A Notion of Spread Spectrum

- Spread spectrum can be considered as a method of *“camouflaging”* the information-bearing signal

- It can provide protection against externally generated interfering (jamming) signals with finite power

- Protection is done by purposely making the information-bearing signal occupy a bandwidth far in excess of the minimum required bandwidth

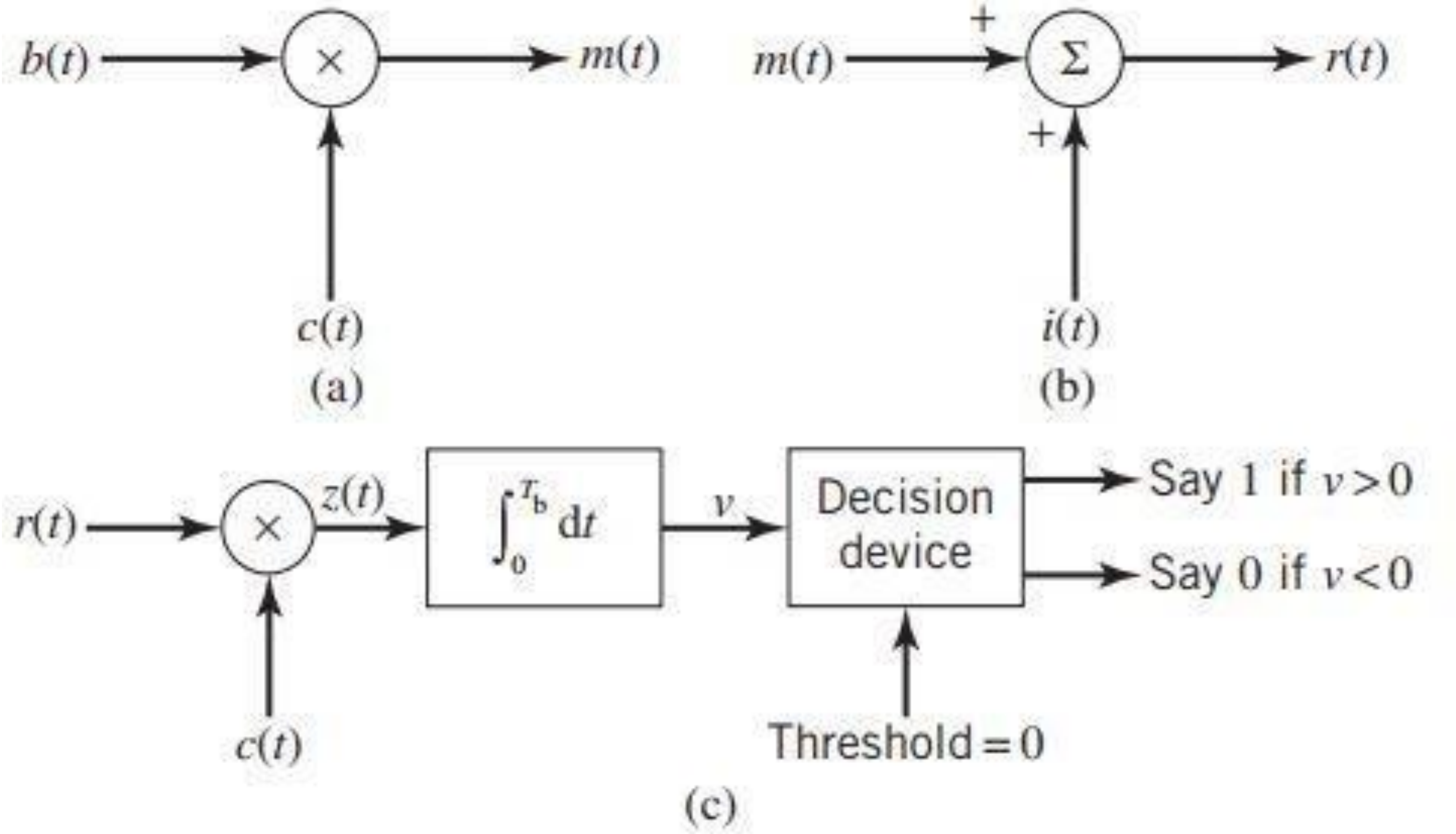
- One method of widening the bandwidth is by the use of *modulation*

