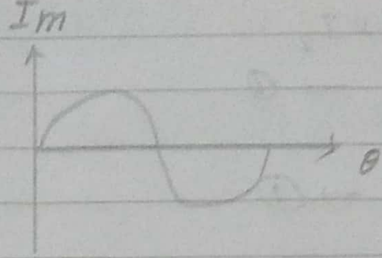
Module 2



→ $\mathcal{E}/V = E_m \sin\omega t$

$i = I_m \sin\omega t$

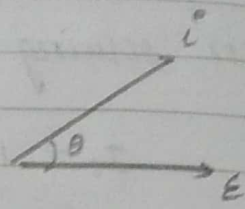
phase angle = 0

→ $\mathcal{E} = E_m \sin\omega t$

$i = I_m \sin(\omega t + \theta)$

↓

current leads voltage by an angle θ.

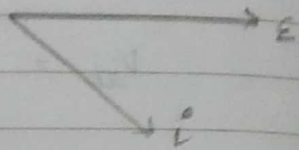


→ $\mathcal{E} = E_m \sin\omega t$

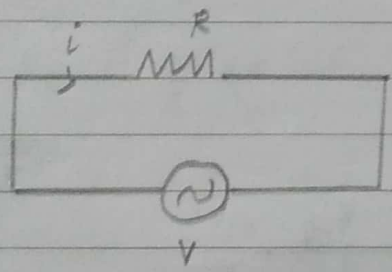
$i = I_m \sin(\omega t - \theta)$

↓

current lags voltage by an angle θ.



pure resistive

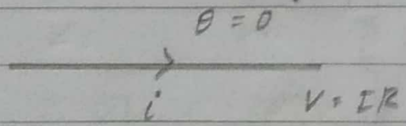


$V = V_m \sin\omega t$

$i = \frac{V}{R}$

$= \frac{V_m \sin\omega t}{R}$

$i = I_m \sin\omega t$



$$Z = \frac{V}{i} = \frac{V_m \cancel{\sin\omega t}}{I_m \cancel{\sin\omega t}} = \frac{\cancel{V_m}}{\cancel{V_m}/R} = R$$

$V_{rms} = \frac{V_m}{\sqrt{2}}$ $\qquad I_{rms} = \frac{I_m}{\sqrt{2}}$

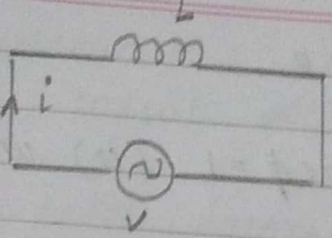
In polar form,

$V = |V| \angle 0^\circ$

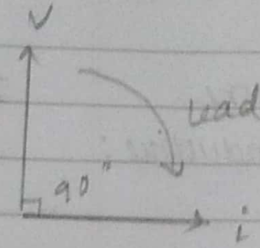
$I = |I| \angle 0^\circ$

$Z = R \angle 0^\circ = Z \angle 0^\circ$

pure inductive



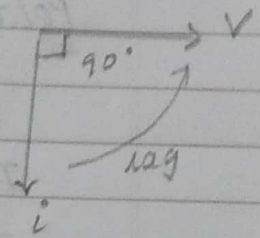
$v = V_m \sin\omega t$
$v = L\,di/dt$



$$i = \frac{1}{L}\int v\,dt$$

$$= \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$



$$i = I_m \sin(\omega t - \pi/2)$$

current lags voltage by $\pi/2$.

Pure resistive:

Instantaneous power $= v \times i$

$$= V_m \sin\omega t \; I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t$$

$$= V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

when $\theta = \omega t = \pi/2$ we get max. power

$$= V_m I_m \left(\frac{1-(-1)}{2}\right)$$

$$= V_m I_m$$

Average power $P = \frac{1}{\pi}\int_0^{\pi} \omega . d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} V_m I_m \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

phase angle $\phi = 0$

power factor $\cos\phi = 1$ (ie) unity

Pure inductive: $Z = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L = X_L$ → inductive reactance

Polar representation,

$V = |V| \angle 0^\circ$
$I = |I| \angle -\pi/2$
$Z = |Z| \angle \pi/2$
$= X_L \angle \pi/2 = jX_L$

$Z_1 = a + ib$ $\quad\quad$ $Z_1 = V_1 \angle \theta_1$
$Z_2 = c + id$ $\quad\quad$ $Z_2 = V_2 \angle \theta_2$

↓ rectangular can be + (or) − $\quad\quad$ ↓ polar can be × (or) ÷

$\frac{Z_1}{Z_2} = \frac{V_1}{V_2} \angle \theta_1 - \theta_2$

$Z_1 Z_2 = V_1 V_2 \angle \theta_1 + \theta_2$

Avg. power = 0

phase angle $\phi = 90^\circ$

Lagging power factor $\cos\phi = 0$

pure capacitive

$v = V_m \sin\omega t$

$i = I_m \cos\omega t$

current leads voltage by 90°

polar form,

$V = |V| \angle 0°$

$I = |I| \angle \pi/2$

$Z = |Z| \angle -\pi/2$

$Z = -jX_c$

Instantaneous power $= \frac{V_m I_m}{2} \sin 2\omega t$

Average power = 0

$\phi = \pi/2$ leading power factor = 0

polar form ($\times$ and $\div$)

$A\angle\theta \times B\angle\phi = AB \angle\theta+\phi$

$A\angle\theta \div B\angle\phi = \frac{A}{B} \angle\theta-\phi$

Rectangular form (+ and −)

$(A+jB) + (C+jD) = (A+C) + j(B+D)$

$(A+jB) - (C+jD) = (A-C) + j(B-D)$

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Phasor algebra:

R, X_L, X_C in rectangular form

$Z_R = R + j0$
$Z_L = 0 + jX_L$
$Z_C = 0 - jX_C$

R, X_L, X_C in polar form

$Z_R = R \angle 0°$
$Z_L = X_L \angle 90°$
$Z_C = X_C \angle -90°$

polar form: shift + pol(10,15) = r value → 18.027
θ value → 56.31

rectangular form: shift − rec(18.03, 56.31) = x = 10
y = 15

Write result in polar form:

$10 \angle 60° + 8 \angle -45°$

Polar forms cannot be added so first convert each into rectangular form, add it and convert the final answer to polar form.

