BEEE101L – Basic Electrical Engineering

Maximum Power Transfer Theorem

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Objective

- To introduce Thevenin's theorem.
- To apply Thevenin's theorem to solve electric circuits and formulate condition for maximum power transfer.

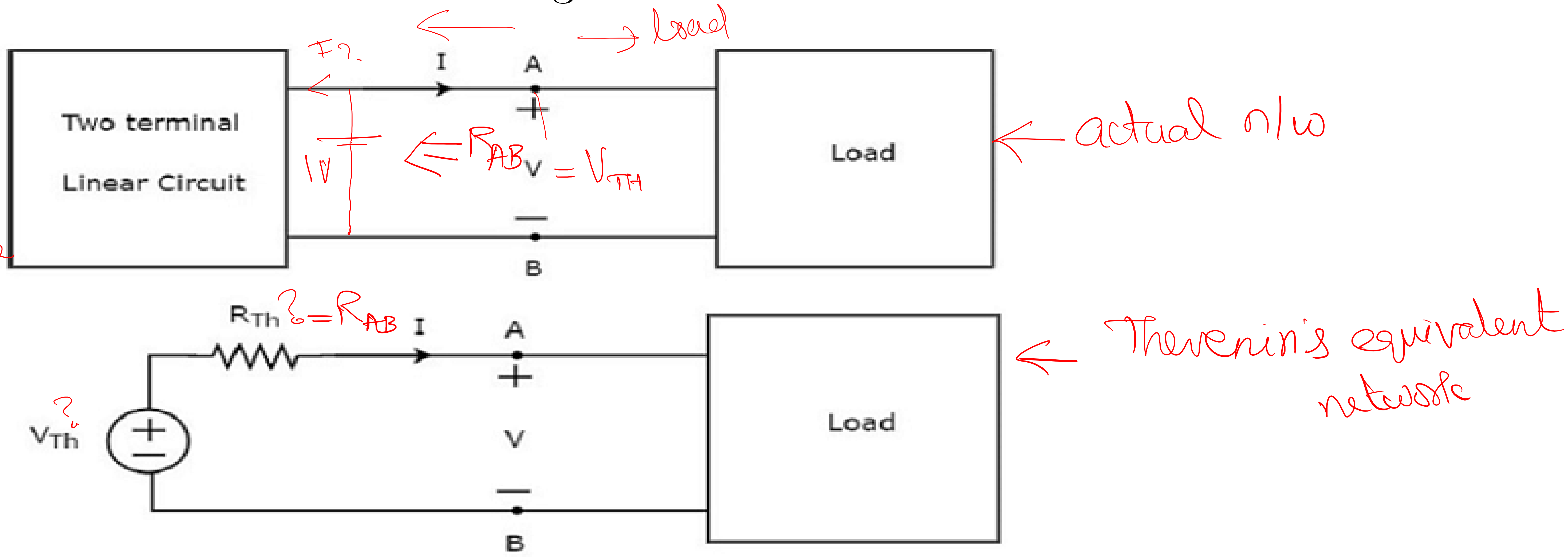
Thevenin's Theorem:

- Any linear active network consisting of independent or dependent voltage and current source and the network elements can be replaced by an equivalent circuit having a voltage source in series with a resistance.
  - Where the voltage source being the open-circuited voltage across the open-circuited load terminals and the resistance being the internal resistance of the source.

$R_{Th} = \frac{V}{I}$

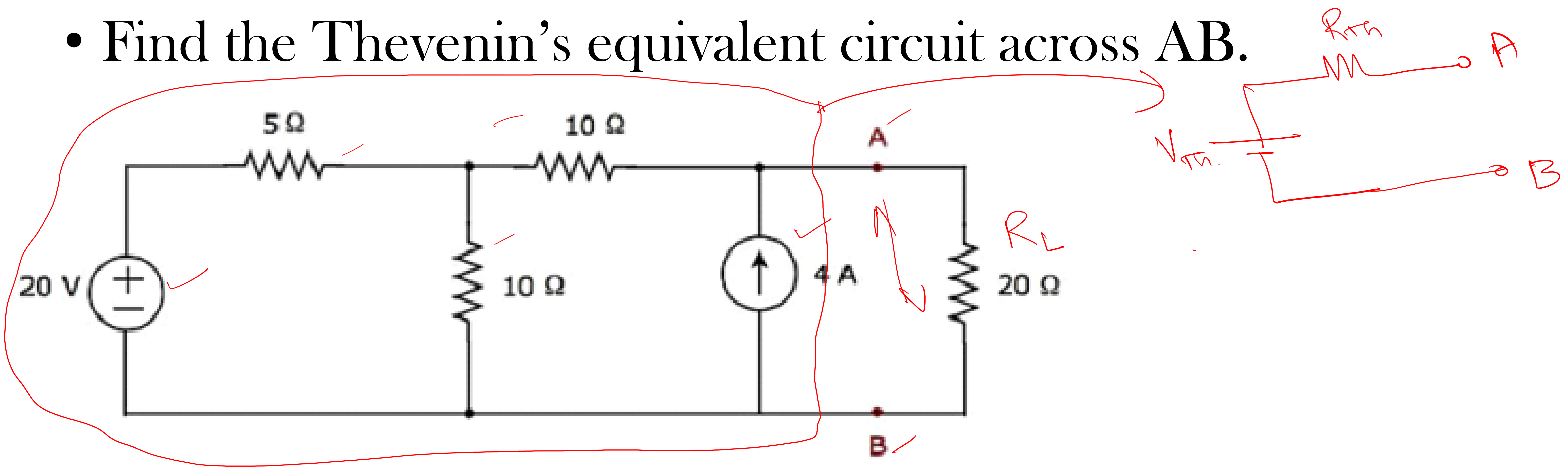
Multimeter → Resistance

ohmmeter →



Example:

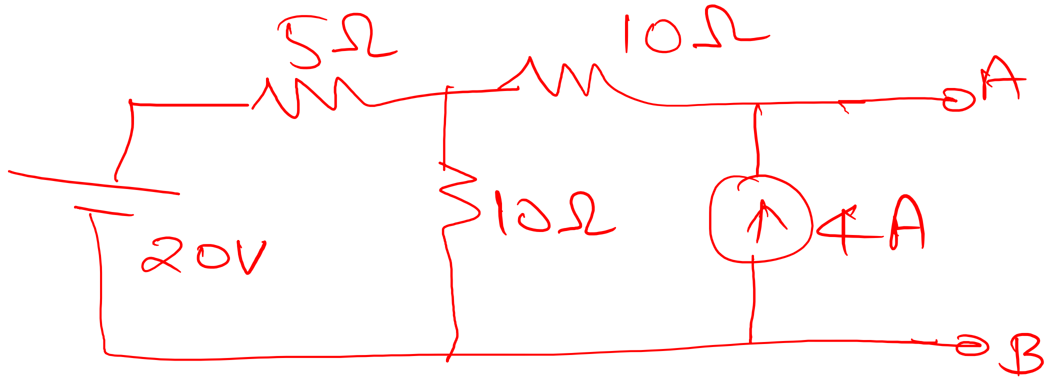
- Find the Thevenin's equivalent circuit across AB.



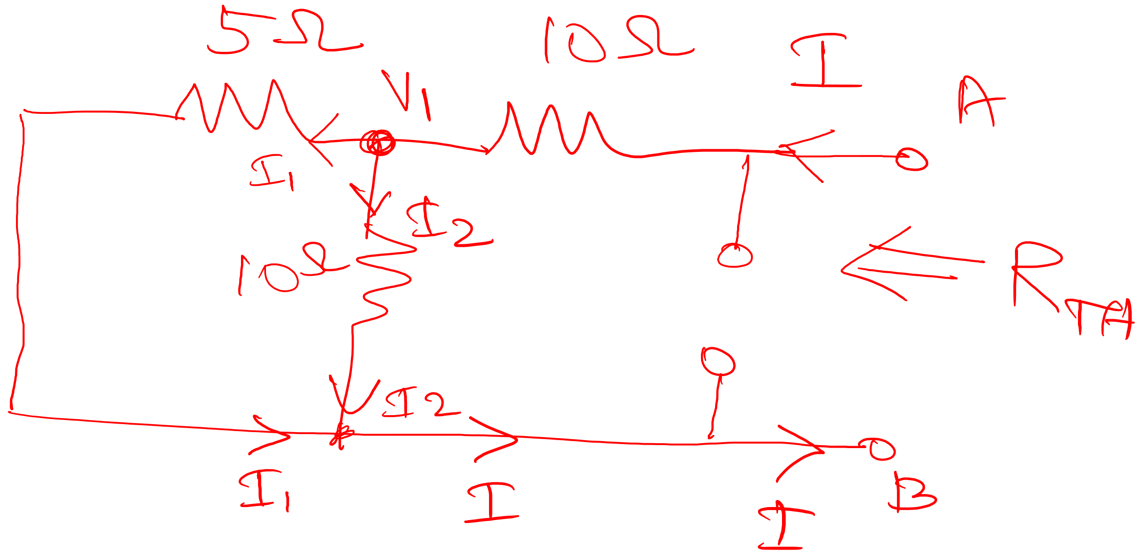
- The solution has 3 steps:
  - To find Thevenin equivalent resistance (R_{TH}).
  - To find open circuit voltage across terminals AB.
  - Draw the equivalent circuit

Simplifying the circuit to find R_{TH}:

