Date 4/6

From ① ⇒ $-2i_1 - 7\left(\frac{-8}{9}\right)i = 1$

$$\frac{-18i_1 + 56i_1}{9} = 1 \Rightarrow i_1 = \frac{9}{38}\,A$$

$$R_{th} = \frac{38}{9}\,\Omega = 4.222\,\Omega$$

For max power ⇒ $R_{Th} = R_L$

$$\boxed{R_L = 4.222\,\Omega}$$

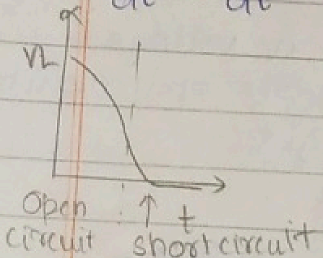
$$P_L = \frac{V_{Th}^2}{4R_L} = \frac{(6.6)^2}{4 \times 4.222} = 2.596\,W$$

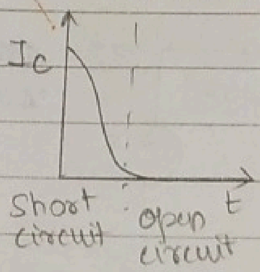
MODULE: 2 : AC circuits

* we dont connect capacitor / inductor in DC circuits,

$V_L = -L\frac{di}{dt}$; $i_c = c\frac{dv}{dt}$

$\frac{di}{dt}, \frac{dv}{dt} = 0$ in DC circuit, hence, $V_L = 0$ & $i_c = 0$



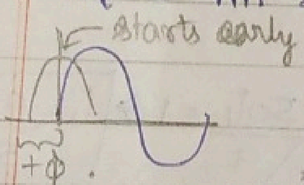


Equalent capacitance

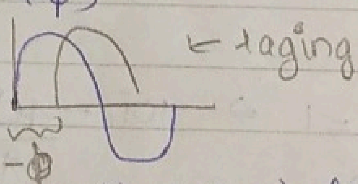
series ⇒ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$; Parallel ⇒ $C_{eq} = C_1 + C_2$

* AC: current & voltage are sinosoidal (should see like which wave reaches peak first / 0 first)

$V_t = V_m \sin(\omega t + \phi)$



$V_t = V_m \sin(\omega t + \phi)$

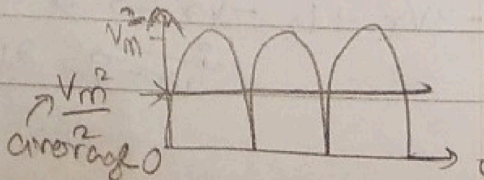


$V_t = V_m \sin(\omega t - \phi)$

*
$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{-V_m}{\pi}[\cos\omega t]_0^{\pi}$$

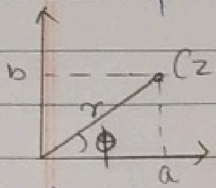
$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$

* $V_{RMS} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$



we cannot add voltage across an AC element scalarly.
hence we convert sinusoids to phasors:-

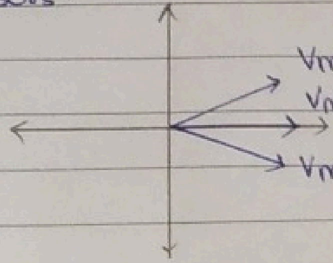
* Phasors: complex number (represented as $a \pm ib \Rightarrow |a| \angle \phi = |a| e^{i\phi}$
  which represents magnitude & direction; rectangular form, polar form, exponential form.



$\phi = \tan^{-1}\left(\frac{b}{a}\right)$; $r = \sqrt{a^2+b^2}$

exp: $re^{i\phi} = r(\cos\phi + i\sin\phi)$

* in phasors:



$V_m \angle 30 = V_m \sin(\omega t + 30^\circ)$

$V_m \angle 0 = V_m \sin \omega t$

$V_m \angle -30 = V_m \sin(\omega t - 30^\circ)$

* All phasors are vector but not all vectors are phasors.

Problem①: $V_1 = 12\sin(\omega t + 30^\circ)$ & $V_2 = 20\cos(\omega t - 30)$

$\Rightarrow V_1 = 12\cos(\omega t - 60^\circ)$

$V_1 = 12 \angle -60^\circ$

$a = 12\cos 60^\circ = 6$

$b = 12 \sin 60^\circ = -\frac{\sqrt{3}}{2} \times 12 = -6\sqrt{3}$

~~$V_1 = 6+6$~~

$V_1 = 6 - 6\sqrt{3}\, i$

$V_2 = 20 \angle -30$

$a = 20\cos 30^\circ$

$b = -20\sin 30^\circ = -10$

$V_2 = 20\cos 30^\circ - 10i$

$V_1 + V_2 = 23.2 - j(20.39)$

$r = \sqrt{a^2+b^2} = \sqrt{(23.2)^2 + (20.39)^2} = 30.88$

$\phi = \tan^{-1}\left(\frac{b}{a}\right) = -41.31^\circ$

$V_1 + V_2 = 30.88 \cos(\omega t - 41.31^\circ)$

* multiplication of 2 phasors $\vec{V_1} \times \vec{V_2} = V_1 V_2 \angle \phi_1 + \phi_2$
* division of 2 phasors, $\vec{V_1} \div \vec{V_2} = \frac{V_1}{V_2} \angle \phi_1 - \phi_2$
* square of phasor $\vec{V_1} = V_1^2 \angle 2\phi_1$
* square root of phasors $\vec{V_1} = \sqrt{V} \angle \frac{1}{2}\phi_1$
* addition of $V_1 = a_1 \pm ib$ & $V_2 = a_2 \pm ib_2$

