BEEE101L - Basic Electrical Engineering

Node Analysis – Simple Problems

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Objective

- To introduce the beginners to node voltage analysis.
- To apply nodal analysis to solve electric circuits.

Node Analysis

- Kirchhoff's current law is the key to apply node analysis.
- **KCL**: *Algebraic sum of currents at any node in a circuit is zero.*
- Sign conventions are important while writing **KCL** equations.
- Identify the nodes and reference node in the circuit.
- At any node, assume the incoming branch currents are negative and outgoing branch currents are positive.
- Write KCL for each node and express the currents as function of node voltages and the resistances (I=V/R form).
- Solve the simultaneous equations to find node voltages.

Node Analysis – Inspection Method

- Assume current flow in all nodes in to the node.
- Form a square matrix, whose size is equal to no. of nodes in the circuit.
- The diagonal elements is the sum of inverse of all resistances in the loop.
- The off-diagonal elements are the sum inverse of resistances common to the neighboring node.
- If no common resistances are present, then corresponding element is zero.

$G_{11} = \frac{1}{R_{11}}$

$G_{12} = \frac{1}{R_{12}}$

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ -G_{31} & -G_{31} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

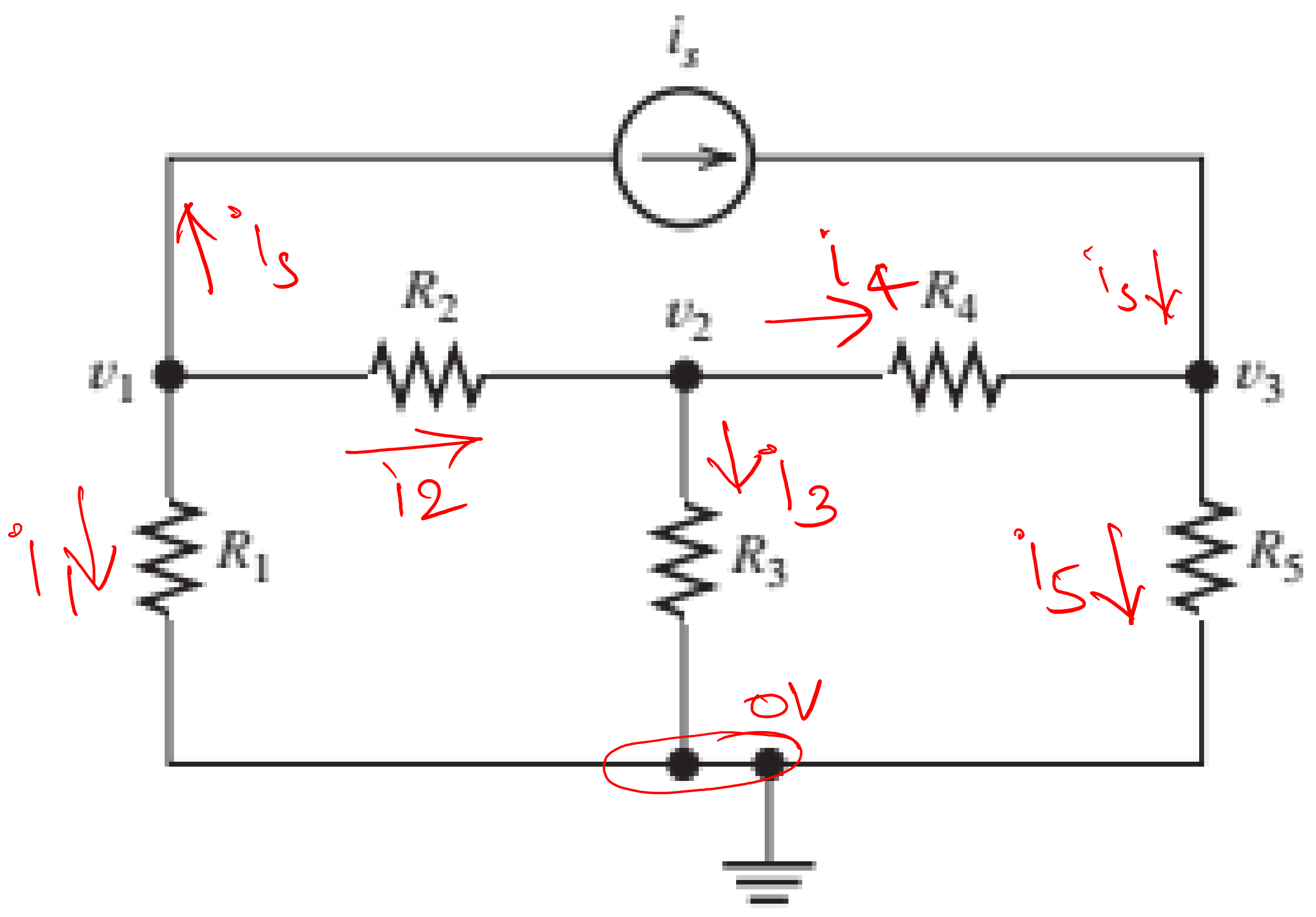
$V = IR$

$I = \frac{V}{R} = VG$

$G = \frac{1}{R}$

- I_1, I_2.... are the branch currents and V_1, V_2, ... are the node voltages

Writing Node Equations



No. of nodes = 3

No. of KCL eqn = 3.

1 Assume the current direction

Applying KCL to node 1

$$i_1 + i_2 + i_s = 0$$

$$\frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} + i_s = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \frac{V_2}{R_2} = -i_s \quad \text{--- (1)}$$

Apply KCL at node 2:

$$-i_2 + i_3 + i_4 = 0$$

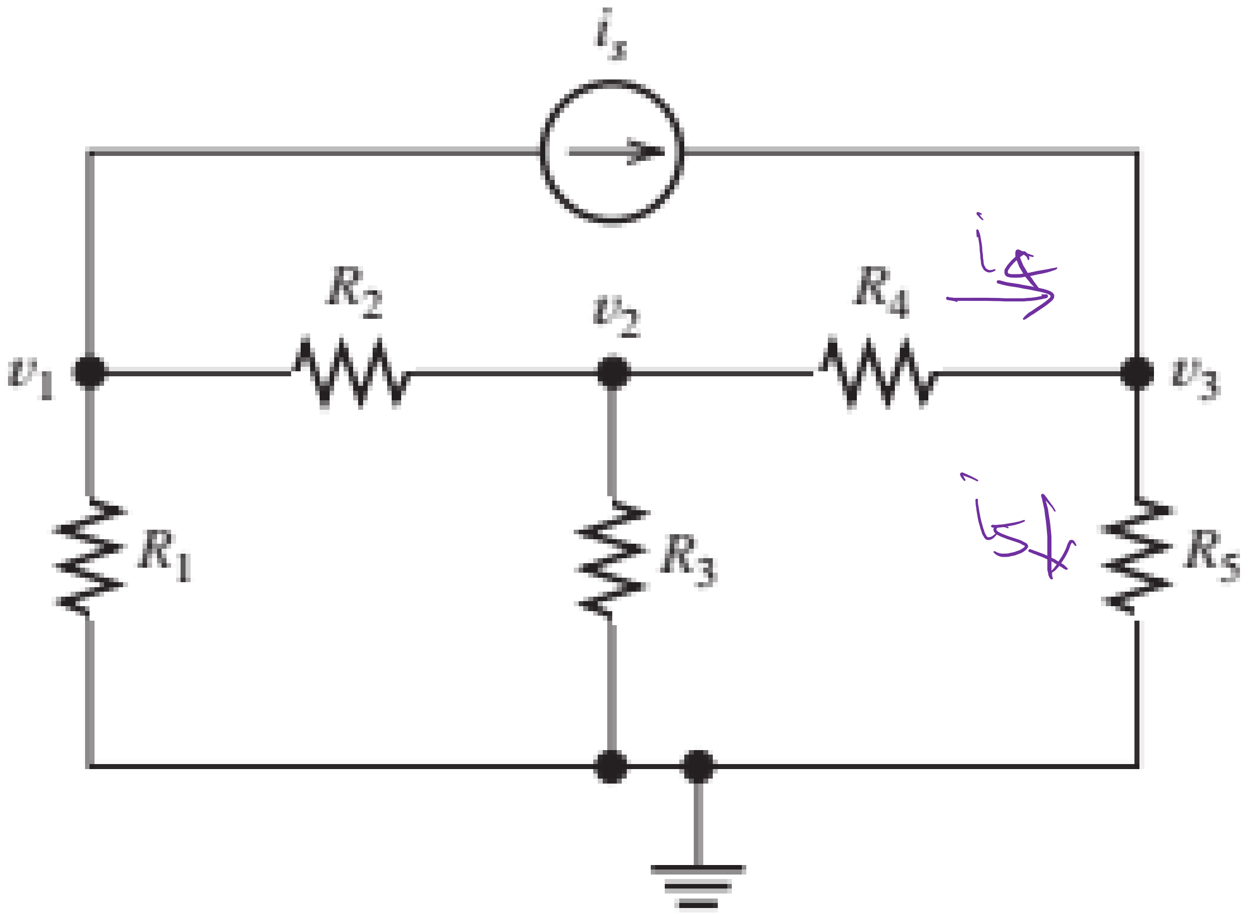
$$-\left(\frac{V_1 - V_2}{R_2}\right) + \frac{V_2}{R_3} + \frac{V_2 - V_3}{R_4} = 0$$

