Classification of Signals & Systems

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Introduction to Signals

- A Signal is the function of one or more independent variables that carries some information to represent a physical phenomenon.

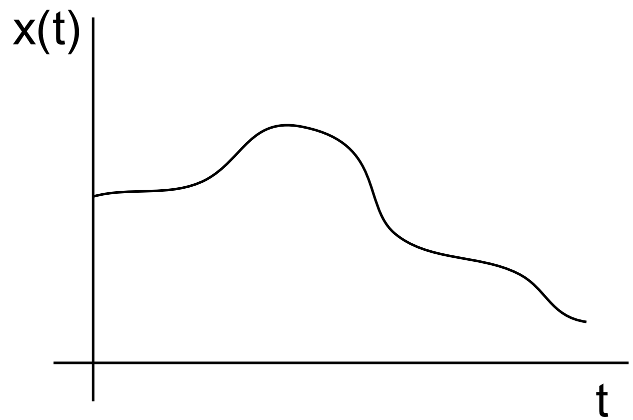
  e.g. ECG, EEG

- Two Types of Signals

1. Continuous-time signals
2. Discrete-time signals

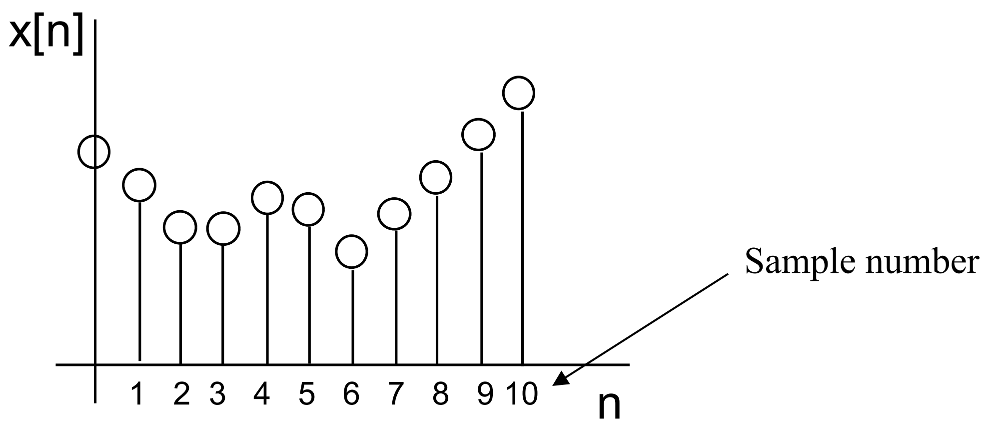
1. Continuous-Time Signals

- Signal that has a value for all points in time
- Function of time
  - Written as x(t) because the signal "x" is a function of time
- Commonly found in the physical world
  - ex. Human speech
- Displayed graphically as a line



2. Discrete-Time Signals

- Signal that has a value for only specific points in time
- Typically formed by “sampling” a continuous-time signal
  - Taking the value of the original waveform at specific intervals in time
- Function of the sample value, n
  - Write as x[n]
  - Often called a sequence
- Commonly found in the digital world
  - ex. wav file or mp3
- Displayed graphically as individual values
  - Called a “stem” plot



Examples: CT vs. DT Signals

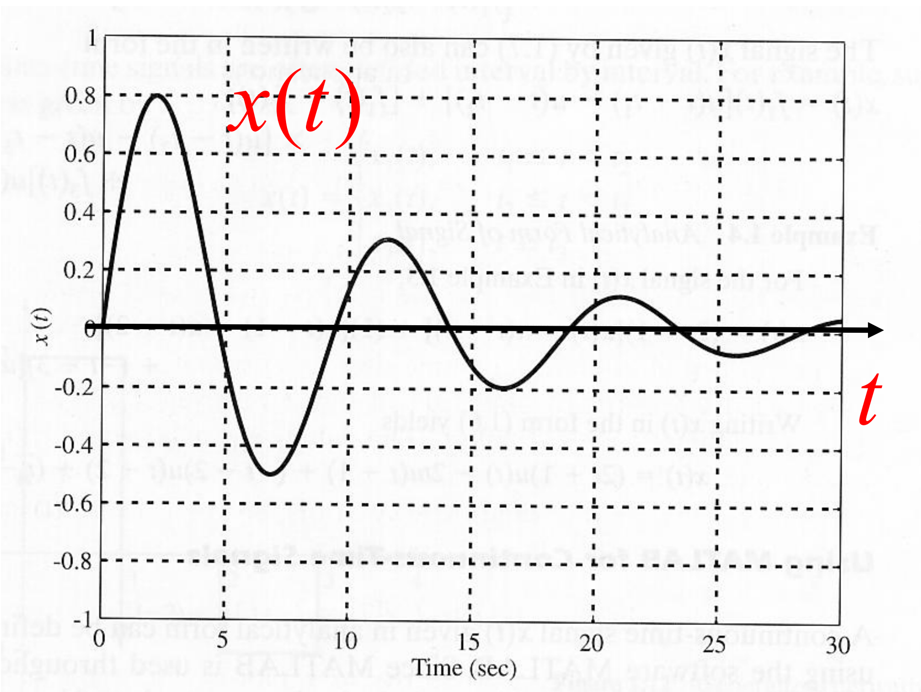


Figure 1.13 MATLAB plot of the signal $x(t) = e^{-0.1t}\sin\frac{2}{3}t$.

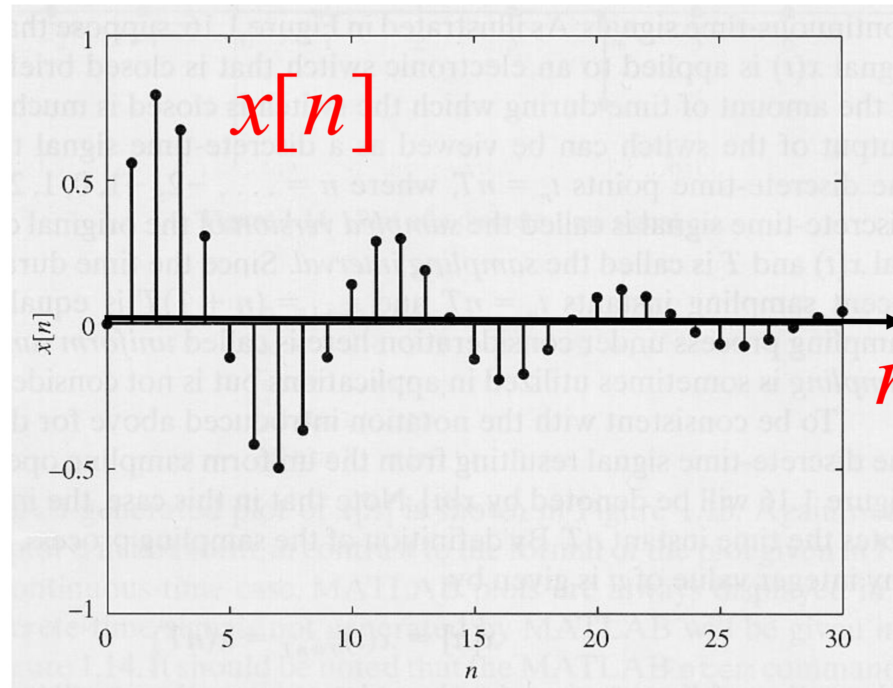
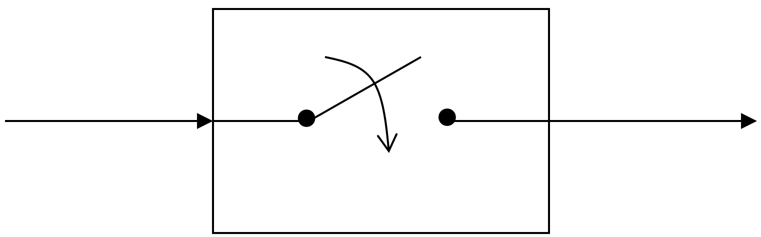


Figure 1.17 Sampled continuous-time signal.

Sampling

- Discrete-time signals are often obtained by sampling continuous-time signals

$x(t)$



$$x[n] = x(t)\Big|_{t=nT} = x(nT)$$

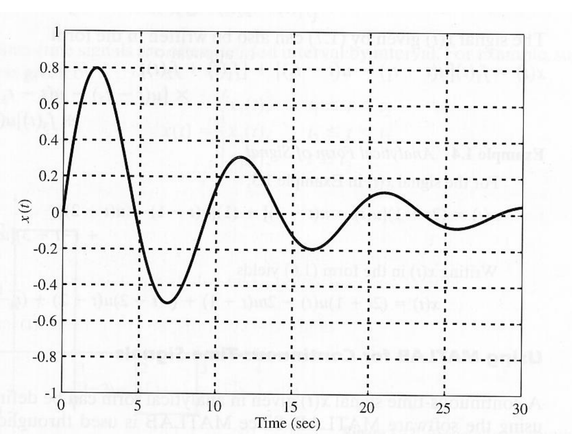


Figure 1.13 MATLAB plot of the signal $x(t) = e^{-0.1t}\sin\frac{2}{3}t$.

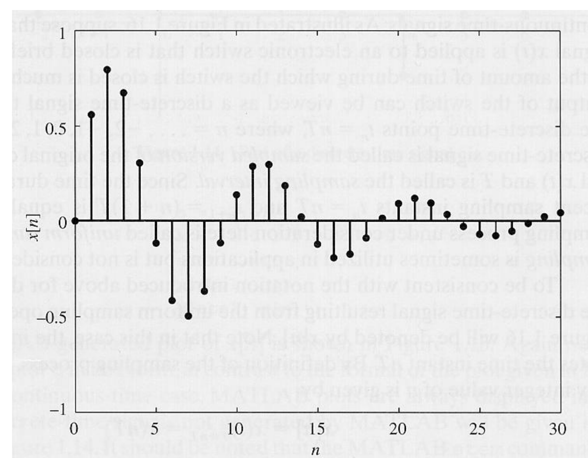
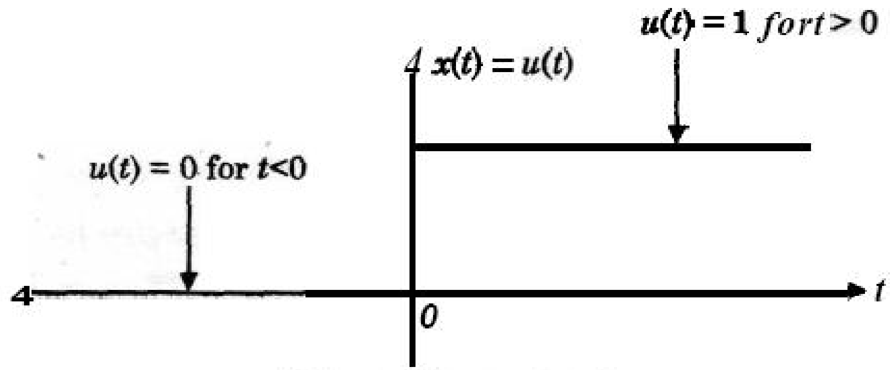


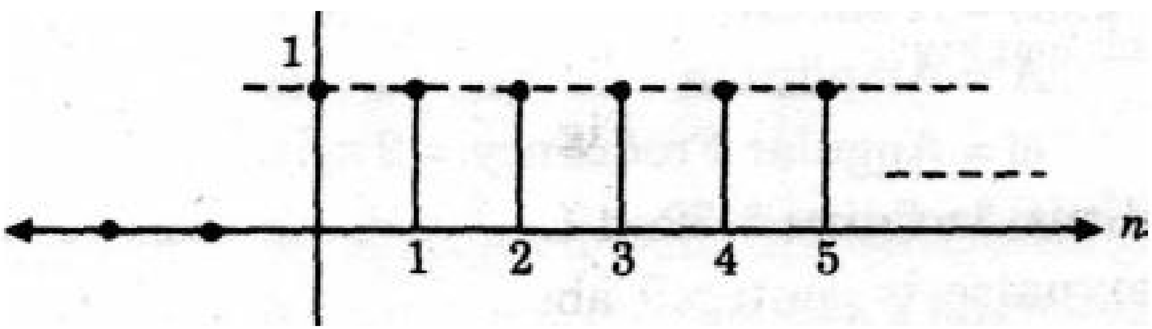
Figure 1.17 Sampled continuous-time signal.

