Problem Solutions for Chapter 2

2-1. $$\mathbf{E} = 100\cos\left(2\pi 10^8 t + 30°\right)\mathbf{e}_x + 20\cos\left(2\pi 10^8 t - 50°\right)\mathbf{e}_y + 40\cos\left(2\pi 10^8 t + 210°\right)\mathbf{e}_z$$

2-2. The general form is:

y = (amplitude) $\cos(\omega t - kz) = A \cos [2\pi(\nu t - z/\lambda)]$. Therefore

(a) amplitude = 8 μm

(b) wavelength: $1/\lambda = 0.8\ \mu m^{-1}$ so that $\lambda = 1.25\ \mu m$

(c) $\omega = 2\pi\nu = 2\pi(2) = 4\pi$

(d) At $t = 0$ and $z = 4\ \mu m$ we have

$$y = 8 \cos [2\pi(-0.8\ \mu m^{-1})(4\ \mu m)]$$

$$= 8 \cos [2\pi(-3.2)] = 2.472$$

2-3. For E in electron volts and λ in μm we have $E = \dfrac{1.240}{\lambda}$

(a) At 0.82 μm, E = 1.240/0.82 = 1.512 eV

At 1.32 μm, E = 1.240/1.32 = 0.939 eV

At 1.55 μm, E = 1.240/1.55 = 0.800 eV

(b) At 0.82 μm, $k = 2\pi/\lambda = 7.662\ \mu m^{-1}$

At 1.32 μm, $k = 2\pi/\lambda = 4.760\ \mu m^{-1}$

At 1.55 μm, $k = 2\pi/\lambda = 4.054\ \mu m^{-1}$

2-4. $x_1 = a_1 \cos (\omega t - \delta_1)$ and $x_2 = a_2 \cos (\omega t - \delta_2)$

Adding x_1 and x_2 yields

$$x_1 + x_2 = a_1 [\cos \omega t \ \cos \delta_1 + \sin \omega t \ \sin \delta_1] + a_2 [\cos \omega t \ \cos \delta_2 + \sin \omega t \ \sin \delta_2]$$

$$= [a_1 \cos \delta_1 + a_2 \cos \delta_2] \cos \omega t + [a_1 \sin \delta_1 + a_2 \sin \delta_2] \sin \omega t$$

Since the a's and the δ's are constants, we can set

$$a_1 \cos \delta_1 + a_2 \cos \delta_2 = A \cos \phi \qquad (1)$$

$$a_1 \sin \delta_1 + a_2 \sin \delta_2 = A \sin \phi \qquad (2)$$

provided that constant values of A and ϕ exist which satisfy these equations. To verify this, first square both sides and add:

$$A^2 (\sin^2 \phi + \cos^2 \phi) = a_1^2\left(\sin^2 \delta_1 + \cos^2 \delta_1\right)$$

$$+\ a_2^2\left(\sin^2 \delta_2 + \cos^2 \delta_2\right) + 2a_1a_2 (\sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2)$$

or

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos (\delta_1 - \delta_2)$$

Dividing (2) by (1) gives

$$\tan \phi = \frac{a_1 \sin \delta_1 + a_2 \sin \delta_2}{a_1 \cos \delta_1 + a_2 \cos \delta_2}$$

Thus we can write

$$x = x_1 + x_2 = A \cos \phi \cos \omega t + A \sin \phi \sin \omega t = A \cos(\omega t - \phi)$$

2-5. First expand Eq. (2-3) as

$$\frac{E_y}{E_{0y}} = \cos (\omega t - kz) \cos \delta - \sin (\omega t - kz) \sin \delta \qquad (2.5\text{-}1)$$

Subtract from this the expression

$$\frac{E_x}{E_{0x}} \cos \delta = \cos (\omega t - kz) \cos \delta$$

to yield

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \delta = - \sin (\omega t - kz) \sin \delta \qquad (2.5\text{-}2)$$

Using the relation $\cos^2 \alpha + \sin^2 \alpha = 1$, we use Eq. (2-2) to write

$$\sin^2(\omega t - kz) = [1 - \cos^2(\omega t - kz)] = \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right] \qquad (2.5\text{-}3)$$

Squaring both sides of Eq. (2.5-2) and substituting it into Eq. (2.5-3) yields

$$\left[\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\delta\right]^2 = \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\delta$$

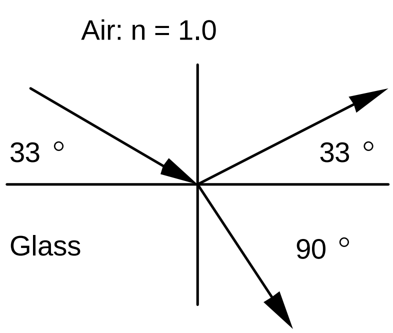
Expanding the left-hand side and rearranging terms yields

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta = \sin^2\delta$$

2-6. Plot of Eq. (2-7).

2-7. Linearly polarized wave.

2-8.



(a) Apply Snell's law

$$n_1 \cos\theta_1 = n_2 \cos\theta_2$$

where $n_1 = 1$, $\theta_1 = 33°$, and $\theta_2 = 90° - 33° = 57°$

$$\therefore\ n_2 = \frac{\cos\ 33°}{\cos\ 57°} = 1.540$$

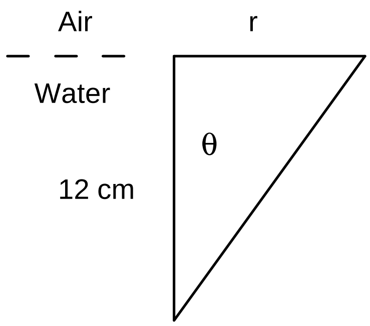
(b) The critical angle is found from

$$n_{glass} \sin\phi_{glass} = n_{air} \sin\phi_{air}$$

with $\phi_{air} = 90°$ and $n_{air} = 1.0$

$$\therefore\ \phi_{critical} = \arcsin \frac{1}{n_{glass}} = \arcsin \frac{1}{1.540} = 40.5°$$

2-9

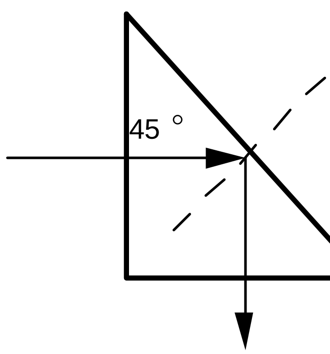


Find θ_c from Snell's law $\quad n_1 \sin\theta_1 = n_2 \sin\theta_c = 1$

When $n_2 = 1.33$, then $\theta_c = 48.75°$

Find r from $\tan\theta_c = \dfrac{r}{12\ \text{cm}}$, which yields r = 13.7 cm.

2-10.



Using Snell's law $\quad n_{glass} \sin\theta_c = n_{alcohol} \sin 90°$

where $\theta_c = 45°$ we have

$$n_{glass} = \frac{1.45}{\sin\ 45°} = 2.05$$

2-11. (a) Use either $NA = \left(n_1^2 - n_2^2\right)^{1/2} = \underline{0.242}$

or

$$NA \approx n_1\sqrt{2\Delta} = n_1\sqrt{\frac{2(n_1 - n_2)}{n_1}} = \underline{0.243}$$

(b) $\theta_{0,max}$ = arcsin (NA/n) = arcsin $\left(\frac{0.242}{1.0}\right)$ = 14°

2-13. $NA = \left(n_1^2 - n_2^2\right)^{1/2} = \left[n_1^2 - n_1^2(1-\Delta)^2\right]^{1/2}$

$$= n_1 \left(2\Delta - \Delta^2\right)^{1/2}$$

Since $\Delta << 1$, $\Delta^2 << \Delta$; $\therefore$ NA $\approx n_1\sqrt{2\Delta}$

2-14. (a) Solve Eq. (2-34a) for jH_ϕ:

$$jH_\phi = j\,\frac{\varepsilon\omega}{\beta}\,E_r - \frac{1}{\beta r}\frac{\partial H_z}{\partial\phi}$$ Substituting into Eq. (2-33b) we have

$$j\,\beta\,E_r + \frac{\partial E_z}{\partial r} = \omega\mu\left[j\frac{\varepsilon\omega}{\beta}E_r - \frac{1}{\beta r}\frac{\partial H_z}{\partial\phi}\right]$$

Solve for E_r and let $q^2 = \omega^2\varepsilon\mu - \beta^2$ to obtain Eq. (2-35a).

(b) Solve Eq. (2-34b) for jH_r:

$$jH_r = -j\,\frac{\varepsilon\omega}{\beta}\,E_\phi - \frac{1}{\beta}\frac{\partial H_z}{\partial r}$$ Substituting into Eq. (2-33a) we have

$$j\,\beta\,E_\phi + \frac{1}{r}\frac{\partial E_z}{\partial\phi} = -\omega\mu\left[-j\frac{\varepsilon\omega}{\beta}E_\phi - \frac{1}{\beta}\frac{\partial H_z}{\partial r}\right]$$

Solve for E_ϕ and let $q^2 = \omega^2\varepsilon\mu - \beta^2$ to obtain Eq. (2-35b).

(c) Solve Eq. (2-34a) for jE_r:

$$jE_r = \frac{1}{\varepsilon\omega}\frac{1}{r}\left(\frac{\partial H_z}{\partial\phi} + jr\beta H_\phi\right)$$

Substituting into Eq. (2-33b) we have

$$\frac{\beta}{\varepsilon\omega}\frac{1}{r}\left(\frac{\partial H_z}{\partial\phi} + jr\beta H_\phi\right) + \frac{\partial E_z}{\partial r} = j\omega\mu\, H_\phi$$

Solve for H_ϕ and let $q^2 = \omega^2\varepsilon\mu - \beta^2$ to obtain Eq. (2-35d).

(d) Solve Eq. (2-34b) for jE_ϕ

$$jE_\phi = -\frac{1}{\varepsilon\omega}\left(j\beta H_r + \frac{\partial H_z}{\partial r}\right)$$

Substituting into Eq. (2-33a) we have

$$\frac{1}{r}\frac{\partial E_z}{\partial\phi} - \frac{\beta}{\varepsilon\omega}\left(j\beta H_r + \frac{\partial H_z}{\partial r}\right) = -j\omega\mu\, H_r$$

Solve for H_r to obtain Eq. (2-35c).

(e) Substitute Eqs. (2-35c) and (2-35d) into Eq. (2-34c)

$$-\frac{j}{q^2}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\beta\frac{\partial H_z}{\partial\phi} + \varepsilon\omega r\frac{\partial E_z}{\partial r}\right) - \frac{\partial}{\partial\phi}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\varepsilon\omega}{r}\frac{\partial E_z}{\partial\phi}\right)\right] = j\varepsilon\omega E_z$$

Upon differentiating and multiplying by $jq^2/\varepsilon\omega$ we obtain Eq. (2-36).

(f) Substitute Eqs. (2-35a) and (2-35b) into Eq. (2-33c)

$$-\frac{j}{q^2}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\beta\frac{\partial E_z}{\partial\phi} - \mu\omega r\frac{\partial H_z}{\partial r}\right) - \frac{\partial}{\partial\phi}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial\phi}\right)\right] = -j\mu\omega H_z$$

Upon differentiating and multiplying by $jq^2/\varepsilon\omega$ we obtain Eq. (2-37).

2-15. For $\nu = 0$, from Eqs. (2-42) and (2-43) we have

$$E_z = AJ_0(ur)\, e^{j(\omega t-\beta z)} \quad \text{and} \quad H_z = BJ_0(ur)\, e^{j(\omega t-\beta z)}$$

We want to find the coefficients A and B. From Eqs. (2-47) and (2-51), respectively, we have

$$C = \frac{J_\nu(ua)}{K_\nu(wa)}\, A \quad \text{and} \quad D = \frac{J_\nu(ua)}{K_\nu(wa)}\, B$$

Substitute these into Eq. (2-50) to find B in terms of A:

$$A\left(\frac{j\beta\nu}{a}\right)\left(\frac{1}{u^2}+\frac{1}{w^2}\right) = B\omega\mu\left[\frac{J'_\nu(ua)}{uJ_\nu(ua)}+\frac{K'_\nu(wa)}{wK_\nu(wa)}\right]$$

For $\nu = 0$, the right-hand side must be zero. Also for $\nu = 0$, either Eq. (2-55a) or (2-56a) holds. Suppose Eq. (2-56a) holds, so that the term in square brackets on the right-hand side in the above equation is not zero. Then we must have that B = 0, which from Eq. (2-43) means that $H_z = 0$. Thus Eq. (2-56) corresponds to TM_{0m} modes.

For the other case, substitute Eqs. (2-47) and (2-51) into Eq. (2-52):

$$0 = \frac{1}{u^2}\left[B\frac{j\beta\nu}{a}J_\nu(ua) + A\omega\varepsilon_1 uJ'_\nu(ua)\right]$$

$$+\ \frac{1}{w^2}\left[B\frac{j\beta\nu}{a}J_\nu(ua) + A\omega\varepsilon_2 w\frac{K'_\nu(wa)J_\nu(ua)}{K_\nu(wa)}\right]$$

With $k_1^2 = \omega^2\mu\varepsilon_1$ and $k_2^2 = \omega^2\mu\varepsilon_2$ rewrite this as

$$B\nu = \frac{ja}{\beta\omega\mu}\left[\frac{1}{\dfrac{1}{u^2}+\dfrac{1}{w^2}}\right]\left[k_1^2 J_\nu + k_2^2 K_\nu\right] A$$

where J_ν and K_ν are defined in Eq. (2-54). If for $\nu = 0$ the term in square brackets on the right-hand side is non-zero, that is, if Eq. (2-56a) does not hold, then we must have that A = 0, which from Eq. (2-42) means that $E_z = 0$. Thus Eq. (2-55) corresponds to TE_{0m} modes.

2-16. From Eq. (2-23) we have

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{1}{2}\left(1 - \frac{n_2^2}{n_1^2}\right)$$

$\Delta << 1$ implies $n_1 \approx n_2$

Thus using Eq. (2-46), which states that $n_2k = k_2 \leq \beta \leq k_1 = n_1k$, we have

$$n_2^2k^2 = k_2^2 \approx n_1^2k^2 = k_1^2 \approx \beta^2$$

2-17.

2-18. (a) From Eqs. (2-59) and (2-61) we have

$$M \approx \frac{2\pi^2a^2}{\lambda^2}\left(n_1^2 - n_2^2\right) = \frac{2\pi^2a^2}{\lambda^2}(NA)^2$$

$$a = \left(\frac{M}{2\pi}\right)^{1/2}\frac{\lambda}{NA} = \left(\frac{1000}{2}\right)^{1/2}\frac{0.85\mu m}{0.2\pi} = 30.25\mu m$$

Therefore, D = 2a =60.5 μm

(b) $M = \dfrac{2\pi^2(30.25\mu m)^2}{(1.32\mu m)^2}(0.2)^2 = 414$

(c) At 1550 nm, M = 300

2-19. From Eq. (2-58),

$$V = \frac{2\pi\ (25\ \mu m)}{0.82\ \mu m}\left[(1.48)^2 - (1.46)^2\right]^{1/2} = 46.5$$

Using Eq. (2-61) $M \approx V^2/2 = 1081$ at 820 nm.

Similarly, M = 417 at 1320 nm and M = 303 at 1550 nm. From Eq. (2-72)

$$\left(\frac{P_{clad}}{P}\right)_{total} \approx \frac{4}{3} M^{-1/2} = \frac{4 \times 100\%}{3\sqrt{1080}} = 4.1\%$$

at 820 nm. Similarly, $(P_{clad}/P)_{total}$ = 6.6% at 1320 nm and 7.8% at 1550 nm.

2-20 (a) At 1320 nm we have from Eqs. (2-23) and (2-57) that V = 25 and M = 312.

(b) From Eq. (2-72) the power flow in the cladding is 7.5%.

2-21. (a) For single-mode operation, we need $V \le 2.40$.

Solving Eq. (2-58) for the core radius a

$$a = \frac{V\lambda}{2\pi}\left(n_1^2 - n_2^2\right)^{-1/2} = \frac{2.40(1.32\mu m)}{2\pi\left[(1.480)^2 - (1.478)^2\right]^{1/2}} = 6.55\ \mu m$$

(b) From Eq. (2-23)

$$NA = \left(n_1^2 - n_2^2\right)^{1/2} = \left[(1.480)^2 - (1.478)^2\right]^{1/2} = 0.077$$

(c) From Eq. (2-23), NA = n sin $\theta_{0,max}$. When n = 1.0 then

$$\theta_{0,max} = \arcsin\left(\frac{NA}{n}\right) = \arcsin\left(\frac{0.077}{1.0}\right) = 4.4°$$

2-22. $n_2 = \sqrt{n_1^2 - NA^2} = \sqrt{(1.458)^2 - (0.3)^2} = 1.427$

$$a = \frac{\lambda V}{2\pi NA} = \frac{(1.30)(75)}{2\pi(0.3)} = 52\ \mu m$$

2-23. For small values of Δ we can write $V \approx \frac{2\pi a}{\lambda}\, n_1 \sqrt{2\Delta}$

For a = 5 μm we have $\Delta \approx 0.002$, so that at 0.82 μm

$$V \approx \frac{2\pi\ (5\ \mu\text{m})}{0.82\ \mu\text{m}}\ 1.45\ \sqrt{2(0.002)} = 3.514$$

Thus the fiber is no longer single-mode. From Figs. 2-18 and 2-19 we see that the LP_{01} and the LP_{11} modes exist in the fiber at 0.82 μm.

2-24.

2-25. From Eq. (2-77) $L_p = \frac{2\pi}{\beta} = \frac{\lambda}{n_y - n_x}$

For L_p = 10 cm $\quad n_y - n_x = \frac{1.3\times10^{-6}\ \text{m}}{10^{-1}\ \text{m}} = 1.3\times10^{-5}$

For L_p = 2 m $\quad n_y - n_x = \frac{1.3\times10^{-6}\ \text{m}}{2\ \text{m}} = 6.5\times10^{-7}$

Thus

$$6.5\times10^{-7} \le n_y - n_x \le 1.3\times10^{-5}$$

2-26. We want to plot n(r) from n_2 to n_1. From Eq. (2-78)

$$n(r) = n_1\left[1-2\Delta(r/a)^{\alpha}\right]^{1/2} = 1.48\left[1-0.02(r/25)^{\alpha}\right]^{1/2}$$

n_2 is found from Eq. (2-79): $\quad n_2 = n_1(1 - \Delta) = 1.465$

2-27. From Eq. (2-81)

$$M = \frac{\alpha}{\alpha+2}\ a^2k^2n_1^2\Delta = \frac{\alpha}{\alpha+2}\left(\frac{2\pi a n_1}{\lambda}\right)^2\Delta$$

where

$$\Delta = \frac{n_1 - n_2}{n_1} = 0.0135$$

At λ = 820 nm, M = 543 and at λ = 1300 nm, M = 216.

For a step index fiber we can use Eq. (2-61)

$$M_{step} \approx \frac{V^2}{2} = \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 \left(n_1^2 - n_2^2\right)$$

At λ = 820 nm, M_{step} = 1078 and at λ = 1300 nm, M_{step} = 429.

Alternatively, we can let $\alpha = \infty$ in Eq. (2-81):

$$M_{step} = \left(\frac{2\pi a n_1}{\lambda}\right)^2 \Delta = \begin{cases} 1086 \text{ at } 820 \text{ nm} \\ 432 \text{ at } 1300 \text{ nm} \end{cases}$$

2-28. Using Eq. (2-23) we have

(a) $NA = \left(n_1^2 - n_2^2\right)^{1/2} = \left[(1.60)^2 - (1.49)^2\right]^{1/2} = 0.58$

(b) $NA = \left[(1.458)^2 - (1.405)^2\right]^{1/2} = 0.39$

2-29. (a) From the Principle of the Conservation of Mass, the volume of a preform rod section of length $L_{preform}$ and cross-sectional area A must equal the volume of the fiber drawn from this section. The preform section of length $L_{preform}$ is drawn into a fiber of length L_{fiber} in a time t. If S is the preform feed speed, then $L_{preform}$ = St. Similarly, if s is the fiber drawing speed, then L_{fiber} = st. Thus, if D and d are the preform and fiber diameters, respectively, then

$$\text{Preform volume} = L_{preform}(D/2)^2 = St\,(D/2)^2$$

and

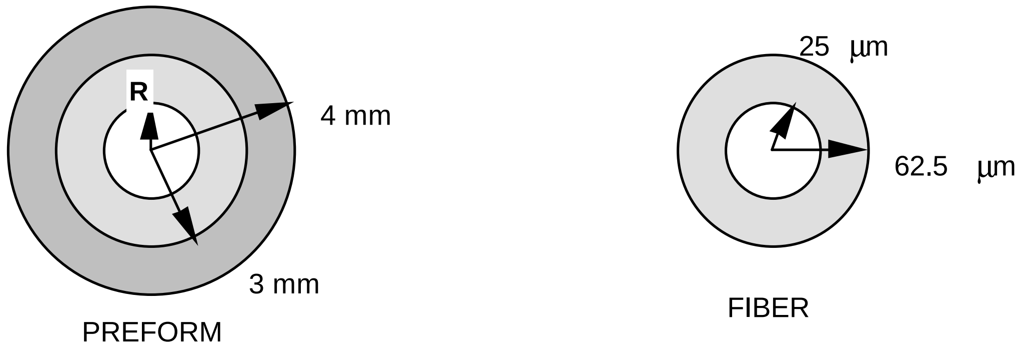
$$\text{Fiber volume} = L_{fiber}\,(d/2)^2 = st\,(d/2)^2$$

Equating these yields

$$St\left(\frac{D}{2}\right)^2 = st\left(\frac{d}{2}\right)^2 \quad \text{or} \quad s = S\left(\frac{D}{d}\right)^2$$

(b) $S = s\left(\frac{d}{D}\right)^2 = 1.2 \text{ m/s}\left(\frac{0.125 \text{ mm}}{9 \text{ mm}}\right)^2 = 1.39 \text{ cm/min}$

2-30. Consider the following geometries of the preform and its corresponding fiber:



We want to find the thickness of the deposited layer (3 mm - R). This can be done by comparing the ratios of the preform core-to-cladding cross-sectional areas and the fiber core-to-cladding cross-sectional areas:

$$\frac{A_{\text{preform core}}}{A_{\text{preform clad}}} = \frac{A_{\text{fiber core}}}{A_{\text{fiber clad}}}$$

or

$$\frac{\pi(3^2 - R^2)}{\pi(4^2 - 3^2)} = \frac{\pi\,(25)^2}{\pi\,\left[(62.5)^2 - (25)^2\right]}$$

from which we have

$$R = \left[9 - \frac{7(25)^2}{(62.5)^2 - (25)^2}\right]^{1/2} = 2.77 \text{ mm}$$

Thus, thickness = 3 mm - 2.77 mm = 0.23 mm.

2-31. (a) The volume of a 1-km-long 50-µm diameter fiber core is

$$V = \pi r^2 L = \pi\,(2.5\times10^{-3}\text{ cm})^2\,(10^5\text{ cm}) = 1.96\text{ cm}^3$$

The mass M equals the density ρ times the volume V:

$$M = \rho V = (2.6\text{ gm/cm}^3)(1.96\text{ cm}^3) = 5.1\text{ gm}$$

(b) If R is the deposition rate, then the deposition time t is

$$t = \frac{M}{R} = \frac{5.1\ \text{gm}}{0.5\ \text{gm/min}} = 10.2\ \text{min}$$

2-32. Solving Eq. (2-82) for χ yields

$$\chi = \left(\frac{K}{Y\sigma}\right)^2 \quad \text{where } Y = \sqrt{\pi} \qquad \text{for surface flaws.}$$

Thus

$$\chi = \frac{(20\ \text{N/mm}^{3/2})^2}{(70\ \text{MN/m}^2)^2\ \pi} = 2.60\times10^{-4}\ \text{mm} = 0.26\ \mu\text{m}$$

2-33. (a) To find the time to failure, we substitute Eq. (2-82) into Eq. (2-86) and integrate (assuming that σ is independent of time):

$$\int_{\chi_i}^{\chi_f} \chi^{-b/2}\, d\chi = AY^b\sigma^b \int_0^t dt$$

which yields

$$\frac{1}{1-\frac{b}{2}}\left[\chi_f^{1-b/2} - \chi_i^{1-b/2}\right] = AY^b\sigma^b t$$

or

$$t = \frac{2}{(b-2)A(Y\sigma)^b}\left[\chi_i^{(2-b)/2} - \chi_f^{(2-b)/2}\right]$$

(b) Rewriting the above expression in terms of K instead of χ yields

$$t = \frac{2}{(b-2)A(Y\sigma)^b}\left[\left(\frac{K_i}{Y\sigma}\right)^{2-b} - \left(\frac{K_f}{Y\sigma}\right)^{2-b}\right]$$

$$\approx \frac{2K_i^{2-b}}{(b-2)A(Y\sigma)^b} \quad \text{if } K_i^{b-2} << K_f^{b-2} \quad \text{or} \quad K_i^{2-b} >> K_f^{2-b}$$

2-34. Substituting Eq. (2-82) into Eq. (2-86) gives

$$\frac{d\chi}{dt} = AK^b = AY^b\chi^{b/2}\sigma^b$$

Integrating this from χ_i to χ_p where

$$\chi_i = \left(\frac{K}{Y\sigma_i}\right)^2 \quad \text{and} \quad \chi_p = \left(\frac{K}{Y\sigma_p}\right)^2$$

are the initial crack depth and the crack depth after proof testing, respectively, yields

$$\int_{\chi_i}^{\chi_p} \chi^{-b/2}\, d\chi = AY^b \int_0^{t_p} \sigma^b\, dt$$

or

$$\frac{1}{1-\frac{b}{2}} \left[\chi_p^{1-b/2} - \chi_i^{1-b/2}\right] = AY^b \sigma_p^b\, t_p$$

for a constant stress σ_p. Substituting for χ_i and χ_p gives

$$\left(\frac{2}{b-2}\right)\left(\frac{K}{Y}\right)^{2-b} \left[\sigma_i^{b-2} - \sigma_p^{b-2}\right] = AY^b \sigma_p^b\, t_p$$

or

$$\left(\frac{2}{b-2}\right)\left(\frac{K}{Y}\right)^{2-b} \frac{1}{AY^b} \left[\sigma_i^{b-2} - \sigma_p^{b-2}\right] = B\left[\sigma_i^{b-2} - \sigma_p^{b-2}\right] = \sigma_p^b\, t_p$$

which is Eq. (2-87).

When a static stress σ_S is applied after proof testing, the time to failure is found from Eq. (2-86):

$$\int_{\chi_p}^{\chi_s} \chi^{-b/2}\, d\chi = AY^b \sigma_s^b \int_0^{t_s} dt$$

where χ_s is the crack depth at the fiber failure point. Integrating (as above) we get Eq. (2-89):

$$B\left[\sigma_p^{b-2} - \sigma_s^{b-2}\right] = \sigma_s^b\, t_S$$

Adding Eqs. (2-87) and (2-89) yields Eq. (2-90).

