DMGT Module 1 Propositional calculus

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Proposition - statement

↳ I am Vidhya working in VIT < True / false } truth values

Do Attend the class regularly - Command/order .

Proposition - A statement which it has truth values either 'true' or 'false' is called Proposition

Some examples

x + y =56

He will go to office by car

Its rainy today

Propositions are denoted by the 'variables' P, Q, R, S….. In which all the variables have two values T or F

If P is a proposition, its truth values are denoted by T/F or 1/0

P	1	0

If P and Q are two variables then its combined truth value is denoted as

P	Q
1	1
1	0
0	1
0	0

Algebra(operations) of propositions

AND

If P and Q are two propositions, then P and Q denoted by $P\wedge Q$ which has truth values

P	Q	$P\wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0

which is defined as if both P and Q are true then $P\wedge Q$ is true otherwise it is false

OR

If P and Q are two propositions, then P and Q denoted by $P\vee Q$ which has truth values

P	Q	$P\vee Q$
1	1	1
1	0	1
0	1	1
0	0	0

which is defined as if both P and Q are false then $P\vee Q$ is false otherwise it is true

Negation

If P has truth value true then its negation denoted by $\sim P$ or $\neg P$ has truth value false

P	$\sim P$
1	0
0	1

Implications: $P \rightarrow Q$ read as If P then Q taking truth value false when P is true and Q is false, otherwise true

P	Q	$P \rightarrow Q$
1	1	1
1	0	0
0	1	1
0	0	1

$P \leftrightarrow Q$ - Biconditional read as P if and only if Q which is true if both P & Q are true or false

P	Q	$P \leftrightarrow Q$
1	0 1	1
0	1	0

p	q	$p \leftrightarrow q$
1	1	1
0	1	0
1	0	0
0	0	1

$p \vee (p \to q)$ $\quad p \vee b$

p	q	$p \to q$	$p \vee (p \to q)$	$q \to p$	$(p \to q) \vee (q \to p)$
1	1	1	1	1	1
1	0	0	1	1	1
0	1	1	1	0	1
0	0	1	1	1	1

} tautology

$p \vee (q \wedge r)$. $\quad p \to (q \to r)$ $\quad p \leftrightarrow (q \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$q \to r$	$p \to (q \to r)$	$q \vee r$	$p \leftrightarrow (q \vee r)$
1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	0	1	1
1	0	1	0	1	1	1	1	1
1	0	0	0	1	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	0	0	0	0	1	1	0
0	0	1	0	0	1	1	1	0
0	0	0	0	0	1	1	0	1

tautology and Contradiction

if ~~for~~ the given statement is true for all values then it is tautology.

" " " " " false Contradiction.

Equivalence:- $P \Leftrightarrow Q$

~~$p \to q$~~

P and Q are statement formula.

for ex. $\quad p \to q \Leftrightarrow \neg p \vee q$

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
1	1	1	0	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$p \to q \Leftrightarrow \neg q \to \neg p$

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
1	1	1	0	0	1
1	0	0	1	0	0
0	1	1	0	1	1
0	0	1	1	1	1

$p \leftrightarrow q \Leftrightarrow ((p \to q) \wedge (q \to p))$ — prove..

Results. $\quad p, p \to q \Leftrightarrow q \qquad$ ' , ' $\Rightarrow \wedge$

$\qquad$ i.e. $p, q \Rightarrow p \wedge q$

$p \wedge (p \to q) \Leftrightarrow q$

De-morgan's law

$\sim (P \wedge Q) \Leftrightarrow \neg P \vee \sim Q$

$\sim (P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

Distributive law

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

$(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

$(p \vee q) \wedge r = (p \wedge r) \vee (q \wedge r)$

Associative law

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$

$p \vee (q \vee r) = (p \vee q) \vee r$

$p \vee q = q \vee p$ } Commutative

$p \wedge q = q \wedge p$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ $\qquad$ $p \vee (q \vee r) = (p \vee q) \vee r$

$(p \wedge q) \vee r = (p \vee r) \wedge (q \vee r)$

$(p \vee q) \wedge r = (p \wedge r) \vee (q \wedge r)$

$\left.\begin{array}{l} p \vee q = q \vee p \\ p \wedge q = q \wedge p \end{array}\right\}$ Commutative

Practice:

