16

A.C. Bridges

16.1 INTRODUCTION

Alternating current bridge methods are of outstanding importance for measurement of electrical quantities. Measurement of inductance, capacitance, storage factor, loss factor may be made conveniently and accurately by employing a.c. bridge networks.

The a.c. bridge is a natural outgrowth of the Wheatstone bridge. An a.c. bridge, in its basic form, consists of four arms, a source of excitation, and a balance detector. In an a.c. bridge each of the four arms is an impedance, and the battery and the galvanometer of the Wheatstone bridge are replaced respectively by an a.c. source and a detector sensitive to small alternating potential differences.

The usefulness of a.c. bridge circuits is not restricted to the measurement of unknown impedances and associated parameters like inductance, capacitance, storage factor, dissipation factor etc. These circuits find other applications in communication systems and complex electronic circuits. Alternating current bridge circuits are commonly used for phase shifting, providing feedback paths for oscillators and amplifiers, filtering out undesirable signals and measuring the frequency of audio signals.

16.2 SOURCES AND DETECTORS

For measurements at low frequencies, the power line may act as the source of supply to the bridge circuits. For higher frequencies electronic oscillators are universally used as bridge source supplies. These oscillators have the advantage that the frequency is constant, easily adjustable, and determinable with accuracy. The waveform is very close to a sine wave, and their power output is sufficient for most bridge measurements. A typical oscillator has a frequency range of 40 Hz to 125 kHz with a power output of 7 W.

The detectors commonly used for a.c. bridges are :

(*i*) Head phones,

(*ii*) Vibration galvanometers, and

(*iii*) Tuneable amplifier detectors.

Head phones are widely used as detectors at frequencies of 250 Hz and over upto 3 or 4 kHz. They are most sensitive detectors for this frequency range.

When working at a single frequency a tuned detector normally gives the greatest sensitivity and discrimination against harmonics in the supply.

Vibration galvanometers are extremely useful for power and low audio frequency ranges. Vibration galvanometers are manufactured to work at various frequencies ranging from 5 Hz to 1000 Hz but are most commonly used below 200 Hz as below this frequency they are more sensitive than the head phones.

Tuneable amplifier detectors are the most versatile of the detectors. The transistor amplifier can be tuned electrically and thus can be made to respond to a narrow bandwidth at the bridge frequency. The output of the amplifier is fed to a pointer type of instrument. This detector can be used, over a frequency range of 10 Hz to 100 kHz.

For ordinary a.c. bridge measurements of inductance and capacitance, a fixed frequency oscillator of 1000 Hz and output of about 1 W is adequate. For more specialised work continuously variable oscillators are preferable with outputs upto 5 W. The high power may be necessary on some occasions, but in practice it is better to limit the power supplied to the bridge. Another practice which is usually followed is to use an untuned amplifier detector. The balance detection is sensed both orally by head phones, and visually by a pointer galvanometer having a logarithmic deflection (to avoid damage to the galvanometer which may be caused by unbalance).

16.3 GENERAL EQUATION FOR BRIDGE BALANCE

Figure 16.1 shows a basic a.c. bridge. The four arms of the bridge are impedances Z_1, Z_2, Z_3 and Z_4.

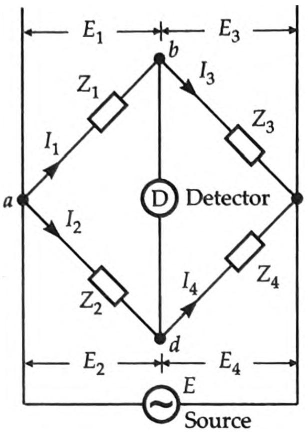


Fig. 16.1 Basic a.c. bridge network.

The conditions for balance of bridge require that there should be no current through the detector. This requires that the potential difference between points b and d should be zero. This will be the case when the voltage drop from a to b equals to voltage drop from a to d, both in magnitude and phase. In complex notation we can, thus, write :

$$E_1 = E_2 \qquad ...(16.1)$$

or $$I_1 Z_1 = I_2 Z_2 \qquad ...(16.2)$$

Also at balance,

$$I_1 = I_3 = \frac{E}{Z_1 + Z_3} \qquad ...(16.3)$$

and $$I_2 = I_4 = \frac{E}{Z_2 + Z_4} \qquad ...(16.4)$$

Substitution of Eqns. 16.3 and 16.4 into Eqn. 16.2 gives,

$$Z_1 Z_4 = Z_2 Z_3 \qquad ...(16.5)$$

or when using admittances instead of impedances

$$Y_1 Y_4 = Y_2 Y_3 \qquad ...(16.6)$$

Equations 16.5 and 16.6 represent the basic equations for balance of an a.c. bridge. Equation 16.5 is convenient to use when dealing with series elements of a bridge while Eqn. 16.6 is useful when dealing with parallel elements.

Equation 16.5 states that the product of impedances of one pair opposite arms must equal the product of impedances of the other pair of opposite arms expressed in complex notation. This means that both magnitudes and the phase angles of the impedances must be taken into account.

Considering the polar form, the impedance can be written as $\mathbf{Z} = Z\angle\theta$, where Z represents the magnitude and θ represents the phase angle of the complex impedance. Now Eqn. 16.5 can be re-written in the form

$$(Z_1 \angle\theta_1)(Z_4 \angle\theta_4) = (Z_2 \angle\theta_2)(Z_3 \angle\theta_3) \qquad ...(16.7)$$

Thus for balance, we must have :

$$Z_1 Z_4 \angle\theta_1 + \theta_4 = Z_2 Z_3 \angle\theta_2 + \theta_3 \qquad ...(16.8)$$

Equation 16.8 shows that two conditions must be satisfied simultaneously when balancing an a.c. bridge. The first condition is that the magnitude of impedances satisfy the relationship :

$$Z_1 Z_4 = Z_2 Z_3 \qquad ...(16.9)$$

The second condition is that the phase angles of impedances satisfy the relationship :

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \qquad ...(16.10)$$

The phase angles are positive for an inductive impedance and negative for capacitive impedance.

If we work in terms of rectangular co-ordinates, we have

$$\mathbf{Z}_1 = R_1 + jX_1\,; \qquad \mathbf{Z}_2 = R_2 + jX_2\,;$$
$$\mathbf{Z}_3 = R_3 + jX_3 \text{ and } \mathbf{Z}_4 = R_4 + jX_4.$$

Thus from Eqn. 16.5, for balance,

$$\mathbf{Z}_1 \mathbf{Z}_4 = \mathbf{Z}_2 \mathbf{Z}_3$$

or $$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3)$$

or $$R_1 R_4 - X_1 X_4 + j(X_1 R_4 + X_4 R_1) = R_2 R_3 - X_2 X_3 + j(X_2 R_3 + X_3 R_2) \qquad ...(16.11)$$

Equation 16.11 is a complex equation and a complex equation is satisfied only if real and imaginary parts of each side of the equation are separately equal. Thus, for balance,

$$R_1 R_4 - X_1 X_4 = R_2 R_3 - X_2 X_3 \qquad ...(16.12)$$

and $$X_1 R_4 + X_4 R_1 = X_2 R_3 + X_3 R_2 \qquad ...(16.13)$$

Thus there are two independent conditions for balance and both of them must be satisfied for the bridge to be balanced.

Example 16.1 *The four impedances of an a.c. bridge shown in Fig. 16.1 are :*

$$\mathbf{Z}_1 = 400\,\Omega \angle 50°\,; \quad \mathbf{Z}_2 = 200\,\Omega \angle 40°\,;$$
$$\mathbf{Z}_3 = 800 \angle -50° \;; \quad \mathbf{Z}_4 = 400\,\Omega \angle 20°$$

Find out whether the bridge is balanced under these conditions or not.

Solution. Applying the first condition of balance for magnitudes,

$$Z_1 Z_4 = Z_2 Z_3$$

Now, $Z_1Z_4 = 400 \times 400 = 1,60,000$

and $Z_2Z_3 = 200 \times 800 = 1,60,000.$

$\therefore \quad Z_1Z_4 = Z_2Z_3.$

Thus the first condition is satisfied.

Applying the second condition for balance required for phase,

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

Now $\angle\theta_1 + \angle\theta_4 = 50° + 20° = 70°$

and $\angle\theta_2 + \angle\theta_3 = 40° - 50° = -10°.$

This indicates that the condition for phase relationship is not satisfied and therefore the bridge is unbalanced even though the condition for equality of magnitudes is satisfied.

Example 16.2 *An a.c. bridge circuit working at 1000 Hz is shown in Fig. 16.1, Arm ab is a 0.2 μF pure capacitance ; arm bc is a 500 Ω pure resistance ; arm cd contains an unknown impedance and arm da has a 300 Ω resistance in parallel with a 0.1 μF capacitor. Find the R and C or L constants of arm cd considering it as a series circuit.*

Solution. Impedance of arm *ab* is :

$$Z_1 = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 1000 \times 0.2 \times 10.6} = 800\ \Omega.$$

and $Z_1 = 800\underline{/-90°}\ \Omega$ since it is a pure capacitance.

Impedance of arm *bc* is

$$Z_3 = 500\ \Omega$$

or $Z_3 = 500\angle 0°$ since it is a pure resistance.

Arm *da* contains a 300 Ω resistance in parallel with a 0.1 μF capacitance.

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2} = \frac{300}{1 + j(2\pi \times 1000 \times 0.1 \times 10^{-6} \times 300)}$$

$$= 294.8 - j55.4 = 300\underline{/-10.6°}$$

For balance, $Z_1 Z_4 = Z_2 Z_3$

$\therefore$ Impedance of *cd* required for balance is,

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{360 \times 500}{800}\underline{/-10.6° + 0° + 90°}$$

$$= 187.5\underline{/78.4°}.$$

The positive angle for impedance indicates that the branch consists of a series *R-L* circuit.

Resistance, $R_4 = 187.5 \cos 78.4° = 37.7\ \Omega.$

Inductive reactance

$$X_4 = 187.5 \sin 78.4° = 83.6\ \Omega.$$

Inductance, $L_4 = 183.6 / 2\pi(1000)\text{H} = 29.2$ mH.

16.4 GENERAL FORM OF AN A.C. BRIDGE

As an example let us consider the bridge circuit of Fig. 16.2. R_3 and R_4 are non-inductive resistances, L_1 and L_2 are inductances of the negligible resistance and R_1 and R_2 are non-inductive resistors.

Therefore at balance,

$$Z_1Z_4 = Z_2Z_3$$

or $\quad (R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$

Equating the real and imaginary parts separately, we have,

$$R_1R_4 = R_2R_3 \quad \text{or} \quad R_1 = \frac{R_3}{R_4}\cdot R_2 \qquad ...(16.14)$$

and $$j\omega L_1 R_4 = j\omega L_2 R_3 \quad \text{or} \quad L_1 = \frac{R_3}{R_4} L_2 \qquad ...(16.15)$$

Thus if L_1 and R_1 are unknown, the above bridge may be used to measure these quantities in terms of R_2, R_3, R_4 and L_2. We may deduce several important conclusions from the above simple example. They are :

1. Two balance equations are always obtained for an a.c. bridge circuit. This follows from the fact that for balance in an a.c. bridge, both magnitude and phase relationships must be satisfied. This requires that real and imaginary terms must be separated, which give two equations to be satisfied for balance.

2. The two balance equations enable us to know two unknown quantities. The two quantities are usually a resistance and an inductance or a capacitance.

3. In order to satisfy both conditions for balance and for convenience of manipulation, the bridge must contain two variable elements in its configuration. For greatest convenience, each of the balance equations must contain one variable element, and one only. The equations are then said to be *independent*. In the bridge of Fig. 16.2, R_2 and L_2 are obvious choice as variable elements since L_2 does not appear in the expression for

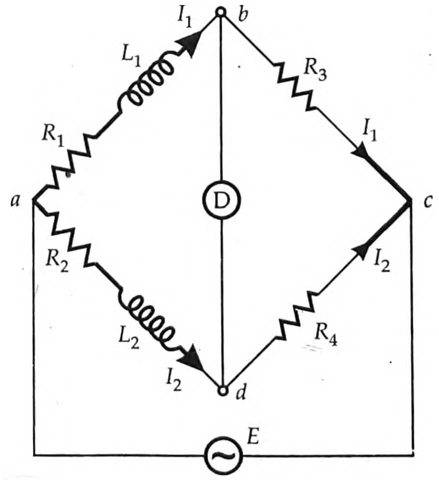


Fig. 16.2 Inductance comparison Bridge.

R_1 and R_2 does not figure in the expression for L_1 and hence the two balance equations are independent. The technique of balancing is to adjust L_2 till a minimum indication is obtained on the detector, then to adjust R_2 until a new smaller minimum indication is obtained. Then L_2 and R_2 are alternately adjusted until the detector shows no indication.

The process of alternate manipulation of two variable elements is rather typical of the general balancing procedures adopted in most a.c. bridges. When two variables are chosen such that the two balance equations are no longer independent, the bridge has a very poor convergence of balance and gives the effect of sliding balance. The term *Sliding Balance* describes a condition of interaction between the two controls. Thus when we balance with R_2, then go to R_3 and back to R_2 for adjustment, we find a new apparent balance. Thus the balance point appears to move, or slide and settles only gradually to its final point after many adjustments. It may be emphasised here that in case the two balance conditions are independent, not more than two or three adjustments of the variable elements would be necessary to obtain balance. In case we choose the two variable components such that the two equations are not independent the balance procedure becomes laborious and time-consuming. For example, if we choose R_2 and R_3 as variable elements, the two equations are no longer independent since R_3 appears in both the equations. Thus R_3 is adjusted to satisfy Eqn. 16.15, its value may be becoming farther removed from that which satisfies Eqn. 16.14. There are two adjustments, one resistive and the other reactive that must be made to secure balance. For the usual magnitude responsive detector, these adjustments must be made alternately until they converge on the balance point.

The convergence to balance point is best when both the variable elements are in the same arms.

4. In this bridge circuit balance equations are independent of frequency. This is often a considerable advantage in an a.c. bridge, for the exact value of the source frequency need not then be known. Also, if a bridge is balanced for a fundamental frequency it should also be balanced for any harmonic and the wave-form of the source need not be perfectly sinusoidal. On the other hand, it must be realized that the effective inductance and resistance for example, of a coil, vary with frequency (see pages 162-163 and 169), so that a bridge balanced at a fundamental frequency is never, in practice, truly balanced for the harmonics. To minimize difficulties due to this the source wave form should be good, and it is often an advantage to use a detector tuned to the fundamental frequency. Further while the disappearance of the frequency factor is of advantage in many bridges, some bridges derive their usefulness from the presence of a frequency factor ; such bridges must then be supplied from a source with very good wave-form and high frequency stability.

Alternatively, they may be used to determine frequency.

16.5 MEASUREMENT OF SELF-INDUCTANCE

16.5.1 Maxwell's Inductance Bridge

This bridge circuit measures an inductance by comparison with a variable standard self-inductance. The connections and the phasor diagrams for balance conditions are shown in Fig. 16.3.

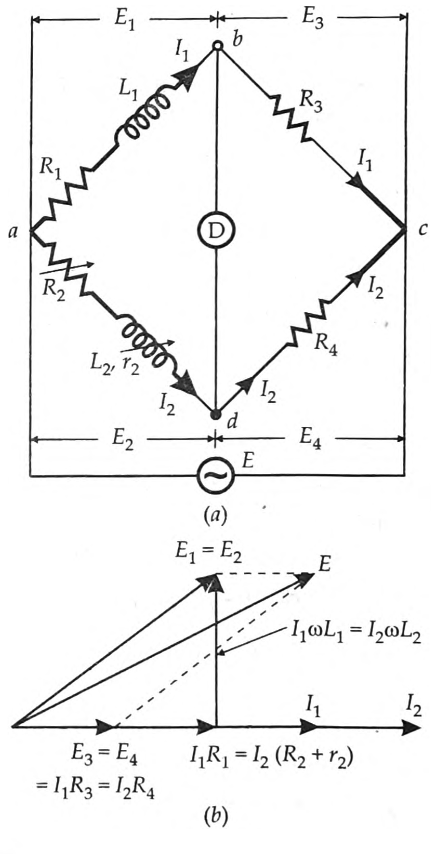


Fig. 16.3 Maxwell's inductance Bridge.

Let

L_1 = unknown inductance of resistance R_1,

L_2 = variable inductance of fixed resistance r_2,

R_2 = variable resistance connected in series with inductor L_2,

and R_3, R_4 = known non-inductive resistances.

The theory of this bridge has been dealt with in Art. 16.4. At balance,

$$L_1 = \frac{R_3}{R_4} L_2 \quad \text{...(16.16)}$$

and
$$R_1 = \frac{R_3}{R_4}(R_2 + r_2) \quad \text{...(16.17)}$$

Resistors R_3 and R_4 are normally a selection of values from 10, 100, 1000 and 10,000 Ω. r_2 is a decade resistance box. In some cases, an additional known resistance may have to be inserted in series with unknown coil in order to obtain balance.

16.5.2 Maxwell's Inductance

Capacitance bridge. In this bridge, an inductance is measured by comparison with a standard variable capacitance. The connections and the phasor diagram at the balance conditions are given in Fig. 16.4.

Let L_1 = unknown inductance,

R_1 = effective resistance of inductor L_1,

R_2, R_3, R_4 = known non-inductive resistances,

and C_4 = variable standard capacitor.

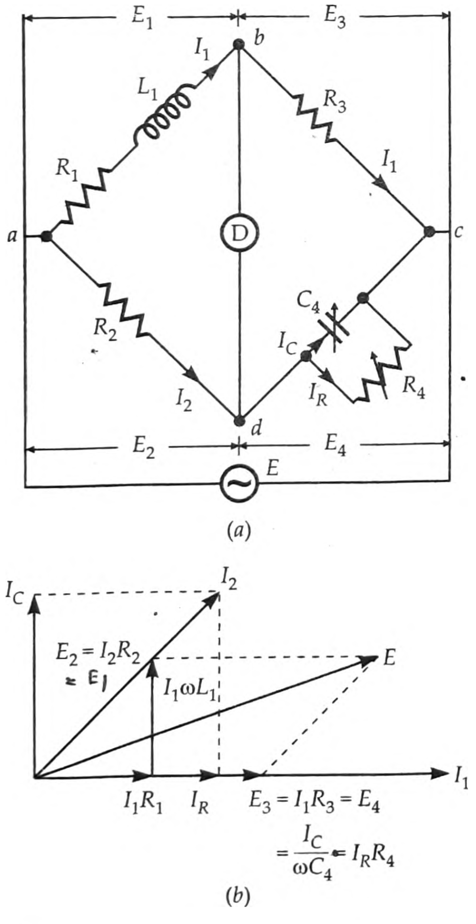


Fig. 16.4 Maxwell's inductance capacitance bridge.

Writing the equation for balance

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = R_2 R_3$$

or
$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 C_4 R_4$$

Separating the real and imaginary terms, we have

$$R_1 = \frac{R_2 R_3}{R_4} \quad \text{...(16.18)}$$

and
$$L_1 = R_2 R_3 C_4 \quad \text{...(16.19)}$$

Thus we have two variables R_4 and C_4 which appear in one of the two balance equations and hence the two equations are independent.

