Term End Examination - November 2012

Course : MAT101 - Multivariable Calculus and Differential Equations **Slot: D2+TD2**

Class NBR : 4357 / 4453 / 4549 / 4552 / 4759 / 4981 / 5044 / 5071 / 5095 / 5140

Time : Three Hours **Max.Marks:100**

PART – A (10 X 3 = 30 Marks)
Answer ALL the Questions

1. Find first and second order partial derivatives of $f(x,y)=ax^2+2hxy+by^2$ and verify $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$.

2. Find the stationary points of x^2y+xy^2-axy.

3. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dydx$.

4. Express $\int_0^\infty e^{-x^2}dx$ in terms of gamma function and hence evaluate.

5. If $\bar{f}=(x+3y)\bar{i}+(y-2z)\bar{j}+(x+pz)\bar{k}$ is solenoidal, find *p*.

6. If $\bar{F}=3xy\bar{i}-y^2\bar{j}$, evaluate $\int_C \bar{F}.d\bar{r}$ where C is the curve $y=2x^2$ in the xy- plane from (0, 0) to (1, 2).

7. Solve $\frac{d^2y}{dx^2}-a^2y=0,\ \ a\neq 0$.

8. Find the particular integral of $y''-3y'+2y=\cos 3x$.

9. Find the Laplace transform of $\{(t+3)^2e^t\}$.

10. Find the inverse Laplace transform of $\frac{4}{(s+1)(s+2)}$.

PART – B (5 X 14 = 70 Marks)
Answer any FIVE Questions

11. a) Expand the function $f(x,y)=e^x\log(1+y)$ in terms of x and y up to the terms of 3 rd degree using Taylor's theorem. **[7]**

b) Using the method of Lagrange's multiplier, find the minimum value of $x^2+y^2+z^2$, given that $xyz=a^3$. **[7]**

12. a) Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same. **[7]**

b) Find the area of the astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, using Gamma functions. **[7]**

13. Verify Gauss divergence theorem for

$\overline{F}=(x^3-yz)\bar{i}-2x^2y\,\bar{j}+z\bar{k}$ taken over the surface of the cube bounded by the planes $x=y=z=a$ and coordinates planes.

14. a) Solve $y''+2y'+y=x\cos x$. **[7]**

b) Solve $\dfrac{d^2y}{dx^2}+y=\operatorname{cosec}x$, using the method of variation of parameters. **[7]**

15. a) Evaluate $L\left\{\dfrac{1-\cos at}{t}\right\}$ **[7]**

b) Using Convolution theorem ,find $L^{-1}\left\{\dfrac{1}{s(s^2+4)}\right\}$ **[7]**

16. a) Evaluate $\iiint_E dV$, where E is the region bounded by the plane $x+y+z=1$ and coordinate planes. **[7]**

b) Find constants a, b and c so that the vector $\overline{F}=(x+2y+az)\bar{i}+(bx-3y-z)\,\bar{j}+(4x+cy+2z)\bar{k}$ is irrotational. For these values of a, b and c, find its scalar potential. **[7]**

17. a) Solve $x^2y''-4xy'+6y=x^2$ **[7]**

b) Solve the differential equation using Laplace transform **[7]**

$$\frac{d^2y}{dt^2}+y=6\cos 2t;\ y(0)=3,\ y'(0)=1.$$

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