IIR Filter

(infinite impulse filter)

consider the first order differential equation of an analog system,

$$\frac{d}{dt} y(t) = x(t) \quad \text{—} \quad ①$$

integrating both sides, we get,

$$\int_{(n-1)T}^{nT} \frac{d\,y(t)}{dt}\, dt = \int_{(n-1)T}^{nT} x(t)\, dt$$

$$[y(t)]_{(n-1)T}^{nT} = \int_{(n-1)T}^{nT} (x(t))\, dt$$

$$y(nT) - y((n-1)T) = \int_{(n-1)T}^{nT} x(t)\, dt$$

Apply trapezoidal rule on RHS,

$$\boxed{\int_a^b f(x)\, dx = \frac{b-a}{2}\,[f(a) + f(b)]}$$

$$\therefore y(nT) - y((n-1)T) = \frac{nT - (n-1)T}{2}\,[x(nT) + x((n-1)T)]$$

$$\therefore y(nT) - y((n-1)T) = \frac{T}{2}\,[x(n) + x(n-1)]$$

taking z-transform,

$$y(z) - z^{-1}(Y(z)) = \frac{T}{2}\,[X(z) + X(z)z^{-1}]$$

$$Y(z)[1 - z^{-1}] = \frac{T}{2}\, x(z)\,[1 + z^{-1}]$$

$$\frac{2}{T}\, \frac{y(z)\,[1 - z^{-1}]}{[1 + z^{-1}]} = X(z)$$

take laplace transform of eqn (1)

$s(y(s)) = X(s)$

$$sy(s) \longrightarrow \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \cdot y(z)$$

by bilinear ~~to~~ transformation, whole T is in the sampling time period.

for analog transfer function,

Q. $H(s) = \frac{\cancel{s}\ 2}{s^2+3s+2}$. determine $H(z)$ for $T=1$ & $T=0.1$

$\Rightarrow$ put $s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$ in $H(s)$ to get $H(z)$

$$\therefore H(z) = \frac{\cancel{s}\ 2}{\left(\frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 3\left(\frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}\right) + 2}$$

$$\frac{2}{\frac{4}{T^2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{6}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 2}$$

$\cancel{\text{put } x = \frac{1-z^{-1}}{1+z^{-1}}}$

$$\therefore \frac{2}{\frac{1-z^{-1}}{1+z^{-1}}\left(\frac{4}{T^2} + \frac{6}{T}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) + 2\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right)}$$

put $T=1$

$$\Rightarrow = \boxed{\frac{2T^2(1-z^{-1})^2}{4(1-z^{-1})^2 + 6T(1-z^{-1}) + 2T^2(1+z^{-1})^2}}$$

for $T = 1$,

$$H(z) = \frac{2(1+z^{-1})^2}{4(1-z^{-1})^2 + 6(1-z^{-1}) + 2(1+z^{-1})^2}$$

$$= \frac{2(1+2z^{-1}+z^{-2})}{4(1-2z^{-1}+z^{-2}) + 6 - 6z^{-1} + 2(1+z^{-2}+2z^{-1})}$$

$$= \frac{2(1+2z^{-1}+z^{-2})}{12 - 10z^{-1} + 8z^{-2}}$$

$$= \frac{1+2z^{-1}+z^{-2}}{6-5z^{-1}+4z^{-2}} = \frac{z^{-2}(z^2+2z+1)}{z^{-2}(\cancel{6}z^2-5z+4)}$$

$$\frac{2(z^{-1}+1)^2}{-4z^{-1}+12} = \boxed{\frac{1+z^{-2}+2z^{-1}}{12-4z^{-1}}}$$

Q. $H(s) = \dfrac{s^3}{(s+1)(s^2+s+1)}$, $T = 1$ sec obtain $H(z)$ from $H(s)$.

$\Rightarrow$ put $s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$

$$\therefore \frac{\frac{2^3}{T^3}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3}{\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}+1\right)\left(\frac{4}{T^2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right)}$$

$$= \frac{\frac{8}{T^3}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3}{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}} + \frac{T}{2}(1+z^{-1})\right)\frac{4}{T^2}\left(\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{T}{2}\left(\frac{1+z^{-1}}{1-z^{-1}}\right) + \frac{T^2}{4}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)\right.}$$

$$= \frac{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3}{\left(\frac{1-z^{-1}+\frac{1+z^{-1}}{2}}{1+z^{-1}}\right)\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2\left[1+\frac{1}{2}\frac{1+z^{-1}}{1-z^{-1}}+\frac{1}{4}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right]}$$

$$= \frac{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{1-z^{-1}}{1+z^{-1}}\left(1+\frac{1}{2}\frac{1+z^{-1}}{1-z^{-1}}\right)\left[1+\frac{1}{2}\frac{1+z^{-1}}{1-z^{-1}}+\frac{1}{4}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right]}$$

$$= \frac{1}{\left(\frac{2-2z^{-1}+1+z^{-1}}{2(1-z^{-1})}\right)\left[\frac{2-2z^{-1}+1+z^{-1}}{2(1-z^{-1})}+\frac{1}{4}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right]}$$

$$= \frac{2(1-z^{-1})}{(3-z^{-1})\left[3-z^{-1}+\frac{1}{2}\left(\frac{1+2z^{-1}+z^{-2}}{1-z^{-1}}\right)\right]}$$

$$= \frac{2\times 2(1-z^{-1})(1-z^{-1})^2}{(3-z^{-1})\left[(3-z^{-1})(2(1-z^{-1}))+1+2z^{-1}+z^{-2}\right]}$$

$$= \frac{4(1-z^{-1})^2}{(3-z^{-1})\left[6-6z^{-1}-2z^{-1}+2z^{-2}+1+2z^{-1}+z^{-2}\right]}$$

$$= \frac{4(1-z^{-1})^2}{(3-z^{-1})\left[7-6z^{-1}+3z^{-2}\right]} = \frac{4-8z^{-1}+4z^{-2}}{\left[21-18z^{-1}+9z^{-2}-7z^{-1}+6z^{-2}-3z^{-3}\right]}$$

