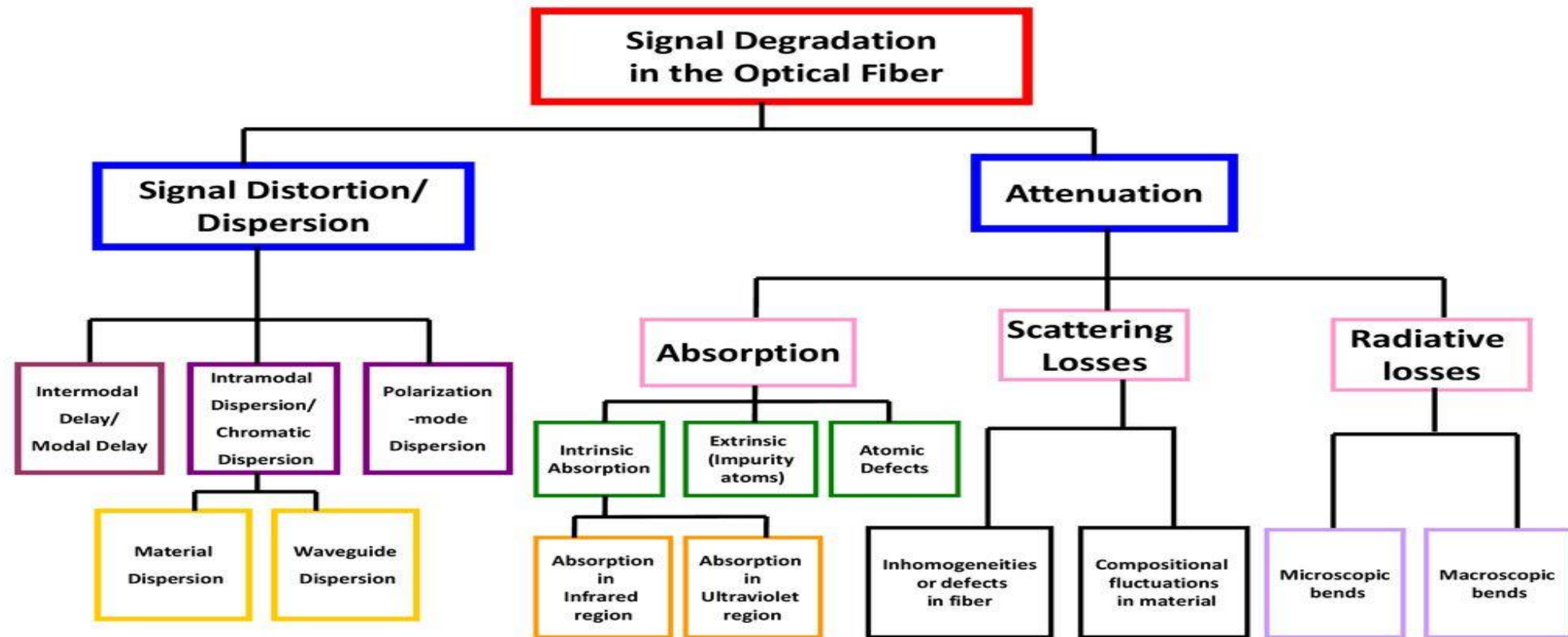


Module - 2

Signal Degradation

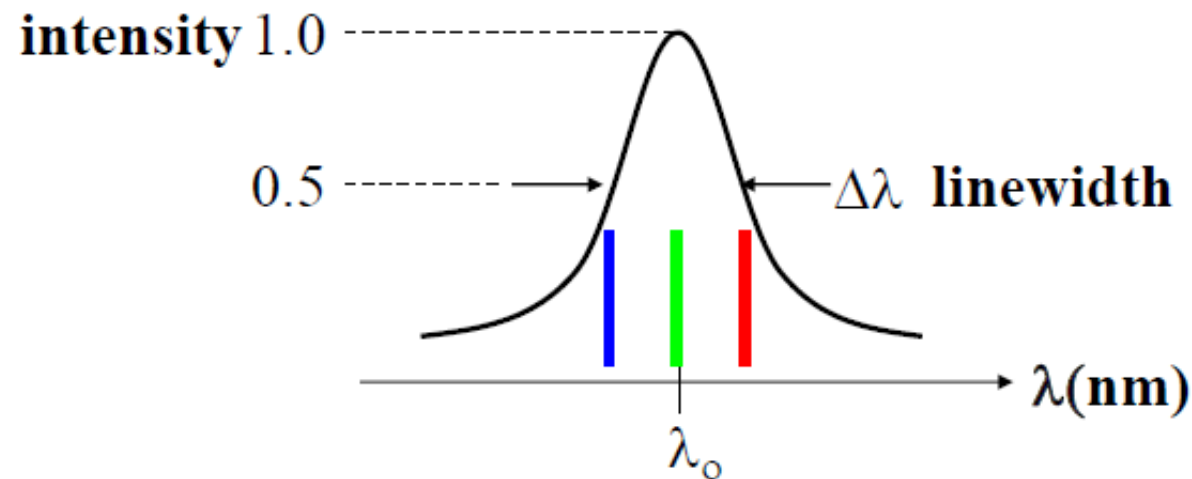


Signal degradation in fiber cable



Chromatic Dispersion

- Chromatic dispersion (CD) may occur in all types of optical fiber. The optical pulse broadening results from the finite spectral linewidth of the optical source and the modulated carrier.
- Origin is **frequency dependence of the refractive index $n(\omega)$**



- In the case of the semiconductor laser, $\Delta\lambda$ corresponds to only a fraction of % of the centre wavelength λ_0 . For LEDs, $\Delta\lambda$ is likely to be a significant percentage of λ_0 .

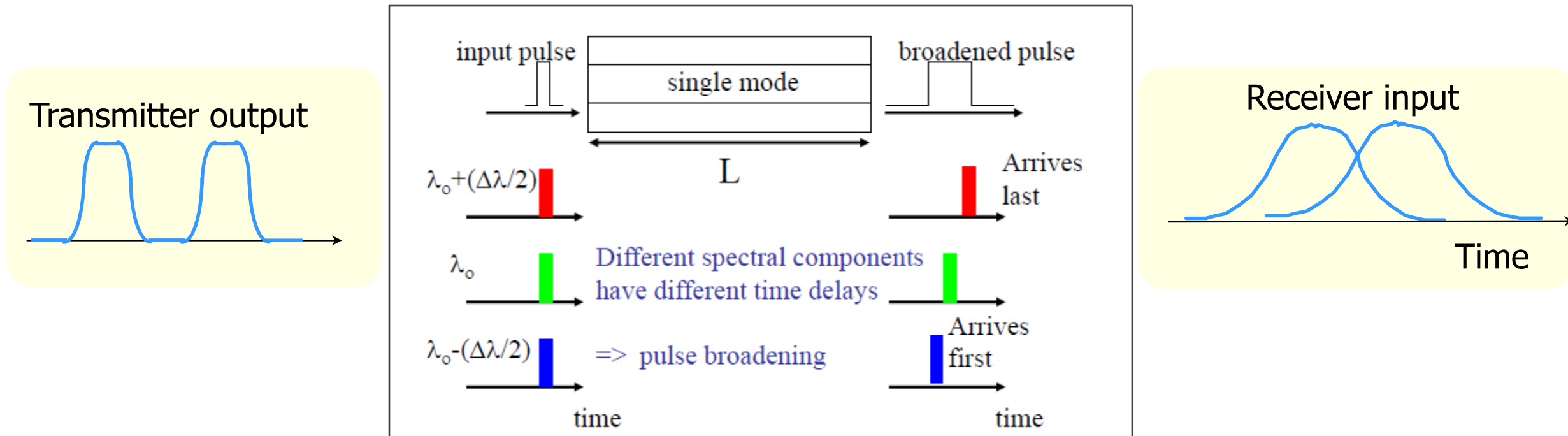
Chromatic Dispersion

- Real sources emit over a range of wavelengths. This range is the **source linewidth or spectral width**.
- The smaller is the linewidth, the smaller is the spread in wavelengths or frequencies, the more coherent is the source.
- An ideal perfectly coherent source emits light at a single wavelength.
- It has **zero linewidth and is perfectly monochromatic**.

