Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E1+TE1**

Class NBR : 4626 / 5052 / 5135 / 5138

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$, find the Jacobian $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

2. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, Prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = \sin 2u$.

3. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$, using polar coordinates.

4. Find the area of the region enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

5. Prove that $\vec{A} = (2x+yz)\vec{i} + (4y+zx)\vec{j} - (6z-xy)\vec{k}$ is solenoidal as well as irrotational.

6. Evaluate $\iint_S (x\,dydz + 2y\,dzdx + 3z\,dxdy)$, where S is the closed surface of the sphere $x^2+y^2+z^2=a^2$, using Gauss divergence theorem.

7. Solve: $(D^5 - D)y = 0$.

8. Find the particular solution of the differential equation $\dfrac{d^3y}{dx^3} - y = \sin x$

9. Find $L\{f(t)\}$ where $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$

10. Find $L^{-1}\left(\dfrac{s+3}{s^2-4s+13}\right)$

PART – B (5 X 14 = 70 Marks)

Answer any <u>FIVE</u> Questions

11. a) If $f(x,y,z)=(x^2+y^2+z^2)^{-1/2}$, then evaluate $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}+\dfrac{\partial^2 f}{\partial z^2}$. **[7]**

b) If the perimeter of a triangle is constant, show that the triangle has maximum area when it is equilateral. **[7]**

12. a) Change the order of integration and hence evaluate $\int_0^2 \int_x^{2x} xy\,dxdy$. **[7]**

b) Using spherical polar coordinates, find the volume of the sphere $x^2+y^2+z^2=a^2$. **[7]**

13. a) Verify Green's theorem for $\int_C \left((xy+y^2)dx+x^2dy\right)$ where C is bounded by $y=x$ and $y=x^2$. **[7]**

b) Using divergence theorem, evaluate $\int\int_S \vec{F}.\vec{N}dS$, where $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ and S is the surface bounding the region $x^2+y^2=4,\ z=0$ and $z=3$. **[7]**

14. a) Solve: $(D^2+5D+4)y=x^2+7x+9$. **[7]**

b) Solve by the method of variation of parameters $(D^2+2D+5)y=e^{-x}\tan x$. **[7]**

15. a) Find $L\left(\dfrac{\cos at-\cos bt}{t}\right)$. **[7]**

b) Using Laplace Transform, solve the differential equation $\dfrac{d^2y}{dt^2}+6\dfrac{dy}{dt}+5y=e^{-2t}$, where $y=0$, $\dfrac{dy}{dt}=1$, when $t=0$. **[7]**

16. a) Expand $e^x\log(1+y)$ in a Taylor's series in the neighbourhood of the origin upto terms of third degree. **[7]**

b) Show that $\int_a^b (x-a)^m(b-x)^n\,dx=(b-a)^{m+n+1}\beta(m+1,n+1)$ **[7]**

17. a) Using Laplace transform, solve $y=\int_0^t \sin u\ \cos(t-u)du$ **[7]**

b) Solve: $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\sin(\log(1+x))$ **[7]**

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