

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	PROBABILITY AND STATISTICS	Course Code	BMAT202L
Faculty Name	Prof. Poulomi De	Slot	F1+TF1
		Class Nbr	CH2022231001617
Time	3 Hours	Max. Marks	100

Part-A (10 X 10 Marks)

Answer any 10 questions

1.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	7	12	18	25	16	14	8

[10]

Find the mean, mean deviation and coefficient of variation for the above data

2. a) The expenditure of 1000 families is given below:

[10]

Expenditure(in Rs)	40-59	60-69	70-79	80-89	90-99
No of families	50	?	500	?	50

The median for the distribution in Rs. 87. Calculate the missing frequencies.

- b) A random variable X has the following probability mass function:

x	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	$3k$

- Find the value of k .
- Compute the variance of X .
- Obtain CDF of X .

3. If the joint probability density function of a bi-variate random variable (X, Y) is

[10]

$$f_{x,y}(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0, y \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

- Find the marginal pdf of X and Y .
- Check whether X and Y are independent random variables or not.
- Compute $E(X)$ and $E(Y)$.
- Evaluate $P(0 < X < 1/Y < 4)$.

4. Consider the following data:

[10]

X	44	45	47	46	48	49
Y	33	31	32	35	34	34
Z	7	8	10	9	11	13

Fit the regression equation of Z on X and Y and hence find the value of Z when X is 35 and Y is 40.

5. a) The coefficient of rank correlation of the marks obtained by 10 students in Mathematics and Statistics was found to be 0.5. It was later discovered that the difference in ranks between the two subjects obtained by one of the students was wrongly taken as 3 instead of 7. Find the correct coefficient of rank correlation.

[10]

- b) Assume that the life of a roller bearing follows a Weibull distribution with shape parameter $\beta = 2$ and scaling parameter $\delta = 10000$ hours.

- i) Determine the probability that a bearing lasts at least 8000 hours.
 ii) Determine the mean time until failure of a bearing.
 iii) If 10 bearings are in use and failures occur independently, what is the probability that all 10 bearings last at least 8000 hours?
6. a) The probability that a patient recovers from COVID-19 is 0.4. If 15 persons have COVID 19, [10]
 determine the probability that
 i) exactly 5 survive.
 ii) at least 10 survive.
 iii) from 3 to 8 survive.
 b) The life of a component is normally distributed with an average life time of 1200 days and standard deviation of 200 days. i) What percentage of this kind of component is expected to fail in the first 800 days of working? ii) What percentage is expected to fail between 800 and 1,000 days?
7. a) A researcher wants to determine the mean time it takes a mechanic to rotate the tires of a car, [10]
 and she wants to assert with 95% confidence that the average of her sample is off by at most 0.50 minutes. If she can presume from past experience that $\sigma^2 = 2.56$ minutes, how large a sample will she have to take?
 b) A manufacturer of light bulbs claims that an average 2% of the bulbs manufactured by his firm are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of this sample, can you support the manufacturer's claim at 5% level of significance?
8. a) A secretary claims that she can take dictation at the rate of 110 words per minute. Can we [10]
 reject her claim on the basis of 100 trials in which she demonstrates a mean of 105 words with a standard deviation of 15 words? Use 5% level of significance.
 b) A random sample of size $n = 100$ is taken from a population with standard deviation 5.1. Given that the sample mean is 21.6. Construct a 95% confidence interval for the population mean.
9. a) In a survey on immunization of COVID-19 vaccination on 200 patients, the following [10]
 contingency table is obtained:

	affected	Not affected	Total
Moderna	60	30	90
Pfizer-BioNtech	40	70	110

Use χ^2 test to test the effectiveness of the vaccine. Use 5% level of significance.

- b) Two random samples drawn from normal population are given by

Sample I : 20,16,26,27,23,22,18,24,25,19

For sample II, the variance is 26.17 with sample size 12.

Test whether the two populations have the same variance with 5% level of significance

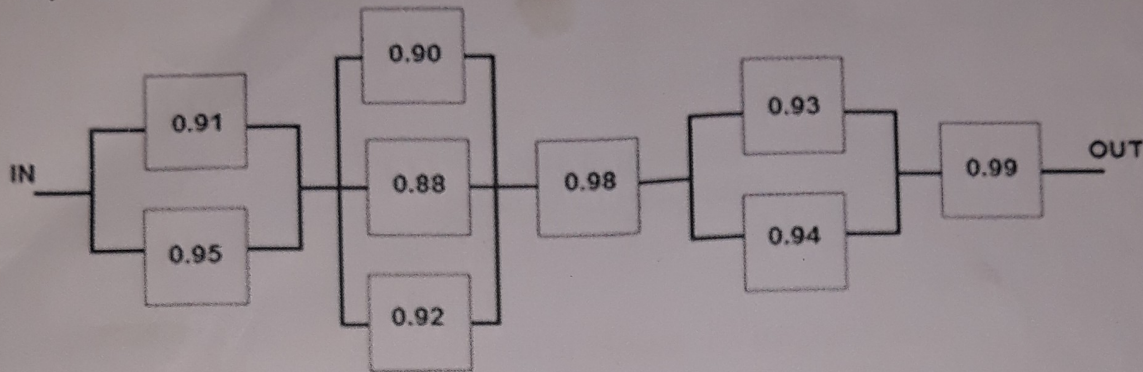
10. The following table gives the number of units of production per day turned out by four different [10]
 types of machines:

Employee / Machines	M1	M2	M3	M4
E1	40	36	45	30
E2	38	42	50	41
E3	36	30	48	35
E4	46	47	52	44

Use Analysis of variance to test the hypotheses that (i) mean production is the same for the four machines and (ii) the employees do not differ with respect to mean productivity at 5% level of significance.

11. a) The weights (in kg) of 10 samples of ice-cream taken from a large consignment are 50, 49, 52, 44, 45, 48, 46, 45, 49, 45. Test if the average packing can be taken to be 50 kgs at $\alpha = 0.05$ level of significance. [10]

b) A block diagram representation of a system is shown below. Determine the overall system reliability



12. A multipurpose automated teller machine that counts the number notes, finds fake currency notes, and identifies the denomination was developed by a company. The machine consists of four important sensors. The machine works if and only if all sensors work. The reliability of those four sensors are 0.98, 0.95, 0.94, and 0.90. [10]
- Find the reliability of the machine.
 - The company wants to improve the reliability of the machine by connecting backup sensors. Due to space constraints of the machine, only one backup component of the same reliability can be added. If so, backup of which sensor has to be added to increase the reliability? Why?
 - If there is a chance to add a backup sensor of reliability 0.92, to which component should it be added to increase the overall reliability? Why?

