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SCHOOL OF ELECTRICAL ENGINEERING

BEEE101L

Basic Electrical Engineering

AC circuits

By,

Meera P. S.

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EXAMPLE For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of C, L, or R if sufficient data are provided

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 40°)$

b. $v = 1000 \sin(377t + 10°)$
   $i = 5 \sin(377t - 80°)$

c. $v = 500 \sin(157t + 30°)$
   $i = 1 \sin(157t + 120°)$

d. $v = 50 \cos(\omega t + 20°)$
   $i = 5 \sin(\omega t + 110°)$

a. Since v and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{100\text{ V}}{20\text{ A}} = \mathbf{5\ \Omega}$$

b. Since v *leads* i by 90°, the element is an *inductor,* and

$$X_L = \frac{V_m}{I_m} = \frac{1000\text{ V}}{5\text{ A}} = 200\ \Omega$$

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377\text{ rad/s}} = \mathbf{0.531\ H}$$

c. Since i *leads* v by 90°, the element is a *capacitor,* and

$$X_C = \frac{V_m}{I_m} = \frac{500\text{ V}}{1\text{ A}} = 500\ \Omega$$

so that $X_C = \dfrac{1}{\omega C} = 500\ \Omega$ or

$$C = \frac{1}{\omega 500\ \Omega} = \frac{1}{(157\text{ rad/s})(500\ \Omega)} = \mathbf{12.74\ \mu F}$$

d. $v = 50\cos(\omega t + 20°) = 50\sin(\omega t + 20° + 90°)$
$= 50\sin(\omega t + 110°)$

Since v and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{50\text{ V}}{5\text{ A}} = \mathbf{10\ \Omega}$$

AVERAGE POWER AND POWER FACTOR

$$v = V_m \sin(\omega t + \theta_v)$$

$$i = I_m \sin(\omega t + \theta_i)$$

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

$$\sin A \sin B = \frac{\cos(A - B) - \cos(A + B)}{2}$$

$$\sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$
$$= \frac{\cos[(\omega t + \theta_v) - (\omega t + \theta_i)] - \cos[(\omega t + \theta_v) + (\omega t + \theta_i)]}{2}$$
$$= \frac{\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)}{2}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2} \cos(\theta_v - \theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)\right]}^{\text{Time-varying (function of } t\text{)}}$$

The first term in the preceding equation, however, has a constant magnitude (no time dependence) and therefore provides some net transfer of energy. This term is referred to as the average power. The average power, or real power as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for dc networks.

$$\boxed{P = \frac{V_m I_m}{2} \cos \theta} \qquad \text{(watts, W)}$$

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) \cos \theta$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = V_{\text{eff}} I_{\text{eff}} \cos \theta}$$

$$\boxed{P = \frac{V_m I_m}{2} = V_{\text{eff}} I_{\text{eff}}}$$

$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R}$$

$$\boxed{P = \frac{V_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R}$$

Inductor

In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = 0 \text{ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos(90°) = \frac{V_m I_m}{2}(0) = 0 \text{ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

EXAMPLE Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 70°)$

b. $v = 150 \sin(\omega t - 70°)$
   $i = 3 \sin(\omega t - 50°)$

$$P = \frac{V_m I_m}{2} \cos\theta = \frac{(100\text{ V})(20\text{ A})}{2} \cos(30°) = (1000\text{ W})(0.866)$$

$$= \mathbf{866\ W}$$

$$P = \frac{V_m I_m}{2} \cos\theta = \frac{(150\text{ V})(3\text{ A})}{2} \cos(20°) = (225\text{ W})(0.9397)$$

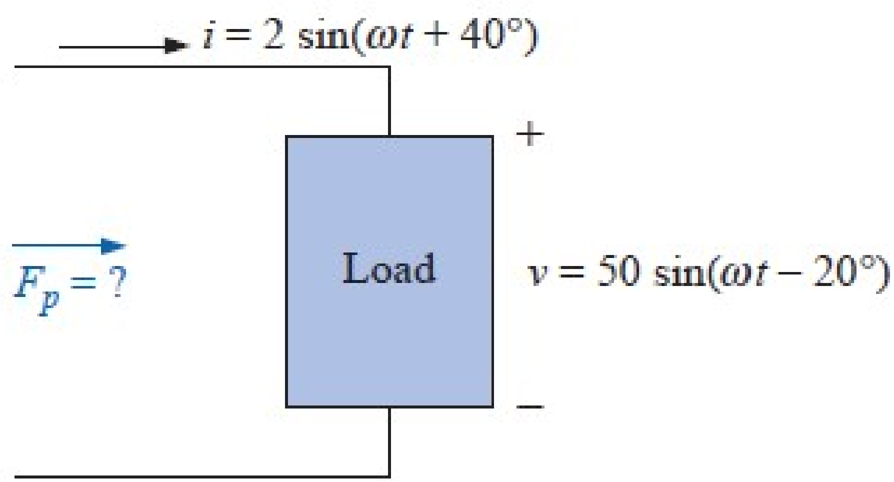
$$= \mathbf{211.43\ W}$$

Power Factor

$$\text{Power factor} = F_p = \cos\theta$$

$$F_p = \cos\theta = \frac{P}{V_{\text{eff}} I_{\text{eff}}}$$

capacitive networks have leading power factors, and inductive networks have lagging power factors.



$$F_p = \cos\theta = \cos|40° - (-20°)| = \cos 60° = \mathbf{0.5\ leading}$$

COMPLEX NUMBERS

RECTANGULAR FORM

The format for the **rectangular form** is

$$\boxed{\mathbf{C} = X + jY}$$

POLAR FORM

The format for the **polar form** is

$$\boxed{\mathbf{C} = Z \angle \theta}$$

Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2}$$

$$j = \sqrt{-1}$$

$$\theta = \tan^{-1} \frac{Y}{X}$$

$$j^2 = -1$$

Polar to Rectangular

$$X = Z \cos \theta$$

$$\frac{1}{j} = -j$$

$$Y = Z \sin \theta$$

Complex Conjugate

$$\mathbf{C} = 2 + j3$$

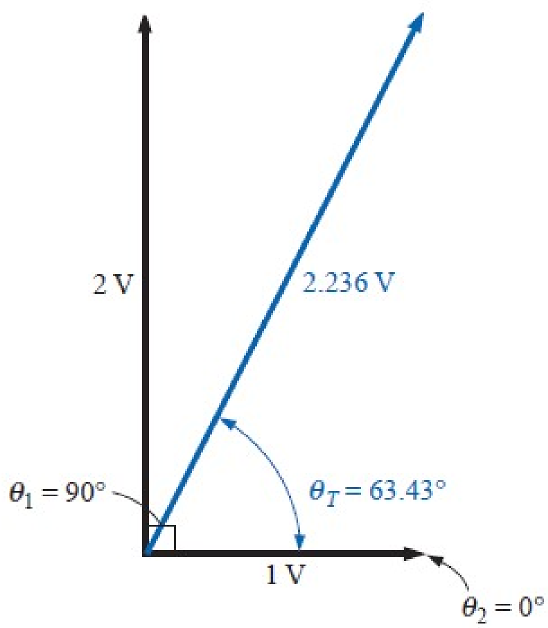
$$2 - j3$$

$$\mathbf{C} = 2 \angle 30^\circ$$

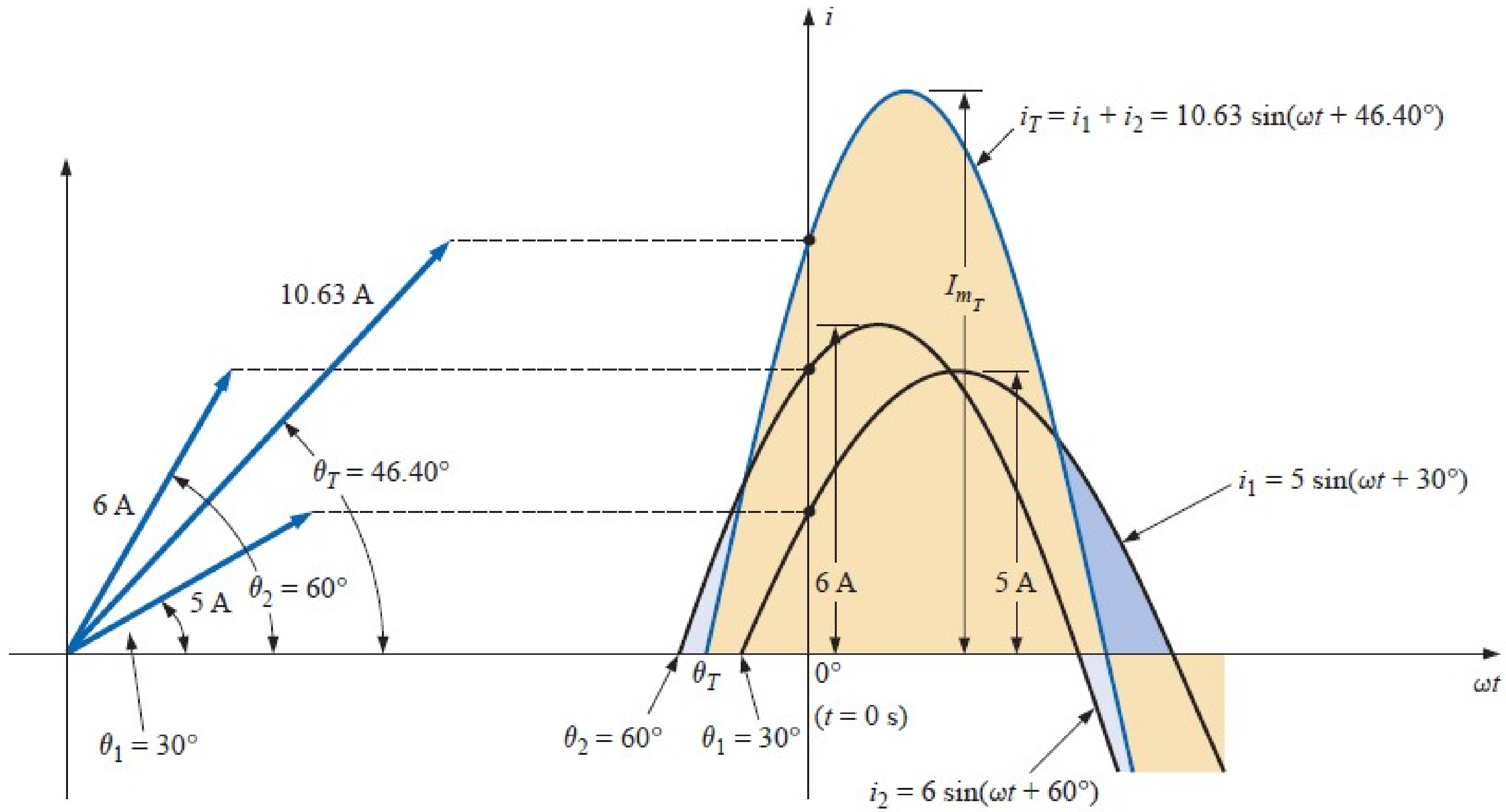
$$2 \angle -30^\circ$$

PHASORS

The *radius vector,* having a *constant magnitude* (length) with *one end fixed at the origin,* is called a phasor when applied to electric circuits.



$$2.236 \sin(\omega t + 63.43^\circ)$$



i
$i_T = i_1 + i_2 = 10.63 \sin(\omega t + 46.40°)$
10.63 A
I_{m_T}
$\theta_T = 46.40°$
6 A
$i_1 = 5 \sin(\omega t + 30°)$
5 A
$\theta_2 = 60°$
6 A
5 A
θ_T
0°
ωt
$(t = 0 \text{ s})$
$\theta_1 = 30°$
$\theta_2 = 60°$
$\theta_1 = 30°$
$i_2 = 6 \sin(\omega t + 60°)$

In general, for all of the analyses to follow, the phasor form of a sinusoidal voltage or current will be

$$\mathbf{V} = V \angle \theta \quad \text{and} \quad \mathbf{I} = I \angle \theta$$

where V and I are rms values and θ is the phase angle. It should be pointed out that in phasor notation, the sine wave is always the reference, and the frequency is not represented.

Phasor algebra for sinusoidal quantities is applicable only for waveforms having the same frequency.

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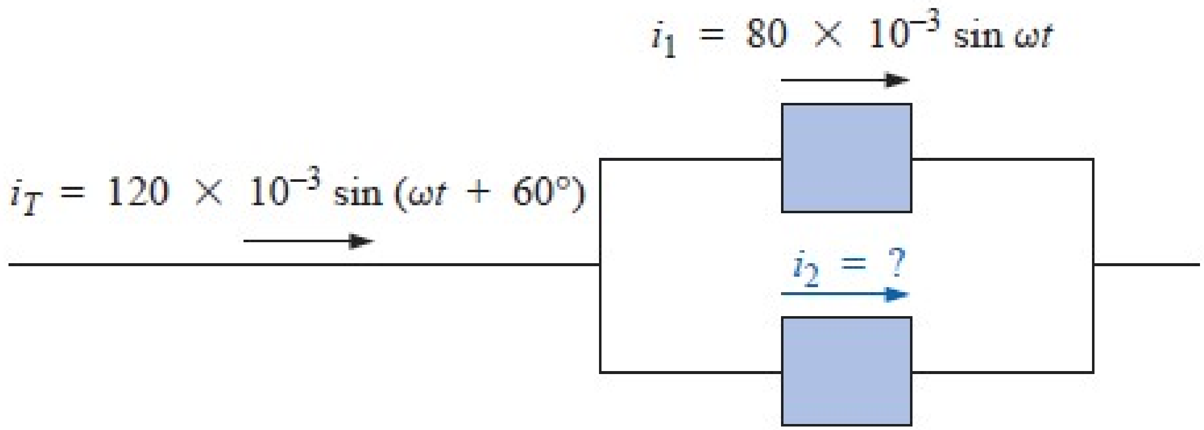
Determine the current i_2 for the network of Fig.



$$i_2 = i_T - i_1$$

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA } \angle 60°$$

$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA } \angle 0°$$

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60^\circ = 42.42 \text{ mA} + j\,73.47 \text{ mA}$$

$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0^\circ = 56.56 \text{ mA} + j\,0$$

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{I}_T - \mathbf{I}_1 \\ &= (42.42 \text{ mA} + j\,73.47 \text{ mA}) - (56.56 \text{ mA} + j\,0)\end{aligned}$$

$$\mathbf{I}_2 = -14.14 \text{ mA} + j\,73.47 \text{ mA}$$

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89^\circ$$

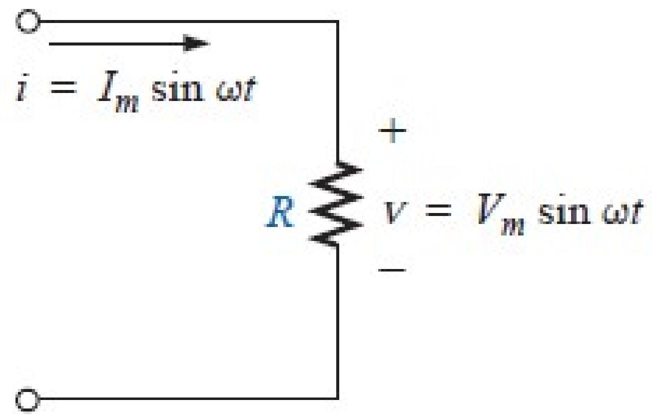
$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89^\circ \Rightarrow$$

$$i_2 = \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89^\circ)$$

$$i_2 = \mathbf{105.8 \times 10^{-3} \sin(\omega t + 100.89^\circ)}$$

SERIES ac CIRCUITS

Resistive Elements



$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0^\circ$$

where $V = 0.707 V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V \angle 0^\circ}{R \angle \theta_R} = \frac{V}{R} \angle 0^\circ - \theta_R$$

$$\mathbf{I} = \frac{V\angle 0^\circ}{R\angle 0^\circ} = \frac{V}{R}\,\underline{/0^\circ - 0^\circ} = \frac{V}{R}\angle 0^\circ \qquad i = \sqrt{2}\left(\frac{V}{R}\right)\sin\omega t$$

$$\boxed{\mathbf{Z}_R = R\ \angle 0^\circ}$$

Inductive Reactance

$$v = V_m \sin\omega t \Rightarrow \text{phasor form } \mathbf{V} = V\angle 0^\circ$$

$$\mathbf{I} = \frac{V\angle 0^\circ}{X_L\angle\theta_L} = \frac{V}{X_L}\,\underline{/0^\circ - \theta_L}$$

$$\mathbf{I} = \frac{V\angle 0^\circ}{X_L\angle 90^\circ} = \frac{V}{X_L}\,\underline{/0^\circ - 90^\circ} = \frac{V}{X_L}\angle -90^\circ$$

$$i = \sqrt{2}\left(\frac{V}{X_L}\right)\sin(\omega t - 90^\circ) \qquad \boxed{\mathbf{Z}_L = X_L\ \angle 90^\circ}$$

Capacitive Reactance

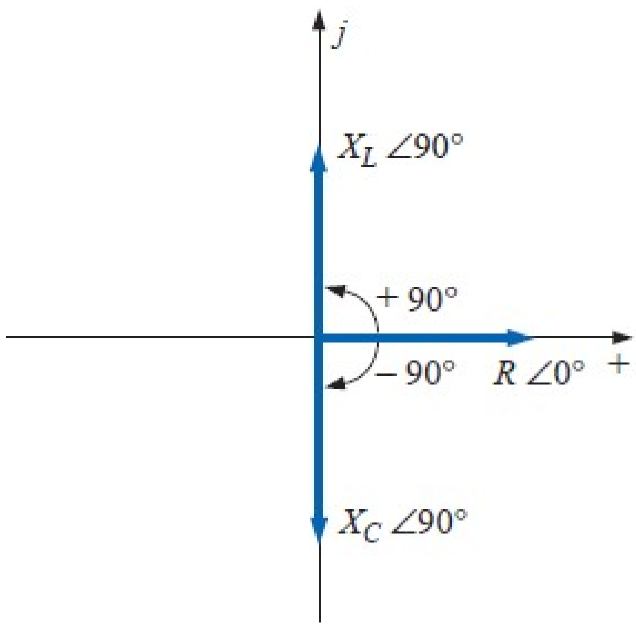
$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle \theta_C} = \frac{V}{X_C} \underline{/0^\circ - \theta_C}$$

$$\mathbf{I} = \frac{V \angle 0^\circ}{X_C \angle -90^\circ} = \frac{V}{X_C} \underline{/0^\circ - (-90^\circ)} = \frac{V}{X_C} \angle 90^\circ$$

$$i = \sqrt{2}\left(\frac{V}{X_C}\right) \sin(\omega t + 90^\circ)$$

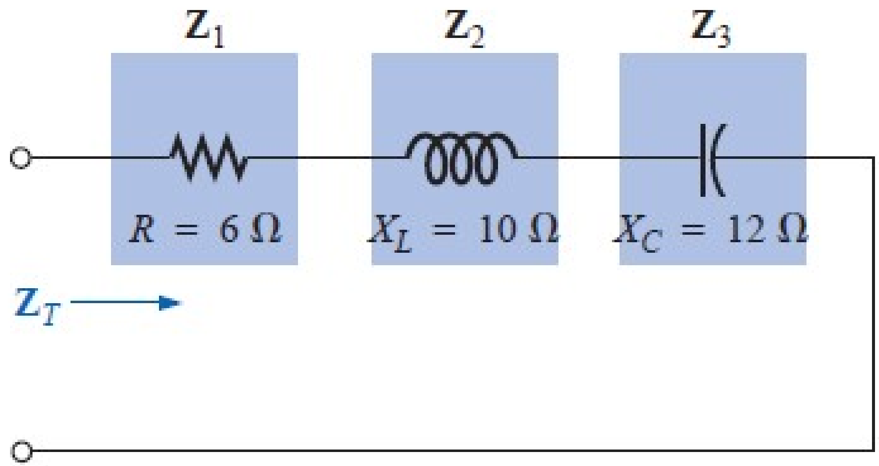
$$\boxed{\mathbf{Z}_C = X_C \ \angle -90^\circ}$$

Impedance Diagram

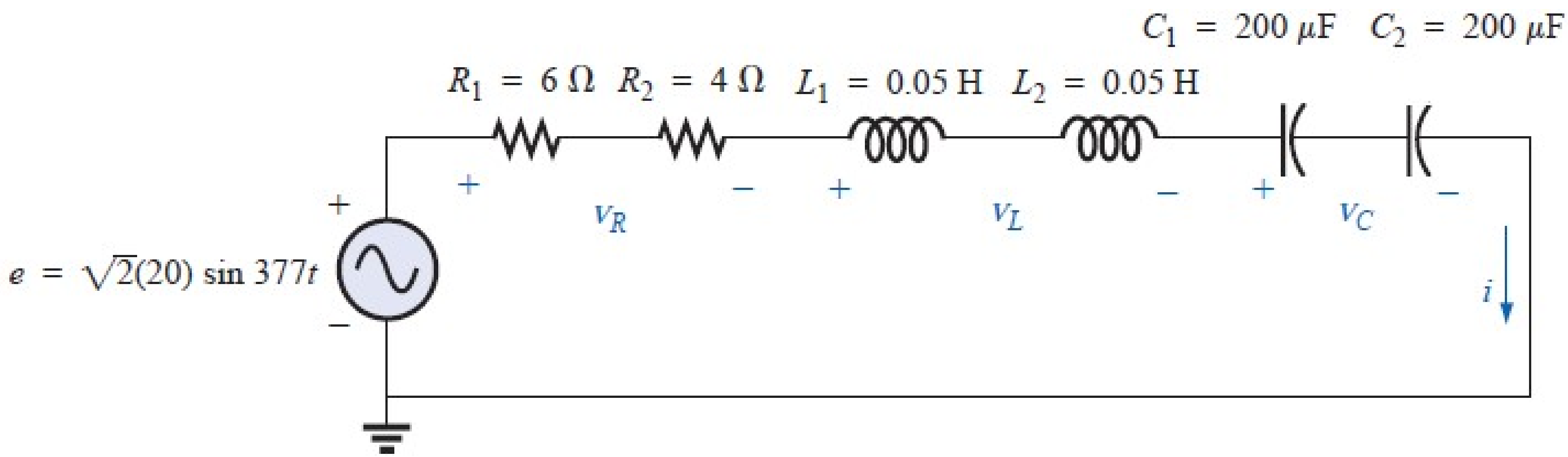


For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.

