Raised-Cosine Spectrum

- Here we try to overcome the practical difficulties encountered with the ideal Nyquist pulse

- In doing so, the bandwidth is extended from the minimum value $W = R_b/2$ to an adjustable value between W and $2W$

- In effect, we are trading off increased channel bandwidth for a more *robust signal design that is tolerant of timing errors*

- Thus, the overall frequency response is designed in the following way:

$$P(f) + P(f - 2W) + P(f + 2W) = \frac{1}{2W}, \qquad -W \le f \le W$$

Raised-Cosine Spectrum

- A particular form of $P(f)$ that embodies many desirable features in provided by a *raised-cosine (RC) spectrum*

- This frequency response consists of a flat portion and a roll-off portion that has a sinusoidal form, given by:

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 \le |f| \le f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi}{2W\alpha}(|f| - f_1)\right]\right\}, & f_1 \le |f| \le 2W - f_1 \\ 0, & |f| \ge 2W - f_1 \end{cases}$$

Raised-Cosine Spectrum

- A new frequency f_1 and a dimensionless parameter α, which are related by:

$$\alpha = 1 - \frac{f_1}{W}$$

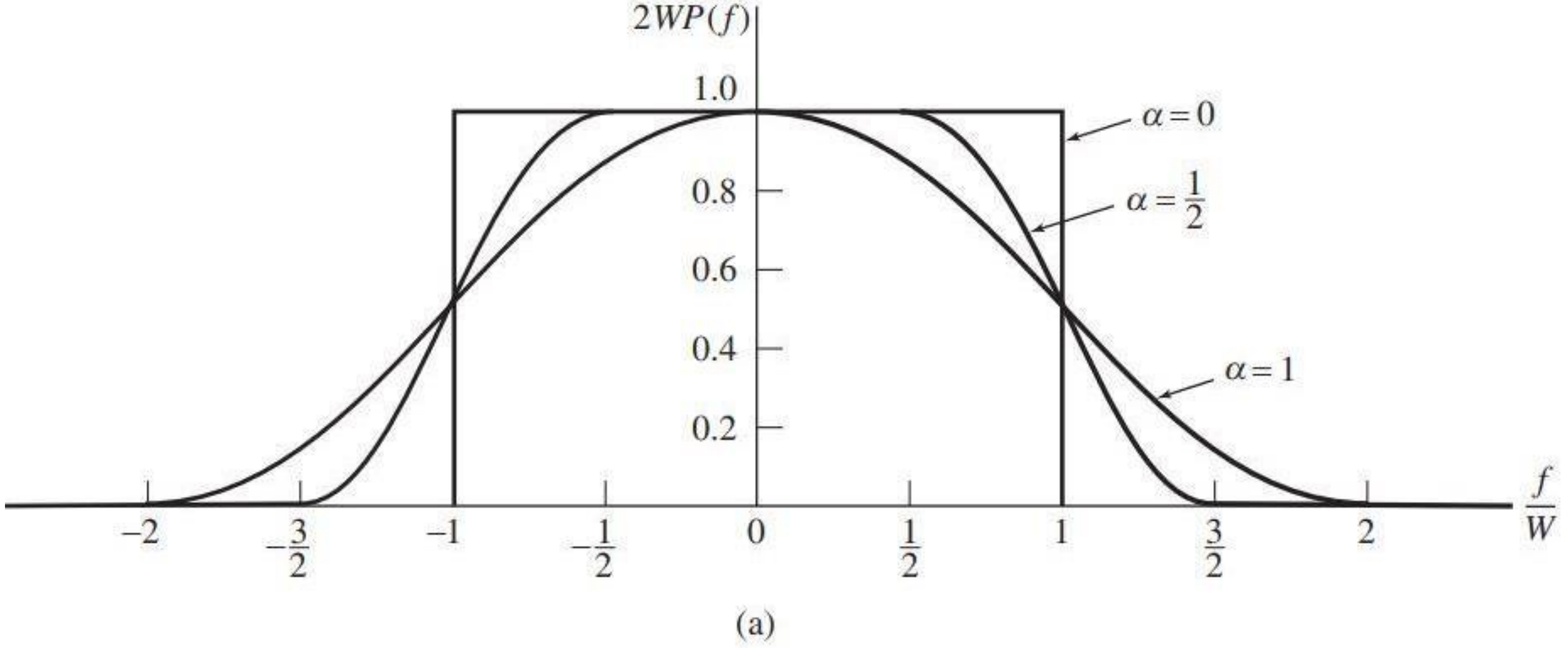
- The parameter α is commonly called the ***roll-off factor***

- It indicates the excess bandwidth over the ideal solution, W

- Specifically, the new *transmission bandwidth* B_T is defined by:

$$B_T = 2W - f_1$$
$$= W(1 + \alpha)$$

Raised-Cosine Spectrum



Frequency response for different roll-off factors

Raised-Cosine Spectrum

- In time domain $p(t)$ is given by:

$$p(t) = sinc(2Wt)\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2W^2t^2}$$

