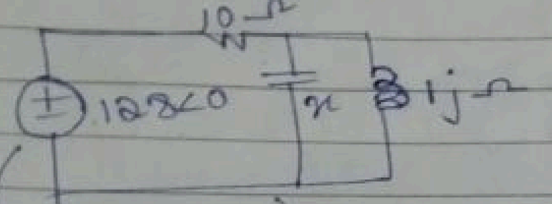
AC (continued)

Power factor correction – technique
→ where capacitor is used to reduce reactive power component of an AC circuit in order to improve its efficiency and reduce current

Q)

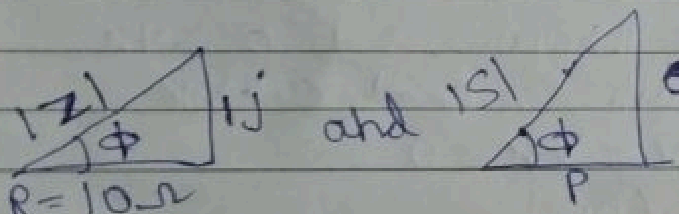


find x if $\cos\phi = 1$

128cos(314t)

$$I = \frac{128\angle 0}{10+j} = 12.6\angle -5.7\,A$$

$$S = P_{supp} = \frac{128\angle 0}{\sqrt{2}} \times \frac{12.6}{\sqrt{2}}\angle 5.7$$
$$= 802.49 \pm 80.09j \quad \{P+Qi\}$$



(b) $\cos\phi = 1 \rightarrow \boxed{\phi = 0}$

$$\text{Power capacitor} = I_{rms}^2 \times X_C = \frac{V_{rms}^2}{X_C} = S_C$$

for $\phi = 0 \rightarrow S_C = Q$

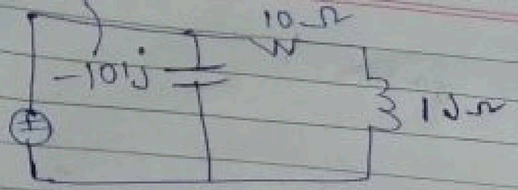
so $$80.09 = \frac{\left(\frac{128}{\sqrt{2}}\right)^2}{\frac{1}{\omega x}}$$

ω → 314 {given}

so $\boxed{x = 31.1\,\mu F}$ (ans)

so $\boxed{X_C = 101.03}$

-ve are for capacitor we use -ve

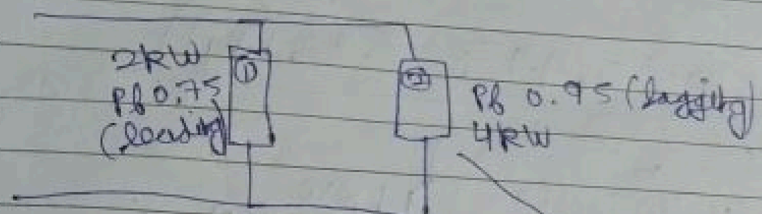


so $Z_{eq} \rightarrow 10.01 + 0j$

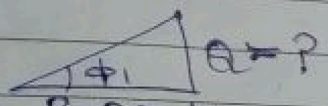
so $I = 12.8\angle 0$

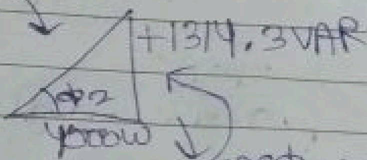
$S_{new} = \frac{128\angle 0}{\sqrt{2}} \times \frac{12.8\angle 0}{\sqrt{2}} = 819.2 + 0j$

Q)



ans)





$P = 2000W$

$\cos\phi_1 = 0.75$ (leading)

$\phi_1 = 41.41^\circ$

$Q_1 = 2000 \tan 41.41$

$= -1763.85$ VAR

$\cos\phi_2 = 0.95$

$\phi_2 = 18.19^\circ$ (lag)

so

$\tan\phi_2 = \frac{Q_2}{4000}$

$Q_2 = 1314.3$ VAR

now lagging power factor means its a inductor and leading power factor means its a capacitor

VVV imp

for inductor $\rightarrow$ Q is +ve

for Capacitor $\rightarrow$ Q is -ve

so $S = (2000 + 4000) + j(1314.3 - 1763.85)$

$= \underset{W}{6000} - \underset{VAR}{449.5\hat{j}}$

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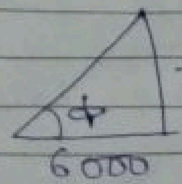
for total power factor →

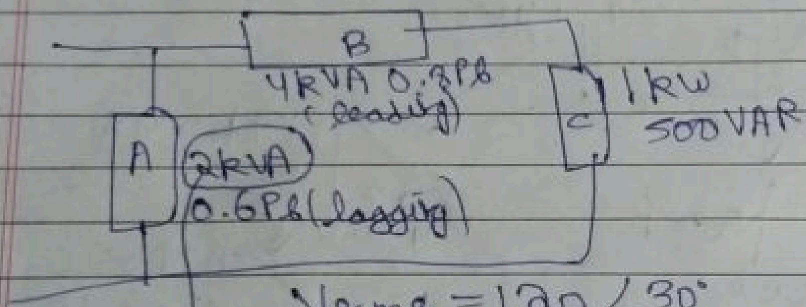


$\tan\phi = \frac{-449.5}{6000}$

$\phi = 4.283^\circ$ (leading (as Q → -ve)
{we can ignore - sign}

$\boxed{\cos\phi = 0.997 \text{ (leading)}}$

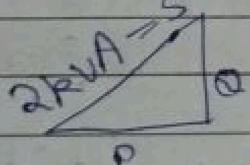
Q)



$V_{rms} = 120\angle 30^\circ$

↓
indicates hypotenus {|S|} of ϕ triangle

ans) (A)

