Chapter 3 Solution to Problems

1. The telemetry system of a geostationary communications satellite samples 100 sensors on the spacecraft in sequence. Each sample is transmitted to earth as an eight-bit word in a TDM frame. An additional 200 bits are added to the frame for synchronization and status information. The data are then transmitted at a rate of 1 kilobit per second using BPSK modulation of a low-power carrier.

a. How long does it take to send a complete set of samples to earth from the satellite?

Answer: The TDM frame consists of $8 \times 100 = 800$ bits of data, plus 200 bits of sync and status information transmitted at 1 kbps. Thus a frame is 1000 bits transmitted in exactly one second.

b. Including the propagation delay, what is the longest time the earth station operator must wait between a change in a parameter occurring at the spacecraft and the new value of that parameter being received via the telemetry link? (Assume a path length of 40,000 km.)

Answer: Using velocity of EM waves, $c = 3 \times 10^8$ m/s and time delay = R / c,

$$T = 40 \times 10^6 / 3 \text{ x } 10^8 = 0.1333 \text{ s.}$$

Longest delay to receive a complete frame is 1.0 + 0.133 = 1.1333 s.

2. A spinner satellite has solar cells wrapped round a cylindrical drum 3.00 m in diameter, with a height of 5.0 m on station. The drum is rotated at 60 rpm to spin-stabilize the satellite. At the end of life, the solar cells are required to deliver 4.0 kW of electrical power.

a. Calculate the efficiency of the solar cells at end of life. Assume an incident solar power of 1.39 kW/m^2, and that the effective solar radiation absorbing area of the solar cells is equal to the cross sectional area of the drum.

Answer: Area of solar cells absorbing sunlight is equivalent to cross sectional area of drum.

$$A = 3.0 \times 5.0 = 15.0 \text{ m}^2.$$

At the end of life the solar cells are producing 4000 watts of electrical power.

Hence efficiency at end of life is

$$\eta_{EoL} = 4000 / (15 \text{ x } 1390) = 19.2\%$$

b. If the solar cells degrade by 15 percent over the lifetime of the satellite, so that the end-of-life output power is 85% of the beginning-of-life output power, what is the output of the solar cells immediately after launch?

Answer: The beginning of life output of the cells is η_{BoL} where

$$\eta_{BoL} = 4000 / 0.85 = 4706 \text{ watts}$$

c. If the drum covered in solar cells of the spinner design had been replaced by solar sails that rotated to face the sun at all times, what area of solar sails would have been needed? Assume that cells on solar sails generate only 90% percent of the power of cells on a spinner due to their higher operating temperature.

Answer: Using the end of life output of 4000 W, the efficiency of the solar cells on the sails is

$$\eta_{ss} = 0.9 \times 19.18 = 17.26 \%$$

The area of solar sails required is A where

$$A = 4000 / (0.1726 \times 1390) = 16.67 \text{ m}^2$$

3. A Direct Broadcast Television (DBS-TV) satellite is in geostationary orbit. The electrical power required to operate the satellite and its transmitters is 4 kW. Two designs of satellite can be used: three axis stabilized with solar cells and a spinner.

a. A three axis stabilized satellite has two solar sails of equal area that rotate to face the sun at all times. The efficiency of the solar cells at end of life is predicted to be 15%. Calculate the area of cells required by the GEO satellite, and the length of each sail if its width is 2.0 m.

Answer: $A_{ss} = 4000 / (0.15 \times 1390) = 19.18 \text{ m}^2$

Each sail is 2 m wide so total length needed = 19.18 / 2 = 9.60 m

Each sail is 4.8 m long.

b. A spinner design of DBS-TV satellite is made up from a drum coated in solar cells. The drum has a diameter of 3.5 m. The efficiency of the solar cells is predicted to be 18 % at end of life. Since half the solar cells are in darkness, and some are weakly illuminated by the sun, the effective area of solar cells on a spinner is equal to the diameter of the satellite multiplied by the height of the solar cells on the drum. Calculate the height of the drum to provide the required

4 kW of electrical power.

Answer: Effective solar cell area for spinner is $A_{eff} = W \times H$

$$A_{eff} = 4000 / (0.18 \times 1390) = 16.0 \text{ m}^2$$

Height of drum $= A_{eff} / W = 16.0 / 3.5 = 4.57$ m.

