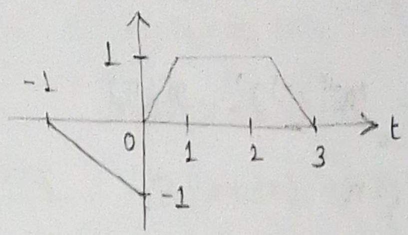
DA-1

1. For the given signals plot (i) $x(2t)\, y(0.5t+1)$

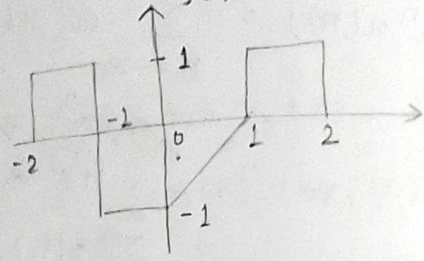
(ii) $x(4-t)\, y(2-t)$

(a)

x(t)

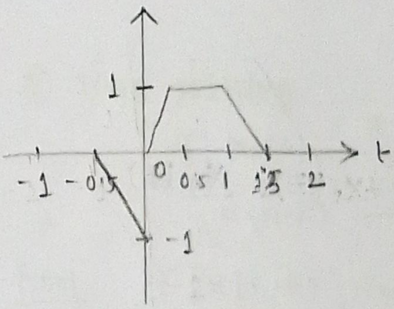


y(t)

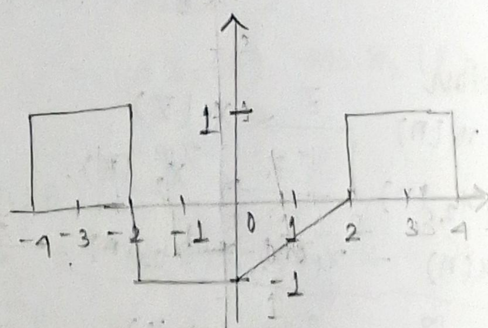


(i) $x(2t)\, y(0.5t+1)$

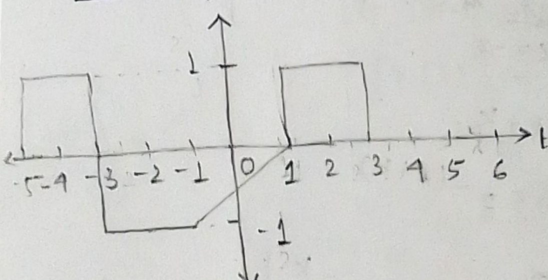
$x(2t)$



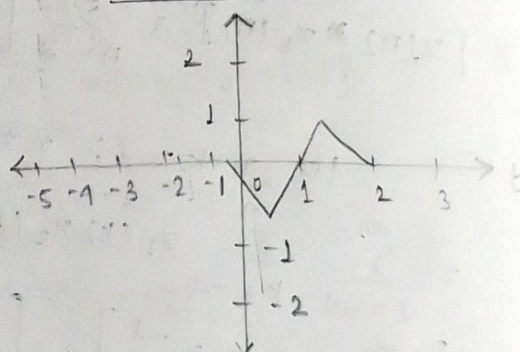
$y(0.5t)$



$y(0.5t+1)$



$x(2t)\, y(0.5t+1)$



$t=0,\ y(0.5t+1) = y(1) = 0$

$t=1,\ y(0.5+1) = y(1.5) =$

$t=2,\ y(0.5\times 2+1) = y(2) = 1$

$t=3,\ y(0.5\times 3+1) = y(2.5)$

$t=4,\ y(0.5\times 4+1) = y(3) = 1$

$t=0,\ x(2.0) = x(0) = -1$

$t=1,\ x(2.1) = x(2) = 1.$

$t=0.5,\ x(2.0.5) = x(1) = 1$

$t=-0.5,\ x(-1) = 0$

$t=1.5,\ x(2.1.5) = x(3) = 0$

$t=-1,\ x(-2)$

$y(t/2)$

$t=0,\ x(0) = -1$

$t=1,\ x(0.5) =$

$t=2,\ x(1) = 1$

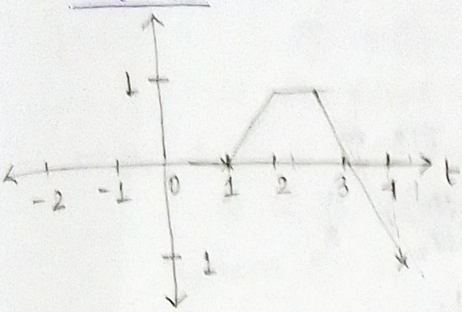
$t=4,\ x(2) = 1$

$t=3,\ x($

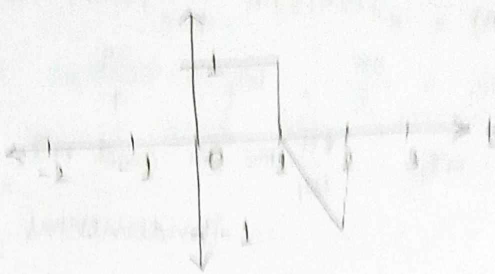
$t=-2,\ x(-1) =$

(ii) $x(4-t)\,y(2-t)$.

