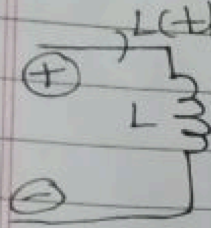
AC

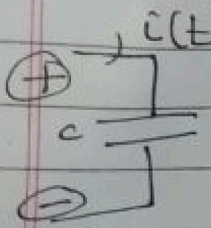
{Plug hole}
N | neutral
Ground/earth → +ve | Phase

Reason of earth → for safety of all people. and to prevent shock.



$\rightarrow v(t) = L(t)\dfrac{di}{dt}$

{ in passive sign → + }
{ in active → − }

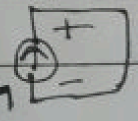


$$v(t) = \frac{1}{c}\int_{-\infty}^{t} i(\xi)\,d\xi$$

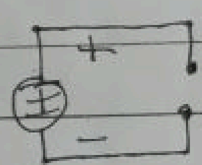
or

$$i(t) = c\frac{dv}{dt}$$

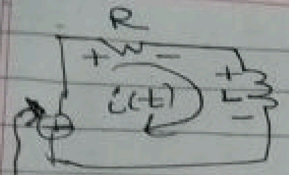
① if DC source added in inductor → { $i(t) = 5A$ }
$v(t) = 0$.
so in DC at steady state inductor acts as short circuit → short



② if DC source in capacitor →
(Say $v(t) = 5$ }
$i(t) = 0$.
so acts as open circuit → open



DC energy in inductor → $\frac{1}{2}Li^2$
" " " Capacitor → $\frac{cv^2}{2}$



$5\cos(2\pi(50)t)$

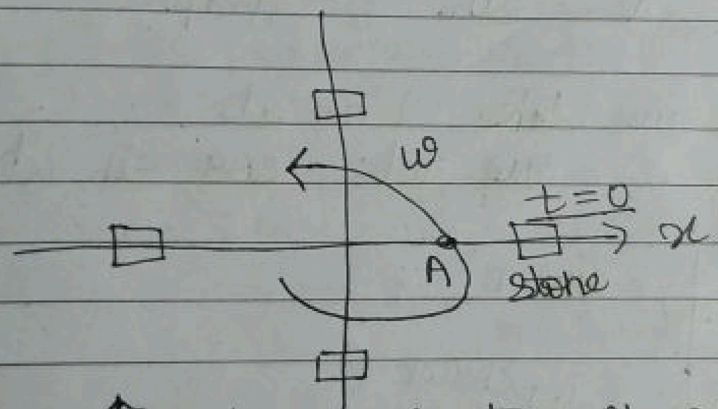
$v(t) = Ri(t) + L\frac{di}{dt}$

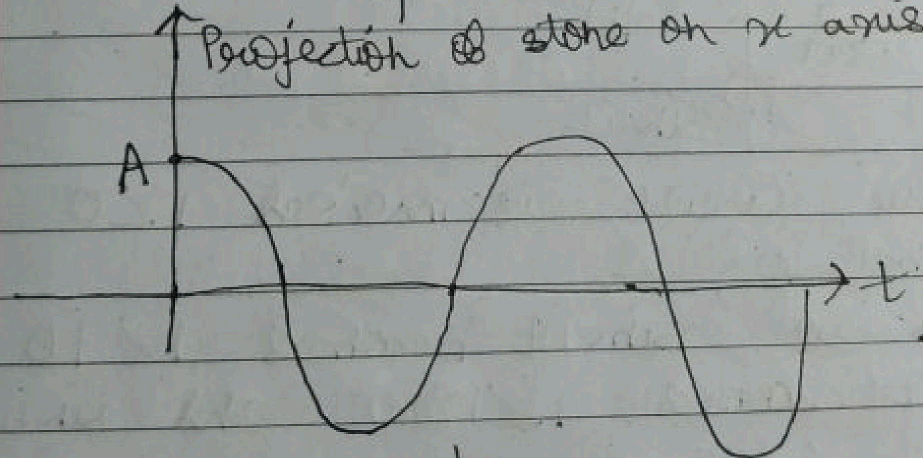
$5\cos(2\pi(50)t) = Ri(t) + L\frac{di}{dt}$

for linear system →

$A\cos\omega t \xrightarrow{I/P}$ $\boxed{\text{linear system}}$ $\longrightarrow B\cos(\omega t+\phi)$