$$-\frac{V_1}{R_2} + V_2\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_2}\right) - \frac{V_3}{R_4} = 0 \quad \text{--- (2)}$$

Applying KCL at node 3:

$$i_s + i_4 = i_5$$

Writing Node Equations



$$i_s + i_4 = i_5$$

$$i_s = i_5 - i_4$$

$$= \frac{v_3}{R_5} - \left(\frac{V_2 - V_3}{R_4}\right)$$

$$i_s = \frac{-V_2}{R_4} + V_3\left(\frac{1}{R_4} + \frac{1}{R_5}\right) \text{ — } 3$$

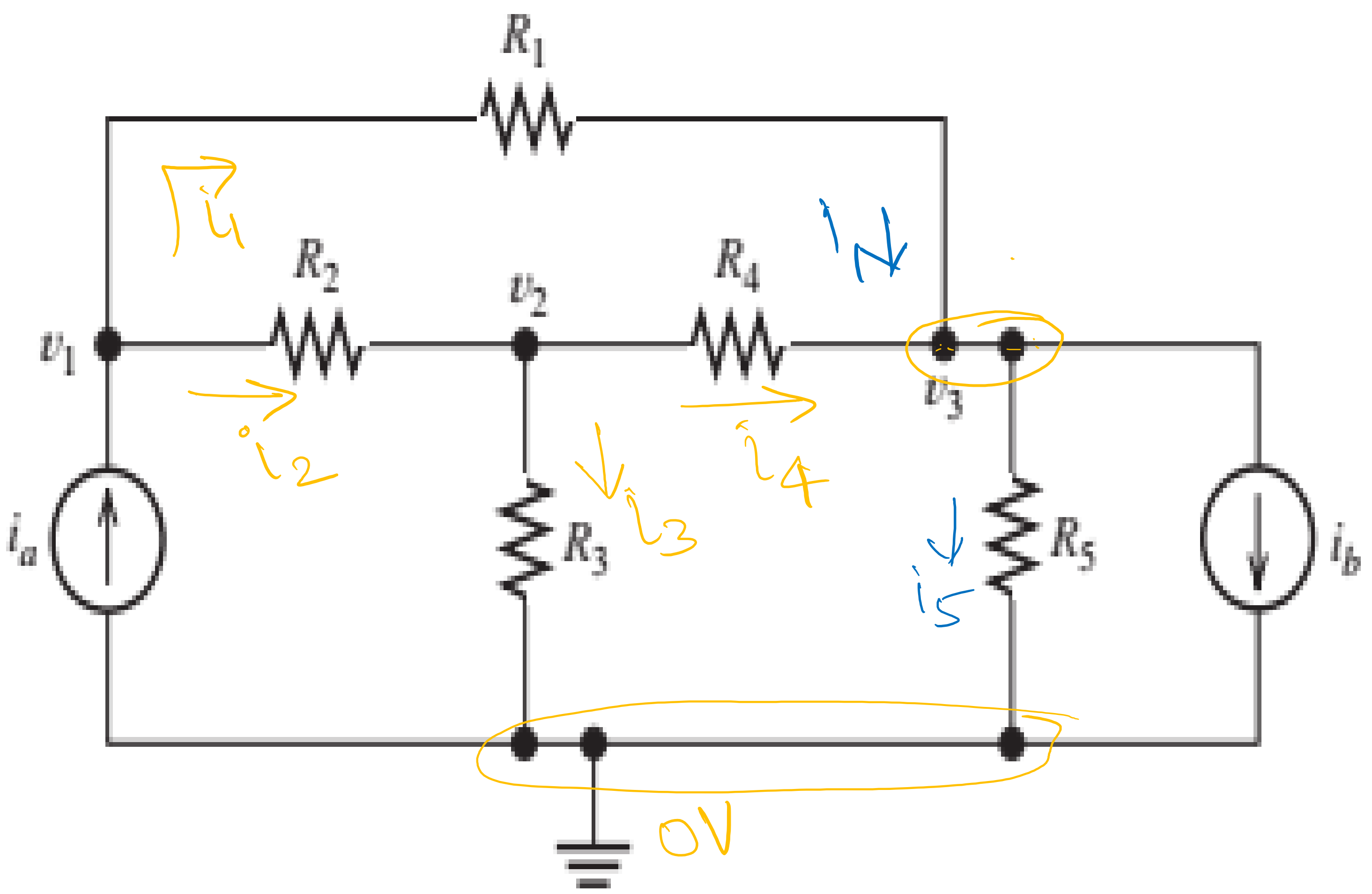
$$1 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{1}{R_2} v_2 = -i_s$$