$\therefore$ Ans: $11.07 \angle 15.72°$

1. $Z_1 = 10 + j10$ and $Z_2 = 20 - j30$ are in series. Find Z_T in polar form.

$$Z_T = Z_1 + Z_2$$
$$= 30 - j20$$
$$= 36.1 \angle -33.7^\circ$$

2. The total impedance of a circuit in which Z_1, Z_2 are in series is equal $(30 + j40)\,\Omega$. $Z_1 = (20 + j60)\,\Omega$. Find Z_2.

$$Z_T = Z_1 + Z_2$$
$$Z_2 = Z_T - Z_1$$
$$= 10 - j20 \text{ (rectangular form)}$$
$$= 22.36 \angle -63.43 \text{ (polar form)}$$

3. In a given circuit $I = 10 \angle 60^\circ$, $Z = 20 \angle 30^\circ$. Find V.

$$V = IZ$$
$$= (10 \times 20) \angle 60 + 30^\circ$$
$$= 200 \angle 90^\circ$$

W.K.T $Z_L = 0 + jX_L$

$x = 0$ $y = 200$ $\therefore Z_L = 0 + j200$

If we have $V = 200 \angle -90^\circ$,

$$Z_L = 0 - j200$$

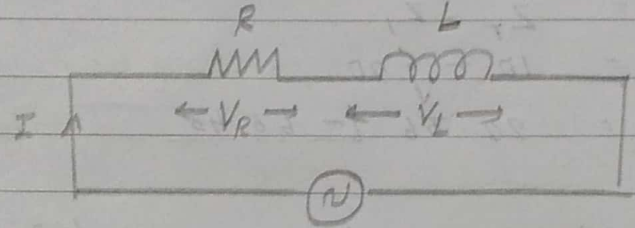
4. In a circuit $V = 200V$ $I = 10\angle 30°$. Find Z both in polar and rectangular form.

$$I = \frac{V}{Z}$$

$$Z = \frac{V}{I} = \frac{200\angle 0°}{10\angle 30°} = 20\angle -30°$$

$$= 17.32 - 10j$$

RL series circuit

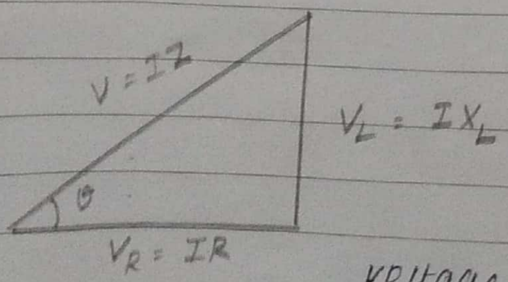


$V = V_m \sin\omega t$ - applied voltage
i = instantaneous current
f = frequency

voltage across resistor $R = V_R = IR$
voltage across inductor $L = jIX_L = V_L$

$$V = V_R + V_L$$
$$= IR + jIX_L$$
$$= I(R + jX_L)$$

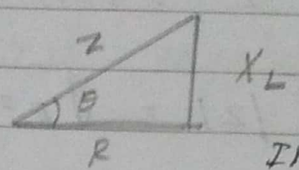
$$\frac{V}{I} = R + jX_L = Z$$



voltage triangle

$Z = R + jX_L$ $|Z| = \sqrt{R^2 + X_L^2}$

$\phi = \tan^{-1} \frac{X_L}{R}$



Impedence Δle

power factor $= \cos\phi = \frac{R}{Z}$

From the impedence Δle,

$$\cos\phi = \frac{R}{Z} \quad (or) \quad \cos\left[\tan^{-1}\left(\frac{X_L}{R}\right)\right]$$

Hence I lags the voltage, power factor is also lagging.

Active power (or) real power $= VI\cos\phi$

$$= VI\left(\frac{R}{Z}\right)$$

$$= \frac{V}{Z} IR = I^2R$$

Reactive power $= VI\sin\phi$

$$= VI \cdot \frac{X_L}{Z} = I^2 X_L$$

1. A coil having a resistance of 10Ω and an inductance of $0.01H$ is connected across a $220V$, $50H_z$ supply. calculate

a) current

b) phase angle b/w current and voltage

c) P.F

d) power

$V = 220V \quad R = 10\Omega \quad f = 50Hz$

$L = 0.01H \quad X_L = 2\pi f L = 3.141\Omega$

$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 3.14^2}$
$= 10.48\Omega$

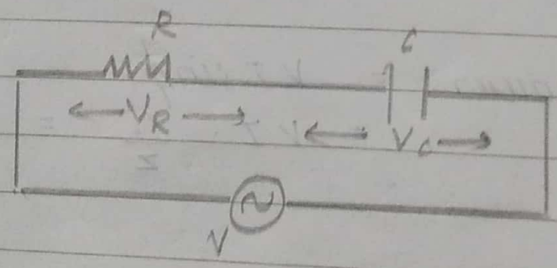
a) current $I = \frac{V}{Z} = \frac{220}{10.48} = 20.99 A$

b) $\phi = \tan^{-1}\frac{X_L}{R} = \frac{3.141}{10} = 17.43$ (lag)

c) $PF = \cos\phi = 0.954$ (lag)

d) power $= VI\cos\phi$
$= 220 \times 20.99 \times 0.954$
$= 4406.2W$

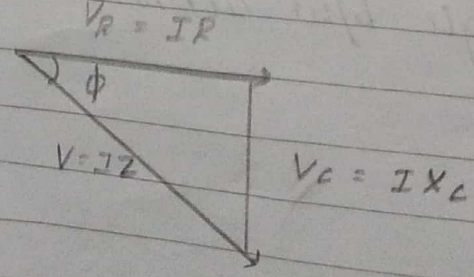
RC series circuit



$V = V_m \sin\omega t$

i = instantaneous current

f = frequency



voltage across R = $V_R = IR$ in phase with I

voltage across C = $V_C = IX_C$ lagging I by 90°

$$V = V_R + V_C$$

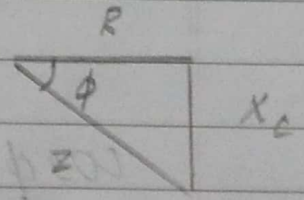
voltage triange,

$$V = IR - jIX_C = I(R - jX_C)$$

$$\frac{V}{I} = R - jX_C = Z$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{1/\omega C}{R} = \tan^{-1}\frac{1}{\omega CR}$$



ϕ = phase angle

$$Z = R - jX_C = |Z| \angle -\phi$$

$$PF = \cos\phi = R/Z$$

Active power = $VI\cos\phi$ (watts)
Reactive power = $VI\sin\phi$ (volt Amp.)

R, C

Find the circuit constants of a two element series circuit which consumes 700 W with 0.707 leading PF. The applied voltage is $V = 141.4 \sin 314t$

