4

Denavit - Hartenberg frame representation

* Homogeneous transformation Matrix (HTM) for single frame
* 3 rotation, 3 position.
* for multiple frames, D-H applies.
* 4 parameter
  - → 2 → rotation → α, θ.
  - → 2 → Linear → r, d.

Steps.

① Assign frames according to the 4 D-H rules.
② Create the DH parameter table. (α, θ, r, d)
③ Plug the table values into generic HTM.
④ Multiply the matrices together.

Assumption ccw → (+)ve

4 DH rules of step 1

- Assumptions.
  ① No of frames for any kinet = No of Joints +
  ② The axes of frames can be ↑ ↓ → ← or 1st & 3rd quadrant... (out or inside page)

4-DH rules

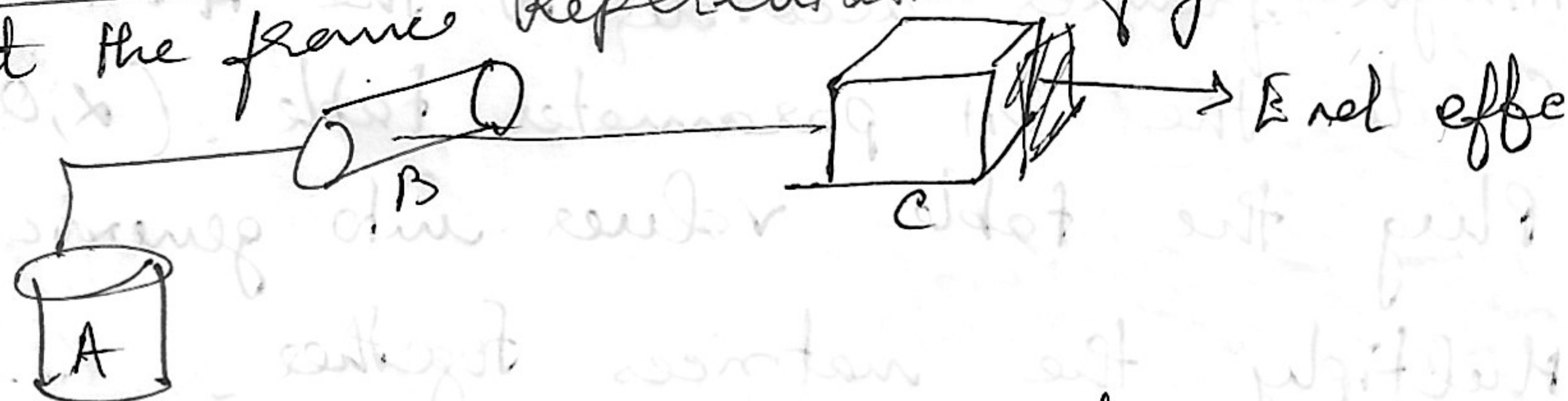
① Z axis must be axis of rotation for revolute joint or direction of motion of prismatic j

② X axis must be ⊥ to Z axis of the frame before it (prev. frame)

③ X axis must intersect the Z axis of P frame

④ Y axis must be drawn so that whole frame follows Right hand rule.

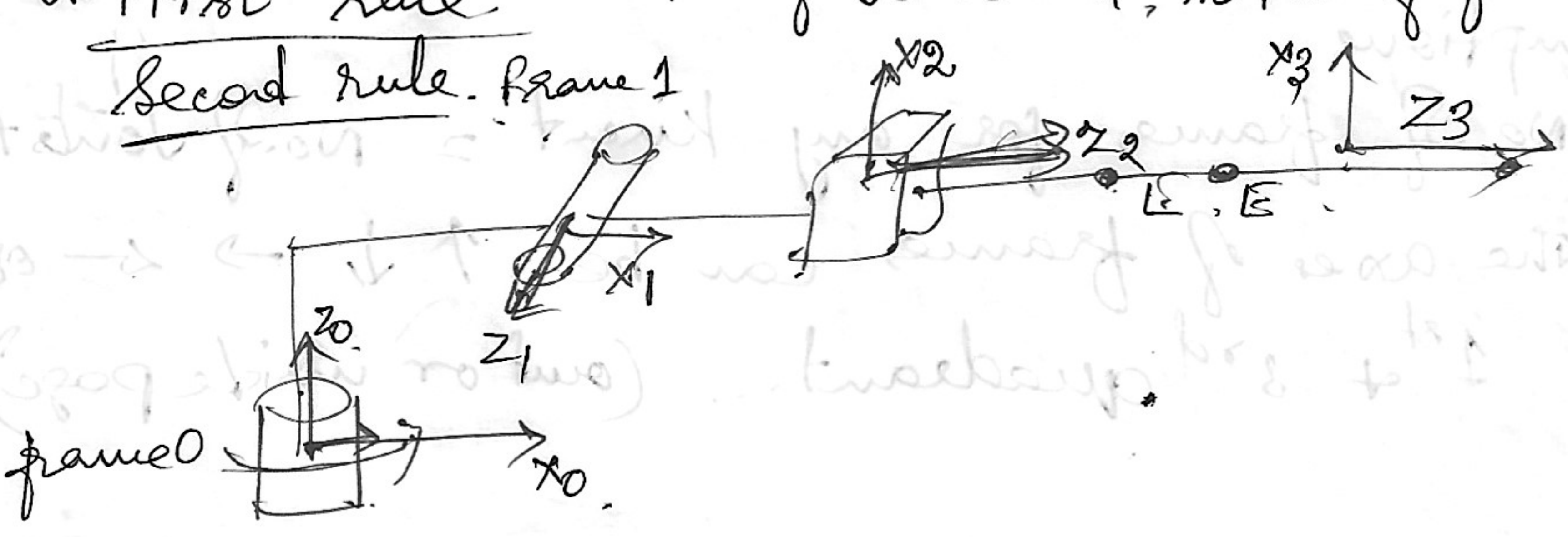
P1) Get the frame Representation of given kinte



