Module 2: Random Variables

Introduction to Probability - Conditional Probability -Bayes Theorem- Random variables-Probability mass Function, distribution and density functions joint Probability distribution and joint density functions- Marginal, conditional distribution and density functions- Mathematical expectation, and its properties Covariance, moment generating function characteristic function.
 Managing Financial Risk -Modelling Sampling Variation(one dimension)

Experiments

Random

Outcome is determined by chance. In other way, the outcomes are known but can not predict the exact outcome

Deterministic

Outcome may be predicted with certainty beforehand, such as combining Hydrogen and Oxygen, or adding two numbers such as 2+3.

Eg: Type 17 No. of	tely home	Cally recei	ved in 12	our
- Las a	numerila	Cally recei	A 1 A . A . A	
	01	· ·	* C.) 400	
Type 2 Toning	a Coin	has a m	m-numerical	value
1 // 1	out come		10 965 89	
		e anign a	numerical	value
		This process		
Variabl		6x x15		
	57-11 U.	punction 1	hat assign	a
		every for		
	experiment.	1		,

The aniqued number strely Actually, P(x=x)=P{A: X(a)=x}.

Random Variables	: One - dimension	al R.V.	(x).
(R.V)	and		
	Two - dimension	al R.V.	(x and y)

Ohe-dimensional	R.v -> Discrete	case.	
cost come and tremes	and	13/	1 1
it fordert the autume	Continuous	Case.	
X -> Kandor	n variable.	6 6 10	1.1

Discrete Vandom Variable	Continuous random	Variable
1) X -> Finite number (or)	1) X -> Takes Values	in
Countably infinite humber.	an interval,	I
die in Mrown.	Eg: length of time	to sete.
in any a major water	1 1 1 (mary	
2) $P(X=x_i)$ (or) $himply$ $P(X=x)$	2) g(x) -> probabil	
Junction. (fing)	function.	(pdg).
function. (fint)		

3) pmg natisfier followings.	3) par sahisfires followings.
(i) P(x=x) ≥0.	(1) fax 20 tx eRx
(ii) 29(x=x)=1.	(ii) f coude =1.
I was and the on the	Rx E (-asa)
Id & Porto the release X	

> 6	reciple x.
(a) Probability distributi	on 4) Protability distribution
n h R.V. X	of a Continuous R.V carlt
Y=x P(x=x)	be represented by a fable
z_1 $P(x=z_1)$	as in decrete R.V.
0(/ 1)	@ Probability distribution
$\frac{\chi_2}{\chi_1}$	in known, of either its
$P(x=x_n)$	pdf (or) cdf is given.
- In Print	(For meetic intervals we can find
	Dr. R. Jayas opal_Module 2

5) Cumulative distribution 5) [Cdf]

Finition (Cdf)

$$F(x) = \int (-\infty \angle X \angle x)$$

$$F(x) = \int f(x) dx$$

$$F(x) = \int f(x) dx$$

$$F(x) = \frac{d}{dx}(F(x))$$

6) Proferties!	6) Propesties
(i) F(x) is hon-decreasing	(i) Same as in descrione
(ive) of x, Lx, Men F(x,) L'F(x,) (ii) Same as in Continuous	(ii) F(-00)=0 (100)=0.
$Case.$ $(iii) P(x=x_i) = F(x_i) - F(x_i)$	and f(0) - d (F(x))
$\frac{(2\pi)}{12} \times (2\pi)^{-1} \times (2$	Note: F(x), is diff at all 2.
	BP(a=x=b)= P(acx=b)

Note!	
DX-) Continuous R.V Men it is impossible the	t a
Continuous ex assumes a specific value.	
(1.e) P(x=a)=P(a=x=a)= f(n) dn	20.
1	
Henry Placx66) = P(a2x66) = P(a6x66)	
$= P(a 1 \times 2b).$	

2) Pr	bability dis	hibubian for inter	mal	I
	Intervals	Probability (1772 .	
40	a = X = b.	3(a=x=b)	7 5	
1994	CEXEd.	P(c ≤ x ≤ d)	14.27	
4	xee	P(x = e) . (3/11:	
	Xzm	P(X Zm)		
	1 000	6) 1924	and the same of th	T

3) Mean =
$$E(x) = \frac{1}{2}x p(n)$$
.

