Outline

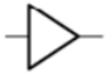
- Rules of Combinational Logic (Laws of Boolean Algebra)

- SOP (Sum of Products) and POS (Product of Sums) forms of logical expressions

- Karnaugh Maps for simplifying logical expressions

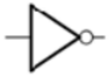
RECAP OF VARIOUS LOGIC GATES

YES



INPUT	OUTPUT
A	
0	0
1	1

NOT



INPUT	OUTPUT
A	
0	1
1	0

AND

INPUT		OUTPUT
A	B	
0	0	0
1	0	0
0	1	0
1	1	1

OR

INPUT		OUTPUT
A	B	
0	0	0
1	0	1
0	1	1
1	1	1

XOR

INPUT		OUTPUT
A	B	
0	0	0
1	0	1
0	1	1
1	1	0

NAND

INPUT		OUTPUT
A	B	
0	0	1
1	0	1
0	1	1
1	1	0

NOR

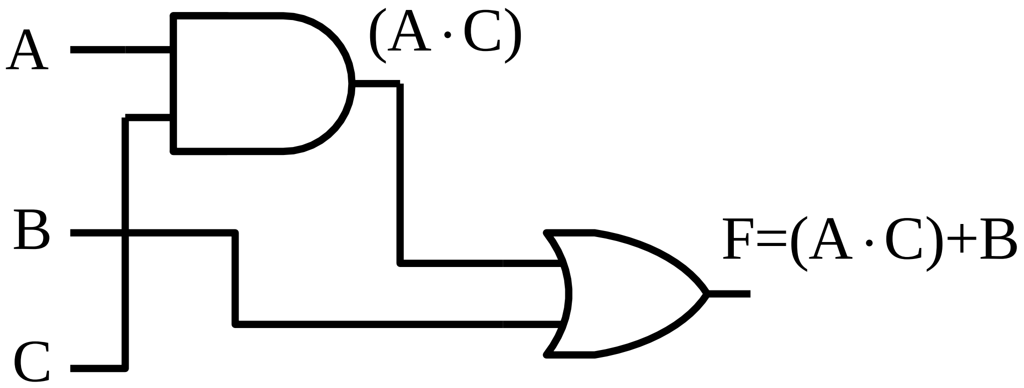
INPUT		OUTPUT
A	B	
0	0	1
1	0	0
0	1	0
1	1	0

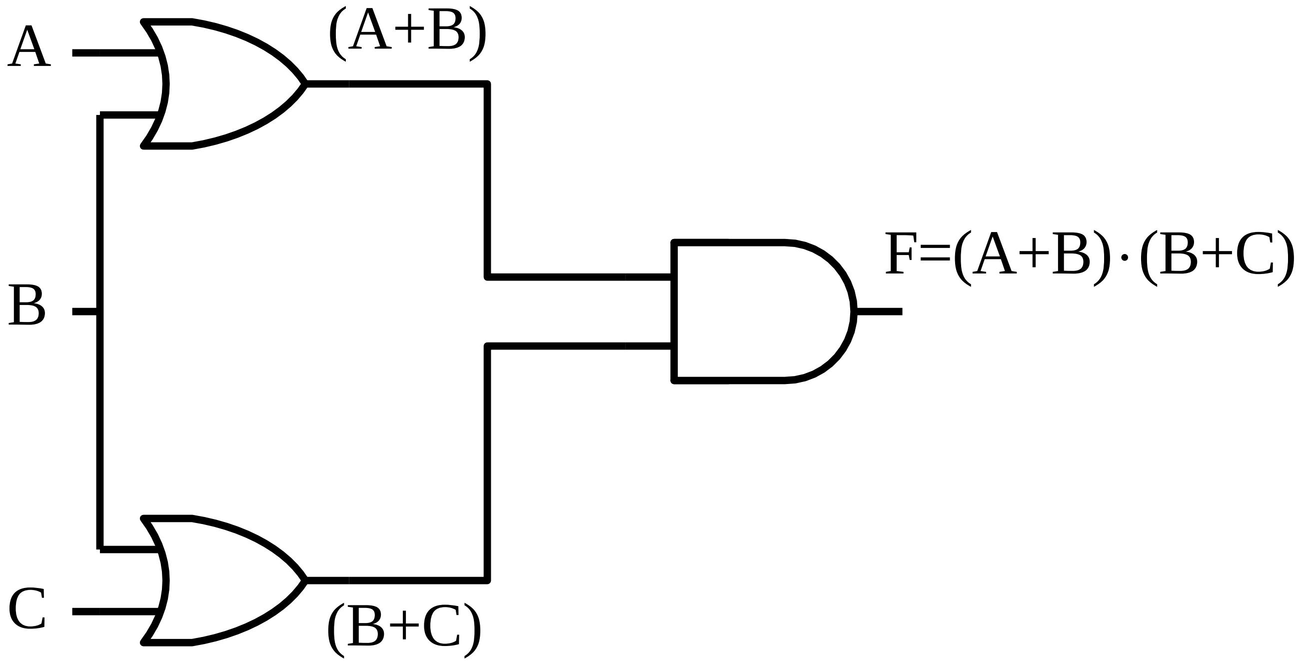
XNOR

INPUT		OUTPUT
A	B	
0	0	1
1	0	0
0	1	0
1	1	1

Logic Networks

Logic gates can be interconnected to give a wide variety of functions.

