ITC

PROBLEM SHEET-3

1Q.

$$V = uG = [m_1\ m_2\ m_3\ m_4]\begin{bmatrix}1&1&0&0&0&1&1\\0&1&0&0&1&0&1\\1&0&1&0&1&0&1\\0&0&0&1&1&1&1\end{bmatrix}$$

m_1	m_2	m_3	m_4	c_1	c_2	c_3	c_4	c_5	c_6	c_7
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	1	1	1
0	0	1	0	0	0	1	0	1	0	1
0	0	1	1	0	0	1	1	0	1	0
0	1	0	0	0	1	0	0	1	0	1
0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	1	1	0	0	0	0
0	1	1	1	0	1	1	1	1	1	1
1	0	0	0	1	0	0	0	0	1	1
1	0	0	1	1	0	0	1	1	0	0
1	0	1	0	1	0	1	0	1	1	0
1	0	1	1	1	0	1	1	0	0	1
1	1	0	0	1	1	0	0	1	1	0
1	1	0	1	1	1	0	1	0	0	1
1	1	1	0	1	1	1	0	0	1	1
1	1	1	1	1	1	1	1	1	0	0

2Q.

$n=8$; $k=4$.

$n-k=4$.

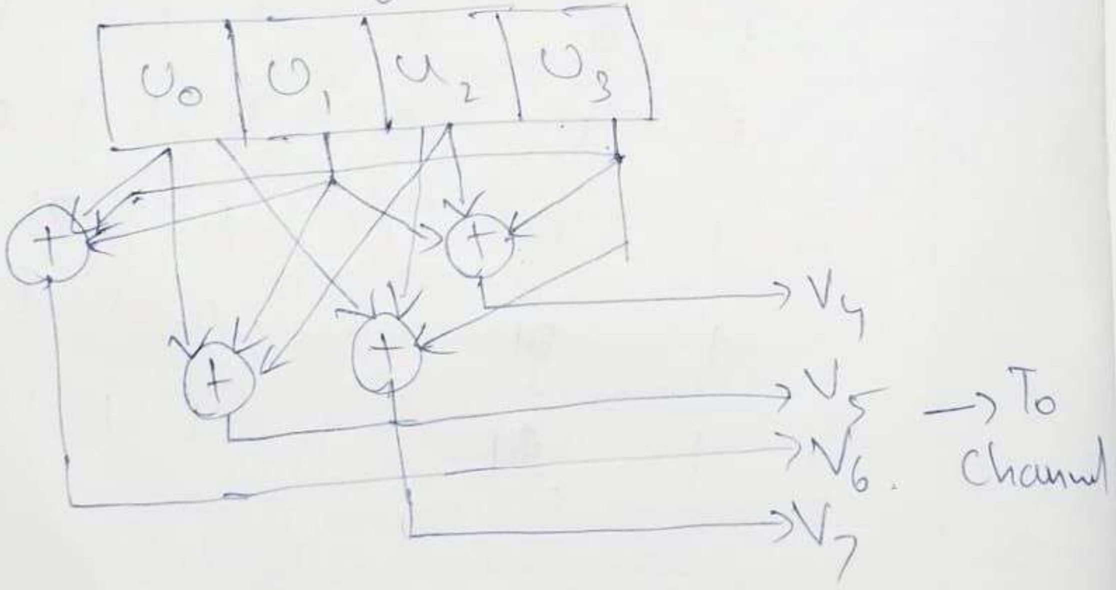
$G=[I_k \mid P_{n-k}]$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right]$$

$H=[P^T \mid I_{n-k}]$

$$= \left[\begin{array}{cccc|cccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Encoder Diagram:



3Q. $d_{min} = 3$, $(10, 5)$

$$V = [m_1\, m_2\, m_3\, m_4\, m_5] \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

