Z-transform:

Z transform of a discrete time signal $x(n)$

~~wkt~~ $$\boxed{x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}}$$

where z is complex variable, $z = re^{jw}$

$$x(re^{jw}) = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-jwn}$$

* Right hand sequence / Causal seq :-

$x(n) = 0$ for all $n < n_0$, $n_0 \to \pm$ ~~infinite~~ finite

if $n_0 \geq 0$, the resulting sequence is causal or a positive time sequence.

ROC (Region of convergence) : entire z-plane except at $z=0$

Ex $x(n) = \{1, 0, 3, -1, 2\}$

~~Apply~~ Z-transform

$$x(z) = x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4}$$

$$= 1 + 0 + 3z^{-2} + -1z^{-3} + 2z^{-4}$$

$x(z)$ converges for all values of z, ~~expect~~ except at $z=0$.

* Left hand sequence / Anticausal seq :-

$x(n) = 0 \;\; \forall\; n \geq n_0$, $n_0 \to \pm$ finite

if $n_0 \leq 0$, the resulting sequence is anticausal seq.

ROC: entire z-plane except $z=0$

$$x(n) = \{-3, -2, -1, 0, \underset{\uparrow}{1}\}$$

$$x(z) = -3z^{4} + -2z^{3} - 1z^{2} + 0 + 1$$

$x(z)$ converges $\forall$ values of z, ~~expect~~ except at $z=0$

Z-transform :

Z transform of a discrete time signal $x(n)$

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ROC (Region of convergence) : entire z-plane except at $z=0$

Ex $x(n) = \{1, 0, 3, -1, 2\}$

Apply z-transform

$$X(z) = x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4}$$

$$= 1 + 0 + 3z^{-2} - 1z^{-3} + 2z^{-4}$$

$x(z)$ converges for all values of z, ~~expect~~ except at $z=0$.

* **Left hand sequence / Anticausal seq :-**

$x(n) = 0$ $\forall$ $n \geq n_0$, $n_0 \rightarrow \pm$ finite

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ROC : entire z-plane except $z=0$

$$x(n) = \{-3, -2, -1, 0, \underset{\uparrow}{1}\}$$

$$x(z) = -3z^{4} - 2z^{3} - 1z^{2} + 0 + 1$$

$x(z)$ converges $\forall$ values of z, ~~expect~~ except at $z=0$

Differentiation of $X(z)$:-

If $x(z) = z\{x(n)\}$ then $z\{n x(n)\} = -z\frac{d}{dz}x(z)$

Proof:-

$$x(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

diff

$$\frac{d}{dz}x(z) = \sum_{-\infty}^{\infty} x(n) z^{-n-1}$$

$$= \frac{-1}{z}\sum_{-\infty}^{\infty} \cancel{x} \, n x(n) z^{-n}$$

multiply $-z$ on both sides

$$-z\frac{d}{dz}x(z) = -\cancel{\infty}\sum_{-\infty}^{\infty} n x(n) z^{-n}$$

$$RHS = z\{n x(n)\}$$

Q. Find the one sided z transform of discrete time signal $x(n) = n a^{(n-1)}$

→ Let $x_1(n) = a^n$

$$x_1(z) = \sum_{-\infty}^{\infty} x_1(n) z^{-n} = \sum_{-\infty}^{\infty} a^n z^{-n}$$

$$= \sum_{-\infty}^{\infty} (az^{-1})^n$$

$$= \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

Let us consider

$$x_1(n-1) = a^{n-1}$$

$$x_1(z) = \sum_{-\infty}^{\infty} z\{x(n-1)\} = z^{-1}x_1(z) \left[\text{time shifting property}\right]$$

$$= z^{-1}\frac{z}{z-a} = \frac{1}{z-a}$$

Given that $x(n) = n.a^{n-1}$

$$z\{x(n)\} = z\{na^{n-1}\} = z\{n x_1(n-1)\}$$

by applying differentiation property

$$\frac{d}{dz}\frac{1}{z-a} = -z\,\frac{-1}{(z-a)^2} = \frac{z}{(z-a)^2}$$

Q. Find the z transform of $x(n) = n^3$.

$x(n) = n^3$

let $x_1(n) = n^3 u(n)$ $\quad [u(n) = 1 \;\forall\, n \geq 0]$

$$Z\{n^m u(n)\} = \left(-z\frac{d}{dz}\right)^m U(z)$$

$$U(z) = Z\{u(n)\} = \frac{z}{z-1}$$

$$\left[\frac{d}{dz}\frac{u}{v} = \frac{v\,du - u\,dv}{v^2}\right]$$

$$-z\frac{d}{dz}U(z) = -z\left[\frac{d}{dz}\frac{z}{z-1}\right]$$

$$= -z\left[\frac{z-1-z}{(z-1)^2}\right]$$

$$= z/(z-1)^2$$

$$\left(-z\frac{d}{dz}\right)^2 U(z) = -z\frac{d}{dz}\left[-z\frac{d}{dz}\cancel{\frac{z}{z-1}}\,U(z)\right]$$

