BECM301L Signal Processing **Slot: F1+TF1** **Dr. Ramesh R**

Digital Assignment 2

1. Find the frequency response of the following causal systems.
   (a) $\mathbf{y(n) - y(n-1) + \frac{3}{16}y(n-2) = x(n) - \frac{1}{2}x(n-1)}$
   (b) $\mathbf{y(n) = \frac{1}{2}x(n) + x(n-1) + \frac{1}{2}x(n-2)}$
   (c) $\mathbf{y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = x(n) + x(n-1)}$
   (d) $\mathbf{y(n) - \frac{1}{2}y(n-1) = x(n)}$

2. Find the 4 point DFT of the following sequences
   (a) $\boldsymbol{x(n) = \{1, 0, -1, 0\}}$
   (b) $\boldsymbol{x(n) = \{1, -2, 3, 4\}}$
   (c) $\boldsymbol{x(n) = \sin(\frac{n\pi}{2})}$
   (d) $\boldsymbol{x(n) = 2^n}$

3. Determine the IDFT of the following
   (a) $\boldsymbol{X(k) = \{1, 1-j2, -1, 1+j2\}}$
   (b) $\boldsymbol{X(k) = \{1, -2-j, 0, -2+j\}}$

4. Compute 4 point DFT and 8 point DFT of causal sequence given by
   $\boldsymbol{x(n) = \frac{1}{8}; 0 \le n \le 3}$
   $= 0$ elsewhere.

5. Compute the following DFT of the sequence,
   (a) $\boldsymbol{x(n) = \{0, 2, 3, -1\}}$
   (b) $\boldsymbol{x(n) = \{1, 3, 3, 3\}}$

6. Compute 8-point DFT of the discrete time signal, $\boldsymbol{x(n) = \{1, 2, 1, 2, 1, 3, 1, 3\}}$ (a) using radix -2 DIT FFT and (b) using radix-2 DIF FFT algorithm.

7. Compute 8-point DFT of the discrete time signal,
   $\boldsymbol{x(n) = \{1\ 1\ 1\ 1 - 1 - 1 - 1 - 1\}}$
   (a) Using radix -2 DIT FFT and (b) using radix-2 DIF FFT algorithm.

8. Find the 8 point DFT of the sequence

$$\boldsymbol{x(n) = (0.707, 1, 0.707, 0, -0.707, -1, -0.707, 0)}$$

9. Find the IDFT of the sequence

$$\boldsymbol{X(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + j0.414, 0, -5.828 + j2.414\}}$$

10. Find the IDFT of the sequence X(k) given below

$$\boldsymbol{X(k) = \{1, 0, 0, j,\ 0, -j, 0, 0\}}$$

11. Find the circular (7 point) and linear convolution of the sequence

$$\boldsymbol{x(n) = \{1, 2, 7, -2, 3 - 1, 5\}\ \ \&\ \ h(n) = \{-1, 3, 5, -3, 1\}}$$

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SEMESTER: WINTER SEMESTER 2022-23 SLOT: F1+TF1

DIGITAL ASSIGNMENT-2

1. a) Find the frequency response of following causal systems.

a) $y(n) - y(n-1) + \frac{3}{16} y(n-2) = x(n) - \frac{1}{2} x(n-1)$

$$Y(e^{j\omega}) - e^{-j\omega} Y(e^{j\omega}) + \frac{3}{16} Y(e^{j\omega}) e^{-j\omega 2} = X(e^{j\omega}) - \frac{1}{2} X(e^{j\omega}) e^{j\omega}$$

$$H(e^{j\omega}) = \frac{1 - \frac{e^{j\omega}}{2}}{1 - e^{-j\omega} + \frac{3}{16} e^{-j\omega 2}}$$

$$H(e^{j\omega}) = \frac{2 - e^{-j\omega}}{2(1 - e^{-j\omega} + \frac{3}{16} e^{-j\omega 2}} = \frac{2 - \cos\omega + j\sin\omega}{2(1 - e^{-j\omega} + \frac{3}{16} e^{j\omega 2})}$$

$$\text{Phase} = \tan^{-1}\left(\frac{\text{Imag}}{\text{Real}}\right) = \tan^{-1}\left(\frac{\sin\omega}{2 - \cos\omega}\right) = \tan^{-1}\left(\frac{\sin\omega}{1 - 2\sin^2\omega/2}\right)$$

$$= \tan^{-1}\left(\frac{\sin\omega}{\cos\omega}\right) = \omega$$

Phase = ω

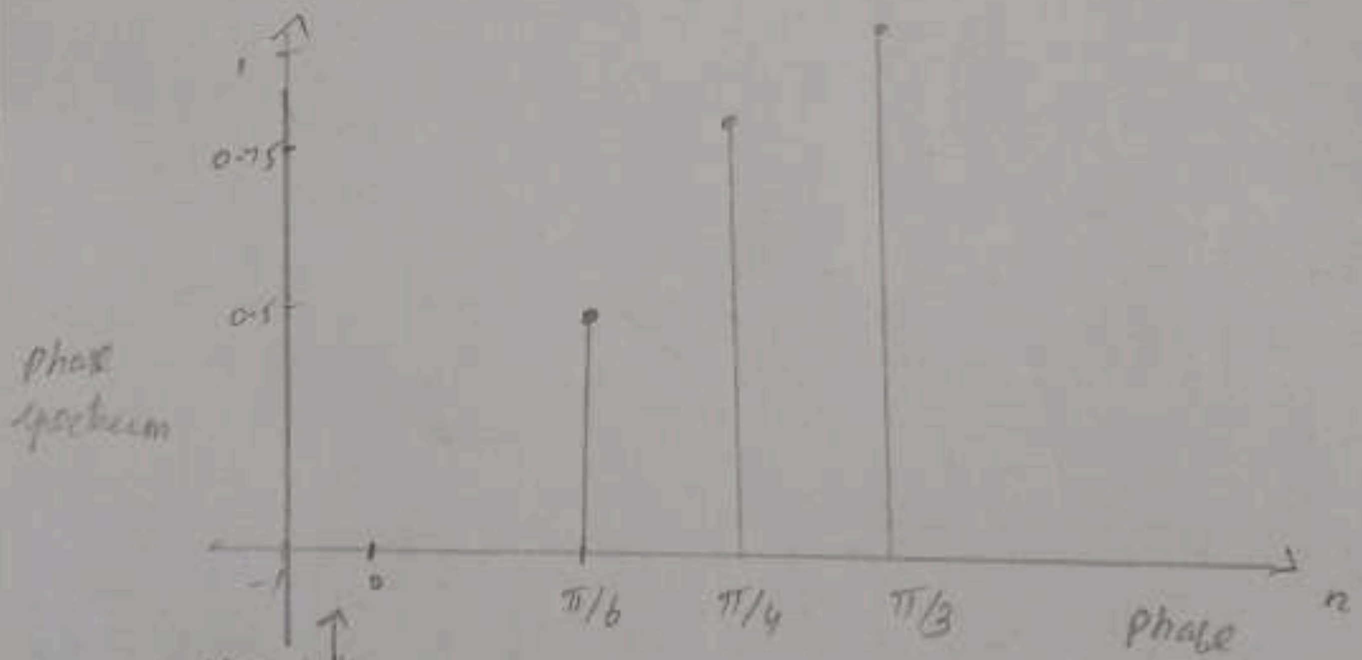
$$H(e^{j\omega}) = \frac{8(2 - \cos\omega + j\sin\omega)}{16 - 16e^{-j\omega} + 3(e^{-j\omega})^2}$$