2-35. (a) Substituting N_S as given by Eq. (2-92) and N_p as given by Eq. (2-93) into Eq. (2-94) yields

$$F = 1 - \exp\left(-\frac{L}{L_0}\left\{ \frac{\left[\left(\sigma_p^b t_p + \sigma_s^b t_s\right)/B + \sigma_s^{b-2}\right]^{\frac{m}{b-2}}}{\sigma_0^m} - \frac{\left(\sigma_p^b t_p / B + \sigma_p^{b-2}\right)^{\frac{m}{b-2}}}{\sigma_0^m} \right\} \right)$$

$$= 1 - \exp\left(-\frac{L}{L_0\sigma_0^m}\left[\sigma_p^b t_p / B + \sigma_p^{b-2}\right]^{\frac{m}{b-2}} \left\{ \left[\frac{\left(\dfrac{\sigma_p^b t_p + \sigma_s^b t_s}{B}\right) + \sigma_s^{b-2}}{\sigma_p^b t_p / B + \sigma_p^{b-2}} \right]^{\frac{m}{b-2}} - 1 \right\} \right)$$

$$= 1 - \exp\left(-LN_p\left\{ \frac{\left[1 + \dfrac{\sigma_s^b t_s}{\sigma_p^b t_p} + \left(\dfrac{\sigma_s}{\sigma_p}\right)^b \dfrac{B}{\sigma_s^2 t_p} \right]^{\frac{m}{b-2}}}{1 + \dfrac{B}{\sigma_p^2 t_p}} - 1 \right\} \right)$$

$$\approx 1 - \exp\left(-LN_p\left\{ \left[\left(1 + \frac{\sigma_s^b t_s}{\sigma_p^b t_p}\right) \frac{1}{1 + \dfrac{B}{\sigma_p^2 t_p}} \right]^{\frac{m}{b-2}} - 1 \right\} \right)$$

(b) For the term given by Eq. (2-96) we have

$$\left(\frac{\sigma_s}{\sigma_p}\right)^b \frac{B}{\sigma_s^2 t_p} = (0.3)^{15} \quad \frac{0.5\ (\mathrm{MN/m^2})^2\ \mathrm{s}}{\left[0.3\ (350\ \mathrm{MN/m^2})\right]^2 10\ \mathrm{s}} = 6.5\times10^{-14}$$

Thus this term can be neglected.

2-36. The failure probability is given by Eq. (2-85). For equal failure probabilities of the two fiber samples, $F_1 = F_2$, or

$$1 - \exp\left[-\left(\frac{\sigma_{1c}}{\sigma_0}\right)^m \frac{L_1}{L_0}\right] = 1 - \exp\left[-\left(\frac{\sigma_{2c}}{\sigma_0}\right)^m \frac{L_2}{L_0}\right]$$

which implies that

$$\left(\frac{\sigma_{1c}}{\sigma_0}\right)^m \frac{L_1}{L_0} = \left(\frac{\sigma_{2c}}{\sigma_0}\right)^m \frac{L_2}{L_0}$$

or

$$\frac{\sigma_{1c}}{\sigma_{2c}} = \left(\frac{L_2}{L_1}\right)^{1/m}$$

If $L_1 = 20$ m, then $\sigma_{1c} = 4.8$ GN/m^2

If $L_2 = 1$ km, then $\sigma_{2c} = 3.9$ GN/m^2

Thus

$$\left(\frac{4.8}{3.9}\right)^m = \frac{1000}{20} = 50$$

gives

$$m = \frac{\log 50}{\log(4.8/3.9)} = 18.8$$

Problem Solutions for Chapter 3

3-1.

$$\alpha(\text{dB/km}) = \frac{10}{z}\log\left[\frac{P(0)}{P(z)}\right] = \frac{10}{z}\log\left(e^{\alpha_p z}\right)$$

$$= 10\alpha_p \log e = 4.343\ \alpha_p (1/\text{km})$$

3-2. Since the attenuations are given in dB/km, first find the power levels in dBm for 100 μW and 150 μW. These are, respectively,

P(100 μW) = 10 log (100 μW/1.0 mW) = 10 log (0.10) = - 10.0 dBm

P(150 μW) = 10 log (150 μW/1.0 mW) = 10 log (0.15) = - 8.24 dBm

(a) At 8 km we have the following power levels:

P_{1300}(8 km) = - 8.2 dBm – (0.6 dB/km)(8 km) = - 13.0 dBm = 50 μW

P_{1550}(8 km) = - 10.0 dBm – (0.3 dB/km)(8 km) = - 12.4 dBm = 57.5 μW

(b) At 20 km we have the following power levels:

P_{1300}(20 km) = - 8.2 dBm – (0.6 dB/km)(20 km) = - 20.2 dBm = 9.55 μW

P_{1550}(20 km) = - 10.0 dBm – (0.3 dB/km)(20 km) = - 16.0 dBm = 25.1 μW

3-3. From Eq. (3-1c) with $P_{out} = 0.45\ P_{in}$

α = (10/3.5 km) log (1/0.45) = 1.0 dB/km

3-4. (a) $P_{in} = P_{out}\ 10^{\alpha L/10} = (0.3\ \mu W)\ 10^{1.5(12)/10} = 18.9\ \mu W$

(b) $P_{in} = P_{out}\ 10^{\alpha L/10} = (0.3\ \mu W)\ 10^{2.5(12)/10} = 300\ \mu W$

3-5. With λ in Eqs. (3-2b) and (3-3) given in μm, we have the following representative points for α_{uv} and α_{IR}:

λ (μm)	α_{uv}	α_{IR}
0.5	20.3	--
0.7	1.44	--
0.9	0.33	--
1.2	0.09	2.2×10^{-6}
1.5	0.04	0.0072
2.0	0.02	23.2
3.0	0.009	7.5×10^{4}

3-6. From Eq. (3-4a) we have

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4}(n^2 - 1)^2 k_B T_f \beta_T$$

$$= \frac{8\pi^3}{3(0.63\ \mu m)^4}\left[(1.46)^2 - 1\right]^2 (1.38\times10^{-16}\ \text{dyne-cm/K})(1400\ \text{K})$$

$$\times(6.8\times10^{-12}\ \text{cm}^2/\text{dyne})$$

$$= 0.883\ \text{km}^{-1}$$

To change to dB/km, multiply by 10 log e = 4.343: α_{scat} = 3.8 dB/km

From Eq. (3-4b):

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 k_B T_f \beta_T = 1.16\ \text{km}^{-1} = 5.0\ \text{dB/km}$$

3-8. Plot of Eq. (3-7).

3-9. Plot of Eq. (3-9).

3-10. From Fig. 2-22, we make the estimates given in this table:

νm	**P_{clad}/P**	**$\alpha_{\nu m} = \alpha_1 + (\alpha_2 - \alpha_1)P_{clad}/P$**	**$5 + 10^3 P_{clad}/P$**
01	0.02	3.0 + 0.02	5 + 20 = 25
11	0.05	3.0 + 0.05	5 + 50 = 55
21	0.10	3.0 + 0.10	5 + 100 = 105
02	0.16	3.0 + 0.16	5 + 160 = 165
31	0.19	3.0 + 0.19	5 + 190 = 195
12	0.31	3.0 + 0.31	5 + 310 = 315

3-11. (a) We want to solve Eq. (3-12) for α_{gi}. With $\alpha = 2$ in Eq. (2-78) and letting

$$\Delta = \frac{n^2(0) - n_2^2}{2n^2(0)}$$

we have

$$\alpha(r) = \alpha_1 + (\alpha_2 - \alpha_1)\,\frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2} = \alpha_1 + (\alpha_2 - \alpha_1)\,\frac{r^2}{a^2}$$

Thus

$$\alpha_{gi} = \frac{\int_0^\infty \alpha(r)\ p(r)\ r\ dr}{\int_0^\infty p(r)\ r\ dr} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{a^2}\,\frac{\int_0^\infty \exp\left(-Kr^2\right) r^3 dr}{\int_0^\infty \exp\left(-Kr^2\right) r\ dr}$$

To evaluate the integrals, let $x = Kr^2$, so that $dx = 2Krdr$. Then

$$\frac{\int_0^\infty \exp\left(-Kr^2\right) r^3 dr}{\int_0^\infty \exp\left(-Kr^2\right) r\ dr} = \frac{\frac{1}{2K^2}\int_0^\infty e^{-x}\ x\ dx}{\frac{1}{2K}\int_0^\infty e^{-x}\ dx} = \frac{\frac{1}{K}\ 1!}{0!} = \frac{1}{K}$$

Thus $\alpha_{gi} = \alpha_1 + \dfrac{(\alpha_2 - \alpha_1)}{Ka^2}$

(b) $p(a) = 0.1\ P_0 = P_0\ e^{-Ka^2}$ yields $e^{Ka^2} = 10$.

From this we have $Ka^2 = \ln 10 = 2.3$. Thus

$$\alpha_{gi} = \alpha_1 + \frac{(\alpha_2 - \alpha_1)}{2.3} = 0.57\alpha_1 + 0.43\alpha_2$$

3-12. With λ in units of micrometers, we have

$$n = \left\{1 + \frac{196.98}{(13.4)^2 - (1.24/\lambda)^2}\right\}^{1/2}$$

To compare this with Fig. 3-12, calculate three representative points, for example, λ = 0.2, 0.6, and 1.0 μm. Thus we have the following:

Wavelength λ	**Calculated n**	**n from Fig. 3-12**
0.2 μm	1.548	1.550
0.6 μm	1.457	1.458
1.0 μm	1.451	1.450

3-13. (a) From Fig. 3-13, $\frac{d\tau}{d\lambda} \approx$ 80 ps/(nm-km) at 850 nm. Therefore, for the LED we have from Eq. (3-20)

$$\frac{\sigma_{mat}}{L} = \frac{d\tau}{d\lambda}\,\sigma_\lambda = [80 \text{ ps/(nm-km)}](45 \text{ nm}) = 3.6 \text{ ns/km}$$

For a laser diode,

$$\frac{\sigma_{mat}}{L} = [80 \text{ ps/(nm-km)}](2 \text{ nm}) = 0.16 \text{ ns/km}$$

(b) From Fig. 3-13, $\frac{d\tau_{mat}}{d\lambda}$ = 22 ps/(nm-km)

Therefore, $D_{mat}(\lambda)$ = [22 ps/(nm-km)](75 nm) = 1.65 ns/km

3-14. (a) Using Eqs. (2-48), (2-49), and (2-57), Eq. (3-21) becomes

$$b = 1 - \left(\frac{ua}{V}\right)^2 = 1 - \frac{u^2a^2}{u^2a^2 + w^2a^2} = \frac{w^2}{u^2 + w^2}$$

$$= \frac{\beta^2 - k^2n_2^2}{k^2n_1^2 - \beta^2 + \beta^2 - k^2n_2^2} = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2}$$

(b) Expand b as $$b = \frac{(\beta/k + n_2)(\beta/k - n_2)}{(n_1 + n_2)(n_1 - n_2)}$$

Since $n_2 < \beta/k < n_1$, let $\beta/k = n_1(1 - \delta)$ where $0 < \delta < \Delta << 1$. Thus,

$$\frac{\beta / k + n_2}{n_1 + n_2} = \frac{n_1(1-\delta) + n_2}{n_1 + n_2} = 1 - \frac{n_1}{n_1 + n_2}\delta$$

Letting $n_2 = n_1(1 - \Delta)$ then yields

$$\frac{\beta / k + n_2}{n_1 + n_2} = 1 - \frac{\delta}{2-\Delta} \approx 1 \text{ since } \frac{\delta}{2-\Delta} << 1$$

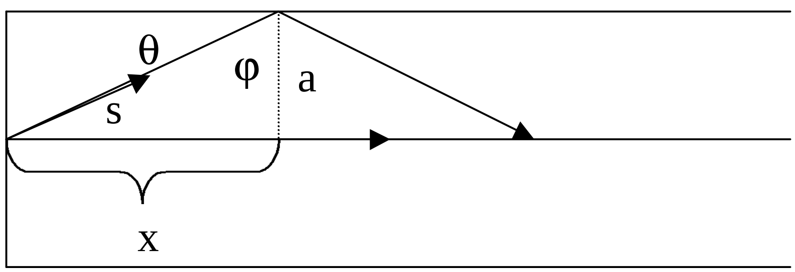
Therefore, $b \approx \dfrac{\beta / k - n_2}{n_1 - n_2}$ or $\beta = k[bn_1\Delta + n_2]$

From $n_2 = n_1(1 - \Delta)$ we have

$$n_1 = n_2(1 - \Delta)^{-1} = n_2(1 + \Delta + \Delta^2 + ...) \approx n_2(1 + \Delta)$$

Therefore, $\beta = k[b\, n_2(1 + \Delta)\Delta + n_2] \approx k\, n_2(b\Delta + 1)$

3-16. The time delay between the highest and lowest order modes can be found from the travel time difference between the two rays shown here.



The travel time of each ray is given by

$$\sin \phi = \frac{x}{s} = \frac{n_2}{n_1} = \frac{n_1(1-\Delta)}{n_1} = (1 - \Delta)$$

The travel time of the highest order ray is thus

$$T_{max} = \frac{n_1}{c}\left[s\left(\frac{L}{x}\right)\right] = \frac{n_1 L}{c}\frac{1}{1-\Delta}$$

For the axial ray the travel time is $T_{min} = \dfrac{Ln_1}{c}$

Therefore

$$T_{min} - T_{max} = \frac{Ln_1}{c}\left(\frac{1}{1-\Delta} - 1\right) = \frac{Ln_1}{c}\frac{\Delta}{1-\Delta} \approx \frac{Ln_1\Delta}{c}$$

3-17. Since $n_2 = n_1(1-\Delta)$, we can rewrite the equation as

$$\frac{\sigma_{mod}}{L} = \frac{n_1\Delta}{c}\left(1 - \frac{\pi}{V}\right)$$

where the first tern is Equation (3-30). The difference is then given by the factor

$$1 - \frac{\pi}{V} = 1 - \frac{\pi\lambda}{2a}\frac{1}{\left(n_1^2 - n_2^2\right)^{1/2}} \approx 1 - \frac{\pi\lambda}{2a}\frac{1}{n_1\sqrt{2\Delta}}$$

At 1300 nm this factor is $1 - \frac{\pi(1.3)}{2(62.5)}\frac{1}{1.48\sqrt{2(0.015)}} = 1 - 0.127 = 0.873$

3-18. For $\varepsilon = 0$ and in the limit of $\alpha \to \infty$ we have

$$C_1 = 1, \quad C_2 = \frac{3}{2}, \quad \frac{\alpha}{\alpha+1} = 1, \quad \frac{\alpha+2}{3\alpha+2} = \frac{1}{3}, \quad \frac{\alpha+1}{2\alpha+1} = \frac{1}{2},$$

and $\frac{(\alpha+1)^2}{(5\alpha+2)(3\alpha+2)} = \frac{1}{15}$

Thus Eq. (3-41) becomes

$$\sigma_{intermodal} = \frac{Ln_1\Delta}{2\sqrt{3}c}\left(1 + 3\Delta + \frac{12}{5}\Delta^2\right)^{1/2} \approx \frac{Ln_1\Delta}{2\sqrt{3}c}$$

3-19. For $\varepsilon = 0$ we have that $\alpha = 2(1 - \frac{6}{5}\Delta)$. Thus C_1 and C_2 in Eq. (3-42) become (ignoring small terms such as Δ^3, Δ^4, ...)

$$C_1 = \frac{\alpha-2}{\alpha+2} = \frac{2\left(1-\frac{6}{5}\Delta\right)-2}{2\left(1-\frac{6}{5}\Delta\right)+2} = \frac{-\frac{3}{5}\Delta}{1-\frac{3}{5}\Delta} \approx -\frac{3}{5}\Delta\left(1+\frac{3}{5}\Delta\right)$$

$$C_2 = \frac{3\alpha-2}{2(\alpha+2)} = \frac{3\left[2\left(1-\frac{6}{5}\Delta\right)\right]-2}{2\left[2\left(1-\frac{6}{5}\Delta\right)+2\right]} = \frac{1-\frac{9}{5}\Delta}{2\left(1-\frac{3}{5}\Delta\right)}$$

Evaluating the factors in Eq. (3-41) yields:

(a) $C_1^2 \approx \frac{9}{25}\ \Delta^2$

(b) $$\frac{4C_1C_2(\alpha+1)\Delta}{2\alpha+1} = \frac{4\Delta\left(-\frac{3}{5}\Delta\right)\left(1-\frac{9}{5}\Delta\right)\left[2\left(1-\frac{6}{5}\Delta\right)+1\right]}{\left(1-\frac{3}{5}\Delta\right)2\left(1-\frac{3}{5}\Delta\right)\left[4\left(1-\frac{6}{5}\Delta\right)+1\right]}$$

$$= \frac{-\frac{18\Delta}{\;}\left(1-\frac{11}{\;}\Delta+\frac{18}{25}\;\right)}{5\left(1\;\;\frac{}{5}\Delta\right)^2\left(\;\;\frac{24}{25}\Delta\right)} \qquad \frac{18}{25}\ \Delta^2$$

(c) $$\frac{16\Delta^2C_2^2(\alpha+1)^2}{(5\alpha+2)(3\alpha+2)}$$

$$= \frac{16\Delta^2\left(1-\frac{9}{5}\Delta\right)^2\left[2(1-\frac{6}{5}\Delta)+1\right]^2}{4\left(1-\frac{3}{5}\Delta\right)^2\left[10(1-\frac{6}{5}\Delta)+2\right]\left[6(1-\frac{6}{5}\Delta)+2\right]}$$

$$= \frac{16\Delta^2\left(1-\frac{9}{5}\Delta\right)^2 9\left(1-\frac{4}{5}\Delta\right)^2}{96(1-\Delta)(1-\frac{9}{10}\Delta)4\left(1-\frac{3}{5}\Delta\right)^2} \approx \frac{9}{24}\ \Delta^2$$

Therefore,

$$\sigma_{\text{int er mod al}} = \frac{Ln_1\Delta}{2c}\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\alpha+2}{3\alpha+2}\right)^{1/2}\left(\frac{9}{25}\Delta^2-\frac{18}{25}\Delta^2+\frac{9}{24}\Delta^2\right)^{1/2}$$

$$= \frac{Ln_1\Delta^2}{2c}\frac{2(1-\frac{6}{5}\Delta)}{\left[2(1-\frac{6}{5}\Delta)+1\right]}\left[\frac{2(1-\frac{6}{5}\Delta)+2}{6(1-\frac{6}{5}\Delta)+2}\right]^{1/2}\frac{3}{10\sqrt{6}} \approx \frac{n_1\Delta^2L}{20\sqrt{3}c}$$

3-20. We want to plot Eq. (3-30) as a function of σ_λ , where $\sigma_{\text{intermodal}}$ and $\sigma_{\text{intramodal}}$ are given by Eqs. (3-41) and (3-45). For ε = 0 and α = 2, we have C_1 = 0 and C_2 = 1/2. Since $\sigma_{\text{intermodal}}$ does not vary with σ_λ , we have

$$\frac{\sigma_{\text{intermodal}}}{L} = \frac{N_1\Delta}{2c}\left(\frac{\alpha}{\alpha+2}\right)\left(\frac{\alpha+2}{3\alpha+2}\right)^{1/2}\frac{4\Delta C_2(\alpha+1)}{\sqrt{(5\alpha+2)(3\alpha+2)}} = 0.070 \text{ ns/km}$$

With C_1 = 0 we have from Eq. (3-45)

$$\sigma_{\text{intramodal}} = \frac{1}{c}\frac{\sigma_\lambda}{\lambda}\left(-\lambda^2\frac{d^2n_1}{d\lambda^2}\right) = \begin{cases} 0.098\sigma_\lambda \text{ ns/km at 850 nm} \\ 1.026\times10^{-2}\sigma_\lambda \text{ ns/km at 1300 nm}\end{cases}$$

3-21. Using the same parameter values as in Prob. 3-18, except with Δ = 0.001, we have from Eq. (3-41) $\sigma_{\text{intermodal}}$ /L = 7 ps/km, and from Eq. (3.45)

$$\frac{\sigma_{\text{intramodal}}}{L} = \begin{cases} 0.098\sigma_\lambda \text{ ns/km at 850 nm} \\ 0.0103\sigma_\lambda \text{ ns/km at 1300 nm}\end{cases}$$

The plot of $\frac{\sigma}{L} = \frac{1}{L}\left(\sigma_{\text{inter}}^2 + \sigma_{\text{intra}}^2\right)^{1/2}$ vs σ_λ :

3-22. Substituting Eq. (3-34) into Eq. (3-33)

$$\tau_g = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\frac{1}{2\beta}\left\{2kn_1^2 + 2k^2n_1\frac{dn_1}{dk}\right.$$

$$- 2\left(\frac{\alpha+2}{\alpha}\frac{m}{a^2}\right)^{\frac{\alpha}{\alpha+2}}\frac{2}{\alpha+2}\left(n_1^2k^2\Delta\right)^{\frac{2}{\alpha+2}-1}$$

$$\left.\times\ \Delta\left[2k^2n_1\frac{dn_1}{dk} + 2kn_1^2 + \frac{n_1^2k^2}{\Delta}\frac{d\Delta}{dk}\right]\right\}$$

$$= \frac{L}{c}\frac{kn_1}{\beta}\left[N_1 - \frac{4\Delta}{\alpha+2}\left(\frac{\alpha+2}{\alpha}\frac{m}{a^2}\frac{1}{n_1^2k^2\Delta}\right)^{\frac{\alpha}{\alpha+2}}\left(N_1 + \frac{n_1k}{2\Delta}\frac{d\Delta}{dk}\right)\right]$$

$$= \frac{LN_1}{c}\frac{kn_1}{\beta}\left[1 - \frac{4\Delta}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\left(1+\frac{\varepsilon}{4}\right)\right]$$

with $N_1 = n_1 + k\,\frac{dn_1}{dk}$ and where M is given by Eq. (2-97) and ε is defined in Eq. (3-36b).

3-23. From Eq. (3-39), ignoring terms of order Δ^2,

$$\lambda\frac{d\tau}{d\lambda} = \frac{L}{c}\frac{dN_1}{d\lambda}\left[1 + \frac{\alpha-\varepsilon-2}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right]$$

$$+ \frac{LN_1}{c}\frac{\alpha-\varepsilon-2}{\alpha+2}\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right]$$

where

$N_1 = n_1 - \lambda\,\frac{dn_1}{d\lambda}$ and $M = \frac{\alpha}{\alpha+2}\,a^2k^2n_1^2\Delta$

(a) $$\frac{dN_1}{d\lambda} = \frac{d}{d\lambda}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) = -\ \lambda\,\frac{d^2n_1}{d\lambda^2}$$

Thus ignoring the term involving $\Delta\,\frac{d^2n_1}{d\lambda^2}$, the first term in square brackets becomes $-\frac{L}{c}\lambda^2\,\frac{d^2n_1}{d\lambda^2}$

(b) $$\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right] = m^{\frac{\alpha}{\alpha+2}}\left[\frac{d\Delta}{d\lambda}\left(\frac{1}{M}\right)^{\frac{\alpha}{\alpha+2}} + \Delta\frac{-\alpha}{\alpha+2}\frac{dM}{d\lambda}\left(\frac{1}{M}\right)^{\frac{\alpha}{\alpha+2}+1}\right]$$

(c) $$\frac{dM}{d\lambda} = \frac{\alpha}{\alpha+2}\,a^2\,\frac{d}{d\lambda}\left(k^2n_1^2\Delta\right)$$

$$= \frac{\alpha}{\alpha+2}\,a^2\left(\frac{d\Delta}{d\lambda}k^2n_1^2 + 2k^2\Delta n_1\frac{dn_1}{d\lambda} + 2kn_1^2\Delta\frac{dk}{d\lambda}\right)$$

Ignoring $\frac{d\Delta}{d\lambda}$ and $\frac{dn_1}{d\lambda}$ terms yields

$$\frac{dM}{d\lambda} = \frac{2\alpha}{\alpha+2} a^2k^2n_1^2\Delta\left(-\frac{1}{\lambda}\right) = -\frac{2M}{\lambda} \qquad \text{so that}$$

$$\frac{d}{d\lambda}\left[\Delta\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}\right] = \frac{\Delta}{\lambda}\frac{2\alpha}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}. \qquad \text{Therefore}$$

$$\lambda\frac{d\tau}{d\lambda} = -\frac{L}{c}\lambda^2\frac{d^2n_1}{d\lambda^2} + \frac{LN_1}{c}\frac{\alpha-\varepsilon-2}{\alpha+2}\frac{2\alpha\Delta}{\alpha+2}\left(\frac{m}{M}\right)^{\frac{\alpha}{\alpha+2}}$$

3-24. Let $a = \lambda^2\frac{d^2n_1}{d\lambda^2}$; $b = N_1C_1\Delta\frac{2\alpha}{\alpha+2}$; $\gamma = \frac{\alpha}{\alpha+2}$

Then from Eqs. (3-32), (3-43), and (3-44) we have

$$\sigma^2_{\text{intramodal}} = L^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\sum_{m=0}^{M}\left(\lambda\frac{d\tau_g}{d\lambda}\right)^2$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\sum_{m=0}^{M}\left[-a+b\left(\frac{m}{M}\right)^\gamma\right]^2$$

$$\approx \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\int_0^M\left[-a+b\left(\frac{m}{M}\right)^\gamma\right]^2 dm$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\frac{1}{M}\int_0^M\left[a^2-2ab\left(\frac{m}{M}\right)^\gamma + b^2\left(\frac{m}{M}\right)^{2\gamma}\right] dm$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\left[a^2-\frac{2ab}{\gamma+1}+\frac{b^2}{2\gamma+1}\right]$$

$$= \left(\frac{L}{c}\right)^2\left(\frac{\sigma_\lambda}{\lambda}\right)^2\left[\left(-\lambda^2\frac{d^2n_1}{d\lambda^2}\right)^2\right.$$

$$- \quad 2\left(\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right) N_1 C_1 \Delta \quad \frac{\alpha}{\alpha+1} + (N_1 C_1 \Delta)^2 \frac{4\alpha^2}{(\alpha+2)(3\alpha+2)} \Bigg]$$

3-25. Plot of Eq. (3-57).

3-26. (a) $D = (\lambda - \lambda_0)S_0 = -50(0.07) = -3.5 \ \text{ps/(nm - km)}$

(b) $$D = \frac{1500(0.09)}{4}\left[1 - \left(\frac{1310}{1500}\right)^4\right] = 14.1 \ \text{ps/(nm - km)}$$

3-27. (a) From Eq. (3-48)

$$\frac{\sigma_{step}}{L} = \frac{n_1 \Delta}{2\sqrt{3}\ c} = \frac{1.49(0.01)}{2\sqrt{3}\ (3\times10^8)} = 14.4 \ \text{ns/km}$$

(b) From Eq. (3-47)

$$\frac{\sigma_{opt}}{L} = \frac{n_1 \Delta^2}{20\sqrt{3}\ c} = \frac{1.49(0.01)^2}{20\sqrt{3}\ (3\times10^8)} = 14.3 \ \text{ps/km}$$

(c) 3.5 ps/km

3-28. (a) From Eq. (3-29)

$$\sigma_{mod} = T_{max} - T_{min} = \frac{n_1 \Delta L}{c} = \frac{(1.49)(0.01)(5\times10^3\,\text{m})}{3\times10^8\,\text{m/s}} = 248 \ \text{ns}$$

(b) From Eq. (3-48)

$$\sigma_{step} = \frac{n_1 \Delta L}{2\sqrt{3}\ c} = \frac{248}{2\sqrt{3}} = 71.7 \ \text{ns}$$

(c) $$B_T = \frac{0.2}{\sigma_{step}} = 2.8 \ \text{Mb/s}$$

(d) $B_T \cdot L = (2.8 \ \text{MHz})(5 \ \text{km}) = 13.9 \ \text{MHz} \cdot \text{km}$

3-29. For $\alpha = 0.95\alpha_{\text{opt}}$, we have

$$\frac{\sigma_{\text{int er}}(\alpha \neq \alpha_{\text{opt}})}{\sigma_{\text{int er}}(\alpha = \alpha_{\text{opt}})} = \frac{(\alpha - \alpha_{\text{opt}})}{\Delta(\alpha + 2)} = -\frac{0.05}{(0.015)(1.95)} = -170\%$$

For $\alpha = 1.05\alpha_{\text{opt}}$, we have

$$\frac{\sigma_{\text{int er}}(\alpha \neq \alpha_{\text{opt}})}{\sigma_{\text{int er}}(\alpha = \alpha_{\text{opt}})} = \frac{(\alpha - \alpha_{\text{opt}})}{\Delta(\alpha + 2)} = +\frac{0.05}{(0.015)(2.05)} = +163\%$$

Problem Solutions for Chapter 4

4-1. From Eq. (4-1), $n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} (m_e m_h)^{3/4} \exp\left(-\frac{E_g}{2k_B T}\right)$

$$= 2\left[\frac{2\pi(1.38\times10^{-23}\,\mathrm{J/K})}{(6.63\times10^{-34}\,\mathrm{J.s})^2}\right]^{3/2} T^{3/2}\left[(.068)(.56)(9.11\times10^{-31}\,\mathrm{kg})^2\right]^{3/4}$$

$$\times \exp\left[-\frac{(1.55-4.3\times10^{-4}\,T)\mathrm{eV}}{2(8.62\times10^{-5}\,\mathrm{eV/K})T}\right]$$

$$= 4.15\times10^{14}\,T^{3/2} \exp\left[-\frac{1.55}{2(8.62\times10^{-5})T}\right]\exp\left[\frac{4.3\times10^{-4}}{2(8.62\times10^{-5})}\right]$$

$$= 5.03\times10^{15}\,T^{3/2} \exp\left(-\frac{8991}{T}\right)$$

4-2. The electron concentration in a p-type semiconductor is $n_P = n_i = p_i$

Since both impurity and intrinsic atoms generate conduction holes, the total conduction-hole concentration p_P is

$p_P = N_A + n_i = N_A + n_P$

From Eq. (4-2) we have that $n_P = n_i^2/p_P$. Then

$p_P = N_A + n_P = N_A + n_i^2/p_P$ or $p_P^2 - N_A p_P - n_i^2 = 0$

so that

$$p_P = \frac{N_A}{2}\left(\sqrt{1+\frac{4n_i^2}{N_A^2}} + 1\right)$$

If $n_i \ll N_A$, which is generally the case, then to a good approximation

$p_P \approx N_A$ and $n_P = n_i^2/p_P \approx n_i^2/N_A$

4-3. (a) From Eq. (4-4) we have $1.540 = 1.424 + 1.266x + 0.266x^2$ or

$x^2 + 4.759x - 0.436 = 0$. Solving this quadratic equation yields (taking the plus sign only)

$$x = \frac{1}{2}\left[- 4.759 + \sqrt{(4.759)^2 + 4(.436)}\ \right] \quad = \quad \underline{0.090}$$

The emission wavelength is $\lambda = \dfrac{1.240}{1.540} \quad = 805$ nm.

