Module 6

Inner Product Space

Tutorial sheet

1. For $\overline{x} = (x_1, x_2, x_3)$, $\overline{y} = (y_1, y_2, y_3)$ in R^3 define $< \overline{x}, \overline{y} > = x_1y_1 + 3x_2y_2 + 5x_3y_3$.Is $<,>$ an inner product on R^3.

5. Find an orthonormal basis to W, which has basis $\{v_1 = (1,1,1,1), v_2 = (1,2,0,1), v_3 = (2,2,4,0)\}$.
6. Consider R^3 with basis $\{e_1, e_2, e_3\}$ and

   $<(x_1, x_2, x_3), (y_1, y_2, y_3)> = x_1y_1 + 3x_2y_2 + x_3y_3$. Find the matrix representation of the above inner product.

8. For $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ in R^3 defined as $<x, y> = x_1y_1 + 3x_2y_2 - x_3y_3$. Is $<,>$ an inner product in R^3.

7. Let $T : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by $T(x, y, z) = (x + y, y + z, x + 2y + z, x - z)$. Find an orthonormal basis for the kernel and range of T.

8. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \langle (x, y, z), (1, 2, 3) \rangle = 0\}$. Find an orthonormal basis for W.