Filter Design:- IIR

IIR → filter (infinite impulse response) → accurate, → stable

FIR → filter (finite impulse response) → not accurate, → unstable

* IIR → infinite bz of feedback

* Analog IIR filters
  - → Butterworth → smooth on both pass & stop band
  - → chebyshev 1 → ripples on pass band & smooth on stop band
  - → chebyshev 2 → smooth on pass band & ripples on stop band
  - → Elliptic filters → ripples on both stop & pass band

* $H(j\Omega) \longrightarrow H(s) \to$ Analog filter

  ↓ → bilinear transformation, IIT

  $H(z) \to$ Digital filter

*

$$|H(j\Omega)| = \frac{1}{\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}}$$

↓

magnitude response of lowpass butterworth fn.

$N \to$ order of filter

$\Omega_c \to$ cutoff frequency.

→ $|H(j\Omega)| = 1$ for all N

→ $|H(j\Omega)| = \frac{1}{\sqrt{2}}$ at $\Omega = \Omega_c$; $20 \log |H(j\Omega_c)| = -3.01\,dB$

→ $|H(j\Omega)| = 0$ as $\Omega \to \infty$

→ $|H(j\Omega)|$ is monotonically decreasing fn of freq

$|H(j\Omega_2)| < |H(j\Omega_1)|$

Butterworth polynomial:-

$\Omega_c = 1 \rightarrow$ normalized.

The magnitude-squared freq response

$$|H_N(j\Omega)|^2 = \frac{1}{1+\Omega^{2N}} \quad \text{--- (1)}$$

$$\text{if } H_N(j\Omega)\, H_N(-j\Omega) = \frac{1}{1+\Omega^{2N}} \quad \text{--- (2)}$$

$j\Omega = s$ replace

$$H_N(s)\, H_N(-s) = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}} \quad \text{--- (3)}$$

eq^n (3) doesn't have zeros.

Poles $\Rightarrow 1+\left(\frac{s}{j}\right)^{2N} = 0 \Rightarrow \frac{s}{j} = (-1)^{1/2N}$

$s = j(-1)^{1/2N} \longrightarrow$ poles.

$-1 = e^{j\pi(2k+1)}$, $k = 0, 1, \ldots, 2N-1$

$j = e^{j\pi/2}$

** $$\boxed{s_k = e^{j\pi\frac{(2k+1)}{2N}} \cdot e^{j\pi/2}}$$

* $$\boxed{H_N(s) = \frac{1}{\pi_{LHP}(s-s_k)} = \frac{1}{B_N(s)}}$$

(transfer fn.)

order (N)	Butterworth Polynomial
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.7$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

* Design of lowpass Butterworth filter

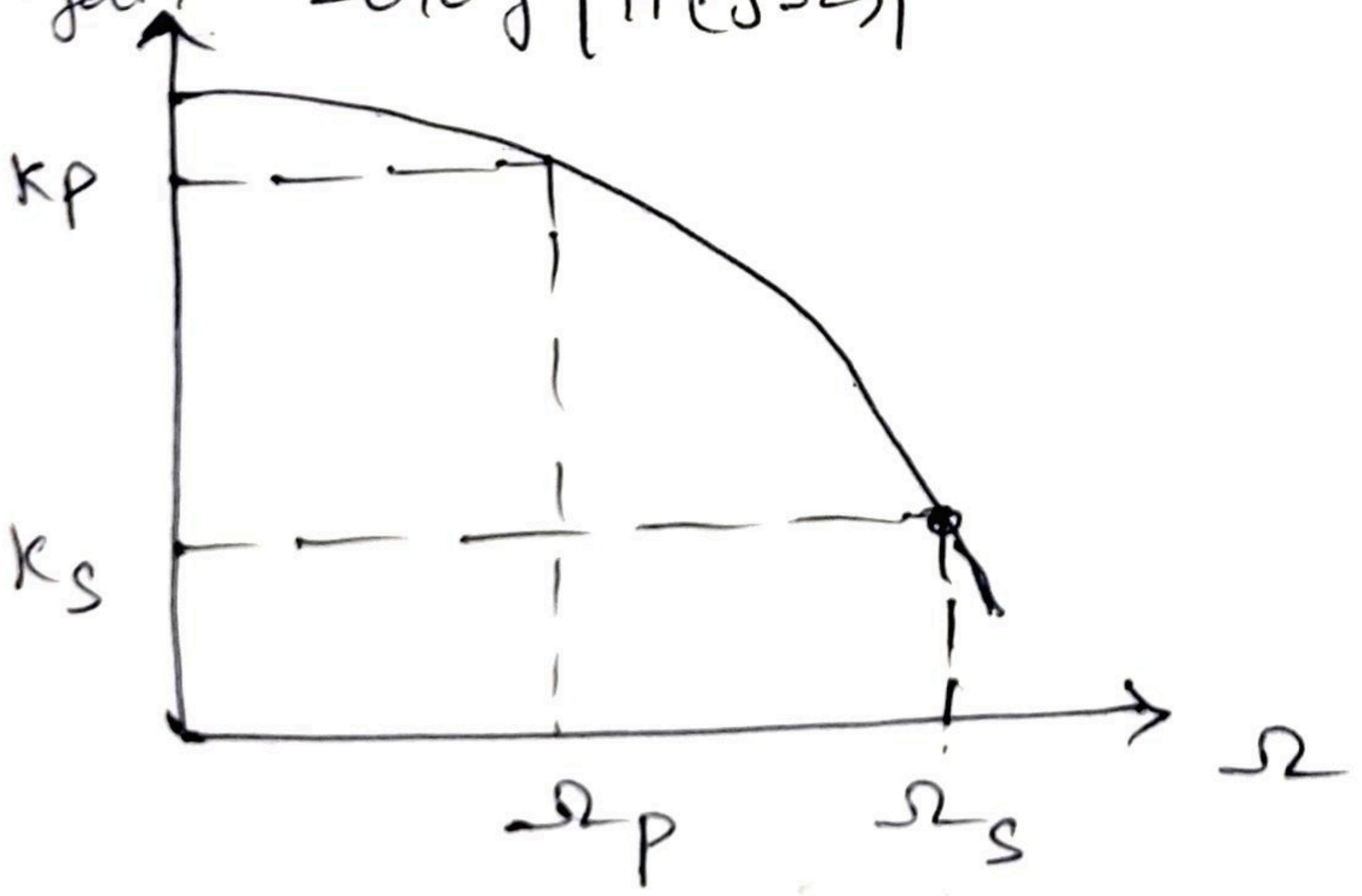
$K_s \rightarrow$ stop band gain

$K_p \rightarrow$ pass band gain

$(\Omega_s)\,\omega_s \rightarrow$ stop band edge frequency

$(\Omega_p)\,\omega_p \rightarrow$ pass band edge frequency.

gain $\rightarrow 20\log|H(j\Omega)|$



$K_p \leq 20\log|H(j\Omega)| \leq 0$ for all $\Omega \leq \Omega_p$

$K_s \geq 20\log|H(j\Omega)|$ for all $\Omega \geq \Omega_s$

Pass band ripple in dB $= A_p = -20\log(1-\delta_p)$

$\delta_p \rightarrow$ pass band tolerance.

