Differential Equations and Transforms
BMAT102L
Module-1
Tutorial Sheet-II,Feb-2022

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1. For a circuit consisting of an inductance L, a resistance R, a capacitance C and an e.m.f, $E(t) = E_0 \sin wt$, find the expression for the steady-state current.

2. Consider an electric circuit with an inductance of 5 henries and a resistance of 12 ohms in series with an e.m.f $120 sin 200t$ volts. Find the current.

3. The electric charge x in a circuit with an inductance, a resistance and a capacitance is given by the equation.

$$L\frac{dx^2}{dt^2} + R\frac{dx}{dt} + \frac{x}{C} = E\cos pt$$

where L, R, C, p, E are constants and t time. Prove that if R is positive, the exponential terms in the solution tend to zero as $t \to \infty$ and find the solution assuming that these terms can be ignored. With this assumption, show that for a given values of E, p, R the largest value of x occurs when $\sqrt{LC} = \frac{1}{p}$

4. Consider an electric circuit with an inductance of 0.05 henry, a resistance of 20 ohms, a condenser of capacitance of 100 micro farads and an e.m.f of $E = 100$ volts. Find i and q given the initial conditions $q = 0, i = 0$ at $t = 0$.

5. Solve $x^2y^{''} + 2xy^{'} - 12y = x^3 \log x$.

6. Solve $(2x+3)^2y^{''} - (2x+3)y^{'} - 12y = 6x$.

7. Solve $x^2y^{''} + xy^{'} - y = \dfrac{1}{x+1}$

8. Solve $x^2y^{''} - 2xy^{'} + 2y = x^4 \sin(4\log x)$

9. The differential equation of motion of a particle, which executes forced oscillations without damping is

$$\frac{dx^2}{dt^2} + 4x = 4a \sin nt.$$

Find the displacement x of the particle at time t,

- when $n = 2$.
- when $n \neq 2$

given that the particle starts from the rest from the origin initially.

10. A body executes damped forced vibration given by equation

$$\frac{dx^2}{dt^2} + 2k\frac{dx}{dt} + b^2 x = e^{-kt} \sin wt.$$

Solve the equation for both the cases when

- $w^2 \neq b^2 - k^2$
- $w^2 = b^2 - k^2$.