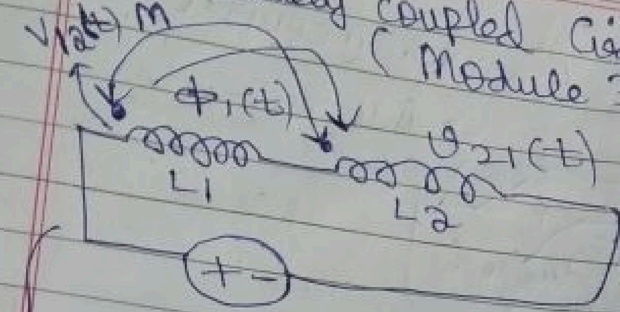
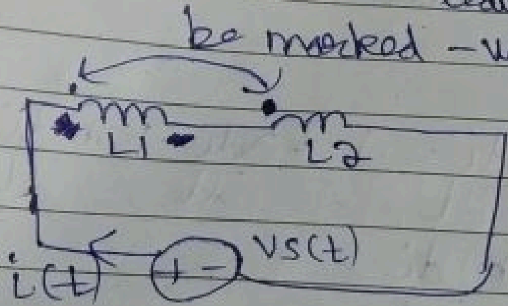
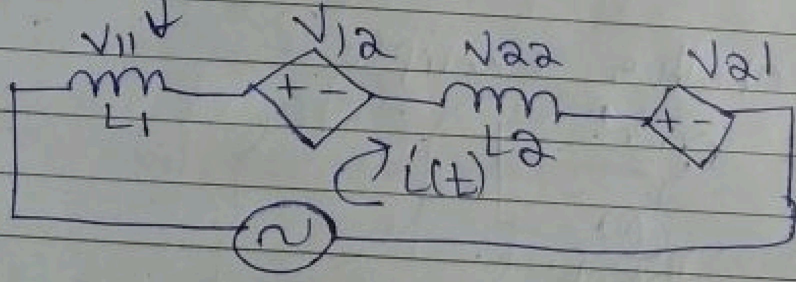
# Mutually coupled Circuits (Module 3)




$V_{21} \to$ voltage in $L_2$ due to flux of $L_1$
$V_{12} \to$ voltage in $L_1$ due to flux of $L_2$

when current enters the dotted terminal $\to$ hence dotted terminal will be +
or $\to$ current leaving dotted $\to$ hence dotted will be marked −ve.



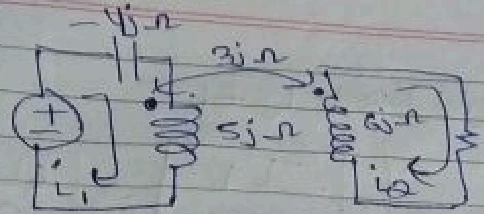




$\Sigma$active = $\Sigma$Passive

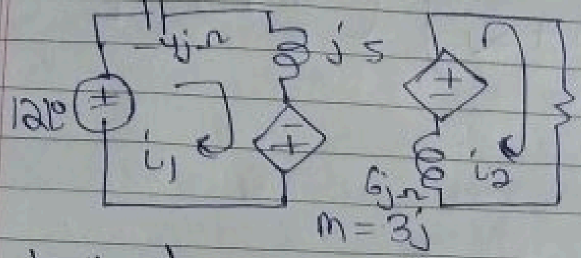
$$V_S(t) = V_{11}(t) + V_{12}(t) + V_{22}(t) + V_{21}(t)$$

$$V_S(t) = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$

$$\boxed{V_S(t) = (L_1 + L_2 + 2M)\frac{di}{dt}}$$

Q) 


find $i_1, i_2$

ans) 


