Property – Poisson:

1) If X_1 and X_2 be independent Random variables that follow Poisson distribution with λ_1 and λ_2, respectively.

Let $X_1 + X_2 = n$ (say).

Then $\underbrace{P(X_1 + X_2)}_{P(X_1 + X_2 = n)} = \dfrac{e^{-(\lambda_1+\lambda_2)} (\lambda_1 + \lambda_2)^n}{n!}$

2). If X_1 and X_2 be two independent Random variables that follow Poisson distribution ~~them~~ with λ_1 & λ_2, resp^tly.

Then, $P(X_1 \mid X_1 + X_2) = nC_r\, p^r q^{n-r}$.

Where, $X_1 + X_2 = n$ (say).
If $X_1 = r$ then $X_2 = n - r$.

Here, $p = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$, $q = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$

(i.e), The Conditional distribution of X is binomial distribution

Probability distribution

⊛ Discrete case
- → Binomial distribution
- → Poisson distribution.

⊛ Continuous case.
- → Exponential distribution
- → Gamma. distribution
- → Normal distribution
- → weibull distribution

Normal Distribution

A Continuous random Variable X with probability density function,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$ is said to

$x \in (-\infty, \infty)$
$\mu \in (-\infty, \infty)$
$\sigma > 0.$

follow normal distribution.

Here, $\mu \to$ Mean and $\sigma^2 \to$ Variance.

Note: *) It is denoted as $N(\mu, \sigma)$ (or) $N(\mu, \sigma^2)$.

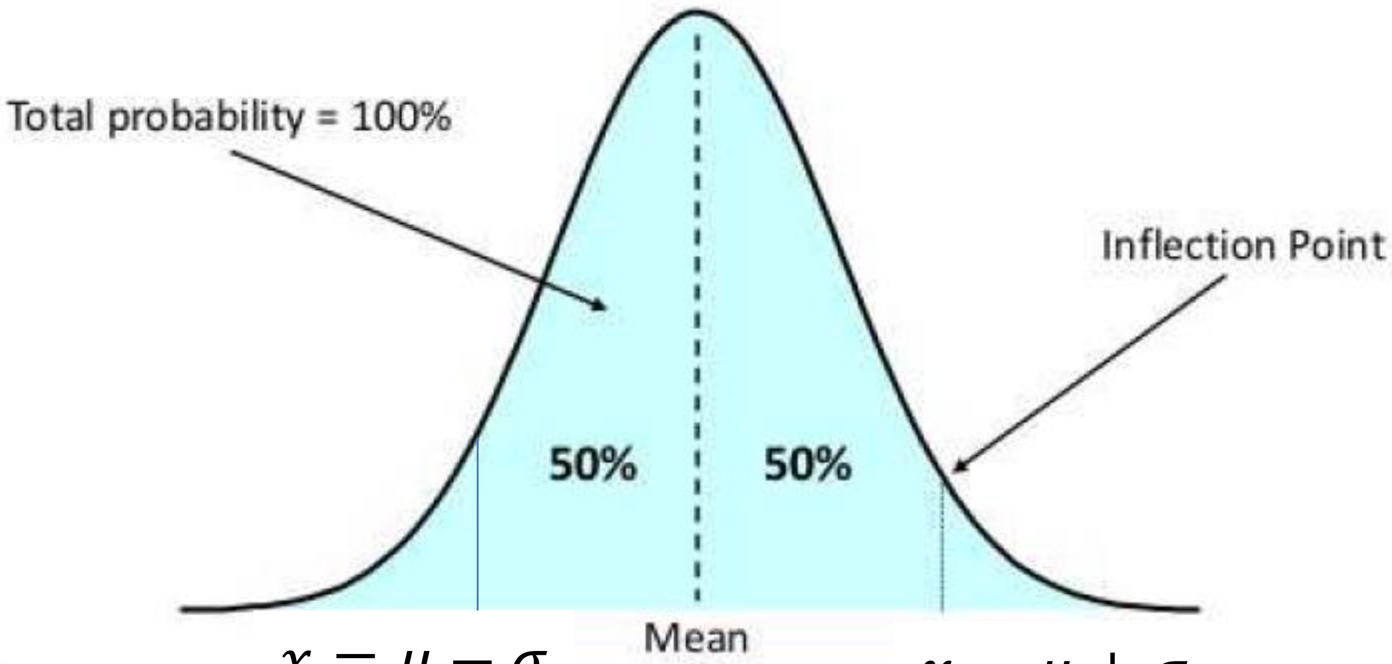
*) It depends on mean (μ) and the Standard deviation (σ).

*) Normal distribution is also refered as Gaussian distribution.

- Normal Distribution

Normal Curve

- Since the normal curve is symmetrical, **50 percent** of the data lie on each side of the curve.



