15.10 EQUATIONS OF THE FORM dx/P = dy/Q = dz/R

(1) Method of grouping

See if it is possible to take two fractions dx/P = dz/R from which y can be cancelled or is absent, leaving equations in x and z only.

DIFFERENTIAL EQUATIONS OF OTHER TYPES

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If so, integrate it by giving $\phi(x, z) = c$.

...(1)

Again see if one variable say: x is absent or can be removed may be with the help of (1), from the equation dy/Q = dz/R.

Then integrate it by giving
$$\psi(y, z) = c'$$

...(2)

...(1)

These two independent solutions (1) and (2) taken together constitute the complete solution required.

(2) Method of multipliers

By a proper choice of the multipliers l, m, n which are not necessarily constants, we write

 $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{ldx + mdy + ndz}{lP + mQ + nR}$ such that lP + mQ + nR = 0.

Then ldx + mdy + ndz = 0 can be solved giving the integral $\phi(x, y, z) = c$

Again search for another set of multipliers λ , μ , γ

so that

$$\lambda P + \mu Q + \gamma R = 0$$

giving

$$\lambda dx + \mu dy + \gamma dz = 0,$$

which on integration gives the solution $\psi(x, y, z) = c'$

...(2)

These two solutions (1) and (2) taken together constitute the required solution.

Example 15.14. Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{y^2x}$

Solution. Taking the first two fractions and cancelling z^2 , we get

$$\frac{dx}{y} = \frac{dy}{x} \quad \text{or} \quad xdx - ydy = 0$$

which on integration gives $x^2 - y^2 = c$.

Its integral is $y^3 - z^3 = c'$.

Again taking the second and third fractions and cancelling x, we have

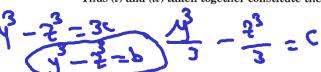
$$\frac{dy}{z^{2}} = \frac{dz}{y^{2}}, i.e., y^{2}dy - z^{2}dz = 0.$$

$$\frac{dy}{z^2} = \frac{dz}{y^2}$$
, i.e., $y^2 dy - z^2 dz = 0$.

...(ii)

...(i)

Thus (i) and (ii) taken together constitute the required solution of the given equations.



y-2)=0 iche neral Solution.

Theorem: The general solution of a quasilinear partial differential equation

P(x, y, z) + + P(x, y, z) = R(x, y, z)is

 $\phi(u,v)=0,$

Where ϕ is an arbitrary function.
The arguments u and v are
obtained from the subsidiary

equations

 $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ by which the two independent Solutions are U(x,y,z) = a and V(x,y,z) = b

Sofre (m2-ny) + + (nx-12) 9 (1) = ly - ma.

Here the subsidiary equations one ऽजः

 $\frac{M5-u\lambda}{4x} = \frac{\mu x-\delta s}{4\lambda} = \frac{3\lambda-\mu x}{4s}$

(i) Using the multipliers x, y and 2, each fraction = Xdxtydyt2d2

ここ か d か ナ y d y ナ そ d を = 0

Integrating, we set

$$\frac{3 + 3 + 5 = 0}{5} = 0$$

· - U= X+4+2

(ii) Using the multipliers I, mand n, each fraction: Idathady + reft

Integrating, We set

[\lambda \tau + my + n2 = b]

·: V= lx+my+nz

General solution of the viven Pde is $\phi(u,v)=0$

i.e., $\phi(x^2+y^2+z^2, 1x+my+nz)=0$ When ϕ is an arbitrary function. 2) Solve (31-y-2) p+2749 The subsidiary equations $\frac{3x^2-y^2-z^2}{2xy}=\frac{3xy}{2xy}=\frac{3xy}{2xy}$ Considering Second and third (i)brackions, we set dy = dz Integrating, We set 2094 = 2092 + 209a 109 (4) = loga = $\frac{y}{2} = \alpha$: U= Y/2

(11)

Choosing multiplies of 3, y and 2,
reach fraction = modertydyt 2 de

2(N+4+2)

25-13-55 = SAA = SX5

= x(x2+42+52) x(x2+42+52)

Now, considerly third and fruth Frachions, we set

2 x f = x d x f y d y + 2 d f

N d x f y d y + 2 d f

$$V = \frac{x_1^2 + x_2^2}{x_1^2 + x_2^2}$$

$$V = \frac{x_1^2 + x_2^2}{x_1^2 + x$$

The general solution of the Fren Pde is VF (u, v)=0 Where U = 4/2, N= - 2 and F is an arbitrary fraction.

Another Soln:

The Sene of Soln. is $V = G(u) \quad \text{where}$ Gris arbitrary function.

solve 2 (4-2) p+ で(モーメ)をこる(オーケ)が solve (~2-42)b+(y-22)q

Ans: $\frac{y-y}{y-z} = \phi\left(xy+yz+zx\right)$