Determine the input impedance to the series network



$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\
&= R\,\angle 0^\circ + X_L\,\angle 90^\circ + X_C\,\angle -90^\circ \\
&= R + jX_L - jX_C \\
&= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega \\
\mathbf{Z}_T &= \mathbf{6.325\ \Omega\ \angle -18.43^\circ}
\end{aligned}
$$



a. Calculate $\mathbf{I}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.
d. Draw the phasor diagram.
e. Obtain the phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$, and show that it equals the input voltage $\mathbf{E}$.
f. Find $\mathbf{V}_R$ and $\mathbf{V}_C$ using the voltage divider rule.

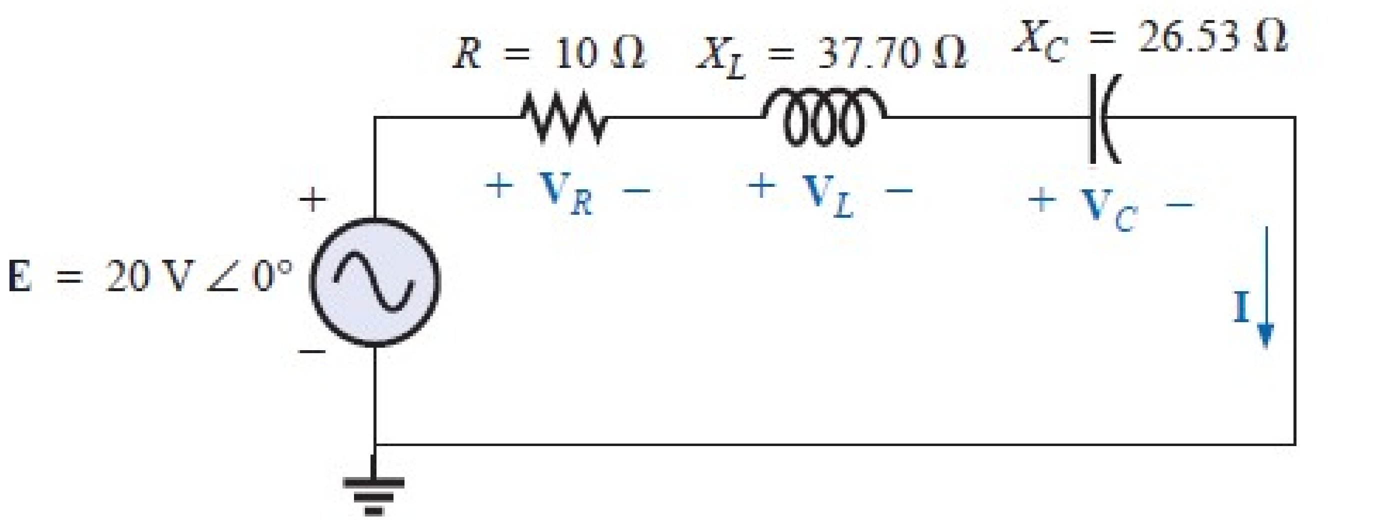
$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$

$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$

$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$

$$X_L = \omega L = (377\ \text{rad/s})(0.1\ \text{H}) = 37.70\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{rad/s})(100 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$



$$\begin{aligned}\mathbf{Z}_T &= R\,\angle 0° + X_L\,\angle 90° + X_C\,\angle -90° \\ &= 10\ \Omega + j\,37.70\ \Omega - j\,26.53\ \Omega \\ &= 10\ \Omega + j\,11.17\ \Omega = \mathbf{15\ \Omega\ \angle 48.16°}\end{aligned}$$

The current **I** is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\ \text{V}\ \angle 0°}{15\ \Omega\ \angle 48.16°} = \mathbf{1.33\ A\ \angle -48.16°}$$

-

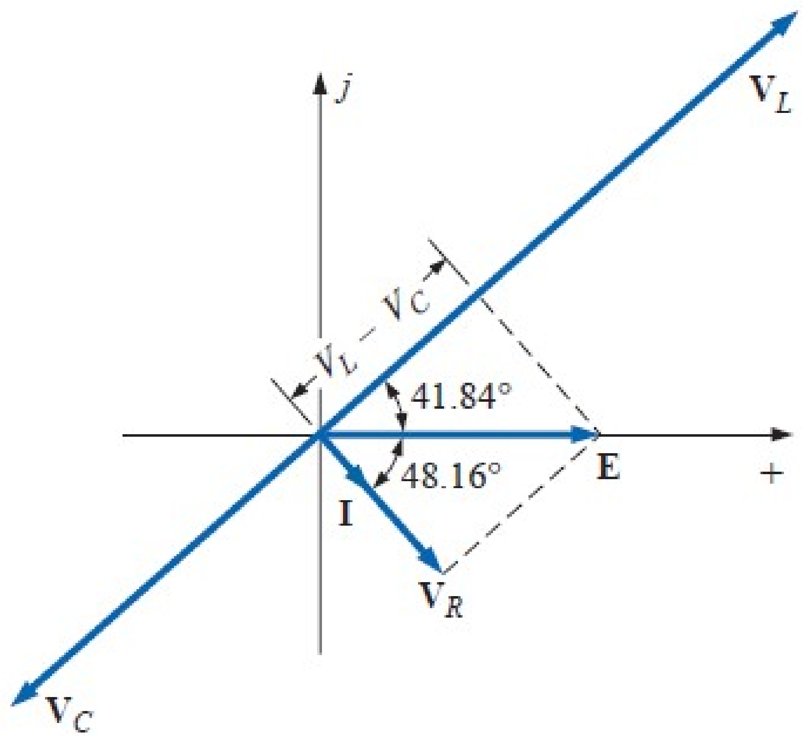
$$\begin{aligned}\mathbf{V}_R = \mathbf{IZ}_R &= (I\,\angle\theta)(R\,\angle 0°) = (1.33\ \text{A}\ \angle -48.16°)(10\ \Omega\ \angle 0°) \\ &= \mathbf{13.30\ V\ \angle -48.16°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_L = \mathbf{IZ}_L &= (I\,\angle\theta)(X_L\,\angle 90°) = (1.33\ \text{A}\ \angle -48.16°)(37.70\ \Omega\ \angle 90°) \\ &= \mathbf{50.14\ V\ \angle 41.84°}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_C = \mathbf{IZ}_C &= (I\,\angle\theta)(X_C\,\angle -90°) = (1.33\ \text{A}\ \angle -48.16°)(26.53\ \Omega\,\angle -90°) \\ &= \mathbf{35.28\ V\ \angle -138.16°}\end{aligned}$$

$$F_p = \cos\theta = \cos 48.16° = \mathbf{0.667\ lagging}$$

$$P_T = EI\cos\theta = (20\ \text{V})(1.33\ \text{A})(0.667) = \mathbf{17.74\ W}$$



$$\begin{aligned}\mathbf{E} &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \\ &= 13.30\ \text{V}\angle -48.16° + 50.14\ \text{V}\angle 41.84° + 35.28\ \text{V}\angle -138.16° \\ \mathbf{E} &= 13.30\ \text{V}\angle -48.16° + 14.86\ \text{V}\angle 41.84°\end{aligned}$$

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\text{ V}}$$

$$\mathbf{E} = 20\ \angle 0°$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\ \angle 0°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{200\text{ V}\angle 0°}{15\ \angle 48.16°}$$

$$= \mathbf{13.3\text{ V}\ \angle -48.16°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\ \angle -90°)(20\text{ V}\ \angle 0°)}{15\ \Omega\ \angle 48.16°} = \frac{530.6\text{ V}\angle -90°}{15\ \angle 48.16°}$$

$$= \mathbf{35.37\text{ V}\ \angle -138.16°}$$

BEEE 101L : Basic Electrical Engineering

Module 2- AC Circuits

Series AC circuits

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Complex power

- The complex power S absorbed by the ac load is the product of the voltage and the complex conjugate of the current,

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* \qquad \mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^* \qquad \mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\angle\theta_v \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\angle\theta_i$$

$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \angle\theta_v - \theta_i$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}}\angle\theta_v - \theta_i$$

$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ$$

$$\boxed{\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*}}$$

$$P = \operatorname{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$
$$Q = \operatorname{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

- The real power P is the average power in watts delivered to a load; it is the only useful power. It is the actual power dissipated by the load.
- The reactive power Q is a measure of the energy exchange between the source and the reactive part of the load. The unit of Q is the volt-ampere reactive(VAR) to distinguish it from the real power.
- The reactive power is being transferred back and forth between the load and the source.

1. $Q=0$ for resistive loads (unity pf).
2. $Q<0$ for capacitive loads (leading pf).
3. $Q>0$ for inductive loads (lagging pf).

- Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$= V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$

$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

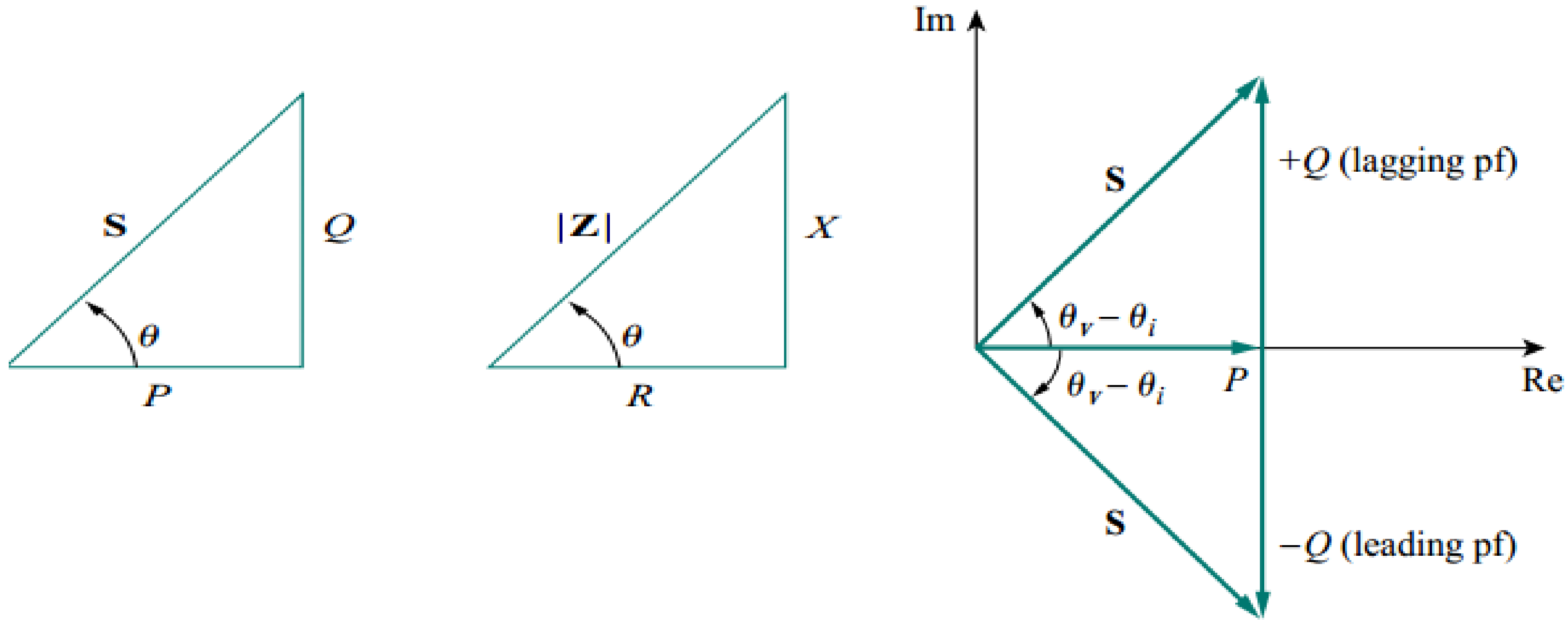
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

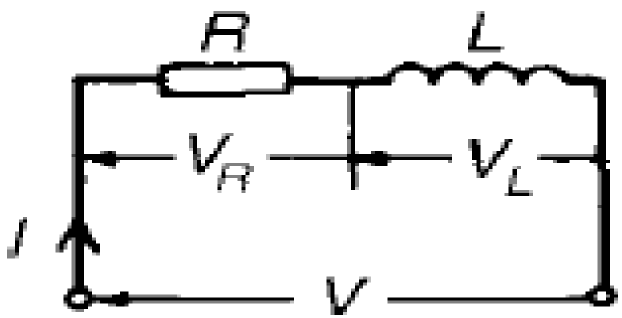
Power Triangle

- It is a standard practice to represent S, P, and Q in the form of a triangle, known as the power triangle.
- The power triangle has four items—the apparent/complex power, real power, reactive power, and the power factor angle.

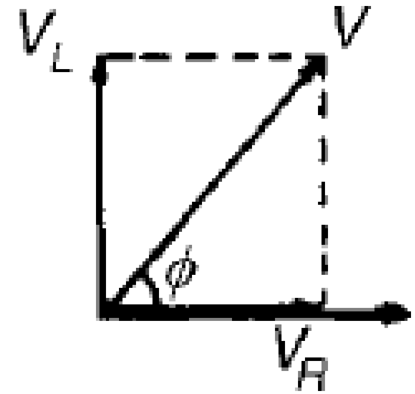


RL Series ac circuit

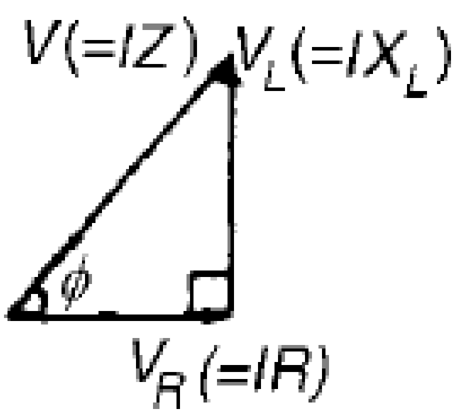
CIRCUIT DIAGRAM



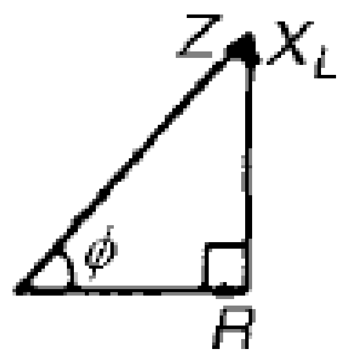
PHASOR DIAGRAM



VOLTAGE TRIANGLE



IMPEDANCE TRIANGLE



For the *R–L* circuit:

$$V = \sqrt{(V_R^2 + V_L^2)} \quad \text{(by Pythagoras' theorem)}$$

and $\tan\phi = \dfrac{V_L}{V_R}$ (by trigonometric ratios)

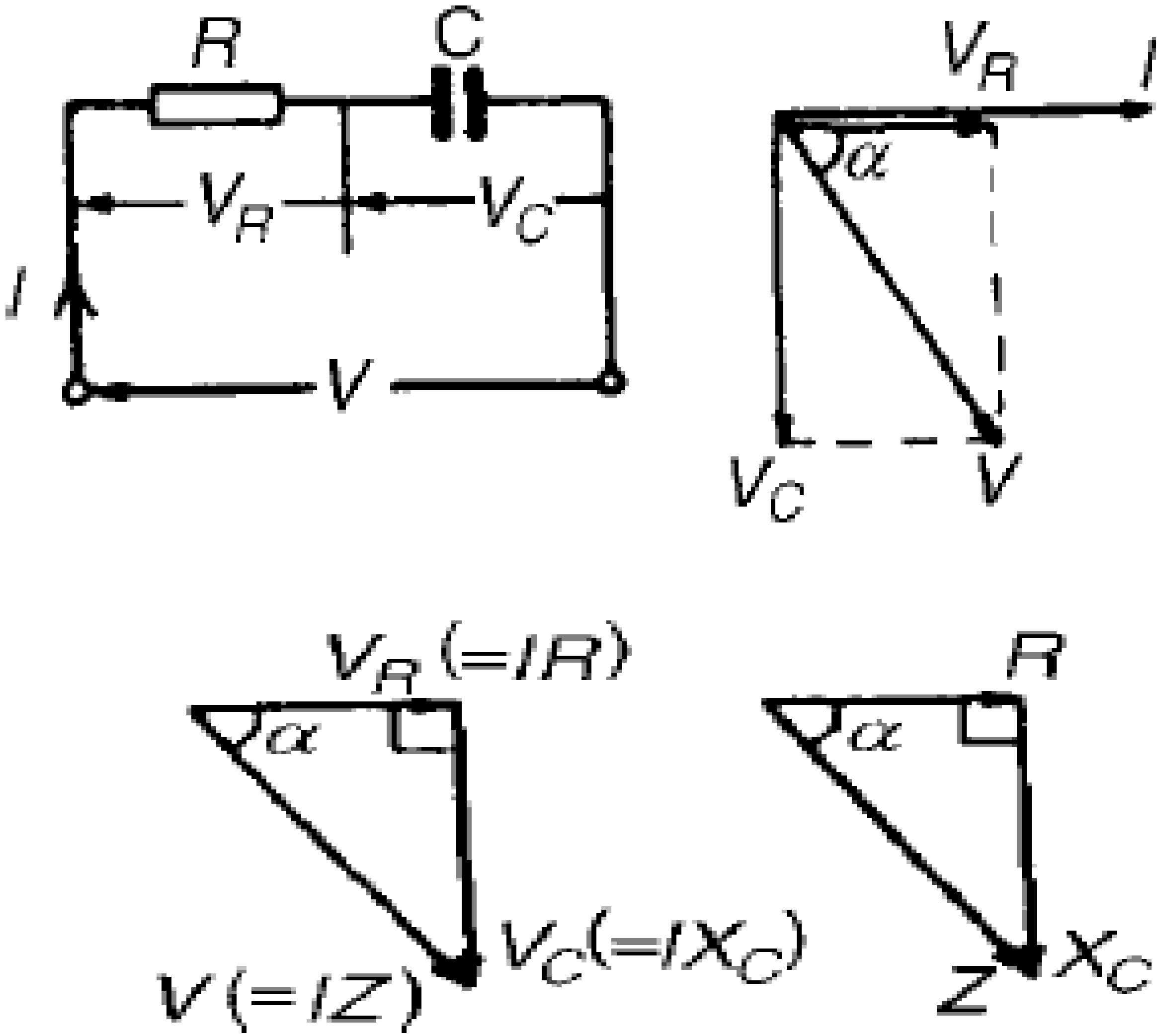
In an a.c. circuit, the ratio $\dfrac{\text{applied voltage } V}{\text{current } I}$ is called the **impedance Z**, i.e.

$$\boxed{Z = \frac{V}{I}\ \Omega}$$

For the *R–L* circuit: $Z = \sqrt{(R^2 + X_L^2)}$

$$\tan\phi = \frac{X_L}{R},\ \sin\phi = \frac{X_L}{Z} \text{ and } \cos\phi = \frac{R}{Z}$$

RC series ac circuit



$$V = \sqrt{(V_R^2 + V_C^2)} \quad \text{(by Pythagoras' theorem)}$$

$$\text{and } \tan\alpha = \frac{V_C}{V_R} \quad \text{(by trigonometric ratios)}$$

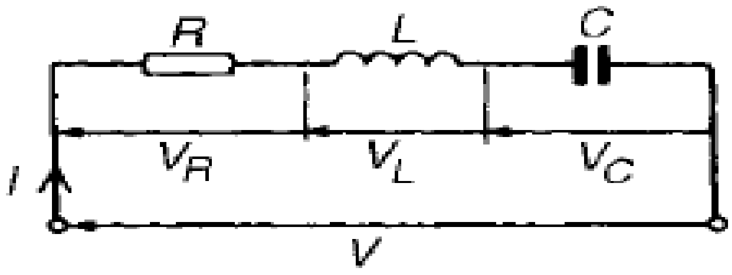
For the R–C circuit: $Z = \sqrt{(R^2 + X_C^2)}$

$$\tan\alpha = \frac{X_C}{R}, \quad \sin\alpha = \frac{X_C}{Z} \quad \text{and} \quad \cos\alpha = \frac{R}{Z}$$

RLC series ac circuit

In an a.c. series circuit containing resistance R, inductance L and capacitance C, the applied voltage V is the phasor sum of V_R, V_L and V_C

V_L and V_C are anti-phase, i.e. displaced by 180°

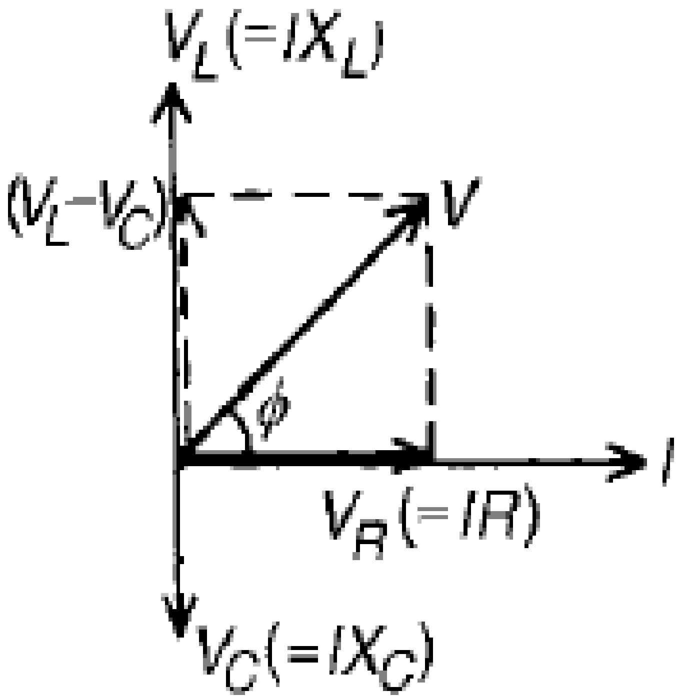


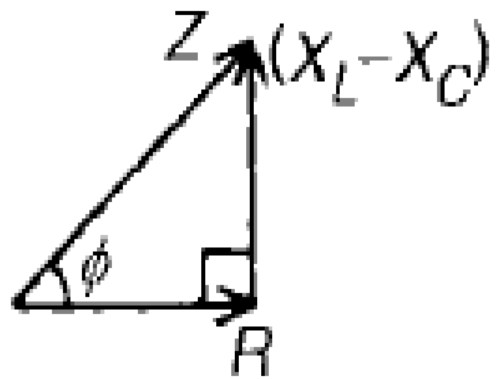
there are three phasor diagrams possible — each depending on the relative values of V_L and V_C.