The expression for Q factor

$$Q = \omega L_1 / R_1 = \omega C_4 R_4 \quad \text{...(16.20)}$$

Advantages :

The advantages of this bridge are :

1. The two balance equations are independent if we choose R_4 and C_4 as variable elements.
2. The frequency does not appear in any of the two equations.
3. This bridge yields simple expression for unknowns L_1 and R_1 in terms of known bridge elements.

   Physically R_2 and R_3 are each, say, 10, 100, 1000 or 10,000 Ω and their value is selected to give suitable value of product $R_2 R_3$ which appears in both the balance equations ; C_4 is decade capacitor and R_4 a decade resistor.

   The simplicity of the bridge can be appreciated by the following example. Suppose the product $R_2 R_3$ is 10^6. Therefore, inductance is $L_1 = C_4 \times 10^6$ (see Eqn. 16.19).

   Thus when balance is achieved the value of C_4 in μF directly gives the value of inductance in H.
4. The Maxwell's inductance – capacitance bridge is very useful for measurement of a wide range of inductance at power and audio frequencies.

Disadvantages :

The main disadvantages of this bridge are :

1. This bridge requires a variable standard capacitor which may be very expensive if calibrated to a high degree of accuracy. Therefore sometimes a fixed standard capacitor is used, either because a variable capacitor is not available or because fixed capacitors have a higher degree of accuracy and are less expensive than the variable ones. The balance adjustments are then done by :

(*a*) either varying R_2 and R_4 and since R_2 appears in both the balance equations, the balance adjustments become difficult ; or

(*b*) putting an additional resistance in series with the inductance under measurement and then varying this resistance and R_4.

2. The bridge is limited to measurement of low Q coils, $(1 < Q < 10)$. It is clear from Eqn. 16.20 that the measurement of high Q coils demands a large value for resistance R_4, perhaps 10^5 or $10^6\,\Omega$. The resistance boxes of such high values are very expensive. Thus for values of $Q > 10$, the Maxwell's bridge is unsuitable.

The Maxwell's bridge is also unsuited for coils with a very low value of Q (*i.e.*, $Q < 1$). Q values of this magnitude occur in inductive resistors, or in an R.F. coil if measured at low frequencies. The difficulty in measurement occurs on account of labour involved in obtaining balance since nominally a fixed capacitor is used and balance is obtained by manipulating resistances R_2 and R_4 alternately. This difficulty is explained as below :

From Eqns. 16.18 and 16.19, it is clear that R_2 enters into both the expressions. A preliminary inductive balance is made with R_2 and then R_4 is varied to give a resistive balance which is dependent on the R_2 setting. Accordingly when R_2 is changed for a second inductive balance, the resistive balance is disturbed and moves to a new value giving slow "*convergence*" to balance. This is particularly true of a low Q coil, for which resistance is prominent (as $Q = \omega L/R$). Thus a sliding balance condition prevails and it takes many manipulations to achieve balance for low Q coils with a Maxwell's bridge.

From the above discussions we conclude that a Maxwell's bridge is suited for measurements of only medium Q coils.

16.5.3 Hay's Bridge

The Hay 's bridge is a modification of Maxwell's bridge. The connection diagram and the phasor diagram for this bridge are shown in Fig. 16.5. This bridge uses a resistance in series with the standard capacitor (unlike the Maxwell's bridge which uses a resistance in parallel with the capacitor).

Let

L_1 = unknown inductance having a resistance R_1.

R_2, R_3, R_4 = known non-inductive resistance,

and C_4 = standard capacitor.

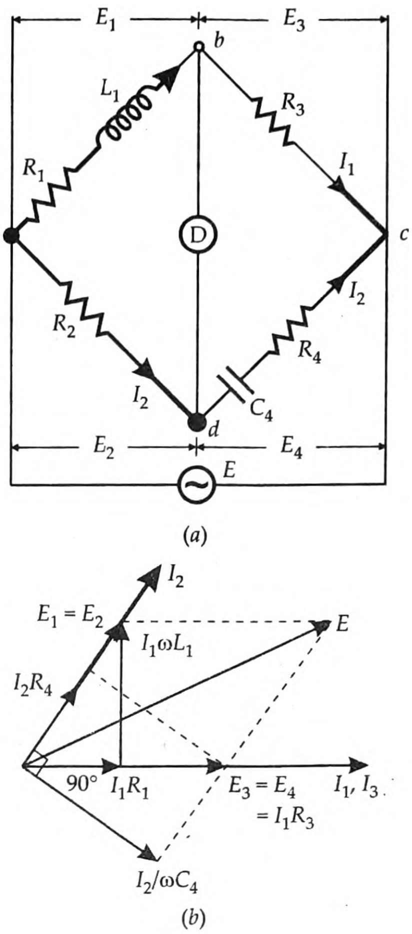


Fig. 16.5 Hay's Bridge.

At balance,

$$(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2 R_3$$

or
$$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3$$

Separating the real and imaginary terms, we obtain,

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \text{ and } L_1 = -\frac{R_1}{\omega^2 R_4 C_4}$$

Solving the above two equations, we have

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2} \qquad ...(16.21)$$

and
$$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 C_4^2 R_4^2} \qquad ...(16.22)$$

The Q factor of the coil is,

$$Q = \omega L_1 / R_1 = 1/\omega C_4 R_4 \qquad ...(16.23)$$

The expressions for the unknown inductance and resistance contain the frequency term. Therefore it appears that the frequency of the source of supply to the bridge must be accurately known. This is not true for the inductance when a high Q coil is being measured, as is explained below :

Now $L_1 = \dfrac{R_2 R_3 C_4}{1+\omega^2 C_4^2 R_4^2}$ but $Q = \dfrac{1}{\omega C_4 R_4}$

and therefore $L_1 = \dfrac{R_2 R_3 C_4}{1+(1/Q)^2}$ ...(16.24)

For a value of Q greater than 10, the term $(1/Q)^2$ will be smaller than 1/100 and can be neglected.

Therefore Eqn. 16.24 reduces to

$$L_1 = R_2 R_3 C_4 \quad \text{...(16.25)}$$

which is the same as for a Maxwell's bridge.

Advantages :

1. This bridge gives very simple expression for unknown inductance for high Q coils, and is suitable for coils having $Q > 10$.
2. This bridge also gives a simple expression for Q factor.
3. If we examine the expression for Q factor :
$$Q = 1/\omega C_4 R_4$$
we find that the resistance R_4 appears in the denominator and hence for high Q coils its value should be small. Thus this bridge requires only a low value resistor for R_4, whereas the Maxwell's bridge requires a parallel resistor, R_4, of a very high value.

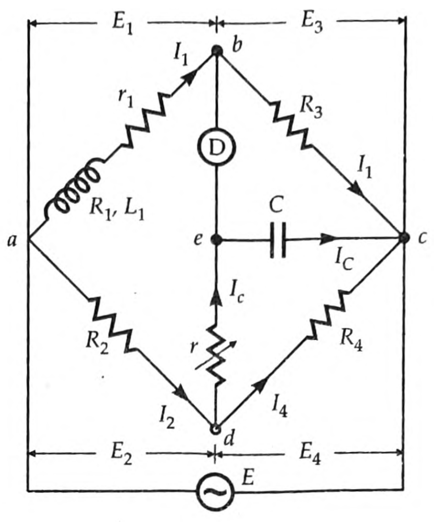
Disadvantages :

The Hay's bridge is suited for the measurement of high Q inductors, especially those inductors having a Q greater than 10. For inductors having Q values smaller than 10, the term $(1/Q)^2$ in the expression for inductance L_1 (Eqn. 16.24) becomes rather important and thus cannot be neglected. Hence this bridge is not suited for measurement of coils having Q less than 10 and for these applications a Maxwell's bridge is more suited.

16.5.4 Anderson's Bridge

This bridge, in fact, is a modification of the Maxwell's inductance capacitance bridge. In this method, the self-inductance is measured in terms of a standard capacitor. This method is applicable for precise measurement of self-inductance over a very wide range of values.

Figure 16.6 shows the connections and the phasor diagram of the bridge for balanced conditions.



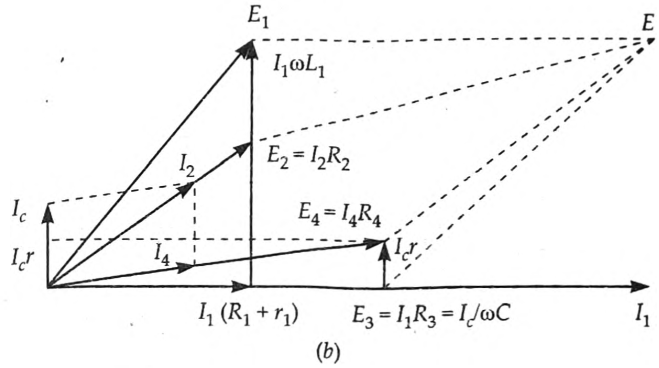


Fig. 16.6 Anderson's Bridge.

Let

L_1 = self-inductance to be measured,

R_1 = resistance of self-inductor,

r_1 = resistance connected in series with self-inductor,

r, R_2, R_3, R_4 = known non-inductive resistances,

and C = fixed standard capacitor.

At balance,

$$I_1 = I_3 \text{ and } I_2 = I_c + I_4$$

Now, $I_1 R_3 = I_c \times \dfrac{1}{j\omega C}$

$\therefore \quad I_c = I_1 j\omega C R_3.$

Writing the other balance equations

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_c r$$

and $$I_c\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_c) R_4.$$

Substituting the value of I_c in the above equations, we have

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 j\omega C R_3 r$$

or $$I_1(r+R_1+j\omega L_1-j\omega CR_3 r)=I_2R_2 \qquad \text{...}(i)$$

and $$j\omega CR_3\, I_1\left(r+\frac{1}{j\omega C}\right)=(I_2-I_1 j\omega CR_3)R_4$$

or $$I_1(j\omega CR_3 r+j\omega CR_3R_4+R_3)=I_2R_4 \qquad \text{...}(ii)$$

From Eqns. (*i*) and (*ii*), we obtain

$$I_1(r_1+R_1+j\omega L_1-j\omega CR_3 r) = I_1\left(\frac{R_2R_3}{R_4}+\frac{j\omega CR_2R_3 r}{R_4}+j\omega CR_3R_2\right)$$

Equating the real and the imaginary parts,

$$R_1=\frac{R_2R_3}{R_4}-r_1 \qquad \text{...(16.26)}$$

and $$L_1=C\frac{R_3}{R_4}[r(R_4+R_2)+R_2R_4] \qquad \text{...(16.27)}$$

An examination of balance equations reveals that to obtain easy convergence of balance, alternate adjustments of r_1 and r should be done as they appear in only one of the two balance equations.

Advantages :

1. In case adjustments are carried out by manipulating control over r_1 and r, they become independent of each other. This is a marked superiority over sliding balance conditions met with low Q coils when measuring with Maxwell's bridge. A study of convergence conditions would reveal that it is much easier to obtain balance in the case of Anderson's bridge than in Maxwell's bridge for low Q-coils.
2. A fixed capacitor can be used instead of a variable capacitor as in the case of Maxwell's bridge.
3. This bridge may be used for accurate determination of capacitance in terms of inductance.

Disadvantages :

1. The Anderson's bridge is more complex than its prototype Maxwell's bridge. The Anderson's bridge has more parts and is more complicated to set up and manipulate. The balance equations are not simple and in fact are much more tedious.
2. An additional junction point increases the difficulty of shielding the bridge.

Considering the above complication of the Anderson's bridge, in all the cases where a variable capacitor is permissible the more simple Maxwell's bridge is used instead of Anderson's bridge.

16.5.5 Owen's Bridge

This bridge may be used for measurement of an inductance in terms of capacitance. Figure 16.7 shows the connections and phasor diagrams, for this bridge, under balance conditions.

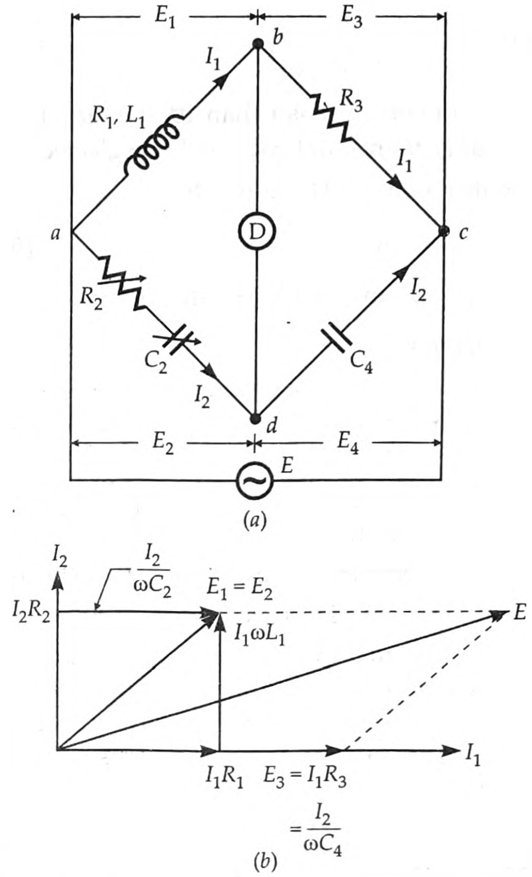


Fig. 16.7 Owen's Bridge.

Let

L_1 = unknown self-inductance of resistance R_1,
R_2 = variable non-inductive resistance,
R_3 = fixed non-inductive resistance,
C_2 = variable standard capacitor,
and C_4 = fixed standard capacitor.

At balance,

$$(R_1+j\omega L_1)\left(\frac{1}{j\omega C_4}\right)=\left(R_2+\frac{1}{j\omega C_2}\right)R_3$$

Separating the real and imaginary terms, we obtain,

$$L_1=R_2R_3C_4 \qquad \text{...(16.28)}$$

and $$R_1=R_3\frac{C_4}{C_2} \qquad \text{...(16.29)}$$

Advantages :

1. Examining the equations for balance, we find that we obtain two independent equations in case C_2 and R_2 are made variable. Since R_2 and C_2, the variable elements, are in the same arm, convergence to balance conditions is much easier.
2. The balance equations are quite simple and do not contain any frequency component.
3. The bridge can be used over a wide range of measurement of inductances.

Disadvantages :

1. This bridge requires a variable capacitor which is an expensive item and also its accuracy is about 1 percent.
2. The value of capacitance C_2 tends to become rather large when measuring high Q coils.

1.6.5.6 Measurements of Incremental Inductance

The magnetic flux linking an ion-cored coil is not in direct proportion to the current flowing in the coil, but varies in a manner usually indicated by a magnetization curve. The flux usually increases fairly rapidly when the current build-up process first begins. However, the flux increment corresponding to a particular size of current increment becomes smaller with continual increase of current as the core approaches the condition of magnetic *"saturation"*. An induced voltage in the coil depends on a change of flux and hence becomes smaller for a given current change at higher values of current.

The basic expression for induced voltage in terms of a changing flux is usually replaced, for purposes of circuit study, by the *"coefficient of self-inductance"* multiplied by the rate of change of current. The new concept must be examined with care when applied to the case of an iron-cored coil. In terms of a flux change, the induced voltage is $e = -Nd\phi / dt$.

In terms of inductance, $e = -Ldi / dt$,

where L = coefficient of self-inductance.

Comparison of the two expressions gives

$$L = Nd\phi / di$$

which helps to explain the nature of this quantity. Inductance may be represented by the slope of the ϕ - i curve, or magnetization curve, of a reactor. For an air-cored coil there is a linear relationship between flux and current and, accordingly, L is a constant. An iron-cored coil does not have a linear flux-current relationship, and L has different values, depending on the portion of the magnetization curve being used and on the manner of defining L That is, for some purpose we may be interested in the average inductance, represented by the slope of a line from the origin to a particular point on the ϕ - i curve. For other purposes we are connected with small variations in the neighbourhood of a point on the curve (for very small variations, L equals N times the slope of the curve at the point in question). It is thus evident that the coefficient of self-inductance is ambiguous for iron-cored coils unless the conditions are carefully defined.

Many iron-cored coils are used as filter reactors in rectifier circuits and in other applications in which a combination of a d.c. component and a superposed a.c. are encountered. The inductance of the reactor to the a.c. in such a case may vary over a wide range, depending on the magnitude of the d.c. term. An air gap in the magnetic core tends to straighten the ϕ - i curve, and make L more nearly constant. An iron-cored coil without an air gap gives an extreme variation of L if the d.c. component becomes high enough to produce a considerable degree of saturation.

The d.c. component of the current fixes the operating point around which changes occur on account of the alternating current. The incremental inductance at this point may be defined as turns time the slope of ϕ - i curve.

$\therefore$ Increment inductance, $L = Nd\phi / di = N\Delta\phi / \Delta i$

Similarly, incremental permeability may be defined as the slope of the B - H curve at the operating point.

Incremental permeability, $\mu = dB / dH = \Delta B / \Delta H$.

The magnitude of the a.c. component also has an effect on the apparent inductance of the reactor, though usually not to so great an extent as the d.c. component in the common type of reactor applications. To make inductance measurements on an iron-cored coil definite, the data should include the amount of d.c. present, and also the frequency and magnitude of the a.c.

The non-linear character of the flux-current curve enters in the measurement problem in another way, by producing distortion of the current and voltage waveforms. That is, a sinusoidal voltage does not produce a sinusoidal current, or vice-versa, in an iron-cored coil. Even though the applied bridge voltage is sinusoidal, the detector voltage is distorted and hence may be analyzed by Fourier method into the fundamental frequency-component plus harmonics. The harmonics cause difficulties in the balancing process if the detector consists of an amplifier, a tuned circuit can be incorporated in it to pass the fundamental frequency and suppress the harmonics. The situation is a little more difficult when a telephone head set is used but a person can train himself to listen for the null point of the fundamental frequency and disregard the harmonics. Due to the difficulties of defining inductance in the first place, and of determining balance in the second place, we cannot expect to measure parameters of an iron-cored coil with the same accuracy as those of an air-cored coil. Fortunately, in most uses of the iron-cored coils, as in filter chokes, a low degree of accuracy can be tolerated.

Many bridge circuits can be modified to permit the simultaneous application of d.c. and a.c. to the reactor. One thing that must be kept in mind in arranging the circuit is the fact that the amount of d.c. specified for the test reactor must also pass through one of the bridge arms, and care must be taken not to overheat the precision resistor.

The incremental inductance can be measured with an Owen's bridge. The original circuit, however, has to be modified in order that the coil under measurement is fed from both d.c. and a.c. This circuit is shown in Fig. 16.8.

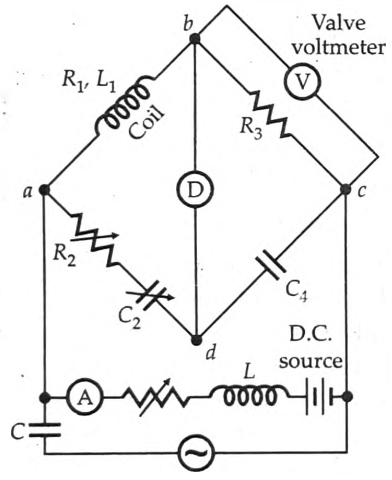


Fig. 16.8 Owen's bridge for measurement of incremental inductance.