$V_m = 141.4$

$V_{rms} = \dfrac{141.4}{\sqrt{2}} = 99.98\,V$

$\cos\phi = 0.707$ (leading)

power $= VI\cos\phi$

$700 = 99.98 \times I \times 0.707$

$I = 9.9\,A$

$Z = \dfrac{V}{I} = \dfrac{99.98}{9.9} = 10.09\,\Omega$

$\cos\phi = \dfrac{R}{Z}$

$R = Z\cos\phi$

$= 10.09 \times 0.707$

$R = 7.13\,\Omega$

$\phi = \cos^{-1} 0.707 = 45°$

$\sin\phi = \dfrac{X_C}{Z} \Rightarrow X_C = Z\sin\phi$

$X_C = 10.09 \times \sin 45°$

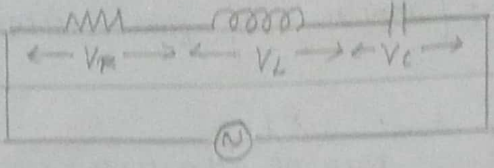
$= 10.09 \times 0.707$

$X_C = 7.13\,\Omega$

$X_C = 1/2\pi f C$

$C = \dfrac{1}{2\pi X_C} = 446.6\,\mu F$

RLC circuit



$$V = V_m \sin\omega t$$

$$V_R = IR$$
$$V_L = jIX_L = IX_L \angle 90^\circ$$
$$V_C = -jIX_C = IX_C \angle -90^\circ$$

$$V = V_R + V_L + V_C$$

$$= IR + IjX_L - IjX_C$$

$$= I(R + j(X_L - X_C))$$

$$\frac{V}{I} = Z = R + j(X_L - X_C)$$

$$= |Z| \angle \phi$$

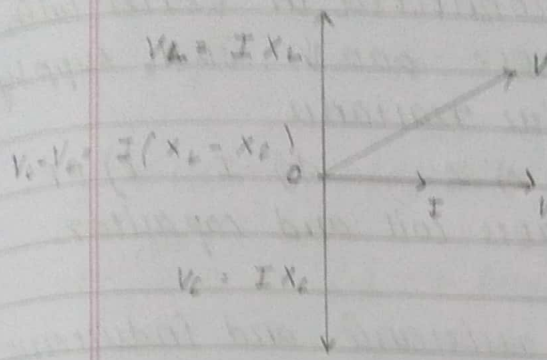
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

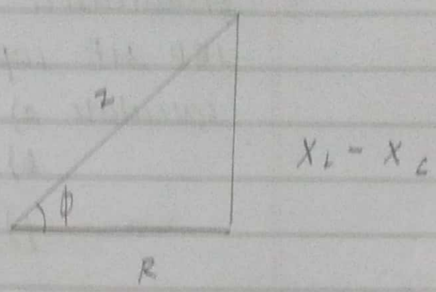
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

2 cases: If $X_L > X_C \rightarrow$ inductive circuit (R_L)

If $X_C > X_L \rightarrow$ capacitive circuit (R_C)

$X_L > X_C$

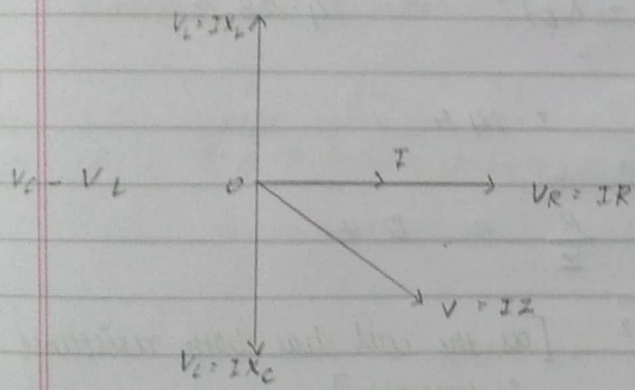


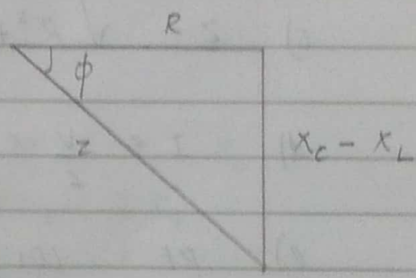


$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

$$\cos\phi = \text{power factor} = \frac{R}{Z}$$

$X_C > X_L$





$$\tan\phi = \frac{X_C - X_L}{R} = \frac{1/\omega C - \omega L}{R}$$

$$pf = \cos\phi = \frac{R}{Z}$$

Actual power = $VI\cos\phi$

Reactive power = $VI\sin\phi$

1. A coil of resistance 10 ohm and an inductor of inductance 0.1 H is connected in series with 150 μF capacitor across 200 V, 50 H_z supply calculate a) the inductive reactance
b) X_c c) Z d) I e) PF
f) voltage across coil and capacitor.

coil - contains both resistance and inductance.

a) $X_L = 2\pi f L = 314 \times 0.1 = 31.4 \Omega$

b) $X_c = \frac{1}{2\pi f c} = \frac{10^6}{314 \times 150} = \frac{10^6}{47100} = \frac{10^4}{471}$

$= 21.23 \Omega$

c) $Z = \sqrt{R^2 + (X_L - X_c)^2} = 14.28 \Omega$

d) $I = \frac{V}{Z} = \frac{200}{14.28} = 14 A$

e) $PF = \cos\phi = \frac{R}{Z} = 0.7$

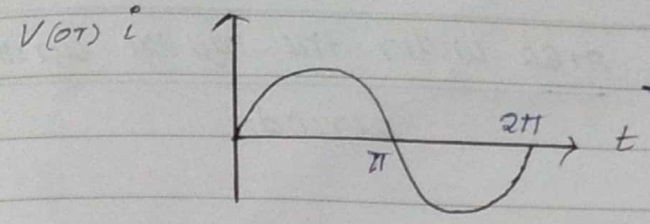
f) $Z_L = \sqrt{R^2 + X_L^2}$ [as the coil has both resistance and inductance]

$Z_L = 32.97$

f) $V_{coil} = IZ_1 = I\sqrt{R^2 + X_L^2} = 461.6 V$

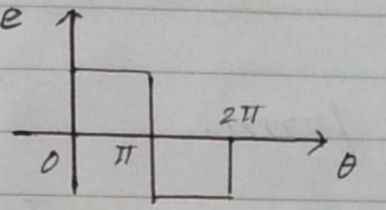
$V_{capa.} = I X_c = 297.08 V$

RMS, Average, Peak:

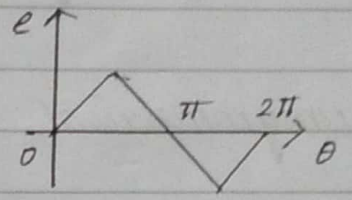


→ sinusoidal / Alternating / periodic wave form

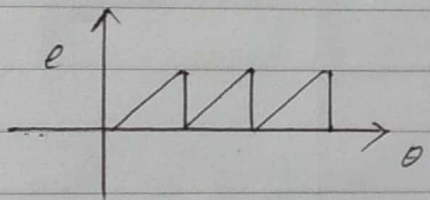
Amplitude: Max. value of the graph.
Frequency: No. of cycles per second $\left(\frac{1}{T}\right)$ (Hertz)



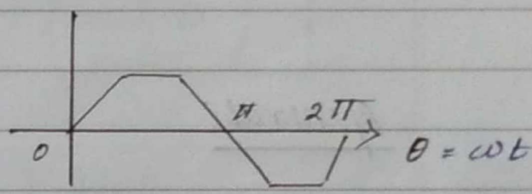
rectangular



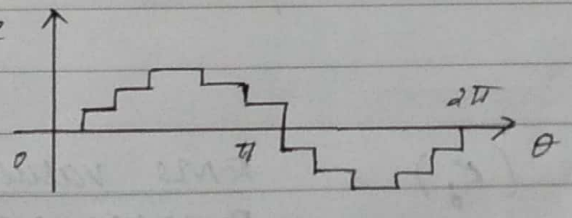
triangular



saw tooth



trapezoidal



→ step

RMS:

Symmetrical wave form: (we consider only half cycle)

$$I_{RMS} = \sqrt{\frac{1}{T/2}\int_0^{T/2} i^2\,dt} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\,d\theta}$$

Unsymmetrical wave form: (we consider full cycle)

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^{T} i^2\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\,d\theta}$$

General:

$$\text{RMS value} = \sqrt{\frac{\text{Area under the square curve for one cycle}}{\text{period}}}$$

Average:

Symmetrical wave form:

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta = \frac{1}{T/2}\int_0^{T/2} i\,d\theta$$

Unsymmetrical wave form:

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d\theta = \frac{1}{T}\int_0^{T} i\,dt$$

General:

$$\text{Ave value} = \frac{\text{Area under the curve for one cycle}}{\text{period}}$$

→ Form factor $(K_f) = \dfrac{\text{RMS value}}{\text{Average value}}$

→ Peak factor $(K_p) = \dfrac{\text{Peak value}}{\text{RMS value}}$

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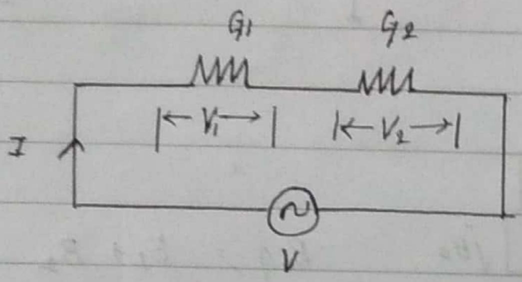
Conductance :

$$G = \frac{1}{R} = \frac{1}{V/I} = \frac{I}{V} \quad ℧ \text{ (mho/ siemens)}$$

$$G = \frac{I}{V} \Rightarrow V = \frac{I}{G}$$

Series :



$$V_1 = I/G_1$$

$$V_2 = I/G_2$$

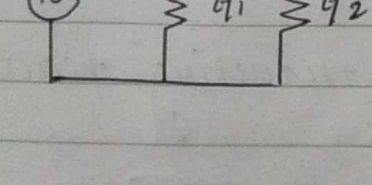
$$V_{net} = V_1 + V_2 = I\left(\frac{1}{G_1} + \frac{1}{G_2}\right)$$

$$\boxed{V = I\left(\frac{G_1 + G_2}{G_1 G_2}\right)}$$

$$I = \frac{V}{\left(\frac{G_1+G_2}{G_1G_2}\right)} = V G_{eq}$$

$$\boxed{\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2}}$$

Parallel :



$$I_1 = VG_1 \qquad I_2 = VG_2$$

$$I = I_1 + I_2$$

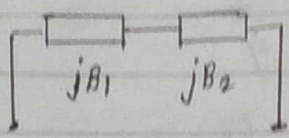
$$I = VG_1 + VG_2$$

$$\boxed{G_{eq} = G_1 + G_2}$$

Susceptance : (B)

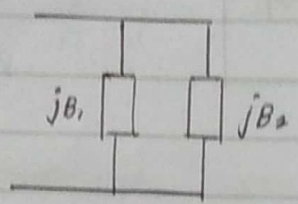
→ Reciprocal of reactance (X) is susceptance

Series:



$$B_{eq} = \frac{B_1 B_2}{B_1 + B_2}$$

Parallel:



$$B_{eq} = B_1 + B_2$$

Impedence : (Z)

Series: $Z_{eq} = Z_1 + Z_2$	combination of resistance and reactance
Parallel: $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$	$Z = \underline{R} + j\underline{X}$ $Z = \sqrt{R^2 + X^2} \tan^{-1}(X/R)$

Admittance : (Y (or) $\bar{Y}$)

→ Reciprocal of impedence $\left(Y = \frac{1}{Z}\right)$

$$Y = G + jB$$

Inductive susceptance ↓ $B_L = \frac{1}{\omega L} = \frac{1}{jX_L}$	Inductive reactance ↓ $X_L = L\omega$
Capacitive susceptance ↓ $B_C = C\omega$	Capacitive reactance ↓ $X_C = \frac{1}{C\omega}$

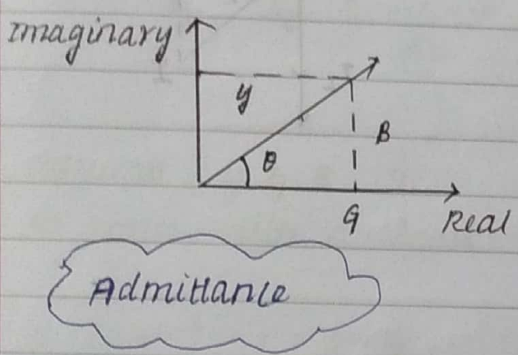
series: $y_{eq} = \frac{y_1 y_2}{y_1 + y_2}$

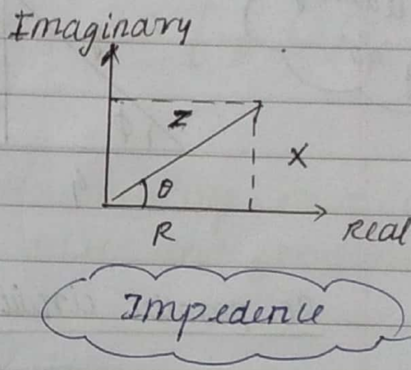
parallel: $y_{eq} = y_1 + y_2$

If circuit is pure L, $B = -B_L$
If circuit is pure C, $B = B_C$
combination of L and C, $B = B_C - B_L$

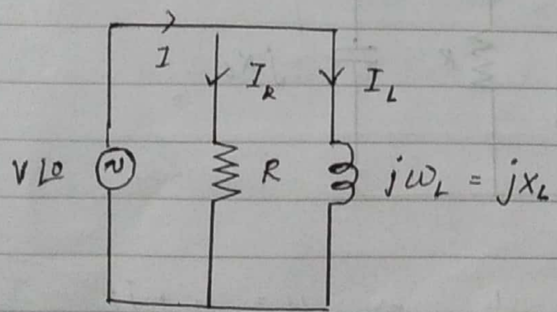
$$y = G + jB = \sqrt{G^2+B^2} \angle \tan^{-1} B/G$$
$$= |y| \angle \theta$$

Graph:





RL Parallel circuit:



circuit
- parallel → admittance
- series → impedence

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} \Rightarrow y = G - jB_L$$
$$= \sqrt{G^2 + B_L^2} \angle \tan^{-1} B_L/G$$
$$= |y| \angle -\phi$$

$$I = \frac{V}{Z} = Vy = V\angle 0 \times y \angle -\phi = I \angle -\phi$$

$$I_R = \frac{V}{Z} = \frac{V}{R} = VG = I_R \angle 0^\circ$$

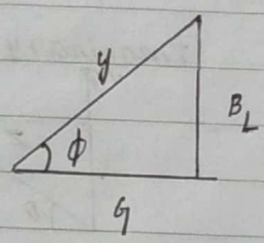
$$I_L = \frac{V}{jX_L} = -jVB_L = VB_L \angle -90^\circ = I_L \angle -90^\circ$$

