Module-3

Fourier transforms:-

fourier ⇒ time ⟶ frequency

- Fourier series
- Discrete fourier series
- CTFT
- DTFT
  - DFT — discrete fourier transform → finite values; ⟵ sampled version of DTFT
  - FFT — fast fourier transform

time	Periodic	non-periodic
cont (t)	FS	CTFT (FT)
discrete (nt) [n]	DFS	DTFT

* $\delta(n)$

$FT(\delta(n)) = 1$

* $\delta(t - t_0)$

$$Ft\left[\delta(t-t_0)\right] = e^{-j\omega t_0}$$

** $$\boxed{x(\omega) = \int_{-\alpha}^{\alpha} x(t)\, e^{-j\omega t}\, dt}$$

$$= e^{-j\omega t}\Big]_{t=0}$$

$$x(\omega) = \int_{-\alpha} \delta(0)\, e^{-0} = 1$$

Discrete - time - fourier transform : (DTFT)

→ $x(n)$
↓
discrete,
non-periodic
signal,

$$DTFT[x(n)] = X(\omega) \text{ (or) } X(e^{j\omega})$$

$$\boxed{X(e^{j\omega}) = X(\omega) = \sum_{n=-\alpha}^{\alpha} x(n)\, e^{-j\omega n}}$$

Note:-

* existence of DTFT

* The F.T for a discrete ~~signal~~ time sequence $x(n)$ is exists if and only if the sequence is absolute summable and which satisfies below conditions:

$$\left(\sum_{n=-\alpha}^{\alpha} |x(n)| < \infty \right)$$

IDTFT

$$\boxed{F^{-1}[X(\omega)] = x[n] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega n}\, d\omega}$$

↳ inverse discrete fourier transform

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$x(n) = \delta(n)$

$DTFT[x(n)] = X(\omega)$

$$= \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} = \delta(0) e^{-0}$$

DT - $\boxed{X(\omega) = 1}$

$\delta(n-m) = x(n)$

$DTFT[x(n)] = X(\omega)$

$$X(\omega) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} = 1 . e^{-j\omega n}\Big|_{n=m}$$

$$\boxed{X(\omega) = e^{-j\omega m}}$$

* relation b/w z & fourier transforms

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$z = re^{j\omega} \Rightarrow X(\omega) = X(z)$ when $r^{-n} = 1$

$x(n) = \delta(n+3) - \delta(n-3)$

$$X(\omega) = e^{j\omega 3} - e^{-3j\omega} = e^{3j\omega} - e^{-3j\omega}$$

$$= \frac{e^{3j\omega} - e^{-3j\omega}}{2j} (2j)$$

$$= \frac{e^{6j\omega} - 1}{e^{3j\omega}}$$

$$\boxed{X(\omega) = 2j \sin 3\omega}$$

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$x(n) = \{\underset{\uparrow}{1}, -2, 2, 3\}$

$X(z) = \cancel{z}\ \cancel{+} \ 1 - 2z^{-1} + 2z^{-2} + 3z^{-3}$

$\cancel{x} \cdot X(\omega) = 1 - 2r^{-1}(e^{j\omega})^{-1} + 2r^{-2}(e^{j\omega})^{-2} + 3r^{-3}(e^{j\omega})^{-3}$

$$\boxed{X(\omega) = 1 - 2e^{-j\omega} + 2e^{-2j\omega} + 3e^{-3j\omega}}$$

$X(\omega) = x(0) + x(1)e^{-j\omega} + x(2)e^{-2j\omega} + \ldots$

$x(n) = (1/4)^n u(n)$

$$X(\omega) = \sum_{-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= \sum_{0}^{\infty} (1/4)^n \cancel{u(n)}\, e^{-j\omega n}$$

$$= \sum_{0}^{\infty} \left(\frac{1}{4} e^{-j\omega}\right)^n = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$$

$$\Rightarrow \boxed{X(\omega) = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}}$$

$x(n) = (1/4)^n u(n+1)$

$$X(\omega) = \sum_{n=-1}^{\infty} (1/4)^n e^{-j\omega n}$$

$$\cancel{X} \left[= \frac{1}{4} e^{-j\omega} \left[\sum_{n=0}^{\infty} (\cancel{1/4}) (1/4\, e^{-j\omega})^n \right] \right.$$

$$= \left(\frac{1}{4} e^{-j\omega} \right)^{-1} \left[\frac{1}{1 - 1/4\, e^{-j\omega}} \right]$$

$$= \left(\frac{1}{\cancel{4}} e^{-j\omega} \right)^{-1} \left[\frac{\cancel{4}}{4 - e^{-j\omega}} \right] \quad \cancel{+} \boxed{\cancel{\frac{e^{-j\omega}}{4 - e^{-j\omega}}}}$$

$$= \left. \boxed{\frac{16}{4 - e^{-j\omega}} = X(\omega)} \right] \times$$

$$X(\omega) = \sum_{n=-1}^{\infty} (1/4)^n e^{-j\omega n}$$

$$= (1/4\, e^{-j\omega})^{-1} + \left[\sum_{n=0}^{\infty} (1/4\, e^{-j\omega})^n \right]$$

$$= \left(\frac{1}{4} e^{-j\omega} \right)^{-1} + \frac{1}{1 - 1/4\, e^{-j\omega}}$$

#

$x(n) = a^n u(n)$

$\text{DTFT}[x(n)]$

$$\text{DTFT} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} a^n u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (a e^{-j\omega})^n$$

$$\Rightarrow \boxed{\frac{1}{1 - a e^{-j\omega}}}$$

#

$x(n) = -a^n u(-n-1)$

$\text{DTFT}[-a^n u(-n-1)]$

$$\Rightarrow \sum_{n=-\infty}^{\infty} -a^n u(-n-1) e^{-j\omega n}$$

$$\Rightarrow \sum_{n=-\infty}^{-1} -a^n e^{-j\omega n}$$

$$= -\sum_{n=1}^{\infty} a^{-n} e^{j\omega n}$$

$$= -\left[\sum_{n=1}^{\infty} (a^{-1} e^{j\omega})^n\right]$$

$$= -\left[a^{-1} e^{j\omega} + (a^{-1} e^{j\omega})^2 + \dots\right]$$

$$= -a^{-1} e^{j\omega}\left[1 + a^{-1} e^{j\omega} + (a^{-2} e^{j\omega})^2 + \dots\right]$$

$$= \frac{-a^{-1} e^{j\omega}}{1 - a^{-1} e^{j\omega}} = \frac{-1}{\frac{1}{a^{-1} e^{j\omega}} - 1}$$

$$= \frac{-1}{a e^{-j\omega} - 1} = \boxed{\frac{1}{1 - a e^{-j\omega}}}$$

Properties :-

* Linearity :-

$$DTFT[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(\omega) + a_2 X_2(\omega)$$

* Ex: $0.55\cos\omega + 0.25\tan\omega = y(n)$

$$Y(\omega) = 0.55\, X_1(\omega) + 0.25\, X_2(\omega)$$

$$X_1(\omega) = \tfrac{1}{2}\,DTFT[e^{j\omega}] - \tfrac{1}{2}\,DTFT[e^{-j\omega}]$$

* Periodicity :-

$$\cancel{DTFT}\,[\; X(\omega + 2n\pi) = X(\omega)$$

* Time shifting :-

$$DTFT[x(n-m)] = e^{-j\omega m}\, X(\omega)$$

* Frequency shifting :-

$$DTFT[e^{+j\omega_0 n}\, x(n)] = X(\omega - \omega_0)$$

$$DTFT[e^{-j\omega_0 n}\, x(n)] = X(\omega + \omega_0)$$

* Time reversal:-

$$DTFT[x(-n)] = X(-\omega)$$

* Differentiation:-

$$DTFT[n.x(n)] = -j \frac{d}{d\omega}[X(\omega)]$$

$$DTFT[n^m.x(n)] = (-j)^m \frac{d^m}{d\omega^m}[X(\omega)]$$

* Time convolution:-

$$DTFT[x_1(n) * x_2(n)] = X_1(\omega) \cdot X_2(\omega)$$

* Frequency convolution:-

$$DTFT[x_1(n) \cdot x_2(n)] = X_1(\omega) * X_2(\omega)$$

prdt modulated signals

* Correlation:-

$$DTFT[R_{xx}(l)] = X_1(\omega) \cdot X_2(-\omega)$$

$\Sigma x(n) x(n-l)$

* Parseval's theorem:-

$$DTFT\left[E = \sum_{n=-\alpha}^{\alpha} |x(n)|^2\right] = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

Problems:-

$x(n) = \left(\frac{1}{4}\right)^{|n-2|}$

$$X(\omega) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} = e^{-2j\omega} X_1(\omega)$$

$$= \cancel{2} \sum_{n=\cancel{2}}^{\alpha} \left(\frac{1}{4}\right)^{n-2} e^{-j\omega n}$$

$$= \cancel{2} \sum_{n=\cancel{2}}^{\alpha} \cancel{\left(\frac{1}{4}\right)^{n-2}} e^{-j\omega n}$$

$$X_1(\omega) = DTFT\left[\left(\frac{1}{4}\right)^{|n|}\right]$$

$$= \sum_{n=-\alpha}^{\alpha} \left(\frac{1}{4}\right)^{|n|} e^{-j\omega n}$$

$$= \sum_{n=-\alpha}^{-1} \left(\frac{1}{4}\right)^{-n} e^{-j\omega n} + \sum_{n=0}^{\alpha} \left(\frac{1}{4}\right)^{n} e^{-j\omega n}$$

$$= \sum_{n=1}^{\alpha} \left(\frac{1}{4}\right)^{n} e^{j\omega n} + \sum_{n=0}^{\alpha} \left(\frac{1}{4}\right)^{n} e^{-j\omega n}$$

$$= \left(\left(\frac{1}{4}\right) e^{j\omega n} + \left(\frac{1}{4}\right)^2 e^{2j\omega n} + \dots\right) + \frac{1}{1 - \frac{1}{4} e^{-j\omega}}$$

$$= \left(\frac{1}{4}\right) e^{j\omega} \left(\frac{1}{1 - \frac{1}{4} e^{j\omega}}\right) + \frac{1}{1 - \frac{1}{4} e^{-j\omega}} \qquad \frac{1}{\frac{1}{4} e^{j\omega}} - 1$$

$$= \left(\frac{1}{4e^{-j\omega} - 1} + \frac{1}{1 - \frac{1}{4} e^{-j\omega}}\right) = X_1(\omega)$$

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$$X(\omega) = e^{-2j\omega}\left[\frac{1}{1-\frac{1}{4}e^{-j\omega}} + \frac{1}{4e^{-j\omega}-1}\right]$$

$$= e^{-2j\omega}\left[\frac{4e^{-j\omega}\cancel{-1}+\cancel{1}-\frac{1}{4}e^{-j\omega}}{(4e^{-j\omega}-1)\left(1-\frac{1}{4}e^{-j\omega}\right)}\right]$$

$$= \frac{e^{-2j\omega}\left[-\frac{1}{4}e^{-j\omega}+4e^{-j\omega}\right]}{4e^{-j\omega}-e^{-2j\omega}-1+\frac{1}{4}e^{-j\omega}}$$

$$= \frac{e^{-2j\omega}\left[-e^{-j\omega}+16e^{-j\omega}\right]}{\frac{\cancel{4}\left(16e^{-j\omega}-4e^{-2j\omega}-4+e^{-j\omega}\right)}{\cancel{4}}}$$

$$= \frac{e^{-2j\omega}\left[-e^{-j\omega}+16e^{-j\omega}\right]}{17e^{-j\omega}-4e^{-2j\omega}-4} = \frac{e^{-2j\omega}\left[15e^{-j\omega}\right]}{17e^{-j\omega}-4e^{-2j\omega}-4}$$

$$= e^{-2j\omega}\left[\frac{15/16}{17/16-\frac{1}{2}\cos\omega}\right]$$

symmetry property:-

$x(\omega) = X_R(\omega) + j X_I(\omega)$

$x(\omega) = |x(\omega)| e^{j\theta(\omega)}$ → phase

↓

magnitude

Magnitude response → graph between $|x(\omega)|$ and ω values.

