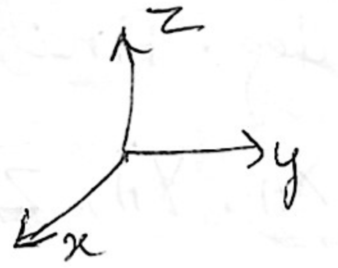
Rotatory Transformation:-

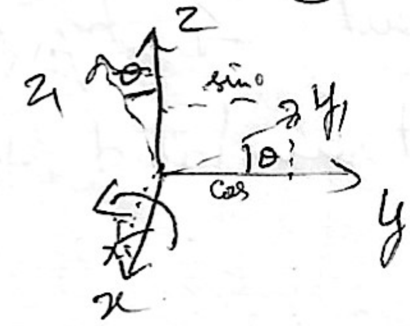
(I) X rotation matrix

phi. φ

(1)



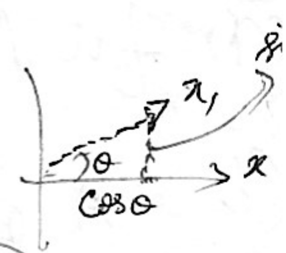
(2)



(2) with (1)

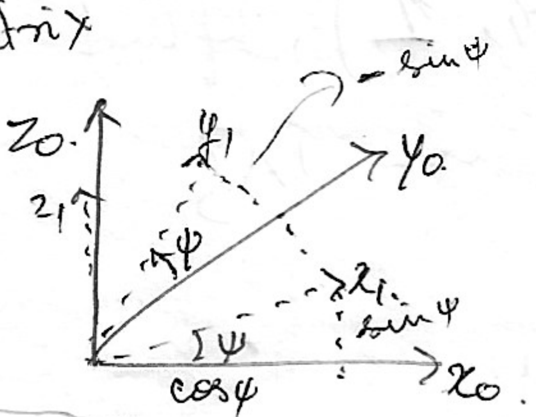
$$R_{x,\varphi} = \begin{bmatrix} x\cdot x & yx & zx \\ x\cdot y & yy & zy \\ x\cdot z & yz & zz \end{bmatrix}$$

$$R_{x,\varphi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$ also called as R_x.



(II) Z rotation matrix

psi. ψ



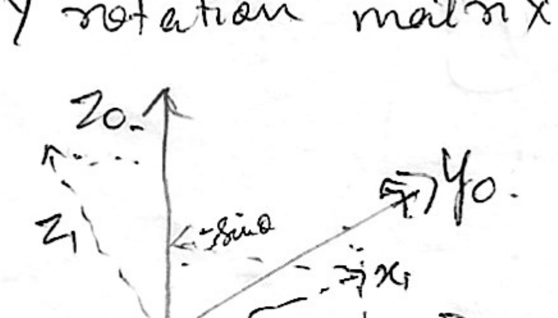
$$R_{z,\psi} = \begin{bmatrix} \underset{x_1 \text{ on } x_0}{\cos\psi} & -\sin\psi & 0 \\ \underset{x_1 \text{ on } y_0}{\sin\psi} & \cos\psi & 0 \\ \underset{x_1 \text{ on } z_0}{0} & 0 & 1 \end{bmatrix}$$

All axes have length of 1

$\boxed{\text{c.c. rotation is (+)ve}}$

(III) Y rotation matrix.

θ theta



$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

(IV) Combination of Rotation.

$${}^{0}_{0}R_{2} = {}^{0}R_{1} \cdot {}^{1}R_{2}$$

Ref.

$$R^{0}_{1} = \begin{array}{c|ccc} \text{on} & x_1 & y_1 & z_1 \\ \hline X_0 & x_1 \text{ on } x_0 & y_1 \text{ on } x_0 & z_1 \text{ on } x_0 \\ Y_0 & x_1 \text{ on } y_0 & y_1 \text{ on } y_0 & z_1 \text{ on } y_0 \\ Z_0 & x_1 \text{ on } z_0 & y_1 \text{ on } z_0 & z_1 \text{ on } z_0 \end{array}$$

(V) No Rotation matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

identity

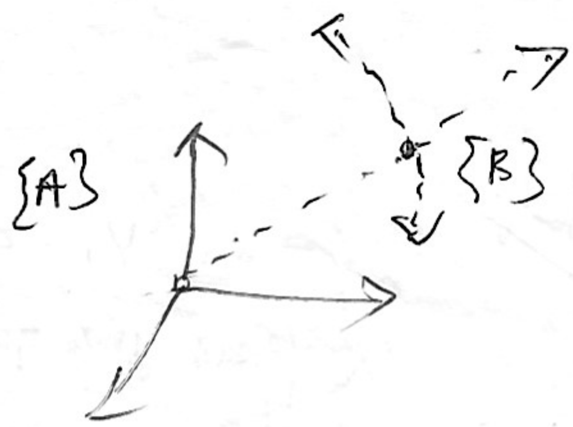
P1) Frame B is initially coincident with frame A. Then B is rotated about Y_B by 30°, then about X_B by 60° then about Z_B by 30 deg. Finally, the origin of $\{B\}$ is translated to $\{X_A, Y_A, Z_A\}^T = [10, -5, 4]^T$.