$$= \frac{4-8z^{-1}+4z^{-2}}{\left[21-25z^{-1}+15z^{-2}-3z^{-3}\right]}$$

Q. $H(s) = \dfrac{2s}{s^2+0.2s+1}$, $T = 1$ sec.

put $s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$

$$H(z) = \frac{2\times\frac{2}{\cancel{T}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{4}{T^{\cancel{2}}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{0.4}{\cancel{T}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \cancel{\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}\, T}$$

$$= \frac{\cancel{4}\left(\frac{\cancel{1-z^{-1}}}{1+z^{-1}}\right)}{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^{\cancel{2}}\left[\cancel{4} + \frac{4}{10}\,\frac{1+z^{-1}}{1-z^{-1}} + \cancel{\frac{4}{4}}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right]}$$

$$\frac{1+z^{-1}}{(\cancel{1-z^{-1}})\left[\frac{4(1-2z^{-1}+z^{-2}) + \frac{0.4}{\cancel{10}}(1+z^{-1})(1-z^{-1}) + 1+2z^{-1}+z^{-2}}{(1-z^{-1})^{\cancel{2}}}\right.}$$

$$\frac{1-z^{-2}}{4-8z^{-1}+4z^{-2}+0.4(1-z^{-2})+1+2z^{-1}+z^{-2}]}$$

$$4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 0.1\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \frac{1}{4}$$

$$= \frac{\cancel{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2\left[1 + 0.1\left(\frac{1+z^{-1}}{1-z^{-1}}\right) + \frac{1}{4}\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2\right]}$$

$$= \frac{1+z^{-1}(1-z^{-1})^2}{(\cancel{1-z^{-1}})\left[(1-z^{-1})^2 + 0.1(1-z^{-2}) + \frac{1}{4}(1+z^{-2}+2z^{-1})\right]}$$

$$= \frac{4-4z^{-2}}{\left[4+4z^{-2}-8z^{-1}+0.4-0.4z^{-2}+1+z^{-2}+2z^{-1}\right]}$$

$$= \boxed{\frac{4-4z^{-2}}{5.4-6z^{-1}+4.6z^{-2}}}$$

IIR filter

Design a butterworth digital IIR low pass filter using bilinear transformation by taking $t = 0.1$ s. to satisfy the following specifications. for $0 \le \omega \le 0.35\pi$

$\cancel{0.6 \le |H(j)|}$ $0.6 \le H(e^{j\omega}) \le 1.0$, $0 \le \omega \le 0.35\pi$

$H(e^{j\omega}) \le 0.1$; for $0.7\pi \le \omega \le \pi$

Draw direct form 1 and direct form 2 structure of the filter.

⇒ Pass band edge digital frequency $\omega_p = 0.35\pi$ rad/s

Stop band edge digital frequency $\omega_s = 0.7\pi$ rad/s

Gain at pass band edge

$A_p = 0.6$

Gain of the stop band

$A_s = 0.1$

$T = 0.1$ sec

pass band edge analog freq, $\boxed{\Omega_p = \frac{2}{T} \tan^{-1}\left(\frac{\omega_p}{2}\right)}$

$= \frac{2}{0.1} \tan^{-1}\left(\frac{0.35\pi}{2}\right)$

$\cancel{1.7476}$ 55.97

$= 12.2560$ r/s

Stop band edge analog freq $= \boxed{\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)}$

$\cancel{1.77}$ 99

$\frac{2}{0.1} = 39.25221$ r/s

order of the filter :- (N)

$$\boxed{N_1 = \frac{1}{1} \times \frac{\log\left[\frac{1}{A_s^2} - 1\right] / \log\left[A_p^2 - 1\right]}{2 \log\left(\frac{\Omega_s}{\Omega_p}\right)}}$$

$$= \frac{1.8819}{\cancel{2.64063}\ 1.0110} = \cancel{0.74428}$$

$$N_1 = 1.726$$

$N > N_1 \Rightarrow N = 2 \rightarrow$ even

Normalized transfer function $H(s_n)$

for N = even

$$\boxed{H(s_n) = \prod_{k=1}^{N/2} \frac{1}{s_n^2 + b_k s_n + 1} \qquad b_k = 2\sin\left(\frac{(2k-1)\pi}{2N}\right)}$$

$$k = \frac{N}{2} = \frac{2}{2} = 1$$

$$b_1 = 2 \sin\left(\frac{(2 \times 1 - 1)\pi}{4}\right)$$

$$= 2 \times \frac{1}{\sqrt{2}} = 1.414$$

$$\boxed{H(s_n) = \prod_{k=1} = \frac{1}{s_n^2 + 1.414\, s_n + 1}}$$

unnormalized transfer function : $H(s)$

$$\boxed{H(s) = H(s_n)\Big|_{s_n = s/\Omega_c} \rightarrow \text{low pass}}$$

$$\Omega_c = \text{cutoff freq} = \frac{\Omega_s}{\left[\frac{1}{A_s^2} - 1\right]^{1/2N}}$$

$$= 12.4439 \text{ r/s}$$

$$H_s = H\left(\frac{1}{\left(\frac{s}{\Omega_c}\right)^2 + 1.414\left(\frac{s}{\Omega_c}\right) + 1}\right)$$

$$= \frac{\Omega_c^2}{s^2 + 1.414 s\Omega_c + \Omega_c^2}$$

$$\boxed{H(s) = \frac{154.8497}{s^2 + 17.5956\, s + 154.8497}}$$

$$H(s) \longrightarrow H(z) \rightarrow \boxed{s = \frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}}}$$

$$H(z) = \frac{154.8506}{\frac{4}{T^2}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)^2 + 17.5956\,\frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}} + 154.8506}$$

$$= \frac{154.8506(1 + z^{-1})^2}{400(1 - z^{-1})^2 + 35.1912(1 - z^{-1})(1 + z^{-1}) + 154.8506(1 + z^{-1})^2}$$

