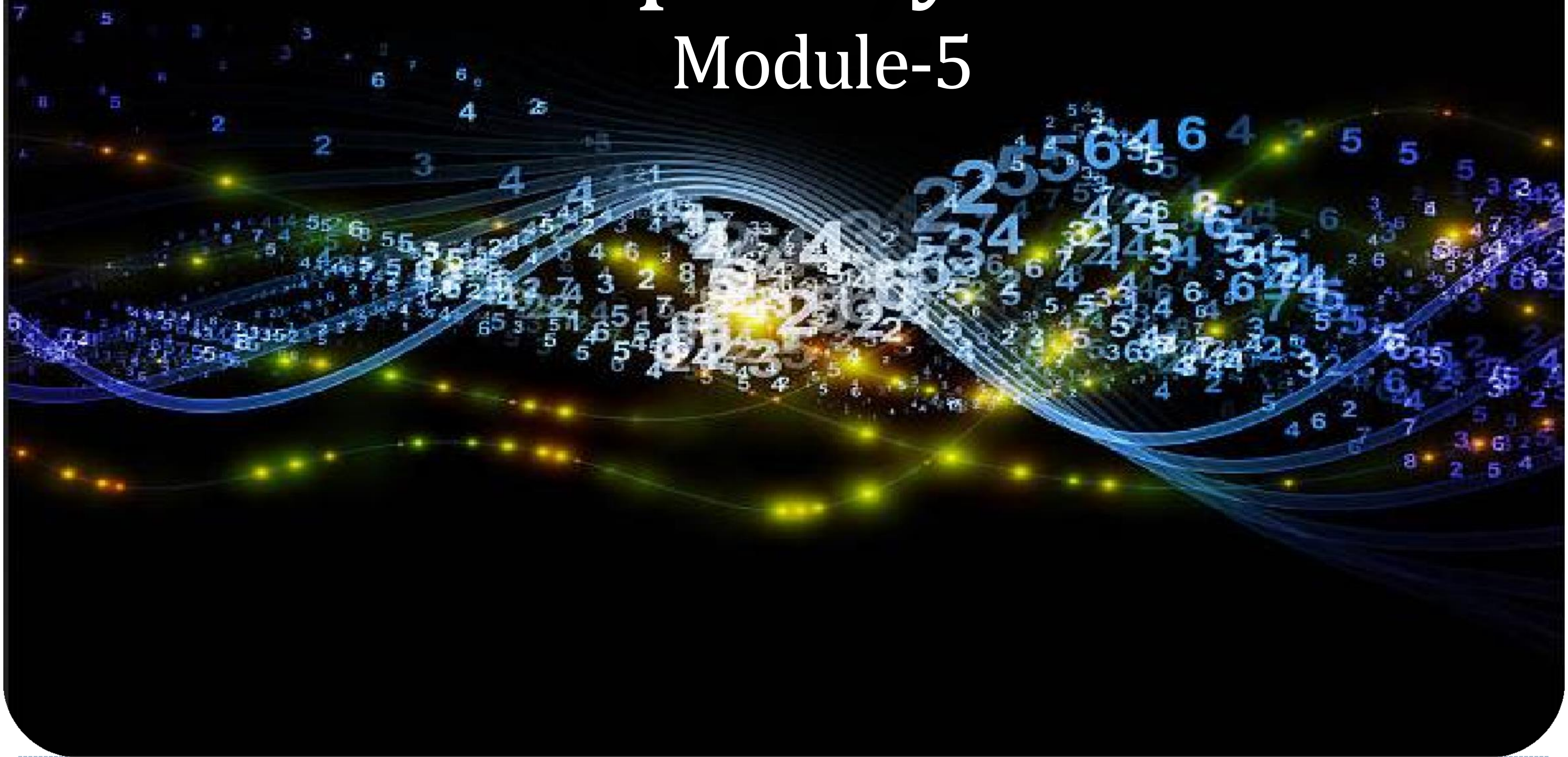
Bandpass System-I

Module-5



Topics to be discussed

- Introduction to representation of signals
- Gram-Schmidt orthogonalization procedure
- Correlation receiver
- QAM
- Generation and detection of coherent system (BASK, BFSK, BPSK, QPSK, MSK)
- Error performance.

