Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	CALCULUS	Course Code	BMAT101L
Faculty Name	Prof. PROSENJIT	Slot	Y11+Y12+Y21
		Class Nbr	CH2023240503676
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer any 10 questions (10 X 10 Marks = 100 Marks)

01. Find the subintervals on which the function, $f(x) = x^4 - 8x^2 + 20$, decreasing or increasing. Also discuss the concavity of the function. [10]

02. (a) Find the maximum value of the function, $f(x) = x^3 - 12x^2 + 36x + 17$, in the interval [1, 10]. [5 Marks] [10]

(b) Check the existence of the limit of the function

$$f(x,y) = \begin{cases} \frac{x^3 y}{x^6+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

at $(x,y) = (0,0)$. [5 Marks]

03. Let $w = xy + yz + zx$, where $x = u + v$, $y = u - v$, $z = uv$. Find $\frac{\partial w}{\partial u}$ *and* $\frac{\partial w}{\partial v}$ at the point $(u,v) = (1,2)$. Also find the total derivative. [10]

04. Classify the stationary point and investigate the maxima and minima of the function $f(x,y) = x^4 + y^4 - 6(x^2 + y^2) + 8xy$. [10]

05. (a) For a rectangle whose perimeter is 20m, use the Lagrange multiplier method to find the dimension that will maximize the area. [5 Marks] [10]

(b) Sketch the region of integration and evaluate $\int_0^1 \int_0^x (x^2 + y^2)\,dy\,dx$ [5 Marks]

06. Evaluate, $\iiint x\,y\,z\,dx\,dy\,dz$ over the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $z = 1$ and the circle $x^2 + y^2 = 1$ [10]

07. Evaluate the following integrals using Beta and Gamma function [10]

(a) $\int_0^1 \frac{x^2}{\sqrt{(1-x^5)}}\,dx$ [5 Marks]

(b) $\int_0^{\frac{\pi}{2}} \left(\sqrt{(\tan\theta)} + \sqrt{(\sec\theta)}\right) d\theta$ [5 Marks]

08. Find the value of $\int\int x^{m-1}y^{n-1}dxdy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in terms of the Gamma function. [10]

09. Find the work done in moving a particle in the force field $F = 3x^2\,\vec{i} + (2xz - y)\,\vec{j} + z\,k$ along the straight line from (0,0,0) to (2,1,3). [10]

10. Find the directional derivative of $\nabla \cdot (\nabla\phi)$ at the point (1,-2,1) in the direction normal to the surfaces [10]

$$xy^2z = 3x + z^2 \text{ where } \phi = 2x^3y^3z^4$$

11. Apply Green's theorem to evaluate $\int_c (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where c is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$ [10]

12. Using Stoke's theorem evaluate $\int_c[(x + y)dx + (2x - z)dy + (y + z)dz]$ where C is the boundary of the triangle with verties (2, 0, 0), (0, 3, 0) and (0, 0, 6). [10]