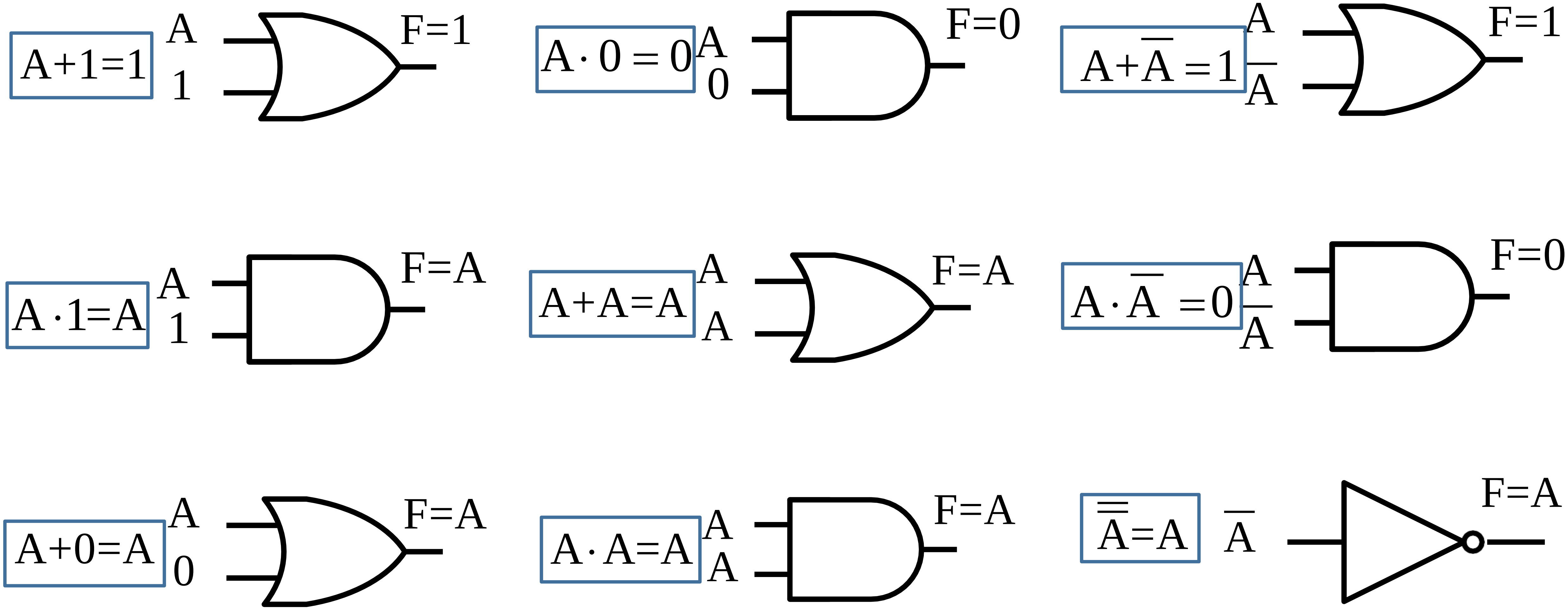




logic function

- ✓ the assignment of "0" or "1" to each possible combination of its input variables

BOOLEAN IDENTITIES



RULES OF COMBINATIONAL LOGIC

Laws of Boolean Algebra ('+' denotes 'OR', '**AB**' denotes '**A** AND **B**')

a) Commutative rules: $A+B = B+A$; $AB = BA$

b) Associative rules: $A+(B+C) = (A+B)+C$; $A(BC) = (AB)C$

c) Distributive rules: $A+BC = (A+B)(A+C)$; $A(B+C) = AB+AC$

$A+AB = A$; $A(A+B) = A$

d) de Morgan's laws: $\overline{A \cdot B \cdot C} = \overline{A} + \overline{B} + \overline{C}$; $\overline{A} \cdot \overline{B} \cdot \overline{C} = \overline{A+B+C}$

RULES OF COMBINATIONAL LOGIC

Verification of distributive rule A+BC = (A+B)(A+C) via truth table

A	B	C	BC	A+BC	A+B	A+C	(A+B)(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Laws of Boolean Algebra used for simplifying logical expressions

($A^l = \bar{A}$ denotes the complement/inverse/NOT of A)

$$A + 0 = A$$

$$A + 1 = A$$

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A + A = A$$

$$A + A^l = 1$$

$$A \cdot A = A$$

$$A \cdot A^l = 0$$

$$(A^l)^l = A$$

$$A + AB = A$$

$$A + A^l B = A + B$$

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

$$(A + B)(A + C) = A + BC$$

SUM OF PRODUCTS (SOP) FORM

- Two or more "product" (AND-gated) terms are "summed" (OR-gated) to form a Boolean expression.

