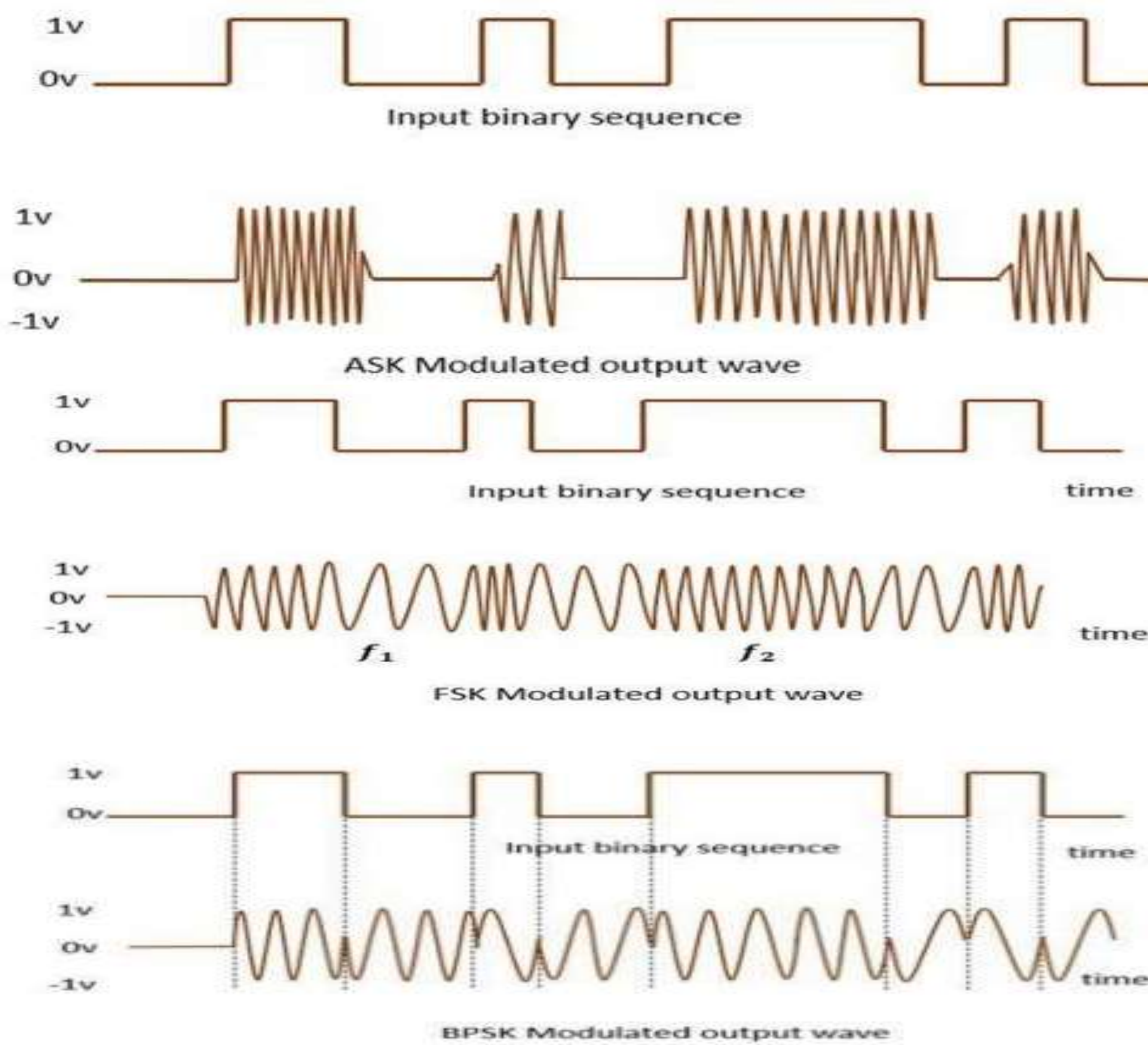
Module 5

Bandpass System

- Gram-Schmidt Orthogonalization Procedure. Correlation and Matched filter receiver.
- Coherent modulation techniques - BASK, BPSK, BFSK, QPSK, MSK, Higher-order PSK and
- QAM, BER and Bandwidth efficiency analysis. Non-coherent modulation techniques –
- BASK, BFSK, DPSK.

Digital Modulation Technique

- Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave. In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it and the carrier is sinusoidal wave.
- Different Shift keying methods that are used in digital modulation techniques are
- Amplitude shift keying [ASK]
- Frequency shift keying [FSK]
- Phase shift keying [PSK]



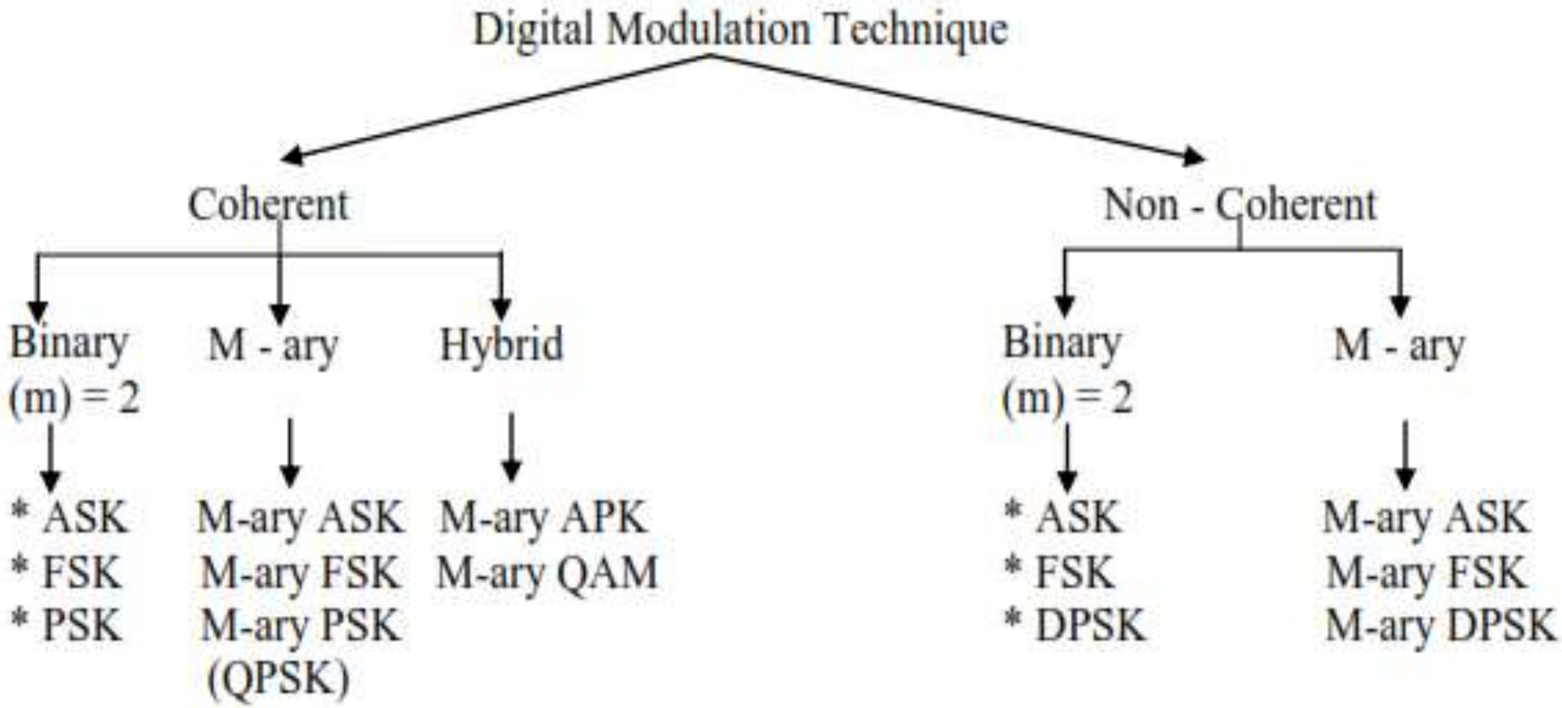
1v
0v
Input binary sequence
1v
0v
-1v
ASK Modulated output wave
1v
0v
Input binary sequence
time
1v
0v
-1v
time
f_1
f_2
FSK Modulated output wave
1v
0v
Input binary sequence
time
1v
0v
-1v
time
BPSK Modulated output wave

- With a sinusoidal carrier ,the feature that is used by the modulator to distinguish one signal from another is a step change in the amplitude,frequency or phase of the carrier.

