Final Assessment Test (FAT) – May 2017

VIT® UNIVERSITY (Estd. u/s 3 of UGC Act 1956)

Course: **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s): **4419 / 4422 / 5327** Slot: **A1+TA1+TAA1**

Time: **Three Hours** Max. Marks: **100**

Answer any <u>FIVE</u> Questions

(5 X 20 = 100 Marks)

1. a) Obtain the pdnf and pcnf of the following formula and hence conclude whether it is a tautology $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ **[10]**

   b) Test the consistency of the following Statements. **[10]**

   i. If Jack studies well he will pass in exams.

   ii. If Jack studies well he will get a job.

   iii. Succeeding in exam and getting a job simultaneously are not possible for him.

   iv. Jack either enjoys or studies well.

   v. Finally, Jack enjoys.

2. a) Show that the conclusion $(x)(F(x) \to \neg S(x))$ follows from **[10]**

   (i) $(x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and

   (ii) $(\exists y)(M(y) \wedge \neg W(y))$

   b) Show that **[10]**

   (i) $\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$

   (ii) $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$

3. a) (i) Prove that for any monoid $\langle M, * \rangle$, no two rows or columns of the composition table are identical. **[4]**

   (ii) Establish the isomorphism between the following two algebraic systems: **[6]**

   I. $\langle F, \circ \rangle$ where $F = \{f^0, f^1, f^2, f^3\}$ with $f = f^1 = \{\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,4 \rangle, \langle 4,1 \rangle\}$ and the composite functions are formed from the equation $f^k = f^{k-1} \circ f,\ k \geq 2$. Further, $f^0 = f^4$ is the identity map.

   II. $\langle Z_4, +_4 \rangle$, the algebraic system of equivalence classes generated by congruence modulo 4 under addition modulo 4.

   b) State and prove Lagrange's theorem. **[10]**

4. a) What is the condition for a code to correct 'k' or fewer errors. Generate a single error correcting code with m = 4 and n = 7. **[10]**

   b) Obtain the Hasse diagrams of the lattices $\langle S_n, D \rangle$ when $n = 30,\ 45$. Which of these are complemented? Are these lattices distributive? Explain. **[10]**

5. a) (i) State and prove the isotonicity property of a lattice $\langle L, \leq \rangle$. **[5]**

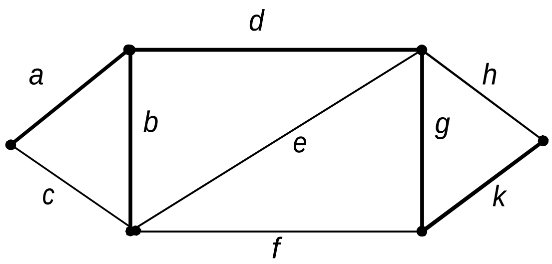
   (ii) Obtain the simplified Boolean expression which is equivalent to the expression $m_0 + m_1 + m_2 + m_3$ **[5]**

   b) Obtain the Karnaugh map for the Boolean function $f = x_1 \circ [x_2 + (x_3 \circ x_4)]$. **[10]**

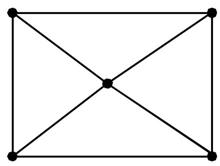
6. a) (i) Prove that any simple graph with *n* vertices has at most $\frac{n(n-1)}{2}$ edges. **[5]**

   (ii) Prove that the graph K_5 is nonplanar. **[5]**

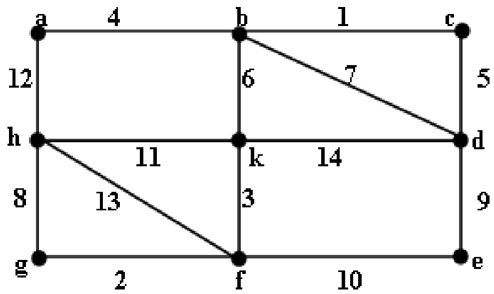
b) Determine all the fundamental circuits and fundamental cut-sets of the following graph with respect to the spanning tree shown by thick lines. Also find its vertex connectivity. **[10]**



