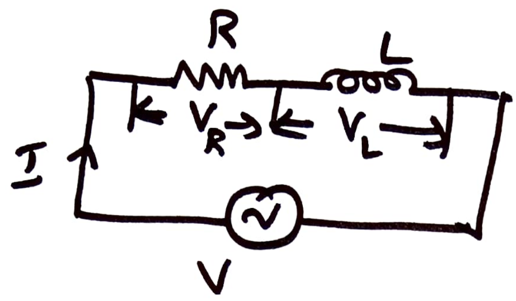
RL Series circuit



$V = V_m \sin\omega t$ - applied voltage

i = instantaneous current

f = frequency.

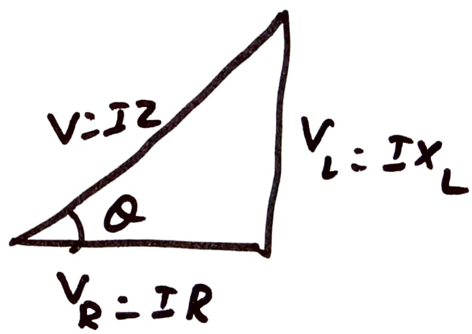
voltage across Resistor $R = V_R = IR$

voltage across inductor $L = jIX_L = V_L$.

$\therefore V = V_R + V_L$.

$= IR + jIX_L$

$= I(R + jX_L)$

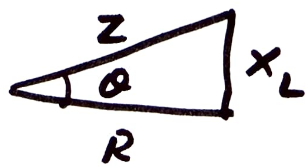


$$\frac{V}{I} = R + jX_L = Z$$

Voltage triangle

$Z = R + jX_L$

$|Z| = \sqrt{R^2 + X_L^2}$



$\phi = \tan^{-1} X_L / R$.

power factor : $p.f = \cos\phi$

From the impedance triangle

$$\cos\phi = \frac{R}{Z} \text{ (or) } \cos\left[\tan^{-1}\left(\frac{X_L}{R}\right)\right]$$

Hence I lags the voltage. ∴ p.f is also lagging.

Active power (or) real power $= VI\cos\phi$

$$= VI \cdot \frac{R}{Z}$$

$$= \frac{V}{Z} IR = I^2 R.$$

Reactive power $= VI\sin\phi$

$$= VI \cdot \frac{X_L}{Z} = I^2 X_L.$$

1. A coil having a resistance of $10\,\Omega$ and an inductance of 0.01 H is connected across a 220V, 50Hz supply. calculate (a) the current b) phase angle between current and voltage c) p.f (d) power.

Soln:.

$V = 220V, \quad R = 10\Omega \quad f = 50Hz \quad L = 0.01H$

$X_L = 2\pi f L = 3.141\Omega.$

$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 3.14^2} = 10.48\Omega$

a) current $I = \frac{V}{Z} = \frac{220}{10.48} = 20.99A.$

b) $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{3.141}{10} = 17.43$ lag.

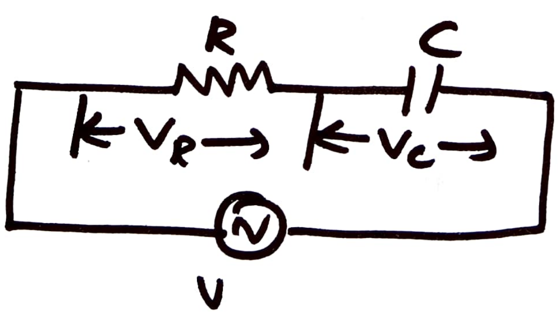
c) $p.f = \cos\phi = 0.954$ lag.

d) power $= VI\cos\phi$

$= 220 \times 20.99 \times 0.954$

$= 4406.2w.$

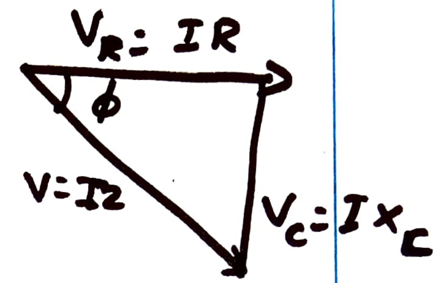
RL series circuit



$V = V_m \sin \omega t$

i = instantaneous current

f = frequency



voltage across R : $V_R = IR$ in phase with I

voltage across C : $V_c = IX_c$ lagging I by 90°.

