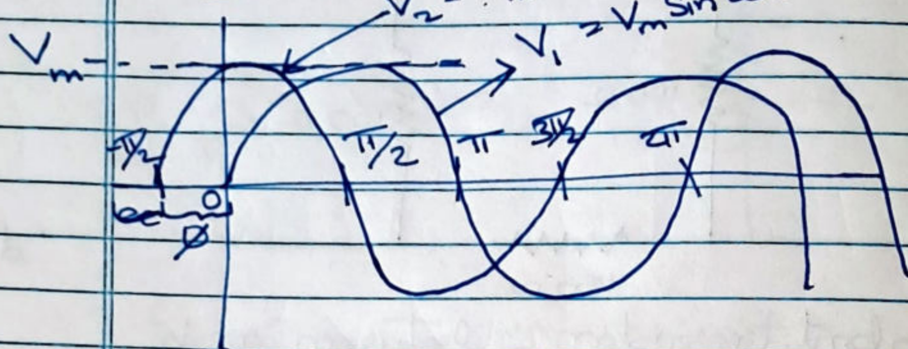
Module 2:

Ac Circuits:



We can find

1. Amplitude, $V_m \rightarrow$ Volts
2. Phase angle, $\phi \rightarrow$ deg
3. Angular frequency $\boxed{\omega = 2\pi f} \rightarrow$ rad/sec
4. Time period $\boxed{T = \frac{2\pi}{\omega}} \rightarrow$ sec
5. frequency $\boxed{f = \frac{1}{T}} \rightarrow$ Hz or cycles/sec

Eg: $V(t) = 10 \sin(50\pi t + 60^\circ)$

Sol: $V_m = 10$ volts

$\phi = 60^\circ$

$\omega = 50\pi$ rad/sec

$T = \frac{2\pi}{\omega} = \frac{1}{25}$ secs

$f = \frac{1}{T} = 25$ Hz

Phasor: (a rotating vector).

→ A scaled line where length of the line represents the magnitude and direction of the line represents the phase at a particular instant of time.

Complex → (i) $Z = x + jy$ [Rectangular coordinate]

(ii) $Z = r\angle\phi$ [polar coordinates]

where $r = \sqrt{x^2+y^2}$

&

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

also can be written,

(iii) $Z = re^{j\phi}$

$Z = \underbrace{r\cos\phi}_{\text{Real}} + j\underbrace{r\sin\phi}_{\text{Imaginary}}$ [exponential form]

Suppose,

$V(t) = V_m \cos(\omega t + \phi)$

In exponential form;

$V(t) = V_m e^{j(\omega t+\phi)}$

$\Rightarrow V_m \cos(\omega t+\phi) + jV_m \sin(\omega t+\phi)$

So,

$V(t) = V_m \cos(\omega t+\phi)$

$= \text{Real}\{V_m e^{j(\omega t+\phi)}\}$

$= \text{Real}\{V_m e^{j\omega t} . e^{j\phi}\}$

$V(t) = \text{Real}\{V_m e^{j\phi}\}$ [$\because e^{j\omega t}$ negligible]

$v(t) = V_m \angle \phi$

So overall;

$v(t) = V_m \cos(\omega t + \phi) = V_m \angle \phi$

Eg: ① $V(t) = 10\cos[50t + 10^\circ]$

$V = 10\angle 10$

$\omega = 50$ rad/s.

② $V(t) = 20\cos(100\pi t - 20^\circ)$

$V = 20\angle 20$

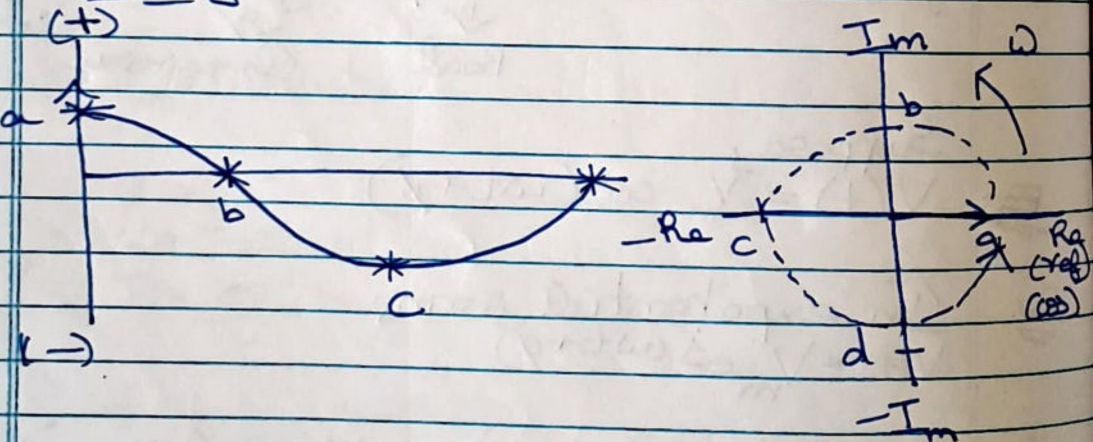
$\omega = 100\pi$ rad/s.

Note:

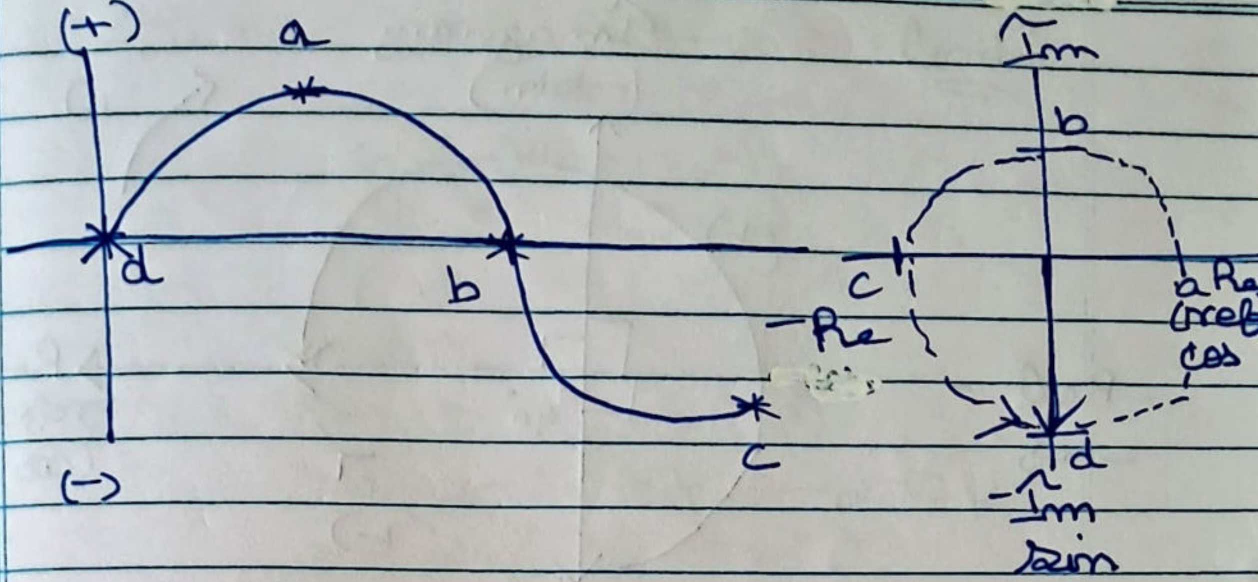
- $V_m \cos(\omega t + \phi) = V_m \angle \phi$
- $V_m \cos(\omega t - \phi) = V_m \angle -\phi$

Plotting:

(i)



(ii)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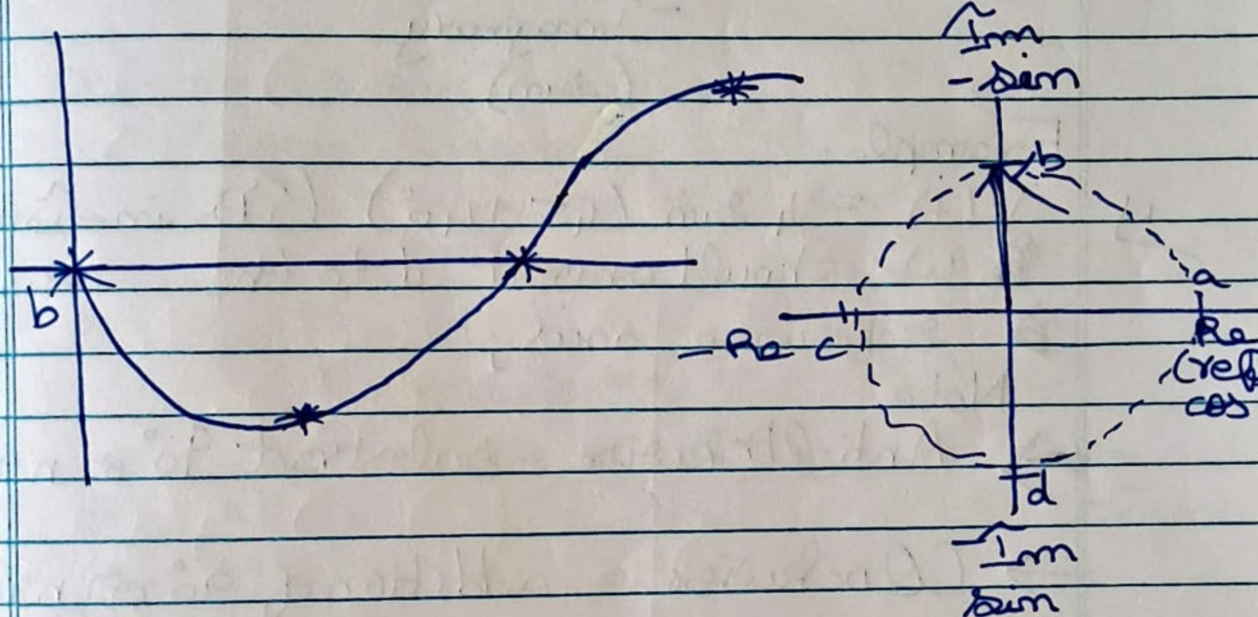
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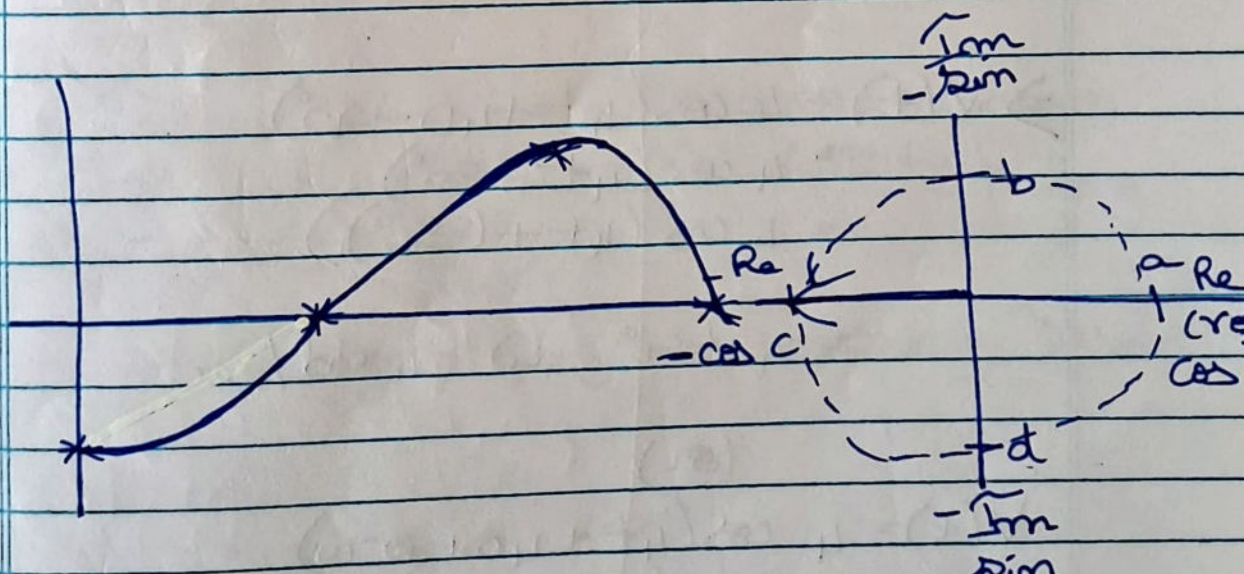
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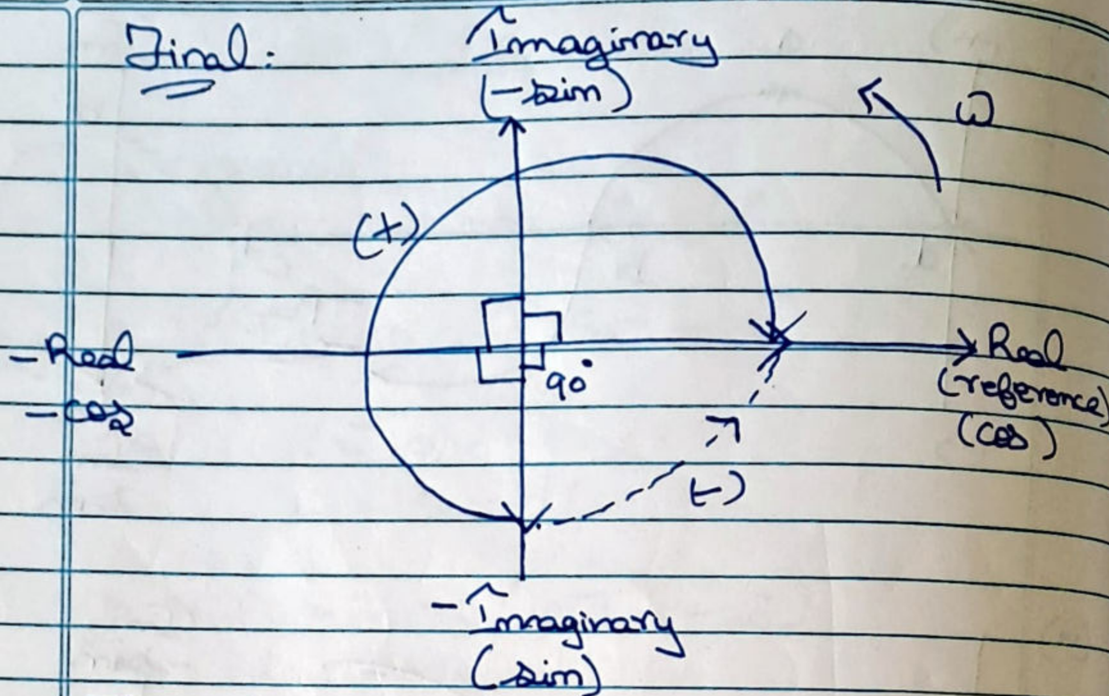
Final:



Example:

1) $V(t) = 4\sin(4t + 40°)$ (It's in −$\hat{I}$m axis).
∴ We should convert it to the reference axis;

Note

→ Anticlockwise = substract 90° or n90°.

→ Clockwise = addition of 90° or n90°.

$\Rightarrow V(t) = 4\cos(4t + 40 - 90°)$
$= 4\cos(4t - 50°)$
$= 4\cos(4t + (-50°))$

$= 4\angle -50°$; $\omega = 4$ rad/sec

(or)

$V(t) = 4\cos(4t + 40 + 270)$
$= 4\cos(4t + 310°)$
$= 4\angle 310°$; $\omega = 4$ rad/sec.

2) $V(t) = -4\sin(\omega t + 40^\circ)$
$= 4\cos(\omega t + 40^\circ + 90^\circ)$
$= 4\cos(\omega t + 130^\circ)$
$= 4\angle 130^\circ$; $\omega = \omega$.

(or)

$V(t) = -4\sin(\omega t + 40^\circ)$
$= 4\cos(\omega t + 40^\circ - 270^\circ)$
$= 4\cos(\omega t - 230^\circ)$
$= 4\angle -230^\circ$; $\omega = \omega$.

So steps simplified;
- Given Sinusoid
  ↓
  Check
  ↓
  $V_m\cos(\omega t + \phi)$ — ✓ ⇒ $V_m\angle\phi$
  ↳ × ⟶ convert using phasor.

Sum of phasors:

- To add sinusoids in time domain;
  → Use cos (A+B) identity } long
  → simplify
  (or)
  → Change to phasor and solve] short

Example:
1) $V_1(t) = -10\cos(\omega t - 30^\circ)$
$V_2(t) = 20\cos(\omega t + 50^\circ)$
Find $V_1(t) + V_2(t)$;

Sol:

First convert $V_1(t)$ to reference axis

$$V_1(t) = -10\cos(\omega t - 30^\circ)$$
$$= 10\cos(\omega t - 30^\circ + 180^\circ)$$
$$= 10\cos(\omega t + 150^\circ)$$

$\Rightarrow V_1(t) = 10\angle 150^\circ$

$V_2(t) = 20\angle 50^\circ$

$V_1 + V_2 = 10\angle 150^\circ + 20\angle 50^\circ$

$\Rightarrow (-8.67 + j5) + (12.85 + j15.32)$

$\Rightarrow 4.18 + j20.32$

$\Downarrow$ To polar

$\Rightarrow 20.74\angle 78.37$

$\Rightarrow V_1(t) + V_2(t)$

$= 20.74\cos(\omega t + 78.37)$

$r\angle\phi \Rightarrow$ polar

$r = x + jy$

$\hookrightarrow$ rectangular (complex)

Addition / substraction

$(x_1 \pm jy_1) \pm (x_2 \pm jy_2)$

$\Rightarrow (x_1 \pm x_2) + j(y_1 \pm y_2)$

Eg: $(2 + j4) + (3 + j5)$

$\Rightarrow (2+3) + j(4+5)$

$= 5 + j9$

Eg: $(2 + j4) - (3 + j5)$

$\Rightarrow (2-3) + j(4-5)$

$\Rightarrow -1 - j1$

Multiplication / Division

$r_1\angle\phi_1 \times r_2\angle\phi_2 = (r_1 \times r_2)\angle\phi_1+\phi_2$

$\dfrac{r_1\angle\phi_1}{r_2\angle\phi_2} = \left(\dfrac{r_1}{r_2}\right)\angle\phi_1-\phi_2$

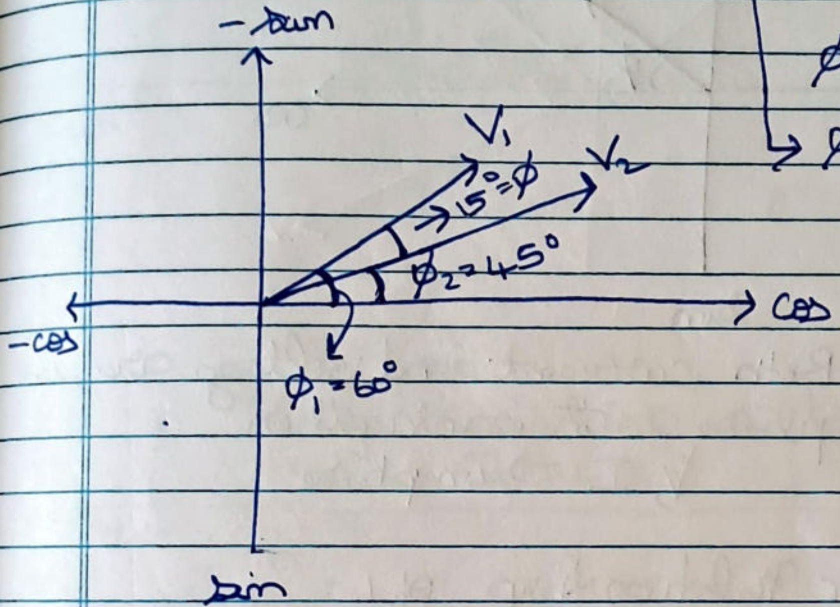
Angle between the phasors:

Eg: $V_1(t) = -10\sin(\omega t - 30^\circ) = 10\cos(\omega t + 60^\circ)$

$V_2(t) = 20\cos(\omega t + 45^\circ)$

$\phi_1 = 60^\circ$

$\phi_2 = 45^\circ$



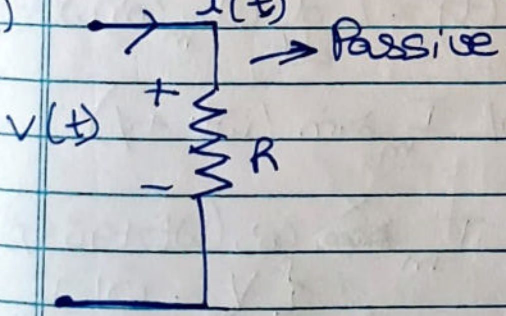
$\Rightarrow \phi = \phi_1 \sim \phi_2 = 15^\circ$.

$\Rightarrow V_1$ leads V_2 by 15°

V_2 lags V_1 by 15°.

Phasor Relationships of R : (RESISTOR)

(i)



$V(t) = R \times i(t)$.

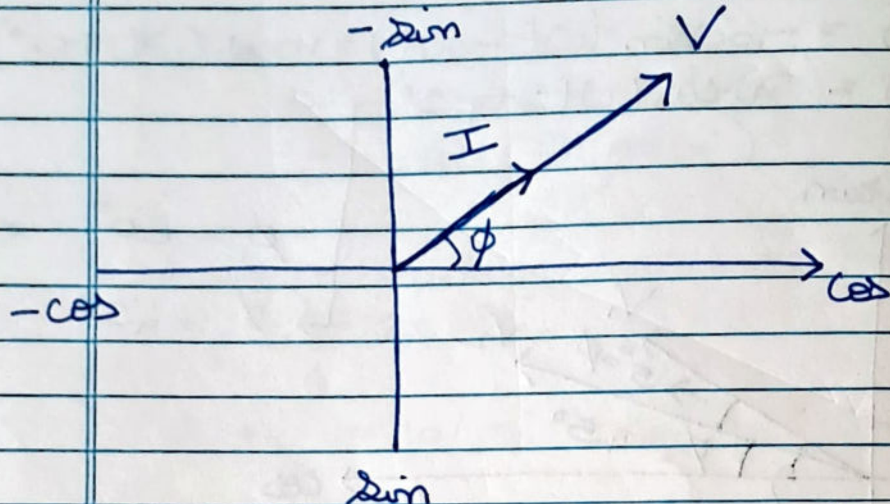
if $i(t) = I_m \cos(\omega t + \phi) \rightarrow (1) \{ I_m \angle \phi$

$\rightarrow V(t) = \underbrace{I_m R}_{V_m} \cos(\omega t + \phi)$

$V_m = I_m R \Rightarrow V(t) = V_m \cos(\omega t + \phi) \rightarrow (2)$

$\hookrightarrow V_m \angle \phi$

Graphical Representation:

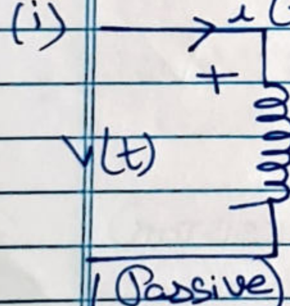


⇒ Both current and voltage are in phase with each other.