(b) $E_g = 1.424 + 1.266(0.15) + 0.266(0.15)^2 = 1.620$ eV, so that

$$\lambda = \frac{1.240}{1.620} \quad = 766 \text{ nm}$$

4-4. (a) The lattice spacings are as follows:

$a(BC) = a(GaAs) = 5.6536$ Å

$a(BD) = a(GaP) \quad = 5.4512$ Å

$a(AC) = a(InAs) = 6.0590$ Å

$a(AD) = a(InP) \quad = 5.8696$ Å

$$a(x,y) = xy\ 5.6536 + x(1\text{-}y)\ 5.4512 + (1\text{-}x)y\ 6.0590 + (1\text{-}x)(1\text{-}y)5.8696$$

$$= 0.1894y - 0.4184x + 0.0130xy + 5.8696$$

(b) Substituting $a(xy) = a(InP) \quad = 5.8696$ Å into the expression for $a(xy)$ in (a), we have

$$y = \frac{0.4184x}{0.1894 - 0.0130x} \quad \approx \frac{0.4184x}{0.1894} \quad = 2.20x$$

(c) With $x = 0.26$ and $y = 0.56$, we have

$$E_g = 1.35 + 0.668(.26) - 1.17(.56) + 0.758(.26)^2 + 0.18(.56)^2$$

$$- .069(.26)(.56) - .322(.26)^2(.56) + 0.03(.26)(.56)^2 = 0.956 \text{ eV}$$

4-5. Differentiating the expression for E, we have

$$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda \quad \text{or} \quad \Delta\lambda = \frac{\lambda^2}{hc}\Delta E$$

For the same energy difference ΔE, the spectral width Δλ is proportional to the wavelength squared. Thus, for example,

$$\frac{\Delta\lambda_{1550}}{\Delta\lambda_{1310}} = \left(\frac{1550}{1310}\right)^2 = 1.40$$

4-6. (a) From Eq. (4-10), the internal quantum efficiency is

$\eta_{int} = \dfrac{1}{1+25/90} = 0.783$, and from Eq. (4-13) the internal power level is

$$P_{int} = (0.783)\frac{hc(35\ \text{mA})}{q(1310\ \text{nm})} = 26\ \text{mW}$$

(b) From Eq. (4-16),

$$P = \frac{1}{3.5(3.5+1)^2}\,26\ \text{mW} = 0.37\ \text{mW}$$

4-7. Plot of Eq. (4-18). Some representative values of P/P_0 are given in the table:

f in MHz	**P/P_0**
1	0.999
10	0.954
20	0.847
40	0.623
60	0.469
80	0.370
100	0.303

4-8. The 3-dB optical bandwidth is found from Eq. (4-21). It is the frequency f at which the expression is equal to -3; that is,

$$10\log\left(\frac{1}{\left[1+(2\pi f\tau)^2\right]^{1/2}}\right) = -3$$

With a 5-ns lifetime, we find $f = \frac{1}{2\pi(5\ \text{ns})}\left(10^{0.6} - 1\right) = 9.5\ \text{MHz}$

4-9. (a) Using Eq. (4-28) with $\Gamma = 1$

$$g_{th} = \frac{1}{0.05\ \text{cm}} \ln\left(\frac{1}{0.32}\right)^2 + 10\ \text{cm}^{-1} = 55.6\ \text{cm}^{-1}$$

(b) With $R_1 = 0.9$ and $R_2 = 0.32$,

$$g_{th} = \frac{1}{0.05\ \text{cm}} \ln\left[\frac{1}{0.9(0.32)}\right] + 10\ \text{cm}^{-1} = 34.9\ \text{cm}^{-1}$$

(c) From Eq. (4-37) $\eta_{ext} = \eta_i\,(g_{th} - \bar{\alpha})/g_{th}$;

thus for case (a): $\eta_{ext} = 0.65(55.6 - 10)/55.6 = 0.53$

For case (b): $\eta_{ext} = 0.65(34.9 - 10)/34.9 = 0.46$

4-10. Using Eq. (4-4) to find E_g and Eq. (4-3) to find λ, we have for x = 0.03,

$$\lambda = \frac{1.24}{E_g} = \frac{1.24}{1.424 + 1.266(0.3) + 0.266(0.3)^2} = 1.462\ \mu\text{m}$$

From Eq. (4-38)

$$\eta_{ext} = 0.8065\ \lambda(\mu\text{m})\,\frac{dP(\text{mW})}{dI(\text{mA})}$$

Taking dI/dP = 0.5 mW/mA, we have $\eta_{ext} = 0.8065\ (1.462)(0.5) = 0.590$

4-11. (a) From the given values, D = 0.74, so that $\Gamma_T = 0.216$

Then $n_{eff}^2 = 10.75$ and W = 3.45, yielding $\Gamma_L = 0.856$

(b) The total confinement factor then is $\Gamma = 0.185$

4-12. From Eq. (4-46) the mode spacing is

$$\Delta\lambda = \frac{\lambda^2}{2Ln} = \frac{(0.80\ \mu\text{m})^2}{2(400\ \mu\text{m})(3.6)} = 0.22\ \text{nm}$$

Therefore the number of modes in the range 0.75-to-0.85 μm is

$$\frac{.85-.75}{.22\times10^{-3}}=\frac{.1}{.22}\times10^{3}=\underline{455\text{ modes}}$$

4-13. (a) From Eq. (4-44) we have $g(\lambda) = (50\text{ cm}^{-1})\exp\left[-\frac{(\lambda - 850\text{ nm})^2}{2(32\text{ nm})^2}\right]$

$$= (50\text{ cm}^{-1})\exp\left[-\frac{(\lambda - 850)^2}{2048}\right]$$

(b) On the plot of $g(\lambda)$ versus λ, drawing a horizontal line at $g(\lambda) = \alpha_t$

$= 32.2\text{ cm}^{-1}$ shows that lasing occurs in the region 820 nm < λ < 880 nm.

(c) From Eq. (4-47) the mode spacing is

$$\Delta\lambda = \frac{\lambda^2}{2Ln} = \frac{(850)^2}{2(3.6)(400\ \mu\text{m})} = 0.25\text{ nm}$$

Therefore the number of modes in the range 820-to-880 nm is

$$N = \frac{880 - 820}{0.25} = \underline{240\text{ modes}}$$

4-14. (a) Let $N_m = n/\lambda = \frac{m}{2L}$ be the wave number (reciprocal wavelength) of mode m. The difference ΔN between adjacent modes is then

$$\Delta N = N_m - N_{m-1} = \frac{1}{2L} \qquad \text{(a-1)}$$

We now want to relate ΔN to the change $\Delta\lambda$ in the free-space wavelength. First differentiate N with respect to λ:

$$\frac{dN}{d\lambda} = \frac{d}{d\lambda}\left(\frac{n}{\lambda}\right) = \frac{1}{\lambda}\frac{dn}{d\lambda} - \frac{n}{\lambda^2} = -\frac{1}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

Thus for an incremental change in wavenumber ΔN, we have, in absolute values,

$$\Delta N = \frac{1}{\lambda^2}\left(n - \lambda\frac{dn}{d\lambda}\right)\Delta\lambda \qquad \text{(a-2)}$$

Equating (a-1) and (a-2) then yields $\Delta\lambda = \dfrac{\lambda^2}{2L\left(n - \lambda\dfrac{dn}{d\lambda}\right)}$

(b) The mode spacing is $\Delta\lambda = \dfrac{(.85\ \mu m)^2}{2(4.5)(400\ \mu m)} = 0.20$ nm

4-15. (a) The reflectivity at the GaAs-air interface is

$$R_1 = R_2 = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{3.6-1}{3.6+1}\right)^2 = 0.32$$

Then $J_{th} = \dfrac{1}{\beta}\left[\overline{\alpha} + \dfrac{1}{2L}\ln\dfrac{1}{R_1R_2}\right] = 2.65\times10^3\ \text{A/cm}^2$

Therefore

$I_{th} = J_{th} \times l \times w = (2.65\times10^3\ \text{A/cm}^2)(250\times10^{-4}\ \text{cm})(100\times10^{-4}\ \text{cm}) = 663$ mA

(b) $I_{th} = (2.65\times10^3\ \text{A/cm}^2)(250\times10^{-4}\ \text{cm})(10\times10^{-4}\ \text{cm}) = 66.3$ mA

4-16. From the given equation

$$\Delta E_{11} = 1.43\ \text{eV} + \frac{\left(6.6256\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8(5\ \text{nm})^2}\left(\frac{1}{6.19\times10^{-32}\ \text{kg}} + \frac{1}{5.10\times10^{-31}\ \text{kg}}\right)$$

$$= 1.43\ \text{eV} + 0.25\ \text{eV} = 1.68\ \text{eV}$$

Thus the emission wavelength is $\lambda = hc/E = 1.240/1.68 = 739$ nm.

4-17. Plots of the external quantum efficiency and power output of a MQW laser.

4-18. From Eq. (4-48a) the effective refractive index is

$$n_e = \frac{m\lambda_B}{2\Lambda} = \frac{2(1570\ \text{nm})}{2(460\ \text{nm})} = 3.4$$

Then, from Eq. (4-48b), for m = 0

$$\lambda = \lambda_B \pm \frac{\lambda_B^2}{2n_eL}\left(\frac{1}{2}\right) = 1570 \text{ nm} \pm \frac{(1.57\ \mu\text{m})(1570\text{ nm})}{4(3.4)(300\ \mu\text{m})} = 1570\text{ nm} \pm 1.20\text{ nm}$$

Therefore for m = 1, $\lambda = \lambda_B \pm 3(1.20\text{ nm}) = 1570\text{ nm} \pm 3.60\text{ nm}$

For m = 2, $\lambda = \lambda_B \pm 5(1.20\text{ nm}) = 1570\text{ nm} \pm 6.0\text{ nm}$

4-19. (a) Integrate the carrier-pair-density versus time equation from time 0 to t_d (time for onset of stimulated emission). In this time the injected carrier pair density changes from 0 to n_{th}.

$$t_d = \int_0^{t_d} dt = \int_0^{n_{th}} \frac{1}{\frac{J}{qd} - \frac{n}{\tau}}\, dn = -\tau \left(\frac{J}{qd} - \frac{n}{\tau}\right)\Bigg|_{n=0}^{n=n_{th}} = \tau\ \ln\left(\frac{J}{J - J_{th}}\right)$$

where $J = I_p/A$ and $J_{th} = I_{th}/A$. Therefore $t_d = \tau \ln\left(\frac{I_p}{I_p - I_{th}}\right)$

(b) At time t = 0 we have $n = n_B$, and at $t = t_d$ we have $n = n_{th}$. Therefore,

$$t_d = \int_0^{t_d} dt = \int_{n_B}^{n_{th}} \frac{1}{\frac{J}{qd} - \frac{n}{\tau}}\, dn = \tau \ln\left(\frac{\frac{J}{qd} - \frac{n_B}{\tau}}{\frac{J}{qd} - \frac{n_{th}}{\tau}}\right)$$

In the steady state before a pulse is applied, $n_B = J_B\tau/qd$. When a pulse is applied, the current density becomes $I/A = J = J_B + J_p = (I_B + I_p)/A$

Therefore, $t_d = \tau \ln\left(\frac{I - I_B}{I - I_{th}}\right) = \tau \ln\left(\frac{I_p}{I_p + I_B - I_{th}}\right)$

4-20. A common-emitter transistor configuration:

4-21. Laser transmitter design.

4-22. Since the dc component of $x(t)$ is 0.2, its range is $-2.36 < x(t) < 2.76$. The power has the form $P(t) = P_0[1 + mx(t)]$ where we need to find m and P_0. The average value is

$$\langle P(t) \rangle = P_0[1 + 0.2m] = 1 \text{ mW}$$

The minimum value is

$$P(t) = P_0[1 - 2.36m] \geq 0 \qquad \text{which implies } m \leq \frac{1}{2.36} = 0.42$$

Therefore for the average value we have $\langle P(t) \rangle = P_0[1 + 0.2(0.42)] \leq 1$ mW, which implies

$$P_0 = \frac{1}{1.084} = 0.92 \text{ mW} \qquad \text{so that } P(t) = 0.92[1 + 0.42x(t)] \text{ mW and}$$

$$i(t) = 10\, P(t) = 9.2[1 + 0.42x(t)] \text{ mA}$$

4-23. Substitute $x(t)$ into $y(t)$:

$$\begin{aligned}
y(t) &= a_1b_1 \cos \omega_1t + a_1b_2 \cos \omega_2t \\
&+ a_2(b_1^2\cos^2 \omega_1t + 2b_1b_2 \cos \omega_1t \cos \omega_2t + b_2^2\cos^2 \omega_2t) \\
&+ a_3(b_1^3\cos^3 \omega_1t + 3b_1^2b_2 \cos^2 \omega_1t \cos \omega_2t + 3b_1b_2^2\cos \omega_1t \cos^2 \omega_2t + b_2^3\cos^3 \omega_2t) \\
&+ a_4(b_1^4\cos^4 \omega_1t + 4b_1^3b_2 \cos^3 \omega_1t \cos \omega_2t + 6b_1^2 b_2^2\cos^2 \omega_1t \cos^2 \omega_2t \\
&+ 4b_1b_2^3\cos \omega_1t \cos^3 \omega_2t + b_2^4\cos^4 \omega_2t)
\end{aligned}$$

Use the following trigonometric relationships:

i) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

ii) $\cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$

iii) $\cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + 3)$

iv) $2\cos x \cos y = \cos (x+y) + \cos (x-y)$

v) $\cos^2 x \cos y = \frac{1}{4}\, [\cos (2x+y) + 2\cos y + \cos (2x-y)]$

vi) $\cos^2 x \cos^2 y = \frac{1}{4} \left[1 + \cos 2x + \cos 2y + \frac{1}{2} \cos(2x+2y) + \frac{1}{2} \cos(2x-2y)\right]$

vii) $\cos^3 x \cos y = \frac{1}{8} \left[\cos (3x+y) + \cos (3x-y) + 3\cos (x+y) + 3\cos (x-y)\right]$

then

$$y(t) = \frac{1}{2}\left[a_2 b_1^2 + a_2 b_2^2 + \frac{3}{4} a_4 b_1^4 + 3a_4 b_1^2 b_2^2 + \frac{3}{4} a_4 b_2^4\right] \qquad \text{constant terms}$$

$$+ \frac{3}{4}\left[a_3 b_1^3 + 2a_3 b_1 b_2^2\right] \cos \omega_1 t + \frac{3}{4} a_3\left[b_2^3 + 2b_1^2 b_2\right] \cos \omega_2 t \qquad \text{fundamental terms}$$

$$+ \frac{b_1^2}{2}\left[a_2 + a_4 b_1^2 + 3a_4 b_2^2\right] \cos 2\omega_1 t + \frac{b_2^2}{2}\left[a_2 + a_4 b_2^2 + 3a_4 b_1^2\right] \cos 2\omega_2 t$$

2nd-order harmonic terms

$$+ \frac{1}{4} a_3 b_1^3 \cos 3\omega_1 t + \frac{1}{4} a_3 b_2^3 \cos 3\omega_2 t \qquad \text{3rd-order harmonic terms}$$

$$+ \frac{1}{8} a_4 b_1^4 \cos 4\omega_1 t + \frac{1}{8} a_4 b_2^4 \cos 4\omega_2 t \qquad \text{4th-order harmonic terms}$$

$$+ \left[a_2 b_1 b_2 + \frac{3}{2} a_4 b_1^3 b_2 + \frac{3}{2} a_4 b_1 b_2^3\right]\left[\cos (\omega_1+\omega_2)t + \cos (\omega_1-\omega_2)t\right]$$

2nd-order intermodulation terms

$$+ \frac{3}{4} a_3 b_1^2 b_2 \left[\cos(2\omega_1+\omega_2)t + \cos(2\omega_1-\omega_2)t\right] + \frac{3}{4} a_3 b_1 b_2^2 \left[\cos(2\omega_2+\omega_1)t + \cos(2\omega_2-\omega_1)t\right]$$

3rd-order intermodulation terms

$$+ \frac{1}{2} a_4 b_1^3 b_2 \left[\cos(3\omega_1+\omega_2)t + \cos(3\omega_1-\omega_2)t\right]$$
$$+ \frac{3}{4} a_4 b_1^2 b_2^2 \left[\cos(2\omega_1+2\omega_2)t + \cos(2\omega_1-2\omega_2)t\right]$$
$$+ \frac{1}{2} a_4 b_1 b_2^3 \left[\cos(3\omega_2+\omega_1)t + \cos(3\omega_2-\omega_1)t\right] \qquad \text{4th-order intermodulation}$$

terms

This output is of the form

$$y(t) = A_0 + A_1(\omega_1) \cos \omega_1 t + A_2(\omega_1) \cos 2\omega_1 t + A_3(\omega_1) \cos 3\omega_1 t$$
$$+ A_4(\omega_1) \cos 4\omega_1 t + A_1(\omega_2) \cos \omega_2 t + A_2(\omega_2) \cos 2\omega_2 t$$

$+ A_3(\omega_2) \cos 3\omega_2 t + A_4(\omega_2) \cos 4\omega_2 t + \sum_m \sum_n B_{mn} \cos(m\omega_1+n\omega_2)t$

where $A_n(\omega_j)$ is the coefficient for the $\cos(n\omega_j)t$ term.

4-24. From Eq. (4-58) $P = P_0 e^{-t/\tau_m}$ where P_0 = 1 mW and $\tau_m = 2(5\times10^4$ hrs) = 10^5 hrs.

(a) 1 month = 720 hours. Therefore:

P(1 month) = (1 mW) $\exp(-720/10^5)$ = 0.99 mW

(b) 1 year = 8760 hours. Therefore

P(1 year) = (1 mW) $\exp(-8760/10^5)$ = 0.92 mW

(c) 5 years = 5×8760 hours = 43,800 hours. Therefore

P(5 years) = (1 mW) $\exp(-43800/10^5)$ = 0.65 mW

4-25. From Eq. (4-60) $\tau_s = K e^{E_A/k_BT}$ or $\ln \tau_s = \ln K + E_A/k_BT$

where $k_B = 1.38\times10^{-23}$ J/°K = 8.625×10^{-5} eV/°K

At T = 60°C = 333°K, we have

$\ln 4\times10^4 = \ln K + E_A/[(8.625\times10^{-5} \text{ eV})(333)]$

or $10.60 = \ln K + 34.82 E_A$ (1)

At T = 90°C = 363°K, we have

$\ln 6500 = \ln K + E_A/[(8.625\times10^{-5} \text{ eV})(363)]$

or $8.78 = \ln K + 31.94 E_A$ (2)

Solving (1) and (2) for E_A and K yields

E_A = 0.63 eV and k = 1.11×10^{-5} hrs

Thus at T = 20°C = 293°K

$$\tau_s = 1.11\times10^{-5} \exp\{0.63/[(8.625\times10^{-5})(293)]\} = 7.45\times10^{5} \text{ hrs}$$

Problem Solutions for Chapter 5

5-3. (a) $\cos^L 30° = 0.5$

$\cos 30° = (0.5)^{1/L} = 0.8660$

L = log 0.5/log 0.8660 = 4.82

(b) $\cos^T 15° = 0.5$

$\cos 15° = (0.5)^{1/T} = 0.9659$

T = log 0.5/log 0.9659 = 20.0

5-4. The source radius is less than the fiber radius, so Eq. (5-5) holds:

$$P_{LED\text{-}step} = \pi^2 r_s^2\ B_0(NA)^2 = \pi^2(2\times10^{-3}\text{ cm})^2(100\text{ W/cm}^2)(.22)^2 = 191\ \mu W$$

From Eq. (5-9)

$$P_{LED\text{-}graded} = 2\pi^2(2\times10^{-3}\text{ cm})^2(100\text{ W/cm}^2)(1.48)^2(.01)\left[1 - \frac{1}{2}\left(\frac{2}{5}\right)^2\right] = 159\ \mu W$$

5-5. Using Eq. (5-10), we have that the reflectivity at the source-to-gel interface is

$$R_{s-g} = \left(\frac{3.600 - 1.305}{3.600 + 1.305}\right)^2 = 0.219$$

Similarly, the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.465 - 1.305}{1.465 + 1.305}\right)^2 = 3.34 \times 10^{-3}$$

The total reflectivity then is $R = R_{s-g}R_{g-f} = 7.30 \times 10^{-4}$

The power loss in decibels is (see Example 5-3)

$$L = -10\ \log\ (1 - R) = -10\ \log\ (0.999) = 3.17 \times 10^{-3}\ \text{dB}$$

5-6. Substituting $B(\theta) = B_0 \cos^m \theta$ into Eq. (5-3) for $B(\theta,\phi)$, we have

$$P = \int_0^{r_m} \int_0^{2\pi} \left[2\pi \int_0^{\theta_{0\text{-max}}} \cos^3 \theta \sin \theta \, d\theta \right] d\theta_s \, r \, dr$$

Using

$$\int_0^{\theta_0} \cos^3 \theta \sin \theta \, d\theta = \int_0^{\theta_0} (1 - \sin^2 \theta) \sin \theta \, d(\sin \theta) = \int_0^{\sin \theta_0} (x - x^3) \, dx$$

we have

$$P = 2\pi \int_0^{r_m} \int_0^{2\pi} \left[\frac{\sin^2 \theta_{0\text{-max}}}{2} - \frac{\sin^4 \theta_{0\text{-max}}}{4} \right] d\theta_s \, r \, dr$$

$$= \pi \int_0^{r_m} \int_0^{2\pi} \left[NA^2 - \frac{1}{2} NA^4 \right] d\theta_s \, r \, dr$$

$$= \frac{\pi}{2} [2NA^2 - NA^4] \int_0^{r_m} r \, dr \int_0^{2\pi} d\theta_s$$

5-7. (a) Let a = 25 μm and NA = 0.16. For $r_s \geq a(NA) = 4$ μm, Eq. (5-17) holds. For $r_s \leq 4$ μm, $\eta = 1$.

(b) With a = 50 μm and NA = 0.20, Eq. (5-17) holds for $r_s \geq 10$ μm. Otherwise, $\eta = 1$.

5-8. Using Eq. (5-10), the relfectivity at the gel-to-fiber interface is

$$R_{g-f} = \left(\frac{1.485 - 1.305}{1.485 + 1.305}\right)^2 = 4.16 \times 10^{-3}$$

The power loss is (see Example 5-3)

$$L = -10 \log (1 - R) = -10 \log (0.9958) = 0.018 \text{ dB}$$

When there is no index-matching gel, the joint loss is

$$R_{a-f} = \left(\frac{1.485 - 1.000}{1.485 + 1.000}\right)^2 = 0.038$$

The power loss is $L = -10 \log (1 - R) = -10 \log (0.962) = 0.17$ dB

5-9. Shaded area = (circle segment area) - (area of triangle) = $\frac{1}{2}$ sa - $\frac{1}{2}$ cy

$$s = a\theta = a\,[2 \arccos (y/a)] = 2a \arccos\left(\frac{d}{2a}\right)$$

$$c = 2\left[a^2 - \left(\frac{d}{2}\right)^2\right]^{1/2}$$

Therefore

$$A_{common} = 2(\text{shaded area}) = sa - cy = 2a^2 \arccos\left(\frac{d}{2a}\right) - d\left[a^2 - \left(\frac{d^2}{4}\right)\right]^{1/2}$$

5-10.

Core/cladding diameters (μm)	**Coupling loss (dB) for Given axial misalignments (μm)**			
	1	**3**	**5**	**10**
50/125	0.112	0.385	0.590	1.266
62.5/125	0.089	0.274	0.465	0.985
100/140	0.056	0.169	0.286	0.590

5-11. $\arccos x = \frac{\pi}{2} - \arcsin x$

For small values of x, $\arcsin x = x + \frac{x^3}{2(3)} + \frac{x^5}{2(4)(5)} + \ldots$

Therefore, for $\frac{d}{2a} << 1$, we have $\arccos\frac{d}{2a} \approx \frac{\pi}{2} - \frac{d}{2a}$

Thus Eq. (5-30) becomes $P_T = \frac{2}{\pi} P\left(\frac{\pi}{2} - \frac{d}{2a} - \frac{5d}{6a}\right) = P\left(1 - \frac{8d}{3\pi a}\right)$

d/a	**P_T/P (Eq.5-30)**	**P_T/P (Eq.5-31)**
0.00	1.00	1.00
0.05	0.9576	0.9576
0.10	0.9152	0.9151
0.15	0.8729	0.8727
0.20	0.8309	0.8302
0.25	0.7890	0.7878
0.30	0.7475	0.7454
0.35	0.7063	0.7029
0.40	0.6656	0.6605

5-12. Plots of mechanical misalignment losses.

5-13. From Eq. (5-20) the coupling efficiency η_F is given by the ratio of the number of modes in the receiving fiber to the number of modes in the emitting fiber, where the number of modes M is found from Eq. (5-19). Therefore

$$\eta_F = \frac{M_{aR}}{M_{aE}} = \frac{k^2NA^2(0)\left(\frac{1}{2} - \frac{1}{\alpha+2}\right)a_R^2}{k^2NA^2(0)\left(\frac{1}{2} - \frac{1}{\alpha+2}\right)a_E^2} = \frac{a_R^2}{a_E^2}$$

Therefore from Eq. (5-21) the coupling loss for $a_R \le a_E$ is $L_F = -10 \log\left(\frac{a_R^2}{a_E^2}\right)$

5-14. For fibers with different NAs, where $NA_R < NA_E$

$$L_F = -10 \log \eta_F = -10 \log \frac{M_R}{M_E} = -10 \log \frac{k^2NA_R^2(0)\left(\frac{\alpha}{2\alpha+4}\right)a^2}{k^2NA_E^2(0)\left(\frac{\alpha}{2\alpha+4}\right)a^2}$$

$$= -10 \log \left[\frac{NA_R^2(0)}{NA_E^2(0)}\right]$$

5-15. For fibers with different α values, where $\alpha_R < \alpha_E$

$$L_F = -10 \log \eta_F = -10 \log \frac{k^2 NA^2(0) \left(\frac{\alpha_R}{2\alpha_R + 4}\right) a^2}{k^2 NA^2(0) \left(\frac{\alpha_E}{2\alpha_E + 4}\right) a^2} = -10 \log \left[\frac{\alpha_R(\alpha_E + 2)}{\alpha_E(\alpha_R + 2)}\right]$$

5-16. The splice losses are found from the sum of Eqs. (5-35) through (5-37). First find NA(0) from Eq. (2-80b).

