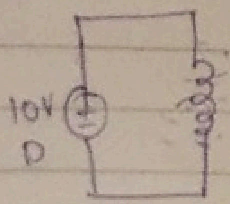
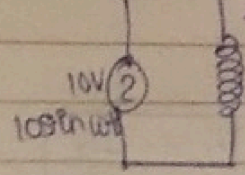
MODULE 3: MAGNETIC CIRCUITS

* $\mathcal{E} = -N\dfrac{d\phi_B}{dt}$ [Faraday's law]

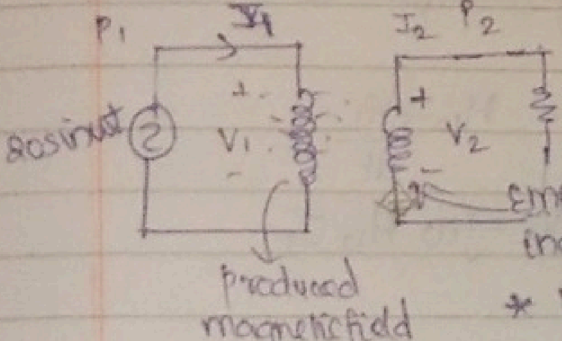


← constant magnetic field



← varying magnetic field.

Transformer

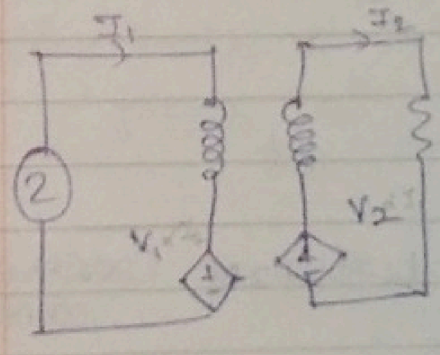


we use $\frac{V^2}{R}$ formula but not I^2R formular

* hence, $V_2 \propto I_1$
  hence can be thought of as current dependent voltage source.

* similarly, due to inductor ~~I_2~~, current V_1 is induced in inductor L_1.
  ~~$V_1 \propto$~~ $V_1 \propto I_2$

*



ϕ_1, ϕ_2 ← magnetic field produced in coil 1 & 2.

ϕ_{21} ← $\vec{B}$ induced in coil 1 due to coil 2.

* $V_1 = N_1\dfrac{d\phi_1}{dt} \Rightarrow V_1 = N_1 \cdot \dfrac{d\phi_1}{di_1} \cdot \dfrac{di_1}{dt}$

hence, $L_1 = N_1 \dfrac{d\phi_1}{di_1}$ = self inductance [$\because \mathcal{E} = L\dfrac{di}{dt}$]

Voltage across L_2 due to I_1 → $V_{12} = N_2\dfrac{d\phi_{12}}{dt} \Rightarrow V_{12} = N_2 \cdot \dfrac{d\phi_{12}}{di_1} \cdot \dfrac{di_1}{dt}$

hence M = Mutual inductance

$$M = N_2\frac{d\phi_{12}}{di_1}$$

* $M_{12} = M_{21}$

  $\Phi_{12} = \Phi_{21}$

* $e_1 = e_2 \in$ No. of turns/length.

* $V_1 = N_1 \times e_1$

  $V_2 = N_2 \times e_2$

hence dividing above 2 eq.