$$= -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z\left[\frac{(z-1)^2 - z * 2(z-1)}{(z-1)^4}\right] = z\left[\frac{(z-1)(z-1-2z)}{(z-1)^4}\right]$$

$$= -z\left[\frac{\cancel{z^2 - 2z + 1 - 2z^2 + 2z}}{(z-1)^4}\right] = z\left[\frac{z+1}{(z-1)^3}\right]$$

$$= -z\left[\frac{\cancel{-z^2 - z + 1}}{(z-1)^4}\right] = \boxed{\frac{z(z+1)}{(z-1)^3}}$$

$$\left(-z\frac{d}{dz}\right)^3 U(z) = -z\frac{d}{dz}\left[\left(-z\frac{d}{dz}\right)^2 U(z)\right]$$

$$= -z\frac{d}{dz}\left[\frac{z(z+1)}{(z-1)^3}\right] = -z\frac{d}{dz}\left[\frac{(z^2+z)}{(z-1)^3}\right]$$

$$= -z\left[\frac{(z-1)^3(2z+1) - (z^2+z)3(z-1)^2}{(z-1)^6}\right]$$

$$= -z\left[\frac{(2z+1)(z-1) - 3(z^2+z)}{(z-1)^4}\right]$$

$$= -z\left[\frac{2z^2-2z+z-1-3z^2-3z}{(z-1)^4}\right]$$

$$= -z\left[\frac{-z^2-4z-1}{(z-1)^4}\right] = z\left[\frac{z^2+4z+1}{(z-1)^4}\right]$$

Q. Find the z-transform of the discrete time signal generated by mathematically sampling the continuous time signal t^2. $x(t) = t^2$

$\Rightarrow$ discrete time signal is generated by replacing $t = nT$. (T: sampling period)

$$x(n) = (nT)^2$$
$$= n^2T^2$$
$$= n^2 g(n)$$

If $Z\{g(n)\} = G(z)$

then $Z\{n^m g(n)\} = -z\frac{d}{dz}G(z)$

$$G(z) = Z\{g(z)\}$$
$$= Z\{T^2\}$$

$$G(z) = T^2 \sum_{n=0}^{\infty} (z^{-1})^n$$
$$= T^2 \frac{1}{1-z^{-1}}$$

$$G(z) = \frac{T^2}{1-z^{-1}} = \frac{zT^2}{z-1}$$

multiply by n^m property of

$x(z) = Z\{x(n)\}$

$= Z\{n^2 g(n)\}$

$= \left(-z\frac{d}{dz}\right)^2 G(z)$

$= \left(-z\frac{d}{dz}\right)\left(-z\frac{d}{dz}\, G(z)\right)$

$= -z\frac{d}{dz}\left(-z\frac{d}{dz}\, \frac{zt^2}{z-1}\right)$

$= -z\frac{d}{dz}\left(-z\left[\frac{(z-1)T^2 - zT^2}{(z-1)^2}\right]\right)$

$= -z\frac{d}{dz}\left(\frac{(-z^2+z)T^2 + z^2T^2}{(z-1)^2}\right)$

$= -z\frac{d}{dz}\left(\frac{+zT^2}{(z-1)^2}\right)$

$= -z\left[\frac{(z-1)^2 T^2 - zT^2 2(z-1)}{(z-1)^4}\right] = -zT^2\left[\frac{z^2-2z+1-2z^2+2z}{(z-1)^4}\right]$

$= -zT^2\left[\frac{z^2-2z+1-z^2+z}{(z-1)^4}\right] = \frac{z^3T^2 - zT^2}{(z-1)^4}$

