BEEE101L : Basic Electrical Engineering

Module 3- Magnetic Circuits

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Introduction

- *Conductively coupled* circuit means that one loop affects the neighboring loop through current conduction.
- *Magnetically coupled* circuit means that two loops, with or without contacts between them, affect each other through the magnetic field generated by one of them.
- T*ransformer* is a static electrical apparatus designed based on magnetic coupling for stepping up or down ac voltages or currents.
  - Transfers energy from one circuit to other circuit.

- **Consider a coil carrying a current i that sets up a flux φ linking the coil. The inductance of the coil can be defined as flux linkages divided by current:**

$$\boxed{L = \frac{\lambda}{i}}$$

Assuming that the flux is confined to the core so that all of the flux links all of the turns, we can write $\lambda = N\phi$. Then, we have

$$L = \frac{N\phi}{i}$$

Substituting $\phi = Ni/\mathcal{R}$, we obtain

$$\boxed{L = \frac{N^2}{\mathcal{R}}}$$

INDUCTANCE AND MUTUAL INDUCTANCE

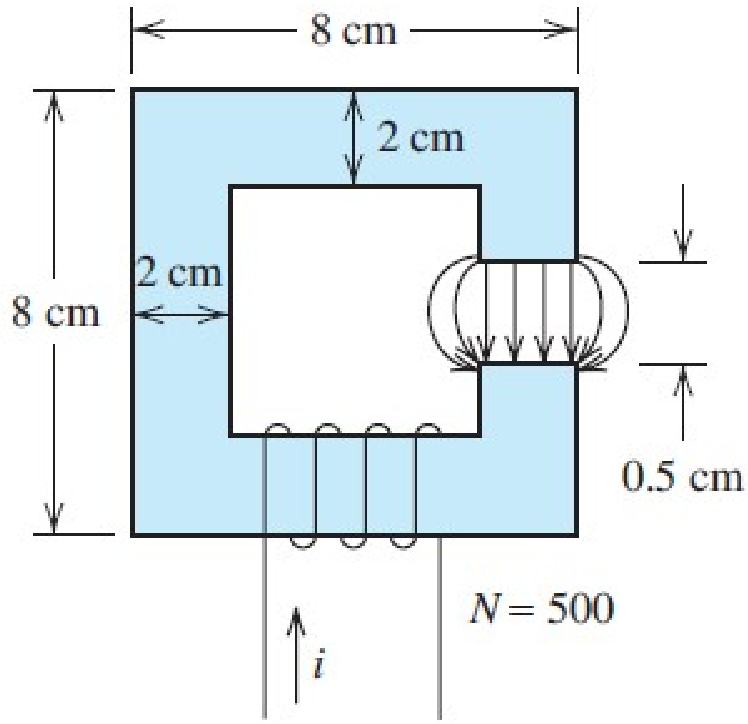
According to Faraday's law, voltage is induced in a coil when its flux linkages change:

$$e = \frac{d\lambda}{dt}$$

$$e = \frac{d(Li)}{dt}$$

$$e = L\frac{di}{dt}$$

Determine the inductance of the 500-turn coil shown in Figure



$$\mathcal{R} = 4.600 \times 10^6 \text{ A·turns/Wb}$$

$$L = \frac{N^2}{\mathcal{R}} = \frac{500^2}{4.6 \times 10^6} = 54.35 \text{ mH}$$

Mutual Inductance

When two coils are wound on the same core, some of the flux produced by one coil links the other coil. We denote the flux linkages of coil 2 caused by the current in coil 1 as λ_{21}. Correspondingly, the flux linkages of coil 1 produced by its own current are denoted as λ_{11}. Similarly, the current in coil 2 produces flux linkages λ_{22} in coil 2 and λ_{12} in coil 1.

The **self inductances** of the coils are defined as

$$L_1 = \frac{\lambda_{11}}{i_1}$$

$$L_2 = \frac{\lambda_{22}}{i_2}$$

The **mutual inductance** between the coils is

$$M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$$

Mutual Inductance

The total fluxes linking the coils are

$$\lambda_1 = \lambda_{11} \pm \lambda_{12}$$

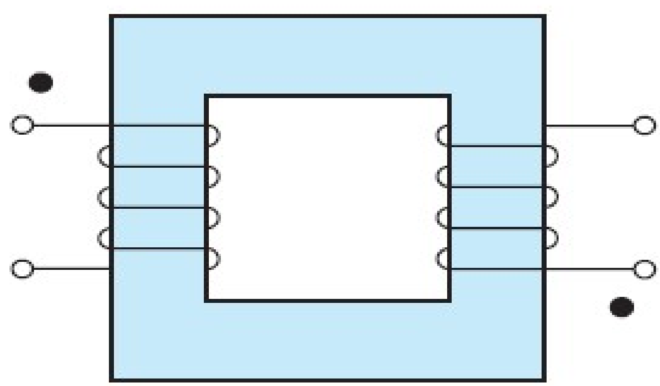
and

$$\lambda_2 = \pm\lambda_{21} + \lambda_{22}$$

where the + sign applies if the fluxes are aiding and the − sign applies if the fluxes are opposing.