4. Batteries make up a significant part of the in-orbit weight of a communications satellite but are needed to keep the communications system operating during eclipses. A direct broadcast TV satellite requires 500 W of electrical power to operate the housekeeping functions of the satellite and 5 kW to operate its 16 high power transponders. The longest duration of an eclipse is 70 minutes, during which time the batteries must provide power to keep the satellite operating, but the batteries must not discharge below 70% of their capacity. The satellite bus operates at 48 volts.

a. What is the current that must be supplied by the power conditioning unit to keep the satellite operating normally?

Answer: Satellite power bus operates at 48 volts, hence $I = P / V = 5500 / 48 = 114.6$ A.

b. Battery capacity is rated in ampere hours, the product of the current (in amps) that the battery can supply multiplied by the length of time that this current can be supplied before the battery is fully discharged. The satellite batteries must not discharge beyond 70% of their rated capacity during eclipse. Find the battery capacity required for this DBS-TV satellite.

Answer: Duration of eclipse = 70 min = 1.167 hours.

Battery capacity required $= 114.6 \text{ A} \times 1.167 \text{ h} / 0.7 = 191$ AH

c. If batteries weigh 1.25 kg per ampere-hour of capacity, how much weight on this satellite is devoted to batteries?

Answer: Weight of batteries $= 191 \times 1.25 = 238.8$ kg.

d. If half of the transponders are shut down during eclipse, what saving in battery weight is achieved?

Answer: Current demand with half transponders off is $114.6 \text{ A} \times 3.0/5.5 = 62.5$ A

Battery capacity required $= 1.167 \times 62.5 / 0.7 = 104.2$ A. Weight $= 130.2$ kg.

5. A geostationary satellite provides service to a region which can be covered by the beam of an antenna on the satellite with a beamwidth of 1.8^o. The satellite carries transponders for Ku band and Ka band, with separate antennas for transmit and receive. For center frequencies of 14.0/11.5 GHz and 30.0/20.0 GHz, determine the diameters of the four antennas on the satellite.

a. Find the diameters of the two transmitting antennas. Specify the diameter and calculate the gain at each frequency.

Answer: Use the approximate relationship for beamwidth: $\theta_{3\,dB} = 75\,\lambda / D$.

Hence $D = 75\,\lambda / \theta_{3\,dB}$. Gain is approximately $33{,}000 / (\theta_{3\,dB})^2$

The transmitting antennas on the satellite operate at the lower frequency (downlink) in each band.

For 11.5 GHz: $\lambda = 0.02609$ m, $D = 75 \times 0.02609 / 1.8 = 1.087$ m

For 20 GHz: $\lambda = 0.015$ m, $D = 75 \times 0.015 / 1.8 = 0.625$ m

$G = 33{,}000 / 1.8^2 = 10.185$ or 40.1 dB

b. Find the diameters of the two receiving antennas. Specify the diameter and calculate the gain at each frequency.

The receiving antennas on the satellite operate at the higher frequency (uplink) in each band.

For 14.0 GHz: $\lambda = 0.02143$ m, $D = 75 \times 0.02413/ 1.8 = 0.893$ m

For 30.0 GHz: $\lambda = 0.010$ m, $D = 75 \times 0.010 / 1.8 = 0.417$ m

Because the beamwidth of each antenna is the same, the gains are all the same: $G = 40.1$ dB

6. A geostationary satellite provides communications within the United States at Ku band. The antennas on the satellite have beamwidths of 6^o in the E-W direction and 3^o in the N-S direction. A separate antenna is used for transmitting in the 11 GHz band and receiving in the 14 GHz band.

a. Find the dimensions and estimate the gain of the transmitting antenna in the N-S and E-W directions.

Answer: Using the approximations $\theta_{3\,dB} = 75\,\lambda / D$, and $G = 33{,}000 / (\theta_{3\,dB})^2$:

N-S dimension $= 75\,\lambda / 3.0 = 25\,\lambda$ E-W dimension $= 75\,\lambda / 6 = 12.5\,\lambda$