Pass band gain at $\Omega = \Omega_p \Rightarrow K_p = -A_p$

Stop band attenuation $= A_s = -20\log(\delta_s)$

$\delta_s \rightarrow$ stop band tolerance

Stop band gain at $\Omega = \Omega_s$, $K_s = -A_s$

$= 20\log(\delta_s)$

** order of filter $= \boxed{N = \dfrac{\log\left[\left(10^{\frac{-K_p}{10}}-1\right)\Big/\left(10^{\frac{-K_s}{10}}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}}$

* $\boxed{\Omega_c = \dfrac{\Omega_p}{\left(10^{\frac{-K_p}{10}}-1\right)^{1/2N}}}$

$\boxed{\Omega_c = \dfrac{\Omega_s}{\left(10^{\frac{-K_s}{10}}-1\right)^{1/2N}}}$

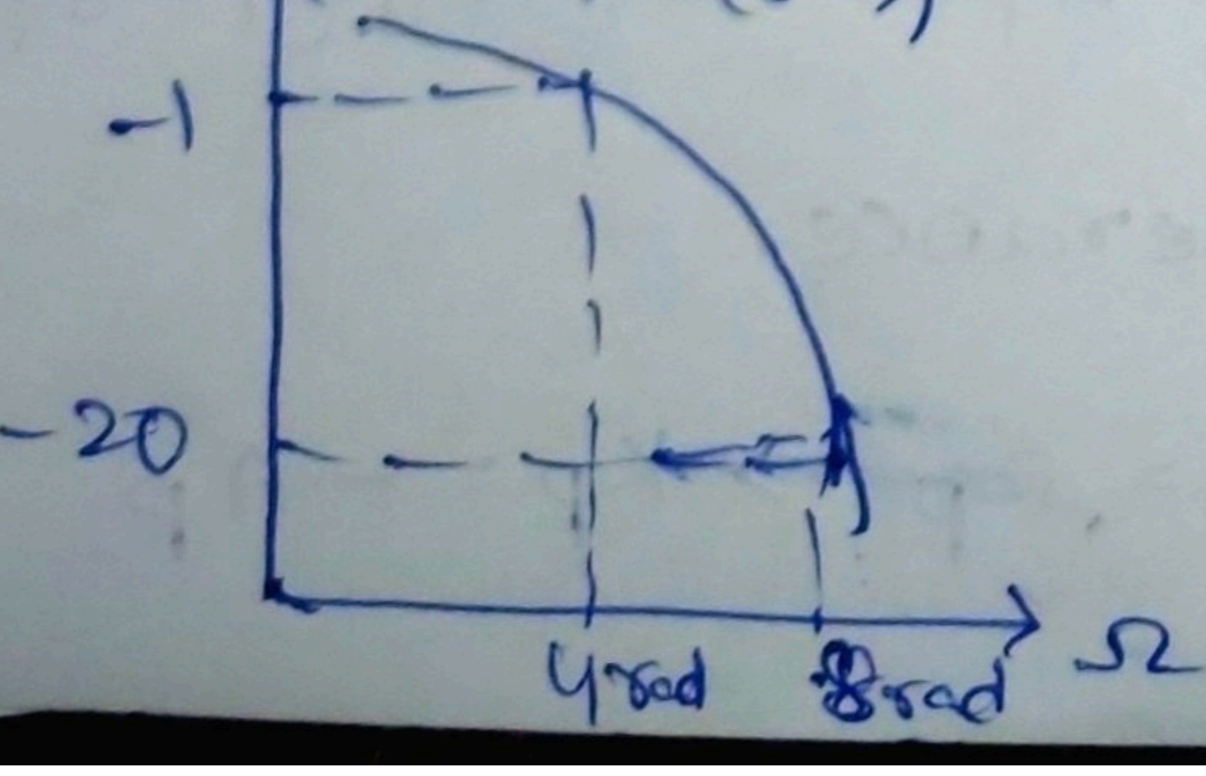
$K_p = -1\,dB$

$\Omega_p = 4\,rad/sec$

$\Omega_s = 8\,rad/sec$

$A_s \geq 20\,dB$; $K_s = -A_s$; $K_s = -20\,dB$

step-1



Step-2

normalized pass band edge frequency

$\Omega_p = 1$ for normalized low pass filter.

$\Omega_s = \frac{\Omega_s}{\Omega_p} = \frac{8}{4} = 2$ for normalized low pass filter

Note:-

* for lowpass $\Rightarrow \Omega_s = \frac{\Omega_s}{\Omega_p}$

* for Highpass $\Rightarrow \Omega_s = \frac{\Omega_p}{\Omega_s}$

* for bandpass $\Rightarrow \Omega_s = \min\{|A|, |B|\}$

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)} \; ; \; B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$$

* for bandstop $\Rightarrow \Omega_s = \min\{|A|, |B|\}$

$$A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u} \; ; \; B = \frac{\Omega_2(\Omega_u - \Omega_l)}{\Omega_2^2 - \Omega_l \Omega_u}$$

$\Omega_p = 1$ for every filter.

Step-3

$$N = \frac{\log\left[\left(10^{-\frac{k_p}{10}} - 1\right) / \left(10^{-\frac{k_s}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$k_p = -1\,dB$

$k_s = -20\,dB$

$\Omega_p = 1$ rad/sec

$\Omega_s = 2$ rad/sec

$N = 4.289 \approx 5$; $N = \dfrac{\log\left[(10^{\cancel{kp}\frac{1}{10}} - 1)/(10^{2} - 1)\right]}{2\log\left(\frac{1}{2}\right)} = \dfrac{\log\left(\frac{0.2589}{99}\right)}{-\cancel{0}0.602}$

$N = \dfrac{-2.582}{-0.602} = 4.289$

step-4

$$S_k = e^{\frac{j\pi(2k+1)}{2N}} \cdot e^{j\pi/2}$$

S_0 to S_9 but ~~so~~ we need 5 poles.

