Continuous Assessment Test (CAT) – II

B1-Slot-Answer Key

Q1

a) Suitable Image Compression Techniques (5 Marks)

A suitable image compression technique for medical MRI scans is **lossless compression**, such as **PNG** or **JPEG 2000 (lossless mode)**. Here are five valid reasons:

1. **Preservation of Image Quality:** Lossless compression ensures no data is lost during compression, which is crucial for accurate medical diagnosis from MRI scans.
2. **Compliance with Medical Standards:** Many healthcare standards, like DICOM, require that medical images maintain their integrity, making lossless compression ideal.
3. **Retention of Diagnostic Features:** Important diagnostic details, like subtle differences in tissue density, are preserved, allowing accurate analysis.
4. **Efficient Storage and Transmission:** While preserving all data, lossless compression reduces file size, helping in efficient storage and transmission of large MRI datasets.
5. **Segmentation Accuracy:** Since no data is discarded, the accuracy of segmentation techniques such as thresholding and region-growing remains unaffected, allowing precise detection of abnormalities like tumors.

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b) Impact of Compression Artifacts on Segmentation Results (5 Marks)

Compression artifacts, particularly in **lossy compression** techniques like **standard JPEG**, may distort image data, which can impact segmentation results. For example, **blockiness** or **blurring** artifacts can occur, especially at higher compression ratios, leading to inaccurate segmentation.

Example: Suppose an MRI scan is compressed using a high-ratio JPEG. When applying region-growing techniques to detect a tumor, the compression artifacts might blur the edges of the tumor, making it harder for the algorithm to accurately distinguish between healthy and abnormal tissue. This could result in missed tumor boundaries, leading to incorrect diagnoses. Compression artifacts can also introduce noise, which may confuse thresholding methods, causing false positives or negatives in the segmentation process.

a) Illustration of Cubic Polynomial in Trajectory Generation (5 Marks)

A **cubic polynomial** is used in trajectory generation to ensure smooth transitions for a robotic joint from an initial position to a final position. A cubic polynomial has the form:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Where:

- $\theta(t)$ is the joint angle at time t.
- a_0, a_1, a_2, a_3 are the coefficients determined by boundary conditions.

Application in Trajectory Generation:

To move a robot joint from an initial position θ_i at time $t = 0$ to a final position θ_f at time $t = t_f$, the cubic polynomial ensures smooth motion, providing continuous position, velocity, and acceleration profiles. The robot's motion will follow this smooth curve, avoiding sudden jumps in velocity or acceleration.

Boundary conditions are set to ensure smooth motion:

- **Position**: $\theta(0) = \theta_i$ and $\theta(t_f) = \theta_f$
- **Velocity**: $\dot{\theta}(0) = 0$ (starting at rest) and $\dot{\theta}(t_f) = 0$ (ending at rest)

The coefficients a_0, a_1, a_2, a_3 are computed to satisfy these conditions, providing a trajectory that smoothly moves the joint from its initial to final position within the given time.

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b) Deriving the General Form of the Cubic Polynomial (5 Marks)

For a smooth trajectory that ensures **zero initial and final velocities**, we use the following boundary conditions:

1. **Initial position**: $\theta(0) = \theta_i$
2. **Final position**: $\theta(t_f) = \theta_f$
3. **Initial velocity**: $\dot{\theta}(0) = 0$
4. **Final velocity**: $\dot{\theta}(t_f) = 0$

Using the general cubic polynomial:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Taking the first derivative to find velocity:

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

Now apply the boundary conditions to derive the coefficients:

- At $t = 0$:
  - $\theta(0) = \theta_i$ gives $a_0 = \theta_i$
  - $\dot{\theta}(0) = 0$ gives $a_1 = 0$
- At $t = t_f$:
  - $\theta(t_f) = \theta_f$ gives:

$$\theta_f = a_0 + a_2 t_f^2 + a_3 t_f^3$$

  - $\dot{\theta}(t_f) = 0$ gives:

$$0 = a_1 + 2a_2 t_f + 3a_3 t_f^2 \Rightarrow 0 = 2a_2 t_f + 3a_3 t_f^2$$

From the above equations, we can solve for a_2 and a_3:

- $a_2 = \frac{3(\theta_f - \theta_i)}{t_f^2}$
- $a_3 = \frac{-2(\theta_f - \theta_i)}{t_f^3}$

Thus, the general form of the cubic polynomial for a smooth trajectory is:

$$\theta(t) = \theta_i + \frac{3(\theta_f - \theta_i)}{t_f^2} t^2 - \frac{2(\theta_f - \theta_i)}{t_f^3} t^3$$

This ensures the robot joint moves smoothly from the initial to the final position with zero velocity at both ends.

a) Identify the link parameters and draw DH (Denavit–Hartenberg) table (7 Marks)

The Denavit-Hartenberg (DH) parameters are used to describe the relative positions of the robot's links. Each joint can be described by 4 parameters:

- θ_i: Joint angle (variable for revolute joints)
- d_i: Offset along the previous z-axis to the common normal
- a_i: Length of the common normal (distance between z-axes)
- α_i: Twist angle (angle between z-axes)

For the **three-link planar arm** in the diagram, we have the following DH parameters:

Link (i)	θ_i	d_i	a_i	α_i
1	θ_1	0	L_1	0
2	θ_2	0	L_2	0
3	θ_3	0	L_3	0

- **Link 1**: Revolute joint connecting the base to the first arm link of length L_1.
- **Link 2**: Revolute joint connecting the second link of length L_2.
- **Link 3**: Revolute joint connecting the final link (end-effector) of length L_3.
- All twist angles α_i are 0 because the robot operates in a 2D plane (no rotation between axes).

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b) Calculate θ_1, θ_2, and θ_3 using inverse kinematics (8 Marks)

Inverse kinematics calculates the joint angles ($\theta_1, \theta_2, \theta_3$) to position the end-effector at a specific point (X_e, Y_e).

Given:

- End-effector position: (X_e, Y_e)
- Link lengths: L_1, L_2, L_3

To calculate θ_1, θ_2, and θ_3, follow these steps:

1. **Calculate the distance from the base to the end-effector**:

$$r = \sqrt{X_e^2 + Y_e^2}$$

2. **Use cosine law to find θ_2**:

$$\cos(\theta_2) = \frac{r^2 - L_1^2 - L_2^2}{2L_1L_2}$$

   From this, θ_2 can be calculated as:

$$\theta_2 = \cos^{-1}\left(\frac{r^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

3. **Find angle θ_1 using geometric relationships**: First, calculate the angle ϕ, which is the angle between the x-axis and the line connecting the base to the end-effector:

$$\phi = \tan^{-1}\left(\frac{Y_e}{X_e}\right)$$

   Then, use the following equation to find θ_1:

$$\theta_1 = \phi - \tan^{-1}\left(\frac{L_2\sin(\theta_2)}{L_1 + L_2\cos(\theta_2)}\right)$$

4. **Find the third joint angle θ_3**:

$$\theta_3 = \theta_{\text{goal}} - (\theta_1 + \theta_2)$$

   Where θ_{goal} is the desired orientation of the end-effector.

The joint angles $\theta_1, \theta_2, \theta_3$ are now calculated, allowing the robotic arm to reach the desired end-effector position (X_e, Y_e).

The joint angles $\theta_1, \theta_2, \theta_3$ are now calculated, allowing the robotic arm to reach the desired end-effector position (X_e, Y_e).

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A two-link manipulator with rotational joints is shown above. Calculate the velocity of the tip of the arm as a function of joint rates. Give the answer in two forms—in terms of frame {3}, and also in terms of frame {0}. answer key:

Answer Key:

To calculate the velocity of the tip of the two-link manipulator as a function of joint rates, we need to use the Jacobian matrix, which relates the joint velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ to the end-effector velocity.