Light sources	Linewidth (nm)
Light-emitting diodes	20 nm – 100 nm
Semiconductor laser diodes	1 nm – 5 nm
Nd:YAG solid-state lasers	0.1 nm
HeNe gas lasers	0.002 nm

Chromatic Dispersion

- Pulse broadening occurs because there may be propagation delay differences among the spectral components of the transmitted signal.



- Chromatic dispersion (CD): Different spectral components of a pulse travel at different group velocities. This is known as **group velocity dispersion (GVD)**.

Phase velocity

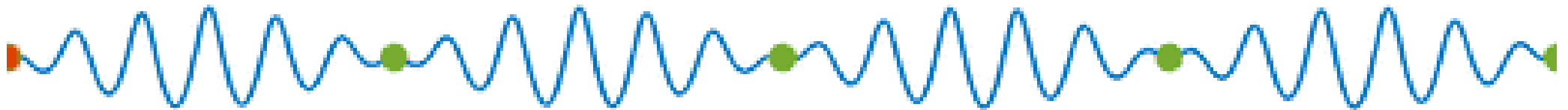
- When the optical waves are propagating through the fiber, there are certain points having constant phase
- These points of constant phase travel with a velocity called as phase velocity

$$v_p = \omega / \beta$$

$$\omega = \frac{2\pi c}{\lambda}$$

$$\beta = \frac{2\pi n}{\lambda}$$

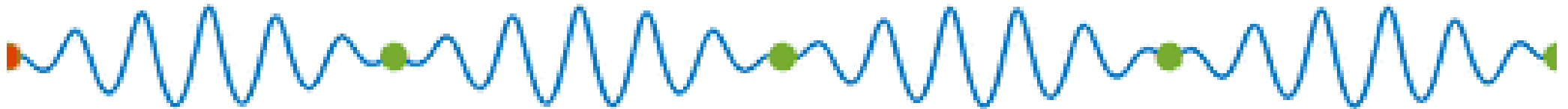
$$v_p = c / n$$



Group velocity

- Optical waves are travelling as wave packets
- A group of waves with closely similar wavelengths (or frequencies) always combine to form a packet of wave
- The rate at which envelope of the waves propagates called as group velocity

$$V_g = \frac{\partial \omega}{\partial \beta} = 2\pi c \frac{\partial}{\partial \beta} \left(\frac{1}{\lambda} \right)$$



Remarks on group velocity

- Because the energy of a harmonic wave is proportional to the square of its field amplitude, the energy carried by a wave packet that is composed of many frequency components is concentrated in regions where the amplitude of the envelope is large.
- Therefore, the energy in a wave packet is transported at group velocity v_g .
- The constant-phase wavefront travels at the phase velocity, but the group velocity is the velocity at which energy (and information) travels.
- Any information signal is a wave packet, and thus travels at the group velocity, not at the phase velocity.

Light pulse in dispersive medium

- When a light pulse with a spread in frequency $\delta\omega$ and a spread in propagation constant $\delta\beta$ propagates in a dispersive medium $n(\lambda)$, the group velocity:

$$V_g = \frac{\partial\omega}{\partial\beta}$$

$$V_g = \frac{\partial\omega}{\partial\lambda} \cdot \frac{\partial\lambda}{\partial\beta}$$

$$\frac{\partial\omega}{\partial\lambda} = -\frac{\omega}{\lambda}$$

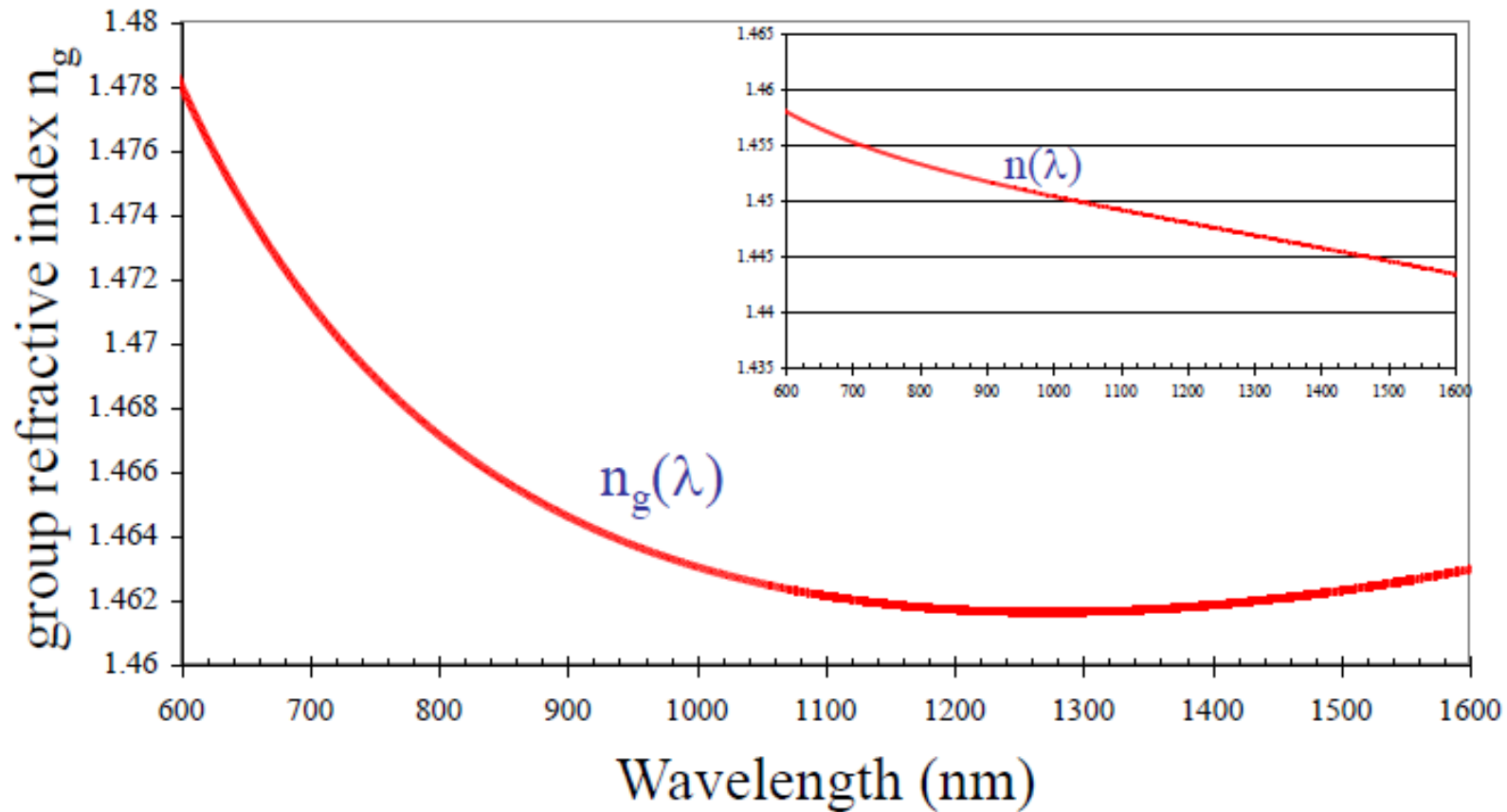
$$\frac{\partial\lambda}{\partial\beta} = \left(\frac{\partial\beta}{\partial\lambda} \right)^{-1}$$

