Module:1 Sampling Process

- Block diagram of a digital communication system, bandwidth of signals.
- Sampling theorem - quadrature sampling of bandpass signals,
- Reconstruction of a message from its samples,
- Practical aspects of sampling and signal recovery.

Advantages of Digital Communication

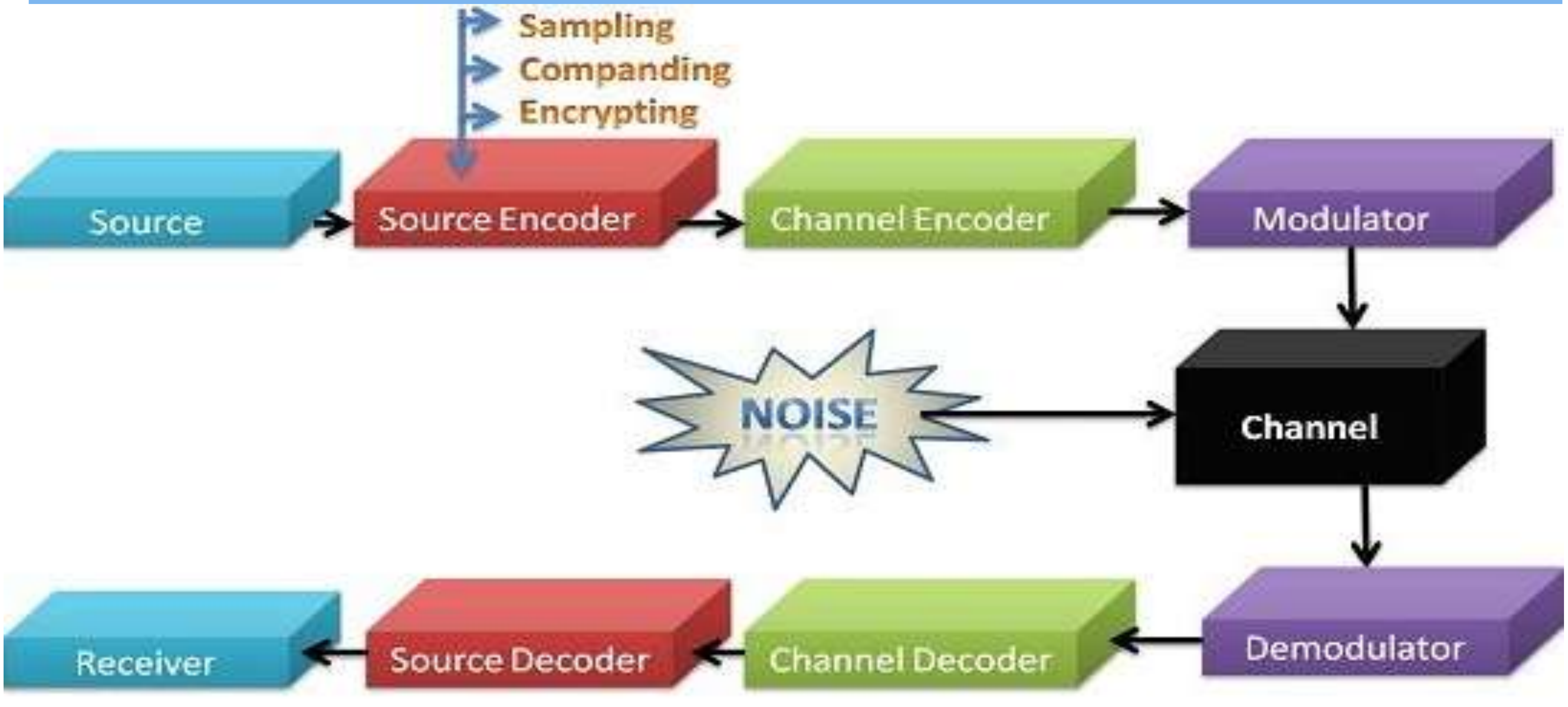
- Noise immunity
- Error detection and correction
- Ease of multiplexing
- Integration of analog and digital data
- Use of signal regenerators
- Data integrity and security
- Ease of evaluation and measurements
- More suitable for processing

Disadvantages of Digital Communication

- More bandwidth requirement
- Need of precise time synchronization
- Additional hardware for encoding/decoding
- Integration of analog and digital data
- Sudden degradation in
- Incompatible with existing analog facilities

Digital communication System Block diagram

Figure 1:Digital communication Block diagram



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Basic Digital Communication Transformations

Formatting/Source Coding

•Transforms source information into digital symbols (digitization)

•Selects compatible waveforms (matching function)

•Introduces redundancy which facilitates accurate decoding despite errors

•It is essential for reliable communication

Modulation/Demodulation

•Modulation is the process of modifying the information signal to facilitate transmission

•Demodulation reverses the process of modulation. It involves the detection and retrieval of the info signal