m_1	m_2	m_3	m_4	m_5	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	Weight
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	1	1	0	1	4
0	0	0	1	0	0	0	0	1	0	1	1	0	1	0	4
0	0	0	1	1	0	0	0	1	1	1	0	1	0	1	5
0	0	1	0	0	0	0	1	0	0	1	0	1	1	1	5
0	0	1	0	1	0	0	1	0	1	1	1	0	1	0	5
0	0	1	1	0	0	0	1	1	0	0	1	1	0	1	5
0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	3
0	1	0	0	0	0	1	0	0	0	0	1	1	1	0	4
0	1	0	0	1	0	1	0	0	1	0	0	0	1	1	4
0	1	0	1	0	0	1	0	1	0	0	0	1	0	0	3
0	1	0	1	1	0	1	0	1	1	1	1	0	0	1	6
0	1	1	0	0	0	1	1	0	0	1	1	0	0	1	5
0	1	1	0	1	0	1	1	0	1	1	0	1	0	0	5
0	1	1	1	0	0	1	1	1	0	0	0	0	1	1	5
0	1	1	1	1	0	1	1	1	1	0	1	1	1	0	7
1	0	0	0	0	1	0	0	0	0	1	1	1	0	1	5
1	0	0	0	1	1	0	0	0	1	1	0	0	0	0	3
1	0	0	1	0	1	0	0	1	0	0	0	1	1	1	5
1	0	0	1	1	1	0	0	1	1	0	1	0	1	0	5
1	0	1	0	0	1	0	1	0	0	0	1	0	1	0	4
1	0	1	0	1	1	0	1	0	1	0	0	1	1	1	5
1	0	1	1	0	1	0	1	1	0	1	0	0	0	0	4
1	0	1	1	1	1	0	1	1	1	1	1	1	0	1	8
1	1	0	0	0	1	1	0	0	0	1	0	0	1	1	5
1	1	0	0	1	1	1	0	0	1	1	1	1	1	0	7
1	1	0	1	0	1	1	0	1	0	0	1	0	0	0	4
1	1	0	1	1	1	1	0	1	1	0	0	1	0	0	5
1	1	1	0	0	1	1	1	0	0	0	0	1	0	0	4
1	1	1	0	1	1	1	1	0	1	0	1	0	0	1	6
1	1	1	1	0	1	1	1	1	0	1	1	1	1	0	8
1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	8

∴ The minimum Weight = 3.

∴ $d_{min} = 3$.

$$H = [P^T | I] = \left[\begin{array}{ccccc|ccccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\therefore$ No. of errors detectable $= d_{min} - 1$

$= 3 - 1$

$= 2$.

No. of correctable errors $= \frac{d_{min} - 1}{2} = 1$

Encoder Circuit:

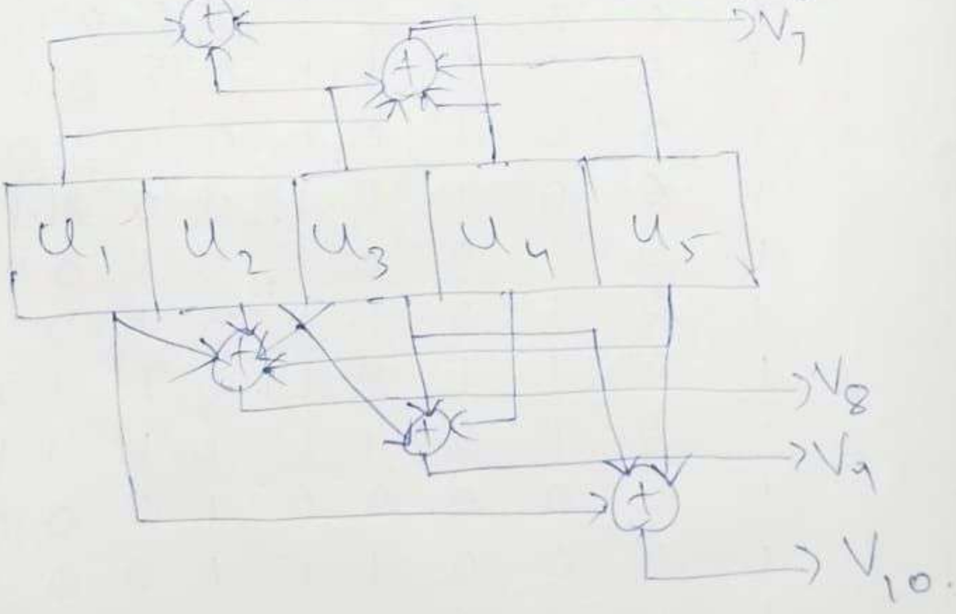
$v_6 = u_1 + u_3 + u_4$

$v_7 = u_2 + u_1 + u_4 + u_5$

$v_8 = u_1 + u_2 + u_3 + u_5$

$v_9 = u_2 + u_3 + u_4$

$v_{10} = u_1 + u_3 + u_5$



4Q: $(7,4)$ $g(x) = 1 + x + x^3$. $u_1 = 1101$, $u_2 = 0110$

(i) $u_1 = 1101 = 1 + x + x^3$

$c(x) = m(x) g(x) = (1 + x + x^3)(1 + x + x^3)$

$= 1 + \cancel{x} + \cancel{x^3} + \cancel{x} + x^2 + \cancel{x^4} + \cancel{x^3} + \cancel{x^4} + x^6$.

