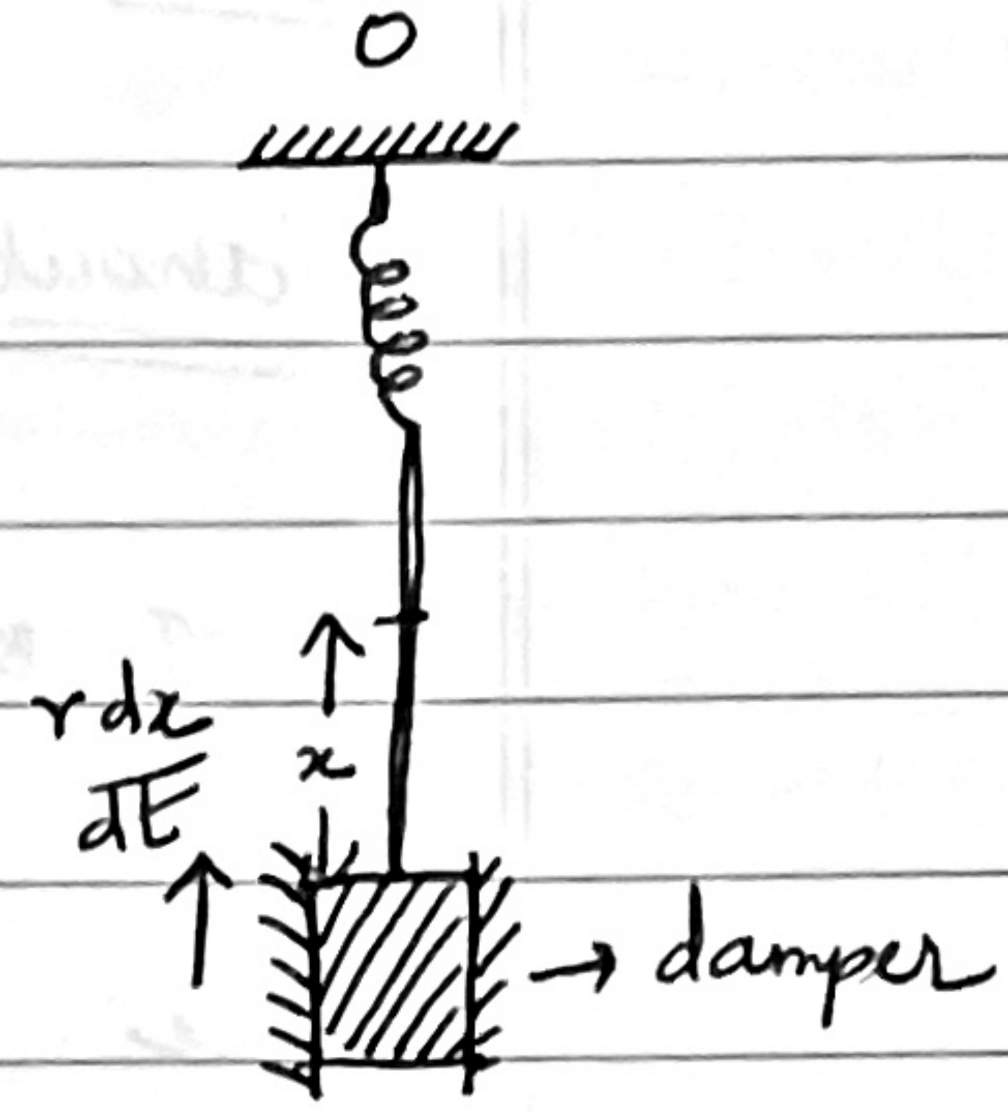
DAMPED OSCILLATIONS

If the mass m be subjected to do damping force proportional to velocity (say: $c\frac{dx}{dt}$)



damping force is proportion to the velocity of the mass and acts in the direction opposite to that of motion.

$$\therefore m\frac{d^2x}{dt^2} = \text{restoring force} + \text{damping force}$$

$$= -kx - c\frac{dx}{dt}$$ c is a positive constant.