V, I → in phase

Phasor Relationship of L:

(i)



$$V(t) = L\frac{di(t)}{dt}$$

$$\text{If } i(t) = I_m \cos(\omega t + \phi) \rightarrow ①$$

$$V(t) = L\frac{d}{dt}\left(I_m \cos(\omega t + \phi)\right)$$

$$\Rightarrow V(t) = -\omega L I_m \sin(\omega t + \phi)$$

$$V(t) = \omega L I_m \cos(\omega t + 90^\circ + \phi)$$

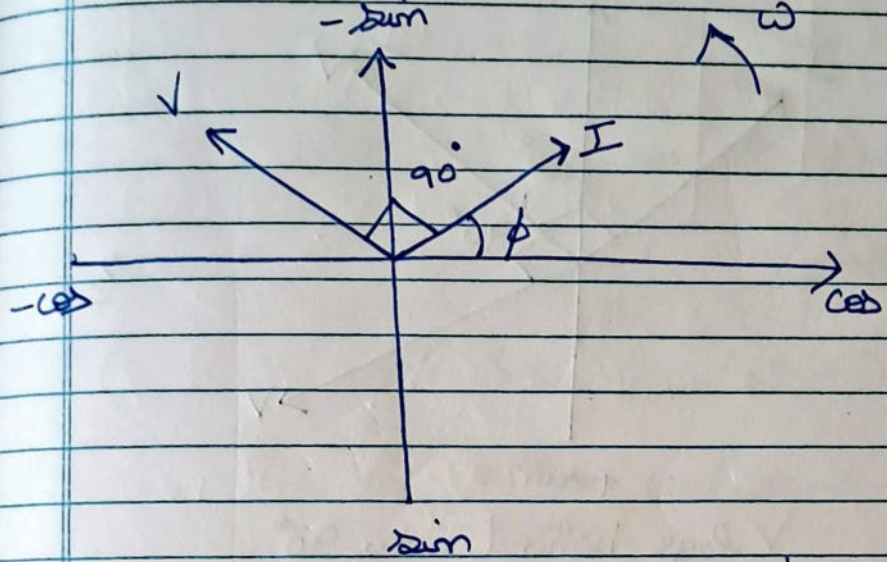
$$\{ V_m = \omega L I_m$$

$$V(t) = V_m \cos(\omega t + \phi + 90^\circ) \rightarrow ②$$

$$\{ V_m \angle \phi + 90^\circ$$

$$\omega L = \frac{V_m}{I_m} = |X_L| = \text{inductive reactance}$$

Graphical Representation:

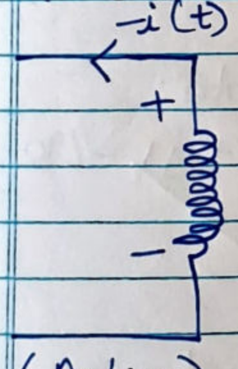


⇒ Voltage leads current by 90°.

V leads I → by 90°

H.W

(ii)



$$V(t) = -L\frac{di(t)}{dt}.$$

If $i(t) = I_m \cos(\omega t + \phi) \rightarrow ①$

$$V(t) = -L\frac{d}{dt}(I_m \cos(\omega t + \phi)).$$

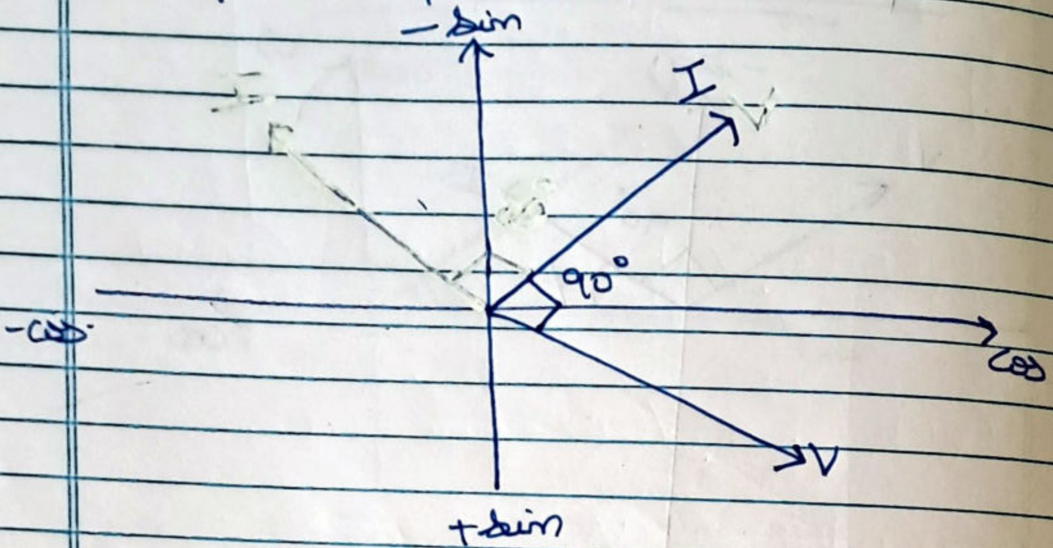
$$V(t) = +L\omega I_m \sin(\omega t + \phi).$$

$$V(t) = \omega L I_m \cos(\omega t + \phi - 90^\circ)$$

$\Rightarrow V(t) = V_m \cos(\omega t + \phi - 90^\circ)$

$= V_m \angle \phi - 90^\circ$

Graphical Representation.



V lags behind I by 90°.

$V = \cancel{\cos}\ V_m \angle \phi + 90°$ (Condt for passive convention)

$= \omega L \boxed{I_m \angle \phi} \angle 90°$

$V = j\omega L\, I$

[Note: $0 + j1 = \angle 90$]

$j\omega L = \dfrac{V}{I}$

Time domain $\quad$ Phasor domain

$V(t) = L\dfrac{di}{dt} \Rightarrow V = j\omega L I$

$$\boxed{\frac{d}{dt} \longrightarrow j\omega}$$

Note:

$S = P + jQ$

$S = VI^*$ $\quad$ [$I = I_m \angle \phi$, $I^* = I_m \angle -\phi$]

$= V_m \angle \phi + 90° \times I_m \angle \phi$

$= V_m I_m \angle \phi + 90° - \phi$

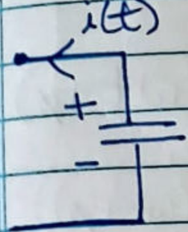
$P + jQ \leftarrow \boxed{S = V_m I_m \angle 90°} \rightarrow$ for an inductor

[let $V_m I_m = 1$] $\Rightarrow \angle 90° = 0 + 1j \Rightarrow P = 0$ & $Q = 1$

$\therefore$ Real Power = 0W

Page

Phasor Relationships of C:

(i) 

(Active)

$i(t) = -C\frac{dv}{dt}$

Let $V(t) = V_m \cos(\omega t + \phi) \rightarrow$ (1)

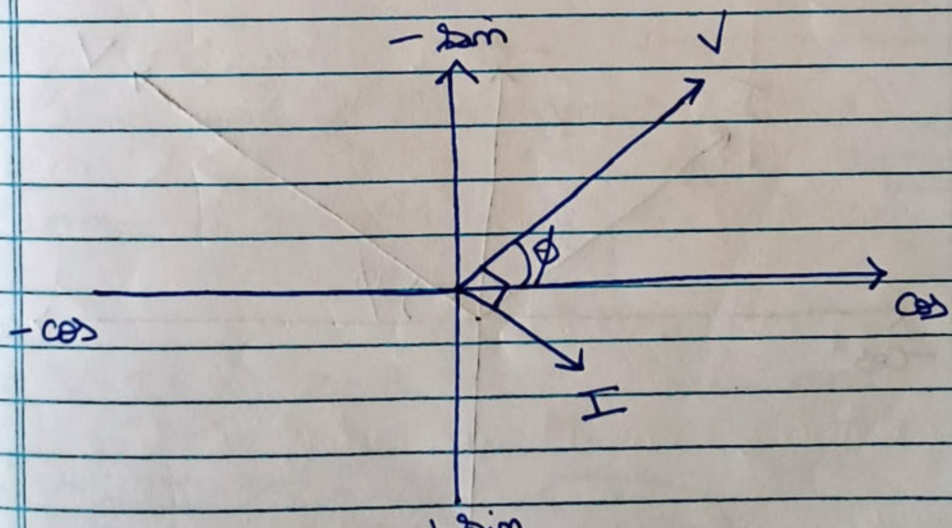
$\rightarrow i(t) = -C\frac{d}{dt}\left[V_m \cos(\omega t + \phi)\right]$

$$i(t) = C\omega V_m \sin(\omega t + \phi).$$

$$= \underbrace{C\omega V_m}_{I_m} \cos(\omega t + \phi - 90^\circ)$$

As a phasor; $V = V_m \angle \phi$; $I = I_m \angle \phi - 90^\circ$

Graphical Representation:



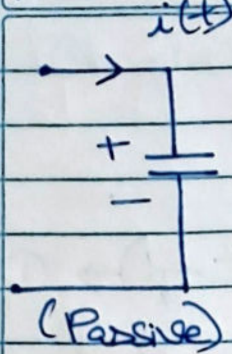
I lags V by 90°

(or)

V leads I by 90°.

H.W

(ii)



$i(t) = C\frac{dv}{dt}$

Let $V(t) = V_m \cos(\omega t + \phi)$

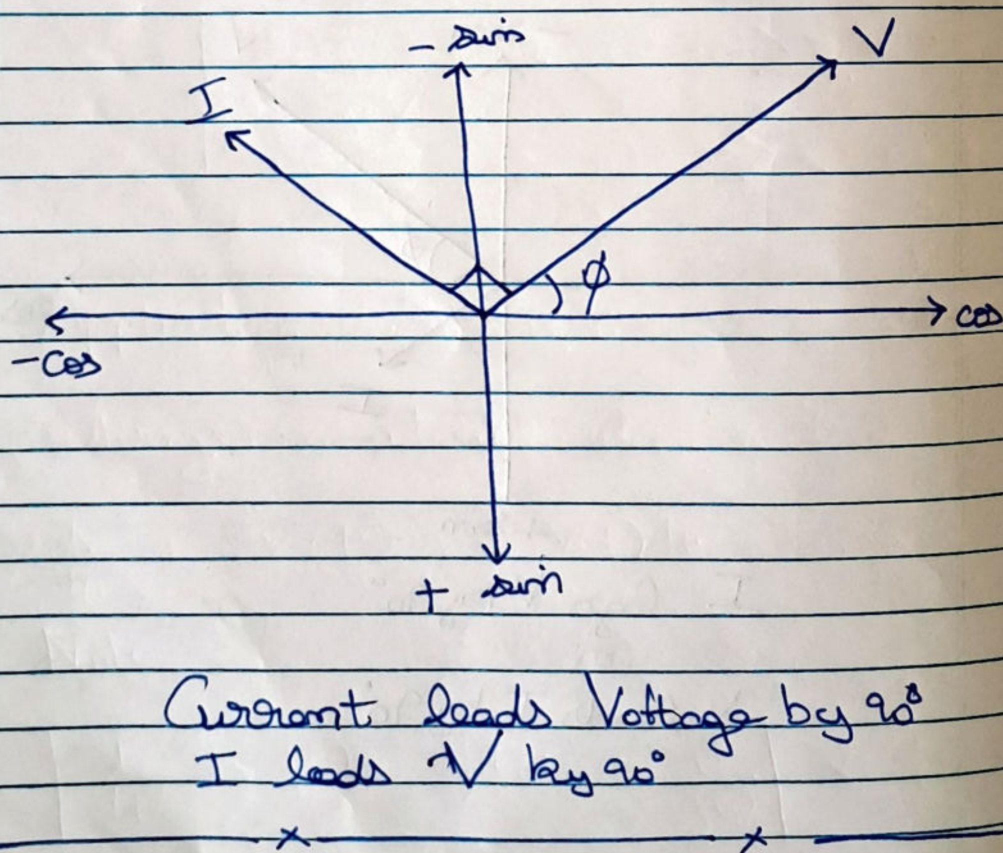
$i(t) = C\frac{d}{dt}(V_m \cos(\omega t + \phi))$

