SCHOOL OF ELECTRICAL ENGINEERING

BEEE 101L

Basic Electrical Engineering

Module 3: Magnetic circuits

By,

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MAGNETIC CIRCUITS

- Many devices such as transformers, motors, and generators contain coils wound on iron cores.
- The magneto motive force (mmf) of an N-turn current-carrying coil is given by

$$\mathcal{F} = Ni$$

- The reluctance of a path for magnetic flux, is given by

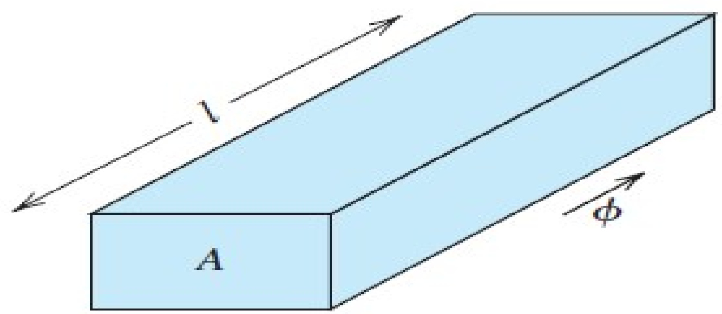
$$\mathcal{R} = \frac{l}{\mu A}$$

in which l is the length of the path (in the direction of the magnetic flux),A is the cross sectional area, and μ is the permeability of the material. Reluctance is analogous to resistance in an electrical circuit.

MAGNETIC CIRCUITS

- Magnetic flux φ in a magnetic circuit is analogous to current in an electrical circuit. Magnetic flux, reluctance, and magneto motive force are related by

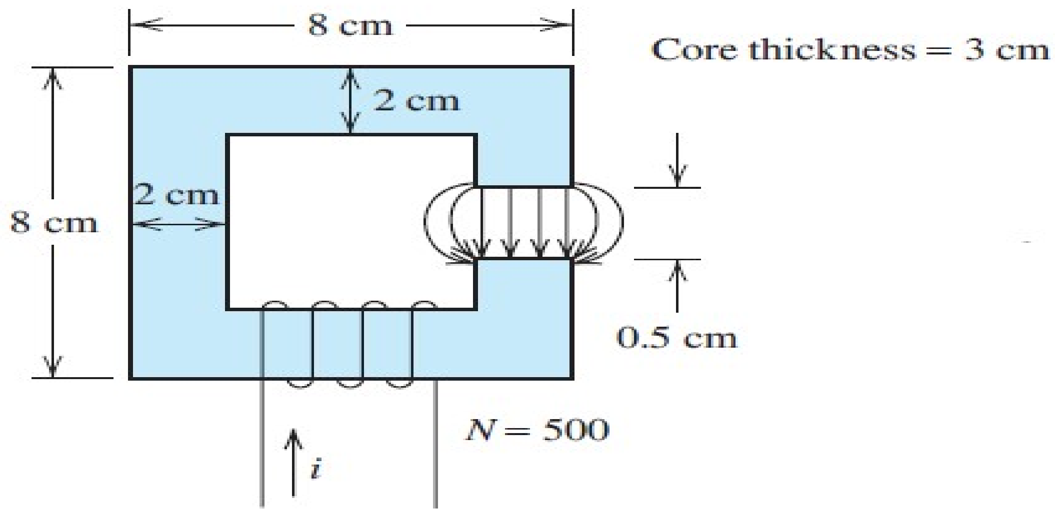
$$\mathcal{F} = \mathcal{R}\phi$$

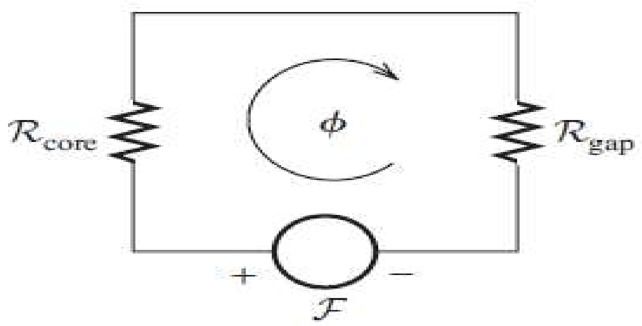


$$\mathcal{R} = \frac{l}{\mu A}$$

MAGNETIC CIRCUITS

- Consider the magnetic core with an air gap as shown in Figure. The core material has a relative permeability of 6000 and a rectangular cross section 2 cm by 3 cm. The coil has 500 turns. Determine the current required to establish a flux density of $B_{gap} = 0.25$ T in the air gap.





