14

Measurement of Resistance

14.1 CLASSIFICATION OF RESISTANCES

The classification of resistances, from the point of view of measurement, is as follows :

(*i*) *Low resistances.* All resistances of the order of 1 Ω and under may be classified as low resistances.

(*ii*) *Medium resistances.* This class includes resistances from 1 Ω upwards to about 0.1 MΩ.

(*iii*) *High resistances.* Resistances of the order of 0.1 MΩ and upwards are classified as high resistances.

The classification outlined above is not rigid, but forms a basis for techniques, followed or measurement, which may be different for different classes.

14.2 MEASUREMENT OF MEDIUM RESISTANCES

The different methods used for measurement of medium resistances are :

(*i*) Ammeter-Voltmeter method.
(*ii*) Substitution method.
(*iii*) Wheatstone bridge method.
(*iv*) Ohmmeter method.

14.2.1 Ammeter-Voltmeter Method

This method is very popular since the instruments required for this test are usually available in the laboratory. The *two* types of connections employed for ammeter-voltmeter method are shown in Figs. 14.1(*a*) and (*b*). In both the cases, if readings of ammeter and voltmeter are taken, then the measured value of resistance is given by :

$$R_m = \frac{\text{voltmeter reading}}{\text{ammeter reading}} = \frac{V}{I} \qquad \text{...(14.1)}$$

The measured value of resistance R_m, would be equal to the true value, R, if the ammeter resistance is zero and the voltmeter resistance is infinite, so that the conditions in the circuit are not disturbed. However, in practice this is not possible and hence both the methods give inaccurate results.

Consider circuit of Fig. 14.1(*a*). In this circuit the ammeter measures the true value of the current through the resistance but the voltmeter does not measure the true voltage across the resistance. The voltmeter indicates the sum of the voltages across the ammeter and the measured resistance.

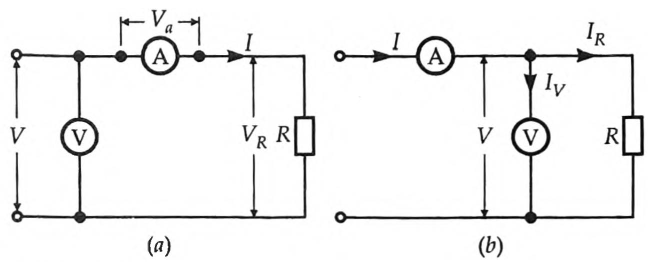


Fig. 14.1 Measurement of resistance by ammeter-voltmeter method.

Let R_a be the resistance of the ammeter.

∴ Voltage across the ammeter, $V_a = IR_a$

Now, measured value of resistance,

$$R_{m1} = \frac{V}{I} = \frac{V_R + V_a}{I} = \frac{IR + IR_a}{I} = R + R_a \qquad \text{...(14.2)}$$

∴ True value of resistance,

$$R = R_{m1} - R_a \qquad \text{...(14.3)}$$

$$= R_{m1}\left(1 - \frac{R_a}{R_{m1}}\right) \qquad \text{...(14.4)}$$

Thus the measured value of resistance is higher than the true value. It is also clear from above that the true value is equal to the measured only if the ammeter resistance, R_a, is zero.

$$\text{Relative error, } \varepsilon_r = \frac{R_{m1} - R}{R} = \frac{R_a}{R} \qquad \text{...(14.5)}$$

It is clear from Eqn. 14.5 that the error in measurements would be small if the value of resistance under measurement is large as compared to the internal resistance of the ammeter. Therefore the circuit of Fig. 14.1(*a*) should be used when measuring high resistance values.

Consider the circuit of Fig. 14.1(*b*). In this circuit the voltmeter measures the true value of voltage but the ammeter measures the sum of currents through the resistance and the voltmeter.

Let R_v be the resistance of the voltmeter.

Current through the voltmeter, $I_v = V / I_v$

Measured value of resistance,

$$R_{m2} = \frac{V}{I} = \frac{V}{I_R + I_v} = \frac{V}{V/R + V/R_v} = \frac{R}{1 + R/R_v}$$

True value of resistance,

$$R = \frac{R_{m2}R_v}{R_v - R_{m2}} = R_{m2}\left(\frac{1}{1 - R_{m2}/R_v}\right) \quad \text{...(14.6)}$$

From Eqn. 14.6 it is clear that the true value of resistance is equal to the measured value only if the resistance of voltmeter, R_v, is infinite. However, if the resistance of voltmeter is very large as compared to the resistance under measurement :

or $R_v >> R_{m2}$, and therefore R_{m2} / R_v is very small.

We have, $R = R_{m2}\left(1 + \frac{R_{m2}}{R_v}\right)$ ...(14.7)

Thus the measured value of resistance is smaller than the true value.

Relative error, $\varepsilon_r = \frac{R_{m2} - R}{R} = \frac{R_{m2}^2}{R_v R}$ ...(14.8)

The value of R_{m2} is approximately equal to R.

$$\therefore \quad \varepsilon_r = -\frac{R}{R_v} \quad \text{...(14.9)}$$

It is clear from Eqn. 14.9 that the error in measurement would be small if the value of resistance under measurement is very small as compared to the resistance of the voltmeter. Hence the circuit of Fig. 14.1(*b*) should be used when measuring low resistance values.

The Ammeter-Voltmeter method, in the two forms explained above, is a simple method but is, essentially a comparatively rough method, the accuracy being limited by accuracy of ammeter and voltmeter used, even if corrections are made for the voltage drop across the ammeter for connections of Fig. 14.1(*a*) and for the shunting effect of voltmeter for connections of Fig. 14.1(*b*). If the two instruments are of 0.5% accuracy and are read near full scale, the instrumental error in the result may be any thing from 0 to 1%. If read near half scale, the percentage error may be twice as great and for lower readings may mount considerably higher. With less accurate instruments the possible error, of course is increased. It is difficult to obtain accuracy much better than 1% in a resistance value under usual conditions and the error sometimes may be considerably higher. However, the method is useful in some laboratory work in which high accuracy is not required.

The suitability of a particular method for resistance measurement depends upon the relative values of resistance under measurement and the resistance of the meters.

The division point between the two methods is at the resistance for which the relative errors given by the two methods are equal.

The relative errors for the two cases are equal when :

$$\frac{R_a}{R} = \frac{R}{R_v} \quad \text{(See Eqns. 14.5 and 14.9)}$$

or when true value of resistance

$$R = \sqrt{R_a R_v} \quad \text{...(14.10)}$$

For resistances greater than the value given by Eqn. 14.10 the method of Fig. 14.1(*a*) is used while for lower resistances method of 14.1(*b*) is used.

Example 14.1 *In the measurement of a resistance, R, by the Ammeter-voltmeter method, connections as in Figs. 14.1(a) and 14.1(b) are used. The resistance of ammeter is 0.01 Ω and that of voltmeter, 2000 Ω. In case of (b) the current measured is 2 A and the voltage 180 V. Find the percentage error in calculating resistance R as a quotient of the readings and the true value of R. Also find the reading of the voltmeter in case of (a) if the current indicated by the ammeter is 2 A.*

Solution. Case (*b*) See Fig. 14.1(*b*)

Measured value of resistance

$$R_{m2} = V / I = 180/2 = 90\ \Omega.$$

Current through the voltmeter

$$I_v = V / R_v = 180/2000 = 0.09\ \text{A}.$$

Current through the resistance

$$I_R = I - I_v = (2.0 - 0.09)\ \text{A}.$$

True value of resistance

$$R = \frac{V}{I_R} = \frac{180}{(2.0 - 0.09)} = 94.24\ \Omega.$$

Percentage error

$$= \frac{R_{m2} - R}{R} \times 100 = \frac{90 - 94.24}{94.24} = -4.5\%.$$

Case (*a*) See Fig. 14.1(*a*)

Reading of voltmeter

$$V = V_a + V_R = I(R_a + R) = 2(0.01 + 94.24)$$
$$= 188.50 \text{ V}.$$

Example 14.2 *A resistance of approximate value of 80 Ω is to be measured by voltmeter-ammeter method using a 1 A ammeter having a resistance of 2 Ω and a 50 V voltmeter having a resistance of 5000 Ω.*

(*a*) *Suggest which one of the two methods should be used ?*

(*b*) *Supposing in the suggested method the following measurements are made :*

$$I = 0.42 \text{ A} \quad and \quad V = 35.5 \text{ V}.$$

What is the resulting error if the accuracy of the instruments is ± 0.5% at full scale and the errors are standard deviations.

Solution. (*a*) Value of resistance for which the errors are equal for the two types of connections

$$R = \sqrt{R_A R_v} = \sqrt{2 \times 5000} = 100\ \Omega$$

Since the resistance to be measured has a value less than 100 Ω, the method of Fig. 14.1 (*b*) should be used as it results in smaller error.

Measured value of resistance

$$R_{m2} = V / I = 35.5/0.42 = 84.52\ \Omega$$

True value of resistance

$$R = R_{m2}\left(\frac{1}{1 - R_{m2}/R_v}\right)$$

$$= 84.52\left(\frac{1}{1 - 84.52/5000}\right) = 86\ \Omega.$$

(*b*) Error in ammeter reading

$$= (0.5/100) \times 1 = 0.005 \text{ A}$$

∴ Percentage error at 0.42 A reading

$$= (0.005/0.42) \times 100 = 1.19\%$$

Error in voltmeter reading

$$= 0.5 \times (50/100) = 0.25 \text{ V}$$

∴ Percentage error at 35.5 V reading

$$= (0.25/35.5) \times 100 = 0.704\%$$

Since the errors correspond to standard deviations, error due to ammeter and voltmeter

$$= \sqrt{(1.19)^2 + (0.704)^2} = \pm 1.38\%$$

Absolute error due to ammeter and voltmeter

$$= (1.38/100) \times 86 \approx \pm 1.2\ \Omega.$$

∴ The resistance is specified as 86 ± 1.2 Ω.

Example 14.3 *A resistance R is measured using the connections of Fig. 14.1(a). The current measured is 10 A on range 10 A and the voltage measured is 125 V on 150 V range. The scales of the ammeter and voltmeter are uniform, the total scale divisions of ammeter are 100 and that of voltmeter are 150. The scales of these instruments are such that 1/10 of a scale division can be distinguished. The constructional error of the ammeter is ± 0.3% and that of voltmeter ± 0.4%. The resistance of the ammeter is 0.25 Ω. Calculate the value of R and the limits of possible error in the results.*

Solution. Reading error of ammeter

$$= \pm\frac{1}{10 \times 100} \times 100 = \pm 0.1\%$$

Reading error of voltmeter

$$= \pm\frac{1}{10 \times 150} \times 100 = \pm 0.087\%$$

Total error of ammeter $\delta I = \pm 0.3 \pm 0.1 = \pm 0.4\%$

Total error of voltmeter

$$\delta V = \pm 0.4 \pm 0.067 = \pm 0.467\%$$

Now, resistance $R = V/I$ and therefore total systematic error in measurement

$$= \pm \delta V \pm \delta A = \pm 0.467 \pm 0.4 = \pm 0.867\%$$

Measured value of resistance

$$R_{m2} = 125/10 = 12.5\ \Omega.$$

True value of resistance

$$R = R_{m1}\left(1 - \frac{R_a}{R_{m1}}\right) \qquad \text{(See Eqn. 14.4)}$$

$$= 12.5\left(1 - \frac{0.25}{12.5}\right) = 12.25\ \Omega.$$

Therefore the value of R is specified as

12.25 Ω ± 0.867% = 12.25 ± 0.11 Ω.

14.2.2 Substitution Method

The connection diagram for this method is shown in Fig. 14.2. *R* is the unknown resistance while *S* is a standard variable resistance. '*A*' is an ammeter and '*r*' is a regulating resistance. There is a switch for putting *R* and *S* into the circuit alternately.

The switch is put at position '*1*' and resistance *R* is connected in the circuit. The regulating resistance *r* is adjusted till the ammeter pointer is at a chosen scale mark. Now, the switch is thrown to position '2' putting the standard variable resistance *S* in the circuit. The value of *S* is varied till the same deflection as was obtained with *R* in the circuit is obtained. The settings of the dials of *S* are read. Since the substitution of one resistance for another has left the current unaltered, and provided that the emf of battery and the position of r are unaltered, the two resistances must be equal. Thus the value of unknown resistance R is equal to the dial settings of resistance S.

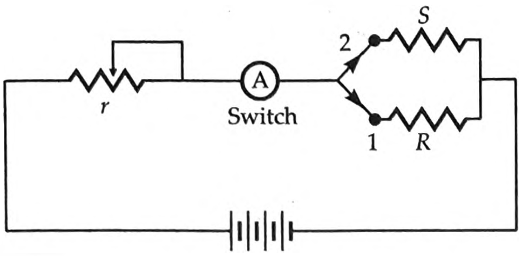


Fig. 14.2 Substitution method.

This is a more accurate method than the Ammeter-Voltmeter method, as it is not subject to the errors encountered in the latter method. However, the accuracy of this method is greatly affected if there is any change in the battery emf during the time the readings on the two settings are taken. Thus in order to avoid errors on this account, a battery of ample capacity should be used so that its emf remains constant.

The accuracy of the measurement naturally depends upon the constancy of the battery emf and of the resistance of the circuit excluding R and S, upon the sensitivity of the instrument, and upon the accuracy with which standard resistance S is known.

This method is not widely used for simple resistance measurements and is used in a modified form for the measurement of high resistances. The substitution principle, however, is very important and finds many applications in bridge methods and in high frequency a.c. measurements.

Example 14.4 *In a measurement of resistance by substitution method a standard 0.5 MΩ resistor is used. The galvanometer has a resistance of 10 kΩ and gives deflections as follows :*

(*i*) *With standard resistor, 41 divisions,*

(*ii*) *With unknown resistance, 51 divisions.*

Find the unknown resistance.

Solution. The deflection of the galvanometer is directly proportional to the current passing through the circuit and hence is inversely proportional to the total resistance of the circuit. Let S, R and G be respectively the resistances of standard resistor, unknown resistor and the galvanometer. Also let θ_1 be the deflection with standard resistor in circuit and θ_2 with unknown resistor in circuit.

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{R+G}{S+G}$$

Hence, unknown resistance

$$R = (S+G)\frac{\theta_1}{\theta_2} - G$$

$$= (0.5\times10^6 + 10\times10^3)\times(41/51) - 10\times10^3$$

$$= 0.4\times10^6\,\Omega = 0.4\ \text{M}\Omega.$$

14.2.3 Wheatstone Bridge

A very important device used in the measurement of medium resistances is the Wheatstone bridge. A Wheatstone bridge has been in use longer than almost any electrical measuring instrument. It is still an accurate and reliable instrument and is extensively used in industry. The Wheatstone bridge is an instrument for making *comparison measurements* and operates upon a *null indication* principle. This means the indication is independent of the calibration of the null indicating instrument or any of its characteristics. For this reason, very high degrees of accuracy can be achieved using Wheatstone bridge. Accuracy of 0.1% is quite common with a Wheatstone bridge as opposed to accuracies of 3% to 5% with ordinary ohmmeter for measurement of medium resistances. Figure 14.3 shows the basic circuit of a Wheatstone bridge. It has four resistive arms, consisting of resistances P, Q, R and S together with a source of emf (a battery) and a null detector, usually a galvanometer G or other sensitive current meter. The current through the galvanometer depends on the potential difference between points c and d. The bridge is said to be balanced when there is no current through the galvanometer or when the potential difference across the galvanometer is zero. This occurs when the voltage from point '*b*' to point '*a*' equals the voltage from point '*d*' to point '*b*'; or, by referring to the other battery terminal, when the voltage from point '*d*' to point '*c*' equals the voltage from point '*b*' to point '*c*'.

For bridge balance, we can write,

$$I_1 P = I_2 R \qquad \text{...(14.11)}$$

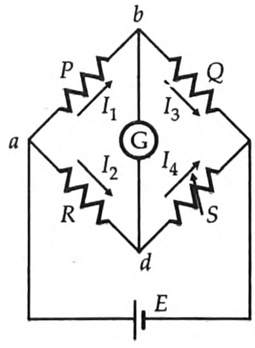


Fig. 14.3 Wheatstone bridge.

For the galvanometer current to be zero, the following conditions also exist :

$$I_1 = I_3 = \frac{E}{P+Q} \quad \text{...(14.12)}$$

and $$I_2 = I_4 = \frac{E}{R+S} \quad \text{...(14.13)}$$

where E = emf of the battery.

Combining Eqns. 14.11, 14.12 and 14.13 and simplifying, we obtain :

$$\frac{P}{P+Q} = \frac{R}{R+S} \quad \text{...(14.14)}$$

from which $QR = PS$ ...(14.15)

Equation 14.15 is the well known expression for the balance of Wheatstone bridge. If three of the resistances are known, the fourth may be determined from Eqn. 14.15 and we obtain :

$$R = S\frac{P}{Q} \quad \text{...(14.16)}$$

where R is the unknown resistance, S is called the *'standard arm'* of the bridge and P and Q are called the *'ratio arms'*.

In the industrial and laboratory form of the bridge, the resistors which make up P, Q and S are mounted together in a box, the appropriate values being selected by dial switches. Battery and galvanometer switches are also included together with a galvanometer and a dry battery in the portable sets. P and Q normally consist of four resistors each, the values being 10, 100, 1000 and 10,000 Ω respectively S consists of a 4 dial or 5 dial decade arrangement of resistors. Figure 14.4 shows the commercial form of Wheatstone bridge.

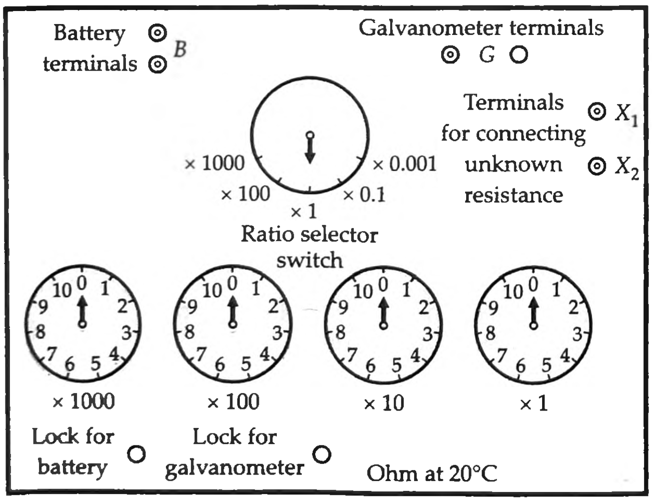


Fig. 14.4 Commercial form of Wheatstone bridge.

