CLAIRAUT'S FORM

$$z = px + qy + f(p,q)$$

Replace p by a and q by b to get the complete solution.

The complete soln is $z = ax + by + f(a,b)$

Singular solution

consider the complete soln

differentiate it partially w.r.to a and b then equate to 0.

$$\frac{\partial z}{\partial a} = 0$$

$$\frac{\partial z}{\partial b} = 0$$

1) Solve $z = px + qy + p^2q^2$

Complete solution is $z = ax + by + a^2b^2$ — ①

General solution put $b = \phi(a)$ in ①

$$z = ax + \phi(a)y + a^2[\phi(a)]^2 \quad \text{— ②}$$

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diff ② w.r.to 'a',

$$\frac{\partial z}{\partial a} = 0 = x + \phi'(a)y + a^2 . 2\phi(a). \phi'(a) + 2a . [\phi(a)]^2 \quad —③$$

Eliminating 'a' using ② and ③ will give the general solution.

Singular solution

differentiate ① partially w.r.to a and b, we get

$$\frac{\partial z}{\partial a} = 0 = x + 2ab^2 \quad — ④ \qquad \Rightarrow 2ab = -\frac{x}{b}$$

$$\frac{\partial z}{\partial b} = 0 = y + 2a^2 b \quad —⑤ \qquad \Rightarrow 2ab = -\frac{y}{a}$$

$$\frac{-x}{a} = \frac{-y}{b} = K$$

$$\frac{x}{a} = K \Rightarrow \boxed{a = \frac{x}{K}}$$

$$\frac{y}{b} = K \Rightarrow \boxed{b = \frac{y}{K}}$$

substituting a and b in ④

$$\Rightarrow x = -2\frac{y}{K} . \frac{x^2}{K^2} \Rightarrow \boxed{K^3 = -2xy}$$

substituting in ①

$$z = \frac{xy}{k} + \frac{xy}{k} + \frac{x^2y^2}{k^4}$$

$$= \frac{2xy}{k} + \frac{x^2y^2}{k^4}$$

$$kz = 2xy + \frac{x^2y^2}{k^3}$$

$$kz = 2xy - \frac{x^2y^2}{2xy} = 2xy - \frac{xy}{2}$$

$$\Rightarrow \quad kz = \frac{3xy}{2}$$

cubing both sides

$$k^3z^3 = \frac{27}{8}x^3y^3$$

$$-2xy\,z^3 = \frac{27}{8}x^3y^3$$

$$\Rightarrow \quad 16z^3 + 27x^2y^2 = 0 \quad //$$

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2) Find the singular integral of the PDE

$$z = px + qy + p^2 - q^2 \rightarrow$$ clairauts' form

The complete integral is

$$z = ax + by + a^2 - b^2 \quad \text{—} \quad ②$$

SINGULAR SOLUTION

Differentiate ① partially a and b, we get

$$0 = x + 2a \Rightarrow a = -\frac{x}{2}$$

$$0 = y + 2b \Rightarrow b = +\frac{y}{2}$$

substituting in ①, we get

$$z = -\frac{x}{2} \cdot x + \frac{y}{2} \cdot y + \left(-\frac{x}{2}\right)^2 - \left(\frac{+y}{2}\right)^2$$

$$= -\frac{x^2}{2} + \frac{y^2}{2} + \frac{x^2}{4} - \frac{y^2}{4}$$

$$z = -\frac{x^2}{4} + \frac{y^2}{4}$$

$4z = y^2 - x^2$ is the singular solution.