Problem – 1- Inference by Enumeration

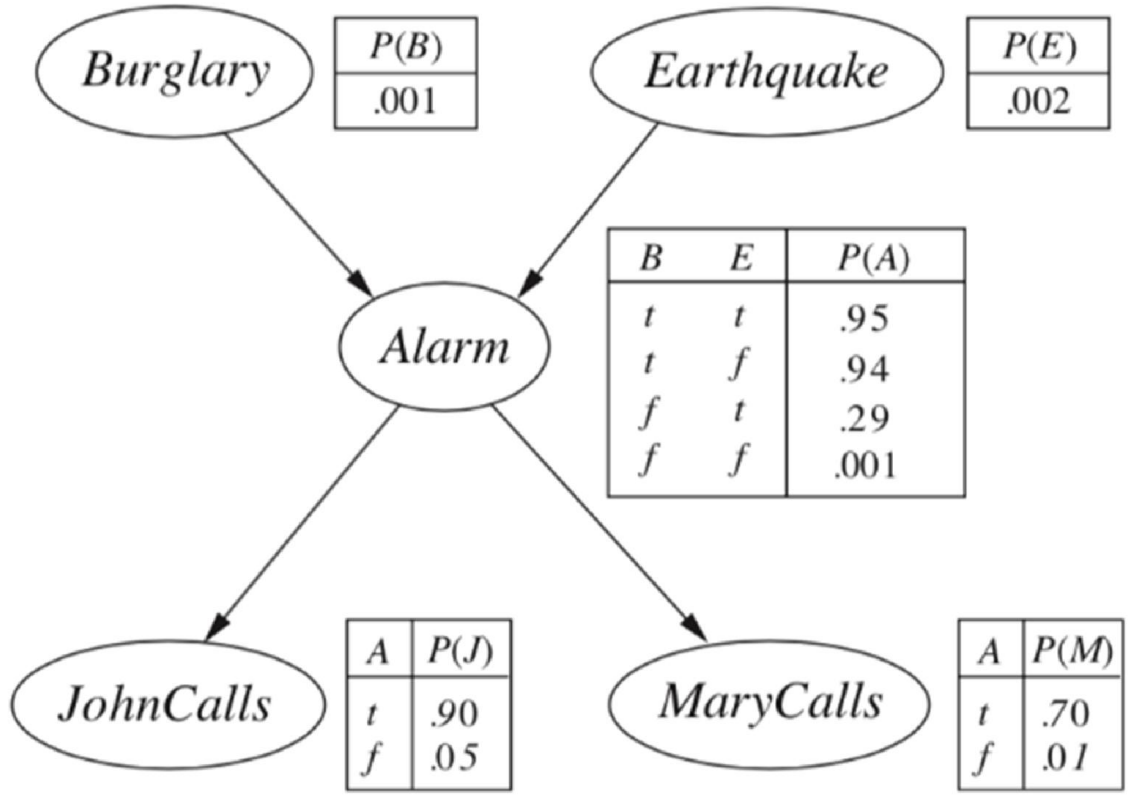


Figure 14.2 A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, and *MaryCalls*, respectively.

P(Burglary | johncalls,marycalls) ?

$P(B \mid j, m) \Rightarrow$ $X \to B$, $y \to earth, a$, $e \to j, m$ } q/p

Eg

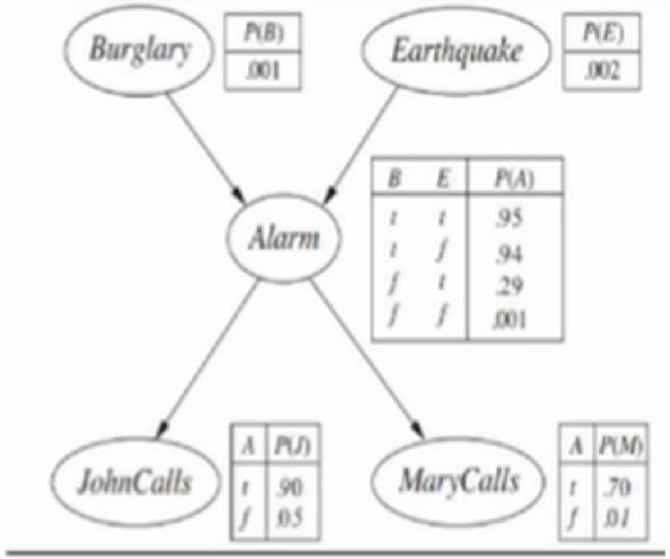
Capital $B \to T$, $\to F$

$$P(B \mid j, m) = \alpha \sum_{earth} \sum_{a} P(B, j, m, earth, a)$$

$X \Rightarrow$ query variable

$y \Rightarrow$ Non-evident variable

$e \Rightarrow$ evidence variable

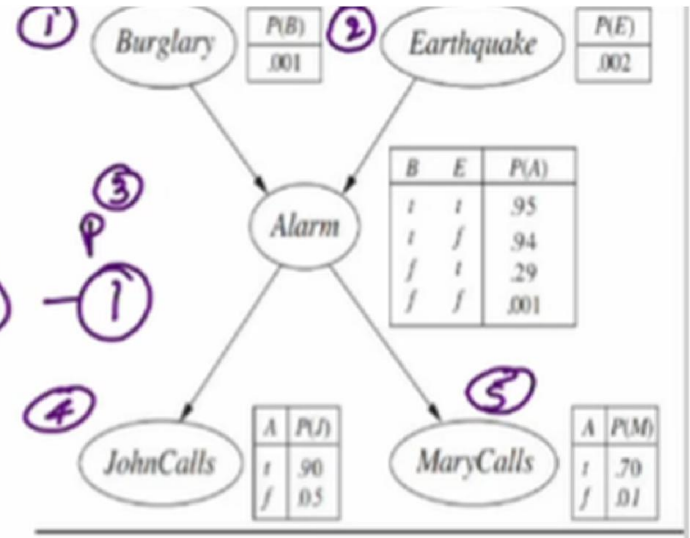


Eg

$P(B/j,m) \Rightarrow$ $X \to B$, $Y \to earth, a$, $e \to j, m$ } $\underline{\underline{I/P}}$

True / False

$$P(\underline{B}/j,m) = \alpha \sum_{earth} \sum_{a} P(B,j,m,earth,a) \quad -(1)$$



for B=true

$$P(G/j,m) = \alpha \sum_{e} \sum_{a} P(G)P(e)P(a/G,e)P(j/a)P(m/a) \quad -(2)$$

$G \Rightarrow$ True

$b \Rightarrow$ True

Frame the eqn., Burglary ✓ $\Rightarrow P(b)$ ✓

earthquake $\Rightarrow$ know nothing regarding this. So keep it.

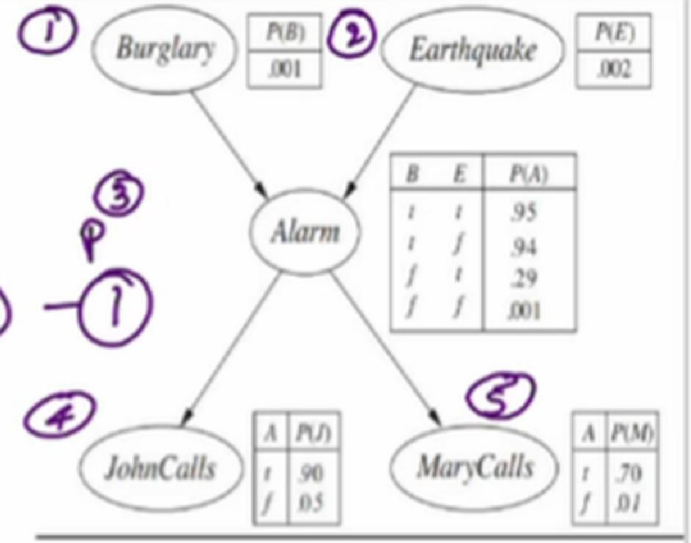
Alarm$(b,e) \Rightarrow b$ ✓ alone is True

Eg

$P(B/j,m) \Rightarrow$ $X \to B$, $y \to earth, a$, $e \to j, m$ } J/P

True / False

$$P(B/j,m) = \alpha \sum_{earth} \sum_{a} P(B, j, m, earth, a) \quad -(1)$$



for B=true

$$P(G/j,m) = \alpha \sum_{e} \sum_{a} P(G) P(e) P(a/G,e) P(j/a) P(m/a) \quad -(2)$$

$G \Rightarrow$ True

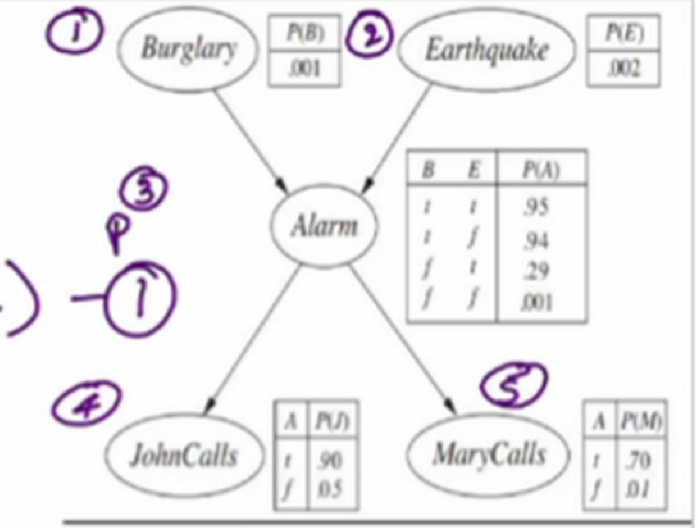
$G \Rightarrow$ True

then, since Alarm has sounded, john calls as well as Mary calls

$P(j/a)$ $P(m/a)$

Eg

$P(B/j,m) \Rightarrow \left.\begin{array}{l} X \to B \\ y \to earth, a \\ e \to j, m \end{array}\right\} \underline{\underline{g/p}}$



True
False

$P(\underline{B}/j,m) = \alpha \sum_{earth} \sum_{a} P(B,j,m,earth,a)$ — (1)

constant

for B=true

$P(G/j,m) = \alpha \sum_{e} \sum_{a} P(G) P(e) P(a/G,e) P(j/a) P(m/a)$ — (2)

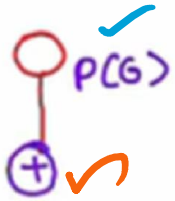
$= \alpha P(G) \sum_{e} P(e) \sum_{a} P(a/G,e) P(j/a) P(m/a)$

So bring outside

Group e

Group a

How to construct Tree?



