11.04.2022. If $L(f(t)) = F(s)$, then **Module 4**

$$\begin{cases} L(f'(t)) = sF(s) - f(0) = sL(f(t)) - f(0). \\ L(f''(t)) = s^2F(s) - sf(0) - f'(0). \end{cases}$$

.....

1. Using L.T, solve $L\frac{di}{dt} + Ri = Ee^{-at}$, $i(0) = 0$, L, R, E & a are consts/.

$$Li' + Ri = Ee^{-at}$$

Taking L.T throughout

$$L(\underline{L}i') + L(Ri) = L(Ee^{-at}).$$

$$L\,L(i'(t)) + RL(i) = E(L(e^{-at}))$$

$L(i'(t))$

$$L(sL(i(t)) - \cancel{i(0)}) + RL(i(t)) = \frac{E}{s+a}.$$

$$L(i(t))(Ls + R) = \frac{E}{s+a}.$$

$$L(i(t)) = \frac{E}{(s+a)(Ls+R)}.$$

$$i(t) = L^{-1}\left(\frac{E}{(s+a)(Ls+R)}\right). \quad \text{—(1).}$$

Let $\dfrac{E}{(s+a)(\mathcal{L}s+R)} = \dfrac{A}{s+a} + \dfrac{B}{\mathcal{L}s+R}$.

$$= \frac{A(\mathcal{L}s+R)+B(s+a)}{(s+a)(\mathcal{L}s+R)}$$

$$\Rightarrow E = A(\mathcal{L}s+R)+B(s+a)$$

Let $s=-a$.

$$E = A(\mathcal{L}(-a)+R) \Rightarrow A = \frac{E}{R-\mathcal{L}a}.$$

Put $s = -\dfrac{R}{\mathcal{L}}$

$$E = A(0)+B\left(\frac{-R}{\mathcal{L}}+a\right) \Rightarrow E = B\left(\frac{a\mathcal{L}-R}{\mathcal{L}}\right)$$

$$\Rightarrow B = \frac{E\mathcal{L}}{a\mathcal{L}-R}.$$

$$L^{-1}\left\{\frac{E}{(s+a)(\mathcal{L}s+R)}\right\} = L^{-1}\left\{\frac{\frac{E}{R-\mathcal{L}a}}{s+a}\right\} + L^{-1}\left\{\frac{\frac{E\mathcal{L}}{a\mathcal{L}-R}}{\mathcal{L}s+R}\right\}.$$

$$= \frac{E}{R-\mathcal{L}a}e^{-at} + \frac{E\cancel{\mathcal{L}}}{(a\mathcal{L}-R)\cancel{\mathcal{L}}} \times L^{-1}\left(\frac{1}{s+\frac{R}{\mathcal{L}}}\right)$$

$$= E\left\{\frac{1}{R-\mathcal{L}a}e^{-at} + \frac{1}{a\mathcal{L}-R}e^{-R/\mathcal{L}\,t}\right\}$$

$$= \frac{E}{a\mathcal{L} - R}\left\{ -e^{-at} + e^{-R/\mathcal{L}\,t} \right\}.$$

2. Solve $y'' - 4y' + 8y = e^{2t}$, $y(0) = 2$ and $y'(0) = -2$.

$$L(y'') - 4L(y') + 8L(y) = L(e^{2t})$$

$$\{s^2 L(y(t)) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-2}\} - 4\{sL(y) - y(0)\} + 8L(y) = \frac{1}{s-2}$$

$$L(y(t))(s^2 - 4s + 8) - 2s + 2 + 8 = \frac{1}{s-2}.$$

$$L(y(t))(s^2 - 4s + 8) - 2s + 10 = \frac{1}{s-2}.$$

$$L(y(t))(s^2 - 4s + 8) = \frac{1}{s-2} + 2s - 10.$$

$$= \frac{1}{s-2} + 2s - 10.$$

$$L(y(t)) = \frac{1}{(s-2)(s^2-4s+8)} + \frac{2s-10}{s^2-4s+8} \quad \text{--- (1).}$$

$$\frac{1}{(s-2)(s^2-4s+8)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-4s+8}.$$

$$1 = A(s^2 - 4s + 8) + (Bs + C)(s - 2).$$

coeffts. of s^2 : $0 = A + B$.

