Module 7: Reliability

Quality	Reliability
*) Associated with the manufacturer.	*) Primarily associated with the design.
*) It is impossible to construct a 'good' quality system from 'poor' quality elements.	*) One can build a reliable complex system using less reliable elements.

Reliability

It is defined as the probability that a given system will successfully perform a required function without break (or) failure under specified environmental conditions, for a specified period of time.

$$R(t) = P(T > t)$$

Where, $T \rightarrow$ life length of the system.

$R(t) \rightarrow$ 1) Reliability function of time 't'.

2) It is the probability that the system does not fail during the time interval $(0, t)$.

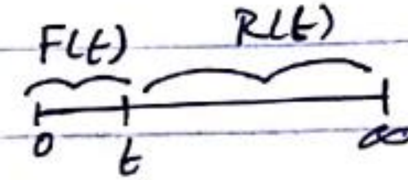
Eg: $R(t_1) = 0.91$, means that 91% of the items of the system will be functioning under given conditions, without failure, at time 't_1'.

Properties:

1) $0 \le R(t) \le 1$.
2) $R(0) = 1$ and $R(\infty) = 0$
3) $R(t)$ is a decreasing function of 't'.
4) If $f(t)$ is the p.d.f of random variable T and $F(t)$ is its Cumulative distr. funct,

We have, $R(t) = \int_t^{\infty} f(t)\cdot dt = P(T > t)$

$$F(t) = \int_0^t f(t)\cdot dt = P(T \le t).$$



Now, $R(t) = 1 - F(t)$

Its derivative is given as,

$$R'(t) = -F'(t)$$

$$\boxed{R'(t) = -f(t)}$$

$$\Rightarrow \boxed{R'(t) + f(t) = 0}$$

$\because F(t) = \int_0^t f(t)\cdot dt$

$F'(t) = f(t)$

Interval Failure rate $[\lambda(t)]$:

It is the conditional probability that failures occurs in the interval (t_1, t_2) given that failures have not happened before 't_1'.

$$\lambda(t) = \frac{\int_{t_1}^{t_2} f(t).dt}{(t_2 - t_1)\int_{t_1}^{\infty} f(t).dt}$$

$$\Rightarrow \boxed{\lambda(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)\, R(t_1)}}$$

//.

Hazard rate $(z(t))$:

It is the proportion that will fail in the next unit of time, out of those units that have survived upto that time.

$$\boxed{f(t) = z(t)\, \underbrace{e^{-\int_0^t z(t)\,dt}}_{R(t)}}$$

↳ $f(t)$ is uniquely determined if the hazard rate $z(t)$ is given.

and

$$\boxed{z(t) = \frac{-R'(t)}{R(t)} = \frac{f(t)}{1-F(t)}}$$

Conditional reliability:

It is the reliability of a system following a wear-in period (burn-in period) (or after a warranty period.

$$R(t/t_0) = \frac{R(t_0+t)}{R(t_0)}$$

$$\boxed{R(t/t_0) = e^{-\int_{t_0}^{t_0+t} z(t)\,dt}}$$

Note:

*) Failure rate $Z(t)$ = $\dfrac{\text{No. of failures during a particular unit interval}}{\text{Average population during that interval.}}$

No.of survival

Mean time to failure (MTTF):

*) It is the mean of the life distribution. (i.e) average life length of all the units in the population.

*) It is used for products that have only one life, that is, not repairable.

*) If the product is repairable, the term mean time btn failures (MTBF) is used to denote the average time btn two successive component failures.

$$\boxed{MTTF = E(T) = \int_0^\infty R(t)\,dt}$$

Relation btn MTTF and MTBF:

Let there are n-components in a system and m_i is the MTTF of the i^{th} component. If each of the component is replaced (not repaired) on failure, then

$$\boxed{\frac{1}{MTBF} = \sum_{i=1}^{n} \frac{1}{m_i}}$$

Note:

1) If the system is first operated with all new components, then the MTTF and MTBF are identical.

2) For the useful life, MTTF = MTBF

3) If the repair time is negligible then MTBF $\simeq$ MTTF.

4) For a constant failure rate λ,

$$MTTF = \frac{1}{\lambda} = MTBF.$$

5) $R(t) = e^{-\lambda t}$, $\lambda \rightarrow$ Failure rate

and $\lambda = \frac{1}{MTTF}$.

Problems

Problem 1: What is the reliability of a component for an operating period of 200 hours, having a failure rate of 0.4×10^{-5} failures per hour? If 5000 such items are tested, how many items fail in 200 hours?

Solution:

Failure rate $\lambda = 0.4 \times 10^{-5}$ failures/hour

Operating period $t = 200$ hours

Reliability of the component $= R(t) = e^{-\lambda t}$

Reliability of the component $= R(t) = e^{-\lambda t}$

$$R\ (200) = e^{-(0.4 \times 10^{-5})(200)}$$

$$= 0.9992$$

Let N_o be the total number of components in the test = 5000 items

N_s be the number of survival components

N_f be the number of failed components

$\therefore N_s = N_o\ R(t)$

$= 5000\ (0.9992)$

$= 4996$ items

also $N_f = N_o - N_s$

$= 5000 - 4996 = 4$ items

Hence, if 5000 items are tested, 4 items will fail in 200 hours.

Problem 2: Determine the MTTF for a mission time of 1000 hours life if the test data on 10 such components gave items to fail as shown in table. Also find the reliability of the system for 1000 hours.

Component number	1	2	3	4	5	6	7	8	9	10
Time to failure in hours	807	820	810	875	900	837	850	790	866	815

Solution:

$$\text{MTTF} = \frac{1}{n}\sum_{i=1}^{n} t_i \text{, since the sample is small}$$

$$\text{MTTF} = \frac{1}{n}\sum_{i=1}^{n} t_i \text{, since the sample is small}$$

$$= \frac{1}{10}\left[807+820+810+875+900+837+850+790+866+815\right]$$

$$= 837$$

Reliability of the component $R(t) = e^{-\lambda t}$

where λ is the failure rate $= \frac{1}{\text{MTTF}} = \frac{1}{837} = 0.0012$ failures/hr

$\therefore$ $R(1000) = e^{-(0.0012)(1000)} = 0.3012$

Problem 3: A company manufactures a laptop for the use of people. The time to failure in years, of this laptop has the pdf

$$f(\mathrm{t}) = \frac{200}{(\mathrm{t}+10)^3}, \mathrm{t} \geq 0.$$

(i) Find the reliability function and determine the reliability for the first year of operation.

(ii) Find the Mean time to failure.

(iii) What is the design life for reliability 0.90?

(iv) Will a one year burn-in period improve the reliability in (i)? If so, find the new reliability.

Solution:

(i) Given that $f(t) = \dfrac{200}{(t+10)^3}, t \geq 0$

$$\text{Reliability} = R(t) = \int_t^{\infty} f(t)\,dt = \int_t^{\infty} \frac{200}{(t+10)^3}\,dt, t > 0 = \left[\frac{-100}{(t+10)^2}\right]_t^{\infty} = \frac{100}{(t+10)^2}$$

Reliability for the first year of operation = R (1)

$$\therefore R(1) = \frac{100}{(1+10)^2} = 0.8264$$

(ii) Mean time to failure $= \text{MTTF} = \int_0^\infty R(t)\, dt$

$$= \int_0^\infty \frac{100}{(t+10)^2}\, dt$$

$$= \left(-\frac{100}{(t+10)} \right)_0^\infty$$

$= 10$ years

(iii) Design life is the time to failure (t_f) that corresponds to a specified reliability. Now we find t_f corresponding to R = 0.90.

$$\therefore \quad \frac{100}{(t_f + 10)^2} = 0.90$$

$$(t_f + 10)^2 = \frac{100}{0.90}$$

$$\therefore \quad t_f = 0.5409 \text{ year} \quad \text{or} \quad 197 \text{ days.}$$

(iv) We have the conditional reliability $R(t/t_0) = \frac{R(t_0+t)}{R(t_0)}$

$$\therefore \quad R(t/1) = \frac{R(t+1)}{R(1)} = \frac{100}{(t+11)^2} \times \frac{11^2}{100} = \frac{121}{(t+11)^2}$$

Now, $R(t/1) > R(t)$, if $\frac{121}{(t+11)^2} > \frac{100}{(t+10)^2}$

i.e., if $\frac{(t+10)^2}{(t+11)^2} > \frac{100}{121}$

i.e., if $\frac{t+10}{t+11} > \frac{10}{11}$

i.e., $11t > 10t$ which is true, as $t \geq 0$.