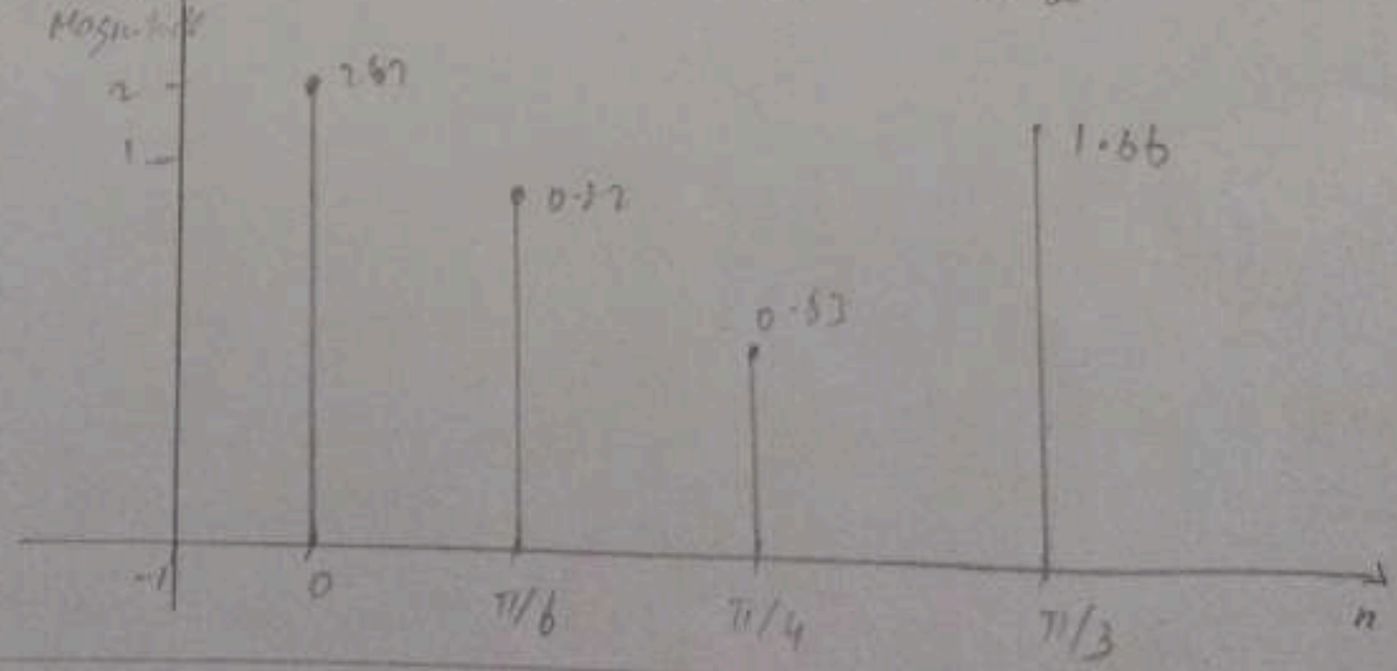
$$H(e^{j\omega}) = \frac{8(2-\cos\omega + j\sin\omega)}{16 - 16\cos\omega + 16j\sin\omega + 3\cos 2\omega + 3j\sin 2\omega}$$

$$H(e^{j\omega}) = \frac{8(2-\cos\omega + j\sin\omega)}{(16-16\cos\omega + 3\cos 2\omega) + j(16\sin\omega + 3\sin 2\omega)}$$

$$|H(e^{j\omega})| = \frac{8\sqrt{(2-\cos\omega)^2 + \sin^2\theta}}{\sqrt{(16-16\cos\omega+3\cos 2\omega)^2 + (16\sin\omega + 3\sin 2\omega)^2}}$$

n	0	$\pi/6$	$\pi/4$	$\pi/3$
Phase	0	0.52	0.785	1.05
Mag.	2.67	0.82	0.53	1.06





b) $y(n) = \frac{1}{2}x(n) + x(n-1) + \frac{1}{2}x(n-2)$

Applying Fourier Transform

$$Y(e^{j\omega}) = \frac{1}{2}X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) + \frac{1}{2}e^{-2j\omega}X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{e^{-2j\omega} + 2e^{-j\omega} + 1}{2}$$

$$H(e^{j\omega}) = \frac{(e^{-j\omega})^2 + 2e^{-j\omega} + 1}{2}$$

$$= \frac{(e^{-j\omega}+1)^2}{2}$$

$$= \frac{[\cos\omega - j\sin\omega + 1]^2}{2}$$

$$= \frac{(\cos\omega - j\sin\omega)^2 + 1 + 2(\cos\omega - j\sin\omega)}{2}$$

$$= \frac{\cos^2\omega - \sin^2\omega - 2\cos\omega\, j\sin\omega + 1 + 2\cos\omega - 2j\sin\omega}{2}$$

$$= \frac{2\cos^2\omega - 2\cos\omega\, j\sin\omega + 2\cos\omega - 2j\sin\omega}{2}$$

$$= (\cos\omega + \cos^2\omega - j\sin\omega - j\sin\omega\cos\omega)$$

$$H(e^{j\omega}) = \cos\omega + \cos^2\omega - j[\sin\omega + \sin\omega\cos\omega]$$

Phase: $\tan^{-1}\left[\frac{\text{Imag}}{\text{Real}}\right] = \tan^{-1}\left[\frac{-\sin\omega[1+\cos\omega]}{\cos\omega[1+\cos\omega]}\right] = \tan^{-1}[-\tan\omega]$

Phase $= \tan^{-1}(\tan(\pi-\omega)) = \pi - \omega$

Magnitude:

$$|H(e^{j\omega})| = \sqrt{(\cos\omega + \cos^2\omega)^2 + (\sin\omega + \sin\omega\cos\omega)^2}$$

$$\Rightarrow \sqrt{\cos^2\omega + \cos^4\omega + 2\cos^3\omega + \sin^2\omega + \sin^2\omega\cos^2\omega + 2\sin^2\omega\cos\omega}$$

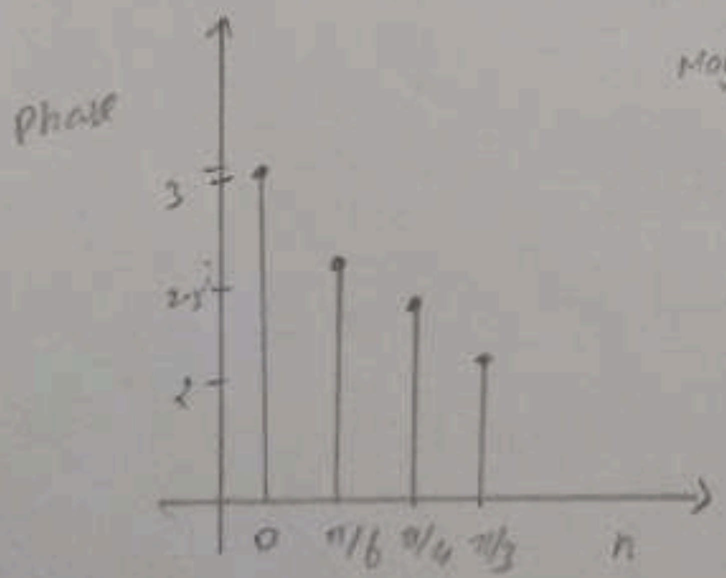
$$\Rightarrow \sqrt{1 + \cos^2\omega[\cos^2\omega + \sin^2\omega] + 2\cos\omega[\cos^2\omega + \sin^2\omega]}$$

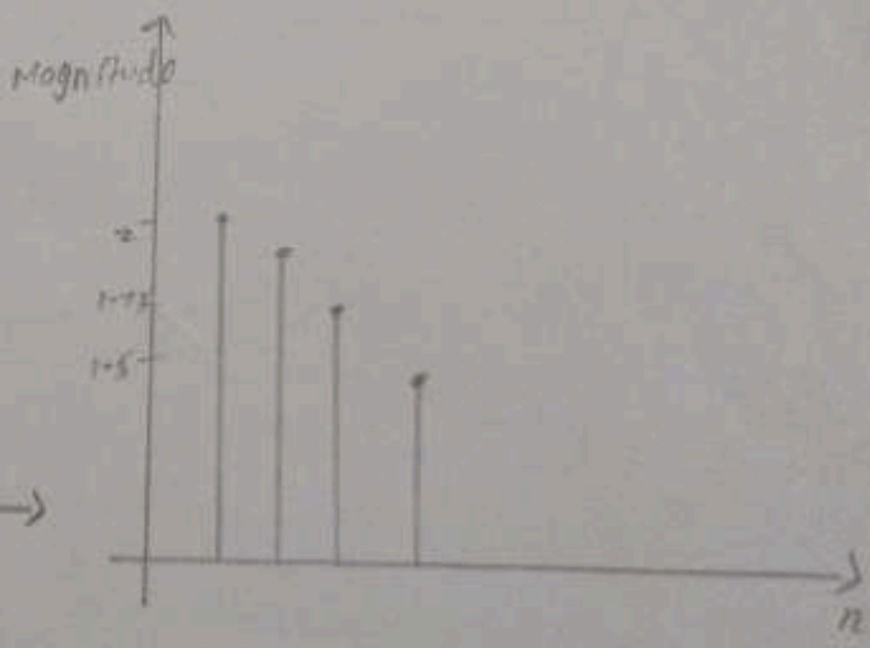
$$\Rightarrow \sqrt{1 + \cos^2\omega + 2\cos\omega}$$

$$\Rightarrow \sqrt{(\cos\omega + 1)^2} \Rightarrow \cos\omega + 1$$

$$|He^{j\omega}| = 1 + \cos\omega$$

n	0	$\pi/6$	$\pi/4$	$\pi/3$
Phase	3.14	2.62	2.355	2.09
Mag.	2	1.865	1.707	1.5





c) $y(n) - \frac{1}{4}y(n-1) - \frac{3}{8}y(n-2) = x(n) + x(n-1)$

$$Y(e^{j\omega}) - \frac{1}{4}e^{-j\omega}Y(e^{j\omega}) - \frac{3}{8}e^{-j\omega 2}Y(e^{j\omega}) = X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left[1 - \frac{e^{-j\omega}}{4} - \frac{3e^{-j\omega 2}}{8}\right] = X(e^{j\omega})\left[1 + e^{-j\omega}\right]$$

$$H(e^{j\omega}) = \frac{1 + e^{-j\omega}}{1 - \frac{e^{-j\omega}}{4} - \frac{3}{8}e^{-j\omega 2}}$$