$P(B \mid j, m) \Rightarrow B = \text{true}$

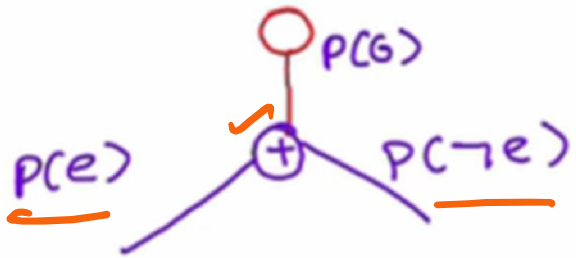
$P(G \mid j, m) = \alpha P(G) \sum_e P(e) \sum_a P(a \mid G, e) P(m \mid a) P(j \mid a)$

If E comes, then find the options of T & F for the corresponding variable

How to construct Tree?

$P(B \mid j, m) \Rightarrow B = \text{true}$

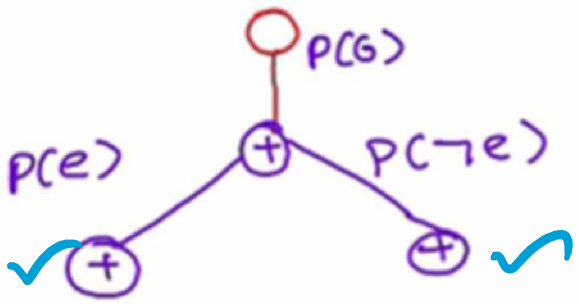
$$P(B \mid j, m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a \mid B, e) P(m \mid a) P(j \mid a)$$



How to construct Tree?

$P(B \mid j, m) \Rightarrow B = \text{true}$

$P(B \mid j, m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a \mid B, e) P(m \mid a) P(j \mid a)$



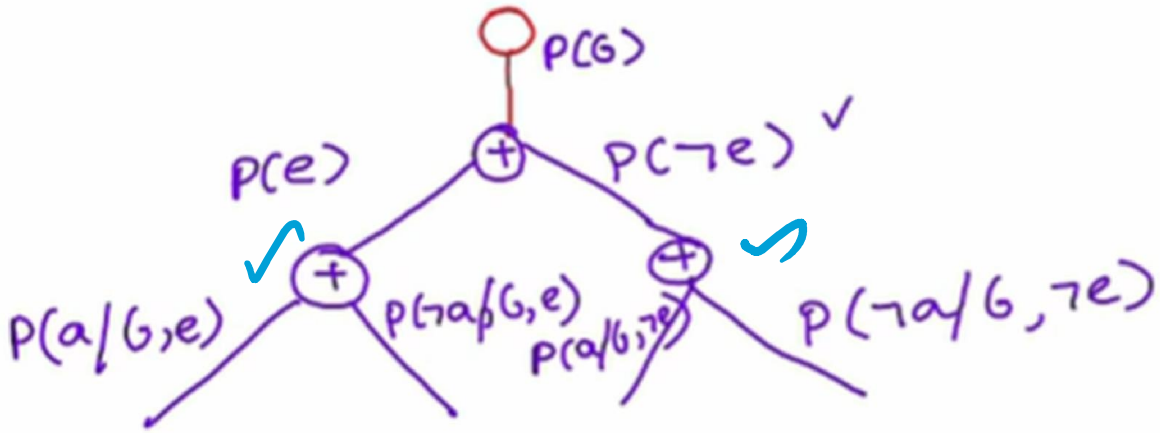
$e \prec_{f}^{T}$

Second time Σ encounters, So put $\oplus$ in all branches

How to construct Tree?

$P(B \mid j, m) \Rightarrow B = \text{true}$

$$P(B \mid j, m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a \mid B, e) P(m \mid a) P(j \mid a)$$



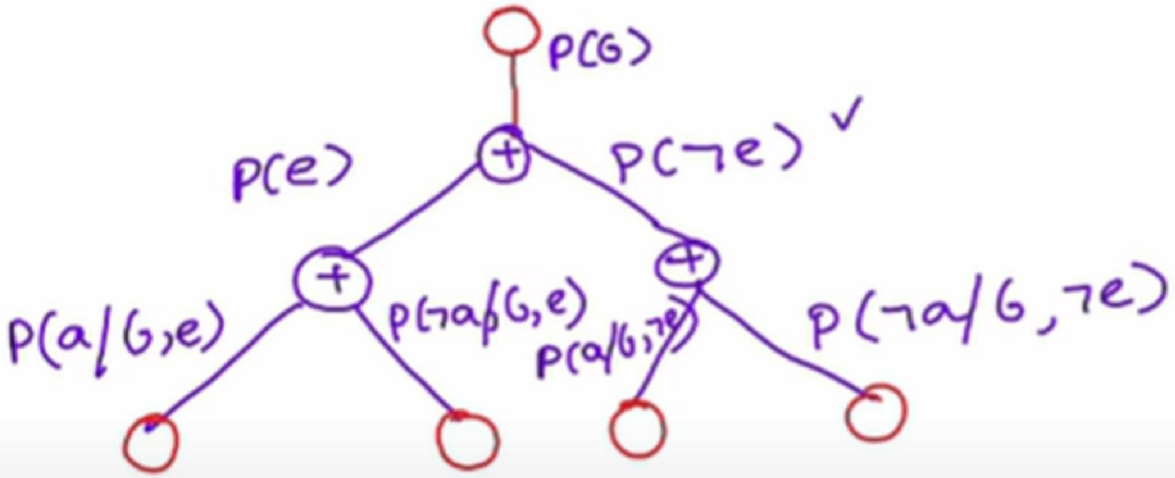
$e \begin{cases} T \\ f \end{cases}$

$a \begin{cases} t \\ f \end{cases}$

How to construct Tree?

$P(B|j,m) \Rightarrow B = \text{true}$

$P(B|j,m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a|B,e) P(m|a) P(j|a)$



$e \begin{cases} T \\ f \end{cases}$

$a \begin{cases} t \\ f \end{cases}$

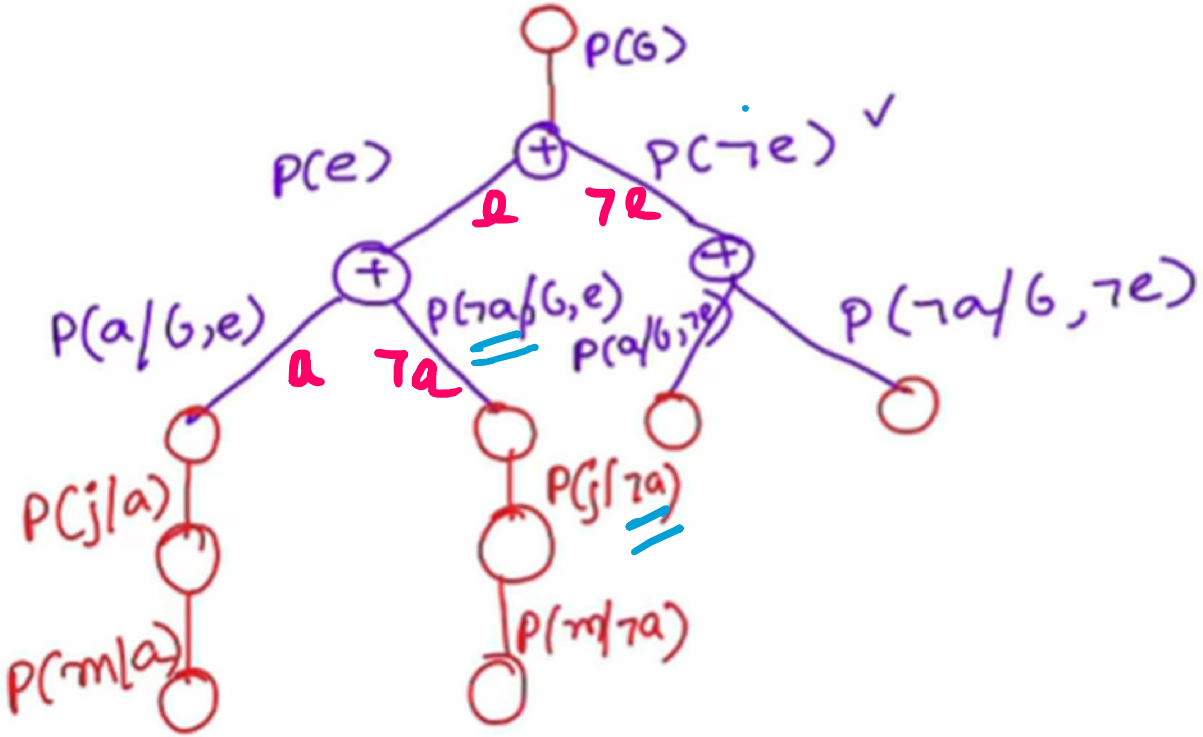
Now remaining Terms :- $P(m|a)$, $P(j|a)$ Here we have

We can take in any order. taken d

How to construct Tree?

