Finite Impulse Response (FIR) filter

- linear phase
- less complexity
- less error

Q. Design a linear phase FIR low pass Filter using rectangular ~~written~~ window by taking 7 samples of window sequence & with a cutoff freq of 0.2 π rad/sample.

⇒ Desired ideal freq response for FIR LPF

$$H_d(e^{j\omega}) = e^{-j\omega\alpha}, \quad -\omega_c \le \omega \le +\omega_c$$
$$= 0, \quad \text{otherwise}$$

Take inverse fourier transform of $H_d(e^{j\omega})$

$$\boxed{h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}).e^{j\omega n}\, d\omega}$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$\boxed{h_d(n) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega}$$

$$h_d(n) = \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\omega(n-\alpha)}}{\quad}\right]^{\omega_c}$$

$\pi(n-\alpha)[\quad\quad]$

$$\boxed{h_d(n) = \frac{\sin w_c(n-\alpha)}{\pi(n-\alpha)}}$$ ~~$\forall n$~~ for all n, except $n=\alpha$

$n = \alpha$

L'hospital rule,

$$\lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A$$

$$\lim_{(n-\alpha)\to 0} \frac{\sin w_c(n-\alpha)}{\pi(n-\alpha)} = \boxed{\frac{w_c}{\pi} = h_d(n)} \text{ (when } n=\alpha)$$

$$\boxed{\alpha = \frac{N-1}{2}} = \frac{7-1}{2} = 3$$

Rectangular window, W_n

$W_R(n) = 1; \; n = 0 \text{ to } (N-1)$

$= 0;$ otherwise

$$\boxed{h(n) = h_d(n) \times W_R(n)}$$

$n=0, \; h(0) = \dfrac{\sin(0.2\pi(0-3))}{\pi(0-3)} = +0.1009$

$n=1, \; h(0) = \dfrac{\sin(0.2\pi(1-3))}{\pi(1-3)} = 0.1513$

$n=2, \; h(2) = \dfrac{\sin(0.2\pi(2-3))}{\pi(2-3)} = 0.1870$

$n=3, \; h(3) = \dfrac{w_c}{\pi} = 0.2$

Symmetry condition,

$$h(N-1-n) = h(n)$$

If $n=4$, $h(4) = h(7-1-4) = h(2) = 0.1870$

$n=5$, $h(5) = h(7-1-5) = h(1) = 0.1313$

$n=6$, $h(6) = h(7-1-6) = h(0) = 0.1009$

$$H(z) = Z\{h(n)\}$$

$$= \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) z^{0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6}$$

$$= h(0)[z^{0} + z^{-6}] + h(1)[z^{-1} + z^{-5}] + h(2)[z^{-2} \ldots + h(3) z^{-3}$$

=

Q Design a linear phase fir ~~fil~~ high pass using hamming window

$\omega_c = 0.8\pi$ r/sample

$N = 7$

→ Desired ideal freq response for FIR HPF,

$$H_d(e^{j\omega}) = e^{-j\omega\alpha} \quad , \quad -\pi \le \omega \le -\omega_c \quad \& \quad \omega_c \le \omega \le \pi$$

$$= 0 \quad ; \text{ otherwise.}$$

inverse fourier transform of $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} H_d(e^{j\omega}) e^{j\omega n} \, d\omega + \int_{\omega_c}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{(n-\alpha)j\omega} \, d\omega + \int_{\omega_c}^{\pi} e^{(n-\alpha)j\omega} \, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-\omega_c} + \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega_c}^{\pi}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{2j}\right]$$

$$= \cancel{\frac{1}{\pi(n-\alpha)}\left[\frac{\sin \omega_c(n-\alpha) - \sin \pi(n-\alpha)}{\pi(n-\alpha)}\right]}$$

$$h_d(n) = \cancel{\frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} - \frac{\sin \pi(n-\alpha)}{\pi(n-\alpha)}}$$

~~for all n, except $n=\alpha$~~

l'hospitals rule.

$$\lim_{n \to \alpha} \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} = \frac{\sin \pi(n-\alpha)}{\pi(n-\alpha)}$$

$$= \frac{\omega_c}{\pi}$$

$$h_d(n) = \frac{\sin \pi(n-\alpha)}{\pi(n-\alpha)} - \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}$$

for all n, exept $n = \alpha$

l'hospitals rule,

$$\lim_{n \to \alpha} \frac{\sin \pi(n-\alpha)}{\pi(n-\alpha)} - \lim_{n \to \alpha} \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}$$

$$\frac{\sin A\theta}{\theta} \qquad \boxed{1 - \frac{\omega_c}{\pi}} \quad \text{for } n = \alpha$$

$$W_{Ham}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \quad \text{for } (0 \to N-1)$$

$$N = 7, \alpha = \frac{N-1}{2} = \frac{7-1}{2} = 3$$

$$\boxed{h(n) = h_d(n) \times W_{ham}(n)}$$

$n=0, h(0) = 0.08[0 - 0.1009]$

$= 8.07 \times 10^{-3} = [-0.0080\cancel{8}]$

$n=1, h(1) = [0.54 - \cancel{0.46(\cos(}\ 0.23]$

$= 0.31\left[\frac{\sin\pi(-2)}{-2\pi} + \frac{\sin\pi(-1.6)}{2\pi}\right]$

$= 0.31[0 + 0.15]$

$= 0.04\cancel{7}$

$= \underline{\underline{0.047}}$

$n=2, h(2) = 0.54 + \cancel{0.46(}\ 0.23$

$= 0.77\left[\frac{\sin(\pi(-1)}{-\pi} + \cancel{\frac{\pi}{\pi}}\ 0.1871\right]$

$(0]$

$= 0.77[-0.1871]$

$= -0.1441$

$n=3, h(3) = \left[\cancel{\sin}\ 1 - \frac{0.8\pi}{\pi}\right] = 0.2$

by symmetric prop.

