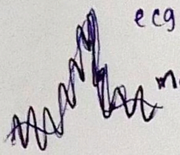
MOD-4

Design of Analog Filters



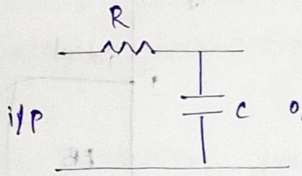
if we use lowpass filter,
it will not get seperated.
↓ advanced soln.

Adaptive filters.

Analog filters → Highpass Filter (HPF), Lowpass Filter (LPF). } **Bandpass Filter (BPF).**

↓

$f = 50\,Hz$
↓
specific R & C value.



For $f = 60\,Hz$, new design.

* Drawbacks: ① Not suitable for changing freq.
② Multiple filtering not possible.

Digital Filters → Programmable filters.
↳ ① No worry for component aging.
② Multiple filtering possible.
③ No external noises interferance.

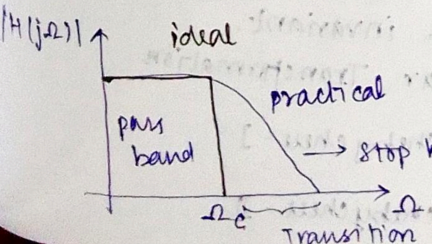
$x(n) \rightarrow$ System (filter) $h(n)$ $\rightarrow$ **$y(n) = x(n) * h(n)$.**

To design a filter, $\boxed{H(z) = \dfrac{Y(z)}{X(z)}}$

Digital filter transfer function.

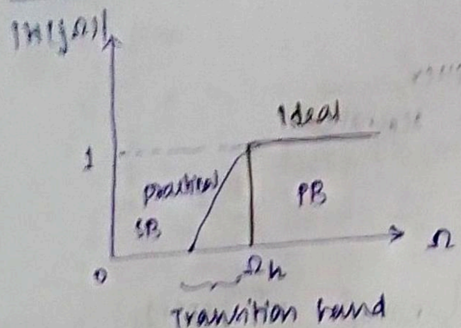
Types of digital filter → LPF, HPF, BPF, BRF. } (same as Analog).

LPF

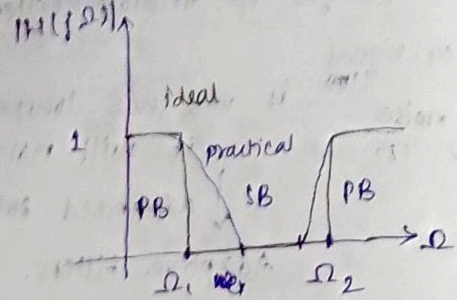


filter characteristics
(or)
Magnitude response

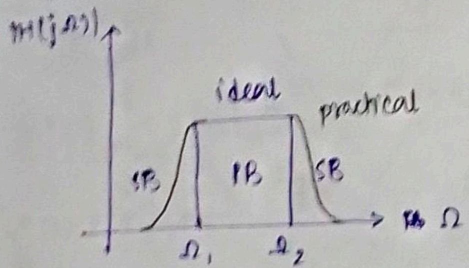
HPF



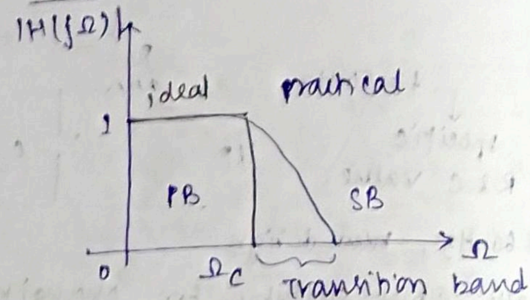
BRF



BPF



LPF



In DSP,

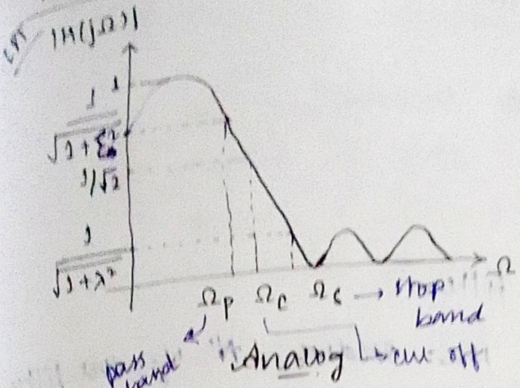
Filter → It will pass desired band of frequency and stop (or) alternate (or) block band of frequency.