$P(B|j,m) \Rightarrow B = \text{true}$

$P(B|j,m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a|B,e) P(m|a) P(j|a)$



$e \begin{cases} T \\ f \end{cases}$ One branch is e, another " is ¬e

$a \begin{cases} t \\ f \end{cases}$ one is a, another is ¬a

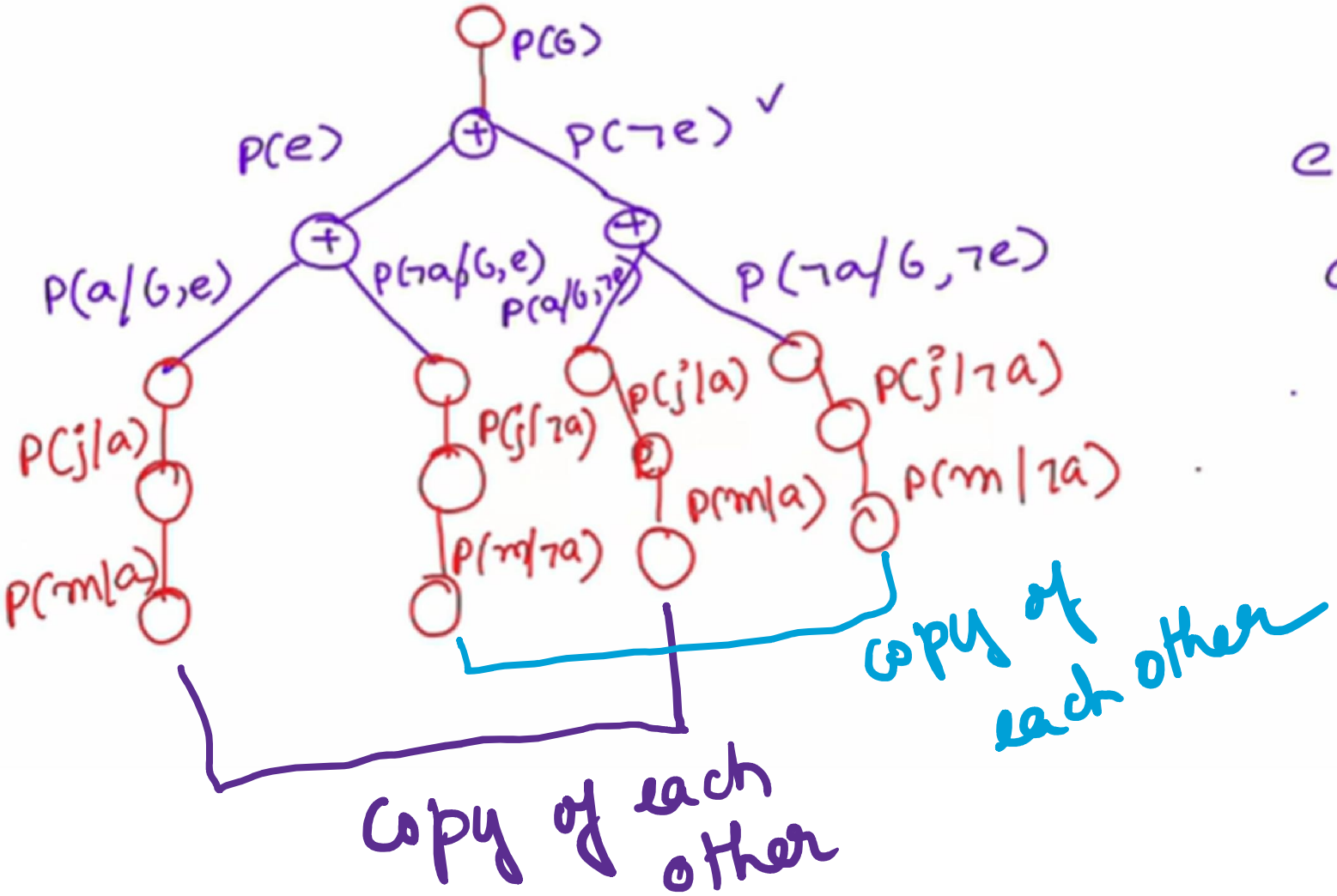
How to construct Tree?

$P(B|j,m) \Rightarrow B = \text{true}$

$P(B|j,m) = \alpha P(B) \sum_e P(e) \sum_a P(a|B,e) P(m|a) P(j|a)$

$e < T, f$

$a < t, f$



Purpose of Tree (construction)

Finding inference from the Tree (constructed).

$P(G/j, m) =$

$\hookrightarrow$ Motivation

Hence, substitute the prob. Values from the table upon the tree

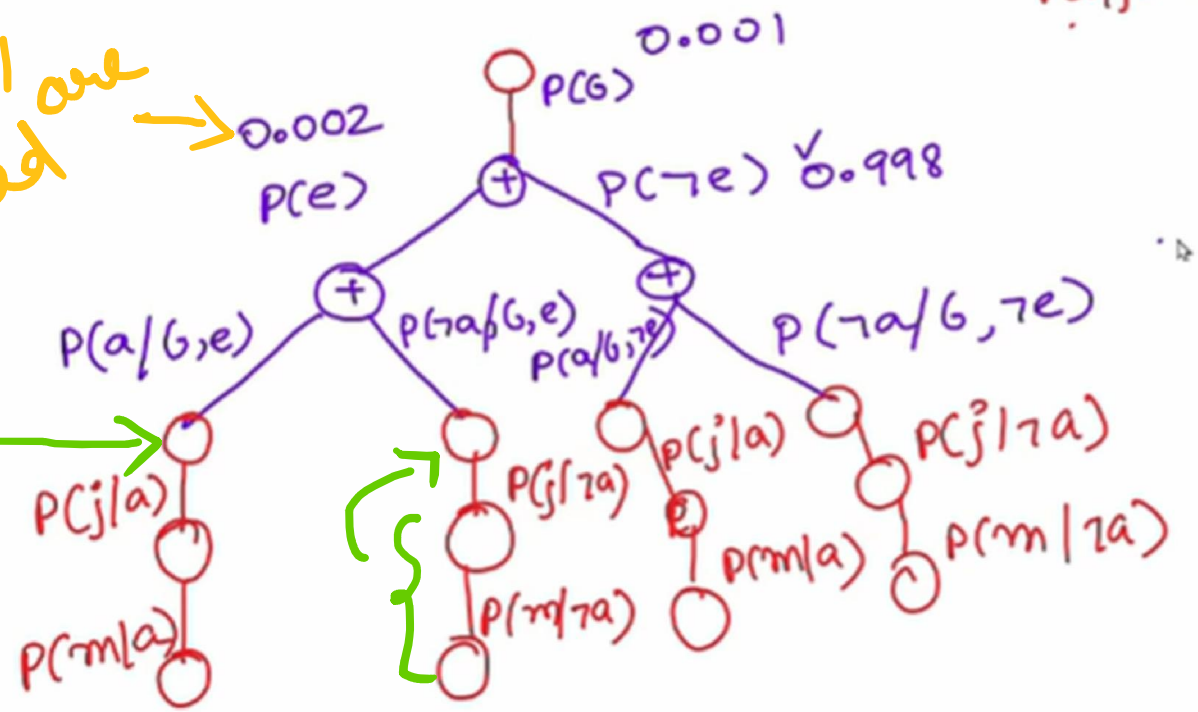
Finding inference from the Tree (constructed)

How to construct Tree?

$P(B|j,m) \Rightarrow B = \text{true}$

$P(G|j,m) = \alpha P(G) \sum_e P(e) \sum_a P(a|G,e) P(m|a) P(j|a)$

from Table / Prob. values / Copied one



P(j|a) × P(m|a)

