Module I: DC Circuits 6 Hrs

Basic circuit elements and sources; Ohms law, Kirchhoff's laws; Series and parallel connection of circuit elements; Source transformation; Node voltage analysis; Mesh current analysis; Maximum power transfer theorem

CO1:

Evaluate DC and AC circuit parameters using various laws and theorems

Module 1

Evaluate DC circuit parameters using various laws and theorems

Linearity property

- Homogeneity

$$kiR = kv$$

- Additive property

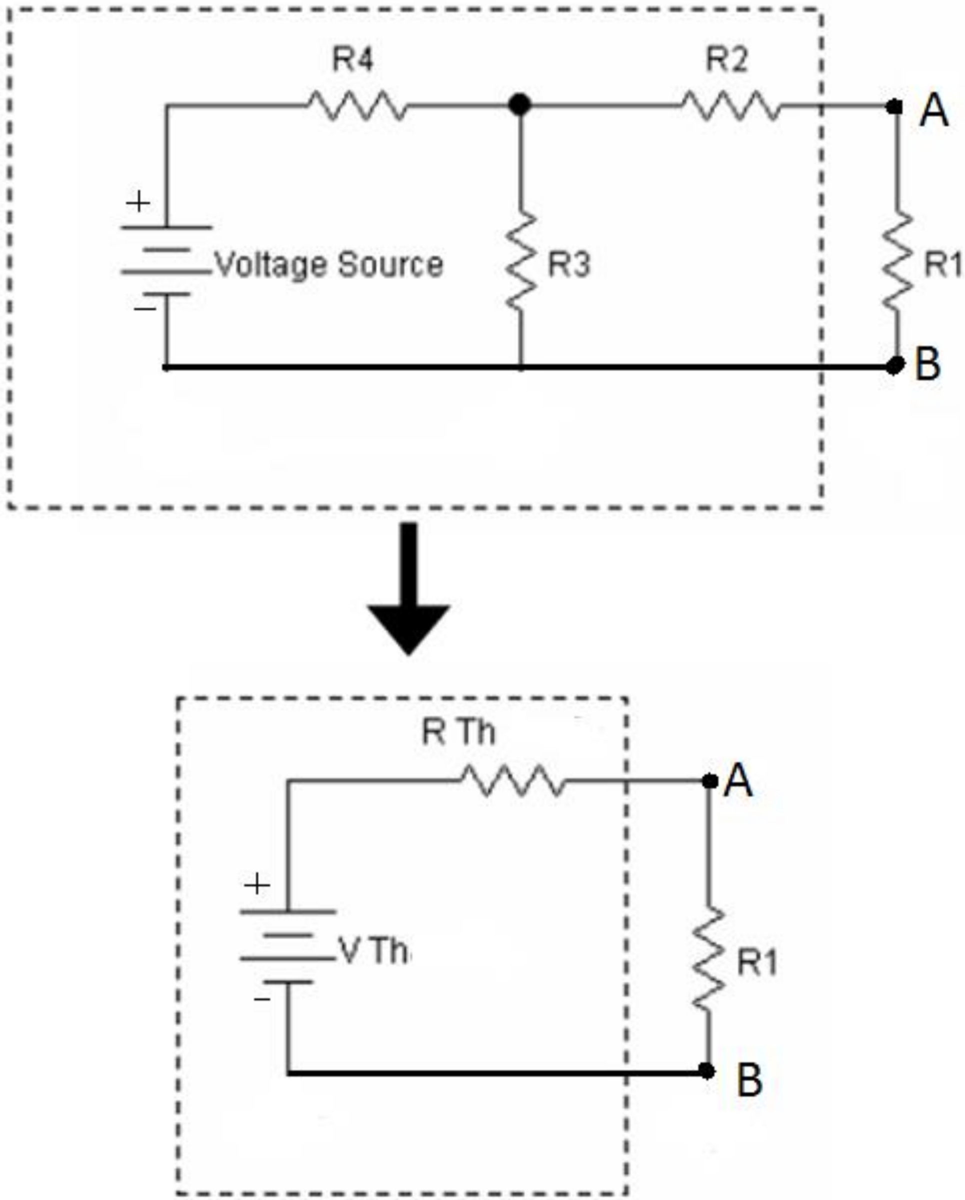
$$v = (i_1 + i_2)R = i_1R + i_2R = v_1 + v_2$$

- A circuit is linear if it is both additive and homogeneous.
- Linear circuit consists of only linear elements, linear dependent and independent sources.