(a) find ${}^A_B T$ in the order given

(b) A point in $\{B\}$ is ${}^B p = [6, -4, 1]^T$, find the coordinates of this point in $\{A\}$

Soln:

$${}^A R_B$$



$R_y \rightarrow 30^\circ$

$R_x \rightarrow 60^\circ$

$R_z \rightarrow 30^\circ$

$${}^A P_{B\,org} = \begin{pmatrix} 10 \\ -5 \\ 4 \end{pmatrix}$$

$${}^A_B T = \left[\begin{array}{c|c} {}^A_B R_{3\times 3} & {}^A P_{B\,org} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$${}^{A}_{B}R = R_y \cdot R_x \cdot R_z$$

WKT, $R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$ $R_y = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$

$$R_z = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore {}^{A}_{B}R = \begin{bmatrix} c30 & 0 & s30 \\ 0 & 1 & 0 \\ -s30 & 0 & c30 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c60 & -s60 \\ 0 & s60 & c60 \end{bmatrix} \begin{bmatrix} c30 & -s30 & 0 \\ s30 & c30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

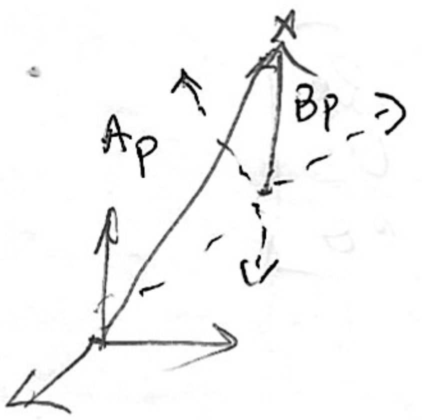
$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{A}_{B}R = \cancel{\begin{bmatrix} 0.967 & -0.050 \end{bmatrix}} \begin{bmatrix} 0.967 & -0.058 & 0.25 \\ 0.25 & 0.433 & -0.86 \\ -0.058 & 0.90 & 0.43 \end{bmatrix}$$

$$\therefore {}^{A}_{B}T = \left[\begin{array}{c|c} {}^{A}_{B}R & P_{Borg} \\ \hline 000 & 1 \end{array}\right] = \begin{bmatrix} 0.0967 & -0.058 & 0.25 \\ 0.25 & 0.433 & -0.866 \\ -0.058 & 0.9 & 0.433 \\ 0 & 0 & 0 \end{bmatrix}$$

is Transformation matrix

(b).



$$^A p = {}^A_B T \times {}^B P$$

$$= \begin{bmatrix} 0.967 & -0.058 & 0.25 & 10 \\ 0.25 & 0.433 & -0.866 & -5 \\ -0.058 & 0.9 & 0.433 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16.28 \\ -6.10 \\ 0.49 \\ 1 \end{bmatrix} \quad \therefore {}^A P = \begin{bmatrix} 16.28 \\ -6.10 \\ 0.49 \end{bmatrix}$$

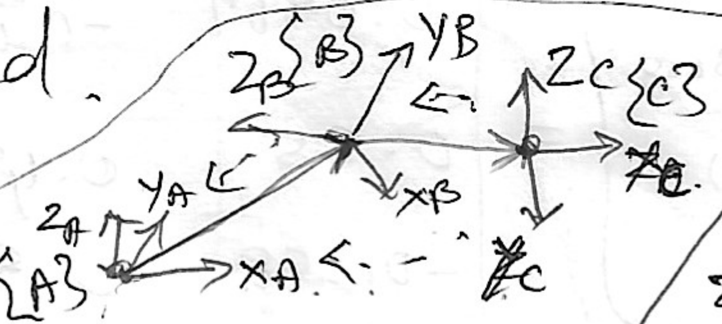
Frames.

