1. Show that for every integer n, there is a multiple of n consisting of 0 and 1 its decimal expansion.

Soln: Let n be positive integer. Consider $n+1$ integers

1, 11, 111, 1111, . . .

when we divide them by n there are n possible remainder, but there are $n+1$ integer.. So one box contains two of them say 11111 and 111 then

$n \mid 11111 - 111 = \underline{\underline{1000}}$. This satisfies the claim.

2. If n objects placed into k boxes There is at least one box containing $\left[n/k \right]$ objects.

How many cards must be selected from 52 to guarantee at least 3 are of the same ~~the~~ suit.

(1) At least one box contains

$\left\lceil \frac{N}{4} \right\rceil \geq 3$ $\quad N = 2 \cdot 4 + 1 = 9$

(5) Show that among $n+1$ positive integers not exceeding $2n$ there must be an integer that divides one another.

write $a_1, \ldots a_{n+1}$ as

$a_j = 2^{k_j} q_j$ k_j is non-negative and q_j is odd. The integers $q_1, \ldots q_{n+1}$ are all odd less than $2n$. They are $n+1$ odd positive integers $< 2n$.

$q_i = q_j$ for some i, j

Every sequence of n^2+1 distinct real nos. contains a subsequence of $n+1$ strictly increasing or strictly decreasing.

Pf: a_1, a_{n^2+1} distinct real no. Associate an ordered pair with each term

(i_k, d_k) to the term a_k

i_k is the length of the longest increasing subseq. at a_k and d_k longest decr. subseq. at a_k.

suppose there is no incr. or decr. subseq. of length $n+1$,

$i_k, d_k \le n$

$k = 1, 2, \ldots n^2+1$

there are n^2 possible pairs (i_k, d_k).

~~there are~~ two of these n^2+1 ordered pairs equal.

a_s, a_t $s<t$ s.t. $i_s = i_t$ $d_s = d_t$

Give a combinatorial pf that

$$\sum k^2 \binom{n}{k} = n(n+1)\, 2^{n-2}$$

$$\binom{2n}{2} = 2\binom{n}{2} + n^2$$

$$\sum k \binom{n}{k} = n\, 2^{n-1}$$

$$\sum k \binom{n}{k}^2 = n \binom{2n-1}{n-1}$$

(n members from a group of n math. prof. n computer sc. prof. such that ~~they~~ chair is computer sc. prof.