Phase: $\tan^{-1}\left(\frac{Imag}{Num}\right)$ $\quad 1 + e^{-j\omega} = 1 + \cos\omega - j\sin\omega$

$$= \tan^{-1}\left(\frac{-\sin\omega}{1+\cos\omega}\right) = \tan^{-1}\left[\frac{-2\sin\omega/2\,\cos\omega/2}{\cancel{1} + 2\cos^2\omega/2 - \cancel{1}}\right]$$

$$= \tan^{-1}(-\tan\omega/2) = \tan^{-1}(\tan(\pi - \omega/2))$$

Phase $= \pi - \omega/2$

$$H(e^{j\omega}) = \frac{8(1+\cos\omega - j\sin\theta)}{8 - 2e^{-j\omega} - 3(e^{-j\omega})^2}$$

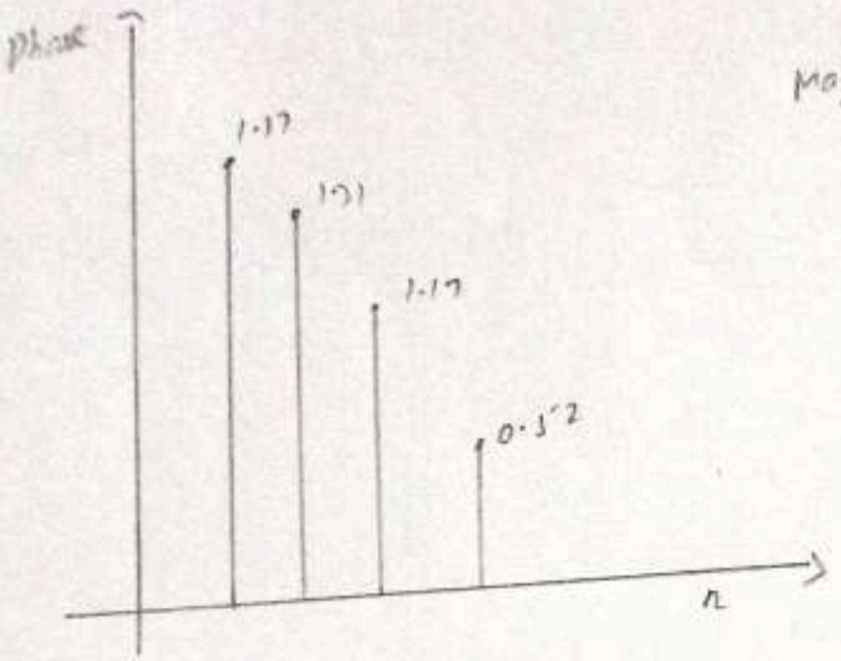
$$= \frac{8(1+\cos\omega - j\sin\omega)}{8 - 2\cos\omega - 2j\sin\omega - 3\cos 2\omega - 3j\sin 2\omega}$$

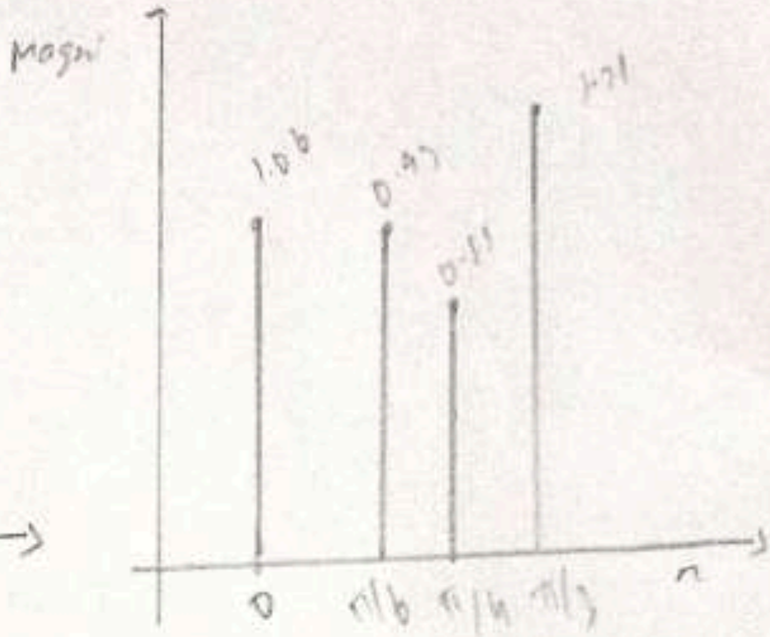
$$= \frac{8(1+\cos\omega - j\sin\omega)}{(8 - 2\cos\omega - 3\cos 2\omega) - j(2\sin\omega + 3\sin 2\omega)}$$

$$|H(e^{j\omega})| = \frac{8\sqrt{(1+\cos\omega)^2 + \sin^2\theta}}{\sqrt{(8-2\cos\omega - 3\cos 2\omega)^2 + (2\sin\omega + 3\sin 2\omega)^2}}$$

$$|H(e^{j\omega})| = \frac{8\sqrt{2}\sqrt{(1+\cos\omega)}}{\sqrt{(8-2\cos\omega-3\cos 2\omega)^2 + (2\sin\omega + 3\sin 2\omega)^2}}$$

n	0	π/6	π/4	π/3
phase	1.57	1.21	1.17	0.52
Magnitude	1.06	0.93	0.85	1.21





d) $y(n) - \frac{1}{2} y(n-1) = x(n)$

$$Y(e^{j\omega}) - \frac{1}{2} e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega})$$

$$\cancel{H(e)}\, Y(e^{j\omega}) \left(1 - \frac{1}{2} e^{-j\omega}\right) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}} = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

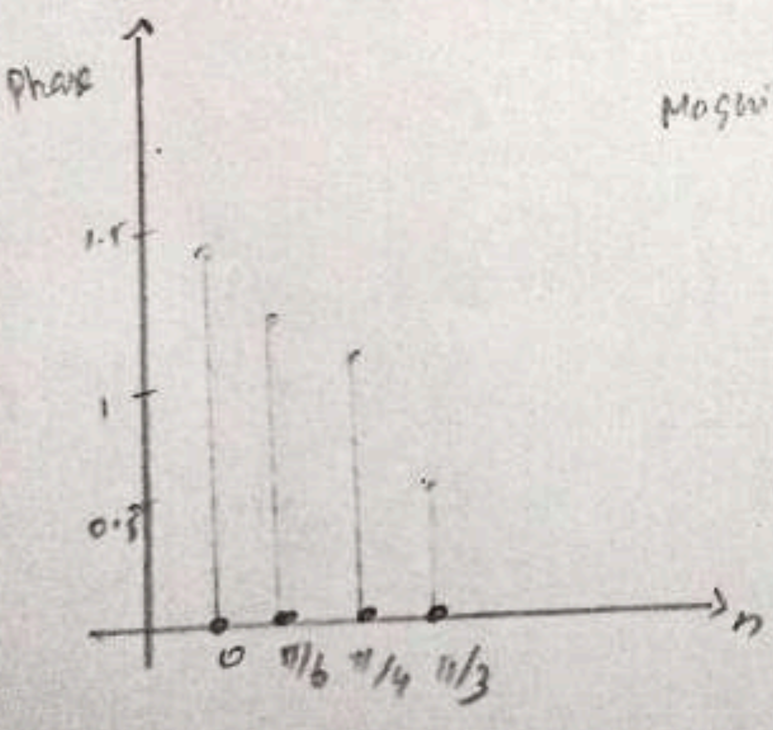
Phase: $\tan^{-1}\left(\frac{\text{Imag}}{\text{Real}}\right) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

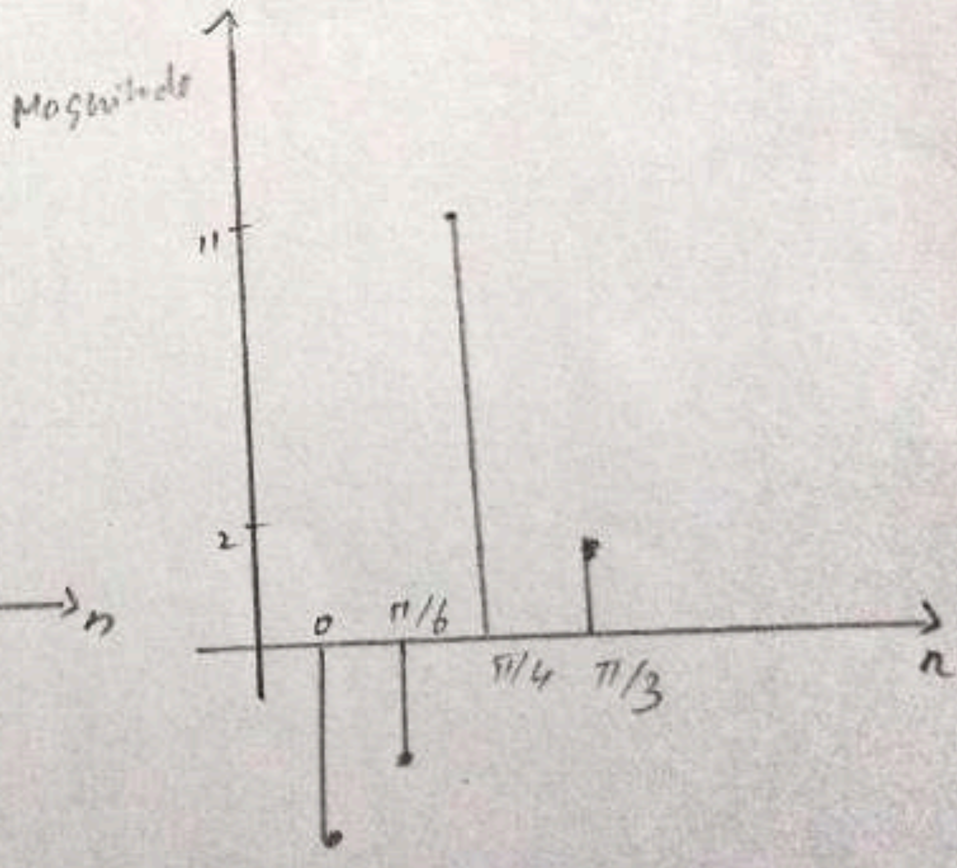
$$|H(e^{j\omega})| = \frac{2}{2 - e^{-j\omega}} = \frac{2}{2 - \cos\omega - j\sin\omega}$$