* A, B revolute, C is prismatic

* First rule No of Joints = 3, so No of frames =

Second rule. Frame 1

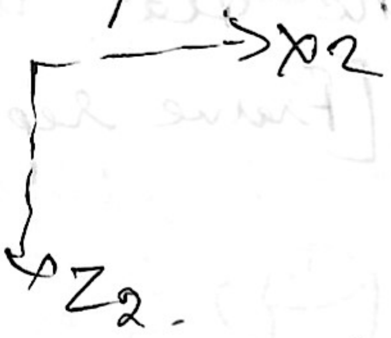


Third rule

$X_1 \rightarrow$ intersects Z_0.

$X_2 \rightarrow$ does not intersect Z_1, so

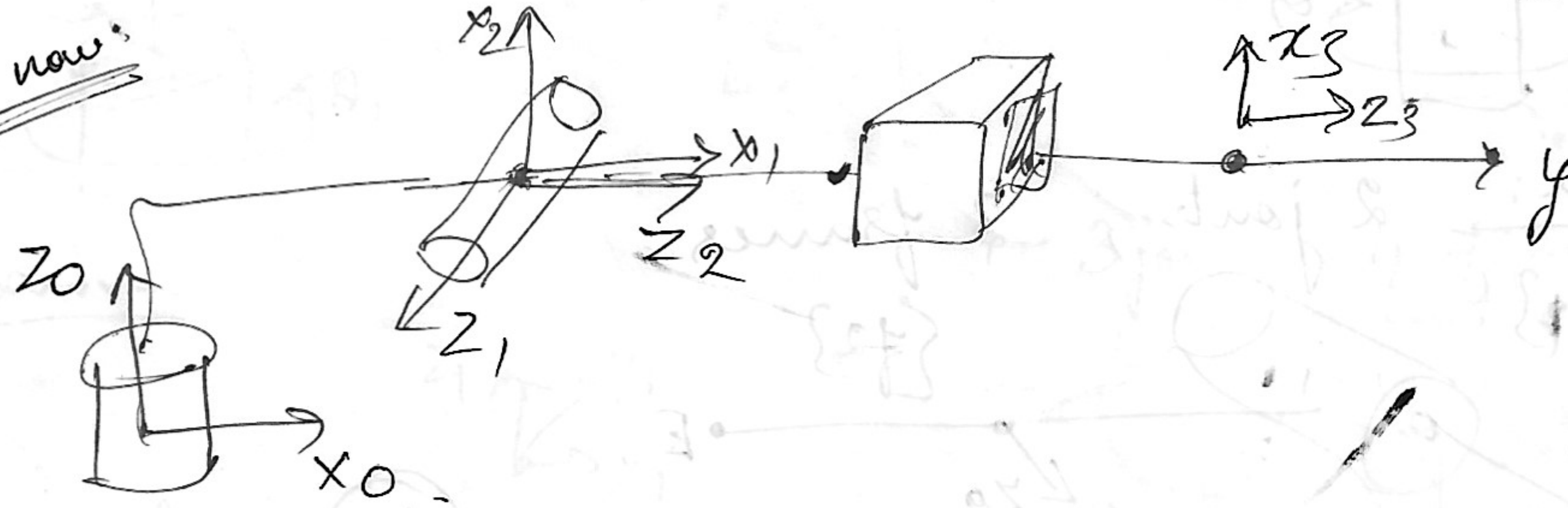
~~try change~~ try to change by rotating,



but here Z_2 is not in axis of revolute.