Dot Convention

- It is standard practice to place a dot on one end of each coil in a circuit diagram to indicate how the fluxes interact.
- The dots are placed such that currents entering the dotted terminals produce aiding magnetic flux.
- If both currents enter (or if both leave) the dotted terminals, the mutual flux linkages add to the self flux linkages.
- On the other hand, if one current enters a dotted terminal and the other leaves, the mutual flux linkages carry a minus sign.



Circuit Equations for Mutual Inductance

$$\lambda_1 = L_1 i_1 \pm M i_2$$

$$\lambda_2 = \pm M i_1 + L_2 i_2$$

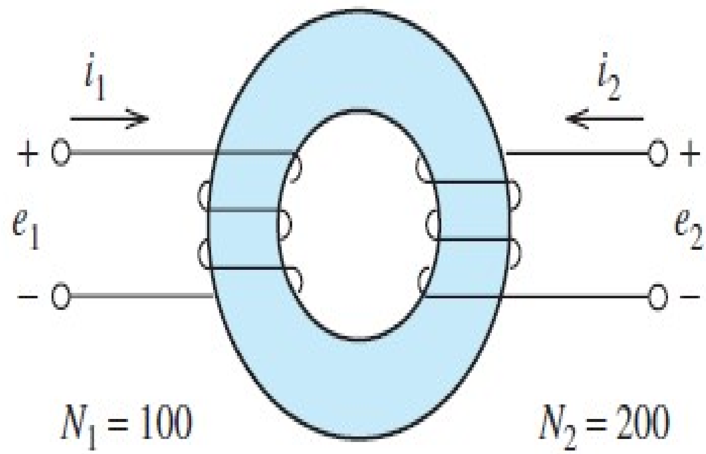
Applying Faraday's law to find the voltages induced in the coils, we get

$$e_1 = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

and

$$e_2 = \frac{d\lambda_2}{dt} = \pm M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Two coils are wound on a toroidal core as illustrated in Figure. The reluctance of the core is 10^7 (ampere-turns) / Wb. Determine the self inductances and mutual inductance of the coils. Assume that the flux is confined to the core so that all of the flux links both coils.



For coil 1,we have

$$L_1 = \frac{N_1^2}{\mathcal{R}} = \frac{100^2}{10^7} = 1 \text{ mH}$$

Similarly, for coil 2 we get

$$L_2 = \frac{N_2^2}{\mathcal{R}} = \frac{200^2}{10^7} = 4\ \text{mH}$$

To compute the mutual inductance, we find the flux produced by i_1:

$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}} = \frac{100 i_1}{10^7} = 10^{-5} i_1$$

The flux linkages of coil 2 resulting from the current in coil 1 are given by

$$\lambda_{21} = N_2 \phi_1 = 200 \times 10^{-5} i_1$$

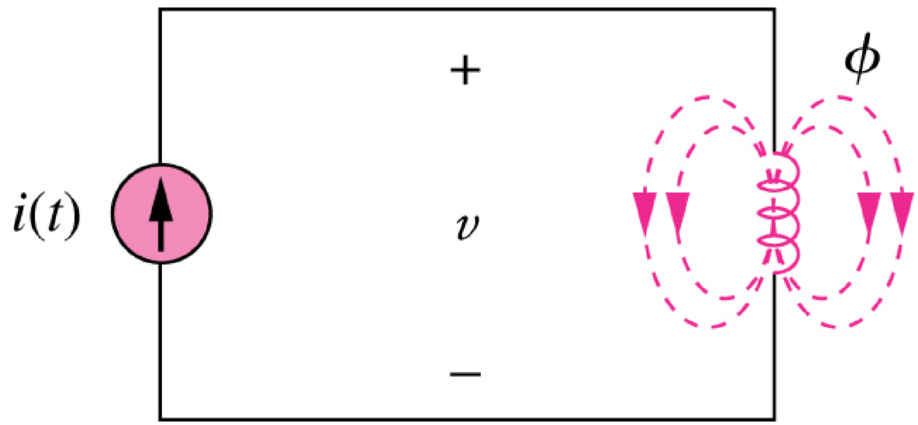
Finally, the mutual inductance is

$$M = \frac{\lambda_{21}}{i_1} = 2\ \text{mH}$$

Self Inductance

An inductor :