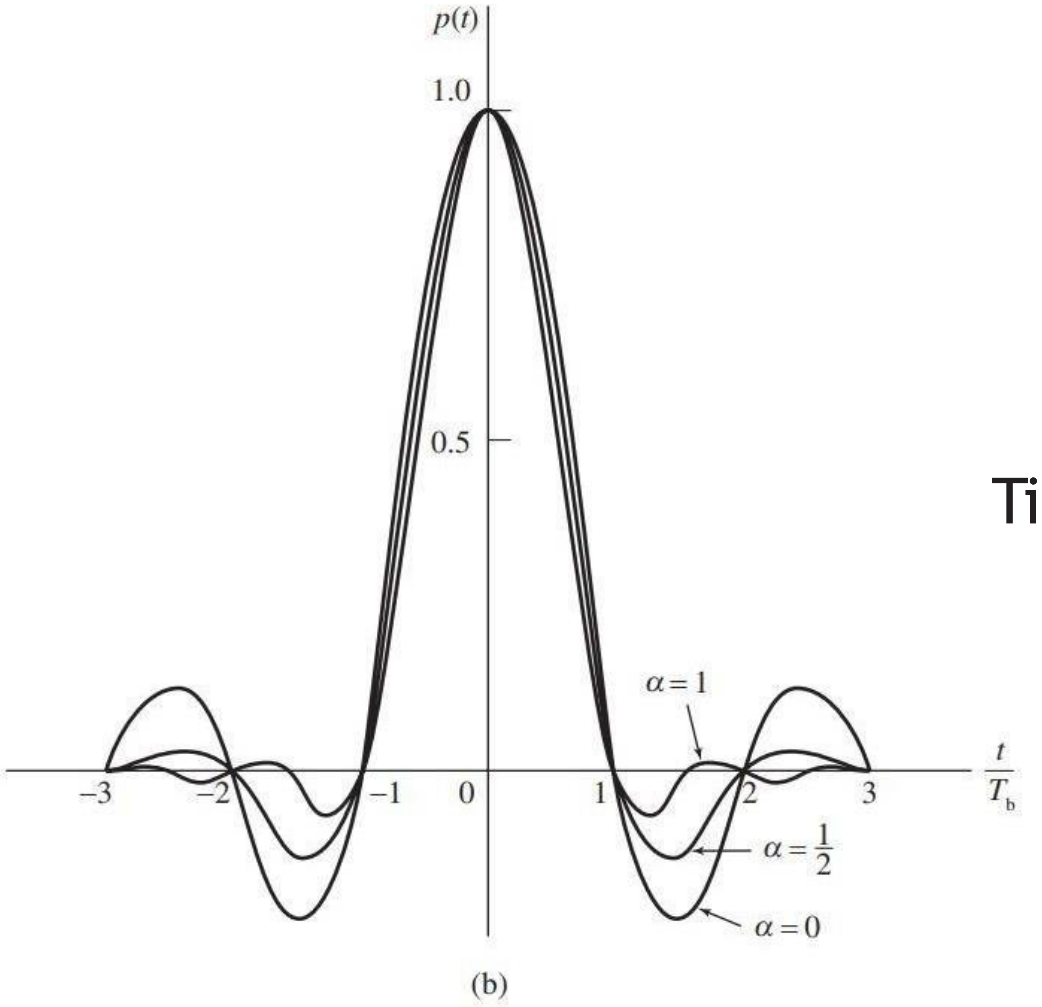
- In the special case with $\alpha = 1$ (i.e., $f_1 = 0$) which is known as *full-cosine roll-off* characteristic, the frequency response simplifies to:

$$P(f) = \{ \begin{matrix} \frac{1}{4W}\left[1 + \cos\left(\frac{\pi f}{2W}\right)\right], & 0 < |f| < W \\ 0, & |f| > 2W \end{matrix}$$

- Correspondingly, the time response simplifies to:

$$p(t) = \frac{sinc(4Wt)}{1 - 16W^2t^2}$$

Raised-Cosine Spectrum



Time response for different roll-off factors

Numerical problem

- A BSNL telephone system carries 24 voice channels in a single cable using TDM technology. Find the bit rate and transmission bandwidth (Ideal Nyquist pulse and Full Raised Cosine pulse)

Correlative-Level Coding

- Here intersymbol interference is added to the transmitted signal in a controlled manner

- Doing so, it is possible to achieve a signaling rate equal to the Nyquist rate of $2W$ symbols per second in a channel of bandwidth W Hertz

- Such schemes are called as *correlative level coding or partial response algorithm*"

