L-C circuits

consider an electric circuit containing an inductance L and a capacitance C. If q is the charge on the capacitor plate and i the current in the circuit at any time t, then

(i) the voltage drops across the inductance

$$= L\frac{di}{dt} = L\frac{d^2q}{dt^2} \quad \because i = \frac{dq}{dt}.$$

(ii) the voltage drop due to capacitance $= \frac{q}{c}$

since there is no applied emf in the circuit, we have

$$L\frac{d^2q}{dt^2} + \frac{q}{c} = 0$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{q}{Lc} = 0 \quad \text{Taking } \frac{1}{Lc} = \omega^2$$

$$\Rightarrow \frac{d^2q}{dt^2} + \omega^2 q = 0$$ (Similar to eqn of free oscillations)

L-C circuit with an emf

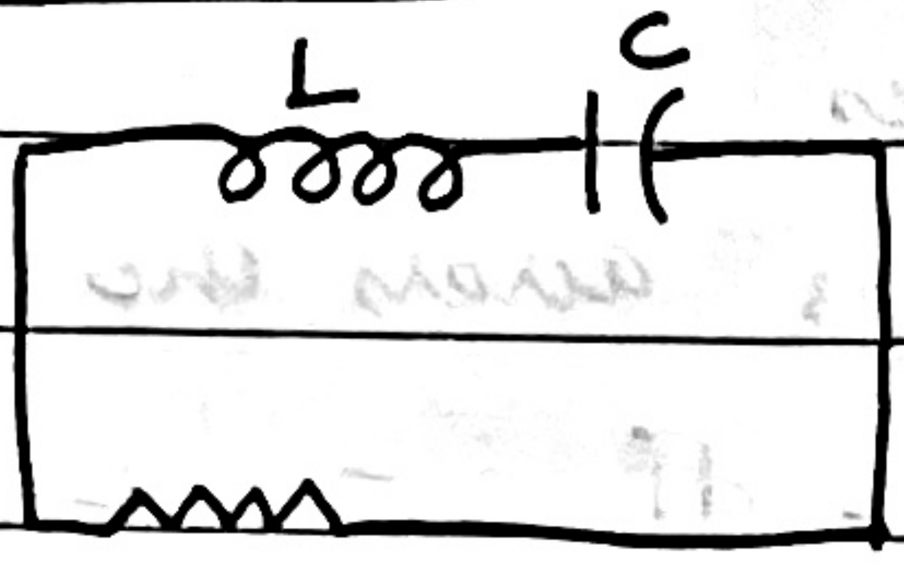
In the previous case, along with inductance L and capacitance C an emf $E_0 \cos nt$ is also applied to the circuit, the sum of voltage drops in the circuit equals to an applied emf.

$$\therefore L\frac{d^2q}{dt^2} + \frac{q}{c} = E_0 \cos nt$$

$$\frac{d^2q}{dt^2} + \omega^2 q = \frac{E_0}{L} \cos nt \qquad \frac{1}{LC} = \omega^2$$

(Similar to forced oscillations)

case(iii) L-C-R circuit



(i) consider a circuit containing an inductance L, capacitance c and a resistance R and q the charge, i the current in the circuit at any time t, then

voltage drops across the inductance $= L \frac{d^2q}{dt^2}$

voltage drops due to capacitance $= \frac{q}{c}$

voltage drop across the resistance $Ri = R\frac{dq}{dt}$

The governing differential eqn is:

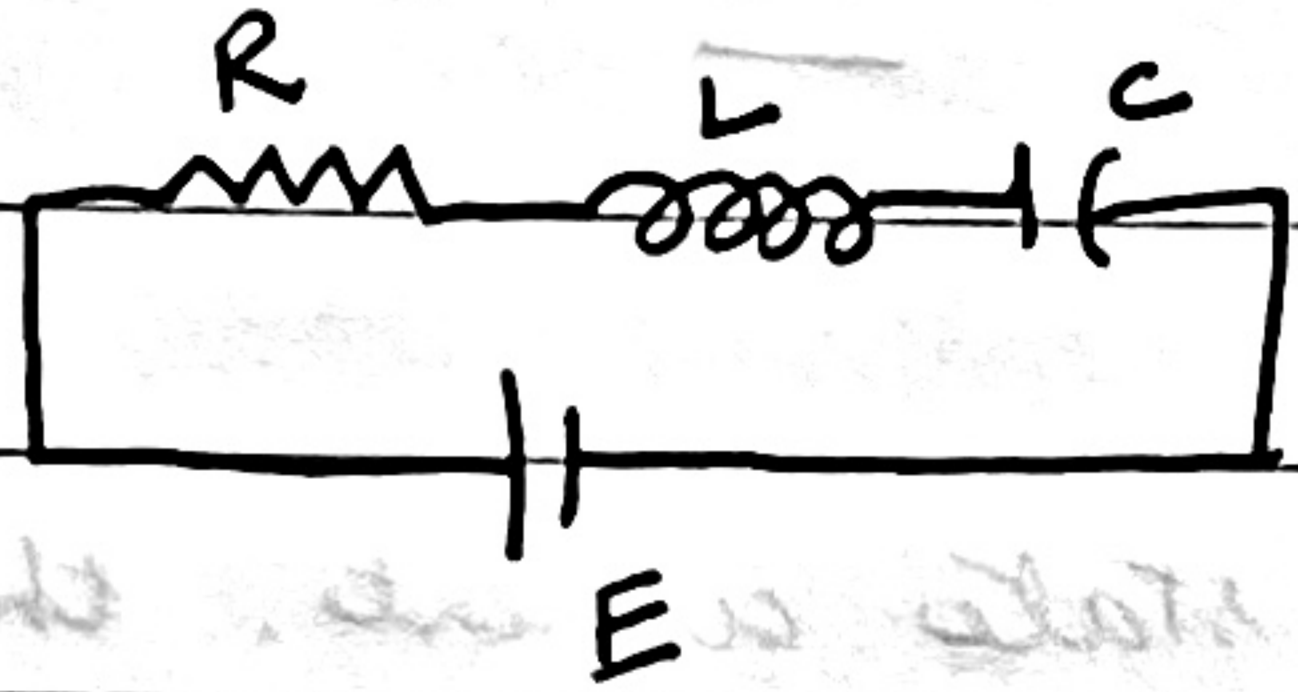
$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{Lc} = 0$$

$$\frac{d^2q}{dt^2} + 2p\frac{dq}{dt} + \omega^2 q = 0 \qquad \omega^2 = \frac{1}{Lc}, \ \frac{R}{L} = 2p$$

(similar to damped oscillations)

case (iv) L-C-R with emf: $E_0 \cos nt$.



By equating the sum of voltage drops to the applied emf the differential eqn can be written as

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0 \cos nt$$

$$\frac{d^2q}{dt^2} + 2p\frac{dq}{dt} + \omega^2 q = \frac{E_0}{L}\cos nt$$

$$\frac{d^2q}{dt^2} + 2p\frac{dq}{dt} + \omega^2 q = F\cos nt$$

$$\left\{ \begin{aligned} \frac{1}{LC} &= \omega^2 \\ \frac{R}{L} &= 2p \\ \frac{E_0}{L} &= F \end{aligned} \right.$$

(similar to forced with damping oscillations)