Note-1: *If passband dB-attenuation,* $\alpha_{p,dB}$ *and stopband dB-attenuation,* $\alpha_{s,dB}$ *are specified, then convert them to* A_p *and* A_s *as shown below.*

$$A_p = 10^{(-\alpha_{p,dB}/20)}$$

$$A_s = 10^{(-\alpha_{s,dB}/20)}$$

$\alpha_{p,dB}$ and $\alpha_{s,dB}$ are positive dB

Note-2: *Sometimes the passband dB ripple* $\delta_{p,dB}$ *is specified instead of passband dB-attenuation,* $\alpha_{p,dB}$. *Remember that* $\alpha_{p,dB}$ *equal to* $\delta_{p,dB}$ *(refer section 7.5).*

Note-3: *If T is not specified then take T = 1 second.*

1. Choose either bilinear or impulse invariant transformation, and determine the specifications of equivalent analog filter. The gain or attenuation of analog filter is same as digital filter. The band edge frequencies are calculated using the following equations.

Let, Ω_p = Passband edge analog frequency corresponding to ω_p.

Ω_s = Stopband edge analog frequency corresponding to ω_s.

For bilinear transformation,

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \qquad .....(7.53)$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} \qquad .....(7.54)$$

Note : *If either T or F_s is not specified then take T = 1 second.*

If F_s is specified, then $T = \dfrac{1}{F_s}$

For impulse invariant transformation,

$$\Omega_p = \frac{\omega_p}{T} \qquad .....(7.55)$$

$$\Omega_s = \frac{\omega_s}{T} \qquad .....(7.56)$$

2. Decide the order N of the filter. In order to estimate the order N, calculate a parameter N_1 using the following equation.

$$N_1 = \frac{1}{2}\,\frac{\log\left[\dfrac{(1/A_s^2)-1}{(1/A_p^2)-1}\right]}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)} \qquad .....(7.57)$$

Choose N such that, $N \geq N_1$. Usually N is chosen as nearest integer just greater than N_1.

3. Determine the normalized transfer function, $H(s_n)$ of the analog lowpass filter.

When N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{1}{s_n^2 + b_k s_n + 1} \qquad .....(7.58)$$

When N is odd,

$$H(s_n) = \frac{1}{s_n + 1}\prod_{k=1}^{\frac{N-1}{2}} \frac{1}{s_n^2 + b_k s_n + 1} \qquad .....(7.59)$$

where, $b_k = 2\sin\left[\frac{(2k-1)\pi}{2N}\right] \qquad .....(7.60)$

4. Calculate the analog cutoff frequency, Ω_c.

$$\text{Cutoff frequency, } \Omega_c = \frac{\Omega_s}{\left[(1/A_s^2) - 1\right]^{\frac{1}{2N}}} \quad \text{.....(7.61)}$$

5. Determine the unnormalized analog transfer function H(s) of the lowpass filter.

$$H(s) = H(s_n)\Big|_{s_n = \frac{s}{\Omega_c}}$$

When the order N is even, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in equation (7.58).

$$\therefore\ H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{1}{s_n^2 + b_k\, s_n + 1}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{\Omega_c^2}{s^2 + b_k\, \Omega_c s + \Omega_c^2} \quad \text{.....(7.62)}$$

When the order N is odd, H(s) is obtained by letting $s_n \rightarrow s/\Omega_c$ in equation (7.59).

$$\therefore\ H(s) = \frac{1}{s_n + 1} \prod_{k=1}^{\frac{N-1}{2}} \frac{1}{s_n^2 + b_k\, s_n + 1}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{\Omega_c}{s + \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{\Omega_c^2}{s^2 + b_k\, \Omega_c s + \Omega_c^2} \quad \text{.....(7.63)}$$

6. Determine the transfer function of digital filter, H(z). Using the chosen transformation in step-1, transform H(s) to H(z). When impulse invariant transformation is employed, if $T < 1$, then multiply H(z) by T to normalize the magnitude.

7. Realize the digital filter transfer function H(z) by a suitable structure.

8. Verify the design by sketching the frequency response $H(e^{j\omega})$.

$$H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}}$$

Note : *The basic filter design is lowpass filter design. The highpass, bandpass or bandstop filters are obtained from lowpass filter design by frequency transformation.*

7.7.5 Design Procedure for Lowpass Digital Chebyshev IIR Filter

Let, ω_p = Passband edge digital frequency in rad/sample.

ω_s = Stopband edge digital frequency in rad/sample.

$T = \frac{1}{F_s}$ = Sampling time in seconds.

where, F_s = sampling frequency in Hz.