$n=4, h(4) = h(2) = -0.1441$

$n=5, h(5) = h(1) = 0.047$

$n=6, h(6) = h(0) = -0.0081$

taking Z transform

$H(z) = Z\{h(n)\}$

$= h(0)z^{0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3}$

$+ h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$

$= h(0)[1+z^{-6}] + h(1)[z^{-1}+z^{-5}]$

$+ h(2)[z^{-2}+z^{-4}] + h(3)z^{-3}$

$H(z) = -0.0081[1+z^{-6}] + 0.0459[z^{-1}+z^{-5}]$

$+ 0.144[z^{-2}+z^{-4}] + 0.2z^{-3}$

FAT

Dit-FFT - 13 marks question

DFT

No. of complex addition $= \frac{N(N-1)}{2}$

No of complex multiplication $= N^2$

FFT.

No. of complex addition $= N \log_2 N$

——— 〃 —— multi $= \frac{N}{2} \log_2 N$

~~For 5 marks~~

For $n = 8$, what is the reduction computational performance in DFT & FFT.

Q. Design an ideal high pass filter with freq. resp.

$H_d(e^{j\omega}) = 1 \; ; \; \pi/4 \le |\omega| \le \pi$

$= 0 \; ; \; |\omega| \le \pi/4$

find $h(n)$ using hanning, $N=11$. find $H(z)$

$H_d(e^{j\omega}) = e^{-j\omega\alpha} \; ; \; -\pi \le \omega \le -\omega_c, \; \omega_c \le \omega \le \pi$

$= 0 \; ; \;$ otherwise

taking inverse fourier transform of $H_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)} + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-\omega_c} + \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega_c}^{\pi}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin \pi(n-\alpha) - \sin \omega_c(n-\alpha)\right]$$

$$\boxed{h_d(n) = \frac{\sin(\pi(n-\alpha))}{\pi(n-\alpha)} - \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)}}$$

for all n, except $n=\alpha$,

For $n = \alpha$ $\qquad$ $N = 11$, $\alpha = \frac{11-1}{2} = 5$

$$\Rightarrow \boxed{h_d(n) = 1 - \frac{\omega_c}{\pi}}$$

$$\boxed{W_{han}(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right)}$$

$$\boxed{h(n) = h_d(n) \times W_{han}(n)}$$

$n=0$, $h(0) = \cancel{0.045}$ 0

$n=1$, $h(1) = \left[\frac{\sin(\pi(-4))}{\pi(-4)} - \frac{\sin(\pi/4(-4))}{\pi(-4)}\right](0.095)$

$= 0$

$n=2$, $h(2) = \left[\frac{\sin(\pi(-3))}{\pi(-3)} - \frac{\sin(\frac{\pi}{4})(-3)}{\pi(-3)}\right](0.345)$

$= -0.02588$

$n=3$, $h(3) = \left[\frac{\cancel{\sin(\pi(-2))}}{\pi(-2)} - \frac{\sin(\frac{\pi}{4}(-2))}{-\pi(2)}\right](0.206)$

$= -0.0327$

$n=4$, $h(4) = \left[\frac{-\sin(\frac{\pi}{4}(-1))}{-\pi}\right][0.904]$

$= -0.20347$

$n=5,\ h(5) = [\ \ \ \ ] \ 1-\omega_c/\pi$

$n=6,\ h(6) = h(4) = -0.20347$

$n=7,\ h(7) = h(3) = -0.0327$

$h(8) = h(2) = -0.0259$

$h(9) = h(1) = 0$

$h(10) = h(0) = 0$

taking z transform

$$H(z) = Z\{h(n)\}$$

$$= \cancel{h(0)z^{0}} + \cancel{h(1)z^{-1}} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8} + \cancel{h(9)z^{-9}} + \cancel{h(10)z^{-10}}$$

$$= h(2)(z^{-2} + z^{-8}) + h(3)(z^{-3} + z^{-7}) + h(4)[z^{-4} + z^{-6}] + \cancel{h(5)}\ 0.75z^{-5}$$

$$\boxed{H(z) = -0.0259(z^{-2} + z^{-8}) - 0.0327(z^{-3} + z^{-7}) - 0.20347(z^{-4} + z^{-6}) + 0.75z^{-5}}$$

Q. Desired freq response of a low pass filter

$$H_d(e^{j\omega}) = e^{-j3\omega}, \quad |\omega| \le 3\pi/4$$
$$= 0, \quad \frac{3\pi}{4} \le |\omega| \le \pi$$

Determine the freq. response of FIR filter using hamming window with $\underline{N=7}$