$k = 0, 1, 2, 3, 4$

$N = 5.$

Step-5

$$H_N(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)(s-s_3)(s-s_4)}$$

$$H_N(s) = \frac{1}{(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)}$$

step-6

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-kp}{10}}-1\right)^{1/2N}} = \frac{4}{\left(10^{+\frac{1}{10}}-1\right)^{1/10}} \neq \cancel{4.5787}\ 6.2184$$

$$= \frac{4}{\left(10^{0.1}-1\right)^{0.1}} = 4.5787$$

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Step - 7

$$H_a(S) = H_5(S)\Big|_{S \to \frac{S}{\Omega_c = 4.5787}}$$

$$= \frac{1}{\left(\frac{S}{4.57}+1\right)\cdot\left(\left(\frac{S}{4.57}\right)^2+0.6180\left(\frac{S}{4.57}\right)+1\right)\cdot\left(\left(\frac{S}{4.57}\right)^2+1.6180\left(\frac{S}{4.57}\right)+1\right)}$$

Note:-

* for lowpass :→ $S \to S/\Omega_c$
* for Highpass : $S \to \frac{\Omega_c}{S}$
* for bandpass : $S \to \frac{S^2+\Omega_u\Omega_l}{S(\Omega_u-\Omega_l)}$
* for band stop : $S \to \frac{S(\Omega_u-\Omega_l)}{S^2+\Omega_u\Omega_l}$

#

−3.0103 dB lower and upper

Design a low pass butterworth filter with 3dB attenuation at pass band frequency of 1.5KHZ, 10dB stopband attenuation at frequ of 3KHZ.

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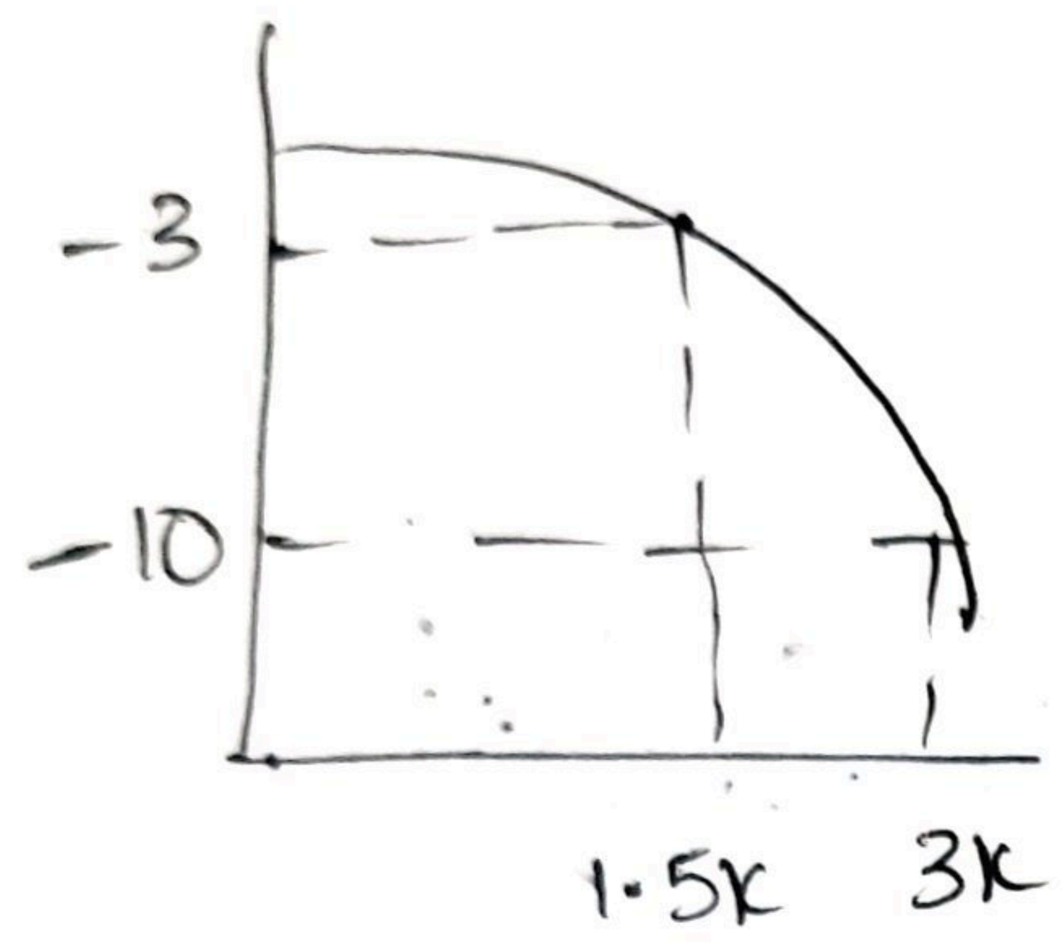
Step-1



$K_p = -3 \quad \Omega_p = 1.5k$

$K_s = -10 \quad \Omega_s = 3k$

Step-2

normalization.

$\Omega_p = 1 \; , \; \Omega_s = \frac{\Omega_s}{\Omega_p} = \frac{3}{1.5} = 2$

$\Omega_s = 2$

Step-3

$$N = \frac{\log\left(\left(10^{\frac{-K_p}{10}} - 1\right) / \left(10^{\frac{-K_s}{10}} - 1\right)\right)}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left(\left(10^{\frac{3}{10}} - 1\right) / \left(10^{1} - 1\right)\right)}{2\log\left(\frac{1}{2}\right)} = \frac{\log(0.99/9)}{-0.602}$$

$$= \frac{\log(0.11)}{-0.602} = \frac{-0.958}{-0.602} = 1.592$$

$N = 1.592 = 2$

step-4

$$S_K = e^{j\pi\left(\frac{2K+1}{2N}\right)} \cdot e^{j\pi/2}$$

$2\times2+1$ $\frac{3\times3.14}{4}$ $\Rightarrow S_0\ S_1\ S_2\ S_3\ S_4$

S_0 to S_5 but we need only S_0, S_1, S_2

~~step-5~~ $S_0 = e^{j\pi\left(\frac{1}{4}\right)} \cdot e^{j\pi/2} = e^{j(3\pi/4)}$

$$= \cos 3\pi/4 + j\sin 3\pi/4 = -\frac{1}{\sqrt{2}} + j/\sqrt{2}$$

$$S_1 = e^{j\pi\left(\frac{3}{4}\right)} \cdot e^{j\pi/2} = e^{j\pi(5/4)}$$

$$= -\frac{1}{\sqrt{2}} - j/\sqrt{2}$$

$$S_2 = e^{j\pi\left(\frac{5}{4}\right)} \cdot e^{j\pi/2} = e^{j\pi(7/4)}$$

$$= \cos 7\pi/4 + j\sin 7\pi/4$$

$$= \frac{1+j}{\sqrt{2}}$$

step-5

$$H_N(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)} = \frac{1}{\left(s-\frac{(j-1)}{\sqrt{2}}\right)\left(s+\frac{(1+j)}{\sqrt{2}}\right)\left(s-\frac{(1+j)}{\sqrt{2}}\right)}$$