→ LTI system (Linear time invariant).

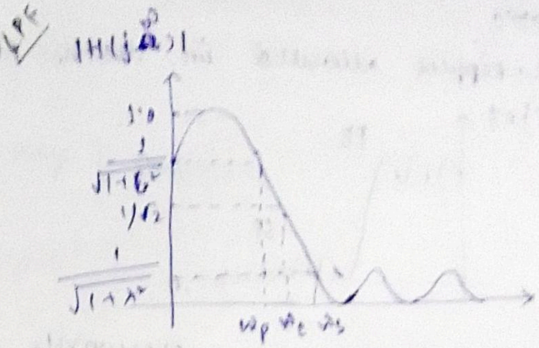
Digital Filters

- **FIR** — Finite impulse response A.K.A. Non recursive filters
  - o/p depends on present i/p and o/p.
  - methods → Windowing techniques
- **IIR** — Infinite impulse response / Recursive filters.
  - o/p depends on present i/p o/p and past i/p & o/p.
  - methods to design
    - **Indirect** — analog prototype (H(s)) From there, make H(z)
      - H(s) → H(z)
      - procedure to design
        - Butterworth
        - Chebychev → Chebychev - I, Chebychev - II
    - **Direct** — Directly H(z).
      - → Impulse invariant.
      - → Bilinear Transformation

Alternate specifications



Analog



digital

Steps to design analog (continous time) IIR filter :-

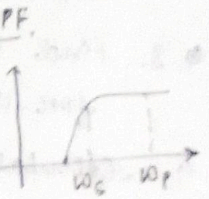
1. From the given analog specifications, find the corresponding digital specifications.
2. derive the transfer function for analog prototype.
3. Transform the analog prototype by using suitable digital transformation.

Types of IIR filter design methods :

1. Butterworth
2. chebbychev → I, II
3. Elliptic filter
4. Bessel filter.

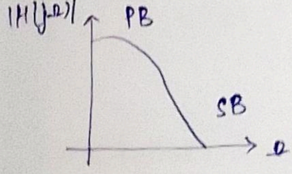
I → LPF, HPF, BPF, BRF.

II → LPF, HPF, BPF, BRF



① Butterworth

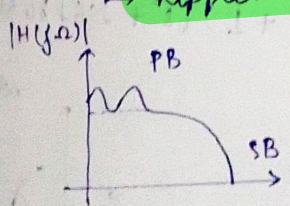
↳ Monotonic passband and stopband.



There are no ripples. (smooth filter)

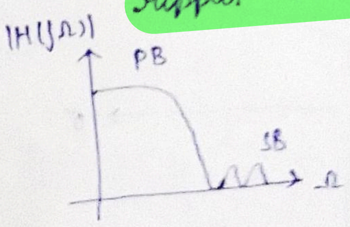
② chebbychev - I

↳ Ripples in passband, monotonic in stopband



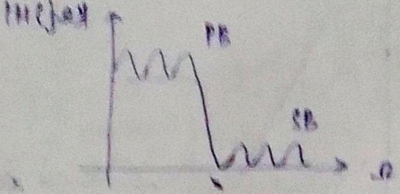
③ chebbychev - II

↳ Monotonic in passband, ripples in stopband.



③ Elliptic

↳ Ripples allowed in both PB & SB.