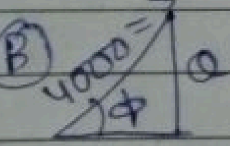


$\cos\phi_A = 0.6$
$\phi_A = 53.13^\circ$ } lagging

$P = S\cos\phi_A,\ Q = S\sin\phi_A$

$P = 1200\,W$ $\quad$ $Q = +1600\,VAR$ {+ve as lagging}

(B)



$\cos\phi = 0.8$
$\phi = 36.86^\circ$ } {leading}

$P = 4000\cos\phi,\ Q = 4000\sin\phi$

$P = 3200\,W$ $\quad$ $Q = -2400\,VAR$

{- as leading}

$P = 1kW$

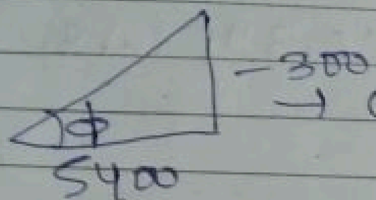
$Q = 500\ VAR$ ← (c)

$S_{tot} = (3200 + 1000 + 1200) + j(500 + 1600 - 2400)$

$= 5400 - 300j$

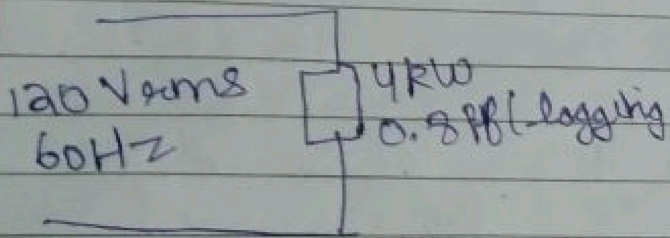
$$I^*_{rms} = \frac{5400 - 300j}{120\angle 30} = 45.06\angle -33.18\ A$$

$$\boxed{I_{rms} = 45.06\ \angle 33.18 A}$$



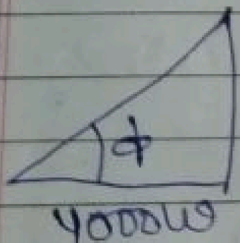
→ $\cos\phi$ = {calculate} and its leading

Q)



find C so that power factor is 0.95 lagging

ans)

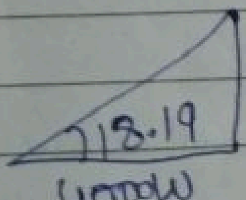


$\cos\phi = 0.8$ {lagging}

$\phi = 36.86$

$Q = +2999\ VAR$ {$P\tan\phi$}

↓ {+ as lagging}



$\cos\phi_{new} = 0.95$ {lagging}

$\phi_{new} = 18.19$

{Real power doesnt change}

↓ $\boxed{\text{Imp}}$

$Q_{new} = +1314.3$ {+ as lagging}

$$\boxed{\begin{aligned} Q_c &= 2999 - 1314.3 \\ &= 1684.6 \text{ VAR} \end{aligned}}$$

$$I_{rms}^2 X_C = \frac{V_{rms}^2}{X_C}$$

$$\frac{V_{rms}^2}{\frac{1}{\omega C}} = 1684.6$$

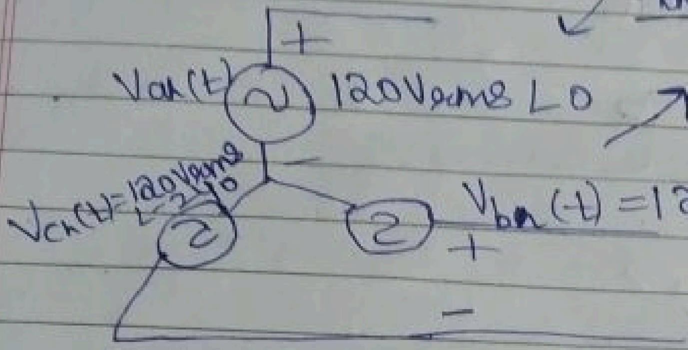
$$C = \frac{1684.6}{120^2 \times 2\pi \times 60} = 310.3\mu F \text{ (ans)}$$

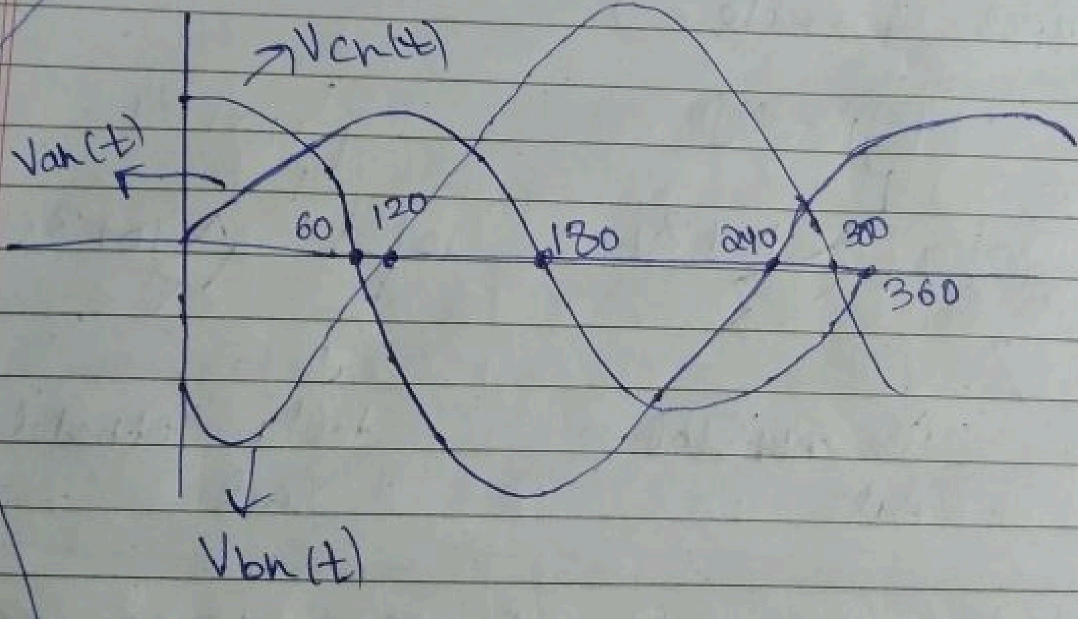
Three phase Systems

drawback of single phase → ① cant connect lots of loads to it ② power is oscillatory and varies wrt time.