$$|H(e^{j\omega})| = \frac{2}{\sqrt{(2-\cos\omega)^2 + \sin^2\omega}} = \frac{2}{\sqrt{3 - 4\cos\omega}}$$

$$|H(e^{j\omega})| = \frac{2}{\sqrt{3 - 4\cos\omega}}$$

n	0	$\pi/6$	$\pi/4$	$\pi/3$
Phase	0 ~~[illegible]~~ 1.57	1.38 0	1.17 0	0.52 0
Magnitude	$-2j$	$-1.07j$	11.63	2





2. Find the 4 point DFT of the following sequences:

a) $x(n) = \{1, 0, -1, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{4}} = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{42}}$$

$$X(k) = x(0)e^{0} + x(1)e^{\frac{-j\pi k}{2}} + x(2)e^{-j\pi k} + x(3)e^{-j\frac{\pi k3}{2}}$$

$$X(k) = 1 - 1(e^{-j\pi k})$$

$$X(0) = 1 - 1 = 0$$

$$X(1) = 1 - e^{-j\pi} = 1 - [-1] = 2$$

$$X(2) = 1 - e^{-j2\pi} = 1 - [1-0] = 0$$

$$X(3) = 1 - e^{-j3\pi} = 1 - [-1-0] = 2$$

$$X(k) = \{0, 2, 0, 2\}$$

b) $x(n) = \{1, -2, 3, 4\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi kn}{2}}$$

$$X(k) = x(0) + e^{0} + x(1)e^{\frac{-j\pi k}{2}} + x(2)e^{-j\pi k} + x(3)e^{-j\frac{\pi k3}{2}}$$

$$X(k) = 1 - 2e^{\frac{-j\pi k}{2}} + 3e^{-j\pi k} + 4e^{-j\frac{3\pi k}{2}}$$

$$X(0) = 1 - 2 + 3 + 4 = 6$$

$$X(1) = 1 - 2e^{-j\pi/2} + 3e^{-j\pi k} + 4e^{-j\frac{3\pi}{2}} = 1 - 2[0-j] + 3[-1-0] + 4[0+j]$$

$$X(1) = 1 + 2j - 3 + 4j = -2 + 6j$$

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$X(2) = 1 - 2e^{-j\pi} + 3e^{-j2\pi} + 4e^{-j3\pi} = 1 - 2[-1-0] + 3[1-0] + 4[-1-0]$

$X(2) = 1 + 2 + 3 - 4 = 2$

$X(3) = 1 - 2e^{-\frac{j\pi 3}{2}} + 3e^{-j3\pi} + 4e^{-j\frac{9\pi}{2}}$

$X(3) = 1 - 2(0 + j) + 3[-1-0] + 4[0-j]$

$= [1 - 2j - 3 - 4j]$

$= -2 - 6j$

$X(k) = \{6, -2+6j, 2, -2-6j\}$

c) $x(n) = \sin\left(\frac{n\pi}{2}\right)$

$X(k) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi k n}{2}}$

$X(k) = x(0)e^{0} + x(1)e^{-\frac{j\pi k}{2}} + x(2)e^{-j\pi k} + x(3)e^{-j\frac{3\pi k}{2}}$

$X(k) = \cancel{\sin 0}^{\,0} + \sin\frac{\pi}{2}e^{-j\frac{\pi}{2}k} + \cancel{\sin\pi}^{\,0}\, e^{-j\pi k} + \sin\frac{3\pi}{2}e^{-j\frac{3\pi}{2}k}$

$\cancel{X(k) = -j \quad -1[0+j] = -2j}$

$X(k) = e^{-j\frac{\pi}{2}k} - e^{-j\frac{3\pi}{2}k}$

$X(0) = 1 - 1 = 0$

$X(1) = e^{-j\pi/2} - e^{-j3\pi/2}$

$= [0 - j] - [0 + j] = -2j$

$X(2) = e^{-j\pi} - e^{-j3\pi} = [-1-0] - [-1-0] = 0$

$X(3) = e^{-j\frac{3\pi}{2}} - e^{-j\frac{9\pi}{2}} = [0+j] - [0-j] = 2j$

$X(k) = \{0, -2j, 0, 2j\}$

d) $x(n) = 2^n$

$$X(k) = \sum_{n=0}^{3} x(n) e^{\frac{-j\pi kn}{2}}$$

$$X(k) = x(0) e^{0} + x(1) e^{\frac{-j\pi k}{2}} + x(2) e^{-j\pi k} + x(3) e^{-j\frac{3\pi k}{2}}$$

$$X(k) = 1 + 2 e^{\frac{-j\pi k}{2}} + 4 e^{-j\pi k} + 8 e^{-j\frac{3\pi k}{2}}$$

$X(0) = 1 + 2 + 4 + 8 = 15$

$X(1) = 1 + 2[0 - j] + 4[-1 + 0] + 8[0 + j]$

$= 1 - 2j - 4 + 8j$

$= -3 + 6j$

$X(2) = 1 + 2e^{-j\pi} + 4e^{-j2\pi} + 8e^{-j3\pi}$

$= 1 + 2[-1 - 0] + 4[1 - 0] + 8[-1 - 0]$

$= 1 - 2 + 4 - 8 = -5$

$X(3) = 1 + 2e^{-j\frac{3\pi k}{2}} + 4e^{-j3\pi} + 8e^{-j\frac{9\pi}{2}}$

$= 1 + 2[0 + j] + 4[-1 - 0] + 8[0 - j]$

$= 1 + 2j - 4 - 8j$

$= -3 - 6j$

$X(k) = \{15, -3+6j, -5, -3-6j\}$

3. Determine the IDFT of the following:

a) $X(k) = \{1, 1-j2, -1, 1+j2\}$

$$x(n) = \frac{1}{4} \sum_{n=0}^{3} X(k) e^{j\pi nk/2}$$

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$$x(n) = \frac{1}{4}\left[x(0)e^{-j\pi} + x(1)e^{-j\pi k/2} + x(2)e^{-j\pi k} + x(3)e^{-j3\pi k/2}\right]$$

$$x(n) = \frac{1}{4}\left[1 + (1-2j)e^{-j\pi k/2} - e^{-j\pi k} + (1+2j)e^{-j3\pi k/2}\right]$$

$$x(0) = \frac{1}{4}\left[1 + 1 - \cancel{2j} - \cancel{1} + \cancel{1} + \cancel{2j}\right]$$

$$x(0) = \frac{1}{2}$$

$$x(1) = \frac{1}{4}\left[1 + (1-2j)(-j) - 1[-1] + [1+2j][j]\right]$$

$$= \frac{1}{4}\left[1 - \cancel{j} - \cancel{2} + \cancel{1} + \cancel{j} - 2\right]$$

$$x(1) = -\frac{1}{2}$$

$$x(2) = \frac{1}{4}\left[1 + (1-2j)e^{-j\pi} - e^{-j2\pi} + (1+2j)e^{-j3\pi}\right]$$

$$= \frac{1}{4}\left[1 - 1 + \cancel{2j} - 1 - 1 - \cancel{2j}\right]$$

$$x(2) = -\frac{1}{2}$$

$$x(3) = \frac{1}{4}\left[1 + (1-2j)e^{-j3\pi/2} - e^{-j3\pi} + (1+2j)e^{-j9\pi/2}\right]$$

$$= \frac{1}{4}\left[1 + j + \cancel{2} + 1 + j - \cancel{2}\right]$$

$$= \frac{1}{4}\left[2 + 2j\right]$$

$$x(3) = \frac{1}{2} + j\frac{1}{2}$$

$$x(n) = \left\{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0.5 + 0.5j\right\}$$

