4. puma 560

(a)

Link i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90	a_3	d_4	θ_4
5	90	0	0	θ_5
6	-90	0	0	θ_6

↳ DH matrix -

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↳ Transformation matrix :-

$$T_1^0 = A_1$$

$$T_1(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

similary

$$T_2^0 = A_1 A_2$$

$$T_2(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3$$

$$T_3(\theta_3) = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1 A_2 A_3 A_4$$

$$T_4(\theta_4) = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_3 \\ 0 & 0 & -1 & -d_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = A_1 A_2 A_3 A_4 A_5$$

$$T_5(\theta_5) = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = A_1 A_2 A_3 A_4 A_5 A_6$$

$$T_6(\theta_6) = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The transformation matrices are:-

By putting the values from the table -

$$T_1(\theta_1) = \begin{bmatrix} \cos 0^\circ & -\sin 0^\circ & 0 & 0 \\ \sin 0^\circ & \cos 0^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2(\theta_2) = \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 & 0 \\ \cancel{\sin(-90^\circ)}\ 0 & 0 & 1 & 0 \\ -\sin(90^\circ) & -\cos(-90^\circ) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2(\theta_2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3(\theta_3) = \begin{bmatrix} \cos 0^\circ & -\sin 0 & 0 & a_2 \\ \sin 0^\circ & \cos 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4(\theta_4) = \begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) & 0 & a_3 \\ 0 & 0 & -1 & -d_4 \\ \sin(-90^\circ) & \cos(-90^\circ) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4(\theta_4) = \begin{bmatrix} 0 & 1 & 0 & a_3 \\ 0 & 0 & -1 & -d_4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5(\theta_5) = \begin{bmatrix} \cos 90 & -\sin 90 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin 90^\circ & -\cos 90^\circ & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

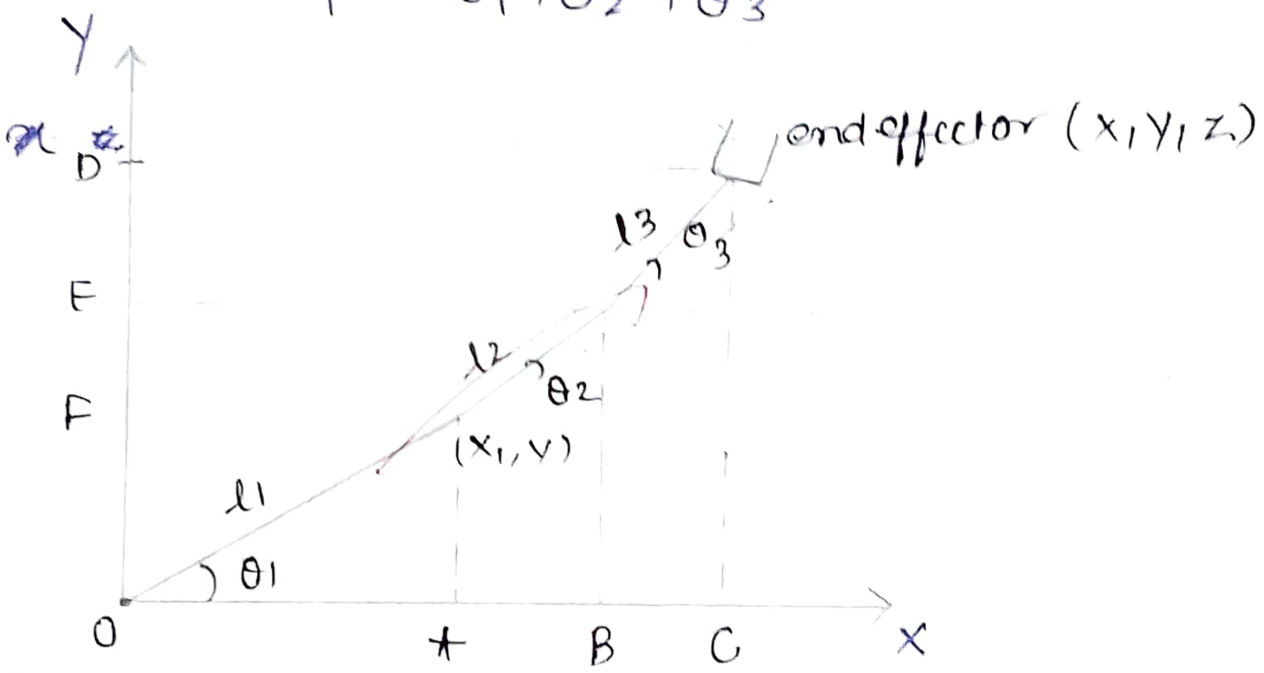
$$T_5(\theta_5) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6(\theta_6) = \begin{bmatrix} \cos -90^\circ & -\sin(-90)^\circ & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin -90^\circ & \cos(-90^\circ) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6(\theta_6) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Equations for end effector position :-
(P_x, P_y and P_z)

