Solutions of difference equations using Z transforms :

Let $Z\{y(n)\} = Y(z)$.

Now, $Z\{y(n+1)\} = z[Y(z) - y(0)]$ ✓

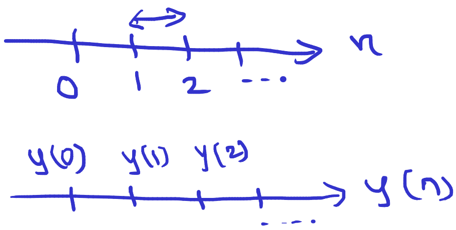
$$Z\{y(n+2)\} = z^2\left[Y(z) - y(0) - \frac{y(1)}{z}\right]$$

$$= z^2[Y(z) - y(0)] - zy(1) ✓$$

$$Z\{y(n+3)\} = z^3\left[Y(z) - y(0) - \frac{y(1)}{z} - \frac{y(2)}{z^2}\right]$$

$$= z^3[Y(z) - y(0)] - z^2y(1) - zy(2) ✓$$

$$\{y_n\} = \{y_n\}_{n=0}^{\infty} = \{y(0), y(1), y(2), \dots, y(n), \dots\}$$



✓ $\boxed{y(2) = y(1) + y(0)}$

$$\text{order} = \frac{2-0}{1} = 2$$

$$\boxed{\frac{dy}{dx} + Py = \varphi} \longrightarrow y(n+1) + P(n)y(n) = \varphi(n)$$

$$\frac{n+1-n}{1} = 1$$

$$\boxed{y'' + py' + qy = r(x)}$$

$$\boxed{y(n+2) + p\,y(n+1) + q\,y(n) = r(n)}$$

(linear) order 2

$$\boxed{y(n)\,y(n-1) + y(n+2) = n^2}$$

(nonlinear)

① Solve the difference equation $y(k+2) - 4\,y(k+1) + 4\,y(k) = 0$,

where $y(0) = 1$, $y(1) = 0$.

Sol: Let $Z\{y(k)\} = Y(z)$.

Applying z transform on equation, we get

$$Z\{y(k+2)\} - 4Z\{y(k+1)\} + 4Z\{y(k)\} = Z\{0\}.$$

$$\Rightarrow \left[z^2(Y(z) - y(0)) - z\,y(1)\right] - 4z(Y(z) - y(0)) + 4Y(z) = 0$$

$$(z^2 - 4z + 4)Y - z^2 + 4z = 0$$

$$(z-2)^2 Y = z^2 - 4z$$

$$\boxed{Y(z) = \frac{z(z-4)}{(z-2)^2}}$$

Applying inverse Z transforms, we get

$$y(k) = Z^{-1}\left[\frac{z(z-4)}{(z-2)^2}\right] \quad \text{--- (1)}$$

$$= Z^{-1}[H(z)], \text{ where}$$

$H(z) = \dfrac{z(z-4)}{(z-2)^2}$. But $H(z)$ is not a proper rational function.

$$\Rightarrow \quad \frac{H(z)}{z} = \frac{z-6}{(z-2)^2}$$ is a proper rational function.

By Partial fraction expansion, let

$$\frac{H(z)}{z} = \frac{A}{z-2} + \frac{B}{(z-2)^2}$$

$$B = \lim_{z\to 2}\left[(z-2)^2 \frac{H(z)}{z}\right] = \lim_{z\to 2}\left[z-6\right] = -2$$

$$A = \lim_{z\to 2} \frac{d}{dz}\left[z-6\right] = 1$$

$$\therefore \quad \frac{H(z)}{z} = \frac{1}{z-2} - 2\cdot\frac{1}{(z-2)^2}$$

$$\Rightarrow \quad \boxed{H(z) = \frac{z}{z-2} - \frac{2z}{(z-2)^2}}$$

$$\therefore \quad y(k) = Z^{-1}\left[H(z)\right]$$

$$= (2)^k - k(2)^k = (1-k)\,2^k \quad //$$

✓ Check: $y(0) = 1$

$y(1) = 0$