Course Code	Course Title	L	T	P	J	C
MAT-1011	Calculus for Engineers	3	0	2	0	4
Pre-requisite	10+2 Mathematics or MAT1001	Syllabus Version				
		1.0				

Course Objectives (CoB):1,2,3

1. To provide the requisite and relevant background necessary to understand the other important engineering mathematics courses offered for Engineers and Scientists.
2. To introduce important topics of applied mathematics, namely Single and Multivariable Calculus and Vector Calculus etc.
3. To impart the knowledge of Laplace transform, an important transform technique for Engineers which requires knowledge of integration

Course Outcome (CO): 1,2,3,4,5,6

At the end of this course the students should be able to

1. apply single variable differentiation and integration to solve applied problems in engineering and find the maxima and minima of functions
2. understand basic concepts of Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution
3. evaluate partial derivatives, limits, total differentials, Jacobians, Taylor series and optimization problems involving several variables with or without constraints
4. evaluate multiple integrals in Cartesian, Polar, Cylindrical and Spherical coordinates.
5. understand gradient, directional derivatives, divergence, curl and Greens', Stokes, Gauss theorems
6. demonstrate MATLAB code for challenging problems in engineering

Student Learning Outcome (SLO): 1, 2, 9

[1] Having an ability to apply mathematics and science in engineering applications
[2] Having a clear understanding of the subject related concepts and of contemporary issues
[9] Having problem solving ability- solving social issues and engineering problems

Module:1	Application of Single Variable Calculus	9 hours	CO: 1

Differentiation- Extrema on an Interval-Rolle's Theorem and the Mean Value Theorem-Increasing and Decreasing functions and First derivative test-Second derivative test-Maxima and Minima-Concavity. Integration-Average function value - Area between curves - Volumes of solids of revolution -

Module:2	Laplace transforms	7 hours	CO: 2

Definition of Laplace transform-Properties-Laplace transform of periodic functions-Laplace transform of unit step function, Impulse function-Inverse Laplace transform-Convolution.

Module:3	Multivariable Calculus	4 hours	CO: 3

Functions of two variables-limits and continuity-partial derivatives –total differential-

Jacobian and its properties.			
Module:4	**Application of Multivariable Calculus**	**5 hours**	**CO: 3**
Taylor's expansion for two variables–maxima and minima–constrained maxima and minima-Lagrange's multiplier method.			
Module:5	**Multiple integrals**	**8 hours**	**CO: 4**
Evaluation of double integrals–change of order of integration–change of variables between Cartesian and polar co-ordinates - Evaluation of triple integrals-change of variables between Cartesian and cylindrical and spherical co-ordinates- Beta and Gamma functions–interrelation -evaluation of multiple integrals using gamma and beta functions.			
Module:6	**Vector Differentiation**	**5 hours**	**CO: 5**
Scalar and vector valued functions – gradient, tangent plane–directional derivative-divergence and curl–scalar and vector potentials–Statement of vector identities-Simple problems			
Module:7	**Vector Integration**	**5 hours**	**CO: 5**
line, surface and volume integrals - Statement of Green's, Stoke's and Gauss divergence theorems -verification and evaluation of vector integrals using them.			
Module:8	**Contemporary Issues:**	**2 hours**	**CO: 1, 2, 3,4,5**
Industry Expert Lecture			
	Total Lecture hours:	**45 hours**	
Text Book(s)			
[1] Thomas' Calculus, George B.Thomas, D.Weir and J. Hass, 13th edition, Pearson, 2014. [2] Advanced Engineering Mathematics, Erwin Kreyszig, 10th Edition, Wiley India, 2015.			
Reference Books			
1. Higher Engineering Mathematics, B.S. Grewal, 43rd Edition ,Khanna Publishers, 2015 2. Higher Engineering Mathematics, John Bird, 6th Edition, Elsevier Limited, 2017. 3. Calculus: Early Transcendentals, James Stewart, 8th edition, Cengage Learning, 2017. 4. Engineering Mathematics, K.A.Stroud and Dexter J. Booth, 7th Edition, Palgrave Macmillan (2013)			
Mode of Evaluation			
Digital Assignments, Quiz, Continuous Assessments, Final Assessment Test			

Module-I

Differentiation-Extrema on an Interval-Rolle’s Theorem and the Mean Value Theorem-Increasing and Decreasing functions and First derivative test-Second derivative test-Maxima and Minima-Concavity.