Elementary Signals

Unit step signal : $u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$

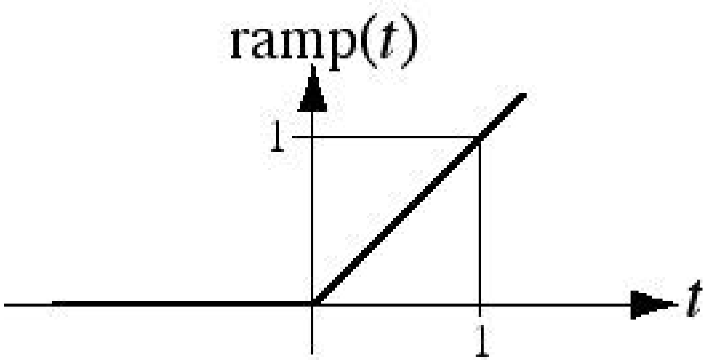


$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



$$u(n) = \{1, 1, 1, 1, \ldots\}$$

Unit Ramp Function

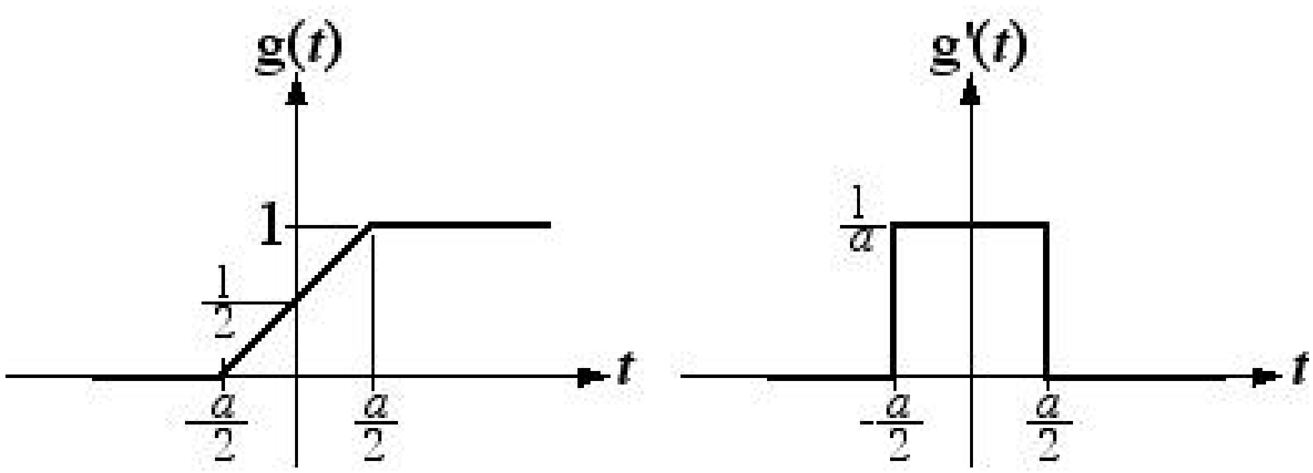


$$\mathrm{ramp}(t) = \begin{Bmatrix} t & , & t > 0 \\ 0 & , & t \le 0 \end{Bmatrix} = \int_{-\infty}^{t} \mathrm{u}(\lambda)\, d\lambda = t\,\mathrm{u}(t)$$

- The unit ramp function is the integral of the unit step function.
- It is called the unit ramp function because for positive t, its slope is one amplitude unit per time.

Unit Impulse Function

As a approaches zero, $\mathrm{g}\left(t\right)$ approaches a unit step and $\mathrm{g}'\left(t\right)$ approaches a unit impulse

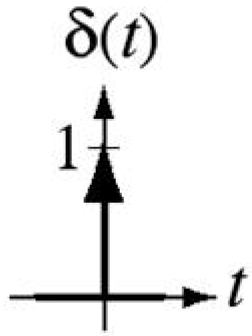


Functions that approach unit step and unit impulse

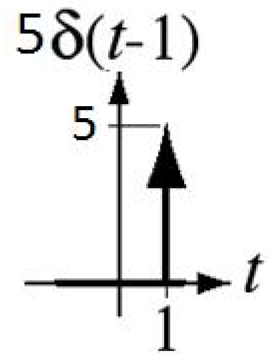
So unit impulse function is the **derivative** of the unit step function or unit step is the integral of the unit impulse function

Representation of Impulse Function

The area under an impulse is called its strength or weight. It is represented graphically by a vertical arrow. An impulse with a strength of one is called a unit impulse.



Representation of Unit Impulse



Shifted Impulse of Amplitude5

Properties of the Impulse Function

The Sampling Property

$$\int_{-\infty}^{\infty} \mathrm{g}(t)\,\delta(t-t_0)\,dt = \mathrm{g}(t_0)$$

The Scaling Property

$$\delta\big(a(t-t_0)\big) = \frac{1}{|a|}\delta(t-t_0)$$

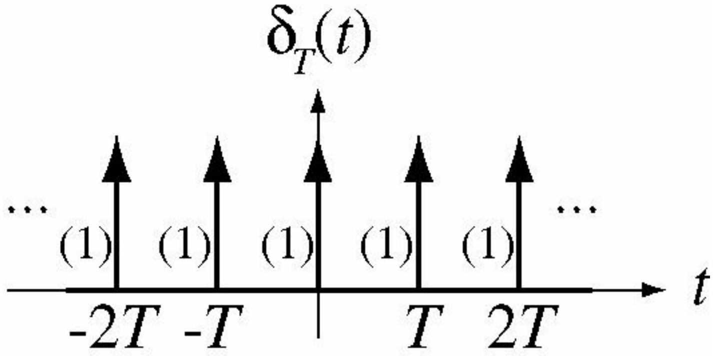
The Replication Property

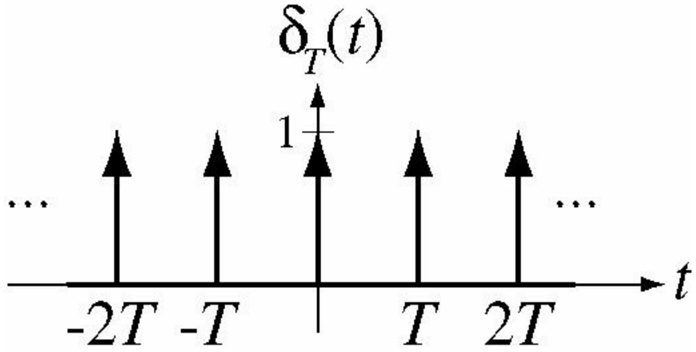
$$g(t) \otimes \delta(t) = g(t)$$

Unit Impulse Train

The unit impulse train is a sum of infinitely uniformly-spaced impulses and is given by

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad , \quad n \text{ an integer}$$





Sinusoidal & Exponential Signals

- Sinusoids and exponentials are important in signal and system analysis because they arise naturally in the solutions of the differential equations.
- Sinusoidal Signals can expressed in either of two ways :

cyclic frequency *form*- $A \sin 2\Pi f_o t = A \sin(2\Pi/T_o)t$

radian frequency form- $A \sin \omega_o t$

$\omega_o = 2\Pi f_o = 2\Pi/T_o$

T_o = Time Period of the Sinusoidal Wave

Sinusoidal & Exponential Signals Contd.

$$\left.\begin{aligned} x(t) &= A \sin(2\Pi f_o t + \theta) \\ &= A \sin(\omega_o t + \theta) \end{aligned}\right\}$$ Sinusoidal signal

$x(t) = Ae^{at}$ Real Exponential

$= Ae^{j\omega_o t} = A[\cos(\omega_o t) + j \sin(\omega_o t)]$ Complex Exponential

θ = Phase of sinusoidal wave

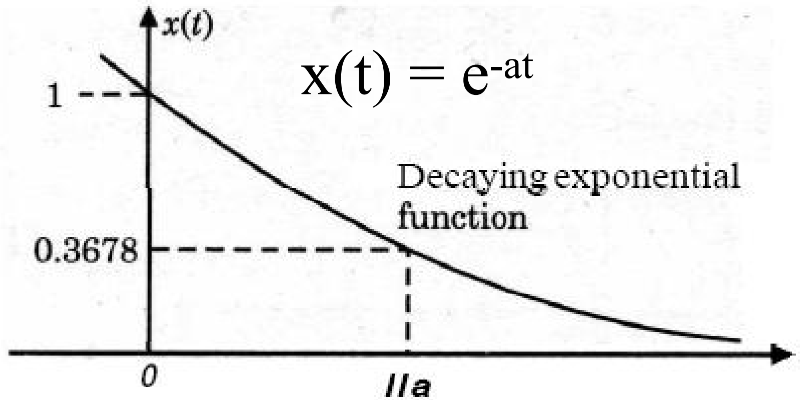
A = amplitude of a sinusoidal or exponential signal

