ITC

(1)

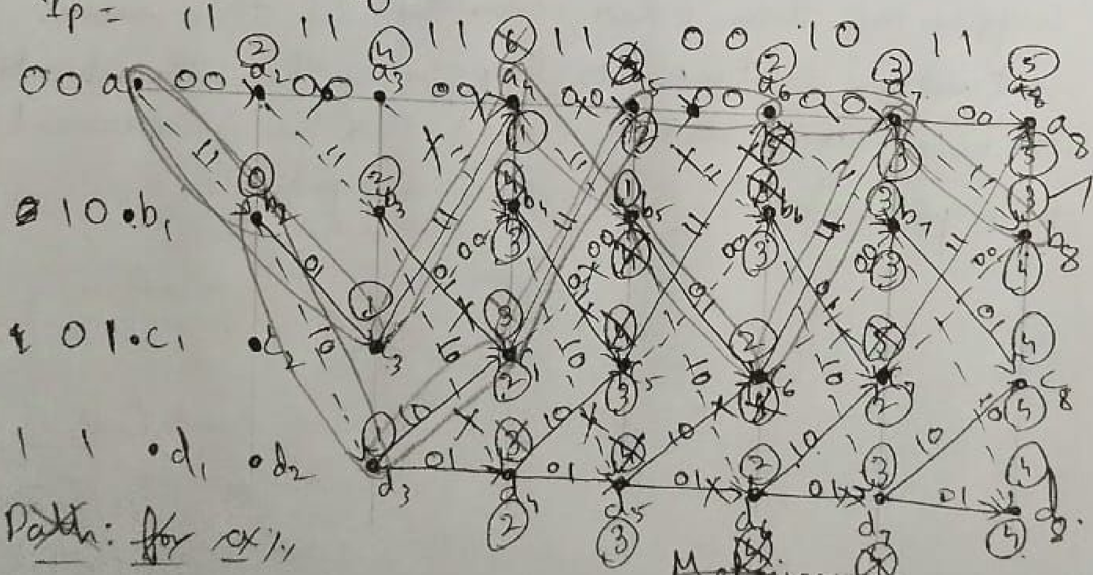
PROBLEM SHEET-4

1Q:

$C_1 = i \oplus m_1$, $C_2 = i \oplus m_0 \oplus m_1$

Input i	Current State m_0	m_1	Output c_1	c_2	Next State m_0	m_1
0	0	0	0	0	0	0
1	0	0	1	1	1	0
0	0	1	1	1	0	0
1	0	1	0	0	1	0
0	1	0	0	1	0	1
1	1	0	1	0	1	1
0	1	1	1	0	0	1
1	1	1	0	1	1	1

~~Verti~~ Viterbi Diagram & Path Metrics:



Path: ~~for ex 1,1~~

~~$a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7$~~

Metrics: ~~11~~

Path:

11 11 11 11 00 10 11

(1) $a_1 b_2 c_3 a_4 b_5 c_6 a_7 b_8$ - 11-01-11-11-01-11-11

(2) $a_1 b_2 d_3 c_4 a_5 a_6 a_7 b_8$ - 11-10-10-11-00-00-11

Optimal Path can be 1 or 2.

Decoding bits based on path 1:

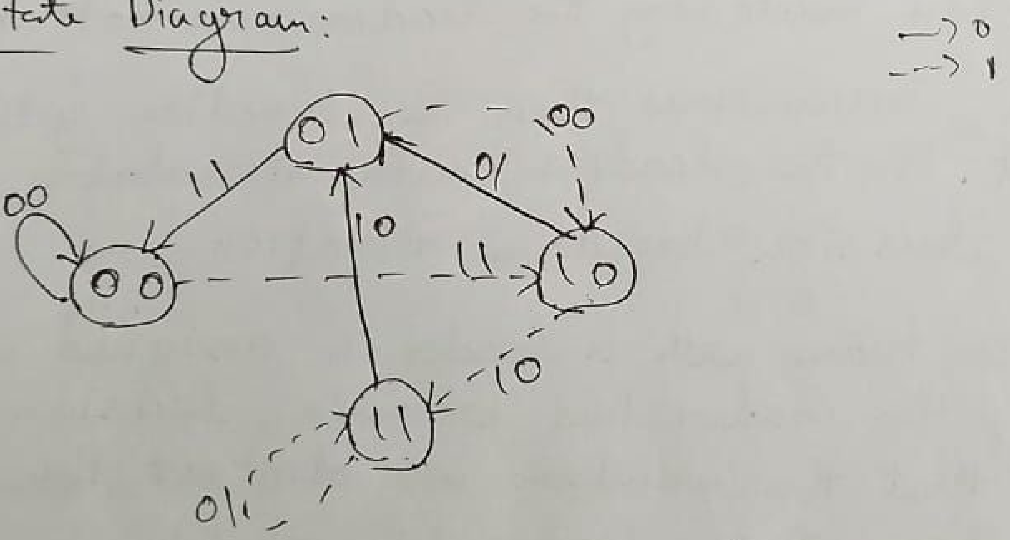
1 0 0 1 0 0 1

Decoding bits based on path 2:

1 1 0 0 0 0 1.

2Q. Taking the previous question: with (2, 1, 2).

(i) State Diagram:



(ii) $u = \underset{u_1}{1}\underset{u_2}{1}\underset{u_3}{0}\underset{u_4}{1}\underset{u_5}{1}\underset{u_6}{0}$.

~~Encoding~~:

~~11 10 01 00 10 01~~

Initial State & Ip u_i	Encoding
$u_1 = 1$ (00) --11--> (10)	11
$u_2 = 1$ (10) --10--> (11)	10
$u_3 = 0$ (11) —10—> (01)	10.
$u_4 = 1$ (01) --00--> (10)	00.

$u_5 = 1$ $(10) \xrightarrow{10} (11)$ 10.

$u_6 = 0.$ $(11) \xrightarrow{10} (01)$ 10.

