$$y'' - 2y' = x^2 + 5x - 2$$

$$D^2y - 2Dy = x^2 + 5x - 2 \qquad \text{where } D = \frac{d}{dx}$$

$$D(D-2)y = x^2 + 5x - 2$$

It is a second order non-homogeneous LDE

The complete soln. is $y = C.F + P.I$

C.F $\quad D(D-2)y = 0$

A.E $\quad m(m-2) = 0$

$$m = 0, 2$$

$$C.F = c_1 e^{0x} + c_2 e^{2x}$$

$$C.F = c_1 + c_2 e^{2x}$$

P.I $\quad$ RHS $X = x^2 + 5x - 2$

$$a_1' + a_2' x + a_3 x^2 + a_1'' + a_2'' x + a_1'''$$
$$= (a_1' + a_1'' + a_1''') + (a_2' + a_2'')x + a_3 x^2$$

The trial soln of the given eqn is

$$y = a_0 + a_1 x + a_2 x^2$$

Rewritten trial soln is $\quad y = a_0 x + a_1 x + a_2 x^2$

$$y = (a_0 + a_1)x + a_2 x^2$$

$$y' = (a_0 + a_1) + 2x a_2$$

$$y'' = 2a_2$$

substitute the above in the given differential eqn.

(ii) $y'' - 2y' = x^2 + 5x - 2$

$$2a_2 - 2[(a_0 + a_1) + 2xa_2] = x^2 + 5x - 2$$

$$2a_2 - 2a_0 - 2a_1 - \underline{4}x\underline{a_2} = x^2 + 5x - 2$$

compare the coefficient of x

$$-4a_2 = 5 \Rightarrow \boxed{a_2 = -\frac{5}{4}}$$

compare the constants

$$2a_2 - 2a_0 - 2a_1 = -2$$

$$\cancel{2}(a_2 - a_0 - a_1) = -\cancel{2}$$

$$-\frac{5}{4} - a_0 - a_1 = -1$$

$$-a_0 - a_1 = -1 + \frac{5}{4}$$

$$-(a_0 + a_1) = \frac{1}{4}$$

$$\Rightarrow \boxed{a_0 + a_1 = -\frac{1}{4}}$$

The trial soln is $y = (a_0 + a_1)x + a_2 x^2$

$$P \cdot I = -\frac{1}{4}x + \left(\frac{-5}{4}\right)x^2$$

$$= -\frac{x}{4}(1 + 5x)$$

$\therefore$ The complete soln. $y = C \cdot F + P \cdot I$

$$y = c_1 + c_2 e^{2x} - \frac{x}{4}(1 + 5x) \;//$$

4(i) $y'' - 7y' + 10y = 20e^{5x} - 10$

It is a second order non-homogeneous LDE

$(D^2 - 7D + 10)\, y = 20e^{5x} - 10$

complete soln is
$y = C \cdot F + P \cdot I$

C.F $(D^2 - 7D + 10)y = 0$

A.E $m^2 - 7m + 10 = 0$

$$\begin{array}{c} 10 \\ -5 \quad -2 \end{array}$$

$m^2 - 5m - 2m + 10 = 0$

$(m - 5)(m - 2) = 0$

$m = 2, 5$

$$C.F = c_1 e^{2x} + c_2 e^{5x}$$

P.I RHS $x = 20e^{5x} - 10$

The trial soln is $y = c_3 e^{5x} + c_4$

Rewrite the trial soln. as $y = c_3 x e^{5x} + c_4$

$$y' = c_3\left[x(5e^{5x}) + e^{5x}.1\right]$$

$$y' = 5c_3 x e^{5x} + c_3 e^{5x}$$

$$y'' = 5c_3\left[x(5e^{5x}) + e^{5x}\right] + c_3 5e^{5x}$$

$$y'' = 25c_3 x e^{5x} + 5c_3 e^{5x} + 5c_3 e^{5x}$$

$$y'' = 25c_3 x e^{5x} + 10c_3 e^{5x}$$

Substitute the above in the given eqn.

$$y'' - 7y' + 10y = 20e^{5x} - 10$$

$$25c_3 x e^{5x} + 10c_3 e^{5x} - 7(5c_3 x e^{5x} + c_3 e^{5x}) + 10(c_3 x e^{5x} + c_4) = 20e^{5x} - 10$$

$$xe^{5x}(\underbrace{25c_3 - 35c_3 + 10c_3}_{0}) + e^{5x}(10c_3 - 7c_3) + 10c_4 = 20e^{5x} - 10$$

