VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

BMAT201L - Complex Variables and Linear Algebra

Module 2 – Conformal and Bilinear transformations

1. Find where the following mappings are conformal and also find the critical points if any.
   (a) $w = z^3$ (b) $w = cos\ z$ (c) $w = sin\ hz$
2. Under the transformation $w = iz + i$ show that the half plane $x > 0$ maps onto the half plane $v > 1$.
3. Find the image of the rectangular region bounded by $x = 0; y = 0; x = 2; y = 1$ under the transformation $w = z + (1 - 2i)$.
4. Find the image of the strip $0 < x < 1$ under the transformation $w = iz$.
5. Find the image of the region $y > 1$ under the transformation $w = iz + 1$.
6. Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.
7. Find the image of the circle $|z - 3i| = 3$ under $w = \frac{1}{z}$.
8. Determine the region of w-plane for the following regions under the mapping $w = z^2$
   a) First quadrant of z-plane
   b) Region bounded by $x = 1, y = 1\ and\ x + y = 1$
   c) $\frac{1}{2} < |z| < 2, Re(z) \geq 0$
9. Find and draw the image of the rectangular region $-1 \leq x \leq 3, -\pi \leq y \leq \pi$ in the z-plane under the transformation $w = e^z$
10. Show that the region between the real axis and a line parallel to real axis at $y = \pi$ transforms into upper half of the w-plane under the transformation $w = e^z$.
11. Determine the bilinear transformations whose fixed points are i, −i.
12. Find the bilinear transformation that maps z1, z2, z3 onto w1, w2, w3 respectively: z = ∞, i, 0 onto w = 0, i,∞
13. Find the bilinear transformation whose fixed points are 1/ 2 and 2 and maps (5+3i)/4 into ∞.
14. Find the bilinear transformation which maps z = 1, i, −1 onto w = i, 0, −i. Find the image of |z| < 1. Determine fixed points.
15. What is the form of a bilinear transformation which has one fixed point ‘a’ and the other fixed point’∞’.

Answer Key:

1. (a) Conformal at all points except $z = 0$. Origin is a critical point.
   (b) Conformal except $z = 0, \pm\pi, \pm 2\pi,$ ....These are the critical points.
   (c) Conformal except at $z = \pm\left(\frac{\pi i}{2}\right), \pm\left(\frac{3\pi i}{2}\right)$,…. These are the critical points.
3. The rectangular region bounded by $u = 1, v = -2, u = 3$ and $v = -1$.

4. The strip $0 < v < 1$.
5. $u + v > 2$.
6. The strip $2 < x < 3$ is mapped onto the region bounded by the circles $u^2 + v^2 = \frac{u}{2}$ and $u^2 + v^2 = \frac{u}{3}$ in the *w*-plane.
7. $6v + 1 = 0$.
8. a) $Im(z) = y > 0$
   b) $v^2 = 4u + 4, v^2 = 4 - 4u, 2v = 1 - u^2$
   c) $\frac{1}{4} < |w| < 4, -\pi \leq \emptyset \leq \pi$

9. $e^{-1} \leq R \leq e^{-3}, -\pi \leq \emptyset \leq \pi$