$$f(t) \longrightarrow \{ f(0), f(T), f(2T), \ldots, f(nT), \ldots \}$$

$$\downarrow$$

$$\{f(nT)\}_{n=0}^{\infty}$$

$$Z[f(t)] = Z\{f(nT)\}$$

$$= \sum_{n=0}^{\infty} f(nT)\, z^{-n}$$

$$\boxed{\begin{aligned} dt &= t_{n+1} - t_n \\ &= (n+1)T - nT \\ &= T \end{aligned}}$$

$t_n = nT$

$dt_n = n\Delta T$

$$L[f(t)] = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$= \sum_{n=0}^{\infty} e^{-snT} f(nT)\, T$$

$$= T \sum_{n=0}^{\infty} \left(e^{sT}\right)^{-n} f(nT)$$

$$= T \sum_{n=0}^{\infty} f(nT)\, z^{-n}, \text{ where}$$

$$z = e^{sT}$$

$$\boxed{L(f(t)) = T\, Z(f(t))}$$

Problems (Using Properties)

① $Z\{n\}$

Sol:

$$Z\{n\} = Z\{n.1\}$$

$$= -z\frac{d}{dz}\left[Z\{1\}\right] \quad \text{(by property)}$$

Now, $Z\{1\} = \dfrac{z}{z-1}$

Hence $\dfrac{d}{dz}\left[Z\{1\}\right] = \dfrac{(z-1)(1) - z(1)}{(z-1)^2} = \dfrac{-1}{(z-1)^2}$

$$\therefore Z\{n\} = -z\left[\frac{-1}{(z-1)^2}\right] = \frac{z}{(z-1)^2}.$$

② $Z\{n^2\} = Z\{n.n\}$

$$= -z\frac{d}{dz}\left[Z\{n\}\right] \quad \text{(Using property)}$$

$$= -z\frac{d}{dz}\left[\frac{z}{(z-1)^2}\right]$$

$$= -z\left[\frac{(z-1)^2(1) - 2z(z-1)}{(z-1)^4}\right]$$

$$\therefore z\{n^2\} = -z\left[\frac{z-1-2z}{(z-1)^3}\right] = \frac{z(z+1)}{(z-1)^3} \quad //$$

③ $Z\{na^n\} = -z\dfrac{d}{dz}\left[Z\{a^n\}\right]$

$$= -z\frac{d}{dz}\left[\frac{z}{z-a}\right]$$

$$= -z\left[\frac{(z-a)(1) - z(1)}{(z-a)^2}\right]$$

$$\boxed{\therefore Z\{na^n\} = \frac{az}{(z-a)^2}}$$

④ $Z\left\{\dfrac{1}{n+1}\right\}$

Sol: Let $f(n+1) = \dfrac{1}{n+1}$.

$\downarrow n \to n-1$

Then $f(n) = \dfrac{1}{n-1+1} = \dfrac{1}{n}$

$\bar{f}(z) = Z\left\{\dfrac{1}{n}\right\} = \log\left(\dfrac{z}{z-1}\right)$

By property,

$$Z\{f(n+1)\} = z\left[\bar{f}(z) - f(0)\right]$$

i.e., $Z\left\{\dfrac{1}{n+1}\right\} = z\log\left(\dfrac{z}{z-1}\right)$ (Since $f(0)$ is undefined and is taken to be zero)

⑤ $Z\left\{\frac{1}{n-1}\right\}$.

Sol: Let $f(n-1) = \frac{1}{n-1}$.

$\downarrow n \to n+1$

Then $f(n) = \frac{1}{n+1-1} = \frac{1}{n}$.

$$\bar{f}(z) = Z\left\{\frac{1}{n}\right\} = \log\left(\frac{z}{z-1}\right).$$

By property,

$$Z\{f(n-1)\} = z^{-1}\,\bar{f}(z)$$

i.e., $Z\left\{\frac{1}{n-1}\right\} = \frac{1}{z}\log\left(\frac{z}{z-1}\right)$ //

⑥ $Z\{a^{n-1}\}$

Sol: Let $f(n-1) = a^{n-1}$

Then $f(n) = a^n$

$$\bar{f}(z) = Z\{a^n\} = \frac{z}{z-a}$$

By property,

$$Z\{f(n-1)\} = z^{-1}.\bar{f}(z)$$

$\therefore$ $Z\{a^{n-1}\} = z^{-1}.\frac{z}{z-a} = \frac{1}{z-a}$

$$Z\{a^{n-1}\} = \frac{1}{z-a}$$

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$$Z\{n(n-1)\} = Z\{n^2 - n\}$$

$$= Z\{n^2\} - Z\{n\} \quad \text{(by linearity prop.)}$$

$$= \frac{z(z+1)}{(z-1)^3} - \frac{z}{(z-1)^2}$$

$$= \frac{1}{(z-1)^3}\left[z(z+1) - z(z-1) \right]$$

$$\therefore \boxed{Z\{n(n-1)\} = \frac{2z}{(z-1)^3}}$$