Hence one year burn-in period will improve the reliability.

New reliability $= R(1/1) = \frac{121}{(1+11)^2} = 0.8403$

Problem 4: The density function of the time to failure of an appliance is $f(t) = \frac{32}{(t+4)^3}, t > 0$ is in years.

(i) Find the reliability function $R(t)$,

(ii) Find the failure rate and

(iii) Find the MTTF.

Solution:

(i) Here $f(t)=\dfrac{32}{(t+4)^3}, t>0$

$$R(t)=\int_t^{\infty} f(t)\,dt=\int_t^{\infty}\frac{32}{(t+4)^3}\,dt$$

$$=\left[-\frac{16}{(t+4)^2}\right]_t^{\infty}=\frac{16}{(t+4)^2}$$

(ii) Failure rate $\lambda = \frac{-R'(t)}{R(t)} = \frac{2}{(t+4)}$

(iii) $$\text{MTTF} = \int_0^\infty \mathrm{R}(t)\, dt$$

$$= \int_0^\infty \frac{16}{(t+4)^2}\, dt$$

$$\left(-\frac{16}{t+4}\right)_0^\infty = 4 \text{ years}$$

System Reliability

All systems are made up of parts and components ~~may~~ assembled to perform a certain function.

$\boxed{\text{Configuration of a system}}$:

- ↳ Series Configuration. } → Simple system.
- → Parallel configuration. }
- → Mixed Configuration. → Complex system.

Simple system:

Reliability of the entire system is called as system reliability. The entire system reliability is calculated by the reliability of each component.

Complex system:

Reliability of the entire system is called as system reliability.

*) In this case, we first decompose them into subsystem, each report a specific function.

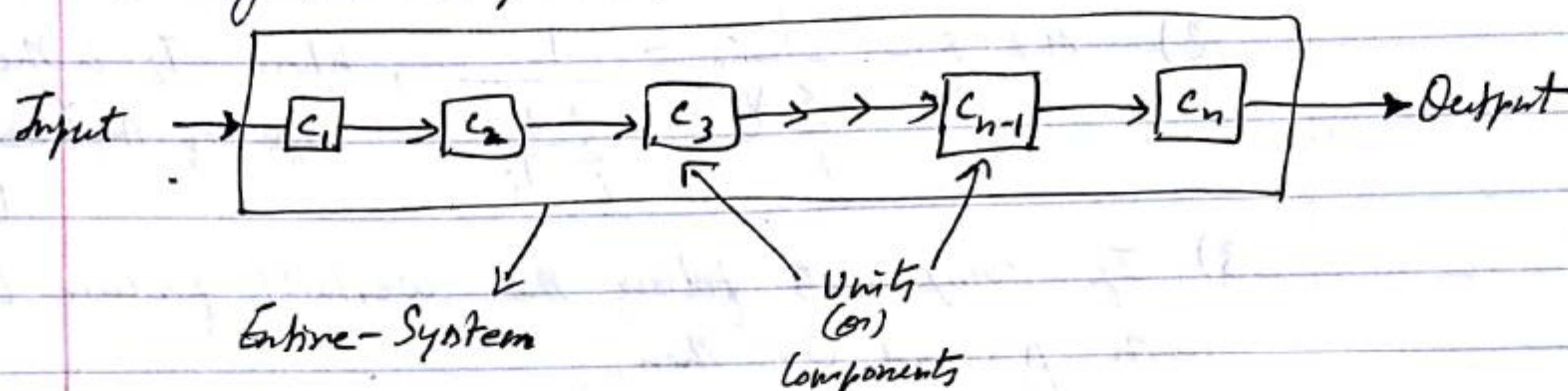
*) Reliabilities of system are then estimated and combined to determine the reliability of the entire system using certain probability laws.

Series Configuration:

The entire system will fail if any one of its component fails.

(or)

The entire system will work if all of its components work.



Consider, units independent of each other.

Let $X_i \rightarrow$ Successful operations of an individual components.

$$P(S) = P(X_1)P(X_2)\cdots P(X_n)$$

(i.e) $$\boxed{R(S) = R(X_1)\cdot R(X_2)\cdots R(X_n).}$$

Let $R_s(t) \rightarrow$ Reliability of entire system

$R_i(t) \rightarrow$ " " i^{th} component.

Then, $$\boxed{R_s(t) = R_1(t)\,R_2(t)\cdots R_n(t)}.$$

(or) $$\boxed{R_s(t) = \prod_{i=1}^{n} R_i(t)}.$$

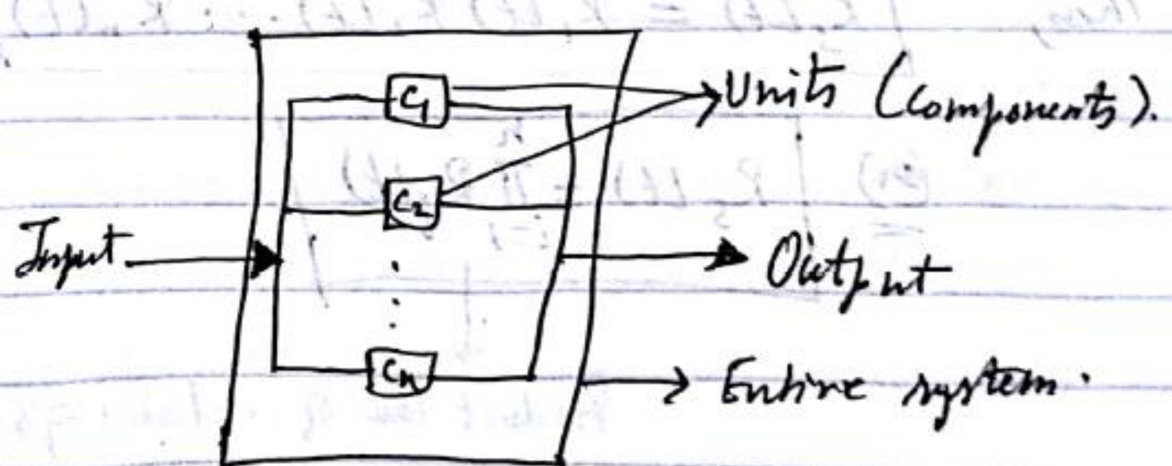
Product law of reliability's.

Parallel configuration:

Entire system will fail only when ~~the~~ all the units in the system fails.

(or)

Entire system will function even when any one of the component (~~unit~~) operates succesfully.



Let $x_i \rightarrow$ successful operation of the individual units.
Let $\overline{x}_i \rightarrow$ ~~~~ unsuccessful operation " " ".

Consider, units are independent to each other.

$$P(\overline{S}) = P(\overline{x}_1) \cdot P(\overline{x}_2) \cdots P(\overline{x}_n).$$

$$\boxed{1 - P(\overline{S}) = P(S)} \rightarrow \text{we know.}$$

$$\therefore\ P(S) = 1 - \left[P(\overline{x}_1) \cdot P(\overline{x}_2) \cdots P(\overline{x}_n)\right].$$

$$P(S) = 1 - \left[(1 - P(x_1)) \cdot (1 - P(x_2)) \cdots (1 - P(x_n))\right].$$

$$\text{(i.e.)}\ R_s(t) = 1 - \left[(1 - R_1(t)) \cdot (1 - R_2(t)) \cdots (1 - R_n(t))\right].$$

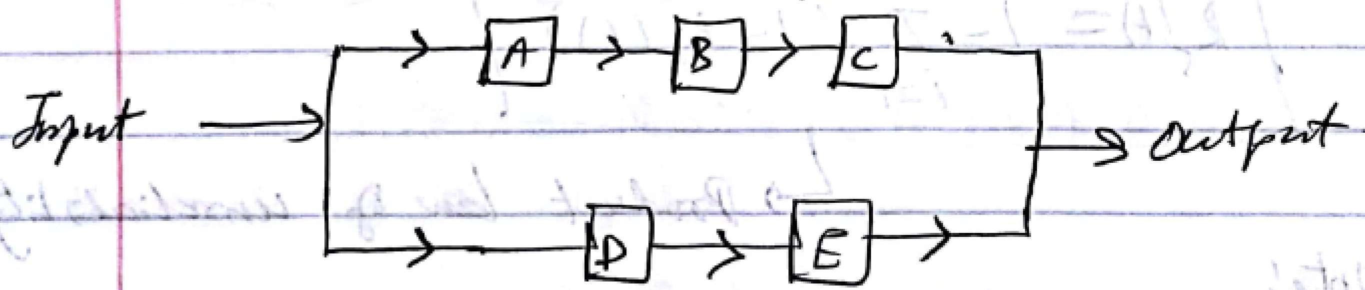
$$\boxed{R_s(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))}.$$

$\rightarrow$ Product law of unreliability.