KVL →

loop 1 $12\angle 0 + 3j\, i_2 = -j4\, i_1 + 5j\, i_1$

$$12\angle 0 = 1j I_1 - 3j I_2 \to (1)$$

loop 2 → $3jI_1 = 6jI_2 + 12I_2$

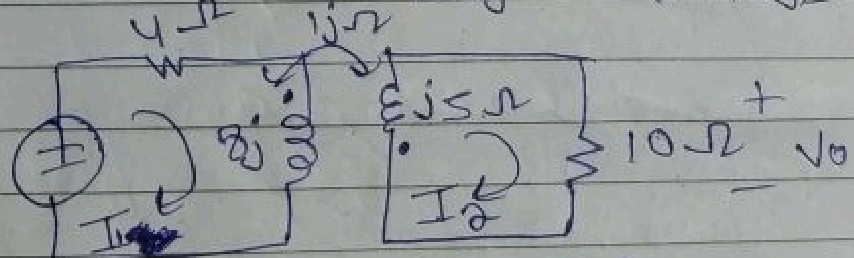
$$3jI_1 - (12+6j)I_2 = 0$$

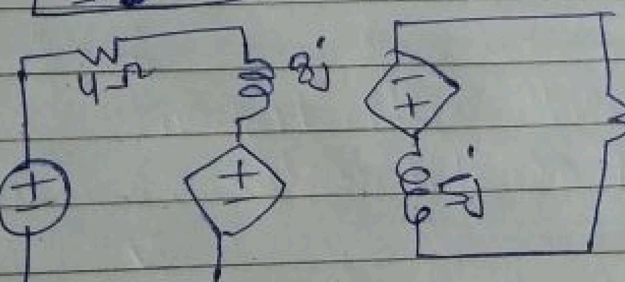
$$I_1 = \frac{(12+6j)}{3j} I_2 \to (2)$$

solving → (1) method → put (2) in (1)

(2) method → cramers rule

$$\begin{bmatrix} 1j & -3j \\ 3j & -(12+6j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0 \\ 0 \end{bmatrix}$$

Q) 


ans) 


$$6\angle 90 = 4I_1 + 8jI_1 + 1jI_2$$
$$= (4+8j)I_1 + 1jI_2 \to (1)$$ loop1

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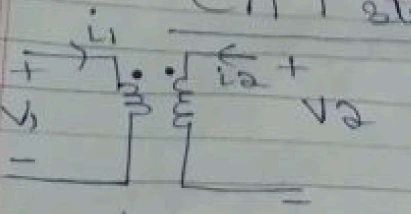
Loop 2 →

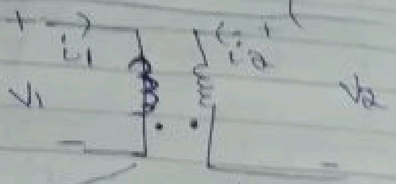
$$0 = 10 + (5j)I_2 + 1j I_1$$

So $\begin{bmatrix} 4+8j & 1j \\ 1j & 10+5j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 6\angle 90 \\ 0 \end{bmatrix}$

$$\boxed{V_0 = 10 i_2}$$

CAT stuff {for CAT 2}

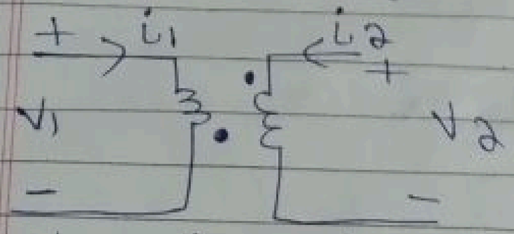


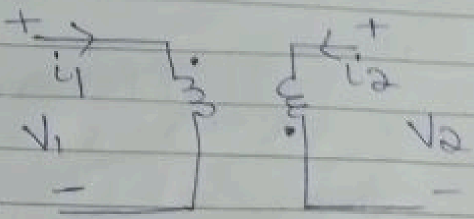




$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

{same for this}



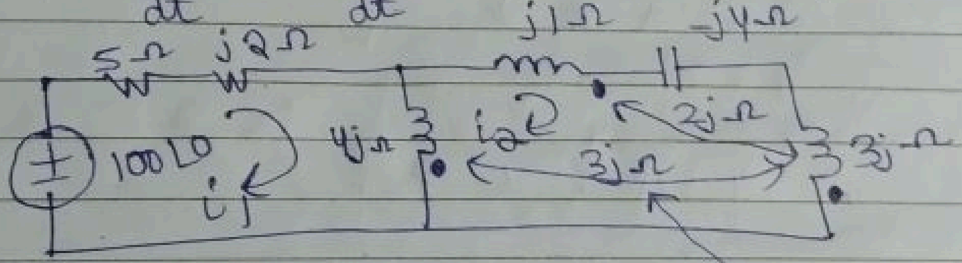




$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

{Same for this}

Q)




ans) $100\angle 0 = 5i_1 + j2i_1 + j4(i_1 - i_2) + 3ji_2$ (1)

