Module - 6

Realization of filters

- Pictorial representation
- Direct form I
- Direct form II
- Cascade
- Parallel
- Lattice - ladder
- Transpose form (X)

Realization should be done using these 3 elements:

(1) Adder or summer → $x_1(n)$ + $x_2(n)$ → $x_1(n) + x_2(n)$.

(2) Multiplier → $x(n)$ → a → $ax(n)$.

(3) delay element

$x(n)$ → z^{-1} → $x(n-1)$

$z^{-1} X(z)$

$z[x(n-1)]$ [delayed by 1 unit].

$= z^{-1} X(z)$.

Q. Obtain the direct form - I (df - I), df - II, cascade and parallel forms for the following difference eqn,

$$y(n) = -0.1\, y(n-1) + 0.2\, y(n-2) + 3\, x(n) + 3.6\, x(n-1) + 0.6\, x(n-2)$$

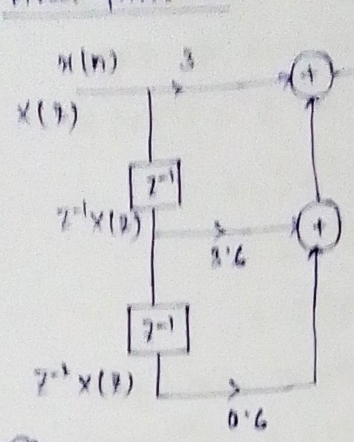
→ Applying Z - transform,

$$Y(z) = -0.1\, z^{-1} Y(z) + 0.2\, z^{-2} Y(z) + 3\, X(z) + 3.6\, z^{-1} X(z) + 0.6\, z^{-2} X(z)$$

$$\Rightarrow Y(z) + 0.1\, z^{-1} Y(z) - 0.2\, z^{-2} Y(z) = W(z) \quad \text{---} (II)$$

Let, $W(z) = 3X(z) + 3.6\, z^{-1} X(z) + 0.6\, z^{-2} X(z)$ --- (I)

Direct form - I



(I) (II)

Direct form - II

$$H(z) = \frac{y(z)}{x(z)} = \frac{3 + 3.6\,z^{-1} + 0.6\,z^{-2}}{1 + 0.1\,z^{-1} - 0.2\,z^{-2}}$$

→ No. of delay elements reduced by 1/2.

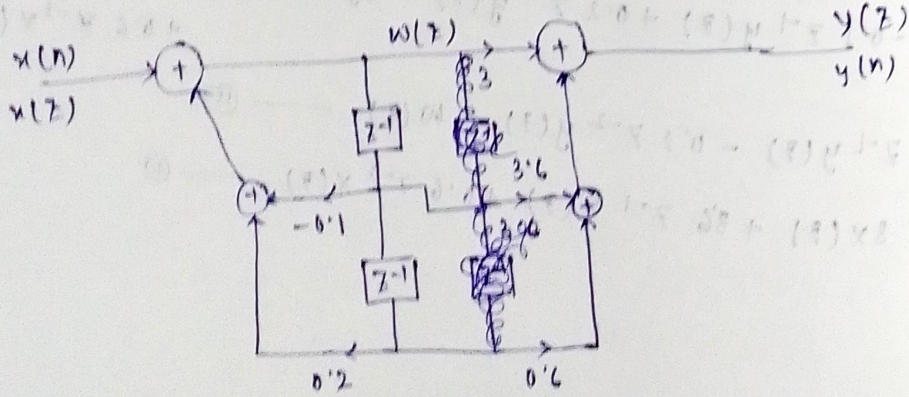
$$H(z) = \frac{y(z)}{x(z)} = \frac{y(z)}{w(z)} \cdot \frac{w(z)}{x(z)}$$

$$\frac{y(z)}{w(z)} = 3 + 3.6\,z^{-1} + 0.6\,z^{-2}$$

$$y(z) = 3w(z) + 3.6\,z^{-1}w(z) + 0.6\,z^{-2}w(z) \quad \text{— (II)}$$

$$\frac{w(z)}{x(z)} = (1 + 0.1\,z^{-1} - 0.2\,z^{-2})^{-1}$$

$$w(z) = x(z) - 0.1\,z^{-1}x(z) + 0.2\,z^{-2}x(z) \quad \text{— (I)}$$



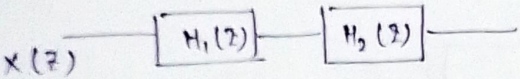
cascade

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$= H_1(z) \cdot H_2(z).$$

$$H_1(z) = \frac{3+0.6z^{-1}}{1+0.5z^{-1}}, \quad H_2(z) = \frac{1+z^{-1}}{1-0.4z^{-1}}$$

realise both using DF-II.



