Module 2

1. A loss for a company has mgf $M(t) = \frac{0.16}{0.16-t}, t < 0.16$. An insurance policy pays a benefit equals to 70% of the loss. What is the mgf of the benefit?
2. Suppose that you have a fair 4-sided die, and let X be the random variable representing the value of the number rolled.
   (a) Write down the mgf
   (b) Use this mgf to compute first and second moments of X

3. For the following bivariate probability distribution of X and Y, find $P(Y \leq 3)$ and $P(X < 3,\ Y \leq 4)$.

Y / X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Also, find the conditional distribution of X given $Y = 4$.

4. Suppose that the random variables X and Y have the joint probability density function:
$$f(x,y) = \begin{Bmatrix} kx(x-y) & 0 < x < 2, -x < y < x \\ 0 & elsewhere \end{Bmatrix}$$
   (i) Evaluate the constant k,
   (ii) Find the marginal and conditional probability density functions of the random variables.

5. let (X, Y) be continuous r.v., with joint p.d.f: $f_{XY}(x,y) = x + y; 0 \leq (x,y) \leq 1$. Find the marginal p.d.f of X and Y.

6. The joint probability distribution of two random variables X and Y is given by:
$$p(x,y) = \frac{1}{(n+1)}, x = 1,2,3, \dots, n; y = 1,2,3, \dots, x.$$
   Examine whether X and Y are independent.

7. Let X and Y be two random variables each taking three values -1, 0 and 1 and having the joint probability distribution:

X / Y	-1	0	1
-1	0	0.1	0.1
0	0.2	0.2	0.2
1	0	0.1	0.1

Prove that X and Y are uncorrelated.

8. Let X be a random variable whose probability density function is given by

$$f(x) = \begin{cases} e^{-2x} + \frac{1}{2}e^{-x}, & x > 0 \\ 0, & otherwise \end{cases}$$

(a) Write down the moment generating function for X

(b) Use this moment generating function to compute the first and second moments of X

9. Let X be a random variable with PDF given by

$$f(x) = \begin{cases} cx^2 & |x| \leq 1 \\ 0, & otherwise \end{cases}$$

(a) Find the constant c

(b) Find $E(X)$and $Var(X)$

(c) Find $P\ (x \geq 12)$.

10. Find k so that $f(x)$ given below may be p.d.f

$$f(x) = \begin{cases} \frac{1}{k} & for\ a \leq x \leq b \\ 0 & otherwise \end{cases}$$

Find the distribution function also.`

11. If the p.d.f of a continuous random variable X is$f(x) = \begin{cases} c(3+2x), if\ 0 < x < 2 \\ 0 \quad otherwise \end{cases}$, then find the value of c and the distribution function $F(x)$.

12. The amount of bread (in hundreds of kilos) that a bakery sells in a day is a random variables with density

$$f(x) = \begin{cases} cx, & for\ 0 \leq x < 3 \\ c(6-x), & for\ 3 \leq x < 6 \\ 0, & otherwise \end{cases}$$

(i) Find the value of c which makes $f(x)$ a pdf.

(ii) What is the probability that the number of kilos of bread that will be sold in a day is

(a) more than 300 kilos (b) between 150 and 450 kilos.