A Notion of Spread Spectrum

- Let $\{b_k\}$ denote a binary data sequence and $\{c_t\}$ denote a pseudo-noise (PN) sequence
- Let the waveforms $b(t)$ and $c(t)$ denote their respective polar nonreturn-to-zero representations namely ± 1
- $b(t)$ is referred as the information-bearing (data) signal and $c(t)$ as the PN signal
- The desired modulation is achieved by applying the data signal $b(t)$ and PN signal $c(t)$ to a *product modulator or multiplier* as shown in figure
- Thus, if $b(t)$ is narrowband and $c(t)$ is wideband, *the product (modulated) signal will have a spectrum that is nearly same as that of the wideband PN signal*

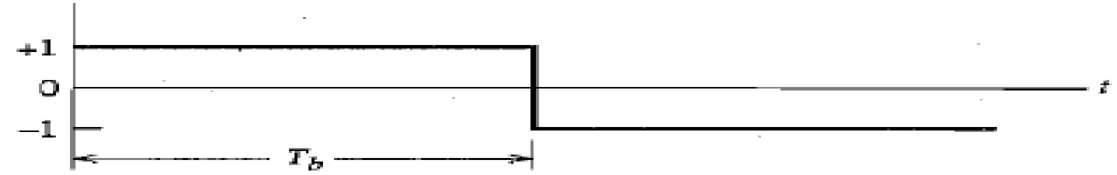
A Notion of Spread Spectrum

- Idealized model of baseband spread-spectrum system: (a) Transmitter, (b) Channel, (c) Receiver

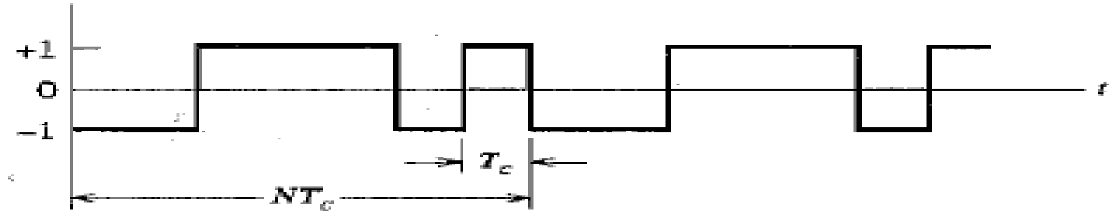


A Notion of Spread Spectrum

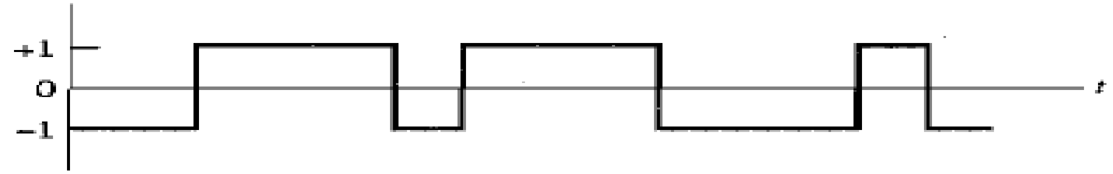
- Illustrating the waveforms in the transmitter



(a) Data signal $b(t)$



(b) Spreading code $c(t)$



(c) Product signal $m(t)$

A Notion of Spread Spectrum

- The transmitted signal is expressed as: $m(t) = c(t)b(t)$
- The received signal $r(t)$ consists of transmitted signal $m(t)$ plus an additive *interference* $i(t)$
- Thus, $r(t) = m(t) + i(t) = c(t)b(t) + i(t)$
- The multiplier output in the receiver is given by:

$$z(t) = c(t)r(t) = c^2(t)b(t) + c(t)i(t)$$

- Thus, the data signal $b(t)$ is multiplied twice by the PN signal $c(t)$ and the unwanted signal $i(t)$ is multiplied only once by $c(t)$