$$V = V_R + V_c .$$

voltage triangle

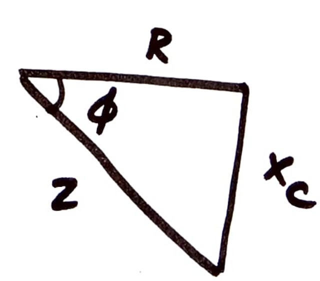
$$V = IR - jIX_c = I(R - jX_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$\phi = \tan^{-1}\frac{X_c}{R} = \tan^{-1}\frac{1/\omega C}{R} = \tan^{-1}\frac{1}{\omega CR}$$

$$X_c = \frac{1}{\omega C}$$



ϕ = phase angle

$$Z = R - jX_c = |Z| \angle -\phi$$

$P.f = \cos\phi = R/Z .$

Active power = $VI\cos\phi$ (watts)

Reactive power = $VI\sin\phi$ (VAR)

Find the circuit constants of a two element series circuits which consumes 700W with 0.707 leading p.f. The applied voltage is $V = 141.4 \sin 314t$.

Soln:

$V_m = 141.4$

$V_{rms} = \frac{141.4}{\sqrt{2}} = 99.98\,V.$

$\cos\phi = 0.707$ leading

power $= VI\cos\phi$

$700 = 99.98 \times I \times 0.707.$

$I = 9.9A.$

$Z = \frac{V}{I} = \frac{99.98}{9.9} = 10.09\,\Omega.$

$\cos\phi = R/Z$

$R = Z\cos\phi$

$= 10.09 \times 0.707 = 7.13\,\Omega.$

$\phi = \cos^{-1} 0.707 = 45^\circ$

$$\sin\phi = \frac{X_c}{Z} \Rightarrow X_c = Z\sin\phi$$

$$X_c = 10.09 \times \sin 45^\circ$$

$$= 10.09 \times 0.707$$

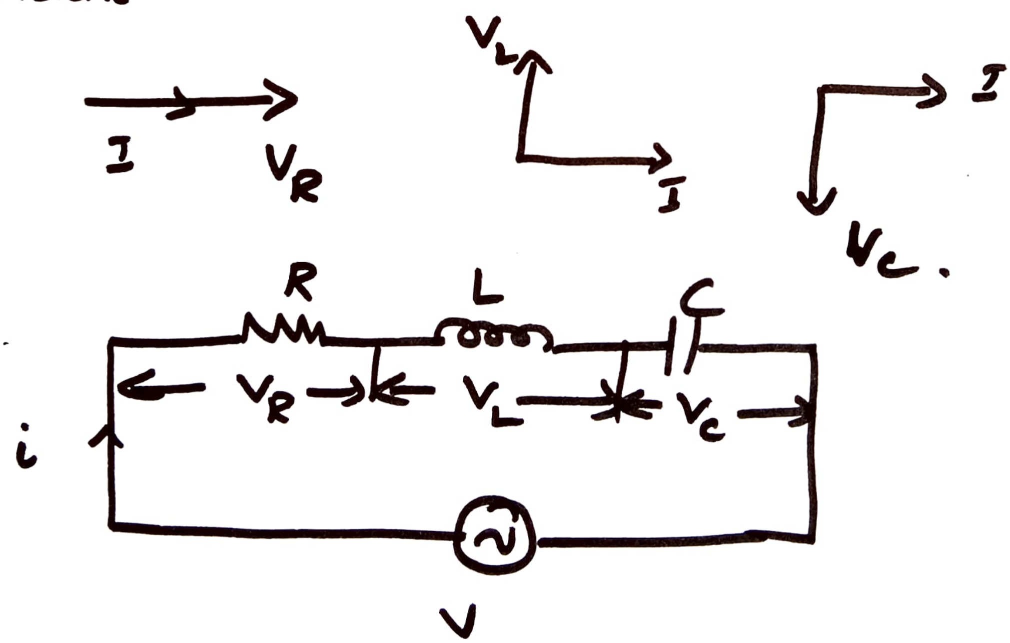
$$= 7.13\,\Omega.$$

$$X_c = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi X_c} = 446.6\,\mu F.$$

RLC series circuits

consider a circuit having Resistance $R\,\Omega$, inductance L, Henrys and capacitance C farads all are connected in series.



$V = V_m \sin\omega t$

i = Instantaneous current

I is taken as reference vector

Voltage across $R = V_R = IR$ in phase with I

Voltage across $L : V_L = IX_L \angle 90^\circ$ V leads I by 90°.

$= jIX_L$

Voltage across $C : V_C = IX_C \angle -90^\circ$ V lags I by 90°.

$= -jIX_C$.

Applied voltage $V = V_R + V_L + V_C$.

