Chapter 5 Solution to Problems

1. A C-band satellite link sends a single NTSC-TV signal through a 36 MHz transponder on a C-band GEO satellite. The NTSC video signal is modulated onto the carrier using wideband frequency modulation, and the bandwidth of the transmitted RF signal is 32 MHz. The baseband bandwidth of the TV signal is 4.2 MHz.

a. Calculate the peak frequency deviation of the FM carrier using Carson's rule.

Answer: Carson's rule gives the bandwidth of an FM signal in terms of the peak frequency deviation, $\Delta f_{pk,}$ and the maximum baseband frequency, f_{max}.

$$B = 2 (\Delta f_{pk} + f_{max})$$

Hence the peak frequency deviation can be found as

$$\Delta f_{pk} = B/2 - f_{max}$$

For $B = 32$ MHz and $f_{max} = 4.2$ MHz

$$\Delta f_{pk} = 16 – 4.2 = 11.8 \text{ MHz}$$

b. Calculate the unweighted FM improvement factor for the video signal.

Answer: The unweighted FM improvement factor is given by

$$\text{Improvement in S/N} = 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} /f_{max}) + 1.8 \text{ dB}$$

Using the results from part (a) above,

$$\text{Improvement} = 10 \log (32 / 4.2) + 20 \log(11.8 / 4.2) + 1.8 \text{ dB}$$

$$= 8.8 + 9.0 + 1.8 = 19.6 \text{ dB}$$

c. The overall C/N in an earth station receiving the FM-TV transmission is 17 dB. What is the unweighted video S/N ratio at baseband?

Answer: The unweighted S/N ratio at baseband is the receiver C/N ratio plus the FM improvement

$$S/N_{unweighted} = C/N + \text{FM improvement}$$

Using the value for improvement obtained in part (b) above and C/N = 17 dB

$$S/N_{unweighted} = 17 + 19.6 = 36.6 \text{ dB}$$

d. De-emphasis and weighting factors improve the S/N of the baseband signal by a subjective factor of 17 dB. What is the weighted S/N of the baseband video signal?

Answer: Adding the weighting factors to the unweighted S/N ratio gives the weighted S/N

$$S/N_{weighted} = 36.6 + 17 = 53.6 \text{ dB}$$

2. When overall C/N is sufficiently high, it is possible to transmits two FM-TV signals in one 36 MHz transponder. The signal to noise ratio improvement is reduced when two TV signals are transmitted rather than one because the frequency deviation must be reduced. Two NTSC FM-TV signals are transmitted through a 36 MHz bandwidth transponder. The bandwidth of each signal is 16 MHz.

a. Calculate the peak frequency deviation of the FM signal using Carson's rule.

Answer: Carson's Rule gives the bandwidth required for transmission of FM signals as

$$B = 2 (\Delta f_{pk} + f_{max})$$

The maximum baseband frequency for a NTSC TV signal is $f_{max} = 4.2$ MHz.

Hence the peak frequency deviation is found from

$$16 \text{ MHz} = 2 (\Delta f_{pk} + 4.2 \text{ MHz})$$

$$\Delta f_{pk} = 8 - 4.2 = 3.8 \text{ MHz}$$

b. Calculate the unweighted S/N in the baseband video bandwidth of 4.2 MHz for an overall C/N ratio in the earth station receiver of $(C/N)_o$.

Answer: Unweighted S/N ratio for an FM TV signal is given by

$$S/N = (C/N)_o + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB}$$

$$= (C/N)_o + 10 \log (16 / 4.2) + 20 \log (3.8 / 4.2) + 1.8 \text{ dB}$$

$$= (C/N)_o + 5.8 - 0.9 + 1.8 \text{ dB}$$

$$= (C/N)_o + 6.7 \text{ dB}$$

c. What value must $(C/N)_o$ have to ensure that the unweighted (S/N) of the video signal is 33 dB?

Answer: The value of overall C/N at the earth station receiver must be

$$(C/N)_o = 33 - 6.7 = 26.3 \text{ dB}$$

d. Use the value of $(C/N)_o$ you found in part (c) above to find the baseband video S/N ratio in clear air conditions. The de-emphasis and subjective weighting factors for the video signal total 17 dB. If the value of $(C/N)_o$ at the earth station receiver falls by 4 dB because of rain in the downlink path, what is the weighted baseband video S/N? How would you rate the quality of the video signal?

Answer: Subjective improvement and pre/de-emphasis improvement adds 17 dB to the unweighted S/N to yield the weighted S/N.

$$\text{Weighted S/N} = 26.3 + 17 = 43.3 \text{ dB}$$

If the $(C/N)_o$ value falls by 4 dB, from 26.3 dB to 22.3 dB, we are still well above the FM threshold. S/N will fall by the same amount, so weighted S/N = 39.3 dB.

The video signal in clear air has S/N = 43.3 dB, which is a reasonable quality signal. There would be some perceptible noise in the TV picture. With S/N = 39 dB, noise would be visible, marginally to an annoying extent, and the TV picture would be rated as acceptable quality.

3. In Problem #2, two NTSC video signals are transmitted as FM carriers in a bandwidth of 36 MHz. Each FM carrier occupies a bandwidth of 16 MHz. A digital T1 carrier with a bandwidth of 2.0 MHz can be sent through the same transponder by using a gap between the two FM carriers. Some of the transponder power must be devoted to the T1 carrier, with the result that the FM-TV carriers have reduced C/N at the earth station and lower video S/N at baseband. This question asks you to determine the reduction in video S/N. You will need to solve problem #2 before attempting this problem.

a. The power at the output of the transponder must be shared between the three RF signals in proportion to bandwidth occupied by each signal. For convenience, assume that the transponder radiates a total power of 20 watts. Calculate the power allocated to each signal when only two FM-TV signals are transmitted, and when all three signals are transmitted.

Answer: When two TV signals are transmitted, power at the transponder output must be shared equally between them. Hence each carrier gets 10 W.

When an additional T1 signal is added, power must be shared in proportion to bandwidth occupied by each signal to keep PSD across the transponder constant.

Total bandwidth occupied = 34 MHz.

Total power radiated = 20 watts

Power per MHz = 20/34 W/MHz = 0.588 W/MHz.

Each TV signal gets 0.588 x 16 = 9.41 W

T1 carrier gets 0.588 x 2 = 1.18 W

b. Using the results from part (a) above, determine the reduction in C/N of the FM-TV signals. Hence find the reduction in baseband video S/N and the new value of unweighted video S/N ratio, based on results from part (d) of Problem #2.

Answer: The TV signals were transmitted at a power of 10 W with two signals in the transponder. When the third signal is added, transmitted power drops by 10 log (9.41/10) dB = 0.3 dB. Hence the unweighted S/N drops by 0.3 dB from 33.0 to 32.7 dB.

c. What is the overall C/N ratio at the earth station receiver for the T1 carrier?

Answer: The overall C/N for the T1 carrier is the same as for the TV carriers, 26.0 dB. The T1 carrier gets less power, but also is received against a lower noise background because of its narrower bandwidth. The effects are in proportion, so the C/N value is the same for both the carrier and the T1 signal.

5. A satellite telemetry link operating in S-band uses frequency modulation to transmit the value of an analog voltage on the satellite to a receiving earth station. The voltage has a range from –1.0 volts to + 1.0 volts, and a maximum frequency of 1000 Hz. The FM modulator on the satellite has a constant of 10,000 Hz per volt. At the receiving earth station the C/N ratio of this signal is 10 dB measured in the Carson's rule bandwidth, and is 3 dB above the FM threshold of the FM demodulator.

a. What is the Carson's rule bandwidth for the FM signal?

Answer: We must first calculate the peak frequency deviation for this signal.

The frequency deviation of an FM signal is given by

$$\Delta f = k_f\, m(t)$$

where k_f is the modulator constant and m(t) is the modulating signal. Hence the peak frequency deviation is given by

$$\Delta f_{pk} = k_f\, m(t)_{max}$$

The question gives $m(t)_{max}$ as ± 1 volt, and k_f as 10 kHz/volt.

Hence $\Delta f_{pk} = 10$ kHz.

Applying Carson's rule with $f_{max} = 1$ kHz

$$B = 2(\Delta f_{pk} + f_{max}) = 2 \times (10 + 1) = 22 \text{ kHz}$$

b. What is the baseband S/N ratio at the earth station receiver output for the recovered analog signal?

Answer: The C/N at the receiver input is given as 10 dB, so the unweighted S/N is

$$S/N = C/N + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB}$$

$$= 10.0 + 10 \log (22/1) + 20 \log (10/1) + 1.8 \text{ dB}$$

$$= 10 + 13.4 + 20 + 1.8 = 45.2 \text{ dB}$$

6. A satellite link has an RF channel with a bandwidth 2.0 MHz. The transmitter and receiver have RRC filters with $\alpha = 0.5$. What is correct symbol rate (pulse rate) for this link?

