DTFT – Discrete time fourier transform:

$$z = e^{j\omega}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

Q. Determine the frequency response $H(e^{j\omega})$ for the system & plot magnitude response and phase response.

→ $y(n) + \frac{1}{4} y(n-1) = x(n) - x(n-1)$

take Fourier-transform

$$Y(e^{j\omega}) + \frac{1}{4} e^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega} X(e^{j\omega})$$

$$Y(e^{j\omega}) \left[1 + \frac{1}{4} e^{-j\omega}\right] = X(e^{j\omega})(1 - e^{-j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$\boxed{H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 + \frac{1}{4} e^{-j\omega}}}$$

$$|H(e^{j\omega})| = \frac{1 - \cos\omega + j\sin\omega}{1 + \frac{1}{4}(\cos\omega - j\sin\omega)}$$

$$= \frac{|(1-\cos\omega)^2 + \sin^2\omega|^{1/2}}{|1 + \frac{1}{16}(\cos\omega - j\sin\omega)^2|^{1/2}}$$

$$= \frac{|1 + \cos^2\omega - 2\cos\omega + \sin^2\omega|^{1/2}}{|1 + \frac{1}{16}(\cos^2\omega - \sin^2\omega - 2\cos\omega j\sin\omega)|^{1/2}}$$

$$= \frac{(2 - 2\cos\omega)^{1/2}}{1 + \frac{1}{16}(\cos^2\omega ...}$$

$$= \frac{|(1-\cos\omega)^2 + \sin^2\omega|^{1/2}}{|(1+\frac{1}{4}\cos\omega)^2 + \frac{1}{16}\sin^2\omega|^{1/2}}$$

$$= \frac{|1+\cos^2\omega - 2\cos\omega + \sin^2\omega|^{1/2}}{|1+\frac{1}{16}\cos^2\omega + \frac{1}{16}\sin^2\omega + \frac{1}{2}\cos\omega|^{1/2}}$$

$$= \frac{|2-2\cos\omega|^{1/2}}{|1.0625 - 0.5\cos\omega|^{1/2}}$$

$$= \boxed{\frac{2\sin\omega/2}{(1.0625 - 0.5\cos\omega)^{1/2}} = |H(e^{j\omega})|}$$

phase ~~of~~ response

$$= \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \cancel{\tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)}$$

$$= \tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)$$

For numerator part

$$\tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)$$

$$\tan^{-1}\left(\frac{2\sin\omega/2\cos\omega/2}{2\sin^2\omega/2}\right)$$

$$= \tan^{-1}\left(\cot\omega/2\right)$$

$$= \tan^{-1}\left(\tan\left(90 - \omega/2\right)\right)$$

$$=$$

For denominator

$$\tan^{-1}\left(\frac{\frac{1}{4}\sin\omega}{1+\frac{1}{4}\cos\omega}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{4}\sin\omega/2\cos\omega/2}{\quad}\right)$$

$\angle H(e^{j\omega})$

$$= 90 - \frac{\omega}{2} - \tan^{-1}\left(\frac{-0.25\sin\omega}{1+0.25\cos\omega}\right)$$

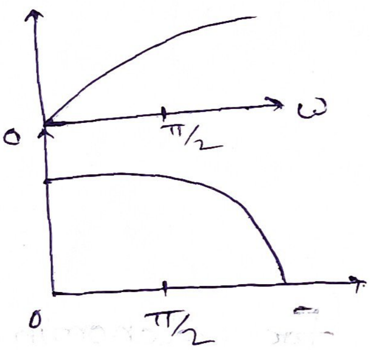
→ if ω is a negative value, then magnitude of

$$\angle H(e^{j\omega}) = \tan^{-1}\left(\frac{\sin(-\omega)}{1-\cos(-\omega)}\right) - \tan^{-1}\left(\frac{0.25\sin(-\omega)}{1+0.25\cos(-\omega)}\right)$$

$$\angle H(e^{j\omega}) = \tan^{-1}\left(\frac{-\sin\omega}{1-\cos\omega}\right) - \tan^{-1}\left(\frac{0.25\sin\omega}{1+0.25\cos\omega}\right)$$

$$\boxed{\angle H(e^{j\omega}) = -90 + \frac{\omega}{2} - \tan^{-1}\left(\frac{0.25\sin\omega}{1+0.25\cos\omega}\right)}$$

ω	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$7\pi/12$	$2\pi/3$	$5\pi/6$	π
$\lvert H(e^{j\omega})\rvert$	0	0.423	0.636	0.873	1.372	1.642	1.92	2.435	2.66
$\angle H(e^{j\omega})$	$\frac{\pi}{2}$	0.449π	0.422π	0.394π	0.328π	0.288π	0.244π	0.1566π	0



Discrete Fourier Transform (DFT)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Q. compute 4-point & 8 point DFT of the sequence given by $x(n) = \begin{cases} 1/3 ; 0 \le n \le 2 \\ 0 ; \text{else} \end{cases}$

→ 4 point DFT (N)

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j2\pi kn/2}$$

$$= x(0) e^{0} + x(1) e^{-j\pi k/2} + x(2) e^{-2jk\pi} + 0$$

$$= \cancel{\frac{1}{3} + \frac{1}{3} e^{-j\pi k/2} + \frac{1}{3} e^{-jk\pi}}$$

(k value lies between 0 to N-1)

take $k=0$, $X(0) = x(0) + x(1) e^{0} + x(2) e^{0}$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \boxed{1 = X(1)}$$

