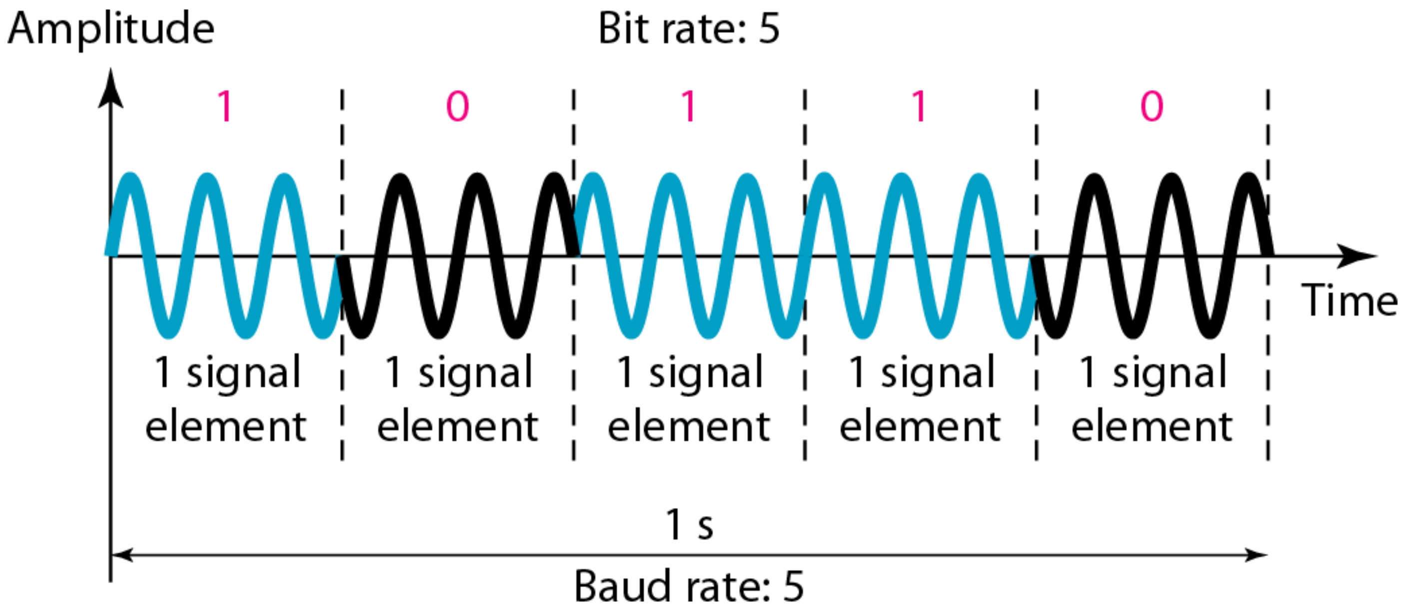
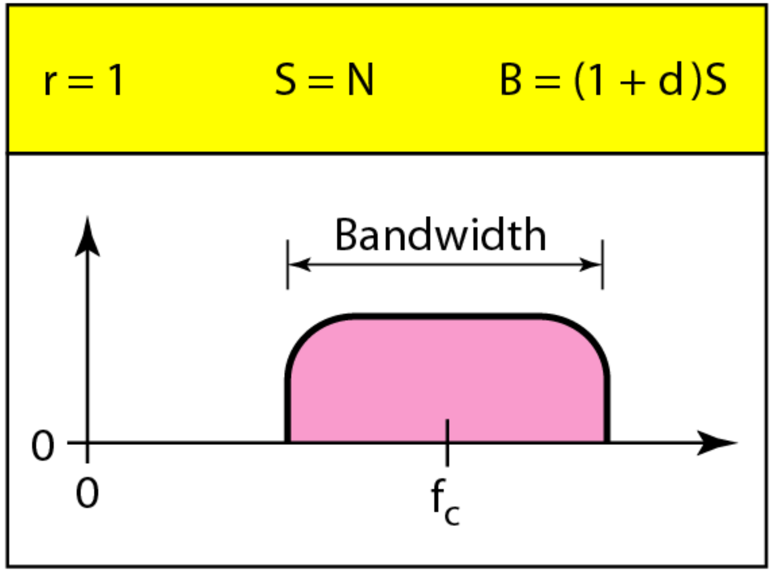
Phase Shift Keying (PSK)

- It is the digital modulation technique in which the phase of the carrier signal is changed or shifted in accordance with the digital input signal '1' or '0'.
- The change is shown by varying the sine and cosine inputs at a particular time.
- PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.
- PSK is much more robust than ASK as it is not that vulnerable to noise, which changes amplitude of the signal.

Phase Shift Keying (PSK)





Phase Shift Keying (PSK)

- PSK is of two types, depending upon the phases the signal gets shifted.

They are – BPSK & QPSK

Binary Phase Shift Keying

- This is also called as 2-phase PSK or Phase Reversal Keying.
- In this technique, the sine wave carrier takes two phase reversals such as 0° and 180°.

Quadrature Phase Shift Keying

- This is the phase shift keying technique, in which the sine wave carrier takes four phase reversals such as 0°,90°,180°,and 270°.

Phase Shift Keying (PSK)

Signal Representation of PSK

- PSK signal is represented as,

$$\boldsymbol{s_1(t) = \sqrt{\frac{2E_b}{T_b}}\ Cos\ (2\pi f_c t) \rightarrow for\ binary\ '1'}$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\ Cos\ (2\pi f_c t + \pi)$$

$$\boldsymbol{s_2(t) = -\sqrt{\frac{2E_b}{T_b}}\ Cos\ (2\pi f_c t) \rightarrow for\ binary\ '0'}$$

- Where, $0 \le t \le T_b$ and E_b is the transmitted signal energy per bit.

$$\therefore \boldsymbol{s_2(t) = -s_1(t)}$$

Phase Shift Keying (PSK)

- These PSK signals are called as **antipodal** signals, defined as a pair of sinusoidal waves that differ only in a relative phase-shift of 180 degrees.

- In BPSK, there is only one basis function of unit energy,

$$\varphi_1(t) = \sqrt{\frac{2}{T_b}} Cos(2\pi f_c t), 0 \le t \le T_b$$

$$\therefore \boldsymbol{s_1(t) = \sqrt{E_b}\varphi_1(t), 0 \le t \le T_b}$$

$$\boldsymbol{s_2(t) = -\sqrt{E_b}\varphi_1(t), 0 \le t \le T_b}$$

Phase Shift Keying (PSK)

Signal Space diagram of BPSK

- A coherent BPSK system has two message points M=2 with one dimensional signal space (N=1).

