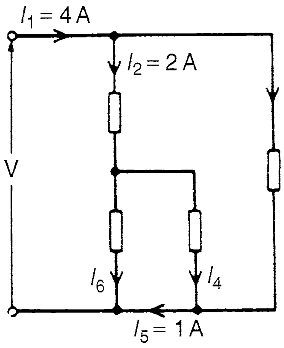
CHAPTER 13 D.C. CIRCUIT THEORY

Exercise 69, Page 193

1. Find currents I_3 , I_4 and I_6 in the circuit below.

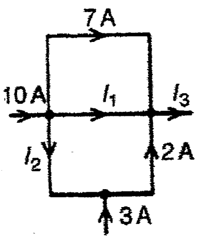


$I_1 = I_2 + I_3$ i.e. $4 = 2 + I_3$ from which, $\mathbf{I_3} = 4 - 2 = \mathbf{2\ A}$

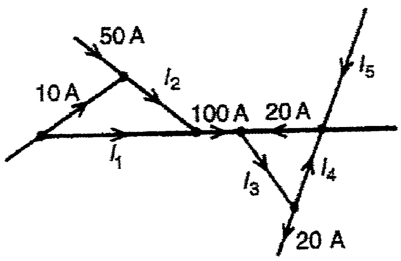
$I_3 + I_4 = I_5$ i.e. $2 + I_4 = 1$ from which, $\mathbf{I_4} = 1 - 2 = \mathbf{-1\ A}$

$I_2 = I_4 + I_6$ i.e. $2 = -1 + I_6$ from which, $\mathbf{I_6} = 2 + 1 = \mathbf{3\ A}$

2. For the networks shown below, find the values of the currents marked.



(a)



(b)

(a) $I_2 + 3 = 2$ from which, $\mathbf{I_2} = 2 - 3 = \mathbf{-1\ A}$

$10 = 7 + I_1 + I_2$ i.e. $10 = 7 + I_1 - 1$ from which, $\mathbf{I_1} = 10 - 7 + 1 = \mathbf{4\ A}$

$7 + I_1 + 2 = I_3$ i.e. $7 + 4 + 2 = I_3$ from which, $\mathbf{I_3} = \mathbf{13\ A}$

(b) $10 + 50 = \mathbf{I_2} = \mathbf{60\ A}$

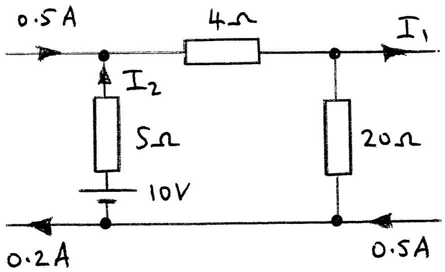
$I_1 + I_2 = 100$ from which, $\mathbf{I_1} = 100 - I_2 = 100 - 60 = \mathbf{40\ A}$

$100 + 20 = \mathbf{I_3} = \mathbf{120\ A}$

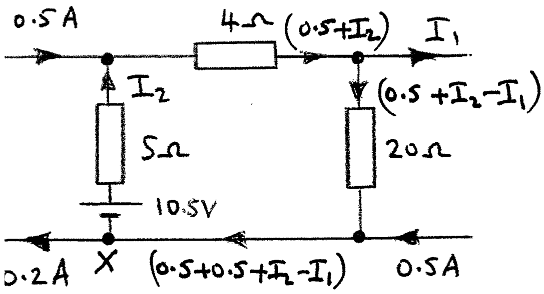
$I_3 = 20 + I_4$ from which, $\mathbf{I_4} = I_3 - 20 = 120 - 20 = \mathbf{100\ A}$

$I_4 + I_5 = 20$ from which, $\mathbf{I_5} = 20 - I_4 = 20 - 100 = \mathbf{-80\ A}$

3. Calculate the currents I_1 and I_2 in the circuit diagram below.



The circuit with its currents is shown below.



By Kirchhoff's current law at node X: $0.5 + 0.5 + I_2 - I_1 = 0.2 + I_2$

i.e. $1 - I_1 = 0.2$

from which, current $\mathbf{I_1 = 0.8\ A}$

Applying Kirchhoff's voltage law in the closed loop, moving clockwise gives:

$$10.5 = 5I_2 + 4(0.5 + I_2) + 20(0.5 + I_2 - I_1)$$

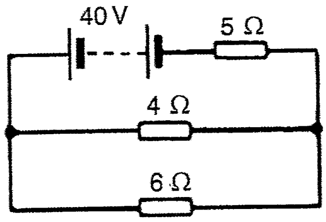
$$10.5 = 5I_2 + 2 + 4I_2 + 10 + 20I_2 - 20I_1$$

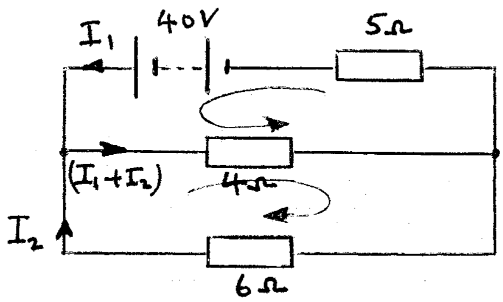
$$10.5 = 29I_2 + 12 - 20(0.8)$$

i.e. $29I_2 = 10.5 - 12 + 16$

Hence, current $\mathbf{I_2} = \dfrac{14.5}{29} = \mathbf{0.5\ A}$

4. Use Kirchhoff's laws to find the current flowing in the 6 Ω resistor in the circuit below and the power dissipated in the 4 Ω resistor.





From the top loop: $40 = 5I_1 + 4(I_1 + I_2)$

From the lower loop: $0 = 6I_2 + 4(I_1 + I_2)$

Hence, $9I_1 + 4I_2 = 40$ (1)

and $4I_1 + 10I_2 = 0$ (2)

4 × (1) gives: $36I_1 + 16I_2 = 160$ (3)

9 × (2) gives: $36I_1 + 90I_2 = 0$ (4)

(3) – (4) gives: $-74I_2 = 160$

and $I_2 = -\frac{160}{74}$ = - 2.162 A

i.e. **the current in the 6 Ω resistor = 2.162 A**

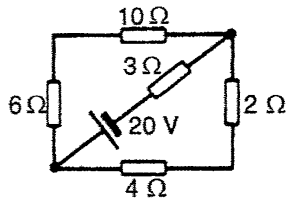
Substituting in (1) gives: $9I_1 + 4(-2.162) = 40$

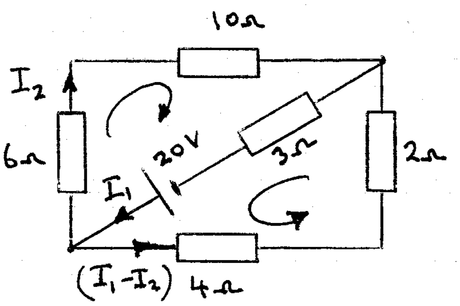
from which, $I_1 = \frac{40 + 4(2.162)}{9}$ = 5.405 A

$I_1 + I_2 = 5.405 - 2.162 = 3.243\,\text{A}$

Hence, **power dissipated in the 4Ω resistor**, $\mathbf{P_{6\Omega}} = \left(I_1 + I_2\right)^2 R = \left(3.243\right)^2 \left(4\right)$ = **42.07 W**

5. Find the current flowing in the 3 Ω resistor for the network shown below. Find also the p.d. across the 10 Ω and 2 Ω resistors.





From the top loop: $20 = 3I_1 + 16I_2$

From the bottom loop: $20 = 3I_1 + 6\left(I_1 - I_2\right)$

Hence, $3I_1 + 16I_2 = 20$ (1)

and $9I_1 - 6I_2 = 20$ (2)

3 × (1) gives: $9I_1 + 48I_2 = 60$ (3)

(3) – (2) gives: $54I_2 = 40$

i.e. $I_2 = \dfrac{40}{54}$ = 0.741 A

Substituting in (1) gives: $3I_1 + 16(0.741) = 20$

and $I_1 = \dfrac{20 - 16\left(0.741\right)}{3}$ = 2.715 A

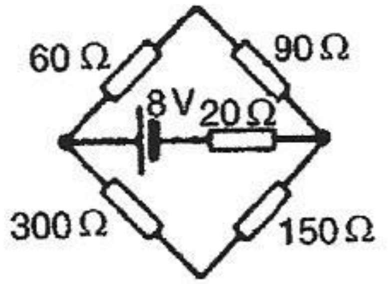
Thus, $I_1 - I_2 = 2.715 - 0.741$ = 1.974 A

Hence, **current in 3 Ω resistor =** $\mathbf{I_1}$ **= 2.715 A**

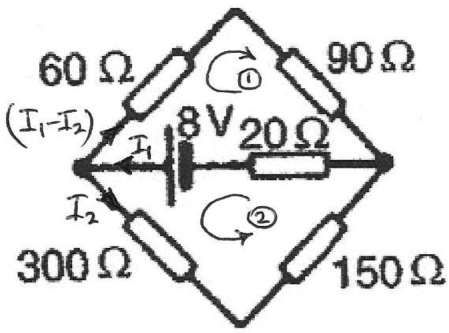
p.d. across 10 Ω resistor = $\mathbf{I_2(10)}$ **= 0.741 × 10 = 7.410 V**

p.d. across 2 Ω resistor = $(I_1 - I_2)(2) = 1.974 \times 2$ = **3.948 V**

6. For the network shown below find: (a) the current in the battery, (b) the current in the 300 Ω resistor, (c) the current in the 90 Ω resistor, and (d) the power dissipated in the 150 Ω resistor.



(a) With the current directions as shown in the diagram below,



from loop 1: $8 = 20 I_1 + (60 + 90)(I_1 - I_2)$

and from loop 2: $8 = 20 I_1 + (300 + 150)(I_2)$

i.e. $170 I_1 - 150 I_2 = 8$ (1)

and $20 I_1 + 450 I_2 = 8$ (2)

3 × equation (1) gives: $510 I_1 - 450 I_2 = 24$ (3)

(2) + (3) gives: $530 I_1 = 32$

from which, **current in the battery,** $\mathbf{I_1} = \frac{32}{530}$ = 0.0603774 A = **60.38 mA**

(b) In equation (1), $170(60.3774 \times 10^{-3}) - 150 I_2 = 8$

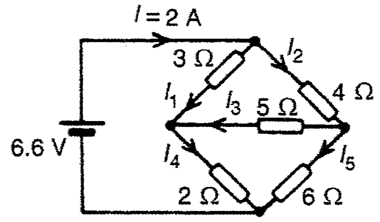
from which, $10.26415 - 8 = 150 I_2$

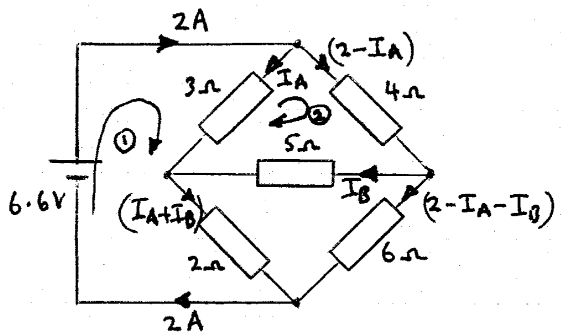
and **current in the 300 Ω resistor**, $\mathbf{I_2} = \dfrac{10.26415-8}{150}$ = 0.015094 A = **15.09 mA**