Phase response → graph between $\theta(\omega)$ and ω values

$$\theta(\omega) = \tan^{-1}\left(\frac{X_I(\omega)}{X_R(\omega)}\right)$$

$$|x(\omega)| = \sqrt{(X_I(\omega))^2 + (X_R(\omega))^2}$$

DTFT $\{n(1/2)^n u(n)\}$ → differentiation.

$$= \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n} \Rightarrow \sum_{n=-\alpha}^{\alpha} \left[n(1/2)^n u(n)\right]$$

$$DTFT\{x(n)\} = -j \frac{d}{d\omega} X(\omega)$$

let $x_1(n) = (1/2)^n u(n)$

$$DTFT[x_1(n)] = X(\omega) = \sum_{n=-\alpha}^{\alpha} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\alpha} (1/2)^n e^{-j\omega n} = \frac{1}{1-1/2 e^{-j\omega}} \Rightarrow X(\omega) = \frac{1}{1-1/2 e^{-j\omega}}$$

$$X(\omega) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

$\cancel{n\,x(n)}$ $$\text{DTFT}[n\,x(n)] = -j\frac{d}{d\omega}[X(\omega)]$$

$$= -j\frac{d}{d\omega}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]$$

$$= -j\left[\frac{0-(-\frac{1}{2}e^{-j\omega})(-j)}{(1-\frac{1}{2}e^{-j\omega})^2}\right]$$

$$= \frac{j\,(\frac{1}{2}e^{-j\omega})}{(1-\frac{1}{2}e^{-j\omega})^2} = \frac{\frac{1}{2}e^{-j\omega}(j)^2}{\frac{1+\frac{1}{4}e^{-2j\omega}-e^{-j\omega}}{\cancel{4}}}$$

$$= \cancel{k} \frac{\frac{\frac{1}{2}e^{-j\omega}(j)^2}{e^{-j\omega}}}{\frac{1}{e^{-j\omega}}+\frac{1}{4}e^{-j\omega}-1} = \frac{\frac{1}{2}j^2}{e^{j\omega}+\frac{e^{-j\omega}}{4}-1}$$

$$= \frac{\frac{1}{2}j^2}{e^{j\omega}+\frac{e^{-j\omega}}{4}-1} = \frac{1}{\frac{e^{j\omega}+\frac{e^{-j\omega}}{4}-1}{\frac{1}{2}j^2}}$$

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Inverse DTFT:-

$$\boxed{x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega}$$

$X(\omega) = e^{-j\omega} \longrightarrow$ find IDTFT

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega}\cdot e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-1)}\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(n-1)}e^{j\omega(n-1)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left(\frac{1}{j(n-1)}\right)\left[e^{j\pi(n-1)} - e^{-j\pi(n-1)}\right]$$

$$= \frac{1}{\pi(n-1)}\left[\sin\pi(n-1)\right]$$

$$\boxed{x(n) = \frac{\sin\pi(n-1)}{\pi(n-1)}}$$

Transfer function:-

$$\boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)}}$$

$$\downarrow$$

$DTFT[h(n)] = H(\omega)$

$$H(\omega) = \underbrace{|H(\omega)|}_{\text{magnitude}}\, e^{j\underbrace{\angle H(\omega)}_{\text{phase}}}$$

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$|H(\omega)|$ vs $\omega \rightarrow$ ~~magni~~ magnitude response

& $\angle H(\omega)$ vs $\omega \longrightarrow$ phase response

Find the magnitude & phase response of the following system given,

① $y(n) - 5y(n-1) = x(n) + 4x(n-1)$ $\quad e^{-j\omega n}$

$$Y(\omega) - 5e^{-j\omega} Y(\omega) = X(\omega) + 4e^{-j\omega} X(\omega)$$

$$Y(\omega)\left[1 - 5e^{-j\omega}\right] = X(\omega)\left[1 + 4e^{-j\omega}\right]$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{1+4e^{-j\omega}}{1-5e^{-j\omega}} = H(\omega)$$

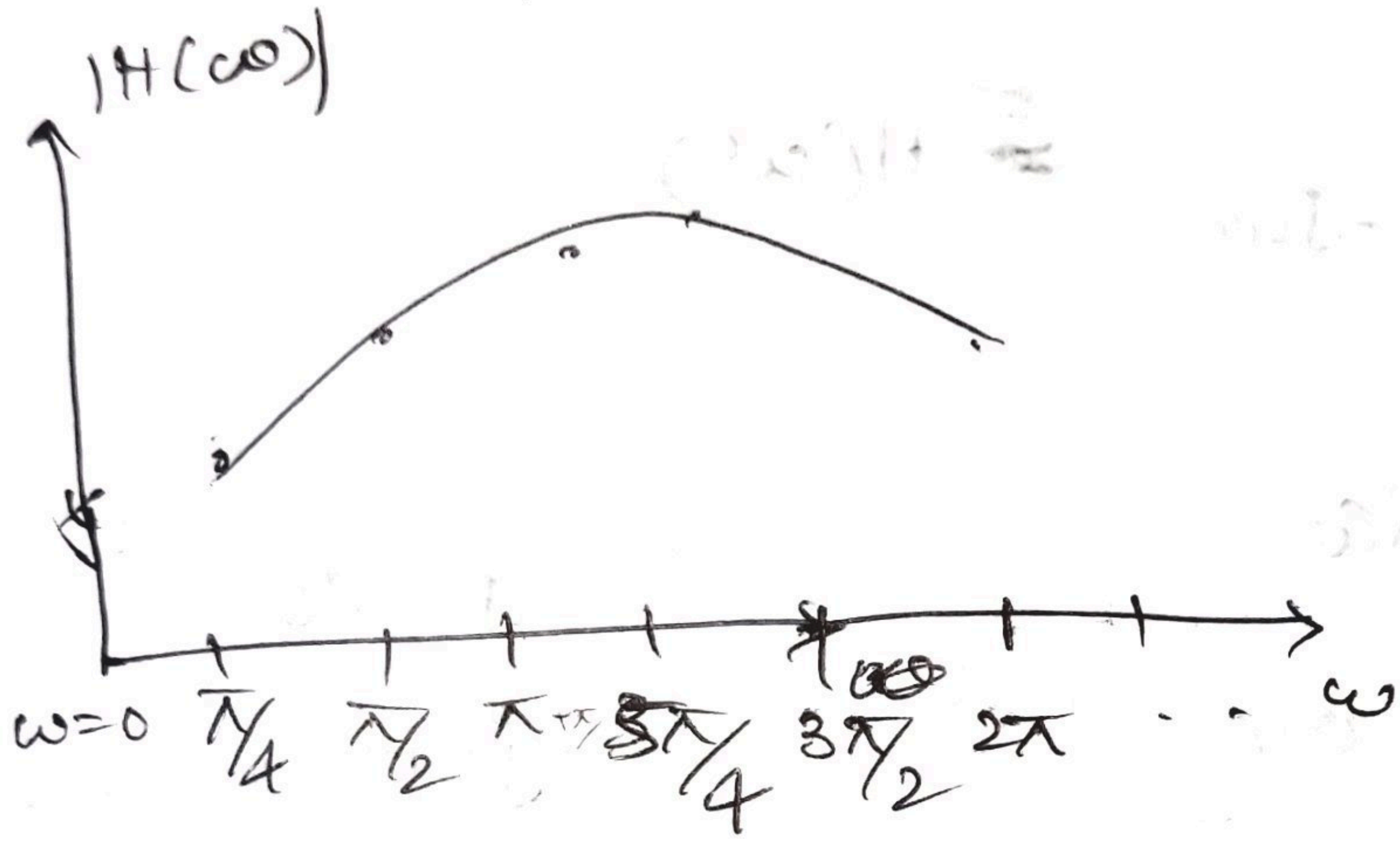
$$H(\omega) = \frac{e^{j\omega}+4}{e^{j\omega}-5} = \frac{\cos\omega + j\sin\omega + 4}{\cos\omega + j\sin\omega - 5}$$

$$= \frac{(\cos\omega+4) + j\sin\omega}{(\cos\omega - 5) + j\sin\omega}$$

$$|H(\omega)| = \frac{\sqrt{(\cos\omega+4)^2 + \sin^2\omega}}{\sqrt{(\cos\omega-5)^2 + \sin^2\omega}} = \frac{\sqrt{1+16+8\cos\omega}}{\sqrt{1+25-10\cos\omega}}$$

$$|H(\omega)| = \frac{\sqrt{17+8\cos\omega}}{\sqrt{26-10\cos\omega}}$$

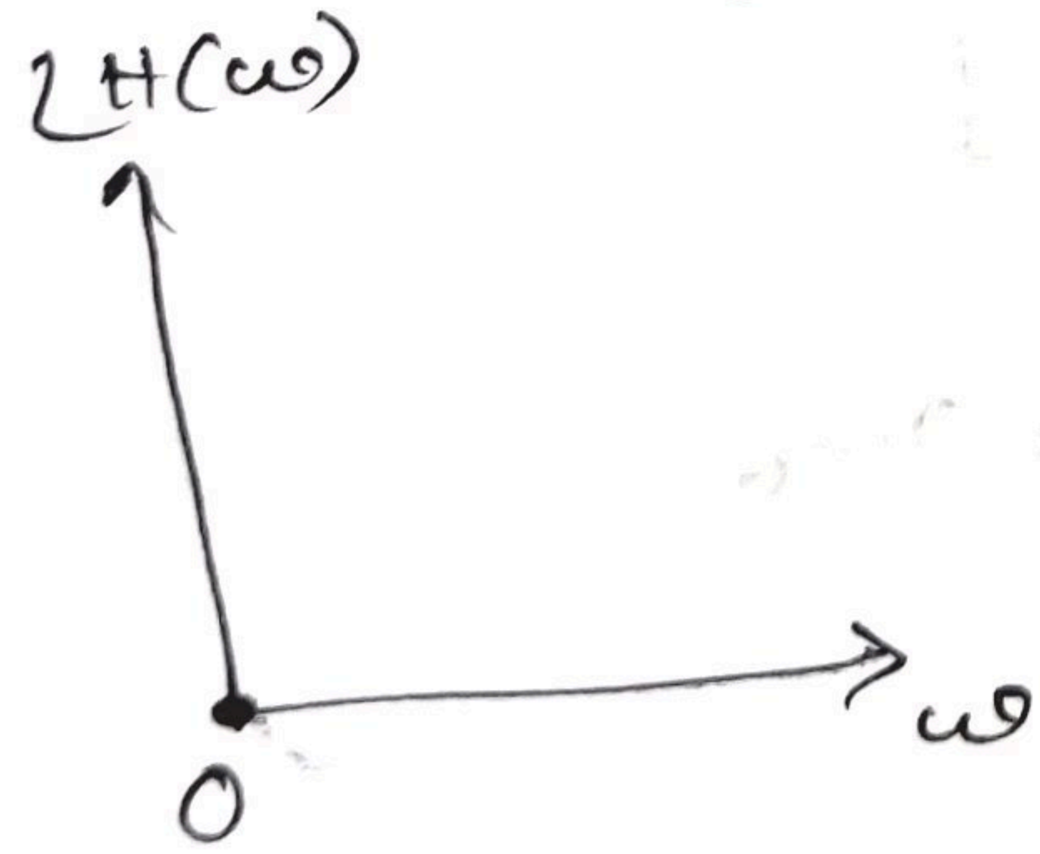
magnitude response:-



Phase response:-

$$\angle H(\omega) = \tan^{-1}(b) - \tan^{-1}(a)$$

$$= \tan^{-1}\left(\frac{\sin\omega}{(\cos\omega+4)}\right) - \tan^{-1}\left(\frac{\sin\omega}{\cos\omega-5}\right)$$



→ phase response

impulse response of LTI $h(n) = (0.6)^n u(n)$

$$\text{DTFT}[h(n)] = \text{DTFT}[(0.6)^n u(n)]$$

$$= \frac{1}{1 - \frac{3}{5} e^{-j\omega}} = H(\omega)$$

$$\cancel{H(\omega) = \frac{e^{j\omega}}{e^{j\omega} - 0.6} = \frac{\cos\omega + j\sin\omega}{\cos\omega + j\sin\omega - 0.6}}$$

$$H(\omega) = \frac{1}{1-0.6e^{-j\omega}} \times \frac{(1-0.6e^{+j\omega})}{(1-0.6e^{j\omega})}$$

$$= \frac{1-0.6e^{j\omega}}{(1-0.6e^{-j\omega})(1-0.6e^{j\omega})}$$

$$= \frac{1-0.6[\cos\omega + j\sin\omega]}{1-0.6[e^{-j\omega}] - 0.6e^{+j\omega} + 0.36}$$

$$= \frac{1-0.6(\cos\omega + j\sin\omega)}{1-1.2(\cos\omega + j\sin\omega) + 0.36}$$

$$= \frac{1-0.6(\cos\omega + j\sin\omega)}{1-0.6(\cos\omega - j\sin\omega) - 0.6(\cos\omega + j\sin\omega) + 0.36}$$

$$= \frac{1-0.6\cos\omega - 0.6j\sin\omega}{1-1.2\cos\omega + 0.36}$$

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$$H(\omega) = \underbrace{\frac{1-0.6\cos\omega}{1.36-1.2\cos\omega}}_{H_R(\omega)} - j\underbrace{\left(\frac{0.6\sin\omega}{1.36-1.2\cos\omega}\right)}_{H_I(\omega)}$$

$$|H(\omega)| = \sqrt{(X_R(\omega))^2 + (X_I(\omega))^2}$$

$$= \sqrt{\frac{(1-0.6\cos\omega)^2}{(1.36-1.2\cos\omega)^2} + \frac{(0.6\sin\omega)^2}{(1.36-1.2\cos\omega)^2}}$$

$$= \sqrt{\frac{1+(0.6)^2\cos^2\omega - 1.2\cos\omega + (0.6)^2\sin^2\omega}{(1.36-1.2\cos\omega)^2}}$$

$$= \sqrt{\frac{1+(0.6)^2 - 1.2\cos\omega}{(1.36-1.2\cos\omega)^2}}$$

$$\angle H(\omega) = \tan^{-1}\left[\frac{\frac{-(0.6\sin\omega)}{1.36-1.2\cos\omega}}{\frac{1-0.6\cos\omega}{1.36-1.2\cos\omega}}\right] = \tan^{-1}\left[\frac{-0.6\sin\omega}{1-0.6\cos\omega}\right]$$

consider LTI system ; $x_1(n) = (1/2)^n u(n)$,

$x_2(n) = (1/3)^n u(n)$, find the convolution sum. in time domain

$$X_1(\omega) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \qquad X_2(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$Y(\omega) = X_1(\omega)\cdot X_2(\omega) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{3}e^{-j\omega}\right)}$$

$$Y(\omega) = \frac{e^{j\omega}}{e^{j\omega}-1/2} \cdot \frac{e^{j\omega}}{e^{j\omega}-1/3}$$

$$\frac{Y(\omega)}{e^{j\omega}} = \frac{e^{j\omega}}{(e^{j\omega}-1/2)(e^{j\omega}-1/3)} = \frac{A}{e^{j\omega}-1/2} + \frac{B}{e^{j\omega}-1/3}$$

$$A\Big|_{e^{j\omega}=1/2} = \cancel{(e^{j\omega}-1/2)} \cdot \frac{e^{j\omega}}{\cancel{(e^{j\omega}-1/2)}(e^{j\omega}-1/3)} = \frac{1/2}{1/2-1/3} = \frac{1/2}{1/6}$$

$$\boxed{A = 3}$$

$$B\Big|_{e^{j\omega}=1/3} = \frac{1/3}{1/3-1/2} = \frac{1/3}{-1/6} = -2$$

$$\frac{Y(\omega)}{e^{j\omega}} = \frac{3}{e^{j\omega}-1/2} - \frac{2}{e^{j\omega}-1/3}$$

$$Y(\omega) = \frac{3e^{j\omega}}{e^{j\omega}-1/2} - \frac{2e^{j\omega}}{e^{j\omega}-1/3} = \frac{3}{1-e^{-j\omega}\frac{1}{2}} - \frac{2}{1-\frac{1}{3}e^{-j\omega}}$$

$$\boxed{y(n) = 3\left(\tfrac{1}{2}\right)^n u(n) - 2\left(\tfrac{1}{3}\right)^n u(n)}$$

