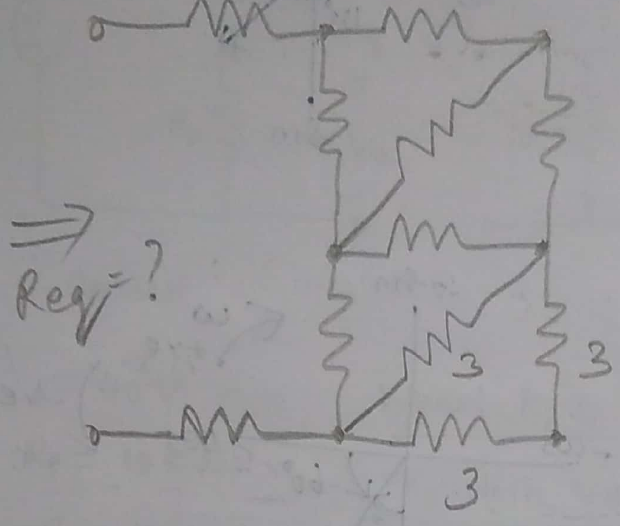
Module1 - solutions

(1) R = 3 Ω for all resistors.



Two resistors in parallel of equal value

$$R_{eq} = \frac{(R)(R)}{R+R} = R/2$$

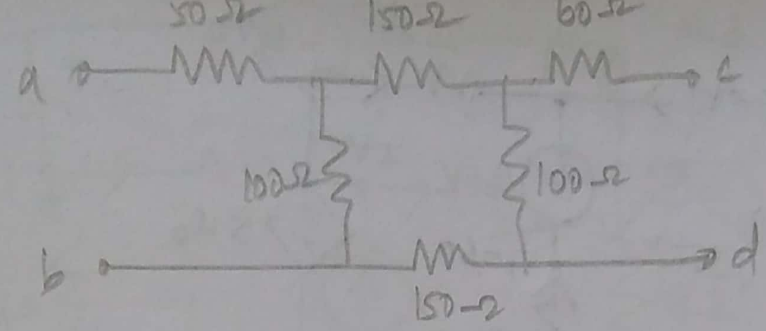
3, 3 series ⇒ 6 Ω

6, 3 parallel ⇒ $\frac{6\times 3}{6+3} = 2\,\Omega$

It can be further reduced using star-delta transformation only

(Ignore this problem)

(2) equivalent resistance



R_{ab}

Since c-d is open, I = 0 through 60 Ω resistor.

150 Ω, 100 Ω, 150 Ω series R = 400 Ω

400 Ω, 100 Ω parallel $R = \frac{400 \times 100}{400 + 100} = 80\ \Omega$

50 Ω, 80 Ω series = 130 Ω

$R_{ab} = 130\ \Omega$

R_{cd}

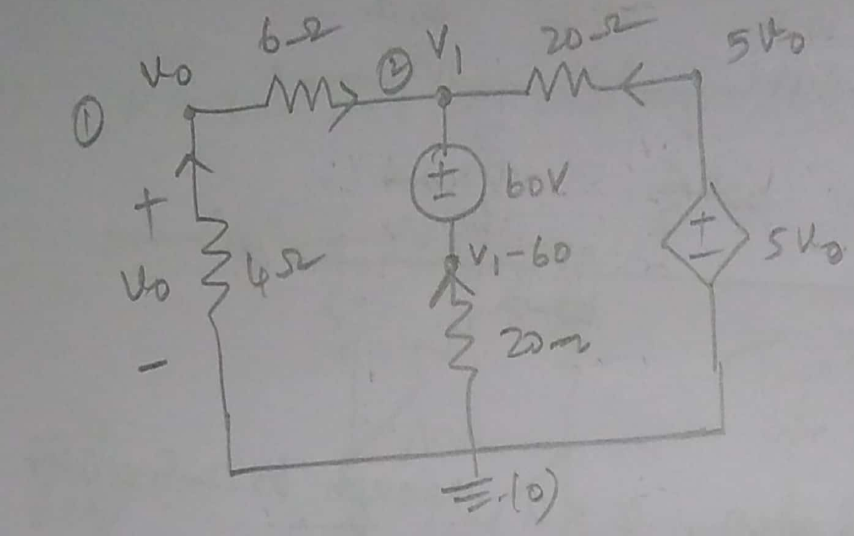
Since a-b is open, I = 0 through 50 Ω resistor.

150, 100, 150 Ω series R = 400 Ω

400 Ω, 100 Ω parallel R = 80 Ω

80 Ω, 60 Ω series R = 140 Ω

③ Nodal analysis



At node ①,

$$\frac{0 - V_0}{4} = \frac{V_0 - V_1}{6}$$

$$V_0\left[\frac{1}{4}+\frac{1}{6}\right] - \frac{1}{6}V_1 = 0$$

$$0.416V_0 - 0.166V_1 = 0$$

At node 2

$$\frac{V_0 - V_1}{6} + \frac{0-(V_1-60)}{20} + \frac{5V_0 - V_1}{20} = 0$$

$$V_0\left[\frac{1}{6}+\frac{1}{4}\right] - V_1\left[\frac{1}{6}+\frac{1}{20}+\frac{1}{20}\right] = -3$$