For fiber 1: $NA_1(0) = n_1\sqrt{2\Delta} = 1.46\sqrt{2(0.01)} = 0.206$

For fiber 2: $NA_2(0) = n_1\sqrt{2\Delta} = 1.48\sqrt{2(0.015)} = 0.256$

(a) The only loss is that from index-profile differences. From Eq. (5-37)

$$L_{1\to 2}(\alpha) = -10 \ \log \ \frac{1.80(2.00+2)}{2.00(1.80+2)} = 0.24 \ \text{dB}$$

(b) The losses result from core-size differences and NA differences.

$$L_{2\to 1}(a) = -20 \ \log \ \left(\frac{50}{62.5}\right) = 1.94 \ \text{dB}$$

$$L_{2\to 1}(NA) = -20 \ \log \ \left[\frac{.206}{.256}\right] = 1.89 \ \text{dB}$$

5-17. Plots of connector losses using Eq. (5-43).

5-18. When there are no losses due to extrinsic factors, Eq. (5-43) reduces to

$$L_{SM;ff} = -10 \log \left[\frac{4}{\left(\frac{W_1}{W_2} + \frac{W_2}{W_1} \right)^2} \right]$$

For $W_1 = 0.9W_2$, we then have $L_{SM;ff} = -10 \log \left[\frac{4}{4.0446} \right] = -0.0482$ dB

5-19. Plot of Eq. (5-44).

5-20. Plot of the throughput loss.

Problem Solutions for Chapter 6

6-1. From Eqs. (6-4) and (6-5) with $R_f = 0$, $\eta = 1 - \exp(-\alpha_s w)$

To assist in making the plots, from Fig. P6-1, we have the following representative values of the absorption coefficient:

λ (μm)	**α_s (cm^{-1})**
.60	4.4×10^3
.65	2.9×10^3
.70	2.0×10^3
.75	1.4×10^3
.80	0.97×10^3
.85	630
.90	370
.95	190
1.00	70

6-2. $$I_p = qA\int_0^w G(x)\,dx = qA\,\Phi_0\,\alpha_s\int_0^w e^{-\alpha_s x}\,dx$$

$$= qA\,\Phi_0\left[1 - e^{-\alpha_s w}\right] = qA\,\frac{P_0(1 - R_f)}{h\nu A}\left[1 - e^{-\alpha_s w}\right]$$

6-3. From Eq. (6-6), $$R = \frac{\eta q}{h\nu} = \frac{\eta q\lambda}{hc} = 0.8044\,\eta\lambda \quad (\text{ in } \mu\text{m})$$

Plot R as a function of wavelength.

6-4. (a) Using the fact that $V_a \approx V_B$, rewrite the denominator as

$$1 - \left(\frac{V_a - I_M R_M}{V_B}\right)^n = 1 - \left(\frac{V_B - V_B + V_a - I_M R_M}{V_B}\right)^n$$

$$= 1 - \left(1 - \frac{V_B - V_a + I_M R_M}{V_B}\right)^n$$

Since $\dfrac{V_B - V_a + I_M R_M}{V_B} << 1$, we can expand the term in parenthesis:

$$1 - \left(1 - \frac{V_B - V_a + I_M R_M}{V_B}\right)^n \approx 1 - \left[1 - \frac{n(V_B - V_a + I_M R_M)}{V_B}\right]$$

$$= \frac{n(V_B - V_a + I_M R_M)}{V_B} \approx \frac{n I_M R_M}{V_B}$$

Therefore, $M_0 = \frac{I_M}{I_p} \approx \frac{V_B}{n(V_B - V_a + I_M R_M)} \approx \frac{V_B}{n I_M R_M}$

(b) $M_0 = \frac{I_M}{I_p} = \frac{V_B}{n I_M R_M}$ implies $I_M^2 = \frac{I_p V_B}{n R_M}$, so that $M_0 = \left(\frac{V_B}{n I_p R_M}\right)^{1/2}$

6-5. $\left\langle i_s^2(t) \right\rangle = \frac{1}{T}\int_0^T i_s^2(t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} R_0^2 P^2(t)\, dt$ (where $T = 2\pi/\omega$),

$$= \frac{\omega}{2\pi} R_0^2 P_0^2 \int_0^{2\pi/\omega} (1 + 2m \cos \omega t + m^2 \cos^2 \omega t)\, dt$$

Using

$$\int_0^{2\pi/\omega} \cos \omega t\, dt = \frac{1}{\omega} \sin \omega t \Big|_{t=0}^{t=2\pi/\omega} = 0$$

and $\int_0^{2\pi/\omega} \cos^2 \omega t\, dt = \frac{1}{\omega}\int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)dx = \frac{\pi}{\omega}$

we have $\left\langle i_s^2(t) \right\rangle = R_0^2 P_0^2 \left(1 + \frac{m^2}{2}\right)$

6-6. Same problem as Example 6-6: compare Eqs. (6-13), (6-14), and (6-17).

(a) First from Eq. (6-6), $I_p = \frac{\eta q \lambda}{hc} P_0 = 0.593\ \mu A$

Then $\sigma_Q^2 = 2qI_pB = 2(1.6 \times 10^{-19}\ C)(0.593\ \mu A)(150 \times 10^6\ Hz) = 2.84 \times 10^{-17}\ A^2$

(b) $\sigma_{DB}^2 = 2qI_D B = 2\left(1.6\times10^{-19}\ \text{C}\right)(1.0\ \text{nA})(150\times10^6\ \text{Hz}) = 4.81\times10^{-20}\ \text{A}^2$

(c) $\sigma_T^2 = \dfrac{4k_B T}{R_L} B = \dfrac{4\left(1.38\times10^{-23}\ \text{J/K}\right)(293\ \text{K})}{500\ \Omega}\left(150\times10^6\ \text{Hz}\right) = 4.85\times10^{-15}\ \text{A}^2$

6-7. Using $R_0 = \dfrac{\eta q\lambda}{hc}$ = 0.58 A/W, we have from Eqs. (6-4), (6-11b), (6-15), and (6-17)

$$\left(\frac{S}{N}\right)_Q = \frac{\frac{1}{2}\left(R_0 P_0 m\right)^2 M^2}{2qI_p BM^{1/2}M^2} = \frac{R_0P_0m^2}{4qBM^{1/2}} = 6.565\times10^{12}\ P_0$$

$$\left(\frac{S}{N}\right)_{DB} = \frac{(R_0P_0m)^2}{4qI_DBM^{1/2}} = 3.798\times10^{22}\ P_0^2$$

$$\left(\frac{S}{N}\right)_{DS} = \frac{(R_0P_0m)^2 M^2}{4qI_LB} = 3.798\times10^{26}\ P_0^2$$

$$\left(\frac{S}{N}\right)_T = \frac{\frac{1}{2}(R_0P_0m)^2 M^2}{4k_BTB/R_L} = 7.333\times10^{22}\ P_0^2$$

where P_0 is given in watts. To convert $P_0 = 10^{-n}$ W to dBm, use $10 \log\left(\dfrac{P_0}{10^{-3}}\right) = 10(3-n)$ dBm

6-8. Using Eq. (6-18) we have

$$\frac{S}{N} = \frac{\frac{1}{2}(R_0P_0m)^2 M^2}{2qB(R_0P_0 + I_D)M^{5/2} + 2qI_LB + 4k_BTB/R_L}$$

$$= \frac{1.215\times10^{-16}M^2}{2.176\times10^{-23}M^{5/2} + 1.656\times10^{-19}}$$

The value of M for maximum S/N is found from Eq. (6-19), with x = 0.5: $M_{optimum}$ = 62.1.

6-9. $0 = \frac{d}{dM}\left(\frac{S}{N}\right) = \frac{d}{dM}\left[\frac{\frac{1}{2} I_p^2 M^2}{2q(I_p + I_D)M^{2+x} + 2qI_L + 4k_BT/R_L}\right]$

$$0 = I_p^2\ M - \frac{(2+x)M^{1+x}\ 2q(I_p + I_D)\frac{1}{2} I_p^2 M^2}{2q(I_p + I_D)M^{2+x} + 2qI_L + 4k_BT/R_L}$$

Solving for M: $M_{opt}^{2+x} = \frac{2qI_L + 4k_BT/R_L}{xq(I_p + I_D)}$

6-10. (a) Differentiating p_n, we have

$$\frac{\partial p_n}{\partial x} = \frac{1}{L_p}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} - \alpha_s Be^{-\alpha_s x}$$

$$\frac{\partial^2 p_n}{\partial x^2} = -\frac{1}{L_p^2}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} + \alpha_s^2\ Be^{-\alpha_s x}$$

Substituting p_n and $\frac{\partial^2 p_n}{\partial x^2}$ into the left side of Eq. (6-23):

$$-\frac{D_p}{L_p^2}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} + D_p\,\alpha_s^2\ Be^{-\alpha_s x}$$

$$+\frac{1}{\tau_p}\left(p_{n0} + Be^{-\alpha_s w}\right) e^{(w-x)/L_p} - \frac{B}{\tau_p}\ e^{-\alpha_s x} + \Phi_0\,\alpha_s\,e^{-\alpha_s x}$$

$$= \left[B\left(D_p\,\alpha_s^2 - \frac{1}{\tau_p}\right) + \Phi_0\,\alpha_s\right] e^{-\alpha_s x}$$

where the first and third terms cancelled because $L_p^2 = D_p\tau_p$.

Substituting in for B:

$$\text{Left side} = \left[\frac{\Phi_0}{D_p}\frac{\alpha_s L_p^2}{1 - \alpha_s^2 L_p^2}\left(D_p\,\alpha_s^2 - \frac{1}{\tau_p}\right) + \Phi_0\,\alpha_s\right] e^{-\alpha_s x}$$

$$= \frac{\Phi_0}{D_p}\left[\frac{\alpha_s L_p^2}{1 - \alpha_s^2 L_p^2}\left(\frac{\alpha_s^2 L_p - 1}{\tau_p}\right) + D_p\alpha_s\right] e^{-\alpha_s x}$$

$$= \frac{\Phi_0}{D_p} (-D_p\alpha_s + D_p\alpha_s) \quad e^{-\alpha_s x} = 0$$ Thus Eq. (6-23) is satisfied.

(b) $J_{diff} = qD_p \left(\frac{\partial p_n}{\partial x}\right)_{x=w}$

$$= qD_p\left[\frac{1}{L_p}\left(p_{n0} + Be^{-\alpha_s w}\right) - \alpha_s Be^{-\alpha_s w}\right]$$

$$= qD_p\, B\left(\frac{1}{L_p} - \alpha_s\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0\left(\frac{\alpha_s L_p^2}{1 - \alpha_s^2 L_p^2}\right)\left(\frac{1 - \alpha_s L_p}{L_p}\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0 \frac{\alpha_s L_p}{1 + \alpha_s L_p} e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

c) Adding Eqs. (6-21) and (6-25), we have

$$J_{total} = J_{drift} + J_{diffusion} = q\Phi_0\left[\left(1 - e^{-\alpha_s w}\right) + \left(\frac{\alpha_s L_p}{1 + \alpha_s L_p}\right)e^{-\alpha_s w}\right] + qp_{n0}\frac{D_p}{L_p}$$

$$= q\Phi_0\left(1 - \frac{e^{-\alpha_s w}}{1 + \alpha_s L_p}\right) e^{-\alpha_s w} + qp_{n0}\frac{D_p}{L_p}$$

6-11. (a) To find the amplitude, consider

$$\left(J_{tot} J_{tot}^*\right)_{sc}^{1/2} = q\Phi_0 \,(S\, S^*)^{1/2} \quad \text{where } S = \frac{1 - e^{-j\omega t_d}}{j\omega t_d}$$

We want to find the value of ωt_d at which $(S\, S^*)^{1/2} = \frac{1}{\sqrt{2}}$.

Evaluating $(S\, S^*)^{1/2}$, we have

$$(S\,S^*)^{1/2} = \left[\left(\frac{1 - e^{-j\omega t_d}}{j\omega t_d}\right)\left(\frac{1 - e^{+j\omega t_d}}{-j\omega t_d}\right)\right]^{1/2}$$

$$= \frac{\left[1 - \left(e^{+j\omega t_d} + e^{-j\omega t_d}\right) + 1\right]^{1/2}}{\omega t_d} = \frac{(2 - 2\cos\omega t_d)^{1/2}}{\omega t_d}$$

$$= \frac{[\,(1 - \cos\omega t_d)/2]^{1/2}}{\omega t_d/2} = \frac{\sin\left(\frac{\omega t_d}{2}\right)}{\frac{\omega t_d}{2}} = \mathrm{sinc}\left(\frac{\omega t_d}{2}\right)$$

We want to find values of ωt_d where $(S\,S^*)^{1/2} = \frac{1}{\sqrt{2}}$.

x	**sinc x**		**x**	**sinc x**
0.0	1.000		0.5	0.637
0.1	0.984		0.6	0.505
0.2	0.935		0.7	0.368
0.3	0.858		0.8	0.234
0.4	0.757		0.9	0.109

By extrapolation, we find sinc x = 0.707 at x = 0.442.

Thus $\frac{\omega t_d}{2} = 0.442$ which implies $\omega t_d = 0.884$

(b) From Eq. (6-27) we have $t_d = \frac{w}{v_d} = \frac{1}{\alpha_s v_d}$. Then

$$\omega t_d = 2\pi f_{3\text{-dB}}\, t_d = 2\pi f_{3\text{-dB}} \frac{1}{\alpha_s v_d} = 0.884 \qquad \text{or}$$

$$f_{3\text{-dB}} = 0.884\,\alpha_s v_d / 2\pi$$

6-12. (a) The RC time constant is

$$RC = \frac{R\varepsilon_0 K_s A}{w} = \frac{(10^4\,\Omega)(8.85\times10^{-12}\,\mathrm{F/m})(11.7)(5\times10^{-8}\,\mathrm{m}^2)}{2\times10^{-5}\mathrm{m}} = 2.59\ \mathrm{ns}$$

(b) From Eq. (6-27), the carrier drift time is

$$t_d = \frac{w}{v_d} = \frac{20\times10^{-6}\,\mathrm{m}}{4.4\times10^{4}\,\mathrm{m/s}} = 0.45\ \mathrm{ns}$$

c) $\dfrac{1}{\alpha_s} = 10^{-3}$ cm = 10 μm = $\dfrac{1}{2}$ w

Thus since most carriers are absorbed in the depletion region, the carrier diffusion time is not important here. The detector response time is dominated by the RC time constant.

6-13. (a) With $k_1 \approx k_2$ and k_{eff} defined in Eq. (6-10), we have

(1) $$1 - \frac{k_1(1 - k_1)}{1 - k_2} = 1 - \frac{k_1 - k_1^2}{1 - k_2} \approx 1 - \frac{k_2 - k_1^2}{1 - k_2} = 1 - k_{eff}$$

(2) $$\frac{(1 - k_1)^2}{1 - k_2} = \frac{1 - 2k_1 + k_1^2}{1 - k_2} \approx \frac{1 - 2k_2 + k_1^2}{1 - k_2}$$

$$= \frac{1 - k_2}{1 - k_2} - \frac{k_2 - k_1^2}{1 - k_2} = 1 - k_{eff}$$

Therefore Eq. (6-34) becomes Eq. (6-38):

$$F_e = k_{eff}M_e + 2(1 - k_{eff}) - \frac{1}{M_e}(1 - k_{eff}) = k_{eff}M_e + \left(2 - \frac{1}{M_e}\right)(1 - k_{eff})$$

(b) With $k_1 \approx k_2$ and k'_{eff} defined in Eq. (6-40), we have

(1) $$\frac{k_2(1 - k_1)}{k_1^2(1 - k_2)} \approx \frac{k_2 - k_1^2}{k_1^2(1 - k_2)} = k'_{eff}$$

(2) $$\frac{(1 - k_1)^2 k_2}{k_1^2(1 - k_2)} = \frac{k_2 - 2k_1k_2 + k_2k_1^2}{k_1^2(1 - k_2)}$$

$$\approx \frac{\left(k_2 - k_1^2\right) - \left(k_1^2 - k_2k_1^2\right)}{k_1^2(1 - k_2)} = k'_{eff} - 1$$

Therefore Eq. (6-35) becomes Eq. (6-39): $F_h = k'_{eff} \; M_h - \left(2 - \frac{1}{M_h}\right)(k'_{eff} - 1)$

6-14. (a) If only electrons cause ionization, then $\beta = 0$, so that from Eqs. (6-36) and (6-37), $k_1 = k_2 = 0$ and $k_{eff} = 0$. Then from Eq. (6-38)

$$F_e = 2 - \frac{1}{M_e} \approx 2 \text{ for large } M_e$$

(b) If $\alpha = \beta$, then from Eqs. (6-36) and (6-37), $k_1 = k_2 = 1$ so that

$k_{eff} = 1$. Then, from Eq. (6-38), we have $F_e = M_e$.

Problem Solutions for Chapter 7

7-1. We want to compare $F_1 = kM + (1 - k)\left(2 - \frac{1}{M}\right)$ and $F_2 = M^x$.

For silicon, k = 0.02 and we take x = 0.3:

M	**F_1(M)**	**F_2(M)**	**% difference**
9	2.03	1.93	0.60
25	2.42	2.63	8.7
100	3.95	3.98	0.80

For InGaAs, k = 0.35 and we take x = 0.7:

M	**F_1(M)**	**F_2(M)**	**% difference**
4	2.54	2.64	3.00
9	4.38	4.66	6.4
25	6.86	6.96	1.5
100	10.02	9.52	5.0

For germanium, k = 1.0, and if we take x = 1.0, then $F_1 = F_2$.

7-2. The Fourier transform is

$$h_B(t) = \int_{-\infty}^{\infty} H_B(f)e^{j2\pi ft}\,df = R\int_{-\infty}^{\infty} \frac{e^{j2\pi ft}}{1 + j2\pi fRC}\,df$$

Using the integral solution from Appendix B3:

$$\int_{-\infty}^{\infty} \frac{e^{jpx}}{(\beta + jx)^n}\,dx = \frac{2\pi(p)^{n-1}e^{-\beta p}}{\Gamma(n)} \qquad \text{for } p > 0\text{, we have}$$

$$h_B(t) = \frac{1}{2\pi C}\int_{-\infty}^{\infty} \frac{e^{j2\pi ft}}{\left(\frac{1}{2\pi RC} + jf\right)}\,df = \frac{1}{C}e^{-t/RC}$$

7-3. Part (a):

$$\int_{-\infty}^{\infty} h_p(t)\,dt = \frac{1}{\alpha T_b}\int_{-\alpha T_b/2}^{\alpha T_b/2} dt = \frac{1}{\alpha T_b}\left(\frac{\alpha T_b}{2} + \frac{\alpha T_b}{2}\right) = 1$$

Part (b):

$$\int_{-\infty}^{\infty} h_p(t)\ dt = \frac{1}{\sqrt{2\pi}}\ \frac{1}{\alpha T_b} \int_{-\infty}^{\infty} \exp\left[-\frac{t^2}{2(\alpha T_b)^2}\right] dt$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\alpha T_b} \sqrt{\pi}\ \alpha T_b\ \sqrt{2} = 1 \quad \text{(see Appendix B3 for integral solution)}$$

Part (c):

$$\int_{-\infty}^{\infty} h_p(t)\ dt = \frac{1}{\alpha T_b} \int_{0}^{\infty} \exp\left[-\frac{t}{\alpha T_b}\right] dt = -\ \left[e^{-\infty} - e^{-0}\right] = 1$$

7-4. The Fourier transform is

$$F[p(t)*q(t)] = \int_{-\infty}^{\infty} p(t) * q(t) e^{-j2\pi ft}\ dt = \int_{-\infty}^{\infty} q(x) \int_{-\infty}^{\infty} p(t-x)\ e^{-j2\pi ft}\ dt\ dx$$

$$= \int_{-\infty}^{\infty} q(x)\ e^{-j2\pi fx} \int_{-\infty}^{\infty} p(t-x)\ e^{-j2\pi f(t-x)}\ dt\ dx$$

$$= \int_{-\infty}^{\infty} q(x)\ e^{-j2\pi fx}\ dx \int_{-\infty}^{\infty} p(y)\ e^{-j2\pi fy}\ dy \qquad \text{where } y = t - x$$

$$= F[q(t)]\ F[p(t)] = F[p(t)]\ F[q(t)] = P(f)\ Q(f)$$

7-5. From Eq. (7-18) the probability for unbiased data ($a = b = 0$) is

$$P_e = \frac{1}{2}\left[P_0(v_{th}) + P_1(v_{th})\right].$$

Substituting Eq. (7-20) and (7-22) for P_0 and P_1, respectively, we have

$$P_e = \frac{1}{2}\ \frac{1}{\sqrt{2\pi\sigma^2}} \left[\int_{V/2}^{\infty} e^{-v^2/2\sigma^2}\ dv + \int_{-\infty}^{V/2} e^{-(v-V)^2/2\sigma^2}\ dv\right]$$

In the first integral, let $x = v/\sqrt{2\sigma^2}$ so that $dv = \sqrt{2\sigma^2}\ dx$.

In the second integral, let $q = v-V$, so that $dv = dq$. The second integral then becomes

$$\int_{-\infty}^{V/2-V} e^{-q^2/2\sigma^2}\,dq = \sqrt{2\sigma^2}\int_{-\infty}^{-V/2\sqrt{2\sigma^2}} e^{-x^2}\,dx \qquad \text{where } x = q/\sqrt{2\sigma^2}$$

Then

$$P_e = \frac{1}{2}\,\frac{\sqrt{2\sigma^2}}{\sqrt{2\pi\sigma^2}}\left[\int_{V/2\sqrt{2\sigma^2}}^{\infty} e^{-x^2}\,dx + \int_{-\infty}^{-V/2\sqrt{2\sigma^2}} e^{-x^2}\,dx\right]$$

$$= \frac{1}{2\sqrt{\pi}}\left[\int_{-\infty}^{\infty} e^{-x^2}\,dx - 2\int_{0}^{V/2\sqrt{2\sigma^2}} e^{-x^2}\,dx\right]$$

Using the following relationships from Appendix B,

$$\int_{-\infty}^{\infty} e^{-p^2x^2}\,dx = \frac{\sqrt{\pi}}{p} \quad \text{and} \quad \frac{2}{\sqrt{\pi}}\int_{0}^{t} e^{-x^2}\,dx = \text{erf}(t), \text{ we have}$$

$$P_e = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

7-6. (a) V = 1 volt and σ = 0.2 volts, so that $\frac{V}{2\sigma}$ = 2.5. From Fig. 7-6 for $\frac{V}{2\sigma}$ = 2.5, we find $P_e \approx 7\times10^{-3}$ errors/bit. Thus there are $(2\times10^5$ bits/second)$(7\times10^{-3}$ errors/bit) = 1400 errors/second, so that

$$\frac{1}{1400 \text{ errors/second}} = 7\times10^{-4} \text{ seconds/error}$$

(b) If V is doubled, then $\frac{V}{2\sigma}$ = 5 for which $P_e \approx 3\times10^{-7}$ errors/bit from Fig. 7-6. Thus

$$\frac{1}{(2\times10^5 \text{ bits/second})(3\times10^{-7} \text{ errors/bit})} = 16.7 \text{ seconds/error}$$

7-7. (a) From Eqs. (7-20) and (7-22) we have

$$P_0(v_{th}) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{V/2}^{\infty} e^{-v^2/2\sigma^2}\,dv = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

and

$$P_1(v_{th}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{V/2} e^{-(v-V)^2/2\sigma^2} dv = \frac{1}{2}\left[1 - \text{erf}\left(\frac{V}{2\sigma\sqrt{2}}\right)\right]$$

Then for $V = V_1$ and $\sigma = 0.20V_1$

$$P_0(v_{th}) = \frac{1}{2}\left[1 - \text{erf}\left(\frac{1}{2(.2)\sqrt{2}}\right)\right] \quad = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\sqrt{2}}{0.8}\right)\right]$$

$$= \frac{1}{2}\,[\,1 - \text{erf}(1.768)\,] \quad = \frac{1}{2}\,(1 - 0.987) \quad = 0.0065$$

Likewise, for $V = V_1$ and $\sigma = 0.24V_1$

$$P_1(v_{th}) = \frac{1}{2}\left[1 - \text{erf}\left(\frac{1}{2(.24)\sqrt{2}}\right)\right] \quad = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\sqrt{2}}{0.96}\right)\right]$$

$$= \frac{1}{2}\,[\,1 - \text{erf}(1.473)\,] \quad = \frac{1}{2}\,(1 - 0.963) \quad = 0.0185$$

(b) $P_e = 0.65(0.0185) + 0.35(0.0065) = 0.0143$

(c) $P_e = 0.5(0.0185) + 0.5(0.0065) = 0.0125$

7-8. From Eq. (7-1), the average number of electron-hole pairs generated in a time t is

$$N = \frac{\eta E}{h\nu} \quad = \frac{\eta P t}{hc/\lambda} \quad = \frac{0.65(25\times10^{-10}\,\text{W})(1\times10^{-9}\,\text{s})(1.3\times10^{-6}\,\text{m})}{(6.6256\times10^{-34}\,\text{Js})(3\times10^{8}\,\text{m/s})} = 10.6$$

Then, from Eq. (7-2)

$$P(n) = N^n \frac{e^{-N}}{n!} \quad = (10.6)^5 \frac{e^{-10.6}}{5!} \quad = \frac{133822}{120} \quad e^{-10.6} = 0.05 = 5\%$$

7-9. $v_N = v_{out} - \langle v_{out} \rangle$

$$\langle v_N^2 \rangle = \left\langle \left[v_{out} - \langle v_{out} \rangle \right]^2 \right\rangle$$

$$= \langle v_{out}^2 \rangle - 2\,\langle v_{out} \rangle^2 + \langle v_{out} \rangle^2$$

$$= \langle v_{out}^2 \rangle - \langle v_{out} \rangle^2$$

7-10. (a) Letting $\phi = fT_b$ and using Eq. (7-40), Eq. (7-30) becomes

$$B_{bae} = \left|\frac{H_p(0)}{H_{out}(0)}\right|^2 \int_0^\infty T_b^2 \left|\frac{H'_{out}(\phi)}{H'_p(\phi)}\right|^2 \frac{d\phi}{T_b} = \frac{I_2}{T_b}$$

since $H_p(0) = 1$ and $H_{out}(0) = T_b$. Similarly, Eq. (7-33) becomes

$$B_e = \left|\frac{H_p(0)}{H_{out}(0)}\right|^2 \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}(1 + j2\pi fRC)\right|^2 df$$

$$= \frac{1}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 \left(1 + 4\pi^2 f^2 R^2 C^2\right) df$$

$$= \frac{1}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 df + \frac{(2\pi RC)^2}{T_b^2} \int_0^\infty \left|\frac{H_{out}(f)}{H_p(f)}\right|^2 f^2\, df = \frac{I_2}{T_b} + \frac{(2\pi RC)^2}{T_b^3} I_3$$

(b) From Eqs. (7-29), (7-31), (7-32), and (7-34), Eq. (7-28) becomes

$$\langle v_N^2 \rangle = \langle v_s^2 \rangle + \langle v_R^2 \rangle + \langle v_I^2 \rangle + \langle v_E^2 \rangle$$

$$= 2q \langle i_0 \rangle \langle m^2 \rangle B_{bae} R^2 A^2 + \frac{4k_B T}{R_b} B_{bae} R^2 A^2 + S_I B_{bae} R^2 A^2 + S_E B_e A^2$$

$$= \left(2q \langle i_0 \rangle M^{2+x} + \frac{4k_B T}{R_b} + S_I\right) B_{bae} R^2 A^2 + S_E B_e A^2$$

$$= \frac{R^2 A^2 I_2}{T_b}\left(2q \langle i_0 \rangle M^{2+x} + \frac{4k_B T}{R_b} + S_I + \frac{S_E}{R^2}\right) + \frac{(2\pi RCA)^2}{T_b^3} S_E I_3$$

7-11. First let $x = (v - b_{off})/(\sqrt{2}\ \sigma_{off})$ with $dx = dv/(\sqrt{2}\ \sigma_{off})$ in the first part of Eq. (7-49):

$$P_e = \frac{\sqrt{2}\sigma_{off}}{\sqrt{2\pi}\sigma_{off}} \int_{\frac{v_{th}-b_{off}}{\sqrt{2}\sigma_{off}}}^{\infty} \exp(-x^2)\,dx = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} \exp(-x^2)\,dx$$

Similarly, let $y = (-v + b_{on})/(\sqrt{2}\ \sigma_{on})$ so that

$dy = -dv/(\sqrt{2}\sigma_{on})$ in the second part of Eq. (7-49):

$$P_e = -\frac{1}{\sqrt{\pi}} \int_{\infty}^{\frac{-v_{th}+b_{on}}{\sqrt{2}\sigma_{on}}} \exp\left(-y^2\right) dy = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} \exp\left(-y^2\right) dy$$

7-12. (a) Let $x = \dfrac{V}{2\sqrt{2}\,\sigma} = \dfrac{K}{2\sqrt{2}}$ For K = 10, x = 3.536. Thus

$$P_e = \frac{e^{-x^2}}{2\sqrt{\pi}\,x} = 2.97\times10^{-7} \text{ errors/bit}$$

(b) Given that $P_e = 10^{-5} = \dfrac{e^{-x^2}}{2\sqrt{\pi}\,x}$ then $e^{-x^2} = 2\sqrt{\pi}\ \ 10^{-5}\,x$.

