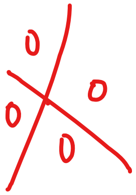
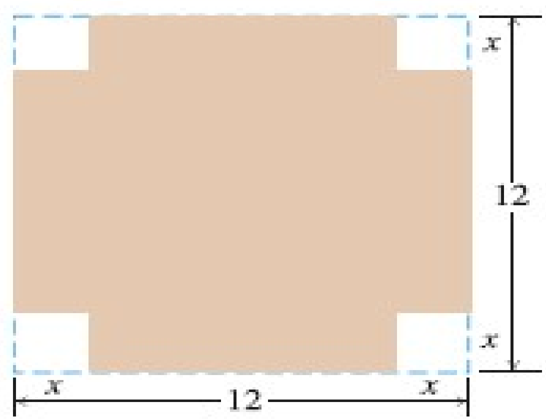
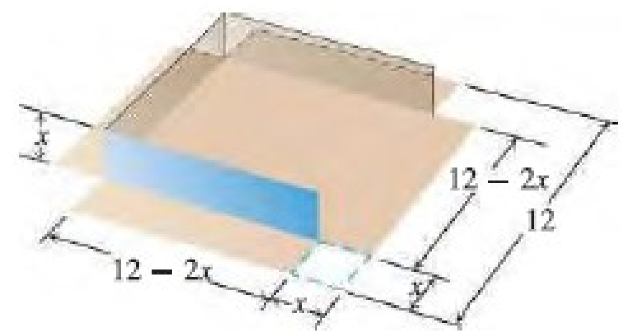


Second Derivative Test for Local Extrema Suppose f'' is continuous on an open interval that contains $x = c$.

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
3. If $f'(c) = 0$ and $f''(c) = 0$, then the test fails. The function f may have a local maximum, a local minimum, or neither.

1) An open-top box is to be made by cutting small congruent squares from the corners of a 12-in.-by-12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?





The volume of the box is a function of this variable:

$$V(x) = x(12 - 2x)^2 = 144x - 48x^2 + 4x^3.$$

Since the sides of the sheet of tin are only 12 in. long, $x \le 6$ and the domain of V is the interval $0 \le x \le 6$.

$$\frac{dV}{dx} = 144 - 96x + 12x^2 = 12(12 - 8x + x^2) = 12(2 - x)(6 - x).$$

Of the two zeros, $x = 2$ and $x = 6$, only $x = 2$ lies in the interior of the function's domain and makes the critical-point list. The values of V at this one critical point and two endpoints are

Critical-point value: $V(2) = 128$

Endpoint values: $V(0) = 0, \quad V(6) = 0.$

The maximum volume is 128 in^3. The cutout squares should be 2 in. on a side.

2) What is the smallest perimeter possible for a rectangle whose area is 16 in^2, and what are its dimensions?

Sol:- Let l & w represents the length and width of the rectangle.

Area A is 16 in^2 $\Rightarrow$ $lw = 16$

$w = 16/l$

$$P = 2l + 2w = 2l + 2\left(\frac{16}{l}\right) = 2l + \frac{32}{l}$$

$$P'(l) = 2 - \frac{32}{l^2}, \quad P'(l) = 0$$

$$2 - \frac{32}{l^2} = 0$$

$$\frac{2l^2-32}{l^2}=0$$

$$\frac{2(l^2-16)}{l^2}=0$$

$$2(l+4)(l-4)=0$$

$$l=-4,4$$

Since $l>0$ we $l=4$

$$w=4$$

$$P''(l)=\frac{64}{l^3}>0$$

The smallest possible perimeter with A 16 in^2 $P=2(4)+2(4)=16$ in $l=4$ & $w=4$

3. You are designing a rectangular poster to contain 50 in^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin at each side ? What overall dimensions will minimize the amount of paper used?

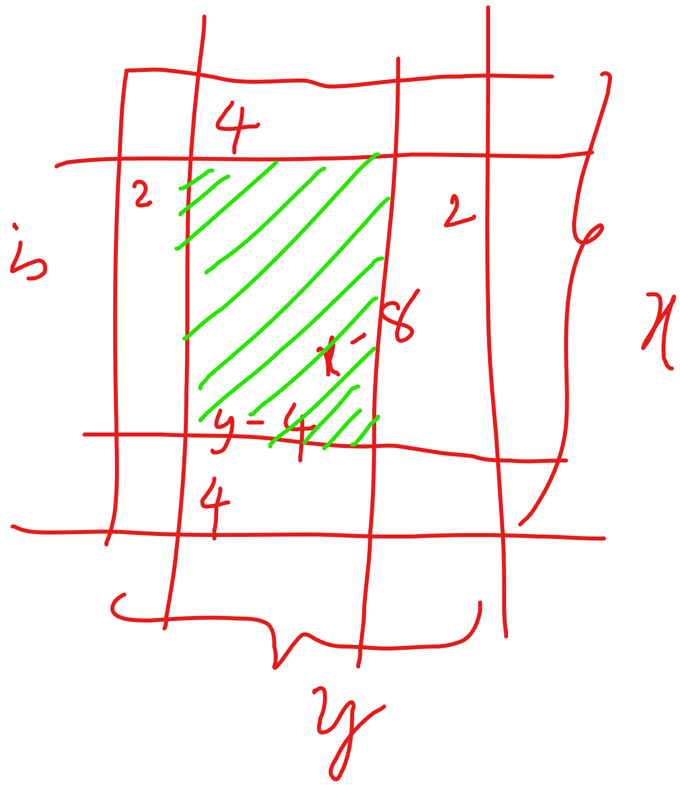
Sol :-

The area of the printing is

$$(y-4)(x-8) = 50$$

$$y-4 = \frac{50}{x-8}$$

ie., $y = \frac{50}{x-8} + 4$



The area of the paper is

$$A(x) = x\left(\frac{50}{x-8}+4\right), \text{ where } x > 8$$

$$A'(x) = \left(\frac{50}{x-8}+4\right)(1) + x\left(\frac{-50}{(x-8)^2}\right)$$

$$= \frac{50}{x-8} + 4 - \frac{50x}{(x-8)^2} = \frac{50(x-8)+4(x-8)^2-50x}{(x-8)^2}$$

$$= \frac{\cancel{50x} - 400 + 4(x-8)^2 - \cancel{50x}}{(x-8)^2}$$

$$= \frac{4(x-8)^2-400}{(x-8)^2}$$

$$A'(x) = 0 \Rightarrow 4(x-8)^2 - 400 = 0$$

$$x^2 - 16x - 36 = 0$$

$$(x+2)(x-18) = 0$$

-36

$2 \quad -18$

The critical pts are -2 & 18

Since -2 is not in the domain, the only value of x is 18.

$$A''(x) = \frac{-50}{(x-8)^2} - \left[\frac{(x-8)^2(50) - 50x\,2(x-8)(1)}{(x-8)^4} \right]$$

$$= \frac{800}{(x-8)^3}$$

$A''(18) > 0$ so we have a minimum

at $x = 18$

$\therefore \quad y = 9$

Therefore the dimension is 18 by 9 inches

minimizes the amount of paper used.

Homework Problem:-

1) Show that among all rectangles with an 8-m perimeter, the one with largest area is a square.
2) You are planning to make an open rectangular box from an 8-in-by-15-in piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is its volume?