## Module 4: Probability Distributions

Discrete Distributions: Binomial distribution - Poisson distributions; Continuous Distributions: Normal distribution - Gamma distribution - Exponential distribution - Weibull distribution.

## Binomial Distribution

$$P(X = x) = nc_x p^x q^{n-x}, x = 0, 1, 2 \cdots n$$

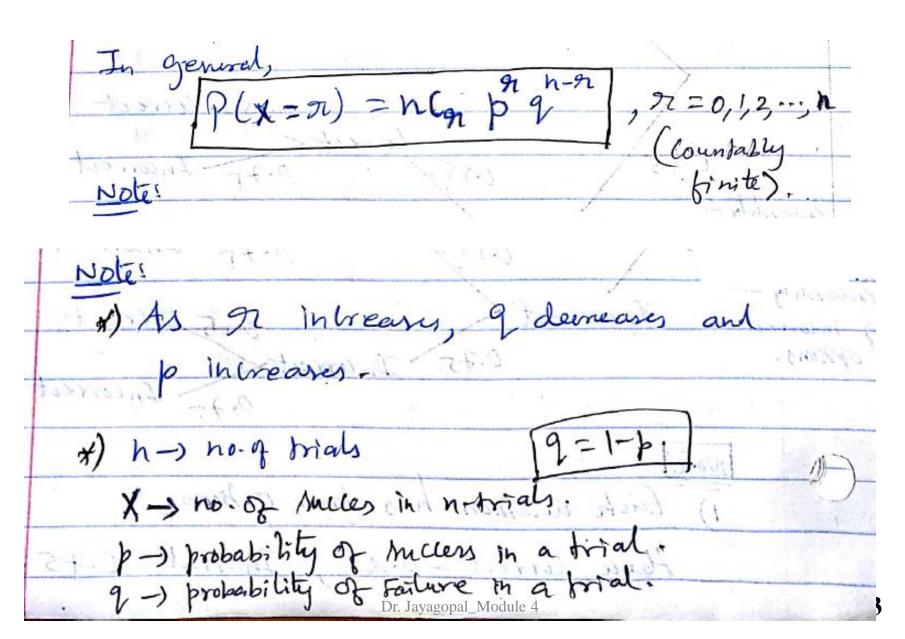
Mean E(X) = np

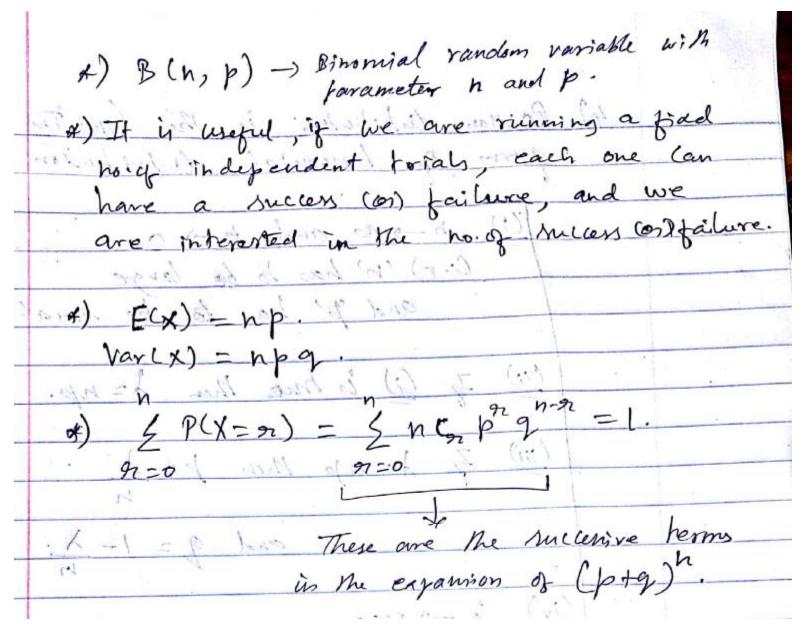
Variance V(X) = npq

Moment generating function

$$M_X(t) = (q + pe^t)^n$$

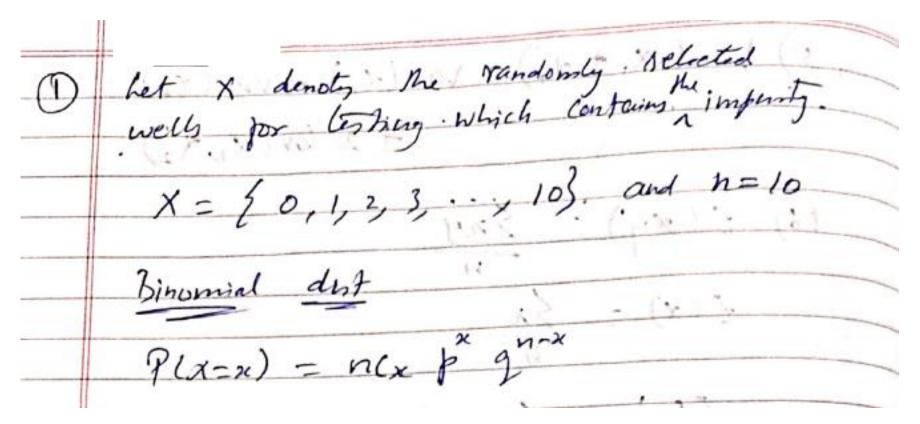
## **Binomial Distribution**





**Problem 1:** It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing.

- (a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?
- (b) What is the probability that more than 3 wells are impure?



$$(ii) \quad P(x>3) = 1 - P(x \le 3)$$

$$= 1 - \left[P(x=0) + P(x=1) + P(x=1) + P(x=1) + P(x=2)\right]$$

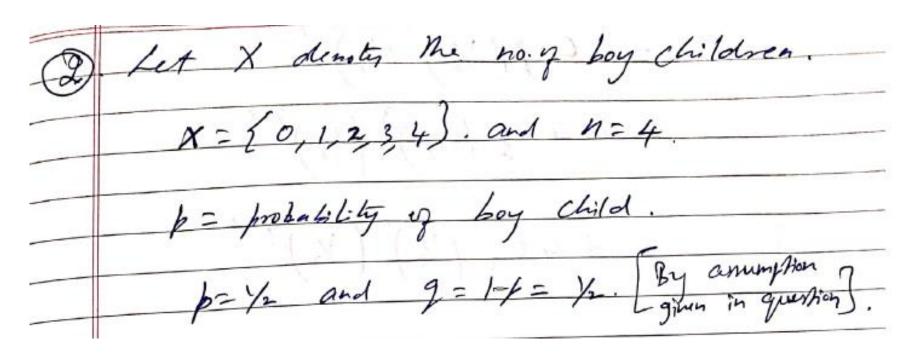
$$= 1 - \left[P(x=0) + P(x=1) + P(x=1) + P(x=1) + P(x=1) + P(x=2)\right]$$

