Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Class NBR : 4444 / 4446 / 4456 / 4463 / 4560 / 4633 / 5157 / 5413

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = xy + yz + zx$ where $x = t, y = t^2$ and $z = \log t$, find $\frac{du}{dt}$.

2. If $u = x^2, v = x + y + z, w = x - 2y + 3z,$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

3. Evaluate $\iint dxdy$ over the positive quadrant of the circle $x^2 + y^2 = 4$ by changing into polar coordinates.

4. Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$ using gamma function.

5. Find the unit normal vector to the surface $x^2 y + 2xz^2 = 8$ at the point (1, 0, 2).

6. Find the value of 'a' so that the vector $\vec{F} = (z+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$ is solenoidal.

7. Find the general solution for $y''' - 3y'' + 3y' - y = 0$

8. Solve $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0.$

9. Find Laplace transform of $t^2 e^{-4t}$.

10. Find the inverse Laplace transform of $\frac{1}{s(s-1)}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Find the Taylor's series expansion of $e^x \sin y$ near the point $(0,0)$ up to second degree terms. **[7]**

    b) Find the extreme values of $f(x,y) = x^3 y^3 - 3x - 3y$. **[7]**

12. a) Change the order of integration in $\int_0^{4a}\int_{y^2/4a}^{2\sqrt{ay}} dxdy$ and hence evaluate it. **[7]**

b) By changing into cylindrical polar co-ordinates, evaluate $\iiint_V dxdydz$ where V is the region of space inside the cylinder $x^2+y^2=a^2$, that is bounded by the planes $z=0$ and z = h. **[7]**

13. a) Show that the vector field $\overline{F}=\left(6xy+z^3\right)\hat{i}+\left(3x^2-z\right)\hat{j}+(3xz^2-y)\hat{k}$ is irrotational and find its scalar potential. **[7]**

b) Evaluate $\iint_S \vec{F}\bullet\hat{n}\,ds$ using Gauss divergence theorem, where $\vec{F}=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S is the surface of the region bounded by $x=0,\ y=0,\ z=0,\ y=3,\ x+2z=6.$ **[7]**

14. a) Solve $\frac{d^2y}{dx^2}+y=\sec x$ by using method of variation of parameters. **[7]**

b) Find the general solution for $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x\sin x$ by the method of undetermined coefficients. **[7]**

15. a) A function *f(t)* is periodic in (*0,2b*) and is defined as **[7]**

$$f(t)=\begin{cases}1, & 0<t<b\\ -1, & b<t<2b\end{cases}$$

Find its Laplace transform.

b) Solve $y''-2y'+y=e^t$ using Laplace transforms given that $y(0)=-2$ and $y'(0)=-3$. **[7]**

16. a) Find the value of $\iint x^{m-1}y^{n-1}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in terms of gamma function. **[7]**

b) Apply Convolution theorem to evaluate $L^{-1}\left[\frac{1}{(s^2+9)^2}\right]$. **[7]**

17. a) Evaluate $\iint_S \vec{F}\bullet\hat{n}ds$ where $\vec{F}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and S is the part of the sphere $x^2+y^2+z^2=1$, which lies in the first octant. **[7]**

b) Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x^3$. **[7]**

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