Find the truth values of

(i) $p \wedge (q \leftrightarrow \neg p) \wedge r$

(ii) $(\neg p \vee q) \leftrightarrow \neg r$

(iii) $(p \wedge q \wedge \neg r) \vee (p \to (q \to r))$

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \wedge \neg r = a$	$q \to r$	$p \to (q \to r) = b$	$a \vee b$
1	1	1	1	0	0	1	1	1
1	0	1	0	0	0	1	1	1
1	1	0	1	1	1	0	0	0
1	0	0	0	1	0	1	1	1
0	1	1	0	0	0	1	1	1

Absorption law: $p \vee (p \wedge q) \Leftrightarrow p$, $p \wedge (p \vee q) \Leftrightarrow p$

Dominance law: $p \vee T = T$, $p \wedge F = F$

Identity law: $p \wedge T = p$, $p \vee F = p$

Complement law: $p \vee \neg p = T$, $p \wedge \neg p = F$

Prove the following without constructing truth table:

(1) Prove $(p \vee q) \wedge \neg p \Leftrightarrow \neg p \wedge q$

LHS $\Rightarrow (p \vee q) \wedge \neg p$

$\Rightarrow (p \vee \neg p) \wedge (q \wedge \neg p)$ - Distributive law

$\Rightarrow T \wedge (q \wedge \neg p)$ - Complement law

$\Rightarrow q \wedge \neg p$ - Identity law

$\Rightarrow$ RHS.

2) $\sim(\sim((p \vee q) \wedge r) \vee \sim q) \Leftrightarrow q \wedge r$

LHS $\Rightarrow \sim(\sim((p \vee q) \wedge r) \vee \sim q)$

$\Rightarrow (p \vee q) \wedge r \wedge q$ - demorgan

$\Rightarrow (p \vee q) \wedge (r \wedge q)$ Associative

$\Rightarrow ((p \vee q) \wedge q) \wedge r$ Commutative

$\Rightarrow q \wedge r$ absorption law

3) $(p \vee q) \wedge (\sim p \wedge (\sim p \wedge q)) \Leftrightarrow \sim p \wedge q$

LHS $= (p \vee q) \wedge (\sim p \wedge q)$ $\qquad$ $\sim p \wedge \sim p = \sim p$

$\Rightarrow (p \vee q) \wedge q \wedge \sim p$ - Commutative

$\Rightarrow ((p \vee q) \wedge q) \wedge \sim p$

$\Rightarrow q \wedge \sim p$

Show that $((p \vee q) \wedge \underbrace{\sim(\sim p \wedge (\sim q \vee \sim r))}_{\text{demorg}}) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$ is a tautology. ($\sim(q \wedge r)$ - demorgan's law)

$= p \vee q \wedge (p \vee q \wedge r) \vee \sim(p \vee q) \vee \sim(p \vee r)$

is a tautology.

"demorg"

$= \underline{p \vee q} \wedge (\underline{p} \vee \underline{q \wedge r}) \vee \sim(p \vee q) \vee \sim(p \vee r)$

$= [(p \vee q \wedge p) \vee (p \vee q \wedge q \wedge r) \vee \sim(p \vee q) \vee \sim(p \vee r)$

$= p \vee (q \wedge r) \vee$.

$= \left(\underline{(p \vee q) \wedge (p \vee r)}\right) \vee \underline{\sim(p \vee q) \wedge \sim(p \vee r)}$

$= \underbrace{(p \vee q) \wedge (p \vee r)}_{a} \vee \sim \left(\underbrace{(p \vee q) \wedge (p \vee r)}_{a}\right)$ - demorgan

$= a \vee \sim a = \underline{T}$

$p \to q \Leftrightarrow \neg p \vee q$

$p \to q \Leftrightarrow \neg q \to \neg p$

S.T 1) $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

2) $(p \to r) \wedge (q \to r) \Leftrightarrow (p \vee q) \to r$

that $(\neg p \vee r) \wedge (\neg q \vee r) \Leftrightarrow \neg(p \vee q) \vee r$

3) $\left((p \vee \neg q) \to q\right) \to \left((p \vee \neg q) \to r\right) \Leftrightarrow q \to r$

Sl. No.	Name of the law	Primal form	Dual form
1.	Idempotent law	$p \vee p \equiv p$	$p \wedge p \equiv p$
2.	Identity law	$p \vee F \equiv p$	$p \wedge T \equiv p$
3.	Dominant law	$p \vee T \equiv T$	$p \wedge F \equiv F$
4.	Complement law	$p \vee \neg p \equiv T$	$p \wedge \neg p \equiv F$
5.	Commutative law	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
6.	Associative law	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
7.	Distributive law	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
8.	Absorption law	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
9.	De Morgan's law	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$