" " s : $0 = -4A - 2B + C$

const. terms

$1 = 8A - 2C$.

$A+B=0 \Rightarrow B=-A$

Sub in

$$A=1/4, \quad B=-1/4, \quad C=1/2$$

$$\frac{1}{(s-2)(s^2-4s+8)} = \frac{1}{4}\cdot\frac{1}{s-2} + \frac{(-1/4)s+1/2}{s^2-4s+8}.$$

Sub in

$$\frac{1}{(s-2)(s^2-4s+8)} + \frac{(2s-10)}{(s^2-4s+8)}$$

$$= \frac{1}{4}\frac{1}{s-2} - \frac{1}{4}\frac{s}{s^2-4s+8} + \frac{\frac{1}{2}}{s^2-4s+8}$$

$$+ \frac{2s}{s^2-4s+8} - \frac{10}{s^2-4s+8}.$$

$$= \frac{1}{4}\cdot\frac{1}{s-2} + \frac{-s+8s}{4(s^2-4s+8)} - \frac{\frac{19}{2}}{s^2-4s+8}.$$

$$= \frac{1}{4}\frac{1}{s-2} + \frac{7}{4}\frac{s}{(s-2)^2+2^2} - \frac{19}{2}\frac{1}{(s-2)^2+2^2}$$

$$L^{-1}\{\ \} = \frac{1}{4}e^{2t} + \frac{7}{4}L^{-1}\left(\frac{\underbrace{s-2}+2}{(s-2)^2+2^2}\right) - \frac{19}{2}L^{-1}\left(\frac{1}{(s-2)^2+2^2}\right)$$

$$= \frac{1}{4}e^{2t} + \frac{7}{4}L^{-1}\left(\frac{s-2}{(s-2)^2+2^2}\right) + \frac{7}{2}L^{-1}\left(\frac{1}{(s-2)^2+2^2}\right)$$

$$= \frac{1}{4}e^{2t} + \frac{7}{4}e^{2t}\cos 2t + \cancel{6}^{3} e^{2t}\frac{\sin 2t}{\cancel{2}}.$$

3. Solve $y'' + y' - 2y = 3\cos 3t - 11\sin 3t$, $y(0)=0$ $y'(0)=6$.

$$L(y''(t)) + L(y'(t)) - 2L(y) = 3L(\cos 3t) - 11L(\sin 3t).$$

$$s^2L(y(t)) - s\underbrace{\cancel{y(0)}}_{0} - y'(0) + sL(y(t)) - \underbrace{\cancel{y(0)}}_{0} - 2L(y)$$

$$= 3\cdot\left(\frac{s}{s^2+9}\right) - 11\times\left(\frac{3}{s^2+9}\right) + 6$$

$$(s^2+s-2)L(y) = \frac{3s-33}{s^2+9} + 6 = \frac{3(s-11)}{s^2+9} + 6$$

$$L(y) = \frac{3(s-11)+6s^2+54}{(s^2+9)(s^2+s-2)}.$$

$$= \frac{6s^2+3s+21}{(s^2+9)(s+2)(s-1)}.$$

$$= \frac{As+B}{s^2+9} + \frac{C}{s+2} + \frac{D}{s-1}$$

$A=0$, $B=3$, $C=-1$ & $D=1$.

$$L(y) = \frac{3}{s^2+9} - \frac{1}{s+2} + \frac{1}{s-1}$$

$$y = \sin 3t - e^{-2t} + e^{t}$$

4). $(D^2 + 4D + 13)y = e^{-t}\sin t$, $y(0)=0$, $Dy(0)=0$.

$D = \frac{d}{dt}$.

$$\left[s^2 L(y) - s\cancel{y(0)} - \cancel{y'(0)}\right] + 4\left[sL(y) - \cancel{y(0)}\right] + 13L(y) = \left(L(\sin t)\right)_{s \leftrightarrow s+1}$$

$\to 0$, $\to 0$, $\to 0$

$$(s^2 + 4s + 13)L(y) = \left(\frac{1}{s^2+1}\right)_{s \leftrightarrow s+1}$$

$$= \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$L(y) = \frac{1}{(s^2+2s+2)(s^2+4s+13)}$$

$$= \frac{As+B}{s^2+2s+2} + \frac{Cs+D}{s^2+4s+13}$$

$A = \frac{-2}{85}$ $B = \frac{7}{85}$ $C = \frac{2}{85}$ & $D = -3/85$

$$L(y) = \frac{1}{85}\left\{\frac{-2s+7}{s^2+2s+2} + \frac{2s-3}{s^2+4s+13}\right\}$$