Thevenin's theorem

A linear active two terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}

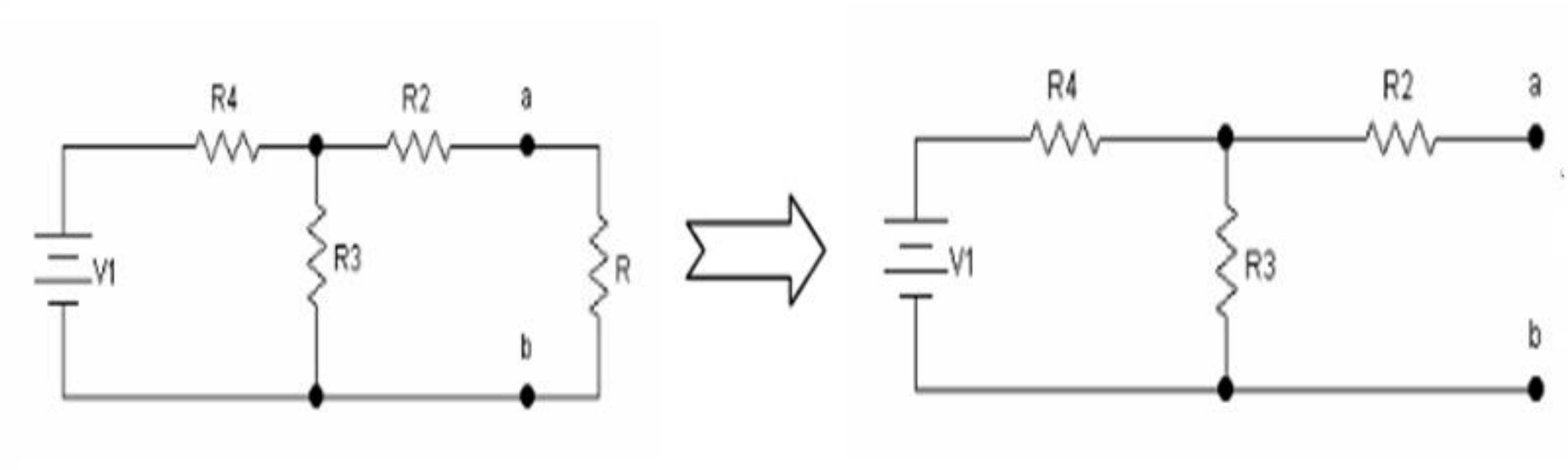
A fixed part of the circuit is replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}


R4
R2
A
+
Voltage Source
R3
R1
−
B
R Th
A
+
V Th
R1
−
B

To find V_{th}:

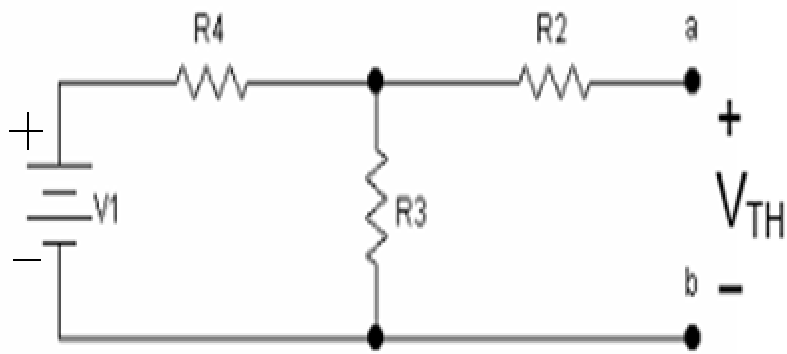
Step 1

Remove the resistor 'R' through which the current or across which the voltage to be determined.



Step 2

Find the voltage across the terminals ab. It is V_{th}.



$$V_{TH} = V_1 \frac{R_3}{R_3 + R_4}$$

Current through R_2 is zero because it is open circuited. The drop across R_3 gives the value V_{th}.

To find R_{th}:

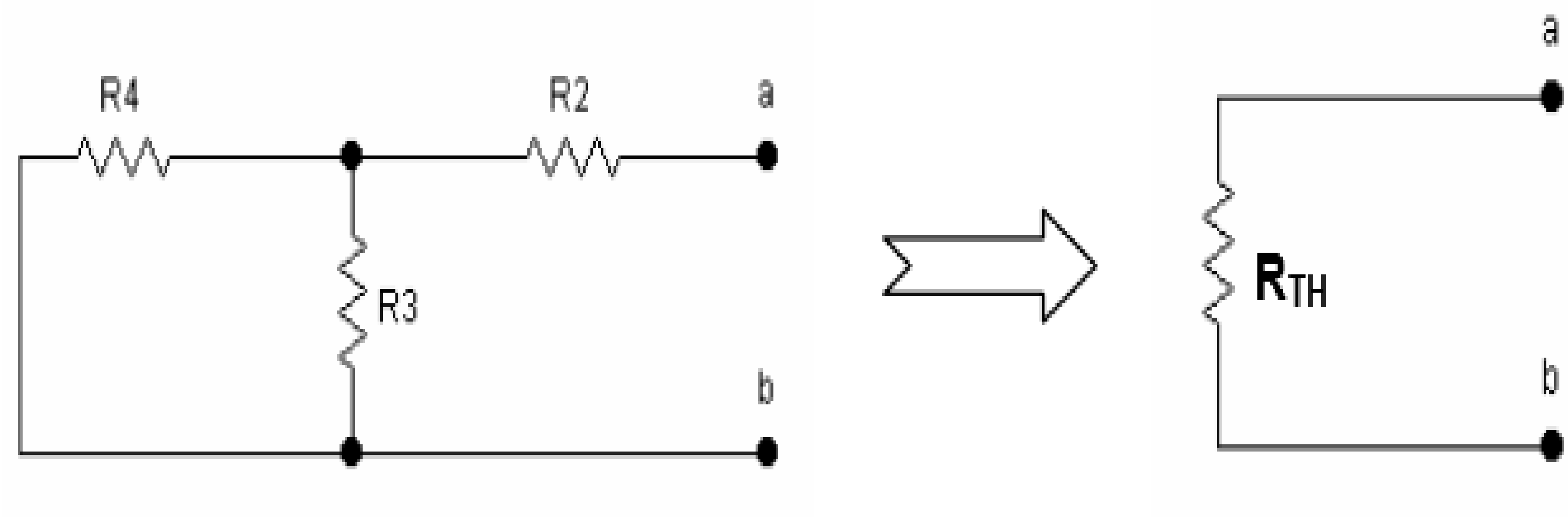
Step 3

Remove all the sources and replace them by their internal resistances

Voltage source – short circuited

Current source – open circuited

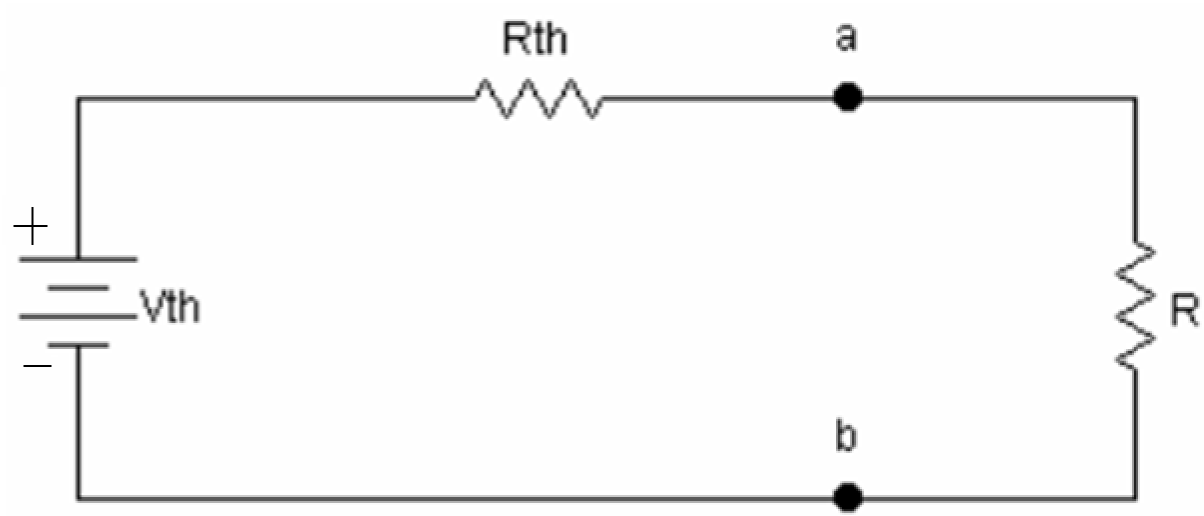
Find the resistance 'looking back' from the terminals 'ab'



$$R_{TH} = \frac{R_3 R_4}{R_3 + R_4} + R_2$$

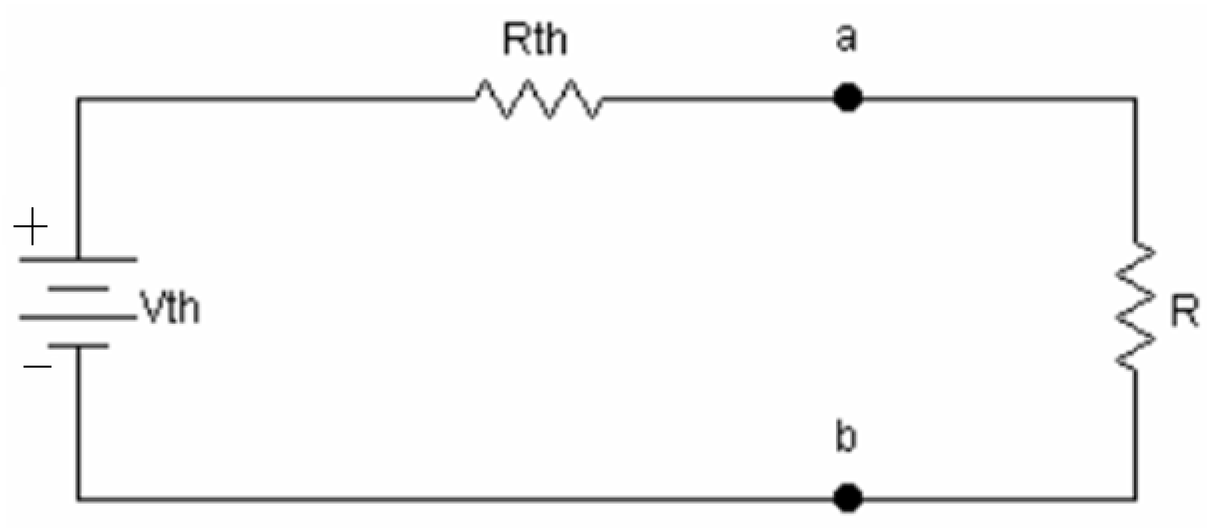
Step 4

Draw the Thevenin's equivalent circuit including the resistance across the terminals 'ab'



