BEEE101L – Basic Electrical Engineering

Parallel RLC Circuits

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Objective

- To introduce the concept of admittance, susceptance in ac circuit.
- To solve simple problems based on admittance.

Admittance (Y): Inverse of impedance, expressed in mho ($\mho$)

$$Y = \frac{1}{Z}$$

$$Z = R + jX$$

$$Y = \frac{1}{R \pm jX} = G \mp jB$$

$$G = \text{Conductance} = \frac{1}{R}$$

$$B = \text{Susceptance} = \frac{1}{X}$$

$$Z = 1 + j2$$

$$Y = \frac{1}{Z} = \frac{1}{1+j2} \times \frac{(1-j2)}{(1-j2)}$$

$$= \frac{1-j2}{1^2+2^2}$$

$$= \frac{1}{5} - j\frac{2}{5}$$

$$\boxed{Y = 0.2 - j0.4\ \mho}$$

$Y = a - jb$ inductive

$a + jb$ Capacitive

$Z = R$

Then $Y = G = \frac{1}{R}$

$Z = +jX$ [L]

$$Y = \frac{-j}{X} = -jB$$

$Z = -jX$

$$Y = +\frac{j}{X} = jB$$

$$Z = 1 - j2$$

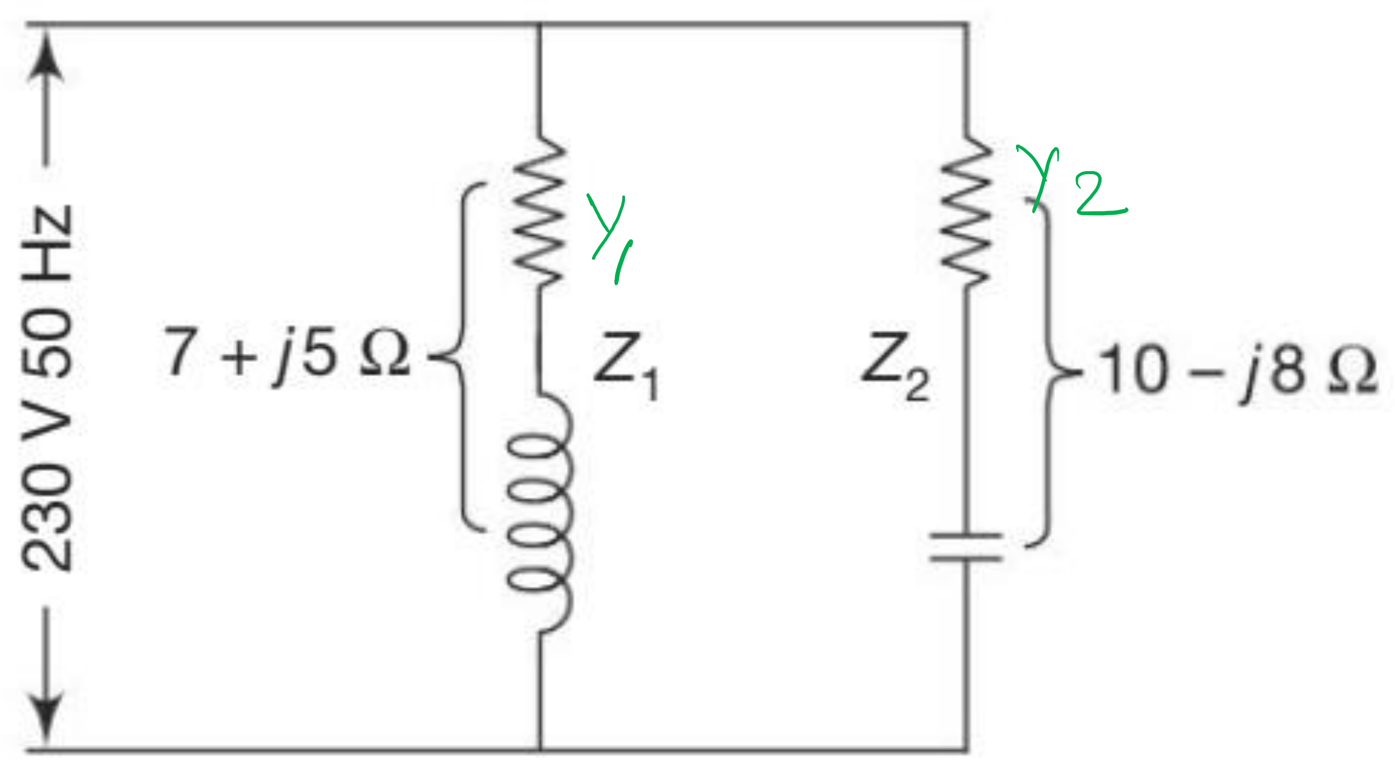
$$Y = \frac{1}{1-j2} \times \frac{1+j2}{1+j2}$$

