VIT

Reg. No.: 22BEC18[illegible]

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Ankit Kumar	Slot	C2+TC2+TCC2
		Class Nbr	CH2022232300621
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)

Answer <u>any 10</u> questions

01. Solve by using the method of variation of parameter $4\frac{d^2y}{dx^2} + y = \frac{1}{4}xe^{(\frac{x}{2})}; y(0) = 1, y'(0) = 0.$ [10]

02. A) Solve the differential equation $\frac{d^2y}{dx^2} + y = 12e^{2x}$ by using the method of undetermined coefficients. [10]

B) Form a partial differential equation by eliminating f and g from the following equation $z = f(x^2 - y) + g(x^2 + y)$.

(5+5 Marks)

03. A. Solve $z^2(p^2z^2 + q^2) = 1$. Does singular solution exist? [10]

B. Obtain the general solution of the following partial differential equation $\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$.

(5+5 Marks)

04. Find $L^{-1}\left[\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right]$. [10]

05. A. Find the Laplace transform of $f(t) = \begin{cases} t & if\ t < 6 \\ -8 + (t-6)^2 & if\ t \geq 6 \end{cases}$ [10]

B. Using the Fourier series of $f(x) = x$ in the interval $(0, 2\pi)$, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(5+5 Marks)

06. Solve the following differential equation using Laplace transform $y'' + 3y' + 2y = \delta(t-1)$ with the initial condition $y(0) = y'(0) = 0$, where $\delta(t-1)$ is the unit impulse at time $t = 1$. [10]

07. Find the bounded solution $y(x,t), x, t > 0$ of following partial differential equation by the method of Laplace transform $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}; x, t > 0$ with $y(x, 0) = 0$. [10]

08. Find the half range sine series for $f(x) = \begin{cases} x & 0 \leq x < \pi/2 \\ \pi - x & \pi/2 \leq x \leq \pi \end{cases}$ [10]

Deduce (i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

(ii) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$.

09. Find $f(x)$, if its Fourier sine transform is $\frac{e^{-a\omega}}{\omega}$. Hence, deduce $F_s^{-1}\left(\frac{1}{\omega}\right)$. [10]

10. Find the Fourier transform of $f(x) = \begin{cases} 4-|x|, & if\ |x| < 4 \\ 0, & if\ |x| > 4 \end{cases}$. Hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$. [10]

11. A. Find the $Z-$ transform and the radius of convergence of $f(n) = 2^n, n < 0$ [10]

B. If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, then evaluate u_2 and u_3.

(5+5 Marks)

12. Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$ using $Z-$ transform. [10]

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