The coil is fed from a.c. and d.c. sources in parallel. A blocking capacitor C is used to block direct current from entering the a.c. source. A high inductance L is used to block alternating current to enter d.c. source. Any direct current must not affect the balance and this condition is automatically satisfied in Owen's bridge because capacitors C_2 and C_4 block any d.c. current flowing through the detector.

As has been mentioned earlier, it is necessary to know the magnetization conditions under which the coil is being worked. The d.c. component of current is measured by a moving coil ammeter A connected in the d.c. supply circuit. The a.c. component of current may be easily obtained from the reading of a valve voltmeter (not sensitive to d.c.) connected across the known resistance R_3. The value of current calculated from this reading is a.c. current through R_3, but this is also, at balance, the a.c. current through the coil.

At balance, incremental inductance

$$L_1 = R_2 R_3 C_4 \qquad \text{...(16.30)}$$

Now inductance, $L_1 = N^2/(l/\mu A)$

$\therefore$ Incremental permeability.

$$\mu = L_1 l / N^2 A \qquad \text{...(16.31)}$$

where N = number of turns,

A = area of flux path,

l = length of flux path,

and L_1 = incremental inductance.

16.6 MEASUREMENT OF CAPACITANCE

16.6.1 De Sauty's Bridge

The bridge is the simplest method of comparing two capacitances. The connections and the phasor diagram of this bridge are shown in Fig. 16.9.

Let C_1 = capacitor whose capacitance is to be measured,

C_2 = a standard capacitor,

and R_3, R_4 = non-inductive resistors.

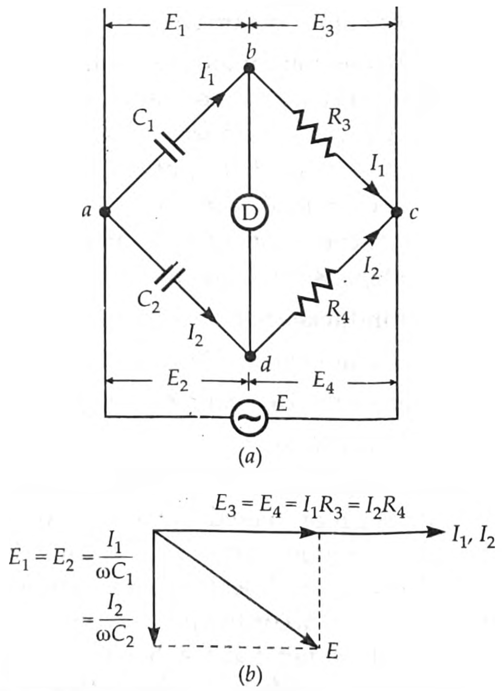


Fig. 16.9 De Sauty's bridge.

At balance,

$$\left(\frac{1}{j\omega C_1}\right)R_4 = \left(\frac{1}{j\omega C_2}\right)R_3$$

or $$C_1 = C_2 R_4 / R_3 \qquad ...(16.23)$$

The balance can be obtained by varying either R_3 or R_4. The advantage of this bridge is its simplicity. But this advantage is nullified by the fact that it is impossible to obtain balance if both the capacitors are not free from dielectric loss. Thus with this method only loss-less capacitors like air capacitors can be compared.

In order to make measurements on imperfect capacitors (*i.e.*, capacitors having dielectric loss), the bridge is modified as shown in Fig. 16.10. This modification is due to Grover.

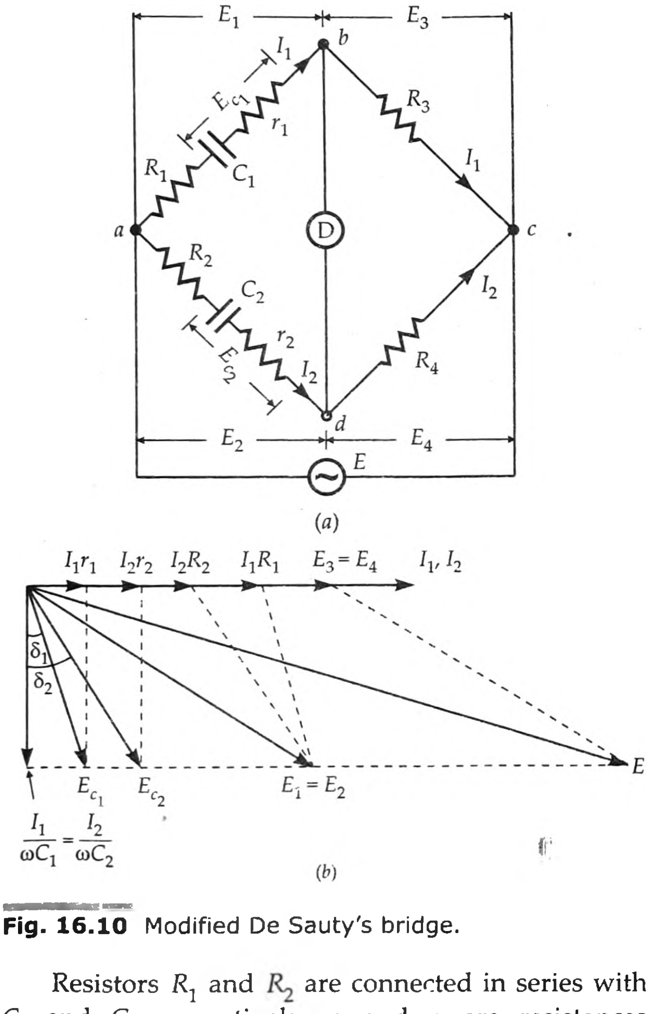


Fig. 16.10 Modified De Sauty's bridge.

Resistors R_1 and R_2 are connected in series with C_1 and C_2 respectively. r_1 and r_2 are resistances representing the loss component of the two capacitors.

At balance,

$$\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right)R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)R_3$$

From which we have

$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1} = \frac{R_4}{R_3} \qquad ...(16.33)$$

The balance may be obtained by variation of resistances R_1, R_2, R_3, R_4.

Figure 16.10(*b*) shows the phasor diagram of the bridge under balance conditions. The angles δ_1 and δ_2 are the phase angles of capacitors C_1 and C_2 respectively.

Dissipation factors for the capacitors are :

$$D_1 = \tan\delta_1 = \omega C_1 r_1 \quad \text{and} \quad D_2 = \tan\delta_2 = \omega C_2 r_2.$$

From Eqn. 16.33, we have

$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1}$$

or $$C_2 r_2 - C_1 r_1 = C_1 R_1 - C_2 R_2$$

or $$\omega C_2 r_2 - \omega C_1 r_1 = \omega(C_1 R_1 - C_2 R_2)$$

$\therefore$ $$D_2 - D_1 = \omega(C_1 R_1 - C_2 R_2)$$

But $$C_1 / C_2 = R_4 / R_3$$

$\therefore$ $$C_1 = C_2 R_4 / R_3$$

Hence $$D_2 - D_1 = \omega C_2\left(\frac{R_1 R_4}{R_3} - R_2\right) \qquad ...(16.34)$$

Therefore, if the dissipation factor of one of the capacitors is known, the dissipation factor for the other can be determined.

This method does not give accurate results for dissipation factor since its value depends on difference of quantities $R_1 R_4 / R_3$ and R_2. These quantities are moderately large and their difference is very small and since this difference cannot be known with a high degree accuracy, the dissipation factor cannot be determined accurately.

16.6.2 Schering Bridge

The connection and phasor diagram of the bridge under balance conditions are shown in Fig. 16.11.

Let

C_1 = capacitor whose capacitance is to be determined,

r_1 = a series resistance representing the loss in the capacitor C_1,

C_2 = a standard capacitor. This capacitor is either an air or a gas capacitor and hence is loss-free. However, if necessary, a correction may be made for the loss angle of this capacitor,

R_3 = a non-inductive resistance,

C_4 = a variable capacitor,

and R_4 = a variable non-inductive resistance in parallel with variable capacitor C_4.

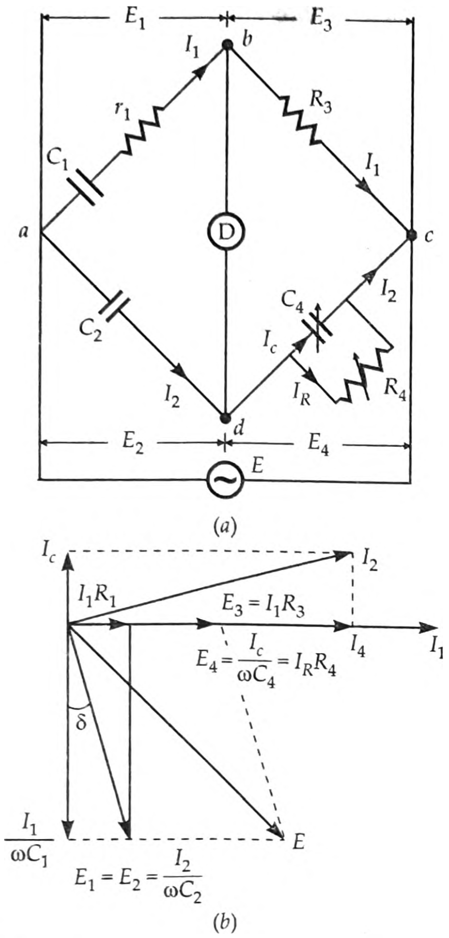


Fig. 16.11 Low voltage Schering bridge.

At balance,

$$\left(r_1 + \frac{1}{j\omega C_1}\right)\left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = \frac{1}{j\omega C_2} \cdot R_3$$

or
$$\left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3}{j\omega C_2}(1 + j\omega C_4 R_4)$$

or
$$r_1 R_4 - \frac{jR_4}{\omega C_1} = -j\frac{R_3}{\omega C_2} + \frac{R_3 R_4 C_4}{C_2}$$

Equating the real and imaginary terms, we obtain

$$r_1 = R_3 C_4 / C_2 \qquad \text{...(16.35)}$$

and
$$C_1 = C_2 (R_4 / R_3) \qquad \text{...(16.36)}$$

Two independent balance equations are obtained if C_4 and R_4 are chosen as the variable elements.

Dissipation factor,

$$D_1 = \tan\delta = \omega C_1 r_1 = \omega(C_2 R_4 / R_3) \times (R_3 C_4 / C_2)$$
$$= \omega C_4 R_4 \qquad \text{...(16.37)}$$

Therefore values of capacitance C_1, and its dissipation factor are obtained from the values of bridge elements at balance.

Permanently set up Schering bridges are sometimes arranged so that balancing is done by adjustment of R_2 and C_4 with C_2 and R_4 remaining fixed. Since R_3 appears in both the balance equations and therefore there is some difficulty in obtaining balance but it has certain advantages as explained below :

The equation for capacitance is $C_1 = (R_4 / R_3) C_2$ and since R_4 and C_2 are fixed, the dial of resistor R_3 may be calibrated to read the capacitance directly.

Dissipation factor $D_1 = \omega C_4 R_4$ and in case the frequency is fixed the dial of capacitor C_4 can be calibrated to read the dissipation factor directly.

Let us say that the working frequency is 50 Hz and the value of R_4 is kept fixed at 3,180 Ω.

∴ Dissipation factor D_1

$$= 2\pi \times 50 \times 3180 \times C_4 = C_4 \times 10^6.$$

Since C_4 is a variable decade capacitance box, its setting in μF directly gives the value of the dissipation factor.

It should, however, be understood that the calibration for dissipation factor holds good for one particular frequency, but may be used at another frequency if correction is made by multiplying by the ratio of frequencies.

16.6.3 High Voltage Schering Bridge

Schering bridge is widely used for capacitance and dissipation factor measurements. In fact Schering bridge is one of the most important of the a.c. bridges. It is extensively used in the measurement of capacitance in general, and in particular in the measurement of the properties of insulators, capacitor

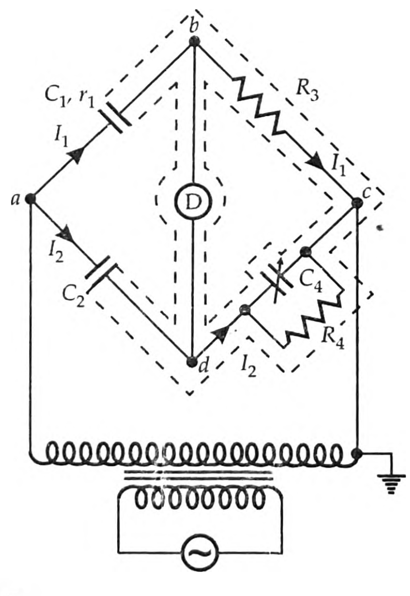


Fig. 16.12 High Voltage Schering bridge.

bushings, insulating oil and other insulating materials. This bridge is particularly suitable for small capacitances, and is then usually supplied from a high frequency or a high voltage source. The measurements done on small capacitances suffer from many disadvantages if carried out at low voltages. High voltage Schering bridge is certainly preferable for such measurements.

The special features of a high voltage Schering bridge shown in Fig. 16.12, are explained below :

1. The high voltage supply is obtained from a transformer usually at 50 Hz. The detector, in this case, is a vibration galvanometer.

2. Arms *ab* and *ad* each contain only a capacitor and these capacitors are designed for high voltage work. The impedances of these two arms are very high in comparison with the other arms, *bc* and *dc*. Thus the major portion of potential drop will be in the arms *ab* and *ad* and very little voltage drop is there across the arms *bc* and *dc*. The point *c* is earthed. Such is the large magnitude of impedances in arms *ab* and *ad*, that even if a voltage as high as 100 kV is applied to the bridge, the voltage across arms *bc* and *dc* is a few volt above earth. This is certainly a great advantage as the controls are located in arms *bc* and *dc* and for the safety of the operator, these controls should be and are at low potential with respect to earth. For the same reason the detector is also at a low potential.

3. It is necessary to prevent dangerous high voltages appearing across arms *bc* and *ac* in the case of breakdown of either of the high voltage capacitors. This is done by connecting a spark gap, (set to breakdown at about 100 V) across each of the arms *bc* and *dc*.

4. The impedances of arms *ab* and *ad* are large and therefore the current drawn from the source is small and hence the power loss is quite small. But this small value of current also necessitates the use of a sensitive detector.

5. The fixed standard capacitor, C_2, has either air or compressed gas as dielectric. The dissipation factor of a dry and clean gas is sensibly zero, loss in the insulating supports cannot be avoided. This loss, however, can be prevented from influencing the measurements by the use of a guard ring from which both electrodes of the capacitor are supported. With this arrangement the current through the high voltage supports passes direct to earth, and as the potential difference between low voltage electrode and the guard ring is very small, the insulation of the low voltage electrode has a negligible effect. For some general applications, mica capacitors are used and in such cases, the dissipation factor of the capacitor must be accurately known.

6. Earthed screens are provided in order to avoid errors caused due to inter-capacitance between high and low arms of the bridge. Instead of earthing one point on the circuit as shown in Fig. 16.12, the earth capacitance effect on the galvanometer and leads is eliminated by means of a "Wagner earth device" (described later in this chapter on page 498).

16.6.4 Measurement of Relative Permittivity with Schering Bridge

Schering bridge is very useful for the measurement of relative permittivity of dielectric materials. The determination of relative permittivity involves the measurement of capacitance of a small capacitor with the specimen as dielectric. The capacitor with specimen as dielectric is formed by using either a parallel plate or a concentric cylinder configuration for the electrodes. Guard circuits are used in order to make the plate area definite.

Many different techniques are used, both in the type of specimen capacitor used and in the measuring circuit. The normal arrangement for solid materials is to use a disc specimen with metal electrodes. The electrodes may consist of thin metal foil attached to the specimen by petroleum jelly, or thin films of silver or aluminium applied by evaporation (both these arrangements normally have solid metal backing electrodes), or mercury. Mercury electrodes are obtained by floating (or supporting) the specimen on the surface of a mercury pool, the upper electrodes consisting of a smaller mercury pool held in place by a metal containing ring. Liquid specimens fill the space between the concentric cylindrical electrodes of a test cell.

The relative permittivity is calculated from the measured value of capacitance and the dimensions of the electrodes. For a parallel plate arrangement, relative permittivity

$$\varepsilon_r = \frac{C_s d}{\varepsilon_0 A} \qquad \text{...(16.38)}$$

where C_s = measured value of capacitance with specimen as dielectric,

d = spacing between electrodes,

A = effective area of electrodes,

and ε_0 = permittivity of free space.

A method which avoids the necessity for close contact between electrodes and specimen uses a pair of solid electrodes (one with a guard ring) between which the specimen (thinner than the space between electrodes) is slipped. The capacitance of the arrangement is measured, the specimen is then removed, and the spacing between the electrodes is adjusted, by means of micrometer adjustments, until the capacitance is the same as before. The relative permittivity of the specimen can be calculated from the thickness of the specimen and the alteration in electrode spacing.

Let

C = capacitance with specimen between electrodes,

A = area of electrodes,

d = thickness of specimen,

t = gap between specimen and electrode,

and x = reduction in separation between the two measurements.

These dimensions are shown in Fig. 16.13.

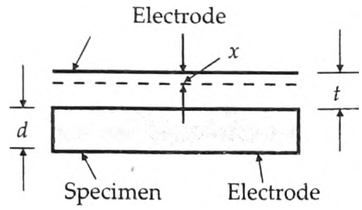


Fig. 16.13 Measurement of relative permittivity.

Let

C_s = capacitance of specimen,

C_0 = capacitance due to space between specimen and electrode,

and C = effective capacitance of C_s and C_0 in series.

$$\therefore\ C = \frac{C_s C_0}{C_s + C_0} = \frac{(\varepsilon_r \varepsilon_0 A/d)\cdot(\varepsilon_0 A/t)}{(\varepsilon_r \varepsilon_0 A/d)+(\varepsilon_0 A/t)} = \frac{\varepsilon_r \varepsilon_0 A}{\varepsilon_r t + d}$$

When the specimen is removed and the spacing readjusted to give the same capacitance, the expression for capacitance is :

$$C = \frac{\varepsilon_0 A}{t+d-x}.$$

Hence
$$\frac{\varepsilon_0 A}{t+d-x} = \frac{\varepsilon_r \varepsilon_0 A}{\varepsilon_r t + d}$$

$$\therefore\quad \varepsilon_r = \frac{d}{d-x} \qquad \text{...(16.39)}$$

16.7 MEASUREMENT OF MUTUAL INDUCTANCE

16.7.1 Uses of Mutual Inductance in Bridge Circuits

Mutual inductance has been used in a great number of bridges for a variety of purposes. Known mutual inductances are used in some circuits for the measurement of unknown mutual inductance. Variable standard mutual inductances have been used as components in bridges for the measurement of self-inductances, capacitance, and frequency. Many such bridges are found in literature, but some of the circuits represent minor modifications of other bridges ; that is, changes to achieve greater accuracy or convenience for a particular type of measurement, or a special range of unknown quantities.

