Inductance, Coupling

Self: $V \text{ (or) } \mathcal{E} \propto \frac{di}{dt}$

$$V = L\frac{di}{dt}$$ $L \rightarrow$ self inductance

$$V \propto \frac{d\phi}{dt}$$

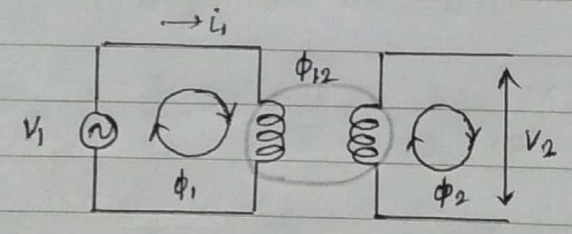
$$V = N\frac{d\phi}{dt}$$ $N \rightarrow$ no. of turns in the coil

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$L = N\frac{d\phi}{di}$$

$$L = \frac{N\phi}{i}$$ $\rightarrow$ when permeability is constant

Mutual:



$$V_2 = N_2\frac{d\phi_{12}}{dt}$$ $$M = N_2\frac{d\phi_{12}}{di_1}$$

coefficient of coupling $K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

ϕ_{12} is produced by i_1 and V_2

$$V_2 = M\frac{di_1}{dt}$$

$$M = N_2 \frac{d\phi_{12}}{di_1} = N_2 \frac{\phi_{12}}{i_1} \quad (\text{or}) \quad N_1 \frac{\phi_{21}}{i_2}$$

multiplying these 2

$$M^2 = \frac{N_1 N_2 \phi_{12} \phi_{21}}{i_1 i_2}$$

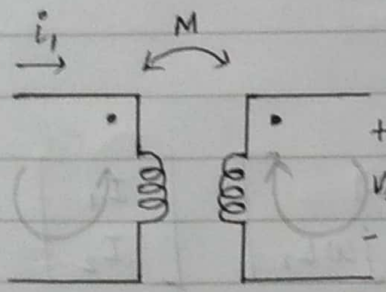
$$= \frac{N_1 N_2 K\phi_1 K\phi_2}{i_1 i_2}$$

$$= \frac{N_1 N_2 K^2 \phi_1 \phi_2}{i_1 i_2}$$

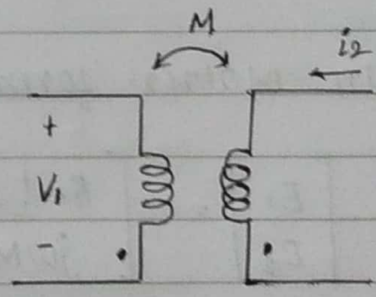
$$M^2 = L_1 L_2 K^2$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

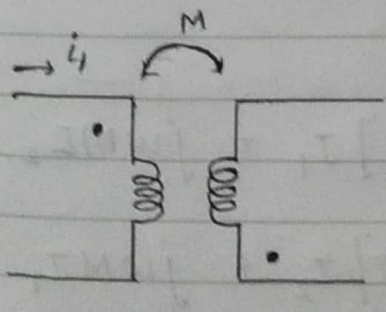
$$L_1 = \frac{N_1 \phi_1}{i_1} \qquad L_2 = \frac{N_2 \phi_2}{i_2}$$



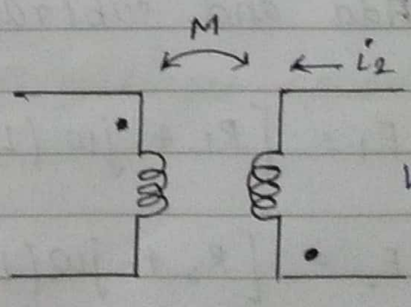
$$V_2 = M\frac{di_1}{dt}$$



$$V_1 = M\frac{di_2}{dt}$$



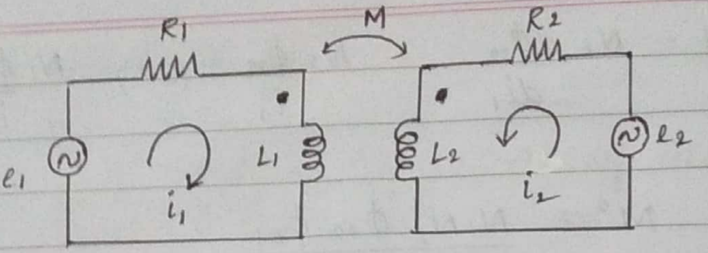
$$V_2 = -M\frac{di_1}{dt}$$



$$V_1 = -M\frac{di_2}{dt}$$

(*) when dots are in alternative side, sign is -ve.

