

Solⁿ c.d.f of $F(x)$ must be defined over 5 intervals

$$\text{for } x < 0 : F(x) = \int_{-\infty}^x 0 dt = 0$$

$$\begin{aligned} \text{for } 0 \leq x < 1 : F(x) &= \int_0^x t dt = \frac{1}{2} \int_0^x t dt \\ &= \frac{1}{2} \left[\frac{t^2}{2} \right]_0^x = \frac{x^2}{4} \end{aligned}$$

$$\text{for } 1 \leq x < 2 : F(x) = \int_0^1 (-kt + 3k) dt + \int_1^x (-kt + 3k) dt$$

$$\begin{aligned} F(x) &= k \int_0^1 dt + k \int_1^x dt = \frac{1}{2} [t]_0^1 + k [t]_1^x \\ &= \frac{1}{2} + \frac{1}{2} (x-1) \\ &= \frac{1}{2} (1+x-1) = \frac{x}{2} \end{aligned}$$

$$\begin{aligned} \text{for } 2 \leq x < 3 : F(x) &= \int_0^1 (-kt + 3k) dt + \int_1^2 (-kt + 3k) dt + \int_2^x (-kt + 3k) dt \\ &= k \int_0^1 (3-t) dt + k \int_1^2 (3-t) dt + k \int_2^x (3-t) dt \\ &= k \left[3t - \frac{t^2}{2} \right]_0^1 + k \left[3t - \frac{t^2}{2} \right]_1^2 + k \left[3t - \frac{t^2}{2} \right]_2^x \\ &= k \left[3 - \frac{1}{2} \right] + k \left[6 - 2 - 3 + \frac{1}{2} \right] + k \left[3x - \frac{x^2}{2} - 6 + 2 \right] \\ &= \frac{1}{2} \left[\frac{5}{2} + \frac{3}{2} + 3x - \frac{x^2}{2} - 4 \right] = \frac{1}{2} \left(3x - \frac{x^2}{2} \right) \end{aligned}$$

$$\text{for } x > 3 : F(x) = \int_{-\infty}^x f(t) dt = 1 \quad [\because \text{all probabilities has been accumulated for } x \text{ beyond } 1]$$

$$\text{Thus c.d.f } F(x): \begin{cases} 0 & \text{for } x < 0 \\ \frac{x^2}{4} & \text{for } 0 \leq x < 1 \\ \frac{x}{2} & \text{for } 1 \leq x < 2 \\ \frac{1}{2} \left(3x - \frac{x^2}{2} \right) & \text{for } 2 \leq x < 3 \\ 1 & \text{for } x > 3 \end{cases}$$