Integration-Average function Value-Area between Curves- Volumes of Solids of revolution.

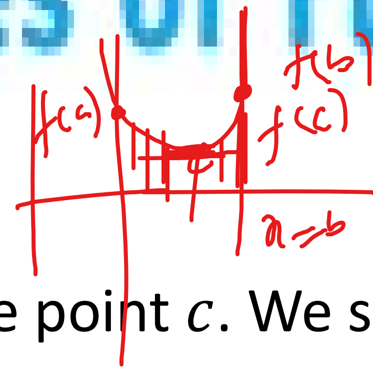
Motivation:-

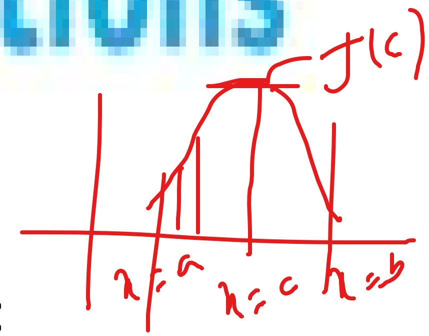
Often in life, we are faced with the problem of finding the *best* way to do something. For example,

- A farmer wants to choose the mix of crops that is likely to produce the largest profit.
- A pharmacist wishes to select the smallest dosage of a drug that will cure certain disease.
- A manufacturer would like to minimize the cost of distributing his products

Problems of above types can be formulated so that it involves *maximizing or minimizing a function* over a *specified set*. Calculus provide a powerful tool for solving the problem.

Extreme Values of Functions





Let D, the domain of f, contain the point c. We say that:

(i) $f(c)$ is the maximum value of f on D if $f(c) \geq f(x)$ for all x in D.

(ii) $f(c)$ is the minimum value of f on D if $f(c) \leq f(x)$ for all x in D.

(iii) $f(c)$ is an extreme value of f on D if it is either the maximum value or the minimum value;

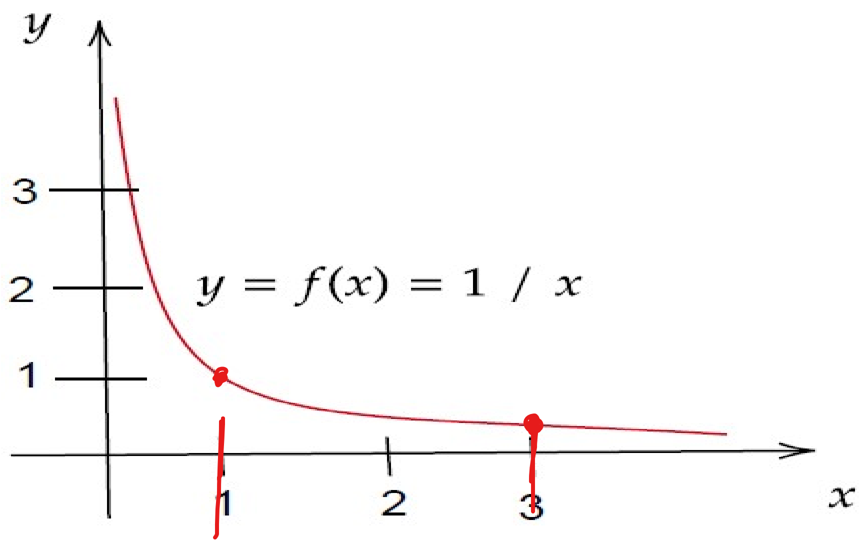
(iv) the function we want to maximize or minimize is the **objective function**.

The Existence Question:-

Does f have a maximum (or minimum) value on D?

The answer depends on

(i) the domain D.



