Test (FAT) – June 2022

Programme	B.Tech	Semester	Winter Semester 2021-22
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Kalyani Desikan	Slot	A1+TA1+TAA1
		Class Nbr	CH2021222300663
Time	3 Hours	Max. Marks	100

PART A (10 X 10 Marks)

Answer any 10 questions

1. Solve $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 12y = 4x$. [10]

2. Solve the partial differential equation $(yz+2y)p - (2x+3z)q + xy - 3y = 0$. [10]

3. (i) If $f(t) = \begin{Bmatrix} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{Bmatrix}$, then find $L[f(t)]$. [10] [10]

   (ii) If $f(t) = \begin{Bmatrix} e^{-2(t-1)}, & 1 < t < 3 \\ t^2, & t \geq 3 \end{Bmatrix}$ and $g(t) = e^{4t}.\delta(t-2)$, then find $L\left(f(t) - \frac{g(t)}{e^{4t}}\right)$.

4. Solve the system of first order differential equation [10]

   $\frac{dx_1}{dt} + x_2 = e^{-t}$

   $\frac{dx_2}{dt} - x_1 = 3e^{-t}$

   using Laplace transform with $x_1(0) = 0$ and $x_2(0) = 1$.

5. Find Fourier series of the function $f(x) = \frac{x}{2}$ for $-\pi \leq x \leq \pi$, where $f(x+2\pi) = f(x)$ and hence prove that $1 + \frac{1}{4} + \frac{1}{9} + \ldots = \frac{\pi^2}{6}$. [10]

6. Find the half range sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$ and hence deduce $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} + \ldots = \frac{\pi^3}{32}$ using Parseval's theorem. [10]

7. (i) Find the Fourier cosine transform of $f(x) = e^{-ax}$, $a > 0$ and hence evaluate $\int_0^\infty \frac{\cos sx}{s^2+a^2}\,ds$. [10]

   (ii) Find the Fourier sine transform of xe^{-ax}.

8. Find the Fourier transform of $f(x) = \begin{Bmatrix} 1-|x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{Bmatrix}$ and hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$. [10]

9. (i) If $u_n = 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n$, $n = 0, 1, 2, 3\ldots$, then find the Z-transform of u_n. [10]

   (ii) Find the inverse Z-transform of $F(z) = \frac{2z}{(z-1)(z^2+1)}$.

10. Solve the difference equation $u_{n+2} - 3u_{n+1} + 2u_n = 0$, given $u_0 = 0$ and $u_1 = 1$ by using Z-transform. [10]

11. (i) Solve $(2D^2 - 7D + 3)y = 3\cos x$. [10]

    (ii) Use the method of partial fraction to find the Inverse Laplace transform of $F(s) = \frac{5s}{(s^2+4s+4)}$

12. (i) Form the partial differential equation by eliminating the arbitrary function from $z = x^2 + xf(x + e^y)$. [10]

    (ii) Solve the first order partial differential equation using Laplace transform $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = x^5$ with the boundary conditions $u(0,t) = 0$ and $u(x,0) = 0$.

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Final Assessment Test (FAT) – June 2022

Programme	**B.Tech**	Semester	**Winter Semester 2021-22**
Course Title	**DIFFERENTIAL EQUATIONS AND TRANSFORMS**	Course Code	**BMAT102L**
Faculty Name	**Prof. Muthunagai**	Slot	**A2+TA2+TAA2**
		Class Nbr	**CH2021222300672**
Time	**3 Hours**	Max. Marks	**100**

