Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Show that the function, $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$

   is not continuous at the point (0,0)

2. If $u=\log\left(\dfrac{x^4+y^4}{x+y}\right)$, then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$.

3. Using double integrals, find the area of a circle of radius a.

4. Evaluate $\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta$ using Beta Gamma function.

5. Find the value of the constants a, b, c so that the vector $\vec{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$ is irrotational.

6. Using Green's theorem, evaluate $\int_C (x^2-y^2)dx+2xydy$ where C is the closed curve bounded by $y=x^2$ and $y^2=x$.

7. Solve $(D^2-4D+13)y=0$

8. Solve $xy''+xy'-3y=0$.

9. Find $L\left[t^2e^{-3t}\right]$

10. Find the Laplace transform of the unit step function $H(t)=\begin{cases}1 & for\ \ t>a\\ 0 & for\ \ t<a\end{cases}$ $where\ \ a\geq 0$

PART – B (5 X 14 = 70 Marks)

Answer any FIVE Questions

11. (a) Expand $e^x \log(1+y)$ in a Taylor's series abount the point (0,0) as far as terms of second degree in x and y . **[7]**

(b) The temperature T at any point x, y, z in space is $T = 400xyz^2$. Find the highest temperature of unit sphere $x^2 + y^2 + z^2 = 1$. **[7]**

12. (a) Evaluate the following integral by changing the order of integration **[7]**

$$\int_0^a \int_y^a \frac{x}{x^2 + y^2}\, dxdy.$$

(b) Evaluate the integral $\iiint (x^2 + y^2 + z^2)\, dx\, dy\, dz$ over the region of space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1$ by transforming in to cylindrical coordinates. **[7]**

13. (a) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2 z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. **[5]**

(b) Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - (2xyz)\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. **[9]**

14. (a) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$ **[9]**

(b) Solve by the method of undetermined coefficients $(D^2 - 6D + 9)y = 4e^x$ **[5]**

15. (a) Solve $y'' + 2y' + 5y = 0$ with the intial conditions $y(0) = 2$ and $y'(0) = -4$. Using Laplace transform technique. **[7]**

(b) Find the Laplace transform of the Half wave rectifier function

$$f(t) = \begin{cases} \sin \omega t, & 0 < t < \dfrac{\pi}{\omega} \\ 0, & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases}$$ **[7]**

16. (a) Using Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where **[7]**

$\vec{F} = (x^3)\vec{i} + (y^3)\vec{j} + (z^3)\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

[7]

(b) If we transform from the three dimensional cartesian coordiantes (x, y, z) to spherical polar coordinates (r, θ, ϕ), show that the Jacobian of x, y, z with respect to r, θ, ϕ is $r^2 \sin\theta$.

17. (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **[9]**

(b) Find $L^{-1}\left[\frac{s}{(s+2)^2}\right]$ **[5]**

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