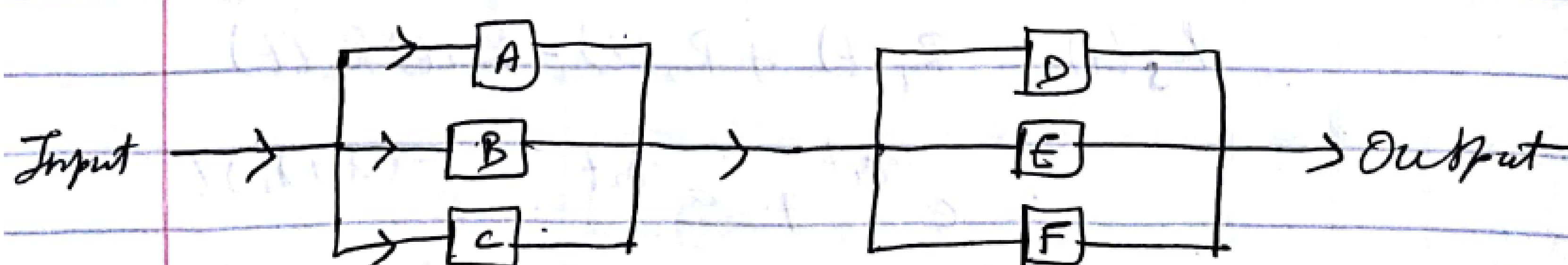
Mixed Configuration:

System connected in series and parallel.

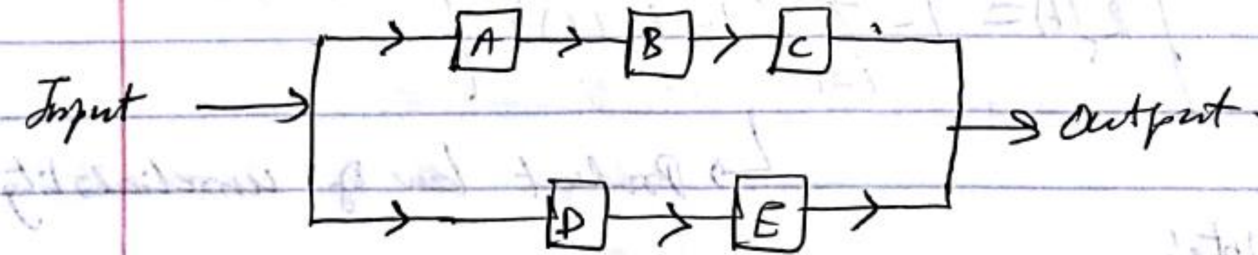
Analysed by considering each subsystem and then the whole system.



Series - parallel (Fig. 1)



Parallel series. (Fig. 2)



Series - parallel (Fig. 1)

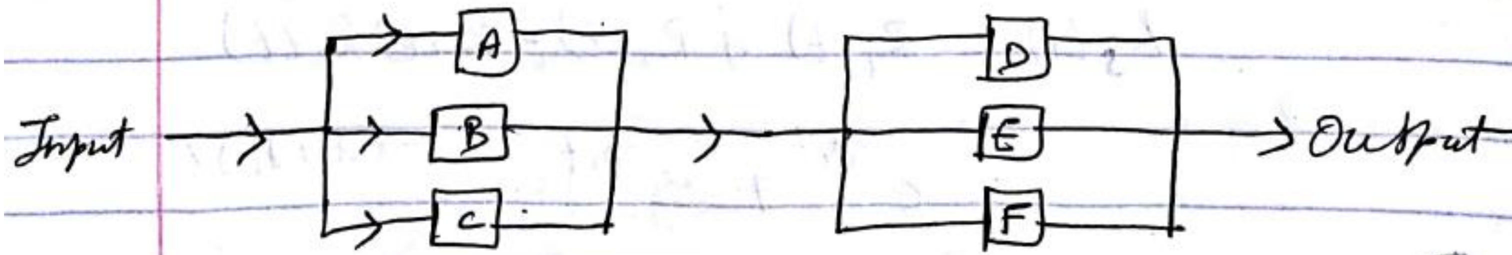
Series - parallel:

X - subsystem [A, B and C - components]
Y - subsystem [D and E - components].

Now X and Y are parallel.

$$R_S = 1 - (1 - R_X)(1 - R_Y).$$

$$= 1 - \left[\underbrace{(1 - \underbrace{R_A R_B R_C}_{\text{series}})}_{\text{Parallel}}\underbrace{(1 - \underbrace{R_D R_E}_{\text{series}})}_{\text{Parallel}}\right].$$



Parallel series. (Fig. 2)

Parallel - series:

$X \rightarrow$ subsystem (A, B and C).

$Y \rightarrow$ subsystem (D, E and F).

Now, $R_s(t) = R_X \cdot R_Y$.

$$= \left[1-(1-R_A)(1-R_B)(1-R_C)\right] \times \left[1-(1-R_D)(1-R_E)(1-R_F)\right].$$

Note:

Fig. 1 $\rightarrow$ High-level redundancy.

Fig. 2 $\rightarrow$ Low-level redundancy.

Problems

Prob 1: A system has 100 units in series, each has a reliability of 0.98. What is the reliability of the system?

Solution:

Since the system has constant reliability,

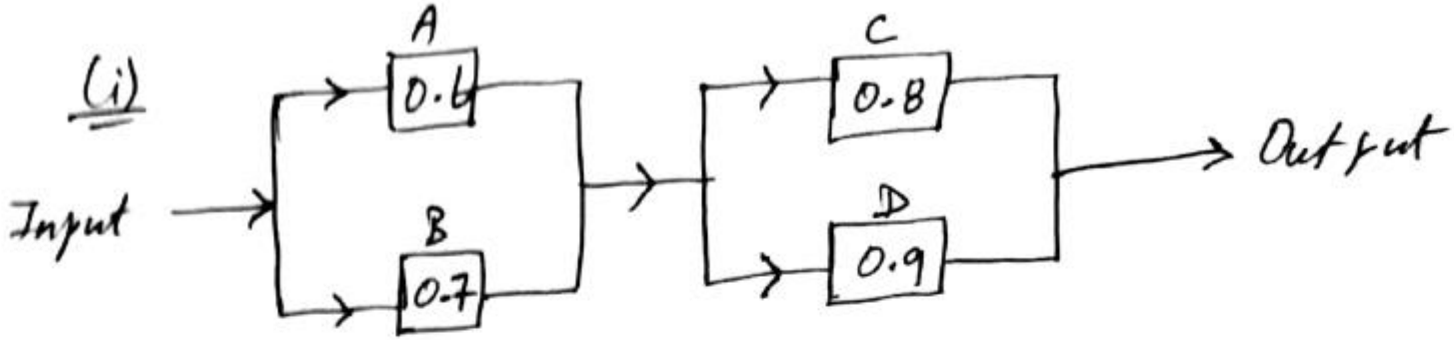
$$R_s(t) = \prod_{i=1}^{n} R_i(t) = p^n$$