$= IR + jIX_L - jIX_C$

$= I(R + jX_L - jX_C)$

$$\frac{V}{I} = Z = (R + jX_L - jX_C)$$

$= R + j(X_L - X_C)$

$= Z \angle \phi$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

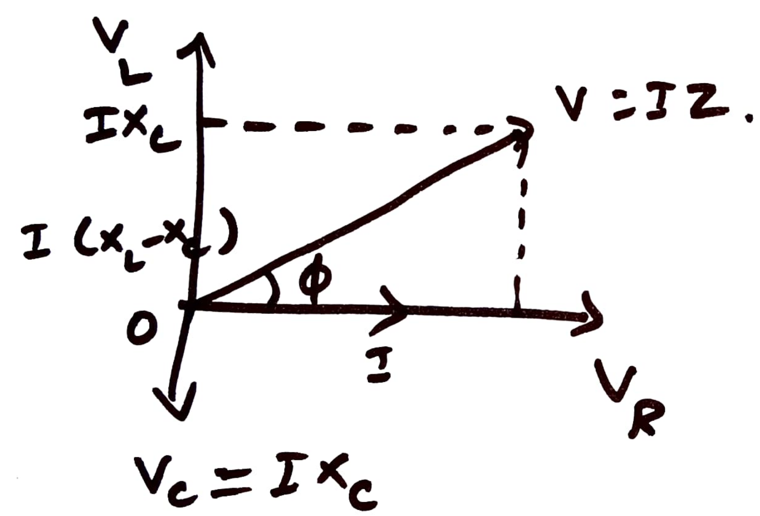
$(X_L - X_C) \rightarrow$ net reactance.

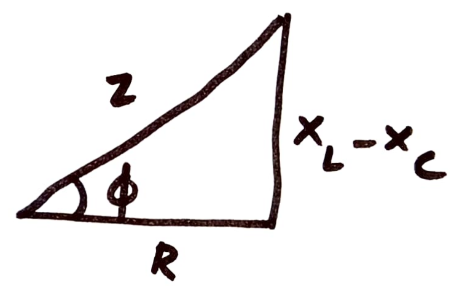
If $X_L > X_C$ the circuit behaves like R-C circuit

If $X_C > X_L$ the circuit behaves like R-L circuit.

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

case (i) If $X_L > X_C$.





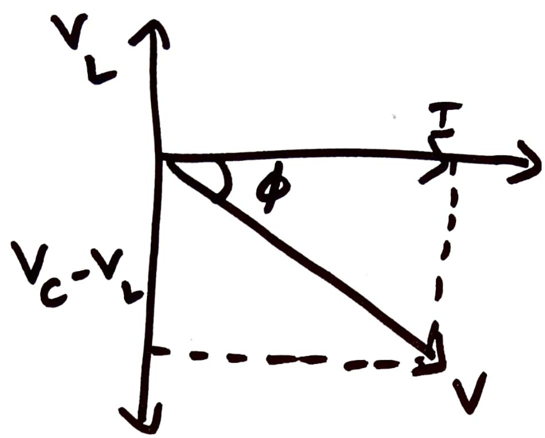
$$\tan\phi = \frac{X_L - X_C}{R}$$

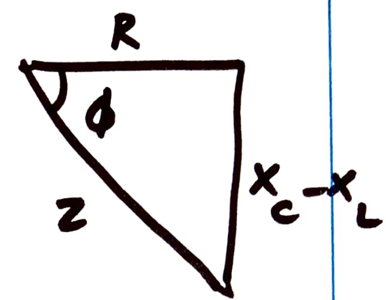
$$= \frac{(\omega L - 1/\omega C)}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)$$

P.f = $\cos\phi = R/Z$ lagging (R-L)

case (ii) If $X_C > X_L$ RC circuit





$V_C = IX_C$

$$\tan\phi = \frac{X_C - X_L}{R} = \frac{\frac{1}{\omega C} - \omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

p.f $= \cos\phi = R/Z$. leading (RC circuit)

Actual power $= VI\cos\phi$ (watts)

Reactive power $= VI\sin\phi$ (VAR)

A coil of resistance 10Ω and L= 0.1H is connected in series with a 150μF capacitor across 200V, 50Hz supply. calculate (a) inductive reactance b) capacitive reactance (c) impedance (d) current (e) p.f (f) voltage across coil and capacitor.

solution:-

a) $X_L = \omega L = 2\pi f L = 31.42\,\Omega$.

b) $X_C = 1/\omega C = 1/2\pi f C = 21.22\,\Omega$.

c) $Z = \sqrt{R^2+(X_L-X_C)^2} = 14.28\,\Omega$

d) $I = \frac{V}{I} = \frac{220}{14.28} = 14A$.

e) $p.f = \cos\phi = R/Z = 0.7$.

f) voltage across coil = I × impedance of the coil

$$Z = \sqrt{R^2+X_L^2}$$

$$= \sqrt{10^2+31.42^2} = 32.97\,\Omega.$$

$V_{coil} = 461.62\,V$.

$V_C = IX_C = 297.08V$.