21/7

MOD-3 FOURIER TRANSFORM → Fourier series (periodic)

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 n}}$$

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\omega n}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$X[jn] = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

↓

Fourier transform (Aperiodic)

Q) $x[n] = 1 + 2\cos\frac{2\pi}{3}n$

$$x[n] = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 n}$$

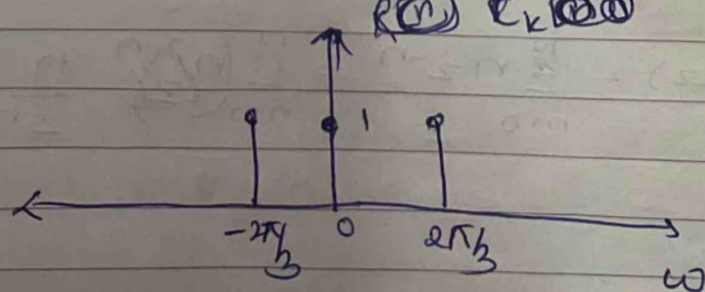
$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\omega n}$$

$$1 + 2\cos\frac{2\pi}{3}n = \overset{c_0}{1} + \overset{c_1}{e^{j\frac{2\pi}{3}n}} + \overset{c_{-1}}{e^{-j\frac{2\pi}{3}n}}$$

$c_0 = 1 \quad c_1 = c_{-1} = 1$

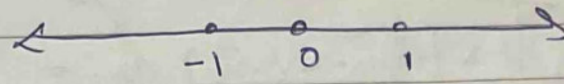
Magnitude response $|C_k|$

$R(n)$ R_k



Phase response

$\angle C_k = \tan^{-1}(b/a)$ $\qquad$ $C_0 = 1 = 1 + 0i$



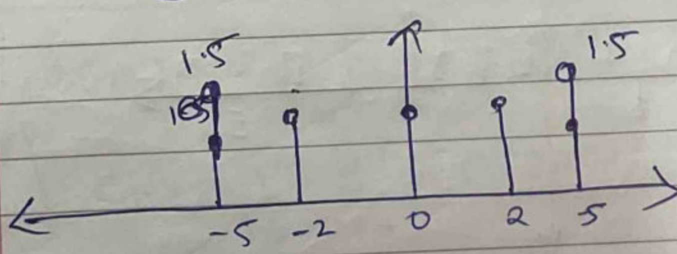
Q) $x[n] = 1 + 2\cos\frac{2\pi}{3}n + 3\sin\frac{5\pi}{3}n$

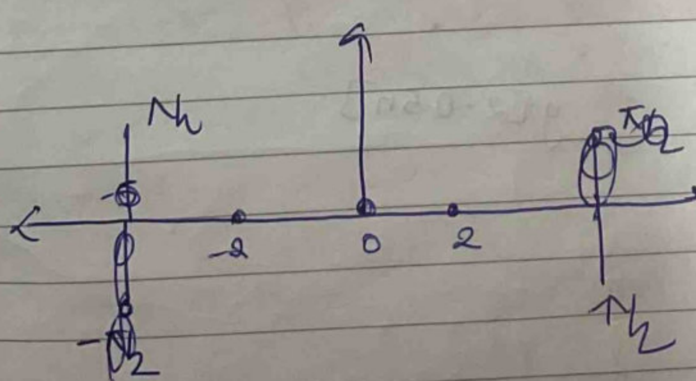
$= 1 + e^{j\frac{2\pi}{3}n} + e^{-j2\pi/3} + \frac{3}{2}\left(\frac{e^{j5\pi/3\, n} - e^{-j5\pi n/3}}{j}\right)$

$= \cancel{1 + \frac{5}{2}e^{j7\pi/3}}$ $\qquad$ $\omega_0 = \pi/3$

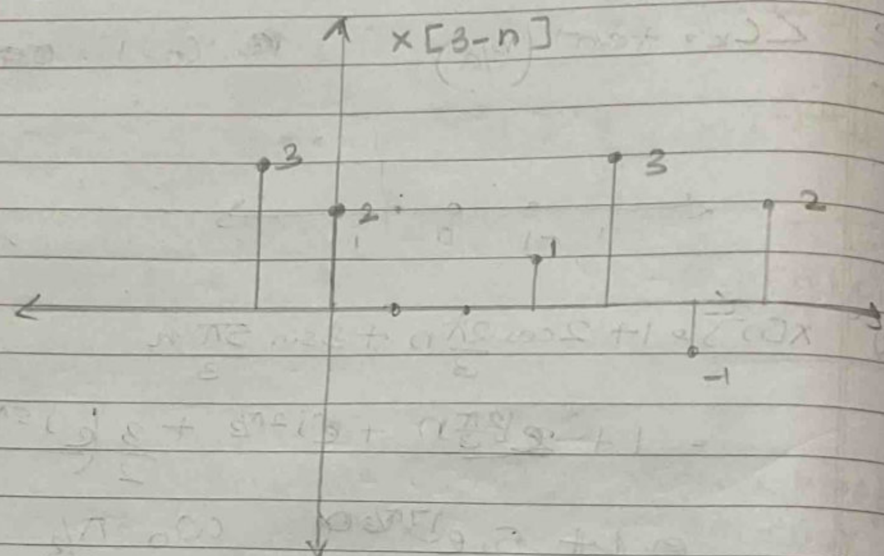
$C_0 = 1$ $\quad C_2 = 1$ $\quad C_{-2} = 1$ $\quad C_{-5} = C_5 = \frac{3}{2j} = +\frac{3}{2}j$

Q. Magnitude response

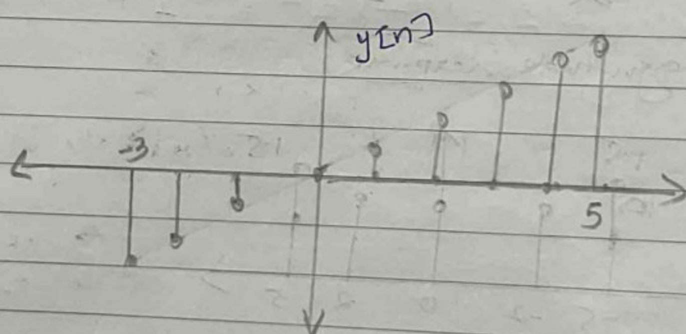


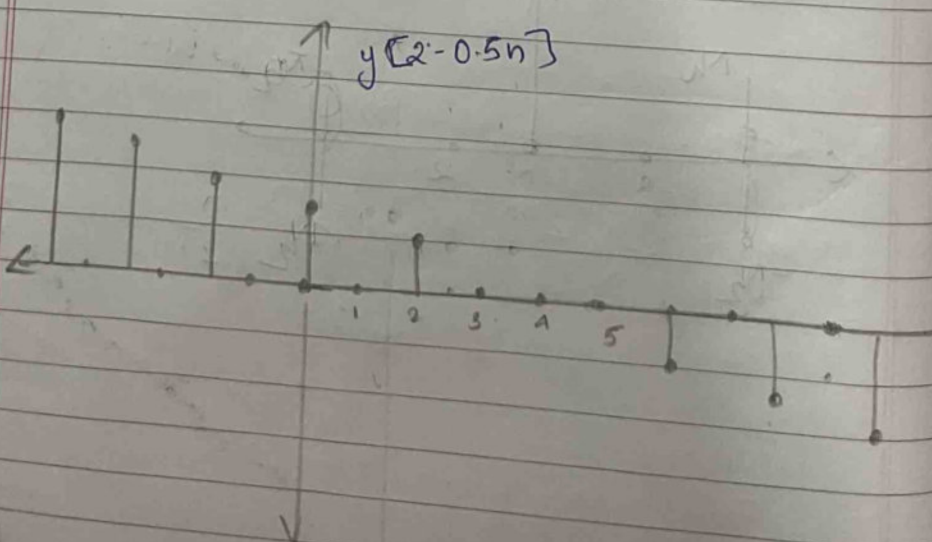


7) ~~y~~ $x[n] = \{2, -1, 3, \underset{\uparrow}{1}, 0, 0, 2, 3\}$



(iii) $y[n] = (r[n] + r[-n])(u[n+3] - u[n-5])$





(c) $y[n] = (x[n] + x[-n]) \{u(n+3) - u(n-5)\}$

$y[-n] = (x[-n] + x[n]) \{u(n+3) - u(n-5)\}$

$y_e[n] = \frac{1}{2}[y[n] + y[-n]]$

$= \frac{1}{2}(2x[n] + 2x[-n])$

$= x[n] + x[-n]$

$y_o[n] = \frac{1}{2}[y[n] - y[-n]]$

$= \underline{\underline{0}}$

$\therefore y[n] = y_e[n] + y_o[n]$

$= (x[n] + x[-n]) \{u(n+3) - u(n-5)\}$

FOURIER SERIES

$x[n] = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega n}$

$c_k = \frac{1}{N_0} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega n}$

$\boxed{\Omega = \frac{2\pi}{N_0}}$

properties: Periodicity $= c_{k+N_0} = c_k$

Duality $= x[n] \xleftrightarrow{DFS} c_k = c[k]$

$c[n] \longleftrightarrow \frac{1}{N_0} x[-k]$

Parseval

$\frac{1}{N_0} \sum_{n=-N_0}^{N_0} |x[n]|^2 = \sum_{k=-N_0}^{N_0} |c_k|^2$

Fourier transform $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$x[n] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega n} d\omega$