b) $x(k) = \{1, -2-j, 0, -2+j\}$

$$x(n) = \frac{1}{4}\sum_{n=0}^{3} x(k)\, e^{j\pi n k/2}$$

$$x(n) = \frac{1}{4}\left[x(0)e^{0} + x(1)\, e^{j\pi k/2} + x(2)\, e^{j\pi k} + x(3)\, e^{j3\pi k/2}\right]$$

$$x(n) = \frac{1}{4}\left[1 + (-2-j)\, e^{j\pi k/2} + (-2+j)\, e^{j3\pi k/2}\right]$$

$$x(0) = \frac{1}{4}\left[1 - 2 - j - 2 + j\right] = \frac{-3}{4}$$

$$x(1) = \frac{1}{4}\left[1 + (-2-j)e^{j\pi/2} + (-2+j)\, e^{j3\pi/2}\right]$$

$$x(1) = \frac{1}{4}\left[1 - 2j - j^{2} + 2j - 2j^{2}\right]$$

$$= \frac{4-4j}{4} = 1-j$$

$$x(2) = \frac{1}{4}\left[1\ (-2-j)\, e^{j\pi}\ (-2+j)\, e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[1 + 2 + \cancel{j} + 2 - \cancel{j}\right] = \frac{5}{4}$$

$$x(3) = \frac{1}{4}\left[1 + (-2-j)\, e^{j3\pi/2}\ (-2+j)\, e^{j9\pi/2}\right]$$

$$x(3) = \frac{1}{4}\left[1 + \cancel{2} + j - \cancel{2} + j\right]\left[1 + 2\cancel{j} + j^{2} - 2\cancel{j} + j^{2}\right]$$

$$x(3) = \frac{-1}{4}$$

$$x(n) = \left\{\frac{-3}{4},\ 1-j,\ \frac{5}{4},\ \frac{-1}{4}\right\}$$

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5. Compute the following DFT of sequence

a) $x(n) = \{0, 2, 3, -1\}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nk}{N}} \qquad X(K) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi nk}{2}}$$

$$X(K) = x(0)e^{0} + x(1)e^{-\frac{j\pi K}{2}} + x(2)e^{-j\pi K} + x(3)\, e^{-\frac{j3\pi K}{2}}$$

$$X(K) = 2e^{-\frac{j\pi K}{2}} + 3e^{-j\pi K} - e^{-j\frac{3\pi}{2}K}$$

$$X(0) = 2+3-1 = 4 \qquad X(1) = 2e^{-\frac{j\pi}{2}} + 3e^{-j\pi} - e^{-\frac{j3\pi}{2}}$$

$$X(1) = 2[0-j] + 3[-1-0] - 1[0+j]$$

$$= -2j - 3 - j$$

$$X(1) = -3 - 3j$$

$$X(2) = 2e^{-j\pi} + 3e^{-j2\pi} - e^{-j\frac{3\pi}{1}} = 2[-1-0] + 3[1-0] - 1[-1]$$

$$X(2) = -2 + 3 + 1 = 2$$

$$X(3) = 2e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} - e^{-j\frac{9\pi}{2}}$$

$$= 2[j] + 3(-1-0) - [-j] = -3 + 3j$$

$$X(K) = \{4, -3-j, 2, -3+3j\}$$

b) $x(n) = \{1, 3, 3, 3\}$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}} = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi nK}{2}}$$

$$X(K) = x(0)e^{0} + x(1)e^{-\frac{j\pi K}{2}} + x(2)e^{-j\pi K} + x(3)e^{-j\frac{3\pi K}{2}}$$

$$X(K) = 1 + 3e^{-\frac{j\pi K}{2}} + 3e^{-j\pi K} + 3e^{-j\frac{3\pi K}{2}}$$

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$X(0) = 1+3+3+3 = 10$

$X(1) = 1 + 3e^{-j\frac{\pi}{2}} + 3e^{-j\pi} + 3e^{-j3\pi/2}$

$= 1 + 3[0 - j] + 3[-1 - 0] + 3[0 + j]$

$= 1 - \cancel{3j} - 3 + \cancel{3j} = -2$

$X(2) = 1 + 3e^{-j\pi} + 3e^{-j2\pi} + 3e^{-j3\pi}$

$= 1 + [3][-1-0] + 3[1-0] + 3[-1-0]$

$= 1 - \cancel{3} + \cancel{3} - 3 = -2$

$X(3) = 1 + 3e^{-j3\pi/2} + 3e^{-j3\pi} + 3e^{-j9\pi/2}$

$= 1 + 3[j] + 3[-1-0] + 3[-j]$

$X(3) = -2$

$X(k) = \{10, -2, -2, -2\}$

4. Compute 4 point DFT and 8 point DFT of causal sequence given by $x(n) =$ [illegible]^{n}, $0 \le n \le 3$ | 0 elsewhere

$x(n) = \{0, 1, 2, 3, 0, 0, 0, 0\}$

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{7} x(n) e^{-j\frac{\pi kn}{4}}$

$X(k) = x(0)e^{0} + x(1)e^{-j\frac{\pi k}{4}} + x(2)e^{-j\frac{\pi k}{2}} + x(3)e^{-j\frac{3\pi k}{4}}$
$+ x(4)e^{-j\pi k} + x(5)e^{-j\frac{5\pi k}{4}} + x(6)e^{-j\frac{3\pi k}{2}} + x(7)e^{-j\frac{7\pi k}{4}}$

$X(k) = e^{-j\frac{\pi k}{4}} + 2e^{-j\frac{\pi k}{2}} + 3e^{-j\frac{3\pi k}{4}}$

$$x(0) = 1+2+3 = 6$$

$$x(1) = e^{-j\frac{\pi}{4}} + 2e^{-j\pi/2} + 3e^{-j\frac{3\pi}{4}}$$

$$= [0.707 - 0.707j] + 2[0 - j] + 3[-0.707 - 0.707j]$$

$$= -1.414 - 4.828j$$

$$x(2) = e^{-j\pi/2} + 2e^{-j\pi} + 3e^{-j3\pi/2}$$

$$= [0 - j] + 2[-1 - 0] + 3[j]$$

$$= -2 + 2j$$

$$x(3) = e^{-j\frac{3\pi}{4}} + 2e^{-j3\pi/2} + 3e^{-j9\pi/2}$$

$$= [-0.707 - 0.707j] + 2[j] + 3[-j]$$

$$= -0.707 - 1.707j$$

$$x(4) = e^{-j\pi} + 2e^{-j2\pi} + 3e^{-j3\pi}$$

$$= -1 + 2 - 3$$

$$x(4) = -2$$

$$x(5) = e^{-j\frac{5\pi}{4}} + 2e^{-j5\pi/2} + 3e^{-j15\pi/4}$$

$$= -0.707 - 0.707j - 2j + 2.12 - 2.12j$$

$$= -1.414 + 4.828j$$

$$x(6) = e^{-j3\pi/2} + 2e^{-j3\pi} + 3e^{-j9\pi/2}$$

$$= (j) - 2 - 3j$$

$$= -2 - 2j$$

$$x(7) = e^{-j\frac{7\pi}{4}} + 2e^{-j7\pi/2} + 3e^{-j21\pi/4}$$

$$= 0.707 - 0.707j + 2j + 3(-0.707 + 0.707j)$$

$$= 1.414 + 3.414j$$

$$X(k) = \{6, -1.414 - 4.828j, -2 + 2j, -0.707 - 1.707j, -2, -1.414 + 4.828j, -2 - 2j, 1.414 + 3.414j\}$$

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$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{\pi k n}{2}}$$

$$X(k) = x(0)e^{0} + x(1)\, e^{-j\frac{\pi}{2}k} + x(2)\, e^{-j\pi k} + x(3)\, e^{-j\frac{3\pi k}{2}}$$

