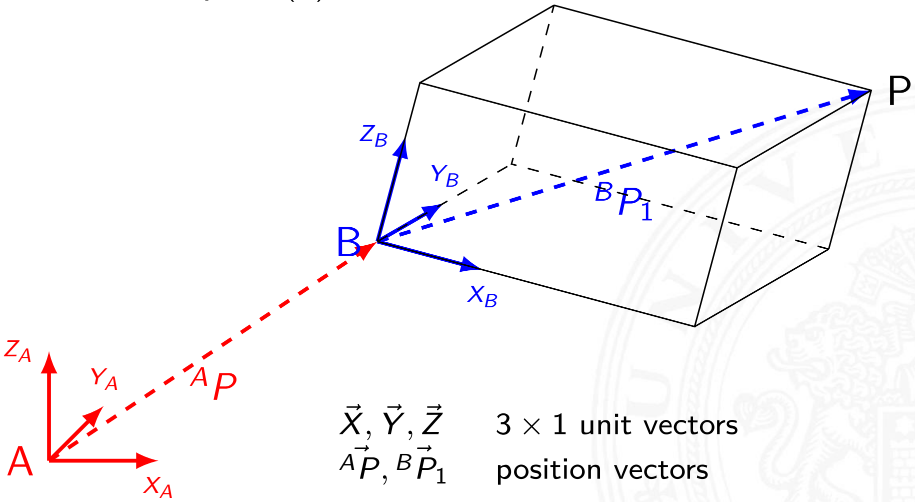
Outline

Outline (cont.)

Coordinate Systems

The **pose** of objects, in other words their **position** and **orientation** in Euclidian space can be described through specification of a cartesian coordinate system (B) in relation to a base coordinate system (A).



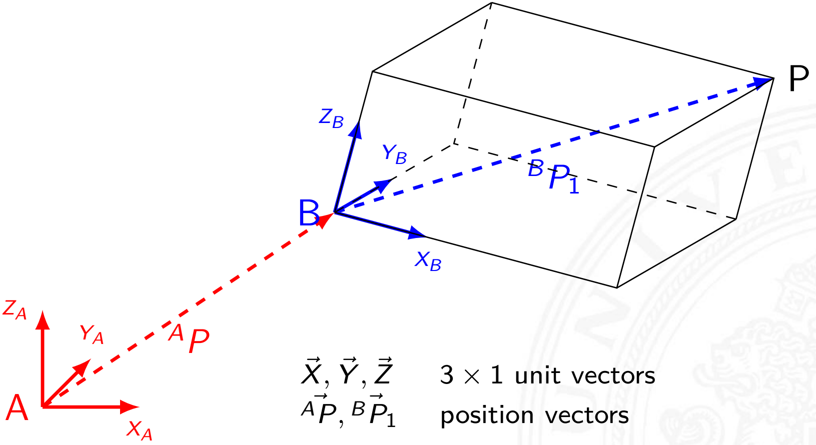
$\vec{X}, \vec{Y}, \vec{Z}$ 3 × 1 unit vectors

$^A\vec{P}, {}^B\vec{P}_1$ position vectors

Specification of position and orientation

Position:

- translation along the axes of the base coordinate system (A)



$\vec{X}, \vec{Y}, \vec{Z}$	3×1 unit vectors
$^A\vec{P}, {}^B\vec{P}_1$	position vectors

- given by position vector $\vec{^A\mathbf{P}} = [^A p_x, ^A p_y, ^A p_z]^T \in \mathcal{R}^3$

`Orientation` (in space):

- given by Rotation matrix $R_B = [\vec{X}_B \ \vec{Y}_B \ \vec{Z}_B] \in \mathcal{R}^{3\times3}$
- given by Rotation matrix ${}^AR_B = [{}^A\vec{X}_B \ {}^A\vec{Y}_B \ {}^A\vec{Z}_B] \in \mathcal{R}^{3\times3}$

- ${}^AR_B$: the orientation of B with respect to A.
  (Latex: `$^{A}R_{B}$`)
- ${}^A\vec{X}_B, {}^A\vec{Y}_B, {}^A\vec{Z}_B$ are projection of $\vec{X}_B$, $\vec{Y}_B$, $\vec{Z}_B$ in A.

