Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D1+TD1**

Class NBR : 4337 / 4366 / 4372 /4977/ 5049 / 5056 / 5069 / 5158 / 5426

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If z is a function of x and y, where $x = e^u + e^v$ and $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

2. If $x = u(1-v),\ y = uv,$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Evaluate $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$ using polar coordinates.

4. Using Beta function, evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$

5. Obtain the directional derivative of $\varphi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$.

6. Prove that for a closed surface S, $\iint_S \vec{r}.\hat{n}\,dS = 3V$ where V is the volume enclosed by S, using Gauss divergence theorem.

7. Find the particular integral of $(D^3 - 3D^2 + 3D - 1)y = x^2e^x$

8. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 5y = 0.$

9. Evaluate $\int_0^\infty te^{-2t}\sin t\,dt$, using Laplace transform.

10. Verify the initial value theorem for the function $2e^{-3t}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. (a) Using Taylor's series expand $e^x \sin y$ in powers of x and y upto third degree terms. **[7]**

(b) Find the shortest and the longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. **[7]**

12. (a) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dxdy$. **[7]**

(b) Find the area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, using Gamma function. **[7]**

13. (a) Prove that $\vec{F} = (2x + yz)\hat{i} + (4y + zx)\hat{j} - (6z - xy)\hat{k}$ is irrotational vector field. Hence find its scalar potential φ.. **[7]**

(b) Verify Green's theorem for $\oint_C (2x - y)dx + (x + y)dy$, where C is the boundary of the circle $x^2 + y^2 = a^2$ in the xy - plane. **[7]**

14. (a) Solve the equation $\frac{d^2y}{dx^2} + y = sec\, x$, by the method of variation of parameters. **[7]**

(b) Solve the differential equation, $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$ by the method of undetermined coefficients. **[7]**

15. (a) Find the Laplace transform of periodic function $f(t) = \begin{cases} a \sin \omega t & in\ 0 \le t \le \frac{\pi}{\omega} \\ 0 & in\ \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$ given that $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$. **[7]**

(b) Using convolution theorem, find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$. **[7]**

16. (a) By transforming into spherical polar coordinates, evaluate $\iiint \frac{dx\,dy\,dz}{x^2 + y^2 + z^2}$, taken throughout the volume of the Sphere $x^2 + y^2 + z^2 = a^2$. **[7]**

(b) Verify Stoke's theorem when $\vec{F} = (2xy - x^2)\hat{i} - (x^2 - y^2)\hat{j}$ and C is the boundary of the region enclosed by the parabolas $y^2 = x$ and $x^2 = y$. **[7]**

17. (a) Solve the differential equation $x^2 \frac{d^2y}{d^2x} - 3x\frac{dy}{dx} - 5y = \sin(\log x)$. **[7]**

(b) Using Laplace transform, solve the differential equation $\frac{d^2y}{d^2x} + \frac{dy}{dx} = t^2 + 2t$, where $y(0) = 4$, $y'(0) = -2$. **[7]**

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