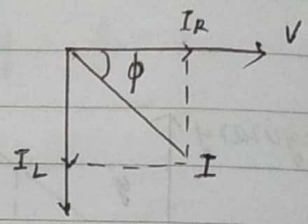
$$V = V_R = V_L = I_R \cdot R = I_L \, jX_L$$

$$I = I_R + I_L = \frac{V}{R} + \frac{V}{jX_L} = Vy$$

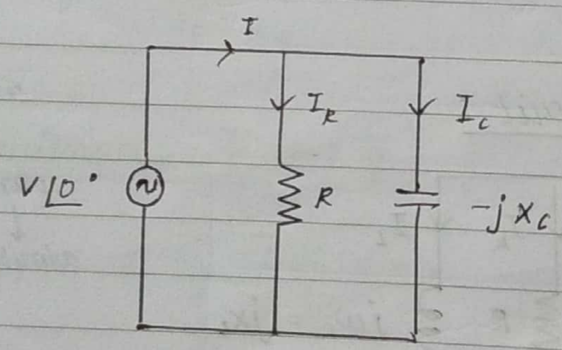
where $y = \frac{1}{R} + \frac{1}{jX_L}$

I_L leading by 90°





RC parallel circuit:



$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{-jX_c} \Rightarrow y = G + jB_c$$

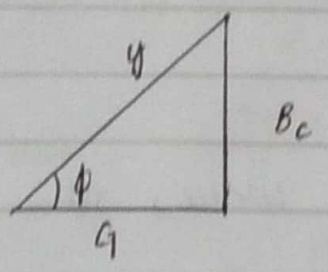
$$y = \sqrt{G^2 + B_c^2} \angle \tan^{-1} B_c/G$$

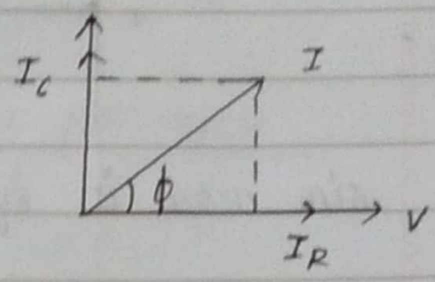
$$y = |y| \angle \phi$$

$$I = \frac{V}{Z} = V\angle 0^\circ \, y \angle \phi = I \angle \phi$$

$I_R = \frac{V}{Z} = \frac{V}{R} = VG = V\angle 0^\circ \, G\angle 0^\circ = I_R \angle 0^\circ$

$I_C = \frac{V}{-jX_C} = jVB_C = V\angle 0^\circ \, B_C \angle 90^\circ = I_C \angle 90^\circ$





$V = V_R = V_C = I_R R = I_C \, jX_C$

$I = I_R + I_C = \frac{V}{R} + VjB_C = V\left[\frac{1}{R} + jB_C\right]$

1. Calculate y, G, B in a circuit consisting 10Ω connected in series with an inductance 0.1 H and frequency = 50 Hz

$R = 10\Omega \qquad L = 0.1H \qquad f = 50 Hz$

In a RL series circuit, $Z = R + jX_L$

$\therefore X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.1 = 31.4$

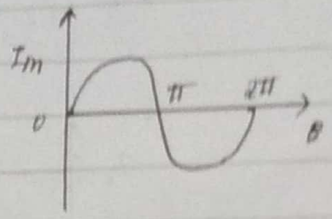
$Z = 10 + j31.4$

$y = \frac{1}{Z} = \frac{1}{10 + j31.4}$

convert this to polar form befor. only in that form we can $\times$ and $\div$

$\theta - x$ axis

2. Find RMS value, Average value, form factor, peak factor.

a) 

sine wave is symmetrical wave, so

Avg. value: $i = I_m \sin\theta$

$$I_{ave} = \frac{\text{Area under one half cycle}}{\text{period}}$$

$$= \frac{1}{\pi}\int_0^{\pi} i \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{2I_m}{\pi} = 0.637\, I_m$$

RMS value: $$I_{rms} = \sqrt{\frac{\text{Area under the squared curve of 1 half cycle}}{\text{period}}}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta}$$

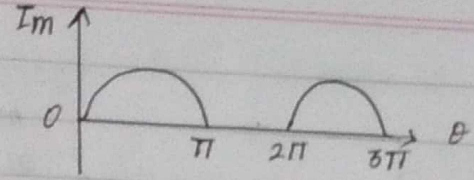
$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta}$$

$$= \frac{I_m}{\sqrt{2}}$$

Form factor: $$FF = \frac{RMS}{Average} = \frac{0.707\, I_m}{0.637\, I_m} = 1.11$$

Peak factor: $$PF = \frac{\text{peak value}}{\text{RMS value}} = \frac{I_m}{0.707\, I_m} = 1.414$$

b)



This is an unsymmetrical wave, so

vg. value: $$I_{ave} = \frac{\text{Area under one full cycle}}{\text{period}}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} i\, d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i\, d\theta + \underbrace{\int_{\pi}^{2\pi} i\, d\theta}_{0}\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}$$

MS value: $$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta}$$

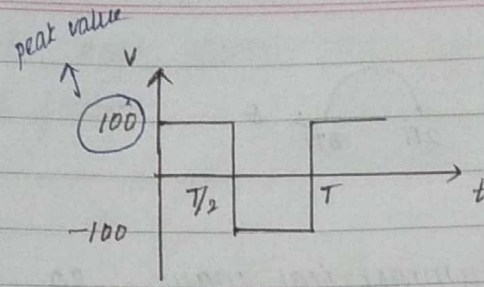
$$= \frac{I_m}{2}$$

orm factor: $$FF = \frac{RMS}{Average} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} =$$

eak Factor: $$PF = \frac{\text{Peak value}}{\text{RMS value}} = \frac{I_m}{I_m/2} = 2$$

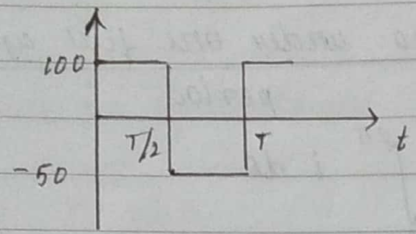
t - x axis

c)



→ magnitude of as amplitude is same (100), it is symmetrical.

d)