$$= \frac{154.8506(1 + z^{-2} + 2z^{-1})}{400 - 800z^{-1} + 400z^{-2} + 351.912 - 351.912z^{-2} + 154.8506 + 309.70123 + 154.8506z^{-2}}$$

unnormalized transfer function : $H(s)$

$$\boxed{H(s) = H(s_n)\Big|_{s_n = s/\Omega_c} \rightarrow \text{low pass}}$$

$$\Omega_c = \text{cutoff freq} = \frac{\Omega_s}{\left[\frac{1}{A_s^2} - 1\right]^{1/2N}}$$

$$= \underline{12.4439 \text{ r/s}}$$

$$H_s = H\left(\frac{1}{\left(\frac{s}{\Omega_c}\right)^2 + 1.414\left(\frac{s}{\Omega_c}\right) + 1} \right.$$

$$= \frac{\Omega_c^2}{s^2 + 1.414 s\Omega_c + \Omega_c^2}$$

$$\boxed{H(s) = \frac{154.8497}{s^2 + 17.5956 s + 154.8497}}$$

$$H(s) \longrightarrow H(z) \rightarrow \boxed{s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

$$H(z) = \frac{154.8506}{\frac{4}{T^2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 17.5956\,\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} + 154.8506}$$

$$= \frac{154.8506(1+z^{-1})^2}{400(1-z^{-1})^2 + 35.1912(1-z^{-1})(1+z^{-1}) + 154.8506(1+z^{-1})^2}$$

$$= \frac{154.8506(1+z^{-2}+2z^{-1})}{400 - 800z^{-1} + 400z^{-2} + 351.912 - 351.912z^{-2} + 154.8506 + 154.8506z^{-2} + 309.7012z^{-1}}$$

$$= \frac{154.8506\,(1+z^{-2}+2z^{-1})}{906.7626 - 490.2988\,z^{-1} + 202.9386\,z^{-2}}$$

$$= \frac{154.8506}{202.9386}\left(\frac{1+z^{-2}+2z^{-1}}{4.4681 - 2.4159\,z^{-1} + z^{-2}}\right)$$

$$= 0.7630$$

Q. Design a butterworth IIR high pass filter using bilinear transformation by taking T = 0.5 sec. to satisfy the following specifications.

S_p (pass band ripple) < 3.01 dB.

S_s (Stop band attenuation) > 13.97 dB

w_p (pass band edge freq) = 0.65π r/s

w_s (stop band —||—) = 0.45π r/s

convert HPF to LPF

$$\Rightarrow \quad A_p = 10^{-(S_p\,dB/20)} = 0.7071$$

$$A_s = 10^{-(S_s\,dB/20)} = 0.2002$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{w_p}{2}\right) = \cancel{6.5274}\ 3.4163 \text{ r/s}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{w_s}{2}\right) = 6.5274 \text{ r/s}$$

$$N_1 = \frac{\frac{1}{2}\left[\log\left[\frac{\frac{1}{A_s^2}-1}{\frac{1}{A_p^2}-1}\right]\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\frac{1}{2}\log\left[\frac{23.95}{1}\right]}{\log[1.9106]}$$

$$= \frac{1.3793}{0.2811 \times 2} = 2.457$$

$N > N_1$

$\Rightarrow \boxed{N = 3} \rightarrow$ odd

Normalized freq $H(s_n)$

for N = Odd

$$H(s_n) = \frac{1}{s_n + 1} \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{s_n^2 + b_n s_n + 1}$$

$$\boxed{b_k = 2\sin\left(\frac{(2k-1)\pi}{2N}\right)}$$

Hsr $\boxed{b_k = 2\sin\left(\frac{\pi}{6}\right) = 1}$

$$H(s_n) = \frac{1}{(s_n+1)} \cdot \frac{1}{(s_n^2 + s_n + 1)}$$

$$= \frac{1}{s_n^3 + s_n^2 + s_n + s_n^2 + s_n + 1}$$

$$\boxed{H(s_n) = \frac{1}{s_n^3 + 2s_n^2 + 2s_n + 1}}$$

$$\Omega_c = \frac{\Omega_s}{\left[\frac{1}{A_s^2} - 1\right]^{1/2N}}$$

$$= 3.8433 \ r/s$$

$$H(s) = \frac{1}{\left(\frac{3.8433}{s}\right)^3 + 2\left(\frac{3.8433}{s}\right)^2 + 2\left(\frac{3.8433}{s}\right) + 1}$$

$$= \frac{1}{\frac{56.76}{s^3} + \frac{29.5419}{s^2} + \frac{7.6866}{s} + 1}$$

$$\boxed{H(s) = \frac{s^3}{s^3 + 7.6866 s^2 + 29.5419 s + 56.76}}$$

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$$

$$\cancel{H}(z) = \frac{\left(4 \frac{1-z^{-1}}{1+z^{-1}}\right)^3}{\left(4 \frac{1-z^{-1}}{1+z^{-1}}\right)^3 + 7.6866\left(4 \frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 29.5419 \left(4 \frac{1-z^{-1}}{1+z^{-1}}\right) + 56.76}$$

$$= \frac{64\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3}{64\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + 122.98\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 118.16\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 56.76}$$

$$= \frac{64}{64 + 122.98\left(\frac{1+z^{-1}}{1-z^{-1}}\right) + 118.16\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^2 + 56.76\left(\frac{1+z^{-1}}{1-z^{-1}}\right)^3}$$

$$= \frac{64(1-z^{-1})^3}{64(1-z^{-1})^3 + 122.98(1+z^{-1})(1-z^{-1})^2 + 118.16(1+z^{-1})^2(1-z^{-1}) + 56.76(1+z^{-1})^3}$$

$$= \frac{64(1-z^{-1})^3}{64(1-3z^{-1}+3z^{-2}-z^{-3}) + 122.98(1-z^{-2})(1-z^{-1}) + 118.16(1-z^{-2})(1+z^{-1}) + 56.76(1+3z^{-1}+3z^{-2}+z^{-3})}$$

$$= \frac{(1-3z^{-1}+3z^{-2}-z^{-3})}{(1+3z^{-1}+3z^{-2}-z^{-3})+0.52(1+z^{-1}-z^{-2}+z^{-3})+0.54(1-z^{-2}+z^{-1}-z^{-3})+1.127(1+3z^{-1}+3z^{-2}-z^{-3})}$$