$$\beta = \frac{2\pi n}{\lambda}$$

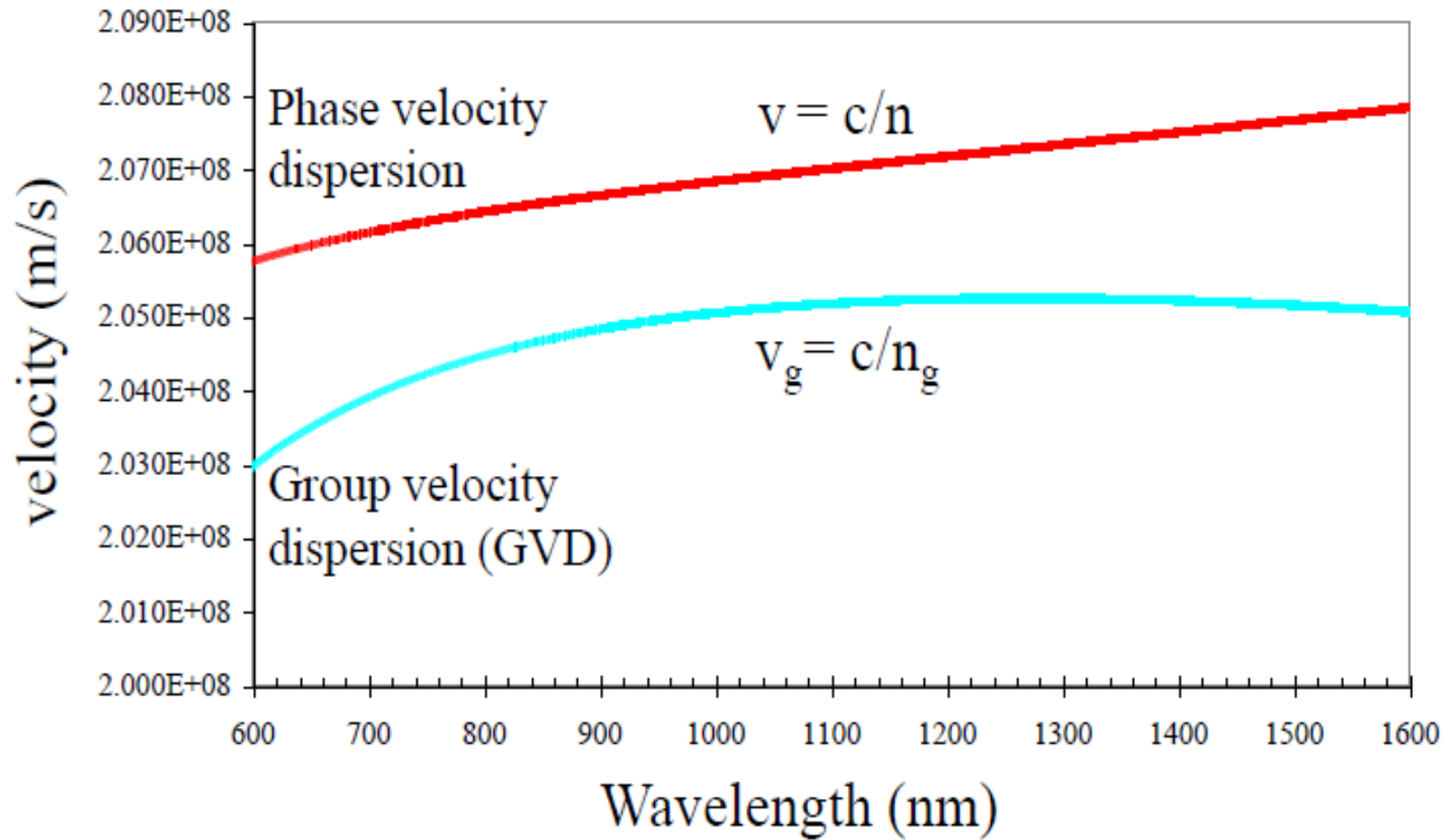
$$V_g = -\frac{2\pi c}{\lambda^2} \left(\frac{\partial\beta}{\partial\lambda} \right)^{-1}$$

$$V_g = \frac{c}{n_g} = \frac{c}{n - \lambda \left(\frac{dn}{d\lambda} \right)}$$

Group refractive index n_g vs. λ for fused silica



Phase velocity c/n and group velocity c/n_g vs. λ for fused silica



Group-Velocity Dispersion (GVD)

- Consider a light pulse propagates in a dispersive medium of length L
- A specific spectral component at the frequency ω (or wavelength λ) would arrive at the output end of length L after a time delay:

$$T = \frac{L}{V_g}$$

- If $\Delta\lambda$ is the spectral width of an optical pulse, the extent of pulse broadening for a material of length L is given by

$$\Delta T = \left(\frac{dT}{d\lambda} \right) \Delta\lambda = L \left[\frac{d\left(\frac{1}{V_g} \right)}{d\lambda} \right] \Delta\lambda$$

Group-Velocity Dispersion (GVD)

- Hence the pulse broadening due to a differential time delay:

$$\Delta T = L |D_{mat}| \Delta \lambda$$

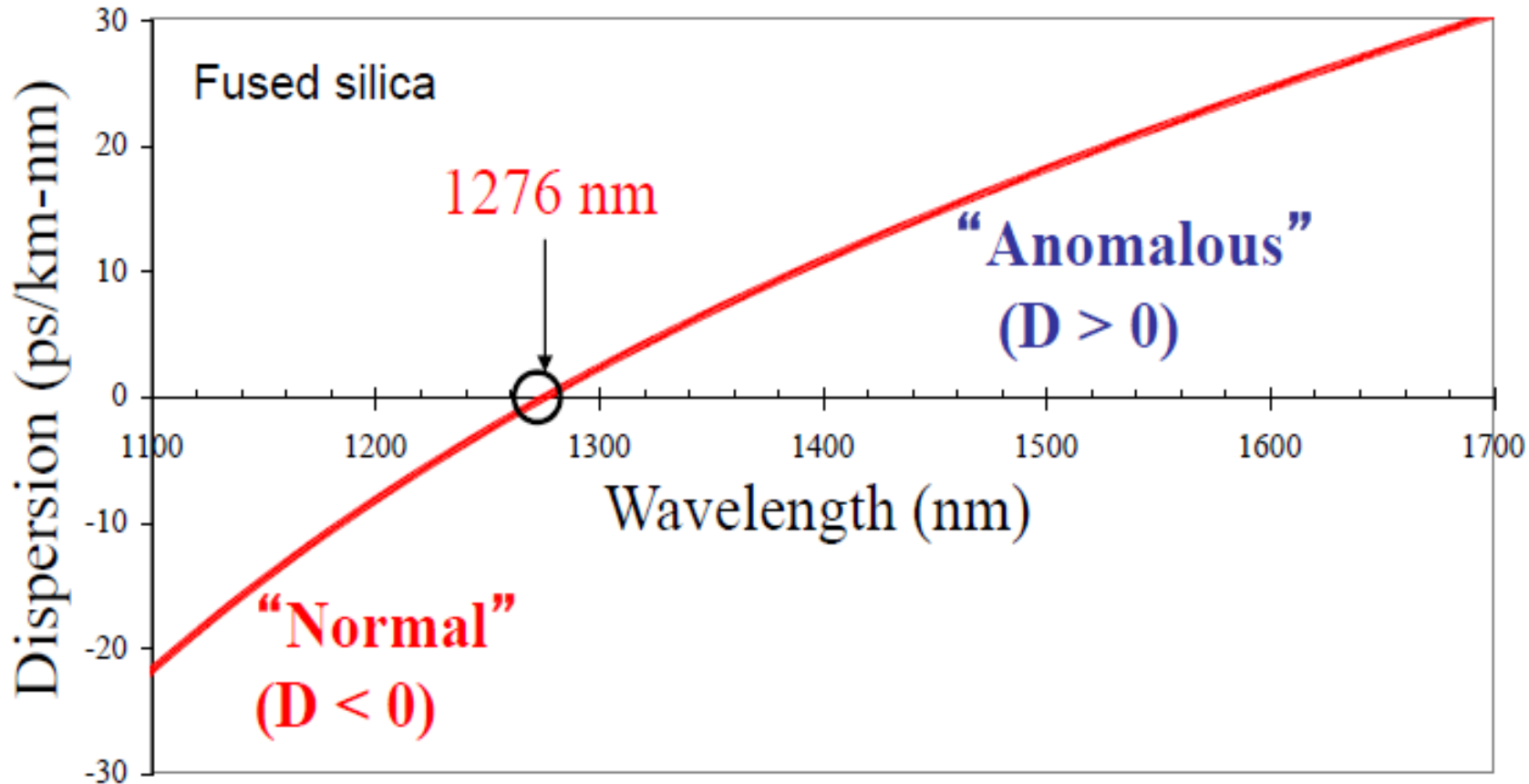
- where $D_{mat} = d(1/v_g)/d\lambda$ is called the **material dispersion parameter** and is expressed in units of ps/(km-nm).

