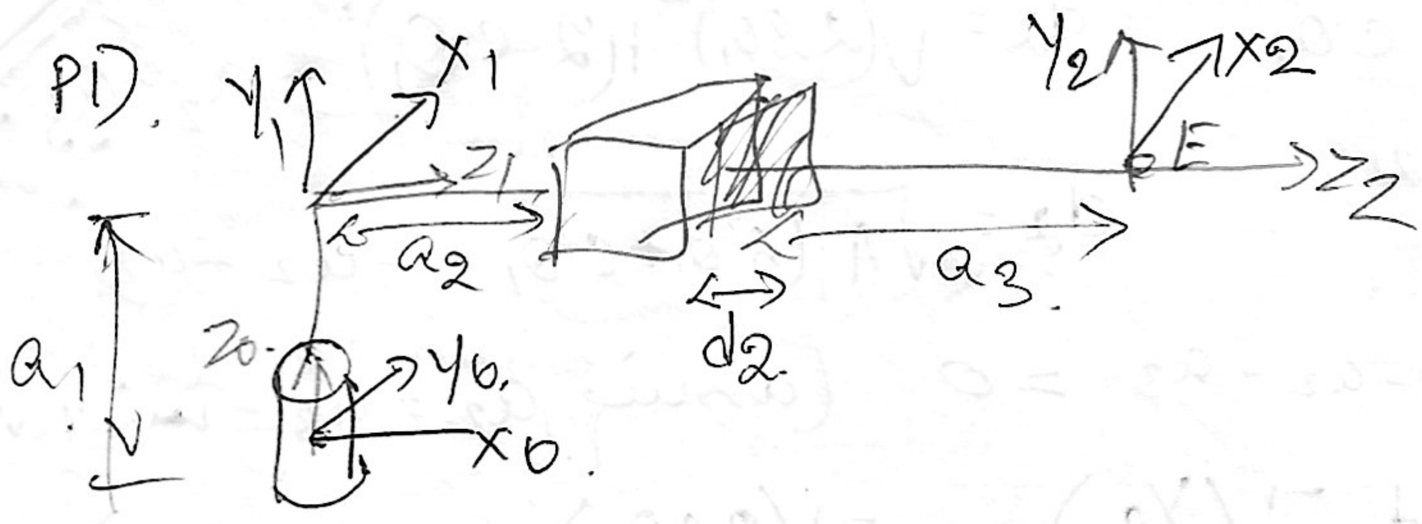
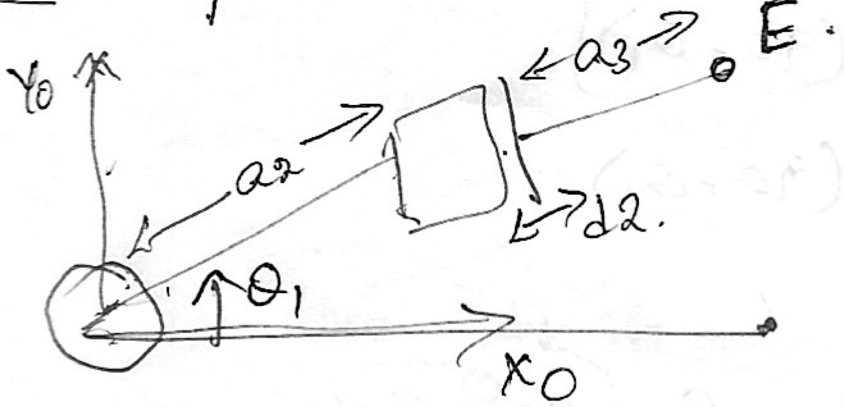
Inverse Kinematics -

PD.

