①

Information Theory & Coding
BECE313L
Digital Assignment - 1

[redacted]

1Q. $H(X) = -\sum p(x) \log_2 x$

$= -(0.1 \log_2 0.1 + 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.1 \log_2 0.1)$

$= -(0.33 + 4 \times 0.464 + 0.33)$

$= 2.623$ bits.

$H(X_1, X_2) = H(X_1) + H(X_2)$

$= 2 \times H(X)$

$= 2 \times 2.623$

$= 5.24$ bits.

$H(X_{fair}) = -\sum p(x) \log_2 p(x)$

$= -\cancel{6}\left(\frac{1}{\cancel{6}} \log_2 \frac{1}{6}\right)$

$= 2.584$ bits.

Entropy of biased die is lower than fair die which indicates that biased die has less uncertainity than fair die which in turn makes the overall system to be predictable.

Hence the predictability is more in case of biased die where as the information content gained on an average is less when compared to a fair die.

2Q: $P(X,Y) = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.4 \end{bmatrix}$

$I(X;Y) = \sum_{x}\sum_{y} p(X=x, Y=y) \log_2\left(\frac{P(x,y)}{P(x) \times P(y)}\right)$

$P(x) = 0.1 + 0.1 + 0.1 = 0.3 = P(X=x_1)$

$P(X=x_2) = 0.1 + 0.2 + 0.4 = 0.7.$

$P(Y=Y_1) = 0.1 + 0.1 = 0.2$

$P(Y=Y_2) = 0.1 + 0.2 = 0.3$

$P(Y=Y_3) = 0.1 + 0.4 = 0.5.$

$\Rightarrow X=0, Y=0;$

$P(x_1, y_1) = 0.1$

$I(X:Y) = 0.1 \log_2\left(\frac{0.1}{0.3 \times 0.2}\right) + 0.1 \log_2\left(\frac{0.1}{0.3 \times 0.3}\right)$

$+ 0.1 \times \log_2\left(\frac{0.1}{0.3 \times 0.5}\right) + 0.1 \log_2\left(\frac{0.1}{0.7 \times 0.2}\right)$

$+ 0.1 \times \log_2\left(\frac{0.2}{0.7 \times 0.3}\right) + 0.1 \log_2\left(\frac{0.4}{0.7 \times 0.5}\right)$

$= 0.073 + 0.0154 - 0.058 - 0.048 - 0.014$
$+ 0.0778$

$= 0.044$ bits.

I(x:Y) suggest a small amount of information about Y can be obtained by knowing x, & vice versa.

In communication system, I(x:Y) is crucial because it quantifies the reduction in uncertainity of one variable given that knowledge of other variable. which is vital for efficient data transmission & error detection.

$$H(Y/X) = -\sum p(x) \sum P(Y=y, X=x) \log_2 P(Y=y | X=x)$$

$$H(Y/X_1) = -\left(\frac{0.1}{0.3} \log_2\left(\frac{1}{3}\right) + \frac{0.1}{0.3} \log_2\left(\frac{1}{3}\right) + \frac{0.1}{0.3} \log_2\left(\frac{1}{3}\right)\right)$$

$$= 1.585 \text{ bits.}$$

$$H(Y/X_2) = -\left(\frac{0.1}{0.7} \log_2\left(\frac{1}{7}\right) + \frac{0.2}{0.7} \log_2\left(\frac{2}{7}\right) + \frac{0.4}{0.7} \log_2\left(\frac{5}{7}\right)\right)$$

$$= 1.19.$$

$$X(Y/x) = 0.3 \times 1.58 + 0.7 \times 1.19$$

$$= 1.32 \text{ bits.}$$

∴ The redundancy is lower as the H(Y/X) is less thus offering efficient data compression.

$$H(X,Y) = \sum_x \sum_y P(x,y)\log_2 P(x,y)$$

$$= -(0.1 \log_2 0.1 + 0.1 \log 0.1 + 0.1 \log 0.1 + 0.1 \log_2 0.1 + 0.2 \log_2 0.2 + 0.4 \log_2 0.4)$$

$$= -(4 \times 0.1 \times -3.32 + 0.2 \times -2.32 + 0.4 \times -1.32)$$

$$= 2.32 \text{ bits}$$

$$H(X) = -(0.3 \log 0.3 + 0.7 \log 0.7)$$

$$= 0.88 \text{ bits}$$

$$H(Y) = -(0.2 \log_2 0.2 + 0.3 \log_2 0.3 + 0.5 \log_2 0.5)$$

$$\simeq 1.48$$

$\therefore$ $H(X,Y)$ tells us about the uncertainity in both X & Y whereas $H(X)$, $H(Y)$ tells us about the uncertainity of X & Y individually.

$$H(X,Y) = H(X) + H(Y/X)$$

Lower mutual information implies less dependancy b/w variables while lower conditional entropy indicates better potential for data compression.