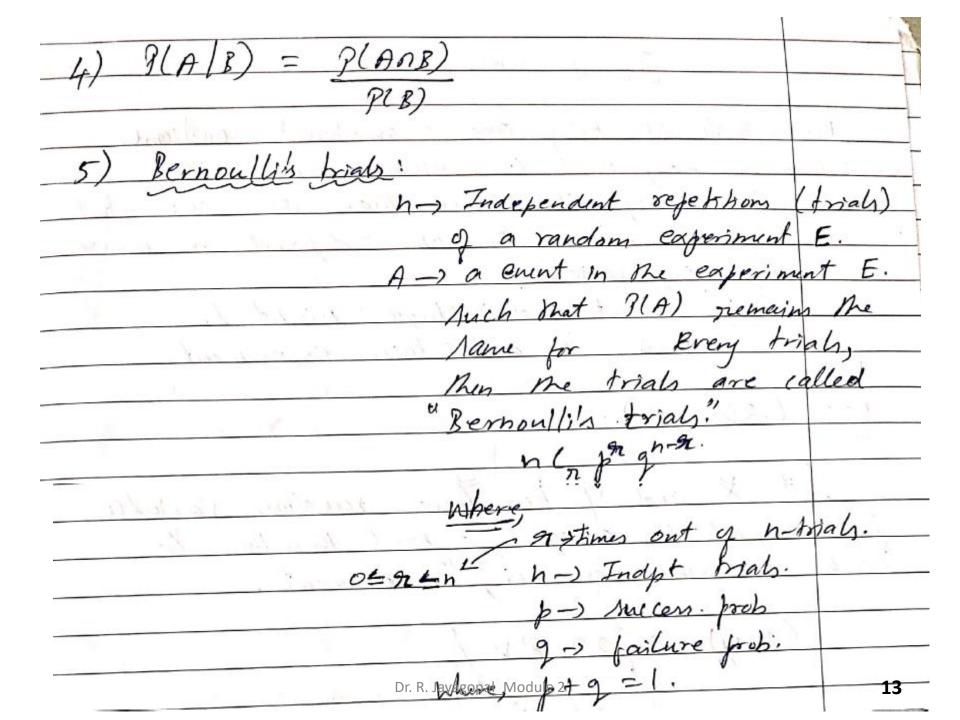
Variance = $E(x^2) - (E(x))^2$ $\Rightarrow Discreticon(RN)$.

$$= \frac{1}{2}n^2p(n) - (\frac{1}{2}xp(n)) .$$

Muan = $E(x) = \frac{1}{2}n^2p(n) dn$

Results of the properties of

12



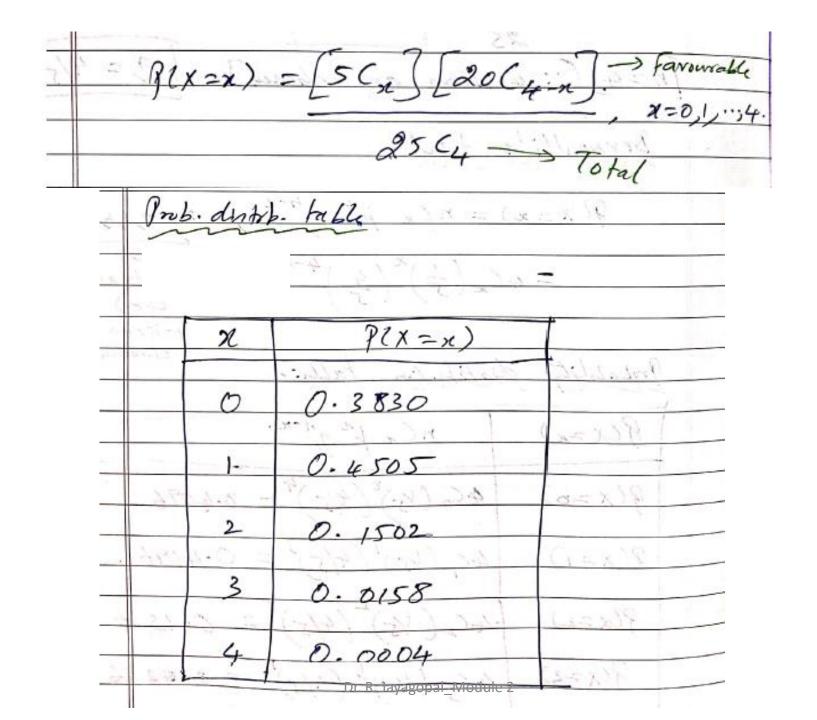
(X) Two coins = { HH, HT, TH, TT3.	Let X be the no. of heads in bowing true wins.
P(exactly one head) = = = = 1	Then X = of 2, 1, 03.
	p= /2 (probability of getting a head).
pp (at least one head) = 3	>P(x=1)=24(1/2)(1/2)
	$= 2 \times \frac{1}{4} = \frac{1}{2}$
= 1 + 1 = 3	P(X=2) = 2(2(Y2)2(1/2)9-
Logical) First take x to be heads out com.	exactly,
2) Find probability of getting head. 3) Then find P(x=2).	Exauty, wo head,

