Laplace Transform

Aim

- To visualize the time and frequency functions using Laplace Transform.

- To find the Laplace transform of periodic function and to visualize periodic function.

laplace(y)	The Laplace transform of the scalar symbol y with default independent variable t. The default return is a function of s.
ilaplace(Y)	The inverse Laplace transform of the scalar symbolic object Y with default independent variable s. The default return is a function of t.
heaviside$(t-a)$	To input either the heaviside or piecewise continuous function.
dirac$(t-value)$	To input the Dirac delta function.

syms t s

f = input ('Enter the function in terms of t :');

F = laplace (f)

Find the Laplace transform of cos(t).

Find $L(1 + 2\sqrt{t} + 3/\sqrt{t})$

Find Laplace Transform of $f(t) = \begin{cases} t^2, t < 2 \\ t - 1, 2 < t < 3 \\ 7, t > 3 \end{cases}$

$t^2 \left[u(t-0) - u(t-2) \right]$

$+ (t-1) \left[u(t-2) - u(t-3) \right] + 7 u(t-3)$

Laplace transform of periodic functions and visualization of periodic function

MATLAB code

$$L[f(t)] = \frac{1}{1-e^{-st}} \int_{0}^{T} e^{-st} f(t)\, dt$$

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases} \qquad T = 2\pi$$

```
clc
clear all
syms t s
T=input('Enter the period of the periodic function: ');   2*pi
n=input('Enter the number of partitions in one period: ')   2
fun = 0;
for i=1:n
a(i)=input('Enter the left end point of the ith sub interval: ');
b(i)=input('Enter the right end point of the ith sub interval: ');
f(i)=input('Enter the functions f(i): ');
fun = fun+f(i)*(heaviside(t-a(i))-heaviside(t-b(i)));
end
ezplot(fun, [a(1) b(n)])
```

$a(1) = 0$

$b(1) = pi$

$f(1) = \sin(t)$

$fun = 0 + (\sin t)(u(t-0) - u(t-pi))$

$a(2) = \pi \qquad f(2) = 0$

$b(2) = 2\pi$

$fun = (\sin t)(u(t-0) - u(t-pi)) +$

$(0)(u(t-\pi) - u(t-2\pi))$

```
sum=0;
for i=1:n
sum=sum+int(f(i)*exp(-s*t)ˆt,a(i),b(i))
end

g = (1/(1-exp(-s*T)))*sum
g1=simplify(g)
figure
ezplot(g1,[0 b(n)])
```

Handwritten annotations (red):

$$SUM = 0 + \int_{0}^{\pi} \sin t \, e^{-st} \, dt + \int_{0}^{2\pi} (0) e^{-st} \, dt$$

2 (above `n`)

$a(l)$ (near `figure`)

Find the Laplace transform of triangular wave of period 2 given by

$$f(t) = \begin{cases} t, 0 < t < 1 \\ 2-t, 1 < t < 2 \end{cases}; \quad f(t+2) = f(t) \text{ , for all } t.$$

Practice problems

1. Find the Laplace transform of a square wave $f(t) = \begin{cases} 1, 0 < t < \pi \\ -1, \pi < t < 2\pi \end{cases}, f(t+2\pi) = f(t)$ for all t.

2. Find the Laplace transform of $f(t) = \begin{cases} \sin t, 0 < t < \pi \\ 0, \pi < t < 2\pi \end{cases}, f(t+2\pi) = f(t)$ for all t.