Answer: For any RF channel, the bandwidth occupied by a digital signal with a symbol rate R_s is $B = R_s (1 + \alpha)$

Hence the symbol rate with a bandwidth of 2.0 MHz and $\alpha = 0.5$ is

$$R_s = B / 1.5 = 1.333 \text{ Mbaud (Msps)}$$

7. A Ku band satellite uplink has a carrier frequency of 14.125 GHz and carries a symbol stream at $R_s = 16$ Msps. The transmitter and receiver have ideal RRC filters with $\alpha = 0.25$. What is bandwidth occupied by RF signal, and what is the frequency range of the transmitted RF signal?

Note: There is a typo in the text in the first printing that gives the frequency as 14.125 MHz instead of GHz.

Answer: Using the same rule as in Q # 6 above

$$B = R_s (1 + \alpha) = 16.0 (1 + 0.25) = 20.0 \text{ MHz}$$

The signal occupies the frequency range 14.115 GHz to 14.135 GHz.

8. A T1 data transmission system transmits data at 1.544 Mbps over a GEO satellite link At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB. The modulation used on the link is BPSK and the implementation margin of the BPSK demodulator is 0.5 dB.

a. Find the BER at the receiver output and the average time between errors.

Answer: For a BPSK link, the BER is given by Equation 5.65

$$P_e = Q [\sqrt{(2\ C/N)_{effective}}]$$

With C/N = 16 dB and an implementation margin of 0.5 dB,

$$(C/N)_{effective} = 16.0 – 0.5 = 15.5 \text{ dB} = 10^{15.5/10} = 35.5 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q [\sqrt{(2 \times 35.5)}] = Q[\sqrt{71.0}] = Q[8.43]$$

Using the Q(z) table in Appendix C, BER < 10^{-16}

Data is delivered at 1.544×10^6 bps, so an error occurs less frequently than 10^{-10} seconds.

There are $3600 \times 24 \times 365 = 3.1536 \times 10^7$ seconds in a year, so errors occur (theoretically) at a rate of about one every 300 years - which means there are no errors on this link.

b. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now?

Answer: With the C/N at 10.0 dB, $(C/N)_{effective} = 9.5$ dB = 8.91 as a ratio.

Hence the probability of a bit error is

$$P_e = Q [\sqrt{(2 \times 8.91)}] = Q[\sqrt{17.82}] = Q[4.22]$$

Using the Q(z) table in Appendix C, and interpolating between entries for z = 4.2 and z = 4.3,

$$\text{BER} \approx 10^{-6}$$

The data rate is 1.544 Mbps, so there are 1.544 bit errors per second, on average.

9. A satellite data transmission system transmits data from two T1 carriers as a single 3.088 Mbps bit stream using QPSK . The symbol rate on the link is 1.544 Msps. The satellite link uses ideal RRC filters with $\alpha = 0.25$. At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB and the implementation margin of the QPSK demodulator is 1 dB.

a. What is the bandwidth occupied by this signal, and the noise bandwidth of the receiver for this signal?

Answer: From equation 5.32, the bandwidth occupied by any digital signal with a symbol rate R_s is

$$B_{occ} = R_s (1 + \alpha)$$

Hence for a symbol rate of 1.544 Msps and $\alpha = 0.25$

$$B_{occ} = 1.544 \times 10^6 \times (1 + 0.25) = 1.93 \text{ MHz}$$

In every case where RRC filters are used in the receiver, the noise bandwidth B_N is equal to the symbol rate. Hence $B_N = 1.544$ MHz

b. Find the BER at the receiver output and the average time between errors.

Answer: For a QPSK link, the BER is given by Equation 5.68

$$P_e = Q [\sqrt{(C/N)_{effective}}]$$

With C/N = 16 dB and an implementation margin of 1.0 dB,

$$(C/N)_{effective} = 16.0 - 1.0 = 15.0 \text{ dB} = 10^{15.0/10} = 31.6 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q [\sqrt{(31.6)}] = Q[5.62]$$

Using the Q(z) table in Appendix C, BER $\approx 1 \times 10^{-8}$

Data is delivered at 3.088×10^6 bps, so an error occurs once every 32.4 seconds.

c. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now?

Answer: For the QPSK link, the BER is given by Equation 5.68

$$P_e = Q\,[\sqrt{(C/N)_{effective}}\,]$$

With C/N = 10 dB and an implementation margin of 1.0 dB,

$$(C/N)_{effective} = 10.0 - 1.0 = 9.0 \text{ dB} = 10^{9.0/10} = 7.94 \text{ as a ratio}$$

Hence the BER is given by

$$P_e = Q\,[\sqrt{(7.94)}\,] = Q[2.82]$$

Using the Q(z) table in Appendix C, and interpolating between entries for z = 2.8 and z = 2.9, BER $\approx 2.4 \times 10^{-3}$

Data is delivered at 3.088×10^6 bps, so there are $3.088 \times 2.4 \times 10^3 = 5488$ bit errors per second. FEC would be needed to maintain a more reasonable error rate on this link.

10 a. A 36 MHz bandwidth transponder is used to carry digital signals. A 20 MHz bandwidth in the transponder is occupied by a QPSK signal generated by a transmitter with ideal Nyquist filters with parameter alpha = 0.25. What is the symbol rate of the QPSK signal in Msps? What is the bit rate of the QPSK signal?

Answer: From equation 5.32, the bandwidth occupied by any digital signal with a symbol rate R_s is $B_{occ} = R_s\,(1 + \alpha)$

Hence for an occupied bandwidth of 20 MHz, and $\alpha = 0.25$, the symbol rate is R_s where

$$B_{occ} = R_s\,(1 + 0.25)$$

Hence $R_s = 16.0$ MHz.

Since we are using QPSK, there are two bits transmitted with every QPSK symbol, so

$$R_b = 2 \times 16 = 32 \text{ Mbps.}$$

b. Under clear air conditions, the overall $(C/N)_o$ ratio in the earth station receiver is 18.0 dB If the QPSK demodulator has an implementation margin of 1.5 dB, what is the Bit Error Rate of the baseband digital signal in clear air conditions? How often does a bit error occur. (Give your answer in days, hours, minutes, or seconds, as appropriate.)

Answer: For a QPSK link, the BER is given by Equation 5.68

$$P_e = Q\,[\sqrt{(C/N)_{effective}}\,]$$

With C/N = 18.0 dB and an implementation margin of 1.5 dB,

$(C/N)_{effective} = 18.0 - 1.5 = 16.5 \text{ dB} = 10^{16.5/10} = 44.7$ as a ratio

Hence the BER is given by

$P_e = Q [\sqrt{(44.7)}] = Q[6.68]$

Using the Q(z) table in Appendix C, BER $\approx 1.2 \times 10^{-12}$

Data is delivered at 32×10^6 bps, so an error occurs once every $32 \times 10^6 \times 1.2 \times 10^{-12}$

$= 3.84 \times 10^{-5}$ seconds.

Put another way, we have one error every 26042 seconds.

An error occurs on the link, on average, every 26042 seconds = 7 hours 14 minutes 2 seconds.

c. Under rain conditions, the overall $(C/N)_o$ ratio of the QPSK signal in part (a) above falls to 14.3 dB at a receiving station. What Bit Error Rate would you expect in the recovered bit stream? How often does a bit error occur?

Answer: For a QPSK link, the BER is given by Equation 5.68

$P_e = Q [\sqrt{(C/N)_{effective}}]$

With C/N = 14.3 dB and an implementation margin of 1.5 dB,

$(C/N)_{effective} = 14.3 - 1.5 = 12.8 \text{ dB} = 10^{1.28} = 19.05$ as a ratio

Hence the BER is given by

$P_e = Q [\sqrt{(19.05)}] = Q[4.37]$

Using the Q(z) table in Appendix C, BER $\approx 6.1 \times 10^{-6}$

Data is delivered at 32×10^6 bps, so an errors occurs at a rate of 195 per second.

11. A satellite communication system is built as a star network with one large hub station and many remote small earth stations. The system operates at Ka band using the K9 geostationary satellite, and carries digital signals which may be voice, data, or compressed video. The K9 satellite has transponders with a bandwidth of 60 MHz that can be operated in either of two modes: as a bent pipe or with a 40 Msps QPSK baseband processor.

The outbound link from the hub to the remote stations has an uplink from the hub station to the satellite that is the input of transponder #1. Signals from the hub are transmitted using a single TDM carrier and QPSK modulation with a symbol rate of 40 Msps. In the initial system design the remote earth stations use receivers capable of receiving 40 Msps QPSK signals. The transponder is operated in bent pipe mode with sufficient back-off to make it linear. The hub transmitter operates at an output power of 100 W, which gives C/N = 30 dB in the transponder in clear air, measured in the correct noise bandwidth of a 40 Msps QPSK receiver equipped with RRC filters having $\alpha = 0.4$. In clear air conditions, the resulting C/N of the earth station receiver, ignoring noise transmitted by the satellite, is 20 dB. The receiver has a QPSK demodulator with an implementation margin of 1.0 dB. For the purposes of this question you may assume that all transmitters and receivers in the network and on the K9 satellite have ideal RRC filters.

a. Find the overall C/N in the earth station receiver in clear air conditions and estimate the bit error rate for the recovered data signal, assuming that FEC is not used. What is the correct noise bandwidth for the earth station receiver that receives the QPSK signal, and what is the bit rate of the link?

