* Signals & systems :-

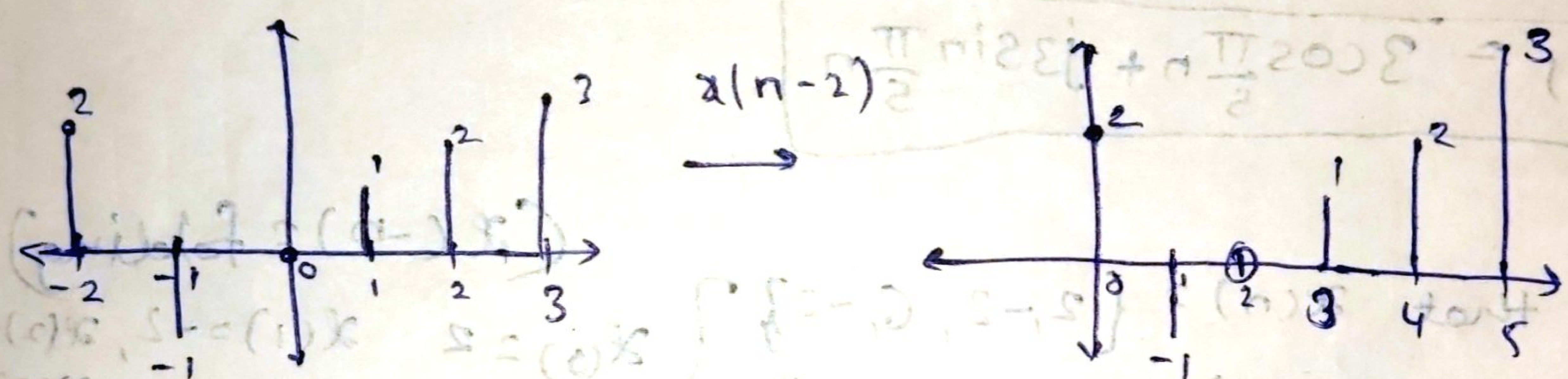
mathematical operation of discrete time signal :-

1. folding
2. Shifting
3. scaling
4. Addition
5. multiplication

Shifting :-

$(x-n) \rightarrow$ shift right (delay)

$(x+n) \rightarrow$ shift left (advance)



* Classification of signals :-

1. Even/odd signals (Symmetric/Assymmetric) :-

ex. even part $= \frac{1}{2}[x(n) + x(-n)] = x_e(n)$

odd part $= \frac{1}{2}[x(n) - x(-n)] = x_o(n)$

ex. $x(n) = 3^n$ even part $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x(-n) = 3^{-n}$

$$\boxed{x_e(n) = \frac{1}{2}[3^n + 3^{-n}]}$$

odd part $= x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$= \frac{1}{2}[3^n - 3^{-n}]$

ex.2

even $= x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x(n) = 3e^{j\pi/5\, n} = 3\cos\frac{\pi}{5}n + j3\sin\frac{\pi}{5}n$

$x(-n) = 3e^{-j\pi/5\, n}$

$= 3\cos\frac{\pi}{5}n - j3\sin\frac{\pi}{5}n$

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$= \frac{1}{2}[6\cos\frac{\pi}{5}n]$

$\boxed{x_e(n) = 3\cos\frac{\pi}{5}n}$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$= \frac{1}{2}[j6\sin\frac{\pi}{5}n]$

$\boxed{x_o(n) = j3\sin\frac{\pi}{5}n}$

$x(n) = x_e(n) + x_o(n)$

$\boxed{x(n) = 3\cos\frac{\pi}{5}n + j3\sin\frac{\pi}{5}n}$

ex.3

($x(-n)$ = folding)

Given that $x(n) = \{2, -2, 6, -2\}$
$x(-n) = \{-2, 6, -2, 2\}$

$x(0) = 2 \quad x(1) = -2, \; x(2) = 6$
$x(3) = -2$
$x(-1) = -2, \; x(-2) = 6, \; x(-3) = -2$

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

for $n = 0$, $x_e(0) = \frac{1}{2}[x(0) + x(0)] = \frac{1}{2}[2+2] = 2$

$n = 1 \quad x_e(1) = \frac{1}{2}[x(1) + x(-1)] = \frac{1}{2}[-2-2] = -2$

$n = 2, \quad x_e(2) = \frac{1}{2}[x(2) + x(-2)] = \frac{1}{2}[6+6] = 6$

$n = 3 \quad x_e(3) = \frac{1}{2}[x(3) + x(-3)] = \frac{1}{2}[-2-2] = -2$

$\boxed{\cancel{x_e(n) = \{2, -2, 6, -2\}}}$

$n = -1 \quad x_e(-1) = \frac{1}{2}[x(-1) + x(1)] = -2$

$n = -2 \quad x_e(-2) = \frac{1}{2}[x(-2) + x(2)] = 6$

$n = -3 \quad x_e(-3) = \frac{1}{2}[x(-3) + x(3)] = -2$

$\boxed{x_e(n) = \{-2, 6, -2, \underset{\uparrow}{2}, -2, 6, -2\}}$

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$x_0(n) = \{0, 0, 0, 0, 0, 0, 0\}$

* **Energy and power signals:** (may or maynot in exams)

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{n \to \infty} \frac{1}{2n+1} \sum_{n=-n}^{n} |x(n)|^2$$

for energy signal $0 < E < \infty$ & $P = 0$

for power signal $0 < P < \infty$ & $E = \infty$

Ex. 1 $x(n) = \left(\frac{1}{4}\right)^n u(n)$, Determine whether its energy or power signal.

$x(n) = (0.25)^n ; \quad n \geq 0$

$\quad = 0 \quad ; \quad n < 0$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{0}^{\infty} (0.25)^{2n}$$

$$= \sum_{0}^{\infty} (0.0625)^n$$

$\sum_{0}^{\infty} C^n = \frac{1}{1-C}$

$$= \frac{1}{1 - 0.0625}$$

$$= \underline{1.0667 \text{ J}}$$

$\sum_{0}^{n} C^n = \frac{C^{n+1} - 1}{C - 1}$

$$P = \lim_{n \to \infty} \frac{1}{2n+1} \sum_{-N}^{N} (x(n))^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{0}^{N} |(0.25)^n|^2$$

$$= \lim_{n \to \infty} \frac{1}{2N+1} \left(\frac{(0.0625)^{N+1} - 1}{0.0625 - 1} \right)$$

$\boxed{P = 0}$

$\therefore$ this is an energy signal.

$\boxed{\begin{array}{l} P = 0 \\ E = 1.067 \end{array}}$

Ex. 2 $x(n) = \sin(\frac{\pi}{3}n)$ $x(n) =$

$$E = \sum_{-\infty}^{\infty} \left|\sin(\frac{\pi}{3}n)\right|^2$$

$$= \sum_{-\infty}^{\infty} \left|\sin^2(\frac{\pi}{3}n)\right|$$

$$= \sum_{-\infty}^{\infty} \left|\frac{1-\cos\frac{2\pi}{3}n}{2}\right|$$

$$\boxed{E = \frac{1}{2}}$$

$$\frac{1}{2}\left(\sum_{-\infty}^{\infty} 1 - \sum_{-\infty}^{\infty} \cos\frac{2\pi}{3}n\right)$$

$$= \frac{1}{2}[\infty - 0]$$

$$\boxed{E = \infty}$$

$$\cancel{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2}$$

$$\cancel{= \lim_{N-\infty} \frac{1}{2N+1} \sum_{-N}^{N} \left|\frac{1-\cos\frac{2\pi}{3}n}{2}\right|}$$

$\cos\frac{2\pi}{3}n$ is periodic with periodicity of 3 samples

$n=0$ $\cos\frac{2\pi}{3}n = 1$

$n=1$ $\cos\frac{2\pi}{3} = -0.5$

$n=2$ $\cos 2\pi/3\, n = -0.5$

$n=3$ $\cos\frac{2\pi}{3}n = 1$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\frac{1-\cos\frac{2\pi}{3}n}{2}\right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\left[\sum_{-N}^{N} \frac{1}{2} - \sum_{-N}^{N}\frac{\cos\frac{2\pi}{3}n}{2}\right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1}\cdot\frac{1}{2}\left[2(N+1) - 0\right]$$

$$= \lim_{N\to\infty} \frac{1}{2}\left[\frac{N(1+1/N)}{N(2+1/N)}\right]$$

$$= \frac{1+1/\infty \to 0}{2+1/\infty \to 0} = \frac{1}{2}\ \text{watt}$$

$$P = \frac{1}{2}\ \text{watt}$$

$\therefore$ this is a power signal

Q. $x(n) = u(n) \to$ unit step signal.

$u(n) = 1 \quad n > 0$

$\quad\;\; = 0 \quad n \le 0$

$$E = \sum_{-\infty}^{\infty} u^2(n)$$

$$= \sum_{0}^{\infty} 1$$

$$E = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} |N|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N\to\infty} \frac{N(1+1/N)}{N(2+1/N)} = \frac{1}{2}$$