Introduction

Digital communication system

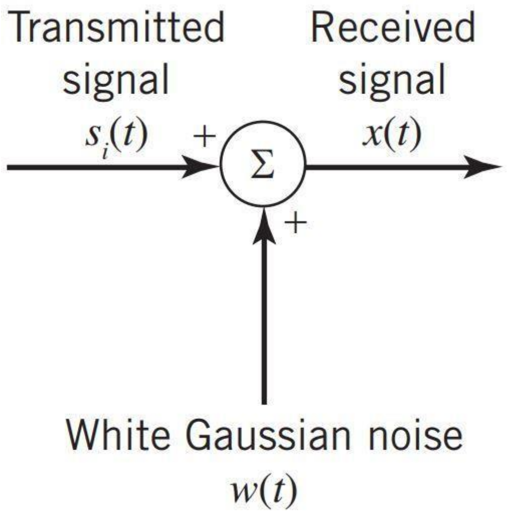
- It is designed for the transmission of information in digital form.
- The source information may be an analogue or digital source.
- An analogue source can be converted into digital form by analogue-to-digital conversion (e.g., pulse-code modulation and delta modulation).

Data communication system

- It is designed for the transmission of information in digital form.
- The source information is already in digital form.

Introduction

- In a data/digital communication system, the digital information is used to modulate a carrier.
- The operation performed with the digital signal is called digital modulation.
- At the receiving end, a process of demodulation is used to recover the original signal.
- We are here concerned with the principles of digital modulation techniques with detection of a digital signal in the presence of additive white Gaussian noise (AWGN).



Introduction

- In general, the digital signal may be transmitted directly (transmission at baseband) or as a modulated-carrier signal (transmission at radio frequency).

- In both transmission cases, the concept of signal space can be used to represent a set of signals in terms of a set of orthonormal functions.

- The Gram-Schmidt orthogonal procedure is therefore discussed.

Introduction

- With the analog channel represented by an AWGN model, the *received signal* is defined by:

$$\boldsymbol{x(t) = s_i(t) + w(t)}, \ \begin{cases} \boldsymbol{0 \le t \le T_b} \\ \boldsymbol{i = 1, 2} \end{cases}$$

- The receiver then makes an *estimate* of the transmitted signal $s_i\,(t)$, or equivalently upto i^{th} symbol, i = 1,2

- However, due to the presence of noise, the receiver may make occasional *errors*

- Therefore, we need to design the receiver such that:

"The average probability of symbol error is minimized"

Introduction

- The average probability of symbol error is given by:

$$\boldsymbol{P_e = \pi_1 P(\hat{m} = 0|1\ sent) + \pi_2 P(\hat{m} = 1|0\ sent)}$$

- Here, π_1 and π_2 are the priori probabilities of transmitting symbols 1 and 0, respectively
- $\hat{m}$ is the estimate of the symbol 1 or 0 sent by the source
- $P(\hat{m} = 0\ |1\ sent)$ and $\pi_2 P\ (\hat{m} = 1\ |0\ sent)$ are conditional probabilities

- Minimizing P_e leads to making the digital communication system more *reliable*

Introduction

- To achieve objective, an $M-ary\ alphabet$ whose symbols are denoted by $m_1, m_2, \dots, m_M$ are involved

- Two basic issues needs to addressed here:

1. How to optimize the design of the receiver so as to minimize P_e

2. How to choose the set of signals $s_1(t), s_2(t), \dots, s_M(t)$ for representing the symbols $m_1, m_2, \dots, m_M$

- The answer to these fundamental questions lies in :

"geometric representation of signals"