Discrete Fourier Transform:- (DFT) → $X(k)$

Sampled Version of DTFT → DFT

$X(K) = DFT[x(n)]$; $K = 0$ to $N-1$

DTFT

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$$

sampling ⇒ $X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}$

DFT

$$= \sum_{n=0}^{N-1} x(n) e^{-j\left(\frac{2\pi k}{N}\right)n}$$

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\left(\frac{2\pi k}{N}\right)n}}$$ → DFT

$K = 0 \rightarrow N-1$; $0 \le K \le N-1$

$n = 0 \rightarrow N-1$; $0 \le n \le N-1$

IDFT

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k) e^{j\left(\frac{2\pi k n}{N}\right)}$$

Twiddle factor:-

$$\boxed{W_N = e^{-j\frac{2\pi}{N}}}$$

At every Period.

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n) (W_N)^{kn}}$$

$$\boxed{x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(k) W_N^{-kn}}$$

#

$W_4^0 \Rightarrow \quad W_N = e^{-j\frac{2\pi}{N}}$

$W_4^0 = \left(e^{-\frac{j2\pi}{4}}\right)^0 = 1$

$W_4^1 = \left(e^{-j2\pi/4}\right)^1 = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = -j$

$W_4^2 = \left(e^{-j2\pi/4}\right)^2 = e^{-j\pi} = \cos\pi - j\sin\pi = -1$

$W_4^3 = \left(e^{-j2\pi/4}\right)^3 = e^{-\frac{3\pi}{2}j} = \cos 3\pi/2 - j\sin 3\pi/2$

$= j$

$W_4^4 = \left(e^{-j2\pi/4}\right)^4 = e^{-2\pi j}$

$= \cos 2\pi - j\sin 2\pi$

$W_N = e^{-j2\pi/N}$

$= 1 = (W_4)^0$ $\quad \cos \pi - \pi/4$

$W_8^0 = 1$

$W_8^1 = \left(e^{-j2\pi/8}\right)^1 = e^{-j\pi/4}$

$W_8^2 = \left(e^{-j2\pi/8}\right)^2 = e^{-j\pi/2} = -j$

$W_8^3 = \left(e^{-j\frac{2\pi}{8}}\right)^3 = e^{-3\pi/4\, j} = \cos 3\pi/4 - j\sin\frac{3\pi}{4}$

$= 1/\sqrt{2} \; -\frac{j}{\sqrt{2}}$

$W_8^8 = \left(e^{-j\frac{2\pi}{8}}\right)^8 = e^{-2\pi j} = \cos 2\pi - j\sin 2\pi = 1$

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Twiddle matrix: for $N=4$

If $N=4$:-
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_N & W_N^2 & W_N^3 \\ 1 & W_N^2 & W_N^4 & W_N^6 \\ 1 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} = W_4$$

Twiddle factor matrix:-

$$W_N = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & W_N^3 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & W_N^6 & \cdots & W_N^{2(N-1)} \\ 1 & W_N^3 & W_N^6 & W_N^9 & \cdots & W_N^{3(N-1)} \\ \vdots & \vdots & \vdots & & & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

* $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$ (or) W_N^{nk}

DFT
* N^2 complex multiplications
$N(N-1)$ complex add^{n}s.

$X(k) = W_N^k \, x(n)$

Find the DFT & IDFT of the following sequence

$x(n) = \{1, 1, 0, 0\}$

DFT

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{4}nk} = \sum_{n=0}^{3} x(n) e^{-j\pi/2\, nk}$$

$$X(0) = \sum_{n=0}^{3} x(n) e^{-j\pi/2\, n(0)} = \sum_{n=0}^{3} x(n) = \{1+1\} = \boxed{2}$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-\frac{j\pi n}{2}} = \sum_{n=0}^{3} x(n) e^{-\frac{j\pi n}{2}} = \{1+e^{-j\pi/2}\}$$
$$= 1+\cos\pi/2 - j\sin\pi/2 = \boxed{1-j}$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi/2 (2) n}$$

e^{-}

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n} = x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-2j\pi} + x(3)e^{-3j\pi}$$

$$= 1 + e^{-j\pi} = 1 + \cos\pi - j\sin\pi$$

$$= 1 - 1 = \boxed{0}$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j\pi/2 (3) n}$$

$$= \sum_{n=0}^{3} x(n) e^{-3j\pi/2 \, n}$$

$$= x(0)e^{-0} + x(1)e^{-3\pi/2 j}$$

$$= 1 + \cos 3\pi/2 - j\sin 3\pi/2$$

$$= \boxed{1+j}$$

$$X(k) = \{2, 1-j, 0, 1+j\}$$

IDFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j\left(\frac{2\pi k}{N} n\right)}$$

$$\cancel{x(0)} = \frac{\cancel{1}}{4} \sum_{k=0}^{3} \cancel{x}$$

$$\text{IDFT}[X(K)] = \{1, 1, 0, 0\}$$

$$x(0) = \frac{1}{4} \sum_{K=0}^{3} X(k) e^{0} = \frac{1}{4}\left[2 + 1 + \cancel{j} + 1 - \cancel{j}\right]$$

$$\boxed{x(0) = 1}$$

$$x(1) = \frac{1}{4} \sum_{K=0}^{3} X(K) e^{\frac{j2\pi K}{4}} = \frac{1}{4} \sum_{K=0}^{3} X(K) e^{\frac{jK\pi}{2}}$$

$$= \frac{1}{4}\left[X(0)e^{0} + X(1)e^{j\pi/2} + X(2)e^{j\pi} + X(3)e^{\frac{3j\pi}{2}}\right]$$

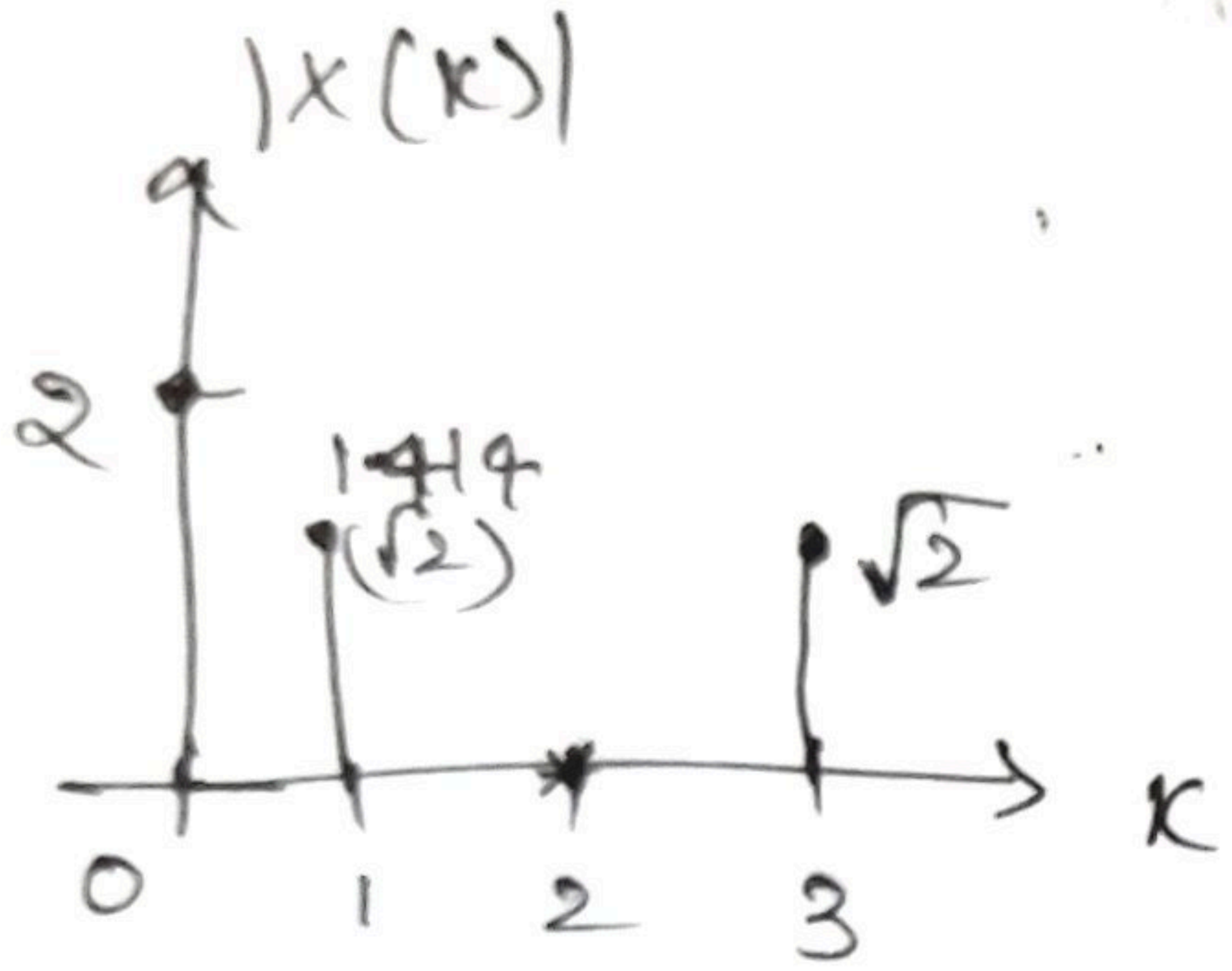
$$= \frac{1}{4}\left[2 + (1-j)e^{\frac{j\pi}{2}} + 0 + (1+j)e^{3j\pi/2}\right]$$

$$= \frac{1}{4}\left[2 + \cancel{(1+j)}(1-j)(j) + (1+j)(-j)\right]$$

$$= \frac{1}{4}\left[2 + \cancel{j} - j^2 \cancel{-j} - j^2\right] = \frac{1}{4}\left[2 - 2j^2\right]$$

$$= \frac{1}{4}\left[2 + 2\right] = \boxed{1 = x(1)}$$

$x(k) =$



$X(1) = 1-j$

$|X(1)| = \sqrt{1+1} = \sqrt{2}$

DFT - matrix method

$X = x \cdot W_N$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

Matlab → FFT[x]

IDFT - matrix method

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = W_N^{-1} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}$$

$$\boxed{W_N^{-1} = \frac{1}{N} W_N^{*}}$$

$$W_4^{-1} = \frac{W_N^{*}}{4} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$W_N^{-1}\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{j}{4} & -\frac{1}{4} & -\frac{j}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{j}{4} & -\frac{1}{4} & \frac{j}{4} \end{bmatrix} \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} + \frac{1-j}{4} + \frac{1+j}{4} \\ \frac{1}{2} + \frac{j(1-j)}{4} - \frac{j}{4}(1+j) \\ \frac{1}{2} - \frac{1}{4}(1-j) - \frac{1}{4}(1+j) \\ \frac{1}{2} - \frac{j}{4}(1-j) + \frac{j}{4}(1+j) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

* FFT complexity.

$$\frac{N}{2}\log_2 N$$

$x(n) = \cos \frac{2\pi r n}{N}$

$$DFT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} nK} \qquad K = 0 \text{ to } N-1$$

$$x(n) = \cos \frac{2\pi r n}{N} = \frac{e^{j\frac{2\pi r n}{N}} + e^{-j\frac{2\pi r n}{N}}}{2}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$x(n) = \frac{W_N^{-rn} + W_N^{rn}}{2}$$

$$DFT[x(n)] = \sum_{n=0}^{N-1} \left(\frac{W_N^{-rn} + W_N^{rn}}{2} \right) e^{-j\frac{2\pi}{N} nK}$$

$$= \frac{1}{2} \sum_{n=0}^{N-1} \left(W_N^{-rn} + W_N^{rn} \right) W_N^{nk}$$

$(W_N)^{(0)} = 1$

$$= \frac{1}{2} \left(\sum_{n=0}^{N-1} W_N^{n(K-r)} + \sum_{n=0}^{N-1} W_N^{n(k+r)} \right)$$

$$K=0 \Rightarrow DFT(x(n)) = \frac{1}{2} \left(\sum_{n=0}^{N-1} W_N^{-nr} + \sum_{n=0}^{N-1} W_N^{nr} \right)$$

$$K = r \Rightarrow DFT(x(n)) = \frac{1}{2} \left[\sum_{n=0}^{N-1} (W_N)^0 + \sum_{n=0}^{N-1} W_N^{n(2K)} \right]$$

$$= \frac{1}{2} \left[N + \sum_{n=0}^{N-1} W_N^{n(2K)} \right]$$

$$= \frac{N}{2} + \frac{1}{2} \sum_{n=0}^{N-1} W_N^{n(2K)} = N/2$$

$K = -r \Rightarrow DFT(x(n)) = N/2$

Properties of DFT :-

① Periodicity

$$x(n+N) = x(n)$$

② $DFT\left[x(n+N)\right] = X(n+k)$.