This holds for $x \approx 3$, so that $K = 2\sqrt{2}\ \ x = 8.49$.

7-13. Differentiating Eq. (7-54) with respect to M and setting $db_{on}/dM = 0$, we have

$$\frac{db_{on}}{dM} = 0$$

$$= -\frac{Q(h\nu/\eta)}{M^2}\left\{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2 + W\right]^{1/2} + \left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2(1-\gamma\right] + W\right\}^{1/2}$$

$$+\frac{Q(h\nu/\eta)}{M(h\nu/\eta)}\left\{\frac{\frac{1}{2}(2+x)M^{1+x}\ b_{on}I_2}{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2 + W\right]^{1/2}} + \frac{\frac{1}{2}(2+x)M^{1+x}\ b_{on}I_2(1-\gamma)}{\left[M^{2+x}\left(\frac{\eta}{h\nu}\right) b_{on}I_2(1-\gamma)+W\right]^{1/2}}\right\}^{1/2}$$

Letting $G = M^{2+x}\left(\dfrac{\eta}{h\nu}\right) b_{on}I_2$ for simplicity, yields

$$\left\{(G + W)^{1/2} + [\,G(1-\gamma) + W]^{1/2}\right\} = \frac{G}{2}(2 + x)\left\{\frac{1}{(G + W)^{1/2}} + \frac{(1-\gamma)}{[\,G(1-\gamma) + W]^{1/2}}\right\}$$

Multiply by $(G + W)^{1/2}\,[\,G(1-\gamma) + W]^{1/2}$ and rearrange terms to get

$$(G + W)^{1/2}\left[W - \frac{Gx}{2}(1-\gamma)\right] = [\,G(1-\gamma) + W]^{1/2}\left(\frac{Gx}{2} - W\right)$$

Squaring both sides and collecting terms in powers of G, we obtain the quadratic equation

$$G^2\left[\frac{x^2\gamma}{4}(1-\gamma)\right] + G\left[\frac{x^2\gamma}{4}W(2-\gamma)\right] - \gamma W^2(1+x) = 0$$

Solving this equation for G yields

$$G = \frac{-\frac{x^2}{4}W(2-\gamma) \pm \left\{\frac{x^4}{16}W^2(2-\gamma)^2 + x^2(1-\gamma)W^2(1+x)\right\}^{\frac{1}{2}}}{\frac{x^2}{2}(1-\gamma)}$$

$$= \frac{W(2-\gamma)}{2(1-\gamma)}\left\{1 - \left[1 + 16\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2}\right]^{\frac{1}{2}}\right\}$$

where we have chosen the "+" sign. Equation (7-55) results by letting

$$G = M_{opt}^{2+x} \quad b_{on} I_2\left(\frac{\eta}{h\nu}\right)$$

7-14. Substituting Eq. (7-55) for $M^{2+x}b_{on}$ into the square root expressions in Eq. (7-54) and solving Eq. (7-55) for M, Eq. (7-54) becomes

$$b_{on} = \frac{Q\left(\frac{h\nu}{\eta}\right)b_{on}^{1/(2+x)}}{\left[\frac{h\nu}{\eta}\,\frac{W(2-\gamma)}{2I_2(1-\gamma)}K\right]^{1/(2+x)}}\left\{\left[\frac{W(2-\gamma)}{2(1-\gamma)}K + W\right]^{\frac{1}{2}} + \left[\frac{W}{2}(2-\gamma)K + W\right]^{\frac{1}{2}}\right\}$$

Factoring out terms:

$$b_{on}^{(1+x)/(2+x)} \quad = Q\left(\frac{h\nu}{\eta}\right)^{(1+x)/(2+x)} \quad W^{x/2(2+x)} \quad I_2^{1/(2+x)}$$

$$\times\left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K + 1\right]^{\frac{1}{2}} + \left[\frac{1}{2}(2-\gamma)K + 1\right]^{\frac{1}{2}}\right\} \quad \div \left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{1/(2+x)}$$

or $$b_{on} = Q^{(2+x)/(1+x)}\left(\frac{h\nu}{\eta}\right) \quad W^{x/2(1+x)} \quad I_2^{1/(1+x)} \quad L$$

7-15. In Eq. (7-59) we want to evaluate

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)L}\right]^{\frac{1+x}{2+x}} = \lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{\frac{1+x}{2+x}} \lim_{\gamma\to 1}\left(\frac{1}{L}\right)^{\frac{1+x}{2+x}}$$

Consider first

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right] = \lim_{\gamma\to 1}\ \frac{(2-\gamma)}{2(1-\gamma)}\left\{-1+\left[1+B\frac{(1-\gamma)}{(2-\gamma)^2}\right]^{\frac{1}{2}}\right\}$$

where $B = 16(1+x)/x^2$. Since $\gamma\to 1$, we can expand the square root term in a binomial series, so that

$$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right] = \lim_{\gamma\to 1}\ \frac{(2-\gamma)}{2(1-\gamma)}\left\{-1+\left[1+\frac{1}{2}B\frac{(1-\gamma)}{(2-\gamma)^2}-\mathrm{Order}(1-\gamma)^2\right]\right\}$$

$$= \lim_{\gamma\to 1}\ \frac{B}{4(2-\gamma)} = \frac{B}{4} \quad = 4\,\frac{1+x}{x^2}$$

Thus $$\lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{\frac{1+x}{2+x}} = \left(4\,\frac{1+x}{x^2}\right)^{\frac{1+x}{2+x}}$$

Next consider, using Eq. (7-58)

$$\lim_{\gamma\to 1}\ \left(\frac{1}{L}\right)^{\frac{1+x}{2+x}}$$

$$= \lim_{\gamma\to 1}\left[\frac{(2-\gamma)K}{2(1-\gamma)}\right]^{1/(2+x)} \div \left\{\left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}}+\left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}}\right\}$$

From the above result, the first square root term is

$$\left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}} = \left[4\,\frac{1+x}{x^2}+1\right]^{\frac{1}{2}} = \left(\frac{x^2+4x+4}{x^2}\right)^{\frac{1}{2}} = \frac{x+2}{x}$$

From the expression for K in Eq. (7-55), we have that $\lim_{\gamma\to 1} K = 0$, so that

$$\lim_{\gamma \to 1} \left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}} = 1 \qquad \text{Thus}$$

$$\lim_{\gamma \to 1} \left\{ \left[\frac{(2-\gamma)}{2(1-\gamma)}K+1\right]^{\frac{1}{2}} + \left[\frac{1}{2}(2-\gamma)K+1\right]^{\frac{1}{2}} \right\} = \frac{x+2}{x} + 1 = \frac{2(1+x)}{x}$$

Combining the above results yields

$$\lim_{\gamma \to 1} \left[\frac{(2-\gamma)K}{2(1-\gamma)L}\right]^{\frac{1+x}{2+x}} = \left(4\,\frac{1+x}{x^2}\right)^{\frac{1+x}{2+x}} \left(4\,\frac{1+x}{x^2}\right)^{\frac{1}{2+x}} \frac{2(1+x)}{x} = \frac{2}{x}$$

so that $$\lim_{\gamma \to 1} M_{opt}^{1+x} = \frac{W^{1/2}}{QI_2}\,\frac{2}{x}$$

7-16. Using $H_p^{'}(f) = 1$ from Eq. (7-69) for the impulse input and Eq. (7-66) for the raised cosine output, Eq. (7-41) yields

$$I_2 = \int_0^{\infty} \left|H_{out}^{'}(\phi)\right|^2 d\phi = \frac{1}{2}\int_{-\infty}^{\infty} \left|H_{out}^{'}(\phi)\right|^2 d\phi$$

$$= \frac{1}{2}\int_{-\frac{1-\beta}{2}}^{\frac{1-\beta}{2}} d\phi + \int_{\frac{1-\beta}{2}}^{\frac{1+\beta}{2}} \frac{1}{8}\left[1-\sin\left(\frac{\pi\phi}{\beta}-\frac{\pi}{2\beta}\right)\right]^2 d\phi + \int_{-\frac{1+\beta}{2}}^{-\frac{1-\beta}{2}} \frac{1}{8}\left[1-\sin\left(\frac{\pi\phi}{\beta}-\frac{\pi}{2\beta}\right)\right]^2 d\phi$$

Letting $y = \frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}$ we have

$$I_2 = \frac{1}{2}(1-\beta) + \frac{\beta}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\,1 - 2\sin y + \sin^2 y\,]\; dy$$

$$= \frac{1}{2}(1-\beta) + \frac{\beta}{4\pi}\left[\pi - 0 + \frac{\pi}{2}\right] = \frac{1}{2}\left(1 - \frac{\beta}{4}\right)$$

Use Eq. (7-42) to find I_3:

$$I_3 = \int_0^{\infty} \left|H'_{out}(\phi)\right|^2 \phi^2 \; d\phi = \frac{1}{2} \int_{-\infty}^{\infty} \left|H'_{out}(\phi)\right|^2 \phi^2 \; d\phi$$

$$= \frac{1}{2} \int_{-\frac{1-\beta}{2}}^{\frac{1-\beta}{2}} \phi^2 \, d\phi \quad + \int_{\frac{1-\beta}{2}}^{\frac{1+\beta}{2}} \frac{1}{8}\left[1\text{-}\sin\left(\frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}\right)\right]^2 \phi^2 d\phi \quad + \int_{-\frac{1+\beta}{2}}^{-\frac{1-\beta}{2}} \frac{1}{8}\left[1\text{-}\sin\left(\frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}\right)\right]^2 \phi^2 d\phi$$

Letting $y = \frac{\pi\phi}{\beta} - \frac{\pi}{2\beta}$

$$I_3 = \frac{1}{3}\left(\frac{1\text{-}\beta}{2}\right)^3 \quad + \frac{\beta}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\,1 - 2\sin y + \sin^2 y]\left[\frac{\beta^2 y^2}{\pi^2} + \frac{\beta y}{\pi} + \frac{1}{4}\right] \; dy$$

$$= \frac{1}{3}\left(\frac{1\text{-}\beta}{2}\right)^3 \quad + \frac{\beta}{4\pi}\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\beta^2 y^2}{\pi^2} + \frac{1}{4}\right)(1 + \sin^2 y)\, dy - \frac{2\beta}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y \sin y \, dy\right]$$

where only even terms in "y" are nonzero. Using the relationships

$$\int_0^{\frac{\pi}{2}} \sin^2 y \quad dy = \frac{\pi}{4} \quad ; \qquad \int y \sin y dy \quad = -y \cos y + \sin y$$

and $\displaystyle \int x^2 \sin x^2 \, dx \quad = \frac{x^3}{6} \quad - \left(\frac{x^2}{4} - \frac{1}{8}\right) \quad \sin 2x - \frac{x \cos 2x}{4}$

we have

$$I_3 = \frac{1}{3}\left(\frac{1\text{-}\beta}{2}\right)^3 \quad + \frac{\beta}{2\pi}\left\{\frac{\beta^2}{\pi^2}\left[\frac{1}{3}\left(\frac{\pi}{2}\right)^3 + \frac{1}{6}\left(\frac{\pi}{2}\right)^3 + \frac{\pi}{8}\right] + \frac{1}{4}\left(\frac{\pi}{2} + \frac{\pi}{4}\right) - \frac{2\beta}{\pi}(1)\right\}$$

$$= \frac{\beta^3}{16}\left(\frac{1}{\pi^2} - \frac{1}{6}\right) - \beta^2\left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{\beta}{32} + \frac{1}{24}$$

7-17. Substituting Eq. (7-64) and (7-66) into Eq. (7-41), with $s^2 = 4\pi^2\alpha^2$ and $\beta = 1$, we have

$$I_2 = \int_0 \left| \frac{H'_{out}(\phi)}{H'_p(\phi)} \right|^2 d\phi = \frac{1}{4}\int_0^1 e^{s^2\phi^2}\left[1\text{-}\sin\left(\pi\phi - \frac{\pi}{2}\right)\right]^2 d\phi$$

$$= \int_0^1 e^{s^2\phi^2}\left(\frac{1 + \cos \pi\phi}{2}\right)^2 d\phi \quad = \int_0^1 e^{s^2\phi^2} \cos^4\left(\frac{\pi\phi}{2}\right) d\phi$$

Letting $x = \frac{\pi\phi}{2}$ yields $\qquad I_2 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} e^{16\alpha^2 x^2} \cos^4 x \; dx$

Similarly, using Eqs. (7-64) and (7-66), Eq. (7-42) becomes

$$I_3 = \int_0^1 e^{s^2\phi^2}\cos^4\left(\frac{\pi\phi}{2}\right)\phi^2 d\phi = \left(\frac{2}{\pi}\right)^3 \int_0^{\frac{\pi}{2}} x^2 e^{16\alpha^2 x^2} \cos^4 x \; dx$$

7-18. Plot of I_2 versus α for a gaussian input pulse:

7-19. Plot of I_3 versus α for a gaussian input pulse:

7-20. Consider first $\lim\limits_{\gamma \to 1} K$:

$$\lim_{\gamma \to 1} K = \lim_{\gamma \to 1} \left\{ -1 + \left[1 + 16\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2} \right]^{\frac{1}{2}} \right\} = -1 + 1 = 0$$

Also $\lim_{\gamma \to 1} (1-\gamma) = 0$. Therefore from Eq. (7-58)

$$\lim_{\gamma \to 1} L = \lim_{\gamma \to 1} \left[\frac{2(1-\gamma)}{(2-\gamma)K} \right]^{1/(1+x)} \left\{ \left[\frac{(2-\gamma)}{2(1-\gamma)} K + 1 \right]^{\frac{1}{2}} + 1 \right\}^{\frac{2+x}{1+x}}$$

Expanding the square root term in K yields

$$\lim_{\gamma \to 1} \frac{2-\gamma}{1-\gamma} K = \lim_{\gamma \to 1} \left(\frac{2-\gamma}{1-\gamma}\right) \left\{ -1 + \left[1 + \frac{16}{2}\left(\frac{1+x}{x^2}\right)\frac{1-\gamma}{(2-\gamma)^2} + \text{order}(1-\gamma)^2 \right] \right\}$$

$$= \lim_{\gamma \to 1} \left[\frac{8(1+x)}{x^2} \frac{1}{2-\gamma} \right] = \frac{8(1+x)}{x^2}$$

Therefore

$$\lim_{\gamma \to 1} L = \left[\frac{2x^2}{8(1+x)} \right]^{1/(1+x)} \left\{ \left[\frac{4(1+x)}{x^2} + 1 \right]^{\frac{1}{2}} + 1 \right\}^{\frac{2+x}{1+x}}$$

$$= \left[\frac{2x^2}{8(1+x)} \right]^{1/(1+x)} \left\{ \frac{x+2}{x} + 1 \right\}^{\frac{2+x}{1+x}} = (1+x) \left(\frac{2}{x}\right)^{\frac{x}{1+x}}$$

7-21. (a) First we need to find L and L'. With $x = 0.5$ and $\gamma = 0.9$, Eq. (7-56) yields $K = 0.7824$, so that from Eq. (7-58) we have $L = 2.89$. With $\varepsilon = 0.1$, we have $\gamma' = \gamma(1 - \varepsilon) = 0.9\gamma = 0.81$. Thus $L' = 3.166$ from Eq. (7-80). Substituting these values into Eq. (7-83) yields

$$y(\varepsilon) = (1+\varepsilon)\left(\frac{1}{1-\varepsilon}\right)^{\frac{2+x}{1+x}} \frac{L'}{L} = 1.1\left(\frac{1}{.9}\right)^{5/3} \frac{3.166}{2.89} = 1.437$$

Then $10 \log y(\varepsilon) = 10 \log 1.437 = 1.57$ dB

(b) Similarly, for $x = 1.0$, $\gamma = 0.9$, and $\varepsilon = 0.1$, we have $L = 3.15$ and $L' = 3.35$, so that

$$y(\varepsilon) = 1.1\left(\frac{1}{.9}\right)^{3/2} \frac{3.35}{3.15} = 1.37$$

Then $10 \log y(\varepsilon) = 10 \log 1.37 = 1.37$ dB

7-22. (a) First we need to find L and L'. With $x = 0.5$ and $\gamma = 0.9$, Eq. (7-56) yields $K = 0.7824$, so that from Eq. (7-58) we have $L = 2.89$. With $\varepsilon = 0.1$, we have $\gamma' = \gamma(1 - \varepsilon) = 0.9\gamma = 0.81$. Thus $L' = 3.166$ from Eq. (7-80). Substituting these values into Eq. (7-83) yields

$$y(\varepsilon) = (1 + \varepsilon)\left(\frac{1}{1 - \varepsilon}\right)^{\frac{2+x}{1+x}} \frac{L'}{L} = 1.1\left(\frac{1}{.9}\right)^{5/3} \frac{3.166}{2.89} = 1.437$$

Then $10 \log y(\varepsilon) = 10 \log 1.437 = 1.57$ dB

(b) Similarly, for $x = 1.0$, $\gamma = 0.9$, and $\varepsilon = 0.1$, we have $L = 3.15$ and $L' = 3.35$, so that

$$y(\varepsilon) = 1.1\left(\frac{1}{.9}\right)^{3/2} \frac{3.35}{3.15} = 1.37$$

Then $10 \log y(\varepsilon) = 10 \log 1.37 = 1.37$ dB

7-23. Consider using a Si JFET with $I_{gate} = 0.01$ nA. From Fig. 7-14 we have that $\alpha = 0.3$ for $\gamma = 0.9$. At $\alpha = 0.3$, Fig. 7-13 gives $I_2 = 0.543$ and $I_3 = 0.073$. Thus from Eq. (7-86)

$$W_{JFET} = \frac{1}{B}\left[\frac{2(.01\text{nA})}{1.6\times10^{-19}\text{C}} + \frac{4(1.38\times10^{-23}\text{J/K})(300\text{K})}{(1.6\times10^{-19}\text{C})^2 10^5\,\Omega}\right] 0.543$$

$$+ \frac{1}{B}\left[\frac{4(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})(.7)}{(1.6\times10^{-19}\ \text{C})^2(.005\ \text{S})(10^5\ \Omega)^2}\right] 0.543$$

$$+ \left[\frac{2\pi(10\ \text{pF})}{1.6\times10^{-19}\ \text{C}}\right]^2 \frac{4(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})(.7)}{(.005\ \text{S})}\ 0.073\ \text{B}$$

or

$$W_{JFET} \approx \frac{3.51\times10^{12}}{B} + 0.026B$$

and from Eq. (7-92) $\qquad W_{BP} = \dfrac{3.39\times10^{13}}{B} + 0.0049B$

7-24. We need to find b_{on} from Eq. (7-57). From Fig. 7-9 we have Q = 6 for a 10^{-9} BER. To evaluate Eq. (7-57) we also need the values of W and L. With $\gamma = 0.9$, Fig. 7-14 gives $\alpha = 0.3$, so that Fig. 7-13 gives $I_2 = 0.543$ and $I_3 = 0.073$. Thus from Eq. (7-86)

$$W = \frac{3.51\times10^{12}}{B} + 0.026B = 3.51\times10^5 + 2.6\times10^5 = 6.1\times10^5$$

Using Eq. (7-58) to find L yields L = 2.871 at $\gamma = 0.9$ and x = 0.5. Substituting these values into eq. (7-57) we have

$$b_{on} = (6)^{5/3}\,(1.6\times10^{-19}/0.7)\,(6.1\times10^5)^{.5/3}\,(0.543)^{1/1.5}\,2.871 = 7.97\times10^{-17}\text{ J}$$

Thus $\qquad P_r = b_{on}B = (7.97\times10^{-17}\text{ J})(10^7\text{ b/s}) = 7.97\times10^{-10}\text{ W}$

or

$P_r(\text{dBm}) = 10 \log 7.97\times10^{-10} = -61.0$ dBm

7-25. From Eq. (7-96) the difference in the two amplifier designs is given by

$$\Delta W = \frac{1}{Bq^2}\frac{2k_BT}{R_f}\; I_2 = 3.52\times10^6 \text{ for } I_2 = 0.543 \text{ and } \gamma = 0.9.$$

From Eq. (7-57), the change in sensitivity is found from

$$10 \log\left[\frac{W_{HZ} + \Delta W}{W_{HZ}}\right]^{\frac{x}{2(1+x)}} = 10 \log\left[\frac{1.0 + 3.52}{1.0}\right]^{\frac{.5}{3}} = 10 \log 1.29 = 1.09 \text{ dB}$$

7-26. (a) For simplicity, let

$$D = M^{2+x}\left(\frac{\eta}{h\nu}\right) I_2 \quad \text{and} \quad F = \frac{Q}{M}\left(\frac{\eta}{h\nu}\right)$$

so that Eq. (7-54) becomes, for $\gamma = 1$,

$b = F[(Db + W)^{1/2} + W^{1/2}]$

Squaring both sides and rearranging terms gives

$$\frac{b^2}{F^2} - Db - 2W = 2W^{1/2}(Db + W)^{1/2}$$

Squaring again and factoring out a "b^2" term yields

$$b^2 - (2DF^2)b + (F^4D^2 - 4WF^2) = 0$$

Solving this quadratic equation in b yields

$$b = \frac{1}{2}\left[2DF^2 \pm \sqrt{4F^4D^2 - 4F^4D^2 + 16WF^2}\right] = DF^2 + 2F\sqrt{W}$$

(where we chose the "+" sign) $\quad = \frac{h\nu}{\eta}\left(M^xQ^2I_2 + \frac{2Q}{M}W^{1/2}\right)$

(b) With the given parameter values, we have

$$b_{on} = 2.286\times10^{-19}\left(39.1M^{0.5} + \frac{1.7\times10^4}{M}\right)$$

The receiver sensitivity in dBm is found from

$$P_r = 10\log\left[b_{on}(50\times10^6\,b/s)\right]$$

Representative values of P_r for several values of M are listed in the table below:

M	**P_r(dBm)**		**M**	**P_r(dBm)**
30	- 50.49		80	-51.92
40	-51.14		90	-51.94
50	-51.52		100	-51.93
60	-51.74		110	-51.90
70	-51.86		120	-51.86

7-27. Using Eq. (E-10) and the relationship

$$\int_0^\infty \frac{1}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{\pi a}{2}$$

from App. B, we have from Eq. (7-97)

$$B_{HZ} = \frac{1}{(AR)^2}\int_0^\infty \frac{(AR)^2}{1+(2\pi RC)^2 f^2}\,df = \frac{\pi}{2}\frac{1}{2\pi RC} = \frac{1}{4RC}$$

where H(0) = AR. Similarly, from Eq. (7-98)

$$B_{TZ} = \frac{1}{1}\int_0^\infty \frac{1}{1+\left(\frac{2\pi RC}{A}\right)^2 f^2}\,df = \frac{\pi}{2}\frac{A}{2\pi RC} = \frac{A}{4RC}$$

7-28. To find the optimum value of M for a maximum S/N, differentiate Eq. (7-105) with respect to M and set the result equal to zero:

$$\frac{d(S/N)}{dM} = \frac{(I_p m)^2 M}{2q(I_p+I_D)M^{2+x}B + \frac{4k_BTB}{R_{eq}}F_T}$$

$$- \frac{q(I_p+I_D)\,(2+x)\,M^{1+x}B\,(I_p m)^2 M^2}{\left[2q(I_p+I_D)M^{2+x}B + \frac{4k_BTB}{R_{eq}}F_T\right]^2} = 0$$

Solving for M,

$$M_{opt}^{2+x} = \frac{4k_BTBF_T/R_{eq}}{q(I_p+I_D)x}$$

7-29. (a) For computational simplicity, let $K = 4k_BTBF_T/R_{eq}$; substituting M_{opt} from Problem 7-28 into Eq. (7-105) gives

$$\frac{S}{N} = \frac{\frac{1}{2}(I_p m)^2 M_{opt}^2}{2q(I_p+I_D)M_{opt}^{2+x}B + KB} = \frac{\frac{1}{2}(I_p m)^2\left[\frac{K}{q(I_p+I_D)x}\right]^{\frac{2}{2+x}}}{\frac{2q(I_p+I_D)K}{q(I_p+I_D)x}B + KB}$$

$$= \frac{xm^2 I_p^2}{2B(2+x)\,[q(I_p+I_D)x]^{2/(2+x)}}\left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

(b) If $I_p >> I_D$, then

$$\frac{S}{N} = \frac{xm^2 I_p^2}{2B(2+x)\,(qx)^{2/(2+x)}\,I_p^{2/(2+x)}}\left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

$$= \frac{m^2}{2Bx(2+x)}\left[\frac{(xI_p)^{2(1+x)}}{q^2(4k_B T F_T/R_{eq})^x}\right]^{1/(2+x)}$$

7-30. Substituting $I_p = R_0 P_r$ into the S/N expression in Prob. 7-29a,

$$\frac{S}{N} = \frac{xm^2}{2B(2+x)}\,\frac{(R_0 P_r)^2}{[q(R_0 P_r + I_D)x]^{2/(2+x)}}\left(\frac{R_{eq}}{4k_B T F_T}\right)^{x/(2+x)}$$

$$= \frac{(0.8)^2(0.5\ A/W)^2 P_r^2}{2(5\times 10^6/s)\ 3\ [1.6\times 10^{-19} C(0.5P_r + 10^{-8})\ A]^{2/3}}\left(\frac{10^4\ \Omega/J}{1.656\times 10^{-20}}\right)^{1/3}$$

$$= \frac{1.530\times 10^{12} P_r^2}{\left(0.5P_r + 10^{-8}\right)^{2/3}}$$ where P_r is in watts.

