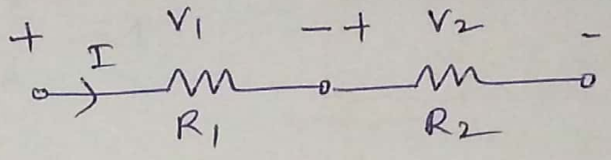
LECTURE # 7

SERIES / PARALLEL RESISTANCES

(1) SERIES

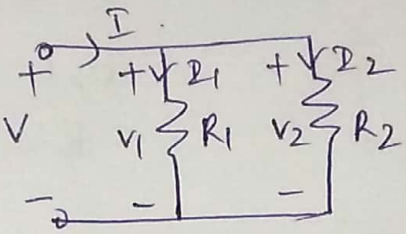


$$R_{eq} = (R_1 + R_2)\ \Omega$$

$$V = V_1 + V_2$$

$$I = \frac{V}{R}$$

(2) PARALLEL



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = V_1 = V_2$$

$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

$$I = I_1 + I_2$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

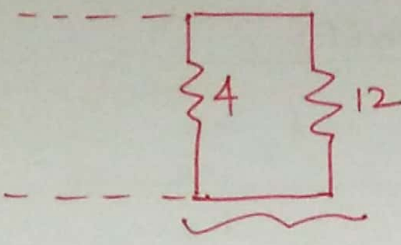
Problems

(1) Combine the resistors and find Req.

Start from the last part of the circuit.

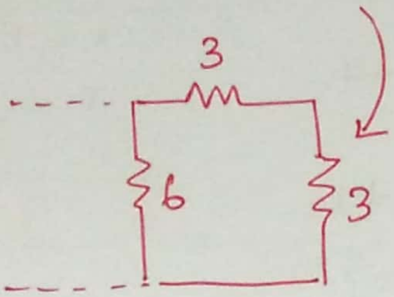
(2)

4, 5, 3 are in series. $R = 4+5+3 = 12\ \Omega$



4, 12 in parallel

$$R = \frac{4 \times 12}{4+12} = 3\ \Omega$$

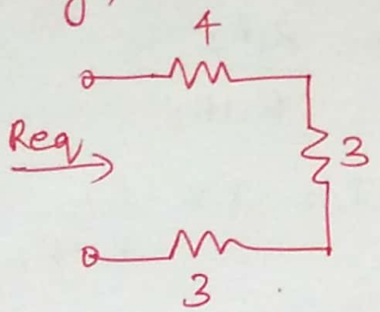


3+3 series $R = 6\ \Omega$

6, 6 parallel

$R = 3\ \Omega$

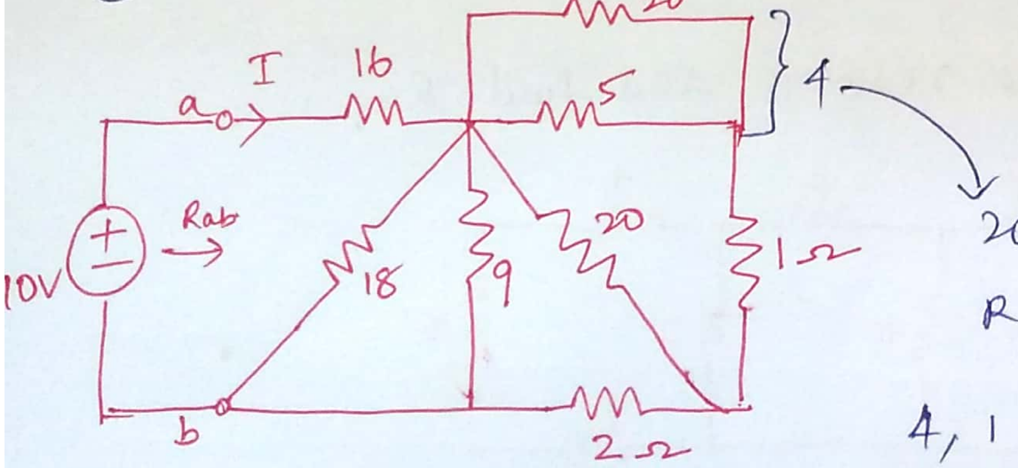
Finally,



$R_{eq} = 4+3+3 = 10.$

$$\boxed{R_{eq} = 10\ \Omega.}$$

(2) Find Rab and the total current I in the circuit.