this is a generator

Called Star connected network denoted by Y → star → generator





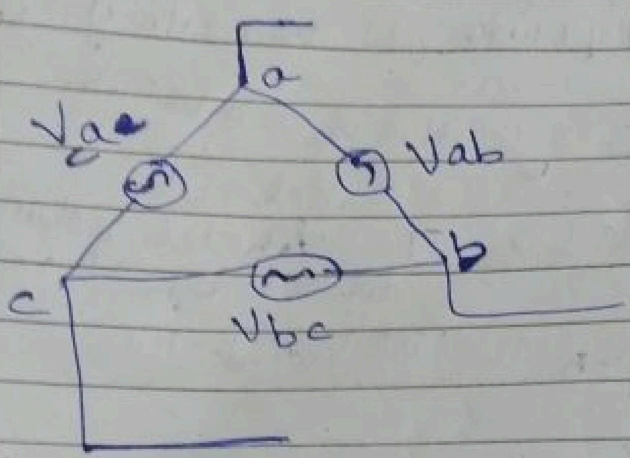
reference → $\cos(2\pi \times 50t)$

$V_{an}(t) = 120\sqrt{2}\cos(2\pi \times 50t)$

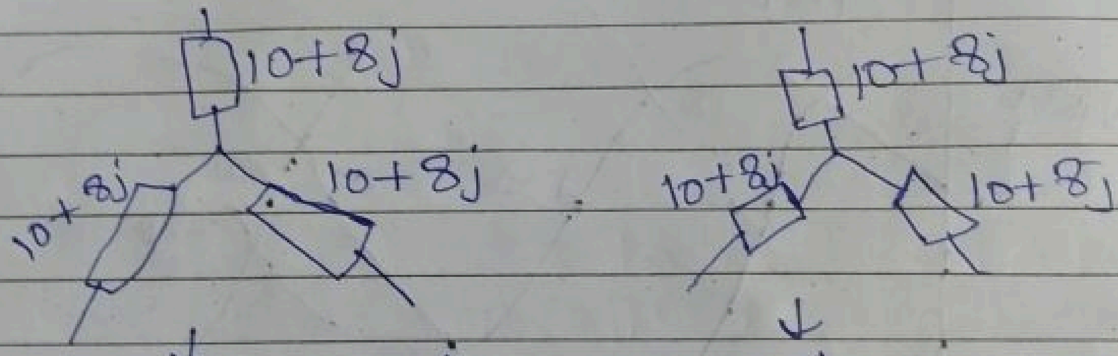
$V_{bn}(t) = 120\sqrt{2}\cos(2\pi \times 50t - 120)$

$V_{cn}(t) = 120\sqrt{2}\cos(2\pi \times 50t - 240)$

Delta connected network



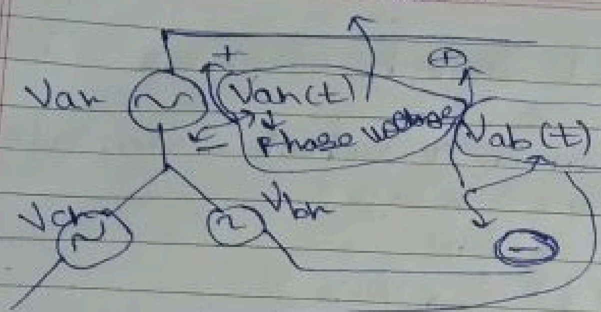
types of loads →



If balanced system → all loads are of same value and voltage source phase differs by 120

only balanced system is in syllabus for us.

Star network can have delta and star load. same with delta network voltage in 1 phase



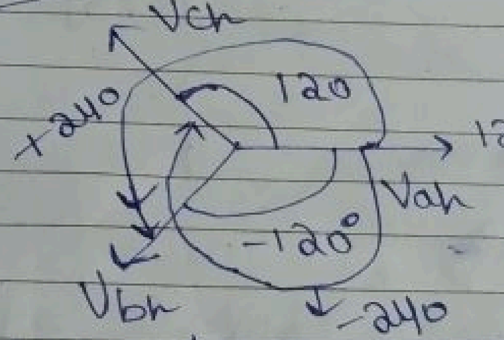
$V_{ab}(t) \rightarrow$ called line voltage → voltage b/w 2 phases

$V_{ab}(t) = V_{an}(t) - V_{bn}(t)$

$V_{ab} = V_{an} - V_{bn}$

$= 120\,V_{rms}\angle 0 - 120\,V_{rms}\angle -120$

$= 120\sqrt{3}\angle 30$



similarly

$V_{bc} = 120\angle -120 - 120\angle -240$

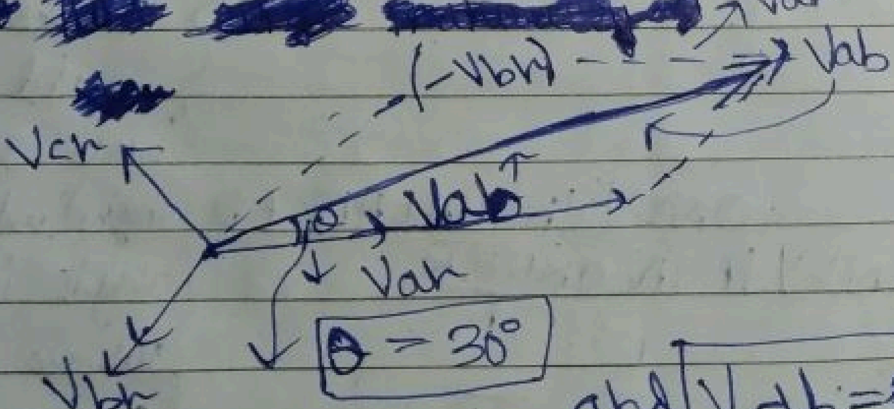
$= 120\sqrt{3}\angle -90$

$V_{ca} = 120\angle -240 - 120\angle 0$

$= 120\sqrt{3}\angle 150$

We can say → line voltage is scaled by $\sqrt{3}$ of phase voltage and some angle.