1. Velocity in terms of frame {3}:

The velocity of the tip in frame {3} can be found by multiplying the Jacobian matrix by the joint velocities:

$$\mathbf{v}_3 = \mathbf{J}_3 \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Where:

- $\mathbf{J}_3$ is the Jacobian matrix in frame {3}.
- $\dot{\theta}_1$ and $\dot{\theta}_2$ are the angular velocities of the first and second joints, respectively.

In frame {3}, the manipulator's velocity is influenced by both the rotational and linear contributions of each joint.

2. Velocity in terms of frame {0}:

To calculate the velocity in the base frame {0}, we use the transformation matrices between the frames. The velocity of the end-effector in frame {0} is:

$$\mathbf{v}_0 = \mathbf{J}_0 \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Where:

- $\mathbf{J}_0$ is the Jacobian matrix in frame {0}.

In the base frame, the velocity calculation will account for the full kinematic chain of the two-link arm.

Jacobian Matrix Calculation:

For a two-link planar manipulator with link lengths L_1 and L_2, the Jacobian in frame {0} is derived as:

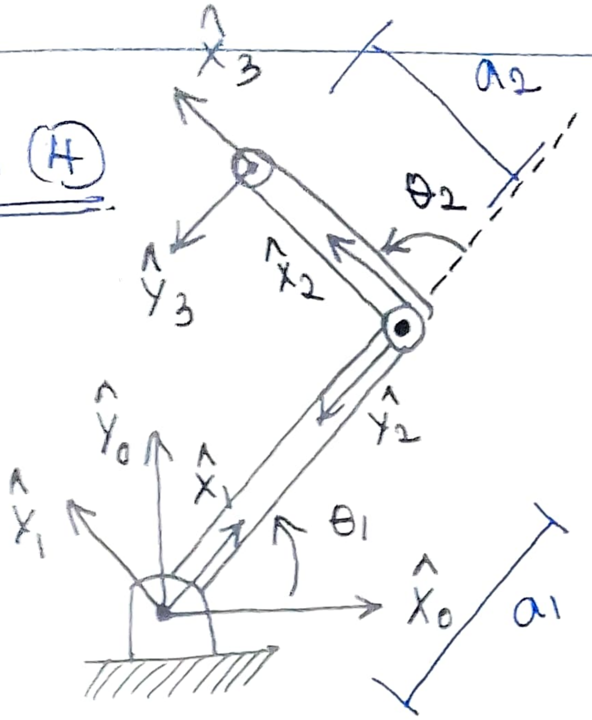
$$\mathbf{J}_0 = \begin{bmatrix} -L_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

Multiplying this Jacobian by the joint velocity vector gives the linear velocity of the end-effector in frame {0}.

Summary:

- The velocity of the tip of the manipulator can be expressed in terms of the joint velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ through the Jacobian matrix.
- In frame {3}, the velocity $\mathbf{v}_3$ is computed using $\mathbf{J}_3$.
- In frame {0}, the velocity $\mathbf{v}_0$ is computed using the Jacobian $\mathbf{J}_0$, considering the lengths of the links L_1 and L_2 and the angles θ_1 and θ_2.

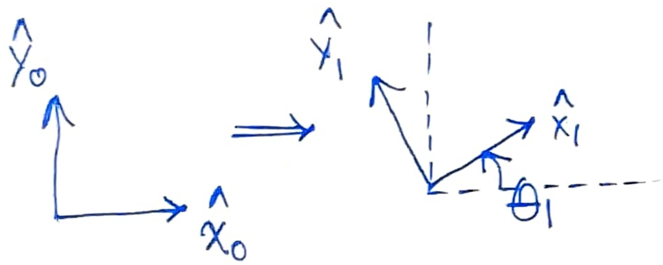
Q. (4)



A two link manipulator with rotational joints.

counterclock (+ve)
clockwise (-ve)

