I Formation of PDE's by eliminating arbitrary constants.

1. $$(x-a)^2 + (y-b)^2 + z^2 = c^2$$

Sol: Diff. PDE w.r.t. x, y, we get

$$2(x-a) + 2zz_x = 0 \Rightarrow x-a = -zz_x$$

$$2(y-b) + 2zz_y = 0 \Rightarrow y-b = -zz_y$$

Substituting in the given PDE, we get

$$z^2 z_x^2 + z^2 z_y^2 + z^2 = c^2$$

$$\Rightarrow \boxed{z^2[p^2 + q^2 + 1] = c^2}$$

where $p = z_x$ and $q = z_y$.

(2) $$z = a \log\left(\frac{b(y-1)}{1-x}\right)$$

Sol: Diff. PDE w.r.t. x, we get

$$p = a\left[\frac{1-x}{\cancel{b(y-1)}}\right]\left[\frac{\cancel{b(y-1)}}{(1-x)^2}\right]$$

$$p = \frac{a}{1-x} \quad \text{— (A)}$$

Diff. PDE w.r.t. y, we get

$$q = a\left(\frac{\cancel{1-x}}{\cancel{b}(y-1)}\right)\left(\frac{\cancel{b}}{\cancel{1-x}}\right)$$

$$q = \frac{a}{y-1} \quad \text{— (B)}$$

Eliminating a from (A) and (B), we get

$$\boxed{p(1-x) = q(y-1)}$$

This is the reqd. PDE as it is free from a and b.