We shall study only a small number of these circuits. Applications to capacitance and self-inductance determinations are of secondary interest because methods using a capacitance standard are generally more convenient and accurate for these measurements than a mutual inductor and they (capacitors) are also cheaper, compact, and more generally available. The mutual inductance circuits may have particular merit in some cases, but this enters the field of special research rather than of general measurements, so we shall not attempt to go into much detail. Much reference material is available on the subject.

The derivation of the balance equations requires a different method when we have mutual coupling between the arms. It is necessary to write equations for the voltage around the loops and then to make solution for the unknown quantities.

16.7.2 Mutual Inductance Measured as Self-Inductance

If the terminals of the two coils whose mutual inductance is to be measured are available for series connection, measurement may be made by one of the self-inductance bridges considered earlier. If the connections are made so that the magnetic fields of the two coils are addative as shown in Fig. 16.14(*a*), the effective inductance of the two coils in series is,

$$L_{e1} = L_1 + L_2 + 2M \qquad \text{...(16.40)}$$

If the connections of one coil are reversed as shown in Fig. 16.14(*b*),

$$L_{e2} = L_1 + L_2 - 2M \qquad \text{...(16.41)}$$

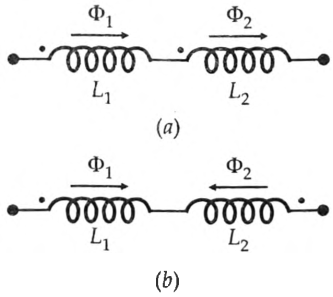


Fig. 16.14 Positive and negative couplings for mutual inductors.

Thus from Eqns. 16.40 and 16.41, we get

Mutual inductance

$$M = \frac{1}{4}(L_{e1} - L_{e2}) \quad \text{...(16.42)}$$

Hence mutual inductance is obtained as one-fourth of difference of self-inductance measured with series addative and series subtractive connections. This method is of advantage only when fairly high coupling between the two coils is obtained otherwise it results in poor accuracy due to nearly equal terms of Eqn. 16.42.

16.7.3 Heaviside Mutual Inductance Bridge

This Bridge (shown in Fig. 16.15) measures mutual inductance in terms of a known self- inductance. The same bridge, slightly modified, was used by Campbell to measure a self-inductance in terms of a known mutual inductance.

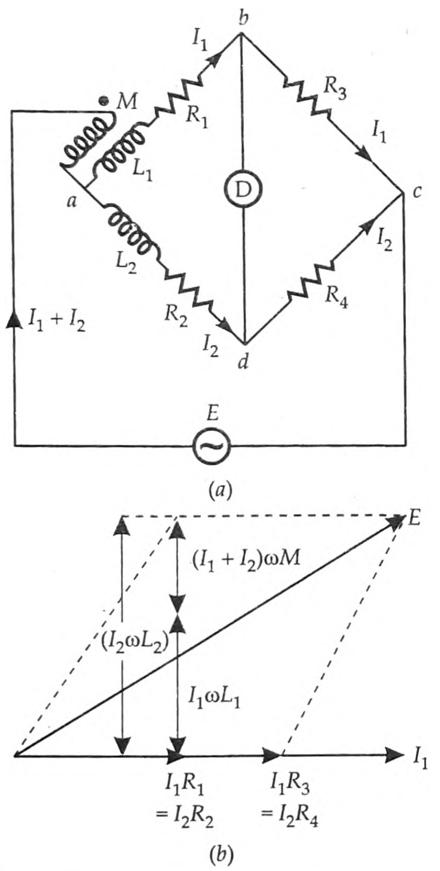


Fig. 16.15 Heaviside mutual inductance bridge.

Let

M = unknown mutual inductance,

L_1 = self-inductance of secondary of mutual inductance,

L_2 = known self-inductance, and

R_1, R_2, R_3, R_4 = non-inductive resistors.

At balance voltage drop between b and c must equal the voltage drop between d and c. Also the voltage drop across a-b-c must equal the voltage drop across a-d-c. Thus we have the following equations at balance :

$$I_1 R_3 = I_2 R_4$$

and $\quad (I_1 + I_2)(j\omega M) + I_1(R_1 + R_3 + j\omega L_1)$

$$= I_2(R_2 + R_4 + j\omega L_2)$$

$$\therefore \quad I_2\left(\frac{R_4}{R_3} + 1\right)j\omega M + I_2\frac{R_4}{R_3}(R_1 + R_3 + j\omega L_1)$$

$$= I_2(R_2 + R_4 + j\omega L_2)$$

or $\quad j\omega M\left(\frac{R_4}{R_3} + 1\right) + \frac{R_4}{R_3}R_1 + R_4 + j\omega L_1\frac{R_4}{R_3} = R_2 + R_4 + j\omega L_2$

Thus $\quad R_1 = R_2 R_3 / R_4 \quad \text{...(16.43)}$

and $\quad M = \dfrac{L_2 - L_1 R_4 / R_3}{R_4 / R_3 + 1} = \dfrac{R_3 L_2 - R_4 L_1}{R_3 + R_4} \quad \text{...(16.44)}$

It is clear from Eqn. 16.44, that L_1, the self- inductance of the secondary of the mutual inductor must be known in order that M be measured by this method.

In case $R_3 = R_4$, we get

$$M = \frac{L_2 - L_1}{2} \quad \text{...(16.45)}$$

and $\quad R_1 = R_2 \quad \text{...(16.46)}$

This method can be used for measurement of self-inductance. Supposing L_2 is the self-inductance to be determined. From Eqns. 16.43 and 16.44, we get

$$L_2 = \frac{M(R_3 + R_4) + R_4 L_1}{R_3}$$

$$= M\left(1 + \frac{R_4}{R_3}\right) + \frac{R_4}{R_3}L_1 \quad \text{...(16.47)}$$

and $\quad R_2 = R_1 R_4 / R_3 \quad \text{...(16.48)}$

In case $R_4 = R_3$, we have

$$L_2 = 2M + L_1 \quad \text{and} \quad R_2 = R_1.$$

16.7.4 Campbell's Modification of Heaviside Bridge

Figure 16.16 shows a modified Heaviside bridge. This modification is due to Campbell. This is used to measure a self-inductance in terms of a mutual inductance. In this case an additional balancing coil L, R is included in arm ad in series with inductor under test. An additional resistance r is put in arm ab. Balance is obtained by varying M and r. A short circuiting switch is placed across the coil R_2, L_2 under measurement. Two sets of readings are taken one with switch being open and the other with switch being closed. Let values of M and r, be M_1 and r_1 with switch open, and M_2 and r_2 with switch closed.

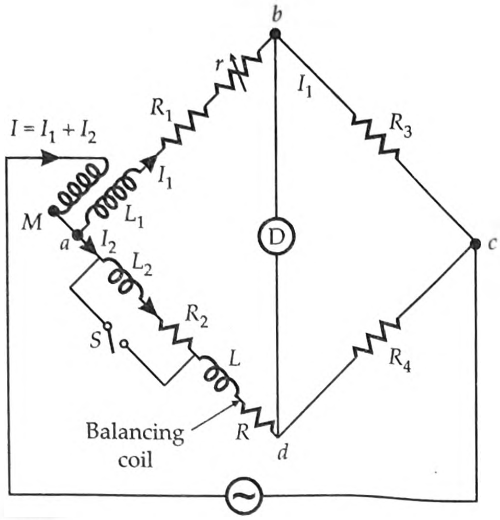


Fig. 16.16 Compbell's modification of Heaviside bridge.

∴ With switch open, from Eqn. 16.47, we have,

$$L_2 + L = \frac{M_1(R_3 + R_4) + R_4 L_1}{R_3}$$

and with switch closed, $L = \dfrac{M_2(R_3 + R_4) + R_4 L_1}{R_3}$

Hence $\quad L_2 = (M_1 - M_2)(1 + R_4 / R_3) \qquad$ ...(16.49)

With switch open, from Eqn. 16.48, we have,

$$R_2 + R = (R_1 + r_1) R_4 / R_3$$

and with switch closed

$$R = (R_1 + r_2) R_4 / R_3$$

∴ $\quad R_2 = (r_1 - r_2) R_4 / R_3 \qquad$ ...(16.50)

This method is a good example of the methods adopted to eliminate the effects of leads etc.

When we have equal ratio arms $R_3 = R_4$.

and therefore from Eqns. 16.49 and 16.50, we get,

$$L_2 = 2(M_1 - M_2) \qquad \text{...(16.51)}$$

and $\quad R_2 = r_1 - r_2 \qquad$ ...(16.52)

16.7.5 Heaviside Campbell Equal Ratio Bridge

The use of balancing coil in the above method reduces the sensitivity of the bridge. Figure 16.17 shows Heaviside Campbell equal ratio bridge. This is a better arrangement which improves sensitivity and also dispenses with the use of a balancing coil.

In this method the secondary of the mutual inductor is made up of two equal coils each having a self-inductance. One of the coils is connected in arm *ab* and the other in arm *ad*. The primary of mutual inductance reacts with both of them. L_2, R_2 is the coil whose self-inductance and resistance is to be determined. The resistances R_3 and R_4 are made equal. Balance is obtained by varying the mutual inductance and resistance *r*.

At balance, $\quad I_1 R_3 = I_2 R_4 \quad$ but $R_3 = R_4$

and therefore $\quad I_1 = I_2 = I/2$ as $\quad I = I_1 + I_2$.

Writing the other equation for balance :

$$I_1(R_1 + r) + I_1 j\omega L + I j\omega M_x = I_2 R_2 + I_2 j\omega(L_2 + L) - I j\omega M_y$$

or $\quad \dfrac{R_1 + r}{2} + j\omega\left(\dfrac{L}{2} + M_x\right) = \dfrac{R_2}{2} + j\omega\left(\dfrac{L_2 + L}{2}\right) - j\omega M_y$

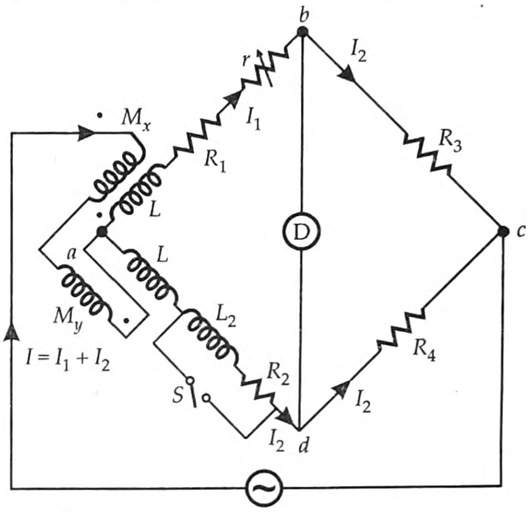


Fig. 16.17 Heaviside Campbell equal ratio bridge.

Equating the real and imaginary terms

$$R_2 = R_1 + r \qquad \text{...(16.53)}$$

and $\quad L_2 = 2(M_x + M_y) = 2M \qquad$ ...(16.54)

Thus the magnitude of inductance which can be measured with this method is twice the range of the mutual inductor. The values calculated above include the effects of leads etc. In order to eliminate these effects, we take two readings with switch open circuited and another with switch closed. Let M_1, r_1 be the readings of *M*, *r* with open circuit and M_2, r_2 with short circuit.

$$R_1 = r_1 - r_2 \qquad \text{...(16.55)}$$

and $\quad L_1 = 2(M_1 - M_2) \qquad$ ...(16.56)

16.7.6 Carey Foster Bridge ; Heydweiller Bridge

This bridge was used basically by Carey Foster but was subsequently modified by Heydweiller for use on a.c. Both names are associated with the bridge and is used for *two* opposite purposes :

(*i*) It is used for measurement of capacitance in terms of a standard mutual inductance. The bridge in this case is known as Carey Foster's bridge.

(*ii*) It can also be used for measurement of mutual inductance in terms of a standard capacitance and is then known as Heydweiller bridge.

Figure 16.18 shows the connection diagram for the bridge under balance conditions.

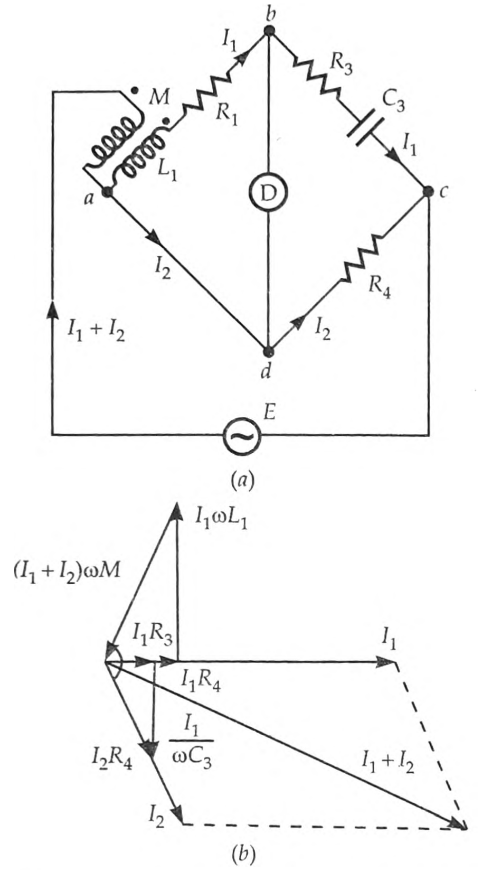


Fig. 16.18 Carey Foster (Heydweiller) Bridge.

This bridge has an unusual feature, one of its arms, *ad*, is short circuited and therefore the potential drop across this arm is zero. In order to achieve balance, the potential drop across arm *ab* should also be equal to zero and for this reason negative coupling is needed for the mutual inductance.

At balance,

$$I_1(R_1 + j\omega L_1) - (I_1 + I_2) j\omega M = 0$$

and
$$I_1\left(R_3 + \frac{1}{j\omega C_3}\right) = I_1 R_4.$$

The solution of the above equation gives :

$$M = R_1 R_4 C_3 \qquad \text{...(16.57)}$$

and
$$L_1 = \frac{M(R_3 + R_4)}{R_4} = R_1 C_3 (R_3 + R_4) \qquad \text{...(16.58)}$$

If the bridge is used for measurement of capacitance, Eqns. 16.57 and 16.58 may be written as :

$$C_3 = M / R_1 R_4 \qquad \text{...(16.59)}$$

$$R_3 = R_4 (L_1 - M) / M \qquad \text{...(16.60)}$$

In the measurement of mutual inductance with this bridge, R_3 is a separate resistance while in the measurement of capacitance R_3 is not a separate unit but represents the equivalent series resistance of the capacitor and thus can be determined in terms of the elements of the bridge.

16.7.7 Campbell's Bridge

This bridge measures an unknown mutual inductance in terms of a standard mutual inductance. Figure 16.19 shows the circuit diagram for the bridge.

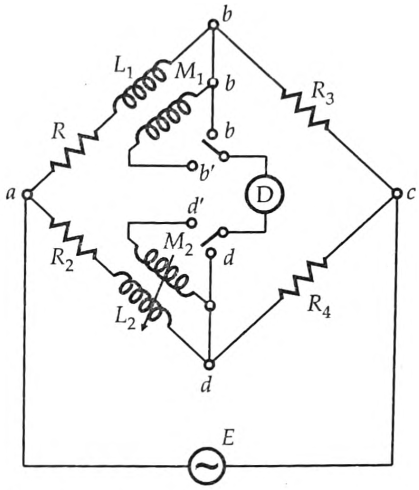


Fig. 16.19 Campbell's Bridge.

Let

M_1 = unknown mutual inductance,

L_1 = self-inductance of secondary of mutual inductance M_1,

M_2 = variable standard mutual inductance,

L_2 = self-inductance of secondary of mutual inductance M_2, and

R_1, R_2, R_3, R_4 = non-inductive resistances.

There are *two* steps required in the balancing process.

1. Detector is connected between *b* and *d*. The circuit now becomes a simple self-inductance comparison bridge. The requirement for balance is :
$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \text{...(16.61)}$$
The bridge may be balanced by adjustment of R_3 (or R_4) and R_1 (or R_2).
2. Detector is connected between *b'* and *d'*. Keeping adjustments as in step 1 above, the variable mutual inductance M_2 is varied to get balance. Then,
$$\frac{M_1}{M_2} = \frac{R_3}{R_4}$$
or
$$M_1 = M_2 R_3 / R_4 \qquad \text{...(16.62)}$$

16.8 MEASUREMENT OF FREQUENCY

Some bridges have balance equations which involve frequency directly even if the performance of individual bridge elements is independent of frequency. These bridges may be used for determination of frequency in terms of values of various bridge elements. We shall describe here the Wien's bridge, which is the most important one for determination of frequency.

16.8.1 Wien's Bridge

The Wien's bridge is primarily known as a frequency determining bridge and is described here not only for its use as an a.c. bridge to measure frequency but also for its application in various other useful circuits. A Wien's bridge, for example, may be employed in a harmonic distortion analyzer, where it is used as notch filter, discriminating against one specific frequency. The Wien's bridge also finds applications in audio and HF oscillators as the frequency determining device.

Figure 16.20 shows a Wien's bridge under balance conditions.

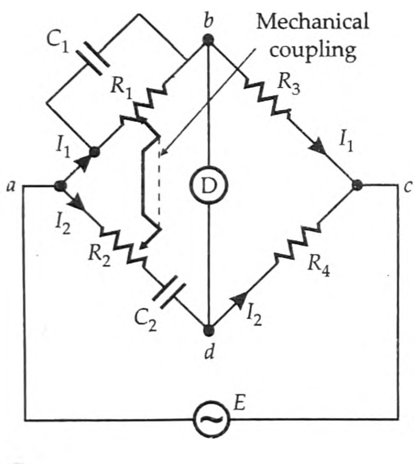


Fig. 16.20 Wien's Bridge.

At balance

$$\left(\frac{R_1}{1+j\omega C_1 R_1}\right)R_4 = \left(R_2 - \frac{j}{\omega C_2}\right)R_3$$

or
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} + j\left(\omega C_1 R_2 - \frac{1}{\omega C_2 R_1}\right)$$

Equating the real and imaginary parts,

or
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \qquad \ldots(16.63)$$

and
$$\omega C_1 R_2 - \frac{1}{\omega C_2 R_1} = 0$$

from which
$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

and frequency,
$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \text{ Hz.}$$

In most Wien bridges, the components are so chosen that

$$R_1 = R_2 = R \text{ and } C_1 = C_2 = C.$$

Then Eqn. 16.63 reduces to,

$$R_4 / R_3 = 2$$

and Eqn. 16.64 reduces to

$$f = 1/2\pi RC \qquad \ldots(16.65)$$

Switches for resistors R_1 and R_2 are mechanically linked so as to fulfil the condition $R_1 = R_2$.

As long as C_1 and C_2 are fixed capacitors equal in value and $R_4 = 2R_3$, the Wien's bridge may be used as a frequency determining device, balanced by a single control. This control may be directly calibrated in terms of frequency as is evident from Eqn. 16.65.

