VIT® UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Class NBR : 4362 / 4459 / 4469 / 4558 / 4583 / 4593 / 4619

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = xy + yz + zx$, where $x = e^t$, $y = e^{-t}$ and $y = \frac{1}{t}$ then find $\frac{du}{dt}$.

2. If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.

3. Using double integrals, find the area of a circle of radius a.

4. Show that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}dx = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)$

5. If $\phi = x^3 + y^3 + z^3 - 3xyz$ then find $\nabla \times \nabla\phi$ at the point (1,2,3).

6. Using Green's theorem, evaluate $\int_C \left[(x^2 - y^2)dx + 2xydy\right]$ where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$.

7. Solve $(D^2 + 2D - 1)y = 0$.

8. Solve $(D^2 - D + 12)y = 0$, given that $y(0) = 3$ and $Dy(0) = 5$.

9. Find $L(1 + te^{-t})^3$.

10. Find the Laplace transform of the unit step function

$$H(t) = \begin{cases} 1 & for\ \ t > a \\ 0 & for\ \ t < a \end{cases} \quad where\ \ a \geq 0$$

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $e^x \sin y$ as a Taylor's series about the point $\left(-1, \frac{\pi}{4}\right)$ up to second degree in x and y. **[7]**

b) The temperature T at any point x, y, z in space is $T = 400xyz^2$. Find the highest temperature of unit sphere $x^2 + y^2 + z^2 = 1$. **[7]**

12. a) Evaluate the following integral by changing the order of integration **[7]**

$$\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$$

b) Prove that $\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx \times \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{4\sqrt{2}}$ **[7]**

13. a) Find the values of the constants a, b and c so that **[5]**

$\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. For these values of a, b, c find the scalar potential of $\vec{F}$.

b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - (2xyz)\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. **[9]**

14. a) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = x^4 + \cos^2 x$. **[9]**

b) Solve by the method of undetermined coefficients $(D^2 - 6D + 9)y = 4e^x$ **[5]**

15. a) The motion of an iron ball attached at the lower end of an elastic string whose upper end is fixed, is given by $y'' + 4y' + 8y = 1$ with the intial conditions $y(0) = 0$ and $y'(0) = 1$. Determine the free vibration $y(t)$ of the ball using Laplace transform technique. **[7]**

b) Find the Laplace transform of the triangular wave function

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$$ **[7]**

16. a) Verify divergence theorem, for $\vec{F} = (x^2)\vec{i} + (z)\vec{j} + (yz)\vec{k}$ over the cube formed by the planes $x = \pm 1$, $y = \pm 1$, and $z = \pm 1$. **[7]**

b) Find the jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if

$y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$. **[7]**

17. a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **[9]**

b) Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$. **[5]**

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