$= -zT^2\left[\frac{-z+1}{(z-1)^4}\right]$

$= T^2\left[\frac{z^2 - z(z-1)}{(z-1)^4}\right]$

$= \cancel{\frac{zT^2}{(z-1)^3}}$

$= \frac{zT^2(z^2-1)}{(z-1)^4}$

$= \boxed{\frac{zT^2(z+1)}{(z-1)^3}}$

ar

Q. Find the z transform of $x(t) = \sin \omega_0 t$

$x(n) = \sin \omega_0 nT \Rightarrow \omega = \omega_0 T$

$x(n) = \sin \omega n$

$Z\{x(n)\} = X(z)$

$= \sum_{n=0}^{\infty} x(n) z^{-n}$

$$\boxed{\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}}$$

$= \sum_{0}^{\infty} (\sin \omega n) z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{e^{j\omega n} - e^{-j\omega n}}{2j} \right) z^{-n}$

$\cancel{\frac{d}{dz} X(z) = \frac{d}{dz} \sum_{0}^{\infty} \left(\frac{e^{j\omega n} - e^{-j\omega n}}{2j} \right) z^{-n}}$

$\cancel{= \frac{d}{dz} \left(\frac{e^{-j\omega n} - e^{-j\omega n}}{2j} \right) z^{-n-1}}$

$= \left[\sum_{n=0}^{\infty} \frac{e^{j\omega n}}{2j} - \sum_{n=0}^{\infty} \frac{e^{-j\omega n}}{2j} \right] z^{-n}$

$= \frac{1}{2j} z^{-n} \left[\sum_{0}^{\infty} e^{j\omega n} - \sum_{0}^{\infty} e^{-j\omega n} \right]$

$= \frac{1}{2j} \cancel{z^{-n}} \left[\sum_{0}^{\infty} z^{-n} e^{j\omega n} - \sum_{0}^{\infty} z^{-n} e^{-j\omega n} \right]$

$= \frac{1}{2j} \left[\frac{1}{1 - z^{-n} e^{j\omega n}} - \frac{1}{1 - z^{-n} e^{-j\omega n}} \right]$

$= \frac{1}{2j} \left[\frac{1 - z^{-n} e^{-j\omega n} - 1 + z^{-n} e^{j\omega n}}{(1 - z^{-n} e^{j\omega n})(1 - z^{-n} e^{-j\omega n})} \right]$

$= \frac{z^{-n}}{2j} \left[\frac{\cancel{\cos\theta}\ \sin\theta}{1 - z^{-n} e^{-j\omega n} - z^{-n} e^{j\omega n} - z^{-2n} \cancel{e^{0}}} \right]$

$= \frac{1}{2j} \left[\frac{\sin\theta}{z^{n} - e^{-j\omega n} - e^{j\omega n} - z^{-n}} \right] = \frac{1}{2j} \left[\frac{\sin\theta}{z^{n} - z^{-n}} - \cos\theta \right]$

ne

$$\sum_{n=0}^{\infty}\left(\frac{e^{j\omega n}-e^{-j\omega n}}{2j}\right)z^{-n}$$

$$=\frac{1}{2j}\left[\sum_{n=0}^{\infty} z^{-n}e^{j\omega n}-\sum_{0}^{\infty} z^{-n}e^{-j\omega n}\right]$$

$$=\frac{1}{2j}\left[\frac{1}{1-z^{-n}e^{j\omega}}-\frac{1}{1-z^{-n}e^{-j\omega}}\right]$$

$$=\frac{\cancel{1}}{2j}\left[\frac{1}{z-e^{j\omega}}-\frac{1}{z-e^{-j\omega}}\right]$$

$$=\frac{z}{2j}\left[\frac{z-e^{-j\omega}-z+e^{j\omega}}{(z-e^{j\omega})(z-e^{-j\omega})}\right]$$

$$=\frac{z}{2j}\left[\frac{e^{j\omega}-e^{-j\omega}}{j}\right]\frac{1}{[z^2-ze^{-j\omega}-ze^{j\omega}+1]}$$

$$=z[\sin\omega]\left[\frac{1}{z^2-\cancel{z}(e^{-j\omega}+e^{j\omega})+1}\right]$$

$$=\sin\omega\left[\frac{z}{z^2-z(e^{-j\omega}+e^{j\omega})+1}\right]$$

$$=\frac{(\sin\omega)z}{\left[z^2-2z\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)+1\right]}$$ multiply & divide by 2.

