Module-1

Signals and Systems

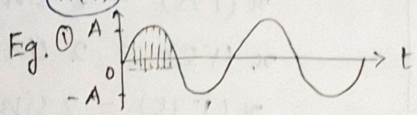
Signal → A physical quantity that varies with time, space or any other independent variable.

- Continuous time (CT)
- Discrete time (DT) * → Signal that is defined at discrete instance of time.

Continuous time (CT) [All signals that occur in nature] → Each & every instance of time, the signal is defined.

$x(t)$

Eg. ①



$x(t) = A\sin(\omega t + \theta)$.

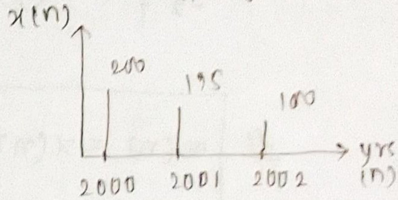
② AC power supply.

③ ECG / EEG signal.

* Also called as analog signal.

Discrete time (DT): $x(n)$

Eg. ① No. of accidents in diff. years.



② Temp. recorded for certain duration.

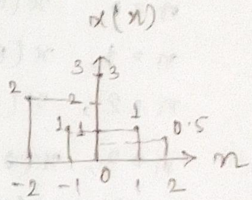
* Sometimes, CT derived as dT by sampling process.

DT representation

*1. Graphical Representation
2. Functional representation
*3. Sequence
4. Tabular

Graphical Representation →

$x(-2) = 2$
$x(-1) = 1$
$x(0) = 3$
$x(1) = 1$
$x(2) = 0.5$



Tabular:

n	x(n)
-2	2
-1	1
0	3
1	1
2	0.5

Sequence:

$x(n) = \{2, 1, \underset{\uparrow}{3}, 1, 0.5\}$

$n = 0$.

[Otherwise first element taken for 0].

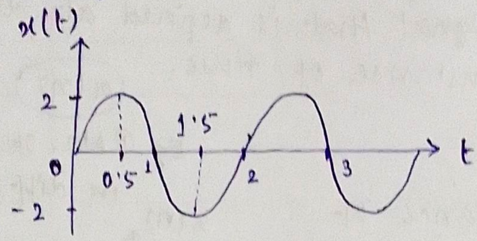
Functional representation:

$$x(n) = \begin{cases} 2, & n = -2 \\ 1, & n = -1 \\ 3, & n = 0 \\ 1, & n = 1 \\ 0.5, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

DT → Digital

Q sketch the CT signal defined as $x(t) = 2\sin \pi t$, $0 \le t \le 2$. Sketch sample the signal for the sampling period $T = 0.2$ sec. Sketch the ~~dt~~ DT signal.

→ CT signal:



$x(0) = 0$

~~$x(0.5) =$~~

$x(0.25) = 2\sin\frac{\pi}{4} = 1.414$

$x(0.5) = 2\sin\frac{\pi}{2} = 2$

$x(0.75) = 2\sin\frac{3\pi}{4} = 1.414$

$x(1) = 2\sin\pi = 0$

$x(1.25) = 2\sin 1.25 = -1.414$

$x(1.5) = 2\sin 1.5 = -2$

$x(1.75) = 2\sin 1.75 = -1.414$

$x(2) = 2\sin 2\pi = 0$

$\boxed{x(n) = x(nT) = x(t)\Big|_{t=nT}}$

$x(n) = x(0.2n) = x(t)\Big|_{t=0.2n}$

$x(0.2n) = 2\sin(n\pi\, 0.2)$

$n=0,\ x(0) = 2\sin(0\times\pi) = 0$

$n=1,\ x(0.2) = 2\sin(0.2\pi) = 1.17$

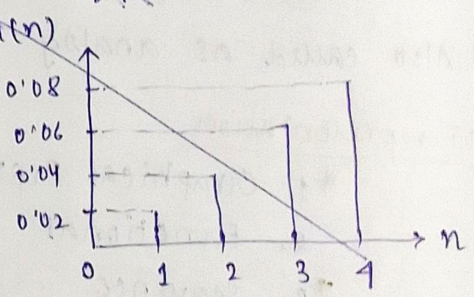
$n=2,\ x(0.4) = 2\sin(0.4\pi) = 1.90$

$n=3,\ x(0.6) = 2\sin(0.6\pi) = 1.90$

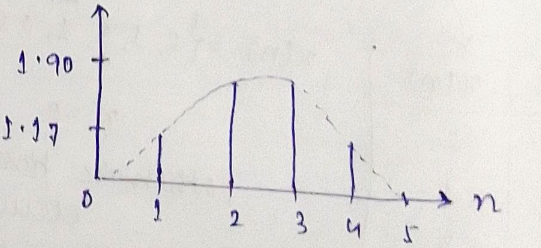
$n=4,\ x(0.8) = 2\sin(0.8\pi) = 1.17$

$n=5,\ x(1) = 0.$

~~DT:~~



DT:



HW:

Sketch the signals:

(a) $x(n) = (0.8)^n$, $0 \le t \le 10$

(b) $x(t) = e^{2t}$, $0 \le t \le 2$.