Step 5

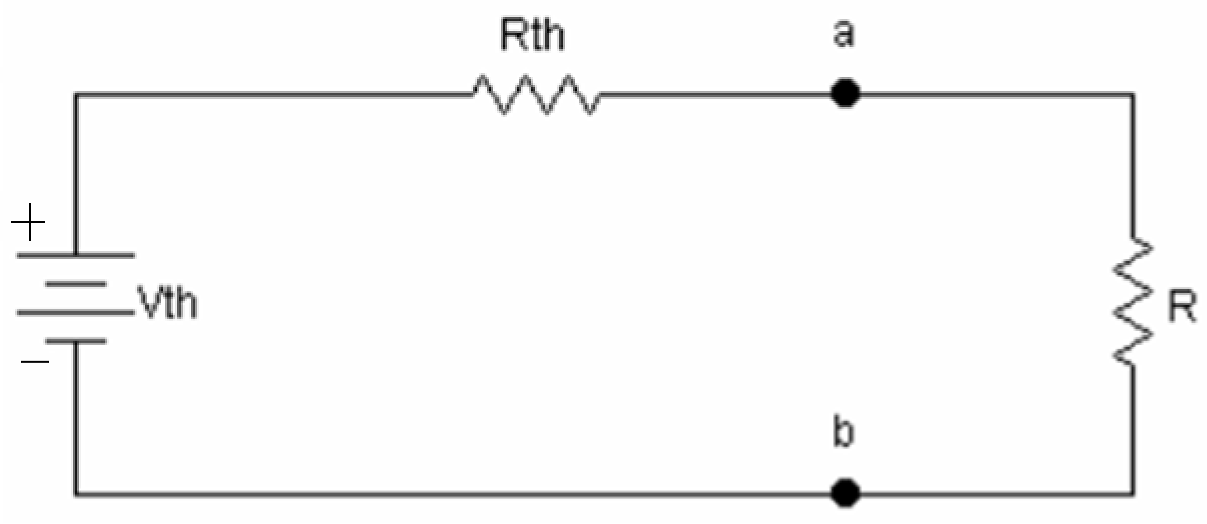
Find the current through the resistance across the terminals 'ab'



$$I_R = \frac{V_{TH}}{R_{TH} + R}$$

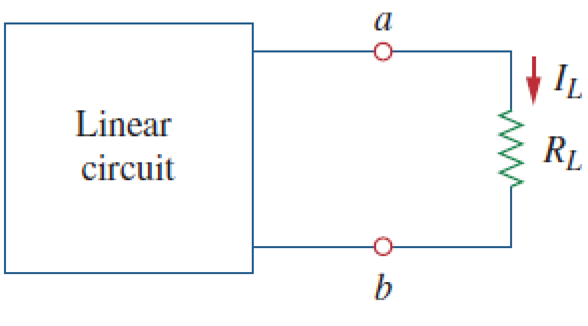
Step 6

Find the voltage across the terminals ab

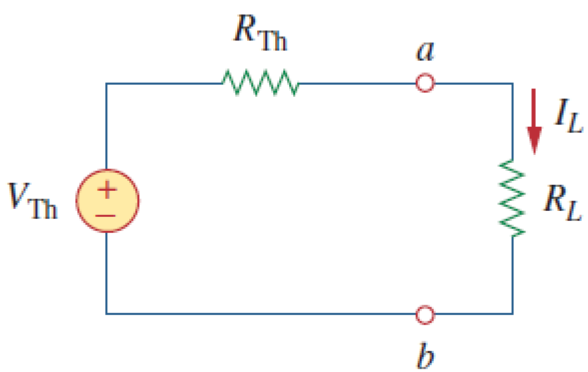


$$V_R = IR = \frac{V_{TH}R}{R_{TH} + R}$$

Load current



(a)



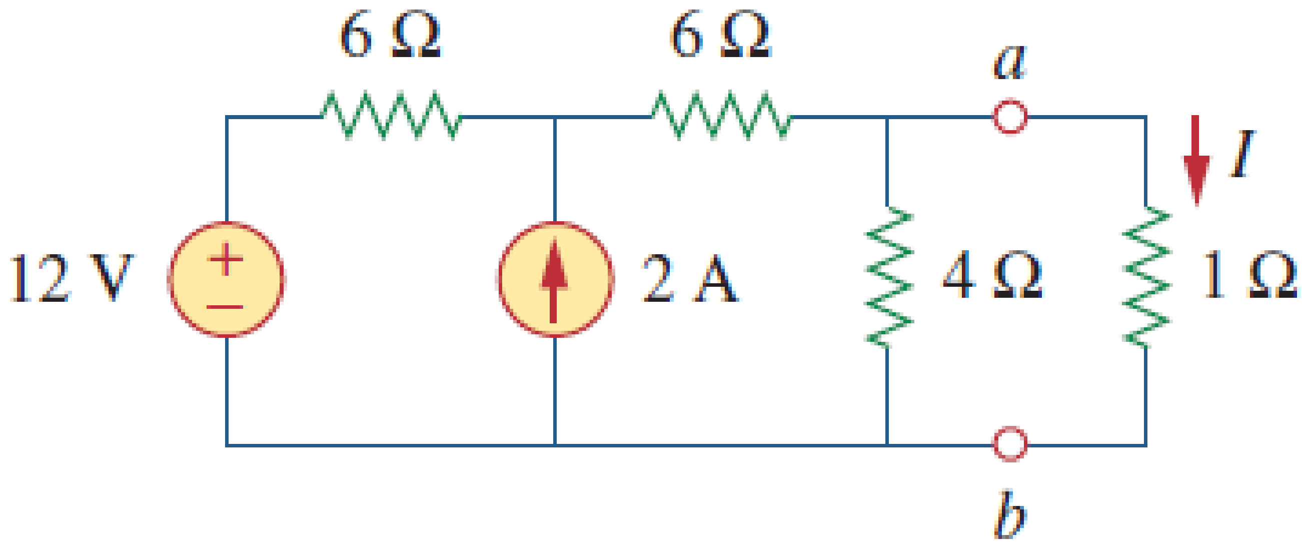
(b)

