**VIT UNIVERSITY**
(Estd. u/s 3 of UGC Act 1956)

**Term End Examination - November 2012**

**Course : MAT101 - Multivariable Calculus and Differential Equations** **Slot: D1+TD1**

**Class NBR : 4354 / 5072**

**Time : Three Hours** **Max.Marks:100**

**Answer any FIVE Questions**
**(5 X 20 = 100 Marks)**

1. (a) Expand $e^x \log(1 + y)$ in powers of $x$ and $y$ up to third degree terms. **[8]**

   (b) Find the extreme values of $f(x, y) = x^3y^3 - 3x - 3y$. **[8]**

   (c) Use the chain rule to find the total derivative of $w = xy$ with respect to $t$ along the path $x = cos\, t$ , $y = sin\, t$. What is the derivative value at $= \frac{\pi}{2}$ ? **[4]**

2. (a) Evaluate $\int_0^1 \int_{x^2}^{2-x} xydydx$ by changing the order of integration. **[10]**

   (b) By transforming into cylindrical polar co-ordinates evaluate $\iiint (x^2 + y^2 + z^2)dxdydz$ taken over the region of space defined by $x^2 + y^2 \leq 1\ and\ 0 \leq z \leq 1$. **[10]**

3. (a) Show that the vector field $\bar{F} = 2x(y^2 + z^3)\hat{\imath} + 2x^2y\hat{\jmath} + 3x^2z^2\hat{k}$ is conservative. Find its scalar potential and the workdone in moving a particle from (-1, 2, 1) to (2, 3, 4). **[10]**

   (b) Evaluate $\iiint \bar{F}.\hat{n}dS$ using Gauss divergence theorem, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ and $\bar{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ . **[10]**

4. (a) Solve $\frac{d^2y}{dx^2} + 4y = cot\, x$ by the method of variation of parameters. **[10]**

   (b) Using the method of undetermined coefficients, solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{3x}$. **[10]**

5. (a) Find the Laplace Transform of the function $f(t) = \begin{cases} sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < 0 < \frac{2\pi}{\omega} \end{cases}$ **[6]**

(b) Using Laplace transform solve y'' + 16y = 32*t* with y(0) = 3, *y*'(0) = -2. **[14]**

6. (a) Find the general solution of $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2$. **[10]**

(b) Find the value of $\iint x^{m-1} y^{n-1}\, dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in terms of Gamma function. **[10]**

7. (a) Evaluate $\int \bar{F} . d\bar{r}$ using Stoke's theorem over the surface of the cube *x = 0, y = 0*, *z* = 0, *x = 2, y = 2, z = 2*, above the XY- plane where $\bar{F} = (y - z + 2)\hat{\imath} + (yz + 4)\hat{\jmath} - xz\hat{k}$ . **[10]**

(b) Verify the initial and final value theorems when $f(t) = (t + 2)^2 \mathrm{e}^{-\mathrm{t}}$ **[5]**

(c) Using Convolution theorem evaluate $L^{-1} \left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$ . **[5]**

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