Phasors





↓ graph of cos f^n.

if we change initial pos^n →

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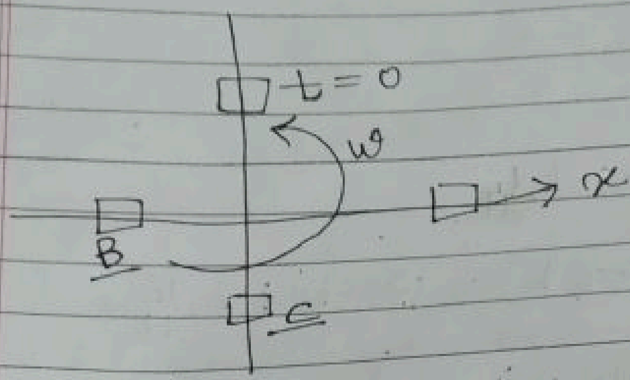
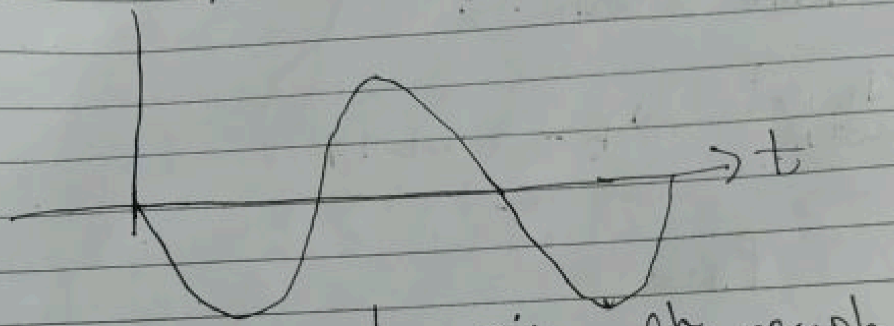
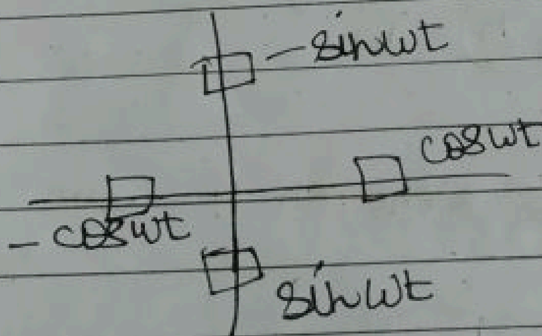




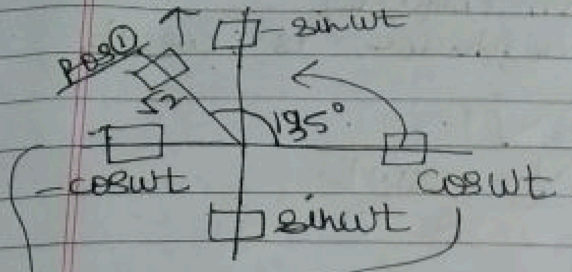
↓ $-\sin$ f^n graph

similarly if we take $t=0$ at B and C we get $-\cos$, $\sin$ f^{n}s



now $\cos\omega t$ is marked $1\angle 0$
{1 angle 0}
so $-\sin\omega t$ becomes $1\angle 90$
$-\cos\omega t$ becomes $1\angle 180$ and $\sin\omega t$ becomes $1\angle -90$