Rule :-
* Anticlock wise (+ve) angle
* $\sin\theta_1 \rightarrow S_1$
* $\cos\theta_1 \rightarrow C_1$
* $\sin(\theta_1+\theta_2) \rightarrow S_{12}$
* $\cos(\theta_1+\theta_2) \rightarrow C_{12}$
* $\sin(\theta_1+\theta_2+\theta_3) \rightarrow S_{123}$
* $\cos(\theta_1+\theta_2+\theta_3) \rightarrow C_{123}$
* $\phi = \theta_1+\theta_2+\theta_3$



x :-

$$x = OA + AB + BC$$

$\therefore OA = l_1 \cos\theta_1$

$\therefore AB = l_2 \cos(\theta_1+\theta_2)$

$\therefore BC = l_3 \cos(\theta_1+\theta_2+\theta_3)$

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$\boxed{x = l_1 C_1 + l_2 C_{12} + l_3 C_{123}}$$

y :-

$Y = OF + FE + ED$

$\therefore$ $OF = l_1 \sin\theta_1$

$FE = l_2 \sin(\theta_1 + \theta_2)$

$ED = l_3 \sin(\theta_1 + \theta_2 + \theta_3)$

$Y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$

$$\boxed{Y = l_1 S_1 + l_2 S_{12} + l_3 S_{123}}$$

Z :-

$Z = l_3 \sin(\theta_3)$

$$\boxed{Z = l_3 \sin\theta_3 = l_3 S_3}$$

Continuous Assessment Test (CAT) – II
Answer key

1. (a) What is feature extraction, and how does the robot use it to identify blue square-shaped boxes from the camera feed? [5 marks]

Feature extraction is the process of identifying important characteristics or attributes from raw data that can be used to recognize or classify objects. In the context of the robot, feature extraction involves identifying the relevant properties of the packages, such as their **shape** (square) and **color** (blue), from the images captured by the camera.

- The robot's camera captures the images of packages on the shelves.
- The feature extraction process analyzes these images and isolates important visual features like **edges**, **corners**, and **color information**.
- To detect the **shape**, the robot might use edge detection techniques to identify a square-shaped object by looking for four connected straight edges.
- To detect the **color**, the robot could use color segmentation methods to isolate blue objects from other colors in the image.

(b) How does the robot use an object recognition algorithm to identify the blue square-shaped boxes based on the features extracted? [5 marks]

An **object recognition algorithm** helps the robot match the extracted features (shape and color) to known patterns or objects it has been trained to recognize.

- The robot has been trained on images of different shapes (squares, circles, triangles) and colors (blue, red, green). This training allows the object recognition algorithm to learn the visual patterns associated with each combination of shape and color.
- Once the robot extracts the **shape** (square) and **color** (blue) features from the camera feed, the object recognition algorithm compares these features to its learned models of different package types.
- If the algorithm detects that the extracted features match the stored model of a **blue square**, it identifies the object as a blue square-shaped box.
- After identification, the robot proceeds to pick up the recognized blue square-shaped box and place it on the conveyor belt for further processing.

This ensures that the robot accurately selects the correct packages based on the features detected.

2. a) Construct a connectivity graph using the exact cell decomposition method to represent this warehouse environment. [3 Marks]

The **exact cell decomposition method** divides the free space of the robot's environment into non-overlapping cells, ensuring that each cell is either fully free or fully occupied by an obstacle. The connectivity graph is then constructed by linking adjacent free cells.

Steps to construct the connectivity graph for the warehouse environment:

1. **Identify free and obstacle spaces:** In Fig. 1, the black polygons represent obstacles, and the white regions are the free spaces where the robot can move.
2. **Decompose the free space into cells:** The free space is divided into smaller cells using vertical and horizontal lines that are tangent to the edges of the obstacles. This ensures that every cell is either fully inside the free space or fully inside the obstacle space.
3. **Connect adjacent cells:** A connectivity graph is constructed by connecting adjacent free cells with edges. Each node in the graph represents a free cell, and an edge between two nodes signifies that the robot can move between those two cells without hitting an obstacle.
4. **Place q_init and q_goal:** Nodes representing the start position (`q_init`) and goal position (`q_goal`) are included in the graph, and they are connected to their respective adjacent cells.