$x(4-t)$



$y(2-t)$



$t=0,\ x(4-0) = x(4)$

$t=1,\ x(4-1) = x(3) = 0$

$t=2,\ x(4-2) = x(2) = 1$

$t=3,\ x(4-3) = x(1) = 1$

$t=4,\ x(4-4) = x(0) = -1$

$t=-1,\ x(4+1) = x(5)$

$t=-2,\ x(4+2) = x(6)$

$t=0,\ y(2-0) = y(2) = 1$

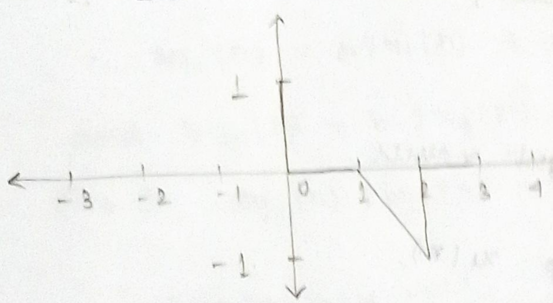
$t=-2,\ y(2+2) = y(4)$

$t=-1,\ y(2+1) = y(3)$

$t=1,\ y(2-1) = y(1) = 1$

$t=2,\ y(2-2) = y(0) = 1$

$x(4-t)\,y(2-t)$



2. Check periodic or not. Find fundemental period if signal is periodic.

(i) $x(n) = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$

$\rightarrow \quad \omega_1 = \frac{2\pi}{3}, \quad \omega_2 = \frac{3\pi}{4}.$

$\therefore N_1 = \frac{2\pi}{\omega_1}.m = 3m$ and $N_2 = \frac{2\pi}{\omega_2}m = 8/3.m$

$\therefore$ Fundemental period $= \frac{N_1}{N_2} = \frac{9}{8}.$

$N = 8N_1 = 9N_2 = 24.$

(ii) $x(n) = 2\cos 3\pi n + 7\cos 9n$

$\rightarrow \quad \omega_1 = 3\pi, \quad \omega_2 = 9$

$\therefore N_1 = \frac{2\pi}{\omega_1}.m = \frac{2}{3}.m$ and $N_2 = \frac{2\pi}{\omega_1}.m = \frac{2\pi}{9}.m$

$\therefore$ Fundemental period $= \frac{N_1}{N_2} = \frac{3}{\pi}.$

$\therefore N = \pi N_1 = 3N_2 = \frac{2}{3}\pi.$

3. Check whether the following systems are static / dynamic, causal / linear / non-linear, time variant / time invariant, causal / non-causal.

① $y(n) = x(n).x(n-1).$

It depends on past input vaules

$\therefore$ It is dynamic.

when o/p $y_1(n)$ and i/p $x_1(n)$,

$y_1(n) = x_1(n).x_1(n-1).$

when o/p is $y_2(n)$ and i/p $x_2(n)$,

$y_2(n) = x_2(n).x_2(n-1)$

$\therefore a y_1(n) = a x_1(n) x_1(n-1)$ —— ①

$b y_2(n) = b x_2(n) x_2(n-1)$ —— ②

① + ②, $a y_1(n) + b y_2(n) = a x_1(n) x_1(n-1) + b x_2(n) x_2(n-1)$ —— (a)

and, $T[a x_1(n) + b x_2(n)]$

$= a^2 x_1(n).x_1(n-1) + ab x_2(n).x_2(n-1)$ —— (b)

$\because$ (a) $\neq$ (b)

$\therefore$ It is a non linear system.

Now, $T[x(n-N)] = x(n-N).\ x(n-N-1)$ —①

and, $y(n-N) = x(n-N)\, x(n-N-1)$ —②

$\therefore$ ① = ②.

$\therefore$ It is time invariant system.

$\because$ It depends on only past values and not future,

$\therefore$ It is a causal system.

$\therefore$ System ① is : Dynamic
non - Linear
Time invariant
causal.

② $y(n) = 2x(n) + \dfrac{1}{x(n-1)}$.

It depends on past input values

$\therefore$ It is dynamic system.

When o/p is $y_1(n)$ and i/p is $x_1(n)$,

$y_1(n) = 2x_1(n) + \dfrac{1}{x_1(n-1)}$

$\therefore\ ay_1(n) = a2x_1(n) + \dfrac{a}{x_1(n-1)}$ —①.

and $by_2(n) = b.2x_2(n) + \dfrac{b}{x_2(n-1)}$ —②.