For transmit antenna at 11.0 GHz, $\lambda = 0.02727$ m

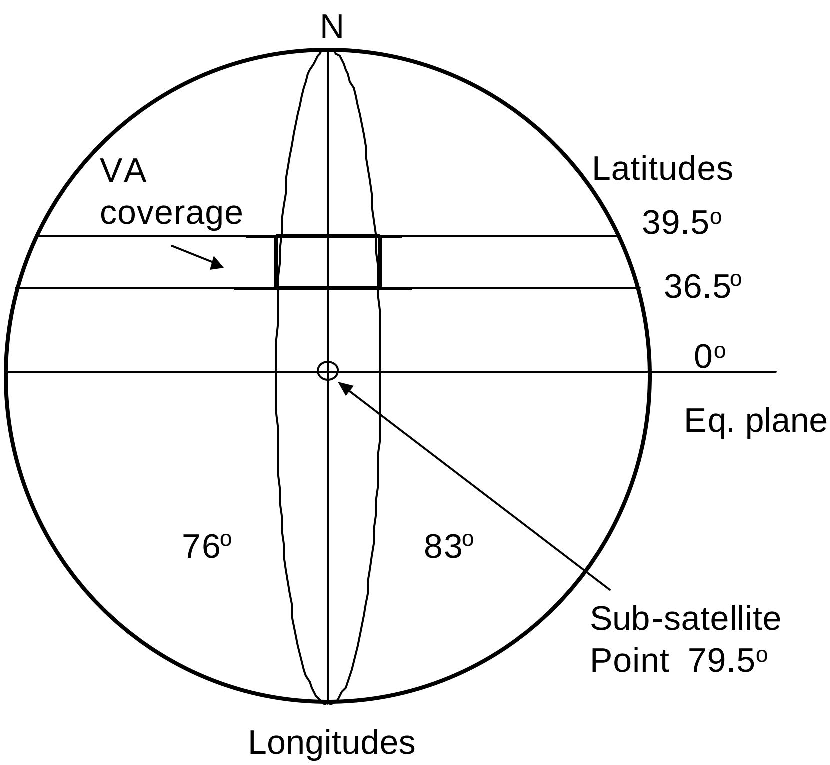
$D_{N\text{-}S} = 25 \times 0.02727 = 0.681$ m $\quad D_{E\text{-}W} = 12.3 \times 0.02727 = 0.341$ m

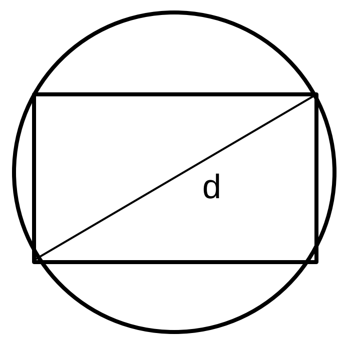
b. Find the dimensions and estimate the gain of the receiving antenna in the N-S and E-W directions.

Answer: For receiving antenna at 14.0 GHz, $\lambda = 0.02143$ m

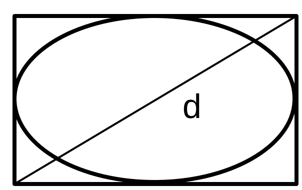
$D_{N\text{-}S} = 25 \times 0.02143 = 0.536$ m $\quad D_{E\text{-}W} = 12.5 \times 0.02143 = 0.268$ m

7. The State of Virginia can be represented approximately on a map as an area bounded by 39.5° N latitude, 36.5° N latitude, 76.0° W longitude, and 83.0° W longitude. A geostationary satellite located at 79.5° W longitude has an antenna with a spot beam that covers all of Virginia at a downlink center frequency of 11.155 MHz. In this problem you will estimate the antenna dimensions subject to two different assumptions. In both cases use an aperture efficiency of 65 percent.





Antenna Footprint in Part (a)



Antenna Footprint in Part (b)

a. The antenna is a circular parabolic reflector generating a circular beam with a 3 dB beamwidth equal to the diagonal of the area bounding the State of Virginia. Estimate the length of the diagonal by measuring the distance on a map of the US, and calculate the beamwidth of the antenna from simple geometry. Hence determine the diameter of the antenna on the satellite in meters and its approximate gain in decibels.