as current enters both dots

and $0 = j1i_2 + j2i_2 - j4i_2 + j3i_2$
$+ 2ji_2 - j3(i_2 - i_1)$

as current enters dot

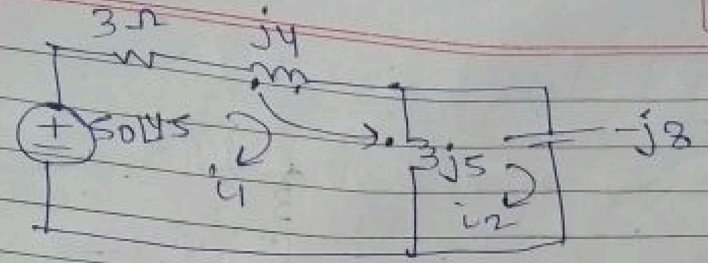
as current enters both

$+ j4(i_2 - i_1) - j3i_2$

$$0 = j1i_2 + j2i_2 - j4i_2 + j3i_2 + j2i_2 - j3(i_2 - i_1)$$
$$+ j4(i_2 - i_1) - j3i_2 \rightarrow (2)$$

solve both.

Q)



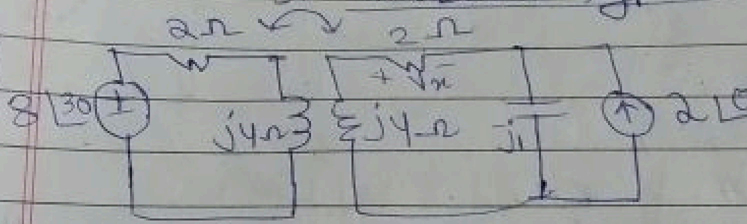

ans) $50\angle 45 = 3i_1 + 4j + j5(i_1 - i_2) + j3(i_1 - i_2) + j3i_1$ —①

$0 = -j8i_2 + j5(i_2 - i_1) - j3i_1$ —②

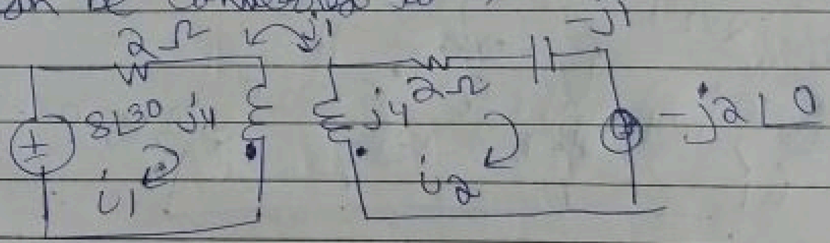
solve both

Probable types

Q)




ans can be converted to →



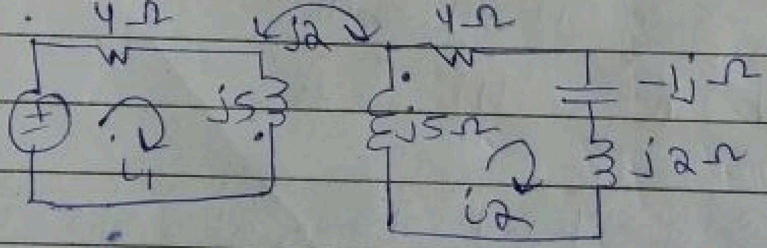

$8\angle 30 + 2i_1 + 4ji_1 - j1i_2 = 0$

and

$j4i_2 + 2i_2 - j1i_2 - j2\angle 0 - j1i_1 = 0$

solve both.

Q)




ans) $Z_S = \frac{V_S}{I_S}$

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- $Z_s = \frac{80\angle 0}{I_1}$

$(4+j5)i_1 + j2i_2 = 80\angle 0 \rightarrow (1)$

$j2i_1 + (j5+4-j1+j2)i_2 = 0 \rightarrow (2)$

find $Z_s = \frac{80\angle 0}{i_1}$.