[redacted]

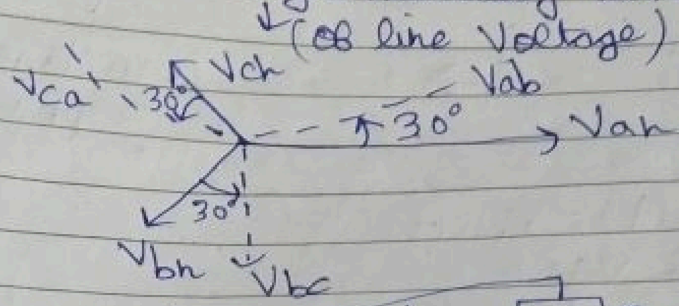


$\boxed{\theta = 30°}$

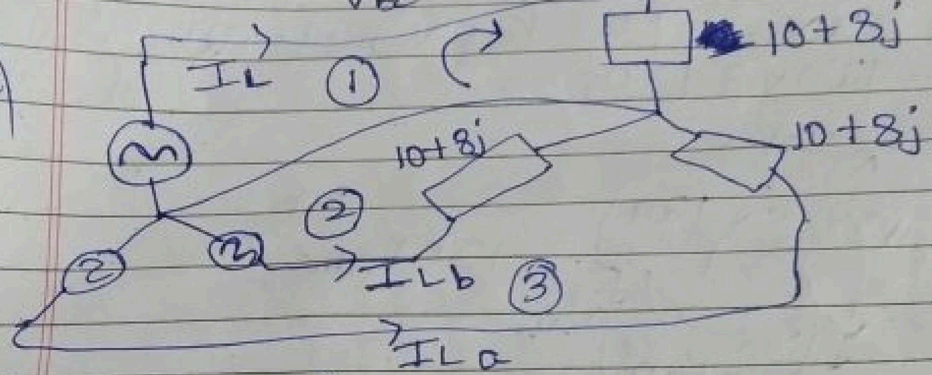
and $\boxed{V_{ab} = [redacted]\, V_{rms_{an}} \times \sqrt{3}}$

In star connected network $I_{phase} = I_{line}$

to find angle manually without calculator
(of line voltage)



a)



ans) in loop (1)

$$I_{L\,rms} = I_{phase} = \frac{120\angle 0}{10+8j} = 9.37\angle -38.6$$

in loop (2)

$$I_{Lb} = \frac{120\,V_{rms}\angle -120}{10+8j} = 9.37\angle -158.6$$

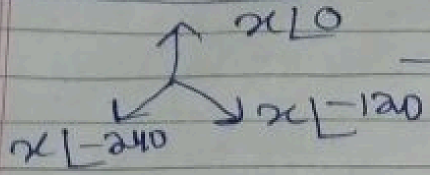
$$I_{La} = \frac{120\angle -240}{10+8j} = 9.37\angle -278.6$$

if you find one current others can be found as they have same magnitude and angle is $(\theta - 120)$ where θ is angle of previous current

ONLY IN BALANCED
AND ONLY IF IN ORDER

meaning I_a, I_b, I_c

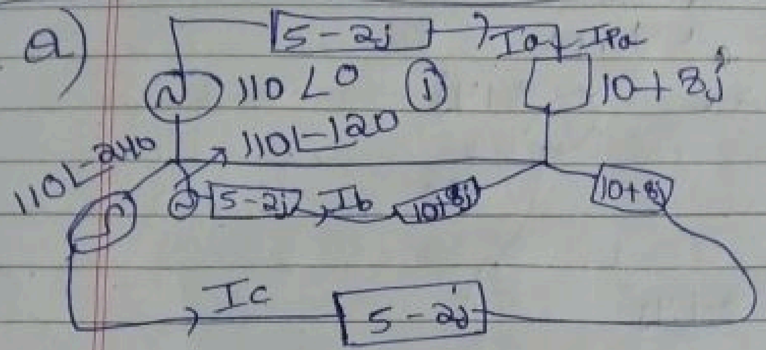
Extra Note



$\rightarrow$ resultant $= x\angle 0 + x\angle -120 + x\angle -240$

$= 0$ (always).

a)

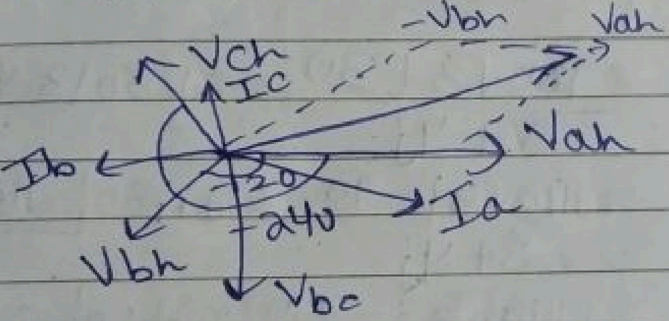


(110 is Vrms)