(5)

3Q: $f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

$$h(x) = -\int_{-\infty}^{\infty} f(x) \log_2 f(x)\, dx.$$

$$= -\int_0^1 1 \cdot \cancel{\log_2}^{\,0} (1)\, dx$$

$$= 0.$$

Discrete entropy: Measures the average info. of a variable with discrete set of values

Differential entropy: It is same as discrete but it deals with continous random variable.

* Discrete is non negative, Differential may be -ve.
* Discrete is invariant, Differential is variant.
* Applications:
    1.) Source coding.
    2.) Channel capacity
    3.) Image processing.
    4.) Estimation theory.

4Q: $f(x) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) e^{-(x-\mu)^2/2\sigma^2}$

$$h(x) = -\int_{-\infty}^{\infty} f(x) \log_2(f(x))\, dx.$$

$$h(x) = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \log_2\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)$$

$$= -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(-\log_2(\sqrt{2\pi\sigma^2}) - \frac{(x-\mu)^2}{2\sigma^2} \log_2 e\right) dx$$

$$= \log_2(\sqrt{2\pi\sigma^2}) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx + \frac{\log_2(e)}{2\sigma^2} \int_{-\infty}^{\infty} \frac{(x-\mu)^2}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \log_2(\sqrt{2\pi\sigma^2}) \times 1 + \frac{\log_2(e)}{2\cancel{\sigma^2}} \times \cancel{\sigma^2}$$

$$= \frac{1}{2} \log_2(2\pi\sigma^2 e).$$

$h(x)$ is independent of mean μ & depends on σ i.e if σ increases $h(x)$ increases leading to greater uncertainity in the value of X.

5Q: $$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_X^2} + \frac{y^2}{\sigma_Y^2} - \frac{2\rho xy}{\sigma_X\sigma_Y}\right)\right)$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

$$H(X) = \frac{1}{2}\log_2(2\pi e\sigma_X^2)$$

for gaussian distribution with variance σ^2

7.

$$H(Y) = \frac{1}{2}\log_2(2\pi e \sigma_Y^2)$$

$$H(X,Y) = \frac{1}{2}\log_2\left((2\pi e)^2 \,|\text{Cov Matrix}|\right)$$

$$\text{Cov Matrix} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_Y \\ \rho\sigma_x\sigma_Y & \sigma_Y^2 \end{bmatrix}$$

$$|\text{Cov}| = \sigma_x^2\sigma_Y^2 - \rho^2\sigma_x^2\sigma_Y^2$$
$$= \sigma_x^2\sigma_Y^2(1-\rho^2)$$

$$H(X,Y) = \frac{1}{2}\log_2\left((2\pi e)^2(\sigma_x^2\sigma_Y^2(1-\rho^2))\right)$$

$$= \log_2(2\pi e) + \frac{1}{2}\log_e\left(\sigma_x^2\sigma_Y^2(1-\rho^2)\right)$$

$$I(X;Y) = \left[\frac{1}{2}\log_2(2\pi e\sigma_x^2) + \frac{1}{2}\log_2(2\pi e\sigma_Y^2)\right]$$
$$- \frac{1}{2}\log_2\left(\sigma_x^2\sigma_Y^2(1-\rho^2)\right) - \log_2(2\pi e)$$

$$= \frac{1}{2}\log_2(\cancel{2\pi e\,\sigma_x^2}) + \frac{1}{2}\log_2\cancel{(2\pi e\sigma_Y^2)}$$
$$- \frac{1}{2}\log_2\cancel{(2\pi e\sigma_x^2)} - \frac{1}{2}\log_2\cancel{(2\pi e\sigma_Y^2)}$$
$$- \frac{1}{2}\log_2(1-\rho^2)$$

$$I(X;Y) = -\frac{1}{2}\log_2(1-\rho^2)$$

When ρ increases, $I(X;Y)$ increases i.e two variable share more info which reflects greater predictability of one variable given the other.

