VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: C1+TC1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer <u>ALL</u> the Questions

1. Verify Euler's theorem for the function $f(x, y, z) = 3x^2yz + 5xy^2z + 4z^4$.

2. If $u = x^2 - y^2, v = 2xy$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,(x+y)dxdy$.

4. Evaluate $\int_0^\infty \sqrt{x}\;\; e^{-4\sqrt{x}}dx$ using beta-gamma functions.

5. Find the angle between the normals to the surface $xy = z^2$ at the point (-2, -2, 2) and (1, 9, -3).

6. Evaluate $\iint_S (xdydz + 2ydzdx + 3zdxdy)$ where S is the closed surface of the sphere $x^2 + y^2 + z^2 = a^2$.

7. Find the solution of $y'' - 4y' + 4y = 0$ which satisfies $y(0) = 3, y'(0) = 1$.

8. Solve $\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 0$.

9. Find the Laplace transforms of $[t^2 \sin^2 t]$.

10. Find $L^{-1}\left(\frac{s+2}{s^2-4s+13}\right)$

PART – B (5 X 14 = 70 Marks)
Answer any <u>FIVE</u> Questions

11. (a) Expand the Taylor's series expansion of $e^x \log(1+y)$ at the point (0, 0) to third approximation. **[7]**

(b) Find the volume of greatest rectangle parallelepiped that can be insecribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **[7]**

12. (a) Change the order of integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$ and then evaluate it. **[7]**

(b) Find the value of $\iint x^{m-1} y^{n-1} dxdy$, over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, in terms of Gamma functions. **[7]**

13. (a) Verify Green's theorem in a plane with respect to $\int_C [(x^2-y^2)dx+2xydy]$, where C is the boundary of the rectangle in the xoy – plane bounded by the lines $x=0, x=a, y=0$ *and* $y=b$ **[7]**

(b) If $\overline{F}=4xz\,\hat{i}-y^2\,\hat{j}+yz\,\hat{k}$, evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$. **[7]**

14. (a) Solve the equation $\frac{d^2y}{dx^2}+y=x\cos x$, by the method of variation of parameters. **[7]**

(b) Solve the differential equation $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=96x$. **[7]**

15. (a) Solve $y''-3y'+2y=4t+e^{3t}$ using Laplace transform given that $y(0)=1$, *and* $y'(0)=-1$. **[7]**

(b) Use convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$ **[7]**

16. (a) Use convolution theorem to evaluate $L^{-1}\left(\frac{1}{(s-2)(s^2+1)}\right)$. **[7]**

(b) Find the maxima and minima of the function $x^3+y^3-3x-12y+20$. Find also the saddle points. **[7]**

17. (a) Evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F}=yz\,\hat{i}+zx\,\hat{j}+xy\,\hat{k}$ and S is the part of the sphere $x^2+y^2+z^2=1$ that lies in the first octant. **[7]**

(b) Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ **[7]**

⇔⇔⇔