This bridge is suitable for measurement of frequencies from 100 Hz to 100 kHz. It is possible to obtain an accuracy of 0.1 to 0.5 per cent.

Because of its frequency sensitivity, the Wien's bridge may be difficult to balance unless the waveform of the applied voltage is sinusoidal. The bridge is not balanced for any harmonics present in the applied voltage, so that these harmonics will sometimes produce an output voltage marking the true balance point. This difficulty can be overcome by connecting a filter in series with the null detector.

A Wien's bridge may be used for measurement of capacitance also.

16.9 UNIVERSAL IMPEDANCE BRIDGE

One of the most useful and versatile laboratory bridges is the *Universal Impedance Bridge.* The set up of this bridge combines several of the bridge configurations described so far into a single portable instrument. This instrument is capable of measuring both d.c. and a.c. resistance, inductance and storage factor Q. factor of an inductor, capacitance and dissipation factor D of a capacitor.

The universal bridge consists of four basic bridge circuits. It has suitable a.c. and d.c. sources, a.c. and d.c. null detectors, and impedance standards. The Wheatstone bridge is used for both d.c. and a.c. resistance measurements. Capacitance and dissipation factors are determined by modified De Sauty's bridge. The Maxwell's bridge is used for medium Q inductors while Hay's bridge configuration is used for inductors having Q more than 10.

For d.c. resistance measurements a suspension type galvanometer having a sensitivity 0.5 μA per scale division is used. The null indicator for a.c. measurements is usually an electron ray tube. Terminals are provided for connection of any external null detectors. High impedance head phones may also be used as a.c. detectors.

The a.c. source of the bridge consists of an oscillator having 10 kHz as the standard frequency.

16.10 SOURCES OF ERRORS IN BRIDGE CIRCUITS

We have assumed in our derivation of basic circuit relationships that a bridge consists of lumped impedance units connected only by the wires that are placed in the circuit for making connections. This idealized condition exists in bridge circuits to a lesser or greater extent. The idealized arrangement works fairly well if the frequency is low, if component impedances are not high and if accuracy desired is not high. But in practice there are certain factors which we have not considered yet and these complicate the behaviour or the bridge circuits considered so far.

Some of these factors are stray couplings between one bridge arm and another and from elements to ground. These stray couplings modify the balance conditions making a definite balance impossible or may lead to false balance. Thus these effects will cause incorrect values of unknown components to be attained.

Factors causing errors. The various factors causing errors in a.c. bridge circuit are listed below :

(*i*) stray-conductance effects, due to imperfect insulation ;

(*ii*) mutual-inductance effects, due to magnetic coupling between various components of the bridge ;

(*iii*) stray-capacitance effects, due to electrostatic fields between conductors at different potentials ;

(*iv*) 'residues' in components – *e.g.*, the existence of small amount of series, inductance or shunt capacitance in nominally non-reactive resistors.

16.10.1 Precautions and Techniques used for Reducing Errors

Errors introduced by the above effects become greater in high frequency and high-voltage bridges, but precautions should always be taken to reduce them to a minimum. Even so, in high-accuracy work it is often necessary to determine the magnitude of some of the effects and allow for them in the bridge calculations. The more important methods of reducing bridge errors are now described.

1. ***Use of high-quality components.*** Good-quality bridge components will normally have the advantages of high-accuracy calibration, freedom from stray conductance effects, and a minimum of residues. Where residues exist, manufacturer supplies information giving the values of the residues.

2. ***Bridge lay-out.*** The conventional bridge circuit diagrams in the present chapter are drawn to represent the potential distribution in the circuit ; they do not indicate the best physical lay-out. In general, it is desirable that the four 'corners' of the bridge *a, b, c* and *d* (Fig. 16.21) should be closed together. Each component is then connected to the appropriate corners by its own leads. The physical lay-out of the bridge will then be as shown in Fig. 16.21. All the leads to a single corner should be brought as nearly as possible to the same point. A pair of leads to a component should not form a large loop ; if possible the leads should be co-axial. If there is more than one inductor in the bridge, leads may have to be quite long. In this manner stray mutual inductance coupling and lead self-inductances are reduced to minimum and the stray capacitances are definitely located across individual arms of bridge ; they can then be measured and compensated for relatively easily.

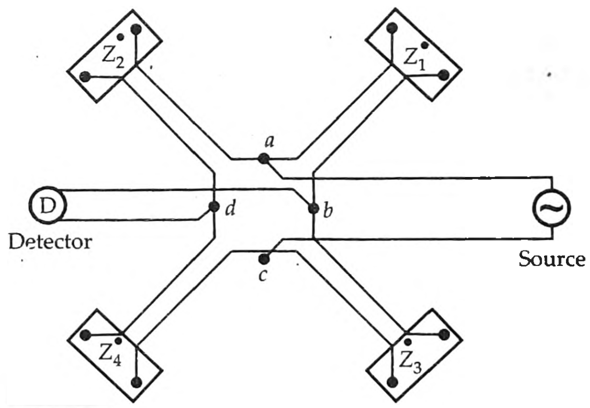


Fig. 16.21 Bridge lay-out.

3. ***Sensitivity.*** To obtain a high degree of accuracy *e.g.*, 1 part in 10^4 or better, it is necessary that the bridge must be operated at sufficient sensitivity which implies that the change in voltage at the detector terminals from the balance condition should be large when the detector is considered open. The treatment is similar as with the Wheatstone bridge except that with the a.c. bridge circuit we have to consider both the magnitude and phase angle of the impedance of the bridge arms.

4. ***Stray conductance effects.*** If the insulation between various components (elements) of a bridge circuit is not good, trouble may arise because of leakage currents from one arm to another. This is specially true in the case of high impedance bridges. To avoid this, various bridge components and other pieces of apparatus may be mounted on insulating stands.

5. ***Eddy current errors.*** Errors may result due to variation in the value of the standards, which may occur because of induced eddy currents in the standard resistors and inductors. In order to avoid such errors the presence of large conducting masses near the bridge network is avoided.

6. ***Residual errors.*** Though the resistors used are taken as non-inductive and non-capacitive but their inductance and capacitance are never zero. Residues mean small inherent inductance or capacitance of the resistors. In precise work it becomes necessary to evaluate them in order to eliminate them or compensate for errors due to these. The self-inductance is important only when the coils used are multi-turn coils and supply used is of high frequency.

7. ***Frequency and wave form errors.*** In case of bridges whose results are independent of frequency, the supply frequency is important only from the point of view of its effects on resistance, inductance and capacitance of the apparatus under test. The presence of harmonics in the supply waveform is also important from the same point of view.

If the case of bridge network whose balance equations involve frequency the variation in supply frequency is very important both from the point of view of balance and evaluation. The waveform of the supply is also important as the bridge cannot be balanced both for fundamental and harmonics in the waveform (if any) simultaneously. If headphones are used, it will not be possible to obtain complete silence at all, but only a point of minimum sound can be achieved.

This difficulty is overcome either by employing wave filters which eliminate the unwanted harmonics from the source or by employing tuned detectors in place of headphones such as vibration galvanometers which do not respond to harmonics and respond readily only to the fundamental for which they are tuned.

16.11 WAGNER EARTHING DEVICE

If each component in a bridge has a defining screen connected to one end, a very high accuracy in measurement is made possible by the addition of a Wagner earthing device. This device removes all the earth capacitances from the bridge network.

Figure 16.22 shows the connections of the device for use in conjunction with the general form of bridge network, in which Z_1, Z_2, Z_3 and Z_4 are the impedances of the bridge arms. Z_5 and Z_6 are the two variable impedances of the Wagner earth branch, the centre point of which is earthed as shown. These impedances may consist of variable resistances and capacitances similar to those used in the arms of the bridge proper, but not necessarily of known value. The two impedances Z_5 and Z_4, must be capable of forming a balanced bridge with Z_1 and Z_3 or Z_2 and Z_4 and can be a duplicate of either of these pairs of arms. C_1, C_2, C_3 and C_4 are the stray earth capacitances appearing at the apexes of the bridge. D is the detector, which in this case is headphones.

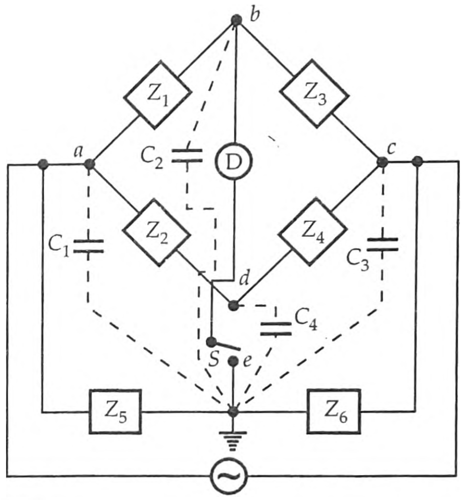


Fig. 16.22 Wagner Earthing Device.

If the switch S is on contact d, balance of the bridge may be obtained by adjustment of the impedances Z_2 and Z_4. The presence of the earth capacitances will prevent a true balance being obtained, but a point of minimum sound can be obtained.

After adjusting the bridge to give minimum sound, the switch S is thrown to contact e so that the headphones are then connected between b and earth ; Z_5 and Z_4 are next adjusted until minimum sound is obtained. The headphones are next reconnected to b, d and Z_2 and Z_4 adjusted to give minimum sound again. The process is repeated until silence is obtained with the switch on d, and silence or the minimum attainable sound, with the switch on e. Then all three points b, d and e are at earth potential. Under these conditions no current flows in the earth capacitances C_2 and C_4 and since C_1 and C_3 shunt the Wagner arms Z_5 and Z_6, these capacitances are eliminated from the bridge network Z_1, Z_2, Z_3 and Z_4.

The capacitances C_1 and C_3, shunting Z_5 and Z_6, complicate these arms, and because of this the combination may not give a true balance against Z_2 and Z_4, only a minimum being attainable during the Wagner balance. The existence of these capacitances should be borne in mind when deciding the form of the Wagner arm, which should preferably be composed of resistances and capacitances. Much of C_1 and C_3 will be due to the earth capacitance of the source transformer and a common cause of difficulty in achieving a good Wagner balance is due to the use of transformers with large and lossy earth capacitance. Poor insulation in the source transformer will cause resistance to appear shunting C_1 and C_3. Some of the disadvantages of the use of Wagner earth devices can be overcome by using double ratio a.c. bridges ; which have two sets of inductively coupled ratio arms.

NOTE The students must attempt these problems from fundamentals although here direct formulae have been used since analysis has been done earlier.

Example 16.3 A Maxwell's inductance comparison bridge is shown in Fig. 16.23. Arm ab consists of a coil with inductance L_1 and resistance r_1 in series with a non-inductive resistance R. Arm bc and coil ad are each a non-inductive resistance of 100 Ω. Arm ad consists of standard variable inductor L of resistance 32.7 Ω. Balance is obtained when L_2 = 47.8 mH and R = 1.36 Ω. Find the resistance and inductance of the coil in arm ab.

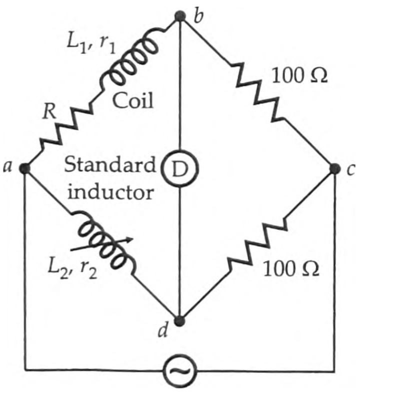


Fig. 16.23 Diagram of Example 16.3.

Solution. At balance

$$[(R_1 + r_1) + j\omega L_1] \times 100 = (r_2 + j\omega L_2) \times 100$$

Equating the real and imaginary terms

$$R_1 + r_1 = r_2 \text{ and } L_2 = L_1$$

∴ Resistance of coil :

$$r_1 = r_2 - R_1 = 32.7 - 1.36 = 31.34\ \Omega.$$

Inductance of coil,

$$L_1 = L_2 = 47.8 \text{ mH}.$$

Example 16.4 A Maxwell's capacitance bridge shown in Fig. 16.4 is used to measure an unknown inductance in comparison with capacitance. The various values at balance,

$$R_2 = 400\Omega\ ;\ R_3 = 600\ \Omega\ ;\ R_4 = 1000\ \Omega\ ;\ C_4 = 0.5\ \mu F.$$

Calculate the values of R_1 and L_1. Calculate also the value of storage (Q) factor of coil if frequency is 1000 Hz.

Solution. At balance,

$$(R_1 + j\omega L_1)\left(\frac{R_4}{1 + j\omega C_4 r_4}\right) = R_2 R_3.$$

Separating the real and imaginary terms, we have

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{400 \times 600}{1000} = 240\ \Omega$$

and $$L_1 = R_2 R_3 C_4$$

$$= 400 \times 600 \times 0.5 \times 10^{-6} = 0.12 \text{ H}.$$

Storage factor,

$$Q = \frac{\omega L_1}{R_1} = \frac{2\pi \times 100 \times 0.12}{240} = 3.14.$$

Example 16.5 An inductance of 0.22 H and 20 Ω resistance is measured by comparison with a fixed standard inductance of 0.1 H and 40 Ω resistance. They are connected as shown in Fig. 16.24(a). The unknown inductance is in arm ab and the standard inductance is arm bc, a resistance of 750 Ω is connected in arm cd and a resistance whose amount is not known is in arm da.

Find the resistance of arm da and show any necessary and practical additions required to achieve both resistive and inductive balance.

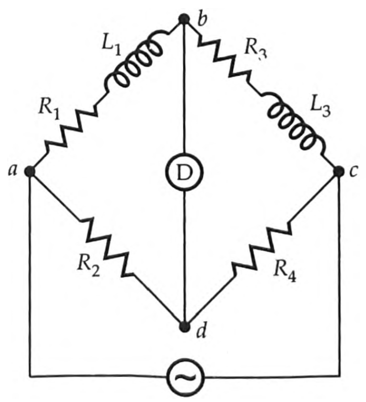


Fig. 16.24(a) Diagram of Example 16.5.

Solution. Now $R_1 = 20\,\Omega$; $L_1 = 0.22$ H ;

$R_4 = 750\,\Omega$; $R_3 = 40\,\Omega$; $L_3 = 0.1$ H.

At balance, $(R_1 + j\omega L_1)R_4 = R_2(R_3 + j\omega L_3)$

Thus the two balance equations are :

$$R_1 = \frac{R_2 R_3}{R_4} \text{ and } L_1 = \frac{R_2 L_3}{R_4}$$

From above, we have

$$\frac{L_1}{L_3} = \frac{R_2}{R_4} = \frac{R_1}{R_3}$$

$\therefore$ Value of R_2 required for balance

$$R_2 = R_4 \frac{L_1}{L_3} = 750 \times \frac{0.22}{0.1} = 1650\,\Omega$$

and $$\frac{L_1}{L_3} = \frac{R_2}{R_4} = 2.2.$$

Now examine the value of ratio $\frac{R_1}{R_3}$ for the existing circuit, we have

$$\frac{R_1}{R_3} = \frac{20}{40} = 0.5$$

The value of this ratio should be 2.2 for both resistive and inductive balance and therefore we must add a series resistance to arm *ab*. Let this series resistance be r_1. Therefore

$$\frac{R_1 + r_1}{R_3} = 2.2 \quad \text{or} \quad r_1 = 2.2 \times 40 - 20 = 68\,\Omega.$$

The modified circuit is shown in Fig. 16.24(*b*).

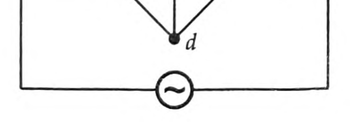


Fig. 16.24(*b*) Diagram of Example 16.5.

Example 16.6 A bridge consists of the following :

Arm ab : a choke coil having a resistance R_1 and inductance L_1.

Arm bc : a non-inductive resistance R_3.

Arm cd : a mica condenser C_4 in series with a non-inductive resistance R_4.

Arm da : a non-inductive resistance R_2

When this bridge is fed from a source of 500 Hz, balance is obtained under following conditions :

$R_2 = 2410\,\Omega$; $R_3 = 750\,\Omega$; $C_4 = 0.35\,\mu F$; $R_4 = 64.5\,\Omega$.

The series resistance of capacitor $= 0.4\Omega$. Calculate the resistance and inductance of the choke coil.

The supply is connected between a and c and the detector is between b and d.

Solution. When we draw the sketch of this bridge, we find that it is Hay's bridge shown in Fig. 16.5. The analysis of Hay's bridge is done in Art. 16.5.3. on page 484.

Now from Eqns. 16.21 and 16.22, inductance and resistance of the coil are :

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2}$$

$$= \frac{2410 \times 750 \times 0.35 \times 10^{-6}}{1 + (2\pi \times 500 \times 0.35 \times 10^{-6} \times 64.9)}$$

$$= 0.63 \text{ H}.$$

and $$R_1 = \frac{\omega^2 R_2 R_3 R_4 R_4^2}{1 + \omega^2 C_4^2 R_4^2}$$

$$= \frac{(2\pi \times 500)^2 \times 2410 \times 750 \times 64.9 \times (0.35 \times 10^{-6})^2}{1 + (2\pi \times 500 \times 0.35 \times 10^{-6} \times 64.9)^2}$$

$$= 141.1\,\Omega$$

(Note that the resistance of arm *cd* is the sum of resistance of the series resistor and the series resistance of capacitor *i.e.*, $R_4 = 64.5 + 0.4 = 64.9\Omega$).

Example 16.7 The arms of a five node bridge are as follows :

arm ab : an unknown impedance (R_1, L_1) in series with a non-inductive variable resistor r_1,

arm bc : a non-inductive resistor $R_3 = 100\,\Omega$,

arm cd : a non-inductive resistor $R_4 = 200\,\Omega$

arm da : a non-inductive resistor $R_2 = 250\,\Omega$,

arm de : a non-inductive variable resistor r

arm ec : a loss-less capacitor $C = 1\,\mu F$, and

arm be : a detector.

An a.c. supply is connected between a and c.

Calculate the resistance and inductance R_1, L_1 when under balance conditions $r_1 = 43.1\,\Omega$ and, $r = 229.7\,\Omega$.

Solution. After drawing the network we find that it is the Anderson's bridge shown in Fig. 16.6 and dealt with in Art. 16.5.4 on page 485.

From Eqns. 16.26 and 16.27,

Resistance,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1 = \frac{250 \times 100}{200} - 43.1 = 81.9\ \Omega.$$

Inductance,

$$L_1 = \frac{CR_3}{R_4}[r(R_4 + R_2) + R_2 R_4]$$

$$= 1 \times 10^{-6} \times \frac{100}{200}[229.7(200+250) + 250 \times 200]$$

$$= 0.0766\ \text{H}.$$

Example 16.8 *The four arms of a bridge are :*

arm ab : an imperfect capacitor C_1 with an equivalent series resistance of r_1

arm bc : a non-inductive resistance R_3,

arm cd : a non-inductive resistance R_4,

arm da : an imperfect capacitor C_2 with an equivalent series resistance of r_2 series with a resistance R_2.

