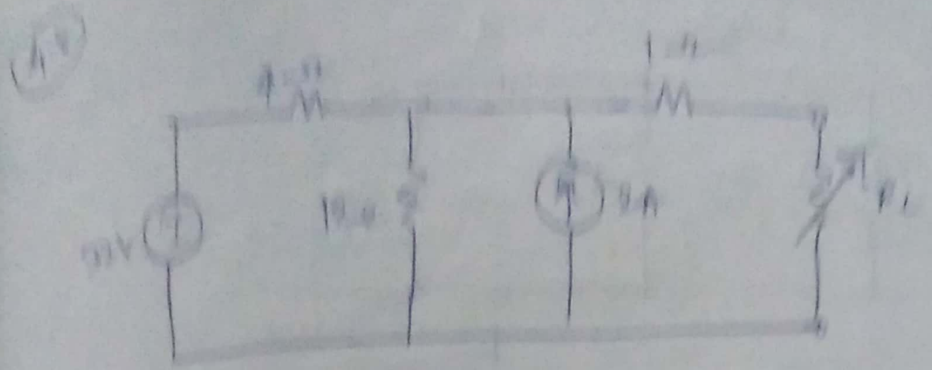
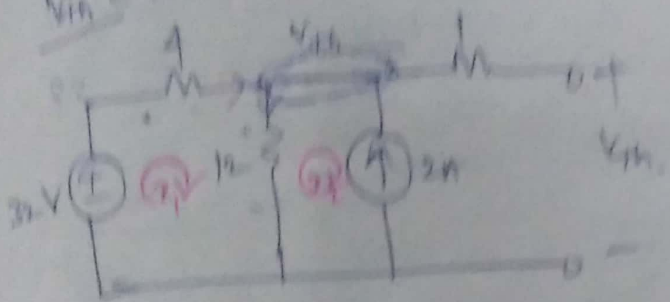