$k=1$, $X(1) = \frac{1}{3} + x(1) e^{-j\pi/2} + x(2) e^{-j\pi}$

$$= \frac{1}{3}\left[1 + e^{-j\pi/2} + e^{-j\pi}\right]$$

$$= \frac{1}{3}\left[1 + \cos\pi/2 - j\sin\pi/2 + \cos\pi - j\sin\pi\right]$$

$$= \frac{1}{3}\left[1 + (0) - j + (-1) - j\right]$$

$k=2,\ X(2) = \frac{1}{3} + \frac{1}{3}e^{-j\pi} + \frac{1}{3}e^{-2\pi j}$

$= \frac{1}{3}\left[1 + \cos\pi - j\sin\pi + \cos 2\pi - j\sin 2\pi\right]$

$= \frac{1}{3}\left[1 - 1 - j(0) + 1 - j \times 0\right]$

$\boxed{X(2) = \frac{1}{3}}$

$k=3,\ X(3) = \frac{1}{3} + \frac{1}{3}e^{-3j\pi/2} + \frac{1}{3}e^{-3\pi j}$

$= \frac{1}{3}\left[1 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi\right]$

$= \frac{1}{3}\left[1 + 0 - j(-1) + (-1) - j(0)\right]$

$\boxed{X(3) = J/3}$

$x(K) = \left\{1, -J/3, 1/3, J/3\right\}$ → magnitude response

$= \left\{1\angle 0, 3\angle{-\pi/2}, 1/3, 3\angle \pi/2\right\}$ → phase response

$\left\{0, -\pi/2, 0, \pi/2\right\}$

8-point DFT

$$X(K) = \sum_{n=0}^{7} x(n) e^{-j2\pi nk/8}$$

$$= \sum_{0}^{7} x(n) e^{-j\pi kn/4}$$

$$X(K) = x(0)e^{0} + x(1)e^{-j\pi k/4} + x(2)e^{-j\pi k/2}$$

$$k=0, X(K) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \boxed{1 = X(1)}$$

$$k=1, X(1) = \frac{1}{3} + \frac{1}{3}e^{-j\pi/4} + \frac{1}{3}e^{-j\pi/2}$$

$$= \frac{1}{3}\left[1 + \cos\frac{\pi}{4} - j\sin\pi/4 + \cos\frac{\pi}{2} - j\sin\pi/2\right]$$

$$= \frac{1}{3}\left[1 + 0.707 - j(0.707) + 0 - j\right]$$

$$= \frac{1}{3}\left[1.707 - 1.707j\right]$$

$$= \frac{1.707}{3}(1-j) = \boxed{0.569(1-j) = X(1)}$$

$$k=2, X(2) = \frac{1}{3} + \frac{1}{3}e^{-j\pi/2} + \frac{1}{3}e^{-j\pi}$$

$$= \frac{1}{3}\left[1 + -j - 1\right] = \boxed{-j/3 = X(2)}$$

$$k=3, X(3) = \frac{1}{3} + \frac{1}{3}e^{-j3\pi/4} + \frac{1}{3}e^{-j3\pi/2}$$

$$= \frac{1}{3}\left[1 + \cos\frac{3\pi}{4} - j\sin 3\pi/4 + \cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right]$$

$$= \frac{1}{3}\left[1 - 0.707 - j(0.707) + +j - 0\right]$$

$$= \frac{1}{3}\left[0.293 + j(0.293)\right] = \boxed{0.097 + 0.097j}$$

$$X(4) = \cancel{e^{0}} \cancel{*} \frac{1}{3} + \frac{1}{3}e^{-j\pi} + \frac{1}{3}e^{-2j\pi}$$

$$= \frac{1}{3}[1+(-1) + \cos 2\pi - j\sin 2\pi$$

$$= \frac{1}{3}[1+0] = \boxed{\frac{1}{3} = X(4)}$$

$$X(5) = \frac{1}{3} + \frac{1}{3}e^{-j\pi 5/4} + \frac{1}{3}e^{-j\pi 5/2}$$

$$= \frac{1}{3}[1+(-0.707)+j0.707 + 0 - j]$$

$$= \frac{1}{3}[0.293 - 0.293j] = \boxed{0.097 - 0.097j}$$

$$X(6) = \frac{1}{3}[1 + e^{-j\pi 3/2} + e^{j3\pi/8}]$$

$$= \frac{1}{3}[1 + j + -1 - \cancel{g} - 0]$$

$$\boxed{X(6) = 0.33}$$

$$X(7) = \frac{1}{3}[1 + e^{-j\pi 7/4} + e^{j7\pi/2}]$$

$$= \frac{1}{3}[1 + 0.707 + j0.707 + 0 + j]$$

$$= \frac{1}{3}[1.707 + 1.707j]$$

$$\boxed{X(7)\ 0.569 + 0.569j}$$

0.569 − 0.569j

$$X(k) = \{1, -j/3, 0.097+0.097j, \frac{1}{3}, 0.097-0.097j, 0.33$$

$$0.569 + 0.569j\}$$

$$x(k) = \left\{ 1,\ 0.569 - 0.569j,\ \frac{-j}{3},\ 0.097 + 0.097j,\ \frac{j}{3},\right.$$

$$\left. 0.097 - 0.097j,\ 0{,}33,\ 0.569 + 0.569j \right\}$$

* Circular Time Shift property of DFT *

If $DFT\{x(n)\} = X(K)$, then DFT of

$DFT\{x(n-m)_N\} = W_N^{mk} X(K)$

Definition of inverse DFT :-

$$\boxed{x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kn}}$$

$W_N^{-kn} = e^{-\frac{j2\pi k}{N}n}$

$= e^{-j2\pi k}$

$W_4^2 = e^{-j\frac{2\pi}{4}}$

$0 \leq K \leq N-1$

$$\Rightarrow x(n-m) = \frac{1}{N}\sum_{k=0}^{N-1} X(K) W_N^{-k(n-m)}$$

time shift is circular,

$$x((n-m))_N = \frac{1}{N}\sum_{K=0}^{N-1} X(k) W_N^{km} W_N^{-kn}$$

$$= IDFT[X(K)] W_N^{km}$$

Q. Find the 4-point DFT of the sequence $x(n)=\{1,-1,1,-1\}$ & Also using time shift property find the DFT of the sequence $y(n) = x((n-2))_4$.