$$= \frac{1}{s^2+\sqrt{2}s+1}$$

step-6

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-k_p}{10}}-1\right)^{1/2N}} = \frac{1.5K}{\left(10^{3/10}-1\right)^{1/4}} = \frac{1.5K}{(0.99)^{1/4}} = \frac{1.5}{0.997}$$

$$\Omega_c = 1.504K$$

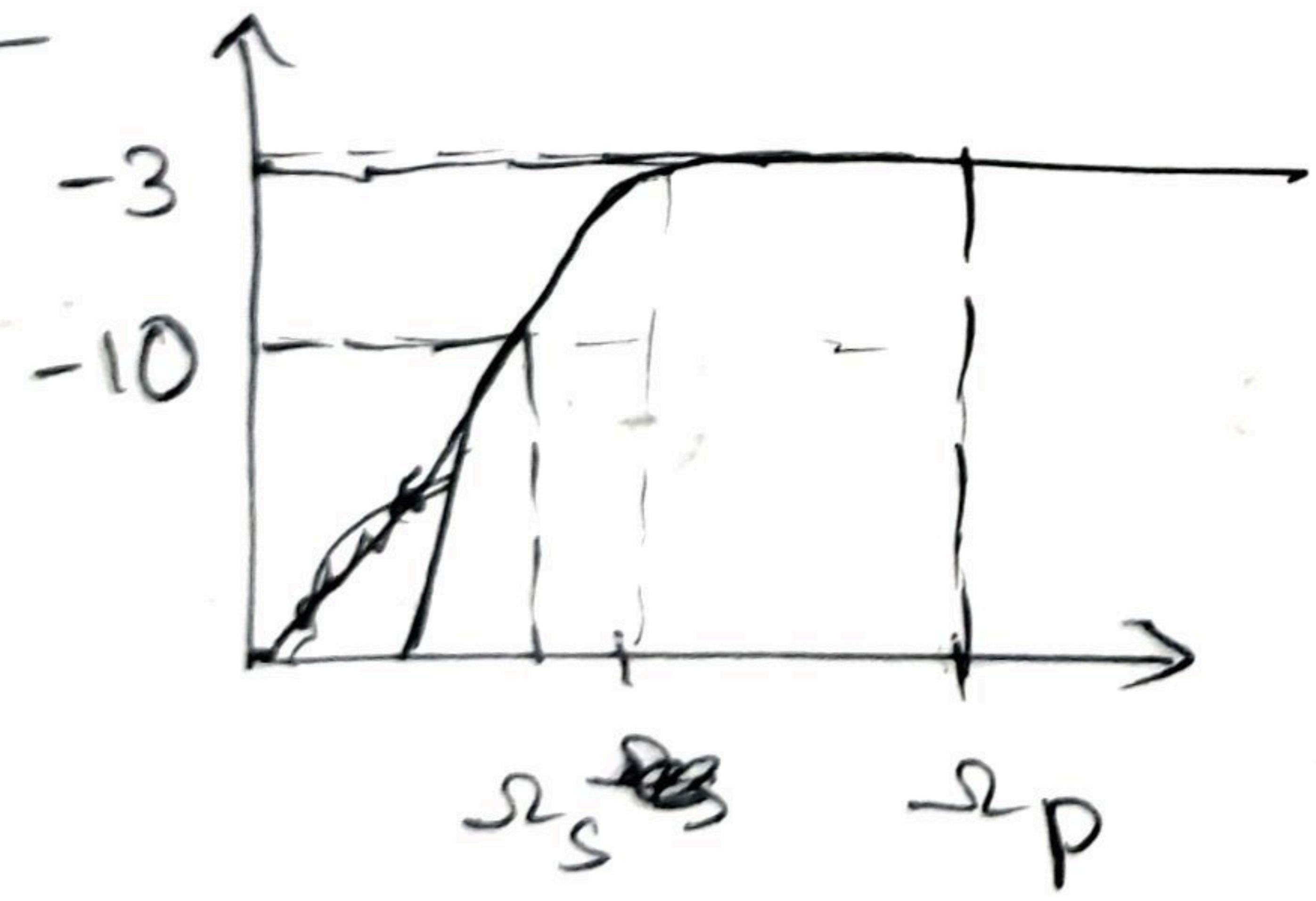
step - 7

$$H_a(s) = H_2(s)\Big|_{s \to \frac{s}{\Omega_c} := \frac{s}{1.504k}}$$

$$H_a(s) = \frac{1}{\left(\frac{s}{1.504k}\right)^2 + \sqrt{2}\left(\frac{s}{1.504k}\right) + 1}$$

Design a high pass butterworth filter with 10dB attenuation at ~~1000~~ stop ~~pass~~ band frequency of ~~1.5dB~~ ~~1.5 KHz~~ 1.5 rad/sec, ~~10~~ 3 dB attenuation at pass band frequency of ~~3KHz~~. 3 rad/sec

step - 1



$\Omega_s = 1.5 KHz$

$\Omega_p = 3 KHz$

$A_s = 10dB \Rightarrow k_s = -10dB$

$A_p = 3\ dB \Rightarrow k_p = -3dB$

step - 2

$\Omega_p = 1$

$\Omega_s = \frac{\Omega_p}{\Omega_s} = 2$ } normalized

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Step-3

$$N = \frac{\log\left(\left(10^{-k_p/10} - 1\right) / \left(10^{-k_s/10} - 1\right)\right)}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left(\left(10^{3/10} - 1\right) / \left(10^{1} - 1\right)\right)}{2\log\left(1/2\right)} = \frac{\log\left(0.99/9\right)}{-0.602}$$

$$= \frac{\log(0.11)}{-0.602} = \frac{-0.958}{-0.602} = 1.592 \approx 2$$

$N = 2$

Step-4

$$H_N(s) = \frac{1}{(s-s_0)(s-s_1)(s-s_2)} = \frac{1}{s^2+\sqrt{2}s+1}$$

Step-5

$$\Omega_c = \frac{\Omega_p}{\left(10^{-k_p/10} - 1\right)^{1/2N}} = \frac{3\,KHz}{\left(10^{3/10} - 1\right)^{1/4}}$$

$$= \frac{3k}{\left(10^{0.3} - 1\right)^{0.25}} = \frac{3k}{(0.995)^{0.25}} = \frac{3k}{0.998}$$

$\Omega_c = 3.006\ kHz$

Step-6

$$H_a(s) = H_2(s)\big|_{s \to \frac{s}{\Omega_c}}\ \frac{\Omega_c}{s} = \frac{3k}{s}$$

