MODULE -1 : DC circuits

* electrical circuits : Interconnection of electrical components
* Active elements : Voltages from the sources changes/provides electric current. Battery

  Passive elements: Values doesn't change irrespective of any voltage change. Resistors.

* Analysing: Calculating I, V etc.,

  Reason for analysing: To know whats happening, power factor & problems in that circuit etc. using laws & theorems.

* magnectically coupled circuits: electrical circuits not connected physically but magnetically. eg: Transformers.

* maxwell eq: $\oint \vec{H} \cdot dl = I_{elec}$

Charge: electrical property of the atomic particles of which materials consists, measured in coulomb(C)

——— x ——— x ———

DC circuits

* current doesn't change with time in DC
* I flows from higher potential ~~to~~ to lower & find low resistance path

Problem 4.

$$i = \begin{cases} 4A, & 0<t<1 \\ 4t^2, & 1<t<2 \end{cases} \quad \text{find } q \text{ from } t=0 \text{ to } t=2$$

Solution :-

$$q = \int i\,dt$$

$$q = \int_0^1 4\,dt + \int_1^2 4t^2\,dt = 4[t]_0^1 + \frac{4}{3}[t^3]_1^2$$

$$= 4(1) + \frac{4}{3}(8-1) = 4 + \frac{4}{3}(7) = 4\left(1+\frac{7}{3}\right)$$

$$q = \frac{40}{3}\,C$$

* Voltage – ~~force~~ workdone that is required to move the charges

$$V = \frac{dw}{dq}$$

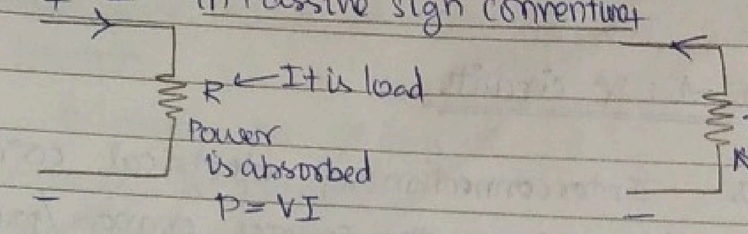
* Power – work done per unit time

$$P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = V \cdot I$$

* law of energy conservation : power lost = power gained

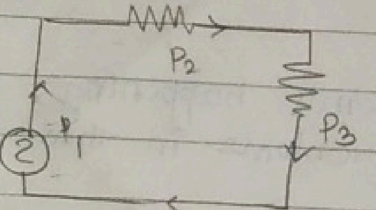
~~Ep~~ Σ Power = 0 ← across any circuit

* In Passive sign convention



* If power supplied is '–'ve, then power is absorbed.
  If power supplied is '+'ve, then power is supplied

  In active sign convention

* 

where P is Power supplied.

P_1 = '+'ve, P_2 = '–'ve, P_3 = '–'ve

hence, $P_1 + P_2 + P_3 = 0$

* In Passive sign convention, P is Power absorbed,

P_1 = '–'ve, P_2 = '+'ve, P_3 = '+'ve.

however, $\Sigma\, P_1 + P_2 + P_3 = 0$

* Energy : Capacity to do work (J)

~~Power~~ $E = P \times t$

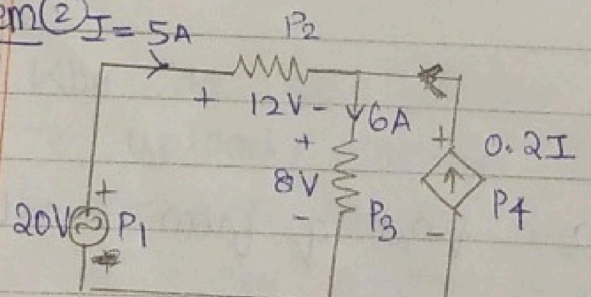
1 unit = 1 kwh = 3600 J

1 HP = 746 W

 – dependent current source

 – dependent Voltage source

Problem ②



Find power absorbed in each elements. (Passive sign convention)

Solution :-

$P_1 = 20(-5) = -100\,W$

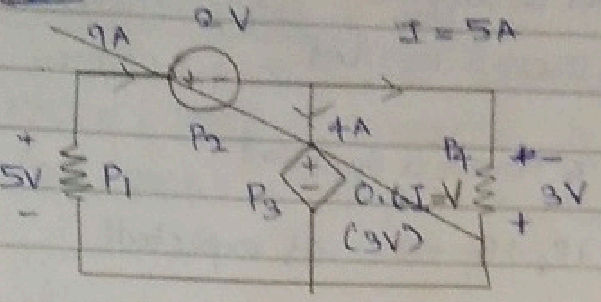
$P_2 = 12(5) = 60\,W$

$P_3 = 8(6) = 48\,W$

$P_4 = 8(0.2)(20) = -8\,W$

Ex POD

(1)



Calculate Power absorbed & Power supplied by each element using both active sign convention & passive sign convention.

Solution :-

(i) using Passive sign convention :-

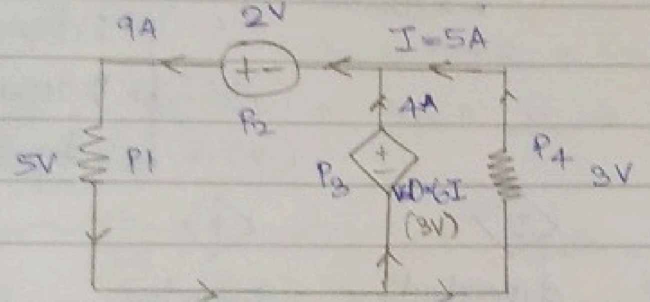
$P_1 = 5(9) = +45W$ ⇐ Power is absorbed

$P_2 = -2(9) = -18W$ } ⇐ Power is supplied

$P_3 = -(0.6)(5) = -12W$ }

$P_4 = 3(5) = 15W$ ⇐ Power is absorbed.

[∵ we dint get the summation of power in this circuit as zero. hence, P_4 must be negative (i.e., element 4 must supply ~~energy~~ power)]



(ii) using Active sign convention :-

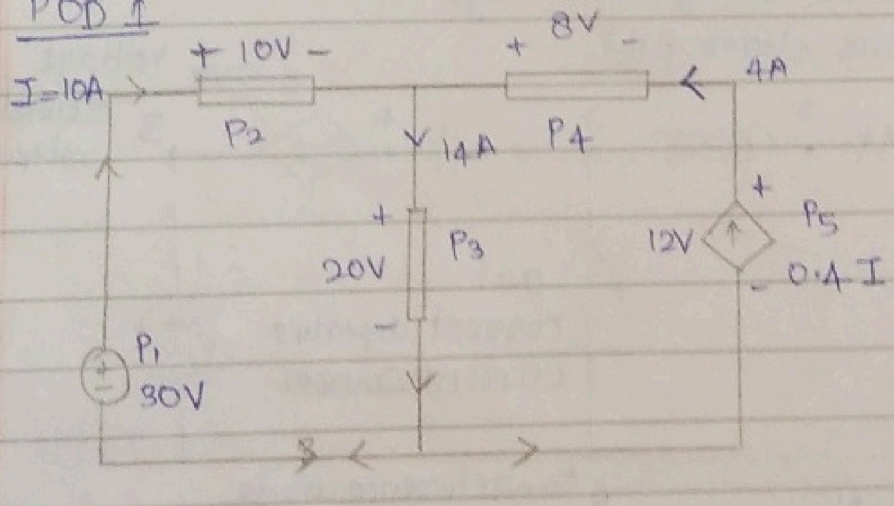
$P_1 = -45W$ ⇐ Power is absorbed

$P_2 = +18W$ } ⇐ Power is supplied.

$P_3 = +12W$ }

$P_4 = +15W$ }

POD 1



compute power supplied / absorbed by each component of the circuit using Passive sign convention:

$P_1 = -30V \times 10A = -300W$ ⇐ Power is supplied

$P_2 = 10V \times 10A = 100W$
$P_3 = 20V \times 14A = 280W$ } ⇐ Power is absorbed

$P_4 = -8V \times 4A = -32W$
$P_5 = -12V \times 4A = -48W$ } ⇐ Power is supplied

total $P = P_1 + P_2 + P_3 + P_4 + P_5 = 0$ as expected.

using active sign convention:

$P_1 = 300W$ ⇐ Power is supplied

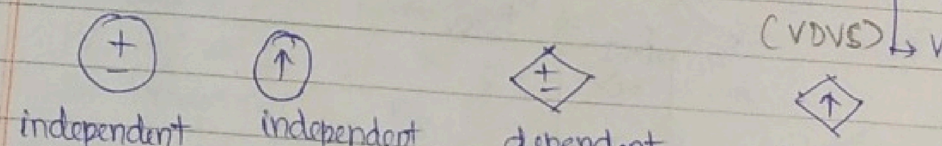
$P_2 = -100W$
$P_3 = -280W$ } ⇐ Power is absorbed

$P_4 = 32W$
$P_5 = 48W$ } ⇐ Power is supplied

* DC circuits → circuit elements → Active → independent sources
→ dependent sources
(CDCS) → I dependent I source
(CDVS) → I dependent V source
(VDCS) → V dependent I source
(VDVS) → V dependent V source

→ Passive

*

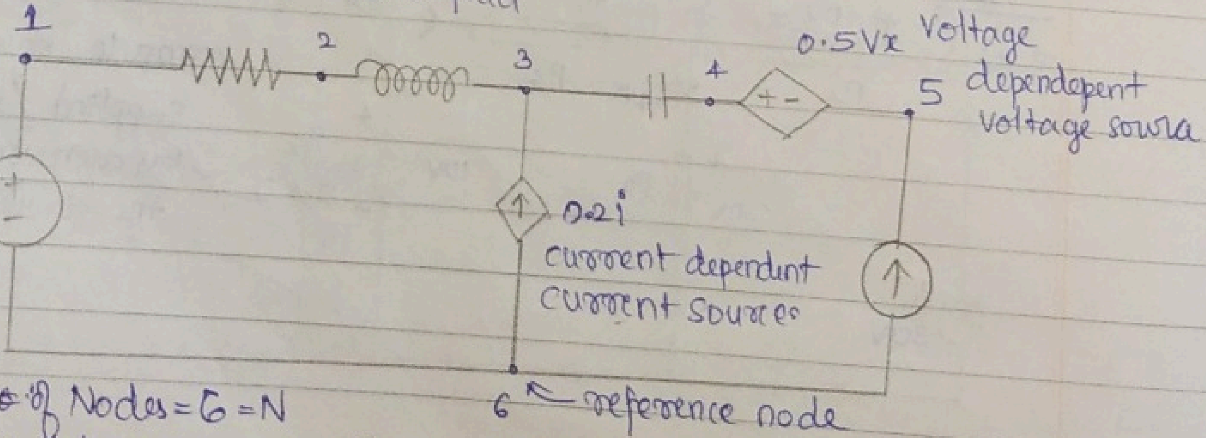


independent voltage source | independent current source | dependent voltage source | dependent current source