f_o = fundamental cyclic frequency of sinusoidal signal

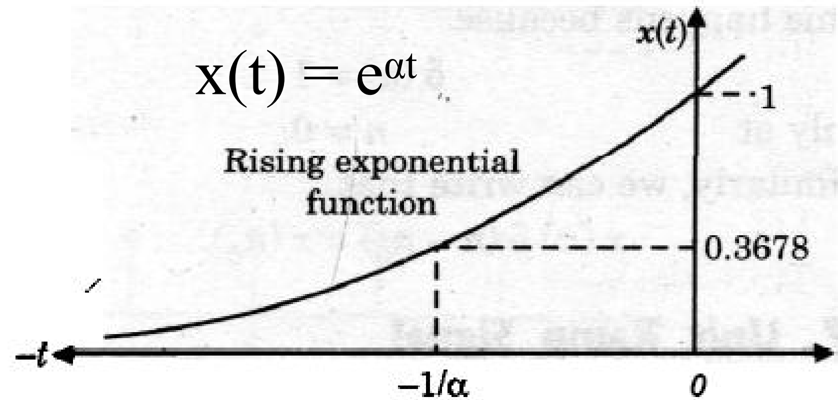
ω_o = radian frequency

Real Exponential Signals and damped Sinusoidal

$x(t) = e^{-at}$

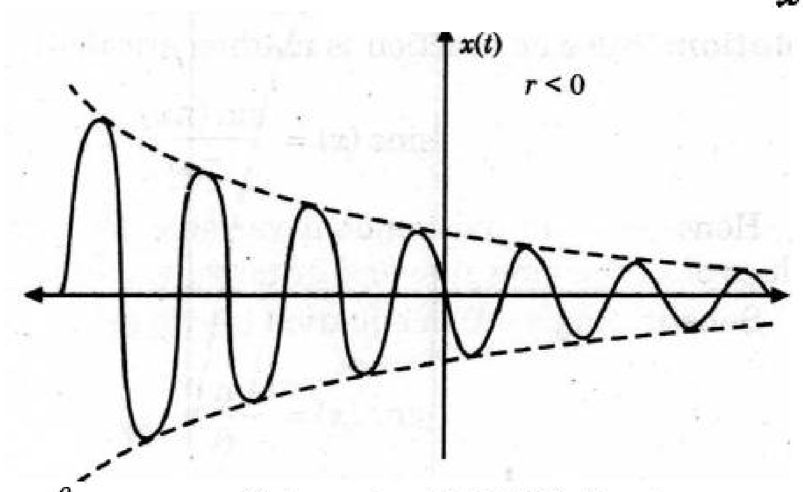


$x(t) = e^{\alpha t}$

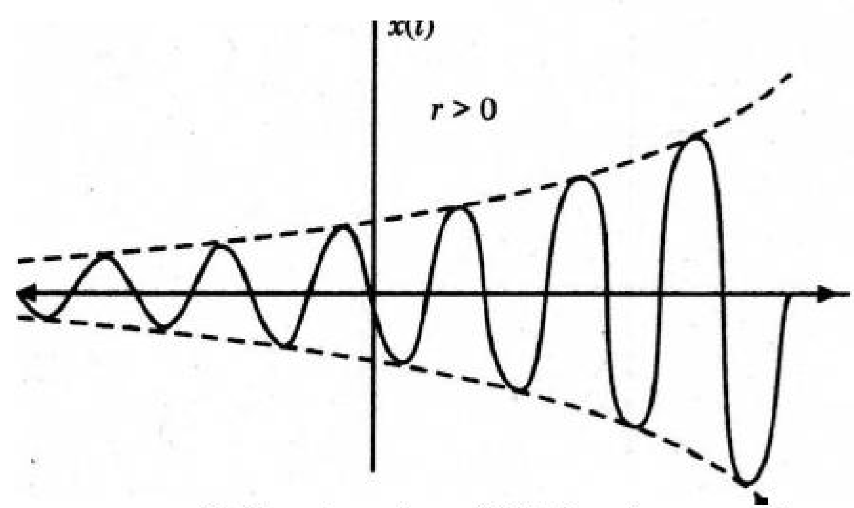


A discrete time exponential signal is expressed as

$$x(n) = a^n$$



(b) Decaying sinusoidal signal



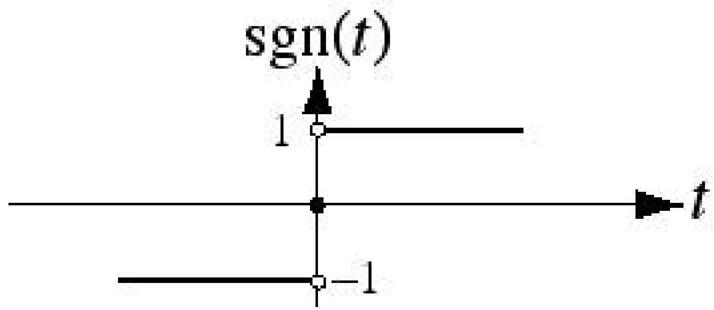
(a) Growing sinusoidal signal

$$e^{rt} \cos(\omega_0 t + \theta)$$

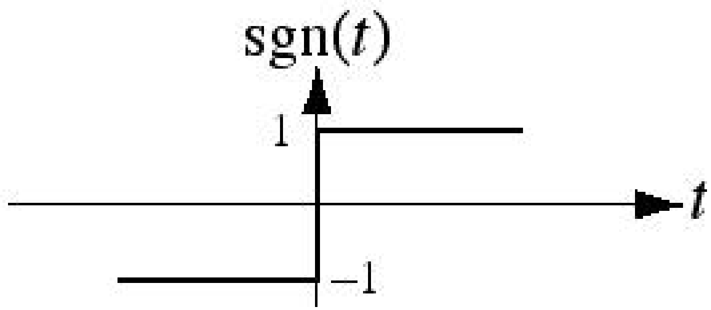
Signum Function

$$\operatorname{sgn}(t) = \begin{cases} 1 & , \; t > 0 \\ 0 & , \; t = 0 \\ -1 & , \; t < 0 \end{cases} = 2\mathrm{u}(t) - 1$$

Precise Graph



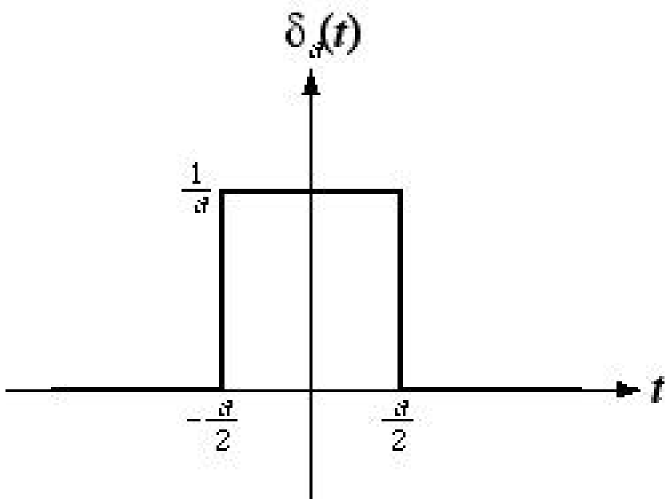
Commonly-Used Graph



The signum function, is closely related to the unit-step function.

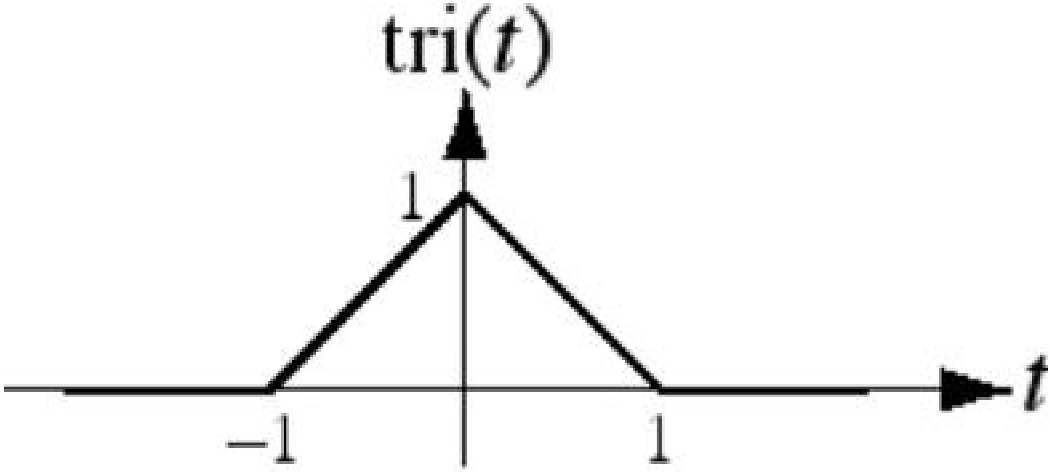
Rectangular Pulse or Gate Function

Rectangular pulse, $\delta_a(t) = \begin{cases} 1/a & , \quad |t| < a/2 \\ 0 & , \quad |t| > a/2 \end{cases}$



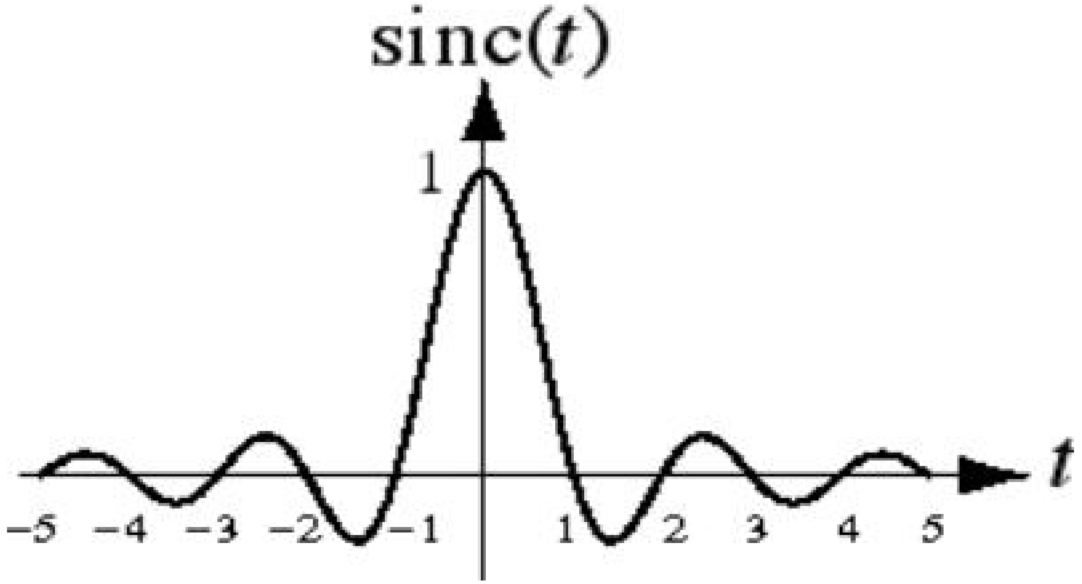
The Unit Triangle Function

A triangular pulse whose height and area are both one but its base width is not, is called unit triangle function. The unit triangle is related to the unit rectangle through an operation called **convolution**.