Types

• Coherent: Requires a reference info for detection

• Non coherent: Does not require reference phase information

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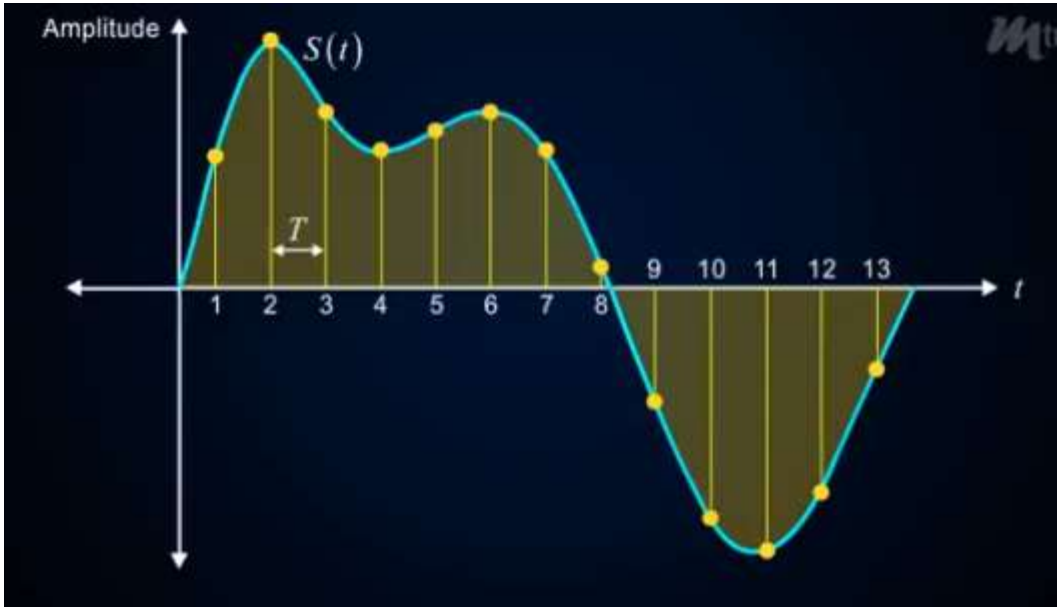
Coding/Decoding

- Translating bits to transmitter data symbols
- Techniques used to enhance information of signal so that they are less vulnerable to channel impairment (e.g. noise, fading, jamming, interference)