$V(an)(t) = 110\sqrt{2}\cos(\omega t - 0)$

$V_{bn}(t) = 110\sqrt{2}\cos(\omega t - 120)$

$V_{cn}(t) = 110\sqrt{2}\cos(\omega t - 240)$



$V_{ab\,rms} = V_{an\,rms} - V_{bn\,rms} = 110\sqrt{3}\angle 30^\circ$

$V_{bc\,rms} = V_{bn\,rms} - V_{cn\,rms} = 110\sqrt{3}\angle -90^\circ$

$V_{ca\,rms} = V_{cn\,rms} - V_{an\,rms} = 110\sqrt{3}\angle -210^\circ$

Ia in loop 1 $\rightarrow$

$$I_{a\,rms} = \frac{110\angle 0}{15+6j} = 6.80\angle -21.80$$

$$I_{b\,rms} = \frac{110\angle -120}{15+6j} = 6.80\angle -141.80$$

$$I_{c\,rms} = \frac{110\angle -240}{15+6j} = 6.80\angle -261.80$$

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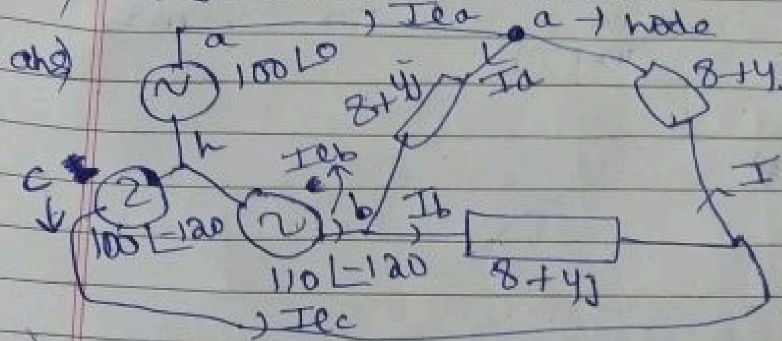
$I_{line} = I_{phase}$

so $I_{pa} = 6.8\angle -21.80$

$I_{pb} = 6.8\angle -141.80$

$I_{pc} = 6.8\angle -261.80$

Q) Y generator — Δ Load

ans) 

ans) $V_{ab} = 100\sqrt{3}\angle 30°$

$V_{bc} = 100\sqrt{3}\angle -90°$

$V_{ca} = 100\sqrt{3}\angle -210 V$

$$I_a = \frac{V_{ab}}{Z} = \frac{100\sqrt{3}\angle 30}{8+4j} = 21.30\angle 3.43$$

$$I_b = \frac{V_{bc}}{Z} = \frac{110\sqrt{3}\angle -90}{8+4j} = 21.30\angle -116.57$$

$$I_c = \frac{V_{ca}}{Z} = \frac{110\sqrt{3}\angle -210}{8+4j} = 21.30\angle -236.57$$

↓

Same $-120°$ trend follows here

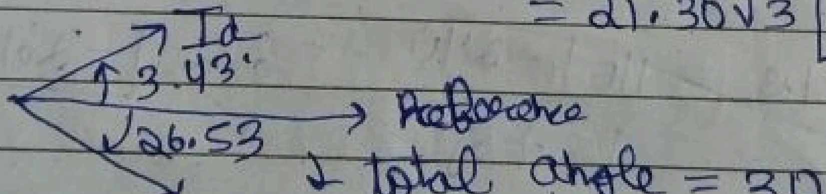
at node a

use KCL → $I_{La} + I_c = I_a$

$I_{La} = I_a - I_c$

$I_{La} = 36.89\angle -26.53$

$= 21.30\sqrt{3}\angle -26.53$



similarly $I_{Lb} = I_b - I_a = 36.89\angle -146.53$

$I_{Lc} = I_c - I_b = 36.89\angle -266.53$

So in delta

$V_{line} = V_{phase}$

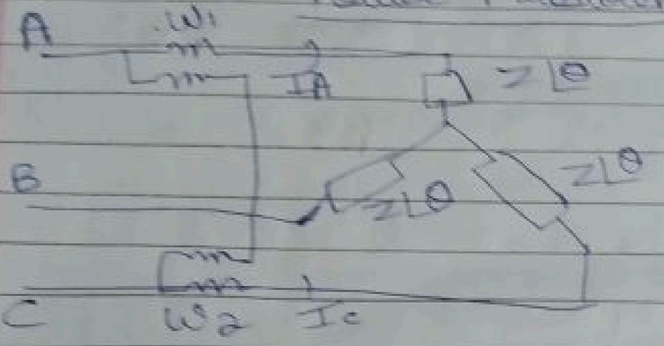
$I_L = I_p\sqrt{3}\angle -30$

In star

$I_L = I_{ph}$

$V_L = V_{ph}\sqrt{3}\angle 30$

Power Measurement



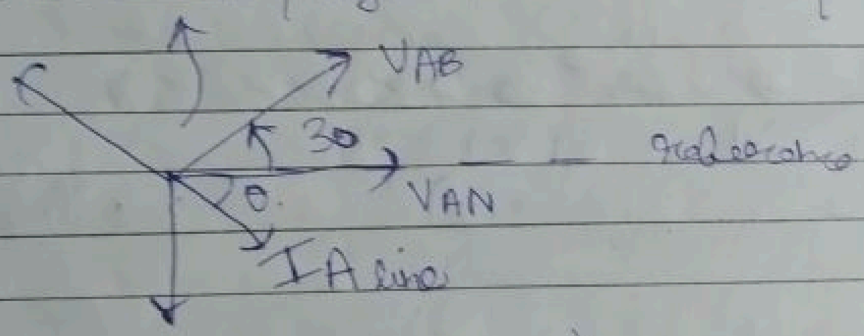
$W_1, W_2 \to$ Wattmeter.

