VIT UNIVERSITY

(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$ then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\sin 2u$.

2. If $x^x.y^y.z^z=c$ then show that $\dfrac{\partial^2 z}{\partial x^2}=(x.\log ex)^{-1}$ at x = y =z.

3. Change into polar coordinates and then evaluate $\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}\,dxdy$

4. Evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\cot\theta\ d\theta}$ by beta-gamma functions.

5. Find div curl $\vec{F}$ where $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$.

6. Prove, by Stokes theorem, that curl (grad Φ) = 0.

7. Find the general solution of the equation $y''+4y=0$.

8. Find the particular integral of $\dfrac{d^3y}{dx^3}+4\dfrac{dy}{dx}=\sin 2x$.

9. Find the Laplace transform of unit step function.

10. Find the Inverse Laplace transform of $\log\left(\dfrac{s^2+1}{s(s+1)}\right)$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) If $u=\log(x^3+y^3+z^3-3xyz)$ then prove that $\left(\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}+\dfrac{\partial}{\partial z}\right)^2 u=\dfrac{-9}{(x+y+z)^2}$.

    b) Find the dimensions of a rectangular box that opens at the top and requires least material for its construction with volume 32 cubic feet.

12. a) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx.dy$ and hence evaluate the same.

b) Evaluate $\iiint_V xyz\, dz\, dy\, dx$ where V is volume of the sphere $x^2+y^2+z^2=1$.

13. (a) Find the values of a, b, c so that

$\vec{F} = (axy + bz^3)\hat{i} + (3x^2 - cz)\hat{j} + (3xz^2 - y)\hat{k}$ may be irrotational. For those values of a, b, c find its scalar potential.

(b)By using Gauss divergence theorem, evaluate $\iint_S (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy)$,

where S is the closed surface of the cylinder $x^2+y^2 = a^2$ and circular disks z=0, z=6.

14. a) Find the general solution of $(D-2)^2 y = 8.(e^{2x} + \sin 2x + x^2)$

b) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = \tan 2x$, using the method of variation of parameters.

15. (a) Using Laplace transform solve

$y''' + 2y'' - y' - 2y = 0 \;\; where \;\; y = 0,\; y' = 0,\; y'' = 6 \;\; at\; x = 0$

(b) State convolution theorem and hence evaluate $L^{-1}\left[\frac{1}{(s^2+9)(s^2+1)}\right]$

16. a) Evaluate by Greens theorem $\oint_C [(y - \sin x)\, dx + \cos x\, dy]$ where C is the triangle enclosed by the lines y = 0 , x = $\frac{\pi}{2}$, $\pi y = 2x$.

b) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ by the method of undetermined co-efficients.

17. (a) Show that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$ and hence find the value of $\mu(\frac{1}{2})$. **[7]**

(b) Find the Laplace transform of **[7]**

$$f(t) = \begin{cases} \sin \omega t & 0 < t < \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}; \quad f\left(t + \frac{2\pi}{\omega}\right) = f(t)$$

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