→ unsymmetrical

<u>c. Ans:</u> Area of rectangle $= 100 \times T/2$

period $= T/2$ [as we consider only the half cycle due to symmetry]

$$I_{av} = \frac{100\,T/2}{T/2} = 100$$

$$I_{Rms} = \sqrt{\frac{\text{Area under squared curve}}{\text{period}}}$$

$$= \sqrt{\frac{100^2 \times T/2}{T/2}}$$

$$= 100$$

FF = 1 and PF = 1

<u>d. Ans:</u> Area of rectange $= 100 \times T/2 + (-50) \times T/2$

$= 25\,T$

period = T [as we consider full cycle due to unsymmetry]

$$I_{av} = \frac{25\,T}{T} = 25$$

$$I_{RMS} = \sqrt{\frac{\text{Area under squared curve}}{\text{period}}}$$

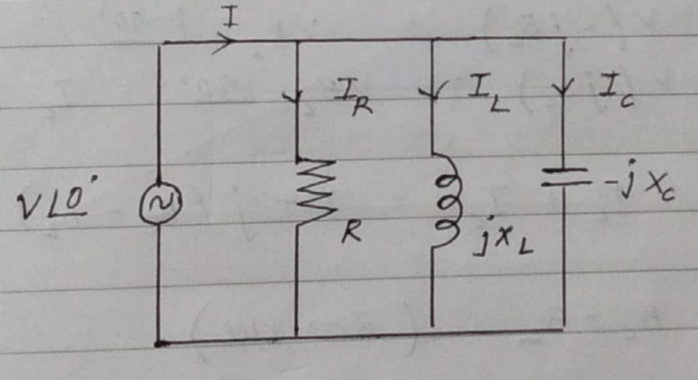
$$= \sqrt{\frac{(100)^2 \times T/2 + (-50)^2 \times T/2}{T}}$$

$$= \sqrt{5000 + 1250} = 79.057$$

$$FF = \frac{RMS}{\text{average}} = \frac{79.057}{25} = 3.16$$

$$\text{Peak } F = \frac{\text{Peak}}{RMS} = \frac{100}{79.057} = 1.26$$

RLC parallel circuit:



$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$y = G - jB_L + jB_C = G + j(B_C - B_L)$$

$$y = G \pm jB$$

B = +ve (capacitive)

B = −ve (inductive)

B = 0 (resistive)

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$I = Vy$

$I_R = VG$

$I_L = V(-jB_L) \qquad I_c = V(jB_c)$

$V = I_R R = I_L\, j\omega_L = I_c\, j\omega_c$

$I_T = I_R + I_L + I_c = \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{jX_c}$

$$I_T = V\left[\frac{1}{R} + \frac{1}{j\omega_L} - \frac{1}{j\omega_c}\right]$$

$y = y\angle\phi \Rightarrow I = Vy = V\angle 0 \times y\angle\phi$

$= I\angle\phi$

$I_R = VG = I_R\angle 0^\circ$

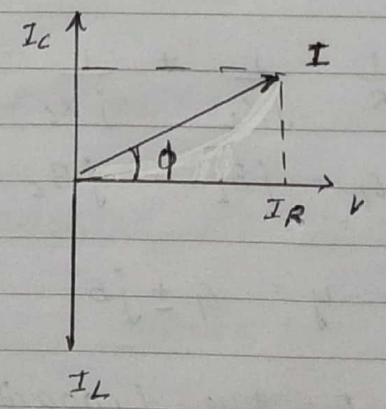
$I_L = V(-jB_L) = VB_L\angle -90^\circ = I_L\angle -90^\circ$

$I_c = V(jB_c) = VB_c\angle 90^\circ = I_c\angle 90^\circ$

$I_B = I_L + I_c = V + j(B_c - B_L) = I_B\angle 90^\circ$

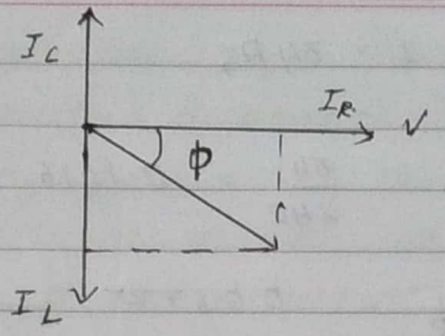
case 1: $B = B_c - B_L \quad (B \to +ve)$

$I_B = I_B\angle 90^\circ$

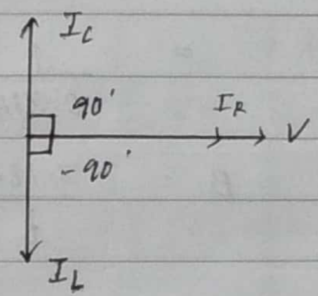


case 2: $B = B_L - B_c \quad (B \to -ve)$

$I_B = I_B\angle -90^\circ$



case 3 : $B = 0$ $\quad I_B = 0$



1. When a 240 V, 50 Hz supply is applied to a resistor of 15Ω is connected in parallel with an inductor of total current 22.1 A. What value must the frequency have for the total current to be 34 A.

$$G = \frac{1}{R} = \frac{1}{15} = 0.067$$

$$B = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L} = \frac{0.00318}{L}$$

$$Y = G - jB$$

$$Y = 0.067 - \frac{0.00318}{L}$$

$$Y = \frac{I}{V} = \frac{22.1}{240} = 0.09$$

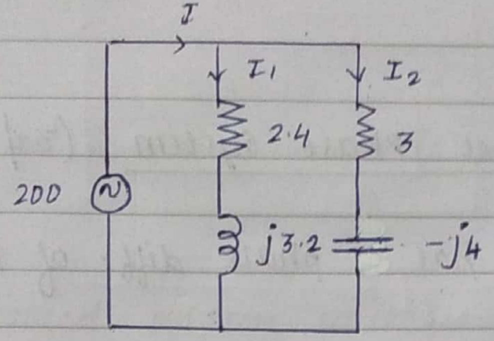
$$\therefore 0.09 = 0.067 - \frac{0.00318}{L}$$

$$L = 0.055$$

this method always won't give correct answer so use universal method.

2. An Inductance $z_1 = (2.4 + j3.2)\,\Omega$ is in parallel with another inductance $z_2 = (3 - j4)\,\Omega$. $V = 200\,V$

Calculate current in each branch, impedence, reactive power, total current, apparent power and power factor.



$$I_1 = \frac{V}{Z_1} = \frac{200\angle 0^\circ}{(2.4 + j3.2)} = \frac{200\angle 0^\circ}{4\angle 53.13^\circ} = 50\angle -53.13^\circ$$

$$I_2 = \frac{V}{Z_2} = \frac{200\angle 0^\circ}{(3 - j4)} = \frac{200\angle 0^\circ}{5\angle -53.13^\circ} = 40\angle 53.13^\circ$$

$$I = I_1 + I_2 = 50\angle -53.13^\circ + 40\angle 53.13^\circ$$
$$= 30 - j40 + 24 + j32$$
$$= 54 - j8$$
$$I = 54.59\angle -8.43^\circ$$

$$Z = \frac{V}{I} = \frac{200\angle 0^\circ}{54.59\angle -8.43^\circ} = 3.66\angle 8.43^\circ = 3.62 + j0.54$$

$*$ when we compare voltage and current eqn,

$\phi = 8.43$ (lagging)

Power factor $= \cos\phi = \cos 8.43 = 0.99$

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True / Real power = $VI\cos\phi$ = 10.761 kW

Reactive power = $VI\sin\phi$ = 1.63 KVAR

Total power / Apparent power = VI

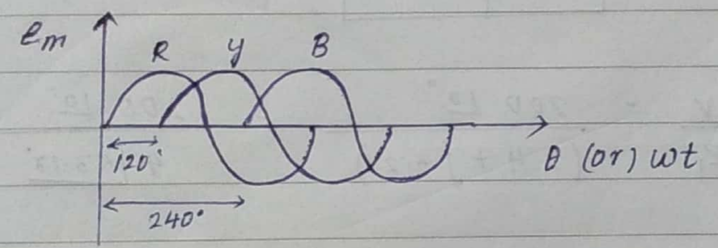
= $10980 \angle -8.43°$

= $(10861.37)^2 + (1609.7)^2$

= 10.98 kW

Three Phase system (3ϕ)

Each phase has a phase diff. of 120°



These 3 phases are interconnected to

→ reduce the no. of conductors and spaces.

