OPTICAL FIBER COMMUNICATION
(15A04701)

SYLLABUS

Optical Fiber Communication

UNIT-I

Introduction to Optical Fibers: Evolution of fiber optic system- Element of an Optical Fiber Transmission link- Ray Optics-Optical Fiber Modes and Configurations –Mode theory of Circular Wave guides- Overview of Modes-Key Modal concepts- Linearly Polarized Modes –Single Mode Fibers-Graded Index fiber structure.

UNIT-II

Signal Degradation Optical Fibers: Attenuation – Absorption losses, Scattering losses, Bending Losses, Core and Cladding losses, Signal Distortion in Optical Wave guides -Information Capacity determination –Group Delay- Material Dispersion, Wave guide Dispersion, Signal distortion in SM fiber`s-Polarization Mode dispersion, Intermodal dispersion, Pulse Broadening in GI fibers-Mode Coupling –Design Optimization of SM fibers-RI profile and cut-off wavelength.

UNIT-III

Fiber Optical Sources and Coupling : Direct and indirect Band gap materials-LED structures –Light source materials –Quantum efficiency and LED power, Modulation of a LED, lasers Diodes-Modes and Threshold condition –Rate equations –External Quantum efficiency –Resonant frequencies –Temperature effects, Introduction to Quantum laser, source-to-fiber Power Launching, Lensing schemes, Fiber –to- Fiber joints, Fiber splicing.

UNIT-IV

Fiber Optical Receivers : PIN and APD diodes –Photo detector noise, SNR, Detector Response time, Avalanche Multiplication Noise –Comparison of Photo detectors –Fundamental Receiver Operation – preamplifiers, Error Sources –Receiver Configuration –Probability of Error – Quantum Limit.

UNIT-V

System Designand Applications: Design of Analog Systems: system specification, power budget, bandwidth budget.

Design of Digital Systems: system specification, rise time budget, power budget, Receiver sensitivity.

Text Books:

1. Gerd Keiser, “Optical Fiber Communication” McGraw –Hill International, Singapore, 3rd ed., 2000.

2. J.Senior, “Optical Communication, Principles and Practice”, Prentice Hall of India,1994.

References:

1. Max Ming-Kang Liu, “Principles and Applications of Optical Communications”,TMH, 2010.
2. S.C.Gupta, “Text book on optical fiber communication and its applications”, PHI,2005.
3. Satish Kumar, “Fundamentals of Optical Fiber communications”, PHI,

UNIT –I

Introduction to Optical Fibers:

1.1. Historical Development

- Fiber optics deals with study of propagation of light through transparent dielectric wageguides. The fiber optics are used for transmission of data from point to point location. Fiber optic systems currently used most extensively as the transmission line between terrestrial hardwired systems.

- The carrier frequencies used in conventional systems had the limitations in handlinmg the volume and rate of the data transmission. The greater the carrier frequency larger the available bandwith and information carrying capacity.

First generation

- The first generation of lightwave systems uses GaAs semiconductor laser and operating region was near 0.8 µm. Other specifications of this generation are as under:

  i) Bit rate : 45 Mb/s

  ii) Repeater spacing : 10 km

Second generation

i) Bit rate : 100 Mb/s to 1.7 Gb/s

ii) Repeater spacing : 50 km

iii) Operation wavelength : 1.3 µm

iv) Semiconductor : In GaAsP

Third generation

i) Bit rate : 10 Gb/s

ii) Repeater spacing : 100 km

iii) Operating wavelength : 1.55 µm

Fourth generation

Fourth generation uses WDM technique.

Bit rate : 10 Tb/s

Repeater spacing : > 10,000 km

Operating wavelength : 1.45 to 1.62 µm

Fifth generation

Fifth generation uses Roman amplification technique and optical solitiors.

Bit rate	: 40 - 160 Gb/s
Repeater spacing	: 24000 km - 35000 km
Operating wavelength	: 1.53 to 1.57 μm

Need of fiber optic communication

- Fiber optic communication system has emerged as most important communication system. Compared to traditional system because of following requirements :

    1. In long haul transmission system there is need of low loss transmission medium
    2. There is need of compact and least weight transmitters and receivers.
    3. There is need of increase dspan of transmission.
    4. There is need of increased bit rate-distrance product.

- A fiber optic communication system fulfills these requirements, hence most widely acception.

1.2 General Optical Fiber Communication System

- Basic block diagram of optical fiber communication system consists of following important blocks.

    1. Transmitter
    2. Information channel
    3. Receiver.

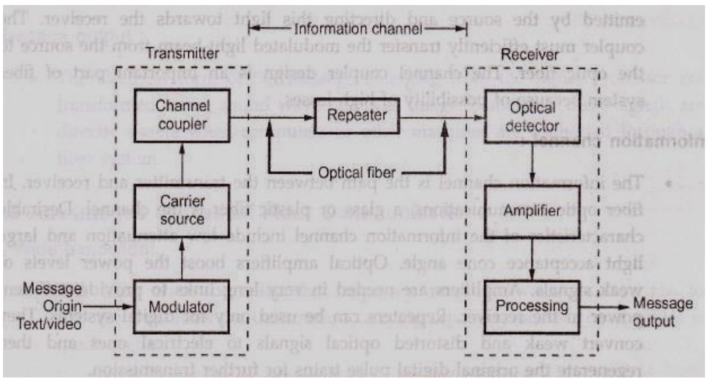


Fig. 1.2.1 shows block diagram of OFC system.

Message origin :

- Generally message origin is from a transducer that converts a non-electrical message into an electrical signal. Common examples include microphones for converting sound waves into currents and video (TV) cameras for converting images into current. For data transfer between computers, the message is already in electrical form.

Modulator :

- The modulator has two main functions.

  1) It converts the electrical message into the proper format.

  2) It impresses this signal onto the wave generated by the carrier source.

  Two distinct categories of modulation are used i.e. analog modulation and digital modulation.

Carrier source :

- Carrier source generates the wave on which the information is transmitted. This wave is called the carrier. For fiber optic system, a laser diode (LD) or a light emitting diode (LED) is used. They can be called as optic oscillators, they provide stable, single frequency waves with sufficient power for long distance propagation.

Channel coupler :

- Coupler feeds the power into the information channel. For an atmospheric optic system, the channel coupler is a lens used for collimating the light emitted by the source and directing this light towards the receiver. The coupler must efficiently transfer the modulated light beam from the source to the optic fiber. The channel coupler design is an important part of fiber system because of possibility of high losses.

Information channel :

- The information channel is the path between the transmitter and receiver. In fiber optic communications, a glass or plastic fiber is the channel. Desirable characteristics of the information channel include low attenuation and large light acceptance cone angle. Optical amplifiers boost the power levels of weak signals. Amplifiers are needed in very long links to provide sufficient power to the receiver. Repeaters can be used only for digital systems. They convert weak and distorted optical signals to electrical ones and then regenerate the original disgital pulse trains for further transmission.

- Another important property of the information channel is the propagation time of the waes travelling along it. A signal propagating along a fiber normally contains a range of optic frequencies and divides its power along several ray paths. This results in a distortion of the propagating signal. In a digital system, this distortion appears as a spreading and deforming of the pulses. The spreading is so great that adjacent pulses begin to overlap and become unrecognizable as separate bits of information.

Optical detector :

- The information being transmitted is detector. In the fiber system the optic wave is converted into an electric current by a photodetector. The current developed by the

- detector is proportional to the power in the incident optic wave. Detector output current contains the transmitted information. This detector output is then filtered to remove the constant bias and thn amplified.

- The important properties of photodetectors are small size, economy, long life, low power consumption, high sensitivity to optic signals and fast response to quick variations in the optic power.

Signal processing :

- Signal processing includes filtering, amplification. Proper filtering maximizes the ratio of signal to unwanted power. For a digital system decision circuit is an additional block. The bit error rate (BER) should be very small for quality communications.

Message output :

- The electrical form of the message emerging from the signal processor are transformed into a soud wave or visual image. Sometimes these signals are directly usable when computers or other machines are connected through a fiber system.

1.3 Advantages of Optical Fiber Communications

1. Wide bandwidth

- The light wave occupies the frequency range between 2 x 1012 Hz to 3.7 x 1012 Hz. Thus the information carrying capability of fiber optic cables is much higher.

2. Low losses

- Fiber optic cables offers bery less signal attenuation over long distances. Typically it is less than 1 dB/km. This enables longer distance between repeaters.

3. Immune to cross talk

- Fiber optic cables has very high immunity to electricaland magnetic field. Since fiber optic cables are non-conductors of electricity hence they do not produce magnetic field. Thus fiber optic cables are immune to cross talk between cables cause dby magnetic induction.

4. Interference immune

- Fiber optic cable sar eimmune to conductive and radiative interferences caused by electrical noise sources such as lighting, electric motors, fluorescent lights.

5. Light weight

- As fiber cables are made of silica glass or plastic which is much lighter than copper or aluminium cables. Light weight fiber cables are cheaper to transport.