A supply of 450 Hz is given between terminals a and c and the detector is connected between b and d. At balance :

$R_2 = 4.8\ \Omega$, $R_3 = 2000\ \Omega$, $R_4 = 2850\ \Omega$ and $C_2 = 0.5\ \mu F$ and $r_2 = 0.4\ \Omega$.

Calculate the value of C_1 and r_1 and also of the dissipating factor for this capacitor.

Solution. The bridge is shown in Fig. 16.25.

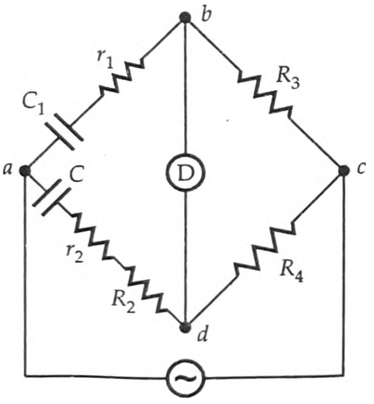


Fig. 16.25 Diagram of Example 16.8.

$$\text{At balance} \left(r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(r_2 + R_2 + \frac{1}{j\omega C_2}\right) R_3$$

Separating the real and imaginary terms

$$r_1 = (r_2 + R_2)\frac{R_3}{R_4} = (0.4 + 4.8) \times \frac{2000}{2850} = 3.65\ \Omega.$$

$$\text{and} \quad C_1 = C_2 \frac{R_4}{R_3} = (0.5 \times 10^{-6}) \times \frac{2850}{2000}\ \text{F} = 0.712\ \mu\text{F}.$$

Dissipating factor,

$$D_1 = \tan \delta_1 = \omega C_1 r_1 = 2\pi \times 450 \times 0.712 \times 10^{-6} \times 3.65$$
$$= 0.00734.$$

Example 16.9 *An Owen's bridge is used to measure the properties of a sample of sheet steel at 2 kHz. At balance, arm ab is test specimen ; arm bc is $R_3 = 100\ \Omega$; arm cd is $C_4 = 0.1\ \mu F$ and arm da is $R_2 = 834\ \Omega$ in series with $C = 0.124\ \mu F$. Derive balance conditions and calculate the effective impedance of the specimen under test conditions.*

Solution. Let R_1 and L_1 be the effective resistance and inductance of the specimen respectively.

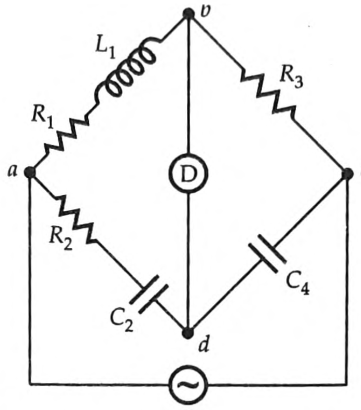


Fig. 16.26 Diagram of Example 16.9.

$$\text{At balance} \ (R_1 + j\omega L_1)\frac{1}{j\omega C_4} = R_3\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$\therefore \quad L_1 = R_2 R_3 C_4 = 834 \times 100 \times 0.1 \times 10^{-6}\ \text{H}$$
$$= 8.34\ \text{mH}$$

$$\text{and} \quad R_1 = R_3 \frac{C_4}{C_2} = 100 \times \frac{0.1}{0.124} = 80.7\ \Omega.$$

Reactance of specimen at 2 kHz,

$$X_1 = 2\pi \times 2 \times 1000 \times 8.34 \times 10^{-3} = 104.5\ \Omega.$$

Impedance of specimen

$$Z_1 = \sqrt{(80.7)^2 + (104.5)^2} = 132\ \Omega.$$

Example 16.10 *A sheet of bakelite 4.5 mm thick is tested at 50 Hz between electrodes 0.12 m in diameter. The Schering bridge employs a standard air capacitor C_2 of 106 pF capacitance, a non-reactive resistance R_4 of $1000/\pi\Omega$ in parallel with a variable capacitor $C_4 = 0.5\ \mu F$, and a non-reactive variable resistance R_3.*

Balance is obtained with $C_4 = 0.5\ \mu F$ and $R_2 = 260\ \Omega$.

Calculate the capacitance, power factor and relative permittivity of sheet.

Solution. The Schering bridge is shown in Fig. 16.11 where C_1 and r_1 represent the capacitance and series resistance of the capacitor. From Eqns. 16.35 and 16.36

$$r_1 = \frac{C_4}{C_2} R_3 = \frac{0.5 \times 10^{-6}}{106 \times 10^{-12}} 260 = 1.23 \times 10^6\ \Omega$$

$$\text{and} \quad C_1 = \frac{R_4}{R_3} C_2 = \frac{1000}{\pi \times 260} \times 106 \times 10^{-12}\ \text{F} = 130\ \text{pF}.$$

Power factor of sheet

$$= \omega C_1 r_1 = 2\pi \times 50 \times 130 \times 10^{-12} \times 1.23 \times 10^6 = 0.05.$$

Now capacitance, $C_1 = \varepsilon_r \varepsilon_0 \dfrac{A}{d}$

∴ Relative permittivity

$$\varepsilon_r = \frac{C_1 d}{\varepsilon_0 A} = \frac{130 \times 10^{-12} \times 4.5 \times 10^{-3}}{8.854 \times 10^{-12} \times \frac{\pi}{4}(0.12)^2} = 5.9$$

where ε_0 = permittivity of free space $= 8.84 \times 10^{-12}$ F/m.

Example 16.11 *In a low-voltage Schering bridge designed for the measurement of permittivity, the branch ab consists of two electrodes between which the specimen under test may be inserted ; arm bc is a non-reactive resistor R_3 in parallel with a standard capacitor C, arm cd is a non-reactive resistor R_4 in parallel with a standard capacitor C_4 ; arm da is a standard air capacitor of capacitance C_2. Without the specimen between the electrodes, balance is obtained with the following values ; $C_3 = C_4 = 120$ pF, $C_2 = 150$ pF, $R_3 = R_4 = 5000\ \Omega$. With the specimen inserted these values become $C_3 = 200$ pF, $C_4 = 1000$ pF, and $R_3 = R_4 = 5000\ \Omega$. In each test $\omega = 5000$ rad/s. Find the relative permittivity of the specimen.*

Solution. The voltage circuit is shown in Fig. 16.27.

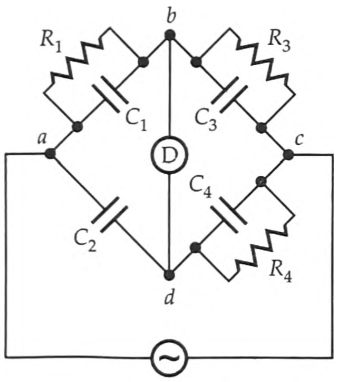


Fig. 16.27 Diagram of Example 16.11.

For balance $Y_1 Y_4 = Y_2 Y_3$

$$\text{or} \quad \left(\frac{1}{R_1} + j\omega C_1\right)\left(\frac{1}{R_4} + j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$\text{or} \quad \left(\frac{1}{R_1 R_4} - \omega^2 C_1 C_4\right) + j\omega\left(\frac{C_4}{R_1} + \frac{C_1}{R_4}\right) = j\omega\frac{C_2}{R_3} - \omega^2 C_2 C_3$$

Equating the real and imaginary parts, we have

$$\frac{1}{R_1 R_4} - \omega^2 C_1 C_4 = -\omega^2 C_2 C_3 \qquad \text{...}(i)$$

and

$$\frac{C_4}{R_1} + \frac{C_1}{R_4} = \frac{C_2}{R_3} \qquad \text{...}(ii)$$

From (*i*) and (*ii*), we have

$$C_1 = \frac{\dfrac{C_2 R_4}{R_3} + \omega^2 C_2 C_3 C_4 R_4^2}{1 + \omega^2 C_4^2 R_4^2}$$

Now $\qquad \omega^2 C_2 C_3 C_4 R_4^2 << \dfrac{C_2 R_4}{R_3}$

and $\qquad \omega^2 C_4^2 R_4^2 << 1.$

Hence we can write

$$C_1 = C_2 \frac{R_4}{R_3}$$

When the capacitor C_1 is without specimen dielectric, let its capacitance be C_0.

$$\therefore \quad C_0 = C_2 \frac{R_4}{R_3} = 150 \times \frac{5000}{5000} = 150 \text{ pF}.$$

When the specimen is inserted as dielectric, let the capacitance be C_s.

$$\therefore \quad C_s = C_2 \frac{R_4}{R_3} = 900 \times \frac{5000}{5000} = 900 \text{ pF}.$$

Now $\quad C_0 = \varepsilon_0 A/d$ and $C_s = \varepsilon_r \varepsilon_0 A/d$.

Hence relative permittivity of specimen

$$\varepsilon_r = \frac{C_s}{C_0} = \frac{900}{150} = 6.$$

Example 16.12 *The arms of a four arm bridge abcd, supplied with sinusoidal voltage, have the following values :*

arm ab : A resistance of 200 Ω in parallel with a capacitance 1 μF.

arm bc : 400 Ω resistance,

arm cd : 1000 Ω resistance.

arm da : A resistance R_2 in series with a 2 μF capacitance.

Determine the value of R_2 and the frequency at which the bridge will balance.

Solution. If we draw the sketch of this bridge, we find that it is Wien's bridge shown in Fig. 16.20.

The analysis of this bridge is given in Art. 16.8.1.

From Eqn. 16.63,

$$R_2 = \left(\frac{R_4}{R_3} - \frac{C_1}{C_2}\right) R_1 = \left(\frac{1000}{400} - \frac{1 \times 10^{-6}}{2 \times 10^{-6}}\right) \times 200$$

$$= 400\ \Omega.$$

From Eqn. 16.64, the frequency at which the bridge balance,

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

$$= \frac{1}{2\pi\sqrt{200 \times 400 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}} = 398 \text{ Hz}.$$

Example 16.13 *Fig. 16.17 shows a Heaviside Campbell bridge for measurement of a self-inductance L_2 with the equal ratios i.e., $R_3 = R_4$, the following results were obtained :*

With switch open $M = 15.8$ mH and $r = 25.7\ \Omega$.

With switch close $M = 0.2$ mH and $r = 1.2\ \Omega$.

Find the resistance and self-inductance of coil.

Solution. The theory of this bridge is dealt in Art. 16.7.4.

From Eqns. 16.49 and 16.50, for equal ratios

Self-inductance

$$L_2 = 2(M_1 - M_2) = 2(15.8 - 0.2) = 31.2 \text{ mH}$$

Resistance

$$R_2 = r_1 - r_2 = 25.7 - 11.2 = 14.5\ \Omega.$$

Example 16.14 *A modified Carey Foster's bridge shown in Fig 16.18 is used for measurement of capacitance. Arm ab contains the secondary winding a mutual inductance of 18.35 mH and a total non-reactive resistance of 200 Ω. The inductance of secondary of mutual inductor in arm ab is 40.6 mH. Arm ad is short circuited. Arm bc contains the unknown capacitor in series with a resistance of 119.5 Ω. Arm cd comprises of a resistor of 100 Ω resistance. The detector is across bd. Determine the capacitance and its equivalent series resistance.*

Solution. From the analysis carried in Art. 16.7.6, we have

$$\text{Capacitance } C_3 = \frac{M}{R_1 R_4} = \frac{18.36 \times 10^{-3}}{200 \times 100} \text{F} = 0.918\ \mu\text{F}.$$

$$R_3 = \frac{R_4(L_1 - M)}{M} = \frac{100(40.6 - 18.36)}{18.36} = 121.2\ \Omega.$$

∴ Series resistance of capacitor

$$= 121.2 - 119.5 = 1.7\ \Omega.$$

Example 16.15 *In a modified Carey-Foster bridge shown in Fig. 16.18. At balance $R_4 = 10\ \Omega$, $R_3 = 25.1 \pm 0.1\ \Omega$, $R_1 = 100\ \Omega$, $L_1 = 22.82$ mH and $M = 4920 \pm 10\ \mu$H. Find the unknown capacitance C_3 and the limits of accuracy determined by the limits of balance measurements indicated.*

Solution. From Eqn. 16.59, we get

$$C_3 = \frac{M}{R_1 R_4} = \frac{4920 \pm 10}{100 \times 10} = 4.92 \pm 0.01\ \mu\text{F}.$$

$$\text{Accuracy limit} = \frac{0.01}{4.92} \times 100 = 0.2\%.$$

Example 16.16 *Suppose in the above example the value of R_3 does not include the series loss resistance of capacitor, find the loss resistance and limits of its accuracy.*

Solution. From Eqn. 16.60,

$$R_3 = \frac{R_4(L_1 - M)}{M} = \frac{228200}{4920(1 \pm 10/4920)}$$

$$= \frac{46.5}{1 \pm 10/4920} = 46.5\left(1 \pm \frac{10}{4920}\right)$$

$$= 46.5(1 \pm 0.00204) = 46.5 \pm 0.095\ \Omega.$$

This resistance includes the series resistance of capacitor.

Series resistance of capacitor

$$= (46.5 \pm 0.095) - (25.1 \pm 0.1)$$

$$= 21.4 \pm 0.195\ \Omega$$

$$\text{Limits of accuracy} = \frac{0.195}{21.4} \times 100 = 0.915\%.$$

16.12 TRANSFORMER RATIO BRIDGE

The *Transformer Ratio Bridges* are becoming increasingly popular and are being used for a wide range of applications. This is on account of versatility and accuracy of *Ratio Transformers*, which are used in the transformer ratio bridges. In fact, transformer ratio bridges are replacing the conventional a.c. bridges at a rapid rate.

A transformer ratio bridge consists of voltage transformer whose performance approaches that of an ideal transformer. An ideal transformer is one that has no resistance, no core loss and no leakage flux (*i.e.*, there is perfect coupling between the windings).

The ratio transformer is provided with a number of tappings in order to obtain voltage division.

Voltage appearing across the windings of a transformer is :

$$E = 4 K_f N \Phi_m f \text{ volt} \qquad \text{...(16.66)}$$

where N = number of turns,

Φ_m = maximum value of flux ; Wb,

f = frequency ; Hz,

K_f = form sector

(Its value is 1.11 for sinusoidal flux).

For a given value of K_f, flux Φ_m and frequency f,

$$E = K_1 N \qquad \text{...(16.67)}$$

Figure 16.28 shows an autotransformer provided with tappings. Suppose an alternating voltage E is applied across the winding. Assuming that the

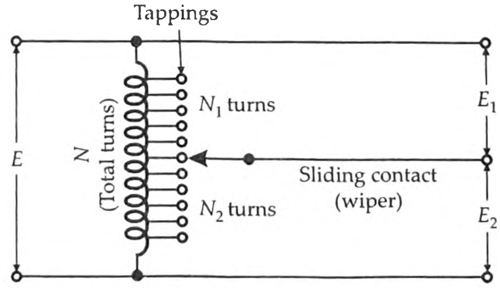


Fig. 16.28 Tapped autotransformer.

autotransformer is ideal type, the division of applied voltage E into output voltages E_1 and E_2 is :

$$E_1 = E \cdot \frac{N_1}{N} \quad \text{and} \quad E_2 = E \cdot \frac{N_2}{N}$$

Different values of E_1 and E_2 may be had by changing the position of wiper on the tappings.

However, in practice it is impossible to construct an ideal transformer. But the ideals of zero winding resistance, zero core loss and perfect coupling can be closely achieved if the design features similar to those for instrument transformers are used. The material used for construction of core should be such that it gives the smallest core losses at the desired operating frequency. The magnetizing current is reduced by using a *Toroidal Core*. The added advantage of toroidal core is that winding put on it has minimum leakage reactance giving an almost perfect coupling. The leakage reactance can be reduced further by using a special type of construction for the windings as shown in Fig. 16.29. This winding takes the form of a *Multiconductor Rope*. In order to obtain a decade of voltage division,the multicon- ductor rope has ten wires with successive sets of turns connected in series and a tapping is taken from each joint.

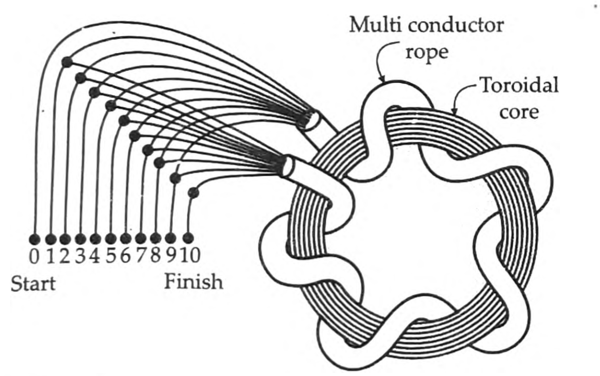


Fig. 16.29 Tapped ratio transformer using toroidal core and multiconductor rope type winding.

The resistance of the windings can be reduced by using copper wire of heavy cross-section.

A 4-decade ratio transformer is shown in Fig. 16.30. The successive decades are obtained by using an arrangement similar to that in a Kelvin Varley slide. This transformer arrangement gives a ratio error of less than 1 part in 10^4.

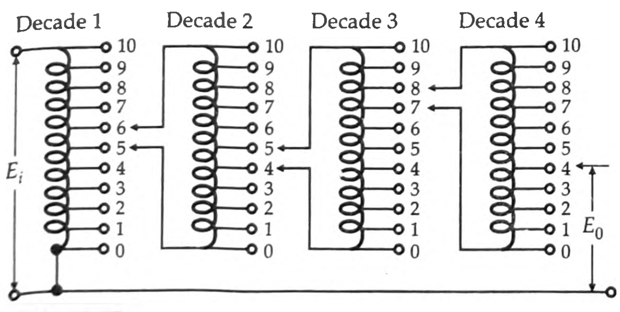


Fig. 16.30 Four decade ratio transformer.

16.12.1 Applications and Features of Ratio Transformers

Applications. The ratio transformers can be used for :

(*i*) Measurement of resistance capacitance and inductance in comparison with standard resistance, standard capacitance and standard inductance respectively,

(*ii*) Measurement of amplifier gain and phase shift, and

(*iii*) Measurement of transformer ratios.

Features. The ratio transformers have the following features :

(*i*) They can be used on a.c. only.

(*ii*) They have very small ratio errors.

(*iii*) They have a wide frequency range extending from 50 Hz to 50 kHz.

(*iv*) They have high input impedance and low input impedance. Thus the loading effects in them are small.

16.12.2 Measurement of Resistance

The circuit used for measurement of an unknown resistance, R, in comparison with a standard resistance, R_s is shown in Fig. 16.31. The position of the wiper is adjusted till the detector D shows null. (The detectors used are the same as for conventional a.c. bridges.)

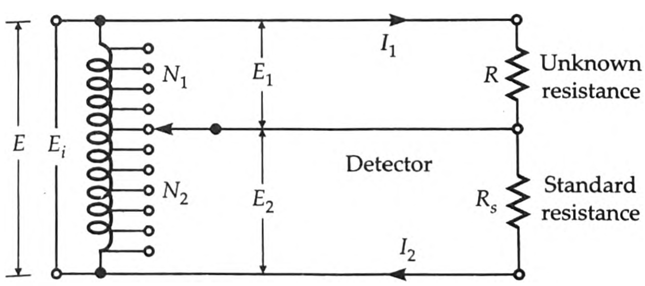


Fig. 16.31 Measurement of resistance using ratio transformer.

