Defn:- Distributive lattice

A lattice $\langle L, *, \oplus \rangle$ is called a distributive lattice, if for any $a, b, c \in L$,

$$a * (b \oplus c) = (a * b) \oplus (a * c)$$

&

$$a \oplus (b * c) = (a \oplus b) * (a \oplus c).$$

Theorem: Every chain is a distributive lattice

Proof:- Let $\langle L, \leq \rangle$ be a totally ordered set or chain & $a, b, c \in L$.

Consider the following cases:

(i) $a \leq b$ or $a \leq c$ (ii) $a \geq b$ and $a \geq c$.

case (i) $a \leq b$ or $a \leq c$

Now $a * (b \oplus c) = a$ — (1)

since $a \leq b$ or $a \leq c$

$\Rightarrow a \leq b \oplus c$

& $b, c \leq b \oplus c$

& $(a * b) \oplus (a * c)$

$= a \oplus a = a$ — (2)

(1) & (2) $\Rightarrow$ $a * (b \oplus c) = (a * b) \oplus (a * c)$

case (ii) $a \geq b$ & $a \geq c$.

$a * (b \oplus c) = b \oplus c$

& $(a * b) \oplus (a * c) = b \oplus c$

$\therefore$ $a * (b \oplus c) = (a * b) \oplus (a * c)$

Hence the theorem.

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