$$3c_3 e^{5x} + 10c_4 = 20e^{5x} - 10$$

compare the coefficient of e^{5x}

$$3c_3 = 20 \Rightarrow \boxed{c_3 = \frac{20}{3}}$$

compare the constants

$$10c_4 = -10 \Rightarrow \boxed{c_4 = -1}$$

The trial soln is $y = c_3 x e^{5x} + c_4$

$$P.I = \frac{20xe^{5x}}{3} - 1 \quad //$$

The complete soln. is $y = C.F + P.I$

$$y = c_1 e^{2x} + c_2 e^{5x} + \frac{20xe^{5x}}{3} - 1 \quad //$$

9)

$$xy'' - (x+1)y' + y = x^2$$

$$x\left[y'' - \left(\frac{x+1}{x}\right)y' + \frac{y}{x}\right] = x^2$$

$$y'' - \left(\frac{x+1}{x}\right)y' + \frac{y}{x} = \frac{x^2}{x}$$

$$\Rightarrow \quad y'' - \left(\frac{x+1}{x}\right)y' + \frac{y}{x} = x$$

which is a second order non-homogeneous LDE.

$$y_1(x) = e^x, \quad y_2(x) = (x+1)$$

$$C.F = c_1 y_1 + c_2 y_2 = c_1 e^{x} + c_2 (x+1)$$

$\underline{\underline{P.I}}$ $\quad X = x, \quad y_1 = e^{x}, \; y_2 = x+1$

$$y_1' = e^{x} \qquad y_2' = 1$$

$$P.I = -y_1 \int \frac{y_2 X}{W}\,dx + y_2 \int \frac{y_1 X}{W}\,dx$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{x} & x+1 \\ e^{x} & 1 \end{vmatrix} = e^{x} - e^{x}(x+1)$$

$$= \cancel{e^{x}} - xe^{x} - \cancel{e^{x}}$$

$$= -xe^{x}$$

$$P.I = -e^{x} \int \frac{(x+1)\cdot x}{-\cancel{x}e^{x}}\,dx + (x+1) \int \frac{\cancel{e^{x}\cdot x}}{\cancel{-xe^{x}}}\,dx$$

$$= e^{x} \int (x+1)e^{-x}\,dx + (x+1)\int (-1)\,dx$$

$$= e^{x}\left[(x+1)\frac{e^{-x}}{-1} - 1\cdot e^{-x} \right] - (x+1)\cdot x$$

$$= e^{x}\left[-xe^{-x} - \underset{-e^{-x}}{e^{-x}} \right] - x^2 - x$$

$$= -x\underbrace{e^{x}\cdot e^{-x}}_{1} - \underbrace{e^{x}\cdot e^{-x}}_{1} - \underbrace{e^{x}\cdot e^{-x}}_{1} - x^2 - x$$

$$= -x - 1 - 1 - x^2 - x = -x^2 - 2x - 2$$

$$P.I = -x^2 - 2x - 2 \quad //$$

1. For a circuit consisting of an inductance L, a resistance R, a capacitance C and an e.m.f, $E(t) = E_0 \sin wt$, find the expression for the steady-state current.

1) $$L \frac{di}{dt} + R \cdot \frac{dq}{dt} + \frac{q}{c} = E_0 \sin \omega t$$

$$i = \frac{dq}{dt}$$

$$\Rightarrow L \frac{d^2 q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{q}{c} = E_0 \sin \omega t$$

$$\frac{d}{dt}\left(L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} \right) = \frac{d}{dt}(E_0 \sin \omega t)$$

$$L \frac{d^3 q}{dt^3} + R \frac{d^2 q}{dt^2} + \frac{dq}{dt} \cdot \frac{1}{c} = \omega E_0 \cos \omega t$$

$$\frac{dq}{dt} = I$$

$$\Rightarrow L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{c} = \omega E_0 \cos \omega t$$

$$\frac{d}{dt} = D$$

$$D^2 I + \frac{R}{L} I + \frac{I}{Lc} = \frac{\omega E_0}{L} \cos \omega t$$

$$\left(D^2+\frac{R}{L}D+\frac{1}{LC}\right)I=\frac{\omega E_0}{L}\cos\omega t$$

$\hookrightarrow$ second order non homogeneous LDE

$\underline{\underline{LHS}}$ $\left(D^2+\frac{R}{L}D+\frac{1}{LC}\right)I=0$

$\underline{\underline{AE}}$ $m^2+\frac{R}{L}m+\frac{1}{LC}=0$

$$m=\frac{-\frac{R}{L}\pm\sqrt{\frac{R^2}{L^2}-4(1)\left(\frac{1}{LC}\right)}}{2(1)}$$

$$m=\frac{-\frac{R}{L}\pm\sqrt{\frac{R^2}{L^2}-\frac{4}{LC}}}{2}=\frac{-\frac{R}{L}\pm\sqrt{4\left(\frac{R^2}{4L^2}-\frac{1}{LC}\right)}}{2}$$

$$=\frac{-\frac{R}{L}\pm 2\sqrt{\frac{R^2}{4L^2}-\frac{1}{LC}}}{2} \rightarrow -p^2$$