Sinc Function

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Discrete-Time Signals

- **Sampling** is the acquisition of the values of a continuous-time signal at discrete points in time
- $x(t)$ is a continuous-time signal, $x[n]$ is a discrete-time signal

$x[n] = x(nT_s)$ where T_s is the time between samples

Discrete Time Exponential and Sinusoidal Signals

- DT signals can be defined in a manner analogous to their continuous-time counter part

$x[n] = A \sin(2\Pi n/N_o+\theta)$

$= A \sin(2\Pi F_o n + \theta)$

Discrete Time Sinusoidal Signal

$x[n] = a^n$

Discrete Time Exponential Signal

n = the discrete time

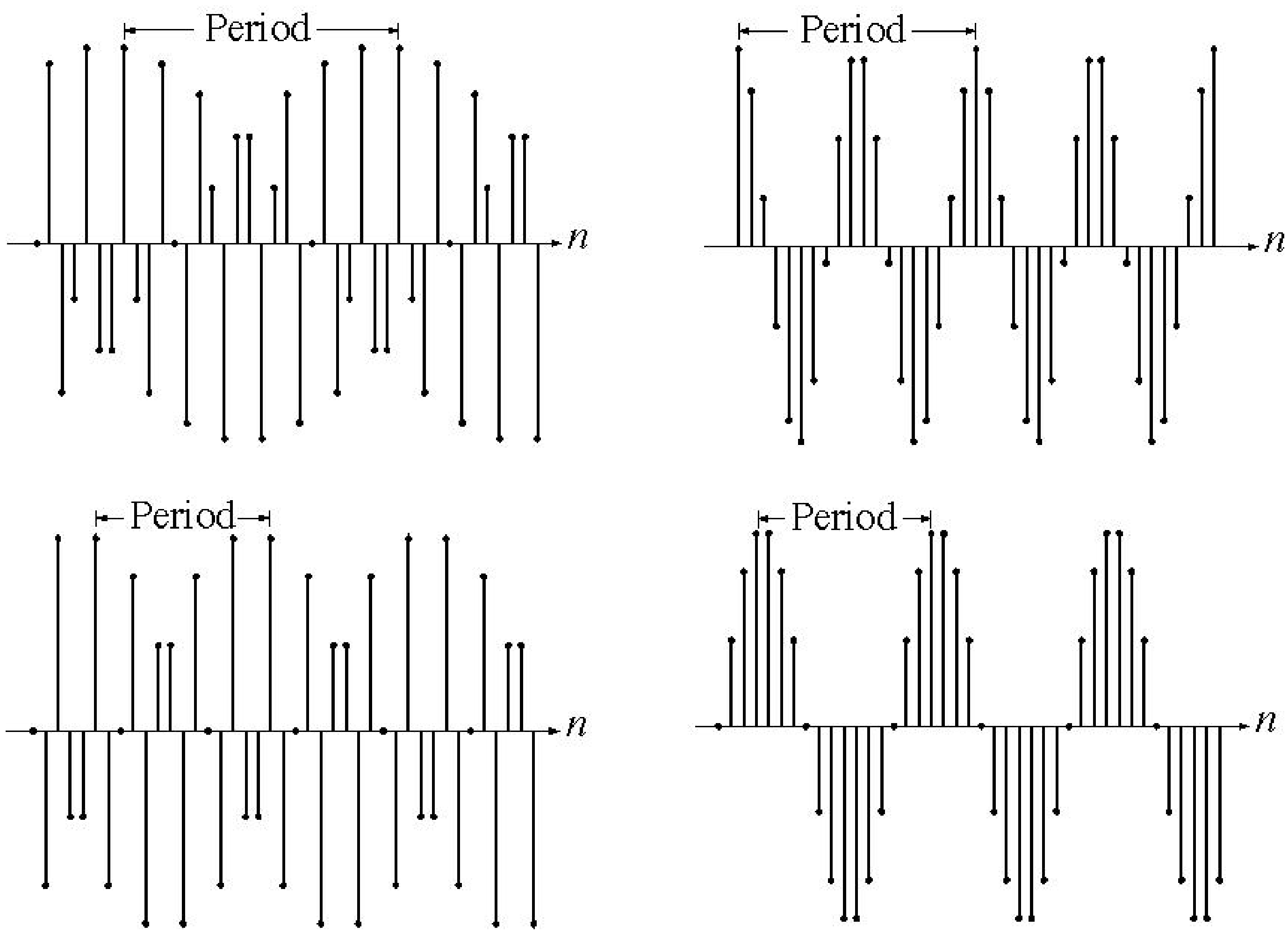
A = amplitude

θ = phase shifting radians,

N_o = Discrete Period of the wave

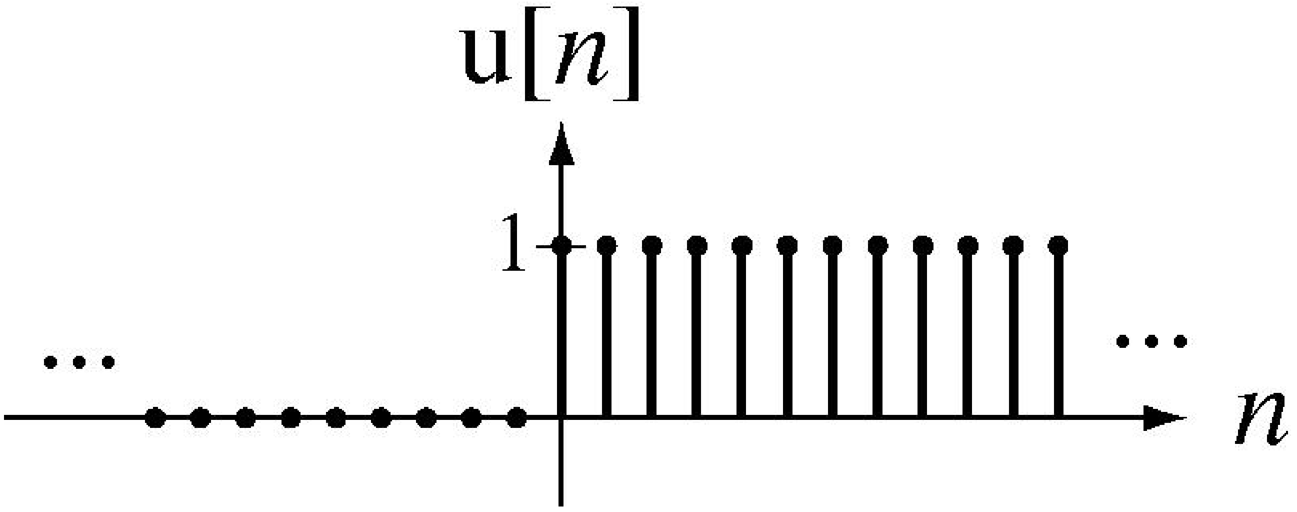
$1/N_0 = F_o = \Omega_o/2\Pi$ = Discrete Frequency

Discrete Time Sinusoidal Signals



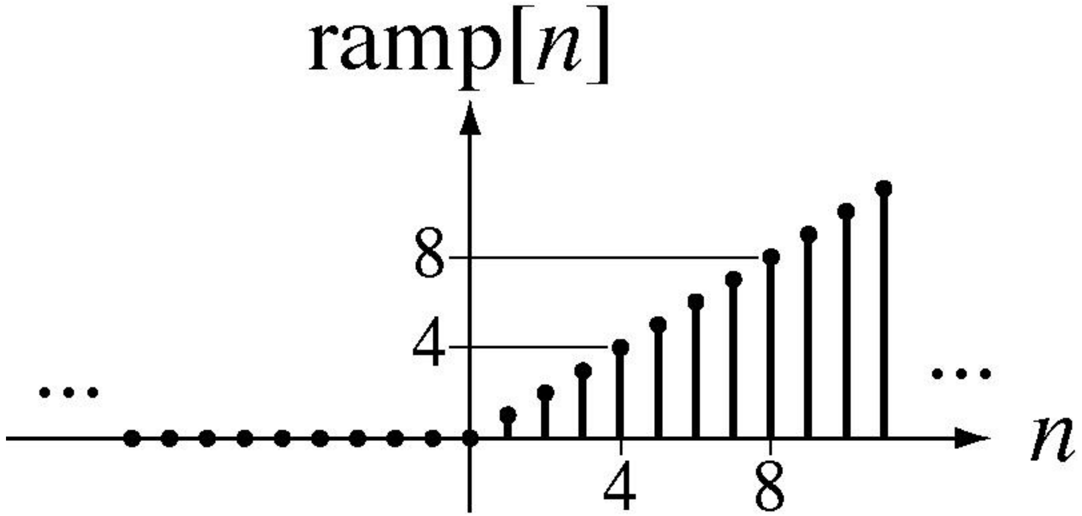
Discrete Time Unit Step Function or Unit Sequence Function

$$\mathrm{u}[n] = \begin{cases} 1 \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases}$$



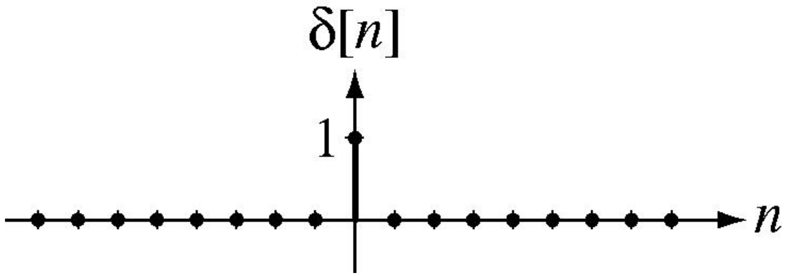
Discrete Time Unit Ramp Function

$$\operatorname{ramp}[n] = \begin{Bmatrix} n & , & n \ge 0 \\ 0 & , & n < 0 \end{Bmatrix} = \sum_{m=-\infty}^{n} \mathrm{u}[m-1]$$



Discrete Time Unit Impulse Function or Unit Pulse Sequence

$$\delta[n] = \begin{cases} 1 \ , & n = 0 \\ 0 \ , & n \neq 0 \end{cases}$$



$\delta[n] = \delta[an]$ for any non-zero, finite integer a.

Unit Pulse Sequence Contd.

- The discrete-time unit impulse is a function in the ordinary sense in contrast with the continuous-time unit impulse.
- It has a sampling property.
- It has no scaling property i.e.

  $\delta[n] = \delta[an]$ for any non-zero finite integer 'a'

Operations of Signals

- Sometime a given mathematical function may completely describe a signal .
- Different operations are required for different purposes of arbitrary signals.
- The operations on signals can be

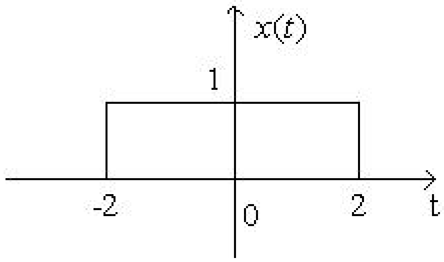
  Time Shifting

  Time Scaling

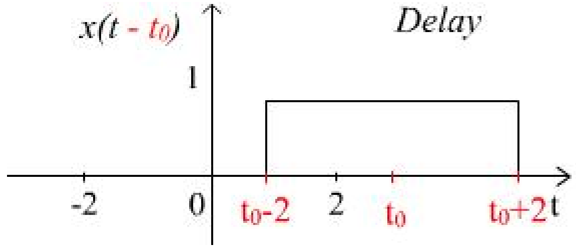
  Time Inversion or Time Folding

Time Shifting

- The original signal $x(t)$ is shifted by an amount t_o.

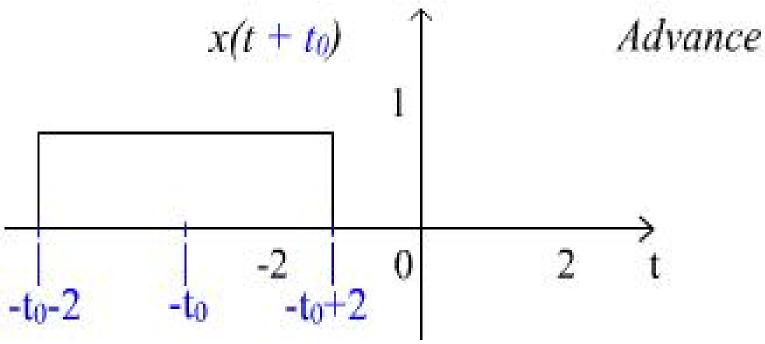


- X(t)→X(t-to) →Signal Delayed→ Shift to the right



Time Shifting Contd.