1. Disconnect the load resistance R_L:



2. Short circuit the Voltage source and open circuit the current source and find the equivalent resistance.



$$R_{TH} = 10 + (10 \| 5)$$

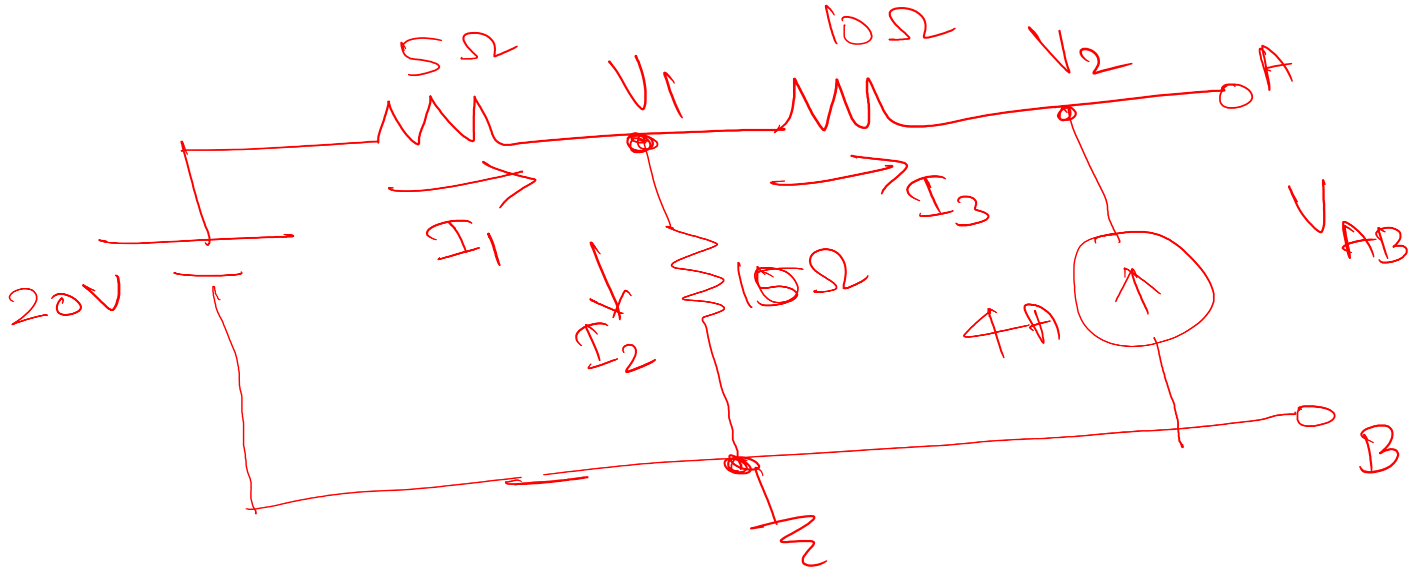
$$= 10 + \frac{10 \times 5}{10 + 5}$$

$$= 10 + \frac{50}{15}$$

$$\boxed{R_{TH} = 13.33\ \Omega}$$

Simplifying the circuit to find R_{TH}:

To find V_{TH}:



$V_{AB} = V_{TH} = V_{4A}$

Applying KCL at node 1, we have

$$I_1 = I_2 + I_3$$

$$\frac{20 - V_1}{5} = \frac{V_1}{10} + \frac{V_1 - V_{TH}}{10}$$

$$4 = \left[\frac{1}{5} + \frac{1}{10} + \frac{1}{10}\right] V_1 - \frac{1}{10} V_{Th} \quad \text{--- (1)}$$

$$4 = \frac{4}{10} V_1 - \frac{1}{10} V_{th} \quad \text{--- (1)}$$

Applying KCL at node 2,

$$I_3 + 4 = 0$$

$$\frac{V_1 - V_{Th}}{10} + 4 = 0$$

$$\frac{V_1}{10} + 4 = \frac{V_{Th}}{10}$$

$$\boxed{\therefore V_{Th} = V_1 + 40} \quad \text{--- (2)}$$

Sub (2) in (1) and solve for V_1

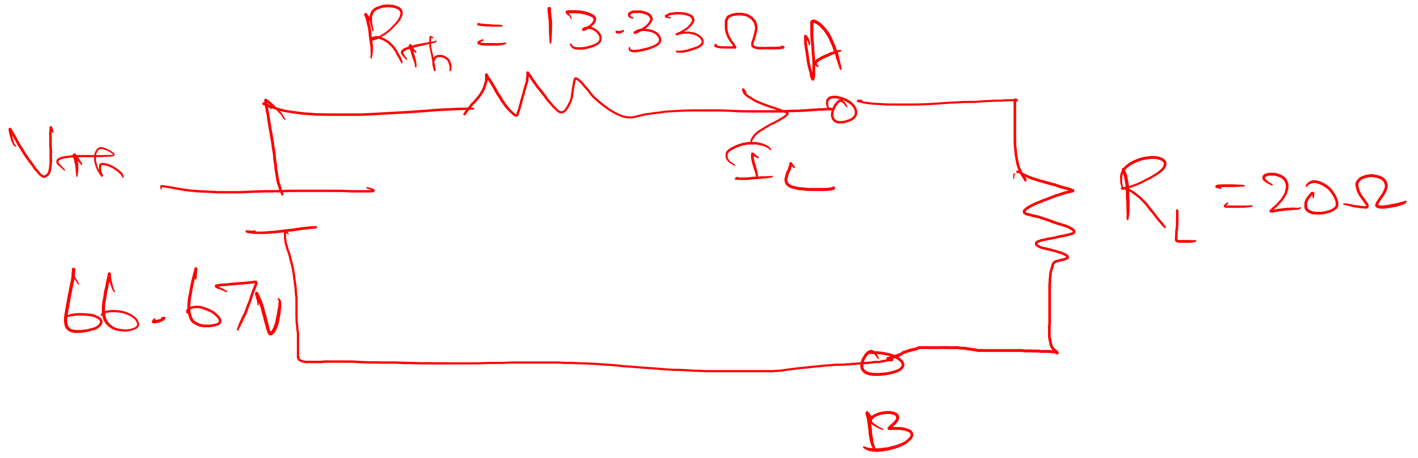
$$4 = \frac{4}{10} V_1 - \frac{1}{10} V_1 - \frac{\cancel{40}\,4}{\cancel{10}}$$

$$8 = \frac{3}{10} V_1 \qquad \therefore V_1 = \frac{80}{3} = 26.67\,V$$

$$\therefore V_{th} = V_1 + 40 = 66.67\,V$$

To find V_{TH}:

To find the load current with Thevenin's equivalent circuit,



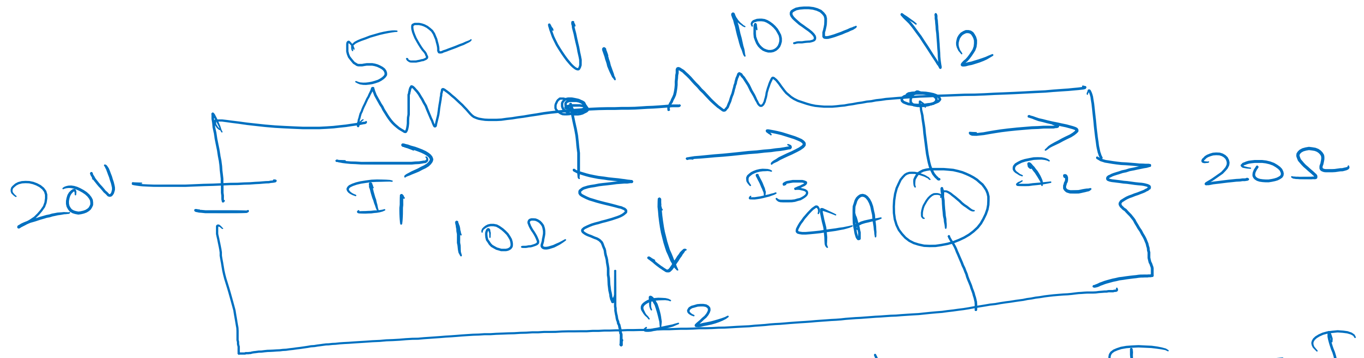
Load current $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$

