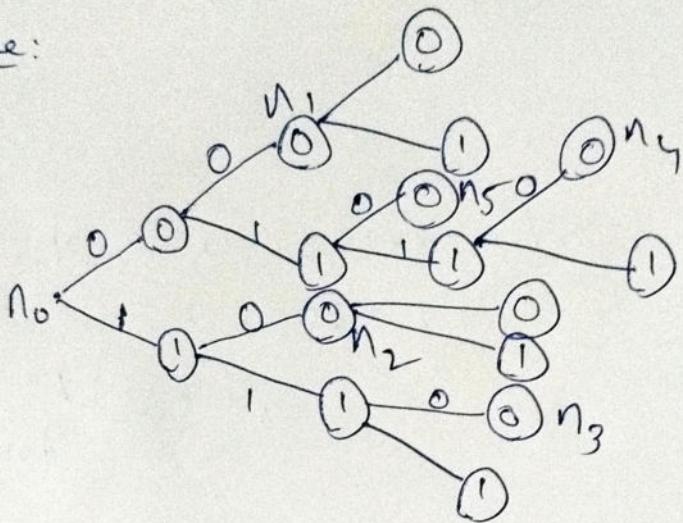


1Q:

Code A:

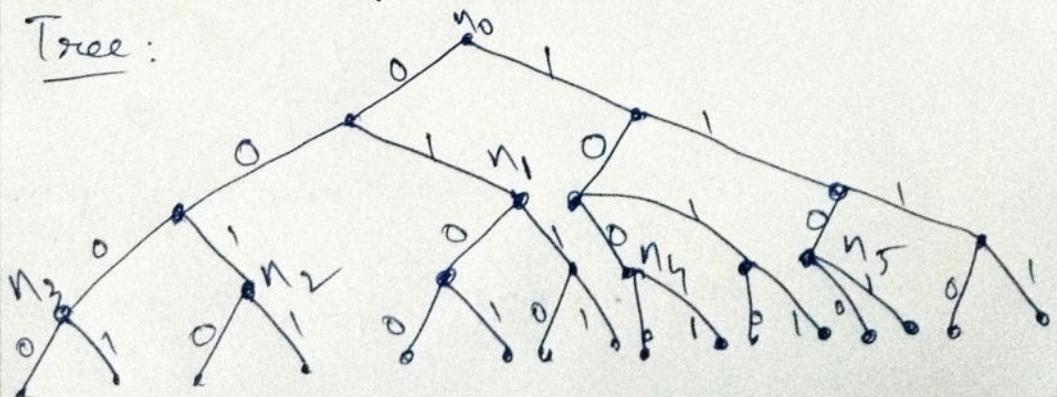
It is a prefix code, as none of codewords has the prefix of another codeword.

Tree:Code B:

Not a prefix code has "110" codeword has the prefix of another codeword "11".

Code C:

It is a prefix. As none of the codewords has the prefix of another codeword.

Tree:

Code D:

It is not a prefix code as the code word "1010" contains "101" codeword as prefix.

Q: Kraft inequality:

$$\sum_{k=0}^n \pi^{w_k} \leq 1.$$

$$\sum_{i=1}^n \frac{L_i}{\pi^{w_i}} \leq 1.$$

$$W = (1, 2, 3, 4, 5)$$

$$L = (1, 2, 3, 4, 5).$$

$$\frac{1}{\pi^1} + \frac{2}{\pi^2} + \frac{3}{\pi^3} + \frac{4}{\pi^4} + \frac{5}{\pi^5} \leq 1.$$

$$\text{Let } \pi = 1,$$

$$1 + \frac{2}{1} + 3 + 4 + 5 \neq 1$$

$$\text{Let } \pi = 2,$$

$$= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32}$$

$$= 0.5 + 0.5 + 0.375 + 0.25 + 0.15625$$

$$= 1.78125 \neq 1$$

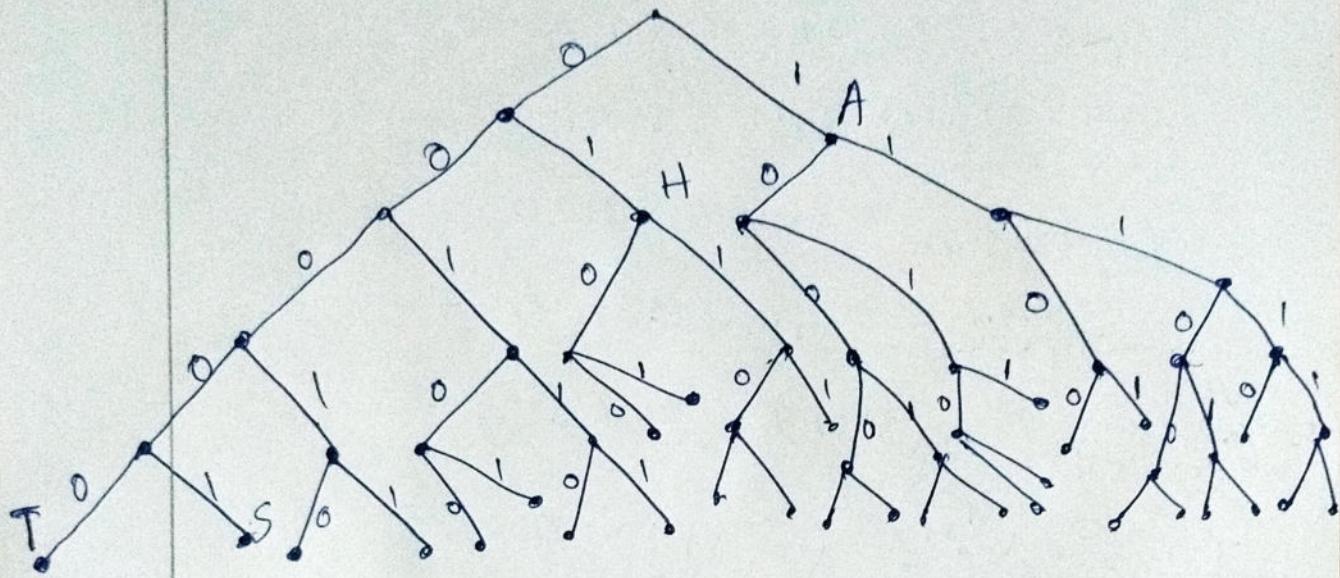
$$\text{Let } \pi = 3,$$

$$= \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \frac{5}{243} = 0.736 \leq 1$$

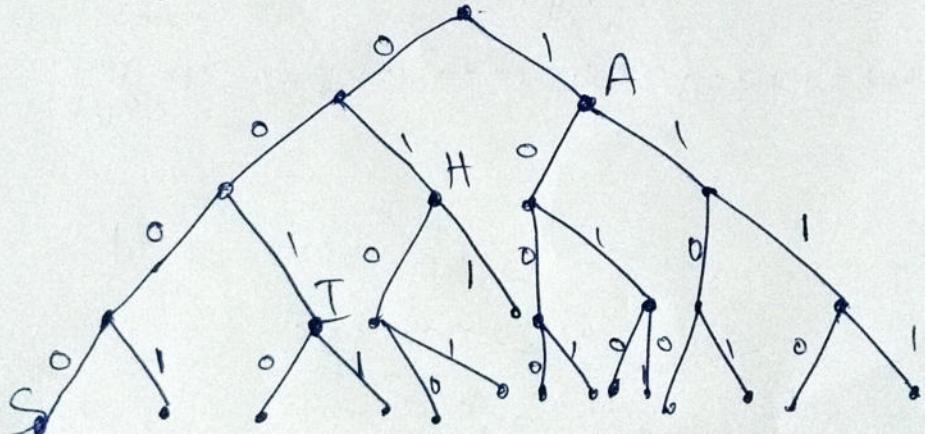
$$\therefore \pi = 3.$$

3Q.