When $X_L > X_C$

$$Z = \sqrt{[R^2 + (X_L - X_C)^2]}$$

$$\text{and } \tan\phi = \frac{(X_L - X_C)}{R}$$



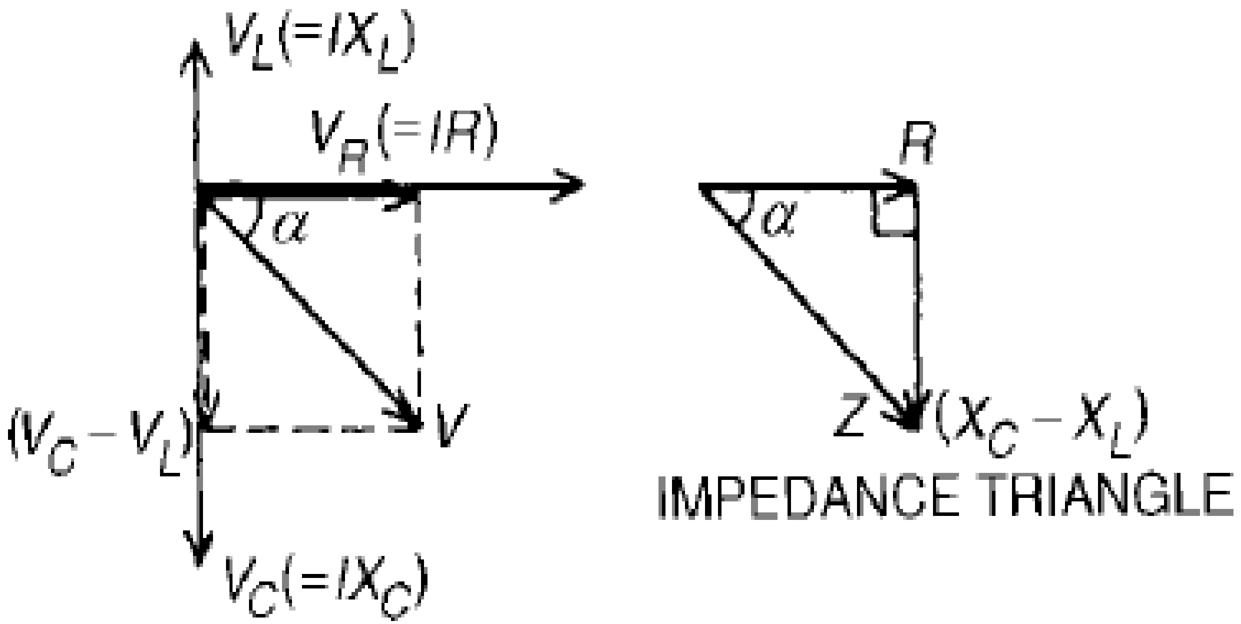


IMPEDANCE TRIANGLE

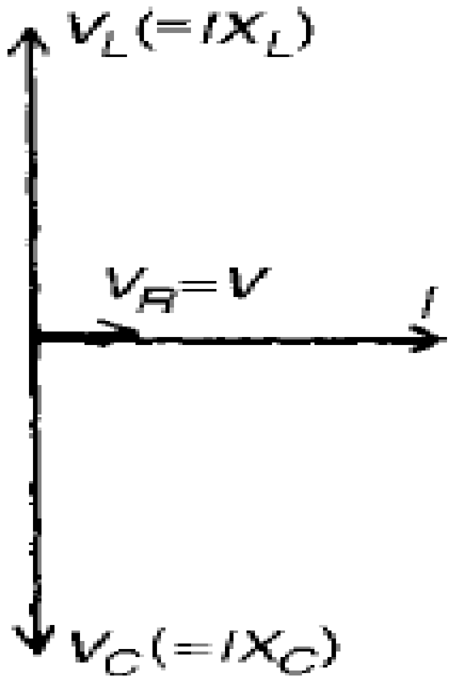
When $X_C > X_L$

$$Z = \sqrt{[R^2 + (X_C - X_L)^2]}$$

$$\text{and } \tan\alpha = \frac{(X_C - X_L)}{R}$$



When $X_L = X_C$ the applied voltage V and the current I are in phase. This effect is called **series resonance**



Problem In a series R–L circuit the p.d. across the resistance R is 12 V and the p.d. across the inductance L is 5 V. Find the supply voltage and the phase angle between current and voltage.

$$V = \sqrt{(12^2 + 5^2)} \text{ i.e. } \mathbf{V = 13\,V}$$

$$\boxed{\tan\phi = \frac{V_L}{V_R} = \frac{5}{12}}, \text{ from which } \phi = \tan^{-1}\left(\frac{5}{12}\right)$$

$$= 22.62^\circ$$

$$= \mathbf{22^\circ 37' \text{ lagging}}$$

Problem A coil consists of a resistance of 100 Ω and an inductance of 200 mH. If an alternating voltage, v, given by $v = 200 \sin 500t$ volts is applied across the coil, calculate (a) the circuit impedance, (b) the current flowing, (c) the p.d. across the resistance, (d) the p.d. across the inductance and (e) the phase angle between voltage and current.

Since $v = 200 \sin 500t$ volts then $V_m = 200\,\text{V}$ and

$$\omega = 2\pi f = 500\,\text{rad/s}$$

Hence rms voltage $V = 0.707 \times 200 = 141.4\,\text{V}$

Inductive reactance, $X_L = 2\pi f L = \omega L$

$$= 500 \times 200 \times 10^{-3} = 100\,\Omega$$

(a) Impedance $Z = \sqrt{(R^2 + X_L^2)}$

$$= \sqrt{(100^2 + 100^2)} = \mathbf{141.4\ \Omega}$$

(b) Current $I = \dfrac{V}{Z} = \dfrac{141.4}{141.4} = \mathbf{1\ A}$

(c) p.d. across the resistance $V_R = IR = 1 \times 100 = \mathbf{100\ V}$

p.d. across the inductance $V_L = IX_L = 1 \times 100 = \mathbf{100\ V}$

(e) Phase angle between voltage and current is given by:

$$\tan\phi = \left(\frac{X_L}{R}\right)$$

from which, $\phi = \tan^{-1}(100/100)$, hence $\boldsymbol{\phi = 45°}$ **or** $\boldsymbol{\frac{\pi}{4}}$ **rads**

Problem A capacitor C is connected in series with a 40 Ω resistor across a supply of frequency 60 Hz. A current of 3 A flows and the circuit impedance is 50 Ω. Calculate: (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.

(a) Impedance $Z = \sqrt{(R^2 + X_C^2)}$

Hence $X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(50^2 - 40^2)} = 30\,\Omega$

$$X_C = \frac{1}{2\pi f C} \text{ hence } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)30}\,\text{F}$$

$$= \mathbf{88.42\,\mu F}$$

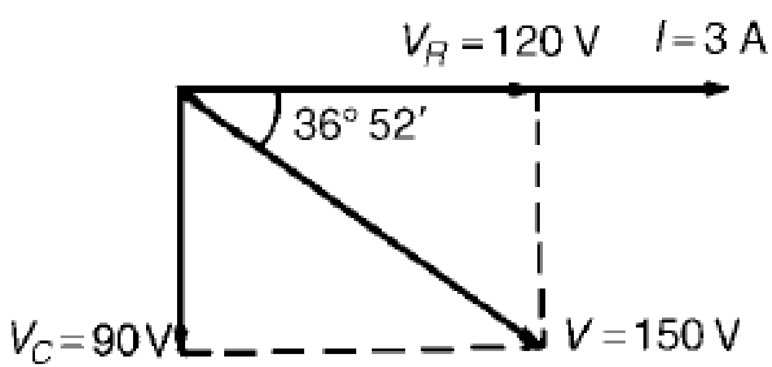
(b) Since $Z = \frac{V}{I}$ then $V = IZ = (3)(50) = \mathbf{150\ V}$

(c) Phase angle, $\alpha = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{30}{40}\right) = 36.87°$

$= \mathbf{36°52'\ leading}$

(d) P.d. across resistor, $V_R = IR = (3)(40) = \mathbf{120\ V}$

(e) P.d. across capacitor, $V_C = IX_C = (3)(30) = \mathbf{90\ V}$

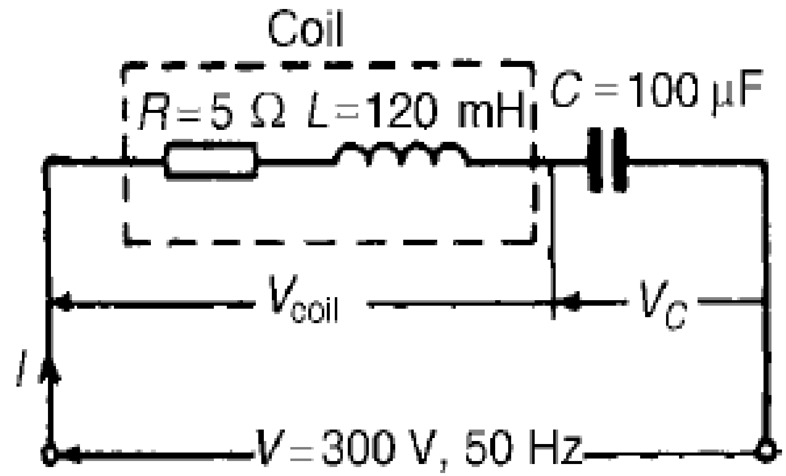


Phasor diagram

Problem A coil of resistance 5 Ω and inductance 120 mH in series with a 100 μF capacitor, is connected to a 300 V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

$$X_L = 2\pi fL = 2\pi(50)(120 \times 10^{-3}) = \mathbf{37.70\ \Omega}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(100 \times 10^{-6})} = \mathbf{31.83\ \Omega}$$



Since X_L is greater than X_C the circuit is inductive.

$$X_L - X_C = 37.70 - 31.83 = 5.87\ \Omega$$

$$\text{Impedance } Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{[(5)^2 + (5.87)^2]}$$
$$= 7.71\ \Omega$$

(a) Current $I = \dfrac{V}{Z} = \dfrac{300}{7.71} = \mathbf{38.91\ A}$

(b) Phase angle $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\dfrac{5.87}{5}$
$$= 49.58^\circ$$
$$= \mathbf{49^\circ 35'}$$

(c) Impedance of coil, Z_{COIL}
$$= \sqrt{(R^2 + X_L^2)} = \sqrt{[(5)^2 + (37.70)^2]} = 38.03\ \Omega$$

$$\text{Voltage across coil } V_{\text{COIL}} = IZ_{\text{COIL}} = (38.91)(38.03)$$

$$= \mathbf{1480\,V}$$

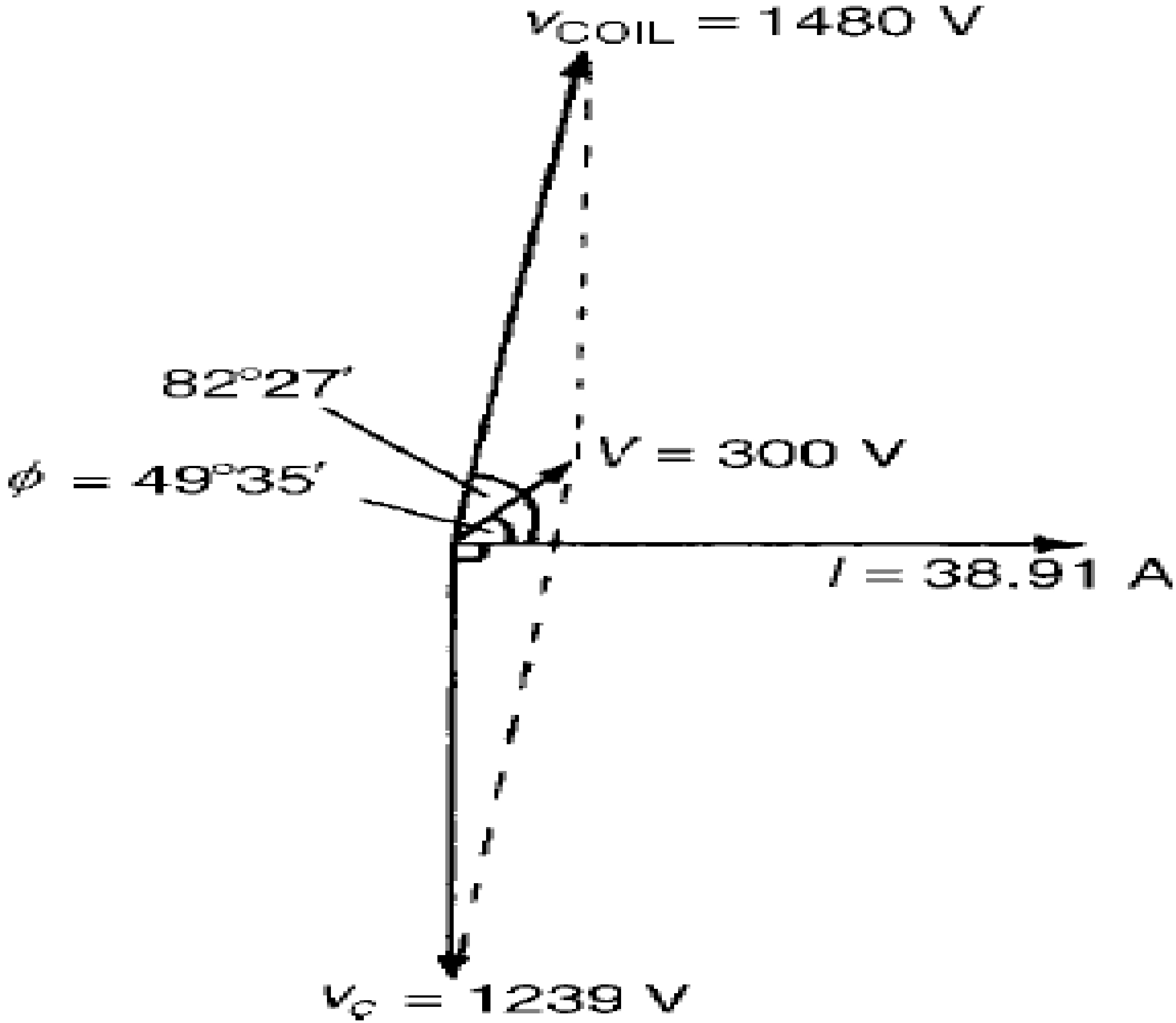
$$\text{Phase angle of coil} = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{37.70}{5}\right)$$

$$= 82.45^\circ$$

$$= 82^\circ 27' \text{ lagging}$$

(d) $$\text{Voltage across capacitor } V_C = IX_C = (38.91)(31.83)$$

$$= \mathbf{1239\,V}$$



$V_{COIL} = 1480$ V
82°27′
$V = 300$ V
$\phi = 49°35′$
$I = 38.91$ A
$V_C = 1239$ V

Thank you!

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ELECTRICAL ENGINEERING

BEEE101L

Basic Electrical Engineering

AC circuits

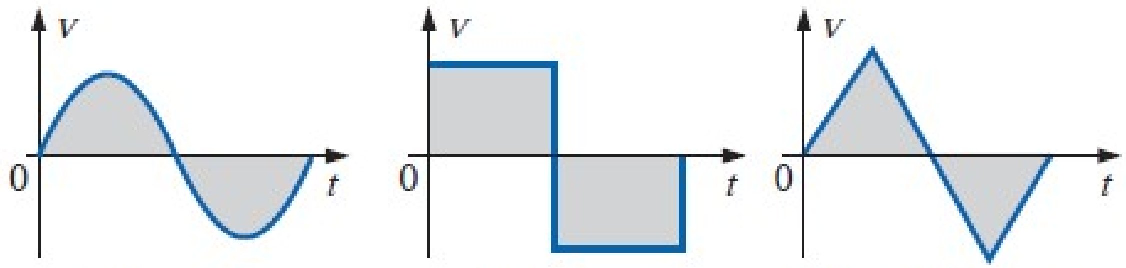
By,

Meera P. S.

Assistant Professor, SELECT

Alternating waveforms

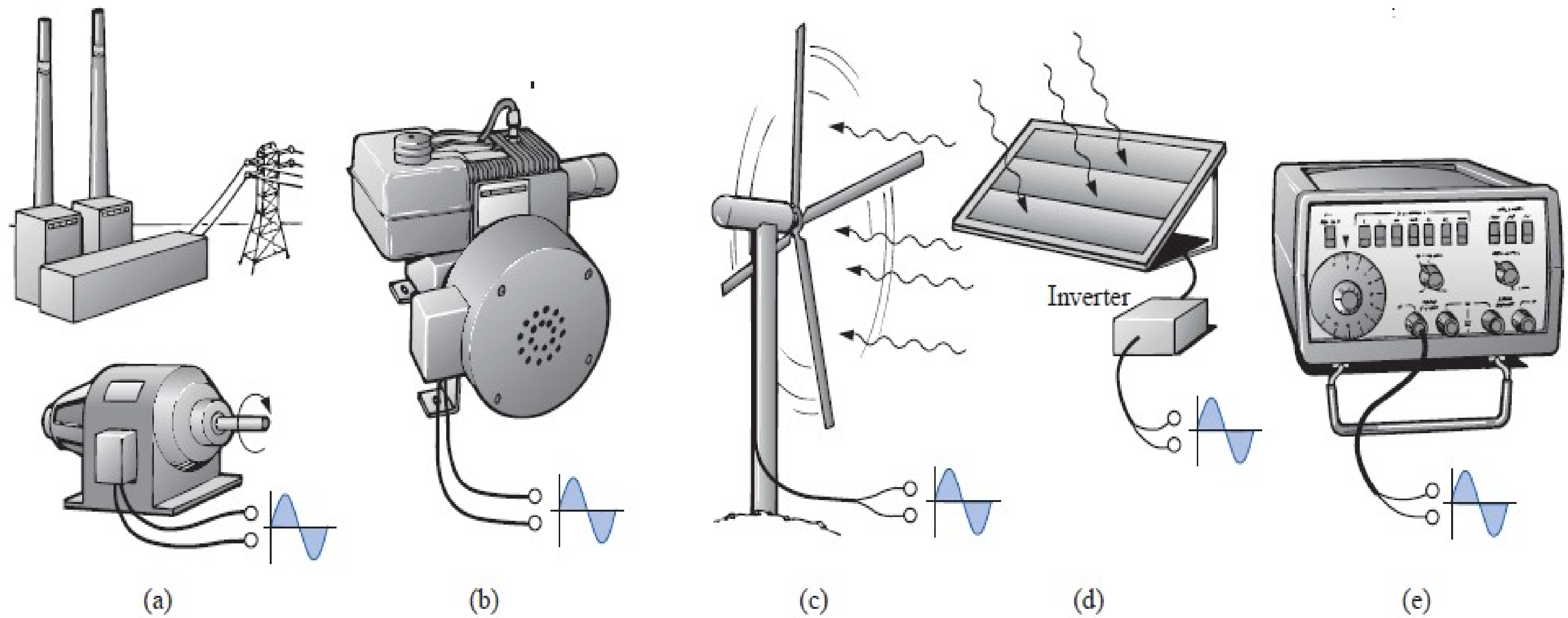
❖ The term **alternating** indicates only that the waveform alternates between two prescribed levels in a set time sequence.



