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DA-3

1. Given, $\omega_p = 0.25\pi$ rad/s.

$\therefore f_p = \frac{\omega_p}{2\pi} \cdot \frac{2}{T_s}$; where T_s is the sampling period.

$\omega_s = 0.75\pi$ rad/s.

$\therefore f_s = \frac{\omega_s}{2\pi} \cdot \frac{2}{T_s}$

Now, $N = \dfrac{\log_{10}\left[\dfrac{10^{A_{max}/10} - 1}{10^{A_{min}/10} - 1}\right]}{2 \cdot \log_{10}\left(\dfrac{\omega_p}{\omega_s}\right)}$

where, A_{max} = max. passband ripple

A_{min} = min. stopband attenuation.

$\therefore N = \dfrac{\log_{10}\left[10^{7.2/10} - 1 / 10^{30/10} - 1\right]}{2 \cdot \log_{10}\left(\dfrac{0.25}{0.75}\right)}$

$= 4.19$ (approx.)

$\therefore \boxed{N = 5}$.

$\therefore H(s) = \dfrac{1}{(s+1)(s^2+0.618035+1)(s^2+1.618035+1)}$

Using Bilinear Transform,

$H(z) = H(s)\Big|_{s \to \frac{2}{T_s}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$= H(s)\Big|_{s \to 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$ [Taking $T_s = 1$ s]

$= \dfrac{1}{\left(\dfrac{2-2z^{-1}}{1+z^{-1}}+1\right)\left[\left(\dfrac{2-2z^{-1}}{1+z^{-1}}\right)^2+0.618035+1\right]\left[\left(\dfrac{2-2z^{-1}}{1+z^{-1}}\right)^2+1.618035+1\right]}$

$= \dfrac{1}{\left(\dfrac{2-2z^{-1}+1+z^{-1}}{1+z^{-1}}\right)\left[\dfrac{4-8z^{-1}+4z^{-2}}{1+2z^{-1}+z^{-2}}+0.61803\left(\dfrac{2-2z^{-1}}{1+z^{-1}}\right)+1\right]\left[\dfrac{4-8z^{-1}+4z^{-2}}{1+2z^{-1}+z^{-2}}+1.61803\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)+1\right]}$

2. $H_d(e^{j\omega}) = 1, \quad \text{for} \quad -\pi/2 \le \omega \le \pi/2$

$= 0, \quad \text{for} \quad -\pi/2 \le |\omega| \le \pi$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1 . e^{j\omega n}\, d\omega .$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}\, d\omega .$$

$$= \frac{1}{\pi n} \sin\left(\frac{\pi}{2} n\right).$$

Using Hamming window, $h(n) = h_d(n) . w(n)$.

$w_n = 0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right), \quad -5 \le n \le 5.$

$w(-1) = w(1) = 0.904$

$w(-2) = w(2) = 0.654$

$w(-3) = w(3) = 0.345$

$w(-4) = w(4) = 0.095$

$w(-5) = w(5) = 0$

$w(0) = 1.$

Also, $h_d(0) = 0$

$h_d(-1) = h_d(1) = 0.318$

$h_d(-2) = h_d(2) = 0$

$h_d(-3) = h_d(3) = -0.106$

$h_d(-4) = h_d(4) = 0$

$h_d(-5) = h_d(5) = 0.064.$

Now, $h(0) = 0$

$h(-1) = h(1) = 0.287$

$h(-2) = h(2) = 0$

$h(-3) = h(3) = -0.036$

$h(-4) = h(4) = 0$

$h(-5) = h(5) = 0.$

$$\therefore H(Z) = \sum_{n=-5}^{5} h(n) Z^{-n}$$

$= h(0) Z^0 + h(-1) Z^1 + h(-2) Z^2 + h(-3) Z^3 + h(-4) Z^4 + h(-5) Z^5 + h(1) Z^{-1} + h(2) Z^{-2} + h(3) Z^{-3} + h(4) Z^{-4} + h(5) Z^{-5}$

$= 0 + 0.287 Z + 0 - 0.036 Z^3 + 0 + 0 + 0.287 Z^{-1} + 0 - 0.036 Z^{-3} + 0 + 0$

$$= 0.287 Z - 0.036 Z^3 + \frac{0.287}{Z} - \frac{0.036}{Z^3}.$$

3. Finite word length effect :

In digital systems, the numbers and coefficients are stored in finite length registers.

$\therefore$ The numbers are quantized by :

① Truncation

(or)

② Rounding off

① Truncation : For eg., if the number is 3·1765423, then if we truncate it by 5 digits, then it becomes 3·17.

② Rounding off : For eg., 2·51529 becomes 2·6.