→ taking inverse fourier transform,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{3\pi/4} e^{j\omega(n-3)}\, d\omega$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\omega(n-3)}}{j(n-3)}\right)_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\frac{3\pi}{4}(n-3)} - e^{-j\frac{3\pi}{4}(n-3)}}{j(n-3)}\right]$$

$$= \frac{1}{\cancel{2\pi 2}\ \pi(n-3)}\left[\sin\frac{3\pi}{4}(n-3)\right]$$

= for all n., except $n = \alpha$

If

$$h_d(n) = \frac{3}{4} = 0.75 \ (n = \alpha)$$

$\alpha = 3$

$$w_{ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$$

$$h(n) = h_d(n) \times w_{ham}(n)$$

~~hto~~ $n=0,\ h(0) = \dfrac{\sin \frac{3\pi}{4}(-3)}{\pi(-3)} \times \cos\left(\dfrac{2\pi n}{6}\right)$

$= 0.07502 = 0.006$

$n=1,\ h(1) = \dfrac{\sin\left(\frac{3\pi}{4}(-\cancel{8}2)\right)}{\pi(-2)} \times \cos\left(\dfrac{2\pi}{6}\right)$

$= -0.049$

$= \cancel{0.1591}\ -0.07955 =$

$n=2,\ h(2) = \left[\dfrac{\sin\left(\frac{3\pi}{4}(-1)\right)}{-\pi} \times \cos\left(\dfrac{4\pi}{6}\right)\right]$

$= 0.225 \times (-0.5) = -0.1125 = 0.173$

$n=3,\ h(3) = 0.75$

$n=4,\ h(4) = h(7-1-4) = h(2) = -0.1125$

$n=5,\ h(5) = h(7-1-5) = h(1) = -0.07955$

$n=6,\ h(6) = h(7-1-6) = h(0) = 0.07502$

$$H(z) = Z\{h(n)\}$$

$$= h(0)z^{0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$\boxed{H(z) = 0.07502(1+z^{-6}) + 0.07955(z^{-1}+z^{-5}) + -0.1125(z^{-2}+z^{-4}) + 0.75z^{-3}}$$

[illegible]

[illegible]

[illegible]

[illegible]

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[illegible]

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

[illegible]

Direct Sum

[illegible]

IIR filter Structures

Obtain the direct form I, direct form II, cascade, parallel form re of LTI governed by the equation $y(n) = \frac{-3}{8} y(n-1) + \frac{3}{32} y(n-2) + \frac{1}{64} y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$