$$X(k) = e^{-j\pi/2 k} + 2e^{-j\pi k} + 3e^{-j3\pi/2 k}$$

$$X(0) = 1+2+3 = 6$$

$$X(1) = [0-j] + 2[-1-0] + 3[j] = -2+2j$$

$$X(2) = e^{-j\pi} + 2e^{-j2\pi} + 3e^{-j3\pi} = -1 + 2 - 1 = 0$$

$$X(3) = e^{-j3\pi/2} + 2e^{-j3\pi} + 3e^{-j9\pi/2} = [j - 2 - 3j] = -2-2j$$

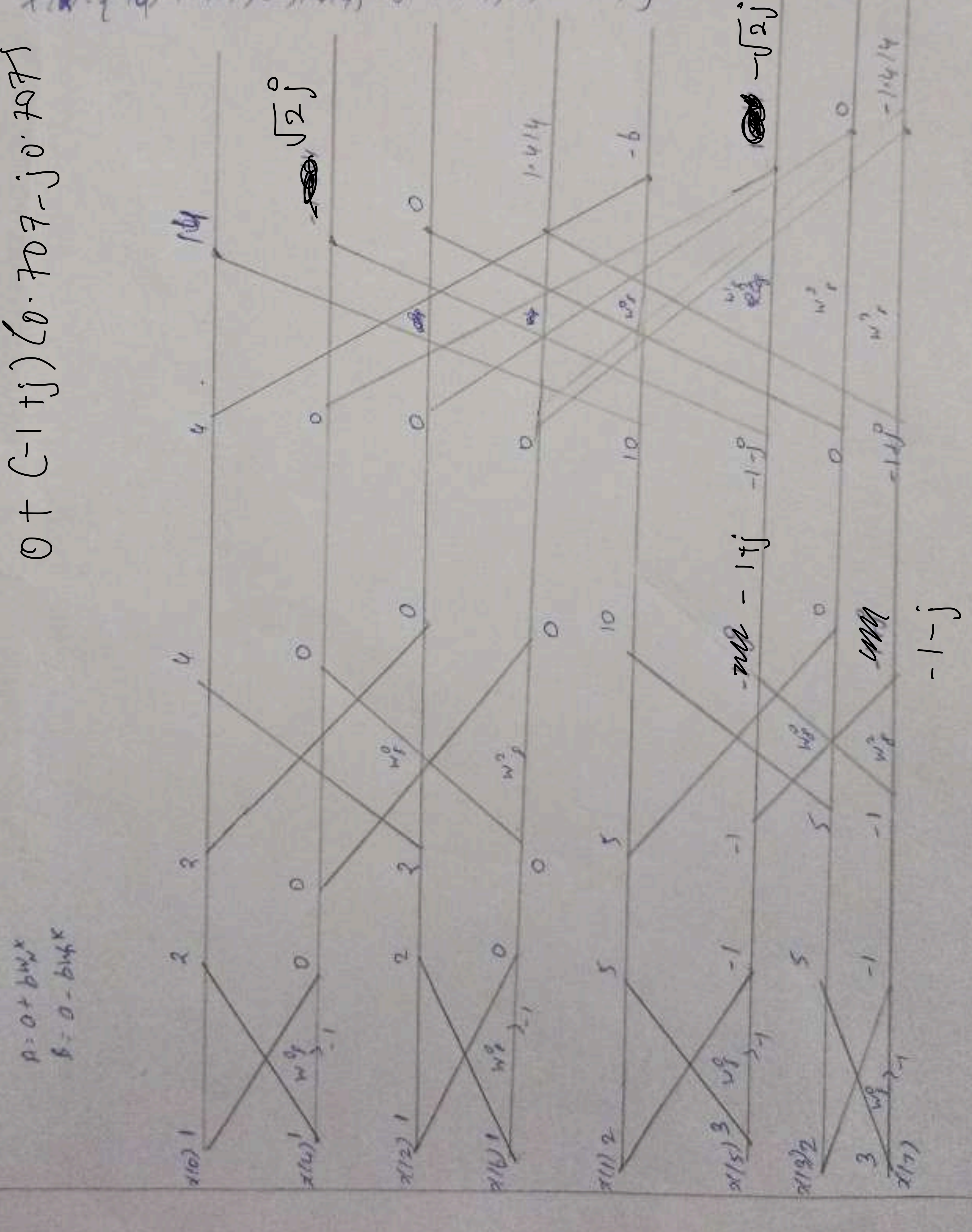
$$X(k) = \{6, -2+2j, 0, -2-2j\}$$

6. compute 8 point DFT of DT $x(n) = \{1, 2, 1, 2, 1, 3, 1, 3\}$ using

a) DIT b) DIF

$W_8^0 = 1$ $W_8^1 = 0.707 - j0.707$ $W_8^2 = -j$ $W_8^3 = -0.707 - j0.707$

$X(k) = \{14, -1.414, 0, 1.414, -6, 1.414, 0, -1.414\}$

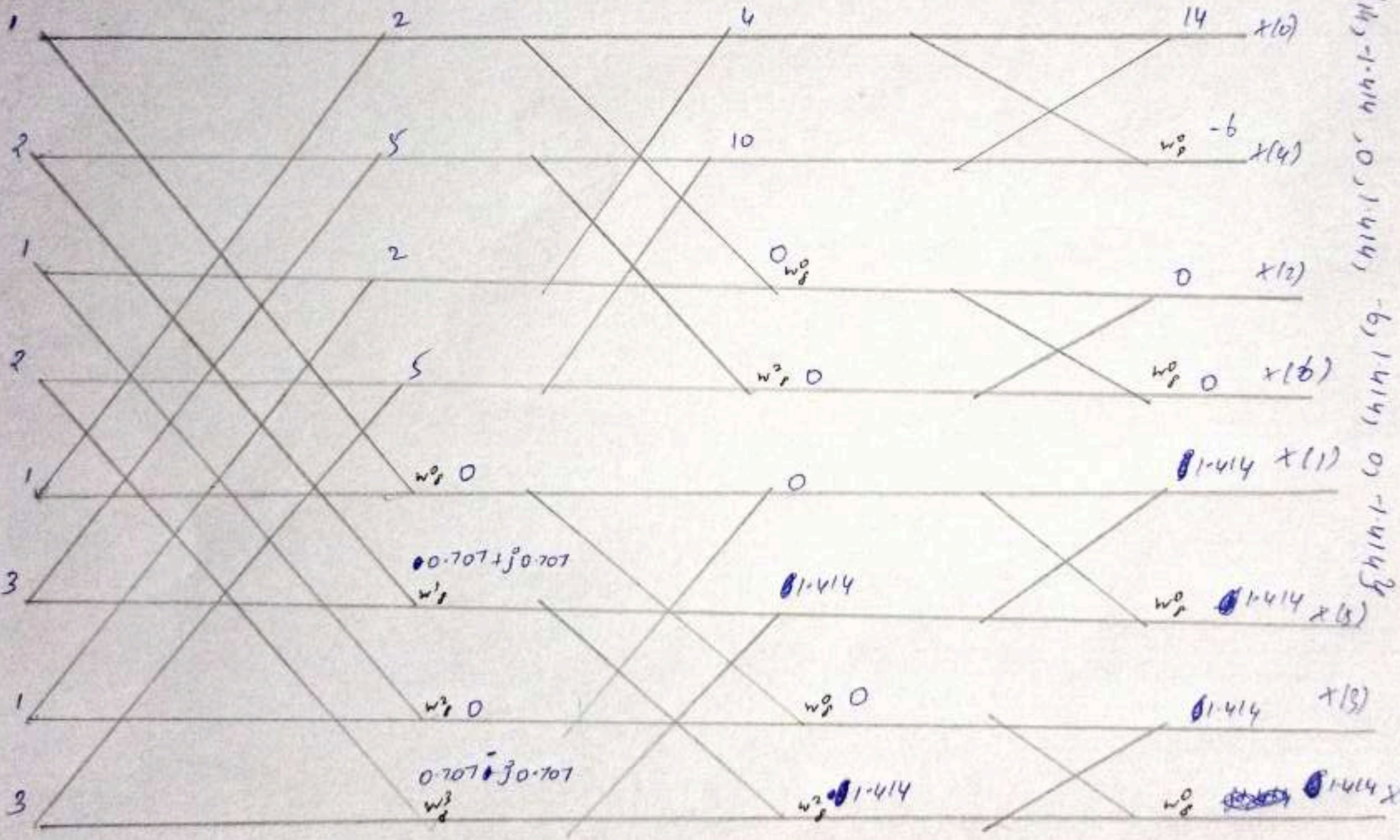


DIF

$X(k) = \{14, -1.414, 0, 1.414, -6, 1.414, 0, -1.414\}$

$A = a + b$

$B = (a-b)W_N^k$

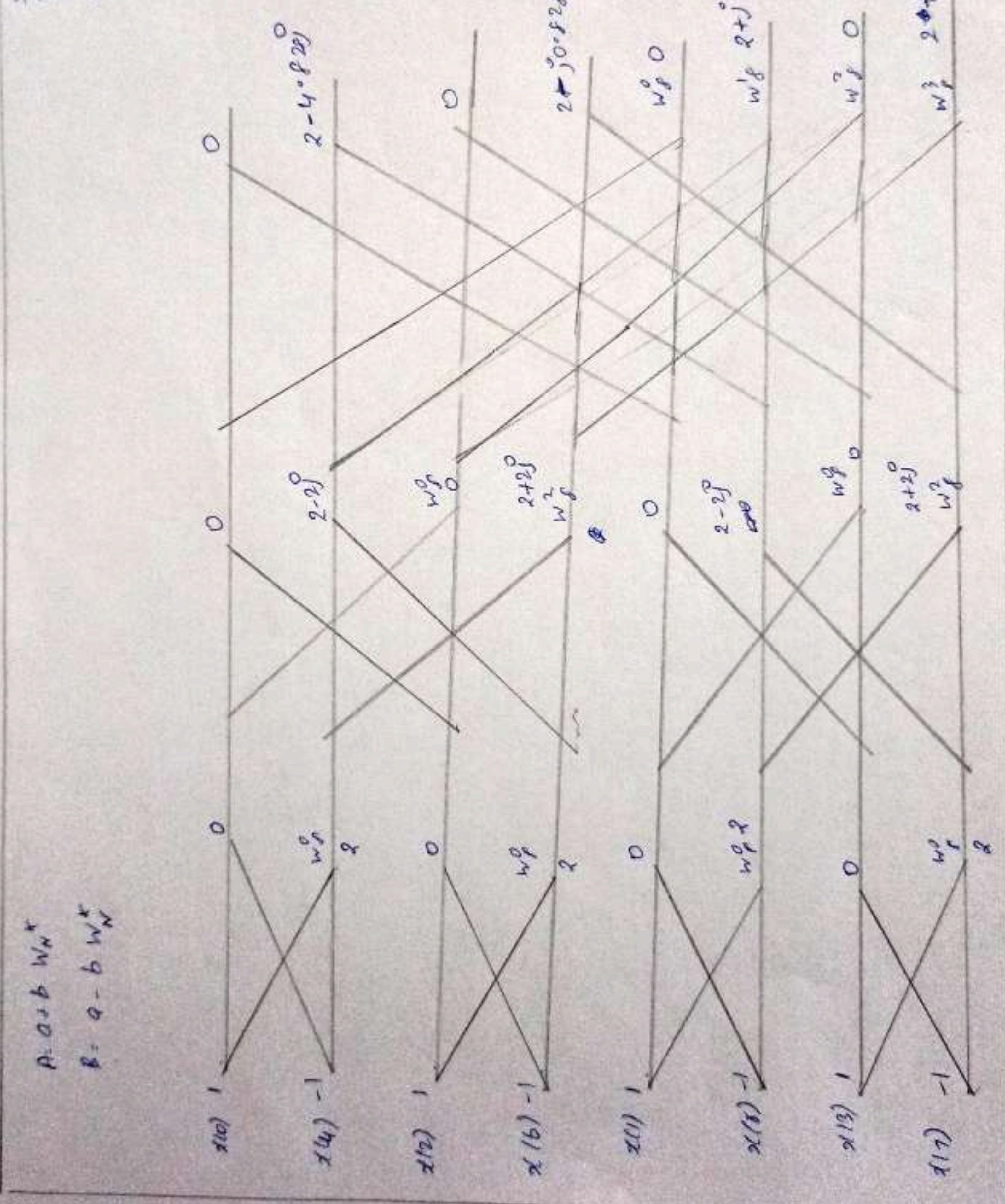


7. Compute 8 point DFT of discrete time

$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$ a) DIT b) DIF

$W_8^0 = 1$ $W_8^1 = 0.707 - j0.707$ $W_8^2 = -j$ $W_8^3 = -0.707 - j0.707$

$X(k) = \{0, 2 - j4.828, 0, 2 + j0.828, 0, 2 + j0.828, 0, 2 + j4.828\}$

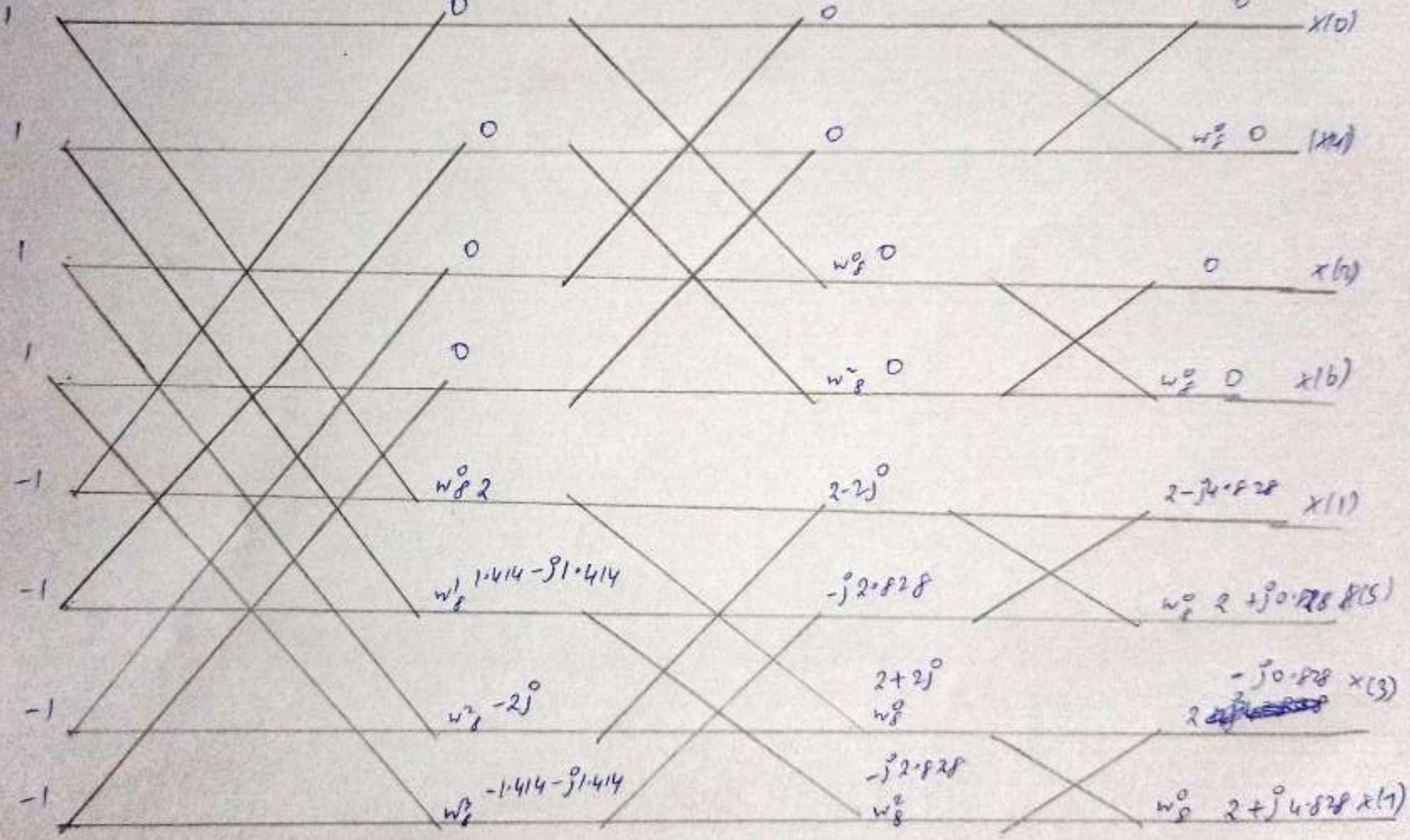


DIF

A: a + b

B: $(a-b)\,W_8^{k}$

$X(k) = \{0,\ 2-j4.828,\ 0,\ 2-j0.828,\ 0,\ 2+j0.828,\ 0,\ 2+j4.828\}$



8. Find 8pt DFT of sequence