HW

(a) $x(n) = (0.8)^n, \quad 0 \leq t \leq 10$

$x(0) = (0.8)^0 = 1.$

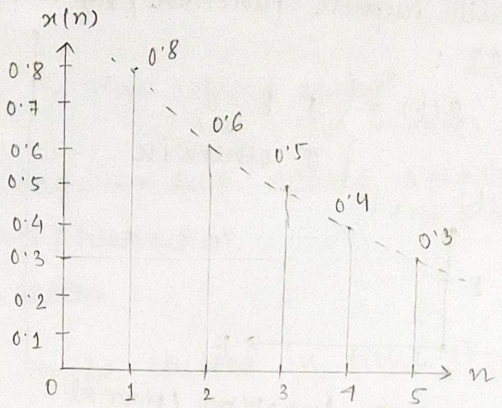
$x(1) = (0.8)^1 = 0.8.$

$x(2) = (0.8)^2 = 0.64.$

$x(3) = (0.8)^3 = 0.512.$

$x(4) = (0.8)^4 = 0.4096.$

$x(5) = (0.8)^5 = 0.32768.$



(b) $x(t) = e^{2t}, \quad 0 \leq t \leq 2.$

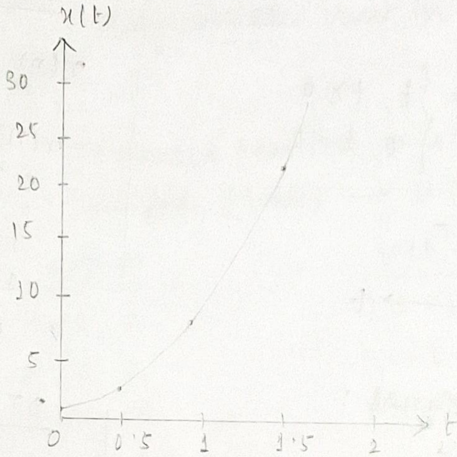
$x(0) = e^0 = 1.$

$x(0.5) = 2.71$

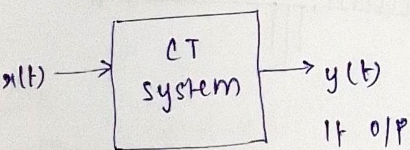
$x(1) = 7.38$

$x(1.5) = 20.08$

$x(2) = 54.5$



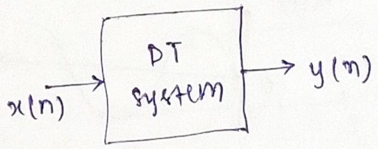
System



If o/p $x(t)$, its not a system. Its a buffer.

filter, amplifier.

$y(t) = T[f(t)].$



$y(n) = T[x(n)].$

Response of the system.

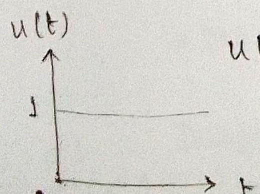
Standard Test Signals

→ Elementary signals.
→ Singularity function.
} → For testing the behaviour of system.

CT:

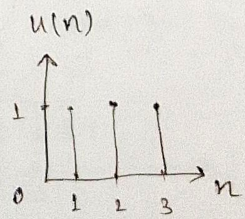
① Unit step signal / Function:

$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$



DT:

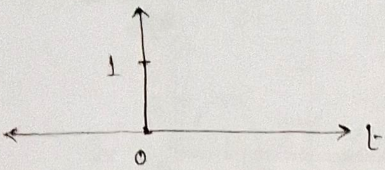
$u(n) = 1, \; n \geq 0$
$\quad\;\; = 0, \; n < 0.$



② Unit impulse function / signal :

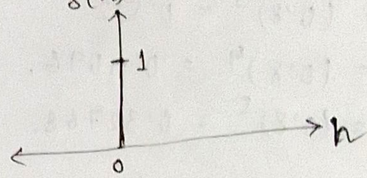
CT :

$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$$



DT :

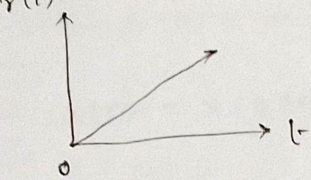
$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$



③ Unit ramp function / signal :

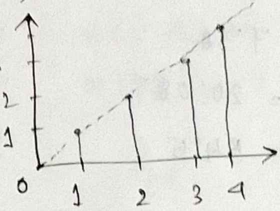
CT :

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



DT :

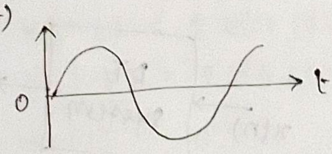
$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



④ Sinusoidal signal :

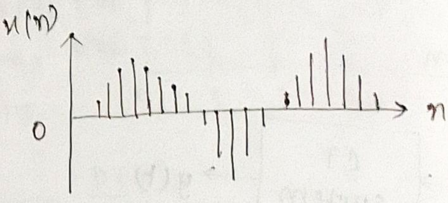
CT :

$$x(t) = A\sin(\omega t + \theta)$$



DT :

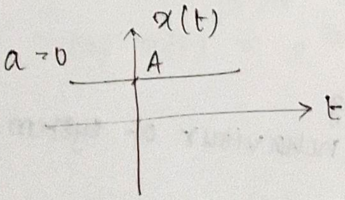
$$x(n) = A\sin(\omega n + \theta)$$

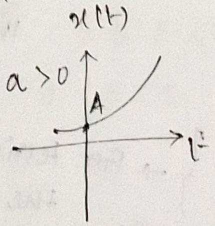


⑤ Exponential signal :

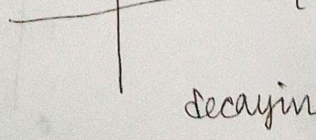
CT :

$$x(t) = Ae^{at}$$





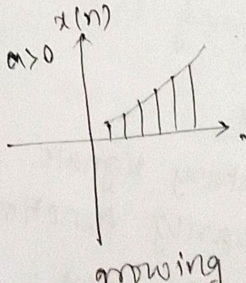
Growing



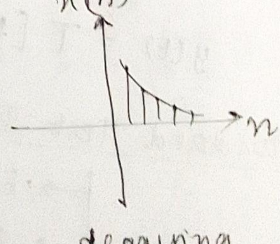
decaying

DT :

$$x(n) = a^n \rightarrow Ae^{an}$$



growing



decaying

Basic Operations on Signals

1. Time shifting.
2. Time reversal.
3. Time scaling.

} Time related operatⁿ (Amp. fixed y-axis)

4. Amplitude scaling. → Amp. related operation. (No change in time)
5. Signal addition / subtraction.
6. Signal multiplication.