6) $y[n] = (x[n] + x[-n])\{u[n+3] - u[n-5]\}$

$y[-n] = (x[-n] + x[n])\{u[-n+3] - u[-n-5]\}$

$y_o[n] = \frac{1}{2}(y[n] - y[-n])$

$= \frac{1}{2}(x[n] + x[-n])(u[n+3] - u[n-5] - u[-n+3] + u[-n-5])$

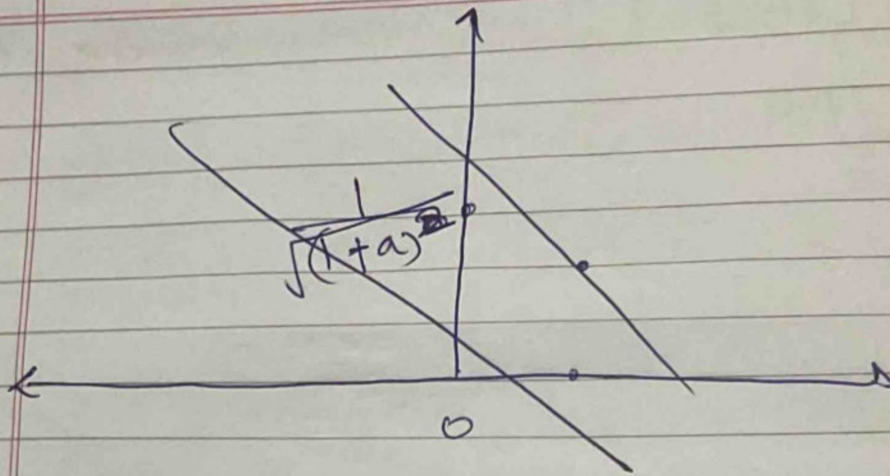
$y_e[n] = \frac{1}{2}(y[n] + y[-n])$

$= \frac{1}{2}(x[n] + x[-n])(u[n+3] - u[n-5] + u[-n+3] - u[-n-5])$

$y[n] = y_o[n] + y_e[n]$

$= \frac{1}{\cancel{2}}(x[n] + x[-n]) \times \cancel{2} \times [u(n+3) - u(n-5)]$

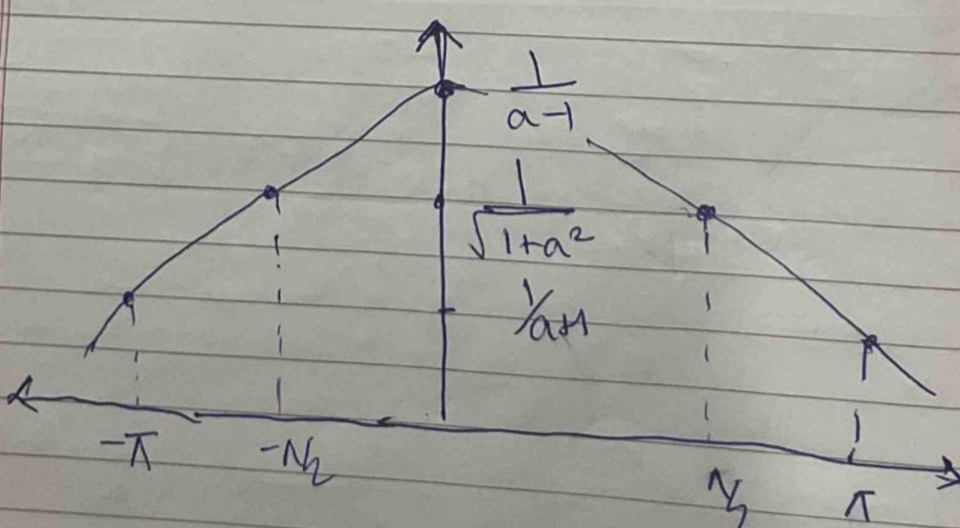
$= [x[n] + x[-n]][u[n+3] - u[n-5]]$



$\omega = 0 = \frac{1}{\sqrt{(a-1)^2}} = \frac{1}{a-1}$

$\omega = \pi/2 = \frac{1}{\sqrt{1+a^2}}$ $\omega = -\pi/2 = \frac{1}{\sqrt{1-a^2}}$

$\omega = \pi = \frac{1}{(1+a)}$ $\omega = -\pi = \frac{1}{(1+a)}$



DTFT

$$X(e^{j\omega}) = X(j\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Q) $x[n] = a^n u[n]$ $\quad -1 < a < 1$

$-\infty \; -1$

$$X[\omega] = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \frac{1}{1 - a/e^{j\omega}} = \frac{1}{1 - ae^{-j\omega}}$$

$|a < e^{j\omega}$

$$= \frac{1}{(1 - a\cos\omega) + ja\sin\omega}$$

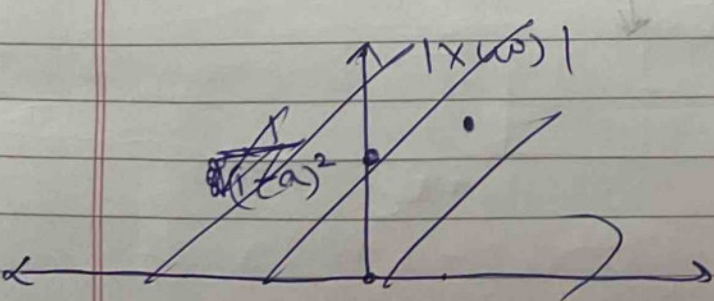
$$= \frac{(1 - a\cos\omega) - ja\sin\omega}{(1 - a\cos\omega)^2 + (a\sin\omega)^2}$$

$$= \frac{1 - a\cos\omega}{(1 - a\cos\omega)^2 + (a\sin\omega)^2} - \frac{ja\sin\omega}{(1 - a\cos\omega)^2 + (a\sin\omega)^2}$$

$$|X(\omega)| = \frac{\sqrt{(1 - a\cos\omega)^2 + (a\sin\omega)^2}}{(1 - a\cos\omega)^2 + (a\sin\omega)^2}$$

$$= \frac{1}{\sqrt{(1 - a\cos\omega)^2 + (a\sin\omega)^2}}$$

$\frac{1}{1-a^2}$



$$= \cancel{\frac{1}{\sqrt{1 - a^2\cos^2\omega - a^2\sin^2\omega}}}$$

$$= \frac{1}{\sqrt{1 - 2a\cos\omega + a^2}}$$

$$= \frac{1}{\;}$$

Q) $x[n] = 1 \quad |n| \leq 1$

$\quad\quad\quad\; 0 \quad$ otherwise

⇒ DTFT

$$X(j\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-1}^{1} e^{-j\omega n}$$

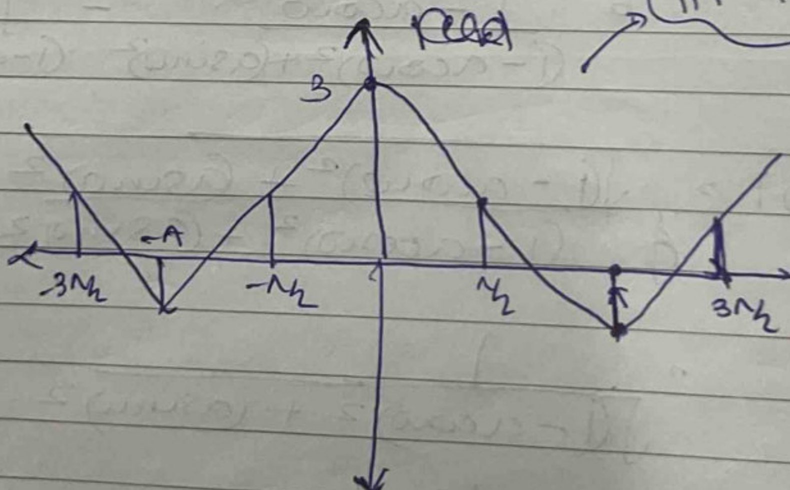
$$= \sum_{n=-1}^{1} e^{-j\omega n}$$

$$= e^{j\omega} + \cancel{e^{0}}^{\,1} + e^{-j\omega}$$

$$= 1 + 2\cos(\omega)$$

→ Magnitude response

11th to sinc fn



→ Phase response

Fourier

$$\text{sinc} = \frac{\sin \pi}{\pi}$$

Q) $x[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$

$$X(j\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{0}^{N-1} e^{-j\omega n}$$

$$\frac{1-r^n}{1-r}$$

$$X(j\omega) = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}$$

$$= \frac{e^{-j\omega N/2}\left[e^{+j\omega N/2} - e^{-j\omega N/2}\right]}{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}$$

$$= e^{-j\omega[N-1]/2} \frac{\left[\cancel{2j}\sin(\omega N/2)\right]}{\left[\cancel{2j}\sin(\omega/2)\right]}$$

$$= e^{-j\omega[N-1]/2} \frac{\left[\sin(\omega N/2)\right]}{\left[\sin(\omega/2)\right]}$$