* Periodic & Aperiodic Signals

when $x(n)$ satisfies the condition $x(n+N)$ for all integer values of n, then $x(n)$ is periodic otherswise its non-periodic.

$$x(n+N) = x(n)$$

Q. $x(n) = \cos\left(\frac{5\pi}{9}n + 1\right)$

let N & m be two integers.

$$x(n+N) = \cos\left(\frac{5\pi}{9}(n+N) + 1\right)$$

$$= \cos\left(\frac{5\pi n}{9} + \frac{5\pi N}{9} + 1\right)$$

$\because \cos(\theta + 2\pi m) = \cos\theta$ for periodicity

$\frac{5\pi}{9}N$ should be integral multiple of 2π.

$$\therefore \frac{5\pi}{9}N = m.2\pi \Rightarrow \boxed{N = \frac{18m}{5}}$$

$\therefore$ N is an iteger if $m = 5, 10, 15 \ldots$

let $m = 5$, $N = 18$

when $N = 18$, $x(n+N) = \cos\left(\frac{5\pi n}{9} + \frac{5\pi}{9}18 + 1\right)$

$$= \cos\left(\frac{5\pi n}{9} + 10\pi + 1\right)$$

$$= \cos\left(\frac{5\pi n}{9} + 1\right) = x(n)$$

[it's fundamental period is 18 samples].

~~Q.~~ $\cancel{x(n) = \sin\left(\frac{n}{9} - \pi\right)}$

$$= -\sin\left(\pi - \frac{n}{9}\right) \qquad \frac{N}{9} = 2\pi m$$

$$= \sin\left(\frac{n}{9}\right)$$

$$= \sin\left(\frac{n+N}{9}\right)$$

Q. $\sin\left(\frac{n}{9} - \pi\right)$

$$x(n+N) = \sin\left(\frac{n+N}{9} - \pi\right)$$

$$= \sin\left(\frac{n}{9} + \frac{N}{9} - \pi\right)$$

$$= \sin\left(\frac{n}{9} - \pi + \frac{N}{9}\right)$$

$\sin(\theta + 2\pi m) = \sin\theta$, periodicity of $N/9$

$$N/9 = 2\pi m$$

$$\underline{\underline{N = 18m\pi}}$$

$\therefore$ N is an integer for every integer value of m

let $m=1$. $\Rightarrow N = 18\pi$

$$= \sin\left(\frac{n}{9} - \pi + \frac{18\pi}{9}\right) = \sin\left(\frac{n}{9} - \pi + 2\pi\right)$$

$$= \cancel{\sin\left(\frac{n}{9} + 18\pi\right)} \quad = \sin\left(\frac{n}{9} - \pi\right) = x(n)$$

$$= \cancel{\sin\left(\frac{n}{9}\right)}$$

* Casusal & Non-causal :-

If the output depends on present & future value of input signals then its called as ~~casual~~ Causal.

Signal depends on previous value of input signal, its called as non-causal signal.

$x(n) = \{\underset{\uparrow}{1}, -1, 2, 3, -3\}$ — causal

$x(n) = \{\underset{\uparrow}{2}, 2, 3, 3\}$ — Causal

$x(n) = \{1, -1, \underset{\uparrow}{2}, 3, 4\}$ — Non causal

$x(n) = \{\ldots, 2, 3, 4, \underset{\uparrow}{5}, 1, 2, 3\}$ — Non-causal

* Classification of discrete time systems :-

1) Static & Dynamic systems :

A discrete time called static or memoryless system if its output at any instant depends on the input sample at the same time but not on the past or futu- sample of input.

Dynamic system are memory system. Memory may be finite or infinite.

$y(n) = a\,x(n)$ — Static

$y(n) = n\,x(n) + 6x^3(n)$ — Static

$y(n) = x(n) + 3x(n-1)$ — dynamic

$y(n) = \sum_{m=0}^{1} x(n-m)$ — dynamic with finite memory

$y(n) = \sum_{m=0}^{\infty} x(n-m)$ — dynamic with infinite memory.

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2) Time invarient & time varient system :-

A system is said to be time invarient if its input output characteristics do not change with time.

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \xrightarrow{y(n-m)}$$

delay/unit element

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n)$$

$$\boxed{y(n) = H[x(n)]}$$

$y(n) = x(n) - b\,x(n-1)$

→ let $y(n-m)$ = Response of the delayed input

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \longrightarrow y(n-m) = x(n-m) - b[x(n-m-1)]$$

Response for delayed input.

→ $y_d(n)$ = Delayed response $\quad [y(n-m) = y_d]$

$$x(n) \longrightarrow \boxed{H} \xrightarrow{y(n)} \boxed{z^{-m}} \longrightarrow$$

$y(n-m) = \cancel{x(n)} - b\,x(n-1)$

$= x(n-m) - b(x(n-m-1))$

$y_d = x(n-m) - b\,x(n-m-1)$

∴ system is time invarient.

Q. $y(n) = 2n\,x(n)$

$y(n-m)$

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \longrightarrow$$

$y(n-m) = \cancel{x(n-m)} - \cancel{b}$

$= 2(n-m)\,x(n-m)$ (Response for delayed input)

$y_d(n)$

$$x(n) \longrightarrow \boxed{H} \xrightarrow{y(n) = 2n\,x(n)} \boxed{z^{-m}} \longrightarrow y_d(n) = 2n\,x(n-m)$$

∵ $y(n-m) \neq y_d(n) \Rightarrow$ System is time varient.

Q. $y(n) = x(n) + x(n-1)$

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \longrightarrow \quad y(n-m) = x(n-m) + x(n-m-1)$$

$$x(n) \longrightarrow \boxed{H} \xrightarrow{y(n) = x(n) + x(n-1)} \boxed{z^{-m}} \longrightarrow \quad y_d = x(n-m) + x(n-m-1)$$

$\Rightarrow \because$ ~~x~~ $y(n-m) = y_d(n) \Rightarrow$ system is time invarient.

Q. ~~2~~ $y(n) = x(-n)$

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \longrightarrow \quad y(n-m) = x(-n+m)$$

$$x(n) \longrightarrow \boxed{H} \xrightarrow{y(n) = x(-n)} \boxed{z^{-m}} \longrightarrow \quad y_d(n) = x(-n-m)$$

$\because y_d \neq y(n-m) \Rightarrow$ time varient.

Q. $y(n) = n\,x^2(n)$

$$x(n) \longrightarrow \boxed{z^{-m}} \xrightarrow{x(n-m)} \boxed{H} \longrightarrow \quad y(n-m) = (n-m)\,x^2(n-m)$$

$$x(n) \longrightarrow \boxed{H} \xrightarrow{y(n) = n\,x^2(n)} \boxed{z^{-m}} \longrightarrow \quad y_d(n) = n\,x^2(n-m)$$

$y(n-m) \neq y_d \Rightarrow$ Time varient

3 Causal & Non-causal system :-

Output of the system at any time depends only on the present, past input & past outputs. [Causal] but does not on future inputs & outputs.

System output at any time depends on future input & output, non-causal system.

ex.

$y(n) = x(n) - x(n-2)$

$$\left.\begin{aligned} n=0,\ & y(0) = x(0) - x(-2) \\ n=1\ & y(1) = x(1) - x(-1) \\ n=-1\ & y(-1) = x(-1) - x(-3) \end{aligned}\right] - \text{Causal.}$$

$y(n) = n x(n)$

$n=0, \; y(0) = 0$
$n=1 \quad y(1) = x(1)$
$n=-1 \quad y(-1) = -x(-1)$
] - Causal

$y(n) = x(n^2)$

$n=0, \; y(0) = x(0)$
$n=2 \quad y(2) = x(4)$
$n=-5 \quad y(-5) = x(25)$
] - Non-causal

$y(n) = x(-n)$

$n=0, \; y(0) = x(0)$
$n=1, \; y(1) = x(-1)$
$n=-1 \quad y(-1) = x(1)$
] - Non-causal

③ Stable & Unstable system

condition for stability of linear time invarient system is given by LTI

$$\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$$

1) $h(n) = 0.2^n \, u(n)$

$u(n) = 1 \quad n > 0$
$\quad\;\; = 0 \quad n < 0$

$$\sum_{0}^{\infty} |0.2^n|$$

$$= \frac{1}{1-0.2} = 1.25 < \infty \Rightarrow \text{Stable system}$$

2) $h(n) = 0.3^n u(n) + 2^n u(n)$ $\quad$ [$u(n) = 1 \quad n > 0$

$$= \sum_{0}^{\infty} |0.3^n| + \sum_{0}^{\infty} 2^n u(n)$$

$$= \frac{1}{1-0.3} + \infty$$

$$= 1.428 + \infty > \not< \infty \Rightarrow \text{unstable}$$

$h(n)$ is unstable.