Current through the unknown resistance

$$I_1 = \frac{E_1}{R} = \frac{K_1 N_1}{R}$$

Current through the standard resistance

$$I_2 = \frac{E_2}{R_s} = \frac{K_1 N_2}{R_s}$$

Under balance conditions, the current through the detector is zero *i.e.*, $I_1 = I_2$

$$\therefore \quad \frac{K_1 N_1}{R} = \frac{K_1 N_2}{R_s}$$

or $$R = \frac{N_1}{N_2} R_s \qquad \ldots(16.68)$$

A circuit used for measurement of low resistance is shown in Fig. 16.32. This circuit is similar to that of Kelvin's double bridge.

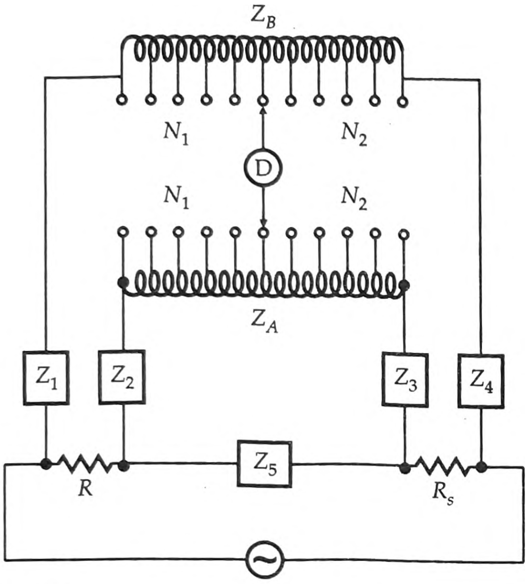


Fig. 16.32 Measurement of low resistance.

By using two transformers a form of Kelvin's double bridge for measurement of low resistances may be devised as shown in Fig. 16.32, where

$$R = R_s\left[\frac{N_1 Z_B + Z_1}{(1-N_1)Z_B + Z_4}\right] + Z_5\left[\frac{(1-Z_2)Z_A + Z_3}{Z_1 + Z_2 + Z_3 + Z_5}\right]$$
$$\times\left[\frac{N_1 Z_B + Z_1}{(1-N_1)Z_B + Z_4} - \frac{N_1 Z_A + Z_3}{(1-N_2)Z_A + Z_3}\right] \qquad \ldots(16.69)$$

If the impedances of leads *i.e.*, Z_1, Z_2, Z_3, Z_4, and Z_5 are small and the resistances R and R_s are of the same order, the unknown resistance is given by :

$$R = \frac{N_1}{N_2} R_s \qquad \ldots(16.70)$$

This form of Kelvin's bridge can, of course, only be used on *ac* but by plotting the values R against frequency, an extrapolation may be used to find the d.c. value of R with an accuracy of a few parts in a million.

16.12.2 Measurement of Capacitance

A circuit for measurement of capacitance is given in Fig. 16.33. An unknown capacitance C is measured in comparison with a standard capacitance C, which is assumed to be perfect. Resistance R represents the loss of the capacitor. Since for balance, the magnitude and phase of the currents passing through the detector should be the same, a variable standard resistance is connected in parallel with the standard capacitor.

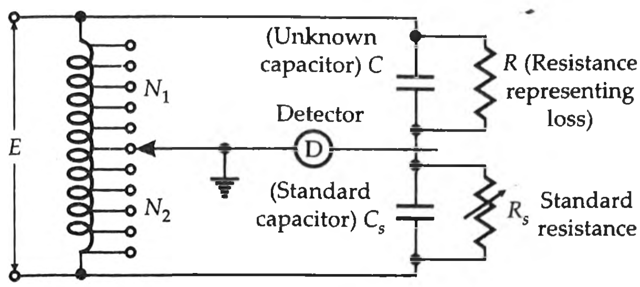


Fig. 16.33 Measurement of capacitance using ratio transformer.

At balance, $$C = \frac{N_2}{N_1} C_s \text{ and } R = \frac{N_1}{N_2} R_s \qquad \ldots(16.71)$$

Dissipation factor

$$D = \tan\delta = \frac{1}{\omega CR} = \frac{1}{\omega C_s R_s} \qquad \ldots(16.72)$$

16.12.3 Measurement of Phase Angle

A circuit for measurement of small phase angles with the help of ratio transformers is shown in Fig. 16.34. An RC circuit is used where the capacitance is variable in order to get phase shift. The value of resistance should be large in order that there are no loading effects on the ratio transformer. The capacitance is changed till the detector indicates null.

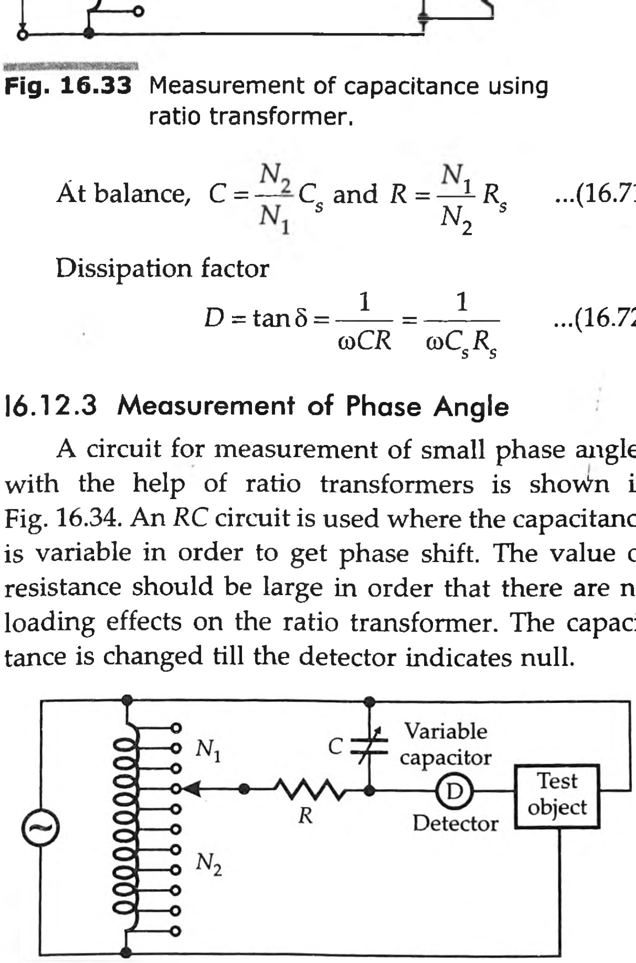


Fig. 16.34 Measurement of phase angle using ratio transformer.

Phase angle $$\phi = \tan^{-1}(-\omega RC) \qquad \ldots(16.73)$$

The magnitude of in-phase component is :

$$\phi_1 = \frac{N_2}{N_1 + N_2}\cos^2\phi \qquad \ldots(16.74)$$

16.12.4 Transformer Double Ratio Bridges

The transformer bridges described till now in this chapter are similar to the conventional a.c. bridges in the sense that the balance conditions are indicated when the voltage across the detector is zero and therefore, the current through the detector is zero.

An alternate way of indication of balance is that the current flowing through the unknown impedance is equal and opposite to that through the standard or known impedance. The detector indicates this condition of equality.

The arrangement of *Transformer Double Ratio Bridges* is shown in Fig. 16.35. It consists of an ideal voltage transformer having a secondary winding of N_1 turns. The secondary winding is tapped at N_2 turns. The resulting voltages E_1 and E_2 are applied across the unknown impedance, Z, and standard impedance, Z_s, respectively.

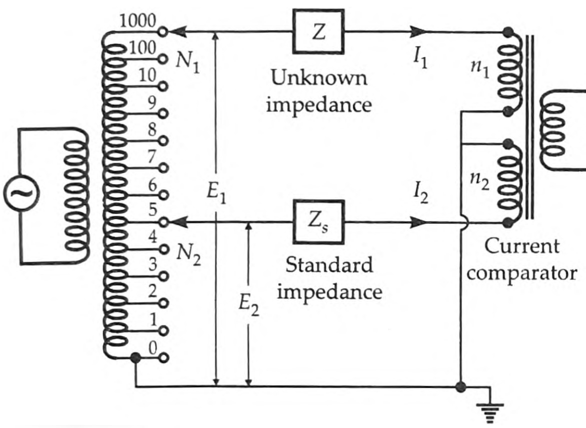


Fig. 16.35 Basic circuit for a double ratio transformer bridge.

The two voltages E_1 and E_2 produce currents I_1 and I_2 respectively. These currents flow in the windings n_1 and n_2 of a current comparator. The two mmfs $n_1 I_1$ and $n_2 I_2$ produce fluxes in the core of the comparator. The two fluxes oppose each other and when there is no flux in the core, no emf is induced in the secondary winding of this comparator and the detector D indicates null condition.

At balance, the two mmfs $I_1 n_1 = I_2 n_2$ ...(16.75)

Now $I_1 = \frac{E_1}{Z} = \frac{K_1 N_1}{Z}$ and $I_2 \frac{E_2}{Z_s} = \frac{K_1 N_2}{Z_s}$

$$\therefore \quad \frac{N_1 n_1}{Z} = \frac{N_2 n_2}{Z_s}$$

or unknown impedance

$$Z = Z_s \frac{N_1 n_1}{N_2 n_2} \qquad \text{...(16.76)}$$

The balance is obtained by changing the setting of N_2.

If the unknown impedance is a pure resistance R, a standard resistance R_s is required for obtaining balance

$$R = \frac{N_1 n_1}{N_2 n_2} R_s \qquad \text{...(16.77)}$$

If the unknown impedance is a pure capacitance C, a standard capacitor C_s is required for balance,

$$C = \frac{N_2 n_2}{N_1 n_1} C_s \qquad \text{...(16.78)}$$

Equation 16.77 can also be converted to a form similar to that of Eqn. 16.78. This is done by using conductance in place of resistance.

Unknown conductance

$$G = \frac{1}{R} = \frac{N_2 n_2}{N_1 n_1} G_s \qquad \text{...(16.79)}$$

where G_s is the standard conductance.

In practice, the capacitors are not perfect, the losses being represented by a conductance in parallel with the capacitance. Therefore, balance has to be obtained both for resistance as well as for capacitance. In other words, mmf equality for both magnitude as well as for phase has to be obtained in order to get balance.

A circuit used of measurement of an imperfect capacitor in terms of standard capacitor and standard conductance is shown in Fig. 16.36.

Under balance,

$$C = \frac{n_2}{N_1 n_1}(C_{s1} N_{21} + C_s N_{22}) \qquad \text{...(16.80)}$$

and $$G = \frac{n_2}{N_1 n_1}(G_{s1} N'_{21} + G_{s2} N'_{22}) \qquad \text{...(16.81)}$$

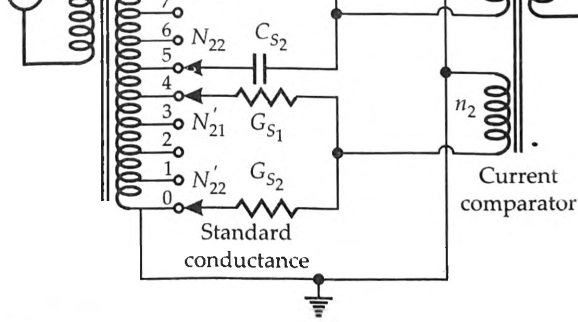


Fig. 16.36 Measurement of imperfect capacitor.

It is clear from Eqns. 16.80 and 16.81 that the double ratio bridge allows the reactive and resistive components of an unknown to be adjusted independently of each other (capacitive balance can be had by changing N_{21} and N_{22} while resistive balance can be obtained by changing N'_{21} and N'_{22}). This property of independent balance facilitates the use of this bridge in a very wide range of applications.

16.12.5 Measurement of Inductance

The circuit for measurement of inductance is the same as that for measurement of capacitance (Fig. 16.36) except that the unknown capacitance is replaced by the unknown inductance.

In order that independent balance of the resistive and reactive mmfs can be obtained any inductive circuit must be considered as a two terminal network, the components of which are in parallel. The mmf due to the resistive component of the unknown may then be opposed by that through the conductance standard. On the other hand, the mmf due to inductive component of the unknown must be balanced due to capacitance standard. In order to obtain mmf balance due to unknown inductance and the standard capacitance, it is necessary to reverse the comparator winding connected to the standard capacitor. This condition of bridge operation is normally indicated by addition of a negative sign to the display of capacitance C_m. Suppose an inductance L_p in parallel with a resistance R_p represents the actual inductor having an inductance L in series with a resistance R.

At balance, $\dfrac{1}{R_p} + \dfrac{1}{j\omega L_p} = G_m - j\omega C_m$

where G_m = indicated conductance, and

C_m = indicated capacitance.

$\therefore$ Series resistance, $$R = \frac{G_m}{G_m^2 + \omega^2 C_m^2} \qquad ...(16.82)$$

and Series inductance, $$L = \frac{C_m}{G_m^2 + \omega^2 C_m^2} \qquad ....(16.83)$$

16.12.6 Measurement of Components in 'SITU'

One of the greatest advantages of double ratio transformer bridges is their capability to measure the values of the components while they remain connected in the circuit. Figure 16.37 shows a circuit for measurement of an unknown impedance **Z**. If the

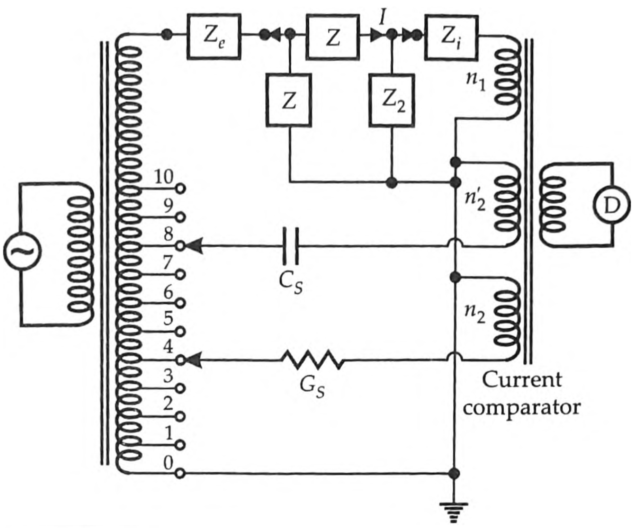


Fig. 16.37 Measurement of components 'IN SITU' using a double ratio transformer bridge.

ratio transformers are considered ideal the values of impedances $\mathbf{Z}_e$ and $\mathbf{Z}_i$ can be assumed to be zero and the voltage applied to the unknown impedance **Z** is not affected by current flowing in impedance $\mathbf{Z}_1$. Also all the current flowing through Z would flow into n_1, winding as :

$$I_2 = \frac{\mathbf{Z}_i \mathbf{I}}{\mathbf{Z}_i + \mathbf{Z}_2}$$

The value of unknown impedance is approximately given by :

$$\mathbf{Z} = \left[\frac{1}{G_m + j\omega C_m}\right]\left[1 - \left(\frac{\mathbf{Z}_e}{\mathbf{Z}_1} + \frac{\mathbf{Z}_i}{\mathbf{Z}_2}\right)\right] \qquad ...(16.84)$$

In practice impedances $\mathbf{Z}_e$ and $\mathbf{Z}_i$ have small finite values and corrections must be applied for this. In order that corrections be applied $\mathbf{Z}_e$ and $\mathbf{Z}_i$ must be determined.

Review Questions

1. Derive the general equations for balance for an a.c. bridge. Prove that two conditions *i.e.,* for magnitude and phase have to be satisfied if an a.c. bridge is to be balanced unlike a d.c. bridge wherein only the magnitude condition is to be satisfied
2. Describe the sources and the null detectors that are used for a.c. bridges.
3. Explain what is meant by sliding balance. How is this condition avoided by choosing variables for

manipulation of balance *i.e.*, why variables are so chosen that the two equations for balance are independent of each other.

4. Why is it preferable in bridge circuits that the equations of balance are independent of frequency ? Explain.
5. Derive the equations for balance in the case of Maxwell's inductance capacitance bridge. Draw the phasor diagram for balance conditions.
6. In Maxwell's inductance-capacitance bridge the dial of variable capacitor can be made to read the value of unknown inductance directly. How is it done ?
7. Explain why Maxwell's inductance-capacitance bridge is useful for measurement of inductance of coils having storage factor between 1 and 10.
8. Describe the working of Hay's bridge for measurement of inductance. Derive the equations for balance and draw the phasor diagram under conditions of balance. Why is this bridge suited for measurement of inductance of high Q coils ?
9. Derive the equations of balance for an Anderson's bridge. Draw the phasor diagram for conditions under balance.

   Discuss the advantages and disadvantages of the bridge.
10. What are incremental inductance and permeability ? Describe how are they measured with Owen's bridge.
11. Describe how an unknown capacitance can be measured with the help of D'Sauty's bridge. What are the limitations of this bridge and how are they overcome by using a modified form of D'Sauty's bridge ? Draw phasor diagrams to illustrate your answer.
12. Describe the working of a low voltage Schering bridge. Derive the equations for capacitance and dissipation factor. Draw the phasor diagram of the bridge under conditions of balance.
13. What are the modifications and additional features incorporated in a low voltage Schering bridge for it to be used on high voltages ? Explain. Describe how relative pemittivity of a specimen of insulating material can be determined using a Schering bridge.
14. Describe the working of Heaviside Mutual Inductance Bridge for determination of mutual inductance of a coil. Explain how the self-inductance of a coil can be determined with the help of a mutual inductance. Also, explain how the use of a balancing coil helps in elimination of errors on account of resistance of leads etc.
15. Explain how Wien's bridge can be used for experimental determination of frequency. Derive the expression for frequency in terms of bridge parameters.
16. What is a Universal Impedance Bridge ? What are the features incorporated in it ? Draw the circuit diagrams of the various bridges used in a Universal Impedance Bridge.
17. What are the different sources of errors in a.c. bridges ? Explain the precautions taken and the techniques used for elimination/minimization of these errors.
18. Explain the function and working of Wagner Earth Devices.
19. Describe what do you understand by the term "Ratio Transformer". Explain the construction of a Ratio transformer and describe its uses.
20. Explain how a Ratio Transformer bridge can be used for measurement of

    (*i*) resistance (*ii*) capacitance, (*iii*) phase angle.
21. What is a "Transformer Double Bridge" ? Explain how can it be used for measurement of

    (*i*) resistance, (*ii*) inductance,

    (*iii*) capacitance and loss angle of a imperfect capacitor.

Unsolved Problems

Note. Students must attempt these problems from fundamentals.

1. A four arm a.c. bridge *a,b,c,d* has the following impedances :

   Arm *ab* : $\mathbf{Z}_1 = 200\angle 60°\Omega$ (inductive impedance).