$x(0)$, 1, 2, 3

$\rightarrow X(K) = DFT\{x(n)\}$ $(N=4 ;\ n=0 \text{ to } 3)$

$$= \sum_{n=0}^{3} x(n) W_4^{kn} \qquad 0 \leq K \leq 3$$

$$= \sum_{k=0} x(0)W_4^0 + x(1) W_4^k + x(2) W_4^{2k} + x(3) W_4^{3k}$$

$$= W_4^0 - \cancel{0} W_4^k + W_4^{2k} - W_4^{3k}$$

$W_4^0 = e^{-j2\pi 0/4} = e^0 = 1$

$W_4^1 = e^{-j2\pi 1/4} = e^{-j\pi/2} = -j$

$W_4^2 = e^{-j2\pi 2/4} = e^{-j\pi} = -1$

$W_4^3 = e^{-j3\pi 2/4} = e^{-j3\pi/2} = j$

$k=0$, $x(0) = W_4^0 - W_4^0 + W_4^0 - W_4^0 = 0$

$k=1$, $X(1) = W_4^0 - W_4^1 + W_4^2 - W_4^3 = 1+j-1-j = 0$

$k=2$, $X(2) = W_4^0 - W_4^2 + W_4^4 - W_4^6 = 1+1+1+1 = 4$

$k=3$, $X(3) = W_4^0 - W_4^3 + W_4^6 - W_4^9 = 1-j-1+j = 0$

$X(K) = \{0, 0, 4, 0\}$

for $y(n) = x((n-2))_4$

$DFT = \{x((n-2))_4\}$

$X(K)\, W_N^{mk} = X(K)\, W_4^{2k}$

$k=0$, $Y(0) = X(0) W_4^0 = 0$

$k=1$, $Y(1) = X(1) W_4^2 = 0$

$k=2$, $Y(2) = X(2) W_4^4 = -4j$

$k=3$, $Y(3) = X(3) W_4^8 = 0$

Q. Find the 4 point DFT of sequence $x(n) = \{-1, 1, 1, 2\}$ & using time shift find DFT $y(n) = x((n-2))_4$.

→ $X(K) = DFT\{x(n)\}$

$= \sum_{k=0}^{3} x(n) W_4^{+kn}$ $\quad 0 \le k \le 3$

$= x(0) W_4^0 + x(1) W_4^K + x(2) W_4^{2K} + x(3) W_4^{3K}$

heaveside

~~f = 0.5^n (heaviside (-n));~~

$\omega = -20\pi : \frac{pi}{20} : 20\pi$

$W_4^0 = e^{-2j\pi 0/4} = 1$

$W_4^1 = e^{-j2\pi/4} = -j$

$W_4^2 = e^{-j\pi} = -1$

$W_4^3 = e^{-j3\pi/2} = j$

$\cancel{X(k)} =$

at $k=0$, $X(0) = -1-j-j+2j = -3$

at $k=1$, $X(1) = -1-j-1+2j = -2+j$

$k=2$, $X(2) = -1-1+1-2 = -3$

$k=3$, $X(3) = -2-j$

* Linearity property of DFT *

$$DFT\{ax_1(n)+bx_2(n)\} = aX_1(k)+bX_2(k), \; k=0,1,2,3$$

$$DFT \; x(n) \triangleq \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$DFT\{ax_1(n)+bx_2(n)\} = \sum_{n=0}^{N-1} [ax_1(n)+bx_2(n)] W_N^{kn}$$

$$= aX_1(k)+bX_2(k)$$

$$ax_1(n)+bx_2(n) \xleftrightarrow{DFT} aX_1(k)+bX_2(k)$$

Q. Find the 4-point DFT of the sequence of $x(n) = \cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{4}n)$ use linearity property.

$x(n) = \cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{4}n)$: $0 \leq n \leq 3$

$x_1(n) = \cos(\frac{\pi}{4}n)$, $x_2(n) = \sin(\frac{\pi}{4}n)$

n	$x_1(n) = \cos(\frac{\pi}{4}n)$	$x_2(n) = \sin(\frac{\pi}{4}n)$
0	1	0
1	$1/\sqrt{2}$	$1/\sqrt{2}$
2	0	1
3	$-1/\sqrt{2}$	$1/\sqrt{2}$

$$X_1(k) = \sum_{n=0}^{3} x_1(n) W_4^{kn}, \; k=0,1,2,3$$

$$= x_1(0)W_4^0 + x_1(1)W_4^k + x_1(2)W_4^{2k} + x_1(3)\cdot W_4^{3k}$$

$$= 1\times 1 + \frac{1}{\sqrt{2}} W_4^k + 0 + -\frac{1}{\sqrt{2}} W_4^{3k}$$

$$= 1 + \frac{1}{\sqrt{2}}(\underset{-j}{W_4^k} - W_4^{3k})$$

$k=0$, $X_1(0) = 1$

$k=1$, $X_1(1) = 1-\sqrt{2}j$

$k=2$, $X_1(2) = 1 - \frac{1}{\sqrt{2}}(-1+1) = 1$

$k=3$, $X_1(3) = 1 - \frac{1}{\sqrt{2}}(-j+j) = 1+j\sqrt{2}$

$$\boxed{X_1(k) = \{1, 1-j\sqrt{2}, 1, 1+j\sqrt{2}\}}$$

$$X_2(k) = \sum_{0}^{3} x_2(n) W_4^{kn}, \quad k=0,1,2,3$$

$$= x_2(0) W_4^{0} + x_2(1) W_4^{k} + x_2(2) W_4^{2k} + x_2(3) W_4^{3k}$$

$$= 0 + \frac{1}{\sqrt{2}} W_4^{k} + W_4^{2k} + \frac{1}{\sqrt{2}} W_4^{3k}$$

$$= \frac{1}{\sqrt{2}} (W_4^{k} + W_4^{3k}) + W_4^{2k}$$

$k=0$, $X(0) = \cancel{\sqrt{2}\,1} = \sqrt{2} - 1$

$k=1$, $X(1) = \frac{1}{\sqrt{2}}(-j+j) + \cancel{j} - 1 = \cancel{j} - 1$

$k=2$, $X(2) = \frac{1}{\sqrt{2}}(\cancel{j} -1 -1) + 1 = +1 - \sqrt{2}$

$k=3$, $X(3) = \frac{1}{\sqrt{2}}(j - j) = -1$

$\Rightarrow X_2(k) = \{\overset{2.414}{\cancel{0.414}}, -1, -0.414, -1\}$

$k=0$, $X(0) = \frac{1}{\sqrt{2}} W_4^{0} + W_4^{0} + \frac{1}{\sqrt{2}} W_4^{0}$