↑ V_{AB}, I_A

W_1 reading $\to P_1 = VI\cos\phi$

$= V_{line} \times I_{line} \cos\phi$

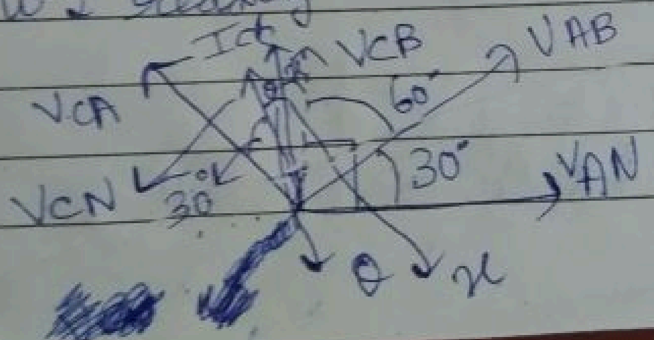
$\phi = 30 + \theta$ {angle b/w V_L and I_L}



so $P_1 = V_L I_L \cos(30+\theta)$

↑ V_{CB}, I_c

W_2 reading $\to$



so $\boxed{\alpha = 30 - \theta}$

so $P_1+P_2 = V_L I_L [\cos(30+\theta) + \cos(30-\theta)]$

$\boxed{P_{3phase} = \sqrt{3} V_L I_L \cos\theta}$

$\lambda \rightarrow V_{ph} = \frac{V_{line}}{\sqrt{3}}$, $I_{ph} = I_L$

$\boxed{P_{3phase} = \sqrt{3} V_L I_L \cos\theta}$

$\Delta \rightarrow V_{ph} = V_L$, $I_{ph} = \frac{I_L}{\sqrt{3}}$

$\boxed{P_{3phase} = \sqrt{3} V_L I_L \cos\theta}$

$\boxed{P_{3phase} = 3 V_{ph} I_{ph} \cos\theta}$

$\theta = \phi$ as phase angle = angle of polar form of impedance θ

total reactive power $\rightarrow$ $\boxed{Q_{3phase} = \sqrt{3} V_L I_L \sin\theta}$

so $\boxed{pf = \tan^{-1}\frac{Q}{P} = \tan^{-1}\left(\frac{\sqrt{3}(P_2-P_1)}{P_2+P_1}\right)}$

$\boxed{Q_{3phase} = (P_2-P_1)\times\sqrt{3} = \sqrt{3} V_L I_L \sin\theta}$

Q) $\lambda - \lambda$, $V_L = 197\angle 0$ (V_{rms})
$Z = 51\angle 22$ per phase

ans) [redacted]

$P_1 = V_L I_L \cos(\theta + 30°)$

$|I_L| = \frac{V_{ph}}{Z} = \frac{\frac{197}{\sqrt{3}}}{51} = 2.23A$

$P_1 = 197 \times 2.23 \times \cos(22+30)$

$P_1 = 270W$

$P_2 = 197 \times 2.23 \cos(8)$

$= 435W$

so $P_{3phase} = 705W$

$Q_{3phase} = \sqrt{3}(P_2 - P_1) = 285\ VAR$

$\phi = \tan^{-1}\left(\frac{285}{705}\right) = 22^\circ$

so $\cos\phi = 0.927$.

or ϕ = angle of phase from of impedance
so $\phi = 22^\circ$
$\cos 22 = 0.927$

Q) $V_L = 208V$ $\quad f = 50hz$
$P_1 = -560W$
$P_2 = 800W$

ans) $P_{3phase} = 240W$

$Q_{3phase} = \sqrt{3}(P_2 - P_1) = 2356VAR$

$\phi = \tan^{-1}\frac{Q}{P} = 84^\circ = \theta_{volt} - \theta_{current}$

$\cos\phi = 0.1014$.

$P_{3phase} = 240 = 3(P_{1phase}) = 80W \times 3$

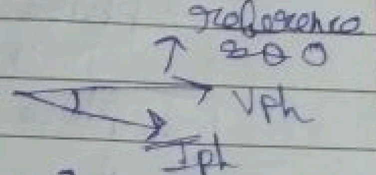
so $P_{1 phase} = 80$

$80 = V_{ph} I_{ph} \cos\phi = 80$

$|I_{ph}| = \frac{80}{120 \times 0.1014}$

$|I_{ph}| = 6.57A$

$I_{ph} = 6.57 \angle \theta_v - \theta_i$
$= 6.57 \angle -84$



$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{120}{6.57\angle -84} = 18.2\angle 84^\circ$ (ans)

$\boxed{Z_{ph} = 1.9 + 18.1j}$

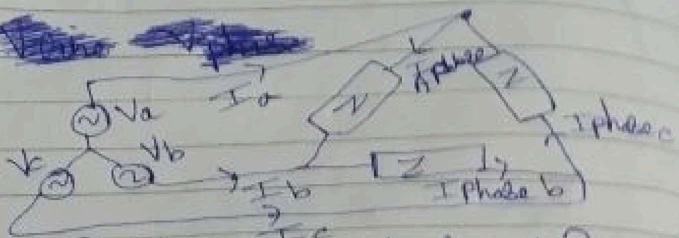
inductive load

so $R + jX_L$.

