

## CHAPTER

## 8

# Principles of Friction



## 8.1. INTRODUCTION

It has been established since long that all surfaces of the bodies are never perfectly smooth. It has been observed that whenever, even a very smooth surface is viewed under a microscope, it is found to have some roughness and irregularities, which may not be detected by an ordinary touch.

It will be interesting to know that if a block of one substance is placed over the level surface of the same or different material, a certain degree of interlocking of the minutely projecting particles takes place. This does not involve any force, so long as the block does not move or tends to move. But whenever one of the blocks moves or tends to move tangentially with respect to the surface, on which it rests, the

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interlocking property of the projecting particles opposes the motion. This opposing force, which acts in the opposite direction of the movement of the block, is called *force of friction* or simply *friction*. It is of the following two types:

1. Static friction.    2. Dynamic friction.

### 8.2. STATIC FRICTION

It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

### 8.3. DYNAMIC FRICTION

It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types :

1. *Sliding friction*. It is the friction, experienced by a body when it slides over another body.
2. *Rolling friction*. It is the friction, experienced by a body when it rolls over another body.

### 8.4. LIMITING FRICTION

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite to the applied force. Thus the force of friction has a remarkable property of adjusting its magnitude, so as to become exactly equal and opposite to the applied force, which tends to produce motion.

There is, however, a limit beyond which the force of friction cannot increase. If the applied force exceeds this limit, the force of friction cannot balance it and the body begins to move, in the direction of the applied force. This maximum value of frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction, the body remains at rest, and the friction is called static friction, which may have any value between zero and limiting friction.

### 8.5. NORMAL REACTION

It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity. The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called normal reaction and is, generally, denoted by  $R$ . It will be interesting to know that the term 'normal reaction' is very important in the field of friction, as the force of friction is directly proportional to it.

### 8.6. ANGLE OF FRICTION

Consider a body of weight  $W$  resting on an inclined plane as shown in Fig. 8.1. We know that the body is in equilibrium under the action of the following forces :

1. Weight ( $W$ ) of the body, acting vertically downwards,
2. Friction force ( $F$ ) acting upwards along the plane, and
3. Normal reaction ( $R$ ) acting at right angles to the plane.

Let the angle of inclination ( $\alpha$ ) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

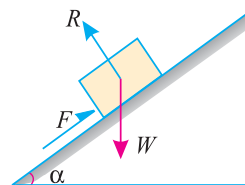


Fig. 8.1. Angle of friction.

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### 8.7. COEFFICIENT OF FRICTION

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is generally denoted by  $\mu$ .

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \phi \quad \text{or} \quad F = \mu R$$

where

$\phi$  = Angle of friction,

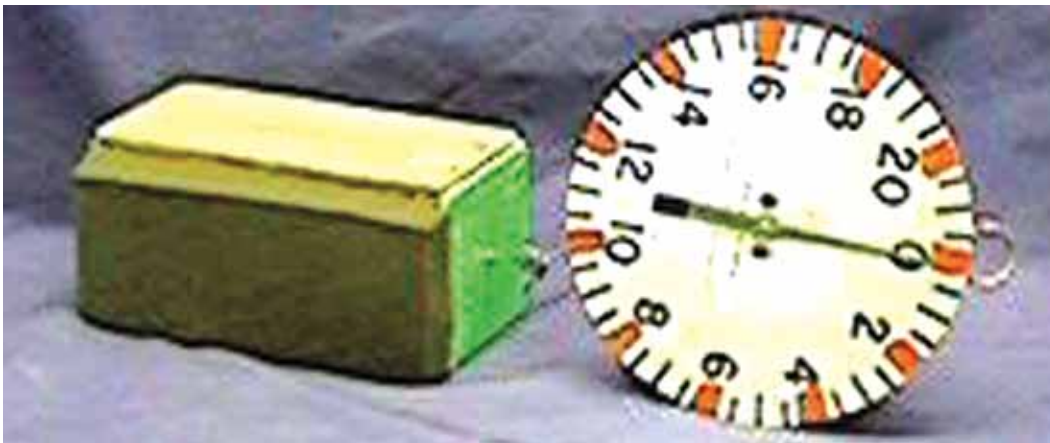
$F$  = Limiting friction, and

$R$  = Normal reaction between the two bodies.