$-2s+7$

$$= \frac{1}{85}\left\{\frac{-2(s+1)+9}{(s+1)^2+1} + \frac{2s+4-4-3}{(s+2)^2+3^2}\right\}$$

$$= \frac{1}{85}\left\{ \frac{-2(s+1)}{(s+1)^2+1} + \frac{9}{(s+1)^2+1} + \frac{2(s+2)}{(s+2)^2+3^2} - \frac{7}{(s+2)^2+3^2} \right\}$$

$$= \frac{-2}{85} e^{-t}\cos t + \frac{9\sin t\,(e^{-t})}{85} + \frac{2}{85} e^{-2t}\cos 3t - \frac{7e^{-2t}\sin 3t}{3\times 85}$$

13.04.2022

1. Write $f(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$ using $u_a(t)$ and then find the L.T

2. $L(\sin\sqrt{t})$.

3. $L(\cosh at \cos at)$

4. $L(e^{-2t}\cos^2 4t)$

5. $L\left(\sinh\left(\frac{t}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)t\right)$

$$f(t) = (t-1)^2 u_1(t)$$

$$L(f(t)) = \int_1^\infty (t-1)^2 e^{-st}\,dt.$$

$$= e^{-s}\left(\frac{2}{s^3}\right)$$

$$\left[(t-1)^2\left(\frac{e^{-st}}{-s}\right) - 2(t-1)\left(\frac{e^{-st}}{(-s)^2}\right) + 2\left(\frac{e^{-st}}{(-s)^3}\right)\right]_1^\infty$$

$$= \frac{-2}{s^3}\left(e^{-\infty} - e^{-s}\right)$$

$$= \frac{2e^{-s}}{s^3}$$

2. $L(\sin\sqrt{t})$.

$$= L\left(\sqrt{t} - \frac{(\sqrt{t})^3}{3!} + \frac{(\sqrt{t})^5}{5!} \cdots\right).$$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

$$= L\left(t^{1/2}\right) - \frac{1}{3!} L\left(t^{3/2}\right) + \frac{1}{5!} L\left(t^{5/2}\right) \dots$$

$$\checkmark = \frac{\Gamma(3/2)}{s^{3/2}} - \frac{1}{3!}\frac{\Gamma(5/2)}{s^{5/2}} + \frac{1}{5!}\frac{\Gamma(7/2)}{s^{7/2}} + \dots$$

$$\boxed{L(t^n) = \frac{t^{n+1}}{s^{n+1}}}$$

$$= \frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}} - \frac{1}{\underset{\substack{\| \\ \cancel{6}\,2}}{3!}}\frac{\cancel{3}/2 \cdot 1/2\sqrt{\pi}}{s^{5/2}} + \frac{1}{5!}\frac{5/2 \times 3/2 \times 1/2\sqrt{\pi}}{s^{7/2}} + \dots$$

$\Gamma(n+1) = n\Gamma(n)$

$\Gamma(5/2) = 3/2\,\Gamma(3/2)$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\left[1 - \frac{1}{1!}\left(\frac{1}{4s}\right) + \frac{1}{2!}\left(\frac{1}{4s}\right)^2 - \dots\right]$$

$\frac{\cancel{15}}{\cancel{120}}$ 8

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\left[e^{-1/4s}\right]$$

3. $L(\cosh at \cos at) = L\left[\left(\frac{e^{at} + e^{-at}}{2}\right)\cos at\right]$

$$= \frac{1}{2}\left[L(e^{at}\cos at) + L(e^{-at}\cos at)\right]$$

$$= \frac{1}{2}\left[L(\cos at)_{s \leftrightarrow s-a} + L(\cos at)_{s \leftrightarrow s+a} \right]$$

$$= \frac{1}{2}\left[\left(\frac{s}{s^2+a^2}\right)_{s \leftrightarrow s-a} + \left(\frac{s}{s^2+a^2}\right)_{s \leftrightarrow s+a} \right]$$

$$= \frac{1}{2}\left[\frac{s-a}{(s-a)^2+a^2}\right] + \frac{1}{2}\left[\frac{s+a}{(s+a)^2+a^2}\right]$$

