Mod 3:

1) Magnetic flux (ϕ) - a measurement of the total magnetic field which passes through a given area (Magnetic lines of force produced by a magnet). (Wb) is the unit.

2) Magnetic flux density (B) - amount of magnetic flux thru unit area taken perpendicular to direction of magnetic flux (flux / unit area). (Wb/m^2).

3) MagnetoMotive Force (mmf) - Cause for producing magnetic flux in a magnetic circuit. ($F = NI$)

4) Magnetic field intensity (H) - mmf / unit length
(NI / l)

5) Reluctance S (or R_m) - the opposition offered by a magnetic circuit to the production of magnetic flux (mmf / flux)

6) Permeability (μ) - property of the material to allow the magnetic line of force to pass through it.

7) Magnetization curve (B-H curve) - plots changes in a magnetic circuit's flux density as the magnetic field strength is gradually increased.

8) Relative permeability - The ratio of the flux density produced in a material to the flux density produced in a vaccum (or free space) by the same magnetic field strength.

Leakage flux:

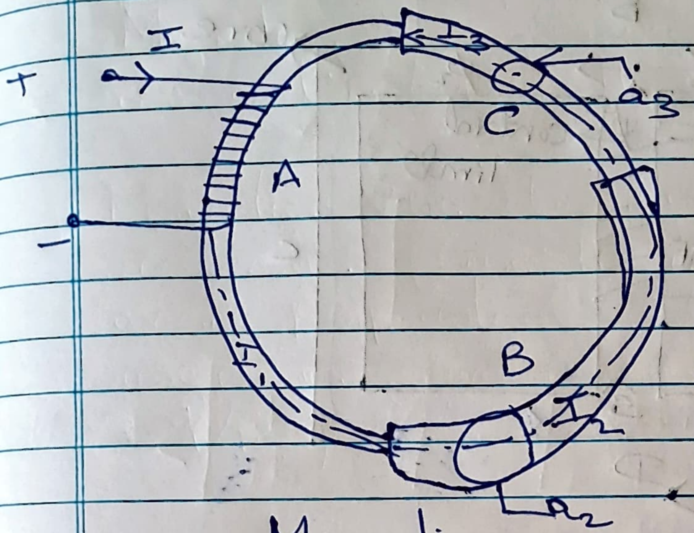
→ The magnetic flux which does not follow the intended path in a magnetic circuit (total flux / useful flux).

Leakage coefficient or leakage factor:

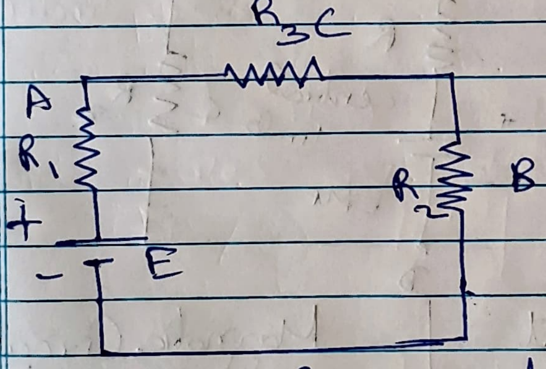
→ The ratio of the total flux produced to the useful flux set up in the air gap of the magnetic circuit.

→ Leakage coefficient = total flux / useful flux.

• Series magnetic circuit - Same magnetic flux flows through each part of the magnetic circuit