### 8.8. LAWS OF FRICTION

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads :

1. Laws of static friction, and
2. Laws of kinetic or dynamic friction.



The coefficient of friction of various surfaces, as well as the difference between static and kinetic friction can be illustrated by pulling objects with large spring scale.

### 8.9. LAWS OF STATIC FRICTION

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically :

$$\frac{F}{R} = \text{Constant}$$

where  $F$  = Limiting friction, and  
 $R$  = Normal reaction.

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

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## 8.10. LAWS OF KINETIC OR DYNAMIC FRICTION

Following are the laws of kinetic or dynamic friction :

1. The force of friction always acts in a direction, opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
3. For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.



This rock climber uses the static frictional force between her hands and feet and the vertical rock walls.

## 8.11. EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.

Now the value of the force of friction is obtained from the relation :

$$F = \mu R$$

where

$\mu$  = Coefficient of friction, and

$R$  = Normal reaction.

**Example 8.1.** A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of  $25^\circ$  with the horizontal.

**Solution.** Given: Weight of the body ( $W$ ) = 300 N; Coefficient of friction ( $\mu$ ) = 0.3 and angle made by the force with the horizontal ( $\alpha$ ) =  $25^\circ$

Let  $P$  = Magnitude of the force, which can move the body, and

$F$  = Force of friction.

Resolving the forces horizontally,

$$F = P \cos \alpha = P \cos 25^\circ = P \times 0.9063$$

and now resolving the forces vertically,

$$\begin{aligned} R &= W - P \sin \alpha = 300 - P \sin 25^\circ \\ &= 300 - P \times 0.4226 \end{aligned}$$

We know that the force of friction ( $F$ ),

$$0.9063 P = \mu R = 0.3 \times (300 - 0.4226 P) = 90 - 0.1268 P$$

$$\text{or} \quad 90 = 0.9063 P + 0.1268 P = 1.0331 P$$

$$\therefore P = \frac{90}{1.0331} = 87.1 \text{ N} \quad \text{Ans.}$$

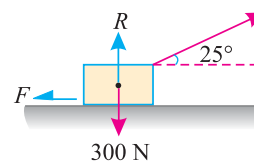


Fig. 8.2.

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**Example 8.2.** A body, resting on a rough horizontal plane, required a pull of 180 N inclined at  $30^\circ$  to the plane just to move it. It was found that a push of 220 N inclined at  $30^\circ$  to the plane just moved the body. Determine the weight of the body and the coefficient of friction.

**Solution.** Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane ( $\alpha$ ) =  $30^\circ$

Let  $W$  = Weight of the body  
 $R$  = Normal reaction, and  
 $\mu$  = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction ( $F_1$ ) will act towards left as shown in Fig. 8.3. (a).

Resolving the forces horizontally,

$$F_1 = 180 \cos 30^\circ = 180 \times 0.866 = 155.9 \text{ N}$$

and now resolving the forces vertically,

$$R_1 = W - 180 \sin 30^\circ = W - 180 \times 0.5 = W - 90 \text{ N}$$

We know that the force of friction ( $F_1$ ),

$$155.9 = \mu R_1 = \mu (W - 90) \quad \dots(i)$$

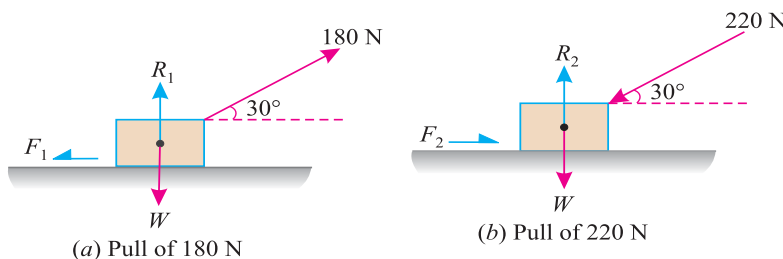


Fig. 8.3.