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

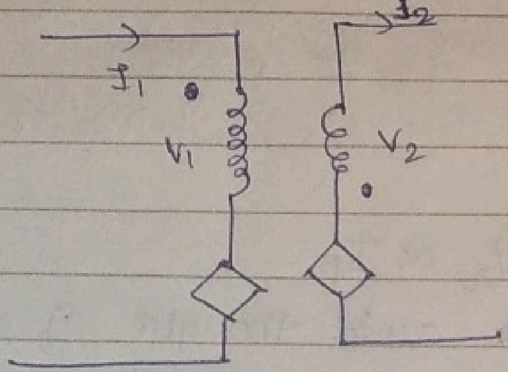
$V_1 I_1 = V_2 I_2$

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}}$$

if $N_1 > N_2 \Rightarrow V_1 > V_2 \in$ step down transform

$N_1 < N_2 \Rightarrow V_1 < V_2 \in$ step up transformer

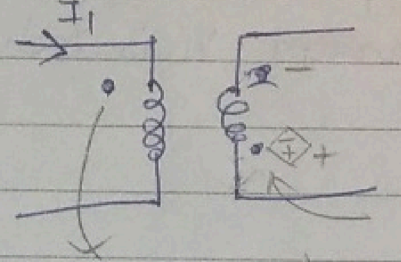
*

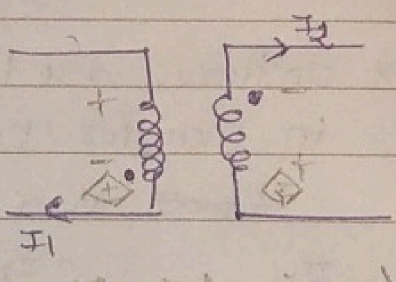


$$V_2 = M_{12} \frac{di_1}{dt}$$

$$V_2 = j\omega M_{12} I_1$$

*

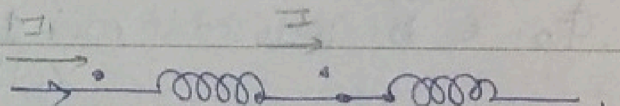




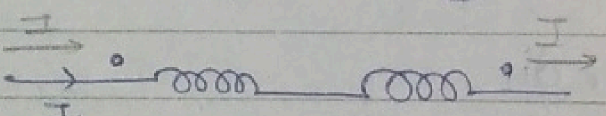
I_1 is entering the dot, hence '+' polarity is in the dot of opposite inductor

I_1 is leaving dot, hence '−' polarity in the opposite dot

*



$L = L_1 + L_2 + 2M$ (series aiding connection)

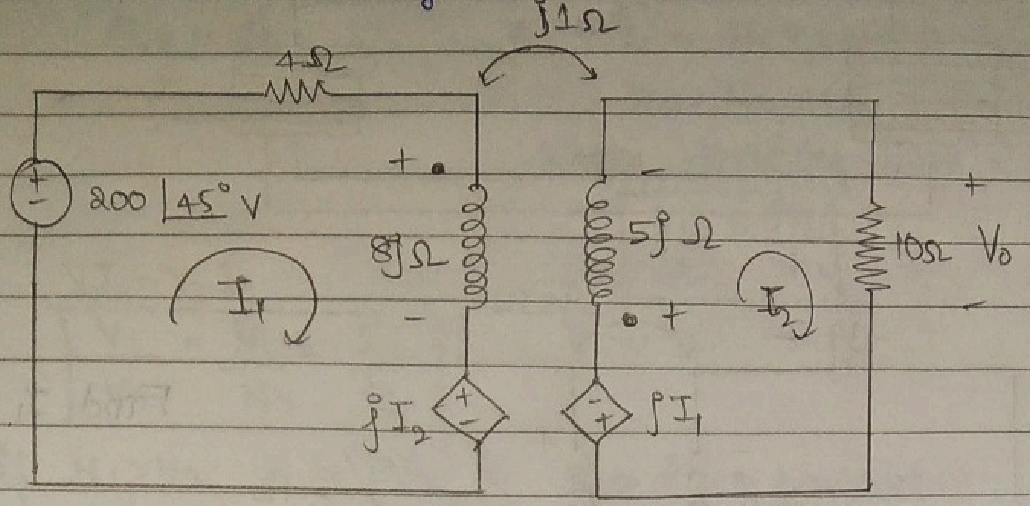


$L = L_1 + L_2 - 2M$

(series opposing connection)

Dot represents that the Inductors are mutually coupled.

H.W Determine the Voltage V_o.



solution: applying KVL along loop 1:-

$$-200\angle 45^\circ + 4I_1 + 8jI_1 + 1jI_2 = 0$$

$$I_1(4+8j) + 1jI_2 = 200\angle 45^\circ \quad \text{—— (i)}$$

applying KVL along loop 2:-

$$10I_2 + jI_1 + 5jI_2 = 0$$

$$\Rightarrow jI_1 + (10+5j)I_2 = 0 \Rightarrow I_1 = -\frac{(10+5j)I_2}{j}$$

From (i) $\Rightarrow$ $-\frac{I_2(10+5j)(4+8j)}{j} + jI_2 = \cancel{141.42+141.42}\,200\angle 45^\circ$

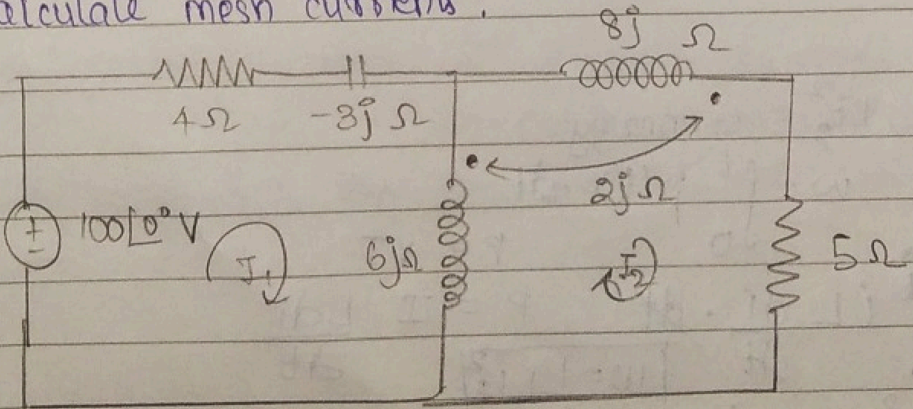
$$-I_2\,100 + jI_2 = 200\angle 45^\circ$$

$$I_2(-100+j) = 200\angle 45^\circ \Rightarrow I_2 = \frac{200\angle 45^\circ}{100\angle 180^\circ}$$