$$= \frac{1+j2}{1^2+2^2}$$

$$\boxed{Y = 0.2 + j0.4 \ \mho}$$

Example 1:

An impedance of (7 + j5) ohms is connected in parallel with another circuit having an impedance of (10 – j8) ohm. The supply voltage is 230 V, 50 Hz. Calculate (i) the admittance(Y), the conductance(G) and susceptance(B) of the combined circuit (ii) the total current (I) taken from mains and its p.f.



Z in series $= Z_1 + Z_2$

Z in parallel $= \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

Y in parallel $= Y_1 + Y_2$

Y in series $= \dfrac{Y_1 Y_2}{Y_1 + Y_2}$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{7+j5} \times \frac{7-j5}{7-j5} \qquad Y_2 = \frac{1}{Z_2} = \frac{1}{10-j8} \times \frac{10+j8}{10+j8}$$

$$= \frac{7-j5}{7^2+5^2} \qquad = \frac{10+j8}{10^2+8^2}$$

$$= \frac{7-j5}{49+25} \qquad = \frac{10}{164} + j\frac{8}{164}$$

$$= \frac{7}{74} - j\frac{5}{74} \qquad = 0.061 + j0.049 \,\mho$$

$$Y_1 = 0.095 - j0.068 \,\mho$$

The net admittance $Y = Y_1 + Y_2 = 0.095 - j0.068 + 0.061 + j0.049$

$$Y = 0.156 - j0.019 \,\mho = G - jB$$

$$\therefore G = 0.156 \,\mho \quad ; B = -j0.019 \,\mho$$

