i) Material Dispersion

↳ variation of n of core as a fn. of λ

also called chromatic dispersion

Analysis:-

Propagation const β for a mode is given as,

$$\beta = n \cdot \frac{2\pi}{\lambda}$$ $n \to$ ref. index, $\lambda = c/f$.

$$= n \frac{2\pi}{c/f} = n \frac{2\pi f}{c} = \frac{n\omega}{c}$$

$$\Rightarrow \boxed{\beta = \frac{n\omega}{c}} = \frac{n_1 \omega}{c}$$ $n = n_1$

The grp. velocity of fiber is given as,

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left[n_1 - \frac{\lambda\, dn_1}{d\lambda}\right] \quad \text{— (1)}$$

Mam Derivation:

$$\beta = \frac{2\pi n(\lambda)}{\lambda}$$ $\because k = \frac{2\pi}{\lambda} \Rightarrow \beta = k n(\lambda)$

grp. delay due to mat. dispersion

$$\tau_{mat} = L \frac{d\beta}{d\omega} = L \frac{d}{d\omega}\left(\frac{2\pi n(\lambda)}{\lambda}\right)$$

$$\omega = 2\pi f$$
$$\lambda = \frac{c}{f}$$
$$c = \lambda f$$
$$\boxed{\frac{\omega}{c} = \frac{2\pi}{\lambda}}$$

$$= L \frac{d}{d\omega}\left[\frac{\omega\, n(\lambda)}{c}\right]$$

$$= \frac{L}{c} \frac{d}{d\omega}\left[\omega\, n(\lambda)\right]$$

$$= \frac{L}{c}\left[\frac{d\omega}{d\omega} + \frac{d\, n(\lambda)}{d\omega}\right]$$

$$= \frac{L}{c}\left[\omega \frac{d\, n(\lambda)}{d\omega} + n(\lambda) \frac{d\omega}{d\omega}\right]$$

$$= \frac{L}{c}\left[\omega \frac{dn(\lambda)}{d\omega} + n(\lambda)\right]$$

$$\omega = \frac{2\pi c}{\lambda}$$

$$d\omega = \cancel{d\lambda}\, 2\pi c\, \lambda^2$$

$$= \frac{L\, n(\lambda)}{c}\left[\omega \frac{dn(\lambda)}{d\omega} + 1\right]$$

$$= \frac{L\, n(\lambda)}{c}\left[\frac{\cancel{2\pi c}}{\cancel{\lambda}} \frac{dn(\lambda) \times \cancel{\lambda^2}}{\cancel{2\pi c}\, d\lambda} + 1\right]$$

$$= \frac{L}{c}\left[n(\lambda) + \lambda \frac{dn(\lambda)}{d\lambda}\right]$$

$$\Rightarrow \quad \tau_{mat} = \frac{L}{c}\left[n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}\right]$$

The RMS pulse spread

$$\cancel{\sigma_{mat} = \left|\frac{d^2 mat}{d\lambda}\right| \sigma_\lambda} = \frac{\sigma_\lambda L}{c}$$

$$\sigma_{mat} = \sigma_\lambda \frac{d\tau_{mat}}{d\lambda}$$

$$= \frac{\sigma_\lambda L}{c} \frac{d}{d\lambda}\left[n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}\right]$$

$$= \frac{\sigma_\lambda L}{c}\left[\cancel{\frac{dn(\lambda)}{d\lambda}} - \lambda \frac{d^2 n(\lambda)}{d\lambda^2} - \cancel{\frac{dn(\lambda)}{d\lambda}} \frac{d\lambda}{d\lambda}\right]$$

$$= -\frac{\sigma_\lambda L}{c}\left[-\lambda \frac{d^2 n(\lambda)}{d\lambda^2}\right]$$

$$= -\frac{\sigma_\lambda \lambda L}{c}\left[\frac{d^2 n(\lambda)}{d\lambda^2}\right]$$

$$\sigma_{mat} = \frac{\sigma_\lambda L}{c}\left|\lambda \frac{d^2 n(\lambda)}{d\lambda^2}\right|$$

$\sigma_{mat} = \sigma_\lambda L \, |D_{mat}(\lambda)|$

$\lambda \propto v_g$

$\lambda \propto n_1$

$$\boxed{D_{mat}(\lambda) = \frac{1}{c} \left| \lambda \frac{d^2 n(\lambda)}{d\lambda^2} \right|}$$

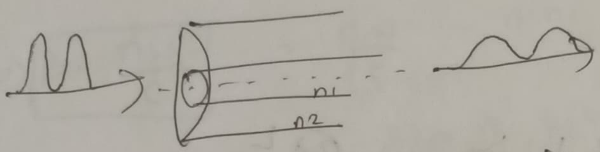
Can be reduced by reducing Spectral width σ_λ

(or)

Longer wavelength.