coupling:



Applying KVL,

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}$$

as dots are same side

$$e_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

Interms of sinusoidal steady state condition,

$$E_1 = (R_1 + j\omega L_1) I_1 + j\omega M I_2$$

$$E_2 = (R_2 + j\omega L_2) I_2 + j\omega M I_1$$

In Matrix form,

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} R_1 + j\omega L_1 & j\omega M \\ j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Add and subtract M,

$$E_1 = [R_1 + j\omega (L_1 + M - M)] I_1 + j\omega M I_2$$

$$E_2 = [R_2 + j\omega (L_2 + M - M)] I_2 + j\omega M I_1$$

Now we draw circuit for above 2 eqns.

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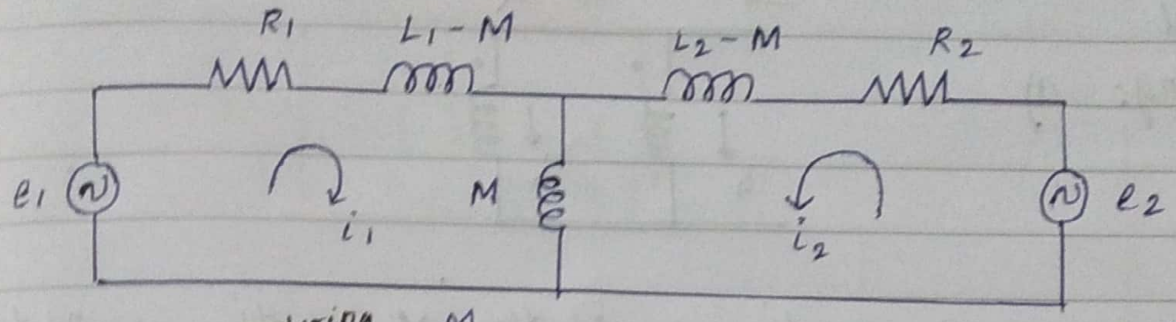
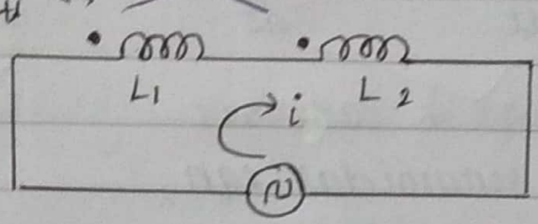


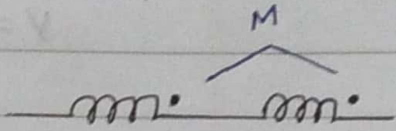
current leaving the dots

Series coupling: a)

current entering the dots



(or)

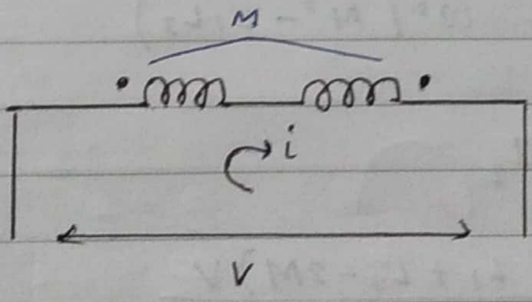


$$V(t) = \underbrace{L_1 \frac{di}{dt} + M\frac{di}{dt}}_{\text{for first inductor}} + \underbrace{L_2 \frac{di}{dt} + M\frac{di}{dt}}_{\text{for second inductor}}$$

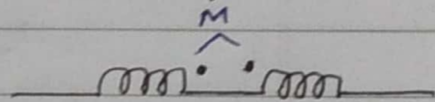
$$V = (L_1 + L_2 + 2M)\frac{di}{dt}$$

$$\therefore L_a = L_1 + L_2 + 2M \rightarrow \text{series aiding}$$

b)



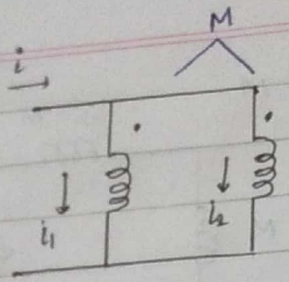
(or)



$$V = L_1\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt}$$

$$\therefore L_a = L_1 + L_2 - 2M \rightarrow \text{series opposing}$$

Parallel coupling: a)



$$V = \underbrace{L_1 \frac{di_1}{dt} + M\frac{di_2}{dt}}_{\text{in inductor 1}} \qquad V = \underbrace{L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}}_{\text{in inductor 2}}$$