$x(n) = \{0.707, 1, 0.707, 0, -0.707, -1, -0.707, 0\}$

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} = \sum_{n=0}^{7} x(n) e^{-j\pi kn/4}$

$X(k) = 0.707 + e^{-j\pi k/4} + 0.707 e^{-j\pi k/2} - 0.707 e^{-j\pi k} - e^{-j5\pi k/4} - 0.707 e^{-j3\pi k/2}$

$X(0) = 0.707 + 0.707 + 1 - 1 - 0.707 - 0.707 = 0$

$X(1) = 0.707 + 0.707 - 0.707j - 0.707j + 0.707 - 0.707 + 0.707j - 0.707j$

$X(1) = 1.414 - 1.414j$

$X(2) = 0.707 - j - 0.707 - 0.707 + j + 0.707 = 0$

$X(3) = 0.707 - 0.707 - 0.707j + 0.707j + 0.707 - 0.707 + 0.707j + 0.707j$

$X(3) = 1.414j$

$X(4) = 0.707 + e^{-j\pi} + 0.707 e^{-j2\pi} - 0.707 e^{-j4\pi} - e^{-j5\pi} - 0.707 e^{-6\pi}$

$X(4) = 0.707 - 1 + 0.707 - 0.707 + 0.707 - 0.707$

$X(4) = -0.293$

$X(5) = 0.707 + e^{-j5\pi/4} + 0.707 e^{-j5\pi/2} - 0.707 e^{-j5\pi} - e^{-j25\pi/4} - 0.707 e^{-j15\pi/2}$

$X(5) = 0.707 - 0.707 + 0.707j - 0.707j + 0.707 - 0.707 + 0.707j - 0.707j$

$X(5) = 0$

$X(6) = 0.707 + e^{-j3\pi/2} + 0.707 e^{-j3\pi} - 0.707 e^{-j6\pi} - e^{-j15\pi/2} - 0.707 e^{-j9\pi}$

$X(6) = 0.707 - j - 0.707 - 0.707 - j + 0.707$

$X(6) = -2j$

$X(7) = 0.707 + e^{-j7\pi/4} + 0.707 e^{-j7\pi/2} - 0.707 e^{-j7\pi} - e^{-j35\pi/4} - 0.707 e^{-j21\pi/2}$

