SCHOOL OF ELECTRONICS ENGINEERING (SENSE)

BECE313L: Information Theory and Coding

PROBLEM SHEET-3
Channel Coding

Instructions:

1. Total Marks: 15
2. Weightage of marks in grades : 4%
3. Last Date for Submission: 15.11.2024
4. All answers must be handwritten
5. Late submission are not allowed
6. Submission must be through teams

Address each problem with thorough analysis and detailed solutions.

Q.No	Question	Marks
1	Obtain all possible code vectors for a (7, 4) linear block code (LBC) in its systematic form for the generator matrix: $$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$	1
2	Given a systematic (8, 4) LBC with parity check equations: $$v_4 = u_1 + u_2 + u_3$$ $$v_5 = u_0 + u_1 + u_2$$ $$v_6 = u_0 + u_1 + u_3$$ $$v_7 = u_0 + u_2 + u_3$$ Write the generator and parity check matrices and draw the encoder diagram.	1

3 A systematic (10, 5) LBC is designed with a minimum Hamming distance of 4. The generator matrix is: 2

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

1. Verify if the minimum Hamming distance criterion is satisfied by finding all codewords and computing their Hamming weights.
2. Construct the corresponding parity check matrix H.
3. Analyze the error detection and correction capabilities of this code based on the minimum Hamming distance.
4. Design the encoder circuit for this code.

4 Consider a (7, 4) cyclic code with generator polynomial $g(x) = 1 + x + x^3$. 1
Given two input sequences $\mathbf{u_1} = 1101$ and $\mathbf{u_2} = 0110$, perform the following tasks:

1. Derive the code polynomial for both input sequences using polynomial multiplication.
2. Construct the full code vector for each input by using the generator polynomial.
3. If the received vector is $r(x) = x + x^2 + x^5 + x^6$, compute the syndrome and determine if any error is detected.
4. Correct the received vector if an error is detected and identify the error location.

5 A (15, 7) cyclic code is defined by the generator polynomial $g(x) = 1 + x + x^4$. The code is used in a communication system where the following received vectors are obtained: 2

$$r_1(x) = x^3 + x^6 + x^9 + x^{12} + x^{14}$$

$$r_2(x) = 1 + x + x^2 + x^7 + x^{11} + x^{14}$$

1. For each received vector, calculate the syndrome and determine if there is any error.
2. Explain the steps required to correct the received vectors if an error exists.
3. Determine the error detection and correction capabilities of the code based on the properties of the generator polynomial.
4. Explain how this cyclic code could be converted into a systematic form, if possible, and discuss the implications on encoding complexity.

6 Given a (6, 3) linear block code with generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$: 1

1. Construct the entire set of codewords and verify the minimum Hamming distance.
2. Design the encoder circuit for this code using basic logic gates.
3. If the received vector is $\mathbf{r} = [1, 0, 1, 1, 0, 1]$, use syndrome decoding to check for errors. If an error exists, identify the corrected codeword.
4. Discuss the theoretical maximum error detection and correction capabilities of this code.

7 Consider a (9, 5) systematic block code designed to detect and correct single-bit errors. The parity check matrix for this code is: 2

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

1. Construct the generator matrix G for this systematic code.
2. Using the syndrome decoding approach, analyze how this code detects and corrects single-bit errors.
3. If a received vector $\mathbf{r} = [1, 0, 1, 0, 1, 1, 0, 1, 0]$ is detected, compute the syndrome and determine if there is an error.
4. Draw the encoder and syndrome decoder circuit diagrams for this code.
5. Provide an analysis on how the minimum Hamming distance impacts the error detection and correction capabilities for this code.

8 For the following (6, 3) systematic LBC with generator matrix: 1

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

1. Find all code vectors.
2. Draw the encoder circuit for the above code.
3. Find the minimum Hamming weight.
4. Find the error detection and correction capabilities.

9 In an LBC, the syndrome is given by: 1

$$S_1 = r_1 + r_2 + r_3 + r_4$$

$$S_2 = r_1 + r_2 + r_4 + r_6$$

$$S_3 = r_1 + r_3 + r_4 + r_7$$

1. Find the parity check matrix.
2. Draw the encoder circuit.
3. Find the codeword for all input sequences.
4. How many errors can it detect and correct?
5. What is the syndrome for the received data 1011011?

10	Consider a (7, 4) cyclic code with $g(x) = 1+x+x^3$. Obtain the code polynomial in non-systematic form for the input sequences 1010 and 1100.	1
11	For a (7, 4) cyclic code with generator polynomial $g(x) = 1 + x + x^3$, obtain the syndrome for the received vector $r(x) = 1 + x^2 + x^3 + x^4 + x^5$.	1
12	Given $n \leq 7$, identify the (n, k) values of cyclic codes generated by the following generator polynomials: 1. $g(x) = 1 + x^2 + x^3$ 2. $g(x) = 1 + x + x^2 + x^4$ 3. $g(x) = 1 + x + x^2 + x^3 + x^4$	1