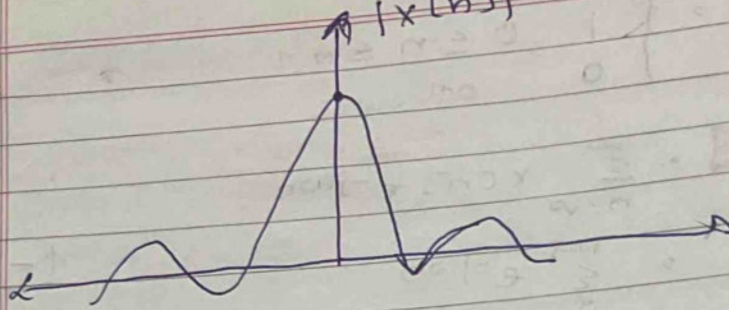
Q) $X(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & |\omega| \le \pi \end{cases}$

$$x[n] = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega = \left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c} \frac{1}{2\pi}$$

$$= \left[\frac{e^{j\omega_c n} - e^{-j\omega_c n}}{jn}\right] \frac{1}{2\pi}$$

$$= \frac{\cancel{2j}\sin \omega_c n}{\cancel{2}\pi \cancel{j} n}$$

$$x[n] = \frac{\sin \omega_c n}{\pi n}$$



Q) $x[n] = \delta(n)$

$X(\omega) = \sum_{n=0} 1$

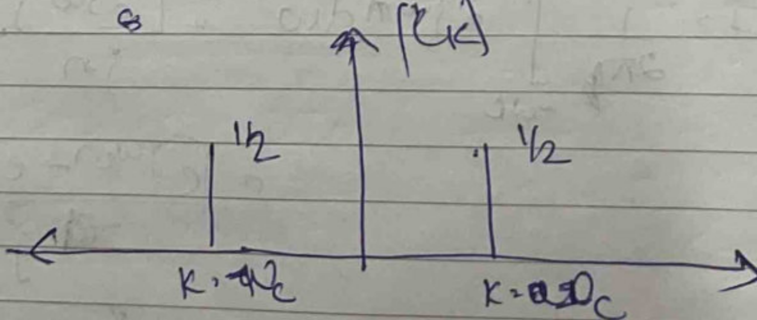
Q) $x[n] = u[n]$

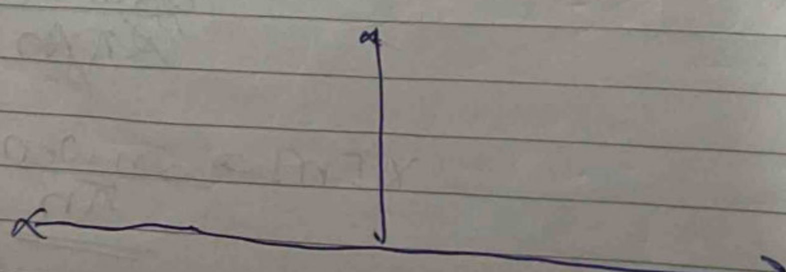
$X(\omega) = \dfrac{1}{1 - e^{-j\omega}}$

Q) $x[n] = \sin \omega_0 n \rightarrow$ ~~Period~~ Periodic signal

$X(\omega) = \sum_{n=-\infty}^{\infty} \sin \omega_0 n \, e^{-j\omega n}$ Apply fourier series

$= \dfrac{1}{2j}\left[e^{j\omega_0 n} - e^{-j\omega_0 n}\right]$





Date
Page

3/2/23 LAB-6 Introductions to CCS

1) Basic Sine wave
2) Multifrequency

21BLC1141

O/P verified

3/2/23

Property

Periodicity

Scaling

$x(at) = \frac{1}{|a|} X(w/a)$

$X(e^{j(\omega+2\pi)}) = X(e^{j\omega}$

$X(\omega + 2\pi) = X(\omega)$

Time shifting

$x(n-n_0) = e^{-j\omega_0 n_0} X(e^{j\omega})$

Linearity

$a x_1[n] + b x_2[n] \rightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$

$\cancel{\sum_{n=-\infty}^{\infty} (a x_1[n] + b x_2[n]) e^{-j\omega_0 n}}$

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n-n_0) e^{-j\omega_0 n}$

$= \sum_{m=-\infty}^{\infty} x(m) e^{-j\omega\cdot(m+n_0)}$

$= \sum_{m=-\infty}^{\infty} x(m) e^{-j\omega m} \cdot e^{-j\omega n_0}$

$= e^{-j\omega n_0} X(+j\omega)$

Frequency shifting

$$e^{j\omega_0 n} x[n] \rightarrow X(e^{j(\omega-\omega_0)})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} e^{j\omega_0 n} x[n]\, e^{-j\omega_0 n}$$

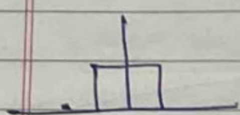
$$= \sum_{n=-\infty}^{\infty} x[n]\, e^{-jn[\omega-\omega_0]}$$

$$= X(e^{j(\omega-\omega_0)})$$

Q) $x[n] = u[n-5]$ (DTFT)

$$X(e^{j\omega}) = e^{-5j\omega} \times \frac{1}{1-e^{-j\omega}}$$

X:



$\Longleftrightarrow$ $\text{sinc} = \dfrac{\sin n\pi}{n\pi}$

Q) Time reversal

$$x[-n] = X(e^{-j\omega})$$

Q) ~~$x[n] = \sin \frac{3\pi}{8} n$~~

Q) First difference

$$x[n] - x[n-1] \leftrightarrow (1-e^{-j\omega})\, X(\omega)$$

Differentiation in Frequency

$$n\, x[n] \leftrightarrow j \frac{dX(\omega)}{d\omega}$$

Impulse train $\Rightarrow x[n] = \delta[x - kN]$

Summation

constant

$$\sum_{k=-\infty}^{\infty} x(k) \leftrightarrow \pi X(0)\,\delta(\omega) + \frac{X(\omega)}{1 - e^{-j\omega}}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega_0 n}$$

$$\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} x[n] \frac{d}{d\omega} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, (-jn)\, e^{-j\omega n}$$

$$j\frac{dX(\omega)}{d\omega} \Leftrightarrow n\,x[n]$$

$$\sum_{k=-\infty}^{\infty} x(k) = \sum_{k=\infty}^{\infty} x(k)\,\delta(n-k)$$

Parsevals Identity

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_0^{2\pi} |X(e^{j\omega})|^2\, d\omega$$

Duality

$$x[n] \leftrightarrow 2\pi\, X(\omega)$$

$$\sum_{n=-\infty}^{\infty} x(n)\, x^*(n) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} x^*(n) \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) \left(\sum_{n=-\infty}^{\infty} x^*(n)\, e^{j\omega n}\right) d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, X^*(\omega)\, d\omega = \frac{1}{2\pi}\int_0^{2\pi} |X(\omega)|^2\, d\omega$$

Duality

Eg $\delta[n] \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(\omega)$

$$\boxed{\begin{aligned} x[n] &\leftrightarrow X(\omega) \\ X(n) &\leftrightarrow 2\pi x(\omega) \end{aligned}}$$

Q) $x[n] = \cos\frac{\pi}{3}n$

$x[n] = \frac{e^{j\pi/3 n} + e^{-j\pi/3 n}}{2}$

→ freq shifting

$e^{j\pi/3 n} \cdot 1 \longleftrightarrow 2\pi\delta(\omega - \pi/3)$

$1 \longleftrightarrow 2\pi\delta(\omega)$

$e^{-j\pi/3 n} \cdot 1 \longleftrightarrow 2\pi\delta(\omega + \pi/3)$

$\Longrightarrow X(\omega) = \pi\delta(\omega - \pi/3) + \pi\delta(\omega + \pi/3)$

Convolution property

$x_1[n] * x_2[n] \leftrightarrow X_1(\omega) \cdot X_2(\omega)$

Q) $x_1[n] = (1/2)^n u[n] \qquad x_2[n] = n(1/3)^n u[n]$

$F[x_1[n] * x_2[n]] = \left[\frac{1}{1 - \frac{e^{-j\omega}}{2}}\right] * j\frac{d}{d\omega}\left[\frac{1}{1 - \frac{e^{-j\omega}}{3}}\right]$

$= \left(\frac{2}{2 - e^{-j\omega}}\right) \times j3 + \frac{j e^{-j\omega}}{(3 - e^{-j\omega})^2}$

$(3 - e^{-j\omega})^{-1} = \frac{-e^{-j\omega}}{(3 - e^{-j\omega})^2} \times -j$

$= \frac{2}{2 - e^{-j\omega}} \times \frac{3e^{-j\omega}}{(3 - e^{-j\omega})^2}$

classmate

Date
Page

$\frac{1}{1-\frac{e^{j\omega}}{3}}$ $\quad$ $\frac{\cancel{e}}{1-\frac{1}{3e^{j\omega}}}$

$\cancel{\frac{d}{d\omega}\frac{e^{j\omega}}{e^{j\omega}-1/3}} = \cancel{(je^{j\omega})(e^{j\omega}-1/3)} - \cancel{e^{j\omega}(je^{j\omega})}$