(c) **Current in the 90 Ω resistor** = $I_1 - I_2$ = 60.38 – 15.09 = **45.29 mA**

(d) **Power dissipated in the 150 Ω resistor** = $I_2^2R = \left(15.09\times10^{-3}\right)^2(150)$ = 0.034156 W = **34.20 mW**

7. For the bridge network shown, find the currents I_1 to I_5





From loop 1: $6.6 = 3I_A + 2(I_A + I_B)$

From loop 2: $0 = 4(2 - I_A) + 5I_B - 3I_A$

i.e. $5I_A + 2I_B = 6.6$ (1)

and $-7I_A + 5I_B = -8$ (2)

5 × (1) gives: $25I_A + 10I_B = 33$ (3)

2 × (2) gives: $-14I_A + 10I_B = -16$ (4)

(3) – (4) gives: $39I_A = 49$

and $I_A = \dfrac{49}{39}$ = 1.256 A

Substituting in (1) gives: 5(1.256) + $2I_B = 6.6$

from which, $I_B = \dfrac{6.6 - 5(1.256)}{2} = 0.160$ A

Hence, correct to 2 decimal places, $\mathbf{I_1} = I_A = \mathbf{1.26\ A}$

$\mathbf{I_2} = 2 - I_A = 2 - 1.256 = \mathbf{0.74\ A}$

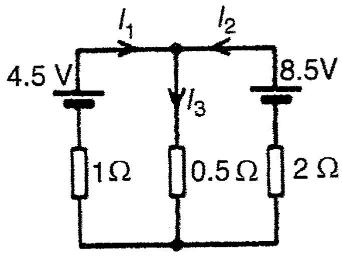
$\mathbf{I_3} = I_B = \mathbf{0.16\ A}$

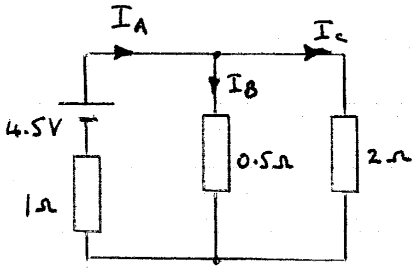
$\mathbf{I_4} = I_A + I_B = 1.256 + 0.160 = \mathbf{1.42\ A}$

$\mathbf{I_5} = 2 - I_A - I_B = 2 - 1.26 - 0.16 = \mathbf{0.58\ A}$

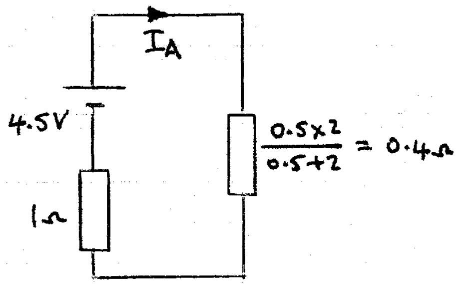
Exercise 70, Page 196

1. Use the superposition theorem to find currents I_1, I_2 and I_3 of the circuit shown.





(a)



(b)

Initially the 8.5 V source is removed as shown in diagram (a). This simplifies to diagram (b) where $I_A = \dfrac{4.5}{1.4}$

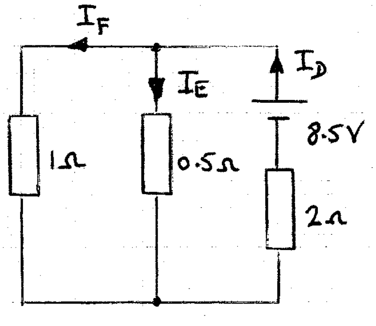
= 3.214 A

From diagram (a), $I_B = \left(\dfrac{2}{2+0.5}\right)(3.214)$ = 2.571 A

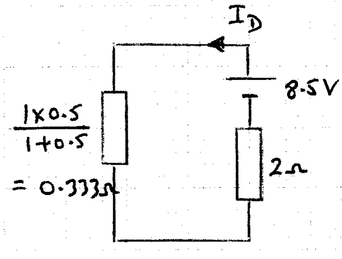
and $I_C = \left(\dfrac{0.5}{2+0.5}\right)(3.214)$ = 0.643 A

Next, the 4.5 V source is removed as shown in diagram (c). This simplifies to diagram (d) where

$I_D = \dfrac{8.5}{2+0.333}$ = 3.643 A



(c)



(d)

From diagram (c), $I_E = \left(\dfrac{1}{1+0.5}\right)(3.643)$ = 2.429 A

and $I_F = \left(\dfrac{0.5}{1+0.5}\right)(3.643)$ = 1.214 A

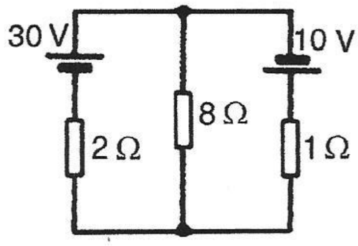
Hence, if diagram (a) is superimposed on to diagram (c), then:

$\mathbf{I_1} = I_A - I_F$ = 3.214 – 1.214 = **2 A**

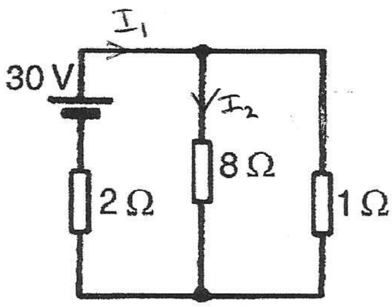
$\mathbf{I_2} = I_D - I_C$ = 3.643 – 0.643 = **3 A**

$\mathbf{I_3} = I_B + I_E$ = 2.571 + 2.429 = **5 A**

2. Use the superposition theorem to find the current in the 8 Ω resistor in the circuit shown.



Initially the 10 V source is removed as shown in diagram (a). 8 Ω in parallel with 1 Ω is given by $\dfrac{8\times1}{8+1} = \dfrac{8}{9}\,\Omega$

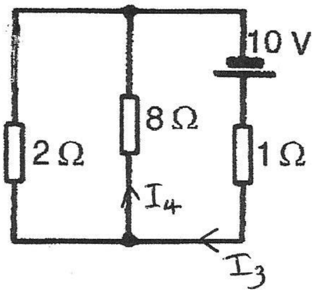


(a)

From diagram (a), $I_1 = \dfrac{30}{2+\dfrac{8}{9}}$ = 10.385 A

and $I_2 = \left(\dfrac{1}{1+8}\right)(10.385)$ = 1.154 A

Next, the 30 V source is removed as shown in diagram (b). 8 Ω in parallel with 2 Ω is given by $\dfrac{8\times2}{8+2} = 1.6\,\Omega$



(b)

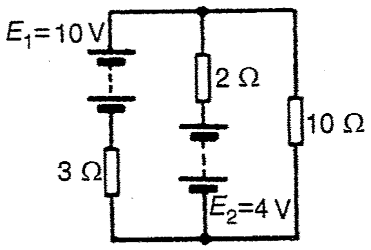
From diagram (b), $I_3 = \dfrac{10}{1+1.6}$ = 3.846 A

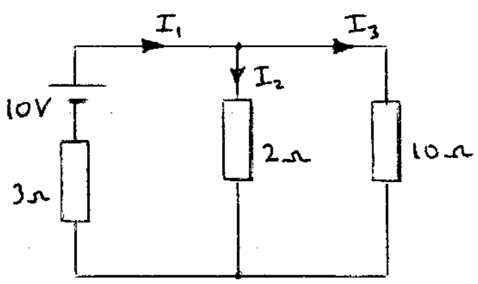
and $I_4 = \left(\dfrac{2}{2+8}\right)(3.846)$ = 0.769 A

Hence, if diagram (a) is superimposed on to diagram (b), then:

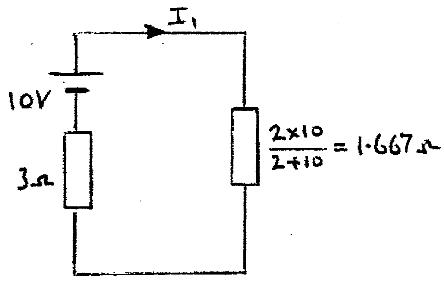
current in 8 Ω resistor = $I_2 - I_4$ = 1.154 – 0.769 = **0.385 A**

3. Use the superposition theorem to find the current in each branch of the network shown.





(a)



(b)

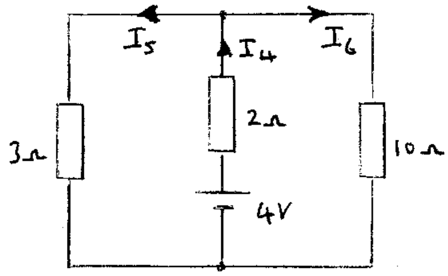
Initially the 4 V source is removed as shown in diagram (a). This simplifies to diagram (b) where

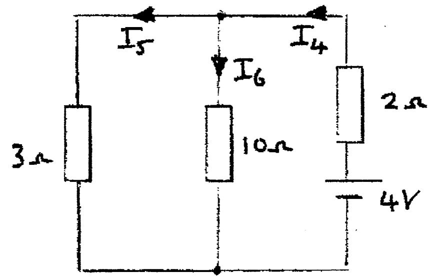
$I_1 = \dfrac{10}{3+1.667}$ = 2.143 A

From diagram (a), $I_2 = \left(\dfrac{10}{2+10}\right)(2.143)$ = 1.786 A

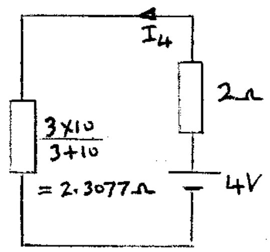
and $I_3 = \left(\dfrac{2}{2+10}\right)(2.143)$ = 0.357 A

Next, the 10 V source is removed as shown in diagram (c). Diagram (d) is the same circuit as diagram (c) and this simplifies to diagram (e) where $I_4 = \dfrac{4}{2 + 2.3077} = 0.9286$ A





(c) (d) (e)



From diagram (d), $I_5 = \left(\dfrac{10}{3+10}\right)(0.9286) = 0.714$ A

and $I_6 = \left(\dfrac{3}{3+10}\right)(0.9286) = 0.214$ A

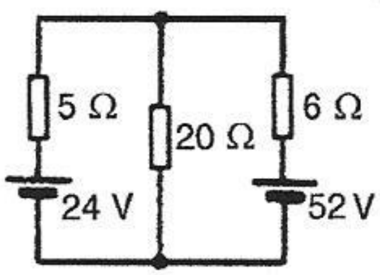
Hence, if diagram (a) is superimposed on to diagram (c), then:

the current discharging from 10 V source = $I_1 - I_5$ = 2.143 – 0.714 = **1.429 A,**

the current charging 4 V source = $I_2 - I_4$ = 1.786 – 0.9286 = **0.857 A**

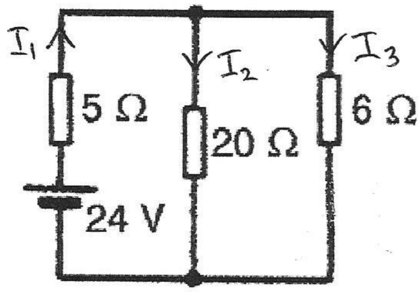
and **the current through the 10 Ω resistor** = $I_3 + I_6$ = 0.357 + 0.214 = **0.571 A**