In terms of sinusoidal eqn,

$$V = j\omega L_1 I_1 + j\omega M I_2$$
$$V = j\omega L_2 I_2 + j\omega M I_1$$

Now solving both eqns, we get

$$I_1 = \frac{j\omega(L_2 - M)V}{\omega^2(M^2 - L_1L_2)}$$

$$I_2 = \frac{j\omega(L_1 - M)V}{\omega^2(M^2 - L_1L_2)}$$

$$I = I_1 + I_2$$

$$I = \frac{j\omega(L_1 + L_2 - 2M)V}{\omega^2(M^2 - L_1L_2)}$$

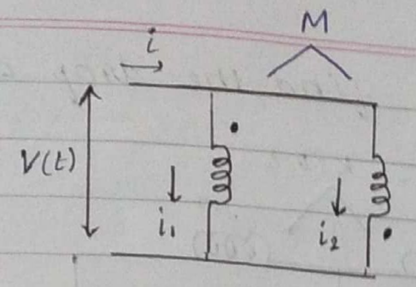
$$\frac{V}{I} = \frac{\omega^2(M^2 - L_1L_2)}{j\omega(L_1 + L_2 - 2M)} = \frac{-\omega^2(L_1L_2 - M^2)}{j\omega(L_1 + L_2 - 2M)}$$

$$= \frac{j\omega(L_1 \cdot L_2 - M^2)}{L_1 + L_2 - 2M}$$

$$\frac{V}{I} = j\omega L_a$$

$$\therefore L_a = \frac{L_1 \cdot L_2 - M^2}{L_1 + L_2 - 2M}$$

b)



$$V = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} \qquad V = L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

on solving like the before sum we get,

$$L_b = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

1. 2 coupled coils $L_1 = 0.04\,H$, $L_2 = 0.02H$ and $K = 0.5$ are connected in four different ways.
   a) series aiding
   b) series opposing
   c) parallel aiding
   d) parallel opposing

$$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow M = K\sqrt{L_1 L_2}$$
$$= 0.5\sqrt{(0.04)(0.02)}$$
$$= 0.014$$

a) $L_a = L_1 + L_2 + 2M$
$= 0.04 + 0.02 + 2(0.014)$
$= 0.088\ H$

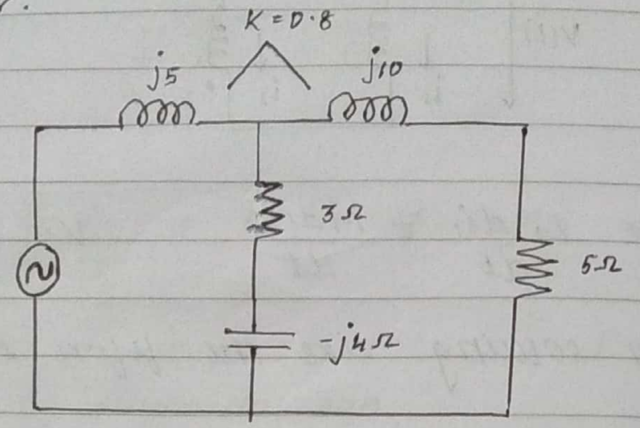
b) $L_b = L_1 + L_2 - 2M$
$= 0.06 - 2(0.014)$
$= 0.032$