① Time shifting:

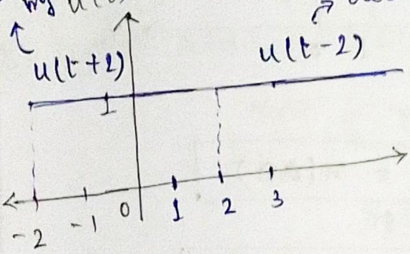
↳ The signal is said to be shifted in time if,

CT: $y(t) = x(t \pm T)$ ↳ ~~time periode~~ unit in which shifting happens.

DT: $y(n) = x(n \pm k)$.

advanced by 2 units u(t)

delayed by 2 units



Advanced (earlier) → $u(t+2)$

Delayed (later) → $u(t-2)$.

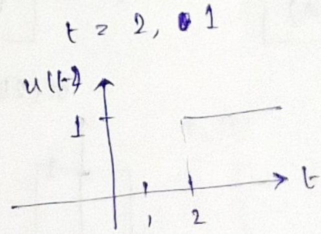
u(t-2)

t = 0, 0

t = 1, 0

t = -1, 0

t = 2, ~~0~~ 1



u(t+2)

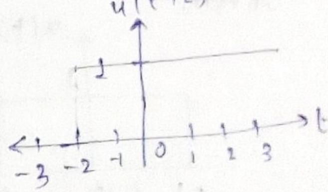
t = -1, ~~0~~ 1

t = -2, 1

t = -3, 0

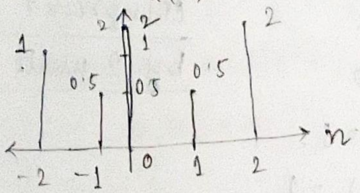
t = 0, 1

t = 1, 1



HW

① $y(n) = x(n-k)$



Find,

$x(n-3)$

$x(n+3)$.

$x(-2) = 1$

$x(-1) = 0.5$

$x(0) = 2$

$x(0.5) = 1$

$x(2) = 2$.

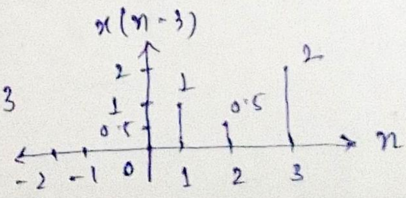
x(n-3)

n = 0, x(n-3) = 0

n = 1, x(n-3) = 1.

n = 2, x(-1) = 0.5

n = -1, 0.



Delayed by ~~8~~ 3 unit

x(n+3)

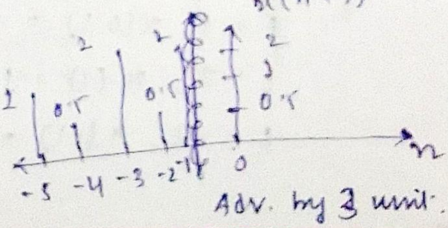
n = -1, x(2) = ~~1~~ 2

n = -2, x(-1) = 0.5

n = -3, x(0) = 2

n = -4, x(-1) = 0.5

n = -5, 1.



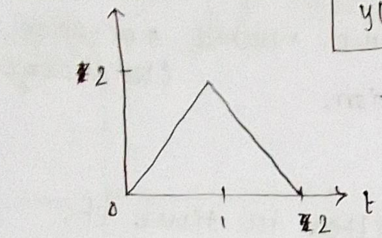
Adv. by 3 unit

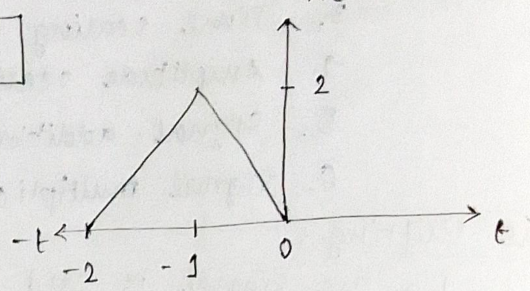
② Time Reversal :

↳ folding of signal.

CT

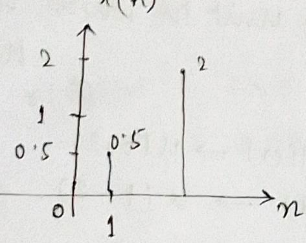
$$\boxed{y(t) = x(-t)}$$

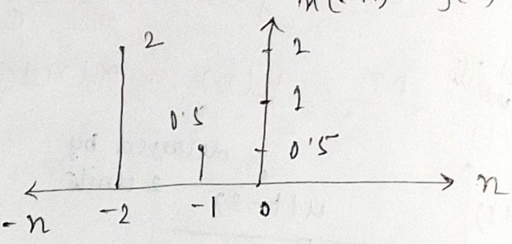




DT

$y(n) = x(-n)$





③ Time Scaling :

CT. $\boxed{y(t) = x(at)}$ compressed

or

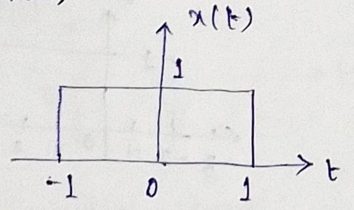
$\boxed{y(t) = x(t/a)}$ Expanded.