$c(x) = 1 + x^2 + x^6 = 1010001$

$u_2 = 0110 = x + x^2$.

$c(x) = (x + x^2)(1 + x + x^3) = x + \cancel{x^2} + x^4 + \cancel{x^2} + x^3 + x^5$

$c(x) = x + x^3 + x^5$.

(ii)

message & $m(x)$	$c(x) = m(x) \times g(x)$	Code Vector
$0000 = 0$	0	0000000
$0001 = 1$	$1 + x + x^3$	0001011
$0010 = x$	$x + x^2 + x^4$	0010110
$0011 = x + 1$	$1 + x^2 + x^4 + x^3$	0011101
$0100 = x^2$	$x^2 + x^3 + x^5$	0101100
$0101 = 1 + x^2$	$1 + x + x^2 + x^5$	0010111
$0110 = x + x^2$	$x + x^3 + x^4 + x^5$	0111010
$0111 = 1 + x + x^2$	$1 + x^4 + x^5$	0110001
$1000 = x^3$	$x^3 + x^4 + x^6$	1011001
$1001 = x^3 + 1$	$1 + x + x^4 + x^6$	1010011
$1010 = x^3 + x$	$x + x^2 + x^3 + x^6$	1001110
$1011 = x^3 + x + 1$	$1 + x^2 + x^6$.	1000101
$1100 = x^3 + x^2$	$x^2 + x^4 + x^5 + x^6$.	1110100
$1101 = x^3 + x^2 + 1$	$x^2 + 1 + x + x^4 + x^5 + x^6 + x^3$	1111111
$1110 = x^3 + x^2 + x$	$x + x^5 + x^6$.	1100010
$1111 = x^3 + x^2 + x + 1$	$1 + x^3 + x^5 + x^6$.	1101001

(iii) $1+x+x^2 \sqrt{x+x^2+x^5+x^6}$

$$
\begin{array}{r}
x^3+x^2+x \\
1+x+x^3 \overline{\big)\, x^6+x^5+x^2+x} \\
x^6+x^4+x^3 \\
\hline
x^5+x^4+x^3+x^2 \\
x^5+\quad +x^3+x^2 \\
\hline
x^4+x \\
x^4+x^2+x \\
\hline
x^2 \\
\hline
\end{array}
$$

$\therefore S = x^2$

$\therefore r(x)$ has errors as $s \neq 0$.

(iv) Error is at 2^{nd} bit position.

$r(x) = 0110011$

$\therefore$ corrected codeword $= 0110001$

5Q: $(15,7)$, $g(x) = 1+x+x^4$

(i) $r_1(x) = x^3+x^6+x^9+x^{12}+x^{14}$.

$$
\begin{array}{r}
x^{10}+x^8+x^7+x^6 \\
1+x+x^4 \overline{\big)\, x^{14}+x^{12}+x^9+x^6+x^3} \\
x^{14}+x^{11}+x^{10} \\
\hline
x^{12}+x^{11}+x^{10}+x^9+x^6 \\
x^{12}+x^9+x^8 \\
\hline
x^{11}+x^{10}+x^8+x^6+x^3 \\
x^{11}+x^8+x^7 \\
\hline
x^{10}+x^7+x^6+x^3 \\
x^{10}+x^7+x^6 \\
\hline
x^3 \\
\hline
\end{array}
$$

$$
\begin{array}{r l}
 & x^{10} + x^{8} + x^{7} + x^{6} \\
x^4+x+1 & \overline{)\, x^{14} + x^{12} + x^{9} + x^{6} + x^{3}} \\
 & x^{14} + x^{11} + x^{10} \\
\hline
 & x^{12} + x^{11} + x^{10} + x^{9} + x^{6} \\
 & x^{12} + x^{9} + x^{8} \\
\hline
 & x^{11} + x^{10} + x^{8} + x^{6} + x^{3} \\
 & x^{11} + x^{8} + x^{7} \\
\hline
 & x^{10} + x^{7} + x^{6} + x^{3} \\
 & x^{10} + x^{7} + x^{6} \\
\hline
 & x^{3} \\
\hline
\end{array}
$$

$\therefore S = x^3$.