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

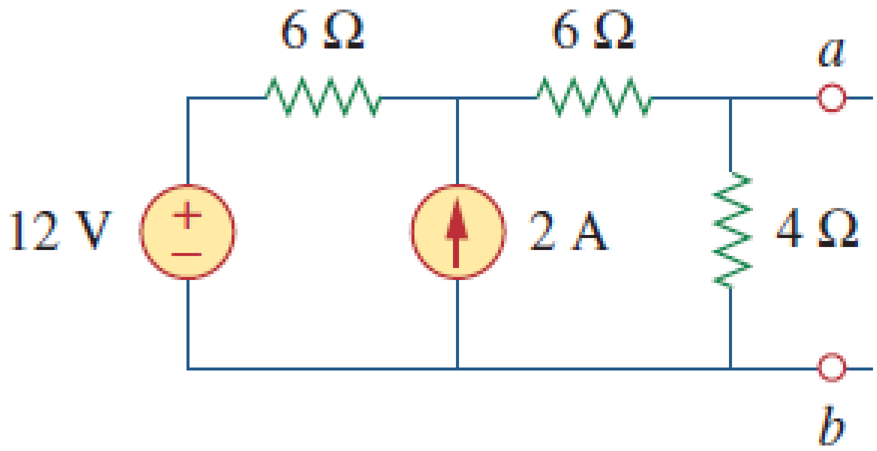
$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Example 1