Introduction-Digital Model

- We consider the following model of a generic transmission system (*digital source*):
  - A message source transmits 1 symbol every T sec
  - Symbols belong to an alphabet M $(m_1, m_2, \ldots m_M)$
    - Binary – symbols are 0s and 1s
    - Quaternary PCM – symbols are 00,01,10,11

Introduction-Digital Model

Transmitter

- Symbol generation (message) is probabilistic, with a priori probabilities $p_1, p_2, .p_M$. or
- Symbols are equally likely
- So, probability that symbol m_i will be emitted:

$$\rho_i = P(m_i)$$

$$= \frac{1}{M} \ for \quad \mathrm{i=1,2,....,M}$$

Introduction-Digital Model

- Transmitter takes the *symbol (data)* ***m_i*** (digital message source output) and encodes it into a *distinct signal* ***$s_i(t)$***.
- The *signal $s_i(t)$* occupies the whole ***slot T*** allotted to *symbol m_i*
- $s_i(t)$ is a real valued energy signal (*signal with finite energy*)

$$E_i = \int_0^T s_i^2(t)\,dt, \quad i=1,2,\ldots,M$$

Introduction-Digital Model

Channel Assumptions

- *Linear*,wide enough to accommodate the signal $s_i(t)$ with no or negligible distortion
- *Channel noise* is w(t) is a zero-mean white Gaussian noise process – AWGN
  - additive noise
  - received signal may be expressed as:

$$x(t) = s_i(t) + w(t), \qquad \begin{Bmatrix} 0 \le t \le T \\ i=1,2,....,M \end{Bmatrix}$$

Introduction-Digital Model

Receiver

- Observes the received signal x(t) for a duration of time T sec
- Makes an estimate of the transmitted signal $s_i(t)$ (eq.symbol m_i).
- Process is statistical
  - presence of noise
  - errors
- So, receiver has to be designed for *minimizing the average probability of error (P_e)*

What is this?

$$P_e = \sum_{i=1}^{M} p_i P(\hat{m} \neq m_i \;/\; m_i)$$

cond. error probability given ith symbol was sent

Symbol sent

Geometric Representation of Signals

- <u>Objective</u>: To represent any set of ***M*** *energy signals* *$\{s_i(t)\}$* as linear combinations of ***N*** *orthonormal basis functions*, where N ≤ M
- Given a set of real-valued energy signals, $s_1(t), s_2(t), \dots, s_M(t)$, each of duration T seconds, we write:

Orthonormal basis function

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \begin{cases} 0 \le t \le T \\ i = 1,2,\dots,M \end{cases}$$

coefficient

Energy signal

Geometric Representation of Signals

- The coefficients are defined by:

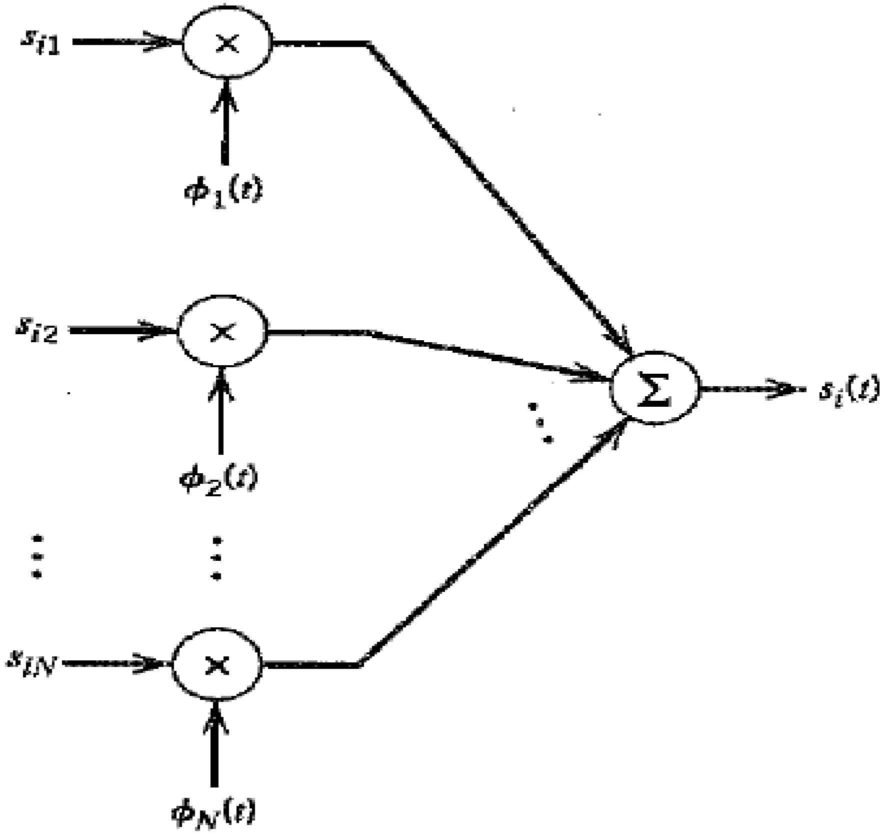
$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \qquad \{ \begin{matrix} i = 1,2,\ldots,M \\ j = 1,2,\ldots,N \end{matrix}$$