We want to plot 10 log (S/N) versus 10 log $\frac{P_r}{1\text{ mW}}$. Representative values are shown in the following table:

P_r (W)	P_r (dBm)	S/N	10 log (S/N) (dB)
2×10^{-9}	- 57	1.237	0.92
4×10^{-9}	- 54	4.669	6.69
1×10^{-8}	- 50	25.15	14.01
4×10^{-8}	- 44	253.5	24.04
1×10^{-7}	- 40	998.0	29.99
1×10^{-6}	- 30	2.4×10^{4}	43.80
1×10^{-5}	- 20	5.2×10^{5}	57.18
1×10^{-4}	- 10	1.13×10^{7}	70.52

Problem Solutions for Chapter 8

8-1. SYSTEM 1: From Eq. (8-2) the total optical power loss allowed between the light source and the photodetector is

$$P_T = P_S - P_R = 0 \text{ dBm} - (-50 \text{ dBm}) = 50 \text{ dB}$$

$$= 2(l_c) + \alpha_f L + \text{system margin} = 2(1 \text{ dB}) + (3.5 \text{ dB/km})L + 6 \text{ dB}$$

which gives L = 12 km for the maximum transmission distance.

SYSTEM 2: Similarly, from Eq. (8-2)

$$P_T = -13 \text{ dBm} - (-38 \text{ dBm}) = 25 \text{ dB} = 2(1 \text{ dB}) + (1.5 \text{ dB/km})L + 6 \text{ dB}$$

which gives L = 11.3 km for the maximum transmission distance.

8-2. (a) Use Eq. (8-2) to analyze the link power budget. (a) For the *pin* photodiode, with 11 joints

$$P_T = P_S - P_R = 11(l_c) + \alpha_f L + \text{system margin}$$

$$= 0 \text{ dBm} - (-45 \text{ dBm}) = 11(2 \text{ dB}) + (4 \text{ dB/km})L + 6 \text{ dB}$$

which gives L = 4.25 km. The transmission distance cannot be met with these components.

(b) For the APD

$$0 \text{ dBm} - (-56 \text{ dBm}) = 11(2 \text{ dB}) + (4 \text{ dB/km})L + 6 \text{ dB}$$

which gives L = 7.0 km. The transmission distance can be met with these components.

8-3. From $g(t) = (1 - e^{-2\pi Bt})\ u(t)$ we have

$$\left(1 - e^{-2\pi Bt_{10}}\right) = 0.1 \quad \text{and} \quad \left(1 - e^{-2\pi Bt_{90}}\right) = 0.9$$

so that

$$e^{-2\pi B t_{10}} = 0.9 \text{ and } e^{-2\pi B t_{90}} = 0.1$$

Then

$$e^{2\pi B t_r} = e^{2\pi B(t_{90}-t_{10})} = \frac{.9}{.1} = 9$$

It follows that

$$2\pi B t_r = \ln 9 \quad \text{or} \quad t_r = \frac{\ln 9}{2\pi B} = \frac{0.35}{B}$$

8-4. (a) From Eq. (8-11) we have

$$\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{t_{1/2}^2}{2\sigma^2}\right) = \frac{1}{2}\frac{1}{\sqrt{2\pi}\,\sigma} \qquad \text{which yields } t_{1/2} = (2 \ln 2)^{1/2}\,\sigma$$

(b) From Eq. (8-10), the 3-dB frequency is the point at which

$$G(\omega) = \frac{1}{2}\,G(0), \qquad \text{or} \qquad \exp\left[-\frac{(2\pi f_{3dB})^2\,\sigma^2}{2}\right] = \frac{1}{2}$$

Using σ as defined in Eq. (8-13), we have

$$f_{3dB} = \frac{(2 \ln 2)^{1/2}}{2\pi\sigma} = \frac{2 \ln 2}{\pi\, t_{FWHM}} = \frac{0.44}{t_{FWHM}}$$

8-5. From Eq. (8-9), the temporal response of the optical output from the fiber is

$$g(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{t^2}{2\sigma^2}\right)$$

If τ_e is the time required for g(t) to drop to g(0)/e, then

$$g(\tau_e) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{\tau_e^2}{2\sigma^2}\right) = \frac{g(0)}{e} = \frac{1}{\sqrt{2\pi}\,\sigma e}$$

from which we have that $\tau_e = \sqrt{2}\ \sigma$. Since t_e is the full width of the pulse at the 1/e points, then $t_e = 2\tau_e = 2\sqrt{2}\ \sigma$.

From Eq. (8-10), the 3-dB frequency is the point at which

$G(f_{3dB}) = \frac{1}{2}\ G(0)$. Therefore with $\sigma = t_e/(2\sqrt{2}\ )$

$$G(f_{3dB}) = \frac{1}{\sqrt{2\pi}}\ \exp\left[-\frac{1}{2}(2\pi f_{3dB}\,\sigma)^2\right] = \frac{1}{2}\frac{1}{\sqrt{2\pi}}$$

Solving for f_{3dB}:

$$f_{3dB} = \frac{\sqrt{2\ln 2}}{2\pi\sigma} = \frac{\sqrt{2\ln 2}}{2\pi}\frac{2\sqrt{2}}{t_e} = \frac{0.53}{t_e}$$

8-6. (a) We want to evaluate Eq. (8-17) for t_{sys}.

Using D_{mat} = 0.07 ns/(nm-km), we have

$$t_{sys} = \left\{(2)^2 + (0.07)^2(1)^2(7)^2 + \left[\frac{440(7)^{0.7}}{800}\right]^2 + \left[\frac{350}{90}\right]^2\right\}^{1/2}$$

= 4.90 ns

The data pulse width is $T_b = \frac{1}{B} = \frac{1}{90\text{ Mb/s}}$ = 11.1 ns

Thus $0.7T_b$ = 7.8 ns > t_{sys}, so that the rise time meets the NRZ data requirements.

(b) For q = 1.0,

$$t_{sys} = \left\{(2)^2 + (0.49)^2 + \left[\frac{440(7)}{800}\right]^2 + \left[\frac{350}{90}\right]^2\right\}^{1/2} = 5.85\text{ ns}$$

8-7. We want to plot the following 4 curves of L vs $B = \frac{1}{T_b}$:

(a) <u>Attenuation limit</u>

$P_S - P_R = 2(l_c) + \alpha_f L + 6$ dB, where $P_R = 9 \log B - 68.5$

so that $L = (P_S - 9 \log B + 62.5 - 2l_c)/\alpha_f$

(b) <u>Material dispersion</u>

$t_{mat} = D_{mat}\, \sigma_\lambda\, L = 0.7T_b$ or

$$L = \frac{0.7T_b}{D_{mat}\, \sigma_\lambda} = \frac{0.7}{BD_{mat}\, \sigma_\lambda} = \frac{10^4}{B} \quad \text{(with B in Mb/s)}$$

(c) <u>Modal dispersion</u> (one curve for q = 0.5 and one for q = 1)

$$t_{mod} = \frac{0.440L^q}{800} = \frac{0.7}{B} \quad \text{or} \quad L = \left[\left(\frac{800}{0.44}\right)\frac{0.7}{B}\right]^{1/q}$$

With B in Mb/s, L = 1273/B for q =1, and L = $(1273/B)^2$ for q = .5.

8-8. We want to plot the following 3 curves of L vs $B = \dfrac{1}{T_b}$:

(a) <u>Attenuation limit</u>

$P_S - P_R = 2(l_c) + \alpha_f L + 6$ dB, where $P_R = 11.5 \log B - 60.5$, $P_S = -13$ dBm, $\alpha_f =$ 1.5 dB/km, and $l_c = 1$ dB,

so that L = (39.5 - 11.5 log B)/1.5 with B in Mb/s.

(b) <u>Modal dispersion</u> (one curve for q = 0.5 and one for q = 1)

$$t_{mod} = \frac{0.440L^q}{800} = \frac{0.7}{B} \quad \text{or} \quad L = \left[\left(\frac{800}{0.44}\right)\frac{0.7}{B}\right]^{1/q}$$

With B in Mb/s, L = 1273/B for q =1, and L = $(1273/B)^2$ for q = .5.

8-9. The margin can be found from

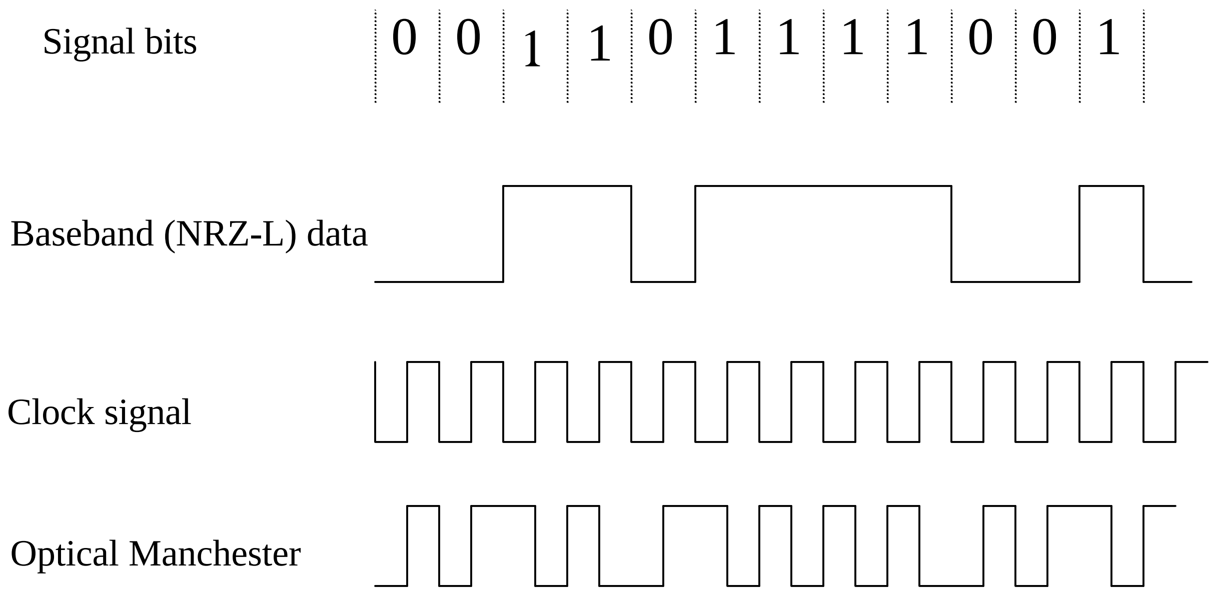
$P_S - P_R = l_c + 49(l_{sp}) + 50\alpha_f$ + noise penalty + system margin

-13 - (-39) = 0.5 + 49(.1) + 50(.35) + 1.5 + system margin

from which we have

system margin = 1.6 dB

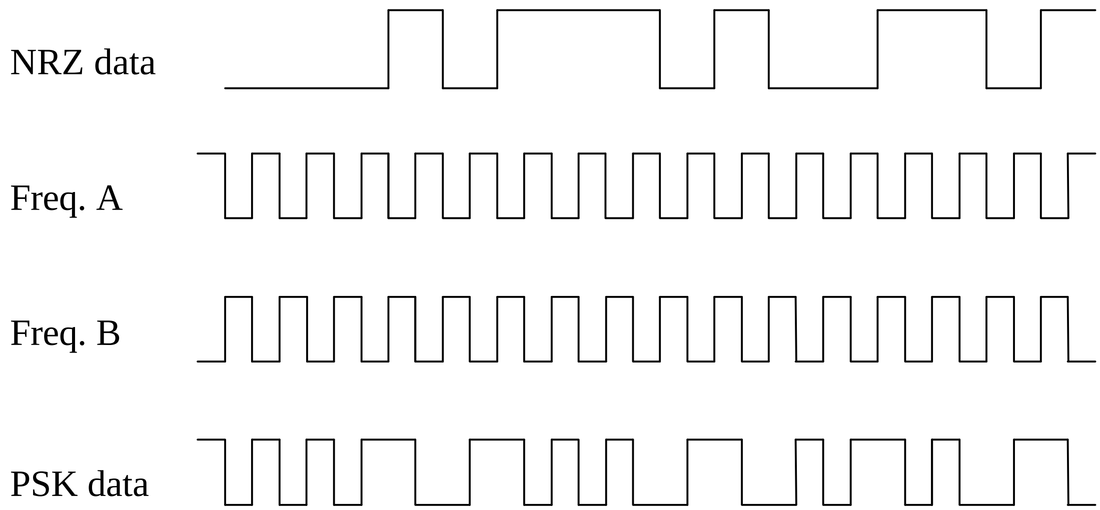
8-10. Signal bits



8-11. The simplest method is to use an exclusive-OR gate (EXOR), which can be implemented using a single integrated circuit. The operation is as follows: when the clock period is compared with the bit cell and the inputs are not identical, the EXOR has a high output. When the two inputs are identical, the EXOR output is low. Thus, for a binary zero, the EXOR produces a high during the last half of the bit cell; for a binary one, the output is high during the first half of the bit cell.

A	B	C
L	L	L
L	H	H
H	L	H
H	H	L

8-12.



8-13.

Original code	010	001	111	111	101	000	000	001	111	110
3B4B encoded	0101	0011	1011	0100	1010	0010	1101	0011	1011	1100

8-14. (a) For x = 0.7 and with Q = 6 at a 10^{-9} BER,

$P_{mpn} = -7.94 \log (1 - 18k^2\pi^4h^4)$ where for simplicity $h = BZD\sigma_\lambda$

(b) With $x = 0.7$ and $k = 0.3$, for an 0.5-dB power penalty at

140 Mb/s = 1.4×10^{-4} b/ps [to give D in ps/(nm.km)]:

$$0.5 = -7.94 \log \{1 - 18(0.3)^2[\pi(1.4\times10^{-4})(100)(3.5)]^4 D^4\}$$

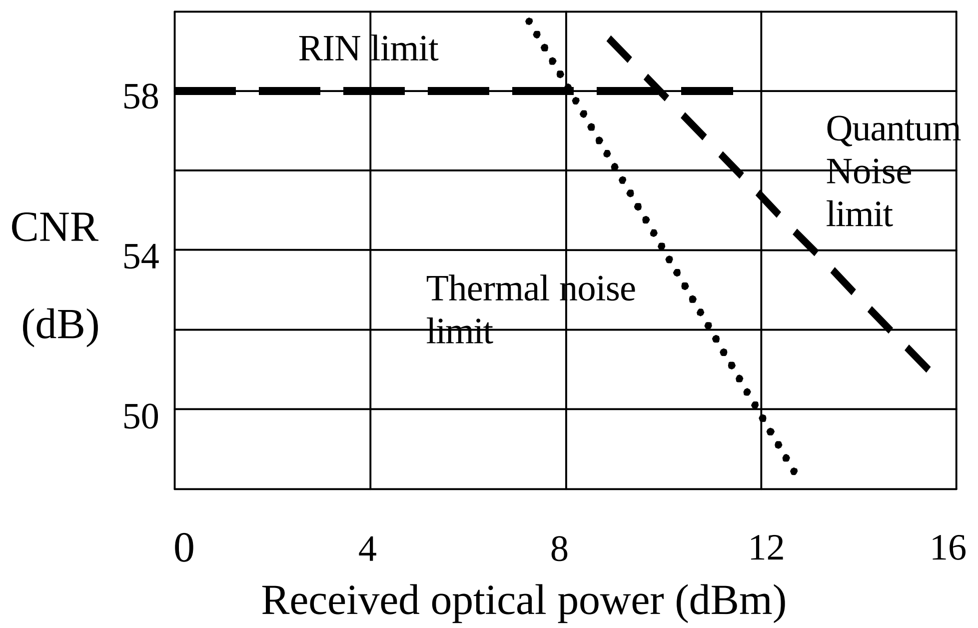
or

$0.5 = -7.94 \log \{1 - 9.097\times10^{-4} D^4\}$ from which D = 2 ps/(nm.km)

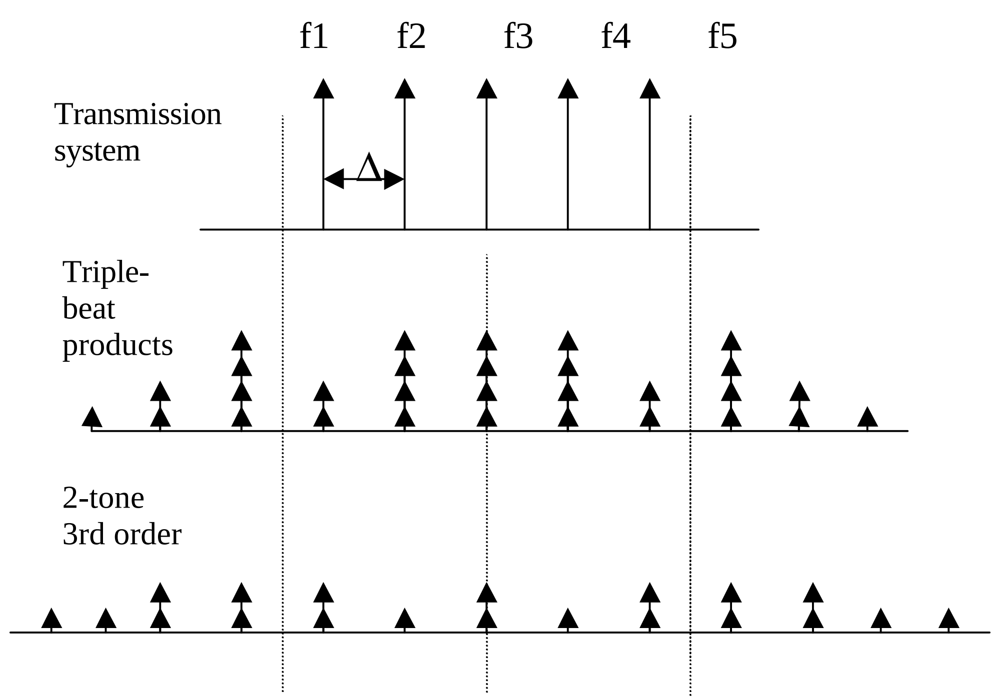
B (Mb/s)	D [ps/(nm.km)]
140	2
280	1
560	0.5

Problem Solutions for Chapter 9

9-1.



9-2.



9-3. The total optical modulation index is

$$m = \left[\sum_i m_i^2 \right]^{1/2} = \left[30(.03)^2 + 30(.04)^2 \right]^{1/2} = 27.4\ \%$$

9-4 The modulation index is $m = \left[\sum_{i=1}^{120} (.023)^2 \right]^{1/2} = 0.25$

The received power is

$P = P_0 - 2(l_c) - \alpha_f L = 3\text{ dBm} - 1\text{ dB} - 12\text{ dB} = -10\text{ dBm} = 100\mu W$

The carrier power is

$$C = \frac{1}{2}(mR_0P)^2 = \frac{1}{2}(15 \times 10^{-6}\,A)^2$$

The source noise is, with RIN = -135 dB/Hz = 3.162×10^{-14} /Hz,

$$\langle i_{source}^2 \rangle = RIN\,(R_0P)^2\,B = 5.69\times10^{-13}A^2$$

The quantum noise is

$$\langle i_Q^2 \rangle = 2q(R_0P + I_D)B = 9.5\times10^{-14}A^2$$

The thermal noise is

$$\langle i_T^2 \rangle = \frac{4k_BT}{R_{eq}}\,F_e = 8.25\times10^{-13}A^2$$

Thus the carrier-to-noise ratio is

$$\frac{C}{N} = \frac{\frac{1}{2}(15\times10^{-6}\,A)^2}{5.69\times10^{-13}A^2 + 9.5\times10^{-14}A^2 + 8.25\times10^{-13}A^2} = 75.6$$

or, in dB, $C/N = 10 \log 75.6 = 18.8$ dB.

9-5. When an APD is used, the carrier power and the quantum noise change.

The carrier power is

$$C = \frac{1}{2}(mR_0MP)^2 = \frac{1}{2}(15\times10^{-5}\,A)^2$$

The quantum noise is

$$\langle i_Q^2 \rangle = 2q(R_0P + I_D)M^2F(M)B = 2q(R_0P + I_D)M^{2.7}B = 4.76\times10^{-10}A^2$$

Thus the carrier-to-noise ratio is

$$\frac{C}{N} = \frac{\frac{1}{2}(15\times10^{-5}\,A)^2}{5.69\times10^{-13}A^2 + 4.76\times10^{-11}A^2 + 8.25\times10^{-13}A^2} = 236.3$$

or, in dB, $\frac{C}{N} = 23.7$ dB

9-6. (a) The modulation index is $m = \left[\sum_{i=1}^{32}(.044)^2\right]^{1/2} = 0.25$

The received power is $P = -10$ dBm $= 100\mu W$

The carrier power is

$$C = \frac{1}{2}(mR_0P)^2 = \frac{1}{2}(15\times10^{-6}\,A)^2$$

The source noise is, with RIN = -135 dB/Hz = 3.162×10^{-14} /Hz,

$$\langle i^2_{source} \rangle = RIN\,(R_0P)^2\,B = 5.69\times10^{-13}A^2$$

The quantum noise is

$$\langle i^2_{Q} \rangle = 2q(R_0P + I_D)B = 9.5\times10^{-14}A^2$$

The thermal noise is

$$\langle i^2_{T} \rangle = \frac{4k_BT}{R_{eq}}\,F_e = 8.25\times10^{-13}A^2$$

Thus the carrier-to-noise ratio is

$$\frac{C}{N} = \frac{\frac{1}{2}(15\times10^{-6}\,A)^2}{5.69\times10^{-13}A^2 + 9.5\times10^{-14}A^2 + 8.25\times10^{-13}A^2} = 75.6$$

or, in dB, C/N = 10 log 75.6 = 18.8 dB.

(b) When m_i = 7% per channel, the modulation index is

$$m = \left[\sum_{i=1}^{32}(.07)^2\right]^{1/2} = 0.396$$

The received power is P = -13 dBm = 50μW

The carrier power is

$$C = \frac{1}{2}(mR_0P)^2 = \frac{1}{2}(1.19\times10^{-5}A)^2 = 7.06\times10^{-11}A^2$$

The source noise is, with RIN = -135 dB/Hz = 3.162×10^{-14} /Hz,

$$\langle i^2_{source} \rangle = RIN\,(R_0P)^2\,B = 1.42\times10^{-13}A^2$$

The quantum noise is

$$\langle i_Q^2 \rangle = 2q(R_0P + I_D)B = 4.8\times10^{-14}A^2$$

The thermal noise is the same as in Part (a).

Thus the carrier-to-noise ratio is

$$\frac{C}{N} = \frac{7.06\times10^{-11}A^2}{1.42\times10^{-13}A^2 + 4.8\times10^{-14}A^2 + 8.25\times10^{-13}A^2} = 69.6$$

or, in dB, C/N = 10 log 69.6 = 18.4 dB.

9-8. Using the expression from Prob. 9-7 with $\Delta\nu\tau = 0.05$, $f\tau = 0.05$, and $\Delta\nu = f = 10$ MHz, yields

$$RIN(f) = \frac{4R_1R_2}{\pi}\ \frac{\Delta\nu}{f^2 + \Delta\nu^2}\left[1 + e^{-4\pi\Delta\nu\tau} - 2\,e^{-2\pi\Delta\nu\tau}\cos(2\pi f\tau)\right]$$

$$= \frac{4R_1R_2}{\pi}\frac{1}{20\text{ MHz}}(.1442)$$

Taking the log and letting the result be less than -140 dB/Hz gives

-80.3 dB/Hz + 10 log $R_1R_2 < -140$ dB/Hz

If $R_1 = R_2$ then 10 log R_1R_2 = 20 log $R_1 < -60$ dB

or 10 log R_1 = 10 log $R_2 < -30$ dB

Problem Solutions for Chapter 10

10-1. In terms of wavelength, at a central wavelength of 1546 nm a 500-GHz channel spacing is

$$\Delta\lambda = \frac{\lambda^2}{c}\ \Delta f = \frac{(1546\ \text{nm})^2}{3\times10^8\ \text{m/s}}\ 500\times10^9\ \text{s}^{-1} = 4\ \text{nm}$$

The number of wavelength channels fitting into the 1536-to-1556 spectral band then is

N = (1556 – 1536 nm)/4 nm = 5

10-2. (a) We first find P_1 by using Eq. (10-6):

$$10\ \log\left(\frac{200\ \mu\text{W}}{P_1}\right) = 2.7\ \text{dB} \quad \text{yields} \quad P_1 = 10^{(\log\ 200 - 0.27)} = 107.4\ \mu\text{W}$$

Similarly, $P_2 = 10^{(\log\ 200 - 0.47)} = 67.8\ \mu\text{W}$

(b) From Eq. (10-5): Excess loss = $10\ \log\left(\frac{200}{107.4 + 67.8}\right) = 0.58\ \text{dB}$

(c) $\frac{P_1}{P_1 + P_2} = \frac{107.4}{175.2} = 61\%$ and $\frac{P_2}{P_1 + P_2} = \frac{67.8}{175.2} = 39\%$

10-3. The following coupling percents are are realized when the pull length is stopped at the designated points:

Coupling percents from input fiber to output 2

Points	**A**	**B**	**C**	**D**	**E**	**F**
1310 nm	25	50	75	90	100	0
1540 nm	50	88	100	90	50	100

10-4. From $A_{out} = s_{11}A_{in} + s_{12}B_{in}$ and $B_{out} = s_{21}A_{in} + s_{22}B_{in} = 0$, we have

$$B_{in} = -\frac{s_{21}}{s_{22}} A_{in} \qquad \text{and} \qquad A_{out} = \left[s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right] A_{in}$$

Then

$$T = \left|\frac{A_{out}}{A_{in}}\right|^2 = \left|s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right|^2 \qquad \text{and} \qquad R = \left|\frac{B_{in}}{A_{in}}\right|^2 = \left|\left(\frac{s_{21}}{s_{22}}\right) \div \left(s_{11} - \frac{s_{12}s_{21}}{s_{22}}\right)\right|^2$$

10-5. From Eq. (10-18)

$$\frac{P_2}{P_0} = \sin^2(0.4z)\exp(-0.06z) = 0.5$$

One can either plot both curves and find the intersection point, or solve the equation numerically to yield z = 2.15 mm.

10-6. Since $\beta_z \propto n$, then for $n_A > n_B$ we have $\kappa_A < \kappa_B$. Thus, since we need to have $\kappa_A L_A = \kappa_B L_B$, we need to have $L_A > L_B$.

10-7. From Eq. (10-6), the insertion loss L_{Ij} for output port j is

$$L_{Ij} = 10\log\left(\frac{P_{i-in}}{P_{j-out}}\right)$$

Let

$$a_j = \frac{P_{i-in}}{P_{j-out}} = 10^{L_{Ij}/10}$$, where the values of L_{Ij} are given in Table P10-7.

Exit port no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Value of a_j	8.57	6.71	5.66	8.00	9.18	7.31	8.02

Then from Eq. (10-25) the excess loss is

$$10\ \log\left(\frac{P_{in}}{\sum P_j}\right) = 10\ \log\left(\frac{P_{in}}{P_{in}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right)}\right) = 10\ \log\left(\frac{1}{0.95}\right) = 0.22\ \text{dB}$$

10-8. (a) The coupling loss is found from the area mismatch between the fiber-core endface areas and the coupling-rod cross-sectional area. If <u>a</u> is the fiber-core radius and R is the coupling-rod radius, then the coupling loss is

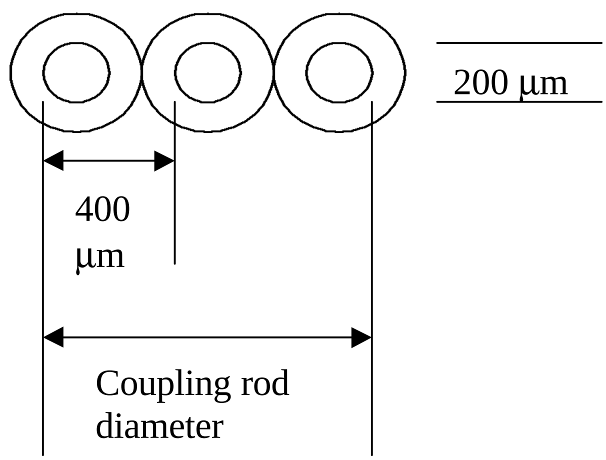
$$L_{coupling} = 10 \log \frac{P_{out}}{P_{in}} = 10 \log \frac{7\pi a^2}{\pi R^2} = 10 \log \frac{7(25)^2}{(150)^2} = -7.11 \text{ dB}$$

(b) Similarly, for the linear-plate coupler

$$L_{coupling} = 10 \log \frac{7\pi a^2}{l\infty w} = 10 \log \frac{7\pi(25)^2}{800(50)} = -4.64 \text{ dB}$$

10-9. (a) The diameter of the circular coupling rod must be 1000 μm, as shown in the figure below. The coupling loss is

$$L_{coupling} = 10 \log \frac{7\pi a^2}{\pi R^2} = 10 \log \frac{7(100)^2}{(500)^2} = -5.53 \text{ dB}$$



(b) The size of the plate coupler must be 200 μm by 2600 μm.