$i(t) = -C\omega V_m \sin(\omega t + \phi)$

$= \underbrace{C\omega V_m}_{I_m} \cos(\omega t + \phi + 90^\circ)$

As a phasor; $V = V_m \angle\phi$, $I = I_m \angle \phi + 90^\circ$

Graphical Representation:



Current leads Voltage by 90°

I leads V by 90°

Contd for active;

$\underline{I} = I_m \angle \phi - 90^\circ$

$= C\omega V_m \angle \phi - 90^\circ$

$= C\omega V_m \dfrac{\angle \phi}{\angle 90^\circ}$ $\left[\dfrac{1}{\angle 90^\circ} = -j = \dfrac{1}{j} \right]$

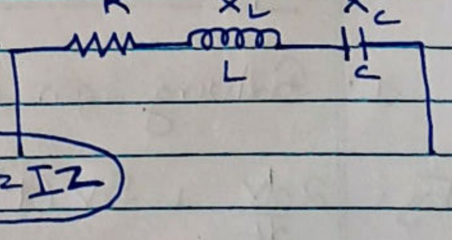
$= -jC\omega V_m \angle \phi$ $\left[\angle 90^\circ = j \right]$

$\underline{I} = -jC\omega \underline{V}$

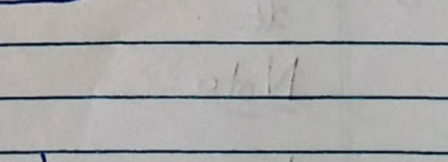
Note:

Time	Phasor
$\int$	$\dfrac{1}{j\omega}$
C	$\dfrac{1}{j\omega C}$

Impedance & Admittance:



Impedance $Z = R + jX$
$= R + j(X_L - X_C)$



Resistor: $V = IR \Rightarrow (Z = R)$

Inductor: $V = j\omega L I \Rightarrow Z = j\omega L$

Capacitor: $V = \dfrac{1}{j\omega C} I \Rightarrow Z = \dfrac{1}{j\omega C}$

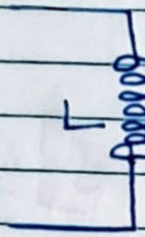
$\boxed{V = IZ}$

Admittance $= \dfrac{1}{Z} =$ 'Y'

Behaviour of Inductor and Capacitor at different frequency (ω):

$\boxed{\omega = 2\pi f}$

f - frequency

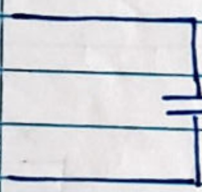


L $X_L = j\omega L$

1. At low frequency ($\omega = 0$)
   $X_L = 0$

   Inductor $\rightarrow$ Short circuited

2. At high frequency ($\omega = \infty$)
   $X_L = \infty$
   Inductor $\rightarrow$ Open circuited



$X_C = \frac{1}{j\omega C}$

1. $\omega = 0 \Rightarrow X_C = \infty$ (open circuited)
2. $\omega = \infty \Rightarrow X_C = 0$ (Short circuited)

Application of Phasors:

1. Solving an integro-differential equation

Eg: $2\frac{dv}{dt} + 5v(t) + 10\int v(t)dt = 50\cos(5t-30)$

Note:

$\therefore$ $\left| \frac{d}{dt} \rightarrow j\omega \; ; \; \int dt \rightarrow \frac{1}{j\omega} \right|$ $\therefore$ $\frac{1}{j} = -j$

Transforming for phasor domain,

$$2j\omega V + 5V + 10 \cdot \frac{1}{j\omega} V = 50\angle{-30^\circ}$$

Given $\omega = 5$;

$$10jV + 5V + \frac{2V}{j} = 50\angle{-30^\circ}$$

$$V\left[j10 + 5 + \frac{2}{j}\right] = 50\angle{-30^\circ}$$

$$V[j10 + 5 - j2] = 50\angle{-30^\circ}$$

$$V[j8 + 5] = 50\angle{-30^\circ}$$

$$V = \frac{50\angle{-30}}{5+j8} \Rightarrow V = \frac{50\angle{-30^\circ}}{9.43\angle{57.99}}$$

$$V = \frac{50}{9.43}\angle{-30^\circ - 57.99} = 5.3\angle{-87.99}$$

$$\Rightarrow V = 5.3\cos(5t - 87.99)$$

Practice H.W:

1. $\frac{2di}{dt} + 3i(t) = 4\cos(2t - 45^\circ)$

$$2j\omega i + 3i = 4\angle{-45^\circ}$$

$$i(3 + j\omega 2) = 4\angle{-45^\circ}$$

Given $\omega = 2$;

$$\Rightarrow i(3 + j4) = 4\angle{-45^\circ}$$

$i = \frac{4\angle -45^\circ}{5\angle 53.13^\circ}$

$i = 0.8\angle -98.13$

$i = 0.8\cos(2t - 98.13)$

2. $10\int i(t)\,dt + \frac{di}{dt} + 6i(t) = 5\cos(5t + 22^\circ)A$

$\frac{10}{j\omega} i + j\omega i + 6i = 5\angle 22^\circ$

Given $\omega = 5$;

$\Rightarrow -2j i + 5ji + 6i = 5\angle 22^\circ$

$i(-j2 + j5 + 6) = 5\angle 22$

$i(6 + j3) = 5\angle 22$

$i = \frac{5\angle 22}{6.708\angle 26.565}$

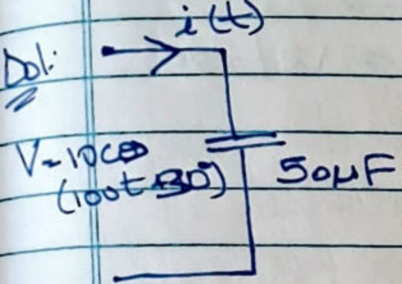
$i = 0.7453\angle -4.565$

$i(t) = 0.7453\cos(5t - 4.565)$

2. Solving circuits with ac sources in phasor domain.

Eg: If $V = 10\cos(100t + 30^\circ)$ is applied to a $50\,\mu F$ capacitor, calculate the current through the capacitor.

Sol:



$$X_c = \frac{1}{j\omega C}$$

$$\omega = 100\,rad/s$$

$$X_c = \frac{1}{j \times 100 \times 50 \times 10^{-6}}$$

$$X_c = -j200\,\Omega$$

$$X_c = 200\angle -90^\circ\,\Omega$$

W.K.T,

$V = IZ$

$$I = \frac{V}{Z} = \frac{V}{\cancel{R}+jX_c}$$

$$\boxed{I = \frac{V}{jX_c}}$$

As a phasor, $V = 10\angle 30^\circ\,V$

$$\Rightarrow I = \frac{V}{Z}$$

$$= \frac{10\angle 30^\circ}{200\angle -90^\circ}$$

$\underline{I = 0.05\angle 120^\circ}$ $\Rightarrow I(t) = 0.05\cos(\omega t + 120^\circ)\,A$

$I(t) = 0.05\cos(100t + 120^\circ)\,A.$

Practice H.W:

1. The voltage $V(t) = -12\cos(600t + 45^\circ)$ is applied to 0.1H inductor. Find the steady state current through the inductor.

$V(t) = 12\cos(600t + 225^\circ) = 12\angle 225^\circ$

$X_L = j\omega L$

$\omega = 600 \text{ rad/s}$

$X_L = j60$

$Z = X_L = j60\ \Omega$

$$\boxed{Z = X_L = 60\angle 90^\circ}$$

$I = \dfrac{V}{Z}$

$= \dfrac{12\angle 225^\circ}{60\angle 90^\circ}$

$I = 0.2\angle 135^\circ$

$\Rightarrow I = 0.2\cos(600t + 135^\circ)$.

Instantaneous Power:

If $V(t) = V_m\cos(\omega t + \theta_v) \Rightarrow V_m\angle\theta_v$

$i(t) = I_m\cos(\omega t + \theta_i) \Rightarrow I_m\angle\theta_i$

$P(t) = V(t) \cdot i(t)$

$= V_m \cos(\underbrace{\omega t + \theta_v}_{A}) \cdot I_m \cos(\underbrace{\omega t + \theta_i})$.

W.K.T.,

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$.

$\Rightarrow P(t) = \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) +$

$\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$.

1. Constant
2. Independent of ω, t
3. depends upon $(\theta_v - \theta_i)$

1. Sinusoidal
2. dependant on twice the frequency of $v(t)$ & $i(t)$

Average Power: (P)

→ Average of the instantaneous power $P(t)$ over one period.

→ Measure using Wattmeter (watts).

$$P = \frac{1}{T}\int_0^T P(t)\,dt.$$

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

As a phasor;

$V = V_m \angle \theta_v$ & $I = I_m \angle \theta_i$

$$\boxed{S = \frac{1}{2} V I^*}$$ → complex power.

$P + jQ$

(or)

$$\boxed{S = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i}$$

Power in AC circuits:

$P \rightarrow$ Real power (True / Active) $\rightarrow$ Watts 'W'.

$Q \rightarrow$ Reactive power $\longrightarrow$ VAr (Volt Ampere reactive)

$S = P + jQ \longrightarrow$ Complex / Total - VA

(Contd) Average Power:

~~Eq~~ Using Average power we can rewrite S as,

$$P + jQ = \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{P} + \underbrace{j\frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)}_{Q}$$

$(\theta_v - \theta_i)$ - angle b/w V & I

(1) circuit $\rightarrow$ resistive ; $\theta_v - \theta_i = 0$.

$$P = \frac{1}{2} V_m I_m \ ; \quad Q = 0 .$$

(2) circuit $\rightarrow$ reactive (inductive or capacitive).

$$(\theta_v - \theta_i) = \pm 90^\circ$$

$$P = 0 \ ; \quad Q = \pm \frac{1}{2} V_m I_m$$

Eg 1: If $V(t) = 330\cos(10t + 20^\circ)$ V and $i(t) = 33\sin(10t + 60^\circ)$ A. Calculate instantaneous and average power.

Sol: $i(t) = 33\cos(10t + 60^\circ - 90^\circ)$

$i(t) = 33\cos(10t - 30^\circ)$

$$P(t) = V(t) \cdot i(t)$$

$$= \frac{1}{2} V_m I_m \cos(2\omega t + \overset{20+(-30)}{\theta_v + \theta_i})$$

$$+ \frac{1}{2} V_m I_m \cos(\overset{20-(-30^\circ)}{\theta_v - \theta_i})$$

$$\Rightarrow P(t) = 5445\cos(20t + 10^\circ) + \underbrace{3499.97854}_{\text{Avg. Power}}$$

Average Power (P) = 3499.97854 Watt

Eg 2: A Current $I = 33\angle 30^\circ$ A flows through an impedance of $Z = 40\angle\cancel{22}$ Ω. Find the average power delivered to the load.