(iii)

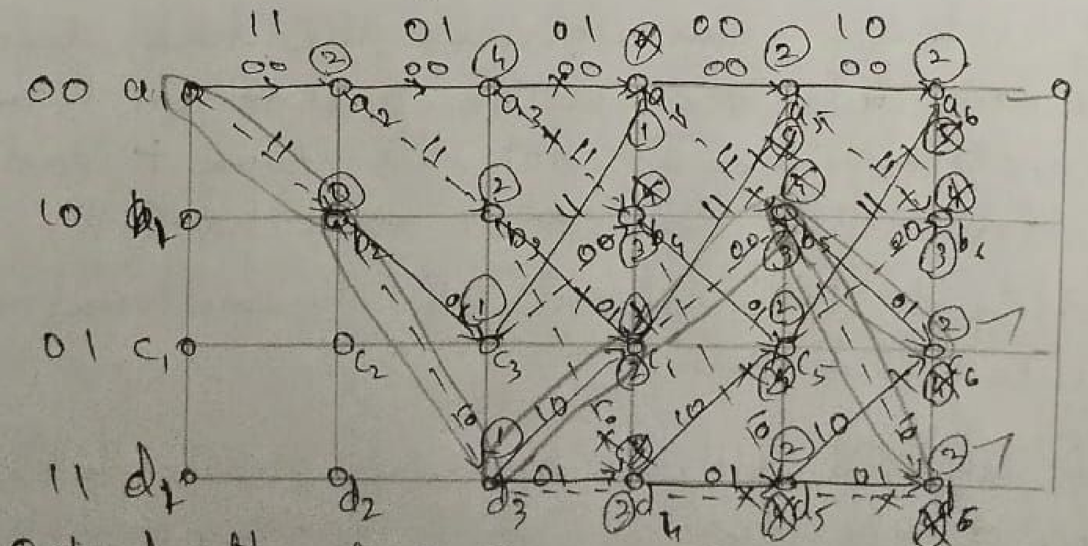
Codeword of the ip u = 11 10 10 00 10 10.

3Q: $C_1 = i \oplus m_1$, $C_2 = i \oplus m_0 \oplus m_1$.

Ip (i)	Current State ($m_0 m_1$)	Op ($C_1 C_2$)	Next State ($m_0 m_1$)
0	0 0	0 0	0 0
1	0 0	1 1	1 0
0	0 1	1 1	0 0
1	0 1	0 0	1 0
0	1 0	0 1	0 1
1	1 0	1 0	1 1
0	1 1	1 0	0 1
1	1 1	0 1	1 1

(ii)

~~Veet~~ Viterbi Diagram & Path Metrics:



Optimal path: least metrics = 2. Ip = 11-01-01-00-10 ...

(1.) $a_1\, b_2\, d_3\, c_4\, b_5\, c_6$ · -11-10-10-00-01

(2) $a_1 b_2 d_3 c_4 b_5 d_6$ - 11-10-10-00-10.
(3) $a_1 b_2 c_3 a_4 a_5 a_6$ - 11-01-11-00-00

Decoding based on (1),

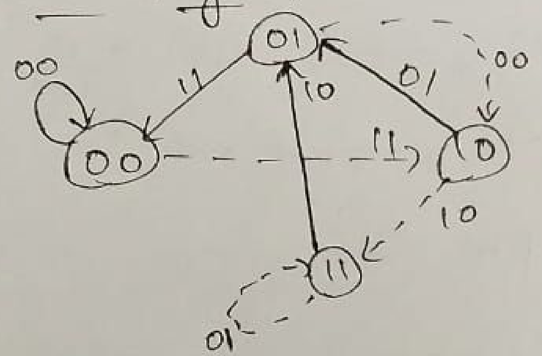
11010

Decoding based on (2),

110110.

(i)

State Diagram:



Encoding 1010. & Let initial state = 00.

$u_1 = 1$ (00) --11--> (10) 11

$u_2 = 0$ (10) ——01——> (01) 01

$u_3 = 1$ (01) --00--> (10) 00

$u_4 = 0$ (10) ——01——> (01) 01

∴ codeword = 11010001

(iii) & (iv)

Based on Path (1),

r = 11 01 01 00 10. } There are errors
(1) = 11 10 10 00 01 } at 5 bit position.

Based on Path (2),

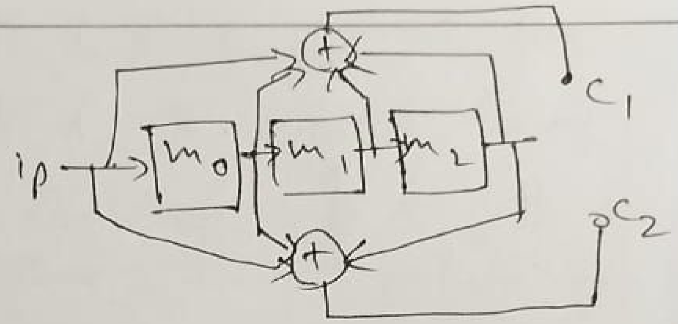
r = 11 01 01 00 10 } There are errors
(2) : 11 10 10 00 10 } at 4 bit position

Based on path 3

$$\left.\begin{aligned} r &= 11\ 01\ \underline{0}1\ 00\ \underline{1}0. \\ 3 &= 11\ 01\ \underline{1}1\ 00\ \underline{0}0 \end{aligned}\right\}$$ 2 places have to corrected.

$\therefore$ decoded message = 1 0 0 0 0.