- Real-valued basis functions:

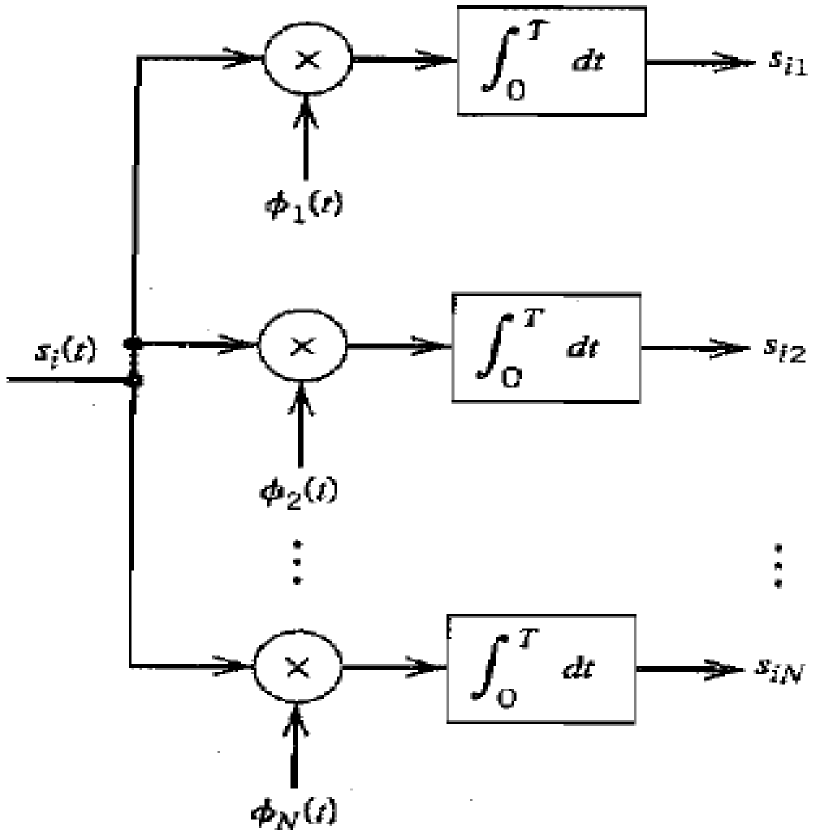
$$\int_0^T \phi_i(t)\phi_j(t)dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Geometric Representation of Signals

Synthesizer for generating the signal $s_i(t)$



Analyzer for reconstructing the signal vector $\{s_i\}$



Geometric Representation of Signals

- For prescribed i, the set of coefficients $\{s_{ij}\}_{j=1}^{N}$ may be viewed as an N-dimensional signal vector, denoted by s_i
- The vector s_i bears a one-to-one relationship with the transmitted signal $s_i(t)$
- Each signal set $\{s_i(t)\}$ is completely determined by the signal vector given by:

