DIVISION OF MATHEMATICS, VIT CHENNAI

BMAT201L Complex Variables and Linear Algebra- Fall Semester 2022-23

Module III Tutorial Sheet-2

1. Evaluate the integral $(i)\oint\limits_C \mid z \mid^2 \, \mathrm{d}z$ and $(ii)\oint\limits_C \frac{1}{z^2}\,\mathrm{d}z$, where the contour C is (a) the line segment with initial point -1 and final point i; (b) the arc of the unit circle $Imz \geq 0$ with initial point -1 and final point i. Do the two results agree?

2. Evaluate $\oint\limits_C \left(z^3 + z^2 + Re(z)\right)\,\mathrm{d}z$, where C is the triangle with vertices $z = 0$, $z = 1 + 2i$ and $z = 1$.

3. If C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining points $(1, 1)$ and $(2, 3)$, the value of $\oint\limits_C \left(12z^2 - 4iz\right)\,\mathrm{d}z$.

4. Estimate an upper bound of the modulus of the integral $\oint\limits_C \frac{Logz}{z - 4i}\mathrm{d}z$, where C is the circle $\mid z \mid = 3$.

5. Evaluate the integral $\oint\limits_\Gamma \frac{1}{(z - z_0)^n}\,\mathrm{d}z$, n is an integer, where Γ is a circle centered at z_0 and of any radius. The path is traced out once in the anticlockwise direction.

6. Evalute $\oint\limits_C \frac{1}{z^2}\,\mathrm{d}z$, where the contour C is the ellipse $(x - 2)^2 + \frac{1}{4}(y - 5)^2 = 1$.

7. Evaluate $\oint\limits_C \frac{sin\pi z^2 + cos\pi z^2}{(z - 1)(z - 2)}\,\mathrm{d}z$, where C is the circle:$\mid z - i \mid = 3$.

8. Evaluate $\oint\limits_C \frac{e^{2z}}{(z + 1)^4}\,\mathrm{d}z$, where C is the circle:$\mid z \mid = 3$.

9. Suppose $f(z)$ is defined by the integral $f(z) = \oint\limits_{|\zeta|=3} \frac{3\zeta^2 + 7\zeta + 1}{(\zeta - z)}\,\mathrm{d}\zeta$, find $f'(1 + i)$.

10. Evaluate $\int\limits_0^\pi \frac{\mathrm{d}\theta}{1 + \sin^2(\theta)}$ using residue theory.

11. Evaluate $\int\limits_0^\infty \frac{x\sin(x)}{x^2 + 9}\,\mathrm{d}x$ using residue theory.

12. Evaluate $\int\limits_{-\infty}^\infty \frac{\sin(x)}{x(x^2 - 2x + 2)}\,\mathrm{d}x$ using an indented contour.