A Notion of Spread Spectrum

- The PN signal $c(t)$ alters between +1 and −1 and is destroyed when squared

$$c^2(t) = 1 \ for\ all\ t$$

- Accordingly, we may simplify $z(t) = b(t) + c(t)i(t)$

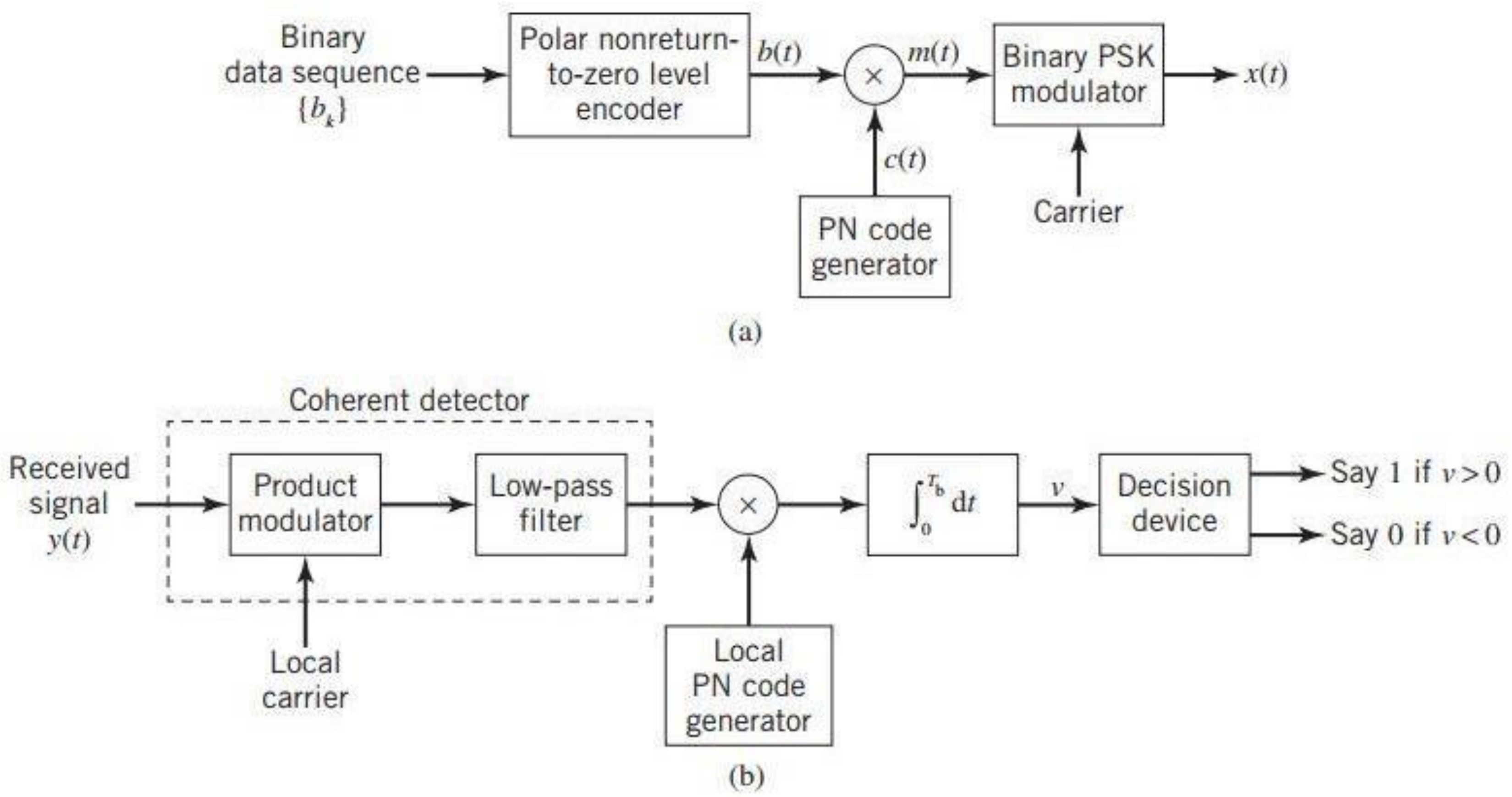
- Thus, the data signal $b(t)$ is reproduced at the multiplier output in the receiver, except for the additive interference $c(t)i(t)$

