VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Class NBR : 4330 / 4969

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $z = f(x,y)$ and $x = u+v$, $y = u-v$ then find $\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}$.

2. If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Calculate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{xdydx}{x^2+y^2}$ by changing in to polar coordinates.

4. Evaluate $\int_0^\infty x^2 e^{-x^4} dx$

5. Find the directional derivative of $\phi = 2xy + z^2$ at (1,-1,3) in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$.

6. Evaluate $\int_c \left\{(2xy - x^2)dx + (x^2+y^2)dy\right\}$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$.

7. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$

8. Reduce $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x + \frac{1}{x}$ in to a linear differential equation with constant coefficients.

9. Find the Laplace transform of $te^{-t}\cos t$.

10. Determine $f(0)$ and $f(\infty)$ if $F(s) = \frac{2}{(s+1)^3} + \frac{4}{(s+1)^2} + \frac{4}{(s+1)}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $e^x \log(1+y)$ at $(0,0)$ as the Taylor's series up to third degree. **[7]**

b) A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. **[7]**

12. a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate it. **[7]**

b) Evaluate $\iiint_D (x+y+z)\,dx\,dy\,dz$ by transforming in to spherical coordinates where D is the volume bounded by the positive octant of the sphere $x^2+y^2+z^2=a^2$. **[7]**

13. a) Verify Green's theorem in the XY- plane for $\int_C \left\{(3x^2-8y^2)dx+(4y-6xy)dy\right\}$ where C is the boundary of the region given by $x=0, y=0, x+y=1$ **[7]**

b) Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ and S is the surface bounding the region $x^2+y^2=4, z=0$ and $z=3$. **[7]**

14. a) Solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$ by using the method of variation of parameters. **[7]**

b) Solve $(D^3+D^2)y=10e^x\cos x$ by the method of undetermined coefficients. **[7]**

15. a) Find the Laplace transform of the triangular wave function **[7]**

$$f(t)=\begin{cases} t & for \quad 0\le t\le a \\ 2a-t & for \quad a\le t\le 2a \end{cases}$$

b) Using Convolution theorem, find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$. **[7]**

16. a) Show that $\vec{F}=(y^2+2xz^2)\vec{i}+(2xy-z)\vec{j}+(2x^2z-y+2z)\vec{k}$ is irrotational and also find its scalar potential. **[7]**

b) Using Laplace transform solve the integral equation $\frac{dy}{dt}+2y+\int_0^t y(t)dt=0$, given that $y(0)=1$. **[7]**

17. a) Using Laplace transform solve $y''(t)-3y'(t)+2y(t)=4t+e^{2t}$ where $y(0)=1, y'(0)=-1$. **[7]**

b) Solve the differential equation **[7]**

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x\log x$$

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