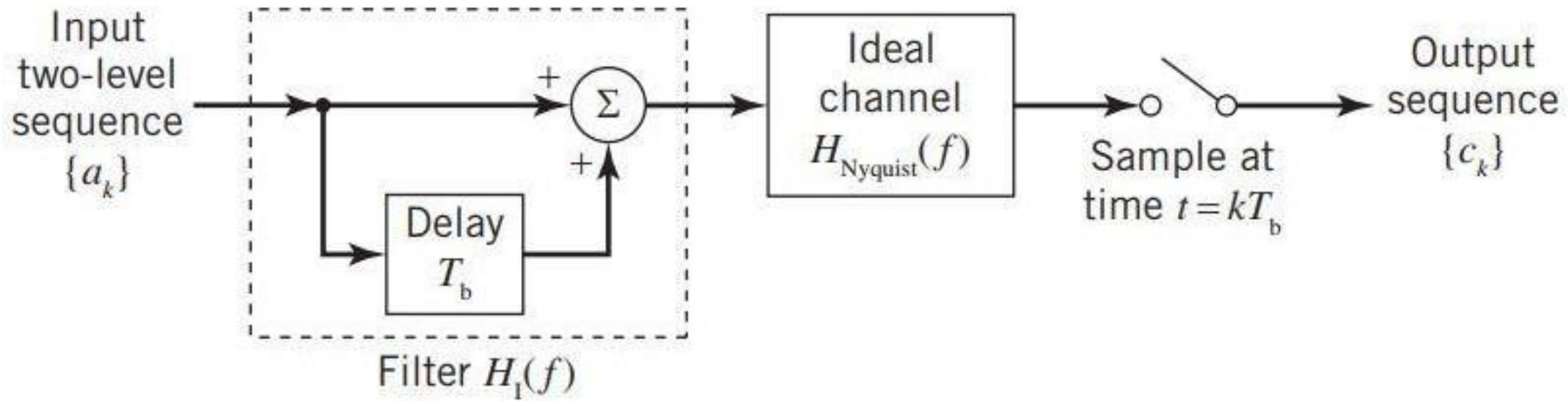
- It is a practical method of achieving theoretical maximum signaling rate

Correlative-Level Coding (Duobinary Signaling)

- Here "duo" implies doubling of the transmission capacity of a straight binary system
- This particular form of correlative-level coding is also called *class I partial response*
- Consider a binary input sequence $\{b_k\}$ consisting of uncorrelated binary symbols 0 and 1
- Each of them has a duration T_b
- This sequence is now applied to a pulse amplitude modulator producing a two-level sequence of short pulses having amplitude

Correlative-Level Coding (Duobinary Signaling)

- When this sequence is applied to a duobinary encoder, it is converted to a three-level output namely −2, 0, and + 2



Correlative-Level Coding (Duobinary Signaling)

- Using the signaling scheme, the output c_k is given by:

$$c_k = a_k + a_{k-1}$$

- An ideal delay element produces a delay of T_b seconds and has the frequency response $\exp(-j2\pi f T_b)$
- Thus, the overall frequency response of this filter connected in cascade with an ideal Nyquist channel is:

$$H_I(f) = H_{Nyquist}(f)[1 + \exp(-j2\pi f T_b)] = 2H_{Nyquist}(f)\cos(\pi f T_b)\exp(-j\pi f T_b)$$

- $H_I(f)$ indicates the pertinent class of partial response

Correlative-Level Coding (Duobinary Signaling)

- Also, $H_{Nyquist}$ is given by:

$$H_{Nyquist}(f) = \begin{cases} 1, & |f| \leq 2/T_b \\ 0, & otherwise \end{cases}$$

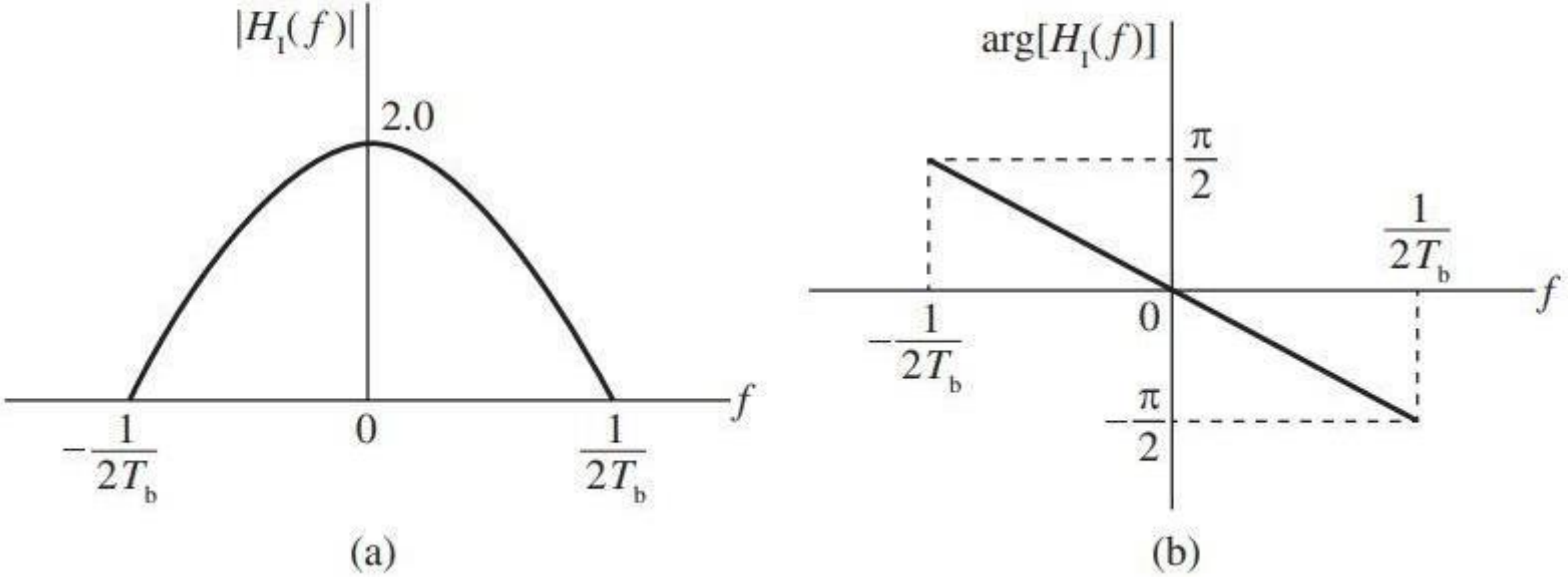
- Thus, the overall frequency response of the duobinary signaling has the form of a half-cycle cosine function given by:

$$H_I(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \leq 1/2T_b \\ 0, & otherwise \end{cases}$$

- The impulse response $h_I(t)$ will be given by:

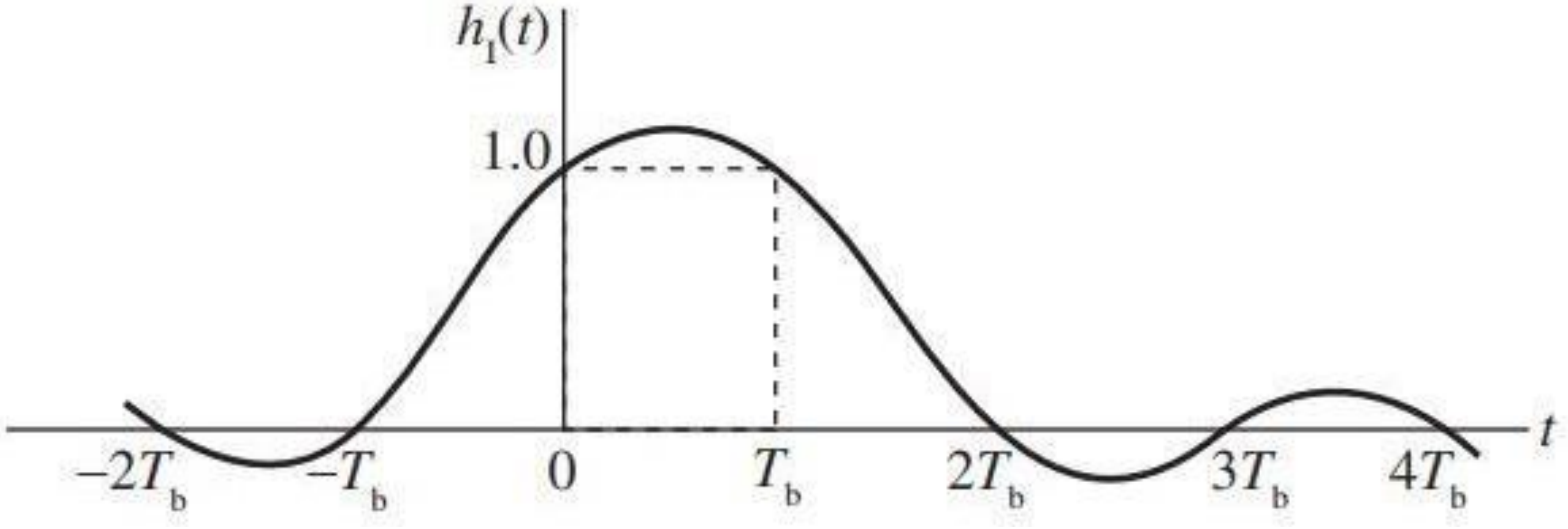
$$h_I(t) = \frac{\sin(\pi t/T_\mathrm{b})}{\pi t/T_\mathrm{b}} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b} = \frac{T_\mathrm{b}{}^2\sin(\pi t/T_b)}{\pi t(T_b - t)}$$

Correlative-Level Coding (Duobinary Signaling)



Frequency response of the duobinary conversion filter:
a) Magnitude response, b) Phase response

Correlative-Level Coding (Duobinary Signaling)