Rotation matrix

Dot product

In terms of the geometric definition, the dot product of two unit vectors $\vec{a}$ and $\vec{b}$ means the projection of the $\vec{a}$ in $\vec{b}$.
$\vec{a} \cdot \vec{b} = \|a\|\|b\| \cos(\theta)$

$$^A\vec{X}_B = \begin{bmatrix} \vec{X}_B \cdot \vec{X}_A \\ \vec{X}_B \cdot \vec{Y}_A \\ \vec{X}_B \cdot \vec{Z}_A \end{bmatrix} \quad \text{and} \quad {}^AR_B = \begin{bmatrix} {}^A\vec{X}_B & {}^A\vec{Y}_B & {}^A\vec{Z}_B \end{bmatrix}$$

$$^AR_B = \begin{bmatrix} \vec{X}_B \cdot \vec{X}_A & \vec{Y}_B \cdot \vec{X}_A & \vec{Z}_B \cdot \vec{X}_A \\ \vec{X}_B \cdot \vec{Y}_A & \vec{Y}_B \cdot \vec{Y}_A & \vec{Z}_B \cdot \vec{Y}_A \\ \vec{X}_B \cdot \vec{Z}_A & \vec{Y}_B \cdot \vec{Z}_A & \vec{Z}_B \cdot \vec{Z}_A \end{bmatrix}$$

Inverse of rotation matrix

$$
\begin{array}{ll}
{}^{A}R_{B} = \begin{bmatrix} \vec{X}_B \cdot \vec{X}_A & \vec{Y}_B \cdot \vec{X}_A & \vec{Z}_B \cdot \vec{X}_A \\ \vec{X}_B \cdot \vec{Y}_A & \vec{Y}_B \cdot \vec{Y}_A & \vec{Z}_B \cdot \vec{Y}_A \\ \vec{X}_B \cdot \vec{Z}_A & \vec{Y}_B \cdot \vec{Z}_A & \vec{Z}_B \cdot \vec{Z}_A \end{bmatrix} & \begin{array}{l} {}^{B}X_A^T \\ \text{the projection of } \vec{X}_A \text{ in B} \\ \\ \end{array}
\end{array}
$$

$$
{}^{A}R_{B} = \begin{bmatrix} {}^{A}\vec{X}_B & {}^{A}\vec{Y}_B & {}^{A}\vec{Z}_B \end{bmatrix} = \begin{bmatrix} {}^{B}\vec{X}_A^T \\ {}^{B}\vec{Y}_A^T \\ {}^{B}\vec{Z}_A^T \end{bmatrix} = \begin{bmatrix} {}^{B}\vec{X}_A & {}^{B}\vec{Y}_A & {}^{B}\vec{Z}_A \end{bmatrix}^T = {}^{B}R_A^T
$$

Inverse of rotation matrix (cont.)

$$^{A}R_{B} = \begin{bmatrix} ^{A}\vec{X}_{B} & ^{A}\vec{Y}_{B} & ^{A}\vec{Z}_{B} \end{bmatrix} = \begin{bmatrix} ^{B}\vec{X}_{A}^{T} \\ ^{B}\vec{Y}_{A}^{T} \\ ^{B}\vec{Z}_{A}^{T} \end{bmatrix} = \begin{bmatrix} ^{B}\vec{X}_{A} & ^{B}\vec{Y}_{A} & ^{B}\vec{Z}_{A} \end{bmatrix}^{T} = {^{B}R_{A}^{T}}$$

The inverse of a rotation matrix is simply its transpose:

$$^{A}R_{B}^{-1} = {^{B}R_{A}} = {^{B}R_{A}^{T}} \quad \text{and} \quad {^{A}R_{B}}{^{B}R_{A}} = I$$

whereas I is the identity matrix.

