VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E2+TE2**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = xy + yz + zx,$ where $x = e^t, y = e^{-t}$ and $z = \frac{1}{t},$ find $\frac{du}{dt}$.

2. If $u = 2xy, v = x^2 - y^2, x = r\cos\theta$ and $y = r\sin\theta$, compute the Jacobian of x , y with respect to r, θ.

3. Prove that $\int_0^\infty e^{-ax}x^{n-1}dx = \frac{\Gamma(n)}{a^n}$ where a and n are positive.

4. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin\phi \; dr\, d\phi\, d\theta.$

5. Show that $\vec{F} = (y^2 + 2xz^2)\,\hat{i} + (2xy - z)\,\hat{j} + (2x^2z - y + 2z)\,\hat{k}$ is irrotational and hence find its scalar potential.

6. Evaluate $\int_c \phi \, d\,\vec{r}$ where C is the curve x = t, y = t^2, z = (1– t) and $\phi = x^2y\,(1 + z)$ from t = 0 to t = 1.

7. Solve $(D^4 - 2D^3 + D^2)\, y = e^x$.

8. Solve $(D^2 + 4)\, y = \cos^2 x$.

9. Find the Laplace transform of $\frac{1+2t}{\sqrt{t}}$.

10. Find $L^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$ using convolution theorem.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (i) Using Taylor's series, verify that **[7]**

$$\log(1 + x + y) = (x + y) - (1/2)(x + y)^2 + (1/3)(x + y)^3 - \dots\dots\dots$$

(ii) Examine f (x, y) = $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **[7]**

12. i) By transforming into cylindrical coordinates, evaluate the integral **[7]**

$\iiint (x^2 + y^2 + z^2)\,dxdydz$ taken over the region of space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1$.

(ii) Change the order of integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dxdy$ **[7]**

13. (i) If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla(r^n) = nr^{n-2}\vec{r}$. **[5]**

(ii) Verify Stokes theorem for $\overrightarrow{F} = -y\hat{i} + 2yz\,\hat{j} + y^2\hat{k}$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and C is the circular boundary on the xoy plane. **[9]**

14. (i) Solve $[(2x + 3)^2D^2 - 2(2x + 3)D - 12]y = 6x$ **[7]**

(ii) Solve the equation $(D^2 + a^2)y = \tan ax$, by the method of variation of parameters. **[7]**

15. (i) Verify the initial and final value theorems when $f(t) = (t + 2)^2 e^{-t}$. **[7]**

(ii) Solve $y'' - 4y' + 8y = e^{2t}$, $y(0) = 2$ and $y'(0) = -2$. **[7]**

16. (i) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$. **[7]**

(ii) Find the Area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ lying in the first quadrant, by double integration. **[7]**

17. (i) Verify Green's theorem in a plane with respect to $\int_C [(x^2 - y^2)dx + 2xydy]$, **[8]**

where c is the boundary of the rectangle in the xoy plane bounded by the lines $x = 0, x = a, y = 0$ *and* $y = b$.

(ii) Find the Laplace transform of the "full – sine wave rectifier" function **[6]**

$f(t) = |\sin \omega t|, t \ge 0.$

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