6. Small size

- The diameter of fiber is much smaller compared to other cables, therefore fiber calbe is small in size, requires less storage space.

7. More strength

- Fiber cables are stronger and rugged hence can support more weight.

8. Security

- Fiber cables are more secure than other cables. It is almost impossible to tap into a fiber cable as they do n ot radiate signals.

No ground loops exist between optical fibers hence they are more secure.

9. Long distance transmission

- Becaujse of less attenuation transmission at a longer distance is possible.

10. Environment immune

- Fiber calbes are more immune to environmental extremes. They can operate over a large temperature varioations. Also they are not affected by corrosive liquids and gases.

11. Sage and easy installation

- Fiber cables are safer and easier to install and maintain. They are non-conductors hence there is no shock hazards as no current or voltage is associated with them. Their small size and light weight feature makes installation easier.

12. Less cost

- Cost of fiber optic system is less compated to any other system.

1.4 Disadvantages of Optical Fiber Communicaitons

1. High initial cost

- The intial cost of installation or setting up cost is very high compared to all other system.

2. Maintenance and repairing cost

- The maintenance and repairing of fiber optic systems is not only difficult but expensive also.

3. Jointing and test procedures

- Since optical fibers are of very small size. The fiber joining process is very constly and requires skilled manpower.

4. Tensile stress

- Optical fibers are more susceptible to buckling, bending and tensile stress than copper cables. This leades to restricted practice to use optical fiber technology to premises and floor backbones with a few interfaces to the copper cables.

5. Short links

- Eventhough optical fiber calbes are inexpensive, it is still not cost effective to replace every small conventional connector (e.g. between computers and peripherals), as the price of optoelectronic transducers are very high.

6. Fiber losses

- The amount of optical fiber available to the photodetector at the end of fiber length depends on various fiber losses such as scattering, dispersion, attenuation and reflection.

1.5 Applications of Optical Fiber Communicaiton

- Applications of optical fiber communications include telecommunications, data communications, video control and protection switching, sensors and power applications.

1. Telephone networks

- Optical waveguide has low attenuation, high transmission bandwidth compated to copper lines, therefore numbers of long haul co-axial trunks l;links between telephone exchanges are being replaced by optical fiber links.

2. Urban broadband service networks

- Optical waveguide provides much larger bandwidth than co-axial calbe, also the number of repeaters required is reduced considerably.

- Modern suburban communications involves videotext, videoconferencing videotelephony, switched broadband communication network. All these can be supplied over a single fiber optic link. Fiber optic calbes is the solution to many of today's high speed, high bandwidth data communication problems and will continue to play a large role in future telecom and data-com networks.

1.6 Optical Fiber Waveguides

- In free space light ravels as its maximum possible speed i.e. 3 x 10^8 m/s or 186 x 10^3 miles/sec. When light travels through a material it exnibits certain behavior explaned by laws of reflection, refraction.

Electromagnetic Spectrum

- The radio waves and light are electromagnetic waves. The rate at which they alternate in
  polarity is called their frequency (f) measured in hertz (Hz). The speed of electromagnetic wave (c) in free space is approximately 3 x 10^8 m/sec. The distance travelled during each cycle is called as wavelength (λ)

$$\text{Wavelength } (\lambda) = \frac{\text{Speed of light}}{\text{Frequency}} = \frac{c}{f}$$

- In fiber optics, it is more convenient to use the wavelength of light instead of the frequency with light frequencies, wavlengfth is often stated in microns or nanometers.

1 micron (μ) = 1

Micrometre (1 x 10^{-6}) 1

nano (n) = 10^{-9} metre

- Fiber optics uses visible and infrared light. Infrared light covers a fairly wide range of wavelengths and is generally used for all fiber optic communications. Visible light is normally used for very short range transmission using a plastic fiber.

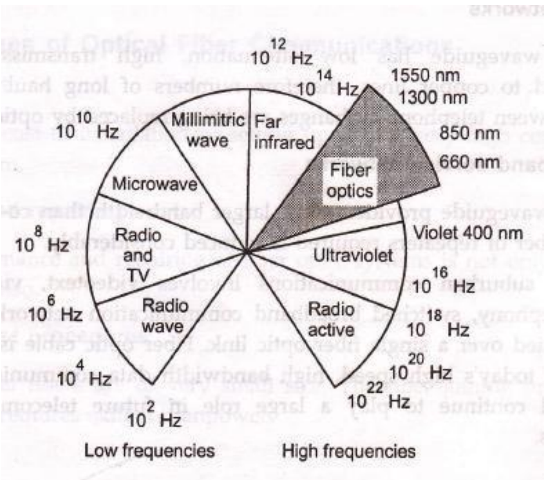


Fig. 1.6.1 shows electromagnetic frequency spectrum

Ray Transmission Theory

- Before studying how the light actually propagates through the fiber, laws governing the nature of light m ust be studied. These was called as **laws of optics (Ray theory).** There is conception that light always travels at the same speed. This fact is simply not true. The speed of light depends upon the material or medium through which it is moving. In free space light travels at its maximum possible speed i.e. 3 x 108 m/s or 186 x 103 miles/sec. When light travels through a material it exhibits certain behavior explained by laws of reflection, refraction.

Reflection

- The law of reflection states that, when a light ray is incident upon a reflective surface at some incident angle ϕ_1 from imaginary perpendicular normal, the ray will be reflected from the surface at some angle ϕ_2 from normal which is equal to the angle of incidence.

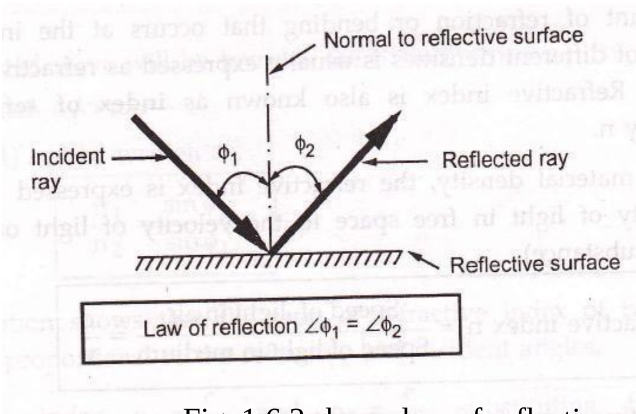


Fig. 1.6.2 shows law of reflection.

Refraction

- Refraction occurs when light ray passes from one medium to another i.e. the light ray changes its direction at interface. Refraction occurs whenever density of medium changes. E.g. refraction occurs at air and water interface, the straw in a glass of water will appear as it is bent.

The refraction can also observed at air and glass interface.

- When wave passes through less dense medium to more dense medium, the wave is refracted (bent) towards the normal. Fig. 1.6.3 shows the refraction phenomena.

- The refraction (bending) takes place because light travels at different speed in different mediums. The speed of light in free space is higher than in water or glass.

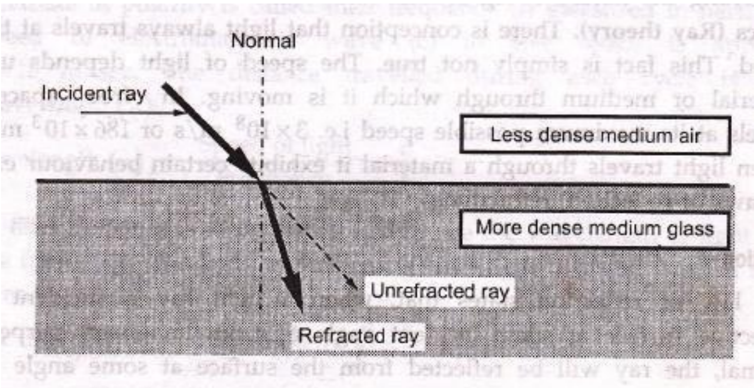


Fig.1.6.3 Refraction

Refractive Index

- The amount of refraction or bending that occurs at the interface of two materials of different densities is usually expressed as refractive index of two materials. Refractive index is also known as **index of refraction** and is denoted by n.

- Based on material density, the refractive index is expressed as the ratio of the velocity of light in free space to the velocity of light of the dielectric material (substance).

$$\boxed{\text{Refractive index } n = \frac{\text{Speed of light in air}}{\text{Speed of light in medium}} = \frac{c}{v}}$$

The refractive index for vacuum and air os 1.0 for water it is 1.3 and for glass refractive index is 1.5.

Snell's Law

- Snell's law states how light ray reacts when it meets the interface of two media having different indexes of refraction.

- Let the two medias have refractive indexes n1 and n_2 where $n_1 > n_2$.

  $\phi 1$ and $\phi 2$ be the angles of incidence and angle of refraction respectively.

Then according to Snell's law, a relationship exists between the refractive index of both materials given by

$$n1 \sin\phi 1 = n2 \sin\phi 2 \qquad \ldots (1.6.1)$$

- A refractive index model for Snell's law is shown in Fig. 1.6.4.

- The refracted wave will be towards the normal when $n_1 < n_2$ and will away from it when $n_1 > n_2$.