- X(t)→X(t+to) →Signal Advanced→ Shift to the left

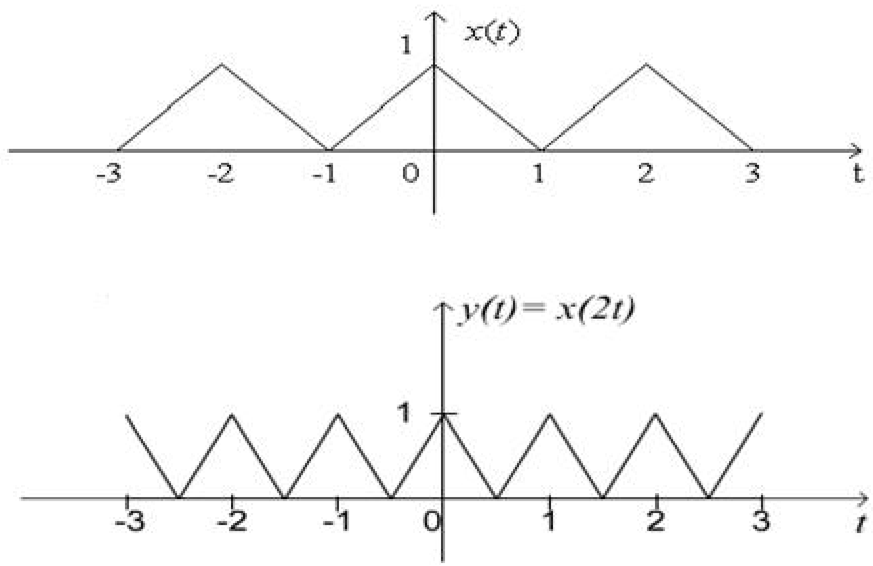


Time Scaling

- For the given function x(t), x(at) is the time scaled version of x(t)
- For $a > 1$,period of function x(t) reduces and function speeds up. Graph of the function shrinks.
- For $a < 1$, the period of the x(t) increases and the function slows down. Graph of the function expands.

Time scaling Contd.

Example: Given $x(t)$ and we are to find $y(t) = x(2t)$.

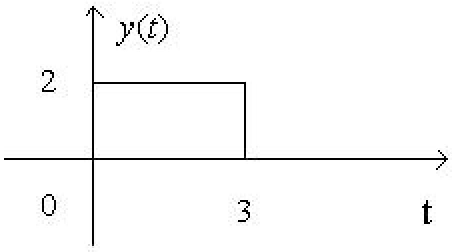


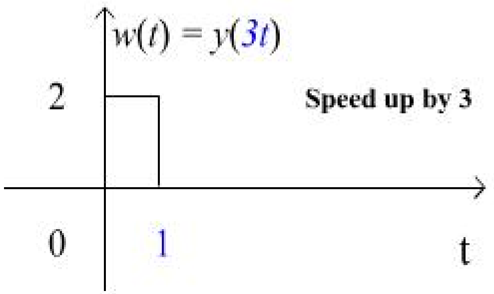
The period of $x(t)$ is 2 and the period of $y(t)$ is 1,

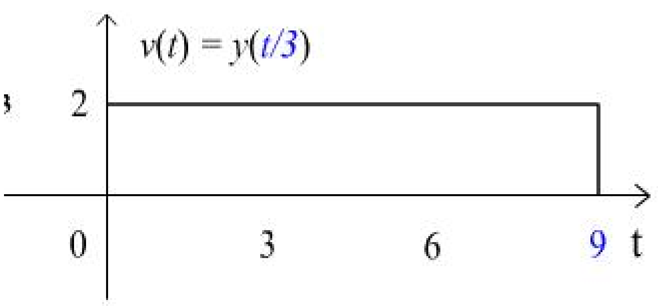
Time scaling Contd.

- Given $y(t)$,
  - find $w(t) = y(3t)$

  and $v(t) = y(t/3)$.





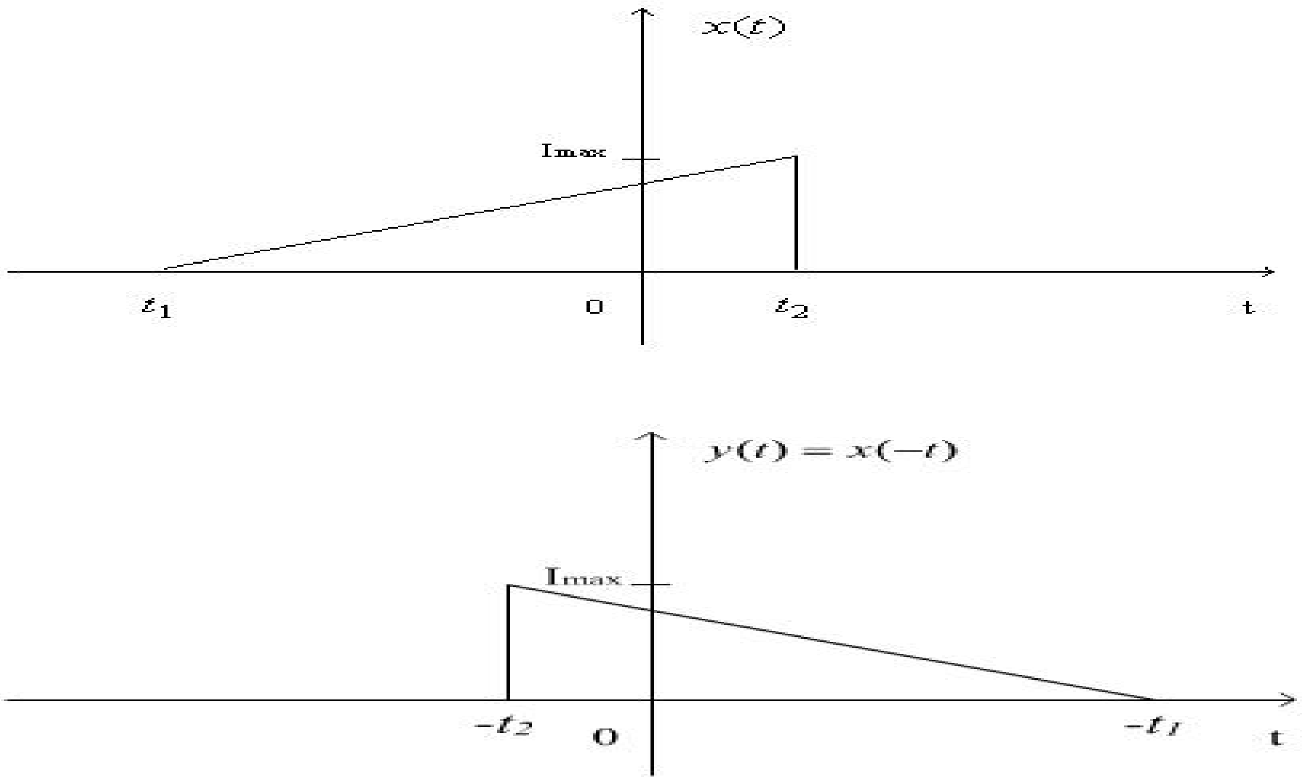


Time Reversal

- Time reversal is also called time folding
- In Time reversal signal is reversed with respect to time i.e.

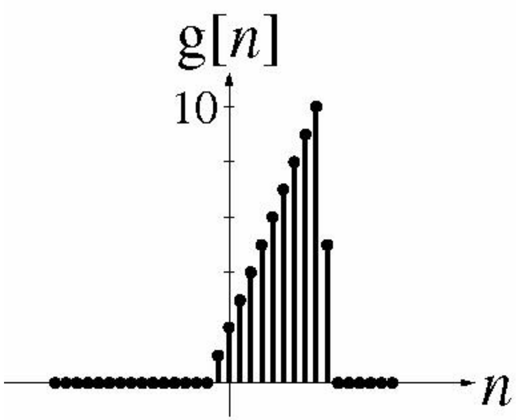
  $y(t) = x(-t)$ is obtained for the given function

Time reversal Contd.

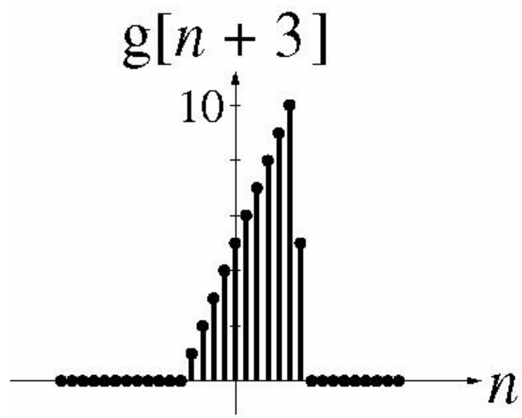


Operations of Discrete Time Functions

Time shifting $\quad n \rightarrow n + n_0,\ n_0$ an integer



n	$g[n]$
–1	1
0	2
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10
9	5
10	0

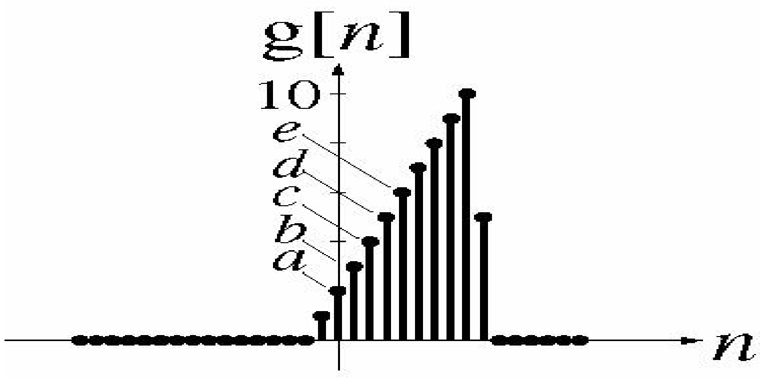


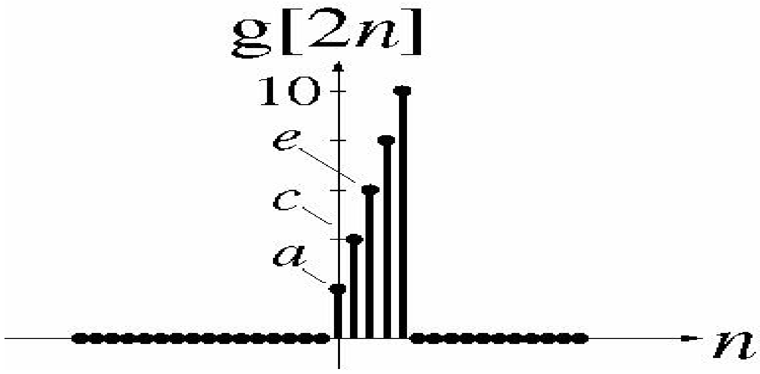
n	$n+3$	$g[n+3]$
–4	–1	1
–3	0	2
–2	1	3
–1	2	4
0	3	5
1	4	6
2	5	7
3	6	8
4	7	9
5	8	10
6	9	5
7	10	0

Operations of Discrete Functions Contd.