$\therefore r_1(x)$ has errors. (at 3rd bit)

$r_2(x) = x^{14} + x^{11} + x^{7} + x^{2} + x + 1$

$$
\begin{array}{r l}
 & x^{10} + x^{6} + x^{2} + 1 \\
x^4+x+1 & \overline{)\, x^{14} + x^{11} + x^{7} + x^{2} + x + 1} \\
 & x^{14} + x^{11} + x^{10} \\
\hline
 & x^{10} + x^{7} + x^{4} \\
 & x^{10} + x^{7} + x^{6} \\
\hline
 & x^{6} + x^{4} + x + 1 \\
 & x^{6} + x^{3} + x^{2} \\
\hline
 & x^{4} + x^{3} + x^{2} + x + 1 \\
 & x^{4} + x + 1 \\
\hline
 & x^{3} + x^{2} \\
\hline
\end{array}
$$

$\therefore r_2(x)$ has error bcos $S \neq 0$. (at 3rd & 2nd bit)

(ii) $r_1(x) = x^{14} + x^{12} + x^{7} + x^{6} + x^{3} = 101000011001000$

Correct Codeword $= 101000011000000$.

$r_2(x) = x^{14} + x^{12} + x^{9} + x^{6} + x^{3} = 101001001001000$

Corrected Codeword $= 101001001001110$.

(iii) $d_{min} = 4$. = highest deg of $g(x)$.

Detectable errors $= 4 - 1 = 3$

Correctable errors $= \frac{3}{2} = 1.5 \simeq 2$.

(iv)

For each message polynomial find the $C(x)$ such that

$$C(x) = u(x)\, x^{n-k} + p(x)$$

$$\hookrightarrow = \text{rem}\left(\frac{u(x)\, x^{n-k}}{g^x}\right)$$

Inorder to covert the cyclic code into systematic code.

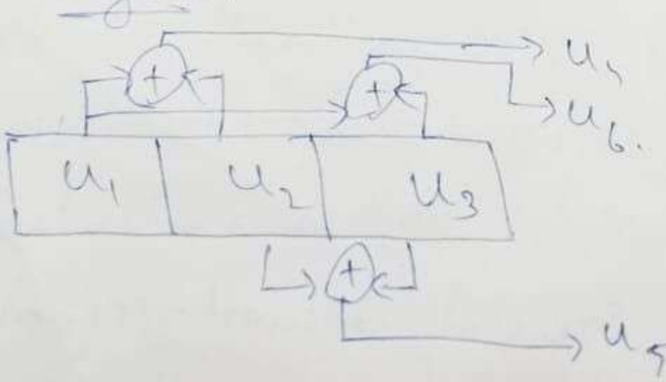
Implication:

It simplifies the encoding process as the message bits are directly present in codeword. The parity bits are computed by $\frac{u(x) . x^{n-k}}{g(x)}$ g(x) & taking its remainder. This increases the computation.

6Q: (ii) $u_4 = u_1 + u_2$ & $u_6 = u_1 + u_3$

$u_5 = u_2 + u_3$

Encoder Diagram:



$$V = [m_1 m_2 m_3] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

m_1 m_2 m_3	Code vector	Weight
0 0 0	000 000	0
0 0 1	001011	3
0 1 0	010110	3
0 1 1	011101	4
1 0 0	100101	3
1 0 1	101110	4
1 1 0	110011	4
1 1 1	111000	3.

$\therefore d_{min} = 3.$

(iii) $H = [P^T | I] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

$S = rH^T = \cancel{[101101] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}$

$= [101101] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

$= [1+0+0+1+0+0 \quad 0+0+1+0+0+0 \quad 1+0+1+0+0+1]$

$= [0 \quad 1 \quad 1] = 3^{rd}$ row of H^T mat

$\therefore$ Corrected codeword $= [100101]$

(iv) Detectable errors = 3 − 1 = 2.