$$=\frac{(\sin\omega)z}{[z^2-2z\cos\omega+1]}$$

Q. $x(t) = \cos \Omega_0 t$

$x(n) = \cos \Omega_0 n T \Rightarrow \omega = \Omega_0 T$

$x(n) = \cos \omega n$

$$Z\{x(n)\} = X(z)$$

$$= \sum_{0}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (\cos \omega n) z^{-n}$$

$$= \sum_{0}^{\infty} \left[\frac{e^{j\omega n} + e^{-j\omega n}}{2}\right] z^{-n}$$

$$= \left[\sum_{0}^{\infty} e^{j\omega n} + \sum_{0}^{\infty} e^{-j\omega n}\right] \frac{z^{-n}}{2j}$$

$$= \frac{1}{2j}\left[\sum_{0}^{\infty} z^{-n} e^{j\omega n} + \sum_{0}^{\infty} z^{-n} e^{-j\omega n}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - z^{-1} e^{j\omega}} + \frac{1}{1 - z^{-1} e^{-j\omega}}\right]$$

$$= \frac{z}{2j}\left[\frac{1}{z - e^{j\omega}} + \frac{1}{z - e^{-j\omega}}\right]$$

$$= \frac{z}{2j}\left(\frac{z - e^{-j\omega} + z - e^{j\omega}}{z^2 - z e^{-j\omega} - z e^{j\omega} + 1}\right]$$

$$= \frac{z[2z - e^{j\omega} - e^{-j\omega}]}{2j[z^2 - z(e^{j\omega} + e^{-j\omega}) + 1]}$$

$$= \frac{2z^2 - z e^{j\omega} - z e^{-j\omega}}{2j[z^2 - 2z \cos \omega n + 1]}$$

$$= \frac{2z^2 - 2z\left(\frac{e^{j\omega} + e^{j\omega}}{2}\right)}{2[z^2 - 2z \cos \omega n + 1]} \qquad \frac{[z^2 - z\cos\omega]}{z^2 - 2z\cos\omega + 1}$$

$$= \boxed{\frac{2(z - \cos\omega)}{z^2 - 2z\cos\omega + 1}}$$

Homework

① $x(t) = e^{-at} \cos \omega_0 t$

② $x(t) = e^{-at} \sin \Omega_0 t$

$$\boxed{\sum_{0}^{\infty} c^n = \frac{1}{1-c}}$$

Q/ Find the inverse z-transform of $x(z) = \frac{2}{(1+z^{-1})(1-z^{-1})^2}$

$$= \frac{2}{z^{-1}(z+1)\, z^{-2}(z-1)^2}$$

$$= \frac{2z^3}{(z+1)(z-1)^2}$$

$$\frac{x(z)}{z} = \frac{2z^2}{(z+1)(z-1)^2}$$

by partial fraction,

$$\frac{x(z)}{z} = \frac{A_1}{(z+1)} + \frac{A_2}{(z-1)^2} + \frac{A_3}{(z-1)}$$

$=$

$$A_1 = \frac{(z+1)\,x(z)}{z}\bigg|_{z=-1}$$

$$= \frac{(z+1)(2z^2)}{(z+1)(z-1)^2}\bigg|_{z=-1}$$

$$= \frac{2}{(4)} = \boxed{+\frac{1}{2} = A_1}$$

$$A_2 = \frac{(z-1)^2\,x(z)}{z}\bigg|_{z=1}$$

$$= \frac{\cancel{(z-1)^2}\, 2z^2}{(z+1)\cancel{(z-1)^2}}\bigg|_{z=1}$$

$$\boxed{A_2 = 1}$$

$$A_3 = \frac{(z-1)\,x(z)}{z}$$

$$= \frac{(z-1)(2z^2)}{(z+1)(z-1)^2}\bigg|$$

$$\boxed{A_3 = \frac{3}{2}}$$

$$\frac{x(z)}{z} = \frac{1}{2(z+1)} + \frac{1}{(z-1)^2} + \frac{3}{2(z-1)}$$

$$x(z) = \frac{0.5z}{(z+1)} + \frac{z}{(z-1)^2} + \frac{3z}{2(z-1)}$$

taking inverse of $x(z)$.