* Resistance – Resists flow of I
– dissipates power in the form of heat

* Inductance – Stores magnetic field
– resists change in I

* capacitance – resists change in voltage
– stores electric field



No of Nodes = 6 = N
No. of branches = B = 7

Nodes, branches & loops

* branch - each circuit elements represents branches (B)
* Nodes : point of intersection of ~~...~~ ≥ 2 branches (N)
* loop / mesh : closed path in an circuit (L)

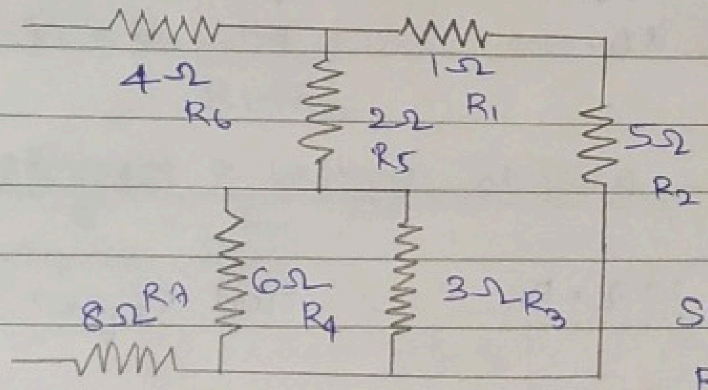
$$L = B - (N-1)$$

combination of resistance

Series : $R_{eq} = R_1 + R_2 + R_3 \cdots$

parallel : $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots$

Problem 1:



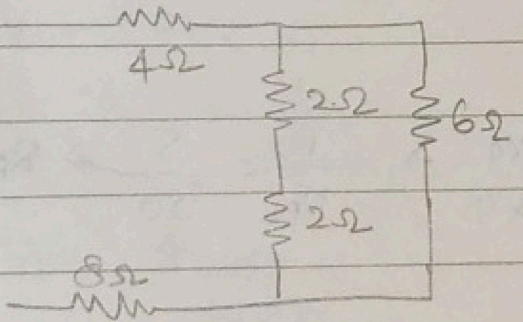
Find Req : ?

Solution :-

R_1 & R_2 are in series

$$R_{S1} = R_1 + R_2 = 1\Omega + 5\Omega = 6\Omega$$

$$\frac{1}{R_{p1}} = \frac{1}{6} + \frac{1 \times 2}{3 \times 2} \Rightarrow R_{p1} = \frac{6}{3} \Rightarrow R_{p_1} = 2\Omega$$



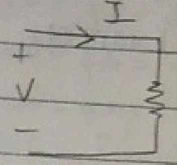
$$\frac{1}{R_{p_2}} = \frac{1 \times 2}{6 \times 2} + \frac{1 \times 3}{4 \times 3}$$

$$R_{p_2} = \frac{12}{5} \Rightarrow 2.4$$

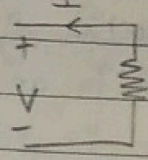
$$R_{eq} = 4\Omega + 2.4\Omega + 8\Omega = 14.4\Omega$$

Laws

* Ohms Law :



$V = IR$



$V = -IR$

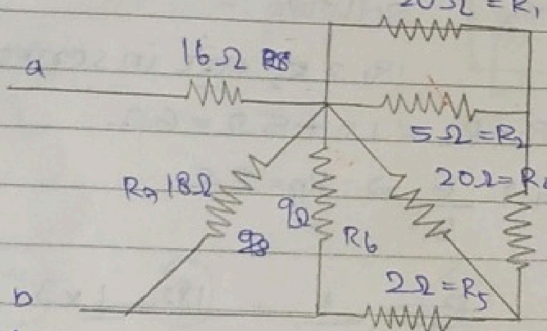
* Kirchoff voltage law: algebric sum of voltage drop across ~~of~~ each element is 0.

Kirchoff current law: sum of current entering = sum of current leaving.

current at a node is 0.

H.W

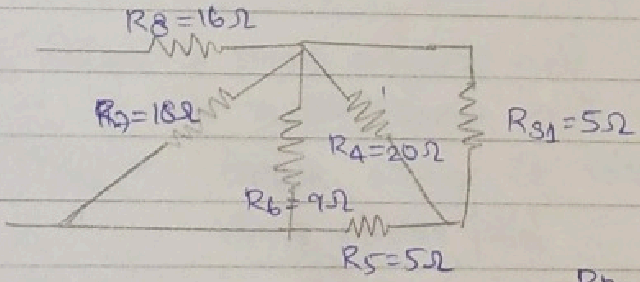
Find R_{eq} = ?



Solution:

$$\frac{1}{R_{p1}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_{p1}} = \frac{1}{20} + \frac{1 \times 4}{5 \times 4} \Rightarrow \frac{1}{R_{p1}} = \frac{5}{20} \Rightarrow R_{p1} = 4\Omega$$

$$R_{s1} = R_{p1} + R_3 = 4\Omega + 1\Omega = 5\Omega$$

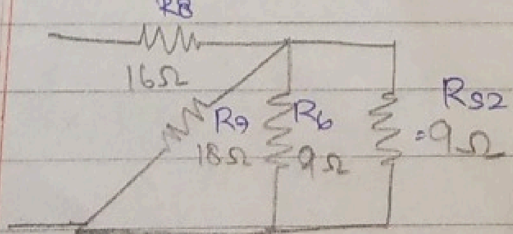


$$\frac{1}{R_{p2}} = \frac{1}{R_{s1}} + \frac{1}{R_4}$$

$$\frac{1}{R_{p2}} = \frac{1}{5} + \frac{1}{20}$$

$$R_{p2} = 4\Omega$$

$$R_{s2} = R_{p2} + R_5 \Rightarrow R_{s2} = 4\Omega + 5\Omega \Rightarrow R_{s2} = 9\Omega$$



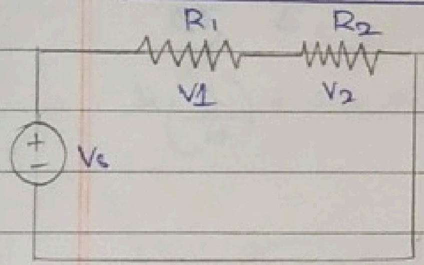
$$\frac{1}{R_{p3}} = \frac{1}{R_{s2}} + \frac{1}{R_6} + \frac{1}{R_7}$$

$$= \frac{1 \times 2}{9 \times 2} + \frac{1 \times 2}{9 \times 2} + \frac{1}{18} = \frac{5}{18}$$

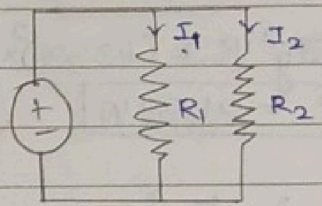
$$R_{p3} = \frac{18}{5}\Omega = 3.6\Omega$$

$$R_{eq} = R_{p3} + R_8 = (16 + 3.6)\Omega = 19.6\Omega$$

current & voltage division



$$V_1 = \frac{R_1 V}{R_1+R_2+R_3 \cdots}$$



* Voltage divides in series circuit

* Current divides in parallel circuits

* $V_1 = I R_1$

$$I = \frac{V_S}{R_{eq}} = \frac{V_S}{R_1+R_2}$$

$$V_1 = \frac{V_S}{R_1+R_2} \cdot R_1$$

$$\boxed{V_1 = \left(\frac{R_1}{R_1+R_2}\right) \cdot V_S}$$

lly $V_2 = \left(\frac{R_2}{R_1+R_2}\right) V_S$

* $I_1 = \frac{V}{R_1}$; $V = I R_{eq}$

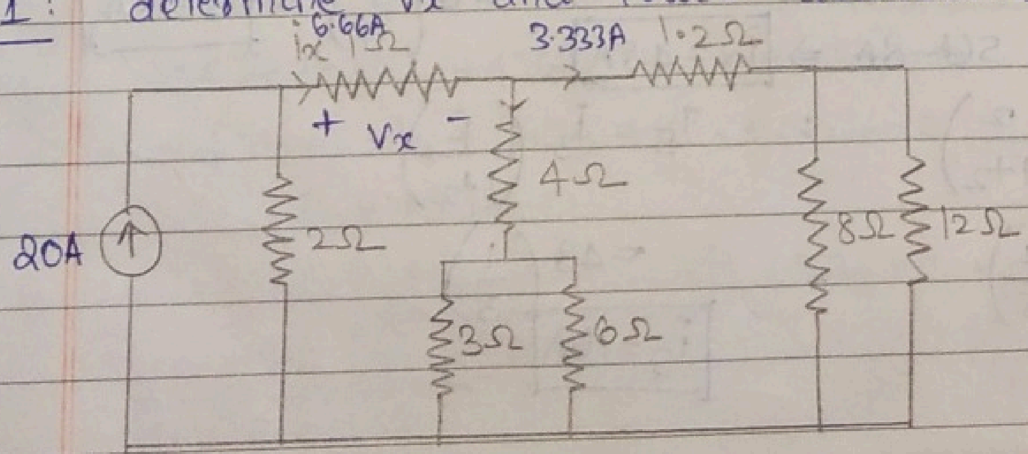
$$= I\left(\frac{R_1R_2}{R_1+R_2}\right)$$

$$\boxed{I_1 = \left(\frac{R_2}{R_1+R_2}\right) V_S} \; ; \; I_1 = \left(\frac{R_2}{R_1+R_2}\right) I$$

lly * $I_2 = \left(\frac{R_1}{R_1+R_2}\right) V_S$

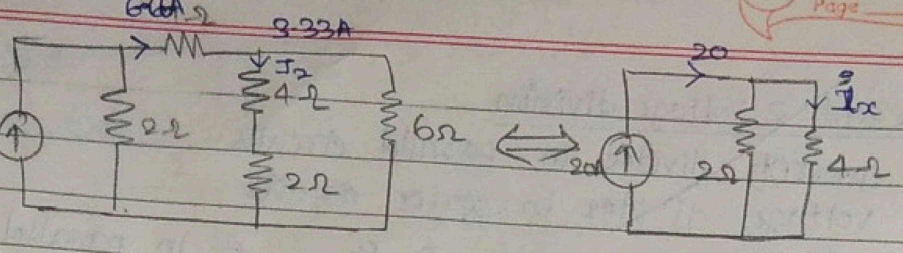
$i_1 = \frac{G_1}{G_1+G_2+G_3+G_4 \cdots} \, i$; G = conductance

exp 1: determine V_x and Power absorbed by the 12Ω resistor



$V_x = i_x (1)$

to find i_x → reducing the circuit to



$$i_x = I\left(\frac{2}{2+4}\right) = 20\left(\frac{\cancel{2}^{1}}{\cancel{6}_{3}}\right) = \frac{20}{3}A = 6.667A$$

$$V_x = i_x(1\Omega) \Rightarrow \boxed{V_x = 6.667V}$$

To calculate Power across 12Ω resistor, we should know the current (I_y) flowing through it.