5) $h(n) = 4^n \cancel{u(n)}\, u(-n)$

$$= \sum_{-\infty}^{0} |4^n|$$

$$= \sum_{0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \frac{1}{1-0.25} = \frac{100}{75} = \underline{\underline{1.333}} < \infty$$

<u>h(n) is stable.</u>

④ $0.2^n u(-n) + 3^n [u(-n)]$

$$= \sum_{-\infty}^{0} |0.2|^n + \sum_{0}^{\infty} \left[\frac{1}{3}\right]^n$$

$$= \sum_{0}^{\infty} \left[\frac{1}{0.2}\right]^n + \frac{1}{1-0.33}$$

$$= \sum [5]^n + 1.49$$

$$= \infty \quad [\text{unstable system}]$$

④ Linear & Non-linear system :-

A system is said to be linear if it satisfies superposition principle.

$$H[a_1 x_1(n) + a_2 x_2(n)] = a_1 H[x_1(n)] + a_2 H[x_2(n)]$$

here a_1 & a_2 are constants.

$x_1(n)$ & $x_2(n)$ are arbitrary signals.

Q. $y(n) = n\,x(n)$

$$x(n) \rightarrow \boxed{H} \rightarrow y(n) = H[x(n)] = n\,x(n)$$

$$x_1(n) \rightarrow \boxed{H} \rightarrow y_1(n) = H[x_1(n)] = n\,x_1(n)$$

$$x_2(n) \rightarrow \boxed{H} \rightarrow y_2(n) = H[x_2(n)] = n\,x_2(n)$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n x_1(n) + a_2 n x_2(n) \quad \text{— (I)}$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$x_3(n) \longrightarrow \boxed{H} \longrightarrow y_3(n) = H[x_3(n)]$$

$$y_3(n) = H[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 n x_1(n) + a_2 n x_2(n)$$

$$\Rightarrow y_3(n) = a_1 y_1(n) + a_2 y_2(n) \quad \text{— (II)}$$

(I) & (II) are equal.

$\Rightarrow$ The given system is linear.

Q. $y(n) = x^2(n)$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n) = H[x(n)] = x^2(n)$$

consider x_1 & x_2

$$x_1(n) \longrightarrow \boxed{H} \longrightarrow y_1(n) = H[x_1(n) = x_1^2(n)$$

$$x_2(n) \longrightarrow \boxed{H} \longrightarrow y_2(n) = H[x_2(n)] = x_2(n)$$

$$\Rightarrow a_1 y_1(n) + a_2 y_2(n) = a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{— (I)}$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$x_3(n) \longrightarrow \boxed{H} \longrightarrow y_3(n) = H[x_3(n)]$$

$$y_3(n) = H[a_1 x_1(n) + a_2 x_2(n)]$$

$$y_3(n) = a_1 x_1^2(n) + a_2 x_2^2(n) \quad \text{— (II)}$$

compare ① & ⑪ $\longrightarrow$ equal.

$\Rightarrow$ the system is <u>linear.</u>

Q $y(n) = e^{x(n)}$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n) = H[x(n)] = e^{x(n)}$$

consider x_1 & x_2,

$$x_1(n) \longrightarrow \boxed{H} \longrightarrow y_1(n) = H[x_1(n)] = e^{x_1(n)}$$

$$x_2(n) \longrightarrow \boxed{H} \longrightarrow y_2(n) = H[x_2(n)] = e^{x_2(n)}$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 e^{x_1(n)} + a_2 e^{x_2(n)} \quad \text{— ①}$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$x_3(n) \longrightarrow \boxed{H} \longrightarrow y_3(n) = H[x_3(n)]$$

$$y_3(n) = H[a_1 x_1(n) + a_2 (x_2(n))]$$

$$y_3(n) = e^{a_1 x_1(n)} + e^{a_2 x_2(n)} \quad \text{— ⑪}$$

① $\neq$ ⑪

the system is Non-linear.

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Q. $y(n) = n x^2(n)$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n) = H[x(n)] = n x^2(n)$$

consider x_1 & x_2

$$x_1(n) \longrightarrow \boxed{H} \longrightarrow y_1(n) = H[x_1(n)] = n x_1^2(n)$$

$$x_2(n) \longrightarrow \boxed{H} \longrightarrow y_2(n) = H[x_2(n)] = n x_2^2(n)$$

$$a_1 y_1(n) + a_2 y_2(n) = a_1 n x_1^2(n) + a_2 n x_2^2(n) \quad \text{(I)}$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$x_3(n) \longrightarrow \boxed{H} \longrightarrow y_3(n) = H[x_3(n)]$$

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

$$y_3(n) = a_1 n x_1^2(n) + a_2 n x_2^2(n) \quad \text{— (I)}$$

$$y_3(n) = H[a_1 x_1(n) + a_2 x_2(n)]$$

$$= a_1 n x_1^2(n) + a_2 n x_2^2(n)$$

$$= n[a_1 x_1(n) + a_2 x_2(n)]^2$$

$$= n[a_1 x_1(n)]^2 + n[a_2 x_2(n)]^2 + 2n[a_1 x_1(n)][a_2 x_2(n)]$$

$\hfill \hookrightarrow$ (II)

(I) $\neq$ (II)

$\Rightarrow$ The system is non-linear

* Convolution theorem:

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$

the above equation gives the response y(n) of the LTI system as a function of input signal x(n) & unit sample (h(n)) is called convolution sum.