- By applying a baseband (low-pass) filter with a bandwidth just occupying $b(t)$, most of the power in the spurious component $c(t)\,i(t)$ is filtered out

- Thus, the effect of $i(t)$ is significantly reduced at the receiver output

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- Direct-sequence spread coherent phase-shift keying: (a) Transmitter, (b) Receiver



Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- The spread spectrum described in the previous section is referred as *direct-sequence spread spectrum*

- In order to use this technique in passband transmission over a satellite channel, we incorporate *coherent binary phase-shift keying* (PSK) as shown in figure

- The first stage consists of a product modulator or a multiplier with data signal $b(t)$ and PN signal $c(t)$

- The second stage consists of a binary PSK modulator

- The transmitted signal $x(t)$ is thus a direct-sequence spread binary phase-shift-keyed (DS/BPSK) signal

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- The phase modulation $\theta(t)$ of $x(t)$ has one of two values, 0 and π

- The values depend on the polarities of the message signal $b(t)$ and PN signal $c(t)$ at time t in accordance to the table given below:

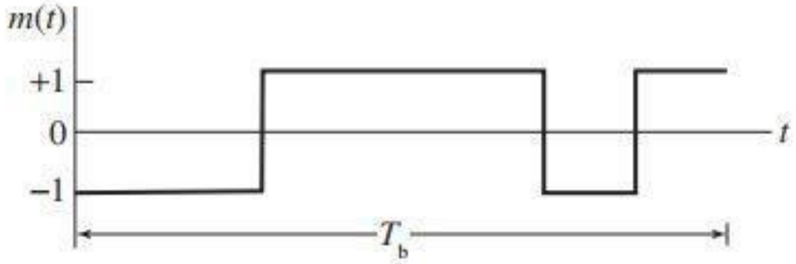
- Truth table for phase modulation $\theta(t)$,radians

		Polarity of data sequence $b(t)$ at time t	
		+	−
Polarity of PN	+	0	π
sequence $c(t)$ at time t	−	Π	0

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

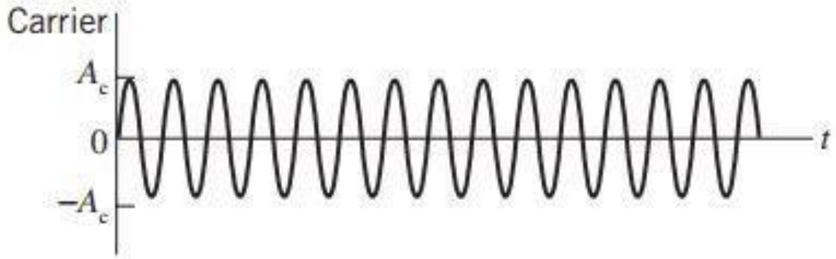
Model for Analysis

- (a) Product signal $m(t) = c(t)\mathrm{b}(t)$



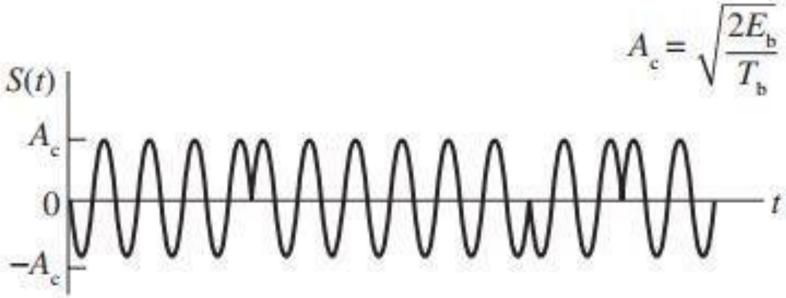
(a)

