VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D1+TD1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Find $\frac{dy}{dx}$ if $x = e^{-t}, y = t^2$.

2. Compute the stationary points of $x^3 + y^3 - 3axy$.

3. Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

4. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ using gamma integrals.

5. Write the physical interpretation of curl (in terms velocity and angular velocity), divergence (related to fluid flows), and gradient (in terms of a surface).

6. Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz-y)\bar{j} + 2\bar{k}$ along the curve $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.

7. Solve $(D^4 + 4)x = 0$, where $D = d/dt$.

8. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

9. By Laplace transforms, evaluate the integral $\int_0^{\infty} \frac{\sin t}{t}\, dt$.

10. If $L\{e^{-t}\cos^2 t\} = F(s)$, find $\lim_{s\to 0}[sF(s)]$ and $\lim_{s\to\infty}[sF(s)]$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Expand $f(x,y) = \tan^{-1}(y/x)$ in powers of $(x-1)$ and $(y-1)$ up to third degree. **[7]**

(b) Prove that the rectangular solid of maximum volume which can be inscribed in a given sphere is a cube. **[7]**

12. a) Change the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\, dy\, dx}{\sqrt{x^2+y^2}}$ and hence or otherwise evaluate. **[7]**

b) Find the area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, using Beta and Gamma functions. **[7]**

13. (a) Show that $\overline{F} = (y^2-z^2+3yz-2x)\ \bar{i} + (3xz + 2xy)\ \bar{j} + (3xy-2xz+2z)\bar{k}$ is both solenoidal and irrotational. **[4]**

(b) Verify Stoke's theorem for $\overline{F} = (2x-y)\ \bar{i} - yz^2\ \bar{j}\ - y^2z\,\bar{k}$ where S is the upper half surface of the sphere $x^2+y^2+z^2 = 1$ and C is its boundary. **[10]**

14. (a) Solve by the method of variation of parameters $\frac{d^2x}{dx^2} + 4y = \tan 2x.$ **[7]**

(b) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x.$ **[7]**

15. a) Solve the following equation by Laplace transform **[7]**

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t \quad with \quad x(0) = 2 \quad and \quad x'(0) = -1.$$

(b) Find the Laplace transform of the periodic function $f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega}, \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}.$ **[7]**

16. (a) Using the method of undetermined coefficients solve the following equation **[7]**

$$(D^2 + D + 1)y = 4e^x - 4\cos 2x.$$

(b) Use Laplace transform solve the equation $y(t) = e^{-t} - 2\int_0^t y(u)\sin(t-u)du$ **[7]**

17. (a) Using Gauss divergence theorem, evaluate $\iint_S (4x\,\hat{i} - 2y^2\,\hat{j} + z^2\,\hat{k}) \bullet \overrightarrow{dS}$, where S is the closed surface bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z=0$ and $z=3$. **[7]**

b) By transforming into spherical polar coordinates, evaluate $\iiint \sqrt{1-x^2-y^2-z^2}\ dxdydz$, taken throughout the volume of the sphere $x^2+y^2+z^2 = 1$. **[7]**

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