Advantage of PSK,FSK over ASK

- PSK and FSK signals have a constant envelope used in microwave radio links and satellite channels.
- Hybrid Technique- Change in both amplitude and phase of the carrier are combined to produce amplitude-phase keying(APK)

Hierarchy of digital modulation scheme



Demodulation at receiver

1. Coherent detection
2. Non coherent detection

- Coherent Detection
- In coherent detection, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. The detection is done by correlating received noisy signal and locally generated carrier. The coherent detection is a synchronous detection.
- Non coherent detection:
- In this method, the receiver carrier need not be phase locked with transmitter carrier. Hence it is called envelope detection.

Design goals of Digital communication

1.Maximum data rate

2. Minimum probability of symbol error

3.Minimum Transmitted power

4.Minimum Channel bandwidth

5.Maximum resistance to interfering signals

6.Minimum circuit complexity.

Gram Schmidt Orthogonalization

Let there be M number of energy signals which form the input signal set i.e.,s_i(t),i=1,……M

Let these signals be represented in terms of N number of orthonormal basis functions φ_j(t) j=1,…N

Then the linear relationship between s_i(t) and φ_j(t) can be written as,

$$s_i(t)=s_{i1}\varphi_1(t)+s_{i2}\varphi_2(t)+\ldots\ldots+s_{iN}\varphi_j(t)$$

$$=\sum_{j=1}^{N} s_{ij}\,\varphi_j(t) \qquad (1)$$

$$s_{ij} = coefficients\,of\,expansion$$

$$s_{ij}=\int_0^T s_i\,(t)\,\varphi_j(t)\,dt$$

T=duration of symbol s_i(t)

Properties of orthonormal basis function

Since basis function φ_1(t),φ_2(t),….φ_N(t) are orthonormal ,they satisfy following property.

$$\int_0^T \varphi_i(t)\varphi_j(t)\,dt = \begin{cases}1, & for\ i = j\\ 0, & for\ i \neq j\end{cases} \qquad (2)$$

When i=j $,\int_0^T \varphi_i^2(t)\,dt$=1

Thus the basis function have energy,E=1 or unit energy when $i = j$

When $i \neq j, \int_0^T \varphi_i(t)\varphi_j(t)\,dt$ =0

This means the basis functions,φ_1(t),φ_2(t),….φ_N(t) are orthogonal to each other over an interval of 0 to T_b

Two dimensional Signal space with Three symbols

Let vector space representation of M=3 message symbols with the help of N=2 orthonormal basis functions.

$$s_i(t)=\sum_{j=1}^{N} s_{ij}\,\varphi_j(t),$$

$$=s_{i1}\varphi_1(t)+s_{i2}\varphi_2(t)$$

$$s_1(t)=s_{11}\varphi_1(t)+s_{12}\varphi_2(t)$$

$$s_2(t)=s_{21}\varphi_1(t)+s_{22}\varphi_2(t)$$

$$s_3(t)=s_{31}\varphi_1(t)+s_{32}\varphi_2(t)$$

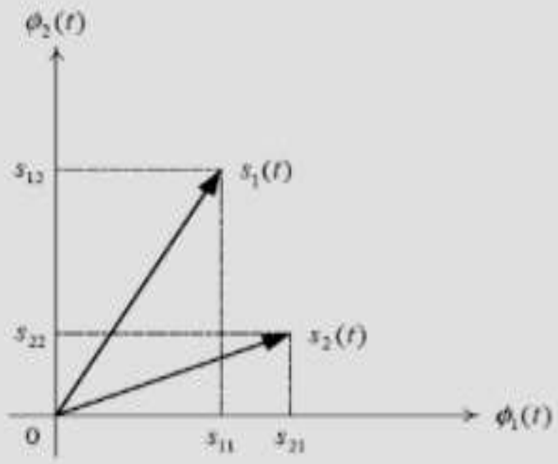


Fig 2: Geometric representation of message vectors

$s_1(t), s_2(t), s_3(t)$ are expressed in terms of $\varphi_1(t)$ and $\varphi_2(t)$, $s_1(t)$ is represented as vector space $\varphi_1-\varphi_2$ in signal space.

The position of the vectors depends upon the coefficients $s_{11}, s_{12}, s_{21}, s_{22}, \ldots\ldots etc$,$\varphi_1 and \varphi_2$ are perpendicular to each other,they are orthogonal, $\varphi_1 - \varphi_2$ signal space is called Euclidean space.

Absolute value or Norm of a vector

Consider the vector S_i,which is completely determined by its coefficients i.e

$$S_i=\begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{iN} \end{bmatrix} \text{ and } i=1,2,\ldots\ldots\ldots M$$

The product $||s_i||^2$ is given as,

$$||s_i||^2 = s_i^T s_i$$

$$=[s_{i1} \quad s_{i2}\ldots\ldots\ldots\ldots \quad s_{iN}]\begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{iN} \end{bmatrix}$$

$$=s_{i1}^2 + s_{i2}^2 + s_{i3}^2 + \ldots\ldots\ldots\ldots\ldots\ldots + s_{iN}^2$$

$$=\sum_{j=1}^{N} s_{ij}^2 \text{ and } i=1,2,\ldots\ldots,M$$

Relationship between Signal Energy and its vector

Energy of the signal s_i(t)is given as

$$E_i=\int_0^T s_i^2 \, dt$$

Substitute s_i(t) from eq(1)

$$E_i=\int_0^T \left[\sum_{j=1}^{N} s_{ij}\,\varphi_j(t)\right]\left[\sum_{k=1}^{N} s_{ij}\,\varphi_k(t)\right] dt$$

Rearranging order of summation and integration

$$E_i=\sum_{j=1}^{N} s_{ij} \sum_{k=1}^{N} s_{ik} \int_0^T \varphi_i(t)\varphi_k(t)\, dt$$

From equ(2) we know that $\int_0^T \varphi_i(t)\varphi_j(t)\, dt = \begin{cases} 1, & for\ i=j \\ 0, & for\ i \neq j \end{cases}$

E_i becomes

...cont

$$E_i = \begin{cases} \sum_{j=1}^{N} s_{ij} \sum_{j=1}^{N} s_{ik}, if\ j = k \\ 0, if\ j \neq k \end{cases}$$

$E_i = \sum_{j=1}^{N} s_{ij}^2$ we know that $\sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2$

$E_i = \|s_i\|^2$, Signal energy is equal to squared length of the signal vector since $\|s_i\|^2 = s_i^T s_i$

$E_i = s_i^T s_i$

Euclidean Distance

Euclidean distance between the two signal vector is given as,

$d_{ik}=\|s_i - s_k\|$

Squared Euclidean distance will be $\|s_i - s_k\|^2=\sum_{j=1}^{N}(s_{ij} - s_{kj})^2$

Since $E_i=\int_0^T s_i^2\,dt=\|s_i\|^2$

$\|s_i - s_k\|^2=\int_0^T [s_i(t) - s_k(t)]^2\,dt$.The angle θ_{ik} between the two vectors s_i and s_k is given as $\mathbf{\cos\theta_{ik}}=\frac{\boldsymbol{s_i^T s_k}}{\|\boldsymbol{s_i}\|\|\boldsymbol{s_k}\|}$