so, translate, (move to prev. frame)

So now:

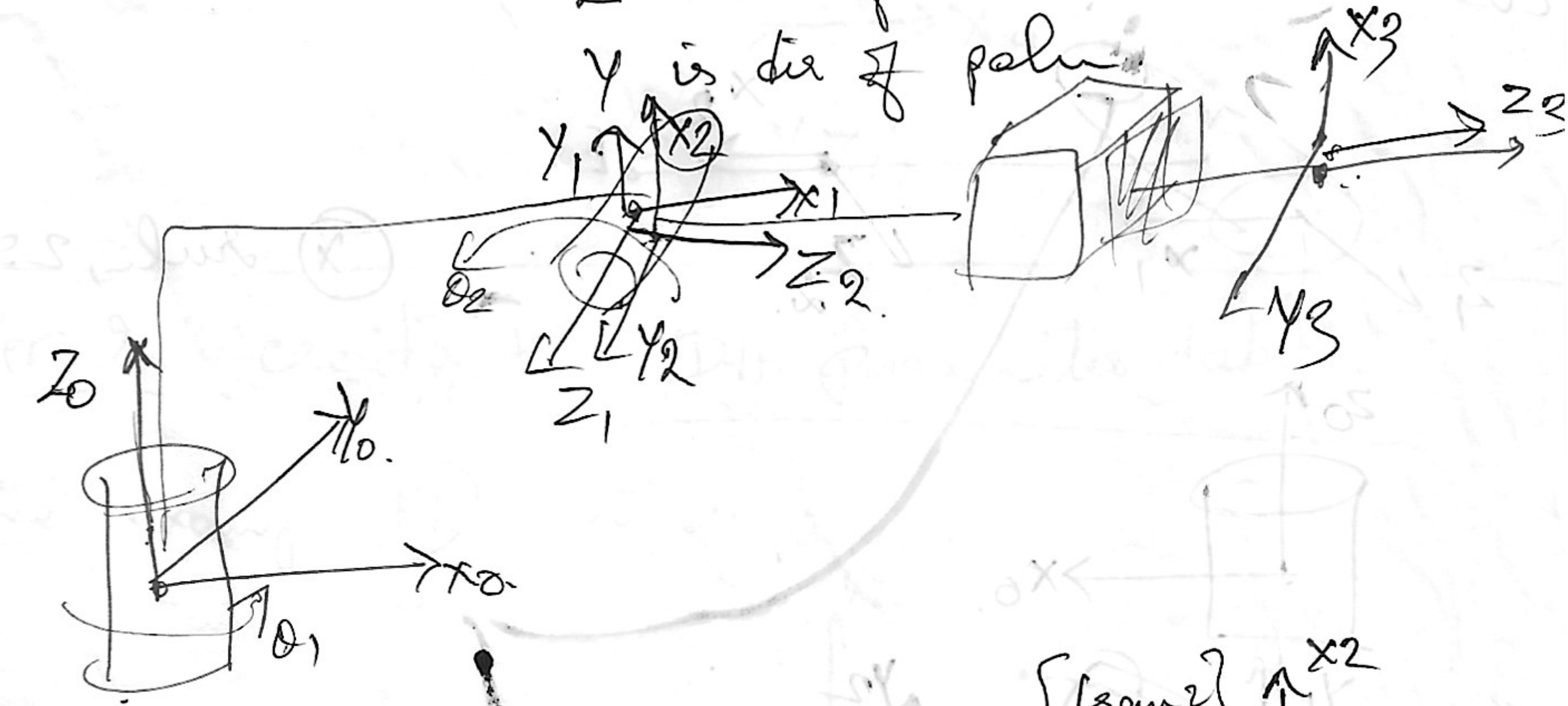


For Y direction → Right Hand Rule (4^{th} rule)