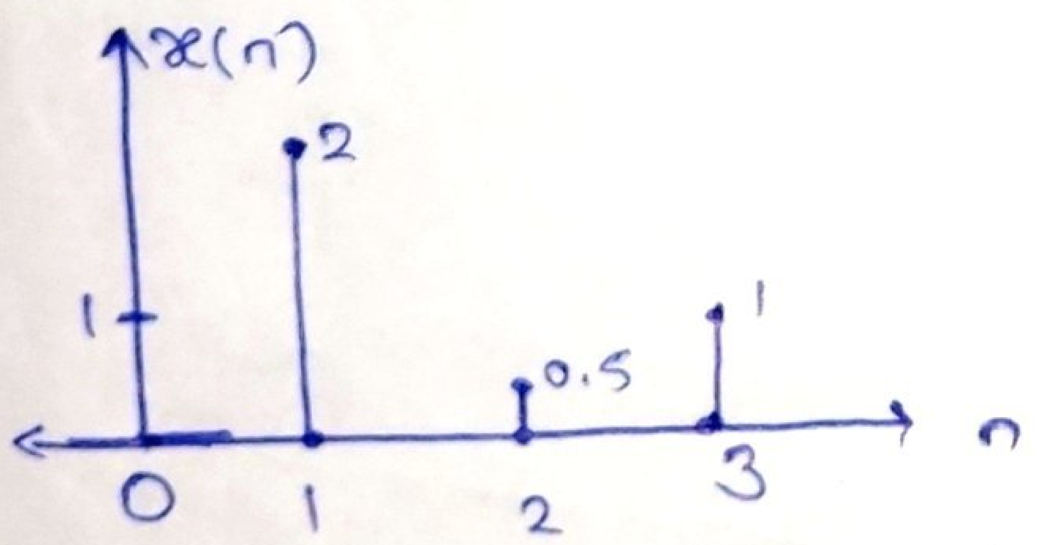
$$\boxed{y(n) = x(n) * h(n)}$$

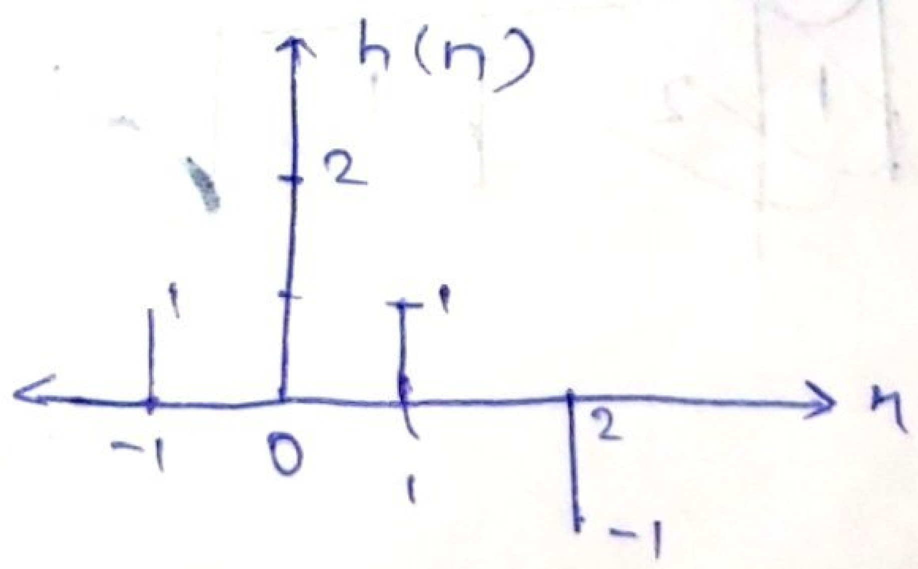
Process:
→ folding
→ shifting
→ multiplication
→ Summation

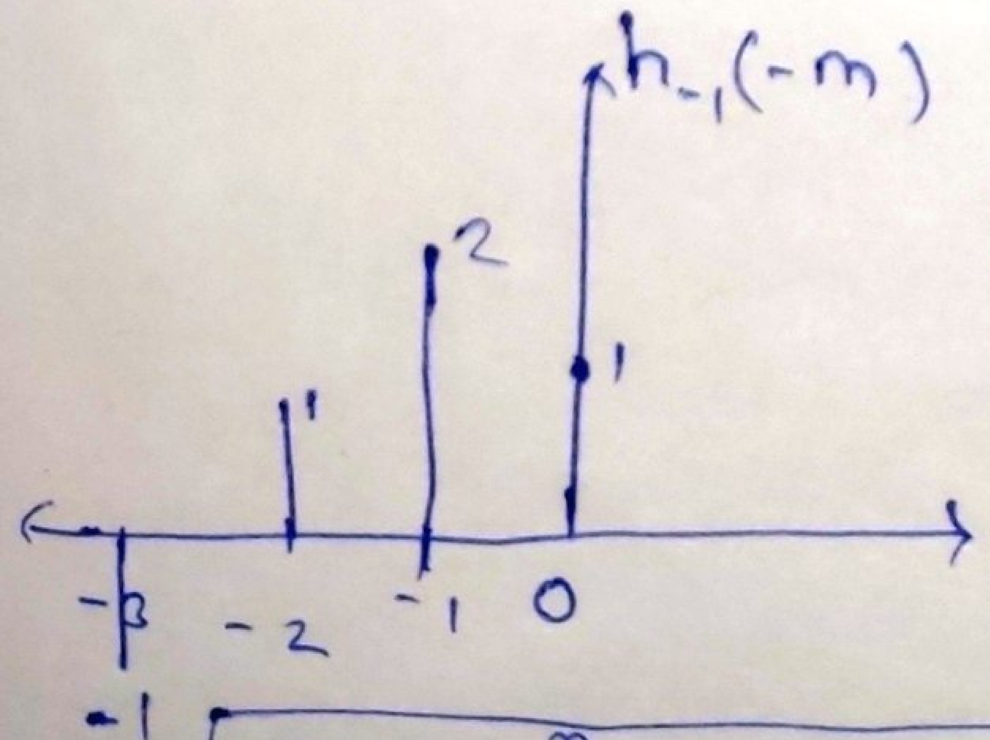
Q. Determine the response of LTI system whose input is $x(n) = \{\underset{\uparrow}{1}, 2, 0.5, 1\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$ [8 to 10 marks]

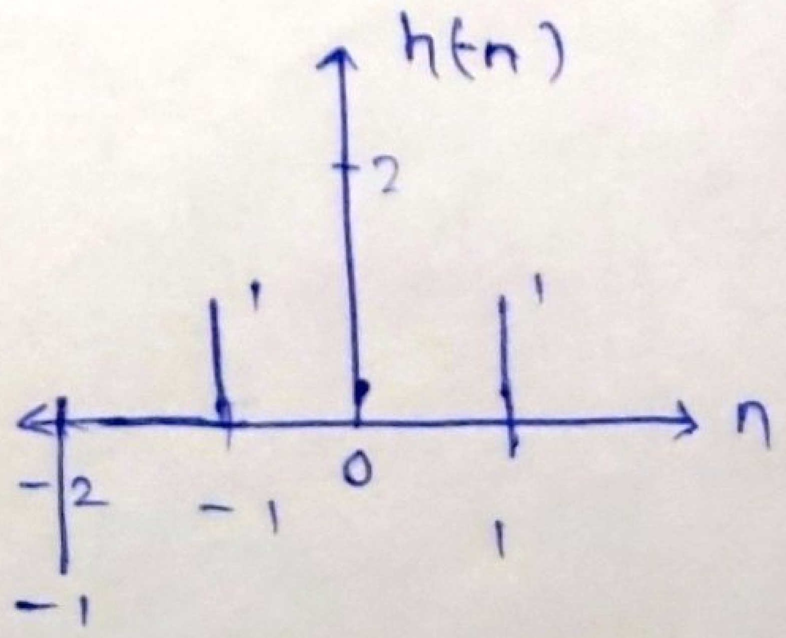
Soln:

$x(n) \rightarrow L_1 \rightarrow 4$ samples
$h(n) \rightarrow L_2 \rightarrow 4$ samples
$y(n) \rightarrow L_1 + L_2 - 1 \rightarrow 7$ samples.







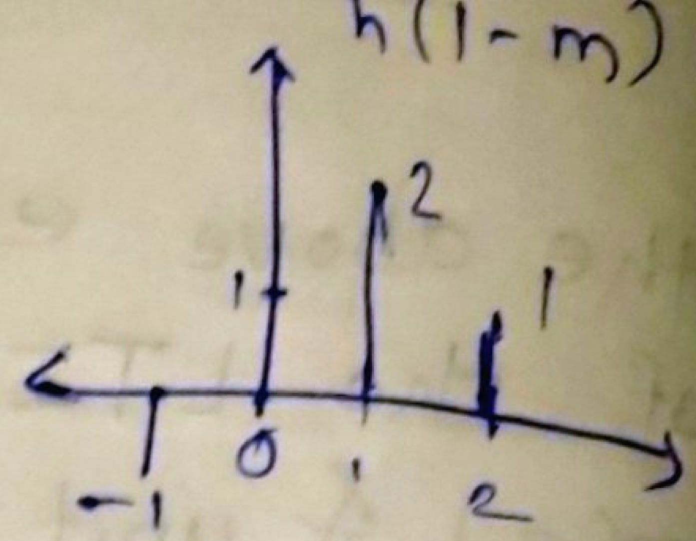


$$\boxed{y(n) = \sum_{-\infty}^{\infty} x(k) h(n-k)}$$

$n = -1,\ y(-1) = 1$

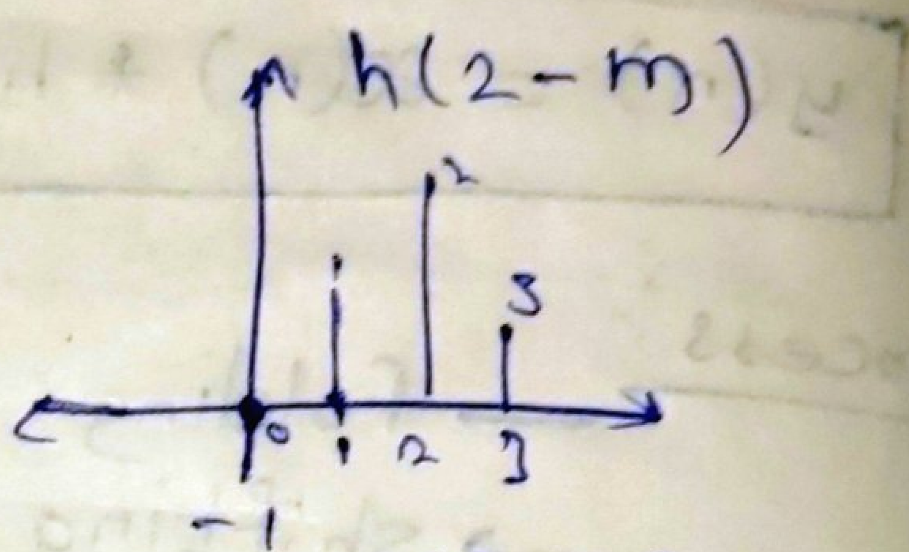
$n=0, \quad y(0) = \sum_{-\infty}^{\infty} x(m) * h(-m)$

