Final Assessment Test (FAT) – November/December 2022

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	DISCRETE MATHEMATICS AND GRAPH THEORY	Course Code	BMAT205L
Faculty Name	Prof. Nathiya N	Slot	D1+TD1+TDD1
		Class Nbr	CH2022231001436
Time	3 Hours	Max. Marks	100

Section-A (10 X 10 Marks)
Answer any 10 questions

1. Determine whether the following argument is valid or not. **[10]**

   "If a leader is able or willing to save people, he would do so. If the leader is unable to save people, he would be inefficient. If he was unwilling to save people, he would be a pessimist. Leader does not save people. If a leader exists, he is neither inefficient nor pessimist. Therefore, the leader does not exist." [10]

2. (a) Translate the following English sentence into a logical expression and write it's negation: **[10]**
   Every child either depends on parents or some person.

   (b) Use the indirect method to prove that the conclusion $\exists z\, Q(z)$ follows from the premises $\forall x\,(P(x) \to Q(x))$ and $\exists y\, P(y)$.   [5+5]

3. (a) If $*$ is the multiplication operation on the set of complex numbers C and if **[10]**
   $H = \{a + ib \in C : a^2 + b^2 = 1\}$, then check whether H is a subgroup of $(C, *)$. Justify your answer.

   (b) If $*$ is defined on $S = N \times N$ by $(a, b) * (c, d) = (a + c,\ b + d)$ and if the mapping $f : (S, *) \to (Z, +)$ is defined by $f(a, b) = a - b$, then show that f is a homomorphism where N is the set of natural numbers and Z is the set of integers. [5+5]

4. (a) For the following graph, find the chromatic number and chromatic polynomial. **[10]**
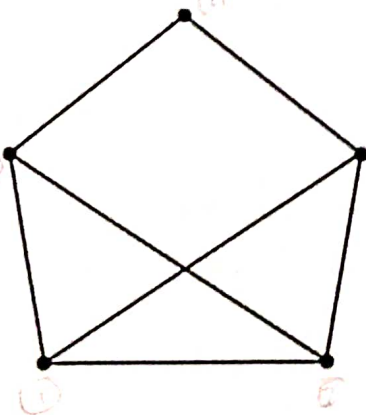
   (b) Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$,

   corresponding to the encoding function $e : B^3 \to B^6$. [5+5]

5. (a) Show that in a party of 6 people, either there are three people who know each other or there are three people who do not know each other. **[10]**

(b) Let $m, n \in Z$, mRn iff $m - n$ is a multiple of 9. Check whether R is a partial ordered relation, where Z is the set of all integers. [5+5]

6. Consider the following graph **[10]**
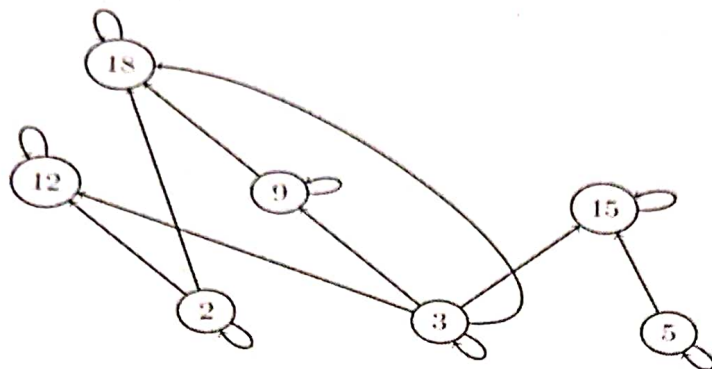
(i) Show that the relation given by the graph is a partial order.

(ii) Get the Hasse diagram from the given graph.

(iii) Find the greatest and least element of the graph if it exists.

(iv) For the subset {2,3} of the graph, find the upper bound, lower bound, LUB and GLB.

(v) Check whether the Hasse diagram is a Lattice or not. [2+2+2+2+2]

7. (a) Among 7 Indians and 4 foreigners, in how many ways a committee of 4 members has to be formed such that **[10]**

(i) Committee will contain at least 2 foreigners?

(ii) Committee will contain exactly 2 foreigners? [2.5+2.5]

(b) Find the first five terms of the sequence defined by each of the following recurrence relations and its initial condition :

(i) $a_n = a_{n-1}^2$, $a_1 = 2$.

(ii) $a_n = na_{n-1} + n^2 a_{n-2}$, $a_0 = 1$, $a_1 = 1$. [2.5+2.5]
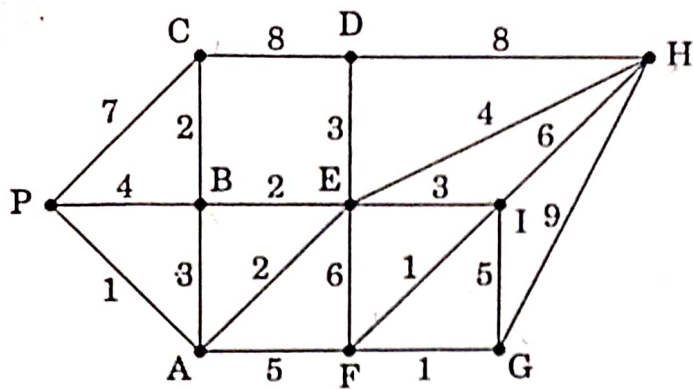
8. (a) Apply the Dijkstra's algorithm to find the shortest path and distance from the vertex " a to all other vertices" in the following weighted graph. **[10]**
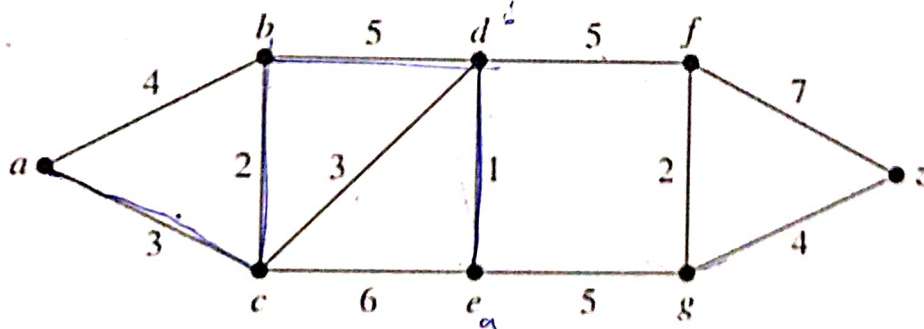
(b) Construct a graph with the degree sequence 7, 7, 7, 5, 4, 3, 2, 1, if possible. If not, justify your answer. [7+3]