14.2.4 Sensitivity of Wheatstone Bridge

It is frequently desirable to know the galvanometer response to be expected in a bridge which is slightly unbalanced so that a current flows in the galvanometer branch of the bridge network. This may be used for :

(*i*) selecting a galvanometer with which a given unbalance may be observed in a specified bridge arrangement,

(*ii*) determining the minimum unbalance which can be observed with a given galvanometer in the specified bridge arrangement, and

(*iii*) determining the deflection to be expected for a given unbalance.

The sensitivity to unbalance can be computed by solving the bridge circuit for a small unbalance. The solution is approached by converting the Wheatstone bridge of Fig. 14.3 to its "*Thevenin Equivalent*" circuit. Assume that the bridge is balanced when the branch resistances are P, Q, R, S so that $P/Q = R/S$. Suppose the resistance R is changed to $R + \Delta R$ creating an unbalance. This will to cause an emf e to appear across the galvanometer branch. With galvanometer branch open, the voltage drop between points a and b is :

$$E_{ab} = I_1 P = \frac{EP}{P+Q}$$

Similarly, $$E_{ad} = I_2(R+\Delta R) = \frac{E(R+\Delta R)}{R+\Delta R+S}$$

Therefore voltage difference between points d and b is :

$$e = E_{ad} - E_{ab} = E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right] \quad \text{...(14.17)}$$

and since $$\frac{P}{P+Q} = \frac{R}{R+S}$$

$$\therefore \quad e = E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+S}\right]$$

$$= \frac{ES\Delta R}{(R+S)^2 + \Delta R(R+S)}$$

$$\approx \frac{ES\Delta R}{(R+S)^2} \quad \text{...(14.18)}$$

as $\Delta R(R+S) << (R+S)^2$

Let S_v be the voltage sensitivity of galvanometer. Therefore, deflection of galvanometer is

$$\theta = S_v e = S_v \frac{ES\Delta R}{(R+S)^2} \quad \text{...(14.19)}$$

The bridge sensitivity S_B is defined as the deflection of the galvanometer per unit fractional change in unknown resistance.

Bridge sensitivity

$$S_B = \frac{\theta}{\Delta R / R} \quad \text{...(14.20)}$$

$$= \frac{S_v ESR}{(R+S)^2} \quad \text{...(14.21)}$$

From Eqn. 14.21, it is clear that the sensitivity of the bridge is dependent upon bridge voltage, bridge parameters and the voltage sensitivity of the galvanometer. Rearranging the terms in the expression for sensitivity,

$$S_B = \frac{S_v E}{(R+S)^2 / SR} = \frac{S_v E}{\frac{R}{S} + 2 + \frac{S}{R}}$$

$$= \frac{S_v E}{\frac{P}{Q} + 2 + \frac{Q}{P}} \quad \text{...(14.22)}$$

From Eqn. 14.22, it is apparent that maximum sensitivity occurs where $R/S = 1$. As the ratio becomes either larger or smaller, the sensitivity decreases. Since the accuracy of measurement is dependent upon sensitivity a limit can be seen to the usefulness for a given bridge, battery and galvanometer combination.

For a bridge with equal arms, $R = S = P = Q$,

Bridge sensitivity $\quad S_B = \frac{S_v E}{4}$ ...(14.23)

As explained above the sensitivity is maximum when the ratio is unity. The sensitivity with ratio $P/Q = R/S = 1000$ would be about 1/250 of that for unity ratio. The sensitivity with $P/Q = R/S = 1000$ would similarly be about 1/250 of that for unity ratio.

Thus the sensitivity decreases considerably if the ratio $P/Q = R/S$ is greater or smaller than unity. This reduction in sensitivity is accompanied by a reduction in accuracy with which a bridge can be balanced.

Galvanometer current. The current through the galvanometer can be found out by finding the Thevenin equivalent circuit. The Thevenin or open circuit voltage appearing between terminals b and d with galvanometer circuit open circuited is,

$$E_0 = E_{ad} - E_{ac} = I_2(R+\Delta R) - I_1 P$$

$$= \frac{E(R+\Delta R)}{R+\Delta R+S} - \frac{EP}{P+Q}$$

$$= E\left[\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right] \quad \text{...(14.24)}$$

For a bridge with equal arms, $R = S = P = Q$.

$$E_0 = E\left[\frac{R+\Delta R}{R+\Delta R+R} - \frac{R}{R+R}\right]$$

$$= E\left[\frac{R+\Delta R}{2R+\Delta R} - \frac{1}{2}\right]$$

$$\approx E\left(\frac{AR}{4R}\right) \text{ as } \Delta R << R. \quad \text{...(14.25)}$$

The resistance of the Thevenin equivalent circuit is found by looking back into terminals c and d (Fig. 14.3) and replacing the battery by its internal resistance. In most cases, however, the extremely low resistance of the battery can be neglected and this simplifies the solution as we can assume that terminals a and b are shorted. The Thevenin equivalent resistance can be calculated by referring to Fig. 14.5.

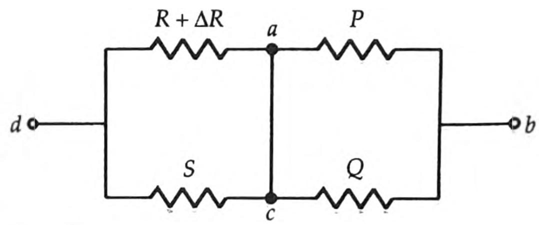


Fig. 14.5 Finding resistance of bridge looking into terminals *d* and *b*.

Thevenin equivalent resistance of bridge

$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q} \quad \text{...(14.26)}$$

Considering $\Delta R << R$

For a bridge with equal arms,

$$P = Q = S = R,$$

$$R_0 = R \quad \text{...(14.27)}$$

The Thevenin equivalent of the bridge circuit therefore reduces to a Thevenin generator with an emf E_0 and an internal resistance R_0. This circuit is shown in Fig. 14.6.

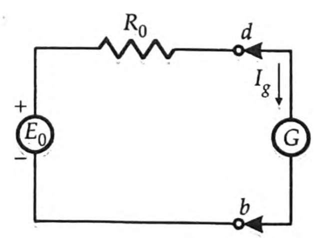


Fig. 14.6 Thevenin equivalent circuit of Wheatstone bridge.

∴ The current in the galvanometer circuit

$$I_g = \frac{E_0}{R_0 + G} \quad \text{...(14.28)}$$

where G = resistance of the galvanometer circuit.

For a bridge with equal arms,

$$I_g = \frac{E(\Delta R/4R)}{(R+G)} \qquad ...(14.29)$$

The deflection of the galvanometer for a small change in resistance in the unknown arm is,

$$\theta = \frac{S_v E S \Delta R}{(R+S)^2}$$

But $\quad S_v = \dfrac{S_i}{R_0 + G}$

where S_i = current sensitivity of the galvanometer.

$$\therefore \quad \theta = \frac{S_i E S \Delta R}{(R_0+G)(R+S)^2} \qquad ...(14.30)$$

For a bridge with equal arms,

$$\theta = \frac{S_i E \Delta R}{4R(R+G)} \qquad ...(14.31)$$

Also bridge sensitivity,

$$S_B = \frac{\theta}{\Delta R/R} = \frac{S_i E S R}{(R_0+G)(R+S)^2} \qquad ...(14.32)$$

For a bridge with equal arms,

Bridge sensitivity,

$$S_B = \frac{S_i E}{4(R+G)} \qquad ...(14.33)$$

14.2.5 Precision Measurement of Medium Resistances with Wheatstone Bridge

It is sometimes necessary to measure resistances to a precision of 1 part in 10,000 or even greater by comparing them with standard resistances. In such cases more than ordinary precautions are necessary in order to secure the required accuracy. The following factors should be taken into consideration.

1. ***Resistance of connecting leads.*** A lead of 22 SWG wire having a length of 25 cm has a resistance of about 0.012 Ω and this represents more than 1 part in 1000 for a 10 Ω resistance or more than one part in 10,000 for a 100 Ω resistance.

2. ***Thermo-electric effects.*** Thermoelectric emfs are often present in the measuring circuit and they must be taken into account since they affect the galvanometer deflection in the same way as an emf occurring because of unbalance. The effect of thermoelectric emfs and other parasite emfs on the measurement may be eliminated by reversing the battery connections to the bridge through a quick acting switch and adjusting the balance until no change in galvanometer deflection can be observed on reversal. The results are obtained by averaging the two readings. This way the effect of thermo-electric emfs is eliminated with the added advantage of doubling the sensitivity of the bridge.

3. ***Temperature effects.*** The errors caused by change of resistance due to change of temperature produces serious errors in measurements especially in the case of resistances made up of materials having a large value of resistance temperature coefficient. For example in the case of copper, which has a resistance temperature coefficient of 0.004/°C, a change in temperature of 1°C will cause an error of 0.4% or 1 part in 250.

4. ***Contact resistance.*** Serious errors may be caused by contact resistances of switches and binding posts. A dial may have a contact resistance of about 0.003 Ω and thus a 4 dial resistance box has a contact resistance of about 0.01 Ω. This value is quite high especially when low resistance measurements are being done. Another aspect of the contact resistance is that error caused by it is difficult to account for since its magnitude *i.e.*, magnitude of contact resistance is uncertain.

In precision resistance measurements, the most accurate comparisons are made on an equal ratio bridge with a fixed standard resistance nominally equal to resistance under test. Then with equal leads, equal currents and equal heating of the ratio arms, the possible errors are minimized. The problem is then reduced to that of determining the exact ratio of the unknown resistance, R_1 to the standard resistance S or the difference between them. The different methods used for this purpose are :

(*i*) *Change in ratio arms.* Small known changes are made in the ratio arms and the exact balance is obtained.

(*ii*) *Using a high resistance shunt.* In this case the bridge is balanced by means of an adjustable high resistance put in parallel with one of the bridge arms. A decade resistance box is used for this purpose. Suppose the test resistance, R, is slightly lower in value than the standard resistance, S and let the balance be obtained with a resistance xS, put in parallel with S.

The resistance of this arm is then

$$xS^2/(S+xS), \; i.e., \; S/(1+1/x).$$

$\therefore$ Value of unknown resistance

$$R = \frac{P}{Q} \cdot \frac{S}{1+1/x}.$$

Since x is very large and therefore $1/x$ is very small and hence we can write,

$$R = \frac{P}{Q} \cdot S(1 - 1/x) \quad ...(14.34)$$

For an equal arm bridge, $P = Q$ and therefore,

$$R = S(1 - 1/x) \quad ...(14.35)$$

The fractional difference, $1/x$ between S and R can be known. Since this method determines the fractional difference and x has a large value, the high resistance put in parallel with S need not be particularly accurate.

(iii) Carey foster slide wire bridge. A slide wire bridge is used for the purpose of determining the difference between the standard and the unknown resistances. The details are explained below.

14.2.6 Carey-Foster Slide-wire Bridge

The connections of this bridge are shown in Fig. 14.7, a slide-wire of length L being included between R and S as shown. This bridge is specially suited for the comparison of two nearly equal resistances.

Resistances P and Q are first adjusted so that the ratio P/Q is approximately equal to the ratio R/S. Exact balance is obtained by adjustment of the sliding contact on the slide-wire. Let l_1 be the distance of the sliding contact form the left-hand end of the slide wire. The resistances R and S are then interchanged and balance again obtained. Let the distance now be l_2.

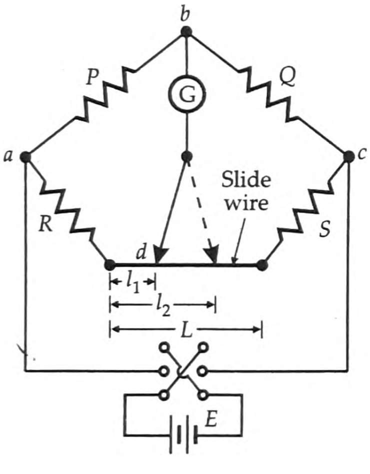


Fig. 14.7 Carey-Foster Slide wire bridge.

Then for the first balance

$$\frac{P}{Q} = \frac{R + l_1 r}{S + (L - l_1) r}$$

where r is the resistance per unit length of the slide-wire.

For the second balance, $\dfrac{P}{Q} = \dfrac{S + l_2 r}{R + (L - l_2)}$

Now, for first balance,

$$\frac{P}{Q} + 1 = \frac{R + l_1 r + S + (L - l_1) r}{S + (L - l_1) r} = \frac{R + S + Lr}{S + (L - l_1) r} \quad ...(i)$$

also, for second balance,

$$\frac{P}{Q} + 1 = \frac{S + l_2 r + R + (L - l_2) r}{R + (L - l_2) r} = \frac{S + R + Lr}{R + (L - l_2) r} \quad ...(ii)$$

From (*i*) and (*ii*), we have,

$$S + (L - l_1) r = R + (L - l_2) r$$

Hence $$S - R = (l_1 - l_2) r \quad ...(14.36)$$

Thus the difference between S and R is obtained from the resistance per unit length of the slide-wire together with the difference $(l_1 - l_2)$ between the two slide-wire lengths at balance.

The slide-wire is calibrated *i.e.*, r is obtained by shunting either S or R by a known resistance and again determining the difference in length $(l_1' - l_2')$.

Suppose that S is known and that S' is its value when shunted by a known resistance ; then

$$S - R = (l_1 - l_2) r \quad \text{and} \quad S' - R = (l_1' - l_2') r$$

$$\therefore \quad \frac{S - R}{l_1 - l_2} = \frac{S' - R}{l_1' - l_2'}$$

from which $$R = \frac{S(l_1' - l_2') - S(l_1 - l_2)}{(l_1' - l_2' - l_1 + l_2)} \quad ...(14.37)$$

Equation 14.37 shows that this method gives a direct comparison between S and R in terms of lengths only, the resistances of P and Q contact resistances, and the resistances of connecting leads being eliminated.

As it is important that the two resistors R and S shall not be handled or disturbed during the measurement, a special switch is used to affect the interchanging of these two resistors during the test.

14.2.7 Kelvin-Varley Slide

A Kelvin-Varley slide is used for voltage division. This method is very precise and finds extensive applications. A Kelvin-Varley slide is shown in Fig. 14.8. It consists of several decades of resistors which are interconnected. The voltage division is carried out successively. Each voltage division decade is made up of eleven equal resistors with successive division decades having a total resistance equal to twice the value of a unit resistor in the previous decade. For example, in the Kelvin Varley Slide shown in Fig. 14.8, there are four decade dividers. This decade is constructed using 11 resistance coils having a resistance of 10 kΩ each. The second decade divider has 11 resistors of 2 kΩ) each*. Similarly, the 3rd decade has 11 resistors of 400 Ω each and the fourth and final decade has 10 resistors of 80 Ω each.

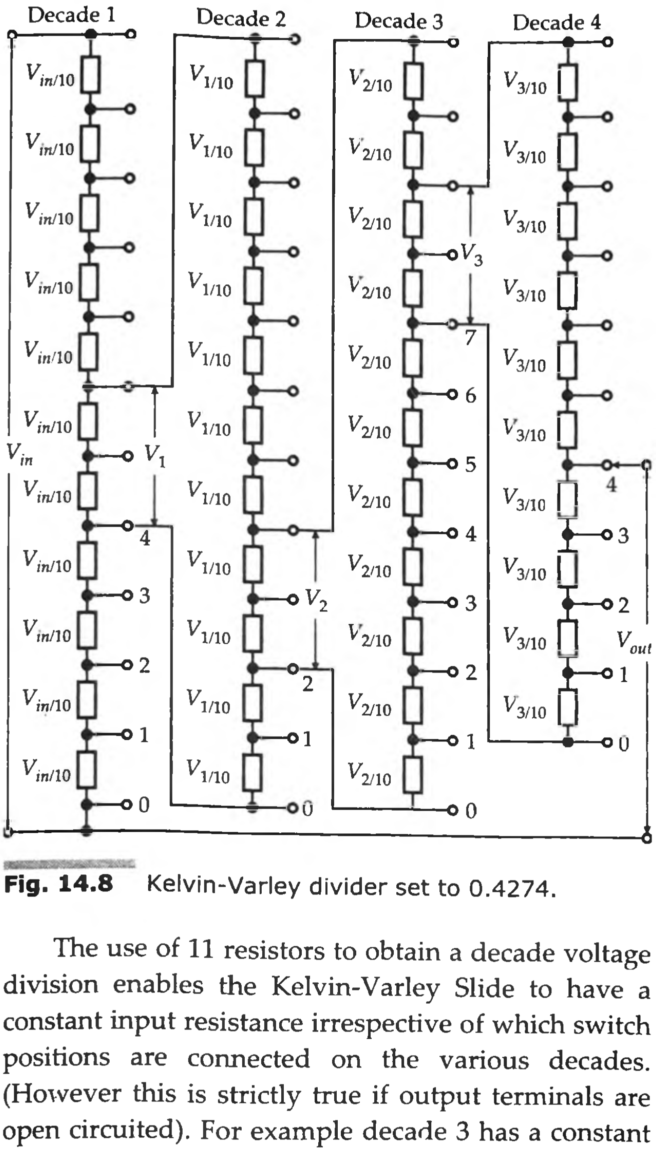


Fig. 14.8 Kelvin-Varley divider set to 0.4274.

The use of 11 resistors to obtain a decade voltage division enables the Kelvin-Varley Slide to have a constant input resistance irrespective of which switch positions are connected on the various decades. (However this is strictly true if output terminals are open circuited). For example decade 3 has a constant input resistance of :

$$9 \times 400 + \frac{1 \times 400 + 10 \times 80}{800 + 800} = 4000\ \Omega = 4\ \text{k}\Omega.$$

Decade 2 has an input resistance of

$$9 \times 2000 + \frac{1 \times 2000 \times 4000}{4000 + 4000} = 20{,}000\ \Omega = 20\ \text{k}\Omega.$$

The input terminal impedance is 100 kΩ irrespective of the switch position if it is assumed that the output current is negligible.

The advantage of using the Kelvin-Varley Slide is the reduction of errors which arise on account of switch contact resistance due to current sharing within the device. The disadvantages of the Kelvin-Varley Slide are its calibration and the errors on account of temperature. The errors on account of temperature are dut to the fact that the resistors carry different accounts and the changes in the value of r ohm resistance is different on account of different self-heating conditions. The temperature effects can be reduced to negligible proportions by using resistors made of materials having a low resistance temperature co-efficient. The error may be reduced to as low a value as ± 0.1 ppm. The principle of Kelvin-Varley Slides is used with advantage in potentio- meters and universal shunts.