Problem 1: From a lot containing 25 items, 5 of which are defective, 4 items are chosen at random. If X is the number of defectives found, then obtained probability distribution of X, when the items are chosen with replacement and without replacement.

1	
(1)	Tare (18): W. Mout reformat.
1	Case (1): With replacement
	- Tepaupunt
	Let X -> the no. of deserting it
	Let X -> the no. of dejective items.
1,041	an an Bernsullin thinks
	Nince probability for an event remain same
	Nince probability for an event remain same for every trials.
The same	= & Colores x- Long from ;
	atti borb = 5 da barri 1 10 a 1 1
	p= 5 => p=1/5 then g=1-p.
320	[n=4] [: 4-2tim are chosen]. [2=4/5.]
10.00	77

92x-~1	$= n(x p^{x} q^{n-x}, x=0,1,3,4$
= 4	(x(5)2(4)4-2 4-trials (i.e.)
7.70	4-items am
Inchability of	Bluken talli:-
BCX=X)	nCx fr ghrx.
Q(x=0)	460 (45)° (4/5)4 = 0.4096
La de a la l	44 (45) (4/5) = 0.4096.
q(x=i)	
Plx=2)	4C2 (15) (4/5) = 0-1536
9(x=3)	4(3 (45)3 (4/5) = 0.0256
	4(4 (1/5) (4/5) = 0.0016

6	ase (ii):-Wilhout replacement.
	Case (1) - Willy replacement
Z	Vinus (toneously.
20	P(x=x) = P (choosing exactly n-dependent
	the court will be
	= P (chosing x-directive stime)
-	B(x=x) = [5(x] [20(4-n] -> favourale, x=0,1,
	25 C4 Total

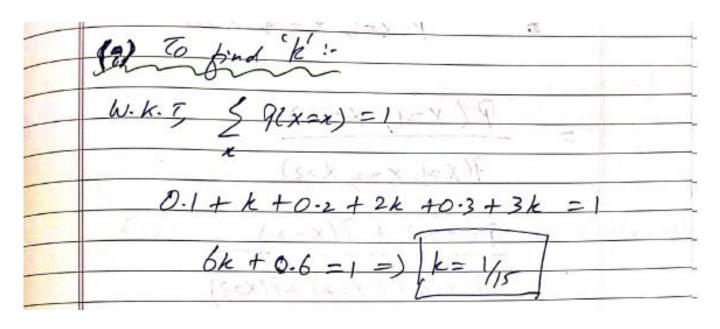


Problem 2: A random variable X has the following probability distribution.

X	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

Find (i) k, (ii) P(X<2), (iii) $P(-1 \le X < 3 / X > 0)$, (iv) cdf of X and

(v) Mean of X



x	-2	-1	0	11-, \	2	3
(4)	0.1	V	0-2	2/15	0.3	3/=1/

$$\frac{(ii) P(x \angle 2)}{= x^{2}} = P(x = -1) + P(x = 0) + P(x = 0) + P(x = 1)$$

$$= (0.1) + (415) + (0.2) + 2/15$$

$$P(x \angle 2) = 42$$

x	-2	-1	0	4	2	3
L(x)	0.1	V	0-2	2/15	0.3	3/5=1/
PIN		113	12 -	1 - 131		1/3