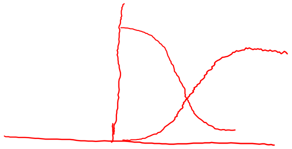


Chebyshev Filter

- Chebyshev–I filter is equiripple in the passband and monotonic in the stopband.

- Chebyshev-II filter is equiripple in the stopband and monotonic in the passband.

- In order to understand the frequency-domain behavior of Chebyshev filters, it is utmost important to define a Chebyshev polynomial and then its properties

- A Chebyshev polynomial of degree N is defined as

$$T_N(x) = \begin{cases} \cos(N\, cos^{-1}x), & |x| \le 1 \\ \cosh(N\, cosh^{-1}x), & |x| > 1 \end{cases} \qquad (1)$$

Also, it is possible to generate Chebyshev polynomials using the following recursive formula.

$$T_N(x) = 2xT_{N-1}(x) - T_{N-2}(x), \qquad N \ge 2 \quad (2)$$
$$T_0(x) = 1 \; and \; T_1(x) = x \qquad (3)$$

Derivative part of equation (2) and (3) will be given in notes

N	$T_N(x)$
0	1
1	x
2	$2x^2$-1
3	$4x^3 - 3x$
4	$8x^4 - 8x^2 + 1$
5	$16x^5 - 20x^3 + 5x$

The first five Chebyshev polynomials

Some of the important properties of the Chebyshev polynomials are follows:

1. For $|x| \leq 1, |T_N(x)| \leq 1$, and it oscillates between -1 and +1 a number of times proportional to N. This evident from equation (1)
2. For $|x| > 1, |T_N(x)| > 1$, and it is monotonically increasing in $|x|$. This is also evident from equation (1)

3. Chebyshev polynomial of odd orders are odd function of x (i.e., they contain only odd powers of x.

4. $T_N(0) = \pm 1, for\ even\ N, and\ T_N(0) = 0$ for odd N.

5. $T_N(-x) = (-1)^N T_N(x)$.

6. $|T_N(\pm 1)| = 1\ for\ all\ N$.

The oscillatory property of Chebyshev polynomials-being oscillatory in the range $|x| \leq 1$ and monotonic outside it – is utilized for constructing Chebyshev filters that are equiripple in either passband or the stopband.

Hence the magnitude response oscillates between the permitted minimum and maximum value in the band a number of times depending upon the order of the filter.

- The equiripple property yields a narrower transition band compared with that obtained when the magnitude response is monotone.

- As a consequence of this, the order of a Chebyshev filter needed to achieve the given frequency-domain specifications is usually lower than that of a Butterworth filter.

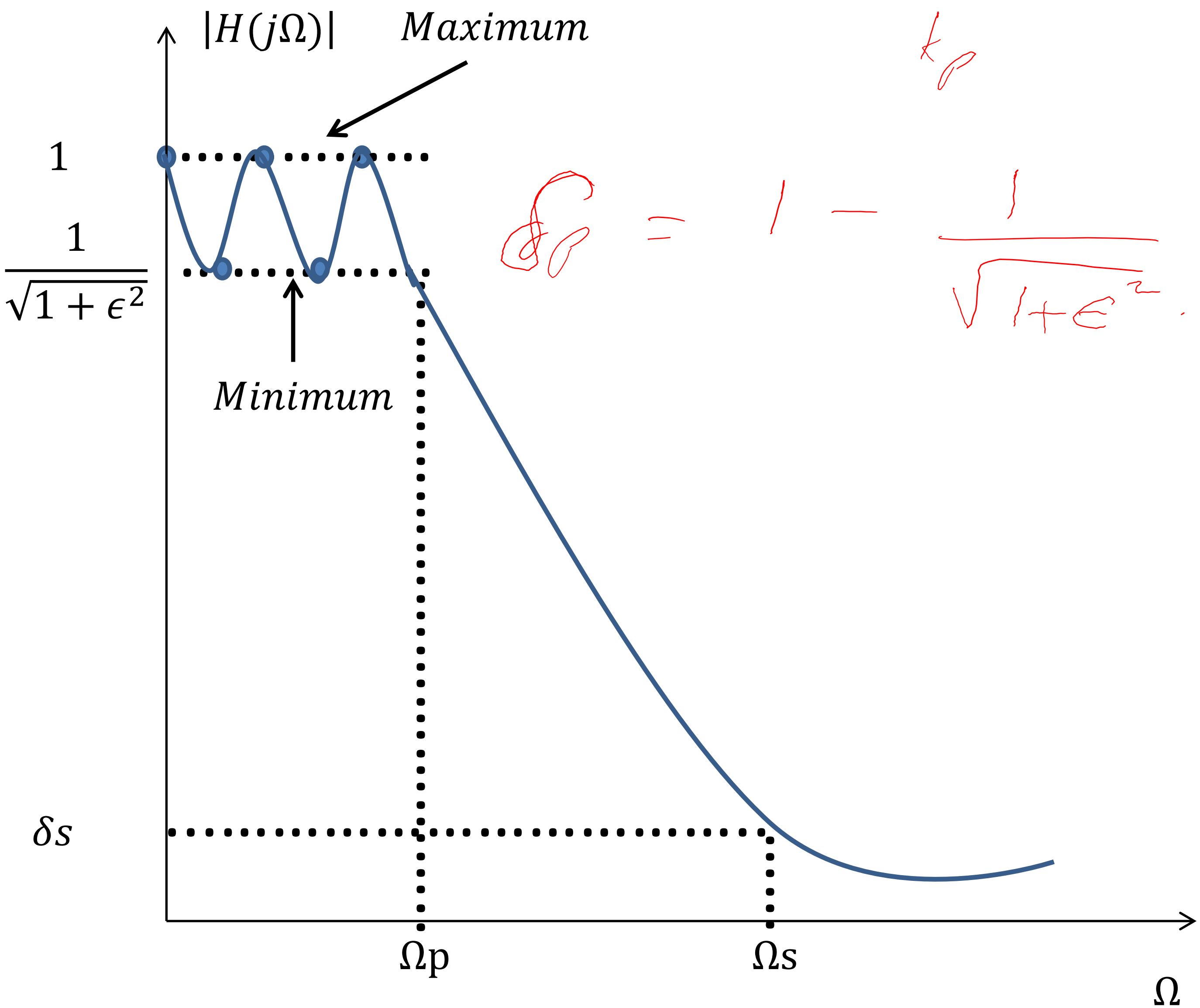
- There are two types of Chebyshev filters. The Chebyshev I filter is equiripple in the pass band and monotonic in the stopband, whereas Chebyshev II is just the opposite.