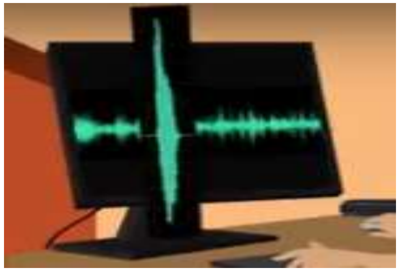
Waveform Coding

Produces new waveforms with better performance

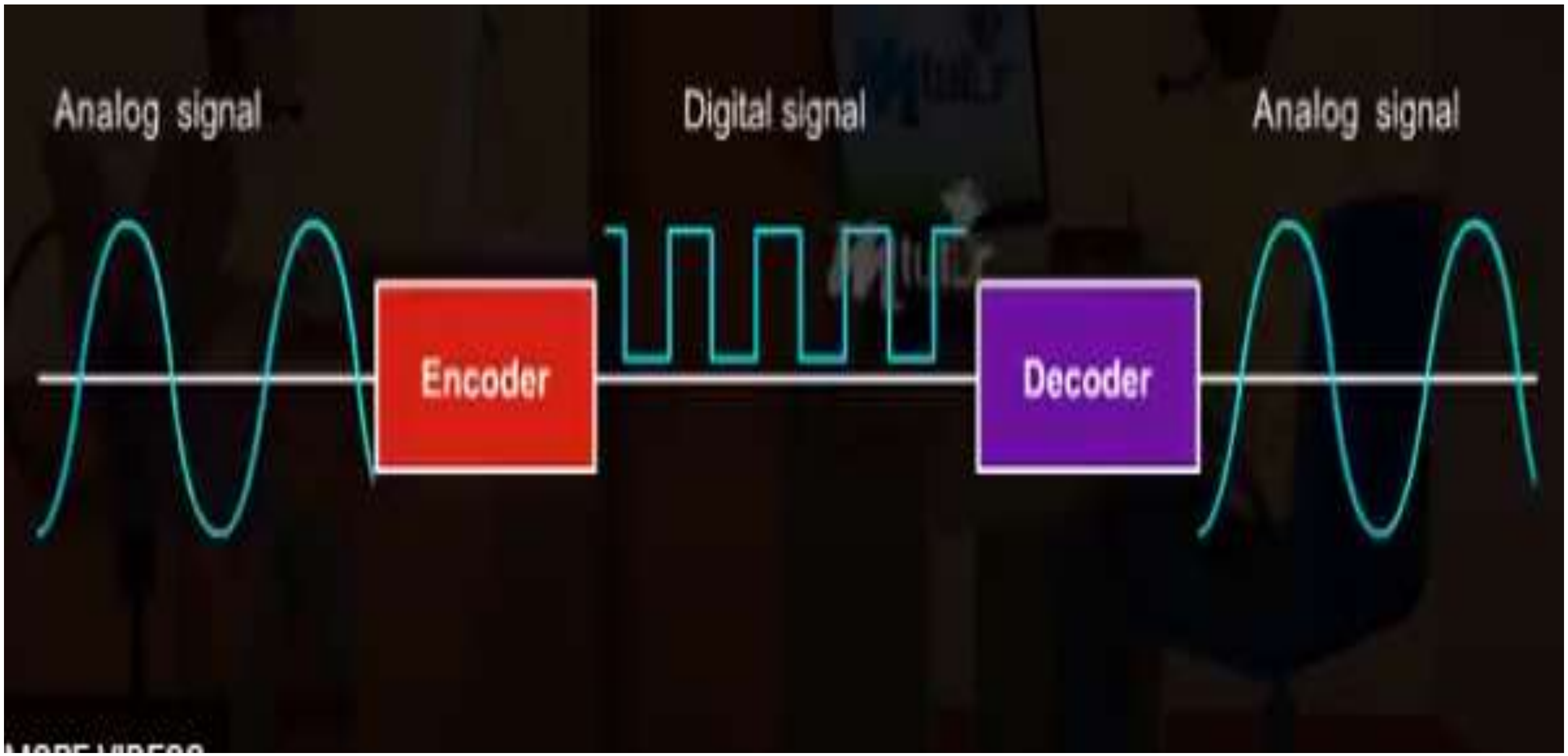
Sampling


Amplitude
S(t)
T
t
1
2
3
4
5
6
7
8
9
10
11
12
13


How are you?




1.0
0.5
0.0
−0.5
−1.0


Analog signal
Digital signal
Analog signal
Encoder
Decoder

Sampling theorem

- A band limited analog signal can be sampled and reconstructed back back from its samples ,if the sampling frequency is greater than or equal to twice the maximum frequency of the baseband signal.

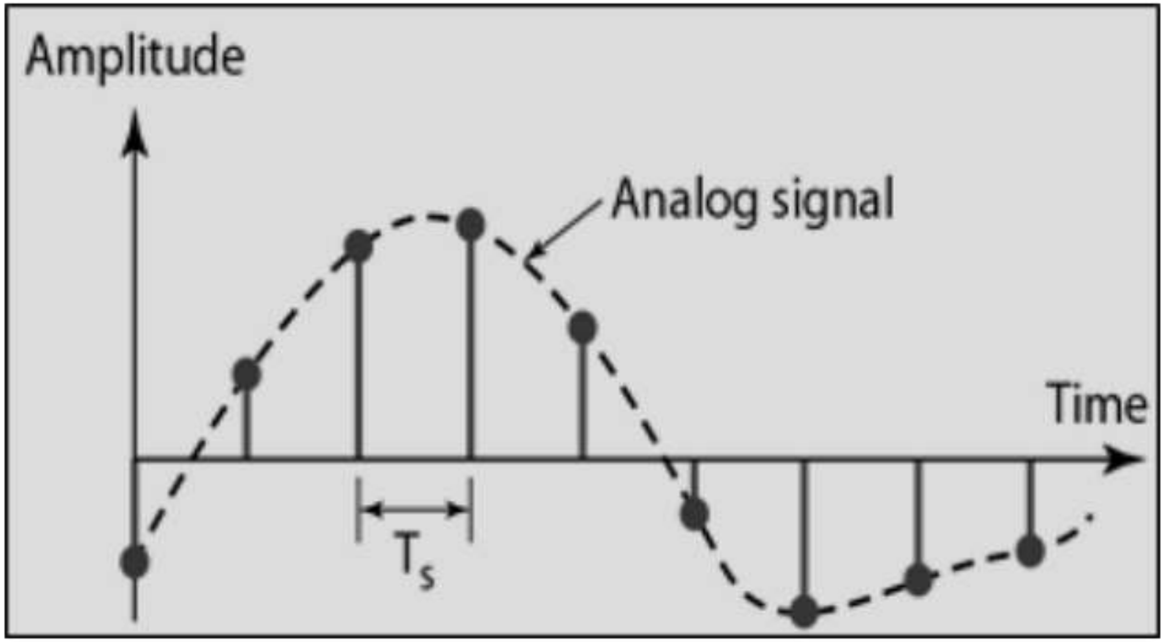
$$Fs \geq 2Fm$$

Nyquist rate

- The minimum rate at which a signal can be sampled without introducing error.
- Discrete signal, the gap between the samples(sampling period) should be fixed.
- Sampling rate denotes the number of samples taken per second