Answer: The overall C/N must be found from the reciprocal formula

$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn}$ Convert the C/N and C/I values to ratios:

$(C/N)_{up} = 30$ dB or 1000, $(C/N)_{dn} = 20$ dB or 100, hence

$$1 / (C/N)_o = 1 / 1000 + 1 / 100 = 0.011$$

$$(C/N)_o = 90.9 \text{ or } 19.6 \text{ dB}$$

The symbol rate on the link is 40 Msps, so the noise bandwidth of the RRC bandpass filter in the receiver must be 40 MHz. The bandwidth occupied by the QPSK signal is $R_s \times (1 + \alpha)$. Hence

$$B_{occ} = 40 \times 10^6 \times (1 + 0.4) \text{ Hz} = 56 \text{ MHz.}$$

The bit error rate for QPSK signals is given by $P_e = Q[\sqrt{(C/N)_{eff}}]$.

$$(C/N)_{eff} = (C/N)_o - \text{implementation margin} = 19.6 - 1.0 = 18.6 \text{ dB or } 72.44$$

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[8.51] \text{ which is a number less than } 10^{-16}.$$

There are no errors on this link in clear air conditions.

Bit rate on the link is $R_b = 2 \times R_s = 80$ Mbps.

b. An uplink fade occurs which causes an attenuation of 10 dB between the hub station and the satellite. The transponder is operated in bent pipe mode. Find the overall C/N in the remote earth station receiver and estimate the BER of the recovered data.

Answer: We will assume a linear transponder, so that the uplink attenuation of 10 dB causes $(C/N)_{up}$ to fall by 10 dB to 20 dB, and the output power from the transponder to fall by 10 dB also. Then $(C/N)_{dn}$ falls by 10 dB to a new value of 10 dB and $(C/N)_o = 9.6$ dB.

The bit error rate for the QPSK receiver is given by $P_e = Q[\sqrt{(C/N)_{eff}}]$.

$(C/N)_{eff} = (C/N)_o$ – implementation margin $= 9.6 - 1.0 = 8.6$ dB or 7.24

$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[2.69] \approx 3.6 \times 10^{-3}$.

There are now thousands of errors every second and the link is in an outage.

(Note: The noise temperature of the satellite receiver is unaffected by uplink attenuation, so $(C/N)_{up}$ falls by 10 dB when there is 10 dB rain attenuation on the uplink.

c. An uplink fade occurs which causes an attenuation of 10 dB between the hub station and the satellite. Transponder #1 is switched to operate with the 40 Msps baseband processor.

The QPSK demodulator in transponder #1 has an implementation margin of 1 dB. Find the overall C/N in the remote earth station receiver and estimate the BER.

Answer: With a baseband processor on the satellite, the uplink and downlink are independent, and the bit error rates on the uplink and downlink add. (This assumes low error rates, say $< 10^{-4}$ on each link). With 10 dB rain attenuation on the uplink $(C/N)_{up} = 20$ dB, and $(C/N)_{dn} = 20$ dB also. On both links the effective C/N ratio is 19.6 dB, so the error rate is the same as in part (a): there are no errors on either link.

d. Rainfall statistics for the location of the hub station show that attenuation at the uplink frequency will exceed 20 dB for 0.01% of an average year. If the hub station uses uplink power control to mitigate the effects of uplink rain attenuation, determine the maximum uplink transmitter power (in watts) that must be transmitted to ensure that the link BER does not exceed 10^{-6} at the remote earth station receiver output for 99.99% of an average year when:

(i) A linear bent pipe transponder is used

(ii) A 40 Msps QPSK baseband processing transponder is used.

Answer: The requirement here is that the bit error rate at the earth station receiver output not exceed 10^{-6} for more than 0.01% of a year (about 52 minutes per year).

(i) Linear bent pipe transponder.

The overall $(C/N)_o$ must not exceed the value that gives BER = 10^{-6}.

From the Q(z) table in Appendix C, $Q(z) = 10^{-6}$ gives $z = 4.76$ and $z^2 = (C/N)_{eff}$ for QPSK. Hence $(C/N)_{eff} = 22.66$ or 13.6 dB and $(C/N)_o = 13.6 + 1.0 = 14.6$ dB. In an uplink fade due to rain attenuation, both $(C/N)_{up}$ and $(C/N)_{dn}$ are affected equally, causing $(C/N)_o$ to fall directly in proportion to the uplink attenuation. Under clear air conditions, $(C/N)_o$ was 19.6 dB, allowing 5.0 dB of uplink fading before the BER limit is reached.

Uplink power control (UPC) must compensate for 15 dB of rain fading to ensure that the overall C/N ratio at the earth station receiver does not fall below 14.6 dB with 20 dB rain attenuation in the uplink. This is a substantial increase in transmitter power – a factor of 31.6.

Check: The limiting condition is when there is 5.0 dB rain attenuation on the uplink. Then $(C/N)_{up} = 25.0$ dB, $(C/N)_{dn} = 15.0$ dB, $(C/N)_o = 28.75$ or 14.6 dB, as required.

(ii) 40 Msps QPSK baseband processing transponder.

A baseband processor on the satellite makes the uplink and downlink independent.

The downlink $(C/N)_{dn} = 20$ dB regardless of uplink fading., and BER for the downlink is zero. We can therefore allow the uplink C/N ratio to fall until $(C/N)_{up+} = 14.6$ dB, when BER for the uplink is 10^{-6}. Thus rain attenuation of 30.0 – 14.6 = 15.4 dB can be tolerated before UPC must be applied, and the dynamic range of the UPC system needs to be only 4.6 dB, a factor of 2.9.

e. Discuss the value of UPC at the hub station transmitter in this application. Would you recommend a linear transponder or a baseband processing transponder be used on the K9 satellite? Give reasons for your answer.

Answer: Uplink power control is effective at preventing the downlink from going into outage because of rain attenuation on the uplink, when a linear transponder is used. Two factors tend to make this happen more often than can be allowed in many Ku band satellite links: the uplink is always at a higher frequency and therefore suffers more rain attenuation than the downlink, and the downlink C/N ratio is often lower than the uplink C/N ratio. Without UPC, a typical Ku band link is likely to fail because of uplink rain attenuation. With UPC applied, this effect can be ameliorated and outages are then caused primarily by rain attenuation on the downlink.

With a linear transponder the dynamic range of the UPC may need to be large – 31.6 in the example above. If clear air conditions require a transmit power level $P_t = 100$ W, uplink rain attenuation of 20 dB requires an increase to 3,160 W. This happens for less than one hour per year, on average, so the expensive high power transmitter is run at a low power setting most of the time.

With a baseband processing transponder, the separation of rain attenuation effects between the uplink and downlink allows the downlink to operate as usual when uplink rain attenuation occurs. This places fewer demands on the uplink power control system and a dynamic range of 4.6 dB meets the specification. The transmitter maximum output power is only 290 W, when P_t = 100 W in clear air conditions. The range of the uplink power control system is now 5 dB, a factor of 3.16. However, it is easier to install a powerful transmitter at an earth station than to build a highly reliable baseband processing transponder for the satellite, so linear transponders are still preferred in most GEO satellites.

12. A Ku-band VSAT station receives a TDM data stream at 1.544 Mbps from a GEO satellite. The modulation is QPSK and under clear air conditions the downlink C/N in the VSAT receiver is 20 dB (ignoring noise from the satellite). The C/N in the satellite transponder is 30 dB. A nearby terrestrial LOS link causes interference with the VSAT such that the carrier to interference ratio C/I in the VSAT receiver is 19.6 dB. All C/N and C/I values are quoted for the optimum noise bandwidth of the VSAT receiver. The receiver uses ideal RRC filters with $\alpha = 0.4$ and its QPSK demodulator has an implementation margin of 1 dB.

a. What is the symbol rate of the QPSK signal and the noise bandwidth of the VSAT receiver?

Answer: Symbol rate $R_s = R_b/2 = 1.544/2 = 0.772$ MHz = 772 kHz

Receiver noise bandwidth with RRC filters always equals the symbol rate.

Hence B_N = 772 kHz.

b. What is the clear air overall C/N ratio in the VSAT receiver, assuming that the interference can be considered AWGN? What BER would you expect at the data output of the VSAT receiver assuming no FEC is applied to the signal?