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, i = 1,2, \dots, M$$

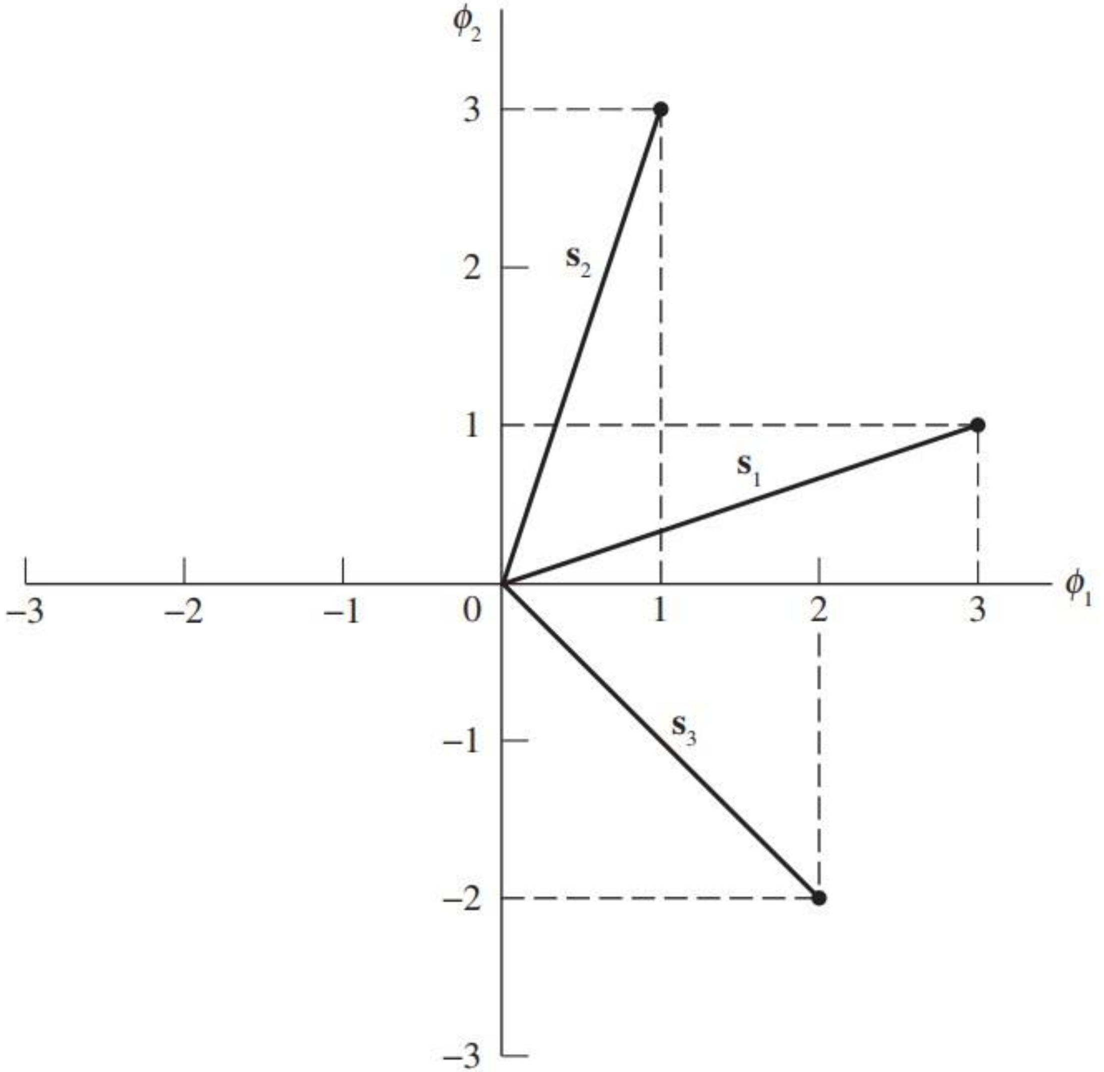
Geometric Representation of Signals

- We may now visualize the set of signal vectors $\{s_i | i = 1,2, \dots, M\}$ as defining a corresponding set of M points in an $N - dimensional\ Eucledian\ space$
- It has N mutually perpendicular axes labeled $\phi_1, \phi_2, \dots, \phi_N$
- This N-dimensional Euclidian space is called the $signal\ space$
- It provides the mathematical basis for the geometric representation of signals
- Allows definition of

  1. Length of vectors (absolute value)
  2. Angles between vectors
  3. Squared value (inner product of s_i with itself)

Geometric Representation of Signals

- Illustrating the geometric representation of signals for the case when N = 2 and M = 3. (two dimensional space, three signals)



Geometric Representation of Signals

- The *lengths* (also called as the *absolute value or norm*) of a signal vector $\boldsymbol{s}_i$ is given by the symbol $||\boldsymbol{s}_i||$
- The squared length of any signal vector s_i is defined to be the inner product or dot product of s_i with itself, given by

$$||s_i||^2 = s_i^T s_i$$

Matrix Transposition

$$= \sum_{j=1}^{N} s_{ij}^2, \quad i = 1,2,\dots,M$$

- The **energy of a signal** $s_i(t)$ of duration T seconds is given by:

$$E_i = \int_0^T s_i^2(t)dt, \qquad i = 1,2,\dots,M \qquad\qquad where, s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t)$$

Geometric Representation of Signals

- What is the relation between the *vector representation* of a signal and its *energy value?*
- After substitution:

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right]dt$$

- After regrouping:

$$E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}s_{ik}\int_0^T \phi_j(t)\phi_k(t)\,dt$$

