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Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D2+TD2**

Class NBR : 2238/2279/2287/2304/2330/2376/2388/5067/5076/5078

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Determine whether the functions $u = e^x \sin y,\ v = e^x \cos y$ are functionally dependent.

2. Prove that $xu_x + yu_y = \frac{5}{2}\tan u$ if $u = \sin^{-1}\left(\frac{x^3 + y^3}{\sqrt{x} + \sqrt{y}}\right)$.

3. Find the value of $\int_0^2 (8 - x^3)^{-\frac{1}{3}} dx$ in terms of Gamma function.

4. Evaluate $\int_0^a \int_0^b xy(x - y)dxdy.$

5. Find the direction in which the directional derivative of $\varphi = x^3 y^2 z$ at (1,2,3) is maximum.

6. Find 'a' and 'b' such that $\vec{F} = 3x^2 y\vec{i} + (ax^3 - 2yz^2)\vec{j} + (3z^2 - by^2 z)\vec{k}$ is irrotational.

7. Solve $(D^4 - 2D^3 + D^2)y = 0.$

8. Find the particular integral of $(D^2 + 4)y = \cos^2 x.$

9. Find $L^{-1}\left(\frac{1}{s+5}\right)^5$.

10. Find $L\left(t\cos\frac{t}{a}\right)$.

PART – B (5X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $e^y \ln(1 + x)$ in powers of x and y. **[7]**
b) Find the minimum value of $x^2 yz^3$ subject to the condition $2x + y + 3z = a$. **[7]**

12. a) Evaluate by changing to polar coordinates $\int_0^a \int_y^a \frac{x^2 dxdy}{\sqrt{x^2 + y^2}}$. **[7]**

b) Evaluate $\iint [xy(1 - x - y)]^{\frac{1}{2}} dxdy$ over the area enclosed by the lines $x = 0, y = 0$ and $x + y = 1$ in the positive quadrant, using Gamma functions. **[7]**

13. Verify Stoke's theorem for $\overline{F} = xy\overline{i} - 2yz\overline{j} - zx\overline{k}$ where S is the open surface of the rectangular parallelepiped formed by the planes $x = 0, x = 1, y = 0, y = 2$ and $z = 3$ above the xy - plane. **[14]**

14. a) Solve $\frac{d^2y}{dx^2} + y = \sin x$ by the method of undetermined coefficients. **[7]**

b) Solve by method of variation of parameters, $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$. **[7]**

15. a) Use convolution theorem to find the inverse Laplace transform of **[7]**

$$\frac{s^2+s}{(s^2+1)(s^2+2s+2)}.$$

b) Solve by using Laplace transforms $(D^2+4D+13)y = e^{-t}\sin t$ given that **[7]**

$y(0)=0$ and $y'(0)=0$ where $D = \frac{d}{dt}$.

16. a) Find the maximum and minimum values of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ **[7]**

b) By changing the order of integration evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\,dxdy$ **[7]**

17. a) Evaluate $\iint_S \vec{F} \bullet \vec{ds}$, if $\overline{F} = yz\overline{i} + 2y^2\overline{j} + xz^2\overline{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between the planes $z = 0$ *and* $z = 2$ **[7]**

b) Solve $(x^2D^2 + 3xD + 5)y = x\cos(\log x)$. **[7]**

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