Figure 14.9 shows the use of Kelvin-Varley slide in a Wheatstone bridge. The device is used to replace the simple slide wire with the advantage that it gives greater accuracy.

For the case shown in Fig. 14.9

$$\frac{R}{S} = \frac{7554}{5346}$$

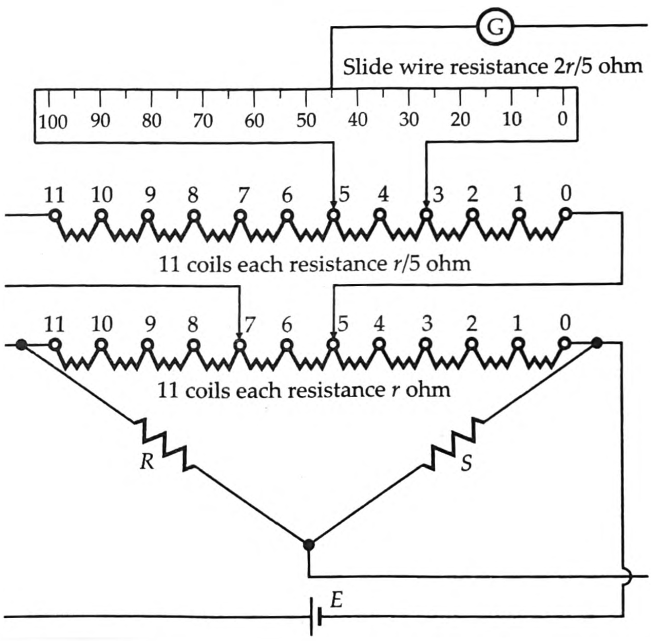


Fig. 14.9 Kelvin-Varley slide.

14.2.8 Limitations of Wheatstone Bridge

The use of Wheatstone bridge is limited to the measurement of resistances ranging from a few ohm to several megohms. The upper limit is set by the reduction in sensitivity to unbalance caused by high resistance values. The upper limit can be extended to a

* Resistance of each unit in first decade is 10 kΩ. The second decade has a total resistance of 2 × 10 = 20 kΩ. Therefore, each unit in the second decade has a resistance of 20/10 = 2 kΩ. In other words, we can say that if the first decade consists of 11 coils of resistance r ohm each, the next decade has 11 coils of $r/5$ ohm each and the third decade has 11 coils of $2r/5$ ohm each.

certain extent by increasing the emf applied to the bridge but in this case care has to be taken to avoid overheating of any arm of the bridge. Inaccuracy may also be introduced on account of leakage over insulation of the bridge arms when measuring very high resistances. For measurement of very high resistances, a megohm bridge (described on page 441) is preferable.

The lower limit for measurement is set by the resistance of the connecting leads and by contact resistance at the binding posts. The error caused by leads may be corrected fairly well, but contact resistance presents a source of uncertainty that is difficult to overcome. The lower limit for accurate measurement is in the neighbourhood of 1 to 5 ohm. For low resistance measurements, therefore, a Kelvin bridge (described in Art. 14.3.2 on page 434) is generally preferred.

Example 14.5 *A resistance of approximately 3000 Ω is needed to balance a bridge. It is obtained on a 5 dial resistance box having steps of 1000, 100, 10, 1 and 0.1 Ω. The measurement is to be guaranteed to 0.1 percent. For this accuracy, how many of these dials would it be worth adjusting ?*

Solution. Limiting value of 3000 Ω resistor

$$= 3000 \pm 3000 \times (0.1/100) = 3000 \pm 3\Omega$$

$$= 2997 \text{ to } 3003\ \Omega$$

Thus it would be worth adjusting the 1000, 100, 10 and 1 Ω dials.

Example 14.6 *Each of the ratio arms of a laboratory type Wheatstone bridge has guaranteed accuracy of ± 0.05%, while the standard arm has a guaranteed accuracy of ± 0.1%. The ratio arms are both set at 1000 Ω and the bridge is balanced with standard arm adjusted to 3,154 Ω. Determine the upper and the lower limits of the unknown resistance, based upon the guaranteed accuracies of the known bridge arms.*

Solution. Value of unknown resistance

$$R = (P/Q) \times S = (1000/1000) \times 3154$$

$$= 3154\ \Omega.$$

∴ Percentage error in determination of R.

$$\frac{\delta R}{R} = \pm\frac{\delta P}{P} \pm \frac{\delta Q}{Q} \pm \frac{\delta S}{S}$$

$$= \pm 0.05 \pm 0.05 \pm 0.01 = \pm\ 0.2\ \%.$$

∴ Limiting values of

$$R = 3154 \pm 0.2\% = 3091 \text{ to } 3217\ \Omega.$$

Example 14.7 *In the Wheatstone bridge of Fig. 14.3, the values of resistances of various arms are $P = 1000\ \Omega$, $Q = 100\ \Omega$, $R = 2{,}005\ \Omega$ and $S = 200\ \Omega$. The battery has an emf of 5 V and negligible internal resistance. The galvanometer has a current sensitivity of 10 mm/μA and an internal resistance of 100 Ω. Calculate the deflection of galvanometer and the sensitivity of the bridge in terms of deflection per unit change in resistance.*

Solution. Resistance of unknown resistor required for balance

$$R = (P/Q)S = (1000/100) \times 200 = 2000\ \Omega.$$

In the actual bridge the unknown resistor has a value of 2005 Ω or the deviation from the balance conditions is $\Delta R = 2005 - 2000 = 5\ \Omega$.

Thevenin source generator emf

$$E_0 = E\left[\frac{R}{R+S} - \frac{P}{P+Q}\right]$$

$$= 5\left[\frac{2005}{2005+200} - \frac{1000}{1000+100}\right]$$

$$= 1.0307 \times 10^{-3}\ \text{V}.$$

Internal resistance of bridge looking into terminals b and d.

$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q}$$

$$= \frac{2005 \times 200}{2005+200} + \frac{1000 \times 100}{1000+100} = 272.8\ \Omega$$

Hence the current through the galvanometer

$$I_g = \frac{E_0}{R_0 + G} = \frac{1.0307 \times 10^{-3}}{272.8 + 100}\ \text{A} = 2.77\ \mu\text{A}.$$

Deflection of the galvanometer

$$\theta = S_i I_g = 10 \times 2.77 = 27.7\ \text{mm}/\Omega.$$

Sensitivity of bridge

$$S_B = \frac{\theta}{\Delta R} = \frac{27.7}{5} = 5.54\ \text{mm}/\Omega.$$

Example 14.8 *A Wheatstone bridge has ratio arms of 1000 Ω and 100 Ω and is being used to measure an unknown resistance of 25 Ω. Two galvanometers are available. Galvanometer 'A' has a resistance of 50 Ω and a sensitivity of 200 mm/μA and galvanometer 'B' has values of 600 Ω and 500 mm/μA. Which of the two galvanometers more is sensitive to a small unbalance on the above bridge, and what is the ratio of sensitivities ? The galvanometer is connected from the junction of the ratio arms to the opposite comers. Comment upon the results.*

Solution. The arrangement of this bridge is shown in Fig. 14.3. Value of standard resistance under balance conditions

$$S = R \cdot \frac{Q}{P} = 25 \times \frac{1000}{100} = 250\ \Omega.$$

Internal resistance of bridge looking into terminals b and d.

$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q}$$

$$= \frac{25 \times 250}{25 + 250} + \frac{100 \times 1000}{100 + 1000} = 113.6\ \Omega.$$

From Eqn. 14.30, the deflection for a small change in resistance

$$\theta = \frac{S_i ES\Delta R}{(R_0 + G)(R+S)^2}$$

$\therefore$ Ratio of deflections for two galvanometers

$$\frac{\theta_A}{\theta_B} = \frac{(S_i)_A ES\Delta R}{(R_0 + G_A)(R+S)^2} \Bigg/ \frac{(S_i)_B ES\Delta R}{(R_0 + G_B)(R+S)^2}$$

$$= \frac{(S_i)_A}{(S_i)_B} \cdot \frac{R_0 + G_B}{R_0 + G_A}$$

$$= \frac{200}{500} \times \frac{(113.6 + 600)}{(113.6 + 50)} = 1.75$$

Thus galvanometer A has a sensitivity of 1.75 times that of galvanometer B as far as this bridge is concerned, even though on its own galvanometer A is less sensitive than galvanometer B.

Example 14.9 *A highly sensitive galvanometer can detect a current as low as 0.1 nA. This galvanometer is used in a Wheatstone bridge as a detector. The resistance of galvanometer is negligible. Each arm of the bridge has a resistance of 1 kΩ. The input voltage applied to the bridge is 20 V. Calculate the smallest change in resistance which can be detected. The resistance of the galvanometer can be neglected as compared with the internal resistance of bridge.*

Solution. Now $P = Q = R = S = 1\ \text{k}\Omega = 1000\ \Omega$.

It is a bridge with equal arms.

$\therefore$ The internal resistance of bridge

$$R_0 = R = 1000\Omega \qquad \text{(See Eqn. 14.27)}$$

Let the change in resistance be ΔR.

$\therefore$ Output voltage of bridge due to unbalance

$$E_0 = E\Delta R / 4R$$

$$= 20 \times \Delta R / (4 \times 1000) = 5 \times 10^{-3}\ \Delta R$$

Current through the galvanometer

$$= \frac{E_0}{R_0} = \frac{5 \times 10^{-3} \Delta R}{1000} = 0.1 \times 10^{-9}.$$

$\therefore$ The smallest change in resistance which can be detected

$$\Delta R = 20 \times 10^{-6} \Omega = 20\ \mu\Omega.$$

Example 14.10 *A regular Wheatstone bridge is used to measure high resistances (in the megohm range). The bridge has ratio arms of 10,000 Ω and 10 Ω.*

The adjustable arm has a maximum value of 10,000 Ω. A battery of 10 V emf and negligible resistance is connected from the junction of ratio aims to the opposite corner ?

(*a*) *What is the maximum resistance that can be measured by this arrangement ?*

(*b*) *If the galvanometer has a sensitivity of 200 mm/μA and a resistance of 50 Ω, how much unbalance is needed to give a galvanometer deflection of 1 mm for the maximum resistance of part (a) ?*

(*c*) *If the galvanometer of part (b) is replaced by a galvanometer of sensitivity of 1000 mm/μA and a resistance of 1000 Ω, calculate the change in resistance to cause a deflection of 1 mm.*

Solution. The arrangement of the bridge is shown in Fig. 14.3.

Now we have,

unknown resistance, $R = (P/Q) \times S$.

The maximum value of

$$R = (\text{maximum value of } P/Q) \times (\text{maximum value of } S)$$

$$= \frac{10000}{10} \times 10000 = 10\ \text{M}\Omega.$$

(*b*) Internal resistance of bridge

$$R_0 = \frac{RS}{R+S} + \frac{PQ}{P+Q}$$

$$= \frac{10 \times 10^6 \times 10 \times 10^3}{10 \times 10^6 + 10 \times 10^3} + \frac{10 \times 10^3 \times 10}{10 \times 10^3 + 10}$$

$$\approx 10{,}000\ \Omega$$

From Eqn 14.30, change in resistance needed for 1 mm deflection

$$\Delta R = \frac{\theta (R_0 + G)(R+S)^2}{S_i ES}$$

$$= \frac{1 \times (10000 + 50)(10 \times 10^6 + 10 \times 10^3)^2}{200 \times 10^6 \times 10 \times 10^3}$$

$$\approx 0.5\ \text{M}\Omega.$$

(*c*) The change in resistance to cause 1 mm deflection with galvanometer having a resistance,

$$G = 1000\ \Omega$$

and current sensitivity of $S_i = 1000$ mm/μA is

$$\Delta R = \frac{1 \times (10{,}000 + 1000)(10 \times 10^6 + 10 \times 10^3)^2}{1000 \times 10^6 \times 10 \times 10^3}$$

$$\approx 0.1\ \text{M}\Omega$$

Example 14.11 *A Wheatstone bridge is shown in Fig. 14.10*

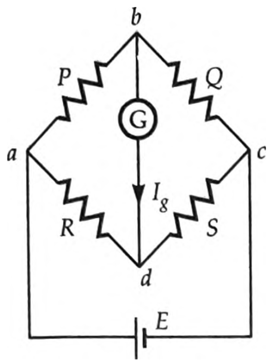


Fig. 14.10 Diagram of Example 14.11.

The values of resistances are :

$$P = 1\ k\Omega,\ R = 1\ k\Omega,\ S = 5\ k\Omega,\ G = 100\ \Omega.$$

The Thevenin source generator voltage $E_0 = 24\ mV$ *and the galvanometer current is* $13.6\ \mu A$. *Calculate the value of Q.*

Solution. The Thevenin equivalent circuit of the bridge is shown in Fig 14.11.

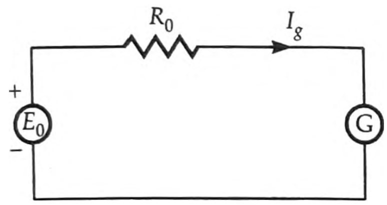


Fig. 14.11 Thevenin source generator of Example 14.11.

R_0 = resistance of circuit looking into terminals d & c with terminals a & b short-circuited.

$$= \frac{RS}{R+S} + \frac{PQ}{P+Q} = \frac{1\times 5}{1+5} + \frac{1\times Q}{1+Q}$$

$$= 0.833 + \frac{Q}{1+Q}\ k\Omega.$$

Now, $R_0 + G = 24 \times 10^{-3} / 13.6 \times 10^{6} = 1.765\ k\Omega$

or $$R_0 = 1765 - 100 = 1665\ \Omega = 1.665\ k\Omega.$$

$\therefore$ $$0.833 + \frac{Q}{1+Q} = 1.665$$

or $$Q = 4.95\ k\Omega.$$

Example 14.12 *A modified form of Wheatstone bridge is shown in Fig 14.12. Calculate the value of unknown resistance,* R_x, *if*

$$R_a = 1200\ \Omega, \qquad R_a = 1600\ R_b,$$

$$R_1 = 800\ R_b,\quad R_1 = 1.25\ R_2 \text{ and } R_3 = 0.5\ R_b$$

are the resistance values under balanced conditions.

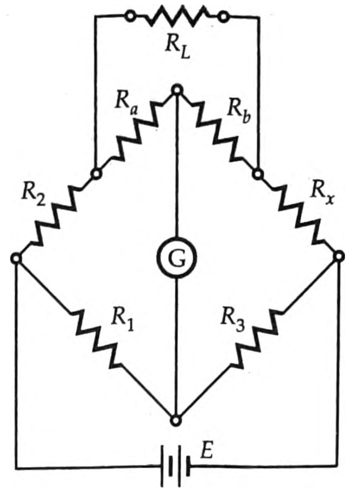


Fig. 14.12 Wheatstone bridge of Example 14.12.

Solution. The calculated values are as under :

$$R_a = 1200\ \Omega,$$
$$R_b = (1/1600) \times 1200 = 0.75\ \Omega$$
$$R_1 = 800\ R_b = 800 \times 0.75 = 600\ \Omega$$
$$R_2 = R_1 / 1.25 = 600/1.25 = 480\ \Omega$$
$$R_3 = 0.5\ R_b = 0.5 \times 0.75 = 0.375\Omega.$$

Delta-star transformation has to be used for the analysis.

This is shown in Fig. 14.13.

$$R_{10} = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{1200 \times 0.75}{1200 + 0.75 + R_L} = \frac{900}{1200.75 + R_L}$$

$$R_{20} = \frac{R_b R_L}{R_a + R_b + R_L} = \frac{0.75\ R_L}{1200 + 0.75 + R_L} = \frac{0.75\ R_L}{1200.75 + R_L}$$

$$R_{30} = \frac{R_a R_L}{R_a + R_b + R_L} = \frac{1200\ R_L}{1200 + 0.75 + R_L} = \frac{1200\ R_L}{1200.75 + R_L}$$

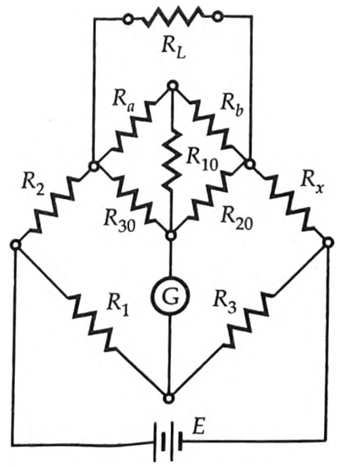


Fig. 14.13 Delta-star transformation for analysis of Fig. 14.12.

The modified circuit is shown in Fig. 14.14.

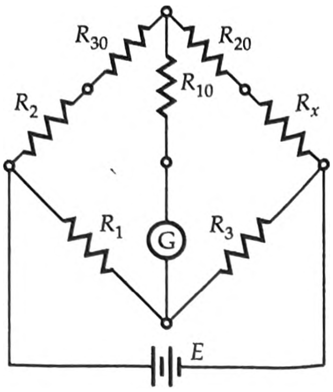


Fig. 14.14 Configuration of bridge of Fig. 14.12 after Delta-star transformation.

For balance,

$$\frac{R_2 + R_{30}}{R_1} = \frac{R_x + R_{20}}{R_3}$$

or
$$\frac{480 + \dfrac{1200\,R_L}{1200.75 + R_L}}{600} = \frac{R_x + \dfrac{0.75\,R_L}{1200.75 + R_L}}{0.375}$$

or
$$R_x = 0.3\,\Omega.$$

Example 14.13 *A Wheatstone bridge is used for measuring the value of change of resistance of a strain gauge which forms one of the arms of the bridge. All the arms of the bridge including the strain gauge have a resistance of 100 Ω each. The maximum allowable power dissipation from the strain gauge is 250 mW. Determine the value of maximum permissible current through the strain gauge and maximum allowable value of bridge supply voltage. Suppose a source of 20 V is available, find the value of series resistance to be connected between the source and the bridge to limit the input voltage of the bridge to permissible level.*

Solution. The resistance of strain gauge $R = 100\,\Omega$

Suppose I is the current through each arm under balanced conditions.

$\therefore\ I^2R = P,$

where P = power dissipation.