.10 Equivalences Involving Conditionals

1.	$p \to q \equiv \neg p \vee q$
2.	$p \to q \equiv \neg q \to \neg p$
3.	$p \vee q \equiv \neg p \to q$
4.	$p \wedge q \equiv \neg(p \to \neg q)$
5.	$\neg(p \to q) \equiv p \wedge \neg q$
6.	$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
7.	$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
8.	$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
9.	$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

11 Equivalences Involving Biconditionals

1.	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
2.	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3.	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4.	$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Table 1.14 Implications

1.	$p \wedge q \Rightarrow p$
2.	$p \wedge q \Rightarrow q$
3.	$p \Rightarrow p \vee q$
4.	$\neg p \Rightarrow p \to q$
5.	$q \Rightarrow p \to q$
6.	$\neg(p \to q) \Rightarrow p$
7.	$\neg(p \to q) \Rightarrow \neg q$
8.	$p \wedge (p \to q) \Rightarrow q$
9.	$\neg q \wedge (p \to q) \Rightarrow \neg p$
10.	$\neg p \wedge (p \vee q) \Rightarrow q$
11.	$(p \to q) \wedge (q \to r) \Rightarrow p \to r$
12.	$(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$

Rule P - given statement formula
given premises
that can be added anywhere
in the derivation

Rule T . derived from P and set of rules

1) Show that $\underbrace{\underline{p \vee q}}_{\text{Rule P}}$, $\underbrace{\underline{p \to r}}_{\text{Rule P}}$ and $\underbrace{\underline{q \to r}}_{\text{Rule P}}$ implies r

Comma $\Rightarrow$ and
, $\Rightarrow \wedge$

, and, $\wedge$ are same

$p \vee q$, $p \to r$ can be written as $p \vee q \wedge p \to r$

given $(p \vee q) \wedge (p \to r) \wedge (q \to r) \to r$

$p \vee q,\ p \to r$ can be written as $p \vee q \wedge p \to r$ | , and, $\wedge$ are same

given $(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$

1) $p \to r$ Rule P
2) $\neg p \vee r$ Rule T
3) $q \to r$ Rule P
4) $\neg q \vee r$ Rule T
5) $(\neg p \vee r) \wedge (\neg q \vee r)$ Rule T from (2) (4)
6) $(\neg p \wedge \neg q) \vee r$ Rule T from 5 Distributive law
7) $\neg(p \vee q) \vee r$ " Demorgan's law
8) $(p \vee q) \to r$ Rule T from 7
9) $p \vee q$ Rule P
10) r Rule T $p,\ p \to q \Rightarrow q$ from (8) (9)

$p,\ p \to q \Rightarrow q$

2) $(p \to q) \wedge \neg q \Rightarrow \neg p$ S.T

1) $p \to q$ Rule P
2) $\neg p \vee q$ Rule T
3) $\neg q$ Rule P
4) $(\neg p \vee q) \wedge \neg q$ Rule T (2) (3)
5) $(\neg p \wedge \neg q) \vee q \wedge \neg q$ Rule T Distr law from 4
6) $(\neg p \wedge \neg q) \vee F$ Rule T Dominant law
7) $\neg p \wedge \neg q$ Rule T Compl
8) $\neg p$ Rule T

$p \wedge q \Rightarrow p$
$\Rightarrow q$

1) $p \to q$ Rule P
2) $\neg q \to \neg p$ Rule T
3) $\neg q$ Rule P
4) $\neg p$ Rule T (2) (3)
$p,\ p \to q \Rightarrow q$

Ram goes either by car or two wheeler.

p : Ram goes by car
q : Ram goes by two wheeler
} proposition

Ram goes either by car or two wheeler $= p \vee q$ — Premises

If Ram does not go by car then he will go by two wheeler $= \neg p \to q$

Ram goes by car iff it rains.

r : It rains $\quad p \leftrightarrow r$

Ram goes by car and it rains. $= p \wedge r$

"If I attend the meeting, I will come to know the rules and regulations"

"Both preparing well for exam and present it neatly will surely lead to good scores"

"I don't follow the rules if and only if either I didn't attend the meeting or didn't go through the brochure"

1) p : I attend meeting
q : I know rules
r : I know regulations
} $p \to (q \wedge r)$

q: know rules & regulations

$p \to q$

2) p : Preparing well
q : Present neatly

$(p \wedge q) \to r$

r : good score

2) p : follow rules
q : attend meeting
r : go through brochure

$\sim p \leftrightarrow (\sim q \text{ or } \sim r)$
$\neg p \leftrightarrow (\neg p \vee \neg r)$

p : don't follow rules
q : not attending
r : not go through brochure
$p \leftrightarrow (q \vee r)$

"Radha works hard", "If Radha works hard, then she is a dull girl" and "If Radha is a dull girl, then she will not get the job" imply the conclusion "Radha will not get the job".