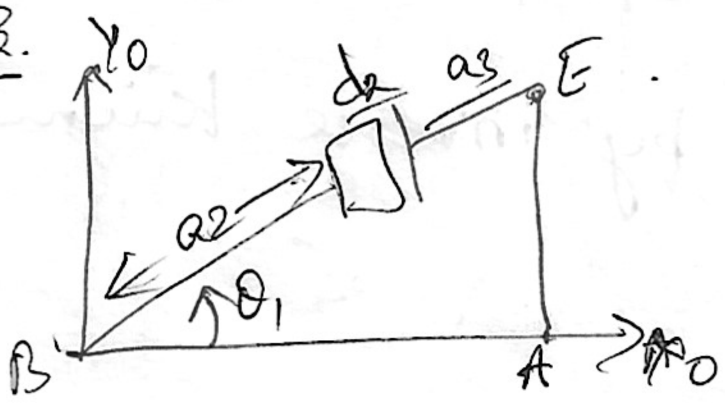


Soln: step 1 - framing.

step 2. Top view.



step 3.



Py. theo.

$a^2+b^2=c^2$

Cosine rule

$a^2 = b^2+c^2-2bc\cos\alpha$

Py. theo... $X_0^2 + Y_0^2 = (a_2+a_3+d_2)^2$.

$\therefore d_2 = \sqrt{X_0^2+Y_0^2} - a_2 - a_3$

$\tan\theta_1 = \frac{Y_0}{X_0}$

$\therefore \theta_1 = \tan^{-1}\left(\frac{Y_0}{X_0}\right)$

step 4: given the HTM, find X_0 Y_0 $\rightarrow$ to get d_2,

$$H = \begin{bmatrix} S\theta_1 & -C\theta_1 & 0 & 2S\theta_1 \\ C\theta_1 & S(\theta_1) & 0 & 2C\theta_1 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \rightarrow X_0 \\ \rightarrow Y_0 \\ \rightarrow Z_0 \\ \\ \end{matrix}$$

from HTM, $X_0 = 2S\theta_1$, $Y_0 = 2C\theta_1$, $Z_0 = d_2$

$$d_2 = \sqrt{(2S\theta_1)^2 + (2C\theta_1)^2} - a_2 - a_3$$

$$d_2 = \sqrt{4(S^2\theta_1 + C^2\theta_1)} - a_2 - a_3$$

$d_2 = 2 - a_2 - a_3 = 0$ (assuming $a_2 = a_3 =$ unit vectors)

$$\theta_1 = \tan^{-1}\left(\frac{Y_0}{X_0}\right) = \tan^{-1}\left(\frac{2C\theta_1}{2S\theta_1}\right) = \tan^{-1}\left(\frac{C\theta_1}{S\theta_1}\right)$$