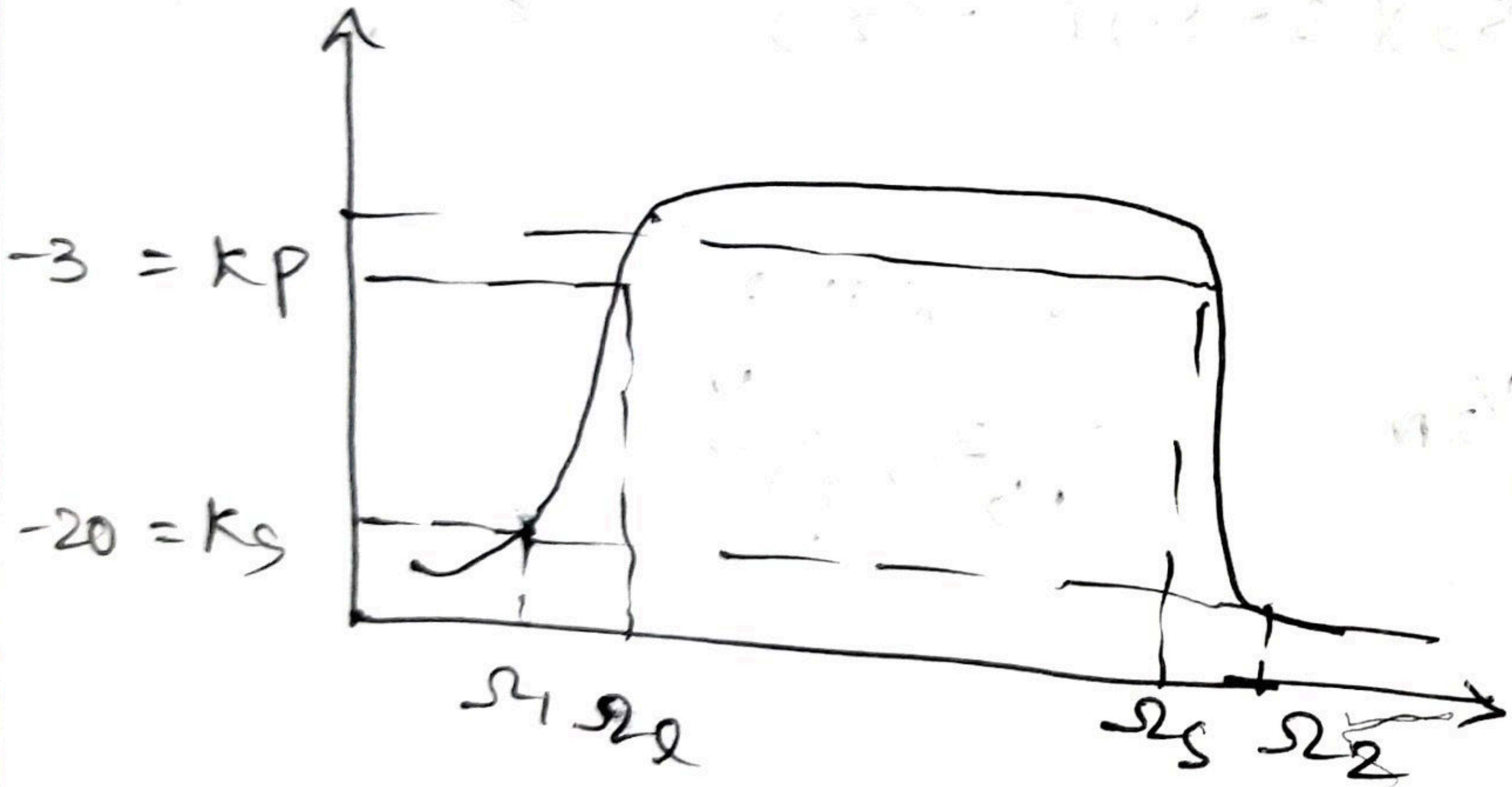
$$H_a(s) = \boxed{\frac{1}{\left(\left(\frac{3K}{s}\right)^2 + \sqrt{2}\left(\frac{3K}{s}\right) + 1\right)}}$$

Design a analog bandpass filter with specifications:-

i) a -3.0103 dB lower & upper cutoff freq of 50HZ and 20KHZ

ii) a stopband attenuation of atleast 20dB at 20HZ and 45KHZ and

iii) a monotonic frequency response.



$\Omega_1 = 20HZ$

$\Omega_2 = 45KHZ$

$\Omega_l = 50HZ$

$\Omega_u = 20KHZ$

$K_p = -3$

$K_s = -20$

$A_s = 20dB$

$K_s = -20dB$

$\Omega_1 = 20HZ \times 2\pi = 125.663\ rad/sec$

$\Omega_2 = 45 \times 10^3 \times 2\pi = 2.827 \times 10^5\ rad/sec$

$\Omega_u = 20 \times 10^3 \times 2\pi = 1.257 \times 10^5\ rad/sec$

$\Omega_l = 2\pi \times 50 = 314.159\ rad/sec$

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Step-2

$\Omega_p = 1$ for normalized.

$\Omega_s = \min\{|A|, |B|\}$

$A = \dfrac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$; $B = \dfrac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$

$= \dfrac{-(125.663)^2 + (314.159)(\cancel{1}1.257\times10^5)}{125.663(1.257\times10^5 - 314.159)}$

$= \dfrac{-15791.19 + 39489786.3}{\cancel{125385.841}\ 15756360.94} = \dfrac{39473995.11}{\cancel{125385.841}\ 15756360.94}$

$\boxed{\cancel{A = 1314.82}}$

$= 2.5052$

$B = \dfrac{-\Omega_2^2 + \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)} = \dfrac{-(2.827\times10^5)^2 + 39489786.3}{\cancel{1256}\ 2.827\times10^5}$

$B = 2.25$ rad/sec.

$\Omega_s = 2.25$ rad/sec

Step-3

$N = \dfrac{\log\left(\left(10^{\frac{-K_P}{10}} - 1\right) / \left(10^{\frac{-K_S}{10}} - 1\right)\right)}{2\log\left(\frac{\Omega_P}{\Omega_S}\right)} = \dfrac{\log(0.995/99)}{2\log\left(\frac{1}{2.25}\right)}$

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$$N = \frac{\log(0.01)}{2\log(0.44)} = \frac{-2}{-0.71309} = 2.8$$

$$N = 2.8 \simeq 3$$

$$\boxed{N=3}$$

step-4

$$S_k = e^{\frac{j\pi(2k+1)}{2N}} \cdot e^{j\pi/2} \qquad k = 0, 1, \ldots, 2N-1$$