$$x(n) = 0.5(-1)^n u(n) + n u(n) + 1.5 \frac{(z)}{z-1}$$

Imp

a) $z\{a^n u(n)\} = z/z-a$

b) $z\{n u(n)\} = z/(z-1)^2$

c) $z\{u(n)\} = z/z-1$

d) $z\{u(n-1)\} = z^{-1}\left(\frac{z}{z-1}\right)$

e) $z\{a^{(n-1)} u(n-1)\} = z^{-1}\left(\frac{z}{z-a}\right)$

f) $z\{n\, a^n u(n)\} = \frac{az}{(z-a)^2}$

Q. Given that

$$x(z) = \frac{2z-4}{(z-1)(z+2)^2}$$

$$\frac{x(z)}{z} = \frac{2z-4}{(z-1)(z+2)^2}$$

by partial fraction,

$$x(z) = \frac{A_1}{(z-1)} + \frac{A_2}{(z+2)^2} + \frac{A_3}{(z+2)}$$

$$A_1 = (z-1)\,x(z)\Big|_{z=1}$$

$$= \frac{\cancel{(z-1)}(2z-4)}{\cancel{(z-1)}(z+2)^2}\Big|_{z=1}$$

$$= \frac{2z-4}{(z+2)^2}\Big|_{z=1}$$

$$\boxed{A_1 = \frac{-2}{9}}$$

$$A_2 = (z+2)^2\, x(z)\Big|_{z=-2}$$

$$= \cancel{(z+2)^2}\frac{2z-4}{(z-1)\cancel{(z+2)^2}}$$

$$= \frac{-8}{-3}$$

$$= 8/3$$

$x(z) = (z+2)^2 A_1 + (z-1)A_2 + A_3(z-1)(z+2)$

$2z-4 = (z+2)^2 A_1 + (z-1)A_2 + A_3(z-1)(z+2)$

$= (z^2+4z+4)\frac{-2}{9} + (z-1)\frac{8}{3} + A_3(z^2-z+2z-2)$

$= (z^2+4z+4)\frac{-2}{9} + (z-1)\frac{24}{9} + A_3(z^2+z-2)$

$= -2z^2-8z-8+24z-24+9A_3(z^2+z-2) = 18z-36$

$= 16z-32-2z^2-18z+36 = -9A_3(z^2+z-2)$

$-2z^2-2z+4 = -9A_3(z^2+z-2)$

$= 2z^2+2z-4 = +9A_3(z^2+z-2)$

$\boxed{\frac{2}{9} = A_3}$

~~Taking in~~

$\Rightarrow x(z) = \frac{-2}{9(z-1)} + \frac{8}{3(z+2)^2} + \frac{2}{9(z+2)}$

$A_1 = -2/9$

$A_2 = 8/3$

$A_3 = 2/9$

~~Taking inverse z-transform of $x(z)$~~

$\cancel{x(n) = -0.22\frac{u(n)}{z} + \frac{2.67(-2)^n u(n)}{z}}$

Taking inverse z-transform of $x(z)$

$x(n) = -0.22\frac{1}{z}\cdot\frac{z}{(z-1)} + \frac{1}{z}\frac{2.67}{(z+2)^2} + 0.22\frac{1}{z}\cdot\frac{z}{(z+2)}$

$= -0.22z^{-1}\frac{z}{(z-1)} + z^{-1}\frac{2.67}{(z+2)^2} + 0.22\,z^{-1}\left(\frac{z}{(z+2)}\right)$

$= -0.22\,z^{-1}\left(\frac{z}{z-1}\right) + \frac{2.67z^{-1}}{-2}\frac{(-2z)}{(z+2)^2} + 0.22z^{-1}\frac{z}{z-(-2)}$

$\boxed{x(n) = -0.22\,u(n-1) - \frac{2.67}{2}n(-2)^n u(n) + 0.22(-2)^{n-1}u(n-1)}$

$\boxed{x(n) = -0.22\,u(n-1) - 1.335(-2)^n u(n) + 0.22(-2)^{n-1}u(n-1)}$

Region of Convergence (ROC):

The range of values of the complex variable z for which, the z-transform converges is called region of convergence (ROC).