  $V = (a_1 + a_2) \pm (b_1 + b_2)i$
* Substraction of $V_1 = a_1 \pm ib$ & $V_2 = a_2 \pm ib_2$

  $V = (a_1 - a_2) \pm i(b_1 - b_2)$
* conjugate of $V = a + ib$ is $V = a - ib$

  $V = V_m \angle \phi$ is $V = V_m \angle -\phi$

  $V = re^{i\phi}$ is $V = re^{-\phi i}$

H.W: Evaluate the complex numbers

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

$V_1\angle\phi_1 + V_2\angle\phi_2 = (a_1+a_2) \pm (b_1+b_2)j$

~~$V_1 = 40\cos 50^\circ$~~

$V_1 = a_1 \pm ib_1$

$a_1 = 40\cos 50^\circ = 25.711$

$b_1 = 40\sin 50^\circ = 30.641$

~~$V_2 = 20\cos(-30^\circ)$~~

$V_2 = a_2 \pm ib_2$

$a_2 = 20\cos(-30^\circ) = 17.32$

$b_2 = 20\sin(-30)^\circ = -10$

$40\angle 50^\circ + 20\angle -30^\circ = (a_1+a_2) \pm (b_1+b_2)i$

$= (25.711 + 17.32) \pm (30.641 - 10)i$

$= 43.031 \pm 20.641i$

$r = \sqrt{(43.031)^2 + (20.641)^2} = \sqrt{1851.66 + 426.05}$

$r = 47.72$

$\phi = 25.62^\circ$

hence $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

$= (47.72\angle 25.62^\circ)^{1/2}$

$= \sqrt{47.72}\angle \frac{25.62^\circ}{2}$

$= 6.907\angle 12.81$

(b) $\dfrac{10\angle -30^\circ + (3+j4)}{(2+j4)(3-j5)}$

$10\angle -30^\circ$: $a = 10\cos(-30^\circ)$; $b = 10\sin(-30^\circ)$

$a = 8.660$; $b = -5$

$10\angle -30^\circ = 8.660 - 5j$

$\dfrac{(5\sqrt{3} - 5j) + (3+4j)}{(2+4j)(3-5j)}$ ~~$= 15\sqrt{3} + 20\sqrt{3}j - 15j - 20j^2$~~

$= \dfrac{(5\sqrt{3}+3) - j}{6 - 10j + 12j - 20j^2} = \dfrac{(5\sqrt{3}+3) - j}{26 + 2j} = \dfrac{1}{2} \times \cancel{\dfrac{(5\sqrt{3}+3) - j}{(13+j)}}$

$= \dfrac{1}{2} \times \cancel{\dfrac{(5\sqrt{3}+3) - j}{(13+j)}} \times \cancel{\dfrac{(13-j)}{(13-j)}} = \dfrac{((5\sqrt{3}+3) - j)}{(26+2j)} \times \dfrac{(26-2j)}{(26-2j)}$

$= \cancel{\dfrac{1}{2}} \times$ $=$ ~~225.16~~ $\dfrac{303.16 - 23.32j - 26j + 2j^2}{676 - 4j^2}$

$= \dfrac{301.16 - 49.32j}{680}$

$= 0.4428 - 0.0725j$

$r = \sqrt{(0.4428)^2 + (0.0725)^2} = \sqrt{0.196 + 0.00525} = \sqrt{0.2012}$

$r = 0.4485$

$\phi = \tan^{-1}\left(\frac{-0.0725}{0.4428}\right) = -9.2985^\circ$

hence the simplified one $0.4485 \cos(-9.2985^\circ)$

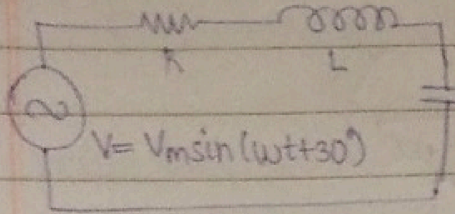
$= 0.4485 \angle -9.2985^\circ$

* In Ac Analysis :-

1. sinosoids to Phasors
2. Technique
3. Phasors to sinosoids.