Q. (2, 1, 3)

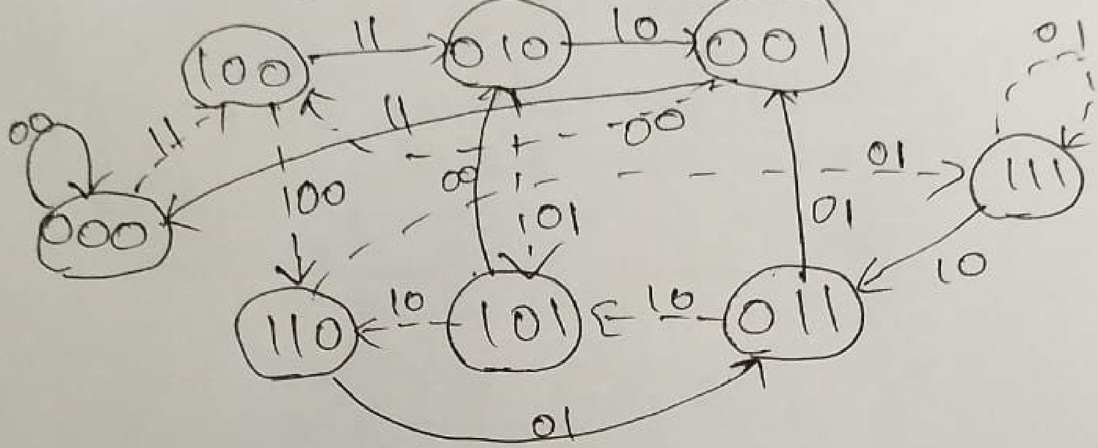


$C_1 = i \oplus m_0 \oplus m_1 \oplus m_2$.

$C_2 = i \oplus m_0 \oplus m_2$.

Input (i) i	Current State ($m_0\, m_1\, m_2$) m_0 m_1 m_2	Output ($c_1 c_2$) c_1 c_2	Next State ($m_0\, m_1\, m_2$)
0	0 0 0	0 0	0 0 0
1	0 0 0	1 1	1 0 0
0	0 0 1	1 1	0 0 0
1	0 0 1	0 0	1 0 0
0	0 1 0	1 0	0 0 1
1	0 1 0	0 1	1 0 1
0	0 1 1	0 1	0 0 1
1	0 1 1	1 0	1 0 1
0	1 0 0	1 1	0 1 0
1	1 0 0	0 0	1 1 0
0	1 0 1	0 0	0 1 0
1	1 0 1	1 1	1 1 0
0	1 1 0	0 1	0 1 1
1	1 1 0	1 0	1 1 1
0	1 1 1	1 0	0 1 1
1	1 1 1	0 1	1 1 1

(ii) State Diagram:

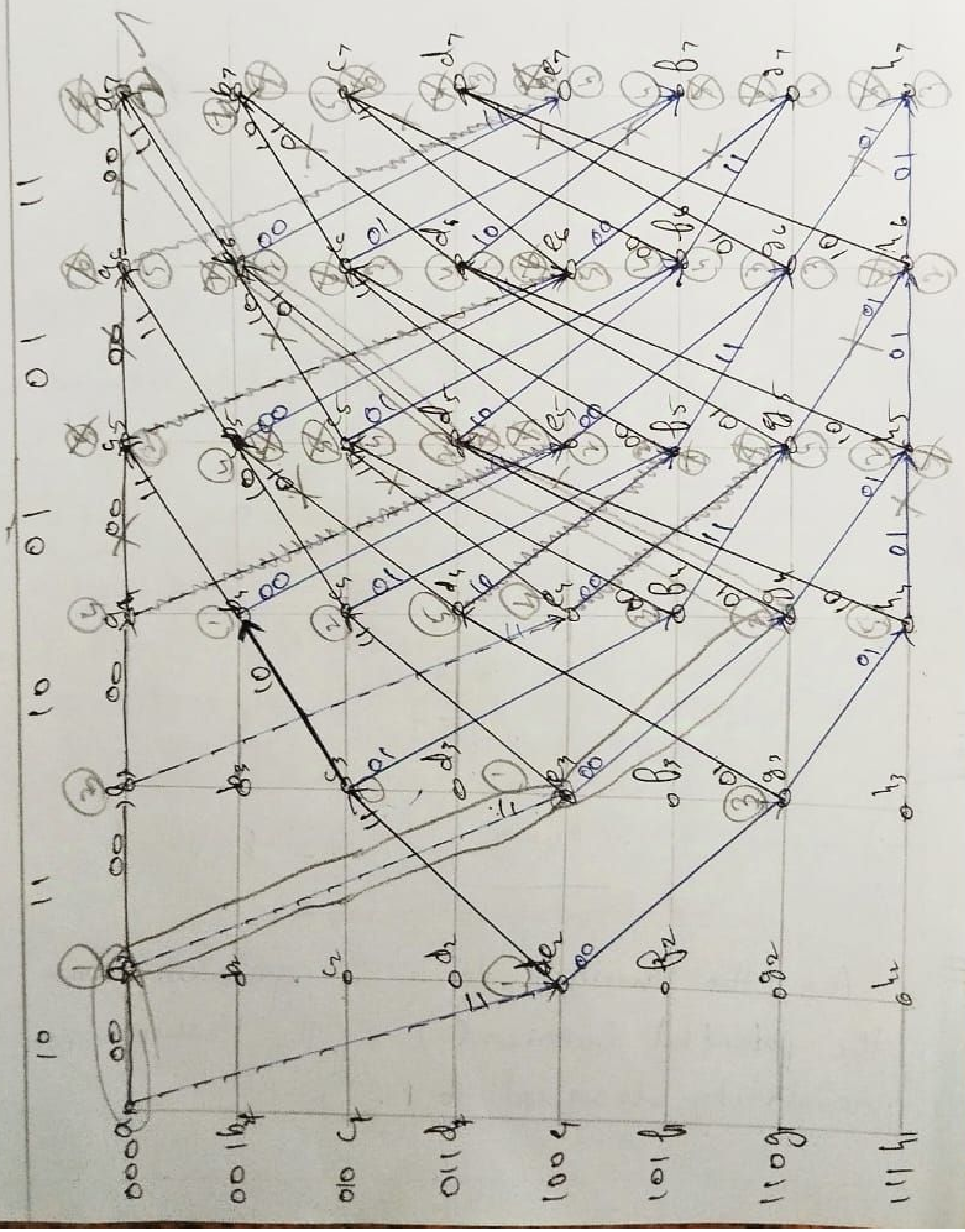


(i) Encoding: u=[1,0,1,1,0,1,0]

Ip	Current State	Op	Next State
1	000	11	100
0	100	11	010
1	010	01	101
1	101	10	110
0	110	01	011
1	011	10	101
0	101	00	010

∴ encoded message = 11 1101100111000.