- The magnitude frequency response of a lowpass Chebyshev I filter is given by Butt

$$|H(j\Omega)| = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}}$$

$$|H(j\Omega| = \frac{1}{\left[1+\epsilon^2 {T_N}^2\left(\frac{\Omega}{\Omega_P}\right)\right]^{\frac{1}{2}}} \qquad (4)$$

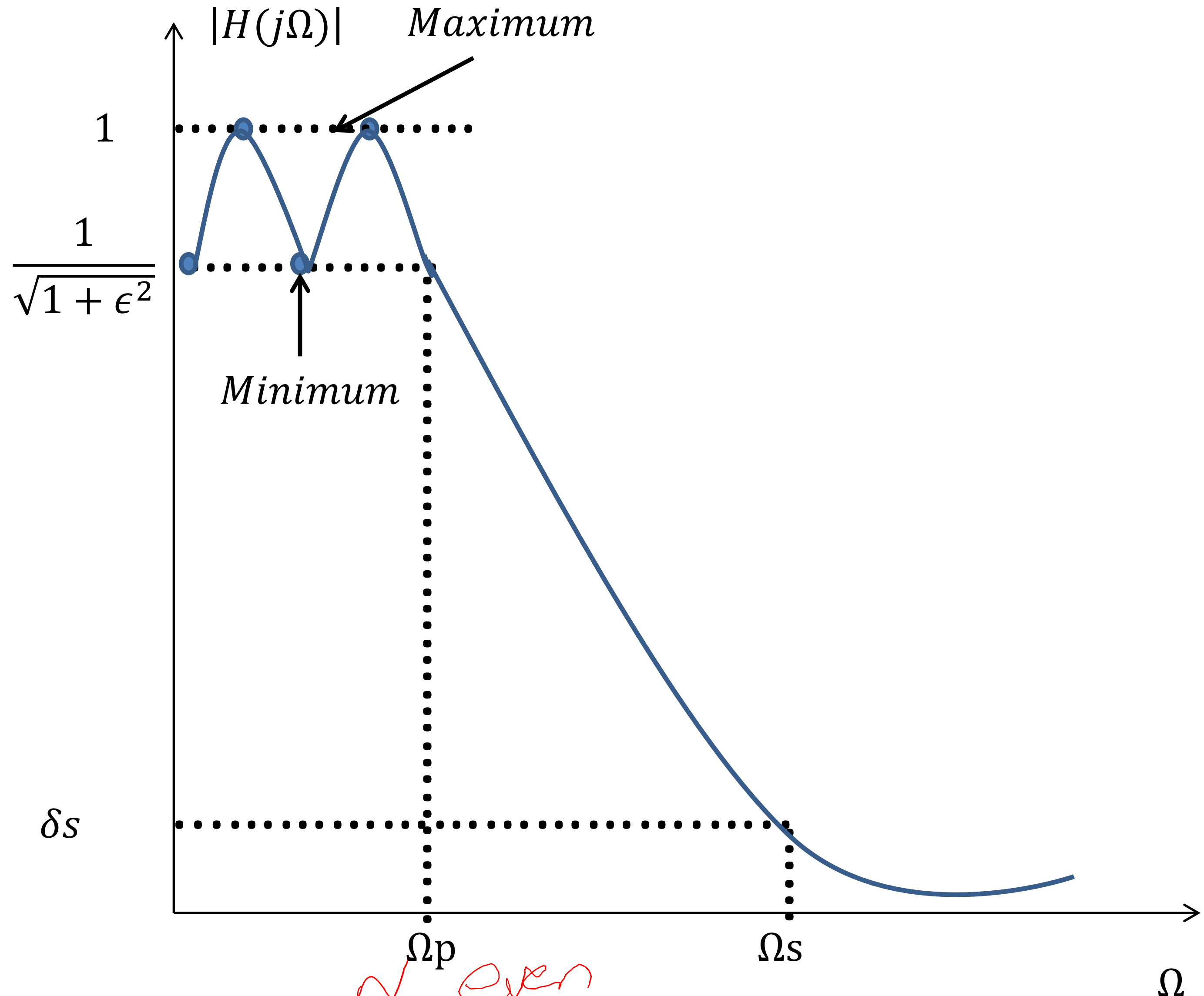
Where ϵ is a parameter of the filter related to the ripple in the passband and $T_N(x) is\ the\ Nth\ order\ Chebyshev\ polynomial$.



$$\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}}$$

N odd (N=5)

Magnitude frequency response of a Chebyshev I filter for N being odd



N even
~~N odd~~ (N=4)

Magnitude frequency response of a Chebyshev I filter for N being even

Salient properties of a lowpass Chebyshev I filter:

- We note that $|H(j0)| = 1$for N odd and $|H(j0)| = \frac{1}{\sqrt{1+\epsilon^2}}$ for N even.

- The filter has uniform ripples in the pass band and is monotonic outside the passband.

- The sum of the number of maxima and minima in the passband equals to the order of the filter.

Chebyshev Normalized Transfer function $H_N(s)$

- The lowpass normalized ($\Omega_P = 1\, rad/sec$) Chebyshev I filter is characterized by the following magnitude squared frequency response:

$$|H_N(j\Omega)|^2 = \frac{1}{1+\epsilon^2 {T_N}^2(\Omega)} \qquad (5)$$

Replacing $j\Omega$ by s and consequently Ω by $\frac{s}{j}$ in the equation (5)

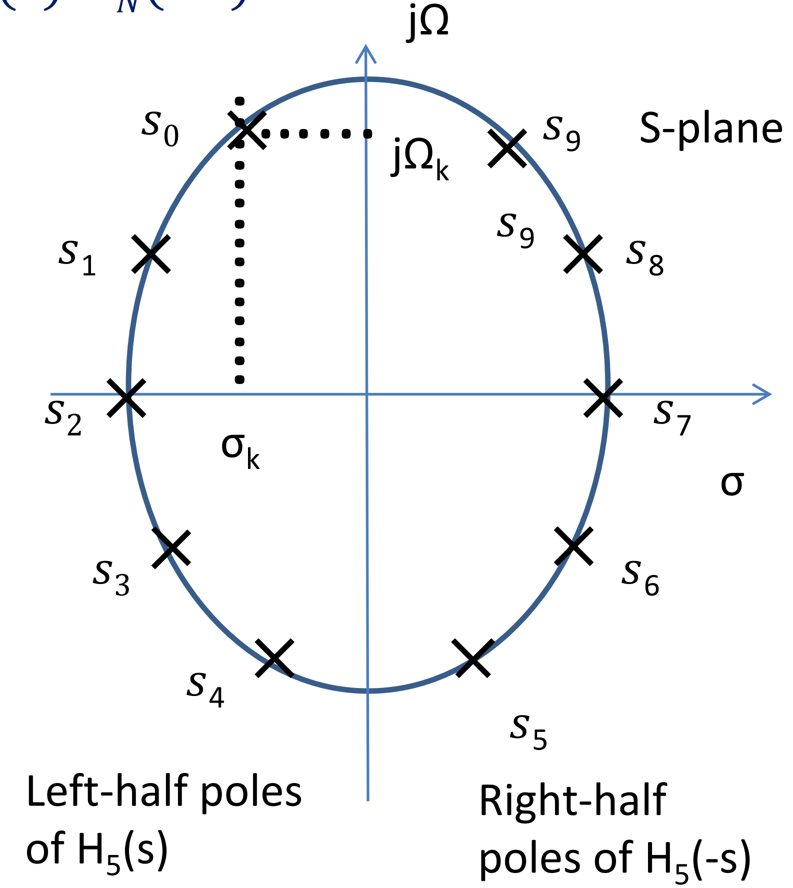
$$|H_N(s)|^2 = \frac{1}{1+\epsilon^2 {T_N}^2\left(\frac{s}{j}\right)} \qquad (6)$$

$$H_N(s)\, H_N(-s) = \frac{1}{1+\epsilon^2 {T_N}^2\left(\frac{s}{j}\right)} \qquad (7)$$

- The transfer function $H_N(s)\ H_N(-s)$ has no finite zeros.
- The poles of the product $H_N(s)\ H_N(-s)$ are determined by equating the denominator of equation (7) to zero.

$$1+\epsilon^2 {T_N}^2\left(\frac{s}{j}\right)=0 \qquad (8)$$

Equation (8) is known as the characteristic equation of the filter. The roots of this equation or poles of $H_N(s)\ H_N(-s)$ are found to lie on an ellipse in the s-plane as shown in figure.



