VIT

Reg. No. :

Final Assessment Test (FAT) - APRIL MAY 2023

Programme	B.Tech		
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Semester	Winter Semester 2022-23
		Course Code	BMAT201L
Faculty Name	Prof. Radha S	Slot	A2+TA2+TAA2
Time	3 Hours	Class Nbr	CH2022235001050
		Max. Marks	100

Part-A (10 X 10 Marks)

Answer any 10 questions

01. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate v. Also find the corresponding analytic function $f(z) = u + iv$ in terms of z. [10]

02. a) Find the bilinear transformation which map the points $-i, 0, 2+i$ in z plane to the points $0, -2i, 4$ in w plane respectively. (5 marks) [10]
b) Find the image of $y = x$ under the transformation $w = \frac{z-1}{z+1}$. (5 marks)

03. a) Find the image of the line $y - x + 1 = 0$ under the transformation $w = \frac{1}{z}$. (5 marks) [10]
b) Find the image of the triangular region bounded by $x = 0, y = 0$ and $x + y = 1$ under the transformation $w = e^{i\pi/4}z$. (5 marks)

04. Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+9)(x^2+4)}dx$. [10]

05. a) Evaluate the integral by Cauchy's Integral formula
$\int_C \frac{z+3}{z^2+2z+5}$ where C is the circle $|z+1+i| = 2$ (7 marks) [10]
b) Obtain the singular points of the function $f(z) = \sin\left(\frac{1}{z-a}\right)$ and classify it. (3 marks)

06. Let $P_2(t)$ be the set of polynomials of degree 2 or less. Let the following vectors be in $P_2(t)$:
$v_1 = t^2 - t - 1$, $v_2 = 6t^2 + 3t - 3$ and $v_3 = t^2 - 5t + 1$. [10]
Determine whether the vector $u = 3t^2 + 2t - 1$ is spanned by $\{v_1, v_2, v_3\}$

07. a) Consider the vector space $V = M_{2\times 2}$, the set of 2 by 2 matrices. Let S be a subset of $M_{2\times 2}$, containing matrices of the form $\begin{pmatrix} 1 & b \\ c & d \end{pmatrix}$. Show that S is not a subspace of V. (5 marks) [10]
b) Let V be the vector space of continuous functions defined on the real line and $u = \cos(x)$, $v = \sin(x)$ and $w = 2$ be vectors in V. Check whether the vectors u, v and w are linearly independent or not. (5 marks)

08. Consider the matrix $A = \begin{bmatrix} -6 & 4 & -4 \\ 1 & -1 & 1 \\ 7 & -4 & 5 \end{bmatrix}$ [10]
and also consider the plane W defined by the equation $x + z = 0$.
We are given that the two vectors $u = (1, 1, -1)^t$, $v = (1, 0, -1)^t$ form a basis for W. Check that the vectors Au and Av also lie in the same plane W. For any $w \in W$ define a linear transformation T by $T(w) = Aw$. Find the 2×2 matrix for T for the above basis.

09. Check whether the following matrices are similar or not. [10]

$$B = \begin{bmatrix} -3 & 4 & -1 \\ 0 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix}, A = \begin{bmatrix} -6 & 4 & -4 \\ 1 & -1 & 1 \\ 7 & -4 & 5 \end{bmatrix}.$$

10. a) Find the values of a and b such that the function $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic. Also, find $f'(z)$. (5 marks) **[10]**

b) For the triangle in 3d-space formed by joining the three vertices, $P = (1, 1, 1), Q = (3, 2, -1), R = (1, 0, 1)$, determine at which vertex the angle is the maximum. (5 marks)

11. Prove that $W = \{(x, y, z, w) : 5x - y + 4z + 2w = 0\}$ is a subspace of $\mathbb{R}^4$ and find an orthogonal basis for the subspace W. **[10]**

12.

For the matrix $B = \begin{bmatrix} -1 & -17 & 7 \\ 0 & 0 & 1 \\ -1 & -11 & 7 \end{bmatrix}$, it is known that the vector $(1, -1, -2)'$ is an eigen vector and then find the remaining eigenvalues and eigenvectors of B by either computing the characteristic polynomial or otherwise. **[10]**

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