$$= \frac{66.67}{13.33 + 20}$$

$$= \frac{66.67}{33.33}$$

$$\boxed{I_L \cong 2\,A}$$

To verify Thevenin's theorem, Let us compute the load current in actual circuit,



Applying KCL @ node 1, $I_1 = I_2 + I_3$

$$4 = \frac{4}{10} V_1 - \frac{1}{10} V_2 \longrightarrow (1)$$

Applying KCL @ node 2, $I_3 + 4 = I_L$

$$\frac{V_1 - V_2}{10} + 4 = \frac{V_2}{20}$$

$$4 = -\frac{1}{10} V_1 + \left(\frac{1}{10} + \frac{1}{20}\right) V_2$$

$$4 = -0.1 V_1 + 0.15 V_2 \longrightarrow (2)$$

Simplified Circuit

The load current $I_L = \frac{V_2}{20}$

To find V_2, write equations ① & ② in matrix form

$$0.4V_1 - 0.1V_2 = 4 \quad -①$$

$$-0.1V_1 + 0.15V_2 = 4 \quad ②$$

$$\begin{bmatrix} 0.4 & -0.1 \\ -0.1 & 0.15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$V_2 = \frac{\Delta_2}{\Delta}$ where $\Delta = \begin{vmatrix} 0.4 & -0.1 \\ -0.1 & 0.15 \end{vmatrix}$ & $\Delta_2 = \begin{vmatrix} 0.4 & 4 \\ -0.1 & 4 \end{vmatrix}$

$\therefore V_2 = \frac{2}{0.05} = 40$

$\therefore$ The load current $I_L = \frac{V_2}{20} = \frac{40}{20} = 2A$

Maximum Power Transfer Theorem:

The power delivered to the load $P_L = I_L^2 R_L$ ✓

By Thevenin's theorem, $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$ ✓

$$\therefore \quad P_L = \frac{V_{Th}^2 \, R_L}{(R_{Th} + R_L)^2} \quad \text{— (3)}$$

The power delivered to the load will be maximum if $\dfrac{dP_L}{dR_L} = 0$

$$\Rightarrow \quad P_{Lmax} \ @ \ R_L = R_{Th}$$

$$P_{Lmax} = \frac{V_{Th}^2 \, R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_{Th}^2}{4 R_{Th}}$$

