Digital Modulation Scheme

Introduction: Analog Transmission

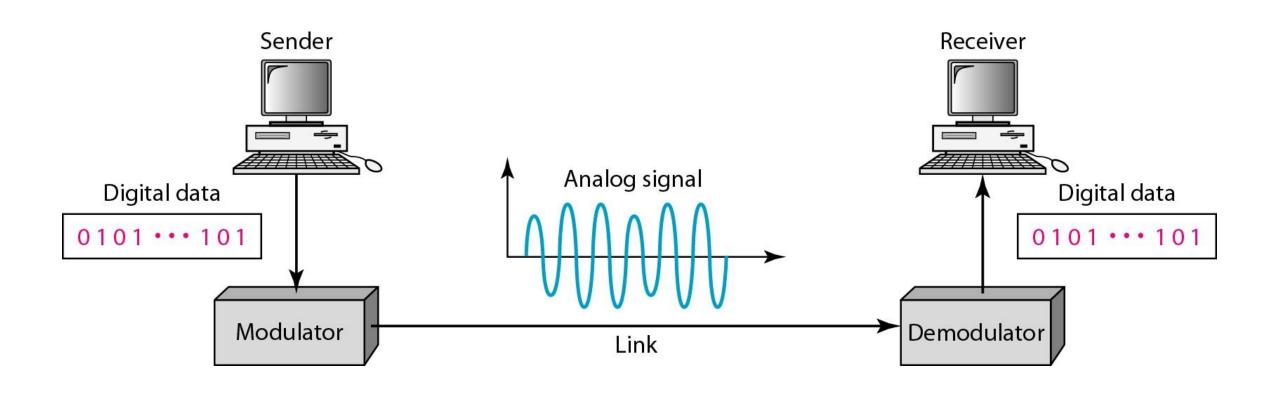
Digital-to-analogue

• Digital-to-analog conversion is the process of changing one of the characteristics of an analog signal(carrier) based on the information in digital data(message).

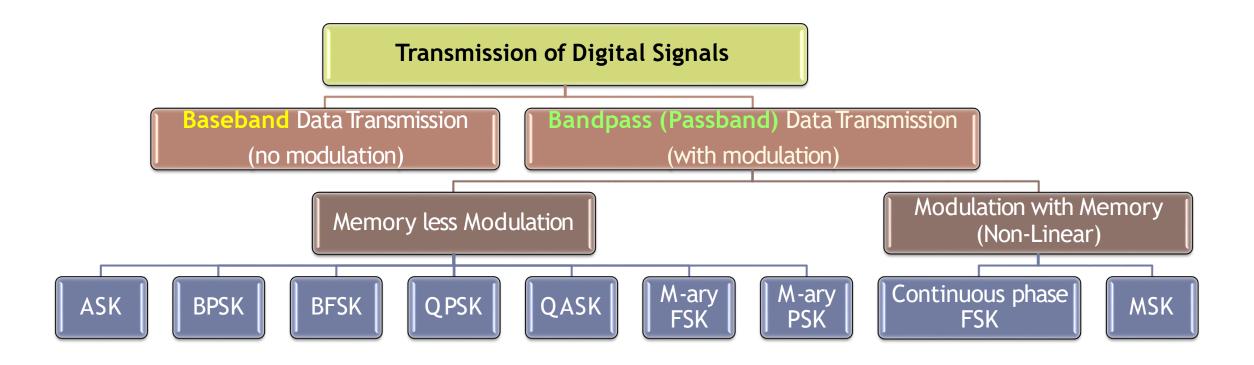
Aspects of Digital to Analogue

- Digital data needs to be carried on an analog signal.
- A carrier signal (frequency f_c) performs the function of transporting the digital data in an analog waveform.
- The analog carrier signal is manipulated to uniquely identify the digital data being carried.