4. $L(e^{-2t}\cos^2 4t) = L\left(e^{-2t}\left(\frac{1+\cos 8t}{2}\right)\right)$

$$= \frac{1}{2} L(e^{-2t}) + \frac{1}{2} L(e^{-2t}\cos 8t).$$

$$= \frac{1}{2}\frac{1}{s+2} + \frac{1}{2}\left(\frac{s}{s^2+64}\right)_{s \leftrightarrow s+2}$$

$$= \frac{1}{2(s+2)} + \frac{1}{2}\frac{(s+2)}{(s+2)^2+64}$$

5. $L\left(\sinh\left(\frac{t}{2}\right)\sin\left(\frac{\sqrt{3}}{2}t\right)\right). = L\left[\left(\frac{e^{t/2}-e^{-t/2}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}\right)t\right]$

$$\frac{\frac{\sqrt{3}}{2}}{s^2+3/4}$$

$$= \frac{1}{2}\times\frac{\sqrt{3}}{2}\left[\frac{1}{(s-1/2)^2+3/4} - \frac{1}{(s+1/2)^2+3/4}\right]$$

6. $L(te^{-4t}\sin 3t) =$

$= L(t\sin 3t)_{s \leftrightarrow s+4}$

$= \left[-\frac{d}{ds} L(\sin 3t)\right]_{s \leftrightarrow s+4}$

$= -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]_{s \leftrightarrow s+4}$

$(s^2+9)^{-1}$

$= -1(s^2+9)^{-2} \cdot 2s$

$= \frac{-2s}{(s^2+9)^2}$

$= -3\left(\frac{-2s}{(s^2+9)^2}\right)_{s \leftrightarrow s+4}$

$f(t-a)u_a(t) = L^{-1}(e^{-as}F(s))$

$= \frac{6(s+4)}{((s+4)^2+9)^2}$

$L(f(t-a)u_a(t))$
$= e^{-as} L(f(t))$
$F(s)$

7. $L^{-1}\left(\frac{e^{-2s}}{s^2(s^2+1)}\right) = L^{-1}\left\{\left(\frac{1}{s^2(s^2+1)}\right) u_2(t)\right\}_{t \leftrightarrow t-2}$

1

$a=2$

$F(s) = \frac{1}{s^2(s^2+1)}$

$\frac{1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s^2+1}$

$1 = A(s^2+1) + Bs^2$

$s^2 = -1 \qquad B = -1$

$s = 0 \rightarrow \quad 1 = A \quad 1$

$$\frac{1}{s^2(s^2+1)} = \frac{1}{s^2} + \frac{(-1)}{s^2+1}.$$

$$L^{-1}\left(\frac{1}{s^2(s^2+1)}\right) = L^{-1}\left(\frac{1}{s^2}\right) - L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= t - \sin t$$

Sub in (1) $L^{-1}\left(\frac{e^{-2s}}{s^2(s^2+1)}\right) = \left[(t-2) - \sin(t-2)\right] u_2(t)$

7. $L^{-1}\left(\frac{2s^2+5s+2}{(s-2)^4}\right)$ $u = s-2.$ $s = u+2.$

$$= L^{-1}\left(\frac{2(u+2)^2+5(u+2)+2}{u^4}\right)$$

$(u+2)^2$
$= u^2+4u+4$
$5u+2$
$+2$

$$= L^{-1}\left(\frac{2u^2+13u+2}{u^4}\right)$$

$$= L^{-1}\left(\frac{2}{u^2}\right) + L^{-1}\left(\frac{13}{u^3}\right) + L^{-1}\left(\frac{2}{u^4}\right)$$

$$= 2L^{-1}\left(\frac{1}{(s-2)^2}\right) + 13L^{-1}\left(\frac{1}{(s-2)^3}\right) + 2L^{-1}\left(\frac{1}{(s-2)^4}\right)$$

