VIT
Vellore Institute of Technology

Continuous Assessment Test II - March 2023

Programme	B.Tech.	Semester	Winter 2022-2023
Course	Probability and Statistics	Code	BMAT202L
Faculty	Dr. Kaliyappan M, Dr. Nathiya N, Dr. Vidhya V, Dr. Devi Yamini S, Dr. Durgaprasad P, Dr. Mythili G Y, Dr. Pulak Konar, Dr. Surath Ghosh, Dr. Avinash Kumar Mittal Dr. Rajivgandhi C, Dr. Ashis Bera, Dr. Sethukumarasamy K	Class ID	CH2022235002640, 2643, 2648, 2650, 2653, 2655, 2657, 2663, 2665, 2668, 2670, 2710
		Slot	E2
		Max.Marks	50
Duration	90 Mins		

Answer all questions
(Non-programmable calculators are permitted)

(a) Find the moment-generating function of the random variable X with probability mass function

$$p(x) = \begin{cases} \dfrac{x^2}{14}, & x = 1,2,3; \\ 0, & \text{otherwise.} \end{cases}$$

[5]

Hence obtain the variance of X.

(b) The average number of articles produced by two machines per day is 200 and 250 with standard deviations 20 and 25 respectively on the basis of records of 21 days production. Can you regard both the machines equally efficient at 1% level of significance? [5]

2. A study was conducted on a specific type of bearing to determine the relationship between the amount of wear y and the oil viscosity x_1 and the load x_2. The following information was collected:

x_1:	17	23	33	15	43	40
x_2:	851	1058	1357	816	1201	1115
y:	193	172	113	230	91	125

Estimate the unknown parameters of the multiple linear regression equation $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ and hence predict the wear when the oil viscosity is 20 and the load is 1200. [10]

3. (a) A random variable X has the Weibull distribution with parameters $\alpha = \frac{1}{5}$ and $\beta = \frac{1}{3}$. Find (i) the mean of X, (ii) $P(X \le 3)$ and (iii) $P(-5 < X < 5)$. [5]

(b) Let X be the random variable denoting the number of persons suffer a bad reaction from a certain injection. If the probability that an individual suffers a bad reaction is 0.001, determine the probability that out of 2000 individuals

    (i) exactly 3 individuals suffer,

    (ii) more than 2 individuals suffer.

Also, find the smallest positive integer k, such that $P(X \le k) > 0.75$. [5]

4. (a) In an examination, it is laid down that a student passes if he secures 30 percent or more marks. He is placed in the first, second, or third division according as he secures 60% or more marks, between 45% and 60% marks, and marks between 30% and 45% respectively. Calculate the percentage of students placed in the third division. Assume that the distribution of marks is random variable X such that $X - N(52, 293.10)$. [5]

(b) The fairness of a coin is suspected. So it was tossed for 8000 times and the tails appeared for 4200 times. Is the coin biased in favor of tails? Use $\alpha = 0.05$ to carry out this test. Does the P-value support your conclusion? [5]

5. (a) To test the claim that engine B's average miles per litre (mpl) is 2 miles higher than engine A's average mpl, 50 experiments were carried out under similar conditions with engine type A and 75 with engine type B. The average mileage for engine A was 36 mpl, and the average mileage for engine B was 42 mpl, with sample standard deviations of 6 and 8 miles, respectively. At the 0.05 level of significance, put the claim to the test. Is your conclusion supported by the 95% confidence interval for $\mu_B - \mu_A$? [5]

(b) Suppose that is known from experience that the standard deviation of the weight of 200 gram packages of cookies made by certain bakery is 8 gram. To check whether its production is under control on a given day, a random sample of 32 packages were selected and their mean weight is 203 grams.

    (i) At 0.05 level of significance is the production under control? Suppose the probability committing Type I error is 0.01, then is the adjustment under control?

    (ii) Let α_1 and α_2 be two levels of significance such that $\alpha_1 < \alpha_2$. Can we state that H_0 will unquestionably be rejected under α_1, if H_0 is rejected under α_2? Explain.

[5]

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