$$0.416V_0 - 0.266V_1 = -3$$

Solving two equations,

$$V_0 = 11.97V \approx 12V$$

$$V_1 = 30V$$

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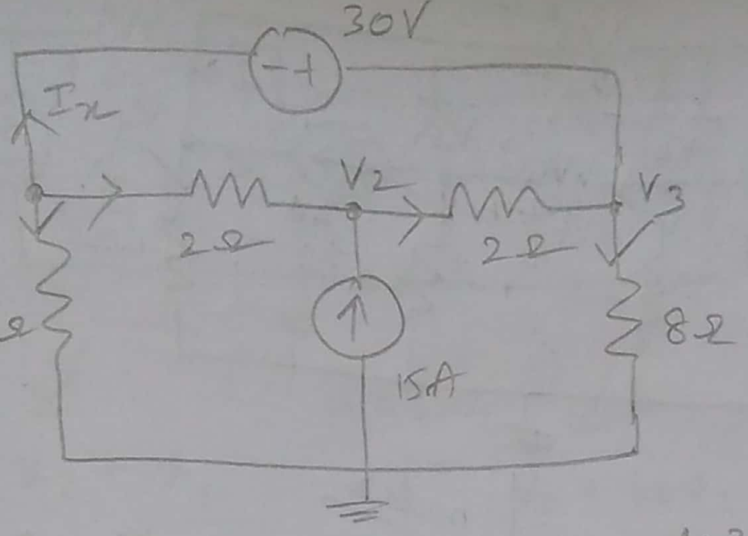
(4) Nodal Analysis.



At node 1,

$$\frac{V_1-0}{4}+\frac{V_1-V_2}{2}+I_x=0$$

sub I_x in node ① eqn.

At node 3,

$$I_x+\frac{V_2-V_3}{2}=\frac{V_3}{8}$$

$$I_x=V_3\left[\frac{1}{8}+\frac{1}{2}\right]-\frac{1}{2}V_2$$

$$\frac{V_1}{4}+\frac{V_1}{2}-\frac{V_2}{2}-\frac{1}{2}V_2+V_3\left(\frac{1}{8}+\frac{1}{2}\right)=0$$

$$0.75V_1-V_2+0.625V_3=0 \longrightarrow ①$$

Between V_1 and V_3 there is a voltage source (super node)

$$\therefore V_3-V_1=30 \longrightarrow ②$$

At node 2,

$$\frac{V_1-V_2}{2}+15=\frac{V_2-V_3}{2}$$

$$0.5V_1-V_2+0.5V_3=-15 \longrightarrow ③$$

Solving eqns ①, ② & ③, we get (calculator)

$V_1=30$ $V_2=60\text{ V}$ $V_3=60\text{ V}$

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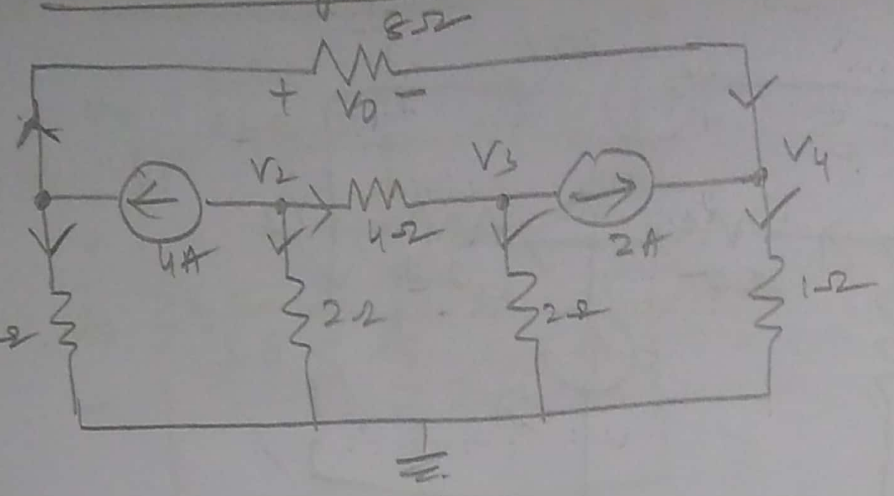
(5) Nodal analysis



