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Term End Examination - November 2013

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: E2+TE2**

Class NBR : 2270/2284/2293/2310/2313/2324/5064

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Find $\frac{du}{dt}$, if $u = xy + yz + zx$, $x = e^t$, $y = e^t$ and $z = \frac{1}{t}$.

2. Compute $\frac{\partial(u,v)}{\partial(r,\theta)}$, if $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$ and $y = r\sin\theta$. By the property of Jacobians.

3. Evaluate $\int_0^a \int_0^b \frac{xy}{\sqrt{1-x^2-y^2}}\, dxdy$ and describe the region of integration.

4. Evaluate $\int_0^\infty e^{-x^2}\, dx$ using Gamma function.

5. Determine the constants a,b,c so that the vector $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

6. Show that the area of the region bounded by a simple closed curve C is $\frac{1}{2}\oint_c (xdy - ydx)$ using Green's theorem.

7. Solve: $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$.

8. Calculate the particular solution of the differential equation $(D^2 - 6D + 9)y = 3^x - \log 2$.

9. Find the Laplace transform of the function $f(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & t < 1 \end{cases}$

10. Find the inverse Laplace transform of $F(s) = \frac{S-3}{S^2+4S+13}$.

PART – B (5 X 14 = 70 Marks)

Answer any FIVE Questions

11. a) Prove that $x^2u_x + y^2u_y + z^2u_z = 0$, if $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ **[7]**

b) Expand $e^x \cos y$ in powers of x and y upto terms of third degree, using Taylor's Expansion. **[7]**

12. a) Change the order of integration in $\int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} x^2 dydx$ and then integrate it. **[7]**

b) Evaluate $\iiint_V (x+y+z)dxdydz$, where 'V' is the region of space inside the cylinder $x^2 + y^2 = a^2$ that is bounded by the planes $z = 0$ and $z = h$. **[7]**

13. a) If $\overline{F} = (x^2 - y^2 + 2xz)\overline{i} + (xz - xy + yz)\overline{j} + (z^2 + x^2)\overline{k}$, find $\nabla \cdot \vec{F}$, $\nabla(\nabla \cdot \vec{F})$, $\nabla \times \vec{F}$ and $\nabla \cdot (\nabla \times \vec{F})$. **[8]**

b) Evaluate $\oint_c (xydx + xy^2dy)$ by Stoke's theorem where 'C' is the square in *xy*-plane with vertices (1,0), (-1,0), (0,1) and (0, -1). **[6]**

14. a) Solve: $x^2 \frac{d^2y}{dx^2} - 7x\frac{dy}{dx} + 12y = x^2$ **[7]**

b) By using method of undetermined coefficients, find the solution of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ **[7]**

15. a) Find the Laplace transform of **[7]**

$$f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases} \text{ with } f(t + 2a) = f(t)$$

b) Solve by using Laplace transform technique $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 0$, given that $y = 2,\ \frac{dy}{dt} = -4$ at $t = 0$. **[7]**

16. a) Examine $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. **[7]**

b) Transforming to polar coordinates, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2y + y^3)dydx$ **[7]**

17. a) Verify Gauss divergence theorem for $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$, where 'S' is the surface of the cuboid formed by the planes $x = 0,\ x = a,\ y = 0,\ y = a, z = 0$ and $z = a$. **[10]**

b) Verify the initial value theorem for the function $f(t) = 1 + e^{-t}(\sin t + cost)$. **[4]**

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