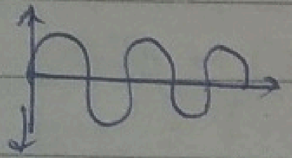
Three phase circuits

* 1ϕ generator - by electromagnetic induction
  - by faraday's law : $\Delta\phi_B$ induces ε.

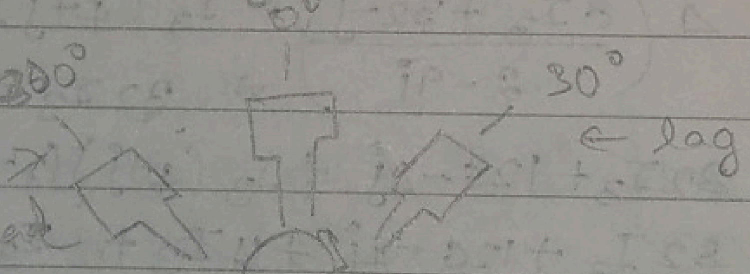


* frequency : $f = \frac{NP}{120°}$

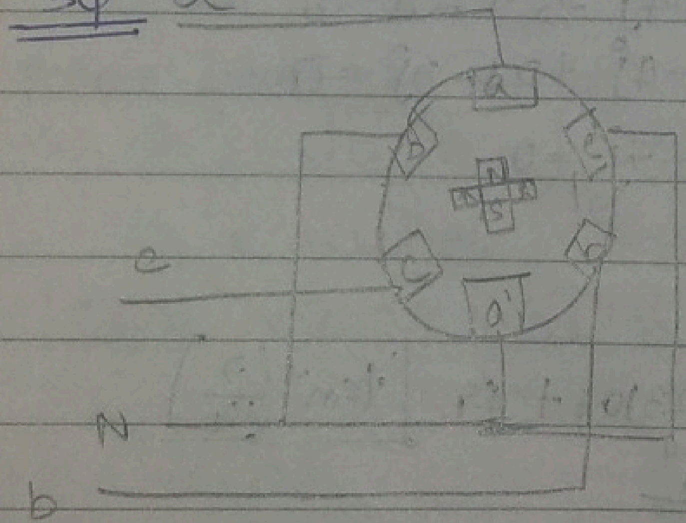
N = speed , P = No. of poles

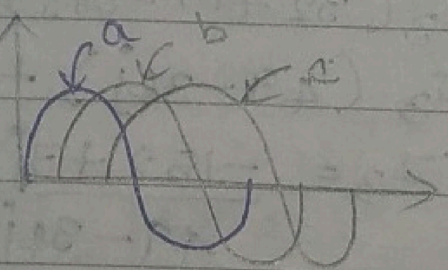
generator : coil - rotated : magnetics - fixed
Alternator : coil - fixed : magnets - rotated

* clockwise - leading 30°
anticlockwise - laging



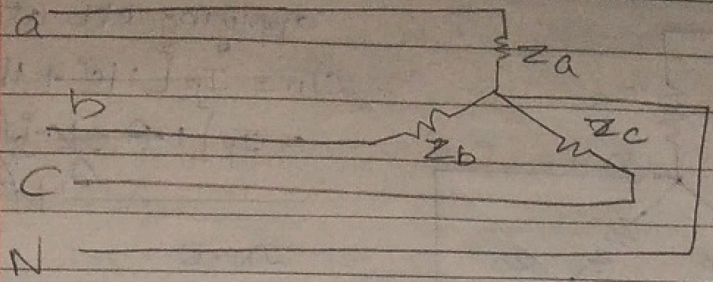
3ϕ



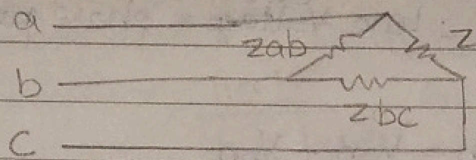


Star connection

star connection



Delta connection



* Phase voltage
  - voltage btwn a & N
* line voltage
  - voltage btwn a-b / b-c / c-a

* Phase sequence in clockwise ⇒ abc
  '+' sequence
  a reaches 1st, b reaches 2nd, c reaches 3rd.

* Phase sequence in anticlockwise ⇒ acb
  '–' sequence.