$I(X;Y)$ quantifies the amount of shared information b/w X & Y.

6Q: $$P = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$$

Stationary distribution $\pi = (\pi_1, \pi_2, \pi_3)$ satisfies:

$$\pi P = \pi_i$$

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix} \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix}$$

$$0.2\pi_1 + 0.3\pi_2 + 0.4\pi_3 = \pi_1$$

$$0.5\pi_1 + 0.4\pi_2 + 0.3\pi_3 = \pi_2$$

$$0.3\pi_1 + 0.3\pi_2 + 0.3\pi_3 = \pi_3$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$-0.8\pi_1 + 0.3\pi_2 + 0.4\pi_3 = \cancel{\pi_1}\, 0$$

$$0.5\pi_1 - 0.6\pi_2 + 0.3\pi_3 = 0$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

(a)

solving these eqs

$\pi_1 = 0.33$

$\pi_2 = 0.33$

$\pi_3 = 0.33$.

$$H(x) = -\sum_{i=1}^{3} \pi_i \sum_{j=1}^{3} P_{ij} \log_2 P_{ij}$$

$$= -\Big[4/13 (0.2 \log 0.2 + 0.5 \log 0.5 + 0.3 \log 0.3) + 5/13 (0.3 \log 0.3 + 0.3 \log 0.3 + 0.4 \log 0.4) + 4/13 (0.4 \log 0.4 + 0.3 \log 0.3 \times 2) \Big]$$

~~for π_1,~~

$\cancel{= 0.2 \log (0.2) + 0.5}$

$$H(x) = -\Big[\frac{4}{13}(-0.4472) + (-0.4730)\left(\frac{5}{13}\right) + (-0.47)\left(\frac{4}{13}\right)\Big]$$

$$= -[-0.1378 - 0.182 - 0.1456]$$

$$= 0.4654.$$

Channel Capacity, entropy rate, error detection & correction are some implications of entropy rate.

7Q: $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$

$\pi = [0.67, 0.33]$.

Information Rate $R = H(X) - H(X/Y)$

$H(X) = -\sum \pi_i \log_2 \pi_i$

$= -[0.67 \log_2 0.67 + 0.33 \log_2 (0.33)]$

$= \cancel{0.673 \text{ bits}}\ 0.92$ bits.

$H(X/Y) = -\sum \pi_i \sum P_{ij} \log_2 P_{ij}$

$= -0.67(0.8 \log_2 0.8 + 0.2 \log_2 0.2) - 0.33(0.4 \log 0.4 + 0.6 \log 0.6)$

$= +0.816$ bits.

$R = H(X) - H(X/Y)$

$= \cancel{[0.673 - 0.86]}\ 0.92 - 0.81$

$= \cancel{+0.186}\ 0.11$ bits.

R represents the average amount of info. Lower R implies more predictable source output whereas higher R implies ~~in~~ less redundancy. Based on R one can select most appropriate coding technique to maximize compression while maintaining data integrity

8Q. $D_{KL}(P||Q) = \sum_i P(i) \log_2 \left(\frac{P(i)}{Q(i)}\right)$

$= \frac{1}{3}\left(\log_2\left(\frac{2}{3}\right) + \log_2\left(\frac{4}{3}\right) + \log_2\left(\frac{4}{3}\right)\right)$

$= \frac{1}{3}\left(-0.585 + 0.415 + 0.415\right)$

$= 0.081$

KL divergence measures the information loss. ∴ it implies that the information lost is less. To be exact KL divergence measures the info loss when the Q distribution is used to approximate the true distribution P. ∴ Distribution of Q is fairly close to P.

9Q. $C = B \log_2 \left(1 + \frac{S}{N}\right)$

$SNR = 15dB = 10^{15/10} = 31.62$

$C = 1 \times 10^6 \log_2 (1 + 31.62)$

$= 5.04 \times 10^6$

$= 5.04$ Mbps.

C represents the theoritical maximum data rate that can be achieved without errors. Thus the system has to designed in such a way that the maximum limit is not exceeded. Efficient Coding schemes has to be used based on the value of C. Bandwidth allocation also depends the capacity.

10Q: $H(S_i) = -P(S_i\ fails)\log_2(P(S_i\ fails))$
$\quad - P(S_i\ works)\log_2(P(S_i\ works))$

$H(S_1) = -0.1 \times \log_2(0.1) - 0.9\log_2(0.9)$
$= 0.467$ bits.