PART A (10 X 10 Marks)
Answer any 10 questions

1. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ by the method of variation of parameters. [10]
2. Reduce the following partial differential equation to one of the standard forms and find the complete integral and singular integral (if it exists) $x^3p^3 - 3yz^2q = z^3$. [10]
3. (i) Suppose $f(t) = |t-3| + |t+3|$. Find $L(f(t))$. [10]
(ii) If $f(t) = \begin{Bmatrix} \sin t, \ 0 < t < 2 \\ t, \ t \geq 2 \end{Bmatrix}$ and $g(t) = \cos t \, . \, \delta(t-5)$, then find $L\left[f(t) + \frac{g(t)}{\cos(5)}\right]$.
4. Solve $y'' + 3y' - 28y = u(t-2)$, $y(0) = 2$, $y'(0) = -3$ by Laplace transform. [10]
5. Find the Fourier series expansion of $f(x) = \begin{Bmatrix} 1 + \frac{2x}{\pi}, \ -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, \ 0 \leq x \leq \pi \end{Bmatrix}$ and hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. [10]
6. Obtain a half range cosine and sine series for $f(x) = 1 + x$ in the interval $0 \leq x \leq \pi$ and hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. [10]
7. Find the Fourier transform of $f(x) = \begin{Bmatrix} a^2 - x^2, \ |x| < a \\ 0, \ |x| > a > 0 \end{Bmatrix}$, hence deduce [10]

(i) $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$,

(ii) $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds = \frac{\pi}{15}$.
8. Find the Fourier transform of $e^{-a^2x^2}$ and hence find the Fourier transform of $xe^{-a^2x^2}$. [10]
9. (i) Find inverse Z-transform of $F(z) = \frac{z^2}{(z-5)^2}$ by using convolution theorem. [10]
(ii) Find $Z(u_{n+1})$ and $Z(u_{n+2})$ if $Z(u_n) = \frac{z}{z-2} + \frac{z}{z^2+2}$.
10. Find the Z-transform of $\sin(2n+3)$. [10]
11. (i) Suppose that the spring with a mass of 2 kg has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m. The spring is immersed in a fluid with damping constant $c = 40$. Find the position of the mass at any time t if it starts from the equilibrium position and is given a push to start with an initial velocity of 0.6 m/s. [10]

(ii) Solve the partial differential equation $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial t} = 3x$, $u(x,0) = 0$, $u(0,t) = 0$ using Laplace transform.
12. (i) Find the complete integral and singular integral (if it exists) of the partial differential equation $z = px + qy - p^3 - 3q^2$. [10]
(ii) Find the inverse Laplace transform using the partial fractions for $G(s) = \frac{2s+12}{(s^2+6s+13)}$.

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Final Assessment Test (FAT) – June 2022

	B.Tech	Semester	Winter Semester 2021-22
	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
...y Name	Prof. Revathi G K	Slot	C1+TC1+TCC1
		Class Nbr	CH2021222300560
...me	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

1. Solve the initial value problem by the method of undetermined coefficients [10]
   $y'' - 5y' + 6y = e^{x}(2x - 3),\ y(0) = 1\ and\ y'(0) = 3.$
2. Find the complete solution of the partial differential equation $p^2 + q^2 = z^2(x^2 + y^2)$. Find the singular solution, if it exists. [10]
3. (a) Find the Laplace transform of the square wave function of a period T defined as: [10]
   $f(t) = \begin{Bmatrix} 1 & for\ 0 < t < \frac{T}{2} \\ -1 & for\ \frac{T}{2} < t < T \end{Bmatrix}$ (5 Marks)
   (b) Find the Laplace transform of $f(t) = \cos(t)u(t-3) - 5\delta(t-3)$ (5 Marks)
4. Solve the following simultaneous differential equations by using Laplace transform: [10]
   $y_1' = 3y_2 - e^t,\ y_2' = -2y_1 + e^t$ with $y_1(0) = 0, y_2(0) = 2$.
5. Find the Fourier sine transform of $e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x\sin(mx)}{(1+x^2)}dx,\ m > 0$ [10]
6. (a) Evaluate the Fourier transform of $xf(x)\ if\ f(x) = \begin{Bmatrix} 1, & for\ |x| < 1 \\ 0, & for\ |x| > 1 \end{Bmatrix}$ (4 marks) [10]
   (b) Evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+1)(x^2+b^2)}$ using Fourier transform . (6 marks)
7. Express $f(x) = \begin{Bmatrix} x\ , & for\ \ 0 < x < \pi \\ 2\pi - x\ , & for\ \pi < x < 2\pi \end{Bmatrix}$ as Fourier series, where $f(x + 2\pi) = f(x)$. [10]
   Hence find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$
8. Find the half range cosine series of $f(x) = (\pi - x)^2,\ 0 < x < \pi$ and hence deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}$. [10]
9. (a) Find the Z-transform of $n + 2$ (5 marks) [10]
   (b). Find the inverse Z transform of $\frac{2z^2+5z}{(z+2)(z-4)}$ (5 marks)
10. Solve the difference equations using Z-transform $u(n+2) + 5u(n+1) + 3u(n) = 3^n$, given that $u(0) = 0\ and\ u(1) = 0$. [10]
11. (a) Using Convolution theorem , find the inverse Laplace transform of $F(s) = \frac{3}{s^3(s^2-3)}$ (5 Marks) [10]
    (b) Solve the partial differential equation $3p^2 - 2q^2 = 4pq$. Find the singular solution, if it exists. (5 Marks)
12. (a) A series of LCR circuit consists of an inductance of 0.05 H, a resistance of 5Ω and a capacitance of $4 \times 10^{-4}\ F$. Find the current flowing through the circuit. (5 Marks) [10]
    (b) Solve the PDE using Laplace transform $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = 2t$ with $u(0,t) = 0, u(x,0) = 0$.(5 marks)