e < T, f

a < t, f

+ → add
→ multiply

$P(G|j,m) =$

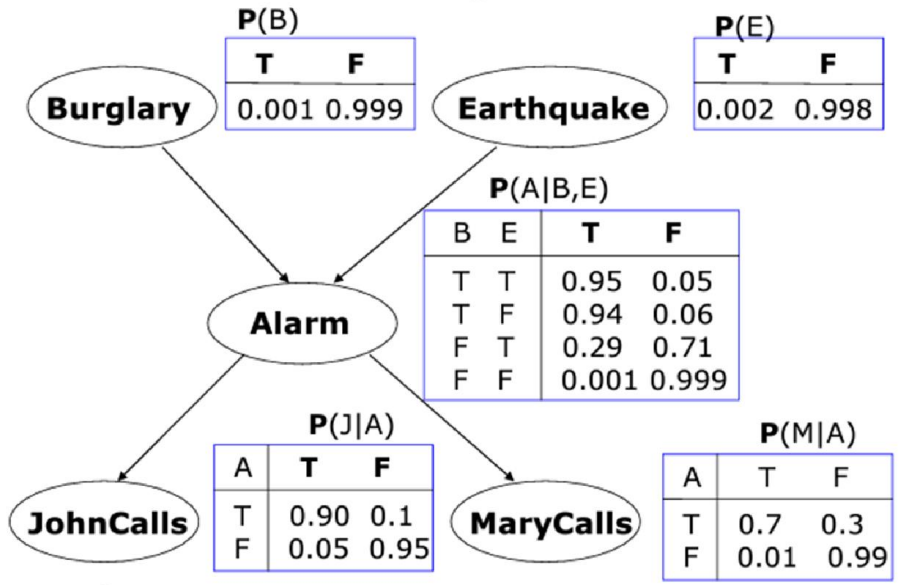
→ Motivation

PTO

Bayesian belief network

2. Local conditional distributions

- relate variables and their parents



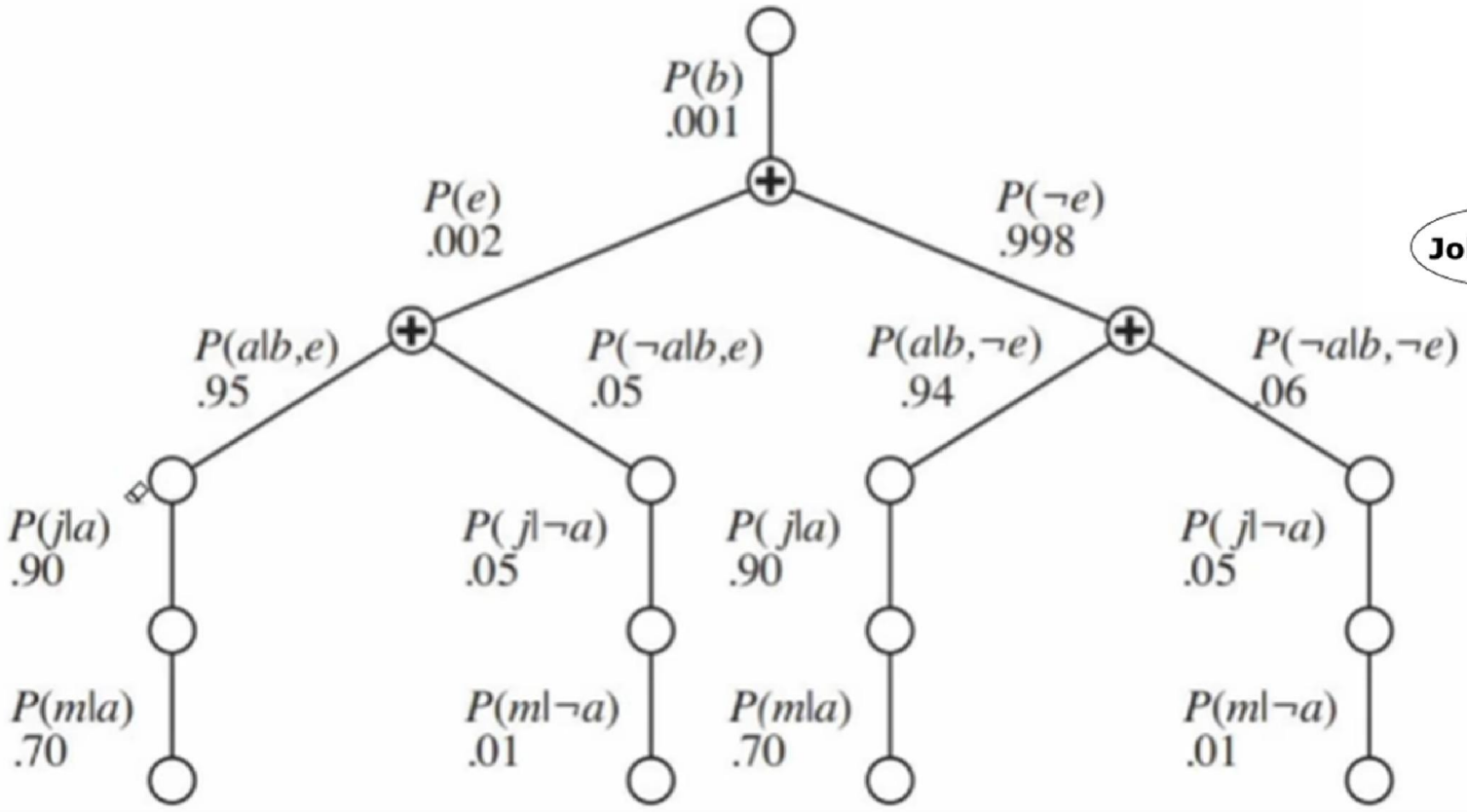


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice the repetition of the paths for j and m.

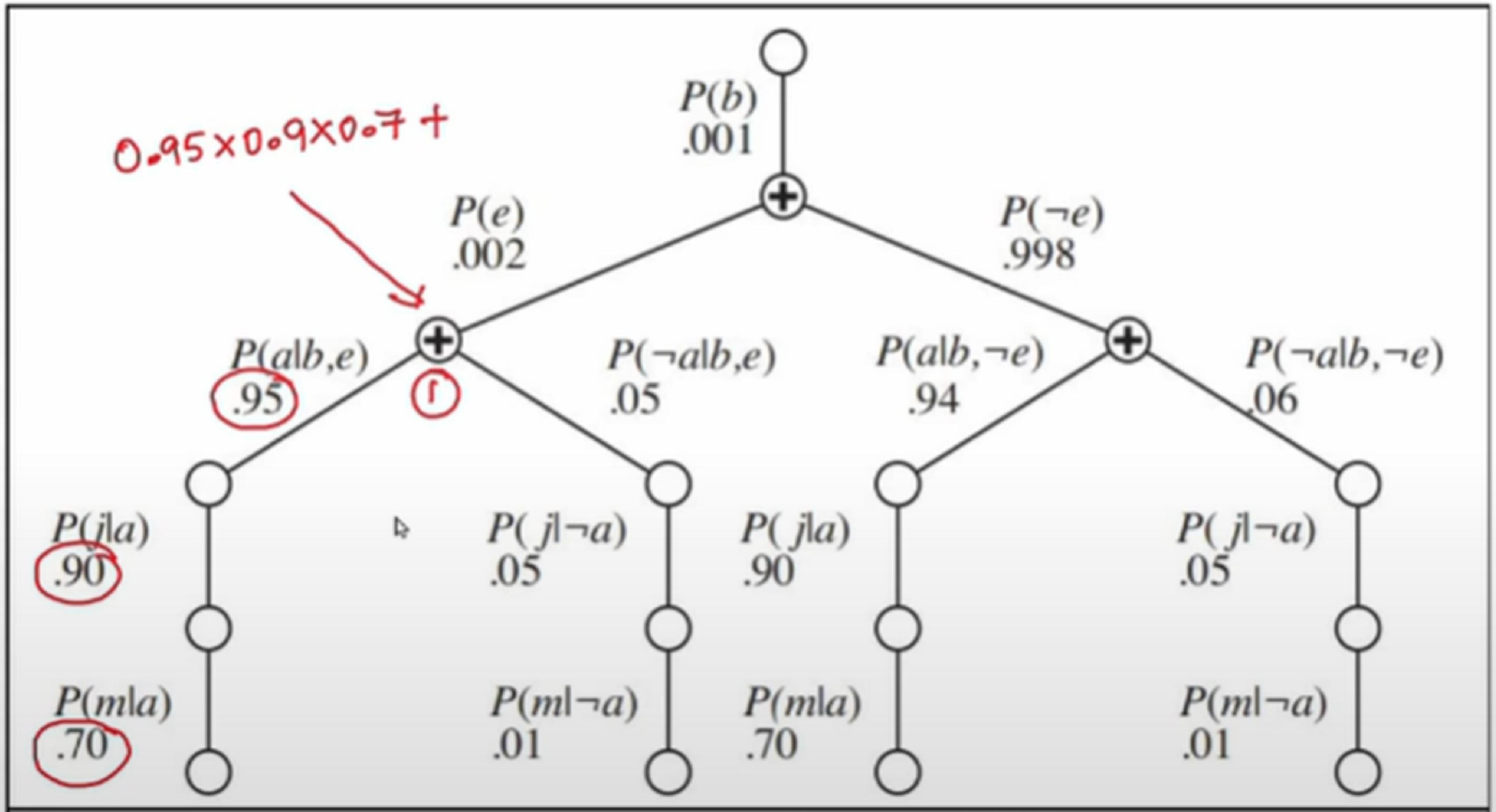


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice the repetition of the paths for j and m.

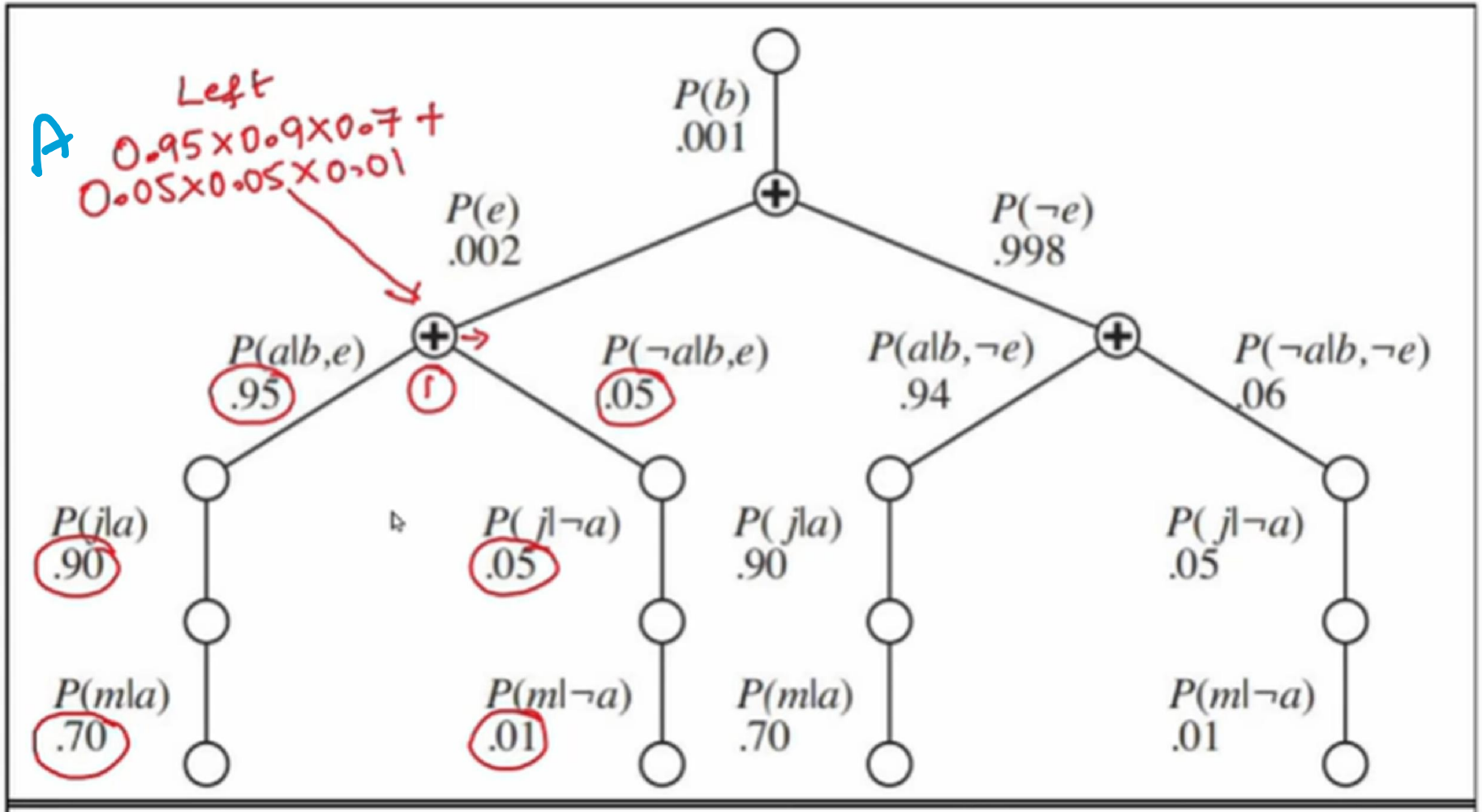


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice the repetition of the paths for j and m.

