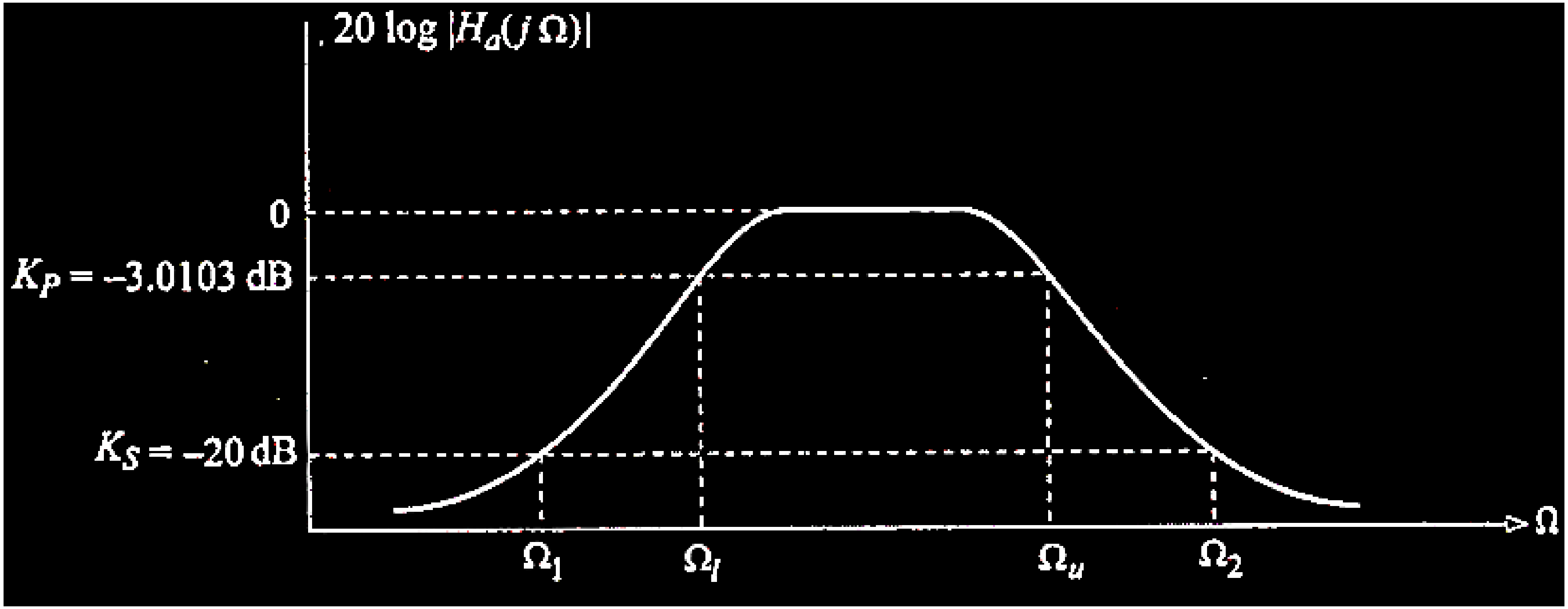
- Design an analog bandpass filter to meet the following frequency-domain specification:
  - a -3.0103 dB lower and upper cutoff frequency of 50 Hz and 20 KHz.
  - a stopband attenuation of atleast 20 dB at 20 Hz and 45 KHz, and
  - a monotonic frequency response.

Solution :

The monotonic frequency response can be achieved by using Butterworth filter.

Step : 1 : Specified magnitude frequency response of the bandpass Butterworth filter.



$\Omega_1 = 2\pi \times 20 = 125.663\ rad/sec$

$\Omega_2 = 2\pi \times 45 \times 10^3 = 2.827 \times 10^5 rad/sec$

$\Omega_u = 2\pi \times 20 \times 10^3 = 1.257 \times 10^5\ rad/sec$

$\Omega_l = 2\pi \times 50 = 314.159\ rad/sec$

- If the given filter is Bandpass then the backward design equation to find the stopband edge frequency