$$2 \Rightarrow -\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_2 - \frac{1}{R_4} v_3 = 0$$

$$3 \Rightarrow -\frac{1}{R_4} v_2 + \left(\frac{1}{R_4} + \frac{1}{R_5}\right) v_3 = i_s$$

$$\begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 & 0 \\ -1/R_2 & 1/R_2 + 1/R_3 + 1/R_4 & -1/R_4 \\ 0 & -1/R_4 & 1/R_4 + 1/R_5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_s \\ 0 \\ i_s \end{bmatrix}$$

Writing Node Equations



$$i_a = \frac{v_1 - v_3}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$i_a = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_1 - \frac{1}{R_2} v_2 - \frac{1}{R_1} v_3 \quad — (1)$$

Apply KCL at node 2:

$$i_2 = i_3 + i_4$$

$$\frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4}$$

$$0 = -\frac{1}{R_2} v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_2 - \frac{1}{R_4} v_3 \quad — (2)$$

No. of nodes : 3

No. of KCL equations : 3

Applying KCL at node v_1:

$$i_a = i_1 + i_2$$

Apply KCL at node 3:

$$i_1 + i_4 = i_5 + i_b$$

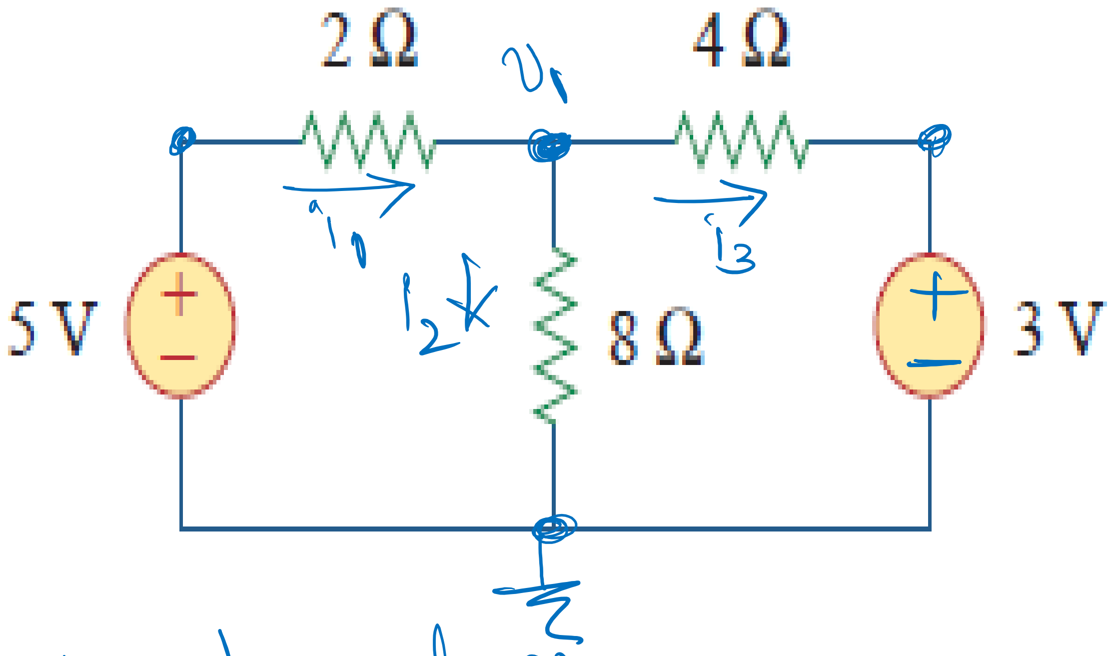
$$-i_b = i_5 - i_4 - i_1$$

$$-i_b = \frac{v_3}{R_5} - \left(\frac{v_2 - v_3}{R_4}\right) - \left(\frac{v_1 - v_3}{R_1}\right)$$

$$-i_b = -\frac{1}{R_1} v_1 - \frac{1}{R_4} v_2 + \left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_5}\right) v_3 \quad — (3)$$

Example 1:

- Solve for current through 8Ω resistor using **KCL**



KCL at node v_1

$$i_1 = i_2 + i_3$$

$$\frac{5-V_1}{2} = \frac{V_1}{8} + \frac{V_1-3}{4}$$

$$\frac{5}{2} + \frac{3}{4} = \frac{V_1}{2} + \frac{V_1}{8} + \frac{V_1}{4}$$

$$2.5+0.75 = V_1\left[\frac{4+1+2}{8}\right]$$

$$\frac{13}{4} = \frac{7V_1}{8}$$

$$\boxed{V_1 = 3.71V}$$

$$i_1 = \frac{5-3.71}{2} = 0.645A$$

$$i_2 = \frac{V_1}{8} = \frac{3.71}{8} = 0.463A$$

$$i_3 = \frac{3.71-3}{4} = 0.177A$$

To check:

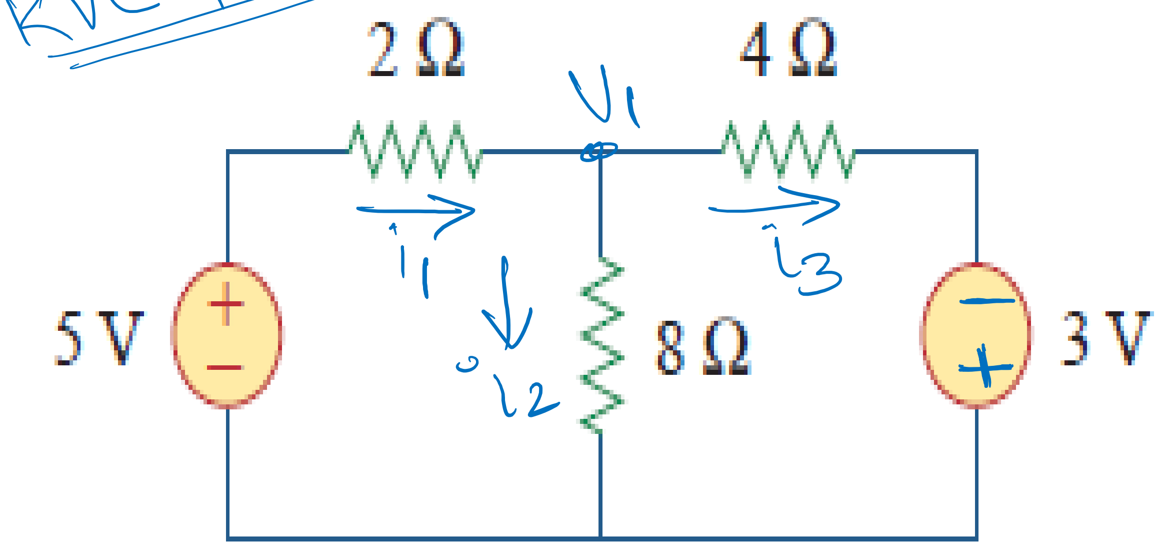
$0.463 \rightarrow i_2$

$0.177 \rightarrow i_3$

$0.640 \rightarrow i_1$

Example 1:

KVL problem



$$i_1 = i_2 + i_3$$

$$\frac{5-V_1}{2} = \frac{V_1}{8} + \frac{V_1-(-3)}{4}$$

$$\frac{5}{2} - \frac{3}{4} = V_1\left(\frac{1}{2} + \frac{1}{8} + \frac{1}{4}\right)$$

$$\frac{7}{4} = V_1\left(\frac{7}{8}\right)$$