Answer: The overall C/N must be found from the reciprocal formula, treating the interfering signal as noise.

$$1 / (C/N)_o = 1 / (C/N)_{up} + 1 / (C/N)_{dn} + 1 / (C/I)$$

Convert the C/N and C/I values to ratios:

$(C/N)_{up}$ = 30 dB = 1000, $(C/N)_{dn}$ = 20 dB = 100, (C/I) = 19.6 dB = 91.2

Hence

$$1 / (C/N)_o = 1 / 1000 + 1 / 100 + 1 / 91.2 = 0.02196$$

$$(C/N)_o = 45.5 \text{ or } 16.6 \text{ dB}$$

Using QPSK with a 1 dB implementation margin, $(C/N)_{eff}$ = 15.6 dB = ratio 36.3

$$BER = Q(\sqrt{36.3}) = Q(6.03) \approx 8.3 \times 10^{-10}$$

c. The system is redesigned and half rate FEC is added to the signal so that the bit rate at the transmitter is doubled, but transmitter power is not increased. In the receiver, the FEC decoder has a coding gain of 6 dB. For the case when FEC is used, determine the overall C/N ratio and the expected BER during a rain fade that causes the C/N ratio of the received signal to fall by 5 dB but which does not attenuate the interfering signal.

Answer: With half rate FEC added to the signal, the bit rate increases to 3.088 Mbps, and the symbol rate is 1.544 Msps. This increases the receiver noise bandwidth by a factor of two, to 1.544 MHz, and lowers the transponder and earth station receiver C/N values by 3 dB.

Let's assume first that the interference is wide band and increases by 3 dB when the receiver bandwidth is doubled.

The new values for C/N on the uplink and downlink are $(C/N)_{up}$ = 30.0 – 3.0 = 27.0, (C/N) = 20.0 – 3.0 = 17.0, and the new overall C/N in clear air = 16.6 dB.

If the downlink signal is attenuated by rain and $(C/N)_{dn}$ falls by 5 dB, the $(C/N)_{dn}$ ratio in rain is $(C/N)_{dn\ rain}$ = 17.0 – 5.0 = 12.0 dB = ratio 15.84.

The C/I ratio for wideband interference is 19.6 – 3.0 = 16.6 dB = ratio 45.7

$$1 / (C/N)_o = 1 / 500 + 1 / 15.84 + 1 / 45.7 = 0.0870$$

$$(C/N)_o = 11.49 \text{ or } 10.6 \text{ dB}$$

[If the interference is narrowband and does not increase when the receiver bandwidth is doubled, C/I = 19.6 = ratio 91.2 and

$$1 / (C/N)_o = 1 / 500 + 1 / 15.84 + 1 / 91.2 = 0.0761$$

$$(C/N)_o = 13.14 \text{ or } 11.2 \text{ dB}].$$

The effective C/N with half rate FEC coding gain of 6 dB and an implementation margin of 1 dB is given by

$$(C/N)_{eff} = (C/N)_o \text{ - Imp. Margin + coding gain}$$

$(C/N)_{eff}$ = 10.6 [or 11.2] - 1 + 6 = 15.6 dB [or 16.2 dB].

The corresponding bit error rate for a QPSK signal is given by $Q(\sqrt{C/N})$

BER = $Q(\sqrt{36.3})$ [or $Q(\sqrt{41.7})$] = Q(6.03) [or Q(6.45)]

The error rate is below 10^{-9} in both cases. The symbol rate is 40 Msps = 4×10^7 sps, so that a symbol error occurs no more often than once every 25 seconds, on average, in both cases.

We should assume the worst case, that the interference is wideband and that interference power into the earth station increases when we use a wider receiver bandwidth for the FEC encoded signal. For QPSK with Gray coding, there is one bit error for each symbol error, so the expected BER when $(C/N)_{dn}$ falls by 5 dB is approximately 10^{-9}.

d. If the extra bandwidth to implement half rate FEC is available at the satellite, would you recommend that FEC be used in this case? Give reasons for your answer.

Answer: The implementation of half rate FEC on the link improves the performance from adequate in clear air conditions to essentially error free when the downlink C/N falls by 5 dB because of rain in the downlink path. This is a significant improvement in performance and justifies the use of half rate FEC, provided bandwidth is available in the transponder. Remember that adding half rate FEC to the transmitted signal doubled the bandwidth that was needed in the transponder.

It is useful to compare the link performance with and without forward error correction.

No FEC, clear air, BER $\approx 8.5 \times 10^{-10}$. With a 5 dB reduction in $(C/N)_{dn}$ due to rain, $(C/N)_o$ = 22.93 or 13.6 dB. With 1 dB implementation margin, $(C/N)_{eff}$ = 12.6 dB or 18.2 and $P_e = Q(\sqrt{18.2}) \approx 10^{-5}$. This is at or below the lower limit for satisfactory operation in most links. With FEC encoding, in clear air, $(C/N)_o$ = 13.6 dB with wideband interference.

The effective overall C/N ratio is $(C/N)_{eff}$ = 13.6 – 1.0 + 5.0 = 17.6 dB or 57.7 giving a symbol error probability of $P_e = 10^{-14}$ and essentially error free operation of the link in clear air. In a rain event that causes $(C/N)_{dn}$ to fall by 5 dB, BER $\approx 8 \times 10^{-10}$.

The link is error free in clear air conditions and almost error free in the stated downlink rain event when half rate FEC is employed. This is a big improvement over the performance without FEC.

e. What are the advantages and disadvantages of using forward error correction in satellite links? Illustrate your answer using the above example of a high data rate signal sent to a small earth terminal.

Answer: Advantages: Lower bit error rates (error free under all clear air conditions), and the ability to withstand fading on the uplink and downlink. In the case above, the link remains essentially error free when downlink C/N falls by 5 dB, corresponding to a typical rain attenuation of 2.5 to 3 dB on the downlink path. In east coast areas of the US, this attenuation occurs about 0.1% of the time (about 8 hours per year).

Disadvantages: More bandwidth is needed in the satellite transponder, which may increase the cost of leasing the satellite channel. Communication capacity is generally sold by the MHz, so doubling the transmission bandwidth will double the cost. Most VSAT systems seem to be willing to pay the price of extra bandwidth and use half rate FEC to improve BER performance in fading conditions.

13. A T1 data transmission system transmits data at 1.544 Mbps over a GEO satellite link At the receiving terminal the clear air value of overall $(C/N)_o$ is 16.0 dB. The modulation used on the link is BPSK and the implementation margin of the BPSK demodulator is 0.5 dB.

a. Find the BER at the receiver output, and the time that elapses, on average, between bit errors.

Answer: With BPSK modulation $R_b = R_s = 1.544$ Mbps in this link.

The effective overall C/N ratio at the input to the BPSK demodulator in the earth station receiver is 16.0 dB – 0.5 = 15.5 dB or a ratio of 35.48. The probability of a bit error for BPSK modulation is $P_e = Q[\sqrt{(2 \times C/N)_{eff}}] = Q[8.42]$, which is less than 10^{-16}, so there are no errors on the link. There is no definable time between errors.

b. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now? What is the average time between bit errors?

Answer: The effective C/N ratio is now 10.0 – 0.5 = 9.5 dB, a ratio of 8.91. Hence the bit error rate is

$$P_e = Q[\sqrt{(2 \times C/N)_{eff}}] = Q[4.22] \approx 10^{-5}.$$

The time between errors is given by $T_{error} = 1 / (R_b \times P_e) = 1 / (1.544 \times 10^6 \times 10^{-5}) = 0.065$s.

c. The modulation method is changed to QPSK and the bit rate is increased to $2 \times$ T1 = 3.088 Mbps, and the symbol rate on the link is 1.544 Msps (Mbaud). What is the bit error rate now? What is the average time between bit errors?

Answer: The symbol rate on the link is unchanged, so the C/N ratios do not change. However, the BER is higher for QPSK than for BPSK at any given C/N ratio. We will assume that the implementation margin for the QPSK receiver is the same as the BPSK receiver, at 0.5 dB.
In clear air conditions the effective C/N ratio is still 15.5 dB, a ratio of 35.48. For QPSK modulation, the bit error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[5.96] \approx 1.4 \times 10^{-9}.$$

The average time between symbol errors is given by

$$T_{error} = 1 / (R_s \times P_e) = 1 / (1.544 \times 10^6 \times 1.4 \times 10^{-9}) = 462 \text{ seconds, or } 7.7 \text{ minutes.}$$

With Gray coding of the QPSK states, there will usually be one symbol error for each bit error, so BER $\approx 1.4 \times 10^{-9}$ and $T_{bit\ error} \approx 7.7$ minutes.

d. Rain affects the downlink from the satellite and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate and average time between errors now for the QPSK link?