$$= 1 - \left[P(x=0) + P(x=1) + P(x=1$$

**Problem 2:** In an apartment there are 500 families with 4 children each. How many families would be expected to have

- i. 2 boys and 2 girls
- ii. at least one boy
- iii. at most two girls
- iv. children of same gender
- v. children of different gender

Assume equal probabilities for boys and girls.



Binomial distribution  $g(x=x) = nc_x p^x q^{n-x}$  n=4, cach child is a trial.

(i) P(2 boys and 2 girls) = 3(x = 2)=  $4 C_2 (1/2)^2 (1/2)^4$ = 3/8 ( at least one boy) at most two girly) = ?(x=4)+?(x=3)+?(x=2)

$$= 4 \frac{4}{12} \left( \frac{1}{12} \right)^{4} \left( \frac{1}{12} \right)^{4-3}$$

$$+ 4 \left( \frac{3}{3} \left( \frac{1}{12} \right)^{3} \left( \frac{1}{12} \right)^{4-3}$$

$$+ 4 \left( \frac{1}{2} \left( \frac{1}{12} \right)^{2} \left( \frac{1}{12} \right)^{4-2}$$

$$= \frac{1}{16}$$

(IV) P( children of Name gender = P(X=4) + P(X=0) = 46(4 (1/2)4/1/2) children of different grades. (x=1) + P(x=2) + P(x=3)chiddren y Name gender Dr. Jayagopal\_Module 4