Equation (1.6.1) can be written as,

$$\boxed{\frac{n_1}{n_2} = \frac{\sin \phi_2}{\sin \phi_1}}$$

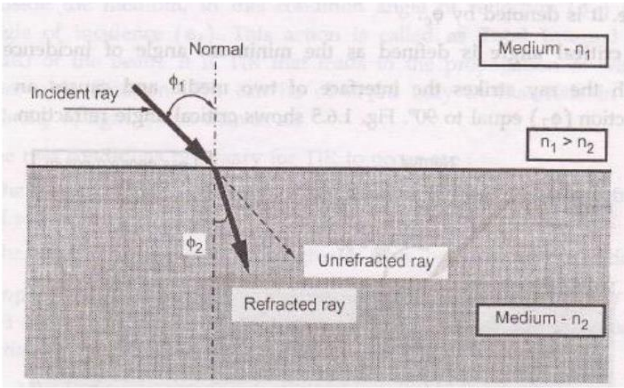


Fig 1.6.4 Refractive model for Snells Law

- This equation shows that the ratio of refractive index of two mediums is inversely proportional to the refractive and incident angles.

As refractive index $n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$ substituting these values in equation (1.6.2)

$$\frac{c / v_1}{c / v_2} = \frac{\sin \phi_2}{\sin \phi_1}$$

$$\frac{v_2}{v_1} = \frac{\sin \phi_2}{\sin \phi_1}$$

Critical Angle

- When the angle of incidence (ϕ_1) is profressively increased, there will be progressive increase of refractive angle (ϕ_2). At some condition (ϕ_1) the refractive angle (ϕ_2) becomes
90o to the normal. When this happens the refracted light ray travels along the interface. The angle of incidence (ϕ_1) at the point at which the refractive angle (ϕ_1) becomes 90^o is called the critical angle. It is denoted by ϕ_c.

- The **critical angle** is defined as the minimum angle of incidence (ϕ_1) at which the ray strikes the interface of two media and causes an agnle of refraction (ϕ_2) equal to 90^o. Fig 1.6.5 shows critical angle refraction.

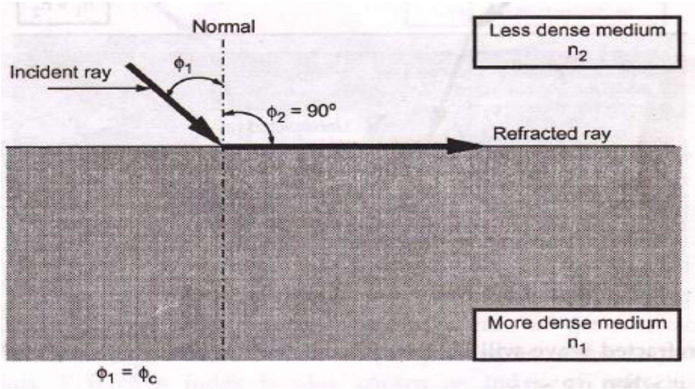


Fig.1.6.5 Critical Angle

Hence at critical angle $\phi_1 = \phi_c$ and $\phi_2 = 90^o$

Using Snell's law : $n_1 \sin \phi_1 = n_2 \sin \phi_2$

$$\sin \phi_c = \frac{n_2}{n_1} \sin 90^o$$

$\because$ $$\sin 90^o = 1$$

Therefore $$\sin \phi_c = \frac{n_2}{n_1}$$

$$\boxed{\text{Critical angle } \phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)} \qquad \ldots (1.6.3)$$

- The actual value of critical angle is dependent upon combination of materials present on each side of boundary.

Total Internal Refleciton (TIR)

- When the incident angle is increase dbeyond the critical angle, the light ray does not pass through the interface into the other medium. This gives the effect of mirror exist at the interface with no possibility of light escaping outside the medium. In this condition angle of reflection (ϕ_2) is equal to angle of incidence (ϕ_1). This action is called as **Total Internal Reflection (TIR)** of the beam. It is TIR that leads to the propagation of waves within fiber-cable medium. TIR can be observed only in materials in which the velocity of light is less than in air.

The refractive index of first medium must be greater than the refractive index of second one.

1. The angle of incidence must be greater than (or equal to) the critical angle.

Example 1.6.1 : A light ray is incident from medium-1 to medium-2. If the refractive indices of medium-1 and medium-2 are 1.5 and 1.36 respectively then determine the angle of refraction for an angle of incidence of 30^{o}.

Solution : Medium-1 n1 = 1.5

Medium-2 n2 = 1.36

Angle of incidence $\phi_1 = 30^{o}$.

Angle of incident ϕ_2 = ?

$$\boxed{\text{Snell's law}: n_1 \sin\phi_1 = n_2 \sin\phi_2}$$

$$1.5 \sin 30^{\circ} = 1.36 \sin\phi_2$$

$$\sin\phi_2 = \frac{1.5}{1.36}\sin 30^{\circ}$$

$$\sin\phi_2 = 0.55147$$

$$\therefore \quad \phi_2 = 33.46^{\circ}$$

Angle of refraction 33.46^{o} from normal. **… Ans.**

Example 1.6.2 : A light ray is incident from glass to air. Calculate the critical angle (ϕ_C).

Solution : Refractive index of glass n_1 = 1.50

Refrative indes of air n_2 = 1.00

$$\boxed{\text{Snell's law}: n_1 \sin\phi_1 = n_2 \sin\phi_2}$$

$$\sin\phi_1 = \frac{n_2}{n_1}\sin\phi_2$$

$$\therefore \quad \sin\phi_1 = \frac{n_2}{n_1}\sin 90^{\circ}$$

Example 1.6.3 : Calculate the NA, acceptance angle and critical angle of the fiber having n_1 (Core refractive index) = 1.50 and refractive index of cladding = 1.45.

Soluiton : n_1 = 1.50, n_2 = 1.45

$$\Delta = \frac{(n_1 - n_2)}{(n_1)} = \frac{1.50 - 1.45}{1.50} = 0.033$$

$$\text{Numerical aperture, NA} = n_1\sqrt{2\Delta}$$

$$\text{NA} = 1.50\sqrt{2 \times 0.033}$$

$$\text{NA} = 0.387$$

$$\text{Acceptance angle } \phi_0 = \sin^{-1}\text{NA} \qquad \phi_0 = \sin^{-1} 0.387$$

$$\phi_0 = 22.78^{\circ}$$

$$\text{Critical angle } \phi_c = \sin^{-1}\frac{n_2}{n_1} \qquad \phi_c = \sin^{-1}\frac{1.45}{1.50}$$

Optical Fiber as Waveguide

- An optical fiber is a cylindrical dielectric waveguide capable of conveying electromagnetic waves at optical frequencies. The electromagnetic energy is in the form of the light and propagates along the axis of the fiber. The structural of the fiver determines the transmission characteristics.
- The propagation of light along the waveguide is decided by the modes of the waveguides, here mode means path. Each mode has distict pattern of electric and magnetic field distributions along the fiber length. Only few modes can satisfy the homogeneous wave

  equation in the fiber also the boundary condition a waveguide surfaces. When there is only one path for light to follow then it is called as single mode propagation. When there is more than one path then it is called as multimode propagation.

Single fiber structure

- A single fiber structure is shown in Fig. 1.6.6. It consists of a solid dielectric cylinder with radius ‘a’. This cylinder is called as **core** of fiber. The core is surrounded by dielectric, called **cladding**. The index of refraction of core (glass fiber) is slightly greater than the index of refraction of cladding.

If refractive index of core (glass fiber) = n_1

and refractive index of cladding = n_2

then $n_1 > n_2$.

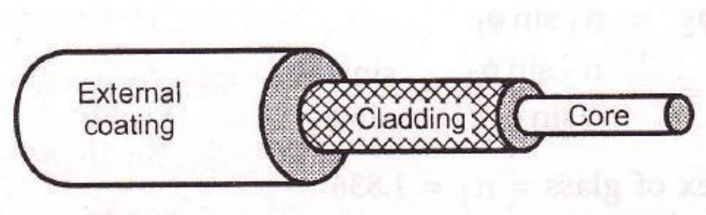


Fig.1.6.6. Single optical Fibre Structure

Propagation in Optical Fiber

- To understand the general nature of light wave propagation in optical fiber. We first consider the construction of optical fiber. The innermost is the glass core of very thin diameter with a slight lower refractive index n2. The light wave can propagate along such a optical fiber. A single mode propagation is illustrated in Fig. 1.6.7 along with standard size of fiber.

Single mode fibers are capable of carrying only one signal of a specific wavelength.

- In multimode propagation the light propagates along the fiber in zigzag fashion, provided it can undergo total internal reflection (TIR) at the core cladding boundaries.
- Total internal reflection at the fiber wall can occur only if two conditions are satisfied.

Condition 1:

The index of refraction of glass fiber must be slightly greater than the index of refraction of material surrounding the fiber (cladding).