Hence, maximum permisible current

$$I = \sqrt{P/R} = \sqrt{\frac{250 \times 10^{-3}}{100}} = 0.05\text{ A} = 50\text{ mA}.$$

The maximum allowable voltage which can be applied to the bridge

$$= 2 \times 50 \times 10^{-3} \times 100 = 10\,\text{V}.$$

$\therefore$ Voltage across the series resistor

$$= 20 - 10 = 10\,\text{V}.$$

Current through the series resistor

$$= 2 \times 50 \times 10^{-3} = 100 \times 10^{-3}\text{ A}.$$

$\therefore$ Resistance of series resistor

$$R_s = \frac{10}{100 \times 10^{-3}} = 100\,\Omega.$$

Example 14.14 *In a Carey-Foster's bridge a resistance of 1.0125 Ω is compared with a standard resistance of 1.0000 Ω, the slide wire has a resistance of 0.250 Ω in 100 divisions. The ratio arms nominally each 10 Ω, are actually 10.05 and 9.95 Ω respectively.*

How far (in scale divisions) are the balance positions from those which would obtain of ratio arms were true to their nominal value ? The slide wire is 100 cm long.

Solution. *Balance with ratio arms equal to nominal value.* Let l_1 be the distance of balance point on slide wire from the unknown resistance end in cm of slide wire. Let r be the resistance per cm length of slide wire.

$$\therefore\quad r = \frac{0.0250}{100}$$

$= 0.00025\ \Omega$/cm as length of wire is 100 cm.

In this case

$P = Q = 10\Omega,\ S = 1.0000\,\Omega$ and $R = 1.0125\,\Omega.$

Under balance conditions :

$$\frac{P}{Q} = \frac{R + l_1 r}{S + (100 - l_1)r}$$

or
$$\frac{10}{10} = \frac{1.0125 + 0.00025\,l_1}{1.0000 + 0.025 - 0.00025\,l_1} \quad \text{or } l_1 = 25\text{ cm}.$$

Thus the balance is obtained at 25 and 75 scale divisions.

Balance arms equal to true values :

Now in this case, $P = 9.05\,\Omega$ and $Q = 10.05\,\Omega.$

Under balance conditions :

$$\frac{9.95}{10.05} = \frac{1.0125 + 0.00025\,l_1}{1.0000 + 0.025 - 0.000025\,l_1}$$

or
$$\frac{1 - 0.005}{1 + 0.005} = \frac{1.0125 + 0.00025\,l_1}{1.0250 - 0.00025\,l_1}$$

or
$$l_1 = 25\text{ cm}.$$

Thus the balance is obtained at 5 and 95 cm.

14.3 MEASUREMENT OF LOW RESISTANCE

The methods used for measurement of medium resistances are unsuitable for measurement of low resistances *i.e.*, resistances having a value under 1Ω. The reason is that the resistance of leads and contacts, though small, are appreciable in comparison in the case of low resistances. For example, a contact resistance of 0.002 Ω causes a negligible error when a resistance of 100 Ω is being measured but the same contact resistance would cause an error of 10% if a low resistance of the value of 0.02 Ω is measured. Hence special type of construction and techniques have to be used for the measurement of low resistances in order to avoid serious errors occurring on account of the factors mentioned above.

Low resistances are constructed with four terminals as shown in Fig. 14.15. One pair of terminals CC' (called the current terminals) is used to lead current to and from the resistor. The voltage drop is measured between the other two terminals PP', called the potential terminals.

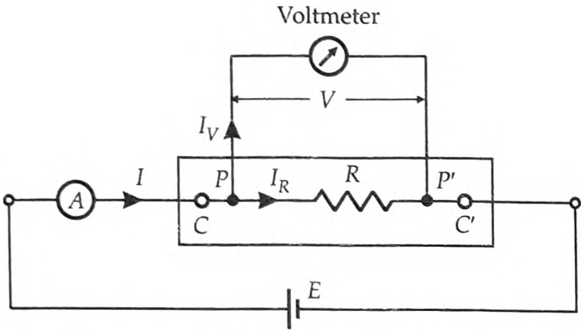


Fig. 14.15 Ammeter voltmeter method for measuring 4 terminal resistances.

The voltage V, indicated in Fig. 14.15, is thus I_R times the resistance R between terminals PP' and does not include any contact resistance drop that may be present at the current terminals CC'.

Resistors of low values are thus measured in terms of resistance, between potential terminals, which becomes perfectly and precisely definite in value and is independent of the contact resistance drop at the current terminals. Contact resistance drop at the potential terminals need not be a source of error, as current crossing at these terminals is usually extremely small or even zero for null methods. Also this contact resistance now becomes a part of the potential circuit and is, therefore, a negligible part of the total resistance of the potential circuit since potential circuits have a high value of resistance.

14.3.1 Methods for Measurement of Low Resistance

The methods for measurement of low resistance are :

1. Ammeter voltmeter method. (This method has been explained in Art. 14.2.1 on page 421)
2. Kelvin's double bridge method.
3. Potentiometer method (This is explained in Chapter 15).

14.3.2 Kelvin Double Bridge Method of Measurement of Low Resistances

The Kelvin bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in measurement of low value resistances. An understanding of the Kelvin bridge arrangement may PV be obtained by a study of the difficulties that arise in a Wheatstone bridge on account of the resistance of the leads and the contact resistances while measuring low valued resistors.

Consider the bridge circuit shown in Fig. 14.15 where r represents the resistance of the lead that connects the unknown resistance R to standard resistance S. Two galvanometer connections indicated by dotted lines, are possible. The connection may he either to point 'm' or to point 'n'. When the galvanometer is connected to point m, the resistance, r, of the connecting leads is added to the standard resistance, S, resulting in indication of too low an indication for unknown resistance R. When the connection is made to point n, the resistance, r, is added to the unknown resistance resulting in indication of too high a value for R.

Suppose that instead of using point m, which gives a low result, or n, which makes the result high, we, make the galvanometer connection to any intermediate point 'd' as shown by full line in Fig. 14.16. If at point 'd' the resistance r is divided into two parts, r_1 and r_2, such that

$$\frac{r_1}{r_2} = \frac{P}{Q} \qquad \text{...(14.36)}$$

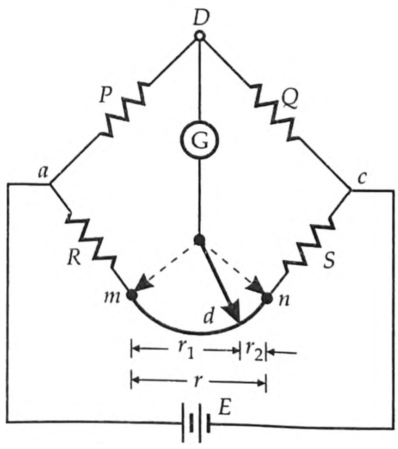


Fig. 14.16 Illustrating principle of Kelvin's bridge.

Then the presence of r_1 the resistance of connecting leads, causes no error in the result. We have,

$$R + r_1 = \frac{P}{Q}\cdot(S + r_2) \quad \text{but} \quad \frac{r_1}{r_2} = \frac{P}{Q} \qquad \text{...(14.37)}$$

or $$\frac{r_1}{r_1+r_2}=\frac{P}{P+Q}$$

or $$r_1=\frac{P}{P+Q}\cdot r$$

as $r_1+r_2=r$ and $r_2=\dfrac{Q}{P+Q}\cdot r$

∴ We can write Eqn. 14.37 as

$$\left(R+\frac{P}{P+Q}r\right)=\frac{P}{Q}\left(S+\frac{Q}{P+Q}r\right)$$

or $$R=\frac{P}{Q}\cdot S \qquad ...(14.38)$$

Therefore we conclude that making the galvanometer connection as at c, the resistance of leads does not affect the result.

The process described above is obviously not a practical way of achieving the desired result, as there would certainly be a trouble in determining the correct point for galvanometer connections. It does, however, suggest the simple modification, that two actual resistance units of correct ratio be connected between points m and n, the galvanometer be connected to the junction of the resistors. This is the actual Kelvin bridge arrangement, which is shown in Fig. 14.17.

The Kelvin double bridge incorporates the idea of a second set of ratio arms – hence the name double bridge – and the use of four terminal resistors for low resistance arms. Figure 14.17 shows the schematic diagram of the Kelvin bridge. The first of ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point d at the appropriate potential between points m and n to eliminate the effect of connecting lead of resistance r between the known resistance, R, and the standard resistance, S.

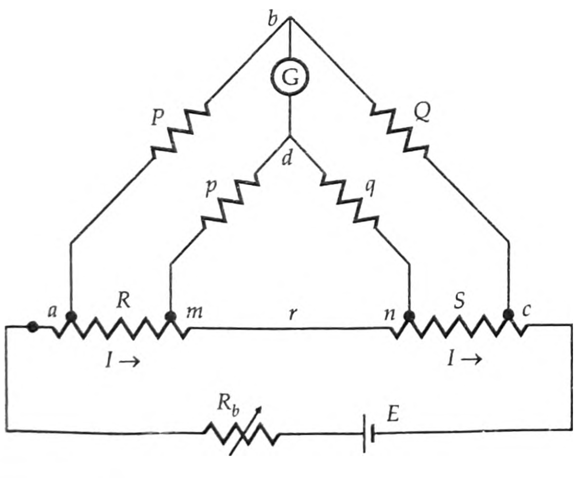


Fig. 14.17 Kelvin double bridge

The ratio p/q is made equal to P/Q. Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E_{ab} is equal to the voltage drop E_{amd} between a and c.

Now $$E_{ab}=\frac{P}{P+Q}E_{ac}$$

and $$E_{ac}=I\left[R+S+\frac{(p+q)r}{p+q+r}\right] \qquad ...(14.39)$$

and $$E_{amd}=I\left[R+\frac{p}{p+q}\left\{\frac{(p+q)r}{p+q+r}\right\}\right]$$

$$=I\left[R+\frac{pr}{p+q+r}\right] \qquad ...(14.40)$$

For zero galvanometer deflection,

$$E_{ab}=E_{amd}$$

or $$\frac{P}{P+Q}I\left[R+S+\frac{(p+q)r}{p+q+r}\right]=I\left[R+\frac{pr}{p+q+r}\right]$$

or $$R=\frac{P}{Q}\cdot S+\frac{qr}{p+q+r}\left[\frac{P}{Q}-\frac{p}{q}\right] \qquad ...(14.41)$$

Now, if $P/Q=p/q$, Eqn. 14.41 becomes,

$$R=\frac{P}{Q}\cdot S. \qquad ...(14.42)$$

Equation 14.42 is the usual working equation for the Kelvin bridge. It indicates that the resistance of connecting lead, r, has no effect on the measurement, provided that the two sets of ratio arms have equal ratios. Equation 14.41 is useful, however, as it shows the error that is introduced in case the ratios are not exactly Ω equal. It indicates that it is desirable to keep r as small as possible in order to minimize the errors in case there is a difference between ratios P/Q and p/q.

The effect of thermo-electric emfs can be eliminated by making another measurement with the battery connections reversed. The true value of R being the mean of the two readings.

In a typical Kelvin bridge, the range of resistance covered is 0.1 μΩ to 1.0 Ω.

The accuracies are as under :

From 1000 μΩ to 1.0 Ω : 0.05%.

From 100 μΩ to 1000 μΩ : 0.2% to 0.05%.

From 10 μΩ to 100 μΩ : 0.5%. to 0.2,

limited by thermoelectric emfs.

In this bridge there are four internal resistance standards of 1 Ω, 0.1 Ω, 0.01 Ω and 0.001 Ω respectively.

14.3.3 Kelvin Bridge Ohmmeter

This is a modified form of the Kelvin bridge and is intended for the rapid measurement of the winding resistances of machines and transformers, and for the measurement of contact and earth conductor resistances. The accuracy is of the order of ± 0.2%. This instrument is direct reading and the balance is obtained by rotating a single dial.

Figure 14.18 gives the circuit diagram of a typical Kelvin bridge ohmmeter. The ratio arms P/Q and p/q of Fig. 14.17 are replaced by a combination of fixed resistors P and p and a double slide wire enabling the bridge ratio to be varied continuously between values 10/1 to 200/1.

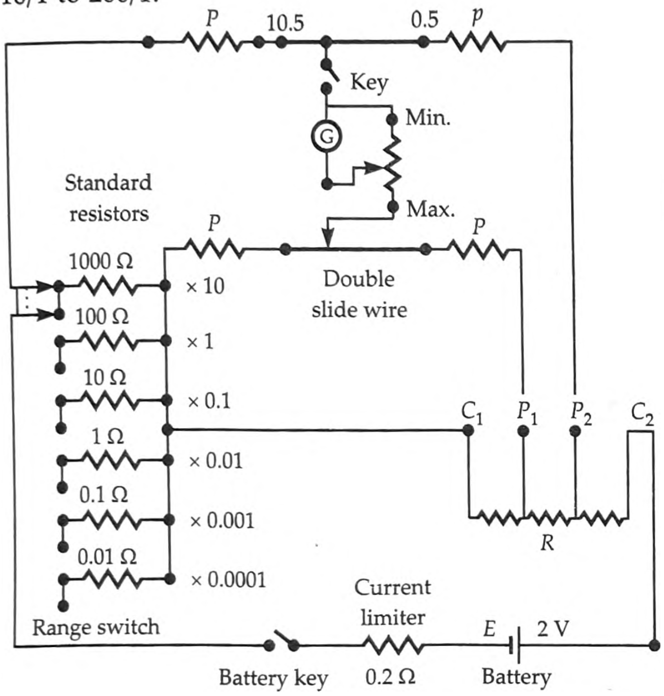


Fig. 14.18 Kelvin bridge ohmmeter.

Six standard resistors are used and these range from 0.01 Ω to 1000 Ω. The ratio dial is calibrated from 0.5 Ω to 10.5 Ω on the × 1 range using 100 Ω standard resistor. The over all range of the instrument is from 0.00005 Ω to 105 Ω.

14.3.3 Unbalanced Kelvin Bridge

The galvanometer current of unbalanced Kelvin bridge can be found by the same Thevenin method as was used for Wheatstone bridge. The Thevenin source generator voltage, referring to Fig. 14.17 is :

$$E_0 = E_{ab} - E_{amd}$$

where $$E_{ab} = \frac{P}{P+Q} \cdot E_{ac} \quad \text{...(14.43)}$$

$$E_{amd} = \frac{R + \dfrac{pr}{p+q+r}}{R + S + \dfrac{(p+q)r}{p+q+r}} E_{ab} \quad \text{...(14.44)}$$

The circuit for the Thevenin equivalent resistance as seen from galvanometer terminals c and d is shown in Fig. 14.19(a) where R_b is the resistance in the battery circuit.

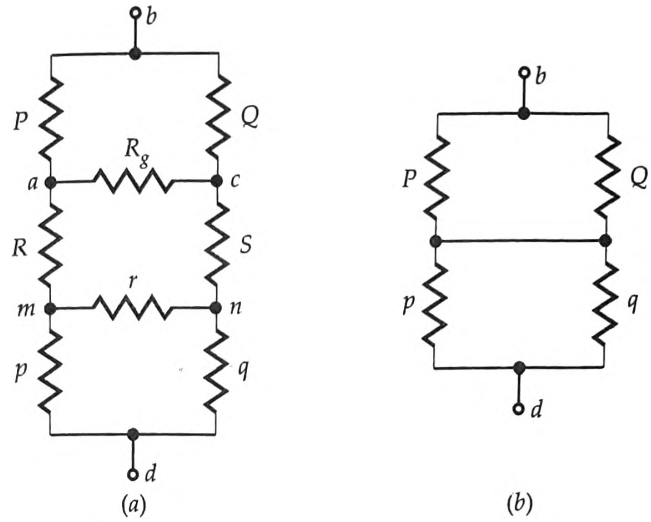


Fig. 14.19 Equivalent circuit for calculation of resistance of Kelvin bridge.

However, R, S, r and R_b, are usually very small and may be neglected with no appreciable-loss, in accuracy. The equivalent circuit then reduces to the one as shown in Fig. 14.19(b). Therefore, Thevenin equivalent resistance

$$R_0 = \frac{PQ}{P+Q} + \frac{pq}{p+q} \quad \text{...(14.45)}$$

The galvanometer current

$$I_g = \frac{E_0}{R_0 + G} \quad \text{...(14.46)}$$

Example 14.15 *A 4 terminal resistor of approximately 50 μΩ resistance was measured by means of a Kelvin bridge having the following component resistances :*

Standard resistor = 100.03 Ω ; Inner ratio arms = 100.31 Ω and 200 Ω ; outer ratio arms = 100.24 Ω and 200 Ω ; resistance of link connecting the standard and the unknown resistance = 700 μΩ. Calculate the unknown resistance to the nearest 0.01 μΩ.

Solution. From Eqn. 14.41 the unknown resistance

$$R = \frac{P}{Q} \cdot S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{p}\right]$$

$$= \frac{100.24}{200} \times 100.03 \times 10^{-6}$$

$$+ \frac{200 \times 700 \times 10^{-6}}{100.31 + 200 + 700 \times 10^{-6}}\left[\frac{100.24}{200} - \frac{100.31}{200}\right]$$

$$= 49.97 \times 10^{-6}\,\Omega = 49.97\ \mu\Omega.$$

Example 14.16 *A Kelvin double bridge (shown in Fig. 14.17) each of the ratio arms $P = Q = p = q = 1000\ \Omega$. The emf of the battery is 100 V and a resistance of $5\ \Omega$ is included in the battery circuit. The galvanometer has a resistance of $500\ \Omega$ and the resistance of the link connecting the unknown resistance to the standard resistance may be neglected. The bridge is balanced when the standard resistance $S = 0.001\ \Omega$.*

(*a*) *Determine the value of unknown resistance.*

(*b*) *Determine the current (approximate value) through the unknown resistance R at balance.*

(*c*) *Determine the deflection of the galvanometer when the unknown resistance, R, is changed by 0.1 percent from its value at balance. The galvanometer has a sensitivity of 200 mm/μA.*

Solution. (*a*) At balance, the value of unknown resistance

$$R = \frac{P}{Q} \cdot S = \frac{1000}{1000} \times 0.001 = 0.001\ \Omega.$$

(*b*) If we examine the Kelvin bridge circuit, we find the resistors P, Q and p, q are in parallel with the resistance of link, r. Since r is negligible and P, Q, p and q have large values, the effect of ratios arms can be neglected for the purpose of calculation of current.