- If $s_k = \sigma_k + j\Omega_k$ represent a pole, then σ_k and Ω_k satisfy the equation

$$\frac{{\sigma_k}^2}{a^2} + \frac{{\Omega_k}^2}{b^2} = 1 \qquad (9)$$

Where $a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}}$ (10)

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} \qquad (11)$$

$$\sigma_k = -asin\left[(2k-1)\frac{\pi}{2N}\right] \qquad (12)$$

$$\Omega_k = b\ cos\left[(2k-1)\frac{\pi}{2N}\right], k = 1, 2, \ldots .2N \qquad (13)$$

- Making use of only Half ($H_N(s)$)on the left-half plane, the transfer function of the stable normalized lowpass Chebyshev I filter is

$$H_{N(s)} = \frac{K_N}{\prod_{LHP}(s-s_k)} = \frac{K_N}{V_N(s)} \qquad (14)$$

- In equation (14) $V_N(s)$ polynomial is given by
$$V_N(s) = s^N + b_{N-1}s^{N-1} + b_2 s^{N-2} + \cdots + b_0$$

Where $K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}}, & N = even \\ b_0 & N = odd \end{cases}$ (15)

Two different forms of table for the coefficients of $V_N(s)$ for various N and given values of $\in$.

Appendix-II

Polynomials $V_N(s)$ used in Chebyshev I filter design for $\frac{1}{2}$, 1, 2, and 3 dB ripples

$$\text{Chebyshev filter } H_N(s) = \frac{K_N}{V_N(s)}, \text{ where } K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N \text{ odd} \end{cases}$$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0$$

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
	a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)									
1	2.8627752									
2	1.5162026	1.4256245								
3	0.7156938	1.5348954	1.2529130							
4	0.3790506	1.0254553	1.7168662	1.1973856						
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909					
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761				
7	0.0447309	0.2820722	0.7556511	1.6479029	1.8694079	2.4126510	1.1512176			
8	0.0236907	0.1525444	0.5735604	1.1485894	2.1840154	2.1492173	2.6567498	1.1460801		
9	0.0111827	0.0941198	0.3408193	0.9836199	1.6113880	2.7814990	2.4293297	2.9027337	1.1425705	
10	0.0059227	0.0492855	0.2372688	0.6269689	1.5274307	2.1442372	3.4409268	2.7097415	3.1498757	1.1400664
	b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)									
1	1.9652267									
2	1.1025103	1.0977343								
3	0.4913067	1.2384092	0.9883412							
4	0.2756276	0.7426194	1.4539248	0.9528114						
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201					
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510				
7	0.0307066	0.2136712	0.5486192	1.3575440	1.4287930	2.1760778	0.9231228			
8	0.0172267	0.1073447	0.4478257	0.8468243	1.8369024	1.6551557	2.4230264	0.9198113		
9	0.0076767	0.0706048	0.2441864	0.7863109	1.2016071	2.3781188	1.8814798	2.6709468	0.9175476	
10	0.0043067	0.0344971	0.1824512	0.4553892	1.2444914	1.6129856	2.9815094	2.1078524	2.9194657	0.9159320

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
					c. 2 dB Ripple ($\epsilon = 0.7647831$, $\epsilon^2 = 0.5848932$)					
1	1.3075603									
2	0.6367681	0.8038164								
3	0.3268901	1.0221903	0.7378216							
4	0.2057651	0.5167981	1.2564819	0.7162150						
5	0.0817225	0.4593491	0.6934770	1.4995433	0.7064606					
6	0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257				
7	0.0204228	0.1660920	0.3825056	1.1444390	1.0392203	1.9935272	0.6978929			
8	0.0128603	0.0729373	0.3587043	0.5982214	1.5795807	1.2117121	2.2422529	0.6960646		
9	0.0051076	0.0543756	0.1684473	0.6444677	0.8568648	2.0767479	1.3837464	2.4912897	0.6946793	
10	0.0032151	0.0233347	0.1440057	0.3177560	1.0389104	1.1585287	2.6362507	1.5557424	2.7406032	0.6936904
					d. 3 dB Ripple ($\epsilon = 0.9976283$, $\epsilon^2 = 0.9952623$)					
1	1.0023773									
2	0.7079478	0.6448996								
3	0.2505943	0.9283480	0.5972404							
4	0.1769869	0.4047679	1.1691176	0.5815799						
5	0.0626391	0.4079421	0.5488626	1.4149847	0.5744296					
6	0.0442467	0.1634299	0.6990977	0.6906098	1.6628481	0.5706979				
7	0.0156621	0.1461530	0.3000167	1.0518448	0.8314411	1.9115507	0.5684201			
8	0.0110617	0.0564813	0.3207646	0.4718990	1.4666990	0.9719473	2.1607148	0.5669476		
9	0.0039154	0.0475900	0.1313851	0.5834984	0.6789075	1.9438443	1.1122863	2.4101346	0.5659234	
10	0.0027654	0.0180313	0.1277560	0.2492043	0.9499208	0.9210659	2.4834205	1.2526467	2.6597378	0.5652218

Selection of N

Order of the filter N is given as

$$N \geq \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)}, \qquad (16)$$

Where d is the discrimination factor

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{{\delta_s}^{-2}-1}} \qquad (17)$$

