Complex Variable and Linear Algebra (BMAT201L)

Tutorial Sheet- 1

1. If $u = x^2 - y^2$ and $v = \dfrac{-y}{x^2+y^2}$ prove that u and v are harmonic but $u + i\, v$ is not an analytic function.
2. Verify that the family of curves u= constant and v=constant cut orthogonally, when $w = u + i\, v = z^3$.
3. If both $f(z)$ and its conjugate are analytic, show that $f(z)$ is a constant function.
4. Find the values of a, b, c, d such that the following functions are analytic

(i) $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$

(ii) $f(z) = cosx(coshy + asinhy) + i\, sinx(coshy + bsinhy)$

5. Show that the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies the Cauchy Riemann equations at origin but $f'(0)$ does not exist?
6. Let $f(z) = u + iv$ is an analytic function. If $u(x,y) = \ln(x^2+y^2)$, then find $f'(1+8i)$.
7. If $f(z)$ is an analytic function in the domain D and $|f(z)|$ is a non-zero constant, then prove that $f(z)$ is a constant function.
8. Find $f'(z)$ when $f(z) = (r^2\cos(2\,\theta) + r\cos(\theta)) + i\,(r^2\sin(2\,\theta)\, r\sin(\theta))$.
9. If $u(x,y) = Re(z^2) = x^2 - y^2$ and $v(x,y) = Im(z^3) = 3\,x^2y - y^3$ are harmonic functions in a domain D. Show that the function $(u_y - v_x) + i\,(u_x + v_y)$ is analytic in D.
10. Discuss the function $f(z) = z\,\bar{z}$ is analytic or not at origin.
11. Examine the nature of the function $f(z) = \begin{cases} \dfrac{x^3y^5 + i\,x^2y^6}{x^4 + y^{10}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$
12. Show that the function $u = e^{x^2-y^2}\cos 2xy$ is harmonic and then determine their harmonic conjugates.
13. Discuss the analyticity of the function $f(z) = \dfrac{x - iy}{x^2+y^2}$.
14. Show that the function f(z) defined by $f(z) = \dfrac{xy(y-ix)}{x^2+y^2}$ $for\ z \neq 0\ and\ f(0) = 0$ is not analytic at the origin even though it satisfies C-R equation at the origin.