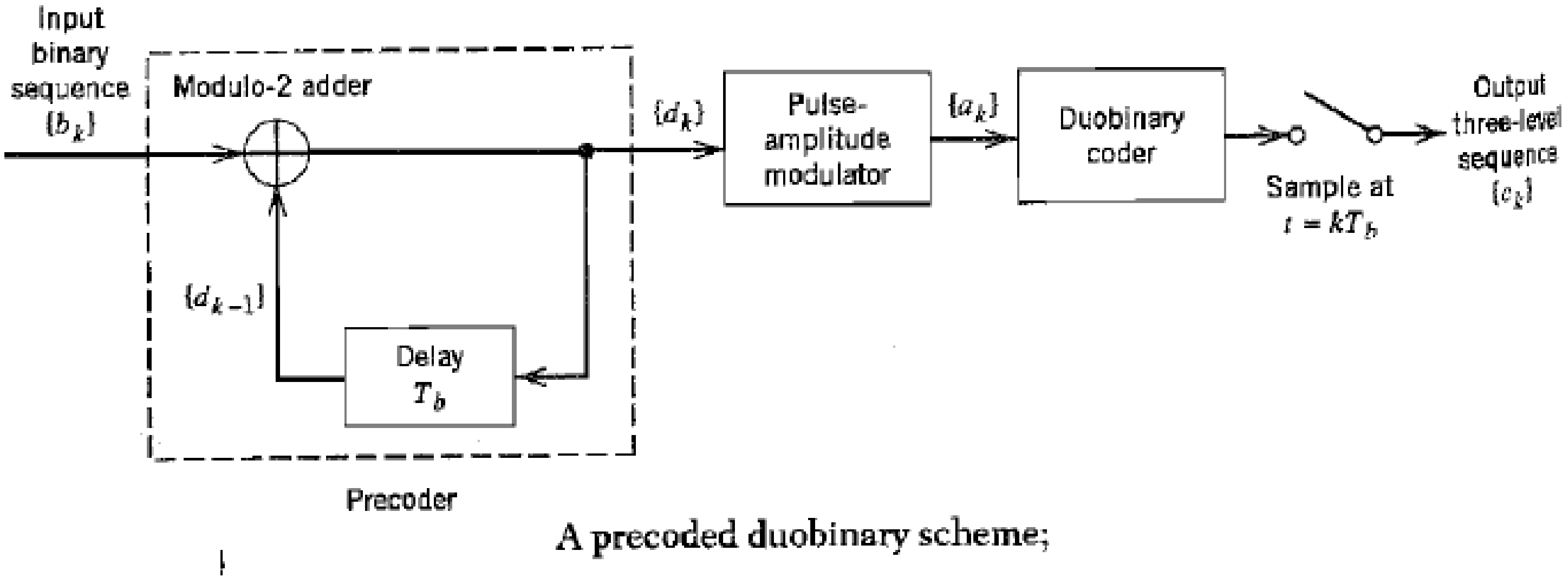
Impulse response of the duobinary conversion filter

Correlative-Level Coding – Duo Binary Signalling

- Generate the duobinary sequence for the given binary data - 0010110 .

Correlative-Level Coding – Precoded Duo Binary Signalling

- To avoid the error propagation phenomenon, we need to use the precoder before doing the duo binary encoding



A precoded duobinary scheme;

Correlative-Level Coding – Precoded Duo Binary Signalling

$$d_k = b_k \oplus d_{k-1}$$

$$d_k = \begin{cases} \text{symbol } 1 & \text{if either symbol } b_k \text{ or symbol } d_{k-1} \text{ (but not both) is } 1 \\ \text{symbol } 0 & \text{otherwise} \end{cases}$$

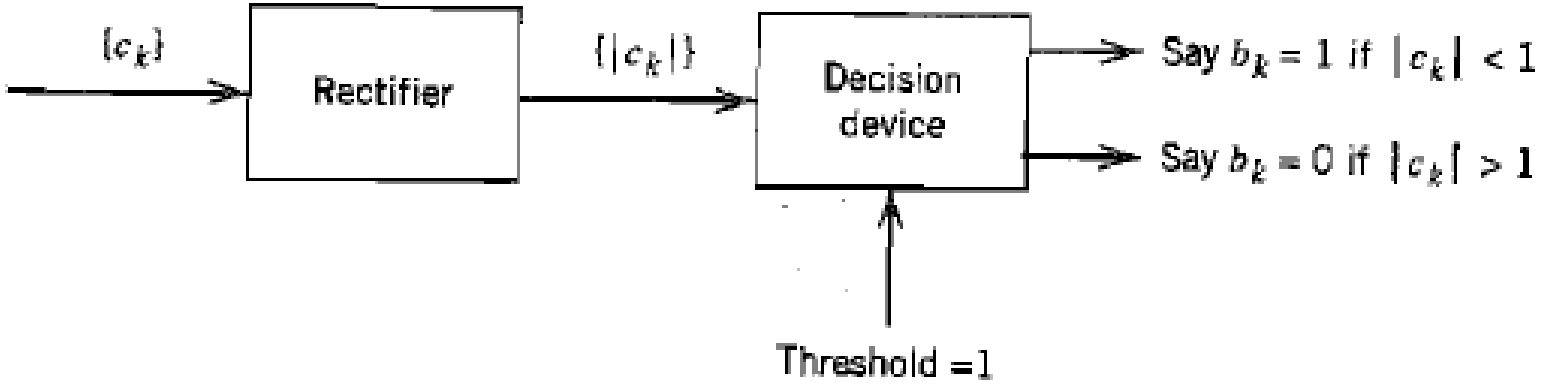
$$c_k = a_k + a_{k-1}$$

$$c_k = \begin{cases} 0 & \text{if data symbol } b_k \text{ is } 1 \\ \pm 2 & \text{if data symbol } b_k \text{ is } 0 \end{cases}$$

Correlative-Level Coding – Precoded Duo Binary Signalling

- Here, the decision is purely based on the present value and not on the previous.