$\underline{R = 1.9}$, $\underline{X_L = 18.1}$

$\omega L = 18.1 \rightarrow L = \frac{18.1}{100\pi (2\pi f)}$ (ans)

Classification



(1) Y–Δ circuit {Y source, Δ load}

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \times \sqrt{3}\angle 30 = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

{phase voltages} {line voltages}

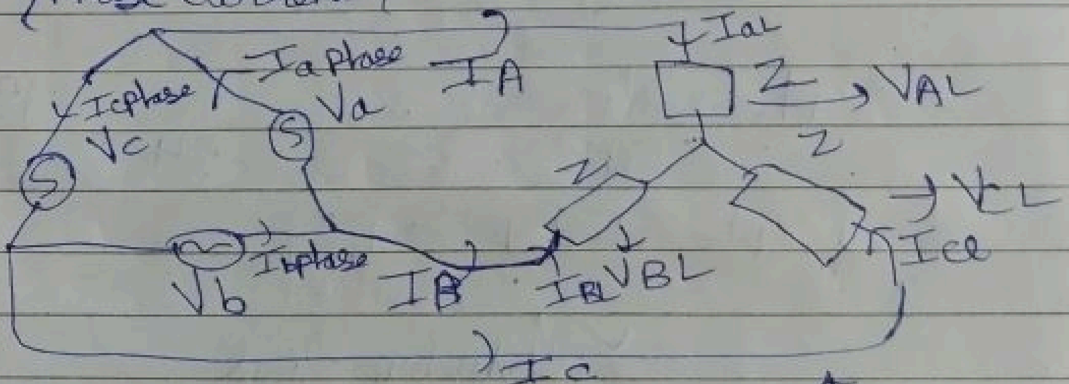
$$\begin{bmatrix} I_{phase\,a} \\ I_{phase\,b} \\ I_{phase\,c} \end{bmatrix} \times \sqrt{3}\angle -30 = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

{phase current} {line current}

$$\begin{bmatrix} I_{phase\,a} \\ I_{phase\,b} \\ I_{phase\,c} \end{bmatrix} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \times \frac{1}{Z}$$

(3)

(2)



Delta - Star

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \times \frac{1}{\sqrt{3}\angle 30} = \begin{bmatrix} V_{aL} \\ V_{BL} \\ V_{CL} \end{bmatrix}$$

$$\begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} = \begin{bmatrix} I_a \\ I_B \\ I_c \end{bmatrix} \times \frac{1}{\sqrt{3}\angle -30}$$

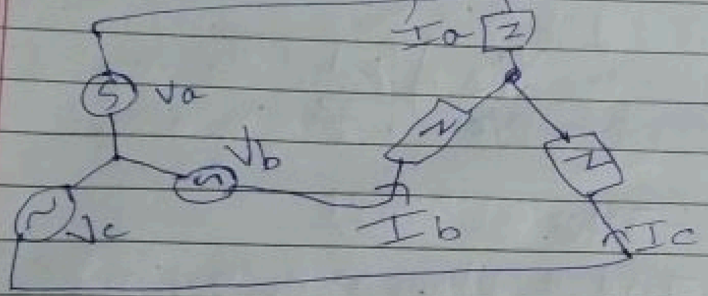
↓ phase current ↓ line current

$I_{as} = I$ phase a
$I_{bs} = I$ phase b
$I_{cs} = I$ phase c

$$\begin{bmatrix} I_{aL} \\ I_{bL} \\ I_{cL} \end{bmatrix} = \begin{bmatrix} V_{aL} \\ V_{BL} \\ V_{cL} \end{bmatrix} \times \frac{1}{Z} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

↓ line current

(3) ~~Stage~~ Star Circuit



$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \times \sqrt{3}\angle 30^\circ = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

↓ phase voltage ↓ line voltage

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \times \frac{1}{Z}$$

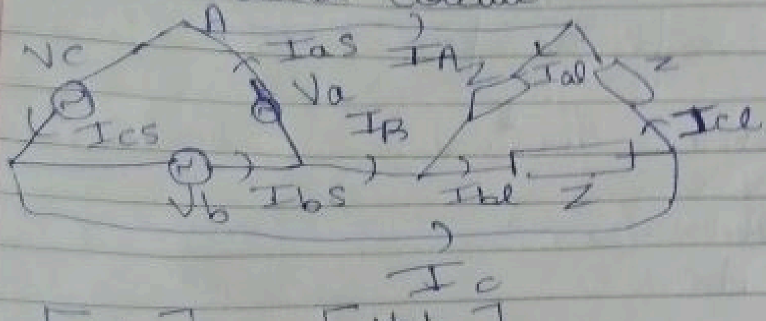
↓
phase current

$\boxed{\text{Phase current} = \text{line current}}$

$I_{La} = I_{Lb} = I_{Lc}$
↓ ↓ ↓
$I_a \quad I_b \quad I_c$ } equal

$\boxed{\text{phase voltage} \times \sqrt{3}\angle 30 = \text{line voltage}}$

(4) Delta - Delta Circuit



$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$\downarrow$ Phase voltage $\qquad$ $\downarrow$ line voltage

$$\begin{bmatrix} I_{al} \\ I_{bl} \\ I_{cl} \end{bmatrix} = \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \times \frac{1}{Z}$$

$I_a = I_{al} - I_{cl}$

$I_b = I_{bl} - I_{al}$

$I_c = I_{cl} - I_{bl}$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} I_{al} \\ I_{bl} \\ I_{cl} \end{bmatrix} \times \sqrt{3}\angle -30$$

$\downarrow$

line current

$$\begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} = \begin{bmatrix} I_{al} \\ I_{bl} \\ I_{cl} \end{bmatrix}$$

$\downarrow$

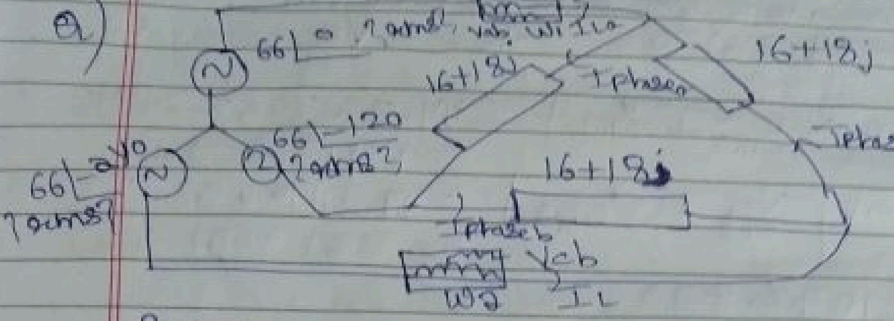
phase current

so $\boxed{\text{phase current} \times \sqrt{3}\angle -30 = \text{line current}}$