Scaling; Signal Compression

$n \rightarrow Kn \qquad K$ an integer > 1





n	$2n$	g[2n]
0	0	2
1	2	4
2	4	6
3	6	8
4	8	10

Classification of Signals

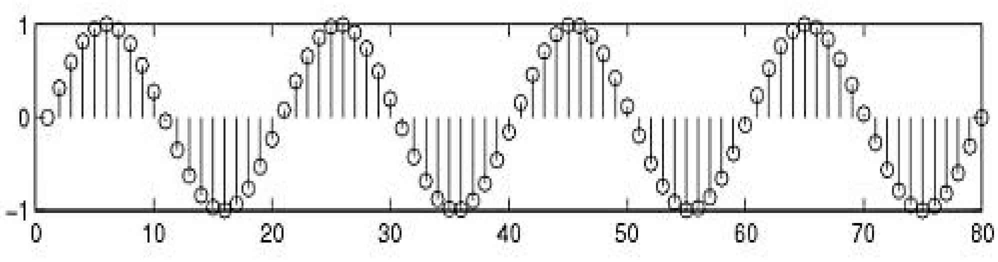
- Deterministic & Non Deterministic Signals
- Periodic & A periodic Signals
- Even & Odd Signals
- Energy & Power Signals

Deterministic & Non Deterministic Signals

Deterministic signals

- **Behavior of these signals is predictable w.r.t time**
- **There is no uncertainty with respect to its value at any time.**
- **These signals can be expressed mathematically.**

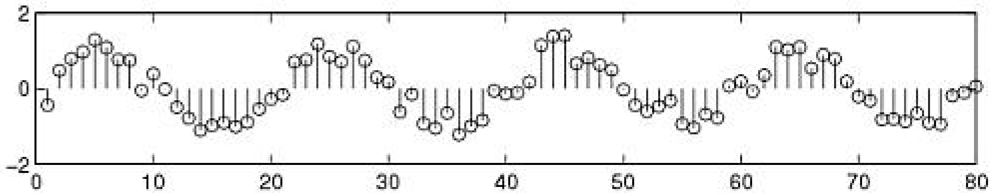
For example $x(t) = \sin(3t)$ is deterministic signal.



Deterministic & Non Deterministic Signals Contd.

Non Deterministic or Random signals

- Behavior of these signals is random i.e. not predictable w.r.t time.
- There is an uncertainty with respect to its value at any time.
- These signals can't be expressed mathematically.
- For example Thermal Noise generated is non deterministic signal.



Periodic and Non-periodic Signals

- Given x(t) is a continuous-time signal
- x (t) is periodic iff $x(t) = x(t+T_o)$ for any T and any integer n
- Example
  - $x(t) = A\cos(\omega t)$
  - $x(t+T_o) = A\cos[\omega(t+T_o)] = A\cos(\omega t+\omega T_o) = A\cos(\omega t+2\pi) = A\cos(\omega t)$
  - Note: $T_o = 1/f_o$; $\omega = 2\pi f_o$

Periodic and Non-periodic Signals Contd.

- For non-periodic signals

$$x(t) \neq x(t+T_o)$$

- A non-periodic signal is assumed to have a period $T = \infty$
- Example of non periodic signal is an exponential signal

Important Condition of Periodicity for Discrete Time Signals

- A discrete time signal is periodic if

$$x(n) = x(n+N)$$

- For satisfying the above condition the frequency of the discrete time signal should be ratio of two integers

$$\text{i.e. } f_o = k/N$$

Sum of periodic Signals

- $X(t) = x1(t) + X2(t)$
- $X(t+T) = x1(t+m_1T_1) + X2(t+m_2T_2)$
- $m_1T_1 = m_2T_2 = T_o$ = Fundamental period
- Example: $\cos(t\pi/3)+\sin(t\pi/4)$
  - $T1=(2\pi)/(\pi/3)=6$; $T2=(2\pi)/(\pi/4)=8$;
  - $T1/T2=6/8 = 3/4$ = (rational number) = $m2/m1$
  - $m_1T_1=m_2T_2 \rightarrow$ Find m1 and m2 $\rightarrow$
  - $6.4 = 3.8 = 24 = T_o$

Sum of periodic Signals – may not always be periodic!

$$x(t) = x_1(t) + x_2(t) = \cos t + \sin \sqrt{2t}$$

T1=(2π)/(1)= 2π; T2 =(2π)/(sqrt(2));

T1/T2= sqrt(2);

– Note: T1/T2 = sqrt(2) is an irrational number

– X(t) is aperiodic

e) Given that, $x(t) = \cos^2\left(2t - \frac{\pi}{3}\right)$

$$x(t) = \cos^2\left(2t - \frac{\pi}{3}\right) = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

$$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$\therefore\ x(t + T) = \frac{1 + \cos\left(4(t + T) - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t + 4T - \frac{2\pi}{3}\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4T\right)}{2}$$

Let $4T = 2\pi$, $\therefore\ T = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\therefore\ x(t + T) = \frac{1 + \cos\left(4t - \frac{2\pi}{3} + 4 \times \frac{\pi}{2}\right)}{2} = \frac{1 + \cos\left(\left(4t - \frac{2\pi}{3}\right) + 2\pi\right)}{2}$$

$$= \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2} = \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \cos^2\left(2t - \frac{\pi}{3}\right) = x(t)$$

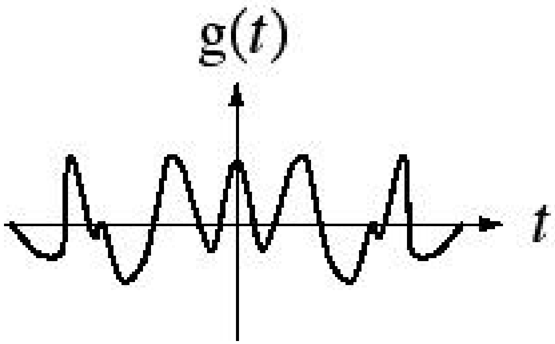
Since $x(t + T) = x(t)$, the signal $x(t)$ is periodic with period, $T = \frac{\pi}{2}$

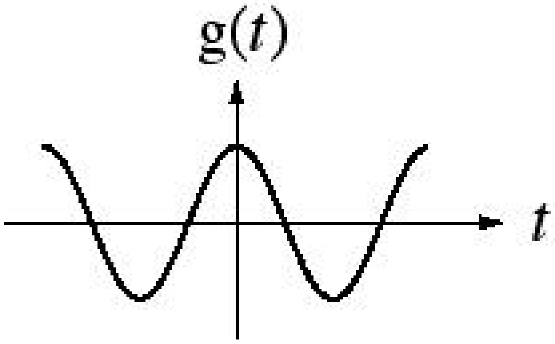
For integer values of M, $\cos(\theta + 2\pi M) = \cos\theta$

Even and Odd Signals

Even Functions

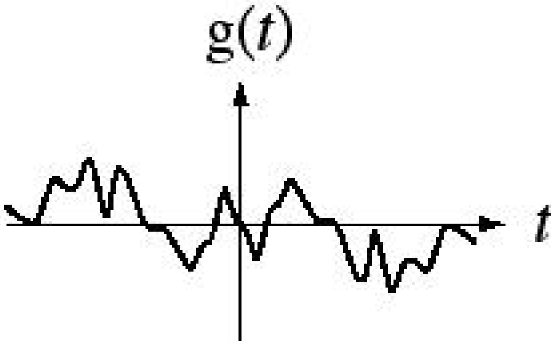
$$g(t) = g(-t)$$

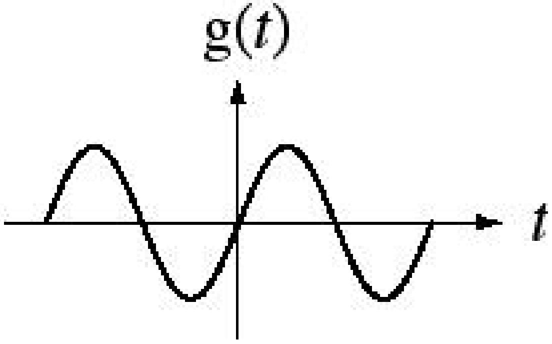




Odd Functions

$$g(t) = -g(-t)$$





Even and Odd Parts of Functions

The **even part** of a function is $g_e(t) = \dfrac{g(t) + g(-t)}{2}$

The **odd part** of a function is $g_o(t) = \dfrac{g(t) - g(-t)}{2}$

A function whose even part is zero, is odd and a function whose odd part is zero, is even.

Various Combinations of even and odd functions

Function type	Sum	Difference	Product	Quotient
Both even	**Even**	**Even**	**Even**	**Even**
Both odd	**Odd**	**Odd**	**Even**	**Even**
Even and odd	**Neither**	**Neither**	**Odd**	**Odd**