7. a) Find the chromatic polynomial of the graph given below. **[10]**



b) Determine a shortest spanning tree for the following graph using Kruskal's algorithm. Can we modify the algorithm to generate a Hamiltonian path. **[10]**



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Final Assessment Test (FAT) – May 2017

VIT® UNIVERSITY (Estd. u/s 3 of UGC Act 1956)

Course : **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s):**4425 / 4427 / 4429** Slot: **A2+TA2+TAA2**

Time: **Three Hours** Max. Marks: **100**

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Obtain the principal conjunctive normal form of the formula S given by $(\sim P \to R)\wedge(Q \leftrightarrow P)$ and hence obtain Principal disjunctive normal form of S. **[10]**

   b) Show that the following sets of premises are inconsistent **[4+4 +2]**
   (i) $A \to (B \to C), D \to (B \wedge \sim C), A \wedge D$.
   (ii) $P \to Q, P \to R, Q \to \sim R$, P
   Hence show that $A \to (B \to C), D \to (B \wedge \sim C), A \wedge D \Rightarrow P$

2. a) Prove that $(x)\left(P(x) \to \left(Q(y) \wedge R(x)\right)\right), (\exists x)(P(x)) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x))$. **[8]**

   b) Show that the set of all the invertible elements of a monoid form a group under the same operation as that of the monoid. **[6]**

   c) Let $(S,*)$ be a given semigroup. Prove that there exists a homomorphism $f: S \longrightarrow S^S$, where $(S^S,\circ)$ is a semi group of functions from S to S under left composition. **[6]**

3. a) Show that if every element in a group is its own inverse, then the group must be abelian. **[4]**

   b) Prove that the order of a subgroup of a finite group divides the order of the group. **[10]**

   c) Let R be the set of real numbers in [0,1] and '≤' be the usual "less than or equal to" in R. Show that (R,≤) is a lattice. What are the operations of meet and join on this lattice? **[6]**

4. a) Prove that every chain is a distributive lattice. **[6]**

   b) Show that in a complemented distributive lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. **[10]**

   c) In any Boolean Algebra, show that $a \leq b \Rightarrow a + bc = b(a + c)$. **[4]**

5. a) Show that a mapping from one Boolean algebra to another which preserves the operations $\oplus$ and' also preserves the operation $*$. **[6]**

   b) Show that the Boolean expression **[4]**

$$\left[a * \left(b' \oplus c\right)\right]' * \left[b' \oplus \left(a * c'\right)'\right]' = a * b * c'$$

   c) Prove that a connected graph G is Euler if and only if all vertices of G is even. **[10]**

6. a) Prove that a tree with n vertices has n-1 edges. **[10]**

   b) Prove that every circuit has even number of edges in common with any cut-set. **[10]**

7. a) Prove that a tree with two or more vertices is 2-chromatic. **[6]**

   b) Prove that the chromatic polynomial of a complete graph with n vertices is **[8]**

$$P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1)$$

   c) Show that if a bipartite graph has any circuits, they must all be of even length. **[6]**

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Final Assessment Test (FAT) – May 2017

Course: **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s):4431 / 4434 /4436 / 4439 / 4444 / 4448 / 4934 Slot: **C1+TC1+TCC1**

Time: Three Hours Max. Marks: **100**

Answer any <u>FIVE</u> Questions

(5 X 20 = 100 Marks)

1. a) If *A* works hard, then either *B* or *C* will enjoy themselves. **[10]**
If *B* enjoys himself, then *A* will not work hard.
If *D* enjoys himself, then *C* will not.
Therefore, if *A* works hard, then *D* will not enjoy himself.
Show that the above statements constitute a valid argument by using the Rule CP.