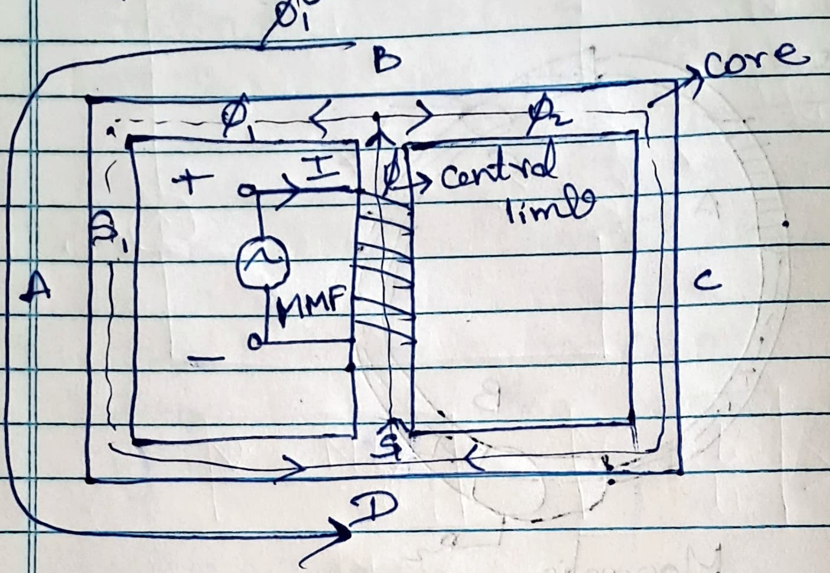


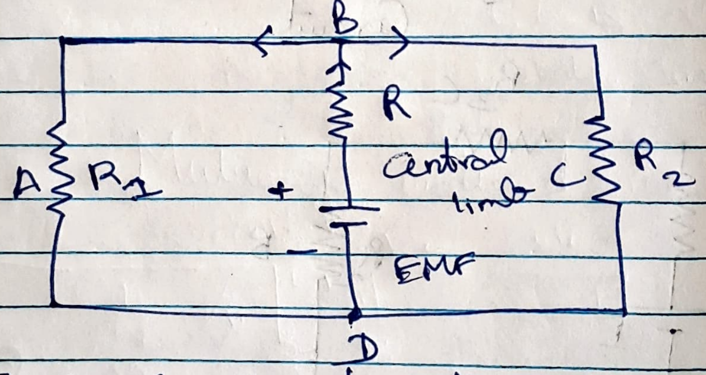
Magnetic Circuit



Equivalent Electrical Circuit.

• **Parallel magnetic circuit** – A magnetic circuit which has more than one path for the magnetic flux.





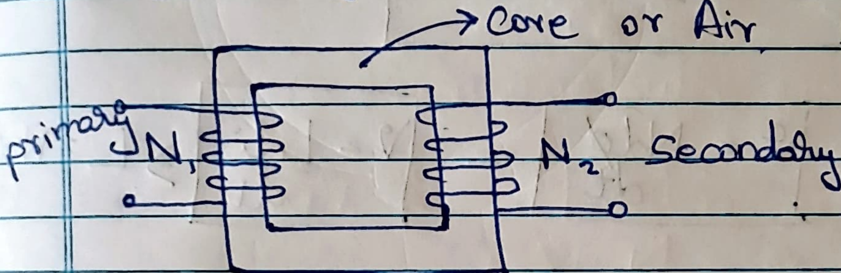
Electric Circuit	Magnetic Circuit
e.m.f E (V)	m.m.f F_m (AT)
current I (A)	flux ϕ (Wb)
Resistance R (Ω)	reluctance S (H^{-1})
$R = \rho l / A$	$S = l/\mu_0 \mu_r A$
$I = E/R$	$\phi = mmf/S$

Magnetically coupled circuits

Magnetically coupled circuit is a combination of two individual circuits which are coupled by magnetic flux.

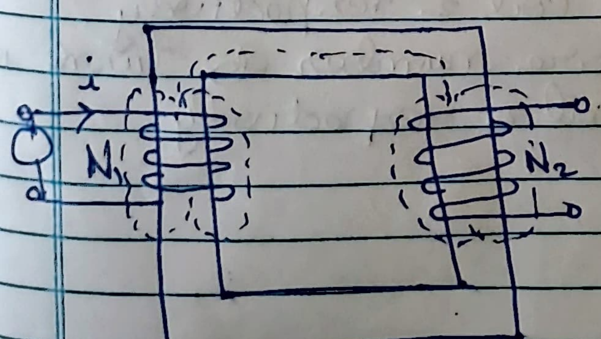
Ex: Transformer

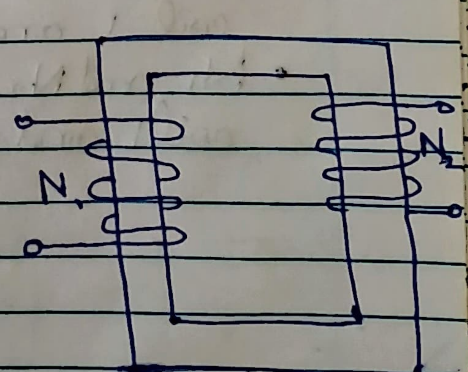
Applications:
Power systems
Radio and television receivers