The total current $I = \frac{V}{Z} = VY$

$$I = 230\angle 0^\circ \times \text{Polar}(0.156 - j0.019)$$

$$= 230\angle 0^\circ \times 0.157\angle -6.94$$

$$I = 36.11\angle -6.94 \text{ A}$$

Powerfactor $\cos\phi = \cos(-6.94)$

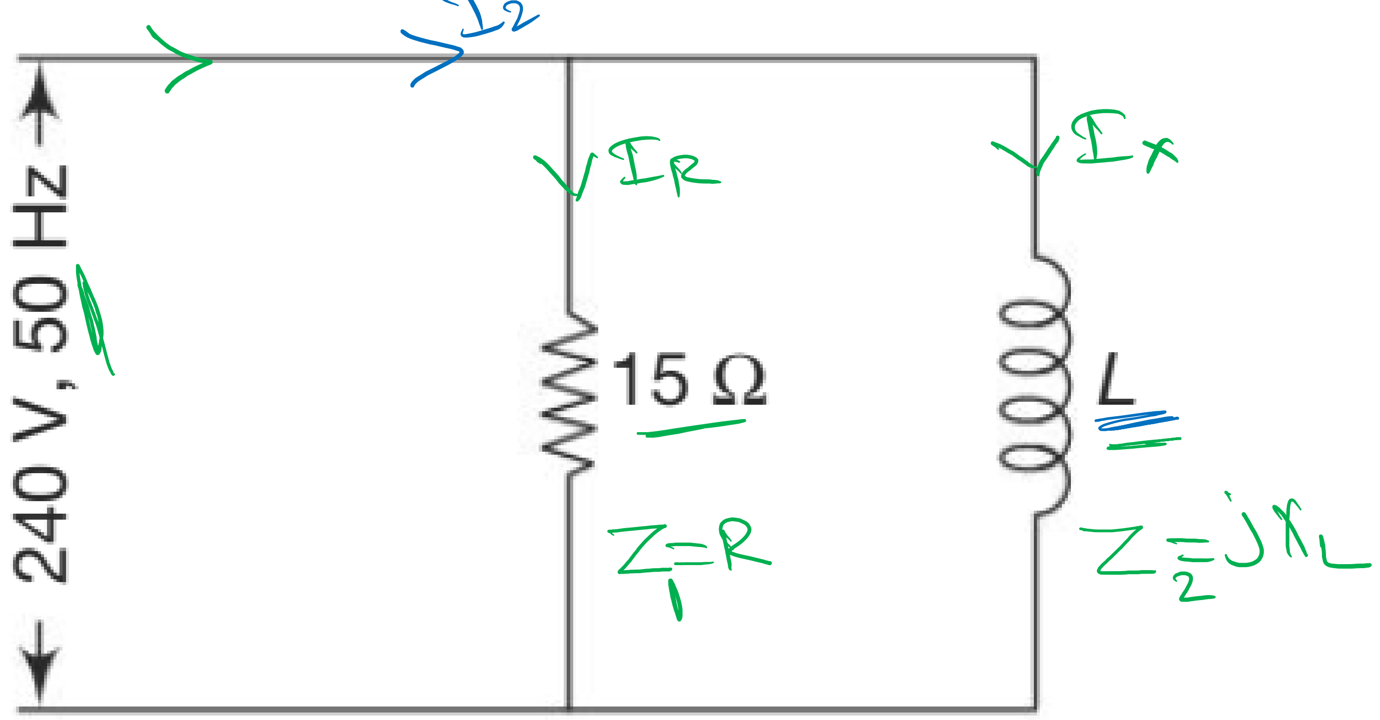
$= 0.992$ lagging.

Example 2:

When a 240 V, 50 Hz supply is applied to a resistor of 15 ohms is parallel with an inductor, the total current is 22.1 A. What value must the frequency have for the total current to be 34 A?

$I_1 = 22.1\,A$

$I_2 = 34\,A, \; f = ?$



$I_1 = 22.1 = \bar{V}\,\bar{Y}_N$

$\bar{Y}_N = |G_1 + jB_2| \Rightarrow G_1 = \frac{1}{R} \quad B_2 = \frac{1}{X_L}$

$Y_N = Y_1 + Y_2 = G - jB$

$\bar{Y}_N = \sqrt{G^2 + B^2} = \frac{I}{V} = \frac{22.1}{240} =$

$\bar{Y}_N = 0.0920\,\mho = \sqrt{G^2 + B^2}$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R} = G$

$Y_2 = \frac{1}{Z_2} = \frac{1}{j\omega L} = \frac{-j}{\omega L} = -jB$

$$G = \frac{1}{R} = \frac{1}{15} = 0.067\ \mho$$

$$B = \frac{1}{\omega L} = \frac{1}{2\pi f L} = \frac{1}{2\pi \times 50 L} = \frac{0.003}{L}\ \mho$$