→ For abc sequence, $V_{an} = V_p \angle 0^\circ$

$V_{bn} = V_p \angle -120^\circ$

$V_{cn} = V_p \angle -240^\circ$

Problem ①

$V_{an} = 200 \cos(\omega t + 10^\circ)$

find V_{bn} & V_{cn} for abc sequence,

solution:-

$V_{an} = 200 \angle 10^\circ$; $V_{bn} = 200 \angle -110^\circ$; $V_{cn} = 200 \angle -230^\circ$

* current flowing through the line = $I_a = I_b = I_b = I_L$
  ⇒ line current

$I_{ab} = I_{bc} = I_{ca} = I_{phase}$ ⇒ Phase current

* balanced circuit ⇒ $Z_{ab} = Z_{bc} = Z_{ac}$

* $Z_D = 3 \times Z_Y$ ⇒

Z_D → impedence of delta connection

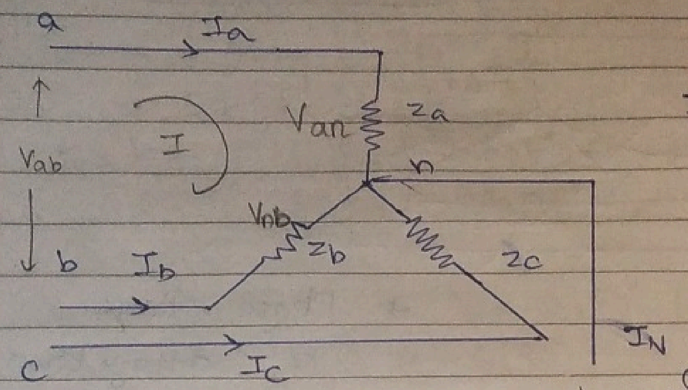
Z_Y → impedence in star connection.

Three pin plug

$\bigcirc$ large radius, $\uparrow A$ - $\downarrow$ Resistance, all I goes into the earth in case of short circuit

$\bigcirc$ o

In star connection (Balanced circuit)



applying KCL at node 1

$I_n = I_p[1\angle 0^\circ + 1\angle -120^\circ + 1\angle -210^\circ]$

$= I_p[1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - 0.5 + j\frac{\sqrt{3}}{2}]$

$I_N = 0$

$I_L = I_{ph}$: Line ~~Voltage~~ current = phase current

applying KVL at loop 1.

$V_{ab} = V_{an} + V_{nb} \Rightarrow V_{ab} = V_{an} - V_{bn}$

$V_{ab} = V_p[1\angle 0^\circ - 1\angle -120^\circ] = V_p(1.5 + j0.83)$

$V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$

$V_{ab} = \sqrt{3}\, V_p \angle 0^\circ \cdot \angle 30^\circ$ $\quad [\because V_{an} = V_p\angle 0^\circ]$

$\boxed{V_{ab} = \sqrt{3}\, V_{an} \angle 30^\circ} \Rightarrow V_L = \sqrt{3}\, V_{ph} \angle 30^\circ$

$\boxed{I_L = I_{ph}}$

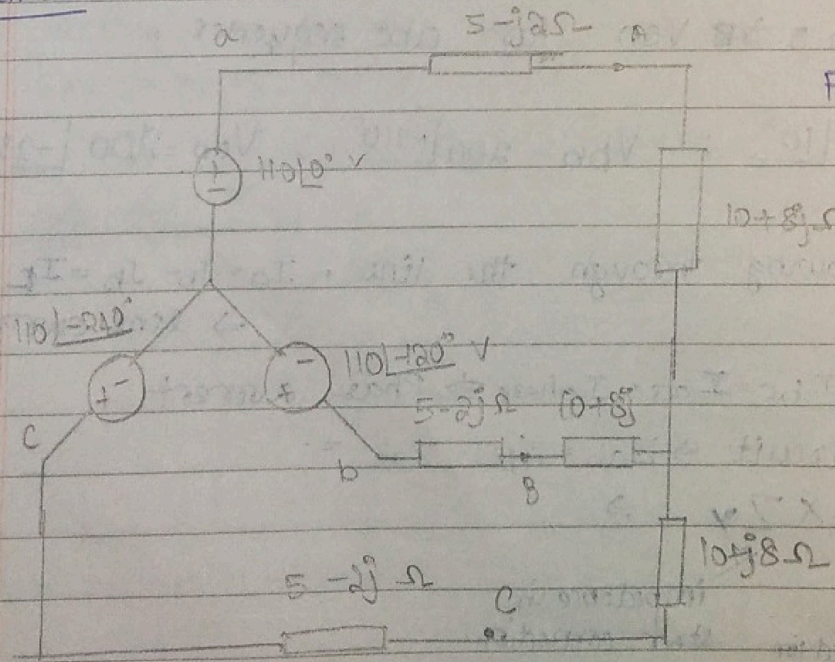
* ~~Va~~ $I_{an} + I_{bn} + I_{cn} = I_n = 0$

$V_{an} + V_{bn} + V_{cn} = 0$ $\leftarrow$ since of same impedence.