Waveguide Dispersion:

↳ Intramodal ↳ Pulse Spreading in a SMF



↳ Due to difference in the RI (n) between core & cladding. [due to diff grp velocity]

↳ Light in cladding propagates faster than core

↳ This loss is negligible for Multimode in range 1.27 μm.

& significant for SM

Analysis:

Results from propagation constant.

For SMF, the waveguide dispersion occurs when,

β as a fn. of $\left(\frac{a}{\lambda}\right)$ radius of core

$$\frac{d^2\beta}{d\lambda^2} \neq 0$$

The normalised propagation const. is defind as,

$$b = 1 - \left(\frac{ua}{v}\right)^2 \quad \text{— (1)}$$

$$k = \frac{2\pi}{\lambda}$$

where $u = a\left(n_1^2 k^2 - \beta^2\right)^{1/2}$

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} \rightarrow$$ normalised freq (or) 'V' number.

$$V = ka\sqrt{n_1^2 - n_2^2}$$

$$b = 1 - \left(\frac{a^2\left(n_1^2 k^2 - \beta^2\right)^{1/2}}{ka\sqrt{n_1^2 - n_2^2}}\right)^2$$

$$b = 1 - \frac{a^2\left(n_1^2 k^2 - \beta^2\right)}{k^2\left(n_1^2 - n_2^2\right)}$$

$$b = 1 - \left(\frac{ua}{v}\right)^2$$

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$$u = a\left(n_1^2 k^2 - \beta^2\right)^{1/2}$$

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Assume $a = 1\,\mu m$.

$$\Rightarrow b = 1 - \frac{n_1^2 k^2 - \beta^2}{\left(n_1^2 - n_2^2\right) k^2}$$

$$b = \frac{k^2\left(n_1^2 - n_2^2\right) - n_1^2 k^2 + \beta^2}{\left(n_1^2 - n_2^2\right) k^2}$$

$$b = \frac{\beta^2 - n_2^2 k^2}{k^2\left(n_1^2 - n_2^2\right)} = \frac{k^2\left[\frac{\beta^2}{k^2} - n_2^2\right]}{k^2\left(n_1^2 - n_2^2\right)}$$

$$\boxed{b = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2}}$$

For small values of index diff $[n_1 \approx n_2]$

$$\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow$$

~~$\Delta n_1 = n_1 - n_2$~~

$$\Delta n_1 = n_1 - n_2$$

$$n_2 = n_1 - \Delta n_1 \Rightarrow n_1(1 - \Delta)$$

$$\Rightarrow \quad b \propto \frac{\beta/k - n_2}{\Delta n_2}$$

$$b \Delta n_2 \propto \beta/k - n_2 = \frac{\beta - n_2 k}{k}$$

$$b \Delta n_2 k = \beta - n_2 k.$$

$$\boxed{\beta = b \Delta n_2 k + n_2 k}$$

Assume that n_2 is independent of λ & diff w.r.to k

$$\frac{d\beta}{dk} = n_2 \frac{dk}{dk} + \Delta n_2 \frac{d(bk)}{dk}$$

$$\boxed{\frac{d\beta}{dk} = n_2 + n_2 \Delta \frac{d(kb)}{dk}} \quad - (2)$$

Grp. Delay

$$\tau_{wg} = \frac{L}{c} \frac{d\beta}{dk}$$

$$\tau_{wg} = \frac{L}{c} \left[n_2 + n_2 \Delta \frac{d(kb)}{dk} \right]$$

$$\Rightarrow \tau_{wg} = \frac{L}{c} \left[n_2 + n_2 \Delta \frac{d(Vb)}{dV} \right]$$

$\frac{L}{c} n_2 \rightarrow$ const.

