OFC :

1. $n = c / v$

2. $n_1 \sin \phi_1 = n_2 \sin \phi_2$ (Snell's Law)

3. $\theta_c = \sin^{-1} (n_2 / n_1)$

4. Power Law for ref. index : (GIF)

$$n(r) = \begin{cases} n_1 \left[1 - 2\Delta \left(\frac{r}{a} \right)^{\alpha} \right]^{1/2} & \boxed{r \in [0, a]} \\ n_1 (1 - 2\Delta)^{1/2} \approx n_1 (1 - \Delta) = n_2 & \boxed{r \geq a} \end{cases}$$

5. $NA = \sin \alpha = \sqrt{n_1^2 - n_2^2}$

when $n_1 \approx n_2$: $NA = n_1 \sqrt{2\Delta}$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

6. $\theta_a = \sin^{-1} (NA)$: Meridional

7. $\theta_a = \sin^{-1} (NA / \cos \gamma)$: Skew

8. Normalized Freq or V-Number :

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$

9. Modes $M \approx V^2/2$. (Step Index)

$M \approx \left[\dfrac{\alpha}{\alpha+2}\right]\dfrac{V^2}{2}$ $\alpha = 1$ (Triangular index)

$\alpha = 2$ (Parabolic)

11) Mode Field Diameter $E(r) = E_0\, e^{(r^2/w^2)}$

12) MFD $2W_0$ width

$$2W_0 = 2\left[\frac{\int_0^\infty r^3 E^2(r)\,dr}{\int_0^\infty r E^2(r)\,dr}\right]$$

13) Fiber Beat Length $L_p = 2\pi/\beta$ $\boxed{\text{Propagation const.}}$

$\beta \downarrow k_0 (n_y - n_x)$

Module 2:

$P(z) = P(0)\, e^{-\alpha_p z}$

$\alpha_{dB} = \dfrac{10}{z} \log\left[\dfrac{P(0)}{P(z)}\right]$

$\alpha_{dB} = \cancel{0.4}\ 4.343\, \alpha_p \ (km^{-1})$

$\Delta T = \dfrac{L n_1 \Delta}{c}$

RMS $\sigma = \dfrac{\cancel{\Delta T_m}\ L n_1 \Delta}{2c\sqrt{3}}$

Optical Derivation :

Phase Velocity , $v_p = \frac{c}{n}$

Group Velocity, $v_g = \frac{d\omega}{d\beta}$

$$\boxed{\omega = \frac{2\pi c}{\lambda}}$$

$$\Rightarrow v_g = \frac{d\omega}{d\lambda} \cdot \frac{d\lambda}{d\beta}$$

$$\boxed{\frac{d\omega}{d\lambda} \Rightarrow \frac{d}{d\lambda}\left[\frac{2\pi c}{\lambda}\right] = -\frac{2\pi c}{\lambda^2}}$$

$$v_g = -\frac{2\pi c}{\lambda^2} \cdot \frac{d\lambda}{d\beta}$$

$$\boxed{\beta = \frac{2\pi n_1}{\lambda}} \qquad \boxed{\frac{d\beta}{d\lambda} = \frac{d(2\pi n_1/\lambda)}{d\lambda}}$$

$$v_g = -\frac{2\pi c}{\lambda^2} \cdot \left(\frac{d\beta}{d\lambda}\right)^{-1}$$

$$= -\frac{2\pi c}{\lambda^2} \cdot \left(2\pi \cdot \frac{d(n_1/\lambda)}{d\lambda}\right)^{-1}$$

$$= -\frac{\cancel{2\pi} c}{\lambda^2} \cdot \frac{1}{\cancel{2\pi}} \left(\frac{d(n_1/\lambda)}{d\lambda}\right)^{-1}$$

$$= \frac{c}{\lambda^2}\left(\frac{\lambda n_1' - n_1(1)}{\lambda^2}\right)^{-1}$$

$$= -\frac{f\cancel{\lambda}}{\lambda^{\cancel{2}}}\left(\frac{\lambda \frac{dn_1}{d\lambda} - n_1}{\lambda^2}\right)^{-1}$$

$$= -\frac{f}{\lambda}\left(\frac{\lambda \cdot \frac{dn_1}{d\lambda} - n_1}{\lambda^2}\right)^{-1}$$

$$= -\frac{f\lambda^{\cancel{2}}}{\cancel{\lambda}}\left(\lambda\frac{dn_1}{d\lambda} - n_1\right)^{-1}$$

$$v_g = -f\lambda\left(\lambda\frac{dn_1}{d\lambda} - n_1\right)^{-1}$$

$$v_g = \frac{c}{n_1 - \lambda \frac{dn_1}{d\lambda}}$$

Group Delay : $\tau_g = \frac{1}{v_g}$

Pulse Delay

$$= \frac{1}{c}\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right)$$

$$\tau_m = \frac{L}{c}\left(n_1 - \lambda \frac{dn_1}{d\lambda}\right)$$

RMS Spectral width

$$\sigma_m = \sigma_\lambda \cdot \frac{d\tau_m}{d\lambda} + \sigma_\lambda \frac{2d^2\tau_m}{d\lambda^2} + \ldots$$

$$\sigma_m = \sigma_\lambda \cdot \frac{d\tau_m}{d\lambda}$$

$$\frac{d\tau_m}{d\lambda} = \frac{L}{c}\left[\cancel{\frac{dn_1}{d\lambda}} - \lambda \frac{d^2 n_1}{d\lambda^2} - \cancel{\frac{dn_1}{d\lambda}(1)}\right]$$

$$\frac{d\tau_m}{d\lambda} = \frac{-L}{c}\left(\frac{d^2 n_1}{d\lambda^2}\right)$$

$$\sigma_m = \sigma_\lambda \cdot \frac{L}{c}\left|\lambda \cdot \frac{d^2 n_1}{d\lambda^2}\right|$$

Material Dispersion Parameter :

$$M = \frac{1}{L} \cdot \frac{d\tau_m}{d\lambda}$$

$$= \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right| \quad ps/(km-nm)$$

Group delay in terms of propagation const :

$$b = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2}$$

$$b = \frac{\beta/k - n_2}{n_1 - n_2}$$

$$b(n_1 - n_2) = \beta/k - n_2$$

$$\beta = bk(n_1 - n_2) + kn_2$$

Multiply & Divide by n_1

$$\beta = n_2 k [b\Delta + 1] \qquad \boxed{\Delta = \frac{n_1 - n_2}{n_1}}$$

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2 \Delta \frac{d(kb)}{dk}\right]$$

$$V = \frac{2\pi a}{\lambda} \cdot NA$$

$$\approx kan_1 \sqrt{2\Delta}$$

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2 \Delta \frac{d(Vb)}{dV}\right]$$

$$\sigma_{wg} = \left|\frac{d\,wg}{d\lambda}\right| \sigma_\lambda$$

$$= L\,|D_{wg}|\,\sigma_\lambda$$

$$= L\left|\frac{d\tau_{wg}}{dV}\right| \sigma_\lambda$$

$$\sigma_{wg} = \frac{n_2 L \Delta \sigma_\lambda}{c\lambda} \cdot \frac{V\,d^2(Vb)}{dV^2}$$