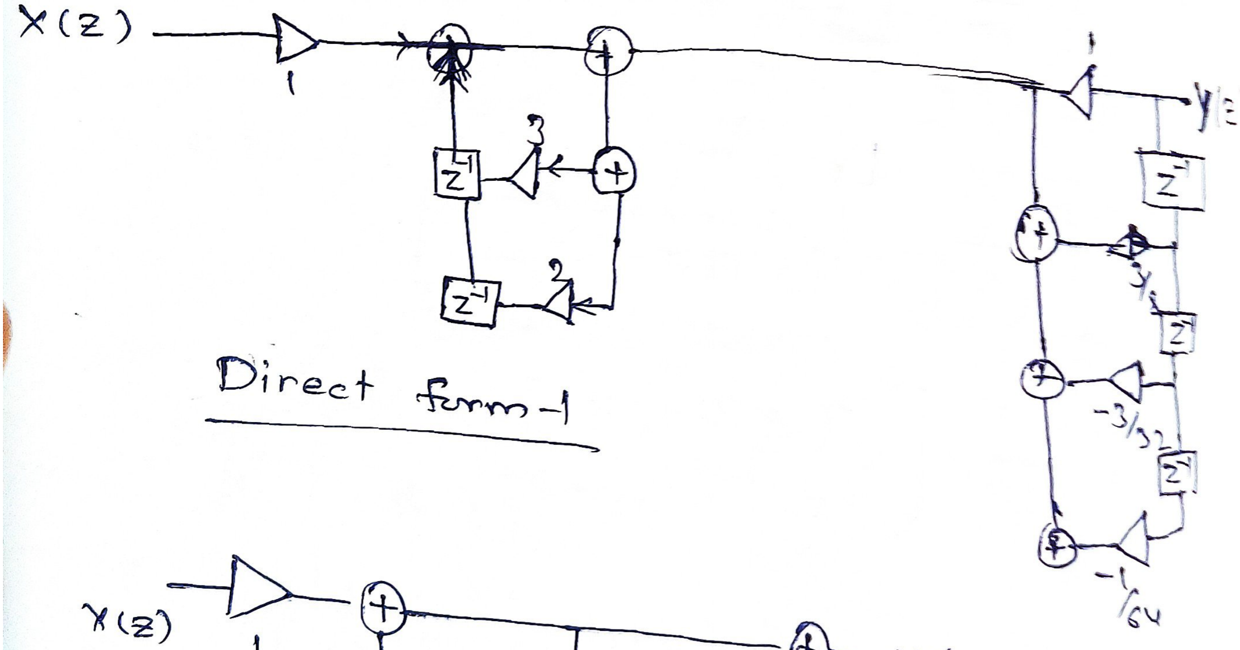
$\Rightarrow$

take z transform on both sides,

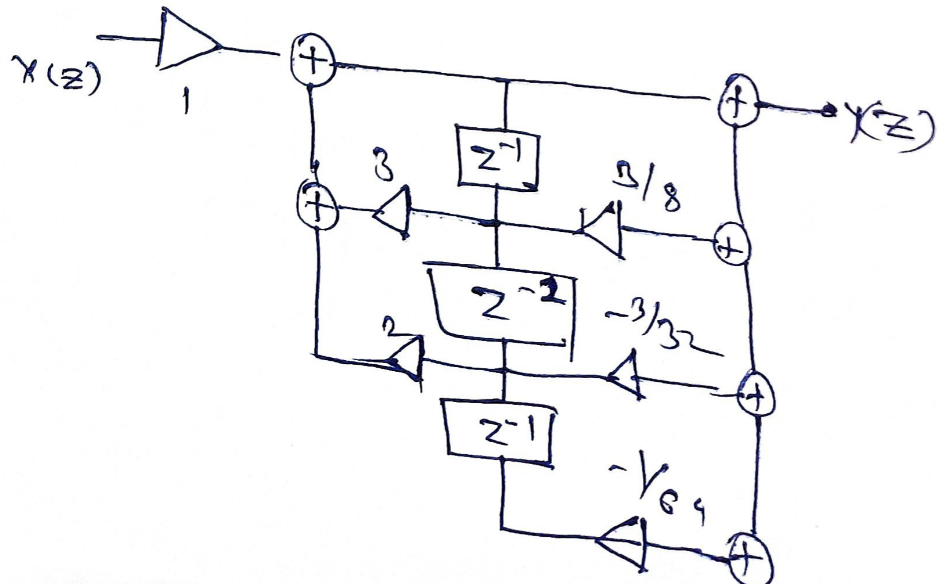
$$Y(z) = \frac{-3}{8} Y(z) z^{-1} + \frac{3}{32} Y(z) z^{-2} + \frac{1}{64} Y(z) z^{-3} + X(z) + 3X(z)z^{-1} + 2X(z)z^{-2}$$

$$\Rightarrow Y(z)\left[1 + \frac{3}{8} z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}\right] = X(z)\left[1 + 3z^{-1} + 2z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{\left[1 + \frac{3}{8} z^{-1} - \frac{3}{32} z^{-2} - \frac{1}{64} z^{-3}\right]}$$



Direct form -1



Q. $H(z) = \dfrac{2z^3 - 4z^2 + 11z - 8}{(z-8)(z^2 - z + 3)} = \dfrac{Y(z)}{X(z)}$

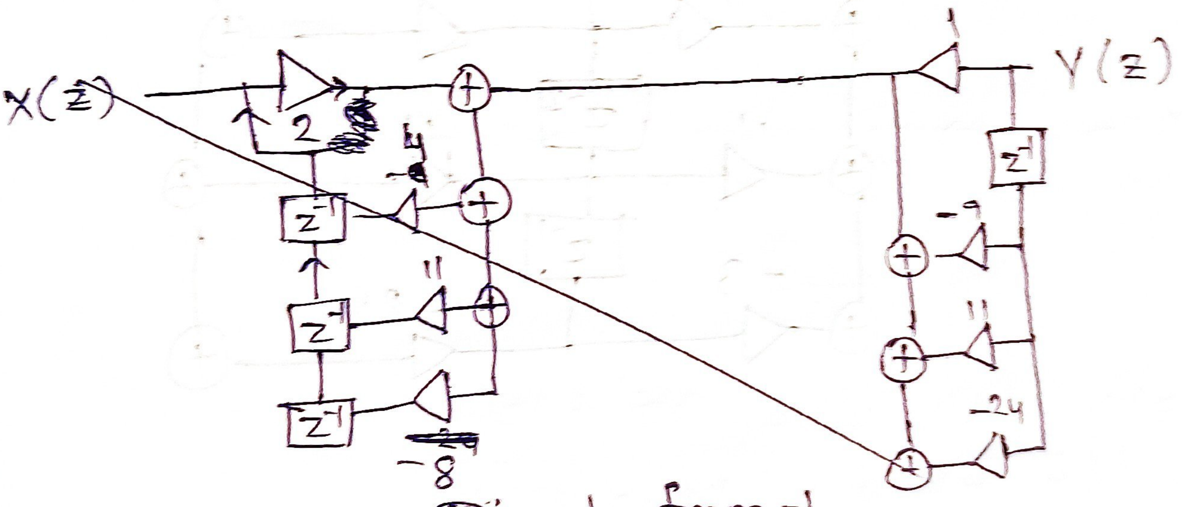
$$= \frac{2z^3 - 4z^2 + 11z - 8}{z^3 - z^2 - 3z - 8z^2 + 8z + 24}$$

$$= \frac{2z^3 - 4z^2 + 11z - 8}{z^3 - 9z^2 + 5z + 24}$$

$$= \frac{z^3[2 - 4z^{-1} + 11z^{-2} - 8z^{-3}]}{z^3[1 - 9z^{-1} + \overset{11}{5}z^{-2} + 24z^{-3}]} = \frac{Y(z)}{X(z)}$$