Correctable errors = $\frac{2}{2}$ = 1

7Q: (i) $H = [P^T \mid I_{n \times k}]$. $G = [I_{n \times k} \mid P_{n-k}]$.

$$P^T = \cancel{\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}} \qquad P = \cancel{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}}$$

$$H = \left[\begin{array}{ccccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \rightarrow R_1 \oplus R_2$, ~~$R_3 \rightarrow R_3 - R_2$~~. $R_4 \rightarrow R_4 - R_3$

$$\cancel{H} = \left[\begin{array}{ccccc|cccc} -1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$H = \left[\begin{array}{ccccc|cccc} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \rightarrow R_2 \oplus R_3$, $R_4 \rightarrow R_4 \oplus R_3$.

$$H = \left[\begin{array}{c|cccc} & 1 & 0 & 0 & 1 \\ & 0 & 1 & 1 & 0 \\ & 1 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 \end{array}\right]$$

7Q. $H = [P^T | I]$. $\quad G = [I | P]$. $\quad k = 5$

(i)

$$P^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$G = \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

(ii) $s = r H^T$.

Based on the syndrome value, & the position of the syndrome in H^T is found which is the bit position of codeword which has to be corrected/flipped. Thereby detecting & correcting the single bit error.

(iii)

$$s = [1\,0\,1\,0\,1\,1\,0\,1\,0] \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+1+0+1+0+0+0+0 \\ 0+0+1+0+0+1+0+0+0 \\ 1+0+0+0+0+1+0+1+0 \\ 0+0+1+0+1+1+0+0+1 \end{bmatrix} = [1\,0\,1\,0]$$

There is error at 1st bit position.
corrected codeword = [001011010].

(iv) $v_6 = u_1 + u_3 + u_5$

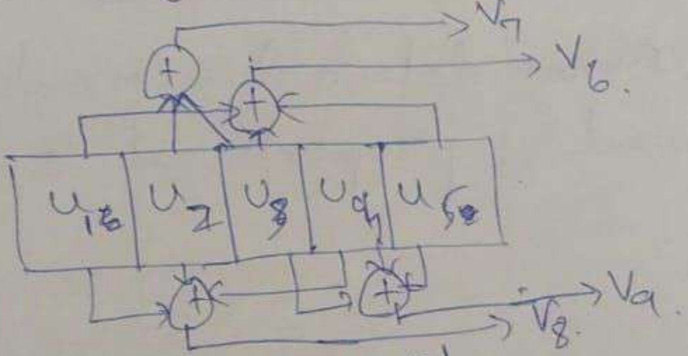
$v_7 = u_2 + u_3$

$v_8 = u_1 + u_2 + u_4$

$v_9 = u_3 + u_4 + u_5$.

$\cancel{v_{10} = u_1 + u_4}$.

Encoder Diagram.



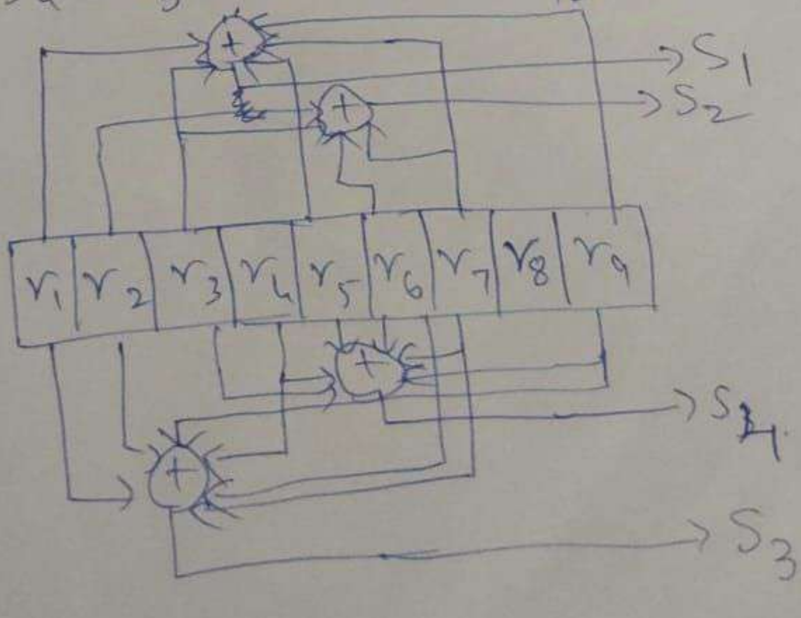
Syndrome Decoder Circuit:

$S_1 = r_1 + r_3 + r_5 + r_7 + r_9$

$S_2 = r_2 + r_3 + r_6 + r_7$

$S_3 = r_1 + r_2 + r_4 + r_6 + r_8$.

