Comparison between butterworth and chebychev filter:

1. The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞, whereas the magnitude response of the Chebyshev filter exhibits ripples in the passband or stopband according to the type.
2. The transition band is more in Butterworth filter when compared to Chebyshev filter.
3. The poles of the Butterworth filter lie on a circle, whereas the poles of the Chebyshev filter lie on an ellipse.
4. For the same specifications, the number of poles in Butterworth are more when compared to the Chebyshev filter i.e., the order of the Chebyshev filter is less than that of Butterworth. This is a great advantage because less number of discrete components will be necessary to construct the filter.

Steps to design an analog chebychev LPF:

5.9 Steps to design an analog Chebyshev lowpass filter

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Using the following formulas find the values of a and b, which are minor and major axis of the ellipse respectively.

$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2}; \quad b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

Ω_p = Passband frequency

α_p = Maximum allowable attenuation in the passband

($\because$ For normalized Chebyshev filter $\Omega p = 1$ rad/sec)

4. Calculate the poles of Chebyshev filter which lie on an ellipse by using the formula.

$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1, 2, \ldots, N$$

$$\text{where } \phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \quad k = 1, 2, \ldots, N$$

5. Find the denominator polynomial of the transfer function using the above poles.

6. The numerator of the transfer function depends on the value of N.

   (a) For N odd substitute $s = 0$ in the denominator polynomial and find the value. This value is equal to the numerator of the transfer function.
   ($\because$ For N odd the magnitude response $|H(j\Omega)|$ starts at 1.)

   (b) For N even substitute $s = 0$ in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$. This value is equal to the numerator.

Example 5.6 Given the specifications $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$; $f_p = 1\,\text{KHz}$ and $f_s = 2\,\text{KHz}$. Determine the order of the filter using Chebyshev approximation. Find $H(s)$.

Solution

From the given data we can find

$$\Omega_p = 2\pi \times 1000\,\text{Hz} = 2000\,\pi\ \text{rad/sec}$$
$$\Omega_s = 2\pi \times 2000\,\text{Hz} = 4000\,\pi\ \text{rad/sec}$$

and $\alpha_p = 3\,\text{dB}$; $\alpha_s = 16\,\text{dB}$.

Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.6}-1}{10^{0.3}-1}}}{\cosh^{-1}\dfrac{4000\pi}{2000\pi}}$$
$$= 1.91$$

Step 2: Rounding N to next higher value we get $N = 2$.

For N even, the oscillatory curve starts from $\dfrac{1}{\sqrt{1+\varepsilon^2}}$.

Step 3: The values of minor axis and major axis can be found as below

$$\varepsilon = (10^{0.1\alpha_p} - 1)^{0.5} = (10^{0.3} - 1)^{0.5} = 1$$
$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414$$
$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} - (2.414)^{-1/2}]}{2} = 910\pi$$
$$b = \Omega_p \frac{[\mu^{1/N} + \mu^{-1/N}]}{2} = 2000\pi \frac{[(2.414)^{1/2} + (2.414)^{-1/2}]}{2} = 2197\pi$$

Step 4: The poles are given by

$$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1,2$$
$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2$$
$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = 135^\circ$$
$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$
$$s_1 = a\cos\phi_1 + j\,b\sin\phi_1 = -643.46\pi + j1554\pi$$
$$s_2 = a\cos\phi_2 + j\,b\sin\phi_2 = -643.46\pi - j1554\pi$$

Step 5: The denominator of $H(s) = (s + 643.46\pi)^2 + (1554\pi)^2$

Step 6: The numerator of $H(s) = \dfrac{(643.46\pi)^2 + (1554\pi)^2}{\sqrt{1+\varepsilon^2}} = (1414.38)^2\pi^2$

The transfer function $H(s) = \dfrac{(1414.38)^2\pi^2}{s^2 + 1287\pi s + (1682)^2\pi^2}$.

Example-2

Example 5.8 Determine the order and the poles of a type I lowpass Chebyshev filter that has a 1 dB ripple in the passband and passband frequency $\Omega_p = 1000\pi$, a stopband frequency of 2000π and an attenuation of 40 dB or more.