$V_{th} = V_{oc}$



$4I_1 + 12(I_1 - I_2) = 32$ $\qquad I_2 = -2$

$4I_1 + 12I_1 + 24 = 32$

$16 I_1 = 8$

$I_1 = \frac{8}{16} = 0.5\ A$

$V_{th} = 12(I_1 - I_2)$

$= 12 \times (0.5 + 2)$

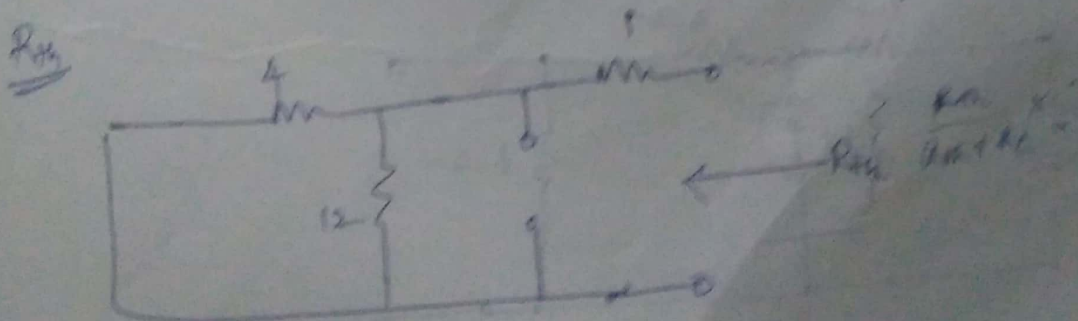
$V_{th} = 30V$

using nodal analysis

$$\frac{32 - V_{th}}{4} + 2 = \frac{V_{th}}{12}$$

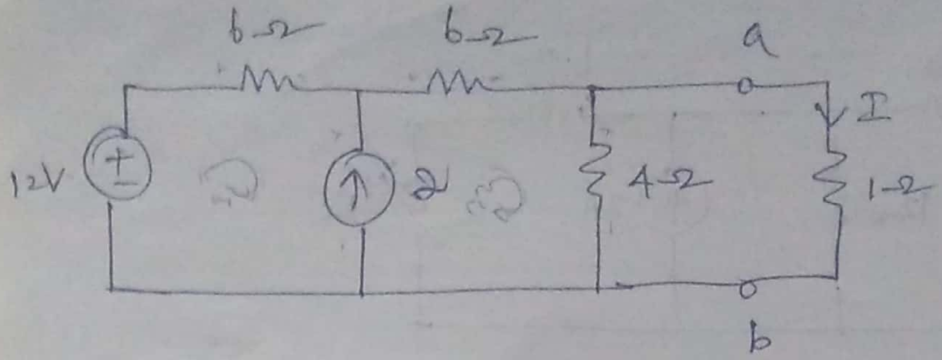
Solving $V_{th} = 30\ V$

$R_{th}$



$R_{th} = 1 + 12 \parallel (4) = 4\ \Omega$

Scanned by CamScanner



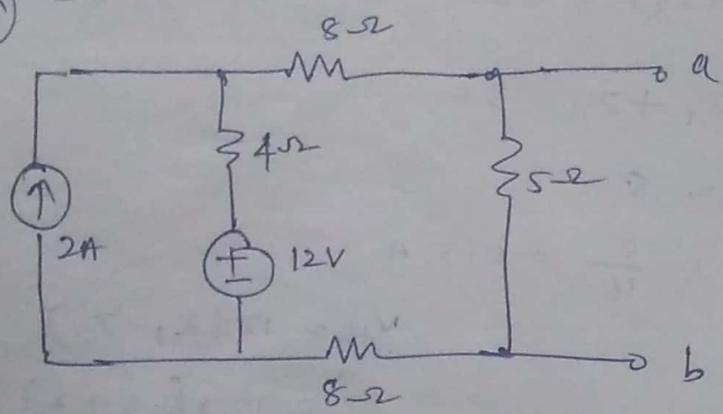

$V_{th} = 6\,V$

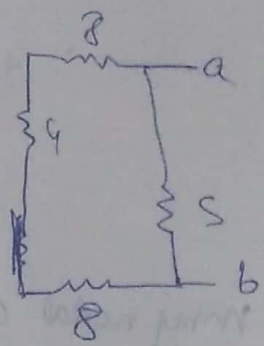
$R_{th} = 3\,\Omega$

$I = 1.5\,A$.

Norton's equivalent

(4.11)