$$\begin{cases} \text{inductance} & L \\ N & \text{turns} \end{cases}$$



For each turn, the induced volatge is

$$v_{1\mathrm{T}} = \frac{d\phi}{dt}$$

For N turns, the induced volatge is

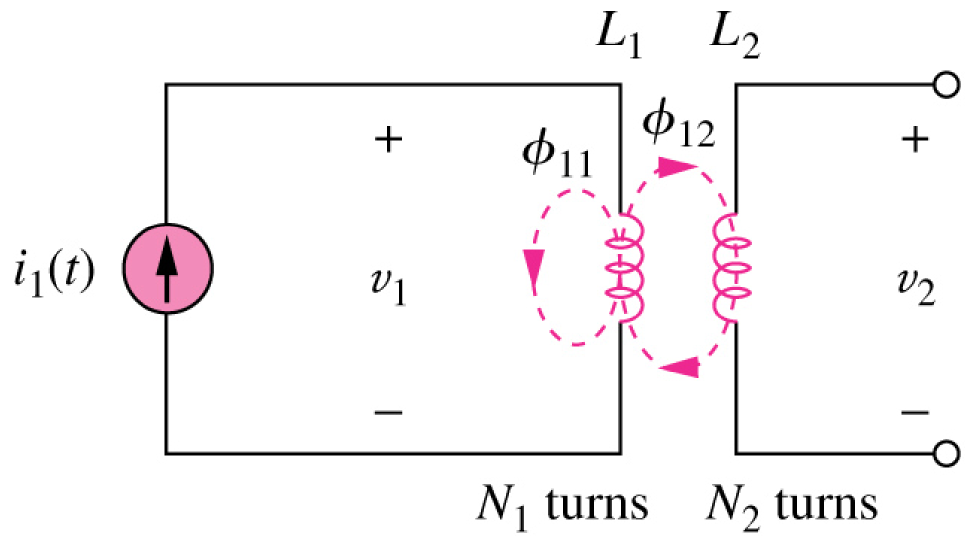
$$v = N\frac{d\phi}{dt} = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$$

$$\Rightarrow L = N\frac{d\phi}{di} \quad \text{(self - inductance)}$$

Mutual Inductance

$$\text{Coil 1}: \begin{cases} \text{self-inductances } L_1 \\ N_1 \text{ turns} \end{cases}$$

$$\text{Coil 2}: \begin{cases} \text{self-inductances } L_2 \\ N_2 \text{ turns} \end{cases}$$



Assuming no current in coil 2,

The flux generated by coil 1 is

$$\phi_1 = \phi_{11} + \phi_{12}$$

$$\Rightarrow v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$\text{where} \quad L_1 = N_1 \frac{d\phi_1}{di_1}$$