b) Obtain the PCNF of the statement $S:(P \wedge Q) \vee (\neg P \wedge R)$. Using this obtain the PCNF of $\neg S$ and hence the PDNF of S. Also determine the unique representation of the PCNF and PDNF. **[10]**

2. a) (i) Prove that the set of idempotent elements of a commutative monoid $\langle M, * \rangle$ forms a submonoid. **[10]**
(ii) Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation $*$ defined by $a*b = \frac{1}{2}(a \cdot b); \forall a,b \in \mathrm{Q}^+$.

b) Find the code generated by the given parity matrix 'H' when the encoding function is $e:\mathrm{B}^3 \to B^6$ **[10]**

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

3. a) Let $(L, \leq)$ be a lattice in which $*$, $\oplus$ denote the operations of meet and join respectively. For any $a,b \in L$, prove that $a \leq b \Leftrightarrow a*b = a \Leftrightarrow a \oplus b = b$. **[10]**

b) Show that every chain is a distributive lattice. Also discuss about the converse of this statement with justification. **[10]**

4. a) State and prove the necessary and sufficient conditions for a connected graph to be an Euler graph. **[8]**

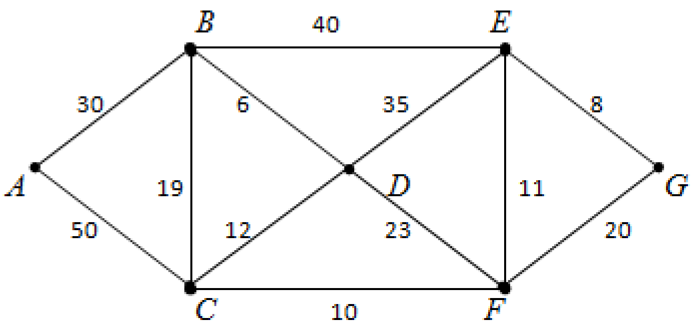
b) Obtain a graph G for the following adjacency matrix. **[6]**

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

Also find (I) the number of vertices in G.
(II) the number of edges in G.
(III) the degree of each vertex of G.
(IV) the number of loops in G and
(V) the number of components in G.

c) Let $\omega(G)$ be the number of components of G, then prove that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$. **[6]**

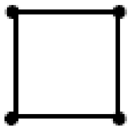
5. a) Use Kruskal's and Prim's algorithm to find a shortest path between vertices A to G in the following weighted network. **[10]**



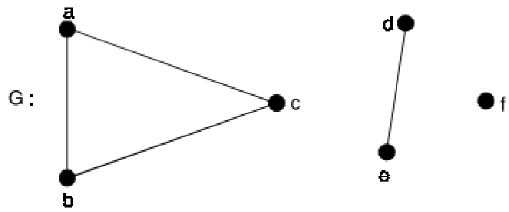
b) Prove that every tree is either unicentral or bicentral but not both. **[10]**

6. a) Find the chromatic polynomial and chromatic number for the following graph G. **[10]**

(i)



(ii)



b) Prove that a simple graph G on n vertices is a tree if and only if $P_n(\lambda) = \lambda(\lambda - 1)^{(n-1)}$ **[10]**

7. a) (i) Use Karnaugh map to simplify the following Boolean expression **[10]**

$$wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z}$$

(ii) Show that in a Boolean algebra $(a \oplus b') * (b \oplus c') * (c \oplus a') = (a' \oplus b) * (b' \oplus c) * (c' \oplus a)$.

b) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow ((\forall x)P(x)) \vee ((\exists x)Q(x))$ by using the indirect method of proof. **[10]**

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VIT UNIVERSITY (Estd. u/s 3 of UGC Act 1956)

Final Assessment Test (FAT) – May 2017

Course: **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s): **4449 / 4452 / 4457 / 4468 / 4471 / 4922 / 4929** Slot: **C2+TC2+TCC2**

Time: **Three Hours** Max. Marks: **100**

Answer any FIVE Questions
(5 X 20 = 100 Marks)