$x = \mu - \sigma$ $\quad$ $x = \mu + \sigma$

Properties:

1) The curve is bell shaped and symmetric about the line $x = \mu$.
2) The curve $f(x)$ is called as normal curve.
3) Mean = Median = Mode.
4) Point of inflection occurs at $x = \mu \pm \sigma$.
5) Concave downward if $\mu - \sigma < X < \mu + \sigma$
6) Concave upward otherwise.

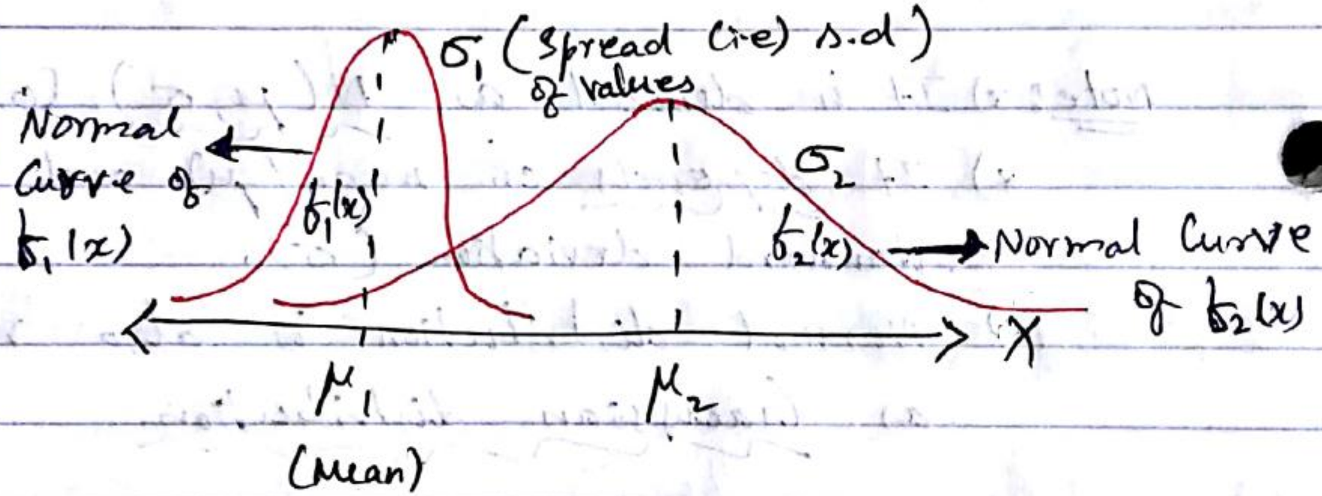
7) $f(x) > 0$, (i.e) the normal curve is asymptotic along the x-axis.

8) Total area under the normal curve is always 1.

Since, Total area = Total probability.

9) $X \sim N(\mu, \sigma^2)$ means X follows a normal distribution with mean (μ) and variance (σ^2).

10) When x increases then $f(x)$ decreases.