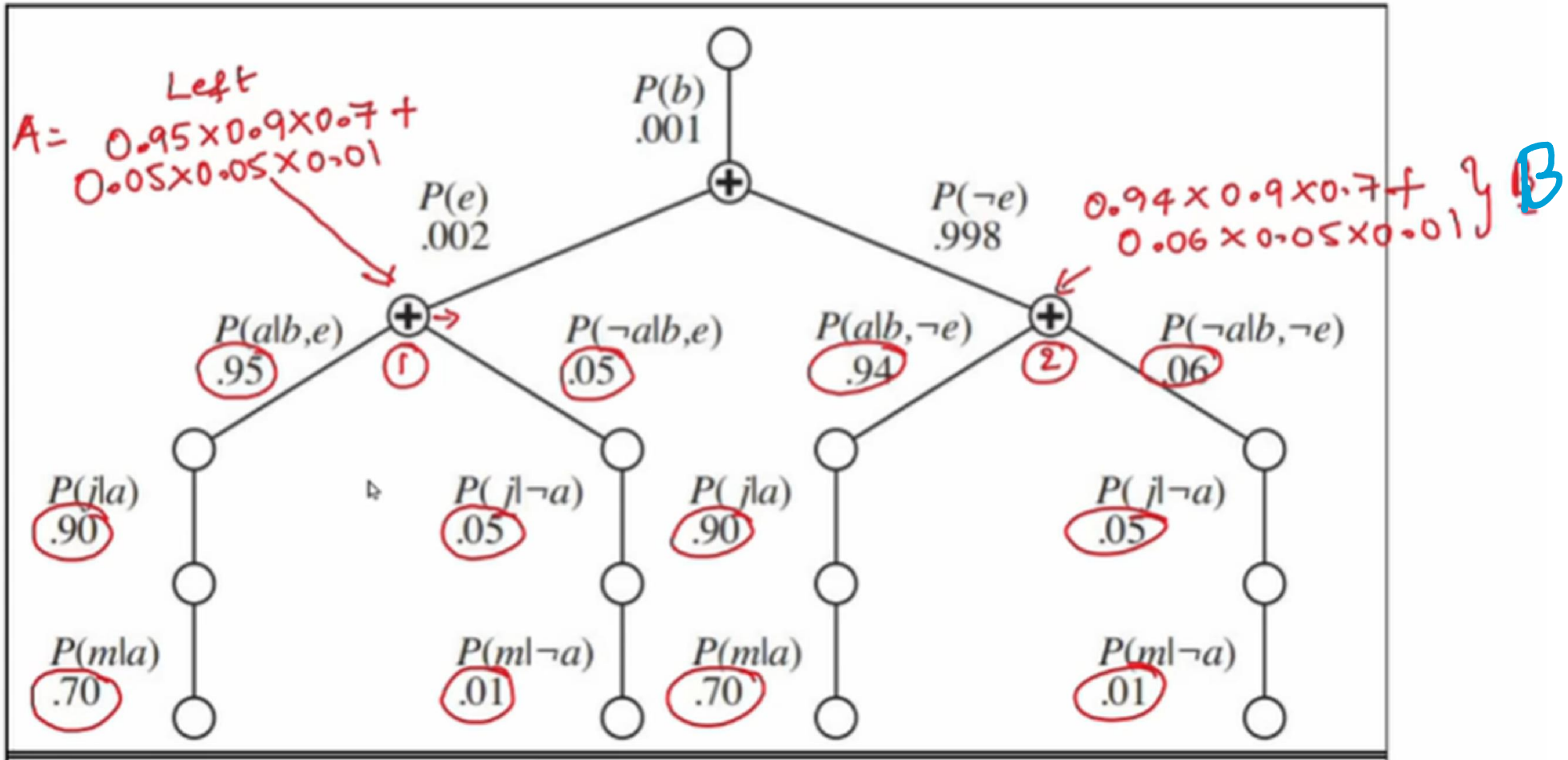


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice

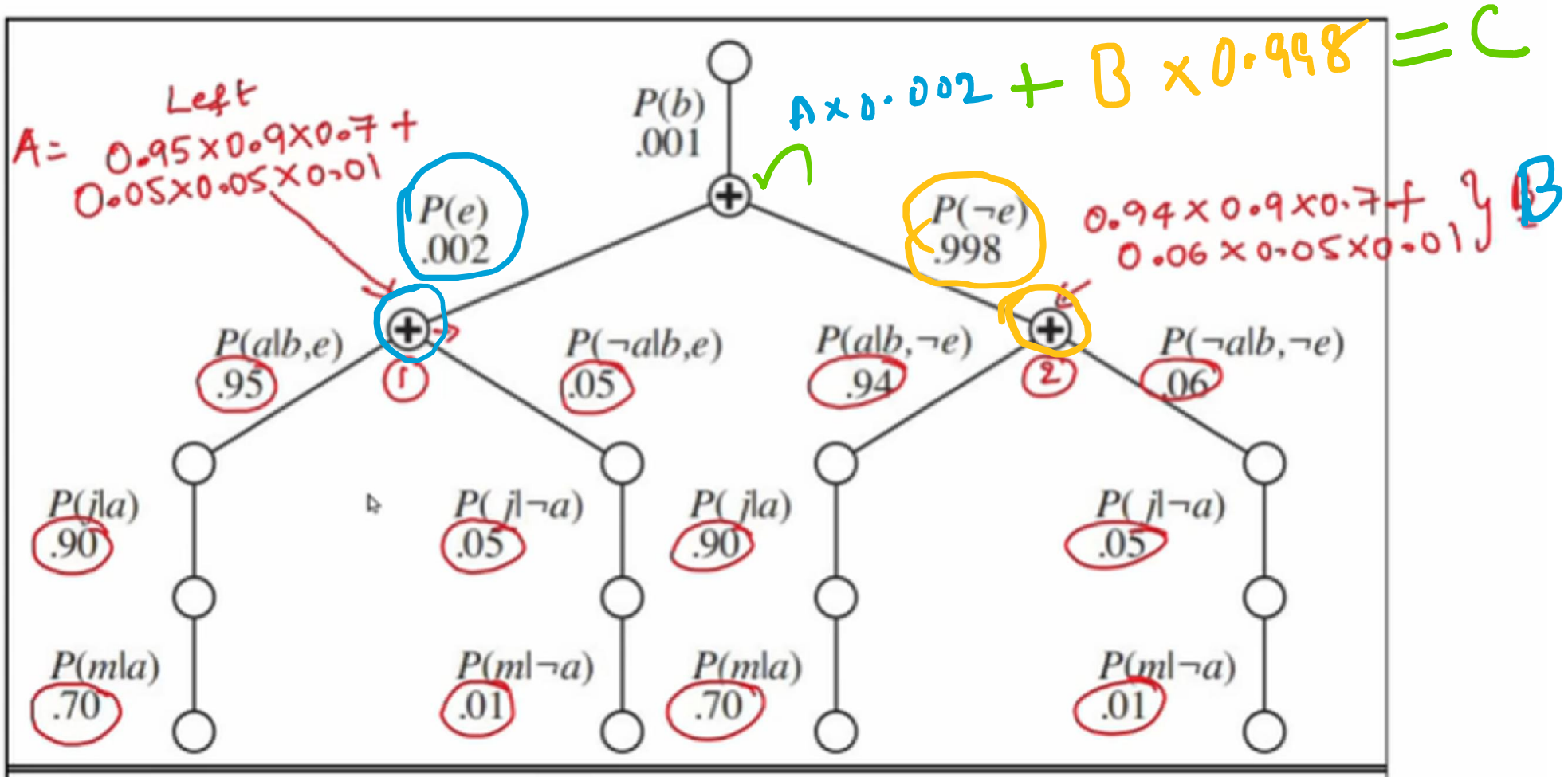


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice

Final ⇒ C × 0.001

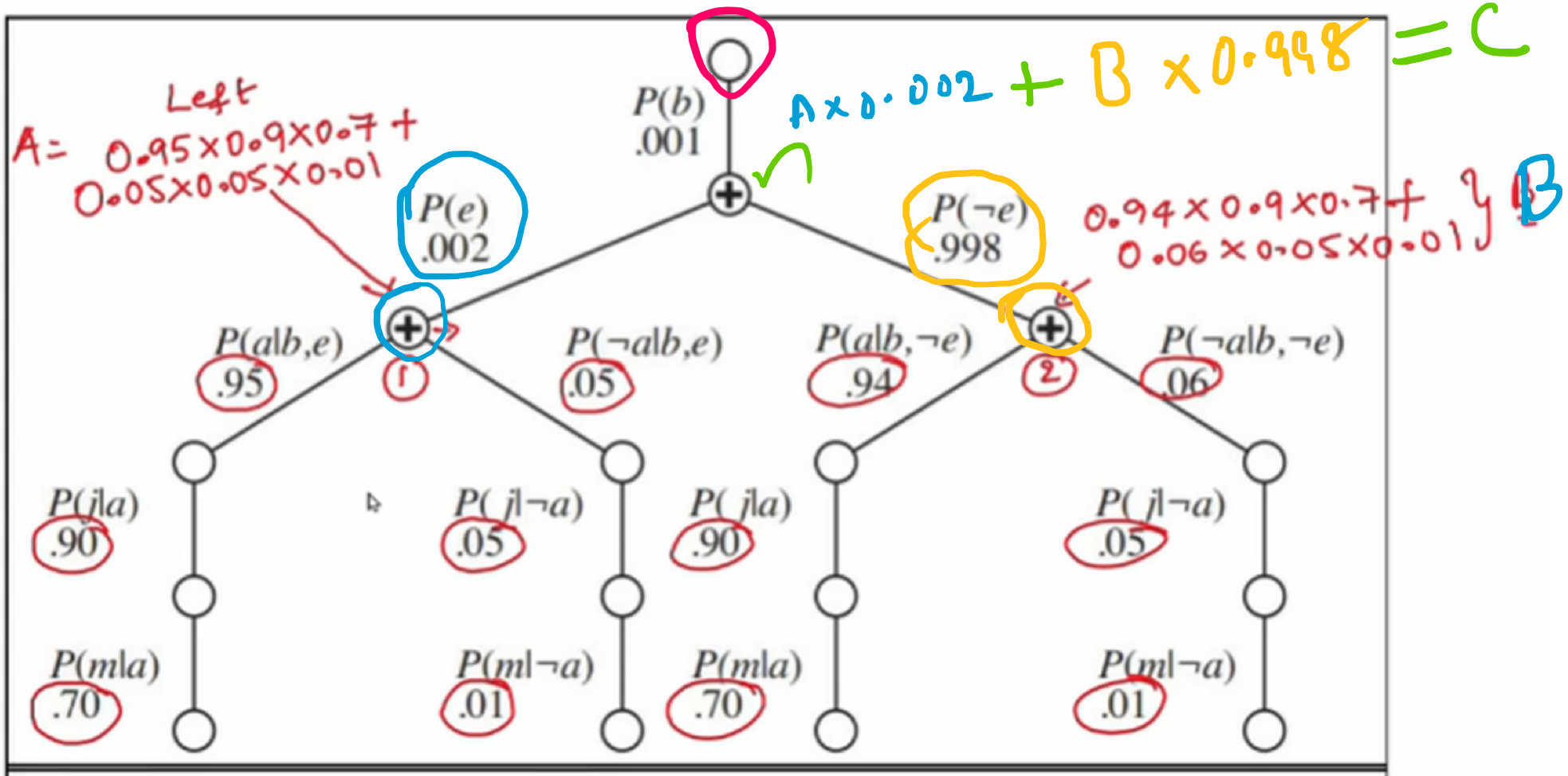
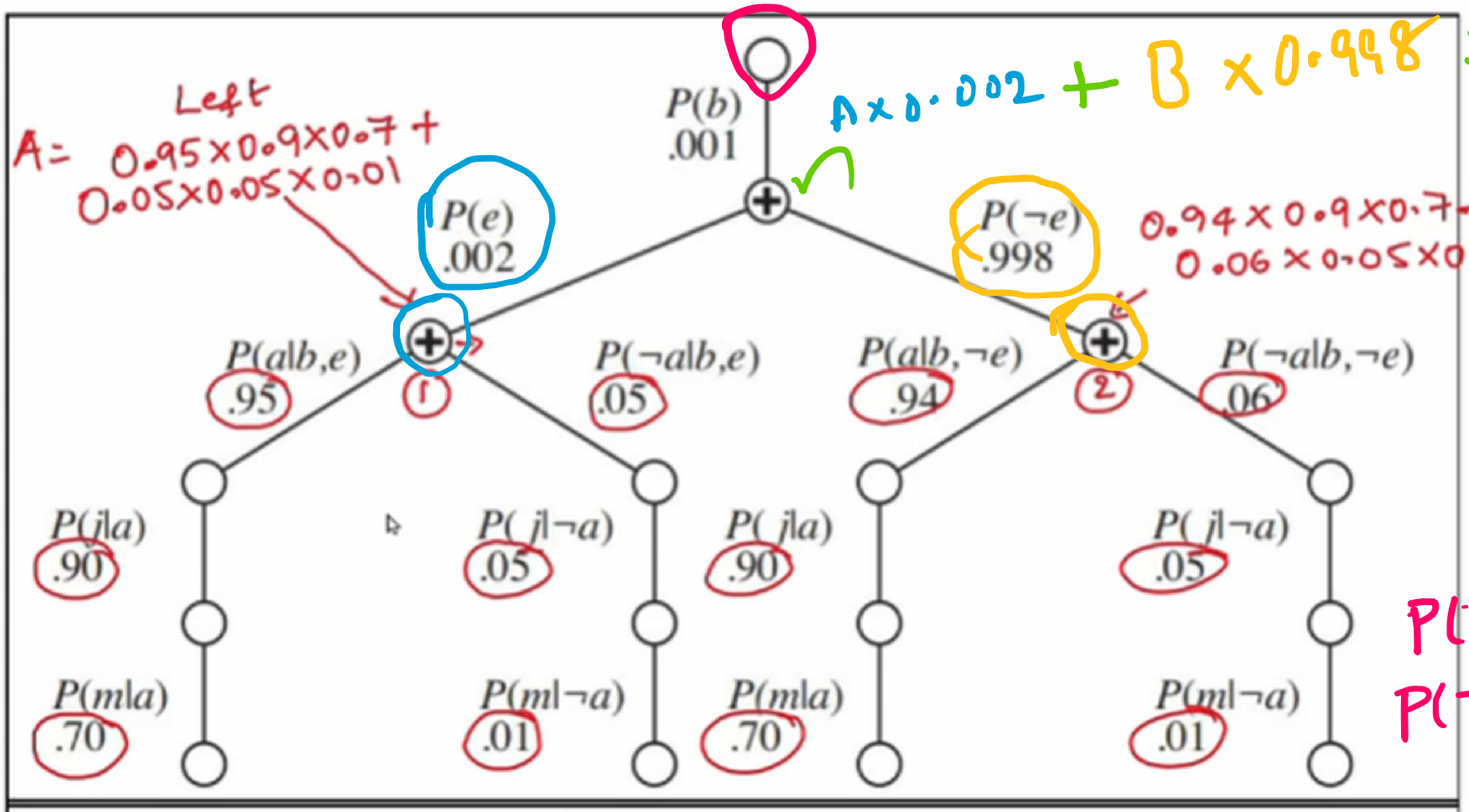


Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice

Final ⇒ $C \times 0.001$



Left

$A = 0.95 \times 0.9 \times 0.7 + 0.05 \times 0.05 \times 0.01$

$A \times 0.002 + B \times 0.998 = C$

$0.94 \times 0.9 \times 0.7 + 0.06 \times 0.05 \times 0.01 \} B$

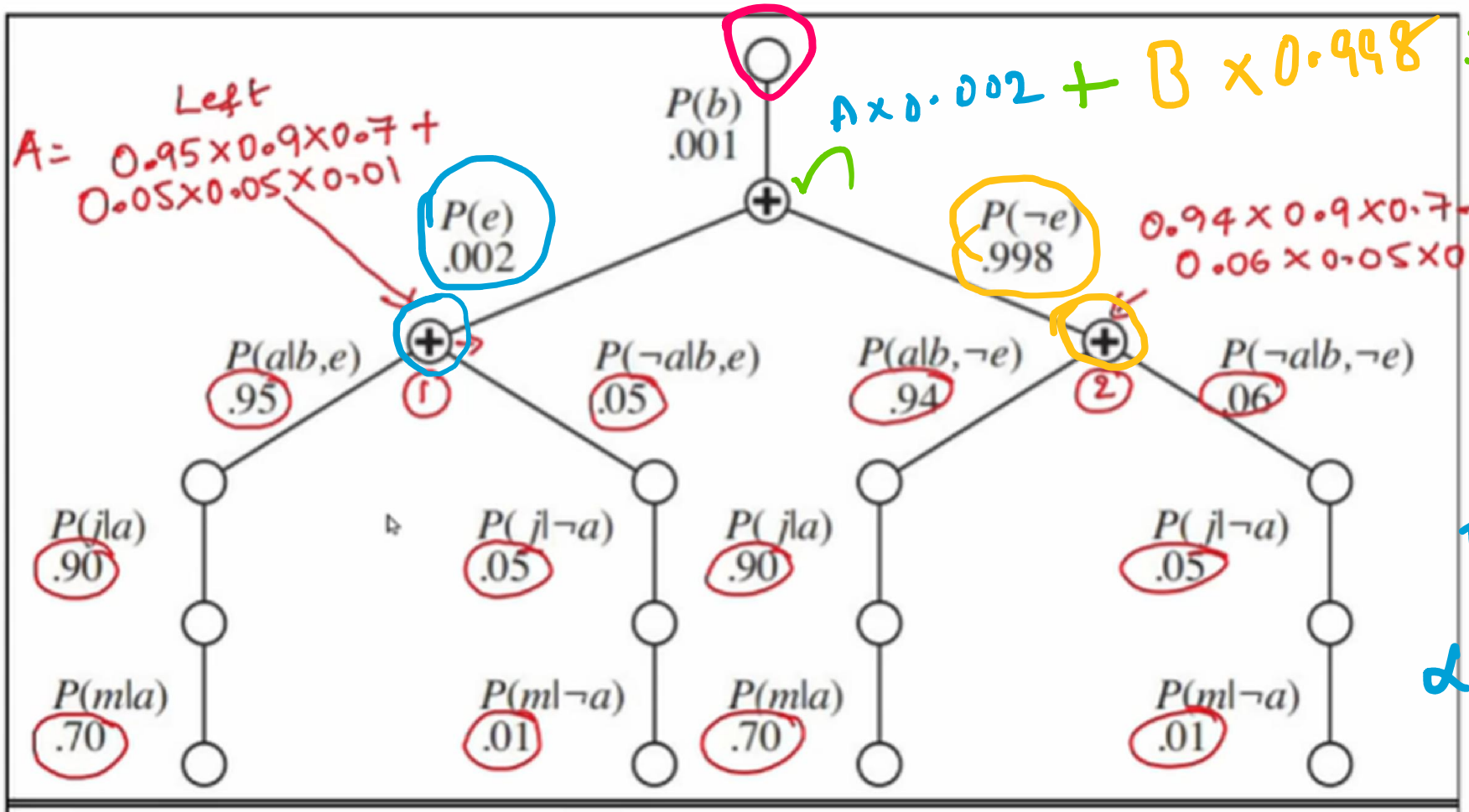
Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice

$P(b) = 0.00059224$

$P(\neg b) = 0.0014919$

need to construct similar tree for $\neg b$

Final => C × 0.001



Left

A = 0.95 × 0.9 × 0.7 + 0.05 × 0.05 × 0.01

A × 0.002 + B × 0.998 = C

0.94 × 0.9 × 0.7 + 0.06 × 0.05 × 0.01 } B

P(b) = α(0.0005922, 0.0014919)

Figure 14.8 The structure of the expression shown in Equation (14.4). The evaluation proceeds top down, multiplying values along each path and summing at the "+" nodes. Notice

$$\alpha = \frac{1}{(P(b,j,m) + P(\neg b,j,m))}$$

$$\alpha = \frac{1}{(.00059 + .0015)}$$

$$= 478.5$$

$$P(b \mid j, m) = \propto * P(b, j, m)$$

$$= 478.5 * .00059$$

$$= 0.28$$

$$P(\neg b \mid j,m) = \propto * P(\neg b, j, m)$$

$$= 478.5 * .0015$$

$$= 0.72$$

How to normalize it?