Code A:



Code B:



Decode Info:

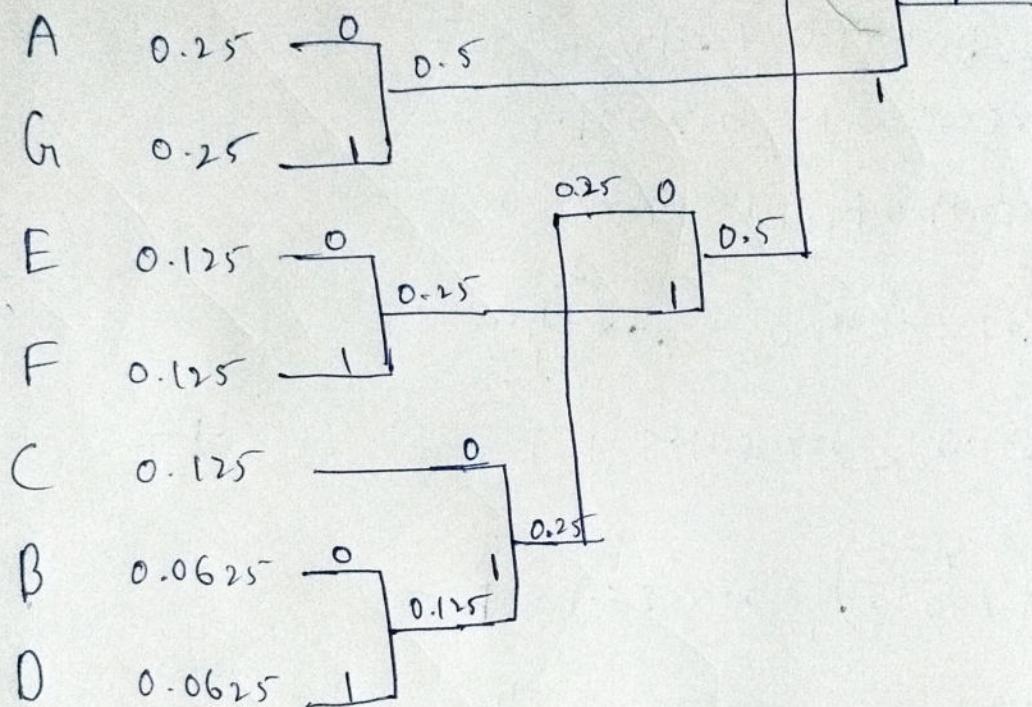
Code A & B:

SHAIITHSTATAHIITHATI
HHITTHATISTHTA

Q:

Sym	A	B	C	D	E	F	G
Prob	0.25	0.0625	0.125	0.0625	0.125	0.125	0.25

Binary Code:



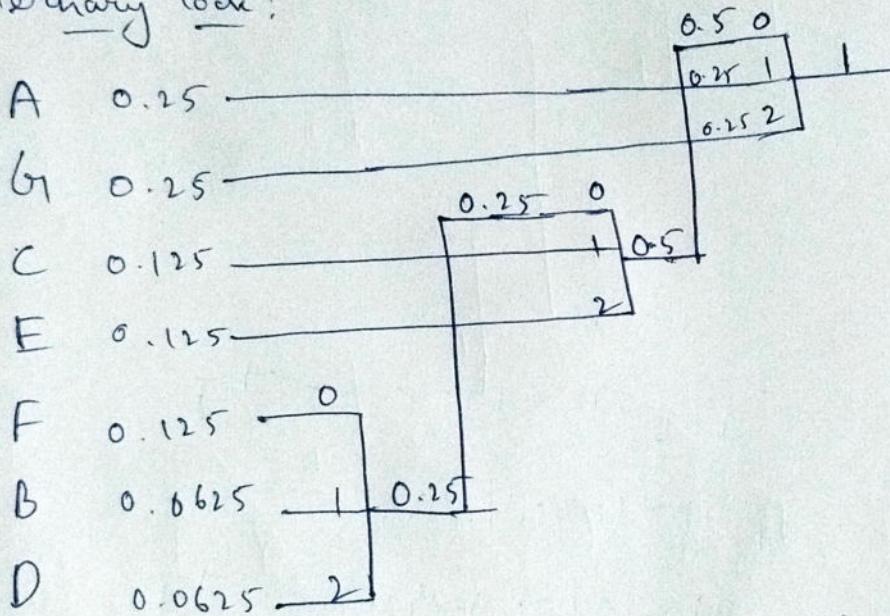
Sym	Prob	Code word	length	$H(x)$ (Self info.)
A	0.25	10	2	0.125
G	0.25	01	2	0.125
E	0.125	010	3	0.375
F	0.125	011	3	0.375
C	0.125	000	3	0.375
B	0.0625	0010	4	0.25
D	0.0625	0011	4	0.25

$$H(x) = 0.125 + 0.125 + 3 \times 0.375 + 0.25 + 0.25 \\ = 0.5 + 1.125 = 1.625$$

$$\bar{R} = \sum_{i=1} p(x_i) l_i = 0.5 + 0.5 + 0.375 + 0.375 + 0.375 + 0.25 + 0.25 \\ = 2.625$$

$$N = \frac{H(x)}{\bar{R}} = \frac{1.625}{2.625} = 0.619 = 61.9\%$$

Ternary code:



Sym	Prob	Codeword	Length	Self information
A	0.25	1	1	0.125
G	0.25	2	1	0.125
C	0.125	01	2	0.375
E	0.125	02	2	0.375
F	0.125	000	3	0.375
B	0.0625	001	3	0.25
D	0.0625	002	3	0.25

$$\bar{R} = \sum p(x_i) d_i = 0.25 + 0.25 + 0.25 + 0.25 + 0.375 + \\ 0.1875 + 0.1875 + 0.18 \\ = 1.75.$$

$$N = \frac{H(x)}{\bar{R}} = \frac{1.625}{1.75} = 0.9285 = 92.85\%.$$

5Q.

Sym Prob.