* Steps to design a butterworth IIR filter:

1. From the given specifications find the order 'N' of the filter.

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

2. Round off to nearest integer value.
(Next higher integer)

3. Find the transfer function $H(s)$ for the given value of N for $\Omega_c = 1$ rad/s.

4. Calculate the cutoff frequency $\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}}$

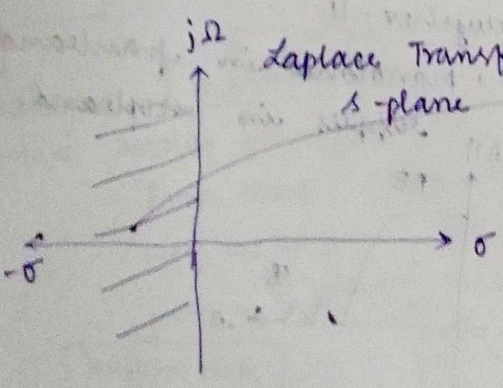
5. Find the transfer function $H_a(s)$ by substituting $s \to \frac{s}{\Omega_c}$ in $H(s)$ for LPF.
($s \to \frac{\Omega_c}{s}$ for HPF)

(Num. always 1)

\# For butterworth, $H(s) = \frac{1}{\ }$

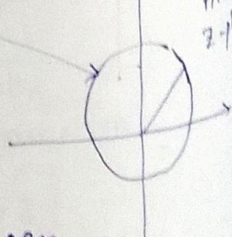
Num	Denom.
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.84773s+1)$
5	$(s+1)(s^2+0.618038s+1)(s^2+1.618035s+1)$

Laplace Transform



If all the poles are in left half plane, then the system is stable

we can map.



$\alpha_s \rightarrow$ max allowable stopband attenuation.
$\alpha_p \rightarrow$ " " passband "

$\Omega_s \rightarrow$ Stopband frequency } analog
$\Omega_p \rightarrow$ PB freq.
$\omega_s \rightarrow$ SB freq. (rad / sec) } digital
$\omega_p \rightarrow$ PB freq.

when α_s and α_p not given,

$$N \geq \frac{\log(\lambda / \varepsilon_0)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}.$$

$$\varepsilon_0 = (10^{0.1\alpha_p} - 1)^{1/2}$$

$$\lambda = (10^{0.1\alpha_s} - 1)^{1/2}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{\Omega_s}{(10^{0.1\alpha_s} - 1)^{1/2N}}$$

$$= \frac{\Omega_p}{(\varepsilon_0)^{1/N}} = \frac{\Omega_s}{\lambda^{1/N}}.$$

$H(s) \rightarrow H(z)$
digital filter design.

chebyshev / butterworth + bilinear tr.
after getting H(s) from butterworth, compute H(z) from here

direct method

① Bilinear Transformation

$$H(z) = H(s)\Big|_{s \rightarrow \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}.$$

* Relationship between Ω and w. (analog freq. and digital frequency).

Pre-warping frequency.
$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2}$$
$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2}$$

Steps to design a Chebyshev LPF:

1. From the given specification, find order 'N'

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

(OR) $$N \geq \frac{\cosh^{-1}(\lambda/\varepsilon_0)}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

2. Round off to next higher integer.

3. Find the values of 'a' and 'b' which are the minor and major axes of ellipse.

$$a: \Omega_p\left[\frac{\mu^{1/N}-\mu^{-1/N}}{2}\right]$$

α_p : Pass band atten.

α_s = SB attenuat^n.

$$b: \Omega_p\left[\frac{\mu^{1/N}+\mu^{-1/N}}{2}\right]$$

where, $\mu = \varepsilon_0^{-1} + \sqrt{\varepsilon_0^{-2}+1}$

$\varepsilon_0 = \sqrt{10^{0.1\alpha_p}-1}$

4. Calculate the poles which lies on the ellipse.

$$s_k = a\cos\phi_k + jb\sin\phi_k$$

$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi$$

$$k = 1, 2, \ldots, N.$$

5. Find the denom. of the transfer function as

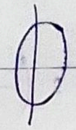
$$(s-s_1)(s-s_2)\ldots$$

6. Find num. of tr. func. by taking denom.,

(a) when N = odd, substitute $s = 0$ in denom.