- Examples: $AB + ABC$, $ABC + CDE + BCD$, $AB + BCD + E$, etc.

- In an SOP expression, a single overbar (denoting inversion) cannot extend over more than one variable e.g. there can be terms like ABC, but terms like $\overline{ABC}$ are not allowed.

- In a "standard" SOP expression, each product term contains all the input variables e.g. for 3 inputs, $AB + ABC$ is not in standard form, whereas $ABC + ABC$ is in standard form.

PRODUCT OF SUMS (POS) FORM

- Two or more "sum" (OR-gated) terms are taken "product" (AND-gated) to form a Boolean expression.

- Examples: $(A+B)(A+B+C)$, $(A+B+C)(B+C+D)(C+D)E$, etc.

- In an POS expression, a single overbar (denoting inversion) cannot extend over more than one variable e.g. there can be terms like $A+B+C$, but terms like $A+B+C$ are not allowed.

- In a "standard" POS expression, each product term contains all the input variables e.g. for 3 inputs, $(A+B)(A+B+C)$ is not in standard form, whereas $(A+B+C)(A+B+C)$ is in standard form.

Solved Examples

Q. Convert the following Boolean expression into standard SOP form: $A\overline{B}C + \overline{A}\overline{B} + AB\overline{C}D$

The domain of this SOP expression A, B, C, D. Take one term at a time.

The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $(D+\overline{D})$ as follows:

$$A\overline{B}C = A\overline{B}C(D+\overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

The second term, $\overline{A}\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $(C+\overline{C})$ as follows:

$$\overline{A}\overline{B} = \overline{A}\overline{B}(C+\overline{C}) = \overline{A}\overline{B}C + \overline{A}\overline{B}\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $(D+\overline{D})$ as follows:

$$\overline{A}\overline{B}C(D+\overline{D}) + \overline{A}\overline{B}\overline{C}(D+\overline{D}) = \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}$$

The third term, $AB\overline{C}D$, is already in standard form.

The complete standard SOP form of the original expression is as follows:

$$ABC + AB + ABCD = ABC(D+D) = ABCD + ABCD + ABCD + ABCD + ABCD + ABCD + ABCD$$

SOLVED EXAMPLES

Q. Convert the following Boolean expression into standard POS form: $\left(\overline{A}+B+C\right)\left(\overline{B}+C+\overline{D}\right)\left(A+\overline{B}+\overline{C}+D\right)$

The domain of this POS expression A, B, C, D. Take one term at a time.

The first term, $\overline{A}+B+C$, is missing variable D or $\overline{\text{D}}$, so add $\text{D}\overline{\text{D}}$ as follows:

$$\overline{A}+B+C=\overline{A}+B+C+D\overline{D}=\left(\overline{A}+B+C+D\right)\left(\overline{A}+B+C+\overline{D}\right)$$

Note, $A+BC=\left(A+B\right)\left(A+C\right)$

The second term, $\overline{B}+C+\overline{D}$, is missing variables A or $\overline{\text{A}}$ and D or $\overline{\text{D}}$, so $\text{A}\overline{\text{A}}$ as follows:

$$\overline{B}+C+\overline{D}=\overline{B}+C+\overline{D}+A\overline{A}=\left(\overline{B}+C+\overline{D}+A\right)\left(\overline{B}+C+\overline{D}+\overline{A}\right)=\left(A+\overline{B}+C+\overline{D}\right)\left(\overline{A}+\overline{B}+C+\overline{D}\right)$$

The third term, $A+\overline{B}+\overline{C}+D$, is already in standard form.

The complete standard POS form of the original expression is as follows:

$$\left(\overline{A}+B+C\right)\left(\overline{B}+C+\overline{D}\right)\left(A+\overline{B}+\overline{C}+D\right)=$$

$$\left(\overline{A}+B+C+D\right)\left(\overline{A}+B+C+\overline{D}\right)\left(A+\overline{B}+C+\overline{D}\right)\left(\overline{A}+\overline{B}+C+\overline{D}\right)\left(A+\overline{B}+\overline{C}+D\right)$$