- So, the error propagation can be avoided.



$$\text{If } |c_k| < 1, \quad \text{say symbol } b_k \text{ is } 1$$
$$\text{If } |c_k| > 1, \quad \text{say symbol } b_k \text{ is } 0$$

Correlative-Level Coding – Precoded Duo Binary Signalling

- Generate the precoded duobinary sequence for the given binary data - 0010110 .

Correlative-Level Coding – Precoded Duo Binary Signalling

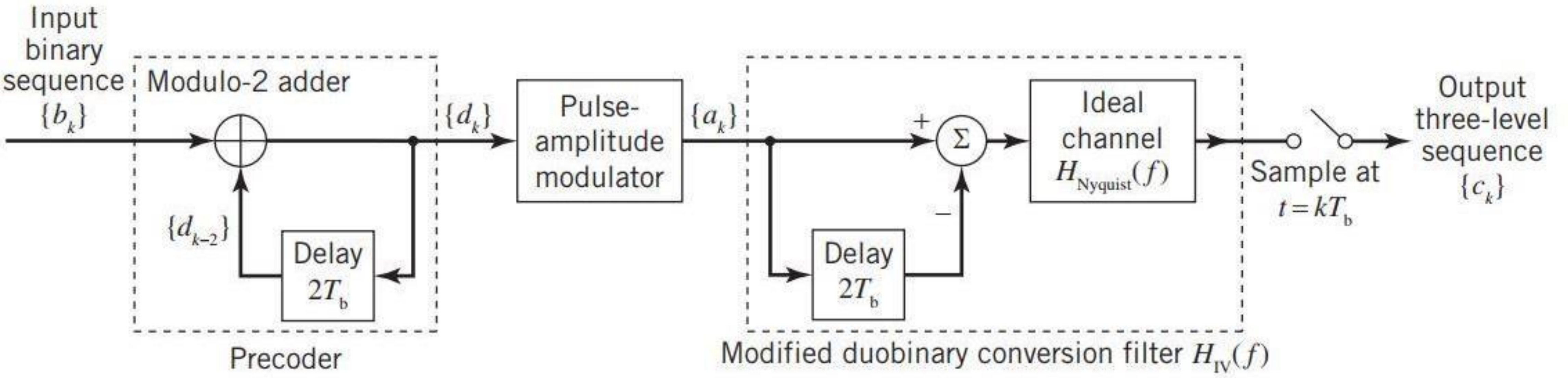
- Generate the precoded duobinary sequence for the given binary data - 0010110 .

Binary sequence $\{b_k\}$		0	0	1	0	1	1	0
Precoded sequence $\{d_k\}$	1	1	1	0	0	1	0	0
Two-level sequence $\{a_k\}$	+1	+1	+1	−1	−1	+1	−1	−1
Duobinary coder output $\{c_k\}$		+2	+2	0	−2	0	0	−2
Binary sequence obtained by applying decision rule		0	0	1	0	1	1	0

Correlative-Level Coding -(Modified Duobinary Signaling)

- Here, frequency response $H(f)$ and power spectral density of the transmitted pulse is nonzero at the origin
- This is considered to be an undesirable feature for many communication channels as they cannot transmit a DC component
- We may correct for this deficiency by using the *class IV partial response* or *modified duobinary* technique
- It involves a correlation span of two binary digits
- It is achieved by subtracting amplitude-modulated pulses spaced $2T_b$ seconds apart

Correlative-Level Coding -(Modified Duobinary Signaling)



Correlative-Level Coding -(Modified Duobinary Signaling)

- The overall frequency response of the delay-line filter connected in cascade with an ideal Nyquist channel is given by:

$$H_{IV}(f) = H_{Nyquist}(f)[1 - \exp(-j4\pi f T_b)]$$

$$= 2jH_{Nyquist}(f)\sin(2\pi f T_b)\exp(-j2\pi f T_b)$$

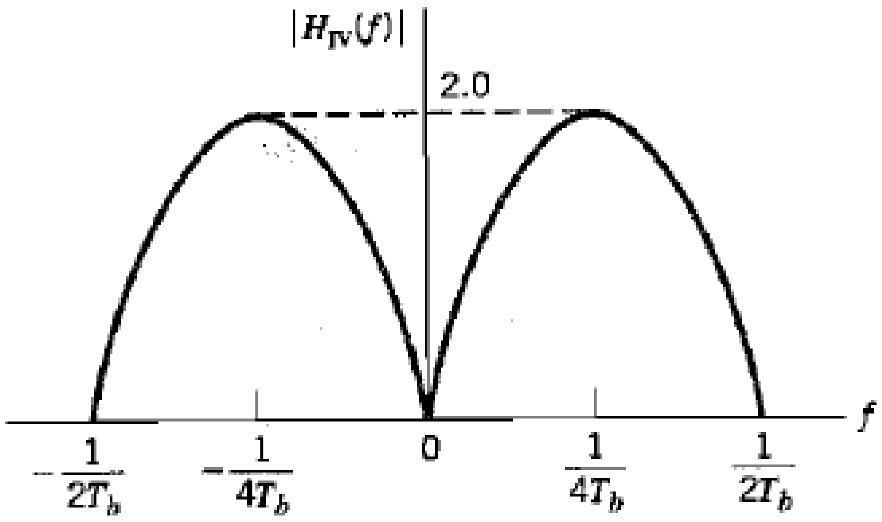
- The subscript IV in $H_{IV}\ (f)$ indicates the pertinent class of partial response and $H_{Nyquist}\ (f)$