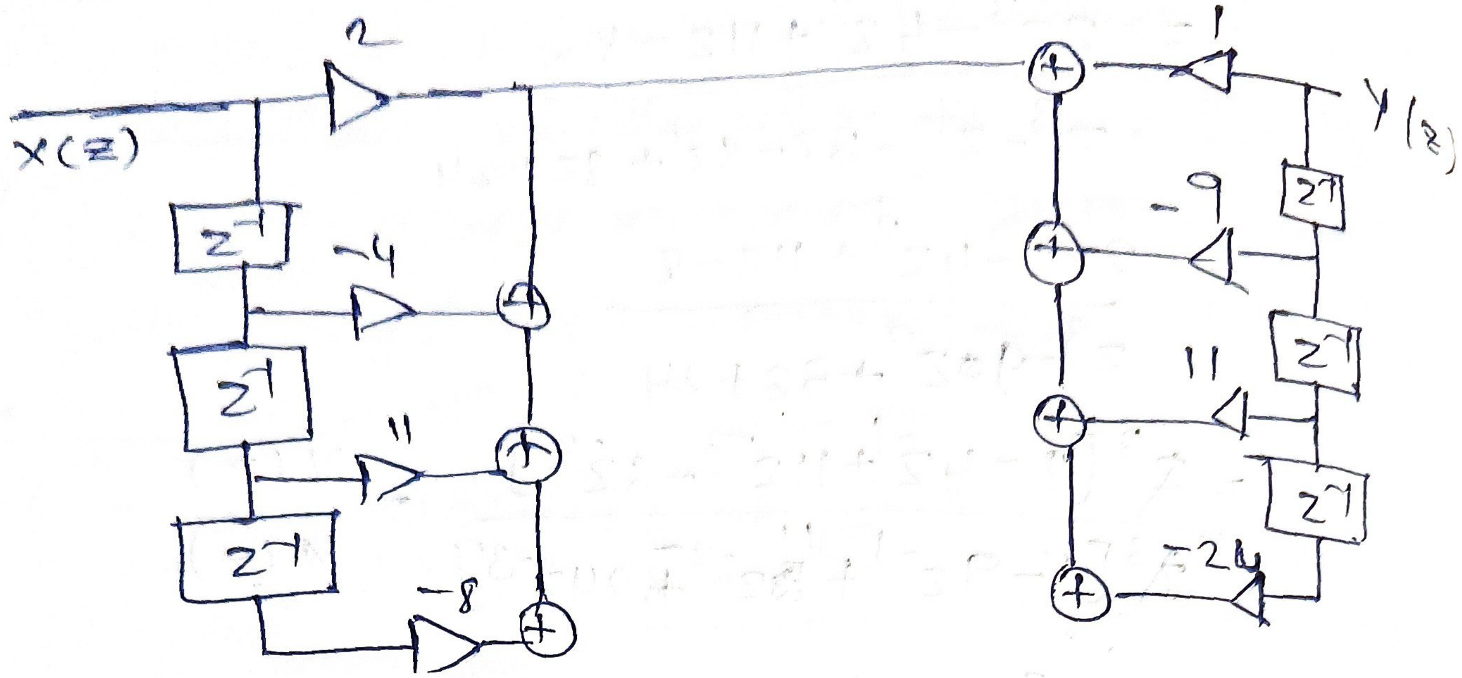
$$= \frac{8z^{-3} - 11z^{-2} + 4z^{-1} - 1}{+24z^{-3} - 11z^{-2} + 9z^{-1} - 1} = \frac{Y(z)}{X(z)}$$

$$Y(z)[1 - 9z^{-1} + 11z^{-2} - 24z^{-3}] = X(z)[2 - 4z^{-1} + 11z^{-2} - 8z^{-3}]$$