1. a) Prove that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ **[5]**

   b) Without using truth table find the Principle conjunctive normal form and Principle disjunctive normal form of $P \to (Q \wedge P) \wedge (\neg P \to (\neg Q \wedge \neg R))$ **[10]**

   c) Show that $R \wedge (P \wedge Q)$ is a valid conclusion from the premises $P \vee Q, Q \to R, P \to M$ and $\neg M$. **[5]**

2. a) Let G denote the set of all matrices of the form $\begin{bmatrix} X & X \\ X & X \end{bmatrix}$ where $X \in R^+$. Prove that G is a group under matrix multiplication. **[8]**

   b) State and prove Lagrange's theorem for groups. **[5]**

   c) Determine the group code (3, 6) using the parity check matrix H is given by **[7]**

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. a) Let $(L, \leq)$ be a lattice in which $*$(meet) and $\oplus$ (join) denote the operations. Prove that for any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **[10]**

   b) S_{42} is the set of all divisors of 42 and D is the relation "divisor of" on S_{42}, prove that (S_{42}, D) is a complemented lattice. **[5]**

   c) State and prove De-Morgan's law for lattice. **[5]**

4. a) Obtain the product of sums canonical form in three variables of the Boolean expression $x_1 * x_2$ **[5]**

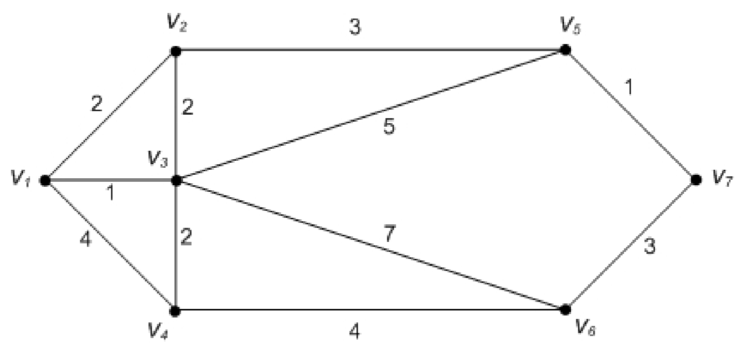
   b) In any Boolean algebra prove that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$ **[5]**

   c) Simplify the following Boolean function by using Quine- McCluskey method **[10]**

$$f(w, x, y, z) = wxyz + wxy\bar{z} + w\bar{x}\bar{y}z + wx\bar{y}z + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z$$

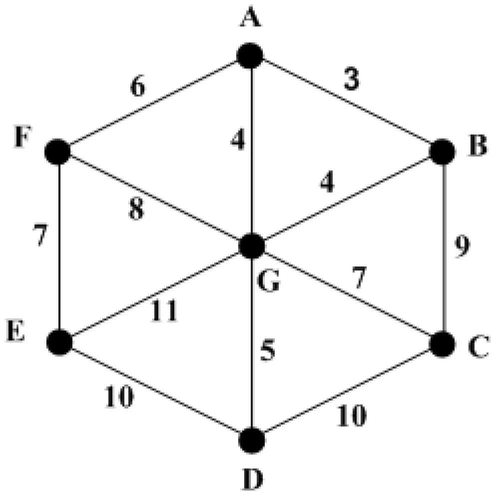
5. a) Prove or disprove the validity of the following argument (i) "All men are fallible" (ii) "all kings are men" The conclusion is "all kings are fallible". **[10]**

   b) Using Dijkstra's algorithm to find the shortest path from v_1 to v_7 **[10]**



6. a) Explain the three kinds of tree traversal in graphs. Give examples. **[10]**

b) Find the minimum spanning tree of the following graph by using kruskal's algorithm. **[10]**



7. a) (i) Define Bipartite graph. Give example. **[4]**

(ii) Define the Chromatic number and write the properties of Chromatic number. **[8]**

b) Define Chromatic Partitioning and the Chromatic Polynomial. **[8]**

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