Propositions, premises, conclusion derivation

p : Radha works hard
q : Radha is a dull girl
r : Radha gets job.

Radha works hard p
If Radha works hard, then she is a dull girl : $p \rightarrow q$
If Radha is dull, she will not get job : $q \rightarrow \neg r$
Conclusion : $\sim r$

$p, \; p \rightarrow q, \; q \rightarrow \neg r \Rightarrow \neg r$

1) p Rule P
2) $p \rightarrow q$ Rule P
3) q Rule T (1)(2)
4) $q \rightarrow \neg r$ Rule P
5) $\underline{\neg r}$ Rule T (3)(4)

$p \rightarrow q, q \rightarrow r \Rightarrow p \rightarrow r$

1) $p \rightarrow q$ Rule P
2) $q \rightarrow \neg r$ Rule P
3) $p \rightarrow \neg r$ Rule T
4) p Rule P
5) $\neg r$ Rule T

$p \wedge (\neg p \vee r)$
$(p \wedge p$
$r \wedge (\neg r \rightarrow t)$
$r \wedge (r \vee t)$
p: not raining
1) p
2) $\sim p \rightarrow r$
3) $p \vee r$
4) r
5) $\neg r \rightarrow s$
6) $s \rightarrow t$
7) $\neg r \rightarrow t$
8) t

"It is not raining today and it is hotter than yesterday. We will go to beach, only if it rains today. If we do not go to beach then we will go to a movie. If we go to a movie then we will be at Sam's place". The conclusion is "We will be at Sam's place"

Propositions: p : It rains q : hotter than yesterday r : go to beach s : go to movie t : at Sam's place

Premises : $\sim p \wedge q, \; p \rightarrow r, \; \neg r \rightarrow s, \; s \rightarrow t \Rightarrow t$

$p, p \rightarrow q \Rightarrow q$

1) $\sim p \wedge q$ P
2) $\sim p$ T
3) $p \rightarrow r$ P
4) $\neg r \rightarrow s$ P
5) $\neg r \rightarrow \neg p$ T
6) $\neg r \rightarrow (s \wedge \neg p)$
7) $r \vee (s \wedge \neg p)$

$\neg q \wedge (p \rightarrow q) \Rightarrow \neg p$

8) $(r \vee s) \wedge (r \vee \neg p)$ T
9) $r \vee s$ T
10) $\neg r \rightarrow s$ T
11) $s \rightarrow t$ P
12) $\neg r \rightarrow t$ T
13)

$\sim p \wedge (p \rightarrow r)$
$\sim p \wedge (\neg p \vee r)$
$(\sim p \wedge \sim p) \vee (\sim p \wedge r)$
$\sim p \vee \sim p$

"Ram is not confident and he is engaged with work all the time. Ram will play chess only if he is confident. If Ram does not play chess then he will visit Chennai. If Ram visits Chennai then he will be happy." The conclusion is "Ram is happy".

Show that the following premises are inconsistent. "If Anita gets her degree, then she will go for higher studies. If she goes for higher studies, then she will go to foreign. If she gets married, then she will not go to foreign. Hence, Anita gets her degree and she gets married".

p : Anitha gets degree
q : " go to higher studies
r : " go to foreign
s : " gets married.

$p \rightarrow q, \; q \rightarrow r, \; s \rightarrow \neg r, \Rightarrow p \wedge s$

1) $p \rightarrow q$ P

$P \to q, \quad q \to r, \quad s \to \neg r, \implies P \wedge s$

1) $P \to q$ P
2) $q \to r$ P
3) $P \to r$ T
4) $s \to \neg r$ P
5) $r \to \neg s$ T
6) $P \to \neg s$ T (3)(5)
7) $\neg P \vee \neg s$ T
8) $\neg(P \wedge s)$ — Inconsistent.