A

0.22

B

0.2

C

0.18

D

0.15

E

0.1

F

0.08

G

0.05

H

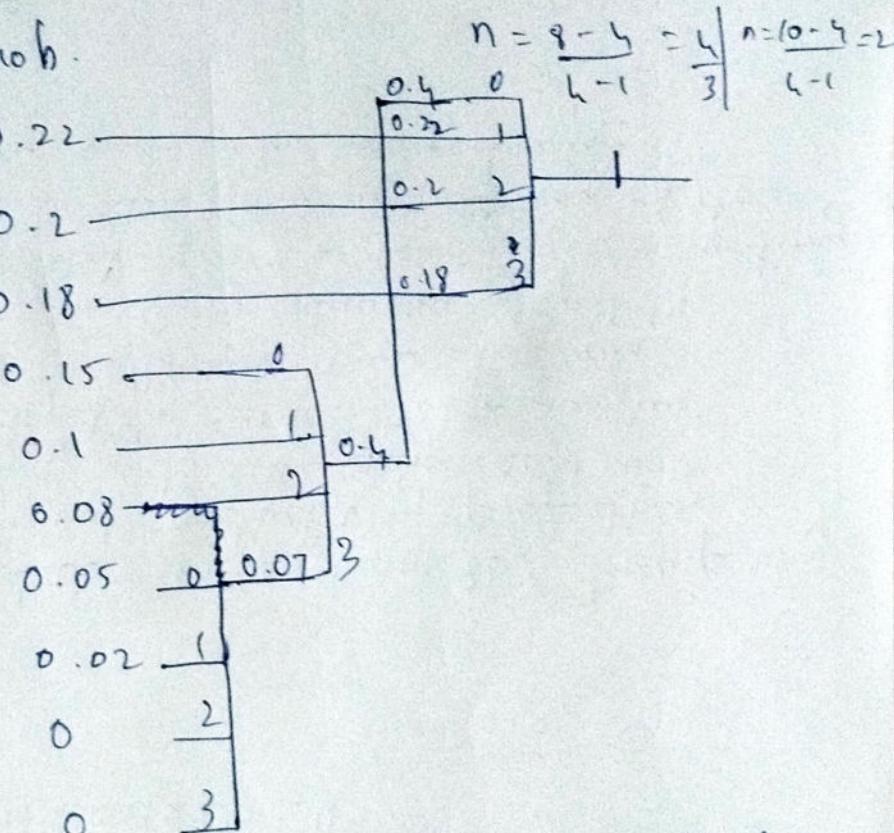
0.02

D₁

0

D₂

0



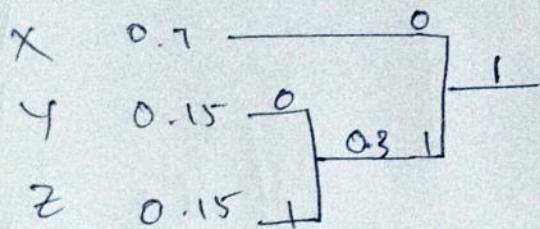
Sym	Prob	Code word	length	Self info.
A	0.22	1	1	0.48
B	0.2	2	1	0.464
C	0.18	3	1	0.445
D	0.15	00	2	0.41
E	0.1	01	2	0.332
F	0.08	02	2	0.291
G	0.05	030	3	0.216
H	0.02	031	3	0.113

$$H(x) = 0.48 + 0.464 + 0.445 + 0.41 + 0.332 + 0.291 + 0.216 + 0.113 \\ = 2.751.$$

$$\bar{R} = 0.22 + 0.2 + 0.18 + 0.3 + 0.2 + 0.16 + 0.15 + 0.06 \\ = 1.47.$$

$$N = \frac{H(x)}{\bar{R}} = 1.871 = 187.1\%.$$

6Q.



Sym	Prob	Codeword	length	$H(n)$
X	0.7	0	1	0.36
Y	0.15	10	2	0.41
Z	0.15	11	2	0.41

$$H(n) = 0.36 + 0.41 + 0.41 \\ = 1.18.$$

$$\bar{R} = 0.7 + 0.3 + 0.3 \\ = 1.3.$$

$$N = \frac{1.18}{1.3} = 0.9077 = 90.77\%.$$

$$\gamma = 1 - N = 1 - 0.9077 = 0.092$$

Second order extension:

Sym Prob.

XX 0.49

XY 0.105

XZ 0.105

YX ~~0.105~~

ZX 0.105

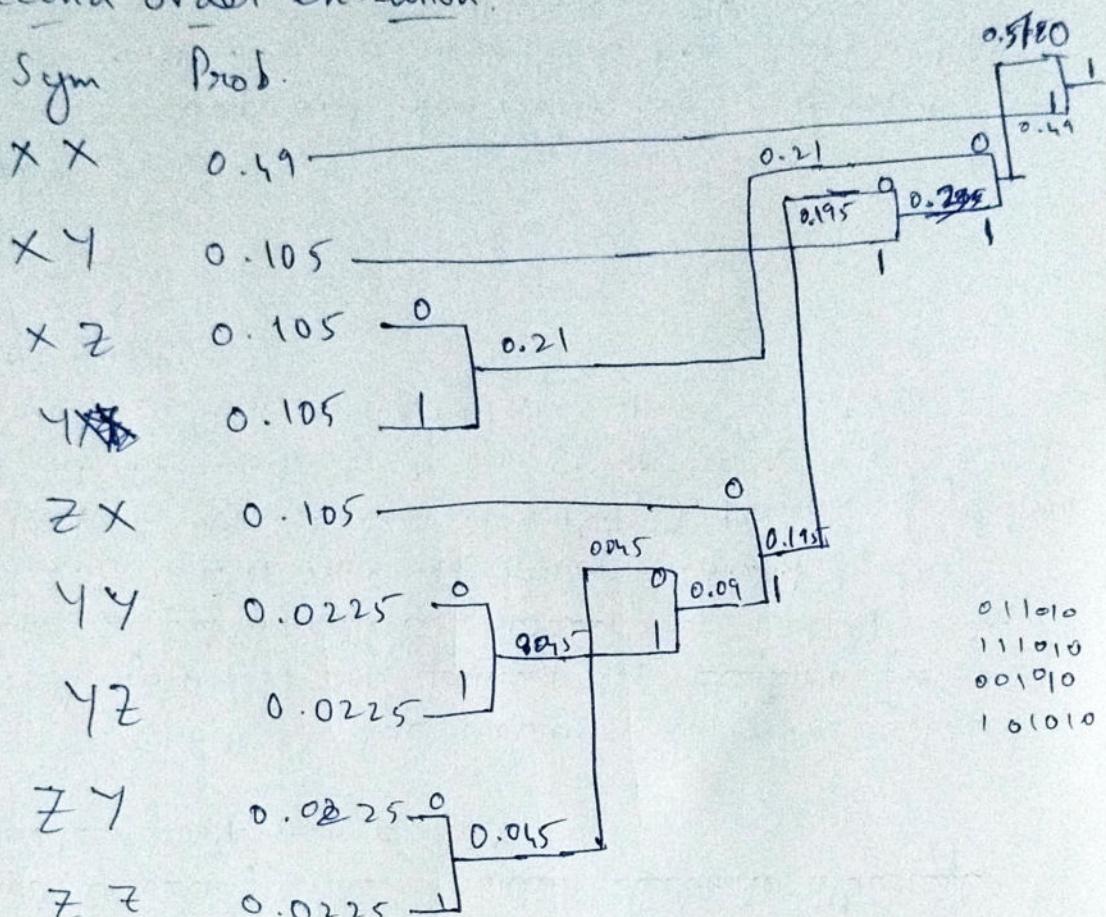
YY 0.0225

YZ

ZY

ZT

Sym	Prob	Codeword	length	Self info.
XX	0.49	1	1	0.5043
XY	0.105	011	3	0.341
XZ	0.105	000	3	0.341
YX	0.105	001	3	0.341
ZX	0.105	0100	4	0.341
YY	0.0225	010110	6	0.123
YZ	0.0225	010111	6	0.123
ZY	0.0225	010100	6	0.123
ZT	0.0225	010101	6	0.123