4. Use the superposition theorem to determine the current in each branch of the arrangement shown.



Initially the 52 V source is removed as shown in diagram (a). 20 Ω in parallel with 6 Ω is given by

$\dfrac{20\times 6}{20+6}=4.615\,\Omega$



(a)

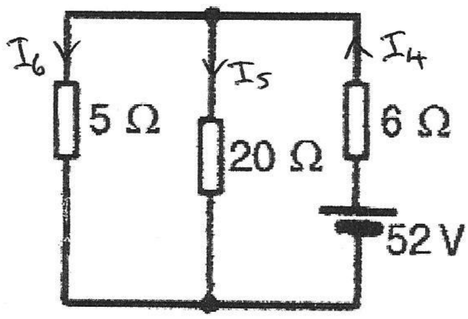
From diagram (a), $I_1=\dfrac{24}{5+4.615}$ = 2.496 A

and $I_2=\left(\dfrac{6}{6+20}\right)(2.496)$ = 0.576 A

and $I_3=\left(\dfrac{20}{6+20}\right)(2.496)$ = 1.920 A

Next, the 24 V source is removed as shown in diagram (b). 5 Ω in parallel with 20 Ω is given by

$\dfrac{5\times 20}{5+20}=4\,\Omega$



(b)

From diagram (b), $I_4=\dfrac{52}{6+4}$ = 5.20 A

and $I_5=\left(\dfrac{5}{5+20}\right)(5.20)$ = 1.040 A

and $I_6=\left(\dfrac{20}{5+20}\right)(5.20)$ = 4.160 A

Hence, if diagram (a) is superimposed on to diagram (b), then:

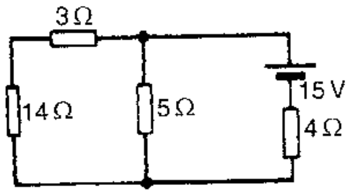
current flowing from 24 V source = I_6-I_1 = 4.160 – 2.496 = **1.664 A (i.e. battery is charging)**

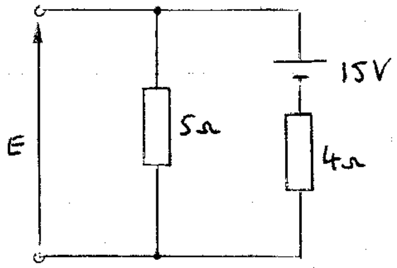
current flowing in the 20Ω resistor = I_2+I_5 = 0.576 + 1.040 = **1.616 A**

current flowing from 52 V source = I_4-I_3 = 5.20 – 1.920 = **3.280 A (i.e. battery is discharging)**

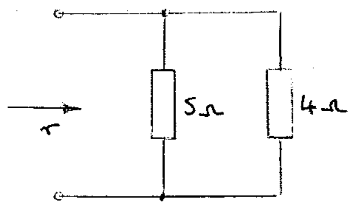
Exercise 71, Page 203

1. Use Thevenin's theorem to find the current flowing in the 14 Ω resistor of the network shown below. Find also the power dissipated in the 14 Ω resistor.





(a)

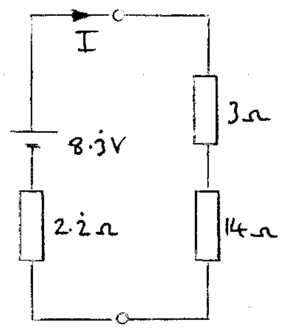


(b)

1. Removing the resistors in the branch containing the 14 Ω gives diagram (a).

2. Open circuit e.m.f., E = $\left(\frac{5}{5+4}\right)(15)$ = 8.333 V by voltage division

3. Resistance 'looking in' at break with source removed = $\frac{5\times4}{5+4}$ = 2.222 Ω from diagram (b).



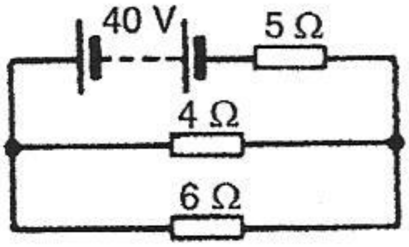
(c)

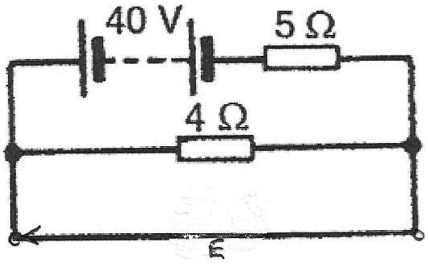
4. From the equivalent Thevenin circuit in diagram (c),

current in 14 Ω resistor, I = $\frac{8.333}{2.222+3+14}$ = **0.434 A**

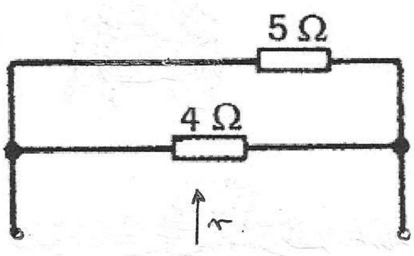
and **power dissipated in 14 Ω resistor, P** = $I^2(14) = (0.434)^2(14)$ = **2.64 W**

2. Use Thévenin's theorem to find the current flowing in the 6 Ω resistor shown below and the power dissipated in the 4 Ω resistor.





(a)

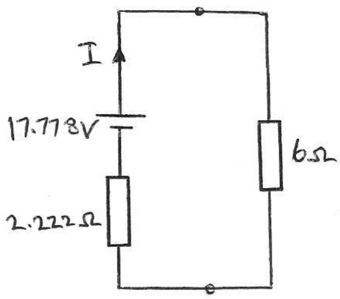


(b)

1. Removing the resistors in the branch containing the 6 Ω gives diagram (a).

2. Open circuit e.m.f., E = $\left(\frac{4}{4+5}\right)(40)$ = 17.778 V by voltage division

3. Resistance 'looking in' at break with source removed = $\frac{5\times4}{5+4}$ = 2.222 Ω from diagram (b).



(c)

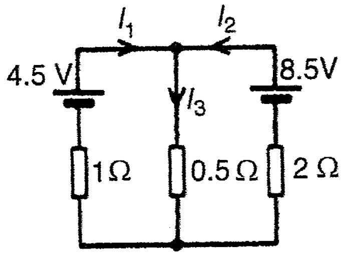
4. From the equivalent Thevenin circuit in diagram (c),

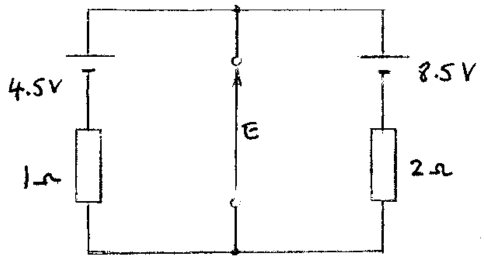
current in 6 Ω resistor, I = $\frac{17.778}{2.222+6}$ = **2.162 A**

If 2.162 A is flowing through the 6 Ω resistor of the circuit shown in the question, then the volt drop across the 6 Ω resistor is 2.162 × 6 = 12.972 V. This is the same voltage as across the 4 Ω resistor. Hence, the current in the 4 Ω resistor is 12.972/4 = 3.243 A

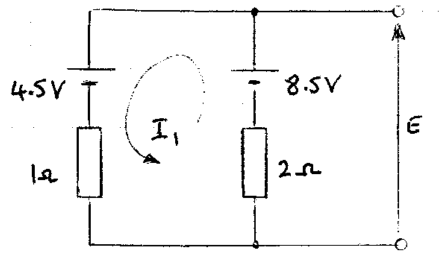
Hence, **power dissipated in 4 Ω resistor, P** = $I^2(4) = (3.243)^2(4)$ = **42.07 W**

3. Q. 1 Exercise 70. Use Thevenin's theorem to find currents I_1, I_2 and I_3 of the circuit shown.





(a)



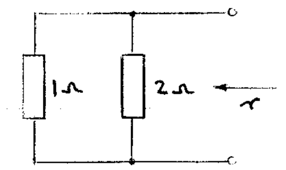
(b)

1. Removing the 0.5 Ω resistor gives diagram (a); diagram (b) is the same circuit as (a).

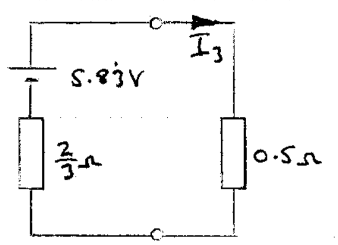
2. From diagram (b), current $I_1 = \dfrac{8.5-4.5}{2+1} = \dfrac{4}{3}$ A

   Hence, open circuit e.m.f., E = 8.5 - $\dfrac{4}{3}(2)$ = 5.833 V

3. Removing the voltage sources, the resistance 'looking in' at the break, r = $\dfrac{1\times 2}{1+2} = \dfrac{2}{3}\ \Omega$ (see diagram (c))



(c)



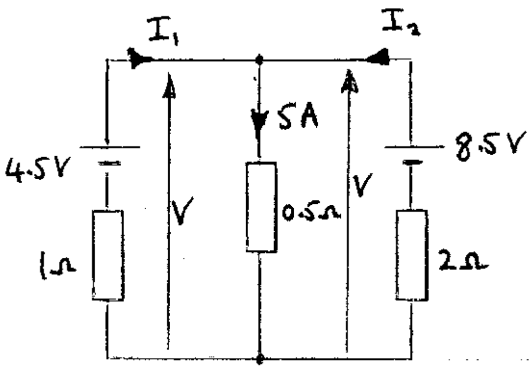
(d)

4. From the equivalent Thevenin circuit in diagram (d),

$$\textbf{current in 0.5 Ω resistor, } \mathbf{I_3} = \frac{5.833}{\frac{2}{3}+0.5} = \mathbf{5\ A}$$

From diagram (e), V = 5 × 0.5 = 2.5 V

Hence, using V = E – Ir, $2.5 = 4.5 - I_1(1)$ from which, $\mathbf{I_1 = 2\,A}$

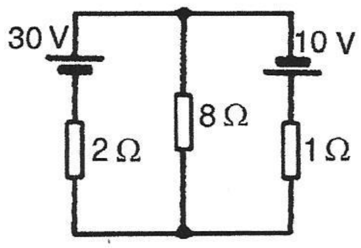


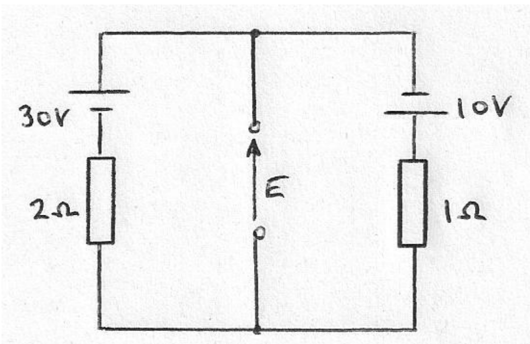
(e)