Pass band gain at $\Omega = \Omega p$,
$Kp = 20 \log(1 - \delta p)$

Stop band gain at $\Omega = \Omega s$,
$Ks = 20 \log(\delta s)$

Where K is the selectivity factor

$$K = \frac{\Omega_P}{\Omega_S} \qquad (18)$$

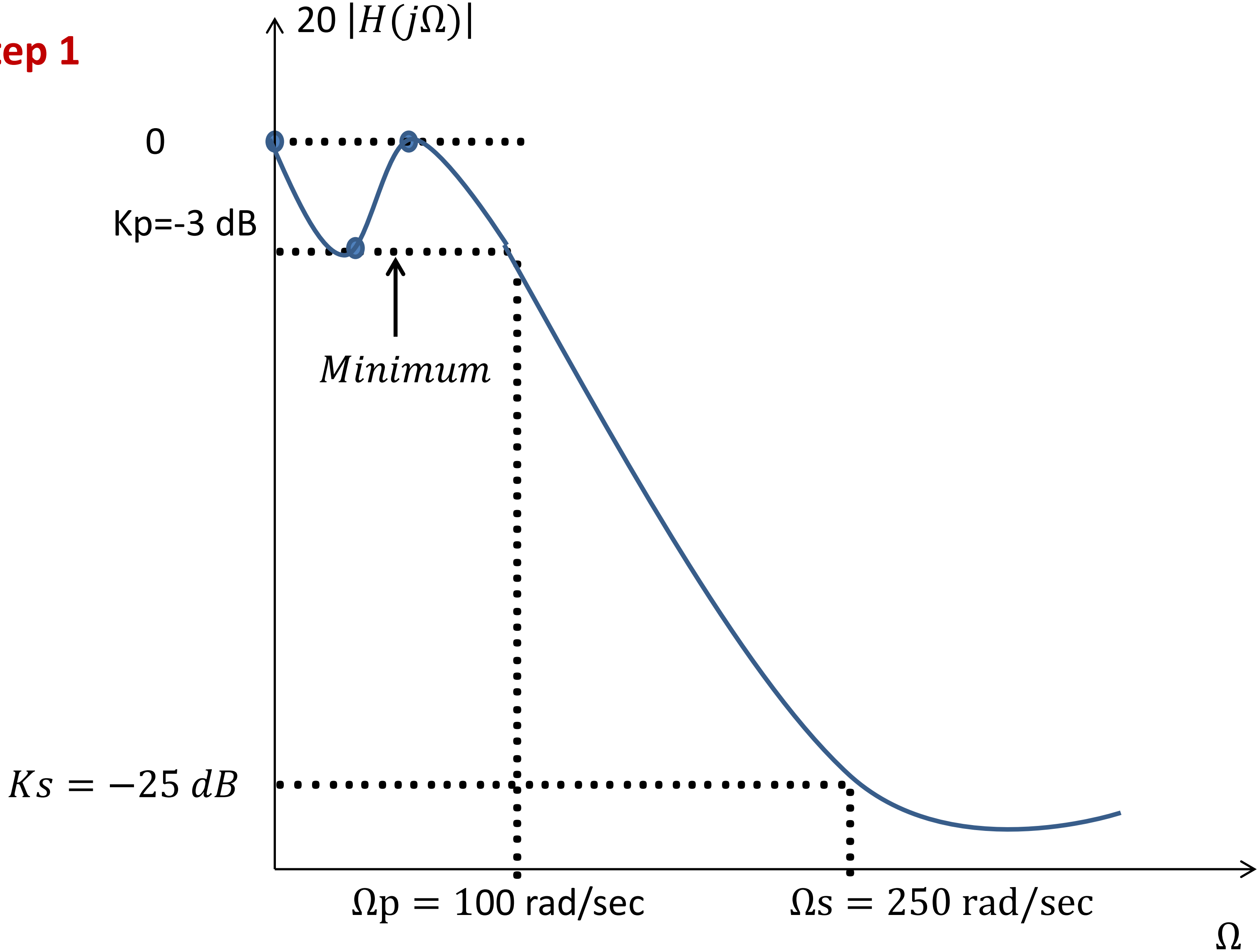
Pass band ripple $\delta_P = 1 - \frac{1}{\sqrt{1+\varepsilon^2}}$ (19)

$$\epsilon^2 = (1-\delta_P)^{-2} - 1 \qquad (20)$$

Problem 1

- Design a Chebyshev I analog low pass filter to meet the following specifications:
  - a. Passband ripple: $\leq -3\ dB$
  - b. Passband edge: 100 rad/sec
  - c. Stopband attenuation: $\geq 25\ dB$
  - d. Stopband edge: 250 rad/sec.

Step 1

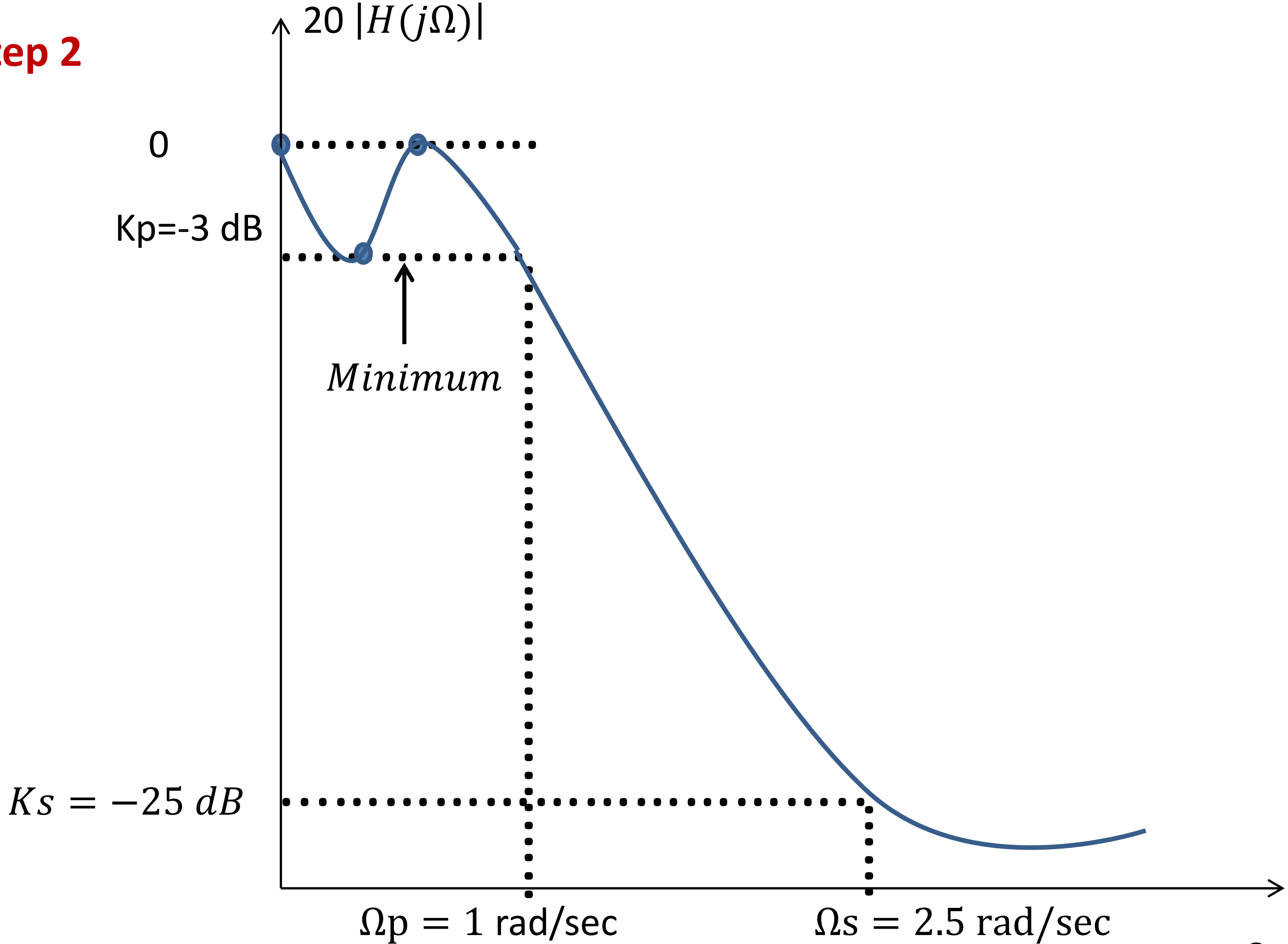


N odd (N=3)