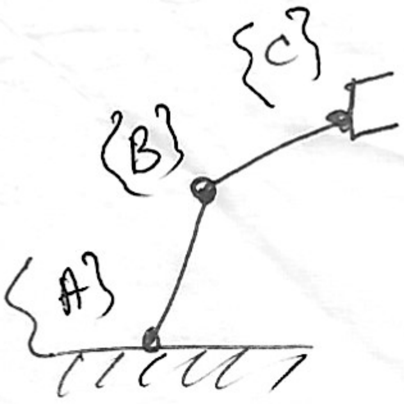
Frame → orientation Matrix & Position Vector

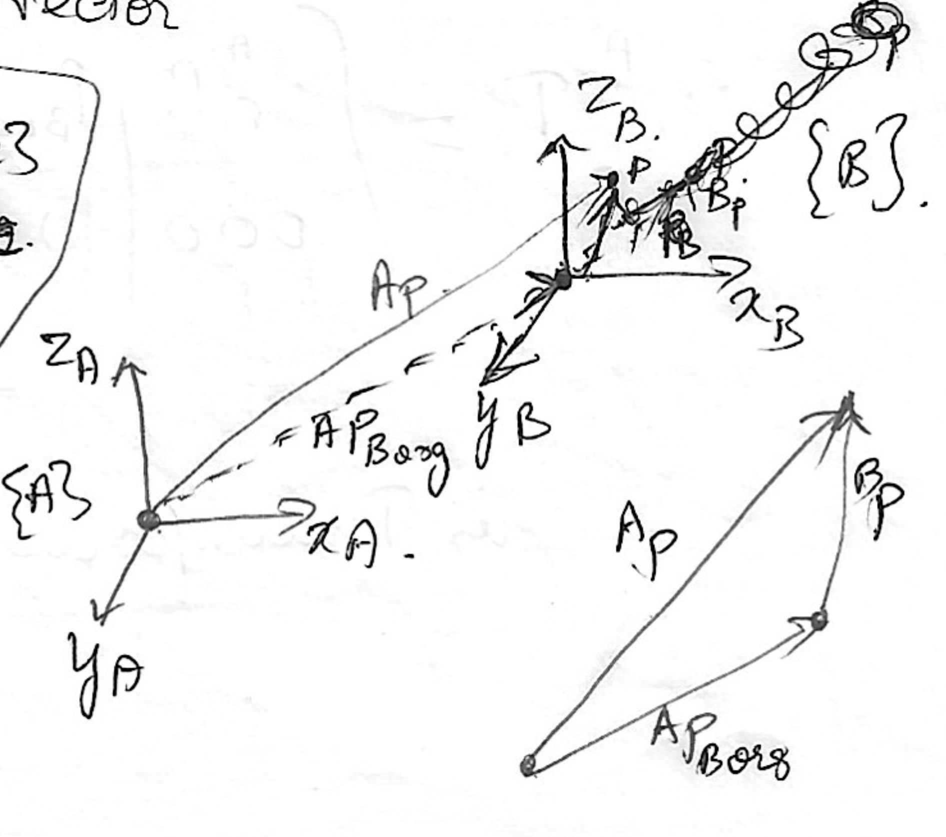
$$\{B\} = \{ {}^A R_B ,\ {}^A P_{Borg} \}.$$

(1) Mapping Translated Frames

1) Translated → position vector
2) Rotated
3) Both



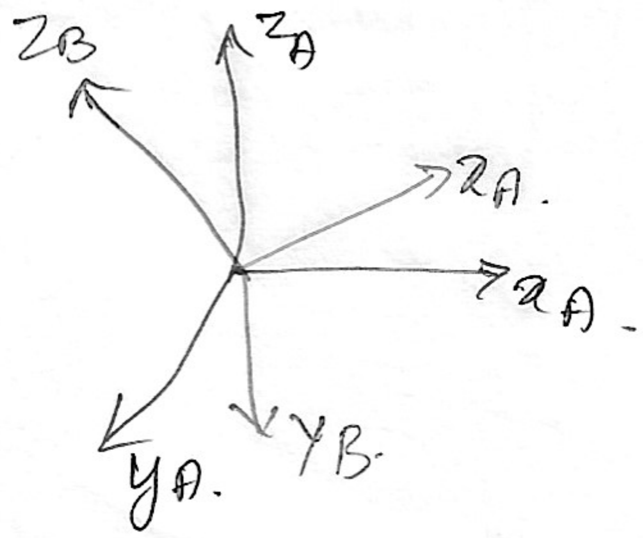




Mapping Rotated Frames.

$R_x,\ R_y,\ R_z \ldots$

$$\therefore\ {}^{A}P = \underbrace{{}^{A}R_{B}}_{\text{Rot matrix}} \cdot \underbrace{{}^{B}P}_{\text{position vector}}$$



③ Mapping General Frames. (Transformation)

* both rotational + Translational.

$$\underbrace{{}^{A}T_{B}}_{\text{Transformation matrix}} = \begin{bmatrix} {}^{A}R_{B}\ (3\times 3) & {}^{A}P_{Borg} \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix}_{4\times 4}$$

position vector translation of ~~matrix~~ origin of B.

To repr. a point in frame B in terms of frame A

${}^{A}T_{B}$

$${}^{A}P = {}^{A}T_{B} \times {}^{B}P$$

→ hint.