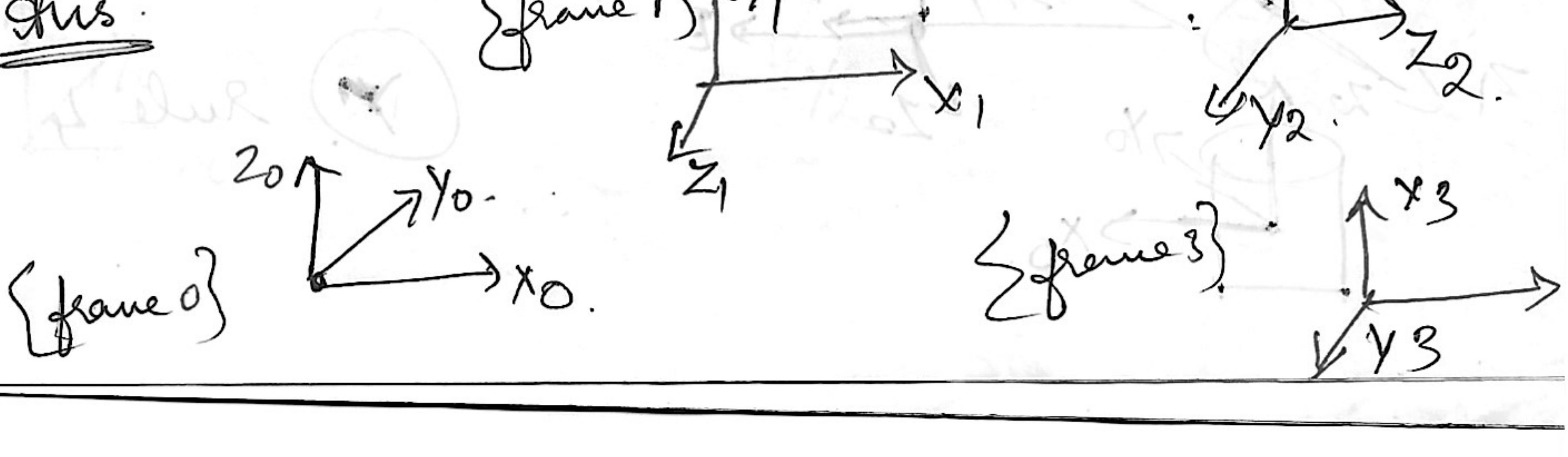
Right hand rule → x is dir of fingers.

Z is dir of thumb.

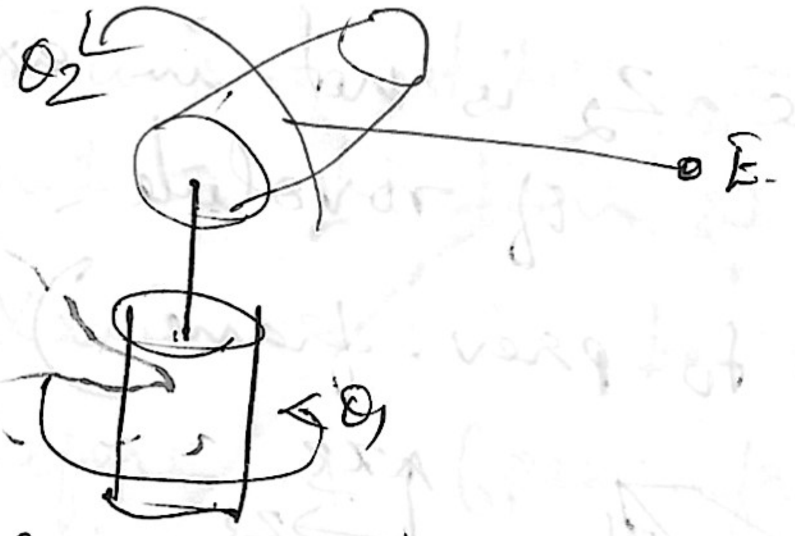
Y is dir of palm.



