AC CIRCUITS

LECTURE # 12, 13

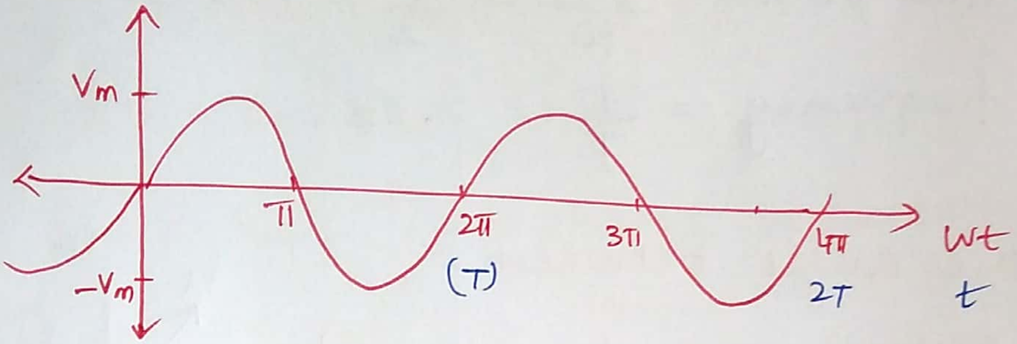
SINUSOIDS & PHASORS

SINUSOID $v(t) = V_m \sin \omega t$.

V_m = amplitude

ω = angular frequency (rad/s)

ωt = argument of the sinusoid.



$T = \frac{2\pi}{\omega}$ = time period of sinusoid.

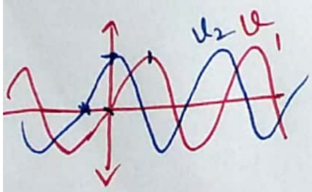
Sinusoid is periodic : $v(t) = v(t+T)$

Time period $T \rightarrow$ number of secs / cycle.

$f = \frac{1}{T}$ = frequency $\rightarrow$ cycles / sec.
↓
Hertz.

$\omega = 2\pi f$ (rad/s)

If we have two sinusoids.



$v_1(t) = V_m \sin \omega t$ $\quad$ $v_2(t) = V_m \sin(\omega t + \phi)$

then ϕ = phase angle between v_1 & v_2

v_1 lags v_2 by ϕ (or) v_2 leads v_1 by ϕ.

examples

1. $v(t) = 12 \cos(4\pi t - 75^\circ)$

   Amplitude $V_m = 12$

   phase $\phi = -75^\circ$.

   angular frequency $\omega = 4\pi = 12.57$ rad/s.

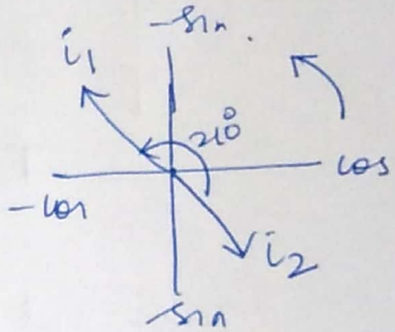
   Time period $= \frac{2\pi}{\omega} = \frac{1}{2} = 0.5$ s.

   frequency $= \frac{1}{T} = 2$ Hz.

2. phase angle between.

   $i_1(t) = -4 \sin(377t + 55^\circ)$

   $i_2(t) = 5 \cos(377t - 65^\circ)$



$i_1(t) = 4 \cos(377t + 55 + 90^\circ)$

$= 4 \cos(377t + 145^\circ)$

$i_2(t) = 5 \cos(377t - 65^\circ)$

$\phi = 210^\circ$ i_1 leads i_2 by 210°.

PHASOR

- complex $z = x + jy$

$$z = r\angle\phi$$

$$z = re^{j\phi}.$$

$$v(t) = V_m \cos(\omega t + \phi)$$

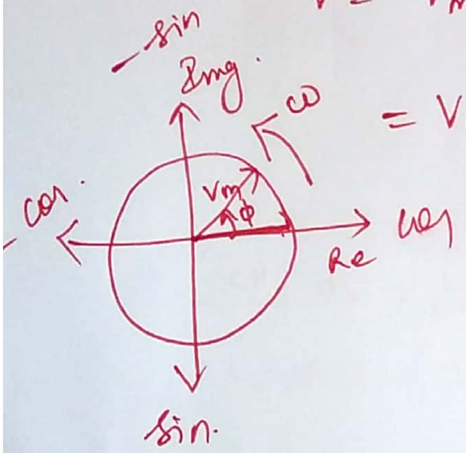
$$= Re\{V_m e^{j(\omega t+\phi)}\}$$

$$= Real\{\underline{V_m} e^{j\omega t} \cdot \underline{e^{j\phi}}\}$$

$$v(t) = Re\{V e^{j\omega t}\}$$

$V = V_m e^{j\phi}$ → phasor representing the sinusoid.

$$= V_m \angle\phi$$



projection Real axis $\cos\omega t$

$v(t) = 7\cos(2t + 40^\circ) \Rightarrow V = 7\angle 40^\circ$ $w = 2$ rad/s.