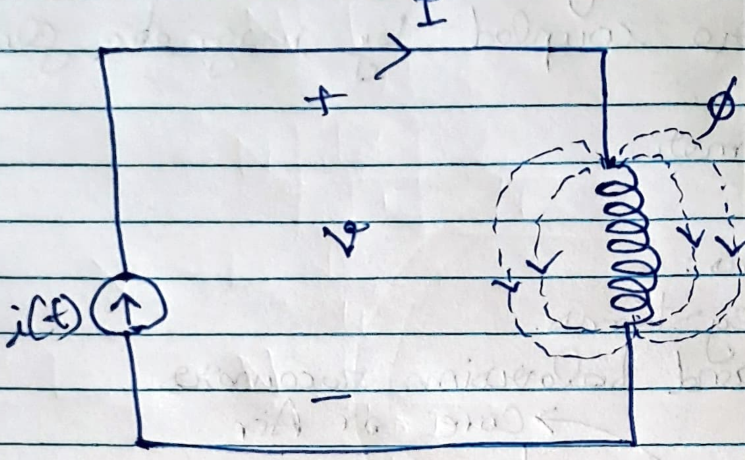
- Self inductance is the property of a coil, which causes a self-induced emf to be produced in the coil itself, when the current through it changes.

- Mutual inductance - ability of one coil to produce an emf in a nearby coil by induction when current in the first coil changes.





Self induced emf

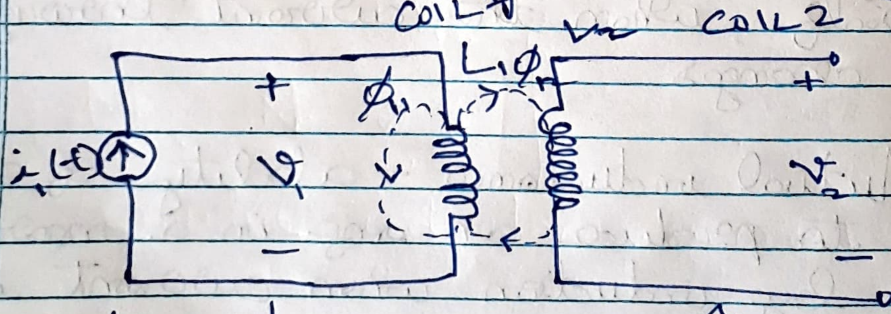


No. of turns - N
Current - I
Flux - φ

$$\boxed{v = N\frac{d\phi}{dt}} = \boxed{v = L\frac{di}{dt}} \Rightarrow L = N\frac{d\phi}{di}$$

(L is called the self Inductance)

Mutually induced emf



- L_1 and L_2 are self Inductance of coil 1 and coil 2 respectively.
- N_1 and N_2 are number of turns in coil 1 and coil 2 respectively.

Voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_{11}}{dt}$$

$$= N_1 \frac{d\phi_{11}}{di_2} \frac{di_1}{dt}$$

$$= L_1 \frac{di_1}{dt}.$$

$L_1 = N_1 \frac{d\phi_{11}}{di_1}$ (self inductance of coil 1)

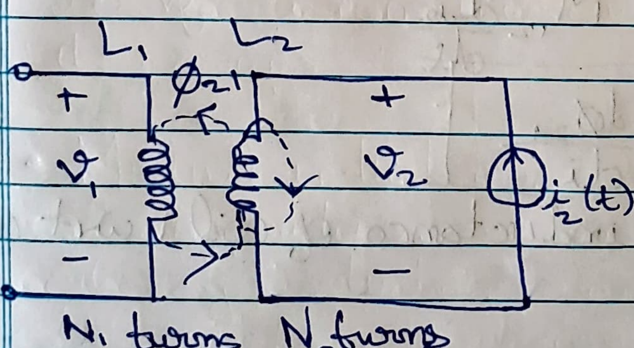
Voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$= N_2 \boxed{\frac{d\phi_{12}}{di_1}} \frac{di_1}{dt}$$

$$= M_{21} \frac{di_1}{dt}.$$

$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$ (mutual inductance of coil 2 w.r.t. coil 1)