When $\boldsymbol{s_i^T s_k}$=0,the two vectors are orthogonal or perpendicular to each other then $\boldsymbol{\theta_{ik}}$**=90**

Numerical(Required equation to solve numericals)

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t) \quad , \quad i = 1, 2, \ldots, M \, , \quad 0 \le t \le T$$

$$\text{where } s_{ij} =< s_i(t), \phi_j(t) >= \int_0^T s_i(t)\phi_j(t)dt$$

$$g_1(t) = s_1(t)$$

$$\phi_1(t) = \frac{g_1(t)}{\|g_1(t)\|} = \frac{s_1(t)}{\sqrt{E_1}}$$

$$\phi_2(t) = \frac{g_2(t)}{\|g_2(t)\|}$$

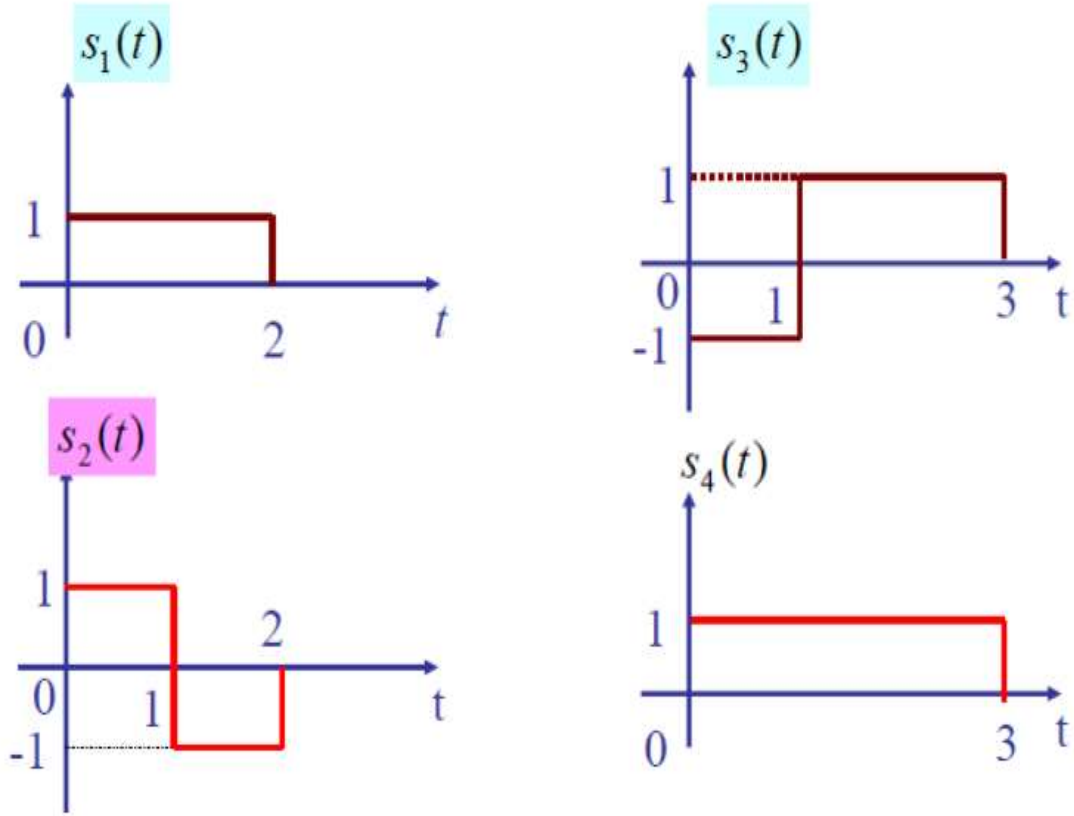
$$\phi_n(t) = \frac{g_n(t)}{\|g_n(t)\|}$$

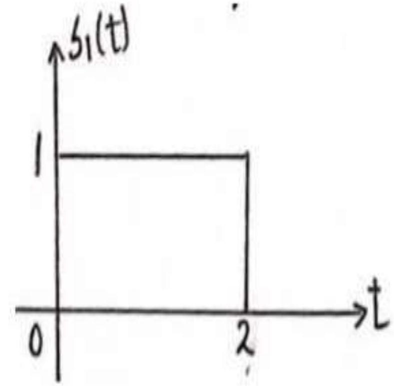
$$g_n(t) = s_n(t) - \sum_{j=1}^{n-1} s_{nj}\phi_j(t)$$

$$\| g_n(t) \| = \sqrt{E_n - \sum_{j=1}^{n-1} s_{nj}^2}$$

$$= \frac{s_n(t) - \sum_{j=1}^{n-1} s_{nj}\phi_j(t)}{\sqrt{E_n - \sum_{j=1}^{n-1} s_{nj}^2}}$$

Example. A set of four waveform is illustrated as below. Find an orthonormal set for this set of signals by applying the Gram-Schmidt procedure.

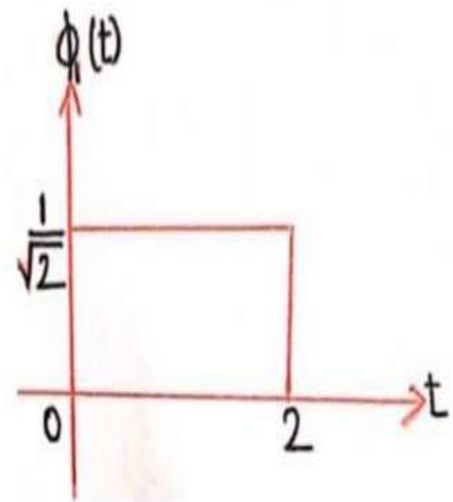




1) To find $\phi_1(t)$

Energy of $s_1(t)$, $E_1 = \int_0^T s_1^2(t)\,dt = \int_0^2 1^2\,dt = \underline{\underline{2}}$

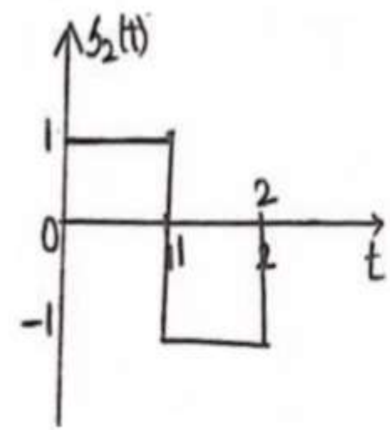
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \underline{\underline{\frac{s_1(t)}{\sqrt{2}}}}$$



2) To find $\phi_2(t)$

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt = \int_0^1 \frac{1}{\sqrt{2}}\,dt + \int_1^2 -\frac{1}{\sqrt{2}}\,dt$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \underline{0}$$

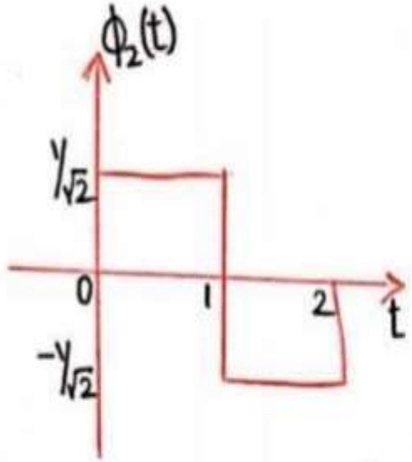


$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t) = s_2(t)$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} = \frac{s_2(t)}{\sqrt{\int_0^T s_2^2(t)\,dt}}$$

