Robotics and Automation: Numeric Problem Answer key:

B1-Slot-Q4)

You are designing a robotic arm with three different-length links L1>L2>L3. The lengths are L1=1.2 meters, L2=1.0 meter, and L3=0.8 meters. Each link weighs 8 kg, 6 kg, and 4 kg respectively. The arm needs to pick up a 7 kg box from a conveyor belt and place it 1.5 meters away. The arm operates in a horizontal plane.

i. Calculate the torque required at each joint when the arm is fully extended and holding the 7 kg box. Assume gravitational acceleration is 9.81 m/s^2 [5 Marks]

ii. Calculate the additional torque needed at each joint if the arm moves the box with an angular acceleration of 2.0 rad/s^2. [5 Marks]

Answer:

Let's solve each part of the problem step by step.

i. Torque Calculation at Each Joint

Given:

- $L_1 = 1.2\,\mathrm{m}$
- $L_2 = 1.0\,\mathrm{m}$
- $L_3 = 0.8\,\mathrm{m}$
- $m_1 = 8\,\mathrm{kg}$
- $m_2 = 6\,\mathrm{kg}$
- $m_3 = 4\,\mathrm{kg}$
- $m_p = 7\,\mathrm{kg}$
- $g = 9.81\,\mathrm{m/s^2}$

Step 1: Calculate the gravitational force acting on each link and the payload

- $F_1 = m_1 \cdot g = 8 \cdot 9.81 = 78.48\,\mathrm{N}$
- $F_2 = m_2 \cdot g = 6 \cdot 9.81 = 58.86\,\mathrm{N}$
- $F_3 = m_3 \cdot g = 4 \cdot 9.81 = 39.24\,\mathrm{N}$
- $F_p = m_p \cdot g = 7 \cdot 9.81 = 68.67\,\mathrm{N}$

Step 2: Calculate the torque at each joint when the arm is fully extended

- Torque at Joint 3 (τ_3):

$$\tau_3 = L_3 \cdot F_p = 0.8 \cdot 68.67 = 54.936\,\mathrm{Nm}$$

- Torque at Joint 2 (τ_2):

$$\tau_2 = (L_2 \cdot F_3) + (L_3 \cdot F_p) = (1.0 \cdot 39.24) + (0.8 \cdot 68.67) = 39.24 + 54.936 = 94.176\,\mathrm{Nm}$$

- Torque at Joint 1 (τ_1):

$$\tau_1 = (L_1 \cdot F_2) + (L_2 \cdot F_3) + (L_3 \cdot F_p)$$

$$\tau_1 = (1.2 \cdot 58.86) + (1.0 \cdot 39.24) + (0.8 \cdot 68.67) = 70.632 + 39.24 + 54.936 = 164.808\,\mathrm{Nm}$$

ii. Dynamic Torque Calculation

Step 1: Calculate the moment of inertia for each link

The moment of inertia for a slender rod rotating about one end is given by:

$$I = \frac{1}{3} \cdot m \cdot L^2$$

- $I_1 = \frac{1}{3} \cdot 8 \cdot (1.2)^2 = \frac{1}{3} \cdot 8 \cdot 1.44 = 3.84\,\mathrm{kg} \cdot \mathrm{m}^2$
- $I_2 = \frac{1}{3} \cdot 6 \cdot (1.0)^2 = \frac{1}{3} \cdot 6 \cdot 1 = 2.0\,\mathrm{kg} \cdot \mathrm{m}^2$
- $I_3 = \frac{1}{3} \cdot 4 \cdot (0.8)^2 = \frac{1}{3} \cdot 4 \cdot 0.64 = 0.8533\,\mathrm{kg} \cdot \mathrm{m}^2$

Step 2: Calculate the additional torque due to angular acceleration

- Additional Torque at Joint 3 ($\tau_{3,\mathrm{inertia}}$):

$$\tau_{3,\mathrm{inertia}} = I_3 \cdot \alpha = 0.8533 \cdot 2.0 = 1.7066\,\mathrm{Nm}$$

- Additional Torque at Joint 2 ($\tau_{2,\mathrm{inertia}}$):

$$\tau_{2,\mathrm{inertia}} = I_2 \cdot \alpha + \tau_{3,\mathrm{inertia}} = 2.0 \cdot 2.0 + 1.7066 = 4.0 + 1.7066 = 5.7066\,\mathrm{Nm}$$

- Additional Torque at Joint 1 ($\tau_{1,\mathrm{inertia}}$):

$$\tau_{1,\mathrm{inertia}} = I_1 \cdot \alpha + \tau_{2,\mathrm{inertia}} = 3.84 \cdot 2.0 + 5.7066 = 7.68 + 5.7066 = 13.3866\,\mathrm{Nm}$$