If refractive index of glass fiber = n_1

and refractive index of cladding = n_2

then $n_1 > n_2$.

Condition 2 :

The angle of incidence (ϕ_1) of light ray must be greater than critical angle (ϕ_c).

- A light beam is focused at one end of cable. The light enters the fibers at different angles.

  Fig. 1.6.8 shows the conditions exist at the launching end of optic fiber. The light source is surrounded by air and the refractive index of air is $n_0 = 1$. Let the incident ray makes an angle ϕ_0 with fiber axis. The ray enters into glass fiber at point P making refracted angle ϕ_1 to the fiber axis, the ray is then propagated diagonally down the core and reflect from the core wall at point Q. When the light ray reflects off the inner surface, the angle of incidence is equal to the angle of reflection, which is greater than critical angle.
- In order for a ray of light to propagate down the cable, it must strike the core cladding interface at an angle that is greater than critical angle (ϕ_c).

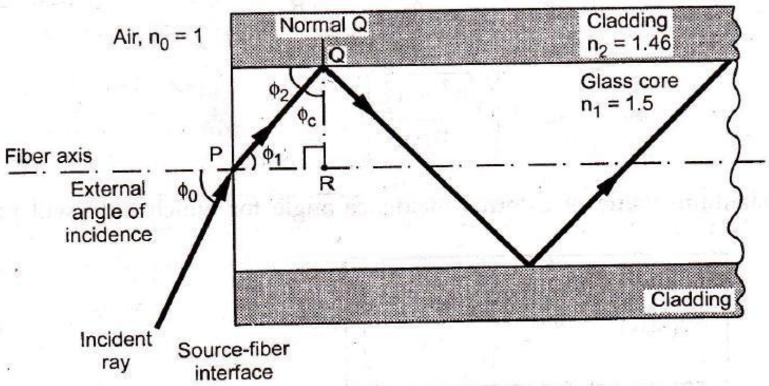


Fig. 1.6.8 Ray propagation by TIR

Acceptance Angle

Applying Snell's law to external incidence angle.

$$n_0 \sin \phi_0 = n_1 \sin \phi_1$$

But $\phi_1 = (90 - \phi_c)$

$$\sin \phi_1 = \text{sing } (90 - \phi_c) = \cos \phi_c$$

Substituting sin ϕ_1 in above equation.

$$n_0 \sin \phi_0 = n_1 \cos \phi_c$$

$$\sin \phi_c = \frac{n_1}{n_0} \cos \phi_c$$

Applying Pythagorean theorem to ΔPQR.

$$\cos \phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin \phi_0 = \frac{n_1}{n_0}\left[\frac{\sqrt{n_1^2 - n_2^2}}{n_1}\right]$$

$$\sin \phi_0 = \left[\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right]$$

The maximum value of external incidence angle for which light will propagate in the fiber.

$$\boxed{\phi_{0(max)} = \sin^{-1}\left[\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right]}$$

When the light rays enters the fivers from an air medium $n_0 = 1$. Then above equation reduces to,

$$\phi_{0(max)} = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$$

The angle ϕ_0 is called as **acceptance angle** and $\phi_{0(max)}$ defines the maximum angle in which the light ray may incident on fiber to propagate down the fiber.

Acceptance Cone

- Rotating the acceptance angle $\phi_{0(max)}$ around the fiber axis, a cone shaped pattern is obtained, it is called as **acceptance cone** of the fiber input. Fig 1.6.10 shows formation of acceptance cone of a fiber cable.

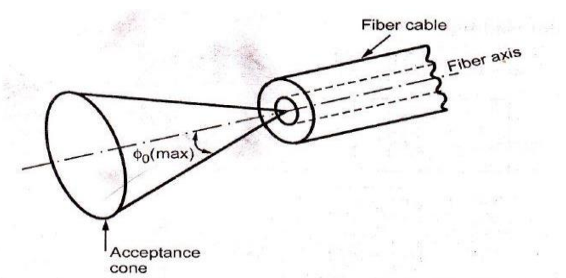


FIG: 1.6.10 shows formation of acceptance cone of a fiber cable.

- The Cone of acceptance is the angle within which the light is accepted into the core and is able to travel along the fiber. The launching of light wave becomes easier for large acceptance come.
- The angle is measured from the axis of the positive cone so the total angle of convergence is actually twice the stated value.

Numerical Aperture (NA)

- The **numerical aperture** (NA) of a fiber is a figure of merit which represents its light gathering capability. Larger the numerical aperture, the greater the amount of light accepted by fiber. The acceptance angle also determines how much light is able to be enter the fiber and hence there is relation between the numerical aperture and the cone of acceptance.

Numerical aperture (NA) = sin $\phi_{0(max)}$

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For air $n_0 = 1$

$$\therefore \quad NA = \sqrt{n_1^2 - n_2^2}$$

$$\boxed{NA = \sqrt{n_{core}^2 - n_{cladding}^2}} \quad \dots (1.6.4)$$

Hence acceptance angle = $\sin^{-1}$ NA

By the formula of NA note that the numerical aperture is effectively dependent only on refractive indices of core and cladding material. NA is not a function of fiber dimension.

- The index difference (Δ) and the numerical aperture (NA) are related to the core and cladding indices:

$$\Delta = \frac{(n_1 - n_2)}{n_1} \qquad \boxed{\Delta = \frac{NA^2}{2n_1^2}}$$

Also $$NA = \sqrt{n_1^2 - n_2^2}$$

Example 1.6.5 : Calculate the numerical aperture and acceptance angle for a fiber cable of which n_{core} = 1.5 and $n_{cladding}$ = 1.48. The launching takes place from air.

Solution $NA = (n_1^2 - n_2^2)^{1/2}$

$$NA = \sqrt{n_{core}^2 - n_{cladding}^2} \qquad NA = \sqrt{1.5^2 - 1.48^2}$$

$$\boxed{NA = n_1 (2\Delta)^{1/2}}$$

NA = **0.244** **…Ans.**

Acceptance angle –

$$\boxed{\sin^{-1}\sqrt{n_{core}^2 - n_{cladding}^2} = \sin^{-1} NA}$$

Acceptance angle = $\sin^{-1}$ 0.244

ϕ_0 = **14.12^{o}** **…Ans.**

Types of Rays

- If the rays are launched within core of acceptance can be successfully propagated along the fiber. But the exact path of the ray is determined by the position and angle of ray at which it strikes the core.
  There exists three different types of rays.
  i) Skew rays ii) Meridional rays iii) Axial rays.

- **The skew rays** does not pass through the center, as show in Fig. 1.6.11 (a). The skew rays reflects off from the core cladding boundaries and again bounces around the outside of the core. It takes somewhat similar shape of spiral of helical path.

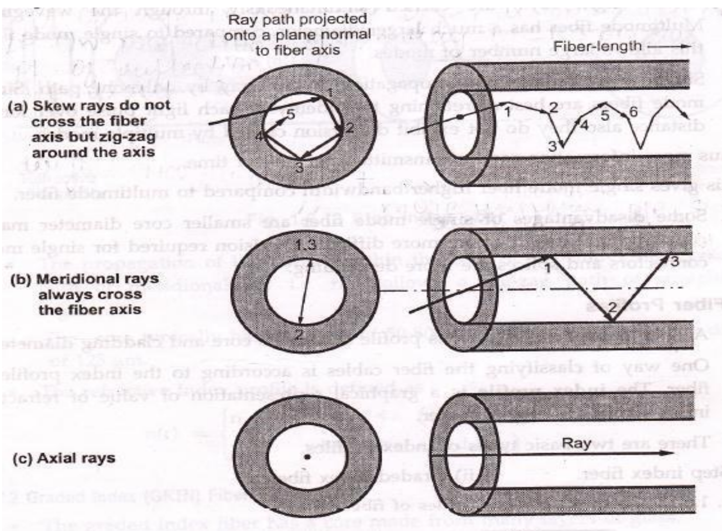


Fig:1.6.11 Different Ray Propagation

- The **meridional** ray enters the core and passes through its axis. When the core surface is parallel, it will always be reflected to pass through the enter. The meridional ray is shown in fig. 1.6.11 (b).
- The **axial ray** travels along the axis of the fiber and stays at the axis all the time. It is shown in fig. 1.6.11 (c).

Modes of Fiber

- Fiber cables cal also be classified as per their mode. Light rays propagate as an electromagnetic wave along the fiber. The two components, the electric field and the magnetic field form patterns across the fiber. These patterns are called **modes** of transmission. The **mode** of a fiber refers to the number of paths for the light rays within the cable. According to modes optic fibers can be classified into two types.
  i) Single mode fiber ii) Multimode fiber.
- Multimode fiber was the first fiber type to be manufactured and commercialized. The term multimode simply refers to the fact that numerous modes (light rays) are carried simultaneously through the waveguide. Multimode fiber has a much larger diameter, compared to single mode fiber, this allows large number of modes.
- Single mode fiber allows propagation to light ray by only one path. Single mode fibers are best at retaining the fidelity of each light pulse over longer distance also they do not exhibit dispersion caused by multiple modes.

