

# **Properties of Expectation, Variance and Covariance**

# Properties of Expectation & Variance:

## Expectations:

$$(1) E(ax) = a E(x)$$

$$(2) E(x+y) = E(x) + E(y)$$

$$(3) E(x-y) = E(x) - E(y)$$

$$(4) E(x+by) = E(x) + bE(y)$$

$$(5) E((x+y)^2) = E(x^2) + E(y^2) + 2E(xy)$$

$$(6) (E(x+y))^2 = (E(x))^2 + (E(y))^2 + 2E(x)E(y)$$

## Variance :-

$$\textcircled{1} \text{ var}(ax) = a^2 \text{ var}(x)$$

$$\begin{aligned} \textcircled{2} \text{ var}(x-y) &= \text{var}(x) + \text{var}(-y) \\ &= \text{var}(x) + (-1)^2 \text{ var}(y) \end{aligned}$$

$$\text{var}(x-y) = \text{var}(x) + \text{var}(y)$$

When X and  
Y are  
independent

$$\textcircled{3} \text{ var}(x+y) = \text{var}(x) + \text{var}(y)$$

$$\textcircled{4} \text{ var}(ax+by) = a^2 \text{ var}(x) + b^2 \text{ var}(y) + 2ab \text{ cov}(x,y)$$

$$\textcircled{5} \text{ var}(ax-by) = a^2 \text{ var}(x) + b^2 \text{ var}(y) - 2ab \text{ cov}(x,y)$$

$$(5) \text{Var}(x_1 \pm x_2) = \text{Var}(x_1) + \text{Var}(x_2) \pm 2 \text{Cov}(x_1, x_2).$$

$$(6) E(xy) = \frac{\sum xy}{n}$$

$$E(x) = \frac{\sum x}{n}$$

$$E(y) = \frac{\sum y}{n}$$

## Problem:

Let  $X$  be a random variable with  $E[X] = 1$ , and  $E[X(X - 1)] = 4$ .

Find  $Var(X)$  and  $Var(2 - 3X)$ .

Sol:-

$X \rightarrow$  Random Variable

$$\boxed{E(X) = 1} \text{ and } E(X(X-1)) = 4$$

$$\Rightarrow E(X^2 - X) = 4$$

$$\Rightarrow E(X^2) - E(X) = 4$$

$$\Rightarrow E(X^2) - 1 = 4.$$

$$\Rightarrow \boxed{E(X^2) = 5}$$

Now  $\text{Var}(x) = E(x^2) - (E(x))^2$

$$= 5 - (1)^2 \Rightarrow \boxed{\text{Var}(x) = 4}$$

Now,

$$\begin{aligned}\text{Var}(2-3x) &= \text{Var}(2 + (-3)x) \\ &= \text{Var}(2) + \text{Var}(-3x) \\ &= 0 + (-3)^2 \text{Var}(x) \\ &= 9 \text{Var}(x) \\ &= 9(4)\end{aligned}$$

$$\boxed{\text{Var}(2-3x) = 36}$$