$$\sqrt{0.067^2 + \left(\frac{0.003}{L}\right)^2} = 0.092$$

On squaring both sides

$$0.067^2 + \frac{0.003^2}{L^2} = 0.092^2$$

$$L = 0.047\ H \simeq 0.05\ H$$

For the total current of 34 A,

$$\overline{Y_2} = \frac{I_2}{V} = \frac{34}{240} = 0.142\ \mho$$

$$Y_2 = G - jB_2 = \frac{1}{R} - j\frac{1}{2\pi f \times 0.05}$$

$$\therefore \overline{Y_2} = \sqrt{G^2 + B_2^2}$$

$$\overline{Y_2} = \sqrt{0.067^2 + \frac{3.18^2}{f^2}} = 0.142$$

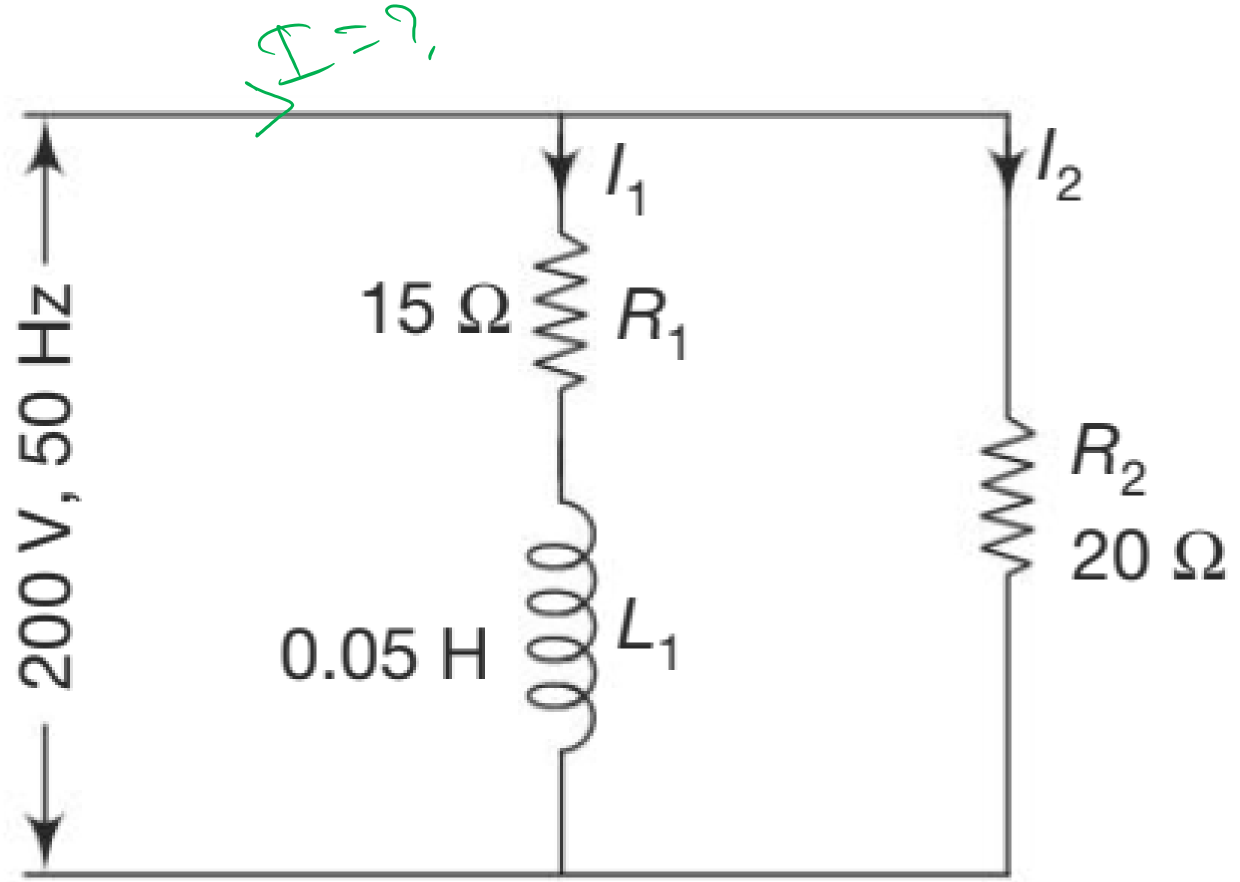
On squaring both sides, we have

$$0.067^2 + \frac{3.18^2}{f^2} = 0.142^2$$

$$\boxed{\therefore f = 25.35\ Hz}$$

Example 3:

A coil of resistance 15 ohms and inductance 0.05 H is connected in parallel with non-inductive resistor of 20 ohms. Find (i) the current in each branch and the total current supplied and (ii) the phase angle of the combination when a voltage of 200 V at 50 Hz is applied. Draw the phasor diagram.