0.5780

1
0.49

0.21
0.195
0.295

1
0.49

0.21
0.195
0.295

1
0.49

0.21
0.195
0.295

1
0.49

0.21
0.195
0.295

1
0.49

0.21
0.195
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0.195
0.295

1
0.49

0.21
0.195
0.295

1
0.49

0.21
0.195
0.295

1
0.49

$$H(n) = 0.5043 + 4 \times 0.341 + 4 \times 0.123 \\ = 2.36.$$

$$\bar{R} = 0.49 + 3(0.105) + 4(0.105) + 6 \times (4 \times 0.0225) \\ = 2.395$$

$$N = \frac{2.36}{2.395} = 0.9854 = 98.54\%$$

7Q:

characters	freq	frequency	character	freq
------------	------	-----------	-----------	------

H	1		'	1
O	10		B	1
P	3		t	4
e	16		b	2
,	20			
f	3			
u	6			
d	4			
s	9			
d	3			
n	5			
a	4			
m	2			
,	3			
g	3			
i	7			
n	8			
w	2			
h	4			
v	3			

① ① ① ② ② ② ③ ③ ③ ③ ③ ④ ④ ④ ⑤ ⑥ ⑦
 B ! H b w m v g ; d f p t h a l r u i

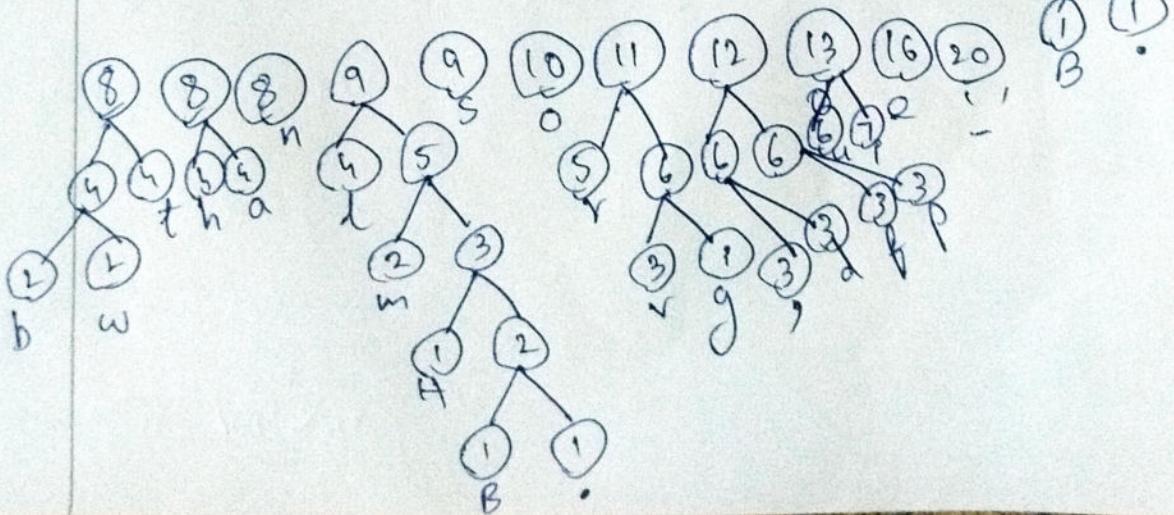
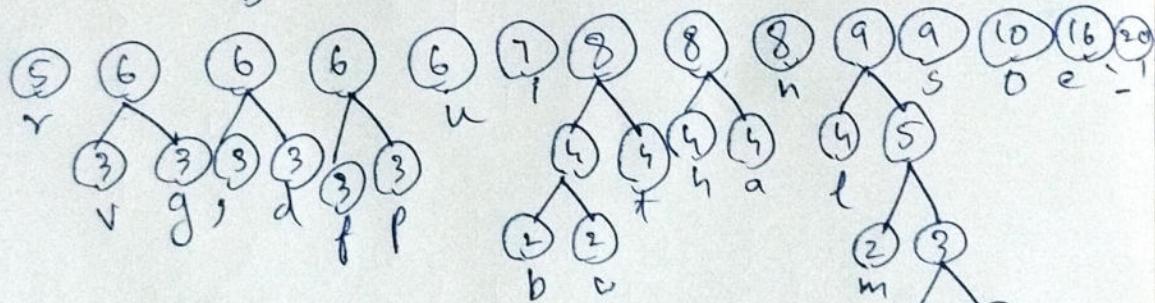
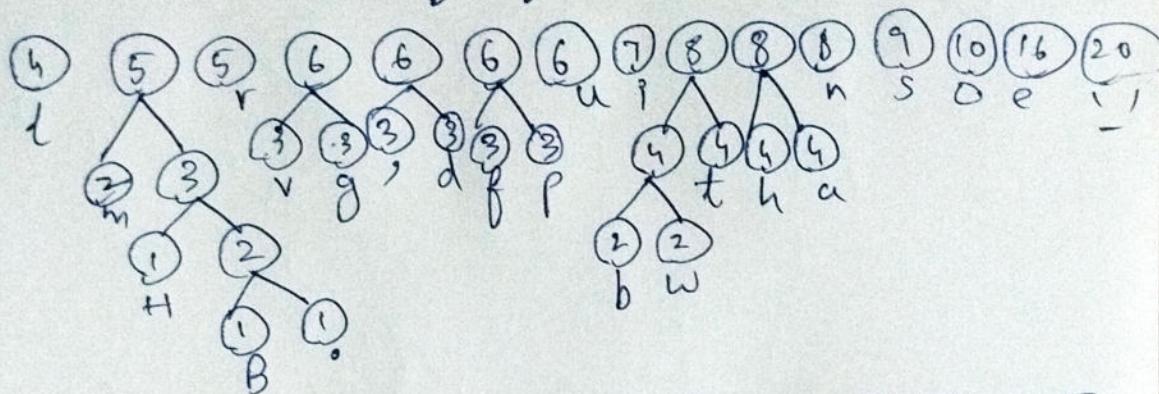
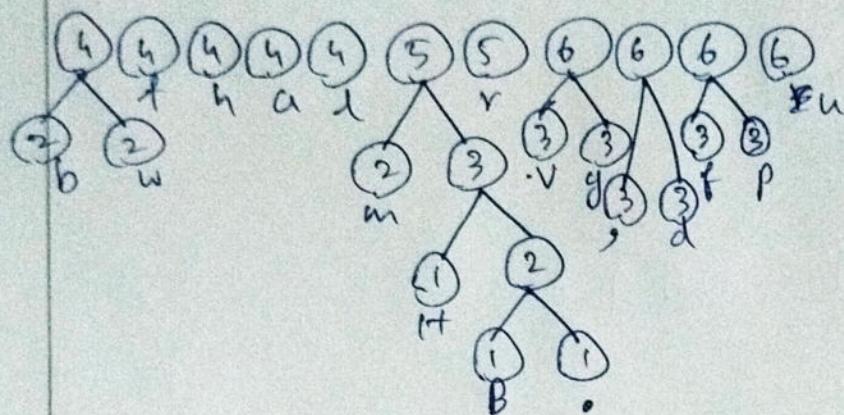
⑧ ⑨ ⑩ ⑪ ⑫ ⑬
 n s o e -