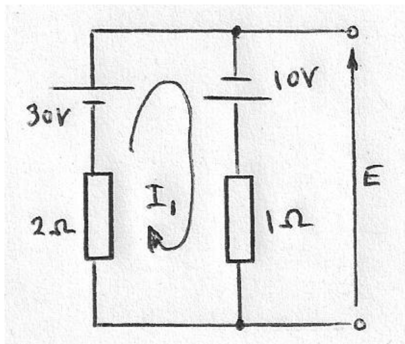
Similarly for the right hand source, $2.5 = 8.5 - I_2(2)$

from which, $\mathbf{I_2} = \dfrac{8.5 - 2.5}{2} = \mathbf{3\ A}$

3. Q. 2 Exercise 70. Use Thevenin's theorem to find the current in the 8 Ω resistor in the circuit shown.



(a)


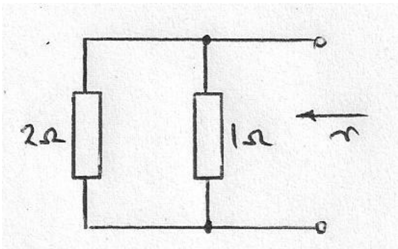
(b)


1. Removing the 8 Ω resistor gives diagram (a); diagram (b) is the same circuit as (a).

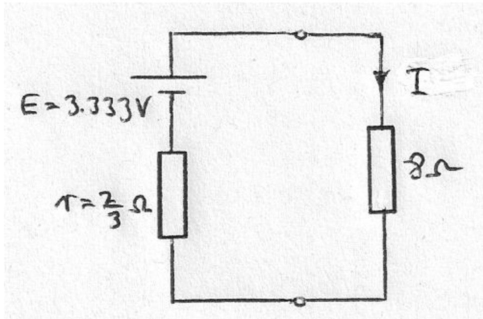
2. From diagram (b), current $I_1 = \dfrac{30+10}{2+1} = \dfrac{40}{3}$ A

   Hence, open circuit e.m.f., E = 30 - $\dfrac{40}{3}(2)$ = 3.333 V

3. Removing the voltage sources, the resistance 'looking in' at the break, r = $\frac{1\times 2}{1+2}=\frac{2}{3}\ \Omega$ (see diagram (c))



(c)



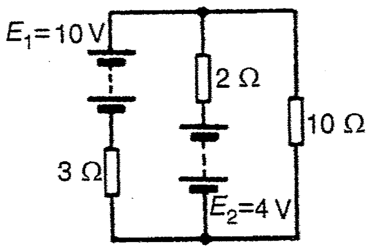
(d)

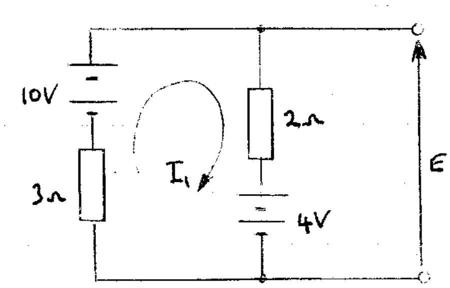
4. From the equivalent Thevenin circuit in diagram (d),

$$\textbf{current in 8}\ \boldsymbol{\Omega}\ \textbf{resistor,}\ \mathbf{I_3} = \frac{3.333}{\frac{2}{3}+8} = \mathbf{0.385\ A}$$

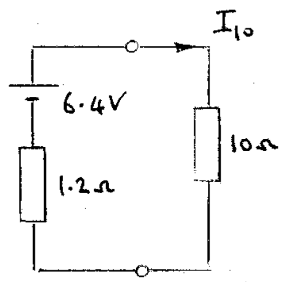
---

3. Q. 3 Exercise 70. Use Thevenin's theorem to find the current in each branch of the network shown.





(a)



(b)

1. Removing the 10 Ω resistor gives diagram (a).

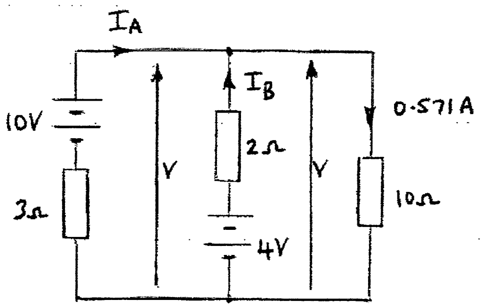
2. From diagram (a), current $I_1=\frac{10-4}{3+2}=\frac{6}{5}=1.2$ A

   Hence, open circuit e.m.f., E = 10 – (1.2)(3) = 6.4 V

3. Removing the voltage sources, the resistance 'looking in' at the break, r = $\frac{3\times 2}{3+2}=\frac{6}{5}=1.2\ \Omega$

4. From the equivalent Thevenin circuit in diagram (b),

$$\textbf{current in 10 Ω resistor, } \mathbf{I_{10}} = \frac{6.4}{1.2+10} = 0.5714 \text{ A} = \mathbf{0.571\ A}\text{, correct to 3 d.p.'s}$$



(c)

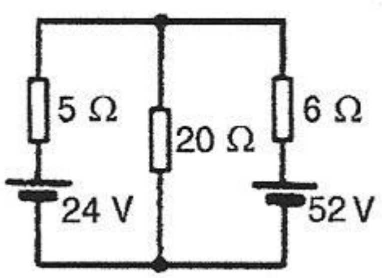
From diagram (c), V = 0.5714 × 10 = 5.714 V

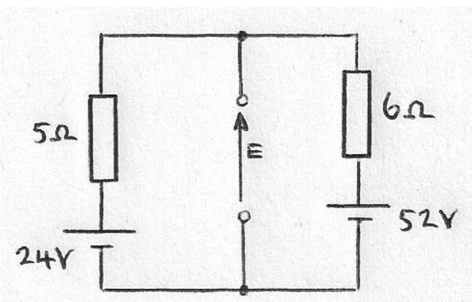
Hence, using V = E – Ir, 5.714 = 10 - I_A (3) from which, $\mathbf{I_A} = \frac{10-5.71}{3} = \mathbf{1.429\ A}$ (i.e. the 10 V source is discharging)

Similarly for the right hand source, 5.714 = 4 - I_B (2)

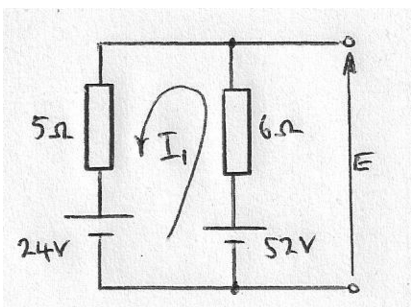
from which, $\mathbf{I_B} = \frac{4-5.714}{2} = \mathbf{-\ 0.857\ A}$ (i.e. the 4 V source is charging)

3. Q. 4 Exercise 70. Use Thevenin's theorem to determine the current in each branch of the arrangement shown.





(a)



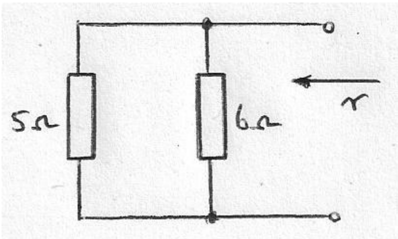
(b)

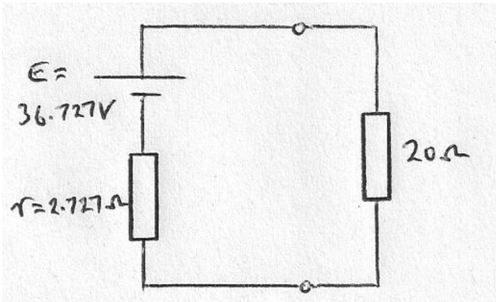
1. Removing the 20 Ω resistor gives diagram (a); diagram (b) is the same circuit as (a).

2. From diagram (b), current $I_1 = \dfrac{52-24}{5+6} = \dfrac{28}{11}$ A

Hence, open circuit e.m.f., E = 52 - $\dfrac{28}{11}(6)$ = 36.727 V

3. Removing the voltage sources, the resistance 'looking in' at the break, r = $\dfrac{5\times 6}{5+6}$ = 2.727 Ω (see diagram (c))

(c) 

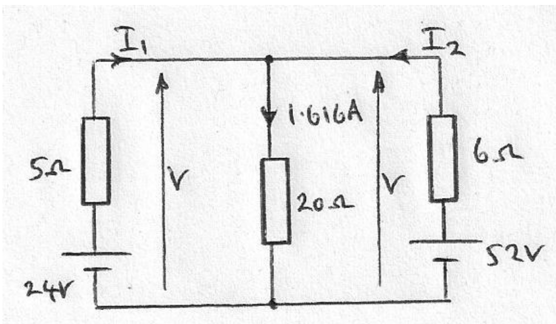
(d) 

4. From the equivalent Thevenin circuit in diagram (d),

current in 20 Ω resistor, $\mathbf{I_3} = \dfrac{36.727}{2.727+20}$ = **1.616 A**

From diagram (e), V = 1.616 × 20 = 32.32 V

Hence, using V = E – Ir, 32.32 = 24 - $I_1(5)$ from which, $\mathbf{I_1 = -1.664\,A}$ (i.e. current is charging)

(e) 

Similarly for the right hand source, 32.32 = 52 - $I_2(6)$

from which, $\mathbf{I_2} = \dfrac{52-32.32}{6}$ = **3.280 A**

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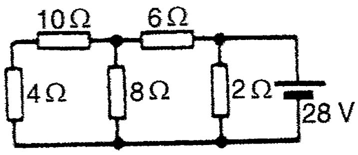
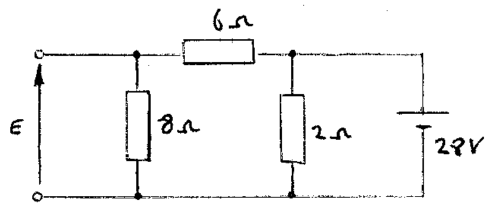
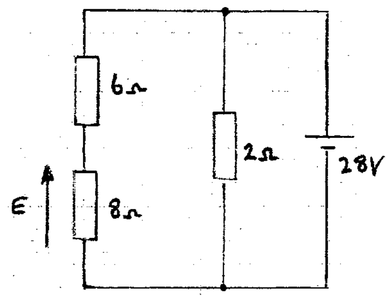
4. In the network shown below, the battery has negligible internal resistance. Find, using Thevenin's theorem, the current flowing in the 4 Ω resistor.





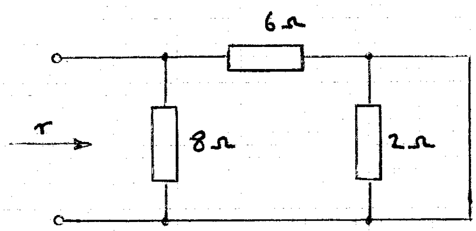
(a)



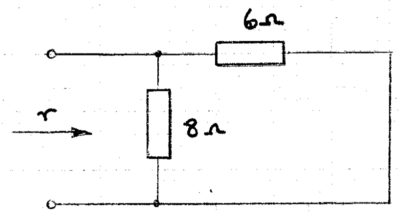
(b)

1. The resistors in the branch containing the 4 Ω resistor are removed as shown in diagram (a). Diagram (b) is diagram (a) redrawn.