Ideal sampling

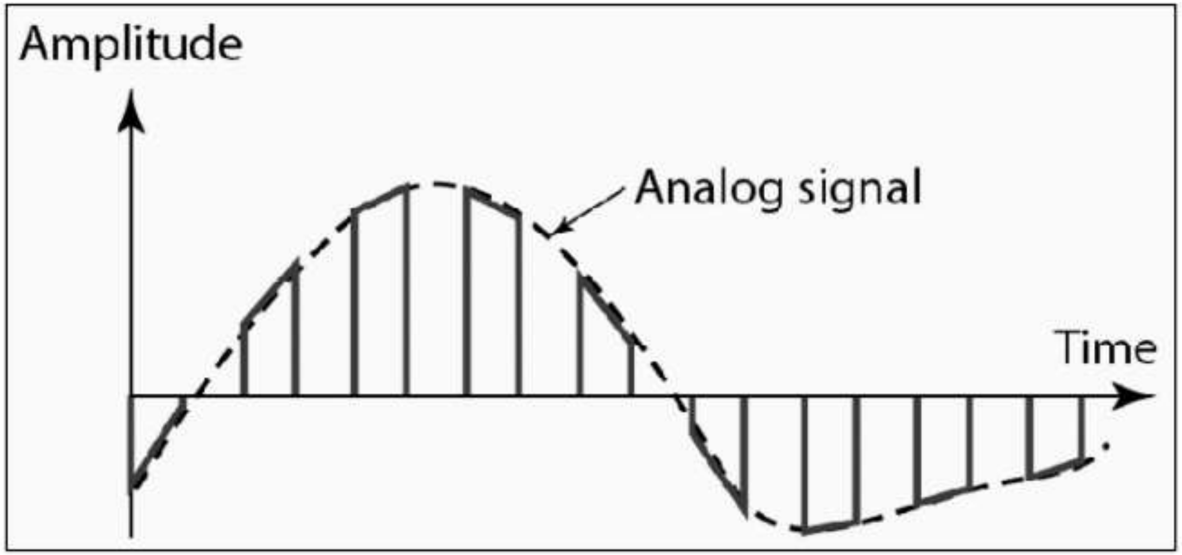
Ideal sampling - an impulse at each sampling instant



Ideal Sampling

Natural Sampling

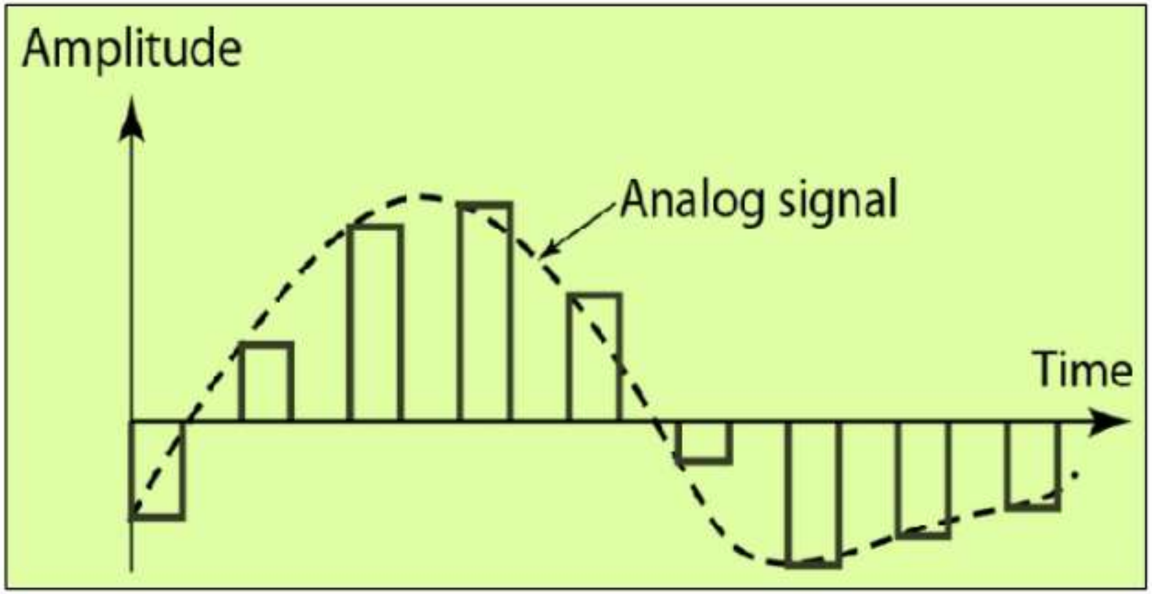
Natural sampling - a pulse of short width with varying amplitude with natural tops



