DAY 4 - SOLUTION OF SIMULTANEOUS EQUATIONS USING L.T

1. Solve $\dfrac{dy}{dt} + 2x = \sin 2t$ —(1)

$\dfrac{dx}{dt} - 2y = \cos 2t$ (2), $x(0) = 1$, $y(0) = 0$

Taking L.T of (1)

$$L(y'(t)) + 2L(x(t)) = L(\sin 2t)$$

$$sL(y(t)) - \underbrace{y(0)}_{0} + 2L(x(t)) = \frac{2}{s^2+4}$$

$$sL(y(t)) + 2L(x(t)) = \frac{2}{s^2+4} \quad \text{—(3).}$$

Taking L.T of (2)

$$L(x'(t)) - 2L(y(t)) = L(\cos 2t)$$

$$sL(x(t)) - x(0) - 2L(y(t)) = \frac{s}{s^2+4}.$$

$$sL(x(t)) - 1 - 2L(y(t)) = \frac{s}{s^2+4}.$$

$$sL(x(t)) - 2L(y(t)) = \frac{s}{s^2+4} + 1 \quad \text{—(4).}$$

$$= \cancel{s^2+s+4}$$

$(3) \times 2 \longrightarrow \cancel{2sL(y(t))} + 4L(x(t)) = \dfrac{4}{s^2+4}$

$(4) \times s \longrightarrow \cancel{-2sL(y(t))} + s^2L(x(t)) = \dfrac{s^2}{s^2+4} + s$

$$(s^2+4)L(x(t)) = \frac{s^2+4}{s^2+4} + s$$

$$= 1 + s$$

$$L(x(t)) = \frac{1}{s^2+4} + \frac{s}{s^2+4} \qquad x(t) = \frac{\sin 2t}{2} + \cos 2t$$

Sub in (2)

$$\frac{dx}{dt} - 2y = \cos 2t.$$

$$\frac{\overset{-2\sin 2t}{2\cos 2t}}{2} - 2y = \cos 2t$$

$$\cos 2t - 2\sin 2t - 2y = \cos 2t.$$

$$2y = -2\sin 2t$$

$$y = -\sin 2t.$$

$$\therefore x = \frac{\sin 2t}{2} + \cos 2t \quad ; \quad y = -\sin 2t.$$

2). Solve $2x' - y' + 3x = 2t$ — (1)

$x' + 2y' - 2x - y = t^2 - t$, $x(0) = 1$, $y(0) = 1$.

(2)

Eqn (1) $2x' - y' + 3x = 2t$

$$2L(x'(t)) - L(y'(t)) + 3L(x(t)) = 2L(t).$$

$$2[\underset{\overset{\|}{1}}{}sL(x(t)) - x(0)] - [sL(y(t)) - \underset{\overset{\|}{1}}{y(0)}] + 3L(x(t)) = \frac{2}{s^2}$$

$$(2s+3)L(x(t)) - sL(y(t)) = \frac{2}{s^2} + 2 - 1 = \frac{2}{s^2} + 1. \quad \text{— (3)}$$

Eqn (2) $x' + 2y' - 2x - y = t^2 - t.$

$$L(x'(t)) + 2L(y'(t)) - 2L(x(t)) - L(y(t)) = L(t^2) - L(t)$$

$$[sL(x(t)) - x(0)] + 2[sL(y(t)) - y(0)] - 2L(x(t)) - L(y(t)) = \frac{2}{s^3} - \frac{1}{s^2}$$

$$(s-2)L(x(t)) + (2s-1)L(y(t)) = \frac{2}{s^3} - \frac{1}{s^2} + 1 + 2$$

$$= \frac{2 - s + 3s^3}{s^3} \quad \text{— (4)}$$

$$(3) \rightarrow \quad L(x(t))(2s+3) - sL(y(t)) = \frac{s^2+2}{s^2} \qquad \times (2s-1)$$

$$(4) \rightarrow \quad L(x(t))(s-2) + L(y(t))(2s-1) = \frac{2-s+3s^3}{s^3} \qquad \times (s)$$

$$(2s+3)(2s-1)L(x(t)) - s(2s-1)L(y(t)) = \frac{s^2+2}{s^2}(2s-1)$$

$$(s-2)s\,L(x(t)) + (2s-1)s\,L(y(t)) = \frac{2-s+3s^3}{s^2}$$

$$L(x(t))\left(4s^2+6s-2s-3+s^2-2s\right) = \frac{2s^3-s^2+4s-2+2-s+3s^3}{s^2}$$

$$L(x(t))\left(5s^2+2s-3\right) = \frac{5s^3-s^2+3s}{s^2}$$

$$= \frac{5s^2-s+3}{s}$$

$$x(t) = L^{-1}\left(\frac{5s^2-s+3}{s(5s-3)(s+1)}\right) \quad \text{—(5)}$$

$$\left(\frac{5s^2-s+3}{s(s+1)(5s-3)}\right) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{5s-3}$$

$$A = -1, \quad B = 9/8, \quad C = 35/8$$

$$x(t) = L^{-1}\left(-\frac{1}{s}\right) + \frac{9}{8}L^{-1}\left(\frac{1}{s+1}\right) + \frac{35}{8}\times\frac{1}{5}L^{-1}\left(\frac{1}{s-3/5}\right)$$

$$= 1 + \frac{9}{8}e^{-t} + \frac{7}{8}e^{3/5\,t}$$

Substituting in (1)

$$2\left(\frac{-9}{8}e^{-t} + \frac{21}{40}e^{3/5\,t}\right) - y' + 3\left(-1 + \frac{9}{8}e^{-t} + \frac{7}{8}e^{3/5\,t}\right) = 2t$$

$$y' = \frac{9}{8}e^{-t} + \frac{147}{40}e^{3/5\,t} - 3 - 2t$$

$$y = -\frac{9}{8}e^{-t} + \frac{49}{8}e^{3/5\,t} - 3t - t^2 + c, \quad y(0)=1 \rightarrow c = -4$$

$$\therefore y = -9/8\,e^{-t} + \frac{49}{8}e^{3/5\,t} - \cancel{3t} - t^2 - 4$$