Introduction: Analog Transmission



Types of transmission of Digital Signals



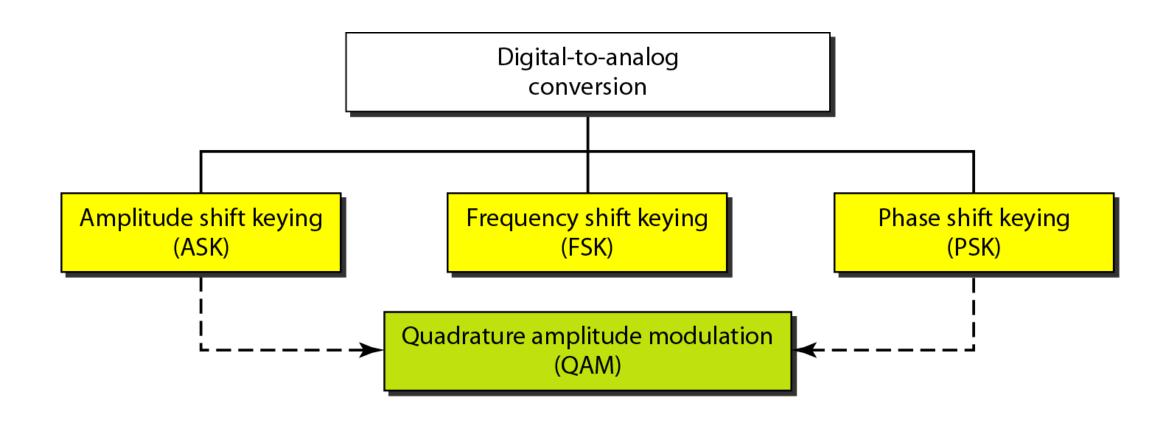
- Memoryless modulation: It is a scheme in which the waveform transmitted in any time interval does not depend on previous digital symbols.
- Modulation with memory: It is a scheme in which the waveform transmitted in any time interval depends on one or more previous digital symbols.

Coherent detection

- Local carrier generated at the receiver is phase locked with the carrier at the transmitter.
 - It is also called synchronous detection.

Non-coherent detection

- Receiver carrier need not be phase locked with the transmitter carrier.
 - It is also called envelope detection.



Bit rate, N, is the number of bits per second (bps). Baud rate is the number of signal elements per second (bauds).

In the analog transmission of digital data, the signal or baud rate is less than or equal to the bit rate.

$$S = N \times \frac{1}{r}$$
 bauds

Where r is the number of data bits per signal element.

• An analog signal carries 4 bits per signal element. If 1000 signal elements are sent per second, find the bit rate.

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Solution:

• In this case, r = 4, S = 1000, and N is unknown. We can find the value of N from

$$S = N \times \frac{1}{r}$$
 or $N = S \times r = 1000 \times 4 = 4000 \text{ bps}$

• An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

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Solution:

- In this example, S = 1000, N = 8000, and r and L are unknown.
- We find first the value of r and then the value of L.

$$S = N \times \frac{1}{r} \longrightarrow r = \frac{N}{S} = \frac{8000}{1000} = 8 \text{ bits/baud}$$

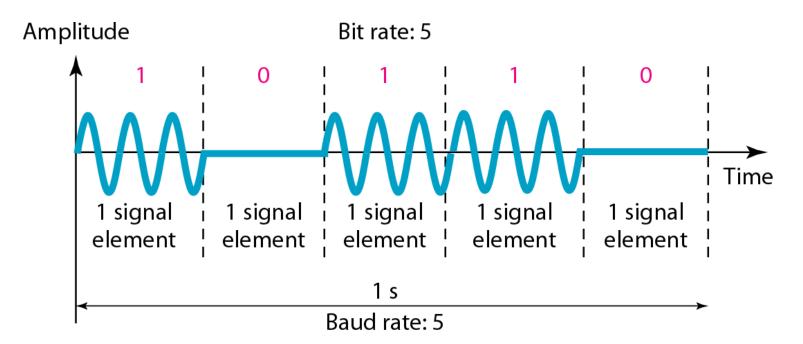
$$r = \log_2 L \longrightarrow L = 2^r = 2^8 = 256$$

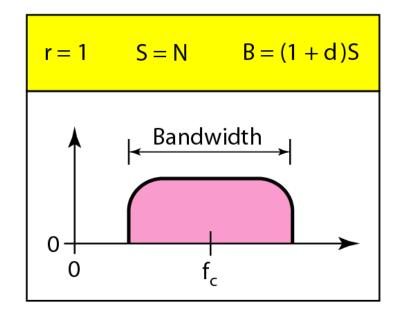
- ASK is implemented by changing the amplitude of a carrier signal to reflect amplitude levels in the digital signal.
- The simplest version is on-off keying (OOK).
- In OOK, either bursts of a carrier wave are transmitted or nothing is transmitted depending whether the input message signal is 1 or 0.
- Transmission of pulse '1' is represented by the presence of a pulse.
- Transmission of pulse '0' is represented by the absence of a pulse.
- It is rarely used since it does not give a satisfactory value for probability of error.
- The line encoding will determine the values of the analog waveform to reflect the digital data being carried.