Answer: The effective C/N ratio is now 10.0 – 0.5 = 9.5 dB, a ratio of 8.91. Hence the symbol error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[2.98] \approx 1.4 \times 10^{-3}.$$

The time between symbol errors is given by

$T_{error} = 1 / (Rs \times P_e) = 1 / (1.544 \times 10^6 \times 1.4 \times 10^{-3}) = 0.000463$ s. Using Gray coding, one symbol error leads to one bit error so there are 2160 errors each second in the link.

e. Changing the modulation method to QPSK and increasing the bit rate to 3.0878 Mbps is likely to lead to an unacceptably high bit error rate when the satellite downlink was affected by rain because the receiver $(C/N)_o$ will fall by 6 dB. We could retain a bit rate on the link of

1.544 Mbps when using QPSK by changing the transmitter and receiver RRC filters to operate at a symbol rate $R_s = 1.544 / 2 = 0.772$ Msps. What is the bit error rate and average time between errors now for the QPSK link?

Answer: Reducing the bit rate on the link to 1.544 Mbps increases the C/N ratios by 3 dB because the receiver bandwidth is halved. In clear air conditions the effective C/N ratio is 15.5 + 3.0 = 18.5 dB, a ratio of 70.8. For QPSK modulation, the bit error rate is given by

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[8.41] .$$

The BER is less than 10^{-16} so there are no errors on the link.

f. Rain affects the downlink from the satellite in part (e) above, and the overall C/N ratio in the receiver falls by 6.0 dB to 10.0 dB. What is the bit error rate now for the QPSK link?

Answer: The effective C/N ratio is now 13.0 – 0.5 = 12.5 dB, a ratio of 17.78. Hence the symbol error rate is

$$P_e = Q[\sqrt{(C/N)_{eff}}] = Q[4.22] \approx 10^{-5}.$$

The time between symbol errors is given by

$T_{error} = 1 / (R_s \times P_e) = 1 / (1.544 \times 10^6 \times 10^{-5}) = 0.065$ s. Using Gray coding, one symbol error leads to one bit error so there are 15 errors each second in the link, on average.

14. The baseband average S/N ratio for a probability of bit error P_e (BER) with N-bit PCM is given by

$$S/N = 10 \log_{10} \left[\frac{2^{2N}}{1 + 4 P_e \times 2^{2N}} \right] \text{ dB}$$

a. The effective C/N in a digital receiver with QPSK modulation is 15.6 dB under clear air conditions. What is the baseband S/N for 8-bit PCM coded speech?

Answer: Quantization S/N = 6 N dB when there are no bit errors, where N = niumber of bits in PCM word. For N = 8, $(S/N)_Q = 48$ dB with linear encoding. The probability of error for QPSK with an effective C/N ratio of 15.6 = ratio 36.3 is $Q[\sqrt{36.3}] = Q[6.02] = 9 \times 10^{-10}$

In the absence of quantization noise, the signal to noise ratio from bit errors is $(S/N)_B = 1/(4 P_e)$.

For BER $= 9 \times 10^{-10}$, $(S/N)_B = 2.77 \times 10^8$ or 84 dB.

The quantization noise is dominant in this case and S/N = 48 dB.

b. In moderate rain conditions the effective C/N falls to 13.6 dB. What is the baseband S/N for the 8-bit PCM signal?

Answer: The effective C/N ratio is 13.6 dB, a ratio of 22.90, giving BER for the link as

$P_e = Q(\sqrt{22.90}) = Q(\sqrt{4.79}) = 10^{-6}$.

The bit errors on their own will cause $(S/N)_B = 1 / 4 \times 10^{-6} = 2.5 \times 10^5$ or 54.0 dB. Since the two S/N ratios are similar, we must use the formula at the beginning of the question to calculate their joint effect.

$$S/N = 10 \log_{10} \left[\frac{2^{2N}}{1 + 4P_e \times 2^{2N}} \right] \text{ dB} = 10 \log [65{,}536 / [\ 1 + (4 \times 10^{-6} \times 65{,}536)\ ]]$$

$$= 10 \log [65{,}536 / 1.262] = 47.1 \text{ dB}$$

c. In heavy rain, the effective C/N falls to 11.6 dB. What is the baseband S/N for the 8-bit PCM signal?

Answer: : The effective C/N ratio is 11.6 dB, a ratio of 14.45, giving BER for the link as

$P_e = Q(\sqrt{14.45}) = Q(\sqrt{3.80}) = 7.25 \times 10^{-5}$.

The bit errors on their own will cause $(S/N)_B = 1 / (4 \times 7.25 \times 10^{-5}) = 3448$ or 35.4 dB, and will therefore be the dominant source of noise; S/N baseband ≈ 35.4 dB.

Check: putting $P_e = 7.25 \times 10^{-5}$ and $2^{2N} = 65{,}536$ in the S/N formula gives

$$S/N = 10 \log [65{,}536 / [\ 1 + (4 \times 7.25 \times 10^{-5} \times 65{,}536)\ ]] = 10 \log [65{,}536 / 19.0] = 35.4 \text{ dB}.$$

d. The minimum acceptable baseband S/N in a speech channel is usually set at 30 dB. What is the corresponding minimum allowable effective C/N for a QPSK link carrying 8-bit PCM coded speech?

Answer: The baseband S/N will be dominated by the bit errors at the demodulator output. Hence $S/N = 30 \text{ dB} = 1000 = 1 / 4 P_e$ and $P_e = 2.5 \times 10^{-4}$.

The probability of error in a QPSK link is $Q[\sqrt{(C/N)_{eff}}]$, so $\sqrt{(C/N)_{eff}} = 3.48$, and $(C/N)_{eff} = 12.11$ or 10.8 dB. Thus when the effective C/N at the QPSK demodulator input is 10.8 dB, the baseband S/N of the recovered PCM signal will be 30 dB. The minimum allowable QPSK effective C/N ratio is 10.8 dB.

15. Direct Broadcast Satellite TV

In this question you are asked to analyze the performance of the TV system using frequency modulation. The uplink station delivers a signal to the satellite which conforms to the following specification:

Transponder and satellite characteristics

Transponder bandwidth	25 MHz
$(C/N)_{up}$ in 20 MHz noise bandwidth	24 dB
Saturated output power	200 W
Downlink frequency	12.5 GHz
Downlink antenna gain, on axis	39.0 dB
Atmospheric clear air loss	0.4 dB
All other losses	0.5 dB

Receive Station parameters

Antenna diameter	18 inches
Aperture efficiency	70 %
Antenna noise temperature (clear air)	40 K
Receiver noise temperature	90 K

a. The uplink master station transmits a NTSC video signal with a baseband bandwidth of 4.2 MHz to one transponder on the satellite using FM. The transponder is operated with 1 dB of output back-off and the FM signal occupies a bandwidth of 24 MHz. For an earth station with a high gain LNA, at a distance of 38,000 km from the satellite, on the – 4 dB contour of the satellite antenna beam. Find:

Answer:

i The peak frequency deviation of the FM signal.

Using Carson's rule: $B = 2\,(\Delta f_{pk} + f_{max})$

Hence $\Delta f_{pk} = B/B - f_{max} = 24 / 2 - 4.2 = 7.8$ MHz

ii The power at the input to the earth station LNA.

Find the path loss and receive antenna gain first. Antenna diameter is 0.457 m.

$L_p = 20 \log (4 \pi R / \lambda)^2 = 20 \log (4 \pi \times 38 \times 10^6 / 0.025) = 205.6$ dB

$G_r = \eta_A \times (\pi D / \lambda)^2 = 0.7 \times (\pi \times 0.457 / 0.025)^2 = 2309$ or 33.6 dB

The downlink budget gives the received power. For 1 dB output back off of the 200 W transponder, atmospheric and miscellaneous loss of 0.9 dB, and the –4 dB contour:

P_r = EIRP + G_r – L_p - losses dBW

= 22 + 39 + 33.6 – 205.6 – 4.0 – 0.4 – 0.5 = -115.9 dBW

iii The downlink $(C/N)_{dn}$ in a noise bandwidth of 24 MHz.

The noise power at the earth station receiver input is

$N = k\, T_s\, B_N$ where T_s = 40 + 90 = 130 K or 21.1 dBK

N = -228.6 + 21.1 + 73.8 = -133.7 dBW

Hence the downlink C/N ratio in the earth station receiver is

C/N = P_r – N = -115.9 + 133.7 = 17.8 dB

iv The overall $(C/N)_o$ in the earth station receiver.

The uplink C/N in the transponder is 24 dB. Using the reciprocal formula

$(C/N)_o$ = 1/(1/251.2 + 1/60.25) = 1 / 0.0206 = 48.60 or 16.9 dB

b. For the FM video signal in part (a) above:

Answer:

i The unweighted video S/N ratio at the baseband output of the receiver

$(S/N)_{unweighted}$ = C/N + 10 log (B/f_{max}) + 20 log ($\Delta f_{pk} / f_{max}$ + 1.8 dB

= 16.9 + 10 log (24.0 / 4.2) + 20 log (7.8 / 4.2) + 1.8

= 31.7 dB

ii The weighted S/N ratio after pre-emphasis and subjective improvements are added

The standard improvement factors for NTSC TV signals transmitted using FM modulation are P = 8 dB, Q = 9 dB giving a weighted S/N in clear air conditions

$(S/N)_{weighted}$ = 31.7 + 8 + 9 = 48.7 dB

iii The link margin for the downlink given an FM threshold at 8.5 dB.