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction ( $F_2$ ) will act towards right as shown in Fig. 8.3 (b).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30^\circ = 220 \times 0.866 = 190.5 \text{ N}$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30^\circ = W + 220 \times 0.5 = W + 110 \text{ N}$$

We know that the force of friction ( $F_2$ ),

$$190.5 = \mu R_2 = \mu (W + 110) \quad \dots(ii)$$

Dividing equation (i) by (ii)

$$\frac{155.9}{190.5} = \frac{\mu (W - 90)}{\mu (W + 110)} = \frac{W - 90}{W + 110}$$

$$155.9 W + 17\,149 = 190.5 W - 17\,145$$

$$34.6 W = 34\,294$$

$$\text{or } W = \frac{34\,294}{34.6} = 991.2 \text{ N} \quad \text{Ans.}$$

Now substituting the value of  $W$  in equation (i),

$$155.9 = \mu (991.2 - 90) = 901.2 \mu$$

$$\therefore \mu = \frac{155.9}{901.2} = 0.173 \quad \text{Ans.}$$

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**Example 8.3.** Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibrium position as shown in Fig. 8.4.

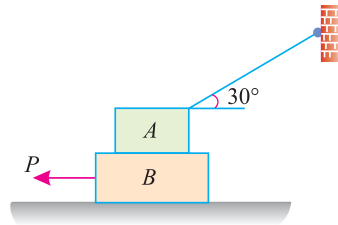


Fig. 8.4.

If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3, find the force ( $P$ ) required to move the block B.

**Solution.** Given: Weight of block A ( $W_A$ ) = 1 kN; Weight of block B ( $W_B$ ) = 2 kN and coefficient of friction ( $\mu$ ) = 0.3.

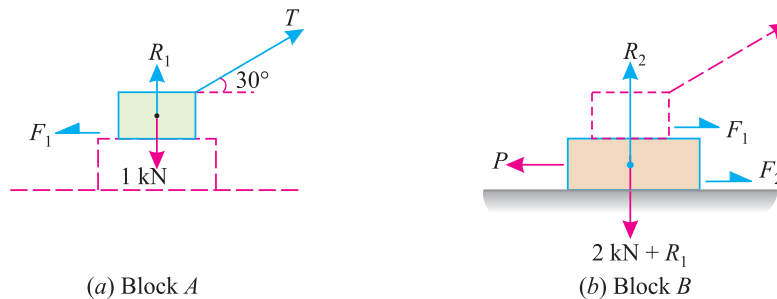


Fig. 8.5.

The forces acting on the two blocks A and B are shown in Fig. 8.5 (a) and (b) respectively. First of all, consider the forces acting in the block A.

Resolving the forces vertically,

$$R_1 + T \sin 30^\circ = 1 \text{ kN}$$

$$\text{or} \quad T \sin 30^\circ = 1 - R_1 \quad \dots(i)$$

and now resolving the forces horizontally,

$$T \cos 30^\circ = F_1 = \mu R_1 = 0.3 R_1 \quad \dots(ii)$$

Dividing equation (i) by (ii)

$$\frac{T \sin 30^\circ}{T \cos 30^\circ} = \frac{1 - R_1}{0.3 R_1} \quad \text{or} \quad \tan 30^\circ = \frac{1 - R_1}{0.3 R_1}$$

$$\therefore \quad 0.5774 = \frac{1 - R_1}{0.3 R_1} \quad \text{or} \quad 0.5774 \times 0.3 R_1 = 1 - R_1$$

$$\text{or} \quad 0.173 R_1 = 1 - R_1 \quad \text{or} \quad 1.173 R_1 = 1$$

$$\text{or} \quad R_1 = \frac{1}{1.173} = 0.85 \text{ kN}$$

$$\text{and} \quad F_1 = \mu R_1 = 0.3 \times 0.85 = 0.255 \text{ kN} \quad \dots(iii)$$

Now consider the block B. A little consideration will show that the downward force of the block A (equal to  $R_1$ ) will also act along with the weight of the block B.

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Resolving the forces vertically,

$$R_2 = 2 + R_1 = 2 + 0.85 = 2.85 \text{ kN}$$

$$\therefore F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855 \text{ kN} \quad \dots(iv)$$

and now resolving the forces horizontally,

$$P = F_1 + F_2 = 0.255 + 0.855 = 1.11 \text{ kN} \quad \text{Ans.}$$

**Example 8.4.** What is the maximum load ( $W$ ) which a force  $P$  equal to 5 kN will hold up, if the coefficient of friction at  $C$  is 0.2 in the arrangement shown in Fig. 8.6. Neglect other friction and weight of the member.