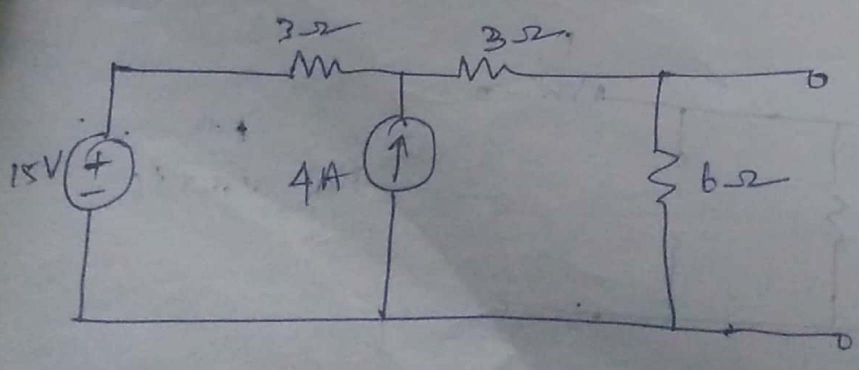



$R_N = 4\,\Omega$

$I_N = 1\,A$

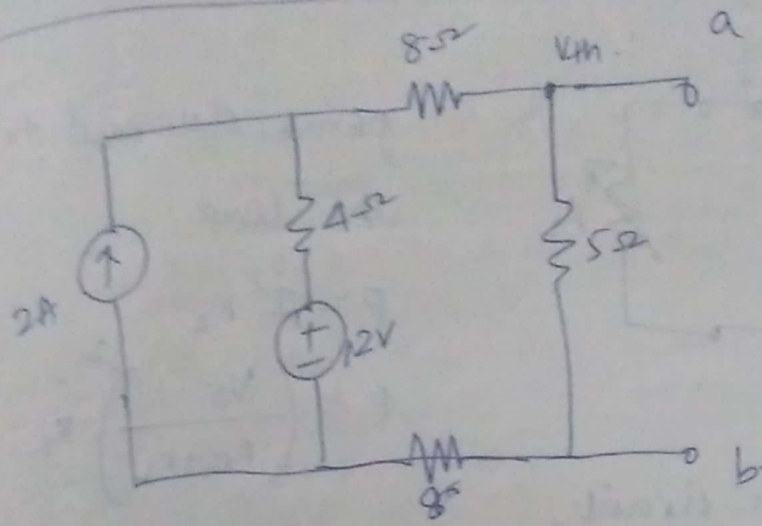
$V_{th} = 4\,V$.



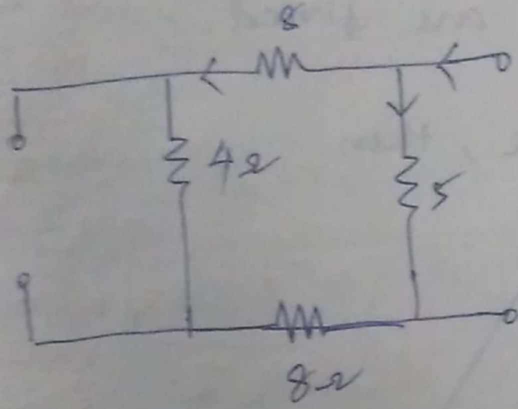

$R_N = 3\,\Omega$ $\quad$ $I_N = 4.5\,A$.

Scanned by CamScanner

# Norton's Equivalent




Rth



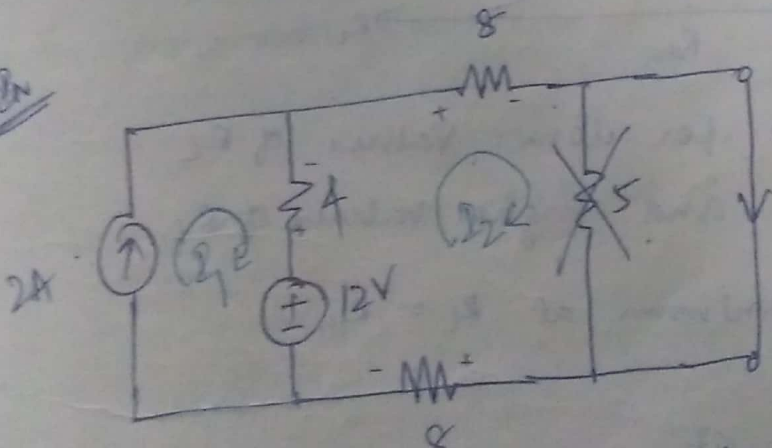

$R_{th} = 20 \parallel 5$

$= \frac{20 \times 5}{20+5}$

$= \frac{100}{25} = 4\,\Omega$

$= \cancel{3.5\,\Omega}$

IN




$4(I_2 - I_1) + 8I_2 + 8I_2 = 12$

$20I_2 - 4I_1 = 12$

$20I_2 = 12 + 8 = 20$

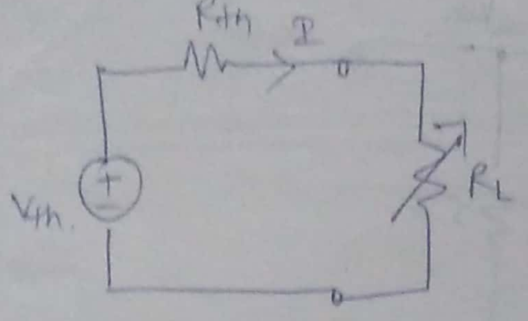
$I_2 = 1A$

$I_1 = 2A$

$I_N = 1A$

$V_{th} =$

## Maximum power Transfer




power delivered to the load

$P = I^2 R_L$

$P = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$

For a given circuit,

$V_{th}$, $R_{th}$ are fixed.