In Inductive circuit, current lags voltage

Ex 1:



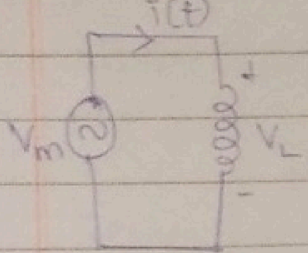
$V_m \sin(\omega t+30^\circ) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$

we have to convert i(t) into phasors before diff / integrating

$V_m \angle -60^\circ = RI + L j\omega I + \frac{1}{C}\frac{I}{j\omega}$

cont next page ⇒

*



$v(t) = V_m \sin\omega t = V_m\angle 0$

$V(t) = L\frac{di(t)}{dt} \Rightarrow i(t) = \frac{1}{L}\int v(t)dt$

$i(t) = \frac{1}{L}\int V_m \sin\omega t\, dt$

$i(t) = \frac{V}{L\omega}(-\cos\omega t) \Rightarrow i(t) = \frac{V_m(-\cos\omega t)}{\omega L}$

$[\because \sin(270^\circ+\omega t) = -\cos\omega t$

$i(t) = \frac{V_m\sin(270^\circ+\omega t)}{\omega L}$ $[\because \sin(360^\circ+\theta) = \sin(\theta-360^\circ)$

$= \frac{V_m}{\omega L}\sin(270^\circ+\omega t-360^\circ) \Rightarrow i(t) = \frac{V_m}{\omega L}\sin(-90^\circ+\omega t)$

$\Rightarrow i(t) = \frac{V_m}{\omega L}\sin(\omega t-90^\circ) \Rightarrow i(t) = \frac{V_m}{\omega L}\angle -90^\circ$

$V = V_m\angle 0$; $I = I_m\angle -90^\circ$ ⇐ current lags in inductive circuit.

$V_m = I\omega L/\angle -90^\circ \Rightarrow V_m = \frac{I\omega L\angle 0^\circ}{\angle -90^\circ}$

$\Rightarrow V_m = I\omega L\angle 90^\circ$ — (1)

$1\angle 90^\circ = 1e^{j90^\circ} = 1(\cos 90^\circ + j\sin 90^\circ) = 1(0+1j) = j$

$\boxed{\angle 90^\circ = j}$ $\boxed{\angle -90^\circ = -j}$

hence eq① ⇒ $\boxed{V = j\omega L I}$

* in sinosoids

$$V = L\frac{di(t)}{dt}$$

In phasors

$$V = j\omega L I$$

comparing these equation : $\boxed{\frac{d}{dt} = j\omega}$

* In sinosoids

$$i(t) = \frac{1}{L}\int v(t) \cdot dt$$

in phasors

$$I = \frac{V}{j\omega L}$$

comparing these equation : $\boxed{\int . dt = \frac{1}{j\omega}}$

cont. of previous page :-

$$V_m \angle -60^\circ = I\left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$V_m \angle -60^\circ = I\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

* in sinsoids

(i) $v(t) = i(t) R$

(ii) $v(t) = L\frac{di(t)}{dt}$

* $i(t) = \frac{1}{L}\int v(t)dt$

(iii) $V = \frac{1}{C}\int i(t)dt$

In phasors

$V = IR$

$V = j\omega L I$

$I = \frac{V}{j\omega L}$

$V = \frac{I}{j\omega C}$

* In ~~RLC~~ series circuit, we have ~~impedence~~,

$$jX_L = \frac{V}{I} = j\omega L$$

* In capacitive circuit :- $-jX_C = \frac{1}{j\omega L}$

Impedendence = $Z = R + jX$ reactance

where $X = \left(\omega L - \frac{1}{\omega C}\right)$

hence eq ① $\Rightarrow$ $\boxed{\mathring{V} = j\omega L \mathring{I}}$

* in sinosoids	In phasors
$v = L\dfrac{di(t)}{dt}$	$\mathring{V} = j\omega L \mathring{I}$

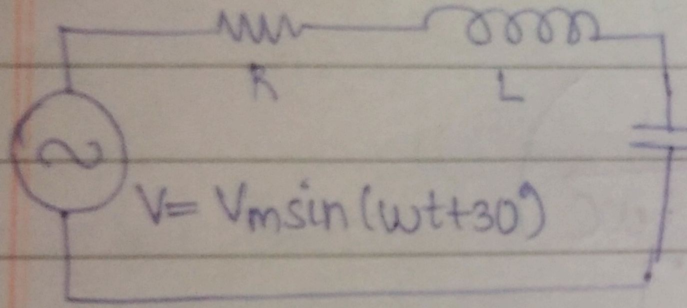
comparing these equation :- $\boxed{\dfrac{d}{dt} = j\omega}$

* In sinosoids	in phasors
$i(t) = \dfrac{1}{L}\int v(t) \cdot dt$	$\mathring{I} = \dfrac{\mathring{V}}{j\omega L}$

comparing these equation : $\boxed{\int \cdot dt = \dfrac{1}{j\omega}}$

3- Phasors to sinosoide.

Ex 1:



C $\quad V_m \sin(\omega t+30^\circ) = R i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int i(t)\,dt.$

we have to convert i(t) into phasors before diff/integrating

$$V_m \angle -60^\circ = RI + L\,j\omega I + \frac{1}{C}\frac{I}{j\omega}$$

$$V_m \angle -60^\circ = I\left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$V_m \angle -60^\circ = I\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]$$

* in sinsoids	In phasors
(i) $v(t) = i(t)R$	$V = IR$
(ii) $v(t) = L\frac{di(t)}{dt}$	$V = j\omega L I$
* $i(t) = \frac{1}{L}\int v(t)dt$	$I = \frac{V}{j\omega L}$
(iii) $V = \frac{1}{C}\int i(t)dt$	$V = \frac{I}{j\omega C}$