- (b) Sinusoidal Carrier



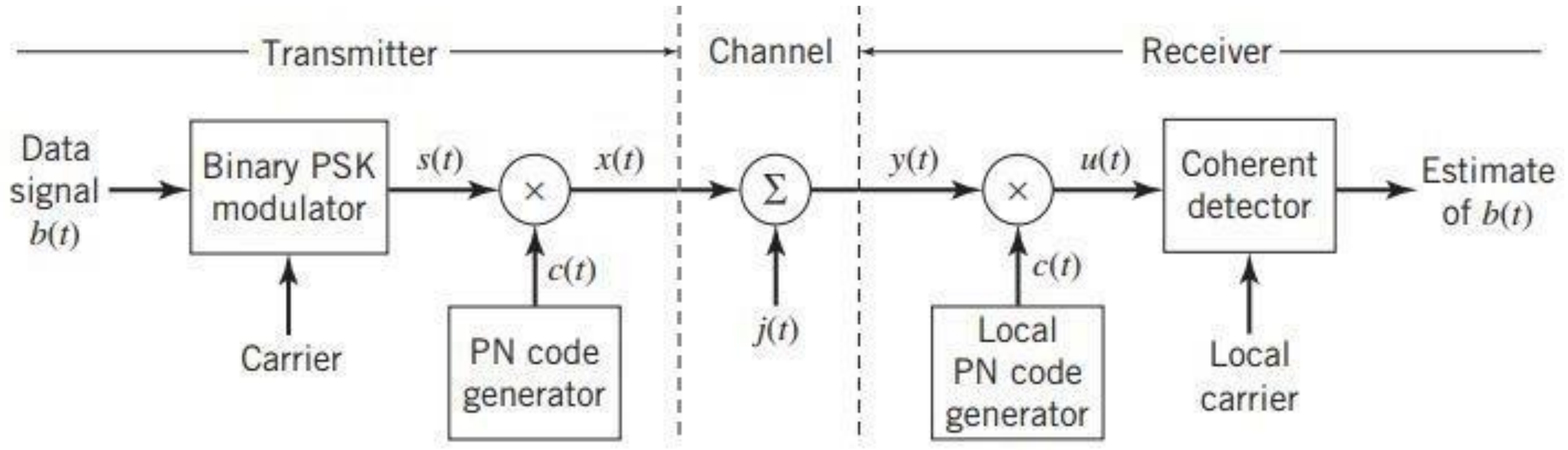
(b)

- (c) DS/BPSK signal



(c)

Model for direct-sequence spread binary PSK system



Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

Synchronization

- For the proper operation of a spread-spectrum communication system, the locally generated PN sequence to despread the received signal needs to be *synchronized*
- The synchronization problem consists of two parts: *acquisition and tracking*
- In acquisition, the two PN codes are aligned to within a fraction of the chip in as short a time as possible
- Once the incoming PN code has been acquired, tracking, or *fine* synchronization takes place

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

Signal-Space Dimensionality Processing Gain

- Here, we develop a signal-space representations of the transmitted signal and the interfering signal (jammer)

- Consider a set of orthonormal basis functions:

$$\phi_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}}\cos(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & otherwise \end{cases}$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}}\sin(2\pi f_c t), & kT_c \le t \le (k+1)T_c \\ 0, & otherwise \end{cases}$$

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- Here, $k = 0,1,\ldots, N-1$, T_c is the chip duration and N is the number of chips per bit
- The transmitted signal $x(t)$ for the interval of an information is given by:

$$x(t) = c(t)s(t)$$

$$= \pm\sqrt{\frac{2E_b}{T_b}}c(t)\cos(2\pi f_c t)$$

$$= \pm\sqrt{\frac{E_b}{\mathrm{N}}}\sum_{k=0}^{N-1} c_k \phi_k(t), \quad 0 \le t \le T_b$$