$I = I_1 + I_2$

$\omega = 2\pi f = 2\pi \times 50$

$= 314$ rad/sec

$I_1 = V Y_1 \; ; \; I_2 = V Y_2$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + j\omega L_1} = \frac{1}{15 + j(314 \times 0.05)}$

$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2} = \frac{1}{20} = 0.05$

$Y_1 = \frac{1}{15 + j15.7} \times \frac{15 - j15.7}{15 - j15.7}$

$$Y_1 = \frac{15 - j15.7}{15^2 + 15.7^2}$$

$$= \frac{15 - j15.7}{471.49}$$

$$= 0.032 - j0.033 \ \mho$$

$$\bar{Y}_1 = \sqrt{0.032^2 + 0.033^2} \angle \tan^{-1}\left(\frac{-0.033}{0.032}\right)$$

$$= 0.046 \angle -45.88^\circ$$

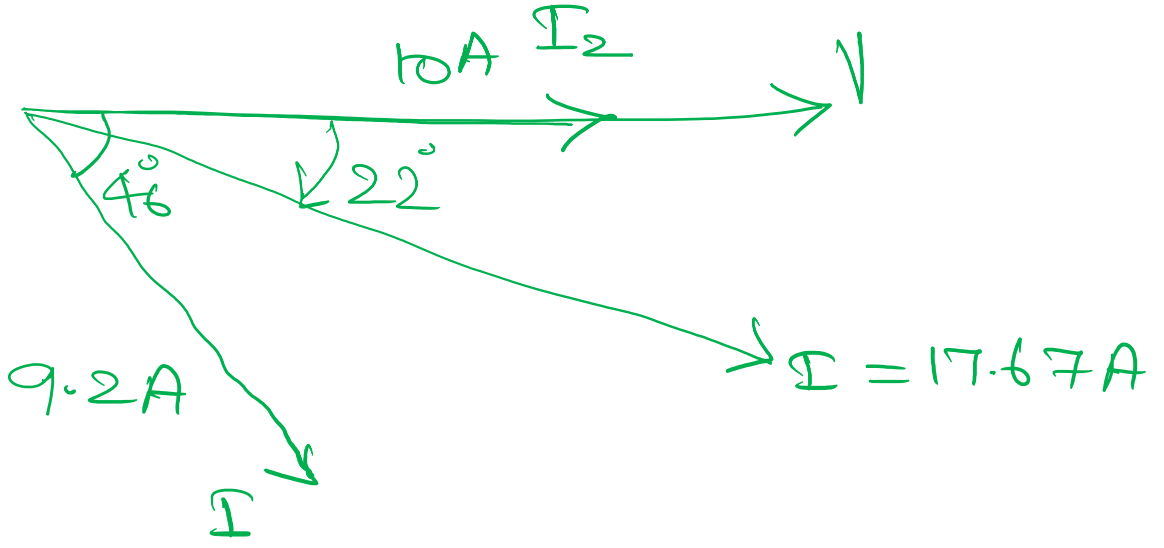
$$I_1 = V\bar{Y}_1 = 200\angle 0 \times 0.046\angle -45.88^\circ$$

$$= 9.2 \angle -45.88^\circ \text{ A}$$

$$I_2 = V\bar{Y}_2 = \frac{200\angle 0}{20\angle 0} = 10\angle 0 \text{ A}$$

$$I = I_1 + I_2$$

$$I = 17.67 \angle -21.9^\circ \text{ A}$$



Example 4:

A coil of inductance 6 mH and resistance 40 ohms is connected across a supply which has a sinusoidal EMF of 100 V and a frequency of 800 Hz. Also across the supply is a circuit consisting of perfect capacitor 4mF in series with resistance of 50 ohms. Find (i) the total current taken from the supply and (ii) the phase angle between the currents in the coil and the capacitor. Draw the phasor diagram.