Sinusoidal Square wave Triangular wave

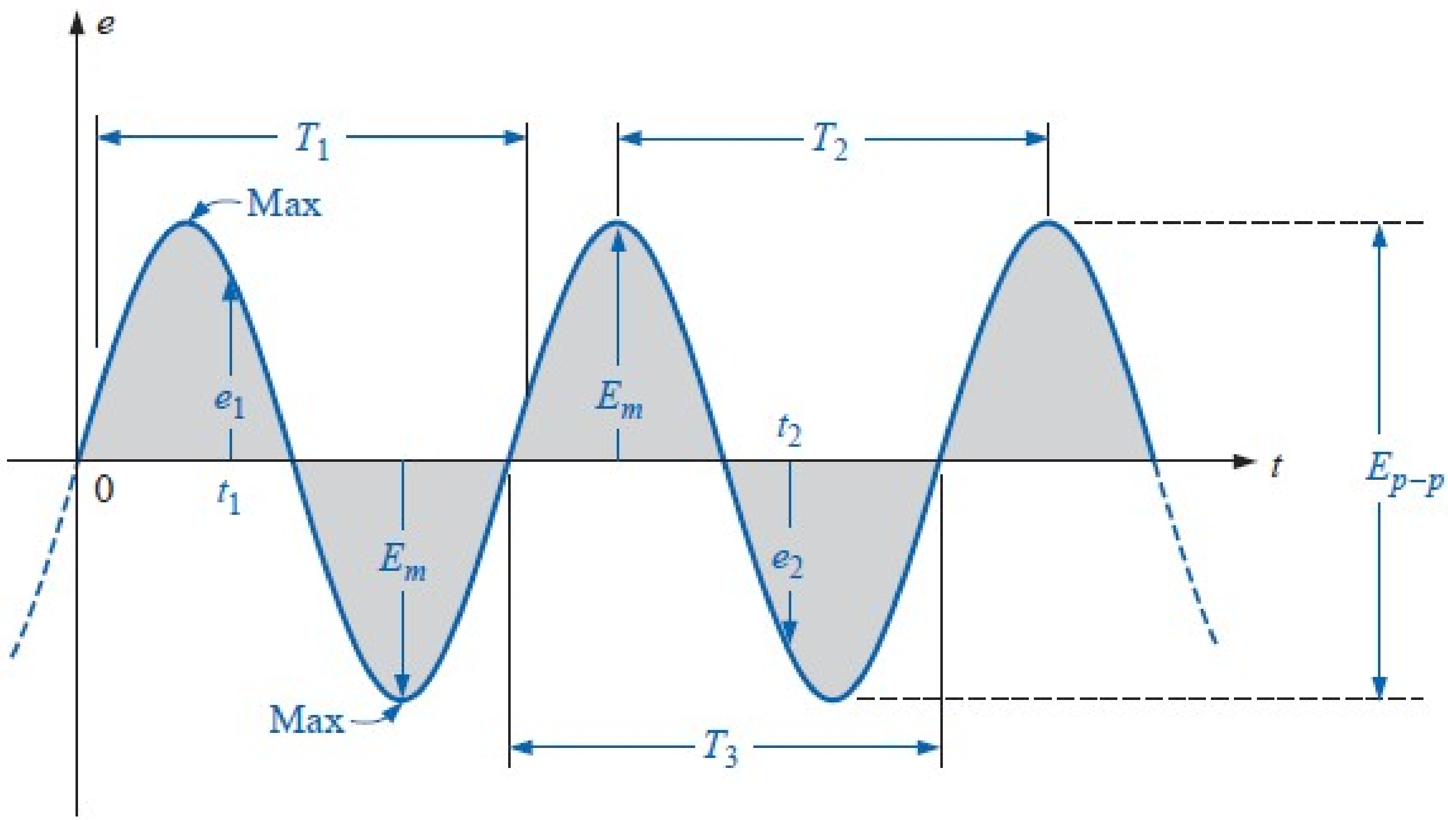
❖ One of the important reasons for concentrating on the sinusoidal ac voltage is that it is the voltage generated by utilities throughout the world. Other reasons include its application throughout electrical, electronic, communication, and industrial systems.

SINUSOIDAL AC VOLTAGE



Various sources of ac power: (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

Definitions



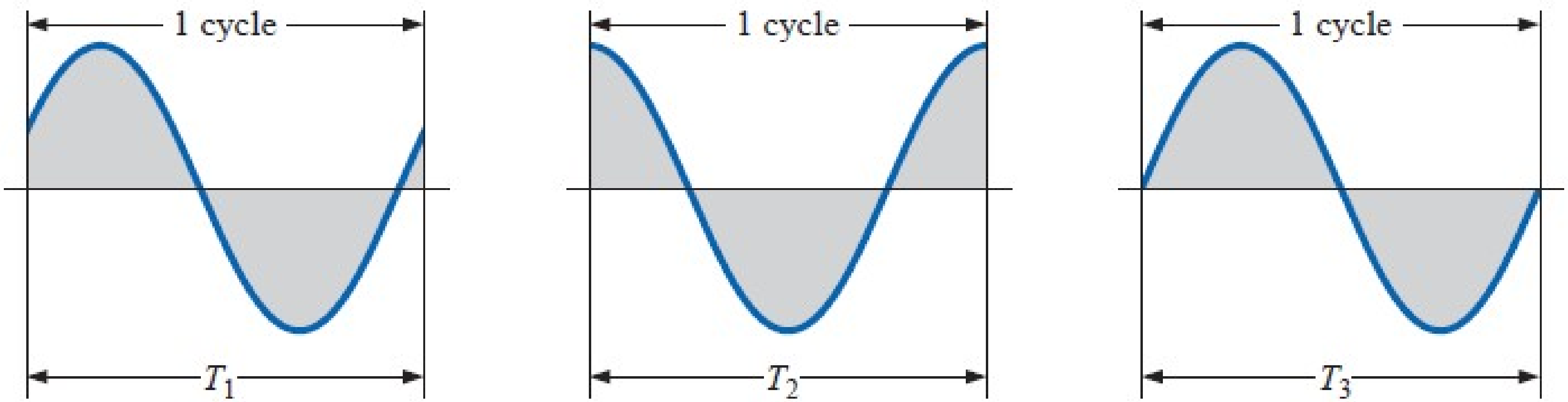
Definitions

- **Instantaneous value:** The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2)
- **Peak amplitude:** The maximum value of a waveform.
- **Peak-to-peak value:** Denoted by E_{p-p}, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.
- **Periodic waveform:** A waveform that continually repeats itself after the same time interval.

Definitions

❖ **Period (T):** The time interval between successive repetitions of a periodic waveform.

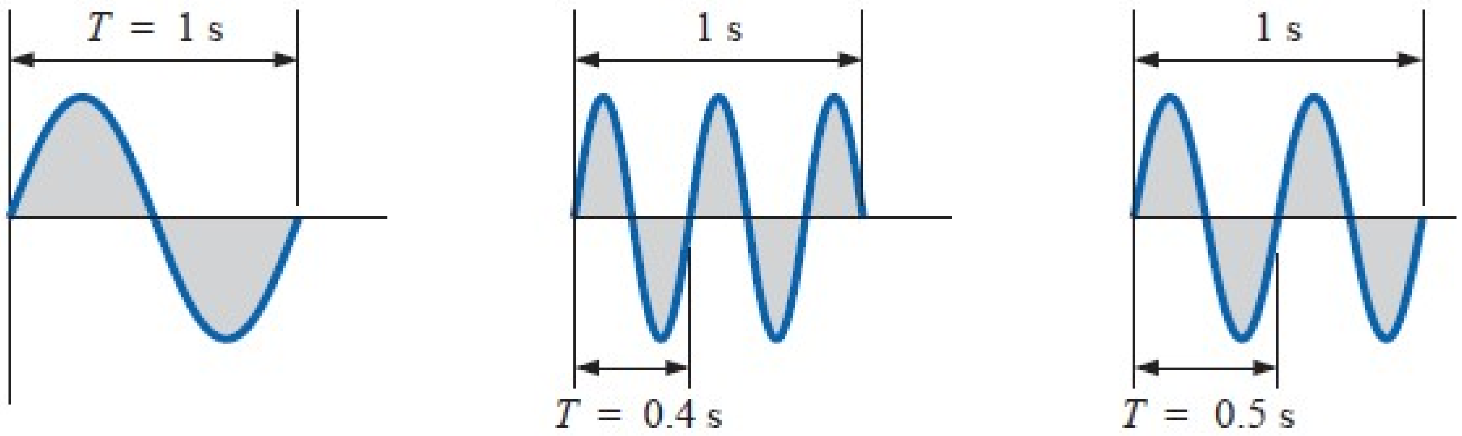
❖ **Cycle:** The portion of a waveform contained in one period of time.



Definitions

❖ **Frequency (f):** The number of cycles that occur in 1 s. The unit of measure for frequency is the hertz (Hz), where

$$\boxed{1 \text{ hertz (Hz)} = 1 \text{ cycle per second (c/s)}}$$

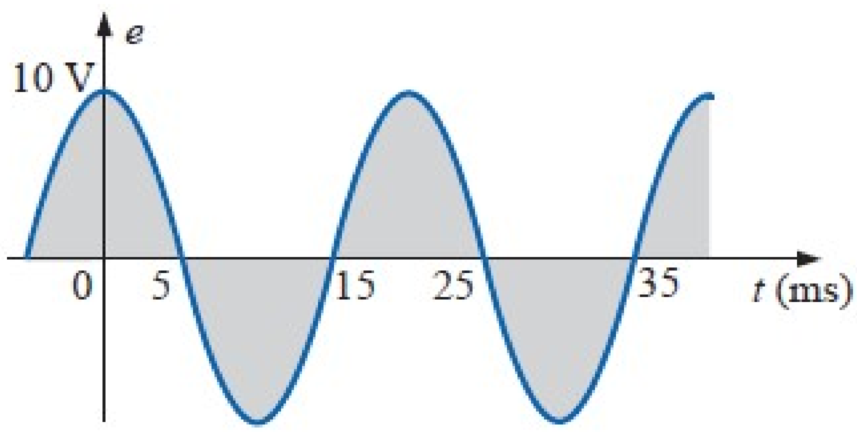


Definitions

$$\boxed{f = \frac{1}{T}}$$

f = Hz
T = seconds (s)

Determine the frequency of the waveform of Fig.



Sine Wave

The **sinusoidal waveform** *is the only alternating waveform whose shape is unaffected by the response characteristics of* ***R, L,*** *and* ***C*** *elements.*

$$2\pi \text{ rad} = 360^\circ$$

$$1 \text{ rad} = 57.296^\circ \cong 57.3^\circ$$

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})$$

Sine Wave

$$\mathbf{90°:} \quad \text{Radians} = \frac{\pi}{180°}(90°) = \frac{\pi}{2}\ \mathbf{rad}$$

$$\mathbf{30°:} \quad \text{Radians} = \frac{\pi}{180°}(30°) = \frac{\pi}{6}\ \mathbf{rad}$$

$$\frac{\pi}{3}\ \mathbf{rad:} \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60°}$$

$$\frac{3\pi}{2}\ \mathbf{rad:} \quad \text{Degrees} = \frac{180°}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270°}$$

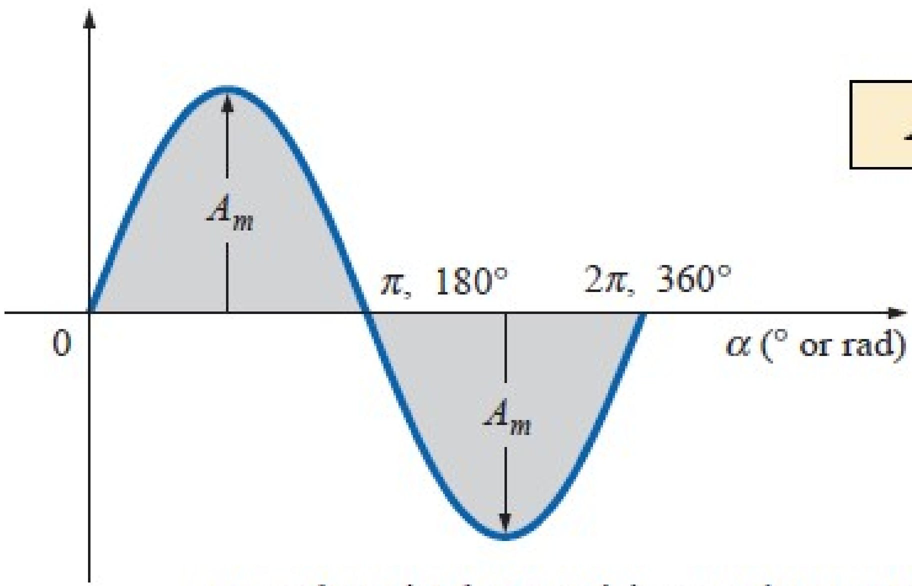
$$\boxed{\omega = \frac{2\pi}{T}} \quad \text{(rad/s)}$$

$$\boxed{\omega = 2\pi f} \quad \text{(rad/s)}$$

Given $\omega = 200$ rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

$$t = \frac{\alpha}{\omega}$$

$$t = \frac{\alpha}{\omega} = \frac{\pi/2 \text{ rad}}{200 \text{ rad/s}} = \frac{\pi}{400} \text{ s} = \mathbf{7.85 \text{ ms}}$$



$$\boxed{A_m \sin \alpha}$$

$$\boxed{A_m \sin \omega t}$$

$$e = E_m \sin \alpha$$

$$\boxed{\alpha = \sin^{-1} \frac{e}{E_m}}$$

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

EXAMPLE

a. Determine the angle at which the magnitude of the sinusoidal function $v = 10 \sin 377t$ is 4 V.

b. Determine the time at which the magnitude is attained.

$$\alpha_1 = \sin^{-1} \frac{v}{E_m} = \sin^{-1} \frac{4\text{ V}}{10\text{ V}} = \sin^{-1} 0.4 = \mathbf{23.578^\circ}$$

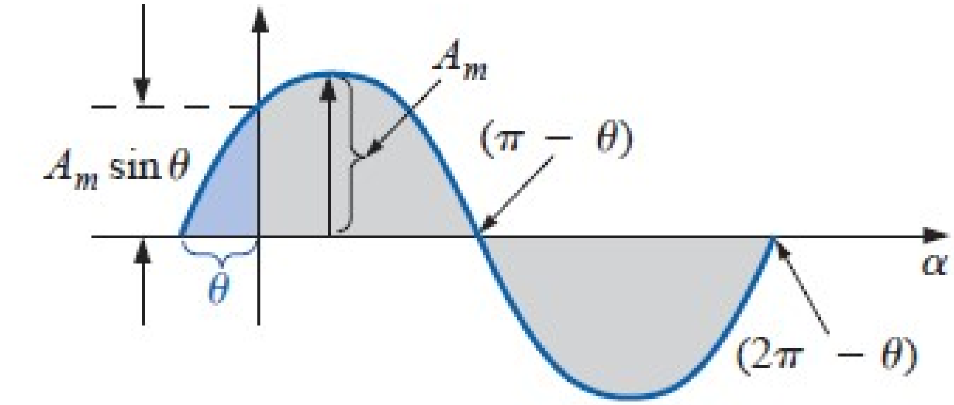
$$\alpha_2 = 180^\circ - 23.578^\circ = \mathbf{156.422^\circ}$$

$$\alpha\ (\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.411\text{ rad}$$

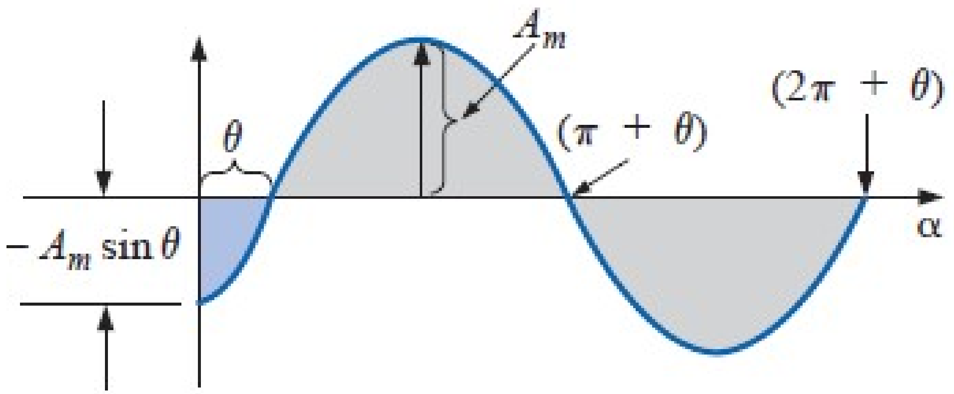
$$t_1 = \frac{\alpha}{\omega} = \frac{0.411\text{ rad}}{377\text{ rad/s}} = \mathbf{1.09\ ms}$$

$$\alpha\ (\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73\text{ rad}$$

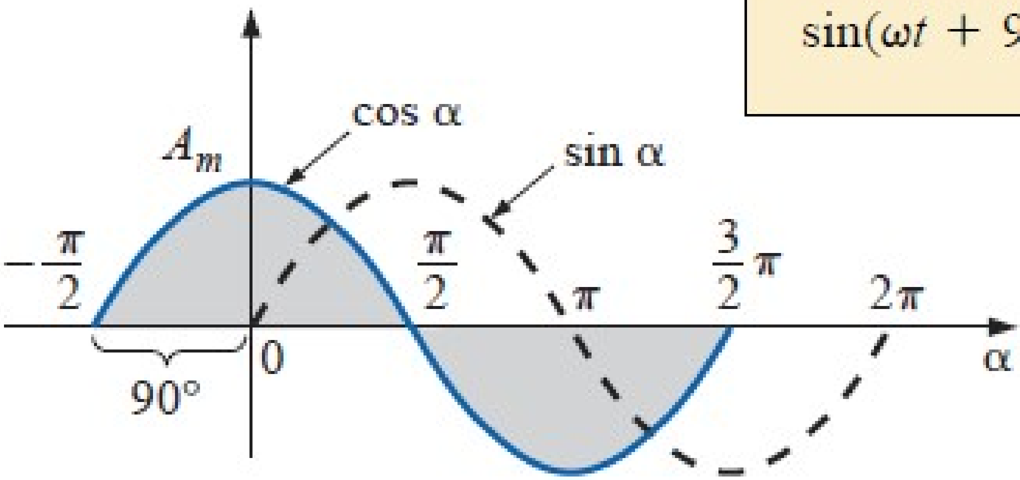
$$t_2 = \frac{\alpha}{\omega} = \frac{2.73\text{ rad}}{377\text{ rad/s}} = \mathbf{7.24\ ms}$$



$$\boxed{A_m \sin(\omega t + \theta)}$$



$$\boxed{A_m \sin(\omega t - \theta)}$$



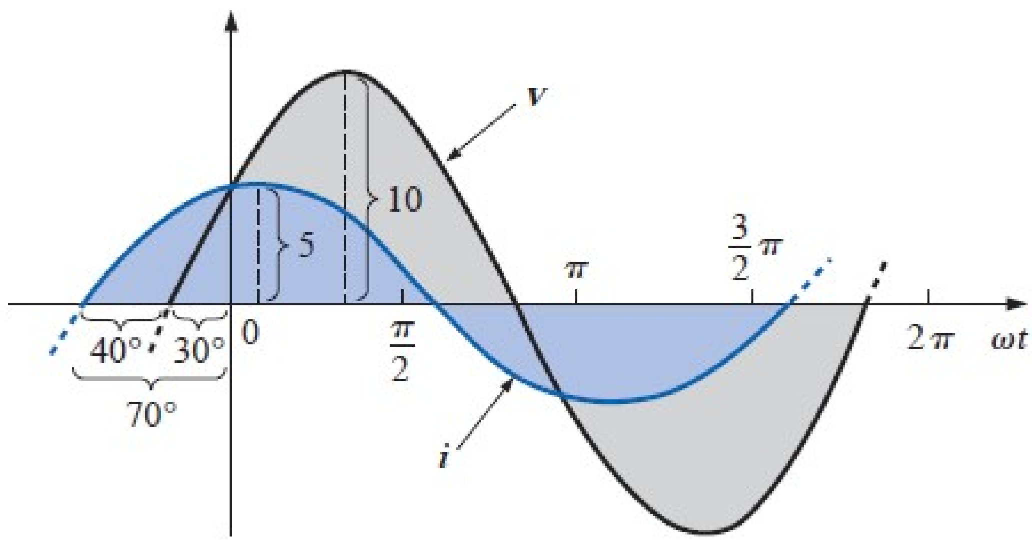
$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

The terms *lead* and *lag* are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes. In Fig., the cosine curve is said to *lead* the sine curve by 90°, and the sine curve is said to *lag* the cosine curve by 90°. Note that the phase angle between the two waveforms is measured between those two points on the horizontal axis through which each passes with the *same slope*. If both waveforms cross the axis at the same point with the same slope, they are *in phase*.