AC Circuits – Phasor Analysis

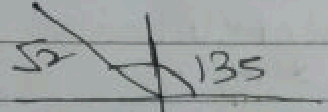


(0 + 1j)

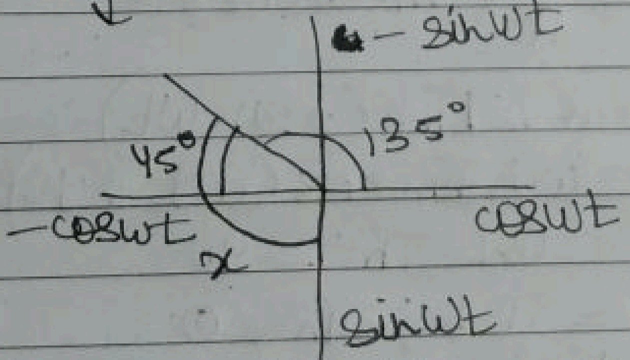
reference phasor is $1\cos\omega t$ or $1\angle 0$
$-\sin\omega t$ in phasor is $1\angle 90$ or $1j$
or in time domain $1\cos(\omega t+90)$
$= -\sin\omega t$.

as reference has cos.

at Pos1 →



ref phasor → $1\cos\omega t$
this becomes $\sqrt{2}\angle 135$
or in time domain → $\sqrt{2}\cos(\omega t+135)$
or $\sqrt{2}\sin(\omega t-135)$



to express in sin we find angle of it
with $\sin\omega t$ or angle x
angle x is 135 as we see from figure
but direction is −ve.
so -135°.
so $\sqrt{2}\sin(\omega t-135)$

$v(t) = V_m \cos(\omega t)$

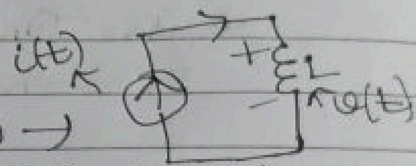
given → ref phasor is $1\cos\omega t$

so $v(t) \to \boxed{V_m \angle 0 = V}$

now $i(t) = \frac{v(t)}{R} = \frac{V_m \cos\omega t}{R}$

so $I \to \frac{V_m}{R} \angle 0$

$\boxed{V = IR}$



Different Circuit now →

$i(t) = I_m \cos\omega t$, $v(t) = L\frac{d\,i(t)}{dt}$

$\downarrow$
$I_m \angle 0$

NOTE → Capital letters denote Phasor Domain stuff

$v(t) = L\frac{d}{dt}(I_m \cos\omega t)$

$= \omega L I_m(-\sin\omega t)$

$= \omega L I_m \cos(\omega t + 90)$

$V = \omega L I_m \angle 90$

$V = \omega L j I_m \angle 0$

$v(t) = L\frac{di}{dt}$, $V = L j\omega I_m$

comparing we get that $\frac{d}{dt}$ can be replaced by $j\omega$.

and

$V = \boxed{j\omega L} I$

$\boxed{V = X_L I}$

$X_L \to$ (in class 12)

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New Circuit

$i(t) = I_m \cos\omega t$

$v(t) = \frac{1}{C}\int i(\tau)\, d\tau$

now $v(t) = \frac{1}{C}\int I_m \cos\omega t \, dt$

$= \frac{1}{C}\frac{I_m}{\omega}\sin\omega t$

$= \frac{I_m}{\omega C}\cos(\omega t - 90)$

$V = \frac{I_m}{\omega C}\angle -90$

$V = \frac{I_m}{\omega C} \times 1\angle 0 \times -j$

$V = + \frac{I_m}{j\omega C}\angle 0$

$\boxed{V = \frac{I_m}{j\omega C}\angle 0}$

as $-j = \frac{-j \times j}{j} = \frac{-j^2}{j} = \frac{1}{j}$

now $v(t) = \frac{1}{C}\int i(t)\,dt$

comparing we see that $\int$ can be replaced by $\frac{1}{j\omega}$ for capacitor $X_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