$y(0) = 2\times1 + 2\times1 = 4$

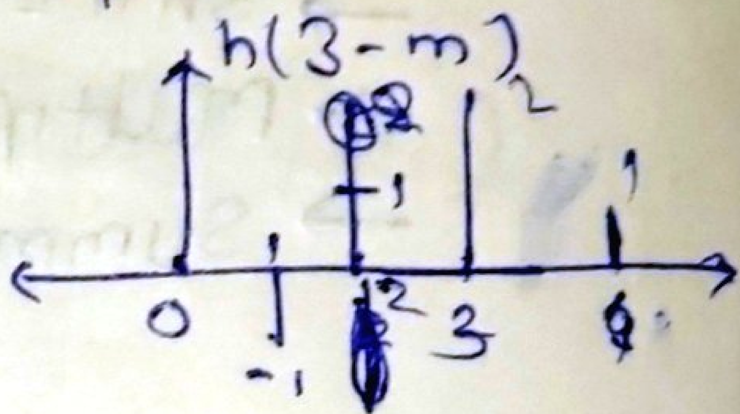


$n=1, \quad y(1) = 1\times1 + 2\times2 + 1\times0.5$
$= 5.5$

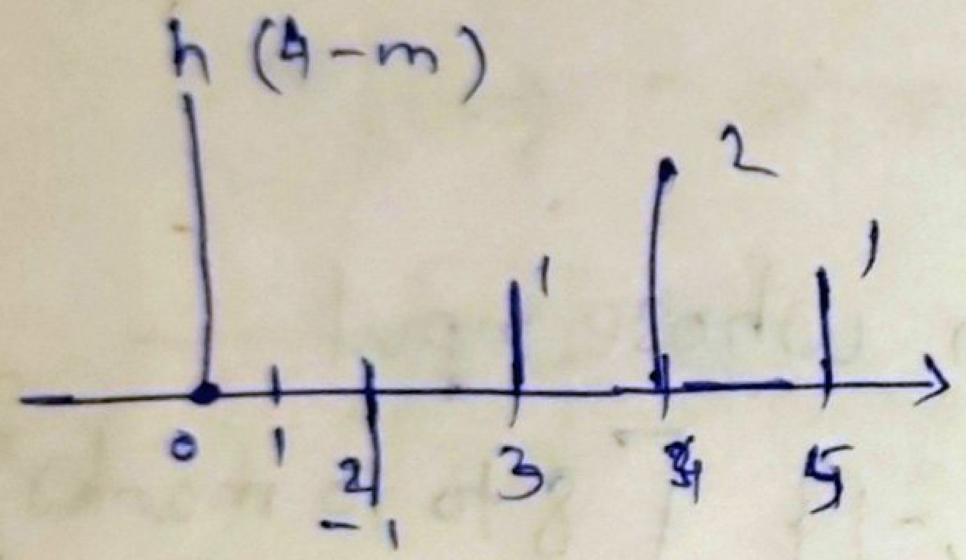
$n=2, \quad y(2) = -1\times1 + 2\times1 + 0.5\times2 + 1\times1$
$= 3$



$n=3, \quad y(3) = 2\times-1 + 1\times1$
$= -2+1$
$= -1$



$n=4, \quad y(4) = 2\times-1 + 0.5\times1 + 2\times1$
$= 0.5$



$x(n)$ \ $h(n)$	1	2	1	-1
1	1	2	1	-1
2	2	4	2	-2
0.5	0.5	1	0.5	-0.5
1	1	2	1	-1

~~correct~~ linear convolution

$x(n) = \{\underset{\uparrow}{1}, 2, 0.5, 2\}$

$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

$y(n) = \{1, \underset{\uparrow}{4}, 5.5, 3, 0.5, 0.5\}$

m	-3	-2	-1	0	1	2	3	4		
x(m)				1						
h(m)				2						
h(-m)			1	2						
h(-m)		-1	1	1						
	-1	1	2	1						
h(1-m)			-1	-1	2	1				
h(2-m)					1	2	1			
h(3-m)					-1	1	2	1		
h(4-m)						-1	1	2	1	
h(5-m)							-1	1	2	1

	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1
1	1	2	1	1

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$x(n) = \{2, \underset{\uparrow}{3}, 4, 5, 6\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)$$

m	-4	-3	-2	-1	0	1	2	3	4	5
x(m)				2	3	4	5	6		
h(m)				1	2	1	-1			
h(-m)			-1	1	2	1				
h(-1-m)		-1	1	2	1					
h(-2-m)	-1	1	2	1						
h(1-m)				-1	1	2	1			
h(2-m)					-1	1	2	1		
h(3-m)						-1	1	2	1	