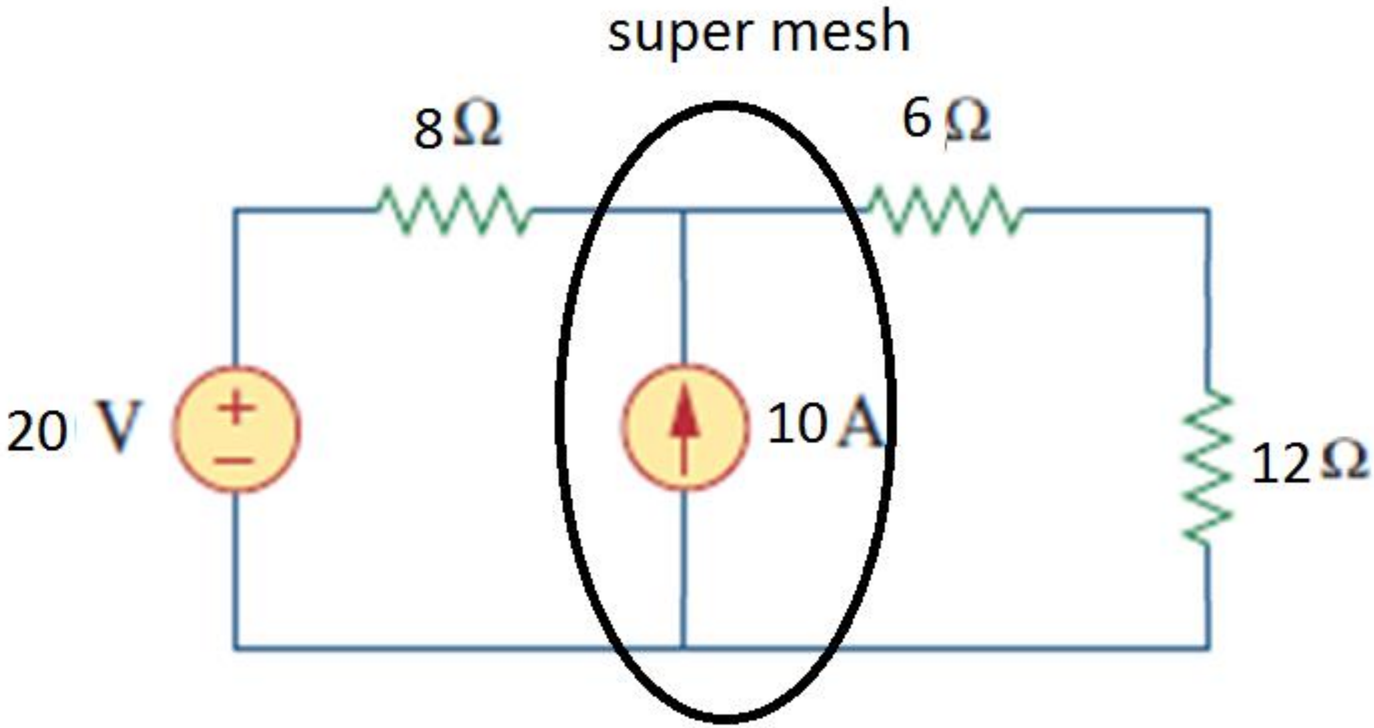
Find I using Thevenin's theorem



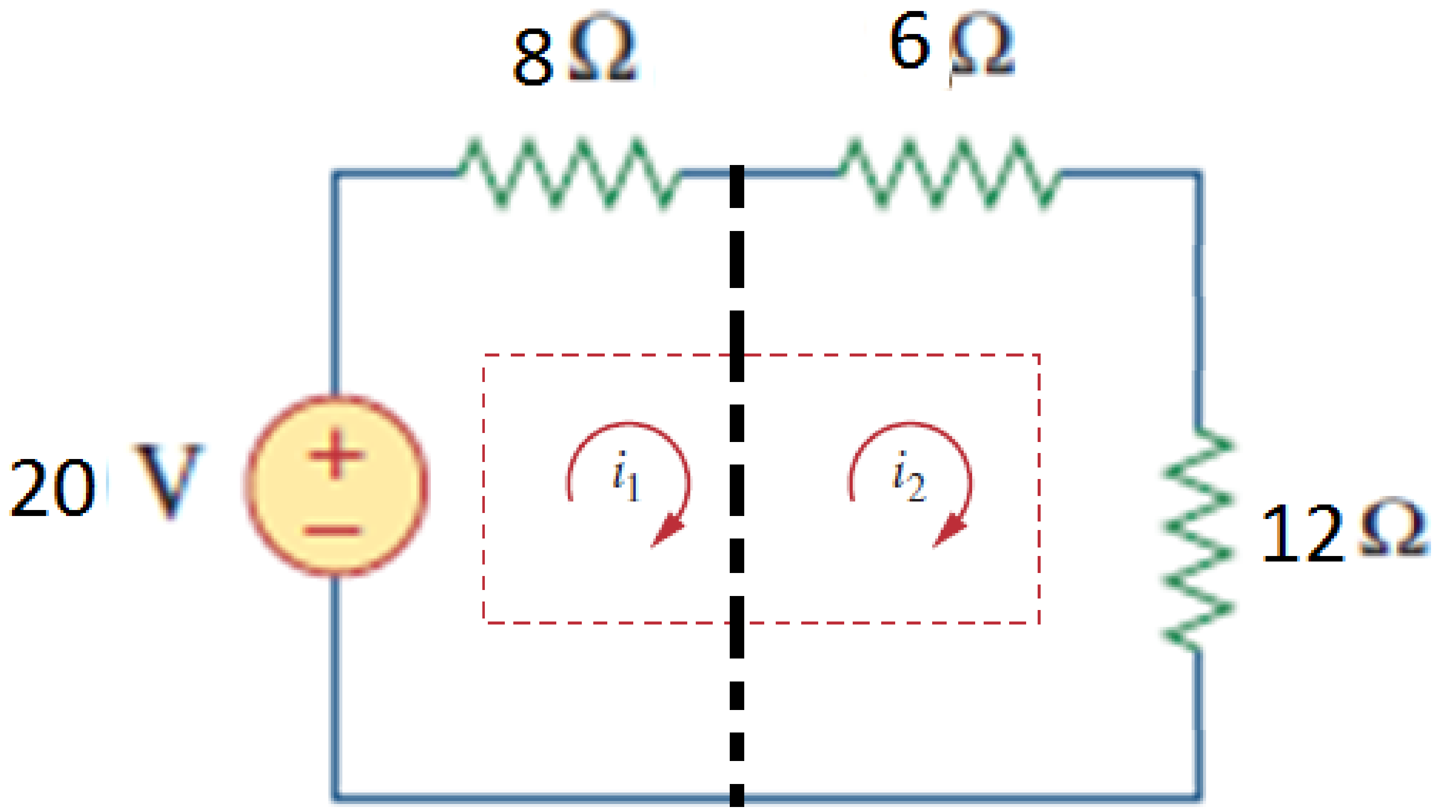
Step 1
Find V_{th} across ab



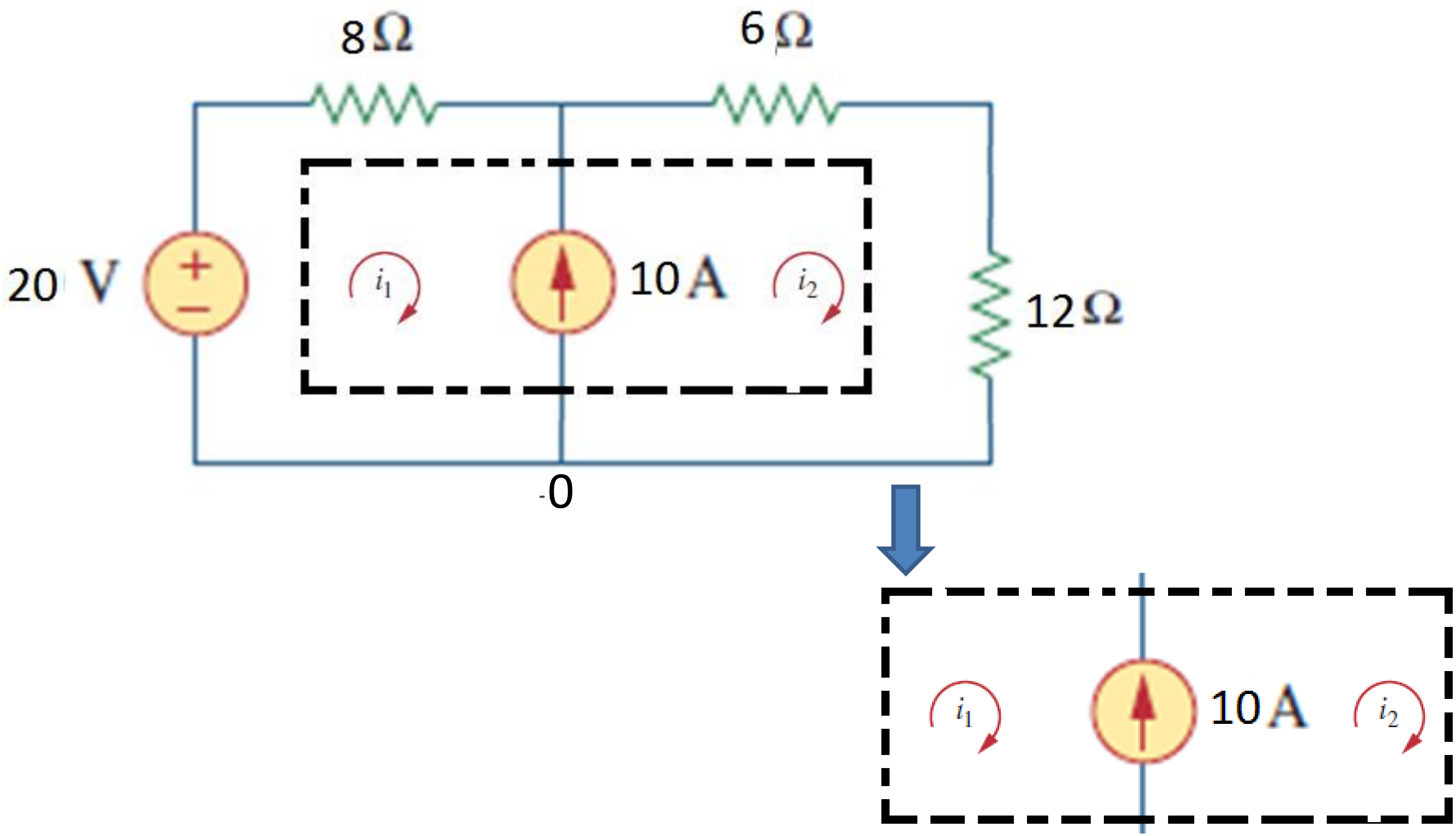
Super mesh

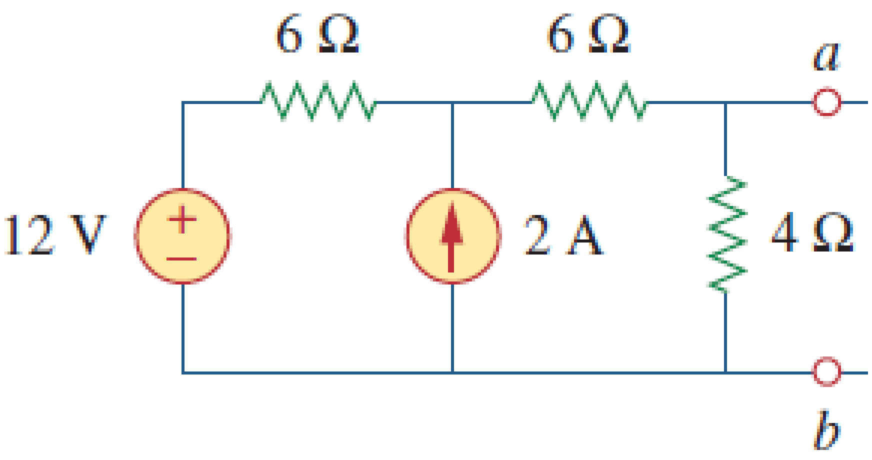


Step 1 – Apply KVL to both meshes

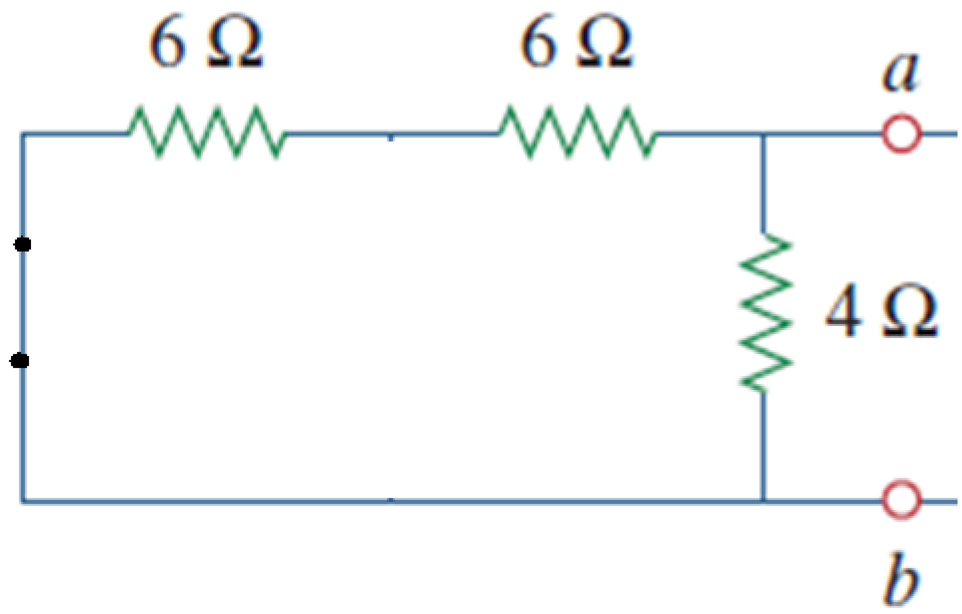


Step 2 - Apply KCL to node 0

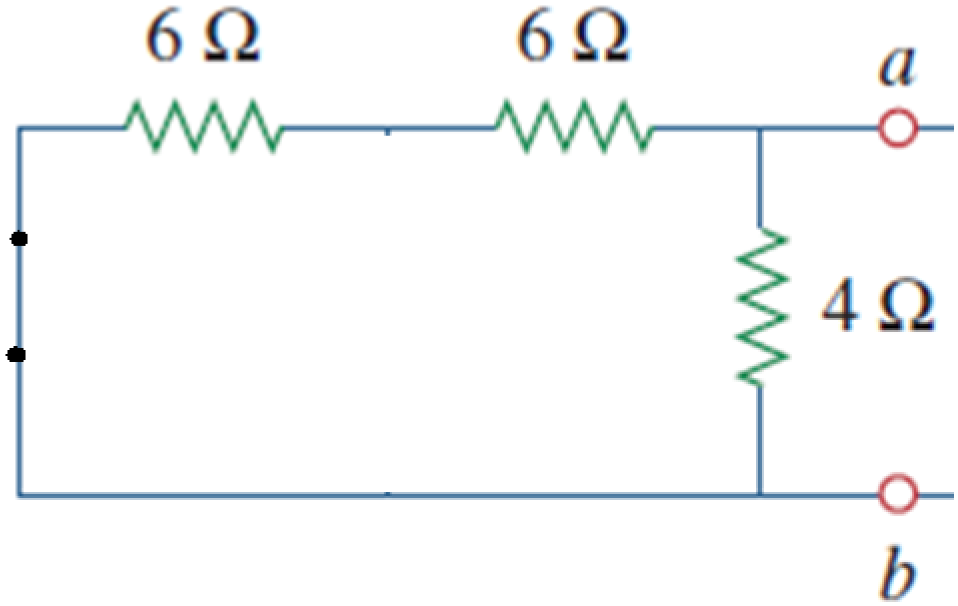



6 Ω
6 Ω
a
12 V
+
−
2 A
4 Ω
b