$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$



***i* leads *v* by 40°, or *v* lags *i* by 40°.**

$$v = 10\sin(\omega t + 30^\circ)$$
$$i = 5\sin(\omega t + 70^\circ)$$

BEEE 101L : Basic Electrical Engineering

Module 2- AC Circuits

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Power in three-phase systems

- The power dissipated in a three-phase load is given by the sum of the power dissipated in each phase.
- If a load is balanced then the total power P is given by:

  $P = 3 \times$ power consumed by one phase.
- The power consumed in one phase $= I_p^2 R_p$ or $V_p I_p \cos\varphi$

  where φ is the phase angle between V_p and I_p

For a star connection, $V_p = \dfrac{V_L}{\sqrt{3}}$ and $I_p = I_L$ hence

$$P = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

For a delta connection, $V_p = V_L$ and $I_p = \dfrac{I_L}{\sqrt{3}}$ hence

$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

Hence for either a star or a delta balanced connection the total power P is given by:

$$P = \sqrt{3}\, V_L I_L \cos\phi \text{ watts} \quad \text{or} \quad P = 3I_p^2 R_p \text{ watts.}$$

Total volt-amperes, $S = \sqrt{3}\, V_L I_L$ volt-amperes

Q1. Three identical coils, each of resistance 10 Ω and inductance 42mH are connected (a) in star and (b) in delta to a 415V, 50Hz, 3-phase supply. Determine the total power dissipated in each case.

(a) Star connection

$$\text{Inductive reactance } X_L = 2\pi f L$$
$$= 2\pi(50)(42 \times 10^{-3})$$
$$= 13.19\,\Omega$$

$$\text{Phase impedance } Z_p = \sqrt{(R^2 + X_L^2)}$$
$$= \sqrt{(10^2 + 13.19^2)}$$
$$= 16.55\,\Omega$$

$$\text{Line voltage } V_L = 415\text{ V and}$$

$$\text{phase voltage, } V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240\text{ V}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = 14.50\,\text{A}$$

$$\text{Line current, } I_L = I_p = 14.50\,\text{A}$$

$$\text{Power factor} = \cos\phi = \frac{R_p}{Z_p} = \frac{10}{16.55}$$
$$= 0.6042 \text{ lagging}$$

$$\textbf{Power dissipated, } P = \sqrt{3}V_L I_L \cos\phi$$
$$= \sqrt{3}(415)(14.50)(0.6042)$$
$$= \mathbf{6.3\,kW}$$

$$\text{(Alternatively, } P = 3I_p^2 R_p = 3(14.50)^2(10)$$
$$= \mathbf{6.3\,kW}\text{)}$$

b. Delta connection

$$V_L = V_p = 415\,\text{V},\ Z_p = 16.55\,\Omega,$$
$$\cos\phi = 0.6042 \text{ lagging (from above).}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08\,\text{A}$$

$$\text{Line current, } I_L = \sqrt{3}I_p = \sqrt{3}(25.08)$$
$$= 43.44\,\text{A}$$

$$\textbf{Power dissipated, } P = \sqrt{3}V_L I_L \cos\phi$$
$$= \sqrt{3}(415)(43.44)(0.6042)$$
$$= \mathbf{18.87\,kW}$$

$$\text{(Alternatively, } P = 3I_p^2 R_p = 3(25.08)^2(10)$$
$$= \mathbf{18.87\,kW})$$

Q2. The input power to a 3-phase a.c. motor is measured as 5 kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system.

Power, $P = 5000\,\text{W}$; Line voltage $V_L = 400\,\text{V}$; Line current, $I_L = 8.6\,\text{A}$

Power, $P = \sqrt{3} V_L I_L \cos\phi$

Hence power factor $= \cos\phi = \dfrac{P}{\sqrt{3} V_L I_L}$

$$= \frac{5000}{\sqrt{3}(400)(8.6)}$$

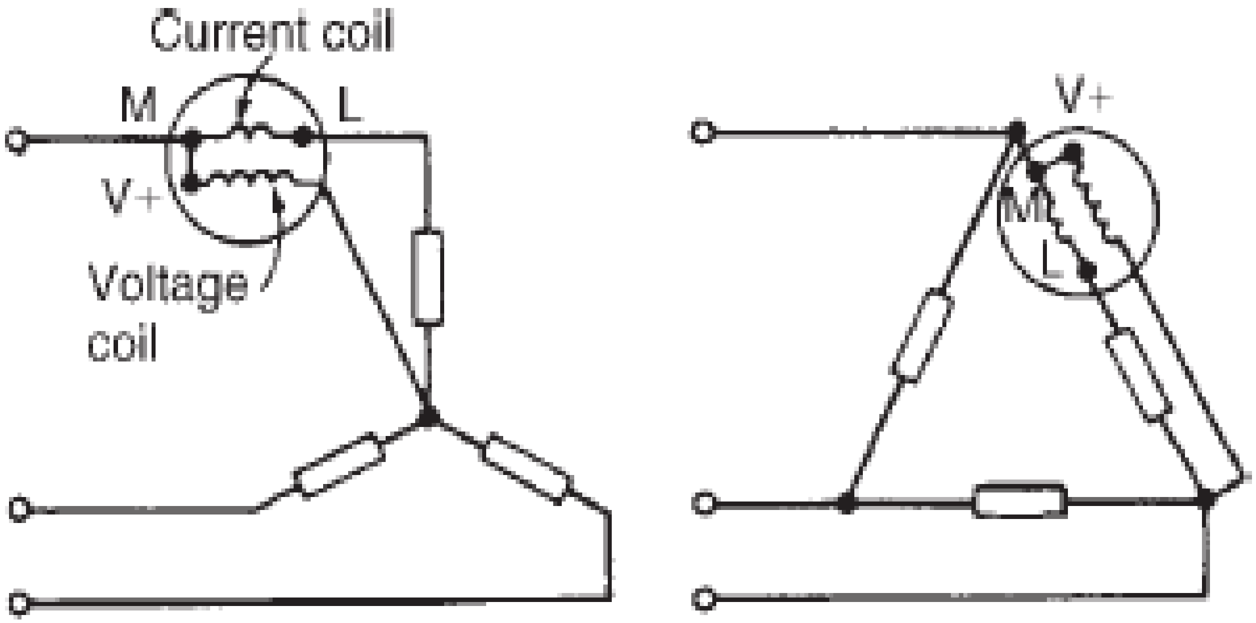
$$= \mathbf{0.839}$$

Measurement of power in three-phase systems

- Power in three-phase loads may be measured by the following methods
  - **One-wattmeter method for a balanced load**
  - **Two-wattmeter method for balanced or unbalanced loads**
  - **Three-wattmeter method for a three-phase, 4-wire system for balanced and unbalanced loads**

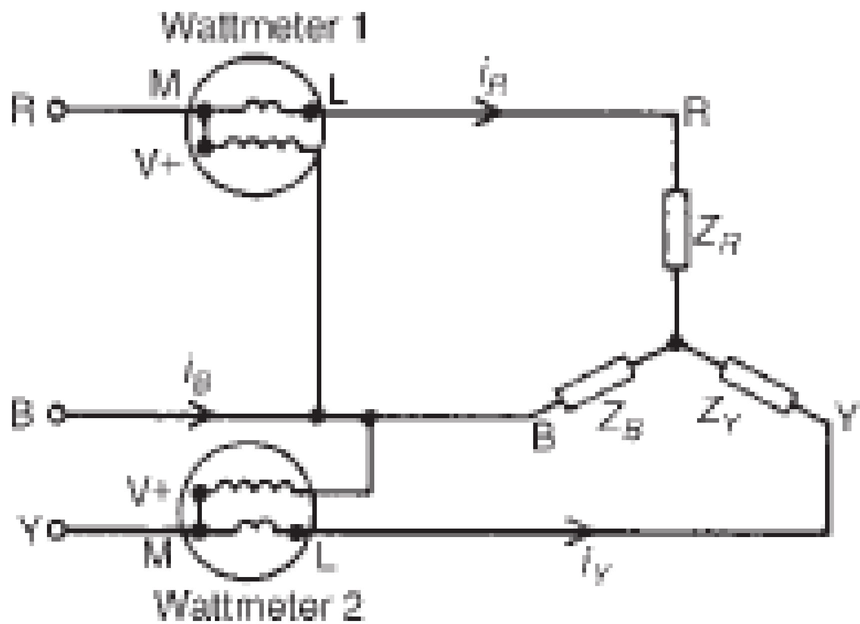
One-wattmeter method

Total power = 3 x wattmeter reading



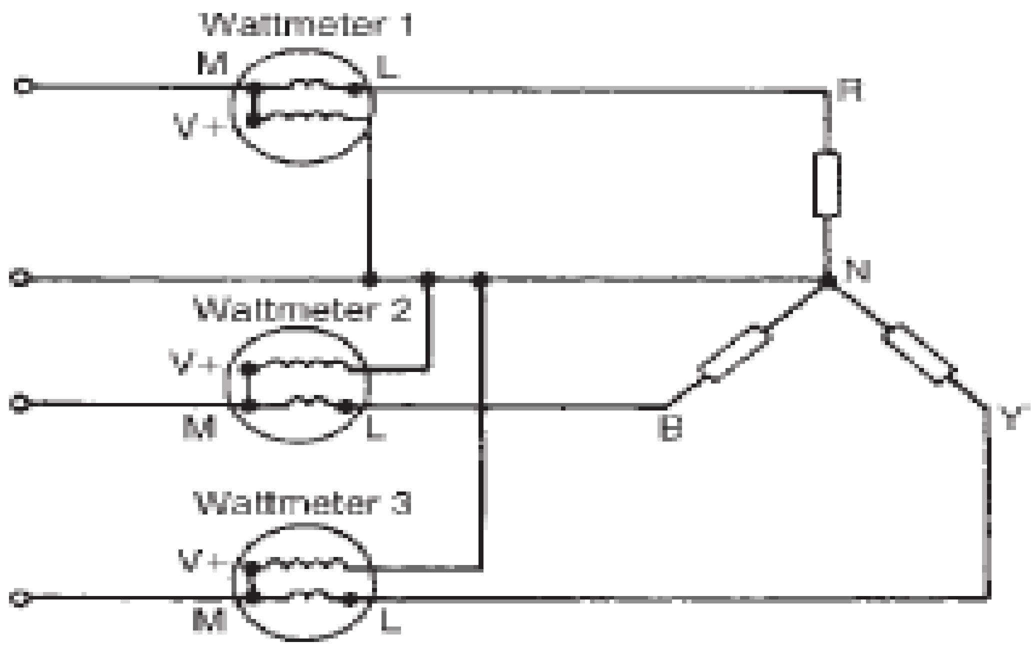
Two-wattmeter method

$$\text{Total power} = \text{sum of wattmeter readings}$$
$$= P_1 + P_2$$



Three-wattmeter method

$$\text{Total power} = P_1 + P_2 + P_3$$



Two-wattmeter method

Total instantaneous power, $p = e_R i_R + e_Y i_Y + e_B i_B$ and in any 3-phase system $i_R + i_Y + i_B = 0$. Hence $i_B = -i_R - i_Y$

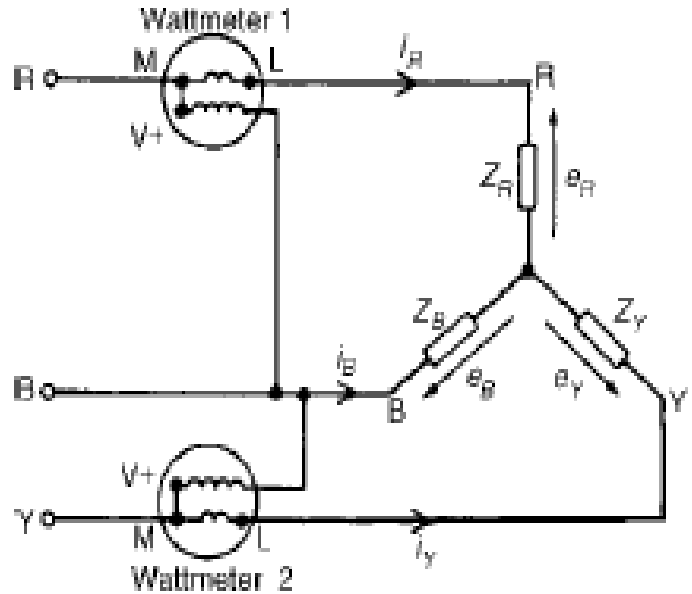
Thus, $p = e_R i_R + e_Y i_Y + e_B(-i_R - i_Y)$

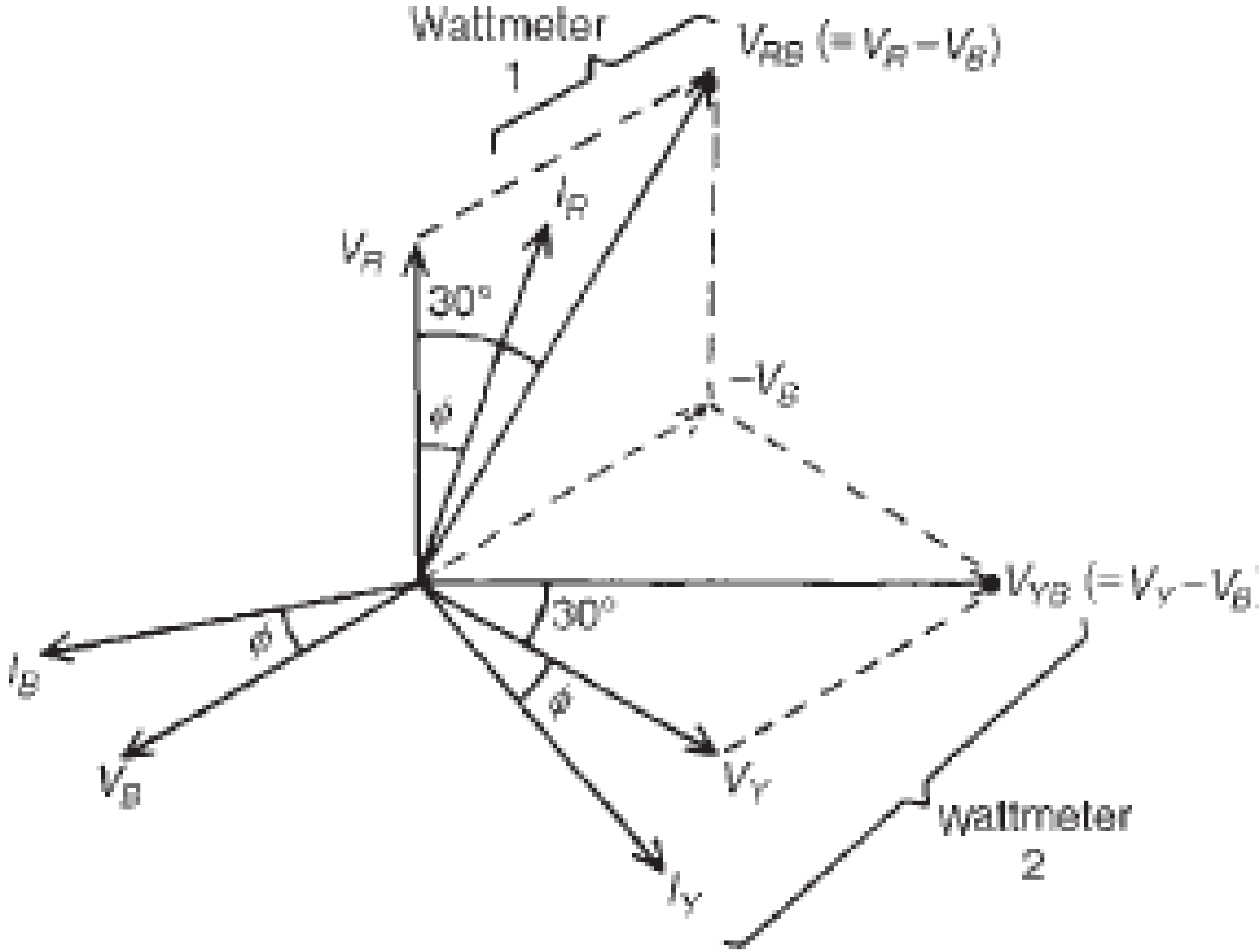
$$= (e_R - e_B) i_R + (e_Y - e_B) i_Y$$

Hence total instantaneous power,

$p = $(wattmeter 1 reading)$+$(wattmeter 2 reading)

$= p_1 + p_2$





Wattmeter
1
V_{RB} (= $V_R - V_B$)
I_R
V_R
30°
ϕ
$-V_B$
V_{YB} (= $V_Y - V_B$)
30°
ϕ
I_B
ϕ
V_B
V_Y
wattmeter
2
I_Y

Wattmeter 1 reads $V_{RB} I_R \cos(30° - \phi) = P_1$

Wattmeter 2 reads $V_{YB} I_Y \cos(30° + \phi) = P_2$

$$\frac{P_1}{P_2} = \frac{V_{RB} I_R \cos(30° - \phi)}{V_{YB} I_Y \cos(30° + \phi)} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

since $I_R = I_Y$ and $V_{RB} = V_{YB}$ for a balanced load.

Hence $\dfrac{P_1}{P_2} = \dfrac{\cos 30° \cos\phi + \sin 30° \sin\phi}{\cos 30° \cos\phi - \sin 30° \sin\phi}$

Dividing throughout by $\cos 30° \cos\phi$ gives:

$$\frac{P_1}{P_2} = \frac{1 + \tan 30° \tan\phi}{1 - \tan 30° \tan\phi}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}\tan\phi}{1 - \frac{1}{\sqrt{3}}\tan\phi} \quad \left(\text{since } \frac{\sin\phi}{\cos\phi} = \tan\phi\right)$$

Cross-multiplying gives:

$$P_1 - \frac{P_1}{\sqrt{3}} \tan \phi = P_2 + \frac{P_2}{\sqrt{3}} \tan \phi$$

Hence $$P_1 - P_2 = (P_1 + P_2)\frac{\tan \phi}{\sqrt{3}}$$

from which $$\tan \phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right)$$

ϕ, $\cos \phi$ and thus power factor can be determined from this formula.

Q3. Each phase of a delta-connected load comprises a resistance of 30 Ω and an 80μF capacitor in series. The load is connected to a 400V, 50Hz, 3-phase supply. Calculate (a) the phase current, (b) the line current, (c) the total power dissipated and (d) the kVA rating of the load. Draw the complete phasor diagram for the load.

(a) Capacitive reactance, $X_C = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2\pi(50)(80 \times 10^{-6})}$$

$$= 39.79\,\Omega$$

Phase impedance, $Z_p = \sqrt{(R_p^2 + X_C^2)}$

$$= \sqrt{(30^2 + 39.79^2)}$$

$$= 49.83\,\Omega$$

$$\text{Power factor} = \cos\phi = \frac{R_p}{Z_p} = \frac{30}{49.83} = 0.602$$

Hence $\phi = \cos^{-1} 0.602 = 52.99°$ leading.