Thus more information can be transmitted per unit of time.

This gives single mode fiber higher bandwidth compared to multimode fiber.

- Some disadvantages of single mode fiber are smaller core diameter makes coupling light into the core more difficult. Precision required for single mode connectors and splices are more demanding.

Fiber Profiles

- A fiber is characterized by its profile and by its core and cladding diameters.
- One way of classifying the fiber cables is according to the index profile at fiber. The **index profile** is a graphical representation of value of refractive index across the core diameter.
- There are two basic types of index profiles.

i) Step index fiber. ii) Graded index fiber.

Fig. 1.6.12 shows the index profiles of fibers.

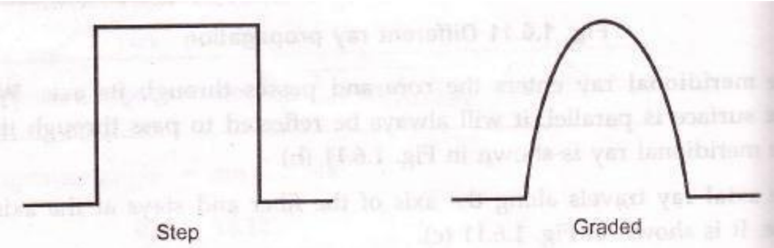


Fig. 1.6.12 Index profiles

Step Index (SI) Fiber

- The step index (SI) fiber is a cylindrical waveguide core with central or inner core has a uniform refractive index of n_1 and the core is surrounded by outer cladding with uniform refractive index of n_2. The cladding refractive index (n_2) is less than the core refractive index (n_1). But there is an abrupt change in the refractive index at the core cladding interface. Refractive index profile of step indexed optical fiber is shown in Fig. 1.6.13. The refractive index is plotted on horizontal axis and radial distance from the core is plotted on vertical axis.

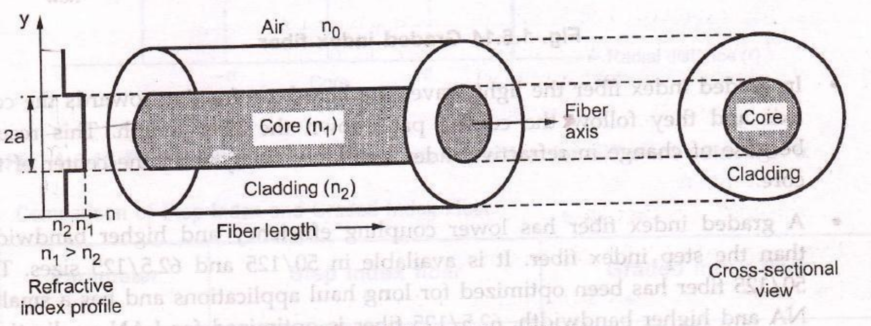


Fig. 1.6.13 Step index fiber

- The propagation of light wave within the core of step index fiber takes the path of meridional ray i.e. ray follows a zig-zag path of straight line segments.

  The core typically has diameter of 50-80 µm and the cladding has a diameter of 125 µm.
- The refractive index profile is defined as –

$$n(r) = \begin{cases} n_1 & \text{when } r < a \text{ (core)} \\ n_2 & \text{when } r \geq a \text{ (cladding)} \end{cases}$$

Graded Index (GRIN) Fiber

- The graded index fiber has a core made from many layers of glass.
- In the **graded index (GRIN)** fiber the refractive index is not uniform within the core, it is highest at the center and decreases smoothly and continuously with distance towards the cladding. The refractive index profile across the core takes the parabolic nature. Fig. 1.6.14 shows refractive index profile of graded index fiber.

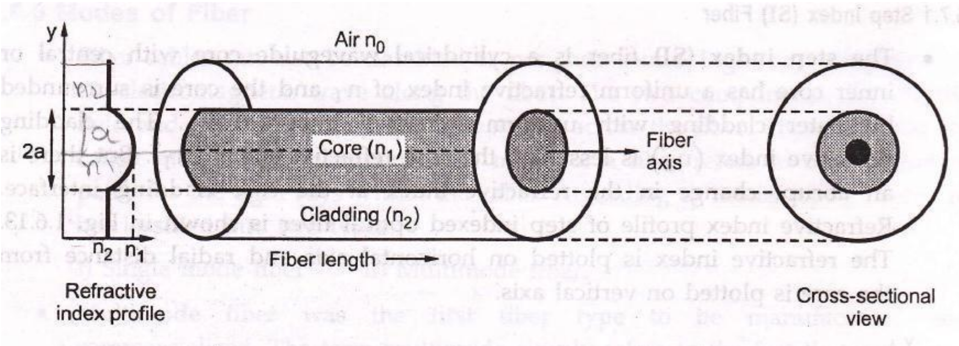


Fig. 1.6.14 Graded index fiber

- In graded index fiber the light waves are bent by refraction towards the core axis and they follow the curved path down the fiber length. This results because of change in refractive index as moved away from the center of the core.
- A graded index fiber has lower coupling efficiency and higher bandwidth than the step index fiber. It is available in 50/125 and 62.5/125 sizes. The 50/125 fiber has been optimized for long haul applications and has a smaller NA and higher bandwidth. 62.5/125 fiber is optimized for LAN applications which is costing 25% more than the 50/125 fiber cable.
- The refractive index variation in the core is giver by relationship

$$n(r) = \begin{cases} n_1\left(1 - 2\Delta\left(\frac{r}{a}\right)^{\alpha}\right) & \text{when } r < a \ (core) \\ n_1(1 - 2\Delta)^{\frac{1}{2}} \approx n_2 & when\ r \geq a \ \text{(cladding)} \end{cases}$$

where,

r = Radial distance from fiber axis

a = Core radius

n_1= Refractive index of core

n_2 = Refractive index of cladding

α = Shape of index profile.

- Profile parameter α determines the characteristic refractive index profile of fiber core. The range of refractive index as variation of α is shown in Fig. 1.6.1

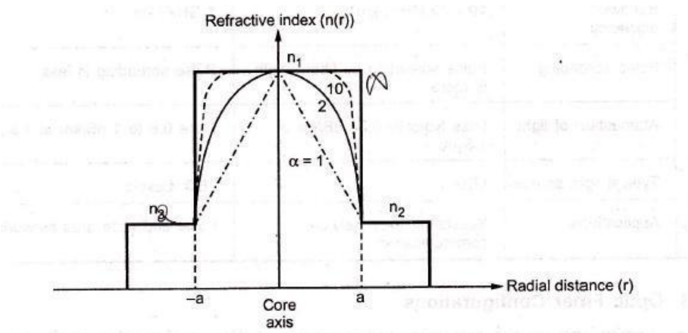


Fig. 1.6.15 Possible fiber refractive index profiles for different values of α

Comparison of Step Index and Graded Index Fiber

Sr. No.	Parameter	Step index fiber	Graded index fiber
1.	Data rate	Slow.	Higher
2.	Coupling efficiency	Coupling efficiency with fiber is higher.	Lower coupling efficiency.
3.	Ray path	By total internal reflection.	Light travelled oscillatory fashion.
4.	Index variation	Constant or Uniform	Non
5.	Numerical aperture	NA remains same.	Changes continuously distance from fiber axis.
6.	Material used	Normally plastic or glass is preferred.	Only glass is preferred.
7.	Bandwidth efficiency	10 – 20 MHz/km	1 GHz/km
8.	Pulse spreading	Pulse spreading by fiber length is more.	Pulse spreading is less
9.	Attenuation of light	Less typically 0.34 dB/km at 1.3 µm.	More 0.6 to 1 dB/km at 1.3 µm.
10	Typical light source	LED.	LED, Lasers.
11	Applications	Subscriber local network communication.	networks.

Optic Fiber Configurations

- Depending on the refractive index profile of fiber and modes of fiber there exist three types of optical fiber configurations.
- These optic-fiber configurations are -

i) Single mode step index fiber.
ii) Multimode step index fiber.
iii) Multimode graded index fiber.

Single mode Step index Fiber

- In single mode step index fiber has a central core that is sufficiently small so that there is essentially only one path for light ray through the cable. The light ray is propagated in the fiber through reflection. Typical core sizes are 2 to 15 µm. Single mode fiber is also known as fundamental or monomode fiber.

Fig. 1.6.16 shows single mode fiber.