$= \cancel{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}}$

$= \underline{\underline{\sqrt{2}+1}} = 1.414 + 1 = 2.414$

$$X(k) = X_1(k) + X_2(k)$$

$= \{1+2.414, 1 - j\sqrt{2} - 1, 1 \overset{-0.414}{\cancel{+1-\sqrt{2}}}, j\sqrt{2}\}$

$$\boxed{= \{3.414, -j\sqrt{2}, 0.586, j\sqrt{2}\}}$$

Q. Compute the 5-point DFT of the sequence $x(n) = (1,0,1,0,1)$ & verify the symmetric property.

$N = 5$,

$W_5^0 = e^{-j2\pi 0/5} = 1$

$W_5^1 = e^{-j2\pi/5} = 0.309 - j0.95$

$W_5^2 = e^{-4\pi/5} = -0.809 - j0.587$

$W_5^3 = e^{-j6\pi/5} = -0.809 + j0.587$

$W_5^4 = e^{-j8\pi/5} = 0.309 + j0.95$

$\cancel{x(n)} = X(k) = \sum_0^4 x(n) W_5^{+kn}$

$= x(0) + \underbrace{x(1)}_{0} W_5^k + x(2) W_5^{2k} + \underbrace{x(3)}_{0} W_5^{3k} + x(4) W_5^{4k}$

$= \cancel{1 + (-0.809 - j0.587)}$

$= 1 + W_5^{2k} + W_5^{4k}$

$k=0$, $X(0) = 1 + 1 + 1 = 3$

$k=1$, $X(1) = 1 + (-0.809 - j0.587) + 0.309 + j0.95 = \cancel{1.618}$ $0.5 + j0.364$

$k=2$, $X(2) = 1 + (0.309 + j0.95) + (-0.809 + j0.587)$

$= 0.5 + j1.537$

$k=3$, $X(3) = 1 + (0.309 - j0.95) - 0.809 - j0.587$

$= 0.5 - j1.537 = 0.5 - j1.537$

$k=4$, $X(4) = 1 + (-0.809 + j0.587) + 0.309 - j0.95$

$= 0.5 - j0.364$

$x(k) = \{3, \cancel{1.618}\ 0.5 + j1.537, 0.5 - j1.537, 0.5 - j0.364\}$

$\boxed{X(k) = \{3, \cancel{1.618}\ 0.5 + j0.36, 0.5 + j1.537, 0.5 - j1.537, 0.5 - j0.364\}}$

Q. The first 5-point of the 8 point DFT of a real valued sequence are {0.25, 0.5-j0.5, 0, 0.5-j0.86, 0}

find the remaining 3 points.

→ $X^*(k) = X(N-k)$

$X^*(5) = X(8-5) = X(3) = 0.5 + j0.86$

$X^*(6) = X(8-6) = X(2) = 0$

$X^*(7) = X(8-7) = X(1) = 0.5 + j0.5$

∴ the 8 point DSP is given by

{0.25, 0.5-j0.5, 0, 0.5-j0.86, 0, 0.5+j0.86, 0, 0.5+j0.5}

Q. compute the circular convolution of the following 2 sequences using DFT.

$x_1(n) = \{0, 1, 0, 1\}$

$x_2(n) = \{1, 2, 1, 2\}$

$x_3(n) = x_1(n) \circledast x_2(n)$

(length $x_1 = x_2$)

→ $X_1(k) = \sum_{n=0}^{3} x(n) e^{-j\pi kn/2}$

$= x(0) e^0 + x(1) e^{-j\pi k/2} + x(2) e^{-j2\pi k/2} + x(3) e^{-j3\pi k/2}$

$= 0 + e^{-j\pi k/2} + 0 + e^{-j3\pi k/2}$

$= e^{-j\pi k/2} + e^{-j3\pi k/2}$

$k=0, X_1(0) = 2$

$k=1, X_1(1) = 0$

$k=2, X_1(2) = -1-1 = -2$

$k=3, X_1(3) = 0+j+0-j = 0$

$\boxed{X_1(k) = \{2, 0, -2, 0\}}$

$$X_2(K) = \sum_{0}^{3} x_2(n)\, e^{-j\pi k n/2}$$

$$= x_2(0)e^{0} + x_2(1)e^{-j\pi k/2} + x_2(2)e^{-j2\pi k/2} + x_2(3)e^{-j k \pi 3/2}$$