$H(S_2) = -0.2\log(0.2) - 0.8(\log_2(0.8))$
$= 0.72$ bits.

$H(S_3) = -0.3\log(0.3) - 0.7\log_2(0.7)$
$= 0.879$ bits.

$H(network) = H(S_1) + H(S_2) + H(S_3)$
$= 0.467 + 0.72 + 0.879$
$= 2.066$ bits.

Hamming Code:

It can correct single bit error & detect two bit error. For a 3 bit data from each sensor a (7, 4) hamming code can be used.

$$\eta = \frac{3}{3+4} = \frac{3}{7} = 0.4286 = 42.86\%$$

Thus using hamming code we can transmit the data with high reliability but with lesser efficiency.

11Q:

a.) $P(X, Y) = P(X)\, P(Y/X)$

$P(1,1) = 0.5 \times 0.8 = 0.4$

$P(1,2) = 0.5 \times 0.1 = 0.05$

$P(1,3) = 0.5 \times 0.1 = 0.05$

$P(2,1) = 0.3 \times 0.3 = 0.09$

$P(2,2) = 0.3 \times 0.4 = 0.12$

$P(2,3) = 0.3 \times 0.3 = 0.09$

$P(3,1) = 0.2 \times 0.2 = 0.04$

$P(3,2) = 0.2 \times 0.2 = 0.04$

$P(3,3) = 0.2 \times 0.6 = 0.12$

14

$$P(X,Y) = \begin{bmatrix} 0.4 & 0.05 & 0.05 \\ 0.09 & 0.12 & 0.09 \\ 0.04 & 0.04 & 0.12 \end{bmatrix}$$

b.) $P(Y) = \sum P(X = x_i, Y = y_i)$

$P(1) = 0.4 + 0.09 + 0.04 = 0.53$

$P(2) = 0.05 + 0.12 + 0.04 = 0.21$

$P(3) = 0.05 + 0.09 + 0.12 = 0.26$

$P(Y) = [0.53, 0.21, 0.26]$

c.) $I(X;Y) = \sum_x \sum_y P(x,y) \log\left(\frac{P(x,y)}{P(x)P(y)}\right)$

$P(1,1) \times \log_2\left(\frac{P(1,1)}{P(1)P(1)}\right) = 0.4 \times \log_2\left(\frac{0.4}{0.5 \times 0.53}\right) = 0.2$

$0.05 \times \log_2\left(\frac{0.05}{0.5 \times 0.21}\right) = -0.034$

$I(1,3) = 0.05 \log_2\left(\frac{0.05}{0.5 \times 0.26}\right) = -0.05$

$I(2,1) = 0.09 \log_2\left(\frac{0.09}{0.3 \times 0.53}\right) = -0.047$

$I(2,2) = 0.12 \log_2\left(\frac{0.12}{0.3 \times 0.21}\right) = 0.114$

$I(2,3) = 0.09 \log_2\left(\frac{0.09}{0.2 \times 0.53}\right) = \cancel{-0.05}\ 0.02$

$I(3,1) = 0.04 \log_2\left(\frac{0.04}{0.2 \times 0.53}\right) = -0.05$

$I(3,2) = 0.04 \times \log_2(0.952) = -0.003$

$I(3,3) = 0.12 \times \log_2\left(\frac{0.12}{0.2 \times 0.26}\right) = 0.152$

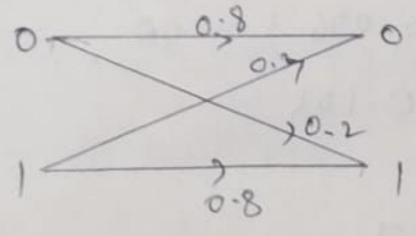
$I(X;Y) = 0.205 - 0.034 - 0.05 - 0.047 + 0.113$
$\qquad + 0.02 - 0.05 - 0.003 + 0.152$

$= 0.307$ bits.

d.) $C = \max(I(X;Y))$

$= 0.307$ bits.

12Q. a.)



b.) $C = 1 - H(p)$.

$= 1 - H(0.2)$

$= 1 - (0.2 \log 0.2 + 0.8 \log 0.8)$

$= 1 - (0.464 + 0.256)$

$= 0.28$ bits.

c.) $I(X;Y) = H(Y) - H(Y/X)$

$H(Y/X) = H(p) = 0.72$

$H(Y) = -p(0)\log_2(p(0)) - p(1)\log_2(p(1))$

$= -(0.8\times0.5 + 0.2\times0.5)\log_2(0.5) - 0.5\log_2(0.5)$

$= 1$ bit.