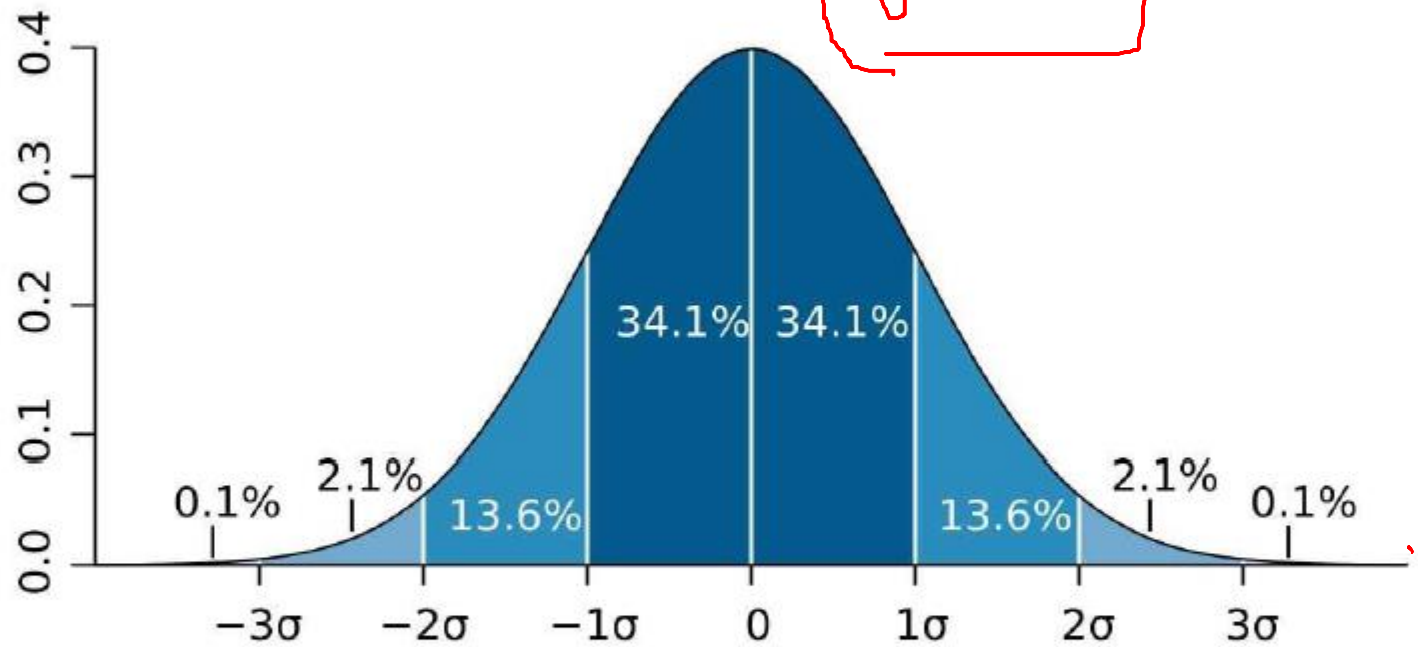


11) If σ increases then the normal curve becomes flatter and wider along x-axis. If σ decreases then the normal curve becomes closer to the mean value (i.e) it is peaked.

12) Left hand side of μ (mean) contains 50% of the area and right hand side of μ (mean) contains the remaining 50% of the area.

$\mu = 0$

$\boxed{\sigma = 1}$



→ x-axis

68%

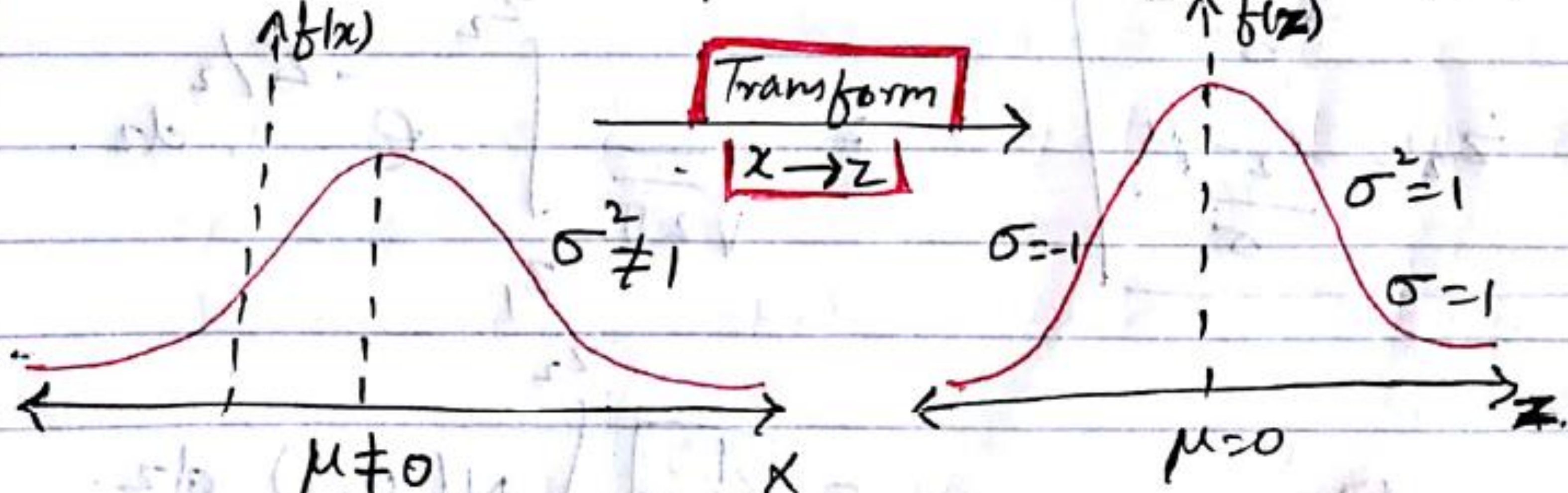
95%

99%

⟹ x-axis

Standard normal distribution:

It is a transformation of $N(\mu, \sigma^2)$ into $N(0,1)$. (i.e) Transformating it to a normal distribution with $\mu = 0$ (mean) and variance, $\sigma^2 = 1$.



