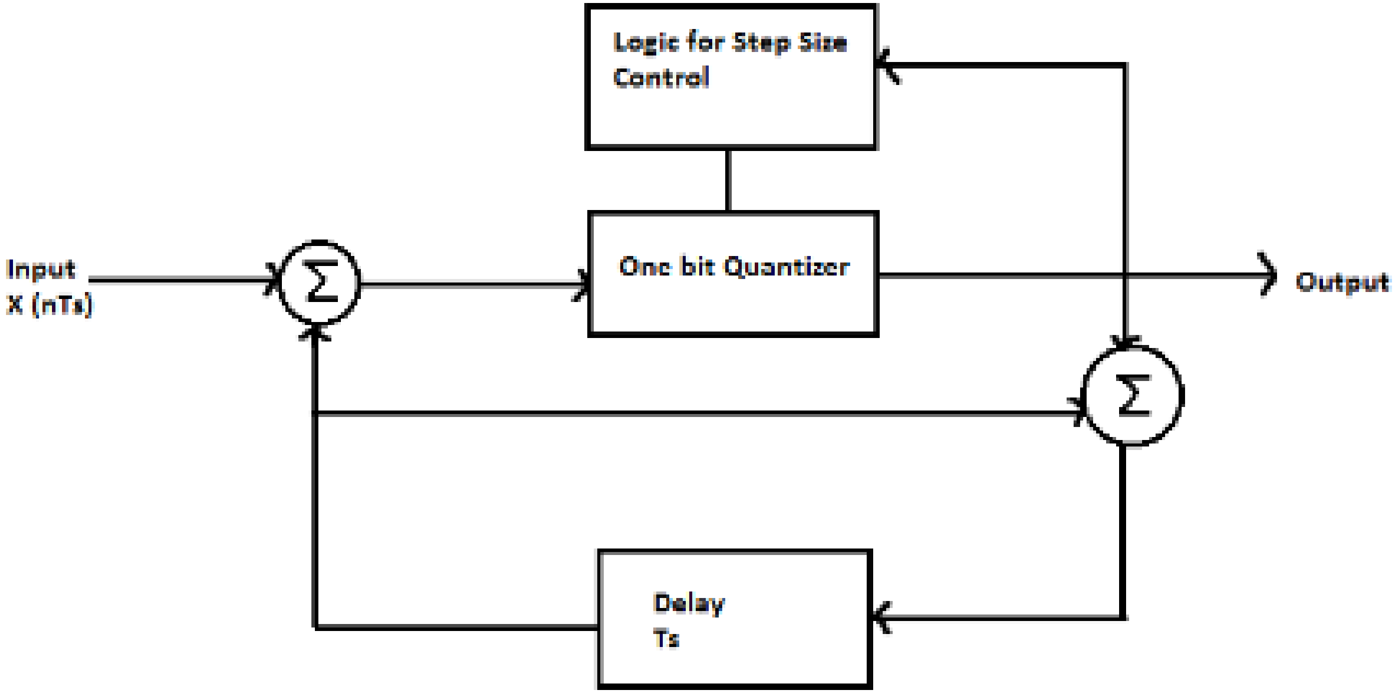
$$= 26.8 \text{ dB}$$

Adaptive Delta Modulation (ADM)

Adaptive Delta Modulation (ADM)

- In delta modulation, we have come across certain problem of determining the step-size, which influences the quality of the output wave.
- A larger step-size is needed in the steep slope of modulating signal and a smaller step size is needed where the message has a small slope.
- So, it would be better if we can control the adjustment of step-size, according to our requirement in order to obtain the sampling in a desired fashion.
- This is the concept of Adaptive Delta Modulation.

Adaptive Delta Modulation (ADM)



$$\Delta(n)=|\Delta(n-1)|e(n)+\Delta(0)e(n-1)$$

Comparison of PCM, DPCM, DM and ADM

S.NO	Parameter of Comparison	Pulse Code Modulation (PCM)	Delta Modulation (DM)	Adaptive Delta Modulation (ADM)	Differential Pulse Code Modulation (DPCM)
1.	Number of bits	It can use 4,8, or 16 bits per sample.	It uses only one bit for one sample	It uses only one bit for one sample	Bits can be more than one but are less than PCM.
2.	Levels and step size	The number of levels depends on number of bits. Level size is fixed.	Step size is kept fixed and cannot be varied.	According to the signal variation, step size varies.	Number of levels is fixed.
3.	Quantization error and distortion	Quantization error depends on number of levels used.	Slope overload distortion and granular noise are present.	Quantization noise is present but other errors are absent.	Slope overload distortion and quantization noise is present.
4.	Transmission bandwidth	Highest bandwidth is required since numbers of bits are high.	Lowest bandwidth is required.	Lowest bandwidth is required.	Bandwidth required is less than PCM.
5.	Feedback	There is no feedback in transmitter or receiver.	Feedback exists in transmitter.	Feedback exists.	Feedback exists.
6.	Complexity of Implementation	System is complex.	Simple	Simple	Simple