Numerical

- Suppose that binary PSK is used for transmitting information over an AWGN with a PSD of $N_0/2 = 1 \times 10^{-10}$W/Hz. The transmitted signal energy is $E_b = 1/2A^2T$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6}, when the data rate is a) 10 kbits/s, b) 100 kbits/s, and c) 1 Mbits/s

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- **<u>Solution:</u>**

$$A = \sqrt{\frac{2E_b}{T_b}} \Rightarrow E_b = \frac{1}{2}A^2 T_b$$

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{0.5 * A^2 T_b}{N_0}}\right)$$

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{0.5 * A^2 T_b}{N_0}}\right) = 10^{-6} \Rightarrow erfc\left(\sqrt{\frac{0.5 * A^2 T_b}{N_0}}\right) = 2 * 10^{-6}$$

Numerical

We know that $erfc(x) = 2 * 10^{-6}$ and from erfc table the value of x=3.36

$\sqrt{0.5A^2T_b/N_0} = 3.36 \Rightarrow 0.5A^2T_b/N_0 = 11.2896$

$A^2T_b = 22.579N_0 \Rightarrow A^2 = 22.5792 * 10^{-10} * (1/T_b) = 4.515 * 10^{-9} * (1/T_b)$

$A^2 = 4.515 * 10^{-9} * (1/T_b) \Rightarrow A = \sqrt{4.515 * 10^{-9} * (1/T_b)}$

If the data rate is 10 Kbps, then

$$A = \sqrt{4.515 \times 10^{-9} \times 10 \times 10^3} = 6.7193 \times 10^{-3}$$

If the data rate is 100 Kbps, then

$$A = \sqrt{4.515 \times 10^{-9} \times 100 \times 10^3} = 21.2 \times 10^{-3}$$

If the data rate is 1 Mbps, then

$$A = \sqrt{4.515 \times 10^{-9} \times 1 \times 10^6} = 67.19 \times 10^{-3}$$

Numerical

- A continuously operating coherent BPSK system makes errors at the average rate of 100 errors per day. The data rate is 1000 bits/s. The single-sided noise power spectral density is $N_0 = 10^{-10}\mathrm{W/Hz}$.

a) what is the average bit error probability?

b) If the value of received average signal power is adjusted to be $10^{-6}\mathrm{W}$ will, this received power be adequate to maintain the error probability found in part (a)?

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Solution:

The total bit detected in one day$= 1000 \times 86400 == 8.64 \times 10^7$

$$P_b = \frac{100}{8.64 \times 10^7} = 1.16 \times 10^{-6}$$

Numerical

$$P_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}} = \frac{1}{2} erfc\sqrt{\frac{P \times T_b}{N_0}}$$

$$E_b = P \times T_b$$

$$= \frac{1}{2} erfc\sqrt{\frac{10^{-6}}{1000 \times 10^{-10}}} = \frac{1}{2} erfc\sqrt{10} = \frac{1}{2}\mathrm{erfc}(3.16)$$

$$\frac{1}{2}\mathrm{erfc}(3.16) = \frac{1}{2} \times 0.00001 = 0.000005 = \mathbf{5 \times 10^{-6}}$$

Numerical

Find the expected number of bit errors made in one day by the following continuously operating coherent BPSK receiver. The data rate is 5000 bps. The input digital waveforms are $s_1(t) = Acos\omega_0 t$ and $s_2(t) = -Acos\omega_0 t$ where A= 1 mV and the the single-sided noise power spectral density is $N_0 = 10^{-11} W/Hz$. Assume that signal power and energy per bit are normalized to a 1 Ω resistive load.

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$$A = \sqrt{\frac{2E_b}{T_b}} \qquad E_b = \frac{A^2 T_b}{2} \qquad T_b = \frac{1}{R_b}$$

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_b}{2N_0}}\right)$$

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{(1 \times 10^{-3})^2}{2 \times 10^{-11} \times 5000}}\right) = \frac{1}{2} erfc(3.1622)$$

$$P_e = \frac{1}{2} \times 0.00000786 = 3.93 \times 10^{-6}$$

Average no of errors in one day $= 5000 bits \times 24 \times 60 \times 60 \times 3.93 \times 10^{-6} \simeq 1698$ bits in error

Numerical

- In DCS, the bitrate of NRZ data stream is 1Mbps and carrier frequency is 1MHz. Find the symbol rate and bandwidth of the channel using i) BPSK; ii) QPSK; iii) 16-PSK

Numerical

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Solution:

- In BPSK, $T_s = T_b = 1/R$ and $BW = 2f_b$
- F_b = 1Mbps and BW = 2MHz,
- $\text{Symbol rate} = \frac{1}{10^{-6}} = 10^6$ symbols/sec
- In QPSK, $T_s = 2T_b = 2/R$ and $BW = f_b$
- F_b = 1Mbps and BW = 1MHz,
- $\text{Symbol rate} = \frac{1}{2\times10^{-6}} = 500 \times 10^3$ symbols/sec

Numerical

- In 16 - PSK,
- $T_s = NT_b$ and $BW = \frac{2f_b}{N}$
- $2^N = M$ in this case, $2^N = 16$, therefore, N=4
- $BW = \frac{2f_b}{4} = \frac{2\times10^6}{4} = 500KHz$
- $\text{Symbol rate} = \frac{1}{4\times10^{-6}} = 250\ K\text{symbols/sec}$

Numerical

- A binary FSK system transmits binary data at a rate of 2 MBPS. Assuming channel AWGN with zero mean and power spectral density of $N_0/2$ = 1 × 10−20W/Hz. The amplitude of the received signal in the absence of noise is $1\mu V$. Determine the average probability of error for coherent detection of FSK.

- A binary data is transmitted over an AWGN channel using antipodal signalling scheme at a rate of 2 Mbps. It is desired to have average probability of error Pe ≤ 10−6. Noise power spectral density is $N_0/2$ = 2 × 10−12 W/Hz. Determine the average carrier power required at the receiver input, if the detector is of coherent type.

Numerical

- An FSK system transmits binary data at a rate of 10^6 bits per second. Assuming channel AWGN with zero mean and power spectral density of $N_0/2 = 2 \times 10^{-20}$W/Hz. Determine the probability of error. Assume coherent detection and amplitude of received sinusoidal signal for both symbol 1 and 0 to be 1.2 microvolt.

- A binary data are transmitted at a rate of 10^6 bits per second over the microwave link. Assuming channel AWGN with zero mean and power spectral density of 1×10^{-10}W/Hz. Determine the average carrier power required to maintain an average probability of error probability of error $P_e \leq 10^{-4}$ for coherent binary FSK. Determine the minimum channel bandwidth required.

Numerical

- A binary data are transmitted at a rate of 10^6 bits per second over the microwave link. Assuming channel AWGN with zero mean and power spectral density of 1×10^{-10}W/Hz. For each of the following pairs, determine which one requires more power than other. Determine the extra average carrier power required to maintain an average probability of error probability of error $P_e \leq 10^{-4}$ for

a) BPSK

b) QPSK

c) FSK