Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D2+TD2**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = xy + yz + zx$ where $x = e^t, y = e^{-t}$ and $z = \frac{1}{t}$. Find $\frac{du}{dt}$.

2. If $u = x^2 - 4y, v = x + y + z, w = x - 2y + 3z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

3. Evaluate $\iint dxdy$ over the positive quadrant of the circle $x^2 + y^2 = 4$ by changing into polar coordinates.

4. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ in terms of gamma function.

5. Find the unit normal vector to the surface $x^2 y + 2xz^2 = 8$ at the point (1, 0, 2).

6. Find the value of b so that the vector
$\vec{F} = (bx + 4y^2 z)\hat{i} + (x^3 \sin z - 3y)\hat{j} - (e^x + 4\cos x^2 y)\hat{k}$ is Solenoidal.

7. Find the particular integral for $y'' - 2y' - 3y = e^t$

8. Find the solution for $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$.

9. Find Laplace transform of $t^2 e^{-3t}$.

10. Find the inverse Laplace transform of $\frac{1}{s(s-1)}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Find the Taylor's series expansion of e^{x+y} near the origin up to second degree terms. **[7]**

    (b) Find the extreme values of $f(x,y) = x^3 + y^3 - 3xy$. **[7]**

12. (a) Change the order of integration in the integral $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$ and hence evaluate**.** **[7]**

(b) By changing into cylindrical co-ordinates, evaluate $\iiint_V (x+y+z)\,dxdydz$ where V is the region of space inside the cylinder $x^2+y^2=1$, that is bounded by the planes $z = 0$ and $z = 1$. **[7]**

13. (a) Find the work done in moving a particle in a force field given by $\overline{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ along the curve $x = t^2+1, y = 2t^2, z = t^3$ from $t=1$ to $t=2$. **[7]**

(b) Evaluate $\iint_S \vec{F} \bullet \hat{n}\, ds$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2+y^2=4$, z = 0 and z = 3 using Gauss divergence theorem. **[7]**

14. a) Solve $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$ by using method of variation of parameters. **[8]**

(b) Find the General solution for $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x \sin x$ by the method of undetermined coefficients. **[6]**

15. (a) A function $f(t)$ is periodic in (0, 2) and is defined as **[4]**

$$f(t) = 1, \quad 0 < t < 1$$
$$= -1, \quad 1 < t < 2$$

Find its Laplace transform.

(b) Solve $y''(t) - 3y'(t) + 2y(t) = e^{3t}$ using Laplace transformation given that $y(0) = 0$ and $y'(0) = 0$. **[10]**

16. (a) A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. **[8]**

(b) Apply Convolution theorem to evaluate $L^{-1}\left[\dfrac{s}{(s^2+4)^2}\right]$ **[6]**

17. (a) Evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the sphere $x^2+y^2+z^2=1$, which lies in the first octant **[7]**

(b) Solve $x^2\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$. **[7]**

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