Consider $f(x) = \frac{1}{x}$ on $D = (0, \infty)$, it has neither a maximum value nor a minimum value.

On $D = [1, 3]$ has the maximum value of $f(1) = 1$ and the minimum value of $f(3) = \frac{1}{3}$.

On $D = (1, 3]$, f has no maximum value and the minimum value of $f(3) = \frac{1}{3}$.

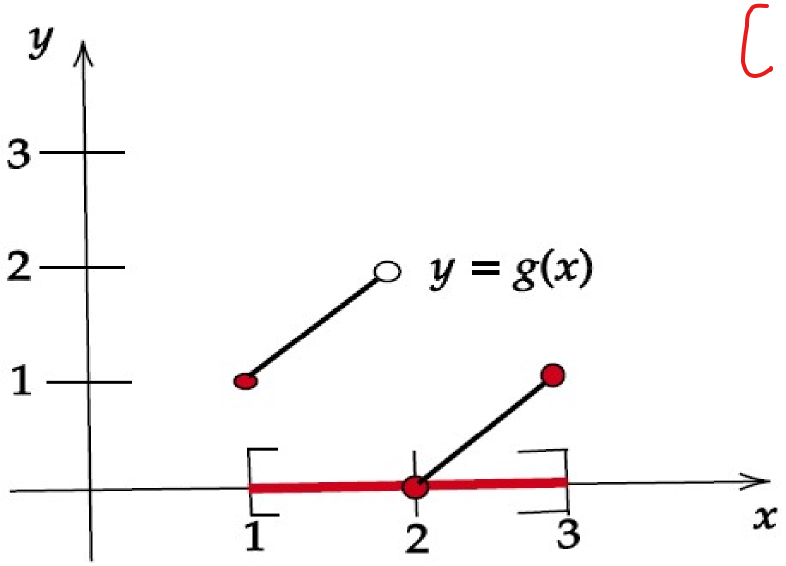
(ii) Type of function

Consider the discontinuous function g defined by

$$g(x) = \begin{cases} x & \text{if } 1 \le x < 2 \\ x - 2 & \text{if } 2 \le x \le 3 \end{cases}$$

$[1, 2)$

$[1, 3]$



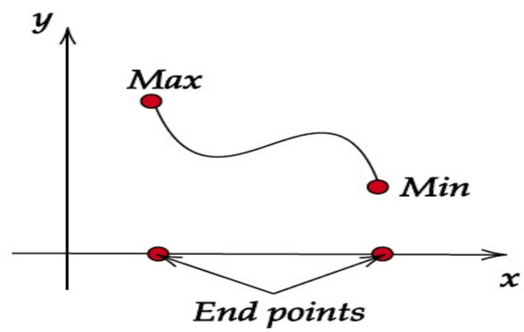
On $D = [1,3]$, g has no maximum value (it gets arbitrarily close to 2 but never attains it). However, g has the minimum value $g(2) = 0$.

Max-Min Existence Theorem

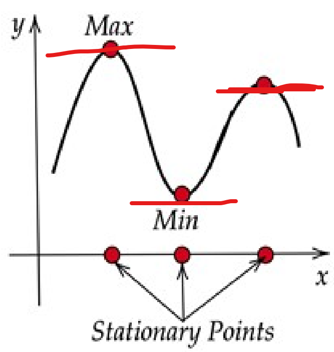
If f is continuous on a closed interval $[a, b]$, then f attains both a maximum value and a minimum value there.

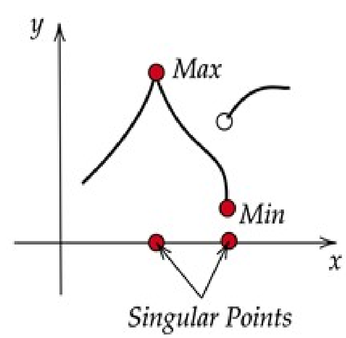
Note:-

- Extreme values of functions defined on closed intervals often occur at end points.