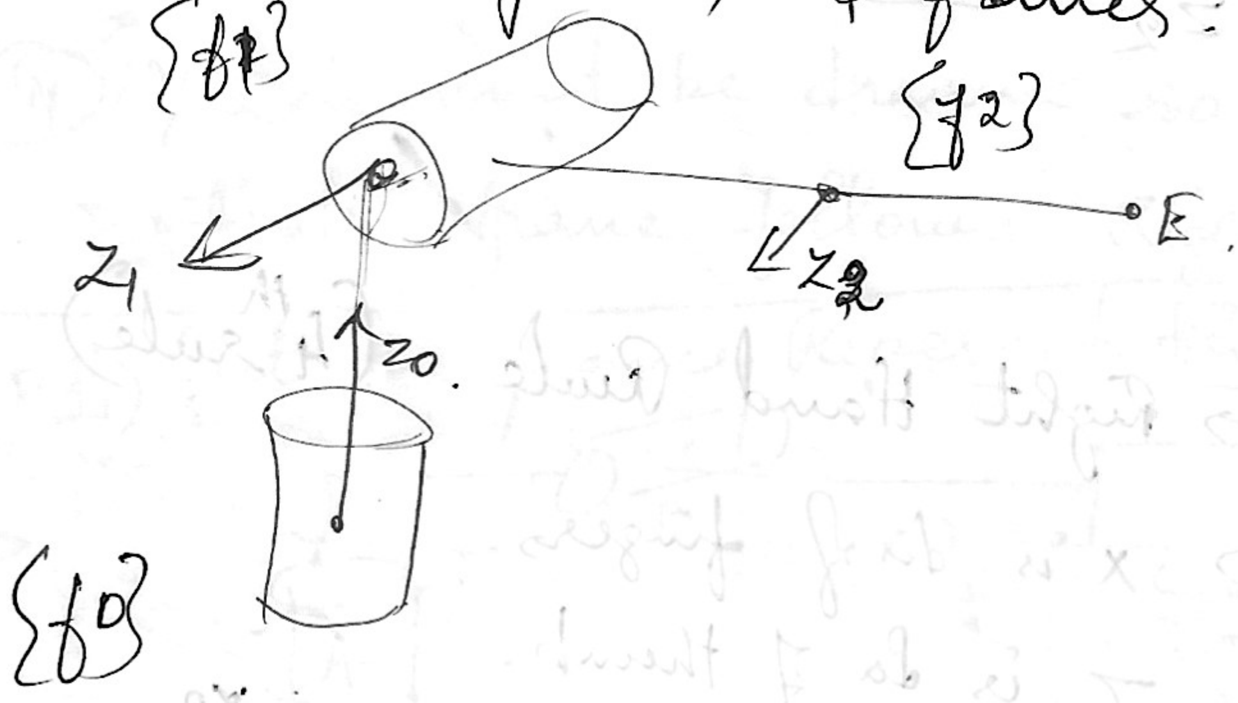
thus



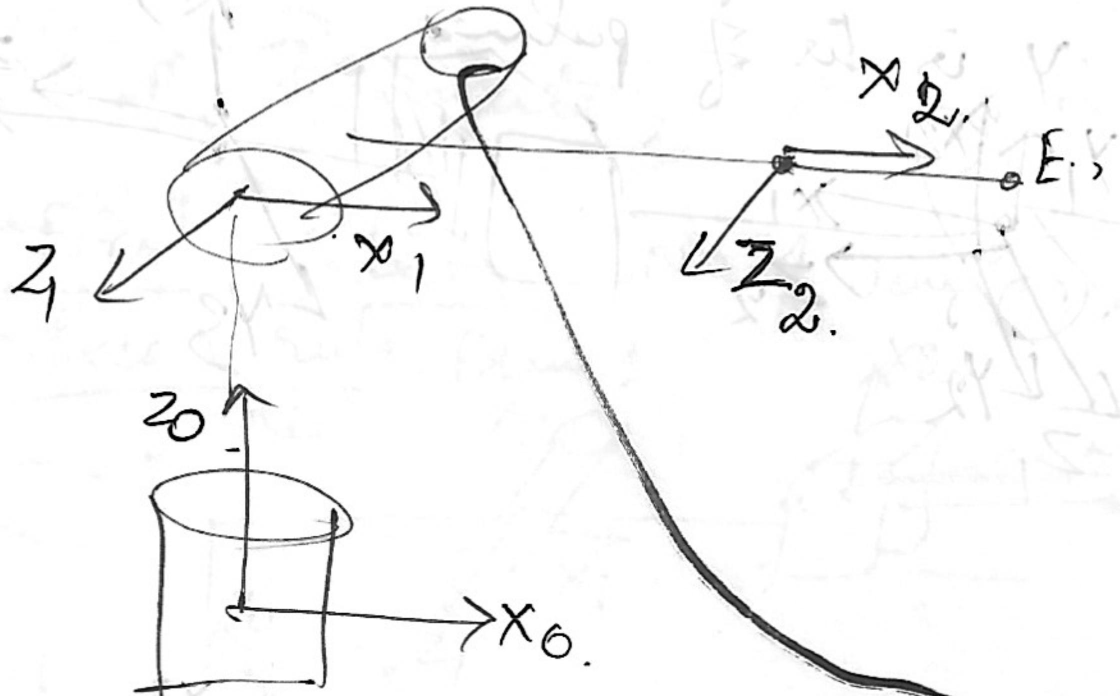
P2) Find frames for given kinematic dia or [Frame repr. eg



