

(Mahesh huddor) Naive Bayes classifier

→ Estimate conditional probabilities of each attribute {Color, legs, height, smelly} for the species classes: {M, H} using the data given in the table.

→ Using these probabilities estimate the probability values for the new instance —

Color = Green, legs = 2, Height = Tall, and smelly = No

No	Color	Legs	Height	Smelly	Species
1	White	3	Short	Yes	M
2	Green	2	Tall	No	M
3	Green	3	Short	Yes	M
4	White	3	Short	Yes	M
5	Green	2	Short	No	H
6	White	2	Tall	No	H
7	White	2	Tall	No	H
8	White	2	Short	Yes	H

New Instance

(Color = Green, legs = 2, Height = Tall & smelly = No)

$$\rightarrow P(M) = 4/8 = 0.5, \quad P(H) = 4/8 = 0.5$$

Color	M	H
White	2/4	3/4
Green	2/4	1/4

Legs	M	H
2	1/4	4/4
3	3/4	0/4

Height	M	H
Tall	2/4	2/4
Short	1/4	2/4

Smelly	M	H
Yes	3/4	1/4
No	1/4	3/4

$$P(\text{New Instance} | M) = P(M) * P(\text{Color} = \text{Green} | M) * P(\text{legs} = 2 | M) * P(\text{Height} = \text{Tall} | M) * P(\text{Smelly} = \text{No} | M) = 0.5 * \frac{2}{4} * \frac{1}{4} * \frac{2}{4} * \frac{1}{4} = 0.047$$

$$P(\text{New Instance} | H) = P(H) * P(\text{Color} = \text{Green} | H) * P(\text{legs} = 2 | H) * P(\text{Height} = \text{Tall} | H) * P(\text{Smelly} = \text{No} | H) = 0.5 * \frac{1}{4} * \frac{4}{4} * \frac{2}{4} * \frac{3}{4} = 0.047$$

Example

Given the data in table, reduce the dimension from 2 to 1 using the PCA Algorithm

F	Ex1	Ex2	Ex3	Ex4
x_1	4	8	10	5
x_2	7	4	2	12

Step-I Calculate Mean

$$\bar{x}_{\text{mean}} = \frac{4+8+10+5}{4}, \quad \bar{x}_{2\text{mean}} = \frac{7+4+2+12}{4}$$

$$\bar{x}_{\text{mean}} = 2\frac{7}{4}, \quad \bar{x}_{2\text{mean}} = \frac{25}{4}$$

$$= 6.75, \quad = 6.25$$

Step-II Calculation of the Covariance Matrix

$$S = \begin{bmatrix} \text{Cov}(x_1, x_2) & \text{Cov}(x_1, x_1) \\ \text{Cov}(x_2, x_1) & \text{Cov}(x_2, x_2) \end{bmatrix} =$$

$$\text{Cov}(x_1, x_1) = 6.75$$

$$= \frac{1}{N-1} \sum_{k=1}^N (x_{1k} - \bar{x}_1)(x_{1k} - \bar{x}_1) \quad (\text{here } N=4)$$

$$= \frac{1}{3} ((4-6.75)^2 + (8-6.75)^2 + (10-6.75)^2 + (5-6.75)^2)$$

$$= \frac{1}{3} ((4-6.75)^2 + (8-6.75)^2 + (10-6.75)^2 + (5-6.75)^2)$$

$$= 7.58$$

$$\text{Cov}(x_1, x_2) = \frac{1}{N-1} \sum_{k=1}^N (x_{1k} - \bar{x}_1)(x_{2k} - \bar{x}_2)$$

$$= \frac{1}{3} \{ (4-6.75)(7-6.25) + (8-6.75)(4-6.25) + (10-6.75)(2-6.25) + (5-6.75)(12-6.25) \}$$

$$= \frac{1}{3} \{ (-2.25)(0.75) + (1.25)(-2.25) + (3.25)(-4.25) + (-1.75)(5.75) \}$$

$$= \frac{1}{3} \{ (-1.68) + (-2.81) + (-13.81) + (-10.0625) \} = -9.45$$

$$\text{cov}(X_1, X_2) = \text{Cov}(X_1, X_2) = -9.45$$

$$\begin{aligned}\text{Cov}(X_1, X_2) &= \frac{1}{N-1} \sum_{k=1}^N (x_{1k} - \bar{x}_1)(x_{2k} - \bar{x}_2) \\ &= \frac{1}{3} \left((7-6.25)^2 + (4-6.25)^2 + (2-6.25)^2 + (12-6.25)^2 \right) \\ &= \frac{1}{3} \left((-0.75)^2 + (-2.25)^2 + (-4.25)^2 + (5.75)^2 \right) \\ &= \frac{1}{3} (56.75) = 18.91\end{aligned}$$

