Module-6

Realization of Digital filters

- ➔ Visual representation of filter(H(z)) helps in implementation in hardware
- ➔ 3 basic building blocks
  Adder
  Multiplier
  Delay element
- ➔ Two types of realization
  (1) Recursive
  (2) Non-recursive
- ➔ For recursive realization, the current output y(n) is a function of past outputs, past and present inputs. This form corresponds to IIR digital filter
- ➔ For non-recursive realization, the current output y(n) is a function of past and present inputs. This form corresponds to FIR digital filter
- ➔ IIR filter can be realized in the following forms:
  - Direct form-I realization
  - Direct form-II realization
  - Cascaded form
  - Parallel form
  - Transposed form
  - Lattice –Ladder

	Multiplication	Addition	Memory location
DF-I	M+N+1	M+N	M+N+1
DF-II	M+N+1	M+N	Max {M,N}

Direct form-I realization

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \tag{5.102}$$

$$\begin{aligned} &= -a_1 y(n-1) - a_2 y(n-2) \ldots - a_{N-1} y(n-N+1) \\ &\quad - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) \end{aligned} \tag{5.103}$$

Let

$$b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) = w(n). \tag{5.104}$$

then $y(n) = -a_1 y(n-1) - a_2 y(n-2) + \ldots - a_N y(n-N) + w(n)$ (5.105)

The Eq. (5.104) can be realized as shown in Fig. 5.28.

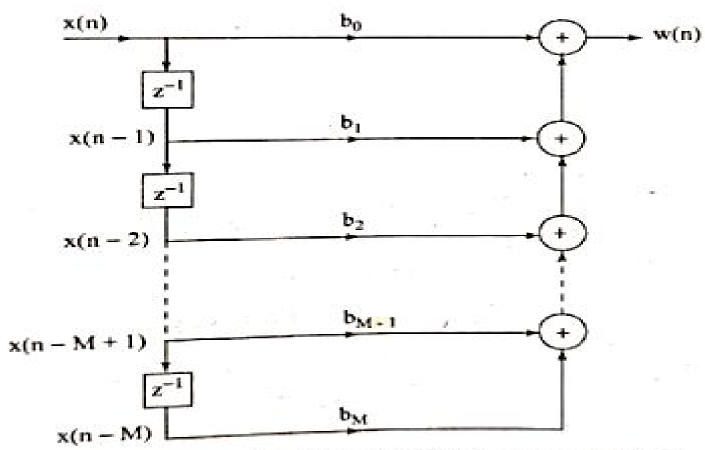


Fig. 5.28 Realization structure of Eq. (5.104)

Example 5.21 Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

Solution

Let

$$x(n) + 0.4x(n-1) = w(n) \tag{5.108}$$

then

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n) \tag{5.109}$$

Realizing Eq. (5.108) and Eq. (5.109) and combining we get

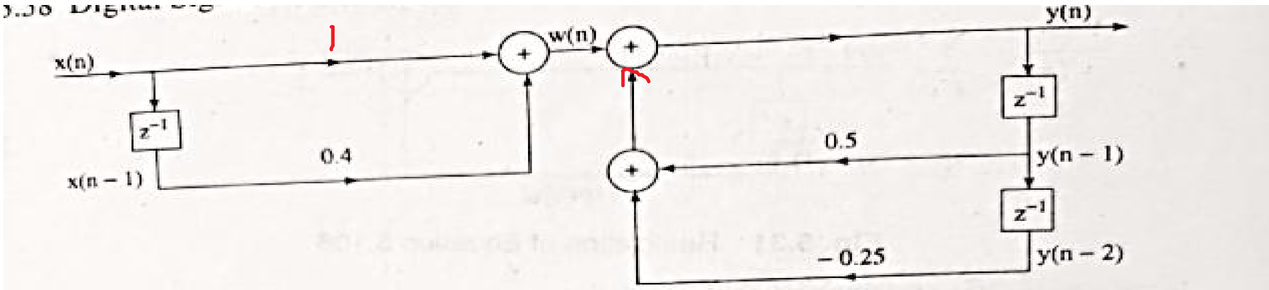


Fig. 5.34 Realization of Example 5.21

DF-I

Practice Problem 5.11 Obtain the direct form-I realization for the systems described by the following difference equations

(i) $y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$ → w(n)

(ii) $y(n) = 0.5y(n-1) + 0.06y(n-2) + 0.3x(n) + 0.5x(n-1)$ → w(n)

Direct form-II realization

Example 5.23 Determine the direct form II realization for the following system $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ ~~(AU ECE May'07)~~

Solution Apply Z-T

The system function is given by

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} \tag{5.119}$$

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

Let

$$\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2}$$
$$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$$

Then

$$y(n) = 0.7w(n) - 0.252w(n-2) \tag{5.120}$$

Similarly let $\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$

$$W(z) = X(z) - 0.1z^{-1}W(z) + 0.72z^{-2}W(z)$$

then $w(n) = x(n) - 0.1w(n-1) + 0.72w(n-2)$ (5.121)

If we realize Eq. (5.120) and Eq. (5.121) and combine them we get direct form II realization of the system shown in Fig. 5.40.

