LECUTRE #15.
POWER IN AC CIRCUITS

1. Instantaneous Power.

If $v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

then $p(t) = v(t) \cdot i(t)$.

$= V_m I_m \cos(\omega t + \theta_v)(\cos(\omega t + \theta_i))$

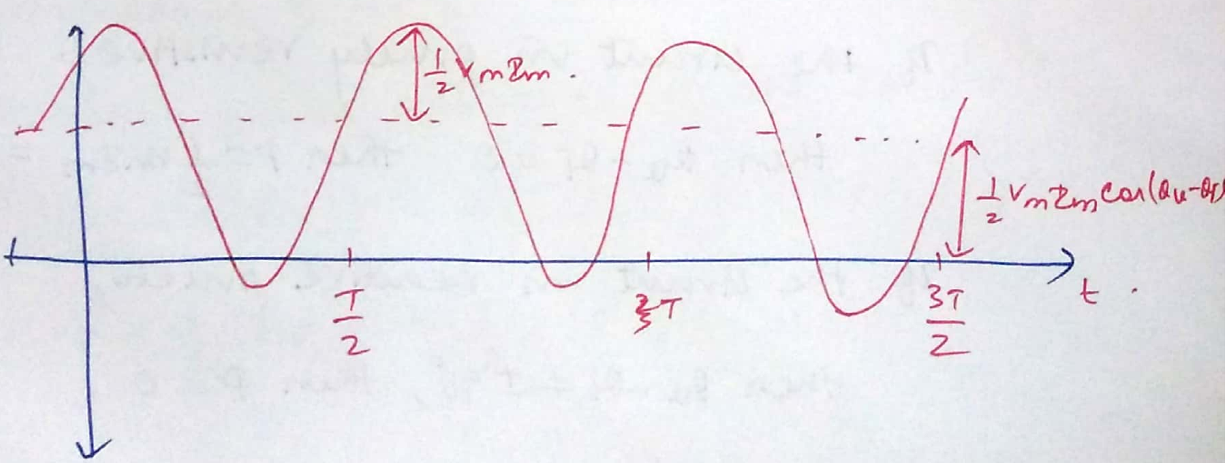
using $\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$$p(t) = \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal.}} + \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant}}$$

Sinusoidal. twice the frequency of $v(t)$, $i(t)$

Constant - independent of ω, t. depends of $(\theta_v - \theta_i)$

Instantaneous power is difficult to measure.



2. Average Power.

- average of the instantaneous power over one period.
- measured in Wattmeter. unit is WATTs.

average power $P = \frac{1}{T}\int_0^T p(t)\,dt$.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i).$$

If P is calculated using phasor.

$$V = V_m \angle \theta_v \qquad I = I_m \angle \theta_i.$$

$$\frac{1}{2} V I^* = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + j\frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)$$

$$P = \frac{1}{2} \text{Real}\{V I^*\}.$$

As $\theta_v - \theta_i$ is the angle between voltage and current,

If the circuit is purely resistive,

then $\theta_v - \theta_i = 0$ then $P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$.

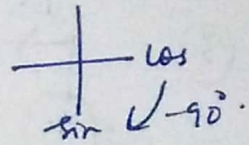
If the circuit is reactive purely,

then $\theta_v - \theta_i = \pm 90^\circ$, then $P = 0$.

Resistive - absorbs real power.

Reactive circuit - absorbs zero real power.

Example.



① $v(t) = 330 \cos(10t + 20^\circ)\,V$

$i(t) = 33 \sin(10t + 60^\circ)\,A = 33 \cos(10t - 30^\circ)$

Instantaneous power.

$$p(t) = v(t)\,i(t).$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

$$= \frac{1}{2} \times 330 \times 33 \times \cos(20 - 30^\circ)$$

$$+ \frac{1}{2}\, 330 \times 33 \times \cos(20t + 20 - 30^\circ)$$

$$= 5445 \cos(-10^\circ) + 5445 \cos(20t - 10^\circ)$$

$$p(t). = \underline{\underline{5362}} + 5445 \cos(2t - 10^\circ).$$

average power $= 5362\ W$.

② A current $I = 33 \angle 30^\circ\ A$ flows through an impedance of $Z = 40 \angle -22^\circ\ \Omega$.

Average power delivered to the load = ?

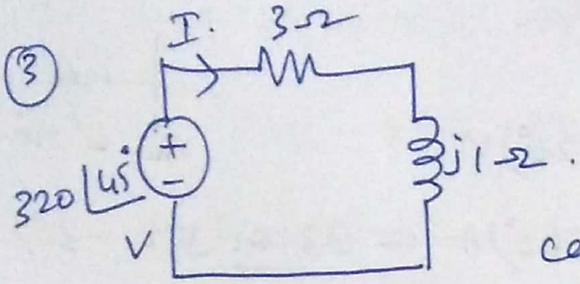
$$Z = R + jX = 40 \angle -22^\circ = 3.7087 - 1.498i$$

$$P = \frac{1}{2} |I_m|^2 R = \frac{1}{2} \times 33^2 \times 3.7087$$

$$P = 2019.387\ W$$

$$P = 2.019\ kW.$$

(3)



calculate power supplied by vt-source.

power absorbed by resistor & inductor = ?

$$I = \frac{V}{Z} = \frac{320\angle 45^\circ}{3+j1} = 101.19\angle 26.56^\circ$$

$V = 320\angle 45^\circ \quad I = 101.19\angle 26.56^\circ$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} \times 320 \times 101.19 \cos(45 - 26.56^\circ)$

power supplied by source $\Rightarrow P = 15359.11$ W.

power absorbed by resistor $= \frac{1}{2}|I|^2 R$

$= \frac{1}{2} \times 101.19^2 \times 3$

$= 15359.12$ W.

power absorbed by inductor = 0 W.