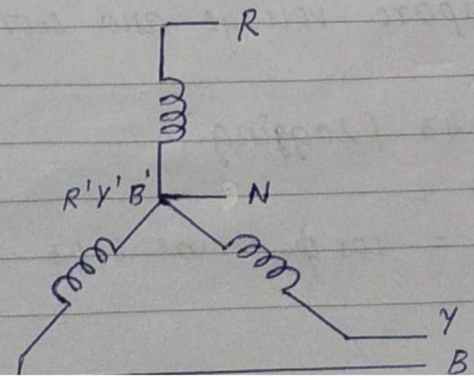
→ reduce the complexity of circuit.

connections: 1. Star (or) Y - connection

2. Delta (or) Mesh connection

Star (or) Y

Each coil is known as phase winding



3 phase, 4 line

Phase voltage: (E_ϕ) voltage across each coil, is called phase voltage (or) voltage b/w a line and neutral.

E_p (or) E_ϕ ← symbol

E_{RN}, E_{YN}, E_{BN} are the three phase voltages.

Balanced system: 3 loads are same, – z, z, z

$z = a \pm jb$

Unbalanced system: 3 loads are unequal → z_1, z_2, z_3

In balanced system, voltages are equal in magnitude, and displaced by 120°.

$$|E_{RN}| = |E_{YN}| = |E_{BN}| = E_\phi \rightarrow (1)$$

Take E_{RN} as referrence voltage

For RYB, $E_{RN} = E_\phi \angle 0°$

$E_{YN} = E_\phi \angle -120°$

$E_{BN} = E_\phi \angle 120°$ (or) $E_\phi \angle -240°$

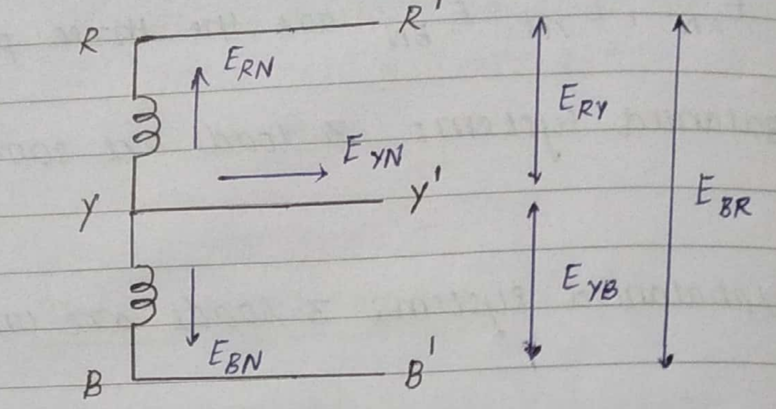
For RBY, $E_{RN} = E_\phi \angle 0°$

$E_{BN} = E_\phi \angle -120°$

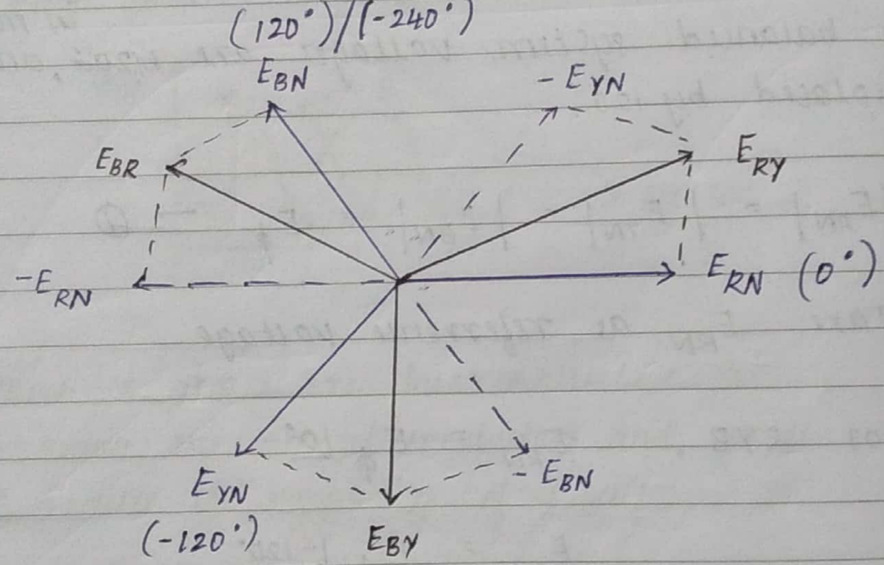
$E_{YN} = E_\phi \angle -240°$

Line voltage: Voltage between 2 lines.
(E_L)

E_{RY}, E_{YB}, E_{BR} → three line voltages



RYB →



In star connection, $E_\phi \neq E_L$

From diag, $E_{RY} = E_{RN} - E_{YN}$

$$= E_\phi \angle 0^\circ - E_\phi \angle -120^\circ$$

$$= E_\phi (1 \angle 0^\circ) - E_\phi (1 \angle -120^\circ)$$

$$= E_\phi (1 + j0) - E_\phi (-0.5 - j0.866)$$

$$= E_\phi (1.55 + j0.866) = 1.753\, E_\phi \angle 30^\circ$$

$$= \sqrt{3} E_\phi \angle 30^\circ$$

$$E_{YB} = E_{YN} - E_{BN}$$

$$= E_\phi \angle -120^\circ - E_\phi \angle -240^\circ$$

$$= E_\phi(-0.5 - j0.866) - E_\phi(-0.5 + j0.866)$$

$$= \sqrt{3} E_\phi \angle -90^\circ$$

$$E_{BR} = E_{BN} - E_{RN}$$

$$= E_\phi \angle 120^\circ - E_\phi \angle 0^\circ$$

$$= \sqrt{3} E_\phi \angle 150^\circ$$

$$E_L = |E_{RY}| = |E_{YB}| = |E_{BR}| = \sqrt{3} E_\phi$$

$$E_{RY} + E_{YB} + E_{BR} = 0$$

$$E_{RN} + E_{YN} + E_{BN} = 0$$

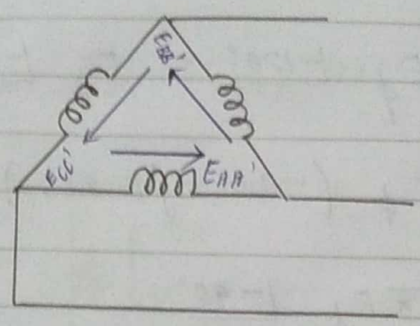
* The phasor sum of line and phase voltages are zero.