- If c is a point at which $f'(c) = 0$, we call c a stationary point. The name derives from the fact that at a stationary point the graph of f levels off, since the tangent line is horizontal. Extreme values often occur at stationary points.
- If c is an interior point on I where f' fails to exist, we call c a singular point. It is a point where the graph of f has a sharp corner, a vertical tangent, or perhaps takes a jump, or near where the graph wiggles very badly.





Note:-

Let f be defined on an interval I containing the point c. If $f(c)$ is an extreme value, then c must be a critical point: that is, either c is:

(i) an end point of I;

(ii) a stationary point of f ; that is point where $f'(c) = 0$;

(iii) a singular point of f; that is point where $f'(c)$ does not exist.

1. Find the maximum and minimum values of $f(x) = -2x^3 + 3x^2$ on $\left[-\frac{1}{2}, 2\right]$.

Sol:- Given $f(x) = -2x^3 + 3x^2$ on $[-1/2, 2]$

The endpts are $-1/2$ and 2.

To find the stationary pts

$f'(x) = -6x^2 + 6x$

$f'(x) = 0 \Rightarrow -6x^2 + 6x = 0$

$-6(x^2 - x) = 0$

$x^2 - x = 0$

$x(x-1) = 0 \Rightarrow x = 0$ & $x = 1$

$f(-1/2) = -2(-1/2)^3 + 3(-1/2)^2 = 1/4 + 3/4 = 1$ ✓

$f(0) = -2(0)^3 + 3(0)^2 = 0$, $f(1) = -2(1)^3 + 3(1)^2 = 1$ ✓

$f(2) = -2(2)^3 + 3(2)^2 = -4$

Maximum value is 1 attained at $-1/2$ & 1, Minimum value is -4 at 2.

2. The function $f(x) = x^{\frac{2}{3}}$ is continuous everywhere. Find its maximum and minimum values on $[-1, 2]$.

Sol:- Given $f(x) = x^{2/3}$ on $[-1, 2]$

The end pts -1 & 2

To find stationary pt

$f'(x) = \frac{2}{3} x^{2/3 - 1} = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$ which is never 0

$f'(0)$ does not exist

The critical pts are $-1, 0, 2$

$f(-1) = (-1)^{2/3} = 1$, $f(0) = 0$, $f(2) = 2^{2/3} = \sqrt[3]{4} = 1.59$

Thus the maximum value is 1.59 at $x = 2$ & minimum value is 0 at $x = 0$.

Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

Solution The function is differentiable over its entire domain, so the only critical point is where $f'(x) = 2x = 0$, namely $x = 0$. We need to check the function's values at $x = 0$ and at the endpoints $x = -2$ and $x = 1$:

Critical point value:	$f(0) = 0$
Endpoint values:	$f(-2) = 4$
	$f(1) = 1$

The function has an absolute maximum value of 4 at $x = -2$ and an absolute minimum value of 0 at $x = 0$.

Find the absolute maximum and minimum values of $g(t) = 8t - t^4$ on $[-2, 1]$.

Solution The function is differentiable on its entire domain, so the only critical points occur where $g'(t) = 0$. Solving this equation gives

$$8 - 4t^3 = 0 \qquad \text{or} \qquad t = \sqrt[3]{2} > 1,$$

a point not in the given domain. The function's absolute extrema therefore occur at the endpoints, $g(-2) = -32$ (absolute minimum), and $g(1) = 7$ (absolute maximum).

Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

Solution We evaluate the function at the critical points and endpoints and take the largest and smallest of the resulting values.

The first derivative

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

has no zeros but is undefined at the interior point $x = 0$. The values of f at this one critical point and at the endpoints are

$$\begin{aligned}
&\text{Critical point value:} && f(0) = 0 \\
&\text{Endpoint values:} && f(-2) = (-2)^{2/3} = \sqrt[3]{4} \\
& && f(3) = (3)^{2/3} = \sqrt[3]{9}.
\end{aligned}$$

We can see from this list that the function's absolute maximum value is $\sqrt[3]{9} \approx 2.08$, and it occurs at the right endpoint $x = 3$. The absolute minimum value is 0, and it occurs at the interior point $x = 0$