Tutorial Sheet : 1
Course Code : BMAT102L
Course Title : Differential Equations and Transforms

Vellore Institute of Technology (Deemed to be University under section 3 of UGC Act, 1956) CHENNAI

Module 1: Ordinary Differential Equations- CO1

1 Easy

1. Solve $y'' - 2y' + 5y = 0$ with $y(0) = -3$ and $y'(0) = 1$

Solution: The roots are $1 \pm 2i$
Particular solution is $y = e^x[-3\cos 2x + 2\sin 2x]$

2. Solve $y'' + 2y' + 4y = 13\cos(4x - 2)$ using method of undetermined coefficients

Solution: $y_c = e^{-x}[C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x]$
$y_p = -\frac{3}{4}\cos(4x-2) + \frac{1}{2}\sin(4x-2)$
The complete solution is $y = y_c + y_p$

3. Solve $y'' - 2y' - 3y = xe^{-x}$ using method of variation of parameters.

Solution: The complete solution is $y = C_1e^{-x} + C_2e^{3x} - \frac{x^2}{8}e^{-x} + e^{3x}(-\frac{x}{16}e^{-4x} - \frac{1}{64}e^{-4x})$

4. Solve $x^2y'' - 2xy' - 4y = x^2 + 2\log x$

Solution: The complete solution is $y = C_1x^{-1} + C_2x^4 + \frac{3}{8} - \frac{\log x}{2} - \frac{x^2}{6}$

5. Solve $(2x+3)^2y'' - (2x+3)y' - 12y = 6x$

Solution: The complete solution is $y = C_1(2x+3)^{\frac{3+\sqrt{57}}{4}} + C_2(2x+3)^{\frac{3-\sqrt{57}}{4}} - \frac{3}{14}(2x+3) + \frac{3}{4}$

2 Moderate

6. Solve the following initial value problems:

$y'' - 2y' + 2y = 0, y(0) = 0, y'(0) = 1$

Solution: $y(x) = e^x \sin x$

7. Find the general solution of the following differential equations using the method of undetermined coefficients

$y'' - 2y' + y = e^x + x^2$

Solution: $y(x) = (a + bx)e^x + \frac{1}{2}x^2 e^x + 6 + 4x + x^2$

8. Solve $(3x+2)^2 y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 3$ using the method of undetermined coefficients.

Solution: $y = a(3x+2)^2 + b(3x+2)^{-2} + (3x+2)^2 \frac{\log x}{108} + \frac{1}{108}$

9. Solve $(x^2D^2 - 3xD + 4)y = x^2 \cos(\log x)$

Solution: $y = (a + b\log x)x^2 - x^2 \cos(\log x)$

10. Solve the differential equation $(x^2D^2 + xD - 1)y = x^2 e^x$

Solution: $y = (ae^x + be^{-x}) + \left(e^x - \frac{e^x}{x}\right)$

3 Hard

11. An LCR circuit connected in series has a resistance of 5hms, an inductance of 0.05 henry, a capacitor of 4×10^4 farad, and an applied alternating emf of $200\cos 100$ tvolts. Find an expression for the current flowing through this circuit if the initial current and the initial charge on the capacitor are both zero.

Solution: $l = -2.35e^{-50t}\cos 50\sqrt{19}t + 22.13e^{-50t}\sin 50\sqrt{19t} + \frac{40}{17}\cos 100t - \frac{160}{17}\sin 100t.$

12. A body executes damped forced vibrations given by the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin nt.$$

Solve the equation for both the cases, when $n^2 \neq b^2 - k^2$ and $n^2 = b^2 - k^2$.

Solution: (i) $x = Ae^{-kt}\cos\left\{t\sqrt{(b^2-k^2)}+B\right\}+\{e^{-kt}/(b^2+k^2-n^2)\}\sin nt$.
(ii) $x = Ae^{-kt}\cos\left\{t\sqrt{(b^2-k^2)}+B\right\}-\left(te^{-kt}/2n\right)\cos nt$.

13. Use variation of parameters to find the general solution of the differential equation $x^2y''-xy' = x^3e^x$ if two solutions to the associated homogeneous problem are known to be 1 and x^2.

Solution: $y = c_1 + c_2x^2 + xe^x - e^x$.

14. Solve $x^2y''+3xy'+y = \frac{1}{(1-x)^2}$.

Solution: $y = \frac{1}{x}(c_1 + c_2\log x) + \frac{1}{x}\log\frac{x}{1-x}$.

15. Solve $(3x+2)^2y''+5(3x+2)y'-3y = x^2+x+1$.

Solution: $y = c_1(3x+2)^{\frac{1}{3}} + c_2(3x+2)^{-1} + \frac{1}{27}\left[\frac{1}{15}(3x+2)^2+\frac{1}{4}(3x+2)-7\right]$.

16. Find the complete solution of the differential equations: (i). $y''-2y' = 6x^2-3e^{\frac{x}{2}}$. (ii). $y''+4y = x\sin 2x+8$.

Solution: (i). $y = c_1 + c_2e^{2x} - x^3 - \frac{3}{2}x^2 - \frac{3}{2}x + 4e^{\frac{x}{2}}$. (ii). $y = c_1\cos 2x + c_2\sin 2x - \frac{1}{8}x^2\cos 2x + \frac{1}{16}x\sin 2x + 2$.