$I(X;Y) = 1 - 0.72 = 0.28$ bits.

d.)

$P(2\ \text{errors}) = 3\,(0.2)^2 \times (0.8)^1 = 0.096$

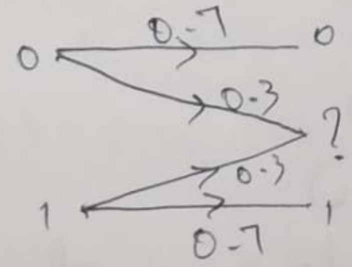
$P(3\ \text{errors}) = (0.2)^3 = 0.008$

$P(\text{error}) = P(2\ \text{er}) + P(3\ \text{errors})$

$= 0.096 + 0.008$

$= 0.104$

13Q: a.)



b.) $C = 1 - p$

$= 1 - 0.3$

$= 0.7$ bits.

c.) $I(X;Y) = H(Y) - H(Y/X) = C$

$= 0.7$ bits.

d.) $P(2 \text{ erasure}) = {}^3C_2\, p^2 (1-p)$

$= 3 \times 0.09 \times 0.7$

$= 0.189$.

$P(3 \text{ erasure}) = p^3 = (0.3)^3$

$= 0.027$

$P(\text{error}) = 0.189 + 0.027$

$= 0.216$.

14Q: a.) $C = B \log_2 (1 + SNR)$

$SNR = 15\,dB = 10^{15/10} = 31.62$

$C = 1 \times 10^6 \log_2 (32.62)$

$= 5.03$ Mbps.

b.) Maximum Data rate $= 5.03$ Mbps.

c.) $R = 8$ Mbps

$SNR = 31.62$

$B = \dfrac{R}{\log_2(1+SNR)} = \dfrac{8 \times 10^6}{\log_2(32.62)}$

$B = 1.59$ MHz.

d.) * When BW is increased, channel Capacity increases.
* When BW is increased data rate also increases.
* Cost & practical difficulties increases as BW increases.

15Q: a.) $C = B \log_2\left(1 + \frac{P}{B N_0}\right)$

$= 2\times10^6 \log_2\left(1 + \frac{1}{2\times10^6 \times 10^{-9}}\right)$

$= 2\times10^6 \log_2\left(1 + \frac{\cancel{10^3}\,500}{\cancel{2\times10^3}}\right)$

$= 2\times10^6 \log_2(501)$

$= 2 \times 8.97 \times 10^6$

$= 17.94$ Mbps

b.) Maximum Data rate = 17.94 Mbps.

c.) $C = 5$ Mbps.

$\cancel{5}^{2.5} \times \cancel{10^6} = \cancel{2} \times \cancel{10^6} \log_2(1+SNR)$

$2^{2.5} = 1 + SNR$

$SNR = 5.656 - 1$

$= 4.656$

$= 10\log(4.656) = 6.67\,dB$

(19a.)

d.) * Increasing transmitted power increases SNR which leads to increase in C.

* When BW is increased C also increases.

* In many cases it is not possible to increase BW than to increase power in BW limited system. In ~~power~~ system where power consumption is limited there only the BW can be increased not the power. Hence there exist a tradeoff b/w Power & BW.

16.Q a.) PDF:

$$f_r(r) = \frac{1}{r_{avg}} \exp\left(-\frac{r}{r_{avg}}\right)$$

$$r_{avg} = 10dB = 10^{10/10} = 10$$

$$f_r(r) = \frac{1}{10} \exp\left(-\frac{r}{10}\right)$$

$$= 0.1 \exp\left(\frac{10}{r}\right) = 0.1\exp(-0.1r)$$

b.) $r_{min} = 5dB = 10^{5/10} = 3.162$

$$P_{out} = P(r < r_{min}) = \int_0^{r_{min}} f_r(r)\,dr$$

$$= \int_0^{3.162} 0.1\, e^{10/r}\, dr$$

$$= \cancel{0.1} \left.\frac{e^{-0.1r}}{-\cancel{0.1}}\right|_0^{3.162}$$

(20)