$$D_{mat} = \frac{d\left(\frac{1}{V_g}\right)}{d\lambda} = -\left(\frac{\lambda}{c}\right) \frac{d^2 n}{d\lambda^2}$$

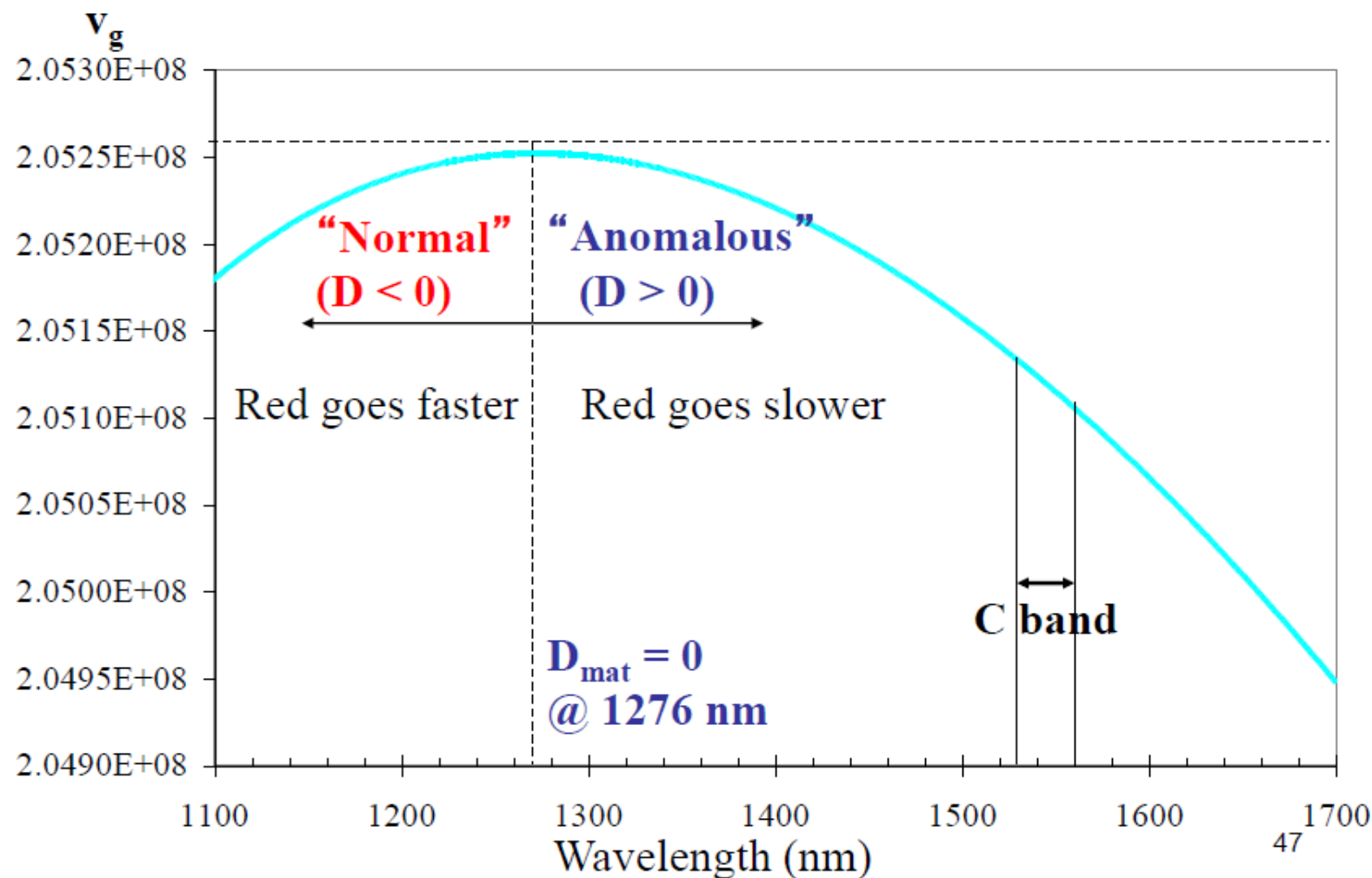
- Suppose, we take, $V_g = -\frac{2\pi c}{\lambda^2} \left(\frac{\partial \beta}{\partial \lambda}\right)^{-1}$ then D_{mat} becomes

$$D_{mat} = \frac{d\left(\frac{1}{V_g}\right)}{d\lambda} = -\left(\frac{2\pi c}{\lambda^2}\right) \frac{d^2 \beta}{d\lambda^2}$$