(iii) Viterbi Alogrithm



Path with Lowest Metrics 2.

$a_1 a_2 e_3 g_4 d_5 b_6 a_7 \rightarrow$ 00 11 00 01 01 11
r = 10 11 10 01 01 11

$\therefore$ Original codeword = 00 11 00 01 01 11
Original Input = 0 1 1 0 0 0.

(IV)

One bit error can be found based on the ~~vert~~ viterbi alogrithm ~~at~~ occuring at any of the places.
Just by flipping the bit of the position which has error can be corrected.
According to the choose path the error correcting & detecting capability depends.

SQ. $C_1 = D_1 \oplus D_2 \oplus D_3$, $C_2 = D_1 \oplus D_3$

Input (i)	Current State D_1	D_2	D_3	Output C_1	C_2	Next State D_1	D_2	D_3
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0
0	0	0	1	1	1	0	0	0
1	0	0	1	1	1	1	0	0
0	0	1	0	1	0	0	0	1
1	0	1	0	1	0	1	0	1
0	0	1	1	0	1	0	0	1
1	0	1	1	0	1	1	0	1
0	1	0	0	1	1	0	1	0
1	1	0	0	1	1	1	1	0
0	1	0	1	0	0	0	1	0
1	1	0	1	0	0	1	1	0
0	1	1	0	0	1	0	1	1
1	1	1	0	0	1	1	1	1
0	1	1	1	1	0	0	1	1
1	1	1	1	1	0	1	1	1

6

(i) Encoding: $u = [1,1,0,1,0]$ with initial status 000.

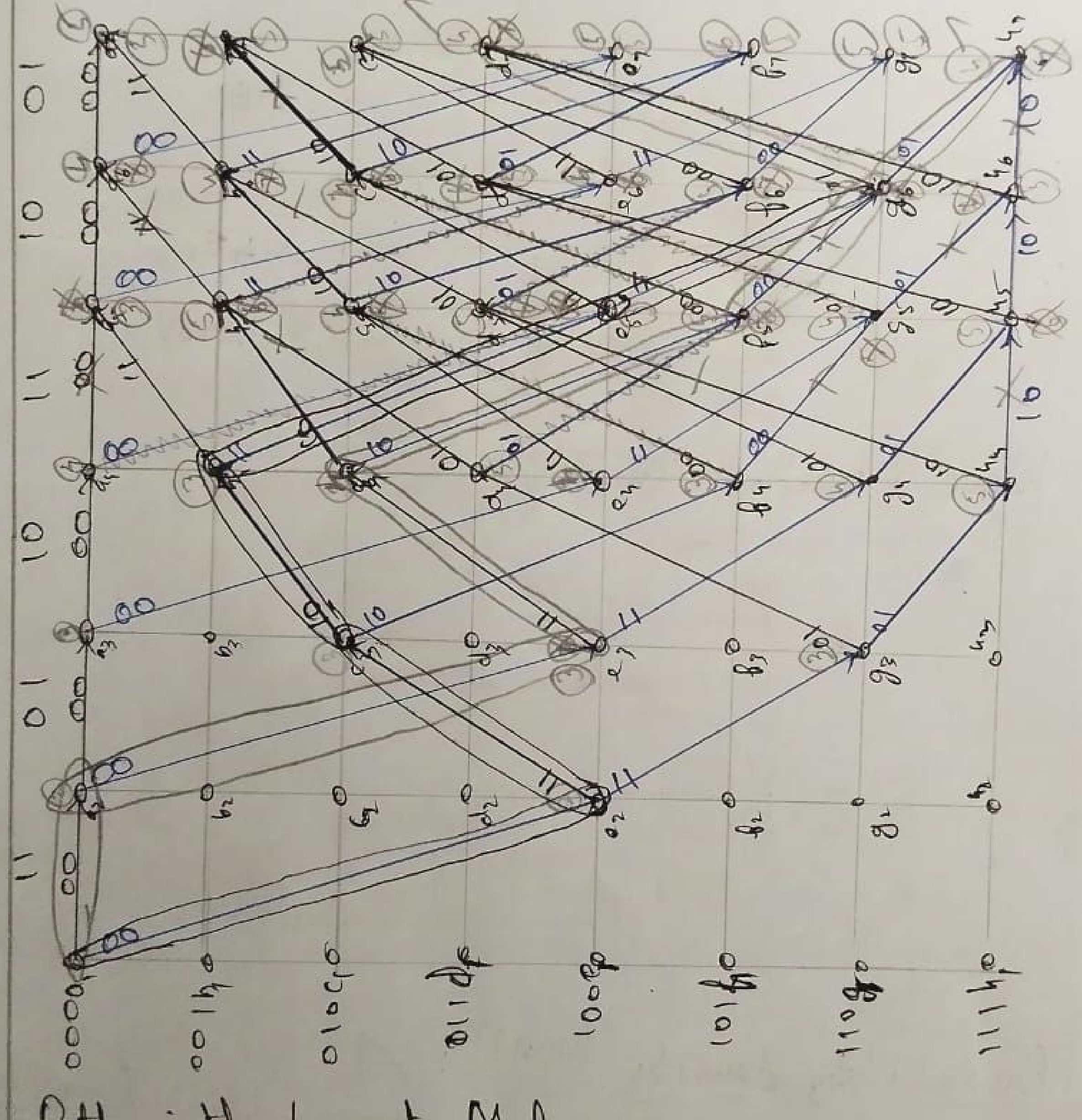
	Current State	Op	Next State
$u_1 = 1$ →	0 0 0	0 0	1 0 0
$u_2 = 1$	1 0 0	1 1	1 1 0
$u_3 = 0$	1 1 0	0 1	0 1 1
$u_4 = 1$	0 1 1	0 1	1 0 1
$u_5 = 0$	1 0 1	0 0	0 1 0

11. 5.2

Encoded message = 0011 0101 00.