Solution

Given data $\alpha_p = 1$ dB; $\Omega_p = 1000\pi$ rad/sec; $\alpha_s = 40$ dB
$\Omega_s = 2000\pi$ rad/sec

$$N \geq \frac{\cos h^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cos h^{-1}\dfrac{\Omega_s}{\Omega_p}} \geq \frac{\cos h^{-1}\sqrt{\dfrac{10^4-1}{10^{0.1}-1}}}{\cos h^{-1}\dfrac{2000\pi}{1000\pi}} = 4.536$$

i.e., $N = 5$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.508; \quad \mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 4.17$$
$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 289.5\pi; \quad b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 1041\pi$$
$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1,2,\ldots 5$$
$$\phi_1 = 180^\circ;\ \phi_2 = 144^\circ;\ \phi_3 = 180^\circ;\ \phi_4 = 216^\circ;\ \phi_5 = 252^\circ$$
$$s_k = a\cos\phi_k + jb\sin\phi_k \quad k = 1,2,\ldots 5$$
$$s_1 = -89.5\pi + j989\pi; \quad s_2 = -234.2\pi + j612\pi; \quad s_3 = -289.5\pi$$
$$s_4 = -234.2\pi - j612\pi; \quad s_5 = -89.5\pi - j989\pi$$

Example-3

Example 5.9 Design a Chebyshev filter with a maximum passband attenuation of 2.5 dB; at $\Omega_p = 20$ rad/sec and the stopband attenuation of 30 dB at $\Omega_s = 50$ rad/sec. (AU ECE May'07)

Solution

Given

$$\Omega_p = 20 \text{ rad/sec}; \quad \alpha_p = 2.5\text{ dB};$$
$$\Omega_s = 50 \text{ rad/sec}; \quad \alpha_s = 30\text{ dB};$$

$$N = \frac{\cosh^{-1}\lambda/\varepsilon}{\cosh^{-1}1/k}$$
$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = 31.607$$
$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.882$$
$$k = \frac{\Omega_p}{\Omega_s} = 0.4$$

Now

$$N \geq \frac{\cosh^{-1}\frac{31.607}{0.882}}{\cosh^{-1}\frac{1}{0.4}} = 2.726$$

i.e., $N = 3$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.65$$
$$a = \Omega_p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2} = 6.6$$
$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 21.06$$
$$s_k = a\cos\phi_k + jb\sin\phi_k; \quad k = 1, 2, 3$$
$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi; \quad k = 1, 2, 3$$
$$\phi_1 = 120°, \phi_2 = 180°, \phi_3 = 240°$$
$$s_1 = -3.3 + j18.23$$
$$s_2 = -6.6$$
$$s_3 = -3.3 - j18.23$$

Denominator of $H(s) = (s + 6.6)(s^2 + 6.6s + 343.2)$

Numerator of $H(s) = (6.6)(343.2) = 2265.27$

Transfer function $H(s) = \dfrac{2265.27}{(s + 6.6)(s^2 + 6.6s + 343.2)}$

Practice Problem 5.4 For the given specifications find the order of the Chebyshev-I filter

$$\alpha_p = 1.5\text{ dB}; \quad \alpha_s = 10\text{ dB}; \quad \Omega_p = 2\text{ rad/sec}; \quad \Omega_s = 30\text{ rad/sec}$$

Obtain the analog Chebyshev filter transfer fn. that satisfies the constraints $\frac{1}{\sqrt{2}} \le |H(j\Omega)| \le 1$ for $0 \le \Omega \le 2$

$|H(j\Omega)| < 0.1$; $\Omega \ge 4$.

