Table 3.3 : Summary of Properties of $\mathcal{Z}$-Transform

Note : $X(z) = \mathcal{Z}\{x(n)\}$; $X_1(z) = \mathcal{Z}\{x_1(n)\}$; $X_2(z) = \mathcal{Z}\{x_2(n)\}$; $Y(z) = \mathcal{Z}\{y(n)\}$

Property		Discrete time signal	$\mathcal{Z}$-transform
Linearity		$a_1 x_1(n) + a_2 x_2(n)$	$a_1 X_1(z) + a_2 X_2(z)$
Shifting ($m \geq 0$)	$x(n)$; for $n \geq 0$	$x(n-m)$	$z^{-m} X(z) + \sum_{i=1}^{m} x(-i)\, z^{-(m-i)}$
		$x(n+m)$	$z^{m} X(z) - \sum_{i=0}^{m-1} x(i)\, z^{m-i}$
	$x(n)$; for all n	$x(n-m)$	$z^{-m} X(z)$
		$x(n+m)$	$z^{m} X(z)$
Multiplication by n^m (or differentiation in z-domain)		$n^m x(n)$	$\left(-z \frac{d}{dz}\right)^m X(z)$
Scaling in z-domain (or multiplication by a^n)		$a^n x(n)$	$X(a^{-1}z)$
Time reversal		$x(-n)$	$X(z^{-1})$
Conjugation		$x^*(n)$	$X^*(z^*)$
Convolution		$x_1(n) * x_2(n) = \sum_{m=-\infty}^{+\infty} x_1(m)\, x_2(n-m)$	$X_1(z) X_2(z)$
Corrrelation		$r_{xy}(m) = \sum_{m=-\infty}^{+\infty} x(n)\, y(n-m)$	$X(z) Y(z^{-1})$
Initial value		$x(0) = \lim_{z \to \infty} X(z)$	
Final value		$x(\infty) = \lim_{z \to 1} (1 - z^{-1}) X(z)$ $= \lim_{z \to 1} \frac{(z-1)}{z} X(z)$ if $X(z)$ is analytic for $\lvert z \rvert > 1$	
Complex convolution theorem		$x_1(n) x_2(n)$	$\frac{1}{2\pi j} \oint_C X_1(v)\, X_2\left(\frac{z}{v}\right) v^{-1}\, dv$
Parseval's relation		$\sum_{n=-\infty}^{+\infty} x_1(n)\, x_2^*(n) = \frac{1}{2\pi j} \oint_C X_1(z)\, X_2^*\left(\frac{1}{z^*}\right) z^{-1}\, dz$	