0 - →
1 - →

(iii) Viterbi Alogrithm.



Path with Lowest Metrices 4:

1. $a_1 e_2 c_3 b_4 e_5 g_6 h_7$ = 00 11 1011 11 01

r = 11 01 1011 1001

The i/p would be = 100111

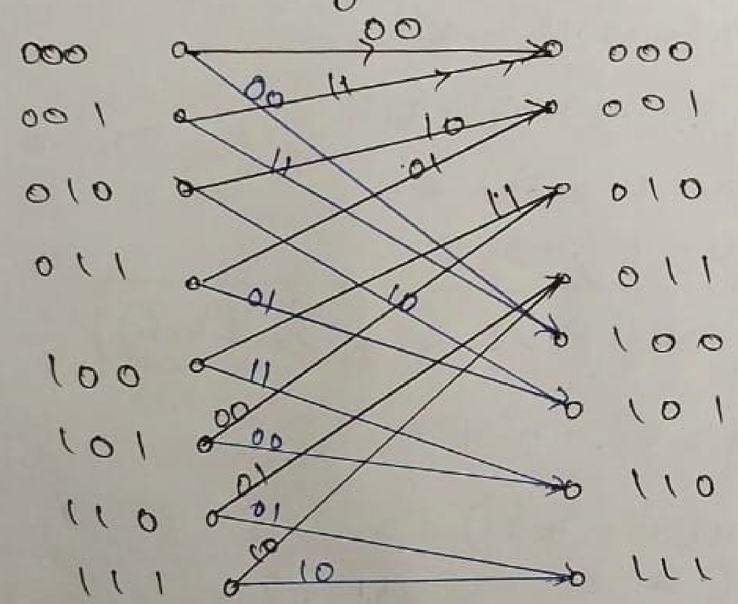
(2) $a_1 e_2 c_3 b_4 e_5 g_6 d_7$ - 00 11 10 11 11 01

Rpr = 11 01 10 11 10 01

Ip = 100110.

(ii). Trellis diagram.

1 - →
0 - →



(iv)
The d_{min} of convolution code is the smallest no. of differing bits b/w any two codewords.
Since the encoder ~~only~~ outputs depends on all the inputs and also the dependence is based on mod 2 operation. the minimum hamming distance will be less when compared to other encoders.
The error correcting capability depends the optimal path metric chosen.

6Q (i) $c_1 = i_1 \oplus m_0 \oplus m_1$

$c_2 = i_1 \oplus i_2 \oplus m_1$

$c_3 = i_1 \oplus m_0$

Input i_1	i_2	Current State m_0	m_1	Next State m_0	m_1	Output c_1	c_2	c_3
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	1	0
0	0	1	0	0	1	1	0	1
0	0	1	1	0	1	0	1	1
0	1	0	0	1	0	0	1	0
0	1	0	1	1	0	1	0	0
0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	0	1
1	0	0	0	0	0	1	1	1
1	0	0	1	0	0	0	0	1
1	0	1	0	0	1	0	1	0
1	0	1	1	0	1	1	0	0
1	1	0	0	1	0	1	0	1
1	1	0	1	1	0	0	0	1
1	1	1	0	1	1	0	0	0
1	1	1	1	1	1	1	1	0

$u = 10, 11, 1 0, 0$.

Code words = 111 011 001 110.

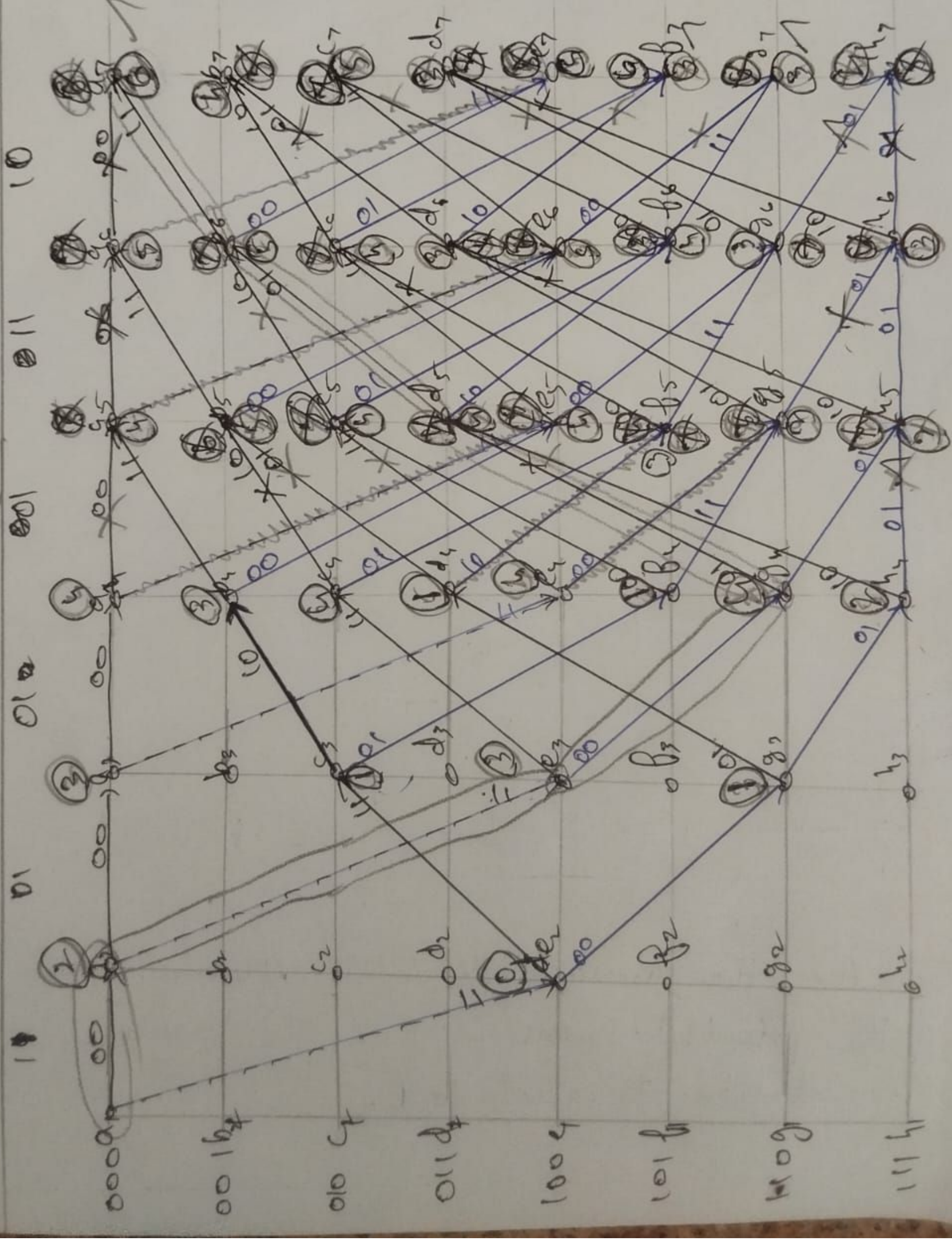
XI ~~8~~
6Q.

