VIT UNIVERSITY

(Estd. u/s 3 of UGC Act 1956)

Term End Examination - May 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F2+TF2**

Class NBR : 3461

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer <u>ALL</u> Questions

1. If $y = f(x+at) + g(x-at)$, prove that $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$.

2. Given $u = \sin\left(\frac{x}{y}\right), x = e^t$ and $y = t^2$, find $\frac{du}{dt}$ as a function of t.

3. Find the area enclosed by the lemniscates $r^2 = a^2 \cos 2\theta$, by double integration.

4. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$.

5. If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla\left(r^n\right) = nr^{n-2}\vec{r}$.

6. Find the work done when a force $\vec{F} = \left(x^2 - y^2 + x\right)\vec{i} - (2xy + y)\vec{j}$ displaces a particle in the - xy -plane from $(0,0)$ to $(1,1)$ along the curve $y^2 = x$.

7. Find the particular integral of $\left(D^2 + 6D + 9\right) y = e^{-2x}x^3$.

8. Match the following:

a)	Homogeneous linear Equation	a)	$y''y - xy' = 0$
b)	Non linear Equation	b)	$y'' + y' = 1$
c)	Linear non-homogeneous Equation	c)	$y'' - y' = 0$

9. If $L\left[f(t)\right] = \phi(s)$, show that $L\left[f(at)\right] = \frac{1}{a}\phi\left(\frac{s}{a}\right)$.

10. Find the Laplace transform of $t \sin at$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $x^2y+3y-z$ in powers of $(x-1)$ and $(y+2)$ up to third degree terms using Taylor's theorem. **[7]**

b) Find the volume of the greatest rectancular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **[7]**

12. a) Change the order of integration $\int_0^1\int_0^{2\sqrt{x}} x^2 dy\,dx$ and hence evaluate. **[7]**

b) Show that $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}=\pi$ using Beta and Gamma functions. **[7]**

13. Verify Stoke's theorem for $\vec{A}=\left(x^2-y^2\right)\vec{i}+2xy\,\vec{j}$ taken around the square region bounded by $x=0, x=2, y=0$ and $y=2$ on the xy plane. **[14]**

14. a) Solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$, by the method of variation parameters. **[7]**

b) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=2x^2+3e^{-x}$, by the method of undetermined co-efficients. **[7]**

15. a) Apply convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right]$. **[7]**

b) Find the Laplace transform of square wave function $E(t)=\begin{cases}1, & 0\le t<1\\ 0, & 1\le t<2\end{cases}$ with $f(t+2)=f(t)$. **[7]**

16. a) Show that $\vec{A}=\left(6xy+z^3\right)\vec{i}+\left(3x^2-z\right)\vec{j}+\left(3xz^2-y\right)\vec{k}$ is irrotational and find the scalar potential of $\vec{A}$. **[7]**

b) Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=3\sin x+4\cos x$ **[7]**

17. a) Apply Divergence theorem to evaluate $\int_S \vec{F}\cdot d\,\vec{s}$, where $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-zx\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ and S is the surface bounded by $0\le x\le a, 0\le y\le b, 0\le z\le c$. **[5]**

b) Solve by the method of Laplace transforms, **[9]**

$(D^3+2D^2-D-2)y=0$ given $y(0)=y'(0)=0$ and $y''(0)=6$.

⇔⇔⇔