

2. For the given specification, find the order of BW Filter.

$$\alpha_p = 3 \text{ dB}, \alpha_s = 18 \text{ dB}, f_p = 1 \text{ kHz}, f_s = 2 \text{ kHz}$$

$$f_p = 1 \text{ kHz} \quad f_s = 2 \text{ kHz} \quad \alpha_p = 3 \text{ dB} \quad \alpha_s = 18 \text{ dB}$$

$$\omega_p = 2\pi \times 10^3 = 2000\pi \text{ rad/sec}$$

$$\omega_s = 2\pi \times 2 \times 10^3 = 4000\pi \text{ rad/sec}$$

$$N \geq \frac{\log A}{\log 1/k}$$

$$A = \left[\frac{(10^{0.1 \times 18} - 1)}{(10^{0.1 \times 3} - 1)} \right]^{1/2} = \left(\frac{10^{0.1 \times 18} - 1}{10^{0.1 \times 3} - 1} \right)^{1/2} = \left[\frac{62.0957}{0.9952} \right]^{1/2}$$

$$A = 7.899$$

$$k = \frac{\omega_p}{\omega_s} = \frac{2000\pi}{4000\pi} = 0.5$$

$$k = 0.5$$

$$1/k = 2$$

$$N \geq \frac{\log A}{\log 1/k} = \frac{\log 7.899}{\log 2} = 2.98$$

$$N = 3$$

1. Design an analog BW filter: $\alpha_p = 0.5 \text{ dB}$, $\alpha_s = 22 \text{ dB}$
 $f_p = 10 \text{ kHz}$, $f_s = 25 \text{ kHz}$.
5. Find the pole locations of the 6th order B.W filter.
 with $\omega_c = 1 \text{ rad/sec}$.

$$\alpha_p = 0.5 \text{ dB}$$

$$\alpha_s = 22 \text{ dB}$$

$$\omega_p = 2\pi \times 10 \times 10^3 = 20000\pi$$

$$\omega_s = 2\pi \times 25 \times 10^3 = 50000\pi$$

$$N \geq \frac{\log A}{\log 1/k}$$

$$A = \left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1} \right)^{1/2} = \left(\frac{10^{0.1 \times 22} - 1}{10^{0.1 \times 0.5} - 1} \right)^{1/2}$$

$$= \left(\frac{157.49}{0.122} \right)^{1/2}$$

$$A = 35.93$$

$$k = \frac{\omega_p}{\omega_s} = \frac{20000\pi}{50000\pi} = \frac{2}{5} = 0.4 \text{ (for } 1 = \omega_c)$$

$$\frac{1}{k} = 2.5$$

$$N \geq \frac{\log A}{\log (1/k)} \geq \frac{\log 35.93}{\log 2.5}$$

$$N \geq 3.91$$

$$N = 4$$

$$H(s) = \frac{1}{(s^2 + 0.765378s + 1)(s^2 + 1.84778s + 1)}$$

$$\omega_c = \frac{\omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{20000\pi}{(10^{0.1 \times 0.5} - 1)^{1/8}} = \frac{20000\pi}{0.7688}$$

$$= 81729.09$$

$$= 81,729.09 \text{ rad/sec}$$

$$H_a(s) = H(s) \Big|_{s \rightarrow \frac{s}{81729.09}}$$

$$= \frac{1}{\left[\left(\frac{s}{81729.09} \right)^2 + \left(\frac{0.765375}{81729.09} \right) + 1 \right] \left[\left(\frac{s}{81729.09} \right)^2 + \left(\frac{1.84718}{81729.09} \right) + 1 \right]}$$

$$= \frac{1}{\left[\frac{s^2}{6.68 \times 10^9} + (9.36 \times 10^{-6})s + 1 \right] \left[\frac{s^2}{6.68 \times 10^9} + (2.26 \times 10^{-5}) + 1 \right]}$$

$$= \frac{(6.68 \times 10^9)^2}{\left[s^2 + (62.52 \times 10^3)s + (6.68 \times 10^9) \right] \left[s^2 + (15.1 \times 10^4)s + (6.68 \times 10^9) \right]}$$

$$H_a(s) = \frac{44.6224 \times 10^{18}}{\left[s^2 + (62.52 \times 10^3)s + (6.68 \times 10^9) \right] \left[s^2 + (15.1 \times 10^4)s + (6.68 \times 10^9) \right]}$$

5. $N = 6$

$$\omega_c = 1 \text{ rad/sec}$$

$$N \rightarrow \text{even} \therefore s_k = \omega_c e^{j \left[\frac{(2k-1)\pi}{2N} + \frac{\pi}{2} \right]}$$

$$s_k = e^{j \left[\frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \right]}$$

$$k = 1, 2, \dots, 6$$

$$s_1 = e^{j \left[\frac{\pi}{2} + \frac{\pi}{12} \right]} = e^{j \frac{7\pi}{12}}$$

$$= -0.2588 + j0.9659$$

$$s_2 = e^{j \left[\frac{\pi}{2} + \frac{3\pi}{12} \right]} = e^{j \frac{9\pi}{12}}$$

$$= -0.707 + j0.707$$

$$s_3 = e^{j \left[\frac{\pi}{2} + \frac{5\pi}{12} \right]} = e^{j \frac{11\pi}{12}}$$

$$= -0.9659 + j0.2588$$

$$s_4 = e^{j \left[\frac{\pi}{2} + \frac{7\pi}{12} \right]} = e^{j \frac{13\pi}{12}}$$

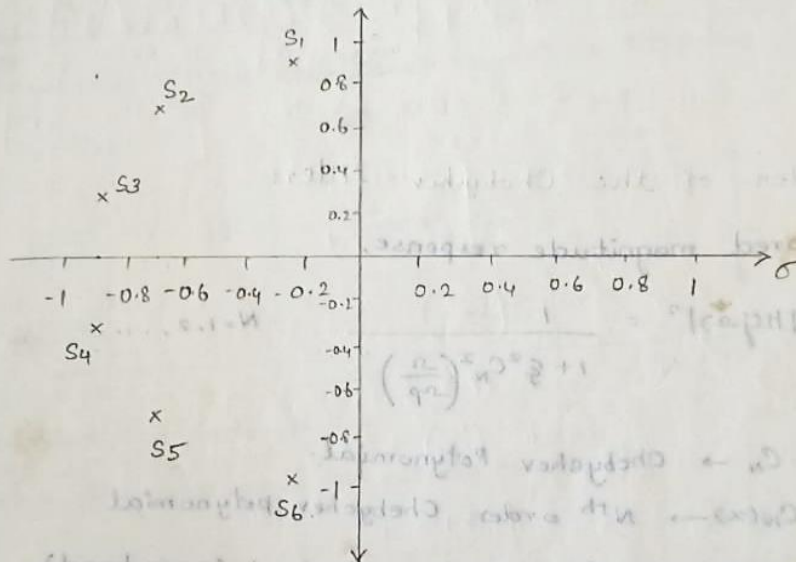
$$= -0.9659 - j0.2588$$

$$s_5 = e^{j[\frac{\pi}{2} + \frac{9\pi}{12}]} = e^{j\frac{15\pi}{12}}$$

$$= -0.707 - j0.707$$

$$s_6 = e^{j[\frac{\pi}{2} + \frac{11\pi}{12}]} = e^{j\frac{17\pi}{12}}$$

$$= -0.2588 - j0.9659$$



All the 6 poles lie on the left half of the s-plane.