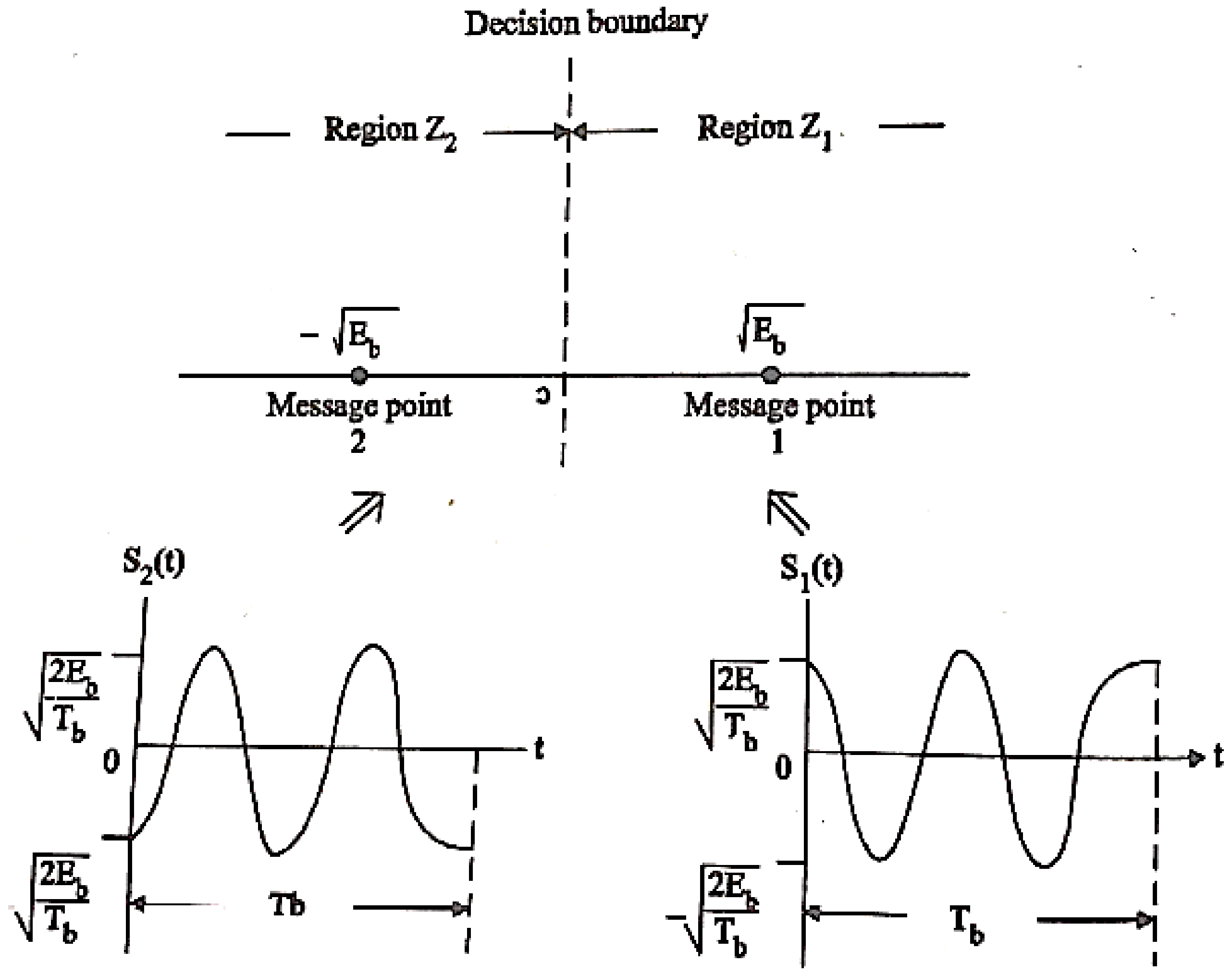
- The message point corresponding to $s_1(t)$ is located at s_{11}

Where, $s_{11} = \int_0^{T_b} s_1(t)\varphi_1(t)dt \rightarrow \boldsymbol{s_{11}} = +\sqrt{\boldsymbol{E_b}}$

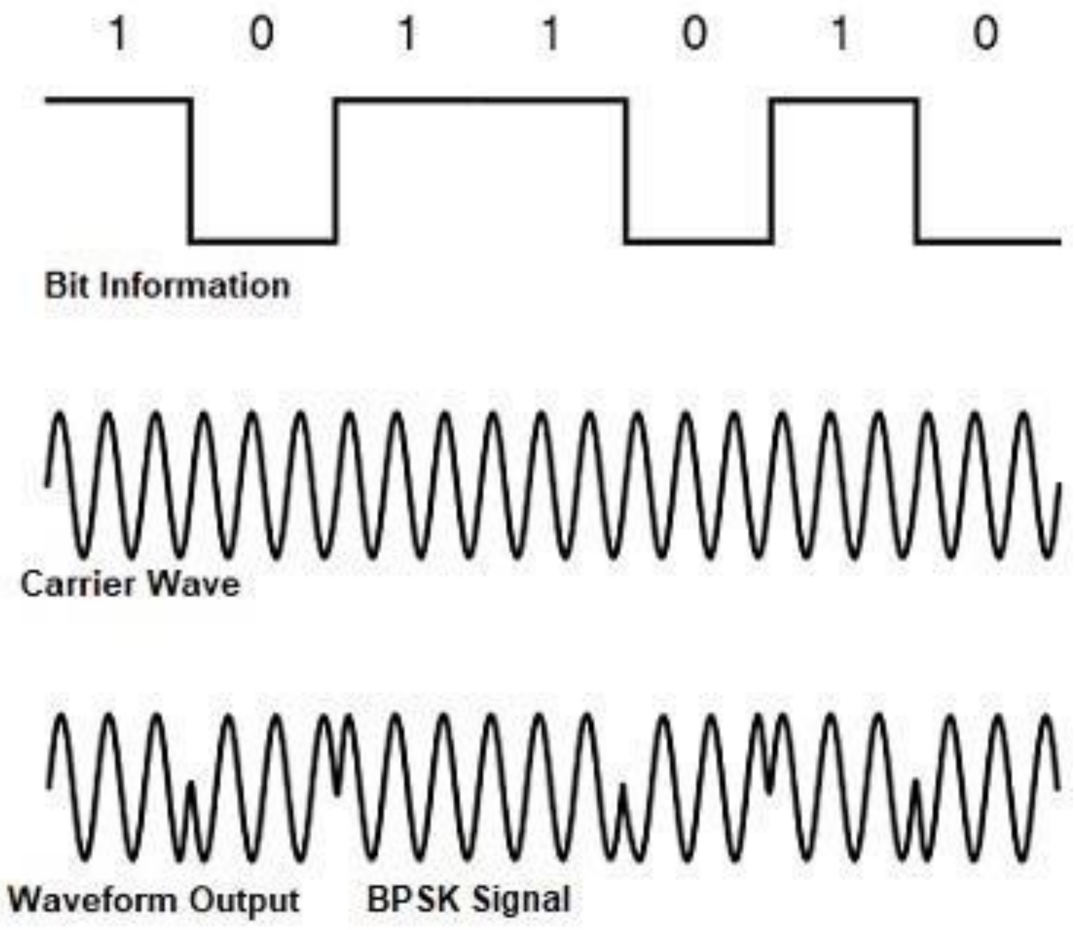
- The message point corresponding to $s_2(t)$ is located at s_{21}

Where, $s_{21} = \int_0^{T_b} s_2(t)\varphi_1(t)dt \rightarrow \boldsymbol{s_{21}} = -\sqrt{\boldsymbol{E_b}}$

Phase Shift Keying (PSK)