Q) find steady state response of $v(t)$

$2\frac{dv(t)}{dt} + 5v(t) + 10\int v(t) = 5\cos(5t - 30)$

ans) as steady state → Phasor

ref phasor $= 1\cos 5t = 1\angle 0$

$\frac{d}{dt} = j\omega$, $\int = \frac{1}{j\omega}$

$\frac{d}{dt} = 5j$ $\quad \int = \frac{1}{5j}$

So →

$2\times 5j\times V + 5V + 10\times\frac{1}{5j}\times V = 5\angle -30$

$\to V(10j + 5 - 2j) = 5\angle -30$

$V = \frac{5\angle -30}{5+8j}$ {we use calculator here}

$= 0.529\angle -87.99$

$v(t) = 0.529\cos(5t - 87.99)$

Power

$v(t) = \omega L I_m \cos(\omega t + 90)$

$V = j\omega L I_m$

(derived earlier)

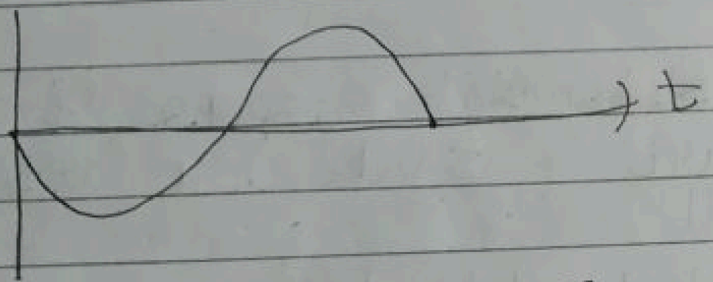
$v(t) = A(-\sin\omega t)$ $\{A = \omega L I_m\}$

$i(t) = B\cos\omega t$ $\{B = I_m\}$

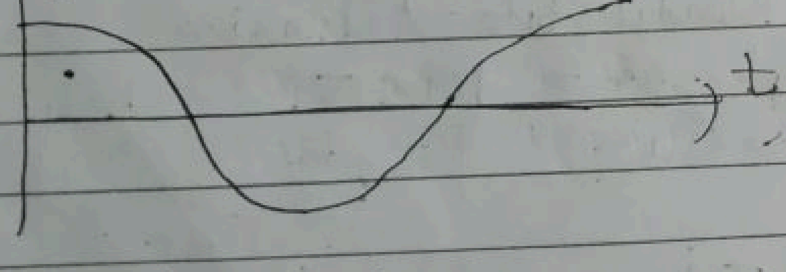
$p(t) = v(t)\times i(t)$

$= AB\cos\omega t \times -\sin\omega t$

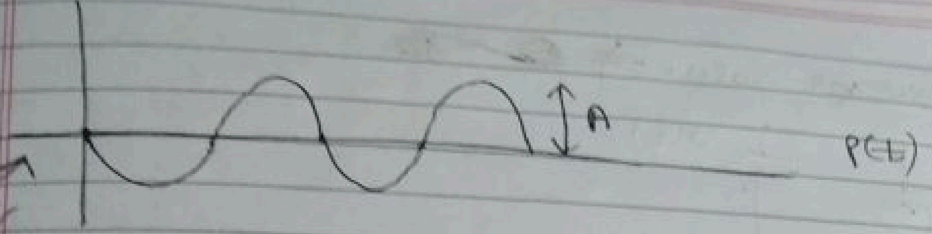
$v(t) =$



$i(t) =$



multiply graphically →



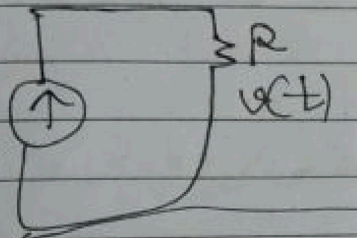
divide into quarter cycle and then multiply values $- \times - = +$, $- \times + = -$ and so on