$\Omega_s = 4$ rad/sec
$\Omega_p = 2$ rad/sec

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{1}{\sqrt{2}} \quad ; \quad \frac{1}{\sqrt{1+\lambda^2}} = 0.1$$

$1 + \varepsilon^2 = 2$ $\qquad \sqrt{1+\lambda^2} = \frac{1}{0.1}$

$\varepsilon^2 = 1$ $\qquad \sqrt{1+\lambda^2} = 10$

$\boxed{\varepsilon = 1}$ $\qquad 1 + \lambda^2 = 100$

$\lambda^2 = 99$

$\boxed{\lambda = 9.9499}$

$$N \ge \frac{\cosh^{-1}(A)}{\cosh^{-1}(1/k)}$$

$$A = \frac{\lambda}{\varepsilon} = \frac{9.9499}{1} = 9.9499$$

$$k = \frac{\Omega p}{\Omega s} = \frac{2}{4} = 0.5.$$

$$N \ge \frac{\cosh^{-1} 9.9499}{\cosh^{-1} 2}$$

$N \ge 2.2689$

$\boxed{N = 3}$

$$\mu = \frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}}$$

$= 1 + \sqrt{1+1}$

$= 1 + \sqrt{2}$

$\boxed{\mu = 2.414}$

$$a = \frac{\Omega_p}{2}\left[\mu^{1/N} - \mu^{-1/N}\right]$$

$$= \frac{2}{2}\left[(2.414)^{1/3} - (2.414)^{-1/3}\right]$$

$$= [1.34 - 0.7454]$$

$$a = 0.596$$

$$b = \frac{\Omega_p}{2}\left[\mu^{1/N} + \mu^{-1/N}\right]$$

$$= 1.34 + 0.7454$$

$$b = 2.087$$

$$s_k = a\cos\phi_k + j\,b\sin\phi_k$$

$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \quad , k = 1,2,3$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{6} = \frac{6\pi}{6} = \pi$$

$$\phi_3 = \frac{\pi}{2} + \frac{5\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

$$s_1 = 0.596\cos\frac{2\pi}{3} + j\,2.087\sin\frac{2\pi}{3}$$

$$= -0.298 + j\,1.807$$

$$s_2 = 0.596\cos\pi + j\,2.087\sin\pi$$

$$= -0.596$$

$$s_3 = 0.596\cos\frac{4\pi}{3} + j\,2.087\sin\frac{4\pi}{3}$$

$$= -0.298 - j\,1.807$$

$$D_r = (s-s_1)(s-s_2)(s-s_3)$$

$$= (s+0.298-j1.807)(s+0.596)(s+0.298+j1.807)$$

$$= \left[(s+0.298)^2 + (1.807)^2\right](s+0.596)$$

$$= (s^2 + 0.596s + 0.088804 + 3.265249)(s+0.596)$$

$D_r = (s^2 + 0.596s + 3.354053)(s + 0.596)$

$N \rightarrow$ odd.

sub $s = 0$ in D_r.

$N_r = (3.354053)(0.596) = 1.999$

$\therefore$ The transfer fn.

$$H(s) = \frac{1.999}{(s^2 + 0.596s + 3.354053)(s + 0.596)}$$

$$= \frac{1.999}{s^3 + 0.596s^2 + 0.596s^2 + 3.354053s + 0.3552168 + 1.999}$$

$$H(s) = \frac{1.999}{s^3 + 1.192s^2 + 3.709269s + 1.999}$$

3. Determine the order & the poles of a Type-I low pass Chebychev filter that has a 1 dB ripple in the pass band and pass band freq. $\Omega_p = 1000\pi$ and stop band freq. of 2000π and an attenuation of 40 dB or more.

4. Design a Chebychev filter with a max. passband attenuation of 2.5 dB at $\Omega_p = 20$ rad/sec and the stop band attenuation of 30 dB at $\Omega_s = 50$ rad/sec.

Frequency transformation in analog domain:

Transformations are:

LP→ LP

LP→ HP

LP→ BP

LP→BR

Frequency Transformati

The freq. trans

design an LPF with

HPF, BPF & BSF from

($\Omega_c = 1$ rad/sec).