Dispersion parameter $D = - (\lambda/c) d^2n/d\lambda^2$



Variation of V_g with wavelength for fused silica



Zero-dispersion wavelength

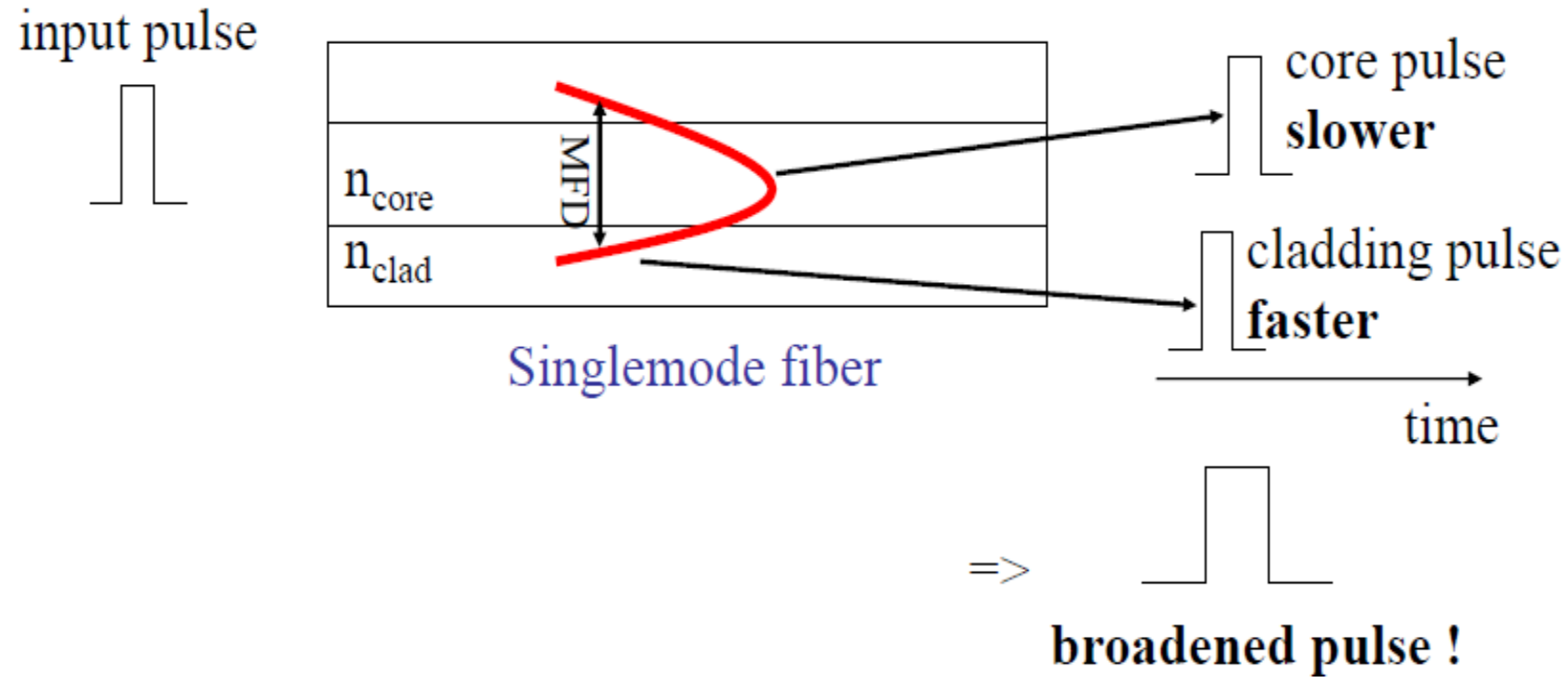
- Material dispersion $D_{\text{mat}} = 0$ at $\lambda \sim 1276$ nm for fused silica.
- This λ is referred to as the zero-dispersion wavelength λ_{ZD} .
- Chromatic (or material) dispersion $D(\lambda)$ can be zero;
 - or negative => longer wavelengths travel faster than shorter wavelengths;
 - or positive => shorter wavelengths travel faster than longer wavelengths.

Material and Waveguide Dispersion

- In fact there are two mechanisms for chromatic dispersion in a fiber:
 - (a) Silica refractive index n is wavelength dependent (i.e. $n = n(\lambda)$) \Rightarrow different wavelength components travel at different speeds in silica. This is known as **material dispersion**.
 - (b) Light energy of a mode propagates partly in the core and partly in the cladding of a fiber. The mode power distribution between the core and the cladding depends on λ . This is known as **waveguide dispersion**.

$$D(\lambda) = D_{mat}(\lambda) + D_{wg}(\lambda)$$

Waveguide Dispersion



Waveguide Dispersion

- The effect of waveguide dispersion on pulse spreading can be approximated by assuming that the refractive index of the material is independent of the wavelength
- The group velocity in terms of normalized propagation constant b:

$$V_{g,eff} = \frac{\partial \omega}{\partial \beta} \quad \beta \approx n_2 k (b\Delta + 1)$$

$$V_g = \frac{c}{n_{g,eff}} = \frac{c}{n_2 + n_2 \Delta \frac{d(Vb)}{dV}}$$

$$V = \frac{2\pi a}{\lambda} NA \approx k a n_2 \sqrt{2\Delta}$$

Waveguide Dispersion

- Consider a light pulse propagates in a dispersive medium of length L
- A specific spectral component at the frequency ω (or wavelength λ) would arrive at the output end of length L after a time delay:

$$T = \frac{L}{V_{g,eff}}$$

- If $\Delta\lambda$ is the spectral width of an optical pulse, the extent of pulse broadening in a waveguide of length L is given by

$$\Delta T = \left(\frac{dT}{d\lambda} \right) \Delta\lambda = L \left[\frac{d \left(\frac{1}{V_{g,eff}} \right)}{d\lambda} \right] \Delta\lambda$$

Waveguide Dispersion

- Hence the pulse broadening due to a differential time delay:

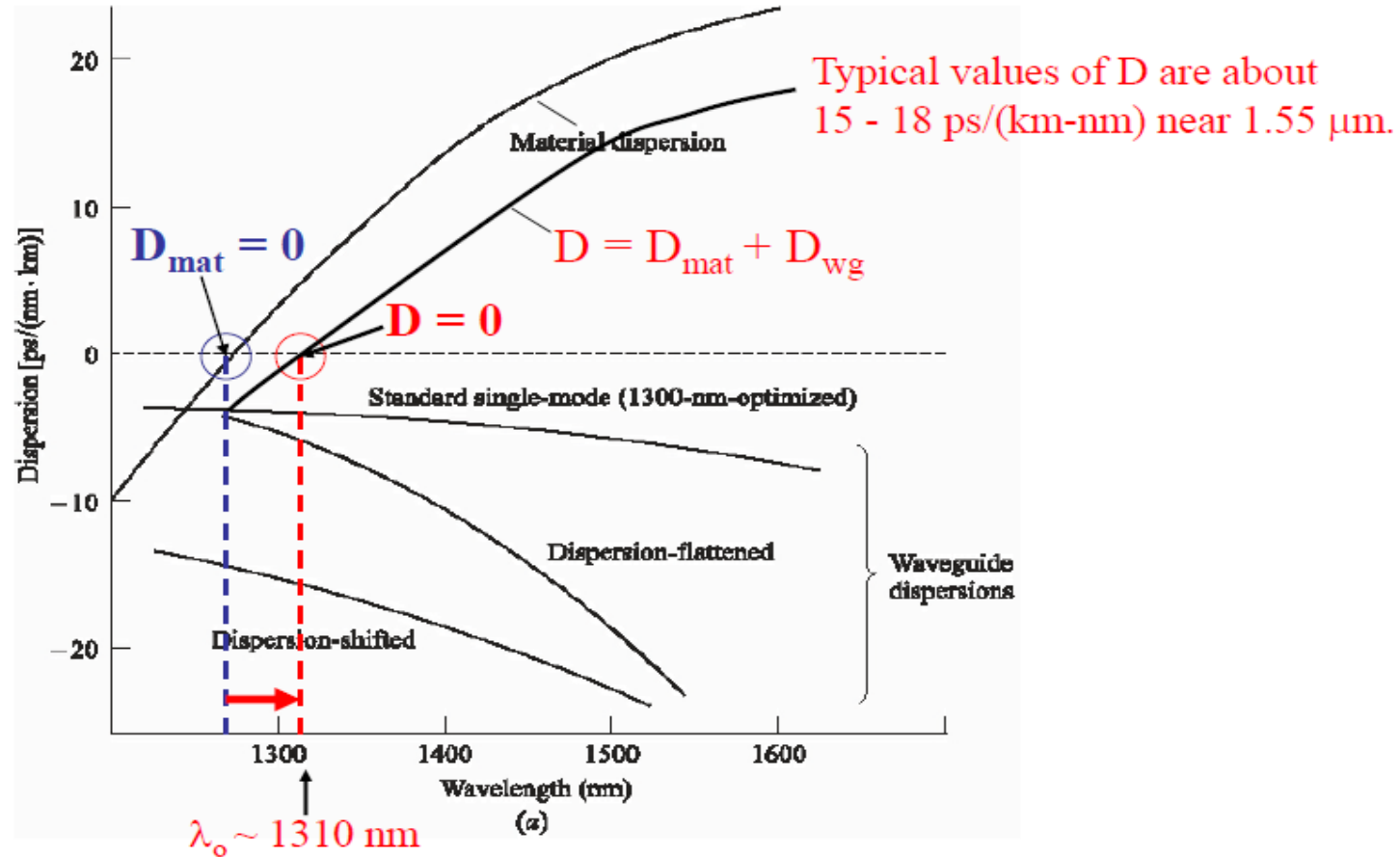
$$\Delta T = L |D_{wg}| \Delta \lambda$$