② Linearity

$$x_1(n) \xleftrightarrow{DFT} X_1(K)$$

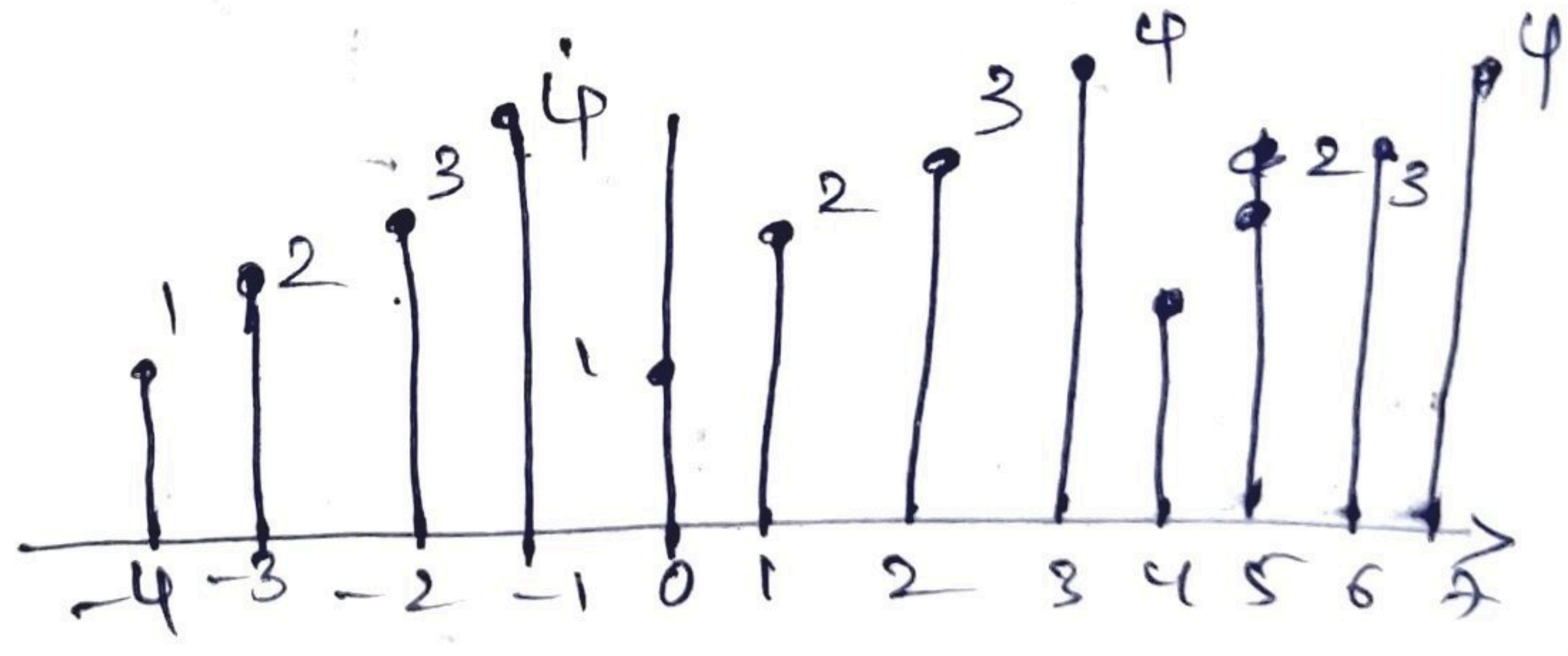
$$x_2(n) \xleftrightarrow{DFT} X_2(K)$$

$$DFT\left[a_1 x_1(n) + a_2 x_2(n)\right] \xleftrightarrow{DFT} a_1 X_1(K) + a_2 X_2(K)$$

③ * circular symmetric of a sequence :-

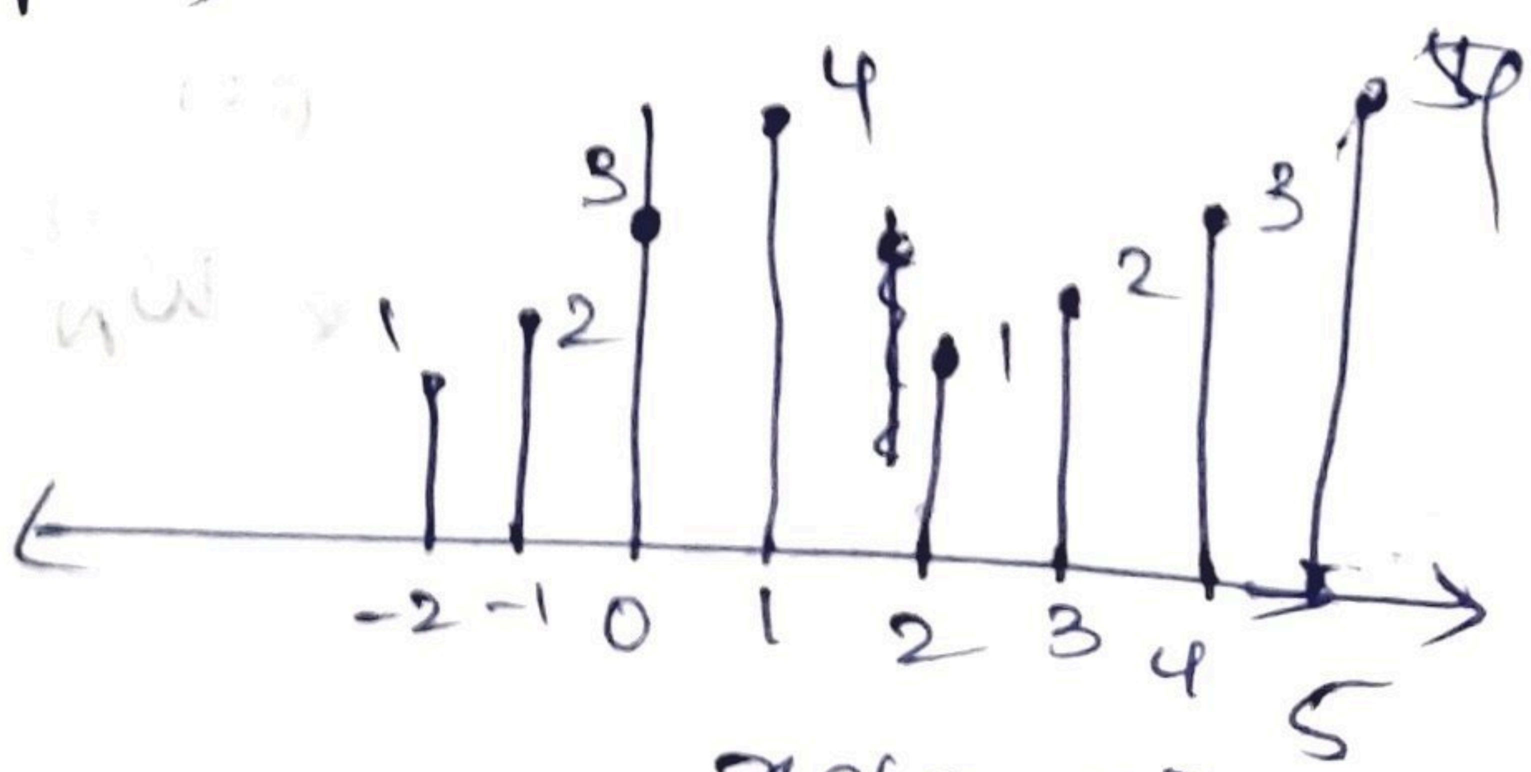
→ In circular convolution lengths of inputs are equal