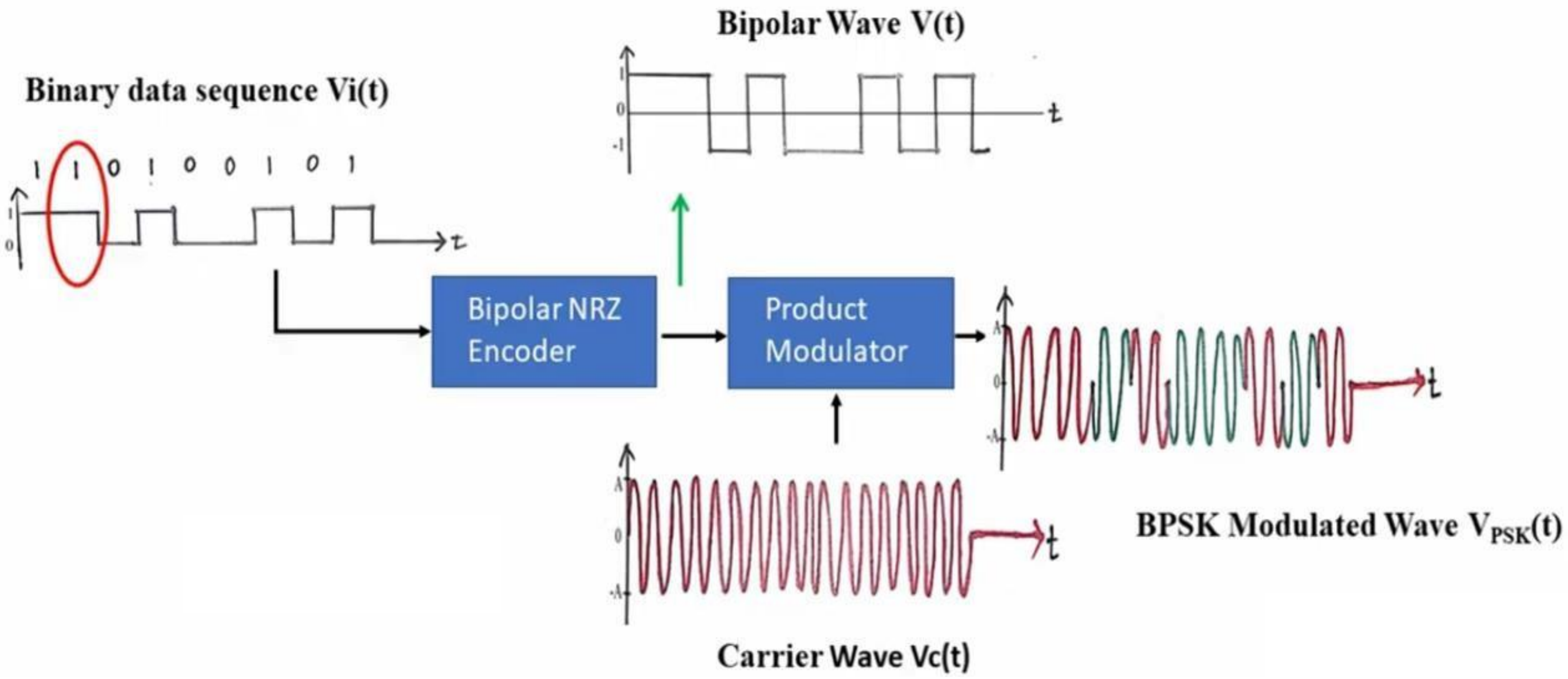


Phase Shift Keying (PSK)



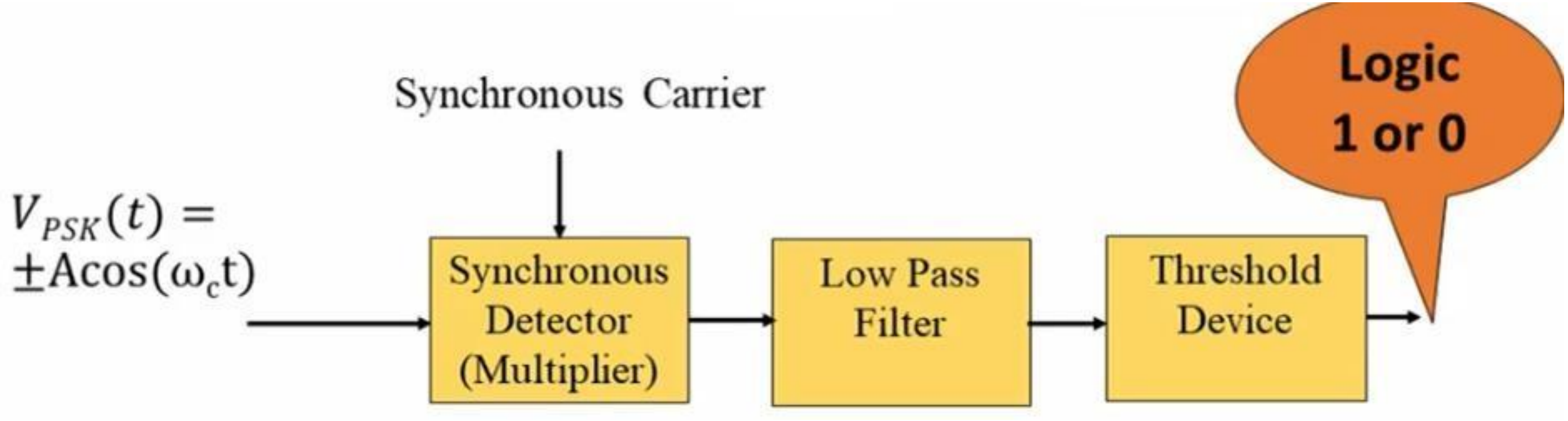
Phase Shift Keying (PSK)

- **BPSK generation (Modulator)**



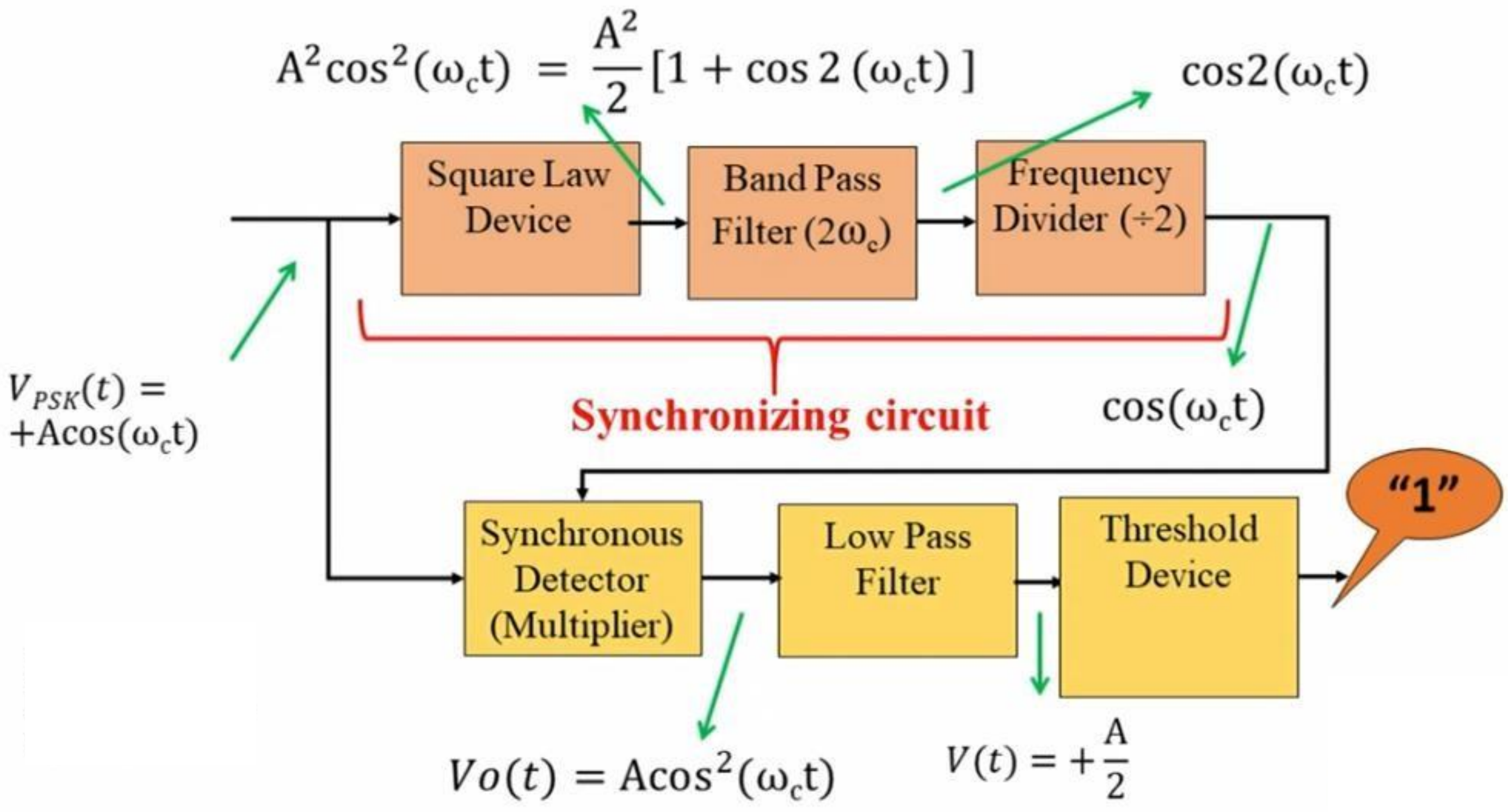
Phase Shift Keying (PSK)