One quarter cycle it absorbs and in next it releases

average value = Real power = 0 = P

Amplitude = Q = peak power of oscillating = A

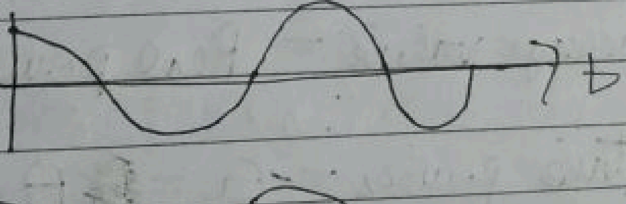
+ as Q in Passive sign for inductor is +ve



$i(t) = I_m \cos\omega t = B\cos\omega t$

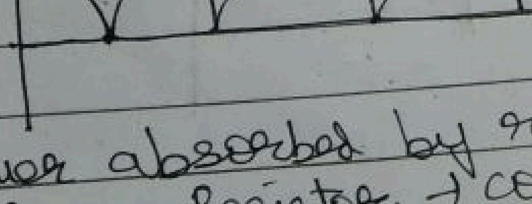
$v(t) = I_m \times R\cos\omega t = A\cos\omega t$

i(t) →



v(t) →

P(t) →



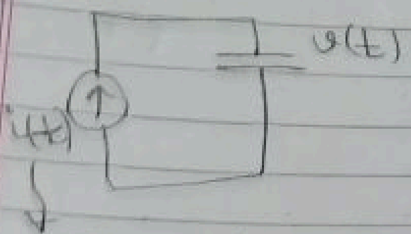
→ pulsating power

power absorbed by resistor

in DC → power Resistor → const

in AC → power resistor → pulsating

Average Value $\neq$ 0

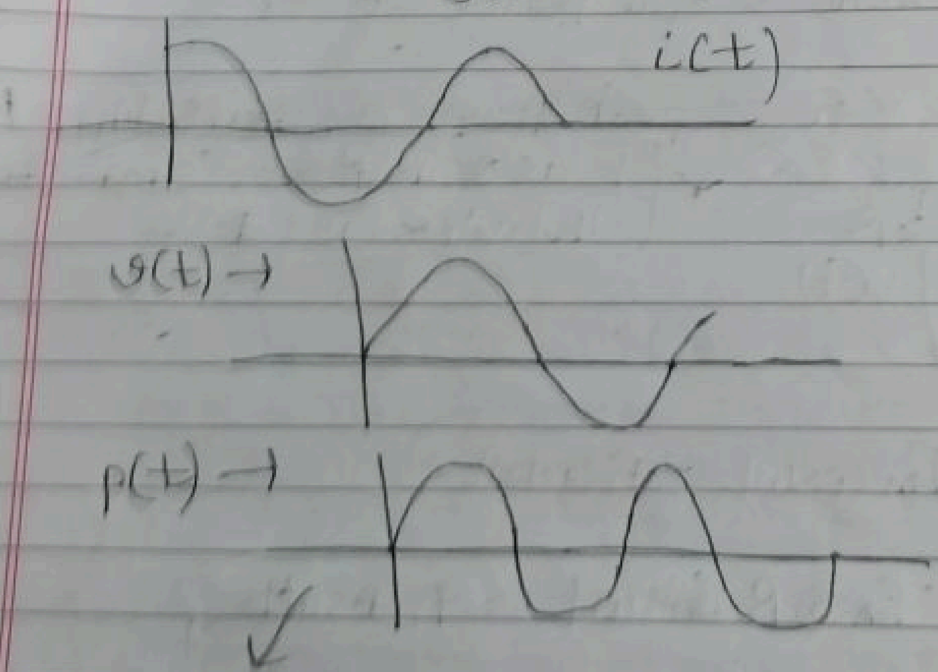


In circuit

$$v(t) = \frac{1}{C}\int i(\tau)\,d\tau = \frac{I_m}{\omega C}\sin\omega t$$

$$= \frac{1}{\omega C} \times I_m \angle -90$$

$$= \frac{I_m}{j\omega C}$$

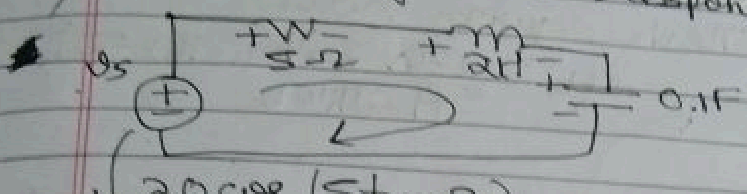


average value = Real power = 0

Reactive power = $-Q$ = A

-ve as Q in passive sign for capacitor → -ve

Q) find steady state response →



$20\cos(5t-30)$

ans) using KVL →

Method 1

$v_s = V_R + V_L + V_C$

$20\cos(5t-30) = 5i(t) + 2\frac{di(t)}{dt} + \frac{1}{0.1}\int i(t)dt$