Phase current, $I_p = \dfrac{V_p}{Z_p}$ and $V_p = V_L$ for a delta connection

Hence $$I_p = \frac{400}{49.83} = \mathbf{8.027\,A}$$

(b) Line current $I_L = \sqrt{3} I_p$ for a delta connection

Hence $$I_L = \sqrt{3}(8.207) = \mathbf{13.90\,A}$$

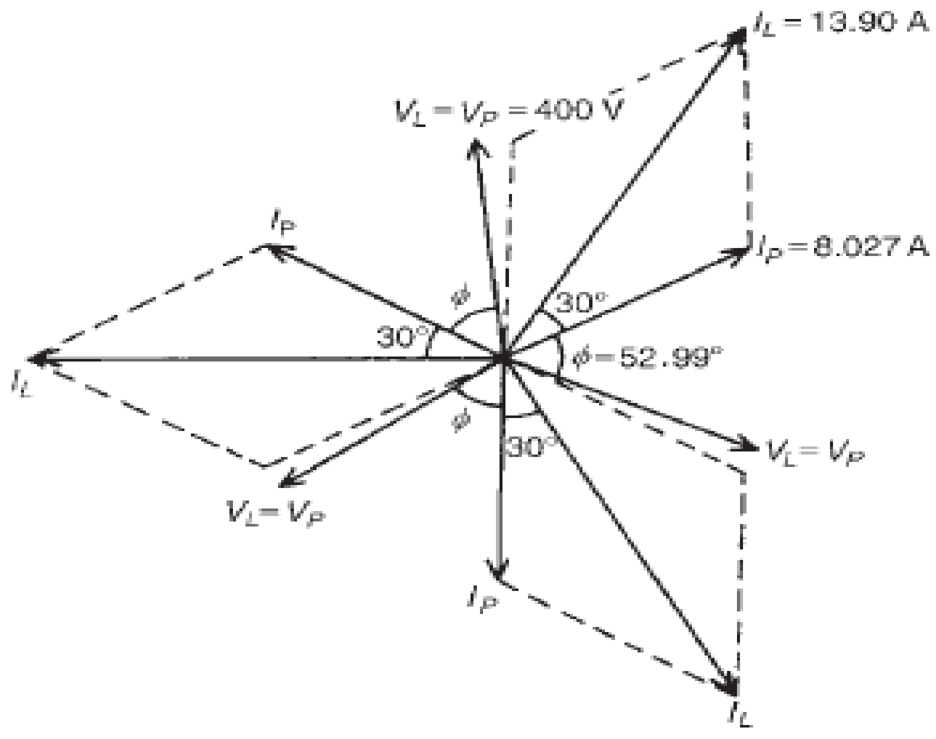
(c) Total power dissipated, $P = \sqrt{3} V_L I_L \cos\phi$

$$= \sqrt{3}(400)(13.90)(0.602)$$

$$= \mathbf{5.797\,kW}$$

(d) Total kVA, $S = \sqrt{3} V_L I_L = \sqrt{3}(400)(13.90)$

$$= \mathbf{9.630\,kVA}$$



Q4. Two wattmeters connected to a 3-phase motor indicate the total power input to be 12kW. The power factor is 0.6. Determine the readings of each wattmeter.

If the two wattmeters indicate P_1 and P_2 respectively

then $P_1 + P_2 = 12\,\text{kW}$ (1)

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) \text{ and power factor} = 0.6 = \cos\phi$$

Angle $\phi = \cos^{-1} 0.6 = 53.13°$ and $\tan 53.13° = 1.3333$

Hence $1.3333 = \dfrac{\sqrt{3}(P_1 - P_2)}{12}$, from which,

$$P_1 - P_2 = \frac{12(1.3333)}{\sqrt{3}}$$

i.e. $P_1 - P_2 = 9.237\,\text{kW}$

Adding equations (1) and (2) gives: $2P_1 = 21.237$

i.e. $$P_1 = \frac{21.237}{2} = 10.62\,\text{kW}$$

Hence **wattmeter 1 reads 10.62 kW**

From equation (1), **wattmeter 2 reads $(12 - 10.62) = 1.38\,\text{kW}$**

Q5 .Three similar coils, each having a resistance of 8 Ω and an inductive reactance of 8Ω are connected (a) in star and (b) in delta, across a 415V, 3-phase supply. Calculate for each connection the readings on each of two wattmeters connected to measure the power by the two-wattmeter method.

(a) Star connection: $V_L = \sqrt{3}V_p$ and $I_L = I_p$

Phase voltage, $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{415}{\sqrt{3}}$ and

phase impedance, $Z_p = \sqrt{(R_p^2 + X_L^2)}$

$= \sqrt{(8^2 + 8^2)} = 11.31\,\Omega$

Hence phase current, $I_p = \dfrac{V_p}{Z_p} = \dfrac{415/\sqrt{3}}{11.31}$

$= 21.18\,\text{A}$

Total power, $P = 3I_p^2 R_p = 3(21.18)^2(8)$

$= 10766\,\text{W}$

If wattmeter readings are P_1 and P_2 then

$$P_1 + P_2 = 10766 \qquad (1)$$

Since $R_p = 8\,\Omega$ and $X_L = 8\,\Omega$, then phase angle $\phi = 45°$ (from impedance triangle)

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right), \text{ hence}$$

$$\tan 45° = \frac{\sqrt{3}(P_1 - P_2)}{10\,766}$$

from which $P_1 - P_2 = \dfrac{10\,766(1)}{\sqrt{3}} = 6216\,\text{W} \quad (2)$

Adding equations (1) and (2) gives:

$$2P_1 = 10\,766 + 6216$$
$$= 16\,982\,\text{W}$$

Hence $P_1 = 8491\,\text{W}$

From equation (1), $P_2 = 10\,766 - 8491 = 2275\,\text{W}$

When the coils are star-connected the wattmeter readings are thus 8.491 kW and 2.275 kW.

(b) **Delta connection:** $V_L = V_p$ **and** $I_L = \sqrt{3} I_p$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{11.31} = 36.69\,\text{A}$$

Total power, $P = 3I_p^2 R_p = 3(36.69)^2(8) = 32\,310\,\text{W}$

Hence $\quad P_1 + P_2 = 32\,310\,\text{W} \quad (3)$

$$\tan\phi = \sqrt{3}\left(\frac{P_1 - P_2}{P_1 + P_2}\right) \text{ thus } 1 = \frac{\sqrt{3}(P_1 - P_2)}{32\,310}$$

$$\text{from which, } P_1 - P_2 = \frac{32\,310}{\sqrt{3}} = 18\,650\,\text{W} \qquad (4)$$

Adding equations (3) and (4) gives:

$2P_1 = 50\,960$, from which $P_1 = 25\,480\,\text{W}$

From equation (3), $P_2 = 32\,310 - 25\,480 = 6830\,\text{W}$

When the coils are delta-connected the wattmeter readings are thus 25.48 kW and 6.83 kW.

Q6 .Three inductive loads, each of resistance 4Ω and reactance 9 Ω are connected in delta. When connected to a 3-phase supply the loads consume 1.2kW. Calculate (a) the power factor of the load, (b) the phase current, (c) the line current and (d) the supply voltage.

Impedance = Z = 4 +j 9 Ω

$Z = 9.848 \angle 66.03^\circ$

Power factor of the load = cos φ = cos (66.03°) = 0.406 (lag)

Power consumed by the load, P= 1.2kW = 1200 W

$P = 3\, I_p^2\, R_p = 1200$

$I_p = 10\ A$

$P = 3\, V_p\, I_p \cos\varphi = 1200$

$V_p = 98.53\ V$

Thank you!

SCHOOL OF ELECTRICAL ENGINEERING

BEEE101L

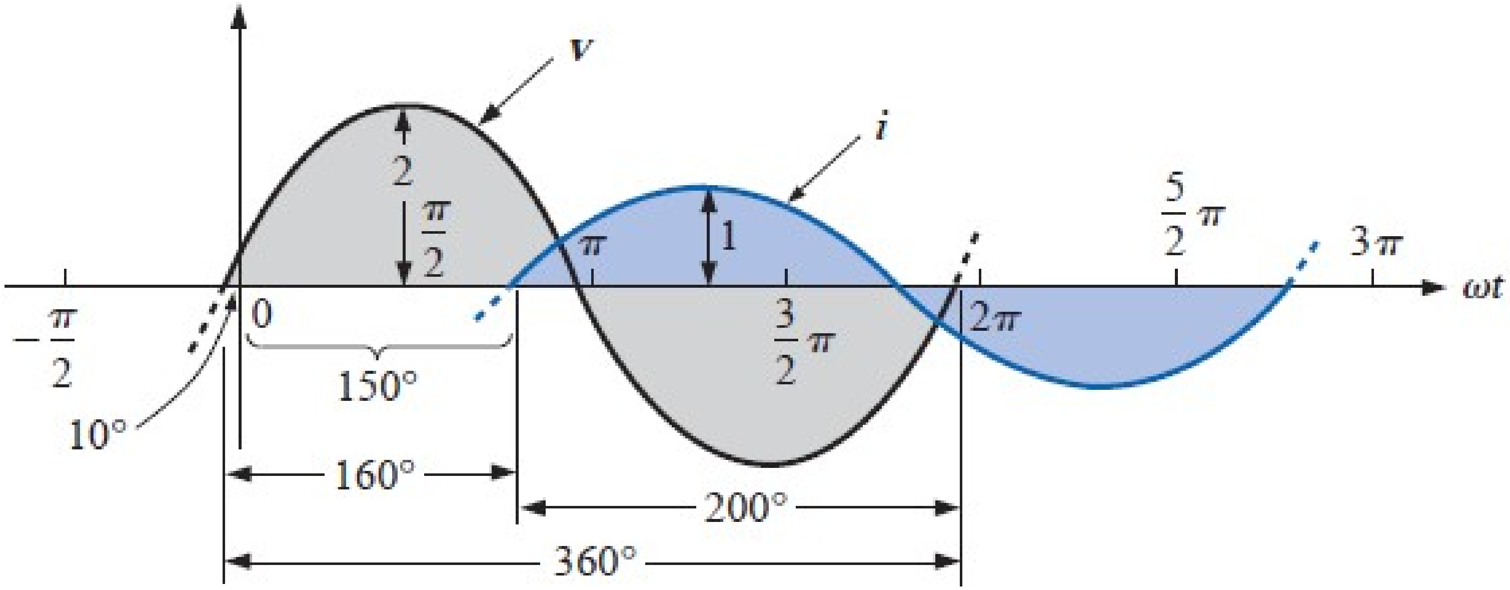
Basic Electrical Engineering

AC circuits

By,

Meera P. S.

Assistant Professor, SELECT

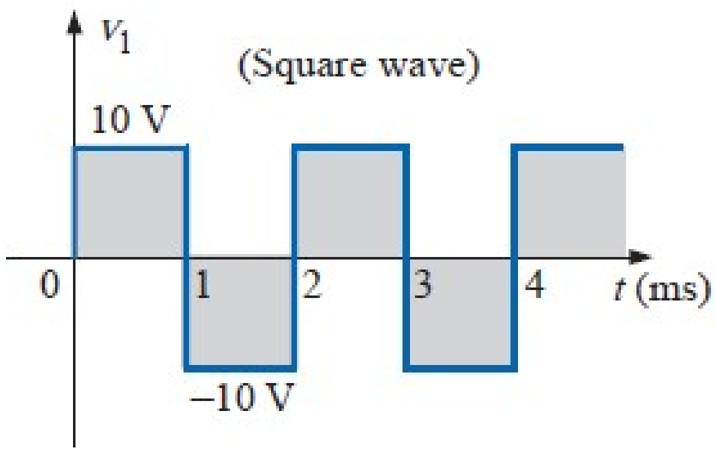


v leads i by 160°, or i lags v by 160°.

$$i = -\sin(\omega t + 30^\circ)$$
$$v = 2\sin(\omega t + 10^\circ)$$

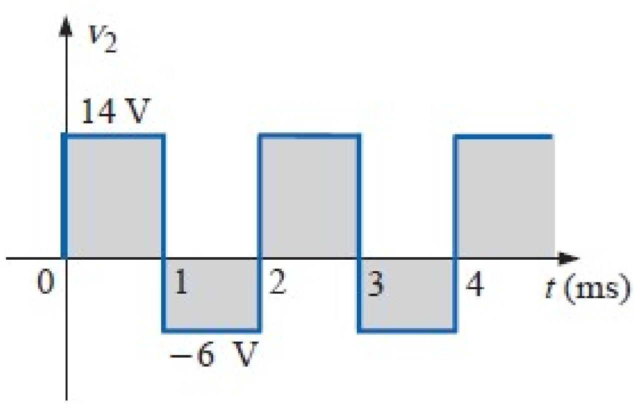
AVERAGE VALUE

$$G \text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$



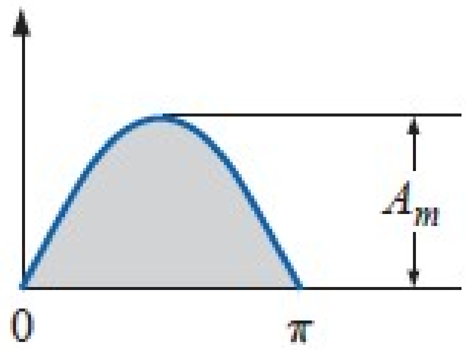
$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$



$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

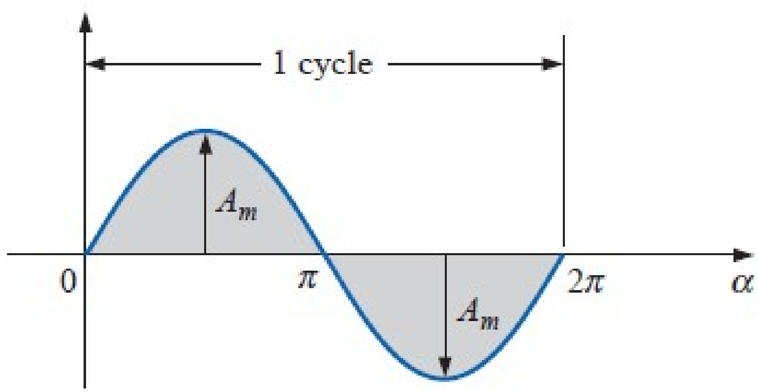


$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

$$\begin{aligned}\text{Area} &= A_m[-\cos \alpha]_0^{\pi} \\ &= -A_m(\cos \pi - \cos 0°) \\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$G = \frac{2A_m}{\pi}$$

$$\boxed{G = 0.637A_m}$$



$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

EFFECTIVE (rms) VALUES

- The amplitude of a sinusoidal ac current required to deliver the same power as a particular dc current.

The power delivered by the ac supply at any instant of time is

$$P_{\mathrm{ac}} = (i_{\mathrm{ac}})^2 R = (I_m \sin \omega t)^2 R = (I_m^2 \sin^2 \omega t) R$$

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\boxed{P_{\mathrm{ac}} = \frac{I_m^2 R}{2} - \frac{I_m^2 R}{2} \cos 2\omega t}$$

EFFECTIVE (rms) VALUES

$$P_{\mathrm{av(ac)}} = P_{\mathrm{dc}}$$

$$\frac{I_m^2 R}{2} = I_{\mathrm{dc}}^2 R \quad \text{and} \quad I_m = \sqrt{2} I_{\mathrm{dc}}$$

$$I_{\mathrm{dc}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

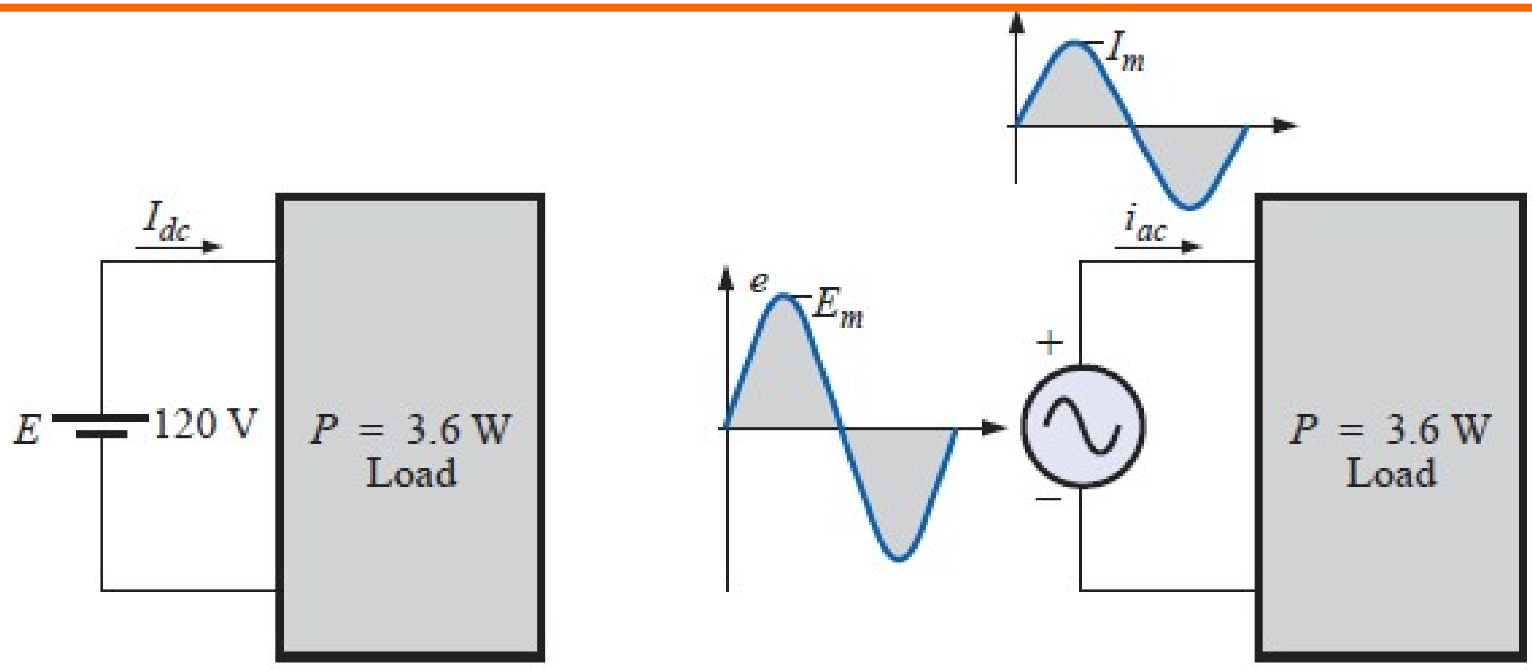
the equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ or 0.707 of its maximum value.

$$\boxed{I_{\mathrm{eq(dc)}} = I_{\mathrm{eff}} = 0.707 I_m}$$

$$\boxed{I_m = \sqrt{2} I_{\mathrm{eff}} = 1.414 I_{\mathrm{eff}}}$$

$$I_{\text{eff}} = \sqrt{\frac{\int_0^T i^2(t)\,dt}{T}}$$

$$I_{\text{eff}} = \sqrt{\frac{\text{area }(i^2(t))}{T}}$$

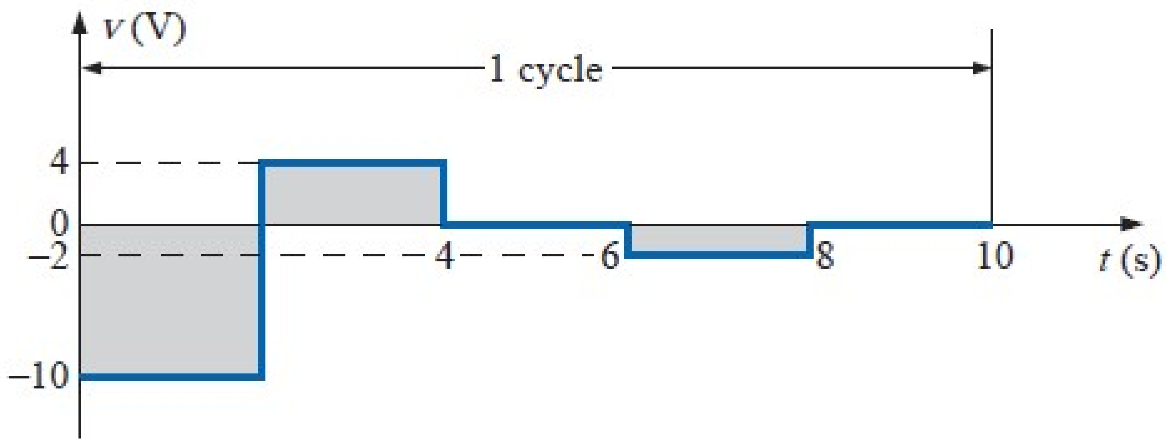


$$P_{dc} = V_{dc} I_{dc}$$

$$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

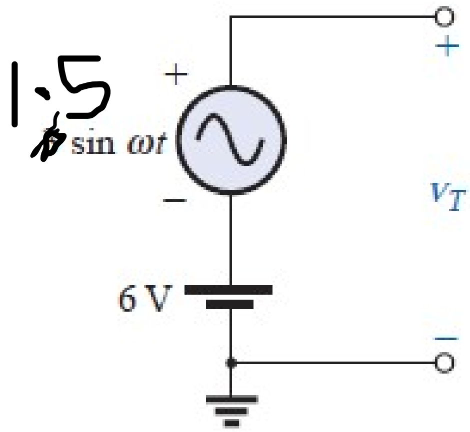
$$I_m = \sqrt{2} I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\ mA}$$

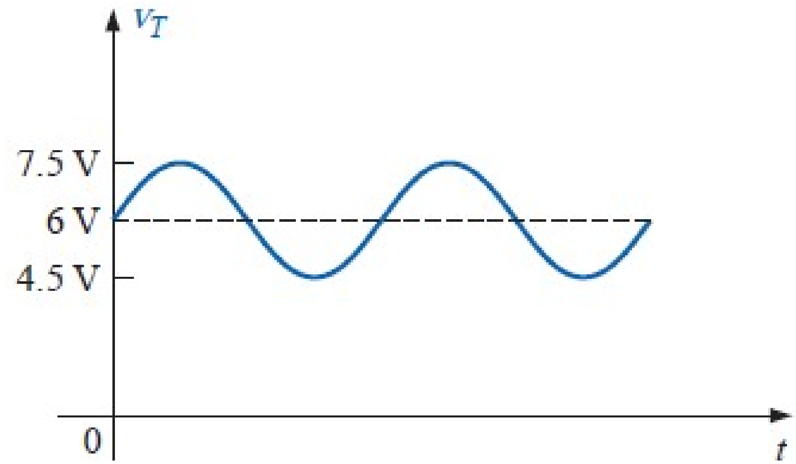
$$E_m = \sqrt{2} E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\ V}$$