$x(n)$



$x_p(n)$

$$x_p(n) = \sum_{l=-\alpha}^{\alpha} x(n-lN)$$



$x_p(n-2)$

$$x((n-2))_4 = x_p(n-2)$$

$$x((0-2))_4 = x((-2))_4 = x(4-2)$$

$$= x(2)$$

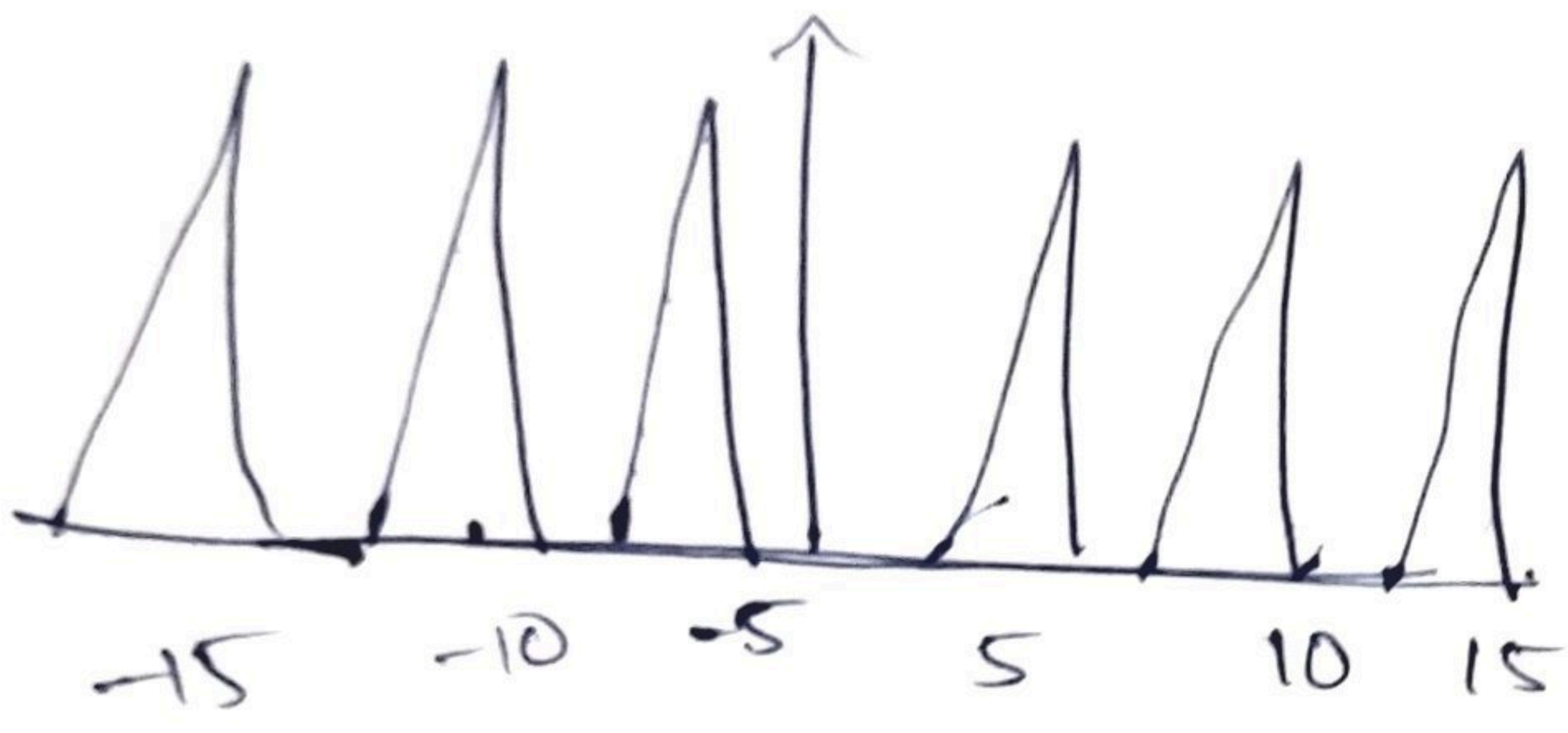
upto period values only

$$x((7-2))_4 = x((5))_4 = x(1)$$

$$x((3-2))_4 = x((1))_4$$

$$= x(1)$$

③ symmetric property :-

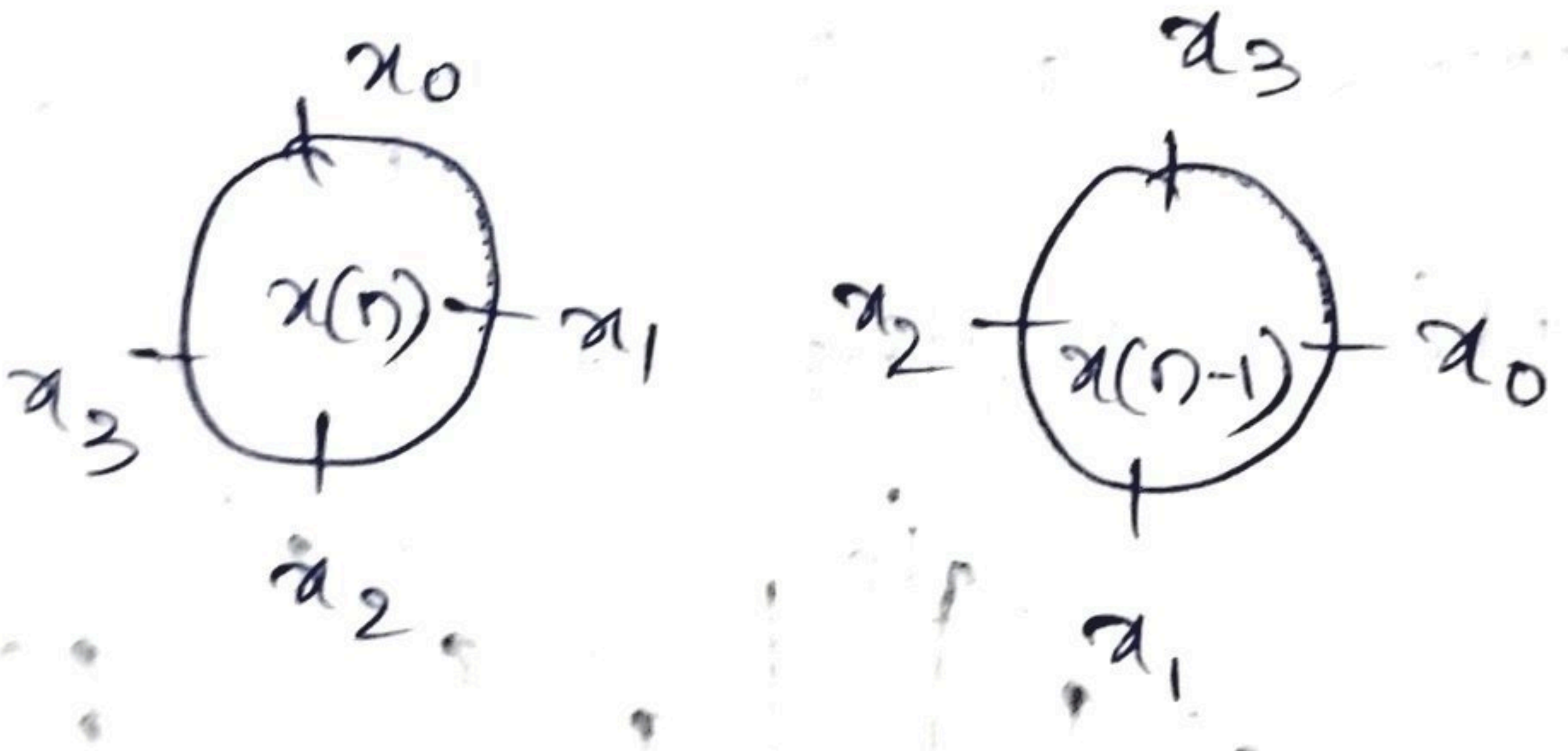


5, 10, 15 – input

$$x(n) = x_R(n) + j\, x_I(n)$$

$$X(K) = X_R(k) + j\, X_I(k)$$

* circular shift :-



④ circular time shift :-

$$x(n-n_0) = e^{-j\frac{2\pi}{N}n_0 k} \cdot X(k)$$

(or)

$$W_N^{n_0 k} \cdot X(k)$$

* $x(n-l)_N = e^{-j\frac{2\pi l k}{N}} X(k)$

⑤ circular frequency shift

$$W_N^{-nk_0}\, x(n) = X(k-k_0)$$

(or)

$$e^{j\frac{2\pi}{N}nk_0}$$

* $e^{j\frac{2\pi}{N}nl} = X(k-l)_N$

6) Time reversal

$$DFT[x((-n))_N] \xleftrightarrow{DFT} X(N-k)$$

$$DFT[x(-n)_N] \xleftrightarrow{DFT} X(N-k)$$

⑥ complex conjugate:-

$$\boxed{DFT[x^*(n)] = X^*(N-K)}$$

$$x(n) = g(n) + jh(n)$$

$$x^*(n) = g(n) - jh(n)$$

$$g(n) = \frac{x(n) + x^*(n)}{2} \quad ; \quad h(n) = \frac{x(n) - x^*(n)}{2j}$$

$$G(x) = \frac{1}{2}\left[X(K) + X^*(N-K)\right]$$

(or)

$$X^*(-K)_N$$

$$H(x) = \frac{1}{2j}\left[X(K) - X^*(N-K)\right]$$

(or)

$$X^*(-K)_N$$

⑦ Parseval's relation:-

$$DFT\left[\sum_{n=0}^{N-1} |x(n)|^2\right] = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2$$

⑧ circular convolution:-

$$y(n) = x(n) \circledast h(n) = \sum_{m=0}^{N-1} x(m)\, h(n-m)$$

$$DFT[y(n)] \longleftrightarrow X(K)\, H(K)$$

⑨ circular correlation:-

$$DFT[r_{xy}(l)] \longleftrightarrow X(K) \cdot Y^*(K)$$

Let $x(K)$ be a 12 pt DFT of a length of 12 real sequence of $x(n)$

$X(K) = \{8, -1+j2, 2+j3, 1-j4, -2+2j, 3+j, -1-j3\}$

$X^*(N-K) = X^*(12-7) = X^*(5) = 3-j$
(or) $X(7)$

$X(8) = X^*(12-8) = X^*(4) = \cancel{1+j4}\ -2-2j$

$X(9) = X^*(12-9) = X^*(3) = 1+j4$

$X(10) = X^*(12-10) = X^*(2) = 2-j3$

$X(11) = X^*(12-11) = X^*(1) = -1-j2$

The values in the sequence are the conjugate of each other because of symmetric propert

$X(K) = \{1, A, -1, B, -7, -j, C, -1+j\}$. Find the value of A, B, C and also find the energ

$A = X^*(8-1) = X^*(7) = -1-j$

$B = X^*(8-3) = X^*(5) = j$

$C = X^*(8-6) = X^*(2) = -1$

$$E = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2 = \frac{1}{8}\sum_{K=0}^{7} |X(K)|^2$$

$E = \frac{1}{8}(1+(\sqrt{2})^2+1+1+49+1+1+2)$

$= \frac{1}{8}(5+4+49) = \frac{1}{8}(58) = 7.25$

\# $DFT\{x(n)\} = X(K) = \{4, -j2, 0, j2\}$

(a) Find $x(n)$ $DFT\{x(n-2)\}$

$DFT\{x(n-2)\} = e^{-j\frac{2\pi}{4}(2)k} \cdot X(K)$

$X'(K) = e^{-j\pi k} X(K)$

$X'(0) = e^{-0} X(0) = 4$

$X'(1) = e^{-j\pi} X(1) = (\cos\pi - j\sin\pi)(-2j)$

$= +2j$

$X'(2) = e^{-2j\pi} X(2) = (\cos 2\pi - j\sin 2\pi)(0) = 0$

$X'(3) = e^{-3j\pi} X(3) = 2j(\cos 3\pi - j\sin 3\pi) = 2j(-1)$

$= -2j$

$X'(K) = \{4, 2j, 0, -2j\}$

(b) Find $DFT\{x(-n)\}$

$DFT\{x(-n)\} = X^*(K) = X^*(N-K) = \{4, +2j, 0, -2j\}$

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(c) $DFT[x^*(n)] = X^*(N-k)$

$X^*(4-0) = X^*(4) = 4$

$X^*(4-1) = X^*(3) = -2j$

$X^*(4-2) = X^*(2) = 0$

$X^*(4-3) = X^*(1) = j2$

(d) Find $DFT[x(n) \circledast x(n)]$

circular convolution

$DFT[x(n) \circledast x(n)] = X(k) \cdot X(k)$

$= \{4, -2j, 0, j2\}\{4, -2j, 0, j2\}$

$= \{16, -4, 0, 4\}$

(e) $E = \frac{1}{4}\sum_{k=0}^{3} |X(k)|^2 = \frac{1}{4}[\cancel{16+2+0+2}]$

$= \frac{32}{4} = \quad = \frac{1}{4}[16+4+0+4] = 6$

* Shortcut for circular convolution:-

$x_1(n) = \{2, 3, 4, 5\}$

$x_2(n) = \{1, 2, 3, 4\}$

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$$\begin{bmatrix} 2 & 5 & 4 & 3 \\ 3 & 2 & 5 & 4 \\ 4 & 3 & 2 & 5 \\ 5 & 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 36 \\ 38 \\ 36 \\ 30 \end{bmatrix}$$

<u>circular</u> <u>convolution</u>:-

$x_1(n) = \{1, 2, 3, 4\}$

$x_2(n) = \{1, 1, 2, 1\}$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

linear convolution $\Rightarrow$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 \\ 2 & 2 & 4 & 6 & 8 \\ 1 & 1 & 2 & 3 & 4 \end{array} = \{1, 3, 7, 12, 12, 14, 4\}$$

$\Rightarrow$ {* The circular convolution is the overlap version of linear convolution.

$\rightarrow$ Do zero pading to first data by adding $n-1$ zeros to first data, where n is length of second data to get linear convolution

$x_1(n) = \{1, 2, 3, 4, 0, 0, 0\}$

$x_2(n) = \{1, 1, 2, 1, 0, 0, 0\}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 7 \\ 12 \\ 12 \\ 11 \\ 4 \end{bmatrix}$$

Graphical method :-

$x_1(n) = \{1, 2, 3, 4\}$

$x_2(n) = \{1, 1, 2, 1\}$

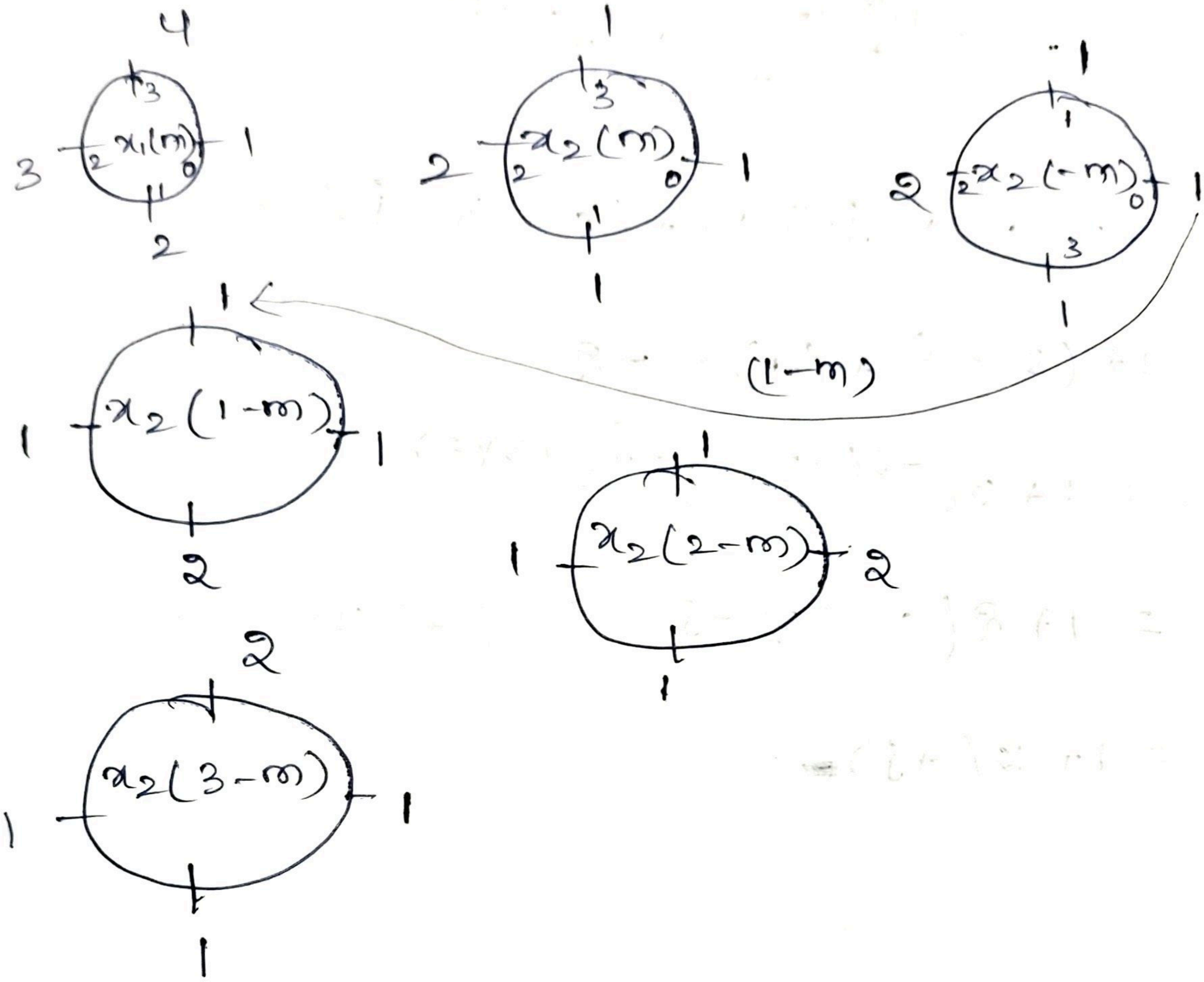
$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)$$

$$x_3(0) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2(-m) = 1+4+6+2 = 13$$

$$x_3(1) = \sum_{m=1}^{N-1} x_1(m)\, x_2(1-m) = 4+1+4+3 = 12$$

$$x_3(2) = \sum_{m=1}^{N-1} x_1(m)\, x_2(2-m) = 4+2+2+3 = 11$$

$$x_3(3) = \sum_{m=1}^{N-1} x_1(m)\, x_2(3-m) = 8+3+2+1 = 14$$



By using properties of DFT :-

$x_1(n) = \{1, 2, 0, 1\}$

$x_2(n) = \{2, 2, 1, 1\}$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) \cdot W_N^{nk} = 1 + 2e^{-j\frac{2\pi}{4}k} + 0 + e^{-j\frac{2\pi}{4}\cdot 3k}$$