node 1

$$4 = \frac{V_1}{1} + \frac{V_1 - V_4}{8}$$

$$1.125V_1 - 0.125V_4 = 4 \rightarrow (1)$$

node 2

$$4 + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$$

$$0.75V_2 - 0.25V_3 = -4 \rightarrow (2)$$

node 3

$$\frac{V_2 - V_3}{4} = \frac{V_3}{2} + 2$$

$$0.25V_2 - 0.75V_3 = 2 \rightarrow (3)$$

node 4

$$\frac{V_1 - V_4}{8} + 2 = \frac{V_4}{1}$$

$$0.125V_1 - 1.125V_4 = -2 \rightarrow (4)$$

Solving (1) and (4) we get,

$V_1 = 3.8\ V$

$V_4 = 2.2\ V$

Solving (2) and (3) we get,

$V_2 = -7V$

$V_3 = -5V$

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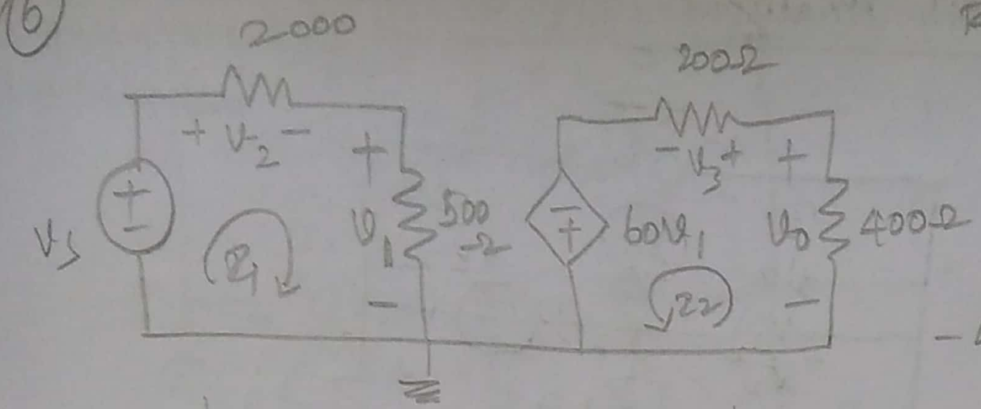
(6) Find $\frac{V_o}{V_s}$



$-400 I_2 = V_o$

$I_2 = \frac{-V_o}{400}$

$V_1 = 500 I_1$

$V_s = V_2 + V_1$

$V_s = 2000 I_1 + 500 I_1$

$V_s = 2500 I_1 \longrightarrow (1)$

$V_o + 60 V_1 = V_3$

$V_o = 200 I_2 - 60(500 I_1)$

$V_o = 200 I_2 - 30000 I_1$

$V_o = \cancel{200}\left(\frac{-V_o}{\cancel{400}_{2}}\right) - 30000 I_1$

$V_o + \frac{V_o}{2} = 30000 I_1$

$\frac{3V_o}{2} = 30000 I_1$

$V_o = 20000 I_1 \longrightarrow (2)$

From (1), (2)

$\frac{V_o}{V_s} = \frac{20000 I_1}{2500 I_1}$

$\frac{V_o}{V_s} = 8$

$V_o = 8 V_s$

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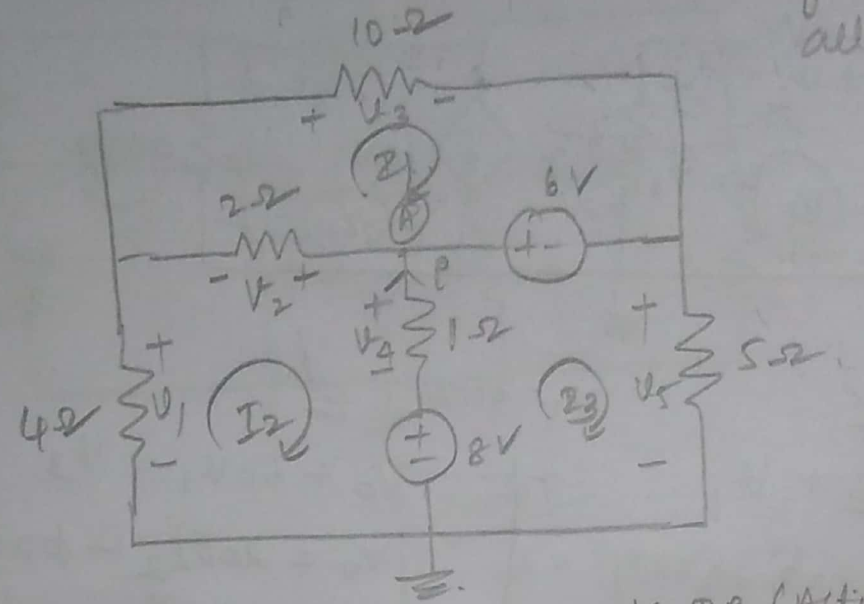
⑦ Mesh Analysis

Mark polarity of voltages across all element



mesh2 Σ Active V = Σ Passive V

V = IR (Active)
V = −IR (passive)