$$= 2e^{+2t}t + 13e^{+2t}\frac{t^2}{2!} + 2e^{+2t}\frac{t^3}{3!}$$

13.04.2022. Module 4 - Solution to ODE & PDE using L.T.

1. $L^{-1}\left(\frac{e^{-2s}}{s-9}\right) = e^{9(t-2)}u(t-2)$

$L(f(t-a)u(t-a)) = e^{-as}L(f(t))$

$\Rightarrow f(t-a)u(t-a)$ ✓

$= L^{-1}\left(e^{-as}L(f(t))\right)$

$= L^{-1}\left(e^{-as}F(s)\right)$ ✓

$a=2$

$F(s) = \frac{1}{s-9}$

$L(f(t)) = \frac{1}{s-9}$

$f(t) = L^{-1}\left(\frac{1}{s-9}\right)$

$= e^{9t}$

2. $L^{-1}\left(\frac{e^{-3s}(2s+7)}{s^2+16}\right)$

$= 2L^{-1}\left(\frac{e^{-3s}s}{s^2+16}\right) + 7L^{-1}\left(\frac{e^{-3s}}{s^2+16}\right).$

$a=3$

$F(s) = \frac{s}{s^2+16} = L(f(t))$

$f(t) = \cos 4t$

$f(t-a) = f(t-3)$

$= \cos 4(t-3)$

$= 2\cos 4(t-3)\,u(t-3)$

$+ \frac{7}{4}\sin 4(t-3)\,u(t-3); t \geq 0.$

$\frac{4}{s^2+16}$

3. Represent $f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 3 \\ 2, & t \geq 3 \end{cases}$ using heaviside step fn.

and hence find LT.

$f(t) = 1.\left(h(t-1) - h(t-3)\right) + 2h(t-3) = h(t-1) + h(t-3)$

$L(f(t)) = L(h(t-1)) + L(h(t-3))$

$= \frac{e^{-s}}{s} + \frac{e^{-3s}}{s}$

4. Use L.T to solve the initial_value problem $y' + 4y = g(t)$,

$y(0) = 2$ where $g(t) = \begin{cases} 0, & 0 \le t < 1 \\ 12, & 1 \le t < 3 \\ 0, & t \ge 3 \end{cases} = 12\left[h(t-1) - h(t-3)\right]$

$$y' + 4y = g(t)$$

$$\boxed{L(y')} + 4L(y) = L(g(t))$$

$$sL(y(t)) - \underbrace{y(0)}_{2} + 4L(y(t)) = 12\left(\frac{e^{-s} - e^{-3s}}{s}\right)$$

$$L(g(t)) = 12L\left[h(t-1) - h(t-3)\right]$$

$$= 12\left(\frac{e^{-s}}{s} - \frac{e^{-3s}}{s}\right)$$

$$L(y'(t)) = sL(y(t)) - y(0).$$

$$\underline{(s+4)}\, L(y(t)) = 12\left(\frac{e^{-s} - e^{-3s}}{s}\right) + 2.$$

$$L(y(t)) = 12\left(\frac{e^{-s} - e^{-3s}}{s(s+4)}\right) + \frac{2}{s+4}.$$

$$y(t) = 12L^{-1}\left(\frac{e^{-s} - e^{-3s}}{s(s+4)}\right) + 2L^{-1}\left(\frac{1}{s+4}\right)$$

$$\frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$A = 1/4 \quad B = -1/4$

$$\frac{1}{s(s+4)} = \frac{1}{4s} - \frac{1}{4(s+4)}.$$

$$12L^{-1}\left(\frac{e^{-s} - e^{-3s}}{s(s+4)}\right) + 2e^{-4t} = 3L^{-1}\left(\frac{e^{-s} - e^{-3s}}{s}\right) - 3L^{-1}\left(\frac{e^{-s} - e^{-3s}}{s+4}\right)$$

$$= 3L^{-1}\left(\frac{e^{-s}}{s}\right) - 3L^{-1}\left(\frac{e^{-3s}}{s}\right) - 3L^{-1}\left(\frac{e^{-s}}{s+4}\right) + 3L^{-1}\left(\frac{e^{-3s}}{s+4}\right) + 2e^{-4t}$$

$a=1.$

$F(s)=\frac{1}{s}$

$L(f(t))=\frac{1}{s}$

$f(t)=L^{-1}\left(\frac{1}{s}\right)=1$

$f(t-a)u(t-a)=L^{-1}\left(e^{-as}F(s)\right)$

$=L^{-1}\left(e^{-as}L(f(t))\right)$

$= 3u(t-1)-3u(t-3)-3u(t-1)e^{-4(t-1)}$

$+3e^{-4(t-3)}\cdot u(t-3).$

$+2e^{-4t}$

19.07.2022.