Specification of position and orientation

- Position:
  - given through ${}^{A}\vec{P} \in \mathcal{R}^3$
- Orientation:
  - given through the projection of $\vec{X}_B, \vec{Y}_B, \vec{Z}_B \in \mathcal{R}^3$ of B to the origin system A
  - summarized to rotation matrix ${}^{A}R_B = [{}^{A}\vec{X}_B \; {}^{A}\vec{Y}_B \; {}^{A}\vec{Z}_B] \in \mathcal{R}^{3\times3}$

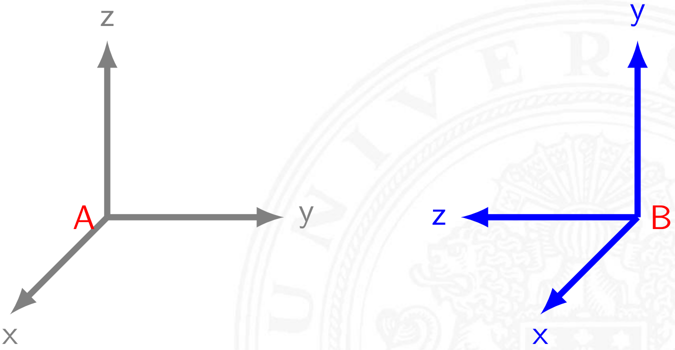
$$
{}^{A}R_B = \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}
$$

  - redundant, since there are 9 parameters for 3 degrees of freedom

Example of rotation matrix

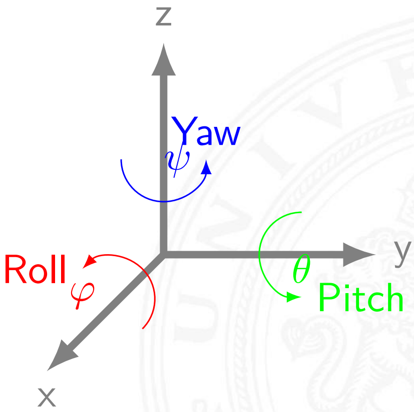
Write the Rotation matrix of ${}^{A}R_{B}$.
${}^{A}R_{B} = [{}^{A}\vec{X}_{B}\ {}^{A}\vec{Y}_{B}\ {}^{A}\vec{Z}_{B}]$

$$
{}^{A}R_{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}
$$



Sequential multiplication of the rotation matrices by order of rotation.

1. rotation φ (*phi*) around the x-axis
   $R_{x,\varphi}$ – Roll
2. rotation θ (*theta*) around the y-axis
   $R_{y,\theta}$ – Pitch
3. rotation ψ (*psi*) around the z-axis
   $R_{z,\psi}$ – Yaw



Rotatory transformation

(shortened representation: S : sin, C : cos)
The rotation matrix corresponding to a rotation around the x-axis with angle φ (*phi*):

$$R_{x,\varphi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$

Rotatory transformation (cont.)

The rotation matrix corresponding to a rotation around the y-axis with angle θ (*theta*):

$$R_{y,\theta} = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

Rotatory transformation (cont.)

The rotation matrix corresponding to a rotation around the *z*-axis with angle ψ (*psi*):

$$R_{z,\psi} = \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Concatenation of rotation matrices

$$R_{\psi,\theta,\varphi} = R_{z,\psi} R_{y,\theta} R_{x,\varphi}$$

$$= \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$

$$= \begin{bmatrix} C\psi C\theta & C\psi S\theta S\varphi - S\psi C\varphi & C\psi S\theta C\varphi + S\psi S\varphi \\ S\psi C\theta & S\psi S\theta S\varphi + C\psi C\varphi & S\psi S\theta C\varphi - C\psi S\varphi \\ -S\theta & C\theta S\varphi & C\theta C\varphi \end{bmatrix}$$

Remark: Matrix multiplication is not commutative:

$$AB \neq BA$$

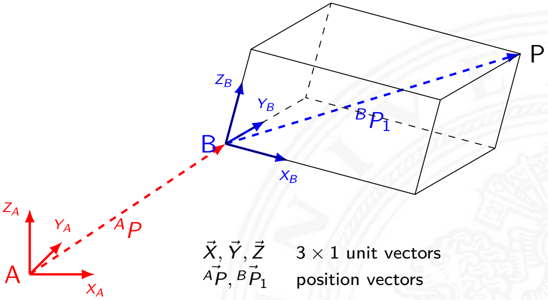
Concatenation of rotation matrices

- `Several rotations` can be multiplied. The following applies:
  - If the rotations are performed in relation to the current, newly defined (or changed) coordinate system, the newly added transformation matrices need to be multiplicatively appended on the right-hand side.
  - If all of them are performed in relation to the fixed reference coordinate system, the transformation matrices need to be multiplicatively appended on the left-hand side.

