BMAT201L-Complex Variables and Linear Algebra

Module-4 Vector Space
Tutorial-1

1. Check whether the following sets form subspace or not.
   (i) $S=\{(x_1,x_2,x_3)\in\mathbb{R}^3 \mid x_1-4x_2+5x_3=2\}$ in the vector space $\mathbb{R}^3$.
   (ii) $S=\{(x_1,x_2)\in\mathbb{R}^2 \mid y=x^2\}$ in the vector space $\mathbb{R}^2$.
   (iii) $S=\{A\in M_{2\times2} \mid \det(A)=0\}$ in the vector space $M_{2\times2}$.
2. If a Vector Space is the set of all real valued continuous function over $\mathbb{R}$, then verify that set W of solutions of differential equation $2\dfrac{d^2y}{dx^2}-9\dfrac{dy}{dx}+2y=0$ is a subspace of V.
3. Let $C^2[-1,1]$ be the Vector space of all functions with continuous second derivative on the domain $[-1,1]$. Which of the following subset is a subspace and which one is not? Justify?
   (i) $W=\left\{f(x)\in C^2[-1,1]: f''(x)+f(x)=0\right\}$
   (ii) $W=\left\{f(x)\in C^2[-1,1]: f''(x)+f(x)=x^2\right\}$
4. Express the first vector as the linear combination of the remaining vectors.
   (i) {(1,-2,5), (1,1,1), (1,2,3), (2,-1,1)}
   (ii) {(2,3,-1), (0,1,3), (2,2,4), (4,2,6)}
5. Verify whether the following set of vectors are linearly independent or dependent.
   (i) $\{1,e^x,e^{2x},e^{3x}\}$ (ii) $\{x,\cos x,\sin x\}$ (iii) $\{x|x|,x^2\}$ (iv) $\{(1,1,1),(1,2,0),(0,-1,2)\}$
   (v){(1,3,-4,2), (2, 2,-4,0), (1,-3,2,-4), (-1,0,1,0)} (vi) $\{x,x+x^2,2x-x^2\}$
6. Determine all the values of k for which the given set of vectors in linearly independent in $\mathbb{R}^4$
   (i) $\{(1,0,1,k),(-1,0,k,1),(2,0,1,3)\}$
   (ii) $\{(1,1.0,-1),(1,k,1,1),(2,1,k,1),(-1,1,1,k)\}$
7. For the given problem, determine a linearly independent set of vectors that spans the same subspace of V as the spanned by the original vectors.
   (i) $V=P_1,\{2-5x,3+7x,4-x\}$
   (ii) $V=\mathbb{R}^3,\{(1,2,3),(-3,4,5),(1,-\frac{4}{3},-\frac{5}{3}\}$
   (iii) $V=\mathbb{R}^3,\{(1,1,1),(1,-1,1),(1,-3,1)(3,1,2)\}$
   (iv) $V=M_{2\times2}(\mathbb{R}),\left\{\begin{bmatrix}1&1\\-1&1\end{bmatrix},\begin{bmatrix}2&-1\\3&1\end{bmatrix},\begin{bmatrix}1&1\\2&1\end{bmatrix},\begin{bmatrix}2&-1\\2&1\end{bmatrix}\right\}$
   (v) $V=M_{2\times2}(\mathbb{R}),\left\{\begin{bmatrix}1&2\\3&4\end{bmatrix},\begin{bmatrix}-1&2\\5&7\end{bmatrix},\begin{bmatrix}3&2\\1&1\end{bmatrix}\right\}$