1) Evaluate $\lim\limits_{\substack{x \to 1 \\ y \to 1}} \dfrac{x(y-2)}{y(x-2)}$

Sol:-

$$\lim_{\substack{x \to 1 \\ y \to 1}} \frac{x(y-2)}{y(x-2)} = \lim_{x \to 1} \left[\lim_{y \to 1} \frac{x(y-2)}{y(x-2)} \right]$$

$$= \lim_{x \to 1} \left[\frac{x(1-2)}{1(x-2)} \right]$$

$$= \lim_{x \to 1} \left[\frac{-x}{x-2} \right]$$

$$= \frac{-1}{1-2} = 1$$

2) If $u = \dfrac{y}{z} + \dfrac{z}{x}$, find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$

Sol:- Given $u = \dfrac{y}{z} + \dfrac{z}{x}$ —— (1)

$$\frac{\partial u}{\partial x} = -\frac{z}{x^2}$$

$$\therefore \quad x\frac{\partial u}{\partial x} = -\frac{z}{x} \quad \text{—— (2)}$$

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$$\frac{\partial u}{\partial y} = \frac{1}{z}$$

$$\therefore \quad y\frac{\partial u}{\partial y} = \frac{y}{z} \quad \text{———} \ (3)$$

$$\frac{\partial u}{\partial z} = -\frac{y}{z^2} + \frac{1}{x}$$

$$\therefore \quad z\frac{\partial u}{\partial z} = -\frac{y}{z} + \frac{z}{x} \quad \text{———} \ (4)$$

$$\therefore \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -\cancel{\frac{z}{x}} + \cancel{\frac{y}{z}} - \cancel{\frac{y}{z}} + \cancel{\frac{z}{x}}$$

$$= 0.$$

3) If $x = r\cos\theta$, $y = r\sin\theta$, find
(i) $\frac{\partial x}{\partial r}$ (ii) $\frac{\partial y}{\partial \theta}$ (iii) $\frac{\partial r}{\partial x}$ (iv) $\frac{\partial \theta}{\partial y}$

Sol:- Given $x = r\cos\theta$, $y = r\sin\theta$

(i) $\frac{\partial x}{\partial r} = \cos\theta$

$r^2 = x^2 + y^2$

(ii) $\frac{\partial y}{\partial \theta} = r\cos\theta$

(iii) $\left(\frac{\partial r}{\partial x}\right) = \frac{1}{\cancel{2}} (x^2+y^2)^{\frac{1}{2}-1} \cdot \cancel{2}x \quad \left(\because r = \sqrt{x^2+y^2}\right)$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

(iv) $\frac{\partial \theta}{\partial y} = \frac{1}{1+y^2/x^2}\left(\frac{1}{x}\right) \quad \left(\because \theta = \tan^{-1} y/x\right)$

$$= \frac{x}{x^2+y^2}$$

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+) If $f(x,y) = \log\sqrt{x^2+y^2}$, show that ②

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$ $\nabla^2 f = 0$

$\log a^m$
$= m\log a$

Sol:- Given $f(x,y) = \log(x^2+y^2)^{1/2}$

$$= \frac{1}{2}\log(x^2+y^2) \quad \text{—— ①}$$

$$\frac{\partial f}{\partial x} = \frac{1}{\cancel{2}} \cdot \frac{1}{x^2+y^2} \cdot \cancel{2}x = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2}$$

$$= \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} \quad \text{—— ②}$$

$$\frac{\partial f}{\partial y} = \frac{1}{\cancel{2}} \frac{1}{x^2+y^2} \cancel{2}y = \frac{y}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2}$$

$$= \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2} \quad \text{—— ③}$$

② + ③ $$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\cancel{y^2} - \cancel{x^2} + \cancel{x^2} - \cancel{y^2}}{(x^2+y^2)^2} = 0$$

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5) If $u = (x-y)(y-z)(z-x)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

Sol:-

Given $u = (x-y)(y-z)(z-x)$ — (1)

$$\frac{\partial u}{\partial x} = (y-z)\left[(x-y)(-1) + (z-x)(1)\right]$$

$$= -(y-z)(x-y) + (z-x)(y-z) \quad \text{— (2)}$$

$$\frac{\partial u}{\partial y} = (z-x)\left[(y-z)(-1) + (x-y)(1)\right]$$

$$= -(z-x)(y-z) + (x-y)(z-x) \quad \text{— (3)}$$

$$\frac{\partial u}{\partial z} = (x-y)\left[(y-z)1 + (z-x)(-1)\right]$$

$$= (x-y)(y-z) - (x-y)(z-x) \quad \text{— (4)}$$

(2) + (3) + (4) $\Rightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = -\cancel{(y-z)(x-y)} + \cancel{(z-x)(y-z)}$$

$$-\cancel{(z-x)(y-z)} + \cancel{(x-y)(z-x)}$$

$$+\cancel{(x-y)(y-z)} - \cancel{(x-y)(z-x)}$$

$$= 0.$$

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