mean length of the iron core is

$$l_{core} = 4 \times 6 - 0.5 = 23.5 \text{ cm}$$

The cross-sectional area of the core is

$$A_{core} = 2 \text{ cm} \times 3 \text{ cm} = 6 \times 10^{-4} \text{ m}^2$$

The permeability of the core is

$$\mu_{core} = \mu_r \mu_0 = 6000 \times 4\pi \times 10^{-7} = 7.540 \times 10^{-3}$$

Finally, the reluctance of the core is $$\mathcal{R}_{core} = \frac{l_{core}}{\mu_{core} A_{core}} = \frac{23.5 \times 10^{-2}}{7.540 \times 10^{-3} \times 6 \times 10^{-4}}$$

$$= 5.195 \times 10^4 \text{ A·turns/Wb}$$

The flux lines tend to bow out as shown in Figure. This is called fringing. *Thus, the effective area of the air gap is larger than that of the iron. Customarily, we take this into account by adding the length of the gap to each of the dimensions of the air-gap cross section.*

Thus, the effective area of the gap is

$$A_{gap} = (2 \text{ cm } + 0.5 \text{ cm}) \times (3 \text{ cm } + 0.5 \text{ cm}) = 8.75 \times 10^{-4} \text{ m}^2$$

Thus, the reluctance of the gap is

$$\mathcal{R}_{gap} = \frac{l_{gap}}{\mu_{gap} A_{gap}} = \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 8.75 \times 10^{-4}}$$

$$= 4.547 \times 10^6 \text{ A·turns/Wb}$$

The total reluctance is the sum of the reluctance of the core and that of the gap:

$$\mathcal{R} = \mathcal{R}_{gap} + \mathcal{R}_{core} = 4.547 \times 10^6 + 5.195 \times 10^4 = 4.600 \times 10^6$$

$$\phi = B_{gap} A_{gap} = 0.25 \times 8.75 \times 10^{-4} = 2.188 \times 10^{-4} \text{ Wb}$$