The clear air downlink C/N in clear air is 17.8 dB. $(C/N)_{up}$ is 24.0 dB.

The limiting value for downlink C/N ratio $(C/N)_{dnmin}$ is set by the FM threshold value of $(C/N)_{oth}$ = 8.5 dB. Using the reciprocal formula

$1 / (C/N)_{oth}) = 1 / (C/N)_{up} + 1 / (C/N)_{dnmin}$

$1 / 7.079 = 1 / 251.2 + 1 / (C/N)_{dnmin}$

$(C/N)_{dnmin}$ = 7.28 or 8.6 dB

The downlink margin is the difference between the clear air downlink C/N ratio of 17.8 dB and the minimum value of 8.6 dB.

Hence downlink margin is 17.8 - 8.6 = 9.2 dB

c. Heavy rain affects the uplink to the satellite causing the C/N in the transponder to fall to 18 dB. Assuming linear bent-pipe operation of the transponder, find:

Answer:

i The overall $(C/N)_o$ ratio in the earth station receiver

When uplink C/N ratio falls from 24 dB to 18 dB, the downlink C/N will also fall by 6 dB to 11.8 dB because we have a linear transponder. The overall C/N will also fall by 6 dB to 10.8 dB. Check using the reciprocal formula

$(C/N)_o = 1 / (1/63.10 + 1/15.14) = 12.21$ or 10.8 dB

ii The video S/N ratio. Is this an acceptable S/N for viewing a television picture?

In clear air conditions the weighted video S/N was 48.7 dB. S/N falls in direct proportion to overall C/N during rain attenuation events, so the weighted S/N will fall by 6 dB to 42.7 dB. The TV picture will have visible noise but would rate as acceptable.

d. Heavy rain affects the downlink from the satellite causing 4 dB of rain attenuation. The uplink is operating in clear air conditions.

Assuming a medium noise temperature of 270 K in rain and 100% coupling of sky noise into antenna noise temperature, find:

i The new value for $(C/N)_{dn}$ in the earth station receiver,

We must first find the increase in sky noise temperature caused by rain in the downlink path.

Answer: The sky temperature increases when rain is in the downlink path. Using the noise model for a lossy device, $T_{sky\ rain} = 270 \times (1 - 0.398) = 162.5$ K.

The new system noise temperature is $T_s = 90 + 162.5 = 252.5$ K

The increase in noise power in the earth station receive is ΔN where

$$\Delta N = 10 \log (252.5 / 130) = 2.9 \text{ dB}$$

The new downlink C/N in rain is

$$(C/N)_{dn\ rain} = 17.8 - 4.0 - 2.9 = 10.9 \text{ dB}$$

ii The corresponding overall $(C/N)_o$ ratio in the earth station receiver

Overall C/N is

$$(C/N)_o = 10 \log (1 / (1 / (C/N)_{up} + 1 / (C/N)_{dn}) = 10.7 \text{ dB}$$

iii The video S/N ratio. Is this an acceptable quality television picture?

Video S/N is proportional to the C/N ratio in the receiver. The weighted S/N ratio is given by

$$\text{S/N weighted} = (C/N)_o + \text{FM Improvement}$$

FM Improvement = 31.8 dB

Hence with C/N overall = 10.7 dB

$$\text{S/N weighted} = 10.7 + 31.8 = 42.5 \text{ dB}$$

e. Draw a block diagram for the earth station receiver, showing only the parts that relate to reception and output of the NTSC video signal. Your block diagram must show the center frequency, gain, and bandwidth of each block, as appropriate.
Do not specify filters with Q exceeding 50.

Answer: Block diagram not included in the Solutions Manual. See text for examples.

16. This problem examines the design and performance of a digital satellite communication link using a geostationary satellite with bent-pipe transponders, used to distribute digital TV signals from one central (hub) earth station to many receiving stations throughout the United States. The link uses QPSK digital transmission at 20 Msps with half rate forward error correction. The half rate FEC gives a coding gain of 5.5 dB.

The design requires that an overall C/N ratio of 9.5 dB be met in the earth station receiver to ensure that noise in the video signal on the TV screen is held to an acceptable level. The uplink transmitter power and the receiving antenna gain and diameter must be determined. The available link margins for each of the systems must be found and the performance of the system analyzed when rain attenuation occurs in the satellite – earth paths.

The system is specified in Table 1.

Table 1. System and Satellite Specification

Ku-band satellite parameters	
Total RF output power	3.2 kW
Antenna gain, on axis, Ku-band (transmit and receive)	31 dB
Receive system noise temperature	500 K
Transponder saturated output power: Ku-band	80 W
Transponder bandwidth: Ku-band	54 MHz
Earth station receiver IF noise bandwidth	20 MHz
Minimum permitted overall C/N in receiver	9.5 dB
Transponder HPA output back-off	1 dB
Transmitting Ku-band earth station	
Antenna diameter	5 m
Aperture efficiency	68 %
Uplink frequency	14.15 GHz
Required C/N in Ku band transponder	30 dB
Miscellaneous uplink losses	0.3 dB
Location: -2 dB contour of satellite receiving antenna	
Receiving Ku-band earth station	
Downlink frequency	11.45 GHz.
Receiver IF bandwidth	20 MHz
Aperture efficiency	68 %
Antenna noise temperature	30 K
LNA noise temperature	110 K
Required overall $(C/N)_o$ in clear air	17 dB
Miscellaneous downlink losses	0.2 dB
Location: -3 dB contour of satellite transmitting antenna	
Rain Attenuation and Propagation Factors at Ku-band	
Clear air attenuation	
Uplink 14.15 GHz	0.7 dB

Downlink	11.45 GHz	0.5 dB

Rain attenuation

Uplink	0.01 % of year	6.0 dB
Downlink	0.01 % of year	5.0 dB

a. Uplink design

Find the uplink transmitter power to achieve the required $(C/N)_{up} = 30$ dB in the transponder in clear air atmospheric conditions. Find the noise power in the transponder for a noise bandwidth of 20 MHz, and then add 30 dB to find the transponder input power level. Calculate the earth station transmit antenna gain, and the path loss at 14.15 GHz. Generate an uplink power budget and find the required power at the transponder input to meet the $(C/N)_{up} = 30$ dB objective in the transponder. Don't forget the various uplink losses.

Answer : Begin with the noise power in the transponder, and then create an uplink power budget. Transponder input noise temperature is 500 K, earth station receiver noise bandwidth is 20 MHz. Noise power is N watts where

$$N = k\, T_s\, B_N = -228.6 + 27.0 + 73.0 = -128.6 \text{ dBW}$$

Uplink earth station transmit antenna gain at 14.15 GHz ($\lambda = 0.02120$ m) is given by

$$G_t = 10 \log (\eta_A \times (\pi\, D / \lambda)^2) = 10 \log (0.68 \times (\pi \times 5 / 0.02120)^2) = 55.7 \text{ dB}$$

Path loss at 14.15 GHz for a typical GEO path length of 38,500 km is given by

$$L_p = 20 \log (\pi\, R / \lambda) = 20 \log (4\, \pi \times 38.5 \times 10^6 / 0.0212) = 207.2 \text{ dB}$$

The uplink power budget includes losses of 2 dB off-axis, 0.7 dB atmospheric, and 0.3 dB misc.

P_t	TBD
G_t	55.7 dB
G_r	31.0 dB
Path loss	-207.2 dB
Off-axis contour loss	-2.0 dB
Other losses	-1.0 dB
Receiver power	P_t - 123.5 dBW

We require $(C/N)_{up} = 30$ dB with N = -128.6 dBW, so the required transmitter power at the uplink earth station is

$$P_t = 30 + 123.5 - 128.6 = 24.9 \text{ dBW or } 309 \text{ W.}$$

b. Downlink design

Assume a high gain LNA and ignore the noise generated in other parts of the receiver. Calculate the downlink $(C/N)_{dn}$ to give overall $(C/N)_o$ = 17 dB when $(C/N)_{up}$ = 30 dB. Hence find the receiver input power to give the required $(C/N)_{dn}$ using a value of receiving antenna gain G_r.

Calculate the path loss at the downlink frequency of 11.15 GHz. Don't forget the downlink losses. The transponder is operated with 1 dB output back off. Find the transponder output power and then generate a downlink power budget .

Hence find the receiving antenna gain G_r and diameter for a frequency of 11.45 GHz. This diameter antenna will provide the required $(C/N)_o$ in the earth station receiver under clear air conditions.

Answer: Find the path loss first by scaling from 14.15 GHz to 11.45 GHz

Path loss L_p = 207.16 + 20 log (11.45 / 14.15) = 205.3 dB

The downlink power budget includes losses of 3 dB off-axis, 0.5 dB atmospheric, and 0.2 dB miscellaneous. Transponder output power is 80 W with 1 dB back-off, giving P_t = 19.0 dBW.