* In ~~RL~~ series circuit, we have ~~impedence~~,

$$j X_L = \frac{V}{I} = j\omega L$$

* In capacitive circuit :- $-jX_C = \frac{1}{j\omega L}$

Impedendence $= Z = R + j\underbrace{X}_{\text{reactance}}$

where $X = \left(\omega L - \frac{1}{\omega C}\right)$

Ex 2: Using the Phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential ~~eq~~ equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ) \quad \text{①}$$

Solution:

$V_m = 50;\ \omega = 2,\ \phi = 75^\circ$

converting sinosoidal ① to phasors

$$4I + \frac{8I}{j(2)} - 3j(2)I = 50\angle 75^\circ$$

$$I\cdot(4 - 4j - 6j) = 50\angle 75^\circ$$

$$I(4 - 10j) = 50\angle 75^\circ$$

$$I = \frac{50\angle 75^\circ}{4 - 10j}$$

$\left[\therefore \text{polar form of } 4 - 10j \Rightarrow \sqrt{4^2 + 10^2}\ \angle \tan^{-1}\left(\frac{-10}{4}\right) = 10.7\angle -68^\circ\right]$

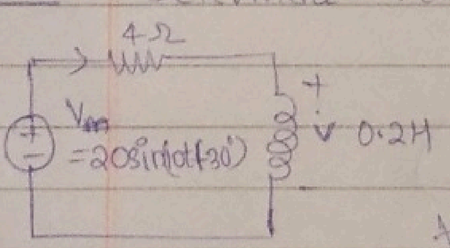
$$I = \frac{50\angle 75^\circ}{10.7\angle -68^\circ}$$

$$I = \frac{50}{10.7}\angle 75^\circ - (-68^\circ)$$

$$\boxed{I = 4.67\angle 143^\circ}$$

$$I = 4.67\cos(2t + 143^\circ)$$

Ex: 3 Determine $v(t)$ and $i(t)$.



$V = 20\sin(10t + 30^\circ)$

$V_m = 20;\ \omega = 10;\ \phi = 30^\circ$

$V = 20\angle 30^\circ$

$\therefore X_L = j\omega L = j \times 10 \times 0.2$

$X_L = 2j$

$$4I + 2jI = 2\angle 30^\circ$$

$$I(4 + 2j) = 2\angle 30^\circ \rightarrow I = \frac{20\angle 30^\circ}{4 + 2j}$$

$\left[\therefore \text{Polar form of } 4 + 2j = \sqrt{4^2 + 2^2}\ \angle \tan^{-1}\left(\frac{1}{2}\right)\right]$

$$I = \frac{20\angle 30^\circ}{4.47\angle 26.5}$$

$$I = 4.47\angle 3.5$$

$$\boxed{I = 4.47\sin(2t + 3.5^\circ)}$$

$V = j\omega L I = V = X_L I$

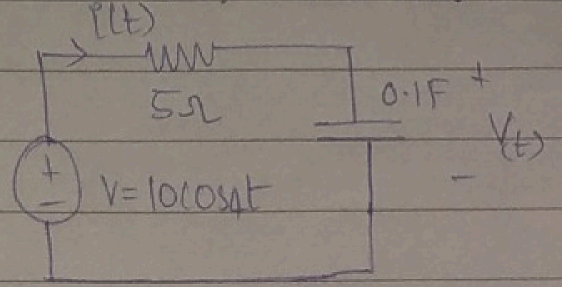
$V = 2j\, I$

$V = \cancel{22}\,2\angle 90^\circ \cdot 4.47\angle 3.5$ $\quad [j = 1\angle 90^\circ]$

$V = 8.94\angle 93.5^\circ$

$V = 8.94 \sin(2t + 93.5^\circ)$

H.W Find $v(t)$ and $i(t)$ in the circuit shown:



$V = 10\cos 4t$; $\omega = 4$; $\phi = 0^\circ$

$V = 5i + iX_c$

$X_c = \dfrac{1}{j\omega C} = \dfrac{1}{j(4)(0.1)} = -j \times 2.5$

$X_c = -2.5j$

In phasor form.

$10\angle 0^\circ = 5I + I(-2.5j)$

$I = \dfrac{10\angle 0^\circ}{5 - 2.5j}$

[∴ $5 - 2.5j$ in polar form $= \sqrt{5^2 + (2.5)^2}\ \angle \tan^{-1}\left(\dfrac{-1}{2}\right)$

$= 5.59\angle -26.5^\circ$

$I = \dfrac{10\angle 0^\circ}{5.6\angle 26.5^\circ} = 1.78\angle +26.5^\circ$

$\boxed{i(t) = 1.78\cos(4t + 26.5^\circ)}$

$V = X_C I$

$V = -2.5j \times (\cancel{+7}\ 1.78\angle +26.5^\circ)$

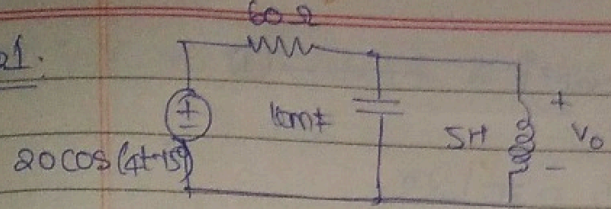
$= (2.5\angle -90^\circ) \times (1.78\angle +26.5^\circ)$

$V = 4.48\ \angle \cancel{-116.5^\circ} -63.5$

$\cancel{V_c = 4.48\cos(4t - 116.5^\circ)}$

$\boxed{V_c = 4.48\cos(4t - 63.5^\circ)}$

Expl.



