VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Class NBR : 5040

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Investigate the continuity of the function $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ at the origin.

2. Find first and second order partial derivatives of $f(x,y)=ax^2+2hxy+by^2$ and verify $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}$

3. If $u=x\sin y, v=y\sin x$, evaluate $\dfrac{\partial(u,v)}{\partial(x,y)}$.

4. Evaluate $\int_0^a \int_0^{\sqrt{a^2+x^2}} yx\,dx\,dy$.

5. Show that $\Gamma(n+1)=n\Gamma(n)$.

6. Find the directional derivative of the function $f(x,y,z)=x^2yz+4xz^2$ at the point (1,-2, 1) in the direction of $2\vec{i}-\vec{j}-2\vec{k}$.

7. Using divergence theorem, prove that $\int_S \vec{R}.d\vec{s}=3V$, where S is any closed surface enclosing the volume V and $\overline{R}=x\vec{i}+y\vec{j}+z\vec{k}$.

8. Find the particular integral of $\dfrac{d^3y}{dx^3}+4\dfrac{dy}{dx}=\sin 2x$

9. Find the Laplace transform of $\sin 2t\sin 3t$.

10. Find the Inverse Laplace Transform of $\dfrac{s^2-3s+4}{s^3}$.

PART – B (5 X 14 = 70 Marks)
Answer any <u>FIVE</u> Questions

11. (a) Expand $e^x \log(1+y)$ in Taylor series at the point (0, 0) up to terms of degree three. **[7]**

(b) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$. **[7]**

12. (a) Change the order of Integration in $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}\,dx\,dy$ and hence evaluate it. **[7]**

(b) Find by triple integration, the volume of the sphere $x^2+y^2+z^2=a^2$. **[7]**

13. (a) Express $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of Gamma function **[7]**

(b) Evaluate $\iint r\sin\theta\, dr\, d\theta$ over the cardioid $r=a(1-\cos\theta)$ above the initial line **[7]**

14. Verify divergence theorem for $\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$ taken over the rectangular parallelepiped $0\le x\le a,\ 0\le y\le b\ \ 0\le z\le c$ **[14]**

15. (a) Using Stoke's theorem evaluate, $\int_c \left[(x+y)dx+(2x-z)dy+(y+z)dz\right]$ where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$ **[7]**

(b) Show that the vector field given by $\vec{F}=(x^2-y^2+x)\vec{i}-(2xy+y)\vec{j}$ is irrotational and find its scalar potential. **[7]**

16. (a) Solve: $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. **[7]**

(b) Solve by using method of variation of parameters **[7]**

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x\log x$$

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