∴ Current under balance conditions

$$I = \frac{E}{R_b + R + S} = \frac{100}{5 + 0.01 + 0.0001} \approx 20\ \text{A}.$$

where R_b = resistance in the battery circuit.

(*c*) The value R is changed by 0.1 per cent.

∴ New value of $R = 1.001 \times 0.001 = 0.001001\ \Omega$.

Voltage between points a and c (Fig. 14.17)

$$E_{ad} = \frac{R + S + r}{R_b + R + S + r} \cdot E = \frac{R + S}{R_b + R + S} E \text{ as } r = 0$$

$$= \frac{R + S}{R_b} = \frac{0.001 + 0.001001}{5} \times 100 = 40 \times 10^{-3}$$

From Eqns. 14.43 and 14.44,

$$E_0 = E_{ab} - E_{amd}$$

$$= \left(\frac{P}{P + Q} - \frac{R}{R + S}\right) E_{ac}$$

$$= \left(\frac{1000}{1000 + 1000} - \frac{0.001001}{0.001 + 0.001001}\right) \times 40 \times 10^{-3}$$

$$= 0.01 \times 10^{-3}\ \text{V} = 0.01\ \text{mV. (considering } r = 0).$$

Since R, S, r and R_b are quite small as compared to P, Q, p and q, we can use circuit of Fig. 14.19(*b*). For the calculation of internal resistance, R_0 as viewed from terminals d and b.

$$R_0 = \frac{PQ}{P + Q} + \frac{pq}{p + q}$$

$$= \frac{1000 \times 1000}{1000 + 1000} + \frac{1000 \times 1000}{1000 + 1000} = 1000\ \Omega$$

Galvanometer current

$$I_g = \frac{E_0}{R_0 + G} = \frac{0.01 \times 10^{-3}}{1000 + 500}$$

$$= 0.0067 \times 10^{-6}\ \text{A} = 0.0067\ \mu\text{A}.$$

∴ Deflection of galvanometer

$$\theta = S_i I_g = 200 \times 0.0067 = 1.34\ \text{mm}.$$

14.4 MEASUREMENT OF HIGH RESISTANCE

High resistance of the order of hundreds or thousands of megohm are often encountered in electrical equipment, and frequently must be measured.

Common examples are :

(*i*) Insulation resistance of components and built up electrical equipment like machines and cables ;

(*ii*) Resistance of high resistance circuit elements like in vacuum tube circuits ;

(*iii*) Leakage resistance of capacitors ;

(*iv*) Volume resistivity of a material, *i.e.*, the resistance between two faces of unit area separated by unit distance with all conduction from face to face being through the body of the material ;

(*v*) Surface resistivity, *i.e.*, the resistance between two lines of unit length and unit distance apart, the lines being on the surface of the material and all conduction being on the surface.

14.4.1 Difficulties in Measurement of High Resistances

High accuracy is rarely required in such measurements, hence simple circuits are used. Since the resistances under measurement have high values, very small currents are encountered in the measurement circuits. This aspect leads to several difficulties :

(*i*) The insulation resistance of the resistor may be comparable with the actual value of the resistor. Thus leakage currents are produced. These leakage currents are of comparable magnitude to the current being measured and must be eliminated from the measurement. Leakage currents no doubt introduce errors, but they generally vary from day to day, depending upon the humidity conditions and therefore cause additional unpredictable complications.

(*ii*) Due to electrostatic effect, stray changes can appear in the measuring circuit causing errors. Alternating fields can also effect the measurements considerably. Therefore. critical points of the measuring circuit must be carefully screened.

(*iii*) In order to obtain definite ratios in the potential distribution with respect to surroundings, one point of the circuit may be connected to earth for accuracy in measurements.

(*iv*) In measurement of insulation resistance the specimen often has considerable capacitance. On application of a direct voltage a large charging current flows initially which gradually decays down after a short interval. Further, insulating materials possess the property of dielectric absorption. *i.e.*, after the main charging current has decayed down, further charge is slowly absorbed over a considerable period of time, perhaps for minutes or even hours. Thus measurement of true conduction current should be delayed until after the cessation of the charging and absorbing currents. But since the absorbing currents take a considerably long time to decay, it is usually inevitable that the conduction current measured includes some absorption in current. The testing conditions, including the time between the application of voltage and observation of the current, must be specified.

(*v*) When measuring the resistance of low conductivity conductors, insulating materials and products, the effect of various factors upon their resistance should be taken into account. Thus, a change in the temperature of cardboard from 20° to 40°C is accompanied by a 13 fold change in its resistance, changes in humidity form 10 to 60 percent cause a 30 fold change in resistance of porcelain. Besides temperature and humidity, the kind of current employed for measurement, the magnitude and duration of the applied voltage, and other factors also effect the resistance being measured.

(*vi*) Fairly high voltages are used in tests in order to raise the currents to reasonable values in order to be measured. So normally a sensitive galvanometer or micro-ammeter is required and adequate steps have to be taken to prevent damage to these delicate instruments.

A voltage supply of 100 V upto a few kV is often used depending upon the nature and breakdown voltage of the test object. The powe supply unit is d.c. transistorized source. Proper smoothing and stabilization circuits are used to ensure constancy of voltage with time.

14.4.2 Use of Guard Circuit

Some form of guard circuits are generally used to eliminate the errors caused by leakage currents over insulation. Figure 14.20 illustrates the operation of a guard circuit. In Fig. 14.20(*a*), a high resistance mounted on a piece of insulating material is measured by the ammeter-voltmeter method. The micro-ammeter measures the sum of the current through the resistor (I_R) and the current through the leakage path around the resistor (I_L). The measured value of resistance computed from the readings indicated on the voltmeter and the micro-ammeter, will not be true value but will be in error. In Fig. 14.20(*b*) guard terminal has been added to resistance terminal block. The guard terminal surrounds the resistance terminal entirely and is connected to the battery side of the micro-ammeter. The leakage current I_L, now bypasses the micro-ammeter which then indicates the current I_R through the resistor and thus allows the correct determination of the resistance value from the readings of voltmeter and micro-ammeter. The guard terminal and resistance terminal are almost at the same potential and thus there will be no flow of current between them.

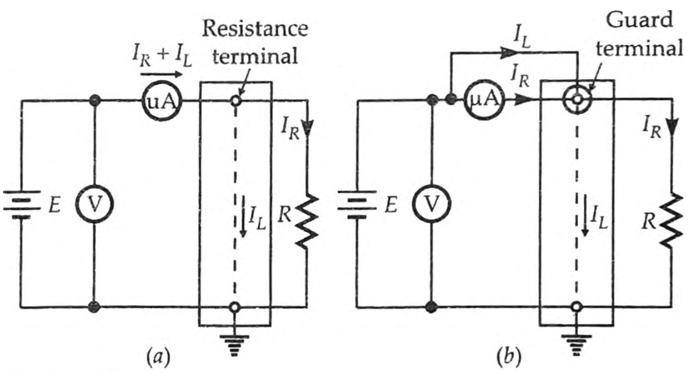


Fig. 14.20 Application of guard circuit for measurement of high resistance.

14.4.3 Methods for Measurement of High Resistance

The different methods employed are :

1. Direct deflection method.
2. Loss of charge method:
3. Megohm bridge.
4. Meggar (This is explained in Art. 9.8, page 256).

Direct deflection method. The direct deflection method is basically that of Fig. 14.20(*b*). For high resistances, such as insulation resistance of cables, a sensitive galvanometer of '*d*' Arsonval type (usually having a current sensitivity of at 1000 mm/μA at a scale distance of 1 metre) is used in place of the microammeter. In fact many sensitive type of galvanometers can detect currents from 0.1 — 1 nA. Therefore, with an applied voltage of 1 kV, resistances as high as 10^{12} to $10 \times 10^{12}\ \Omega$ can be measured.

An illustration of the direct deflection method used for measuring insulation resistance of a cable is shown in Fig. 14.21. The galvanometer G, measures the current I_R between the conductor and the metal Sheath. The leakage current I_L, over the insulating material is carried by the guard wire wound on the insulation and therefore does not flow through the galvanometer.

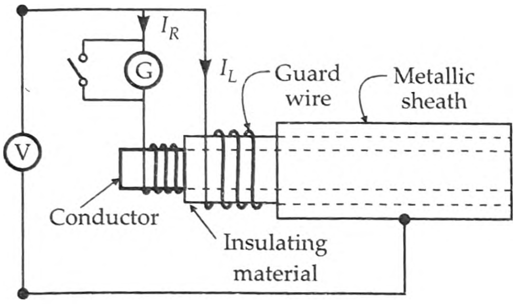


Fig. 14.21 Measurement of insulation resistance of cable having sheath.

Cables without metal sheaths can be tested in a similar way if cable, except the end or ends on which corrections are made, is immersed in water in a tank. The water and the tank then form the return path for the current. The cable is immersed in slightly saline water for about 24 hours and the temperature is kept constant (at about 20°C) and then the measurement is taken as in Fig. 14.22.

The insulation resistance of the cable $R = V / I_R$.

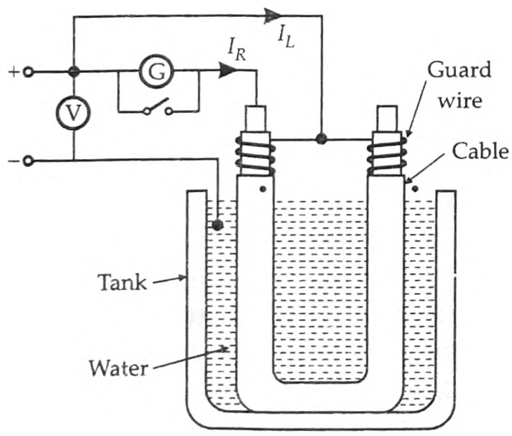


Fig.14.22 Measurement of insulation resistance of cable having no conducting sheath.

In some cases, the deflection of the galvanometer is observed and its scale is afterwards calibrated by replacing the insulation by a standard high resistance (usually 1 MΩ), the galvanometer shunt being varied, as required to give a deflection of the same order as before.

In tests on cables the galvanometer should be short-circuited before applying the voltage. The short circuiting connection is removed only after sufficient time has elapsed so that charging and absorption currents cease to flow. The galvanometer should be well shunted during the early stages of measurement, and it is normally desirable to include a protective series resistance (of several megohm) in the galvanometer circuit. The value of this resistance should be subtracted from the observed resistance value in order to determine the true resistance. A high voltage battery of 500 V emf is required and its emf should remain constant throughout the test.

Measurement of volume and surface resistivity. The direct deflection method is often used for measurement of insulation resistance of insulating material samples available in sheet form. In such cases we are interested in the measurement of volume resistivity and the surface resistivity of the material.

Figure 14.23 shows the schematic diagram for measurement of volume and surface resistivities of a specimen of insulating material. The specimen is provided with tin foil or colloidal graphite electrodes ; the upper electrode having a guard ring. For measurement of volume resistivity (which in fact is the specific resistance) readings of voltage applied and the current through the galvanometer are taken. Leakage currents over the edge of the specimen will flow between the guard ring and the lower electrode and hence will not introduce error into the measurement. The volume resistivity, ρ, can be calculated as follows :

Let d_1 = diameter of the upper electrode d_1,
r = thickness of the specimen sheet,
V_1 = reading of voltmeter,
and I_1 = current through galvanometer G_1.

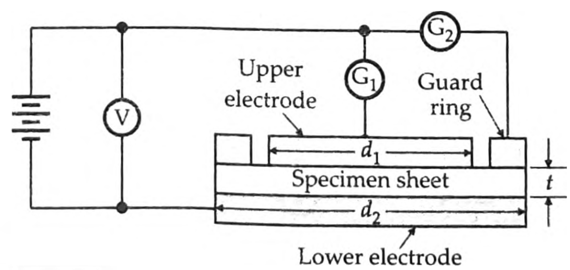


Fig.14.23 Measurement of volume and surface resistivities of insulating material specimen.

$\therefore$ Resistance of specimen

$$R = V_1 / I_1. \quad \text{But } R = \rho t / \pi d_1^2$$

$\therefore$ Volume resistivity of specimen,

$$\rho = \frac{\pi d_1^2}{t} = \frac{\pi d_1^2 V_1}{t I_1} \quad ...(14.47)$$

The resistivity of a thin layer of dielectric materials is different from volume resistivity, not only because of an adherent humidity layer but also because of contamination, chemical alterations, absorption of gases, or structural modification. The resistance R_t between two electrodes embedded in or attached to a dielectric medium is composed of volume resistance R_v and surface resistance R_s with $1/R_t = 1/R_v + 1/R_s$.

The volume resistance, R_v can be measured separately from surface resistance R_s by means of guard rings as shown in Fig 14.20.

If we want to measure surface resistivity, the galvanometer is placed in position G_2. In this position the galvanometer measures the leakage current and current flowing between upper and lower electrodes will be eliminated from measurement. Let

d_2 = diameter of lower electrode disc,

V_2 = reading of voltmeter

and I_2 = current through galvanometer G_2.

$\therefore$ Surface resistance

$$R_s = V_2 / I_2.$$

The leakage current flows along a path of length t and width πd_2 and therefore, surface resistivity,

$$\rho_s = \frac{R_s \times \pi d_2}{t} = \frac{\pi d_2}{t} \cdot \frac{V_2}{I_2} \quad ...(14.48)$$

Other forms of specimen and electrodes are also used. For example, the electrodes and guard ring may be mercury, either placed in specially machined recesses, in moulded insulating materials, or retained by metal rings on the surface of sheet materials.

Loss of charge method. In this method, (Fig. 14.24) the insulation resistance R to be measured is connected in parallel with a capacitor C and a electrostatic voltmeter. The capacitor is charged to some suitable voltage, by means of a battery having

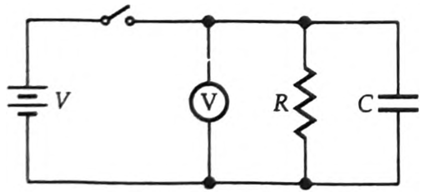


Fig. 14.24 Loss of charge method.

voltage V and is then allowed to discharge through the resistance. The terminal voltage is observed over a considerable period of time during discharge.

The voltage across the capacitor at any instant t after the application of voltage is

$$V = V \exp(-t/CR)$$

or $\quad V/v = \exp(-t/CR)$

or Insulation resistance

$$R = \frac{t}{C \log_e V/v} = \frac{0.4343\,t}{C \log_{10} V/v} \quad ...(14.49)$$

The variation of voltage v with time shown in Fig. 14.25.

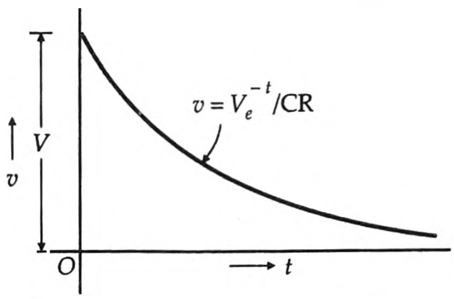


Fig. 14.25 Variation of voltage with time.

From Eqn. 14.49 it follows that if V, v, C and t are known the value of R can be computed.

If the resistance R is very large the time for an appreciable fall in voltage is very large and thus this process may become time-consuming. Also the voltage-time curve will thus be very flat and unless great care is taken in measuring voltages at the beginning and end of the time r, a serious error may be made in the ratio V/v causing a considerable corresponding error in the measured value of R. More accurate results may be obtained by change in the voltage V-v directly and calling this change as e, the expression for R becomes :

$$R = \frac{0.4343\,t}{C \log_{10} \dfrac{V}{V-e}} \quad ...(14.50)$$

This change in voltage may be measured by a galvanometer.

However, form the experimental point of view, it may be advisable to determine the time t from the discharge curve of the capacitor by plotting curve of $\log_e v$ against time t. This curve is linear as shown in Fig. 14.26 and thus determination of time t from this curve for the voltage to fall from V to v yields more accurate results.

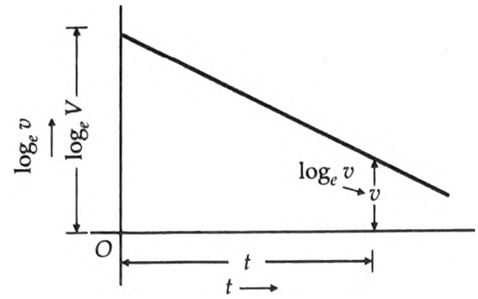


Fig. 14.26 Plot of $\log_e v$ versus time.

This method is applicable to some high resistances, but it requires a capacitor of a very high leakage resistance as high as the resistance being measured. The method is very attractive if the resistance being measured is the leakage resistance of a capacitor as in this case auxiliary R and C units are not required.

Actually in this method we do not measure the true value of resistance since we assume here that the value of resistance of electrostatic voltmeter and the leakage resistance of the capacitor have infinite value. But in practice corrections must be applied to take into consideration the above two resistances. Figure 14.27 shows the actual circuit of the test where R_1 represents the leakage resistance of capacitor. Then if R' is the resistance of R_1 and R in parallel the discharge equation for capacitance gives

$$R' = \frac{0.4343\,t}{C\log_{10} V/v} \qquad \text{...(14.51)}$$

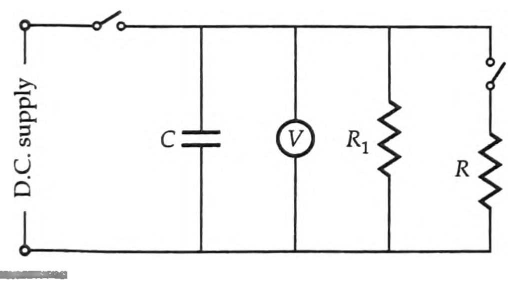


Fig. 14.27 Loss of charge method considering effects of leakage resistance of capacitor.

The test is then repeated with the unknown resistance R, disconnected and the capacitor discharging through R_1. The value of R_1 obtained from this second test and substituted into the expression

$$R' = \frac{R\,R_1}{R + R_1} \qquad \text{...(14.52)}$$

in order to get value of R.

The leakage resistance of the voltmeter, unless very high should also be taken into consideration.

Megohm bridge method. Figure 14.28(*a*) shows a very high resistance R with its two main terminals A and B, and a guard terminal, which is put on the insulation. This high resistance may be diagramatically represented as in Fig. 14.28(*b*). The resistance R is between main terminals A and B and the leakage resistances R_{AG} and R_{BG} between the main terminals A and B of from a "Three terminal resistance".

