Module 4

1) The probability that a certain kind of component will survive a shock test is $3/4$. Find the probability that exactly 2 of the next 4 components tested survive.
2) A company generally purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units.
   a) What is the probability of rejecting a lot that is 1% defective?
   b) What is the probability of accepting a lot that is 5% defective?
3) Ten is the average number of oil tankers arriving each day at a certain port city. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?
4) If X and Y are independent Poisson random variables such that $\text{Var}(X+Y)=9$ and $P[X=3/(X+Y=6)]=5/54$ than find $E(X)$.
5) Fit a Poisson distribution for the following distribution:

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

   Fitting a Poisson distribution for a given distribution means assuming that the given distribution is approximately Poisson.
6) The life length X of an electronic component follows an exponential distribution. There are two processes by which the components may be manufactured. The expected life length of the component is 100 hours, if process I is used to manufacture, while it is 150 hours if process II is used. The cost of manufacturing a single component by process I is Rs. 10, while it is Rs. 20 for process II. Moreover, if the component lasts less than the guaranteed life of 200 hours, a loss of Rs. 50 is to be borne by the manufacturer. Which process is advantageous to the manufacturer?
7) If the life X (in years) of a certain type of a car has a weibull distribution with parameter $\beta=2$, find the value of the parameter α, given that the life of the car exceeds 5 years is $e^{-0.25}$. Find the mean and variance of X, for these values of α and β.
8) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and S.D. 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75?
9) If X and Y are independent R. V.'s following $N(8,2)$ and $N(12,4\sqrt{3})$ respectively, find the value of λ such that $P(2X-Y\leq 2\lambda)=P(X+2Y\geq\lambda)$.

10) Family income follows a normal distribution with the mean $Rs.50{,}000$ and standard deviation $Rs.10{,}000$. If the poverty level is below $Rs.10{,}000$, what percentage of the population lives in poverty?
11) Suppose 50% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. Solve the following, using both the binomial distribution and the normal approximation to the binomial.
    a) What is the probability that exactly 7 people will support the governor?
    b) What is the probability that 7 or fewer people will support the governor?
    c) What is the probability that exactly 11 will support the governor?
    d) What is the probability that 11 or fewer will support the governor?
12) If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective. Given that e^{-3} = 0.0498.
13) It is known from the past experience that in a certain plant there are on the average of 4 industrial accidents per month. Find the probability that in a given year will be less that 3 accidents. Given e^{-4} = 0.0183
14) The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 inches and the standard deviation is 0.005 inches. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 inches, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.
15) The tensile strength of a certain metal component is normally distributed with a mean of 10,000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter. Measurements are recorded to the nearest 50 kilograms per square centimeter.
    a) What proportion of these components exceed 10,150 kilograms per square centimeter in tensile strength?
    b) If specifications require that all components have tensile strength between 9800 and 10,200 kilograms per square centimeter inclusive, what proportion of pieces would we expect to scrap?
16) The random variable X can modeled by a Weibull distribution with $\beta = ½$ and $\theta = 1000$. The spec time limit is set at $x = 4000$. What is the proportion of items not meeting spec?