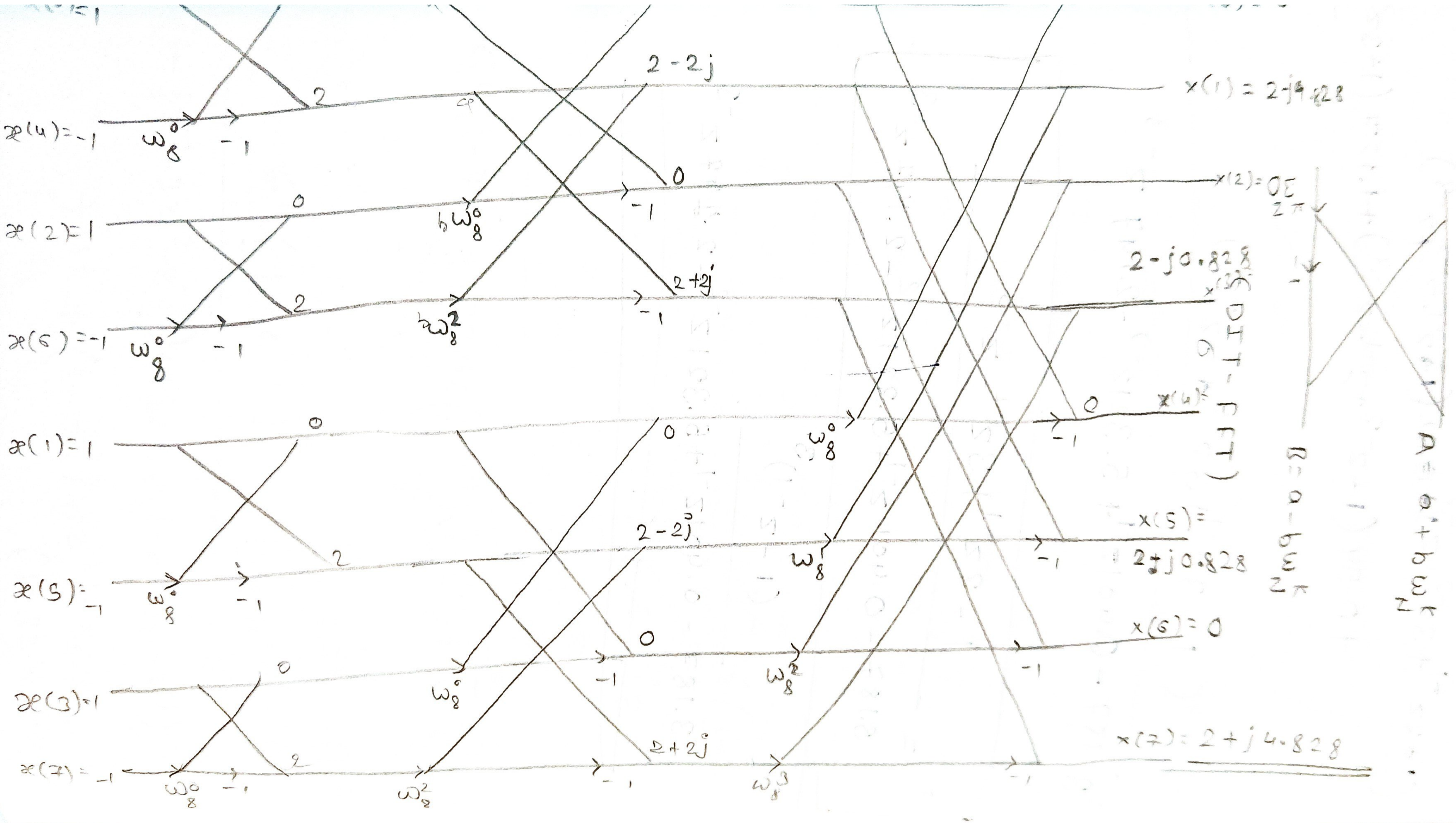
$$= \frac{(1-3z^{-1}+3z^{-2}-z^{-3})}{3.187-0.401z^{-1}+5.321z^{-2}-2.147z^{-3}}$$

$$\boxed{= \frac{1-3z^{-1}+3z^{-2}-z^{-3}}{3.187-0.401z^{-1}+5.321z^{-2}-2.147z^{-3}}}$$

$$= \frac{(1-z^{-1})^3}{3.187-0.401z^{-1}+5.321z^{-2}-2.147z^{-3}}$$

$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$ (Too)

(DIT-FFT)



$W_8^0 = e^{-j2\pi k/N}$
$= e^{-j2\pi 0/N 8}$
$= 1$

$W_8^2 = e^{-j2\pi 2/8}$
$= e^{-j\pi/2}$
$= -j$

$W_8^1 = e^{-j2\pi 1/8}$
$= e^{-j\pi/4}$
$= 0.707 - j0.707$

$W_8^3 = e^{-j2\pi 3/8}$
$= e^{-j\pi 3/4}$
$= -0.707 - j0.707$

$A = 1 - 1 \times 1$
$= 1 - 1$
$= 0$
$B = 1 + 1 \times 1$
$= 2$

$A = 0 + 0 \times 1$
$= 0$
$B = 0 - 0 \times 1$
$= 0$

$A = 2 + 2(-j)$
$= 2 - 2j$
$B = 2 - 2(-j)$
$= 2 + 2j$

$A = 0 + 0(1)$
$= 0$
$B = 0 - 0(1)$

$A = 2 - 2j + (2-2j)(0.707 - j0.707)$
$= 2 - 2j + \cancel{1.414} - j1.414 - j1.414 - \cancel{1.414}$
$(2-j2) + 1.414(\cancel{1} - 2j - \cancel{1})$
$= 2 - 2j - 2.828$
$= 2 - 4.828j$