$$\Rightarrow v_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

The mutual-inductance of coil 2with respect to coil 1 is

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

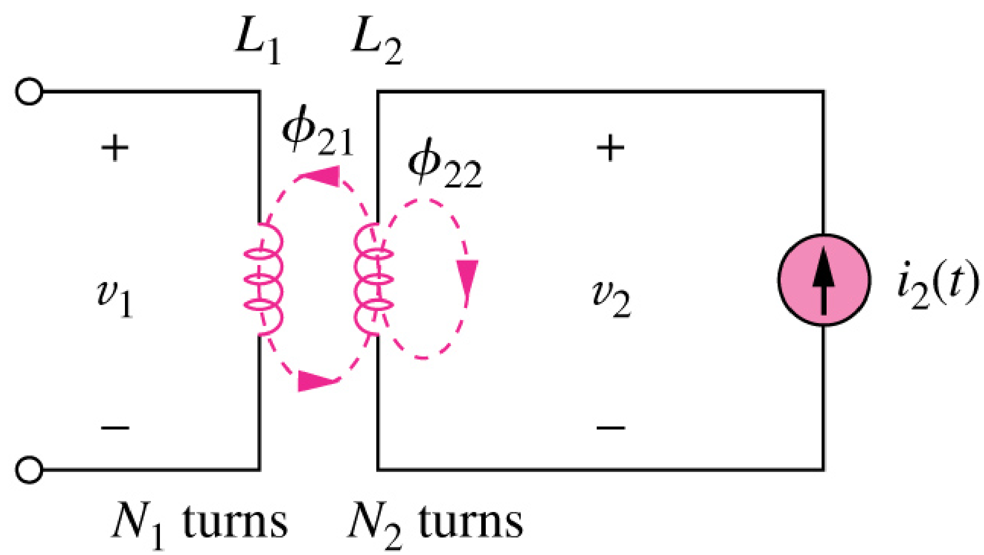
The open-circuit mutual voltage is

$$v_2 = M_{21}\frac{di_1}{dt}$$

Mutual Inductance

$$\text{Coil 1}: \begin{cases} \text{self-inductances } L_1 \\ N_1 \text{ turns} \end{cases}$$

$$\text{Coil 2}: \begin{cases} \text{self-inductances } L_2 \\ N_2 \text{ turns} \end{cases}$$



Assuming no current in coil 1,

The flux generated by coil 2 is

$\phi_2 = \phi_{22} + \phi_{21}$

$$\Rightarrow v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$\text{where} \quad L_2 = N_2 \frac{d\phi_2}{di_2}$$

$$\Rightarrow v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

The mutual-inductance of coil 1 with respect to coil 2 is

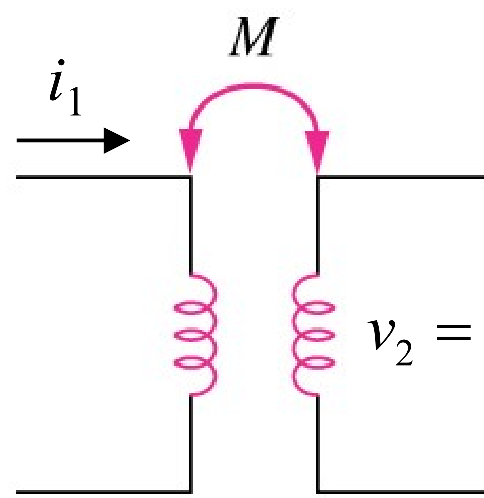
$$M_{12} = N_1 \frac{d\phi_{21}}{di_2} \qquad (= M_{21})$$