- **BPSK Demodulator**



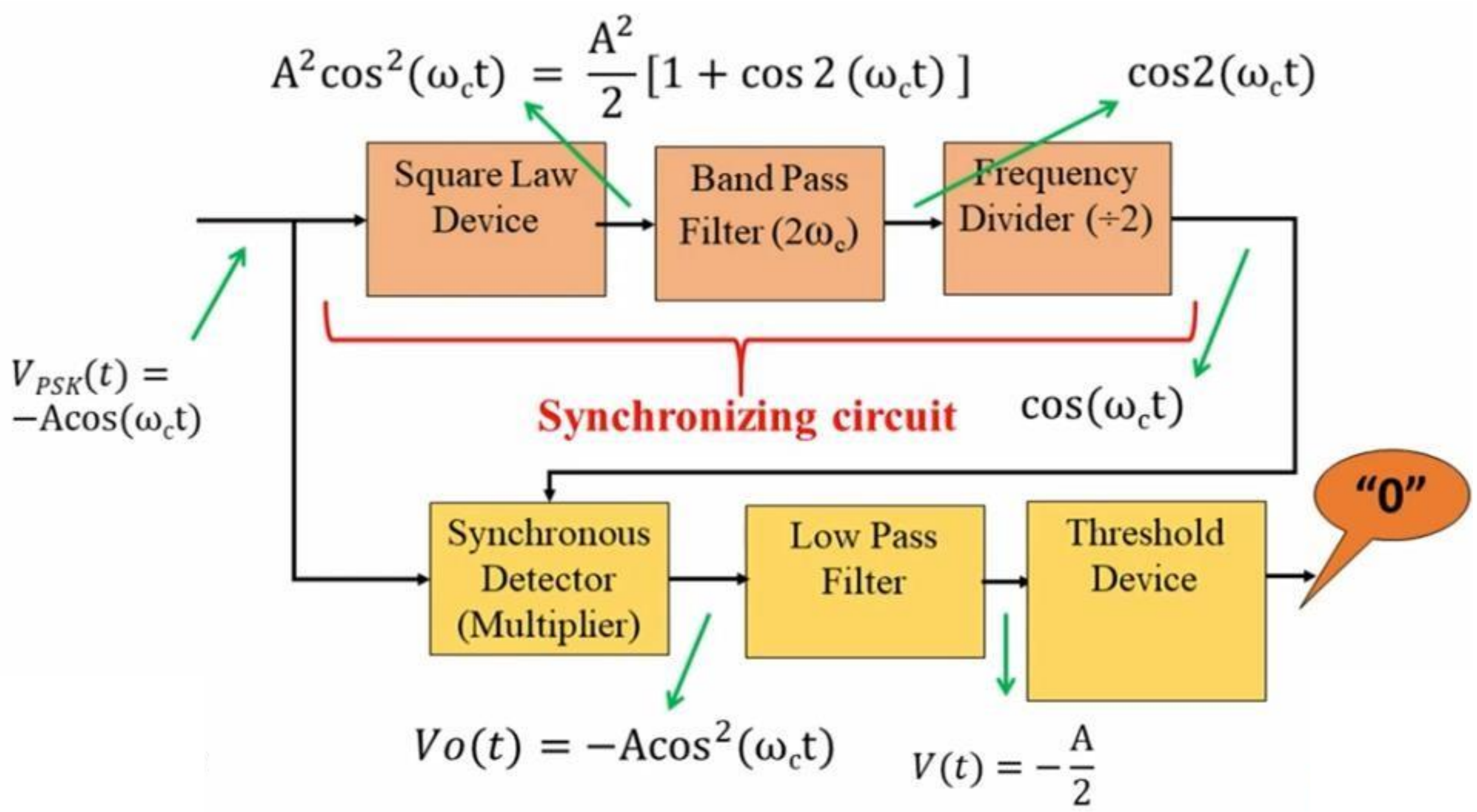
Phase Shift Keying (PSK)

- **BPSK Demodulator (Binary-1)**



Phase Shift Keying (PSK)

- **BPSK Demodulator (Binary-0)**



Phase Shift Keying (PSK)

- **Probability of error in BPSK**

In PSK, binary '1' is represented as

$$s_1(t) = A\cos(2\pi f_c t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

and binary '0' is represented as

$$s_2(t) = -A\cos(2\pi f_c t) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

i.e $\quad s_2(t) = -s_1(t)$

$$P_e = \frac{1}{2}\,erfc\left[\frac{s_1(t) - s_2(t)}{2\sqrt{2}\,\sigma}\right]$$

$$\left[\frac{s_1(t) - s_2(t)}{\sigma}\right]^2_{max} = \int_0^{T_b} \frac{g_m^2(t)}{\text{PSD of noise}}\,dt$$

$$= \int_0^{T_b} \frac{g_m^2(t)}{N_o/2}\,dt$$

Phase Shift Keying (PSK)

$$\int_0^{T_b} \frac{g_m^2(t)}{N_o/2}\,dt = \frac{2}{N_o}\int_0^{T_b}\left[s_1(t) - (-s_1(t))\right]^2 dt$$

$$= \frac{2}{N_o}\int_0^{T_b} [2\,s_1(t)]^2\,dt$$

$$= \frac{2}{N_o}\int_0^{T_b} 4\,s_1^2(t)\,dt$$

$$= \frac{8}{N_o}\int_0^{T_b} A^2\cos^2(2\pi f_c t)\,dt$$

$$= \frac{8A^2}{N_o}\int_0^{T_b} \frac{1+\cos(4\pi f_c t)}{2}\,dt = \frac{8A^2}{N_o}\int_0^{T_b}\frac{1}{2}\,dt$$

$$[\because \int_0^{T_b} \frac{\cos(4\pi f_c t)}{2}\,dt = 0]$$

$$\left[\frac{s_1(t) - s_2(t)}{\sigma}\right]^2 = \frac{4A^2\,T_b}{N_o} = \frac{4}{N_o}\cdot\frac{2E_b}{T_b}\cdot T_b = \frac{8E_b}{N_o}$$

Phase Shift Keying (PSK)

$$\left[\frac{s_1(t) - s_2(t)}{\sigma}\right]_{max} = \sqrt{\frac{8E_b}{N_o}}$$

$$= \frac{1}{2} erfc\left[\frac{1}{2\sqrt{2}} \cdot \sqrt{\frac{8E_b}{N_o}}\right]$$

$$\boxed{Pe_{(PSK)} = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_o}}}$$

Phase Shift Keying (PSK)

Advantages

- BPSK has a good noise immunity
- Even in the presence of noise the performance of BPSK is good
- Requires simple circuit for generation and detection

Disadvantages

- Sometimes its difficult to detect the binary bits without error
- Low bandwidth efficiency

Applications

- It is employed in communication systems with higher bit rates

Quadrature Phase Shift Keying (QPSK)

Two main goals in the design of digital communication systems are,

- Providing reliable performance with very low probability of error
- Efficient utilization of channel Bandwidth

Principle

- The **Quadrature Phase Shift Keying** is a variation of BPSK, and it is also a Double Side Band Suppressed Carrier DSBSC modulation scheme, which sends two bits of digital information at a time, called as **dibits**.
- Instead of the conversion of digital bits into a series of digital stream, it converts them into bit pairs.
- This decreases the data bit rate to half, which allows space for the other users.