$B = 2 - 2j - (2-2j)(0.707 - j0.707)$
$= 2 - 2j - 2(0.707(1-j)(1-j))$
$= 2 - 2j + j2.828$
$= \underline{2 + 0.828j}$

$A = 0 + 0(-j)$
$= 0$
$B = 0 - 0(-j)$
$= 0$

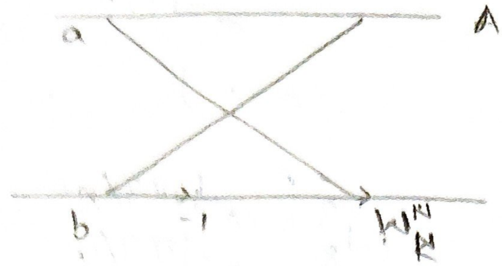
$A = 2 + 2j \mp (2+2j)(0.707 + j0.707)$
$= 2 + 2j - \cancel{2}1.414(\cancel{1} + 2j - \cancel{1})$
$= 2 + 2j - j2.828$
$= 2 - j0.828$

$B = 2 + 2j + (2+2j)(0.707 + j0.707)$
$= 2 + 2j + 1.414(2j)$
$= 2 + 4.828j$

$\therefore X(k) = \{0, 2 - j4.828, 0, 2 - j0.828, 0, 2 + j0.828, 0, 2 + j4.828\}$