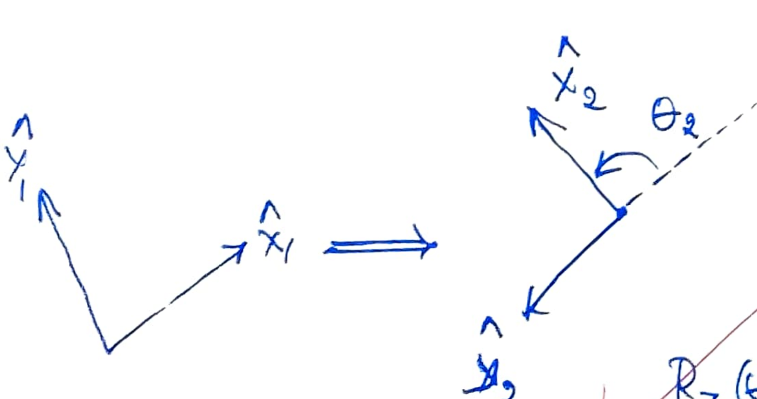
In terms of frame {0}:



Rotational Matrix:

* Rotating in Z-axis with an angle 'θ_1'. (+ve)

$$R_Z(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



(θ_2) +ve

$$R_Z(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & a_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta_3) = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \theta_3 = 0^\circ$$

$$P_0 = R_z(\theta_1)\, R_z(\theta_2)\, R_z(\theta_3)$$

$$R_z(\theta_1)\, R_z(\theta_2) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & a_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 + 0 = \cos(\theta_1+\theta_2) \\ -\cos\theta_1 \sin\theta_2 - \sin\theta_1 \sin\theta_2 + 0 = -\sin\theta_2(\cos\theta_1 + \sin\theta_1 \\ -\sin\theta_1 (a_1) + 0 = -a_1 \sin\theta_1 \end{cases}$$

$$\begin{cases} \sin\theta_1 \cos\theta_2 + \cos\theta_1 + \cos\theta_2 + 0 = \cos\theta_2(\sin\theta_1 + \sin\theta \\ -\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 + 0 = \cos(\theta_1+\theta_2) \\ a_1 \cos\theta_1 = a_1 \cos\theta_1 \end{cases}$$

$$\begin{cases} 0 \\ 0 \\ 1 \end{cases}$$

$$R_Z(\theta_1)\, R_Z(\theta_2) \cdot R_Z(\theta_3)$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin\theta_2(\cos\theta_1 + \cancel{\cos}\,\sin\theta_1) & -a_1\sin\theta_1 \\ \cos\theta_2(\cos\theta_1+\sin\theta_1) & \cancel{\sin}\,\cos(\theta_1+\theta_2) & a_1\cos\theta_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & a_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} \cos(\theta_1+\theta_2) \\ -\sin\theta_2(\cos\theta_1+\sin\theta_1) \\ a_2\cos(\theta_2+\theta_1) - a_1\sin\theta_1 \end{cases}$$

$$\begin{cases} \cos\theta_2\cos(\theta_1+\theta_2) \\ \cos(\theta_1+\theta_2) \\ (a_2\cos\theta_2\cos(\theta_1+\theta_2) + a_1\cos\theta_1) \end{cases}$$

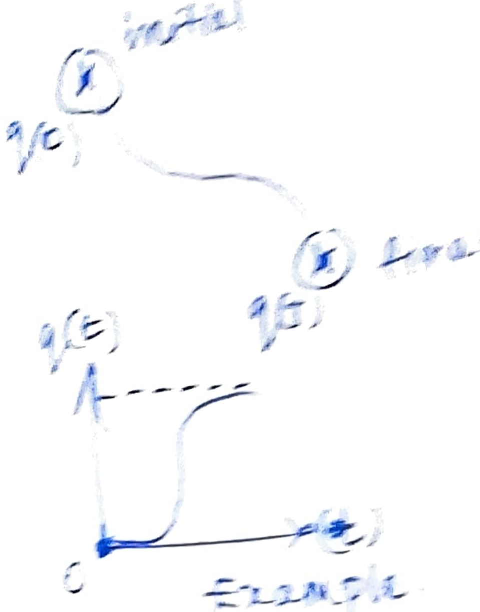
$$\begin{cases} 0 \\ 0 \\ 1 \end{cases}$$

