LAPLACE TRANSFORM PAIRS

S. No.	$f(t)$	$F(s)=L[f(t)]$
1	K	$\frac{K}{s}$
2	t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$
3	e^{at}	$\frac{1}{s-a}$
4	e^{-at}	$\frac{1}{s+a}$
5	$\sin(at)$	$\frac{a}{s^2+a^2}$
6	$\cos(at)$	$\frac{s}{s^2+a^2}$
7	$\sinh(at)$	$\frac{a}{s^2-a^2}$
8	$\cosh(at)$	$\frac{s}{s^2-a^2}$
9	$u(t-a)$ or $u_a(t)$	$\frac{e^{-as}}{s}$
10	1	$\frac{1}{s}$
	$u(t)$ or $u_0(t)$	

S. No.	$f(t)$	$F(s)=L[f(t)]$
11	$\delta(t-a)$ or $\delta_a(t)$	e^{-as}
12	$\delta(t)$ or $\delta_0(t)$	1
13	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
14	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
15	$e^{-at}\sin(bt)$	$\frac{b}{(s+a)^2+b^2}$
16	$e^{-at}\cos(bt)$	$\frac{s+a}{(s+a)^2+b^2}$
17	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$
18	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
19	$e^{-at}\sinh(bt)$	$\frac{b}{(s+a)^2-b^2}$
20	$e^{-at}\cosh(bt)$	$\frac{s+a}{(s+a)^2-b^2}$
21	$\frac{e^{at}t^n}{\Gamma(n+1)}$	$\frac{1}{(s-a)^{n+1}}$

Unit Step Function (or **Heaviside's Unit function**) : $u(t-a)=\begin{cases}1 & \text{if } t\ge a\\ 0 & \text{else}\end{cases}$

Dirac Delta Function : $\delta(t-a)=\underset{\varepsilon\to 0}{\mathrm{Lt}}\,\delta_\varepsilon(t-a)$, where $\delta_\varepsilon(t-a)=\begin{cases}\frac{1}{\varepsilon} & \text{for } a-\frac{\varepsilon}{2}<t<a+\frac{\varepsilon}{2}\\ 0 & \text{else}\end{cases}$

Gamma Function : $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ defined for $x>0$.

Properties of Gamma Function :

- For +ve integer 'n', $\boxed{\Gamma(n+1)=n!}$. If 'n' is zero or a –ve integer, $\Gamma(n)$ is undefined.
- For a +ve non-integer 'x', $\boxed{\Gamma(x)\Gamma(1-x)=\frac{\pi}{\sin(\pi x)}}$. Example : $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.
- For a positive real number 'x', $\boxed{\Gamma(x+1)=x\Gamma(x)}$.
- For a negative real number 'x', $\boxed{\Gamma(x)=\frac{\Gamma(x+1)}{x}}$. Example : $\Gamma\left(\frac{-1}{2}\right)=-2\sqrt{\pi}$.

PROPERTIES OF LAPLACE TRANSFORM

S. No.	Name of the Property	$f(t)$	$F(s) = L[f(t)]$
1	Linearity	$af(t) + bg(t)$	$aF(s) + bG(s)$
2	Change of Scale	$f(at)$	$\frac{1}{a}F(s/a)$
3	First Shifting (or) Frequency Shift	$e^{at}f(t)$	$F(s-a)$
		$e^{-at}f(t)$	$F(s+a)$
4	Second Shifting (or) Time Shift	$u(t-a)f(t-a)$	$e^{-as}F(s)$
5	Derivatives of the transform	$t\,f(t)$	$-\frac{d}{ds}[F(s)]$
		$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}[F(s)]$
6	Transform of the derivatives	$f'(t)$	$sF(s) - f(0)$
		$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
		$f^{n}(t)$	$s^{(n)}F(s) - s^{(n-1)}f(0) - s^{(n-2)}f'(0) - \ldots - f^{(n-1)}(0)$
7	Integrals of the Transform	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$, provided $\underset{t\to 0}{Lt}\left[\frac{f(t)}{t}\right]$ exists.
8	Transform of the integrals	$\int_0^t f(t)dt$	$\frac{F(s)}{s}$
9	Convolution	$f(t)*g(t)$	$F(s)G(s)$
10	Periodicity	$f(t) = f(t+T)$ T- fundamental period	$\frac{1}{\left(1-e^{-sT}\right)}\left[\int_0^T e^{-st}f(t)dt\right]$
11	Initial Value Theorem	$f(0) = \underset{t\to 0}{Lt}[f(t)] = \underset{s\to\infty}{Lt}[sF(s)]$	
12	Final Value Theorem	$f(\infty) = \underset{t\to\infty}{Lt}[f(t)] = \underset{s\to 0}{Lt}[sF(s)]$	