$i(t) = -4\sin(10t + 10^\circ) \Rightarrow I =$

$= 4\cos(10t + 10 + 90^\circ)$

$= 4\cos(10t + 100^\circ) \Rightarrow I = 4\angle 100^\circ$ $w = 2$ rad/s.

PRACTICE

$V = -25\angle 40^\circ$

$I = j(12 - j5)$

$I_r =$

sum of two phasor:

$v_1 = -10\sin(\omega t - 30^\circ)$

$v_2 = 20\cos(\omega t + 45^\circ)$

$v = v_1 + v_2 = ?$

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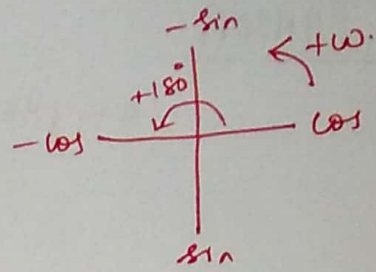
Sum of phasors



$v_1(t) = -10\cos(\omega t - 30^\circ)$

$v_2(t) = 20\cos(\omega t + 45^\circ)$

Find $v = v_1 + v_2$.

To add this in time domain, we need to use cos(A+B) identity and simplify.

Hence change to phasor domain and solve.

$v_1(t) = +10\cos(\omega t - 30 + 180^\circ)$

$= 10\cos(\omega t + 150^\circ)$

$\therefore V_1 = 10\angle 150^\circ$

$v_2(t) = 20\cos(\omega t + 45^\circ) \quad \therefore V_2 = 20\angle 45^\circ$

$V = V_1 + V_2$

$V = 10\angle 150^\circ + 20\angle 45^\circ$

$V = 19.91\angle 74^\circ$ V.

PRACTICE

$v_1(t) = -10\sin(\omega t - 20^\circ)$

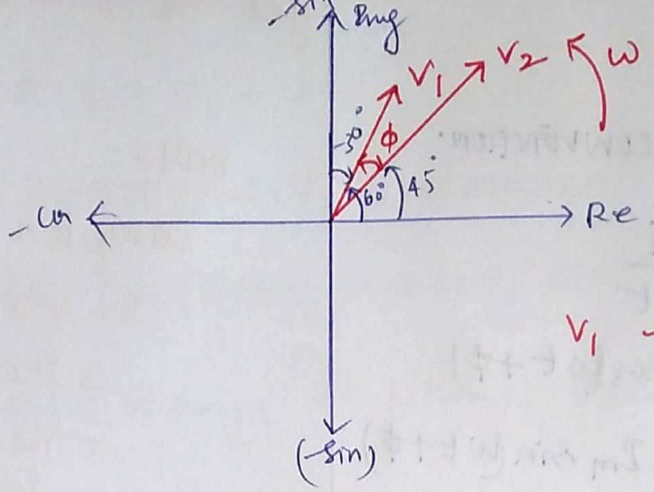
$v_2(t) = \sin(\omega t + 10^\circ)$

Find $v_1(t) + v_2(t)$.

Angle between phasors

$v_1(t) = -10\sin(\omega t - 30^\circ) = 10\cos(\omega t + 60^\circ)$

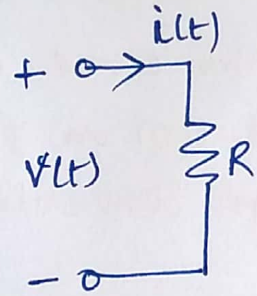
$v_2(t) = 20\cos(\omega t + 45^\circ)$

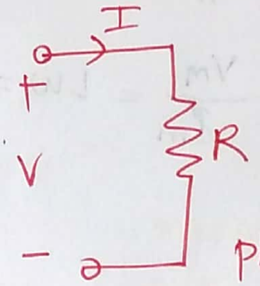


$\phi = 60 - 45 = 15^\circ$

V_1 leads V_2 by ~~45°~~ 15°

PHASOR RELATIONSHIPS OR R, L, C





phasor equivalent of R.