Answer: The diagram above shows the boundaries of the rectangle enclosing the State of Virginia. Measurement on a map shows that the diagonal pf the rectangle is approximately 800 km. The diagonal can be calculated from the lengths of the sides of the rectangle using the given altitude and longitude boundaries.

$$D_{N\text{-}S} = 6378 \times 3.0/57.3 = 334 \text{ km}$$

$$D_{E\text{-}W} = 6378 \times 7.0/57.3 = 779 \text{ km}$$

$$\text{Diagonal } d = [\, D_{E\text{-}W}{}^2 + D_{N\text{-}S}{}^2]^{1/2} = 848 \text{ km}$$

Working in the N-S longitude plane at 79.5° W, the distance from the satellite to the center of the rectangle at Lat 37.5° N, Long 79.5° W is s where a = GEO radius, r_e = earth radius

$$s^2 = a^2 + r_e{}^2 - 2\, a\, r_e \cos 37.5^\circ = 1.3918 \times 10^9 \qquad \text{hence} \quad s = 37{,}307 \text{ km}$$

The beamwidth of the antenna beam at the satellite can be found as a first approximation from the length of the diagonal of the rectangle. Using the map value of d = 800 km:

$$\theta_{3\,dB} = d / s \text{ radians} = 57.3 \times 800 / 37{,}307 = 1.23^\circ.$$

For an antenna operating at 11.155 GHz, $\lambda = 0.02689$ m, the antenna diameter is 75 λ/D giving

$$D = 75 \times 0.02689 / 1.23 = 1.640 \text{ m.}$$

The gain of the antenna, with aperture efficiency of 65%, can be found from

$$G = \eta_A \times (\pi\, D / \lambda)^2 = 0.65 \times (\pi \times 1.640 / 0.02689)^2 = 23{,}862 \text{ or } 43.8 \text{ dB}$$

b. The antenna is an elliptical parabolic reflector with 3 dB beamwidths in the N-S and E-W directions are equal to the height and the width of the area bounding the State of Virginia. Find the N-S and E-W dimensions from a map of the US, and use geometry to calculate the required 3 dB beamwidths of the satellite antenna. Calculate the approximate gain of the antenna.

Answer: Dimensions of the rectangle from a map are approximately 330 km in the N-S direction and 750 km in the E-W direction. Similar dimensions based on the latitudes and longitudes are given in part (a) above. In this case, the dimensions of the antenna must create an elliptical footprint that fits inside the rectangle. Ignoring earth curvature and using the distance from the center of the rectangle to the satellite, s = 37,345 km, the N-S and E-W beamwidths are approximately

$\theta_{N\text{-}S} = y_{N\text{-}S} / s$ radians $= 57.3 \times 330 / 37{,}307 = 0.51^o$

$\theta_{E\text{-}W} = x_{E\text{-}W} / s$ radians $= 57.3 \times 750 / 37{,}307 = 1.15^o$

The antenna dimensions are

$D_{N\text{-}S} = 75 \times 0.02689 / 0.51 = 3.95$ m

$D_{E\text{-}W} = 75 \times 0.02689 / 1.15 = 1.75$ m

The gain of the antenna can be found from

$G = 33{,}000 / (0.51 \times 1.15) = 56{,}273$ or 47.5 dB

Curvature of the earth in the N-S direction makes the N-S angle at the satellite smaller than the result in the calculation above where earth curvature is ignored. A more accurate result can be obtained by recalculating the distance from the satellite to the upper and lower edges of the rectangle and then using the rule of sines to find the angle at the satellite.

$s_1^2 = a^2 + r_e^2 - 2\, a\, r_e \cos 39.5^o = 1.4034 \times 10^9$ hence $s = 37{,}423$ km

$s_2^2 = a^2 + r_e^2 - 2\, a\, r_e \cos 36.5^o = 1.3861 \times 10^9$ hence $s = 37{,}230$ km

The angle between the line from the satellite to the earth's surface and from the satellite to the center of the earth is α where

$\sin \alpha / r_e = \sin \text{Lat} / s$ or $\sin \alpha = r_e \times \sin \text{Lat} / s$

For the two latitudes 39.5^o and 36.5^o we have

$\alpha_1 = 6378 / 37{,}423 \times \sin 39.5^o = 0.10841$ and $\alpha_1 = 6.223^o$

$\alpha_2 = 6378 / 37{,}230 \times \sin 36.5^o = 0.10191$ and $\alpha_2 = 5.849^o$

The satellite antenna beamwidth in the N-S direction is $\theta_{3\,dB} = \alpha_1 - \alpha_2 = 0.374^o$

The difference from the approximate result is significant because it makes the antenna even larger in the N-S direction. The approximation of a flat earth is reasonable for the E-W direction. Using the new N-S beamwidth, the N-S dimension of the antenna is

$D_{N\text{-}S} = 75 \times 0.02689 / 0.374 = 5.392$ m

The gain of the antenna increases to $33{,}000 / (0.374 \times 1.15) = 76{,}726$ or 48.8 dB

8. The state of Pennsylvania is approximately one degree wide (E-W) by one half degree high (N-S) when viewed from geostationary orbit at a longitude of 75 degrees west. Calculate:

a. The dimensions of a downlink Ku-band antenna on a geostationary satellite with 3 dB beamwidths equal to the width and height of Pennsylvania. Use a frequency of 11.0 GHz. Identify the dimensions as E-W and N-S.