Low pass to Low pass :

$$s \rightarrow \frac{s}{\Omega_c}$$

Low pass to High pass :

$$s \rightarrow \frac{\Omega_c}{s}$$

Low pass to Band pass :

The transformation for converting a normalised LPF to a BPF with cut-off frequencies Ω_l, Ω_u can be accomplished by

$$s \rightarrow \frac{s^2 + \Omega_l \Omega_u}{s(\Omega_u - \Omega_l)}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$$

$$s \rightarrow \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{-\Omega_1^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)}$$

$$B = \frac{\Omega_2^2-\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)}$$

Low Pass to Band stop :

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l \Omega_u}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u} \qquad B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_l \Omega_u}$$

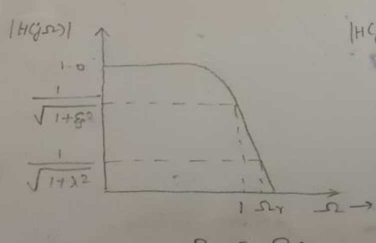
$$s \rightarrow \frac{s(\Omega_u-\Omega_l)}{s^2+\Omega_l\Omega_u}$$

$$\Omega_r = \min\{|A|, |B|\}$$

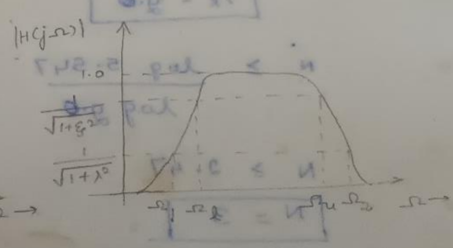
$$A = \frac{\Omega_1(\Omega_u-\Omega_l)}{-\Omega_1^2+\Omega_l\Omega_u}$$

$$B = \frac{\Omega_2(\Omega_u-\Omega_l)}{\Omega_2^2-\Omega_u\Omega_l}$$

Low pass to Band Pass Transformation :

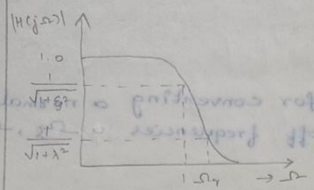


$$\Omega_r = \frac{\Omega_s}{\Omega_p}$$

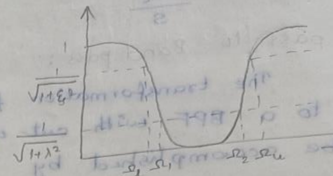


$$\Omega_r = \min\{|A|, |B|\}$$

Low pass to Band Stop :



$\Omega_r = \frac{\Omega_s}{\Omega_p}$



$\Omega_r = \min\{|A|, |B|\}$

1. For the given specifications, $\alpha_p = 3$ dB , $\alpha_s = 15$ dB, $\Omega_s = 1000$ rad/sec, $\Omega_p = 500$ rad/sec . Design a HPF.

$$N \geq \frac{\log A}{\log(1/k)}$$

$$A = \left[\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}\right]^{1/2}$$

$$= \left(\frac{10^{0.1 \times 15} - 1}{10^{0.1 \times 3} - 1}\right)^{1/2}$$

$$\boxed{A = 5.547}$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{500}{1000} = \frac{1}{2} = 0.5$$

$$\boxed{1/k = 2.0}$$

$$N \geq \frac{\log 5.547}{\log 2.0}$$

$$N \geq 2.47$$

$$\boxed{N = 3}$$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}}$$

$$= \frac{500}{(10^{0.1\times 3}-1)^{1/6}}$$

$$\Omega_c = 500.3959 \text{ rad/sec}$$

$$\boxed{H_a(s) = H(s)\Big|_{s \to \frac{\Omega_c}{s}}}$$ for HPF

HPF

$$H_a(s) = \frac{1}{\left(\frac{500.3959}{s}+1\right)\left(\frac{500.3959^2}{s^2}+\frac{500.3959}{s}+1\right)}$$

$$= \frac{s \cdot s^2}{(500.3959+s)(250396.0587+500.3959s+s^2)}$$

$$H_a(s) = \frac{s^3}{(s+500.3959)(s^2+500.3959s+250396.0587)}$$