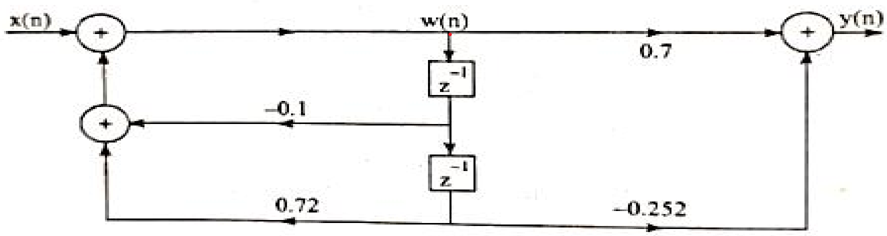


Fig. 5.40 Direct form II realization of example (5.23)

Practice Problem 5.12 Obtain the direct form-II realization for the systems described by the following difference equation

(i) $y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-2)$

(ii) $y(n) = \dfrac{3}{4}y(n-1) - \dfrac{1}{8}y(n-2) + x(n) + \dfrac{1}{2}x(n-1)$

Cascaded form

5.14.5 *Cascade Form*

Let us consider an IIR system with system function

$$H(z) = H_1(z)H_2(z)\ldots H_k(z) \tag{5.122a}$$

This can be represented using block diagram as shown in Fig. 5.46.

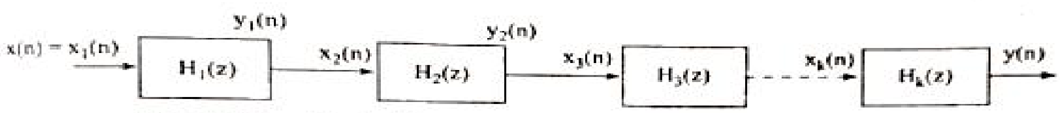


Fig. 5.46 Block diagram representation of Eq. (5.122a)

Now realize each $H_k(z)$ in direct form II and cascade all structures.
For example let us take a system whose transfer function

$$\begin{aligned} H(z) &= \frac{(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2})(b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2})}{(1 + a_{k1}z^{-1} + a_{k2}z^{-2})(1 + a_{m1}z^{-1} + a_{m2}z^{-2})} \\ &= H_1(z)H_2(z) \end{aligned} \tag{5.122b}$$

where $H_1(z) = \dfrac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$ and

$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

Realizing $H_1(z)$ and $H_2(z)$ in direct form II, and cascading we obtain cascade form of the system function.

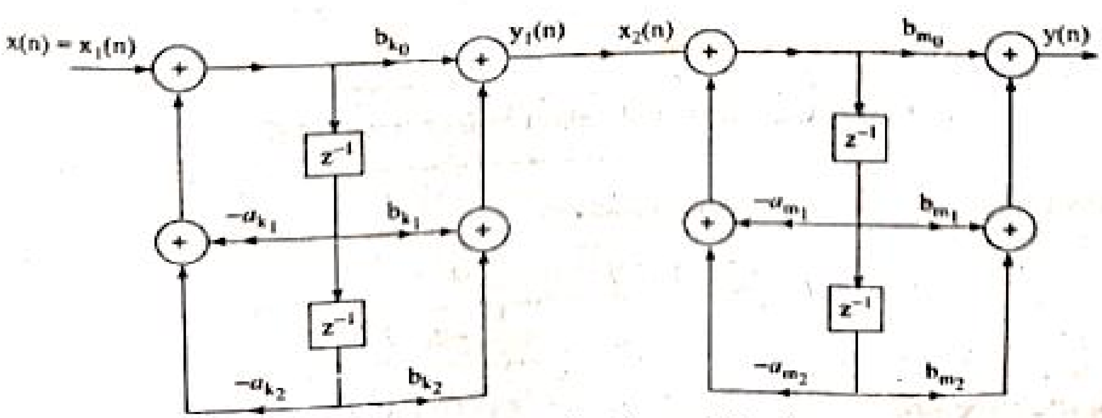


Fig. 5.47 Cascade realization of Eq. (5.122b)

Example 5.25 Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade form.