$$\boxed{V_1 = 2V}$$

$$i_1 = \frac{5-V_1}{2} = \frac{5-2}{2} = 1.5A$$

$$i_2 = \frac{V_1}{8} = \frac{2}{8} = 0.25A$$

$$i_3 = \frac{V_1+3}{4} = \frac{5}{4} = 1.25A$$

To verify KCL:

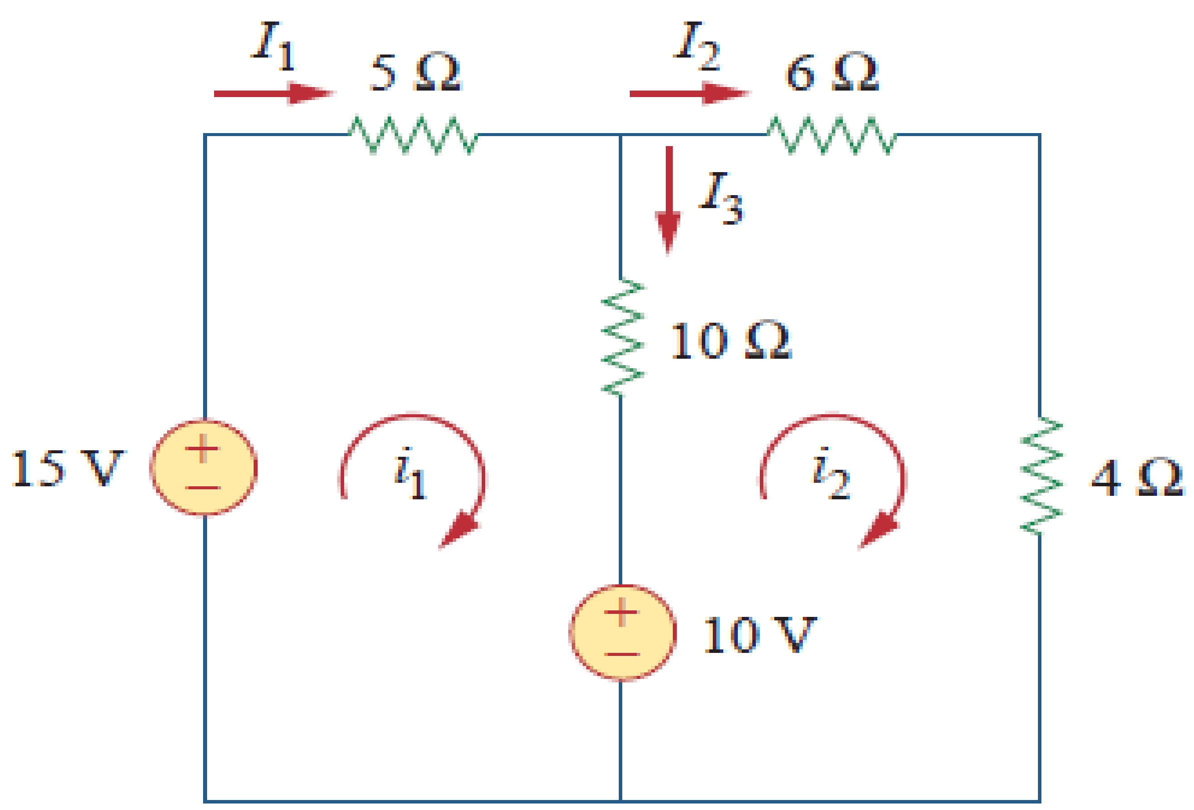
$$i_1 = i_2 + i_3$$

$$1.5 = 0.25 + 1.25$$

Hence proved: Also, the currents are identical as we solve using KVL.

