The average power of a discrete-time signal $x(n)$ is ...

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \tag{1.61}$$

A signal is an energy signal, if and only if the total energy of the signal is finite. For an energy signal $P = 0$. Similarly the signal is said to be power signal if the average power of the signal is finite. For a power signal $E = \infty$. The signals that do not satisfy above properties are neither energy nor power signals.

Example 1.4 Determine the values of power and energy of the following signals. Find whether the signals are power, energy or neither energy nor power signals.

(i) $x(n) = \left(\frac{1}{3}\right)^n u(n)$ (ii) $x(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)}$

(iii) $x(n) = \sin\left(\frac{\pi}{4}n\right)$ (iv) $x(n) = e^{2n}u(n)$

Solution

(i) Given $x(n) = \left(\frac{1}{3}\right)^n u(n)$

The energy of the signal

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} \left[\left(\frac{1}{3}\right)^n\right]^2$$

$$\boxed{\because u(n) = 1 \text{ for } n \geq 0 \\ = 0 \text{ for } n < 0}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$\boxed{1 + a + a^2 + \ldots + \infty = \frac{1}{1-a}}$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

The power $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{1 - \left(\frac{1}{9}\right)^{N+1}}{1-\frac{1}{9}}\right]$$

$$= 0$$

The energy is finite and power is zero. Therefore, the signal is an energy signal.

(ii) $x(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}$

$$E = \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)} \right|^2 \qquad \because |e^{j(\omega+\theta)}| = 1$$

$$= \sum_{n=-\infty}^{\infty} 1 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)} \right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}(2N+1) = 1 \qquad \boxed{\sum_{n=-N}^{N} 1 = 2N+1}$$

The energy is infinite and power is finite. Therefore, the signal is a power signal.

(iii) $x(n) = \sin\left(\frac{\pi}{4}n\right)$

$$E = \sum_{n=-\infty}^{\infty} \left|\sin^2\left(\frac{\pi}{4}n\right)\right| = \sum_{n=-\infty}^{\infty} \left[\frac{1-\cos\left(\frac{\pi}{2}n\right)}{2}\right] = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin^2\left(\frac{\pi}{4}n\right)\right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos\frac{\pi}{2}n}{2} = \frac{1}{2} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \frac{1}{2} \qquad \boxed{\sum_{n=-N}^{N} 1 = 2N+1}$$

The energy is infinite and the power is finite. Therefore, the signal is a power signal.

(iv) $x(n) = e^{2n}u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} e^{4n} = 1 + e^4 + e^8 + \cdots + \infty = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Example 2.9 Find the z-transform of the sequence $x(n) = \left(\frac{1}{3}\right)^{n-1} u(n-1)$

Solution We know the z-transform of the sequence $\left(\frac{1}{3}\right)^{n} u(n)$ is

$$Z\left[\left(\frac{1}{3}\right)^{n} u(n)\right] = \frac{z}{z-\frac{1}{3}}$$

Using time shifting property we have

$$Z[x(n-1)] = z^{-1}X(z)$$

Similarly

$$Z\left[\left(\frac{1}{3}\right)^{n-1} u(n-1)\right] = z^{-1}\frac{z}{z-\frac{1}{3}} = \frac{1}{z-\frac{1}{3}}$$

i.e.,
$$Z\left[\left(\frac{1}{3}\right)^{n-1} u(n-1)\right] = \frac{1}{z-\frac{1}{3}}$$

(iii) $x(n) = (-1)^n \cos\left(\frac{n\pi}{3}\right) u(n)$

$$= (-1)^n \left[\frac{e^{j(\pi/3)n} + e^{-j(\pi/3)n}}{2}\right] u(n)$$

$$X(z) = \sum_{n=0}^{\infty} \frac{(-1)^n e^{j(n\pi/3)} z^{-n}}{2} + \sum_{n=0}^{\infty} \frac{(-1)^n e^{-jn\pi/3} z^{-n}}{2}$$

$$= \sum_{n=0}^{\infty} \frac{(-e^{j\pi/3} z^{-1})^n}{2} + \sum_{n=0}^{\infty} \frac{(-e^{-j\pi/3} z^{-1})^n}{2} \qquad \boxed{\because \sum_{n=0}^{\infty} (-a)^n = \frac{1}{1+a}}$$

$$= \frac{1}{2}\left[\frac{1}{1+e^{j\pi/3} z^{-1}} + \frac{1}{1+e^{-j\pi/3} z^{-1}}\right]$$

$$= \frac{1}{2}\left[\frac{z(2z + e^{j\pi/3} + e^{-j\pi/3})}{z^2 + z(e^{j\pi/3} + e^{-j\pi/3}) + 1}\right] = \frac{z(z+0.5)}{z^2+z+1}$$

Example 2.42 Determine the z-transform and sketch the ROC of the following signal

$$x(n) = \begin{cases} \left(\frac{1}{3}\right)^n, & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n}, & n < 0 \end{cases}$$

Solution

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{2} z\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n$$

The first power series converges for $\left|\frac{z}{2}\right| < 1$, i.e., $|z| < 2$ and the second power series converges for $\left|\frac{1}{3} z^{-1}\right| < 1$, i.e., $|z| > 1/3$.

$$X(z) = \frac{-z}{z-2} + \frac{z}{z-1/3} \text{ and ROC is } \frac{1}{3} < |z| < 2$$

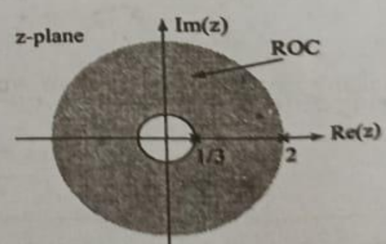

Fig. 2.13 ROC of example 2.42