The specified Magnitude frequency response of a Chebyshev I filter is shown above

- Step 2:
- The pass band edge frequency $\boldsymbol{\Omega}_{\mathbf{P}}$ of the normalized low pass filter is 1 rad/sec
- Let us the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega}_{s}$ of the normalized low pass filter $\boldsymbol{\Omega}_{s} = \frac{\boldsymbol{\Omega}_{s}}{\boldsymbol{\Omega}_{\mathbf{P}}} = \frac{250}{100} = 2.5\, rad/sec$.
- This backward equation is for Low pass to Lowpass transformation

Step 2



N odd (N=3)

Normalized Magnitude frequency response of a Chebyshev I Low pass filter for N being odd

- If the given filter is high pass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s}$ is $\frac{\boldsymbol{\Omega_P}}{\boldsymbol{\Omega_S}}$.

- If the given filter is Bandpass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} =$ $Min\{|A|, |B|\}$. Where $A = \frac{-\Omega_1{}^2 + \Omega_l \Omega_u}{\Omega_1\,(\Omega_u - \Omega_l)}$, $B = \frac{\Omega_2{}^2 - \Omega_l \Omega_u}{\Omega_2\,(\Omega_u - \Omega_l)}$

- If the given filter is Bandstop then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} =$ $Min\{|A|, |B|\}$. Where $A = \frac{\Omega_1\,(\Omega_u - \Omega_l)}{-\Omega_1{}^2 + \Omega_l \Omega_u}$, $B = \frac{\Omega_2\,(\Omega_u - \Omega_l)}{\Omega_2{}^2 - \Omega_l \Omega_u}$

- The normalized passband edge frequency $\boldsymbol{\Omega_P}$ is always equal to 1 rad/sec irrespective of the given filter.

- **Step 3**

$$K_P = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$-3 = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\epsilon = 0.99762834$$

$$\delta_P = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.2920542138$$

$$K_S = -25\ dB$$

$$20 \log \delta_S = -25$$

$$\delta_S = 0.056$$

Pass band gain at $\Omega = \Omega p$,
Kp= 20 log(1- δp)

Stop band gain at $\Omega = \Omega s$,
Ks=20 log(δs)

$$K = \frac{\Omega_P}{\Omega_S} = \frac{1}{2.5} = 0.4$$

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S{}^{-2}-1}}=0.056$$

Step 4:

Minimum filter order (of normalized filter) is

$$N \geq \frac{cosh^{-1}\left(\frac{1}{d}\right)}{cosh^{-1}\left(\frac{1}{K}\right)}= 2.28$$

Rounding off to the next integer we get, N=3

Step 5: Find normalized low pass filter transfer function of order 3

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 0.2986202$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 1.043635$$

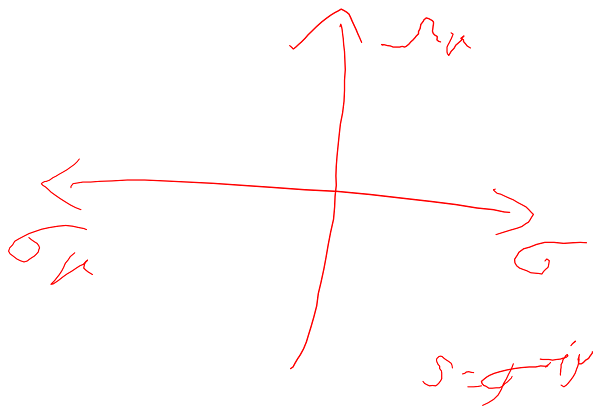
$$\sigma_k = -a sin\left[(2k-1)\frac{\pi}{2N}\right] \qquad \Omega_k = b\ cos\left[(2k-1)\frac{\pi}{2N}\right], k = 1, 2, \ldots .2N$$

When N=3, we get

$$\sigma_k = -0.2986202\, sin\left[(2k-1)\frac{\pi}{6}\right] \qquad \Omega_k = 1.043635\, cos\left[(2k-1)\frac{\pi}{6}\right], \qquad k = 1, 2,3,4,5,6$$

Right

The value of σ_k and Ω_k for k= 1, 2 and 3 gives the left-half poles of $H_3(s)$.



k	σ_k	Ω_k
1	-0.1493101	0.9038144
2	-0.2986202	0
3	-0.1493101	-0.9038144

$$H_{3(s)} = \frac{K_N}{\prod_{LHP}(s-s_k)} = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)}$$

- $H_3(s) = \frac{K_N}{(s+0.1493101-j0.9038144)\times(s+0.298602)\times(s+0.1493101+j0.9038144)}$

$$= \frac{K_N}{(s^3 + 0.5972404s^2 + 0.928348s + 0.2505943)}$$

Where $K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}}, & N = even \\ b_0 & N = odd \end{cases}$

Since N is odd, $K_N = b_0 = 0.2505943$

$$H_3(s) = \frac{0.2505943}{(s^3 + 0.5972404s^2 + 0.928348s + 0.2505943)}$$

Step 6 Freq. Transformation

- If the specified filter is Low pass then apply lowpass to Lowpass transformation on the normalized lowpass filter by replacing $s \rightarrow \frac{s}{\Omega_P}$.

- If the specified filter is high pass then apply lowpass to highpass transformation on the normalized low pass filter by replacing
$s \rightarrow \frac{\Omega_P}{s}$.

- If the specified filter is Bandpass then apply lowpass to Bandpass transformation on the normalized low pass filter by replacing

$$\mathrm{s} \rightarrow \frac{s^2+\Omega_u\Omega_l}{s(\Omega_{u-}\Omega_l)}.$$

- If the specified filter is Bandstop then apply lowpass to Bandstop transformation on the normalized low pass filter by replacing

$$\mathrm{s} \rightarrow \frac{s(\Omega_{u-}\Omega_l)}{s^2+\Omega_u\Omega_l}.$$

- The required lowpass filter $H_a(s)$ is obtained by applying a lowpass-to lowpass transformation to $H_3(s)$