$$X_1(0) = 1 + 2 + 0 + 1 = 4$$

$$X_1(1) = 1 + 2e^{-j\pi/2} + e^{-\frac{j\pi 3}{2}} = 1 + 2(\cos\pi/2 - j\sin\pi/2) + (\cos 3\pi/2 - j\sin 3\pi/2)$$

$$= 1 - 2j + j = 1 - j$$

$$X(2) = 1 + 2e^{-j\frac{2\pi}{4}(2)} + e^{-j\frac{2\pi}{4}\cdot 3\times 2}$$

$$= 1 + 2(\cos\pi - j\sin\pi) + (\cos 3\pi - j\sin 3\pi)$$

$$= 1 + (2(-1)) + (-1) = -2$$

$$X(3) = 1 + 2e^{-j\frac{\pi}{2}(3)} + e^{-j\pi/2(3)(3)}$$

$$= 1 + 2(\cos 3\pi/2 - j\sin 3\pi/2) + (\cos 9\pi/2 - j\sin 9\pi/2)$$

$3\pi + 3\pi/2$

$$= 1 + 2(+j)$$

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k)\, e^{j2\pi/N\, nk} \qquad H(k) = \{24, -2j, 0, 2j\}$$

$$y(0) = \frac{1}{4}\left[24 - 2j + 2j\right] = 6$$

$$y(1) = \frac{1}{4}\left[24e^{j2\pi/4(0)} - 2je^{j2\pi/4(1)} + 0 + 2je^{j2\pi/4(3)}\right]$$

$$= \frac{1}{4}\left(24 - 2j(\cos\pi/2 + j\sin\pi/2) + 2j(\cos 3\pi/2 + j\sin 3\pi/2)\right)$$

$$= \frac{1}{4}\left[24 - 2j(j) + 2j(-j)\right] = 1/4\left[24 - 4j^2\right] = 28/4$$

$$= 7$$

$$y(2) = \frac{1}{4}\left[24e^{j2\pi/4(0)} - 2je^{2\pi/4(1)(2)} + 0 + 2je^{j2\pi/4}\right.$$

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Find the four pt DFT of the sequence

$x(n) = \{1, -1, 1, -1\}$ also find

(a) $DFT[x(n)]$

(b) $DFT[y(n)] \Rightarrow y(n) = x((n-2))_4$.

$y(n) = x((n-2))_4 \rightarrow$ circular time shift

(a) $$DFT[x(n)] = \sum_{n=0}^{N-1} x(n) e^{-j2\pi/N \, nk}$$

$X(k) = DFT[x(n)] = \{0, 0, 4, 0\}$

$X(0) = [1-1+1-1] = 0$

(b) $x(n-2)_4 = y(n)$

$$X(k) \cdot e^{-j2\pi/N \, n_0 k} = Y(k)$$

$n_0 = 2, \quad N = 4$

$\cancel{X(0)}$ $\quad Y(0) = 0$ $\qquad\qquad Y(2) = 4 e^{-j2\pi/4 (2)(2)}$

$Y(1) = 0$ $\qquad\qquad = 4e^{-2j\pi} = 4.(\cos 2\pi)$

$= 4$

$y(3) = 0$

$y(k) = \{0, 0, 4, 0\}$

$x(n) = \{1, 0, 1, 0\}$

Find the $y(n)$ $Y(k) = X((k-2))_4$

$Y(k)$

$$y(n) = e^{-j\frac{2\pi}{N} l n} x(n)$$

$x(n+2)$

Fast fourier transform (FFT) :-

Radix-2 fft :-

$N = r^m \rightarrow$ radix $= 2^m$

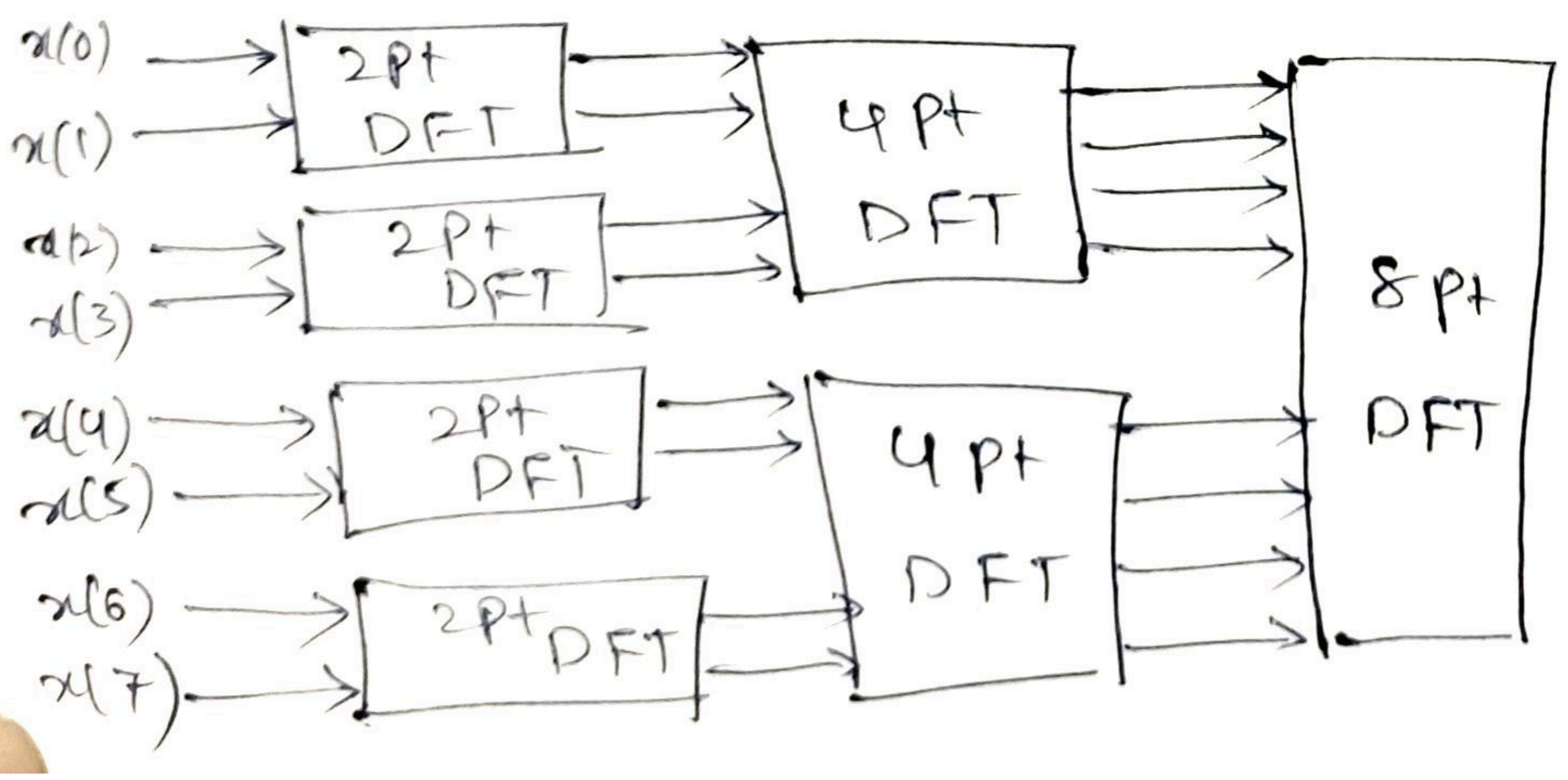
$\rightarrow N(N-1)$

	N	DFT Add	DFT Mul (N^2)	Radix-2 FFT Add ($N\log_2 N$)	Radix-2 FFT Mul ($(N/2)\log_2 N$)
2^2	4	12	16	8	4
2^3	8	56	64	24	12
2^4	16	240	256	64	32
2^5	32	992			
2^6					

DIT - radix 2 FFT

DIT $\rightarrow$ decimation in time

N=8



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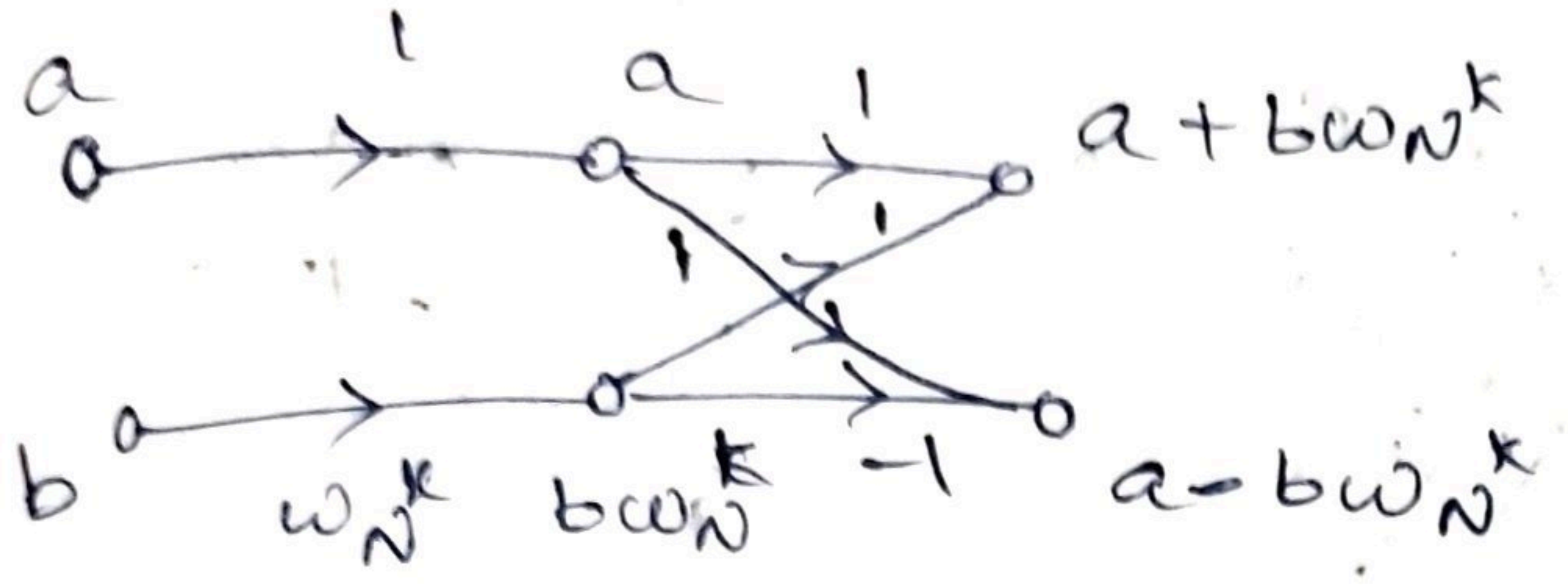

for 1^{st} stage $\rightarrow W_{N/4}^0$

2^{nd} stage $\rightarrow W_{N/2}^0$, $W_{N/2}^1$

3^{rd} stage $\rightarrow$

DIT - radix - 2 - fft

$N = 8$

normal	bit reversed
$x(0) = x(000)$	$x(000) = x(0)$
$x(1) = x(001)$	$x(100) = x(4)$
$x(2) = x(010)$	$x(010) = x(2)$
$x(3) = x(011)$	$x(110) = x(6)$
$x(4) = x(100)$	$x(001) = x(1)$
$x(5) = x(101)$	$x(101) = x(5)$
$x(6) = x(110)$	$x(011) = x(3)$
$x(7) = x(111)$	$x(111) = x(7)$

$x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$, compute 8 pt DFT of $x(n)$ by using radix-2-DITFFT

