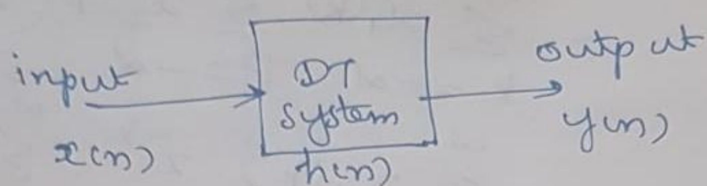
Analysis of DT System using z-transform

Consider a DT System



output of system, $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

In frequency domain

$$Y(z) = X(z) \cdot H(z)$$

Transfer function (or) system function $\boxed{H(z) = \frac{Y(z)}{X(z)}}$

$$h(n) = z^{-1}[H(z)]$$

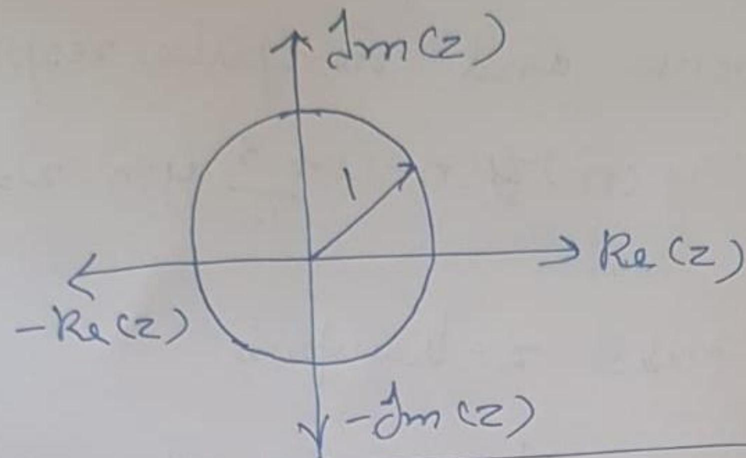
where $h(n) \rightarrow$ Impulse response

$\Rightarrow$ frequency response can be obtained by substituting

$$z = e^{j\omega}$$

$$\therefore \boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}}$$

Region of Convergence (ROC)



If all poles lies within the unit circle then the system is STABLE

If any one poles lies outside the unit circle then the system is UNSTABLE

plot the pole zero pattern and determine stability of the system

(a) $y(n) = 1.8y(n-1) - 0.72y(n-2) + x(n) + 0.5x(n-1)$

Apply Z-transform

$$Y(z) = 1.8z^{-1}Y(z) - 0.72z^{-2}Y(z) + X(z) + 0.5z^{-1}X(z)$$

$$Y(z)\left[1 - 1.8z^{-1} + 0.72z^{-2}\right] = X(z)\left[1 + 0.5z^{-1}\right]$$

$$H(z) = \frac{1 + 0.5z^{-1}}{1 - 1.8z^{-1} + 0.72z^{-2}}$$

$$= \frac{z(z+0.5)}{z^2 - 1.8z + 0.72}$$

$$= \frac{z(z+0.5)}{(z-1.2)(z-0.6)}$$

Equating numerator to Zero $\Rightarrow$ Zeros

$$z(z+0.5) = 0$$

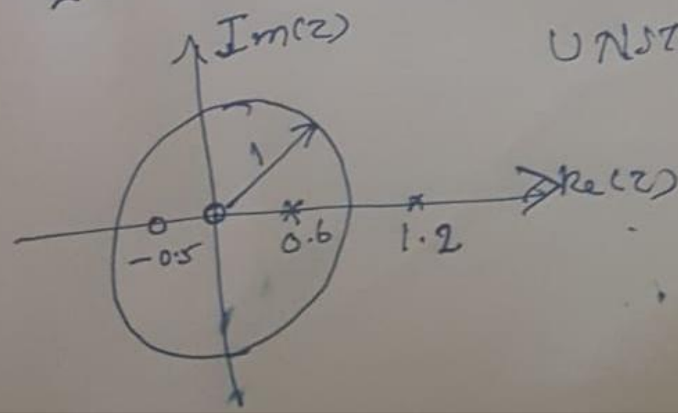
$$z = 0,\ z = -0.5$$

Equating denominator to Zero $\Rightarrow$ poles

$$(z-1.2)(z-0.6) = 0$$

$$z = 1.2,\ z = 0.6$$

UNSTABLE



Find the transfer function, frequency response and impulse response

$$y(n)-y(n-1)+\frac{3}{16}y(n-2)=x(n)-\frac{1}{2}x(n-1)$$

Take z-transform

$$Y(z)-z^{-1}Y(z)+\frac{3}{16}z^{-2}Y(z)=$$

$$X(z)-\frac{1}{2}z^{-1}X(z)$$

$$Y(z)\left[1-z^{-1}+\frac{3}{16}z^{-2}\right]=X(z)\left[1-\frac{1}{2}z^{-1}\right]$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+\frac{3}{16}z^{-2}}$$

$$=\frac{z(z-\frac{1}{2})}{(z-\frac{1}{4})(z-3/4)}$$

Apply partial fraction

$$\frac{H(z)}{z}=\frac{A_1}{z-1/4}+\frac{A_2}{z-3/4}$$

Solve and find A_1 and A_2

$$A_1=\frac{1}{2},\quad A_2=\frac{1}{2}$$

$$H(z)=\frac{1}{2}\left(\frac{z}{z-1/4}\right)+\frac{1}{2}\left(\frac{z}{z-3/4}\right)$$

Take inverse

Impulse response, $h(n)=z^{-1}[H(z)]=\frac{1}{2}(1/4)^n u(n)+\frac{1}{2}(3/4)^n u(n)$