$$= (0.98)^{100}$$

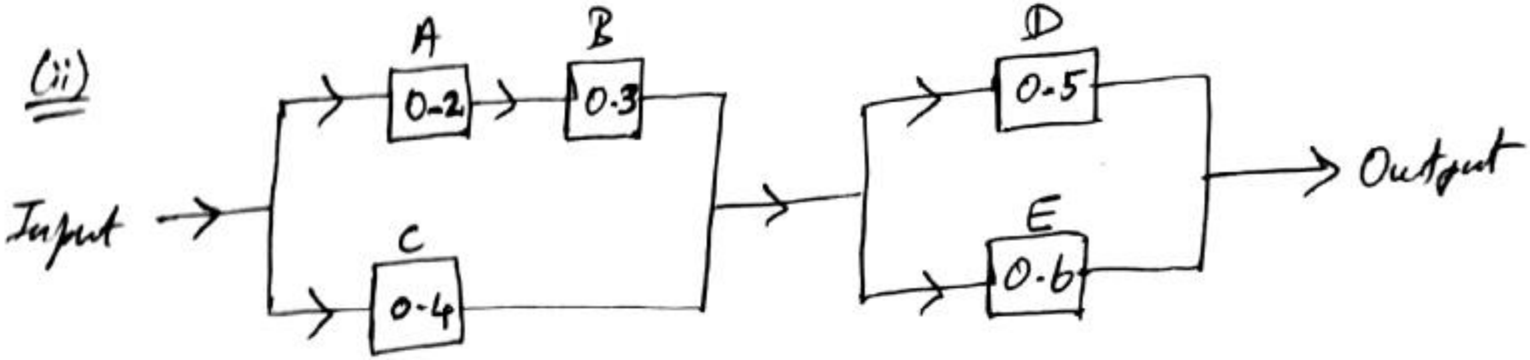
$$= 0.1326$$

Prob 2: Obtain the reliability of the following systems whose block diagrams are given below:

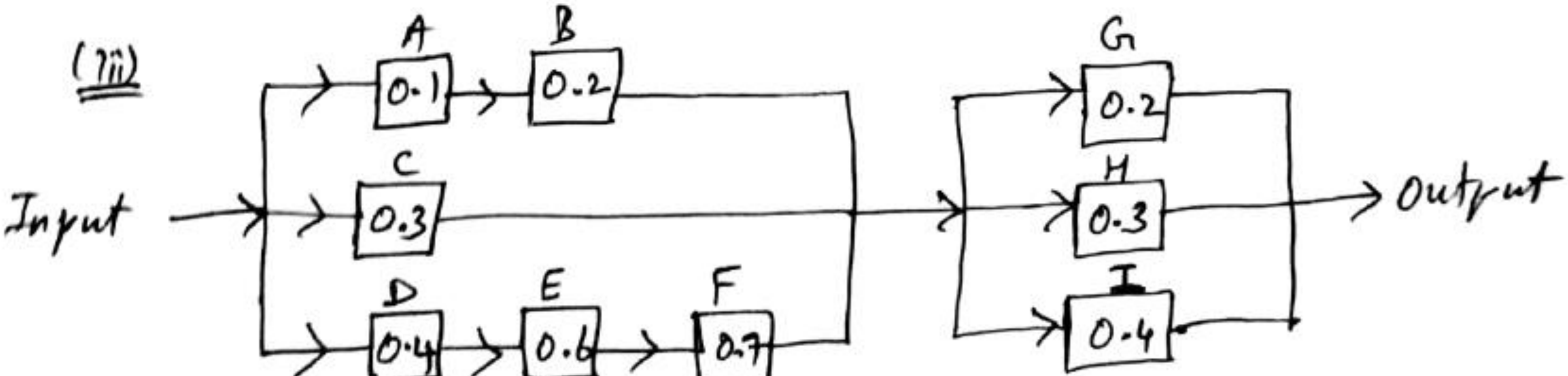
(i)



(ii)

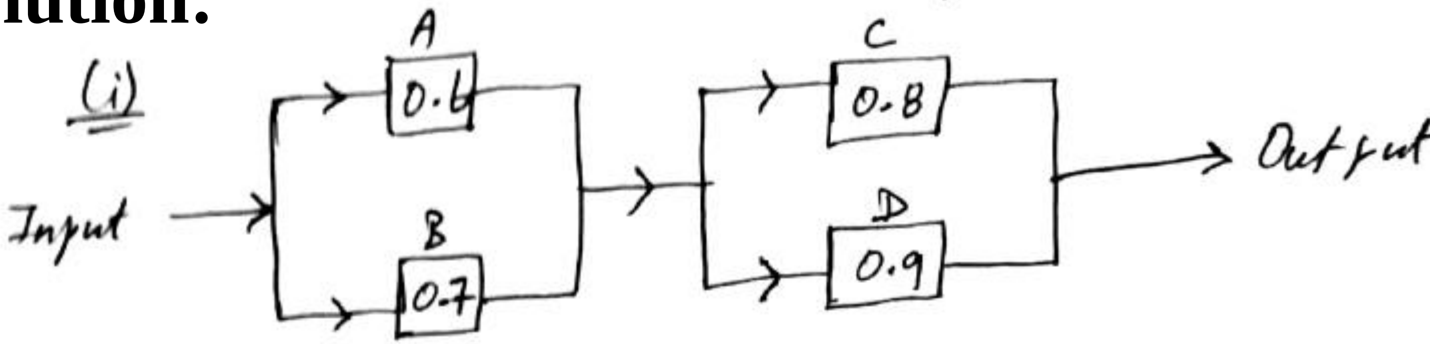


(iii)



Solution:

(i)



(i) $R_{ABCD} = (R_{AB})(R_{CD})$

$$= \left[1-(1-R_A(t))(1-R_B(t))\right]$$

$$\left[1-(1-R_C(t))(1-R_D(t))\right]$$

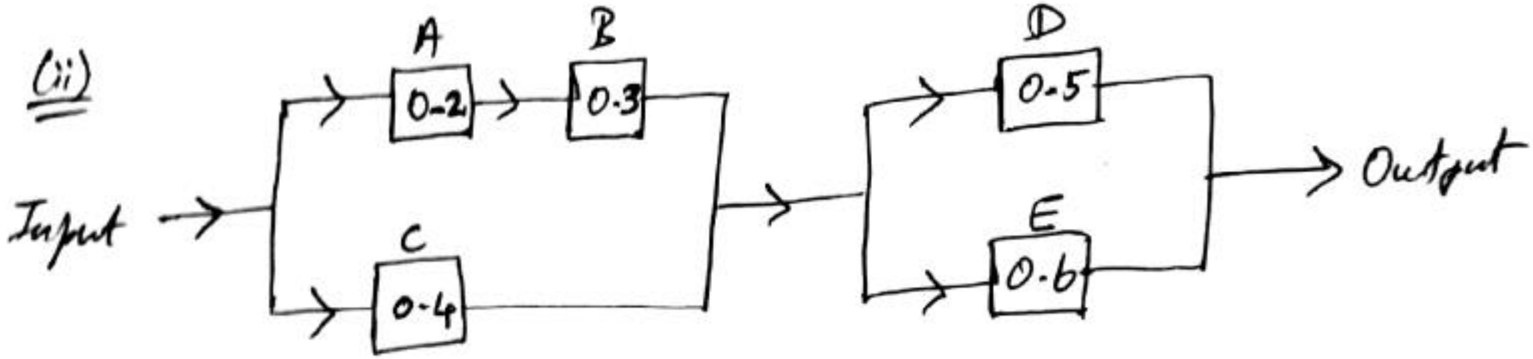
$$= \left[1-(1-0.6)(1-0.7)\right]\left[1-(1-0.8)(1-0.9)\right].$$

$$= \left[1-(0.94)(0.93)\right]\left[1-(0.92)(0.91)\right].$$

$$= (0.88)(0.98)$$

$$\boxed{R_{ABCD} = 0.8624}$$

(ii)



(ii) $R_{ABCDE} = (R_{ABC})(R_{DE}).$

$$= \left[1 - (1 - R_{AB})(1 - R_C(t))\right]$$
$$\left[1 - (1 - R_D(t))(1 - R_E(t))\right]$$