## Force fields

$\varepsilon_0 \rightarrow$ unit $\rightarrow$ Farad / metre or $\frac{Nm}{Volt^2}$

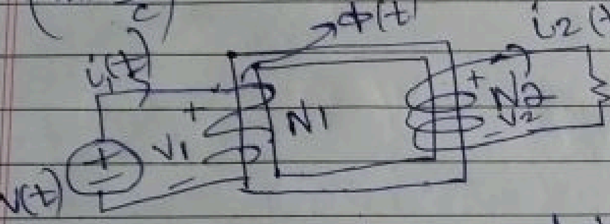
$\mu_0 \rightarrow$ Henry / metre or $Nm/ampere^2$

$\phi = \int_S \vec{B}\cdot\vec{ds}$ $\{\frac{J}{A}$ or $\frac{Nm}{A}\}$
{flux}

$v(t) = N\frac{d\phi}{dt}$ {faraday eqn}

$v(t) = N\frac{d\phi}{di} \times \frac{di}{dt} = L\frac{di}{dt}$

(unit $\frac{J}{C}$)



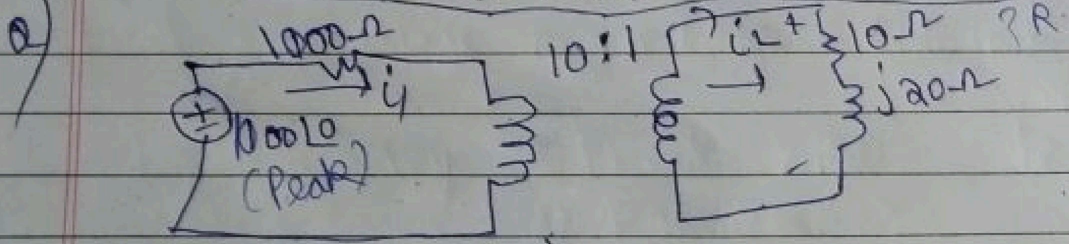

input power = output power

$$V_1(t) = N_1\frac{d\phi}{dt}$$
$$V_2(t) = N_2\frac{d\phi}{dt}$$
{$N_1, N_2 \rightarrow$ no of turns}

$$\boxed{\frac{V_1(t)}{V_2(t)} = \frac{N_1}{N_2} = \frac{i_2}{i_1}}$$

$$P_1(t) = P_2(t)$$
$$V_1(t)\, i_1(t) = V_2(t)\, i_2(t)$$

Q)




$\downarrow$ step down transformer

ans) $i_1^2 R_{L_{eq}} = i_2^2 R_L$

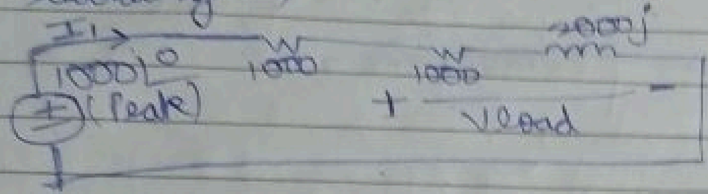
(RMS) $R_{L_{eq}} = \frac{I_2^2 R_L}{I_1^2}$

$R_L$ equivalent $= \left(\frac{I_2}{I_1}\right)^2 R_L = \left(\frac{N_1}{N_2}\right)^2 R_L$.

$I_1^2 X_{L\,eq} = I_2^2 X_L$

{ Basically we replace resistor and inductor and put equivalent of it in 1st circuit }

So accordingly →

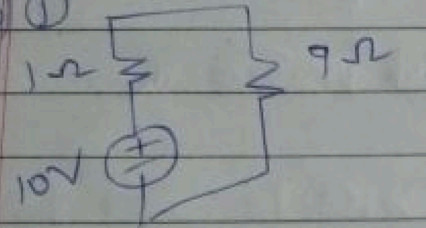



$I_1$ peak $= \dfrac{1000\angle 0}{2000 + 2000j} = 0.35\angle -45$

Vload $= 0.35\angle -45\,(1000 + 2000j)$

Q) Speaker of 9Ω effective R connected to →
case1 → power of 10V with internal R of 1Ω
case2 → transformer of 1:3 b/w source and load
find P absorbed

