Module-6
Problems on Channel Coding
-Linear Block Codes

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Example 1: Problem

Problem:

Obtain the encoded information for the given message and generator matrix:

- $\mathbf{u} = [1\ 0\ 1\ 1]$
- $\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

Example 1: Solution

The code vector **v** for the given data vector can be obtained by:

$$\mathbf{v} = \mathbf{u} \cdot \mathbf{G} = [1\ 0\ 1\ 1] \cdot \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [0\ 1\ 0\ 1\ 0\ 1\ 1]$$

Problem:

Obtain all possible code vectors for a (7, 4) LBC in its systematic form for the generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Example 2: Solution

First, reduce the generator matrix to its systematic form using elementary row operations:

$$r_1 \rightarrow r_1 + r_2$$

$$r_3 \rightarrow r_3 + r_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Codewords can be obtained by multiplying the respective message vector with $\mathbf{G}$.

Example 3: Problem

Problem:

Find matrix H for the given generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Example 3: Solution

From the given generator matrix $\mathbf{G}$, we write H in the form:

$$\mathbf{H} = \left[\mathbf{P}^T : I_{n-k}\right]$$

For $\mathbf{G} = [P|I_k]$, where P is the parity matrix, we can write the parity check matrix as:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Problem:

Given a systematic (8, 4) LBC whose parity check equations are:

$$v_4 = u_1 + u_2 + u_3, \quad v_5 = u_0 + u_1 + u_2, \quad v_6 = u_0 + u_1 + u_3, \quad v_7 = u_0 + u_2 + u_3$$

Write the generator and parity check matrices and draw the encoder diagram.

Example 4: Solution

From the given parity equations, we write the parity matrix as:

$$P = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Hence, the generator matrix is:

$$G = [I_k : P] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Example 5.9: Problem

Problem:

Given the generator matrix for an LBC:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find n and k values, write G in its systematic form, and find all codewords.

Example 5.9: Solution

As there are 8 columns and 4 rows in G, we have $n = 8$ and $k = 4$. It is a (8, 4) LBC.

Reduce G to the form $[P|I_k]$ using row operations:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Example 5.10: Problem

Problem:

For the following (6, 3) systematic LBC with generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(a) Find all code vectors.

(b) Draw the encoder circuit for the above code.

(c) Find the minimum Hamming weight.

Example 5.10: Solution (Part 1)

(a) Find all code vectors:

Code vectors can be obtained by multiplying the respective message vector **u** with **G**:

$$[v] = [u] \cdot [G]$$

For each possible combination of message bits, the resulting code vectors are:

$$\begin{bmatrix} 000 \\ 011 \\ 101 \\ 110 \end{bmatrix} \quad \text{(Output code vectors)}$$

(b) Draw the encoder circuit:

The encoder circuit is based on the generator matrix. The connections represent the summation of the inputs for each respective bit.

encoder_diagram.png

Figure 1: Encoder circuit for the given (6,3) LBC

Example 5.10: Solution (Part 3)

(c) Minimum Hamming weight:

From Table 5.1, the minimum Hamming weight is:

$$d_{\min} = 3$$

The Hamming weight is defined as the number of 1's in the codeword with the least number of 1's.

Example 5.6: Problem

Problem:

Given a systematic (8, 4) LBC whose parity check equations are:

$$v_4 = u_1 + u_2 + u_3, \quad v_5 = u_0 + u_1 + u_2, \quad v_6 = u_0 + u_1 + u_3, \quad v_7 = u_0 + u_2 + u_3$$

(a) Write the generator matrix and parity check matrices.

(b) Draw the encoder diagram.

Example 5.6: Solution (Part 1)

(a) Write the generator and parity check matrices:

From the given parity check equations, we can construct the parity matrix P:

$$P = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

The generator matrix can be written as:

$$G = [I_k : P] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(b) Draw the encoder diagram:

Using the generator matrix, we can draw the encoder circuit, representing how the parity bits v_4, v_5, v_6, v_7 are generated from the message bits u_0, u_1, u_2, u_3.

encoder_diagram_example_5_6.png

Figure 2: Encoder circuit for the (8,4) LBC

Example 5.9: Problem

Problem:

Given the generator matrix for an LBC:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find n and k values.

(b) Write G in its systematic form.

(c) Find all codewords.

Example 5.9: Solution (Part 1)

(a) Find n and k values:

The number of columns is 6 and the number of rows is 4. Therefore, $n = 6$ and $k = 3$, which defines the code as a (6, 3) LBC.

Example 5.9: Solution (Part 2)

(b) Write G in its systematic form:

Perform row operations to convert G into the systematic form $[I_k : P]$:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Example 5.9: Solution (Part 3)

(c) Find all codewords:

The codewords can be obtained by multiplying the message vectors by the systematic generator matrix:

$$\mathbf{v} = \mathbf{u} \cdot \mathbf{G}$$

For all possible message vectors, the corresponding codewords are:

$$\begin{bmatrix} 000 \\ 011 \\ 101 \\ 110 \end{bmatrix} \quad \text{(Output codewords)}$$