2. By voltage division, open circuit e.m.f., E = $\left(\dfrac{8}{6+8}\right)(28)$ = 16 V

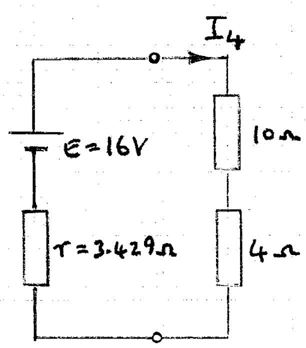


(c)



(d)

3. Replacing the 28 V source with a short circuit, the resistance r 'looking in' at the break is shown in diagram (c). The equivalent circuit of (c) is shown in (d), where r = $\dfrac{8\times 6}{8+6} = \dfrac{48}{14}$ = 3.429 Ω

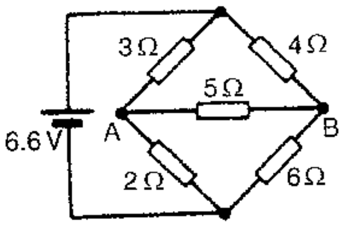


(e)

4. The Thevenin equivalent circuit is shown in diagram (e) where

current in 4 Ω resistor, $I_4 = \dfrac{16}{3.429+10+4}$ = **0.918 A**

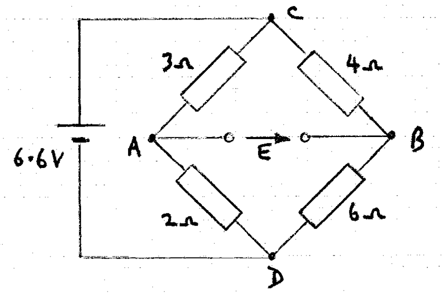
5. For the bridge network shown below, find the current in the 5 Ω resistor, and its direction, by using Thevenin's theorem.



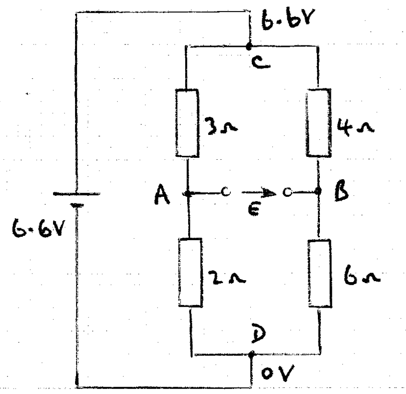
1. The 5 Ω branch is open circuited as shown in diagram (a). Diagram (b) is diagram (a) redrawn.

2. In diagram (b), $V_{CA} = \left(\dfrac{3}{3+2}\right)(6.6)$ = 3.96 V and $V_{CB} = \left(\dfrac{4}{4+6}\right)(6.6)$ = 2.64 V

   Hence, V_A = 6.6 – 3.96 = 2.64 V and V_B = 6.6 – 2.64 = 3.96 V

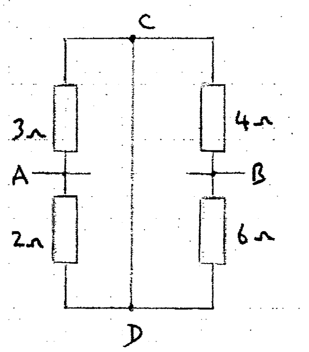


(a)

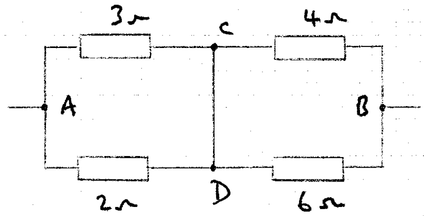


(b)

Thus, the open circuit e.m.f., E = V_{BA} = 3.96 – 2.64 = 1.32 V



(c)

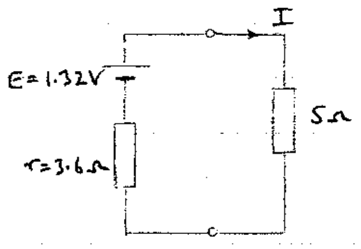


(d)

3. The 6.6 V source is short circuited as shown in diagram (c), which is then redrawn as shown in

diagram (d), where resistance 'looking in' at break = resistance between points A and B, i.e.

$$r = \frac{2\times3}{2+3}+\frac{4\times6}{4+6}=\frac{6}{5}+\frac{24}{10}=1.2+2.4 = 3.6\,\Omega$$



(e)

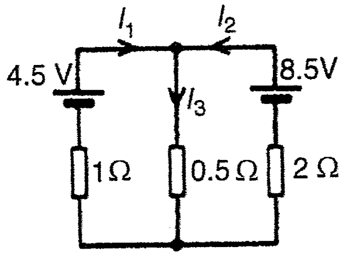
4. The Thevenin equivalent circuit is shown in diagram (e) where

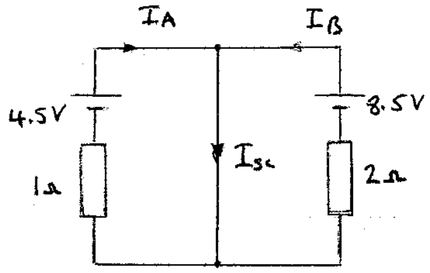
$$\textbf{current in 5 }\boldsymbol{\Omega}\textbf{ resistor, I} = \frac{1.32}{3.6+5} = \mathbf{0.153\ A}$$

which flows from point B to point A (since voltage at B is greater than the voltage at point A).

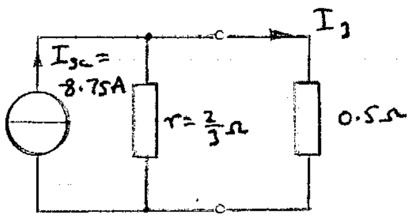
Exercise 72, Page 206

1. Q.1 Exercise 70. Use Norton's theorem to find currents I_1, I_2 and I_3 of the circuit shown.





(a)



(b)

1. The 0.5 Ω resistor is short circuited as shown in diagram (a).

2. From diagram (a), $I_{SC} = I_A + I_B = \frac{4.5}{1} + \frac{8.5}{2}$ = 8.75 A

3. With the voltage sources removed, the resistance 'looking in' at a break in the short circuit is

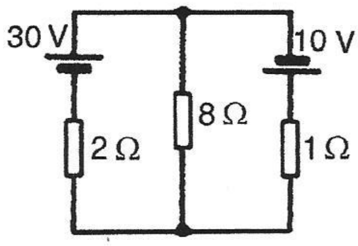
   given by 1 Ω in parallel with 2 Ω, i.e. r = $\frac{1 \times 2}{1+2} = \frac{2}{3}$ Ω

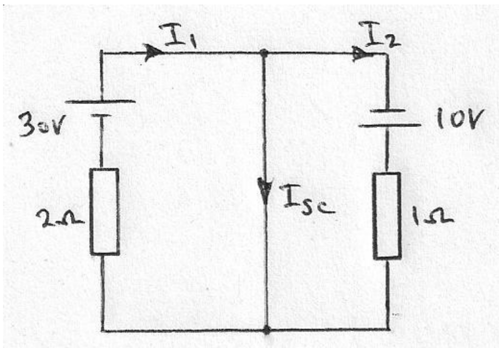
4. The Norton equivalent circuit is shown in diagram (b), where

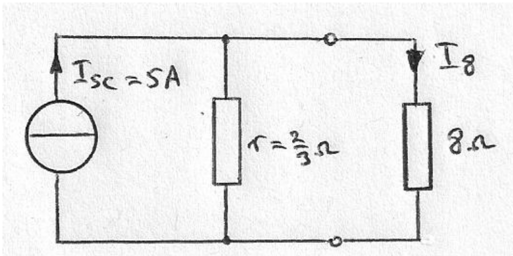
the current in the 0.5 Ω resistor, $\mathbf{I_3}$ = $\left(\frac{\frac{2}{3}}{\frac{2}{3}+0.5}\right)(8.75)$ = **5 A**

The remaining currents are calculated as on page 129/130.

1. Q.2 Exercise 70. Use Norton's theorem to find the current in the 8 Ω resistor in the circuit shown.



(a) 

(b) 

1. The 8 Ω resistor is short circuited as shown in diagram (a).

2. From diagram (a), $I_{SC} = I_1 - I_2 = \frac{30}{2} - \frac{10}{1}$ = 5 A

3. With the voltage sources removed, the resistance 'looking in' at a break in the short circuit is

   given by 2 Ω in parallel with 1 Ω, i.e. r = $\frac{2\times 1}{2+1} = \frac{2}{3}$ Ω

4. The Norton equivalent circuit is shown in diagram (b), where

   the current in the 8 Ω resistor, $\mathbf{I_8}$ = $\left(\frac{\frac{2}{3}}{\frac{2}{3}+8}\right)(5)$ = **0.385 A**

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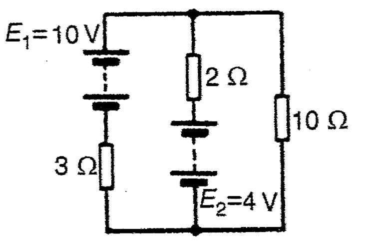
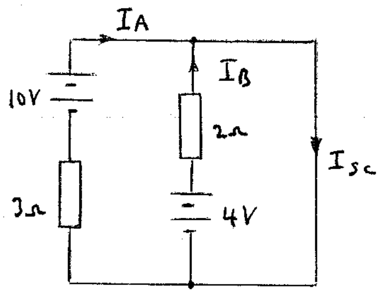
1. Q. 3 Exercise 70. Use Norton's theorem to find the current in each branch of the network shown.



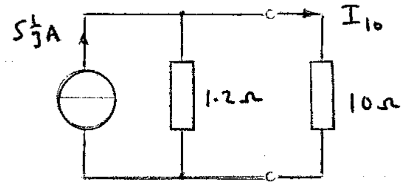
---

1. The 10 Ω resistor is short circuited as shown in diagram (a).

2. From diagram (a), $I_{SC} = I_A + I_B = \frac{10}{3} + \frac{4}{2} = 5\frac{1}{3}$ A



(a)



(b)

3. With the voltage sources removed, the resistance 'looking in' at a break in the short circuit is

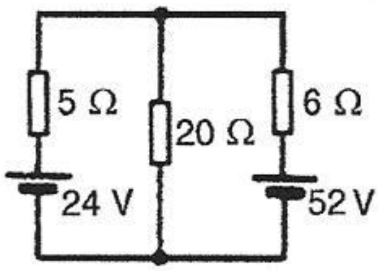
   given by 3 Ω in parallel with 2 Ω, i.e. $r = \dfrac{3\times 2}{3+2} = \dfrac{6}{5} = 1.2\ \Omega$

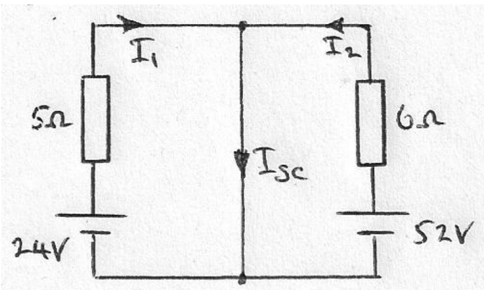
4. The Norton equivalent circuit is shown in diagram (b), where

   the current in the 10 Ω resistor, $\mathbf{I_{10}} = \left(\dfrac{1.2}{1.2+10}\right)\left(5\dfrac{1}{3}\right)$ = **0.571 A**

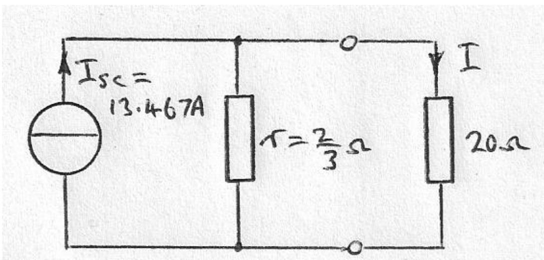
The remaining currents are calculated as on page 132.