P_t	80 W - 1 dB back-off	19.0 dBW
G_t		31.0 dB
G_r		TBD
Path loss		-205.3 dB
Off-axis contour loss		-3.0 dB
Other losses		-0.7 dB
	Receiver power	G_r - 159.0 dBW

We require $(C/N)_o$ = 17.0 dB with $(C/N)_{up}$ = 30 dB. Using the reciprocal formula

$(C/N)_{dn} = 1 / (1/(C/N)_o – 1 / (C/N)_{up})$ = 52.76 or 17.2 dB

The noise power in the receiver is $N = k\, T_s\, B_N$ watts. T_s = 30 K + 110 K = 140 K.

N = -228.6 + 21.5 + 73.0 = -134.1 dBW

Hence G_r - 159.0 dBW = 17.2 - 134.1 dBW, G_r = 42.1 dB

The gain or the receiving antenna is given by

G_r = 42.1 dB = $10 \log (\eta_A \times (\pi\, D / \lambda)^2)$ = $10 \log (0.68 \times (\pi \times D / 0.02620)^2)$

Hence $16{,}218 = 0.68 \times (\pi \times D / 0.02620)^2$ and D = 1.288 m.

c. Rain effects

A practical system must continue to operate under adverse weather conditions, so we need a margin for rain attenuation and increase in sky noise temperature during rain. In the following section you will determine the margins available on the uplink and downlink to combat rain attenuation and increase in sky noise temperature.

d. Uplink rain attenuation

Under conditions of heavy rain, the Ku-band path to the satellite suffers an attenuation of 6 dB for 0.01 % of the year. We must find the uplink attenuation margin and decide whether uplink power control would improve system performance at Ku-band.

The uplink C/N was 30 dB in clear air. With 6 dB uplink path attenuation, the C/N in the transponder falls to 24 dB. (Rain on the earth has no effect on the satellite transponder system noise temperature.) Assume linear transponder characteristic and no uplink power control. Find the transponder output power with 6 dB of rain attenuation in the uplink.

Hence find the overall $(C/N)_o$ in an uplink rain fade of 6 dB, and the link margin available on the uplink. Is this an adequate uplink margin, given the rain attenuation for most of the US?

Answer: We assume a linear transponder, so the output power of the transponder falls by 6 dB when rain attenuation of 6 dB occurs on the uplink. When both $(C/N)_{up}$ and $(C?N)_{dn}$ fall by the same amount, say X dB, the overall $(C/N)_o$ ratio also falls by X dB.

Hence, with an uplink fade of 6 dB, $(C/N)_o = 17.0 – 6.0 = 11.0$ dB. The system objectives require a minimum overall $(C/N)_o$ of 9.5 dB. This leaves an uplink margin of 1.5 dB with 6 dB uplink attenuation, i.e. maximum uplink attenuation of 7.5 dB. This attenuation is exceeded about 0.01% of an average year for a typical GEO satellite link at 11.45 GHz in the East Coast region of the US. If the location of the uplink station is the West Coast or central parts of the United States, an uplink margin of 7.5 dB at 11.45 GHz will ensure satisfactory operation of the receiving terminals for more than 99.99% of an average year.

e. Downlink attenuation and increase in sky noise in rain

The 11.45 GHz path between the satellite and the receive station suffers rain attenuation exceeding 5 dB for 0.01% of the year. Assuming 100% coupling of sky noise into antenna noise, and 0.5 dB clear air gaseous attenuation, calculate the overall C/N under these conditions.

Assume that the uplink station is operating in clear air. Calculate the available downlink fade margin.

Find the sky noise temperature that results from a total excess path attenuation of 5.5 dB (clear air attenuation plus rain attenuation); this is the new antenna temperature in rain, because we assumed 100 % coupling between sky noise temperature and antenna temperature. Evaluate the change in received power and increase in system noise temperature in order to calculate the change in C/N ratio for the downlink.

In clear air, the atmospheric attenuation on the downlink is 0.5 dB. The corresponding sky noise temperature is approximately 0.5 x 7 = 35 K, which leads to the antenna temperature of 30 K given in the Ku-band system specification. When the rain causes 5 dB attenuation, the total path attenuation from the atmosphere and the rain is 5.5 dB. The sky noise will be much higher in rain. Find the increase in noise power caused by the increase in sky temperature.

Hence find the new $(C/N)_{dn\ rain}$ value with 5.5 dB attenuation on the downlink path. Find the overall C/N by combining the clear air uplink $(C/N)_{up}$ of 30 dB with the rain faded downlink $(C/N)_{dn\ rain}$ to give overall $(C/N)_o$ in rain.

Is the downlink link margin acceptable? If not, calculate the gain and diameter of an earth station antenna that will ensure an overall C/N value that does meet the specification.

Answer: When there is 5 dB of rain attenuation on the downlink to the earth station, total path attenuation is 5.5 dB. Assuming a medium temperature of 290 K, sky noise temperature for 5.5 dB path loss is given by

$$T_{sky} = 290 \times (1 - 0.282) = 208 \text{ K.}$$

The corresponding system noise temperature is $T_{s\ rain} = 208 + 110 = 318$ K.

In clear sky conditions, the system noise temperature was 140 K. Hence the increase in earth station receiver noise power is

$$\Delta N = 10 \log (318 / 140) = 3.6 \text{ dB}$$

The downlink has 5 dB rain attenuation, so total reduction in $(C/N)_{dn}$ is 8.6 dB, giving $(C/N)_{dn\ rain} = 17.2 - 8.6 = 8.6$ dB. Overall C/N ratio with $(C/N)_{up} = 30$ dB is 8.6 dB, rounded to the nearest 0.1 dB. This is below the requirement of 9.5 dB for satisfactory signal quality at the video output of the earth station receiver. To improve the overall C/N ratio at the receiver output to 9.5 dB, we require $(C/N)_{dn} = 9.5$ dB. Receiving antenna gain must be increased by 0.9 dB to meet this objective, requiring an increase in antenna diameter by a factor of

$10^{0.9/20}$ = 1.109 to 1.288 $\times$ 1.134 = 1.429 m.

Because there are many steps in the calculation here, it is wise to check that the system design meets the specification. Here are the revised downlink budget and noise power figures.

Antenna gain for a 1.46 m antenna with 68% aperture efficiency and diameter 1.429 m is 43.0 dB, an increase over the original gain of 0.9 dB, as required. Received power increases by 0.9 dB, giving $P_r = G_r - 159.0$ dBW = -116.0 dBW in clear air, and –121.0 dBW with 5 dB rain attenuation in the downlink. With $T_s = 318$ K, the noise power referred to the earth station receiver input is –130.6 dBW. This gives $(C/N)_{dn\ rain}$ = 9.6 dB. Uplink $(C/N)_{up}$ is 30 dB, giving an overall C/N ratio in the earth station receiver of 9.56 dB. This is very close to the reuirement of 9.5 dB – the difference is due to round-off errors in all the many decibel values used in the calculation.

f. Summarize your design for the Ku band earth station and uplink and downlink.

Compare the earth station receiving antenna diameter for the Ku band system with the antenna in the notes for a similar C band system.

If the Ku band antenna is larger (and therefore has a much higher gain) explain why.

Answer: The receiving antenna at Ku band that meets the 9.5 dB overall $(C/N)_o$ requirement for 99.99% of an average year has a diameter of 1.429 m. The uplink will provide better than 99.99% availability of signals provided that it is located away from the east and south of the United States, in a region where heavy rainfall is less frequent. The downlink meets the 99.99% availability criterion over most of the United States. In the south eastern states, a larger antenna would be needed to guarantee 99.99% availability of the downlink. A system of this type could be used for distribution of a digital TV signal to cable TV companies; however, a compressed digital signal at a higher bit rate carrying multiple TV signals, and a wider bandwidth transponder would be used, requiring a receiving antenna about 2 m in diameter.

A typical C-band receiving antenna for FM-TV signals had a diameter of 3 m, but was used with satellites having much lower output power, in the 10 – 30 W range. The Ku-band antenna in this problem is larger than the antennas used for direct broadcast satellite television because of the lower satellite transponder output power and slightly lower satellite antenna gain assumed in this example. See Chapter 11 for details of DBS-TV systems.

17. A satellite communication system uses a single 54 MHz bandwidth Ku-band transponder to carry 400 two way telephone conversations (800 RF channels) using analog modulation with single channel per carrier frequency modulation (SCPC-FM). The parameters of any one channel are:

Voice channel bandwidth :	100 - 3,400 Hz
RF channel bandwidth:	45 kHz
RF channel spacing:	65 kHz
Downlink path loss (inc. atmos. loss)	206.5 dB
Satellite downlink antenna gain (on axis)	29 dB
Demodulator FM threshold:	5 dB

The transponder has a saturated power output of 40 watts, but is run with 3 dB output backoff to achieve near-linear operation. The uplink stations which transmit the SCPC-FM signals to the transponder achieve $(C/N)_{up} = 25$ dB in the 45 kHz channel noise bandwidth of the earth station receiver. The system noise temperature of the receiving earth station is 120 K in clear air.

a. Calculate the power per RF channel at the transponder output.

