VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - May 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: C1+TC1**

Class NBR : 2843 / 2849 / 3452

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. If $u = xy + yz + zx$,where $x = e^t$, $y = e^{-t}$ and $z = \frac{1}{t}$, find $\frac{du}{dt}$.

2. Examine if $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$ are functionally dependent.

3. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$, using polar coordinates.

4. Evaluate $\int_0^\infty x^6 e^{-3x}\, dx$ using Gamma function.

5. Obtain the directional derivative of $\varphi = x^2yz + 4xz^2$ at the point $(1, -2 - 1)$ in the direction of $2\hat{\imath} - \hat{\jmath} - \hat{k}$.

6. Using Stoke's theorem, prove that $\int_C \varphi\nabla\varphi \cdot \overrightarrow{dr} = 0$, where C is a simple closed curve and φ is a scalar point function.

7. Find the particular integral of $(D^2 + D + 1)y = \sin 2x$.

8. Solve $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$.

9. Find the Laplace transform of $\frac{1-e^t}{t}$.

10. Find the inverse Laplace transform of $log\left(1 + \frac{\omega^2}{s^2}\right)$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Using Taylor's series expansion expand $\tan^{-1}\frac{y}{x}$ in the neighbourhood of (1, 1). **[7]**

b) Examine $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **[7]**

12. a) By changing the order of integration, evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx\, dy$. **[7]**

b) Using Gamma function, prove that $\left(\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}}\right)\left(\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta\right) = \pi$. **[7]**

13. a) Prove that $\vec{F} = (z^2 + 2x + 3y)\hat{\imath} + (3x + 2y + z)\hat{\jmath} + (y + 2zx)\hat{k}$ is irrotational vector. Hence find its scalar potential φ . **[7]**

b) Verify Green' s theorem in the plane for $\int_C (2x - y)dx + (x + y)dy$ where C is the boundary of the circle $x^2 + y^2 = a^2$. **[7]**

14. a) Solve the differential equation $\frac{d^2y}{dx^2} + y = cosec\, x$, by the method of variation of parameters. **[7]**

b) Solve the differential equation, $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = x^2$ by the method of undetermined coefficients. **[7]**

15. a) Find the Laplace transform of the periodic function **[7]**

$$f(t) = \begin{Bmatrix} t & in\ 0 < t < 1 \\ 2 - t & in\ 1 < t < 2 \end{Bmatrix} \quad \text{given that } f\,(t + 2) = f(t).$$

b) Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. **[7]**

16. a) By transforming into cylindrical coordinates, evaluate the integral **[7]**

$\int\int\int(x^2 + y^2 + z^2)dxdydz$ taken over the region of space defined by $x^2 + y^2 \leq 1\ and\ 0 \leq z \leq 1$.

b) If $= |\vec{r}|$, where $\vec{r}$ is the position vector of the point (x, y, z), prove that $\nabla^2(r^n) = n(n + 1)r^{n-2}$ and hence deduce that $\frac{1}{r}$ satisfies Laplace equation. **[7]**

17. a) Solve the differential equation $x^2\frac{d^2y}{d^2x} + 4x\frac{dy}{dx} + 2y = x^2 + \frac{1}{x^2}$. **[7]**

b) Using Laplace transform, solve the integral equation **[7]**

$$\frac{dy}{dx} + 2y + \int_0^t y\, dt = 2\cos t\, , y(0) = 1\, .$$

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