Natural Sampling

Flat-top Sampling

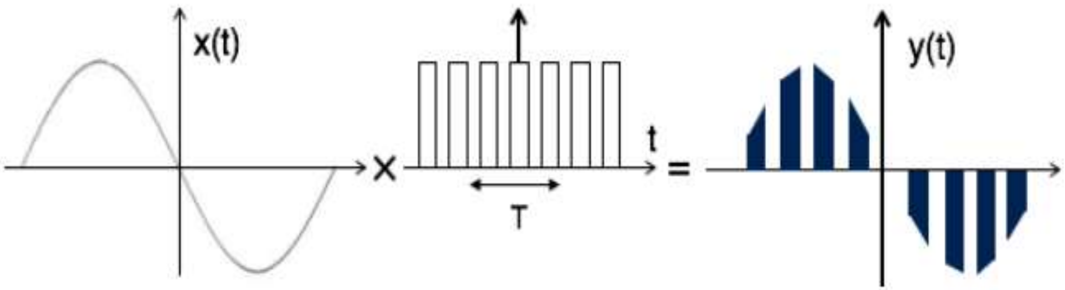
Flat-top sampling - a pulse of short width with varying amplitude with flat tops



Flat-top Sampling

Natural Sampling

Natural sampling is similar to impulse sampling, except the impulse train is replaced by pulse train of period T. i.e. you multiply input signal x(t) to pulse train $\Sigma_{n=-\infty}^{\infty} P(t - nT)$ as shown below



The output of sampler is

$$y(t) = x(t) \times \text{pulse train}$$

$$= x(t) \times p(t)$$

$$= x(t) \times \Sigma_{n=-\infty}^{\infty} P(t - nT) \ldots \ldots (1)$$

The exponential Fourier series representation of p(t) can be given as

$$p(t) = \Sigma_{n=-\infty}^{\infty} F_n e^{jn\omega_s t} \ldots \ldots (2)$$

$$= \Sigma_{n=-\infty}^{\infty} F_n e^{j2\pi n f_s t}$$

Where $F_n = \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} p(t) e^{-jn\omega_s t} dt$

=1/T $P(n\omega_s)$

Substitute F_n value in equation 2

$$\therefore p(t) = \Sigma_{n=-\infty}^{\infty} \frac{1}{T} P(n\omega_s) e^{jn\omega_s t}$$

$$= \frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s) e^{jn\omega_s t}$$

Substitute p(t) in equation 1

$$y(t) = x(t) \times p(t)$$

$$= x(t) \times \frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s)\, e^{jn\omega_s t}$$

$$y(t) = \frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s)\, x(t)\, e^{jn\omega_s t}$$

To get the spectrum of sampled signal, consider the Fourier transform on both sides.

$$F.T\,[y(t)] = F.T[\frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s)\, x(t)\, e^{jn\omega_s t}]$$

$$= \frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s)\, F.T\,[x(t)\, e^{jn\omega_s t}]$$

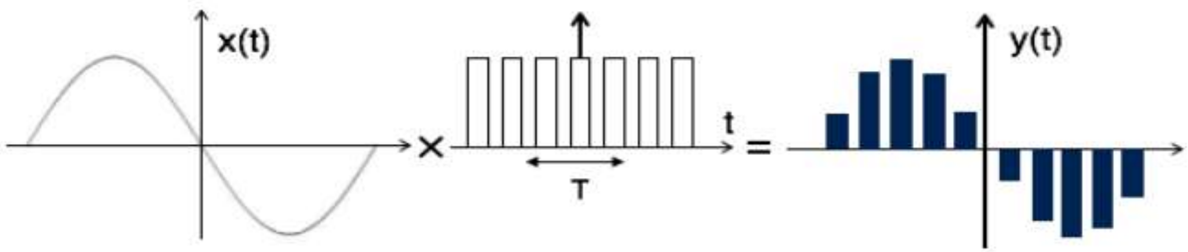
According to frequency shifting property

$$F.T\,[x(t)\, e^{jn\omega_s t}] = X[\omega - n\omega_s]$$

$$\therefore\ Y[\omega] = \frac{1}{T} \Sigma_{n=-\infty}^{\infty} P(n\omega_s)\, X[\omega - n\omega_s]$$

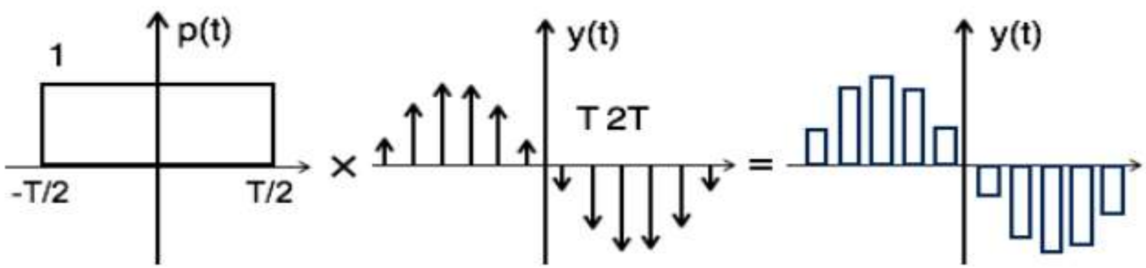
Flat Top Sampling

During transmission, noise is introduced at top of the transmission pulse which can be easily removed if the pulse is in the form of flat top. Here, the top of the samples are flat i.e. they have constant amplitude. Hence, it is called as flat top sampling or practical sampling. Flat top sampling makes use of sample and hold circuit.