$S_4 = r_3 + r_4 + r_5 + r_6 + \cancel{r_8} + r_9$.



(v) Impacts of d_{min} on detection & correction.

1.) No of errors detectable is one less than d_{min} i.e it directly depends on d_{min} .. $(d_{min} - 1)$.

2.) No. of errors correctable is $\frac{d_{min} - 1}{2}$. Thus it depends directly on d_{min}, as other values are constant.

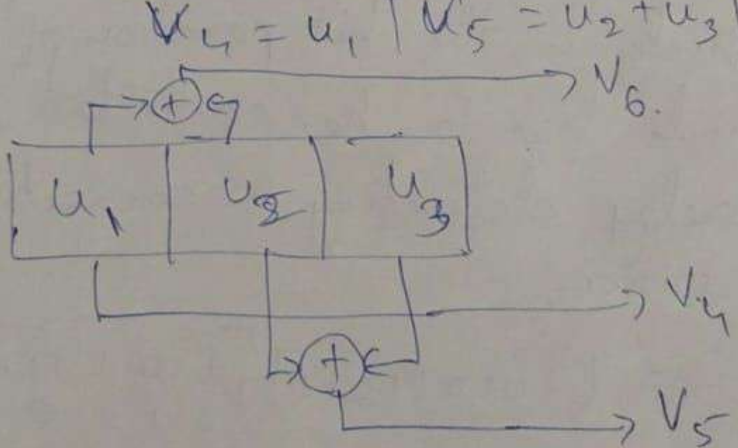
3.) ∴ Errors detectable & Correctable are directly proportional to d_{min}.

8Q (i) $v = uG = [m_1 m_2 m_3] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$

Message: (m_1, m_2, m_3)	Codevector $c_1 c_2 c_3 c_4 c_5 c_6$	Weigth.
0 0 0	0 0 0 0 0 0	0
0 0 1	0 0 1 0 1 0	2
0 1 0	0 1 0 0 1 1	3
0 1 1	0 1 1 0 0 1	3
1 0 0	1 0 0 1 0 1	3
1 0 1	1 0 1 1 1 1	5
1 1 0	1 1 0 1 1 0	4
1 1 1	1 1 1 1 0 0	4

(ii) Encoder Circuit:

$V_4 = u_1 \mid V_5 = u_2 + u_3 \mid V_6 = u_1 + u_2$



(iii) d_{min} = minimum Weight.

$d_{min} = 2$.

(iv) Detectable errors $= d_{min} - 1 = 1$

Correctable errors $= 0$.

$\therefore$ can't correct errors.

9Q. (i) $S_1 = r_1 + r_2 + r_3 + r_4$

$S_2 = r_1 + r_2 + r_4 + r_6$

$S_3 = r_1 + r_3 + r_4 + r_7$

$$P_3^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{7\times3}$$

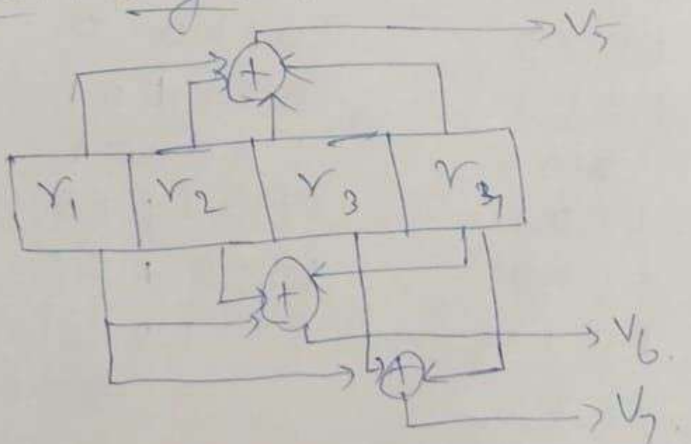
(ii)

$v_5 = r_1 + r_2 + r_3 + r_4$

$v_6 = r_1 + r_2 + r_4$

$v_7 = r_1 + r_3 + r_4$

Encoder Diagram:



(iv) ~~Rate~~ $d_{min} = 2$ as minimum weight $= 2$.

