First-order PDE : $f(x, y, z, p, q) = 0$

Solution: $z = z(x, y)$

Classification of I order PDE

① **Linear PDE**

$$P(x,y)\,p + Q(x,y)\,q = R(x,y)\,z + S(x,y)$$

② **Semi-linear PDE**

$$P(x,y)\,p + Q(x,y)\,q = R(x,y,z)$$

✓ ③ **Quasi-linear PDE** ((Lagrange's linear eqn)

$$P(x,y,z)\,p + Q(x,y,z)\,q = R(x,y,z)$$

✓ ④ **Nonlinear PDE**

A first order PDE that does not fall under any of the above three types.

(1) $yp + xq = xyz + y - x$

(linear PDE)

(2) $xp + yq = \sqrt{z}$

(semi-linear PDE)

(3) $(x^2 - yz)p + (y^2 - xz)q = x^2 + y^2 - z$

(quasi-linear PDE)

(4) $p^2 + q^2 = 1$

(nonlinear PDE)

Solutions of I order PDE

PDE: $f(x, y, z, p, q) = 0$

(1) Complete integral [or Complete solution]

A solution of the PDE involving only arbitrary constants.

(2) General Integral [or general solution]

A solution of the PDE involving arbitrary function(s).