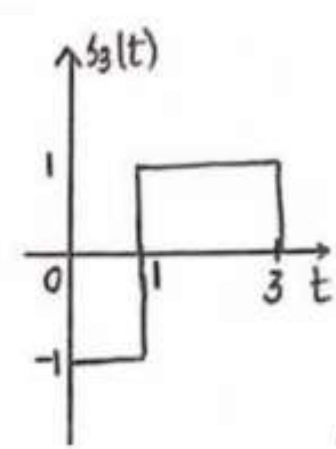
Energy of $s_2(t)$, $E_2 = \int_0^T s_2^2(t)\,dt = \int_0^1 1^2\,dt + \int_1^2 1^2\,dt = 2$

$$\phi_2(t) = \underline{\underline{\frac{s_2(t)}{\sqrt{2}}}}$$



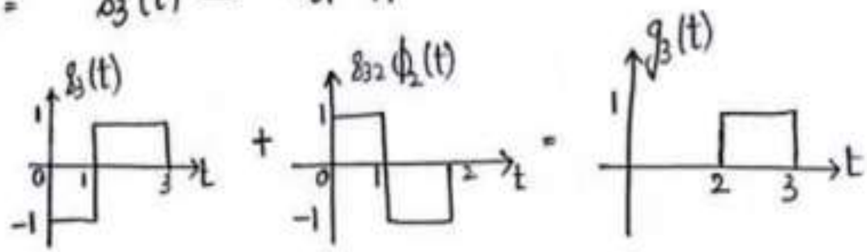
3) To find $\phi_3(t)$

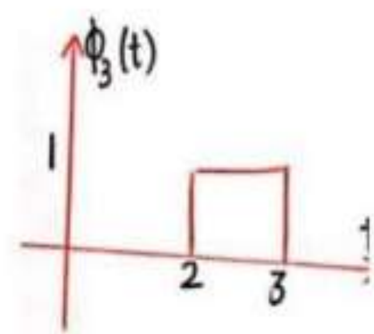
$$s_{31} = \int_0^T s_3(t)\,\phi_1(t)\,dt = \int_0^1 -\tfrac{1}{\sqrt{2}}\,dt + \int_1^2 \frac{1}{\sqrt{2}}\,dt = 0$$



$$s_{32} = \int_0^T s_3(t)\,\phi_2(t)\,dt = \int_0^1 -\frac{1}{\sqrt{2}}\,dt + \int_1^2 -\frac{1}{\sqrt{2}}\,dt = -\sqrt{2}$$

$$g_3(t) = s_3(t) - s_{31}\,\phi_1(t) - s_{32}\,\phi_2(t)$$





$$\phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)\,dt}} = \frac{g_3(t)}{\sqrt{\int_2^3 1^2\,dt}} = g_3(t)$$

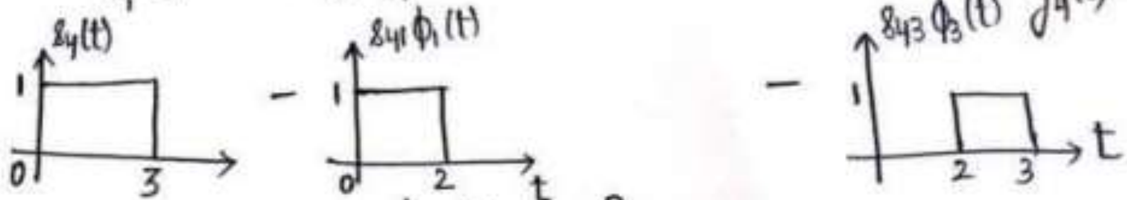
4) To find $\phi_4(t)$

$$s_{41} = \int_0^T s_4(t)\,\phi_1(t)\,dt = \int_0^2 1/\sqrt{2}\,dt = \underline{\sqrt{2}}$$

$$s_{42} = \int_0^T s_4(t)\,\phi_2(t)\,dt = \int_0^1 \frac{1}{\sqrt{2}}\,dt - \int_1^2 \frac{1}{\sqrt{2}}\,dt = \underline{0}$$

$$s_{43} = \int_0^T s_4(t)\,\phi_3(t)\,dt = \int_2^3 1\,dt = \underline{1}$$

$$g_4(t) = s_4(t) - s_{41}\,\phi_1(t) - s_{42}\,\phi_2(t) - s_{43}\,\phi_3(t) = \underline{0}$$



$g_4(t) = \underline{0}$

$\therefore\ \phi_4(t) = \underline{0}$

Coherent Techniques

COHERENT BPSK MODULATION TECHNIQUE

Generation Process

Input binary sequence is represented in polar form with symbol 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$ respectively.

This binary wave and sinusoidal carrier wave $\varphi_1(t)$ are applied to product modulator.

The carrier and the timing pulses used to generate the binary wave are usually extracted from a common master clock. The desired PSK wave is obtained at the modulator output.

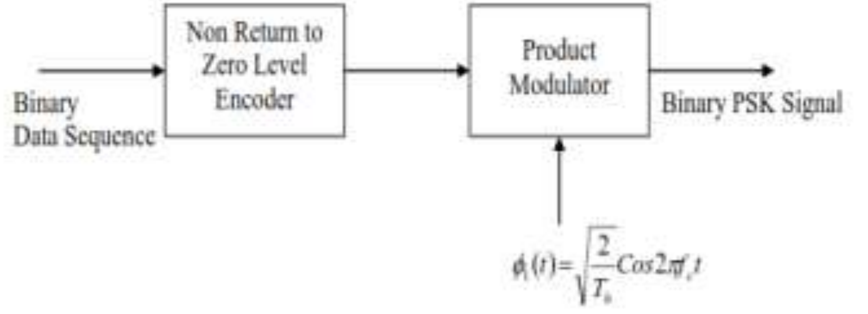


Fig : BPSK Transmitter

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Detection

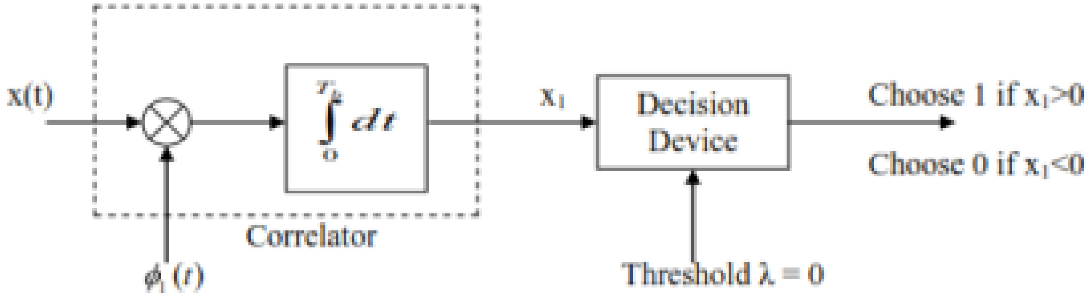


Fig BPSK Receiver

To detect the original binary sequence 0f 1s and 0s,Noisy PSK wave x(t) is applied to a correlator,which is also supplied with a locally generated coherent reference signal $\varphi_1(t)$.The correlator output is compared with threshold of zero volts .If $x_1>0$,the receiver decides in favor of symbol 1 ,if $x_1<0$.it decides in favour of symbol 0.

Probability of error (BER-Bit error rate) of BPSK

In a coherent BPSK the pair of signals $S_1(t)$ and $S_2(t)$ are used to represent binary symbol 1 and 0

$$S_1(t)=\sqrt{\frac{2E_b}{T_b}}\,Cos2\pi f_c t \qquad \text{---------} \qquad \text{for Symbol '1'}$$

$$S_2(t)=\sqrt{\frac{2E_b}{T_b}}\,Cos(2\pi f_c t+\pi)=-\sqrt{\frac{2E_b}{T_b}}\,Cos2\pi f_c t \quad \text{-------} \quad \text{for Symbol '0'}$$

E_b=Transmitted signal energy per bit

Each transmitted bit contains an integral number of cycles of the carrier wave $f_c=n_c/T_b$.