Answer: The wavelength for 11.0 GHz is 0.02727 m, so the antenna dimensions are

$$D_{N-S} = 75 \times 0.02727 / 0.50 = 4.010 \text{ m}$$

$$D_{E-W} = 75 \times 0.02727 / 1.00 = 2.005 \text{ m}$$

The gain of the antenna can be found from

$$G = 33{,}000 / (0.50 \times 1.0) = 66{,}000 \text{ or } 48.2 \text{ dB}$$

b. The dimensions of an uplink Ka-band antenna on a geostationary satellite with 3 dB beamwidths equal to the width and height of Pennsylvania. Use a frequency of 30.0 GHz. Identify the dimensions as E-W and N-S.

Answer: The wavelength for 30.0 GHz is 0.010 m, so the antenna dimensions are

$$D_{N-S} = 75 \times 0.010 / 0.50 = 1.500 \text{ m}$$

$$D_{E-W} = 75 \times 0.010 / 1.00 = 0.750 \text{ m}$$

The gain of the antenna does not change because the beamwidths are the same as for 11 GHz

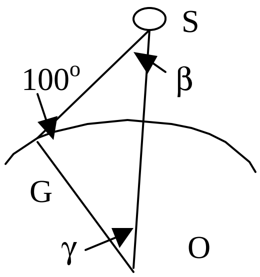
$$G = 33{,}000 / (0.50 \times 1.0) = 66{,}000 \text{ or } 48.2 \text{ dB}$$

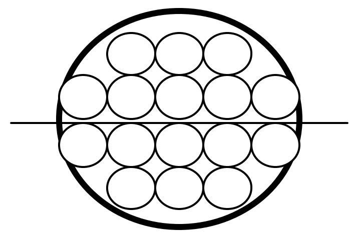
c. Suppose that the maximum dimension of the satellite at launch is 3 m wide, determined by the shroud of the ELV. Describe in a paragraph how you would launch the satellites in (a) and (b) above carrying: (a) the Ku band antenna, and (b) the Ka band antenna.

Answer: The dimensions of the antenna for 30 GHz in part (b) are below the diameter of the ELV shroud, so the antenna will fit inside the shroud. The antenna might need to be folded down against the satellite body for launch. In (a), the 11 GHz antenna dimension exceeds the shroud diameter, so the reflector of the antenna would have to be folded. This is difficult to do mechanically and is avoided whenever possible. Various methods have been used to make reflectors that can be folded or collapsed for launch – read Chapter 3 for the details.

9. A constellation of low earth orbit satellites has an altitude of 1000 km. Each satellite has two multiple beam antennas that generate 16 beams. One antenna is used to transmit at 2.4 GHz and the other antenna receives at 1.6 GHz.

a. Using simple geometry, find the coverage angle of the satellite antenna when the lowest elevation angle for an earth station is 10^o. (Hint: draw a diagram of the earth and the satellite and use the law of sines to solve the angles in a triangle.)





Geometry to find angle at satellite. Coverage zone with 16 beams

O is the center of the earth, G is the earth station, and S is the satellite.

Answer: The tiangle SGO in the above figure is used to solve for the angle β, which is one half of the coverage angle of the satellite. The angle OGS is $90^o + 10^o = 100^o$.

Lengths in triangle SGO are: OG $= r_e = 6378$ km, SO $= r_e + h = 7378$ km

From the law of sines: $\sin 100^o / (R_e + h) = \sin \beta / r_e$

Hence $\sin \beta = r_e / (r_e + h) \times \sin 100^o = 0.8513$; $\beta = 58.35^o$.

The coverage angle is 116.7^o.

b. Estimate the coverage area over the surface of the earth, in km.

Answer: The arc length across the coverage zone is given by $d = 2\, r_e\, \gamma$ with γ in radians.