ROC of infinite duration sequences:

① Positive type exponential sequence.

$$x(n) = a^n u(n)$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= \sum_{0}^{\infty} a^n z^{-n}$$

$$= \sum_{0}^{\infty} (a z^{-1})^n$$

$$\sum_{0}^{\infty} a^n = \frac{1}{1-a} \; ; \; |a| < 1$$

$$\Rightarrow X(z) = \frac{1}{1 - a z^{-1}} = \frac{z}{z-a} \; ; \; \text{ROC } |z| > |a|$$

② Negative type exponential series:

$$x(n) = -b^n u(-n-1)$$

$$X(z) \triangleq \sum_{n=-\infty}^{\infty} x^n z^{-n}$$

$$u(-n-1) = \begin{cases} 1, & n \leq 1 \\ 0, & n > -1 \end{cases}$$

$$X(z) = -\sum_{-\infty}^{-1} b^n z^{-n}$$

$$= -\sum (b z^{-1})^n$$

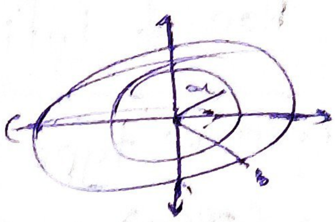
$$= \frac{-1}{1-b} = \frac{z}{-b+z} \; ; \; \text{ro } |z| < |b|$$

* double sided exponential sequence :-

$$x(n) = a^n u(n) - b^n u(-n-1)$$

$$x(z) = \sum_{n=\infty}^{\infty} \left[a^n u(n) - b^n u(n-1) \right] z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n} - \sum_{n=0}^{\infty} b^n z^{-1}$$



$$= \frac{z}{z-a} + \frac{z}{z-b}$$

$$= \frac{\cancel{z^2} - zb - \cancel{z^2} - az}{(z-a)(z-b)}$$

ROC : $|z| > |a| \cap |z| < |b|$

$$= \frac{-(az+bz)}{(z-a)(z-b)}$$

ROC & Stability :-

let $h(n)$ be impulse response of ~~a~~ a causal or ~~a~~ non-causal system $H(z)$ be the z-transform of $h(n)$.

if a discrete LTI system is BIBO (bounded input bounded output) ~~is called~~ stable then ROC of the system function $H(z)$ must contain the unit circle $|z| = 1$.

Q. consider a discrete time LTI system ~~give~~ with the impulse response $h(n)$ given by $h(n) = a^n u(n)$ is this system BIBO stable?

$$H(z) = \frac{z}{z-a} \quad , \quad \text{ROC : } |z| > |a|$$

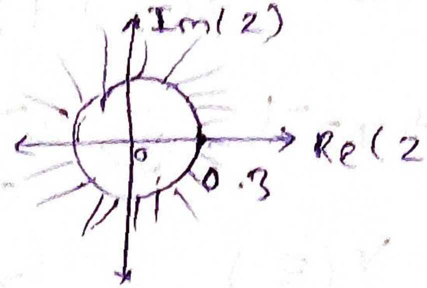
$\therefore$ Not - stable

Q. Determine the z-transform & their ROC of discrete time signal $x(n) = 0.3^n u(n)$

$\therefore$ ~~k h(~~ $x(n) = 0.3^n u(n)$

$$X(z) = \sum_{-\infty}^{\infty} (0.3^n u(n)) x^{-z}$$

$$= \frac{z}{z-0.3}$$



ROC: $|z| > 0.3$.

Q. $x(n) = 0.3^n u(n) + -0.8^n u(-n-1)$

$$X(z) = \frac{z}{z-0.3} + \frac{z}{z+0.8}$$

$$\therefore \boxed{\text{ROC: } |z| > 0.3 \cap |z| < 0.8}$$

Determine the impulse response $h(n)$ for the system described by the second order differential eqn

$$y(n) + 4y(n-1) + 3y(n-2) = x(n-1)$$

taking z-transform

$$Y(z) + 4z^{-1}Y(z) + 3z^{-2}Y(z) = z^{-1}X(z)$$

$$Y(z)[1 + 4z^{-1} + 3z^{-2}] = z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 + 4z^{-1} + 3z^{-2}}$$