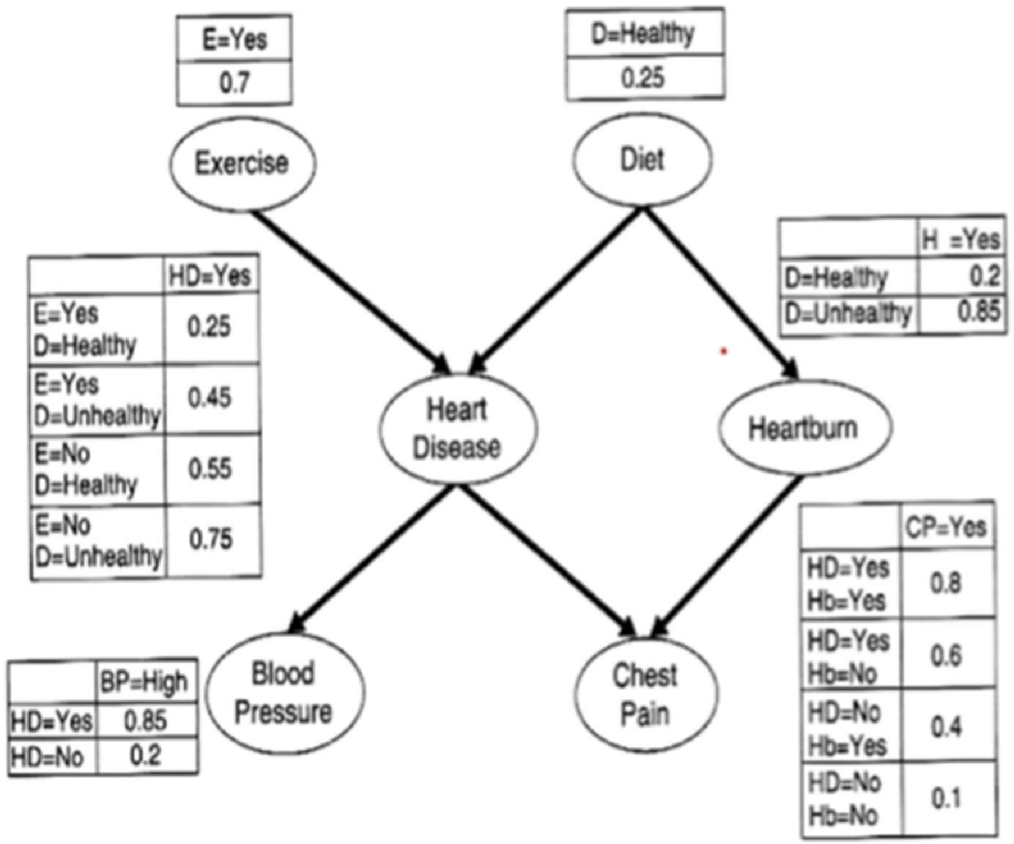
$$P(h) = \frac{0.00059224}{0.00059224 + 0.0014919} = 0.284$$

$$P(\neg h) = \frac{0.0014919}{0.00059224 + 0.0014919} = 0.716$$

final value $\Rightarrow \langle 0.284, 0.716 \rangle$

Problem – 2- Variable Elimination

Bayesian Belief Network



Heart Disease Problem

Solved Example #6

Given problem

E=Yes
0.7

D=Healthy
0.25

	BP=High
HD=Yes	0.85
HD=No	0.2

	HD=Yes
E=Yes D=Healthy	0.25
E=Yes D=Unhealthy	0.45
E=No D=Healthy	0.55
E=No D=Unhealthy	0.75

	CP=Yes
HD=Yes Hb=Yes	0.8
HD=Yes Hb=No	0.6
HD=No Hb=Yes	0.4
HD=No Hb=No	0.1

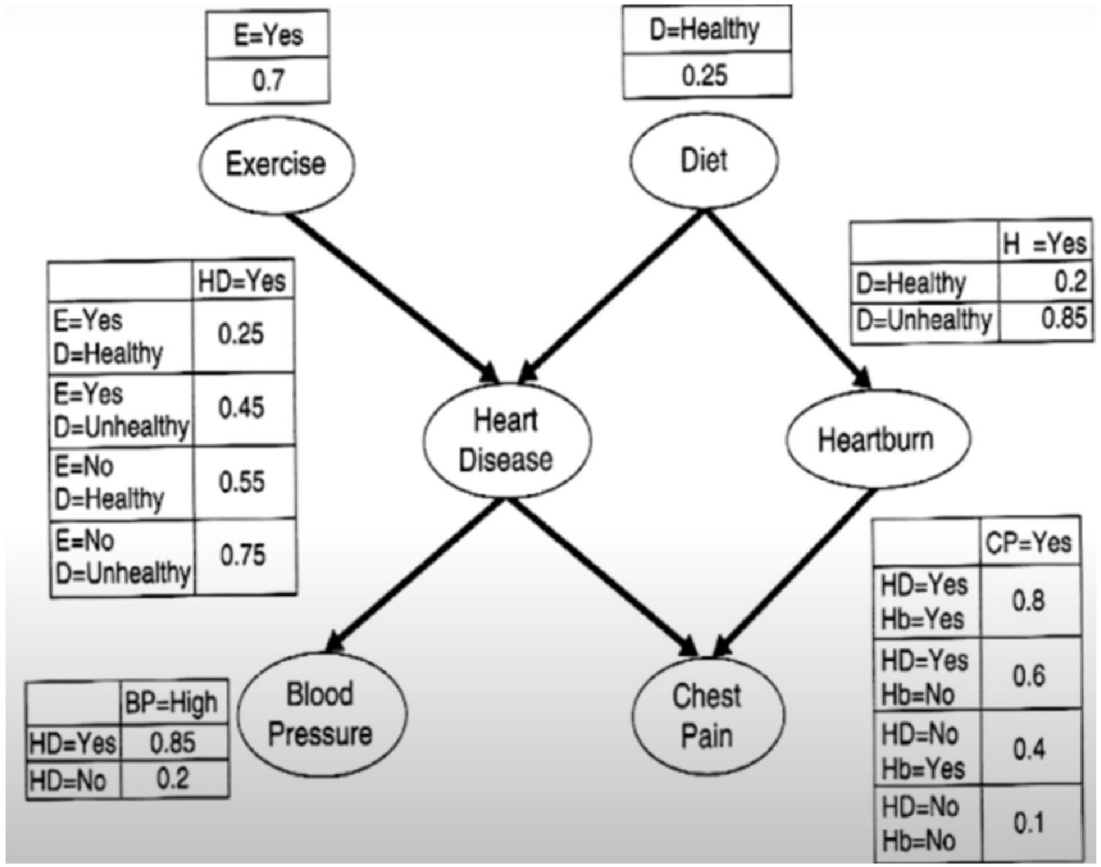
	H =Yes
D=Healthy	0.2
D=Unhealthy	0.85

- Suppose we get to know that the new patient has **high blood pressure.**
- What's the probability he has **heart disease** under this condition?