$\frac{L}{c} \left[n_2 \Delta \frac{d(Vb)}{dV} \right] \rightarrow$ Grp delay due to W.G dispersion

$$\frac{d(Vb)}{dV} = b\left[1 - \frac{2J_V^2(ua)}{J_{V+1}(ua)\, J_{V-1}(ua)}\right]$$

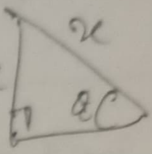
$J_{ua} \rightarrow$ Bessel fn.

$$\boxed{\Delta T = \frac{\Delta n_1^2 L}{C n_2} \quad \frac{\Delta T}{L} = \frac{\Delta n_1^2}{C n_2}}$$

Modal Dispersion :

$$T_{min} = \frac{n_1 L}{c}$$

$$\Delta T = T_{max} - T_{min} \qquad T_{max} = \frac{n_1}{c}(x)$$



$$\sin\phi_c = \frac{L}{x}$$

$$\Rightarrow T_{max} = \frac{n_1}{c}\left[\frac{L}{\sin\phi_c}\right] \qquad \Rightarrow x = \frac{L}{\sin\phi_c}$$

$$\Delta T = T_{max} - T_{min} = \frac{n_1 L}{c}\left[\frac{1}{\sin\phi} - 1\right]$$

$$= \frac{n_1}{c}\left[\frac{L}{\sin\phi_c}\right] - \frac{n_1 L}{c} \qquad \sin\phi_c = \frac{n_2}{n_1}$$

$$= \frac{n_1 L}{c}\left[\frac{n_1}{n_2} - 1\right]$$

$$= \frac{n_1 L}{c}\left[\frac{n_1 - n_2}{n_2}\right] = \frac{n_1 L}{C}\left(\frac{\Delta n_1}{n_2}\right)$$

$$\Delta T = \frac{\Delta n_1^2 L}{C n_2}$$

$$\boxed{\Delta T = \frac{\Delta n_1^2 L}{C n_2}} \qquad \frac{\Delta n_1^2 L}{C n_2}$$

1) $\phi_A = 22^\circ$; $\Delta = 3\%$; $NA = ?$; ϕ_c

$\phi_A = \sin^{-1}(NA)$

$NA = \sqrt{n_1^2 - n_2^2}$;

$n_2 = n_1(1-\Delta)$

$n_2 = n_1 - n_1\Delta$

$n_1\Delta = n_1 - n_2$

$$\boxed{NA = 0.375}$$

$(1-\Delta)$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$555 \times 10^{-9}$$

$$E = hf = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} J \times 3 \times 10^{8}}{555 \times 10^{-9}}$$

$$E = 0.03582 \times 10^{-17}$$

$$E \approx 3.58 \times 10^{-19} J \Rightarrow E = 3.58 \text{ eV}$$

21/3/24

Quantum Efficiency & LED Power:

$$\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau}$$

$\frac{J}{qd}$ ⇒ externally supplied rate

$\frac{n}{\tau}$ ⇒ Thermal generated rate.

role of carrier recombination

$$n = n_0 e^{-t/\tau}$$

⇒ n_0 - initial injected electron density

τ ⇒ time consumes in carrier life time.

$$\eta_{int} = \frac{R_{r}}{R_{nr} + R_{r}}$$

$$\boxed{\tau_r = n / R_r}$$

R_r ⇒ Radiative recombination rate

$$\tau_{nr} = \frac{n}{R_{nr}}$$

R_{nr} ⇒ non radiative recombination rate

$$\eta_{int} = \frac{n/\tau_r}{n/\tau_r + \frac{n}{\tau_{nr}}}$$

$$= \frac{\frac{n}{\tau_r}}{\frac{n\tau_{nr} + n\tau_r}{\tau_r \tau_{nr}}} = \frac{n\tau_{nr}}{n\tau_{nr} + n\tau_r}$$

$$= \frac{\tau_{nr}}{\tau_{nr} + \tau_r}$$

$$\Rightarrow \frac{1}{\eta} = \frac{1}{n\tau_r} + \frac{1}{n\tau_{nr}}$$

External Efficiency:

$$\frac{\text{No. of photons generated}}{\text{No of photons emitted}}$$

$$\eta_{ext} = \frac{1}{4\pi} \int_0^{\phi_c} T(\phi)\,(2\pi \sin \phi)\, d\phi$$

