Problems

① In Boolean algebra, if $a+b=1$ and $a.b=0$, s.t $b=a'$ via the complement of every elt a is unique.

Proof:-

$$b = b.1$$
$$= b.(a+a')$$
$$= b.a + b.a'$$
$$= a.b + b.a' \quad \text{Commutative law}$$
$$= 0 + b.a' \quad \because a.b=0 \text{ Given}$$
$$= a.a' + a'.b$$
$$= a'.(a+b) \quad \text{Distributive law}$$
$$= a'.1 \quad \text{given } a+b=1$$
$$= a'$$

② Simplify the Boolean expression $a'.b'.c + a.b'.c + a^*.b'.c'$ using Boolean algebra identities

Soln:- $a'.b'.c + a.b'.c + a^*.b'.c' = a'.b'.c + (a.b').(c+c')$

$$= a'.b'.c + a.b'.1$$
$$= \cancel{b'.(a+a'.c)}\ b'.(a'.c+a)$$
$$= b'.(a+a').(a+c)$$
$$= b'.(a+c)$$
$$= b'.a + b'.c$$

③ In any Boolean algebra, show that $ab'+a'b=0$ if and only if $a=b$.

Soln:- (i) Let $a=b$, Then

$$ab'+a'b = aa'+a'a \quad (\text{or}) \quad bb'+a'a$$
$$= 0+0 = 0 \qquad = 0.$$

(ii) $ab' + a'b = 0$

$\Rightarrow \quad a + ab' + a'b = a$

$\Rightarrow \quad a + a'.b = a \quad$ by absorption law

$\Rightarrow \quad (a+a').(a+b) = a$

$\quad 1.(a+b) = a$

$\quad a+b = a \quad$ —— ①

$a.(a+b) = a$

$a + a.b = a$

III'ly $\quad ab' + a'b + b = b$

$ab' + b = b \quad$ by absorption law

$(a+b).(b'+b) = b$

$(a+b) = b \quad$ —— ②

$\therefore$ ① & ② $\Rightarrow \quad a = b.$

④ In any Boolean algebra, show that

$(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

Soln:-

LHS $\quad (a+b'+0)(b+c'+0)(c+a'+0)$

$= (a+b'+c.c')(b+c'+a.a')(c+a'+b.b')$

$= (a+b'+c).(a+b'+c')(b+c'+a)(c'+b+a')$
$\quad (c+a'+b)(c+a'+b')$

$= \{(a'+b+c)(a'+b+c')\}\{(b'+c+a)(b'+c+a')\}$
$\quad \{(c'+a+b)(c'+a+b')\}$

$= (a'+b+cc')(b'+c+aa')(c'+a+bb')$

$= (a'+b+0)(b'+c+0)(c'+a+0)$

$= (a'+b)(b'+c)(c'+a)$

$=$ RHS.

⑤ In any Boolean algebra prove that

(i) $x + wy + uvz = \{(x+u+w)(x+u+y)\}\{(x+v+w)(x+v+y)\}\{(x+w+z)(x+y+z)\}$

(ii) $ab + abc + a'b + ab'c = b + ac$

Soln:-

(i) $RHS = \{(x+u+wy)(x+v+wy)\}(x+z+wy)$

$= ((x+wy)+uv)(x+wy+z)$

$= x + wy + uvz$

$= L.HS$

(ii) $LHS \quad (ab + a'b) + (abc + ab'c)$

$= (a+a').b + ac(b+b')$

$= 1.b + ac.1$

$= b + ac$

H.W

⑥ Simplify (i) $(x+y+xy)(x+z) = x + yz$

(ii) $x(y + z(xy+xz)') = xy$

(iii) $xy' + z + (x'+y)z' = 1.$

⑦ Simplify (i) $a'b(a'+c) + ab'(b'+c) = a'b + ab'$

(ii) $a + a'bc' + (b+c)' = a + c'.$

⑧ (i) $(x+y)(x'+z) = xz + x'y + yz = xz + x'y$

(ii) $(xy'z' + xy'z + xyz + xyz')(x+y) = x.$

(1) Find the distinctive normal forms of the following Boolean expressions by (i) truth table method and (ii) algebraic method.

