VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test I – September 2022

Programme	:	B.Tech.	Semester	:	FALLSEM 2022-23
Course Title	:	Discrete Mathematics and Graph Theory	Code	:	BMAT205L
			Slot	:	D2+TD2+TDD2
Faculty(s)	:	Dr. Balamurugan, Dr. Kalyan, Dr. Uma Maheshwari, Dr. Berin Greeni, Dr. Nathiya, Dr. Somnath Bera, Dr. Devi Yamini, Dr. Durga, Dr. Prasannalakshmi, Dr. Dhivya, Dr. Pavithra, Dr. Karan Kumar Pradhan, Dr. Kamalesh, Dr. Amit Kumar Rahul	Class Nos.		CH2022231001488; CH2022231001464; CH2022231001466; CH2022231001468; CH2022231001470; CH2022231001477; CH2022231001480; CH2022231001482; CH2022231001484; CH2022231001490 CH2022231001493; CH2022231001495; CH2022231001497; CH2022231001500
Time	:	90 Minutes	Max. Marks	:	50

Answer ALL the Questions (5 X 10 = 50 Marks)

Q.No.	Sub. Sec.	Question Description	Marks
1.	a.	Show that the conclusion $C: r$ follows from the premises $H_1: p \vee q$, $H_2: p \rightarrow r$, $H_3: q \rightarrow r$ using the truth table technique.	5
	b.	(i) Consider the statement "Given any positive integer, there is a greater positive integer". Symbolize this statement with and without using set of positive integers as the universe of discourse. (2 Marks) (ii) Consider these propositional functions: p(n) – n is prime q(n) – n is even r(n) – n>2 Express these formulas in proper English sentences: (a) $\exists n \in Z(p(n) \wedge q(n))$ (b) $\forall n \in Z(r(n) \Rightarrow (p(n) \vee q(n)))$ (c) $\exists n \in Z(p(n) \wedge q(n) \vee r(n))$ (3Marks)	5
2.		Without constructing the truth table, (i) Find the principal conjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$ (5 Marks) (j) Find the principal disjunctive normal form of $(\neg a \rightarrow b) \wedge (b \leftrightarrow a)$ (5 Marks)	10
3.	a.	Show that the premises "There exist some students in this class and they know how to write programs in Python", "Everyone who knows how to write programs in Python gets a high-paying job" imply the conclusion "Some students in the class can get a high-paying job"	5

	b.	Show that the premises "If Claghorn has wide support, then he will be asked to run for the senate. If Claghorn yells "Eureka" in lowa, he will not be asked to run for the senate. Claghorn yells "Eureka" in lowa" imply the conclusion "Claghorn does not have wide support".	5
4.	a.	Show that b can be derived from the premises $a \rightarrow b, c \rightarrow b, d \rightarrow (a \vee c), d$ by the indirect method.	5
	b.	(i) If α and β are elements of the symmetric group S_4, given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ Find $\beta\alpha$, $\alpha\beta$ and α^{-1} (3 Marks) (ii) Obtain all the distinct left cosets of $\{(0), (3)\}$ in the group $(Z_6, +_6)$.(2 Marks)	5
5.	a.	Let $*$ be an operation defined as $x * y = x + y + 2xy, \forall x, y \in \mathbb{R}$ (real numbers). (i) Check whether $(\mathbb{R}, *)$ is a monoid or not (ii) Is it commutative? (iii) Find the inverse of each element in $\mathbb{R}$ if it exists.	3
	b.	(i) Assume that G is a finite group with subgroups H of order 12 and K of order 30. If the order of G is less than 200, what are the possible values for the order of G. (3 Marks) (ii) Let $S = \{a \in \mathbb{Q} / a \neq 0\}$ be closed under the commutative binary operation $*$ defined by $a * b = \frac{ab}{3}$. Prove that the set S with the binary operation is an abelian group.(4 Marks)	7

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