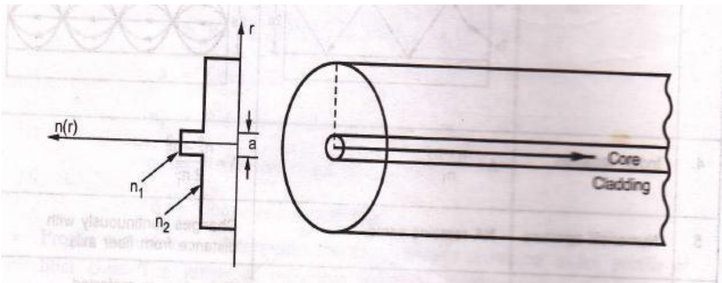


Fig. 1.6.16 The refractive index profile and ray transmission in single mode step index fiber

- Single mode fiber will permit only one mode to propagate and does not suffer from mode delay differences. These are primarily developed for the 1300 nm window but they can be also be used effectively with time division multiplex (TDM) and wavelength division multiplex (WDM) systems operating in 1550 nm wavelength region.
- The core fiber of a single mode fiber is very narrow compared to the wavelength of light being used. Therefore, only a single path exists through the cable core through which light can travel. Usually, 20 percent of the light in a single mode cable actually

  travels down the cladding and the effective diameter of the cable is a blend of single mode core and degree to which the cladding carries light. This is referred to as the 'mode field diameter', which is larger than physical diameter of the core depending on the refractive indices of the core and cladding.

- The disadvantage of this type of cable is that because of extremely small size interconnection of cables and interfacing with source is difficult. Another disadvantage of single mode fibers is that as the refractive index of glass decreases with optical wavelength, the light velocity will also be wavelength dependent. Thus the light from an optical transmitter will have definite spectral width.

Multimode step Index Fiber

- **Multimode step index fiber** is more widely used type. It is easy to manufacture. Its core diameter is 50 to 1000 µm i.e. large aperture and allows more light to enter the cable. The light rays are propagated down the core in zig-zag manner. There are many many paths that a light ray may follow during the propagation.

- The light ray is propagated using the principle of total internal reflection (TIR). Since the core index of refraction is higher than the cladding index of refraction, the light enters at less than critical angle is guided along the fiber.

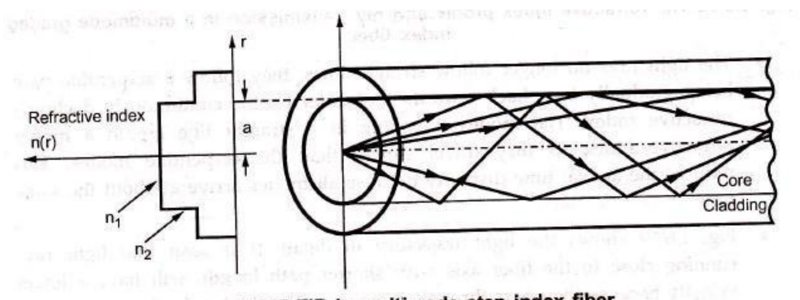


Fig. 1.6.17 TIR in multimode step index fiber

- Light rays passing through the fiber are continuously reflected off the glass cladding towards the centre of the core at different angles and lengths, limiting overall bandwidth.
- The disadvantage of multimode step index fibers is that the different optical lengths caused by various angles at which light is propagated relative to the core, causes the

  transmission bandwidth to be fairly small. Because of these limitations, multimode step index fiber is typically only used in applications requiring distances of less than 1 km.

Multimode Graded Index Fiber

- The core size of **multimode graded index fiber** cable is varying from 50 to 100 µm range. The light ray is propagated through the refraction. The light ray enters the fiber at

  many different angles. As the light propagates across the core toward the center it is intersecting a less dense to more dense medium. Therefore the light rays are being constantly being refracted and ray is bending continuously. This cable is mostly used for long distance communication.

Fig 1.6.18 shows multimode graded index fiber.

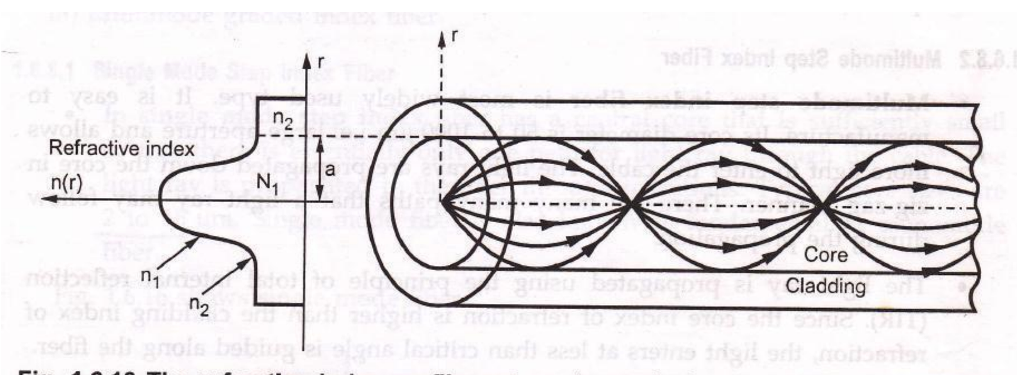


Fig. 1.6.18 The refractive index profile and ray transmission in a multimode graded index fiber

- The light rays no longer follow straight lines, they follow a serpentine path being gradually bent back towards the center by the continuously declining refractive index. The modes travelling in a straight line are in a higher refractive index so they travel slower than the serpentine modes. This reduces the arrival time disparity because all modes arrive at about the same time.
- Fig 1.6.19 shows the light trajectory in detail. It is seen that light rays running close to the fiber axis with shorter path length, will have a lower velocity because they pass through a region with a high refractive index.

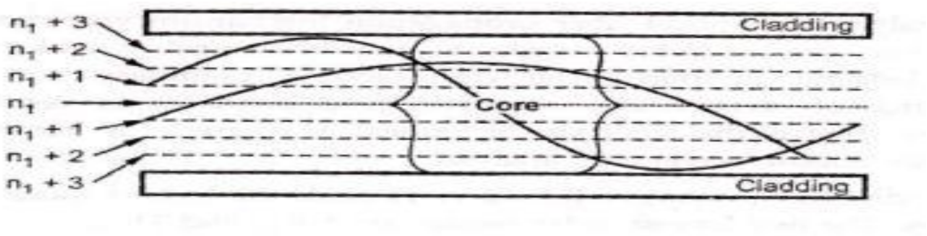


Fig. 1.6.19 Light trajectories in a graded index fiber

- Rays on core edges offers reduced refractive index, hence travel more faster than axial rays and cause the light components to take same amount of time to travel the length of fiber, thus minimizing dispersion losses. Each path at a different angle is termed as

  'transmission mode' and the NA of graded index fiber is defined as the maximum value of acceptance angle at the fiber axis.

- Typical attenuation coefficients of graded index fibers at 850 nm are 2.5 to 3 dB/km, while at 1300 nm they are 1.0 to 1.5 dB/km.
- The main advantages of graded index fiber are:

1. Reduced refractive index at the centre of core.
2. Comparatively cheap to produce.

Standard fibers

Sr. No.	**Fiber type**	**Cladding Diameter (µm)**	**Core diameter (µm)**		**Applications**
1.	Single mode (8/125)	125	8	0.1% to 0.2%	1. Long distance 2. High data rate
2.	Multimode (50/125)	125	50	1% to 2%	1. Short distance 2. Low data rate
3.	Multimode (62.5/125)	125	62.5	1% to 2%	LAN
4.	Multimo	140	100	1% to	LAN

	de (100/140)				2%	

1.7 Mode Theory for Cylindrical Waveguide

- To analyze the optical fiber propagation mechanism within a fiber, Maxwell equations are to solve subject to the cylindrical boundary conditions at core-cladding interface. The core-cladding boundary conditions lead to coupling of electric and magnetic field components resulting in hybrid modes. Hence the analysis of optical waveguide is more complex than metallic hollow waveguide analysis.
- Depending on the large E-field, the hybrid modes are HE or EH modes. The two lowest order does are HE_{11} and TE_{01}.

Overview of Modes

- The order states the number of field zeros across the guide. The electric fields are not completely confined within the core i.e. they do not go to zero at core-cladding interface and extends into the cladding. The low order mode confines the electric field near the axis of the fiber core and there is less penetration into the cladding. While the high order mode distribute the field towards the edge of the core fiber and penetrations into the cladding. Therefore cladding modes also appear resulting in power loss.
- In leaky modes the fields are confined partially in the fiber core attenuated as they propagate along the fiber length due to radiation and tunnel effect.
- Therefore in order to mode remain guided, the propagation factor β must satisfy the condition

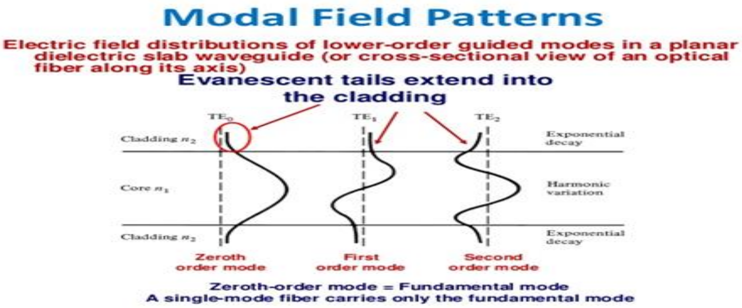
$$n_2 k < \beta < n_1 k$$

where, n_1 = Refractive index of fiber core

n_2 = Refractive index of cladding

k = Propagation constant = $2\pi / \lambda$

- The cladding is used to prevent scattering loss that results from core material discontinuities. Cladding also improves the mechanical strength of fiber core and reduces
surface contamination. Plastic cladding is commonly used. Materials used for fabrication of optical fibers are silicon dioxide (SiO_2), boric oxide-silica.



