Term End Examination – November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Class NBR : 4556

Time : Three Hours **Max.Marks:100**

PART - A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

2. Find the Jacobian of x, y, z w.r.t. r, θ and z, where $x = r\cos\theta$, $y = r\sin\theta$, $z = z$.

3. Express $\int_0^\infty e^{-x^2}\,dx$ in terms of gamma function and hence evaluate.

4. Evaluate $\int_0^\infty \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$ by changing to polar coordinates.

5. Find the unit normal to the surface $z = x^2 + y^2$ at the point (-1, -2, -5).

6. If $\overline{F} = ax\,\hat{i} + by\,\hat{j} + cz\,\hat{k}$, where a, b, c, are constants and S is the surface of a unit sphere, show that $\int_s \overline{F}.\hat{n}\,ds = \frac{4}{3}\pi(a+b+c)$.

7. Find the particular integral of the equation $\frac{dy}{dt} + y = e^{-t}$.

8. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 8y = 0$.

9. Find $L\ \{t^3 e^{-3t}\}$.

10. If $L\{e^{-t}\cos^2 t\} = F(s)$, find $\lim_{s\to 0}[s\,F(s)]$ and $\lim_{s\to\infty}[s\,F(s)]$.

PART – B (5 X 14 = 70 Marks)

Answer any <u>FIVE</u> Questions

11. a) Expand the function $f(x,y)=\sin x\sin y$ in Taylor series expansion about the point $(0,0)$ upto third degree terms. **[7]**

b) The temperature T at any point *(x, y, z)* in space is $400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. **[7]**

12. a) Change the order of integration in $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ and hence evaluate it. **[7]**

b) Evaluate $\iiint_V (x+y+y)\,dxdydz$ where the region V is bounded by the planes $x+y+z=a, x=0, y=0, z=0$ **[7]**

13. a) Apply divergence theorem to evaluate $\int_s \vec{F}\bullet\hat{N}ds$, where $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the surface of the cylinder $x^2+y^2=4$, $z=0$ and $z=3$. **[7]**

b) A vector field is given by $A=\left(x^2+xy^2\right)\hat{i}+\left(y^2+yx^2\right)\hat{j}+3z^2\vec{k}$. Show that the field is irrotational, and find the scalar potential. **[7]**

14. a) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2}+4y=\tan 2x.$ **[7]**

b) Solve $(2x+3)^2\dfrac{d^2y}{dx^2}-(2x+3)\dfrac{dy}{dx}-12y=6x.$ **[7]**

15. a) Solve the following equation by Laplace transform **[7]**

$$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t \quad \text{with} \quad x(0)=2 \text{ and } x'(0)=-1.$$

b) Evaluate $L^{-1}\left(\dfrac{6s+20}{s^2-12s+32}\right)$. **[7]**

16. a) Find the Laplace transform of $f(t)=\begin{cases}\sin\omega t, & 0<t<\dfrac{\pi}{\omega},\\ 0, & \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega}\end{cases}$. **[7]**

b) Verify Stoke's theorem for $\vec{F} = y\,\hat{i} + xz^3\,\hat{j} - zy^3\,\hat{k}$, C is the circle $x^2 + y^2 = 4$ oriented counterclockwise, and $z = -3$. **[7]**

17. a) By transforming into spherical polar coordinates, evaluate $\iiint \sqrt{1 - x^2 - y^2 - z^2}\; dxdydz$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = 1$. **[7]**

b) Solve by the method of undetermined coefficients $(D^2 + 1)y = \sin x$. **[7]**

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