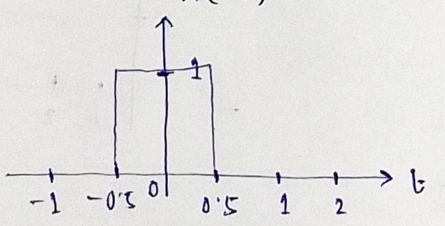
DT $\boxed{y(n) = x(an)}$

or,

$\boxed{y(n) = x(n/a)}$

Given,





$a = 2$

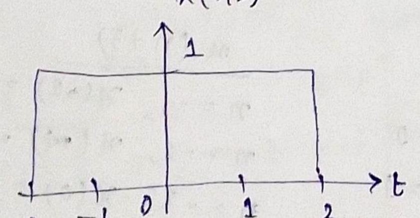
$t = 0, \ x(0) = 1.$

$t = 1, \ x(2) = 0$

$t = 0.5, \ x(1) = 1.$

$t = -0.5, \ x(-1) = 1.$

compressed by 2 units.



Expanded by 2 units.

$t = 0, \ x(0) = 1$

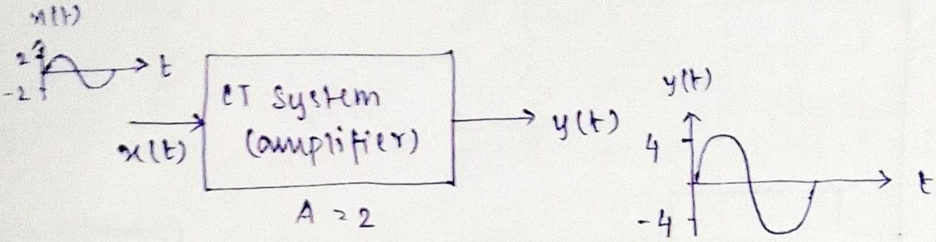
$t = 1, \ x(0.5) = 1.$

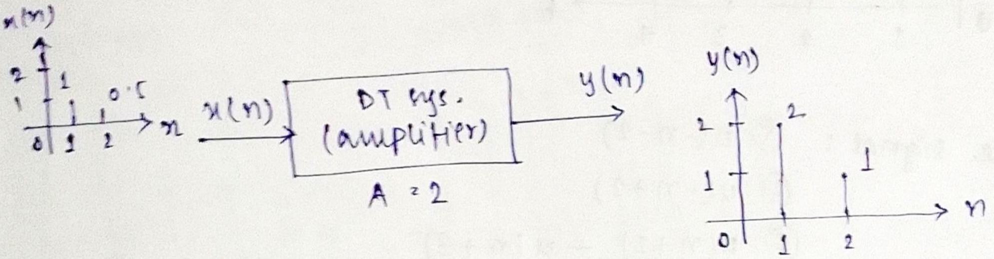
$t = 2, \ x(1) = 1.$

$t = -2, \ x(-1) = 1.$

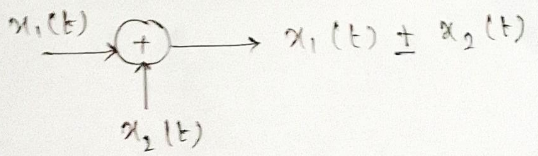
4. Amplitude Scaling :

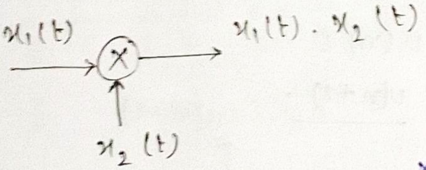
CT $\boxed{y(t) = A x(t)}$ DT $\boxed{y(n) = A x(n)}$



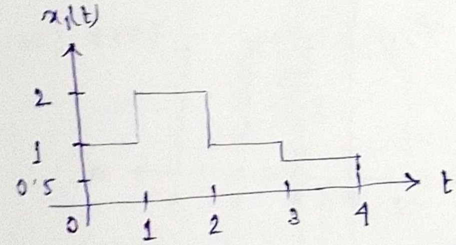


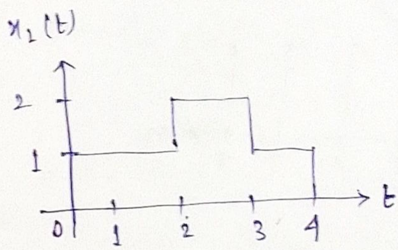
5. Signal Addition / subtraction / Multiplication :





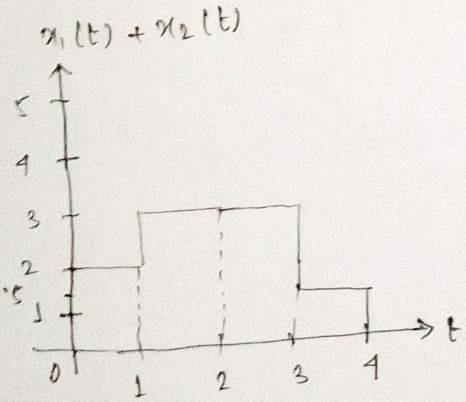
Eg.