A_p = Gain at a passband frequency ω_p.

A_s = Gain at a stopband frequency ω_s.

Note-1: If passband dB-attenuation, $\alpha_{p,dB}$ and stopband dB-attenuation, $\alpha_{s,dB}$ are specified, then convert them to A_p and A_s as shown below.

$$A_p = 10^{(-\alpha_{p,dB}/20)}$$

$$A_s = 10^{(-\alpha_{s,dB}/20)}$$

$\alpha_{p,dB}$ and $\alpha_{s,dB}$ are positive dB

2: Sometimes the passband dB ripple $\delta_{p,dB}$ is specified instead of passband dB-attenuation, $\alpha_{p,dB}$. Remember that $\alpha_{p,dB}$ equal to $\delta_{p,dB}$. (refer section 7.5).

3: If T is not specified then take T = 1 second.

1. Choose either bilinear or impulse invariant transformation, and determine the specifications of equivalent analog filter. The gain or attenuation of analog filter is same as digital filter. The band edge frequencies are calculated using the following equations.

Let, Ω_p = Passband edge analog frequency corresponding to ω_p.

Ω_s = Stopband edge analog frequency corresponding to ω_s.

For bilinear transformation,

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} \quad .....(7.83)$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} \quad .....(7.84)$$

Note : If either T or F_s is not specified then take T = 1 sec.

If F_s is specified, then $T = \frac{1}{F_s}$

For impulse invariant transformation,

$$\Omega_p = \frac{\omega_p}{T} \quad .....(7.85)$$

$$\Omega_s = \frac{\omega_s}{T} \quad .....(7.86)$$

2. Decide the order N of the filter. In order to estimate the order N, calculate a parameter N_1 using the following equation. Choose N such that $N \geq N_1$. Usually N is chosen as nearest integer just greater than N_1.

$$N_1 = \frac{\cosh^{-1}\left[\left(\frac{(1/A_s^2)-1}{(1/A_p^2)-1}\right)^{\frac{1}{2}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} \quad .....(7.87)$$

3. Determine the normalized transfer function $H(s_n)$, of the filter.

When the order N is even,

$$H(s_n) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k} \qquad .....(7.88)$$

When the order N is odd,

$$H(s) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s^2 + b_k s + c_k} \qquad .....(7.89)$$

$$\text{where, } b_k = 2\, y_N \sin\left(\frac{(2k-1)\pi}{2N}\right) \qquad .....(7.90)$$

$$c_k = y_N^2 + \cos^2\left(\frac{(2k-1)\pi}{2N}\right) \qquad .....(7.91)$$

$$c_0 = y_N \qquad .....(7.92)$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{\frac{1}{2}} + \frac{1}{\epsilon}\right]^{-\frac{1}{N}}\right\} \qquad .....(7.93)$$

$$\epsilon = \left[\left(1/A_p^2\right) - 1\right]^{\frac{1}{2}} \qquad .....(7.94)$$

For even values of N, find B_k such that,

$$H(0) = \frac{1}{(1+\epsilon^2)^{\frac{1}{2}}} \qquad .....(7.95)$$

For odd values of N, find B_k such that,

$$H(0) = 1 \qquad .....(7.96)$$

(It is normal practice to take $B_0 = B_1 = B_2 ..... = B_k$).

4. Determine the unnormalized analog transfer function H(s) of the lowpass filter.

$$H(s) = H(s_n)\Big|_{s_n = \frac{s}{\Omega_c}}$$

Here, $\Omega_c = \Omega_p$ = Passband edge frequency.

When the order N is even, H(s) is obtained by letting $s_n \to s/\Omega_c$ in equation (7.88).

$$\therefore H(s) = \prod_{k=1}^{\frac{N}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \prod_{k=1}^{\frac{N}{2}} \frac{B_k\, \Omega_c^2}{s^2 + b_k\, \Omega_c\, s + c_k\, \Omega_c^2} \qquad .....(7.97)$$

When the order N is odd, H(s) is obtained by letting $s_n \to s/\Omega_c$ in equation (7.89).

$$\therefore\ H(s) = \frac{B_0}{s + c_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{s_n^2 + b_k s_n + c_k}\Bigg|_{s_n = \frac{s}{\Omega_c}} = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2} \qquad .....(7.98)$$

5. Determine the transfer function of digital filter, H(z). Using the chosen transformation, in step-1 transform H(s) to H(z). When impulse invariant transformation is employed, if $T < 1$, then multiply H(z) by T to normalize the magnitude.