* $I_{phase} = \dfrac{V_{phase}}{Z/phase}$ ✓ ; $I_L \neq \dfrac{V_L}{Z/phase}$

Problem 1

Find I_a, I_b, I_c



-21.8
$\cancel{1298}$
141.8
120.8
262.6

Given :-

$Zy = Za = (10+j8)\Omega$, $Z_L = 5-2j\ \Omega$

$V_{an} = 110\angle 0^\circ$; $V_{bn} = 110\angle -120^\circ$

$V_{cn} = 110\angle 240^\circ$ } abc sequence

To find :- $I_a = ?$; $I_b = ?$; $I_c = ?$

solution :-

$$I_a = I_{ab} = \frac{110\angle 0^\circ}{5-2j+10+8j} = \frac{110\angle 0^\circ}{15+6j}$$

Polar form of $15+6j = \sqrt{6^2+15^2}\ \angle \tan^{-1}\frac{6}{15}$

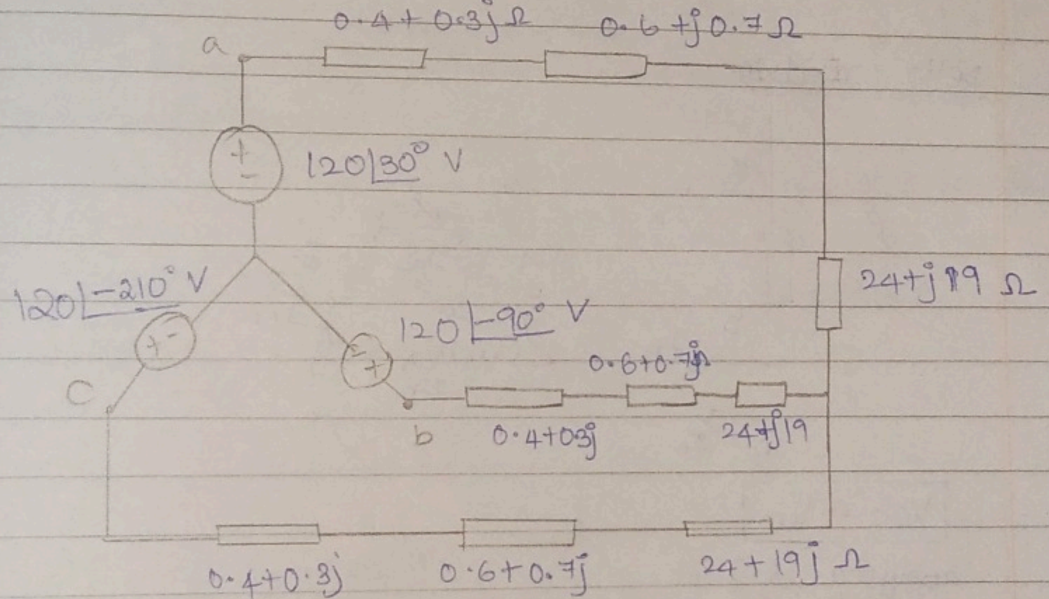
$= 16.15\angle 21.8$

$$I_a = \frac{110\angle 0^\circ}{16.15\angle 21.8} \Rightarrow I_a = 6.81\angle -21.8^\circ$$

$I_b = 6.81\angle -141.8^\circ$

$I_c = 6.81\angle -261.8^\circ$

H.W



To find : line voltages : V_{ab}, V_{bc}, $V_{ac} = ?$

I_a, I_b, I_c

solution :-

$$I_a = I_{ab} = \frac{120\angle 30^\circ}{0.4+0.3j+0.6+0.7j+24+19j}$$

$$= \frac{120\angle 30^\circ}{25+20j} = \frac{120\angle 30^\circ}{32.015\angle 38.66^\circ}$$

$I_a = 3.748\angle -8.66^\circ$

$I_b = 3.748\angle -128.66^\circ$; $I_c = 3.748\angle -248.66$

$V_L = \sqrt{3}\, V_{ph} \angle 30^\circ$

$V_{ab} = \sqrt{3}\, V_{an} \angle 30^\circ \Rightarrow V_{ab} = \sqrt{3} \times 120 \angle 30^\circ \times \angle 30^\circ$

