Extra Problems

1. $\mathcal{L}\left[u(t-7)\,t^2\right]$

Sol:-

Rewriting t^2 in terms of $t-7$

$$t^2 = \left[(t-7)+7\right]^2 = (t-7)^2 + 49 + 14(t-7)$$

$$\therefore \mathcal{L}\left[u(t-7)\,t^2\right]$$
$$= \mathcal{L}\left[u(t-7)(t-7)^2\right] + 14\mathcal{L}\left[u(t-7)(t-7)\right] + 49\mathcal{L}\left[u(t-7)\right]$$
$$= e^{-7s}\left(\frac{2}{s^3} + \frac{14}{s^2} + \frac{49}{s}\right)$$

2. Find $\mathcal{L}\left[u(t-4)\sin 2t\right]$

Sol:-

Rewriting $\sin 2t$ in terms of $t-4$, we have

$$\sin 2t = \sin\left(2(t-4)+8\right)$$

$$= \sin 2(t-4)\cos 8 + \cos 2(t-4)\sin 8$$

$$\therefore L\left[u(t-4)\sin 2t\right]$$

$$= L\left[u(t-4)\sin 2(t-4)\cos 8 + u(t-4)\cos 2(t-4)\sin 8\right]$$

$$= e^{-4s}\left[\cos 8\left(\frac{2}{s^2+4}\right) + \sin 8\left(\frac{s}{s^2+4}\right)\right]$$

3. Find $L^{-1}\left[e^{-4s}\frac{1}{s^4}\right]$

Sol:- WK $F(s) = \frac{6}{s^4}$, then

$$f(t) = t^3$$

$$L^{-1}\left[e^{-4s}\frac{1}{s^4}\right] = \frac{1}{6}L^{-1}\left[e^{-4s}\frac{6}{s^4}\right]$$

$$= \frac{1}{6}u(t-4)(t-4)^3$$

4. Find $L^{-1}\left[\dfrac{e^{-2s}}{s^2+s-2}\right]$

Sol:-

Consider $\dfrac{1}{s^2+s-2}$

ie $\dfrac{1}{(s-1)(s+2)} = \dfrac{A}{(s-1)} + \dfrac{B}{(s+2)}$

$$= \frac{A(s+2)+B(s-1)}{(s-1)(s+2)}$$

ie $1 = A(s+2)+B(s-1)$

when $s=1$, $1 = 3A$

$\Rightarrow A = 1/3$

when $s=-2$, $-3B = 1$

$B = -1/3$

$$L^{-1}\left[\frac{e^{-2s}}{s^2+s-2}\right] = \frac{1}{3}L^{-1}\left[e^{-2s}\frac{1}{s-1}\right] - \frac{1}{3}L^{-1}\left[e^{-2s}\frac{1}{s+2}\right]$$

$$= \frac{1}{3} u(t-2) e^{(t-2)} - \frac{1}{3} u(t-2) e^{-2(t-2)}$$

5. Find $L^{-1}\left[\frac{2e^{-3s}}{s^2-4}\right]$

Sol:-

$$L^{-1}\left[e^{-3s} \frac{2}{s^2-4}\right]$$

$$= u(t-3) \sinh(2(t-3))$$

6. Given $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ t^2, & 2 \le t \end{cases}$

Find $L[f(t)]$ using unit step function.

Sol:-

$f(t)$ in terms of unit step function

$u_2(t)$

$$f(t) = 1\cdot[u(t-0) - u(t-2)]$$
$$+ t^2 u(t-2)$$
$$= 1\cdot(1 - u(t-2))$$
$$+ t^2 u(t-2)$$
$$= 1 + u(t-2)(t^2-1)$$

Now

$$t^2-1 = (t-2+2)^2 - 1$$
$$= (t-2)^2 + 4 + 4(t-2) - 1$$
$$= (t-2)^2 + 4(t-2) + 3$$
$$\therefore f(t) = 1 + u(t-2)(t-2)^2$$
$$+ 4u(t-2)(t-2) + 3u(t-2)$$
$$L[f(t)] = \frac{1}{s} + e^{-2s}\frac{2}{s^3} + 4\frac{e^{-2s}}{s^2} + 3e^{-2s}\frac{1}{s}$$
$$= \frac{1}{s} + e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{3}{s}\right)$$ //