- L_1 and L_2 are self Inductance of coil 1 and coil 2 respectively.
- N_1 and N_2 are number of turns in coil 1 and coil 2 respectively

Voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

$$= N_2 \frac{d\phi_{22}}{di_2} \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt}$$

$$L_2 = N_2 \frac{d\phi_{22}}{di_2}$$

(self inductance of coil 2)

Voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_{21}}{dt}$$

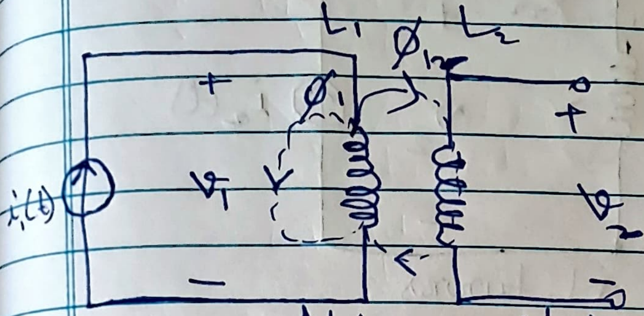
$$= \boxed{N_1 \frac{d\phi_1}{di_2}} \frac{di_2}{dt}$$

$$= M_{12} \frac{di_2}{dt}$$

$$\boxed{M_{12} = N_1 \frac{d\phi_{21}}{di}}$$

(mutual inductance of coil 1 wrt coil 2)

Coefficient of coupling (k)



N_1 turns N_2 turns

$$L_1 = N_1 \frac{d\phi_{11}}{di_1}$$

(self inductance of coil 1)

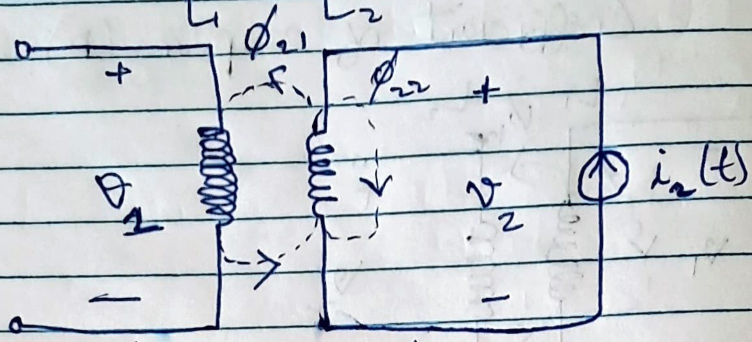
$$\boxed{M_{12} = N_1 \frac{d\phi_{21}}{di_2}}$$

(mutual inductance of coil 1 w.r.t coil 2).

$$M = N_1 \frac{\phi_{21}}{i_2} = N_1 \frac{k\phi_{22}}{i_2} \quad [\phi_{21} = k\phi_{22}]$$

$$M^2 = N_2 \frac{k\phi_{11}}{i_1} \, N_1 \frac{k\phi_{22}}{i_2} = k^2 L_1 L_2$$

$$\Rightarrow M = k\sqrt{L_1 L_2} \Rightarrow \boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$



N_1 turns N_2 turns

$$L_2 = N_2 \frac{d\phi_{22}}{di_2}$$

(self inductance of coil 2)

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

(mutual inductance of coil 2 wrt coil 1)

