VIT

Vellore Institute of Technology

Final Assessment Test (FAT) - November/December 2023

Programme	**B.Tech.**	Semester	**FALL SEMESTER 2023 - 24**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Mohana N**	Slot	**A2+TA2+TAA2**
		Class Nbr	**CH2023240101011**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)

Answer <u>any 10</u> questions

01. Find the analytic function $f(z) = u + iv$, where $2u + v = e^x[\cos y - \sin y]$. **[10]**

02. (a) Find the constants a, b if $f(z) = (x^2 - y^2 - axy) + i(x^2 - y^2 + bxy)$ is analytic. **(5 Marks)** **[10]**
(b) Find the image of the circle with radius 2 and the centre at $(3, -5)$ under the transformation $f(z) = 2iz + 3 - i$. **(5 Marks)**

03. Find the bilinear transformation that interchanges the points 0 and i and sends ∞ to $i/2$. Determine its invariant points. Also find the image of the horizontal line $y = \frac{1}{2}$. **[10]**

04. Evaluate $\int_0^{2\pi} \frac{1}{5+4cos\theta} d\theta$, using contour integration. **[10]**

05. (a) Classify the singularities of the function $f(z) = \frac{cot\pi z}{(z-a)^3}$. Also find the residue at $z = a$. **[10]**
(5 Marks)
(b) Evaluate the integral $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$, where C is $|z| = 3$ using Cauchy's integral formula.
(5 Marks)

06. (a) Find the basis and dimension of the solution space W of this homogeneous system: **[10]**
(5 Marks)

$$x + 2y + 2z - s + 3t = 0$$

$$x + 2y + 3z + s + t = 0$$

$$3x + 6y + 8z + s + 5t = 0$$

(b) Does $W = \{(a, b, c, d) \mid a + b - c + d = 0)\}$ is subspace of $\mathbb{R}^4$. If yes, find the basis.
(5 Marks)

07. Verify Rank Nullity theorem for $A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$. **[10]**

08. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by **[10]**

$$T(x, y) = (2x + 3y, 4x - 5y).$$

Find the matrix representation of T
(a) with respect to the standard basis $E = \{(1, 0), (0, 1)\}$. **(5 Marks)**
(b) with respect to the basis $S = \{(1, 2), (2, 5)\}$. **(5 Marks)**

09. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$ and let $\alpha = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $\beta = \{(1,3), (2,5)\}$ be the bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Then find the associated matrix T with respect to α and β. **[10]**

10. Consider $\mathbb{R}^4$ with the usual dot product. Find an orthonormal basis for the subspace spanned by $\{(1,1,1,0), (1,1,0,0), (1,0,0,0), (2,1,3,0)\}$. **[10]**

11. (a) Use the standard inner product on P_2 to find the angle between vectors $f(x) = 1 - x$ and $g(x) = x^2$. **(5 Marks)** **[10]**

(b) Two eigenvalues of a matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double of third eigenvalue, hence find the eigenvalues of A^{-1} and A^4. **(5 Marks)**

12. Solve the following system by Gaussian Elimination method **[10]**

$$x + y + z - w = -2$$
$$2x - y + z + w = 0$$
$$3x + 2y - z - w = 1$$
$$x + y + 3z - 3w = -8$$