$$= -e^{-0.1 \times 3.162} + e^{0}$$

$$= 1 - 0.729$$

$$= 0.271$$

$$= 27.1\%$$

c.) $C_{avg} = B\, E[\log_2(1+\gamma)]$

$$= B \frac{1}{\ln(2)} \exp\left(\frac{1}{\gamma_{avg}}\right) E_1\left(\frac{1}{\gamma_{avg}}\right)$$

$$= B \frac{1}{\ln(2)} \exp(0.1)\, E_1(0.1)$$

$$= B \frac{1}{\ln(2)} \cancel{0.1}\, 1.1 \times 1.82$$

$$= 2.908\, B \text{ bps}$$

17Q: a.) $C = BW \log_2\left(1 + \frac{P}{BW N_0}\right)$

$$= 2 \times 10^6 \log_2\left(1 + \frac{1}{2 \times 10^{-3}}\right)$$

$$= 2 \times 10^6 \log_2(501)$$

$$= 17.94 \text{ Mbps}$$

b.) $C = 10$ Mbps

$$\cancel{10}^{5} \times \cancel{10^6} = \cancel{2} \times \cancel{10^6} \log_2\left(1 + \frac{P}{2 \times 10^{-3}}\right)$$

21.

$$1 + P \times 0.5 \times 10^3 = 2^5$$
$$P(500) = 33$$
$$P = 0.066 W$$

c.) $B = 5 MHz$.

$$C' = 5 \times 10^6 \log_2 \left(1 + \frac{1}{5 \times 10^{-3}}\right)$$
$$= 5 \times 10^6 \log_2 (\cancel{2}201)$$
$$= 38.25 \text{ Mbps}.$$

$$\Delta C = 38.25 - 17.94 = 20.31 \text{ Mbps}.$$

18Q. $M_T = 2, M_R = 2, SNR = 20dB = 10^{20/10} = 100$.

a.) $$C = M_R \log_2 \left(1 + \frac{SNR}{M_T}\right)$$
$$= 2 \log_2 \left(1 + \frac{100}{2}\right)$$
$$= 2 \times 5.672$$
$$= 11.344 \text{ bps}.$$

b.) $M_T = 4, M_R = 2, SNR = \cancel{100}\ 100$

$$C = 2 \times \log_2 \left(1 + \frac{\overset{25}{\cancel{100}}}{\cancel{4}}\right)$$
$$= 2 \times 4.7$$
$$= 9.4 \text{ bps}$$

c.) SNR = 10 dB

$= 10^{10/10} = 10.$

$C = 2 \times \log_2\left(1 + \frac{\cancel{10}^{5}}{\cancel{2}}\right)$

$= 2 \times 2.585$

$= 5.17$ bps.

Capacity is decreased as SNR is reduced.

19Q: $B = 1MHz$, $SNR = 15 dB = 10^{15/10} = 31.62$

a.) $C = B \log_2(1 + SNR)$

$= 10^6 \cdot \log_2(32.62)$

$= 5.035$ Mbps

b.) $B = 2MHz$.

$C = 2 \times 10^6 \log_2(32.62)$

$= 10.07$ Mbps

c.) $C = 10Mbps$, $W = 1MHz$.

$10 \times \cancel{10^6} = \cancel{10^6} \log_2(1 + SNR)$

$SNR = 2^{10} - 1$

$= 1023$

$= 10\log(1023) = 30.1$ dB

20Q $e = 0.25$

$n = 7$

$P(k \text{ erasures}) = {}^{n}C_{k}\, e^{k} (1-e)^{n-k}$

a.) $P(\text{correctly decoded}) = P(0 \text{ erasures}) + P(1 \text{ erasure})$

$P(0 \text{ erasure}) = {}^{7}C_{0} (0.25)^{0} (0.75)^{7} = 1 \times 0.75^{7}$

$P(1 \text{ erasure}) = {}^{7}C_{1} (0.25)^{1} (0.75)^{6} = 1.75 \times 0.75^{6}$

$P(\text{correctly decoded}) = 0.75^{6}(0.75 + 1.75)$

$= 2.5 \times 0.75^{6}$

$= 0.445$

b.) Expected no. of erasure $= n \times e$

$= 7 \times 0.25 = 1.75$

c.) $e = 0.1$

$P(\text{correctly decoded}) = {}^{7}C_{0} (0.1)^{0} (0.9)^{7} + {}^{7}C_{1} (0.1)(0.9)^{6}$

$= 0.9^{7} + 0.7 \times 0.9^{6}$

$= \cancel{1.7}\, 1.6 \times 0.9^{6}$

$= 0.85$