Quadrature Phase Shift Keying (QPSK)

Signal representation of QPSK

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t + \dfrac{(2i-1)\pi}{4}), & 0 \leq t \leq T_b \\ 0 & elsewhere \end{cases}$$

- Where, $i = 1,2,3,4$ and $E_b =$ transmitted signal energy per bit

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{(2i-1)\pi}{4}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{(2i-1)\pi}{4}\right) \sin(2\pi f_c t)$$

- Using $Cos(A+B) = CosA\ CosB - Sin\ A\ SinB$

Quadrature Phase Shift Keying (QPSK)

- The two quadrature carriers represented in terms of basis functions are,

$$\varphi_1(t) = \sqrt{\frac{2}{T_b}} Cos(2\pi f_c t), 0 \le t \le T_b$$

$$\varphi_2(t) = \sqrt{\frac{2}{T_b}} Sin(2\pi f_c t), 0 \le t \le T_b$$

$$\therefore \boldsymbol{s_1(t)} = \sqrt{\boldsymbol{E_b}} \cos\left(\frac{(2i-1)\pi}{4}\right) \boldsymbol{\varphi_1(t), 0 \le t \le T_b}$$

$$\boldsymbol{s_2(t)} = \sqrt{\boldsymbol{E_b}} \sin\left(\frac{(2i-1)\pi}{4}\right) \boldsymbol{\varphi_2(t), 0 \le t \le T_b}$$

Quadrature Phase Shift Keying (QPSK)

Signal Space Diagram of QPSK

- QPSK has four signal elements (M=4) with two dimensional orthonormal basis function (N=2)
- The four signal elements and their associated signal vectors are defined by

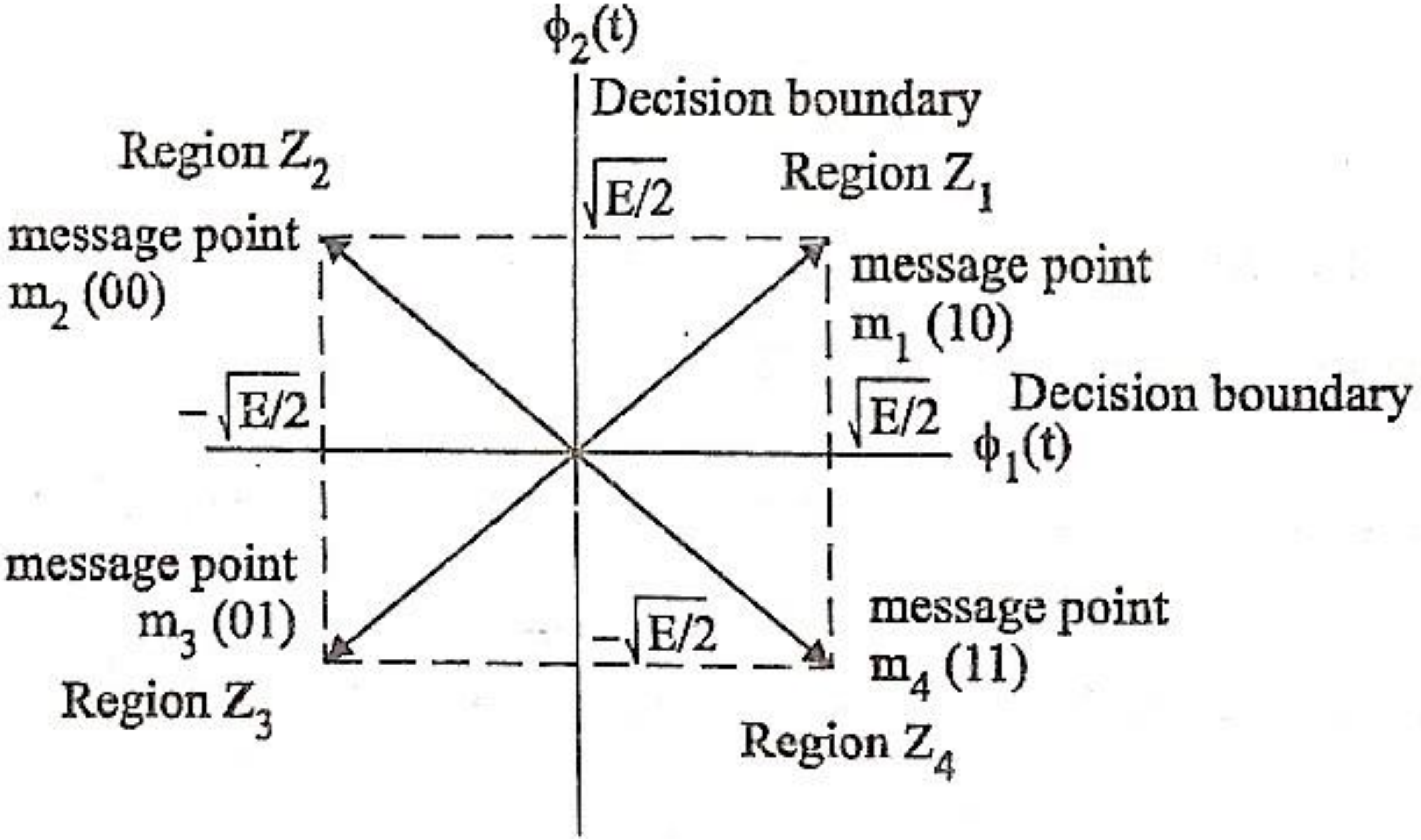
$$s_i = \begin{bmatrix} \sqrt{\boldsymbol{E_b}}\cos\left(\frac{(2i-1)\pi}{4}\right) \\ -\sqrt{\boldsymbol{E_b}}\sin\left(\frac{(2i-1)\pi}{4}\right) \end{bmatrix} \quad where, i = 1,2,3,4$$

Quadrature Phase Shift Keying (QPSK)

- The values of signal vectors are summarised below,

Gray-encoded input Dibit	Phase of QPSK signal (radians)	Coordinates of Message points	
		s_{i1}	s_{i2}
10	$\frac{\pi}{4}$	$+\sqrt{\frac{E_b}{2}}$	$-\sqrt{\frac{E_b}{2}}$
00	$\frac{3\pi}{4}$	$-\sqrt{\frac{E_b}{2}}$	$-\sqrt{\frac{E_b}{2}}$
01	$\frac{5\pi}{4}$	$-\sqrt{\frac{E_b}{2}}$	$+\sqrt{\frac{E_b}{2}}$
11	$\frac{7\pi}{4}$	$+\sqrt{\frac{E_b}{2}}$	$+\sqrt{\frac{E_b}{2}}$

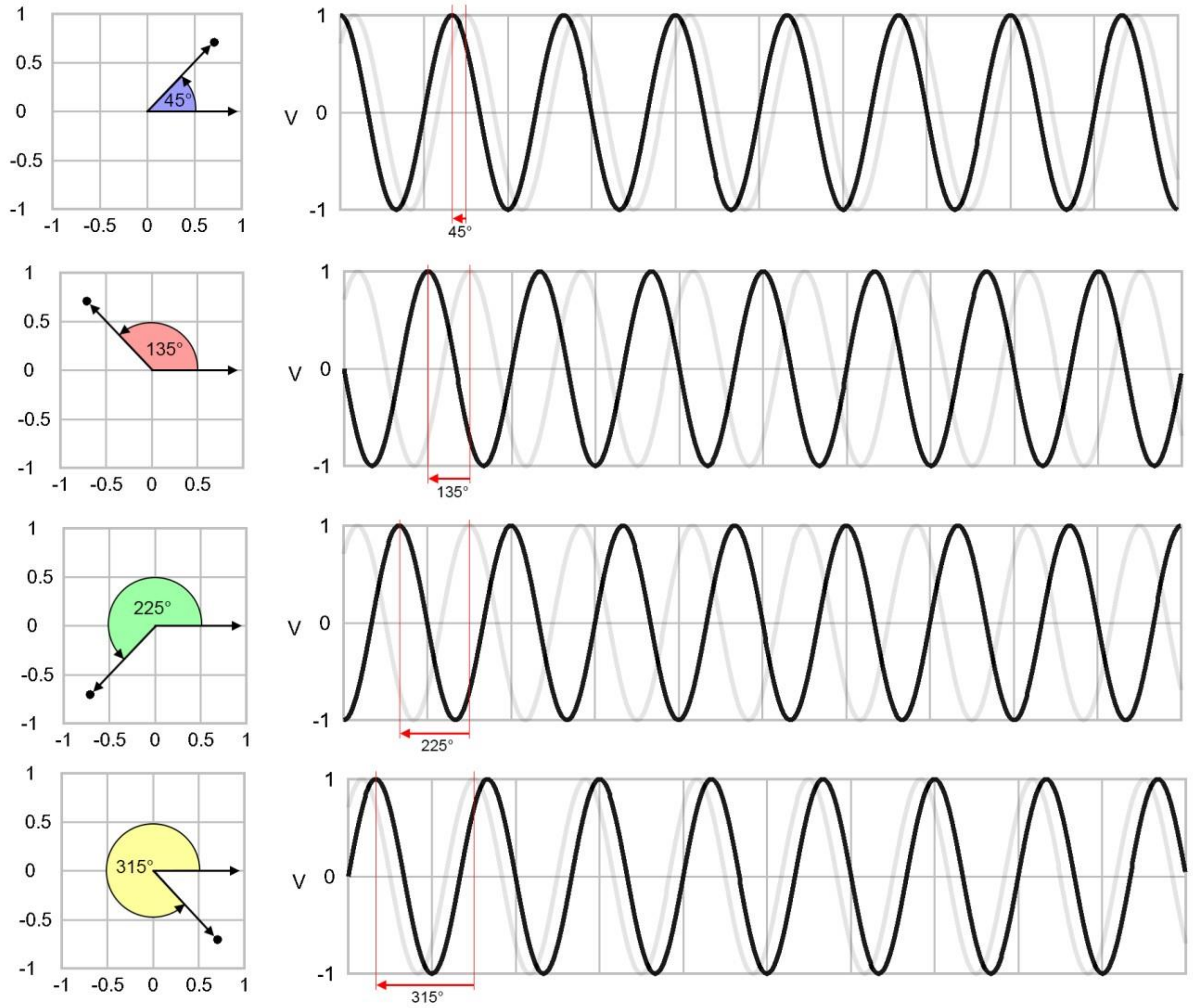
Quadrature Phase Shift Keying (QPSK)



