VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E1+TE1**

Class NBR : 4555

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. If $u = x^2 - y^2, v = 2xy$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

2. Verify Euler's theorem for the function u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

3. Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2)dydx$

4. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dx\,dy$ by changing to polar coordinates.

5. Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$

6. Find the value of the constants a, b, c so that the vector $\vec{F} = (x+2y+az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.

7. Find the solution of $\frac{d^3y}{dx^3} + y = 0$

8. Reduce the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$ into a differential equation with constant coefficients.

9. Find the Laplace transforms of t^3e^{-3t}.

10. If $L\{e^{-t}\cos^2 t\} = F(s)$, find $\lim_{s\to 0}[sF(s)]$ and $\lim_{s\to\infty}[sF(s)]$.

PART – B (5 X 14 = 70 Marks)

Answer any <u>FIVE</u> Questions

11. a) Expand the Taylor's series expansion of $e^{x} \log(1+y)$ at the point (0, 0) to third degree terms. **[7]**

b) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **[7]**

12. a) Change the order of integration in $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{x^2+y^2}\, dx\, dy$ and hence evaluate it. **[7]**

b) Find the area of astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ using gamma function. **[7]**

13. a) Using Divergence theorem evaluate $\int_S \vec{F}.\vec{ds}$ where $\vec{F} = 4x\hat{i} - 2y^2\ \hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4,\ z = 0\ and\ z = 3.$ **[7]**

b) Use Green's theorem to evaluate $\int_c$ [(xy+y^2) dx +x^2 dy] where C is bounded by $y = x$ and $y = x^2$. **[7]**

14. a) Solve $(D^2 - 2D+1)y = xe^x \sin x$, where $D = \frac{d}{dx}$. **[7]**

b) Solve by the method of variation of parameters $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ **[7]**

15. a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dt} - 3y = \sin t$ using Laplace transform, given that $y = \frac{dy}{dt} = 0$ when $t = 0$. **[7]**

b) Use convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. **[7]**

16. a) Solve the differential equation $(3x+1)^2\frac{d^2y}{dx^2} - 2(3x+1)\frac{dy}{dx} - 12y = 96x$ **[7]**

b) Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} + 4y = 5\sin 5x$ **[7]**

17. a) Evaluate $\int_1^{e}\int_1^{\log y}\int_1^{e^x} \log z\ dzdydx$ **[7]**

b) Show that the vector $\vec{F} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational and find its scalar potential. **[7]**

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