$$= 1 + 2e^{-j\pi k/2} + e^{-j2\pi k/2} + 2e^{-j3\pi k/2}$$

$k=0$, $X_2(0) = 1+2+1+2 = 6$

$k=1$, $X_2(1) = 0$

$k=2$, $X_2(2) = -2$

$k=3$, $X_2(3) = 0$

$$X_2(K) = \{6, 0, -2, 0\}$$

$$X_3(K) = \{12, 0, 4, 0\}$$

<u>IDFT</u> of $X_3(k)$ $= \frac{1}{4}\sum_{k=0}^{3} X_3(k)\, e^{j2\pi k n/2}$

$$= \frac{1}{4}\Big[X(0)e^{0} + X(1)e^{j\pi k/2} + X(2)e^{j\pi k}$$

$$+ X(3)e^{3\pi k/2}\Big]$$

Q. Find the circular convolution of the two sequences using DFT. $x(n) = \{1, 2\}$

$h(n) = \{2, 1\}$

$\rightarrow X(K) = \sum_{n=0}^{1} x(n) e^{-j2\pi kn/2}$

$= \sum_{n=0}^{1} x(n) e^{-j\pi kn}$

$= x(0)e^{0} + x(1)e^{-j\pi k} = 1 + 2e^{-j\pi k}$

$K = 0, 1$

$k=0,\ X(0) = 1+2 = 3$

$K=1,\ X(1) = 1+2(-1) = -1$ $\quad X(K) = \{3, -1\}$

$\rightarrow H(K) = \sum_{n=0}^{1} x(n) e^{-j2\pi kn/2}$

$= \sum_{n=0}^{1} x(n) e^{-j\pi kn}$

$= x(0)e^{0} + x(1)e^{-j\pi k}$

$= 2 + e^{-jk\pi}$

$k = 0, 1$

$k=0,\ x(0) = 2+1 = 3$

$k=1,\ X(1) = 2+1(-1) = 1$

$H(K) = \{3, 1\}$

$X_3(K) = X(K) \cdot H(K)$

$= \{9, -1\}$

IDFT of the $X_3(K)$.

$$= \frac{1}{N} \sum_{K=0}^{2} X_3(K) e^{j2\pi kn/2}$$

$$= \frac{1}{2} \sum_{0}^{1} X_3(K) e^{j\pi kn}$$

$$= \frac{1}{2} \left[X(0) e^0 + X(1) e^{j\pi k} \right]$$

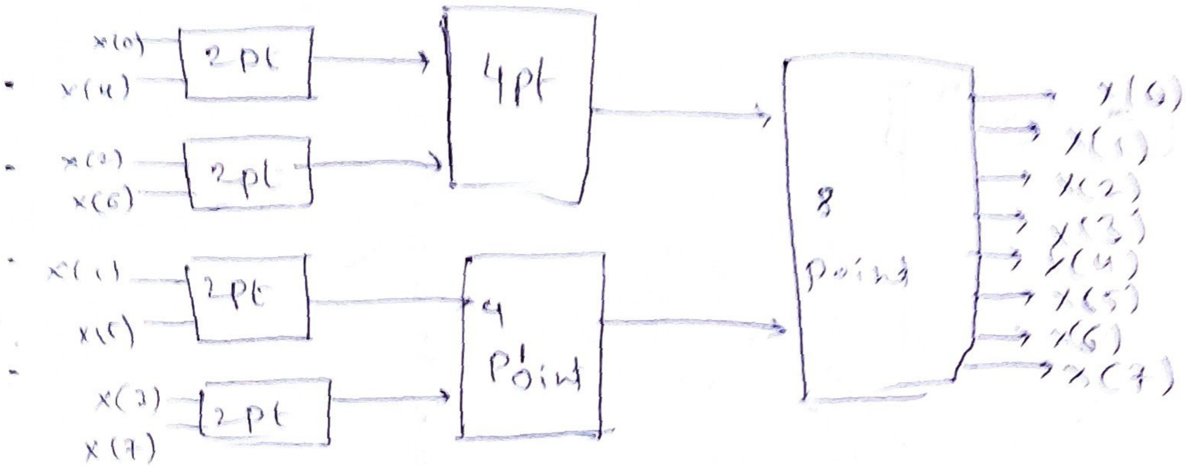
$$= \frac{1}{2} \left[9 - 1 e^{j\pi k} \right]$$

$$k=0, \ X(0) = \frac{1}{2} \left[9 - e^{j\pi} \right]$$

* Fast Fourier Transform * (FFT)

If N-point sequence is given, first we find, decom... the N-point into 2 point DFT and from 2 point we fi... 4point DFT & from 4point we find 8 point until we reach N point in order to reduce the complexity.