Answer: Transponder saturated output power is 40 watts = 16 dBW. With 3 dB backoff, P_t = 13 dBW or 20 W. Each voice channel in the satellite transponder occupies 65 kHz with its guard bands, so we can fit 54,000 / 65 = 831 channels into the transponder. Hence, downlink transmit power per channel is 20 W / 831 = 0.0241 W/channel = -16.2 dBW per channel.

b. The gain of the antenna at a receiving earth station that is located on the -3 dB contour of the satellite footprint which will provide an overall C/N = 10 dB in a receiver for a single RF channel with a noise bandwidth of 45 kHz, in clear air conditions.

Answer: We know the downlink path loss, so we can establish a downlink power budget using a receiving antenna gain of G_r dB. The downlink power budget includes a losses of 3 dB for the receiving station location on the –3 dB contour of the satellite footprint.

The downlink budget is:

P_t	24 mW per channel	-16.2 dBW
G_t		29.0 dB
G_r		TBD
Path loss		-206.5 dB
Off-axis contour loss		-3.0 dB
Receiver power		G_r - 196.7 dBW

Noise power in a single earth station receiver noise bandwidth of 45 kHz, for one FM voice channel, with T_s = 120 K in clear air is

$$N = k\, T_s\, B = -228.6 + 20.8 + 46.5 = -161.3 \text{ dBW}$$

We require $(C/N)_o$ = 10.0 dB with $(C/N)_{up}$ = 25 dB. Using the reciprocal formula

$$(C/N)_{dn} = 1 / (1/(C/N)_o - 1 / (C/N)_{up}) = 10.14 \text{ or } 10.1 \text{ dB.}$$

Hence the earth station receive antenna gain is

$$G_r = 196.7 + 10.1 - 161.3 = 45.5 \text{ dB.}$$

c. Calculate the diameter of the receiving antenna with a circular aperture having 65% aperture efficiency at a frequency of 11.5 GHz.

Answer: The gain or the receiving antenna with λ = 0.02608 m is given by

$$G_r = 45.5 \text{ dB} = 10 \log (\eta_A \times (\pi\, D / \lambda)^2) = 10 \log (0.65 \times (\pi \times D / 0.02608)^2)$$

Hence $35{,}481 = 0.65 \times (\pi \times D / 0.02608)^2$ and D = 1.940 m.

d. The receiver applies a de-emphasis weighting of 6 dB to the recovered voice signal and a psophometric weighting of 2.5 dB.

Calculate the weighted S/N at the baseband output of the receiver.

Answer: The weighted baseband S/N for a single channel FM signal is given by Equation 5.18, with an added subjective improvement factor Q dB:

$$(S/N)_w = C/N + 10 \log (B_{RF} / f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 + P + Q \text{ dB}$$

For the voice signal in this problem, f_{max} = 3.4 kHz, Δf_{pk} = 45/2 – 3.4 = 19.1 kHz, P = 6 dB and Q = 2.5 dB. The overall C/N ratio in the earth station receiver is 10.0 dB.

$$(S/N)_w = 10 + 10 \log (45 / 3.4) + 20 \log 19.1 / 3.4) + 1.8 + 6 + 2.5 = 46.5 \text{ dB.}$$

e. Comment on the performance of the system. Is the S/N adequate in clear air.
If the downlink fades by 5 dB because of rain, what is the S/N at baseband?
Is this acceptable for voice communications?

Answer: The baseband S/N ratio of 45.6 dB would be rated as good, slightly below the wireline telephone system objective of 50 dB. With a 5 dB reduction in earth station receiver overall C/N ratio, the overall C/N ratio in the earth station receiver will be 5 dB. This will be at the threshold of the FM demodulator, assuming a threshold extension design, so operation is marginal and we should deduct 1 dB from the baseband S/N ratio. Hence baseband S/N = 39.6 dB in this rain condition, at the lower limit for acceptable operation.

The antenna diameter, close to 2 m, is similar to many older VSAT systems that used lower power Ku-band transponders. Newer satellites have higher transponder powers in the 50 – 100 W range, and transmission is digital using QPSK, not analog using FM.

18. In problem #5, an analog voltage was transmitted from a satellite to earth using frequency modulation. The signal could have been sent digitally using a digital to analog converter and PSK modulation. This problem compares the performance of the digital link to the analog link of problem #5.

The digital link is allocated an RF bandwidth of 25 kHz, and uses BPSK modulation. At the receiving terminal, the C/N ratio is 10 dB. The link has ideal RRC filters with $\alpha = 0.25$ and the BPSK demodulator has an implementation margin of 0.5 dB.

a. The analog voltage is sampled at 2.5 kHz and converted to a series of digital words with an analog to digital converter. Determine the maximum number of bits in each word and the average quantization signal to noise ratio of the recovered analog signal.

Answer: The available bandwidth of the RF channel is 25 kHz. With RRC filters of $\alpha = 0.25$, the symbol rate in the channel is 25 / 1.25 = 20 ksps, and thus R_b = 20 kbps for BPSK modulation. The analog signal is sampled at 2.5 kHz, so we can send 8 bits per sample as a 20 kbps bit stream. The average quantization signal to noise ratio is $(S/N)_Q$ = 6 N dB = 48 dB.

b. Find the BER for the recovered bit stream at the output of the BPSK demodulator, and hence calculate the average S/N ratio in the analog voltage due to bit errors.

Answer: BER $= Q[\sqrt{(2\ C/N)_{eff}}]$ for a BPSK signal. The C/N ratio in the receiver is 10 dB in a noise bandwidth of 20 kHz. The RRC filter in the BPSK receiver has a noise bandwidth equal to the symbol rate of 20 ksps.

The effective C/N is 10.4 – 0.5 = 9.5 dB or 8.91 as a ratio.

BER is then $Q[\sqrt{(2\ C/N)_{eff}}] = Q[\sqrt{2 \times 8.91}] = Q[4.22] = 1.2 \times 10^{-5}$.

The average baseband S/N for a PCM signal with a bit error rate P_e is $1 / (4\ P_e)$, giving

$$S/N_{\text{bit errors}} = 20{,}833 \text{ or } 43.2 \text{ dB.}$$

The lower S/N ratio caused by the 10^{-5} bit error rate will dominate over the quantization S/N ratio, so the baseband S/N will be around 43 dB.

c. Solve problem #5 for the FM version of this link. Which link has the better performance? What changes should be made to the link with the poorer performance to make the S/N ratios approximately equal for the FM and BPSK links? If a RF bandwidth of 50 kHz could be used for the BPSK signal, would the addition of half rate forward error correction with a coding gain of 6 dB improve the performance of the BPSK link?

Answer: The FM version of the link was analyzed in problem #5. The peak deviation was found to be $\Delta f_{pk} = 10$ kHz giving a Carson's rule bandwidth with $f_{max} = 1$ kHz

$$B = 2(\Delta f_{pk} + f_{max}) = 2 \times (10 + 1) = 22 \text{ kHz}$$

Using this RF bandwidth as the FM receiver noise bandwidth, and a receiver C/N ratio of 10 dB, the baseband S/N ratio at the earth station receiver output for the recovered analog signal is

$$S/N = C/N + 10 \log (B/f_{max}) + 20 \log (\Delta f_{pk} / f_{max}) + 1.8 \text{ dB}$$

$$= 10.0 + 10 \log (22/1) + 20 \log (10/1) + 1.8 \text{ dB}$$

$$= 10 + 13.4 + 20 + 1.8 = 45.2 \text{ dB}$$

This is slightly better than the S/N for the BPSK signal transmitted in a bandwidth of 25 kHz, and would have more graceful degradation than the BPSK receiver output during a rain fade.

If we have a wider RF bandwidth of 5 kHz available and add half rate FEC with a coding gain of 6 dB, the receiver C/N falls by 3 dB because of the wider RF bandwidth and effective C/N ratio becomes

$$(C/N)_{eff} = 7.0 + 6.0 - 0.5 = 12.5 \text{ dB}$$

The bit error rate with BPSK modulation and $(C/N)_{eff} = 12.5$ dB = ratio 17.78 is

$$\text{BER} = Q[\sqrt{2 \times 17.78}] = Q[5.96] \approx 10^{-10}.$$

Quantization noise will now dominate giving S/N = 48 dB, and the BPSK link has a downlink margin of several decibels before the S/N ratio due to bit errors equals the quantization signal to noise ratio of 48 dB. This happens when z = 4.49 and C/N effective = 10.0 dB, C/N = 4.5 dB, giving a downlink C/N ratio margin of 2.5 dB. Performance of the two links is therefore comparable, with the digital link giving better performance for 99% of the time. Because of the many advantages of digital transmission, the BPSK (or a QPSK equivalent) system would be preferred.

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