BMAT202L PROBABILITY AND STATISTICS

Module 2: One-dimensional random variables

Tutorial sheet-I

1. Suppose a discrete random variable X has the following probability mass function:

$$P(X = 1) = \frac{1}{2}, \quad P(X = 2) = \frac{1}{4}, \quad P(X = 3) = k, \quad P(X = 4) = \frac{1}{8}.$$

   Find

   (i) the value of k

   (ii) $P(X \leq 1)$, $P(1 \leq X \leq 3)$, $P(1 < X < 3)$, $P(X > 3)$

   (iii) the cumulative distribution function $F(x)$ of X

   (iv) $E(X)$ and $\mathrm{Var}(X)$

2. Does $P(X = x) = \dfrac{2x}{k(k+1)}$, for $x = 1, 2, 3, \ldots, k$, serve as a probability mass function of a random variable X with the given range? Justify.

3. For what values of k the given functions can serve as probability distributions:

   (i) $P(X = x) = k \begin{pmatrix} 5 \\ x \end{pmatrix}$ for $x = 0, 1, 2, 3, 4, 5$

   (ii) $P(X = x) = k \left(\dfrac{1}{4} \right)^x$ for $x = 1, 2, 3, \ldots..$

4. Let X be continuous random variable with probability density function

$$f(x) = \begin{cases} ke^{-\frac{x}{3}}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

   where k is a constant. Find

   (i) the value of k

   (ii) $E(X)$, $E(X^2)$ and $\mathrm{Var}(X)$

   (iii) the cumulative distribution function, $F(x)$ of X

   (iv) $P(X > 2)$

5. If a contractor's profit on a construction job can be looked upon as a continuous random variable X having the probability density function

$$f(x) = \begin{cases} \dfrac{x+1}{18}, & -1 < x < 5, \\ 0, & \text{otherwise,} \end{cases}$$

   where the units are in Rs.10,000, then find

(i) $P(|X| < 1)$

(ii) $P(1 < X < 2)$

(iii) $P(X > 1)$

(iv) $E(X)$.

6. The total lifetime (in years) of five-year-old dogs of a certain breed is a random variable X whose cumulative distribution function is given by

$$F(x) = \begin{cases} 0, & x \le 5, \\ 1 - \dfrac{25}{x^2}, & x > 5. \end{cases}$$

Find the probability density function and $E(X)$. Also find the probabilities that such a five-year-old dog will live

(i) beyond 10 years;

(ii) less than 8 years;

(iii) anywhere from 12 to 15 years.