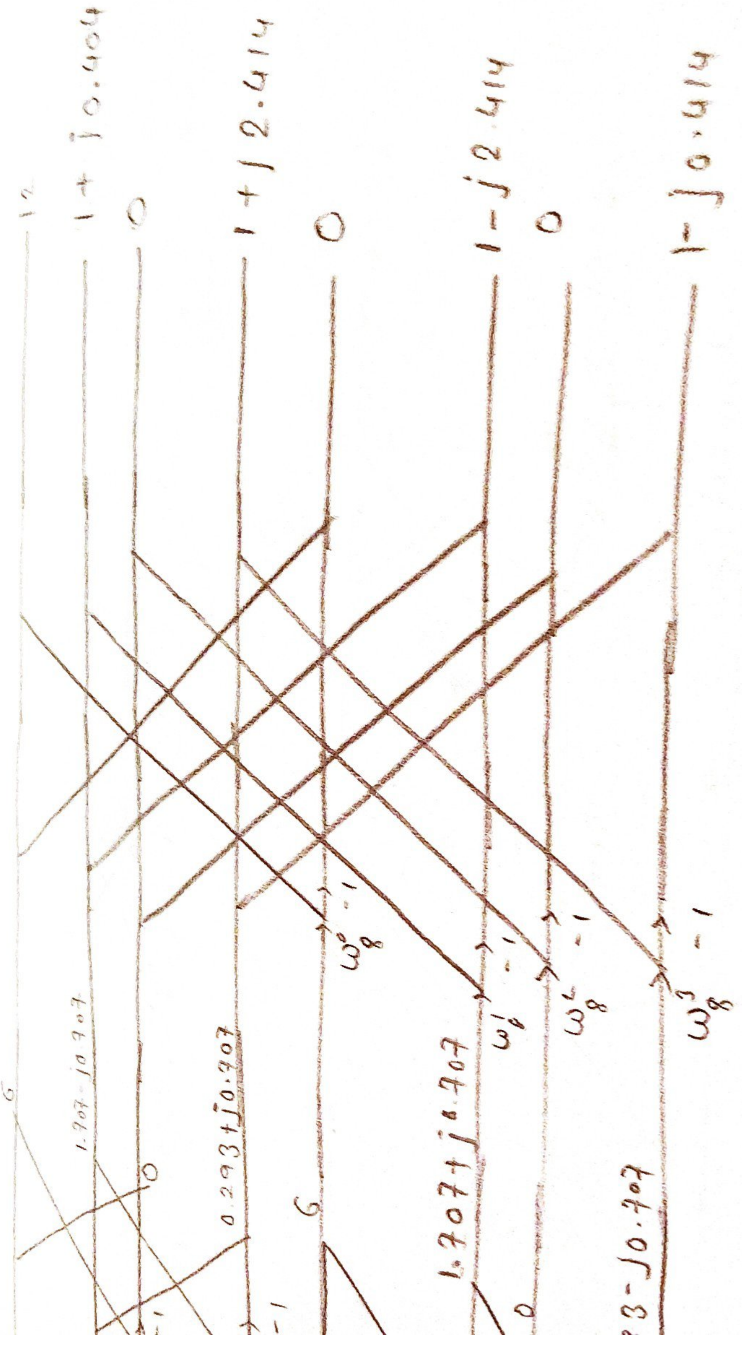
8-point DFT



0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

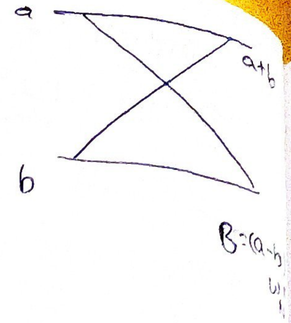
Radix-2 Algorithm:
- DIT-FFT (decimation in time)
- DIF-FFT (decimation in freq)

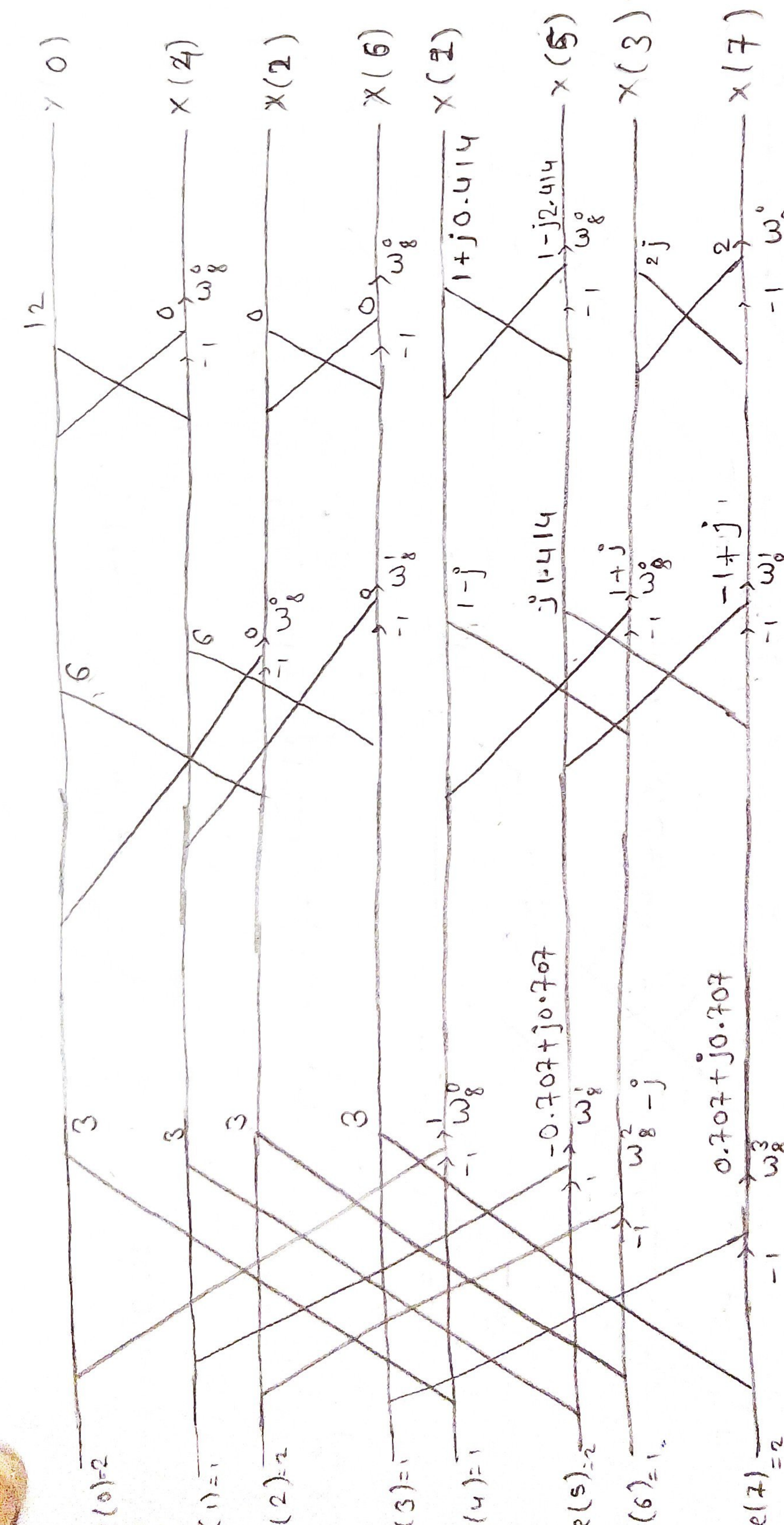
Q) compute 8 point DFT of $x(n) = \{2, 1, 1, 1, 1, [illegible]\}$ using DIT-FFT radix-2 algorithm.