$$v_1 + v_2 = v_4 + 8$$

$$-4I_2 - 2(I_2 - I_1) = 1(I_2 - I_3) + 8$$

$$-4I_2 - 2I_2 + 2I_1 - I_2 + I_3 = 8$$

$$2I_1 - 7I_2 + I_3 = 8 \longrightarrow ①$$

mesh1

$$6 = v_2 + v_3 \Rightarrow 6 = 2(I_1 - I_2) + 10(I_1)$$

$$12I_1 - 2I_2 = 6 \longrightarrow ②$$

mesh3

$$8 + v_4 = 6 + v_5$$

$$8 - 1(I_3 - I_2) = 6 + 5(I_3)$$

$$8 - 6 = 5I_3 + I_3 - I_2$$

$$2 = 6I_3 - I_2 \longrightarrow ③$$

At node Ⓐ

$$I_2 + i = I_3$$

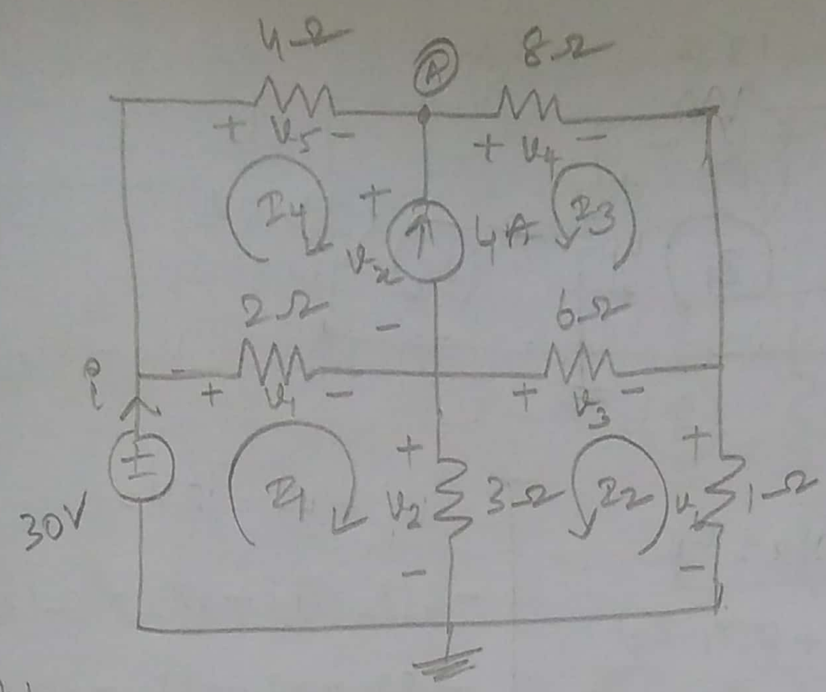
$$i = I_3 - I_2$$

$$\boxed{i = 1.1879}$$

Solving eqns ①, ②, ③ we get

$I_1 = 0.329$ $I_2 = -1.0256$, $I_3 = 0.1623$

(8)



mesh 1

$$30 = v_1 + v_2$$

$$30 = 2(I_1 - I_4) + 3(I_1 + I_2)$$

$$= 2I_1 - 2I_4 + 3I_1 + 3I_2$$

$$5I_1 + 3I_2 - 2I_4 = 30 \longrightarrow (1)$$

mesh 2

$$v_6 + v_3 = v_2$$

$$-(1)I_2 - 6(I_2 - I_3) = 3(I_2 + I_1)$$

$$-I_2 - 6I_2 + 6I_3 = 3I_2 + 3I_1$$

$$3I_1 + 10I_2 - 6I_3 = 0 \longrightarrow (2)$$

mesh 3 & mesh 4 (super mesh)

mesh 4

$$v_1 = v_5 + v_x$$

$$-2(I_4 - I_1) = 4I_4 + v_x$$

sub v_x,

$$-2I_4 + 2I_1 - 4I_4 + 14I_3 - 6I_2 = 0$$

$$2I_1 - 6I_2 + 14I_3 - 6I_4 = 0 \longrightarrow (3).$$

mesh 3

$$v_4 = v_x + v_3$$

$$-8I_3 = v_x + 6(I_3 - I_2)$$

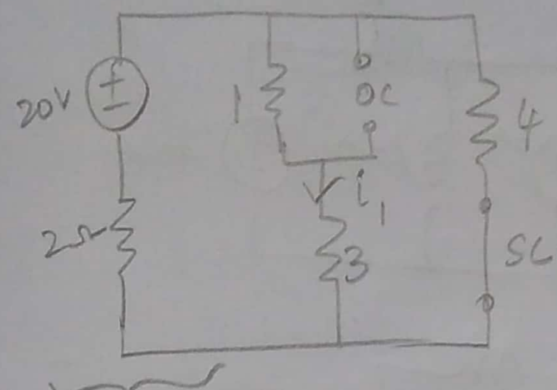
$$v_x = -14I_3 + 6I_2$$

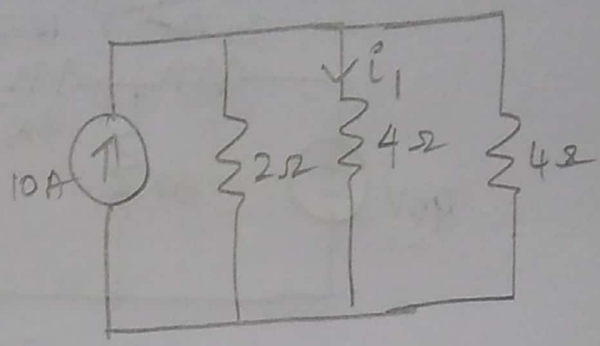
At node (A) $I_3 + I_4 = -4 \longrightarrow (4)$

(No need to solve further)

(10) Superposition, find $i = i_1 + i_2 + i_3$

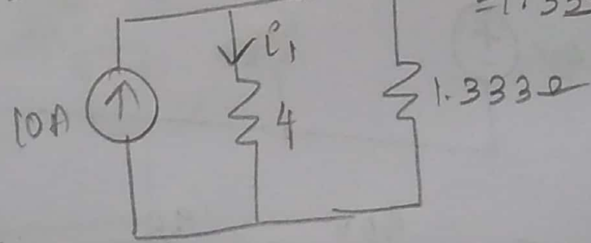
(1) Considering 20V voltage source. (i_1)