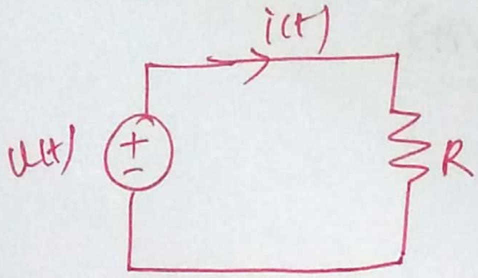
power supplied by the vt. source = power absorbed by the resistor.

No real power absorbed by Inductor.

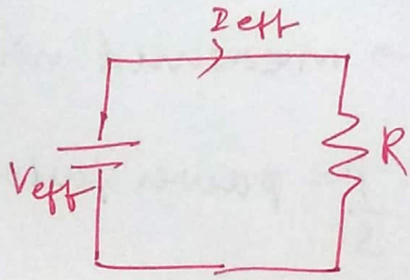
3. RMS / EFFECTIVE VALUE :

RMS – Root Mean Square Value

It is the dc current that delivers the same average power to the resistor as the periodic current.



ac



dc.

$$P = \frac{1}{T}\int_0^T i^2 R \, dt. \qquad P = I_{eff}^2 R.$$

equating both we get.

$$I_{eff}^2 = \frac{1}{T}\int_0^T i^2 \, dt.$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 \, dt} = I_{rms}$$

Similarly

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T v^2 \, dt} = V_{rms}.$$

For a Sinusoid,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1+\cos 2\omega t}{2} \, dt} = \frac{I_m}{\sqrt{2}}$$

and $V_{rms} = \dfrac{V_m}{\sqrt{2}}$ $\qquad P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$.

$$P = V_{rms} I_{rms} \cos(\theta_u - \theta_i).$$

4. Apparent power & POWER FACTOR

defined as $S = V_{rms} I_{rms}$.

angle $\cos(\theta_u - \theta_i)$ = power factor.

$P = S \times$ power factor $= V_{rms} I_{rms} \cos(\theta_u - \theta_i)$.

$S \rightarrow$ measured in (VA)

$\frac{P}{S}$ = power factor.

Impedance $Z = \frac{V}{I} = \frac{V_m}{I_m} \angle \theta_u - \theta_i$.

$\therefore$ load angle = power factor angle.

5. complex power.

$S = V_{rms} I_{rms}^*$ (defined quantity)

Where, $V_{rms} = \frac{V_m}{\sqrt{2}}$, and $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$S = \underbrace{V_{rms} I_{rms}}_{\text{apparent power (magnitude of S)}} \cos(\theta_u - \theta_i) + j V_{rms} I_{rms} \sin(\theta_u - \theta_i)$$

$S = P + jQ$.

$P = V_{rms} I_{rms} \cos(\theta_u - \theta_i) \Rightarrow$ Real power (W)

$Q = V_{rms} I_{rms} \sin(\theta_u - \theta_i) \Rightarrow$ Reactive power. (VAR)

In terms of impedance

$$Z = R + jX.$$

$$S = I_{rms}^2 Z = I_{rms}^2 R + j\, I_{rms}^2 X.$$

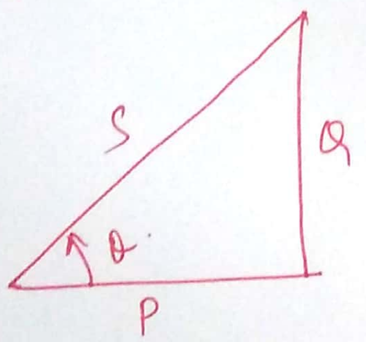
$$P = I_{rms}^2 R \qquad Q = I_{rms}^2 X.$$

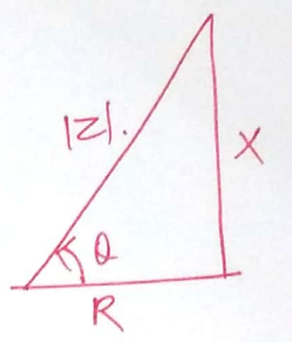
Note:

$P = 0$ for reactive loads. $\quad V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$Q = 0$ for resistive loads. $\quad V_{rms} I_{rms} \cos(\theta_v - \theta_i)$.

All these quantities can be related via a power/impedance triangle.





$$\cos\theta = \frac{P}{S} \qquad \cos\theta = \frac{R}{|Z|}.$$