Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Winter Semester 2021-2022

BMAT102L–Differential Equations and Transforms

Module:4 Solution to ODE and PDE by Laplace transform

Tutorial Sheet – 2

1. Using Laplace transform, solve the following simultaneous equations:

$$\begin{aligned}(D-2)x-(D+1)y &= 6e^{3t},\\ (2D-3)x+(D-3)y &= 6e^{3t}, \quad \text{given} \quad x=3, y=0 \quad \text{when} \quad t=0.\end{aligned}$$

2. Using Laplace transform, solve the following simultaneous equations:

$$\begin{aligned}y_1' &= 4y_2-8\cos 4t,\\ y_2' &= -3y_1-9\sin 4t, \quad y_1(0)=0, \quad y_2(0)=3.\end{aligned}$$

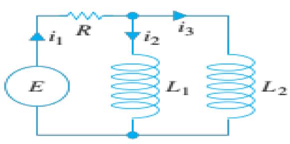
3. Using Laplace transform, solve the following simultaneous equations:

$$\begin{aligned}y_1'-5y_1+4y_2 &= 2t-9t^2,\\ y_2'-10y_1+7y_2 &= -17t^2-2t, \quad y_1(0)=2, \quad y_2(0)=0.\end{aligned}$$

4. Using Laplace transform, solve the following simultaneous equations:

$$\begin{aligned}y_1'+3y_1-y_2 &= u(t-1)e^t,\\ y_2'+4y_1-2y_2 &= u(t-1)e^t, \quad y_1(0)=0, \quad y_2(0)=3.\end{aligned}$$

5. a). Show that the system of differential equations for the currents $i_2(t)$ and $i_3(t)$ in the following electrical network is



$$L_1\frac{di_2}{dt}+R(i_2+i_3)=E(t),$$
$$L_2\frac{di_3}{dt}+R(i_2+i_3)=E(t).$$

b). Solve the system in part (a) if $R=5\Omega, L_1=0.01h, L_2=0.0125h, E=100V, \quad i_2(0)=0$, and $i_3(0)=0$.

6. Solve the following pde by using Laplace transform: $\frac{\partial w}{\partial x}+2x\frac{\partial w}{\partial t}=2x$, $w(x,0)=1, w(0,t)=1$.

7. Solve the following pde by using Laplace transform: $x\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}=xt$, $w(x,0)=0$ if $x\geq 0$ and $w(0,t)=0$ if $t\geq 0$.

8. Solve the following pde by using Laplace transform: $u_t+xu_x=x$, $u(x,0)=0$ and $u(0,t)=0$.

9. Solve the following pde by using Laplace transform: $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=x$, $u(x,0)=0, u(0,t)=0$.

10. Solve the following pde by using Laplace transform: $\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial t}=0$, $u(x,0)=0, u(0,t)=t$.