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b) Apply Dijkstra's algorithm to perform a graph search in order to find the shortest path between the robot's start node q_init and the destination node q_goal, ensuring that obstacles are avoided. [7 Marks]

Dijkstra's algorithm finds the shortest path between two nodes in a weighted graph. The algorithm works by exploring nodes in increasing order of their distance from the start node and updating their neighbors with the shortest known distance.

Steps to apply Dijkstra's algorithm:

1. **Initialization:**
   - Mark all nodes as unvisited.
   - Set the distance to the start node (`q_init`) as 0, and all other nodes as infinity.
   - Set `q_init` as the current node.
2. **Exploration:**
   - For each unvisited neighbor of the current node, calculate the tentative distance (the distance from `q_init` to the neighbor through the current node).
   - If this tentative distance is smaller than the known distance to the neighbor, update the neighbor's distance.
   - After checking all neighbors, mark the current node as visited (it will not be checked again).
3. **Selecting the next node:**
   - Select the unvisited node with the smallest tentative distance and make it the new current node.
   - Repeat the exploration process until the goal node (`q_goal`) is marked as visited, or all reachable nodes have been visited.
4. **Reconstruct the shortest path:**
   - Once `q_goal` has been reached, backtrack from the goal node using the recorded shortest distances to reconstruct the path.

Answer Key: Deriving the Lagrangian Dynamics for the Manipulator [15 Marks]

Given the image of the manipulator, we can derive its equation of motion using the Lagrangian dynamics approach. The Lagrangian is defined as:

$$L = T - V$$

Where:

- T is the **kinetic energy** of the system.
- V is the **potential energy** of the system.

Step-by-Step Derivation:

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1. Define the System Variables:

From the image:

- θ_1: Generalized coordinate representing the angular position of the first link.
- l_1: Length of the first link (rotating).
- d_2: Length from the joint to the center of mass of the second link (translational part).
- m_1: Mass of the first link.
- m_2: Mass of the second link.
- g: Acceleration due to gravity.

2. Kinetic Energy (T):

The kinetic energy is the sum of the kinetic energies of the individual links.

- For link 1 (rotating part with mass m_1):
  - The kinetic energy of a rotating link is given by:

    $$T_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 \dot{\theta}_1^2$$

    where:

    - v_1 is the velocity of the center of mass of link 1.
    - I_1 is the moment of inertia of link 1 about the axis of rotation.
    - The linear velocity of link 1 can be derived from the angular velocity $\dot{\theta}_1$.

    Since $v_1 = l_1 \dot{\theta}_1$, we get:

    $$T_1 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2$$

- For link 2 (translating part with mass m_2):
  - The kinetic energy for the second link is purely translational:

    $$T_2 = \frac{1}{2} m_2 \dot{d}_2^2$$

    Since d_2 changes with time, we account for $\dot{d}_2$, the velocity of the second link.
- Total kinetic energy:

$$T = T_1 + T_2 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{d}_2^2$$

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3. Potential Energy (V):

The potential energy is the sum of the gravitational potential energies of the links.

- For link 1:

$$V_1 = m_1 g h_1$$

  where $h_1 = l_1 \cos(\theta_1)$.
- For link 2:

$$V_2 = m_2 g h_2$$

  where $h_2 = d_2 \cos(\theta_1)$ (assuming the second link is vertically aligned with the first one).
- Total potential energy:

$$V = V_1 + V_2 = m_1 g l_1 \cos(\theta_1) + m_2 g d_2 \cos(\theta_1)$$

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4. Lagrangian (L):

The Lagrangian L is given by:

$$L = T - V$$

Substituting the expressions for T and V:

$$L = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \dot{d}_2^2 - (m_1 g l_1 \cos(\theta_1) + m_2 g d_2 \cos(\theta_1))$$

---

5. Euler-Lagrange Equations:

To obtain the equations of motion, we apply the Euler-Lagrange equation for each generalized coordinate q_i:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

For θ_1:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\frac{d}{dt}(m_1 l_1^2 \dot{\theta}_1) + (m_1 g l_1 \sin(\theta_1) + m_2 g d_2 \sin(\theta_1)) = 0$$

This yields the equation of motion for the rotational part.

For d_2:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{d}_2}\right) - \frac{\partial L}{\partial d_2} = 0$$

$$\frac{d}{dt}(m_2 \dot{d}_2) - m_2 g \cos(\theta_1) = 0$$

This gives the equation of motion for the translational part.