$$= \frac{z^{-1}}{(z^2 + 4z + 3)z^{-2}}$$

$$H(z) = \frac{z}{z^2 + 4z + 3} = \frac{z}{(z+1)(z+3)}$$

$$\frac{H(z)}{z} = \frac{A}{(z+1)} + \frac{B}{(z+3)}$$

$$A = \left.\frac{H(z)(z+1)}{z}\right|_{z=-1}$$

$$= \left.\frac{1}{\cancel{(z+1)}(z+3)}\cancel{(z+1)}\right|_{z=-1}$$

$$= \frac{1}{2} = \underline{0.5}$$

$$B = \left.\frac{H(z)}{z}(z+3)\right|_{z=-3}$$

$$= \left.\frac{\cancel{(z+3)}}{(z+1)\cancel{(z+3)}}\right|_{z=-3}$$

$$= \frac{1}{-2} = \underline{-0.5}$$

$$\frac{H(z)}{z} = \frac{0.5}{(z+1)} + \frac{-0.5}{(z+3)}$$

$$H(z) = 0.5z \quad 0.5z$$

take inverse z-transform

$$\boxed{h(n) = 0.5(-1)^n u(n) - 0.5(3)^n u(n)}$$

Q. $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

taking z-transform

$$Y(z) - 3z^{-1}Y(z) - 4z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$$

$$Y(z)\left[1 - 3z^{-1} - 4z^{-2}\right] = X(z)\left[1 + 2z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1-3z^{-1}-4z^{-2}}$$

$$= \frac{z^{-1}(z+2)}{z^{-2}[z^2 - 3z - 4]}$$

$$= \frac{z^2+2z}{z^2-3z-4}$$

$$= \frac{z^2+2z}{(z+1)(z-4)} = \frac{z(z+2)}{(z+1)(z-4)}$$

$$\frac{H(z)}{z} = \frac{z+2}{(z+1)(z-4)}$$

$$\frac{H(z)}{z} = \frac{A}{z+1} + \frac{B}{z-4}$$

$$A = (z+1)\frac{H(z)}{z}\Bigg|_{z=-1}$$

$$= \frac{(z+1)(z+2)}{(z+1)(z-4)}\Bigg|_{z=-1}$$

$$= \frac{-1}{+5}$$

$$\boxed{A = -0.2}$$

$$B = \frac{(z-4)(z+2)}{(z+1)(z-4)}\Bigg|_{z=4}$$

$$B = \frac{6}{5} = 1.2$$

$$\frac{H(z)}{z} = \frac{-0.2}{(z+1)} + \frac{1.2}{(z-4)}$$

$$H(z) = \frac{-0.2z}{z+1} + \frac{1.2z}{z-4}$$

taking inverse z-transform

$$\boxed{h(n) = -0.2(-1)^n u(n) + 1.2(4)^n u(n)}$$

Q: A causal LTI System is de
$y(n) = y(n-1) + y(n-2) + x(n-1$
where $x(n)$ is the input
① System function $H(z)$ fo
② plot the poles and zeroes of
② Find the unit sample/impulse
③ is the system stable or

⇒ taking z-transform

$$Y(z) = z^{-1} Y(z) + z^{-2} Y(z) +$$

$$Y(z)[1 - z^{-1} - z^{-2}] = X(z) z$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

$$H(z) = \frac{z}{z^2 - z - 1}$$

$$\frac{H(z)}{z} = \frac{1}{z^2 - z - 1}$$

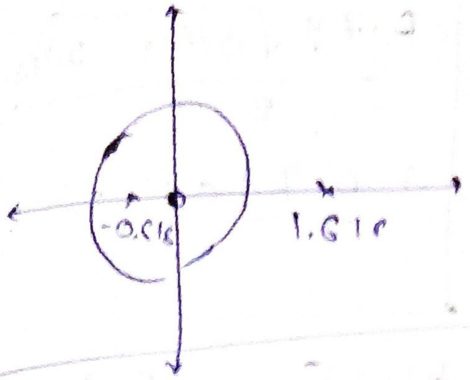
$$\frac{H(z)}{z} = \frac{A}{\left(z - \frac{1+\sqrt{5}}{2}\right)} + \frac{B}{\left(z - \frac{1-\sqrt{5}}{2}\right)}$$

$$A = \frac{H(z)}{z}\left(z - \frac{1+\sqrt{5}}{2}\right)\Bigg|_{z = \frac{1+\sqrt{5}}{2}}$$

$$= \frac{1 \cdot \cancel{(z-1.618)}}{(z+0.618)\cancel{(z-1.618)}}\Bigg|_{z=1.618}$$