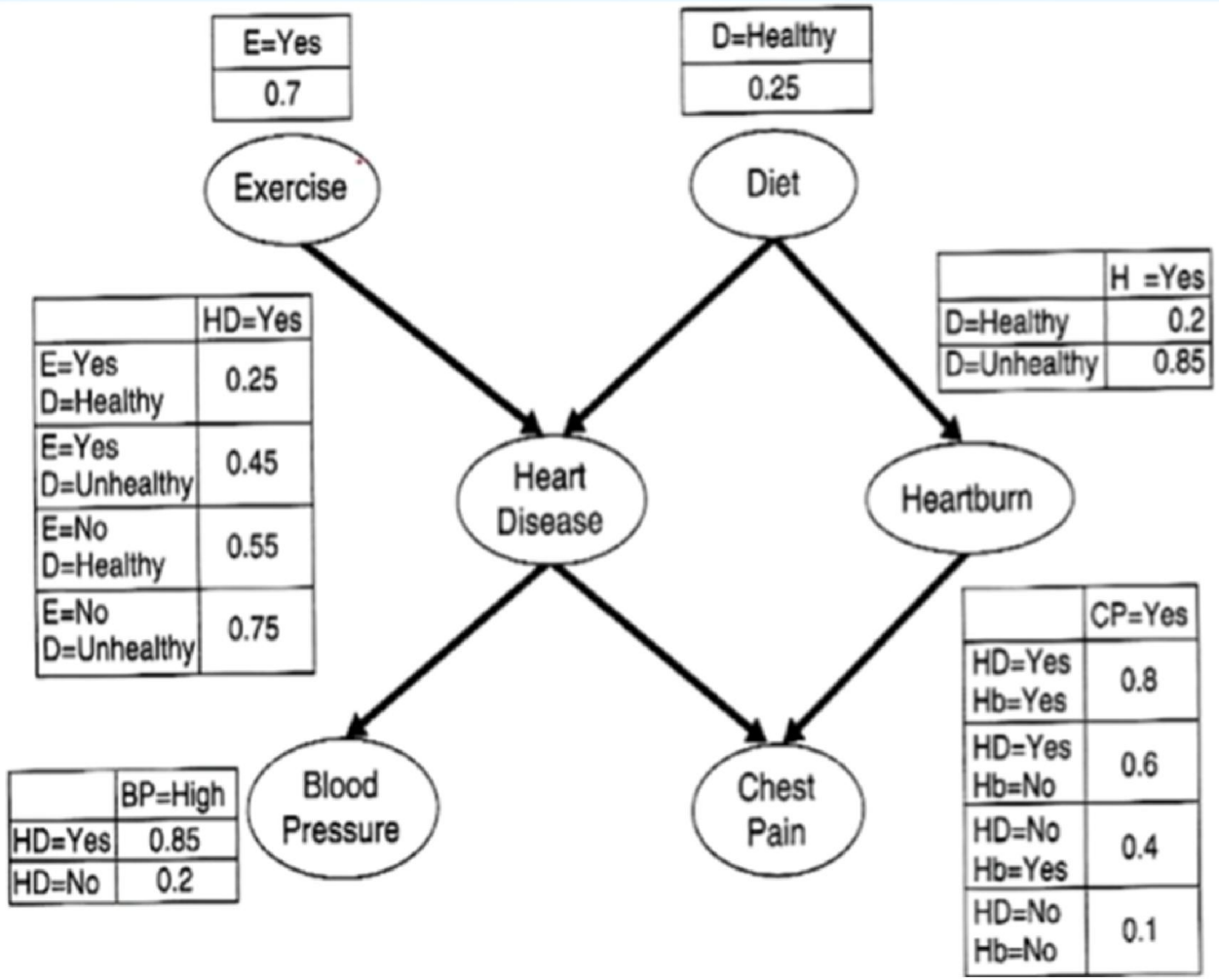
Solution

Construct a Bayesian Network.

- Using conditional Probability in CPT tables, Construct it



Bayesian Belief Network – Heart Disease Problem Solved Example - 6



- Suppose we get to know that the new patient has **high blood pressure.**
- What's the probability he has **heart disease** under this condition?

$P(hd \mid bp)$ $\quad$ $P(\neg hd \mid bp)$

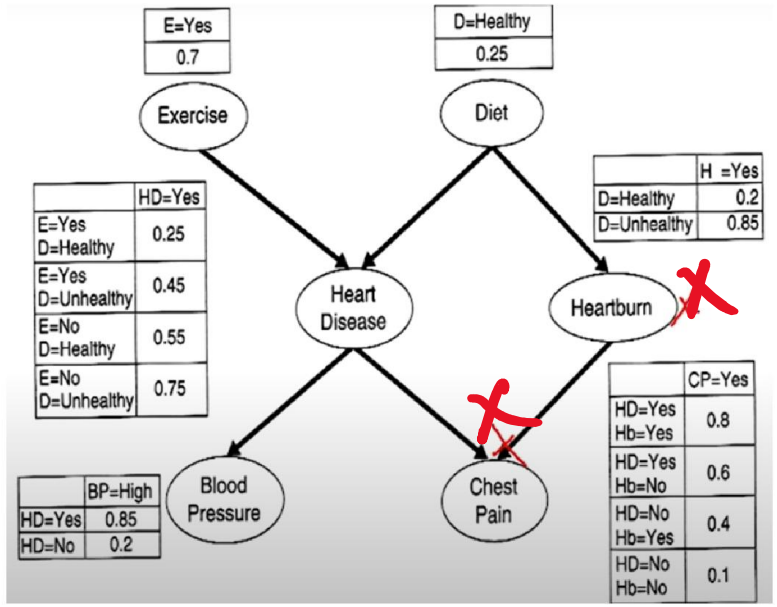
$$P(hd \mid bp) = \alpha \sum_{e} \sum_{d} \sum_{h} \sum_{cp} P(hd, e, d, h, cp, bp)$$

$$P(hd \mid bp) = \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd, e, d, h, cp, bp)$$

$$= \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd \mid e, d) P(h \mid d) P(cp \mid hd, h) P(bp \mid hd) P(e) P(d)$$

Leave these 2 Terms

$$= \alpha P(bp \mid hd) \sum_e P(e) \sum_d P(hd \mid e, d) P(d) \sum_h P(h \mid d) \sum_{cp} P(cp \mid hd, h)$$



We want to find $P(hd \mid bp)$

hd depends on < Exercise, Diet

WKT, Person with bp does not depend on < chestpain, Heartburn

$$P(hd \mid bp) = \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd, e, d, h, cp, bp)$$

$$= \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd \mid e, d) P(h \mid d) P(cp \mid hd, h) P(bp \mid hd) P(e) P(d)$$

$$= \alpha P(bp \mid hd) \sum_e P(e) \sum_d P(hd \mid e, d) P(d) \sum_h P(h \mid d) \sum_{cp} P(cp \mid hd, h)$$

$$= \alpha P(bp \mid hd) \sum_e P(e) \sum_d P(hd \mid e, d) P(d)$$

$$= \alpha P(bp \mid hd) P(e) \left(P(hd \mid e, d) P(d) + P(hd \mid e, \neg d) P(\neg d) \right)$$

$$+ \alpha P(bp \mid hd) P(\neg e) \left(P(hd \mid \neg e, d) P(d) + P(hd \mid \neg e, \neg d) P(\neg d) \right)$$

$$P(hd \mid bp) = \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd, e, d, h, cp, bp)$$

$$= \alpha \sum_e \sum_d \sum_h \sum_{cp} P(hd \mid e, d) P(h \mid d) P(cp \mid hd, h) P(bp \mid hd) P(e) P(d)$$

$$= \alpha P(bp \mid hd) \sum_e P(e) \sum_d P(hd \mid e, d) P(d) \sum_h P(h \mid d) \sum_{cp} P(cp \mid hd, h)$$

$$= \alpha P(bp \mid hd) \sum_e P(e) \sum_d P(hd \mid e, d) P(d)$$

$$= \alpha P(bp \mid hd) P(e) \big(P(hd \mid e, d) P(d) + P(hd \mid e, \neg d) P(\neg d)\big)$$
$$+ \alpha P(bp \mid hd) P(\neg e) \big(P(hd \mid \neg e, d) P(d) + P(hd \mid \neg e, \neg d) P(\neg d)\big)$$

$$= \alpha * 0.85 * 0.7 * (0.25 * 0.25 + 0.45 * 0.75) + \alpha * 0.85 * 0.3 * (0.55 * 0.25 + 0.75 * 0.75)$$

$$= \alpha * 0.4165$$

We found $p(hd \mid bp) = \alpha * 0.4165$

$$P(\neg hd \mid bp)$$

?

$$P(\neg hd \mid bp) = \alpha \sum_e \sum_d \sum_h \sum_{cp} P(\neg hd, e, d, h, cp, bp)$$

$$= \alpha \sum_e \sum_d \sum_h \sum_{cp} P(\neg hd \mid e, d) P(h \mid d) P(cp \mid \neg hd, h) P(bp \mid \neg hd) P(e) P(d)$$

$$= \alpha P(bp \mid \neg hd) \sum_e P(e) \sum_d P(\neg hd \mid e, d) P(d) \sum_h P(h \mid d) \sum_{cp} P(cp \mid \neg hd, h)$$

$$= \alpha P(bp \mid \neg hd) \sum_e P(e) \sum_d P(\neg hd \mid e, d) P(d)$$

$$= \alpha P(bp \mid \neg hd) P(e) \left(P(\neg hd \mid e, d) P(d) + P(\neg hd \mid e, \neg d) P(\neg d) \right)$$
$$+ \alpha P(bp \mid \neg hd) P(\neg e) \left(P(\neg hd \mid \neg e, d) P(d) + P(\neg hd \mid \neg e, \neg d) P(\neg d) \right)$$

$$= \alpha * 0.2 * 0.7 * (0.75 * 0.25 + 0.55 * 0.75) + \alpha * 0.2 * 0.3 * (0.45 * 0.25 + 0.25 * 0.75)$$

$$P(\neg hd \mid bp) = \alpha \sum_e \sum_d \sum_h \sum_{cp} P(\neg hd, e, d, h, cp, bp)$$

$$= \alpha \sum_e \sum_d \sum_h \sum_{cp} P(\neg hd \mid e, d) P(h \mid d) P(cp \mid \neg hd, h) P(bp \mid \neg hd) P(e) P(d)$$

$$= \alpha P(bp \mid \neg hd) \sum_e P(e) \sum_d P(\neg hd \mid e, d) P(d) \sum_h P(h \mid d) \sum_{cp} P(cp \mid \neg hd, h)$$

$$= \alpha P(bp \mid \neg hd) \sum_e P(e) \sum_d P(\neg hd \mid e, d) P(d)$$

$$= \alpha P(bp \mid \neg hd) P(e) \big(P(\neg hd \mid e, d) P(d) + P(\neg hd \mid e, \neg d) P(\neg d) \big)$$
$$+ \alpha P(bp \mid \neg hd) P(\neg e) \big(P(\neg hd \mid \neg e, d) P(d) + P(\neg hd \mid \neg e, \neg d) P(\neg d) \big)$$

$$= \alpha * 0.2 * 0.7 * (0.75 * 0.25 + 0.55 * 0.75) + \alpha * 0.2 * 0.3 * (0.45 * 0.25 + 0.25 * 0.75)$$

$$= \alpha * 0.102$$

$$P(hd \mid bp) = \alpha * 0.4165$$

$$P(\neg hd \mid bp) = \alpha * 0.102$$

$$\alpha = \frac{1}{0.4165 + 0.1020} = \frac{1}{0.5185}$$

$$P(hd \mid bp) = \alpha * 0.4165$$

$$P(\neg hd \mid bp) = \alpha * 0.102$$

$$\alpha = \frac{1}{0.4165 + 0.1020} = \frac{1}{0.5185}$$

$$P(hd \mid bp) = \alpha * 0.4165 = 0.8033$$

$$P(\neg hd \mid bp) = \alpha * 0.102 = 0.1967$$