VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E1+TE1**

Class NBR : 2269 \ 2283 \ 2292 \ 2309 \ 2317 \ 2323 \ 5063

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Discuss the continuity of $f(x,y)=\begin{cases}\dfrac{x}{\sqrt{x^2+y^2}}, & x\neq 0, y\neq 0\\ 2, & x=0, y=0\end{cases}$ at the origin.

2. If $u=x^2-y^2, v=2xy,$ $x=r\cos\theta$ and $y=r\sin\theta$ then find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$.

3. Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$.

4. Show that $erf(x)=\dfrac{2}{\sqrt{\pi}}\left(x-\dfrac{x^3}{3}+\dfrac{x^5}{10}-\dfrac{x^7}{42}+....\right)$.

5. Find the directional derivative of $f=xy+yz+zx$ in the direction of $\bar{i}+2\bar{j}+2\bar{k}$ at the point (1, 2, 0).

6. Prove that $\text{curl grad}\,\phi=0,$ where ϕ is a scalar point function.

7. Solve $4y'''+4y''+y'=0$.

8. Find the particular integral of $(D^2+6D+9)y=2e^{-3x}$.

9. Find the Laplace transform of e^{at}.

10. Find the inverse Laplace transform of $\dfrac{2s-5}{s^2-4}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $e^x\sin y$ in powers of x and y. **[6]**

b) Find the minimum value of $x^2+y^2+z^2$, given that $xyz=a^3$. **[8]**

12. a) Change the order of integration and evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$. **[7]**

b) Find the volume bounded by the elliptic paraboloids $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$ **[7]**

13. a) Find $\operatorname{div}(\operatorname{grad} r^m)$ **[7]**

b) If $\overline{F} = (2x^2 - 3z)\overline{i} - 2xy\,\overline{j} - 4x\,\overline{k}$ then evaluate $\int_V \nabla \times \overline{F}\, dv$ where V is the region bounded by the planes $x = 0,\ y = 0,\ z = 0,\ 2x + 2y + z = 4.$ **[7]**

14. a) Verify divergence theorem for $\overline{F} = x^2\,\overline{i} + y^2\,\overline{j} + z^2\,\overline{k}$ over the surface S of the solid cut off by the plane $x + y + z = a$ in the first octant. **[9]**

b) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **[5]**

15. a) Solve $(D^2 + 2D - 3)y = x^2 e^{-3x}$. **[7]**

b) Solve $(D^2 + a^2)y = \tan ax$, by the method of variation of parameters. **[7]**

16. a) Solve $(x^2D^2 - 3xD + 1)y = \log x\left(\frac{\sin(\log x) + 1}{x}\right)$. **[7]**

b) Find the Laplace transform of $f(t) = \begin{cases} \sin\omega t, & 0 < t < \pi/\omega \\ 0, & t > \pi/\omega \end{cases}$. **[7]**

17. a) Find $L\left(\frac{\cos 2t - \cos 3t}{t}\right)$. **[7]**

b) Using Convolution theorem, evaluate $L^{-1}\left(\frac{1}{s^2(s^2 + 2s + 2)}\right)$. **[7]**

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