   Arm *ad* : $\mathbf{Z}_2 = 400\angle -60°\Omega$

   (purely capacitive impedance)

   Arm *bc* : $\mathbf{Z}_3 = 300\angle 0°\Omega$ (purely resistive),

   Arm *cd* : $\mathbf{Z}_4 = 600\angle 30°\Omega$ (inductive impedance).

   Determine whether it is possible to balance the bridge under above conditions.

   [**Ans.** No as $\angle\theta_1 + \theta_4 \neq \angle\theta_2 + \angle\theta_3$]

2. A 1000 Hz bridge has the following constants : arm ab, $R_1 = 1000\ \Omega$ in parallel with $C_1 = 0.5\ \mu F$; arm bc, $R_3 = 1000\ \Omega$ in series with $C_3 = 0.5\ \mu F$; arm cd $L_4 = 30$ mH in series with $R_4 = 200\ \Omega$. Find the constants of arm da to balance the bridge. Express the result as a pure resistance R in series with a pure inductance L or capacitance C.

[**Ans.** $79.4\angle{-11.1}\Omega$, $R = 77.9\ \Omega$ in series with $C = 10.4\ \mu F$]

3. An a.c. bridge has in arm ab, a pure capacitance of 0.2 pF ; in arm bc, a pure resistance of 500 Ω ; in arm cd, a series combination of a 50 Ω resistance and of 0.1 H inductance. Arm da consists of a capacitor of 0.4 μF in series with a resistance R_2, $\omega = 5000$ rad/s. (a) Find the value of R_2 to give bridge balance. (b) Can complete balance be obtained by adjustment of R_2 ? If not, specify the position and the value of an adjustable resistance to complete the balance.

[**Ans.** 1000 Ω, complete balance cannot be obtained by adjustment of R_2 only as reactive conditions are not satisfied. If an adjustable resistance of 200 Ω is connected in series in branch $c\,d$ capacitor reactive balance can be obtained]

4. The four arms of a Maxwell's capacitance bridge at balance are : arm ab, an unknown inductance L_1, having an inherent resistance R_1 ; arm bc, a non-inductive resistance of 1000 Ω ; arm cd, a capacitor of 0.5 μF in parallel with a resistance of 1000 Ω ; arm da, a resistance of 1000 Ω.

Derive the equations of balance for the bridge and determine the value of R_1 and L_1. Draw the phasor diagram of the bridge under balance conditions.

[**Ans.** $R_1 = 1000\ \Omega$; $L_1 = 0.5$ H]

5. In an Anderson Bridge for the measurement of inductance the arm AB consists of an unknown impedance with inductance L and R, a known variable resistance in arm BC, fixed resistance of 600 Ω each in arms CD and DA, a known variable resistance in arm DE, and a capacitor with fixed capacitance of 1 microfarad in the arm CE.

The a.c. supply of 100 Hz is connected across A and C, and the detector is connected between B and E. If the balance is obtained with a resistance of 400 Ω in the ami DE and a resistance of 800 Ω in the arm BC, calculate the value of unknown R and L. Derive the conditions for balance and draw the phasor diagram under balanced conditions.

[**Ans.** $R = 800\ \Omega$, $L = 1.12$ A]

6. The four arms of a Hay's alternating current bridge are arranged as follows : AB is a coil of unknown impedance ; BC is a non-reactive resistor of 1000 Ω ; CD is a non-reactive resistor of 833 Ω in series with a standard capacitor of 0.38 μF : DA is non-reactive resistor of 16800 Ω. If the supply frequency is 50 Hz determine the inductance and the resistance at the balanced conditions. Derive the conditions for balance and draw the phasor diagram under balanced conditions.

[**Ans.** $R_1 = 210\ \Omega$, $L_1 = 6.38$ H]

7. The a.c. bridge shown in Fig. 16.38 used to measure an unknown inductance L_x, that has inherent resistance R_x. The bridge parameters are $R_1 =$

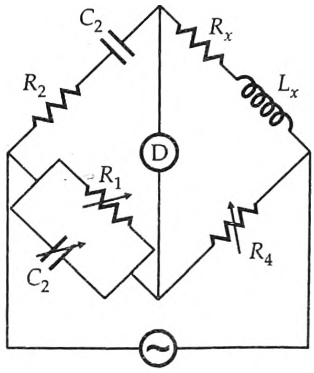


Fig. 16.38 Diagram of Problem 7.

20,000 Ω, $R_2 = 50{,}000\ \Omega$, $C_2 = 0.003\ \mu F$, $\omega = 10^6$ rad/s. C_1 is adjustable from 10 pF to 150 pF and R_4 is adjustable from 0 to 10,000 Ω.

(a) Show that the equations for resistive and reactive balance are independent of each other. Derive expressions for R_x and L_x in terms of ω, R_1, R_2, R_4, C_1 and C_2.

(b) Determine the largest values of R_x and L_x that can be measured with the given parameters.

[**Ans.** (a) $R_x = R_2 R_4 / R_1 + C_1 R_4 / C_2$;
$L_x = R_2 R_4 C_1 - R_4 / \omega^2 R_1 C_2$;
(b) $2.55 \times 10^4\ \Omega$; 58.3 mH]

8. The four arms of a bridge network are made up as follows : ab, a resistor of 50 Ω in parallel with an inductor of 0.1 H ; bc, a resistor of 100 Ω ; cd, and unknown resistor R in parallel with an unknown capacitor C ; da, a resistor of 1000 Ω. A 50 Hz voltage supply is applied across ac. Find R and C when a vibration galvanometer connected across bd is undeflected. [**Ans.** $R = 7060\ \Omega$; $C = 0.72\ \mu F$]

9. A four branch bridge network $ABCD$ balanced at 1000 Hz has branches AB and BC of pure resistance of 1000 Ω and 1250 Ω respectively. An unknown impedance forms the arm CD and the branch DA consists of a standard capacitor of 0.1 μF capacity

and negligible resistance, connected in series with a non-inductive resistance of 10 Ω to give balance. The supply voltage is 15 V and the supply is given at the points *B* and *D*. Find the components of unknown impedance and draw the necessary phasor diagram. [**Ans.** 12.5 Ω ; 0.08 μF]

10. The circuit for measurement of effective resistance and self-inductance of an iron cored coil is as follows : arm *ab*, the unknown impedance ; arm *bc*, a pure resistance R_3 ; arm *cd*, a lossless capacitor C_2 ; arm *da*, a capacitor C_2 in series with a resistance. Under balance conditions.

$R_3 = 10\,\Omega$, $R_2 = 842\,\Omega$, $C_2 = 0.135\,\mu$F and $C_4 = 1\mu$F.

Calculate the value of effective resistance and self-inductance at a supply frequency of 100 Hz.

Derive the equations of balance and draw the phasor diagram under balanced condition.

[**Ans.** 74 Ω, 8.42 mH]

11. A capacitor bushing forms arm *ab* of a Schering bridge and a standard capacitor of 500 pF capacitance and negligible loss, forms arm *ad*. Arm *bc* consists of a non-inductive resistance of 300 Ω. When the bridge is balanced arm *cd* has a resistance of 72.6 Ω in parallel with a capacitance of 0.148 μF. The supply frequency is 50 Hz. Calculate the capacitance and dielectric loss angle of capacitor.

Derive the equations for balance and draw the phasor diagram under conditions of balance.

[**Ans.** 121 pF ; 0.00338 rad]

12. An a.c. bridge was made up as follows : arms *AB* and *BC* equal ratio arms ; *CD* a variable capacitor *C* in series with a variable resistor *R* : *DA* a standard air capacitor of 0.001 μF in series with a fixed standard resistance of 500 Ω. The supply at 796 Hz was connected across *AC* and the detector across *BD*. Balance of the above bridge was obtained with $C = 0.001\,\mu\text{F} \pm 10$ pF and $R = 500 \pm 5\,\Omega$. When an unknown capacitor was connected across *CD*, the balance conditions changed to $C = 360 \pm 10$ pF and $R = 0$. Calculate the capacitance and power factor of the unknown capacitor and the limits of accuracy.

[**Ans.** 650 pF ± 3.2% ; 0.0039 ± 0.002%]

13. A Schering bridge is used for measuring the power loss in dielectrics. The specimens are in the form of discs 0.3 cm thick and have a dielectric constant of 2.3. The area of each electrode is 314 cm^2 and the loss angle is known to be 9 for a frequency of 50 Hz. The fixed resistor of the network has a value of 1,000 Ω and the fixed capacitance is 50 pF. Determine the values of the variable resistor and capacitor required.

Derive the equations for balance and draw the phasor diagram under balanced conditions.

[**Ans.** 4200 Ω, 0.00196 μF]

14. In a balanced bridge network, *AB* is a resistance of 500 Ω in series with an inductance of 0.18 H ; *BC* and *DA* are non-inductive resistances of 1000 Ω ; and *CD* consists of a resistance *R* in series with a capacitance *C*. A potential difference of 5 volt at a frequency of 5,000/2π Hz is established between the points *A* and *C*. Draw to scale a phasor diagram showing the currents and potential difference in the bridge and from it determine the values of *R* and *C*. Check the result algebraically.

[**Ans.** 472 Ω ; 0.235 μF]

15. A 4-arm unbalanced a.c. bridge is supplied from a source having negligible impedance. The bridge has non-reactive resistors of equal resistance in adjacent arms. The third arm has an inductor of resistance *R* and reactance *X*, where *X* is numerically equal to *R*. The fourth arm has a variable non-inductive resistor. The detector is connected between the junction of first and second arms and the junction of third and fourth arms and has a resistance *R* and negligible reactance. Determine the magnitude of the variable resistor where the detector current is in quadrature with the supply current. [**Ans.** 1.37 *R*]

16. An a.c. bridge consists of the following constants : arm *ab*, a resistance of 800 Ω in parallel with a capacitance 0.4 μF ; *bc*, an unknown resistance ; *cd*, a known resistance of 1200 Ω and *da*, a resistance of 500 Ω in series with a capacitance of 1 pF. Find the resistance required in arm *bc* to give balance and also the frequency for which the bridge is balanced. [**Ans.** 384 Ω ; 890 Hz]

17. A 4-arm bridge network, adjusted to balance conditions consists of :

Arm *AB* a standard resistor known to be within 0.1% of 100 Ω.

Arm *BC* a variable capacitor adjusted to 0.362 μF in parallel with a variable resistor adjusted to 2380 Ω.

Arm *CD* a standard resistor known to be within 0.1% of 1000 Ω.

Arm *DA* a coil of inductance *L* and series loss resistance *R*.

Evaluate *L* and *R* deriving the equations used.

Determine the maximum percentage error that can be tolerated in each of the variable components if L is to be determined to within 0.5% and R to within 1% of the correct value. If the variable components are known to be accurate within the limits so determined how accurately is the ratio L/R known?

[**Ans.** 00.362 H ; 42 Ω ; 0.3% for capacitor ; 0.8% for resistor ; 1.1%]

18. A low-resistance coil AB of 500 turns is wound on a ring of high permeability magnetic alloy, and a tapping is made at a point C such $AC = 100$ turns. A secondary winding on the same is connected to an amplifier and headphones, and a supply at 796 Hz is connected between C and a point D, so that $ACBD$ forms a bridge circuit in which the two parts AC and BC of the winding are the ratio arms. Balance is achieved when arm AD comprises a capacitance of 0.16 pF in series with a resistance of 0.4 Ω, and arm BD is the capacitor under test.

Evaluate the capacitance and the equivalent shunt loss resistance of the capacitor at supply frequency.

Mention the special advantages of this form of bridge circuit over the more conventional forms.

[**Ans.** 0.04 μF ; 976 kΩ]

19. A balanced Hay's bridge shown in Fig. 16.5 has $R_2 = R_3 = 1000\,\Omega$, $R_4 = 8120\,\Omega$, $C_4 = 980$ pF, and the frequency = 4000 Hz.

Calculate L_1 and R_1.

If each of the resistors is accurate within ± 0.5% the capacitor within ± 1 pF, and the frequency within 5 Hz, determine the limits of accuracy of determination of L.

[**Ans.** $L = 942.3 \times 10^{-6}(1 \pm 10.00181)$ H ; accuracy limit ± 0.18%]

20. A resonance bridge is shown in Fig. 16.39.

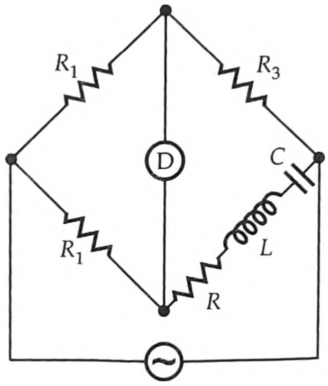


Fig. 16.39 Resonance bridge. (Diagram of Problem 16.20)

Prove that

$$C = \frac{1}{\omega^2 L}, \qquad L = \frac{1}{\omega^2 C}$$

and $$f = \frac{1}{2\pi\sqrt{LC}}$$

Objective Type Questions

Tick (√) the most appropriate answer :

Answer questions 1–3, referring to Fig. 16.40.

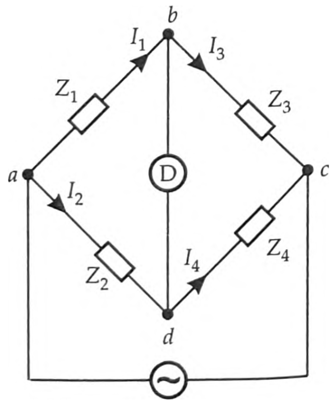


Fig. 16.40

1. In order that the bridge shown in Fig. 16.40, be balanced :
   (*a*) $I_1 = I_3$ and $I_2 = I_4$
   (*b*) $Z_1 Z_4 = Z_2 Z_3$
   (*c*) $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$
   (*d*) all the above.

2. For the bridge shown in Fig 16.40, $Z_1 = 200\,\Omega\,\angle 30°$, $Z_2 = 150\,\Omega\,\angle 0°$, $Z_3 = 250\,\Omega\,\angle -40°$. In order that the bridge be balanced Z_4 should be :
   (*a*) 187.5 Ω ∠ – 70° (*b*) 187.5 Ω ∠ – 10°
   (*c*) 333.3 Ω ∠ 10° (*d*) 120 Ω ∠ 70°.

3. For the bridge shown in Fig. 16.40, arm *ab* consists of resistance in series with an inductance, arms *bc* and *ad* consist of pure resistance. In order to achieve balance arm *cd* should consist of

(a) a variable resistance in series with an variable inductance

(b) a variable resistance

(c) a variable resistance in series or parallel with a variable capacitance

(d) a variable capacitance.

4. The equations under balance conditions for a bridge are :

$R_1 = R_2 R_3 / R_4$ and $L_1 = R_2 R_3 C_4$

where R_1 and L_1 are respectively unknown resistance and inductance.

In order to achieve converging balance

(a) R_2 and R_3 should be chosen as variables

(b) R_2 and C_4 should be chosen as variables

(c) R_4 and C_4 should be chosen as variables

(d) R_3 and C_4 should be chosen as variables.

5. Maxwell's inductance-capacitance bridge is used for measurement of inductance of :

(a) low Q coils

(b) medium Q coils

(c) high Q coils

(d) low and medium Q coils.

6. The advantage of Hay's bridge over Maxwell's inductance-capacitance bridge is because

(a) its equations for balance do not contain any frequency term

(b) it can be used for measurement of inductance of high Q coils

(c) it can be used for measurement of inductance of low Q coils

(d) none of the above.

7. In D'Sauty's bridge (unmodified form) it is :

(a) possible to obtain balance even if both the capacitors are imperfect

(b) possible to obtain balance if one of the capacitors is perfect

(c) possible to obtain balance only if both the capacitors are perfect

(d) all the above.

8. Frequency can be measured by using

(a) Maxwell's bridge

(b) Schering bridge

(c) Heaviside Campbell bridge

(d) Wien's bridge.

9. A bridge circuit works at a frequency of 2 kHz. The following can be used as detectors for detection of null conditions in the bridge

(a) vibration galvanometers and headphones

(b) headphones and tunable amplifiers

(c) vibration galvanometers and tunable amplifiers

(d) vibration galvanometers, headphones and tunable amplifiers.

10. Wagner's Earth Devices are used in a.c. bridge circuits for :

(a) eliminating the effect of earth capacitances

(b) eliminating the effect of inter-component capacitances

(c) eliminating the effect of stray electrostatic fields

(d) shielding the bridge elements.

Answers

1. (d) **2.** (a) **3.** (c) **4.** (c) **5.** (b) **6.** (b) **7.** (c) **8.** (d) **9.** (b) **10.** (a)

17

High Voltage Measurements and Testing

17.1 TYPES OF TESTS

The growing extension, interconnections and use of higher voltages in electric power systems has been only possible due to extensive research work carried in the sphere of high voltage measurements and testing. Due to this research work in the high voltage field, it has been possible to reduce the size of the equipment to economic and manageable proportions. Thus it is logical that the high voltage testing of electric equipment has come into prominence and sometimes it is the sole design criterion for the determination sizes of high voltage electrical apparatus.

High voltage testing includes a large number of methods. Due to limitations of space, we will be considering a few important methods only. The high voltage tests are classified as :

- Sustained low frequency tests.
- Constant direct current tests.
- High frequency tests.
- Surge or Impulse tests.

1. Sustained Low Frequency Tests

These tests are most commonly used. The frequency employed is 50 Hz in India. These tests are carried out on :

(*i*) Motors, switchgear and other electrical apparatus for routine voltage testing after manufacture or in some cases after installation. The voltage normally used is 2 to 3 kV.

(*ii*) Specimens of insulation for the determination of dielectric constant and dielectric loss.

(*iii*) Supply mains for routine testing.

(*iv*) High voltage transformers, porcelain insulators and high voltage cables, etc. for works testing. The test voltage in such cases may be as high as 2000 kV.

2. Constant Direct Current Tests

Modern trend in electric power transmission is to use as high voltages as possible. This is because increase in trans- mission voltage results in an increase in efficiency of transmission. Overhead lines are in actual operation which employ voltages as high as 750 kV while cables for 275 kV have been made and tested.

Before high voltage lines are energised, the insulation of every part connected with the lines must withstand continuously for half an hour, a voltage as specified below :

Table 17.1 *Test Voltages*

Normal system voltage	*Test voltage*
Below 10 kV	Twice the normal voltage
Above 10 kV	Normal voltage + 20 kV

From above it is clear that the transmission lines and high voltage cables would be subjected to very high voltages for a considerably long interval of time, (*i.e.*, 30 minutes). If such voltage tests are carried out with an a.c. voltage supply, the high voltage transformer required for the purpose would have to be of a very large capacity owing to the heavy charging (capacitive) currents drawn by the lines or the cables. This means that a large expenditure has to be incurred for procuring the high voltage testing transformer. Also there would be great difficulties involved in transporting the testing transformer to the site of testing owing to its large size.

Thus in order to overcome the difficulties encountered above, high voltage d.c. testing is done in place of a.c. testing.

3. High Voltage High Frequency Tests

The break-down and flash over of porcelain insulators used on power transmission lines is often due to high frequency disturbances in the transmission lines. These high frequency disturbances are