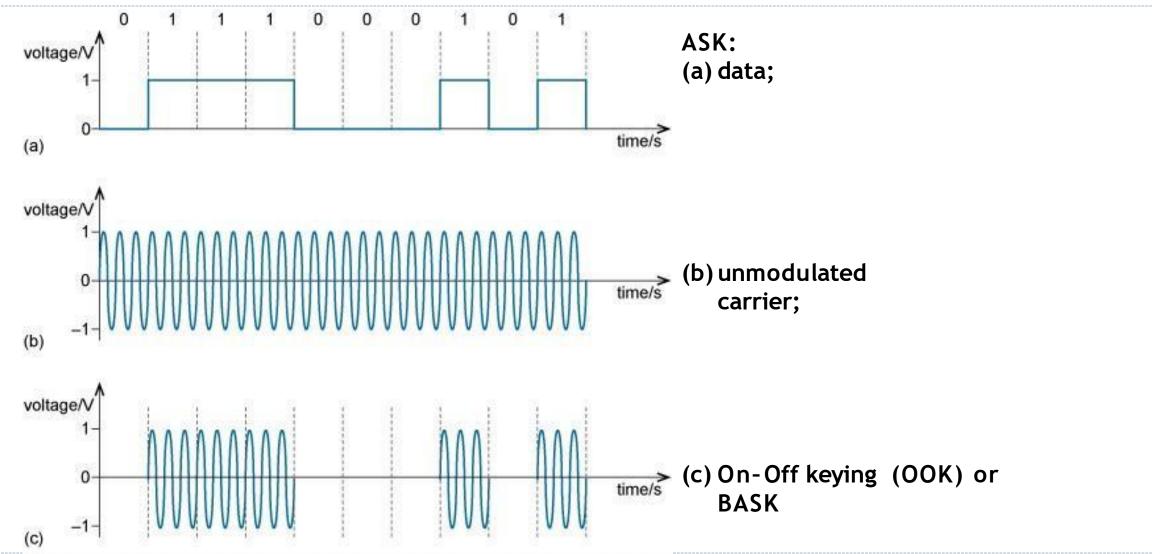
The bandwidth B of ASK is proportional to the signal rate S.

$$B = (1+d)S$$

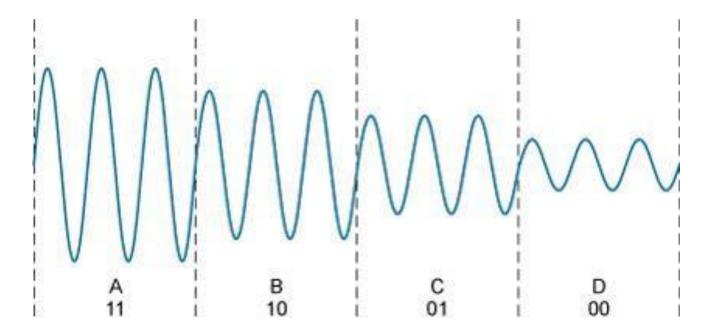
• "d" is due to modulation and filtering, lies between 0 and 1.





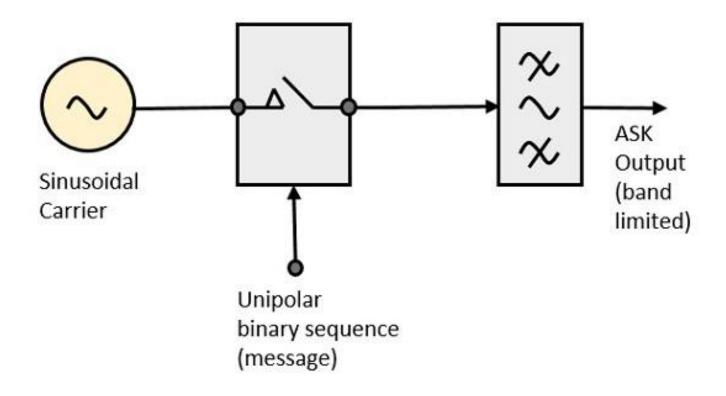


- The figure below shows ASK with four possible amplitude levels, or four symbols.
- With four symbols available, each symbol can be uniquely represented with a two-bit binary number:
- This is because there are just four possible two-bit binary numbers: 11,10,01 and 00.