If $V_{th} = 15V$

$R_{th} = 5\Omega$ $R_L = 10\Omega$

$I = \frac{V_{th}}{R_{th}+R_L} = 1A.$

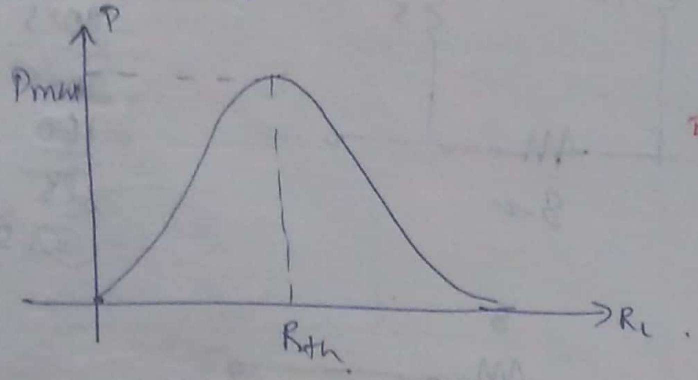
$P_L = I^2 R_L = 5W$

If $R_L = 0$ $P = 0$

$R_L = 2$ $P = 2$

$R_L = 5$ $P =$

If we vary $R_L$, then




P = low for lower values of $R_L$

= " " and higher values of $R_L$.

= maximum at $R_L = R_{th}$.

$\frac{dP}{dR_L} = 0$

$\frac{d}{dR_L}\left[\frac{V_{th}^2 R_L}{(R_{th}+R_L)^2}\right] = 0$

Scanned by CamScanner

$$V_{th}^2 \left[ \frac{(R_{th}+R_L)^2 (1) - R_L \, 2(R_{th}+R_L)}{(R_{th}+R_L)^4} \right] = 0$$

$$\frac{(R_{th}+R_L)^2 - 2R_L(R_{th}+R_L)}{(R_{th}+R_L)^4} = 0.$$

$$\frac{(R_{th}+R_L)(R_{th}+R_L-2R_L)}{(R_{th}+R_L)^4} = 0$$

$$\frac{(R_{th}+R_L)(R_{th}-R_L)}{(R_{th}+R_L)^4} = 0$$

$$\cancel{R_{th} - R_L = 0}$$

$$\frac{R_{th}-R_L}{(R_{th}+R_L)^3} = 0$$

$$R_{th} - R_L = 0$$

$$\boxed{R_{th} = R_L}$$

$\therefore$ max power transfer occurs when $R_{th} = R_L$

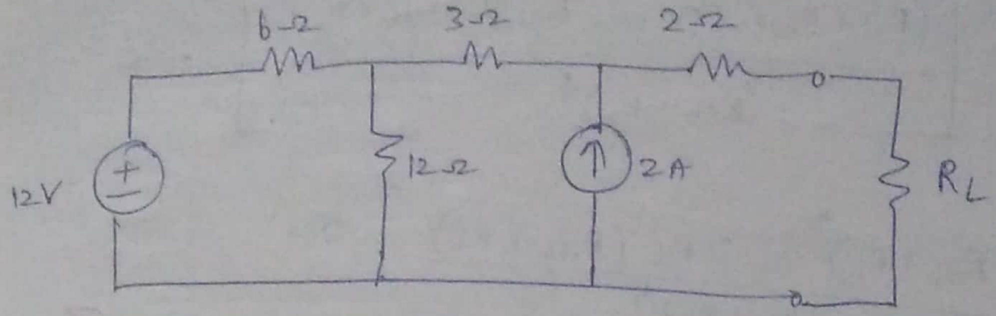
Source impedance = load resistance

max power at that point is

$$P_{max} = \frac{V_{th}^2}{(R_{th}+R_{th})^2} R_{th}$$

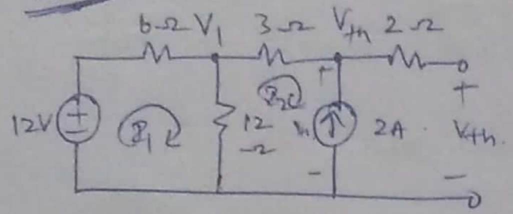
$$= \frac{V_{th}}{4R_{th}^2} \times R_{th}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$