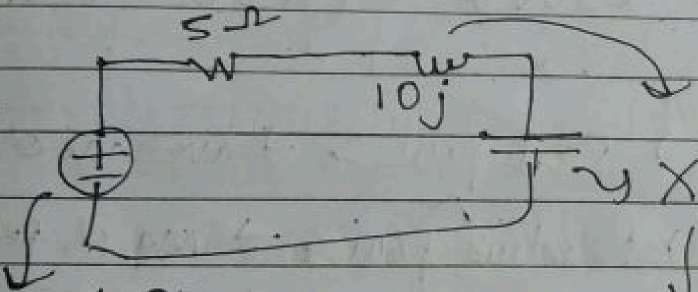
$20\angle -30 = 5i + 2j\omega \times I + \frac{I}{j\omega}$

$20\angle -30 = (5+8j)I$

$I = \frac{20\angle -30}{5+8j} = 2.119\angle -87.99$

$= 2.119\cos(5t-87.99)$

Method 2



$20\angle -30$

$X_L = j\omega L = 10j$

$X_C = \frac{-j}{\omega C} = -2j$

$\left\{ X_C = \frac{1}{j\omega C} = -j/\omega C \right\}$

$Z_{eq} = 5 + 10j - 2j = 5 + 8j.$

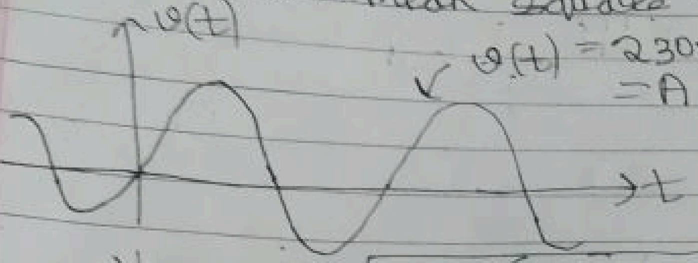
$I = \frac{20\angle -30}{5+8j}$ (ans)

$= 2.119\angle -87.99$

$= 2.119\cos(5t-87.99)$

AC Circuits

RMS → Root Mean square value



$v(t) = 230\sqrt{2}\sin(2\pi \times 50t)$
$= A\sin\omega t$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \sin^2\omega t\,dt \times A^2}$$

$$= \sqrt{\frac{A^2}{T}\int_0^T \frac{1-\cos 2\omega t}{2}}$$

$$= A\sqrt{\frac{1}{T}\int_0^T \frac{1}{2}\,dt}$$

$$= \frac{A}{\sqrt{2}} \Rightarrow = \frac{230\sqrt{2}}{\sqrt{2}} = 230 = V_{rms}$$

$P = V_{rms} I_{rms} \times \cos\phi$
$Q = V_{rms} I_{rms} \sin\phi$

→ AC Circuit → Real power → average of $P(t)$
↳ Reactive power → mag of power osci