$\boxed{X^*(k) = X(N-k)}$

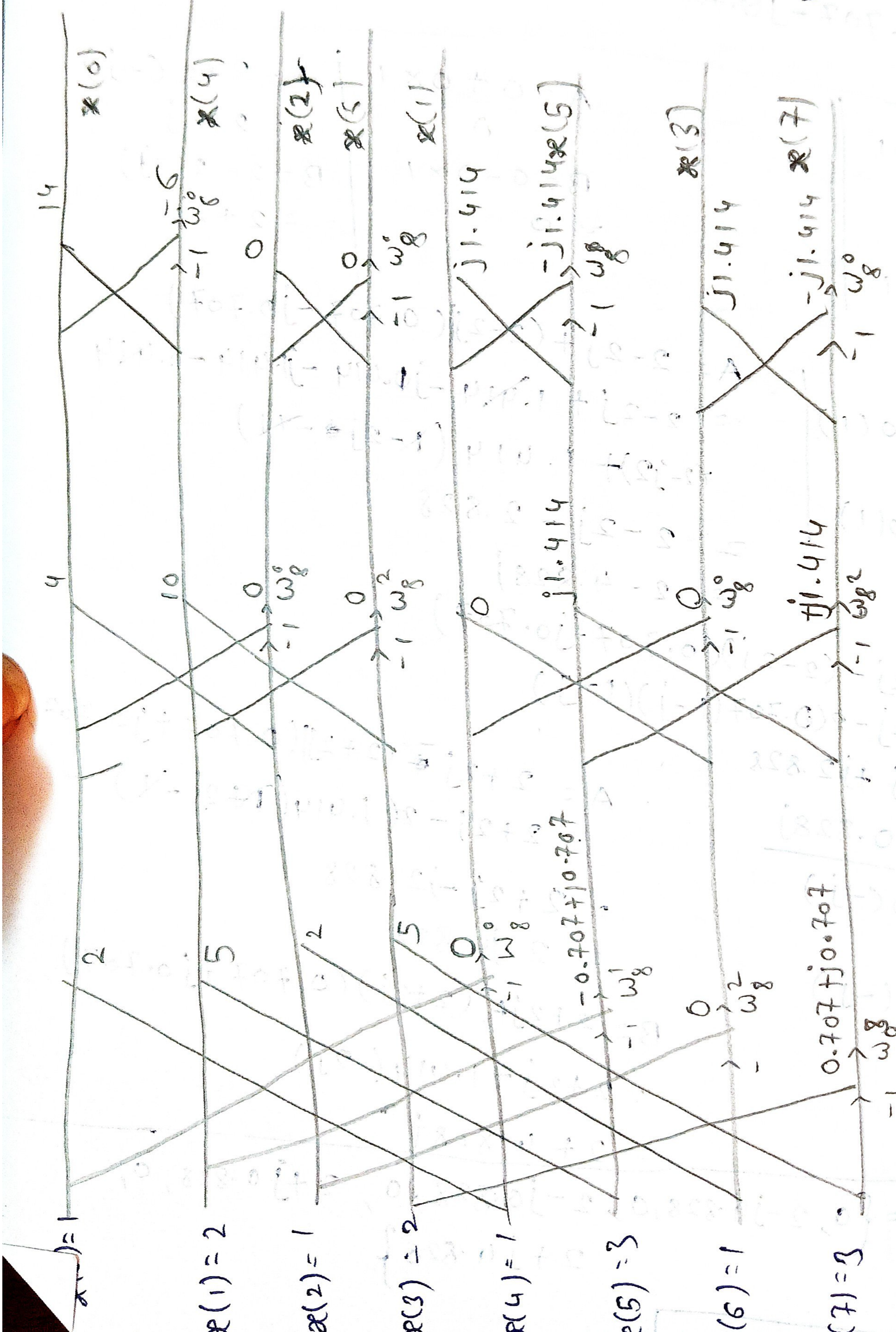
DIF-FFT



$A = a + b$

$B = (a-b)W_N^k$

$W_8^0 = 1$ $\qquad x(n) = \{1, 2, 1, 2, 1, 3, 1, 3\}$



$W_8^0 = 1$ $\quad$ $W_8^2 = -j$

$W_8^1 = 0.707 - j0.707$ $\quad$ $W_8^3 = -0.707 - j0.707$

$A = 1+1 = 2$
$B = (1-1)W_8^0$
$= 0$

$A = 2+3 = 5$
$B = (2-3)W_8^1$
$= -0.707 + j0.707$

$A = 1+1 = 2$
$B = (1-1)W_8^2$
$= 0$

$A = 2+3 = 5$
$B = (2-3)W_8^3$
$= -1(-0.707 - j0.707)$
$= 0.707 + j0.707$

$A = 2+2 = 4$
$B = (2-2)W_8^0 = 0$

$A = 5+5 = 10$
$B = (5-5)W_8^2 = 0$

$A = 0+0 = 0$
$B = (0-0)W_8^0 = 0$

$A = -0.707 + j0.707 + 0.707 + j0.707$
$= j1.414$

$B = (-0.707 + j0.707 - 0.707 - j0.707)W_8^2$
$= (j1.414)(-j)$
$= +j1.414$

$A = 4+10 = 14$
$B = (4-10) = -6$

$A = 0$
$B = 0$

$A = 0 + j1.414$
$B = (0 - j1.414)W_8^0$
$= -j1.414$

$A = 0 + j1.414$
$B = (0 - j1.414)W_8^0$

$$\Rightarrow \boxed{X(K) = \{14, j1.414, 0, j1.414, -6, -j1.414, 0, -j1.414\}}$$

DIT - IFFT

DIF - IFFT

$$\boxed{W_N^k = e^{j2\pi nk/N}}$$

$W_8^0 = e^{j2\pi nk/N} = e^0 = 1$

$W_8^1 = 0.707 + j0.707$

$W_8^2 = j$

$W_8^3 = \cancel{-0.707} \cancel{+j0.7} + j0.707$

$= -0.707 + j0.707$

In the end divide everything by N.

$\therefore \boxed{x(1) = \frac{1}{N}(x(n))}$

~~x(~~

Q.9. DIF-IFFT
Q.10, DIT-IFFT
} before 3.30

$e^{\theta} = \cos\theta + j\sin\theta$

$e^{-\theta} = \cos\theta - j\sin\theta$

1.6842

Q: Determine the poles of low pass butterworth filter for $n=2$. Sketch the location of poles on s-plane & hence determine the normalized transfer function of low pass filter.

$N =$ even $= 2$

(Stable on LHS side)

$$\boxed{S_n = e^{\frac{j(2k-1)\pi}{2N}}} \quad ; \text{ for } k = (1, 2, 3 \ldots, 2N)$$