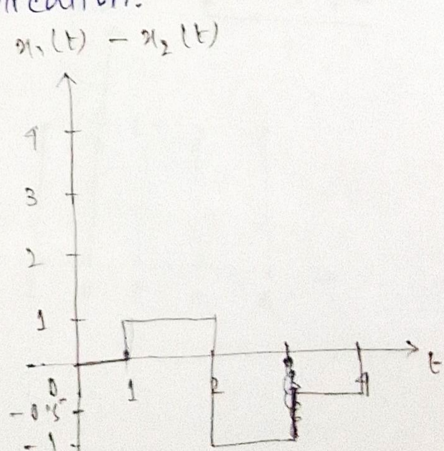


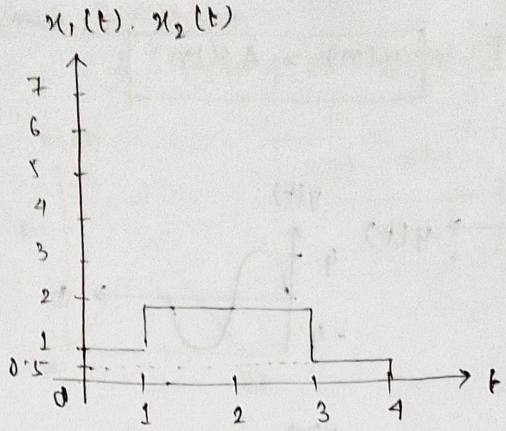


Perform addition, subtraction & multiplication.

→







HW

Sketch the signal : ① $u(-n-2)$

② $u(-n+2)$

③ $u(n+2) - u(n+3)$

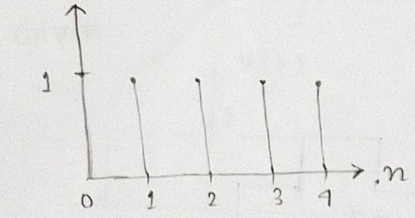
④ $u(n+2) \cdot u(n)$.

Ans-

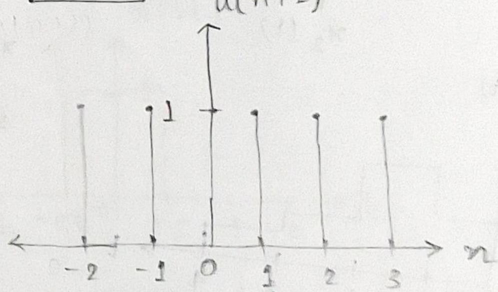
1. $u(-n-2)$

$= u[-(n+2)]$

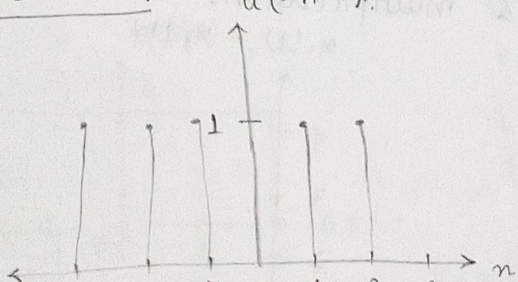
First, we draw the graph for $u(n)$:



$u(n+2)$:

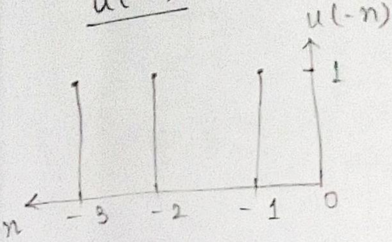


$u[-(n+2)]$:

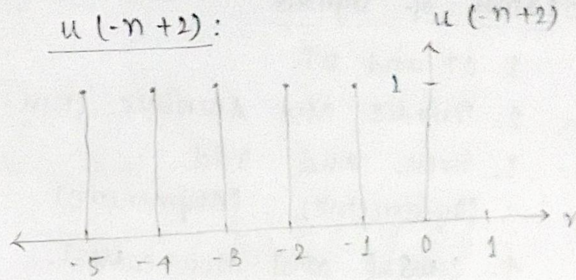


2. $u(-n+2)$

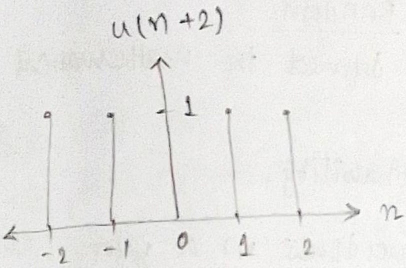
$\underline{u(-n)}$:

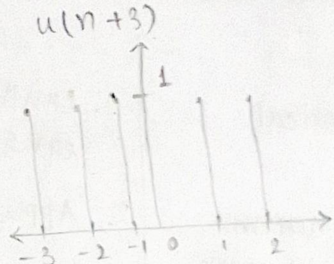


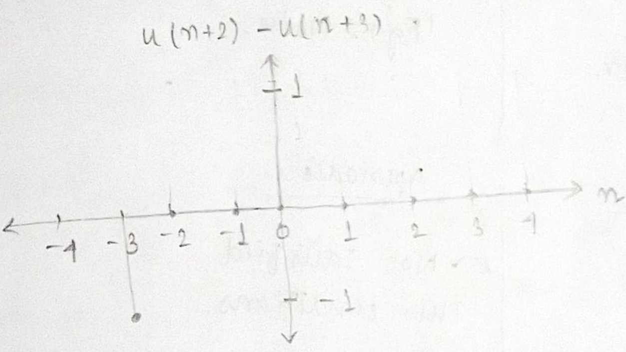
$\underline{u(-n+2)}$:



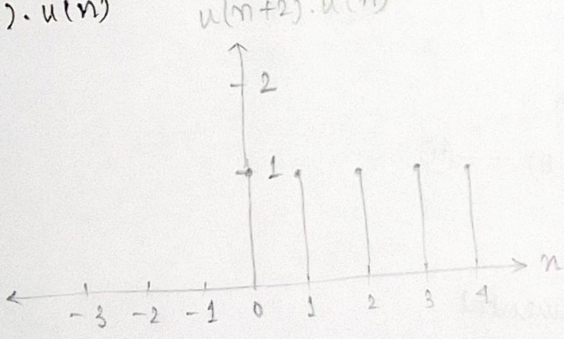
3. $u(n+2) - u(n+3)$.