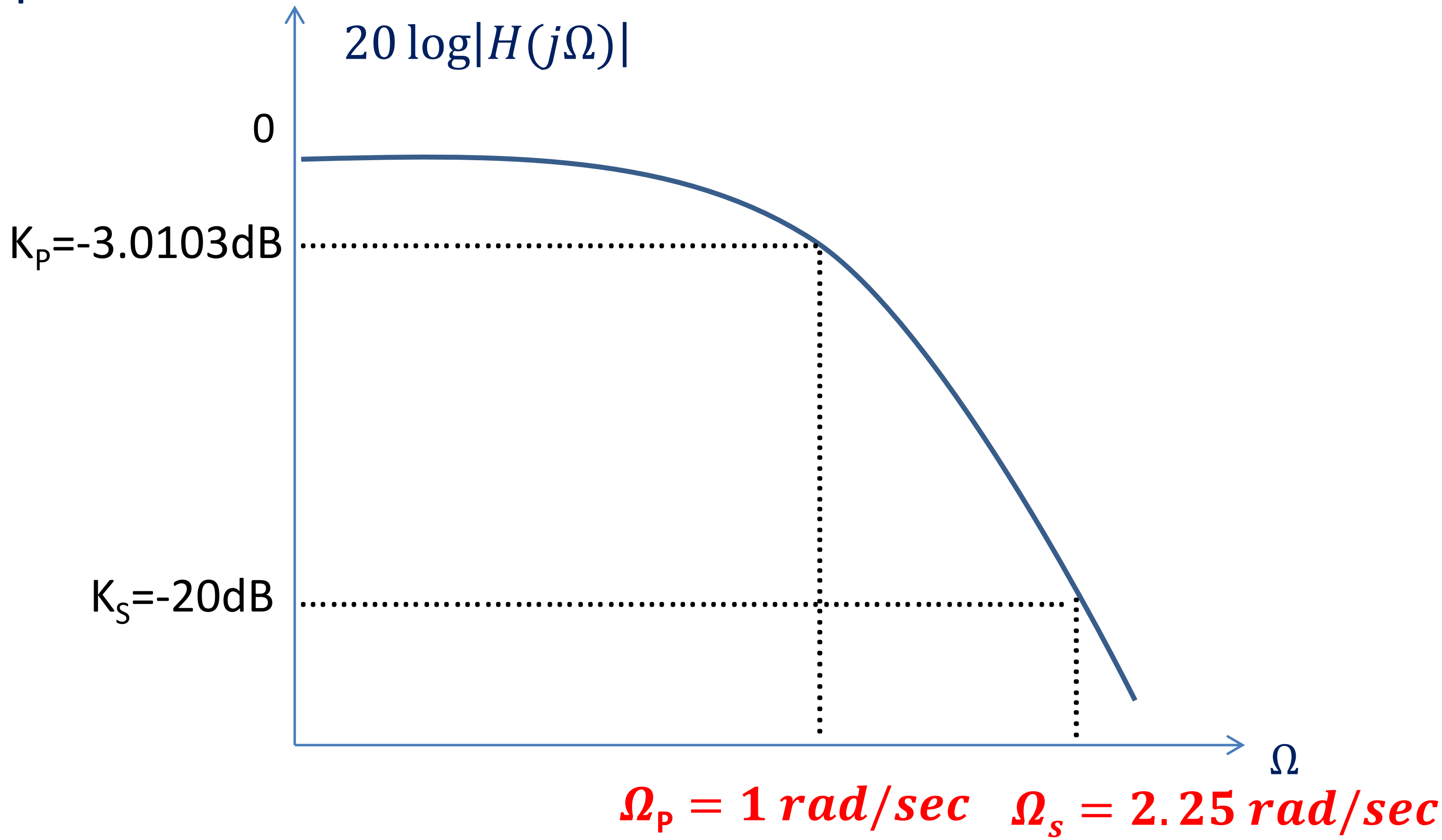
- $\boldsymbol{\Omega_s} = Min\{|A|, |B|\}.$

Where $A = \frac{-\Omega_1{}^2 + \Omega_l \Omega_u}{\Omega_1\,(\Omega_u - \Omega_l)} = 2.51$

$$B = \frac{\Omega_2{}^2 - \Omega_l \Omega_u}{\Omega_2\,(\Omega_u - \Omega_l)} = 2.25$$

$$\boldsymbol{\Omega_s} = Min\{|A|, |B|\} = 2.25$$

- **Step 2**: Magnitude frequency response of the normalized lowpass filter



The pass band edge frequency $\boldsymbol{\Omega_P}$ of the normalized low pass filter is 1 rad/sec

Step 3: Find the order N of the filter using equation (14).

Sub K_P=-3.0103 dB, K_S=-20 dB, Ω_P=1 rad/sec, Ω_S=2.25 rad/sec.