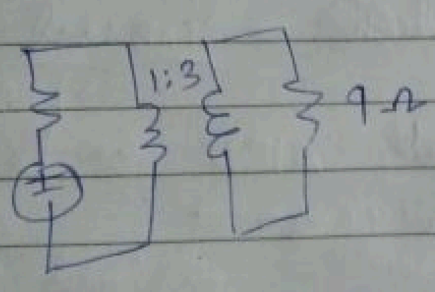
ans) ①



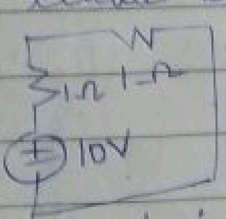

→ $I = \dfrac{10V}{10\Omega} = 1A$

$P = i^2 R = 9W$

②




So circuit becomes




↓ $i = 5A$

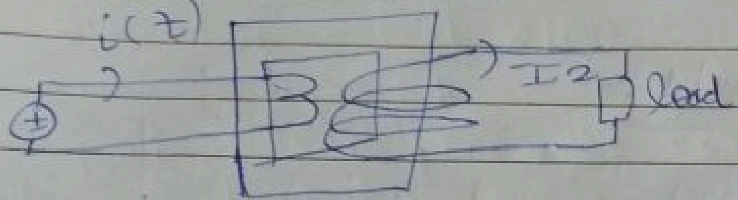
So $P = 25W$.

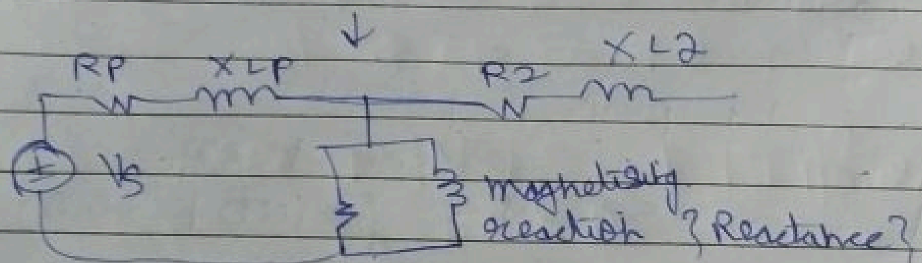
$R_{L,1} = \left(\frac{N_1}{N_2}\right)^2 R_L$

$= \left(\frac{1}{3}\right)^2 9 = 1\Omega$

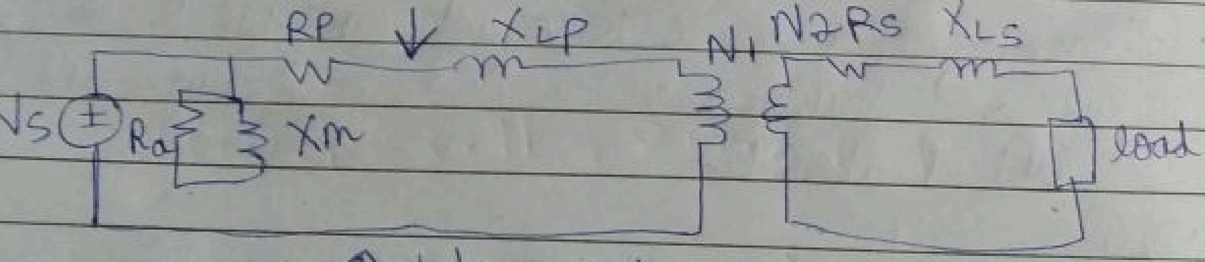
~~to~~ shift all to left side we use prev method.

to shift all to right side multiply voltage source by higher turns
R becomes 9 Ω, voltage becomes 30V
and you get same power.




↑ this is a realistic model




↑ takes lot of losses in account

$$\% \text{ voltage regulation} = \frac{V_{no\ load} - V_{load}}{V_{load}} \times 100$$

$$\text{efficiency} = \frac{P_{Load}}{P_{in}} \times 100$$

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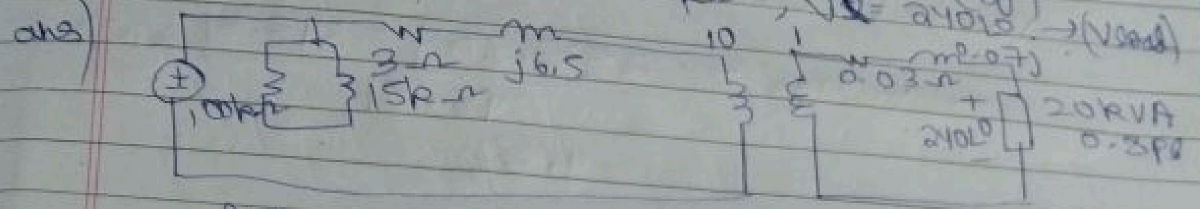
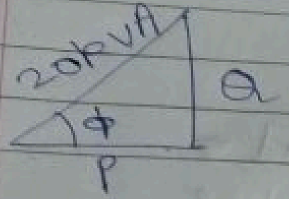
a) $R_P = 3\Omega$, $X_{LP} = 6.5j$, $N_1 : N_2 = 10:1$, $R_S = 0.03\Omega$, $X_{LS} = 0.07j$, $R_a = 100k\Omega$, load $= 20kVA$, $0.8\,pf$ (lag), $X_m = 15k\Omega$, $V_2 = 240\angle 0$ → ($V_{load}$)