$$H_a(s) = H_3(s)|\mathrm{s} \rightarrow \frac{s}{\Omega_P}$$

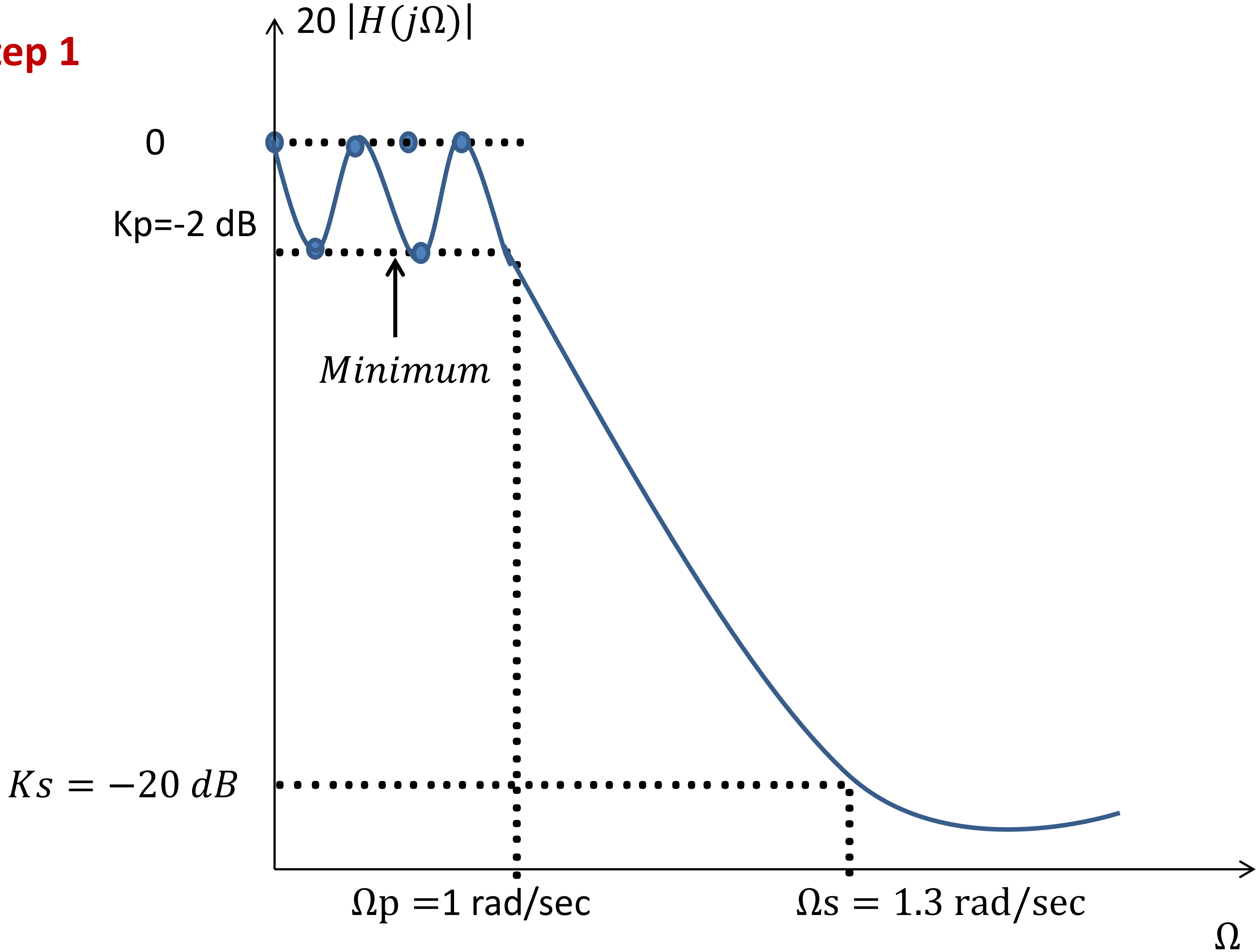
$$= \frac{0.2505943}{s^3+0.5972404s^2+0.928348s+0.2505943}\Bigg|\,\mathrm{s} \rightarrow \frac{s}{100}$$

$$= \frac{250594.3}{s^3+59.72404s^2+9283.48s+250594.3}$$

Problem 2

- Design a Chebyshev I analog low pass filter to meet the following specifications:
  - a. Passband ripple: $\leq 2\ dB$
  - b. Passband edge: 1 rad/sec
  - c. Stopband attenuation: $\geq 20\ dB$
  - d. Stopband edge: 1.3 rad/sec.

Step 1

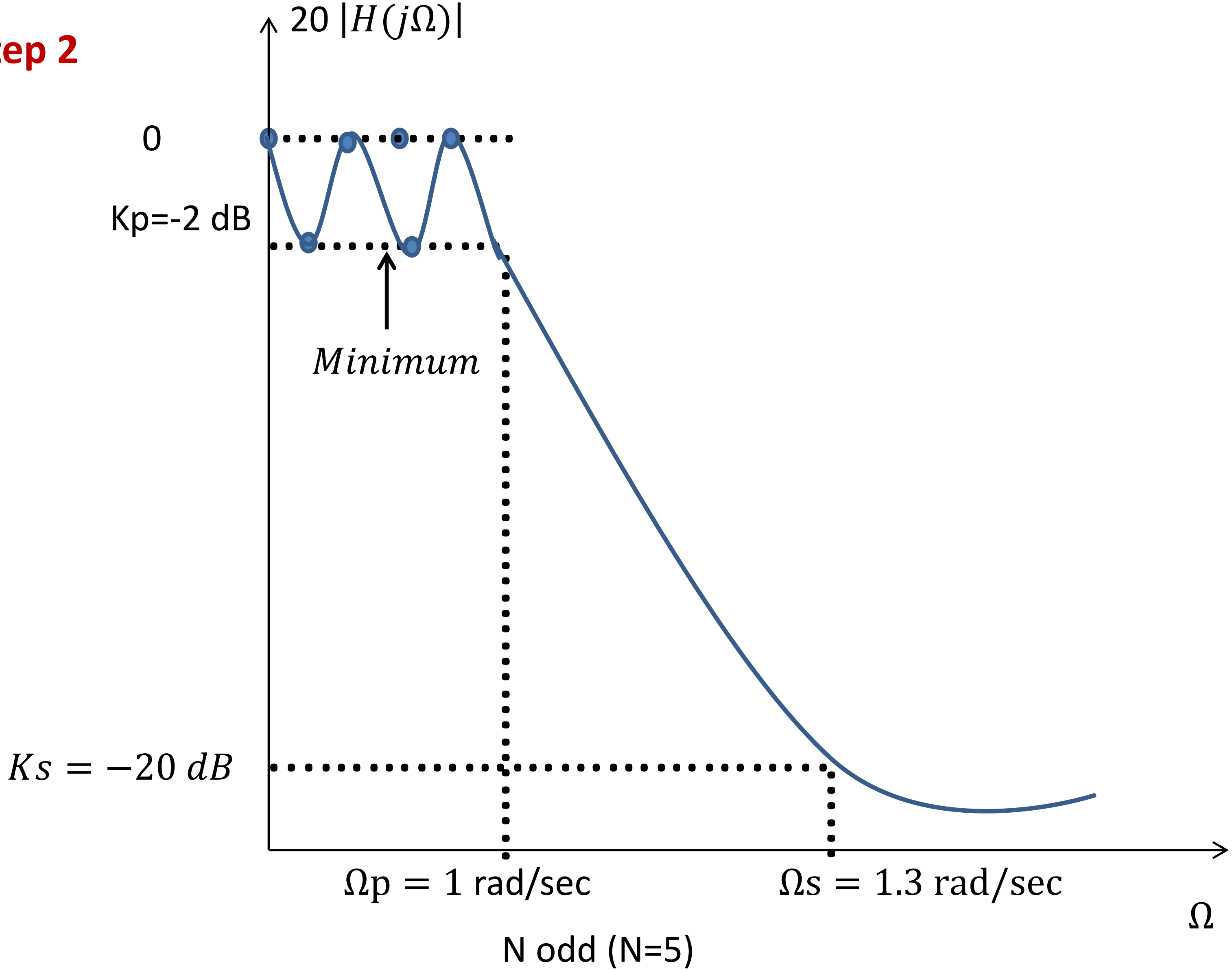


N odd (N=5)