$= \cancel{-\frac{1}{3}je^{j\omega}}$

Q) $y[n] - \frac{1}{2}y[n-1] = 3x[n]$

$$H(\omega) = \frac{3}{1-\frac{1}{2}e^{-j\omega}}$$

IDTFT $\Downarrow$

$$h(n) = 3\left(\frac{1}{2}\right)^n u(n)$$

$$H(\omega) = \frac{3}{1-\frac{1}{2}[\cos(\omega) - j\sin(\omega)]}$$

$$= \frac{3}{\left[\left(1-\frac{1}{2}\cos\omega\right) + \frac{1}{2}\sin\omega\right]}$$

$$|H(\omega)| = \frac{3}{\sqrt{\left(1-\frac{1}{2}\cos\omega\right)^2 + \left(\frac{\sin\omega}{2}\right)^2}}$$

$$= \frac{3}{\sqrt{1 - \cancel{2}\times\frac{1}{\cancel{2}}\cos\omega + \frac{\cos^2\omega}{4} + \frac{\sin^2\omega}{4}}}$$

$$= \frac{3}{\sqrt{\frac{5}{4} - \cos\omega}}$$

$\omega = 0$

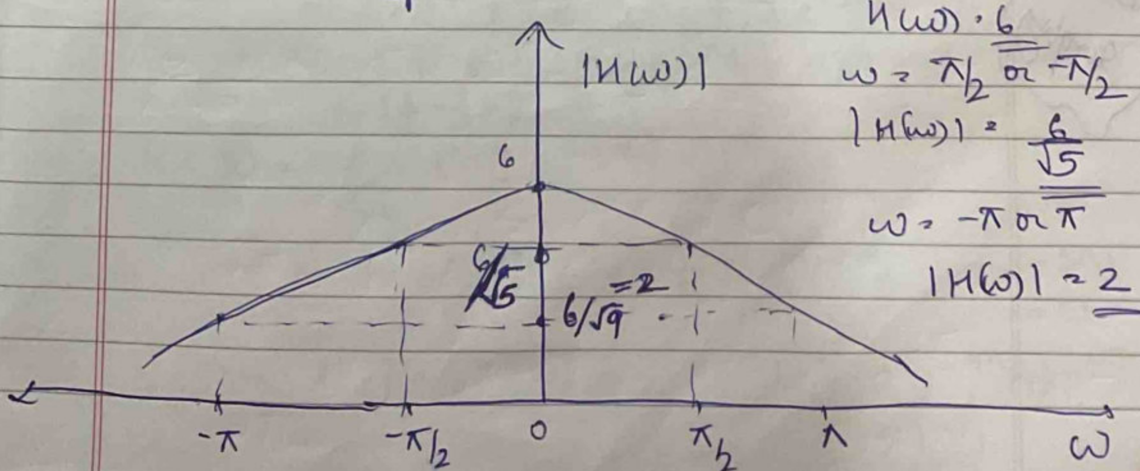
$H(\omega) = 6$

$\omega = \pi/2$ or $-\pi/2$

$|H(\omega)| = \frac{6}{\sqrt{5}}$

$\omega = -\pi$ or π

$|H(\omega)| = 2$

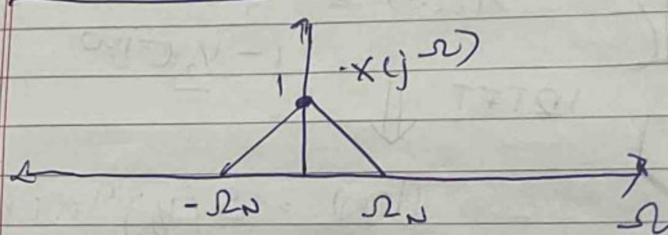


Aperiodic → Continous (Mag spectrum)
periodic → Discrete (Mag Spectrum)

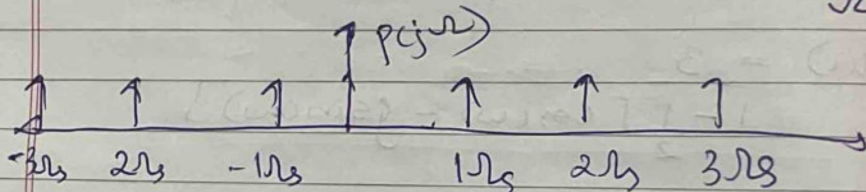
Sampling :- To convert continous signal to discrete

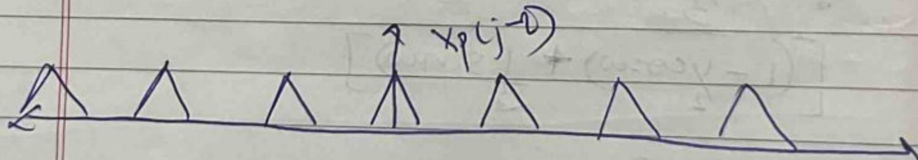
$x(t = n/f_s)$

Band Limited



$\Omega_s > 2\Omega_N$





Nyquist Rate

Let x(t) be a band limited signal with $X(j\omega) = 0$, $|\omega| > W_m$ then x(t) determined by its samples

$$\boxed{\Omega_s \geq 2\Omega_N}$$

Learn sampling

DFT

Problem of DTFT → Mag spectra is continuous

$$x[n] \xleftrightarrow{DFT} X(\Omega) \xleftrightarrow{DFT} X\left(\frac{2\pi}{N}k\right)$$

$\underbrace{\qquad\qquad}_{\text{sampling}}$

$$f(t) = f(f_s/N)$$

$$\boxed{X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}nk} \quad (k = 0 \ldots N-1) \quad \text{Analysis eqn}}$$

$x[n] = \{1, 1, 2, 2\}$ $\quad N = 4$ $\quad \therefore k \to 0 - 3$

$\Rightarrow X(0) = \sum_{n=0}^{N-1} x[n] = 6$

$\Rightarrow X(1) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{4}n\cdot 1}$

$= \sum_{n=0}^{N-1} x[n]\, e^{-j\pi/2\, n}$ $\quad 1e^{0} + 1e^{-j\pi/2} + 2e^{-j\pi/2 \cdot 2}$

$= \cancel{e^{-j\pi/2} + e^{-j\pi/2} + 2e^{-j\pi/2} + 2e^{-j\pi/2}}$

$= \cancel{6e^{-j\pi/2}}$

$\cancel{X(2) = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi n}}$

$= 1\cdot e^{0} + 1e^{-j\pi/2} + 2e^{-j\pi/2 \times 2} + 2e^{-j\pi/2 \times 3}$

$= 1 + e^{-j\pi/2} + 2e^{-j\pi} + 2e^{-3\pi/2 j}$

$= 1 - j - 2 + 2j = -1 + j$

$\Rightarrow X(2) = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi}{4}n\cdot 2}$

$= 1\cdot e^{0} + 1e^{-j\pi} + 2e^{-2\pi j} + 2e^{-3\pi j}$

$= 0$

$\Rightarrow X(3) = 1 + e^{-3\pi/2 j} + 2e^{-3\pi j} + 2e^{-9\pi/2 j}$

$= -1 - j$

Sythesis analysis

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, e^{j 2\pi/N\, nk}$$

$n = 0, 1 \ldots N-1$

$X(k) = \{ \cancel{\ }, 6, -1+j, 0, -1-j \}$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, \omega_N^{nk*}$$

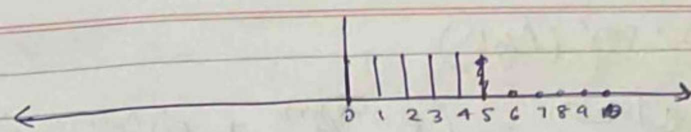
$$x[n] \Rightarrow \frac{1}{N} \left[\sum_{k=0}^{N-1} x^*(k)\, \omega_N^{nk} \right]^*$$

$\omega_N^{nk} = e^{-j\frac{2\pi}{N}}$

Eg 64

$\rightarrow$

Eg 64



Plote $|c_k|$?

$\rightarrow \omega = \frac{2\pi}{N_0} = \frac{2\pi}{10} = \pi/5$

$c_k = \frac{1}{N} \sum_{n=0}^{9} x[n]\, e^{-j\omega n k}$

$= \frac{1}{10} \sum_{n=0}^{4} x[n]\, e^{-jn\pi/5\, k}$

$= \frac{1}{10}\left[\cancel{1 + e^{-j\pi/5 k} + e^{-j2\pi/5 k} + e^{-j3\pi/5 k} + e^{-j4\pi/5 k}}\right]$

$= \cancel{\frac{1}{10}\left[4\right.}$ $\boxed{= \frac{1}{10}\, \frac{1-\left(e^{-j\pi/5 k}\right)^5}{1-e^{-j\pi/5}} \Leftrightarrow \left(\frac{1-e^{N+1}}{1-e}\right)}$

$= \frac{1}{10}\left[\frac{1-e^{-j\pi k}}{1-e^{-j\pi/5}}\right]$

$= \frac{1}{10}\left[\frac{e^{-j\pi/2 k}}{e^{-j\pi/10 k}}\left[\frac{e^{j\pi/2 k} - e^{-j\pi/2 k}}{e^{j\pi/10 k} - e^{-j\pi/10 k}}\right]\right]$

$= \frac{1}{10}\left[e^{-2\pi/5 k}\left[\frac{\sin(\pi/2 k)}{\sin(\pi/10 k)}\right]\right]$

