Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: C2+TC2**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Verify Euler's theorem for the function $f(x, y, z) = 3x^2yz + 5xy^2z + 4z^4$.

2. If $u = x^2 - y^2, v = 2xy$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Evaluate $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}$.

4. Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)dxdy$.

5. Show that the vector field given by $\vec{A} = 3x^2y\vec{i} + (x^3 - 2yz^2)\vec{j} + (3z^2 - 2y^2z)\vec{k}$ is irrotational but not solenoidal.

6. Use Green's theorem in a plane to evaluate $\oint_c [(2x-y)dx + (x+y)dy]$ where c is the boundary of the circle $x^2 + y^2 = a^2$ in the xoy – plane.

7. Find the solution of $y'' - 4y' + 4y = 0$ which satisfies $y(0) = 3, y'(0) = 1$.

8. Find the general solution of $\frac{d^3y}{dx^3} - y = 0$.

9. Find $L[t\cos t]$.

10. Find $L^{-1}\left(\frac{s+2}{s^2 - 4s + 13}\right)$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand the Taylor's series expansion of $e^x \cos y$ at the point (0, 0) to third approximation.

(b) The temperature T at any point *(x, y, z)* in space is $400x^2yz$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

12. (a) Evaluate $\iint_R \left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)dxdy$ over the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Change the Order of integration in $\int_0^{4a}\int_{y=\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ and hence evaluate it.

13. (a) Verify Green's theorem for, $\oint_c [(xy+y^2)dx+x^2dy]$, where c is bounded by $y=x$ and $y=x^2$.

(b) If $\overline{F}=4xz\,\hat{i}-y^2\,\hat{j}+yz\,\hat{k}$, evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

14. (a) Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2}+4y=5\sin 3x$.

(b) Solve the differential equation $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=96x$.

15. (a) Use convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$.

(b) Solve $\frac{d^2y}{dx^2}+9y=\cos 2x$ using Laplace transform given that $y(0)=1, y\left(\frac{\pi}{2}\right)=-1$

16. (a) Discuss the maxima and minima of the function

$$f(x,y)=x^4+y^4-2x^2+4xy-2y^2$$

(b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

17. (a) Verify Stoke's theorem for the vector field $\overline{F}=(x^2-y^2)\,\vec{i}+2xy\,\vec{j}$ integrated round the rectangle in the plane $z=0$ and bounded by the lines $x=0, y=0, x=a, y=b$.

(b) Evaluate $L^{-1}\left(\frac{8s+29}{s^2-12s+32}\right)$.

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