BMAT201L-Complex Variables and Linear Algebra

Module-4 Vector Space
Tutorial-2

1. Find the basis for the subspace $W = \{(a_1, a_2, a_3, a_4, a_5) \in R^5\,; a_1 + a_3 + a_5 = 0, a_2 = a_4\}$ and find it's and dimension.
2. Find a basis of $M_{2\times 2}(R)$ that contains a vector $\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$.
3. Find a basis of $P_3(R)$ that contains the vector$1 + 2x - x^2$.
4. Does $W = \{p(x) \in P_3(R) |\, p(1) = p(2) = 0\}$ a subspace of $P_3(R)$. If so, find its basis.
5. Let $V_1 = \{(x, y, z, w) \in \mathbb{R}^4 | x + 3y + z + w = 0\}$ and $V_2 = \{(x, y, z, w) \in \mathbb{R}^4 | y = -3z - 6w\}$ be subspaces of $\mathbb{R}^4$. Find a basis for $V_1 + V_2$ and $V_1 \cap V_2$.
6. Check whether the following set B is a basis for the corresponding vector space V
   (i) $B = \{(0,0,0), (1, 2, 3), (-1, 0, 1)\}$ for R^3.
   (ii) $B = \left\{ \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \right\}$ for $M_{2\times 2}$.
   (iii) $B = \{1, 1 + x^3, 1 + x^2\}$ for $P_3(x)$.
7. Find bases for row space, column space and null space of A. Also, verify the rank-nullity theorem

   (i) $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$

   (iii) $A = \begin{bmatrix} 1 & 2 & 4 & -5 & 3 \\ 3 & -1 & 5 & 2 & 4 \\ 5 & -4 & 6 & 9 & 2 \\ -6 & 2 & -10 & -4 & -2 \end{bmatrix}$