

Final Assessment Test (FAT) - July/August 2023

Programme	B.Tech.	Semester	Fall Inter Semester 22-23
Course Title	PROBABILITY AND STATISTICS	Course Code	BMAT202L
Faculty Name	Prof. P Durgaprasad	Slot	A1- IAI
		Class Nbr	CH2022232500610
Time	3 Hours	Max. Marks	100

Section I (10 X 10 Marks)

Answer any 10 questions

01. Find the mean, standard deviation, and quartile deviation of the following data [10]

125-175	175-225	225-275	275-325	325-375	375-425	425-475	475-525	525-575
2	22	19	14	3	4	6	1	1

02. i) The Median and Mode of the following wage distribution are known to be Rs. 33.50 and Rs. 34 respectively. Find the values of f_3 , f_4 and f_5 . (5 Marks) [10]

Wages (in Rs.)	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	4	16	f_3	f_4	f_5	6	4	230

- ii) Obtain the rank correlation coefficient for the following data: (5 Marks)

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

03. If a random variable X has a probability density function [10]

$$f(x) = \frac{1}{2}e^{-|x|}; -\infty < x < \infty$$

Show that momenta generating function of X is $\phi(t) = \frac{1}{1-t^2}$. Hence find the variance of X .

04. Let (X, Y) be a bivariate continuous random variable with joint PDF [10]

$$f_{XY}(x, y) = \begin{cases} cx^2y & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the constant c .

b) Find marginal PDFs, $f_X(x)$ and $f_Y(y)$.

c) Find $P(Y < \frac{X}{2})$

d) Find $P(Y \leq \frac{X}{4} | Y \leq \frac{X}{2})$

05. i) For 50 students of a class the regression equation of marks in Statistics (X) on marks in Mathematics (Y) is $4Y - 6X + 180 = 0$. The mean mark in Mathematics is 46 and the variance of marks in Statistics is $4/25$ th of the variance of marks in Mathematics. Find the mean marks in Statistics and the coefficient of correlation between marks in two subjects. (7 marks) [10]

ii) Given two regression lines $X + 3Y - 7 = 0$ and $3X + 4Y - 8 = 0$, find the regression line of X on Y (3 marks)

06. The time spent waiting between the events is often modeled using the exponential distribution [10]
 Suppose that an average of 30 customers per hour arrive at a store and the time between arrivals is exponentially distributed

a) On average, how many minutes elapse between two successive arrivals?

- b. After a customer arrives, find the probability that it takes less than one minute for the next customer to arrive.
- c. After a customer arrives, find the probability that it takes more than five minutes for the next customer to arrive.
- d. Seventy percent of the customers arrive within how many minutes of the previous customer?
- e. What is the probability that in any 2 minutes interval, three customers arrive at the store?

07. A manufacturer of balloons produces 40 percent that are oval and 60 percent that are round. Packets of 10 balloons may be assumed to contain random samples of balloons. Determine the probability that such a packet contains
- An equal number of oval balloons and round balloons. (2 Marks)
 - Fewer oval balloons than round balloons. (3 Marks)

[10]

08. A mining company needs to estimate the average amount of copper per ton mined. A random sample of 50 tons gives a sample mean of 146.75 pounds. The population standard deviation is assumed to be 35.2 pounds. Give a 95% confidence interval for the average amount of copper in the population of tons mined. (5 Marks)

09. i) A car manufacturer aims to improve the quality of the products by reducing defects and also increasing customer satisfaction. Therefore, he monitors the efficiency of two assembly lines on the shop floor. In line A there are 18 defects reported out of 200 samples. While line B shows 25 defects out of 600 cars. At $\alpha = 5\%$, is the differences between the two assembly procedures significant? (5 Marks)

[10]

ii) The Educational Testing Service conducted a study to investigate the difference between the scores of female and male students on the Mathematics Aptitude Test. The study identified a random sample of 562 female and 852 male students who had achieved the same high score on the mathematics portion of the test. That is, female and male students are viewed as having similar high abilities in mathematics. The verbal scores for the two samples are given below.