$$\Rightarrow \boxed{R = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin\theta_2(\cos\theta_1+\sin\theta_1) & a_2\cos(\theta_2+\theta_1) - a_1\sin\theta_1 \\ \cos\theta_2(\cos\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & a_2\cos\theta_2\cos(\theta_1+\theta_2) + a_1\cos\theta_1 \\ 0 & 0 & 1 \end{bmatrix}}$$

Velocity,
$$\boxed{\begin{aligned} \hat{x} &= a_2\cos(\theta_2+\theta_1) - a_1\sin\theta_1 \\ \hat{y} &= a_2\cos\theta_2\cos(\theta_1+\theta_2) + a_1\cos\theta_1 \end{aligned}}$$

Q.2 (a) Cubic polynomial:

$$\boxed{\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3}$$



Let $q(0) = q_{initial}$

$q(T) = q_{final}$

$\Delta q = q_{initial} - q_{final}$

where q is in joint space.

the curve parameter,

$$\tau = t/T, \quad \tau \in [0, 1].$$

$q(\tau)$ can be written as:

$$q(\tau) = q_{int} + \Delta q\,(a\tau^3 + b\tau^2 + c\tau + d).$$

Using a 'doubly normalized' polynomial

$q_N(t)$ such that:

$q_N(0) = 0 \iff d = 0$

$q_N = 1 \iff a + b + c = 1$

$$\dot{q}_N(0) = \left.\frac{d q_N}{d\tau}\right|_{\tau=0} = \Delta q\,(3a\tau^2 + 2b\tau + c)\Big|_{\tau=0}$$

$$(v_{int} T) = \Delta q\,(c)$$

$$\Rightarrow \boxed{c = \frac{V_{init} T}{\Delta q}}$$

$$\dot{q}_N(1) = \left.\frac{dq_N}{d\tau}\right|_{\tau=1} = \Delta q(3a+2b+c) = \frac{V_{init} T}{\cancel{\Delta q}}$$

$$\Rightarrow 3a+2b+c = \frac{V_{init} T}{\Delta q}.$$

$$3a+2b = \cancel{\frac{V_{init} T}{\Delta q}} - \cancel{c}$$

$$3a+2b=0$$

and $a+b=1$ when $q_N=1$

$$\therefore \quad 3a+2b=0$$

$$3a+3b=3$$

$$-b=-3$$

$$\boxed{b=3} \quad \text{and} \quad \boxed{a=2}$$

Therefore

$$a=-2, \quad b=3, \quad c=\frac{V_{init} T}{\Delta q}, \quad d=0$$

$$\Rightarrow \boxed{\theta(t) = -2\tau^3 + 3\tau^2 + \frac{V_{init} T}{\Delta q}\tau}$$

(b). General form of cubic polynomial zero initial and final velocities:

initial velocity

At $t = t_i = 0$; $\theta = \theta_i$ and $\dot{\theta}_i = 0$

At $t = t_f$; $\theta = \theta_f$ and $\dot{\theta}_f = 0$

displacement: final velocity.