Sol: Average Power (P) $= \frac{1}{2} I_m^2 R$ (or) $\frac{V_m^2}{2R}$

↳ Use this for now

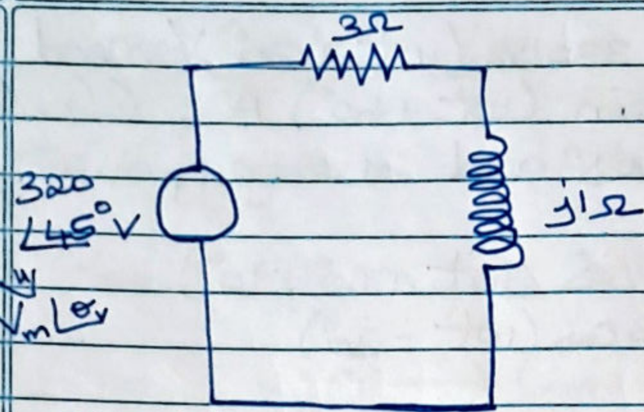
$Z = 40\angle -22$ Ω

↳ $R + jX \Rightarrow Z = \underbrace{37.08}_{R} - j14.98$

$\Rightarrow P = \frac{1}{2} \times 33^2 \times 37.08 = 20190.06$ W

$$\boxed{P = 20.19 \text{ kW}}$$

Eg:3



Calculate power supplied by voltage source, power absorbed by resistor and inductor.

Sol:

$$P = \frac{1}{2} \underset{320}{V_m} \underset{101.26}{I_m} \cos(\underset{45^\circ}{\theta_v} - \underset{26.57^\circ}{\theta_i})$$ {power supplied by source} → Power absorbed by resistance

$$I = \frac{V}{Z} = \frac{320\angle 45^\circ}{(3+j1)} \qquad (\because Z = R + jX_L)$$

$$= \frac{320\angle 45^\circ}{3.16\angle 18.43} = 101.26\angle 26.57\ A.$$

$$P = \frac{1}{2} \times 320 \times 101.26 \cos(45-26.57)$$

$$= 15.35\ KW$$

Power absorbed by Resistor $= \frac{1}{2} I_m^2 R$

$$= \frac{1}{2} \times (101.26)^2 \times 3$$

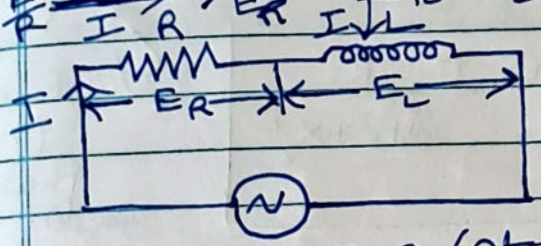
$$= 15.35\ kW.$$

Power absorbed by inductor $= 0W$.

* NO REAL POWER ABSORBED BY INDUCTOR *.

Circuit with Resistance & Inductance in series:

$I = \frac{E}{R}$ $\xrightarrow{I}$ E_R $\quad$ $I \xrightarrow{90^\circ} E_L \Rightarrow I = \frac{E}{\omega L}$



$e = E_m \sin(\omega t + 0)$

R – Resistance → Ω

L – Inductance → H

X_L → Inductive reactance — Ω

$X_L = \omega L = 2\pi f L$

Voltage drops accross resistance,

$E_R = IR$

Voltage drop accross reactance,

$E_L = I \cdot \omega L = I X_L$

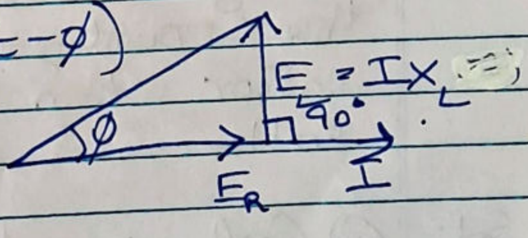
E → Effective value of applied voltage

I → Effective value of current in circuit

$$\boxed{\begin{aligned} Z &= R + jX_L = \sqrt{R^2 + X_L^2} \,\angle \tan^{-1}\frac{X_L}{R} \\ Z &= |Z| \angle \phi \end{aligned}}$$

$$I = \frac{E}{Z} = \frac{E\angle \overset{0}{\phi}}{|Z|\angle\phi} = \frac{E}{Z}\angle{-\phi} = I_m \angle{-\phi}$$

$$i = I_m \sin(\omega t - \phi)$$



I lags V by $\phi = \tan^{-1}\frac{X_L}{R}$

$$E = \sqrt{E_R^2 + E_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow E = IZ \quad \Rightarrow I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X_L^2}}$$

Page :

Power → consumed by 'R' only

$$P = I^2 R = I \cdot IR$$

$$= \frac{E}{\sqrt{R^2+X_L^2}} \cdot IR$$

$$\boxed{P = E.I \frac{R}{\sqrt{R^2+X_L^2}}}$$

ϕ → angle b/w E and I (or) Power factor angle

Power factor $(\cos\phi) = \frac{E_R}{E} = \frac{IR}{I\sqrt{R^2+X_L^2}}$

$$\Rightarrow \cos\phi = \frac{R}{\sqrt{R^2+X_L^2}} = \frac{R}{Z}$$

$$\boxed{P = E.I\cos\phi}$$

$$i = I_m \sin(\omega t - \phi)$$

Eg: (i) Voltage across a coil is 4.5V when it carries direct current of 9A. If the same coil carries an alternating current of 9A at 25Hz then the voltage across the coil is 24V. Find the current, active power, and power factor when it is supplied by 50V at 50 Hz.

Sol: ① Coil → inductor and small resistance in series

$$Z = (R + jX_L)$$

Page :

(2) DC $\Rightarrow R = \frac{V}{I}$

(i) $V_{coil} = 4.5V$; $I_{coil} = 9A$; $R_{COIL} = 0.5\,\Omega$

$\therefore \left(\frac{V}{I} = \frac{4.5}{9}\right)$

(ii) $V_{COIL} = 24V$ at $25 Hz$; $I_{COIL} = 9A$

$$Z_{COIL} = \frac{V_{COIL}}{I_{coil}} = \frac{24}{9} = 2.66\,\Omega$$

$$\underset{2.66\Omega}{\underset{\downarrow}{Z}} = \sqrt{\underset{0.5\Omega}{\underset{\downarrow}{R}}^2 + X_L^2}$$

$$\Rightarrow Z_{coil} = \sqrt{R_{coil}^2 + X_{L_{coil}}^2}$$

$$X_{L_{coil}} = \sqrt{(2.66)^2 - (0.5)^2} \quad [f = 25 Hz]$$

$$X_{L_{coil}} = 2.62\,\Omega \Rightarrow 2\pi f L = 2.6 \Rightarrow L = \frac{2.62}{2\pi \times 25}$$

$$\Rightarrow \boxed{L = 0.0167 H}$$

(iii) New AC circuit :

$$f = 50 Hz, \quad X_{coil\ new} = 2 \times \pi \times f \times L$$
$$= 2 \times \pi \times 50 \times 0.0167$$
$$= 5.24\,\Omega$$

$$Z_{coil\ new} = \sqrt{(R_{coil}^2) + (X_{L_{coil\ new}})^2}$$
$$= \sqrt{(0.5)^2 + (5.24)^2}$$

$$\boxed{Z_{coil\ new} = 5.26\,\Omega}$$

(a) $I_{new} = \dfrac{V}{Z_{coil\ new}} = \dfrac{50}{5.26} = 9.50\ A.$

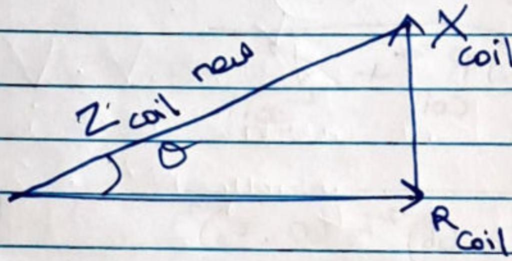
$$\boxed{I_{new} = 9.5A}$$

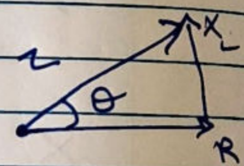
(b) $\boxed{P = I^2 R_{coil}} = (9.5)^2 \times (0.5) = 45.1W$

(c) $\cos\phi = pf =$

$Z = R + jX_L$

$Z = \sqrt{R^2 + X_L^2}$





$\cos\theta = \dfrac{R_{coil}}{Z_{coil,new}} = \dfrac{0.5}{5.26} = 0.095$ (lagging)

$\theta = \cos^{-1}(0.095) = 84.54°$ //

Till CAT-1

Eg2: In a series R-L circuit, the instantaneous voltage $V = 141.4 \sin(\omega t - \frac{2\pi}{3})$ and instantaneous current $i = 0.707 \sin(\omega t - \frac{\pi}{3})$ where $\omega = 314$ rad/sec.

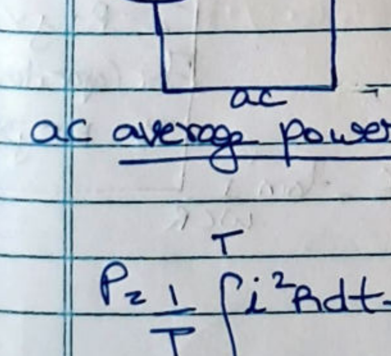
(a) $Z = ?$

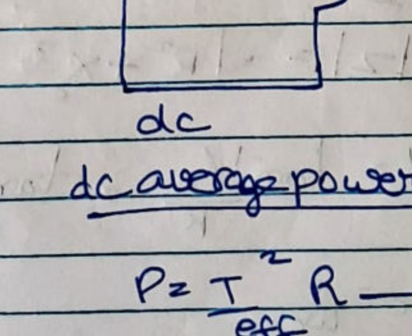
(b) Active Power & Power factor ($\cos\phi$)

(c) Draw the phasor diagram b/w $\bar{V}$ & $\bar{I}$.

(a) $Z = \dfrac{V_{rms}}{I_{rms}}$

RMS / Effective Value (Root Mean Square):





dc

ac average power

dc average power

$$P = \frac{1}{T}\int_0^T i^2 R\,dt \rightarrow (1) \qquad P = I_{eff}^2 R \longrightarrow (2)$$

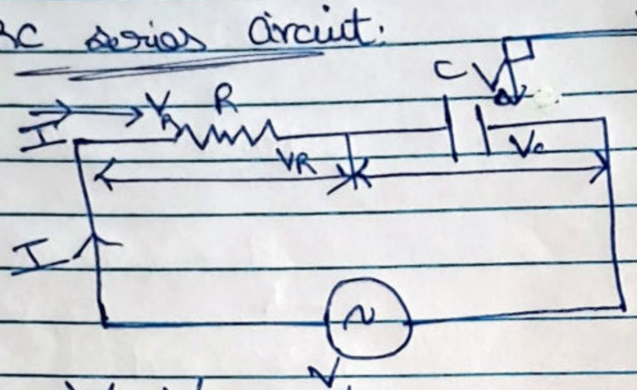
(1) = (2)

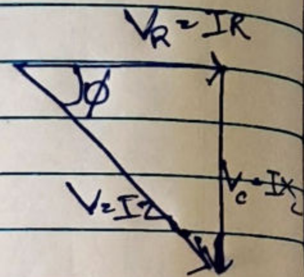
$$I_{eff}^2 = \frac{1}{T}\int_0^T i^2\,dt \Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = I_{rms}}$$

$$\boxed{V_{eff} = \sqrt{\frac{1}{T}\int_0^T V^2\,dt} = V_{rms}}$$

$\Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$ ||ly $I_{rms} = \frac{I_m}{\sqrt{2}}$

RC series circuit:





$V = V_m \sin\omega t$.
i = ~~instantan~~ instantaneous current.
f = Frequency.