1. Q.4 Exercise 70. Use Norton's theorem to find the current in each branch of the arrangement shown.





(a)



(b)

1. The 20 Ω resistor is short circuited as shown in diagram (a).

2. From diagram (a), $I_{SC} = I_1 + I_2 = \dfrac{24}{5} + \dfrac{52}{6}$ = 13.467 A

3. With the voltage sources removed, the resistance 'looking in' at a break in the short circuit is

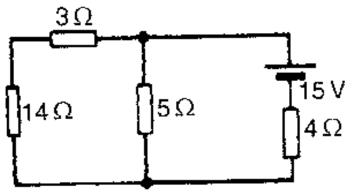
given by 5 Ω in parallel with 6 Ω, i.e. $r = \frac{5 \times 6}{5+6} = 2.727\ \Omega$

4. The Norton equivalent circuit is shown in diagram (b), where

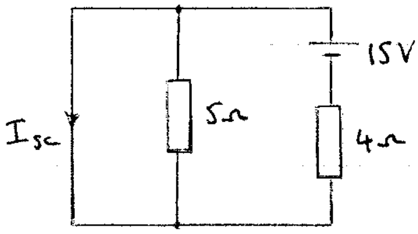
the current in the 20 Ω resistor, I $= \left(\frac{2.727}{2.727+20}\right)(13.467)$ = **1.616 A**

The remaining currents are calculated as on page 133.

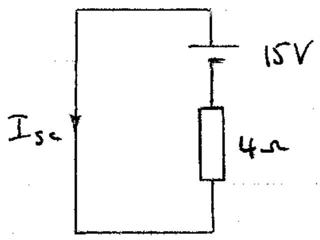
2. Q. 1 Exercise 71. Use Norton's theorem to find the current flowing in the 14 Ω resistor of the network shown below. Find also the power dissipated in the 14 Ω resistor.



1. The branch containing the 14 Ω resistor is short circuited as shown in diagram (a).



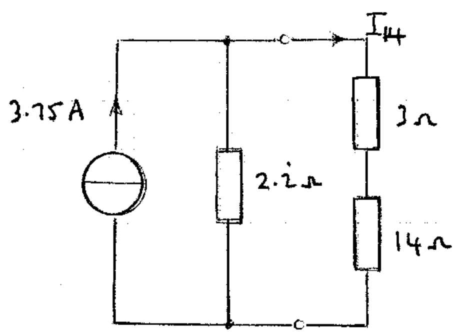
(a)



(b)

2. Diagram (b) is equivalent to diagram (a). From diagram (b), $I_{SC} = \frac{15}{4} = 3.75$ A

3. With the voltage source removed, the resistance 'looking in' at a break in the short circuit is given by 5 Ω in parallel with 4 Ω, i.e. $r = \frac{5 \times 4}{5+4} = \frac{20}{9} = 2.2222\ \Omega$



(c)

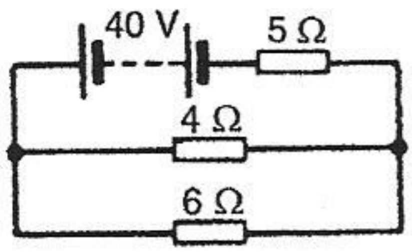
4. The Norton equivalent circuit is shown in diagram (c), where

$$\textbf{the current in the 14}\,\boldsymbol{\Omega}\textbf{ resistor},\ \mathbf{I_{14}} = \left(\frac{2.2222}{2.2222+3+14}\right)(3.75) = \mathbf{0.434\ A}$$

and **power dissipated in 14 Ω resistor, P** = $I^2(14) = (0.434)^2(14)$ = **2.64 W**

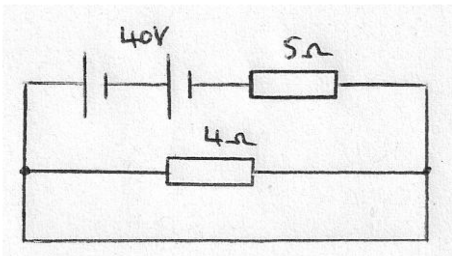
---

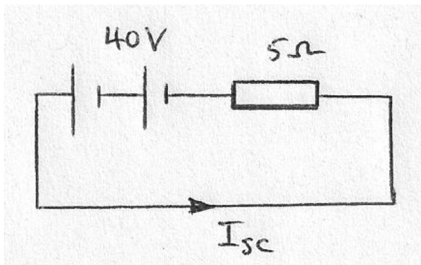
2. Q. 2 Exercise 71. Use Norton's theorem to find the current flowing in the 6 Ω resistor shown below and the power dissipated in the 4 Ω resistor.



---

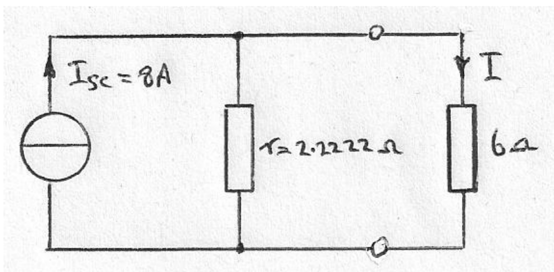
1. The branch containing the 6 Ω resistor is short circuited as shown in diagram (a).

(a) 

(b) 

2. Diagram (b) is equivalent to diagram (a). From diagram (b), $I_{SC} = \frac{40}{5} = 8$ A

3. With the voltage source removed, the resistance 'looking in' at a break in the short circuit is given by 4 Ω in parallel with 5 Ω, i.e. $r = \frac{4\times 5}{4+5} = \frac{20}{9} = 2.2222\ \Omega$



(c)

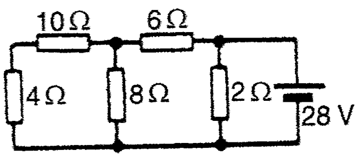
4. The Norton equivalent circuit is shown in diagram (c), where

the current in the 6 Ω resistor, $\mathbf{I_6} = \left(\frac{2.2222}{2.2222+6}\right)(8)$ = **2.162 A**

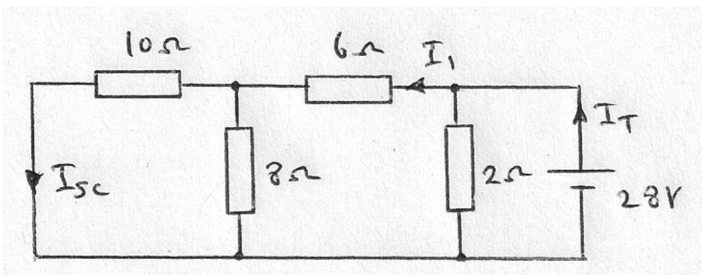
If 2.162 A is flowing through the 6 Ω resistor of the circuit shown in the question, then the volt drop across the 6 Ω resistor is 2.162 × 6 = 12.972 V. This is the same voltage as across the 4 Ω resistor. Hence, the current in the 4 Ω resistor is 12.972/4 = 3.243 A

Hence, **power dissipated in 4 Ω resistor, P** = $I^2(4) = (3.243)^2(4)$ = **42.07 W**

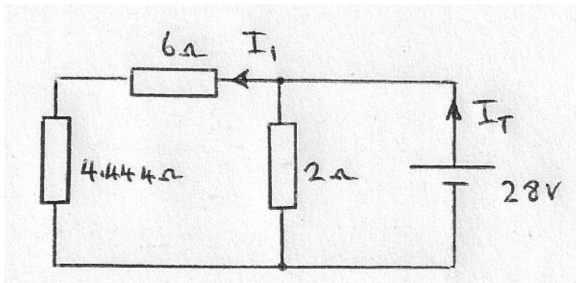
2. Q. 4 Exercise 71. In the network shown below, the battery has negligible internal resistance. Find, using Norton's theorem, the current flowing in the 4 Ω resistor.



1. The branch containing the 4 Ω resistor is short circuited as shown in diagram (a).



(a)

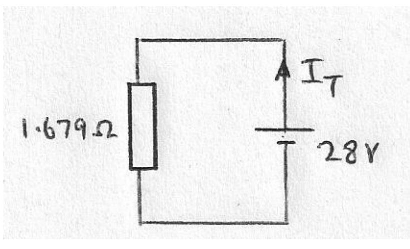


(b)

2. Diagram (b) is equivalent to diagram (a). 10 Ω in parallel with 8 Ω is: $\frac{10\times 8}{10+8} = 4.444\,\Omega$.

Then (4.444 Ω + 6 Ω) in parallel with 2 Ω is given by: $\frac{10.444\times 2}{10.444+2} = 1.679\,\Omega$. Hence diagram (c) results.

$I_T = \frac{28}{1.679}$ = 16.677 A



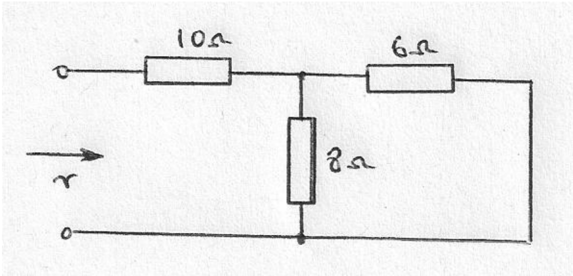
(c)

From diagram (b), $I_1 = \left(\frac{2}{2+6+4.444}\right)(16.677)$ = 2.680 A by current division

From diagram (a), $I_{SC} = \left(\frac{8}{8+10}\right)(2.680) = 1.191$ A by current division

3. With the voltage source removed, the resistance 'looking in' at a break in the short circuit is

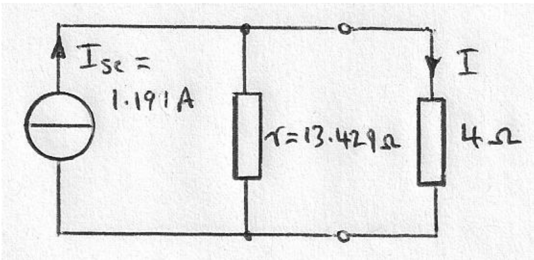
given by 10 Ω + (6 Ω in parallel with 8 Ω), i.e. $r = 10 + \frac{6\times8}{6+8} = 13.429\ \Omega$ from diagram (d).



(d)

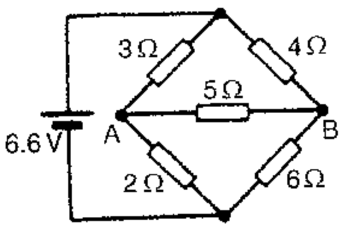
4. The Norton equivalent circuit is shown in diagram (e), where

the current in the 4 Ω resistor, $\mathbf{I_4} = \left(\frac{13.429}{13.429+4}\right)(1.191)$ = **0.918 A**



(e)

2. Q. 5 Exercise 71. For the bridge network shown below, find the current in the 5 Ω resistor, and its direction, by using Norton's theorem.