Simultaneous eqns/:

Solution using L.T.

1. Solve $\underbrace{2x' - y' + 3x = 2t}_{(1)}$; $\overbrace{x' + 2y' - 2x - y = t^2 - t}^{(2)}$

$x(0) = 1$ & $y(0) = 1$.

$$2x' - y' + 3x = 2t$$

$$2L(x') - L(y') + 3L(x) = 2L(t)$$

$$2[sL(x(t)) - 1] - [sL(y(t)) - 1] + 3L(x(t)) = \frac{2}{s^2}$$

$L(x'(t)) = sL(x(t)) - x(0)$

$= sL(x(t)) - 1$

$L(y'(t)) = s[L(y(t)) - y(0)]$

$= sL(y(t)) - 1$

$$L(x(t))(2s+3) - L(y(t))(s) - 1 = \frac{2}{s^2}$$

$$L(x(t))(2s+3) - s\,L(y(t)) = 1 + \frac{2}{s^2}. \quad \text{--- (3)}$$

II eqn/:

$$x'(t) + 2y'(t) - 2x - y = t^2 - t$$

$$L(x'(t)) + 2L(y'(t)) - 2L(x(t)) - L(y(t)) = L(t^2) - L(t)$$

$$[sL(x(t)) - 1] + 2[sL(y(t)) - 1] - 2L(x(t)) - L(y(t)) = \frac{2}{s^3} - \frac{1}{s^2}$$

$$L(x(t))(s-2) + L(y(t))(2s-1) - 3 = \frac{2}{s^3} - \frac{1}{s^2}$$

$$L(x(t))(s-2) + L(y(t))(2s-1) = \frac{2 - s + 3s^3}{s^3}$$

--- (4).

③ & ④

$$L(x(t))(2s+3) - sL(y(t)) = \frac{s^2+2}{s^2} \quad \times(2s-1)$$

$$L(x(t))(s-2) + L(y(t))(2s-1) = \frac{2-s+3s^3}{s^3} \times s.$$

$1+\frac{2}{s^2}$

$$(2s+3)(2s-1)L(x(t)) - s(2s-1)\cancel{L(y(t))} = \frac{(2s-1)(s^2+2)}{s^2}$$

$$s(s-2)L(x(t)) + s\cancel{(2s-1)}L(y(t)) = \frac{2-s+3s^3}{s^2}.$$

$4s^2+6s-2s$
-3

$$L(x(t))(4s^2+4s-3+s^2-2s) = \frac{2s^3-s^2+4s-\cancel{2}+\cancel{2}-s+3s^3}{s^2}$$

$$L(x(t))(5s^2+2s-3) = \frac{5s^3-s^2+3s}{s^2}$$

$$= 5s-1+\frac{3}{s}$$

$$L(x(t))(5s-3)(s+1) = 5s-1+\frac{3}{s}.$$

$5s^2+5s-3s$
-3
$5s(s+1)$
$-3(s+1)$
$(5s-3)(s+1)$

$$L(x(t)) = \frac{5s^2-s+3}{s(s+1)(5s-3)}$$

$$x(t) = L^{-1}\left(\frac{5s^2-s+3}{s(s+1)(5s-3)}\right) \quad -(5).$$

Consider $\left(\dfrac{5s^2-s+3}{s(s+1)(5s-3)}\right) = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{5s-3}$

$5s^2 - s + 3 = A(s+1)(5s-3) + Bs(5s-3) + Cs(s+1).$

$5+1+3$

$s=0 \longrightarrow 3 = A(1)(-3) \Rightarrow A = -1.$

$s=-1 \longrightarrow 9 = B(-1)(-5-3).$

$\Rightarrow 9 = 8B \Rightarrow B = 9/8$

$s = 3/5 \longrightarrow \dfrac{21}{5} = C\left(\dfrac{3}{5}\right)\left(\dfrac{3}{5}+1\right).$

$5\left(\dfrac{9}{25}\right) - 3/5 + 3$

$= \dfrac{9}{5} - 3/5 + 3 = \dfrac{6}{5} + 3$

$\Rightarrow \dfrac{21}{5} = C\left(\dfrac{3}{5}\right)\left(\dfrac{8}{5}\right)$

$\Rightarrow \dfrac{21 \times 5}{24} = C \Rightarrow \dfrac{35}{8} = C.$

Substituting in (5).