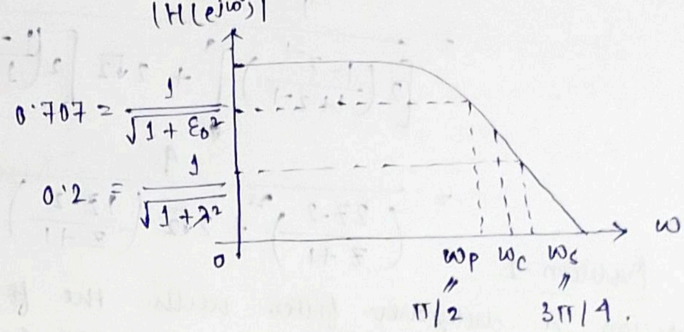
(b) when N = even, substitute $s = 0$ and divide by $\sqrt{1+\varepsilon_0^2}$.

In chebbychev,

$z \longrightarrow s$:  (Not a circle).

Problem 1 :

design a digital butterworth filter satisfying the constraints $0.707 \le |H(e^{j\omega})| \le 1$ for $0 \le \omega \le \pi/2$ and $|H(e^{j\omega})| \le 0.2$ for $\frac{3\pi}{4} \le \omega \le \pi$, with $T = 1$ sec using bilinear transformat^n



Ans - Given for LPF,

$$\frac{1}{\sqrt{1+\varepsilon_0^2}} = 0.707$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\omega_p = \pi/2, \quad \omega_s = \frac{3\pi}{4}.$$

$$\therefore \frac{1}{1+\varepsilon_0^2} = (0.707)^2 \Rightarrow 1+\varepsilon_0^2 = (0.707)^2$$

$$\Rightarrow \varepsilon_0^2 = (0.707)^2 - 1.$$

$$\Rightarrow \varepsilon_0 = \sqrt{(0.707)^2 - 1}$$

$$\Rightarrow \varepsilon_0 = 1.$$

and, $\lambda = 4.89$.

$$\text{Now, } \Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = \frac{2}{1}\tan\frac{3\pi}{8}$$

$$= 2\tan\frac{3\pi}{8} = 4.82 \text{ rad/s}$$

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\frac{\pi}{4} = 2 \text{ rad/s.}$$

$$N = \frac{\log(\lambda/\varepsilon_0)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = 1.804$$

$$\approx 2 \text{ (Rounded off).}$$

$$\therefore H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}, \text{ for } \Omega_c = 1 \text{ rad/s.}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{\Omega_p}{\varepsilon_0^{1/N}} = \frac{2}{1} = 2$$

$\therefore$ Here, $H(s)$ for $\Omega_c = 2$ rad/s.

$$H_a(s) = H(s)\Big|_{s \to \frac{s}{2}} = \frac{1}{\frac{s^2}{4} + \frac{s}{\sqrt{2}} + 1} = \frac{}{\frac{s^2 + 2\sqrt{2}s + 4}{4}}$$

By Bilinear Transformation,

$$H(Z) = H_a(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)} = 2\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right).$$

$$= \frac{4}{s^2 + 2\sqrt{2}s + 4}\Bigg|_{s \to 2\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)}$$

$$\frac{1-\frac{1}{Z}}{1+\frac{1}{Z}} \qquad \frac{\frac{Z-1}{Z}}{\frac{Z+1}{Z}} \qquad \frac{Z-1}{Z+1}$$

$\therefore$

$$= \frac{4}{\left[2\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)\right]^2 + 2\sqrt{2}\left[2\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)\right] + 4}$$

$$= \frac{4}{\left(\frac{2Z-2}{Z+1}\right)^2 + 2\sqrt{2}\left(\frac{2Z-2}{Z+1}\right) + 4}$$

Problem-2

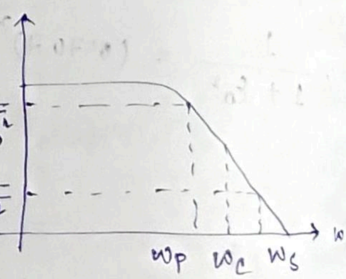
design a chebyshev filter with the following specifications using bilinear transformation $T = 1$ sec, $0.8 \le |H(e^{j\omega})| \le 1$ for $0 \le \omega \le 0.2\pi$

$|H(e^{j\omega})| \le 0.2$ for $0.6\pi \le \omega \le \pi$.

