VIT

Reg. No.

Final Assessment Test (FAT) November/December 2022

Programme	B.Tech	Semester	Fall Semester 2022-23
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
		Slot	C1+TC1+TCC1
Faculty Name	Prof. Mini Ghosh	Class Nbr	CH2022231001814
Time	3 Hours	Max. Marks	100

Part-A (10 X 10 Marks)

Answer <u>any 10</u> questions

1. If $u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$ and $f(z)=u+iv$ is an analytic function of z. Find the function $f(z)$ in terms of z. [10]

2. (a) Show that the transformation $w=\frac{1}{z}$ transforms, in general, circles and straight lines into circles and straight lines. [10]
(b) For the transformation $w=z^2$, which lies in the area of the first quadrant of the z-plane bounded by the axes and circles $|z|=a$ and $|z|=b$, where $(a>b>0)$, discuss the transformation in the w-plane and check whether it is a conformal mapping.

3. Find the bilinear transformation which maps $z=1,\ i,\ -1$ onto $w=i,\ 0\ -i$. Also determine the fixed points. [10]

4. Classify the singularity of the function $f(z)=\frac{\cos z}{z(z-\pi)^4}-\frac{\pi\cos z}{z^2(z-\pi)^4}$ and find the residue if it exists. [10]

5. (a) Using the Cauchy's integral formula, evaluate the integral $\oint_C \frac{e^z}{z^2+4}dz$, where C is the circle $|z-i|=2$. [10]
(b) Evaluate $\oint_C \frac{\sin z}{(z^2-25)(z^2+9)}dz$, where C is the unit circle.

6. Show that $W=\{A\in M_{3\times3}(\mathbf{R}):\ tr(A)=0\}$, where $tr(A)$ denotes trace of A i.e. the sum of the diagonal entries of A, is a subspace of $M_{3\times3}(R)$, the set of all 3×3 matrices with real entries. Also find the basis and the dimension of W. [10]

7. Express the vector u in each of the following cases as the linear combination of the given set of vectors : (i) $u=\left(-\frac{1}{4},\ \frac{9}{4},\ \frac{31}{4},\ \frac{13}{8}\right)$, $S=\left\{(1,\ 2,\ -1,\ 1),\ \left(0,\ \frac{1}{2},\ 3,\ \frac{1}{4}\right),\ (1,\ 1,\ 2,\ 0)\right\}$. [10
(ii) $u=3t^2+6t-2$, $S=\{t-1,\ -t^2+t+1,\ t^2\}$.

8. Let A be the matrix of linear transformation $T:\ R^3\to R^4$ defines by [1
$T(x,\ y,\ z)=(x,\ y+z,\ x-z,0)$ with respect to domain basis
$\{(1,\ 1,\ 1),\ (1,\ 1,\ 0),\ (1,\ 0,\ 0)\}$ and codomain basis
$\left\{\left(\frac{1}{2},\ 0,\ 0,\ 0\right),\ \left(0,\ \frac{1}{2},\ 0,\ 0\right),\ \left(0,\ 0,\ \frac{1}{2},\ 0\right),\ \left(0,\ 0,\ 0,\ \frac{1}{2}\right)\right\}$. What is
Ax for $x=(1,\ 0,1)^T\in\mathbf{R}^3$.

9. Let $(\alpha_1,\ \alpha_2,\ \alpha_3)$, $(\beta_1,\ \beta_2)$ be ordered basis of the real vector space V and $\mathbf{W}$ respectively. A linear mapping $T\ :V\ \to W$ maps the basis vectors as [
$T(\alpha_1)=\beta_1+\beta_2$, $T(\alpha_2)=3\beta_1-\beta_2$, $T(\alpha_3)=\beta_1+3\beta_2$. Find the matrix T relative to the ordered basis
(i) $(\alpha_1,\ \alpha_2,\ \alpha_3)$ of V and $(\beta_2,\ \beta_1)$ of W.
(ii) $(\alpha_1+\alpha_2,\ \alpha_2,\ \alpha_3)$ of V and $(\beta_1,\ \beta_1+\beta_2)$ of W.

10. Apply the Gram Schmidt process to orthonormalize the basis vectors u_1, u_2 and u_3 of the space $\mathbf{R}^3$ with respect to the Euclidean inner product, where $u_1 = (1, 1, 1)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 0, 0)$. [10]

11. Solve the following system by Gaussian Elimination method

$$x + y + z - w = -2,$$
$$2x - y + z + w = 0,$$
$$3x + 2y - z - w = 1,$$
$$x + y + 3z - 3w = -8.$$

[10]

12. Find all the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.