Tutorial Sheet-2

BMAT201L-Complex Variables and Linear Algebra

Module-5 Linear Transfromation

1. Let $B = \{v_1, v_2, v_3, v_4\}$ be a basis for a vector space V. Find the matrix with respect to B of the linear operator T on V defined by $T(v_1) = v_2, T(v_2) = v_4, T(v_3) = v_1, T(v_4) = v_3$.

2. Let $T: P_1 \to P_2$ be a linear transformation defined by $T\big(p(x)\big) = (x+1)p(x)$. Find the matrix for T with respect to basis $\{1, x\}$.

3. Let $A = \begin{bmatrix} 3 & -2 & 1 & 0 \\ 1 & 6 & 2 & 1 \\ -3 & 0 & 7 & 1 \end{bmatrix}$ be a matrix for $T: R^4 \to R^3$ relative to the bases $B = \{v_1, v_2, v_3, v_4\}$ and $B' = \{w_1, w_2, w_3\}$, where

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 4 \\ -1 \\ 2 \end{bmatrix}, v_4 = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 2 \end{bmatrix}, w_1 = \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}, w_2 = \begin{bmatrix} -7 \\ 8 \\ 1 \end{bmatrix}, w_3 = \begin{bmatrix} -6 \\ 9 \\ 1 \end{bmatrix}$$

   (a) Find $[T(v_1)]_{B'}, [T(v_2)]_{B'}, [T(v_3)]_{B'}$ and $[T(v_4)]_{B'}$.

   (b) Find $T(v_1), T(v_2), T(v_3)$ and $T(v_4)$.

   (c) Find the formula for $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right)$.

   (d) Use the formula obtained in (c) to compute $T\left(\begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}\right)$.

4. Let $B = \{u_1, u_2, u_3\}$ be a basis for a vector space V, and let $T: V \to V$ be a linear operator for which

$$[T]_B = \begin{bmatrix} -3 & 4 & 7 \\ 1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}.$$

   Find $[T]_{B'}$, where $B' = \{v_1, v_2, v_3\}$ is a basis for V defined by

$$v_1 = u_1, \qquad v_2 = u_1 + u_2, \qquad v_3 = u_1 + u_2 + u_3.$$

5. Let $B = \{u_1, u_2, u_3\}$ and $B' = \{v_1, v_2, v_3\}$ are two bases of R^3, where

$$u_1 = (1,0,0),\ u_2 = (1,2,4), u_3 = (2,5,7), v_1 = (2,4,5), v_2 = (0,5,1), v_3 = (4,2,1).$$

   (a) Find the change of basis matrix from B to B'.

   (b) Find the change of basis matrix from B' to B.

5. Let $B = \{u_1, u_2, u_3\}$ and $B' = \{v_1, v_2, v_3\}$ are two bases of R^3, where

$$u_1 = (1,0,0),\ u_2 = (1,2,4), u_3 = (2,5,7), v_1 = (2,4,5), v_2 = (0,5,1), v_3 = (4,2,1).$$

   (a) Find the change of basis matrix from B to B'.
   (b) Find the change of basis matrix from B' to B.

6. Let $T: R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - x_2 \\ x_1 \end{bmatrix}$ and $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2\}$, where

$$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$$

   Find the matrix for T relative to the basis B and use it to compute the matrix relative to B'.