Source transformation. $I = \frac{20}{2} = 10A$

$\frac{2 \times 4}{2+4} = 1.333$

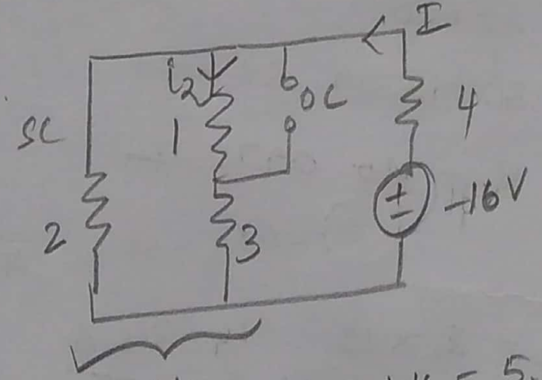


$i_1 = 10 \times \frac{1.333}{5.333}$

$i_1 = 2.4995 A$

$\simeq 2.5 A$

(2) with 16V voltage source (i_2)



$R_T = \frac{2 \times 4}{2+4} = 1.333 + 4 = 5.333 \Omega$ $\quad I = \frac{-16}{5.3333} = -3A.$

By current division $i_2 = -3 \times \frac{2}{2+4}$

$i_2 = -1 A$

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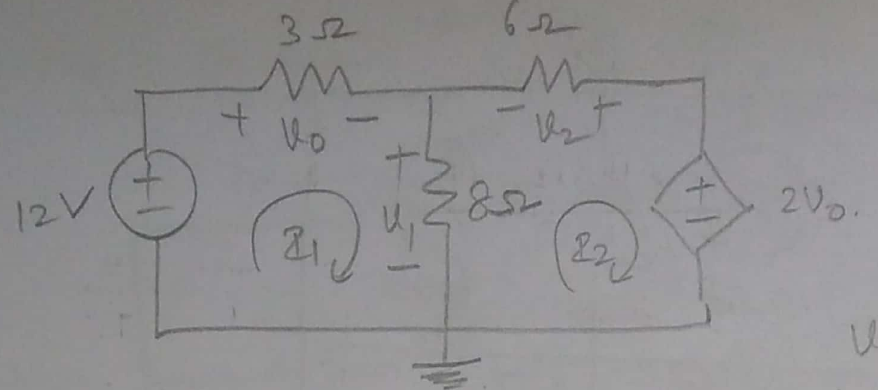
(9)



$V_0 = 3I_1$.

mesh 1

$$12 = V_0 + V_1$$
$$12 = 3I_1 + 8(I_1 - I_2)$$
$$12 = 11I_1 - 8I_2 \longrightarrow (1)$$

$$V_1 + V_2 = 2V_0.$$
$$-8(I_2 - I_1) - 6I_2 = 2(3I_1)$$
$$-8I_2 + 8I_1 - 6I_2 - 6I_1 = 0$$
$$2I_1 - 14I_2 = 0$$
$$I_1 - 7I_2 = 0$$
$$I_1 = 7I_2$$

$$11(7I_2) - 8I_2 = 12$$
$$77I_2 - 8I_2 = 12$$
$$69I_2 = 12$$
$$I_2 = 0.1739A$$
$$I_1 = 1.21739A.$$

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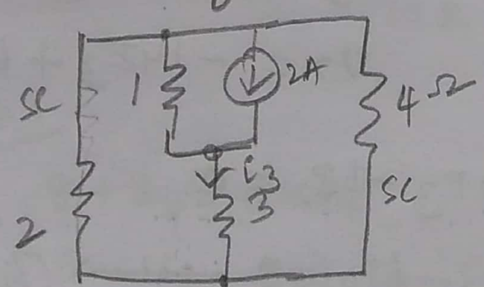
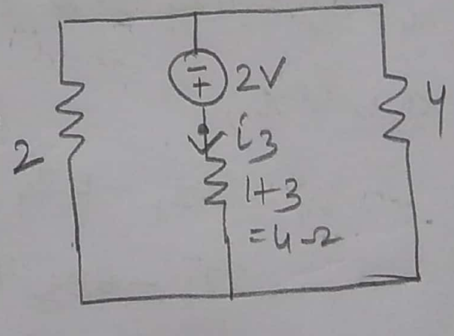
(10) considering 2A current source.