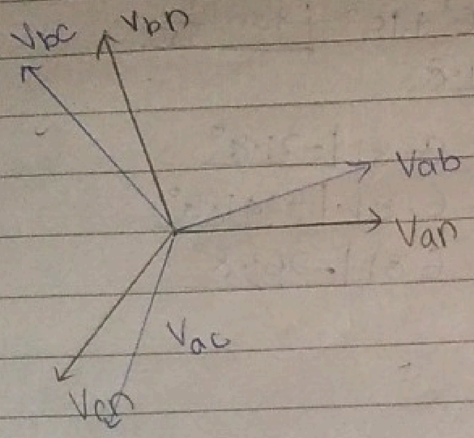
$\boxed{V_{ab} = 207.85 \angle 60^\circ\, V}$

$V_{bc} = \sqrt{3}\, V_{bn} \angle 30^\circ \Rightarrow V_{bc} = \sqrt{3} \times 120 \angle -90^\circ \times \angle 30^\circ$

$\boxed{V_{bc} = 207.85 \angle -60^\circ\, V}$

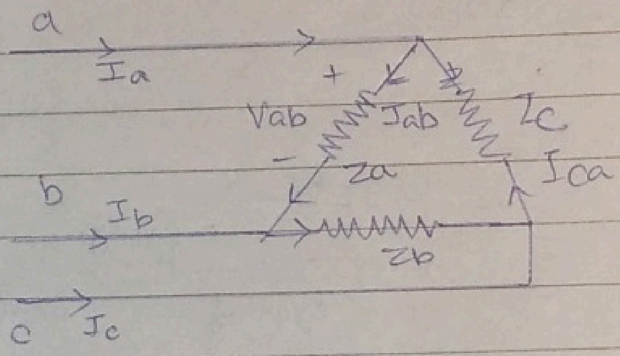
$V_{ca} = \sqrt{3}\, V_{cn} \angle 30^\circ \Rightarrow V_{ca} = \sqrt{3} \times 120 \angle -210^\circ \times \angle 30^\circ$

$\boxed{V_{ca} = 207.85 \angle -180^\circ\, V}$



Phasor diagram.

<u>Delta connection</u>



$\boxed{V_{ab} = V_{an}}$

applying kcl

$I_a + I_{ca} = I_{ab} \Rightarrow I_a = I_{ab} - I_{ca}$

$I_b = I_{bc} - I_{ab}$

$I_c = I_{ac} - I_{bc}$

assumption:

$I_{ab} = I_p \angle 0^\circ$

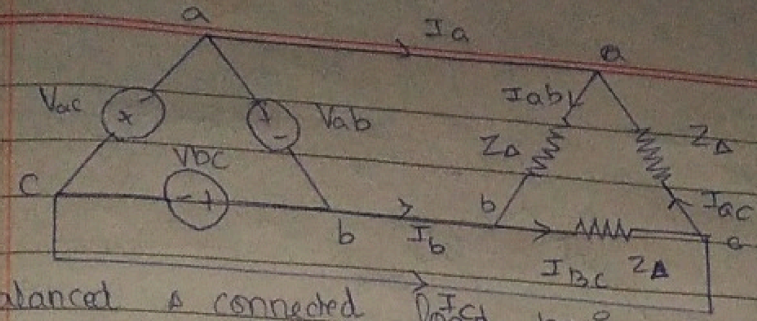
$I_{bc} = I_p \angle -120^\circ$

$I_{ca} = I_p \angle -240^\circ$

$I_a = I_p \angle 0^\circ - I_p \angle -240^\circ$

$\boxed{I_a = I_p \sqrt{3} \angle -30^\circ}$

Problem:



A balanced Δ connected load having an impedence 20-15j is connected to a Δ connected, positive sequence generator having $V_{ab} = 330\angle 0^\circ$ V. Calculate the phase currents of the load & the line currents.