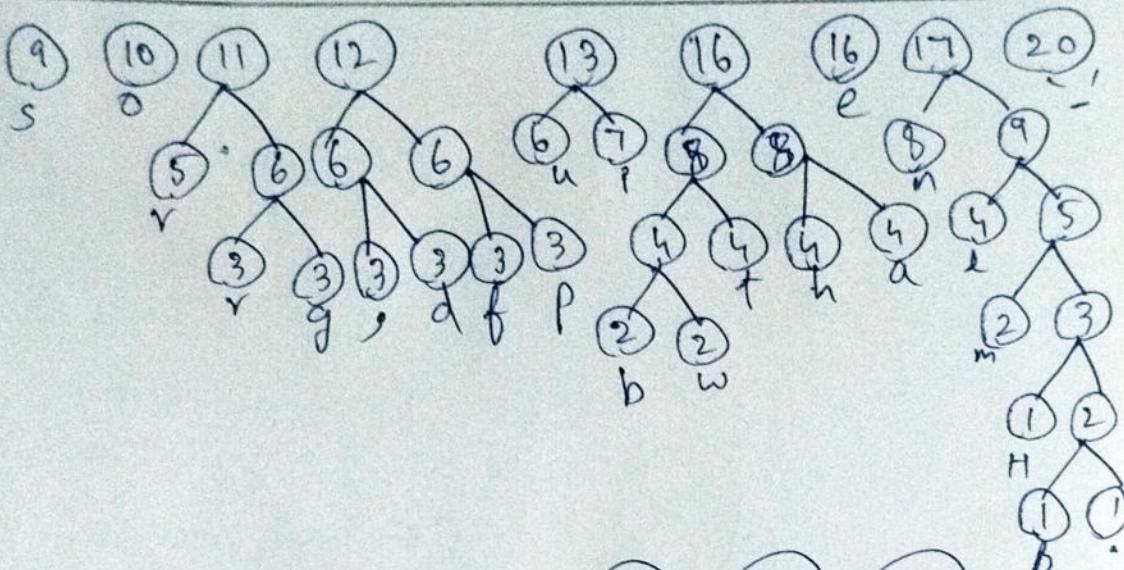
①
 H
 B
 ② ② ② ③ ③ ③ ③ ③ ③ ④ ④ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩
 w m v g ; d f p t h a l r u i n s o
 ⑬ ⑭

② ② ② ③ ③ ③ ③ ③ ③ ④ ④ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩
 b w m v g ; d f p t h a l r u i n s o
 " " "
 ①
 H
 B
 ② ② ② ③ ③ ③ ③ ③ ③ ④ ④ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩
 " " "

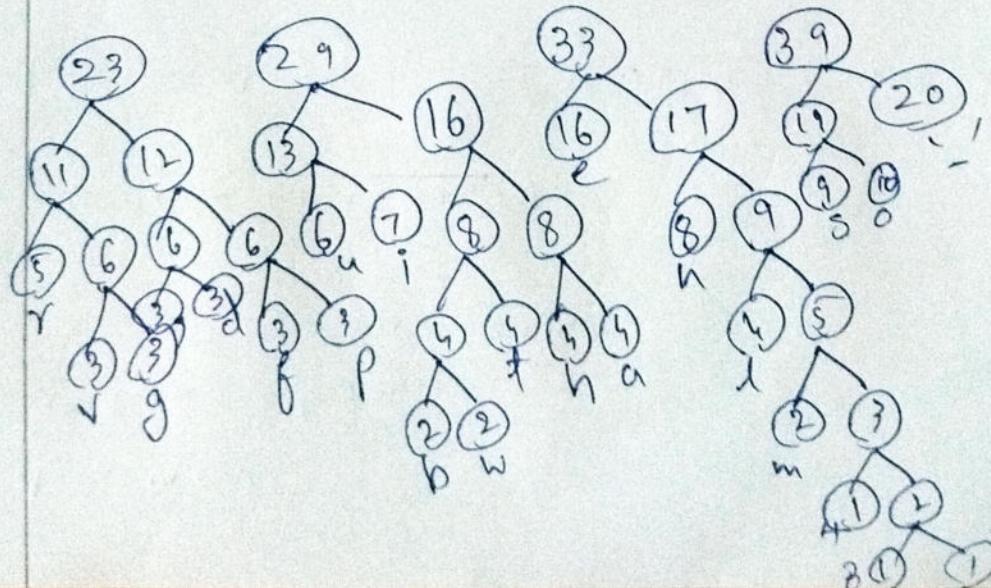
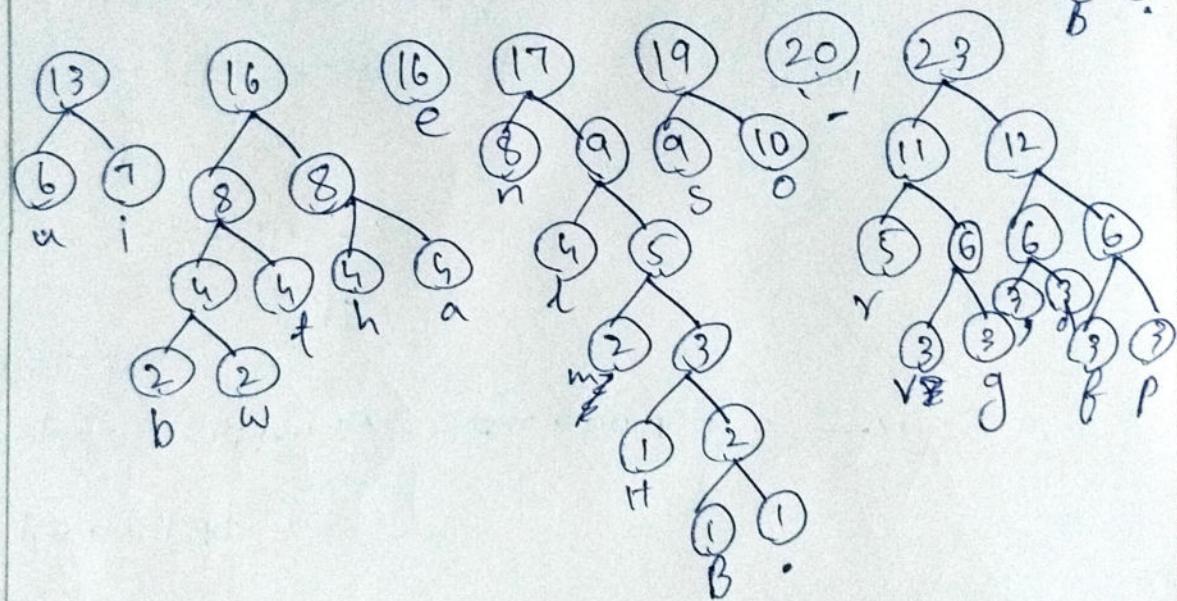
② ③ ③ ③ ③ ③ ③ ③ ③ ④ ④ ④ ④ ④ ④ ④ ④ ④
 m v g ; d f p t h a l r u i n s o
 ①
 H
 B
 ② ② ② ③ ③ ③ ③ ③ ③ ④ ④ ④ ④ ④ ④ ④ ④ ④
 " " "