c) $L_a = \dfrac{(0.06)(0.02) - (0.014)^2}{0.06 + 0.02 - 2(0.014)} = 0.01\,H$

d) $L_b = \dfrac{(0.06)(0.02) - (0.014)^2}{0.06 + 0.02 + 2(0.014)} = 0.018\,H$

2. For a given fig. find the drop across $j10\,\Omega$ inductor.

conductively coupled circuit



conductively coupled circuit to mutually coupled circuit,

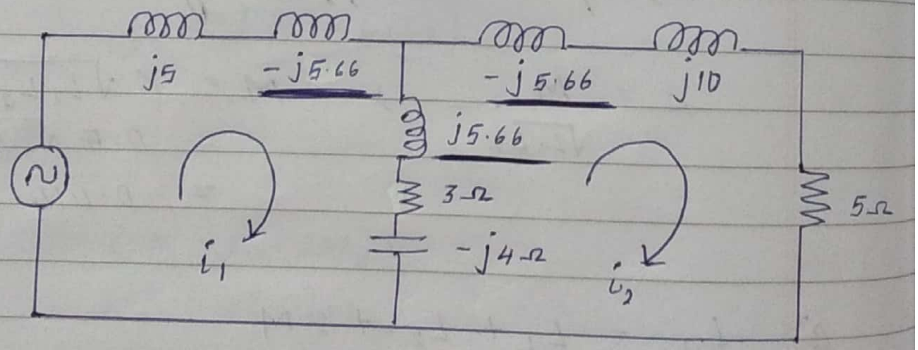
$$M = K\sqrt{L_1 L_2}$$

$$= 0.8\sqrt{5 \times 10}$$

$$M = 5.66\,\Omega$$

converting the circuit,

Mutually coupled circuit



Loop 1: $75\angle 0° = j5 I_1 - j5.66 I_1 + j5.66 I_1 - j5.66 I_2 + 3(I_1 - I_2) - j4(I_1 - I_2)$

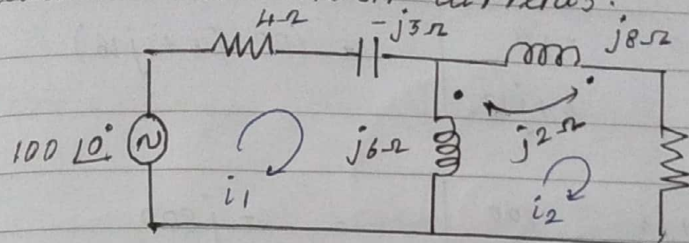
Loop 2: $0 = -j4(I_2 - I_1) + 3(I_2 - I_1) + j5.66(I_2 - I_1) - j(5.66 I_2) + j10 I_2 + 5 I_2$

on solving we get,

$$I_2 = 12.93\,\angle 24.7°$$

$$V_{j10\Omega} = 10\angle 90^\circ \times 12.93\ \angle 24.7^\circ$$
$$= 129.3\ \angle 65.3^\circ\ V$$

3. calculate the mesh currents.



additional due to mutual coupling

In Loop 1: $100\angle 0^\circ = 4I_1 - j3I_1 + j6(I_1 - I_2) - \underline{j2I_2}$

$100\angle 0^\circ = (4+j3)I_1 - j8I_2$

how here -ve sign?
current i_1 is entering at point near $j6\Omega$ and i_2 is leaving at point near $j8\Omega$. so they are opposite. so -ve.

In Loop 2: $0 = -j2I_1 - j6I_1 + (j6 + j8 + 2\times j2)I_2 + 5I_2$

$(L_1 + L_2 \pm 2M)$

+ve sign becoz i_2 is leaving at both points in loop 2.

$0 = -j8I_1 + (5+j18)I_2$

solving these 2 eqns,

$$\begin{vmatrix} 100\angle 0^\circ \\ 0 \end{vmatrix} = \begin{vmatrix} 4+j8 & -j8 \\ -j8 & 5+j18 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4+j8 & -j8 \\ -j8 & 5+j18 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 100\angle 0^\circ & -j8 \\ 0 & 5+j18 \end{vmatrix} \qquad \Delta_2 = \begin{vmatrix} 4+j3 & 100\angle 0^\circ \\ -j8 & 0 \end{vmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = (4+j3)(5+j18) - (j^2)(8^2) \qquad \boxed{j^2 = -1}$$
$$= 20 + j15 + j72 - 54 + 64$$
$$= 30 + j87$$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5+j18 \end{vmatrix} = 100(5+j18)$$

$$\Delta_2 = \begin{vmatrix} 4+j3 & 100 \\ -j8 & 0 \end{vmatrix} = +j800$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{500 + j1800}{30 + j87} = 20.3 \angle 3.5^\circ$$

convert into polar and do ÷

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{0 + j800}{30 + j87} = 8.7 \angle 19.03^\circ$$

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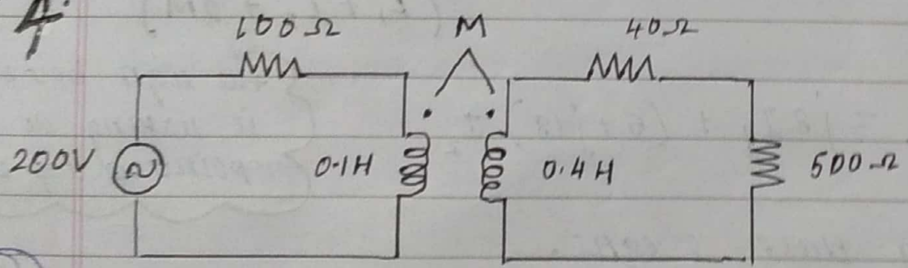
4.



