VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - May 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: C2+TC2**

Class NBR : 2840/2844

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Find the extreme points of the function $f(x,y) = xy + \frac{9}{x} + \frac{3}{y}$

2. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ $find$ $\frac{\partial(x,y)}{\partial(u,v)}$

3. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$, using polar coordinates.

4. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ \ d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$ using Gamma functions.

5. Determine the constant a ,so that the vector $\vec{v} = 3x\vec{i} + (x+y)\vec{j} - az\vec{k}$ is solenoidal.

6. Find the directional derivative of $\phi = 2xy + z^2$ at the point(1,-1,3) in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$.

7. Solve: $x^2y'' + xy' - y = 0$.

8. Find the particular integral of $(D^2 + 4)y = sin2x$

9. Find $L[sin(2t+3)]$

10. Find $L^{-1}\left(\frac{1}{S^2+4S+4}\right)$

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Find the first three terms of the expansion of the function $e^x cosy$ in a Taylor Series at the point $(0,0)$. **[7]**

(b) Find the maximum value of $x^m y^n z^p$, $when\ x + y + z = a$ using Lagrange Multiplier Method. **[7]**

12. (a) Change the order of Integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$ and hence evaluate the same. **[7]**

(b) Find the area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, using Gamma functions. **[7]**

13. Verify Divergence theorem for $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$, where S is a surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$.

14. (a) Solve $(D^2 - 4D + 3)y = x^2$, Using the Method of Undetermined coefficients. **[7]**

(b) Solve by using Method of Variation of Parameters **[7]**

$$\frac{d^2y}{dx^2} + 4y = tan2x$$

15. (a) Find the Laplace transform of the square wave given by **[7]**

$$f(t) = \begin{cases} E; & o < t < \frac{T}{2} \\ -E; & \frac{T}{2} < t < T \end{cases} \quad and \;\; f(t+T) = f(t).$$

(b) Solve the Differential Equation using the Laplace transform **[7]**

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \; given \; that \; y = \frac{dy}{dx} = 1 \; at \; x = 0$$

16. (a) By transforming into cylindrical coordinates, evaluate **[7]**

$\int\int\int(x^2 + y^2 + z^2)dxdy$, Taken over the region of space defined by

$x^2 + y^2 \leq 1 and \; 0 \leq z \leq 1$

(b) Show that $\vec{F} = (2x + yz)\hat{\imath} + (4y + xz)\hat{\jmath} - (6z - xy)\hat{k}$ is irrotational and hence find its scalar potential. **[7]**

17. (a) Find $L^{-1}\left(\frac{2}{(S+1)(S^2+2^2)}\right)$ using convolution theorem. **[7]**

(b) Using Green's theorem evaluate $\int_c [(xy - x^2)dx + x^2ydy]$ along the closed curve C formed by $y = 0, x = 1 \; and \; y = x$. **[7]**

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