Theoretically, the sampled signal can be obtained by convolution of rectangular pulse p(t) with ideally sampled signal say $y_\delta(t)$ as shown in the diagram:

i.e. $y(t) = p(t) \times y_\delta(t) \ldots\ldots (1)$



To get the sampled spectrum, consider Fourier transform on both sides for equation 1

Quadrature Sampling of Band – Pass Signals

This scheme represents a natural extension of the sampling of low – pass signals.

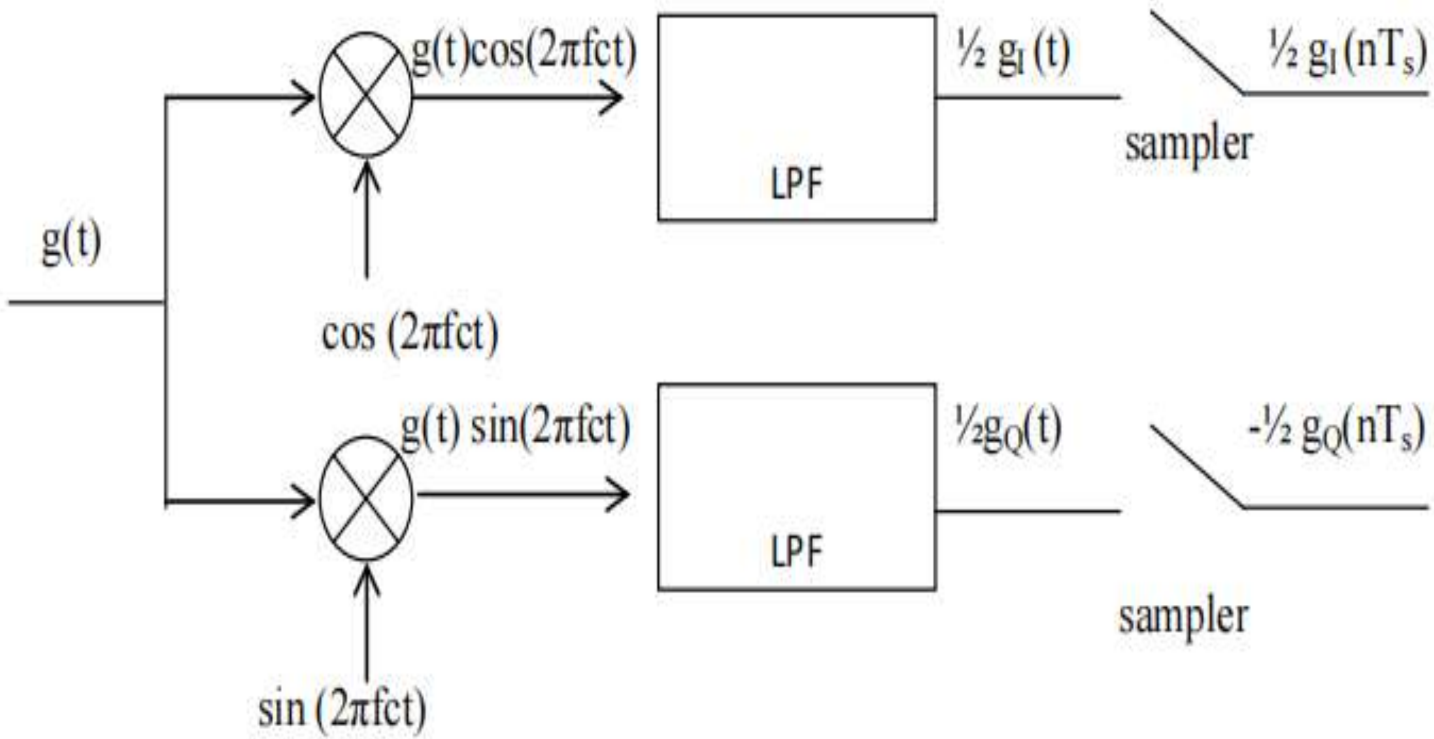
In this scheme, the band pass signal is split into two components, one is in-phase component and other is quadrature component. These two components will be low–pass signals and are sampled separately. This form of sampling is called quadrature sampling.