⇒



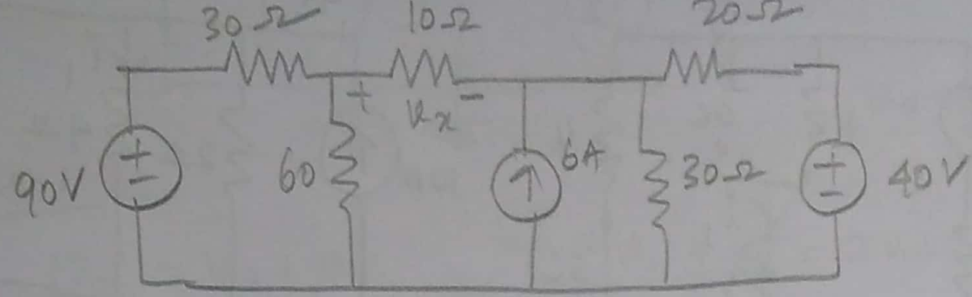
$$i_3 = 0.375A$$

$$R_T = \frac{2 \times 4}{2+4} = 1.333 + 4$$
$$= 5.333\Omega$$

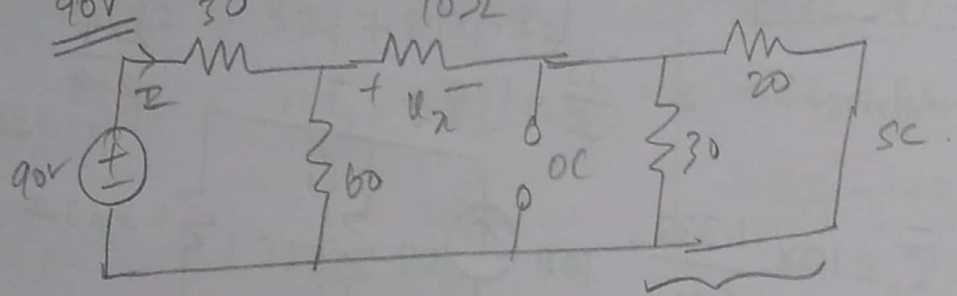
$$I = \frac{2}{5.333} = 0.375A$$

$$\therefore i = 2.5 - 1 + 0.375$$
$$i = 1.875A$$

(11) Find V_x in the circuit using superposition principle



90V



$$\frac{20 \times 30}{20+30} = 12\ \Omega$$

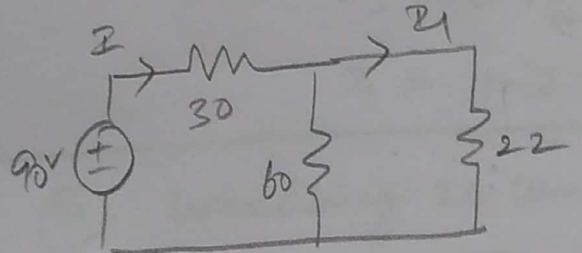
$$I = \frac{90V}{R_T} = \frac{90}{46}$$

12, 10 Ω series. R = 22 Ω

$I = 1.9565\ A$

22, 60 parallel

$$\frac{22 \times 60}{22+60} = 16.0975\ \Omega$$



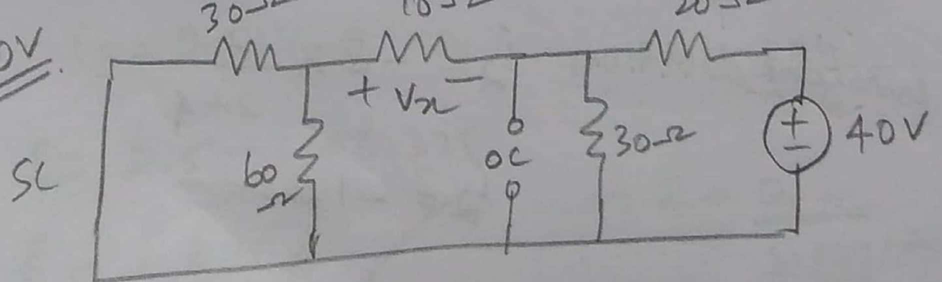
30, 16.09 series

$R_T = 46.0975\ \Omega$

$$I_1 = 1.9565 \times \frac{60}{60+22}$$

$I_1 = 1.4316\ A$ ∴ $V_x = I_1 \times 10 = 14.316\ A$

40V.



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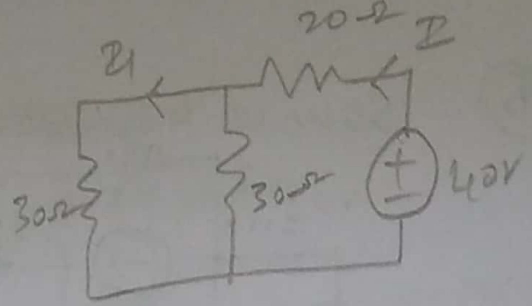
30, 60 parallel $R = 20\,\Omega$

