VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Class NBR : 2204 \ 2216 \ 2217 \ 2223 \2244 \2258 \2272 \2277 \2280 \2337 \ 2463 \ 3604 \ 5498

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. If $U = 2x - 3y$, $V = 5x + 4y$, then find $\frac{\partial(U,V)}{\partial(x,y)}$.

2. Locate the stationary points of $f = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

3. Change into polar coordinates and then evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$

4. Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ by Beta-Gamma functions.

5. Find the constants a, b, c if the vector
$\vec{F} = (2x + 3y + az)\vec{i} + (bx + 2y + 3z)\vec{j} + (2x + cy + 3z)\vec{k}$ is irrotational.

6. Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $\vec{u} = 2\hat{i} - \hat{j} - 2\hat{k}$.

7. Solve $(D^3 - 6D^2 + 11D - 6)y = 0$

8. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

9. Find $L(te^{-4t}\sin 3t)$

10. Find the inverse Laplace transform of $\frac{1}{s(s+a)}$

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $f(x, y) = e^{x+2y}$ in a Taylor's series about $(x, y) = (0, 0)$ up to second degree terms. **[7]**

b) Find the volume of the greatest rectangular parallelopied that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **[7]**

12. a) Change the order of integration in $\int_0^a \int_y^a \frac{x\,dy\,dx}{x^2 + y^2}$ and hence evaluate it. **[7]**

b) Find the value of $\iint x^{m-1} y^{n-1} dxdy$, over the positive quadrant of the ellipse **[7]**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in term of Gamma functions.

13. a) Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$ $0 \le z \le c$. **[10]**

b) Find the work done by the force $\vec{F} = x^2\vec{i} + yx\vec{j}$ along the line joining the points (0,0) to (1,1). **[4]**

14. a) Use method of undetermined coefficients to solve the differential equation **[7]**

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = e^{3x} + \sin x$

b) Solve the differential equation $\frac{d^2y}{dx^2} + y = sec\, x$, using the method of variation of parameters. **[7]**

15. a) Using Laplace Transform, solve **[10]**

$y'' + y = 6\cos 2x$ where $y = 3,\ y' = 1$ at $x = 0$.

b) Find the Laplace Transform of the periodic function known as saw tooth wave $f(t) = \frac{kt}{T}$ $0 < t < T$, $f(t+T) = f(t)$. **[4]**

16. a) Find the maximum value of $x^m y^n z^p$, when $x + y + z = a$ **[7]**

b) Evaluate $\iiint xyz\, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming in to spherical coordinates. **[7]**

17. a) Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2\,dy$ where C is bounded by $y = x$ and $y = x^2$. **[7]**

b) Solve $\left(D^2 + 2\right) y = e^x \cos x$ **[7]**

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