Solution:- Given -

$\cancel{Z_\Delta = 2}$ $V_{ab} = 330\angle 0^\circ$; $V_{bc} = 330\angle -120^\circ$ - $V_{ca} = 330\angle -240^\circ$

$Z_\Delta = 20 - j15\ \Omega$

To find:

$I_{ab} = ?$; $I_{bc} = ?$; $I_{ca} = ?$

$I_a = ?$; $I_b = ?$; $I_c = ?$;

Solution:-

$$I_{ab} = \frac{V_{ab}}{Z_\Delta} = \frac{330\angle 0^\circ}{20 - j15}$$

polar form of $20 - j15 = 25\angle -36.8^\circ$

$I_{ab} = 13.2\angle 36.8^\circ$; $I_a = \sqrt{3}\,ph\angle -30^\circ \Rightarrow I_a = 22.86\angle 6.8^\circ$

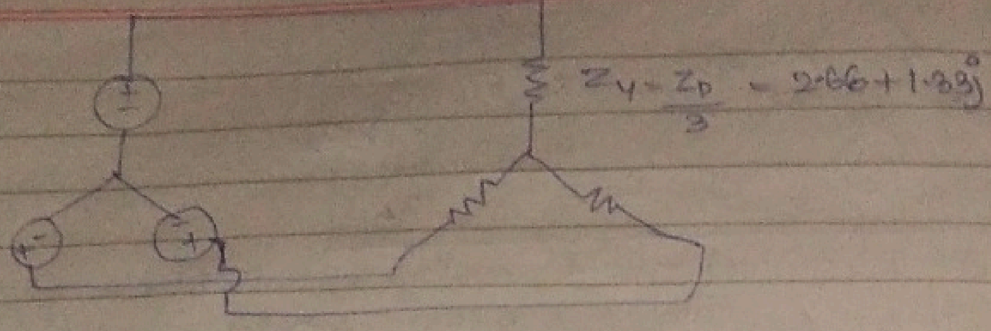
$I_{bc} = 13.2\angle -83.2^\circ$; $I_b = 22.86\angle -113.2^\circ$

$I_{ca} = 13.2\angle -203.2^\circ$; $I_c = 22.86\angle -233.2^\circ$

~~Hw~~
Problem 2

A balanced abc-sequence Y connected source with $V_{an} = 100\angle 10^\circ$ V is connected to a Δ connected load $8 + 4j$ per phase. Calculate phase & line currents.

Solution:-



$$I_{ab} = \frac{V_{ab}}{Z_Y} = \frac{100\angle 10^\circ}{2.66 + j1.33} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ}$$

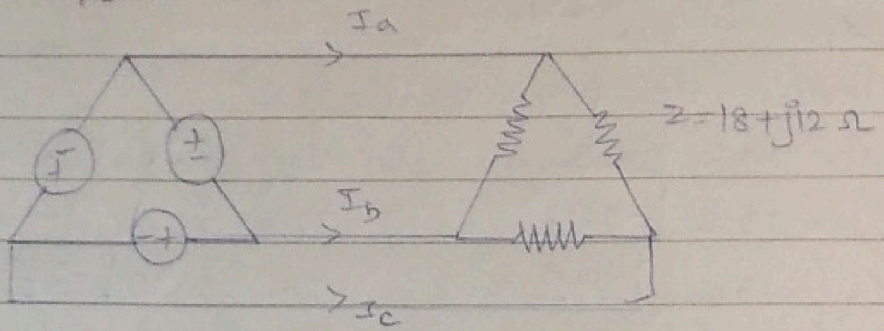
$I_{ab} = 33.5\angle -16.57^\circ = I_a$

$I_{bc} = 33.5\angle -136.57^\circ = I_{bc}$

$I_{ca} = 33.5\angle -256.57^\circ = I_{ca}$

H.W A positive sequenced balanced Δ connected source supplies a balanced Δ connected load. If the impedence per phase of the load is $18+12j\ \Omega$ and $I_a = 9.609\angle 35^\circ$. Find I_{AB} & V_{AB}

Solution.



Given:-

$I_a = 9.609\angle 35^\circ$; $Z = 18 + 12j\ \Omega$

To find: $I_{ab} = ?$ $V_{ab} = ?$

Solution:-

$I_a = I_{ab}\sqrt{3}\angle -30^\circ$

$$I_{ab} = \frac{I_a\angle 30^\circ}{\sqrt{3}} \Rightarrow I_{ab} = \frac{9.609\angle 35^\circ\angle 30^\circ}{\sqrt{3}}$$

