(i) $z^{-1}\left[\dfrac{2z^2+3z}{(z+2)(z-4)}\right]$

consider $U(z) = \dfrac{2z^2+3z}{(z+2)(z-4)}$

$= \dfrac{z(2z+3)}{(z+2)(z-4)}$

$\Rightarrow \dfrac{U(z)}{z} = \dfrac{2z+3}{(z+2)(z-4)}$

Apply partial fraction for RHS

$$\frac{2z+3}{(z+2)(z-4)} = \frac{A}{(z+2)} + \frac{B}{(z-4)}$$

$$2z+3 = A(z-4) + B(z+2)$$

Put $z=4$

$$\Rightarrow \qquad 11 = 6B \Rightarrow \boxed{B = \frac{11}{6}}$$

Put $z = -2$

$$-1 = -6A$$

$$\Rightarrow \boxed{A = \frac{1}{6}}$$

$$\therefore \quad \frac{2z+3}{(z-4)(z+2)} = \frac{1}{6(z+2)} + \frac{\frac{11}{6} \cdot \frac{1}{(z-4)}}$$

(ii)

$$\frac{U(z)}{z} = \frac{1}{6} \cdot \frac{1}{(z+2)} + \frac{11}{6} \cdot \frac{1}{(z-4)}$$

$$U(z) = \frac{1}{6} \cdot \frac{z}{z+2} + \frac{11}{6} \cdot \frac{z}{z-4}$$

$$\Rightarrow z^{-1}[U(z)] = \frac{1}{6} z^{-1}\left[\frac{z}{z+2}\right] + \frac{11}{6} z^{-1}\left[\frac{z}{z-4}\right]$$

$$\Rightarrow u_n = \frac{1}{6}(-2)^n + \frac{11}{6} 4^n$$