



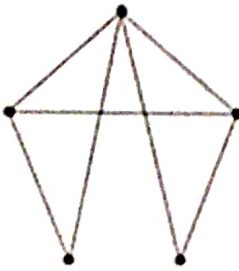
Final Assessment Test (FAT) – November/December 2022

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	DISCRETE MATHEMATICS AND GRAPH THEORY	Course Code	BMAT205L
Faculty Name	Prof. Devi Yamini S	Slot	D2+TD2+TDD2
		Class Nbr	CH2022231001480
Time	3 Hours	Max. Marks	100

PART-A (10 X 10 Marks)

Answer any 10 questions

- (a) Determine the PCNF of the Boolean expression [5 Marks] [10]
 $f(p, q, r) = (p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$.

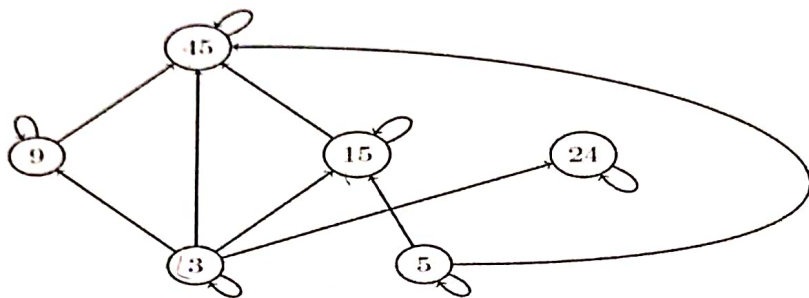
(b) Consider the following premises,
 (i) If he takes dinner, he does not drink tea.
 (ii) He eats fruits only if he drinks tea.
 (iii) He does not take the juice unless he eats fruits.
 Can we conclude that he has taken the juice whenever he takes the dinner? If not, what is the correct conclusion?. [5 Marks]
- (a) Establish the validity of the argument: "All integers are rational numbers. Some integers are a power of 2. Therefore, some rational numbers are a power of 2". [5 Marks] [10]
 (b) Write the inverse and contrapositive of the statement:
 "If every person wears a mask then there is at least one person who can be saved from the transmission of disease". [5 Marks]
- (a) Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ [10]
 corresponding to the encoding function $e : B^2 \rightarrow B^5$
 [5 Marks]
 (b) For the following graph, find the chromatic polynomial and the chromatic number.
 [5 Marks]
- (a) Consider an operation on the set of integers Z defined by $m * n = m + n + 1$. Prove that $(Z, *)$ is a monoid. Is it a group? [5 Marks] [10]
 (b) Prove that $\{0, 4, 8\}$ is a subgroup of the group $(Z_{12}, +_{12})$. Also, find the distinct right cosets of the subgroup $\{0, 4, 8\}$. [5 Marks]

5. (a) How many different words can be formed using the letters of the word "COSTING" where
- Two vowels remain together?
 - Two vowels do not remain together?
 - Two vowels remain in odd positions ? [5 Marks]
- (b) Solve the following recurrence relation

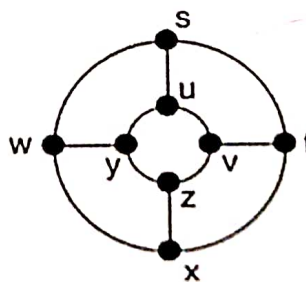
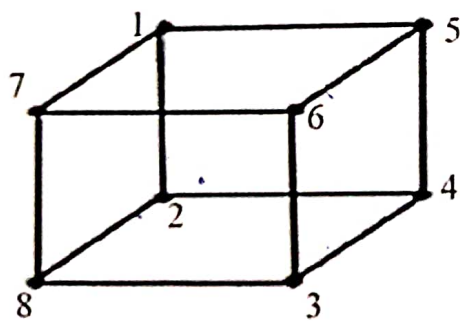
$$\begin{aligned} t_n &= 0 & \text{if } n &= 0 \\ t_n &= 5 & \text{if } n &= 1 \\ t_n &= 3t_{n-1} + 4t_{n-2} & \text{other wise} \end{aligned} \quad [5 \text{ Marks}]$$

6. (a) How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 = a_2 \bmod 5$ and $b_1 \bmod 5 = b_2 \bmod 5$ [5 Marks]
- (b) A relation R on the set $\{1, 2, 3, 4, 5\}$ is defined by aRb if 3 divides $a - b$. Determine whether R is a partial order relation or not. [5 Marks]

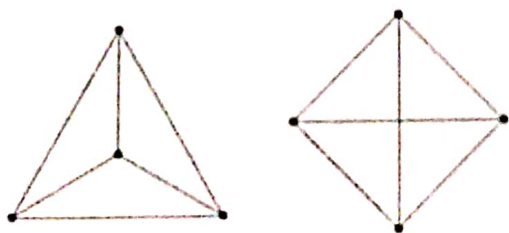
7. [10]



- Show that the relation corresponding to the above directed graph is a partial order relation.
 - Obtain its corresponding Hasse diagram.
 - Find the greatest and least element of the graph if it exists.
 - For the subset $\{5, 9\}$ of the relation find the upper bound, lower bound, LUB and GLB.
 - Check whether the Hasse diagram is a Lattice or not.
8. (a) Check whether the following graphs are isomorphic. Justify your answer. [5 Marks] [10]