$f = 159.2\,Hz$, $K = 0.1$, Find loop currents and load current

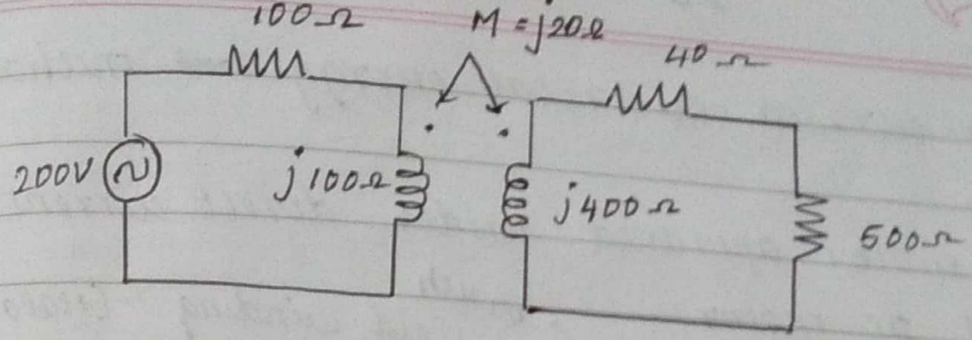
First convert everything in ohm,

Given $L_1 = 0.1H$ $\qquad L_2 = 0.4H$

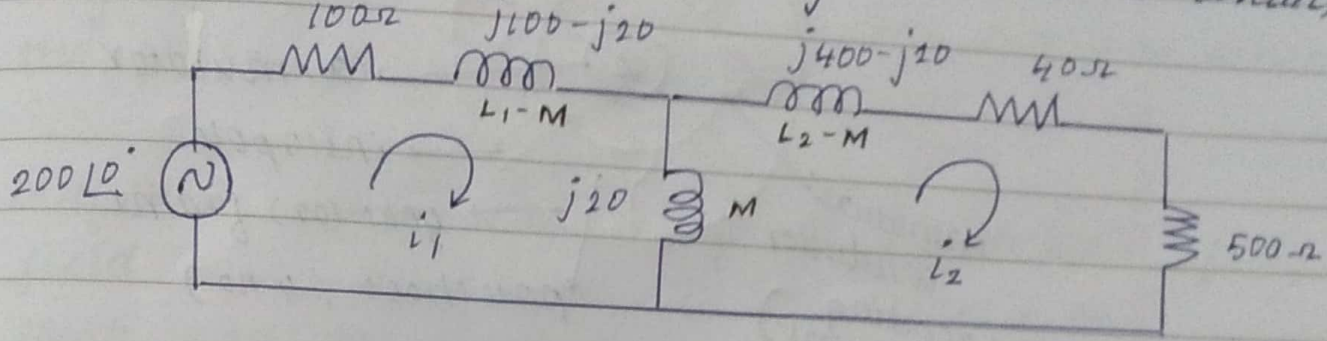
$$X_1 = 2\pi f L_1 = 2 \times 3.14 \times 159.2 \times 0.1 = 100\,\Omega$$
$$X_2 = 2\pi f L_2 = 2 \times 3.14 \times 159.2 \times 0.4 = 400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 0.1\sqrt{0.1 \times 0.4} = 0.02H$$

$$X_M = 2\pi f M = 2 \times 3.14 \times 159.2 \times 0.02 = 20\,\Omega$$



Converting into electrically equivalent circuit,



Loop 1: $200\angle 0 = 100I_1 + j80I_1 + j20I_1 - j20I_2$

$200 = (100 + j100)I_1 - j20I_2$

Loop 2: $0 = 500I_2 + j20I_2 - j20I_1 + j380I_2 + 40I_2$

$0 = (540 + j400)I_2 - j20I_1$

on solving we get, $I_1 = 1.41\angle -44.7$, $I_2 = 0.042\angle 8.7$

$I_1 = 44.8 - j88.2 \qquad I_2 = 8.7 + j90$

current through load,

$I_{RL} = 500 \times I_2$

$= 500(8.7 + j90)$

$I_{RL} = 4350 + j45000$

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