Problem 4: A continuous random variable X that can assume any value between $x = 2\ and\ x = 5$ has a density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

Solution:

Given: $f(x) = k(1+x), \quad 2 < x < 5.$

W.K.T

$$\int_2^5 f(x).dx = 1$$

$$\int_2^5 k(1+x)\, dx = 1$$

$$\Rightarrow \boxed{k = 2/27}$$

Now,

$$P(X<4) = \int_{2}^{4} f(x)\,dx$$

$$= \int_{2}^{4} \frac{2}{27}(1+x)\,dx = \frac{16}{27} \text{ Ans.}$$

Problem 5: A continuous random variable X has a pdf $f(x) = kx^2e^{-x}$, $0 < x < \infty$. Find k, mean and variance.

Solution:

Given: $f(x) = kx^2e^{-x}, \quad 0 < x < \infty.$

W.K.T,

$$\int_0^\infty f(x)\,dx = 1$$

$$\int_0^\infty (kx^2e^{-x})\,dx = 1$$

$$\Rightarrow \boxed{k = 1/2}$$

$$\text{Mean} = E(x) = \int_{R_x} (x) f(x)\, dx.$$

$$= \int_0^\infty x \cdot \left(\frac{1}{2} \cdot x^2 e^{-x}\right) dx.$$

$$\boxed{E(x) = 3}$$

$$\text{Variance} = E(x^2) - \left(E(x)\right)^2$$

$$E(x^2) = \int_0^\infty (x^2) f(x)\, dx.$$

$$= \int_0^\infty x^2 \left(\frac{1}{2} x^2 e^{-x}\right) dx$$

$$\boxed{E(x^2) = 12}$$

$\therefore$ Variance $= E(x^2) - (E(x))^2$

$= 12 - (3)^2$

$\boxed{\text{Variance} = 3}$

Problem 6: If a random variable X has a cumulative distribution function say

$$F(x) = \begin{cases} 0, & x \le 0 \\ c(1-e^{-x}), & x > 0 \end{cases}$$

Find the p.d.f. $f(x)$ value 'c'and $P(1 < X < 2)$

Solution:

Given: cdf, $F(x) = \begin{cases} 0, & x \le 0 \\ c(1-e^{-x}), & x > 0. \end{cases}$

W.K.T,

$$f(x) = \frac{d}{dx}\left(F(x)\right).$$

$$= \frac{d}{dx}\left(c(1-e^{-x})\right)$$

$$\boxed{f(x) = ce^{-x}}, \quad x > 0.$$

(i) To find 'c':

W.K.T. $\int_0^\infty (c \cdot e^{-x})\, dx = 1$

$\Rightarrow \boxed{c = 1}$

(ii) $P(1 < X < 2) = \int_1^2 f(x)\,dx = \int_1^2 (1) \cdot e^{-x}\, dx.$

$= (e-1)/e^2$ 4.

2D - Random variables

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Discrete	Continuous.
* $P(X = x_i, Y = y_j) = p(x_i, y_j) = p_{ij} \geq 0$ is called joint prob. mass function.	* $f(x, y) \geq 0$. is called as joint prob. density function.
* $\sum_i \sum_j (P(X = x_i, Y = y_j)) = \sum \sum p(x_i, y_j) = 1$	* $\int_{R_y} \int_{R_x} f(x, y)\, dx\, dy = 1$.

*) cdf of (x,y)

$$F(x,y) = \sum_{y_j \le y} \sum_{x_i \le x} P(X = x_i, Y = y_j)$$

$$= \sum_{y_j \le y} \sum_{x_i \le x} p_{ij}$$

Note:

*) $F(-\infty, y) = 0 = F(x, -\infty)$

*) $F(\infty, \infty) = 1$.

Same.

*) cdf of (x,y)

$$F(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(x,y)\, dx\, dy.$$

Note:

For the points of continuity of $F(x,y)$

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} \left(F(x,y)\right).$$

Marginal probability distribution of X and Y.

$$P(X=x_i)=\sum_j p_{ij}=p_{i*}$$

$$P(Y=y_j)=\sum_i p_{ij}=p_{*j}$$

Marginal probability distribution of X and Y.

$$f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy.$$

(or)

$$\int_{R_y} f(x,y)\,.dy.$$

$$f_Y(y)=\int_{-\infty}^{\infty} f(x,y)\,dx$$

(or)

$$\int_{R_x} f(x,y)\,dx.$$

(*) Conditional probability distribution:

$$P(X = x_i / Y = y_j)$$

$$= \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)}$$

$$= \frac{p_{ij}}{p_{*j}}$$

(*) Conditional probability distribution:

$$f(x/y) = \frac{f(x,y)}{f_Y(y)}$$

$$P(Y=y_j \mid X=x_i)$$

$$= \frac{P(Y=y_j, X=x_i)}{P(X=x_i)}$$

$$= \frac{p_{ij}}{p_{i*}}$$

$$f(y|x) = \frac{f(x,y)}{f_X(x)}.$$

(*) Independent random variable (X, Y):

X and Y are independent if $p_{ij} = (p_{i*})(p_{*j})$

(*) Independent random variable (X, Y):

X and Y are independent if $f(x,y) = f_X(x) \cdot f_Y(y)$