$\frac{\pi}{2} - \frac{k}{10}$

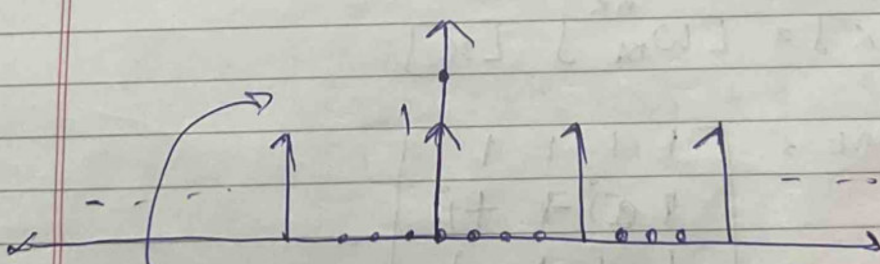
$\frac{10\pi - 2\pi}{20}$

$\frac{8\pi}{20}$

$\frac{\pi}{2} - \frac{\pi}{10}$

$\frac{10\pi - \pi}{20}$

$\frac{5\pi}{10}$

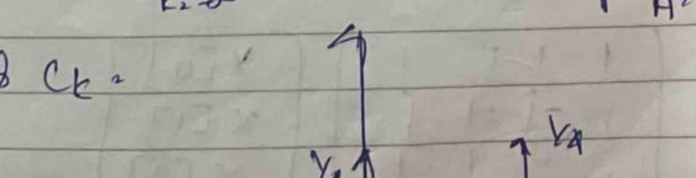


Eg 65 $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

$\frac{1}{4} \sum_{n=0}^{3} x[n]\, e^{j\pi/2 k n}$

$\boxed{\omega = \frac{2\pi}{4} = \pi/2}$

$c_k =$



learn sinc function DTFT & IDTFT

6.6 c) $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$

$\sum c_k e^{j\omega_0 kn} = \left(\frac{e^{j\pi/8 n} + e^{-j\pi/8 n}}{2}\right)^2$

$= \frac{1}{4}\left[2 + e^{j\pi/4 n} + e^{-j\pi/4 n}\right]$

$\therefore C_0 = \frac{1}{2} \quad C_1 = \frac{1}{4} = C_{-1}$

DFT continuation

$e^{-j\frac{2\pi}{N}nk} \Rightarrow$ Twiddle factor

$\omega_N^{nk} \Rightarrow \omega_N = e^{-j\frac{2\pi}{N}}$

$$\boxed{\text{DFT } X(k) = \sum_{n=0}^{\infty} x[n]\, \omega_N^{nk}}$$

MATRIX method

$$\omega_4^{nk} = \begin{array}{c|c} n \backslash k & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} \omega_N^0 & \omega_N^0 & \omega_N^0 & \omega_N^0 \\ \omega_N^0 & \omega_N^1 & \omega_N^2 & \omega_N^3 \\ \omega_N^0 & \omega_N^2 & \omega_N^4 & \omega_N^6 \\ \omega_N^0 & \omega_N^3 & \omega_N^6 & \omega_N^9 \end{bmatrix} \end{array}$$

$$[X] = [\omega_N^{nk}]\,[x]$$

$$\omega_4^{nk} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$X =$

$[x[n$

LAB-6 D

$f_1 = i$

(c) $y[n$

(b) $y[$

$y[$

$Y($

$X($

8p×8rB

6pEKDQ

Date
Page

$$X = \begin{bmatrix} 6 \\ -1+j \\ 0 \\ -1-j \end{bmatrix} \qquad x[n] = \frac{1}{4}\left[W_N^{nk}\, X(k) \right]^*$$

$$[x[n]] = \left[\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 6 \\ -1-j \\ 0 \\ -1+j \end{bmatrix}\right]^*$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

?

LAB-6 DTF Analysis

fi = input ("Enter the freq")

(i) $y[n] = x[n] + 2x[n-2] + 4x[n-4]$

(ii) $y[n] - y[n-1] + 0.5y[n-2] = x[n] + 2x[n-2] + 3x[n-3]$

~~y[n] = x + 2~~

$Y(z) = X(z) + 2z^{-2}X(z) + 4z^{-4}X(z)$

$\frac{Y(z)}{X(z)} = 1 + 2e^{-2j\omega} + 4e^{-4j\omega}$

21BLC1141 ~~dp verified~~ 10/2/23

Properties of DFT (theorem)

Periodicity

$x(n+N) = x(n)$
$X(k+N) = X(k)$

DFT of real & even sequence

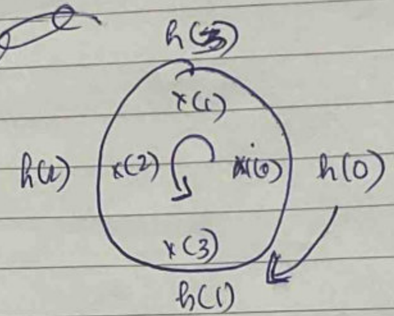
→ If $x[n]$ = real & even (is)
$x(n) = x(N-n)$, $n = 0$ to $N-1$

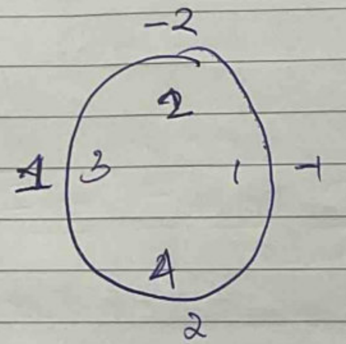
⇒ Circular convolution

Circular Convolution

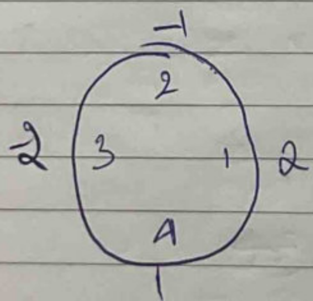
$$y[n] = \sum_{n=-\infty}^{\infty} x(k)\, h((n-k))_N \rightarrow \text{circular shift}$$

$x[n] = \{1, 2, 3, 4\}$
$h[n] = \{-1, 2, 1, -2\}$ } length is 'N' → circular
for linear $= N_2 + N_1 - 1$

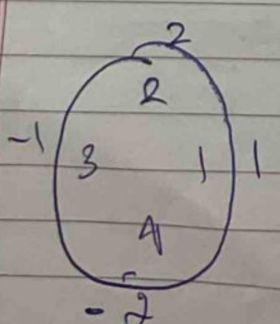




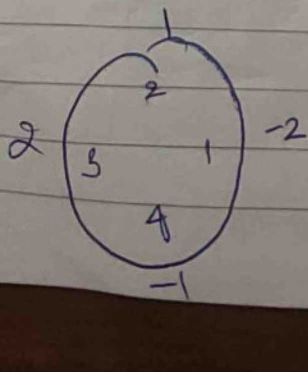
$y[0] = -1 + 8 + 3 - 4$
$= \underline{\underline{6}}$



$y[1] = 2 + 4 - 6 - 2 = -2$ //



$y[2] = 1 + 4 - 3 - 8$
$= -6$ //



$y[3] = -4 - 2 + 2 + 6$
$= 2$ //

Using Matrix

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & \cdots & \cdots \\ h(1) & h(0) & \cdots & - \\ h(2) & h(1) & \cdots h(0) & \cdots \\ \vdots & & & \\ h(N-1) & \cdots & & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} -1 & -2 & 1 & -2 \\ 2 & -1 & -2 & 1 \\ 1 & 2 & -1 & -2 \\ -2 & 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 -4 +3 +8 \\ 2 -2 -6 +4 \\ 1 \quad 4 \quad -3 \quad -8 \\ -2+2+6-4 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ -6 \\ 2 \end{bmatrix}$$

*

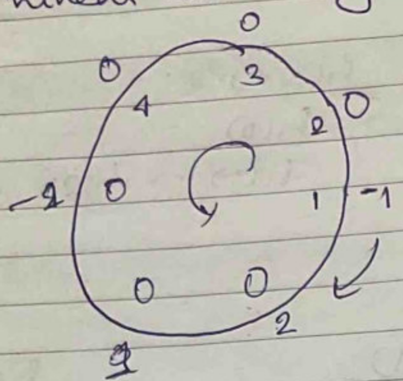
$\Rightarrow$ y {1, 2, 3, 4}
{1, 2, 3} $\Rightarrow$ add 0 to make it equal length

$\Rightarrow$ For linear convolution using circular conv

y {1, 2, 3, 4} $\Rightarrow$ {1, 2, 3, 4, 0, 0, 0}
{1, 2, 3, 4} $\Rightarrow$ {1, 2, 3, 4, 0, 0, 0}

$\because N_1 + N_2 - 1 = 4 + 4 - 1 = \underline{\underline{7}}$

Linear conv using circle method



$y(0) = 1$

$y(1) = -2 + 2 = 0$

$y(2) = -3 + 4 + 1$
$= 2$

$y(3) = -4 + 6 + 2 - 2$
$= 2$

$y(4) = 8 + 3 - 4$
$= 7$

$y(5) = 4 - 6$
$= -2$

$y(6) = -8$

<u>Fast Fourier Transform</u>

$x[n] = \{1, 3, -1, 0, 2, 4, 1, -2\}$

$X(0) = \sum_{n=0}^{N-1} x[n] = 8$

(DFT)

$X(1) = \sum_{n=0}^{7} x[n] e^{-j\frac{2\pi}{8} \cdot n}$

$= 1 + 3e^{-j\pi/4} - 1e^{-j2\pi/4}$
$+ 0 + 2e^{-j4\pi/4} + 4e^{-j5\pi/4} + 1e^{-j6\pi/4}$
$- 2e^{-j7\pi/4}$