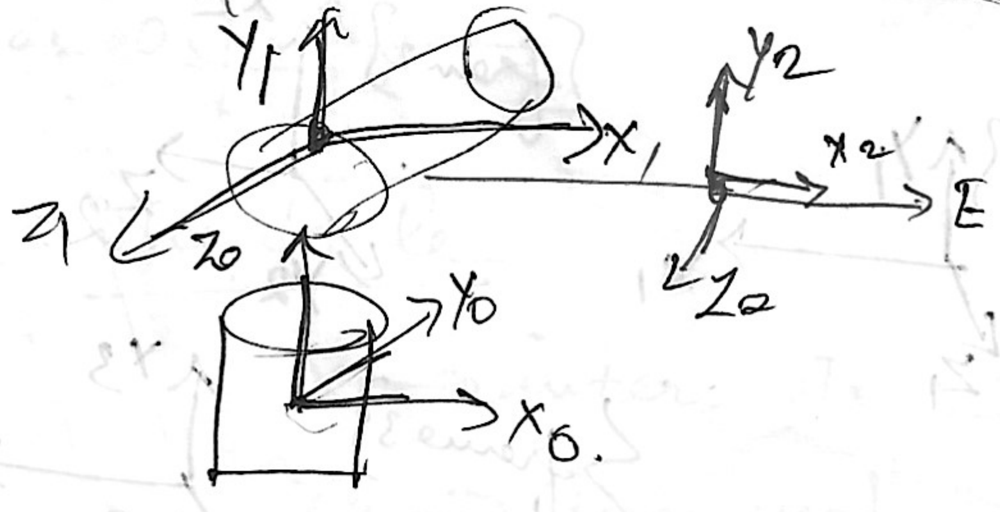
Soln: 2 joints, 3 frames:



(z) rule 1

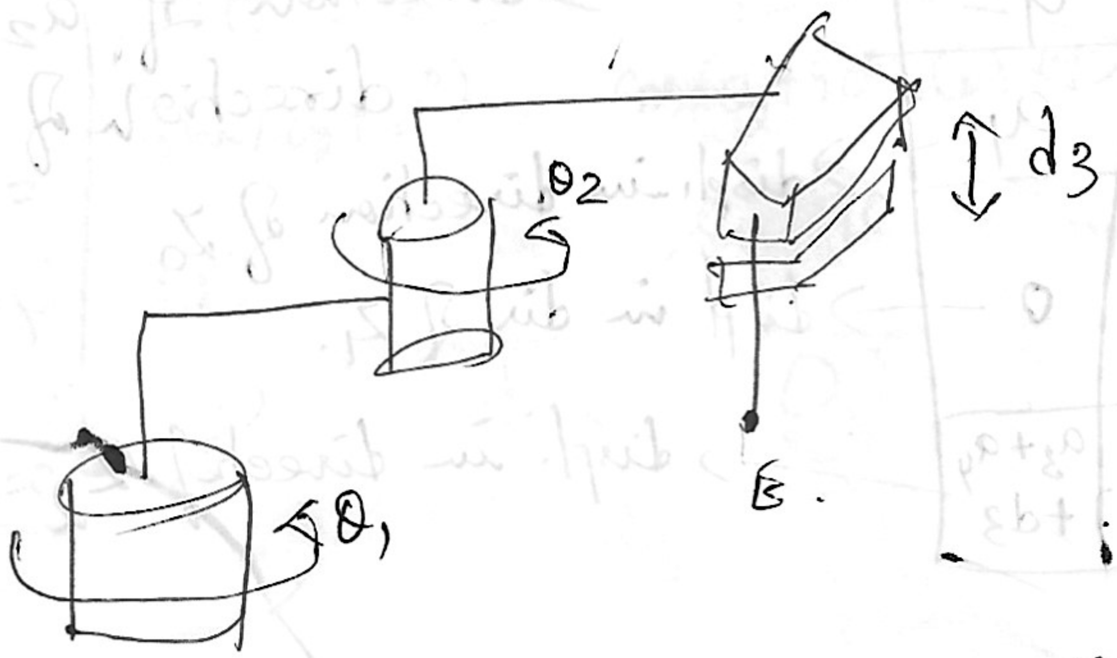


(x) rule, 2 &



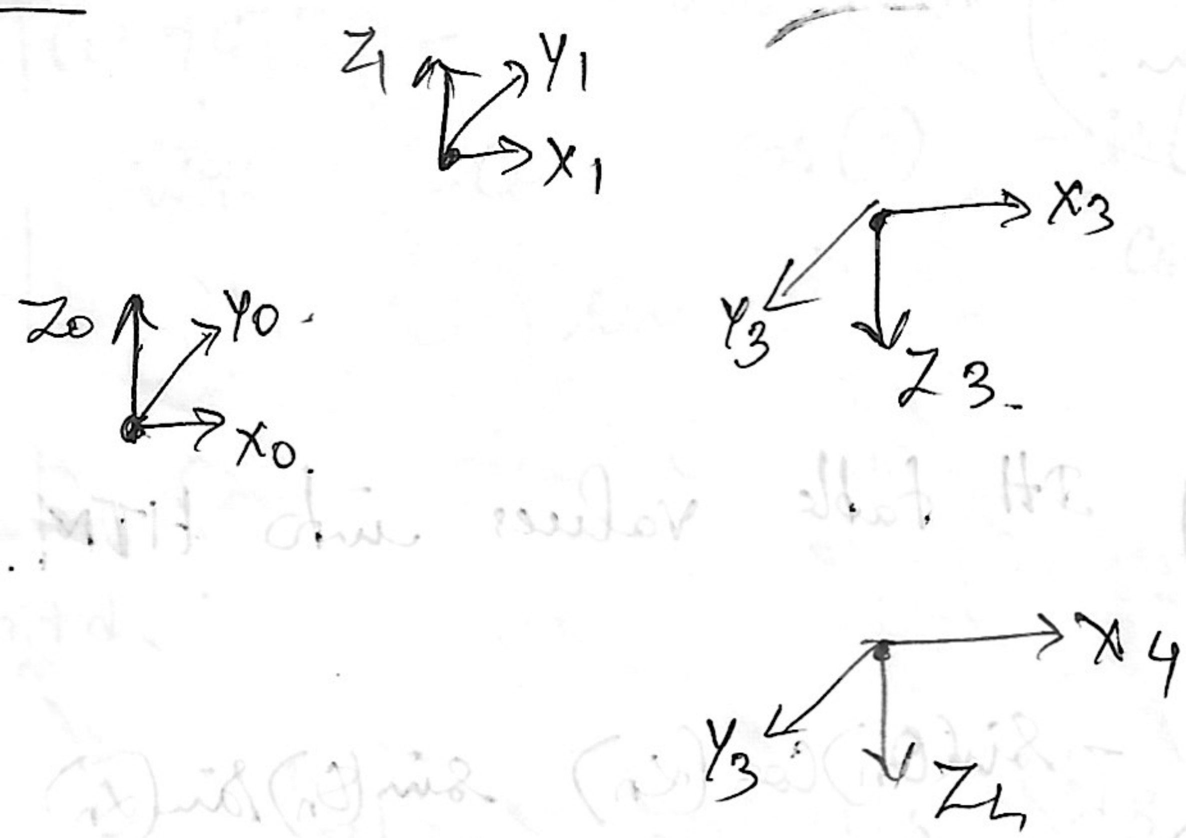
(y) rule 4

P3) Find frames of given kinematic dia.



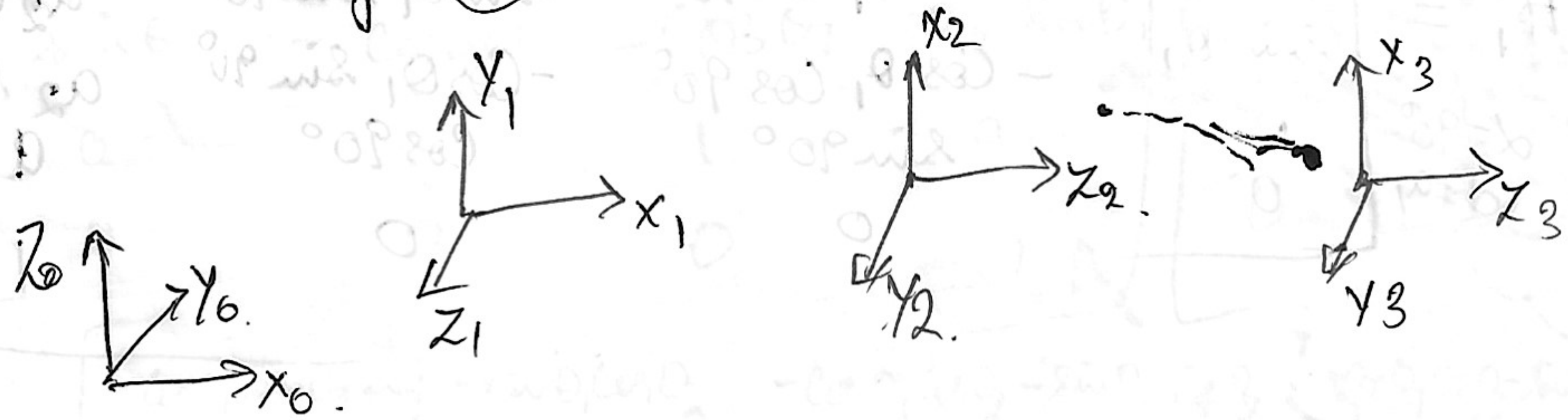
Soln.