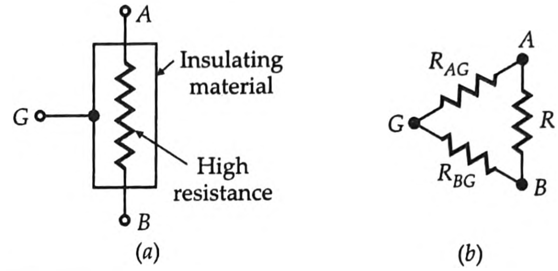


Fig. 14.28 Three Terminal Resistances.

Let us consider the hypothetical case of a 100 MΩ resistance. We assume that each of the leakage resistances is 100 MΩ *i.e.*, $R_{AG} = R_{BG} = 100$ MΩ. Let this resistance be measured by an ordinary Wheatstone bridge as shown in Fig. 14.29(*a*). It is clear that the Wheatstone bridge will measure a resistance of $\frac{100 \times 200}{100 + 200} = 67\,\text{M}\Omega$ instead of 100 MΩ thus giving an error of 33 percent.

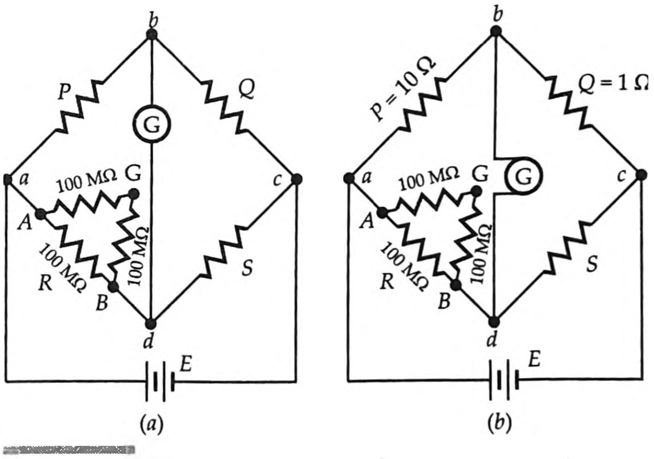


Fig. 14.29 Use of guard circuit for measurement of high resistances.

However if the same resistance is measured by a modified Wheatstone bridge as shown in Fig. 14.29(*b*) with the guard connection G connected as indicated, the error in measurement is considerably reduced. For the arrangement shown in Fig. 14.29(*b*) resistance R_{BG} is put in parallel with the galvanometer and thus it has no effect on the balance and only effects the sensitivity of the galvanometer slightly. The resistance $R_{AG} = 100\,\text{M}\Omega$ is put in parallel with a resistance $P = 100\,\text{k}\Omega$ and therefore for the arrangement shown the measured value has an error of only 0.01 percent and this error is entirely negligible for measurements of this type.

The arrangement of Fig. 14.30 illustrates the operation of a Megohm bridge.

Figure 14.30 shows the circuit of a completely self-contained Megohm bridge which includes power supplies, bridge a members, amplifiers, and indicating, instrument. It has a range from 0.1 MΩ to 10^6 MΩ. The accuracy is within 3% for the lower part of the range to possible 10% above 10,000 MΩ.

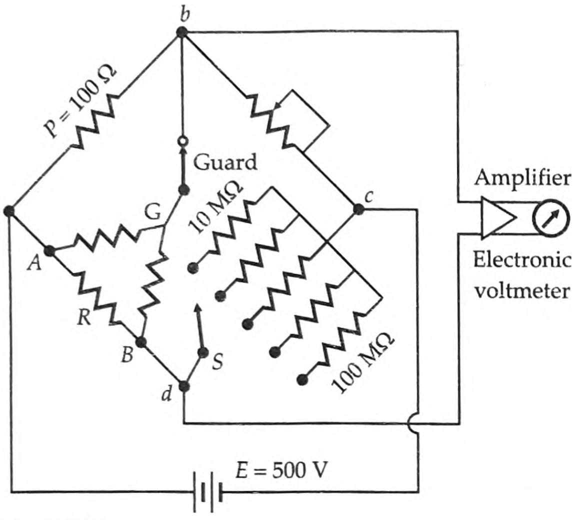


Fig. 14.30 Megohm bridge.

Sensitivity for balancing against high resistance is obtained by use of adjustable high voltage supplies of 500 V or 1000 V and the use of a sensitive null indicating arrangement such as a high gain amplifier with an electronic voltmeter or a C.R.O. The dial on Q is calibrated 1 – 10 – 100 – 1000 MΩ, with main decade 1 – 10 occupying greater part of the dial space. Since unknown resistance $R = PS/Q$, the arm Q is made, tapered, so that the dial calibration is approximately logarithmic in the main decade, 1 – 10. Arm S gives five multipliers, 0.1, 1, 10, 100 and 1000.

The junction of ratio arms P and Q is brought on the main panel and is designated as 'Guard' terminal.

Example 14.17 *Derive an expression for insulation resistance of single core cable. The conductor of a cable has a diameter of 5 mm and the overall diameter of the cable is 25 mm. If the insulation resistance of the cable is 16,000 Ω/km, calculate the specific resistance of insulating material.*

Solution. Let

d = diameter of conductor,

D = diameter of insulated cable,

L = length of cable, and

ρ = resistivity of insulating material.

Let us consider an annular ring of width dx at a radius x from the centre as shown in Fig. 14.31. Insulation resistance of this annular ring is :

$$dR_x = \frac{\rho\, dx}{2\pi x L}$$

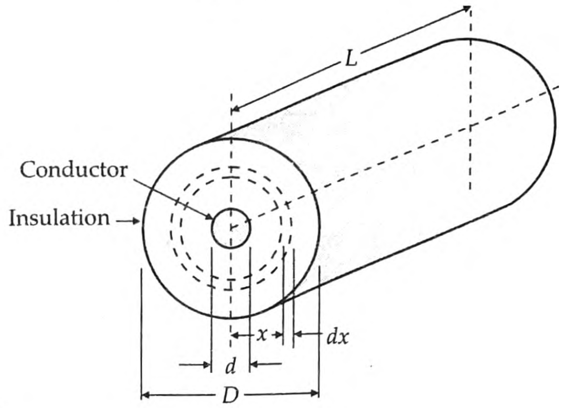


Fig. 14.31 Insulation resistance of cable.

Total insulation resistance of cable

$$R = \int_{d/2}^{D/2} \frac{\rho\, dx}{2\pi x L}$$

$$= \frac{\rho}{2\pi L}\log_e \frac{D}{d} = \frac{2.303\rho}{2\pi L}\log_{10}\frac{D}{d}$$

$$= \frac{0.367}{L}\rho \log_{10}\frac{D}{d}$$

Now in this example we have,

$$D = 25\times10^{-3}\text{ m} ;\ d = 5\times10^{-3}\text{ m} ;$$

$$L = 1\text{ km} = 1000\text{ m and } R = 16{,}000\ \Omega.$$

Specific resistance

$$\rho = \frac{RL}{0.367\log_{10} D/d} = \frac{16000\times1000}{0.367(\log_{10}2.5/0.5)}$$

$$= 62.5\times10^{-6}\ \Omega\text{m}.$$

Example 14.18 *A test voltage is applied for several minutes between the conductor of a 400 metre length of cable and earth. The galvanometer connected in series reads 250 divisions, the value of universal shunt being 2.5 with a standard resistance of 1 MΩ in circuit, the scale reading is 350, the value of shunt being 1000. Calculate the insulation resistance of the cable. What would be the insulation resistance of the same cable of length 100 metre ?*

Solution. Since we have a universal shunt with the galvanometer and therefore the current through the circuit is proportional to the deflection of the galvanometer times universal shunt multiplier, Deflection of galvanometer with unknown resistance 'R' is circuit 350 divisions and the shunt multiplier is 2.5.

$\therefore$ Current through the circuit with unknown resistance connected is :

$$I_R K \times 250 \times 2.5 = 625 \text{ K}$$

where K is a constant.

Deflection of galvanometer with standard resistance 'S' is circuit is 250 divisions and the shunt multiplier is 1000.

$\therefore$ Current through the circuit with standard resistance connected is :

$$I_s = K \times 350 \times 1000 = 350000 \text{ K}.$$

Now $I_R R = I_s S = E,$

where E = voltage of the source.

$\therefore$ Insulation resistance of 400 metre long cable.

$$R = \frac{I_S}{I_R} \cdot S = \frac{3{,}50{,}000\,K}{625\,K} \times 1 = 560 \text{ M}\Omega.$$

The insulation resistance is inversely proportional to length of cable and therefore insulation resistance of 1000 metre long cable

$$= 560 \times (400/1000) = 224 \text{ M}\Omega.$$

Example 14.19 *A length of cable is tested for insulation resistance by the loss of charge method. An electrostatic voltmeter of infinite resistance is connected between the cable conductor and earth, forming therewith a joint capacitance of 600 pF. It is observed that after charging the voltage falls from 250 V to 92 V in 1 minute. Calculate the insulation resistance of the cable.*

Solution. From Eqn. 14.49 insulation resistance of cable,

$$R = \frac{0.4343\,t}{C(\log_{10} V/v)} = \frac{0.4343 \times 60}{600 \times 10^{-12}(\log_{10} 250/92)}$$

$$= 100 \times 10^9\,\Omega = 100{,}000 \text{ M}\Omega.$$

Example 14.20 *A cable is tested by loss of charge method using a ballistic galvanometer, with following results : Discharged immediately after electrification. deflection 200 divisions. Discharged after 30 s and after electrification : (i) deflection 126 divisions, (ii) when in parallel with a resistance of 10 MΩ, deflection 100 divisions. Calculate the insulation resistance of the cable.*

Solution. Suppose R is the insulation resistance of the cable and let R' be the resultant resistance of parallel combination of insulation resistance, R and the 10 MΩ resistance.

$$\therefore\ R = \frac{0.4343 \times 30}{C \log_{10} 200/126} = \frac{0.4343 \times 30}{0.201\,C}$$

$$\text{and}\quad R' = \frac{0.4343 \times 30}{C \log_{10} 200/100} = \frac{0.4343 \times 30}{0.301\,C}$$

Thus we have,

$$\frac{R'}{R} = \frac{0.201}{0.301} = 0.667 \quad \text{or} \quad R' = 0.667\,R.$$

$$\text{Now}\quad R' = \frac{R \times 10}{R + 10} \quad \text{or} \quad 0.667\,R = \frac{R \times 10}{R + 10}$$

$$\therefore\quad R = 5 \text{ M}\Omega$$

14.5 MEASUREMENT OF EARTH RESISTANCE

The provision of an earth electrode for an electrical system is necessitated by the following reasons :

1. All the parts of electrical equipment, like casings of machines. switches and circuit breakers, lead sheathing and armouring of cables, tanks of transformers, etc. which have to be at earth potential, must be connected to an earth electrode. The purpose of this is to protect the various parts of the installation, as well as the persons working against damage in case the insulation of a system fails at any point. By connecting V these parts to an earthed electrode a continuous low resistance path is available for leakage currents to flow to earth. This current operates the protective devices and thus the faulty circuit is isolated in case a fault occurs.

2. The earth electrode ensures that in the event of over voltage on the system due to lightning discharges or other system faults, those parts of equipment which are normally dead as far as voltages are concerned, do not attain dangerously high potentials.

3. In a three phase circuit the neutral of the system is earned in order to stabilize the potential of the circuit with respect to earth.

An earth electrode will only be effective so long it has a low resistance to the earth and can carry large currents without deteriorating. Since the amount of current which an earth electrode will carry is difficult to measure, the resistance value of the earth electrode is taken as sufficiently reliable indication of its effectiveness. The resistance of earth electrode should be low to give good protection and it must be measured.

The main factors on which the resistance of any earthing system depends are :

1. Shape and material of earth electrode or electrodes used.

2. Depth in the soil at which the electrodes are burried.

3. Specific resistance of soil surrounding and in the neighbourhood of electrodes. The specific resistance of the soil is not constant but varies from one type of soil to another. The amount of moisture present in the soil affects its specific resistance and hence the resistance of earth electrode is not a constant factor but suffers seasonal variations. This calls for periodic testing to ensure that the earthing system remains reasonably effective.

The specific resistance of soils varies between wide limits and is very much dependent upon its moisture content. Approximate figures for specific resistance of soil are $80 \times 10^3\ \Omega$m in for moist clay to $80 \times 10^6\ \Omega$m for sand of normal moisture content. A decrease of moisture content of 30% is capable of producing an increase of 300% to 400% in specific resistance. Thus it is necessary to make regular checks for earth resistance during the year round.

14.5.1 Methods of Measuring Earth Resistance

1. ***Fall of potential method.*** Figure 14.32 shows the circuit for measurement of earth resistance with fall of potential method. A current is passed through earth electrode an auxiliary electrode *B* (which is usually an iron spike) inserted in earth at a distance away from the earth electrode. A second auxiliary electrode *A* is inserted in earth between *E* and *B*. The potential difference *V* between *E* and *A* is measured for a given current *I*. The flow of ground currents is shown in Fig. 14.33(*c*).

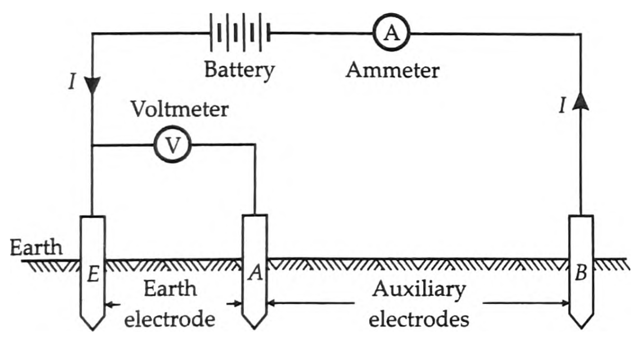


Fig. 14.32 Measurement of earth resistance by fall of potential method.

The lines of the first electrode current diverge and those of the second electrode current converge. As a result the current density is much greater in the vicinity of the electrodes than at a distance from them. The potential distribution between the electrodes is shown in Fig. 14.33(*b*). It is apparent from this curve that the potential rises in the proximity of electrodes *E* and *B* and is constant along the middle section. The resistance of earth, therefore, is

$$R_E = V / I \quad \text{or} \quad V_{EA} / I.$$

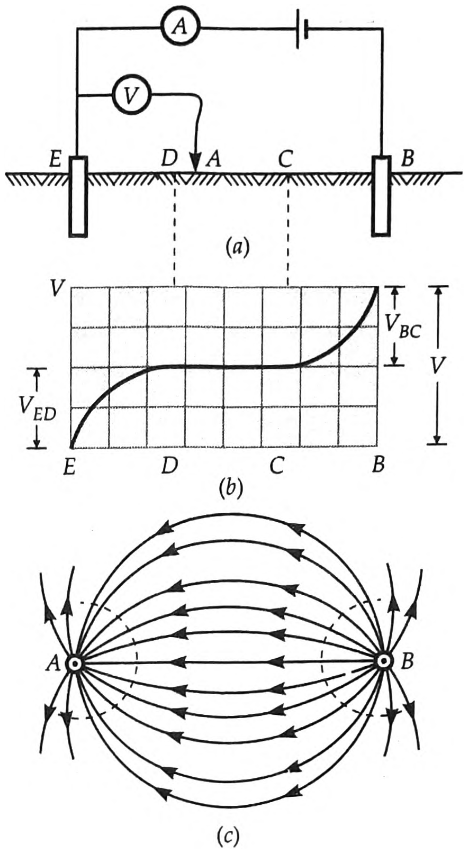


Fig. 14.33 Distribution of potentials between two earthing electrodes.

The position of electrodes *E* and *B* is fixed and the position of electrode *A* is changed and resistance measurements are done for various positions of electrode *A*.

A graph is plotted between earth resistance against the distance between electrode *E* and *A*. This graph is shown in Fig. 14.34.

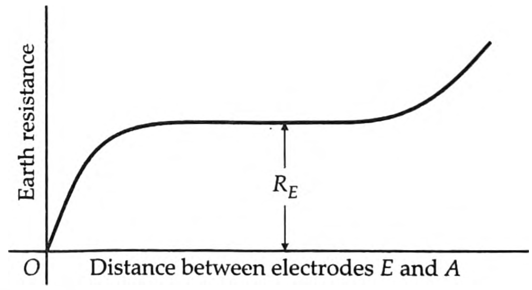


Fig. 14.34 Variation of earth resistance with distance between electrode *E* and *A*.

From Fig. 14.34, it is clear that the measured value of earth resistance depends upon the position of the auxiliary electrode *A*. The earth resistance rises rapidly initially. When the distance between earth electrode *E* and auxiliary electrode *A* is increased, it then becomes constant, and when the auxiliary electrode A approaches the auxiliary electrode B, the resistance rises again. The placing of electrodes is thus very important and serious error may be caused by incorrect placing of the electrodes. The correct value of resistance of earth, R_E, is when the auxiliary electrode A is at such a distance that the resistance lies on the flat part of curve of Fig. 14.30.

The spacing between the earth electrode E and the auxiliary electrodes A, B should be large so as to get proper results. The distance may be a few hundred metres in case the earth resistance is low.

2. Earth tester. The resistance of earth can be measured by an earth tester shown in Fig. 14.35. The "Earth Tester" is a special type of Meggar (See Art. 9.8 page 256) and it has some additional constructional features additional constructional features and they are :

(*i*) a rotating current reverser, and

(*ii*) a rectifier

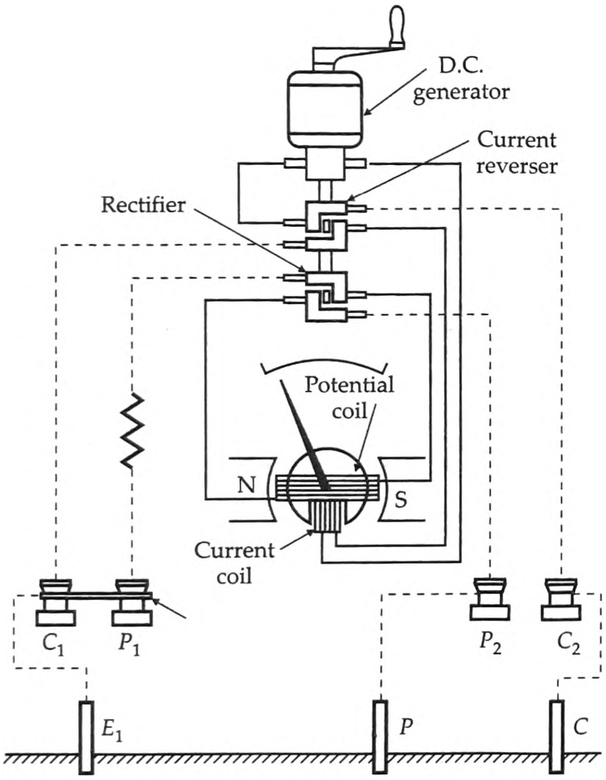


Fig. 14.35 Earth tester.