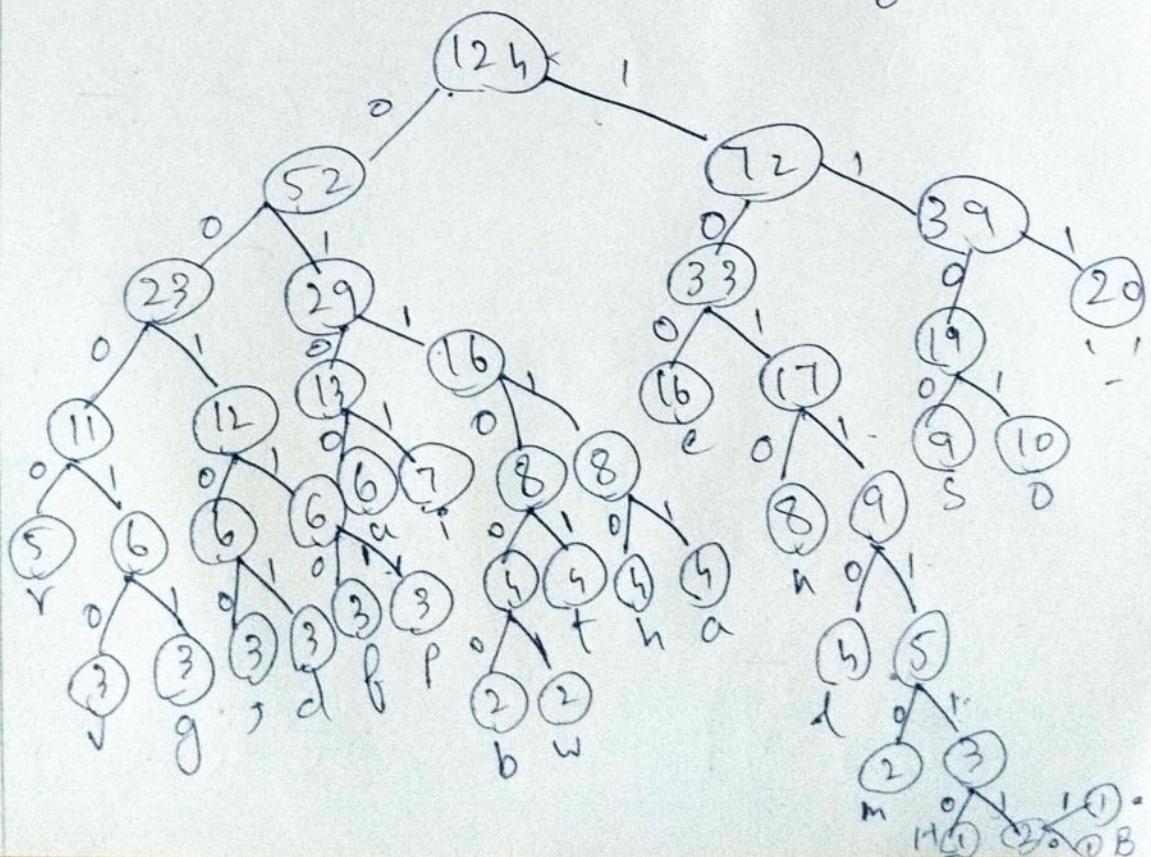
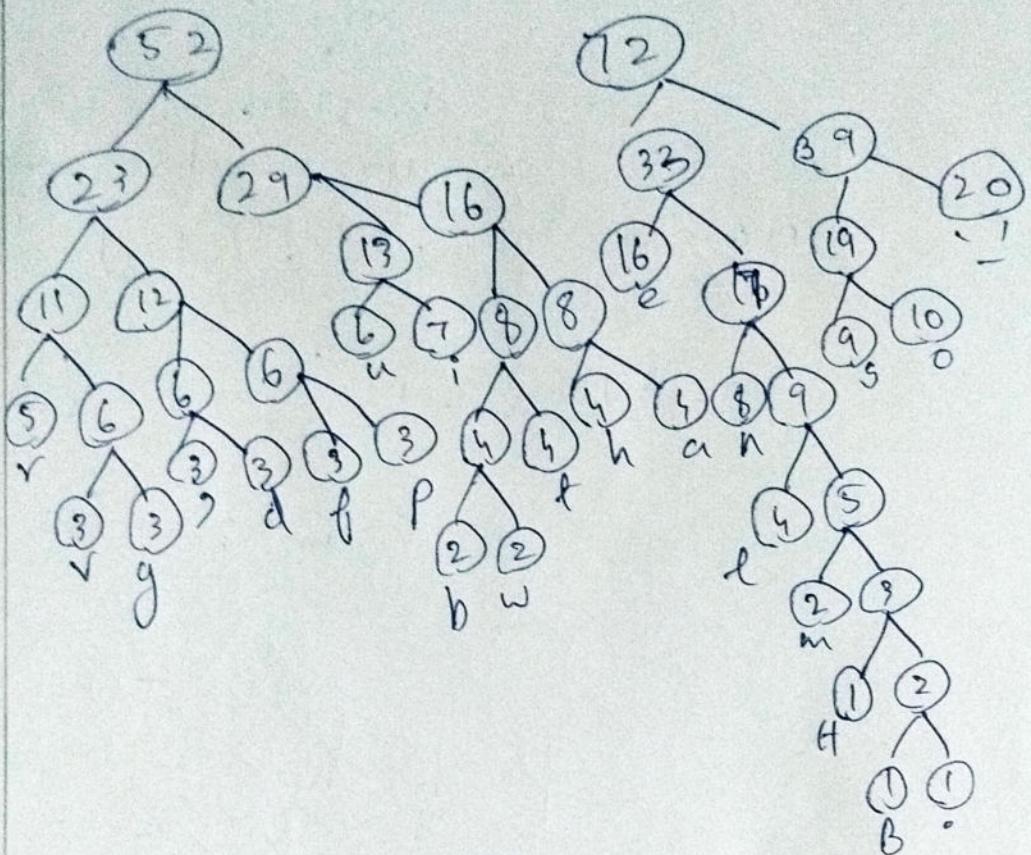
③ ③ ③ ③ ③ ③ ④ ④ ④ ④ ④ ④ ④ ④ ④ ④ ④ ④
 v g ; d f p t h a l r u i n s o
 ② ② ② ③ ③ ③ ③ ③ ③ ④ ④ ④ ④ ④ ④ ④ ④ ④
 " " "