9. Every day a postman delivers mail to the blocks A, B, C, D, E, F, G, and I. The connectivity among the blocks is shown in the following weighted graph where the weight of each edge denotes the time (minutes) taken by the postman to travel between two blocks. In the graph, P denotes the post office and H denotes the home. Which streets should be walked by the postman so that there is a path between each pair of blocks and a minimum time is taken to walk? **[10]**
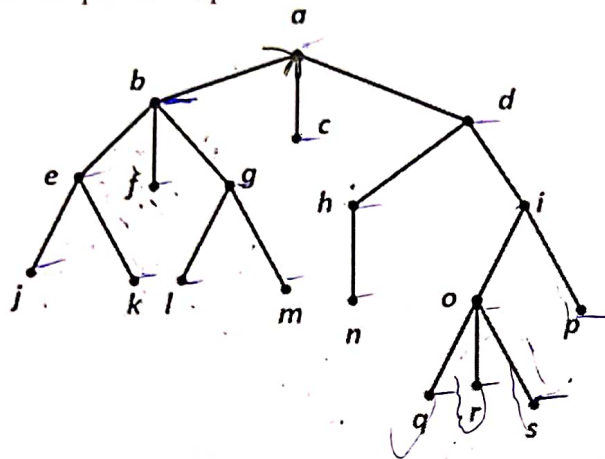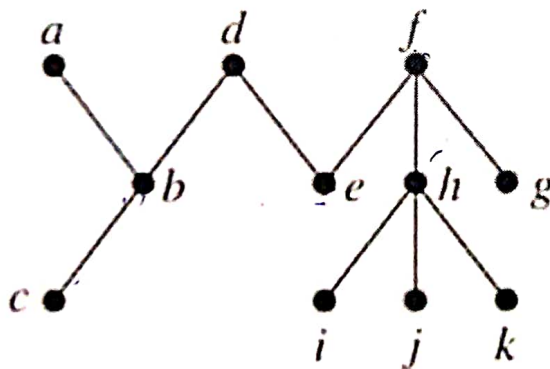
[10]

(a) Find the pre order, post order and in order traversal of the following tree. **[10]**
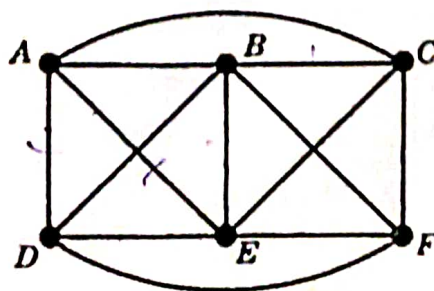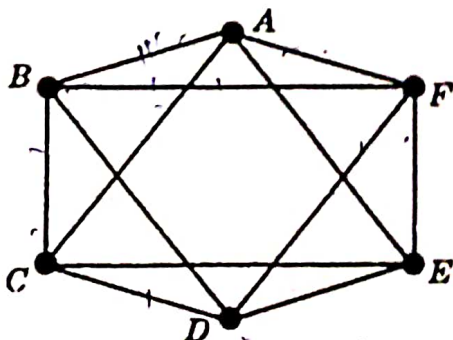

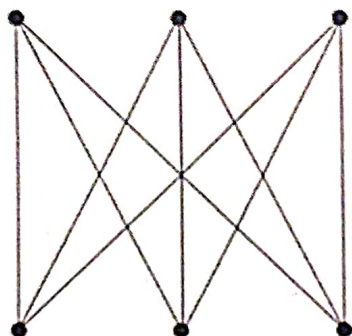**(b)** Find the distance, centre, and eccentricity of each vertex of the following tree.

[6+4]

(a) Check whether the following graphs are planar or not. Justify your answer with proper reasoning. **[5]** **[10]**

(b) Is the following graph G (i)Eulerian?(ii) Hamiltonian? Justify your answer with proper reasoning. **[3]**

(c) Find the number of vertices of degree 3 in an undirected graph with 13 edges, in which two vertices are of degree 4 and three are pendant vertices [2]

12. A schedule for finals is to be drawn up for a group of 7 classes, "a to g". Two classes may not be   [10]
scheduled at the same time if there exist a student in both classes. The table below shows the
classes which may not be scheduled at the same time (marked with dot •).

	a	b	c	d	e	f	g
a		•	•	•			•
b	•		•	•	•		•
c	•	•		•		•	
d	•	•	•			•	
e		•					
f			•	•			•
g	•	•				•	

(i)   What is the minimum number of time slots needed to schedule all 7 classes?

(ii) Is the graph in (i) bipartite? Justify.

(iii) Find a matching for the graph obtained in (i). [10]

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