Both these additional features consist of simple commutators made up of '*L*' shaped segments. They are mounted on the shaft of the hand driven generator. Each commutator has four fixed brushes. One pair of each set of brushes is so positioned that they make contact alternately with one segment and then with the other as the commutator rotates. The second pair of each of set of brushes is positioned on the commutator so that continuous contact is made with one segment whatever the position of the commutator.

The earth tester has four terminals P_1, P_2. Two terminals P_1 and C_1 are shorted to form a common point to be connected to the earth electrode. The other two terminals P_2 and C_2 are connected to auxiliary electrodes P and C respectively.

The indication of the earth tester depends upon the ratio of the voltage across the pressure coil and the current through the coil. The deflection of its pointer indicates the resistance of earth directly. Although the "Earth Tester", which is a permanent magnet moving coil instrument and can operate on d.c. only, yet by including the reverser and the rectifying device it is possible to make measurements with a.c. flowing in the soil.

The sending of a.c. current through the soil has many advantages and therefore this system is used. The use of a.c. passing through the soil eliminates unwanted effects due to production of a back emf in the soil on account of electrolytic action. Also the instrument is free from effects of alternating or direct currents presents in the soil.

Example 14.21 *On a 250 V supply a fault having a resistance of 20 Ω develops between the unearthed end of the winding of an electric heater and the frame. If the resistance of the substation earth electrode is 4 Ω that of human body 2000 Ω, and the safe maximum current through the body is 25 mA, what is the safe maximum resistance of consumer's earth electrode ?*

Solution. Figure 14.36 shows the diagrammatic representation of the problem.

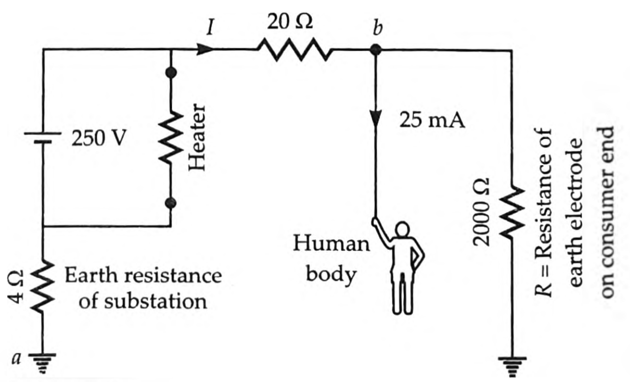


Fig. 14.36 Diagram of Example 14.20.

Let R be the resistance of earth electrode at the consumer end.

Voltage drop across the human body

$$= 25 \times 10^{-3} \times 2000 = 50 \text{ V}.$$

$\therefore$ Voltage between points a and b

$$= 250 - 50 = 200 \text{ V}.$$

Current $$I = \frac{200}{20+4} = \frac{200}{24} \text{ A}.$$

Current through human body

$$= I \frac{R}{R+2000} = 25 \times 10^{-3} \text{ A}.$$

or $$\frac{200}{24} \cdot \frac{R}{R+2000} = 25 \times 10^{-3}$$

or $$200\,R = 0.6 + 1200$$

or resistance of earth electrode on consumer end $R \approx 6\,\Omega$.

14.6 LOCALIZATION OF CABLE FAULTS

In this section, faults occurring in cables which are in use on lower distribution voltages are considered. The common faults which are likely to occur in such cables are :

1. *Ground fault.* The insulation of the cable may breakdown causing a flow of current from the core of the cable to the lead sheath or to the earth. This is called "Ground Fault" .
2. *Short circuit fault.* If the insulation between two conductors is faulty, a current flows between them. This is called a "Short Circuit Fault".

14.6.1 Methods Used for Localizing Ground and Short Circuit Faults

The methods used localizing ground and short circuit faults differ from those used for localizing open circuit faults.

In the case of multicore cables it is advisable, first of all, to measure insulation resistance of each core to earth and also between cores. This enables us to sort out the core that is earthed in case of ground fault ; and to sort out the cores that are shorted in case of a short circuit fault. Loop tests are used for location of ground and short circuit faults. These tests can only be used if a sound cable runs along with the faulty cable or cables. The loop tests work on the principle of a Wheatstone bridge. The advantage of these tests is that their set up is such that the resistance of fault is connected in the battery circuit and therefore does not effect the result. However, if the fault resistance is high, the sensitivity is adversely affected. In this section only two types of tests viz., Murray and Varley loop tests are being described.

Murray loop test. The connections for this test are shown in Fig. 14.37. Figure 14.37(a) relates to the ground fault and Fig. 14.37(b) relates to the short circuit fault.

In both cases, the loop circuit formed by the cable conductors is essentially a Wheatstone bridge consisting of resistances P, Q, R and X. G is a galvanometer for indication of balance.

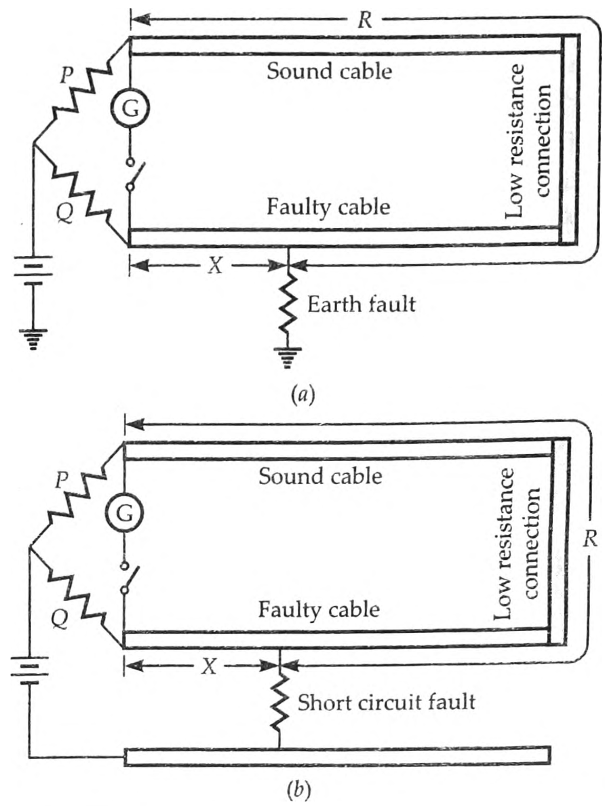


Fig. 14.37 Murray loop test.

The resistors P, Q forming the ratio arms may be decade resistance boxes or slide wires.

Under balance conditions :

$$\frac{X}{R} = \frac{Q}{P} \quad \text{or} \quad \frac{X}{R+X} = \frac{Q}{P+Q}$$

$$\therefore \qquad X = \frac{Q}{P+Q}(R+X) \qquad \text{...(14.53)}$$

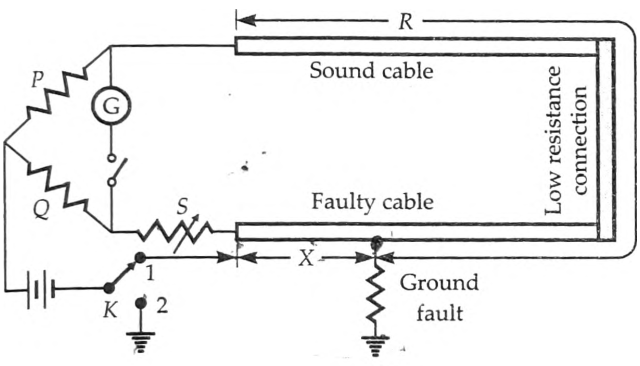
where $(R+X)$ is total loop resistance formed by the sound cable and the faulty cable. When the conductors have the same cross-sectional area and the same resistivity, the resistances are proportional to lengths. If l_1 represents the length of the fault from the test end and l the length of each cable.

Then

$$l_1 = \frac{Q}{P+Q} \cdot 2l \qquad \text{...(14.54)}$$

The above relation shows that the position of the fault may be located when the length of the cable is known. Also, the fault resistance does not alter the balance condition because its resistance enters the battery circuit hence affects only the sensitivity of the bridge circuit. However, if the magnitude of the fault resistance is high, difficulty may be experienced in obtaining the balance condition on account of decrease in sensitivity and hence accurate determination of the position of the fault may not be possible. In such a case, the resistance of the fault may be reduced by applying a high direct or alternating voltage – in consistance with the insulation rating of the cable – on the line so as to carbonize the insulation at the point of the fault.

Varley loop test. In this test we can determine experimentally the total loop resistance instead of calculating it from the known lengths of the cable and its resistance per unit length. The necessary connections for the ground fault are shown in Fig. 14.38(*a*) and for the short circuit fault in Fig. 14.38(*b*). The treatment of the problem, in both cases, is identical.



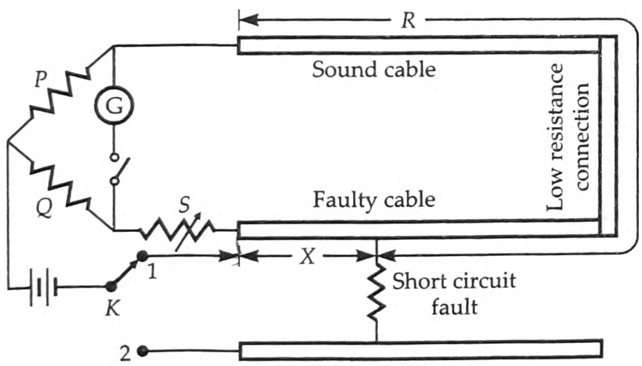


Fig. 14.38 Varley loop test.

A single pole double throw switch K is used in this circuit. Switch K is first thrown to position '1' and the resistance S is varied and balance obtained.

Let the value of S for balance be S_1. The four arms of the Wheatstone bridge are $P, Q, R+X, S_1$.

At balance :

$$\frac{R+X}{S_1} = \frac{P}{Q} \qquad \text{...(14.55)}$$

This determines $R+X$ *i.e.*, the total loop resistance as P, Q and S_1 are known.

The switch K is then thrown to position '2' and the bridge is rebalanced. Let the new value of S for balance be S_2. The four arms of the bridge now are $P, Q, R, X+S_2$.

At balance,

$$\frac{R}{X+S_2} = \frac{P}{Q}.$$

or
$$\frac{R+X+S_2}{X+S_2} = \frac{P+Q}{Q}$$

or
$$X = \frac{(R+X)Q - S_2 P}{P+Q} \qquad \text{...(14.56)}$$

Hence X is known from the known values of P, Q, S_2 from this equation and $R+X$ (the total resistance of 2 cables) as determined from Eqn. 14.55. Knowing the value of X, the position of the fault is determined.

Now

$$\frac{X}{R+X} = \frac{l_1}{2l} \text{ or } l_1 = \frac{X}{R+X} 2l \qquad \text{...(14.57)}$$

where l_1 = length of fault from the test end

and l = total length of conductor.

Equations 14.54 (for Murray loop test) and 14.56 (for Varley loop test) are valid only when the cable sections are uniform throughout the loop. Corrections must be applied in case the cross-sections of faulty and sound cables are different or when the cross-section of the faulty cable is not uniform over its entire length. Since temperature affects the value of resistance, corrections must be applied on this account if the temperatures of the two cables are different. Corrections may also have to be applied in case the cables have a large number of joints.

Example 14.22 *In a test for a fault to earth by Murray loop test, the faulty cable has a length of 5.2 km. The faulty cable is looped with a sound cable of the same length and cross-section. The resistances of ratio arms are 100 Ω and 41.2 Ω at balance. Calculate the distance of the fault from the test end.*

If the decade resistance boxes forming the ratio arms have limits of error of ± 0.5% (standard deviation) of the dial reading, what is the limit of error in the above calculated result ?

Solution. Let X be the resistance of cable from the test end to place of fault and $R+X$ be the total resistance of loop. P and Q are the ratio arms.

$$\therefore \quad X/R = Q/P$$

or
$$X = \frac{Q}{P+Q}(X+R)$$
$$= \frac{41.2}{141.2}(X+R) = 0.292\,(X+R)$$

Let l_1 be the distance of fault from test end l be the length of each cable and r resistance per unit length.

$$\therefore \quad X = rl_1 \text{ and } X+R = 2rl$$

$\therefore$ Distance of fault from test end :

$$l_1 = 0.292 \times 2 \times 5.2 = 3.03 \text{ km.}$$

Limiting fractional error of unknown resistance

$$\frac{\delta X}{X} = \sqrt{\left(\frac{\delta P}{P}\right)^2 + \left(\frac{\delta P + \delta Q}{P+Q}\right)^2} = \sqrt{\left(\frac{\pm 0.5}{100}\right)^2 + \left(\frac{\pm 0.5}{100}\right)^2}$$
$$= 0.707\%.$$

Therefore limiting error of distance

$$= 3.03 \times 0.707/100 \times 1000 = 21.4 \text{ m.}$$

Example 14.23 *In a test by Murray loop method for a fault to earth on a 520 metre length of cable having a resistance of 1.1 Ω per 1000 metre, the faulty cable is looped with a sound cable of the same length but having a resistance 2.29 Ω per 1000 metre. The resistances of the other two arms of the testing network, at balance, are in the ratio of 2.7 : 1. Calculate the distance of fault from the testing end of test cable.*

Solution. Suppose r_1 and r_2 are the resistances per unit length of faulty and sound cable respectively.

Let l be the length of each cable and l_1 be the distance of fault from the test end.

Referring to Fig. 14.37(*a*),

$$\frac{X}{R} = \frac{Q}{P}$$

or
$$X = \frac{Q}{P+Q}(X+R) \quad \ldots(i)$$

Now
$$X = r_1 l_1 = \frac{1.1}{1000} l$$

and
$$X + R = r_1 l + r_2 l$$
$$= \left(\frac{1.1}{1000} + \frac{2.92}{1000}\right) \times 520 = 1.76\ \Omega.$$

Substituting these values in (*i*), we have

$$\frac{1.1}{1000} l_1 = \frac{l}{2.7+1} \times 1.76.$$

$$\therefore \quad l_1 = 432 \text{ m.}$$

Example 14.24 *A short circuit fault is located by Varley loop test. The circuit of Fig. 14.38 (b) is used for the purpose. The ratio arms are set at $P = 5\ \Omega$ and $Q = 10\ \Omega$ and the values of variable resistance S are 16 Ω for position 1 of switch K and 7 Ω for position 2. The sound and faulty cables are identical and have a resistance of 0.4 Ω/km. Determine the length of each cable and the distance of fault from the test end.*

Solution. Let S_1 be the value of resistance S_2 for position 1 and S_2 for position 2.

$$\therefore \quad \frac{R+X}{S_1} = \frac{P}{Q}$$

Hence resistance of loop

$$R + X = \frac{P}{Q} S_1 = \frac{5}{10} \times 16 = 8\ \Omega.$$

Resistance of each cable = 8/2 = 4 Ω

$\therefore$ Length of each cable = 4/0.4 = 10 km.

At positron 2 we have,

$$\frac{P}{X+S_2} = \frac{P}{Q}$$

or
$$\frac{R+X+S_2}{X+S_2} = \frac{P+Q}{Q}$$

or
$$\frac{8+7}{X+7} = \frac{5+10}{10}$$

or
$$X = 3.0\ \Omega.$$

$\therefore$ Distance of fault from testing end

$$= 3/0.4 = 7.5 \text{ km.}$$

Example 14.25 *A Wheatstone bridge is connected for a Varley loop test as shown in Fig. 14.39 when the switch is in position 1, the bridge is balanced with $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$, $R_3 = 100\ \Omega$. When switch is in position 2, the bridge is balanced with $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$ and $R_3 = 99\ \Omega$. If the resistance of the earthed wire is 0.15 km, how many metres from the bridge has the ground fault occurred ?*

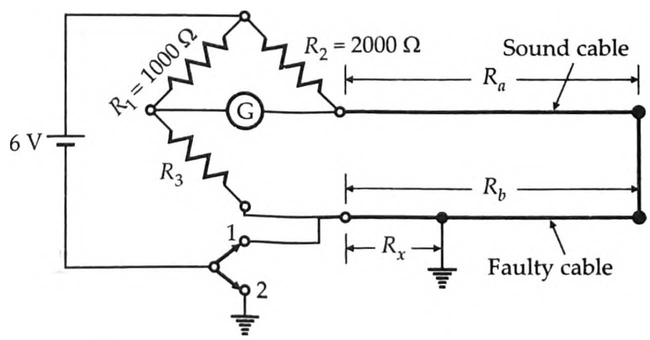


Fig. 14.39 Diagram of Example 14.23.

Solution. When the switch is at position 1

$$2000/1000 = (R_a + R_b)/100$$

or $$R_a + R_b = 200\ \Omega$$

When the switch is at position 2

$$\frac{2000}{1000} = \frac{R_a + R_b - R_x}{99 + R_x}$$

$$198 + 2R_x = 200 - R_x$$

or $$R_x = 0.67\ \Omega.$$

$\therefore$ Resistance of fault from bridge

$$= \frac{0.67}{0.15} \times 1000$$

$$= 4466.7 \text{ m.}$$

Review Questions

1. (*a*) Classify the resistances from the point of view of measurements.

   (*b*) Describe in brief the different methods used for measurement of medium resistances.

2. Describe the ammeter-voltmeter method of measurement of resistances. There are two ways in which the circuit of ammeter voltmeter method can be used

   (*i*) ammeter connected to the side of unknown resistance and

   (*ii*) voltmeter connected to the side of unknown resistance.

   Derive the condition which decides which circuit is to be used for a particular set of ammeter, voltmeter and unknown resistance. Assume equal relative error in both the cases.

3. Describe the substitution method of measurement of medium resistances. List the factors on which the accuracy of the method depends.

4. Draw the circuit of a Wheatstone bridge and derive the conditions of balance.

5. Derive the expression for bridge sensitivity for a Wheatstone bridge with equal arms. Find also the expression for current through the galvanometer for a small unbalance.

6. What are the different factors which affect the precision measurement of medium resistances with Wheatstone bridge ? Explain how their effects are minimized/eliminated.