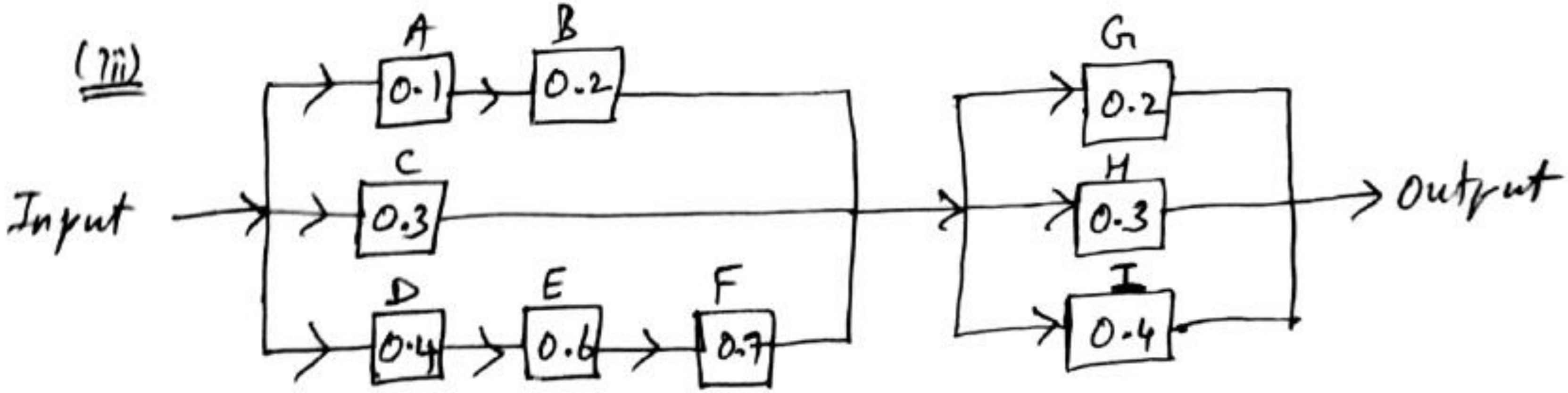
$$= \left[1 - (1 - R_A(t) \cdot R_B(t))(1 - R_C(t))\right]$$
$$\left[1 - (1 - R_D(t))(1 - R_E(t))\right]$$

$$= \left[1 - (1 - (0.2)(0.3))(1 - 0.4)\right]$$

$$\left[1 - (1 - 0.5)(1 - 0.6)\right]$$

$$= (0.436)(0.8)$$

$$\boxed{R_{ABCDE} = 0.3488}$$



(iii) $R_{ABCDEFGHI} = R_S(t)$

Now

$$R_S(t) = (R_{ABCDEF})(R_{GHI})$$

$$= \left[1 - (1-R_{AB})(1-R_C)(1-R_{DEF})\right]\left[1-(1-R_G)(1-R_H)(1-R_I)\right].$$

$$= \left[1 - (1 - R_A(t)\cdot R_B(t))(1-R_C(t))(1-R_D(t)R_E(t)R_F(t))\right]$$

$$\left[1 - (1-R_G(t))(1-R_H(t))(1-R_I(t))\right].$$

$$= \left[1 - (1-(0.1)(0.2))(1-0.3)(1-(0.4)(0.6)(0.7))\right]$$

$$\left[1 - (1-0.2)(1-0.3)(1-0.4)\right].$$

$$= (0.4293)(0.664)$$

$$\boxed{R_{S}(t) = 0.2851}$$

u.

Reliability based on Constant Failure Rate in Series Configuration

If each component has a constant failure rate λ_i, then

$$R_S(t) = e^{-\lambda_1 t}\, e^{-\lambda_2 t} \cdots \cdot e^{-\lambda_n t}$$

$$= e^{-(\lambda_1 + \lambda_2 + \cdots + \lambda_n) t}$$

$$= e^{-\lambda_S t}$$

and pdf $f(t) = -R'(t) = -\lambda_S e^{-\lambda_S t}$

Reliability based on Constant Failure Rate in Parallel Configuration

If two component system in Parallel having constant failure rate λ_1 & λ_2 then

$$R_p(t) = 1 - (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})$$

or

$$R_p(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2) t}$$

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

Problems:-

1) An electronic circuit consists of 5 silicon transistors, 3 silicon diodes, 10 composition resistors and 2 ceramic capacitors connected in series configuration. The hourly failure rate of each component is given below:

Silicon transistor	:	$\lambda_t = 4 \times 10^{-5}$
Silicon diode	:	$\lambda_d = 3 \times 10^{-5}$
Composition resistor	:	$\lambda_r = 2 \times 10^{-4}$
Ceramic capacitor	:	$\lambda_c = 2 \times 10^{-4}$

Calculate the reliability of the circuit for 10 hours, when the components follow exponential distribution.

Sol:-

The components are connected in Series

∴ The system reliability is

$$R_s(t) = R_1(t) R_2(t) R_3(t) R_4(t)$$

$$= e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t} e^{-\lambda_4 t}$$

$$= e^{-5\lambda_t t} \cdot e^{-3\lambda_d t} \cdot e^{-10\lambda_r t} \cdot e^{-2\lambda_c t}$$

$$= e^{-(5\lambda_t + 3\lambda_d + 10\lambda_r + 2\lambda_c) t}$$

$$R_s(10) = e^{-(20\times10^{-5} + 9\times10^{-5} + 20\times10^{-4} + 4\times10^{-4})\times10}$$

$$R_s(10) = e^{-(20+9+200+40)\times 10^{-4}}$$

$$= e^{-0.0269}$$

$$R_s(10) = 0.9735$$

2) A system consists of 4 identical components connected in parallel. find the reliability of each component if the overall reliability of the system is to be 99%.

Sol:-

Let r be the reliability of each component

then $R_p = 1 - (1-r)(1-r)(1-r)(1-r)$

$= 1 - (1-r)^4$

$$1-(1-r)^4 = 0.99$$

$$(1-r)^4 = 0.01$$

$$1-r = 0.316$$

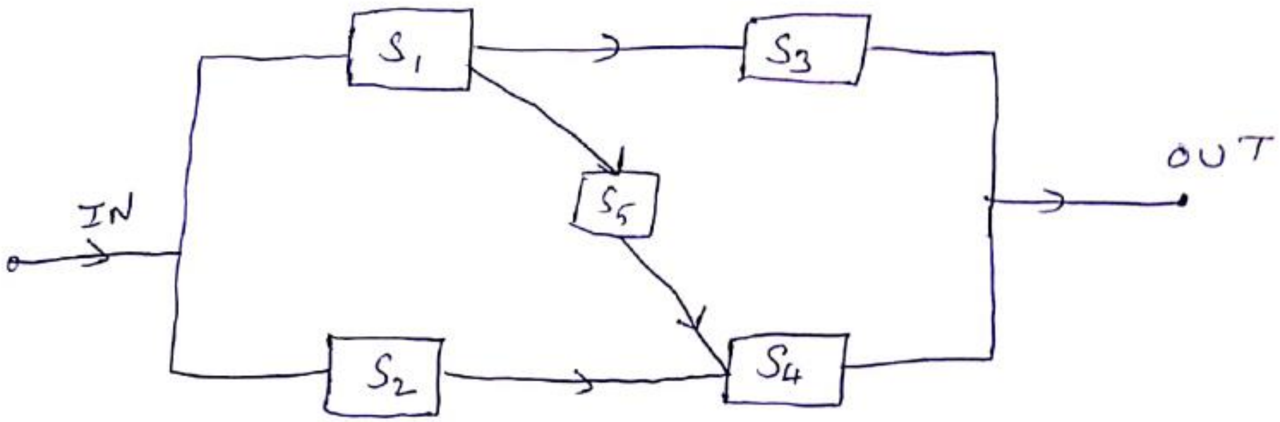
$$r = 0.684$$

or

$$r = 68.4\%$$

Non-Series Parallel System

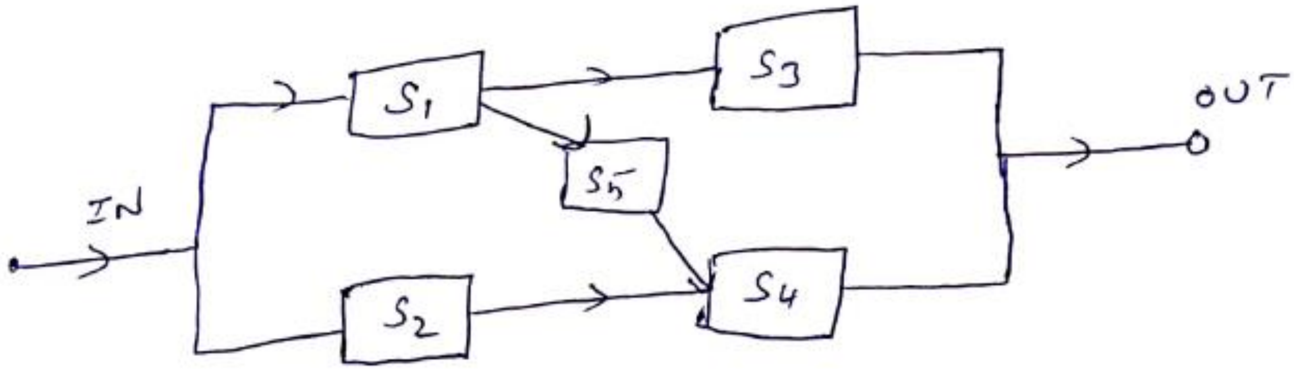
In practice, systems are not always simple series or parallel structures. On simplification of complex system can produce a non-series parallel structure. The most simple non-series parallel structure is a bridge configuration shown in following figure.