PROPERTIES OF INVERSE LAPLACE TRANSFORM

S. No.	Name of the Property	$F(s) = L\left[f(t)\right]$	$f(t) = L^{-1}[F(s)]$
1	Linearity	$aF(s) + bG(s)$	$aL^{-1}[F(s)] + bL^{-1}[G(s)]$
2	Change of Scale	$F\left(\dfrac{s}{a}\right)$	$a\,L^{-1}[F(s)]\Big\vert_{t \to at}$
		$F(as)$	$\dfrac{1}{a} L^{-1}[F(s)]_{t \to \frac{t}{a}}$
3	First Shifting	$F(s-a)$	$e^{at} L^{-1}[F(s)]$
		$F(s+a)$	$e^{-at} L^{-1}[F(s)]$
4	Second Shifting	$e^{-as} F(s)$	$u(t-a) f(t-a)$
5	Inverse LT of the derivatives	$F'(s)$	$-t\,L^{-1}[F(s)]$
		$F^{(n)}(s)$	$(-1)^n t^n L^{-1}[F(s)]$
6	Multiplication by s	$sF(s)$	$f'(t) + f(0)\delta(t)$
7	Inverse LT of the integrals	$\int_s^{\infty} F(s)ds$, provided $\underset{t \to 0}{Lt}\left[\dfrac{f(t)}{t}\right]$ exists.	$\dfrac{1}{t} L^{-1}[F(s)]$
8	Division by s	$\dfrac{F(s)}{s}$	$\int_0^t L^{-1}[F(s)]dt$
9	Convolution	$F(s)G(s)$	$L^{-1}[F(s)] * L^{-1}[G(s)]$

Note :

➢ From the ILT of the derivatives property, $L^{-1}\left[F'(s)\right] = -t f(t) = -t L^{-1}\left[F(s)\right]$. Thus, $\boxed{L^{-1}\left[F(s)\right] = -\dfrac{1}{t} L^{-1}\left[F'(s)\right]}$. This result is very important, if the transform function $F(s)$ involves logarithmic functions and / or inverse trigonometric functions.

INVERSE LAPLACE TRANSFORM by PARTIAL FRACTION METHOD

Consider the transform function $\mathbf{F(s)}$, expressed as a proper rational function in the form $\mathbf{F(s)=\frac{P(s)}{Q(s)}}$. We can find $\mathbf{f(t)=L^{-1}[F(s)]}$ by the method of partial fractions.

Q(s)	By Partial Fraction method	Coefficients (or) Residues
$(s-s_1)(s-s_2)\ldots(s-s_n)$	$F(s)=\sum_{i=1}^{n}\frac{A_i}{(s-s_i)}$	$A_i=\underset{s\to s_i}{Lt}\left[(s-s_i)F(s)\right]$
$(s-r)^n$	$F(s)=\sum_{i=1}^{n}\frac{A_i}{(s-r)^i}$	$A_i=\frac{1}{(n-i)!}\underset{s\to r}{Lt}\left\{\frac{d^{(n-i)}}{ds^{(n-i)}}\left[(s-r)^n F(s)\right]\right\}$ $i=n,(n-1),\ldots,1$
s^2+b^2	$F(s)=A_1\left[\frac{s}{s^2+b^2}\right]+A_2\left[\frac{b}{s^2+b^2}\right]$	Obtain A_1 and A_2 by comparing the numerator terms
s^2-b^2	$F(s)=A_1\left[\frac{s}{s^2-b^2}\right]+A_2\left[\frac{b}{s^2-b^2}\right]$	-do-
$(s-a)^2+b^2$	$F(s)=A_1\left[\frac{(s-a)}{(s-a)^2+b^2}\right]+A_2\left[\frac{b}{(s-a)^2+b^2}\right]$	-do-
$(s-a)^2-b^2$	$F(s)=A_1\left[\frac{(s-a)}{(s-a)^2-b^2}\right]+A_2\left[\frac{b}{(s-a)^2-b^2}\right]$	-do-
$(s+a)^2+b^2$	$F(s)=A_1\left[\frac{(s+a)}{(s+a)^2+b^2}\right]+A_2\left[\frac{b}{(s+a)^2+b^2}\right]$	-do-
$(s+a)^2-b^2$	$F(s)=A_1\left[\frac{(s+a)}{(s+a)^2-b^2}\right]+A_2\left[\frac{b}{(s+a)^2-b^2}\right]$	-do-