- $\Phi_j(t)$ is orthogonal, so finally we have:

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2$$

The *energy of a signal* is equal to the *squared length of its vector*

Geometric Representation of Signals

Formulas for two signals

- In case of a pair of signals $s_i(t)$ and $s_k(t)$ represented by the signal vectors s_i and s_k, we have:

$$\int_0^T s_i(t)s_k(t)dt = s_i^T s_k$$

Inner product of the signals is equal to the inner product of their vector representations [0,T]

Inner product is invariant to the selection of basis functions

Geometric Representation of Signals

Euclidian Distance

- The Euclidean distance between two points represented by vectors (signal vectors) is equal to $||s_i - s_k||$ and the squared value is given by:

$$||\boldsymbol{s}_i - \boldsymbol{s}_k||^2 = \sum_{j=1}^{N} \left(s_{ij} - s_{kj}\right)^2 = \int_0^T \left(s_i(t) - s_k(t)\right)^2 dt$$

- Here, $||s_i - s_k||$ is the Euclidian distance d_{ik} between the points represented by the signal vectors $\boldsymbol{s}_i$ and $\boldsymbol{s}_k$

Geometric Representation of Signals

Angle between two signals

- The angle θ_{ik} subtended between two signals vectors s_i and s_k provides a complete geometric representation
- The *cosine of the angle* θ_{ik} is equal to the inner product of these two vectors divided by the product of their individual norms
- Mathematically, $\cos(\theta_{ik}) = \dfrac{s_i^T s_k}{||s_i||||s_k||}$
- If their inner product $s_i^T s_k$ is zero, it means that the two vectors s_i and s_k are orthogonal to each other

Geometric Representation of Signals

Schwartz Inequality

- Defined as:

$$\left(\int_{-\infty}^{\infty} s_1(t)s_2(t)dt\right)^2 = \left(\int_{-\infty}^{\infty} s_1^2(t)dt\right)\left(\int_{-\infty}^{\infty} s_2^2(t)dt\right)$$