Summary of Key Modal Concepts

- Normalized frequency variable, V is defined as

$$\boxed{V = \frac{2\pi a(n_1^2 - n_2^2)^{1/2}}{\lambda}} \quad \ldots (1.7.1)$$

where, a = Core radius

λ = Free space wavelength

$$\boxed{V = \frac{2\pi a}{\lambda} NA}$$

Since $(n_1^2 - n_2^2)^{1/2}$ = NA … (1.7.2)

- The total number of modes in a multimode fiber is given by

$$M = \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2)$$

$$M = \frac{1}{2}\left[\frac{2\pi\, a}{\lambda} NA\right]^2 = \frac{[V]^2}{2}$$

$$M = \frac{1}{2}\left[\frac{\pi\, d}{\lambda}.NA\right]^2$$

Example 1.7.1 : Calculate the number of modes of an optical fiber having diameter of 50 µm, n_1 = 1.48, n_2 = 1.46 and λ = 0.82 µm.

Solution : d = 50 µm

n_1 = 1.48

$n_2 = 1.46$
$\lambda = 0.82$
μm

$$NA = (n_1^2 - n_2^2)^{1/2}$$

$$NA = (1.48^2 – 1.46^2)^{1/2}$$

$$NA = 0.243$$

Number of modes are given by,

$$M = \frac{1}{2}\left[\frac{\pi d}{\lambda}.NA\right]^2$$

$$M = \frac{1}{2}\left[\frac{\pi (50 X 10^{-6})}{0.82 X 10^{-6}} X 0.243\right]^2$$

M = 1083 …Ans.

Example 1.7.2 : A fiber has normalized frequency V = 26.6 and the operating wavelength is 1300nm. If the radius of the fiber core is 25 μm. Compute the numerical aperture.

Solution : $V = 26.6$
$\lambda = 1300$ nm $= 1300 X 10^{-9}$ m
$a = 25$ μm $= 25 X 10^{-6}$ m

$$\boxed{V = \frac{2\pi a}{\lambda} NA}$$

$$NA = V.\frac{\lambda}{2\pi a}$$

$$NA = 26.6 \frac{1300 X 10^{-9}}{2\pi X 25 X 10^{-6}}$$

NA = **0.220** **… Ans.**

Example 1.7.3 : A multimode step index fiber with a core diameter of 80 μm and a relative index difference of 1.5 % is operating at a wavelength of 0.85 μm. If the core refractive index is 1.48, estimate the normalized frequency for the fiber and number of guided modes.

Solution : Given : MM step index fiber, 2 a = 80 μm

∴ Core radians a = 40 μm

Relative index difference, = 1.5% = 0.015

Wavelength, λ = 0.85μm

Core refractive index, n1 = 1.48

Normalized frequency, V = ?

Number of modes, M = ?

Numerical aperture

$$\boxed{NA = n_1 (2\Delta)^{1/2}}$$

$$= 1.48 \, (2 \text{ X } 0.015)^{1/2}$$

$$= 0.2563$$

Normalized frequency is given by,

$$\boxed{V = \frac{2\pi a}{\lambda} NA}$$

$$V = \frac{2\pi \text{ X } 40}{0.85} \text{ x } 0.2563$$

$$V = 75.78$$ **… Ans.**

Number of modes is given by,

$$\boxed{M = \frac{V^2}{2}}$$

$$M = \frac{(75.78)^2}{2} = \mathbf{2871.50}$$ **Ans**

Example 1.7.4 : A step index multimode fiber with a numerical aperture of a 0.20 supports approximately 1000 modes at an 850 nm wavelength.

i) What is the diameter of its core?
ii) How many modes does the fiber support at 1320 nm?
iii) How many modes does the fiber support at 1550 nm? **[Jan./Feb.-2007, 10 Marks]**

Solution : i) Number of modes is given by,

$$M = \frac{1}{2}\left[\frac{\pi\ a}{\lambda}.NA\right]^2$$

$$1000 = \frac{1}{2}\left[\frac{\pi\ a}{850\ X\ 10^{-9}}\ X\ 0.20\right]^2$$

$$2000 = 5.464\ X\ a^2$$

a = **60.49 µm … Ans.**

ii)

$$M = \frac{1}{2}\left[\frac{\pi\ X\ 60.49\ X\ 10^{-6}}{1320\ X\ 10^{-9}}\ X\ 0.20\right]^2$$

M = $(14.39)^2$ = **207.07**

iii)

$$M = \frac{1}{2}\left[\frac{\pi\ X\ 6.49\ X\ 10^{-6}}{1320\ X\ 10^{-9}}\ X\ 0.20\right]^2$$

M = **300.63**

Wave Propagation

Maxwell's Equations

Maxwell's equation for non-conducting medium:

∇ X E = - ∂B /

∇ X H = - ∂D /

$\nabla . D = 0$

$\nabla . B \ 0$

where,

E and H are electric and magnetic field vectors.

- The relation between flux densities and filed vectors:

$$D = \varepsilon_0 E + P$$

$$B = \mu_0 H + M$$

where,

ε_0 is vacuum permittivity.

μ_0 is vacuum permeability.

P is induced electric polarization.

M is induced magnetic polarization (M = 0, for non-magnetic silica glass)

- P and E are related by:

$$P(r, t) = \varepsilon_0 \int_{-\infty}^{\infty} X\,(r, t - t')E\,(r, t')dt'$$

Where,

X is linear susceptibility.

- Wave equation:

$$\nabla \times \nabla \times E = \frac{-1}{c^2}\frac{\partial^2 E}{\partial E^2} - \mu_0 \frac{\partial^2 P}{\partial t^2}$$

Fourier transform of E (r, t)

$$\tilde{E}\,(r,\omega) = \int_{-\infty}^{\infty} E\,(r,t)e^{i\omega t}\,dt$$

$$\nabla \times \nabla \times \tilde{E} = -\,\varepsilon\,(r,\omega)\frac{\omega^2}{c^2}\tilde{E}$$

where,

$$\varepsilon = \left(\mathrm{n} + \frac{\mathrm{i}\alpha\mathrm{c}}{2\omega}\right)^2$$

n is refractive index.

α is absorption coefficient.

$$\mathrm{n} = \sqrt{(1 + R_e\chi)}$$

$$\alpha = \left(\frac{\omega}{\mathrm{nc}}\right) \mathrm{I_m}\ \chi$$

- Both n and α are frequency dependent. The frequency dependence of n is called as chromatic dispersion or material dispersion.
- For step index fiber,

$$\nabla \times \nabla \times \tilde{\mathrm{E}} = \nabla\,(\nabla . \tilde{\mathrm{E}}) - \nabla^2 .\ \tilde{\mathrm{E}} = -\nabla^2\,\tilde{\mathrm{E}}$$

Fiber Modes

Optical mode : An optical mode is a specific solution of the wave equation that satisfies boundary conditions. There are three types of fiber modes.

a) Guided modes
b) Leaky modes
c) Radiation modes

- For fiber optic communication system guided mode is sued for signal transmission. Considering a step index fiber with core radius ‘a’.
  The cylindrical co-ordinates ρ, ϕ and can be used to represent boundary conditions.

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} . \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} . \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 k_0^2 E_z = 0$$

- The refractive index ‘n’ has values

$$\mathrm{n} = \begin{cases} \mathrm{n_1}; & \rho \le a \\ \mathrm{n_2}; & \rho > a \end{cases}$$

- The general solutions for boundary condition of optical field under guided mode is infinite at $\rho = 0$ and decay to zero at $\rho = \infty$. Using Maxwell’s equation in the core region.

$$E_\rho = \frac{i}{p^2}\left(\beta \frac{\partial E_z}{\partial \rho} + \mu_0 \frac{\omega}{\rho} . \frac{\partial H_z}{\partial \phi}\right)$$

- The **cut-off condition** is defined as –

$$V = k_0 a \sqrt{(n_1^2 - n_2^2)}$$

It is also called as **normalized frequency**.

$$V = \left(\frac{2\pi}{\lambda}\right) a\, n_1 \sqrt{2\Delta}$$

Graded Index Fiber Structure

- The Refractive index of graded index fiber decreases continuously towards its radius from the fiber axis and that for cladding is constant.
- The refractive index variation in the core is usually designed by using power law relationship.

$$n(r) = \begin{cases} n_1 \left[1 - 2\Delta \left(\frac{r}{a}\right)^{\propto}\right]^{\frac{1}{2}}, & when\ 0 \le r \le a \\ n_1 (1 - 2\Delta)^{\frac{1}{2}} \approx n_1 (1 - \Delta) = n_2, & when\ r \ge a \end{cases}$$

Where, r = Radial distance from fiber axis

a = Core radius

n_1 = Refractive index core

n_2 Refractive index of cladding and

α = The shape of the index profile

- For graded index fiber, the index difference is given by,

- In graded index fiber the incident light will propagate when local numerical aperture at distance r from axis, NA is axial numerical aperture NA(0). The local numerical aperture is given as,

$$NA(r) = \begin{cases} [n^2(r) - n_2^2]^{\frac{1}{2}} \approx NA(0) \sqrt{1 - \left(\frac{r}{a}\right)^{\propto}}, & for\ r \le a \\ 0, & for\ r > a \end{cases}$$

- The axial numerical aperture NA(0) is given as,

$$NA(0) = [n^2(0) - n_2^2]^{1/2}$$

$$NA(0) = [n_1^2 - n_2^2]^{1/2}$$

$$NA(0) = n_1\sqrt{2\Delta} \approx n_1(2\Delta)^{1/2}$$

Hence Na for graded index decreases to zero as it moves from fiber axis to core-cladding boundary.