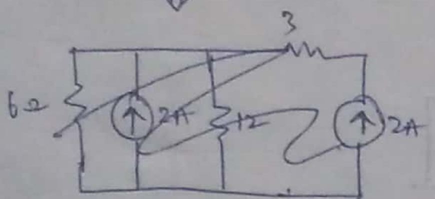


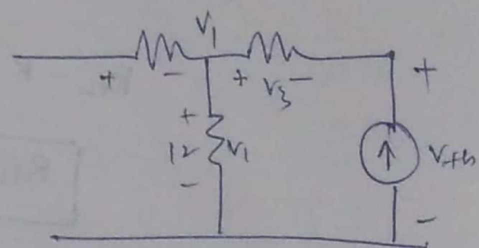

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

$V_{oc} = V_{th}$




⇓




$I_2 = -2A$

$12 = 6I_1 + 12(I_1 - I_2)$

$12 = 6I_1 + 12I_1 + 24$

$18I_1 = -12$

$I_1 = \frac{-2}{3}$

$V_1 = 12(I_1 - I_2)$

$= 12\left(-\frac{2}{3} + 2\right)$

$= 12\left(\frac{6-2}{3}\right)$

$= \cancel{12}^{4} \times \frac{4}{\cancel{3}}$

$V_1 = 16$

$V_3 = I_2 \times 3 = -2 \times 3 = -6$

$V_{th} - V_3 = V$

$-V_1 + V_3 + V_{th} = 0$

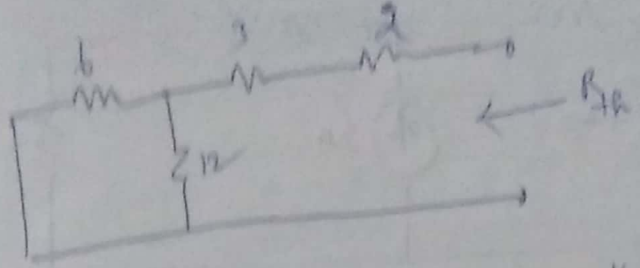
$V_{th} = V_1 - V_3$

$= 16 - (-6)$

$V_{th} = 22$

$\therefore \cancel{R_{th}} = R_L$

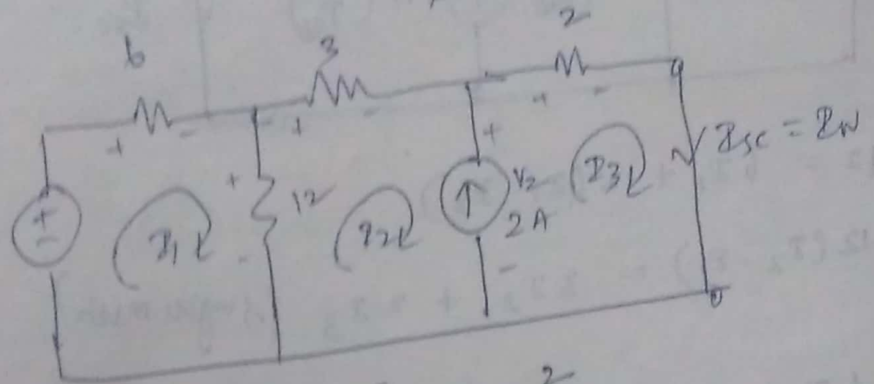
$R_{th}$



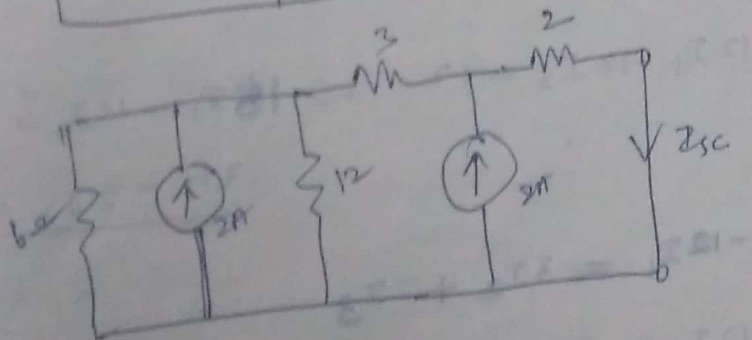

$R_{th} = (6 \parallel 12) + 6 = \frac{6 \times 12}{6+0} + 6 = \frac{\cancel{6} \times \cancel{12}^{4}}{\cancel{18}_{3}} + 5$