$v(t) = i(t) \cdot R$

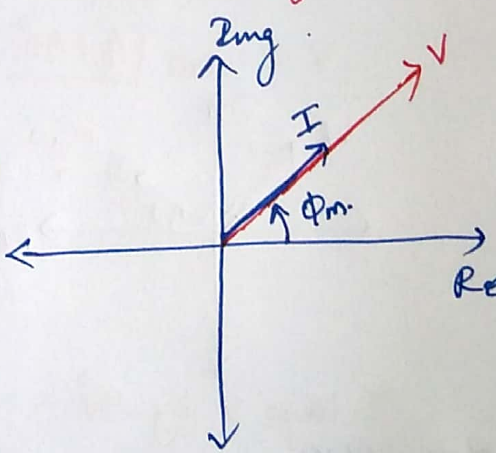
If $i(t) = I_m \cos(\omega t + \phi)$

$v(t) = \underbrace{R I_m}_{V_m} \cos(\omega t + \phi)$

As phasors,

$I = I_m \angle \phi$

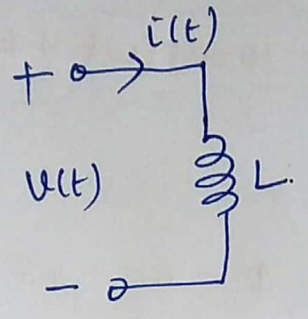
$V = V_m \angle \phi$



V, I in phase for a resistor.

⑥

Inductor (L).



PASSIVE SIGN CONVENTION.

$$v(t) = L\frac{di}{dt}$$

If $i(t) = I_m \cos(\omega t+\phi)$

$$v(t) = -\omega L I_m \sin(\omega t+\phi)$$

$$= V_m \cos(\omega t+\phi+90^\circ)$$

where $V_m = \omega L I_m$.

Inductive reactance. $\Leftarrow \frac{V_m}{I_m} = L\omega = |X_L|$

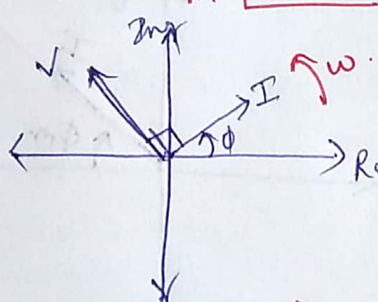
In phasor form.

$$I = I_m \angle \phi_0$$

$$V = V_m \angle \phi+90^\circ \Rightarrow L\omega I_m \angle\phi \angle 90^\circ$$

$$\therefore V = j\omega L I. \quad v(t) = L\frac{di}{dt}$$

$$\frac{V}{I} = j\omega L$$



I lags V by 90°.

We see that

$$\frac{d}{dt} \to j\omega$$

in phasor domain

$$L \to j\omega L \ \Omega$$

complex power

$$S = VI^* = V_m I_m \angle \phi+90-\phi$$

$$S = P+jQ = V_m I_m \angle 90^\circ$$

$$S = P+jQ = 0 + jV_m I_m$$

To do:

Try and repeat the same for inductor in active sign convention and get the similar conclusions.

Note:

complex power.

$$S = VI^* \text{(defined)}$$

$$= P+jQ$$

= real power + reactive power.

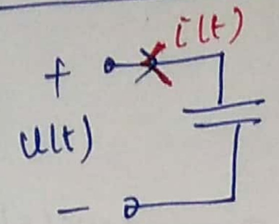
$P = 0$ (no real power)

$Q = V_m I_m$ (+ve)

In this case of Inductor in passive sign convention.

Todo:
Repeat for capacitor in passive sign convention.

Capacitor (C).



Note:

$$i(t) = -C\frac{dv}{dt}.$$

$$I = C\frac{V}{(1/j\omega)}$$

(or) $v(t) = \frac{1}{C}\int i(t)\,dt.$

$$V = \frac{1}{C}\frac{I}{j\omega}$$

active sign convention

$$i(t) = -C\frac{dv}{dt}.$$

let $v(t) = V_m \cos(\omega t + \phi)$

$$i(t) = +C\omega V_m \sin(\omega t + \phi)$$

$$= C\omega V_m \cos(\omega t + \phi - 90^\circ)$$

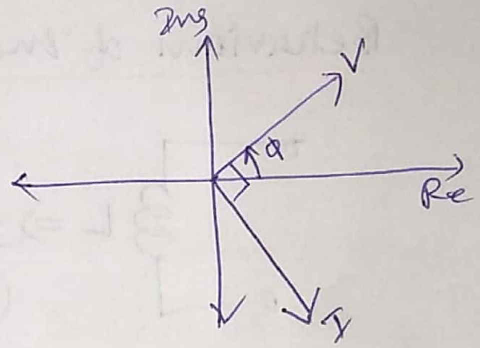
In phasor form,

$$V = V_m \angle \phi$$

$$I = C\omega V_m \angle \phi - 90^\circ = -jC\omega V$$

$$= I_m \angle \phi - 90^\circ \Rightarrow X_c = \frac{1}{j\omega C}$$

X_c = capacitive reactance.