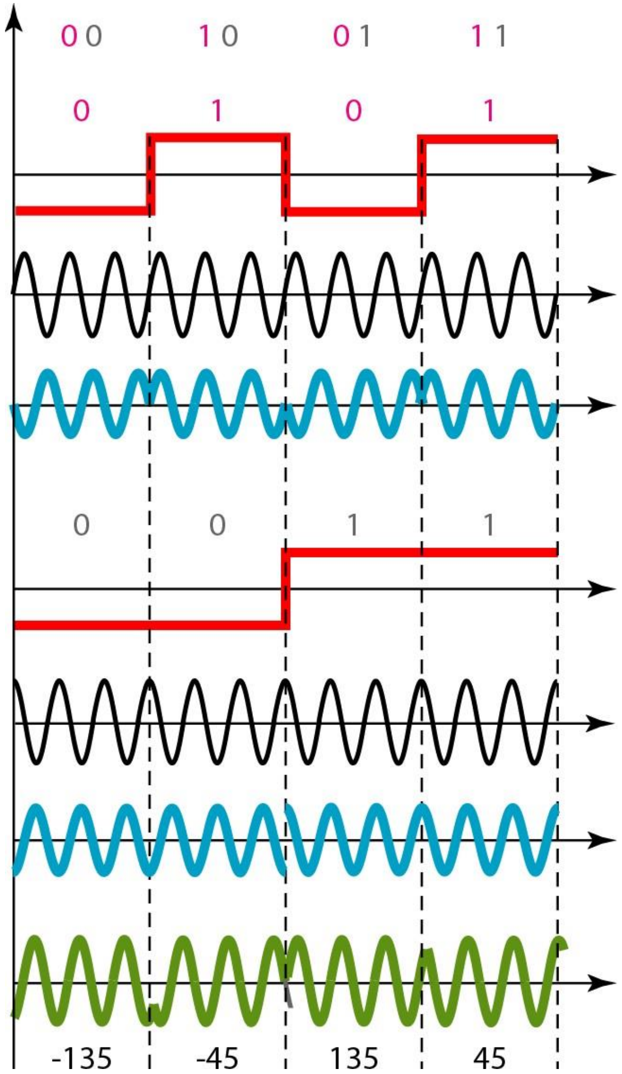
Quadrature Phase Shift Keying (QPSK)

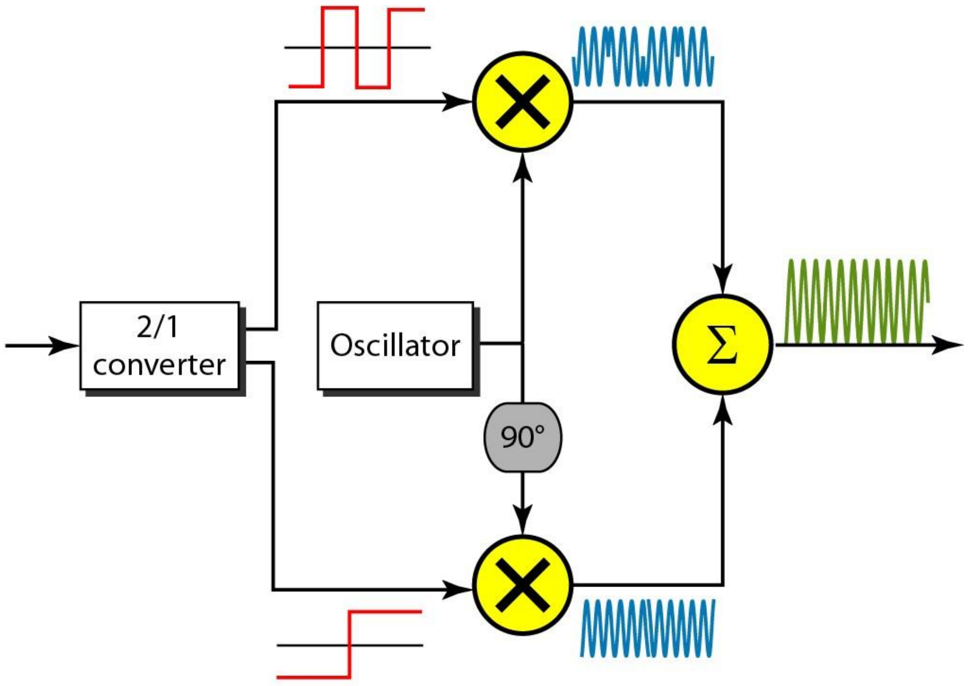
- QPSK phase shifted waveforms

- https://www.etti.unibw.de/labalive/ex



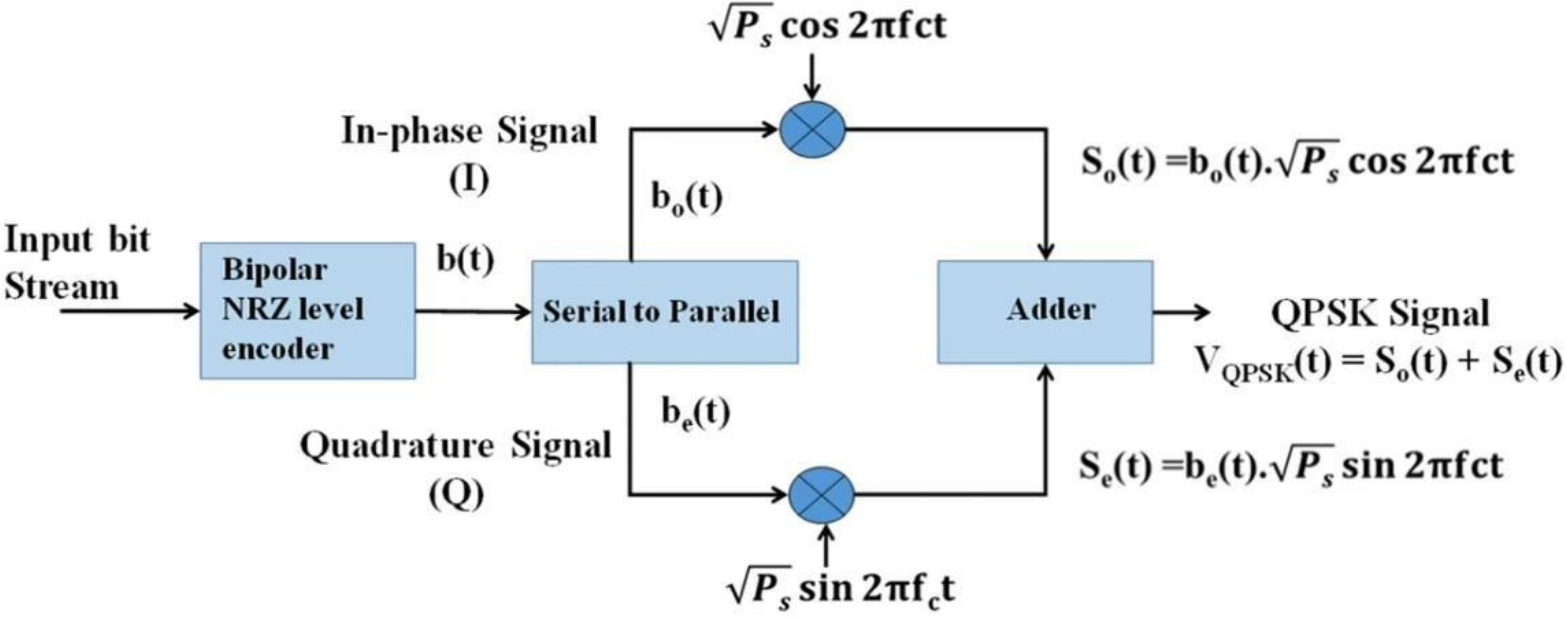
Quadrature Phase Shift Keying (QPSK)