Relation b/w E_L and E_ϕ: $E_L = \sqrt{3} E_\phi$

* Relation b/w line current (I_L) and phase current (I_ϕ):

$$I_L = I_\phi$$

Delta connection

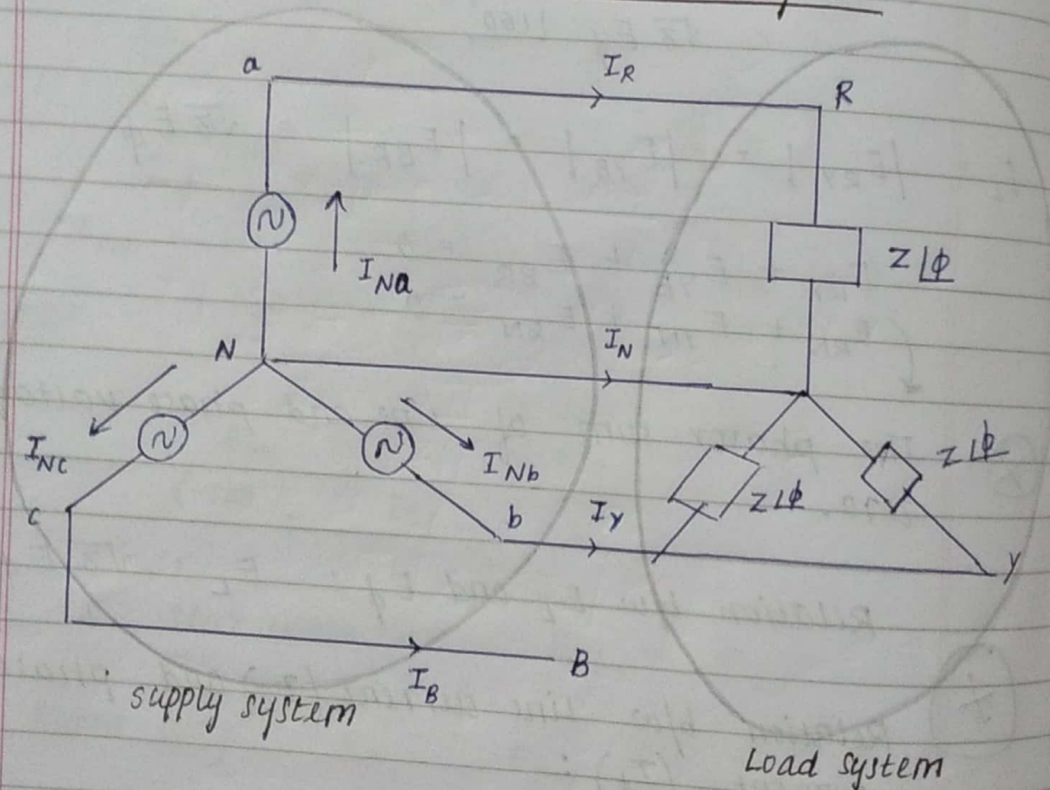


120 phase shift

3 phase, 3 wire system

$$E_L = E_\phi \qquad I_L = \sqrt{3}\, I_\phi$$

Balanced 3 phase star connected system



$I_{Na} = \underset{\text{red}}{I_R} = I_{RS}$

$I_{Nb} = \underset{\text{yellow}}{I_y} = I_{yS}$

$I_{NC} = \underset{\text{blue}}{I_B} = I_{BS}$

$$I_R = \frac{E_{AN}}{Z} = \frac{E_\phi \angle 0^\circ}{Z \angle \phi^\circ} = I_\phi \angle -\phi$$

$$I_Y = \frac{E_{bN}}{Z} = \frac{E_\phi \angle -120^\circ}{Z \angle \phi} = I_\phi \angle -120 - \phi$$

$$I_B = \frac{E_{CN}}{Z} = \frac{E_\phi \angle -240^\circ}{Z \angle \phi} = I_\phi \angle -240^\circ - \phi$$

By applying KCL at neutral point,

$$I_N = I_R + I_Y + I_B$$

1. A balanced star connected load of $(8+j6)\,\Omega$/phase, is connected to a 3 phase, 230V, 50 Hz supply. Find the line current, power factor, reactive volt ampere and total volt ampere.

line voltage $E_L = 230V$

$Z_\phi = 8 + j6\ \Omega = 10 \angle 36.86\ V$ ϕ

W.K.T in star connection, $E_L = \sqrt{3}\, E_\phi$

$$E_\phi = \frac{E_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 133\ V$$

$$I_L = I_\phi = \frac{|E_\phi|}{|Z_\phi|} = \frac{133}{10} = 13.3\ A$$

$PF = \cos(36.86) = 0.8$ lag — how we decided lag?

NOTE: $z = a + jb \rightarrow$ inductive load $\rightarrow$ lag
$z = a - jb \rightarrow$ capacitive load $\rightarrow$ lead
$+ \rightarrow$ lag
$- \rightarrow$ lead

E_L / V_L

Total volt ampere (total power) = $\sqrt{3}\,|V_L|\,|I_L|$

Real volt ampere / power = $\sqrt{3}\,|V_L|\,|I_L|\cos\phi$

Reactive volt ampere / power = $\sqrt{3}\,|V_L|\,|I_L|\sin\phi$

Total VA = $\sqrt{3} \times 230 \times 13.3 = 5298$

Real VA = 4239

Reactive VA = 3181

2. A balanced star connected load of $3-j4$ impedance with a 400V supply. Calculate Real power.

$E_L = 400\,V$

$Z_\phi = 3-j4 = 5\angle{-53.13}$

$E_\phi = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \frac{400}{1.732} = 231\,V$

$I_L = I_\phi = \frac{|E_\phi|}{|Z_\phi|} = \frac{231}{5} = 46.2$

Real power = $\sqrt{3}\,|V_L|\,|I_L|\cos\phi$

$\cos(-53.13)$

$= \sqrt{3} \times 400 \times 46.2 \times 0.6$

$= 19204$

3. Three impedences each of 10Ω resistance and 5Ω inductive reactance are connected in delta to a 400 V, 50 Hz, 3 phase supply. Determine the current in each phase and each line. Calculate the total power drawn from the supply and power factor of load.

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Given: $R_\phi = 10\Omega$ $X_\phi = 5\Omega$

$|E_L| = |E_\phi| = 400V$

$Z_\phi = 10 + j5 = 11.2 \angle 26.6$

$I_\phi = \frac{|E_\phi|}{|Z_\phi|} = \frac{400}{11.2} = 35.7 = 36\ A$

$I_L = \sqrt{3}\, I_\phi = 1.732 \times 36 = 62.35\ A$

Total power $= \sqrt{3}\,|V_L|\,|I_L| = 1.732 \times 400 \times 62.35$
$= 43,197.5$

power factor $= \cos\phi = \cos 26.6 = 0.9$ lag