4. $u(n+2) \cdot u(n)$



Classification of signals

1. CT and DT.
2. Periodic and Aperiodic (Non-periodic)
3. Even and Odd
   (Symmetric) (Asymmetric)
4. Causal and Non-causal.
5. Energy and Power.
6. Deterministic and random.

Deterministic

1. Defined with mathematical equations.
2. Future value can be determined from knowledge of past & present values.
3. Eg. sine, cosine, triangular.

Random

1. Can't be defined in mathematical equation.
2. Apply probability.
3. No. of accidents in a year.
   (Eg) → Noise.

Periodic

CT

$x(t) = x(t+T)$, for all t.

DT

$x(n) = x(n+N)$, for all n.

Aperiodic

Not satisfied This conditions.

Consider,

CT $\quad x(t) = A\sin(\omega t + \theta)$ — (i)

$x(t+T) = A\sin(\omega t + \omega T + \theta)$ — (ii)

$\omega T = 2\pi$

$$\boxed{T = \frac{2\pi}{\omega}}$$ ⇒ fundemental period.

DT

$x(n) = A\sin(\omega n + \theta)$

$x(n+N) = A\sin(\omega n + \omega N + \theta)$

$\omega N = 2\pi$

$$\boxed{N = \frac{2\pi}{\omega}\cdot m}$$, m = integer

* Complex Exponential :

$$x(t) = e^{j\omega t} \quad \text{———} \ (1)$$

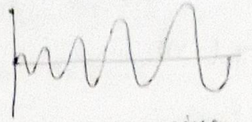
$$x(t+T) = e^{j\omega t} \cdot e^{j\omega T} \quad \text{———} \ (2)$$

(1) & (2) will become equal when,

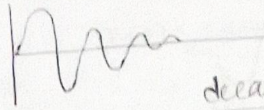
$$e^{j\omega T} = 1.$$

$$\Rightarrow \omega T = 2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}}$$



growing



decaying

Q1. Find the fundamental period for the following signals :

(a) $j5e^{j5t}$

(b) $\frac{5}{2}\cos(2\pi/5\, n + 1)$

(c) $10\sin\left(\frac{4\pi}{7}n + \frac{\pi}{6}\right)$.

$\rightarrow$ (a) $\omega = 5$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$$

(b) $\omega = \frac{2\pi}{5}m$

$N = \frac{2\pi}{\omega}m = \frac{10\pi}{2\pi} = 5m$ (Rational no., then periodic)

$m = 1,\ N = 5.$

(c) $\omega = \frac{4\pi}{7}m$

$$N = \frac{2\pi}{\omega}m = \frac{7 \cancel{4\pi}}{\cancel{2\pi}}m$$

$= 7\cancel{m}$; $m = 2$.

Q2. $x(n) = \frac{5}{2}\cos\left(\frac{2\pi}{5}n + 1\right) + 10\sin\left(\frac{4\pi}{7}n + \frac{\pi}{6}\right)$.

$\hookrightarrow N_2 = 7m$; 7

$N_1 = 5m$; 5

$\therefore N = \frac{N_1}{N_2} = \frac{\cancel{5m}}{\cancel{7m}}\ \frac{5}{7}$

$N = 7N_1 = 5N_2 = 35$

DT
when, $x(n) = N_1 + N_2$ $N = \frac{N_1}{N_2}$. $N_1, N_2 \in I$.
CT
$T = \frac{T_1}{T_2}$

Even

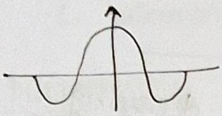
CT

$x(t) = x(-t)$

DT

$x(n) = x(-n)$

Eg. cosine



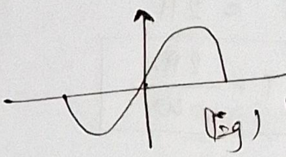
Odd

CT

$x(-t) = -x(t)$

DT

$x(-n) = -x(n)$



Eg) sine

Q. $x(t) = x_e(t) + x_o(t)$ —— ①

$x(-t) = x_e(-t) + x_o(-t)$

$= x_e(t) - x_o(t)$ —— ②

① + ②, $x(t) + x(-t) = 2x_e(t)$ $\Rightarrow \boxed{x_e(t) = \frac{1}{2}[x(t) + x(-t)]}$

① − ②, $x(t) - x(-t) = 2x_o(t)$.

$\Rightarrow \boxed{x_o(t) = \frac{1}{2}[x(t) - x(-t)]}$

Q. Find the even and odd component.

$x(t) = \cos t + \sin t + 2\sin t \cos t$ —— ①

$x(-t) = \cos(-t) + \sin(-t) + 2\sin(-t)\cos(-t)$

$= \cos(t) - \sin t - 2\sin t \cos t$ —— ②.

$x_e(t) = \frac{1}{2}[2\cos t] = \cos t.$

$x_o(t) = \frac{1}{2}[2\sin t] = \sin t.$

Causal | Non-causal

CT

$x(t) = 0, \; t < 0$ | $x(t) \neq 0, \; t < 0.$

DT

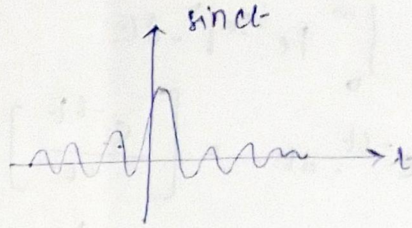
$x(n) = 0, \; n < 0$ | $x(n) \neq 0, \; n < 0.$

① $x(t) = e^{at} \cdot u(t)$ → 1, $t \geq 0$.
↳ causal

② $x(t) = e^{-at} \cdot u(-t)$
non-causal.

