VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CAT- I May 2023

Programme	**B. Tech**	Semester	**FALL Inter Semester 2022 – 23**
Course	**Discrete Mathematics and Graph Theory**	Code	**BMAT205L**
Faculty	Dr. Kalyan Manna, Dr. Avinash Kumar Mittal, Dr. Durga Nagrajan, Dr. Vidhya V, Dr. Devi Yamini S, Dr. Om Namah Shivay, Dr. Uma Maheswari S, Dr. Rajesh Kumar Mohapatra, Dr. Manigandla Prasannalakshmi, Dr. Sandip Dalui, Dr. Pulak Konar, Dr. Surath Ghosh, Dr. Lakshmanan S	Slot(s) Class No.	**C1+TC1+TCC1** CH2022232500280 - CH2022232500287, CH2022232500292 - CH2022232500297
Time	**90 Mins**	Max. Marks	**50**

Answer ALL the Questions

1\. (a) Symbolize the statements using proposition and/or predicates. (2)

i) All apples need not be red. Some apples are green.

ii) Either he goes to movie or to hotel never to temple.

(b) Show that $[(a \to b) \wedge (c \to d)] \wedge [(b \to e) \wedge (d \to f)] \wedge [\neg (e \wedge f) \wedge (a \to c)] \to (\neg a)$ is a tautology. (Without truth table) (8)

2\. (a) Translate the following sentence into predicate logic: "Some student in this class has taken a course in Java" for (i) the domain is the whole class (ii) the domain is all the people. (4)

(b) Use the indirect method to prove that the conclusion $\exists z\, Q(z)$ follows from the premises $\forall x\, (P(x) \to Q(x))$ and $\exists y\, P(y)$. (6)

3\. Without using truth tables, find the PDNF and DNF of the following logical expression: (10)

$$[p \wedge (p \to q)] \to q.$$

(Note that DNF should not be the same expression as PDNF).

4. (a) If $S = \{1,2,3,6\}$ and $\ast$ is defined by $a \ast b = lcm\,(a,b)$, where $a, b \in S$. Show that $\{S, \ast\}$ is a monoid. (4)

(b) Prove that $Z_6 = \{0,1,2,3,4,5\}$ is a group under addition modulo 6. Find all the subgroups of Z_6. (6)

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5. (a) Let $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ be the parity check matrix corresponding to the encoding function $e: B^3 \to B^6$. (3+2)

(i) Find the code words generated by H.

(ii) Decode the received words (a) 111000 and (b) 001110.

(b) Let $(R,+)$ and $(R^+, \times)$ be two groups. Show that the mapping $f: R \to R^+$ defined by $f(x) = 2^x\ \forall\ x \in R$ is a homomorphism. (5)