$n = 0$, $y(0) = \sum x(m)\, h(0-m)$

$= 2\times1 + 2\times3 + 4\times1$

$= 12$

$n = -1$, $y(-1) = \sum x(m)\, h(-1-m)$

$= 4 + 3 = 7$

$n(-2)$ $y(-2-m) = 2$

$n(1);\ y(1-m) = -2 + 3 + 8 + 5 = 14$

$n(2);\ y(2-m) = -3 + 4 + 10 + 6 = 17$

$n(3);\ y(3-m) = -4 + 5 + 12 = 13$

$n(4);\ y(4-m) = -5 + 6 = 1$

$n(5);\ y(5-m) = -6$

$\therefore\ y(n) = \{2, 7, \underset{\uparrow}{12}, 14, 17, 13, 1, -6\}$

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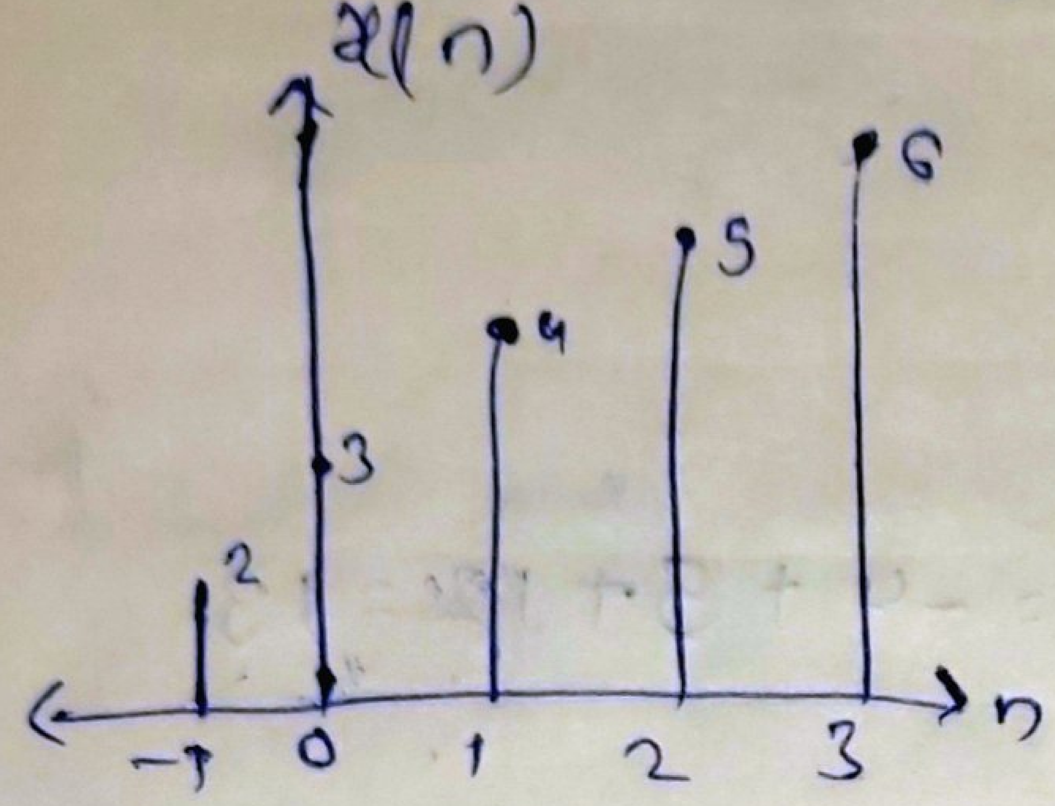
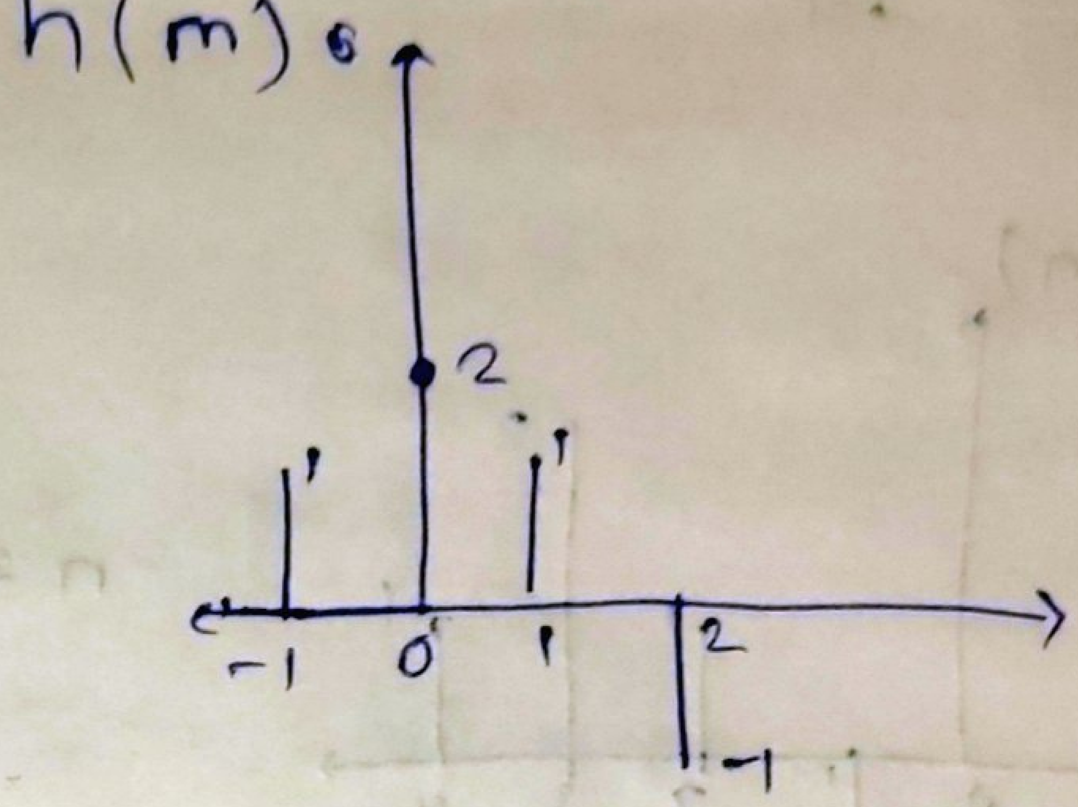
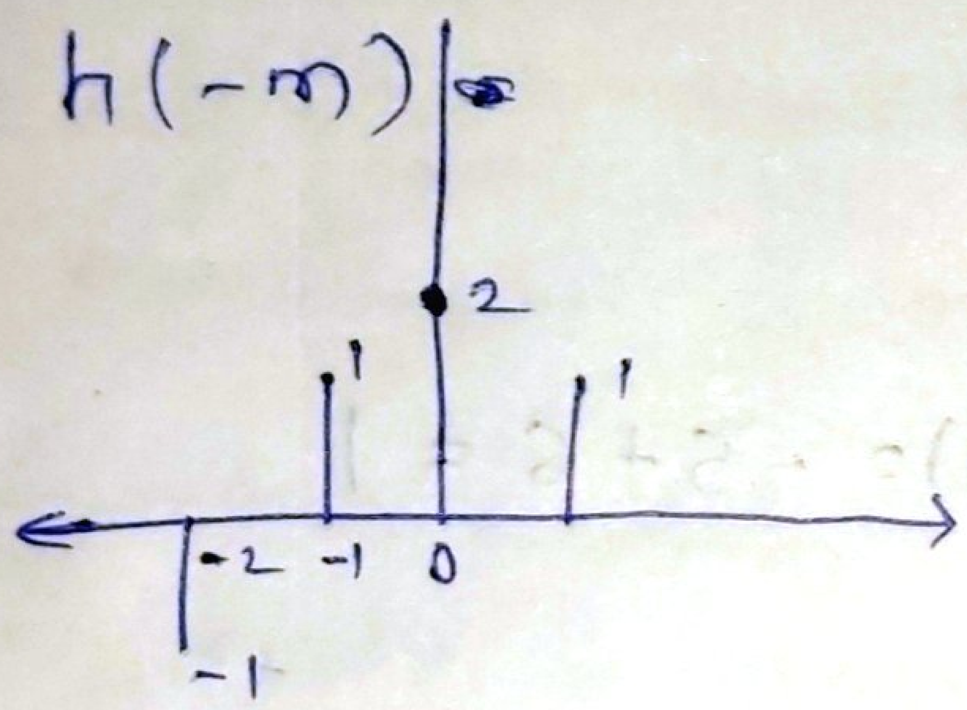
Graphical Method :-

