Reg. No.:

Name :

VIT

Vellore Institute of Technology

Continuous Assessment Test (CAT)- I– January 2023

Programme	:	B.Tech.	Semester	:	Winter 2022-2023
Course Title	:	Complex Variables and Linear Algebra	Code	:	BMAT201L
			Slot	:	D1+TD1+TDD1
Faculty	:	Dr. P. Vanchinathan, Dr. M. Dhivya, Dr. M. Prasannalakshmi, Dr. S. Devi Yamini, Dr. B. J. Balamurugan, Dr. V. Prabhakar	Class Nbr	:	CH2022235001052, 53, 54, 55, 56, 57
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	Show that the function $v(x,y) = \sin x \cos hy + 2\cos x \sin hy + x^2 - y^2 + 4xy$ is harmonic. Also find an analytic function $f(z)$ in terms of z with the given v as its imaginary part.	10
2.	Prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}(z \neq 0), f(0) = 0$ is not analytic at the origin, although continuity condition and the Cauchy-Riemann equations are satisfied at the origin.	10
3.	Consider the velocity potential of an incompressible and irrotational fluid given by $\phi(r,\theta) = -r^3 \sin 3\theta$. Find the harmonic conjugate of ϕ .	5
4.	Determine the region in the w-plane in which the rectangle bounded by the lines $x = 0, y = 0, x = 1, y = 2$ is mapped under the transformation $w = (1+i)z + 2$.	10
5.	Find the bilinear transformation $w = f(z)$ which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence, find the invariant points and critical points of this transformation.	10
6.	Find the image of the rectangular region $-1 \leq x \leq 1, -\pi < y \leq \pi$ under the transformation $w = e^z$ and sketch the regions.	5