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Final Assessment Test (FAT) – June 2022

Programme	B.Tech	Semester	Winter Semester 2021-22
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Dr.Balaji S	Slot	C2+TC2+TCC2
		Class Nbr	CH2021222300655
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

1. Solve $(D^2+2D+1)y=\frac{e^{-x}}{x^2}$ by the method of variation of parameter [10]
2. Solve $(z^2-2yz-y^2)p+(xy+zx)q=xy-zx$ [10]
3. (a)Find the Laplace transform of the saw tooth wave function of a period T defined as $f(t)=\frac{100t}{T},\ for\ 0<t<T$. (5 marks) [10]
   (b) Find the Laplace transform of $f(t)=e^{t}u(t-2)+t\delta(t-3)$ (5 marks)
4. Solve the following initial value problem by using Laplace transform $2\frac{d^2y}{dt^2}+5\frac{dy}{dt}+2y=e^{-2t}$ with $y(0)=1\ and\ y'(0)=1$ [10]
5. Verify Parseval's identity for $f(x)=e^{-x}, x>0$ in Fourier transform [10]
6. Using transform techniques, evaluate $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+25)(x^2+49)}$. [10]
7. Express the function $f(x)=\begin{Bmatrix}0, & for\ -\pi<x<0\\ 1 & for\ 0<x<\pi\end{Bmatrix}$ as Fourier series where $f(x+2\pi)=f(x)$ .Hence find the value of $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$ [10]
8. Express the function $f(x)=x^2$ in $(0,a)$as a Fourier series and h ence find the value of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots$ [10]
9. (a) Find the Z-transform of $f(k)=\sin(\alpha k),\ k\geq 0$. (5 Marks) [10]
   (b) Find the inverse Z-transform of the function $\frac{z}{(z+3)^2(z-2)}, |z|>3$ (5 Marks)
10. Solve the difference equations $y(k+2)-5y(k+1)+6y(k)=k$ by using using Z-transform with $y(0)=0, y(1)=0$ [10]
11. (a) Solve the following partial differential equations using Laplace transform $\frac{\partial y}{\partial t}+\frac{\partial y}{\partial x}+y=0,\ for\ x>0,\ t>0\ with\ y(0,t)=\sin(t)\ and\ y(x,0)=0$ (5 marks) [10]
    (b) Using convolution theorem find $f(t)$ if $L(f(t))=\frac{1}{(s+1)(s^2+4)}$ (5 marks)
12. (a) Solve $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=0$ (5 Marks) [10]
    (b) Form the partial differential equation by eliminating f from $z^2-2xy=f(x^2+y^2+z^2)$ (5 marks)

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