---

6. Final Equations of Motion:

1. **For θ_1 (rotational motion):**

$$m_1 l_1^2 \ddot{\theta}_1 + (m_1 g l_1 + m_2 g d_2) \sin(\theta_1) = 0$$

2. **For d_2 (translational motion):**

$$m_2 \ddot{d}_2 - m_2 g \cos(\theta_1) = 0$$

These are the equations of motion for the given manipulator using Lagrangian dynamics.

Marks Distribution:

- Correct definition and setup of Lagrangian [3 marks].
- Deriving kinetic energy (for both links) [4 marks].
- Deriving potential energy (for both links) [3 marks].

- Correct application of Euler-Lagrange equations [3 marks].
- Final equations of motion [2 marks].

Answer Key: Deriving Transformation Matrices and End Effector Position for PUMA 560 [15 Marks]

To derive the transformation matrices and end-effector position for the PUMA 560 manipulator, we will use the Denavit-Hartenberg (DH) parameters and the frame assignments given in the diagram. The DH parameters give us a systematic way to represent the robot's joint configuration and calculate the forward kinematics.

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1. DH Parameter Table:

Assume the DH parameter table (which is not explicitly shown but implied) follows the standard form:

Link i	θ_i (rotation)	d_i (offset)	a_i (link length)	α_i (twist angle)
1	θ_1	d_1	a_1	α_1
2	θ_2	d_2	a_2	α_2
3	θ_3	d_3	a_3	α_3
4	θ_4	d_4	a_4	α_4
5	θ_5	d_5	a_5	α_5
6	θ_6	d_6	a_6	α_6

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2. Transformation Matrix for Each Joint:

The transformation matrix between two consecutive frames is given by the general DH transformation formula:

$$T_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Base to Joint 1 (T1):

Using the DH parameters for link 1:

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos\alpha_1 & \sin\theta_1\sin\alpha_1 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos\alpha_1 & -\cos\theta_1\sin\alpha_1 & a_1\sin\theta_1 \\ 0 & \sin\alpha_1 & \cos\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 1 to Joint 2 (T2):

$$T_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\alpha_2 & \sin\theta_2\sin\alpha_2 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos\alpha_2 & -\cos\theta_2\sin\alpha_2 & a_2\sin\theta_2 \\ 0 & \sin\alpha_2 & \cos\alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 2 to Joint 3 (T3):

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos\alpha_3 & \sin\theta_3\sin\alpha_3 & a_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos\alpha_3 & -\cos\theta_3\sin\alpha_3 & a_3\sin\theta_3 \\ 0 & \sin\alpha_3 & \cos\alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 3 to Joint 4 (T4):

$$T_4^3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4\cos\alpha_4 & \sin\theta_4\sin\alpha_4 & a_4\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4\cos\alpha_4 & -\cos\theta_4\sin\alpha_4 & a_4\sin\theta_4 \\ 0 & \sin\alpha_4 & \cos\alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 4 to Joint 5 (T5):

$$T_5^4 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5\cos\alpha_5 & \sin\theta_5\sin\alpha_5 & a_5\cos\theta_5 \\ \sin\theta_5 & \cos\theta_5\cos\alpha_5 & -\cos\theta_5\sin\alpha_5 & a_5\sin\theta_5 \\ 0 & \sin\alpha_5 & \cos\alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint 5 to Joint 6 (T6):

$$T_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6\cos\alpha_6 & \sin\theta_6\sin\alpha_6 & a_6\cos\theta_6 \\ \sin\theta_6 & \cos\theta_6\cos\alpha_6 & -\cos\theta_6\sin\alpha_6 & a_6\sin\theta_6 \\ 0 & \sin\alpha_6 & \cos\alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

---

3. Final Transformation Matrix:

The overall transformation matrix from the base to the end-effector (denoted as T_6^0) is obtained by multiplying the individual transformation matrices from the base to each joint.

$$T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5$$

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4. End-Effector Position (Px, Py, Pz):

The position of the end effector (P_x, P_y, P_z) can be extracted from the final transformation matrix T_6^0. The last column of the matrix gives the position of the end effector in the form:

$$\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

Hence:

$$P_x = T_{6(1,4)}$$

$$P_y = T_{6(2,4)}$$

$$P_z = T_{6(3,4)}$$

Where $T_{6(i,4)}$ represents the element in the i^{th} row and the 4th column of the matrix T_6^0.

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5. Marks Distribution:

- Correct identification of DH parameters [3 marks].
- Correct transformation matrix derivation [3 marks].
- Correct multiplication of transformation matrices to get the base-to-end-effector transformation [3 marks].
- Correct derivation of end-effector position (Px, Py, Pz) from the final matrix [6 marks].