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BMAT102L- Differential Equations and Transforms
Module-2 (Partial Differential Equations]- - Tutorial sheet 1

Form the PDE of the following by eliminating arbitrary constants:

1. $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$

[Ans.: $p^2+q^2=\tan^2\alpha$]

2. $z=(x^2+a^2)(y^2+b^2)$

[Ans.: $2z=xp+yq$]

3. $2z=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$

[Ans.: $pq=4xyz$]

II Find the PDE

1. of all planes passing through the origin.

[Ans.: $px+qy-z=0$]

2. of all planes having equal intercepts on the x and y axis.

[Ans.: $p-q=0$]

III Form the PDE of the following by eliminating arbitrary functions from:

1. $z=f_1(x)\,f_2(y)$

[Ans.: $z\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{\partial z}{\partial x}\dfrac{\partial z}{\partial x}$]

2. $z=f(xy/z)$

[Ans.: $px=qy$]

3. $z=f(x^2+y^2)$

[Ans.: $py=qx$]

4. $\phi\left[z^2-xy,\ \dfrac{x}{z}\right]=0$ [Ans.: $x^2p+(2z^2-xy)q=xz$]

5. $f(x^2+y^2,\ z-xy)=0$

[Ans.: $xq-yp=x^2-y^2$]

6. $xyz=\varphi\,(x^2+y^2-z^2)$

[Ans: $(yz+xyp)(2y-2zp)=(xz+xyq)(2x-2zp)$]

7. $xy + yz + zx = f\left(\frac{z}{x+y}\right)$

[Ans: $p(x+y)(x+2z) - q(x+y)(y+2z) = z(x-y)$]

8. $z = (x+y)f(x^2 - y^2)$ [Ans: $py + qx = z$]

IV Solve the following equations:

1. $p + q = pq$

[Ans.: $z = ax + \left(\frac{a}{a-1}\right)y + c$]

2. $\sqrt{p} + \sqrt{q} = 1$ [Ans.: $z = ax + (1-\sqrt{a})^2 y + c$]

3. $z^2 = 1 + p^2 + q^2$ [Ans.: $z = \cosh\left\{\frac{x+ay}{\sqrt{1+a^2}} + b\right\}$]

4. $3p^2 - 2q^2 = 4pq$ [Ans: $z = ax \pm a\left[-1 + \frac{\sqrt{10}}{2}\right]y + c$]

5. $p^2 + q^2 - 4pq = 0$ [Ans: $= \left(2 \pm \sqrt{3}\right)x + by + c$]

6. $p(1+q) = qz$ [Ans.: $\log(az-1) = x + ay + b$]

7. $z = px + qy + p^2q^2$ [Ans: $z^3 = \frac{-27}{16}x^2y^2$]

8. $q^2 = z^2p^2(1-p^2)$ [Ans.: $a^2z^2 = (y + ax + c)^2 + 1$]

9. $p - y^2 = q + x^2$ [Ans.: $z = kx + \frac{x^3}{3} + ky - \frac{y^3}{3} + c$]

10. $\left(\frac{p}{2} + x\right)^2 + \left(\frac{q}{2} + y\right)^2$ [Ans: $z = -(x-k)^2 + 2\left[\sqrt{1-k^2}\ y - \frac{y^2}{2}\right] + a$]

11. $q = px + p^2$ [Ans : $z = \frac{-x^2}{4} \pm \frac{1}{2}\left[\frac{k^2}{2}\sin h^{-1}\frac{x}{k} + \frac{x\sqrt{x^2+k^2}}{2}\right] + \frac{k^2y}{4} + b$]

12. $p^2 + q^2 = x + y$ [Ans.: $z = \frac{2}{3}(a+x)^{\frac{3}{2}} + \frac{2}{3}(y-a)^{\frac{3}{2}} + b$]

13. $z = px + qy + \sqrt{1 + p^2 + q^2}$ [Ans.: $x^2 + y^2 + z^2 = 1$]

14. $z = px + qy + p^2 + pq + q^2$ [Ans.: $3z + x^2 + y^2 - xy = 0$]

15. $z = px + qy + p^2 - q^2$ [Ans.: $4z = y^2 - x^2$]

16. $z = px + qy + p^2q^2$ [Ans.: $16z^3 + 27x^2y^2$]

17. $pz = 1 + q^2$

[Ans: $\frac{1}{4a^2}\left[\frac{z^2}{2} - \frac{z\sqrt{z^2-4a^2}}{2} + \frac{4a^2}{2}cosh^{-1}\left(\frac{z}{2a}\right)\right] = \frac{x+ay}{2a^2} + b$]

18. $p^3 + q^3 = 8z$

[Ans: C.I : $(1 + a^3)z^2 = \frac{64}{27}(x + ay + b)^3$ S.I : $z = 0$]

V Solve the following equations:

1. $x^2p^2 + y^2q^2 = z^2$ [Ans.: $\log z = a\log x + \sqrt{1-a^2}\log y + c$]
2. $pqxy = z^2$ [Ans.: $\log z = a\log x + (1/a)\log y + c$]
3. $z^2(p^2x^2 + q^2) = 1$ [Ans.: $z^2\sqrt{1+q^2} = \pm 2(\log x + ay) + b$]
4. $z^2(p^2 + q^2) = x^2 + y^2$

   [Ans.: $z^2 = x\sqrt{x^2 + a} + a\sinh^{-1}\frac{x}{\sqrt{a}} + y\sqrt{y^2 - a} - a\cosh^{-1}\frac{y}{\sqrt{a}} + b$]
5. $p^2y(1+x^2) = qx^2$ [Ans.: $z = \sqrt{a(1+x^2)} + (1/2)ay^2 + b$]
6. $x^2p^2 + y^2q^2 = z^2$ [Ans : $\log z = \frac{\log x + a\log y}{\sqrt{1+a^2}} + b$]
7. $z^2(p^2x^2 + q^2) = 1$ [Ans : $z^2 = \frac{2}{\sqrt{1+a^2}}[\log x + ay] + 2b$]
8. $z^2(p^2 + q^2) = x^2 + y^2$

   [Ans : $z^2 = x\sqrt{x^2 + a^2} + y\sqrt{y^2 - a^2} + a^2\left[\sinh^{-1}\frac{x}{a} - \cosh^{-1}\frac{y}{a}\right]$]
9. $(zp + x)^2 + (zq + y)^2 = 1$

[Ans: $z^2 = 2ax - x^2 + 2\sqrt{1 - a^2}\ y - y^2 + b$]