To verify max. power transfer theorem,

R_L (Ω)	I_L (A)	P_L (W)
10	2.85	82
12	2.63	83
13.33	2.5	83.31
15	2.35	82.83
20	2	80

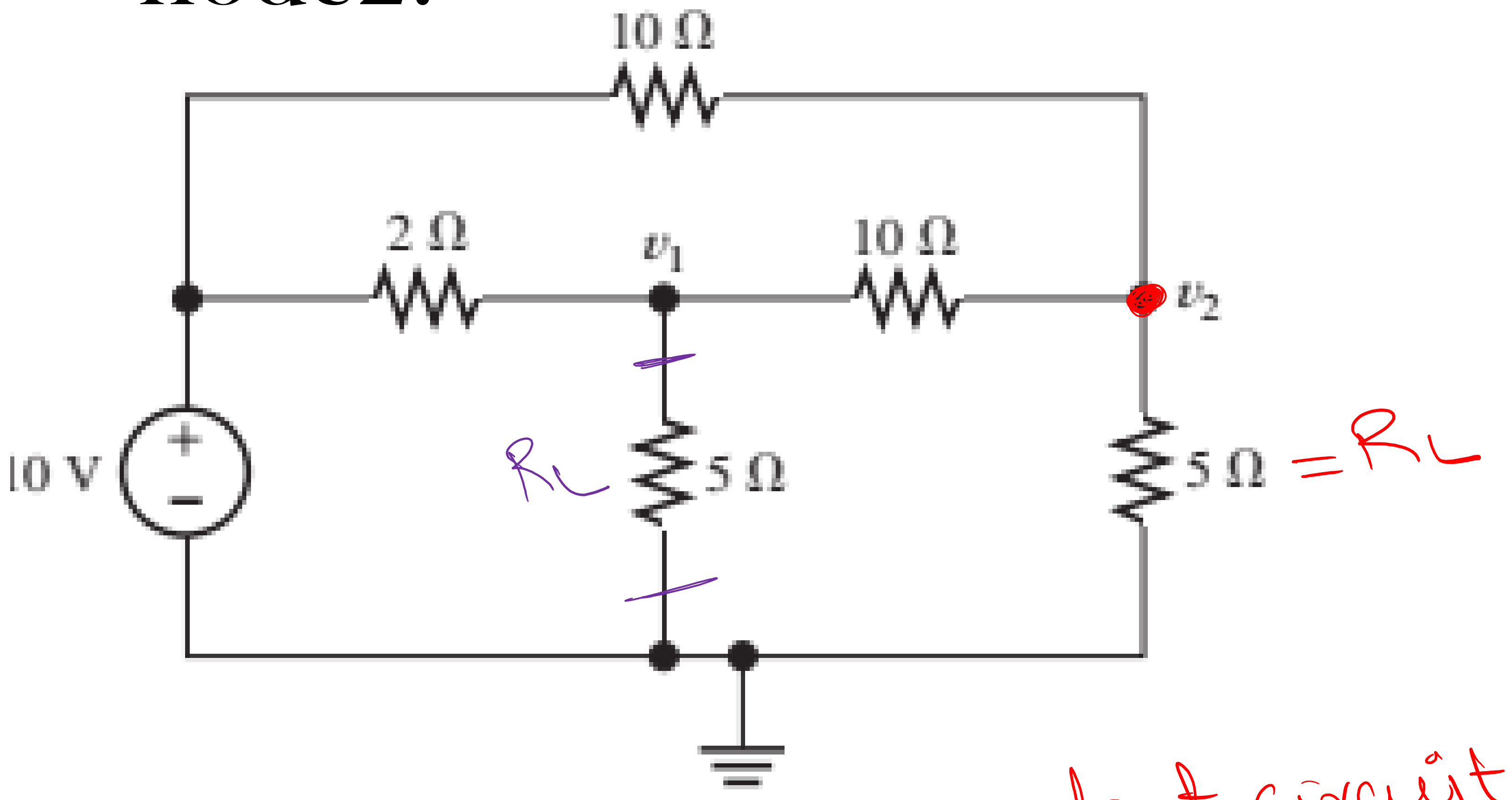
R_{Th} → 13.33

Applied for implementing MPPT techniques in Renewable energy sources

Max. Power Point Tracking

Maximum Power Transfer Theorem:

Prove the power delivered by the 10V source is maximum for $R_l = R_{th}$ at node2.



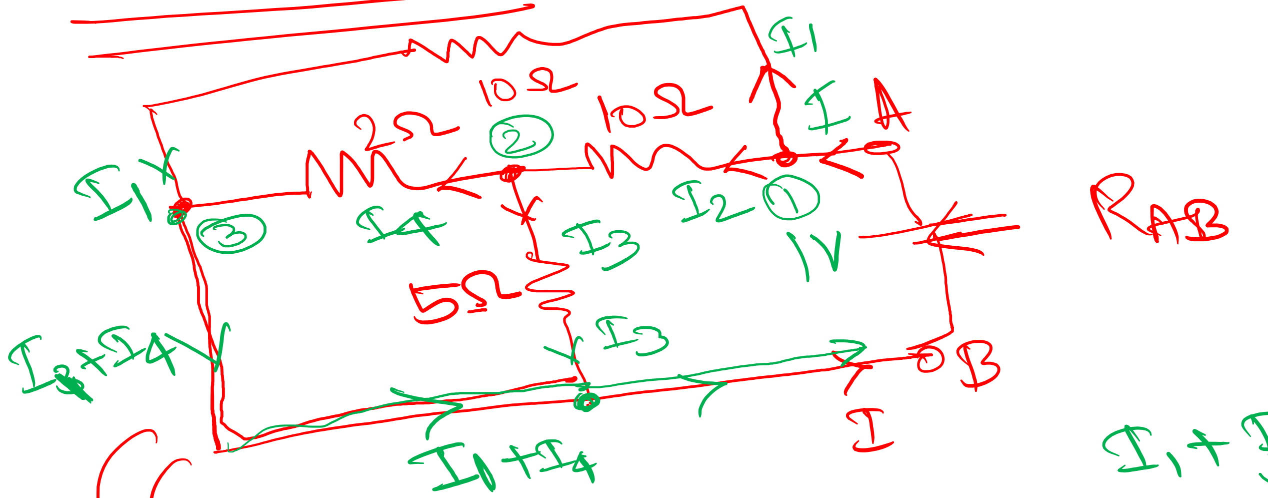
1) To find Thevenin's equivalent circuit

↳ a) To find R_{Th}

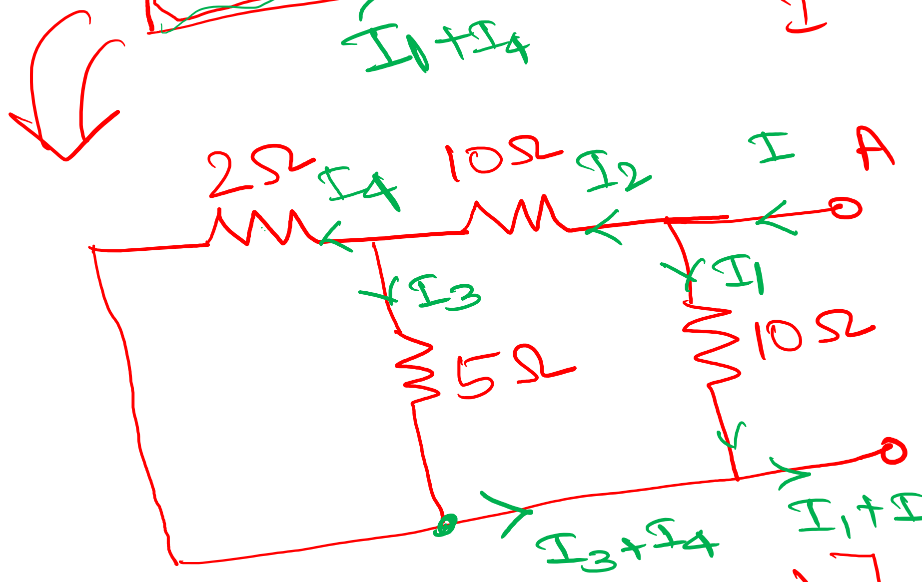
b) To find V_{Th}

2) Apply Max. power transfer

1a) To find R_{Th}:



$I_1 + I_3 + I_4 = I_1 + I_2 = I$



$I_1 + I_3 + I_4 = I_2 + I_1 = I$

$R_{Th} = 10 \parallel [10 + (5 \parallel 2)]$

$= 10 \parallel [10 + \frac{10}{7}] = 10 \parallel 11.43$

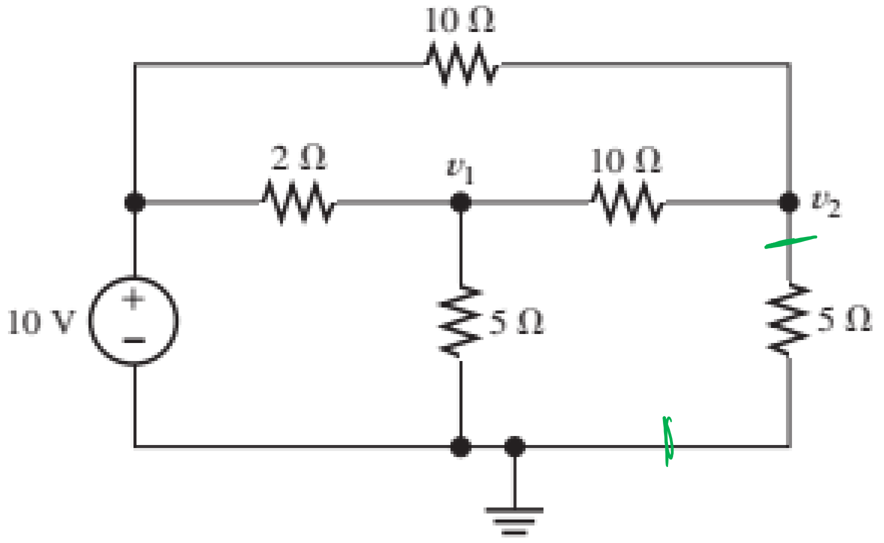
$= \frac{114.3}{21.43} = 5.33\ \Omega$

$\boxed{R_{Th} = 5.33\Omega}$

$\frac{10}{7}$

1.4

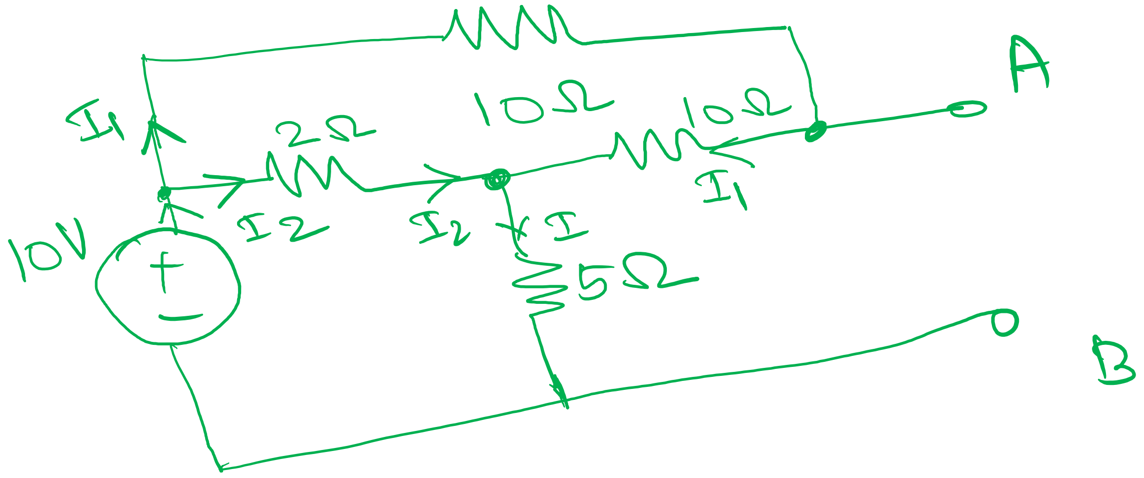
Maximum Power Transfer Theorem:

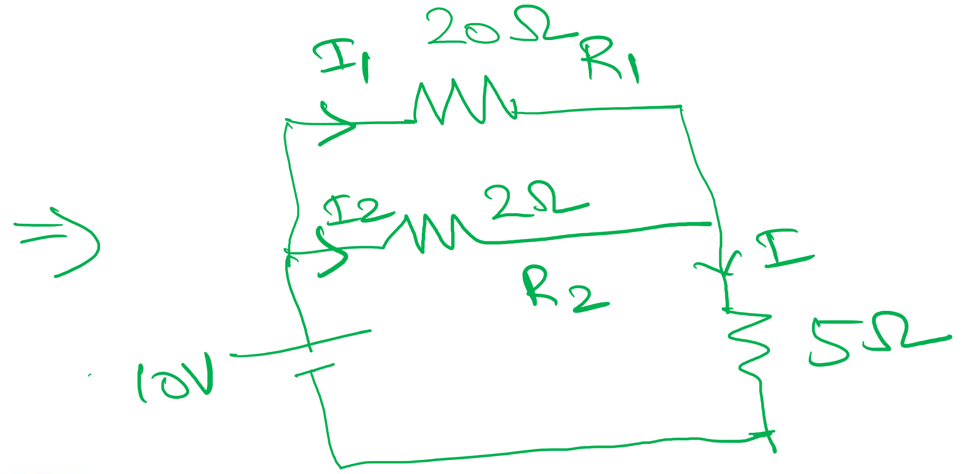


From the circuit

$$V_{AB} = 10 I_1 + 5 I$$

To find V_{Th}:





Now, $I = \dfrac{10}{R_{eq}}$

$$R_{eq} = (20 \| 2) + 5$$

$$= \frac{40}{22} + 5$$

$$= 1.82 + 5$$

$$= 6.82\,\Omega$$

$$I = \frac{10}{6.82}$$

$$\boxed{I = 1.47\,A}$$

By current division Rule,

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

$$= \frac{2}{20+2} \times 1.47$$

$$\boxed{I_1 = 0.13\,A}$$

∴ The Thevenin's voltage is given by

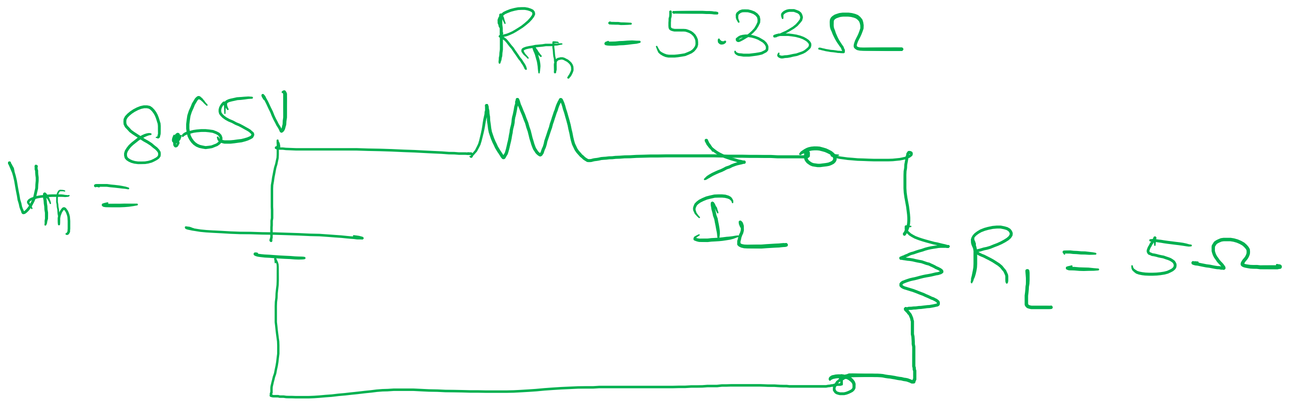
$$V_{Th} = V_{AB} = 10 I_1 + 5 I$$

$$= (10 \times 0.13) + (5 \times 1.47)$$

$$= 1.3 + 7.35$$

$$\boxed{V_{Th} = 8.65\,V}$$

To find the load current and power delivered to the load using Thevenin's equivalent circuit,



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{8.65}{5.33 + 5}$$

$$= 0.837A$$

The power delivered to the load is

$$P_L = I_L^2 R_L = (0.837)^2 \times 5 = \underline{\underline{3.50W}}$$

To verify maximum power transfer theorem, Let us consider the following:

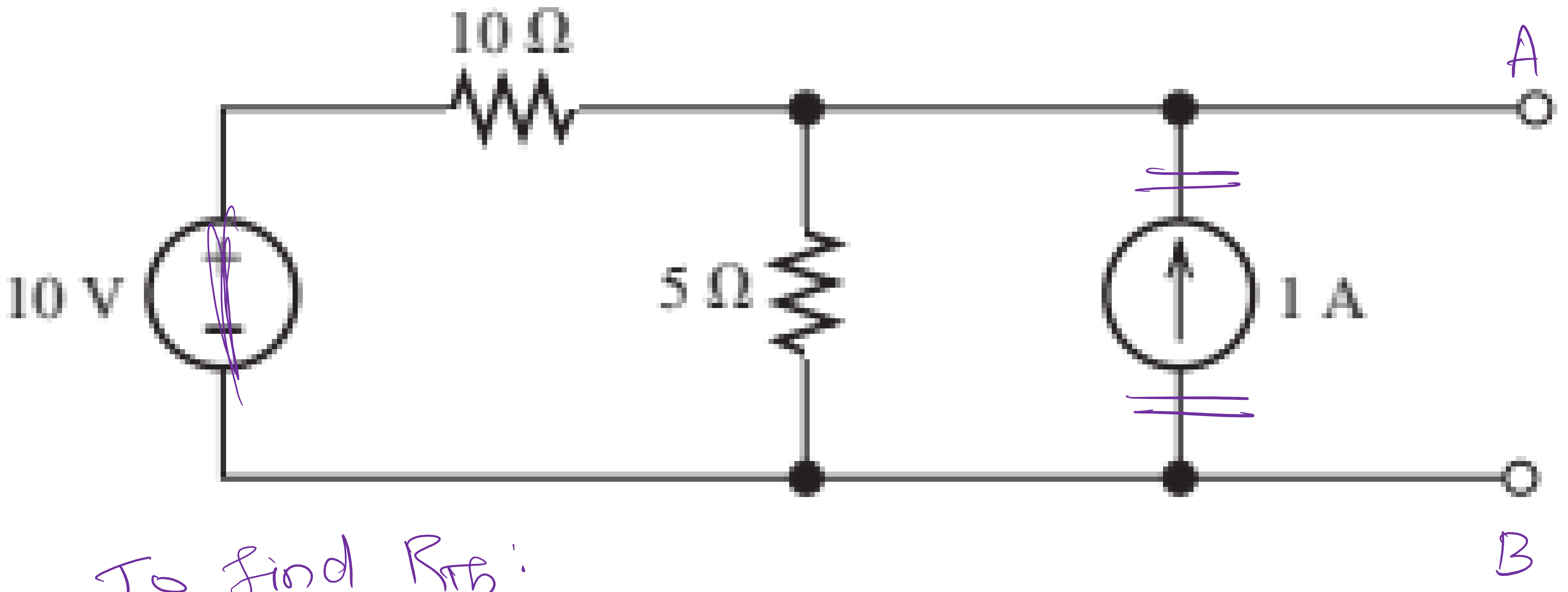
$R_L(\Omega)$	$I_L(A)$	$P_L(W)$
4	0.927	3.44
5	0.837	3.50
5.33	0.811	3.51
6	0.763	3.49

$R_L = R_{Th}$.