Step 3: Calculate the total torque at each joint

- Total Torque at Joint 3:

$$\tau_3 = 54.936 + 1.7066 = 56.6426\,\mathrm{Nm}$$

- Total Torque at Joint 2:

$$\tau_2 = 94.176 + 5.7066 = 99.8826\,\mathrm{Nm}$$

- Total Torque at Joint 1:

$$\tau_1 = 164.808 + 13.3866 = 178.1946\,\mathrm{Nm}$$

B2-Slot

Q4)

You are designing a robotic arm with three equal-length links L1, L2 and L3 each 1 meter long and weighing 10 kg. The arm needs to pick up a 5 kg box from a conveyor belt and place it 2 meters away. The arm operates in a horizontal plane.

i. Calculate the torque required at each joint when the arm is fully extended and holding the 5 kg box. Assume gravitational acceleration is 9.81 m/s2 [5 Marks]
ii. Calculate the additional torque needed at each joint if the arm moves the box with an angular acceleration of 1.5 rad/s2. [5 Marks]

Answer:

Given:

- Length of each link ($L_1 = L_2 = L_3$) = 1 meter
- Mass of each link ($m_1 = m_2 = m_3$) = 10 kg
- Mass of the payload (box) = 5 kg
- Gravitational acceleration (g) = $9.81\ \mathrm{m/s^2}$
- Angular acceleration (α) = $1.5\ \mathrm{rad/s^2}$
- The arm is fully extended horizontally.

a) Torque Calculation:

Step 1: Calculate the forces due to gravity:

- Force due to gravity on each link $F_1 = F_2 = F_3 = m \cdot g = 10 \cdot 9.81 = 98.1\,\mathrm{N}$
- Force due to gravity on the payload $F_p = 5 \cdot 9.81 = 49.05\,\mathrm{N}$

Step 2: Calculate the torque at each joint:

- Torque at Joint 3 (τ_3):

$$\tau_3 = L_3 \cdot F_p = 1 \cdot 49.05 = 49.05\,\mathrm{Nm}$$

- Torque at Joint 2 (τ_2):

$$\tau_2 = (L_2 \cdot F_3) + (L_3 \cdot F_p) = (1 \cdot 98.1) + (1 \cdot 49.05) = 98.1 + 49.05 = 147.15\,\mathrm{Nm}$$

- Torque at Joint 1 (τ_1):

$$\tau_1 = (L_1 \cdot F_2) + (L_2 \cdot F_3) + (L_3 \cdot F_p)$$

$$\tau_1 = (1 \cdot 98.1) + (1 \cdot 98.1) + (1 \cdot 49.05) = 98.1 + 98.1 + 49.05 = 245.25\,\mathrm{Nm}$$

b) Dynamic Torque:

Step 1: Calculate the moment of inertia for each link:

- Moment of Inertia for a link (I)

$$I = \frac{1}{3} \cdot m \cdot L^2 = \frac{1}{3} \cdot 10 \cdot (1)^2 = \frac{10}{3}\,\mathrm{kg \cdot m^2} = 3.33\,\mathrm{kg \cdot m^2}$$

Step 2: Calculate the additional torque due to angular acceleration:

- Additional Torque at Joint 3 ($\tau_{3,\mathrm{inertia}}$):

$$\tau_{3,\mathrm{inertia}} = I \cdot \alpha = 3.33 \cdot 1.5 = 4.995\,\mathrm{Nm}$$

- Additional Torque at Joint 2 ($\tau_{2,\mathrm{inertia}}$):

$$\tau_{2,\mathrm{inertia}} = (I \cdot \alpha) + (\tau_{3,\mathrm{inertia}}) = 3.33 \cdot 1.5 + 4.995 = 4.995 + 4.995 = 9.99\,\mathrm{Nm}$$

- Additional Torque at Joint 1 ($\tau_{1,\mathrm{inertia}}$):

$$\tau_{1,\mathrm{inertia}} = (I \cdot \alpha) + (\tau_{2,\mathrm{inertia}}) = 3.33 \cdot 1.5 + 9.99 = 4.995 + 9.99 = 14.985\,\mathrm{Nm}$$

Step 3: Calculate the total torque at each joint:

- Total Torque at Joint 3:

$$\tau_3 = 49.05 + 4.995 = 54.045\,\mathrm{Nm}$$

- Total Torque at Joint 2:

$$\tau_2 = 147.15 + 9.99 = 157.14\,\mathrm{Nm}$$

- Total Torque at Joint 1:

$$\tau_1 = 245.25 + 14.985 = 260.235\,\mathrm{Nm}$$