STEPS TO DESIGN DIGITAL FILTER USING BILINEAR TRANSFORMATION TECHNIQUE

Therefore, we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \qquad (5.96a)$$

and

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} \qquad (5.96b)$$

Steps to design digital filter using bilinear transform technique.

1. From the given specifications, find prewarping analog frequencies using formula $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$
2. Using the analog frequencies find $H(s)$ of the analog filter.
3. Select the sampling rate of the digital filter, call it T seconds per sample.
4. Substitute $s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$ into the transfer function found in step 2.

$$\boxed{\Omega = \frac{2}{T} \tan\left(\frac{\omega T}{2}\right)}$$

$$\frac{\omega T}{2} = \tan^{-1}\frac{\Omega T}{2}$$

$$\boxed{\omega = 2\tan^{-1}\frac{\Omega T}{2}}$$

$$\omega = \frac{2}{T}\tan^{-1}\frac{\Omega T}{2}$$

.2 Steps to design digital filter using Bilinear Transformation Technique:

(i) From the given specifications find the pre-warping analog freq. using the formula,

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega T}{2}\right)$$

(ii) Using the analog freq. find $H(s)$ of the analog filter

(iii) Select the sampling rate of the digital filter, T sec/sample.

(iv) Sub $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ into the transfer fn. found in step 2.

Apply Bilinear Transformation to $H(s) = \frac{2}{(s+1)(s+2)}$ with $T = 1$ sec. and find $H(z)$.

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$H(z) = H(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{2}{\left[\frac{2}{1}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right]\left[\frac{2(1-z^{-1})}{1+z^{-1}}+2\right]}$$

$$= \frac{2(1+z^{-1})^2}{(2-2z^{-1}+1+z^{-1})(2-2z^{-1}+2+2z^{-1})}$$

$$= \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)} = \frac{\cancel{2}[1+2z^{-1}+z^{-2}]}{\cancel{4}_2(3-z^{-1})}$$

$$= \frac{1+2z^{-1}+z^{-2}}{6-2z^{-1}} = \frac{2z+1+z^2}{(6z-2)z}$$

$$= \frac{2(z+1)}{4(3z-1)} = \frac{z+1}{2(3z-1)} = \frac{z^2+2z+1}{6z^2-2z}$$

2. Design a HPF, monotonic in pass band with cut-off frequency 1000 Hz, down 10 dB at 350 Hz. Sampling frequency is 5000 Hz. using a Bilinear Transformation.

$\alpha_p = 3$ dB $\qquad \omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/sec

$\alpha_s = 10$ dB $\qquad \omega_s = 2 \times \pi \times 350 = 700\pi$ rad/sec

$f = 5000$ Hz

Since the filter is monotonic in SB & PB, it is a Butterworth filter.

By bilinear design technique, $\qquad T = \frac{1}{f} = \frac{1}{5 \times 10^{+3}}$

$$\Omega = \frac{2}{T} \tan \frac{\omega T}{2} \qquad = 0.2 \text{ ms}$$

$$= 0.0002$$

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p T}{2} = \frac{2}{0.0002} \tan\left(\frac{2000\pi \times 0.0002}{2}\right)$$

$$= 10000 \tan |6.28$$

$$\Omega_p = \cancel{109.66}\ 7265.42 \text{ rad/sec}$$

$\boxed{\text{use } \pi = 180}$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s T}{2}\right)$$

$$= \frac{2}{0.0002} \tan\left(\frac{700\pi \times 0.0002}{2}\right)$$

$$= 10000 \tan 2.199$$

$$\Omega_s = \cancel{38.4}\ 2235.26 \text{ rad/sec}$$

$$N \geq \frac{\log A}{\log(1/k)}$$

$$A = \left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}\right)^{1/2}$$

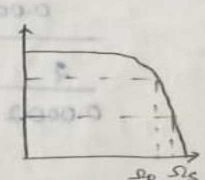
$$= \left(\frac{10^{0.1 \times 10} - 1}{10^{0.1 \times 3} - 1}\right)^{1/2}$$