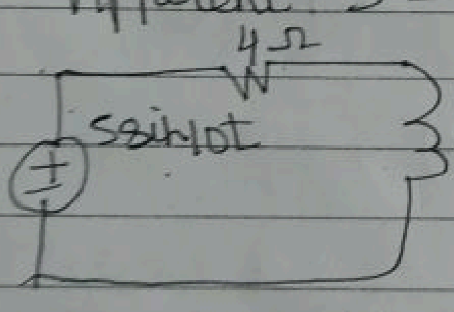
resistor

↓ inductor, capacitor

Apparent $S = V_{rms} \times I_{rms}^* = P \pm jQ$

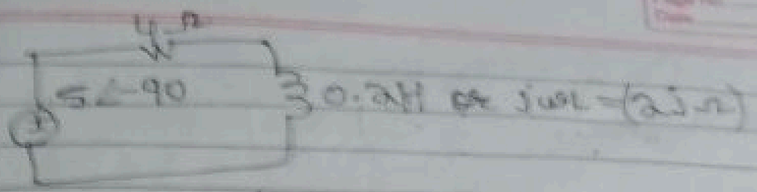
↓ conjugate

Q)



reference → $1\cos 10t \to 1\angle 0$

Ans) →



$Z_{eq} \rightarrow (4+2j)\,\Omega$

$$I = \frac{5\angle -90}{4+2j}$$

$= 1.118033989 \angle -116.565052$

$i(t) = 1.118\cos(10t - 116.56)\,A$

power supplied $= V_{rms} \times I_{rms}^{*}$ → conjugate

$$= \frac{5\angle -90}{\sqrt{2}} \times \left(\frac{1.12\angle -116.56}{\sqrt{2}}\right)^{*}$$

$$= \frac{5\angle -90}{\sqrt{2}} \times \frac{1.12}{\sqrt{2}} \angle 116.56$$

$\{(x+jy)^{*} = x - jy\}$

conjugate

$= (2.5 + 1.25j)$ } we use cartesian for this

$= P + jQ$

Watts — Volt ampere Reactive or VAR

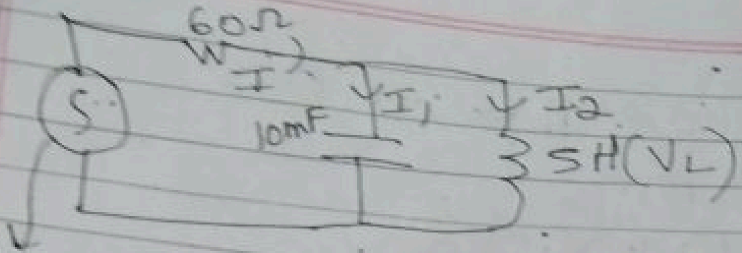
power factor = polar form of Z angle's cosine

cos(angle form ($4.472\angle 26.56505$)

$\cos(26.56505) = 0.894$

or cos(phase diff of voltage and current)

Q)



$20\cos(4t-15)$

$20\angle -15$ ref → $1\cos 4t$ → $1\angle 0$

ans) $10mF = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \underline{-25j}$

$5H \to j\omega L = 20j$

$\frac{1}{Z_{eq}} = \frac{1}{Z_{cap}} + \frac{1}{Z_{ind}}$

$Z_{eq} = \left(\frac{1}{Z_C} + \frac{1}{Z_L}\right)^{-1}$

$Z_{eq} = 100j$

$Z_{final} = 60 + 100j$

so $I = \frac{20\angle -15}{60+100j} = 0.171\angle -74.03$

Power Supp $= \frac{20\angle -15}{\sqrt{2}} \left(\frac{0.171\angle 74.03}{\sqrt{2}} \right)$

$= 0.88\,W + 1.46\,VAR$

V across inductor $= Z_{eq} = 100j$ and

$I = 0.174\angle -74.02$

$V_L = 0.174 \times \angle -74.02 \times 100j$

$= 17.15\angle 15.78$ V.