$= 1 + 3\left[\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right] - 1[j]$
$+ 2[-1] + 4\left[-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right] +$

↳ Hard to do

Ryam

FILTERS

IIR

Analog IIR FILTER design

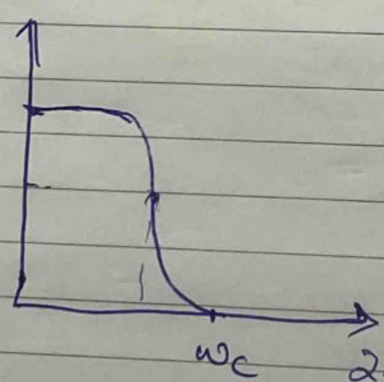
⇒ Less stability compared to FIR

$x[n] \Rightarrow \boxed{* \, h(n)} \longrightarrow y[n]$ → Time domain

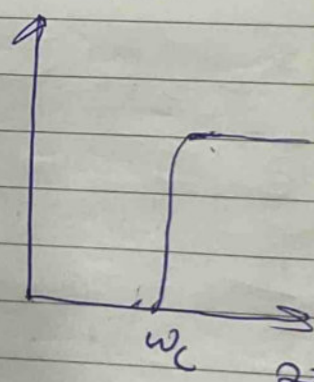
$X(S) \times H(S) \Rightarrow Y(S)$ → analog

$X(z) \times H(z) \Rightarrow Y(z)$ → digital

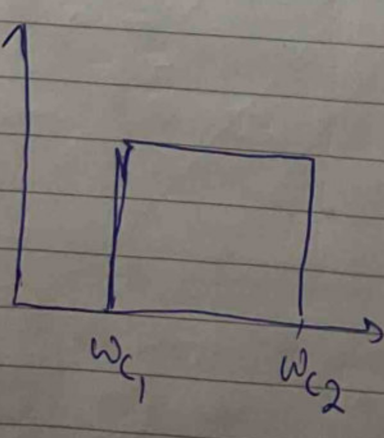
Low pass



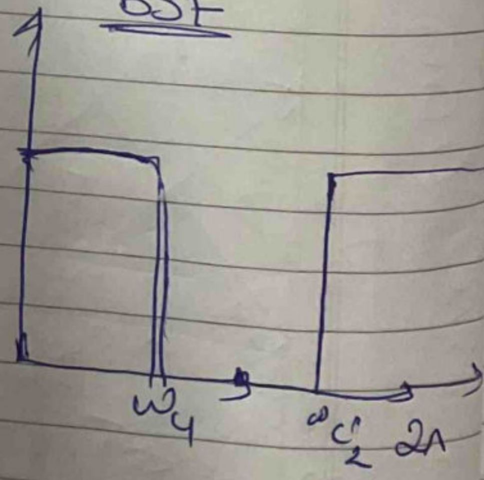
High pass



BPF



BSF



$1 + z^{-1} + z^{-2}$ → order → 2 ; z^{-2} → delay elements

∴ 2 delay elements (z^{-1}) & (z^{-2})

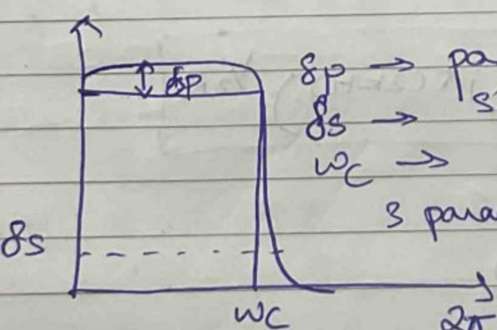
Analog IIR types

(i) Butterworth
(ii) chebshev I
(iii) chebshev II
(iv) Elliptic

Butterworth Filter

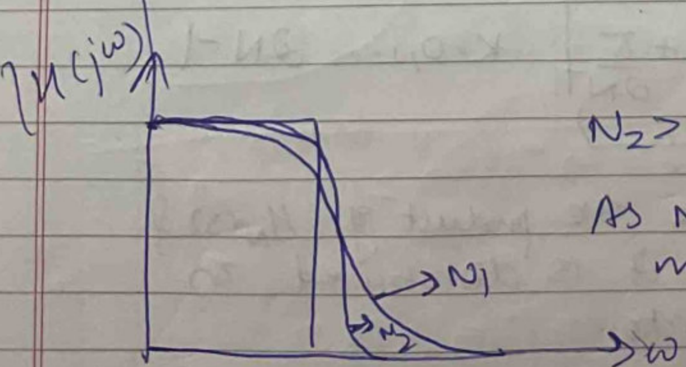
$$|H(j\Omega)| = \frac{1}{\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}}$$ → low pass butterworth

⇒ where N is the order of filter
⇒ ω_c is defined as cut off freq
⇒ Filter mag is $\frac{1}{\sqrt{2}}$ times dc gain $(\Omega = 0)$



δp → pass band ~~attenuation~~ ripple
δs → stop band attenuation
ω_c →
3 parameters for filter specifications

As N ↑ Ω_s ↓ towards Ω_c



$N_2 > N_1$ more smooth

As N ↑ signal becomes more closer to ideal

$$W_c = \frac{\omega_p + \omega_s}{2}$$

If ω_p is not mentioned $\omega_p = \omega_c$

$|H(j\omega)| = 1$ for all N $\omega = 0$

$|H(j\omega)| = 1/\sqrt{2}$ $\omega_c = \omega$

$|H(j\omega)| = 0$ $\omega \to \infty$

$$|H_s(j\omega)|^2 = \frac{1}{1+(\omega)^{2N}} \quad \omega_c = 1$$

$s = j\omega$ $\omega = s/j$

$$= \frac{1}{1+(s/j)^{2N}}$$

To find poles

$$1+(s/j)^{2N} = 0$$

$$(s/j)^{2N} = -1$$

$$s/j = (-1)^{1/2N}$$

$$\therefore s_k = j(-1)^{1/2N}$$

$-1 = e^{j\pi(2k+1)}$ $k = 0, 1, \ldots 2N-1$

$j = e^{j\pi/2}$

$$\therefore \boxed{s_k = e^{j\pi/2}\left(e^{j\pi(2k+1)}\right)^{1/2N}}$$

$$= e^{j\theta_k}$$

$$\theta_k = \frac{\pi}{2} + \frac{\pi(2k+1)}{2N}$$

$$\boxed{\theta_k = \frac{\pi}{2} + \frac{\pi k}{N} + \frac{\pi}{2N}} \quad k = 0, 1 \ldots 2N-1$$

The poles are the product of $H_N(-s)$ & $H_N(s)$ and it is distributed on the unit circle.

$\therefore$ The system is stable & causal

For $N = 1$ $k = 0, 1$ $2N-1 = 2\times1-1$
$= 1$

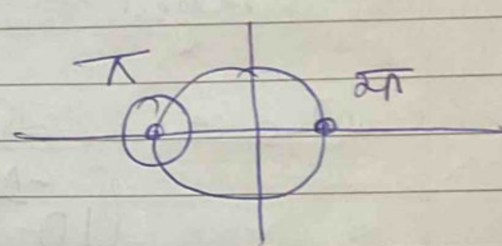
$\theta_k = \pi k + \pi$

If $k=0$ $\theta_k = \pi$
If $k=1$ $\theta_k = 2\pi$

$S = e^{j\pi}$
$= -1$

$$\boxed{H(S) = \frac{1}{\prod_{k=0}^{2N-1} (S - S_k)}}$$

$$H(S) = \frac{1}{1+S}$$



$N = 2$ $k = 0, 1, 2, 3$

$$\theta_k = \frac{\pi k}{N} + \frac{\pi}{2N} + \frac{\pi}{2} = \frac{\pi k}{2} + \frac{\pi}{4} + \frac{\pi}{2}$$

If $k=0$ $\frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

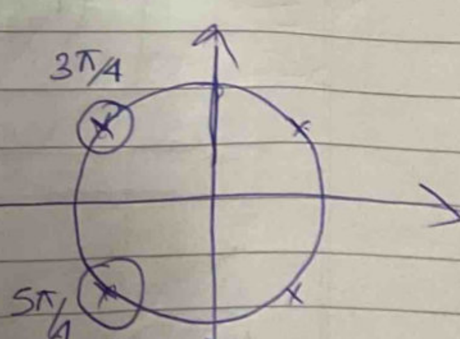
If $k=1$ $\frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$

If $k=2$ $\frac{\pi}{\cancel{2}} + \frac{\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$

If $k=3$ $\frac{3\pi}{2} + \frac{\pi}{4} + \frac{\pi}{2} = \frac{9\pi}{4}$

$\therefore S_0 = e^{j3\pi/4} = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$

$S_1 = e^{j5\pi/4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$



$$N = \frac{\log\left[\left(10^{-\frac{A_p}{10}} - 1\right) \Big/ \left(10^{\frac{-A_s}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$ next

Find $[N]$ round to greatest integer

(either of Ω_p or Ω_s)

$$\Omega_c = \frac{\Omega_p / \Omega_s}{\left(10^{\frac{-A_p/A_s}{10}} - 1\right)^{1/2N}} \qquad \Omega_c = \frac{\Omega_p + \Omega_s}{2}$$