7. Describe the working of a Carey-Foster Slide-wire bridge.

8. Draw the circuit of a Kelvin-Varley slide and explain its working and advantages.

9. What are the different problems associated with measurement of low resistances ? Explain the principle of working a Kelvin's Double Bridge and explain how the effect of contact resistance and resistance of leads is eliminated.

10. Draw the circuit of a Kelvin's Double Bridge used for measurement of low resistances. Derive the condition for balance.

11. What are the different difficulties encountered in the measurement of high resistances ? Explain how these difficulties are overcome.

12. Explain the loss of charge method for measurements of insulation resistance of cables.

13. Differentiate between volume and surface resistivities. Explain the method of measuring them.

14. Describe the working of a Megohm bridge.

15. What is the importance of the value of Earth's resistance. What are the factors which influence its value ? Describe the fall of potential method for measurement of earth resistance.

16. Describe the construction and working of an Earth Tester. Explain how it can be used for measurement of resistance of an earthing electrode.

17. Describe the Murray loop test for localization of ground and short circuit faults in cables.

18. Describe the Varley loop test for localization of ground and short circuit faults in cables.

Unsolved Problems

1. A voltmeter of resistance 500 Ω and a milliammeter of 1.0 Ω resistance are used to measure a resistance by ammeter voltmeter method. If the voltmeter reads 20 V and milliammeter 100 mA.

   Calculate the value of measured resistance

   (*i*) if the voltmeter is put across the resistance and the milliammeter connected in series with the unknown resistance

   (*ii*) if the voltmeter is put across the unknown resistance with ammeter connected on the supply side.

   [**Ans.** 199 Ω ; 333 Ω]

2. In a laboratory a voltmeter of 200 Ω resistance and an ammeter of 0.02 Ω resistance are available. Calculate the value of resistance that can be measured by the ammeter voltmeter method for which the two different circuit measurements give equal errors. [**Ans.** 2 Ω]

3. In a measurement of resistance by the substitution method a standard resistance of 100 kΩ is used. The galvanometer has a resistance of 2000 Ω, and gives the following deflections :

   (*i*) With unknown resistance : 46 divisions,

   (*ii*) With standard resistance : 40 divisions.

   Find the value of unknown resistance

   [**Ans.** 86.7 kΩ]

4. The four arms of a Wheatstone bridge are as follows :

   $AB = 100\,\Omega$; $BC = 10\,\Omega$;

   $CD = 4\,\Omega$ and $DA = 50\,\Omega$.

   The galvanometer has a resistance of 20 Ω and is connected across *BD*. A source of 10 V d.c. is connected across *AC*. Find the current through the galvanometer. What should be the resistance in the arm *DA* for no current through the galvanometer ?

   [**Ans.** 5.15 mA ; 40 Ω]

5. The four arms of a Wheatstone bridge are as follows :

   $AB = 100\,\Omega$;

   $BC = 1000\,\Omega$;

   $CD = 4000\,\Omega$ and

   $DA = 400\,\Omega$.

   The galvanometer has a resistance of 100 Ω, a sensitivity of 100 mm/μA and is connected across *AC*. A source of 4 V d.c. is connected across *BD*. Calculate the current through the galvanometer and its deflection if the resistance of ami *DA* is changed from 400 Ω to 401 Ω.

   [**Ans.** 1.63 μA ; 16.3 mm]

6. In a Wheatstone bridge, the ratio arms $AB = 10\,\Omega$ and $BC = 100\Omega$; standard-resistance across $CD = 10\,\Omega$. The shunt across 10 Ω ratio arm has to be changed from 22310 to 27670 Ω, when the resistor R_2 was changed for R_1 in the arm *DA*. Calculate in magnitude the difference between the resistances of R_1 and R_2. The bridge is balanced in both the cases. [**Ans.** 86.8 μΩ]

7. A modified Wheatstone bridge network is constituted as follows :

   AB is a resistance *P* in parallel with resistance *p* ;

   BC is a resistance *Q* in parallel with a resistance *q* ;
   CD and *DA* are resistances *R* and *S* respectively.

   The nominal values of *P*, *Q* and *S* are each 10 Ω. With resistance *R* in circuit, balance is obtained with $p = 30{,}000\,\Omega$ and $q = 25{,}000$. With *R* replaced by a standard resistance of 10 Ω, balance is obtained when p = 15,000 Q and q = 40,000 Ω. Calculate the value of *R*. [**Ans.** 9.99952 Ω]

8. A Wheatstone bridge is used to measure the resistance of a resistor. The bridge was balanced with the values shown in Fig. 14.40. It is found that, due to presence of chemical impurities emfs are set

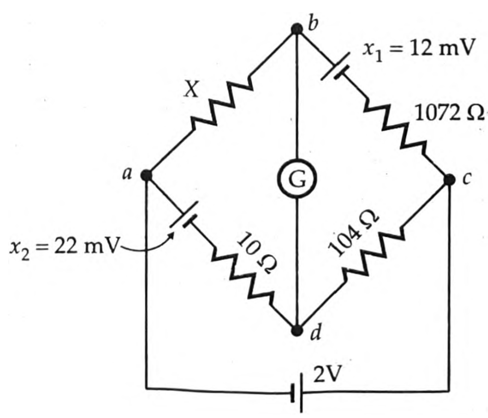


Fig. 14.40 Diagram of Problem 14.8.

up at points x_1 and x_2 of respective values 12 mV and 22 mV with polarities being shown in the diagram. Calculate ;

(i) the apparent value of X,

(ii) the true of X,

(iii) the % error in the apparent value of X.

[**Ans.** 103 Ω ; 115.7 Ω ; – 12.3%]

9. A Kelvin bridge is balanced with the following constants :

Outer ratio arm 100 Ω and 1000 Ω ;

Inner arms, ratio 99.92 Ω and 1000.6 Ω ;

Resistance of link, 0.1 Ω ;

Standard resistance, 0.00377 Ω.

Calculate the value of unknown resistance.

[**Ans.** 0.000467 Ω]

10. The ratio arms of a kelvin bridge are 100 Ω each. The galvanometer has an internal resistance of 500 Ω and a current sensitivity of 200 mm/μA. The unknown resistance is 0.1002 and the standard resistance is set at 0.1000 Ω. A d.c. current of 10 A is passed through the standard and the unknown resistance from a 2.2 V battery in series with a rheostat.

Calculate the deflection of the galvanometer. Neglect the resistance of the link. Find also the resistance unbalance to produce a deflection of 1 mm.

[**Ans.** 3320 mm ; 0.6 μΩ]

11. Calculate insulation resistance of a cable in which the voltage falls from 100 to 80 V in 20 s. The capacitance is 300 pF. [**Ans.** 29800 MΩ]

12. A cable immersed in a testing tank is charged to a voltage of 200 V with an electrostatic voltmeter connected between core and tank. After one minute's electrification, the cable and voltmeter are isolated and is found that in 20 s the voltage falls to 150 V. The test is repeated with a resistance of 20 MΩ between core and tank and the voltage is found to fall to 100 V in the same time. Calculate the insulation resistance of the cable, that of the voltmeter is given as 50 MΩ.

13. The following observations were made for a loss of charge method for the determination of a high resistance *R*. The charged capacitor of capacitance 12.5 μF was connected across an electrostatic voltmeter and *R* in parallel and the voltage measured after intervals of time.

time, s	*voltage, V*
0	150
100	121
200	97
300	83
400	65
500	57
600	340

The further set of readings was taken with resistor *R* removed from circuit, as follows :

time, s	*voltage, V*
0	150
200	143
400	133
600	121

The readings of voltmeter were subject to large random errors. Calculate the value of resistance *R*.

[**Ans.** 52.5 MΩ]

14. A high resistance of 200 MΩ has a leakage resistance of 400 MΩ between each of its main terminals and the guard terminal. Find the percentage error in measurement if the above resistance is measured by an ordinary Wheatstone bridge without providing guard circuit.

[**Ans.** 20% low]

15. Two mains are working at potential difference of 220 V. A 250 V voltmeter having 10,000 Ω/V when connected between positive main and earth reads 149 V and the reading is 42 V when connected between negative main the earth. Calculate the insulation resistance of each main with respect to earth. [**Ans.** 1.73 and 0.486 MΩ]

16. A feeder cable 250 metre long has a fault to earth. The fault is localized by the following resistance measurements between earth and one end of cable (*a*) distant end insulated, 6.95 Ω (*b*) distant end earthed, 1.71 Ω. The cable has a total resistance of 1.80 Ω. Find the resistance of fault and its distance from test end.

17. In a Murray loop test for ground fault on a 500 metre long cable having a resistance of 1.6 Ω/km, the faulty cable is looped with a sound cable of same length and cross-section. If resistances of ratio arms are 3 : 1, calculate the distance of the fault from the test end. [**Ans.** 250 m]

18. A telephone line, 5 km long, has an earth fault 2.3 km from test end. If the resistance of the lines per km is 4.0 Ω, what value of variable resistance will give balance in a Varley loop test ? The ratio arms are equal. **[Ans.** 21.6 Ω]

19. A telephone wire having a resistance of 14.6 Ω km, develops a fault to earth. When looped with a sound wire of the same length, the total resistance is found to be 56 Ω. If the value of variable resistance is 16.3 at balance in a Varley loop test with ratio arms equal, calculate the distance of fault. **[Ans.** 1.36 km]

20. A Wheatstone bridge is connected for a Varley loop test as shown in Fig. 14.41. When the switch is in position *a*, the bridge is balanced with $R_1 = 1000\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 53\,\Omega$. When switch *S* is in position *b*, the bridge is balanced with $R_1 = 1000\,\Omega$, $R_2 = 100\,\Omega$ and $R_3 = 52.9\,\Omega$. If the resistance of the shorted wire is 0.015 Ω/m. What is the distance between the place of fault and the bridge ? **[Ans.** 2353 m]

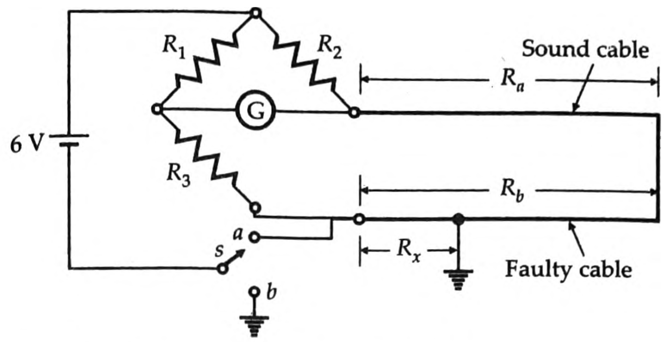


Fig. 14.41 Diagram of Problem 14.20

Objective Type Questions

Tick (√) the most appropriate answer :

1. A resistance of value 10 Ω approximately is to be measured by ammeter-voltmeter method with resistance of ammeter is 0.02 Ω and that of voltmeter is 5000 Ω. The resistance should be measured
   (*a*) by connecting the ammeter on the side of unknown resistance as this connection gives better accuracy
   (*b*) by connecting the voltmeter on the side of unknown resistance as this connection gives better accuracy
   (*c*) by any of the two connections, as both of them give equal accuracy
   (*d*) none of the above.

2. A unknown resistance is measured by substitution method. First a standard known resistance of 100 Ω is connected in series with a circuit having a rheostat and a galvanometer. The battery voltage is 10 V and the setting of the rheostat is 500 Ω and the galvanometer shows a deflection of 60°. After this test, the battery voltage goes down to 9 V and when the unknown resistance is substituted for the known resistance, the galvanometer again shows a deflection of 60° with the same setting of the rheostat. The value of unknown resistance
   (*a*) 100 Ω
   (*b*) 54 Ω
   (*c*) 90 Ω
   (*d*) none of the above.

3. Equal resistances of 100 Ω each are connected in each arm of a Wheatstone bridge which is supplied by a 2 V battery source. The galvanometer of negligible resistance connected to the bridge can sense as low current as 1 μA. The smallest value of resistance that can be measured is :
   (*a*) 20 μΩ
   (*b*) 2 μΩ
   (*c*) 20 μΩ
   (*d*) none of the above.

4. A Wheatstone bridge has ratio arms of 1000 Ω and 100 Ω resistance, the standard resistance arms consists 4 decade resistance boxes of 1000, 100, 10, 1 Ω steps. The maximum and minimum values of unknown resistance which can be determined with this set up is
   (*a*) 111100 Ω,1 Ω

(b) 11110 Ω, 10 Ω

(c) 111100 Ω, 10 Ω

(d) none of the above.

5. A Wheatstone bridge cannot be used for precision measurements because errors are introduced into on account of

(a) resistance of connecting leads

(b) thermo-electric emfs

(c) contact resistances

(d) all the above.

6. A Kelvin-Varley Slide consists of 4 decade dividers. The first decade is constructed by having 11 coils of 10 kΩ resistance each. The subsequent decades will have coils of :

(a) 11 coils of 20 kΩ each, 11 coils of 40 kΩ each, 11 coils of 80 kΩ each

(b) 11 coils of 10 kΩ each, 11 coils of 5 kΩ each, 10 coils of 1 kΩ each

(c) 11 coils of 2 kΩ each, 11 coils of 400 Ω each, 11 coils of 80 Ω each

(d) 11 coils of 2 kΩ each, 11 coils of 400 Ω each, 10 coils of 80 Ω each.

7. Low resistances are provided with four terminals

(a) to facilitate the connection of current and potential circuits

(b) in order that the resistance value becomes definite irrespective of the nature of contacts at the current terminals

(c) to eliminate the effect of thermo-electric emfs

(d) to eliminate the effect of leads.

8. In a Kelvin's Double Bridge two sets of readings are taken when measuring a low resistance, one with the current in one direction and the other with direction of current reversed.

This is done to

(a) eliminate the effect of contact resistance

(b) eliminate the effect of resistance of leads

(c) correct for changes in battery voltage

(d) eliminate the effect of thermo-electric emfs.

9. High resistances are provided with a guard terminal.

This guard terminal is used to :

(a) bypass the leakage current

(b) guard the resistance against stray electrostatic fields

(c) guard the resistance against overloads

(d) none of the above.

10. When measuring insulation resistance of cables using d.c. source, the galvanometer used should be initially short circuited because,

(a) cables have a low value of initial resistance

(b) cables have a high value of capacitance which draws a high value of charging current

(c) cables have a low value of capacitance which draws a high value of charging current

(d) none of the above.

11. A circular piece of specimen has a surface resistance R_s. Its diameter is d and the thickness is t. The surface resistivity ps of the specimen is given by :

(a) $\frac{\pi d^2 R_s}{t}$

(b) $\frac{\pi d R_s}{t}$

(c) $\frac{R_s t}{\pi d^2}$

(d) $\frac{R_s t}{\pi d}$

12. The value of resistance of an earthing electrode depends upon :

(a) shape and material of electrode

(b) depth to which electrode is driven into earth.

(c) specific resistance of soil

(d) all the above.

13. From the point of view of safety, the resistance of earthing electrode should be :

(a) low

(b) high

(c) medium

(d) the value of resistance of earth electrodes does not affect the safety.

14. The advantage of Varley loop tests over Murray loop tests is

(a) they can be used for localizing of short circuit faults

(b) they can be used for localizing of earth fault (*c*) the loop resistance can be experimentally determined

(*d*) their accuracy is higher.

15. When localizing ground fault with the help of loop tests, the resistance of the fault :

(*a*) affects the balance conditions

(*b*) affects the value of cable resistance

(*c*) affects the sensitivity of the bridge

(*d*) all the above.

Answers

1. (*c*) **2.** (*b*) **3.** (*a*) **4.** (*c*) **5.** (*d*) **6.** (*d*) **7.** (*b*) **8.** (*d*) **9.** (*a*) **10.** (*b*)
11. (*b*) **12.** (*d*) **13.** (*a*) **14.** (*c*) **15.** (*c*)

15 Potentiometers

D.C. POTENTIOMETERS

15.1 INTRODUCTION

A potentiometer is an instrument designed to measure an unknown voltage by comparing it with a known voltage. The known voltage may be supplied by a standard cell or any other known voltage-reference source (See pages 149-153). Measurements using comparison methods are capable of a high degree of accuracy because the result obtained does not depend upon the actual deflection of a pointer, as is the case in deflectional methods, but only upon the accuracy with which the voltage of the reference source is known.

Another advantage of the potentiometers is that since a potentiometer makes use of a balance or null condition, no current flows and hence no power is consumed in the circuit containing the unknown emf when the instrument is balanced. Thus the determination of voltage by a potentiometer is quite independent of the source resistance.

Since a potentiometer measures voltage, it can also be used to determine current simply by measuring the voltage drop produced by the unknown current passing through a known standard resistance.

The potentiometer is extensively used for a calibration of voltmeters and ammeters and has in fact become the standard for the calibration of these instruments. For the above mentioned advantages the potentiometer has become very important in the field of electrical measurements and calibration.

15.1.1 Basic Potentiometer Circuit

The principle of operation of all potentiometers is based on the circuit of Fig. 15.1, which shows the schematic diagram of the basic slide wire potentiometer.

With switch 'S' in the *"operate"* position and the galvanometer key K open, the battery supplies the *"working current"* through the rheostat R and the slide wire. The working current through the slide wire may be varied by changing the rheostat setting. The method of measuring the unknown voltage, E, depends upon finding a position for the sliding contact such the galvanometer shows zero deflection, *i.e.*, indicates null condition, when the galvanometer key, K, is closed. Zero galvanometer deflection or a null means that the unknown voltage, E, is equal to the voltage drop E_1, across portion ac of the slide wire. Thus determination of the value of unknown voltage now becomes a matter of evaluating the voltage drop E_1 along the portion ac of the slide wire.

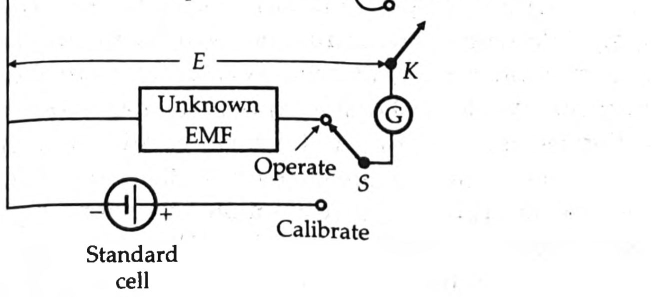


Fig. 15.1 Circuit diagram of a basic slide wire potentiometer.