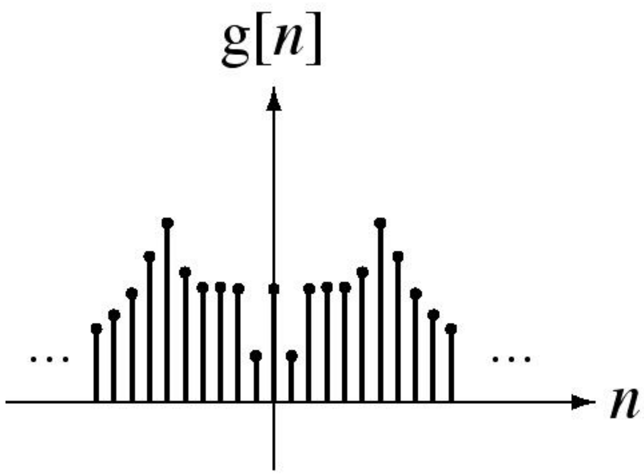
Derivatives and Integrals of Functions

Function type	Derivative	Integral
Even	Odd	Odd + constant
Odd	Even	Even

Discrete Time Even and Odd Signals

$$\mathrm{g}[n] = \mathrm{g}[-n]$$

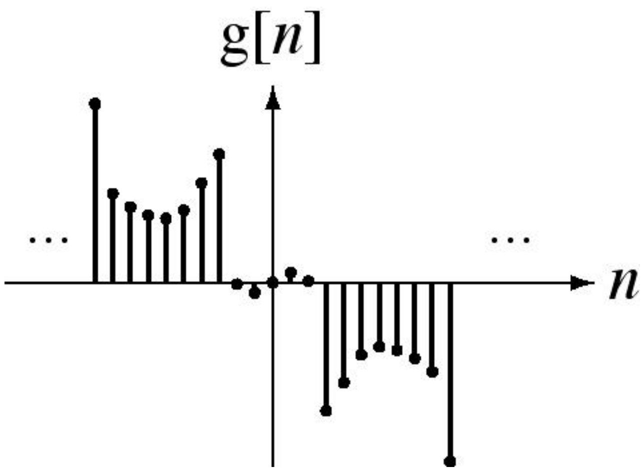
Even Function



$$\mathrm{g}_e[n] = \frac{\mathrm{g}[n] + \mathrm{g}[-n]}{2}$$

$$\mathrm{g}[n] = -\mathrm{g}[-n]$$

Odd Function



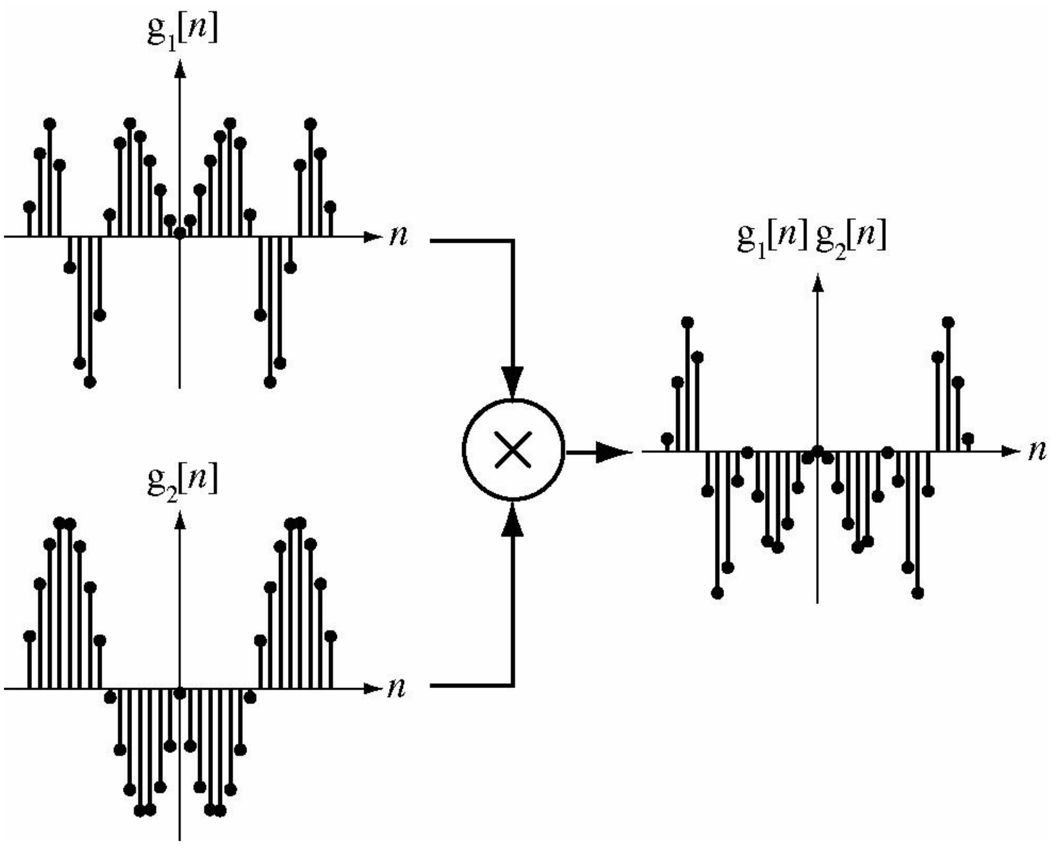
$$\mathrm{g}_o[n] = \frac{\mathrm{g}[n] - \mathrm{g}[-n]}{2}$$

Combination of even and odd function for DT Signals

Function type	Sum	Difference	Product	Quotient
Both even	Even	Even	Even	Even
Both odd	Odd	Odd	Even	Even
Even and odd	Even or Odd	Even or odd	Odd	Odd

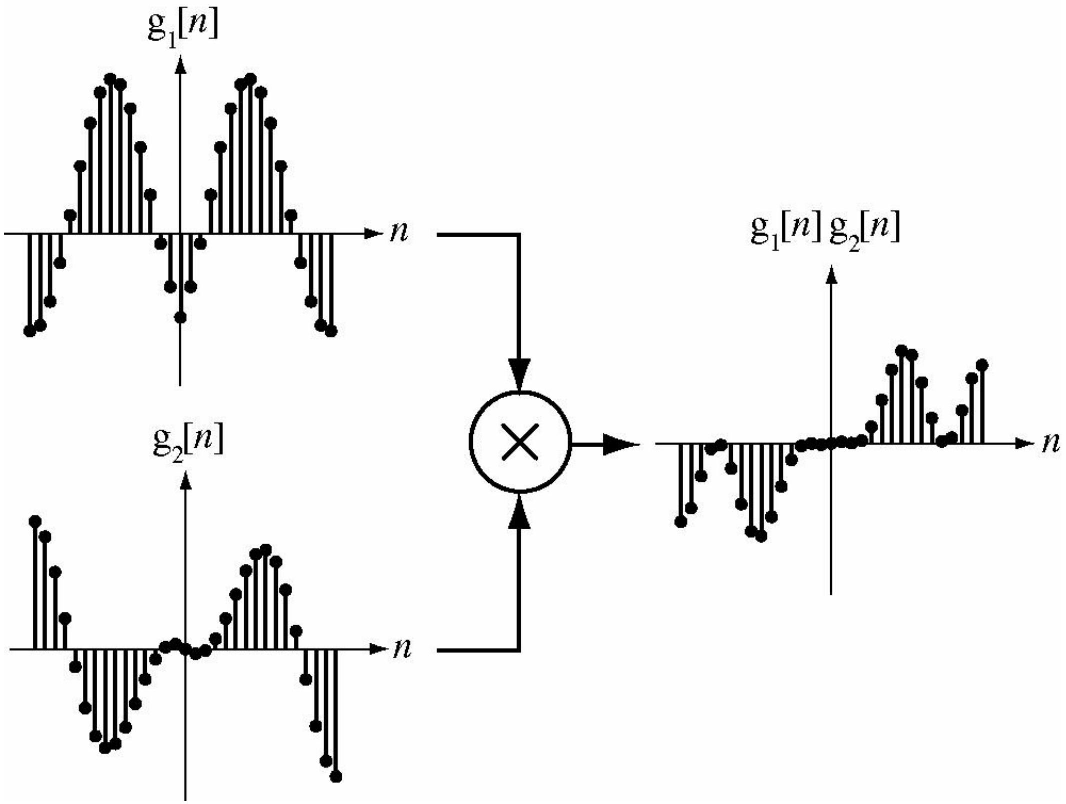
Products of DT Even and Odd Functions

Two Even Functions



Products of DT Even and Odd Functions Contd.

An Even Function and an Odd Function



Proof Examples

- Prove that product of two even signals is even.

Change t→ -t

$$x(t) = x_1(t) \times x_2(t) \rightarrow$$
$$x(-t) = x_1(-t) \times x_2(-t) =$$
$$x_1(t) \times x_2(t) = x(t)$$

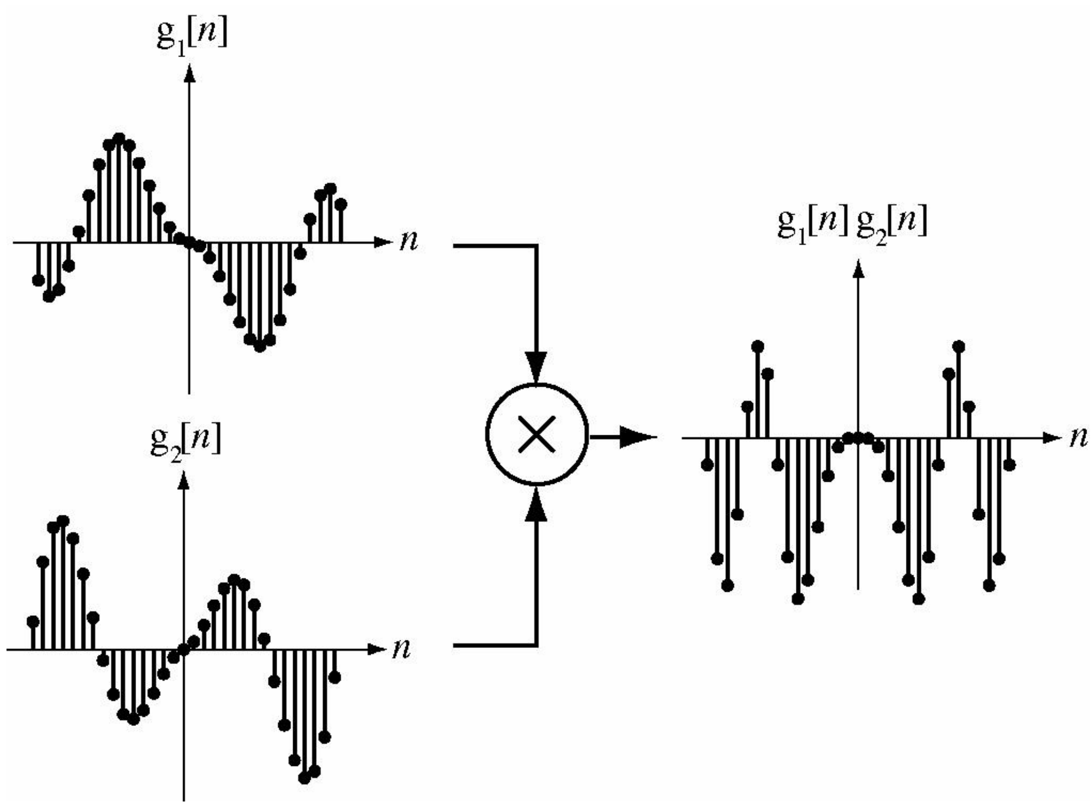
- Prove that product of two odd signals is odd.

- What is the product of an even signal and an odd signal? Prove it!

$$x(t) = x_1(t) \times x_2(t) \rightarrow$$
$$x(-t) = x_1(-t) \times x_2(-t) =$$
$$x_1(t) \times -x_2(t) = -x(t) =$$
$$x(-t) \leftarrow Even$$

Products of DT Even and Odd Functions Contd.

Two Odd Functions



Energy and Power Signals

Energy Signal

- A signal with finite energy and zero power is called Energy Signal i.e.for energy signal

$$0<E<\infty \text{ and } P=0$$

- Signal energy of a signal is defined as the *area under the square of the magnitude of the signal*.

$$E_x = \int_{-\infty}^{\infty} \left| x(t) \right|^2 dt$$

- The units of signal energy depends on the unit of the signal.

Energy and Power Signals Contd.

Power Signal

- Some signals have infinite signal energy. In that caseit is more convenient to deal with **average signal power.**
- For power signals

$$0<P<\infty \text{ and } E=\infty$$

- Average power of the signal is given by

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \left|x(t)\right|^2 dt$$

Energy and Power Signals Contd.

- For a periodic signal x(t) the average signal power is

$$P_x = \frac{1}{T}\int_T \left|\mathrm{x}(t)\right|^2 dt$$

- T is any period of the signal.
- Periodic signals are generally power signals.

Signal Energy and Power for DT Signal

- A discrtet time signal with finite energy and zero power is called Energy Signal i.e.for energy signal

$$0<E<\infty \text{ and } P=0$$

- The **signal energy** of a for a discrete time signal x[*n*] is

$$E_x = \sum_{n=-\infty}^{\infty} \left| x[n] \right|^2$$

Signal Energy and Power for DT Signal Contd.