$$\theta_1 = \tan^{-1}(\cot\theta_1)$$

$$= \tan^{-1}[\tan(90 - \theta_1)]$$

$$= \tan^{-1}\tan(90 - \theta_1)$$

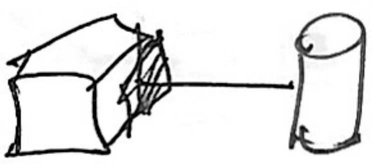
$$\theta_1 = 90 - \theta_1$$

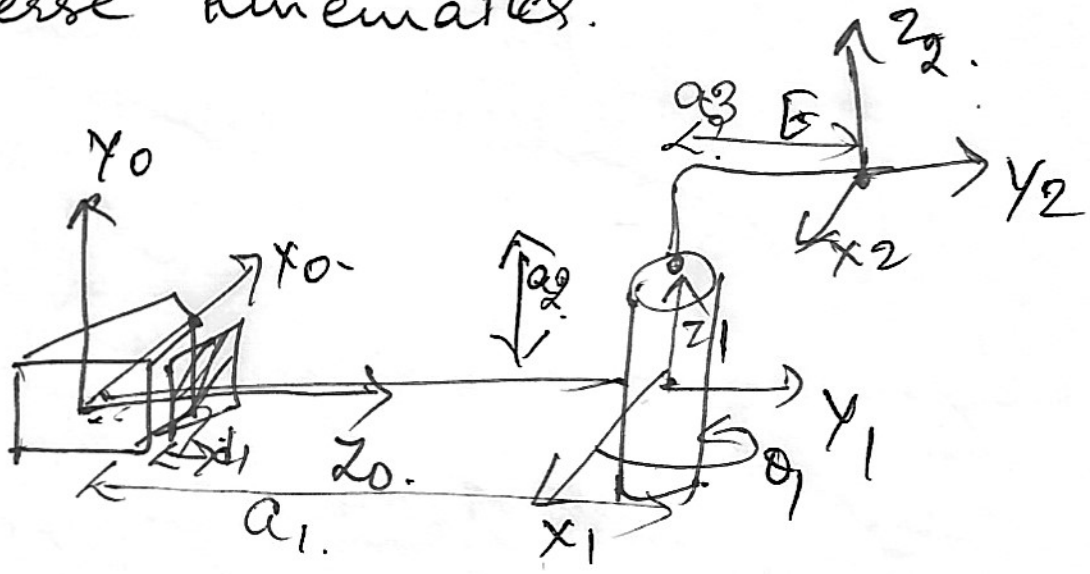
$$2\theta_1 = 90 \Rightarrow \theta_1 = 45^\circ.$$

$\therefore$ Link parameters by inverse kinematics

are $d_2 = 0$, $\theta_1 = 45^\circ$.
$[a_2 = a_3 = 1]$.

P2) Find the inverse Kinematics.

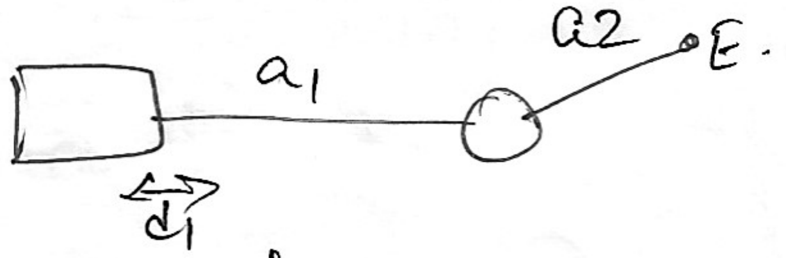




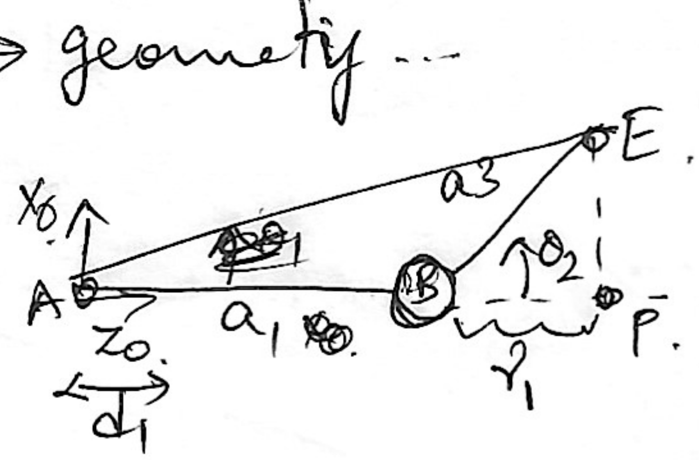
Soln:

step 1 → framing

step 2 → top view.



step 3 → geometry ...



PE ⇒ x_0 AB ⇒ Z_0

BP ⇒ r_1.

ΔABE, ΔBPE.

$\tan\theta_2 = \frac{x_0}{r_1} \Rightarrow \theta_2 = \tan^{-1}\left(\frac{x_0}{r_1}\right)$

$r_1^2 + x_0^2 = a_3^2 \Rightarrow r_1 = \sqrt{a_3^2 - x_0^2}$.

$\therefore \theta_2 = \tan^{-1}\left(\frac{x_0}{r_1}\right)$

Also, $Z_0 = a_1 + d_1 + r_1 \Rightarrow d_1 = Z_0 - a_1 - r_1$