Find $V_o = ?$

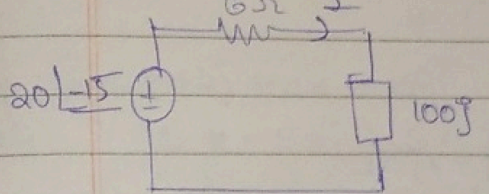
solution:-

$\omega = 4$; $V = 20 \angle -15$; $X_C = \frac{1}{j\omega C} = \frac{-j \times 100}{4 \times 10} = -25j$

$X_L = j\omega L = j \times 4 \times 5 = 20j$

capacitor & inductor are in parallel. hence its ~~induced~~ reactance.

$$Z_{eq} = \frac{X_L X_C}{X_L + X_C} = \frac{-25j \cdot 20j}{-25j + 20j} \quad \frac{+500j^2}{+5j} = 100j$$



; $V_o = 100j \times I$

$(60 + 100j) I = 20 \angle -15$

$$I = \frac{20 \angle -15}{60 + 100j}$$

Polar form of $60 + 100j = \sqrt{13600} \angle 59.03^\circ$

$= 116.61 \angle 59.03^\circ$

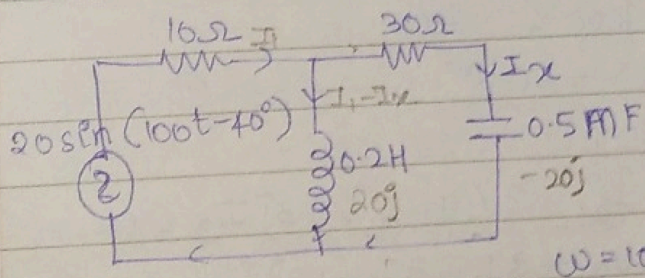
$I = 0.171 \angle -74.03$

$V_o = 100j \times 0.171 \angle -74.03$

$V_o = 100 \angle 90^\circ \times 0.171 \angle -74.03$

~~$V_o = 17.1 \angle 0$~~ $V_o = 17.1 \cos(4t + 15.97^\circ)$

Ex 2!



Find I_x

solution:-

$V = 20 \angle -40^\circ$

$\omega = 100$;

$X_L = j\omega L = j \times 100 \times 0.2H = 20j$

$X_C = \frac{1}{j\omega C} = \frac{-j}{100 \times 0.5 \times 10^{-3}} = -20j$

applying mesh analysis on loop 1:-

$10 I_1 + 20j (I_1 - I_x) = 20 \angle -40^\circ$ —— (1)

in Loop 2: $30 I_x - 20j I_2 + 20j (I_x - I_1) = 0$

$30 I_2 = 20j I_1$

$I_x = \frac{2}{3} j I_1 \Rightarrow I_x = 0.667j\, I_1$ ——— ②

Substituting ② in ①:

$I_1 (10+20j) - 20j\,(0.667j) I_1 = 20\angle{-40}$

$\cancel{I_1 f}\; I_1 = \frac{20\angle{-40}}{23.34 + 20j}$

Polar form of $23.34 + 20j = \sqrt{944.757}\,\angle 40.6^\circ$

$= 30.72\,\angle 40.6^\circ$

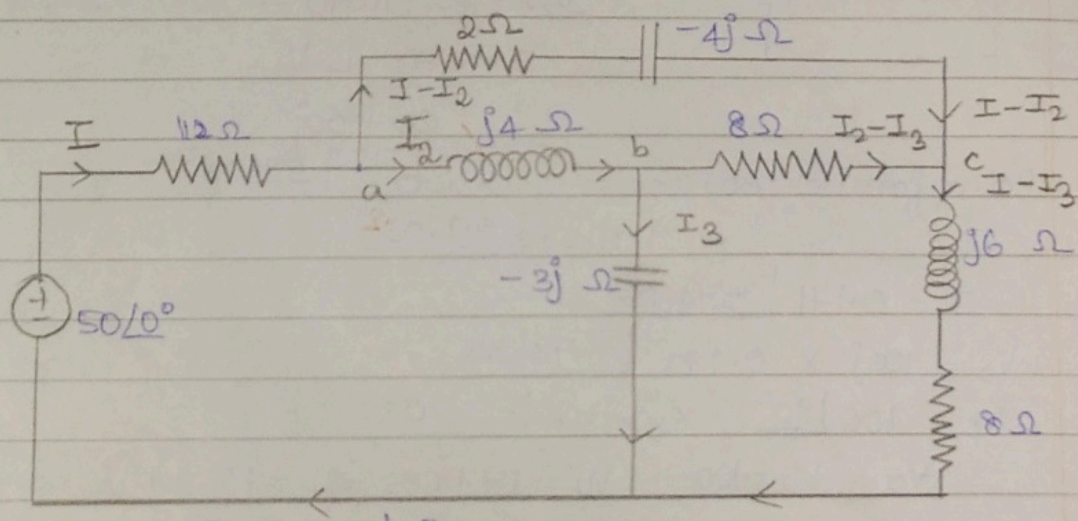
$I_1 = \frac{20\,\angle{-40^\circ}}{30.72\,\angle 40.6^\circ} = 0.651\,\angle{-80.6}$