$k = 0, 1, 2, 3, 4, 5,$ ~~6, 7~~

but we need only S_0, S_1, S_2

$$S_0 = e^{j\pi/6} \cdot e^{j\pi/2} = e^{j4\pi/6} = e^{j2\pi/3} = -0.5 + j0.866$$

$$S_1 = e^{j\pi/2} \cdot e^{j\pi/2} = e^{j\pi} = e^{j\pi} = \cos\pi + j\sin\pi = -1$$

$$S_2 = e^{\frac{j\pi(5)}{6}} \cdot e^{j\pi/2} = e^{8j\pi/6} = e^{4j\pi/3} = -0.5 - j0.866$$

step-5

$$H_3(s) = \frac{1}{(s-(-0.5+j0.866))(s+1)(s+0.5+j0.866)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

step-6

$$H_a(s) = H_3(s)\Big|_{s \to \frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}}$$

$$\Omega_u\Omega_l = 39489786.3$$

$$\Omega_u - \Omega_l = 125385.85$$

$$s \to \frac{s^2+3.947\times10^7}{s(1.2538\times10^5)}$$

$$H_a(s) = \frac{1}{\left[\left(\frac{s^2+3.947\times10^7}{s(1.2538\times10^5)}\right)^3 + 2\left(\frac{s^2+3.947\times10^7}{s(1.2538\times10^5)}\right)^2 + 2\left(\frac{s^2+3.947\times10^7}{s(1.2538\times10^5)}\right)+1\right]}$$

* $N=6$ for bandpass as $H_a(s)$ is in order '6'.

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Design of IIR digital filter:-

$H_a(S) \rightarrow H(z)$

* For digital filter even if it is analog or digital we have to change to digital & then pre warping

Bilinear transformation:-

i) * $s \rightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

$T \rightarrow$ Sampling period.

* $\sigma < 0 \rightarrow$ inside unit circle

$\sigma = 0 \rightarrow$ on the unit circle

$\sigma > 0 \rightarrow$ outside unit circle.

* $\omega = \frac{2}{T} \tan^{-1}\left(\frac{\Omega T}{2}\right)$

$\omega \rightarrow$ digital frequency

$\Omega \rightarrow$ analog freq.

$\Omega = \frac{2}{T} \tan\left(\frac{\omega}{2}\right)$

* $\Omega_p' = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$

$\Omega_s' = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right)$

Apply BLT to $H(s) = \frac{2}{(s+1)(s+2)}$ with $T=1$ and find $H(z)$

$$H(z) = \frac{2}{\left[\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right)\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right)\right]}$$

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$$H(z) = \frac{1+2z^{-1}+z^{-2}}{6-2z^{-1}}$$

Design low pass filter with cutoff frequency $\cancel{0.25}$ 0.2π for analog $H(S) = \frac{\Omega_c}{S+\Omega_c}$ using bilinear transformation.

$\omega_c = 0.2\pi$

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_c}{2}\right)$$

$$\Omega_c = \frac{2}{T}\tan\left(\frac{0.2\pi}{2}\right) = \frac{2}{T}\tan(\pi/10)$$

$$\Omega_c = \frac{2}{T}\tan(\pi/10) = \frac{0.65}{T}$$

$$H(S) = \frac{\Omega_c}{S+\Omega_c} = \frac{2\tan(0.1\pi)}{S+\tan(0.1\pi)} = \frac{0.65/T}{S+0.65/T}$$

$$= \frac{0.65}{T(S)+0.65} = \frac{0.65}{TS+0.65}$$

$$S \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

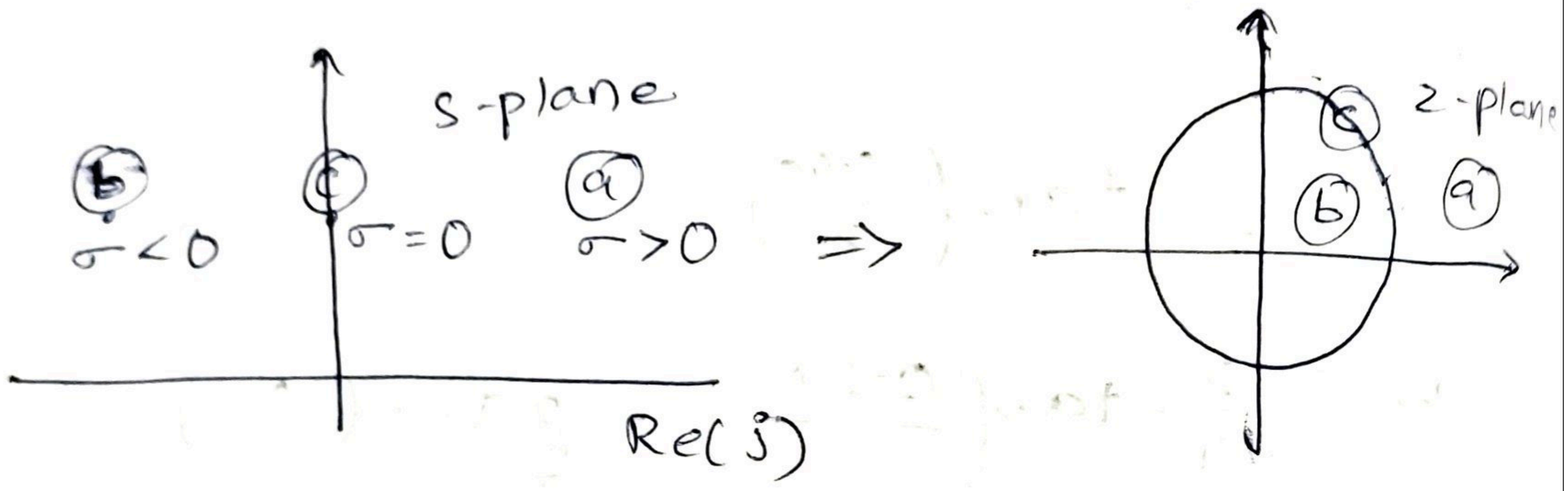
$$H(z) = \frac{0.65}{\frac{2}{\cancel{T}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]\cdot\cancel{T}+0.65} = \frac{0.65(1+z^{-1})}{2-2z^{-1}+0.65+0.65z^{-1}}$$

$$= \frac{0.65+0.65z^{-1}}{2.65-1.35z^{-1}} = \frac{0.65z+0.65}{2.65z-1.35}$$

Mapping of s-plane to z-plane with bilinear transformation.