$x(n) = \{2, \underset{\uparrow}{3}, 4, 5, 6\}$

$h(m) = \{1, \underset{\uparrow}{2}, 1, -1\}$

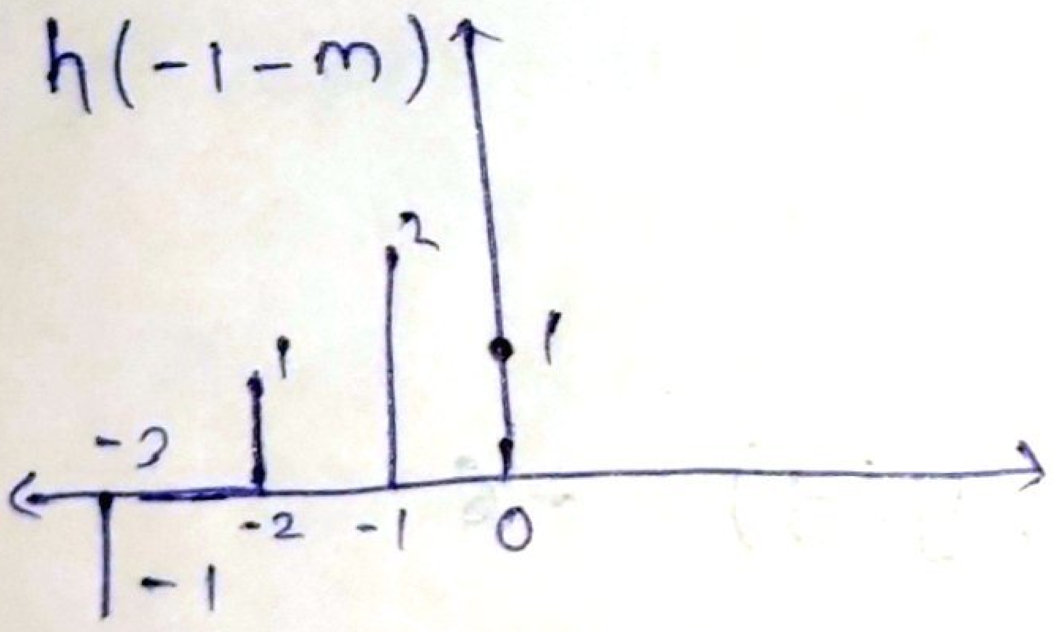






$n = 0, \; y(0) = 2 + 2\times 3 + 4$
$= 12$

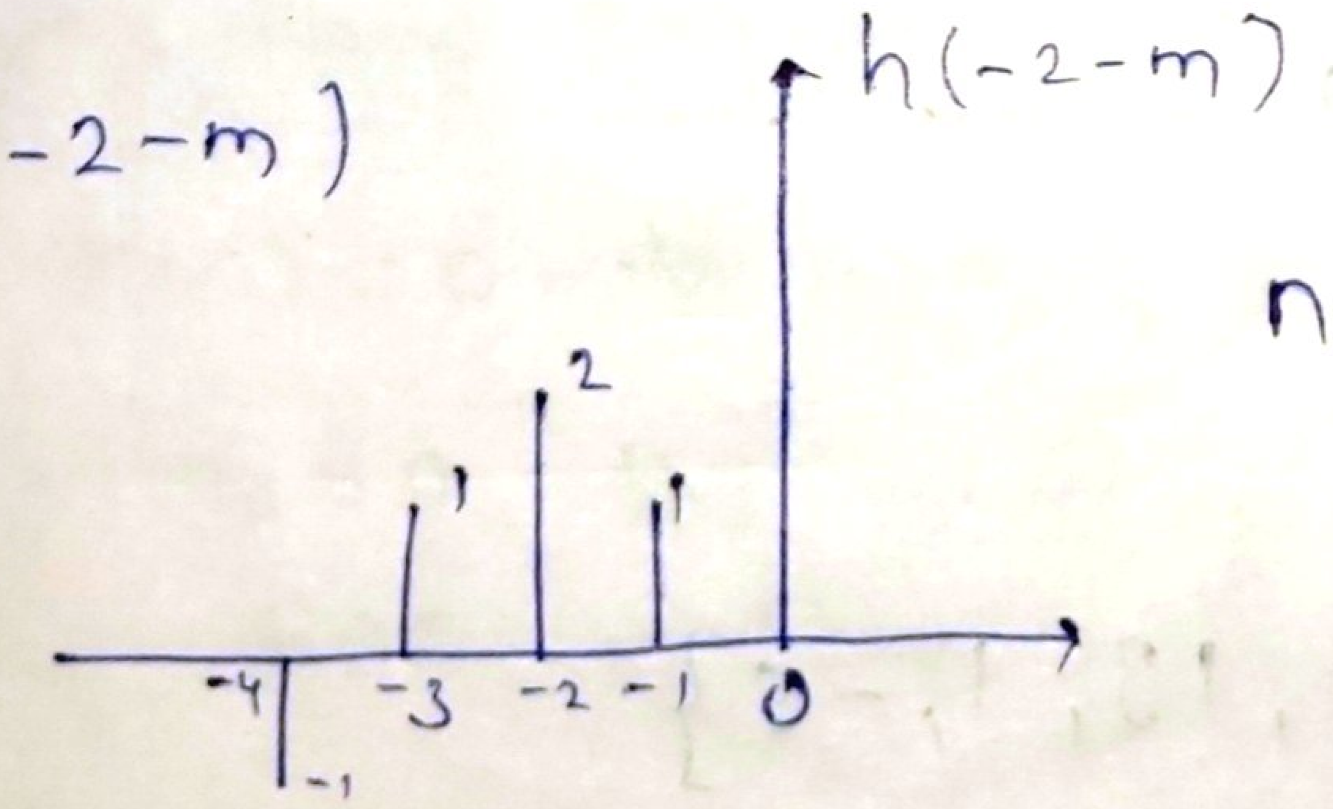
~~n = -1~~



$n = -1, \; y(-1) = 4 + 3 = 7$

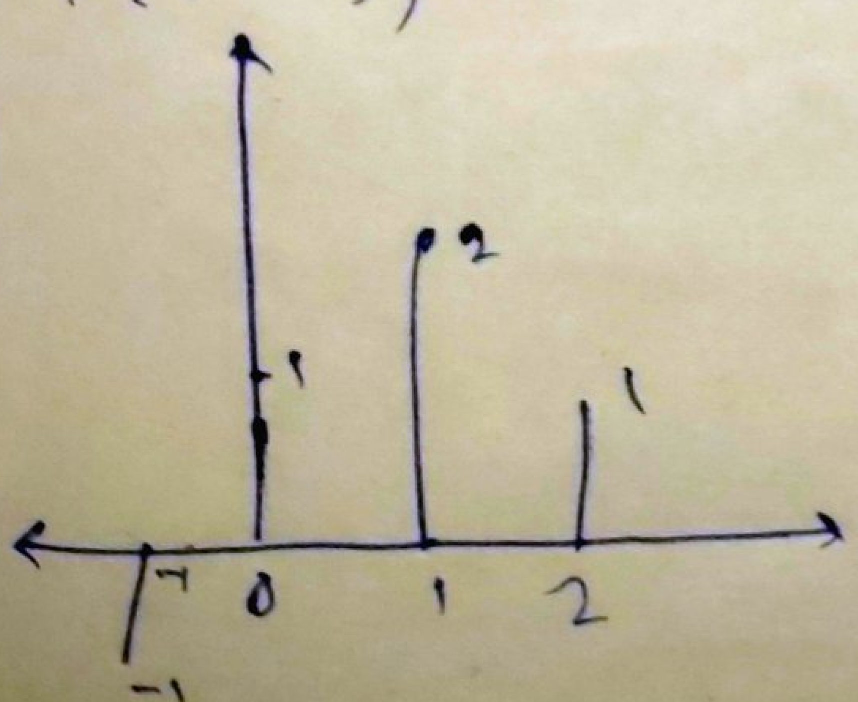
~~h(-1-m)~~

h(-2-m)