$I_x = 0.667\,\angle 90^\circ \times 0.651\,\angle{-80.6}$

$I_x = 0.434\,\angle 9.4^\circ$

$I_x = 0.434 \sin(100t + 9.4^\circ)$

H.W



$V_m = 50\cos 0^\circ = 50\,\angle 0^\circ \quad ; \; I = ?$

applying KVL along loop 1; loop

$12I + 4j\,I_2 - 3j\,I_3 = 50\,\angle 0^\circ$ ——— ①

applying KVL along loop 2:-

$8(I_2 - I_3) + 6j\,(I - I_3) + 8(I - I_3) + 3j\,I_3 = 0$

$8I_2 - 8I_3 + 6jI - 6I_3 + 8I - 8I_3 + 3jI_3 = 0$

$(8+6j)I + 8I_2 + (-22+3j)I_3 = 0$ ——— ②

applying KVL along loop 3:-

$2(I - I_2) - 4j\,(I - I_2) - 8(I_2 - I_3) - 4j\,I_2 = 0$

$2I - 2I_2 - 4jI + 4I_2 - 8I_2 + 8I_3 - 4jI_2 = 0$

$(2-4j)I + (-6-4j)I_2 + 8I_3 = 0$ ~~③~~

$I_3 = \frac{(6+4j)\,I_2 + (-2+4j)\,I}{8}$ ——— ③

③ in ① $\Rightarrow 12I + 4j\, I_2 - 3j\, I_3 = 50\angle 0^\circ$

$$12I + 4j\, I_2 - 3j\left[\frac{(6+4j)\, I_2 + (-2+4j)\, I}{8}\right] = 50\angle 0^\circ$$

$$96I + 32j\, I_2 + (-18j+12)\, I_2 + (6j+12)\, I \} = 400\angle 0^\circ$$

$$I_2(\cancel{44}\ 12 + 14j) + I(108 + 6j) = 400\angle 0^\circ \quad ④$$

③ in ② $\Rightarrow (8+6j)I + 8I_2 + (-22+3j)\left[\frac{(6+4j)\, I_2 + (-2+4j)\, I}{8}\right] = 0$

$$= (8+6j)\, I + 8I_2 \cancel{+} -22\left[\frac{(6+4j)I_2 + (-2+4j)\, I}{8}\right] + 3j\left[\frac{(6+4j)\, I_2 + (-2+4j)I}{8}\right] = 0$$

$$= (64+42j)I + 64I_2 + (-132+88j)\, I_2 + (44-88j)\, I + (18j-12)\, I_2 + (-6j-12)\, I = 0$$

$$= (64+42j+44-88j-6j-12)\, I + I_2(64-132+88j+18j-12) = 0$$

$$= (96 \cancel{+} -52j)\, I + I_2(-80+106j) = 0$$

$$\Rightarrow (48-26j)\, I = (40-53j)\, I_2$$

$$\Rightarrow I_2 = \frac{48-26j}{40-53j}\, I$$

$$= \frac{\sqrt{48^2+26^2}\ \angle \tan^{-1}\left(\frac{-26}{48}\right)}{\sqrt{40^2+53^2}\ \angle \tan^{-1}\left(\frac{-53}{40}\right)} = \frac{\sqrt{2980}\ \angle -28.442^\circ}{\sqrt{4409}\ \angle -52.957^\circ}$$

$$I_2 = (0.822\angle 24.515^\circ)\, I$$

From ④ $\Rightarrow I_2(12+14j) + I(108+6j) = 400\angle 0^\circ$

~~I_2~~ Polar form of $12+14j = \sqrt{12^2+14^2}\ \angle \tan^{-1}\left(\frac{14}{12}\right)$

$= 18.44\angle 49.39^\circ$

Polar form of $108+6j = \sqrt{(108)^2+6^2}\ \angle \tan^{-1}\left(\frac{6}{108}\right)$

$= 108.166\angle 3.18^\circ$

From ④ $\Rightarrow I_2(18.44\angle 49.39^\circ) + I(108.166\angle 3.18^\circ) = 400\angle 0^\circ$

$(0.822\angle 24.515^\circ)(18.44\angle 49.39^\circ) + I\ (108.166\angle 3.18^\circ) = 400\angle 0^\circ$

$15.158\angle 73.90^\circ + I\ (108.166\angle 3.18^\circ) = 400\angle 0^\circ$

$I\ (108.166\angle 3.18^\circ) = 400\angle 0^\circ - 15.158\angle 73.90^\circ \quad ⑤$