- where $D_{wg} = d(1/v_{g,eff})/d\lambda$ is called the **waveguide dispersion parameter** and is expressed in units of ps/(km-nm).

$$D_{wg} = \frac{d\left(\frac{1}{V_{g,eff}}\right)}{d\lambda} = -\frac{n_2 \Delta}{c \lambda} \left[V \frac{d^2(Vb)}{dV^2} \right]$$

- The dependence of D_{wg} on λ may be controlled by altering the core radius, the NA, or the V number

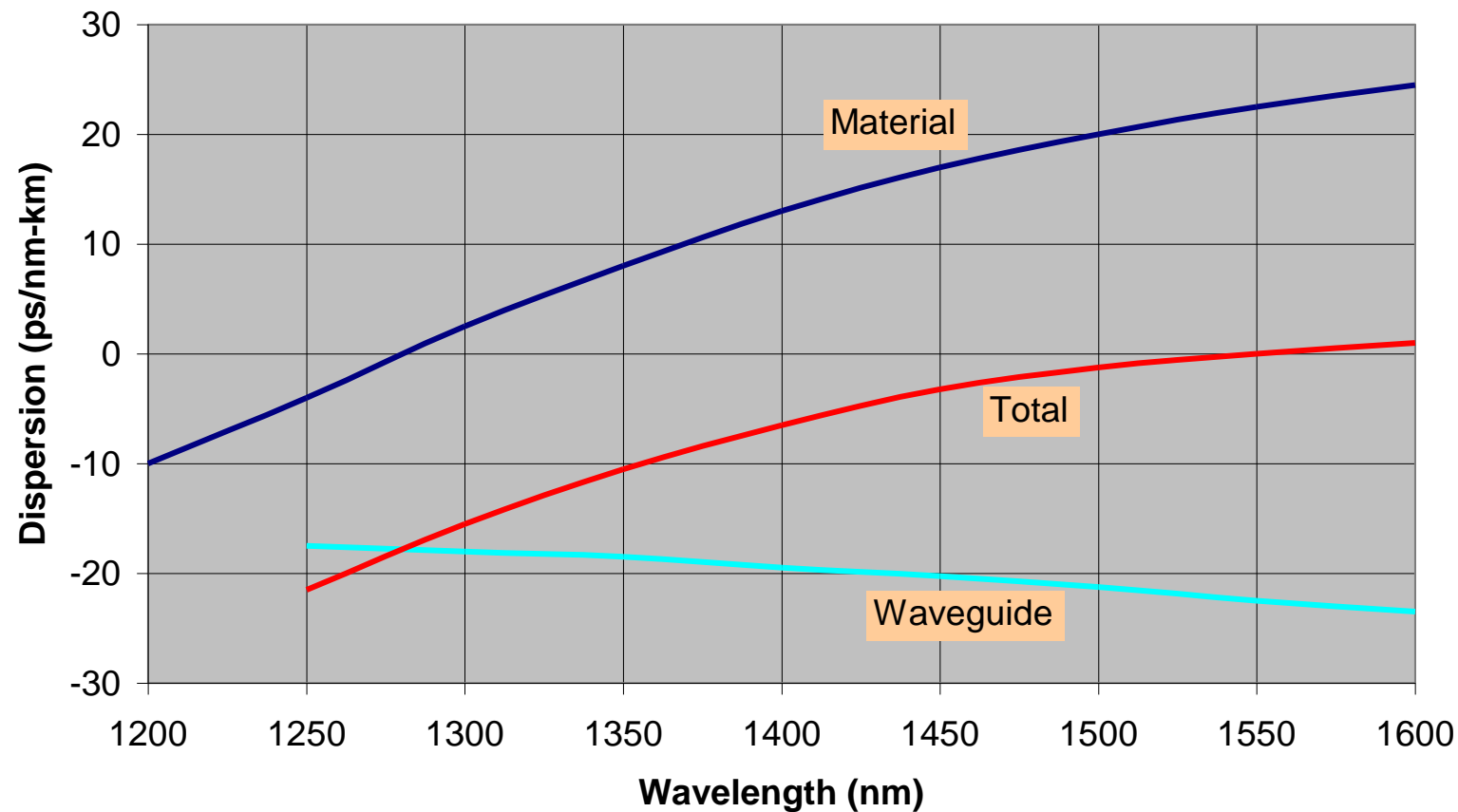
Zero Dispersion @ 1310 nm



Dispersion tailored fibers

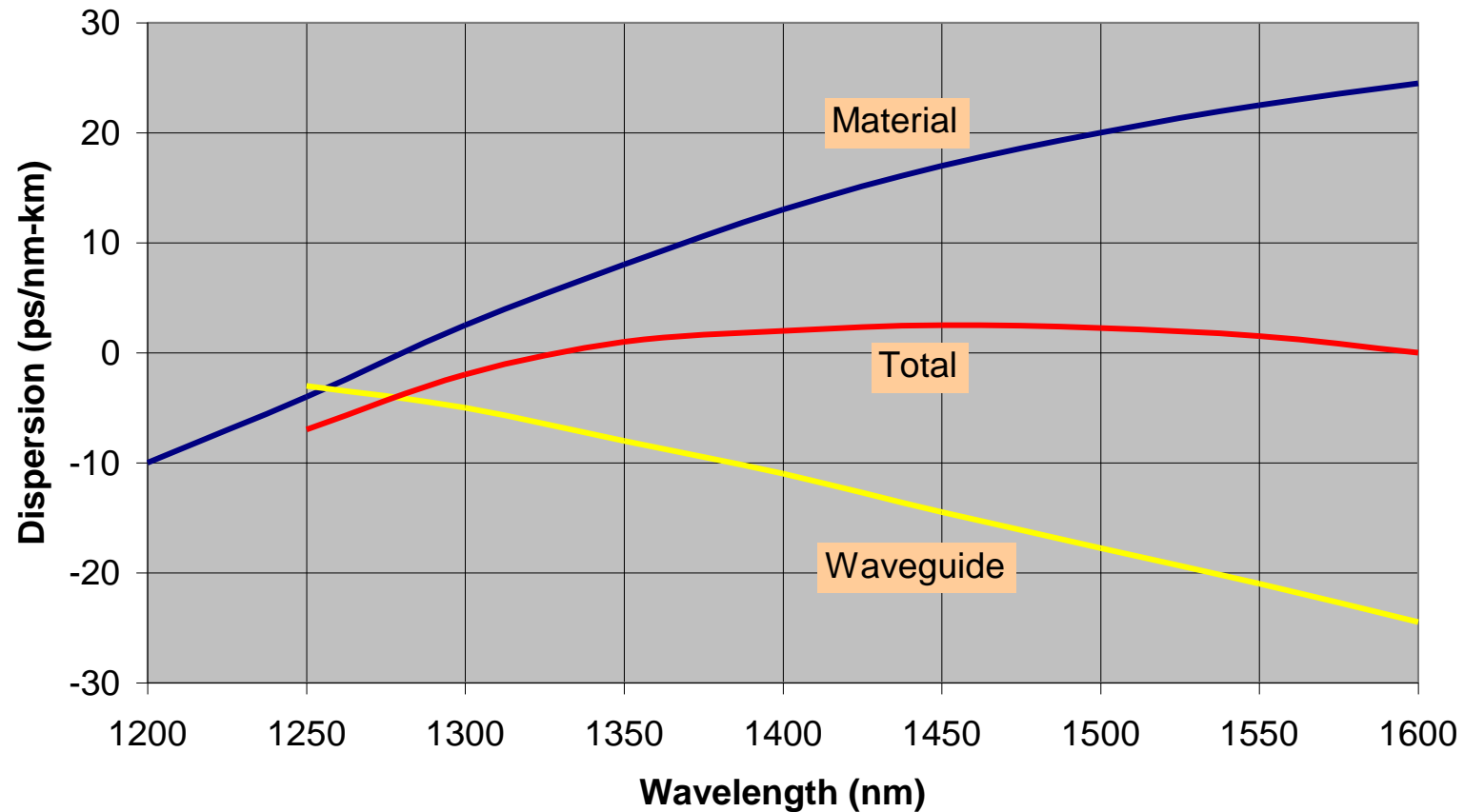
- As the waveguide contribution D_{wg} depends on the fiber parameters such as the core radius a and the index difference D , it is possible to design the fiber such that λ_{zD} is shifted into the neighborhood of 1.55 μm . Such fibers are called **dispersion-shifted fibers**.
- It is also possible to tailor the waveguide contribution such that the total dispersion D is relatively small over a wide wavelength range extending from 1.3 to 1.6 μm . Such fibers are called **dispersion-flattened fibers**.