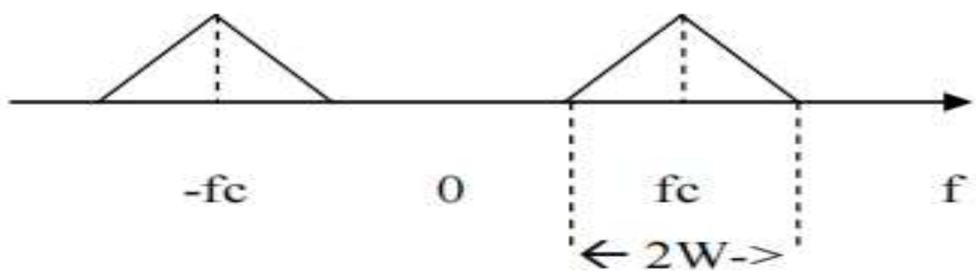
Let g(t) be a band pass signal, of bandwidth '2W' centered around the frequency, fc, (fc>W). The in-phase component, $g_I(t)$ is obtained by multiplying g(t) with $\cos(2\pi fct)$ and then filtering out the high frequency components. Parallelly a quadrature phase component is obtained by multiplying g(t) with $\sin(2\pi fct)$ and then filtering out the high frequency components..

The band pass signal g(t) can be expressed as,

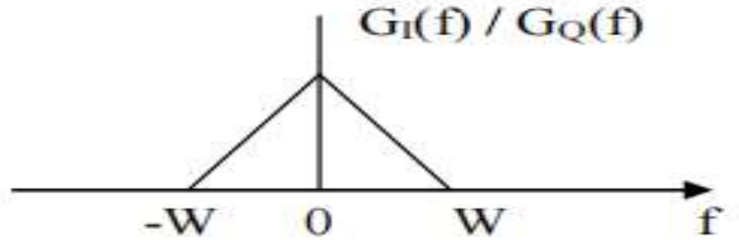
$$g(t) = g_I(t).\ \cos(2\pi fct) - g_Q(t)\ \sin(2\pi fct)$$

The in-phase, $g_I(t)$ and quadrature phase $g_Q(t)$ signals are low–pass signals, having band limited to $(-W < f < W)$. Accordingly each component may be sampled at the rate of 2W samples per second.


g(t)
g(t)cos(2πfct)
cos (2πfct)
LPF
½ gI (t)
½ gI(nTs)
sampler
g(t) sin(2πfct)
sin (2πfct)
LPF
½gQ(t)
-½ gQ(nTs)
sampler



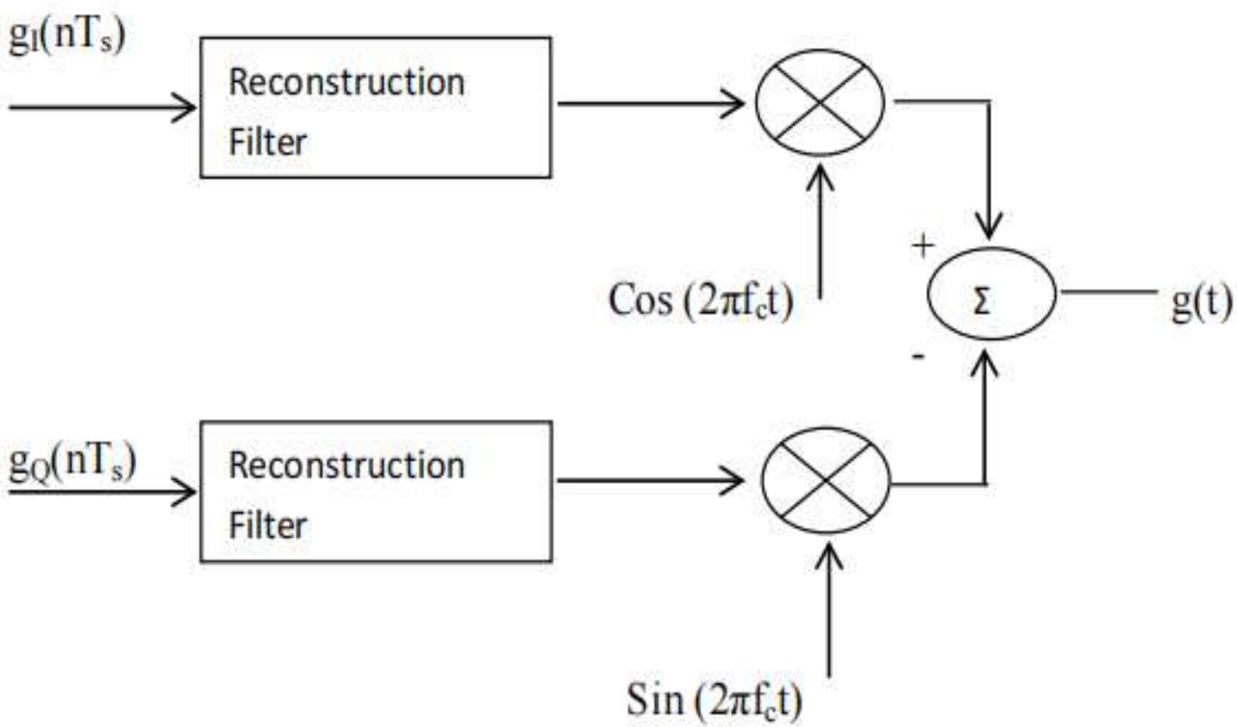
a) Spectrum of a Band pass signal.