$400\angle 0^\circ$

$a_1 = 400\cos 0^\circ = 400$

$b_1 = 400\sin 0^\circ = 0$

$r_1 = 400 + 0j$

$15.158\angle 73.90^\circ$

$a_2 = 15.158\cos(73.90^\circ) = 4.204$

$b_2 = 15.158\sin(73.90^\circ) = 14.565$

$r_2 = 4.204 + 14.565j$

$r = 404.204 + 14.565j$

Polar form $\Rightarrow \sqrt{(404.204)^2 + (14.565)^2}\ \angle \tan^{-1}\left(\frac{14.565}{404.204}\right)$

$= \sqrt{163380.874 + 212.14}\ \angle 2.064^\circ$

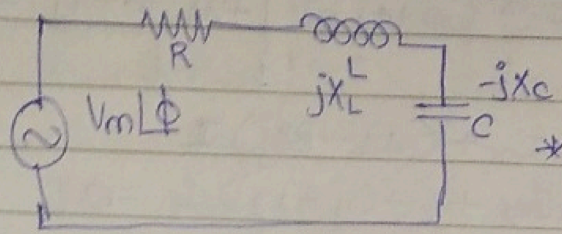
$= \sqrt{163{,}593.014}\ \angle 2.064^\circ$

$= 404.47\ \angle 2.064^\circ$

$I = \dfrac{404.47\ \angle 2.064^\circ}{108.166\ \angle 3.18}$

$I = 3.74\ \angle -1.116 \Rightarrow I = 3.74 \cos\theta$

Instantaneous & Average Power



* Instantaneous power = power at any instant of time

* $P_{av} = \dfrac{1}{T}\int_0^T P(t)\,dt$

$$V(t) = V_m \cos(\omega t + \theta_v)$$
$$I(t) = I_m \cos(\omega t + \theta_i)$$

$P(t) = V(t)\cdot I(t) = V_m I_m \cos(\omega t + \theta_v)\cdot\cos(\omega t + \theta_i)$

$\left[\cos A \cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]\right]$

$P(t) = \dfrac{1}{2} V_m I_m \cos(\omega t + \theta_v - \omega t - \theta_i) + \dfrac{1}{2} I_m V_m \cos(\omega t + \theta_v + \theta_i)$

$$\boxed{p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$

↳ Instantaneous power

integrating

* $P_{av} = \dfrac{1}{T}\int_0^T p(t)$

$$\boxed{P_{av} = \frac{1}{2} I_m V_m \cos(\theta_v - \theta_i)} \Rightarrow P_{av} = I_{rms}\cdot V_{rms}\cdot\cos(\theta_v - \theta_i)$$

Power factor ←

$\left[I_{rms} = \dfrac{I_m}{\sqrt{2}},\quad V_{rms} = \dfrac{V_m}{\sqrt{2}}\right]$

* If 230V DC voltages is supplied to a load & thus it generates a power P, then 230V V_{rms} AC voltage produces exactly the same amount of power P.

* V_{rms}, I_{rms} does not depend on phase but only on peak value.

* Power factor $= \cos\phi = \cos(\theta_v - \theta_i)$

$\phi = \theta_v - \theta_i$

(i) In resistance circuit, $P_{av} = V_{rms} I_{rms} \cos(0^\circ)$
$(\because \theta_v - \theta_i = 0)$

$P_{av} = V_{rms} I_{rms}$ (which is of same as DC circuit)

(ii) In inductive circuit, $P_{av} = V_{rms} I_{rms} \cos(0-(-90^\circ))$

$P_{av} = 0$

av power consumed by an inductor is 0.

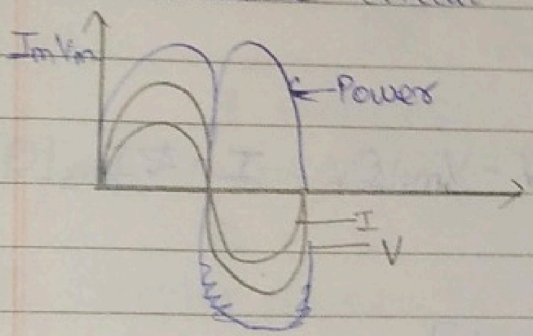
(iii) which is similar to inductor in DC circuit.
(Inductor in DC circuit will be short circuit)

(iii) In capacitive circuit, $P_{av} = V_{rms} I_{rms} \cos(0-90^\circ)$

$P_{av} = 0$

which is similar to ~~inductor~~ capacitor in DC circuit.
(Capacitor in DC circuit will be open circuited)

* In Resistive circuit

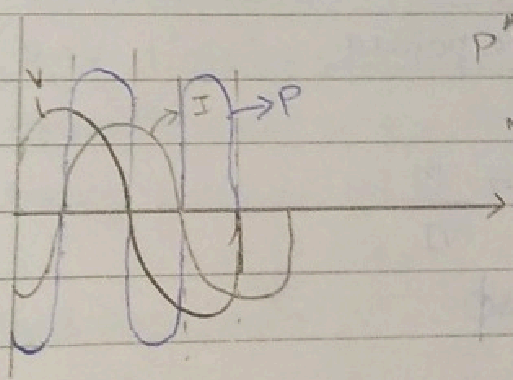


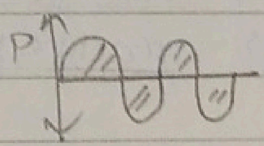
$P = I_m V_m$

$P_{av} = \frac{I_m V_m}{2} = \frac{I_m}{\sqrt{2}} \cdot \frac{V_m}{\sqrt{2}}$

$P_{av} = I_{rms} \cdot V_{rms}$

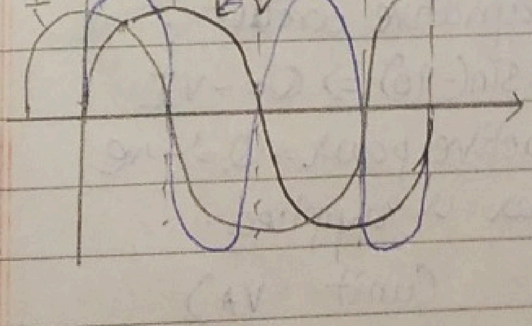
* In Inductive load.





