DA-2.

1. Specifications :

Passband - 0-500 Hz.

stopband - 2-4 KHz.

Passband ripple - 3 dB.

stopband attenuation - 20 dB.

(i) Passband edge frequency, $\omega_p = \frac{2\pi f_p}{F_s}$; where f_p = passband freq
F_s = Sampling Freq.

$\therefore \omega_p = \frac{2\pi(500)}{F_s}$

$= \frac{\pi}{3}$ radians / sample.

similarily, stopband edge frequency, $\omega_s = \frac{2\pi f_s}{F_s}$

$= \frac{4\pi}{3}$ radians / sample

(ii) order $N = \frac{\log_{10}[10^{A/10} - 1 / 10^{S/10} - 1]}{2 \times \log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$; where, A = passband ripple
S = stopband attenuation

$= \frac{\log_{10}[10^{3/10} - 1 / 10^{20/10} - 1]}{2 \times \log_{10}\left(\frac{4\pi/3}{\pi/3}\right)}$

$= 3.96$

$\therefore \boxed{N = 4}$.

(iii) $\varepsilon_0 = \sqrt{10^{A/10} - 1} = 1.955$

$\therefore H(s) = \frac{1}{1 + 1.955 s^4}$

$\therefore H(z) = H(s) \Big|_{\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$ [using bilinear transformation

$= \frac{(2z^4 + 8z^3 + 12z^2 + 8z + 2)}{(z^4 + 1.586 z^3 + 1)}$

2. Given, $0.89 \leq |H(\omega)| \leq 1.0$; $0 \leq \omega \leq 0.2\pi$

$|H(\omega)| \leq 0.18$; $0.3\pi \leq \omega \leq \pi$

$\omega_{c1} = 0.1\pi$ and $|H(\omega)| = 0.89$ to 1.0 for $0 \leq \omega \leq 0.2\pi$

$\omega_{c2} = 0.4\pi$ and $|H(\omega)| \leq 0.18$ for $0.3\pi \leq \omega \leq \pi$

$\therefore \omega_{p1} = \frac{2}{\tan \frac{\omega_{c1} T}{2}} = 0.126$ radians / sample

$\omega_{p2} = \frac{2}{T} \tan \frac{\omega_{c2} T}{2} = 0.511$ radians / sample.

For $\omega_{p1} = 0.126$, $N_1 = 3$ and $\omega_{c1} = 0.250\pi$

For $\omega_{p2} = 0.511$, $N_2 = 5$ and $\omega_{c2} = 0.743\pi$

We use higher order filter,

$$\boxed{N = 5}.$$

$$\varepsilon_0 = \frac{1}{\sqrt{1-\delta^2}} \quad ; \quad \delta = 1 - 0.89 = 0.11$$

$$\therefore \varepsilon_0 = 1.016.$$

$$\therefore H(s) = \frac{1}{\left(1 + 1.016\left(\frac{s}{\omega_{c2}}\right)^5\right)}.$$

using bilinear transformation,

$$H(z) = H(s) \Big| \frac{2}{T}\left(\frac{z-1}{z+1}\right).$$

$$= \frac{1}{\left[1 + 1.016\left\{\tan \frac{\omega_c T}{2}\right\}\right]\left(2\,\frac{z-1}{z+1}\right)^5}$$

3. Given, $0.9 \leq |H(\omega)| \leq 1.0 \;;\; 0 \leq \omega \leq 0.25\pi$

$$|H(\omega)| \leq 0.24 \;;\; 0.5\pi \leq \omega \leq \pi$$

$\omega_{c1} = 0.125\pi$ and $|H(\omega)| = 0.9$ to 1.0 for $0 \leq \omega \leq 0.25\pi$

$\omega_{c2} = 0.75\pi$ and $|H(\omega)| \leq 0.24$ for $0.5\pi \leq \omega \leq \pi$

$$\therefore \omega_{p1} = \frac{2}{T}\tan\frac{\omega_{c1} T}{2} = 0.198 \text{ radians / sample.}$$

$$\omega_{p2} = \frac{2}{T}\tan\frac{\omega_{c2} T}{2} = 0.930 \text{ radians / sample.}$$

For $\omega_{p1} = 0.198$, $N_1 = 5$ and $\omega_{c1} = 0.383\pi$

For $\omega_{p2} = 0.930$, $N_2 = 8$ and $\omega_{c2} = 0.860\pi$

We choose, $\boxed{N = 8}$.

$$\varepsilon_0 = \sqrt{10^{0.1\alpha^n} - 1} \quad ; \quad \alpha^n = 1 - 0.9 = 0.1$$

$$= 0.470.$$

$$\therefore H(s) = \frac{1}{1 + 0.470\left(\frac{s}{\omega_{c_2}}\right)^{16}}$$

using bilinear transformation, $H(s) \Big| \frac{2}{T}\left(\frac{z-1}{z+1}\right)$.

$$\therefore H(z) = \frac{1}{\left(1 + 0.470\left\{\tan \frac{\omega_{c2} T}{2}\right\}\right)\left(2\left(\frac{z-1}{z+1}\right)^{16}\right)}$$

4. (a) $x(n) = a^n u(n) + b^n u(-n-1)$.

DTFT of $x(n)$ is:

$$X(e^{j\omega}) = \Sigma [a^n u(n) + b^n u(-n-1)] e^{-j\omega n}$$

$$= \Sigma a^n u(n) e^{-j\omega n} + \Sigma b^n u(-n-1) e^{-j\omega n}$$

$$= \Sigma a^n u(n) e^{-j\omega n} + \Sigma b^n e^{j\omega(n+1)}$$

$$= \Sigma (ae^{-j\omega})^n + e^{j\omega} \Sigma (be^{j\omega})^n.$$

Magnitude response is,

$$|X(e^{j\omega})| = |1/(1-ae^{-j\omega})| + |e^{j\omega}/(1-be^{j\omega})|$$

Phase response is,

$$\angle X(e^{j\omega}) = \angle[(ae^{-j\omega})^n] + \angle[e^{j\omega}(be^{j\omega})^n]$$

$$= \omega + \omega n(\arg(b) - \arg(a)).$$

(b) $x(n) = \{1, 3, 5, 2\}$.

$$\text{DTFT}[x(n)] = X(e^{j\omega}) = \Sigma x(n) e^{-j\omega n}$$

$$= 1 \cdot e^{-j\omega 0} + 3e^{-j\omega 1} + 5e^{-j\omega 2} + 2e^{-j\omega 3}$$

Magnitude response,

$$|X(e^{j\omega})| = \sqrt{(1^2+3^2+5^2+2^2)} = 39.$$

Phase response,

$$\angle X(e^{j\omega}) = \tan^{-1}(-3\sin\omega - 5\sin 2\omega - 2\sin 3\omega,\ 1 + 3\cos\omega + 5\cos(2\omega) + 2\cos(3\omega)).$$