③ $x(t) = \text{sinc}\, t$
non-causal

sinct



$x(t) = \text{sinc}\, t \cdot u(t)$
causal.

④ $x(n) = \left(\frac{1}{2}\right)^n u(n+2)$
non-causal.

HW
$x(n) = u(n+2) - u(n+3)$

Energy

R
i(t)
+v(t) -

$P = v(t) \cdot i(t) = \frac{v^2(t)}{R} = i^2(t) \cdot R$

$R = 1\,\Omega$,

$P = i^2(t)$

Total energy.

$$E = \int_{-\infty}^{\infty} i^2(t)\, dt \text{ joules}$$

CT
* For a signal, R → Normalized.

$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \cdot dt$$

DT $E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2$

Power

Avg. power $P = \frac{1}{2T} \int_{-\infty}^{\infty} i^2(t)\, dt$ watts.

CT
* For a signal.

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \cdot dt.$$

DT $\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

* Any signal is said to be energy signal, if
$E \Rightarrow 0 < E < \infty$ ($E \rightarrow$ finite) & $P = 0$.

* If its power signal, $0 < P < \infty$ ($P \Rightarrow$ finite) & $E = \infty$

Q ① $x(t) = e^{-3t} u(t)$ find P & E.

$\rightarrow$ $$E = \lim_{T \to \infty} \int_{-T}^{T} |e^{-3t}|^2 . dt$$

$$= \lim_{T \to \infty} \int_{0}^{\infty} |e^{-3t}|^2 . dt$$

$$= \int_0^{\infty} e^{-6t} . dt = \left[\frac{e^{-6t}}{-6} \right]_0^{\infty}$$

$$= \frac{1}{-6} [e^{-\infty} - e^0] = \frac{1}{-6} [0 - (1)]$$

$$= \frac{1}{-6} . (-1) = \frac{1}{6}.$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |e^{-3t}|^2 dt$$

$$= \frac{1}{2T} \int_0^{\infty} |e^{-3t}|^2 . dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} . \frac{1}{6} = 0.$$

This is an energy signal.

② $x(n) = e^{j(\frac{\pi}{2} n + \frac{\pi}{8})}$.

$\rightarrow$ $$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |e^{j(\frac{\pi}{2} n + \frac{\pi}{8})}|^2$$

$$= \lim_{N \to \infty} \sum_{n=-N}^{N} 1. = \lim_{N \to \infty} (N + N + 1)$$

$$\boxed{|e^{j\theta}| = 1}$$

$$= \lim_{N \to \infty} (2N + 1)$$

$$= \infty.$$

$$\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1.$$

$$\sum_{n=0}^{N} = N + 1.$$

$$P = \lim_{n \to \infty} \frac{1}{2N+1} \cdot 2N+1$$

$= 1.$

This is a power signal.

<u>HW</u> $x(t) = e^{j\alpha t} \cdot \cos 5t.$ $\int uv\,dt$

$$\Rightarrow E = \lim_{T \to \infty} \int_{t=-T}^{T} |(e^{j\alpha t} \cdot \cos 5t)|^2 dt$$

$1 + \cos$

$\frac{2\cos 10t}{2}$

Classification of Systems

Systems → CT, DT

$x(n) \to$ DT sys. $h(n)$ $\to y(n)$

$y(n) = T[x(n)]$.

* when both x & y terms present, its a system

$x(t) \to$ CT system $h(t)$ $\to y(t) = T[x(t)]$.

$h(t)$ → Impulse response.

System can be: filter, amplifier, etc.

filter → noisy to clean.

* output $y(t)$
  can be found by convolution *
  of $x(t)$ & $h(t)$.

$y(t) = x(t) * h(t)$. ← CT → $y(t) = \int_{-\infty}^{\infty} x(T)\, h(T-\tau)\, dt$

DT → $y(n) = x(n) * h(n)$.

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

Classification:

1. CT and DT.
2. Static and dynamic
3. Linear & Non-linear
4. Time Invariant & Time Variant.
5. Causal and Non-causal.
6. Stable & unstable.

(2) Static & dynamic system:

Static

1. Sys. is static if it depends only on present i/p and not on future or past i/p.
2. Memoryless system.

Eg. $y(n) = n\,x(n)$.

present value dependency

$n = 1, \quad y(1) = 1\,x(1)$

$n = -1, \quad y(-1) = -1\,x(-1)$

Dynamic

1. Depends on present, past and future i/p.
2. Memory based system.

Eg. $y(n) = x(n-1)$.

$n = 1, \quad y(1) = x(0)$

$n = 0, \quad y(0) = x(-1)$

$n = -1, \quad y(-1) = x(-2)$

} depends on past values.

Q. check whether the system is static or dynamic:

$y(t) = x^2(t)$ → static

$y(n) = x(2n)$ → dynamic

$y(n) = x(n) + x(n-2)$ → dynamic

$y(t) = x(t) + x(t+1) + x^2(t)$. → dynamic

} one term enough to classify.

* $y(t) = \frac{d}{dt} x^2(t) + x(t)$

↳ dynamic.

(5) **Causal**

If it depends on present, past instant of time but NOT ON FUTURE values.

Non-causal

Depends on present, past and future also.

Eg.

Eg. $y(t) = x(t-1)$ → causal

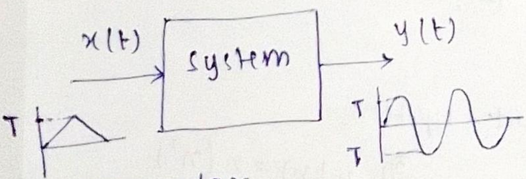
$y(n) = x(2n)$ → non-causal

$y(n) = x(n^2)$ → non-causal.