$\gamma = 180^o - 100^o - 58.35^o = 21.65^o$

Hence $d = 2 \times 6378 \times 21.65 / 57.3 = 4820$ km

c. Assuming that all 16 beams from the satellite antennas have equal beamwidths, determine the beamwidth of one beam. (Hint: draw a circle representing the coverage area and fit 16 circles representing the 3 dB beamwidths of the beams inside the first circle.)

Answer: Between four and five of the multiple beams must fit across the coverage circle. Hence the multiple beams have a beamwidth in the range 116.7 /4 to 116.7 / 5 $= 29^o$ to 23^o.

d. Find the gain and the dimensions of each antenna on the satellite.

Answer: At the uplink frequency of 1.6 GHz, $\lambda = 0.1875$ m, the diameter of the satellite antenna is in the range 75×0.1875 (29^o to 23^o) $= 0.485$ m to 0.611 m.

At the downlink frequency of 2.4 GHz, $\lambda = 0.125$ m, the diameter of the satellite antenna is in the range 75×0.125 (29^o to 23^o) $= 0.323$ m to 0.408 m.

At both frequencies the gain is the same because the beamwidths are the same.

Using $G = 33000 / \theta_{3\,dB}{}^2$, gain is 39.2 to 62.4 or 15.9 to 18.0 dB

10. A geostationary satellite carries a C-band transponder which transmits 15 watts into an antenna with an on-axis gain of 32 dB. An earth station is in the center of the antenna beam from the satellite, at a distance of 38,500 km. For a frequency of 4.2 GHz:

a. Calculate the incident flux density at the earth station in watts per square meter and in dBW/m^2.

Answer: $F = P_t\, G_t / 4\pi\, R^2$ $W/m^2 = 15 + 32 - 11 – 20 \log 38.50 \times 10^6 = -115.7$ dBW / m^2

Converting to ratios

$F = 10^{-11.57} = 2.69 \times 10^{-12}$ W / m^2

b. The earth station has an antenna with a circular aperture 3 m in diameter and an aperture efficiency of 62%. Calculate the received power level in watts and in dBW at the antenna output port.

Answer: Received power is $F \times A_{eff} = F \times \eta \times \pi\, r^2 = 2.69 \text{ x } 10^{-12} \times 0.62 \times \pi \times 1.5^2$

$= 1.179 \times 10^{-11}$ watts or -109.3 dBW

c. Calculate the on-axis gain of the antenna in decibels.

Answer: At 4.2 GHz, $\lambda = 0.07143$:

$$G = \eta\, (\pi\, D / \lambda)^2 = 0.62 \times (\pi \times 3.0 / 0.07143)^2 = 10{,}793 \text{ or } 40.3 \text{ dB}$$

d. Calculate the free space path loss between the satellite and the earth station.

Calculate the power received, Pr, at the earth station using the link equation:

Pr = Pt Gt Gr / Lp where Pt Gt is the EIRP of the satellite transponder and Lp is the path loss. Make your calculation in dB units and give your answer in dBW.

Answer: Path loss = 20 log (4 π R / λ) = 20 log (4 π 38.5 $\times$ 10^6 / 0.07143) = 196.6 dB

The received power at the earth station is calculated from

$$P_r = P_t G_t G_r / L_p$$

Ignoring any losses. In dB units

$$P_r = \text{EIRP} + G_r - \text{path loss} = 15 + 32 + 40.3 - 196.6 = -109.3 \text{ dBW}$$

This is the same answer as obtained in part (b) using flux density.

12. Calculate the total power radiated by the sun in watts and in dBW.

Hint: The sun is 93 million miles (about 150 million kilometers) from the earth. At that distance, the sun produces a flux density of 1.39 kW/m^2. This power density is present over all of a sphere with a radius of 150 million km.

Answer: The entire light output of the sun passes through the surface of an imaginary sphere with a radius of 150 x 10^6 km. At that distance, the flux density crossing the sphere is 1.39×10^3 W/m^2. Hence the output of the sun is $1.39 \times 10^3 \times 4\pi R^2$ where $R = 1.5 \times 10^{11}$ m.

Hence $P = 1.39 \times 10^3 \times 4\pi \times (1.5 \times 10^{11})^2 = 3.93 \times 10^{26}$ watts or 266 dBW.

If calculations show any transmitter power to approach 266 dBW, the calculations are wrong.