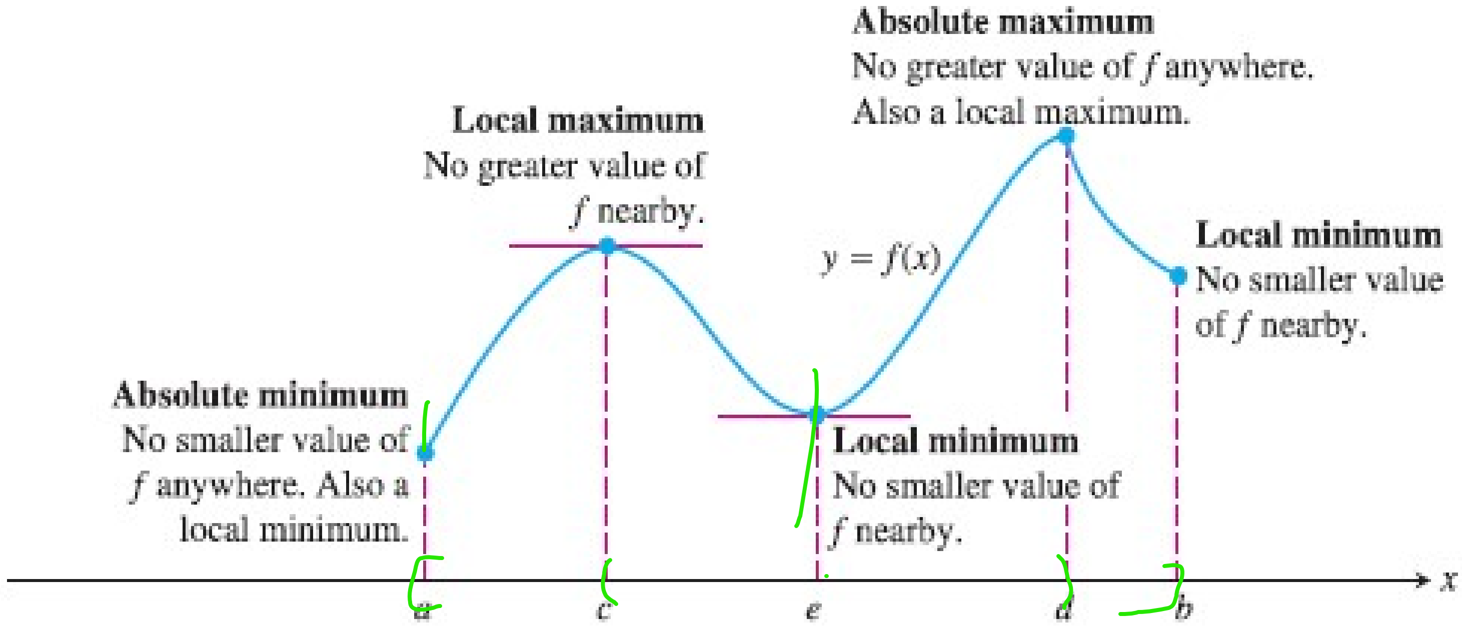
Local (Relative) Extreme Values

DEFINITIONS A function f has a **local maximum** value at a point c within its domain D if $f(x) \leq f(c)$ for all $x \in D$ lying in some open interval containing c.

A function f has a **local minimum** value at a point c within its domain D if $f(x) \geq f(c)$ for all $x \in D$ lying in some open interval containing c.



The First Derivative Theorem for Local Extreme Values If f has a local maximum or minimum value at an interior point c of its domain, and if f' is defined at c, then

$$f'(c) = 0.$$

1. Find the local extreme values of the function $f(x) = x^2 - 6x + 5$ on $(-\infty, \infty)$.

Sol:- Given $f(x) = x^2 - 6x + 5$

$f'(x) = 2x - 6$

$f'(x) = 0 \Rightarrow 2x - 6 = 0, \quad x = 3$

The intervals are $(-\infty, 3)$ & $(3, \infty)$

Interval	Sample	f'	Increasing/decreasing
$(-\infty, 3)$	0	$-6 < 0$	decreasing
$(3, \infty)$	4	$2 > 0$	increasing

Therefore by first derivative test $f(3) = -4$ is a local minimum value.

2. Find the local extreme values of $f(x) = \frac{x^3}{3} - x^2 - 3x + 4$ on $(-\infty, \infty)$

Sol:- Given $f(x) = \frac{x^3}{3} - x^2 - 3x + 4$ on $(-\infty, \infty)$

$$f'(x) = x^2 - 2x - 3$$

$$f'(x) = 0 \Rightarrow x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1 \ \& \ 3$$

The intervals are $(-\infty, -1)$, $(-1, 3)$ & $(3, \infty)$

Interval	Sample	f'	Increasing/Decreasing
$(-\infty, -1)$	-2	$5 > 0$	Increasing

$(-1,3)$	0	$-3<0$	Decreasing
$(3,\infty)$	4	$5>0$	Increasing

By First derivative test, we conclude that $f(-1) = 17/3$ is a local maximum value.

$f(3) = -5$ is a local minimum value.