Voltage across R = $V_R = IR$ in phase I.
Voltage across C = $V_c = IX_c$ lagging I by 90°.
$V = V_R + V_c$ $\quad X_c \rightarrow$ Capacitive Reactance.

Voltage triangle;
$V = IR - jIX_c = I(R - jX_c)$
$\frac{V}{I} = R - jX_c = Z$
$|Z| = \sqrt{R^2 + X_c^2}$

Note,
For Inductors, RL
$Z = R + jX_L$
For Capacitor, RC
$Z = R - jX_c$

$\phi = \tan^{-1}\frac{X_c}{R} = \tan^{-1}\frac{1/\omega C}{R} = \tan^{-1}\frac{1}{\omega CR}$

$X_c = \frac{1}{\omega c}$

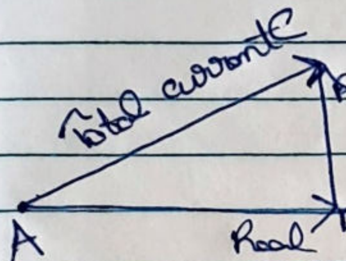
ϕ = phase angle, $Z = R - jX_c = |Z|\angle{-\phi}$
$\cos\phi$ (P.f) = R/Z; Active power (True power) $P = VI\cos\phi$ (Watts)
Reactive power $Q = VI\sin\phi$ (VAR)

POWER FACTOR:

Load
- Real → useful work.
- Reactive → flux (rotating machine)

Total current = $\sqrt{(\text{Real current})^2 + (\text{Reactive current})^2}$

Fan ↓ Motor ↓ Stator & Rotor ↓ rotate ↓ flux ↓ reactive.



Reactive current × V ⇒ Q = VAR

Real current ⇓ V ⇓ R → W

Power factor $= \frac{AB}{AC} = \frac{KW}{KVA} = \frac{P}{S}$

eg: Find the circuit constants of 2 elements series circuits which consumes 700W with 0.707 leading p.f. The applied voltage is $V = \underbrace{141.4}_{V_m} \sin \underbrace{314}_{\omega} t$. Find R & C.

Soln: $V_m = 141.4\,V$.

$V_{rms} = \frac{141.4}{\sqrt{2}} = 99.98\,V$

$\cos\phi = 0.707$ leading

$\phi = 45^\circ$

Power $= VI\cos\phi$

$700 = 0.707 \times 99.98 \times I$

$I_{rms} = 9.902\,A$

$Z = \dfrac{V_{rms}}{I_{rms}} = 10.09\,\Omega$

$\cos\phi = \dfrac{R}{Z}$

$R = \cos\phi \times Z$

$\boxed{R = 7.137\,\Omega}$

$\sin\phi = \dfrac{X_C}{Z}$

$\sin 45^\circ = \dfrac{X_C}{10.09}$

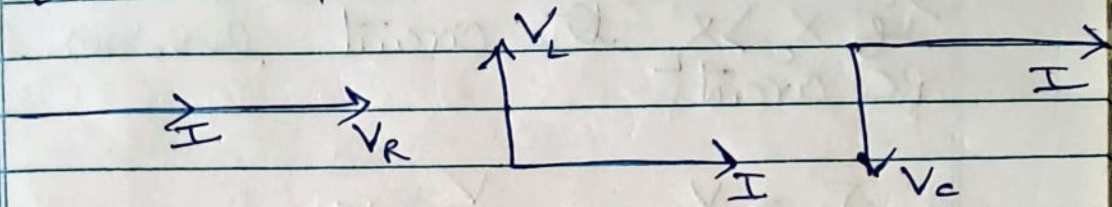
$\boxed{X_C = 7.134\,\Omega}$

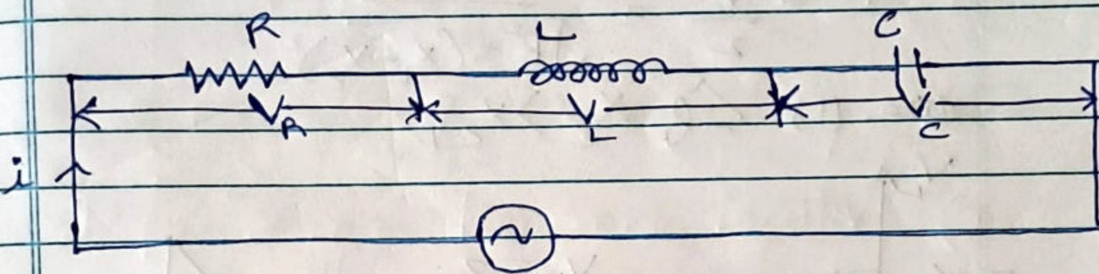
$X_C = \dfrac{1}{\omega C}$

$C = \dfrac{1}{314 \times 7.134} = 446.6\,\mu F$

RLC series circuits:

$R\,\Omega$, inductance L Henrys and Capacitance C Farads all are connected in series.





$V = V_m \sin\omega t$

i = instantaneous current

I is taken as reference vector

Voltage across $R = V_R = IR$ in phase with I

Voltage across $L = V_L = IX_L \angle 90^\circ$ V leads by 90°.

Voltage across $C = V_c = IX_c \angle -90^\circ$ V lags I by 90°.

$= -jIX_C$

Applied Voltage $V = V_R + V_L + V_C$

$= IR + jIX_L - jIX_C$

$$\frac{V}{I} = R + j(X_L - X_C)$$

$$Z = R + j(X_L - X_C)$$

$$= Z\angle\phi$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

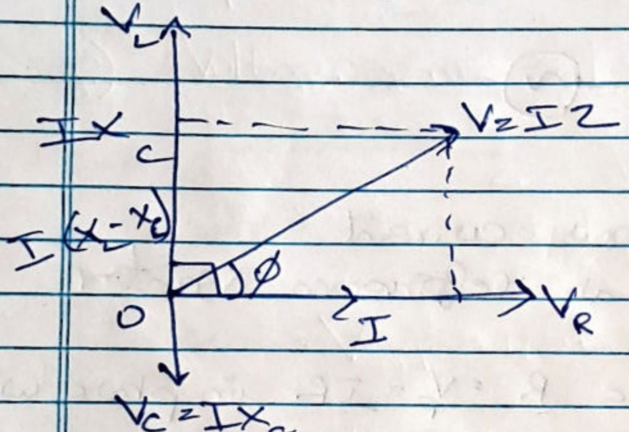
$(X_L - X_C) \longrightarrow$ net reactance.

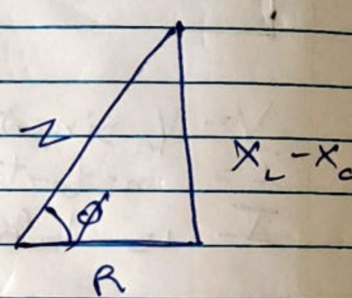
If $X_L > X_C$ the circuit behaves like RL circuit.

If $X_C > X_L$ the circuit behaves like RC circuit.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

case (i) If $X_L > X_C$;





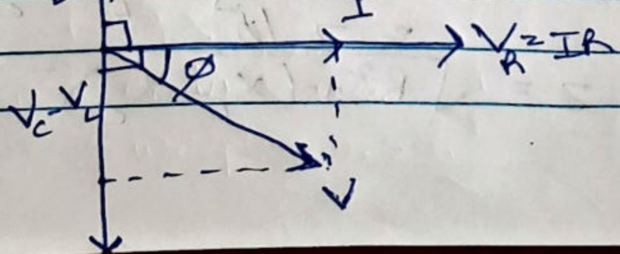
$$\tan\phi = \frac{X_L - X_C}{R}$$

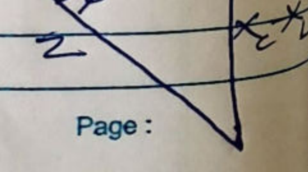
$$= \frac{(\omega L - 1/\omega C)}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

$$P.f = \cos\phi = \frac{R}{Z} \text{ lagging (R-L)}.$$

case (ii) If $X_C > X_L$ RC circuit





Page :

Here I leads V by $\phi°$ ∴ capacitive

$V_c = IX_c$

$$\tan\phi = \frac{X_c - X_L}{R} = \frac{1/\omega C - \omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{1/\omega C - \omega L}{R}\right)$$

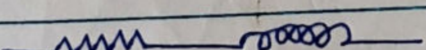
$P.f = \cos\phi = \frac{R}{Z}$ leading (RC circuits).

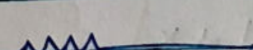
Actual power $= VI\cos\phi$ (watts)

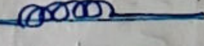
Reactive power $= VI\sin\phi$ (VAR)

eg: A coil of resistance $10\,\Omega$ and $L = 0.1\,H$ is connected in series with a $150\,\mu F$ capacitor across 200V, 50Hz supply. calculate (a) inductive reactance (b) capacitive reactance (c) impedance (d) current (e) p.f (f) voltage across coil and capacitor.

Sol: General:

Coil → 

Non inductive coil → 

Non resistive coil → 

a) $X_L = \omega L = 2\pi f L = 31.42\Omega$.

b) $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 21.23\Omega$.

Here $X_L > X_c \Rightarrow$ Inductive circuit.

c) $Z = \sqrt{R^2 + (X_L - X_c)^2} = 14.28\Omega$.

d) $I = \frac{V}{Z} = \frac{220}{14.28} = 14A$.

e) $p.f = \cos\phi = R/Z = 0.7$ lagging

f) Voltage across coil $= I *$ impedance of the coil.

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{10^2 + 31.42^2} = 32.97\Omega$$

$$V_{coil} = 461.62V.$$

$$V_c = IX_c = 297.22V.$$

$$V_L = IX_L = 439.88V$$

Admittance (Y) : Inverse of impedance, expressed in mho ($\mho$). (Ω)

$$Y = \frac{1}{Z}$$

$Z = R + jX$

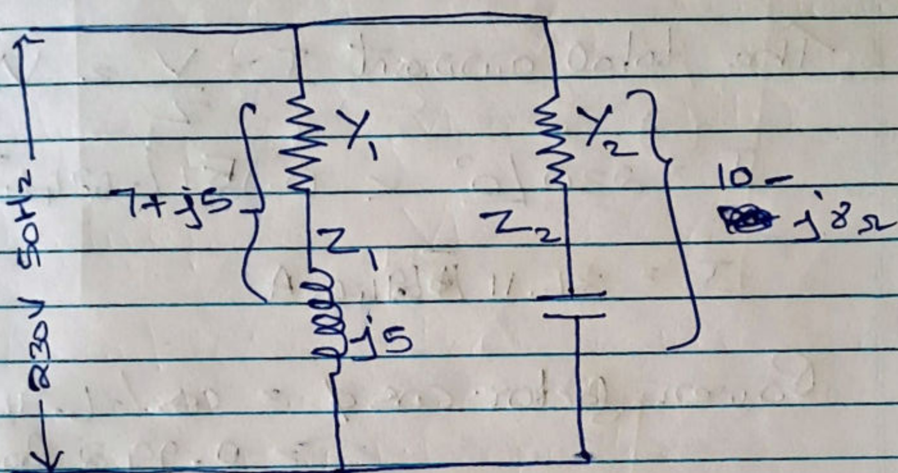
$$Y = \frac{1}{R \pm jX} = G \mp jB$$

$G =$ Conductance $= \frac{1}{R}$

$B =$ Susceptance $= \frac{1}{X}$.

eg: An impedance of $(7+j5)\,\Omega$ is connected in ||el with another circuit having an impedance of $(10 - j8)\,\Omega$. The supply voltage is 230V, 50Hz. Calculate (i) the admittance (Y), the conductance (G) and susceptance (B) of the combined circuit (ii) the total current (I) taken from mains and its p.f.