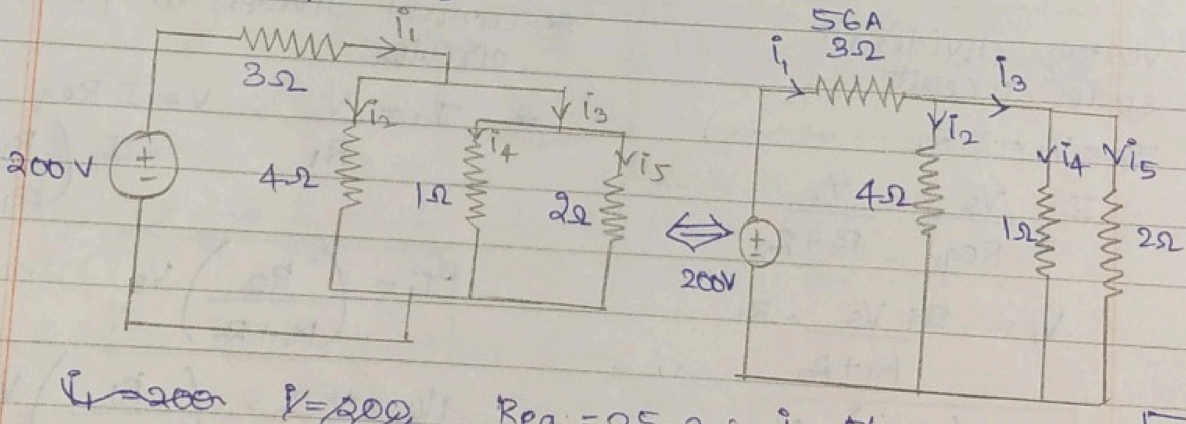
$$I_2 = 6.666A\left(\frac{6}{6+6}\right) = \frac{20}{3}\left(\frac{1}{2}\right) = \frac{10}{3} = 3.33A$$

$$\text{lly}, \; I_y = 3.333\left(\frac{8}{12+8}\right) = 3.333 \times \frac{8}{20} = \frac{2\cancel{10}}{3} \times \frac{\cancel{8}4}{\cancel{20}} = 1.333$$

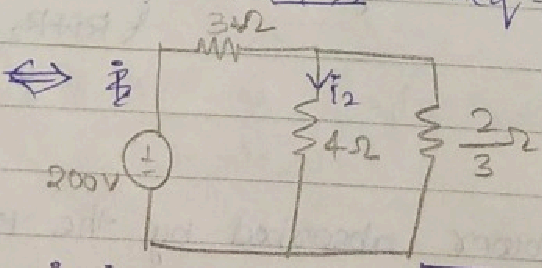
$$\text{Power} = I_y^2 R = (1.333)^2 \times 12$$

$$\boxed{P = 21.33W}$$

exp 2 determine i_1 to i_5



~~$i_1 = 200$~~ ~~$V = 200$~~ $R_{eq} = \frac{25}{7}\Omega$; $i_1 = \frac{V}{R_{eq}} = \frac{200}{25} \times 7 \Rightarrow \boxed{i_1 = 56A}$



$$i_2 = i\left(\frac{2/3}{4+\frac{2}{3}}\right) = 56 \times \left[\frac{2}{3} \div \frac{14}{3}\right]$$

$$i_2 = 56\left(\frac{1}{7}\right) \Rightarrow \boxed{i_2 = 8A}$$

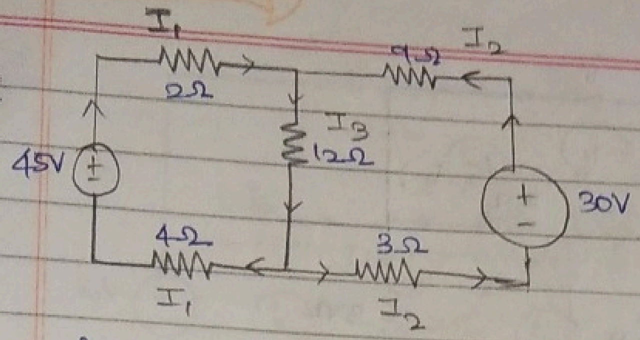
$$i_3 = i_1 - i_2 = 56A - 8A \Rightarrow \boxed{i_3 = 48A}$$

$$i_4 = i_3\left(\frac{2}{1+2}\right) \quad ; \quad i_5 = i_3\left(\frac{1}{1+2}\right)$$

$$= 48\left(\frac{2}{3}\right) \qquad = 48\left(\frac{1}{3}\right)$$

$$\boxed{i_4 = 32A} \qquad \boxed{i_5 = 16A}$$

exp 3:



for loop 1: $-45+25I_1+12I_3+4I_1=0$

$-45+25I_1+12I_1+12I_2+4I_1=0$

$41I_1+12I_2=45$ —— (1)

For loop 2: $-30+9I_2+12I_3+3I_2=0$

$-30+12I_2+12I_1+12I_2=0$

$12I_1+24I_2=30 \Rightarrow 2I_1+4I_2=5$ —— (2)

$\overset{3}{\cancel{4}}\overset{11}{\cancel{1}}I_1+12I_2=45$

$6I_1+12I_2=15$

$35I_1 \quad = 30 \quad \therefore \boxed{I_1=\frac{6}{7}A}=0.857A$

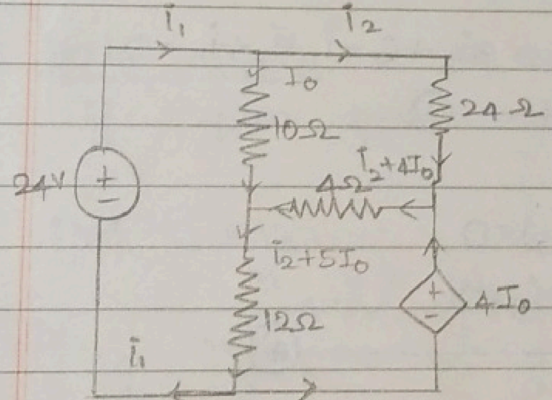
$2\left(\frac{6}{7}\right)+4I_2=5 \Rightarrow 4I_2=5-\frac{12}{7} \Rightarrow 4I_2=\frac{35-12}{7}$

$\boxed{I_2=\frac{23}{28}A}=0.8214$

$\boxed{I_3=1.678A}$

35
12
23

exp 4:



In loop 1:— $I_0=i_1-i_2 \Rightarrow i_1=I_0+i_2$

$-24=10I_0+12(i_2+5I_0)$

$-24=10I_0+12i_2+60I_0$

$-24=70I_0+12i_2$

$35I_0+6i_2+12=0$ —— (1)

In loop 2:—

$24i_2+4(i_2+4I_0)-10I_0=0$

$24i_2+4i_2+4I_0-10I_0=0$

$28i_2-6I_0=0 \Rightarrow 14i_2-3I_0=0$ —— (2)

In loop 3:—

$-4I_0+4(i_2+4I_0)+12(i_2+5I_0)=0$

$-4I_0+4i_2+16I_0+12i_2+60I_0=0$

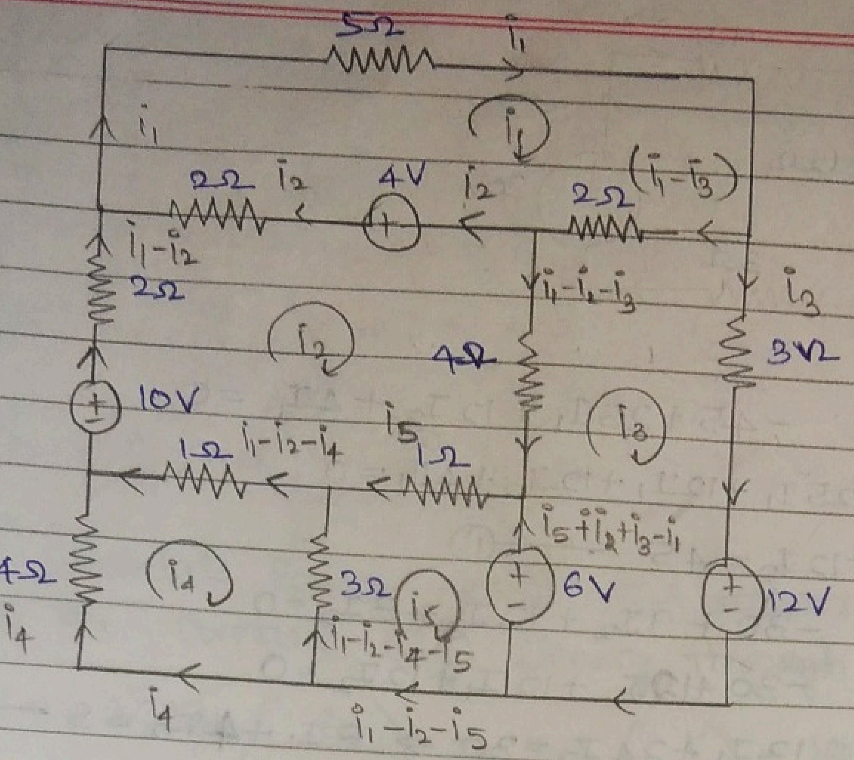
$16i_2+72I_0=0$

$4i_2+18I_0=0 \Rightarrow 2i_2+9I_0=0$ —— (3)

$\boxed{I_0=-0.3307A}$ $\therefore i_2=\frac{3}{12}\left(\frac{-42}{127}\right) \Rightarrow \boxed{i_2=-0.0708A}$

$\boxed{i_1=-0.4015A}$

Q5.