b) Spectrum of $g_I(t)$ and $g_Q(t)$

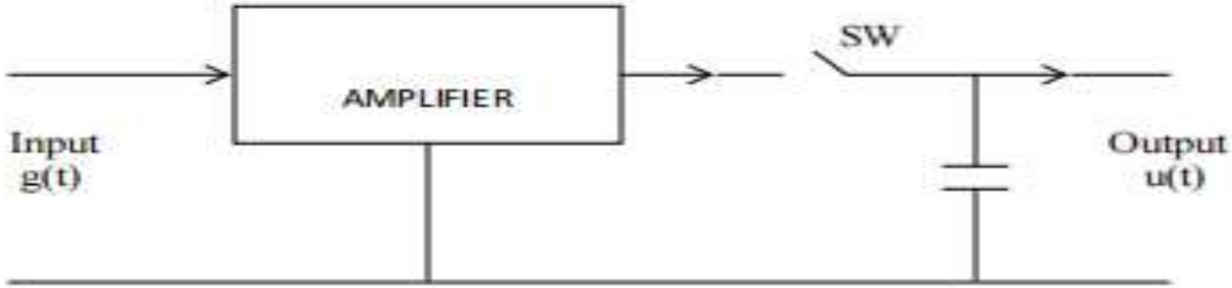
Reconstruction

From the sampled signals $g_I(nTs)$ and $g_Q(nTs)$, the signals $g_I(t)$ and $g_Q(t)$ are obtained. To reconstruct the original band pass signal, multiply the signals $g_I(t)$ by $\cos(2\pi fct)$ and $\sin(2\pi fct)$ respectively and then add the results.

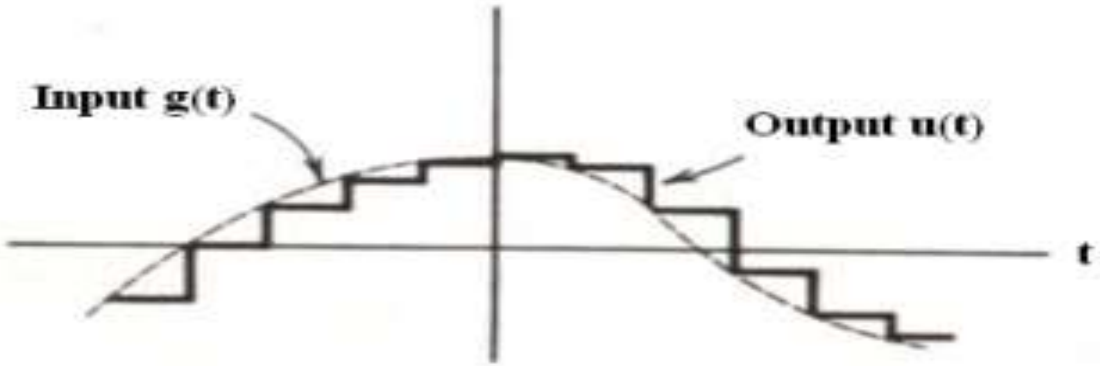


Sample and hold circuit for signal recovery

In both the natural sampling and flat-top sampling methods, the spectrum of the signals are scaled by the ratio τ/Ts, where τ is the pulse duration and Ts is the sampling period. Since this ratio is very small, the signal power at the output of the reconstruction filter is correspondingly small. To overcome this problem a sample-and-hold circuit is used .



a) Sample and Hold Circuit



b) Idealized output waveform of the circuit

$$u(t) = \sum_{n=-\infty}^{+\infty} g(nTs)\, h(t - nTs)$$

where h(t) is the impulse response representing the action of the Sample-and-Hold circuit; that is

$$h(t) = \begin{cases} 1 & \text{for } 0 < t < Ts \\ 0 & \text{for } t < 0 \text{ and } t > Ts \end{cases}$$

Correspondingly, the spectrum for the output of the Sample-and-Hold circuit is given by,

$$U(f) = f_s \sum_{n=-\infty}^{+\infty} H(f)G(f - nf_s))$$

where G(f) is the FT of g(t) and

$H(f) = Ts\ \mathrm{Sinc}(fTs)\ \exp(-j\pi fTs)$

To recover the original signal g(t) without distortion, the output of the Sample-and-Hold circuit is passed through a low-pass filter and an equalizer.