Sol:



Z in series $= Z_1 + Z_2$

Z in parallel $= \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

Y in ||el $= Y_1 + Y_2$

Y in series $= \dfrac{Y_1 Y_2}{Y_1 + Y_2}$

$Y_1 = \dfrac{1}{Z_1} = \dfrac{1}{7+j5} = \dfrac{1}{8.6\angle 35.53}$

$= 0.1162790\angle -35.53 = 0.095 - j0.068\,℧$

Page :

$Y_2 = \frac{1}{Z_2} = \frac{1}{10-j8} = \frac{1}{12.806\angle -38.660}$

$= 0.061 + j\,0.04975$

The net admittance in // lo $= Y_1 + Y_2$

$= 0.156 - j0.019℧$

$Y = G - jB$

$\therefore \underbrace{G = 0.156℧}_{\text{conductance}}, \; \underbrace{B = 0.019℧}_{\text{Susceptance}}$

The total current $I = \frac{V}{Z} = VY$

$I = 230\angle 0^\circ \times 0.157\angle -6.944^\circ$

$I = 36.11\angle -6.944$ A.

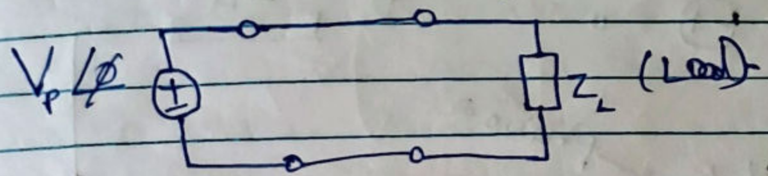
Power factor $\cos\phi = \cos(-6.94)$

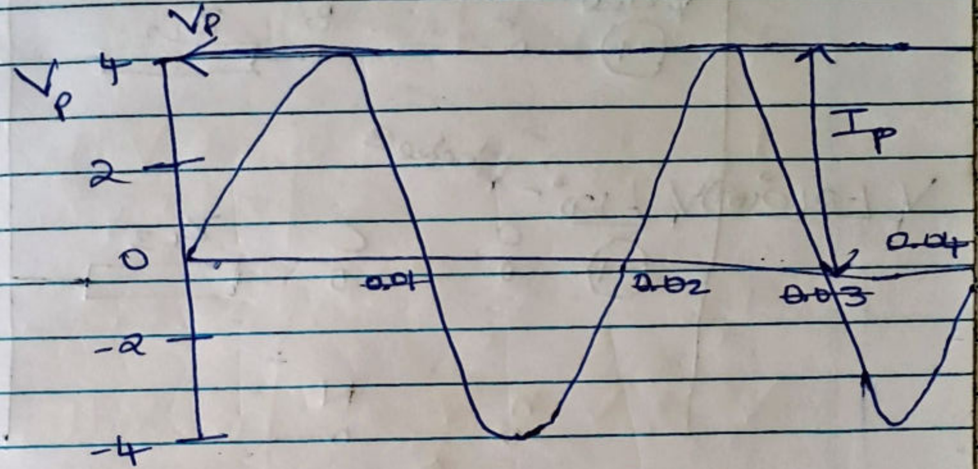
$= 0.992$ lagging

Since $\angle -\phi$

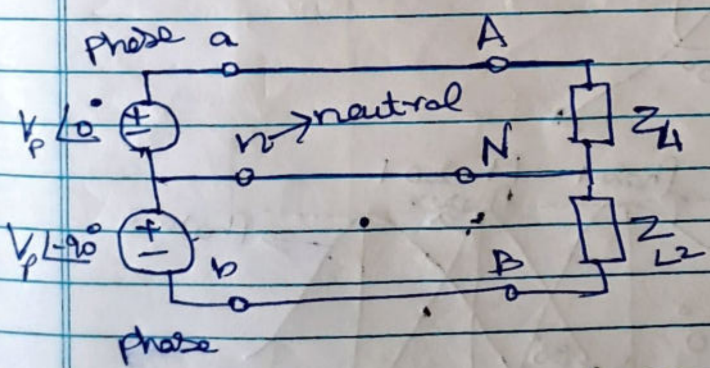
Poly Phase System (Same frequency but different phase)

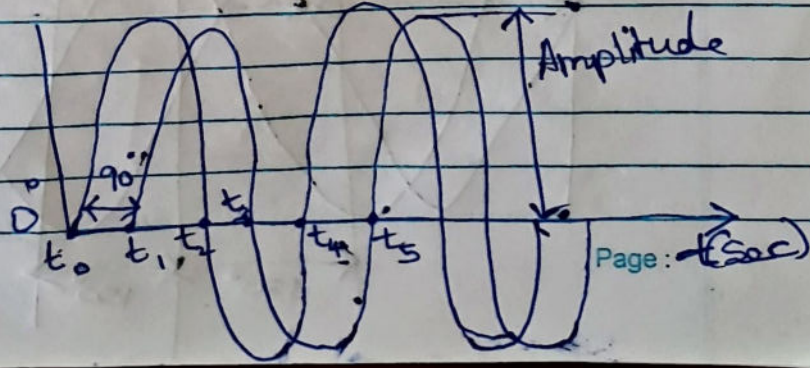
(i) • Single phase - two wire system
→ Phase (P)
→ Neutral (N)



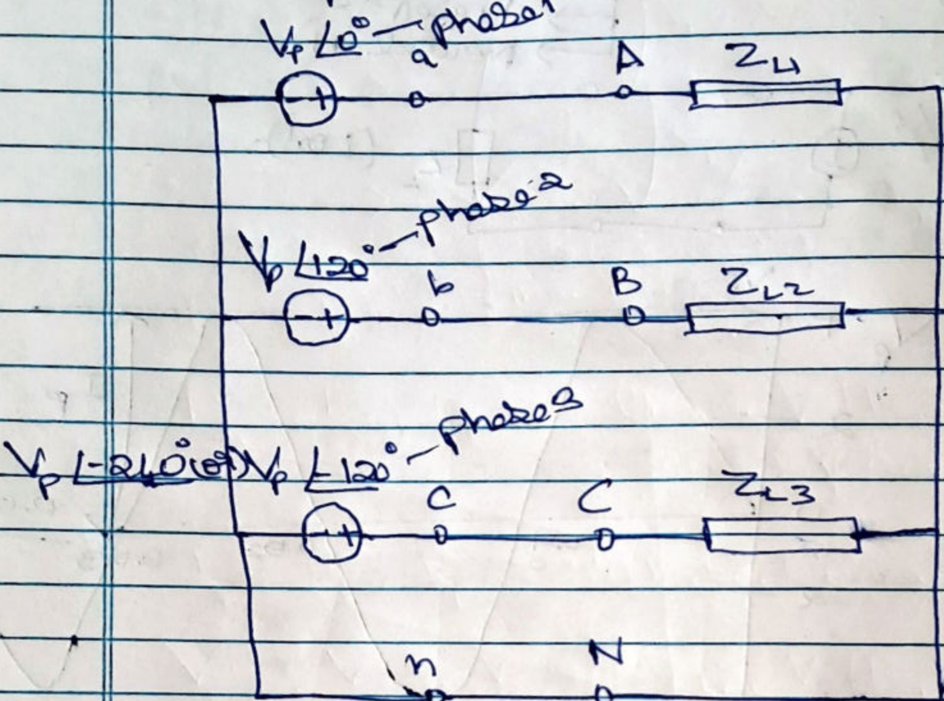


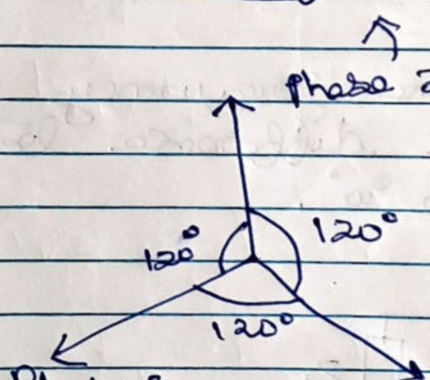
(ii) Two phase - Same frequency and the phase angle difference between the phases are 90°.

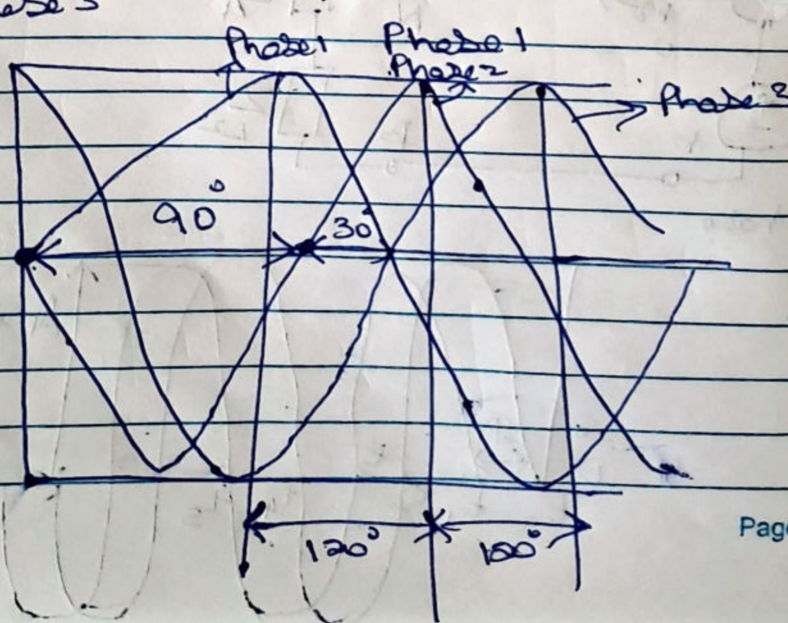




(iii) Three phase - Same frequency and the phase angle difference between the phases are 120°.