$x(7) = 0.707 + 0.707 - 0.707j + 0.707j + 0.707 + 0.707 + 0.707j + 0.707j$

$x(7) = 2.121 + 2.121j$

$x(k) = \{0, 1.414 - 1.414j, 0, 1.414j, -0.293, 0, -2j, 2.121 + 2.121j\}$

9. Find IDFT of the sequence $x(k)$ given below

$$x(k) = \{20, -5.828 - j2.414, 0, -0.172 - j0.414, 0, -0.172 + 0.414, 0, -5.828 + j2.414\}$$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(k)\, e^{j\frac{2\pi nk}{N}} = \frac{1}{8} \sum_{n=0}^{7} x(k)\, e^{j\frac{2\pi nk}{4}}$$

$$x(n) = \frac{1}{8} \left[20 - (5.828 + j2.414) e^{j\frac{\pi k}{4}} - (0.172 + j0.414)\left(e^{j\frac{3\pi k}{4}}\right) - (0.172 - j0.414) e^{j\frac{5\pi k}{4}} - (5.828 - j2.414) e^{j\frac{7\pi}{4}k} \right]$$

$$x(0) = \frac{1}{8} \left[20 - 5.828 - j2.414 - 0.172 - j0.414 - 0.172 + j0.414 - 5.828 + j2.414 \right)$$

$x(0) = 1$

$$x(1) = \frac{1}{8} \left[20 - (5.828 + j2.414) e^{j\pi/4} - (0.172 + j0.414) e^{j\frac{3\pi}{4}} - (0.172 - j0.414) e^{j\frac{5\pi}{4}} - (5.828 - j2.414) e^{j\frac{7\pi}{4}}) \right]$$

$$x(1) = \frac{1}{8} \left[20 - (5.828 + j2.414)(0.707 + j0.707) - (0.172 + j0.414)(-0.707 + 0.707j) - (0.172 - j0.414)(-0.707 - 0.707j) - (5.828 - j2.414)(0.707 - 0.707j) \right]$$

$x(1) = 1.91$

$$x(2) = \frac{1}{8} \left[20 - (5.828 + j2.414) e^{j\pi/2} - (0.172 + j0.414) e^{j3\pi/2} - (0.172 - j0.414) e^{j5\pi/2} - (5.828 - j2.414) e^{j\frac{7\pi}{2}} \right]$$

$x(2) = 2.96$

$$x(3) = \frac{1}{8}\left[20 - (5.828 + j2.414)(e^{j\frac{\pi 3}{4}}) - (0.172 + j0.414)(e^{j\frac{9\pi}{4}}) - (0.172 - j0.414)e^{j\frac{15\pi}{4}} - (5.828 - j2.414)e^{j\frac{21\pi}{4}}\right]$$

$x(3) = 3.92$

$$x(4) = \frac{1}{8}\left[20 - (5.828 + j2.414)e^{j\pi} - (0.172 + j0.414)e^{j3\pi} - (0.172 - j0.414)e^{j5\pi} - (5.828 - j2.414)e^{j7\pi}\right]$$

$x(4) = 4.13$

$$x(5) = \frac{1}{8}\left[20 - (5.828 + j2.414)e^{j5\pi/4} - (0.172 + j0.414)e^{j\frac{15\pi}{4}} - (0.172 - j0.414)e^{j\frac{25\pi}{4}} - (5.828 - j2.414)e^{j\frac{35\pi}{4}}\right]$$

$x(5) = 2.92$

$$x(6) = \frac{1}{8}\left[20 - (5.828 + j2.414)e^{j3\pi/2} - (0.172 + j0.414)e^{j9\pi/2} - (0.172 - j0.414)e^{j15\pi/2} - (5.828 - j2.414)e^{j21\pi/2}\right]$$

$x(6) = 1.98$

$$x(7) = \frac{1}{8}\left[20 - (5.828 + j2.414)e^{j\frac{7\pi}{4}} - (0.172 + j0.414)e^{j\frac{21\pi}{4}} - (0.172 - j0.414)e^{j\frac{35\pi}{4}} - (5.828 - j2.414)e^{-j\frac{49\pi}{4}}\right]$$

$x(7) = 1.23$

$$x(n) = \{\underset{\uparrow}{1}, 1.97, 2.96, 3.92, 4.13, 2.92, 1.98, 1.23\}$$

10. Find IDFT of sequence $x(k)$ given below:

$x(k) = \{1, 0, 0, j, 0, -j, 0, 0\}$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(k) e^{j\frac{2\pi n k}{N}} = \frac{1}{8} \sum_{n=0}^{7} x(k) e^{j\frac{\pi n k}{4}}$$

$x(n) = 1 + j e^{j\frac{3\pi k}{4}} - j e^{j\frac{5\pi k}{4}}$

$x(0) = 1 + j - j = 1/8$

$x(1) = 1 + j e^{j3\pi/4} - j e^{j5\pi/4} = 1 + 0.707j - 0.707 + 0.707j - 0.707$

$x(1) = 1 + j(0.707 + 0.707j) - j(-0.707 - 0.707j)$

$x(1) = 1 + 0.707j - 0.707 + 0.707j - 0.707$

$x(1) = -0.414 + 1.414j / 8$

$x(2) = 1 + je^{j3\pi/2} - je^{j5\pi/2} = 1 - j^2 - j^2$

$= 1 + 1 + 1 = 3/8$

$x(3) = 1 + je^{+j9\pi/4} - je^{j15\pi/4}$

$= 1 + j(0.707 + 0.707j) - j(0.707 - 0.707j)$

$= 1 + j\cancel{0.707} - 0.707 - j\cancel{0.707} - 0.707$

$= -0.414 / 8$

$x(n) = \{1, -0.414 + 1.414j, 3, -0.414 \ldots$

$x(4) = 1 + je^{j3\pi} - je^{j5\pi} = 1 - j + j = 1/8$

$x(5) = 1 + je^{j15\pi/4} - je^{j25\pi/4} = 1 + 0.707j + 0.707 - 0.707j + 0.707$

$x(5) = 2.414/8$

$x(6) = 1 + je^{j9\pi/2} - je^{j15\pi/2} = 1 + j + j = 1 + 2j$

$x(7) = 1 + je^{j21\pi/4} - je^{j35\pi/4} = 1 + j(0.707 - j0.707)$

$= 1 - j0.707 - j^2 0.707 + j0.707 - 0.707j^2$

$x(7) = 2.414/8 = 0.3$

$x(n) = \{0.125, -0.05 + 0.176j, 0.375, -0.05, 0.125, 0.30, 0.125 + 0.25j, 0.3\}$

11. Find the circular (7-point) and linear convolution of the sequence $x(n) = \{1, 2, 7, -2, 3, -1, 5\}$ $h(n) = \{-1, 3, 5, -3, 1\}$

$$\begin{bmatrix} -1 & 0 & 0 & 1 & -3 & 5 & 3 \\ 3 & -1 & 0 & 0 & 1 & -3 & 5 \\ 5 & 3 & -1 & 0 & 0 & 1 & -3 \\ -3 & 5 & 3 & -1 & 0 & 0 & 1 \\ 1 & -3 & 5 & 3 & -1 & 0 & 0 \\ 0 & 1 & -3 & 5 & 3 & -1 & 0 \\ 0 & 0 & 1 & -3 & 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 7 \\ -2 \\ 3 \\ -1 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 32 \\ -12 \\ 35 \\ 21 \\ -19 \\ 10 \end{bmatrix}$$

$x(0) = -1 - 2 - 9 - 5 + 15 = -2$

$x(1) = 3 - 2 + 3 + 3 + 25 = 32$

$x(2) = 5 + 6 - 7 - 1 - 15 = \cancel{28}\ -12$

$x(3) = -3 + 10 + 21 + 2 + 3 = 35$

$x(4) = 1 - 6 + 35 - 6 - 3 = 21$

$x(5) = 2 - 21 - 10 + 9 + 1 = -19$

$x(6) = 7 + 6 + 5 - 3 - 5 = 10$

$x(n) = \{-2, 32, -12, 35, 21, -19, 10\}$

Linear convolution

$x(n) = \{1, 2, 7, -2, 3, -1, 5\}$

$h(n) = \{-1, 3, 5, -3, 1\}$

$h(n)$ \ $x(n)$	1	2	7	-2	3	-1	5
-1	-1	-2	-7	2	-3	1	-5
3	3	6	21	-6	9	-3	15
5	5	10	35	-10	15	-5	25
-3	-3	-6	-21	6	-9	3	-15
1	1	2	7	-2	3	-1	5

$y(0) = -1$ $y(1) = 3-2 = 1$ $y(2) = 5+6-7 = 4$ $y(3) = -3+10+21+2 = 30$

$y(4) = 1-6+35-6-3 = 21$ $y(5) = 2 \cancel{} -21 -10 - 9 + 1 = -19$

$y(6) = 7+6+15-3-5 = 20$ $y(7) = -2-9-5+15 = -1$ $y(8) = 3+3+25 = 31$

$y(9) = -16$ $y(10) = 5$

$y(n) = \{-1, 1, 4, 30, 21, -19, 20, -1, 31, -16, 5\}$

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