$\{12, 1+j0.414, 0, 1+j2.414, 0, 1-j2.414, 0, 1-j0.414\}$

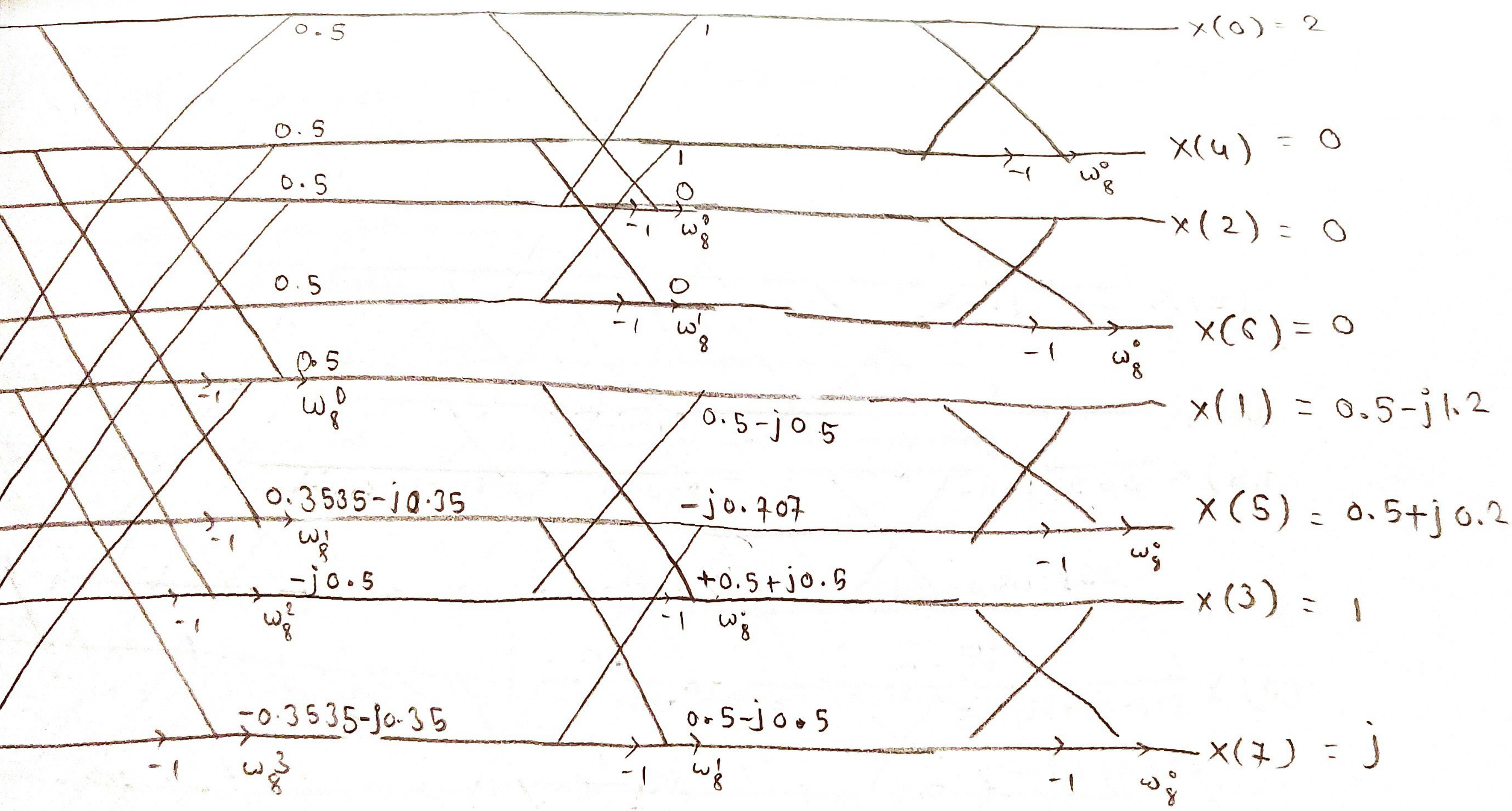
DIF - FFT :- $x(n) = \{2, 1, 2, 1, 1, 2, 1, 2\}$





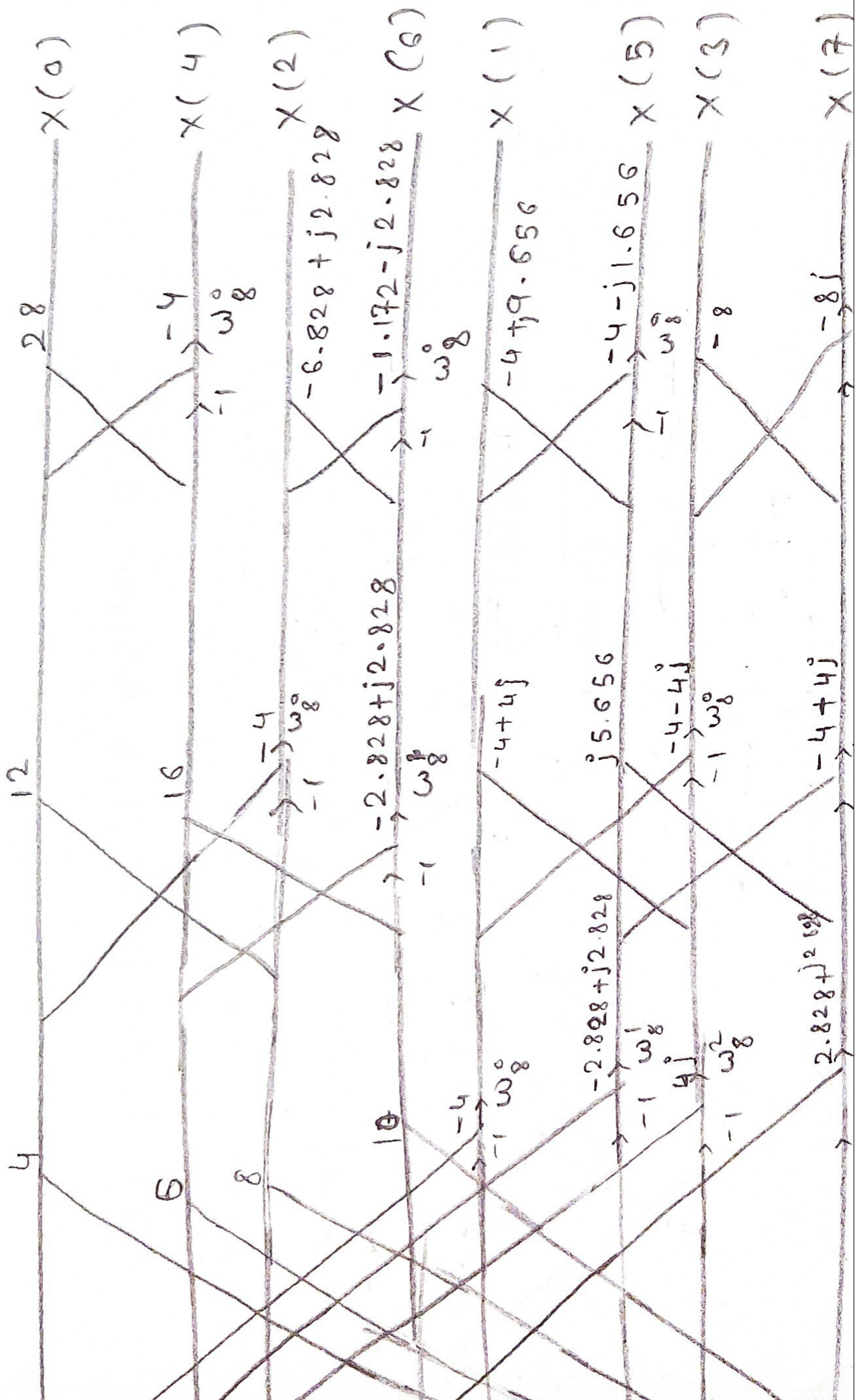
$X(k) = \{12, 1+j0.414, 0, 2j, 0, 1-j2.414, 0, 2\}$