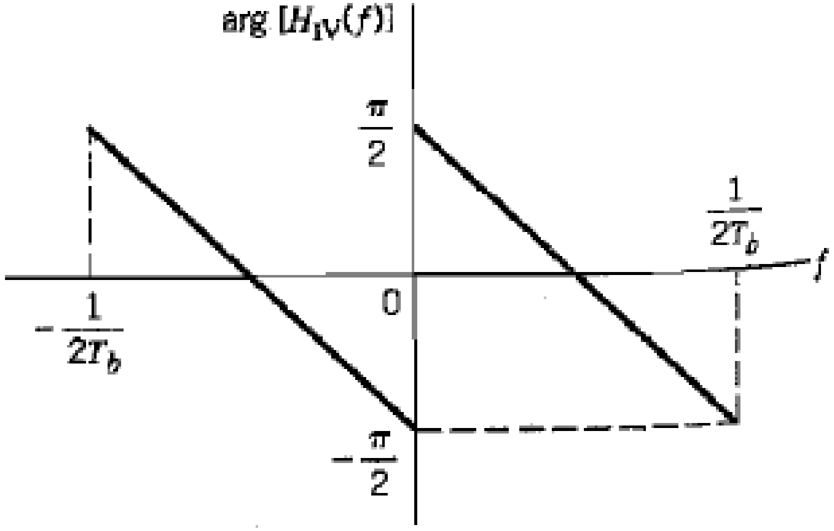
Correlative-Level Coding -(Modified Duobinary Signaling)

$$d_k = b_k \oplus d_{k-2}$$
$$= \begin{cases} \text{symbol } 1 & \text{if either symbol } b_k \text{ or symbol } d_{k-2} \text{ (but not both) is } 1 \\ \text{symbol } 0 & \text{otherwise} \end{cases}$$

$$\text{If } |c_k| > 1, \quad \text{say symbol } b_k \text{ is } 1$$
$$\text{If } |c_k| < 1, \quad \text{say symbol } b_k \text{ is } 0$$

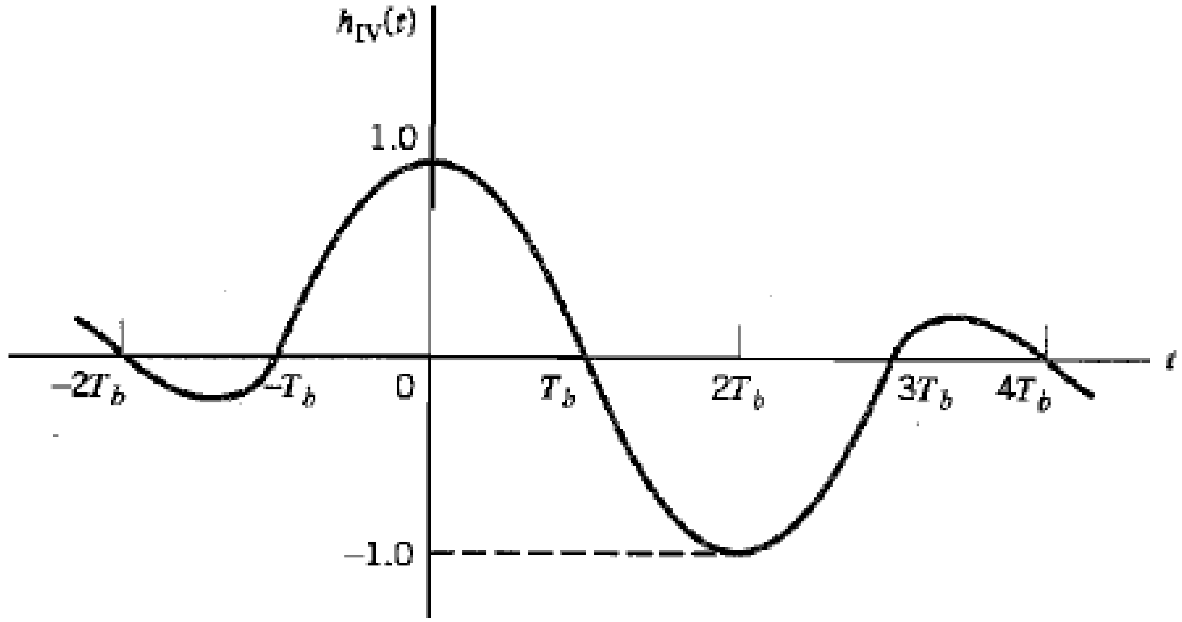
Correlative-Level Coding -(Modified Duobinary Signaling)





Frequency response of the modified duobinary conversion filter:
a) Magnitude response, b) Phase response

Correlative-Level Coding -(Modified Duobinary Signaling)



Impulse response of the modified duobinary conversion filter

Correlative-Level Coding – -(Modified Duobinary Signaling)

- Generate the modified duobinary sequence for the given binary data - 0010110 .