Dispersion-Shifted Fibers



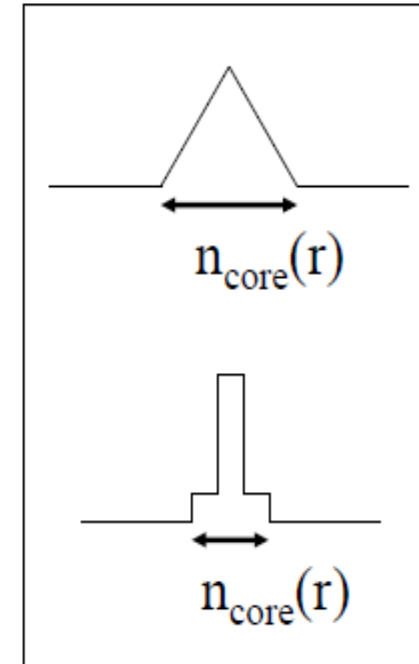
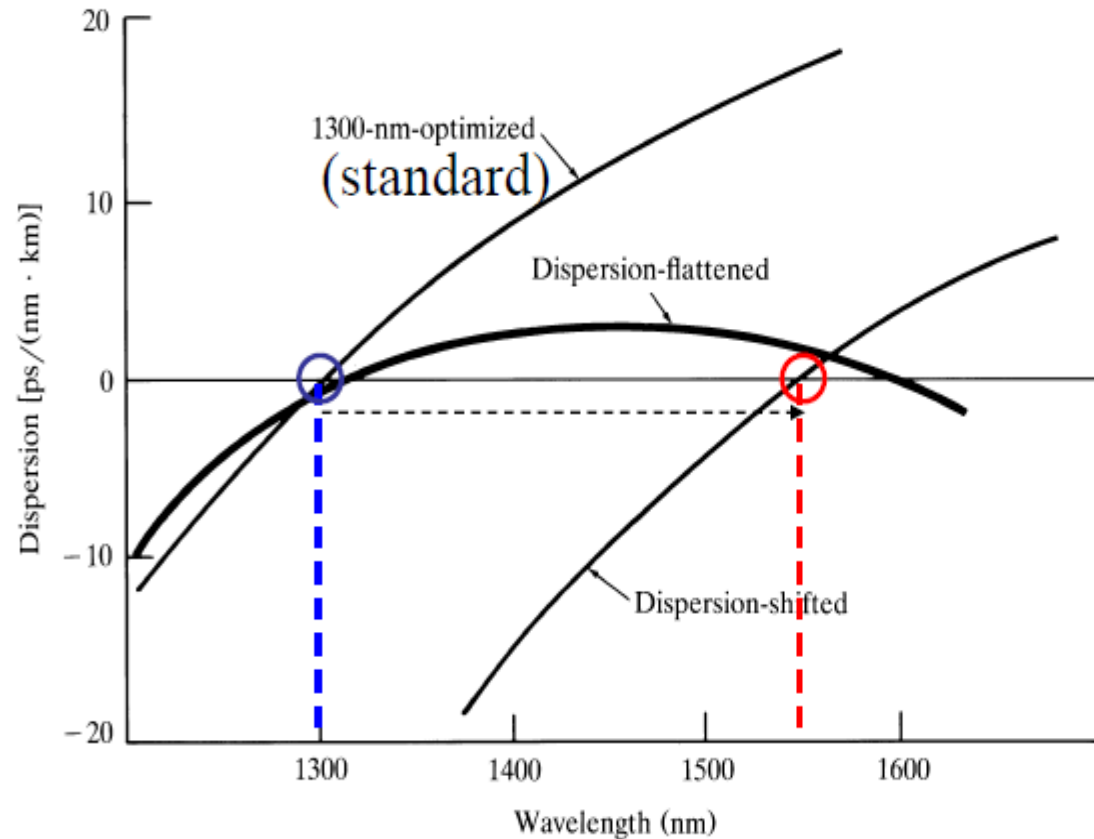
Zero dispersion around 1550 nm!

Dispersion-Flattened Fibers



Near-Zero dispersion around 1300-1550 nm!

Dispersion tailored fibers



- The design of dispersion-modified fibers often involves the use of multiple cladding layers and a tailoring of the refractive index profile.

Chromatic dispersion in low-bit-rate systems

- Broadening of the light pulse due to Chromatic Dispersion

$$\Delta T = LD\Delta\lambda$$

- Consider the maximum pulse broadening equals to the bit time period $1/B$, then the dispersion-limited distance:

$$L_D = \frac{1}{DB\Delta\lambda}$$

- e.g. For $D = 17 \text{ ps}/(\text{km}\cdot\text{nm})$, $B = 2.5 \text{ Gb/s}$ and $\Delta\lambda = 0.03 \text{ nm}$

$$L_D = 784 \text{ km}$$

- (It is known that dispersion limits a 2.5 Gbit/s channel to roughly 800 km! Therefore, chromatic dispersion is not much of an issue in low-bit-rate systems deployed in the early 90s!)

Chromatic dispersion in low-bit-rate systems

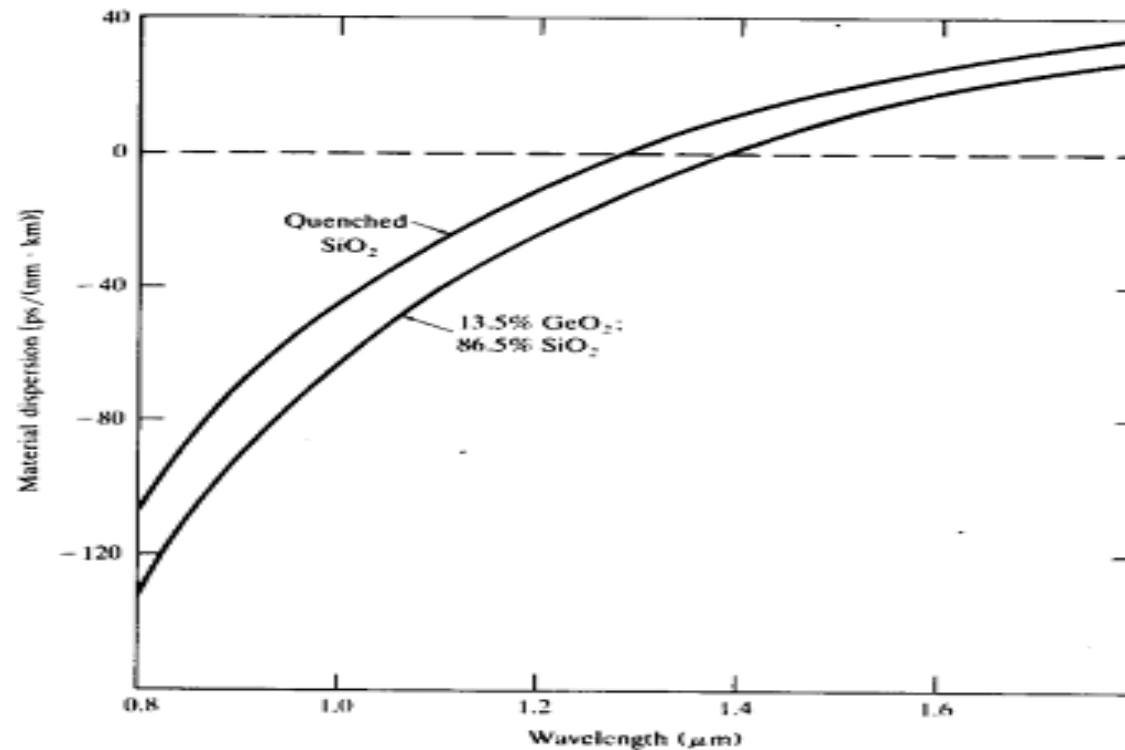
- When upgrading from 2.5- to 10-Gbit/s systems, most technical challenges are less than four times as complicated and the cost of components is usually much less than four times as expensive.
- However, when increasing the bit rate by a factor of 4, the effect of chromatic dispersion increases by a factor of 16!
- For 10 Gb/s system,