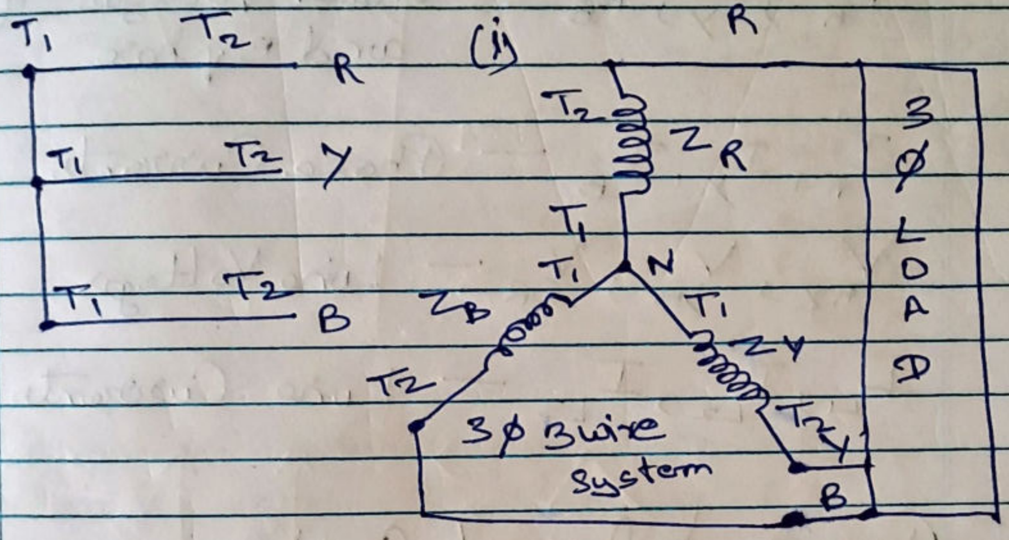


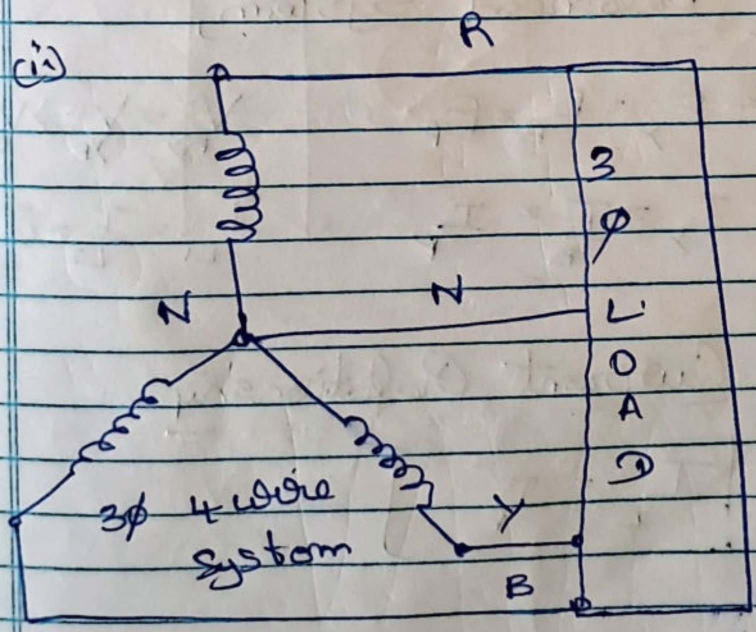




Balanced three phase systems.
Star (wye) connection (Y)

- Three wire system.
- Four wire system.
- The phase angle difference b/w each phase is 120°.
- All the three phase voltages have the same amplitude, same period and frequency.

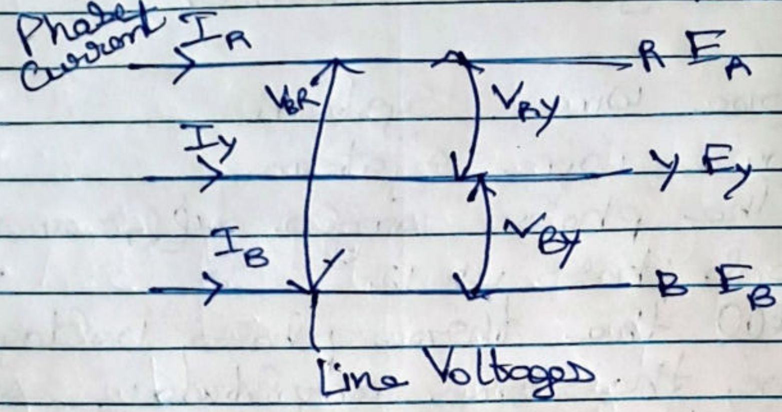




where
N – Neutral point (or) star point.

Voltage & Current relationships

(i) Y — 3 ϕ system



E_R, E_Y, E_B — Phase Voltages of R, Y and B phase

I_R, I_Y, I_B — Phase Currents.

V_{RY}, V_{YB}, V_{BR} — Line Voltages.

I_{L1}, I_{L2}, I_{L3} — Line Currents (Load)

In a balanced System,

$E_R = E_Y = E_B = E_p$ $\qquad$ $V_{RY} = V_{YB} = V_{BR} = V_L$

$I_R = I_Y = I_B = I_p$ $\qquad$ $I_{L1} = I_{L2} = I_{L3} = I_L$

Current Relationship:

$$\boxed{I_L = I_p}$$

Voltage Relationship:

$$\boxed{V_L = \sqrt{3}\, E_p}$$

Power Relationship:
Let $\cos\phi$ be the power factor:

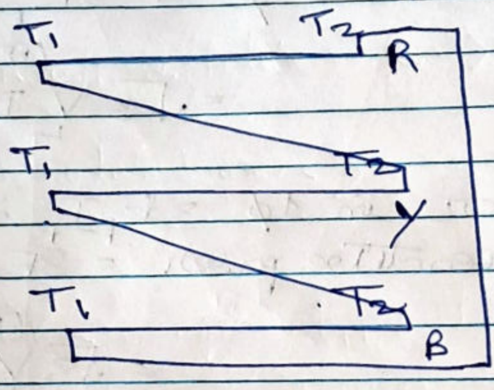
Power consumed in $1\phi = E_p I_p \cos\phi$
(True/Real/Active) Power consumed in $3\phi = 3 E_p I_p \cos\phi$
(or)
$= \sqrt{3}\, V_L I_L \cos\phi$
(watts)

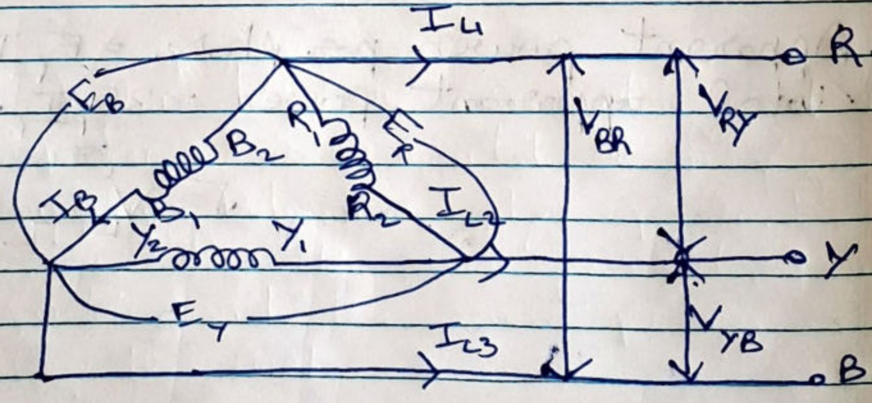
Reactive power in $1\phi = E_p I_p \sin\phi$
Total reactive power $= 3 E_p I_p \sin\phi$
(or)
$= \sqrt{3}\, V_L I_L \sin\phi$
(VAR)

(complex/Total) Apparent power per phase $= E_p I_p$
Total apparent power $= 3 E_p I_p$
(or)
$\sqrt{3}\, V_L I_L$ (Volt-Amp)

Balanced three phase Systems.
Delta (mesh) connection (Δ)

- Three wire System.
- The phase angle difference b/w each phase is 120°.
- All the three phases voltages have the same amplitude, same period and frequency.





Here;

$E_R = E_Y = E_B = E_P$; phase voltage
$I_R = I_Y = I_B = I_P$; phase current.
$V_{RY} = V_{YB} = V_{BR} = V_L$; line Voltage
$I_{L1} = I_{L2} = I_{L3} = I_L$; line Current.

Voltage Relationship:

$$\boxed{E_p = V_L}$$

Current Relationship:

$$\boxed{I_L = \sqrt{3}\, I_p}$$

Power Relationship:

Let $\cos\phi$ be the power factor;

Power for $1\phi = E_p I_p \cos\phi$

Total power for $3\phi = 3 E_p I_p \cos\phi$

(or)

$\sqrt{3}\, V_L I_L \cos\phi$ (Watts)

Reactive power in $1\phi = E_p I_p \sin\phi$

Total reactive power for $3\phi = 3 E_p I_p \sin\phi$

(or)

$\sqrt{3}\, V_L I_L \sin\phi$ (VAR)

Apparent power for $1\phi = E_p I_p$

Total apparent power $= 3 E_p I_p$.

(or)

$\sqrt{3}\, V_L I_L$ (Volt Amp)

Star Y	Delta (Mesh) Δ
1) Current $I_L = I_P$	$I_L = \sqrt{3} \times I_P$
2) Voltage $V_L = \sqrt{3} \times E_P$	$V_L = E_P$

Similarities are:

3) True/Real/Active Power $\Rightarrow 3 E_P I_P \cos\phi$ (Or) $\sqrt{3} V_L I_L \cos\phi$ (Watts)

4) Reactive Power $\Rightarrow 3 \times E_P \times I_P \sin\phi$ (Or) $\sqrt{3} V_L \times I_L \sin\phi$ (VAR)

5) Total/Complex/Apparent Power $\} = E_P \times I_P$ (Or) $\sqrt{3} \times V_L \times I_L$ (VA)

Ex: A balanced star connected load of $3+j4$ (Z) impedance is connected to $400\,V$ (V_L), 3-phase supply. Determine the phase voltage (E_P), phase current (I_P), line current (I_L), power factor $(\cos\phi)$ and the real power (P) consumed by the load.

Sol: Y-connected system,

$V_L = \sqrt{3} \times E_P \Rightarrow 400 = \sqrt{3} \times E_P \Rightarrow \boxed{E_P = 230.9\,V}$

$I_P = \dfrac{E_P}{Z}$; $Z = 3 + j4 \Rightarrow Z = 5 \angle 53.1^\circ\ \Omega$

$$I_p = \frac{230.9\angle 0}{5\angle 53.1^\circ} = \underbrace{46.18\angle -53.1^\circ}_{|I|\angle\phi} A$$

$\cos\phi = \cos 53.1 = 0.6$ lagging.

$I_L = I_p = 46\angle -53.1^\circ$ A

$P = \sqrt{3} \times V_L \times I_L \times \cos\phi = \sqrt{3} \times 400 \times 46.18 \times 0.6 = 19196.66$ W

Ex 2) A 3-phase balanced wye connected load has 400V line to line voltage and 10A line current. Determine the line to neutral voltage and phase current.

(N = Neutral)

($V_{RN} = V_{YN} = V_{BN}$ = Phase Voltage)

Sol:

$V_L = 400V$

$E_p = 230.9V$ (E_p is also known as line to Neutral voltage)

$I_L = 10A$

$I_p = 10A$

Ex 3) A balanced delta connected load of $(4 + j6)\Omega$ impedance is connected to 415V, 50Hz, 3-phase supply. Determine the phase voltage (E_p), phase current (I_p), line current (I_L), power factor and the real power consumed by the load.

(R+jX)

Sol:

$E_p = V_L = 415V$

$I_p = \frac{I_L}{\sqrt{3}} \Rightarrow I_p = \frac{E_p}{Z} = \frac{415}{4+j6} = 57.5\angle -56.3^\circ$ A

$I_L = \sqrt{3} \times I_p = \sqrt{3} \times 57.5 = 99.6A$

$$\cos\phi = \cos 56.3 = 0.5$$
$$P = \sqrt{3} \times V_L \times I_L \cos\phi = 35.7\,kW$$

Eny When 3 balanced impedances are connected in delta accross a 3 phase, 400V, 50Hz supply, the line current drawn is 20A at a lagging power factor of 0.3. Determine the impedance connected in each phase.

Sol:
$$Z = \frac{E_p \angle 0^\circ}{I_p \angle \phi}$$

$$\phi = \cos^{-1} 0.3 = -72.542^\circ$$

$$Z = \frac{400 \angle 0^\circ}{11.547 \angle -72.542^\circ}$$

$$Z = 34.641 \angle 72.542^\circ$$

$$Z = 10.393 + j\,33.045$$