The specified Magnitude frequency response of a Chebyshev I filter is shown above

- Step 2:
- The pass band edge frequency Ω_{P} of the normalized low pass filter is 1 rad/sec
- Let us the backward design equation to find the stopband edge frequency Ω_s of the normalized low pass filter $\Omega_s = \frac{\Omega_s}{\Omega_{P}} = \frac{1.3}{1} = 1.3\, rad/sec$.
- This backward equation is for Low pass to Lowpass transformation

Step 2



Normalized Magnitude frequency response of a Chebyshev I Low pass filter for N being odd

- If the given filter is high pass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s}$ is $\frac{\boldsymbol{\Omega_P}}{\boldsymbol{\Omega_S}}$.

- If the given filter is Bandpass then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} =$ $Min\{|A|, |B|\}$. Where $A = \frac{-\Omega_1{}^2 + \Omega_l\Omega_u}{\Omega_1\,(\Omega_u - \Omega_l)}$, $B = \frac{\Omega_2{}^2 - \Omega_l\Omega_u}{\Omega_2\,(\Omega_u - \Omega_l)}$

- If the given filter is Bandstop then the backward design equation to find the stopband edge frequency $\boldsymbol{\Omega_s} =$ $Min\{|A|, |B|\}$. Where $A = \frac{\Omega_1\,(\Omega_u - \Omega_l)}{-\Omega_1{}^2 + \Omega_l\Omega_u}$, $B = \frac{\Omega_2\,(\Omega_u - \Omega_l)}{\Omega_2{}^2 - \Omega_l\Omega_u}$

- The normalized passband edge frequency $\boldsymbol{\Omega_P}$ is always equal to 1 rad/sec irrespective of the given filter.

- **Step 3**

$$K_P = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$-2 = 20\ log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\epsilon = 0.76478$$

$$\delta_P = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.20567$$

$$K_S = -20\ dB$$

$$20 \log \delta_S = -20$$

$$\delta_S = 0.1$$

$$K = \frac{\Omega_P}{\Omega_S} = \frac{1}{1.3} = 0.769$$

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{{\delta_S}^{-2}-1}}=0.077$$

Step 4:

Minimum filter order (of normalized filter) is

$$N \geq \frac{cosh^{-1}\left(\frac{1}{d}\right)}{cosh^{-1}\left(\frac{1}{K}\right)}= 4.3$$

Rounding off to the next integer we get, N=5

Step 5: Find normalized low pass filter transfer function of order 5

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 0.21830398$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 1.0235520$$

$$\sigma_k = -a sin\left[(2k-1)\frac{\pi}{2N}\right] \quad \Omega_k = b\ cos\left[(2k-1)\frac{\pi}{2N}\right], k = 1, 2, \dots .2N$$

When N=5, we get

$$\sigma_k = -0.21830398\, sin\left[(2k-1)\frac{\pi}{10}\right] \qquad \Omega_k = 1.0235520\, cos\left[(2k-1)\frac{\pi}{10}\right], \quad k = 1, 2,3,4,5,6,7,8,9,10$$

The value of σ_k and Ω_k for k= 1, 2,3,4 and 5 gives the left-half poles of $H_5(s)$.

k	σ_k	Ω_k
1	-0.0674610	0.9734557
2	-0.1766151	0.6016287
3	0.2183083	0
4	-0.1766151	-0.6016287
5	-0.0674610	-0.9734557

$$H_5 = \frac{K_N}{\prod_{LHP}(s - s_k)} = \frac{K_N}{(s - s_1)(s - s_2)(s - s_3)(s - s_4)(s - s_5)}$$

$$H_5(s)$$
$$= \frac{K_N}{(s + 0.0.0674610 - j0.9734557) \times (s + 0.0674610 + j0.9734557) \times (s + 0.1766151 - j0.6016287) \times (s + 0.1766151 + j0.6016287) \times (s + 0.2183083)}$$
$$= \frac{K_N = b_0 = 0.08172}{(s^5 + 0.70646s^4 + 1.4995s^3 + 0.6934s^2 + 0.459349s + 0.08172)}$$

Where $K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}}, & N = even \\ b_0 & N = odd \end{cases}$

Since N is odd, $K_N = b_0 = 0.08172$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + b_2 s^{N-2} + \cdots + b_0$$

$$= (s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0)$$

From the chart

$$b_0 = 0.08172$$

$$b_1 = 0.459349$$

$$b_2 = 0.6934$$

$$b_3 = 1.4995$$

$$b_4 = 0.70646$$

Appendix-II

Polynomials $V_N(s)$ used in Chebyshev I filter design for $\frac{1}{2}$, 1, 2, and 3 dB ripples

$$\text{Chebyshev filter } H_N(s) = \frac{K_N}{V_N(s)}, \text{ where } K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N \text{ odd} \end{cases}$$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0$$

a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	2.8627752									
2	1.5162026	1.4256245								
3	0.7156938	1.5348954	1.2529130							
4	0.3790506	1.0254553	1.7168662	1.1973856						
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909					
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761				
7	0.0447309	0.2820722	0.7556511	1.6479029	1.8694079	2.4126510	1.1512176			
8	0.0236907	0.1525444	0.5735604	1.1485894	2.1840154	2.1492173	2.6567498	1.1460801		
9	0.0111827	0.0941198	0.3408193	0.9836199	1.6113880	2.7814990	2.4293297	2.9027337	1.1425705	
10	0.0059227	0.0492855	0.2372688	0.6269689	1.5274307	2.1442372	3.4409268	2.7097415	3.1498757	1.1400664

b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	1.9652267									
2	1.1025103	1.0977343								
3	0.4913067	1.2384092	0.9883412							
4	0.2756276	0.7426194	1.4539248	0.9528114						
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201					
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510				
7	0.0307066	0.2136712	0.5486192	1.3575440	1.4287930	2.1760778	0.9231228			
8	0.0172267	0.1073447	0.4478257	0.8468243	1.8369024	1.6551557	2.4230264	0.9198113		
9	0.0076767	0.0706048	0.2441864	0.7863109	1.2016071	2.3781188	1.8814798	2.6709468	0.9175476	
10	0.0043067	0.0344971	0.1824512	0.4553892	1.2444914	1.6129856	2.9815094	2.1078524	2.9194657	0.9159320