A pair of sinusoidal waves that differ only in a relative phase shift 180 is referred to as antipodal signals

In case BPSK there is only one basis function with unit energy is given by,

$$\phi_1(t)=\sqrt{\frac{2}{T_b}}\,Cos2\pi f_c t \qquad 0\le t\le T_b$$

Therefore transmitted signals are given by,

$$S_1(t)=\sqrt{E_b}\,\phi_1(t) \qquad 0\le t\le T_b \qquad for\ \ Symbol\,1$$

$$S_2(t)=-\sqrt{E_b}\,\phi_1(t) \qquad 0\le t\le T_b \qquad for\ \ Symbol\,0$$

A coherent BPSK having a signal space that is one dimensional (N=1)with two message points(M=2)

$$S_{11}=\int_0^{T_b} S_1(t)\phi_1(t)\,dt=+\sqrt{E_b}$$

$$S_{21}=\int_0^{T_b} S_2(t)\phi_1(t)\,dt=-\sqrt{E_b}$$

$S_1(t)$ is located at $S_{11}=+\sqrt{E_b}$ and $S_2(t)$ is located at $S_{21}= -\sqrt{E_b}$

Partition of signal space into two region

1.The set of point closest to the message point at $+\sqrt{E_b}$

2.The set of point closest to message point at $-\sqrt{E_b}$

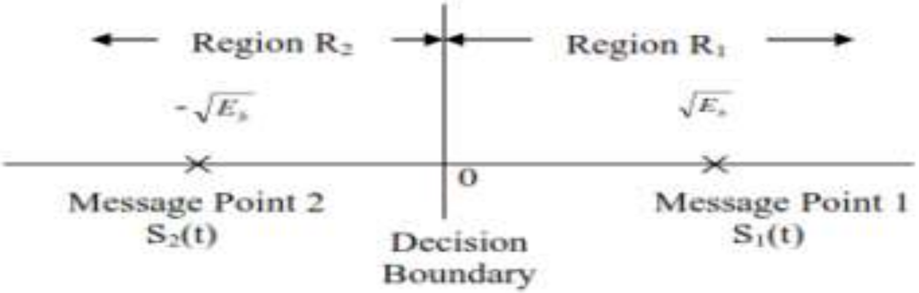


Fig :Signal space representation of BPSK

Decision Rule

Symbol 1 or $S_1(t)$ was transmitted if the received signal point falls in region Z_1 and Symbol 0 or $S_2(t)$ was transmitted if the received signal point falls in region Z_2.

Noise

$S_2(t)$ is transmitted but due to the noise,the received signal point falls inside Z_1 and receiver decides in favour of signal $S_1(t)$

Probability of error

The decision region for symbol 1or signal $S_1(t)$ is $Z_1 : 0 < x_1 < 1$. x_1 is the observation scalar

$$x_1 = \int_0^T x(t)\phi_1(t)\,dt$$

x(t) is received signal.

The error is of two types

1) $P_e(0/1)$ i.e. transmitted as '1' but received as '0' and
2) $P_e(1/0)$ i.e. transmitted as '0' but received as '1'.

Error of first kind

$$P_e(1/0)=\frac{1}{\sqrt{2\pi\sigma^2}}\int_0^{\infty}\exp\left[\frac{(x_1-\mu)^2}{2\sigma^2}\right]dx_1$$

Assuming gaussian distribution

Where μ=mean value=- $\sqrt{E_b}$ for symbol 0,σ^2=variance=$\frac{N_0}{2}$=variance of additive white gaussian noise

Thershold value= 0 [indicates lower limits integration]

Threshold value= 0 [indicates lower limits integration]

$$P_{e0} = P_e(1/0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{N_0}\right] dx_1$$

Put $Z = \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}}$

$$P_{e0} = P_e(1/0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{(E_b/N_0)}}^\infty \exp[(-Z)^2] dz$$

$$P_e(1/0) = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

$$\boxed{\operatorname{erfc}(x) = 2\int_x^\infty \frac{1}{\sqrt{\pi}} \exp(-u^2)\, \mathrm{d}u. \qquad \operatorname{erf}(x) = 1 - \operatorname{erfc}(x).}$$

$$P_e(0/1) = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

Similarly

The total probability of error –assuming probability of error for symbol 1 and symbol 0 are equal

$$P_e = P_e(1/0)\,P_e(0) + P_e(0/1)\,P_e(1)$$

$$P_e = \frac{1}{2}[P_e(1/0) + P_e(0/1)]$$

$$P_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

Coherent BFSK

Generation

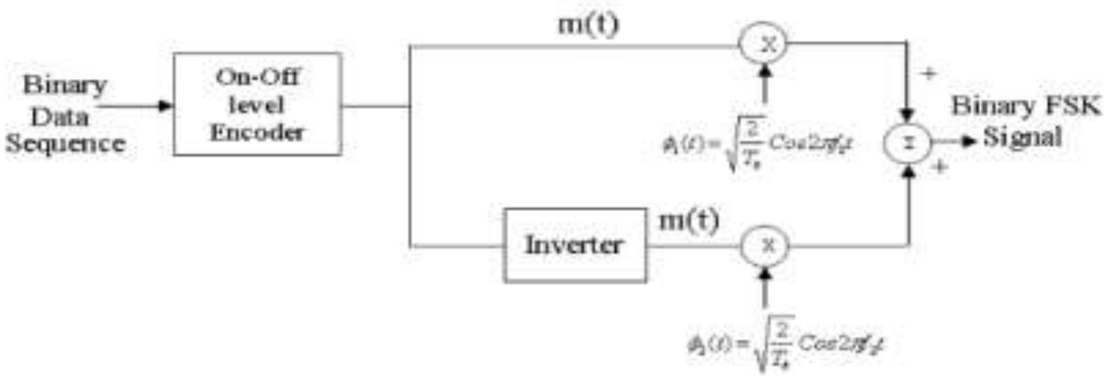


Fig BFSK Transmitter

A binary FSK Transmitter is as shown, the incoming binary data sequence is applied to on-off level encoder.

The output of encoder is $\sqrt{E_b}$ volts for symbol 1 and 0 volts for symbol"0".

When we have symbol 1 the upper channel is switched on with oscillator frequency f1.

For symbol "0", because of inverter the lower channel is switched on with oscillator frequency f2.

These two frequencies are combined using an adder circuit and then transmitted. The transmitted signal is nothing but required BFSK signal.

Detection

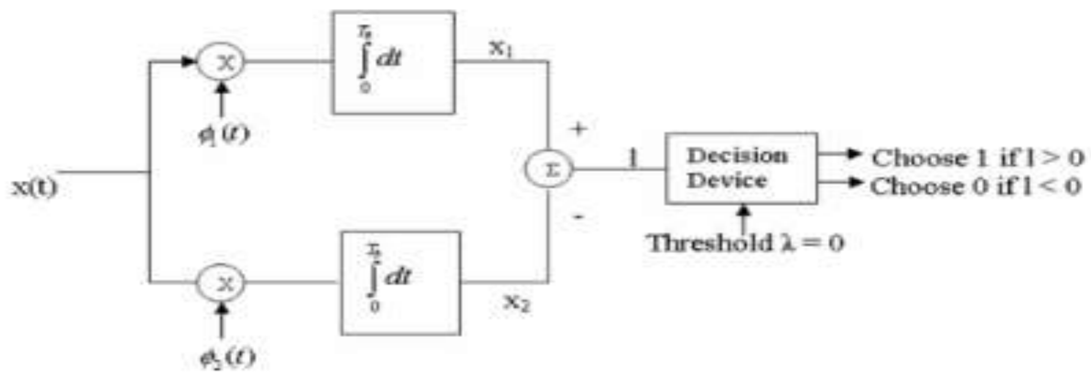


Fig :BFSK Receiver

The detector consists of two correlators. The incoming noisy BFSK signal x(t) is common to both correlator.