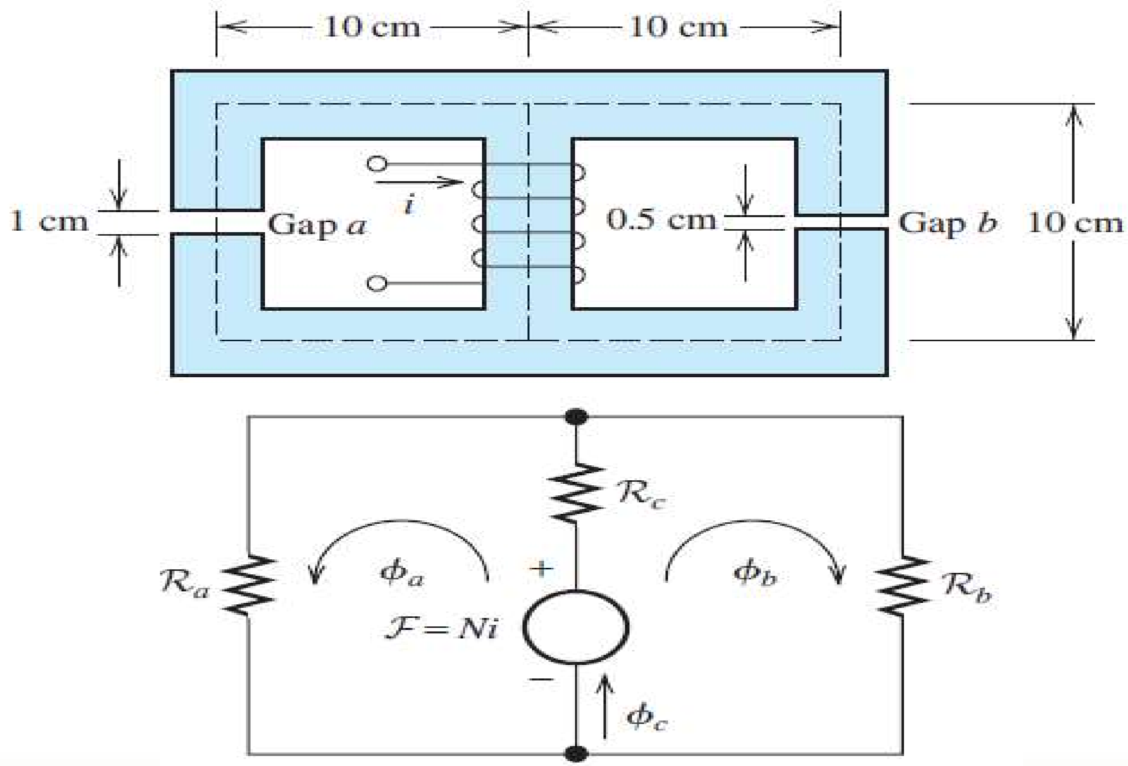
$$\mathcal{F} = \phi\mathcal{R} =$$

$$4.600 \times 10^6 \times 2.188 \times 10^{-4} = 1006\ \text{A}\cdot\text{turns}$$

$$\mathcal{F} = Ni$$

$$i = \frac{\mathcal{F}}{N} = \frac{1006}{500} = 2.012\ \text{A}$$

The iron core shown in Figure has a cross section of 2 cm by 2 cm and a relative permeability of 1000. The coil has 500 turns and carries a current of i = 2A. Find the flux density in each air gap.



For the center path, we have

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_{\text{core}}} = \frac{10 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 1.989 \times 10^5 \text{ A·turns/Wb}$$

For the left-hand path, the total reluctance is the sum of the reluctance of the iron core plus the reluctance of gap a. We take fringing into account by adding the gap length to its width and depth in computing area of the gap. Thus, the area of gap a is $A_a = 3 \text{ cm} \times 3 \text{ cm} = 9 \times 10^{-4} \text{ m}^2$. Then, the total reluctance of the left-hand path is

$$\mathcal{R}_a = \mathcal{R}_{\text{gap}} + \mathcal{R}_{\text{core}}$$

$$= \frac{l_{\text{gap}}}{\mu_0 A_a} + \frac{l_{\text{core}}}{\mu_r \mu_0 A_{\text{core}}}$$

$$= \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} + \frac{29 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 9.420 \times 10^6 \text{ A·turns/Wb}$$

Similarly, the reluctance of the right-hand path is

$$\begin{aligned}\mathcal{R}_b &= \mathcal{R}_{\text{gap}} + \mathcal{R}_{\text{core}} \\ &= \frac{l_{\text{gap}}}{\mu_0 A_b} + \frac{l_{\text{core}}}{\mu_r \mu_0 A_{\text{core}}}\end{aligned}$$

$$= \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} + \frac{29.5 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 6.953 \times 10^6 \text{ A}\cdot\text{turns/Wb}$$

Next, we can combine the reluctances $\mathcal{R}_a$ and $\mathcal{R}_b$ in parallel. Then, the total reluctance is the sum of $\mathcal{R}_c$ and this parallel combination:

$$\mathcal{R}_{\text{total}} = \mathcal{R}_c + \frac{1}{1/\mathcal{R}_a + 1/\mathcal{R}_b}$$

$$= 1.989 \times 10^5 + \frac{1}{1/(9.420 \times 10^6) + 1/(6.953 \times 10^6)}$$

$$= 4.199 \times 10^6 \text{ A}\cdot\text{turns/Wb}$$

Now, the flux in the center leg of the coil can be found by dividing the magnetomotive force by the total reluctance:

$$\phi_c = \frac{Ni}{\mathcal{R}_{\text{total}}} = \frac{500 \times 2}{4.199 \times 10^6} = 238.1\ \mu\text{Wb}$$

Fluxes are analogous to currents. Thus, we use the current-division principle to determine the flux in the left-hand and right-hand paths, resulting in

$$\phi_a = \phi_c \frac{\mathcal{R}_b}{\mathcal{R}_a + \mathcal{R}_b}$$

$$= 238.1 \times 10^{-6} \times \frac{6.953 \times 10^6}{6.953 \times 10^6 + 9.420 \times 10^6}$$

$$= 101.1\ \mu\text{Wb}$$

Similarly, for gap b we have

$$\phi_b = \phi_c \frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_b}$$

$$= 137.0\ \mu\text{Wb}$$

$$B_a = \frac{\phi_a}{A_a} = \frac{101.1\ \mu\text{Wb}}{9 \times 10^{-4}\ \text{m}^2} = 0.1123\ \text{T}$$

$$B_b = \frac{\phi_b}{A_b} = \frac{137.0\ \mu\text{Wb}}{6.25 \times 10^{-4}\ \text{m}^2} = 0.2192\ \text{T}$$