VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - November 2011

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: C1+TC1**

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Find $\left(\dfrac{\partial f}{\partial y}\right)_{(1,0)}$ where $f(x,y) = x^3y^2 + e^{xy^2}$

2. Prove that $\int_0^\infty e^{-x^2}dx = \dfrac{\sqrt{\pi}}{2}$

3. Find the directional derivative of $f(x,y,z) = x^2yz + 4xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

4. Solve the differential equation $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = e^{3x}$

5. Find $L\left[\left(t^2 - 3t + 2\right)e^{5t}\right]$

6. Find $\dfrac{dy}{dx}$ from $xe^{-y} - 2ye^x = 1$

7. Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta = \dfrac{\Gamma\left(1/4\right)\Gamma\left(3/4\right)}{2}$

8. Find the equation of the tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2,1,-3)$.

9. Solve $\left(D^2 - 6D + 9\right)y = 4e^x$ by the method of undetermined coefficients.

10. Applying the second shifting property, find the value of $L^{-1}\left(\dfrac{8e^{-3s}}{s^2+4}\right)$

PART – B (5 X 14 = 70 Marks)
Answer ALL the Questions

11. (a) If $u = \log(\tan x + \tan y)$, then prove that $\sin 2x\dfrac{\partial u}{\partial x} + \sin 2y\dfrac{\partial u}{\partial y} = 2$

(b) Find the point on the parabola $x^2 = 2y$ nearest to the point $(0,3)$, using the method of Lagrange's Multiplier.

12. (a) Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(b) Evaluate $\int_0^1\int_0^3\int_0^5 (x+y+z)\,dx\,dy\,dz$

13. (a) If $r=|\vec{r}|$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, prove that

(i) $\vec{\nabla}\left(\dfrac{1}{r}\right)=-\dfrac{\vec{r}}{r^3}$ (ii) $\vec{\nabla}\left(r^n\right)=nr^{n-2}\vec{r}$

(b) Show that $\vec{A}=\left(6xy+z^3\right)\hat{i}+\left(3x^2-z\right)\hat{j}+\left(3xz^2-y\right)\hat{k}$ is irrotational. Find the scalar functions φ such that $\vec{A}=\vec{\nabla}\phi$

14. (a) Solve the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=xe^x\sin x$

(b)Verify Green's theorem for $\int_C\left[\left(3x-8y^2\right)dx+\left(4y-6xy\right)dy\right]$ where C is the boundary of the region bounded by $x=0, y=0$ and $x+y=1$.

15. (a) Solve, by Laplace transform, the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-t}\sin t,\ y(0)=0,\ y'(0)=1$$

(b) If $x=r\cos\theta, y=r\sin\theta$, then show that $\dfrac{\partial(r,\theta)}{\partial(x,y)}=\dfrac{1}{r}$.

16. (a) Discuss the maximum and minimum of the given function $f=x^3y^3-3x-3y$.

(b) State the Convolution theorem and using it prove that

$$L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}=\frac{r\sin at}{2a}$$

17. (a) Solve the differential equation $\dfrac{d^2y}{dx^2}+y=\sec^2 x$ by the ethod of variation of parameters.

(b) Show that $\beta(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta.\cos^{2n-1}\theta d\theta$.

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