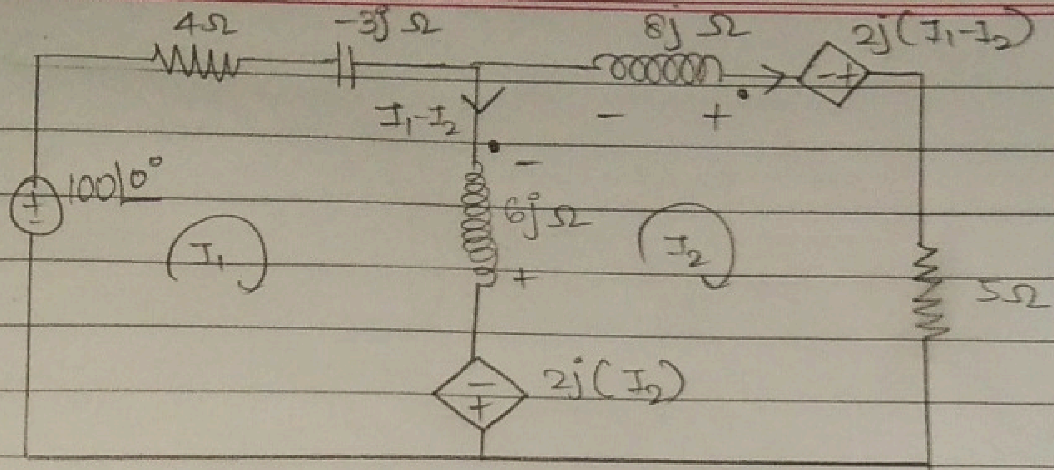
$$I_2 = 2\angle -135^\circ \text{ A}$$

$$V_o = 10I_2 = 20\angle -135^\circ \text{ V}$$

Example1 calculate mesh currents:



redrawing the circuit.



solution applying KVL across loop I :-

$$-100\angle 0^\circ + 4I_1 - 3jI_1 - 6j(I_1 - I_2) - 2jI_2 = 0$$

$$-100 + 4I_1 - 3jI_1 - 6jI_1 + 6jI_2 - 2jI_2 = 0$$

$$-100 + I_1(4 - 9j) + I_2(4j) = 0 \quad \text{——— (1)}$$

applying KV across loop 2 :-

$$2jI_2 + \cancel{6jI_2} - 6j(I_1 - I_2) - 8jI_2 - 2j(I_1 - I_2) + 5I_2 = 0$$

$$2jI_2 + 6jI_1 - 6jI_2 - 8jI_2 - 2jI_1 + 2jI_2 + 5I_2 = 0$$

$$I_2(2j - 6j - 8j + 2j + 5) + I_1(6j - 2j) = 0$$

$$I_2(-10j + 5) + I_1(4j) = 0$$

$$I_1 = \frac{(-5 + 10j)}{4j} I_2$$

From (1) $\Rightarrow$ $$-\frac{(5 - 10j)(4 - 9j)}{4j} I_2 + I_2(4j) = 100$$

$$(70 + 85j - 16) I_2 = 100 \Rightarrow I_2 = \frac{100\angle 0^\circ}{54 + 85j}$$

$$I_2 = \frac{100\angle 0^\circ}{100\angle 57.572^\circ} \Rightarrow \boxed{I_2 = 1\angle -57.572^\circ \text{ A}}$$

$$I_1 = \frac{11.18\angle 116.56^\circ \times 1\angle -57.572^\circ}{4\angle 90^\circ}$$

$$\boxed{I_1 = 2.795\angle -31.012^\circ \text{ A}}$$

* $w = \frac{1}{2} L i^2$ ∈ energy

$$w = \int_0^t p(t)\, dt$$

$$P = VI$$

$$P = I \frac{L\, di}{dt}$$

$$w = \int_0^i i L \frac{di}{\cancel{dt}} \cdot \cancel{dt}$$

$$\boxed{w = \frac{1}{2} L i^2}$$

* $w = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V^2$

* coupling coefficient = k ; $M = k\sqrt{L_1 L_2}$

  k if $k = 1$: 100% efficiency.