VIT
Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - November/December 2023

gramme	B.Tech.	Semester	FALL SEMESTER 2023 - 24
urse Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
aculty Name	Prof. Abhishek Kumar Singh	Slot	Y11+Y12+Y21+Z21
		Class Nbr	CH2023240101881
Time	3 Hours	Max. Marks	100

PART-A (10 X 10 Marks)
Answer any 10 questions

01. Find general solution of given differential equation using variation of parameters [10]
$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}x^{-2}$.

02. Find the general solution of the given differential equation using method of undetermined coefficients [10]
$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 10e^x - 5xe^{-8x}$.

03. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ [10]

04. Find the singular solution of the given partial differential equations [10]

(a) $p^2 + y^2 = q^2 + x^2$
(b) $z = pqxy$

05. (a) Find Laplace transform of the function, $f(t) = e^{10t}\, t^2 \cos 4t + e^{5t}u(t-2)$ [10]
(b) Find inverse Laplace transform of the function, $F(s) = \frac{(s^2+1)}{s^2(s+2)}$

06. Solve the initial value problem using Laplace Transform [10]
$\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 18y = 54x - 9u(x)$ with $y(0)=3$ and $\frac{dy}{dx}(0) = 0$.

07. Solve $\frac{\partial u(x,t)}{\partial t} + \frac{\partial u(x,t)}{\partial x} + 2u(x,t) = 0$ with the initial and boundary conditions [10]
$u(0,t) = -\cos 2t$ and $u(x,0) = -e^{-2x}\cos(2x)$

08. Find the Fourier series expansion of $f(x) = x^2$ in $(-1,1)$ and hence deduce the value of [10]
$\sum_{n=1}^{\infty} \frac{1}{n^2}$.

09. Find the Fourier sine and cosine transform of the function [10]
$$f(x) = \begin{cases} x & 0 < x < 2 \\ 3 - x & 2 < x < 3 \\ 0 & x \geq 3 \end{cases}$$

10. Find the Fourier transform of $F(e^{-x^2})$ and hence find the Fourier transform of $e^{-7(x-3)^2}$ [10]

11. Evaluate [10]

$$Z\left(\frac{1}{n(n+1)}\right) \text{ and } Z^{-1}\left(\frac{4-8z^{-1}+6z^{-2}}{(1+z^{-1})(1-2z^{-1})^2}\right)$$

12. Solve the difference equation $y(n+2)-5y(n+1)+6y(n)=n$ using Z-transform with the initial conditions $y_0=1$ and $y_1=2$ [10]