(iii) Viterbi Alogrithm

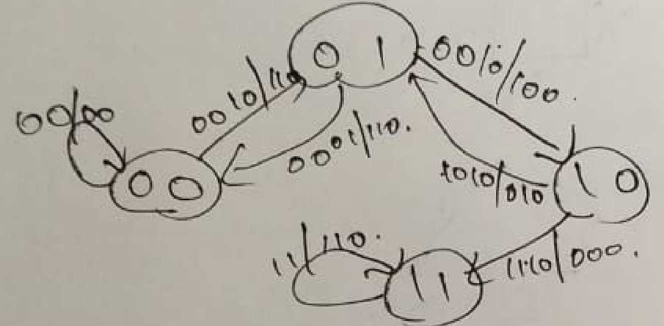
Path → $a_1 a_2 a_3 e_4 c_5 d_6 g_7$ - 00 11 11 01 11 10

Original input sequence I_p = 1 1 01 01 01 11 10

→ 000101



(iii) State Diagram :



(iv) g(x) is degree determines the encoding process & error correcting capability & detection. Error detection is found based on syndrome if the syndrome is zero then there are no errors else there are errors at the bits of the coefficients in remainder.

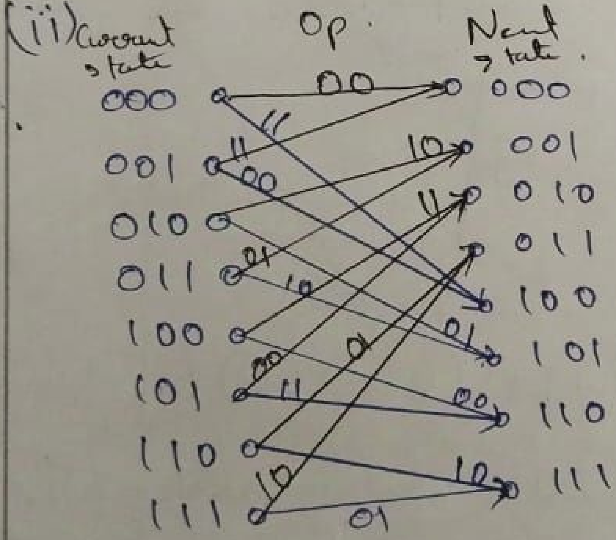
7Q: From 4Q:

(i) $u = [1\,0\,0\,1\,1]$.

Ip	Current State	Op	Next State.
1	000	11	100
0	100	11	010
0	010	10	001
1	001	11	100
1	100	00	110.

Encoded Message = 11 11 10 11 00

(ii)



(iii)

Path with Lowest metrics:

