Module 4:
Probability Distributions

Discrete Distributions: Binomial distribution - Poisson distributions; Continuous Distributions: Normal distribution - Gamma distribution - Exponential distribution - Weibull distribution.

Binomial Distribution

$$P(X = x) = nc_x p^x q^{n-x}, \;\; x = 0, 1, 2 \cdots n$$

Mean $E(X) = np$

Variance $\mathrm{V}(X) = npq$

Moment generating function

$$M_X(t) = (q + pe^t)^n$$

Binomial Distribution

In general,

$$\boxed{P(X=r) = nC_r \, p^r q^{n-r}} \quad , r = 0,1,2,\dots,n$$

(Countably finite).

Note:

Note:

*) As r increases, q decreases and p increases.

*) $n \to$ no. of trials $\qquad \boxed{q = 1-p}$

$X \to$ no. of succes in n-trials.

$p \to$ probability of success in a trial.

$q \to$ probability of failure in a trial.

*) $B(n, p) \to$ Binomial random variable with parameter n and p.

*) It is useful, if we are running a fixed no. of independent trials, each one can have a success (or) failure, and we are interested in the no. of success (or) failure.

*) $E(x) = np$.

$Var(x) = npq$.

*) $$\underbrace{\sum_{r=0}^{n} P(X=r) = \sum_{r=0}^{n} nC_r\, p^r q^{n-r}}_{} = 1.$$

These are the successive terms in the expansion of $(p+q)^n$.

Problem 1: It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing.

(a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?

(b) What is the probability that more than 3 wells are impure?

① Let X denote the randomly selected wells for testing which contains the impurity.

$X = \{0, 1, 2, 3, \dots, 10\}$. and $n = 10$

Binomial dist

$P(X=x) = nC_x \, p^x \, q^{n-x}$

Given :- $p \to$ probability of success
(Here, impurity exists in
30% of all drinking wells.

$\boxed{p = 0.30}$ and $q = 1-p$
$\boxed{q = 0.70}$

$10-3$.

(i) $P(X=3) = {}^{10}C_3\,(0.30)^3\,(0.70)$

$= {}^{10}C_3\,(0.30)^3\,(0.70)^7$

$\boxed{P(X=3) = 0.27}$

(ii) $P(X>3) = 1 - P(X \leq 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$.

$= 1 - \sum_{x=0}^{3} 10C_x (0.30)^x (0.70)^{10-x}$

$= 1 - [0.03 + 0.12 + 0.23 + 0.27]$

$\boxed{P(X>3) = 0.35}$

Problem 2: In an apartment there are 500 families with 4 children each. How many families would be expected to have

i. 2 boys and 2 girls
ii. at least one boy
iii. at most two girls
iv. children of same gender
v. children of different gender

Assume equal probabilities for boys and girls.

(2). Let X denote the no. of boy children.

$X = \{0, 1, 2, 3, 4\}$. and $n = 4$.

p = probability of boy child.

$p = 1/2$ and $q = 1-p = 1/2$. [By assumption given in question].

Binomial distribution

$$P(X=x) = nC_x \, p^x q^{n-x}$$

$n=4$, each child is a trial.

(i) $P(2 \text{ boys and } 2 \text{ girls}) = P(X=2)$

$$= 4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^{4-2}$$

$$= 3/8$$

(ii) P(at least one boy) $= P(X \geq 1)$

$= 1 - P(X=0)$

$= 1 - 4C_0 (1/2)^0 (1/2)^{4-0}$

$= 15/16.$

(iii) P(at most two girls) $= P(X=4) + P(X=3) + P(X=2)$

zero girls   1-girl   2-girls.

$(\because X \to$ no. of boys$)$.

children

$$= {}^4C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{4-4}$$

$$+ {}^4C_3 \left(\tfrac{1}{2}\right)^3 \left(\tfrac{1}{2}\right)^{4-3}$$

$$+ {}^4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^{4-2}$$

$$= 11/16.$$

(iv) P(children of same gender)

$$= P(X=4) + P(X=0)$$

All 4 - boys All 4 - girls.

$$= 4C_4 \left(\tfrac{1}{2}\right)^4 \left(\tfrac{1}{2}\right)^{4-4} + 4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^{4-0}$$

$$= \tfrac{1}{8}.$$

(v) P(children of different gender).

$$= P(X=1) + P(X=2) + P(X=3)$$

(or)

$= 1 - P($children of same gender$)$.

$$= 1 - \tfrac{1}{8} = \tfrac{7}{8}$$

Cases	No. of family
$P(X=2) = 3/8$	$500(3/8) = 188$
$P(X \geq 1) = \frac{15}{16}$	$500\left(\frac{15}{16}\right) = 469$
P(at most 2 girls) $= \frac{11}{16}$	$500\left(\frac{11}{16}\right) = 344$
P(children of same gender) $= 1/8$	$500(1/8) = 63$
P(children of different gender) $= 7/8$	$500(7/8) = 438$