VIT
Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	**B.Tech**	Semester	**Winter Semester 2022-23**
Course Title	**PROBABILITY AND STATISTICS**	Course Code	**BMAT202L**
Faculty Name	**Prof. PROSENJIT**	Slot	A2+TA2
		Class Nbr	**CH2022235000995**
Time	3 **Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)
Answer any 10 questions

01. An incomplete frequency distribution is given as follows. **[10]**

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	a	65	b	25	18

Given that the median is 46 and the total frequency is 200, determine the missing frequencies.

02. (a) Find the mode for the following distribution: **[10]**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

(5Marks)

(b) The diameter of an electric cable says X is assumed to be a continuous random variable with function.

$$f(X) = \begin{cases} \frac{3}{4}x(x+2) & \text{for} \quad 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Show that it is a probability density function and determine the value of b such that $P(X < b) = P(X > b)$. (5Marks)

03. Let a bivariate random variable (X, Y) has the following probability distribution: **[10]**

X \ Y	0	1	2	3
2	k	2k	k	6k
4	2k	k	3k	6k
6	k	2k	2k	3k

a. Determine the value of k.

b. Calculate $E(X)$

c. Find the conditional probability $P(X < 5/Y \le 1)$

d. Find the probability mass function and cumulative distribution function of Y. (10 Marks)

04. The sales of TVs in lakhs of rupees (X) are expected to be influenced by two variables namely the advertising expenditure (Y) and the number of salespersons (Z) in a region. Sample data of 5 regions are given as follows **[10]**

Region	X	Y	Z
1	20	15	10
2	25	20	9
3	15	18	8
4	30	22	12
5	35	16	6

Fit a multiple linear regression model of X on Y and Z. (10 Marks)

05. (a) The equations of lines of regression are given by $3x+12y=19$ and $3y+9x=46$. Obtain the following. [10]

(i) Correlation coefficient of X and Y.

(ii) The most appropriate value of X when Y=8. (5 Marks)

(b) Find the maximum n such that the probability of getting no head in tossing a fair coin n times is greater than 0.1. (5 Marks)

06. (a) A hospital switchboard receives an average of 4 emergency calls in a 10-minute interval. What is the probability that [10]

(i) there are at most 2 emergency calls in a 10-minute interval

(ii) there are exactly 3 emergency calls in a 10-minute interval. (5 Marks)

(b) The speeds of cars are measured using a radar unit, on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. Find the probability that a car selected at chance is moving

(i) more than 100 km/hr

(ii) less than 80 km/hr. (5 Marks)

07. (a) In a random sample of 800 people, 25% of people love to order food in Swiggy. In another sample of 950 people, 27% of people love to order food in Zomato. Is this difference between the two proportions significant at the 5% level? [10]

(5 Marks)

(b) In a sample of 400 parts manufactured by a factory, the number of defective parts was found to be 30. The company however claims that only 5% of their product are defective. Is the claim tenable?

(5 Marks)

08. (a) Two groups A and B, each consisting of 100 people who have a disease. A serum is given to both groups and observed that 75 and 65 people were recovered respectively, Test the hypothesis that the serum helps to cure the disease using at 1% level of significance. (5 Marks) [10]

(b) A normal population has an average of 6.5 and a standard deviation of 1.4. Is the difference significant in a sample of 400 having an average of 6.25 at 5% level of significance? (5 Marks)

09. Two cars were tested in terms of their finishing time (in minutes) to run a particular race. [10]

Car 1	8	10	12	13	13	9	14
Car 2	9	10	10	4	7	9	

Test whether is there any significant difference between their means.

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10. To study the performance of three detergents and three different water temperatures, the following 'whiteness' readings were obtained with designed equipment. **[10]**

Water temperature	Detergent A	Detergent B	Detergent C
Cold	57	55	67
Warm	49	52	68
Hot	54	46	58

Perform a two-way analysis of variance at a 5% level of significance.

11. (a) The density function of the time to failure (in years) of an appliance is **[10]**

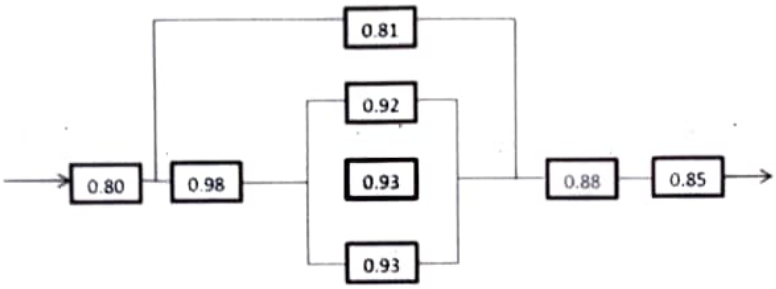
$f(t) = \frac{32}{(t+4)^3}, \quad t > 0$

(i) Find the reliability function R(t)

(ii) Find the failure rate

(iii) Find the MTTF (6 Marks)

(b)) Calculate the reliability of the system,



(4 Marks)

12. The probability density function of the time to failure in years of the machine is **[10]**

$f(t) = \frac{t}{45000}, \quad 0 \le t \le 300$

i) Compute R (t) and R (1).

ii) Compute MTTF.

iii) Find the design life for reliability 0.92?

iv) Will a one-year burn-in period improve the reliability in part (i)? If so what is the new reliability? (10 Marks)

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