Mapping by rotation matrix

Mapping: changing descriptions from frame to frame.
For example, change the reference frame of ${}^{B}\vec{P}_1$?

$$
\begin{aligned}
{}^{A}\vec{P}_1 &= \begin{bmatrix} {}^{B}\vec{X}_A \cdot {}^{B}\vec{P}_1 \\ {}^{B}\vec{Y}_A \cdot {}^{B}\vec{P}_1 \\ {}^{B}\vec{Z}_A \cdot {}^{B}\vec{P}_1 \end{bmatrix} \\
&= \begin{bmatrix} {}^{B}\vec{X}_A^{T} \\ {}^{B}\vec{Y}_A^{T} \\ {}^{B}\vec{Z}_A^{T} \end{bmatrix} \cdot {}^{B}\vec{P}_1 \\
&= {}^{A}R_B {}^{B}\vec{P}_1
\end{aligned}
$$



$\vec{X}, \vec{Y}, \vec{Z}$ 3 × 1 unit vectors
${}^{A}\vec{P}, {}^{B}\vec{P}_1$ position vectors

Summary: three common uses of a rotation matrix

Three common uses of a rotation matrix:

- represent an orientation
- rotate a vector or frame
- change the frame of reference of a vector or frame

Homogenous transformation

- `Homogeneous transformation matrix:`

$$T = \begin{bmatrix} R & \vec{p} \\ P & S \end{bmatrix}$$

where P depicts the perspective transformation and S the scaling.

- In robotics, $P = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ and $S = 1$. Other values are used for computer graphics.

Homogenous transformation (cont.)

- Combination of $\vec{p}$ and R to $T = \begin{bmatrix} R & \vec{p} \\ \vec{0} & 1 \end{bmatrix} \in \mathcal{R}^{4\times4}$
- Concatenation of several T through matrix multiplication
  - ${}^{A}T_{B}\,{}^{B}T_{C} = {}^{A}T_{C}$
- not commutative, in other words ${}^{B}T_{C}\,{}^{A}T_{B} \neq {}^{A}T_{B}\,{}^{B}T_{C}$

They are represented as four vectors using the elements of homogeneous transformation.

$$T = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{21} & r_{31} & p_x \\ r_{12} & r_{22} & r_{32} & p_y \\ r_{13} & r_{23} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Inverse transformation

The inverse of a rotation matrix is simply its transpose:

$$R^{-1} = R^T \text{ and } RR^T = I$$

whereas I is the identity matrix.
The inverse of (1) is:

$$T^{-1} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{p}^\mathbf{T} \cdot \mathbf{r}_1 \\ r_{21} & r_{22} & r_{23} & -\mathbf{p}^\mathbf{T} \cdot \mathbf{r}_2 \\ r_{31} & r_{32} & r_{33} & -\mathbf{p}^\mathbf{T} \cdot \mathbf{r}_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

whereas $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ and $\mathbf{p}$ are the four column vectors of (1) and $\cdot$ represents the dot product of vectors.

Translatory transformation

A translation with a vector $[p_x, p_y, p_z]^T$ is expressed through a transformation:

$$T_{(p_x,p_y,p_z)} = \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotatory transformation

The transformation corresponding to a rotation around the x-axis with angle φ (*phi*):

$$T_{x,\varphi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\varphi & -S\varphi & 0 \\ 0 & S\varphi & C\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation corresponding to a rotation around the y-axis with angle θ (*theta*):

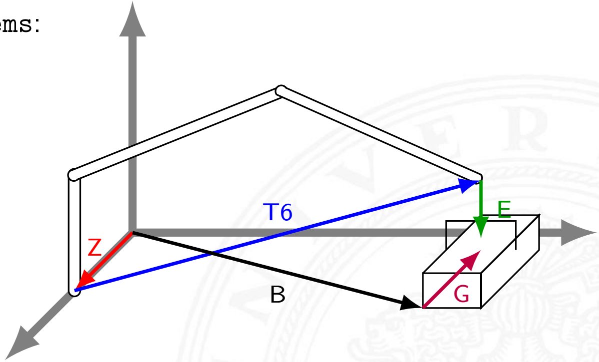
$$T_{y,\theta} = \begin{bmatrix} C\theta & 0 & S\theta & 0 \\ 0 & 1 & 0 & 0 \\ -S\theta & 0 & C\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotatory transformation (cont.)