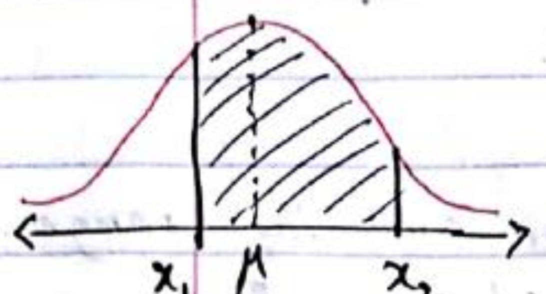
Normal distribution $N(\mu, \sigma^2)$

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

Standard normal distribution. $N(0,1)$

Area under the normal Curve :

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} N(\mu, \sigma^2) \cdot dx$$



$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx .$$

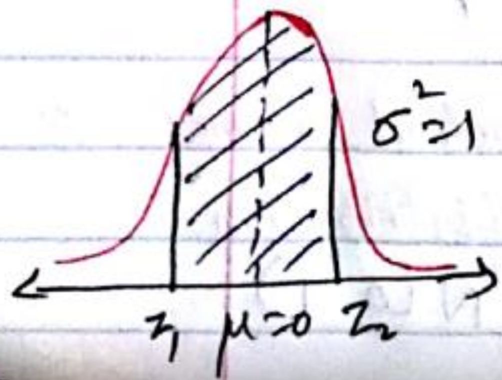
Area under the Standard normal curve:

$$P(x_1 < X < x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$\boxed{\begin{array}{l} \text{Put, } z = \frac{x-\mu}{\sigma} \\ \Rightarrow \sigma . dz = dx \\ \text{Also,} \\ z_1 = \frac{x_1-\mu}{\sigma} \\ \text{and } z_2 = \frac{x_2-\mu}{\sigma} \end{array}}$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2}(z)^2} \cancel{\sigma} . dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2} . dz.$$



$$= \int_{z_1}^{z_2} N(0,1)\, dz.$$

$$= P(z_1 < Z < z_2)$$

Where $N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad -\infty < z < \infty$

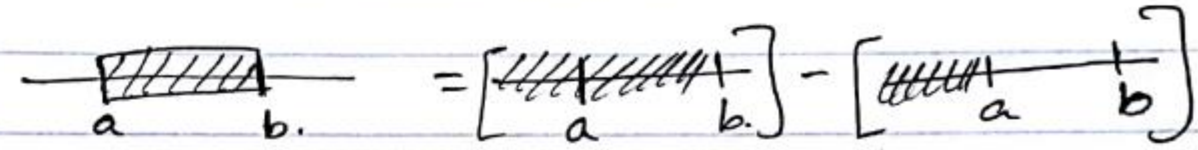
(or)

$\phi(z)$

Note:

1) $\left.\begin{matrix}\text{The area under the} \\ \text{normal curve}\end{matrix}\right\} = \left\{\begin{matrix}\text{The area under} \\ \text{the standard curve}\end{matrix}\right.$

2) $P(a < Z < b) = P(Z < b) - P(Z < a)$.



3) $P(Z > a) = \underbrace{1}_{\text{Total area} = 1.} - P(Z < a)$

4) $z = \dfrac{x - \mu}{\sigma} \Rightarrow x = \sigma z + \mu.$

(i.e) Value of x can be found if σ, z and μ are known.

X and Y are independent

5) If $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$.

then $X \pm Y \sim N(\mu_x \pm \mu_y, \sigma_x^2 + \sigma_y^2)$.

6) If $X \sim N(\mu, \sigma^2)$ then $aX+b \sim N(a\mu+b, a^2\sigma^2)$.

7) If $X_1, X_2, \ldots, X_n$ are independent observations of $X \sim N(\mu, \sigma^2)$, then $X_1 + X_2 + \cdots + X_n \sim N(n\mu, n\sigma^2)$.

Moment generating function:

$$M_X(t) = e^{\mu t + \frac{t^2\sigma^2}{2}}$$

Method to find $N(\mu, \sigma)$:

Step 1: Get the distribution and the range.

Step 2: Change normal distribution into Standard normal distribution.

(i.e) $N(\mu, \sigma) \rightarrow N(0, 1)$.

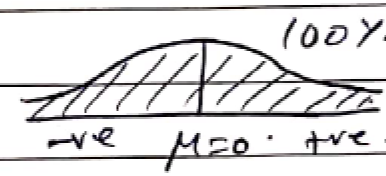
Step 3: Look up the probability using the Standard Normal Distribution table

Normal Distribution to Standard Normal Distribution

(1) Normal distribution ⟶ Standard normal distribution.

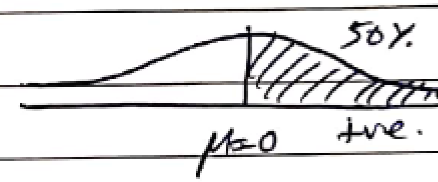
(2) Standard normal distribution.

↳ $\boxed{\text{Full table}}$ ⇒



$\int_{-z}^{z}$

⟶ Half table. ⇒



$\int_{0}^{z}$

(3) Using Half table.