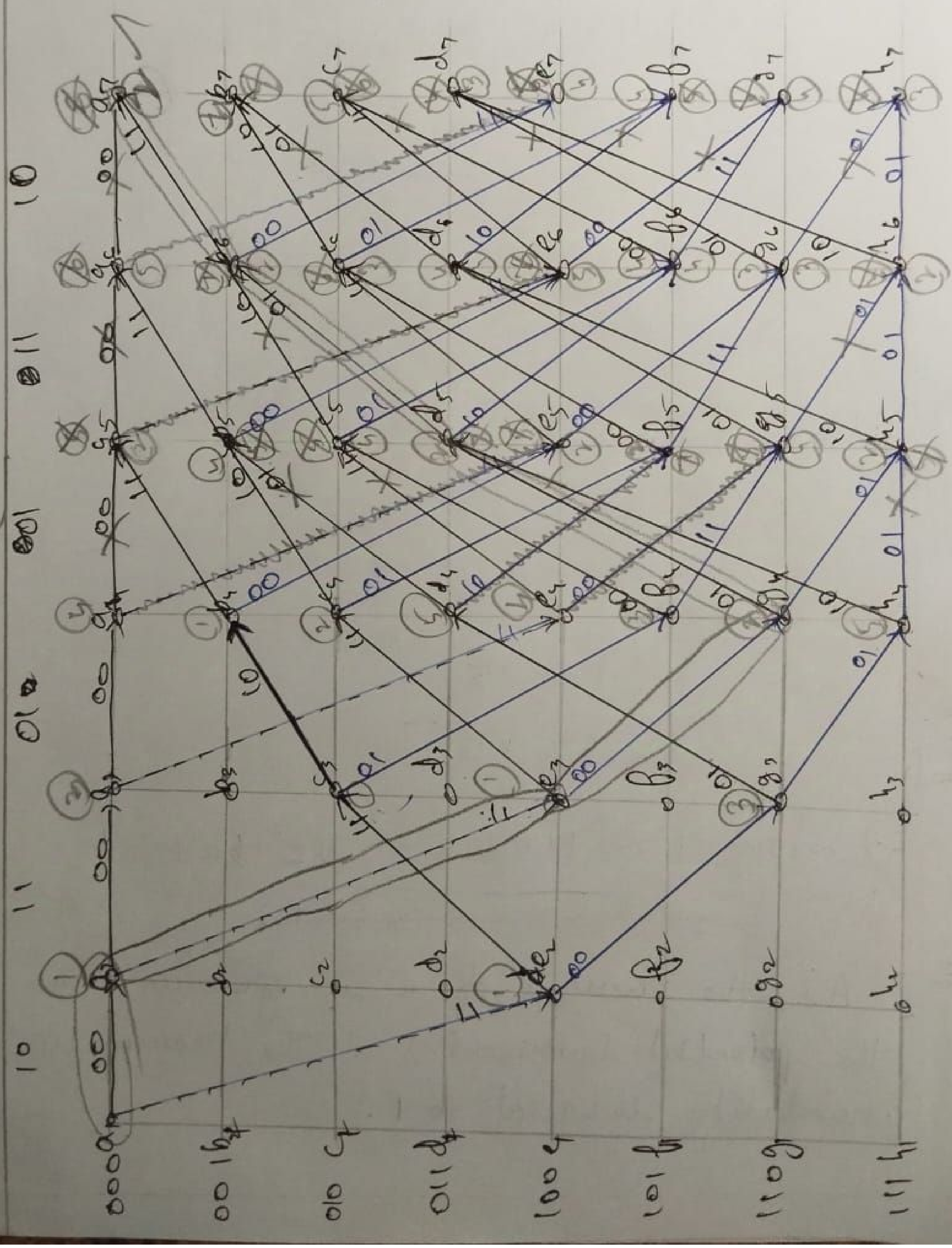
$a_1 a_2 e_3 g_4 d_5 b_6 a_7 \rightarrow$ 00 11 00 01 01 11

Ip $\rightarrow$ 1 0 1 1 0 1 0 1 1 1 1 0

∴ Transmitted Codeword } = 00 11 01 01 10 11

7.1 8

(111) Viterbi Alogrithm



(iv) $d_{min} = 2$

error detectable ~~correctable~~ $= \frac{2}{2} = 1$

error correctable $= \frac{1}{2} = 0$.

8\.1

8Q: $C_1 = D_1 \oplus D_2 \oplus D_3$, $C_2 = D_1 \oplus D_3$.

Ip (i)	Current State $D_1\ D_2\ D_3$	Output $C_1 C_2$	Next State $D_1\ D_2\ D_3$
0	0 0 0	0 0	0 0 0
1	0 0 0	0 0	1 0 0
0	0 0 1	1 1	0 0 0
1	0 0 1	1 1	1 0 0
0	0 1 0	1 0	0 0 1
1	0 1 0	1 0	1 0 1
0	0 1 1	0 1	0 0 1
1	0 1 1	0 1	1 0 1
0	1 0 0	1 1	0 1 0
1	1 0 0	1 1	1 1 0
0	1 0 1	0 0	0 1 0
1	1 0 1	0 0	1 1 0
0	1 1 0	0 1	0 1 1
1	1 1 0	0 1	1 1 1
0	1 1 1	1 0	0 1 1
1	1 1 1	1 0	1 1 1

Encoding:

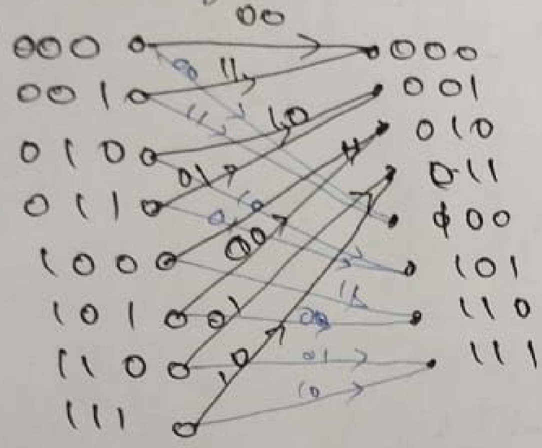
u = [1 1 0 0 1].

	Current State	Op	Next State
$u_1 = 1 \rightarrow$	0 0 0	0 0	1 0 0
$u_2 = 1 \rightarrow$	1 0 0	1 1	1 1 0
$u_3 = 0 \rightarrow$	1 1 0	0 1	0 1 1
$u_4 = 0 \rightarrow$	0 1 1	0 1	0 0 1
$u_5 = 1 \rightarrow$	0 0 1	1 1	1 0 0