From the above table, the power delivered by 10V source at node 2 is maximum for $R_L = R_{Th}$ of the given network.

Maximum Power Transfer Theorem:

Find the maximum power delivered by the 10V source in the circuit.

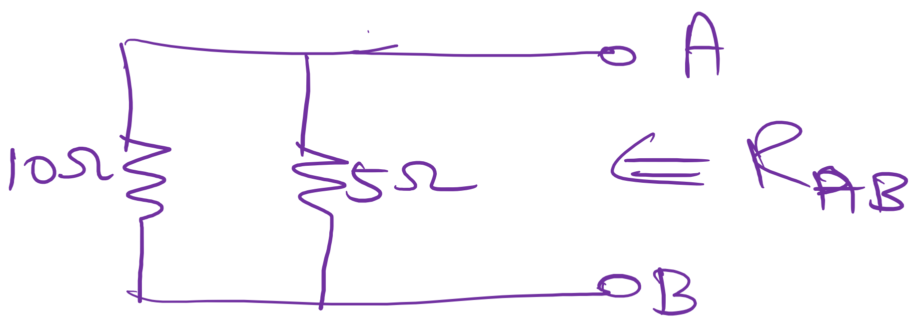


$R_{Th} = ?$

$V_{Th} = ?$

$P_{max} = ?$

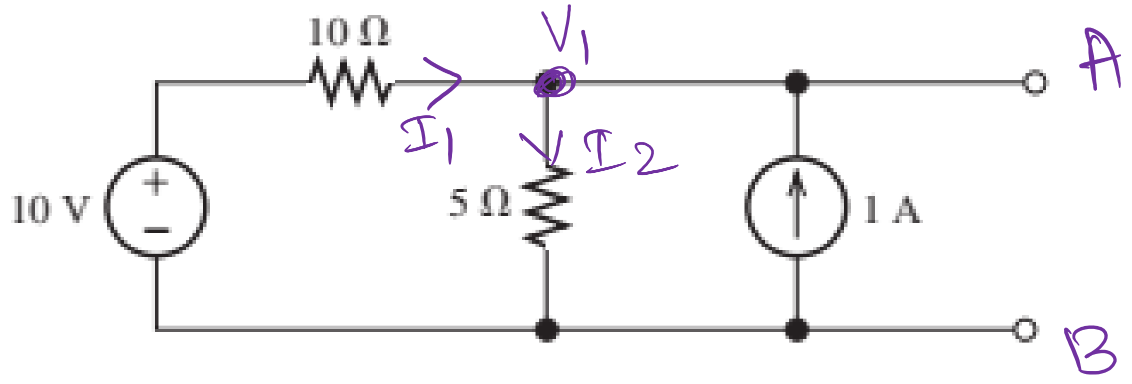
<u>To find R_{Th}:</u>



$R_{Th} = 10 \| 5 = \frac{50}{15} = 3.33\Omega$

$\boxed{R_{Th} = 3.33\Omega}$

Maximum Power Transfer Theorem:



To find V_{Th}:

Apply KCL at node V_1,

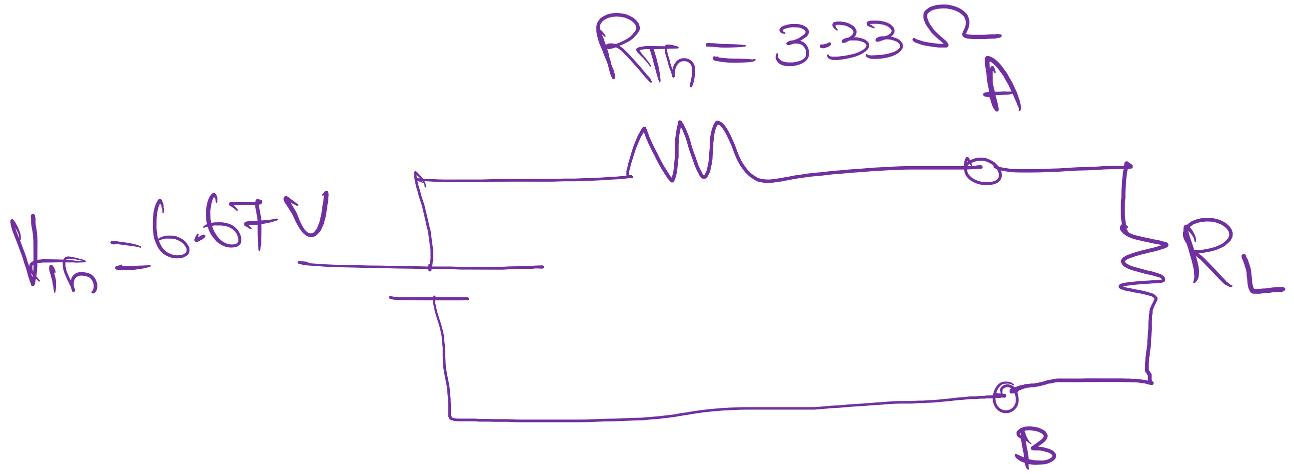
$$I_1 + 1 = I_2$$

$$\frac{10 - V_1}{10} + 1 = \frac{V_1}{5}$$

$$1 + 1 = 0.2V_1 + 0.1V_1$$

$$V_1 = \frac{2}{0.3} = 6.67\,V$$

$\therefore V_{Th} = 6.67\,V$



For the source to deliver maximum power, $R_L = R_{Th}$

$$\therefore P_{Lmax} = \frac{V_{Th}^2}{4R_{Th}} = \frac{6.67^2}{4 \times 3.33}$$

$$\boxed{P_{Lmax} = 3.34\,W}$$