$$S_n = e^{\frac{j(2k-1)\pi}{4}} \quad ; \text{ for } k = 1, 2, 3, 4$$

$k=0$; $S_1 = e^{\pm\left(\frac{j\pi}{4}\right)}$

$$S_n = 0.707 + j0.707 = P_1$$

$K=2$, $S_n = e^{3j\pi/4}$

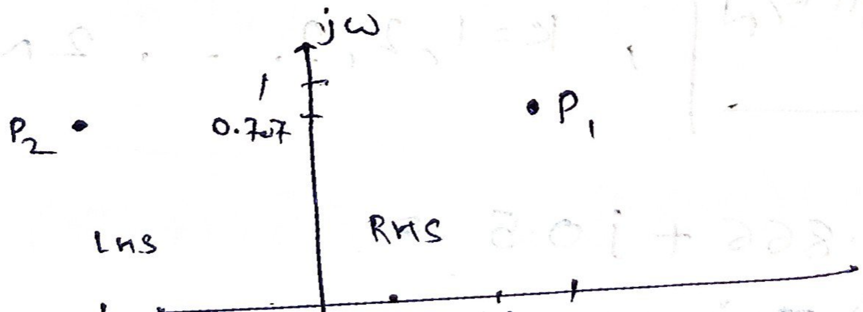
$$S_n = -0.707 + j0.707 = P_2$$

$k=3$, $S_n = e^{5j\pi/4}$

$$S_n = -0.707 - j0.707 = P_2^*$$

$k=4$, $S_n = e^{j7\pi/4}$

$$S_n = 0.707 - j0.707 = P_1^*$$



$$H(S_n) = \frac{1}{(S_n - P_2)(S_n - P_2^*)}$$

$$= \frac{1}{(S_n + 0.707 - j0.707)(S_n + 0.707 + j0.707)}$$

$$= \frac{1}{(S_n^2 + 0.5 + 1.414 S_n + 0.5)}$$

$$\boxed{H_2(S_n) = \frac{1}{S_n^2 + 1.414 S_n + 1}}$$

Q. for $n = 3$ sketch the location of poles on s-plane hence determine normalized transfer function.

$N = 3 \rightarrow$ odd

$$\boxed{S_n = e^{j\pi k/N}} \quad , \quad k = 1, 2, 3, \ldots 2N$$

$k=1$, $S_n = \cancel{0.866 + j0.5}$ =

$k=2$, $S_n = 0.5 + j0.866$

$k=3$, $S_n = j$

$k=4$, $S_n = -0.5 + j0.866$

$k=5$, $S_n = -0.866 + j0.5$

$k=6$, $S_n = -1$

$k=1,\ S_n = 0.5 + j0.866$ $\quad P_1$

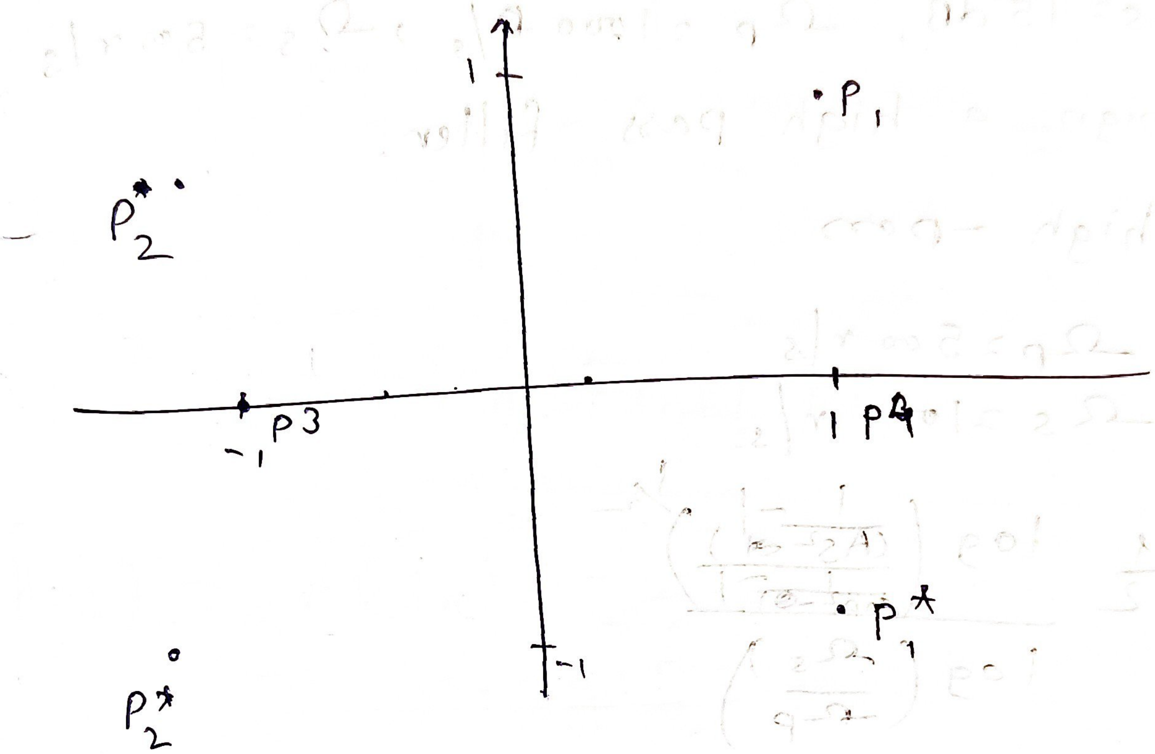
$k=2,\ S_n = -0.5 + j0.866$ $\quad P_2$

$k=3,\ S_n = -1$ $\quad P_3$

$k=4,\ S_n = -0.5 - j0.866$ $\quad P_2^*$

$k=5,\ S_n = 0.5 - j0.866$ $\quad P_1^*$

$k=6,\ S_n = 1$ $\quad P_4$



$$H(S_n) = \frac{1}{(S_n-P_3)(S_n-P_2)(S_n-P_2^*)}$$

$$= \frac{1}{(S_n+1)(\underbrace{S_n+0.5}_{a}-j0.866)(S_n+0.5+j0.866)}$$

$$= \frac{1}{(S_n+1)(S_n^2+0.25+S_n+0.75)}$$

$$= \frac{1}{(S_n+1)(S_n^2+S_n+1)} = \frac{1}{S_n^3+S_n^2+S_n+S_n^2+S_n+1}$$

$$\boxed{H(s_n) = \frac{1}{s_n^3 + 2s_n^2 + 2s_n + 1}}$$

Q.

For the given specification $\alpha_p = 3\,dB$, $\alpha_s = 15\,dB$, $\Omega_p = 1000\ r/s$, $\Omega_s = 500\ r/s$

Design a high pass filter.

$\Rightarrow$

for high-pass

$\Omega_p = 500\ r/s$

$\Omega_s = 1000\ r/s$

$$N = \frac{\frac{1}{2}\log\left(\dfrac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1}\right)}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{1}{2}\log\left(\frac{(0.0044 - 1)}{-0.888}\right)$$

$$\frac{1}{S_n+1} \quad \sin\left(\frac{\pi}{6}\right) \left[\frac{(2k-1)\pi}{2N}\right] \quad \frac{1}{S_n+1}$$

$$= \frac{1}{S_n+1} \times \frac{1}{S_n^2+S_n+1}$$

$$\frac{1}{S_n^3+2S_n^2+2S_n+1}$$

$$H(s) = H(S_n)\Big|_{S_n=\frac{\Omega_c}{s}}$$

DIGITAL IIR butterworth

$$A_p = 10^{(-\delta p\, dB/20)}$$

$$A_s = 10^{(-\delta s\, dB/20)}$$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$$

$$N_1 = \frac{1 \cdot \log\left(\dfrac{\frac{1}{A_s^2} - 1}{\frac{1}{A_p^2} - 1}\right)^{1/2}}{2 \cdot \log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

For N = even

$$H(S_n) = \prod_{k=1}^{N/2} \frac{1}{S_n^2 + b_k S_n + 1}$$

$$b_k = 2 \cdot \sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$H(S_n) = \frac{1}{S_n + 1} \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{S_n^2 + b_k \ldots + 1}$$