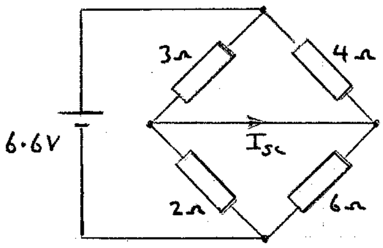


1. The branch containing the 5 Ω resistor is short circuited as shown in diagram (a).

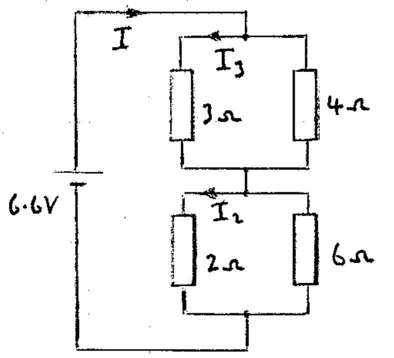
2. Diagrams (b) and (c) are equivalent to diagram (a). Current, $I = \frac{6.6}{\frac{12}{7}+\frac{12}{8}}$ = 2.053 A

$I_3 = \left(\frac{4}{4+3}\right)(2.053) = 1.173$ A and $I_2 = \left(\frac{6}{2+6}\right)(2.053) = 1.540$ A

Hence, $I_{SC} = I_2 - I_3 = 1.540 - 1.173 = 0.367$ A

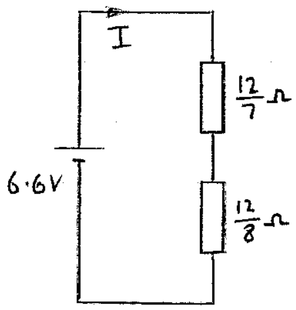


(a)

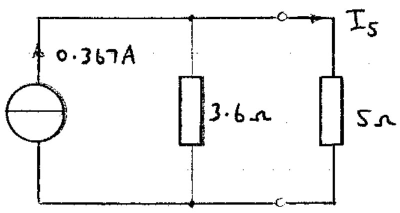


(b)

3. Resistance ‘looking in’ at break in short circuit, r = 3.6 Ω (see page 108)



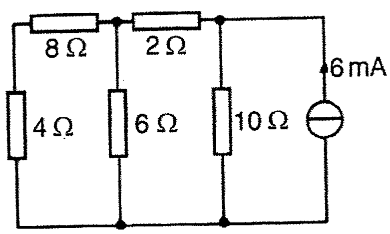
(c)



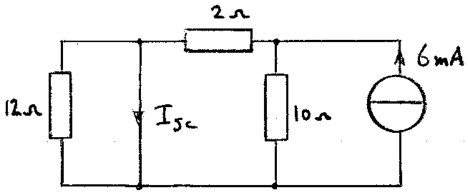
(d)

4. From equivalent Norton circuit of diagram (d), $I_5 = \left(\frac{3.6}{3.6+5}\right)(0.367)$ = **0.154 A flowing from B to A**, since $I_2 > I_3$

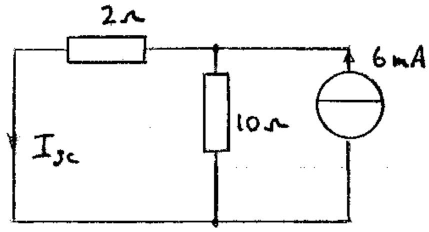
3. Determine the current flowing in the 6 Ω resistance of the network shown below by using Norton’s theorem.



1. Short circuiting the 6 Ω resistor branch gives diagram (a).

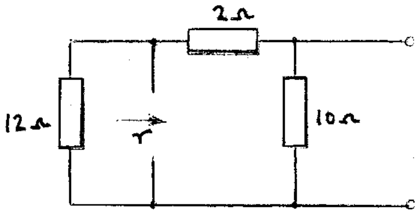


(a)

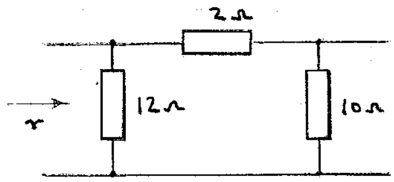


(b)

2. Diagram (b) is equivalent to diagram (a). From diagram (b), $I_{SC} = \left(\frac{10}{2+10}\right)(6) = 5$ mA

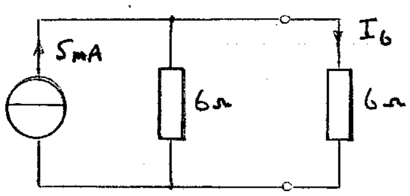


(c)



(d)

3. Open circuiting the 6 mA current source gives the circuit of diagram (c), which is equivalent to diagram (d). Hence, resistance, r , is 12 Ω in parallel with 12 Ω giving r = $\frac{12\times 12}{12+12}$ = 6 Ω

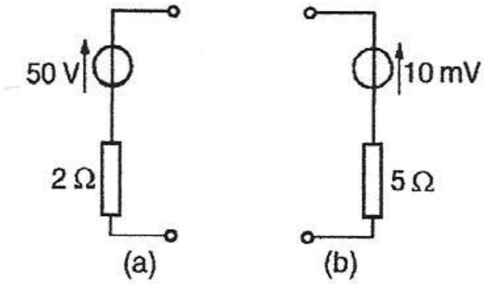


(e)

4. From the equivalent circuit of diagram (e), $I_6 = \left(\frac{6}{6+6}\right)(5)$ = **2.5 mA**

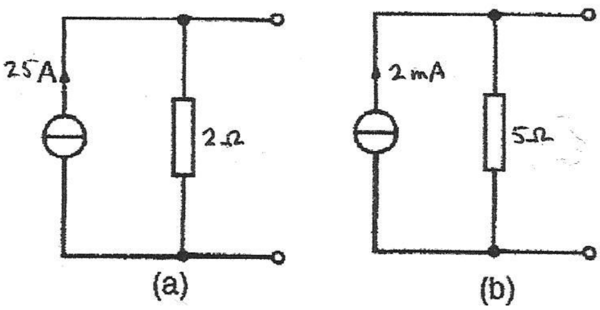
Exercise 73, Page 209

1. Convert the circuits shown below to Norton equivalent networks.



(a) If terminals in Figure (a) are short-circuited, the short-circuit current, $\mathbf{I_{SC}} = \frac{50}{2} = \mathbf{25\ A}$

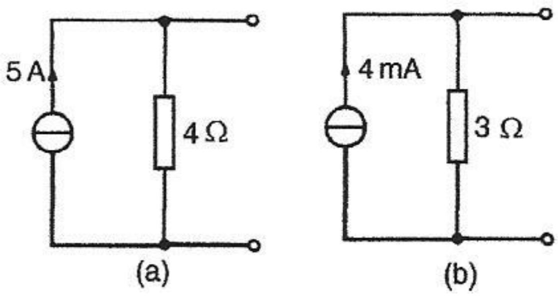
The resistance 'looking-in' at the terminals is **2 Ω**. Hence the equivalent Norton network is as shown below.



(b) If terminals in Figure (b) are short-circuited, the short-circuit current, $\mathbf{I_{SC}} = \frac{10 \times 10^{-3}}{5} = \mathbf{2\ mA}$

The resistance 'looking-in' at the terminals is **5 Ω**. Hence the equivalent Norton network is as shown above.

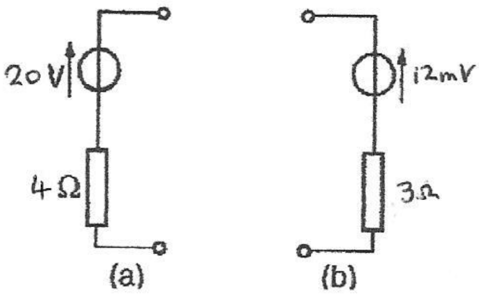
2. Convert the networks shown below to Thévenin equivalent circuits



(a) The open-circuit voltage E across the terminals in Figure (a) is given by:

$$\mathbf{E} = (I_{SC})(r) = (5)(4) = \mathbf{20\ V}$$

The resistance ‘looking-in’ at the terminals is **4 Ω**. Hence the equivalent Thévenin circuit is as shown below.



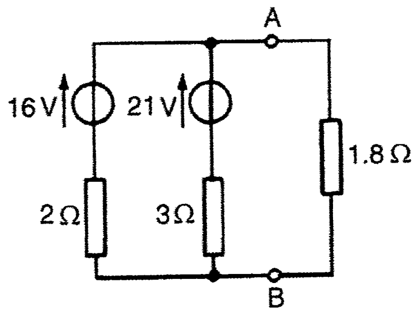
(b) The open-circuit voltage E across the terminals in Figure (b) is given by:

$$\mathbf{E} = (I_{SC})(r) = (4)(3) = \mathbf{12\ mV}$$

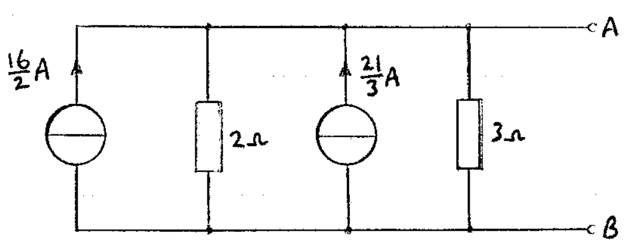
The resistance ‘looking-in’ at the terminals is **3 Ω**. Hence the equivalent Thévenin circuit is as shown above.

3. (a) Convert the network to the left of terminals AB in the diagram below to an equivalent Thevenin circuit by initially converting to a Norton equivalent network.

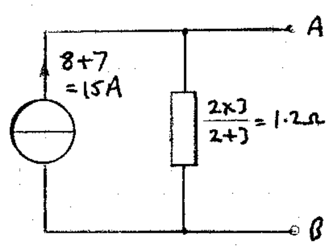
(b) Determine the current flowing in the 1.8 Ω resistance connected between A and B in the circuit shown.



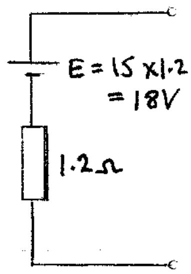
(a) Converting the two Thevenin branches to Norton equivalent circuits gives diagram (i) which is equivalent to diagram (ii). The Thevenin circuit equivalent to diagram (ii) is shown in diagram (iii), i.e. **E = 18 V and r = 1.2 Ω**



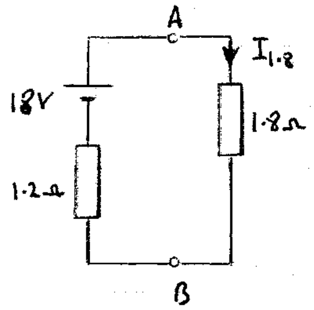
(i)



(ii)



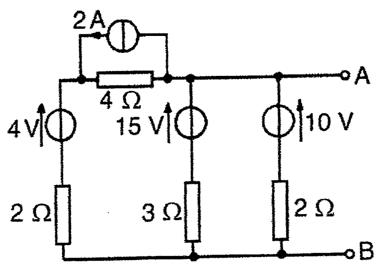
(iii)

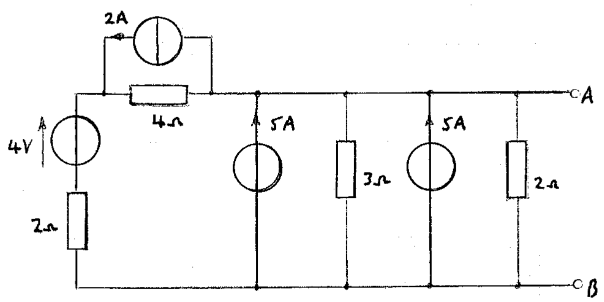


(iv)

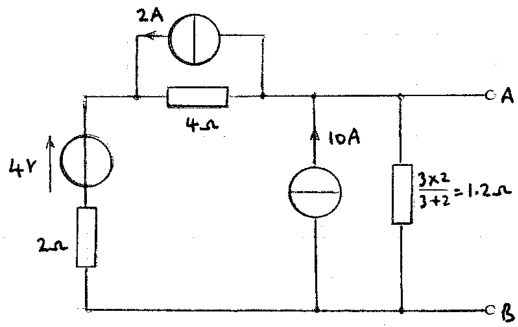
(b) From the circuit of diagram (iv), $I_{1.8} = \dfrac{18}{1.2 + 1.8} = \mathbf{6\ A}$

4. Determine, by successive conversions between Thevenin and Norton equivalent networks, a Thevenin equivalent circuit for terminals AB of the circuit shown. Hence determine the current flowing in a 6 Ω resistor connected between A and B.