$s = \sigma + j\omega \Rightarrow Re(s) = \sigma$

$Im(s) = j\omega$



$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \text{ (or) } s = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$$

Substitute $s = \sigma + j\Omega$ & $z = re^{j\omega}$

$$\Rightarrow \sigma + j\Omega = \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right] = \frac{2}{T}\left[\frac{(r\cos\omega-1)+jr\sin\omega}{(r\cos\omega+1)+jr\sin\omega}\right]$$

A digital lowpass filter is required to meet the following specifications.

a) monotonic pass & stop band

b) -3.01 dB cutoff freq of 0.5π rad

c) stopband attenuation atleast 15 dB at 0.75π rad

find $H(z)$

$K_p = -3.01\,dB$

$\omega_p = 0.5\pi$ rad

$K_s = -15\,dB$

$\omega_s = 0.75\pi$ rad.

use $T = 1$ sec if nothing mentioned.

step-1

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{T}\tan\left(\pi/4\right) = 2$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{T}\tan\left(\frac{3\pi}{8}\right) = 4.8282$$

step-2

$\Omega_p = 1$ rad/sec

$$\Omega_s = \frac{\Omega_s'}{\Omega_p'} = \frac{4.8282}{2} = 2.4141$$

step-3

$$N = \frac{\log\left[\left(10^{-K_p/10}-1\right)/\left(10^{-\frac{K_s}{10}}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} = 1.92 = 2$$

Step-4

$$S_k = e^{\frac{j\pi(2k+1)}{2N}} \cdot e^{j\pi/2}$$

$K = 0, 1, 2, (2(2)-1)$ 3

$$S_0 = e^{\frac{j\pi}{4}} \cdot e^{j\pi/2} = e^{3j\pi/4} = -1/\sqrt{2} + j/\sqrt{2}$$

$$S_1 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

Step-5

$$H_2(S) = \frac{1}{(S-S_0)(S-S_1)} = \frac{1}{(S^2+\sqrt{2}S+1)}$$

Step-6

$$\Omega_c = \frac{\Omega p'}{\left(10^{\frac{-kp}{10}}-1\right)^{1/2N}} = 2\ rad/sec$$

Step-7

$$Ha(S) = H_2(S)\Big|_{S \to \frac{S}{\Omega c}}$$

$$= \frac{1}{\left(\frac{S}{2}\right)^2+\sqrt{2}\left(\frac{S}{2}\right)+1} = \frac{4}{S^2+2\sqrt{2}S+4}$$

Step-8

$$H(z) = Ha(S)\Big|_{S \to \frac{2}{T}\left[\frac{z-1}{z+1}\right]} \quad (T=1)$$

$$= \frac{4}{\left[2\left(\frac{z-1}{z+1}\right)\right]^2+2\sqrt{2}\left[2\left(\frac{z-1}{z+1}\right)\right]+4} = \frac{1+2z^{-1}+z^{-2}}{0.5858z^{-2}+3.4142}$$

step-9

$$H(z) = \frac{Y(z)}{X(z)}$$

cross multiply $\Rightarrow$

$$Y(z)\,0.5858\,z^{-2} + Y(z)\,3.4142 = X(z) + 2z^{-1}X(z) + z^{-2}(X(z))$$

$$\Rightarrow \quad y(n-2)[0.5858] + 3.4142\,y(n) = x(n) + 2x(n-1) + x(n-2)$$

$$\boxed{y(n)[3.4142] = x(n) + 2x(n-1) + x(n-2) - 0.5858\,y(n-2)}$$

* chebyshev filter:-

→ A chebyshev polynomial of degree N is defined as:-

$$T_N(x) = \begin{cases} \cos(N\cos^{-1}x), & |x| \le 1 \\ \cosh(N\cosh^{-1}x), & |x| > 1 \end{cases}$$

using reccursive formula:- analog → digi

$$T_N(x) = 2x\,T_{N-1}(x) - T_{N-2}(x)$$

$$T_0(x) = 1 \quad \text{and} \quad T_1(x) = x$$

Important properties of chebyshev polynomials:-

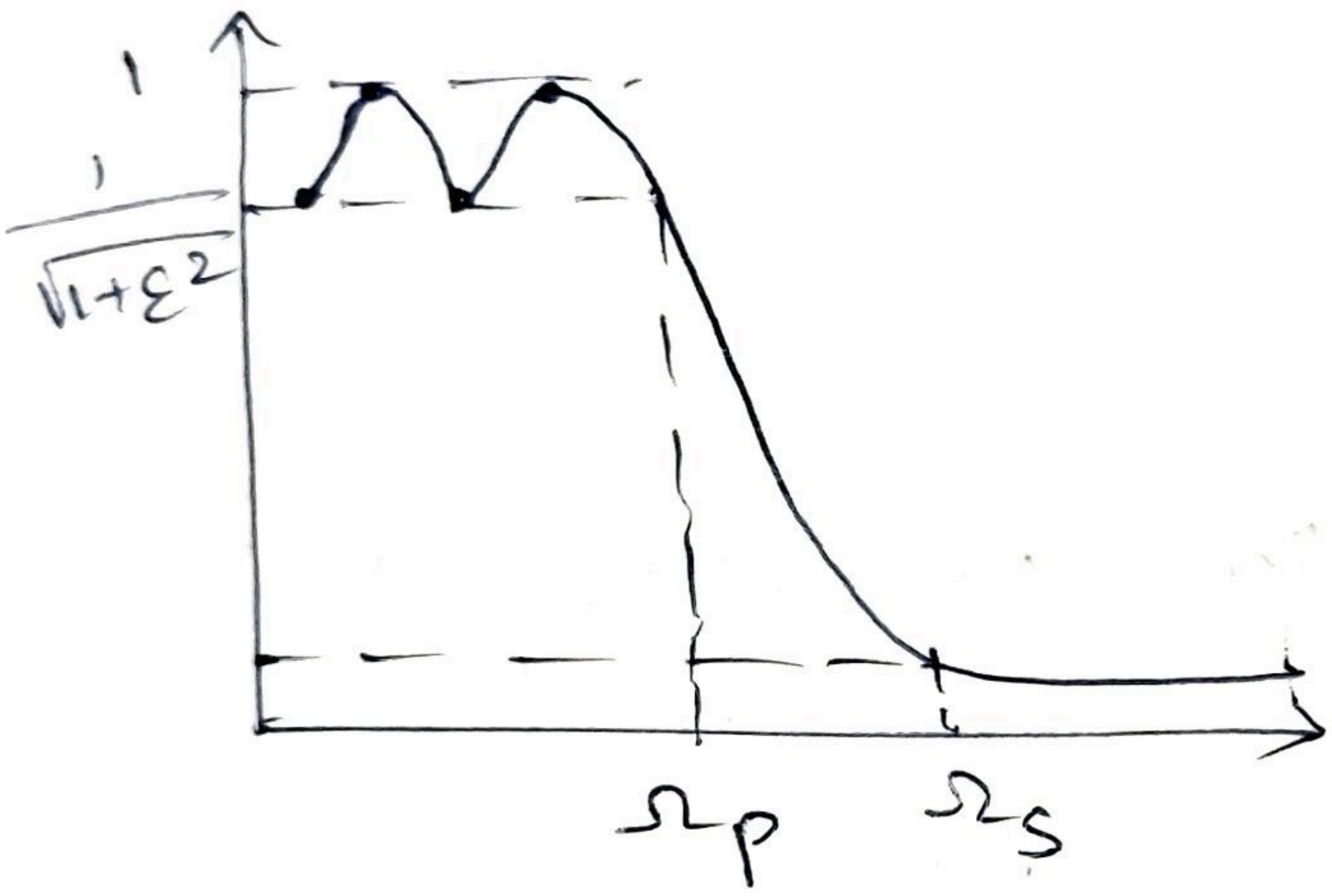
* For $|x| \le 1$, $|T_N(x)| \le 1$ & it oscillates b/w -1 & 1
* for $|x| > 1$, $|T_N(x)| > 1$, it is monotonically ↑ in $|x|$
* chebyshev polynomial of odd orders contains only odd powers of x.

