BMAT201L - Complex Variables and Linear Algebra
Module - 1
Tutorial Sheet 2

1. Let a, b, c are real constants. Determine the relation among the coefficients that will guarantee that the function $\phi(x) = ax^2 + bxy + cy^2$ is harmonic.
2. Does an analytic function $f(z) = u(x, y) + iv(x, y)$ exists for which $v(x, y) = x^3 + y^3$? Why?
3. Let $u_1(x, y) = x^2 - y^2$ and $u_2(x, y) = x^3 - 3xy^2$. Show that u_1 and u_2 are harmonic functions and their product $u_1(x, y)u_2(x, y)$ is not harmonic function.
4. Use polar form of Laplace equation to show that $u(r, \theta) = (r + 1/r)\cos(\theta)$ and $v(r, \theta) = (r - 1/r)\sin(\theta)$ are harmonic functions. (**Hint :** The polar form of Laplace equation is given by $r^2 r_{rr} + ru_r + u_{\theta,\theta}$=0).
5. The function $F(z) = 1/z$ is used to determine a field known as dipole. Express $F(z)$ in the form $F(z) = \phi(x, y) + i\psi(x, y)$ and sketch the equipotentials $\phi = 1, 1/2, 1/4$ and the streamlines $\psi = 1, 1/2, 1/4$.
6. Show that $\phi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$ can represent the velocity potential in an incompressible fluid flow. Also find the corresponding stream function and complex potential.
7. Show that the equation $x^3y - xy^3 + xy + x + y = c$ can represent the path of electric current flow in an electric field. Also find the complex electric potential and the equation of the potential lines.
8. Find the analytic function $P + iQ$, if $P - Q = \dfrac{\sin(2x)}{\cosh(2y) - cos(2x)}$.
9. Find the analytic function $f(z) = u + iv$ if $v = \dfrac{\sin(x)\sinh(y)}{\cos(2x) + \cosh(2y)}$ if $f(0) = 1$.
10. Verify that the family of curves $u = c_1$ and $v = c_2$ cut orthogonally, when $w = 1/z$.