The open-circuit mutual voltage is

$$v_1 = M_{12} \frac{di_2}{dt}$$

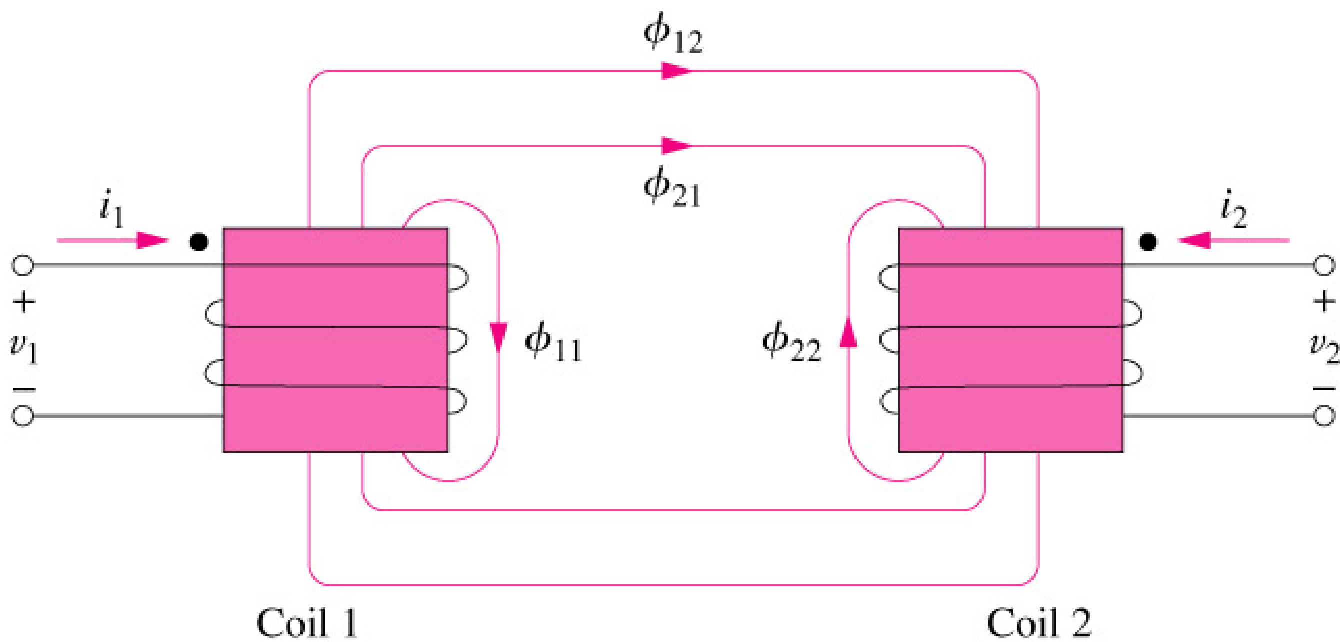
Mutual Inductance

- Here $M_{12}=M_{21}=M$.
- *Mutual coupling* only exists when the inductors or coils are *in close proximity*, and the circuits are driven by *time-varying sources*.
- *Mutual inductance* is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).
- The *dot convention* states that a current entering the dotted terminal induces a positive polarity of the mutual voltage at the dotted terminal of the second coil.



$$v_2 = M\frac{di_1}{dt}$$

Mutual Inductance



i_1 induces ϕ_{11} and ϕ_{12},

i_2 induces ϕ_{21} and ϕ_{22}.

$$v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d(\phi_{11} + \phi_{12})}{dt} + N_1 \frac{d\phi_{21}}{dt} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

$$\phi_1 = (\phi_{11} + \phi_{12}) + \phi_{21}$$

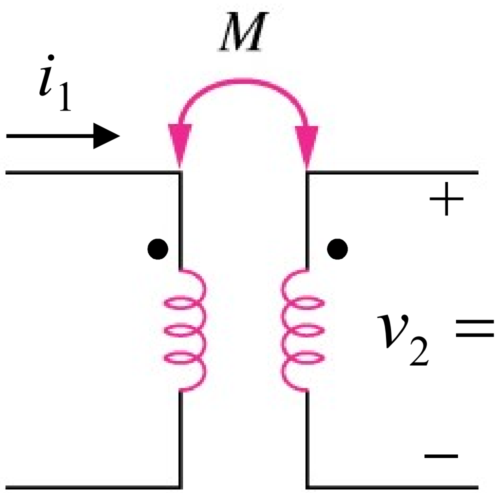
$$\phi_2 = \phi_{12} + (\phi_{21} + \phi_{22})$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d(\phi_{21} + \phi_{22})}{dt} + N_2 \frac{d\phi_{12}}{dt} = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

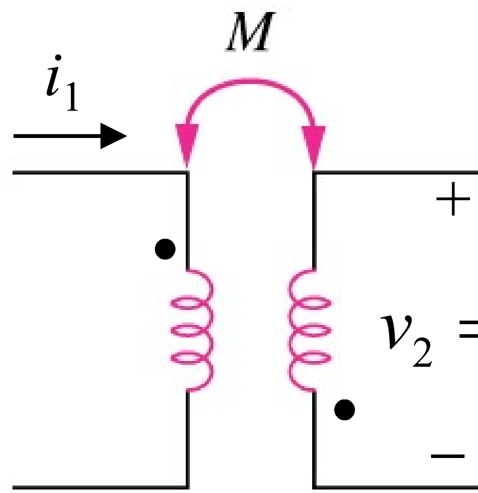
- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