Let, $\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

$$\frac{d\theta(t)}{dt} = \dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$\dot{\theta}(0) = c_1 + 0 + 0 = 0$ | $c_0 = \theta_i$

$\Rightarrow c_1 = 0$

Therefore, $\theta(t_f) = c_2(t_f)^2 + c_3(t_f)^3$

$\dot{\theta}(t_f) = 0 + 2c_2(t_f) + 3c_3(t_f)^2$

$2c_2 t_f = -3c_3 t_f^2$ | $c_2 = \frac{3(\theta_f - \theta_i)}{t_f^2}$

$c_2 = \frac{-3c_3 t_f}{2}$ | $c_3 = -\frac{2(\theta_f - \theta_i)}{t_f^3}$

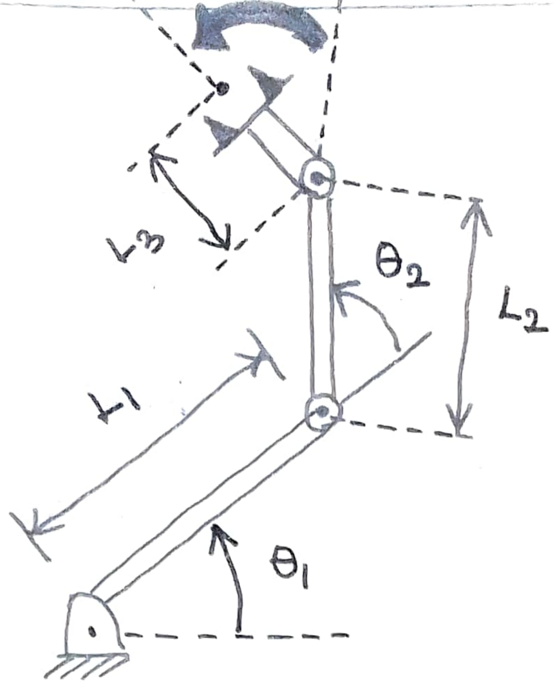
$$\boxed{\theta(t) = \theta_i + \left(3\frac{\theta_f - \theta_i}{t_f^2}\right)t^2 - \frac{2(\theta_f - \theta_i)}{t_f^3}t^3}$$

$\Downarrow$

$\rightarrow$ general form of cubic polynomial.

Q. 3

(a).



Given three-link planar arm.

Link parameters :

$a_i \longrightarrow$ link length

$\alpha_i \longrightarrow$ link twist

$d_i \longrightarrow$ link offset

$\theta_i \longrightarrow$ joint angle

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

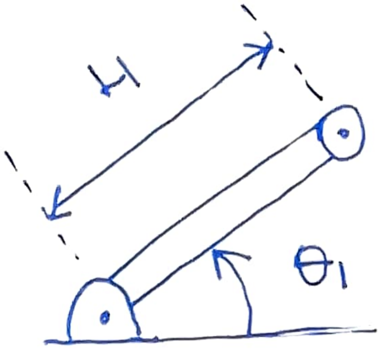
$\Downarrow$

no translation

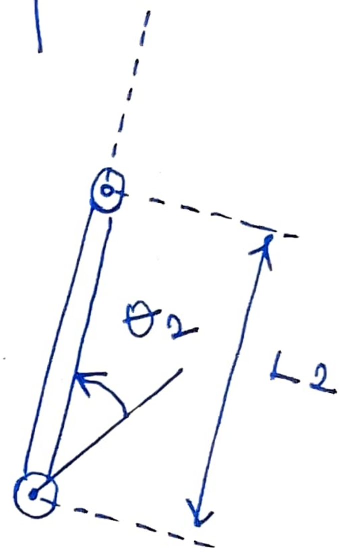
Link lengths : L_1, L_2 and L_3

Joint angles : θ_1, θ_2 and θ_3.

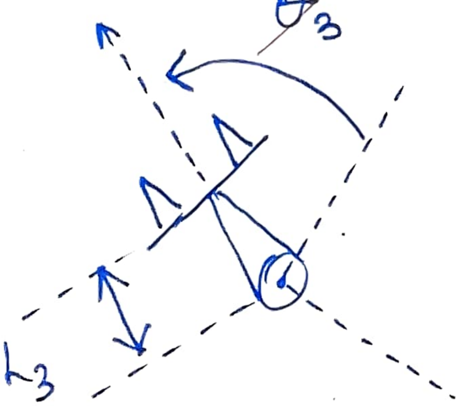
$\Rightarrow$ There is no link offset and no joint twist.