EXERCISES

Q. Convert the following SOP expression into standard SOP form:

$a.\quad AB + \overline{A}BD + \overline{A}C\overline{D}$

$b.\quad \overline{A}BC + AB\overline{C}$

Q. Convert the following POS expression into standard POS form:

$\alpha\left(A + \overline{B} + C\right)\left(A + B + \overline{C}\right)$

$b.\quad A\left(A + \overline{C}\right)(A + B)$

Canonical Forms of Boolean Expressions

- The various possible standard product (AND) terms are called “minterms”, while the standard sum (OR) terms are called “maxterms”.
- N input variables (bits) can be combined to form 2^N minterms or 2^N maxterms, designated by their decimal equivalents (bits are assigned values such that each minterm yields value 1 while each maxterm yields value 0)

X	Y	Z	minterm	designation	maxterm	designation
0	0	0	XYZ	m_o	$X+Y+Z$	M_o
0	0	1	XYZ	m_1	$X+Y+Z$	M_1
0	1	0	XYZ	m_2	$X+Y+Z$	M_2
0	1	1	XYZ	m_3	$X+Y+Z$	M_3
1	0	0	XYZ	m_4	$X+Y+Z$	M_4
1	0	1	XYZ	m_5	$X+Y+Z$	M_5
1	1	0	XYZ	m_6	$X+Y+Z$	M_6
1	1	1	XYZ	m_7	$X+Y+Z$	M_7

Note that each maxterm is the complement (inverse) of the corresponding minterm

i.e. $\boldsymbol{m_i = M_i{}'}$

(from de Morgan’s laws)

Canonical Forms of Boolean Expressions

Example: We can write the expression for function F using the truth table below

X	***Y***	***Z***	***F***
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

$$F = \overline{X}\,\overline{Y}Z + X\overline{Y}\,\overline{Z} + XYZ$$

$$F = m_1 + m_4 + m_7$$

(**SOP** form using **minterms**: collect all the terms which give ***F* = 1**)

$$F = (X+Y+Z)(X+\overline{Y}+Z)(X+\overline{Y}+\overline{Z})(\overline{X}+Y+\overline{Z})(\overline{X}+\overline{Y}+Z)$$

$$F = M_0 M_2 M_3 M_5 M_6$$

(**POS** form using **maxterms**: collect all the terms which give ***F* = 0**)

Any Boolean function can be expressed as a sum of minterms (SOP form) or product of maxterms (POS form).

SOLVED EXAMPLES

Q. Express the Boolean function $F = A + \overline{B}C$ as a sum of minterms (SOP).

First term missing B and C variables,

$$A = A(B+\overline{B})(C+\overline{C}) = \left(AB + A\overline{B}\right)\left(C+\overline{C}\right) = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C}$$

Second term missing variable A,

$$\overline{B}C = \overline{B}C(A+\overline{A}) = A\overline{B}C + \overline{A}\,\overline{B}C$$

$$F = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + \overline{A}\,\overline{B}C = ABC + AB\overline{C} + A\overline{B}C + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

$$\boxed{F = m_7 + m_6 + m_5 + m_4 + m_1}$$

In short notation

$$F(A,B,C) = \sum(1,4,5,6,7)$$

$$\overline{F}(A,B,C) = \sum(0,2,3)$$

SOLVED EXAMPLES

Q. Express $F = XY + \overline{X}Z$ as a product of maxterms (POS form).

$$F = XY + \overline{X}Z$$

$$F = \left(XY + \overline{X}\right)\left(XY + Z\right) = \left(X + \overline{X}\right)\left(Y + \overline{X}\right)\left(X + Z\right)\left(Y + Z\right)$$

$$F = \left(Y + \overline{X}\right)\left(X + Z\right)\left(Y + Z\right) \qquad \left(\therefore X + \overline{X} = 1\right)$$

$$F = \left(\overline{X} + Y + Z\overline{Z}\right)\left(X + Y\overline{Y} + Z\right)\left(X\overline{X} + Y + Z\right) \qquad (\therefore\ X\overline{X} = 0)$$

$$F = \left(\overline{X} + Y + Z\right)\left(\overline{X} + Y + \overline{Z}\right)\left(X + Y + Z\right)\left(X + \overline{Y} + Z\right)\left(X + Y + Z\right)\left(\overline{X} + Y + Z\right) \qquad \left(\therefore X + Y\overline{Y} = (X + Y)(X + \overline{Y})\right)$$

$$F = \left(\overline{X} + Y + Z\right)\left(\overline{X} + Y + \overline{Z}\right)\left(X + Y + Z\right)\left(X + \overline{Y} + Z\right) \qquad (\therefore\ XX = X)$$

$$F = M_4 M_5 M_0 M_2$$

$$F(X, Y, Z) = \Pi(0, 2, 4, 5)$$

$$\overline{F}(X, Y, Z) = \Pi(1, 3, 6, 7)$$

EXERCISES

Q. Express the Boolean function $F = X + YZ$ as a sum of minterms.

Q. Express the Boolean function $F = X + YZ$ as a product of maxterms.