Fresnel's co-efficient / Transmissing

$$T(0) = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

$\cancel{\eta_{ext} = \frac{1}{2\pi}}$

$$\eta_{ext} = \frac{1}{4\pi} \int_0^{\phi_c} \frac{4n}{(1+n)^2} (2\pi \sin\phi)\, d\phi.$$

$$= 2\pi \times \frac{1}{4\pi} \left[\int \frac{4n}{(1+n)^2}\right] [-\cos\phi_c - \cos 0]$$

$$= \frac{2n}{(n+1)^2} (-\cos\phi_c + \cos 0)$$

$$= \int .$$

Laser Rate Equation:

$$\frac{d\phi}{dt} = cn\phi + R_{sp} - \frac{\phi}{\tau_{ph}}$$

Photons produced by simulated emission

src of phonons

photons produced.

$$\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau_{sp}} - cn\phi$$

current density

Spon recombination

emission of simulated emission photon

Simulated emission + Spontaneous emission + Photon loss

Insertion carrier density + Spontaneous recombination + Simulated emission of photon

ϕ - No. of photons.

Under steady state current.

① $\Rightarrow \frac{d\phi}{dt} = 0 \Rightarrow cn\phi + R_{sp} - \frac{\phi}{\tau_{ph}} = 0$

negligible

$$cn\phi - \frac{\phi}{\tau_{ph}} = 0$$

$$\phi\,(cn - 1/\tau_{ph}) = 0$$

no. of photon small ($\phi = 0$)

② $\rightarrow \frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau_{sp}} - Cn\phi = 0$

$$\frac{J}{qd} - \frac{n}{\tau_{sp}} - Cn\phi = 0$$

Steady state no. of photons $= 0$ (no emission)

$$\frac{J}{qd} = \frac{n}{\tau_{sp}}$$

At lasing Threshold

adding

$$0 = Cn_{th}\phi_s + R_{sp} - \frac{\phi_s}{\tau_{ph}}$$

$$0 = J/qd - n_{th}/\tau_{sp} - Cn_{th}\phi_s$$

ϕ_s = steady state photon density.

$$0 = R_{sp} - \phi_s/\tau_{ph} + J/qd - n_{th}/\tau_{sp}$$

$$\Rightarrow \frac{\tau_{ph} R_{sp} - \phi_s}{\tau_{ph}} + \frac{J}{qd} - \frac{n_{th}}{\tau_{sp}}$$

$$\Rightarrow \frac{\tau_{ph} R_{sp} - \phi_s}{\tau_{ph}} + \frac{1}{qd}(J - J_{th})$$

$$\Rightarrow \tau_{ph} R_{sp} - \phi_s + \tau_{ph}/qd\,(J - J_{th})$$

$$\boxed{\phi_s = \frac{\tau_{ph}}{qd}(J - J_{th}) + \tau_{ph} R_{sp}}$$

Laser Resonant Frequencies:

$\cos\theta - j\sin\theta = 1$

$\cos\theta = 1$

$\Rightarrow \theta = 0°$

$\cos(0) = 1$

$$\exp(-j\beta 2L) = 1$$

$$2\beta L = 2m\pi \quad ; \; m = 1, 2, 3$$

$$\beta = \frac{2\pi n}{\lambda}$$

$$2 \times \frac{2\pi n}{\lambda} \;\&\; L = 2m\pi$$

$\lambda = \frac{c}{\nu}$ $\quad e^{-jn 2L} = 1$

$$\frac{2nL}{\lambda} = m$$

$$\frac{2nL\nu_m}{\lambda} = \nu m$$

$$\nu_{m-1} = \frac{(m-1)c}{2nL}$$

$$\Delta\nu\,(\nu_m - \nu_{m-1}) = \cancel{\frac{mc}{2nL}} - \cancel{\frac{mc}{2nL}} + \frac{c}{2nL}$$

$$\Delta\nu = \frac{c}{2nL}$$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\lambda}{\lambda}$$

$$\Delta\lambda = \frac{\lambda\Delta\nu}{\nu}$$

$$\Delta\lambda = \frac{\lambda\Delta\nu}{\nu} = \frac{\lambda}{\nu}\left(\frac{c}{2nL}\right)$$

$$\Delta\lambda = \lambda\left(\frac{c}{\nu}\right)\frac{1}{2nL}$$

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2nL}}$$