(a) $f(x,y,z) = xy + yz'$

(b) $f(x,y,z) = y' + [z' + x + (yz)'](z + x'y)$

(c) $f(x,y,z,w) = xy + yzw'$.

Soln:- (a)

(i)

x	y	z	xy	yz'	f
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	1	0	1

The minterms corresponding to the 3 rows for which 1 occurs in the f column are $x'yz'$, xyz' & xyz.

$\therefore$ DNF of $f(x,y,z) = x'yz' + xyz' + xyz$.

(ii) Algebraic method

$$f = xy + yz' = xy(z+z') + (x+x')yz'$$

$$= xyz + xyz' + xyz' + x'yz'$$

$$= xyz + xyz' + x'yz'.$$

(b)

(i)

x	y	z	yz	$(yz)'$	$g = z'+x+(yz)'$	$x'y$	$h = z+x'y$	gh	$f = y'+gh$
0	0	0	0	1	1	0	0	0	1
0	0	1	0	1	1	0	1	1	1
0	1	0	0	1	1	1	1	1	1
0	1	1	1	0	0	1	1	0	0
1	0	0	0	1	1	0	0	0	1
1	0	1	0	1	1	0	1	1	1
1	1	0	0	1	1	0	0	0	0
1	1	1	1	0	1	0	1	1	1

The minterms corresponding to all the rows except the 4th & 7th rows.

$\therefore$ DNF of $f(x,y,z) = x'y'z' + x'y'z + x'yz' + xy'z' + xy'z + xyz.$

(ii)

$$f(x,y,z) = y' + [z'+x+y'+x'](z+x'y)$$
$$= y' + (x+y'+z')(z+x'y) \quad \because z'+x' = z'$$
$$= y' + xz + y'z + x'y'z \quad \because xx' = 0 = yy' = zz'$$
$$= y'(x+x') + xz(y+y') + y'z(x+x') + x'y'z$$
$$= xy'(z+z') + x'y'(z+z') + xyz + xzy' + y'zx + y'zx' + x'y'z$$
$$= xy'z + xy'z' + x'y'z + x'y'z' + xyz + xy'z + xy'z + x'y'z + x'y'z$$
$$= xy'z + xy'z' + x'y'z + x'y'z' + xyz + x'yz'.$$

(c)

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	1	0	1
w	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
xy	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
yzw'	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
f	0	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1

$\therefore$ DNF is $x'yzw + xyz'w' + xyz'w + xyzw' + xyzw.$

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(ii) Algebraic

$$f(x,y,z) = xy(z+z') + (x+x')yzw'$$

$$= xyz(w+w') + xyz'(w+w') + xyzw' + x'yzw'$$

$$= xyzw + xyzw' + xyz'w + xyz'w' + xyzw' + x'yzw'$$

$$= xyzw + xyzw' + xyz'w + xyz'w + x'yzw'.$$

(2) Find the conjective normal forms of the following Boolean expression using (i) truth table (ii) algebraic method.

(a) $f(x,y,z) = (x+z)y$

(b) $f(x,y,z) = x$ (H.W)

(c) $f(x,y,z) = (yz+xz')(xy'+z)'$ (H.W)

Soln:-
(a) (i)

x	y	z	$x+z$	$(x+z)y$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

The Maxterms corresponding to the rows for which 0 occurs in the f column are

$$(x+y+z)\cdot(x+y+z')(x+y'+z)(x'+y+z)(x'+y+z').$$

(ii)

$$f = (x+z)y = (x+z+yy')y$$

$$= (x+y+z)(x+y'+z)(y+xx')$$

$$= (x+y+z)\cdot(x+y'+z)(x+y)(x'+y)$$

$$= (x+y+z)(x+y'+z)(x+y+zz')(x'+y+zz')$$

$$= (x+y+z)(x+y'+z)(x+y+z)(x+y+z')(x'+y+z)(x'+y+z')$$

$$= (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)(x'+y+z') //.$$