- E_b is the signal energy per bit, T_b is the bit duaration, $+$ sign corresponds to information bit 1, and the $-$ sign corresponds to information bit 0

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- The code sequence $\{c_o, c_1, \ldots, c_k\}$ denotes the PN sequence, with $c_k = \pm 1$
- Now, consider an interfering signal or a jammer $j(t)$ which likes to place all its available energy in exactly same N-dimensional signal space as of $x(t)$
- The jammer $j(t)$ is given by:

$$j(t) = \sum_{k=0}^{N-1} \mathrm{j}_k \phi_k(t) + \sum_{k=0}^{N-1} \tilde{j_k}\, \tilde{\phi}_k(t), \qquad 0 \le t \le T_b$$

where, $j_k = \int_0^{T_b} j(t)\phi_k(t)dt, \;\; k = 0,1,\ldots,N$

and, $\tilde{j_k} = \int_0^{T_b} j(t)\tilde{\phi}_\mathrm{k}(\mathrm{t})dt, \; k = 0,1,\ldots,N$

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- The average power of the interference $j(t)$ will be given by:

$$J = \frac{1}{T_b}\int_0^{T_b} j^2(t) \quad = \frac{1}{T_b}\sum_{k=0}^{N-1} j_k^2 + \frac{1}{T_b}\sum_{k=0}^{N-1} \tilde{j}_k^2 \quad = \frac{2}{T_b}\sum_{k=0}^{N-1} j_k^2$$

- On further calculation and simplifications, we get, the *output signal-to-noise ratio* as:

$$(SNR)_O = \frac{2E_b}{JT_c}$$

- And, the *input signal-to-noise ratio* is expressed as:

$$(SNR)_I = \frac{E_b/T_b}{J}$$

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- We may thus express the output signal-to-noise ratio in terms of input signal-to-noise ratio as:

$$(SNR)_O = \frac{2T_b}{T_c}(SNR)_I$$

- The term *processing gain* (PG) is defined as the gain in SNR obtained by the use of spread spectrum is given by: $PG = \frac{T_b}{T_c}$

- PG represents the gain achieved by processing a spread-spectrum signal over an unspread signal

- We may finally write the equivalent form (in terms of dB) given by:

$$10\log_{10}(SNR)_O = 10\log_{10}(SNR)_I + 3 + 10\log_{10}(PG)dB$$

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

Probability of error

- The probability of error for a direct sequence BPSK (DS/BPSK) binary system is given by:

$$P_e \cong \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

- This formula is appropriate for DS/BPSK binary systems with large spread factor N
- *Antijam Characteristics:* We compare the formula for P_e for a coherent binary PSK system with $P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$
- Thus, on comparing, we have: $\frac{N_0}{2} = \frac{JT_c}{2}$

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- Also, we know, the signal energy per bit $E_b = PT_b$, where P is the average signal power and T_b is the bit duration
- Thus, the signal energy per bit-to-noise spectral density ratio will be:

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right)$$

- Also, the processing gain PG can be reformulated as:

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

- This ratio $\frac{J}{P}$ is termed as the *jamming margin*

Direct-Sequence Spread Spectrum With Coherent Binary Phase-Shift Keying

- The jamming margin and the processing gain, both expressed in decibels are related by:

$$(Jamming\ margin)_{dB} = (Processing\ gain)_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{min}$$

- $\left(\frac{E_b}{N_0}\right)_{min}$ is the minimum value needed to support a prescribed average probability of error

Numerical

- A spread spectrum communication system has the following parameters

  Bit duration = 4.095 ms, Chip duration = 1 μs

For satisfactory reception, assume the average error probability is approximately equal to 10^{-5}

Calculate the PG, Length of the PN sequence, Number of flip-flops in the shift register and the Jamming margin(dB).