in loop 1:

$$5i_1+2(i_1-i_3)-4+2i_2=0$$

$$5i_1+2i_1-2i_3-4+2i_2 \Rightarrow 7i_1+2i_2-2i_3-4=0 \quad \text{——— (1)}$$

in loop 2:-

$$-10+2(i_1-i_2)-2i_2+4+4(i_1-i_2-i_3)+i_5+i_1-i_2-i_4=0$$

$$-6+2i_1-2i_2-2i_2+4i_1-4i_2-4i_3+i_5+i_1-i_2-i_4=0$$

$$7i_1-9i_2-4i_3-i_4+i_5=0 \quad \text{——— (2)}$$

in loop 3:-

$$3i_3+12-6-(i_1-i_2-i_3)=0 \Rightarrow 3i_3+6-i_1+i_2+i_3=0$$

$$6-i_1+i_2+4i_3=0 \quad \text{——— (3)}$$

in loop 4:-

$$4i_4-1(i_1-i_2-i_4)-3(i_1-i_2-i_4-i_5)=0$$

$$4i_4-i_1+i_2+i_4-3i_1+3i_2+3i_4+i_5=0$$

$$-4i_1+4i_2+8i_4+i_5=0 \quad \text{——— (4)}$$

in loop 5:-

$$-6+1i_5-3(i_1-i_2-i_4-i_5)=0$$

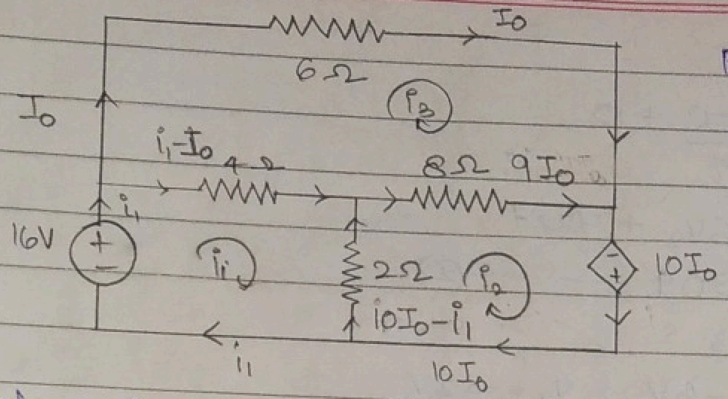
$$-6+i_5-3i_1+3i_2+3i_4+3i_5=0$$

$$-3i_1+3i_2+3i_4+4i_5=6 \quad \text{——— (5)}$$

$\boxed{i_1=-0.1065A}$; $\boxed{i_2=0.6742A}$; $\boxed{i_3=-1.695A}$

$\boxed{i_4=-0.357A}$; $\boxed{i_5=-0.32337A}$

POD 2



Find current through the circuit.

along loop 1:- $-16+4(i_1-I_o)-2(10I_o-i_1)=0$

$-16+4i_1-4I_o-20I_o+2i_1=0 \Rightarrow 6i_1-24I_o-16=0$

$3i_1-12I_o-8=0$ —— (1)

along loop 2 :-

$-10I_o+2(10I_o-i_1)+8(9I_o)=0$

$-10I_o+20I_o-2i_1+72I_o=0$

$82I_o-2i_1=0 \Rightarrow i_1=41I_o$ —— (2)

$3(41I_o)-12I_o-8=0 \Rightarrow 123I_o-12I_o-8=0$

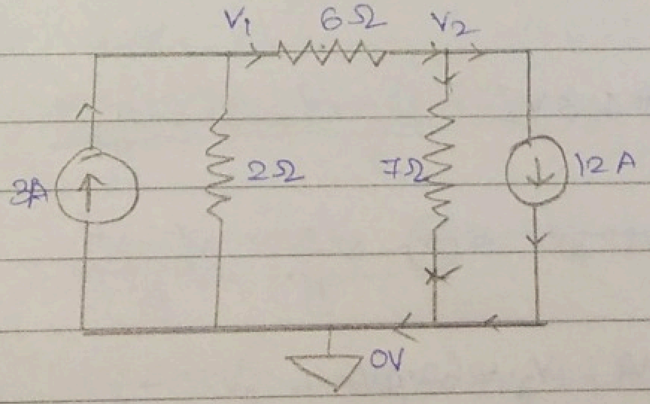
$\Rightarrow 111I_o-8=0 \Rightarrow \boxed{I_o=\frac{8}{111}A} = 0.0720A$

$i_1 = 41\left(\frac{8}{111}\right) \Rightarrow \boxed{i_1=2.954A}$

Nodal analysis - we use KCL

if I flows from A to B, then $i=\frac{V_A-V_B}{R}$

1) Find V_1 & V_2



applying KCL, at node 1

$3=\frac{V_1-0}{2}+\frac{V_1-V_2}{6}$

$8V_1-2V_2=36$

$4V_1-V_2=18$ —— (1)

at node 2:-

$$\frac{V_1 - V_2}{6} = \frac{V_2 - 0}{7} + 12$$

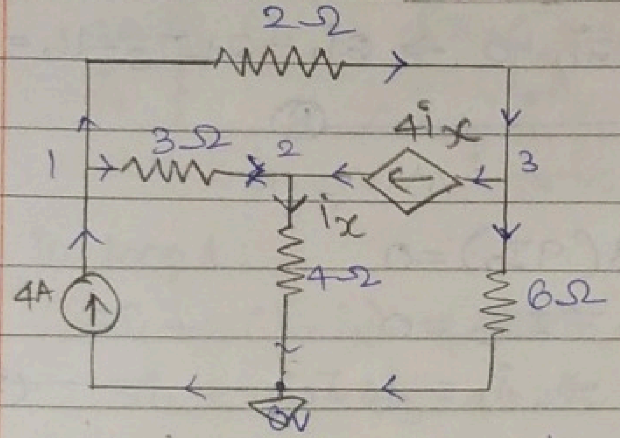
$$7V_1 - 7V_2 = 6V_2 + 72 \times 7$$

$$7V_1 - 13V_2 = 509 \quad \text{—— (2)}$$

solving (1) 2 (2),

$$V_1 = -6V \;;\; V_2 = -42V$$

2)



Find i_x = ?

at node 2:

$$i_x = \frac{V_2 - 0}{4}$$

Applying KCL at nodes 1

$$4 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2} \Rightarrow 24 = 2V_1 - 2V_2 + 3V_1 - V_3$$

$$\Rightarrow 5V_1 - 2V_2 - 2V_3 = 24 \quad \text{—— (1)}$$

At node 2:-

$$\frac{V_1 - V_2}{3} + 4i_x = i_x \Rightarrow \frac{V_1 - V_2}{3} + \frac{3V_2}{4} = 0$$

$$4V_1 + 5V_2 = 0 \quad \text{—— (2)}$$

at node 3:-

$$\frac{V_1 - V_3}{2} = \frac{V_3 - 0}{6} + 4i_x$$

$$24V_1 - 24V_3 = 8V_3 + 48V_2$$

$$24V_1 - 48V_2 - 32V_3 = 0$$

$$3V_1 - 6V_2 - 4V_3 = 0 \quad \text{—— (3)}$$

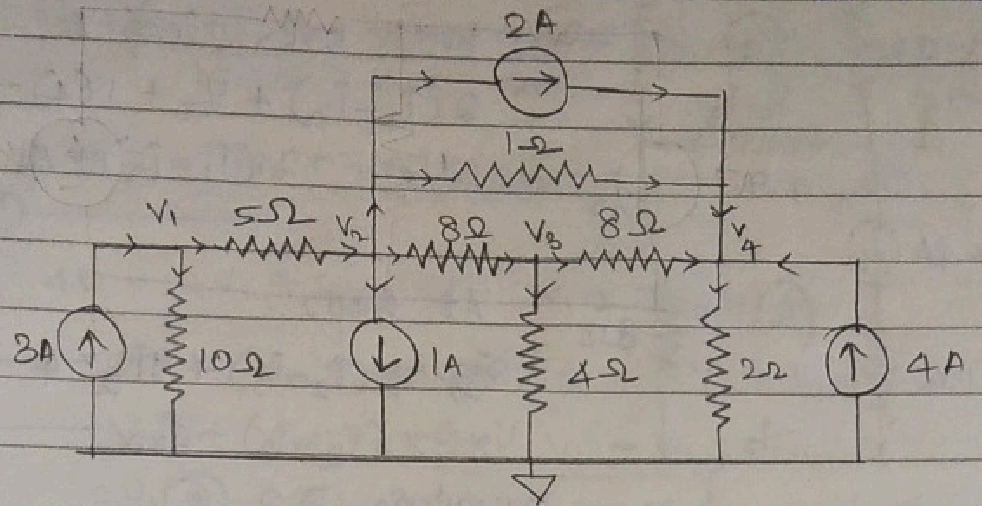
solving (1), (2), (3)

$$V_1 = 32V \;;\; V_2 = -25.6V \;;\; V_3 = 62.4V$$

$$\boxed{i_x = -6.4\,A}$$

3) H.W

write nodal equations.



at node 1 :- $3 = \frac{V_1 - 0}{10} + \frac{(V_1 - V_2)\times 2}{5 \times 2}$

$30 = V_1 + 2V_1 - 2V_2 \Rightarrow 3V_1 - 2V_2 = 30$ ——— (1)

at node 2 :- $\frac{V_1 - V_2}{5} = 1 + \frac{V_2 - V_3}{8} + 2 + \frac{V_2 - V_4}{1}$

$\frac{8(V_1 - V_2)}{5 \times 8} + \frac{(V_3 - V_2)5}{8 \times 5} = 3 + V_2 - V_4$

$8V_1 - 8V_2 + 5V_3 - 5V_2 = 40(3 + V_2 - V_4)$

$8V_1 - 13V_2 + 5V_3 = 120 + 40V_2 - 40V_4$

$8V_1 - 53V_2 + 5V_3 + 40V_4 = 120$ ——— (2)

at node 3 :-

$\frac{V_2 - V_3}{8} = \frac{V_3 - V_4}{8} + \frac{2V_3}{8} \Rightarrow V_2 - V_3 = V_3 - V_4 + 2V_3$

$V_2 - 4V_3 + V_4 = 0$ ——— (3)

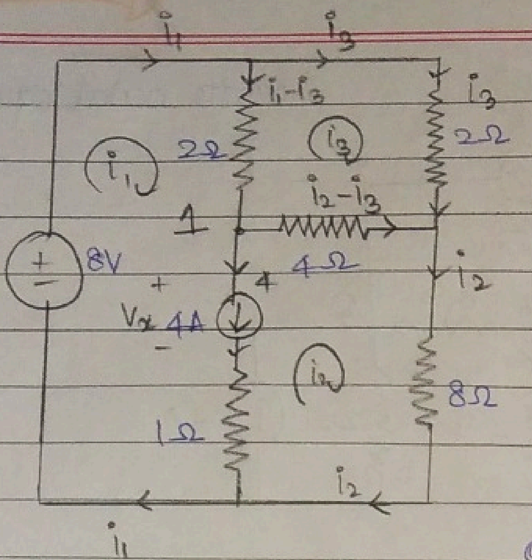
at node 4 :-

$\frac{V_3 - V_4}{8} + \frac{V_2 - V_4}{1} + 2 + 4 = \frac{V_4 \times 4}{2 \times 4}$

$\frac{V_3 - V_4}{8} - \frac{4V_4}{8} + V_2 - V_4 + 6 = 0$

$V_3 - 5V_4 + 8V_2 - 8V_4 + 48 = 0$

$8V_2 + V_3 - 3V_4 + 48 = 0$ ——— (4)