$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}} = \sqrt{\frac{240}{10}}$$

$$= \mathbf{4.899\ V}$$

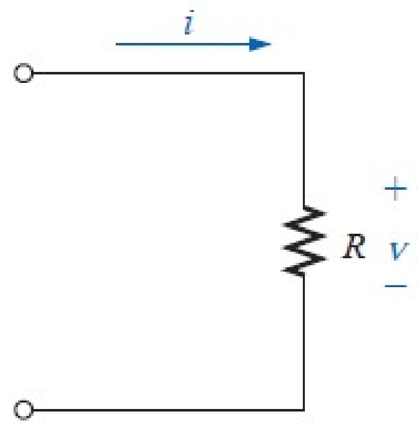




$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^2 + V_{\text{ac(rms)}}^2}$$

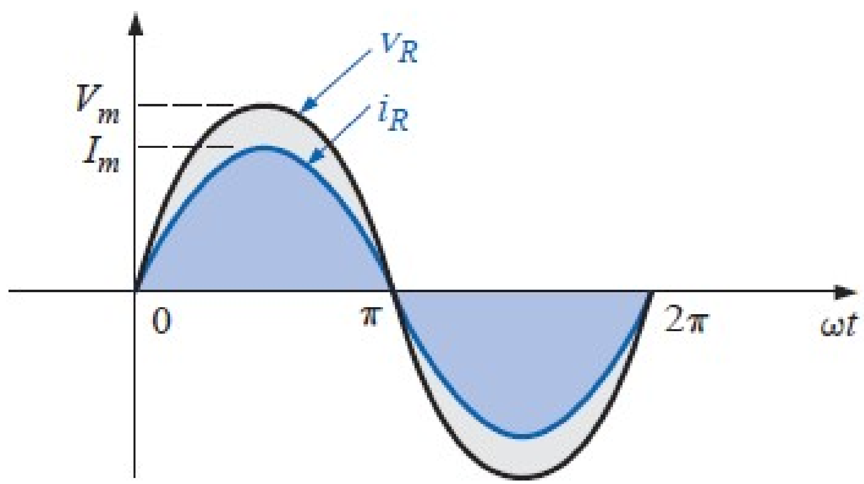
$$\begin{aligned} V_{\text{rms}} &= \sqrt{(6\text{ V})^2 + (1.06\text{ V})^2} \\ &= \sqrt{37.124}\text{ V} \\ &\cong \mathbf{6.1\ V} \end{aligned}$$

Resistor



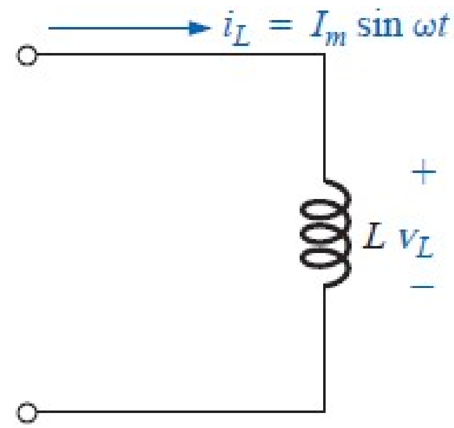
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$\boxed{I_m = \frac{V_m}{R}}$$



for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

Inductor



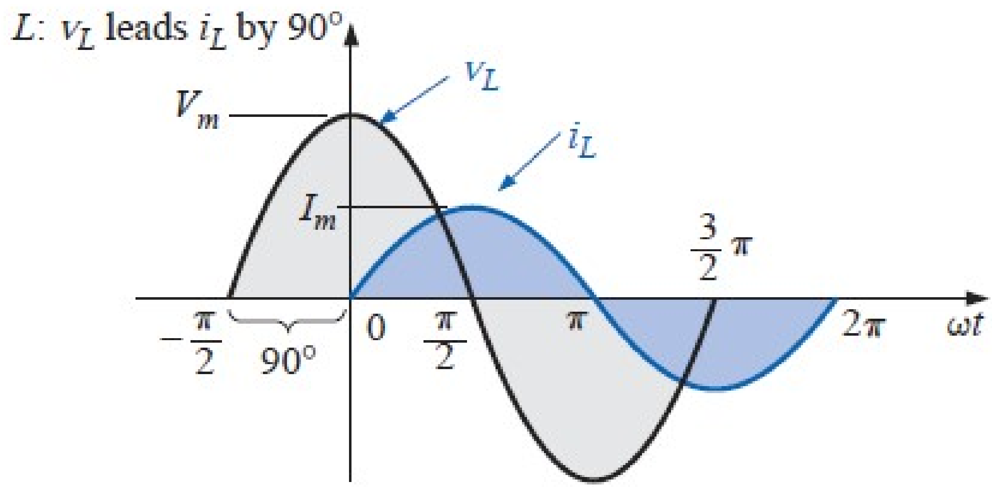
$$v_L = L\,\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

$$v_L = V_m \sin(\omega t + 90°)$$

$$V_m = \omega L I_m$$

for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.



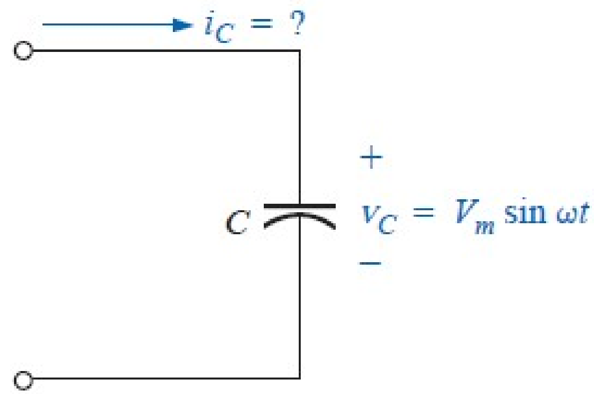
$$\boxed{X_L = \omega L} \qquad \text{(ohms, } \Omega\text{)}$$

$$\boxed{X_L = \frac{V_m}{I_m}}$$

Capacitor

$$i_C = C\,\frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

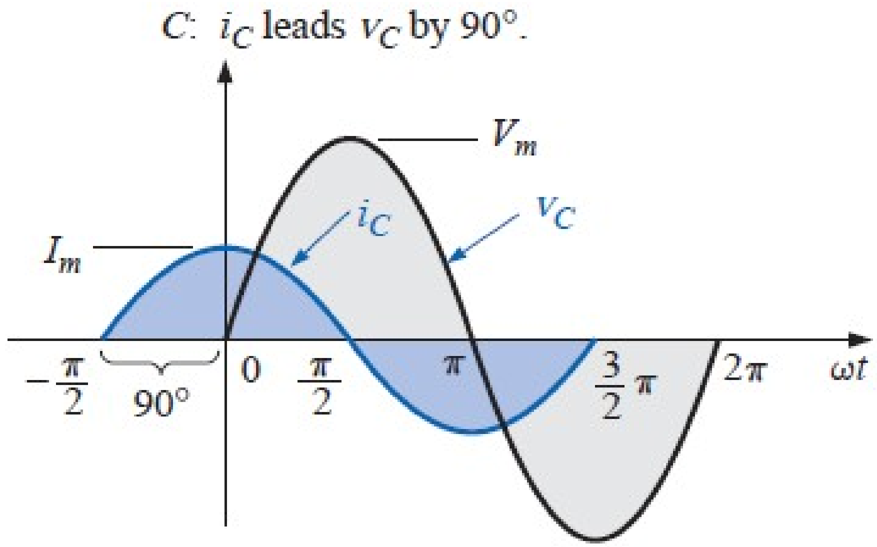


$$i_C = C\,\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

$$i_C = I_m \sin(\omega t + 90°)$$

$$I_m = \omega C V_m$$

for a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.



$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega\text{)}$$

$$X_C = \frac{V_m}{I_m}$$

$$i_L = \frac{1}{L}\int v_L \, dt$$

$$v_C = \frac{1}{C}\int i_C \, dt$$

EXAMPLE The current through a 100-μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60°)$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \ \Omega}{5 \times 10^4} = \frac{10^2 \ \Omega}{5} = 20 \ \Omega$$

$$V_m = I_m X_C = (40 \text{ A})(20 \ \Omega) = 800 \text{ V}$$

for a capacitor, v lags i by 90°. Therefor

$$v = 800 \sin(500t + 60° - 90°)$$

$$\boldsymbol{v = 800 \sin(500t - 30°)}$$

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ELECTRICAL ENGINEERING

BEEE101L

Basic Electrical Engineering

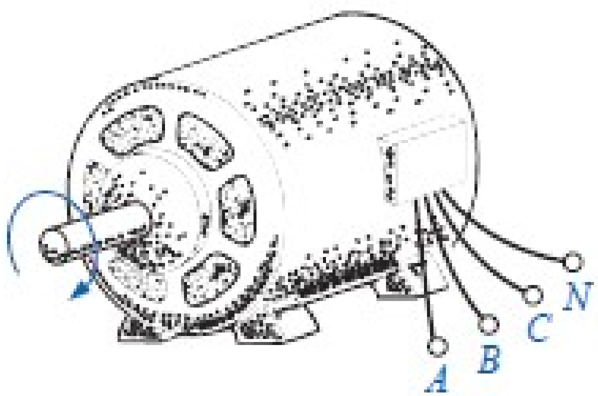
Three phase systems

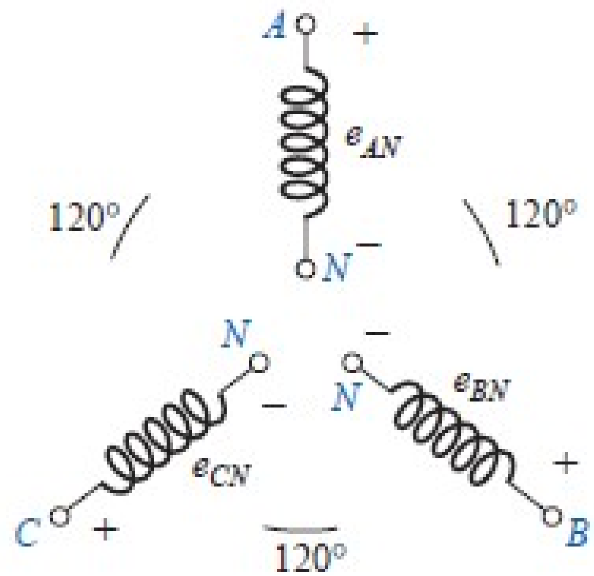
By,

Meera P. S.

Assistant Professor, SELECT

THE THREE-PHASE GENERATOR

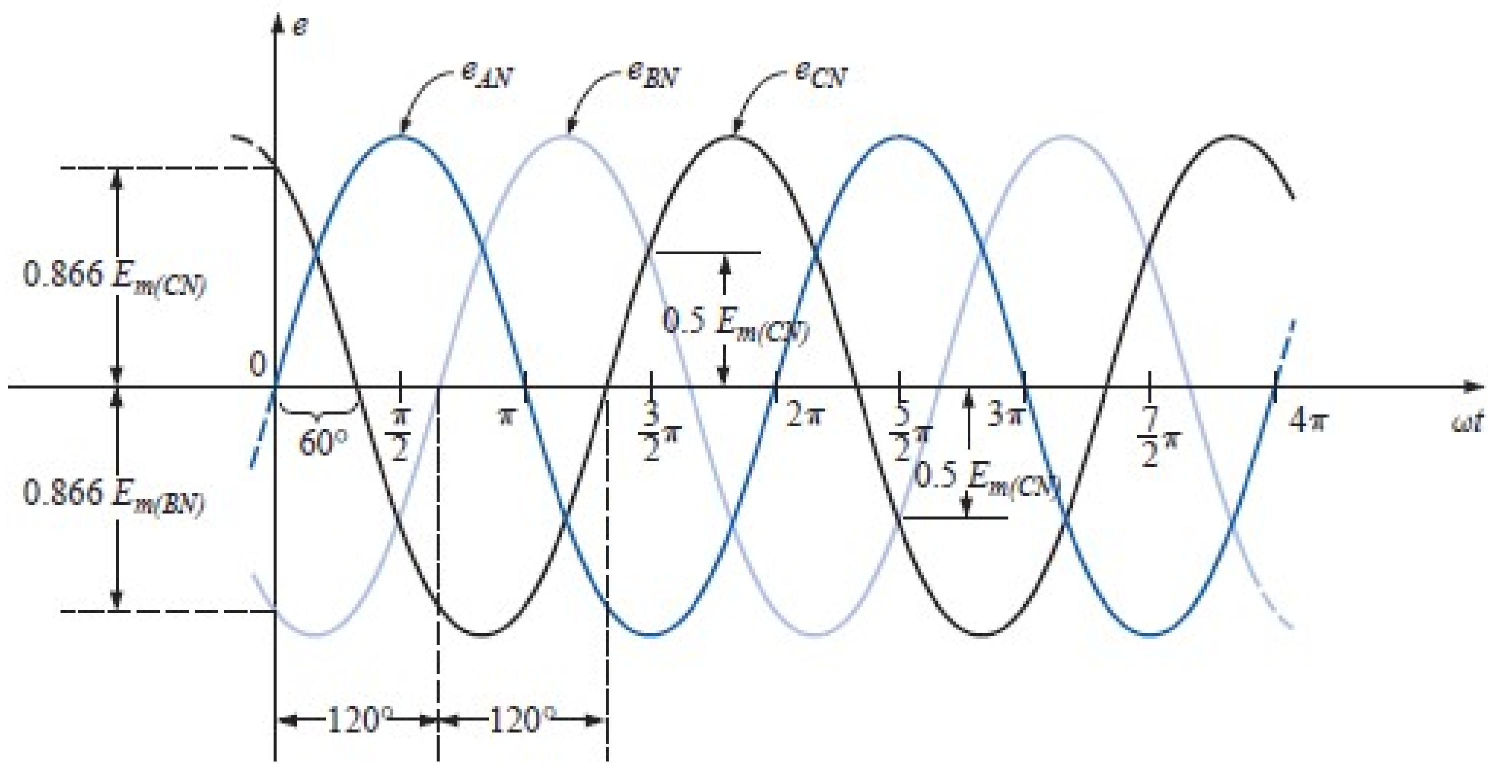




In particular, note that

at any instant of time, the algebraic sum of the three phase voltages of a three-phase generator is zero.

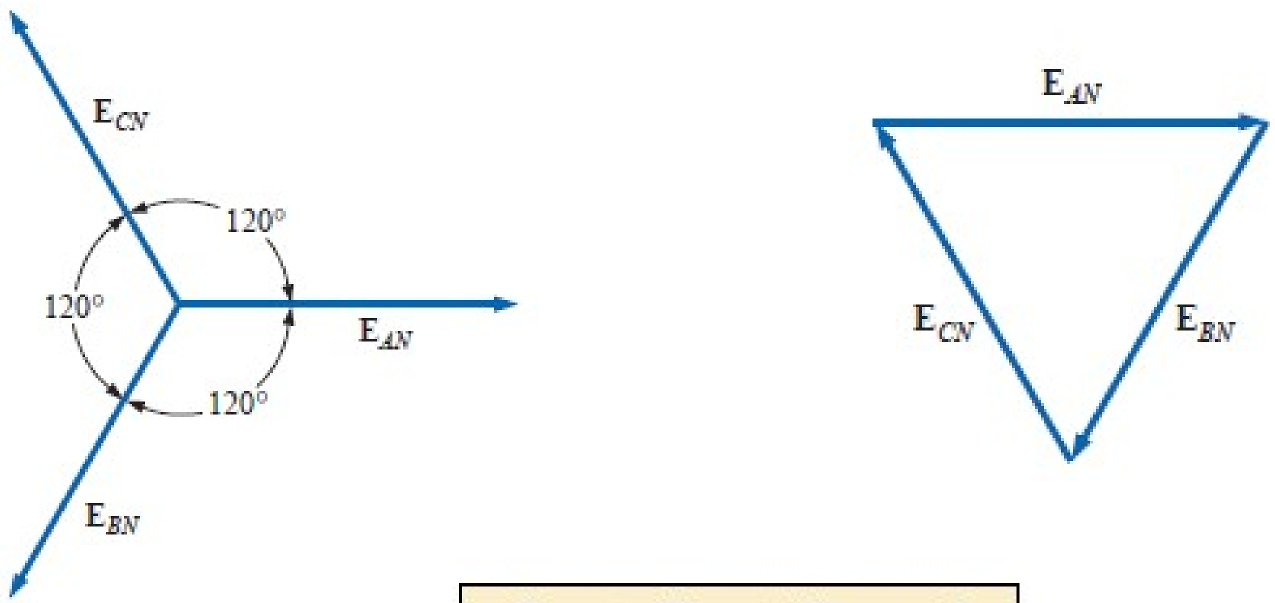
Three phase voltage



$$e_{AN} = E_{m(AN)} \sin \omega t$$

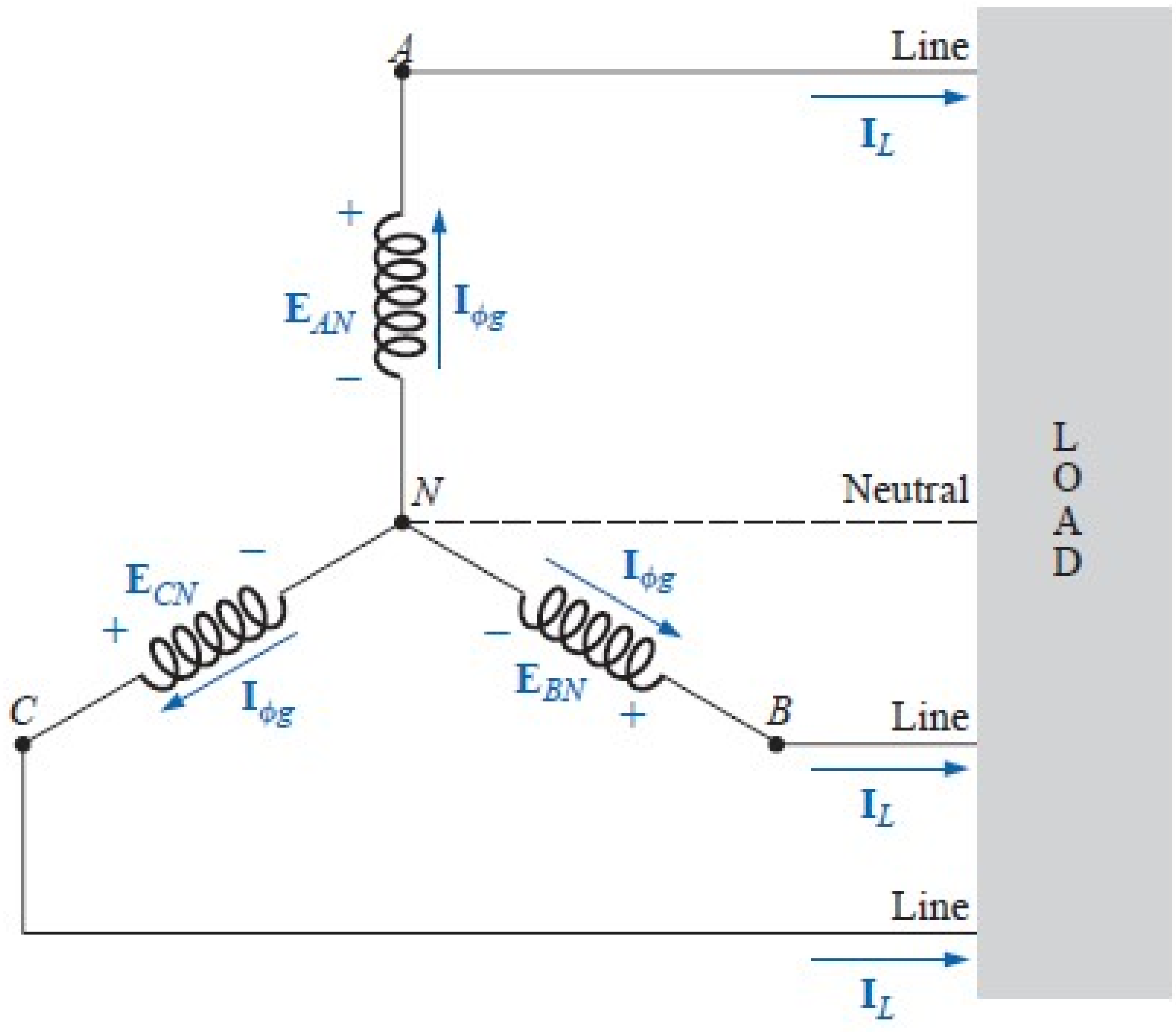
$$e_{BN} = E_{m(BN)} \sin(\omega t - 120°)$$

$$e_{CN} = E_{m(CN)} \sin(\omega t - 240°) = E_{m(CN)} \sin(\omega t + 120°)$$



$$\mathbf{E}_{AN} + \mathbf{E}_{BN} + \mathbf{E}_{CN} = 0$$

THE Y-CONNECTED GENERATOR

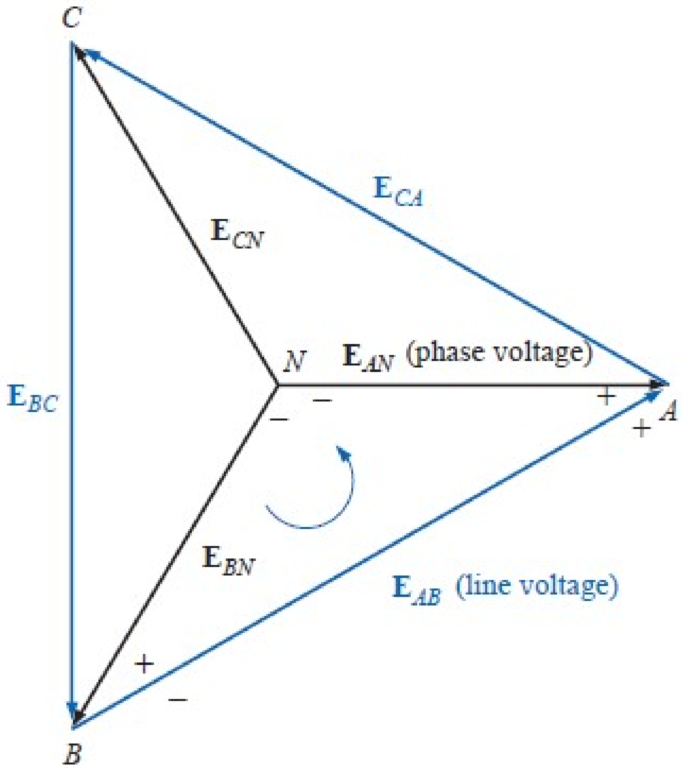


Y-CONNECTION

- The line current equals the phase current for each phase.