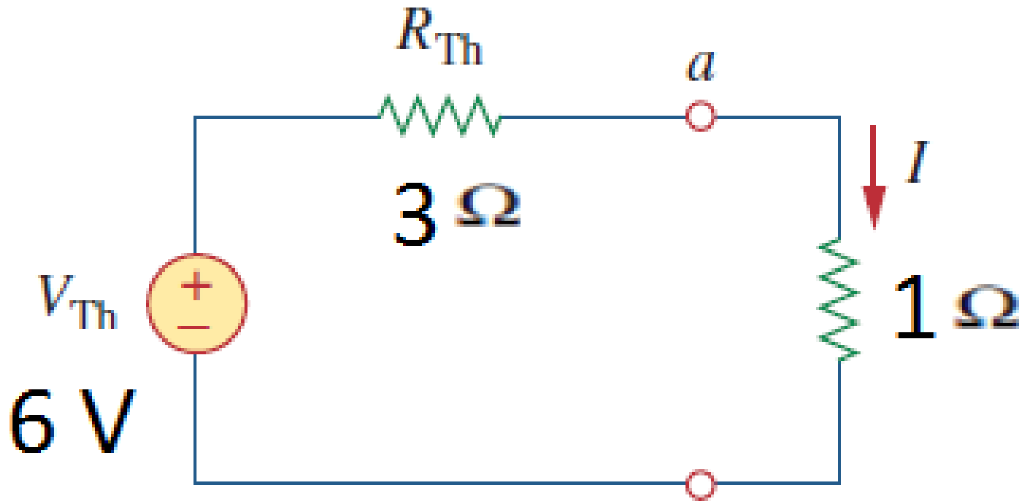
Step 2
Find R_{th} across ab
short circuit the voltage source
open circuit the current source



Step 2
Find R_{th} across ab



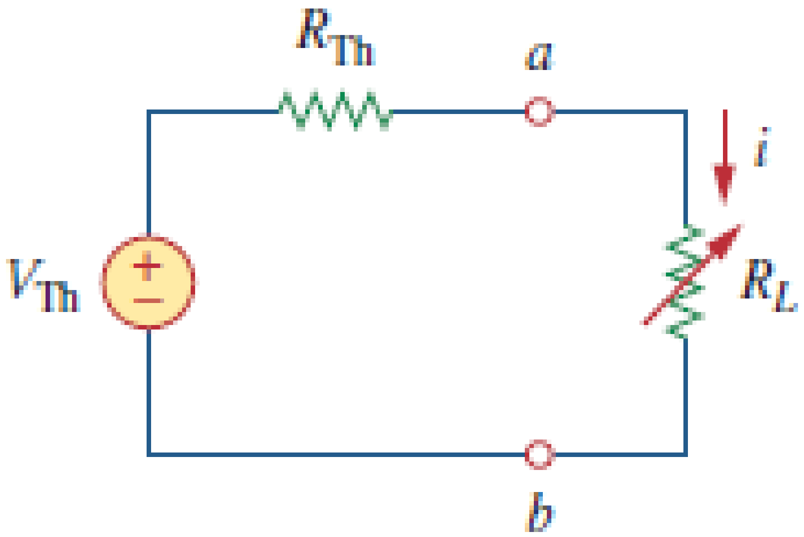
Step 3
Find I through 1 Ω

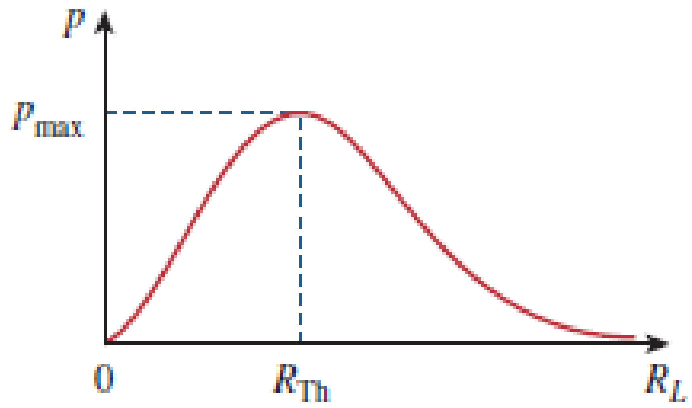


Maximum Power transfer theorem

Maximum power is transferred to the load when the load resistance is equal to the Thevenin's resistance.

$$R_L = R_{th}$$





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4} \right]$$

$$= V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3} \right] = 0$$

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L)$$

$$\boxed{R_L = R_{\text{Th}}}$$

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}}$$

Find the current through the load using network reduction method, mesh analysis, nodal analysis, Thevenin's theorem and find the value of load resistance for maximum power transfer