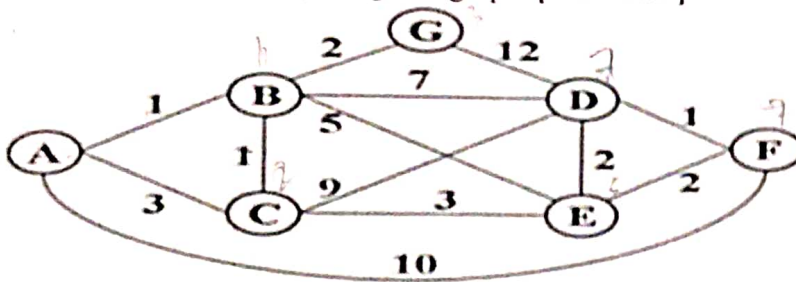


- (b) (I) Identify the planar graph and plane graph from the graphs given below with proper justification: [2 Marks]

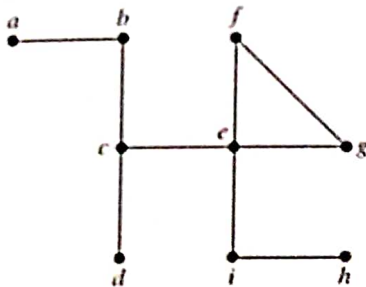


- (II) Which of the following graphs has an Eulerian circuit? Justify.
- A complete graph on 90 vertices .
 - A complete bipartite graph $K_{20,20}$.
- [3 Marks]

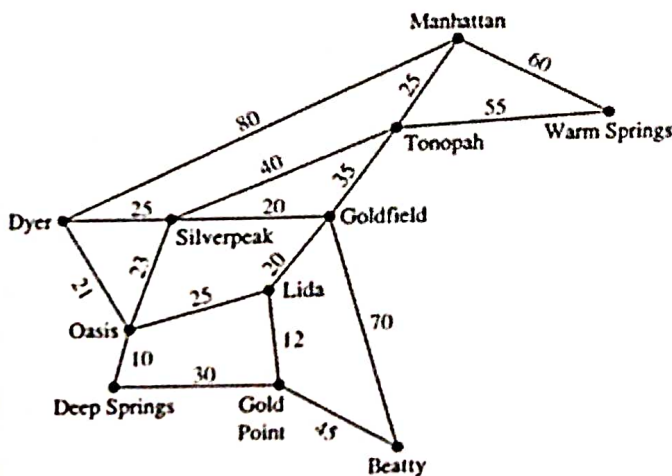
9. (a) Apply the Dijkstra's algorithm to find the shortest path and distance from A to all other vertices in the following weighted graph. [7 Marks] [10]



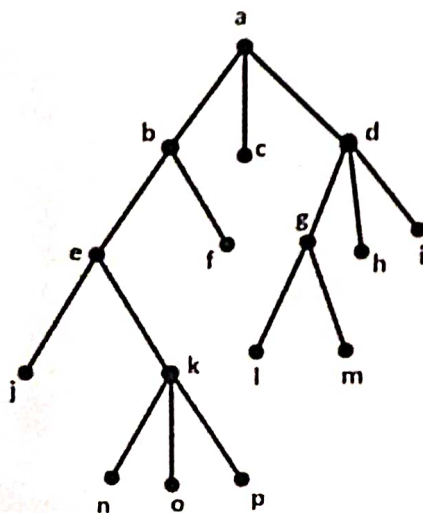
- (b) Find all the cut vertices and cut edges in the following graph: [3 Marks]



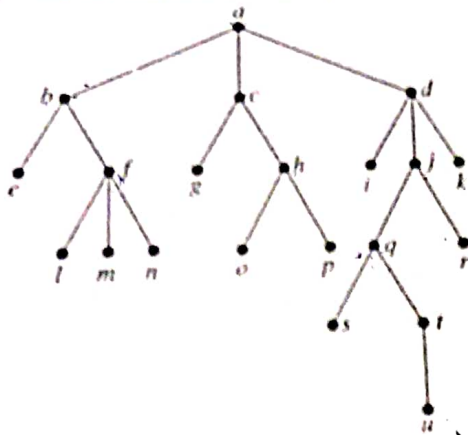
10. The roads represented by the following graph are all unpaved. The lengths (in kms) of the roads between pairs of towns are represented by edge weights. Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved? [10]



11. (a) Find pre order, post order and in order traversal of the following tree [6 Marks] [10]



- (b) Find the distance, centre, and eccentricity of each vertex of the following tree. [4 Marks]



12. Suppose that a new company has five employees: Rahul, Sam, Smith, Raj, and Mani. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Rahul could do planning, sales, marketing, or industry relations; Sam could do planning or development; Smith could do publicity, sales, or industry relations; Raj could do planning, sales, or industry relations; and Mani could do planning, publicity, sales, or industry relations. [10]
- Model the capabilities of these employees through a graph.
 - Find an assignment such that each employee is assigned one responsibility.
 - Find a matching for the graph obtained in (i)