∴ Encoded Message = 0011010111

State Diagram

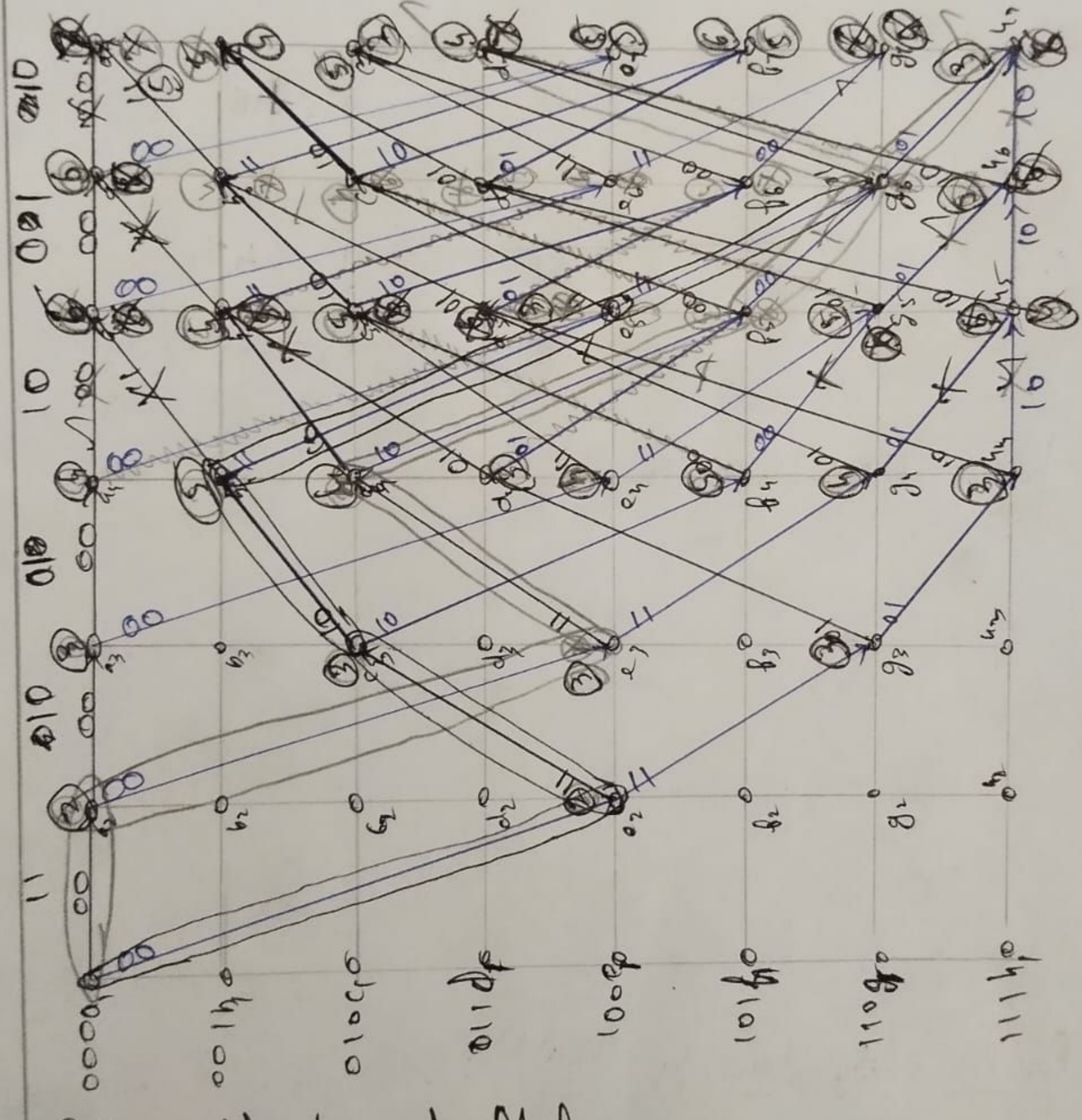
(ii)



(iv) The memory elements does not contribute to the error correcting capability of the encoder. ~~It directly affects the~~ It also doesn't affect the redundancy of the encoded output as the codeword depends on the input as well.

Encoded message = 0011 010100.

(iii) Viterbi Alogrithm.



Path with Lowest Metrices 4:

(1.) $a_1 a_2 e_3 b_4 e_5 g_6 h_7$ = 00 0 0 0 11 11 0 1

r = 11 1 0 1 0 1 1001 0 0 0

The ip would be = 110 01

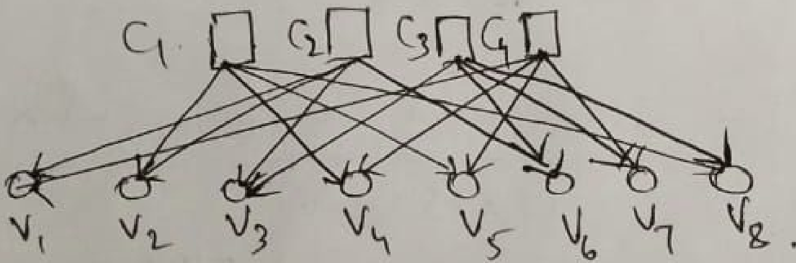
9Q.

$$H = \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}_{4 \times 8} \end{array}$$

$\rho = 4$, $\gamma = 2$. = ~~no. of~~

4 = no. of check nodes, 8 = no. of variable nodes.

Tanner Graph:



10Q.

$\rho = 4$, $\gamma = 2$.

It is a irregular LDPC code because $\rho \neq \gamma$.

Minimum distance of the code will be 2 as there only 2 ones in each row so while multiplying each input with parity check matrix the maxim ~~one~~ no. of ones will in the op will be one as there are only 2 one's & rest all are zeros.

11Q ~~(iii)~~

1.) Let $\rho = 6, \gamma = 3$.

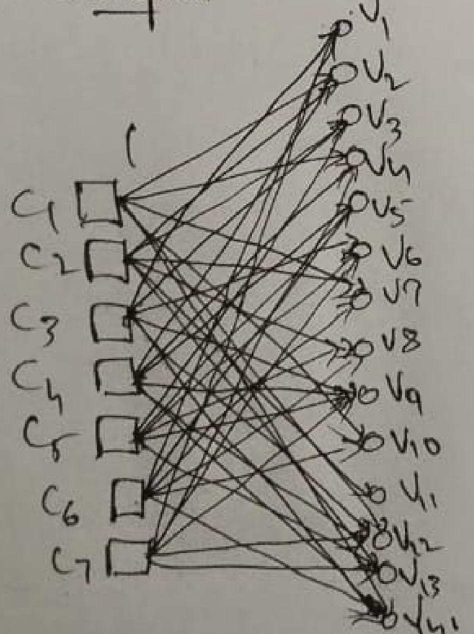
$\rho \neq \gamma$

$\therefore$ It doesn't qualify regular LDPC.

2.)

$$H = \begin{array}{c} \begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad 7 \times 14.$$

Tanner Graph.



3.) Since the $\gamma = 3$, the d_{min} will be 3 at the maximum i.e for 0 as input.

$\therefore$ errors correctable $= 2$

errors detectable $= \frac{2}{2} = 1$