- The variation of NA for different values of α is shown in Fig. 1.7.1.

- The number of modes for graded index fiber in given as,

$$\boxed{M = \frac{\alpha}{\alpha + 2} a^2 k^2 n_1^2 \Delta} \quad \ldots$$

1.8 Single Mode Fibers

- Propagation in single mode fiber is advantageous because signal dispersion due to delay differences amongst various modes in multimode is avoided. Multimode step index fibers cannot be used for single mode propagation due to difficulties in maintaining single mode operation. Therefore for the transmission of single mode the fiber is designed to allow propagation in one mode only, while all other modes are attenuated by leakage or absorption.
- For single mode operation, only fundamental LP_{01} mode many exist. The single mode propagation of LP_{01} mode in step index fibers is possible over the range.

$$0 \leq V < 2405$$

- The normalized frequency for the fiber can be adjusted within the range by reducing core radius and refractive index difference < 1%. In order to obtain single mode operation with maximum V number (2.4), the single mode fiber must have smaller core diameter than the equivalent multimode step index fiber. But smaller core diameter has problem of launching light into the fiber, jointing fibers and reduced relative index difference.

- Graded index fibers can also be sued for single mode operation with some special fiber design. The cut-off value of normalized frequency Vc in single mode operation for a graded index fiber is given by,

$$V_c = 2.405\left(1+\frac{2}{\alpha}\right)^{\frac{1}{2}}$$

Example 1.8.1 : A multimode step index optical fiber with relative refractive index difference 1.5% and core refractive index 1.48 is to be used for single mode operation. If the operating wavelength is 0.85μm calculate the maximum core diameter.

Solution : Given :

$n_1 = 1.48$

$\Delta = 1.5\ \% = 0.015$

$\lambda = 0.85\ \mu m = 0.85 \times 10^{-6}$ m

Maximum V value for a fiber which gives single mode operations is 2.4.

Normalized frequency (V number) and core diameter is related by expression,

$$V = \frac{2\pi}{\lambda} a\ (NA)$$

$$\boxed{V = \frac{2\pi}{\lambda} a\, n_1 (2\Delta)^{\frac{1}{2}}}$$

$$a = \frac{V\lambda}{2\pi\, n_1 (2\Delta)^{\frac{1}{2}}}$$

$$a = \frac{2.4 \times (0.85 \times 10^{-6})}{2\pi \times (1.48) \times (0.03)^{\frac{1}{2}}}$$

a = **1.3 μm** **… Ans.**

Maximum core diameter for single mode operation is 2.6 μm.

Example 1.8.2 : A GRIN fiber with parabolic refractive index profile core has a refractive index at the core axis of 1.5 and relative index difference at 1%. Calculate maximum possible core diameter that allows single mode operations at λ = 1.3 μm.

Solution : Given :

$n_1 = 1.5$

$\Delta = 1\ \% = 0.01$

$\lambda = 1.3\ \mu m = 1.3 \times 10^{-6} m$

for a GRIN

Maximum value of normalized frequency for single mode operation is given by,

$$V = 2.4\left(1 + \frac{2}{\alpha}\right)^{\frac{1}{2}}$$

Maximum core radius is given by expression,

$$a = \frac{V\lambda}{2\pi\, n_1 (2\Delta)^{\frac{1}{2}}}$$

$$a = \frac{24\sqrt{2}\text{ x } 1.3 \text{ x } 10^{-6}}{2\pi \text{ x } 1.5 \text{ x } (0.02)^{\frac{1}{2}}}$$

a = 3.3 µm **… Ans.**

∴ Maximum core diameter which allows single mode operation is 6.6 µm.

Cut-off Wavelength

- One important transmission parameter for single mode fiber us cut-off wavelength for the first higher order mode as it distinguishes the single mode and multim0de regions.
- The effective cut-off wavelength λ_C is defined as the largest wavelength at which higher order $(L_{P_{11}})$ mode power relative to the fundamental mode $(L_{P_{01}})$ power is reduced to 0.1 dB. The range of cut-off wavelength recommended to avoid modal noise and dispersion problems is : 1100 to 1280 nm (1.1 to 1.28µm) for single mode fiber at 1.3 µm.
- The cut-off wavelength λ_C can be computed from expression of normalized frequency.

$$V = \frac{2\pi}{\lambda} a\, n_1 (2\Delta)^{\frac{1}{2}} \Rightarrow \lambda = \frac{2\pi a n_1}{V}(2\Delta)^{\frac{1}{2}} \qquad \text{…. (1.8.1)}$$

∴

$$\lambda = \frac{2\pi a n_1}{V}(2\Delta)^{\frac{1}{2}} \qquad \text{.... (1.8.2)}$$

where,

V_C is cut-off normalized frequency.

- λ_C is the wavelength above which a particular fiber becomes single moded. For same fiber dividing λ_C by λ we get the relation as:

$$\frac{\lambda_c}{\lambda} = \frac{V}{V_C}$$

$$\lambda = \frac{v\lambda}{v_C} \qquad \dots (1.8.3)$$

But for step index fiver $V_C = 2.405$ then $\lambda_c = \frac{v\lambda}{2.405}$

Example 1.8.3 : Estimate cut-off wavelength for step index fiber in single mode operation. The core refractive index is 1.46 and core radius is 4.5 µm. The relative index difference is 0.25 %.

Solutions : Given :

$n_1 = 1.46$

$a = 4.5$ µm

$\Delta = 0.25\ \% = 0.0025$

Cut-off wavelength is given by,

$$\lambda_c = \frac{2\pi a n_1 (2\Delta)^{\frac{1}{2}}}{V_c}$$

For cut-off wavelength, $V_C = 2.405$

$$\lambda_c = \frac{2\pi \text{ x } 4.5 \text{ x } 1.46\ (0.005)^{\frac{1}{2}}}{2.405} \qquad \lambda_c = 1.214\ \mu\text{m}$$

Mode Field Diameter and Spot Size

- The mode filed diameter is fundamental parameter of a single mode fiber. This parameter is determined from mode field distributions of fundamental LP_{01} mode.
- In step index and graded single mode fibers, the field amplitude distribution is approximated by Gaussian distribution. The **mode Field diameter** (MFD) is distance between opposite 1/e – 0.37 times the near field strength)amplitude) and power is $1/e^2 = 0.135$ times.

- In single mode fiber for fundamental mode, on field amplitude distribution the mode filed diameter is shown in fig. 1.8.1.

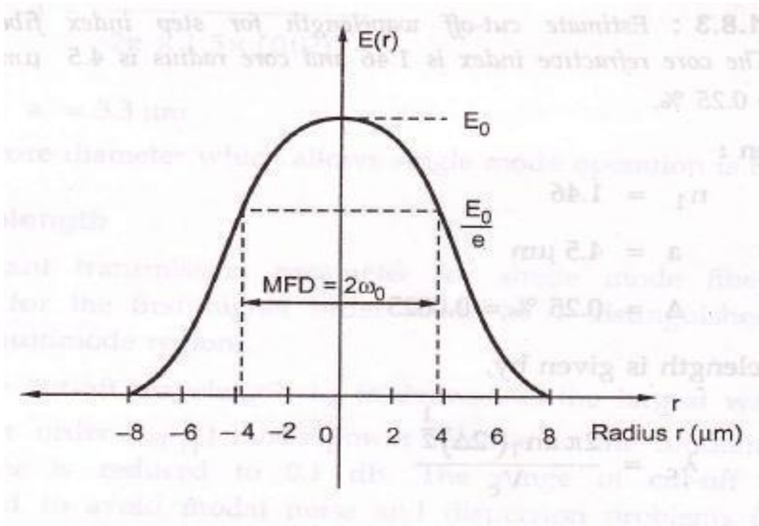


Fig. 1.8.1 Mode field diameter

The spot size ω_0 is gives as –

$$\omega_0 = \frac{MFD}{2}$$

$$MFD = 2\ \omega_0$$

The parameter takes into account the wavelength dependent filed penetration into the cladding.