~~For N = odd~~

~~$H(S_n)$~~

$$H(s) = H(S_n)\Big|_{S_n = s/\Omega_c} \ (\text{LPF})$$

$$= \frac{\Omega_c}{s} \ (\text{HPF})$$

$$H(z) = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$$

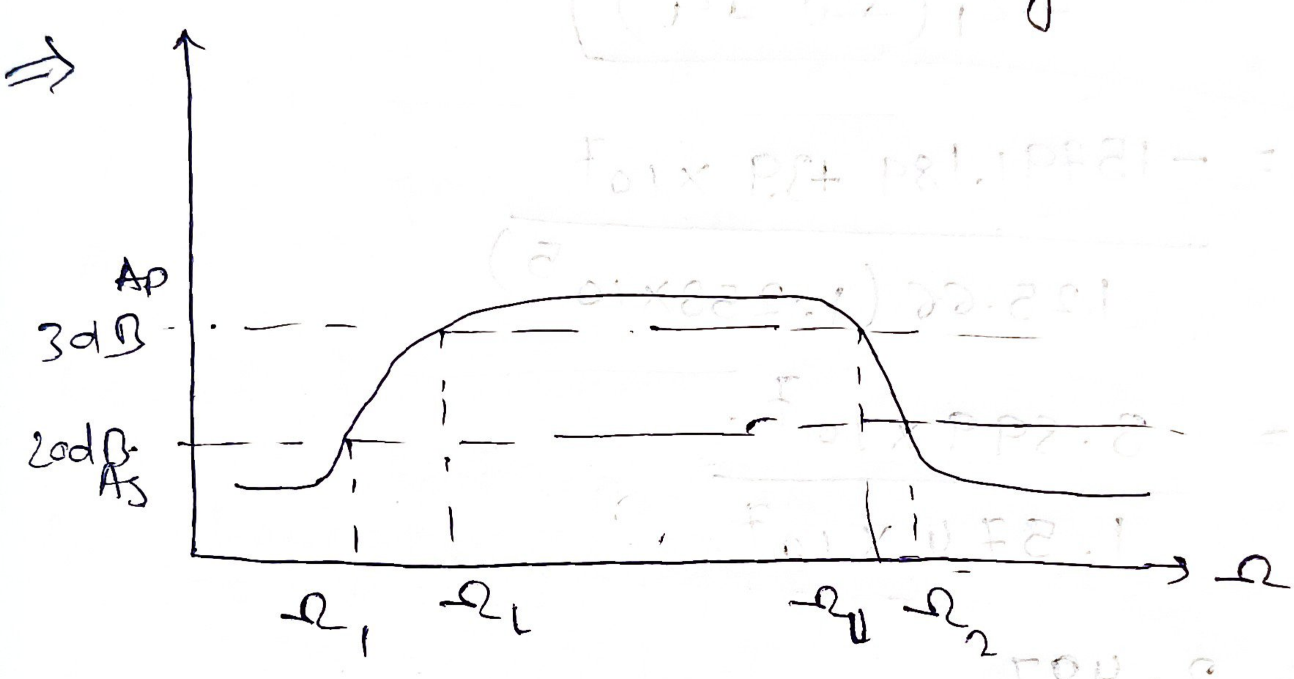
Q. Design an analog band pass Chebyshev filter to meet the following specifications

(i) − 3 dB upper & lower freq of 50 Hz & 20 kHz

ii) Stop band attenuation of at least 20 dB at 20 Hz & 45 kHz

iii) Find the Monotonic frequency response

⇒



$A_p = -3\,dB$

$A_s = 20\,dB$

~~for~~ for −3 dB

$f_u = 20\,kHz$

$f_L = 50\,Hz$

for 20 dB

$f_u = 45\,kHz$

$f_L = 20\,Hz$

$\Omega_1 = 2\pi f = \cancel{62.8}$ 125.663 r/s

$\Omega_U = 2 \times \pi \times 20 \times 10^3 = 1.256 \times 10^5$

$\Omega_2 = 2 \times 3.14 \times 45$

$\Omega_L = 2 \times \pi \times 50 = 314\,r/s$

to find stop band edge freq of Ω_s of the normalized LPF,

$$\Omega_s = \text{Minimum}\left(|A|, |B|\right)$$

$$\boxed{A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}}$$

$$= \frac{-15791.189 + 3.9 \times 10^7}{125.66\,(1.253 \times 10^5)}$$

$$= \frac{3.899 \times 10^7}{1.574 \times 10^7}$$

$$= 2.4\cancel{8}7$$

$$\simeq \boxed{2.5 = A}$$

$$\boxed{B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}}$$

$$= \frac{7.986 \times 10^{10} - 3.9 \times 10^7}{3.54 \times 10^{10}}$$

$$= \frac{7.982}{3.54}$$

$$\boxed{B = 2.25}$$

$\Rightarrow \Omega_s = \min(|A|, |B|)$

$\Rightarrow \boxed{\Omega_s = 2.25}$

$\Omega_p = 1$ (for normalized filter is always 1).

$\delta_p = -3\,dB$, $\delta_s = 20\,dB$

$A_p = 10^{-(3/20)} = 0.7079$

$A_s^2 = 0.1$

for chebyshev filter

$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{1}{A_s^2} - 1\right) / \left(\frac{1}{A_p^2} - 1\right)\right]^{1/2}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{\cosh^{-1}(100)}{\cosh^{-1}(2.25)}$$

$$= \frac{5.2982}{1.4505} = \cancel{}$$

$$= \frac{5.29829}{1.4505} = \underline{\underline{3.6526}}$$

$|N, 0 < N|$

$N = 4 \rightarrow$ even

$$S_n = e^{j(2k-1)\pi/2N}$$
$$= e^{j(2k-1)\pi/8}$$

for $k = 1, 2, 3, 4, 5, 6, 7, 8$

for $k = 0$, $S_n = -0.5 + j0.8660$

$k = 1$, $S_n = -1$

$k = 2$, $-0.5 - j0.866$

$$H(s_n) = \frac{1}{(S_n+1)(S_n+0.5+j0.8660)(S+0.5-j0.866)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

for band pass filter,

$$\boxed{S = \frac{s^2 + \Omega_U \Omega_L}{s(\Omega_U - U_L)}}$$

$$H(s) = \frac{1.9699 \times 10^{15} s^3}{(s^6 + 2.51 \times 10^5 s^5 + 3.154 \times 10^4 s^4 + 1.989 \times 10^{16} s^3 + 1.2451 \times 10^{18} s^2 + 3.9073 \times 10^{20} s + 6.1529 \times 10^{22})}$$

$$A = \frac{\Omega_1(\Omega_0-\Omega_L)}{-\Omega_1^2+\Omega_0\Omega_L}$$

$$B = \frac{\Omega_2(\Omega_0-\Omega_L)}{-\Omega_2^2+\Omega_0\Omega_L}$$

$$\cosh^{-1}(x) = \ln\left(x+\sqrt{x^2-1}\right)$$