$x(t) = -1\, L^{-1}\left(\dfrac{1}{s}\right) + \dfrac{9}{8} L^{-1}\left(\dfrac{1}{s+1}\right) + \dfrac{35}{8} L^{-1}\left(\dfrac{1}{5s-3}\right)$

$= \dfrac{\cancel{35}\,7}{8} \times \dfrac{1}{\cancel{5}} L^{-1}\left(\dfrac{1}{s-3/5}\right)$

$= \dfrac{7}{8} e^{3/5 t}$

$x(t) = -1 + \dfrac{9}{8} e^{-t} + \dfrac{7}{8} e^{3/5 t}$

To find $y(t)$

① $\rightarrow 2x' - y' + 3x = 2t$

$2\left(-\dfrac{9}{8} e^{-t} + \dfrac{21}{40} e^{3/5 t}\right) - y' + 3\left(-1 + \dfrac{9}{8} e^{-t} + \dfrac{7}{8} e^{3/5 t}\right)$

$= 2t$

$$y' = \frac{9}{8}e^{-t} + \frac{147}{40}e^{3/5 t} - 3 - 2t$$

Integrating

$$y(t) = -\frac{9}{8}e^{-t} + \frac{\cancel{147}^{49}}{\cancel{40}_{8}} \times \frac{\cancel{5}}{\cancel{3}} e^{3/5 t} - 3t - t^2 + C.$$

$\frac{21}{20} + \frac{21}{8}$

$21\left(\frac{2+5}{40}\right)$

$$= -\frac{9}{8}e^{-t} + \frac{49}{8}e^{3/5 t} - 3t - t^2 + C \quad \text{---(6)}$$

Given $y(0) = 1$

$$(6) \rightarrow \quad 1 = -\frac{9}{8} + \frac{49}{8} + C$$

$$1 = 5 + C \Rightarrow C = -4$$

Sub $$y(t) = -\frac{9}{8}e^{-t} + \frac{49}{8}e^{3/5 t} - 3t - t^2 - 4$$

$$x(t) = -1 + 9/8\, e^{-t} + 7/8\, e^{3/5 t}$$

2). Solve
$$\frac{dy}{dt} + 2x = \sin 2t$$
$$\frac{dy}{dt} + 2x = \cos 2t, \quad x(0) = 1$$
$$y(0) = 0.$$

20.04.2020. Solution of PDE - using L.T.

$L(f(t)) = F(s)$ ✓ ODE

$L(y'(t)) = sL(y(t)) - y(0)$

$L(y''(t)) = s^2 L(y(t)) - sy(0) - y'(0)$.

$\mathcal{L}(u(x,t)) = U(s,x) = U$

$\mathcal{L}(u_t(x,t)) = \mathcal{L}\left(\frac{\partial u}{\partial t}\right) = sU(x,s) - u(x,0)$

$\mathcal{L}(u_{tt}(x,t)) = \mathcal{L}\left(\frac{\partial^2 u}{\partial t^2}\right) = s^2 U(x,s) - su(x,0) - u_t(x,0)$

$\mathcal{L}\left(\frac{\partial u}{\partial x}\right) = \frac{dU}{dx}$ ✓

$\mathcal{L}\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{d^2 U}{dx^2}$

1. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = x$, $x>0, t>0$, $u(x=0,t)=0$. ✓

$u(x, t=0) = 0$ ✓

$L\left(\frac{\partial u}{\partial x}\right) + L\left(\frac{\partial u}{\partial t}\right) = L(x)$.

$\frac{dU}{dx} + sU(x,s) - \cancel{u(x,0)}_{\to 0} = xL(1)$.

$\frac{dU}{dx} + sU(x,s) = \frac{x}{s}$ ——— (1).

$\frac{dU}{dx} + PU = Q$.

$P = s \qquad Q = \frac{x}{s}$.