$\therefore$ Detectable errors $= 1$

Correctable errors $= 0$.

$\therefore$ Can't correct the errors

(iii) $V = [m_1\ m_2\ m_3\ m_4] \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$

Message ($m_1\, m_2\, m_3\, m_4$)	Code word $v_1 v_2 v_3 v_4 v_5 v_6 v_7$	Weight
0000	0000000	0
0001	0001111	4
0010	0010101	3
0011	0011010	3
0100	0100110	3
0101	0101001	3
0110	0110011	4
0111	0111100	4
1000	1000111	4
1001	1001000	2
1010	1010010	3
1011	1011101	5
1100	1100001	3
1101	1101110	5
1110	1110100	4
1111	1111011	6

(v) $S = r\, H^T$

$= [1011011] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= [\,1+0+1+1+0+0+0 \quad 1+0+0+0+1+0 \quad 1+0+1+1+0+0+1\,]$

$S_1 = [1\ 0\ 0]$

10Q $m_1 = 1010 = 1 + x^2$. $g(x) = 1 + x + x^3$

$c(x) = m(x)\, g(x) = 1 + x + \cancel{x^3} + x^2 + \cancel{x^3} + x^5$

$= 1 + x + x^2 + x^5 = 1110010$

$m_2 = 1100 = 1 + x$.

$c(x) = 1 + x + x^3 + x + x^2 + x^4$

$= 1 + x^2 + x^3 + x^4 = 1011100$

11Q $g(x) = 1 + x + x^3$, $r(x) = x^5 + x^4 + x^3 + x^2 + \cancel{x}\, 1$

$$
\begin{array}{r|l}
 & x^2 + x \\
1 + x + x^3 & x^5 + x^4 + x^3 + x^2 + 1 \\
 & x^5 + x^3 + x^2 \\
\hline
 & x^4 + 1 \\
 & x^4 + x^2 + x \\
\hline
 & x^2 + x + 1 \\
\hline
\end{array}
$$

$\therefore s = x^2 + x + 1 = 1110000$

12Q: (i) $g(x) = 1 + x^2 + x^3$

$\deg(g(x)) = 3$.

$x^n - 1 \div g(x) = 0$.

n = smallest value that satisfies the the above condition.

$x^7 - 1 = (x-1)(x^3 + x^2 + 1)(x^3 + x + 1)$

$\therefore n = 7$. as $x^7 - 1$ is divisible by $g(x)$

$k = 7 - 3 = 4$.

(ii) $g(x) = 1 + x + x^2 + x^4$.

If $n = 4$.

$$\begin{array}{r|l} & 1 \\ 1+x+x^2+x^4 & x^4 - 1 \\ & x^4 + 1 + x^2 + x \\ \hline & \cancel{2} + x^2 + x \end{array}$$

Let $n = 6$,

$$\begin{array}{r|l} & x^2+1 \\ 1+x+x^2+x^4 & x^6 - 1 \\ & x^6 + x^4 + x^3 + x^2 \\ \hline & x^4 + x^3 + x^2 - 1 \\ & x^4 + x^2 + x \\ \hline & x^3 - 1 + x. \end{array}$$

$n = 7$,

$$\begin{array}{r|l} & x^3+x+1 \\ 1+x+x^2+x^4 & x^7 - 1 \\ & x^7 + x^5 + x^4 + x^3 \\ \hline & x^5 + x^4 + x^3 - 1 \\ & x^5 + x^3 + x^2 + x \\ \hline & x^4 + x^2 + x - 1 \\ & x^4 + x^2 + x + 1 \\ \hline & 0 \end{array}$$

$\therefore n = 7$.

$k = 7 - 4 = 3$.

$(n, k) = (7, 3)$.

(iii) $g(x) = 1 + x + x^2 + x^3 + x^4$

$x^5 - 1 = (x-1)(1 + x + x^2 + x^3 + x^4)$.

$\therefore n = 5$ as $x^5 - 1$ is divisible by $g(x)$

$(n, k) = (5, 5-4) = (5, 1)$