21, 33, 39





Symbol	Code word	length	frequency.
H	1011110	7	1
O	1101	4	10
P	00111	5	3
e	100	3	16
'	111	3	20
-	00110	5	3
f	0100	4	6
u	10110	5	4
l	1100	4	9
s	00101	5	3
d	00000	4	5
a	01111	5	4
m	10111 0	6	2
,	00100	5	3
g	00011	5	3
i	0101	4	7
n	1010	4	8
w	011001	6	2
h	01110	5	4
v	00010	5	3
.	10111111	8	1
B	10111110	8	1
t	01101	5	4
b	011000	6	2

$$\text{Avg Codeword len} = \frac{\sum \text{freq} \times \text{length}}{\sum \text{freq}}$$

$$\sum \text{freq} = 124.$$

$$\begin{aligned}\sum \text{freq} \times \text{length} &= 7 + 40 + 15 + 48 + 60 + 24 + 20 + 36 + 15 \\ &\quad + 20 + 20 + 12 + 15 + 15 + 28 + 32 + 12 + 20 \\ &\quad + 15 + 8 + 8 + 20 + 12 \\ &= 502.\end{aligned}$$

$$\begin{aligned}\text{Avg Codeword length} &= \frac{502}{124} \\ &= 4.048 \text{ bits}.\end{aligned}$$

8Q:

	D	0.35	0.6	0	0.35	0	
	C	0.25		0	0.25	1	
0.15	B	0.15	0.4	1	0.15	0	0.15 0
0.15	A	0.1		1	0.1	0	0.1 1
	F	0.08	0.4	1	0.08	1	0.08 0
	F	0.07		1	0.07	1	0.07 1

	Symbol Prob	Codeword	Length	Selfinfo
D	0.35	0 0	2	0.53
C	0.25	0 1	2	0.5
B	0.15	1 0 0	3	0.41
A	0.1	1 0 1	3	0.332
E	0.08	1 1 0	3	0.2915
F	0.07	1 1 1	3	0.2685

(16)

$$H(n) = 0.53 + 0.5 + 0.41 + 0.332 + 0.2915 + 0.2685 \\ = 2.332$$

$$\bar{R} = 0.7 + 0.5 + 0.45 + 0.3 + 0.24 + 0.21 \\ = 2.4$$

$$N = \frac{2.332}{2.4} = 0.9716 = 97.16\%.$$

$$Y = 1 - N = 0.0283$$

Q:

B	0.5	0				
A	0.25	1	0.25	0		
C	0.125	1	0.125	0	0.125	0
D	0.125	1	0.125	0	0.125	1

Symbol	Probability	Code word	length	Self info
B	0.5	0	1	0.5
A	0.25	10	2	0.5
C	0.125	100	3	0.375
D	0.125	101	3	0.375

$$H(n) = 0.5 + 0.5 + 0.375 \times 2 \\ = 1.75$$

$$\bar{R} = 0.5 + 0.5 + 0.375 + 0.375 \\ = 1.75$$

$$N = 1 = 100\%.$$

$$Y = 1 - 1 = 0.$$

Shannon Fano Elia Coding.

Symbol	Probability	$F(x)$	$\bar{F}(n)$	Codeword
B	0.5	0.5	0.25	$(0.01)_2$
A	0.25	0.75	0.625	$(0.101)_2$
C	0.125	0.875	0.8125	$(0.1101)_2$
D	0.125	1	0.9375	$(0.1111)_2$

$$l_i = \lceil \log \frac{1}{P_i} \rceil + 1$$

$l_1 = 2$	\therefore	Sym	Codeword
		B	01
$l_2 = 3$		A	101
$l_3 = 4$		C	1101
$l_4 = 4$		D	1111

$$H(n) = 1.75$$

$$\begin{aligned} \bar{R} &= 1 + 0.75 + 0.5 + 0.5 \\ &= 2.75 \end{aligned}$$

$$N = \frac{1.75}{2.75} = 0.6363 = 63.63\%$$

$$Y = 1 - N = 1 - 0.6363 = 0.3636.$$

10Q

Sym	Prob.		
X	0.5	0	
Y	0.3	1	0.3 0
Z	0.2	1	0.2 1

Sym	Prob	Codeword	length	self info.
X	0.5	0	1	0.5
Y	0.3	10	2	0.521
Z	0.2	11	2	0.4644

$$H(n) = 1.485$$

$$\bar{R} = 0.5 + 0.6 + 0.4 \\ = 1.5$$

$$N = \frac{1.485}{1.5} = 0.9902 = 99.02\%.$$

$$V = 1 - 0.9902 = 0.0098.$$

→ Second Order Extension.

Sym	Prob.		
XX	0.25	0	0.25 0
XY	0.15	0	0.15 1 0.15 0
XZ	0.1		
YX	0.15	0	0.15 1 0.15 1
XZ	0.1	1	0.1 0 0.1 0
ZX	0.1	1	0.1 0 0.1 1
YY	0.09	1	0.09 1 0.09 0 0.09 0
YZ	0.06	1	0.06 1 0.06 0 0.06 1
ZY	0.06	1	0.06 1 0.06 1 0.06 0
ZZ	0.06	1	0.06 1 0.06 1 0.06 1

Symbol	Prob	Codeword	length	self info.
XX	0.25	00	2	0.5
XY	0.15	010	3	0.41
ZYX	0.15	011	3	0.41
XZ	0.1	100	3	0.332
ZX	0.1	101	3	0.332
YY	0.09	1100	4	0.2435
YZ	0.06	1101	4	0.2435
ZY	0.06	1110	4	0.2435
ZZ	0.06	1111	4	0.2435

$$H(n) = 3.0265$$

$$\bar{R} = 0.5 + 0.45 + 0.45 + 0.3 + 0.3 + 0.36 + 0.21 \times 3 \\ = 3.08$$

$$n = \frac{3.0265}{3.08}$$

$$= 0.91263 = 91.263\%$$

$$Y = 1 - n = 0.01737.$$

Shannon's 1st encoding Algorithm

WQ.