The transformation corresponding to a rotation around the *z*-axis with angle ψ (*psi*):

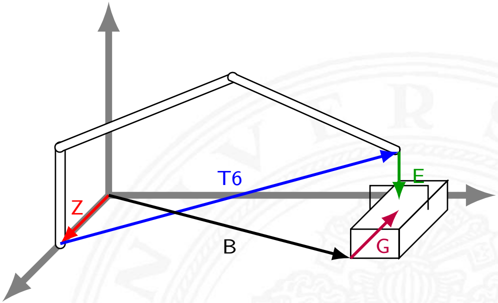
$$T_{z,\psi} = \begin{bmatrix} C\psi & -S\psi & 0 & 0 \\ S\psi & C\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- `Transform of Coordinate systems`:
  frame: a reference S
  typical frames:
  - robot base
  - end effector
  - table (world)
  -
  - object
  - camera
  - ...



One has the following transformations:

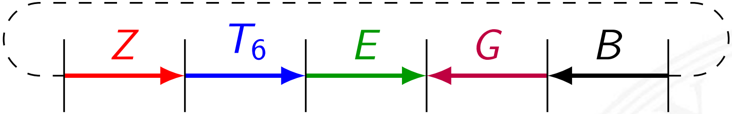
- Z :
  World $\rightarrow$ Manipulator base
- T_6 :
  Manipulator base $\rightarrow$ Manipulator end
- E :
  Manipulator end $\rightarrow$ End effector
- B :
  World $\rightarrow$ Object
- G :
  Object $\rightarrow$ End effector



Transformation equation

There are two descriptions for the desired end effector pose, one in relation to the object and the other in relation to the manipulator. Both descriptions should equal to each other for grasping:

$$Z T_6 E = B G$$



In order to find the manipulator transformation:

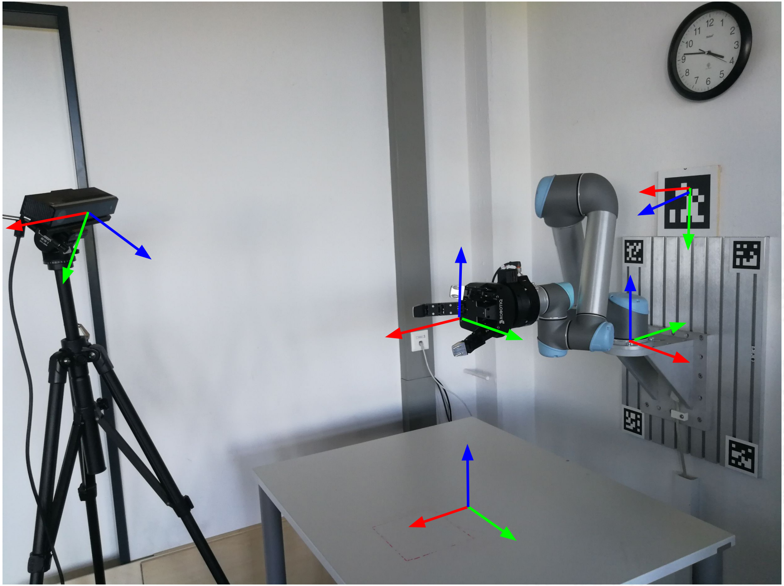
$$T_6 = Z^{-1} B G E^{-1}$$

In order to determine the pose of the object:

$$B = Z T_6 E G^{-1}$$

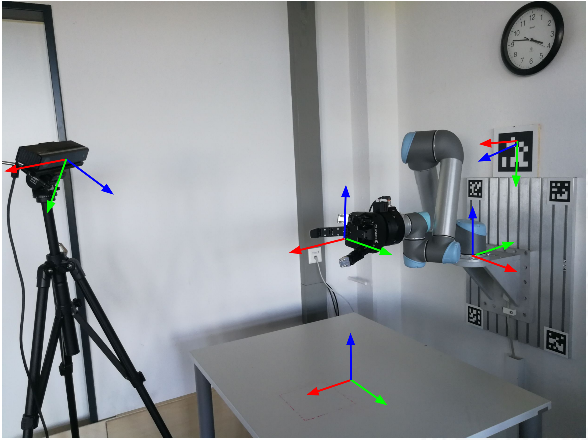
This is also called kinematic chain.

Example: coordinate transformation



Example: coordinate transformation

Given $T_{Base-Apriltag}$, $T_{Camera-Apritag}$, $T_{Camera-Object}$, calculate $T_{Base-Object}$.



$$T_{Base-Object} = T_{Base-Apriltag} T^{-1}_{Camera-Apritag} T_{Camera-Object}$$

Summary of homogeneous transformations

- `A homogeneous transformation` depicts the position and orientation of a coordinate frame in space.
- `If the coordinate frame` is defined in relation to a solid object, the position and orientation of the solid object is unambiguously specified.
- Three common uses of a transformation matrix: to represent a rigid-body configuration; to change the frame of reference of a vector or a frame; to displace a vector or a frame.

Summary of homogeneous transformations (cont.)

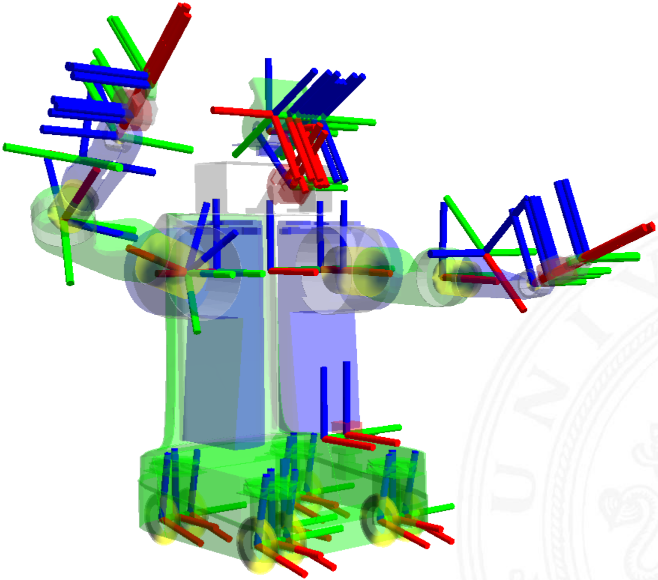
- Several translations and rotations can be multiplied.
  - right-hand multiplication → in relation to the current, newly defined (or changed) coordinate system.
  - left-hand multiplication → in relation to the fixed reference coordinate system.

Coordinates of a manipulator

- Joint coordinates:
  A vector $\mathbf{q}(t) = (q_1(t), q_2(t), ..., q_n(t))^T$
  (a robot configuration)
- End effector coordinates
  (Object coordinates):
  - A vector $\mathbf{p} = [p_x, p_y, p_z]^T$
  - Rotation matrix:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Outlook



Outlook (cont.)

- Can we use less of 9 parameters to represent the orientation?
- How to construct the transformation matrix of the manipulator's end-effector relative to the base of the manipulator?

Suggestions

- Read (available on google & library):
  - J. F. Engelberger, *Robotics in service*. MIT Press, 1989
  - K. Fu, R. González, and C. Lee, *Robotics: Control, Sensing, Vision, and Intelligence*. McGraw-Hill series in CAD/CAM robotics and computer vision, McGraw-Hill, 1987
  - R. Paul, *Robot Manipulators: Mathematics, Programming, and Control: the Computer Control of Robot Manipulators*. Artificial Intelligence Series, MIT Press, 1981
  - J. Craig, *Introduction to Robotics: Pearson New International Edition: Mechanics and Control*. Always learning, Pearson Education, Limited, 2013
- Repeat your linear algebra knowledge, especially regarding elementary algebra of matrices.

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