$\therefore$ $a_1 = L_1$
$\alpha_1 = 0$
$d_1 = 0$
$\theta_1 = \theta_1$



$\therefore$ $a_2 = L_2$
$\alpha_2 = 0$
$d_2 = 0$
$\theta_2 = \theta_2$



$\therefore$ $a_3 = L_3$
$\alpha_3 = 0$
$d_3 = 0$
$\theta_3 = \theta_3$

We know that,

$$\cancel{A_i = \cos\theta_i \quad -\sin\theta_1 \quad 0 \quad a_1 \cos\theta_1}$$
$$\cdot \cos\alpha_i$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1 \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1 \sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, $A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2 \cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2 \sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and $A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3 \cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3 \sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

To find Forward Kinematics

$$T_1^0 = A_1$$

$$T_2^0 = A_1 A_2$$

$$T_3^0 = A_1 A_2 A_3$$

(b).

Inverse Kinematics

$$T_3^0 = A_1 A_2 A_3$$

$$A_1 A_2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_1 A_2 A_3$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We need to find 1x4 and 2x4 elements to find x and y.

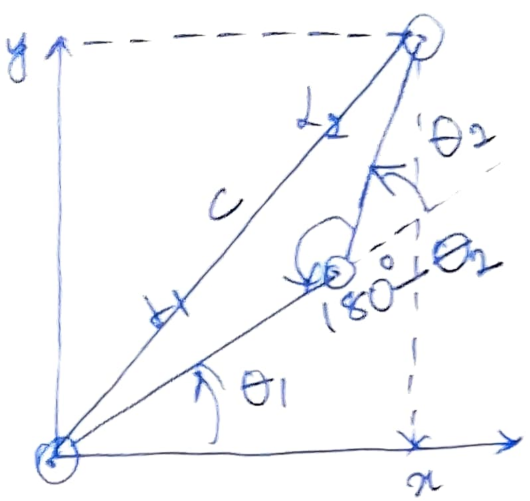
$$\therefore X_e = L_3\cos\theta_3\cos(\theta_1+\theta_2) - L_3\sin\theta_3\sin(\theta_1+\theta_2) + L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$

$$Y_e = L_3\cos\theta_3\sin(\theta_1+\theta_2) + L_3\sin\theta_3\cos(\theta_1+\theta_2) + L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

$$\boxed{\begin{aligned} X_e &= L_3\cos(\theta_1+\theta_2+\theta_3) + L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ Y_e &= L_3\sin(\theta_1+\theta_2+\theta_3) + L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \end{aligned}}$$

$\{\because \sin A\cos B + \cos A\sin B = \sin(A+B)$
and $\cos A\cos B - \sin A\sin B = \cos(A+B)\}$.

Therefore,



$$c^2 = x^2 + y^2$$

$$\Rightarrow c^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(180 - \theta_2)$$

$$\Rightarrow c^2 = L_1^2 + L_2^2 - 2L_1L_2\cos(180 - \theta_2)$$

$$\Rightarrow \cos(\theta_2) = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\boxed{\theta_2 = \cos^{-1}\left(\frac{x_e^2 + y_e^2 - L_1^2 - L_2^2}{2L_1L_2}\right)}$$

$$\sin\theta_2 = \pm\sqrt{1 - \cos^2\theta_2}$$

$$\tan\theta_1 = \left(\frac{y}{x} - \frac{L_2\sin\theta_2}{L_2\cos\theta_2 + L_1}\right)$$

$$\Rightarrow \cancel{\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\frac{L_2\sin\theta_2}{}}$$

$$\Rightarrow \boxed{\theta_1 = \tan^{-1}\left[\frac{y(L_1 + L_2\cos\theta_2) - xL_2\sin\theta_2}{x(L_1 + L_2\cos\theta_2) + yL_2\sin\theta_2}\right]}$$

Good