VIT
Vellore Institute of Technology

Reg. No. :

## Final Assessment Test (FAT) - APRIL/MAY 2023

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|---|---|---|---|
| Programme | **B.Tech** | Semester | **Winter Semester 2022-23** |
| Course Title | **COMPLEX VARIABLES AND LINEAR ALGEBRA** | Course Code | **BMAT201L** |
| Faculty Name | **Prof. Dr.Manigandla Prasannalakshmi** | Slot | **D1+TD1+TDD1** |
| | | Class Nbr | **CH2022235001054** |
| Time | **3 Hours** | Max. Marks | **100** |

### Part A (10 X 10 Marks)
### Answer <u>any 10</u> questions

01. Show that the real part of a complex valued function $u(x, y) = e^x(x \cos y - y \sin y)$ is harmonic and also find its harmonic conjugate. **[10]**

02. If $f(z) = U + iV$ is an analytic function of $z$ and $U - V = (x - y)(x^2 + 4xy + y^2)$ then find $f(z)$ in terms of $z$. **[10]**

03. Find the bilinear transformation $w = f(z)$ which maps the points $z = 0,\ 1,\ \infty$ into the points $w = -5,\ -1,\ 3$ respectively. Hence find the invariant points and critical points of this transformation. **[10]**

04. (i) Find the image of the strip $1 < x < 2$ under the transformation $w = \frac{1}{z}$ and sketch the regions. **(5 Marks)** **[10]**
(ii) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $T(x, y) = (x + 2y, x - y)$. Show that $\ker(T) = \ker(T^2)$. **(5 Marks)**

05. Evaluate $\int_P \bar{z}^2\, dz$ where $P$ represents the path comprises of two segments $L_1$ and $L_2$. Let $L_1$ be a line segment along the real axis from $z = 0$ to $z = 2$ and $L_2$ be a line parallel to $y - axis$ from $z = 2$ to $z = 2 + i$. **[10]**

06. Evaluate $\oint_C \frac{7z-1}{z^2-3z-4}\, dz$, where $C$ is the ellipse $x^2 + 4y^2 = 4$ using (i) Cauchy Integral formula (ii) Cauchy Residue theorem **[10]**

07. Let $u = (1, 4, 3, 1), v = (3, 8, 5, 2)$ and $w = (2, 0, 1, 0)$ **[10]**
(i) Show that $b = (3, 16, 10, 4)$ is a linear combination of $u, v, w$. **(5 Marks)**
(ii) Show that the vector $b' = (3, 2, 1, 1)$ is not a linear combination of $u, v, w$.**(5 Marks)**

08. (i) Construct a $5 \times 5$ matrix of rank 2 without repeating a row or column and with all non-zero real entries. Also describe a general procedure for constructing such matrices. **(5 Marks)** **[10]**
(ii) Show that there are infinitely many non-zero vectors with 4 real entries satisfying simultaneously the following two properties:
**Property 1:** The sum of real entries in even positions is equal to the sum of real entries in the odd positions.
**Property 2:** The sum of first two real entries is double the sum of the remaining real entries: **(5 Marks)**

09. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (x + y, y - z, x + y - z)$. Show that $T$ is an invertible linear transformation and also find the inverse linear transformation. **[10]**

10. Apply Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors: $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 1, 2, 4)$ and $v_3 = (1, 2, -4, -3)$. **[10]**

11. Solve the following system of equations by Gauss- Elimination method: **[10]**

$$
\begin{aligned}
3p - q + 2r - 2s &= 0 \\
2p + 2q - r + s &= 1 \\
2p - q - 2r - s &= 2 \\
p + 3q - 2r + 4s &= 2
\end{aligned}
$$

12. Find the eigen values and eigen vectors of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$. [10]

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