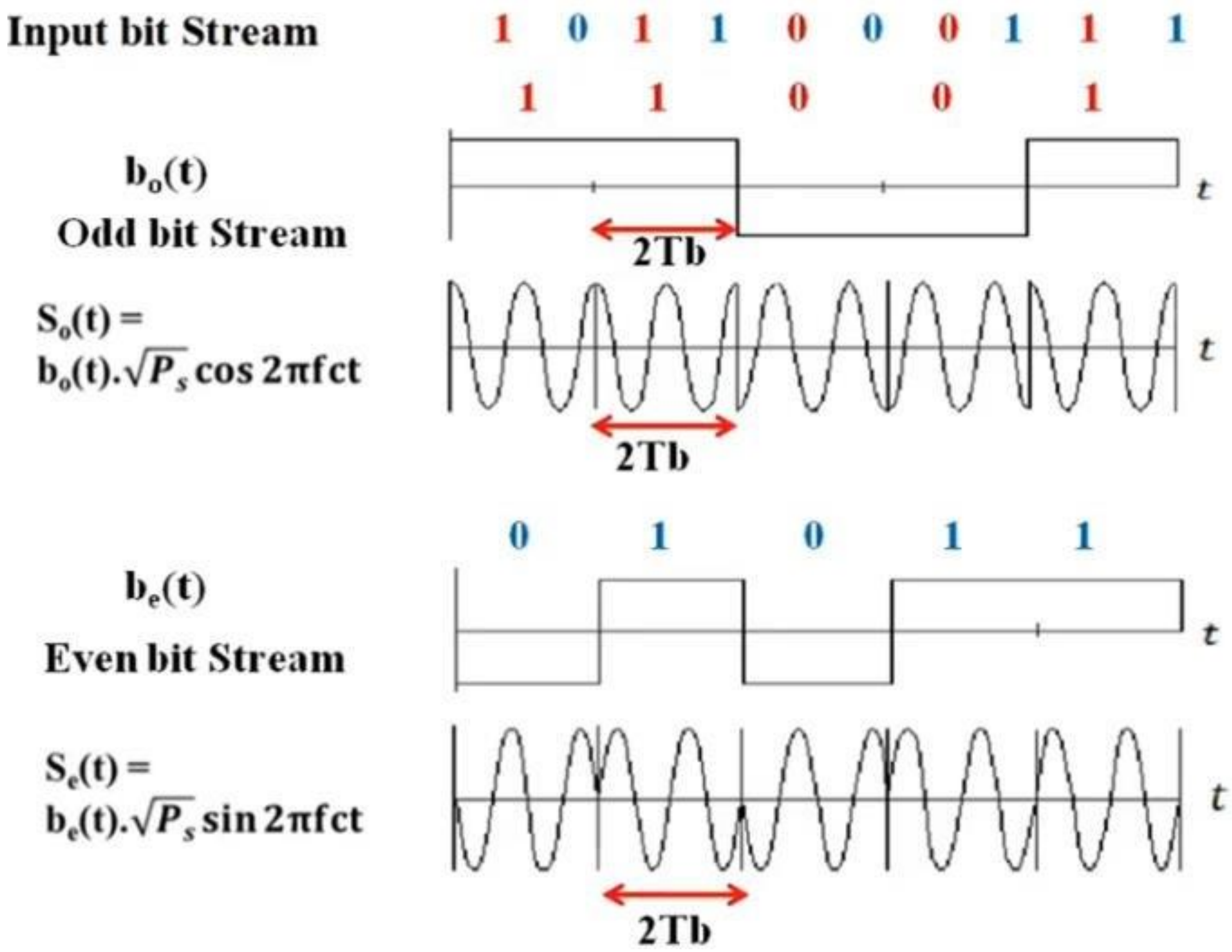


Quadrature Phase Shift Keying (QPSK)

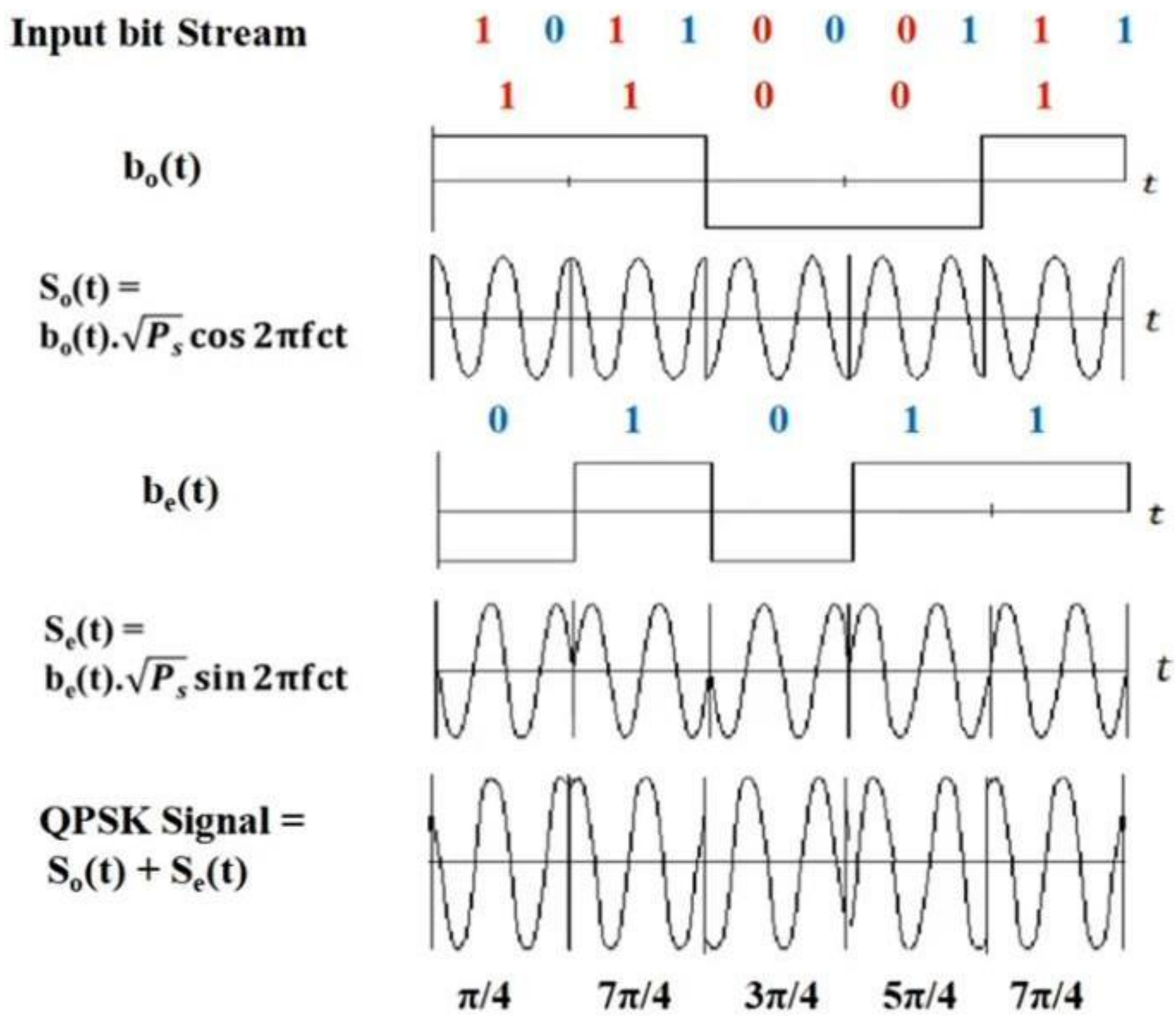
- **QPSK**



Quadrature Phase Shift Keying (QPSK)