First stage 2pt DFT

$x(0)$

$x(4)$

$x(2)$

$x(6)$

$x(1)$

$x(5)$

$x(3)$

$x(7)$

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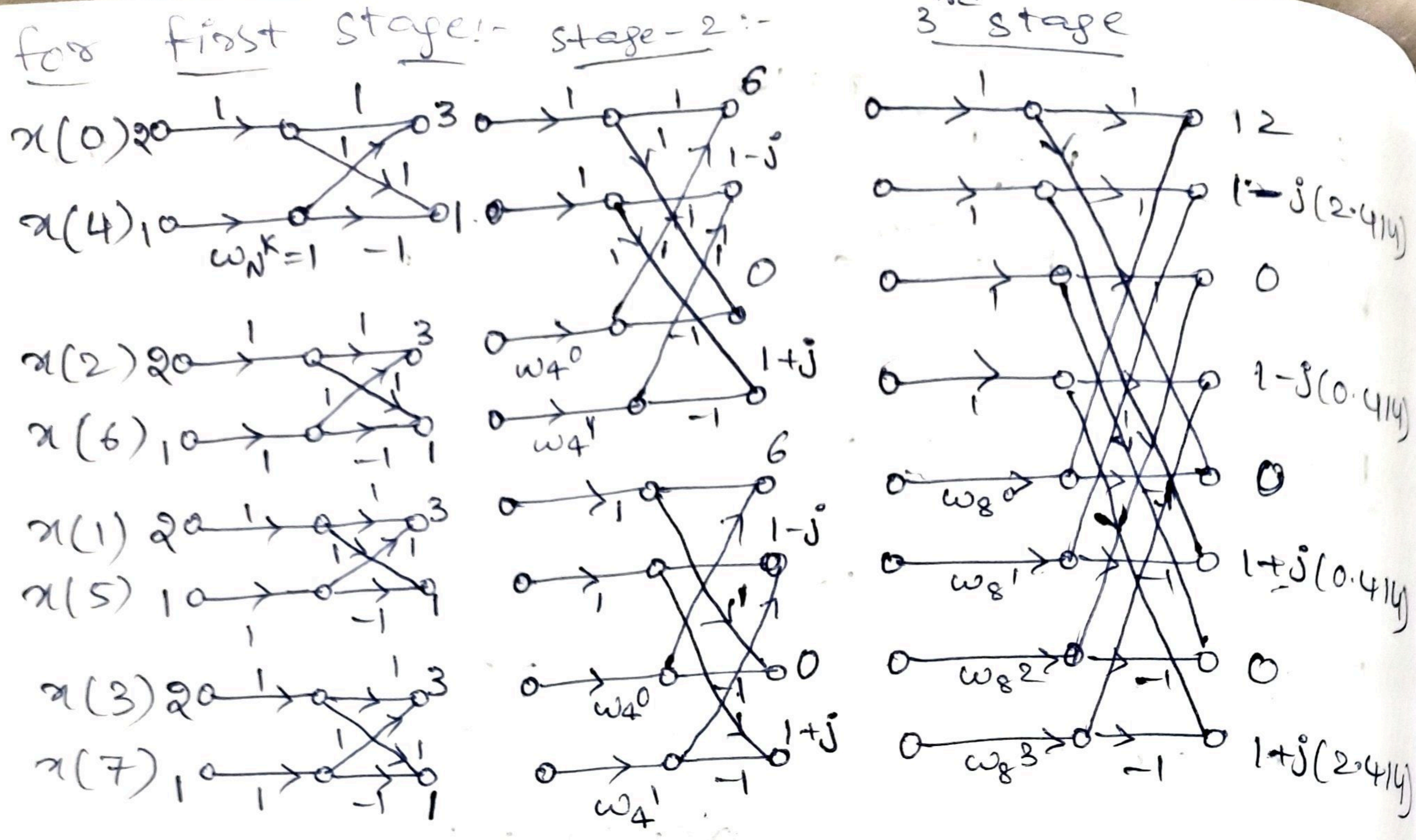


$\omega_{(N/4)}^{k} = \omega_{(N/4)}^{0} = (\omega_2)^0 \rightarrow$ for first stage

$\left(\omega\left(\frac{N}{2}\right)\right)^0, \left(\omega\left(\frac{N}{2}\right)\right)^1 \rightarrow$ for 2nd stage

$(\omega(N))^0,$ ~~ω(N~~ $\omega_N^1, \omega_N^2, \omega_N^3 \rightarrow$ for 3rd stage

$= \{12, (1-j(2.414)), 0, 1-j(0.414), 0, 1+j(0.414), 0,$

$1+j(2.414)\}$

$\omega_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} = \frac{(1-j)}{\sqrt{2}}$

ω_8^1

$\omega_8^2 = e^{\left(\frac{-j2\pi}{8\,4}\right)^2} =$

$\omega_8^2 = -j$

$e^{-j2\pi}$

$\omega_8^3 = e^{-\left(\frac{j2\pi}{8}\right)^3} = e^{-3j\pi/4}$

$=$

$= -j/\sqrt{2} - 1/\sqrt{2}$

IDFT

$y(k) = \{-6, 1+j, 0, 1-j\}$ $N = 4$

$y^*(k) = \{-6, 1-j, 0, 1+j\}$

bit reversed

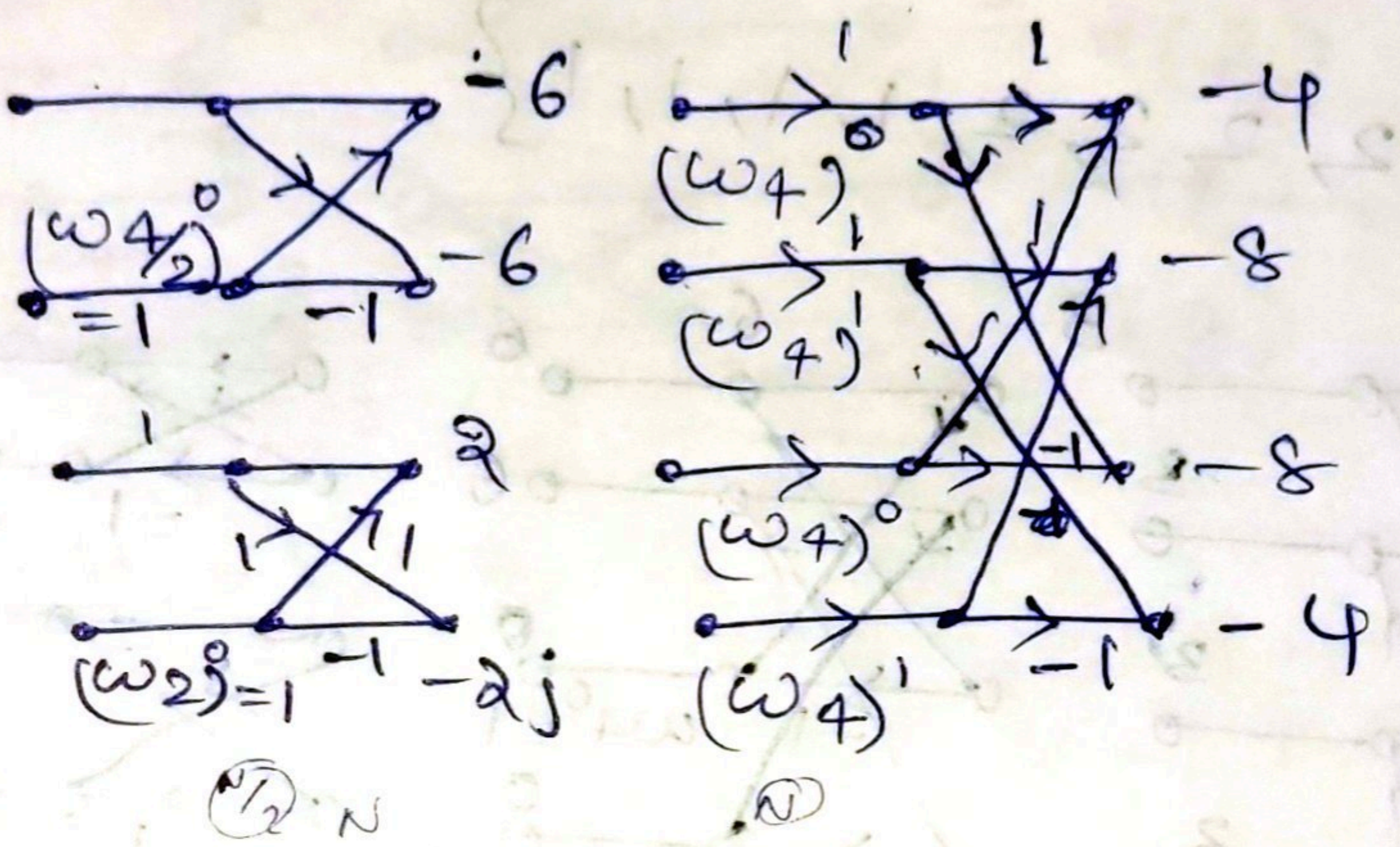
normal

$y^*(0) = -6$

$y^*(2) = 0$

$y^*(1) = 1-j$

$y^*(3) = 1+j$



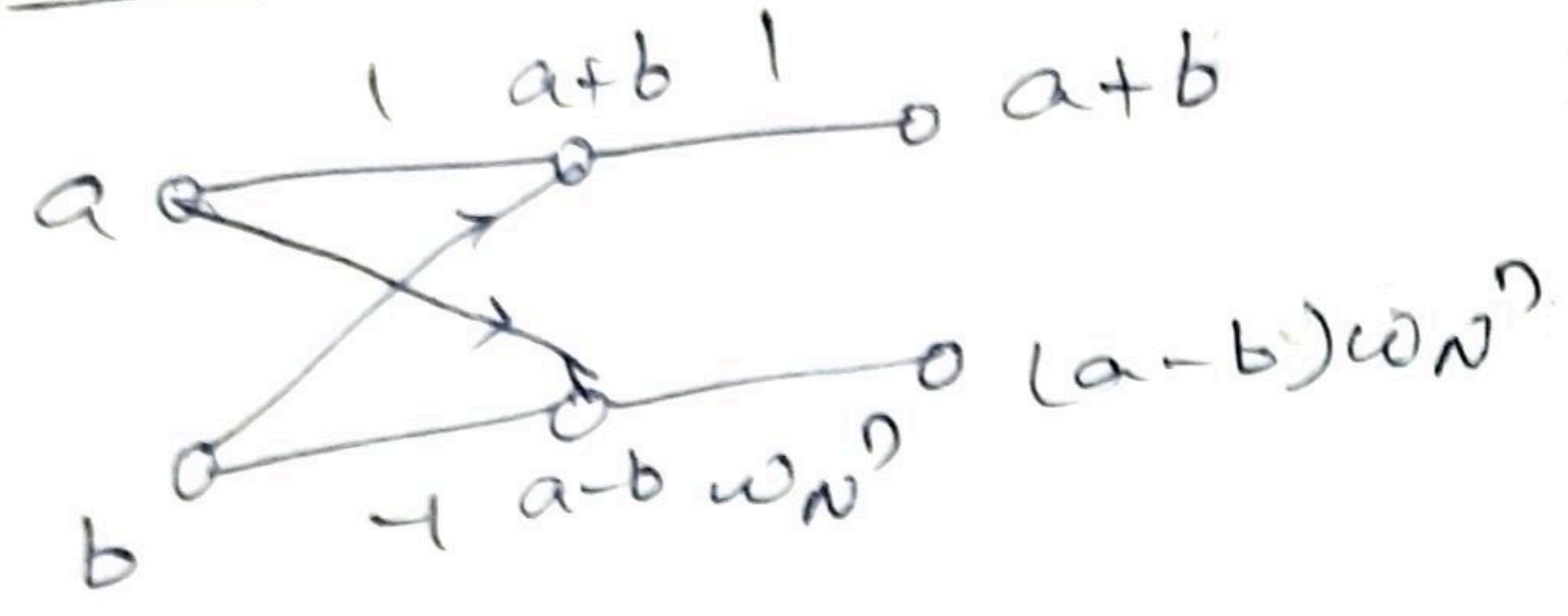
$y^*(n) = \{-4, -8, -8, -4\}$

$\downarrow$ conjugate

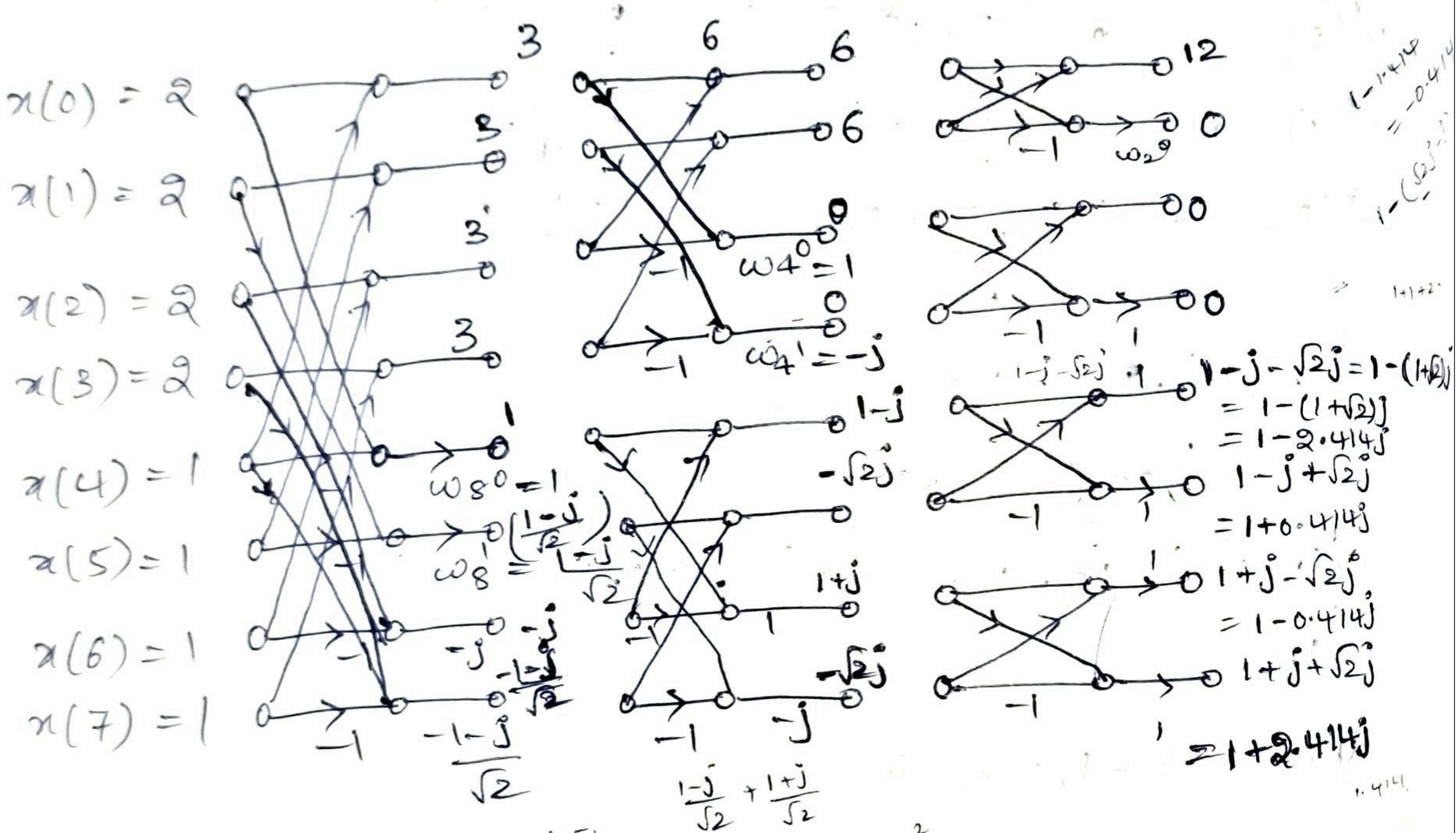
$\frac{y(n)}{N} = \{-4, -8, -8, -4\} = y(n)$