20, 10 series $R = 30\,\Omega$

30, 30 parallel $R = 15\,\Omega$

20, 15 series $\boxed{R_T = 35\,\Omega}$

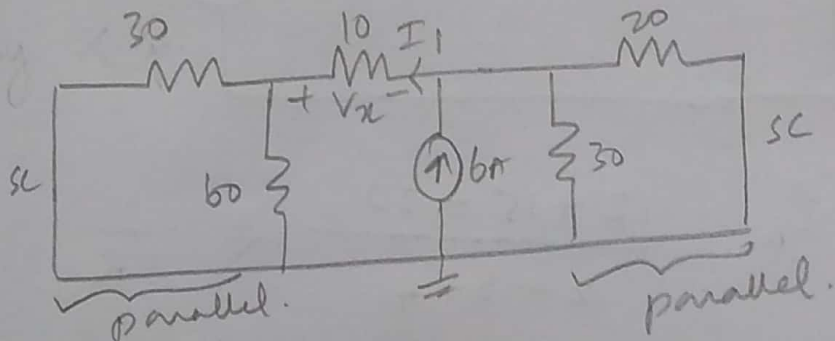


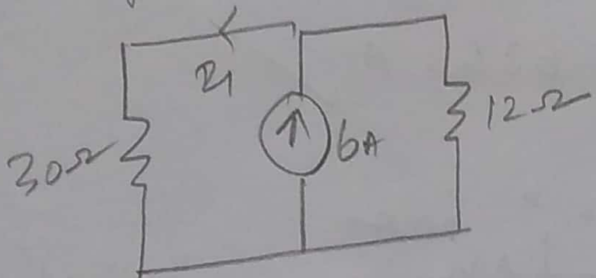
$$I = \frac{40}{35} = 1.1428$$

$$I_1 = \frac{1.1428}{2} = 0.5714\text{ A}$$

$$V_x = -10 \times 0.5714 = \underline{-5.714\text{ V}}$$

6A





$$I_1 = 6 \times \frac{12}{30+12}$$

$$I_1 = 1.714\text{ A}$$

$$V_x = -10 \times I_1$$

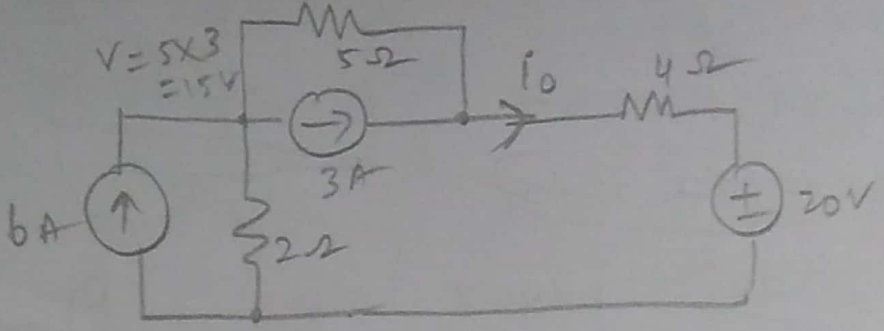
$$V_x = -17.143\text{ V}$$

By superposition.