$$S = \begin{bmatrix} 7.58 & -9.45 \\ -9.45 & 18.91 \end{bmatrix}$$

Step 3: - Eigenvalues of the Covariance matrix

The characteristic Equation of the covariance matrix is,

$$0 = \det(S - \lambda I)$$

$$0 = \begin{vmatrix} 7.58 - \lambda & -9.45 \\ -9.45 & 18.91 - \lambda \end{vmatrix}$$

$$0 = (7.58 - \lambda)(18.91 - \lambda) - (-9.45 \times -9.45) \rightarrow \textcircled{1}$$

Assume (after solⁿ of equation ①) $\Rightarrow \lambda_1 = 7.09$ $\lambda_2 = 6.40$

Please calculate λ with calculator, as I don't have calculator so

I just assumed λ values.

Step 4: Computation of eigenvectors

$$\begin{aligned}\text{let } U &= \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = (S - \lambda I) \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \\ &= \begin{bmatrix} 7.58 - \lambda & -9.45 \\ -9.45 & 18.91 - \lambda \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}\end{aligned}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (7.58 - \lambda)U_1 - 9.45U_2 \\ -9.45U_1 + (18.91 - \lambda)U_2 \end{bmatrix}$$

$$(7.58 - \lambda)U_1 - 9.45U_2 = 0 \rightarrow \textcircled{2}$$

$$(-9.45)U_1 + (18.91 - \lambda)U_2 = 0 \rightarrow \textcircled{3}$$

One component of energy transfer when a high energy ion enters a wafer is collision with lattice nuclei. Many of these atoms are ejected from the lattice during the process. Some displaced substrate atoms have sufficient energy to collide with other substrate atoms to produce additional displaced atoms. As a result, the implantation process produces considerable substrate damage. What steps are needed to be taken in order to repair this damage?

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from (2)

$$\frac{U_1}{9.45} = \frac{U_2}{7.58 - \lambda} = t(\text{let})$$

$$U_1 = 9.45t$$

$$U_2 = (7.58 - \lambda)t$$

$$U = \begin{bmatrix} 9.45 \\ 7.58 - \lambda \end{bmatrix} \text{ we will use } \lambda_1 \text{ as we want only one principle component.}$$

To find a unit eigenvector, we compute the length of U_1 which is given by,

$$U_1 = \begin{bmatrix} 9.45 \\ 7.58 - \lambda_1 \end{bmatrix}$$

$$\begin{aligned} \|U_1\| &= \sqrt{(9.45)^2 + (7.58 - \lambda_1)^2} \\ &= \sqrt{(9.45)^2 + (7.58 - 10)^2} \\ &\approx 10 \end{aligned}$$

$$e_1 = \begin{bmatrix} 9.45 / \|U_1\| \\ (7.58 - \lambda_1) / \|U_1\| \end{bmatrix} \approx \begin{bmatrix} 0.94 \\ -0.25 \end{bmatrix}$$


$$e_2 = \begin{bmatrix} 9.45 / \|U_2\| \\ (7.58 - \lambda_2) / \|U_2\| \end{bmatrix} \rightarrow \text{we are not interested in } U_2 \text{ for our analysis}$$

Steps: Computation of first PCs

$$\begin{aligned} e_1^T \begin{bmatrix} X_{1k} - \bar{X}_1 \\ X_{2k} - \bar{X}_2 \end{bmatrix} &= \begin{bmatrix} 0.94 & 0.25 \end{bmatrix} \begin{bmatrix} X_{11} - \bar{X}_1 \\ X_{21} - \bar{X}_2 \end{bmatrix} \\ &= 0.94(X_{11} - \bar{X}_1) + 0.25(X_{21} - \bar{X}_2) \end{aligned}$$

$$\approx -4.3 \text{ (Assumption) please calculate correct value with}$$

Feature	x_1	x_2	x_2	x_3
x_1	4	8	10	5
x_2	7	4	2	12
First PCs	-4	-5	-2	-1


 I have assumed these values please calculate correct values by calculator.