Q. Compute 8-point DFT of the sequence $x(n) = \{0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0\}$ Using DIF-FFT



$$X(K) = \{2, 0.5-j1.2, 0, 1, 0, 0.5+j0.2, 0, j\}$$

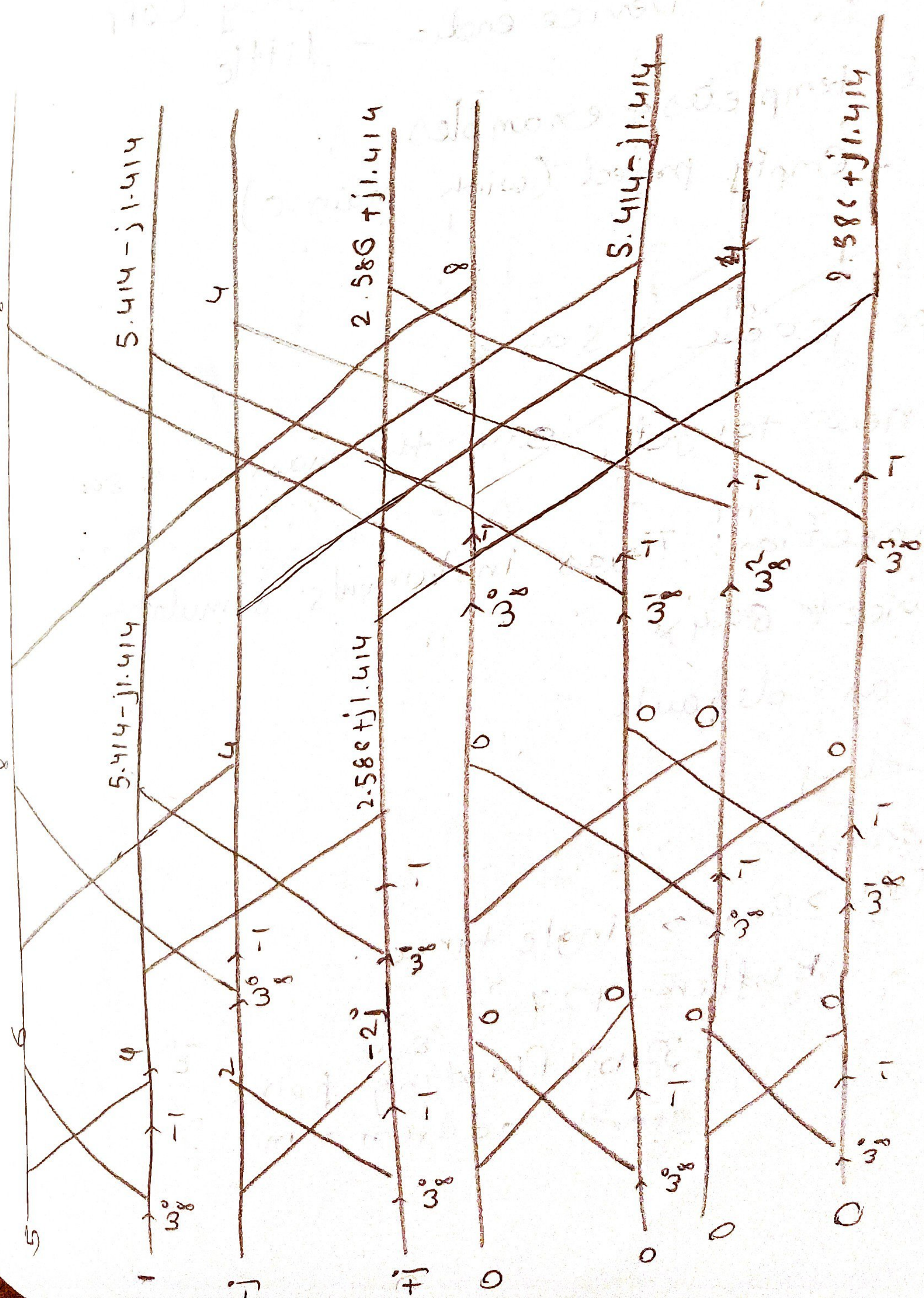
DIF - FFT $x(n) = \{0, 1, 2, 3, 4, 5,$



Find the IDFT of sequence ~~x~~ X(k) = {5, 0, 1-j, 0, 1, 0, 1+j, 0} ~~...~~ using DIT-FFT

$x(n) \rightleftharpoons X(k)$

a + b



$4 + (2j)(0.707 - j0.707)$

$4 + 1.414 + j1.414$