Second Tutorial Sheet: Module 7

1. Discuss the consistency of the system and if consistent, solve the equations:

(i)
$$\begin{aligned} 2x - y + z &= 9 \\ 3x - y + z &= 6 \\ 4x - y + 2z &= 7 \\ -x + y - z &= 4 \end{aligned}$$

(ii)
$$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 6x_1 - 6x_2 + 6x_3 + 12x_4 &= 36 \\ 4x_1 + 3x_2 + 3x_3 - 3x_4 &= 1 \\ 2x_1 + 2x_2 - x_3 + x_4 &= 10 \end{aligned}$$

2. Investigate for what values of λ and μ the equations
$$\begin{aligned} x + 2y + z &= 8 \\ 2x + 2y + 2z &= 13 \\ 3x + 4y + \lambda z &= \mu \end{aligned}$$
have (i) no solution, (ii) unique solution, and (iii) many solutions.

3. Investigate for what values of k the equations
$$\begin{aligned} x + y + z &= 1 \\ 2x + y + 4z &= k \\ 4x + y + 10z &= k^2 \end{aligned}$$
have infinite number of solutions.

4. Using Gauss elimination/ Gauss-Jordan method solve the following system of equations

(i)
$$\begin{aligned} x + y + z &= 2 \\ x + 2y + 3z &= 5 \\ 2x + 3y + 4z &= 11 \end{aligned}$$

(ii)
$$\begin{aligned} 2x - 2y + 3z &= 2 \\ x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \end{aligned}$$

5. Using Gauss elimination/ Gauss-Jordan method solve the following problems:
   (i) A boy is walking along the path $y = ax^2 + bx + c$ through the points (-6, 8), (-2, -12), and (3, 8). He wants to meet his friend at P(7, 60) . Will he meet his friend?
   (ii) The circle given by the equation $x^2 + y^2 + ax + by + c = 0$ passes through the points (-2, 0), (-1, 7), and (5, -1). Find the constants a, b and c.