I lags V by 90°.

complex power

$$S = VI^* = V_m\angle\phi \cdot I_m \angle 90 - \phi.$$

$$S = P + jQ = V_m I_m \angle 90^\circ.$$

$$P + jQ = 0 + jV_m I_m.$$

$P = 0$ (real power is zero)

$Q = V_m I_m$ (reactive power).

Hence,

$\int \longrightarrow \frac{1}{j\omega}$ in phasor domain

$C \longrightarrow \frac{1}{j\omega C}$

Impedance and Admittance

In phasor domain,

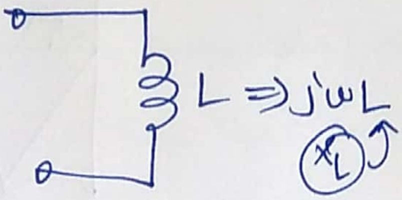
$V = RI \Rightarrow Z = R$ — Impedance (Z)

$V = j\omega L I \Rightarrow Z = j\omega L$ — Admittance $Y = \frac{1}{Z}$.

$V = \frac{1}{j\omega C} I \Rightarrow Z = \frac{1}{j\omega C}$

Impedance $Z = R + jX$. (Resistance + reactance)

Behaviour of Inductor and Capacitor at diff frequency

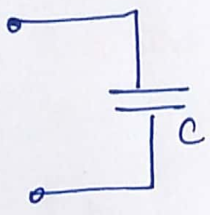


As $\omega \to 0$ $X_L \to 0$ (dc)

Inductor → SC

As $\omega \to \infty$ $X_L \to \infty$ (high freq ac)

Inductor → OC



$X_C = \frac{1}{j\omega C}$

As $\omega \to 0$, $X_C \to \infty$ (dc)

Capacitor → open circuited (OC)

As $\omega \to \infty$, $X_C \to 0$ (ac)

Capacitor → short circuited. (SC).

Note: Capacitor is open circuit to dc. Hence it acts as a filter, allowing dc (low freq) to pass through and block high frequency ac signals (acts as a sc V=0).

Application of Phasors

(1) Solving an integro-differential equation.

In phasor domain

$$\frac{d}{dt} \rightarrow j\omega \qquad \int \rightarrow \frac{1}{j\omega}$$

Consider,

$$2\frac{dv}{dt} + 5v(t) + 10\int v(t)\,dt = 50\cos(5t - 30^\circ)$$

Transform to phasor domain, $\omega = 5$ rad/s.

$$2j\omega V + 5V + 10\frac{V}{j\omega} = 50\angle -30^\circ.$$

$$V\left[5 + j10 + \frac{10}{5j}\right] = 50\angle -30^\circ$$

$$V\left[5 + j10 - j2\right] = 50\angle -30^\circ$$

$$V = \frac{50\angle -30^\circ}{5 + j8}$$

$$V = 0.1855 - 5.2967i$$

$$V = 5.299\angle -87.99^\circ$$

$$V \simeq 5.3\angle -88^\circ.$$

$$\therefore v(t) = 5.3\cos(5t - 88^\circ)$$

PRACTICE (9.25) sadiku.

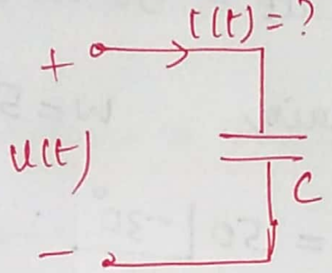
(a) $2\frac{di}{dt} + 3i(t) = 4\cos(2t - 45^\circ)$

(b) $10\int i(t)\,dt + \frac{di}{dt} + 6i(t) = 5\cos(5t + 22^\circ)$ A.

(2) Solving circuits with ac sources in phasor domain:

Example

If $v = 10\cos(100t + 30^\circ)$ is applied to a $50\mu F$ capacitor, calculate the current through the capacitor.



$v(t) = 10\angle 30^\circ$

$\omega = 100$ rad/s.

$C = 50\mu F$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j \times 100 \times 50 \times 10^{-6}}$$

$$X_C = -j200\ \Omega.$$
$$= 200\angle -90^\circ.$$

$V = IZ = IX_C.$

$$I = \frac{V}{Z}$$
$$= \frac{10\angle 30^\circ}{200\angle -90^\circ}$$

$$I = 0.05\angle 120^\circ.$$

$\therefore i(t) = 0.05\cos(100t + 120^\circ)$ A.

PRACTICE

The voltage $v(t) = -12\cos(60t + 45^\circ)$ is applied to a 0.1 H inductor. Find the steady state current through the inductor.