DIFFERENTIAL EQUATIONS AND TRANSFORMS (BMAT102L)
(WINTER SEMESTER 2021-2022)

MODULE – 6 - FOURIER TRANSFORM

TUTORIAL SHEET - 1

1. Find the Fourier transform of the following.
   (i) $\delta(t-a)$ (ii) $\frac{1}{\sqrt{|x|}}$

2. Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|\le a \\ 0, & |x|>a \end{cases}$
   Hence evaluate $\int_0^\infty \left(\frac{x\cos x-\sin x}{x^3}\right)^2 dx$.

3. Find the Fourier transform of $e^{-a|x|}\ if\ \ a>0$. Deduce that $\int_0^\infty \frac{1}{(x^2+a^2)^2}dx=\frac{\pi}{4a^3}$ if $a>0$.

4. Given $F\left(e^{-x^2}\right)=\sqrt{\pi}e^{-\frac{s^2}{4}}$, find the Fourier transform of $e^{-4(x-3)^2}$.

5. Find the Fourier transform of $f(x)=\begin{cases} a-|x|, & |x|<a \\ 0, & |x|>a \end{cases}$ hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$.

6. Find the Fourier transform of $e^{-a|x|}$ and hence deduce that

(i) $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)^2}$

(ii) $\int_0^\infty \frac{\cos xt}{a^2+t^2}dt=\frac{\pi}{2a}e^{-a|x|}$

7. Find the Fourier transform of $e^{-|x|}$ and hence find the Fourier transform of $e^{-|x|}\cos 2x$.

8. Find the Fourier transform of $f(x) = \begin{cases} \cos x, & |x| < \frac{\pi}{2} \\ 0, & |x| > \frac{\pi}{2} \end{cases}$ and hence evaluate $\int_0^\infty \frac{cos^2\left(\frac{\pi x}{2}\right)}{(1-x^2)^2} dx$

9. Show that the Fourier transform of $f(x) = \begin{cases} \frac{\sqrt{2\pi}}{2a}, & |x| \leq a \\ 0, & |x| > a \end{cases}$ is $\frac{\sin sa}{sa}$.