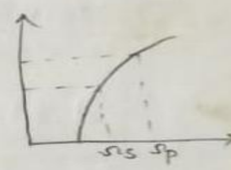
$$= \left(\frac{9}{0.9953}\right)^{1/2}$$

$$A = 3.007$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{7265.42}{2235.26} = 3.25$$

$$\boxed{\cancel{\frac{1}{k} = 0.3077}}$$





$$N \geq \frac{\log 3.007}{\log \overset{3.25}{\cancel{0.3077}}}$$

$$\geq 0.95$$

$$\boxed{N = 1}$$

For $N = 1$, $H(s) = \frac{1}{(s+1)}$

For HPF $s \rightarrow \frac{\Omega_c}{s} = \frac{7282}{s}$

$$H(s) = \frac{1}{\frac{7282}{s}+1} = \frac{s}{7282+s}$$

$$= \frac{s}{s+7282}$$

$$H(z) = H(s) \Big/ _{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{\frac{2}{0.0002}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{0.0002}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7282}$$

$$= \frac{\frac{10000(1-z^{-1})}{\cancel{(1+z^{-1})}}}{\frac{10000(1-z^{-1}) + 7282(1+z^{-1})}{\cancel{(1+z^{-1})}}}$$

$$= \frac{10000(1-z^{-1})}{10000(1-z^{-1}) + 7282(1+z^{-1})}$$

$$= \frac{10000 - 10000z^{-1}}{10000 - 10000z^{-1} + 7282 + 7282z^{-1}}$$

$$= \frac{10000 - 10000z^{-1}}{17282 - 2718z^{-1}}$$

$$= \frac{10000(1-z^{-1})}{17282(1-0.1573z^{-1})} = \frac{0.5786(1-z^{-1})}{(1-0.1573z^{-1})}$$

Determine $H(z)$ that results when B.T. is applied to

$$H_a(s) = \frac{s^2 + 4.525}{s^2 + 0.692s + 0.504}.$$

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Assume $T = 1$ sec

$$H(z) = \frac{\left[\frac{2(1-z^{-1})}{1+z^{-1}}\right]^2 + 4.525}{\left(\frac{2(1-z^{-1})}{1+z^{-1}}\right)^2 + \frac{0.692 \times 2(1-z^{-1})}{1+z^{-1}} + 0.504}$$

$$= \frac{\frac{4(1-z^{-1})^2}{(1+z^{-1})^2} + 4.525}{\frac{4(1-z^{-1})^2}{(1+z^{-1})^2} + \frac{1.384(1-z^{-1})(1+z^{-1})}{(1+z^{-1})^2} + \frac{0.504(1+z^{-1})^2}{(1+z^{-1})^2}}$$

$$= \frac{4(1-z^{-1})^2 + 4.525(1+z^{-1})^2}{4(1-z^{-1})^2 + 1.384(1-z^{-2}) + 0.504(1+z^{-1})^2}$$

$$= \frac{4\left(\frac{z-1}{z}\right)^2 + 4.525\left(\frac{z+1}{z}\right)^2}{4\left(\frac{z-1}{z}\right)^2 + 1.384\left(\frac{z^2-1}{z^2}\right) + 0.504\left(\frac{z+1}{z}\right)^2}$$

$$= \frac{\frac{4}{z^2}(z^2 - 2z + 1) + \frac{4.525}{z^2}(z^2 + 2z + 1)}{\frac{4}{z^2}(z^2 + 1 - 2z) + \frac{1.384}{z^2}(z^2 - 1) + \frac{0.504}{z^2}(z^2 + 2z + 1)}$$

$$= \frac{4 - \frac{8}{z} + \frac{4}{z^2} + 4.525 + \frac{9.05}{z} + \frac{4.525}{z^2}}{4 - \frac{8}{z} + \frac{4}{z^2} + 1.384 - \frac{1.384}{z^2} + 0.504 + \frac{1.008}{z} + \frac{0.504}{z^2}}$$

$$= \frac{\dfrac{4z^2 - 8z + 4 + 4.525z^2 + 9.05z + 4.525}{\cancel{z^2}}}{\dfrac{4z^2 - 8z + 4 + 1.384z^2 - 1.384 + 0.504z^2 + 1.008z + 0.504}{\cancel{z^2}}}$$

$$= \frac{8.525z^2 + 1.05z + 8.525}{5.888z^2 - 6.992z + 3.12}$$

HW:

$H(s) = \dfrac{1}{s^2 + 6s + 9}$. Design using B.T.