$A_p \leq 20\log|H(j\Omega)| \leq 0$ for all $\Omega \leq \Omega_p$

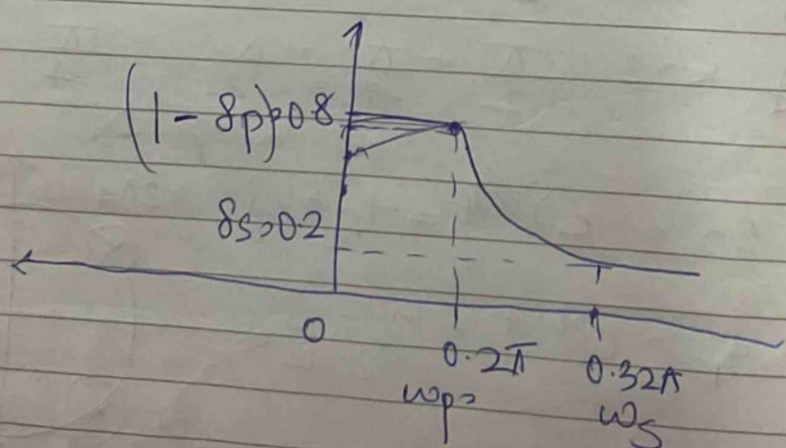
$20\log|H(j\Omega)| \leq A_s$ " " $\Omega \geq \Omega_s$

If $\Omega_p = \Omega$ $\quad A_p = 20\log(1-\delta_p)$

$\Omega_s = \Omega$ $\quad A_s = 20\log(\delta_s)$

Q) $0.8 \leq |H(j\omega)| \leq 1$ $\qquad 0 < \omega < 0.2\pi$

$|H(\omega)| \leq 0.2$ $\qquad 0.32\pi < \omega < \pi$



~~A_p~~ $A_s = 20\log(0.2)$

$A_p = 20\log(1-\delta p)$

$= 20\log(0.8) = -1.9382$

$A_s = 20\log(0.2) = -13.9794$

$$N = \frac{\log\left(\left(10^{\frac{1.9382}{10}} - 1\right) / \left(10^{\frac{13.9794}{10}} - 1\right)\right)}{2\log\left(\frac{0.2\pi}{0.32\pi}\right)}$$

$N = \underline{\underline{4}}$

$\theta_k = \frac{\pi k}{N} + \frac{\pi}{2N} + \frac{\pi}{2}$

$= \frac{\pi k}{4} + \frac{\pi}{8} + \frac{\pi}{2}$

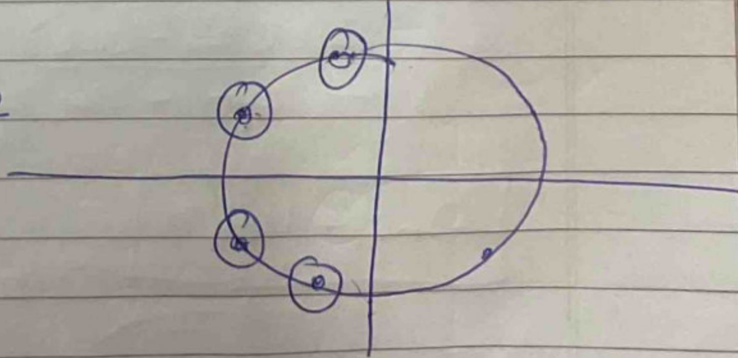
$\theta_0 = \frac{\pi}{8} + \frac{\pi}{2} = \frac{5\pi}{8}$

$\theta_1 = \frac{\pi}{4} + \frac{\pi}{8} + \frac{\pi}{2} = \frac{7\pi}{8}$

$\theta_2 = \frac{9\pi}{8}$ $\quad \theta_3 = \frac{11\pi}{8}$ $\quad \theta_4 = \frac{13\pi}{8}$ $\quad \theta_5 = \frac{15\pi}{8}$

$\theta_6 = \frac{17\pi}{8}$ $\quad \theta_7 = \frac{19\pi}{8}$

$s_0 = e^{j5\pi/8} = 1\underline{\lfloor -1.96}$

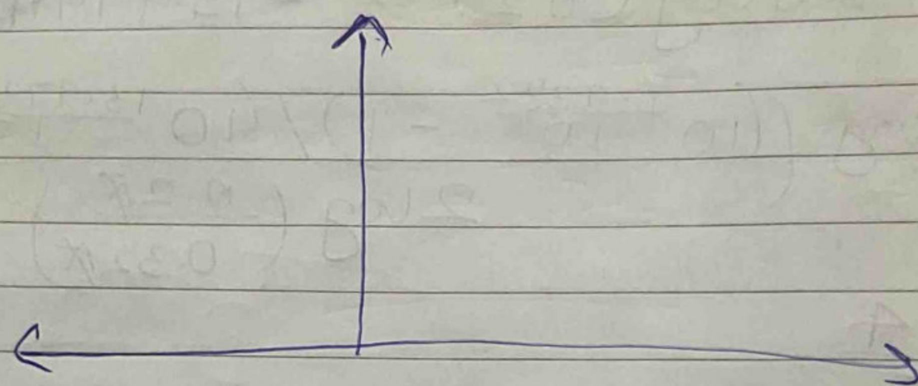


$s_1 = e^{j7\pi/8}$

$s_2 = e^{j9\pi/8}$

$s_3 = e^{j11\pi/8}$

Q) Determine order & pole coeff of butter. low pass filter that has 3dB bandwidth of 500 Hz & attenuation of 40dB at 100 Hz



$K_p = -3\,dB \quad f_p = 500\,Hz$

Frequency Transformation

* If low pass then apply low → low

replace $\boxed{s \rightarrow \frac{s}{\Omega_c}}$

* If high pass then apply low → high

replace $\boxed{s \rightarrow \frac{\Omega_c}{s}}$

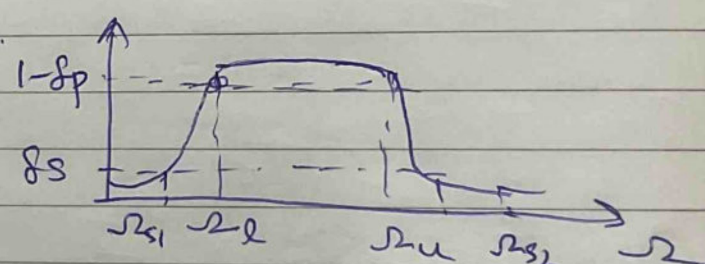
* Low to Bandpass

$$s \rightarrow \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$$

Band stop →

* Low to stop

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$$



- If LPF $\Omega_s = \frac{\Omega_p}{\Omega_s}$
- If BPF $\Omega_s = \{|A|, |B|\}$ Min

$$A = \frac{\Omega_l(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l \Omega_u} \qquad B = \frac{\Omega_2(\Omega_u - \Omega_l)}{\Omega_2^2 - \Omega_1 \Omega_u}$$

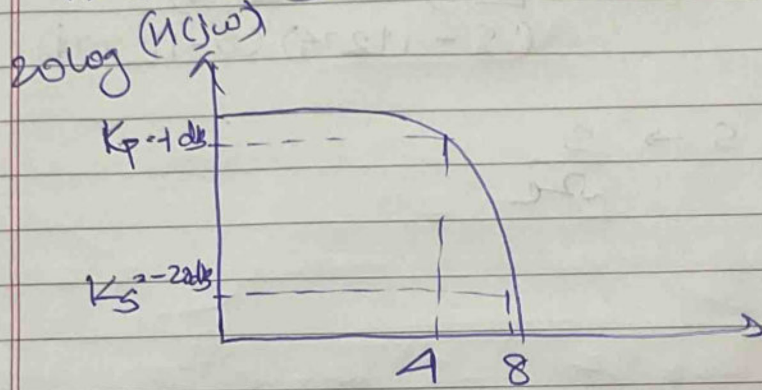
$\boxed{\Omega_p = 1 \text{ rad/sec}}$

Q) Butterworth low pass filter has to meet following specifications

$K_p = -1dB$ at $\Omega_p = 4$ rad/sec

$\Omega_s = 8$ rad/sec at $20dB$

Find $H(s)$



$K_p = -1dB$ $\quad K_s = -20dB$ $\quad \Omega_s = 8$ rad/se

$\Omega_p = 4$ rad/sec

Step. $N = \log\left[\dfrac{(10^{-Ap/10} - 1)(10^{-As/10} - 1)}{2\log(\Omega p/\Omega s)}\right]$

$= \log\left[\dfrac{(10^{3/10} - 1)/(10^{+20/10} - 1)}{2\log(4/8)}\right]$

$= \dfrac{-2}{2\log(1/2)} = 3.32 = \underline{\underline{4}}$

For $N = 4$ $\quad k = 0, 1, \ldots 7$

$\theta_k = \dfrac{\pi}{2} + \dfrac{\pi k}{N} + \dfrac{\pi}{2N} = \dfrac{\pi}{2} + \dfrac{\pi k}{4} + \dfrac{\pi}{8}$

$\theta_0 = 5\pi/8 \quad \theta_1 = 7\pi/8 \quad \theta_2 = 9\pi/8 \quad \theta_3 = 11\pi/8$

$\theta_4 = 13\pi/8 \quad \theta_5 = 15\pi/8 \quad \theta_6 = 19\pi/8 \quad \theta_7 = 19\pi/8$