Find i_1, i_2, i_3:-

From KVL, at loop 1:-

$2(i_1-i_3)+V_x+1(4)-8=0$

$V_x=-2(i_1-i_3)+4$ —— (1)

At loop 2:-

$4(i_2-i_3)+8i_2-4-V_x=0$

$V_x=4(i_2-i_3)+8i_2$ —— (2)

equating (1) & (2)

$\cancel{2i_3-4(i_2-i_3)-2(i_1-i_3)}$

$-2(i_1-i_3)+8=4(i_2-i_3)+8i_2$

$8=2i_1-6i_3+12i_2$ —— (3) $8=$

At loop 3:-

$\cancel{8i_2}\ 2i_3-4(i_2-i_3)-2(i_1-i_3)=0$

$\cancel{i_1-i_2=4}\ 2i_3-4i_2+4i_3-2i_1+2i_3=0$

$\Rightarrow -2i_1-4i_2+8i_3=0 \Rightarrow -i_1-2i_2+4i_3=0$ —— (4)

From KCL, $i_1=i_2+4$

(3) $\Rightarrow 2(i_2+4)+12i_2-6i_3=8 \Rightarrow 2i_2+\cancel{8}+12i_2-6i_3=\cancel{8}$

$\Rightarrow 14i_2-6i_3=0 \Rightarrow 7i_2-3i_3=0$

(4) $\Rightarrow -(i_2+4)-2i_2+4i_3=0 \Rightarrow -i_2-4-2i_2+4i_3=0$

$\Rightarrow 3(-3i_2+4i_3=4) \Rightarrow -9i_2+12i_3=12$

$28i_2-12i_3=0$

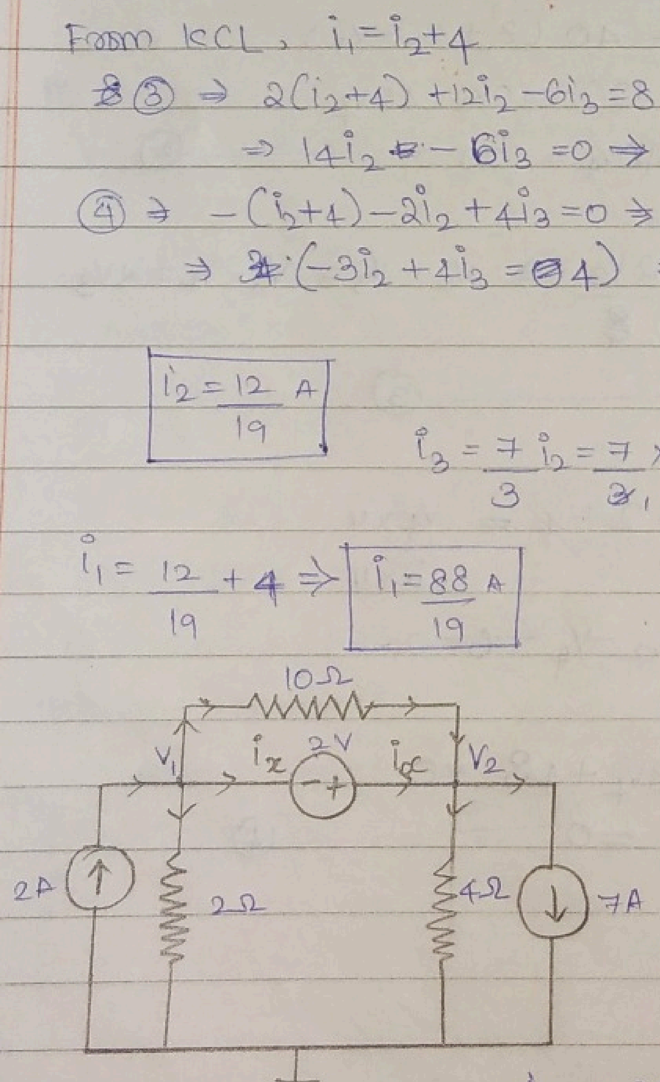
$19i_2=12$

$\boxed{i_2=\frac{12}{19}A}$

$i_3=\frac{7}{3}i_2=\frac{7}{3}\times\frac{12}{19} \Rightarrow \boxed{i_3=\frac{28}{19}A}$

$i_1=\frac{12}{19}+4 \Rightarrow \boxed{i_1=\frac{88}{19}A}$

4.



Find V_1, V_2

solution:-

$2=\frac{V_1\times5}{2\times5}+i_x+\frac{V_1-V_2}{10}$

$i_x=\frac{20}{10}-\frac{5V_1+V_2-V_1}{10}$

$i_x=\frac{20-6V_1+V_2}{10}$ —— (1)

* Linearity is a property of an element describing a linear relationship btwn input & output.
For ex: doubling voltage accross an elements, doubles the I flowing through it. Such circuit is called linear circuit

at node 2:-

$$i_x + \frac{2(V_1 - V_2)}{10 \times 2} = \frac{V_2 \times 5}{4 \times 5} + \frac{7 \times 20}{20} \Rightarrow i_x = \frac{5V_2 + 140 + 2V_2 - 2V_1}{20}$$

$$i_x = \frac{-2V_1 + 7V_2 + 140}{20} \quad \text{(2)}$$

$$40 - 12V_1 + 2V_2 = -2V_1 + 7V_2 + 140$$

$$10V_1 + 5V_2 + 100 = 0 \Rightarrow 2V_1 + V_2 + 20 = 0 \quad \text{(3)}$$

$$\cancel{V_2 \quad V_1 - V_2 = 2} \quad V_1 - V_2 = -2 \quad \text{(4)}$$

$$3V_1 = -22 \Rightarrow \boxed{V_1 = \frac{-22}{3} V}$$

$$V_2 = 2 - \frac{22}{3} \Rightarrow V_2 = \frac{6-22}{3} \Rightarrow \boxed{V_2 = \frac{-16}{3} V}$$

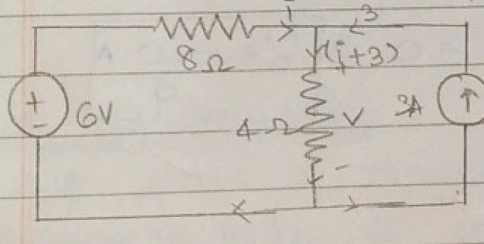
Superposition Principle

Voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or I through) that element due to each independent source acting alone.

Steps to apply Superposition principle:

1) Turn off all independent sources except one source. Find the output (voltage/current) due to that active source using the techniques covered in ch 2 & 3.
2) Repeat step 1 for each of the other independent sources.
3) Find the total contribution by adding algebrically all the contribution due to independent sources

Ex 1:



Find V using superposition principle.

solution:-

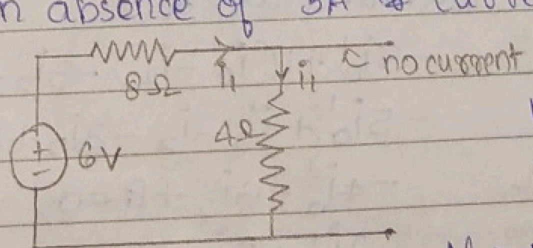
on applying KVL on loop 1:

$$8i + 4(i+3) - 6 = 0$$

$$12i = -6 \Rightarrow i = -0.5$$

$$V = 4(i+3) \Rightarrow V = 4(3 - 0.5) \Rightarrow \boxed{V = 10V}$$

in absence of 3A current source:-



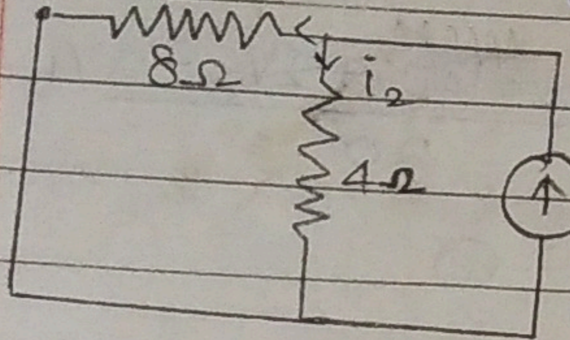
8Ω & 4Ω resistors are in series,

$$R_{eq} = 8\Omega + 4\Omega = 12\Omega$$

$$i_1 = \frac{6}{12} A \Rightarrow 0.5A = i_1$$

$$V_1 = 4i_1 = 4(0.5) \Rightarrow V_1 = 2V \quad \text{(1)}$$

in the absence of voltage source:-



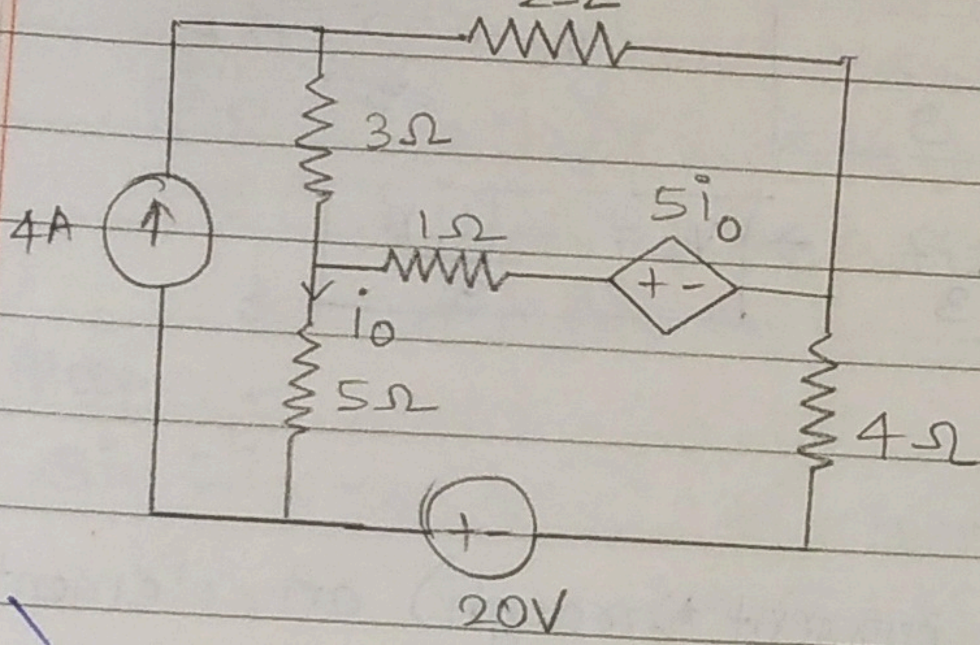
according to current division rule in parallel circuit,

$$i_2 = \frac{8}{4+8}(3) = \frac{2}{3} \times 3 = 2A$$

$V_2 = 4\Omega\,(2A) \Rightarrow V_2 = 8V$ ——— (2)