The average signal power of a discrete time power signal x[n] is

$$P_x = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N-1} \left| x[n] \right|^2$$

For a periodic signal x[n] the average signal power is

$$P_x = \frac{1}{N} \sum_{n=\langle N \rangle} \left| x[n] \right|^2$$

$\left(\text{The notation } \sum_{n=\langle N \rangle} \text{ means the sum over any set of consecutive } n\text{'s exactly } N \text{ in length.} \right)$

a) Given that, $x(t) = e^{-2t}\, u(t)$

Here, $x(t) = e^{-2t}\, u(t)$; for all t

$\therefore\ x(t) = e^{-2t}$; for $t \geq 0$

$$\therefore \int_{-T}^{T} |x(t)|^2\, dt = \int_{0}^{T} \left(\left|e^{-2t}\right|\right)^2 dt = \int_{0}^{T} \left(e^{-2t}\right)^2 dt = \int_{0}^{T} e^{-4t}\, dt = \left[\frac{e^{-4t}}{-4}\right]_0^T$$

$$= \left[\frac{e^{-4T}}{-4} - \frac{e^{0}}{-4}\right] = \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$\text{Energy, } E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt} \left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{4} - \frac{e^{-\infty}}{4} = \frac{1}{4} - \frac{0}{4} = \frac{1}{4} \text{ joules}$$

$$\text{Power, } P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt = \underset{T\to\infty}{Lt} \frac{1}{2T}\left[\frac{1}{4} - \frac{e^{-4T}}{4}\right]$$

$$= \frac{1}{\infty}\left[\frac{1}{4} - \frac{e^{-\infty}}{4}\right] = 0 \times \left[\frac{1}{4} - 0\right] = 0$$

Since energy is constant and power is zero, the given signal is an energy signal.

What is System?

- Systems process input signals to produce output signals
- A system is combination of elements that manipulates one or more signals to accomplish a function and produces some output.

Types of Systems

- Causal & Anticausal
- Linear & Non Linear
- Time Variant &Time-invariant
- Stable & Unstable
- Static & Dynamic
- Invertible & Inverse Systems

Causal & Anticausal Systems

- Causal system : A system is said to be *causal* if the present value of the output signal depends only on the present and/or past values of the input signal.
- Example: $y[n]=x[n]+1/2x[n-1]$

Causal & Anticausal Systems Contd.

- Anticausal system : A system is said to be *anticausal* if the present value of the output signal depends only on the future values of the input signal.

- Examples of causal systems:

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

- Examples of non-causal systems:

$$y(t) = x(-t) \qquad \text{or} \qquad y[n] = \frac{1}{3}(x[n-1] + x[n] + x[n+1]).$$

Linear & Non Linear Systems

1.7.1 Linear and Nonlinear Systems

A system is said to be linear if Superposition theorem applies to that system. Consider the two systems defined as follows :

$y_1(t) = f[x_1(t)]$ i.e. $x_1(t)$ is excitation and $y_1(t)$ is response.

$y_2(t) = f[x_2(t)]$ i.e. $x_2(t)$ is excitation and $y_2(t)$ is response.

Then for the linear system,

$$f[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t) \quad \text{... (1.7.1)}$$

Here a_1 and a_2 are constants.

(ii) $y(t) = x^2(t)$

The output of the system to two inputs $x_1(t)$ and $x_2(t)$ becomes,

$$y_1(t) = f[x_1(t)] = x_1^2(t)$$

$$y_2(t) = f[x_2(t)] = x_2^2(t)$$

Hence linear combination of these outputs become,

$$\begin{aligned} y_3(t) &= a_1 y_1(t) + a_2 y_2(t) \\ &= a_1 x_1^2(t) + a_2 x_2^2(t) \end{aligned}$$

Now let us find the response of the system to linear combination of inputs. i.e.,

$$\begin{aligned} y'_3(t) &= f[a_1 x_1(t) + a_2 x_2(t)] \\ &= [a_1 x_1(t) + a_2 x_2(t)]^2 \\ &= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2 a_1 a_2 x_1(t) x_2(t) \end{aligned}$$

Here note that $y'_3(t) \neq y_3(t)$. Hence this is not linear system.

Time Invariant and Time Variant Systems

- A system is said to be *time invariant* if a time delay or time advance of the input signal leads to a identical time shift in the output signal.

$$y_i(t) = H\{x(t-t_0)\}$$

$$= H\{S^{t0}\{x(t)\}\} = HS^{t0}\{x(t)\}$$

$$y_0(t) = S^{t0}\{y(t)\}$$

$$= S^{t0}\{H\{x(t)\}\} = S^{t0}H\{x(t)\}$$

Solution : (i) $y(t) = \sin x(t)$

Let us determine the output of the system for delayed input $x(t-t_1)$. i.e.,

$$y(t, t_1) = f[x(t-t_1)]$$

$$= \sin x(t-t_1) \qquad \text{... (1.7.6)}$$

Here $y(t, t_1)$ represents output due to delayed input.

Now delay the output $y(t)$ by t_1. Hence we have to replace t by $t-t_1$ in $y(t) = \sin x(t)$. i.e.,

$$y(t-t_1) = \sin x(t-t_1)$$

On comparing the above equation with equation 1.7.6 we find that,

$$y(t, t_1) = y(t-t_1)$$

This satisfies equation 1.7.5. Hence the system is time invariant.

Stable & Unstable Systems

- A system is said to be *bounded-input bounded-output stable* (BIBO stable) iff every bounded input results in a bounded output.

i.e.

$$\forall t \quad |x(t)| \leq M_x < \infty \rightarrow \forall t \quad |y(t)| \leq M_y < \infty$$

Stable & Unstable Systems Contd.

Example

- $y[n]=1/3(x[n]+x[n-1]+x[n-2])$

$$
\begin{aligned}
y[n] &= \frac{1}{3}\left|x[n]+x[n-1]+x[n-2]\right| \\
&\le \frac{1}{3}(|x[n]|+|x[n-1]|+|x[n-2]|) \\
&\le \frac{1}{3}(M_x+M_x+M_x)=M_x
\end{aligned}
$$

Stable & Unstable Systems Contd.

Example: The system represented by

$$y(t) = A\,x(t) \text{ is unstable ; } A>1$$

Reason: let us assume x(t) = u(t), then at every instant u(t) will keep on multiplying with A and hence it will not be bonded.

Static & Dynamic Systems

- A static system is memoryless system
- It has no storage devices
- its output signal depends on present values of the input signal
- For example

$$i(t) = \frac{1}{R} v(t)$$

Static & Dynamic Systems Contd.

- A dynamic system possesses memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal

- Examples of memoryless systems:

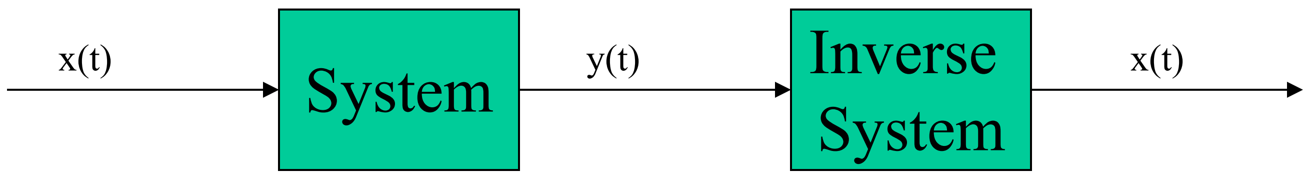
$$y(t) = Rx(t) \qquad \text{or} \qquad y[n] = \left(2x[n] - x^2[n]\right)^2.$$

- Examples of systems with memory:

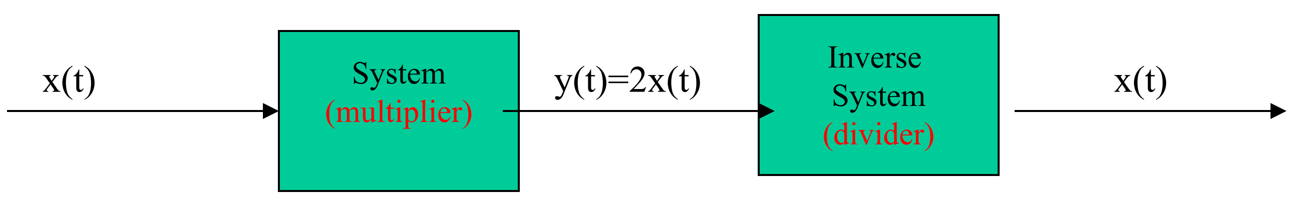
$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

Invertible & Inverse Systems

- If a system is invertible it has an Inverse System



- Example: y(t)=2x(t)
  - System is invertible→ must have inverse, that is:
  - For any x(t) we get a distinct output y(t)
  - Thus, the system must have an Inverse
    - x(t)=1/2 y(t)=z(t)



Discrete-Time Systems

- A Discrete-Time System is a mathematical operation that maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\} \qquad x[n] \longrightarrow \boxed{T\{.\}} \longrightarrow y[n]$$

Example:

Moving (Running) Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

Maximum $\quad y[n] = \max\{x[n], x[n-1], x[n-2]\}$

Ideal Delay System $\quad y[n] = x[n - n_o]$

Memoryless System

A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n

Example :

Square $$y[n] = (x[n])^2$$

Sign $$y[n] = \text{sign}\{x[n]\}$$

counter example:

Ideal Delay System $$y[n] = x[n - n_o]$$

Linear Systems

- Linear System: A system is linear if and only if

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} \quad \text{(additivity)}$$

and

$$T\{ax[n]\} = aT\{x[n]\} \quad \text{(scaling)}$$

Example: Ideal Delay System

$$y[n] = x[n - n_o]$$

$$\begin{aligned}
T\{x_1[n] + x_2[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{x_2[n]\} + T\{x_1[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{ax[n]\} &= ax_1[n - n_o] \\
aT\{x[n]\} &= ax_1[n - n_o]
\end{aligned}$$

Time-Invariant Systems

Time-Invariant (shift-invariant) Systems

$$y[n] = T\{x[n]\} \Rightarrow y[n - n_o] = T\{x[n - n_o]\}$$

A time shift at the input causes corresponding time-shift at output

Example: Square

$$y[n] = (x[n])^2$$

Delay the input the output is $y_1[n] = (x[n - n_o])^2$

Delay the output gives $y[n - n_o] = (x[n - n_o])^2$

Counter Example: Compressor System

$$y[n] = x[Mn]$$

Delay the input the output is $y_1[n] = x[Mn - n_o]$

Delay the output gives $y[n - n_o] = x[M(n - n_o)]$

Causal System

A system is causal iff it's output is a function of only the current and previous samples

Examples: Backward Difference

$$y[n] = x[n] - x[n-1]$$

Counter Example: Forward Difference

$$y[n] = x[n+1] + x[n]$$

Stable System

Stability (in the sense of bounded-input bounded-output BIBO). A system is stable iff every bounded input produces a bounded output

$$|x[n]| \le B_x < \infty \Rightarrow |y[n]| \le B_y < \infty$$

Example: Square $\qquad y[n] = (x[n])^2$

if input is bounded by $|x[n]| \le B_x < \infty$

output is bounded by $|y[n]| \le B_x^2 < \infty$

Counter Example: Log $\qquad y[n] = \log_{10}(|x[n]|)$

even if input is bounded by $|x[n]| \le B_x < \infty$

output not bounded for $x[n] = 0 \Rightarrow y[0] = \log_{10}(|x[n]|) = -\infty$

THANKS