The coupling loss is $10 \log \frac{7\pi(100)^2}{200(2600)} = -3.74$ dB

10-10. The excess loss for a 2-by-2 coupler is given by Eq. (10-5), where $P_1 = P_2$ for a 3-dB coupler. Thus,

$$\text{Excess loss} = 10 \log\left(\frac{P_0}{P_1 + P_2}\right) = 10 \log\left(\frac{P_0}{2P_1}\right) = 0.1 \text{ dB}$$

This yields

$$P_1 = \left(\frac{P_0}{2}\right) \div 10^{0.01} = 0.977 \left(\frac{P_0}{2}\right)$$

Thus the fractional power traversing the 3-dB coupler is $F_T = 0.977$.

Then, from Eq. (10-27),

$$\text{Total loss} = -10 \log\left(\frac{\log F_T}{\log 2} - 1\right)\log N = -10 \log\left(\frac{\log 0.977}{\log 2} - 1\right)\log 2^n \le 30$$

Solving for n yields

$$n \le \frac{-3}{\log 2 \left(\frac{\log 0.977}{\log 2} - 1\right)} = 9.64$$

Thus, $n = 9$ and $N = 2^n = 2^9 = 512$

10-11. For details, see Verbeek et al., Ref. 34, p. 1012

For the general case, from Eq. (10-29) we find

$$M_{11} = \cos(2\kappa d) \cdot \cos(k\Delta L/2) + j \sin(k\Delta L/2)$$

$$M_{12} = M_{21} = j \sin(2\kappa d) \cdot \cos(k\Delta L/2)$$

$$M_{22} = \cos(2\kappa d) \cdot \cos(k\Delta L/2) - j \sin(k\Delta L/2)$$

The output powers are then given by

$$P_{out,1} = \left[\cos^2(2\kappa d) \cdot \cos^2(k\Delta L/2) + \sin^2(k\Delta L/2)\right]P_{in,1}$$

$$+\left[\sin^2(2\kappa d) \cdot \cos^2(k\Delta L/2)\right]P_{in,2}$$

$$P_{out,2} = \left[\sin^2(2\kappa d)\cdot\cos^2(k\Delta L/2)\right]P_{in,1}$$

$$+\left[\cos^2(2\kappa d)\cdot\cos^2(k\Delta L/2)+\sin^2(k\Delta L/2)\right]P_{in,2}$$

10-12. (a) The condition $\Delta\nu$ = 125 GHz is equivalent to having $\Delta\lambda$ = 1 nm. Thus the other three wavelengths are 1549, 1550, and 1551 nm.

(b) From Eqs. (10-42) and (10-43), we have

$$\Delta L_1 = \frac{c}{2n_{eff}(2\Delta\nu)} = 0.4 \text{ mm} \quad \text{and} \qquad \Delta L_3 = \frac{c}{2n_{eff}\Delta\nu} = 0.8 \text{ mm}$$

10-13. An 8-to-1 multiplexer consists of three stages of 2×2 MZI multiplexers. The first stage has four 2×2 MZIs, the second stage has two, and the final stage has one 2×2 MZI. Analogous to Fig. 10-14, the inputs to the first stage are (from top to bottom) ν, $\nu + 4\Delta\nu$, $\nu + 2\Delta\nu$, $\nu + 6\Delta\nu$, $\nu + \Delta\nu$, $\nu + 5\Delta\nu$, $\nu + 3\Delta\nu$, $\nu + 7\Delta\nu$.
In the first stage

$$\Delta L_1 = \frac{c}{2n_{eff}(4\Delta\nu)} = 0.75 \text{ mm}$$

In the second stage

$$\Delta L_2 = \frac{c}{2n_{eff}(2\Delta\nu)} = 1.5 \text{ mm}$$

In the third stage

$$\Delta L_3 = \frac{c}{2n_{eff}(\Delta\nu)} = 3.0 \text{ mm}$$

10-14. (a) For a fixed input angle ϕ, we differentiate both sides of the grating equation to get

$$\cos\theta\, d\theta = \frac{k}{n'\Lambda}\, d\lambda \qquad \text{or} \qquad \frac{d\theta}{d\lambda} = \frac{k}{n'\Lambda\cos\theta}$$

If $\phi \approx \theta$, then the grating equation becomes $2\sin\theta = \dfrac{k\lambda}{n'\Lambda}$.

Solving this for $\frac{k}{n'\Lambda}$ and substituting into the $\frac{d\theta}{d\lambda}$ equation yields

$$\frac{d\theta}{d\lambda} = \frac{2 \sin\theta}{\lambda \cos\theta} = \frac{2 \tan\theta}{\lambda}$$

(b) For S = 0.01,

$$\tan\theta = \left[\frac{S\lambda}{2\Delta\lambda\,(1+m)}\right]^{1/2} = \left[\frac{0.01(1350)}{2(26)(1+3)}\right]^{1/2} = 0.2548$$

or $\theta = 14.3°$

10-15. For 93% reflectivity

$R = \tanh^2(\kappa L) = 0.93$ yields $\kappa L = 2.0$, so that L = 2.7 mm for $\kappa = 0.75\ \text{mm}^{-1}$.

10-16. See Bennion et al., Ref. 42, Fig. 2a.

10-17. Derivation of Eq. (10-49).

10-18. (a) From Eq. (10-45), the grating period is

$$\Lambda = \frac{\lambda_{uv}}{2\ \sin\frac{\theta}{2}} = \frac{244\ \text{nm}}{2\ \sin(13.5°)} = \frac{244}{2(0.2334)}\ \text{nm} = 523\ \text{nm}$$

(b) From Eq. (10-47), $\lambda_{Bragg} = 2\Lambda n_{eff} = 2(523\ \text{nm})\ 1.48 = 1547\ \text{nm}$

(c) Using $\eta = 1 - 1/\sqrt{2} = 0.827$, we have from Eq. (10-51),

$$\kappa = \frac{\pi\ \delta n\ \eta}{\lambda_{Bragg}} = \frac{\pi\left(2.5\times10^{-4}\right)(0.827)}{1.547\times10^{-4}\ \text{cm}} = 4.2\ \text{cm}^{-1}$$

(d) From Eq. (10-49), $\Delta\lambda = \frac{(1.547\ \mu\text{m})^2}{\pi\ (1.48)\ 500\ \mu\text{m}}\ \left[(2.1)^2 + \pi^2\right]^{1/2} = 3.9\ \text{nm}$

(e) From Eq. (10-48), $R_{max} = \tanh^2(\kappa L) = \tanh^2(2.1) = (0.97)^2 = 94\%$

10-19. Derivation of Eq. (10-55).

10-20. (a) From Eq. (10-54),

$$\Delta L = m\,\frac{\lambda_0}{n_c} = 118\,\frac{1.554\ \mu m}{1.451} = 126.4\ \mu m$$

(b) From Eq. (10-57),

$$\Delta\nu = \frac{x}{L_f}\,\frac{n_s cd}{m\lambda^2}\,\frac{n_c}{n_g}$$

$$= \frac{25\ \mu m}{9.36\times10^3\ \mu m}\,\frac{1.453\ (3\times10^8\ m/s)(25\times10^{-6}\ m)}{118\ (1.554\times10^{-6}\ m)^2}\,\frac{1.451}{1.475} = 100.5\ GHz$$

$$\Delta\lambda = \frac{\lambda^2}{c}\,\Delta\nu = \frac{(1.554\times10^{-6}\ m)^2}{3\times10^8\ m/s}\ 100.5\ GHz = 0.81\ nm$$

(c) From Eq. (10-60),

$$\Delta\nu_{FSR} = \frac{c}{n_g\Delta L} = \frac{3\times10^8\ m/s}{1.475(126.4\ \mu m)} = 1609\ GHz$$

Then

$$|\Delta\lambda| = \frac{\lambda^2}{c}\,\Delta\nu_{FSR} = \frac{(1.554\times10^{-6}\ m)^2}{3\times10^8\ m/s}\ 1609\ GHz = 12.95\ nm$$

(d) Using the conditions

$$\sin\theta_i \approx \theta_i = \frac{2(25\ \mu m)}{9380\ \mu m} = 5.33\times10^{-3}\ \text{radians}$$

and

$$\sin\theta_o \approx \theta_o = 21.3\times10^{-3}\ \text{radians}$$

then from Eq. (10-59),

$$\Delta\nu_{FSR} \approx \frac{c}{n_g[\Delta L + d(\theta_i + \theta_o)]}$$

$$= \frac{3\times10^8 \ \text{m/s}}{1.475\left[(126.4\times10^{-6} \ \text{m}) + (25\times10^{-6} \ \text{m})(5.33+21.3)\times10^{-3}\right]} = 1601 \text{ GHz}$$

10-21. The source spectral width is

$$\Delta\lambda_{signal} = \frac{\lambda^2\nu}{c} = \frac{(1550 \ \text{nm})^2(1.25\times10^9 \ \text{s}^{-1})}{(3\times10^8 \ \text{m/s})\left(10^9 \ \text{nm/m}\right)} = 1\times10^{-2} \ \text{nm}$$

Then from Eq. (10-61)

$$\Delta\lambda_{tune} = \lambda\frac{\Delta n_{eff}}{n_{eff}} = (1550 \ \text{nm})(0.5\%) = 7.75 \ \text{nm}$$

Thus, from Eq. (10-63)

$$N = \frac{\Delta\lambda_{tune}}{10 \ \lambda_{signal}} = \frac{7.75 \ \text{nm}}{10(0.01 \ \text{nm})} = 77$$

10-22. (a) From Eq. (10-64), the grating period is

$$\Lambda = \frac{\lambda_{Bragg}}{2n_{eff}} = \frac{1550 \ \text{nm}}{2(3.2)} = 242.2 \ \text{nm}$$

(b) Again, from the grating equation,

$$\Delta\Lambda = \frac{\Delta\lambda}{2n_{eff}} = \frac{2.0 \ \text{nm}}{2(3.2)} = 0.3 \ \text{nm}$$

10-23. (a) From Eq. (10-43)

$$\Delta L = \frac{c}{2n_{eff}\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}\frac{1}{2n_{eff}} = 4.0 \ \text{mm}$$

(b) $\Delta L_{eff} = \Delta n_{eff} L$ implies that $\Delta n_{eff} = \dfrac{4 \ \text{mm}}{100 \ \text{mm}} = 0.04 = 4\%$

10-24. For example, see C. R. Pollock, *Fundamentals of Optoelectronics*, Irwin, 1995, Fig. 15.11, p. 439.

10-25. (a) The driving frequencies are found from

$$f_a = \nu_o \frac{v_a \Delta n}{c} = \frac{v_a \Delta n}{\lambda}$$

Thus we have

Wavelength (nm)	1300	1546	1550	1554
Acoustic frequency (MHz)	56.69	47.67	47.55	47.43

(b) The sensitivity is (4 nm)/(0.12 MHz) = 0.033 nm/kHz

Problem Solutions for Chapter 11

11-1. (a) From Eq. (11-2), the pumping rate is

$$R_p = \frac{I}{qwdL} = \frac{100 \text{ mA}}{(1.6\times10^{-19} \text{ C})(5 \ \mu\text{m})(0.5 \ \mu\text{m})(200 \ \mu\text{m})}$$

$$= 1.25\times10^{27} \text{ (electrons/cm}^3\text{)/s}$$

(b) From Eq. (11-8), the maximum zero-signal gain is

$$g_0 = 0.3(1\times10^{-20}\text{m}^2)(1 \text{ ns})\left[1.25\times10^{33} \text{ (electrons/ m}^3\text{)/s} - \frac{1.0\times10^{24}\text{ /m}^3}{1 \text{ ns}}\right]$$

$$= 750 \text{ m}^{-1} = 7.5 \text{ cm}^{-1}$$

(c) From Eq. (11-7), the saturation photon density is

$$N_{ph;sat} = \frac{1}{0.3 \ (1\times10^{-20}\text{m}^2)\left(2\times10^8 \text{ m/s}\right)(1 \text{ ns})} = 1.67\times10^{15} \text{ photons/cm}^3$$

(d) From Eq. (11-4), the photon density is

$$N_{ph} = \frac{P_{in} \ \lambda}{v_g \ hc \ (wd)} = 1.32\times10^{10} \text{ photons/cm}^3$$

11-2. Carrying out the integrals in Eq. (11-14) yields

$$g_0L = \ln\frac{P(L)}{P(0)} + \frac{P(L) - P(0)}{P_{amp,sat}}$$

Then with P(0) = P_{in}, P(L) = P_{out}, G = P_{out}/P_{in}, and $G_0 = \exp(g_0L)$ from Eq. (11-10), we have

$$\ln G_0 = g_0L = \ln G + \frac{GP_{in}}{P_{amp,sat}} - \frac{P_{in}}{P_{amp,sat}} = \ln G + (1-G)\frac{P_{in}}{P_{amp,sat}}$$

Rearranging terms in the leftmost and rightmost parts then yields Eq. (11-15).

11-3. Plots of amplifier gains.

11-4. Let G = $G_0/2$ and $P_{in} = P_{out}/G = 2P_{out,sat}/G_0$. Then Eq. (11-15) yields

$$\frac{G_0}{2} = 1 + \frac{G_0 P_{amp.sat}}{2P_{out.sat}} \ln\ 2$$

Solving for $P_{out,sat}$ and with $G_0 >> 1$, we have

$$P_{out\,sat} = \frac{G_0 \ln\ 2}{(G_0 - 2)}\ P_{amp.sat} \approx (\ln\ 2)\ P_{amp.sat} = 0.693\ P_{amp.sat}$$

11-5. From Eq. (11-10), at half the amplifier gain we have

$$G = \frac{1}{2} G_0 = \frac{1}{2} \exp(g_0 L) = \exp(gL)$$

Taking the logarithm and substituting into the equation given in the problem,

$$g = g_0 - \frac{1}{L} \ln\ 2 = \frac{g_0}{1 + 4(\nu_{3dB} - \nu_0)^2 / (\Delta\nu)^2}$$

From this we can find that

$$\frac{2(\nu_{3dB} - \nu_0)}{\Delta\nu} = \left[\frac{g_0}{g_0 - \frac{1}{L}\ln\ 2} - 1\right]^{1/2} = \left[\frac{1}{g_0 L/\ln\ 2 - 1}\right]^{1/2} = \frac{1}{\left[\log_2\left(\frac{G_0}{2}\right)\right]^{1/2}}$$

11-6. Since

$$\ln\ G = g(\lambda)L = g_0 \exp\ \left[-(\lambda - \lambda_0)^2 / 2(\Delta\lambda)^2\right] = \ln\ G_0\ \exp\ \left[-(\lambda - \lambda_0)^2 / 2(\Delta\lambda)^2\right]$$

we have

$$\ln\left[\frac{\ln\ G_0}{\ln\ G}\right] = \frac{(\lambda - \lambda_0)^2}{2(\Delta\lambda)^2}$$

The FWHM is given by $2(\lambda - \lambda_0)$, so that from the above equation, with the 3-dB gain G = 27 dB being 3 dB below the peak gain, we have

$$\text{FWHM} = 2|\lambda - \lambda_0| = 2\left[2\ \ln\left(\frac{\ln\ G_0}{\ln\ G}\right)\right]^{1/2}\Delta\lambda$$

$$= 2\left[2\ \ln\left(\frac{\ln\ 30}{\ln\ 27}\right)\right]^{1/2}\Delta\lambda = 0.50\ \Delta\lambda$$

which is the expected result for a gaussian gain profile.

11-7. From Eq. (11-17), the maximum PCE is given by

$$\text{PCE} \le \frac{\lambda_p}{\lambda_s} = \frac{980}{1545} = 63.4\% \text{ for 980-nm pumping, and by}$$

$$\text{PCE} \le \frac{\lambda_p}{\lambda_s} = \frac{1475}{1545} = 95.5\% \text{ for 1475-nm pumping}$$

11-8. (a) 27 dBm = 501 mW and 2 dBm = 1.6 mW.

Thus the gain is

$$G = 10\ \log\left(\frac{501}{1.6}\right) = 10\ \log\ 313 = 25\ \text{dB}$$

(b) From Eq. (11-19),

$313 \le 1 + \frac{980}{1542}\frac{P_{p,in}}{P_{s,\,in}}$. With a 1.6-mW input signal, the pump power needed is

$$P_{p,in} \ge \frac{312(1542)}{980}\ (1.6\ \text{mW}) = 785\ \text{mW}$$

11-9. (a) Noise terms:

From Eq. (6-17), the thermal noise term is

$$\sigma_T^2 = \frac{4k_BT}{R_L}B = \frac{4\ (1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{1000\ \Omega}\ 1\ \text{GHz} = 1.62\times10^{-14}\ \text{A}^2$$

From Eq. (11-26), we have

$$\sigma^2_{shot-s} = 2q\mathcal{R}\, GP_{s,\,in}B$$

$$= 2\left(1.6\times10^{-19}\ \text{C}\right)(0.73\ \text{A/W})(100)\left(1\ \mu\text{W}\right)1\ \text{GHz}$$

$$= 2.34\times10^{-14}\ \text{A}^2$$

From Eqs. (11-26) and (11-24), we have

$$\sigma^2_{shot-ASE} = 2q\mathcal{R}\, S_{ASE}\Delta\nu_{opt}B = 2q\mathcal{R}\ \frac{hc}{\lambda} n_{sp}G\Delta\nu_{opt}B$$

$$= 2\left(1.6\times10^{-19}\ \text{C}\right)(.73\ \text{A/W})\left(6.626\times10^{-34}\ \text{J/K}\right)$$

$$\times\left(3\times10^{8}\,\text{m/s}\right)2(100)(3.77\ \text{THz})(1\ \text{GHz})/1550\text{nm}$$

$$= 2.26\times10^{-14}\ \text{A}^2$$

From Eq. (11-27) and (11-24), we have

$$\sigma^2_{s-ASE} = 4\left[(0.73\ \text{A/W})(100)(1\ \mu\text{W})\right]$$

$$\times\left[(.73\ \text{A/W})\frac{\left(6.626\times10^{-34}\ \text{J/K}\right)\left(3\times10^{8}\,\text{m/s}\right)}{1550\,\text{nm}}2(100)(1\ \text{GHz})\right]$$

$$= 5.47\times10^{-12}\ \text{A}^2$$

From Eq. (11-28), we have

$$\sigma^2_{ASE-ASE} = (.73\text{A/W})^2\left[\frac{\left(6.626\times10^{-34}\ \text{J/K}\right)\left(3\times10^{8}\,\text{m/s}\right)}{1550\text{nm}}2(100)\right]^2$$

$$\times\left[2(1\ \text{THz}) - 1\ \text{GHz}\right](1\ \text{GHz})$$

$$= 7.01\times10^{-13}\ \text{A}^2$$

11-10. Plot of penalty factor from Eq. (11-36).

11-11. (a) Using the transparency condition $G\exp(-\alpha L) = 1$ for a fiber/amplifier segment, we have

$$\langle P \rangle_{path} = \frac{1}{L}\int_0^L P(z)\,dz = \frac{P_{in}}{L}\int_0^L e^{-\alpha z}\,dz$$

$$= \frac{P_{in}}{\alpha L}\left[1 - e^{-\alpha L}\right] = \frac{P_{in}}{\alpha L}\left[1 - \frac{1}{G}\right] = \frac{P_{in}}{G}\left(\frac{G-1}{\ln\, G}\right)$$

since $\ln G = \alpha L$ from the transparency condition.

(b) From Eq. (11-35) and using Eq. (11-24),

$$\langle P_{ASE} \rangle_{path} = \frac{NP_{ASE}}{L}\int_0^L e^{-\alpha z}\,dz = \frac{NP_{ASE}}{\alpha L}\left(1 - e^{-\alpha L}\right)$$

$$= \frac{\alpha\,(NL)}{(\alpha L)^2}\,P_{ASE}\left(1 - \frac{1}{G}\right) = \frac{\alpha L_{tot}}{(\ln\,G)^2}\,h\nu n_{sp}(G-1)\Delta\nu_{opt}\left(1 - \frac{1}{G}\right)$$

$$= \alpha L_{tot}\,h\nu n_{sp}\Delta\nu_{opt}\,\frac{1}{G}\left(\frac{G-1}{\ln\,G}\right)^2$$

11-12. Since the slope of the gain-versus -input power curve is –0.5, then for a 6-dB drop in the input signal, the gain increases by +3 dB.

1. Thus at the first amplifier, a –10.1-dBm signal now arrives and experiences a +10.1-dB gain. This gives a 0-dBm output (versus a normal +3-dBm output).
2. At the second amplifier, the input is now –7.1 dBm (down 3 dB from the usual –4.1 dBm level). Hence the gain is now 8.6 dB (up 1.5 dB), yielding an output of

   –7.1 dBm + (7.1 + 1.5) dB = 1.5 dBm

3. At the third amplifier, the input is now –5.6 dBm (down 1.5 dB from the usual –4.1 dBm level). Hence the gain is up 0.75 dB, yielding an output of
   –5.6 dBm + (7.1 + 0.75) dB = 2.25 dBm
4. At the fourth amplifier, the input is now –4.85 dBm (down 0.75 dB from the usual –4.1 dBm level). Hence the gain is up 0.375 dB, yielding an output of
   –4.85 dBm + (7.1 + 0.375) dB = 2.63 dBm
   which is within 0.37 dB of the normal +3 dBm level.

11-13. First let $2\pi\nu_i t + \phi_i = \theta_i$ for simplicity. Then write the cosine term as

$\cos\ \theta_i = \dfrac{e^{j\theta_i} + e^{-j\theta_i}}{2}$, so that

$$P = E_i(t)E_i^*(t) = \left[\sum_{i=1}^{N}\sqrt{2P_i}\,\frac{e^{j\theta_i} + e^{-j\theta_i}}{2}\right]\times\left[\sum_{k=1}^{N}\sqrt{2P_k}\,\frac{e^{j\theta_k} + e^{-j\theta_k}}{2}\right]$$

$$= \frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j\theta_i}e^{-j\theta_k} + e^{j\theta_k}e^{-j\theta_i} + e^{j\theta_i}e^{j\theta_k} + e^{-j\theta_i}e^{-j\theta_k}\right]$$

$$= \frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j(\theta_i-\theta_k)} + e^{-j(\theta_i-\theta_k)} + e^{j(\theta_i+\theta_k)} + e^{-j(\theta_i+\theta_k)}\right]$$

$$= \sum_{i=1}^{N}P_i + \frac{1}{2}\ \sum_{i=1}^{N}\sum_{k\neq i}^{N}\sqrt{2P_i}\sqrt{2P_k}\ \left[e^{j(\theta_i-\theta_k)} + e^{-j(\theta_i-\theta_k)}\right]$$

where the last two terms in the second-last line drop out because they are beyond the response frequency of the detector. Thus,

$$P = \sum_{i=1}^{N}P_i + \sum_{i=1}^{N}\sum_{k\neq i}^{N}2\sqrt{P_iP_k}\ \left[\cos(\theta_i - \theta_k)\right]$$

11-14. (a) For N input signals, the output signal level is given by

$$P_{s,out} = G\sum_{i=1}^{N}P_{s,in}(i) \le 1\ \text{mW}\,.$$

The inputs are 1 μW (-30 dBm) each and the gain is 26 dB (a factor of 400).

Thus for one input signal, the output is (400)(1 μW) = 400 μW or –4 dBm.

For two input signals, the total output is 800 μW or –1 dBm. Thus the level of each individual output signal is 400 μW or –4 dBm.

For four input signals, the total input level is 4 μW or –24 dBm. The output then reaches its limit of 0 dBm, since the maximum gain is 26 dB. Thus the level of each individual output signal is 250 μW or –6 dBm.

Similarly, for eight input channels the maximum output level is o dBm, so the level of each individual output signal is 1/8(1 mW) = 125 μW or –9 dBm.
(b) When the pump power is doubled, the outputs for one and two inputs remains at the same level. However, for four inputs, the individual output level is 500 μW or –3 dBm, and for 8 inputs, the individual output level is 250 μW or –6 dBm.

11-15. Substituting the various expressions for the variances from Eqs. (11-26) through (11-30) into the expression given for Q in the problem statement, we find

$$Q = \frac{AP}{\left(HP + D^2\right)^{1/2} + D}$$

where we have defined the following terms for simplicity

$$A = 2\mathcal{R}\,G$$

$$H = 4q\mathcal{R}\,GB + 8\mathcal{R}^{\,2}GS_{ASE}B \quad \text{and} \qquad D^2 = \sigma^2_{off}$$

Rearrange terms in the equation for Q to get

$$Q^2\left(HP + D^2\right)^{1/2} = AP - QD$$

Squaring both sides and solving for P yields $P = \frac{2QD}{A} + \frac{Q^2H}{A^2}$

Substituting the expressions for A, H, and D into this equation, and recalling the expression for the responsivity from Eq. (6-6), then produces the result stated in the problem, where

$$F = \frac{1 + 2\eta n_{sp}(G - 1)}{\eta G}$$

Problem Solutions for Chapter 12

12-1. We need to evaluate P_{in} using Eq. (12-11). Here F_c = 0.20,

C_T = 0.05, F_i = 0.10, P_0 = 0.5 mW, and $A_0 = e^{-2.3(3)/10} = 0.933$

Values of P_{in} as a function of N are given in the table below. P_{in} in

dBm is found from the relationship $P_{in}(\text{dBm}) = 10 \log \dfrac{P_{in}(\text{mW})}{1 \text{ mW}}$

N	P_{in}(nW)	P_{in}(dBm)		N	P_{in}(nW)	P_{in}(dBm)
2	387	-34.1		8	5.0	-53.0
3	188	-37.3		9	2.4	-56.2
4	91	-40.4		10	1.2	-59.2
5	44.2	-43.5		11	0.6	-62.2
6	21.4	-46.7		12	0.3	-65.5
7	10.4	-49.8				

(b) Using the values in the above table, the operating margin for 8 stations is

-53 dBm - (-58 dBm) = 5 dB

(c) To have a 6-dB power margin, we can transmit over at most seven stations.