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
c. 2 dB Ripple ($\epsilon = 0.7647831,\ \epsilon^2 = 0.5848932$)									
1.3075603									
0.6367681	0.8038164								
0.3268901	1.0221903	0.7378216							
0.2057651	0.5167981	1.2564819	0.7162150						
0.0817225	0.4593491	0.6934770	1.4995433	0.7064606					
0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257				
0.0204228	0.1660920	0.3825056	1.1444390	1.0392203	1.9935272	0.6978929			
0.0128603	0.0729373	0.3587043	0.5982214	1.5795807	1.2117121	2.2422529	0.6960646		
0.0051076	0.0543756	0.1684473	0.6444677	0.8568648	2.0767479	1.3837464	2.4912897	0.6946793	
0.0032151	0.0233347	0.1440057	0.3177560	1.0389104	1.1585287	2.6362507	1.5557424	2.7406032	0.6936904
d. 3 dB Ripple ($\epsilon = 0.9976283,\ \epsilon^2 = 0.9952623$)									
1.0023773									
0.7079478	0.6448996								
0.2505943	0.9283480	0.5972404							
0.1769869	0.4047679	1.1691176	0.5815799						
0.0626391	0.4079421	0.5488626	1.4149847	0.5744296					
0.0442467	0.1634299	0.6990977	0.6906098	1.6628481	0.5706979				
0.0156621	0.1461530	0.3000167	1.0518448	0.8314411	1.9115507	0.5684201			
0.0110617	0.0564813	0.3207646	0.4718990	1.4666990	0.9719473	2.1607148	0.5669476		
0.0039154	0.0475900	0.1313851	0.5834984	0.6789075	1.9438443	1.1122863	2.4101346	0.5659234	
0.0027654	0.0180313	0.1277560	0.2492043	0.9499208	0.9210659	2.4834205	1.2526467	2.6597378	0.5652218

Step 6

- If the specified filter is Low pass then apply lowpass to Lowpass transformation on the normalized lowpass filter by replacing $s \to \frac{s}{\Omega_P}$.

- If the specified filter is high pass then apply lowpass to highpass transformation on the normalized low pass filter by replacing $s \to \frac{\Omega_P}{s}$.

- If the specified filter is Bandpass then apply lowpass to Bandpass transformation on the normalized low pass filter by replacing

$$\text{s} \rightarrow \frac{s^2+\Omega_u\Omega_l}{s(\Omega_{u-}\Omega_l)}.$$

- If the specified filter is Bandstop then apply lowpass to Bandstop transformation on the normalized low pass filter by replacing

$$\text{s} \rightarrow \frac{s(\Omega_{u-}\Omega_l)}{s^2+\Omega_u\Omega_l}.$$

- The required lowpass filter $H_a(s)$ is obtained by applying a lowpass-to lowpass transformation to $H_3(s)$

$$H_a(s) = H_3(s)|\mathrm{s} \rightarrow \frac{s}{\Omega_P}$$

$$= \left.\frac{K_N = b_0 = 0.08172}{(s^5 + 0.70646s^4 + 1.4995s^3 + 0.6934s^2 + 0.459349s + 0.08172)}\right| \mathrm{s} \rightarrow \frac{s}{1}$$

$$= \frac{0.08172}{(s^5 + 0.70646s^4 + 1.4995s^3 + 0.6934s^2 + 0.459349s + 0.08172)}$$

Appendix-II

Polynomials $V_N(s)$ used in Chebyshev I filter design for $\frac{1}{2}$, 1, 2, and 3 dB ripples

$$\text{Chebyshev filter } H_N(s) = \frac{K_N}{V_N(s)}, \quad \text{where} \quad K_N = \begin{cases} \frac{b_0}{\sqrt{1+\epsilon^2}} & \text{for } N \text{ even} \\ b_0 & \text{for } N \text{ odd} \end{cases}$$

$$V_N(s) = s^N + b_{N-1}s^{N-1} + \cdots + b_1 s + b_0$$

N	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
					a. $\frac{1}{2}$ dB Ripple ($\epsilon = 0.3493114$, $\epsilon^2 = 0.1220184$)					
1	2.8627752									
2	1.5162026	1.4256245								
3	0.7156938	1.5348954	1.2529130							
4	0.3790506	1.0254553	1.7168662	1.1973856						
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909					
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761				
7	0.0447309	0.2820722	0.7556511	1.6479029	1.8694079	2.4126510	1.1512176			
8	0.0236907	0.1525444	0.5735604	1.1485894	2.1840154	2.1492173	2.6567498	1.1460801		
9	0.0111827	0.0941198	0.3408193	0.9836199	1.6113880	2.7814990	2.4293297	2.9027337	1.1425705	
10	0.0059227	0.0492855	0.2372688	0.6269689	1.5274307	2.1442372	3.4409268	2.7097415	3.1498757	1.1400664
					b. 1 dB Ripple ($\epsilon = 0.5088471$, $\epsilon^2 = 0.2589254$)					
1	1.9652267									
2	1.1025103	1.0977343								
3	0.4913067	1.2384092	0.9883412							
4	0.2756276	0.7426194	1.4539248	0.9528114						
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201					
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510				
7	0.0307066	0.2136712	0.5486192	1.3575440	1.4287930	2.1760778	0.9231228			
8	0.0172267	0.1073447	0.4478257	0.8468243	1.8369024	1.6551557	2.4230264	0.9198113		
9	0.0076767	0.0706048	0.2441864	0.7863109	1.2016071	2.3781188	1.8814798	2.6709468	0.9175476	
10	0.0043067	0.0344971	0.1824512	0.4553892	1.2444914	1.6129856	2.9815094	2.1078524	2.9194657	0.9159320

b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
c. 2 dB Ripple ($\epsilon = 0.7647831,\ \epsilon^2 = 0.5848932$)									
1.3075603									
0.6367681	0.8038164								
0.3268901	1.0221903	0.7378216							
0.2057651	0.5167981	1.2564819	0.7162150						
0.0817225	0.4593491	0.6934770	1.4995433	0.7064606					
0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257				
0.0204228	0.1660920	0.3825056	1.1444390	1.0392203	1.9935272	0.6978929			
0.0128603	0.0729373	0.3587043	0.5982214	1.5795807	1.2117121	2.2422529	0.6960646		
0.0051076	0.0543756	0.1684473	0.6444677	0.8568648	2.0767479	1.3837464	2.4912897	0.6946793	
0.0032151	0.0233347	0.1440057	0.3177560	1.0389104	1.1585287	2.6362507	1.5557424	2.7406032	0.6936904
d. 3 dB Ripple ($\epsilon = 0.9976283,\ \epsilon^2 = 0.9952623$)									
1.0023773									
0.7079478	0.6448996								
0.2505943	0.9283480	0.5972404							
0.1769869	0.4047679	1.1691176	0.5815799						
0.0626391	0.4079421	0.5488626	1.4149847	0.5744296					
0.0442467	0.1634299	0.6990977	0.6906098	1.6628481	0.5706979				
0.0156621	0.1461530	0.3000167	1.0518448	0.8314411	1.9115507	0.5684201			
0.0110617	0.0564813	0.3207646	0.4718990	1.4666990	0.9719473	2.1607148	0.5669476		
0.0039154	0.0475900	0.1313851	0.5834984	0.6789075	1.9438443	1.1122863	2.4101346	0.5659234	
0.0027654	0.0180313	0.1277560	0.2492043	0.9499208	0.9210659	2.4834205	1.2526467	2.6597378	0.5652218