$$L_D = 784km / 16 = 50km$$

- This is why chromatic dispersion compensation must be employed for systems operating at 10 Gbit/s (now at 40 Gbit/s and beyond.)

Numerical Problems

- An LED operating at 850 nm has a spectral width of 45 nm. What is the pulse spreading in ns/km due to material dispersion in a silica fiber ? What is the pulse spreading when a laser diode having a 2 nm spectral width is used ?



Numerical Problems

- An LED operating at 850 nm has a spectral width of 45 nm. What is the pulse spreading in ns/km due to material dispersion in a silica fiber ? What is the pulse spreading when a laser diode having a 2 nm spectral width is used ?

Solution:

$|D_{\text{mat}}(\lambda)|$ at 850 nm = 100 ps / (nm.km) from graph

$L = 1$ km

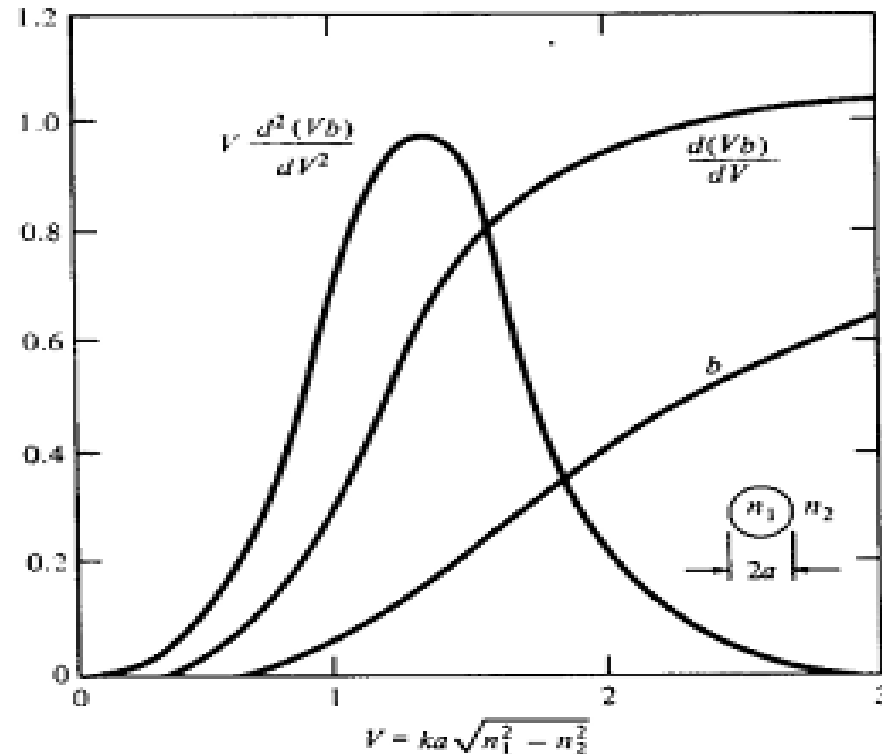
$\sigma_{\text{mat}} = \sigma_{\lambda} L |D_{\text{mat}}(\lambda)| = 4.5$ ns / km for LED

$\sigma_{\text{mat}} = \sigma_{\lambda} L |D_{\text{mat}}(\lambda)| = 0.2$ ns / km for LD

•

Numerical Problems

- Calculate the waveguide dispersion at 1320 nm in units of ps / (nm . km) for a single mode fiber with core and cladding diameters of 9 μm and 125 μm , respectively. Let the core index $n_1 = 1.48$ and the relative index difference $\Delta = 0.22\%$.



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$$D_{wg} = \frac{d\left(\frac{1}{V_{g,eff}}\right)}{d\lambda} = -\frac{n_2\Delta}{c\lambda} \left[V \frac{d^2(Vb)}{dV^2} \right]$$

$$n_2 \approx n_1 (1 - \Delta) = 1.477$$

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = 1.86$$

$$V d^2(Vb)/dV^2 = 0.50 \quad ; \text{ from graph}$$

$$D_{wg}(\lambda) = -4.1 \text{ ps / (nm.km)}$$

Numerical Problems

- A step-index single-mode fiber exhibits material dispersion of $6 \text{ ps nm}^{-1} \text{ km}^{-1}$ at an operation wavelength of $1.55 \text{ }\mu\text{m}$. Assume that $n_1 = 1.45$ and $\Delta = 0.5 \%$. Calculate the $Vd^2(Vb)/dV^2$ value to make the total dispersion of the fiber zero at this wavelength.

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$$D(\lambda) = D_{mat}(\lambda) + D_{wg}(\lambda)$$

$$D_{wg} = -\frac{n_2\Delta}{c\lambda} \left[V \frac{d^2(Vb)}{dV^2} \right]$$

$$\left[V \frac{d^2(Vb)}{dV^2} \right] = 0.3869$$

Numerical Problems

- Consider an optical link consisting of a 5 Km long step index fiber with core index $n_1 = 1.49$ and relative index difference $\Delta = 1\%$.
 - a. Find the delay difference at the fiber end between the slowest and the fastest modes and also the rms pulse broadening caused by intermodal dispersion.
 - b. If the fiber is coupled to an LED operating at 1550 nm having a 75 nm spectral width, calculate the rms pulse spreading due to material dispersion. $D_{mat} = 20 \text{ ps / nm.Km}$.
 - c. Calculate the rms pulse spreading due to waveguide dispersion if $V \frac{d^2(Vb)}{dV^2} = 0.1$.
 - d. Calculate the maximum bit rate B_T that can be transmitted over the fiber without significant errors.
 - e. What is the Bandwidth - Distance product for this fiber ?

Numerical Problems

$$\sigma_s = \Delta T_{\text{mod}} / (2 \sqrt{3}) = L n_1 \Delta / (c 2 \sqrt{3}) = 72 \text{ ns}$$

$$\sigma_{\text{mat}} = \sigma \lambda L |D_{\text{mat}}(\lambda)| = 7.5 \text{ ns/km}$$

$$D_{wg} = -\frac{n_2 \Delta}{c \lambda} \left[V \frac{d^2(Vb)}{dV^2} \right] \quad \sigma_{wg} = \left| -\frac{n_2 \Delta}{c \lambda} \left[V \frac{d^2(Vb)}{dV^2} \right] \right| \sigma_\lambda L = 1.2 \text{ ns}$$

$$B_T = \frac{0.2}{\sigma} = 2.76 \text{ Mbps}$$

$$\sigma = \sqrt{\sigma_{\text{mod}}^2 + \sigma_{\text{mat}}^2 + \sigma_{wg}^2}$$

$$B \cdot L = 13.812155 \approx 14 \text{ MHz} \cdot \text{Km}$$