VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E1+TE1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = f(x-y,\ y-z, z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$.

2. If $x = r\cos\theta,\ \ y = r\sin\theta$, show that $\dfrac{\partial(x,y)}{\partial(r,\theta)}\times\dfrac{\partial(x,y)}{\partial(r,\theta)} = 1.$

3. Evaluate $\displaystyle\int_0^1\int_0^{\sqrt{1+x^2}}\frac{dx\,dy}{1+x^2+y^2}$.

4. Find the value of $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{\sqrt{\sin\theta}}\,d\theta$.

5. Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at the point P(1, -2, -1) in the direction of PQ where Q is (3, -3, 2).

6. Find the work done by the force $\vec{F} = z\,\vec{i} + x\,\vec{j} + y\,\vec{k}$ when it moves a particle along the curve $\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$ from t = 0 to t = 2π.

7. Find the particular integral of $(D^2 + 5D + 4)y = e^{-x}\sin 2x$

8. Solve the equation $(x^2D^2 + 4\,xD + 2)y = 0$.

9. Find the inverse Laplace transform of $\dfrac{s^2+s-2}{s(s+3)(s-2)}$.

10. Use Laplace transform to evaluate $\displaystyle\int_0^{\infty}\frac{e^{-t}\sin\sqrt{3}\,t}{t}\,dt$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand the Taylor's series expansion of $e^x\cos y$ at the point (0, 0) up to third degree terms. **[7]**

b) If $u = x\sqrt{1-y^2} + y\sqrt{1-x^2}$, $v = \sin^{-1}x + \sin^{-1}y$, show that u and v are functionally related and find the relationship. **[7]**

12. a) Change the order of integration in $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ and hence evaluate it. **[7]**

b) Evaluate the integral $\iiint (x^2+y^2+z^2)\,dx\,dy\,dz$ over the region of space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1$ by transforming in to cylindrical coordinates. **[7]**

13. a) Verify Stoke's theorem when $\vec{F} = (2xy - x^2)\vec{i} - (x^2 - y^2)\vec{j}$ and C is the boundary of the region enclosed by the parabolas $y^2 = x$ and $x^2 = y$. **[10]**

b) Find the values of the constants *a, b, c* so that

$\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ may be irrotational. **[4]**

14. (a) Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} + 4y = 5\sin 3x$. **[7]**

(b) Solve the differential equation $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 96x$. **[7]**

15. (a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} - 3y = \sin t$ by using Laplace transform, given that **[7]**

$y = \frac{dy}{dt} = 0$ when $t = 0$.

(b) Using convolution theorem, evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$ **[7]**

16. a) The temperature T at any point *(x, y, z)* in space is $400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. **[7]**

b) Show that volume of the region of space bounded by the co-ordinate planes and the surface $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{z}{c}} = 1$ is $\frac{abc}{90}$. **[7]**

17. a) Verify Gauss divergence theorem for $\overline{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **[10]**

b) Find the inverse Laplace transform of $\frac{1}{s^2(s^2+a^2)}$. **[4]**

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