VIT

Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	**B.Tech**	Semester	**Winter Semester 2022-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Kalyan Banerjee**	Slot	A1+TA1+TAA1
		Class Nbr	**CH2022235001032**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)
Answer any 10 questions

01. Find the analytic function $f(z) = u + iv$, if $u + v = \frac{x}{(x^2+y^2)}$ and $f(1) = 0$. [10]
[8+2=10]

02. (a) Find the image of the region of the half-plane $x > c$ when $c > 0$ under the transformation $w = 1/z$. [10]
(b) Is the given mapping $f(z) = z^2 - 2z - 3$ conformal everywhere? Discuss in detail.
[5+5=10]

03. Find a bilinear transformation $f(z)$ which maps $f(\infty) = 1$, $f(i) = i$ and $f(-i) = -i$. Find the image of unit disc $\{z \in C : |z| < 1\}$ under $f(z)$. [10]
[5+5=10]

04. Find Taylor's series and Laurant's series expansion which represents the function [10]
$f(z) = \frac{z^2-1}{z^2+5z+6}$ in the following regions.
(a) $2 < |z| < 3$,
(b) $0 < |z| < 2$,
(c) $0 < |z+3| < 1$,
(d) $1 < |z+1| < 2$.
[2.5+2.5+2.5+2.5=10]

05. Evaluate $\int_0^{2\pi} \frac{2\cos^2\theta}{5+3\sin\theta}\, d\theta$. [10]

06. Find a homogeneous system whose solution set W is spanned by [10]
$\{u_1 = (1, -2, 0, 3), u_2 = (1, -1, -1, 4), u_3 = (1, 0, -2, 5)\}$.

07. Find the basis of row space, column space and null space of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$. [10]
[4+4+2=10]

08. In the vector space M consisting of all 2×2 matrices [10]
define T: $M \to M$ by (here $*$ denotes matrix multiplication

$$T(A) = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} * A * \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$$

For this linear transformation find the matrix with respect to the standard basis of M.
If A is a diagonal matrix, is $T(A)$ also diagonal? Justify your answer.

09. Let α be the standard basis for R^2 and **[10]**

$\beta = \left\{ \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \end{pmatrix} \right\}$ a non-standard basis.

If $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+y \\ y-x \end{pmatrix}$ find the matrices

$[T]_\alpha$ and $[T]_\beta$.

[5+5=10]

10. (a) Verify whether the families of curves $u = c_1$ and $v = c_2$ cut orthogonally, when $w = e^{[illegible]}$. **[10]**

(b) Define an inner product on R^2 by $\langle u, v \rangle = u^t A v$, where A is the matrix $A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$.

Find the matrix of this inner product for the basis $\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \end{pmatrix} \right\}$.

[5+5=10]

11. Find an orthogonal basis by Gram-Schmidt procedure obtained from **[10]**

the basis $u_1 = (1, 2, 1), u_2 = (-1, 3, 7), u_3 = (14, -4, 18)$.

12. Find the eigen values and eigen vectors of the matrix **[10]**

$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Also, compute $A^{2023} + A^{2024} + A^{2025}$.