VIT® UNIVERSITY (Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Class NBR : 2210/2245/2259/2273/2278/2281/2331/2338/2354/2374/2389

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Investigate the continuity of the function:

$$f(x,y) = x^2 - 3y, \ if (x,y) \neq (1,2)$$
$$= 0 \ if \ (x,y) = (1,2)$$

2. Find the derivative of $f(x, y) = x^2 + 3xy$ with respect to x, given that $y=\sin^{-1}x$.

3. If $x = r\cos\theta$ and $y = r\sin\theta$ then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

4. Prove that $\beta(m+1,n)+\beta(m,n+1)=\beta(m,n)$.

5. Find the value of 'K' if $\overline{F} = (x+2y)\overline{i} + (Ky+4z)\overline{j} + (5z+6x)\overline{k}$ is solenoidal.

6. Find a unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,2,-1).

7. Show that e^x and xe^x are independent solutions of $(D^2 - 2D + 1)u = 0$ in any interval.

8. Find the general solution of $(D^4 - 1)y = 0$.

9. Evaluate $\int_0^\infty t\, e^{-2t} \sin t \, dt$.

10. If $L\{f(t)\} = \frac{e^{-1/s}}{s}$, find $L\{e^{-t} f(3t)\}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Prove that $u = x^2 + y^2 - 3axy$ is maximum or minimum at x=y=a according as a is negative or positive. **[4]**

(b) Expand $\ln\left(x^2 + y^2\right)$ about (1, 1) up to quadratic terms. **[4]**

(c) The temperature of a point (x, y) on a unit circle is given by T(x,y)=1+xy. Find the temperature of the two hottest points on the circle. **[6]**

12. (a) Find by double integration, the area between the parabola $y=4x-x^2$ and the line $y=x$. **[4]**

(b) Show that if $a>1, \int_0^\infty \frac{x^a}{a^x}dx = \frac{\Gamma(a+1)}{(\log a)^{a+1}}$. **[5]**

(c) Prove that $\int_0^\infty e^{-x^2-2ax}dx = \frac{\sqrt{\pi}}{2}e^{a^2}[1-\operatorname{erf}(a)]$. **[5]**

13. (a) Show that the area bounded by any simple closed curve C is given by **[7]**

$\frac{1}{2}\oint_C (xdy - ydx)$ and hence find the area of the ellipse $x = a\cos\theta;\ y = b\sin\theta i$

(b) Using Stroke's theorem, evaluate $\oint_C zdx + xdy + ydz,$ where C is the trace of the **[7]**

Cylinder $x^2+y^2=1$ in the plane $y+z=2$. Orient C counter clockwise as viewed from above.

14. (a) Let D be the region bounded by the hemisphere $x^2+y^2+(z-1)^2=9, 1\le z\le 4$, and the plane z = 1. Verify the divergence theorem if $F = xi + yj + (z-1)k$. **[7]**

(b) Apply the method of variation of parameters to solve $y'' + 3y' + 2y = x^2$. **[7]**

15. (a) Solve by the method of undetermined coefficients: $\frac{d^2y}{dx^2} + 4y = x^2\sin 2x$. **[7]**

(b) Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \log x$, subject to the condition that when x = 1, y = 0 and $\frac{dy}{dx} = 0$. **[7]**

16. (a) Show that $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\} = \ln\left(\frac{s+b}{s+a}\right)$ and hence evaluate $\int_0^\infty \frac{e^{-2t}-e^{-8t}}{t}dt$. **[4]**

(b) Solve $\frac{dy}{dt} + 3y + 2\int_0^t y\,dt = t$ for which $y(0) = 0$. **[5]**

(c) Find the Laplace inverse of $\frac{s}{(s^2+1)(s+1)^2}$ using convolution theorem. **[5]**

17. (a) By triple integration, determine the volume of sphere $x^2+y^2+z^2=c^2$. **[7]**

(b) Solve $\frac{dx}{dt} + x = \sin\omega t$; x(0)=2 using Laplace transform technique. **[7]**

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