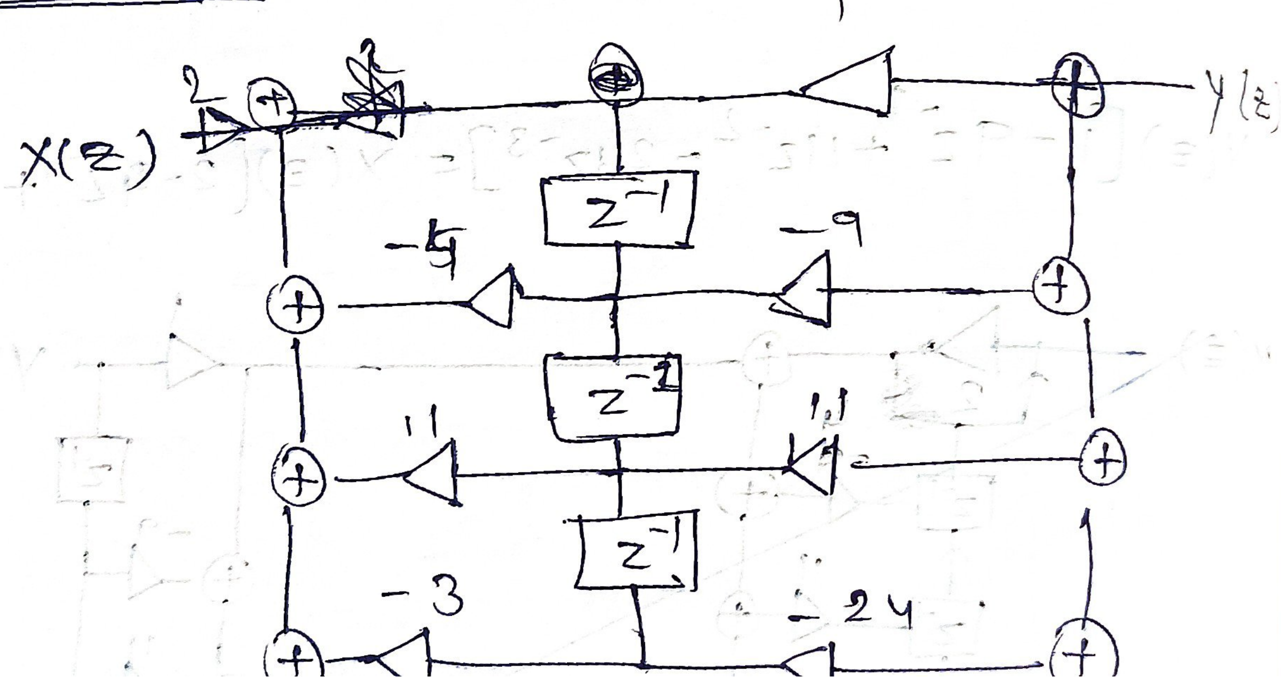


Direct form-1

Direct form-1



Direct form 2



Q. Find the direct form 1 & 2 for the difference equation

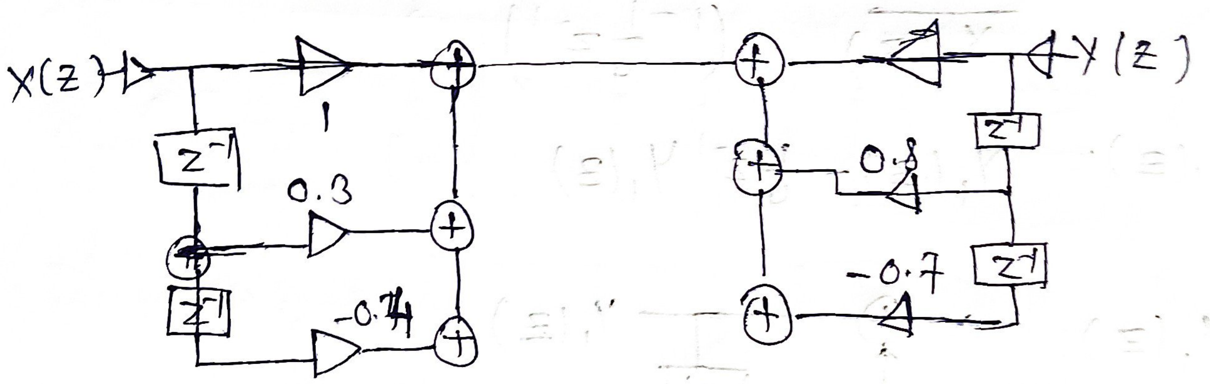
$$y(n) = x(n) + 0.3x(n-1) - 0.4x(n-2) - 0.8y(n-1) + 0.7y(n-2)$$

$$y(n) + 0.8y(n-1) - 0.7y(n-2) = x(n) + 0.3x(n-1) - 0.4x(n-2)$$

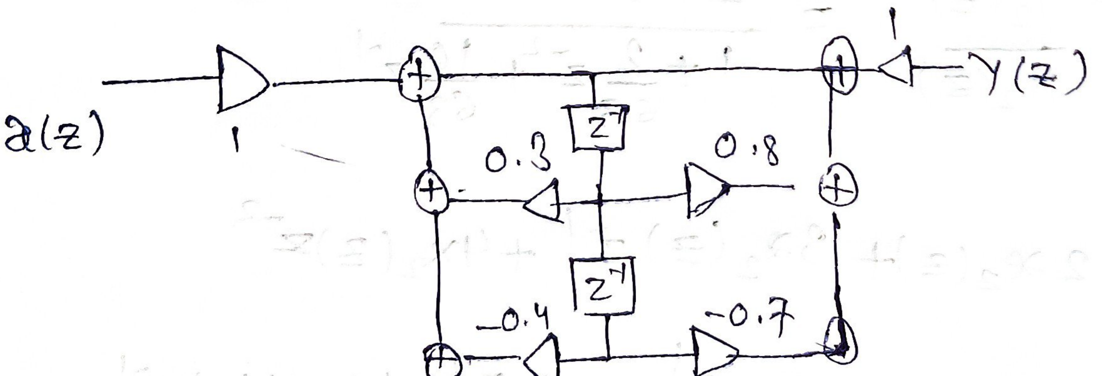
taking inverse z transform

$$Y(n)\left[1 + 0.8z^{-1} - 0.7z^{-2}\right] = X(n)\left[1 + 0.3x^{-1} - 0.4x^{-2}\right]$$

Direct form -1



Direct form-2



Q. open the cascade realisation of the system

$$H(z) = \frac{2+3z^{-1}+4z^{-2}}{\left(1+\frac{1}{7}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)\left(1+\frac{1}{9}z^{-1}\right)}$$

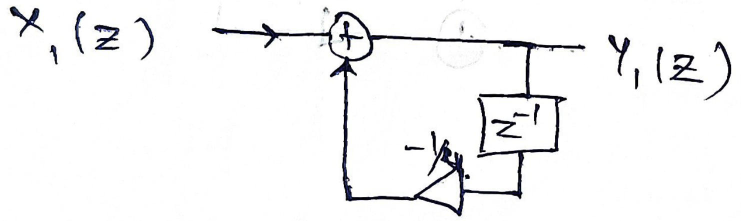
find the roots

$$\Rightarrow \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}\left[\frac{2+3z^{-1}+4z^{-1}}{\left(1+\frac{1}{7}z^{-1}\right)\left(1+\frac{1}{9}z^{-1}\right)}\right]$$