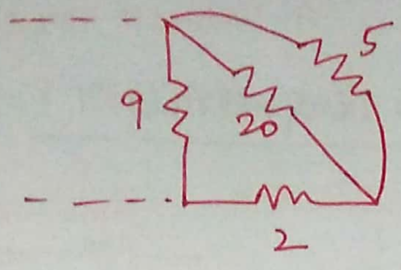


20, 5 in parallel.

$$R = \frac{20 \times 5}{25} = 4.$$

4, 1 are in series.

$R = 5\ \Omega$



again 20,5 in parallel

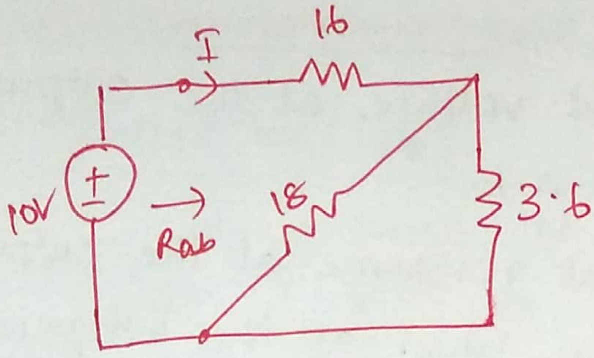
$R = 4\,\Omega$

4, 2 in series

$R = 6\,\Omega$.

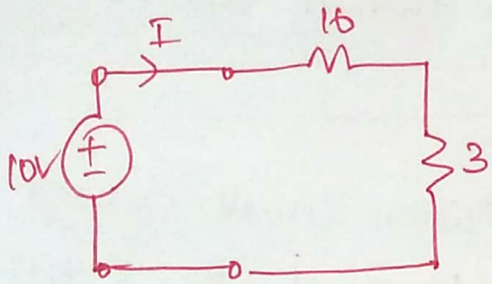
9, 6 in parallel.

$$R = \frac{9 \times 6}{9+6} = 3.6\,\Omega$$



Here 18, 3.6 in parallel.

$$R = \frac{18 \times 3.6}{18+3.6} = 3$$



$R_{ab} = 16 + 3 = 19\,\Omega$

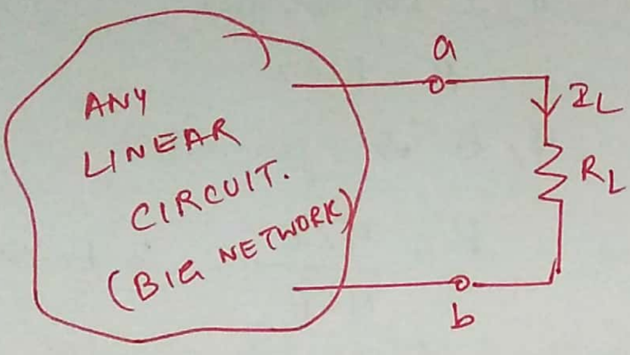
$$I = \frac{V}{R_{ab}} = \frac{10}{19}$$