- If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

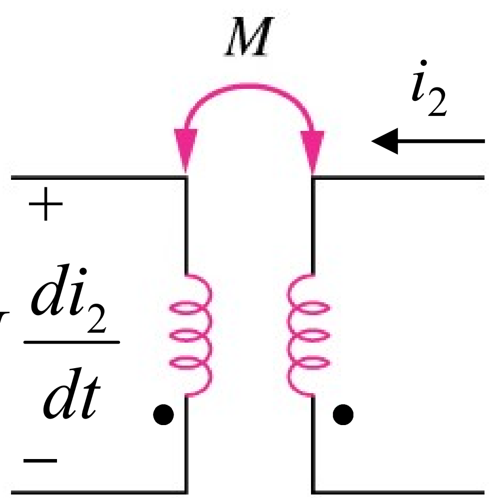
Mutual Inductance



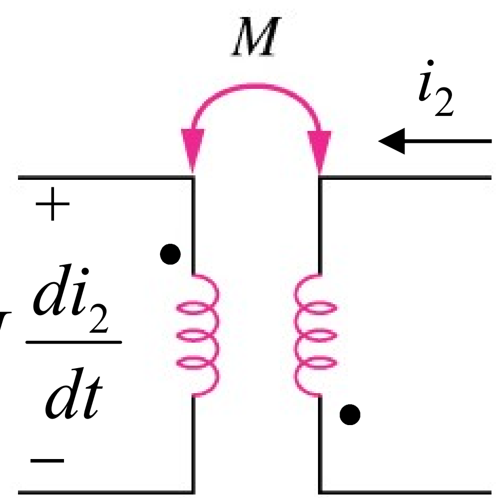
$$v_2 = M\frac{di_1}{dt}$$



$$v_2 = -M\frac{di_1}{dt}$$

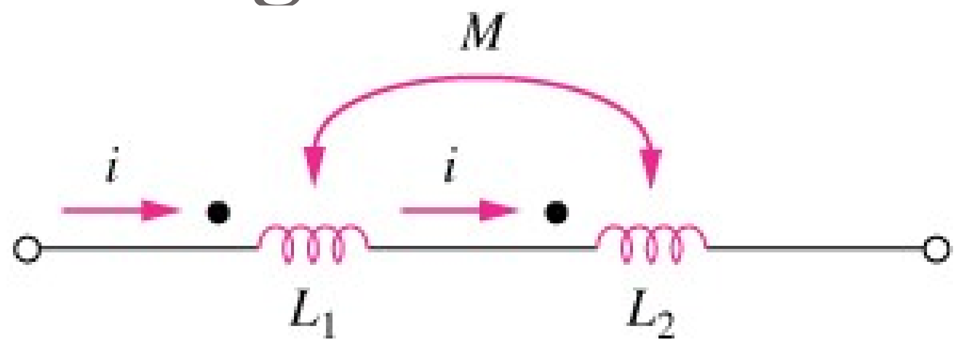


$$v_1 = M\frac{di_2}{dt}$$



$$v_1 = -M\frac{di_2}{dt}$$

Series-Aiding Connection



$$v_1 = L_1 \frac{di}{dt} + M_{12} \frac{di}{dt}$$

$$v_2 = L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1 \frac{di}{dt} + M_{12} \frac{di}{dt} + L_2 \frac{di}{dt} + M_{21} \frac{di}{dt}$$

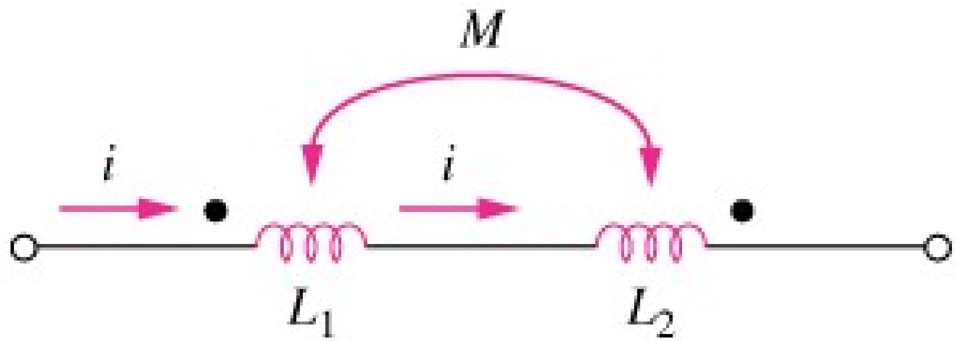
$$= (L_1 + L_2 + M_{12} + M_{21}) \frac{di}{dt}$$

But $M_{12} = M_{21} = M$,

$$\Rightarrow v = (L_1 + L_2 + 2M) \frac{di}{dt}$$

$$\Rightarrow L_{\mathrm{eq}} = L_1 + L_2 + 2M$$

Series-Opposing Connection



$+\ v_1\ -\qquad +\ v_2\ -$

$$v_1 = L_1\frac{di}{dt} - M_{12}\frac{di}{dt}$$

$$v_2 = L_2\frac{di}{dt} - M_{21}\frac{di}{dt}$$

$$v = v_1 + v_2$$

$$= L_1\frac{di}{dt} - M_{12}\frac{di}{dt} + L_2\frac{di}{dt} - M_{21}\frac{di}{dt}$$

$$= \left(L_1 + L_2 - M_{12} - M_{21}\right)\frac{di}{dt}$$

But $M_{12} = M_{21} = M$,

$$\Rightarrow v = \left(L_1 + L_2 - 2M\right)\frac{di}{dt}$$

$$\Rightarrow L_{\text{eq}} = L_1 + L_2 - 2M$$

Thank you!