BMAT201L: Complex Variables and Linear Algebra

Tutorial Sheet 1 (Module 3)

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1. Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$ and determine the region of convergence.
2. Expand ze^{2z} in a Taylor series about $z = -1$ and determine the region of convergence.
3. Find the Taylor series representation of $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| < 2$.
4. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent series valid for (i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$, (iv) $|z-1| > 1$, and (v) $0 < |z-2| < 1$.
5. Find the Laurent series expansions of $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in (i) $0 < |z-1| < 4$ and (ii) $|z-1| > 4$.
6. Expand $f(z) = \frac{e^{2z}}{(z-1)^3}$ about $z = 1$ in a Laurent series. Also indicate the region of convergence of the series.
7. Determine and classify the singular points of $f(z) = \frac{z}{e^z-1}$.
8. Determine and classify the singularities of $f(z) = \sin\left(\frac{1}{z}\right)$.
9. Determine and classify the singular points of $\frac{1}{(2\sin z-1)^2}$.
10. Find the residue of $f(z) = \frac{1+e^z}{z\cos z+\sin z}$ at $z = 0$.
11. Find the poles of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$ and determine the residues at the poles.
12. Find the residues of $\frac{e^z}{z^2(z^2+9)}$ at its poles.
13. Find the residue of $\frac{1}{z-\sin z}$ at its pole.