PRACTICE PROBLEM - MODULE - 4

Solve the following using laplace transform:

1. $y^{''} + 4y = f(t);\ y(0) = 1,\ y^{'}(0) = 0,\ with$

$$f(t) = \begin{cases} 0 & \text{for } 0 \leq t < 4 \\ 3 & \text{for } t \geq 4 \end{cases}$$

2. $y^{''} - 2y^{'} - 3y = f(t);\ y(0) = 1,\ y^{'}(0) = 0,\ with$

$$f(t) = \begin{cases} 0 & \text{for } 0 \leq t < 4 \\ 12 & \text{for } t \geq 4 \end{cases}$$

3. $y^{''} + 5y^{'} + 6y = f(t);\ y(0) = y^{'}(0) = 0,\ with$

$$f(t) = \begin{cases} -2 & \text{for } 0 \leq t < 3 \\ 0 & \text{for } t \geq 3 \end{cases}$$

4. $y^{''} - 4y^{'} + 4y = f(t);\ y(0) = -2, y^{'}(0) = 1,\ with$

$$f(t) = \begin{cases} t & \text{for } 0 \leq t < 3 \\ t+2 & \text{for } t \geq 3 \end{cases}$$

5. $y^{''} + 5y^{'} + 6y = 3\delta(t-2) - 4\delta(t-5);\ y(0) = y^{'}(0) = 0$

6. $y^{''} - 4y^{'} + 13y = 4\delta(t-3);\ y(0) = y^{'}(0) = 0$

7. $y^{''} + 16y^{'} = 12\delta(t - 5\pi/8);\ y(0) = 3,\ y^{'}(0) = 0$

8. $y^{''} + 5y^{'} + 6y = B\delta(t);\ y(0) = 3,\ y^{'}(0) = 0$

9. $y^{''} - 3y^{'} + 2y = u_1(t),\ y(0) = 1,\ y^{'}(0) = 1$, where $u_1(t)$ is unit step function and $u_1(t) = u(t-1)$.

10. $y^{''} + 5y^{'} + 6y = 1 - tu_3(t) - t^2 u_5(t),\ y(0) = 0,\ y^{'}(0) = 0.$

11. $y^{'''} = \delta(t-5),\ y(0) = y^{'}(0) = y^{"}(0) = 0$, where δ is impulse function.

12. $y^{''} + 4y^{'} + 3y = f(t);\ y(0) = 0, y^{'}(0) = 1$, where

$$f(t) = \begin{cases} -1 & \text{for } 0 \leq t < 3 \\ 0 & \text{for } t \geq 3 \end{cases}$$

13. $y^{''} - 3y^{'} + 2y = e^{3t};\ y(0) = 1, y^{'}(0) = 0.$

14. $y^{''} - 10y^{'} + 9y = 5t;\ y(0) = -1, y^{'}(0) = 2.$

15. $y^{''} + 25y = 10\cos 5t$ given that $y(0) = 2, y^{'}(0) = 0.$

16. $y^{''} - 6y^{'} + 15y = 2\sin 3t,\ y(0) = -1, y^{'}(0) = -4.$

17. $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 5y = e^{-t}\sin t$, where $y(0) = 0$ and $y^{'}(0) = 1.$