$$\boxed{I_{ab} = 5.548\angle 65^\circ\ A}$$

$V_{ab} = I_{ab} \times Z_y$

Polar form of $18+12j\ \Omega = 21.63\angle 33.69^\circ$

$V_{ab} = 5.548\angle 65^\circ \times 21.63\angle 33.69^\circ$

$$\boxed{V_{ab} = 120\angle 98.69^\circ\ V}$$

3Phase power

Power in single phase:-

~~P~~ real power : $P = V_{ph} I_{ph} \cos\phi$

complex power : $Q = V_{ph} I_{ph} \sin\phi$

apparent power $= |S| = \sqrt{P^2 + Q^2} = V_{ph} I_{ph}$

$S = P + Qj$

Power in 3phasese :-

real power $= 3 V_{ph} I_{ph} \cos\phi = P$

~~complex power~~ $= 3 V_{ph} I_{ph} \sin\phi = Q$ = reactive power

apparent power $= 3 V_{ph} I_{ph}$

$\cdot S = 3P + 3Qj$ ← complex power

In Y connection (In terms of V_L, I_L) :- $V_L = V_{ph}\sqrt{3}$

real power $= P_{3\phi} = 3 \times \frac{V_L}{\sqrt{3}} \cdot I_L \cos\phi = \sqrt{3} V_L \cdot I_L \cos\phi$

$Q_{3\phi} = \sqrt{3} V_L \cancel{V_L} \sin\phi$

apparent power $= \frac{V_L}{3} I_L$

In Δ connection :- $I_L = I_{ph}\sqrt{3}$

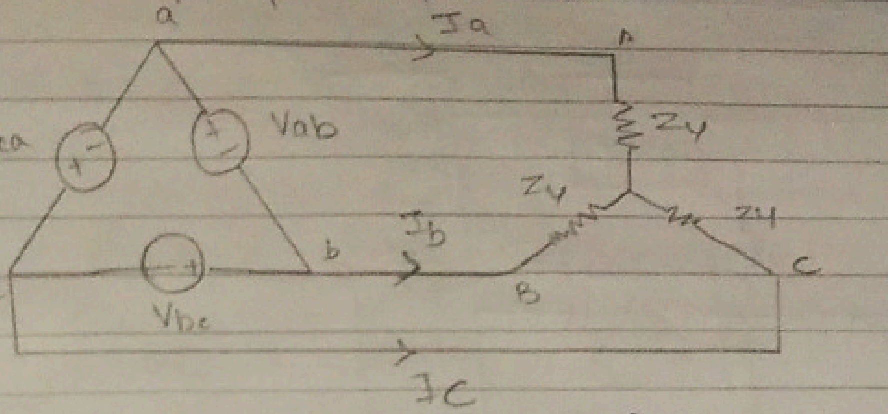
$P_{3\phi} = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi = \sqrt{3} \times V_L \times I_L \cos\phi$

$Q_{3\phi} = \sqrt{3} V_L I_L \sin\phi$

apparent power $= \frac{V_L I_L}{3}$

$Z_Y = Z \angle\phi$ ← power factor $= \cos\phi$

Problem 1: A balanced Y-connected load with a phase impedence of $40+j25$ is supplied by a 3φ balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate real power, reactive power, complex power, phase currents.



Given:- $V_{ab} = 210V$; $Z_Y = 40 + j25$

To find : $P = ?$; $Q = ?$; $S = ?$

solution:-

$$V_{ab} = \sqrt{3}\, V_{an} \angle 30^\circ \Rightarrow V_{an} = \frac{210}{\sqrt{3} \angle 30^\circ}$$

$\cancel{V_{ph} = 69.28 \angle -30^\circ \; V}$ $\qquad V_{ph} = 121.24 \angle -30^\circ$

$\cancel{|V_{ph}| = 69.28 V}$ $\qquad |V_{ph}| = 121.24$

$$Z_Y = 40 + j25 = 47.17 \angle 32^\circ$$

Rotor, $\quad I_{ab} = I_{an} = \frac{V_{an}}{Z_Y} = \frac{121.24 \angle -30^\circ}{47.17 \angle 32^\circ} = 2.57$

$P \; \cancel{I_{ab}} = \sqrt{3}\, V_L I_L \cos\phi$

$\cancel{P_{3\phi}}$ $P_{3\phi} = \sqrt{3}\,(210)(2.57) \cos 32^\circ = 792.74 \text{ W}$

$Q_{3\phi} = \sqrt{3}\,(210)(2.57) \sin 32^\circ = 495.36$

~~1st~~ $S = 792.74 + 495.36i$

Problem 2: A 3 phase motor can be regarded as a balanced Y-load. A three phasor motor draws 5.6 KW when the line voltage is 220 V & the line current is 18.2 A. Determine power factor of the motor.