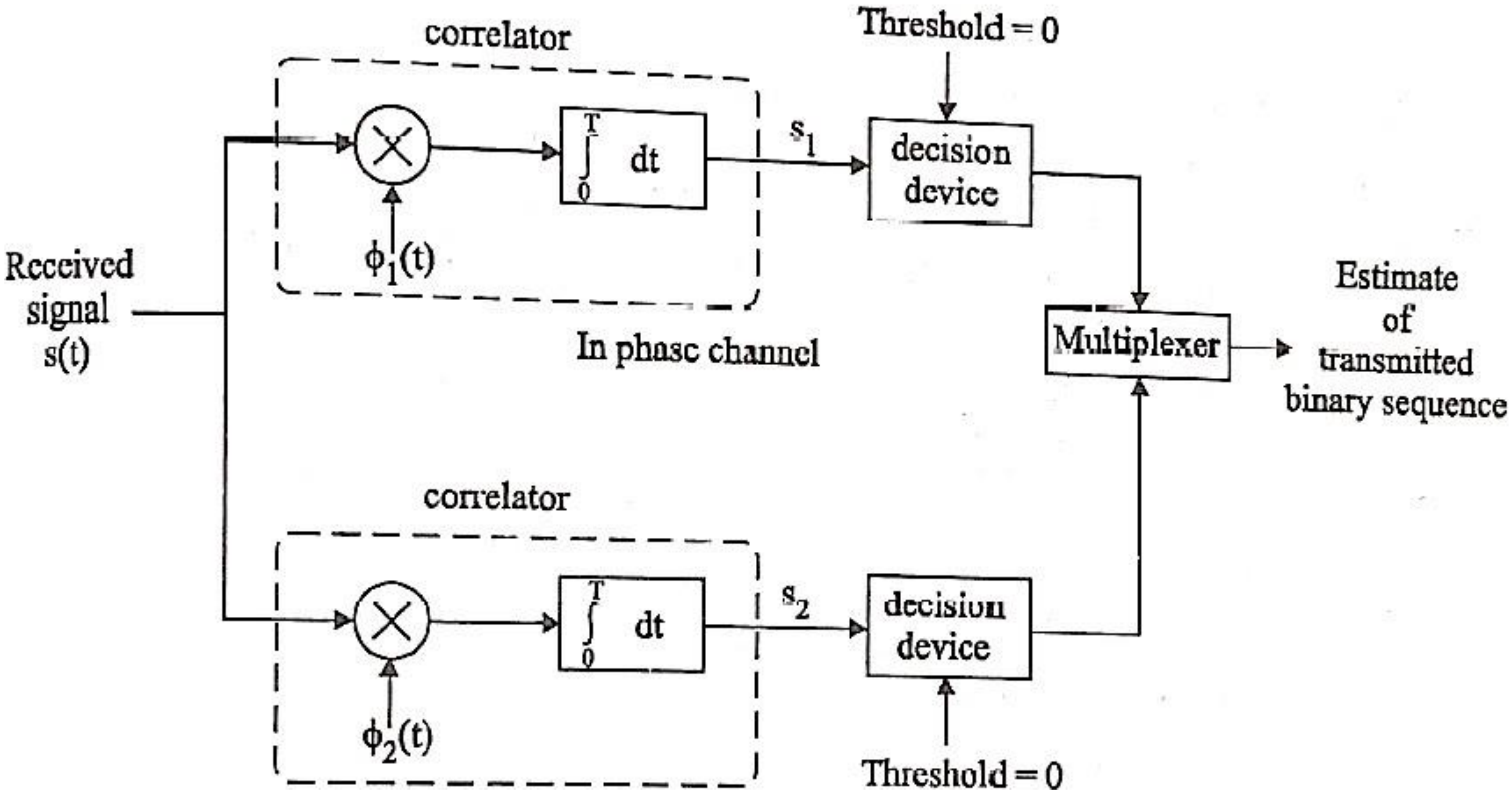


Quadrature Phase Shift Keying (QPSK)



Quadrature Phase Shift Keying (QPSK)

- **QPSK Detection/Demodulation**



Quadrature Phase Shift Keying (QPSK)

- **Probability of error in QPSK**

$$s_1 = \sqrt{E}\cos[(2i-1)\pi/4] + w_1$$

$$= \pm\sqrt{\frac{E}{2}} + w_1$$

$$s_2 = -\sqrt{E}\sin[(2i-1)\pi/4] + w_2$$

$$= \pm\sqrt{\frac{E}{2}} + w_2$$

- Where ω_1 and ω_2 are the Gaussian noise process of zero mean

Quadrature Phase Shift Keying (QPSK)

- We know that a coherent QPSK system is equivalent to two coherent BPSK system
- These two coherent BPSK system is characterised by

1. The signal energy per bit $\frac{E_b}{2}$
2. The Noise power spectral density $\frac{N_0}{2}$

- Probability of error in QPSK can be determined using 3 steps

1. By finding average probability of bit error
2. By finding average probability of correct decision
3. By finding average probability of symbol error

Quadrature Phase Shift Keying (QPSK)

Average probability of error

- The average probability of error in each channel of QPSK is

$$P' = \frac{1}{2} erfc\sqrt{\frac{E/_2}{N_0}}$$

- Where, P' = probability of error in channel
- $N_0/2$ = Noise spectral density
- E_b = Energy of a single bit

Quadrature Phase Shift Keying (QPSK)

Average Probability of correct decision

The Average Probability of correct decision resulting from both the channels working together is

$$P'_e = (1 - P')^2$$

$$P'_e = \left(1 - \frac{1}{2} erfc\sqrt{\frac{E/_2}{N_0}}\right)^2$$

$$P'_e = 1 + \frac{1}{4} erfc^2\sqrt{\frac{E/_2}{N_0}} - erfc\sqrt{\frac{E/_2}{N_0}}$$

Quadrature Phase Shift Keying (QPSK)

Average Probability of symbol error

- The Average Probability of symbol error in QPSK system is

$$P_e = 1 - P_e'$$

$$P_e = 1 - 1 - \frac{1}{4} erfc^2\sqrt{\frac{E/2}{N_0}} + erfc\sqrt{\frac{E/2}{N_0}}$$

$$P_e = erfc\sqrt{\frac{E/2}{N_0}} - \frac{1}{4} erfc^2\sqrt{\frac{E/2}{N_0}} \qquad since, \frac{E/2}{N_0} \gg 1, the\ quadratic\ term\ is\ neglected$$

$$\boldsymbol{P_e = erfc\sqrt{\frac{E/2}{N_0}}}$$

Quadrature Phase Shift Keying (QPSK)

- In QPSK since there are two bits per symbol, the transmitted energy per symbol is twice the signal energy per bit

$$E = 2E_b$$

$$\boldsymbol{P_e(QPSK) = erfc\left[\sqrt{\frac{E_b}{N_0}}\right]}$$

Quadrature Phase Shift Keying (QPSK)

- Advantages

❖ Bandwidth required by QPSK is reduced to half as compared to BPSK, BW of QPSK $= f_b$.

❖ Since there is not much amplitude fluctuations, carrier power almost remains constant.

❖ Information transmission rate is higher because of reduced bandwidth.

- Disadvantages

❖ Low noise immunity.

❖ Waveforms are not smoothen than BPSK.

❖ Abrupt phase shift at $\pi/2$ or π occurs.

❖ Interchannel interference is large due to sidelobes.