The Coherent reference signal $\varphi_1(t) \& \varphi_2(t)$ (are supplied to upper and lower correlators respectively.

The correlator outputs are then subtracted one from the other and resulting a random vector"l" (l=x1 - x2). The output "l" is compared with threshold of zero volts.

If $l > 0$, the receiver decides in favour of symbol 1.
$l < 0$, the receiver decides in favour of symbol 0.

Probability of error (BER-Bit error rate) for BFSK

In binary FSK the two basis function are given by,

$$\varphi_1(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi f_1 t \qquad 0 \leq t \leq T_b$$

$$\varphi_2(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi f_2 t \qquad 0 \leq t \leq T_b$$

The transmitted signals S_1 (t) and S_2(t) are given by,

S_1 (t) = $\sqrt{E_b}\varphi_1(t)$ for symbol 0

S_2(t) = $\sqrt{E_b}\varphi_2(t)$ for symbol 1

Therefore BFSK system has two dimensional signal space with two message points S_1 (t) and S_2(t). {N=2, M=2

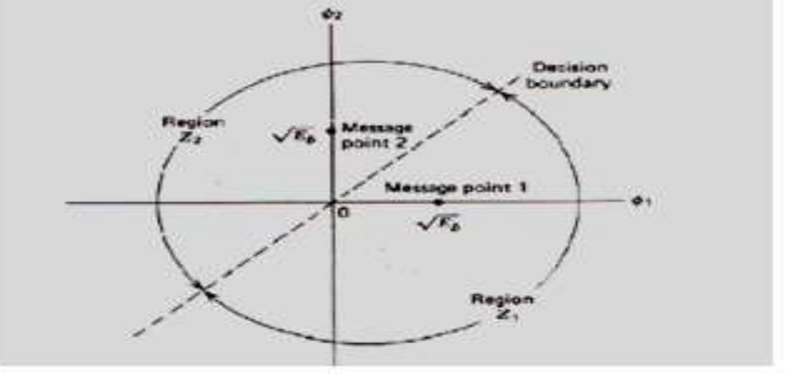


Fig :Signal space diagram of coherent BFSK

The two message point are represented by signal vector

$$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad \text{and} \quad S_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

The observation vector x_1 and x_2 (output of upper and lower correlator) are related to the input signal x (t) as,

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)dt \quad \text{and}$$

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)dt$$

Mean=zero, PSD=σ^2=variance=$\frac{N_0}{2}$ of AWGN noise.

The new observation vector L is the difference of two random variable $L=x_1-x_2$

When symbol 1 was transmitted x_1 and x_2 has 0 and $\sqrt{E_b}$ respectively. Therefore conditional mean of random variable L for symbol 1 is given by

$$\begin{aligned} E\left[\frac{l}{1}\right] &= E\left[\frac{x_1}{1}\right] - E\left[\frac{x_2}{1}\right] \\ &= \sqrt{E_b} - 0 \\ &= \sqrt{E_b} \end{aligned}$$

Conditional mean of random variable L for symbol 0 is given by,

$$E\left[\frac{l}{0}\right] = -\sqrt{E_b}$$

The total variance of L is given by

$$\begin{aligned} Var[l] &= Var[x_1] + Var[x_2] \\ &= N_0 \end{aligned}$$

The probability of error is given by,

$$P_e(1/0) = P_{e0} = \frac{1}{\sqrt{2\pi N_0}} \int_0^{\infty} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] dl$$

Put $Z = \frac{l+\sqrt{E_b}}{\sqrt{2N_0}}$

$$P_{e0} = \frac{1}{\pi} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \exp(-z^2) dz$$

$$= \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{2N_0}}\right]$$

$$\boxed{\operatorname{erfc}(x) = 2\int_x^{\infty} \frac{1}{\sqrt{\pi}} \exp(-u^2)\, \mathrm{d}u. \qquad \operatorname{erf}(x) = 1 - \operatorname{erfc}(x).}$$

$$P_{e1} = \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{2N_0}}\right]$$

The total probability of error is given by,

$$P_e = \frac{1}{2}[P_e(1/0) + P_e(0/1)]$$

Assuming 1 and 0 has equal probability of error

$$P_e = \frac{1}{2}[P_{e0} + P_{e1}]$$

$$P_e = \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{2N_o}}\right]$$

QPSK

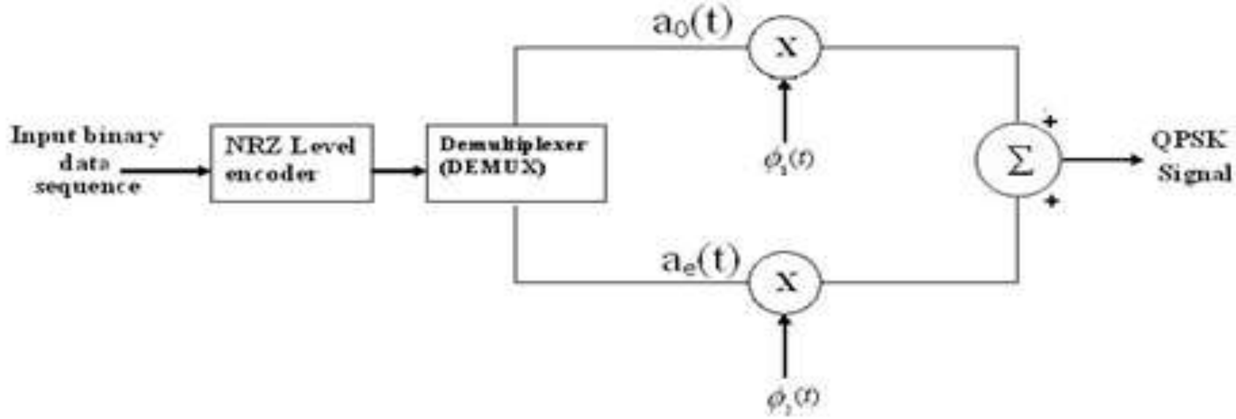


Fig a: QPSK Transmitter

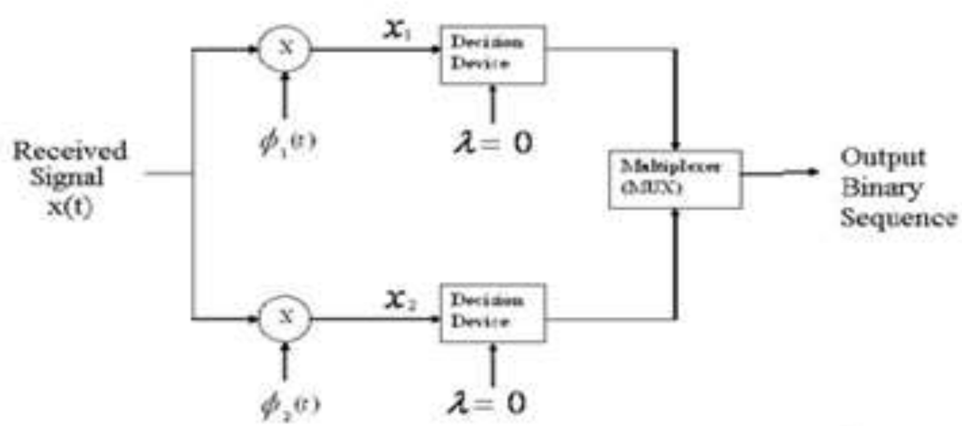


Fig b: QPSK Receiver

Generation:-

Fig(a) shows a block diagram of a typical QPSK transmitter, the incoming binary data sequence is first transformed into polar form by a NRZ level encoder. Thus the symbols 1& 0 are represented by $+\sqrt{E_b}$ *and* $-\sqrt{E_b}$ respectively. This binary wave is next divided by means of a demultiplexer [Serial to parallel conversion] into two separate binary waves consisting of the odd and even numbered input bits. These two binary waves are denoted by $a_o(t)$ and $a_e(t)$