	Female	Male
Sample mean	547	525
Sample standard deviation	83	78

Do the data support the conclusion that given populations of female and male students with similar high abilities in mathematics, the female students will have a significantly high verbal ability? Test at significance level $\alpha = 5\%$. What is your conclusion? (5 Marks)

10. Three experimenters determine the water content of samples of food, each man taking a sample from each of the consignments. The results are given below

[10]

Experiment	Consignment			
	I	II	III	IV
A	9	10	9	10
B	12	11	9	11
C	11	12	10	12

Perform an analysis of variance on these data and discuss whether there is any significant difference between consignments or between experiments.

Given $F(2, 6) = 5.14$, $F(3, 6) = 4.76$ at 0.05 level of significance.

11. Samples of two types of electric light bulbs were tested for length of life (in months) are given by: Type A: 25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25.
Type B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22.
Test if the mean life length of the two type of electric light bulbs differ significantly. Use the level of significance $\alpha = 0.05$.

[10]

11. The time of failure in operating hours of a critical solid-state power unit has hazard rate function (10)

$$\lambda(t) = 0.003 \left(\frac{t}{500} \right)^{0.5}, \text{ for } t \geq 0$$

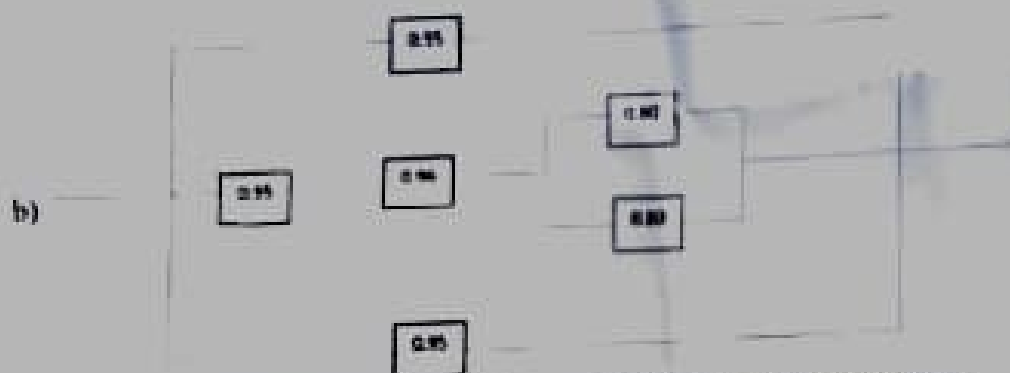
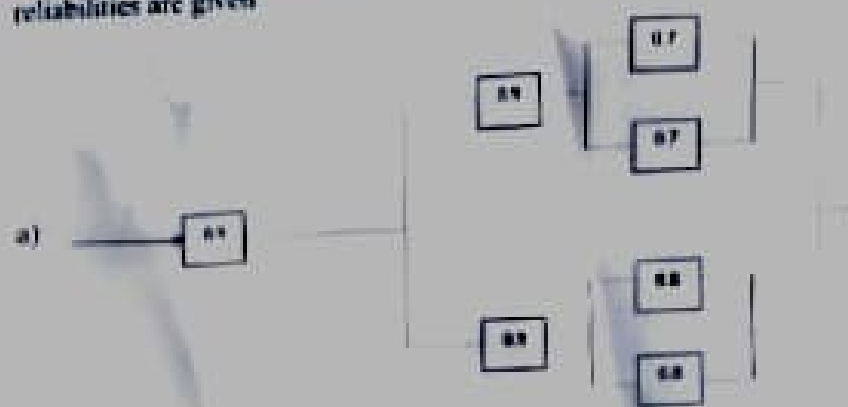
a) What is the reliability of the power unit must operate continuously for 50 hours?

b) Determine the design life if a reliability of 0.90 is desired

c) Compute the MTTF

d) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours operation?

12. Find the system reliability of the following series-parallel configuration. Component reliabilities are given (10)



Instruction : Non-programmable calculators and statistical tables are permitted.

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