$\boxed{\text{phase voltage} = \text{line voltage}}$

Class stuff contd.

a)



Power

$S_{3phase} = 3 V_{phase} I_{phase}^*$

for $Y \rightarrow \sqrt{3} V_L I_L^*$ $\quad$ $* \rightarrow$ conjugate

for $\Delta \rightarrow \sqrt{3} V_L I_L^*$

$W_1 \rightarrow V_{ab\,rms}, I_{La\,rms}$ $\qquad$ $W_2 \rightarrow V_{cb}, I_{LC}$

$W_2 = \text{Real}[V_{ca\,rms} I_{LC\,rms}^*]$

$W_1 = \text{Real}[V_{ab\,rms} I_{La\,rms}^*]$

$P_{3phase} = W_1 + W_2$, $Q_{3phase} = \sqrt{3}(W_2 - W_1)$

$V_{ab\,rms} = 66\sqrt{3}\angle 30 = 114.32\angle 30$

$V_{bc\,rms} = 114.32\angle -90$

$V_{ca\,rms} = 114.32\angle -210$

$I_{phase\,a} = \dfrac{V_{ab}}{Z} = \dfrac{114.32\angle 30}{16+18j} = 4.47\angle -18.36$

$I_{phase\,b} = 4.47\angle -138.36$

$I_{phase\,c} = 4.47\angle -258.36$

$I_{La} + I_{ph\,c} = I_{ph\,a}$

$I_{La} = I_{ph\,a} - I_{ph\,c} = 8.20\angle -48.36$

$I_{Lb} = 8.20\angle -168.36\,A$

$I_{Lc} = 8.20\angle -288.36\,A$

$W_2 \rightarrow$ voltage is V_{bc} but we know $V_{cb} \rightarrow$

$V_{bc} = 114.32\angle 90$ { angle of V_{bc} reversed }

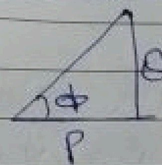
$W_1 = \text{Real}\left[V_{ab_{rms}} I^{*}_{a_{rms}}\right] = 114.32\angle 30 \times 8.20\angle 48.36$
$= 189.13W \rightarrow (1)$

$W_2 = \text{Real}\left[V_{ca_{rms}} I_{Lc}{}^{*}{}_{rms}\right] = 114.32\angle 90 \times 8.20\angle 288.36$
$= 889.70W \rightarrow (2)$

$P_{3phase} = 1078.83W$

$Q_{3Phase} = 1213.4$ VAR

Checking power triangle $\rightarrow$



$\tan\phi = \frac{Q}{P}$

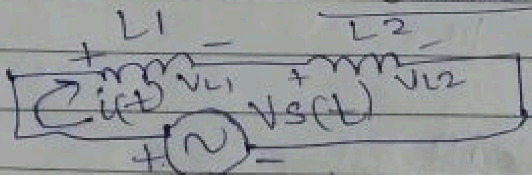
$\phi = 48.35°$ { lagging as inductive }

$\cos\phi = 0.67$

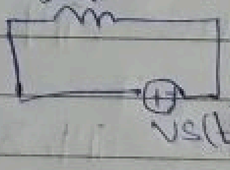
$16 + 18j = 24.08\angle 48.36$

which matches phase angle we got.

Mutually coupled Circuits



$\Rightarrow$



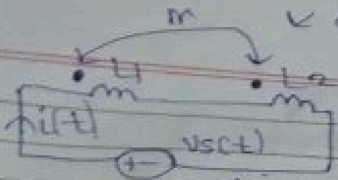
$V_S(t) = V_{L1} + V_{L2}$ { active = Passive }

$V_{L1}(t) = L_1 \frac{di}{dt}$, $V_{L2} = L_2 \frac{di}{dt}$

$V_S(t) = (L_1 + L_2)\frac{di}{dt}$

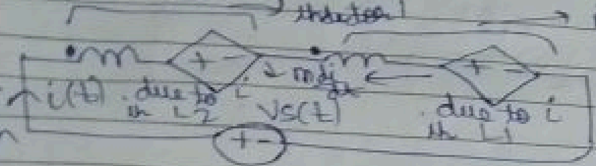
$V_{L1}(t) = L_1 \frac{di}{dt} = N\frac{d\phi}{di}\frac{di}{dt}$

so $V(t) = N\frac{d\phi}{di}\frac{di}{dt}$



← denotes coupling (mutual)

← mutually coupled circuit

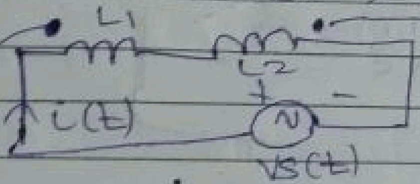


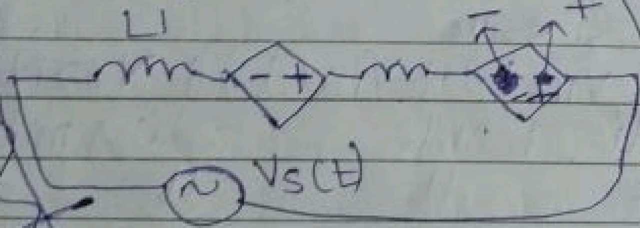
in mutually coupled → when current passes through one inductor voltage is produced in other inductor

dot convention →
when current enters dotted terminal → other (next) is positive

$$V(t) = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$
$$= (L_1 + L_2 + 2M)\frac{di}{dt}$$

dot another example





current enters this dot
so other dot is +

current leaves this dot
so other dot is −

in this
$$Vs(t) = (L_1 + L_2 - 2M)\frac{di}{dt}$$