Assignment # 1

Vellore-632 014, Tamil Nadu, India.
www.vit.ac.in

School of Advanced Sciences
Department of Mathematics
Fall Semester - 2017-18
MAT1711: Calculus for Engineers

---

1. a) Determine the continuity and differentiability of $f(x)=|x-1|$ at $x=1$.

   b) Find whether $f(x)=\begin{cases} \dfrac{x}{\sin x} & x\neq 0 \\ 0 & x=0 \end{cases}$ is differentiable at the origin. In case of differentiability, find the derivative at the origin.

   c) Find the values of the constants mentioned such that $f(x)=\begin{cases} ax+b & x>-1 \\ bx^2-3 & x\leq -1 \end{cases}$ is differentiable for all real values.

   d) Find the first two derivatives of the following

   (i) $4x\sqrt{x+\sqrt{x}}$, (ii) $x^2\sin^2\left(2x^2\right)$, (iii) $\dfrac{\sqrt{x}e^{\sqrt{\sin x}}(\tan x+\sec x)}{x^4+2x^2+1}$.

   e) If $x\,y=\cot\left(x\,y\right)$ then find $\dfrac{dy}{dx}$ using implicit differentiation.

2. a) Find all the critical points for the function $f(z)=\dfrac{z^2+1}{z^2-z-6}$.

   b) Determine all the numbers *c* which satisfy the conclusions of the Mean Value theorem for the function in $f(t)=\dfrac{2}{3}x^3-6x^2+10x$ the interval [0, 6].

   c) If $f(x)=\sin|x|$ for $I:\dfrac{-\pi}{2}\leq x\leq\dfrac{\pi}{2}$, determine the subintervals of *I* on which *f* is concave down and concave up. What about the points of inflection for f in *I*?

   d) Find the values of constants *a*, *b*, and *c* so that the graph of $y=\dfrac{x^2+a}{bx+c}$ has a local minimum at $x=3$ and a local maximum at $(-1,-2)$.

   e) Determine the following for the function $f(x)=\dfrac{(x+1)^2}{1+x^2}$. (i) Domain, (ii) Critical points, (iii) Increasing and decreasing intervals, (iv) Extreme points, (v) Inflection points, (vi) Asymptotes.

Assignment # 1

3. a) Find the angle of intersection of curves $x^2 - y^2 = a^2$ and $x^2 + y^2 = \sqrt{2}\,a^2$ also verify the orthogonality of intersection of curves.
   b) The position of a particle moving along a number line is given by $f(t) = \frac{2}{3}t^3 - 6t^2 + 10t$ where t is time in seconds. The particle moves both left and right in the first six seconds. What is the total distance travelled by the particle for $0 \le t \le 6$.
   c) The equations for free fall at the surfaces of Mars and Jupiter are $s = 1.86t^2$ on Mars and $s = 11.44t^2$ on Jupiter. How long does it take a rock falling from west to each a velocity of $27.8\,m/\sec$ on each planet?
   d) If a mercury thermometer took 14 seconds to rise from $-19^{\circ}C$ to $100^{\circ}C$ when it was taken drawn from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at the rate of $8.5^{\circ}C/\sec$.
   e) A vessel is in the form of an inverted cone of semi-vertical angle 45°. Water is poured into this vessel at the rate of 100 cc per second. Find the rate of rise in the water level when it is 2 cm deep?

4. a) Evaluate the following integrals:

   (i) $\int x\sin\left(2x^2\right)dx$, (ii) $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$, (iii) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$, (iv) $\int \frac{1}{x\log^2 x}\,dx$
   b) A hemispherical bowl of radius *a* meters contains water to a depth *h* meters. Find the volume of water in the bowl. Suppose that the water runs into a sunken concrete hemispherical bowl of radius 5 m at the rate of 0.2 m^3/sec. How fast is the water level in the bowl rising when the water is 4m deep?
   c) Find the area of the region enclosed between the curves $y = -x^4$ and $y = 8x$.
   d) Using Washer Method, find the volume of solid of revolution about the *x*-axis generated by revolving the region *R*, enclosed by the curve $y = \cos x$ and the line $y = 1$ between the ordinates $x = \pm\frac{\pi}{2}$.
   e) Using Disk Method, find the volume of solid of revolution about the *x*-axis generated by revolving the region *R*, enclosed by the triangle with edges $x = 0,\ x + 2y = 2$ and $y = 0$.
   f) The solid lies between planes perpendicular to the *x*-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the axis on the $0 \le x \le 4$ are squares whose bases run from the semi-circle $y = -\sqrt{1 - x^2}$ to the parabola $y = \sqrt{1 - x^2}$. Find the volume of solid by Slicing**.**

Assignment # 1

5. a) Evaluate $\int_0^1 \left(y^5 \log\left(\frac{1}{y}\right)\right)^3 dy$ in terms of Gamma function.

b) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ c) Evaluate $\int_0^\infty \frac{x^c}{c^x} dx$ d) Evaluate $\int_0^\infty a^{-bx^2}\, dx$

e) Prove that $\int_0^1 \frac{x^2\, dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ f) d) Evaluate $\int_0^\infty \sqrt{\tan x}\, dx$

6. a) Find the Laplace Transforms of the following: (i) $t\sin at$, (ii) $f(t) = |t-1| + |t+1|,\quad t>0$,

(iii) $f(t) = [t]$ where $[\ ]$ stands for the greatest integer function.

(iv) Find $\frac{d}{dt}\left(\int_0^{t^4} \sqrt{x}\, dx \right)$, by mentioning the appropriate rule.

(iv) $f(t) = \begin{cases} t & 0<t<\pi \\ \pi t & \pi < t < 2\pi \end{cases}$.

b) Find the Laplace transform of the periodic half-wave rectified signal f which is given in one period as $f(t) = \begin{cases} \sin at & for \quad 0<t<\frac{\pi}{a} \\ 0 & for \quad \frac{\pi}{a} < t < \frac{2\pi}{a} \end{cases}$.

c) Find the Laplace Transforms of the following: (i) $t\sqrt{1+\sin t}$, (ii) $t^2 \sin at$,

(iii) $t e^{-t} \sin 3t$, (iv) $e^{-t} \int_0^t \frac{\sin t}{t}\, dt$, (v) $\int_0^t \int_0^t \int_0^t t \sin t \; dt\, dt\, dt$, (vi) $\int_0^\infty \frac{e^{-t} \sin t}{t}\, dt$.

d) Express the function $f(t) = \begin{cases} t-1 & 1<t<2 \\ 3-t & 2<t<3 \end{cases}$ in terms of unit step function and find its Laplace transform.

e) Obtain the Laplace transform of $e^{-t}\left[1-u(t-2)\right]$.

f) Using Laplace transform, evaluate $\int_0^\infty e^{-t}\left[1+2t-t^2+t^3\right] H(t-1)\, dt$.

g) Evaluate $L\left\{ \frac{1}{t} \delta(t-a) \right\}$.