$$\boxed{I = 0.526\text{ A}, \; R_{ab} = 19\,\Omega}$$

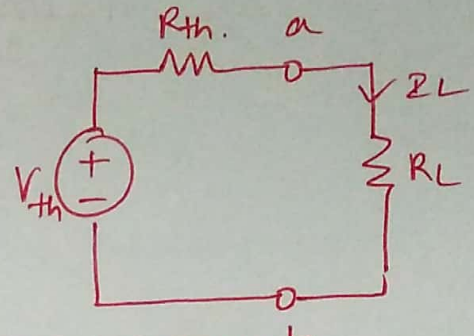
NETWORK REDUCTION

M. LEON THEVENIS (1857–1926)
A FRENCH TELEGRAPH ENGINEER

THEVENIN & NORTON EQUIVALENT



$\Rightarrow$

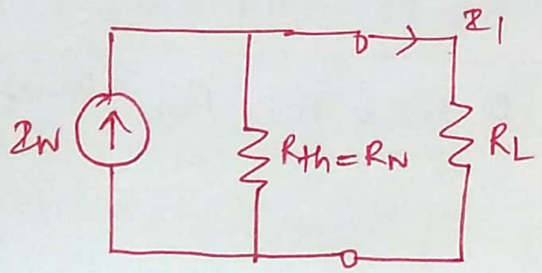


THEVENIN EQUIVALENT.

Where, V_{th} = open-circuit voltage at the output terminals a-b.

R_{th} = equivalent resistance at the output terminals when all the independent sources are turned off.

NORTON EQUIVALENT.



E.L. NORTON (1926)
AN AMERICAN ENGINEER AT BELL TELEPHONE LABORATORIES.

Where,

I_N = short circuit current through the output terminals a-b

We will see that

$$V_{th} = I_N R_{th}.$$

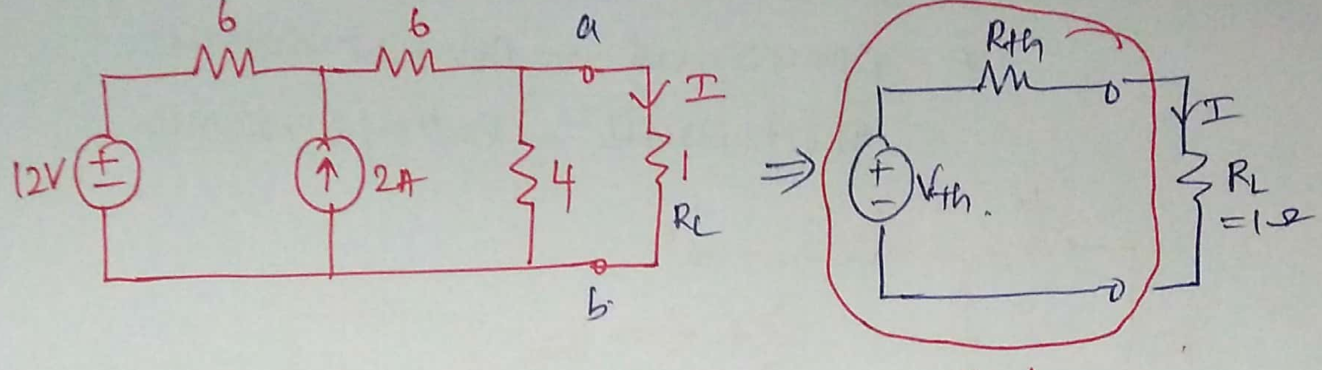
(or)

$$V_{oc} = I_{sc} R_{th}.$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = R_N.$$

Problem

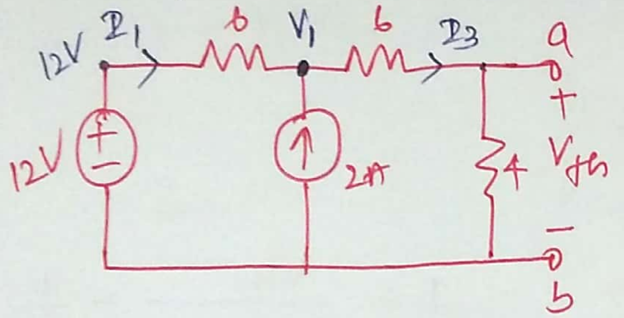
(1) Find the thevenin equivalent, then find I.