$P_{av} = 0$

* In capacitive load:-



$P_{av} = 0$

Ex 1: $v(t) = 120\cos(377t + 45^\circ)$; $i(t) = 10\cos(377t - 10^\circ)$

find instantaneous power = ?

average power = ?

Ans: $p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$

$= \frac{1}{2}(120)(10)\cos(45^\circ - (-10^\circ)) + \frac{1}{2}(120)(10)\cos(2(377)t + 45^\circ \ldots$

$P(t) = 344.1 + 600\cos(754t + 35^\circ)$

$P_{av} = \frac{1}{2}(120)(10)\cos(45^\circ - (-10^\circ))$

$P_{av} = 344.1$

Complex Power

$P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$P_{av} = V_m \angle\theta_v \cdot I_m \angle -\theta_i$

$P_{av} = \frac{1}{2} V_m I_m \angle \theta_v - \theta_i$

if $V = V_{rms}\angle\theta_v$; $I = I_{rms}\angle\theta_i$

$P = VI^*$ ← I conjugate

$P = VI\angle\phi \Rightarrow S = VIe^{j\phi}$

$S = VI[\cos\phi + j\sin\phi]$

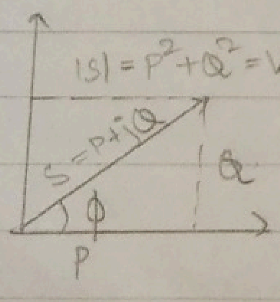
* $S = P + jQ$ ⇒ complex power

$P = VI\cos\phi$ ⇒ real power (unit: Watt)

$Q = VI\sin\phi$ ⇒ reactive power (unit VAR)

volt amp Reactive

*



$\cos\phi = \frac{P}{|S|} \Rightarrow \frac{P}{VI}$

$P = VI\cos\phi$

$\phi = 90^\circ$

in inductor circuit.

$Q = VI\sin 90^\circ \Rightarrow Q = VI$

reactive power = Q = +ve

reactive power is absorbed

$\phi = -90^\circ$

in capacitive circuit.

$Q = VI\sin(-90^\circ) \Rightarrow Q = -VI$

reactive power = Q = −ve

reactive power is supplied.

* Apparent power $= |S| = VI$ (unit : VA)

$= \sqrt{P^2 + Q^2}$

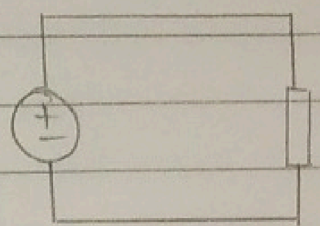
Problem 1: The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V & the current through the element in the direction of voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find ~~the~~

a) complex & apparent powers.

b) real & reactive powers

c) Power factor & load impedance

Solution:



Given:-

$v(t) = 60\cos(\omega t - 10^\circ) = 60\angle -10^\circ$

$i(t) = 1.5\cos(\omega t + 50^\circ) = 1.5\angle 50^\circ$

~~a) complex power~~ $= VI\cos\phi + j\,VI\sin\phi$

b) real power $= VI\cos\phi = V_{rms} \cdot I_{rms} \cdot \cos\phi$

$$= \frac{60}{\sqrt{2}} \cdot \frac{1.5}{\sqrt{2}} \times \cos(-10^\circ - 50^\circ)$$

$$P = 22.5\,W$$

reactive power $= VI\sin\phi = \frac{60}{\sqrt{2}} \cdot \frac{1.5}{\sqrt{2}} \cdot \sin(-60^\circ)$

$$Q = -38.97\ \text{VAR}$$

a) complex power $= S = P + iQ$

$$= 22.5 + i(-38.97)$$

$$= 22.5 - 38.97i$$

apparant power $= |S| = \sqrt{P^2 + Q^2} = \sqrt{(22.5)^2 + (38.97)^2}$

$$= \sqrt{506.25 + 1518.66} = \sqrt{2024.9109}$$

apparent power $= 44.99\,W$

c) Power factor $= \cos\phi = \cos(\theta_v - \theta_i) = \cos(-10^\circ - 50^\circ)$

$$= \frac{1}{2} = 0.5$$

load impedence $= \frac{V}{I} = \frac{60\angle -10^\circ}{1.5\angle 50^\circ} = 40\angle -60^\circ$

$a = 40\cos(-60^\circ) = 20$; $b = 40\sin(-60^\circ) = -34.641$

$Z = 20 - 34.641i$