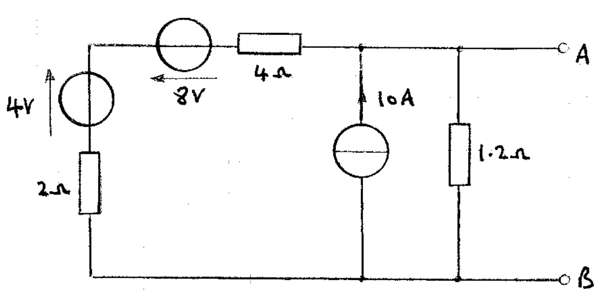




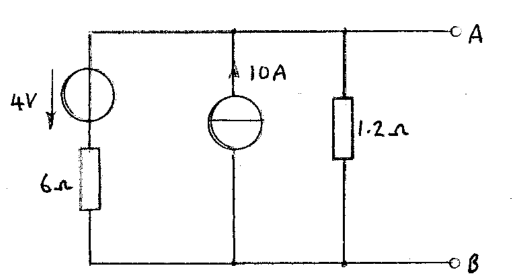
(a)



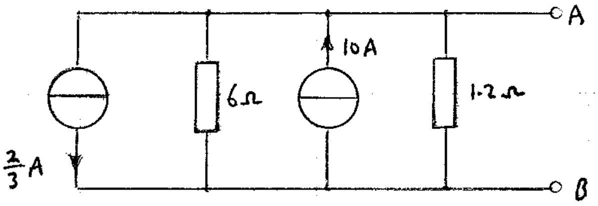
(b)



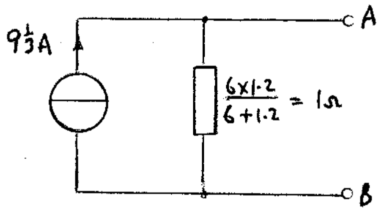
(c)



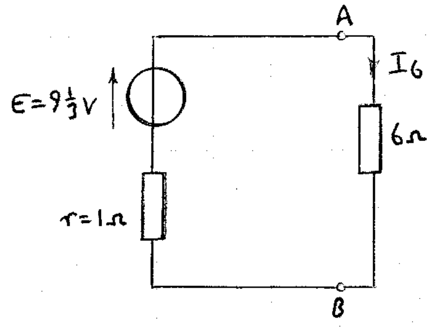
(d)



(e)



(f)

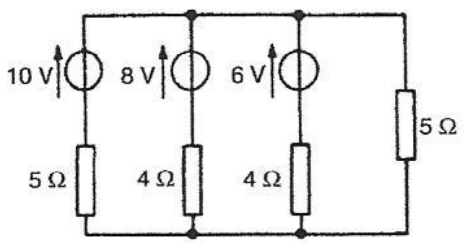


(g)

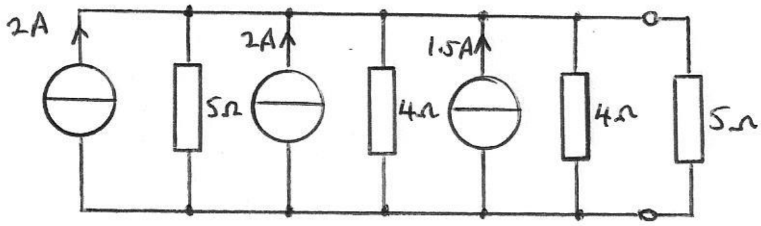
The equivalent Thevenin circuit of diagram (f) is $\mathbf{E = 9\frac{1}{3}\ \Omega}$ and $\mathbf{r = 1\ \Omega}$

and from diagram (g), $I_6 = \dfrac{9\frac{1}{3}}{1+6} = \mathbf{1\frac{1}{3}\ A}$

5. For the network shown below, convert each branch containing a voltage source to its Norton equivalent and hence determine the current flowing in the 5 Ω resistance.



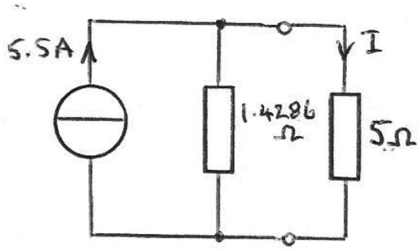
The circuit shown below is equivalent to the circuit shown in the question.



Combining the current sources gives total current = 2 + 2 + 1.5 = 5.5 A

and the total resistance is obtained from $\dfrac{1}{R} = \dfrac{1}{5} + \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{7}{10}$ from which, R = 10/7 = 1.4286 Ω

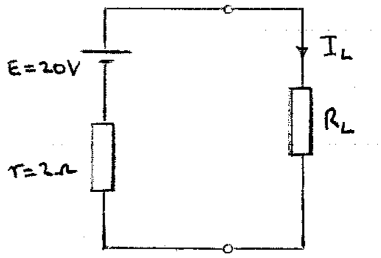
Hence, the circuit reduces to the following:



Current in 5 Ω resistor, I = $\left(\frac{1.4286}{1.4286+5}\right)(5.5)$ = **1.22 A**

Exercise 74, Page 211

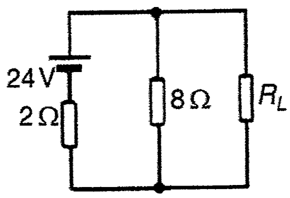
1. A d.c. source has an open-circuit voltage of 20 V and an internal resistance of 2 Ω. Determine the value of the load resistance that gives maximum power dissipation. Find the value of this power.

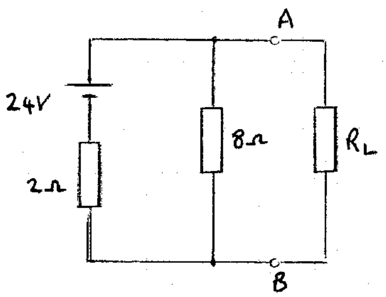


For maximum power transfer, load resistance, $\mathbf{R_L}$ = r = **2 Ω**

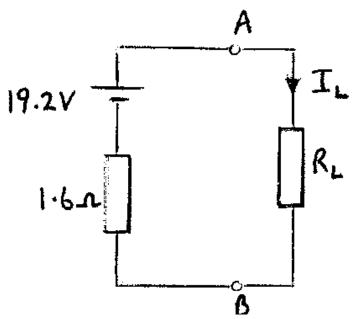
Current, $I_L = \dfrac{20}{2+2}$ = 5 A and **power dissipated**, $\mathbf{P_L} = I_L^2 R_L = (5)^2(2)$ = **50 W**

2. Determine the value of the load resistance R_L shown in the diagram that gives maximum power dissipation and find the value of the power.





(a)



(b)

For the circuit to the left of terminals AB in diagram (a), using Thevenin's theorem,

$$E = \left(\frac{8}{8+2}\right)(24) = 19.2 \text{ V}$$

and resistance 'looking in' at AB when 24 V source is removed, r = $\frac{2 \times 8}{2+8}$ = 1.6 Ω.

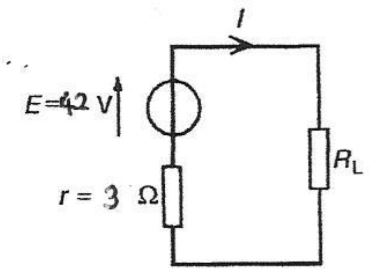
Hence the equivalent Thevenin circuit to the left of AB is shown in diagram (b).

For maximum power transfer, $\mathbf{R_L}$ **= 1.6 Ω**

Current, $I_L = \frac{19.2}{1.6+1.6}$ = 6 A and **power dissipated**, $\mathbf{P_L} = I_L{}^2 R_L = (6)^2 (1.6)$ = **57.6 W**

3. A d.c. source having an open circuit voltage of 42 V and an internal resistance of 3 Ω is connected to a load of resistance R_L. Determine the maximum power dissipated by the load.

The circuit is shown below.



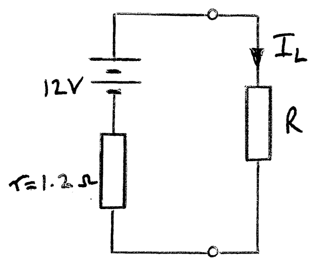
For maximum power transfer R_L = 3 = 3 Ω

Current, I = $\frac{42}{3+3}$ = 7 Ω

Hence, maximum power dissipated in the load, $\mathbf{P_L} = I^2 R_L = (7)^2 (3)$ = **147 W**

4. A voltage source comprising six 2 V cells, each having an internal resistance of 0.2 Ω, is connected to a load resistance R. Determine the maximum power transferred to the load.

The circuit is shown below, where the source voltage is 6 × 2 = 12 V and the internal resistance, r = 6 × 0.2 = 1.2 Ω



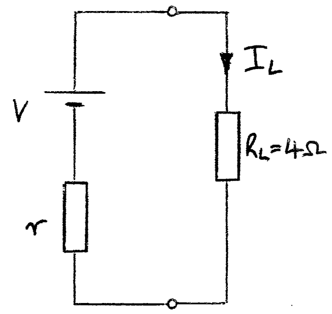
For maximum power transfer, R = r = 1.2 Ω

Hence, current in load, $I_L = \frac{12}{1.2 + 1.2}$ = 5 A

Maximum power transferred to the load, $P_L = I_L{}^2R = (5)^2(1.2)$ = **30 W**

5. The maximum power dissipated in a 4 Ω load is 100 W when connected to a d.c. voltage V and internal resistance r. Calculate (a) the current in the load, (b) internal resistance r, and (c) voltage V.

The circuit is shown below.



(a) Power in load, $P_L = 100\,W = I_L{}^2R_L = I_L{}^2(4)$

from which, $I_L{}^2 = \frac{100}{4}$ = 25 and **load current**, $I_L = \sqrt{25}$ = **5 A**

(b) For maximum power, **internal resistance, r** = R_L = **4 Ω**

(c) From the above circuit, $I_L = \frac{V}{r + R_L}$ from which, **voltage, V** = $I_L(r + R_L) = 5(4 + 4)$ = **40 V**