Generalized Form of Correlative-Level Coding (Partial-Response Signaling)

- The duobinary and modified duobinary techniques have correlation space of 1 binary digit and 2 binary digits,respectively

- It is a straightforward matter to generalize these two techniques to other schemes

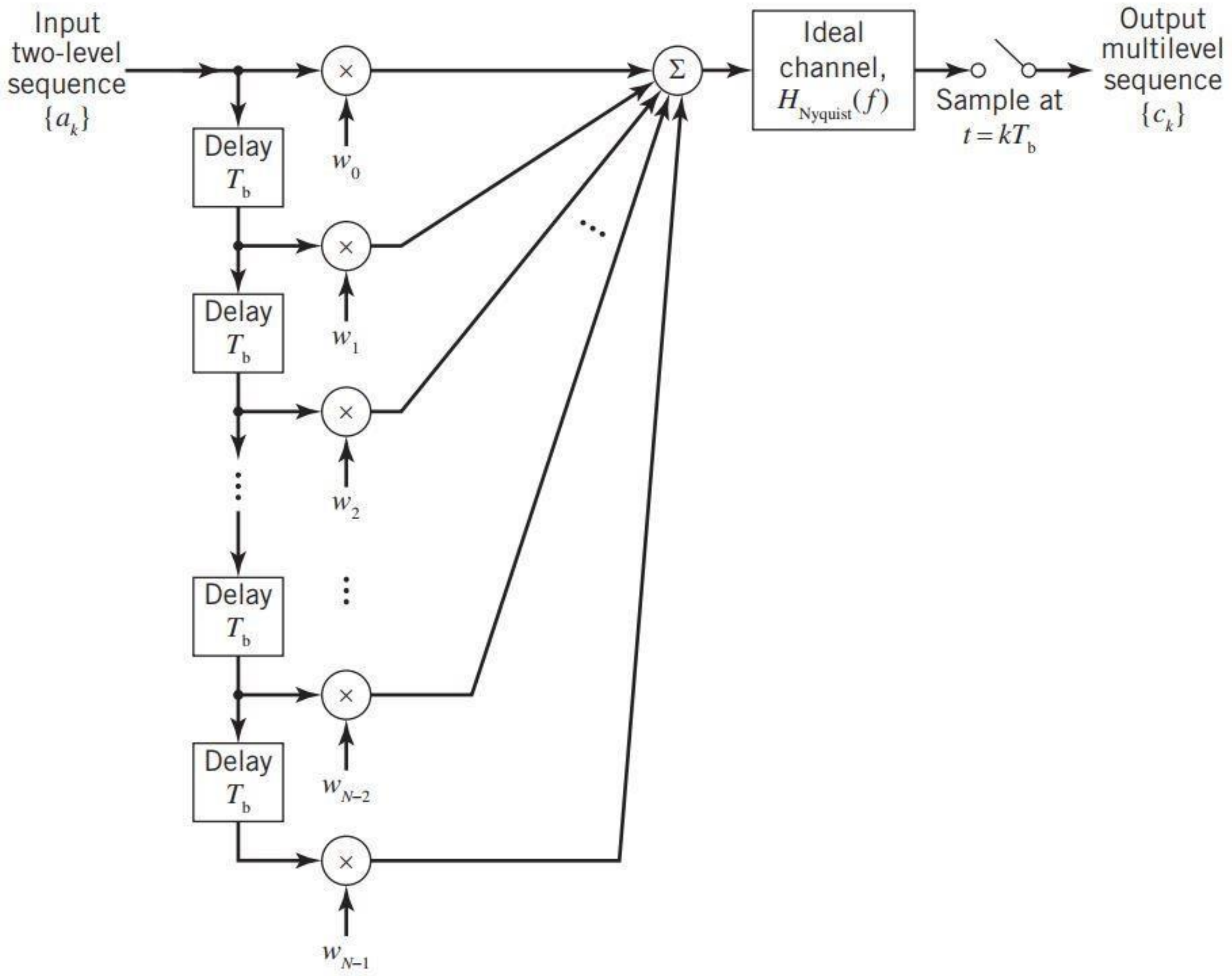
- It is collectively known as correlative-level coding or partial-response signaling schemes

- It involves the use of tapped-delay-line filter with tap weights $w_0, w_1, \dots, w_{N-1}$

- Different classes of partial-response signaling schemes may be achieved by using linear combination of N ideal Nyquist (sine) pulses, given by:

$$h(t) = \sum_{n=0}^{N-1} w_n sinc(\frac{t}{T_b} - n)$$

Generalized Form of Correlative-Level Coding (Partial-Response Signaling)



Generalized Form of Correlative-Level Coding (Partial-Response Signaling)

- Different class of partial-response signaling schemes referring to Figure above

Type of Class	N	w_0	w_1	w_2	w_3	w_4	*Comments*
I	2	1	1				Duobinary coding
II	3	1	2	1			
III	3	2	1	−1			
IV	3	1	0	−1			Modified duobinary coding
V	5	−1	0	2	0	−1	