The two binary waves $a_o(t)$ and $a_e(t)$ are used to modulate a pair of quadrature carriers or orthonormal basis functions $\phi_1(t)$ & $\phi_2(t)$ which are given by

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$$

&

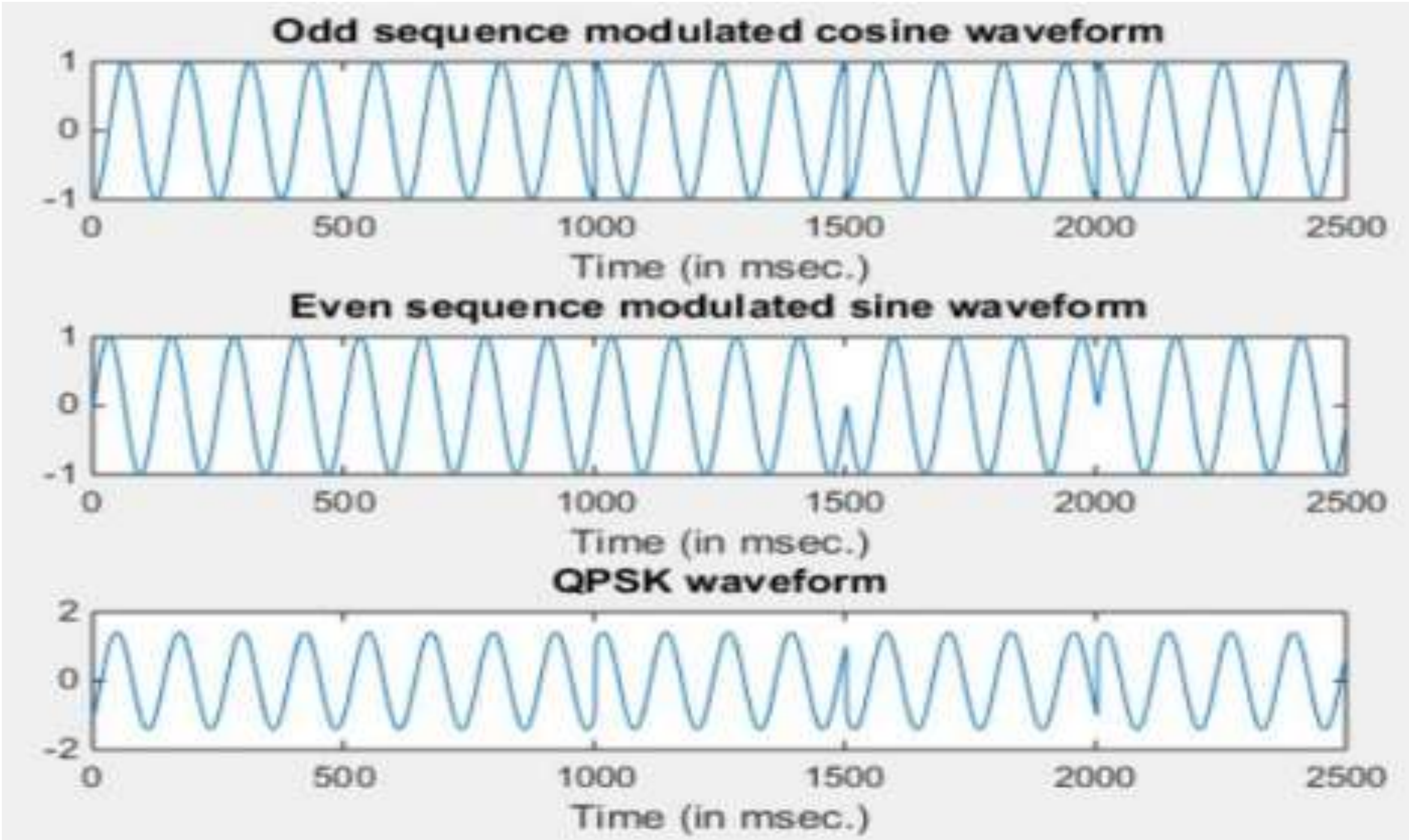
$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$

The result is a pair of binary PSK signals, which may be detected independently due to the orthogonality of $\phi_1(t)$ & $\phi_2(t)$.

Finally the two binary PSK signals are added to produce the desired QPSK signal.

Detection:-

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals $\phi_1(t)$ & $\phi_2(t)$ as shown in fig(b).The correlator outputs x_1 and x_2 produced in response to the received signal x(t) are each compared with a threshold value of zero.


Odd sequence modulated cosine waveform
1
0
-1
0
500
1000
1500
2000
2500
Time (in msec.)
Even sequence modulated sine waveform
1
0
-1
0
500
1000
1500
2000
2500
Time (in msec.)
QPSK waveform
2
0
-2
0
500
1000
1500
2000
2500
Time (in msec.)

In QPSK signal the information carried by the transmitted signal is contained in the phase. The transmitted signal is given by,

$$S_1(t)=\sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t+\frac{\pi}{4}\right] \qquad --- \quad for input \quad dibit \quad 10$$

$$S_2(t)=\sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t+\frac{3\pi}{4}\right] \qquad --- \quad for input \quad dibit \quad 00$$

$$S_3(t)=\sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t+\frac{5\pi}{4}\right] \qquad --- \quad for input \quad dibit \quad 01$$

$$S_4(t)=\sqrt{\frac{2E}{T}}\cos\left[2\pi f_c t+\frac{7\pi}{4}\right] \qquad --- \quad for input \quad dibit \quad 11$$

Where the carrier frequency $f_c=\frac{n_c}{7}$ for some fixed integer n_c

E = the transmitted signal energy per symbol.

T = Symbol duration.

The basic functions $\phi_1(t)$ *and* $\phi_2(t)$ *are given by*

$$\phi_1(t)=\sqrt{\frac{2}{T_b}}\cos\left[2\pi f_c t\right] \qquad 0\le t<T$$

$$\phi_2(t)=\sqrt{\frac{2}{T_b}}\sin\left[2\pi f_c t\right] \qquad 0\le t<T$$

There are four message points and the associated signal vectors are defined by

$$Si=\begin{bmatrix}\sqrt{E}\ \cos\left[(2i-1)\frac{\pi}{4}\right]\\ -\sqrt{E}\ \sin\left[(2i-1)\frac{\pi}{4}\right]\end{bmatrix} \qquad i=1,2,3,4$$

The table shows the elements of signal vectors, namely S_{i1} & S_{i2}

Table:-

Input dibit	Phase of QPSK signal(radians)	Coordinates of message points	
		S_{i1}	S_{i2}
10	$\frac{\pi}{4}$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$\frac{3\pi}{4}$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$\frac{5\pi}{4}$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$\frac{7\pi}{4}$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

Therefore a QPSK signal is characterized by having a two dimensional signal constellation(i.e.N=2)and four message points(i.e. M=4) as illustrated in fig(d)

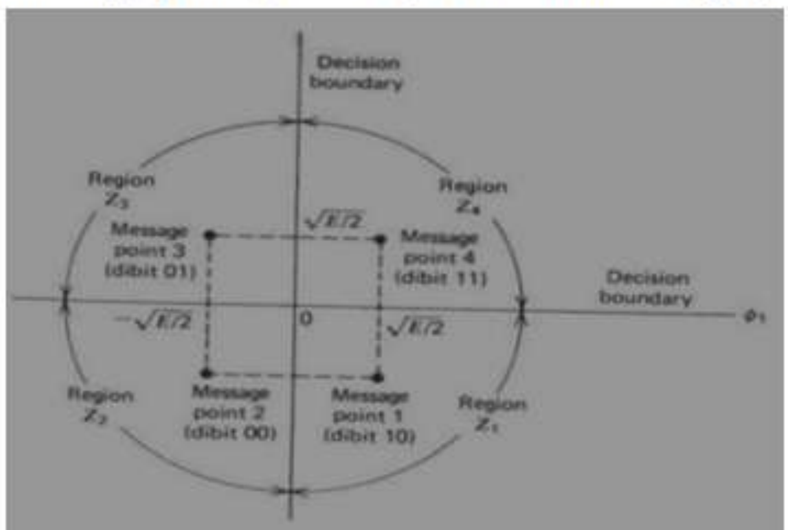


Fig: Signal space diagram of QPSK

Probability of Error

A QPSK system is in fact equivalent to two coherent binary PSK systems working in parallel and using carriers that are in-phase and quadrature.