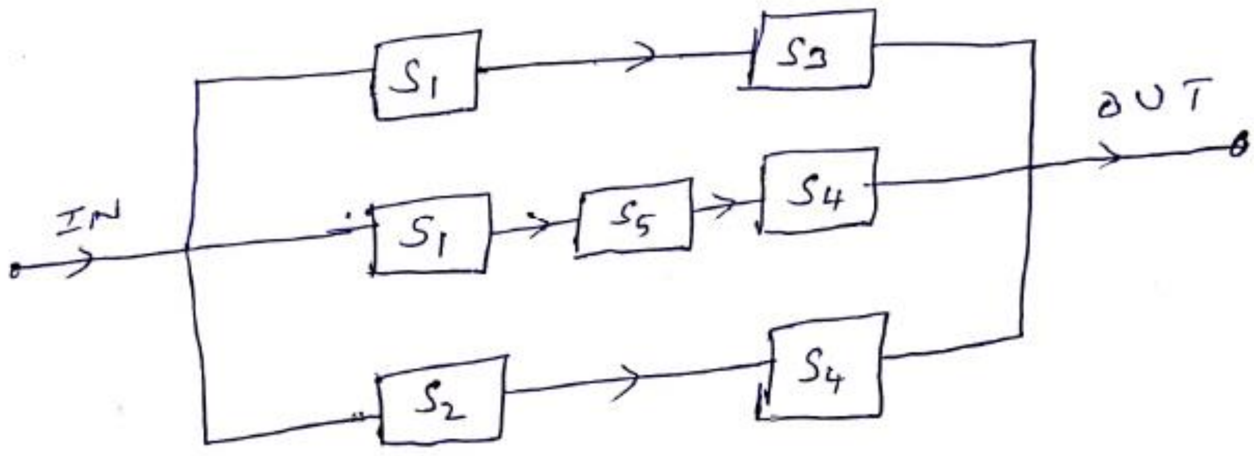
where $S_1, S_2, S_3, \ldots$ are the subsystems and the arrows indicate the flow of information. Such cases can be handled by using another approach known as the logic-diagram technique.

The logic-diagram approach converts the system diagram into a logic diagram which consists of a number of simple parallel paths between IN and OUT Terminals.

Bridge network



Logic diagram for Bridge network



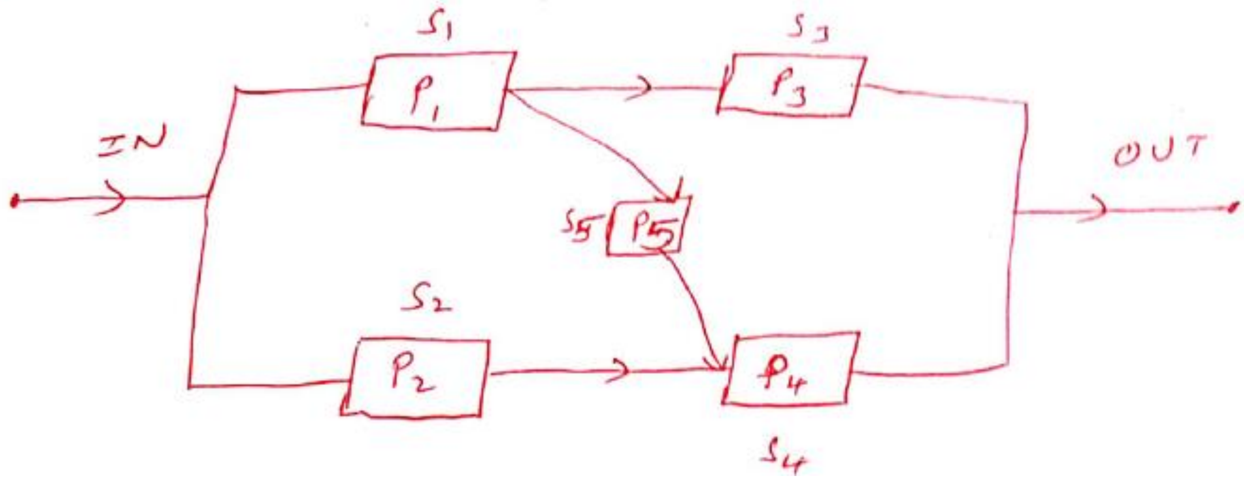
The logic diagram is a simple series parallel system whose reliability can be evaluated by combining the models. The failure of each subsystem is assumed to be independent, but failure of the paths are not independent, since some subsystems find place in more than one path. The use of the following rule takes care of this problem.

If $P_r(E_1) = P_1 P_2$ and $P_r(E_2) = P_2 P_3$ then

$$P_r(E_1) \times P_r(E_2) = P_1 P_2 P_3$$

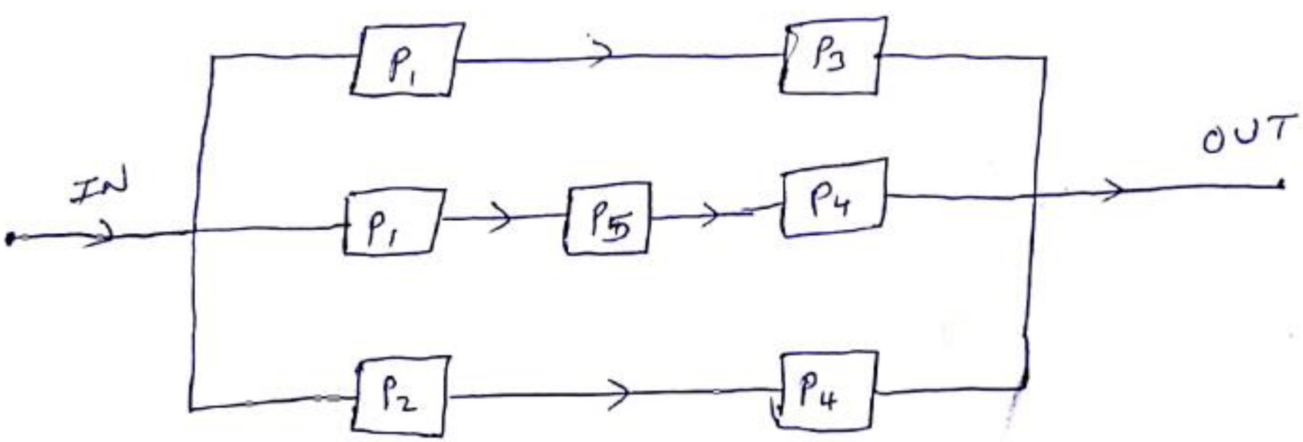
(i.e.) $P_i \times P_i = P_i \quad \forall i$

1) Calculate the reliability of given network.



