Complex Variable and Linear Algebra (BMAT201L)

Module-5 (Linear Transformations)

Tutorial sheet-1

Note: We use short form 'LT' for 'Linear transformation'

1. In each part, determine whether T is LT.
   (a) $T(x, y, z) = (0, 0)$ (b) $T(x, y, z, w) = (1, -1)$
   (c) $T(x, y, z) = (x - y + z, 0)$ (d) $T(x, y, z) = (z, yz, x + y + z)$
   (e) $T(x, y, z) = (2y, x + z, -3y)$.

2. Find the standard matrix for the operator $T : R^3 \to R^3$ defined by

$$w_1 = 4x_1 - 3x_2 + x_3$$
$$w_2 = 2x_1 - x_2 + 5x_3$$
$$w_3 = x_1 + 2x_2 - 2x_3$$

   and then calculate $T(-1, 2, 4)$ by directly substituting in the equation and also by matrix multiplication.

4. Let $L : P_1 \to P_2$ be a LT for which, $L(x + 1) = x^2 - 1$ and $L(x - 1) = x^2 + x$ then,

   (i) what is $L(7x + 3)$?

   (ii) what is $L(ax + b)$?

5. Let $L : V \to W$ be a LT, then prove that kernel of L is a subspace of V and range(L)is a subspace of W.

6. Let $L : V \to W$ be a LT, then prove that L is one - one if and only if kernel(L) = 0_V.

7. Let $L : R^3 \to R^3$ be defined by $L(x, y, z) = (x - y, x + 2y, z)$, then:

   (i) Show that L is a LT .

   (ii) Find a basis for kernel(L).

   (iii) Find a basis for range(L).

   (iv) Is L one-one?

   (v) Is L onto?

   (vi) Is L invertible?

   (vii) Find the matrix corresponding to the standard basis of R^3.

8. Let $L : P_2 \to P_2$ be a LT defined by $L(ax^2 + bx + c) = (a + 2b)x + (b + c)$, then:

   (i) Check whether, $x^2 + 2x + 1 \in$ range(L).

   (ii) Check whether, $-4x^2 + 2x - 2 \in$ null(L).

9. Let $L : P_2 \to R^2$ be defined by $L(p(x)) = (p(1), p'(1))$, then:

   (i) Show that L is a LT .

   (ii) Find a basis for null(L) .

   (iii) Find a basis for range(L) .

   (iv) Find the matrix of L corresponding to $S = 1 + x^2, 1, x - 1$.

10. Let $L : P_2 \to P_1$ be a LT defined by $L(p(x)) = xp(x)+p(0)$. Let $S = \{x + 1, x - 1\}$ and $T = \{x^2 + 1, x - 1, x + 1\}$ be the bases for P_1, P_2 respectively. Find the matrix of the LT with respect to S and T. Compute $L(-3x+3)$ using the definition of L *as well as using the matrix of L.*