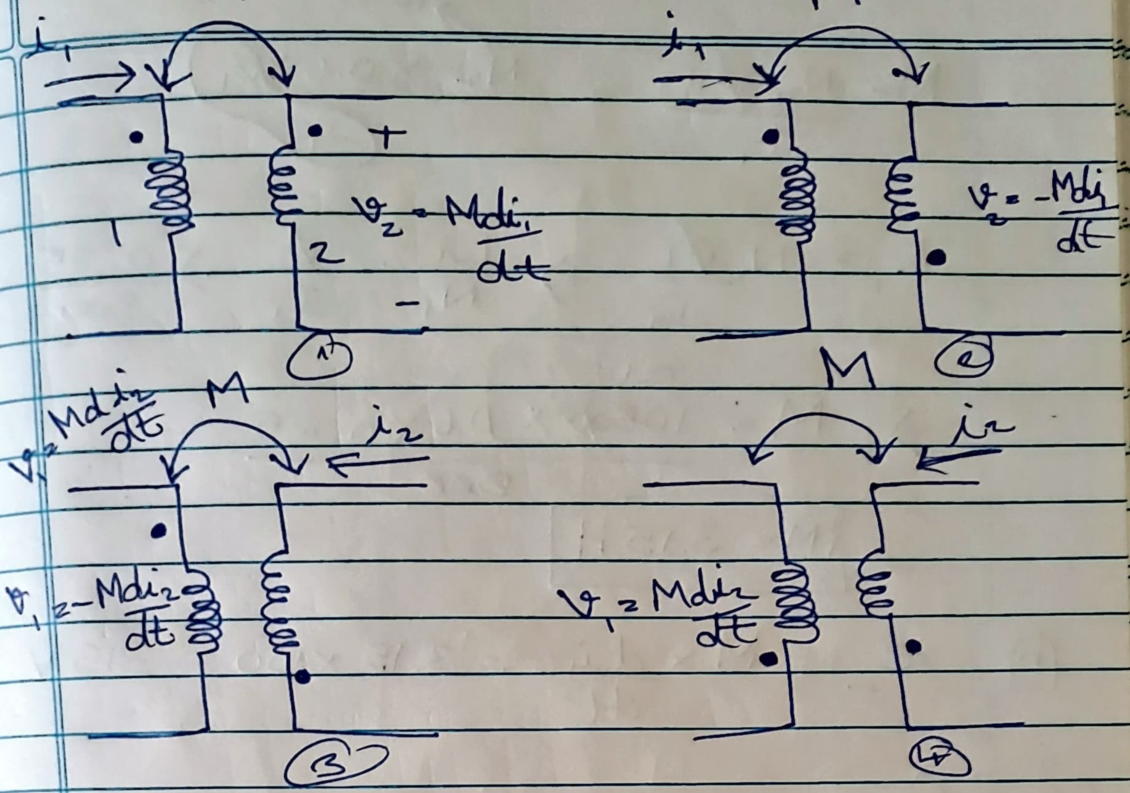
$$M = N_2 \frac{\phi_{12}}{i_1} = N_2 \frac{K\phi_{12}}{i_1}$$

Dot Convention

- If the current enters at the dotted terminal of one coil (or inductor), then it induces a voltage at another coil (or inductor), which is having positive polarity at the dotted terminal.

- If the current leaves from the dotted terminal of one (or inductor), then it induces a voltage at another (or inductor) which is having negative polarity at the dotted terminal.

$M = K\sqrt{L_1 L_2}$

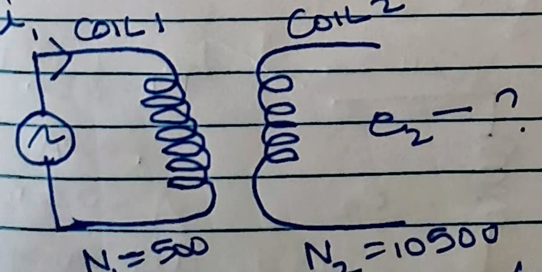


Problems:

1) The self inductance of a coil of 500 turns in 0.25 H. If 60% of the flux is linked with a second coil of 10500 turns, calculate.

(a) the mutual inductance b/w the coils.
(b) emf induced in the second coil when current in the first coil changes at the rate of 100 A/sec.