$s_0 = e^{j5\pi/8} = 1 \angle 1.96$

$s_1 = e^{j7\pi/8} = 1 \angle 2.74$

$s_2 = e^{j9\pi/8} = 1 \angle -2.74$

$s_3 = e^{j11\pi/8} = 1 \angle -1.96$

$$H(s) = \frac{1}{(s - 1\angle 1.96)(s - 1\angle -1.96)(s - 1\angle 2.74)(s - 1\angle -2.74)}$$

sub $s \to \dfrac{s}{\Omega_c}$

Monotonic freq resp ≡ Butterworth

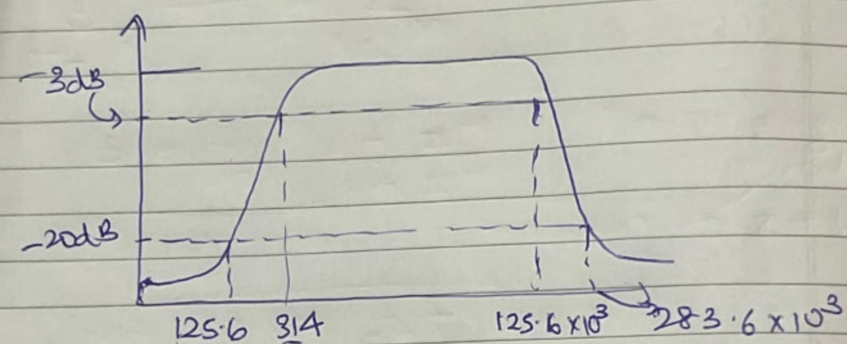
Q) Design Bandpass filter

$-3.0103 \rightarrow 50\,Hz$ & $20\,kHz$

$-20\,dB \rightarrow 20\,Hz$ & $45\,kHz$

2×10

⇒



$A_p = -3\,dB \quad A_s = -20\,dB$

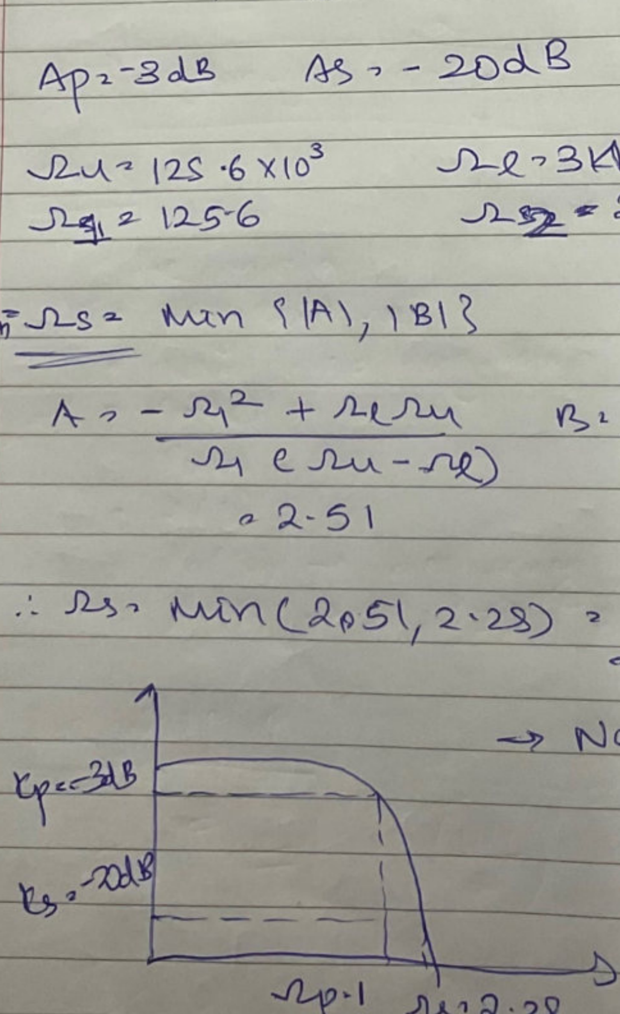
$\Omega_u = 125.6\times10^3 \qquad \Omega_l = 314$

$\Omega_1 = 125.6 \qquad \Omega_2 = 283.6\times10^3$

Normalization = $\Omega_s = \min\{|A|, |B|\}$ $\qquad \Omega_p = 1$

$$A = \frac{-\Omega_1^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)} = 2.51 \qquad B = \frac{\Omega_2^2 - \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)} = 2.25$$

$\therefore \Omega_s = \min(2.51, 2.25) = 2.25$



→ Normalized low pass

Order

N = log

= 3

if N = 3

if θ_k =

θ_0 =

θ_1 =

θ_5 = 14

$s_0 = e^{4\pi/6}$

$s_1 = e^{6\pi/6}$

$s_2 = e^{8\pi/6}$

S →

classmate
Date
Page

$\rightarrow N=3$

Order

$$N = \log \left(\frac{(10^{-A_p/10} - 1)/(10^{-A_s/10} - 1)}{2\log(\Omega_p/\Omega_s)} \right)$$

$= \underline{\underline{3}}$

if $N=3$ $\quad K = (0, 1, \cdots 5)$

if $\theta_k = \frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi k}{N} = \frac{\pi}{2} + \frac{\pi}{6} + \frac{\pi k}{3}$

$\theta_0 = \frac{\pi}{2} + \pi/6 = \frac{4\pi}{6}$ ✓

$\theta_1 = \pi/2 + \pi/6 + \pi/3 = 6\pi/6$ ✓

~~poles~~

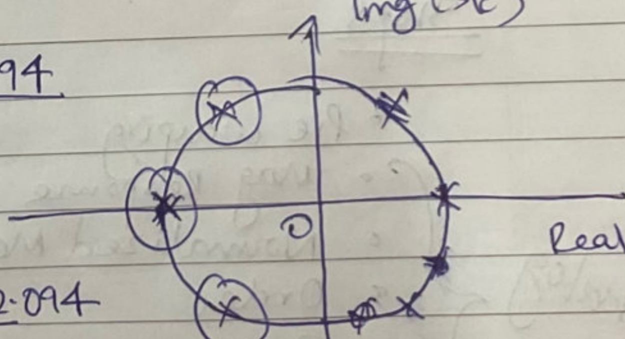
$\theta_2 = 8\pi/6$ ✓ $\quad \theta_3 = 10\pi/6 \quad \theta_4 = 12\pi/6$

$\theta_5 = 14\pi/6$

$s_0 = e^{4\pi/6} = 1\angle 2.094$

$s_1 = e^{6\pi/6} = -1$

$s_2 = e^{8\pi/6} = 1\angle -2.094$



$\therefore H(s) = \frac{1}{(s - 1\angle 2.094)(s+1)(s - 1\angle -2.094)}$

$$s \mapsto \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)} = \frac{s^2 + 3.9438 \times 10^{10}}{s(1.2528 \times 10^5)}$$

After $H(S) \rightarrow$ freq transformation $H_a(S)$

$H_d(z) \leftarrow H_a(S)$

ryen

$\cancel{H(s)} = \cancel{H_{LP}(s - s_k)}$

Analog IIR to Digital IIR

Bilinear Transformation (BLT)

$S \longrightarrow Z$

$\sigma + j\omega \qquad re^{j\omega}$

$$\boxed{S = \frac{2}{T}\left[\frac{1 - z^{-1}}{1 + z^{-1}}\right]}$$ $T \Rightarrow$ sampling period

↳ Mapping poles in $s \rightarrow$ domain to $z \rightarrow$ domain

$$\boxed{\omega = \frac{2}{T}\tan^{-1}\left(\frac{\Omega T}{2}\right)}$$ $\Longrightarrow$ analog to digital freq

↳ frequency wraping

Steps

- Pre warping
- Mag Response
- Normalized Mag Resp
- Order
- Poles
- $H(S)$
- $H_a(S) \rightarrow$ frequency transformation
- $H_a(S) \rightarrow H_d(z)$ (BLT)

analog filter

Pre warping

$$\Omega'_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega'_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

Q) Apply Bilinear transformation to $H(S)$

$H(S) = \frac{2}{(S+1)(S+2)}$ at $T=1$

$S = 2\left[\frac{Z-1}{Z+1}\right] \longrightarrow$ write in terms of z^{-1}

$$H(z) = \frac{2}{\left(2\left[\frac{z-1}{z+1}\right]+1\right)\left(2\left[\frac{z-1}{z+1}\right]+2\right)}$$

$$= \frac{2}{\frac{(2z-2+z+1)}{(z+1)}\frac{(2z-2+2z+2)}{(z+1)}}$$

$$= \frac{2(z+1)^2}{(3z-1)(4z)} = \frac{(z+1)^2}{2(z)(3z-1)}$$

$$= \frac{(1+z^{-1})^2}{2(3-z^{-1})} = \frac{(1+z^{-1})^2}{2(3-z^{-1})}$$

Q) Design low pass filter with 3dB bandwidth of 0.2π using bilinear trans to analog filter $H(s) = \frac{\Omega_c}{s+\Omega_c}$

- $3dB \longrightarrow 0.2\pi$
$\downarrow$ $\quad \hookrightarrow \omega_c$
(Always 3dB)

$$\Omega_c = \frac{2}{T}\tan\left(\frac{\omega_c}{2}\right) = \frac{2}{T}\tan(0.1\pi)$$

$$H(s) = \frac{0.645/T}{s+0.645/T} \qquad s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$