ans)




$\cos\phi = 0.8$ {lag}
$\phi = 36.8°$

$P = S\cos\phi = 16014.627$
$Q = S\sin\phi = 11980.47$
$S = V_{rms} \times I^*_{rms}$

$$I^*_{rms} = \frac{16014.627 + 11980.47j}{240\angle 0}$$

$I^*_{rms} = 83.333\angle 36.799$
$I_{rms} = 83.33\angle -36.799$ A $\{I_2\}$

using mesh analysis in (2)
$V_2 = (0.03 + 0.07j)\,83.28\angle -36.82 + 240\angle 0$
$V_2 = 245.51\angle 0.73$ V

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$ we find $V_1 = 2455.1\angle 0.7$

$I_1 = 8.328\angle -36.83$ A

$V_S = I_1(3 + 6.5j) + 2455.1\angle 0.7$

$V_S = 2508.04\angle 11.333$

$V_{no\ load}$ = scaled value of $V_S$ {by trans}

$\%$ voltage $= \dfrac{258.804 - 245.51}{245.51} \times 100$

$= 4.5\%$

Efficiency $= \dfrac{P_{out}}{P_{in}} \times 100$

$P_{out}$ = Real power of load

$P_{in} = P_{out}$ + Ploss in all resistors

$P_{in} = P_{out}$ + loss in power by R

$$P_{in} = 16000 + \frac{|V_S|^2}{100k\Omega} + |I_1|^2_{rms} \times 3 + |I_2|^2_{rms} \times 0.03$$

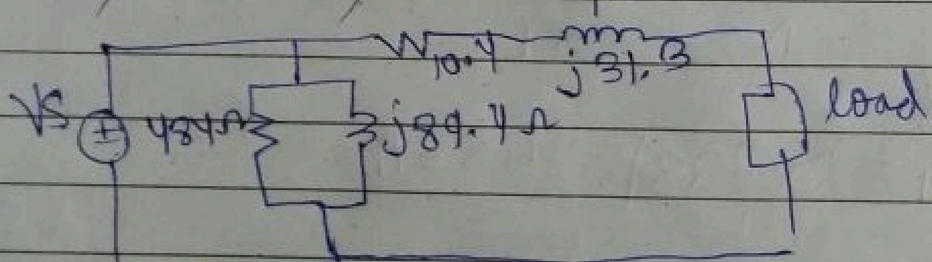
$$= 16000 + \frac{(2508.2)^2}{100{,}000} + (8.33)^2 \times 3 + (83.33)^2 \times 0.03$$

$$= 16000 + 62.91 + 208.17 + 208.316$$

so $P_{in} = 16000 + 479.58$

so efficiency $= \dfrac{16000}{16479.58} \times 100\% = 97.01\%$

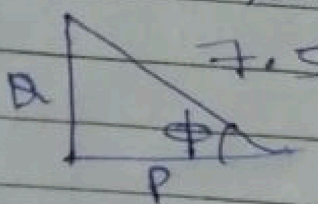
2) $1\Phi$, 10kVA, 2200V/220V 60Hz




(1) 75% of full load, 0.6 PF lag

find % voltage → {ans is 4.86}

ans) load $\rightarrow \frac{75}{100} \times 10000 = 7500VA$




$\phi = \cos^{-1} 0.6$

$\phi = 53.13^\circ$

so $P = 4500W$

$Q = 6000W$

so $S = 4500 + 6000i$

so $i_{rms}^{*} = \frac{4500 + 6000i}{2200}$

$i_{rms}^{*} = 3.4090 \angle 53.130$

$i_{rms} = 3.4090 \angle -53.130$ — 1