$$H_1(z) = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X_1(z)}$$

$$\frac{Y_1(z)}{W_1(z)} = 3 + 0.6z^{-1}$$

$$\therefore Y_1(z) = 3W_1(z) + 0.6z^{-1}W_1(z) \quad \text{(II)}$$

$$\frac{W_1(z)}{X_1(z)} = \frac{1}{1+0.5z^{-1}}$$

$$\therefore W_1(z) = X_1(z) - 0.5z^{-1}W_1(z) \quad \text{(I)}$$

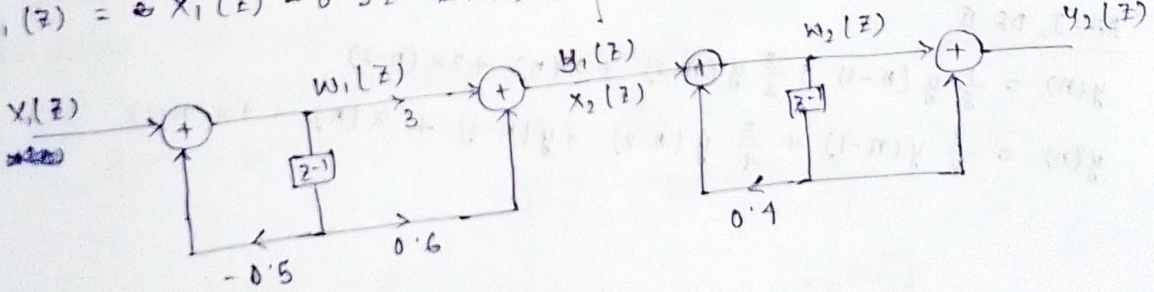
$$H_2(z) = \frac{Y_2(z)}{W_2(z)} \cdot \frac{W_2(z)}{X_2(z)}$$

$$\frac{Y_2(z)}{W_2(z)} = 1 + z^{-1}$$

$$\therefore Y_2(z) = W_2(z) + z^{-1}W_2(z) \quad \text{(II)}$$

$$\frac{W_2(z)}{X_2(z)} = \frac{1}{1-0.4z^{-1}}$$

$$\therefore W_2(z) = X_2(z) + 0.4z^{-1}W_2(z) \quad \text{(I)}$$



Parallel

$$H(z) = H_1(z) + H_2(z) + \dots + H_n(z)$$

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

Dividing num. by denom.

$$\begin{array}{r|l} & -3 \\ \hline -0.2z^{-2} + 0.1z^{-1} + 1 & 0.6z^{-2} + 3.6z^{-1} + 3 \\ & 0.6z^{-2} - 0.3z^{-1} - 3 \\ \hline & 3.9z^{-1} + 6 \end{array}$$

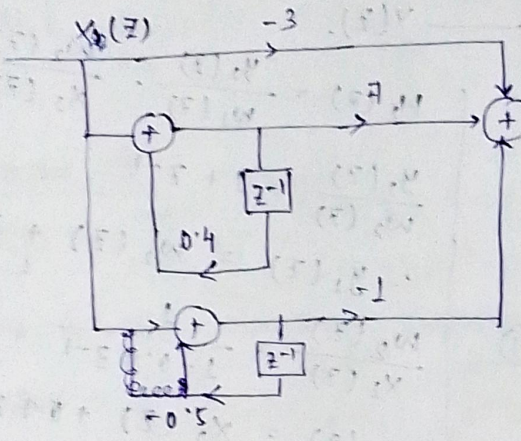
$$\therefore H(z) = -3 + \frac{(3.9z^{-1}+6)}{(1-0.4z^{-1})(1+0.5z^{-1})}$$

Applying partial fractions,

$$\frac{A}{1-0.4z^{-1}} + \frac{B}{1+0.5z^{-1}} = 3.92z^{-1} + 6$$

$$A = 7, \quad B = -1$$

$$\therefore H(z) = -3 + \underbrace{\frac{7}{1-0.4z^{-1}}}_{\text{DF-I}} + \underbrace{\frac{(-1)}{1+0.5z^{-1}}}_{\text{DF-II}}$$



This is not summer (only to represent addition of terms).

DF-I, DF-II

$$y(n) = \frac{1}{8}y(n-1) + \frac{3}{2}y(n-2) + x(n) + 3x(n-2)$$

$$y(n) = \frac{9}{8}y(n-1) + \frac{3}{4}y(n-2) + y(n-3) + x(n) + 4x(n-2)$$