Ans:



L - self inductance
M - Mutual inductance

$$\boxed{M = \frac{N_2 \phi_2}{I_1}}$$

(a) $N_1 = 500$ $N_2 = 10500$

$L_1 = 0.25H$ $\phi_2 = 0.6\phi_1$

$\frac{di_1}{dt} = 100 A/s$

$$M = \frac{N_2 \phi_2}{I_1} = \frac{\overset{10500}{N_2} \times 0.6 \times \phi_1}{I_1}$$

$$L_1 = \frac{N_1 \phi_1}{I_1} \Rightarrow \frac{L_1}{N_1} = \frac{\phi_1}{I_1} \Rightarrow \frac{0.25}{500} = \frac{\phi_1}{I_1}$$

$$\Rightarrow M = \frac{\overset{21}{\cancel{10500}} \times 0.6 \times 0.25}{\cancel{500}}$$

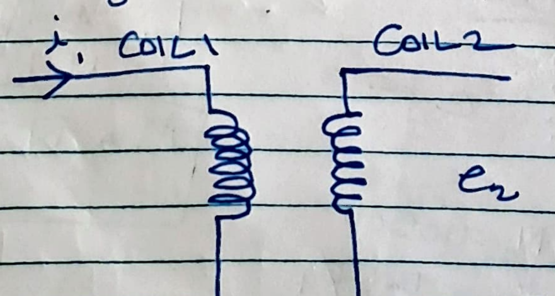
$$\boxed{M = 3.15 H}$$

(b) $e_2 = M \times \frac{di}{dt} = 3.15 \times 100 = 315 V$

Q) The number of turns in a coil is 250. When a current of 2A flows in this coil, the flux in the coil is 0.3m Wb. When this current is reduced to zero in 2ms, the voltage induced in a coil lying in the vicinity of coil is 63.75V. The coupling coefficient is 0.75.

Find mutual inductance b/w both the coils, self inductance of coil 2, number of turns in coil 2, flux linkage in coil 2.

Ans:



$N_1 = 250$

$I_1 = 2A$

$\phi = 0.3 \ mWb$

$dt = 2m \ sec$

To find: M, L_2, N_2, ϕ_2.

Page :

(a) To find M;

$$e_2 = \frac{M\,di_1}{dt}$$

$$63.75 = M \times \frac{\cancel{2}}{\cancel{2} \times 10^{-3}}$$

$$\boxed{M = 63.75\ mH}$$

(b) To find L_2;

$$M = k\sqrt{L_1 L_2}$$

$$M^2 = k^2 L_1 L_2$$

$$L_2 = \frac{M^2}{k^2 L_1}$$

$$L_2 = \frac{(63.75 \times 10^{-3})^2}{(0.75)^2 \times 37.5 \times 10^{-3}}$$

$$L_2 = 192.66\ mH \approx 193\ mH$$

$$\boxed{L_2 = 193\ mH}$$

$$\boxed{L_2 = N_2 \frac{d\phi_2}{di_1}}$$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{250 \times 0.3}{2}$$
$$L_1 = 37.5\ mH$$

(c) To find ϕ_2;

$$\phi_2 = k \times \phi_1$$
$$= 0.75 \times 0.3 \times 10^{-3}$$

$$\boxed{\phi_2 = 0.225\ mWb}$$

(d) To find N_2,

$$e_2 = L_2 \frac{di_2}{dt}$$

also

$$\boxed{e_2 = N_2 \frac{d\phi_2}{dt}}$$

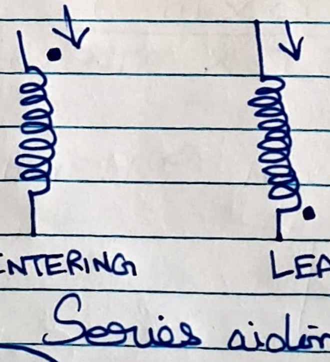
$$63.75 = N_2 \times \frac{0.225 \times \cancel{10^{-3}}}{2 \times \cancel{10^{-3}}}$$

$$N_2 = 566.67 \text{ turns}$$

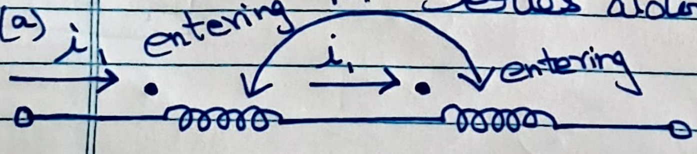
$$\Rightarrow \boxed{N_2 = 567 \text{ turns}}$$

__Coupled coils in Series:__

__General:__



(a) M Series aiding.



