VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

2. If $x^x.y^y.z^z = c$ then show that $\frac{\partial^2 z}{\partial x^2} = (x.\log ex)^{-1}$ at x = y =z.

3. Change into polar coordinates and then evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$

4. Evaluate $\int_0^{\pi/2} \sqrt{Cot\,\theta}\, d\theta$ by Beta-Gamma functions

5. Find div curl $\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.

6. Prove that by stokes theorem Curl(Grad Φ) = 0

7. Find the general solution of the equation $\frac{d^4x}{dt^4} + 4x = 0$.

8. Find the particular integral of $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

9. Find the Laplace transform of unit step function.

10. Find the inverse Laplace transform of $\log\left(\frac{s^2+1}{s(s+1)}\right)$

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

b) Find the dimensions of a rectangular box that opens at the top and requires least material for its construction with volume 32 cubic feets.

12. a) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx.dy$ and hence evaluate the same.

b) Evaluate $\iiint_v xyz\, dz.dy.dx$ where v is volume of the sphere $x^2+y^2+z^2=1$

13. (a) Show that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$

(b) Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$ using Beta and Gamma functions

14. (a) Evaluate $\int_S F.N\, ds$, where F = zi +xj -3y^2zk and S is the surface $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5.

(b)By using Gauss divergence theorem, Evaluate $\iint_S x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy$

Where S is the closed surface of the cylinder $x^2+y^2 = a^2$ and circular disks z=0, z=6.

15. a) Evaluate by greens theorem $\oint_C (y - \sin x)\, dx + Cosx\, dy$ where C is the triangle enclosed by the line y = 0 , x = $\frac{\pi}{2}$, $\pi y = 2x$

b) Solve $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ by the method of undetermined co-efficients

16. a) Find the general solution of $(D-2)^2 y = 8.(e^{2x} + \sin 2x + x^2)$

b) Solve the differential equation $\frac{d^2 y}{dx^2} + 4y = Tan\ 2x$, using the method of variation of parameters.

17. (a) Using Laplace Transform solve

$$y^{111} + 2y^{11} - y^1 - 2y = 0 \quad where \quad y = 0, y' = 0, y^{11} = 6 \quad at \quad x = 0$$

(b) State convolution theorem and hence evaluate $L^{-1}\left[\frac{1}{(s^2+9)(s^2+1)}\right]$

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