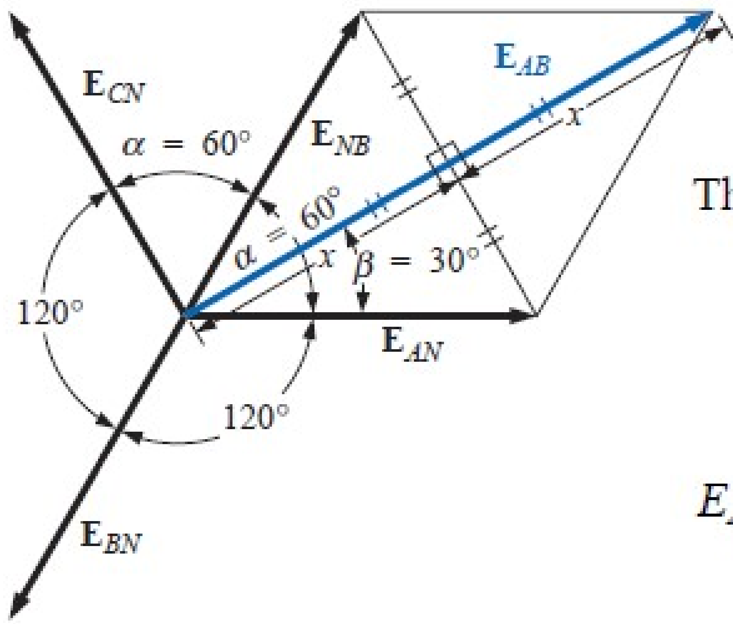
$$\boxed{\mathbf{I}_L = \mathbf{I}_{\phi g}}$$

- The voltage from one line to another is called a line voltage.



$$\mathbf{E}_{AB} - \mathbf{E}_{AN} + \mathbf{E}_{BN} = 0$$

$$\mathbf{E}_{AB} = \mathbf{E}_{AN} - \mathbf{E}_{BN} = \mathbf{E}_{AN} + \mathbf{E}_{NB}$$



The length x is

$$x = E_{AN} \cos 30° = \frac{\sqrt{3}}{2} E_{AN}$$

$$E_{AB} = 2x = (2)\,\frac{\sqrt{3}}{2} E_{AN} = \sqrt{3} E_{AN}$$

Noting from the phasor diagram that θ of $\mathbf{E}_{AB} = \beta = 30°$, the result is

$$\mathbf{E}_{AB} = E_{AB} \angle 30° = \sqrt{3} E_{AN} \angle 30°$$

and

$$\mathbf{E}_{CA} = \sqrt{3} E_{CN} \angle 150°$$

$$\mathbf{E}_{BC} = \sqrt{3} E_{BN} \angle 270°$$

$$\boxed{E_L = \sqrt{3} E_{\phi}}$$

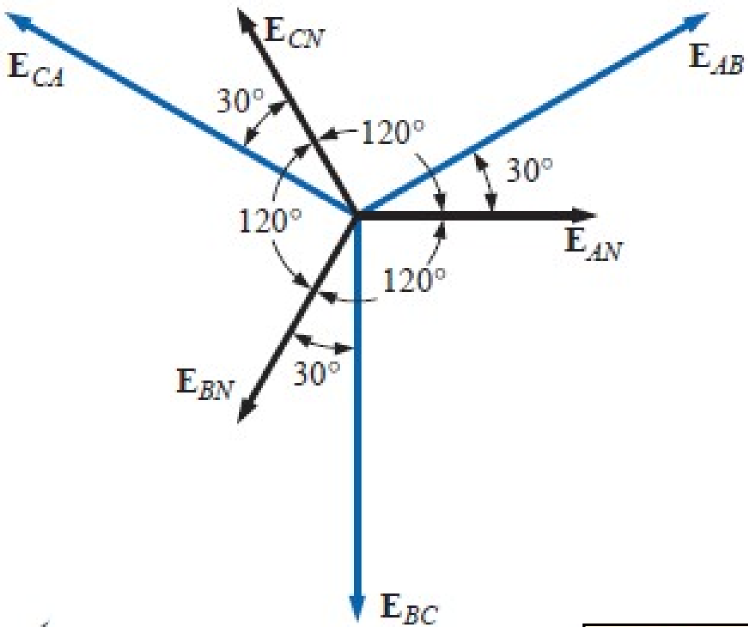
In sinusoidal notation,

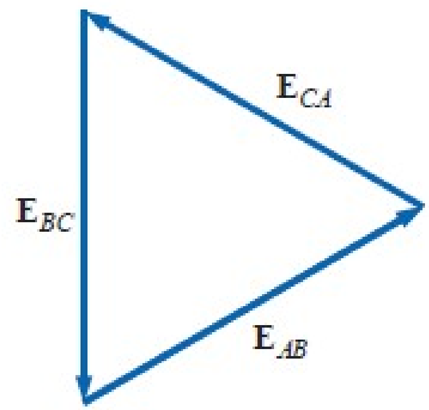
$$e_{AB} = \sqrt{2}E_{AB}\sin(\omega t + 30°)$$

$$e_{CA} = \sqrt{2}E_{CA}\sin(\omega t + 150°)$$

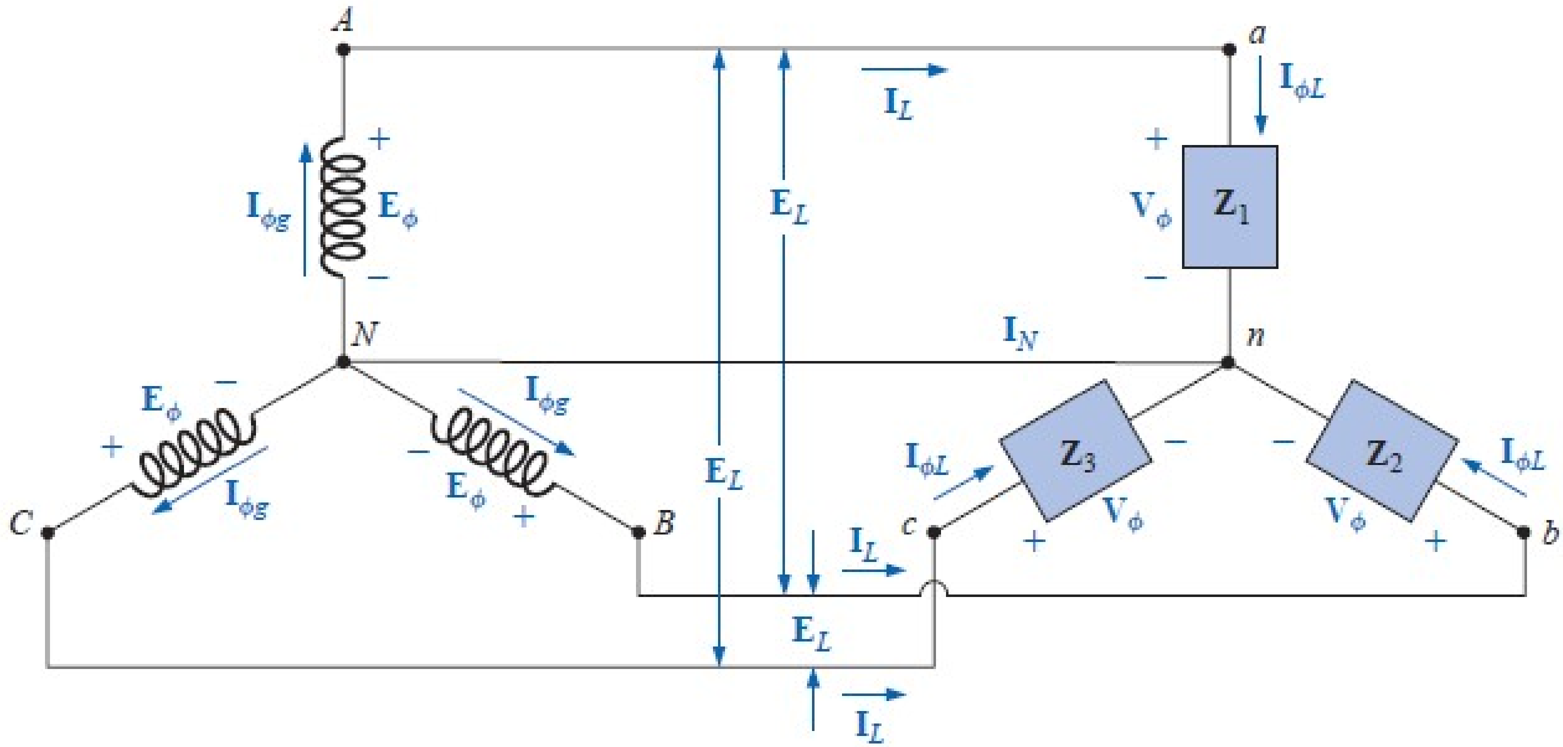
and

$$e_{BC} = \sqrt{2}E_{BC}\sin(\omega t + 270°)$$



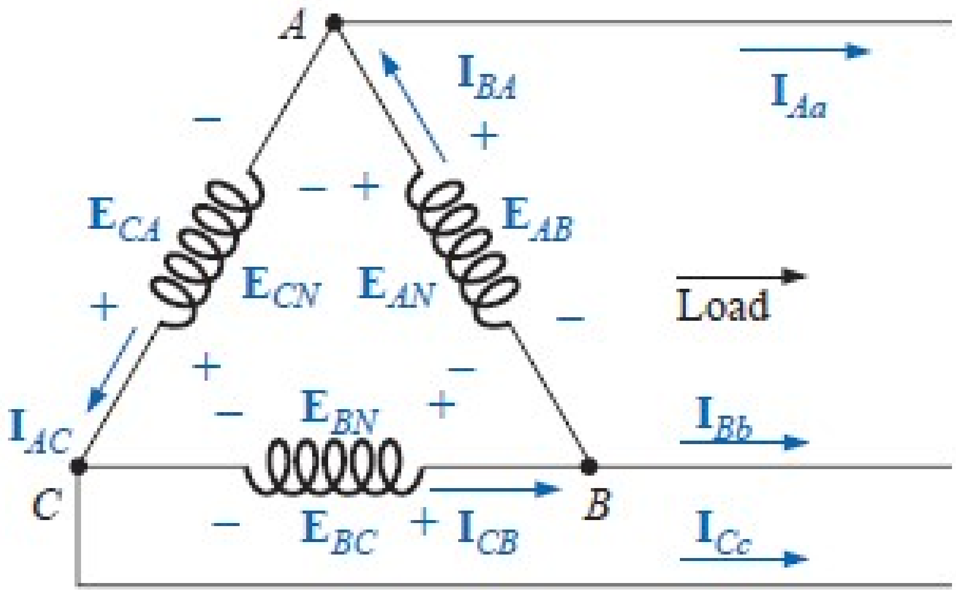


$$\boxed{\mathbf{E}_{AB} + \mathbf{E}_{CA} + \mathbf{E}_{BC} = 0}$$



Y-connected generator with a Y-connected load.

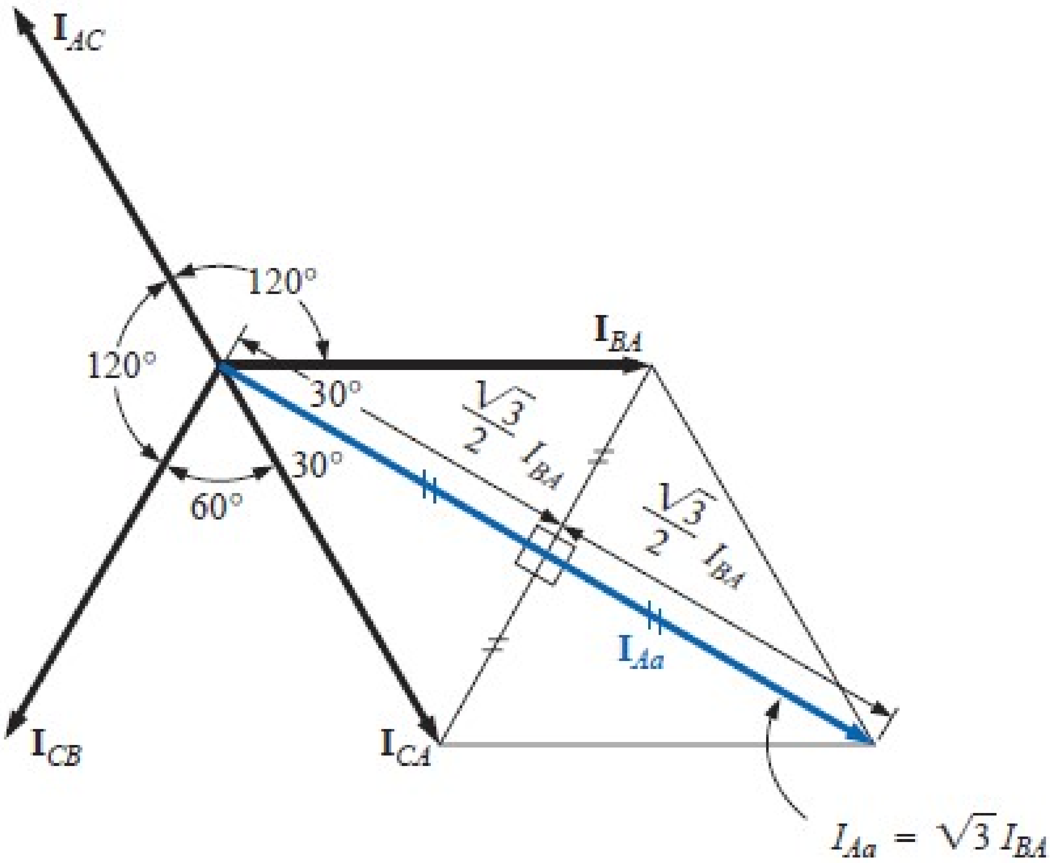
DELTA CONNECTION



The phase and line voltages are equivalent

$$\boxed{\mathbf{E}_L = \mathbf{E}_{\phi g}}$$

DELTA CONNECTION



$$\boxed{I_L = \sqrt{3} I_{\phi g}}$$

Problem 1. Three loads, each of resistance 30 Ω, are connected in star to a 415 V, 3-phase supply. Determine (a) the system phase voltage, (b) the phase current and (c) the line current.

For a star connection, $V_L = \sqrt{3} V_p$

Hence phase voltage, $V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = \mathbf{239.6\ V\ or\ 240\ V}$

Phase current, $I_p = \frac{V_p}{R_p} = \frac{240}{30} = \mathbf{8\ A}$

For a star connection, $I_p = I_L$

Hence the line current, $I_L = \mathbf{8\ A}$

Problem 2. A star-connected load consists of three identical coils each of resistance 30 Ω and inductance 127.3 mH. If the line current is 5.08 A, calculate the line voltage if the supply frequency is 50 Hz

Inductive reactance $X_L = 2\pi f L = 2\pi(50)(127.3 \times 10^{-3}) = 40\ \Omega$

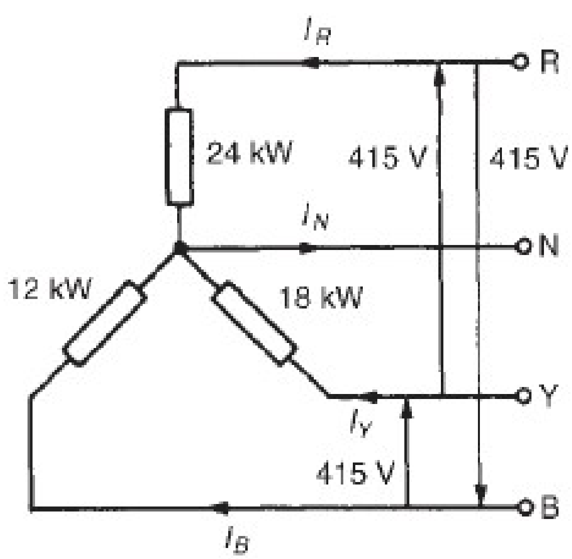
Impedance of each phase $Z_p = \sqrt{(R^2 + X_L{}^2)} = \sqrt{(30^2 + 40^2)} = 50\ \Omega$

For a star connection $I_L = I_p = \dfrac{V_p}{Z_p}$

Hence phase voltage $V_p = I_p Z_p = (5.08)(50) = 254\ \text{V}$

Line voltage $V_L = \sqrt{3}V_p = \sqrt{3}(254) = \mathbf{440\ V}$

Problem . A 415 V, 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in Figure 19.7. Determine (a) the current in each line and (b) the current in the neutral conductor.



For a star-connected system $V_L = \sqrt{3} V_p$

Hence $$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

Since current $I = \dfrac{\text{Power } P}{\text{Voltage } V}$ for a resistive load

then $$I_R = \frac{P_R}{V_R} = \frac{24\,000}{240} = \mathbf{100\ A}$$

$$I_Y = \frac{P_Y}{V_Y} = \frac{18\,000}{240} = \mathbf{75\ A}$$

$$I_B = \frac{P_B}{V_B} = \frac{12\,000}{240} = \mathbf{50\ A}$$

Alternatively, by calculation, considering I_R at 90°, I_B at 210° and I_Y at 330°:

$$\text{Total horizontal component} = 100\cos 90^\circ + 75\cos 330^\circ + 50\cos 210^\circ$$
$$= 21.65$$

$$\text{Total vertical component} = 100\sin 90^\circ + 75\sin 330^\circ + 50\sin 210^\circ$$
$$= 37.50$$

$$\text{Hence magnitude of } I_N = \sqrt{(21.65^2 + 37.50^2)} = \mathbf{43.3\ A}$$

Problem . Three identical coils each of resistance 30 Ω and inductance 127.3 mH are connected in delta to a 440 V, 50 Hz, 3-phase supply. Determine (a) the phase current, and (b) the line current.

Phase impedance, $Z_p = 50\ \Omega$

Phase current, $$I_p = \frac{V_p}{Z_p} = \frac{V_L}{Z_p} = \frac{440}{50} = \mathbf{8.8\ A}$$

For a delta connection, $I_L = \sqrt{3} I_p = \sqrt{3}(8.8) = \mathbf{15.24\ A}$

Problem . Three coils each having resistance 3 Ω and inductive reactance 4 Ω are connected (i) in star and (ii) in delta to a 415 V, 3-phase supply. Calculate for each connection (a) the line and phase voltages and (b) the phase and line currents.

For a star connection: $I_L = I_p$ and $V_L = \sqrt{3}V_p$

A 415 V, 3-phase supply means that the

line voltage, $V_L = \mathbf{415\ V}$

$$\text{Phase voltage, } V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = \mathbf{240\ V}$$

$$\text{Impedance per phase, } Z_p = \sqrt{(R^2 + X_L{}^2)} = \sqrt{(3^2 + 4^2)}$$

$$= 5\ \Omega$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{240}{5} = \mathbf{48\ A}$$

$$\text{Line current, } I_L = I_p = \mathbf{48\ A}$$

For a delta connection: $V_L = V_p$ and $I_L = \sqrt{3} I_p$

$$\text{Line voltage, } V_L = 415\ \text{V}$$

$$\text{Phase voltage, } V_p = V_L = \mathbf{415\ V}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{5} = \mathbf{83\ A}$$

$$\text{Line current, } I_L = \sqrt{3} I_p = \sqrt{3}(83) = \mathbf{144\ A}$$