ASK Modulator

ASK Generation method



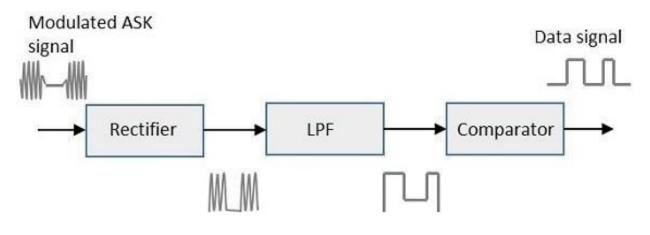
- 1. The carrier generator, sends a continuous high-frequency carrier.
- 2. The binary sequence from the message signal makes the unipolar input to be either High or Low.
- 3. The high signal closes the switch, allowing a carrier wave.
- 4. Hence, the output will be the carrier signal at high input.
- 5. When there is low input, the switch opens, allowing no voltage to appear.
- 6. Hence,the output will be low.
- 7. The band-limiting filter, shapes the pulse depending upon the amplitude and phase characteristics of the band-limiting filter or the pulse-shaping filter.

ASK Demodulator:

- There are two types of ASK Demodulation techniques. They are –
- 1. Asynchronous ASK Demodulation/Non-Coherent detection
- 2. Synchronous ASK Demodulation/Coherent detection

- The clock frequency at the transmitter when matches with the clock frequency at the receiver, it is known as a **Synchronous method**, as the frequency gets synchronized.
- Otherwise, it is known as Asynchronous.

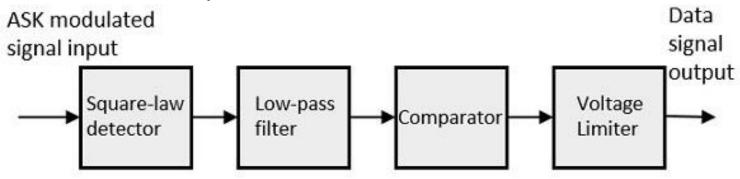
Asynchronous ASK Demodulation/Non-Coherent detection



Asynchronous ASK detector

- The modulated ASK signal is given to the half-wave rectifier, which delivers a positive half output.
- The low pass filter suppresses the higher frequencies and gives an envelope detected output from which the comparator delivers a digital output.

Synchronous ASK Demodulation/Coherent detection



Synchronous ASK detector

- The ASK modulated input signal is given to the Square law detector.
- A square law detector is one whose output voltage is proportional to the square of the amplitude modulated input voltage.
- The low pass filter minimizes the higher frequencies.
- The comparator and the voltage limiter help to get a clean digital output.

W aveform Amplitude Coefficient

Generally an anlog signal is represented by

$$s(t) = A \cos \omega_c t$$

- Where, A is the peak value of the signal
- But peak value of a sinusoidal waveform $=\sqrt{2}$ times the root mean square value (rms).

$$s(t) = \sqrt{2}A_{rms} \cos \omega_c t$$

$$s(t) = \sqrt{\frac{2A^2}{rms}} \cos \omega_c t$$

• Where, A_{rms}^2 represents the average power 'P' normalised to 1Ω

$$s(t) = \sqrt{2P} \cos \omega_c t$$

$$s(t) = \sqrt{2\frac{E_b}{T_b}} \cos \omega_c t$$

• Since,
$$P_{watts} = \frac{E_b Joules}{T_b seconds}$$

Signal representation of ASK

ASK signal is represented as

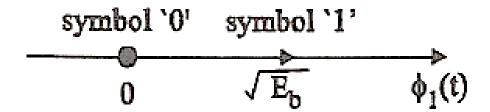
$$s(t) = \sqrt{2\frac{E_b}{T_b}} \cos \omega_c t$$

$$= \sqrt{E_b}.\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t = \sqrt{E_b} \varphi_1(t) \text{ where, } 0 \le t < T_b$$

$$S_{ASK}(t) = \sqrt{E_b} \varphi_1(t)$$

• Where $\varphi_1(t)$ is the basis function

- Signal Space diagram of ASK
- Signal space diagram or signal constellation diagram represents the possible message symbols that may be selected by a given modulation scheme as points on a complex plane.
- Measured constellation diagrams can be used to determine the type of interference and distortion in a signal.
- It is a collection of 'M' message points in 'N' dimensional Euclidean space.