Solution

From the difference equation we obtain

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

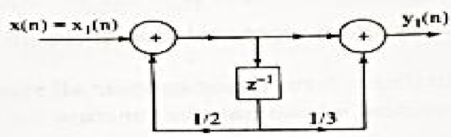
$$= \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} = H_1(z)H_2(z)$$

DF-II

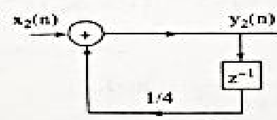
where $H_1(z) = \dfrac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$ and $H_2(z) = \dfrac{1}{1 - \frac{1}{4}z^{-1}}$.

DF-II

$H_1(z)$ can be realized in direct form II as



Similarly, $H_2(z)$ can be realized in direct form II as



Cascading the realization of $H_1(z)$ and $H_2(z)$ we have

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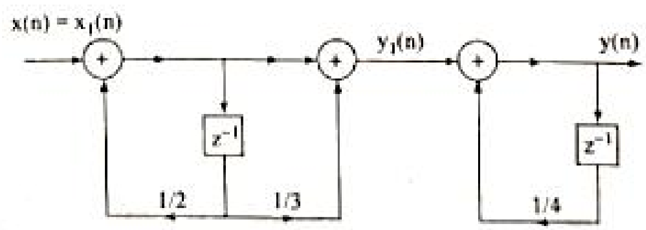


Fig. 5.48 Cascade realization of Example 5.25

Practice Problem 5.14 For the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

obtain cascade structure.

Parallel form

5.14.6 Parallel form structure

A parallel form realization of an IIR system can be obtained by performing a partial expansion of

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - p_k z^{-1}} \tag{5.123}$$

where $\{p_k\}$ are the poles

The Eq. (5.123) can be written as

$$H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \ldots + \frac{c_N}{1 - p_N z^{-1}} \tag{5.124}$$

$$H(z) = \frac{Y(z)}{X(z)} = c + H_1(z) + H_2(z) + \ldots + H_N(z) \tag{5.125}$$

Now

$$Y(z) = cX(z) + H_1(z)X(z) + H_2(z)X(z) + \ldots + H_N(z)X(z) \tag{5.126}$$

The Eq. (5.126) can be realized in parallel form as shown in Fig. 5.49.

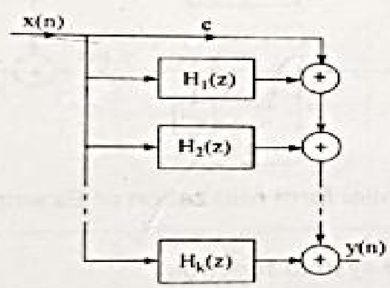


Fig. 5.49 Parallel form realization of Eq. 5.126

Example 5.26 Realize the system given by difference equation $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in parallel form.

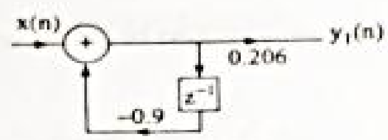
Solution

The system function of the difference equation is

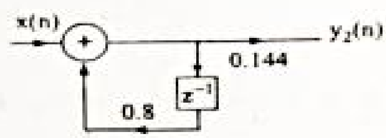
$$
\begin{aligned}
H(z) &= \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} \\
&= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.1z^{-1} - 0.72z^{-2}} \\
&= 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}} \\
&= c + H_1(z) + H_2(z)
\end{aligned}
$$

PF

$H_1(z)$ can be realized in direct form II as



$H_2(z)$ can be realized in direct form II as



Now the realization of $H(z)$ is shown in Fig. 5.50.

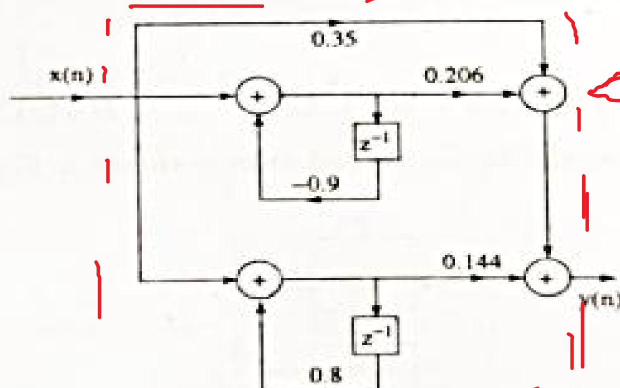


Fig. 5.50 Parallel form realization of Example 5.26

Practice Problem 5.15 For the system function

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{6}z^{-1}\right)}$$

obtain parallel structure.

Example 5.27 Obtain the direct form I, direct form II, cascade and parallel form realization for the system $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

Solution

Direct form I

Let $3x(n) + 3.6x(n-1) + 0.6x(n-2) = w(n)$

$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$

By inspection, The direct form I realization is shown in Fig. 5.51.

