VIT Vellore Institute of Technology

Reg. No. : 21BPS1364

Final Assessment Test (FAT) - July/August 2023

Programme	B.Tech.	Semester	Fall Inter Semester 22-23
Course Title	PROBABILITY AND STATISTICS	Course Code	BMAT202L
Faculty Name	Prof. Ashis Bera	Slot	A2+TA2
		Class Nbr	CH2022232500570
Time	3 Hours	Max. Marks	100
Statistical Table will be provided.			

Part A (10 X 10 Marks)
Answer any 10 questions

01. Find the median and mode of the following data and hence find the arithmetic mean using the empirical relation. [10]

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	10	18	30	20	12	5

02. a) Find the quartile deviation and the coefficient of dispersion for the following data. [10]

Class	0-5	5-10	10-15	15-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	5	8	12	34	46	28	14	10

b) If $r_{12} = 0.6$ and $r_{23} = r_{31} = 0.8$, find $r_{23.1}$ and $R_{1.23}$.

03. a) A discrete random variable X has the probability mass function [10]

$P(X = k) = \frac{c}{k}, k = 1, 3, 5, 7.$

i) Find out the value of c.

ii) Find the following probabilities $P(X \leq 3)$ and $E(X^2)$.

b) Let the probability density function of a random variable X be described as

$$f(x) = \begin{cases} A\sin(\frac{\pi x}{8}) & 0 \leq x \leq 8 \\ 0 & otherwise \end{cases}$$

(i) Find A such that $f(x)$ is a probability density function.

(ii) Find the probabilities $P(0 < X \leq \pi)$ and $P(X \geq 2\pi)$.

04. Find the moment generating function of a random variable X, whose probability density function is as follows: $f(x) = \frac{1}{2}x^2e^{-x}, 0 < x < \infty$. [10]

Hence deduce the value of mean and variance of X.

05. A sample survey of 6 families was taken and the figures were obtained with respect to their annual savings (Rs. in 1000's), annual income (Rs. in 1000's). The data is summarized in the table given below: [10]

Family	Annual Savings (X)	Annual Income (Y)
1	10	16
2	5	13
3	10	21
4	4	10
5	8	13
6	9	15

Obtain the two equations of regression. Find regression coefficients and correlation coefficient. Also, estimate the annual savings of a family whose annual income is Rs.16000.

06. a) The marks obtained in the course probability and statistics by 1000 students are normally distributed with a mean of 78 and a standard deviation of 11 determine [10]

i) How many students got marks above 90.

ii) What was the highest mark obtained by the lowest 10 of students.

b) The amount of time that a surveillance camera will see without having to be reset is a random variable having an exponential distribution with a parameter of 50 days. Find the probability that such a camera will not have to be reset in at least 60 days.

07. a) A manufacturer claims that the average diameter of their ball bearings is 12 mm. A sample of 25 ball bearings is randomly selected, and their diameters are measured. The sample mean is found to be 11.8 mm. The population variance is known to be 0.36 mm². Test the manufacturer's claim at a significance level of 0.05. [10]

b)) Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, at least four samples contain the pollutant.

08. a) In a certain city, 125 men in a sample of 500 are found to be self-employed. In another city, the number of self-employed is 375 in a random sample of 1000. Does this indicate that there is a greater population of self-employed in the second city than in the first? [10]

b) A car company decided to introduce a new car whose mean petrol consumption is claimed to be lower than that of the existing car. A sample of 50 new cars were taken and tested for petrol consumption. It was found that mean petrol consumption for the 50 cars was 30 km per litre with a standard deviation of 3.5 km per litre. Test at 5% level of significance whether the company' s claim that the new car petrol consumption is 28 km per litre on the average is acceptable.

09. A genetics engineer was attempting to cross a Giraffe and a Zebra. She predicted a phenotypic outcome of the traits and she was observing that the cross is to be in the following ratio 4 stripes only: 3 spots only: 9 both stripes and spots. When the cross was performed and she counted the individuals she found 50 with stripes only, 41 with spots only and 85 with both. According to the Chi-square test, did she get the predicted outcome at 1% level of significance. [10]

10. It is suspected that 4 machines used in a canning operation fills cans to different levels on the average. Random samples of cans produced by each machine were taken and the fill (in ounces) was measured. The results are tabulated below. [10]

A	B	C	D
10.20	10.22	10.17	10.15
10.18	10.27	10.22	10.27
10.36	10.26	10.34	10.28
10.21	10.25	10.27	10.40

Do the machines appear to be filling the cans at different average levels?

11. The density function of the time to failure in years of a machine manufactured by a certain company is given by $f(t) = \frac{15}{(t+5)^4}, t \geq 0$. [10]

(a) Derive the reliability function and determine the reliability for the first two years of operation.

(b) Compute MTTF.

(c) What is the design life for reliability 0.93?

(d) Will a two-year burn-in-period improve the reliability in part (a)? If so, what is the new reliability

12. Find the system reliability of the following series-parallel configurations. Component reliabilities are given as follows: [10]

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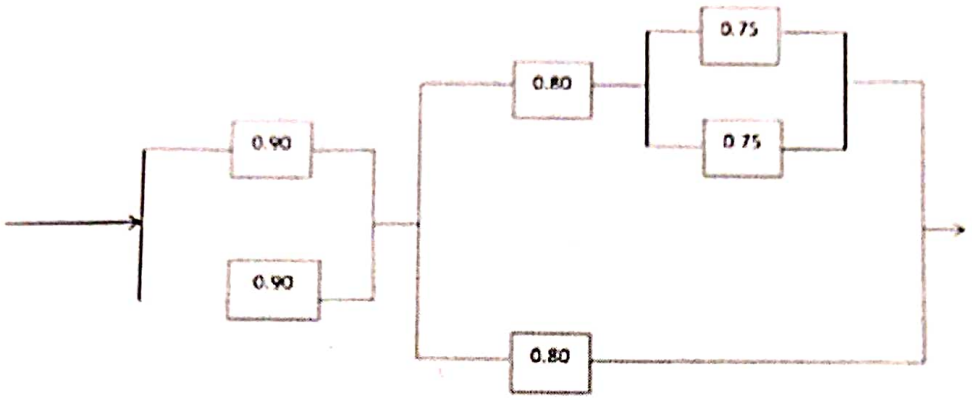
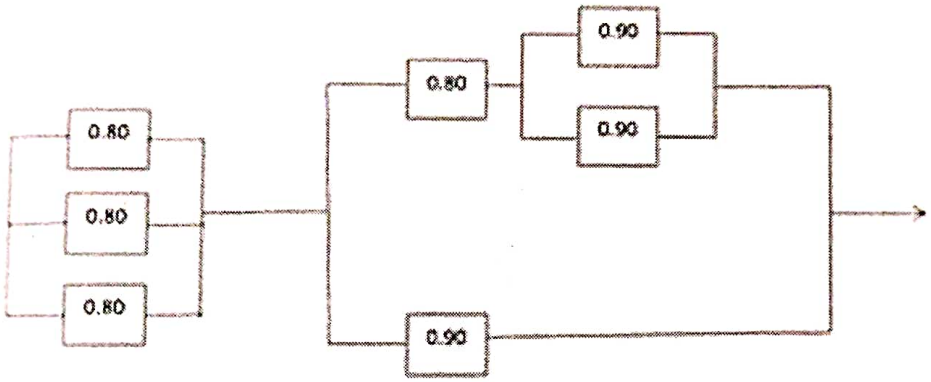


a



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