$\boxed{R_{th} = 9\,\Omega}$

$I_{sc}$




$I = \frac{12}{6}$




$12 I_1 - 8 I_2 = 8$

$-12 I_1 - 5 I_2 = 0$

$-13 I_2 = 8$

$I_2 = \frac{-8}{13}$

$I_3 = 2 + I_2$

$= 2 - \frac{8}{13}$

$= \frac{26-8}{13}$

$= \frac{14}{13}$

$\frac{22}{9}$

$-12 + 6I_1 + 12(I_1 - I_2) = 0$

$18 I_1 - 12 I_2 = 12$

$3I_1 - 2I_2 = 2 \longrightarrow (1)$

$I_3 - I_2 = 2A$

$I_3 = 2 + I_2$

$-12(I_2 - I_1) + 3I_2 + V_2 = 0$

$-V_2 + 2I_3 \neq 0$

$V_2 = 2I_3$

$-12(I_2 - I_1) + 3I_2 + 2I_3 = 0$

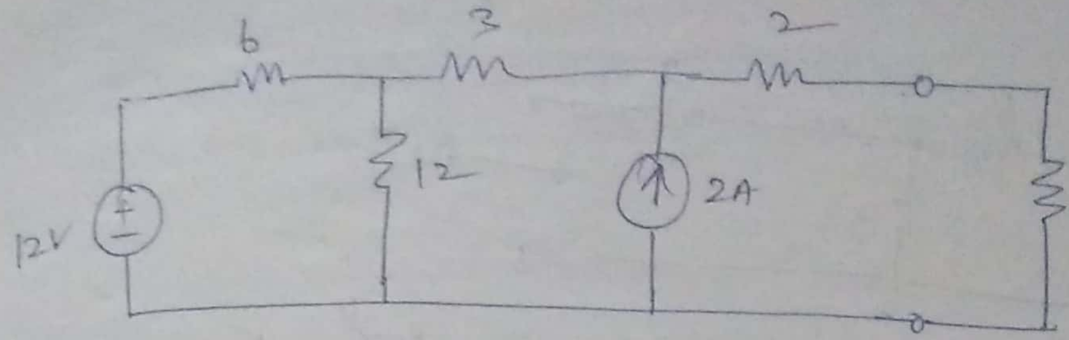
$-12I_2 + 12I_1 + 3I_2 + 4 + 2I_2 = 0$

$7I_2 = -4$

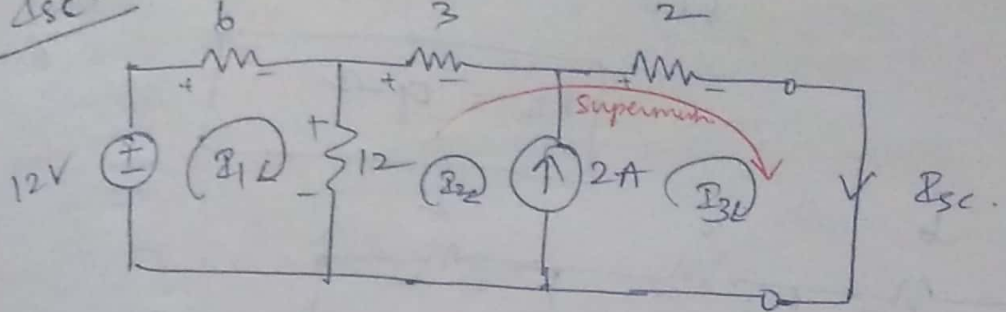
$-12I_1 - 5I_2 = 0$

$+12I_1 - \cancel{9I_2}\ \cancel{+0} = 0$

Scanned by CamScanner




Isc




$$12 = 6I_1 + 12(I_1 - I_2)$$

$$12(I_2 - I_1) = 3I_2 + 2I_3 \quad \text{(super mesh)}$$

① $6I_1 + 12I_1 - 12I_2 = 12 \Rightarrow 18I_1 - 12I_2 = 12$

$$3I_1 - 2I_2 = 2$$

② $12I_2 - 12I_1 = 3I_2 + 2I_3$

$$+12I_1 - 9I_2 + 2I_3 = 0.$$

③ $I_3 - I_2 = 2 \Rightarrow I_3 = 2 + I_2$

② $\Rightarrow 12I_1 - 9I_2 + 2(2 + I_2) = 0$

$$12I_1 - 9I_2 + 4 + 2I_2 = 0.$$

$$12I_1 - 7I_2 = -4.$$

$\therefore I_3 = -10$

$$\cancel{12I_1} - 8I_2 = 8$$

$$\cancel{12I_1} - 7I_2 = -4$$

$$- \quad + \quad +$$

$$-I_2 = 12$$

$I_2 = -12$

when $V_L = \frac{V_s}{2}$

$R_s = R_L$

$\therefore$ Internal resistance can be found.

$\frac{V_{th}}{2}$

$V_L = \frac{R_L}{R_s+R_L} V_s$

If $R_s = R_L$