$V = V_1 + V_2 = 2V + 8V \rightarrow \boxed{V = 10V}$

<u>H.W</u> Find i_o using superposition principle,



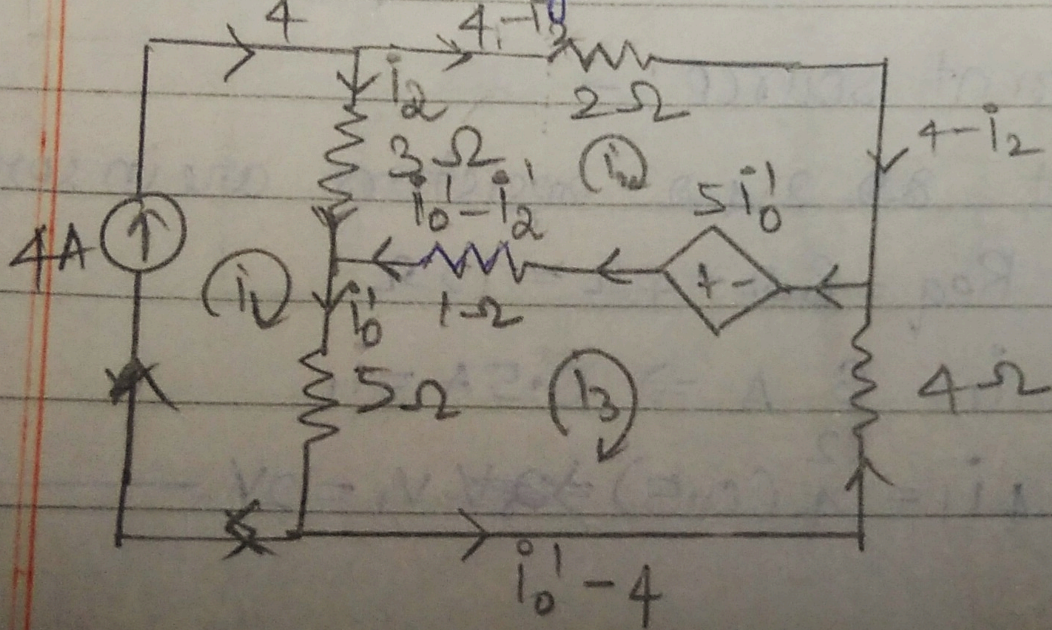
Prove that : $i_o = i_o' + i_o''$

where i_o'' is current through 5Ω resistor when current source is removed &

i_o' is the current through 5Ω resistor when voltage source is removed.

solution :-

when voltage source is removed :-



around loop 2 :-

$$-5i_0' + 1(i_0' - i_2) - 3i_2 + 2(4 - i_2) = 0$$

$$-5i_0' + i_0' - i_2 - 3i_2 + 8 - 2i_2 = 0$$

$$-4i_0' - 6i_2 + 8 = 0$$

$$2i_0' + 3i_2 = 4 \quad \text{--- (1)}$$

around loop 3:- $-4(i_o'-4) - 5i_o' - 1(i_o'-i_2) + 5i_o' = 0$

$-4i_o' + 16 - \cancel{5i_o'} - i_o' + i_2 + \cancel{5i_o'} = 0$

$-5i_o' + i_2 + 16 = 0$ —— (3)

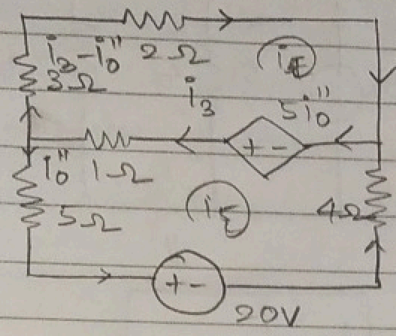
→ $-15i_o' + \cancel{3i_2} = -48$

$2i_o' + \cancel{3i_2} = 4$

$-17i_o' = -52$

$$\boxed{i_o' = \frac{52}{17}\,A}$$

when current source is removed :-



along loop 4:-

$-5i_o'' + i_3 + 3(i_3 - i_o'') + 2(i_3 - i_o'') = 0$

$-5i_o'' + i_3 + 5i_3 - 5i_o'' = 0$

~~4i3~~ $6i_3 - 10i_o'' = 0$

$3i_3 - 5i_o'' = 0$ —— (3)

along loop 5:-

$-i_3 + \cancel{5i_o''} - 4i_o'' - 20 - \cancel{5i_o''} = 0$

$i_3 + 4i_o'' = -20$ —— (4)

$3i_3 - 5i_o'' = 0 \Rightarrow 3(-4i_o'' - 20) - 5i_o'' = 0$

$-12i_o'' - 60 - 5i_o'' = 0 \Rightarrow -17i_o'' = 60$

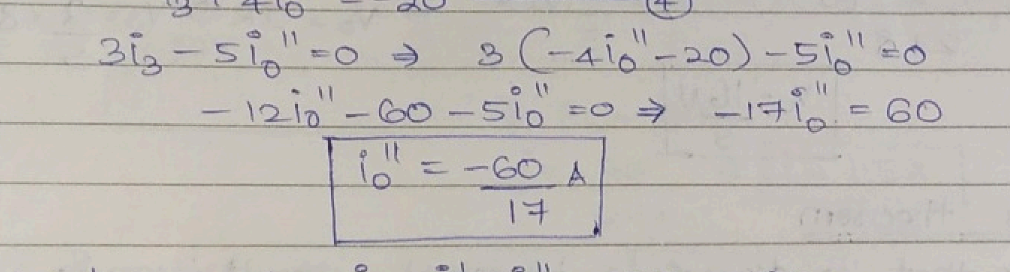
$$\boxed{i_o'' = \frac{-60}{17}\,A}$$

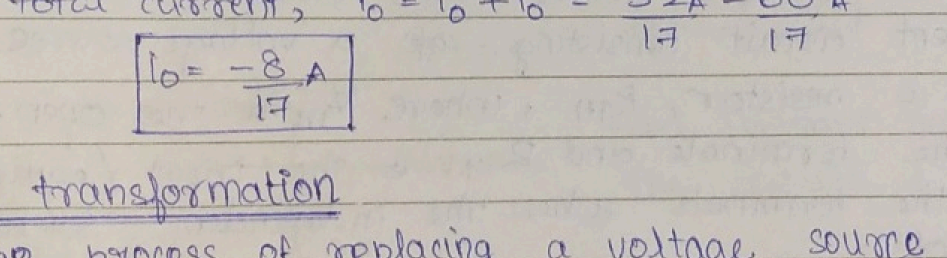
total current, $i_o = i_o' + i_o'' = \frac{52}{17}A - \frac{60}{17}A$

$$\boxed{i_o = \frac{-8}{17}\,A}$$

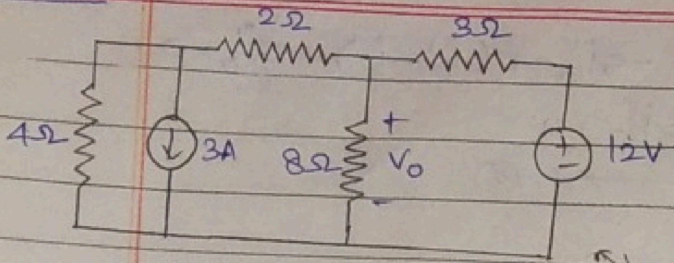
Source transformation

is a process of replacing a voltage source V_s in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.





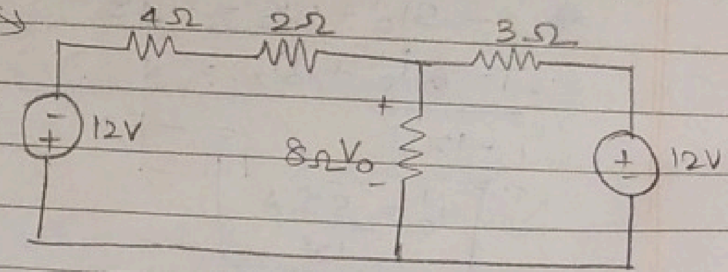
Ex: 1

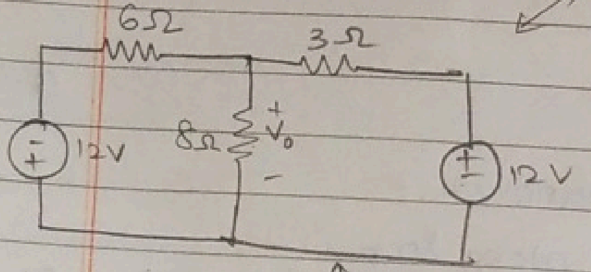


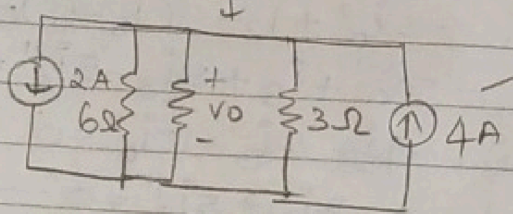
Find V_o = ?

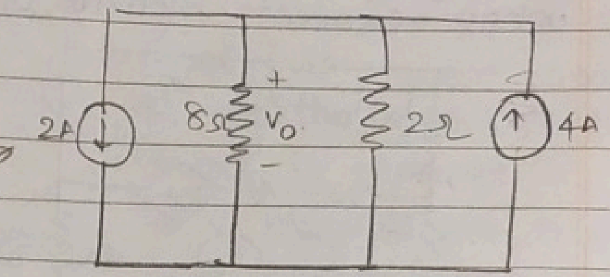
solution:-

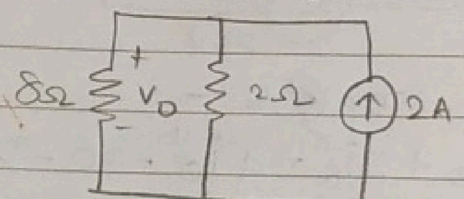
according to source transformation











according to current division rule,

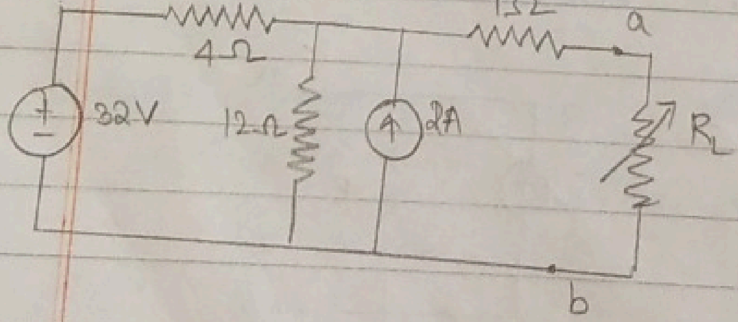
$I_2 = \frac{2}{2+8}(I) \Rightarrow I_2 = \frac{1(2)}{5}\,A \Rightarrow V_o = IR \Rightarrow V_o = \frac{2}{5} \times 8$

$$\boxed{V_o = \frac{16}{5} V}$$

Thevenin's theorem

States that a linear 2-terminal circuit can be replaced by an equivalent circuit consisting of a voltage sourse V_{TH} in series with a resistor R_{TH}, where V_{TH} is the open-circuit voltage at the terminals and R_{TH} is the input / equivalent resistance at the terminals when the independent sources are turned off.