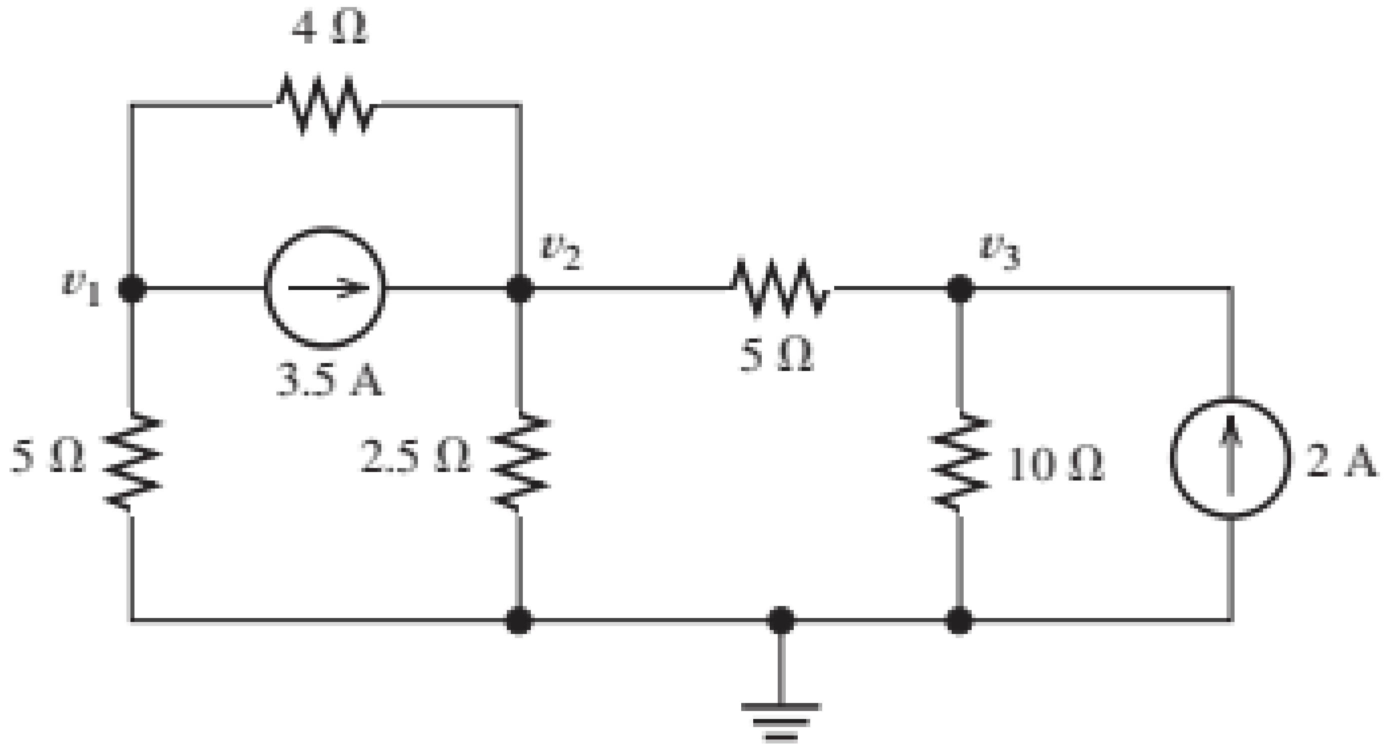
Example 2:

Find the currents using node analysis.



Example 3:

Find the voltages using node analysis.



Example 4:

Use node analysis to find i_x.