$\rightarrow$ $\omega_p = 0.2\pi$, $\omega_s = 0.6\pi$

$$\frac{1}{\sqrt{1+\varepsilon_0^2}} = 0.8, \quad \varepsilon_0 = 0.75$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2, \quad \lambda = 4.89$$



$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2\tan\frac{0.6\pi}{2}$$

$$= 2.75 \text{ rad/s}$$

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\frac{0.2\pi}{2}$$

$$= 0.64 \text{ rad/s}$$

Now, $N \geq \dfrac{\cosh^{-1}(\lambda/\varepsilon_0)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$

$$\boxed{\cosh^{-1}x = \ln\left[x + \sqrt{x^2-1}\right]}$$

$$= \frac{\ln\left[\lambda/\varepsilon_0 + \sqrt{(\lambda/\varepsilon_0)^2 - 1}\right]}{\ln\left[\frac{\Omega_s}{\Omega_p} + \sqrt{\left(\frac{\Omega_s}{\Omega_p}\right)^2 - 1}\right]}$$

$$= 1.208.$$

$$\therefore \boxed{N = 2}$$

$\mu = \varepsilon_0^{-1} + \sqrt{\varepsilon_0^{-2} + 1}$

$= 1.34 + \sqrt{1.78 + 1}$

$= 1.34 + \sqrt{2.78}$

$= 1.34 + 1.667 = 3.007.$

$a : \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right]$

$= 0.64 \left[\frac{3.007^{1/2} - 3.007^{-1/2}}{2} \right]$

$= 0.37.$

$b : \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right]$

$= 0.64 \left[\frac{\sqrt{3} + 3^{-1/2}}{2} \right]$

$= 0.73.$

$s_k = a \cos \phi_k + jb \sin \phi_k$

$\phi_k = \frac{\pi}{2} + \left(\frac{2.k-1}{2.2} \right) \pi$

$k = 1, 2.$

$\phi_1 = \frac{\pi}{2} + \left(\frac{1}{4} \right) \pi = \frac{3\pi}{4}$

$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4}$

$s_1 = a \cos \frac{3\pi}{4} + jb \sin \frac{3\pi}{4}$

$= -0.26 + j0.53$

$s_2 = a \cos \frac{5\pi}{4} + jb$

$= -0.26 - j0.53.$

denom. $= (s - s_1)(s - s_2)$

$= (s - 0.26 - j0.53)(s - 0.26 + j0.53) = s^2 + 0.5306s + 0.3516$

num. = substitute $s = 0$ and divide by $\sqrt{1 + \varepsilon_0^2}$

$= \frac{(-0.26 - j0.53)(-0.26 + j0.53)}{1.25}$

$= 0.2788. \approx 0.28$

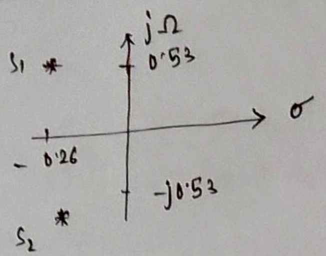
By bilinear Transformation,

$H(s) = \frac{0.28}{s^2 + 0.5306s + 0.3516}$

$H(z) = H(s) \Big|_{s \to \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)} = \frac{0.28}{\left[2\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)\right]^2 + 0.5306 \times 2 \left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) + 0.3516}$

$= \frac{0.28}{\left(\frac{2z - 2}{z + 1}\right)^2 + 0.5306 \left(\frac{2z - 2}{z + 1}\right) + 4}$

$$\boxed{H(Z) = \frac{0.052\,(1+Z^{-1})^2}{1 - 1.3480\,Z^{-1} + 0.608\,Z^{-2}}}$$



All poles lie on the left side of the plane so its a stable filter.

HW

① ⇄ ②.

③ $\frac{1}{\sqrt{2}} \leq |H(e^{j\omega})| \leq 1$ for $0 \leq \omega \leq 0.2\pi$, $0 \leq |H(e^{j\omega})| \leq 0.1$, $0.5\pi \leq \omega \leq \pi$ design using chebyshev filter with $T = 1$ s.