Consider the circuit, (excluding R_L)

To find V_{th} (V_{oc})

open circuit the output terminals, a-b



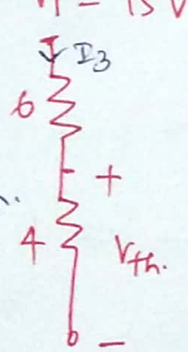
6, 4 series.
Combine $R = 10\Omega$.

Apply KCL at node V_1, $I_1 + 2 = I_3$.

$$\frac{12 - V_1}{6} + 2 = \frac{V_1 - 0}{10}.$$

$$2 + 2 = \frac{V_1}{10} + \frac{V_1}{6} = \frac{16 V_1}{60}.$$

$$V_1 = \frac{4 \times 60}{16} = 15\,V.$$



Voltage division.

$$V_{th} = V_1 \times \frac{4}{4+6} = 6\,V.$$

$$\boxed{V_{th} = 6V}$$

(or) $I_3 = \frac{V_1}{10} = 1.5\,A$ $\therefore V_{th} = 1.5 \times 4 = 6\,V$

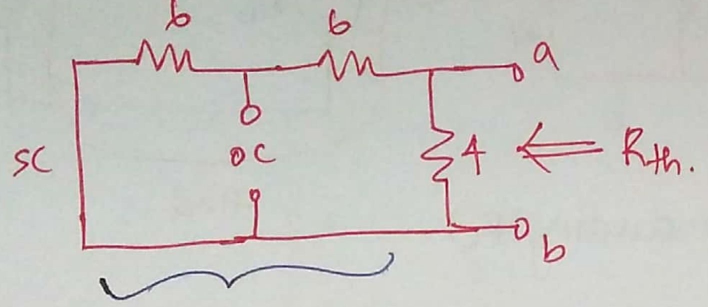
6

To find R_{th}

turn off all independent sources

i.e open circuit – current sources.

short circuit – voltage sources.

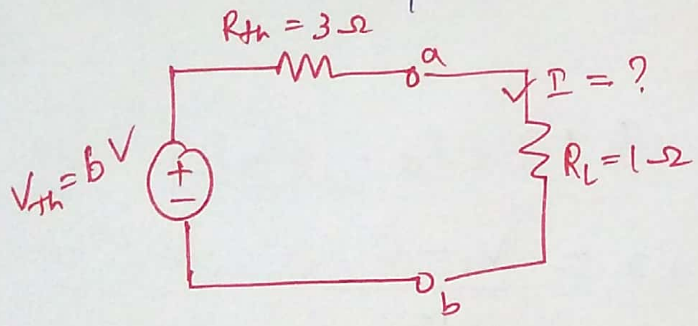


6, 6 series. $R = 12\ \Omega$.

12 and 4 parallel $R_{th} = \dfrac{12 \times 4}{12+4} = 3\ \Omega$.

$$\boxed{R_{th} = 3\ \Omega}$$

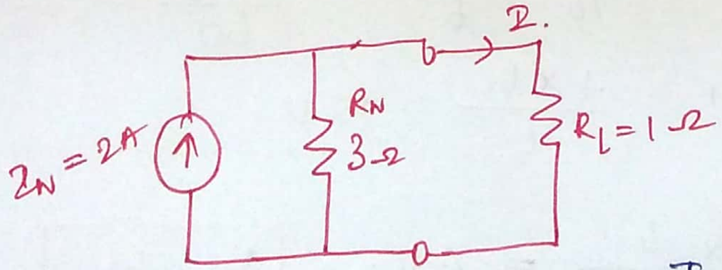
Thevenin Equivalent:



$$I = \frac{V_{th}}{R_L + R_{th}} = \frac{6}{3+1}$$

$$\boxed{I = 1.5A}$$

Norton Equivalent.



$$I_N = \frac{V_{th}}{R_{th}} = \frac{6}{3} = 2A$$

$$R_N = R_{th} = 3\ \Omega$$

current division → $I = 2 \times \dfrac{3}{3+1} = 1.5A$