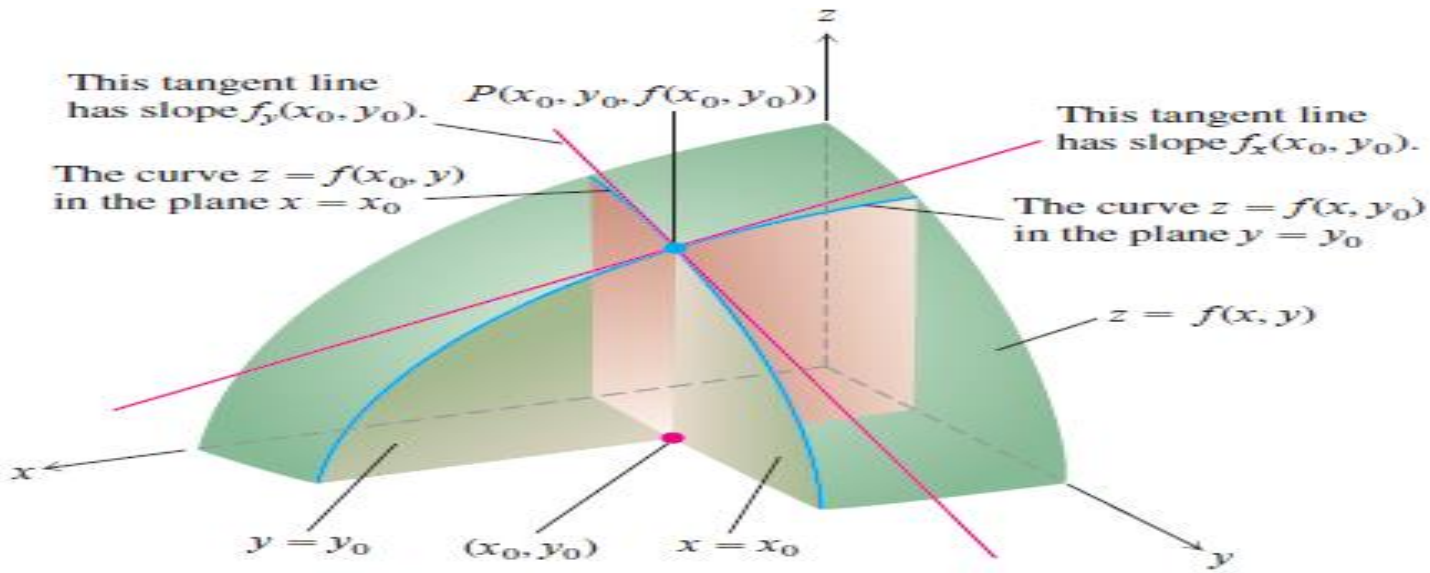
TANGENT PLANES



The tangent lines at the point $(x_0, y_0, f(x_0, y_0))$ determine a plane that, in this picture at least, appears to be tangent to the surface.

Defn: Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

```
clc
clear all
syms x y
 z = input('Enter f(x, y): ');      % e.g. 4 - x^2 - 2y^2
pt = input('Enter [x0, y0] : ');  % e.g. [1,1]
x0 = pt(1);
y0 = pt(2);
D = [x0-2 x0+2 y0-2 y0+2];
ezsurf(z,D);
hold on
z0=subs(z,[x,y],[x0,y0]);
z0 = double(z0);
plot3(x0,y0,z0,'r*','markersize',20);
```

```
zx=diff(z,x);
zy=diff(z,y);
zx0=subs(zx,[x,y],[x0,y0]);
zy0=subs(zy,[x,y],[x0,y0]);

% Graph of the plane y = y0

[x2,z2] = meshgrid([D(1) D(2)],[z0-2 z0+2]);
y2 = y0*ones(size(x2));
surf(x2,y2,z2);
surf(x2,y2,z2,'FaceColor',[0.3,0.1,0.3],'EdgeColor','none
');
```

```
t=linspace(-2,2,10);
x3=x0+t;
y3=y0*ones(size(t));
z3=z0+zx0*t;    % Tangent line is z - z0 = zx(x0,y0)*(x-x0);
line(x3,y3,z3,'color','b','linewidth',2);

% Graph of the plane x = x0

[y4,z4]=meshgrid([D(3) D(4)],[z0-2 z0+2]);
x4=x0*ones(size(y4));
surf(x4,y4,z4);
surf(x4,y4,z4,'FaceColor',[0.1,0.3,0.1],'EdgeColor','none');
```

```
t=linspace(-2,2,10);
x5=x0*ones(size(t));
y5=y0+t;
z5=z0+zy0*t;   % Tangent line is z - z0 = zy(x0,y0)*(y-y0);
line(x5,y5,z5,'color','g','linewidth',2);  %  You can also use plot3

% Tangent plane passing through (x0,y0) on the surface z = f(x,y)

[X,Y]=meshgrid([D(1) D(2)], [D(3) D(4)]);
Z=z0+zx0*(X-x0)+zy0*(Y-y0);
Z = double(Z);
surf(X,Y,Z)
```