Solution: Given: Power = 5600 W : $V_L = 220V$; $I_L = 18.2 A$;

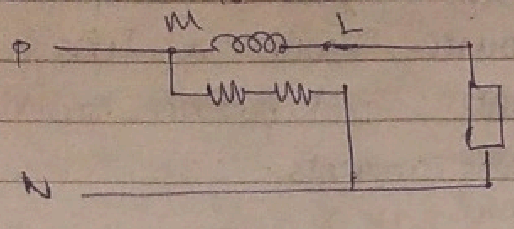
$\cos\phi = ?$

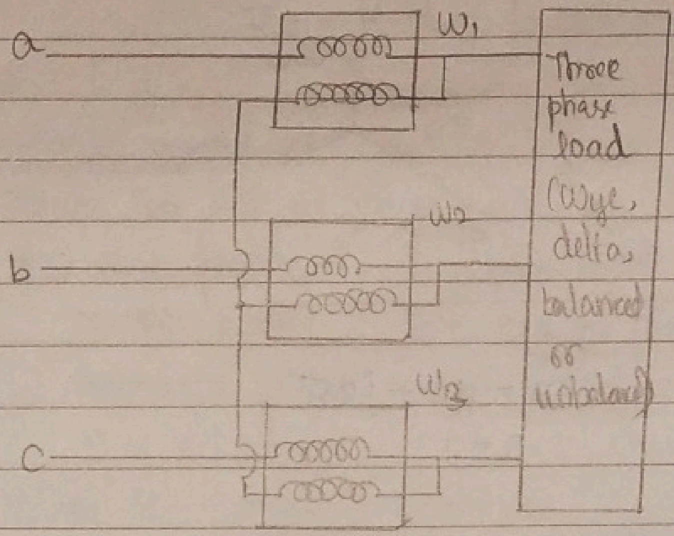
$$\text{Power} = \sqrt{3}\, V_L I_L \cos\phi \Rightarrow \cos\phi = \frac{5600}{\sqrt{3} \times 220 \times 18.2}$$

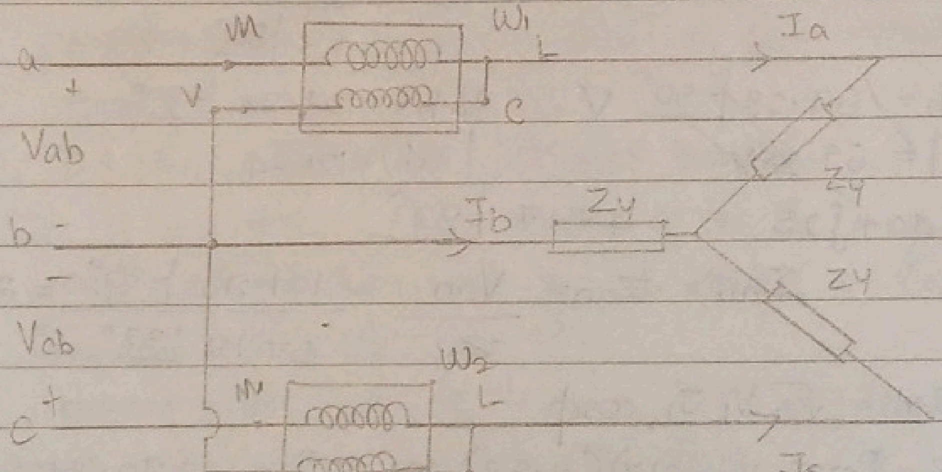
$$\boxed{\cos\phi = 0.807}$$

3 Phase measurement

* Wattmeter - to measure Power.





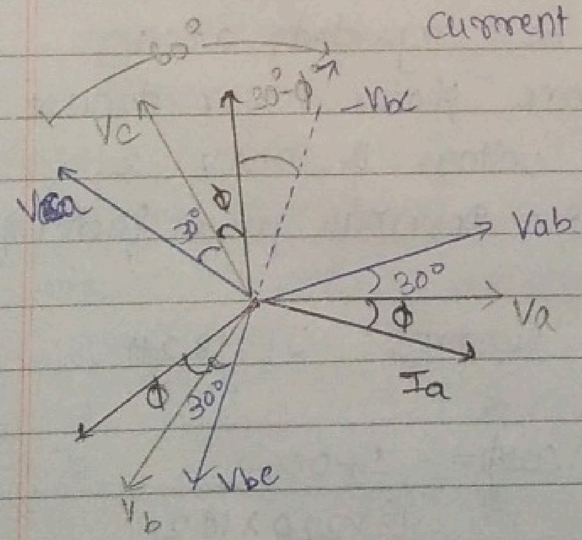


Wattmeter 1 :- Voltage measured = V_{ab}

current measured = I_a

angle btwen I_a & V_{ab} = $30^\circ + \phi$

$P_1 = V_{ab} I_a \cos(30 + \phi)$



Wattmeter 2:

Voltage measure = V_{cb}

current measured = I_c

angle btwn I_c & V_{cb} = $30^\circ - \phi$

$P_2 = V_{cb} \times I_c \times \cos(30^\circ - \phi)$

$P_T = P_1 + P_2 = V_L I_L [\cos(30+\phi) + \cos(30-\phi)]$

$= \cancel{2\cos}\ V_L I_L [2\cos 30^\circ \cos\phi]$

$P_T = \sqrt{3} V_L I_L \cos\phi = P_{3\phi}$ ⇐ 3phase real power

* reactive power ⇒ $Q_{3\phi} = \sqrt{3} V_L I_L \sin\phi$

$P_2 - P_1 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$

$= 2 V_L I_L \sin 30^\circ \sin\phi \Rightarrow V_L I_L \sin\phi$

$Q_{3\phi} = \sqrt{3}(P_2 - P_1)$

* $\phi = \tan^{-1}\left(\frac{\sqrt{3}(P_2-P_1)}{P_1+P_2}\right)$

(i) if $\phi = 0 \Rightarrow P_1 = P_2$ ⇐ resistive load

(ii) Power factor $\phi > 0$: $P_2 > P_1$ ⇐ Inductive load.
I lagging V

(iii) Power factor $\phi < 0$: $P_2 < P_1$ ⇐ capacitive load.
I leading V

Problem1 Two wattmeter methods produces wattmeter readings $P_1 = 1560W$ and $P_2 = 2100W$ when calculated to a delta connected load, If the line voltage is 220V. calculate power perphase ....?

Given: $P_1 = 1560W$

$P_2 = 2100W$

$V_L = 220V$

To find: a) Per phase average power

b) per phase reactive power

c) power factor: d) phase impedence.

Solution:-

a) $P_{3\phi} = P_1 + P_2 = 1560 + \cancel{156}\ 2100 \Rightarrow P_{3\phi} = 3660W$

$P_{1\phi} = \frac{P_{3\phi}}{3} = \frac{3660W}{3} \Rightarrow P_{1\phi} = 1220W$ ⇐ real power/phase

b) $Q_{3\phi} = \sqrt{3}(P_2 - P_1)$

$= \sqrt{3}(2100 - 1560) = 935.30$ VAR

$\frac{Q_{3\phi}}{3} = Q_{1\phi} = \cancel{311.7}$ 311.7 VAR ⇐ reactive power/phase

c) $\cancel{\cos\phi} = \phi = \tan^{-1}\left(\frac{Q}{P}\right) = 14.332^\circ$

$\cos\phi = 0.9688$ ⇐ power factor

d) $\vec{Z} = |Z| \angle\phi$

$$|Z| = \frac{V_{ph}}{I_{ph}} = \frac{220}{I_L}$$

$$I_L = \frac{P_{3\phi}}{\sqrt{3} V_L \cos\phi} = 9.91A$$

$\cancel{|Z| = 220}$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{9.91}{\sqrt{3}} = 5.723 A$$

$$|Z| = \frac{220}{5.723} = 38.44$$

$$Z = 38.44 \angle 14.332^\circ \ \Omega$$

Problem2 If the load ~~is~~ is delta connected with impedence perphase of $Z_{ph} = 30 - 40j$ & $V_L = 440V$. Predict the readings of the wattmeters W_1 & W_2. Calculate P_T & Q_T.

Given :- $Z_\Delta = 30 - 40j$

$V_L = 440V$

To find : $P_1 = ?$, $P_2 = ?$, $P_T = ?$; $Q_T = ?$

Solution:-

$$Z = 50 \angle -53.13^\circ \quad ; \phi = -53.13^\circ$$

$$I_{ph} = \frac{V_{ph}}{|Z|} = \frac{440}{50} = 8.8A \qquad I_L = \sqrt{3} \times 8.8$$

$$I_L = 15.24 A$$

$$P_1 = \sqrt{3} V_L I_L \cos(30 - \phi) = \sqrt{3} \times 15.24 \times 440 \times \cos(+83.13^\circ)$$

$$= 6166.57$$

$$P_2 = \sqrt{3} V_L I_L \cos(30^\circ + \phi) = \sqrt{3} \times 15.24 \times 440 \times \cos(30 - 53.13)$$

$$P_2 = 8021 W$$

$$P_T = P_1 + P_2 = 6968.67 W$$

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(8021 - 666.5)$$

$$Q_T = -9291.41 VAR$$