$y(n) = \{-1, -2, -2, -1\}$

8 pt - Radix-2 DIF FFT



$x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$



$\omega_2^0 = $
$\omega_2^1 = e^{-j2\pi/2} = e^{-j\pi} = \cos\pi - j\sin\pi$

$= \{12, 0, 0, 0, 1-2.414j, 1+0.414j, 1-0.414j, 1+2.414j\}$

bit reversed order

$X(k) = \{\cancel{12}, \cancel{0}, 1-j0.414, \cancel{0}, 1-j2.414, 0, \cancel{1}$

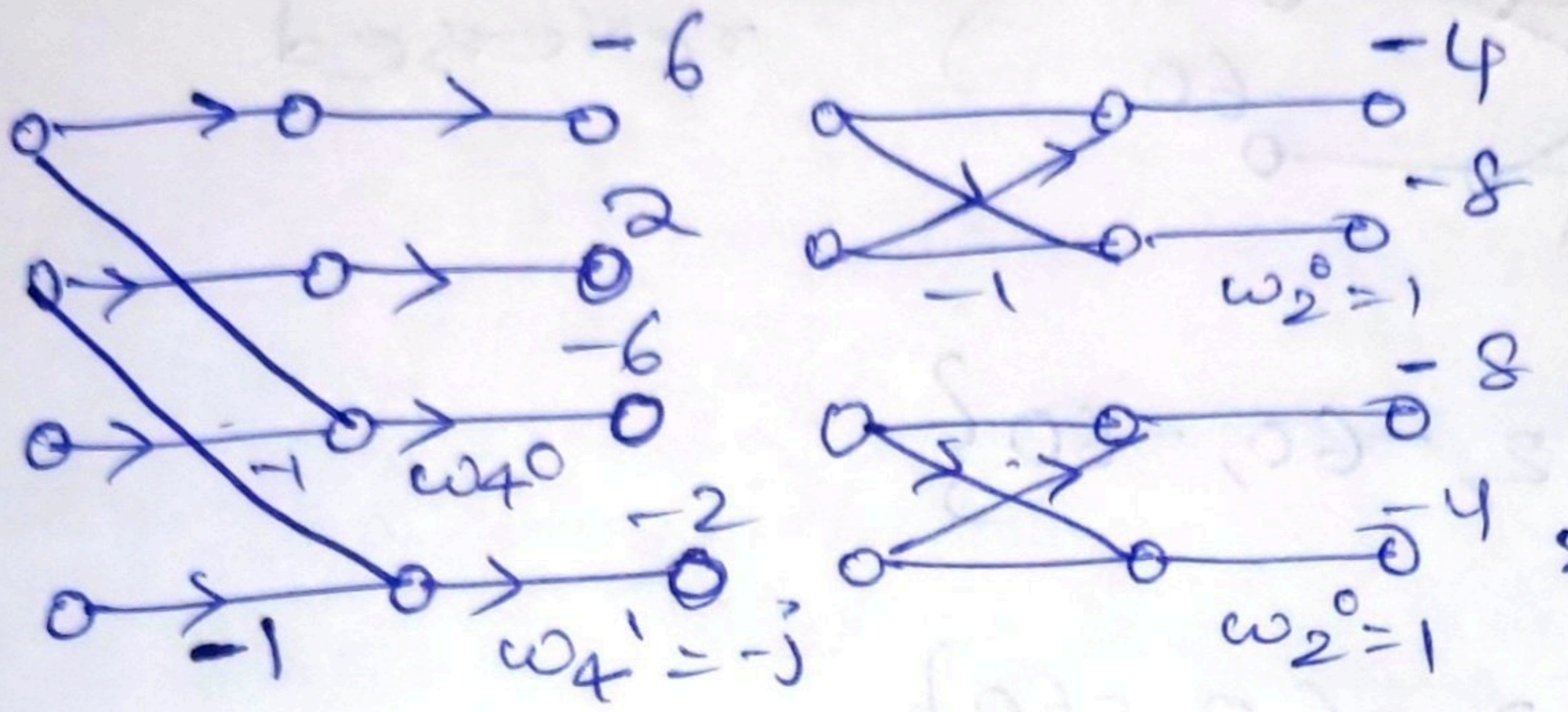
normal order

$X(k) = \{12, 1-2.414j, 0, 1-0.414j, 0, 1+0.414j,$
$0, 1+2.414j\}$

IDIF

$y(k) = \{-6, 1+j, 0, 1-j\}$

$y^*(k) = \{-6, 1-j, 0, 1+j\}$



$y^*(n) = \{-4, -8, -8, -4\}$

$yj(n) = \{-4, -8, -8, -4\}$

$y(n) = \frac{\quad}{4}$

$y(n) = \{-1, -2, -2, -1\}$

$(1+j)(2j-2)$

$2j - 2 - 2 - 2j = -4$

$1-j(-2-2j)$

$-2-2j+2j$

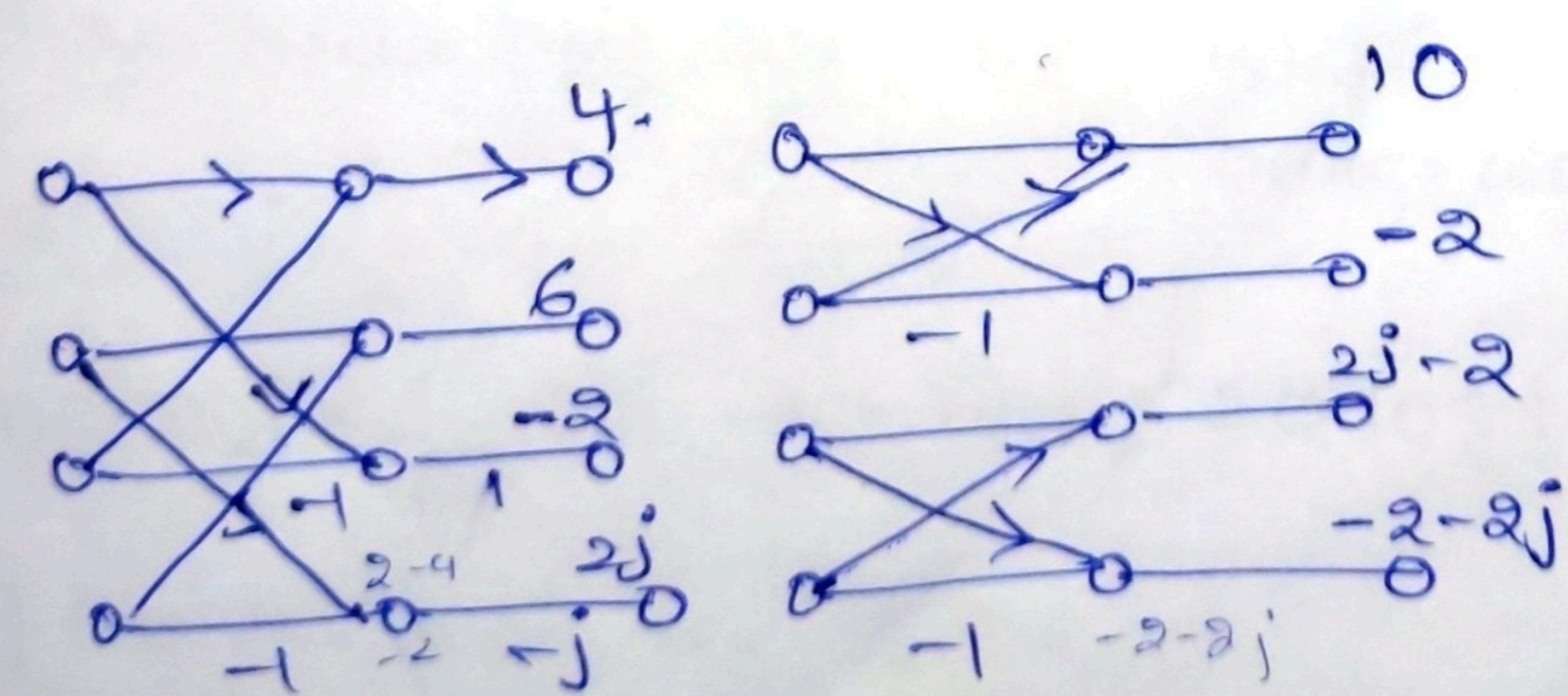
-2

\# $x(n) = \{1, 2, 3, 4\}$, $h(n) = \{-1, -2, -2, -1\}$

$X(k) \cdot H(k) = Y(k)$

$H(k) = \{-6, 1+j, 0, 1-j\}$

X(k)

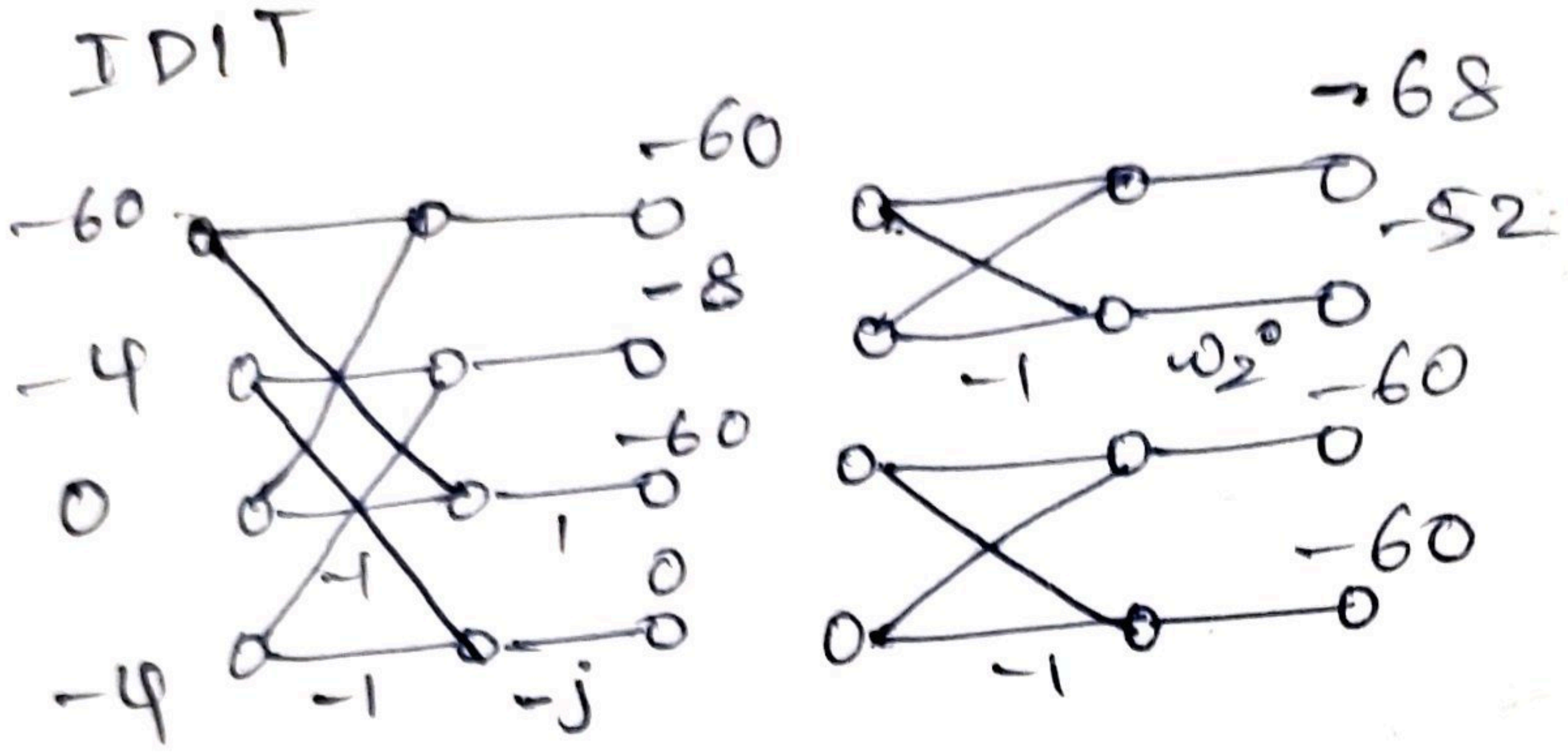


$X(k) = \{10, -2+2j, -2, -2-2j\}$

$Y(k) = \{-60, -4, 0, -4\}$

$Y(k) = \{-60, -4, 0, -4\}$

IDIT



→ bit reversed

$$y^*(n) = \{-68, -52, -60, -60\}$$

$$y(n) = \{-68, -52, -60, -60\}$$

$$\frac{y(n)}{4} = \{-17, -15, -13, -15\}$$

↙

circular convolution

stock ham method

	1	2	3	4
-1	-1	-2	-3	-4
-2	-2	-4	-6	-8
-2	-2	-4	-6	-8
-1	-1	-2	-3	-4

$\{-1, -4, -9, -15, -16, -11, -4\}$

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