Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Module 7 Tutorial Sheet 1

BMAT201L-Complex Variables and Linear Algebra

1. Find all the eigen values and eigen vectors of

   (i) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$

   (iii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

2. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, find eigen values and eigen vectors for the following matrices:

   (i) A^T (ii) A^{-1} (iii) A^θ (iv) $4A^{-1}$ (v) A^2 (vi) $A^2 - 2A + I$ (vii) $A^3 + 2I$ (viii) adj A.

3. Find the values of μ which satisfy the equation $A^{100}X = \mu X$ where $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$.

4. Determine algebraic and geometric multiplicity of the following matrices:

   (i) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

5. Find orthogonal eigen vectors for the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

6. Verify Caley-Hamilton theorem for the following matrices and hence find A^{-1}, A^{-2}, A^4

   (i) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$

7. Show that the matrix $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ satisfies Cayley-Hamilton theorem and hence find A^{-1}, if it exists.

8. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Cayley-Hamilton theorem for this matrix. Find A^{-1} and also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

9. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, prove by induction that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$. Hence, find A^{50}.