3 joints, 4 frames.



Step 2. creating the DH parameter table

Same example (P1)



x_0 x_1 match (z) → z_0 z_1 match (x_n)

	θ	α	r	d
1	$0+\theta_1$	90°	a_2	a_1
2	$90^\circ+\theta_2$	90°	0	0
3	0	0	0	$a_3+a_4+d_3$

d → in z_{n-1} direction

a_2 → direction of a_2 — direction of x_1

a_1 → displ. in direction of z_0

0 → displ. in dir. of z_1.

$a_3+a_4+d_3$ → displ. in direction of z_2

disp. in x_n direction

No displ in direction of x_3

no disp. in dir of x_2.

step 3. Plug DH table values into HTM.

$$^{N-1}H_N = \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & \sin(\theta_n)\sin(\alpha_n) & r_n\cos(\theta_n) \\ \sin(\theta_n) & -\cos(\theta_n)\cos(\alpha_n) & -\cos(\theta_n)\sin(\alpha_n) & r_n\sin(\theta_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}H_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos 90^\circ & \sin\theta_1\sin 90^\circ & a_2\cos\theta_1 \\ \sin\theta_1 & -\cos\theta_1\cos 90^\circ & -\cos\theta_1\sin 90^\circ & a_2\sin\theta_1 \\ 0 & \sin 90^\circ & \cos 90^\circ & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta=\theta_1$ $\alpha=90^\circ$

$r=a_2$ $d=a_1$

$$
{}^1H_2 = \begin{bmatrix} \cos(\theta_2+90^\circ) & -\sin(\theta_2+90^\circ)\cdot\cos(90^\circ) & \sin(\theta_2+90^\circ)\sin 90^\circ & 0 \\ \sin(\theta_2+90^\circ) & -\cos(\theta_2+90^\circ)\cdot\cos(90^\circ) & -\cos(\theta_2+90^\circ)\sin 90^\circ & 0 \\ 0 & \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$\theta = (\theta_2+90^\circ)$
$\alpha = 90^\circ$
$r = 0 \quad d = 0$

$$
{}^2H_3 = \begin{bmatrix} \cos 90^\circ & -\sin(90^\circ)\cos(0^\circ) & \sin(90^\circ)\sin(0) & 0 \\ \sin 90^\circ & -\cos(90^\circ)\cos(0) & -\cos(90^\circ)\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & a_2+a_4+d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$\theta = 90^\circ$
$\alpha = 0$
$r = 0$
$d = a_2+a_4+d_3$

step 4: ${}^0H_3 = {}^0H_1 \times {}^1H_2 \times {}^2H_3$

${}^0H_1 \times {}^1H_2$

$$
\begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cdot 1 & \cancel{\sin\theta_1\cdot 0} & a_2\cos\theta_1 \\ \sin\theta_1 & -\cos\theta_1\cdot 1 & -\cos\theta_1 & a_2\sin\theta_1 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} -\sin\theta_2 & -\cos\theta_2 & \cos\theta_2 & [illegible] \\ [illegible] & [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 & -\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 \\ -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 & -\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 & \sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0H_1 \times {}^1H_2 = \begin{bmatrix} \sin(\theta_1-\theta_2) & -\cos(\theta_1-\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & \sin(\theta_1-\theta_2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$\Downarrow$

${}^0H_2$

No

in

$$
{}^0H_2 \times {}^2H_3 = \begin{bmatrix} \sin(\theta_1-\theta_2) & -\cos(\theta_1-\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & \sin(\theta_1-\theta_2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [illegible] & 1 & 0 \\ [illegible] & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

$\Downarrow$

${}^0H_3$

$$
{}^0H_3 = \begin{bmatrix} -\cos(\theta_1-\theta_2) & -\sin(\theta_1-\theta_2) & \cos(\theta_1+\theta_2) & \cos(\theta_1+\theta_2)\,[a_3+a_4+d \\ -\sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & \sin(\theta_1-\theta_2) & \sin(\theta_1-\theta_2)\,[a_3+a_4+d \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$