$e^{\int s\,dx} = e^{sx}$

$U \cdot e^{\int s\,dx} = \int \frac{x}{s} \cdot e^{\int s\,dx}\,dx + c$.

$Ue^{sx} = \int \frac{x}{s} e^{sx}\,dx + c$.

To solve linear ODE.

$$\frac{dy}{dx} + \underline{P}y = Q.$$

Integrating factor $- e^{\int \underline{P}dx}$

Soln: $y.e^{\int Pdx} = \int Q e^{\int Pdx} dx + C$

$$Ue^{sx} = \frac{1}{s}\left\{x\frac{e^{sx}}{s} - 1.\frac{e^{sx}}{s^2}\right\} + c$$

$$= (x-1)e^{sx}\left(\frac{s-1}{s^3}\right)$$

$$U = (x-1)\frac{(s-1)}{s^3} + ce^{-sx}$$

① $y' - y = xe^x$ $\qquad y' + Py = Q$

Integrating factor $- e^{\int Pdx}$

$P = -1 \qquad Q = xe^x.$

$IF = e^{\int Pdx} = e^{-\int dx} = e^{-x}.$

Soln. is $y.e^{-x} = \int xe^x . e^{-x} dx + c$

$$= \int x dx + c.$$

$$ye^{-x} = \frac{x^2}{2} + c \Rightarrow y = e^x\left(\frac{x^2}{2} + c\right).$$

2. $xy' = y + 2x^3 \Rightarrow xy' - y = 2x^3 \div x$ $\qquad y' + Py = Q$

$$\Rightarrow y' \underbrace{- \frac{1}{x}}_{P} y = \underbrace{2x^2}_{Q}$$

$$IF = e^{\int P dx} = e^{\int -1/x \, dx} = e^{-\log x} = e^{\log x^{-1}} = e^{\log 1/x}$$

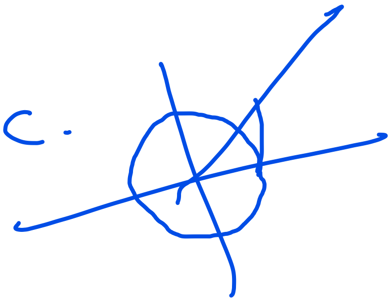
$$= \frac{1}{x}$$

$n \log a = \log a^n$

Soln is

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + c.$$

$$y \cdot \frac{1}{x} = \int 2x^2 \cdot \frac{1}{x} dx + C.$$



$$\frac{y}{x} = x^2 + C.$$

$\sec(2\pi + t)$
$= \sec(4 \times \pi/2 + t)$
$= \sec t$

Fourier Series (Module 5).

$f(x+t) = f(x)$ $\forall x$ for some t, (least)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin\left(\frac{n\pi x}{l}\right) \right)$$

$(c, c+2l)$, $f(x)$ in $[c, c+2l]$

$\cos t = \cos(2\pi + t)$
$= \cos(4\pi + t)$
$= \cos(6\pi + t)$
$\vdots$

$f(x)$ periodic

$\sin t$

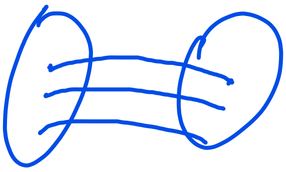
$f(x)$ is defined in $[c, c+2l]$. In addition to periodicity it has to satisfy 3 conditions in order to be expanded as a F.S. Dirichlet's condition

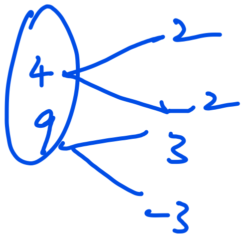
1. $f(x)$ is single-valued & finite in $(c, c+2l)$
2. $f(x)$ is continuous or piece-wise continuous with finite no of finite discontinuities in $(c, c+2l)$

3. It has no or finite no. of max or min in $(c, c+2)$.

$y = x \qquad y = \sqrt{x^2}$ at $x = 2$

$f(x) = \sqrt{4} = \pm 2$





$\lim_{x \to a} f(x) = f(a)$

f is continuous.

$\tan x$ is not cont at $x = \pi/2$

infinite dis

$f(x) = e^{1/(x-2)}$ at $x = 2$.

$f(2) = e^{1/(2-2)} = e^{1/0} = e^{\infty}$

$\to \infty$.