Group delay

As the signal propagates along the fiber each spectral component can be assumed to travel independently and undergo a time delay or group delay per unit length

$$\frac{\tau_g}{L} = \frac{1}{V_g} = \frac{1}{c}\cdot\frac{d\beta}{dk}$$

$$K = \frac{2\pi}{\lambda}$$

$$V_g = c\left(\frac{dk}{d\beta}\right) \qquad \beta -$$

$$K = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{\lambda}c = K.c$$

$$\partial\omega = c\,.\,\partial k$$

$$\partial k = \frac{\partial\omega}{c}$$

Now $V_g = \cancel{c}\left(\frac{\partial\omega/\cancel{c}}{\partial\beta}\right) = \frac{\partial\omega}{\partial\beta}$

$$\boxed{V_g = \left(\partial\beta/\partial\omega\right)^{-1}}$$

$$k = \frac{2\pi}{\lambda}$$

$$dk = -2\pi\lambda^{-2}d\lambda$$

$$\frac{1}{V_g} = \frac{\tau_g}{L} = \frac{1}{c}\left(\frac{d\beta}{-2\pi\lambda^{-2}d\lambda}\right)$$

$$= \frac{1}{c}\left(\frac{d\beta}{-2\pi\lambda^{-2}d\lambda}\right)$$

$$\boxed{\frac{1}{V_g} = \frac{\tau_g}{L} = -\frac{\lambda^2}{2\pi c}\left(\frac{d\beta}{d\lambda}\right)}$$

$$\text{Spectral width} = \frac{d\tau_g}{d\lambda}\cdot\delta\lambda$$

$$\frac{1}{v_g} = \tau_g = -\frac{L\lambda^2}{2\pi c}\left(\frac{d\beta}{d\lambda}\right)$$

$$\frac{d\tau_g}{d\lambda} = -\frac{L}{2\pi c}\cdot\frac{d}{d\lambda}\left(-\lambda^2\frac{d\beta}{d\lambda}\right)$$

$$\boxed{\text{spectral width} = -\frac{L}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)\delta\lambda}$$

interms of angular frequency GVD

$$\delta\tau = \frac{d\tau_g}{d\omega}\,\delta\omega = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\delta\omega$$

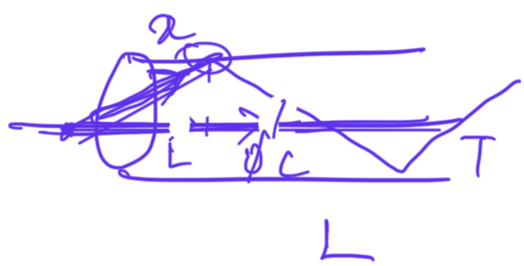
Pulse spreading $\delta\lambda$ is characterised in terms Rms

$$\boxed{\sigma_g = \left|\frac{d\tau_g}{d\lambda}\right|\sigma_\lambda = \frac{L\sigma_\lambda}{2\pi c}\left|2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right|}$$

$$\boxed{D = \frac{1}{L}\cdot\frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda}\left|\frac{1}{v_g}\right| = \frac{2\pi c}{\lambda^2}\beta}$$

Dispersion — ps / (nm - km)

Modal dispersion



$\Delta T = T_{max} - T_{min}$

$T_{min} = \dfrac{n_1 L}{c}$

$\dfrac{L}{c} =$ Time taken to travel 'L' with speed of c

$T_{max} = \dfrac{n_1}{c}(x)$

$T_{max} = \dfrac{n_1}{c}\left(\dfrac{L}{\sin\phi_c}\right)$

$\sin\phi_c = \dfrac{L}{x}$

$$\Delta = T_{max} - T_{min}$$

$$= \frac{n_1}{c} L \left(\frac{1}{\sin\phi_c} - 1 \right) \qquad \sin\phi_c = \frac{n_2}{n_1}$$

$$\Delta = \frac{n_1 L}{c} \left(\frac{n_1}{n_2} - 1 \right)$$

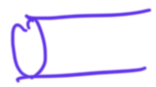
$$= \frac{n_1 L}{c} \left(\frac{n_1 - n_2}{n_2} \right) \qquad \Delta n_1$$

$$= \frac{n_1 L}{c} \cdot \frac{\Delta n_1}{n_2} = \frac{n_1^2 L \Delta}{c n_2}$$

$$\boxed{\Delta T = \frac{n_1^2 L \Delta}{c n_2}}$$

Material dispersion

$$\beta = \frac{2\pi n(\lambda)}{\lambda} \qquad k = 2\pi/\lambda \qquad n(\lambda)$$



infinite dielectric medium

$$\tau_{max} = L \frac{d\beta}{d\omega}$$

$$= L \frac{d}{d\omega} \left(\frac{2\pi n(\lambda)}{\lambda} \right) \qquad \frac{\tau_g}{L} = \frac{1}{v_g}$$

$$= L \, d/d\omega \left(\frac{\omega}{c} \cdot n(\lambda) \right) \qquad \omega = \frac{2\pi c}{\lambda} \quad kc \quad k = \frac{2\pi}{\lambda}$$

$$= \frac{L}{c} \left(n(\lambda) \cdot \frac{d\omega}{d\omega} + \frac{d n(\lambda)}{d\omega} \cdot \omega \right)$$

$$= \frac{L}{c} \left(n(\lambda) + \frac{d n(\lambda)}{d\omega} \cdot \frac{2\pi c}{\lambda} \right) \qquad \omega = \frac{2\pi c}{\lambda}$$

$$= \frac{L}{c} n(\lambda) + \left(-d\, n(\lambda) \frac{\lambda^2}{\quad} \, 2\pi \right) \qquad d\omega = 2\pi c \lambda^{-2}$$

$c \qquad\qquad\qquad 2\pi c \quad d\lambda \quad c)$

$$\boxed{\tau_{mat} = \frac{L}{c}\left(n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}\right)}$$

Pulse Spread

$$\sigma_{mat} = \left|\frac{d\tau_{mat}}{d\lambda}\right| \sigma_\lambda = \frac{\sigma_\lambda L}{c} \left|\lambda \frac{d^2 n}{d\lambda^2}\right|$$

$$\boxed{\sigma_{mat} = \sigma_\lambda L \,|D_{mat}(\lambda)|.}$$

$\uparrow$

Effects due to waveguide dispersion:-

Group delay normalised propagation constant

$$\boxed{b = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2}}$$

$n_1 - n_2$

For small values of index difference

$\Delta = 0.01\%$

$$\Delta = (n_1 - n_2/n_1)$$

$$b = \frac{\beta/k - n_2}{n_1 - n_2}$$

$$\boxed{\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk}\left(\frac{\frac{\beta}{k} - n_2}{n_1 - n_2}\right)}$$

Modal dispersion

V defined $\quad \frac{\tau_g}{L} = \frac{1}{V_g} = \frac{1}{c}\frac{d\beta}{dk}$

$$= -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda}$$

$$\boxed{V = ka\,(n_1^2 - n_2^2)^{1/2}}$$

$$\approx ka\,n_1\sqrt{2\Delta}$$

$\Delta =$

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2 \Delta b\right]$$

Group delay assuming waveguide dispersion