① Solve $y'' + 5y' + 6y = 0$

Sol: Auxiliary equation of the given DE is

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\lambda = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm 1}{2}$$

$$\boxed{\lambda = -3, -2}$$

General solution of DE is

$$y(x) = c_1 e^{-3x} + c_2 e^{-2x}$$

where c_1 and c_2 are arbitrary constants.

② Solve $y'' + 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = 15$

Sol:

$y(0) = 0 \Rightarrow y(0) = c_1 + c_2$

$$\boxed{0 = c_1 + c_2} \quad -①$$

$$y'(x) = -3c_1 e^{-3x} - 2c_2 e^{-2x}$$

$y'(0) = 15 \Rightarrow y'(0) = -3c_1 - 2c_2$

$$\boxed{15 = -3c_1 - 2c_2} \quad -②$$

Solving ① & ②, we get

$$c_1 = -15$$

$$c_2 = +15$$

∴ Reqd. soln. is

$$y(x) = -15e^{-3x} + 15e^{-2x} \quad //$$

② Solve $y'' + 6y' + 9y = 0$, $y' = \frac{dy}{dx}$

Sol:

Auxiliary equation is

$$\lambda^2 + 6\lambda + 9 = 0$$

$$(\lambda + 3)^2 = 0$$

$$\boxed{\lambda = -3, -3}$$

General Soln. of DE is

$$y(x) = C_1 e^{-3x} + C_2 x e^{-3x}$$, where

C_1 and C_2 are arbitrary constants.

③ Solve $y'' + 4y = 0$

Sol:

Auxiliary equation is

$$m^2 + 4 = 0 \Rightarrow m = \pm 2i$$

$$\alpha = 0, \quad \beta = 2$$

General solution of DE is

$$y(x) = e^{0x}\left[C_1 \cos(2x) + C_2 \sin(2x) \right]$$

$$y(x) = C_1 \cos 2x + C_2 \sin 2x,$$

where C_1 and C_2 are arbitrary constants.

(4) Solve $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = 0.$

Sol: Aux. eqn. is $\lambda^2 - 2\lambda + 2 = 0$

$$\lambda = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2}$$

$$\lambda = 1 \pm i \longrightarrow \boxed{\alpha = 1,\ \beta = 1}$$

Fundamental Solns of DE are $e^x \cos x$ and $e^x \sin x$.

General Soln. of DE is

$$y(x) = e^x \left[C_1 \cos x + C_2 \sin x \right]$$

Non-homogeneous DE (linear)

DE: $f(D)y = r(x)$

$$\begin{cases} f(D)y = 0 \; ; \Rightarrow & y_c(x) \text{ [complementary function]} \\ f(D)y = r(x) \Rightarrow & y_p(x) \text{ [Particular solution or Particular integral]} \end{cases}$$

✓ $y(x) = y_c(x) + y_p(x)$ is the general solution of non-homogeneous linear DE.

Particular soln.

1. Method of UC
2. Method of Variation of parameters.