The dynamic range with N = 7 is found from Eq. (12-13):

$$DR = -10(N-2)\ \log\ \left[.933(.8)^2(.95)^2(.9)\right] = -50\ \log\ (0.485) = 15.7\ \text{dBm}$$

12-2. (a) Including a power margin, we have from Eq. (12-16)

$$P_S - P_R - \text{power margin} = L_{excess} + \alpha(2L) + 2L_c + 10\log N$$

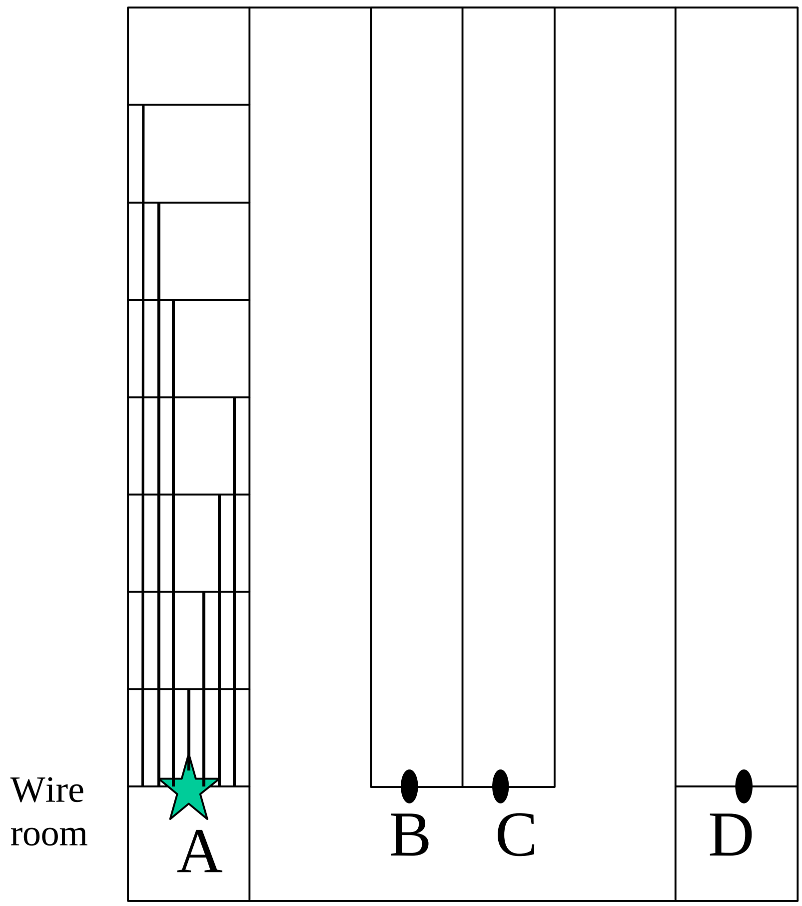
Thus

0 – (-38 dBm) – 6 dB = 3 dB + (0.3 dB/km)2(2 km) + 2(1.0 dB) + 10 log N

so that 10 log N = 25.8. This yields N = 380.1, so that 380 stations can be attached.

(b) For a receiver sensitivity of –32 dBm, one can attach 95 stations.

12-3. (b) Let the star coupler be located in the ceiling in the wire room, as shown in the figure below.



For any row we need seven wires running from the end of the row of offices to each individual office. Thus, in any row we need to have (1+2+3+4+5+6+7)x15 ft = 420 ft of optical fiber to connect the offices. From the wiring closet to the second row of offices (row B), we need 8(10 + 15) ft = 200 ft; from the wiring closet to the third row of offices (row C), we need 8(10 + 30) ft = 320 ft; and from the wiring closet to the fourth row of offices (row D), we need 8(20 + 45) ft = 520 ft of cable. For the 28 offices we also need 28x7 ft = 196 ft for wall risers.

Therefore for each floor we have the following cable needs:

(1) 4 x 420 ft for row runs

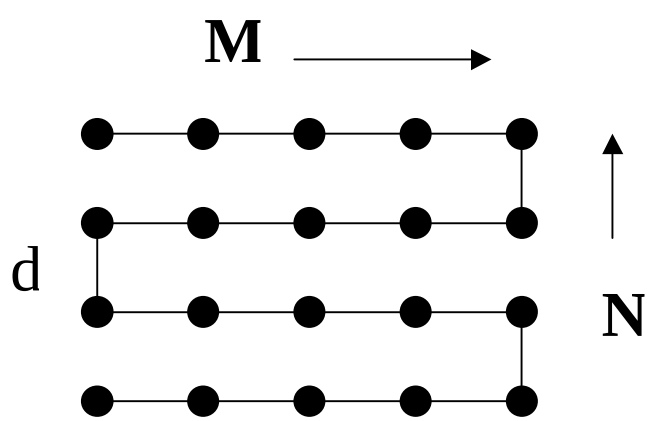
(2) 200 + 320 + 520 ft = 1040 for row connections

(3) 196 ft for wall risers

Thus, the total per floor = 2916 ft

<u>Total cable in the building</u>: 2x9 ft risers + 2916 ft x 2 floors = 5850 ft

12-4. Consider the following figure:



(a) For a bus configuration:

Cable length = N rows×(M-1)stations/row + (N-1) row interconnects

$= N(M-1)d + (N-1)d = (MN-1)d$

(b) The ring is similar to the bus, except that we need to close the loop with one cable of length d. Therefore the cable length = MNd

(c) In this problem we consider the case where we need individual cables run from the star to each station. Then the cable length is

L = cables run along the M vertical rows + cables run along the N horizontal rows:

$$= \; Md\sum_{i=1}^{N-1} i + Nd\sum_{j=1}^{M-1} j = \; M\,\frac{N(N-1)}{2}d + N\,\frac{M(M-1)}{2}d \; = \; \frac{MN}{2}(M+N-2)d$$

12-5. (a) Let the star be located at the relative position (m,n). Then

$$L = \left[N\sum_{j=1}^{m-1} j + N\sum_{j=1}^{M-m} j + M\sum_{i=1}^{n-1} i + M\sum_{i=1}^{N-n} i\right]d$$

$$= \left\{N\left[\frac{m(m-1)}{2} + \frac{(M-m)(M-m+1)}{2}\right] + M\left[\frac{n(n-1)}{2} + \frac{(N-n)(N-n+1)}{2}\right]\right\}d$$

$$= \left[\frac{MN}{2}(M+N+2) - Nm(M-m+1) - Mn(N-n+1)\right]d$$

(b) When the star coupler is located in one corner of the grid, then

m = n= 1, so that the expression in (a) becomes

$$L = \left[\frac{MN}{2}(M+N+2) - NM - MN\right]d = \frac{MN}{2}(M+N-2)d$$

(c) To find the shortest distance, we differentiate the expression for L given in (a) with respect to m and n, and set the result equal to zero:

$$\frac{dL}{dm} = N(m - 1 - M) + Nm = 0 \qquad \text{so that} \quad m = \frac{M+1}{2}$$

Similarly

$$\frac{dL}{dn} = M(n - 1 - N) + nM = 0 \qquad \text{yields} \quad n = \frac{N+1}{2}$$

Thus for the shortest cable runs the star should be located in the center of the grid.

12-6. (a) For a star network, one cannot reuse wavelengths. Thus, since each node must be connected to N – 1 other nodes through a central point, we need N – 1 wavelengths.

For a bus network, these equations can easily be verified by drawing sample diagrams with several even or odd stations.

For a ring network, each node must be connected to N – 1 other nodes. Without wavelength reuse one thus needs N(N – 1) wavelengths. However, since each wavelength can be used twice in the network, the number of wavelengths needed is N(N-1)/2.

12-7. From Tables 12-4 and 12-5, we have the following:

OC-48 output for 40-km links: –5 to 0 dBm; α = 0.5 dB/km; P_R = -18 dBm

OC-48 output for 80-km links: –2 to +3 dBm; α = 0.3 dB/km; P_R = -27 dBm

The margin is found from: $\text{Margin} = (P_s - P_R) - \alpha L - 2L_c$

(a) Minimum power at 40 km:

Margin = [-2 – (-27)] – 0.5(40) –2(1.5) = +2 dB

(b) Maximum power at 40 km:

Margin = [0 – (-27)] – 0.5(40) –2(1.5) = +4 dB

(c) Minimum power at 80 km:

Margin = [-2 – (-27)] – 0.3(80) –2(1.5) = -2 dB

(d) Maximum power at 80 km:

Margin = [3 – (-27)] – 0.3(80) –2(1.5) = +3 dB

12-8. Expanding Table 12-6:

# of λs	P_1(dBm)	$P = 10^{P_1/10}$ (mW)	P_{total}(mW)	P_{total}(dBm)
1	17	50	50	17
2	14	25	50	17
3	12.2	16.6	49.8	17
4	11	12.6	50.4	17
5	10	10	50	17

6	9.2	8.3	49.9	17
7	8.5	7.1	49.6	17
8	8.0	6.3	50.4	17

12-9. See Figure 20 of ANSI T1.105.01-95.

12-10. See Figure 21 of ANSI T1.105.01-95.

12-11. The following wavelengths can be added and dropped at the three other nodes:

Node 2: add/drop wavelengths 3, 5, and 6

Node 3: add/drop wavelengths 1, 2, and 3

Node 4: add/drop wavelengths 1, 4, and 5

12-12. (b) From Eq. (12-18) we have

$$N_\lambda = kp^{k+1} = 2(3)^3 = 54$$

(c) From Eq. (12-20) we have

$$\overline{H} = \frac{2(3)^2(3-1)(6-1) - 4(3^2-1)}{2(3-1)[2(3^2)-1]} = 2.17$$

(d) From Eq. (12-21) we have

$$C = \frac{2(3)^3}{2.17} = 8.27$$

12-13. See Hluchyj and Karol, Ref. 25, Fig. 6, p. 1391 (*Journal of Lightwave Technology*, Oct. 1991).

12-14. From Ref. 25:

In general, for a (p,k) ShuffleNet, the following spanning tree for assigning fixed routes to packets generated by any given user can be obtained:

h	Number of users h hops away from the source
1	p

2	p^2
.	
.	
.	
k – 1	p^{k-1}
k	$P^k - 1$
k + 1	$P^k - p$
k + 2	$P^k - p^2$
.	
.	
.	
2k – 1	$P^k - p^{k-1}$

Summing these up results in Eq. (12-20).

12-15. See Li and Lee (Ref 40) for details.

12-16. The following is one possible solution:

(a) Wavelength 1 for path A-1-2-5-6-F

(b) Wavelength 1 for path B-2-3-C

(c) Wavelength 2 for the partial path B-2-5 and Wavelength 1 for path 5-6-F

(d) Wavelength 2 for path G-7-8-5-6-F

(e) Wavelength 2 for the partial path A-1-4 and Wavelength 1 for path 4-7-G

12-17. See Figure 4 of Barry and Humblet (Ref. 42).

12-18. See Shibata, Braun, and Waarts (Ref. 67).

(a) The following nine 3^{rd}-order waves are generated due to FWM:

$\nu 113 = 2(\nu 2 - \Delta\nu) – (\nu 2 + \Delta\nu) = \nu 2 - 3\Delta\nu$

$\nu 112 = 2(\nu 2 - \Delta\nu) – \nu 2 = \nu 2 - 2\Delta\nu$

$\nu 123 = (\nu 2 - \Delta\nu) + \nu 2 – (\nu 2 + \Delta\nu) = \nu 2 - 2\Delta\nu$

$\nu223 = 2\nu2 - (\nu2 + \Delta\nu) = \nu2 - \Delta\nu = \nu1$

$\nu132 = (\nu2 - \Delta\nu) + (\nu2 + \Delta\nu) - \nu2 = \nu2$

$\nu221 = 2\nu2 - (\nu2 - \Delta\nu) = \nu2 + \Delta\nu = \nu3$

$\nu231 = \nu2 + (\nu2 + \Delta\nu) - (\nu2 - \Delta\nu) = \nu2 + 2\Delta\nu$

$\nu331 = 2(\nu2 + \Delta\nu) - (\nu2 - \Delta\nu) = \nu2 + 3\Delta\nu$

$\nu332 = 2(\nu2 + \Delta\nu) - \nu2 = \nu2 + 2\Delta\nu$

(b) In this case the nine 3^{rd}-order waves are:

$\nu113 = 2(\nu2 - \Delta\nu) - (\nu2 + 1.5\Delta\nu) = \nu2 - 1.5\Delta\nu$

$\nu112 = 2(\nu2 - \Delta\nu) - \nu2 = \nu2 - 2\Delta\nu$

$\nu123 = (\nu2 - \Delta\nu) + \nu2 - (\nu2 + 1.5\Delta\nu) = \nu2 - 2.5\Delta\nu$

$\nu223 = 2\nu2 - (\nu2 + 1.5\Delta\nu) = \nu2 - 1.5\Delta\nu$

$\nu132 = (\nu2 - \Delta\nu) + (\nu2 + 1.5\Delta\nu) - \nu2 = \nu2 + 0.5\Delta\nu$

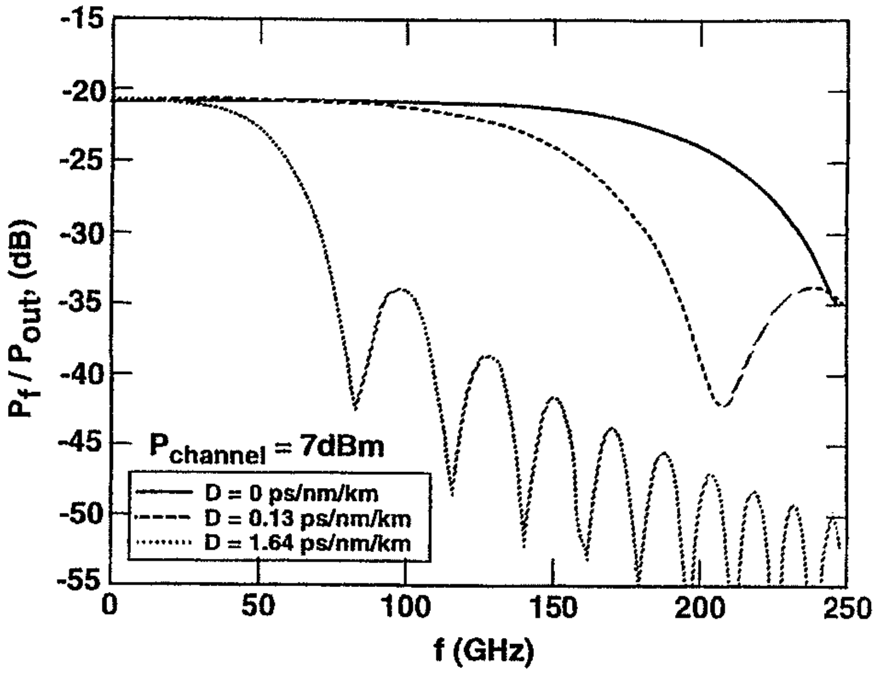
$\nu221 = 2\nu2 - (\nu2 - \Delta\nu) = \nu2 + \Delta\nu$

$\nu231 = \nu2 + (\nu2 + 1.5\Delta\nu) - (\nu2 - \Delta\nu) = \nu2 + 2.5\Delta\nu$

$\nu331 = 2(\nu2 + 1.5\Delta\nu) - (\nu2 - \Delta\nu) = \nu2 + 4\Delta\nu$

$\nu332 = 2(\nu2 + 1.5\Delta\nu) - \nu2 = \nu2 + 3\Delta\nu$

12-19. Plot: from Figure 2 of Y. Jaouën, J-M. P. Delavaux, and D. Barbier, "Repeaterless bidirectional 4x2.5-Gb/s WDM fiber transmission experiment," Optical Fiber Technology, vol. 3, p. 239-245, July 1997.



12-20. (a) From Eq. (12-50) the peak power is

$$P_{peak} = \left(\frac{1.7627}{2\pi}\right)^2 \frac{A_{eff}\lambda^3}{n_2 c} \frac{D}{T_s^2} = 11.0 \text{ mW}$$

(b) From Eq. (12-49) the dispersion length is

L_{disp} = 43 km

(c) From Eq. (12-51) the soliton period is

$$L_{period} = \frac{\pi}{2} L_{disp} = 67.5 \text{ km}$$

(d) From Eq. (12-50) the peak power for 30-ps pulses is

$$P_{peak} = \left(\frac{1.7627}{2\pi}\right)^2 \frac{A_{eff}\lambda^3}{n_2 c} \frac{D}{T_s^2} = 3.1 \text{ mW}$$

12-21. Soliton system design.

12-22. Soliton system cost model.

12-23. (a) From the given equation, L_{coll} = 80 km.

(b) From the given condition, $L_{amp} \le \frac{1}{2} L_{coll} = 40$ km

12-24. From the equation and conditions given in Prob. 12-23, we have that

$$\Delta\lambda_{max} = \frac{T_s}{DL_{amp}} = \frac{20 \text{ ps}}{[0.4 \text{ ps/(nm} \cdot \text{km)](25 km)}} = 2 \text{ nm}$$

Thus 2.0/0.4 = 5 wavelength channels can be accommodated.

12-25 Plot from Figure 3 of Ref. 103.

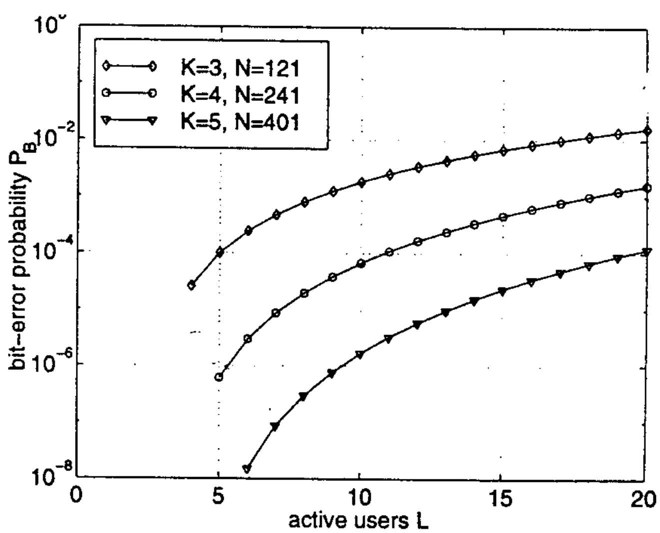


Fig. 3. BER vs. active users for different K, $M = 20$

Problem Solutions for Chapter 13

13-1 (a) From the given equation, n_{air} = 1.000273. Thus,

$$\lambda_{vacuum} = \lambda_{air} n_{air} = 1.000273(1550.0 \text{ nm}) = 1550.42 \text{ nm}$$

(b) From the given equation,

$$n(T, P) = 1 + \frac{(1.000273 - 1)(0.00138823)640}{1 + 0.003671(0)} = 1.000243$$

Then $n(T, P)(1550 \text{ nm}) = 1550.38 \text{ nm}$

13-2 Since the output voltage from the photodetector is proportional to the optical power, we can write Eq. (13-1) as

$$\alpha = \frac{10}{L_1 - L_2} \log \frac{V_2}{V_1}$$

where L_1 is the length of the current fiber, L_2 is the length cut off, and V_1 and V_2 are the voltage output readings from the long and short lengths, respectively. Then the attenuation in decibels is

$$\alpha = \frac{10}{1895 - 2} \log \frac{3.78}{3.31} = 0.31 \text{ dB/km}$$

13-3 (a) From Eq. (13-1)

$$\alpha = \frac{10}{L_N - L_F} \log \frac{P_N}{P_F} = \frac{10}{L_N - L_F} \log \frac{V_N}{V_F} = \frac{10 \log e}{L_N - L_F} \ln \frac{V_N}{V_F}$$

From this we find

$$\Delta\alpha = \frac{10 \log e}{L_N - L_F}\left[\frac{\Delta V_N}{V_N} + \frac{\Delta V_F}{V_F}\right] = \frac{4.343}{L_N - L_F}(\pm 0.1\% \pm 0.1\%) = \pm \frac{8.686}{L_N - L_F} \times 10^{-3}$$

(b) If $\Delta\alpha$ = 0.05 dB/km, then

$$L = L_N - L_F \geq \frac{8.868 \times 10^{-3}}{0.05} \text{ km} = 176 \text{ m}$$

13-4 (a) From Eq. (8-11) we have

$$\frac{1}{\sqrt{2\pi}\,\sigma}\ \exp\left(-\frac{t_{1/2}^2}{2\sigma^2}\right) = \frac{1}{2}\frac{1}{\sqrt{2\pi}\,\sigma} \qquad \text{which yields } t_{1/2} = (2\ \ln 2)^{1/2}\ \sigma$$

(b) From Eq. (8-10), the 3-dB frequency is the point at which

$$G(\omega) = \frac{1}{2}\ \ G(0), \qquad \text{or} \qquad \exp\left[-\frac{(2\pi f_{3dB})^2\ \sigma^2}{2}\right] = \frac{1}{2}$$

Using σ as defined in Eq. (8-13), we have

$$f_{3dB} = \frac{(2\ \ln 2)^{1/2}}{2\pi\sigma} = \frac{2\ \ln 2}{\pi\ t_{FWHM}} = \frac{0.44}{t_{FWHM}}$$

13-5 From Eq. (13-4), $P_{out}(f)/P_{in}(f) = |H(f)|$. To measure the frequency response, we need a constant input amplitude, that is, $P_{in}(f) = P_{in}(0)$. Thus,

$$\frac{P(f)}{P(0)} = \frac{P_{out}(f)/P_{in}(f)}{P_{out}(0)/P_{in}(0)} = \left|\frac{H(f)}{H(0)}\right| = |H(f)|$$

The following table gives some representative values of H(f) for different values of 2σ:

f (MHz)	**2σ = 2 ns**	**2σ = 1 ns**	**2σ = 0.5 ns**
100	0.821	0.952	0.988
200	0.454	0.821	0.952
300	0.169	0.641	0.895
500	0.0072	0.291	0.735
700		0.089	0.546
1000		0.0072	0.291

13-6 To estimate the value of D, consider the slope of the curve in Fig. P13-6 at λ = 1575 nm. There we have Δτ = 400 ps over the wavelength interval from 1560 nm to 1580 nm, i.e., Δλ = 20 nm. Thus

$$D = \frac{1}{L}\frac{\Delta\tau}{\Delta\lambda} = \frac{1}{10\ \text{km}}\frac{400\ \text{ps}}{20\ \text{nm}} = 2\ \ \text{ps}/(\text{nm}\cdot\text{km})$$

Then, using this value of D at 1575 nm and with λ_0 = 1548 nm, we have

$$S_0 = \frac{D(\lambda)}{\lambda - \lambda_0} = \frac{2\ \mathrm{ps/(nm \cdot km)}}{(1575 - 1548)\ \mathrm{nm}} = 0.074\ \mathrm{ps/(nm^2 \cdot km)}$$

13-7 With k = 1, λ_{start} = 1525 nm, and λ_{stop} = 1575 nm, we have N_e = 17 extrema.

Substituting these values into Eq. (13-14) yields 1.36 ps.

13-8 At 10 Gb/s over a 100-km link, the given equation yields:

$$P_{ISI} \approx 26 \frac{(1\ \mathrm{ps})^2\, 0.5(1 - 0.5)}{(100\ \mathrm{ps})^2} = 6.5 \times 10^{-4}\ \mathrm{dB}$$

Similarly, at 10 Gb/s over a 1000-km link, $P_{ISI} \approx 0.065\ \mathrm{dB}$.

This is the same result at 100 Gb/s over a 100-km link.

At 100 Gb/s over a 1000-km link, we have 6.5 dB.

13-9 For a uniform attenuation coefficient, β is independent of y. Thus, Eq. (13-16) becomes

$$P(x) = P(0) \exp\left[-\beta \int_0^x dy\right] = P(0) e^{-\beta x}$$

Writing this as $\exp(-\beta x) = P(0)/P(x)$ and taking the logarithm on both sides yields

$\beta x\ \log\ e = \log\ \dfrac{P(0)}{P(x)}$. Since $\alpha = \beta(10\ \log\ e)$, this becomes

$$\alpha x = 10\ \log\ \frac{P(0)}{P(x)}$$

For a fiber of length x = L with P(0) = P_N being the near-end input power, this equation reduces to Eq. (13-1).

13-10 Consider an isotropically radiating point source in the fiber. The power from this point source is radiated into a sphere that has a surface area $4\pi r^2$. The portion of this power captured by the fiber in the backward direction at a distance r from the point source is the ratio of the area $A = \pi a^2$ to the sphere area $4\pi r^2$. If θ is the acceptance angle of the fiber core, then $A = \pi a^2 = \pi(r\theta)^2$. Therefore S, as defined in Eq. (13-18), is given by

$$S = \frac{A}{4\pi r^2} = \frac{\pi r^2 \theta^2}{4\pi r^2} = \frac{\theta^2}{4}$$

From Eq. (2-23), the acceptance angle is

$$\sin\ \theta \approx \theta = \frac{NA}{n}, \text{ so that} \qquad S = \frac{\theta^2}{4} = \frac{(NA)^2}{4n^2}$$

13-11 The attenuation is found from the slope of the curve, by using Eq. (13-22):

$$\text{Fiber a:}\quad \alpha = \frac{10\ \log\ \dfrac{P_D(x_1)}{P_D(x_2)}}{2(x_2 - x_1)} = \frac{10\ \log\ \dfrac{70}{28}}{2(0.5\ \text{km})} = 4.0\ \text{dB/km}$$

$$\text{Fiber b:}\quad \alpha = \frac{10\ \log\ \dfrac{25}{11}}{2(0.5\ \text{km})} = 3.6\ \text{dB/km}$$

$$\text{Fiber c:}\quad \alpha = \frac{10\ \log\ \dfrac{7}{1.8}}{2(0.5\ \text{km})} = 5.9\ \text{dB/km}$$

To find the final splice loss, let P_1 and P_2 be the input and output power levels, respectively, at the splice point. Then for

$$\text{For splice 1:}\quad L_{splice} = 10\ \log\ \frac{P_2}{P_1} = 10\ \log\ \frac{25}{28} = -0.5\ \text{dB}$$

$$\text{For splice 2:}\quad L_{splice} = 10\ \log\ \frac{7}{11} = -2.0\ \text{dB}$$

13-12 See Ref. 42, pp. 450-452 for a detailed and illustrated derivation.

Consider the light scattered from an infinitesimal interval dz that is located at L = Tv_{gr}. Light scattered from this point will return to the OTDR at time t = 2T. Upon inspection of the pulse of width W being scattered form the point L, it can be deduced that the back-scattered power seen by the OTDR at time 2T is the integrated sum of the light scattered from the locations z = L – W/2 to z = L.

Thus, summing up the power from infinitesimal short intervals dz from the whole pulse and taking the fiber attenuation into account yields

$$P_s(L) = \int_0^W S\alpha_s P_0 \exp\left[-2\alpha\left(L + \frac{z}{2}\right)\right] dz$$

$$= S\ \frac{\alpha_s}{\alpha}\ P_0\ e^{-2\alpha L}\ \left(1 - e^{-\alpha W}\right)$$

which holds for L ≥ W/2. For distances less than W/2, the lower integral limit gets replaced by W – 2L.

13-13 For very short pulse widths, we have that $\alpha W << 1$. Thus the expression in parenthesis becomes

$$\frac{1}{\alpha}\left(1 - e^{-\alpha W}\right) \approx \frac{1}{\alpha}\left[1 - (1 - \alpha W)\right] = W$$

Thus

$$P_s(L) \approx S\ \alpha_s\ W\ P_0\ e^{-2\alpha L}$$

13-14 (a) From the given equation, for an 0.5-dB accuracy, the SNR is 4.5 dB.

The total loss of the fiber is (0.33 dB/km)(50 km) = 16.5 dB.

The OTDR dynamic range D is

$$D = SNR + \alpha L + \text{splice loss}$$

$$= 4.5\ dB + 16.5\ dB + 0.5\ dB = 21.5\ dB$$

Here the splice loss is added to the dynamic range because the noise that limits the achievable accuracy shows up after the event.

(b) For a 0.05-dB accuracy, the OTDR dynamic range must be 26.5 dB.

13-15 To find the fault-location accuracy dL with an OTDR, we differentiate Eq. (13-23):

$$dL = \frac{c}{2n} dt$$

where is the accuracy to which the time difference between the original and reflected pulses must be measured. For dL ≤ 1 m, we need

$$dt = \frac{2n}{c} dL \leq \frac{2(1.5)}{3 \times 10^{8} \text{ m/s}} (0.5 \text{ m}) = 5 \text{ ns}$$

To measure dt to this accuracy, the pulse width must be ≤ 0.5dt (because we are measuring the time difference between the original and reflected pulse widths). Thus we need a pulse width of 2.5 ns or less to locate a fiber fault within 0.5 m of its true position.