$$A = \frac{1}{2.236}$$

$$= 0.447$$

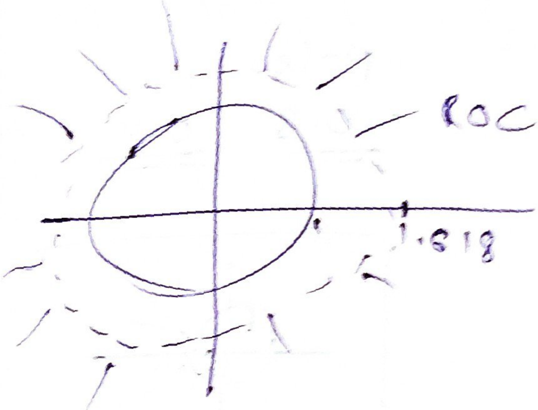


System is unstable

$$H(z) = \frac{-0.447z}{(z+0.618)} + \frac{0.447z}{(z-1.618)}$$

taking inverse z transform

$$\boxed{H(n) = -0.447(-0.618)^n u(n) + 0.447(1.618)^n u(n)}$$



$\boxed{ROC: z > |1.618|}$

Q. compute the response of the system
$y(n) = 0.7\, y(n-1) - 0.12\, y(n-2) + x(n-1) + x(n-2)$
to the input $x(n) = u(n)$. Is the system stable?

$\Rightarrow$ taking z transform

$$Y(z) = 0.7z^{-1}Y(z) - 0.12z^{-2}Y(z) + X(z)z^{-1} + X(z)z^{-2}$$

$$Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] = X(z)\left[z^{-1} + z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} + z^{-2}}{[1 - 0.7z^{-1} + 0.12z^{-2}]}$$

$$= \frac{z^{-2}[z+1]}{z^{-2}[z^2 - 0.7z + 0.12]}$$

$$\boxed{H(z) = \frac{z+1}{[z^2 - 0.7z + 0.12]}}$$

$$H(z) = \frac{z+1}{[z^2 - 0.7z + 0.12]}$$

$$\frac{0.7 \pm \sqrt{0.49 - 0.48}}{2}$$

$$= \frac{0.7 \pm 0.1}{2} =$$

$$H(z) = \frac{A[z+1]}{(z-0.4)} + \frac{B[z+1]}{(z-0.3)}$$

$$A = H(z)[z-0.4]\Big|_{z=0.4}$$

$$= \frac{z+1\,[\cancel{z-0.4}]}{(\cancel{z-0.4})[z-0.3]}$$

$$= \frac{1.4}{0.1}$$

$$\boxed{A = 14}$$

$$B = H(z)[z-0.3]\Big|_{z=0.3}$$

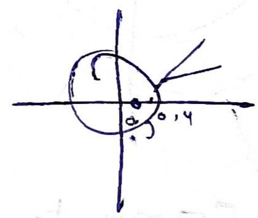
$$\frac{z+1\,[\cancel{z-0.3}]}{[z-0.4]\,[\cancel{z-0.3}]}\Bigg|_{z=0.3}$$

$$= \frac{1.3}{-0.1} = -13$$

poles

$= 0.3 \qquad 0.4$

$$H(z) = \frac{14}{[z-0.4]} - \frac{13}{[z-0.3]}$$



System is Stable

taking inverse z transform

$$\frac{y(z)}{z} = \frac{z+1}{(z-0.3)(z-0.4)(z-1)^2}$$

$$\frac{y(z)}{z} = \frac{A}{(z-0.3)} + \frac{B}{(z-0.4)} + \frac{C}{(z-1)} + \frac{D}{(z-1)^2}$$

$$A = \frac{Y(z)[z-0.3]}{z}\Bigg|_{0.3}$$

$$A = \frac{1.3}{-0.1\,(0.49)} = \frac{-1.3}{0.081} = -16.053$$

$$B = \frac{Y(z)}{z}(z-0.4)\Big|_{z=0.4}$$

$$= \frac{1.4}{0.1[0.36]} = \frac{1.4}{0.036} = \underline{\underline{38.88}}$$

$$D = \frac{Y(z)}{z}(z-1)^2\Big|_{z=1}$$

$$= \frac{2}{0.42} = \underline{\underline{4.76}}$$

$$C = \frac{Y(z)}{z}(z-1)\Big|_{z=1}$$

$$= \frac{z+1}{(z-0.3)(z-0.4)(z-1)}$$

$$C = -12.36$$

$$Y(z) = \frac{-26.53z}{(z-0.3)} + \frac{38.89z}{(z-0.4)} - \frac{12.36z}{(z-1)} + \frac{4.76}{(z-1)^2}$$

taking inverse z transform

$$\boxed{\begin{aligned} y(n) = {} & -26.53(0.3)^n u(n) + 38.89(0.4)u(n) - 12.36(1)^n u(n) \\ & + 4.76\, n\, u(n) \end{aligned}}$$