Sol:-

The logical diagram of the given network is:



The reliability of first path is.

$$R_1 = P_1 P_3$$

Similarly, the reliability of 2^{nd} & 3^{rd} path are

$$R_2 = P_1 P_5 P_4$$

$$R_3 = P_2 P_4$$

$\therefore$ The reliability of the entire structure is

$$R = 1 - (1-R_1)(1-R_2)(1-R_3)$$

$$= R_1 + R_2 + R_3 - R_1R_2 - R_2R_3 + R_1R_2R_3$$

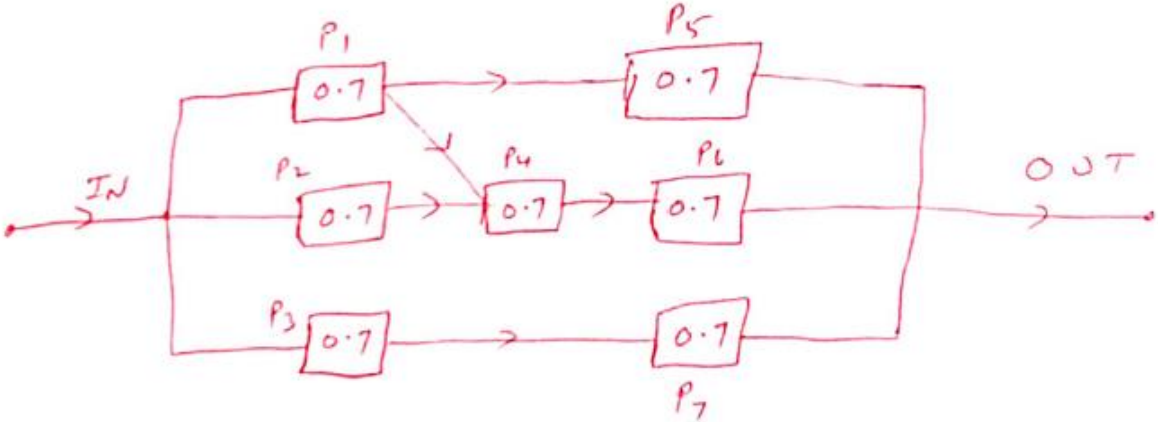
$$= P_1P_3 + P_1P_5P_4 + P_2P_4 - (P_1P_3)(P_1P_5P_4)$$

$$- (P_1P_5P_4)(P_2P_4) + (P_1P_3)(P_1P_5P_4)(P_2P_4)$$

$$R = P_1P_3 + P_1P_5P_4 + P_2P_4 - P_1P_3P_5P_4$$

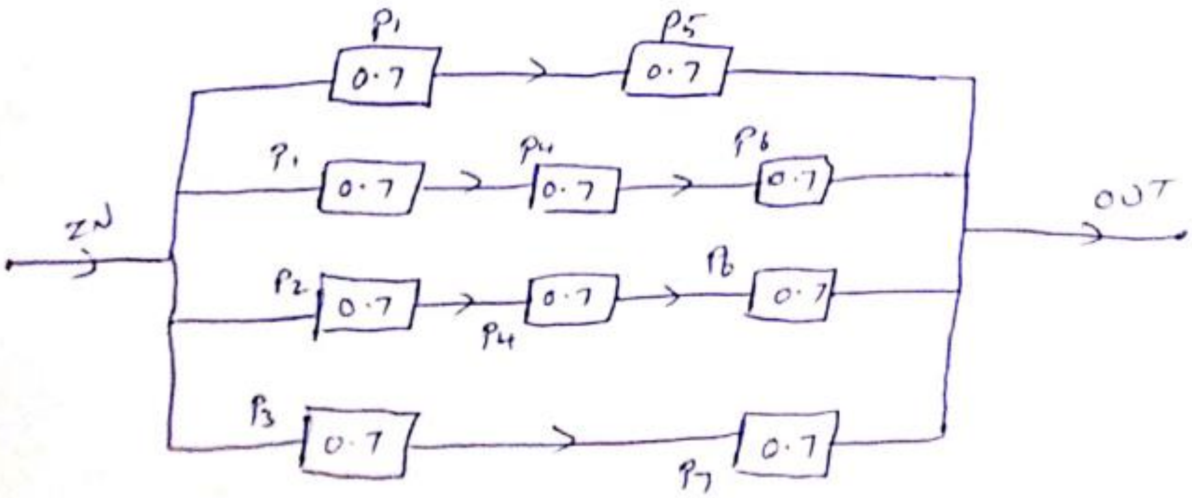
$$- P_1P_2P_4P_5 + P_1P_2P_3P_4P_5$$

2) Calculate the reliability of given network:-



Sol:-

The logical diagram of given network is



The reliability of each path is

$$R_1 = P_1 P_5$$

$$R_2 = P_1 P_4 P_6$$

$$R_3 = P_2 P_4 P_6$$

$$P_4 = P_3 P_7$$

$$R_S = 1 - (1 - R_1)(1 - R_2)(1 - R_3)(1 - R_4)$$

$$= R_1 + R_2 + R_3 + R_4 - R_1 R_2 - R_1 R_3 - R_1 R_4 - R_2 R_3$$

$$- R_2 R_4 - R_3 R_4 + R_1 R_2 R_3 + R_1 R_2 R_4$$

$$+ R_1 R_3 R_4 + R_2 R_3 R_4 - R_1 R_2 R_3 R_4$$

$$
\begin{aligned}
= {} & P_1P_5 + P_1P_4P_6 + P_2P_4P_6 + P_3P_7 - P_1P_4P_5P_6 \\
& - P_1P_2P_4P_5P_6 - P_1P_3P_5P_7 - P_1P_2P_4P_6 \\
& - P_1P_3P_4P_6P_7 - P_2P_3P_4P_6P_7 + P_1P_4P_5P_6 \\
& + P_1P_3P_4P_5P_6P_7 + P_1P_2P_3P_4P_5P_6P_7 \\
& + P_1P_2P_3P_4P_6P_7 - P_1P_2P_3P_4P_5P_6P_7
\end{aligned}
$$

$$= p^2 + p^3 + p^3 + p^2 - p^4 - p^5 - p^4 - p^4 - p^5 - p^5 + p^4$$

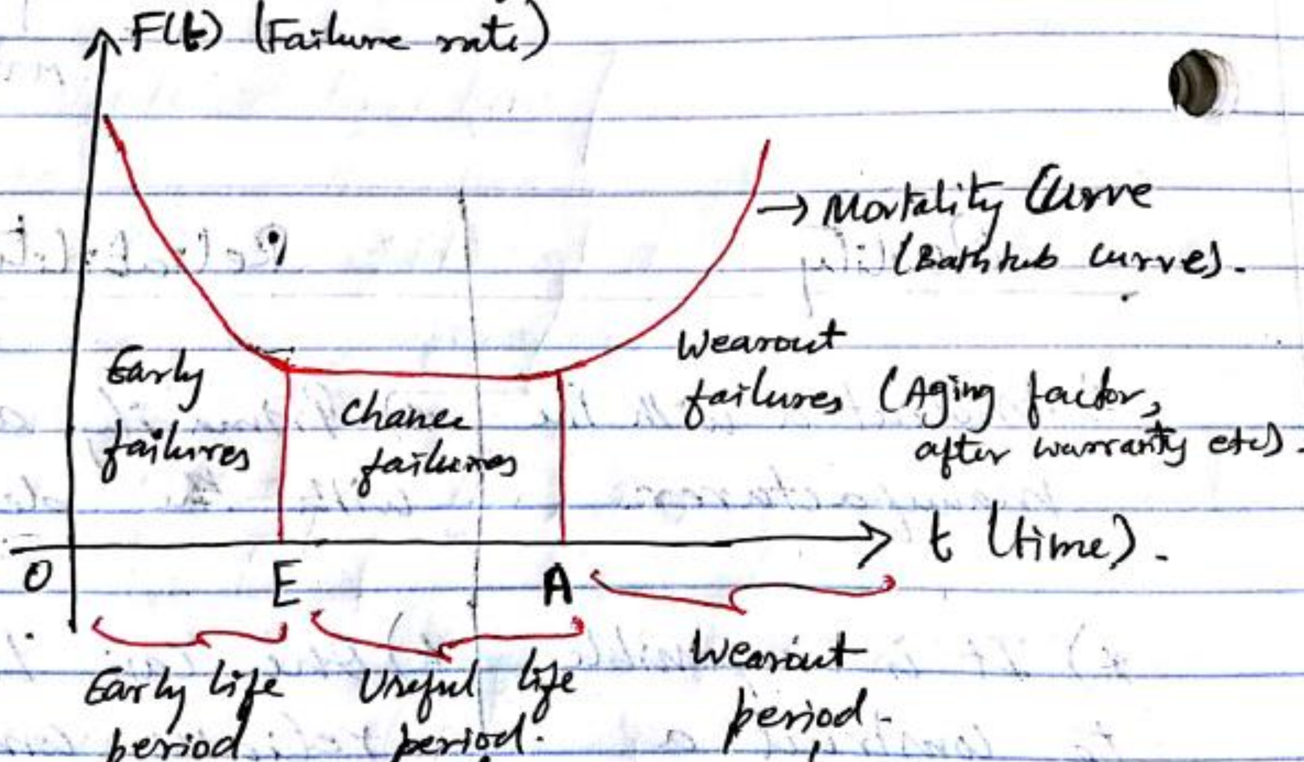
$$+ p^6 + p^7 + p^6 - p^7$$

$$= 2p^2 + 2p^3 - 2p^4 - 3p^5 + 2p^6$$

$$= 2(0.7)^2 + 2(0.7)^3 - 2(0.7)^4 - 3(0.7)^5 + 2(0.7)^6$$

=0.91

Failure → It is a function of time.