$$\boxed{L = L_1 + L_2 + 2M}$$

__Derivation:__

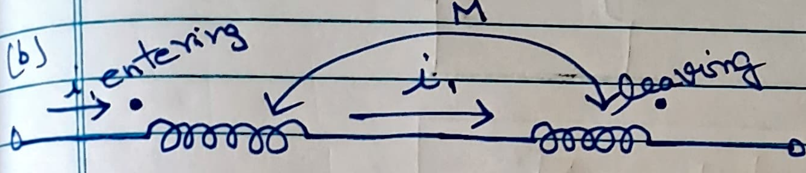
$$L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$\Rightarrow L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt}(L_1 + L_2 + 2M) = \frac{di}{dt} L_{eq}.$$

$$\Rightarrow \boxed{L_{eq} = L_1 + L_2 + 2M}$$

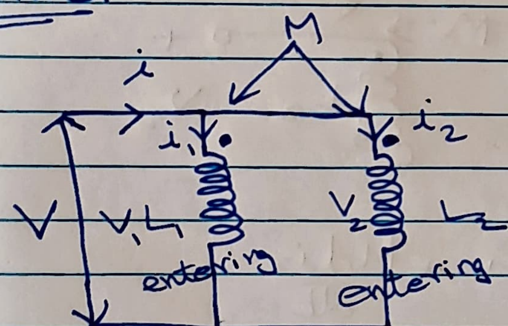
(b)



$$\boxed{L = L_1 + L_2 - 2M}$$

Coupled coils in parallel:

(a) **AIDING:**



Derivation:

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

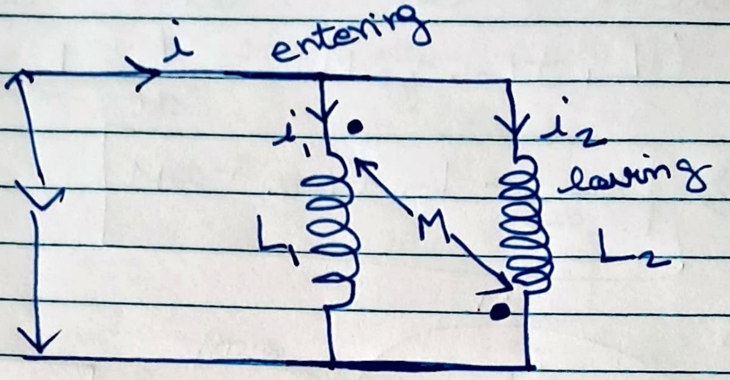
$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt};$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Finally:

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

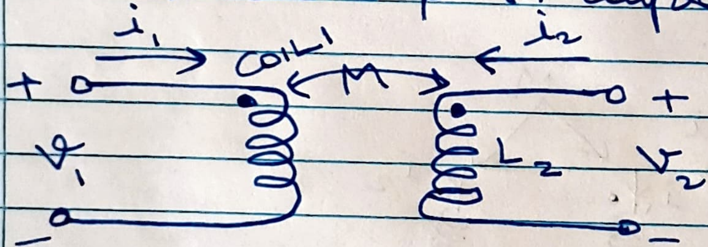
(b) <u>OPPOSING</u> :



$$\boxed{L_{eq} = \frac{L_1L_1 - M^2}{L_1 + L_2 + 2M}}$$

VI relationship in coupled circuits:

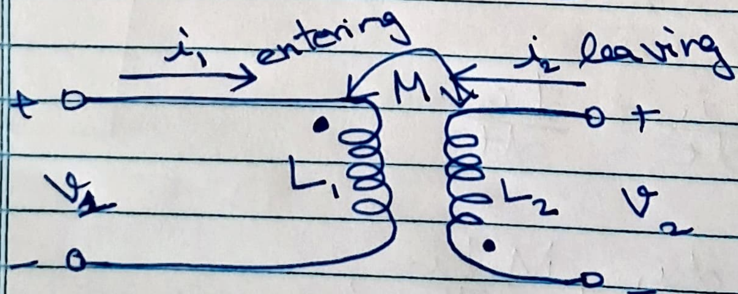
[a]



$$v_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + \frac{Mdi_1}{dt}.$$

(b)



$$v_1 = L_1\frac{di_1}{dt} - \frac{Mdi_2}{dt}$$

$$v_2 = -\frac{Mdi_1}{dt} + \frac{L_2 di_2}{dt}.$$