Probability of error in ASK

In ASK, binary '1' is represented as

$$s_1(t) = A \cos w_c t$$

$$s_1(t) = \sqrt{2P} \cos 2 \pi f_c t$$

and binary '0' is represented as

 $s_2(t) = 0$ (no signal is transmitted)

where P is the normalized power = $\frac{A^2}{2}$

The probability of error of optimum filter is given by

$$P_{e} = \frac{1}{2} \operatorname{erfc} \left[\frac{s_{1}(t) - s_{2}(t)}{2\sqrt{2} \sigma} \right]$$

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$$P_e = \frac{1}{2} \operatorname{erfc} \left[\frac{s_1(t) - s_2(t)}{2\sqrt{2} \sigma} \right]$$

where
$$\left[\frac{s_1(t) - s_2(t)}{\sigma}\right]_{\text{max}}^2 = \int_{0}^{T_b} \frac{g_m^2(t)}{\text{PSD of noise}} dt$$

where $g_m(t)$ is the modulated signal

$$= \int_{0}^{T_{b}} \frac{g_{m}^{2}(t)}{N_{o}/2} dt$$

$$= \frac{2}{N_{o}} \int_{0}^{T_{b}} g_{m}^{2}(t) dt$$

$$= \frac{2}{N_{o}} \int_{0}^{T_{b}} s_{1}^{2}(t) dt$$

$$= \frac{2}{N_o} \int_{0}^{T_b} [A \cdot \cos(2\pi f_c t)]^2 dt$$

$$= \frac{2}{N_o} \int_{0}^{T_b} A^2 \cos^2(2\pi f_c t) dt$$

$$= \frac{2A^2}{N_o} \int_{0}^{T_b} \cos^2(2\pi f_c t) dt$$

$$= \frac{2A^2}{N_o} \int_{0}^{T_b} \frac{1 + \cos 4\pi f_c t}{2} dt$$

$$= \frac{2A^2}{N_o} \int_0^{T_b} \frac{1}{2} + \frac{\cos 4\pi f_c t}{2} dt$$

$$= \frac{2A^2}{2N_o} \int_0^{T_b} dt$$

$$= \frac{2A^2}{2N_o} \int_0^{T_b} dt$$

$$= \frac{s_1(t) - s_2(t)}{\sigma} \Big]^2 = \frac{A^2 T_b}{N_o}$$
We know that $P = \frac{A^2}{2}$

$$\frac{E_b}{T_b} = \frac{A^2}{2}$$

$$A^2 = \frac{2E_b}{T_b}$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

$$\therefore \left[\frac{s_1(t) - s_2(t)}{\sigma} \right]^2 = \frac{2E_b}{T_b} \cdot \frac{T_b}{N_o} = \frac{2E_b}{N_o}$$

$$\frac{s_1(t) - s_2(t)}{\sigma} = \sqrt{2E_b/N_o}$$

The probability of error of optimum filter is given by

$$P_{e} = \frac{1}{2} \operatorname{erfc} \left[\frac{s_{1}(t) - s_{2}(t)}{2\sqrt{2} \sigma} \right]$$

$$= \frac{1}{2} \operatorname{erfc} \left[\frac{1}{2\sqrt{2}} \cdot \sqrt{\frac{2E_b}{No}} \right]$$

$$= \frac{1}{2} \operatorname{erfc} \left[\frac{1}{2} \sqrt{\frac{E_b}{N_o}} \right]$$

$$\therefore P_{e_{(ASK)}} = \frac{1}{2} erfc \left[\sqrt{\frac{E_b}{4No}} \right]$$

Advantages

- ASK method is very simple to implement
- Complexity of the circuit is minimum

Disadvantages

- Since ASK has fluctuations in amplitude, it is not suitable for passband wireless communications.
- It is very much sensitive to noise

Amplitude Shift Keying (ASK)- Example

• A digital communication system having a bandwidth of 100 kHz which spans from 200 kHz to 300 kHz. What are the carrier frequency, bit rate and baud rate, if we use BASK technique with d=1?