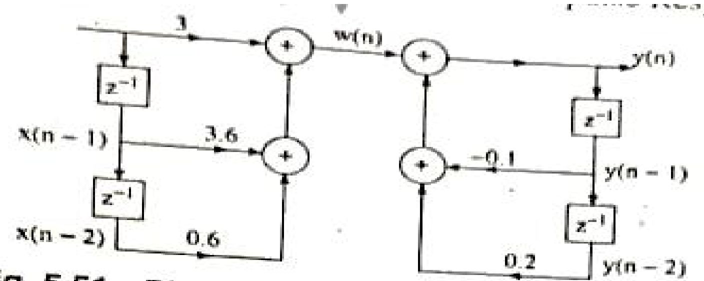


Fig. 5.51 Direct form I realization of example 5.27

Direct form II

From the given difference equation we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

The above system function can be realized in direct form II as shown in Fig. 5.52.

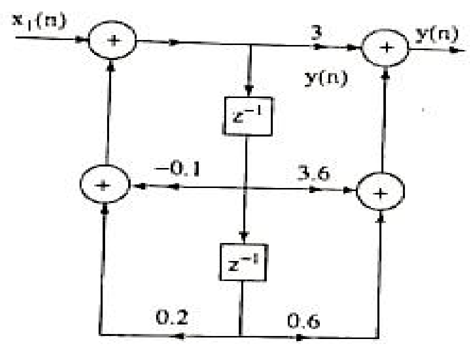


Fig. 5.52 Direct form II realization of example 5.27

Cascade form

$$\text{we have} \quad \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$= \frac{(3 + 0.6z^{-1})(1 + z^{-1})}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})}$$

$$\text{Let} \quad H_1(z) = \frac{3 + 0.6z^{-1}}{1 + 0.5z^{-1}} \quad \text{and}$$

$$H_2(z) = \frac{1 + z^{-1}}{1 - 0.4z^{-1}}$$

Now we realize $H_1(z)$ and $H_2(z)$ and cascade both to get realization of $H(z)$

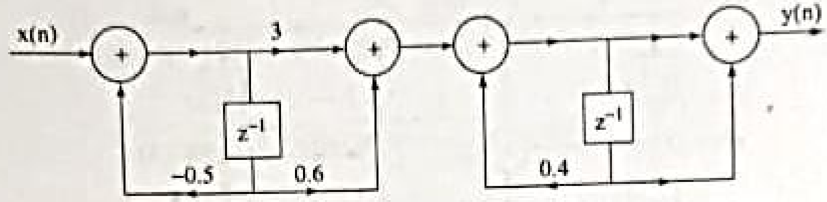


Fig. 5.53 Cascade form realization of example 5.27

Parallel form

$$\begin{aligned}H(z) &= \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}\\ &= -3+\frac{7}{1-0.4z^{-1}}-\frac{1}{1+0.5z^{-1}}\\ &= c+H_1(z)+H_2(z)\end{aligned}$$

$$\begin{array}{r|l} & -3 \\ -0.2z^{-2}+0.1z^{-1}+1 & 0.6z^{-2}+3.6z^{-1}+3 \\ & 0.6z^{-2}-0.3z^{-1}-3 \\ \hline & 3.9z^{-1}+6 \end{array}$$

$$\rightarrow \quad H(z) = -3+\frac{3.9z^{-1}+6}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3+\frac{A}{1-0.4z^{-1}}+\frac{B}{1+0.5z^{-1}}$$

where $A=7, B=-1$

Now we realize $H(z)$ in parallel form as shown in Fig. 5.54.

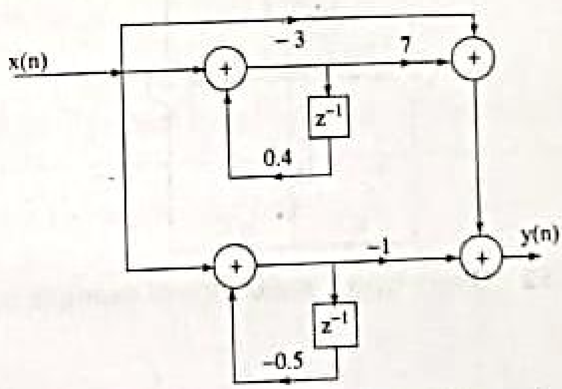


Fig. 5.54 Parallel form realization of example 5.27

Example 5.28 Obtain the cascade realization for the following systems

(a) $H(z) = \dfrac{\left(1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}\right)\left(1-\frac{3}{2}z^{-1}+z^{-2}\right)}{\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}$

(b) $H(z) = \dfrac{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}{\left(1+\frac{1}{4}z^{-1}\right)\left(1+z^{-1}+\frac{1}{2}z^{-2}\right)\left(1-\frac{1}{4}z^{-1}+\frac{1}{2}z^{-2}\right)}$