VIT UNIVERSITY
(Estd. u/s 3 of UGC Act 1956)

Term End Examination - May 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: F1+TF1**

Class NBR : 2636

Time : Three Hours **Max.Marks:100**

PART - A (10 X 3 = 30 Marks)
Answer ALL Questions

1. If $u=\left(x^2+y^2+z^2\right)^{-\frac{1}{2}}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$

2. If $u=2x-3y, v=5x+4y,$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.

3. Transform the following integral in to polar coordinates and hence evaluate.
$$\int_0^a\int_0^{\sqrt{a^2-x^2}}\sqrt{x^2+y^2}\,dy\,dx$$

4. Evaluate.
$$\int_0^\infty e^{-x^2}\,dx$$

5. Find the directional derivative of $f=xyz$ at (1,1,1) in the direction of the vector $\vec{i}+\vec{j}+\vec{k}$.

6. Find $div\, curl\vec{F}$ where $\vec{F}=x^2y\vec{i}+xz\vec{j}+2yz\vec{k}$.

7. Solve $\frac{d^2y}{dx^2}+4y=0$.

8. Reduce the equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+4y=x^2+2logx$ into a linear differential equation.

9. Find the Laplace transform of $e^{-3t}sin^2t$.

10. Find $f(0)$ and $f(\infty)$ if $F(s)=\frac{1}{s(s+1)(s+2)}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand $e^{x}\log(1+y)$ at $(0,0)$ as the Taylor's series up to second degree. **[7]**

b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of unit sphere $x^2 + y^2 + z^2 = 1$. **[7]**

12. a) Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2)\ dy\,dx$ and hence evaluate it. **[7]**

b) Evaluate $\iiint xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming in to spherical coordinates. **[7]**

13. a) Verify Green's theorem in the xy-plane for $\int_C \{(3x^2 - 8y^2)dx + (4y - 6xy)dy\}$, where C is the boundary of the region given by $x = 0, y = 0, x + y = 1$. **[7]**

b) Show that $\bar{F} = (y^2 + 2xz^2)\bar{\imath} + (2xy - z)\bar{\jmath} + (2x^2z - y + 2z)\bar{k}$ is irrotational and hence find its scalar potential. **[7]**

14. a) Solve $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by using the method of variation of parameters. **[7]**

b) Solve the differential equation $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = 2x^2$. **[7]**

15. a) Find the Laplace transform of the function **[7]**

$$f(t) = \begin{cases} \sin\omega t, & for \quad 0 < t < \frac{\pi}{\omega} \\ 0, & for \quad \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases} \quad \text{with} \quad f(t + \frac{2\pi}{\omega}) = f(t).$$

b) Using Convolution theorem find $L^{-1}\left[\dfrac{s}{(s^2 + a^2)^2}\right]$. **[7]**

16. a) Evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 3$. **[7]**

b) Solve: $(D^3 - D)y = 2x$ by the method of undetermined coefficients. **[7]**

17. a) Use the Laplace transform to solve the differential equation **[7]**

$y''(t) + 4y'(t) + 3y(t) = e^{-t}$ given $y(0) = 1$ *and* $y'(0) = 1$.

b) Find the area of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$, using Gamma function. **[7]**

⇔⇔⇔