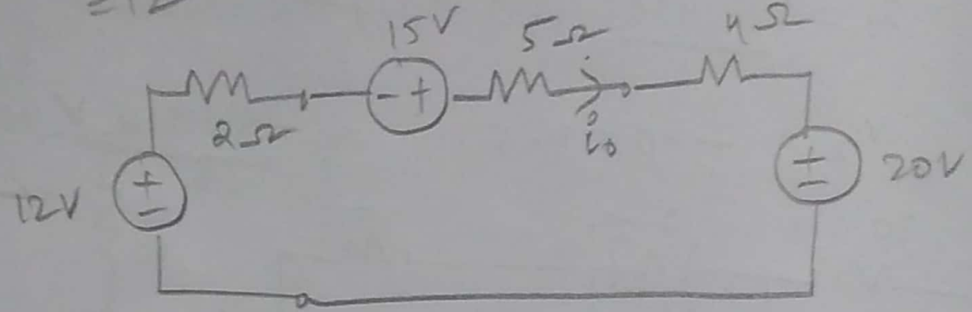
$$V_x = 14.316 - 5.714 - 17.143$$

$$\underline{V_x = -8.541\text{ V}}$$

(12) Source transformation $i_0 = ?$



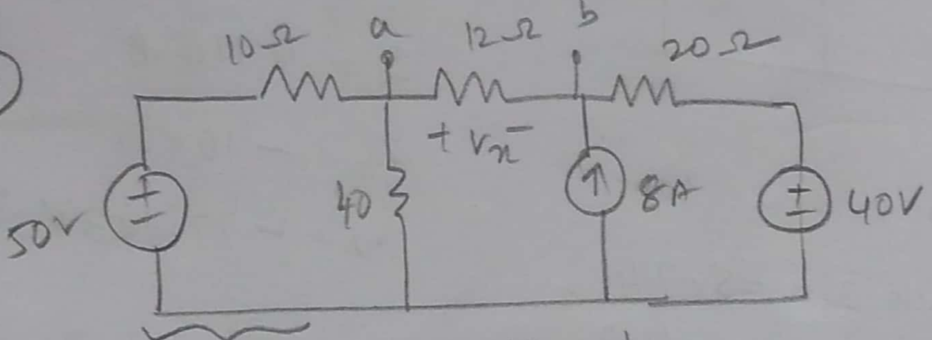
$V = 6 \times 2$
$= 12$



$$i_0 = \frac{12 + 15 - 20}{2 + 5 + 4} \Leftarrow \frac{V}{R}$$

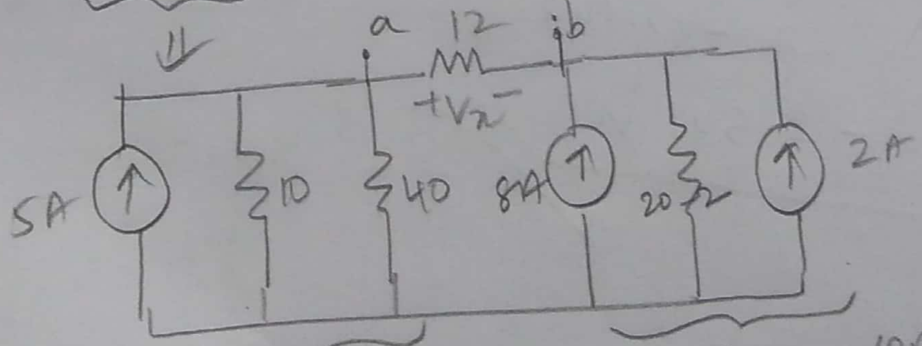
$i_0 = 0.6363$ A ✓

(13) Find V_x
Source transform



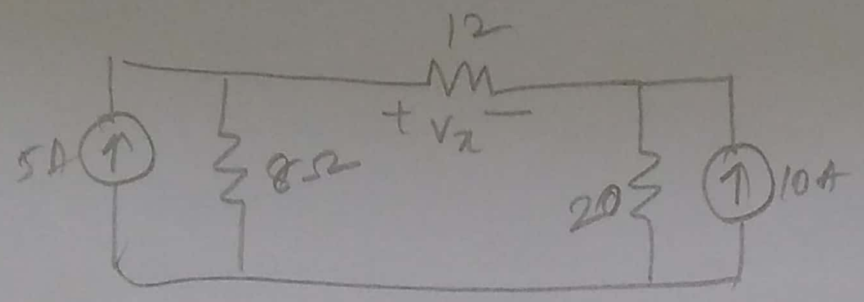
$I = \frac{V}{R}$

⇓



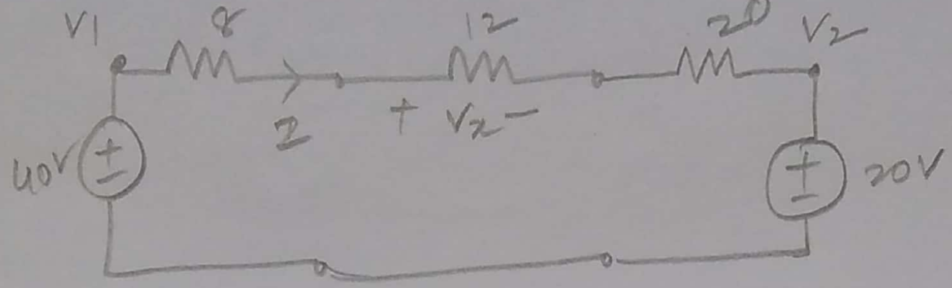
$\frac{10 \times 40}{10 + 40} = 8\Omega$

$8 + 2 = 10A$



$V = 10 \times 2$
$= 20V$

5×8

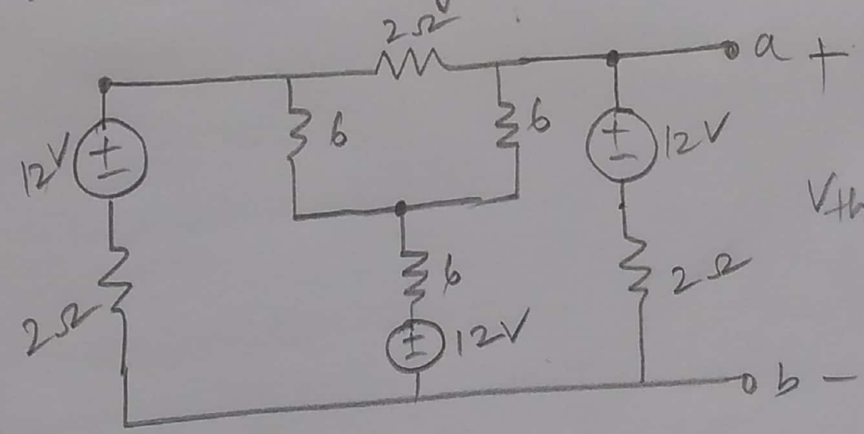


$$I = \frac{40-20}{20+8+12} = \frac{20}{40}\,A = 0.5A$$

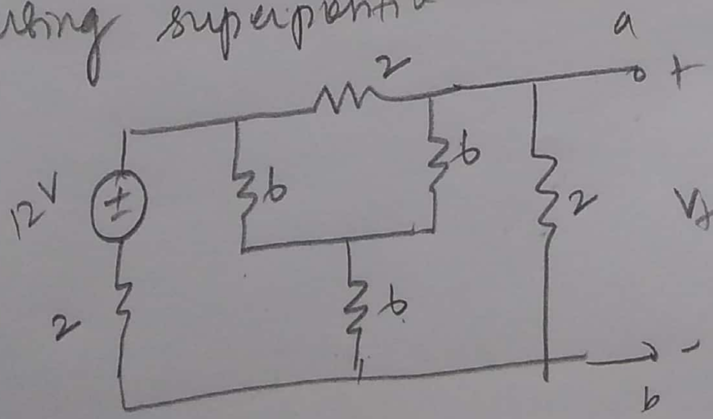
$$V_x = 12 \times \frac{20}{40} = 6\,V$$

Ans
−48V.

(14) Thevenin voltage



Using superposition



(need star delta transformation)

Ans

1.2Ω
9.6V
8A