The in-phase channel output x_1 and the Q-channel output x_2 may be viewed as the individual outputs of the two coherent binary PSK systems. Thus the two binary PSK systems may be characterized as follows.

The signal energy per bit $\sqrt{E/2}$

The noise spectral density is $\frac{N_0}{2}$

$$P^1 = \frac{1}{2}\ erfc\left[\sqrt{\frac{E/2}{N_0}}\right] \qquad \left(E = E/2\right)$$

$$= \frac{1}{2}\ erfc\left[\sqrt{\frac{E}{2N_0}}\right]$$

The bit errors in the I-channel and Q-channel of the QPSK system are statistically independent . The I-channel makes a decision on one of the two bits constituting a symbol (d_i bit) of the QPSK signal and the Q-channel takes care of the other bit.

Therefore, the average probability of a direct decision resulting from the combined action of the two channels working together is

p_c= probability of correct reception

p^1= probability of error

$$P_c = \left[1 - P^1\right]^2$$

$$= \left[1 - \frac{1}{2} erfc\left[\sqrt{\frac{E}{2No}}\right]\right]^2$$

$$= 1 - erfc\left[\sqrt{\frac{E}{2No}}\right] + \frac{1}{4} erfc^2\left[\sqrt{\frac{E}{2No}}\right]$$

In the region where $\frac{E}{2N_o} >> 1$ We may ignore the second term and so the approximate formula for the average probability of symbol error for coherent QPSK system is

$$P_e = erfc\sqrt{\frac{E}{2No}}$$

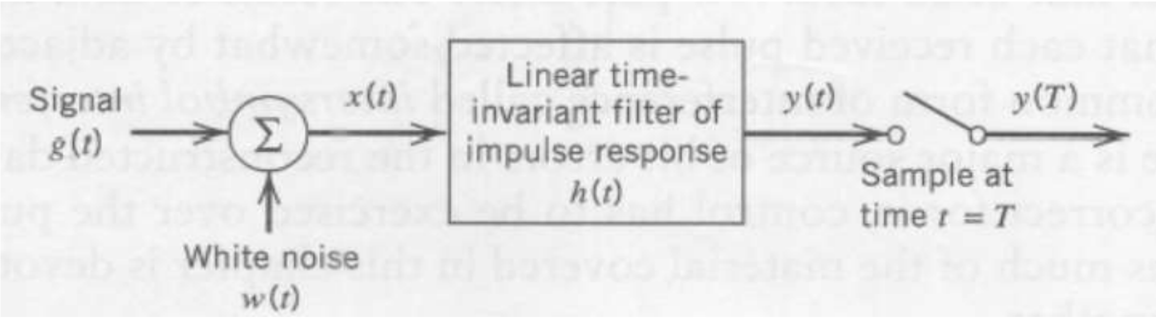
TABLE A6.6 *The error function*[a]

u	$erf(u)$	u	$erf(u)$
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

Matched filter

- The matched filter is the optimal linear filter for maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise.
- Matched filters are commonly used in radar, in which a signal is sent out, and we measure the reflected signals, looking for something similar to what was sent out.
- Two-dimensional matched filters are commonly used in image processing, e.g., to improve SNR for X-ray pictures

- Detecting the pulse over the channel that is corrupted by the channel noise



- The filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive channel noise w(t), as shown by

$$x(t) = g(t) + w(t), \qquad 0 \leq t \leq T$$

- where T is an arbitrary observation interval. The pulse signal *g(t)* may represent a binary symbol I or 0 in a digital communication system.
- The *w(t) is* the sample function of a white noise process of zero mean and power spectral density No/2.
- The source of uncertainty lies in the noise w(t).
- The function of the receiver is to detect the pulse signal *g(t)* in an optimum manner, given the received signal x(t).
- To satisfy this requirement, we have to optimize the design of the filter so as to minimize the effects of noise at the filter output in some statistical sense, and thereby enhance the detection of the pulse signal g(t).

- Since the filter is linear, the resulting output *y(t)* may be

$$y(t) = g_o(t) + n(t)$$

- where $g_o(t)$ and *n(t)* are produced by the signal and noise components of the input *x(t)*, respectively.
- A simple way of describing the requirement that the output signal component $g_o(t)$ be considerably greater than the output noise component *n(t) is* to have the filter make the instantaneous power in the output signal $g_o(t)$, measured at time $t = T$, as large as possible compared with the average power of the output noise n(t). This is equivalent to maximizing the *peak pulse signal-to-noise ratio,* defined as

$$\eta = \frac{|g_o(T)|^2}{E[n^2(t)]}$$

- G(f) is the fourier transform of the known signal
- H(f) is the frequency response of the impulse filter. Thus the fourier transform of output signal is

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f) \exp(j2\pi f t)\, df$$

$$|g_o(T)|^2 = \left| \int_{-\infty}^{\infty} H(f)G(f) \exp(j2\pi f T)\, df \right|^2$$

Power spectral density of the input noise w(t) is equal to the power spectral density of output noise n(t)

$$S_N(f) = \frac{N_0}{2} |H(f)|^2$$

The average power of the output noise $n(t)$ is therefore

$$\begin{aligned} E[n^2(t)] &= \int_{-\infty}^{\infty} S_N(f)\, df \\ &= \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df \end{aligned}$$

$$\eta = \frac{\left| \int_{-\infty}^{\infty} H(f) G(f) \exp(j2\pi f T)\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df}$$

- Schwarz inequality

$$\left|\int_{-\infty}^{\infty} \phi_1(x)\phi_2(x)\, dx\right|^2 \le \int_{-\infty}^{\infty} |\phi_1(x)|^2\, dx \int_{-\infty}^{\infty} |\phi_2(x)|^2\, dx$$

$$\phi_1(x) = k\phi_2^*(x)$$

$$\phi_1(x) = H(f) \text{ and } \phi_2(x) = G(f)\exp(j\pi f T)$$

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi f T)\, df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |G(f)|^2\, df$$

$$\eta \le \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\, df$$

$$\eta_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2 \, df$$

Optimum value of the filter is given by

$$H_{opt}(f) = kG^*(f)\exp(-j2\pi fT)$$

where $G^*(f)$ is the complex conjugate of the Fourier transform of the input signal $g(t)$

k is a scaling factor of appropriate dimensions.

This relation states that expect for the factor $k\exp(-j2\pi fT)$,
The frequency response of the optimum filter is same as the complex conjugate of Fourier transform of input signal

- In time Domain

$$h_{\text{opt}}(t) = k \int_{-\infty}^{\infty} G^*(f) \exp[-j2\pi f(T - t)]\, df$$

Since for a real signal $g(t)$ we have $G^*(f) = G(-f)$, we may rewrite Equation

$$\begin{aligned} h_{\text{opt}}(t) &= k \int_{-\infty}^{\infty} G(-f) \exp[-j2\pi f(T - t)]\, df \\ &= k \int_{-\infty}^{\infty} G(f) \exp\,[j2\pi f(T - t)]\, df \\ &= kg(T - t) \end{aligned}$$