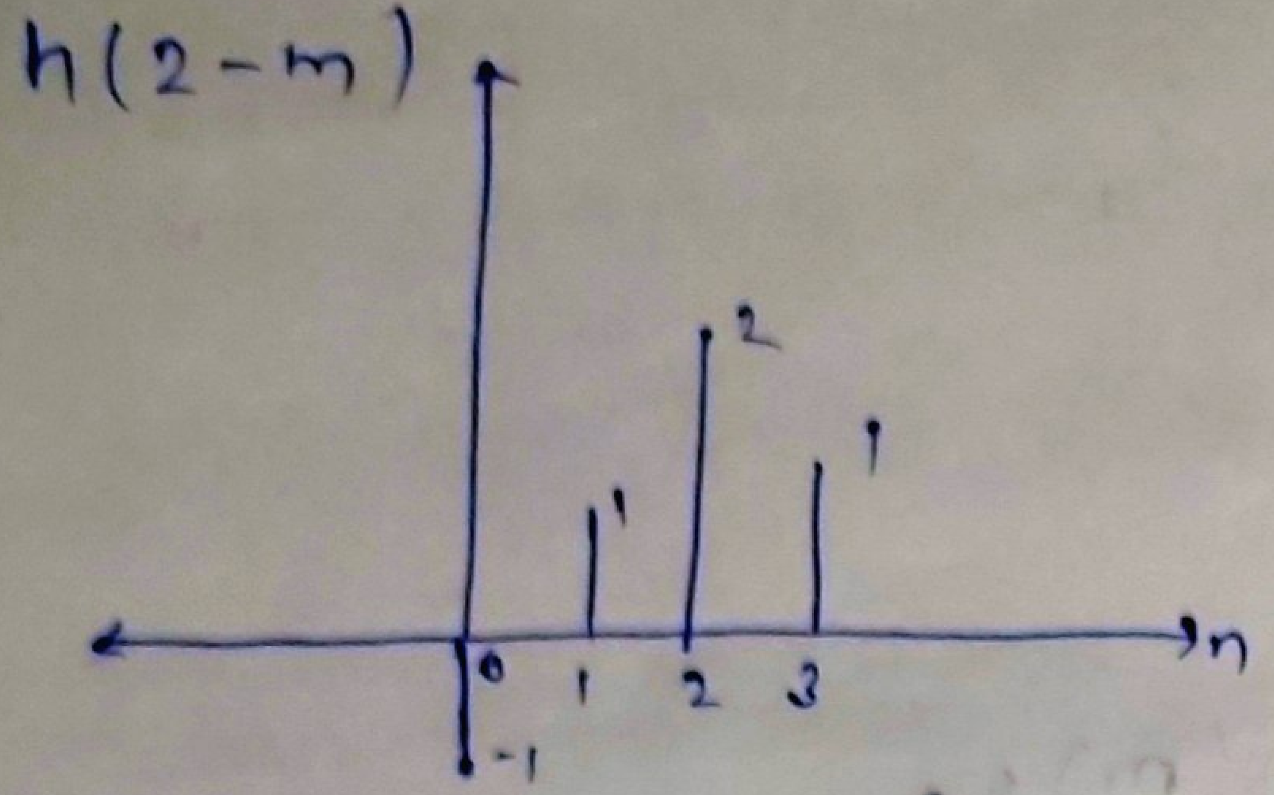


$n = -2, \; y(-2) = 2$

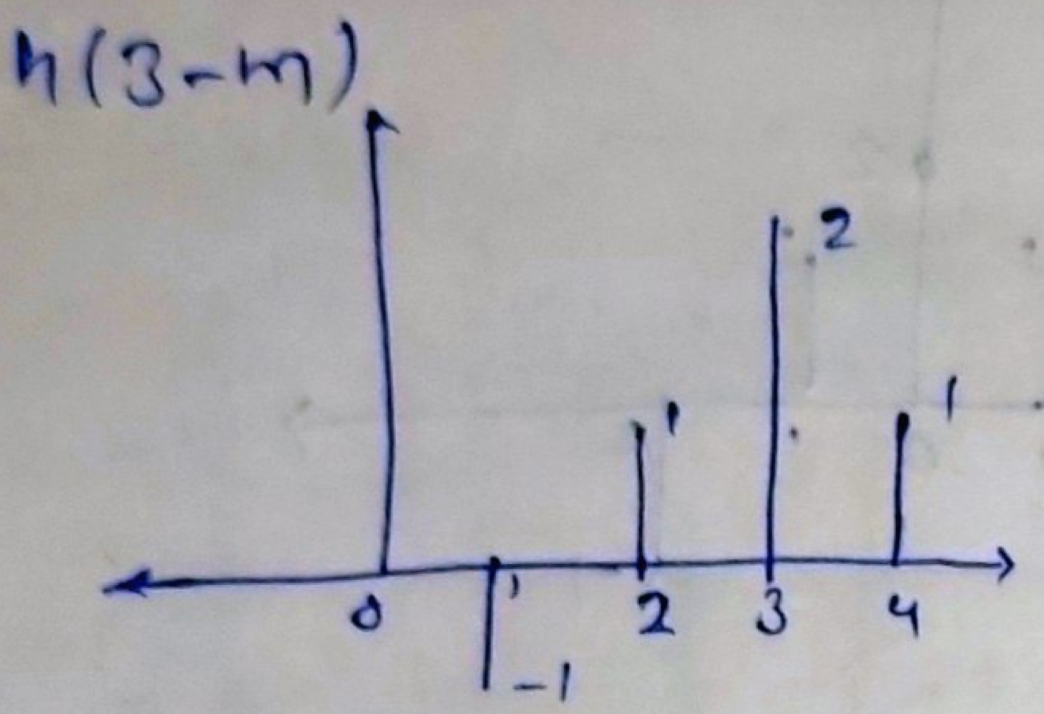
h(1-m)



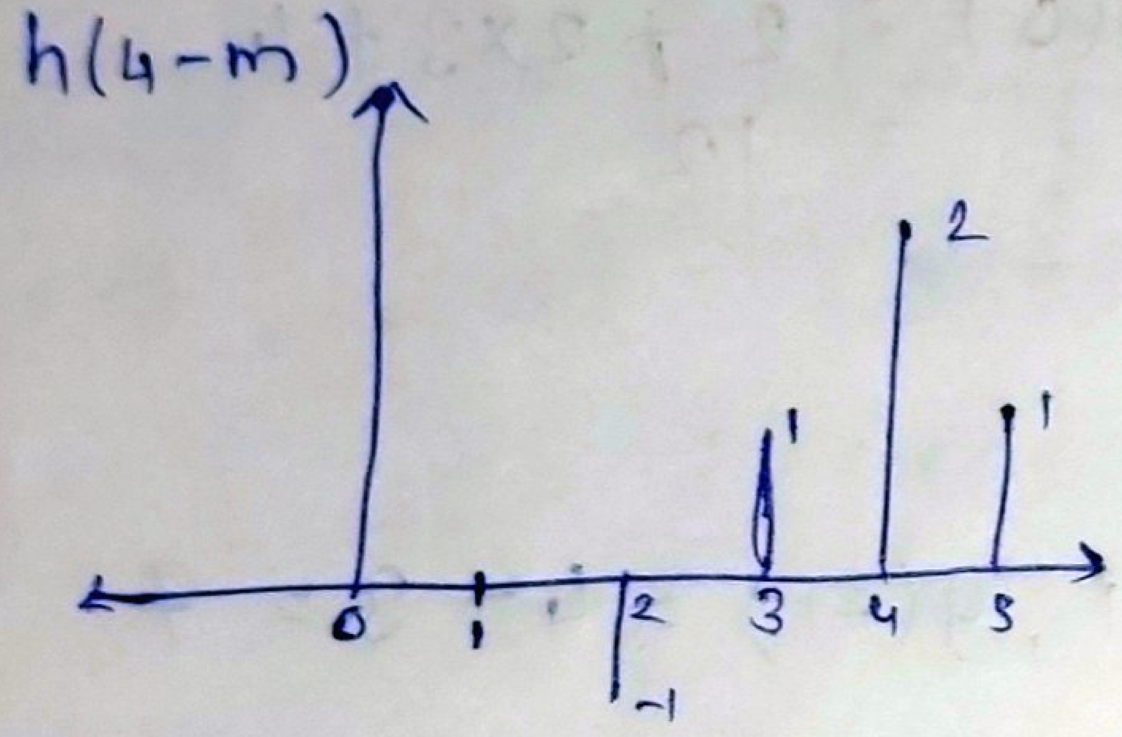
$n(1), \; y(1) = -2 + 3 + 8 + 5 = 14$



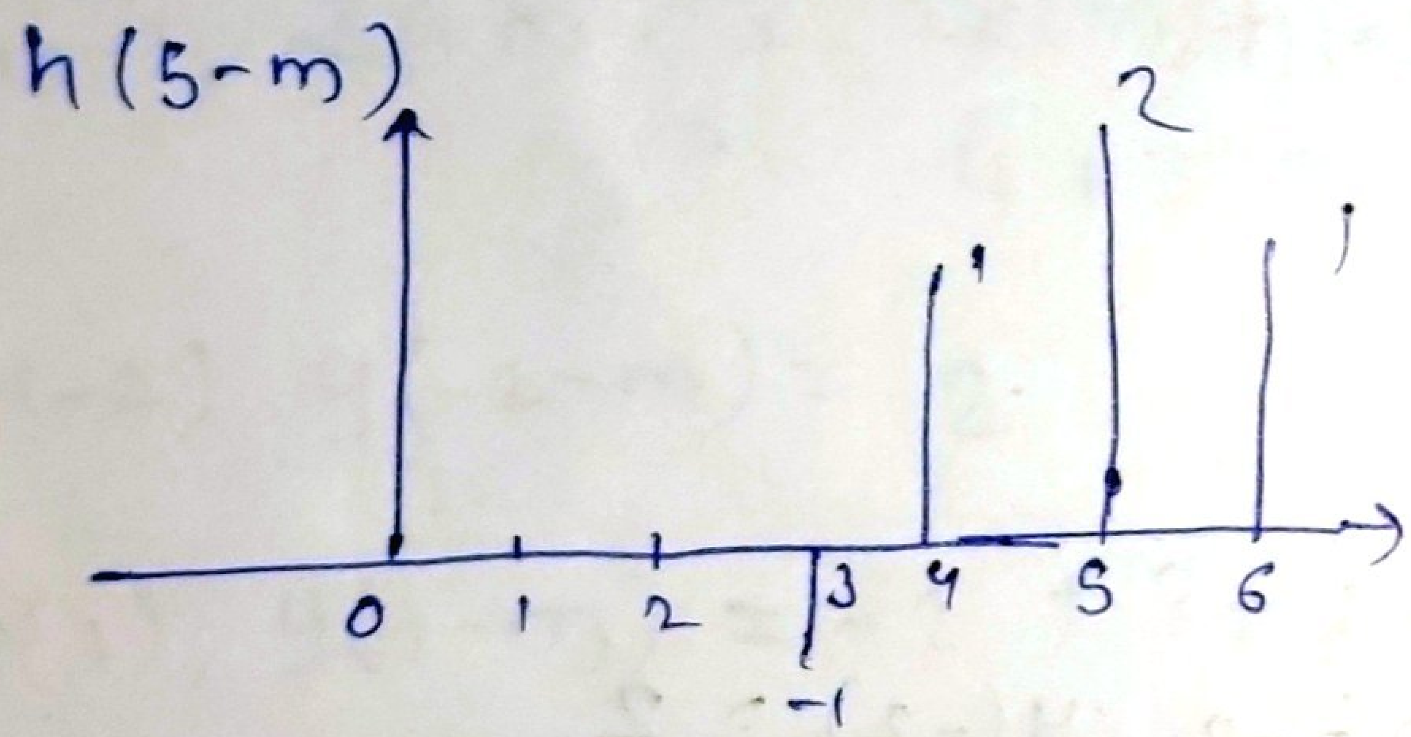
$n=2,\ y(2) = -3+4+10+6$
$= 17$



$n=3,\ y(3) = -4+5+12 = 13$



$n=4,\ y(4) = -5+6 = 1$



$n=5,\ y(5) = -6$

$$y(n) = \{2, 7, \underset{\uparrow}{12}, 14, 17, 13, 1, -6\}$$