$$\Rightarrow m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

A.E $\quad m{m^*}^2 + cm^* + k = 0 \qquad m^* = r$

$$mr^2 + cr + k = 0$$

$$r_1 = \frac{-c + \sqrt{c^2 - 4mk}}{2m}, \quad r_2 = \frac{-c - \sqrt{c^2 - 4mk}}{2m}$$

If $c^2 - 4mk > 0$ OVER DAMPING

$c^2 - 4mk < 0$ UNDER DAMPING

$c^2 - 4mk = 0$ CRITICAL DAMPING

10.0 Newtons = 1.01972

kilograms ~~force~~

FORCED OSCILLATIONS.

without damping

If the point of the support of the spring is also vibrating with some external periodic force, the the resulting force with motion is called the forced oscillatory motion

$$m \cdot \frac{dx^2}{dt^2} = \text{restoring force} + \text{external force}$$

$$m\frac{d^2x}{dt^2} = -Kx + mp\cos nt$$

$$\frac{d^2x}{dt^2} = -\frac{K}{m}x + p\cos nt$$

Take $\boxed{\frac{K}{m} = \mu^2}$

$$\frac{d^2x}{dt^2} + \mu^2 x = p\cos nt$$

Forced oscillations (with damping)

$$m\frac{d^2x}{dt^2} = \text{restoring force} + \text{damping force} + \text{external periodic force}$$

$$m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} + mp\cos nt$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{c}{m}\frac{dx}{dt} + p\cos nt$$

Take $\frac{k}{m} = \mu^2$ and $\frac{c}{m} = 2\lambda$

$$\Rightarrow \frac{d^2x}{dt^2} + \mu^2 x + 2\lambda\frac{dx}{dt} = p\cos nt$$

$$\Rightarrow \frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \mu^2 x = p\cos nt$$

ELECTRICAL CIRCUIT PROBLEMS

A simple electric circuit use to have a electrical energy source such as a battery, a resistance, a generator .. etc. This energy is used by the electrical appliances. The source of energy providing an electromotive force (e.m.f) E, makes a current (I) flow through the circuit.

The e.m.f (E), the current (I) and the resistance (R) in a circuit are connected by ohm's law, which states that $I \propto E$ (or $E = IR$, R is the resistance.

	PHYSICAL QUANTITY	SYMBOL	UNIT	DIAGRAMATIC REPRESENTATION
1.	e.m.f (or voltage	E or V	volt	or
2.	current	I or i	amphere	$\rightarrow i$
3.	charge	Q or q	coulomb	
4.	Resistance	R	ohm	
5.	Inductance	L	Henry	
6.	capacitance	C	Farad	

L-C Circuits

Consider an electric circuit with an inductance I, a capacitance C. If q is the charge on the capacitor plane and i the current is the circuit at any time t, the voltage drops across the inductance

$$L = L\frac{di}{dt} = L\frac{d^2q}{dt^2} \quad \because i = \frac{dq}{dt}$$

(ii) The voltage drop due to capacitance $= \frac{q}{c}$.
Since there is no applied E.M.F in the circuit,

$$L \frac{d^2q}{dt^2} + \frac{q}{c} = 0$$

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = 0 \qquad \frac{1}{LC} = \omega^2$$

$$\Rightarrow \frac{d^2q}{dt^2} + \omega^2 q = 0.$$

L-C circuit with an E.M.F

In the previous case, along with inductance L and capacitance C, an e.m.f $E_0 \cos nt$ is also applied to the circuit, the sum of voltage drops in the circuit equal to the applied e.m.f.

$\therefore$ The differential equation becomes.

$$L\frac{d^2q}{dt^2} + \frac{q}{c} = E_0 \cos nt$$

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{E_0}{L} \cos nt$$

Consider $\frac{1}{LC} = \omega^2$

$$\Rightarrow \frac{d^2q}{dt^2} + \omega^2 q = \frac{E_0}{L} \cos nt$$

Mech. System	Series circuit
Displacement	current i
Force	voltage E
Mass m	Inductance L
Damping force	Resistance R
Spring modulus	Elastance $1/c$