$\therefore$ ① + ②, $ay_1(n) + by_2(n) = a\left(2x_1(n) + \dfrac{1}{x_1(n-1)}\right) + b\left(2x_2(n) + \dfrac{1}{x_2(n-1)}\right)$ —(a)

and, $T[ax_1(n) + bx_2(n)]$

$= 2[a x_1(n) + \dfrac{b}{x_1(n-1)}] + [\dfrac{1}{x_1(n)} + \dfrac{1}{b x_2(n-1)}]$. —(b)

$\because$ (a) $\neq$ (b) non-linear

$\therefore$ It is a ~~time invariant~~ system.

Now, $T[x(n-N)] = 2x(n-N) + \dfrac{1}{x(n-N-1)}$ —①

and $y(n-N) = 2x(n-N) + \dfrac{1}{x(n-N-1)}$ —②

$\because$ ① = ②.

$\therefore$ It is a time invariant system.

$\because$ It depends only on past and present values and not future,

$\therefore$ It is a causal system.

$\therefore$ System ② is : Dynamic
non-Linear
Time invariant
causal

③ $y(n) = nx(n)$

$\because$ It depends on present values.

$\therefore$ It is a static system.

Now, $ay_1(n) = anx_1(n)$ —— ①

and, $by_2(n) = bnx_2(n)$ —— ②.

① + ②, $ay_1(n) + by_2(n) = anx_1(n) + bnx_2(n)$ —— (a)

also, $T[ax_1(n) + bx_2(n)]$

$= anx_1(n) + bnx_2(n)$ —— (b)

$\because$ (a) = (b).

$\therefore$ It is a ~~time invariant~~ linear system.

Now, $T[x(n-N)] = (n-N)\,x(n-N)$

and $y(n-N) = (n)\,x(n-N)$

$\therefore$ It is a time variant system.

$\because$ It depends on present values,

$\therefore$ It is a causal system.

$\therefore$ System ③ is : Static
Linear
Time variant
causal.

④ $y(n) = x^2(n)$.

$\because$ It depends on only present values,

$\therefore$ It is a static system.

Now, $ay_1(n) = ax_1^2(n)$ —— ①

$by_2(n) = bx_2^2(n)$ —— ②

$\therefore$ ① + ②, $ay_1(n) + by_2(n) = ax_1^2(n) + bx_2^2(n)$

also, $T[ax_1(n) + bx_2(n)]$

$= ax_1^2(n) + bx_2^2(n)$

$\therefore$ It is a linear system.

Now, $T[x(n-N)] = x^2(n-N)$

and $y(n-N) = x^2(n-N)$

$\therefore$ It is time invariant system.

$\because$ It depends only on present values

It is a causal system.

$\therefore$ System ④ is : Static

Linear

Time invariant

Causal.

4. Determine Z-transform :

① $x(n) = n^2 u(n)$.

$$Z[x(n)] = X(z) = \sum_{n=0}^{\infty} n^2 . z^{-n} \quad \left[u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}\right].$$

$$= [0 + 1^2 + 2^2 + 3^2 + \dots][z^{-1} + z^{-2} + \dots]$$

$$= [1^2 + 2^2 + 3^2 + \dots][z^{-1} + z^{-2} + \dots]$$

$$= \left[\frac{n(n+1)(2n+1)}{6}\right]\left[z^{-1} + z^{-2} + \dots\right]$$

② $x(n) = na^n u(n)$. $\qquad Z[a^n u(n)] = \frac{z}{z-a}, \; |z| > |a|$

$$\cancel{Z[x(n)] = X(z) = \sum_{n=0}^{\infty} na^n z^{-n}}$$

$$\cancel{= \sum_{n=0}^{\infty} n(az^{-1})^n}$$

$$\therefore Z[na^n u(n)] = -z . \frac{d}{dz}\left(\frac{z}{z-a}\right) \quad \text{[using differentiation property].}$$

$$= -z . \left(\frac{-a}{(z-a)^2}\right)$$

$$= \frac{az}{(z-a)^2}.$$

③ $x(n) = nu(n)$

$$\cancel{Z[x(n)] = X(z) = \sum_{n=0}^{\infty} nz^{-n}}$$

$$\cancel{= \sum_{n=0}^{\infty} 0 + z^{-1} + 2z^{-2} + 3z^{-3} + \dots}$$

$$Z[u(n)] = 1 + z^{-1} + z^{-2} + \dots$$

$$\therefore Z[nu(n)] = -z . \frac{d}{dz} . (1 + z^{-1} + z^{-2} + \dots) \quad \text{[using differentiation property].}$$

$$= -z[0 - z^{-2} - 2z^{-3} - \dots]$$

$$= 0 + z^{-1} + 2z^{-3} + \dots = z^{-2} + 2z^{-3} + \dots$$