Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D1+TD1**

Class NBR : 2206\ 2219\ 2237 \2240 \2246 \2254 \2303 \2336 \2355 \2372 \2375 \2387 \2853 \5499

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. If $u = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

2. If $x = u(1-v)$, $y = uv$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Evaluate $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}\, dxdy$ using polar coordinates.

4. Find the value of $\int_0^\infty \frac{x^3(1-x^6)}{(1+x)^{14}}\, dx$, using Beta function

5. Obtain the directional derivative of $\varphi = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

6. If S is the surface of the sphere $x^2 + y^2 + z^2 = 1$, evaluate $\iint_S \left(x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}\right)\cdot \overrightarrow{dS}$ using Gauss divergence theorem.

7. Find the particular integral of $(D^2 + D + 1)y = \sin 2x$.

8. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$.

9. Evaluate $\int_0^\infty te^{-2t} sint\, dt$, using Laplace transform.

10. Find the inverse Laplace transform of $\log\left(\frac{s^2+a^2}{s^2-b^2}\right)$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Using Taylor's series, expand $e^x \log(1+y)$ in powers of x and y up to third degree terms. **[7]**

    (b) Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **[7]**

12. (a) By changing the order of integration, evaluate $\int_0^1 \int_x^1 \frac{x}{x^2+y^2}\, dydx$. **[7]**

(b) Find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, using Gamma function. **[7]**

13. (a) Prove that $\vec{F} = (z^2 + 2x + 3y)\hat{\imath} + (3x + 2y + z)\hat{\jmath} + (y + 2xz)\hat{k}$ is irrotational vector. Hence find its scalar potential φ. **[7]**

(b) Using Green's theorem, find the area between $y^2 = 4x$ and $x^2 = 4y$. **[7]**

14. (a) Solve $\frac{d^2y}{dx^2} + y = x$ by the method of variation of parameters. **[7]**

(b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$ by the method of undetermined coefficients. **[7]**

15. (a) Using convolution theorem, find the inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$. **[7]**

(b) Using Laplace transform determine $'y'$ which satisfies the equation **[7]**

$\frac{dy}{dt} + 2y + \int_0^t y\, dt = 2\cos t\, , y(0) = 1$.

16. (a) Evaluate $\int\int\int \frac{dx\, dy\, dz}{x^2+y^2+z^2}$, taken throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$. **[7]**

(b) Verify Stoke's theorem when $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ and C is the boundary of the rectangular region in the xoy plane bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. **[7]**

17. (a) Solve the differential equation $x^2\frac{d^2y}{d^2x} + 4x\frac{dy}{dx} + 2y = xlogx$. **[7]**

(b) Find the Laplace transform of periodic function $f(t) = \begin{cases} t & in \;\; 0 < t < \frac{\pi}{2} \\ \pi - t & in \;\; \frac{\pi}{2} \le t \le \pi \end{cases}$ **[7]**

given that $f\,(t + \pi\,) = f(t)$.

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