Consider $\frac{R^2}{4L^2}-\frac{1}{LC}=-p^2$

$$C.F = e^{-\frac{R}{2L}t}\left(c_1 \cos pt + c_2 \sin pt\right)$$

$$= \frac{-R}{2L} \pm \frac{2}{2} pi$$

$$\boxed{m = \frac{-R}{2L} \pm pi}$$

To find steady-state current

we require P.I

because P.I is the steady state solution

C.F is the solution for transient state

P.I

$$RHS = \frac{\omega E_0}{L} \cos \omega t$$

$$P.I \quad I = c_3 \cos \omega t + c_4 \sin \omega t$$

$$I' = -c_3 \omega \sin \omega t + c_4 \omega \cos \omega t$$

$$I'' = -c_3 \omega^2 \cos \omega t - c_4 \omega^2 \sin \omega t$$

diff equation is $\left(D^2+\frac{R}{L}D+\frac{1}{LC}\right)I = \frac{\omega E_0}{L}\cos\omega t$

$$-c_3\omega^2\cos\omega t - c_4\omega^2\sin\omega t + \frac{R}{L}(-c_3\omega\sin\omega t + c_4\omega\cos\omega t) + \frac{1}{LC}(c_3\cos\omega t + c_4\sin\omega t) = \frac{\omega E_0}{L}\cos\omega t$$

$$\left(-\omega^2 c_3 + c_4\frac{R}{L}\omega + \frac{c_3}{LC}\right)\cos\omega t + \sin\omega t\left(-c_4\omega^2 - \frac{c_3 R}{L}\omega + \frac{c_4}{LC}\right) = \frac{\omega E_0}{L}\cos\omega t$$

compare the coefficient of $\cos\omega t$

$$-\omega^2 c_3 + c_4\frac{R}{L}\omega + \frac{c_3}{LC} = \frac{\omega E_0}{L}$$

$$c_4\frac{R}{L}\omega + c_3\left(-\omega^2+\frac{1}{LC}\right) = \frac{\omega E_0}{L} \quad -(1)$$

compare the coefficient of $\sin\omega t$

$$-c_4\omega^2 - c_3\frac{R}{L}\omega + \frac{c_4}{LC} = 0$$

$$c_4\left(-\omega^2+\frac{1}{LC}\right) - c_3\frac{R}{L}\omega = 0 \quad -(2)$$

solving (1) & (2) we will the values of c_3 and c_4

$$(1)\times\left(-\omega^2+\frac{1}{LC}\right) \Rightarrow c_4\frac{R}{L}\left(-\omega^2+\frac{1}{LC}\right)\omega + c_3\left(-\omega^2+\frac{1}{LC}\right)^2 = \frac{\omega E_0}{L}\left(-\omega^2+\frac{1}{LC}\right)$$

$$(2)\times\frac{R}{L}\omega \Rightarrow \overset{(-)}{c_4}\frac{R}{L}\left(-\omega^2+\frac{1}{LC}\right)\omega \overset{(+)}{-} c_3\frac{R^2}{L^2}\omega^2 = \overset{(-)}{0}$$

$$c_3\left(\left(-\omega^2+\frac{1}{LC}\right)^2 - \frac{R^2}{L^2}\omega^2\right) = \omega\frac{E_0}{L}\left(-\omega^2+\frac{1}{LC}\right)$$

c_-

$$c_3 = \frac{\frac{\omega E_0}{L}\left(\frac{1}{LC}-\omega^2\right)}{\left(\frac{1}{LC}-\omega^2\right)^2 - \frac{R^2\omega^2}{L^2}} \quad //$$

steady state p. $I = c_3 \cos\omega t + c_4 \sin\omega t$

2. Consider an electric circuit with an inductance of 5 henries and a resistance of 12 ohms in series with an e.m.f $120sin200t$ volts. Find the current.

Inductance + Resistance = emf

$$L.\frac{di}{dt} + R.\frac{dq}{dt} = 120\sin 200t$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = 120\sin 200t$$

$$5\frac{d^2q}{dt^2} + 12\frac{dq}{dt} = 120\sin 200t$$

4. Consider an electric circuit with an inductance of 0.05 henry, a resistance of 20 ohms, a condenser of capacitance of 100 micro farads and an e.m.f of $E = 100$ volts. Find i and q given the initial conditions $q = 0, i = 0$ at $t = 0$.

4) $L.\frac{d^2q}{dt^2} + R.\frac{dq}{dt} + \frac{q}{c} = E_0 \cos nt$

$q=0$ at $t=0$
$i=0$ at $t=0$

$(0.05)\frac{d^2q}{dt^2} + 20\frac{dq}{dt} + \frac{q}{100} = 100 \cos nt$

$\left.\begin{array}{l} q(0)=0 \\ q'(0)=0 \end{array}\right\}$ I.c's