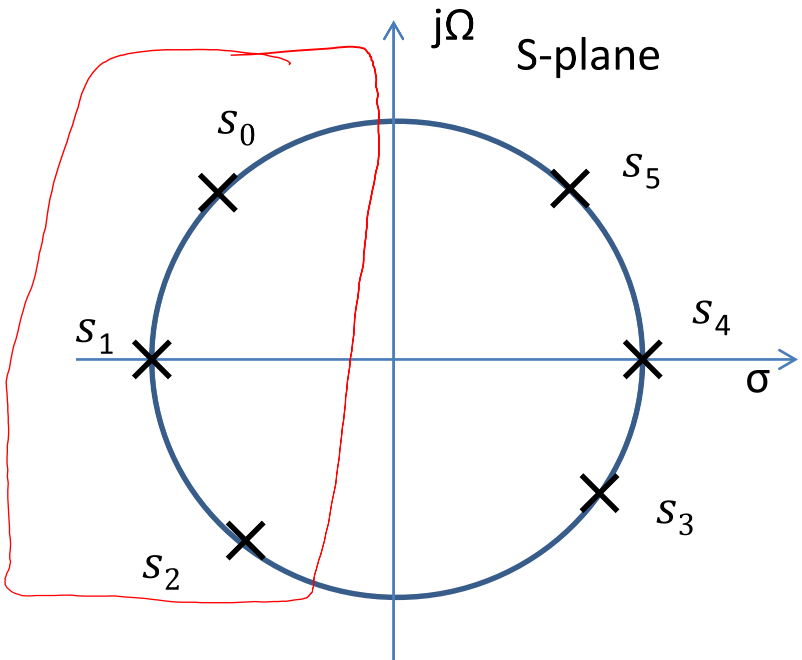
$$N = \frac{log\left[\left(10^{\frac{-K_P}{10}}-1\right)\Big/\left(10^{\frac{-K_S}{10}}-1\right)\right]}{2log\left(\frac{\Omega_P}{\Omega_S}\right)} = 2.83 = 3$$

Step 4: Now proceed to find the transfer function of the 3rd order normalized lowpass filter. Find the poles of the 3^{rd} order normalized low pass filter using equation (5)

$$s_k = e^{j\pi\frac{(2k+1)}{2N}} e^{j\frac{\pi}{2}} \qquad k = 0,1,2, \dots \dots 2N-1$$

N=3, so, $K = 0, 1, 2,3,4,5$ $\theta = \frac{\pi k}{N} + \frac{\pi}{2N} + \frac{\pi}{2}$

Total 6 poles from S_0 to S_5



Left-half poles of $H_3(s)$

Right-half poles of $H_3(-s)$

Poles	σ+jΩ
s_0	-0.5+j0.866
s_1	-1
s_2	-0.5-j0.866
s_3	0.5-j0.866
s_4	1
s_5	0.5+j0.866

Step 5:

Hence, the transfer function of the 3rd order normalized lowpass Butterworth filter is

$$H_{N(s)} = \frac{1}{\prod_{LHP}(s - s_k)}$$

- $H_3(s) = \dfrac{1}{(s-s_0)(s-s_1)(s-s_2)}$

- $H_3(s) = \dfrac{1}{s^3+2s^2+2s+1}$

- **STEP 6: Applying the lowpass to bandpass transformation to $H_3(s)$**

$$H_a(s) = H_3(s)|s \to \frac{s^2 + \Omega_u\Omega_l}{s(\Omega_{u-}\Omega_l)}$$

$$= H_3(s)|s \to \frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}$$

$$H_a(s) = \frac{1.9695 \times 10^{15} s^3}{(s^6+2.51 \times 10^5 s^5 + 3.154 \times 10^{10} s^4 + 1.989 \times 10^{15} s^3 + 1.2453 \times 10^{18} s^2 + 3.907 \times 10^{20} s + 6.152 \times 10^{22})}$$

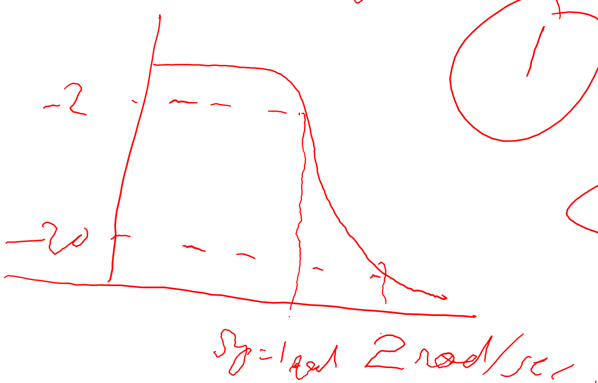
3) Design a monotonous filter for foll. specification.

Passband Attn = 2dB

Passband edge freq = 200 rad/sec.

Stop band Attn = 20 dB

Stopband edge freq = 600 rad/sec.



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