* $T_N(-x) = (-1)^N T_N(x)$
* $|T_N(\pm 1)| = 1$ for all N

* Magnitude frequency of a lowpass chebyshev

$$|H(j\Omega)| = \frac{1}{\left[1+\varepsilon^2 T_N^2\left(\frac{\Omega}{\Omega_p}\right)\right]^{1/2}}$$

ε - ripple factor
T_N - chebyshev polynomial



Chebyshev normalized transfer fn $H_N(s)$:-

$$s_k = \sigma_k + j\Omega_k \Rightarrow \frac{\sigma_k^2}{a^2} + \frac{\Omega_k^2}{b^2} = 1$$

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{1/N}$$

Selection of N:-

$$\boxed{N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}}$$

d - discrimination factor

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$$

where k is selectivity factor $\Rightarrow k = \frac{\Omega_p}{\Omega_s}$

pass band ripple $\delta_p = 1 - \frac{1}{\sqrt{1+\varepsilon^2}}$

$$\boxed{\varepsilon^2 = (1-\delta_p)^{-2} - 1}$$

$$k_p = 20\log(1-\delta_p) = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$k_s = 20\log(\delta_s)$$

Design a chebyshev-1, analog lowpass with

pass band ripple ≤ -3dB

Pass band edge $= 100$ rad/sec

stopband attenuation ≥ 25dB

Stopband edge $= 250$ rad/sec

Step-1

$k_p = -3$dB ; $k_s = -25$dB

$\Omega_p = 100$ rad/sec ; $\Omega_s = 250$ rad/sec

Step-2

$\Omega_p = 1$ rad/sec

$\Omega_s = \frac{\Omega_s}{\Omega_p} = 2.5$ rad/sec

Step-3

$$K_p = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$-3 = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$10^{\frac{-3}{20}} = \frac{1}{\sqrt{1+\varepsilon^2}} \Rightarrow \sqrt{1+\varepsilon^2} = 10^{3/20}$$

$$1+\varepsilon^2 = 10^{\frac{6}{20}} \Rightarrow \varepsilon^2 = 10^{\frac{6}{20}} - 1$$

$$\varepsilon = 0.9976$$

$$1-\delta_p = \frac{1}{\sqrt{1+\varepsilon^2}} \Rightarrow 0.2921 = \delta_p$$

$$K_s = 20\log\delta_s \Rightarrow \delta_s = 0.056$$

Step-4

$$K = \frac{\Omega_p}{\Omega_s} = \frac{1}{2.5} = 0.4$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = 0.056$$

Step-5

$$N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/K)} = 2.28 = 3$$

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step - 6

from chart:-

$N = 3, \quad \varepsilon = 0.9976.$

$b_0 = 0.2505943$

$b_1 = 0.9283480$

$b_2 = 0.5972404.$

$$H_3(s) = \frac{K_N}{s^3 + b_2 s^2 + b_1 s + b_0} \quad ; \text{where } K = \begin{cases} \frac{b_0}{\sqrt{1+\varepsilon^2}}, & N\text{-even} \\ b_0, & N = \text{odd} \end{cases}$$

$$= \frac{0.2505943}{s^3 + 0.5972404 s^2 + 0.9283480 s + 0.2505943}$$

step-7

$H_a(s) \Big|_{s \to \frac{s}{\Omega_p}} \qquad \Omega_p = 100 \text{ rad/sec}$

$$H_a(s) = \frac{0.2505943}{\left(\frac{s}{100}\right)^3 + 0.5972404\left(\frac{s}{100}\right)^2 + 0.9283480\left(\frac{s}{100}\right) + 0.2505943}$$

Determine the system for H(z) of the lowest order chebyshev I analog low pass filter

→ a. 3 dB ripple in passband $0 \le |\omega| \le 0.3\pi$

b. Atleast 20 dB attenuation in stopband

$0.6\pi \le |\omega| \ge \pi$

c. use the bilineer transformation.

$A_p = 3dB$; $K_p = -3 dB$

$A_s = 20dB$; $K_s = -20dB$

$\omega_p = 0.3\pi$

$\omega_s = 0.6\pi$

Step-1

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{1}\tan\left(\frac{0.3\pi}{2}\right) = 1.019 \, rad/sec$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 2\tan\left(\frac{0.6\pi}{2}\right) = 2.75 \, rad/sec$$

Step-2

$\Omega_p = 1$

$$\Omega_s = \frac{\Omega_s'}{\Omega_p'} = 2.698 \, rad/sec$$

Step-3

$$K_p = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$$

$$\delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}}$$

find δ_p $\qquad \delta_p = 0.29205\text{4}1$

$$K_s = -20\,dB$$

$$20\log\delta_s = -20 \Rightarrow \delta_s = 0.1$$

Step-4

$$k = \frac{\Omega p'}{\Omega s'} = \frac{1}{2.698} = 0.3705$$

$$d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = 0.1$$

$$N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = 1.8$$

$N = 2$

Step-5

$$H_N(s) = \frac{K_N}{s^2+b_1 s+b_0}$$

$b_0 = 0.7079$

$b_1 = 0.6448$

$$K_N = \frac{b_0}{\sqrt{1+\epsilon^2}} \text{ as } N = \text{even} \Rightarrow K_N = 0.50119$$

$$H_N(S) = \frac{0.50119}{S^2 + 0.64489S + 0.7079478}$$

Step-6

$$Ha(S) = H_N(S) \Big|_{S \rightarrow \frac{S}{\Omega_p} = \frac{S}{1.019}}$$

$$Ha(S) = \frac{0.50119}{\left(\frac{S}{1.019}\right)^2 + 0.64489\left(\frac{S}{1.019}\right) + 0.7079478}$$

Step-7

$$Ha(z) = Ha(S) \Big|_{S \rightarrow \frac{2}{T}\left[\frac{z-1}{z+1}\right]}$$

$$Ha(z) = \frac{0.50119}{\left(\frac{2\left(\frac{z-1}{z+1}\right)}{1.019}\right)^2 + 0.64489\left(\frac{2\left(\frac{z-1}{z+1}\right)}{1.019}\right) + 0.7079478}$$