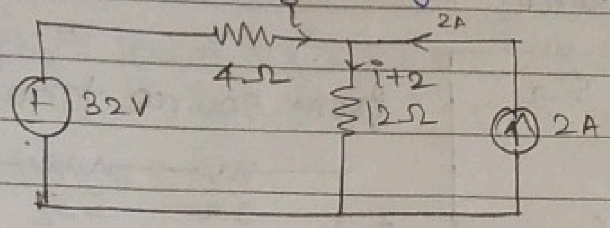
Ex 1: Find



Find the Thevenin equivalent circuit of the given circuit. Find the current through R_L=6, 16 and 36Ω.

To find V_{Th} :-

~~use open circuit~~ ~~use open circuit~~, we remove R_L, hence no current flows through 1Ω resistor.



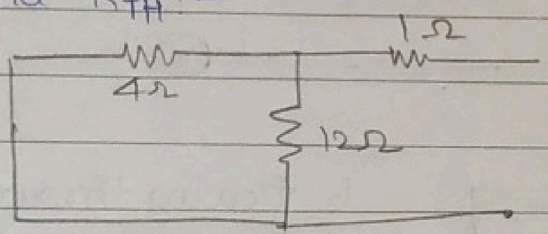
V_{TH} = voltage across 12Ω resistor for which we need to find the current flowing through it.

according to KVL rule :-

$$4i + 12(i+2) = 32 \Rightarrow i = \frac{8}{16} \Rightarrow i = 0.5A$$

voltage across 12Ω = ~~0.5 ×~~ $12(0.5+2) \Rightarrow \boxed{V_{TH} = 30V}$

To find R_{TH} :-

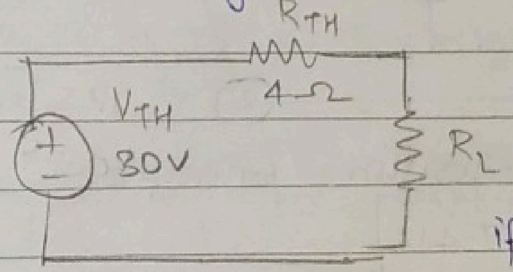


4Ω & 12Ω are in parallel

$$R_p = \frac{(12)(4)}{16} \Rightarrow R_p = 3\Omega$$

$$R_{TH} = 3\Omega + 1\Omega \Rightarrow \boxed{R_{TH} = 4\Omega}$$

Finding I through R_L :-



if $R_L = 6\Omega$

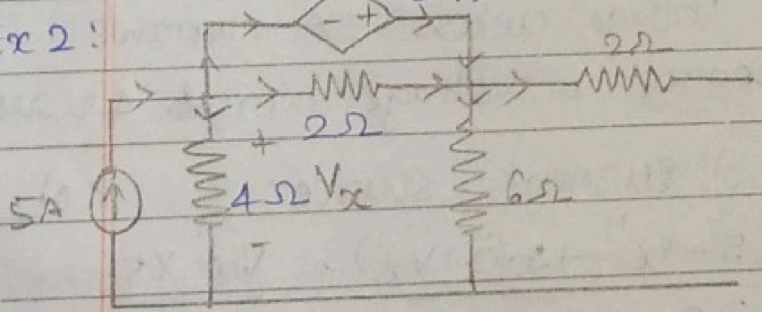
$$i = \frac{30}{6+4} A \Rightarrow \boxed{i = 3A}$$

if $R_L = 16\Omega$

$$i = \frac{30}{16+4} \Rightarrow \boxed{i = 1.5A}$$

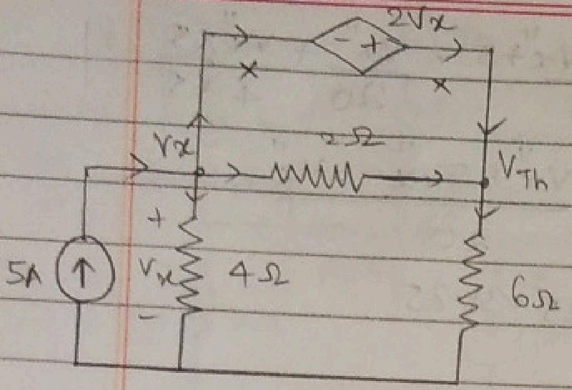
if $R_L = 36\Omega \Rightarrow i = \frac{30}{36+4} \Rightarrow \boxed{i = 0.75A}$

Ex 2 :



Find Thevanin equalent of the circuit.

on calculating V_{th} :-



Voltage across a and b
= Voltage across 6Ω resistor

applying KCL at nodes :-
at node V_x :-

$$5 = \frac{V_x}{4} + \frac{V_x - V_h}{2} + x$$

$$x = 5 - \frac{V_x}{4} + \frac{V_h - V_x}{2}$$

at V_{Th} :-

$$\frac{V_{Th}}{6} = \frac{V_x - V_{th}}{2} + x \Rightarrow x = \frac{V_{th}}{6} + \frac{V_{th} - V_x}{2} = X$$

equating two x values,

$$2V_{th} + 3V_x = 60 \quad \text{——— (1)}$$

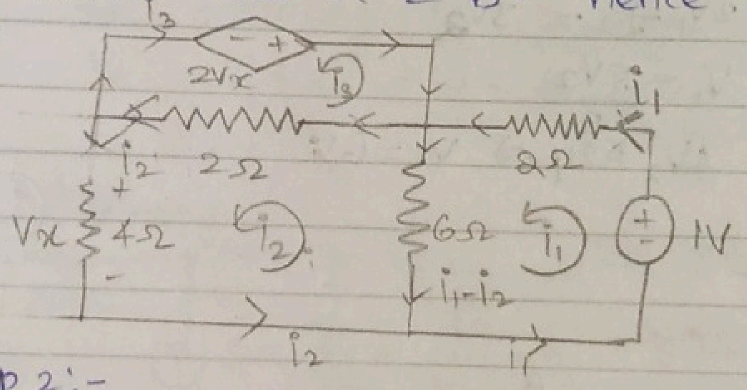
applying KVL across loop 1, $-2V_x + (V_{Th} - V_x) = 0$ ——— (2)

solving (1) & (2)

$$\therefore \boxed{V_{th} = 20V}$$

on calculating R_{Th} :-

Due to the presence of Dependent ~~voltage~~ voltage source which cannot be short circuited, we place an ~~1A~~ 1V voltage source across ~~the~~ a & b. hence $R_{th} = \frac{1}{i_1}$.



applying KVL in loop1 :-

$$-1 + 2i + 6(i_1 - i_2) = 0$$

$$8i_1 - 6i_2 = 0 \quad \text{——— (1)}$$

in loop 2 :-

$$6i_2 - 3i_1 - i_3 = 0 \quad \text{——— (2)}$$

in loop3 :-

$$2(i_2 - i_3) + 2V_x = 0 \qquad V_x = 4i_2$$

$$2(i_2 - i_3) + 2(4i_2) = 0 \quad \text{——— (3)}$$

solving (1), (2), (3), $\quad i_1 = \frac{1}{6} A$

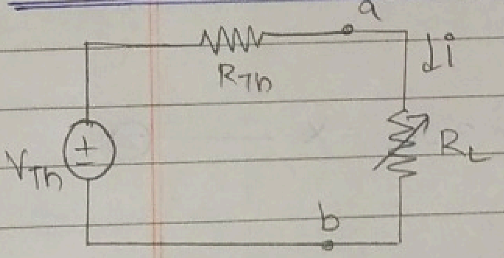
$$R_{Th} = \frac{1}{i_1} = \frac{1}{1/6} \Rightarrow \boxed{R_{Th} = 6\Omega}$$

Norton's theorem : a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit I through the terminals & R_N is the input or equivalent resistance at the terminals when the independent sources ~~of~~ ~~tu~~ are turned off.

* Can do the previous sum ~~using~~ by using Norton's theorem connecting 1A current source in ~~parallel to~~ ~~serie~~ series with 2Ω resistor to calculate I_N.

hence $\frac{V_{Th}}{I_N} = R_{Th}$

Maximum Power transfer theorem



Power of $R_L \Rightarrow P_L = i^2 R_L$

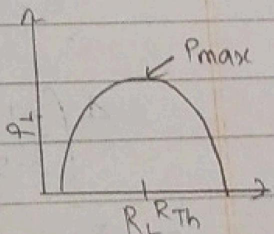
$i = \frac{V_{Th}}{R_{TH}+R_L}$

$P_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$

maximum current is drawn, when

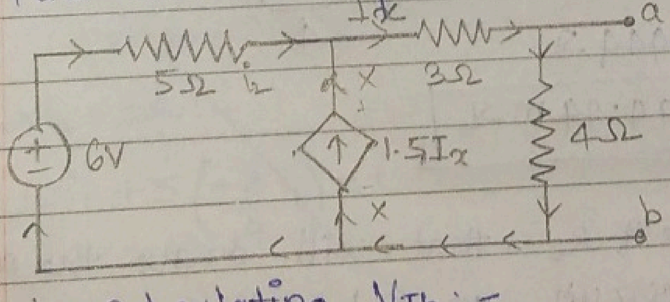
$R_{Th} = R_L$

hence $P_L = \left(\frac{V_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{V_{Th}^2}{4R_{Th}^2} \cdot R_{Th}$



$P_L = \frac{V_{Th}^2}{4R_{Th}}$

H.W : Find Thevenins equivalent of the circuit



solution : calculating V_{Th}:-

V_{TH} = Voltage across 4Ω resistor.

applying KVL along loop1:-

$3I_x + 4I_x - X = 0 \Rightarrow X = 7I_x$

applying KVL along loop2:-

$-6 + 5(-0.5I_x) + \cancel{1.5I_x}\ X = 0 \Rightarrow -6 - 2.5I_x + \cancel{I_x}\ X = 0$

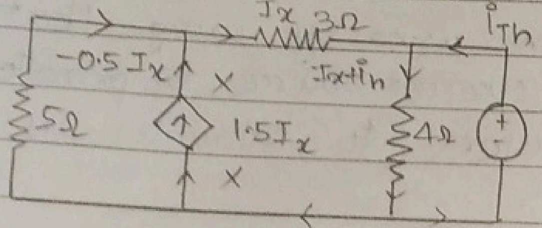
$X = 6 + 2.5I_x \Rightarrow 7I_x - 2.5I_x = 6 \Rightarrow 4.5I_x = 6$

$I_x = \frac{6}{\cancel{4.5}} = \frac{\cancel{12}\,4}{\cancel{9}\,3} A$. Voltage across 4Ω resistor:

$= \frac{\cancel{12}\,4}{\cancel{9}\,3} A \times 4\Omega = \cancel{6.85V} = \frac{16}{3} V$

$\boxed{V_{Th} = 5.333\,V}$

Finding R_{Th}:- removing voltage source & adding an 1V voltage source across a & b, so that $i_{Th} = \frac{1}{R_{Th}}$



applying KVL on loop 1 :-

$4(I_x + i_h) - 1 = 0 \Rightarrow 4I_x + 4i_{Th} = 1$ —— (1)

applying along loop 2 :-

$4(I_x + i_{Th}) - x + 3I_x = 0$

$4I_x + 4i_{Th} + 3I_x = x \Rightarrow 1 + 3I_x = x$ —— (2)

along loop 3 :-

$+x = 5(0.5 I_x) \Rightarrow x = +2.5 I_x$

$1 + 3I_x = +2.5 I_x \Rightarrow \cancel{+5.5 I_x = 1}\ 0.5 I_x = -1$

$\cancel{I_x = \frac{-1}{5.5} A}$ $I_x = -2A$

$\cancel{4\left(\frac{-1}{5.5}\right) + 4i_{Th} = 1 \Rightarrow 4i_{Th} = 1 + \frac{40}{55}}$

$\cancel{4i_{Th} = \frac{19}{11} \Rightarrow i_{Th} = \frac{19}{44} A}$

$\cancel{R_{Th} = \frac{44}{19}\Omega}$

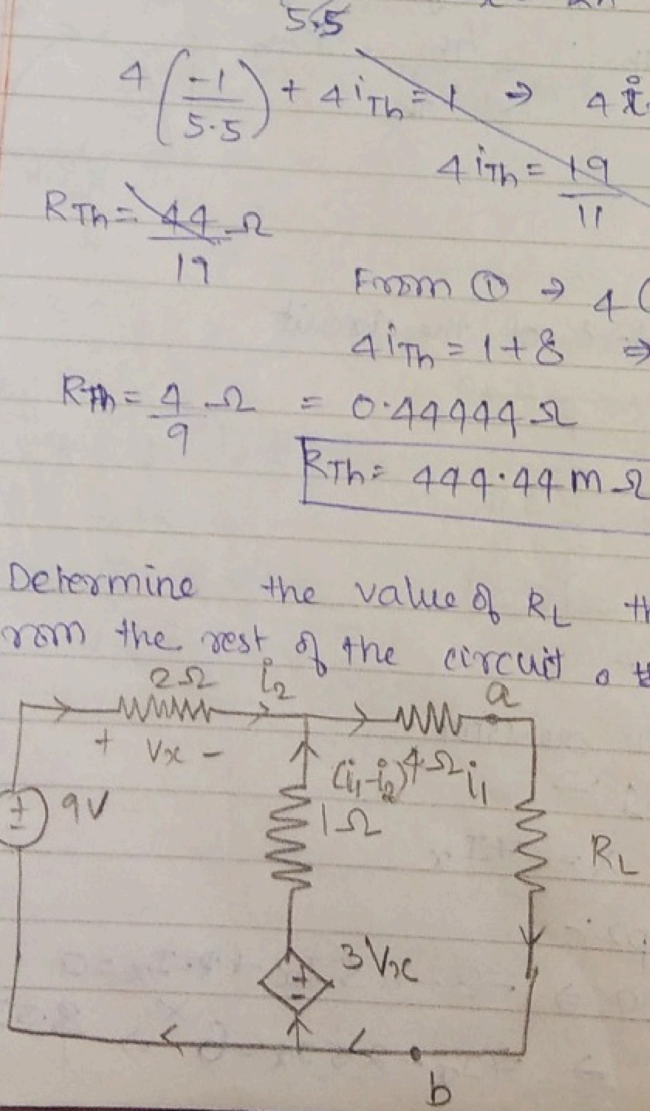
From (1) $\Rightarrow 4(-2) + 4i_{Th} = 1$

$4i_{Th} = 1 + 8 \Rightarrow i_{Th} = \frac{9}{4} A$

$R_{Th} = \frac{4}{9}\Omega = 0.44444\,\Omega$

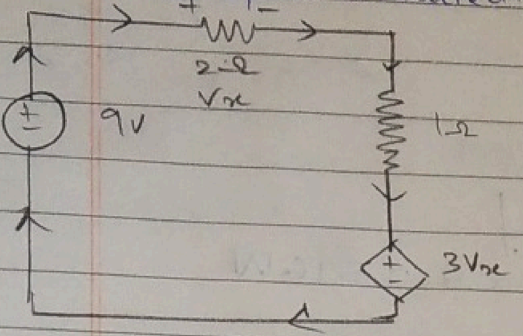
$\boxed{R_{Th} = 444.44\,m\Omega}$

H.W: Determine the value of R_L that will draw the max power from the rest of the circuit & Calculate maximum power



Calculating V_{Th}:- Voltage across R_L = V_{Th} = (voltage across 1Ω resistor & dependent source).

If R_L is open circuited.



$-9 + V_x + 1i + 3V_x = 0$

$i = \frac{3V_x - 9}{3}$

$-9 + V_x + \frac{3V_x - 9}{3} + 3V_x = 0$

$4V_x - 9 + \frac{3x - 9}{3} = 0$

$12V_x - 27 + 3V_x - 9 = 0$

$15V_x = 36 \Rightarrow \boxed{V_x = 2.4V}$

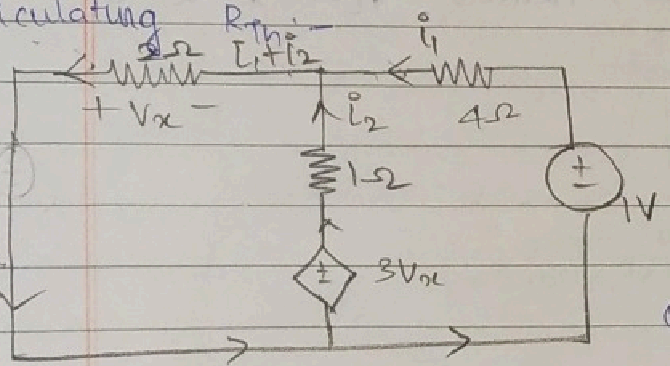
27
9
36

~~Voltage across them~~ $V_{Th} = \frac{3(2.4) - 9}{3} + 3(2.4)$

$= \cancel{7.2}\ -0.6 + 7.2 =$

$\boxed{V_{Th} = 6.6V}$

Calculating R_{Th}:-



On connecting 1V voltage source across the open circuited load, we get

$R_{Th} = \frac{1}{i_1}$

On applying KVL on loop1:-

$-1 + 4i_1 - 1i_2 + 3V_x = 0 \Rightarrow 4i_1 - i_2 + 6i_1 + 6i_2 = 1$

$\Rightarrow 10i_1 + 5i_2 = 1$

loop 2:-

$-3V_x + i_2 - V_x = 0$; $V_x = -2(i_1 + i_2)$

$-4V_x + i_2 = 0 \Rightarrow -8i_1 - 8i_2 = i_2 \Rightarrow -8i_1 = 9i_2$

$\Rightarrow i_2 = \frac{-8}{9} i_1$

(1) $\Rightarrow 10i_1 + 5\left(\frac{-8}{9} i_1\right) = 1 \Rightarrow 10i_1 - \frac{40i_1}{9} = 1 \Rightarrow \frac{50i_1}{9} = 1 \Rightarrow \boxed{i_1 = \frac{9}{50} A}$

$R_{Th} = \frac{50}{9}\Omega = 5.55$

$V_x = -2(i_1 + i_2)$

~~sol.~~ loop 1:- $-1 + 4i_1 - i_2 + 3V_x = 0 \Rightarrow -1 + 4i_1 - i_2 - 6i_1 - 6i_2 =$

$-2i_1 - 7i_2 = 1$ —— (1)

loop2:- $-3V_x + i_2 - V_x = 0 \Rightarrow -4V_x + i_2 = 0 \Rightarrow 8i_1 + 8i_2 + i_2 = 0$

~~$i_1 = 0$~~ $8i_1 = -9i_2 \Rightarrow i_2 = \frac{-8}{9} i_1$

Date 4/6

From ① ⇒ $-2i_1 - 7\left(\frac{-8}{9}\right) i = 1$

$\frac{-18i_1 + 56i_1}{9} = 1 \Rightarrow i_1 = \frac{9}{38} A$

$R_{th} = \frac{38}{9} \Omega = 4.222 \Omega$

For max power ⇒ $R_{Th} = R_L$

$\boxed{R_L = 4.222 \Omega}$

$P_L = \frac{V_{Th}^2}{4R_L} = \frac{(6.6)^2}{4 \times 4.222} = 2.596 W$

$\frac{18}{38}$

$P = IV$

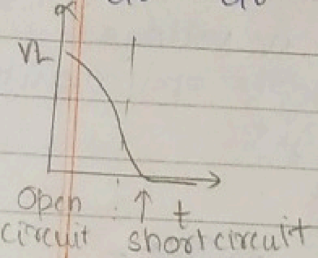
$P = \frac{V}{R} \cdot V$

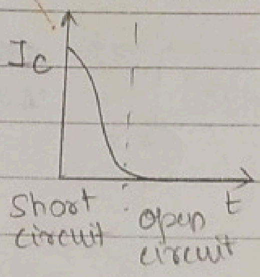
MODULE : 2 : AC circuits

* We dont connect capacitor / inductor in DC circuits,

$V_L = -L \frac{di}{dt}$; $i_c = C \frac{dv}{dt}$

$\frac{di}{dt}, \frac{dv}{dt} = 0$ in DC circuit, hence, $V_L = 0$ & $i_c = 0$





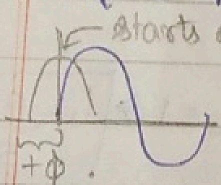
Equalent capacitance

series ⇒ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$; Parallel ⇒ $C_{eq} = C_1 + C_2$

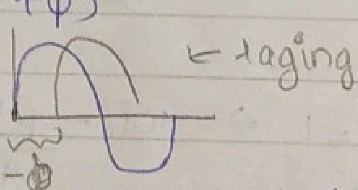
* AC : current & Voltage are sinosoidal

$V_t = V_m \sin(\omega t + \phi)$

(should see like which wave reaches peak first / 0 first)



$V_t = V_m \sin(\omega t + \phi)$



$V_t = V_m \sin(\omega t - \phi)$

* $V_{av} = \frac{1}{\pi} \int_0^{\pi} V_m \sin\omega t \, d\omega t$

$= \frac{-V_m}{\pi} [\cos\omega t]_0^{\pi}$

$\boxed{V_{av} = \frac{2V_m}{\pi}}$

* $V_{RMS} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$