$V_L = \frac{R_L}{2R_L} V_s$

$\boxed{V_L = \frac{V_s}{2}}$

Problems

(4.16) The terminal voltage of a voltage source is 12V when connected to a 2-W load. When the load is disconnected, the terminal voltage rises to 12.4 V.

(a) Calculate the source voltage $V_s$ and internal resistance $R_s$.

(b) Determine the voltage when an 8 Ω load is connected to the source.

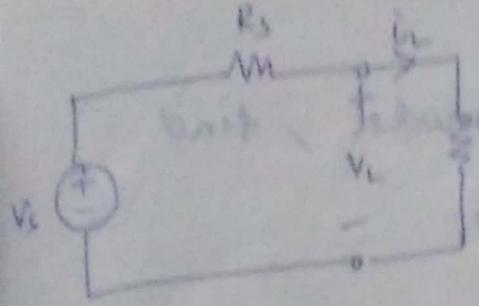
Soln

$V_{oc} = 12.4\ V = V_{th} = V_s$

When loaded (2 W.) $V_L = 12\ V$

$P_L = 2\ W$

$P_L = \frac{V_L^2}{R_L}$ $\quad \therefore R_L = \frac{12^2}{2} = 72\ \Omega$



$i_L = \frac{V_L}{R_L} = \frac{12}{72} = \frac{1}{6}$ A

$V_s - V_L = R_s I_L$

$12.4 - 12 = R_s\left(\frac{1}{6}\right)$

$R_s = 6 \times 0.4$

$\boxed{R_s = 2.4\ \Omega}$

(b) when $R_L = 8\ \Omega$

$V_L = \frac{R_L}{R_s + R_L} V_s$

$V_L = \frac{8}{8+2.4}\ 12.4$

$V_L = 9.538$ V

(4.16) The measured DC voltage across a certain amplifier is 9V. The voltage drops to 8V when a 20-Ω loudspeaker is connected to the amplifier. Calculate the voltage when a 10-Ω loudspeaker is used instead

$V_{th} = V_{oc} = V_s = 9V$ when $R_L = 20\ \Omega$

$V_L = 8$ V

$V_L = \frac{R_L}{R_s + R_L} V_s \Rightarrow \frac{20}{20 + R_s}(9) = 8$

$180 = 160 + 8R_s$

$8R_s = 20$

$R_s = \frac{20}{8} = 2.5$

$\boxed{R_s = 2.5\ \Omega}$

∴ when $R_L = 10\ \Omega$

$V_L = \frac{10}{2.5 + 10} \times 9$

$\boxed{V_L = 7.2V}$

Scanned by CamScanner