A	B	C	D	E	F	Symbol
0.4	0.2	0.2	0.1	0.08	0.02	Prob
2	3	3	4	4	6	$l_i = -\log_2 p_i$

$$q_1 = 0. = (0.00)_2$$

$$0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$q_2 = 0.4 = (0.0110)_2$$

$$0.6 \times 2 = 1.2$$

$$q_3 = 0.6 = (0.10011)_2$$

$$0.2 \times 2 = 0.4$$

$$q_4 = 0.8 = (0.1100)_2$$

$$\frac{1}{4} + \frac{1}{8}$$

$$q_5 = 0.9 = (0.11100)_2$$

$$0.6 \times 2 = \frac{6}{8} 1.2$$

$$0.2 \times 2 = 0.4$$

$$q_6 = 0.98 = (0.1111101011)_2$$

$$0.4 \times 2 = 0.8$$

$$0.8 \times 2 = 1.6$$

$$0.6 \times 2 = 1.2$$

$$q_7 = 1$$

$$0.98 \times 2 = 1.96$$

$$0.96 \times 2 = 1.92$$

$$0.92 \times 2 = 1.84$$

$$0.84 \times 2 = 1.68$$

$$0.68 \times 2 = 1.36$$

$$0.36 \times 2 = 0.72$$

$$0.72 \times 2 = 1.44$$

$$0.44 \times 2 = 0.88$$

$$0.88 \times 2 = 1.76$$

$$0.76 \times 2 =$$

Sym	Prob	Codeword	Self info.
A	0.4	00	0.529
B	0.2	011	0.464
C	0.2	0100	0.464
D	0.1	1100	0.332
E	0.08	1110	0.291
F	0.02	111110	0.113

$$H(x) = 0.529 + 0.464 + 0.464 + 0.332 + 0.291 + 0.113 \\ = 2.193$$

$$R = 0.8 + 0.6 + 0.6 + 0.4 + 0.32 + 0.12$$

$$= 2.84$$

$$n = \frac{2.193}{2.84} = 0.772 = 77.2 \%$$

Shannon Fano Coding:

Prob Sym	Prob.				
A	0.4	0	0.4	0	
B	0.2	0	0.2	1	
C	0.2	1	0.2	0	
D	0.1	1	0.1	1	0.1 0
E	0.08	1	0.08	1	0.08 1 0.08 0
F	0.02	1	0.02	1	0.02 1 0.02 1

Sym	Prob	Codeword	length	H(2)
A	0.4	00	2	0.529
B	0.2	01	2	0.464
C	0.2	10	2	0.464
D	0.1	110	3	0.332
E	0.08	1110	4	0.291
F	0.02	1111	4	0.113

$$H(C) = 2.193.$$

$$\bar{R} = 0.8 + 0.6 + 0.4 + 0.3 + 0.32 + 0.08 \\ = 2.3.$$

$$N = \frac{2.193}{2.3} = 0.953 = 95.3\%.$$

Shannon Fano Elias Coding.

Sym	Prob	$F(n)$	$\bar{F}(n)$	Codeword.	$l_i = \lceil \log_2 p_i \rceil + 1$
A	0.3	0.4	$0.2 = (0.01)_2$	001	3
B	0.2	0.6	$0.5 = (0.1)_2$	1000	4
C	0.2	0.8	$0.7 = (0.1010)_2$	1011	4
D	0.1	0.9	$0.85 = (0.11010)_2$	11010	5
E	0.08	0.98	$0.94 = (0.11110)_2$	11110	5
F	0.02	1	$0.99 = (0.111110)_2$	1111101	7

$$H(n) = 2.193$$

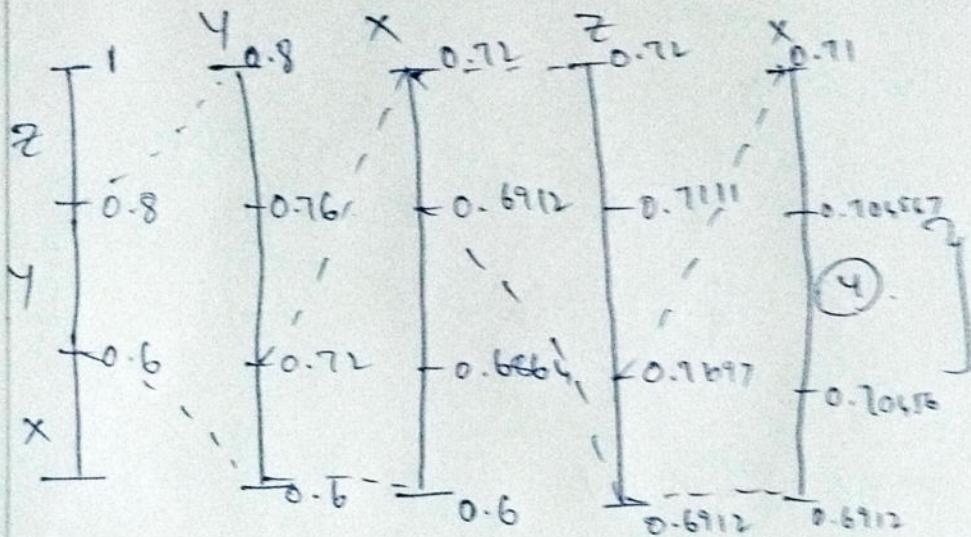
$$\begin{aligned} \bar{R} &= 0.1 \cdot 2 + 0.8 + 0.8 + 0.5 + 0.4 + 0.1 \cdot 4 \\ &= 3.84. \end{aligned}$$

$$N = \frac{2.193}{3.84} = 0.571 = 57.1\%.$$

∴ The efficiency of Shannon Fano coding is found to be the highest amongst the other coding algorithm. Hence, Shannon Fano coding algorithm is best. The average codeword length is also found to be lowest.

<u>12Q</u>	Sym	Prob.	CDF Menage = $Y \times Z \times Y$
X		0.6	0.6
Y		0.2	0.8
Z		0.2	1

$$UL = LL + D \times p(d), \quad D = UL - LL.$$



$$D = 0.8 - 0.6 = 0.2.$$

$$UL = LL + D \times p(d).$$

$$p(X) = 0.6 + 0.2 \times 0.6 \\ = 0.72$$

$$p(Y) = 0.6 + 0.2 \times 0.8 \\ = 0.76$$

$$D = 0.72 - 0.6 = 0.12$$

$$p(Z) = 0.6 + 0.12 \times 0.72 \\ = 0.6864.$$

$$p(Y) = 0.6 + 0.12 \times 0.76 \\ = 0.6912.$$

$$D = 0.72 - 0.6912$$

$$= 0.0288$$

$$p(X) = 0.6912 + 0.0288 \times 0.6864$$

$$= 0.71$$

$$p(Y) = 0.7111$$

$$D = 0.71 - 0.6912 \\ = 0.0188$$

$$p(Z) =$$

∴ Arithmetic code word

$$= \underbrace{0.704567}_{2} + 0.70456$$

$$= \boxed{0.70456}$$

13Q.

From the 12 Q arithmetic coding.

the message for the codeword = 0.70466 is
"Y X Z X." → message.