*) Failure rate is decreasing but it is high.

*) Due to poor design, poor materials etc.

*) Failure rate is approx constant

*) Failure is random and upredicable.

*) Failure rate increases due to aging of the system.

Aim:

To Reduce the early life period, by proper design, manufacture etc., and increase the useful life period by reducing the wear-out period.

Failure distributions:

The following probability distributions are more appropriate for failure process.

*) Exponential distribution.

*) Weibull "

*) Normal "

*) Gamma " (Erlang distribution).

General.

1) $\boxed{\text{Exponential distribution:}}$

*) p.d.f, $f(t) = \begin{cases} \lambda e^{-\lambda t}, & t \geq 0, \lambda > 0. \\ 0, & \text{otherwise.} \end{cases}$

Now, $R(t) = \int_t^\infty f(t) \cdot dt = e^{-\lambda t}.$

$\Rightarrow \boxed{R(t) = e^{-\lambda t}}$

*) Hazard rate, $z(t) = \dfrac{f(t)}{R(t)} = \lambda$

*) If $z(t)$ is known, then $f(t)$ can be found by,

$$f(t) = z(t)\, e^{-\int_0^t z(t)\,dt} \qquad [\because z(t) = \lambda]$$

$$\Rightarrow \quad f(t) = \lambda e^{-\lambda t}, \quad t \geq 0.$$

*) $MTTF = MTBF = \dfrac{1}{\lambda}$

*) $Var(T) = \dfrac{1}{\lambda^2}$

*) $R(t/t_0) = e^{-\lambda t}$

2) Weibull distribution:

p.d.f is, $f(t) = \begin{cases} \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}, & t \geq 0, \ \alpha > 0, \ \beta > 0. \\ 0, & \text{otherwise}. \end{cases}$

$\beta \to$ shape parameter

$\theta \to$ scaled parameter

Let $\boxed{\alpha = \frac{1}{\theta^{\beta}}}$ Then, $f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} e^{-(t/\theta)^{\beta}}$

$\beta, \theta > 0, \quad t \geq 0.$

*) $R(t) = e^{-(t/\theta)^{\beta}}$.

*) $z(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$

*) $R(t|t_0) = \exp\left[-\left(\frac{t+t_0}{\theta}\right)^{\beta} + \left(\frac{t_0}{\theta}\right)^{\beta}\right]$.

*) $MTTF = \theta\,\Gamma\left(1+\frac{1}{\beta}\right)$

*) $Var(T)$

$= \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \left\{\Gamma\left(1+\frac{1}{\beta}\right)\right\}^2\right]$

3) Normal distribution:

$$\text{p.d.f}, \; f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}, \; -\infty < t < \infty$$

*) $z(t) = f(t) / R(t)$.

*) $R(t) = \int_t^{\infty} f(t)\,dt$ is found by expressing the integral in terms of standard normal integral and using the normal tables.

*) $MTTF = \mu$. and $Var(T) = \sigma^2$.

*) For Reliability of a normal failure law,

$$R(t) = 1 - P(T \leq t)$$

$$R(t) = 1 - \phi(z) = 1 - \phi\left(\frac{t-\mu}{\sigma}\right).$$

4) $\boxed{\text{General gamma distribution:}}$

p.d.f, $f(t) = \begin{cases} \dfrac{t^{\alpha-1}\lambda^{\alpha} e^{t/\beta}}{\Gamma\alpha}, & t, \alpha, \beta \geq 0. \\ 0, & \text{otherwise} \end{cases}$

*) $R(t) = \sum_{i=0}^{\alpha-1} \dfrac{(\lambda t)^i}{i!} e^{-\lambda t}$

*) $MTTF = \alpha/\lambda$ *) $z(t) = f(t)/R(t)$.

Note:

1) If n-components (Independt) having exponential failures with $\lambda_1, \lambda_2, \ldots, \lambda_n$ then

$$MTTF = \frac{1}{\sum_i \lambda_i}$$

2) $$MTTF = \frac{1}{\sum_i \lambda_i} = \frac{1}{\sum_i \frac{1}{T_i}}$$, where T_i is the mean life of the i^{th} component.

3) If components follow the weibull failure law, with β_i and θ_i then,

$$R_s(t) = e^{-\sum_i (t/\theta_i)^{\beta_i}}$$

Note!

1) If failure rate is exponentially distb. Then,

$$R_s(t) = R_1(t) + R_2(t) - R_1(t)R_2(t)$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

for the case of 2-components.

2). $MTTF = E(T) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1+\lambda_2}$

for the case of 2-components.

3) The property of parallel system is called "redundancy."

↳ A system operates in case of failure by the help of existence of additional components. (Parallel paths in a system).

Problems

Problem 1: If T, the time to failure, is exponentially distributed, with hazard rate equal to 0.01 and if $R(T)$ equals to 0.90, find approximately the hours of operation.

Solution:

$f(t) = \lambda e^{-\lambda t}$ $\quad = e^{-\lambda t}$

If $\cancel{f(t) = \frac{1}{\beta} e^{-\frac{t}{\beta}}}$ for $t \geq o$, $R(t) = \cancel{e^{-\frac{t}{\beta}}}$ and hazard rate $= 0.01$

$$\boxed{\lambda = \frac{1}{\beta}}$$

$$R(t) \doteq e^{-(0.01)t}$$

i.e., $\boxed{0.9 = e^{-(0.01)t}}$

$-(0.01)t = \log(0.9) = -0.1054$

$\therefore \quad \boxed{t = 10.54 \text{ hours}}$

This means, that if each of 100 such components are operating for 10.54 hours, roughly 90 will not fail during that period of time.

Problem 2: 18 The life time in hours of a component is a random variable X which follows a Weibull distribution with $\alpha = 0.1,\ \beta = 0.5$. Obtain (i) mean life time of these components (ii) the probability that such a component will last more than 300 hours.

Solution: (i) For Weibull distribution, Mean = MTTF = $E(T)$

$$= \alpha^{-\frac{1}{\beta}}\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$= (0.1)^{-2}\ \Gamma\ (3)$$

$$= 100 \times 2 = 200 \text{ hours}$$

(ii) We have $R\,(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$ where $\alpha = \dfrac{1}{\theta^{\beta}}$

$\therefore\quad R\,(t) = e^{-\alpha t^{\beta}}$

Hence $R\,(300) = e^{-(0.1)\,(300)^{0.5}}$

$= 0.177$

Problem 3: Determine the average life length of a component obeying normal failure law with standard deviation of 10 hours if the operational reliability is to be 0.99 for an operation period of 100 hours.

Solution: Here $R(t) = 0.99$, $t = 100$ hours, $\sigma = 10$ hours

Average life $= m$ is required.

$m = \mu = \text{mean}.$

We have $R(t) = 1 - z\left(\frac{t - m}{\sigma}\right)$

$z = \frac{x - \mu}{\sigma}$

$\therefore z\left(\frac{t-m}{\sigma}\right) = 1 - 0.99$

$= 0.01$

From normal tables we have,

$$\boxed{\frac{t - m}{\sigma} = -2.33}$$

i.e., $\frac{100 - m}{10} = -2.33$

$\therefore$ $m = 123.3$ hours