* $y(n) = x(-n)$ → ~~causal~~ non-causal

$y(t) = e^{x(t)}$ → causal.

(6) **Stable**



Triangular pulse.

Both the limits of i/p & o/p are same.

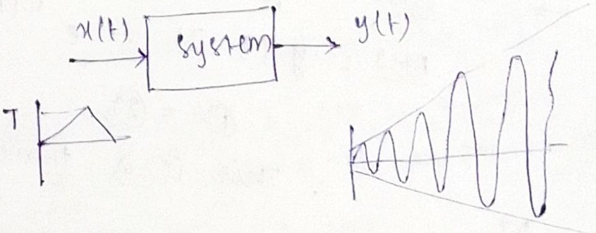
Bounded by limits.

$\boxed{\text{BIBO}}$ criteria

↓

Bounded i/p bounded o/p.

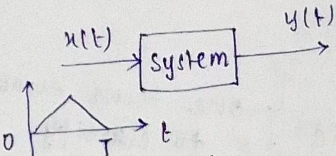
Unstable

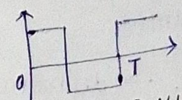


Not bounded.

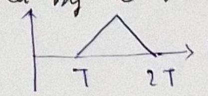
Unbounded o/p for bounded i/p

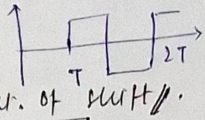
④ Time Invariant





shifted by 2 units: shifted by 2 units:





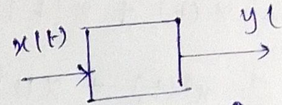
CT same amt. of shift.

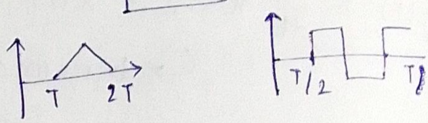
If, $\boxed{y(t-T) = T[x(t-T)]}$

then, time invariant system.

Time Variant

If, $y(t-T) \neq T[x(t-T)]$ then time invariant system.





Q. $y(t) = t\,x(t)$.

Check whether it is time variant or time invariant.

→ If i/p is delayed by 't' sec., $T[x(t-T)]$

$= T x(t-T)$ — ①

If o/p is delayed by 't' sec. $y(t-T) = (t-T)\,x(t-T)$ — ②

① ≠ ②.

∴ This is a time variant system.

Q. $y(t) = e^{x(t)}$

→ LHS : $T[x(t-T)] = e^{x(t-T)}$ — ①

RHS : $y(t-T) = e^{x(t-T)}$ — ②.

① = ②.

∴ This is a time invariant system.

DT

$\boxed{y(n-k) = T[x(n-k)]}$

③ $y(n) = x(n^2)$

$T[x(n-k)] = x(n^2-k)$

$y(n-k) = x((n-k)^2)$

Time variant.

Q. ① $y(n) = x(2n)$

$T[x(n-k)] = x[2n-k]$

$y(n-k) = x[2(n-k)]$

$= x(2n-2k)$

Here, subtract k only

Replace n by n-k

Time variant.

② $y(n) = x(-n)$.

$T[x(n-k)] = x(-n-k)$

$y(n-k) = x(-(n-k))$

$= x(-n+k)$.

Time variant.

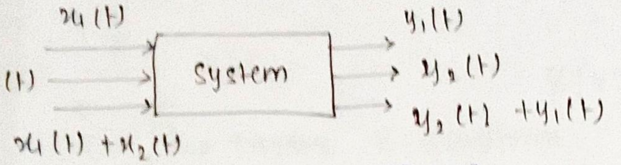
(a) $y(t) = x(t) \cos 50\pi t$.

$T[x(t-T)] = x(t-T) \cos 50\pi t$

$y(t-T) = x(t-T) \cos 50\pi (t-T)$

Time variant.

(3) Linear and non-linear :



DT

$T[ax_1(n) + bx_2(n)]$

$= aT[x_1(n)] + bT[x_2(n$

$y(t) = T[x(t)]$.

CT

$T[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$.

$= ay_1(t) + by_2(t)$.

Linear system

↳ obeys superposition principle.

Q. Check if (1) static & dynamic (2) linear & non-linear (3) Time variant & invariant (4) causal & non-causal

→ (1) $y(n) = x(n+1) - x(n-1)$

Dynamic, time invariant, non-causal, linear

$y(n-k) = x(n-k+1) - x(n-k-1)$

$x(n-k) = x(n+1-k) - x(n-1-k)$

$y(1) = x(2) - x(0)$

$T[x(n+1) - x(n-1)] = T[x(n+1)] + T[x(n-1)] = ay_1(n) + by_2(n)$

(2) $y(n) = x(n+1) + nx(n)$

(3) $y(n) = x(-n)$.

Dynamic

Non-causal.

$x(n+1-N) + (n-N)x(n-N)$

Non-linear.

when i/p $x_1(n)$ & o/p is $y_1(n)$

$y_1(n) = x_1(n+1) - x_1(n-1)$

$a.y_1(n) = ax_1(n+1) - ax_1(n-1)$ — (1)

when $x_2(n) \to y_2(n)$

$y_2(n) = x_2(n+1) - x_2(n-1)$

$b.y_2(n) = b.x_2(n+1) - b.x_2(n$ — (2)

Add (1) & (2).

RHS $ay_1(n) + by_2(n) = a[x_1(n+1) - x_1(n-1)] + b[x_2(n+1) - x_2(n-1)$

LHS $T[ax_1(n) + bx_2(n)]$

$= ay_1(n) + by_2(n)$

$= a[x_1(n+1) - x_1(n-1)] + b[x_2(n+1) - x_2(n-1)]$.

LHS = RHS.