$$H(z) = H_1(z).H_2(z)$$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}$$

$$X_1(z) = Y_1(z) - \frac{1}{4}z^{-1}Y_1(z)$$

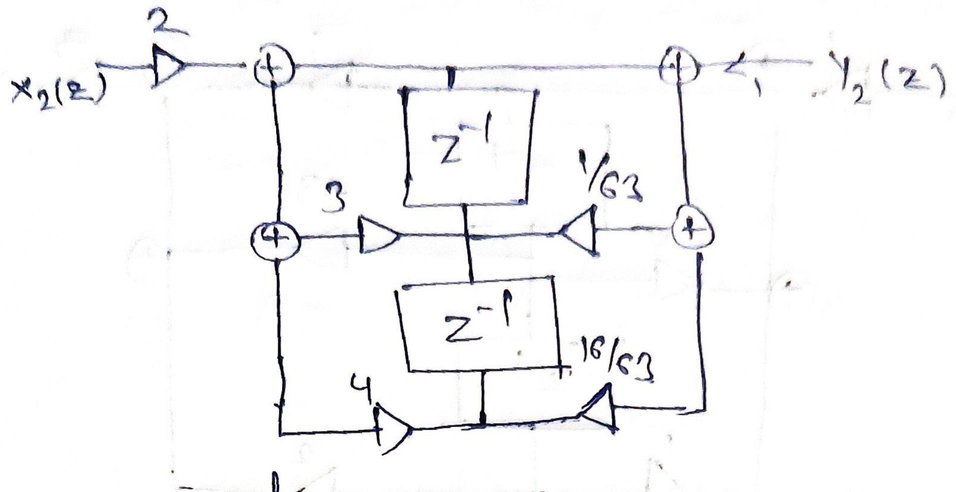


$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = \frac{2+3z^{-1}+4z^{-2}}{1+\frac{2}{63}z^{-2}+\frac{16}{63}z^{-1}}$$

$$\Rightarrow 2x_2(z) + 3x_2(z).z^{-1} + 4X_2(z)z^{-2}$$

$$= Y_2(z) + \frac{2}{63}Y_2(z)z^{-2} + \frac{16}{63}Y(z)z^{-1}$$

direct form 2 :



form 2

Q. The transfer function of a system is given by

$$H(z) = \frac{(2-z^{-1})(1-z^{-1})^2}{(1-2z^{-1})(5-3z^{-1}+2z^{-2})}$$

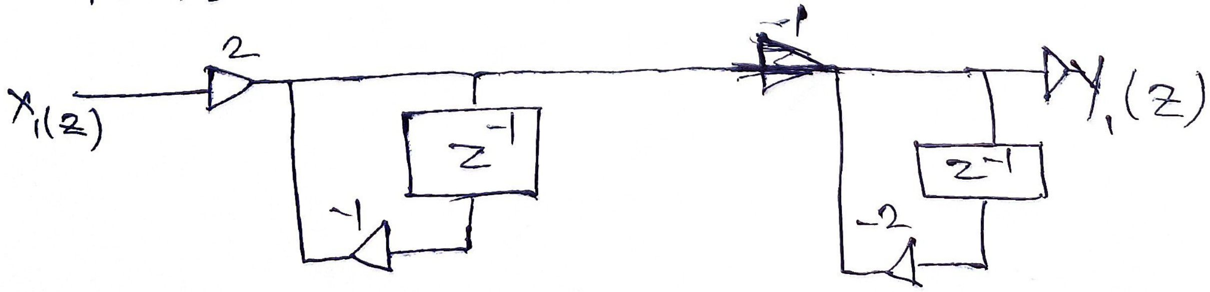
realise the system in cascade & parallel.

$$H(z) = \frac{2-z^{-1}}{(1-2z^{-1})} \cdot \frac{(1-z^{-1})^2}{(5-3z^{-1}+2z^{-2})}$$

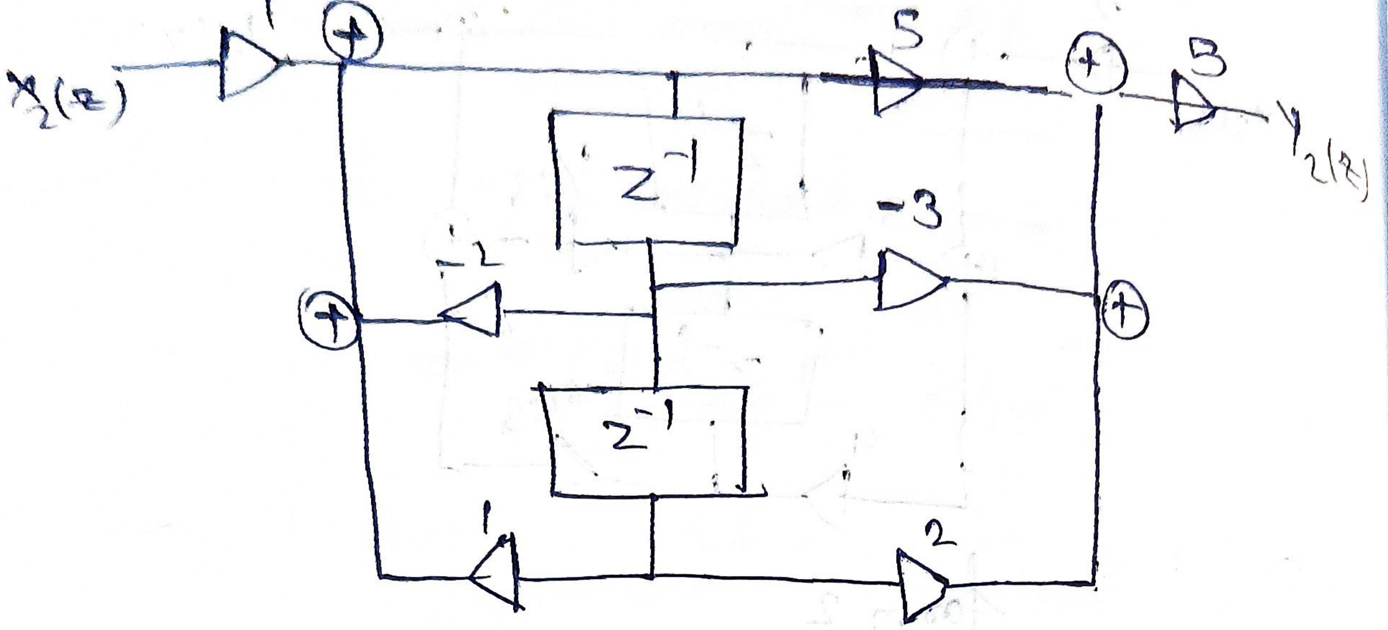
$(H_1(z))$ $\quad$ $H_2(z)$

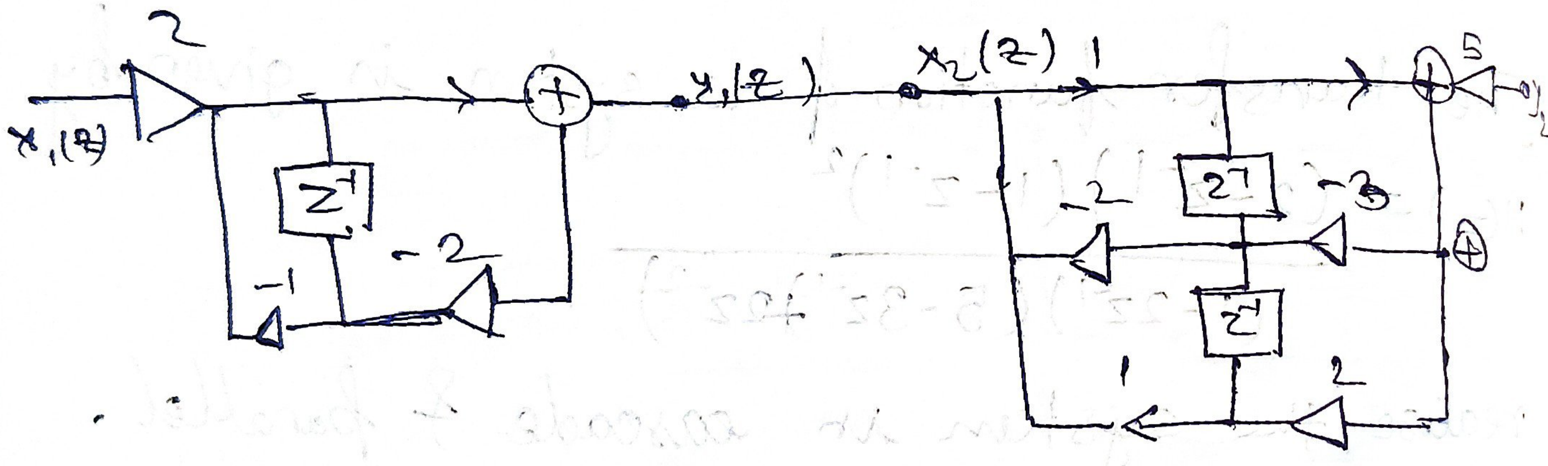
$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = \frac{2-z^{-1}}{1-2z^{-1}}$$

$$X_1(z)[2-z^{-1}] = Y_1(z)[1-2z^{-1}]$$



$$H_2(z) = \frac{1 + z^{-2} - 2z^{-1}}{5 - 3z^{-1} + 2z^{-2}} = \frac{Y_2(z)}{X_2(z)}$$





$$H(z) = \frac{(2-z^{-1})(1+z^{-2}-2z^{-1})}{(1-2z^{-1})(5-3z^{-1}+2z^{-2})}$$

$$= \frac{(2+2z^{-2}-4z^{-1}-z^{-1}-z^{-3}+2z^{-2})}{(5-3z^{-1}+2z^{-2}-10z^{-1}+6z^{-2}-4z^{-3})}$$

$$\frac{Y(z)}{X(z)} = \frac{(2 \cancel{+} -5z^{-1}+4z^{-2}-z^{-3})}{(5-13z^{-1}+8z^{-2}-4z^{-3})}$$

$$\begin{array}{r|l} & 0.4 \\ \hline 5-13z^{-1}+8z^{-2}-4z^{-3} & 2-5z^{-1}+4z^{-2}-z^{-3} \end{array}$$

$$H(z) = 0.4 + z^{-1}\frac{(0.2+0.8z^{-1}+0.6z^{-2})}{(1-2z^{-1})(5-3z^{-1}+2z^{-2})}$$

$$\frac{(0.2+0.8z^{-1}+0.8z^{-2})}{(1-2z^{-1})(5-3z^{-1}+2z^{-2})} = \frac{A}{1-2z^{-1}} + \frac{B+Cz^{-1}}{5-3z^{-1}+2z^{-2}}$$

$$A = \frac{3}{16} \quad ; \quad B = \frac{-59}{80} \quad ; \quad C = \frac{-9}{80}$$

$$H(z) = 0.4 + \frac{3}{16} z^{-1} + \frac{-\frac{59}{80} - \frac{9}{80} z^{-1}}{5 - 3z^{-1} + 2z^{-2}}$$

$$= 0.4 + H_1(z) + H_2(z)$$