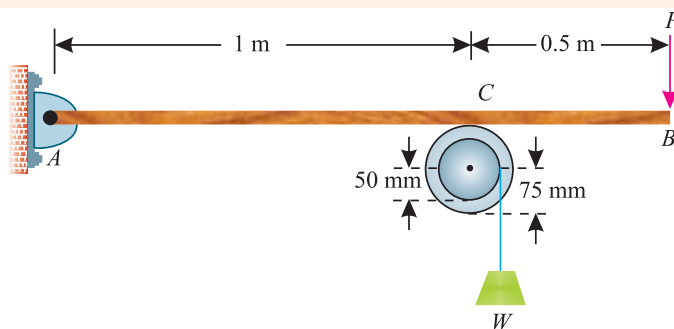


Fig. 8.6.

If  $W = 3 \text{ kN}$  and  $P = 4.5 \text{ kN}$ , what are the normal and tangential forces transmitted at  $C$  ?

**Solution.** Given: Force ( $P$ ) = 5 kN and coefficient of friction at  $C$  ( $\mu$ ) = 0.2

Maximum load  $W$

Let  $R$  = Normal reaction of the pulley on the beam at  $C$ .

First of all, consider the equilibrium of the beam  $AB$ . Taking moments about the hinge  $A$  and equating the same,

$$R \times 1 = 5 \times 1.5 = 7.5 \quad \text{or} \quad R = 7.5 \text{ kN}$$

Now consider the equilibrium of the pulley. It is subjected to a normal reaction of 7.5 kN (as calculated above). The load ( $W$ ) tends to rotate it. A little consideration will show that the rotation of the pulley is prevented by the frictional force between the pulley and beam at  $C$ . We know that maximum force of friction at  $C$

$$= \mu \cdot R = 0.2 \times 7.5 = 1.5 \text{ kN}$$

Now taking moments about the centre of the pulley and equating the same,

$$W \times 50 = 1.5 \times 75 = 112.5$$

$$\text{or} \quad W = \frac{112.5}{50} = 2.25 \text{ kN} \quad \text{Ans.}$$

Normal and tangential forces transmitted at  $C$

Now consider a weight  $W$  equal to 3 kN suspended from the pulley and a force  $P$  equal to 4.5 kN applied at  $B$ .

Let  $R_1$  = Normal force or normal reaction at  $C$ , and

$F_1$  = Tangential force at  $C$ .

Again consider equilibrium of the beam. Taking moments about the hinge  $A$  and equating the same,

$$R_1 \times 1 = 4.5 \times 1.5 = 6.75$$

$$\text{or} \quad R_1 = 6.75 \text{ kN} \quad \text{Ans.}$$



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We know that the tangential force at  $C$  will be the frictional force between the pulley and beam. Again taking moments about the centre of the pulley and equating the same,

$$F_1 \times 75 = W \times 50 = 3 \times 50 = 150$$

$$\text{or} \quad F_1 = \frac{150}{75} = 2 \text{ kN} \quad \text{Ans.}$$

### 8.12. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE

Consider a body, of weight  $W$ , lying on a rough plane inclined at an angle  $\alpha$  with the horizontal as shown in Fig. 8.7 (a) and (b).

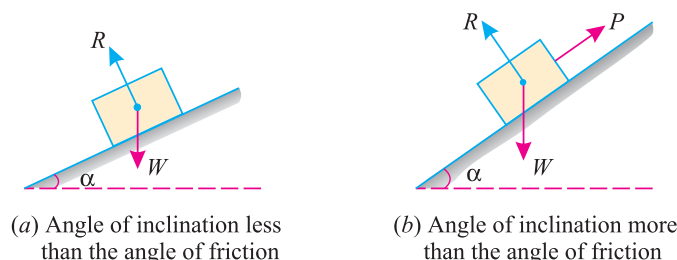


Fig. 8.7.

A little consideration will show, that if the inclination of the plane, with the horizontal, is less than the angle of friction, the body will be automatically in equilibrium as shown in Fig. 8.7 (a). If in this condition, the body is required to be moved upwards or downwards, a corresponding force is required, for the same. But, if the inclination of the plane is more than the angle of friction, the body will move down. And an upward force ( $P$ ) will be required to resist the body from moving down the plane as shown in Fig. 8.7 (b).

Though there are many types of forces, for the movement of the body, yet the following are important from the subject point of view :

1. Force acting along the inclined plane.
2. Force acting horizontally.
3. Force acting at some angle with the inclined plane.

**Note.** In all the above mentioned three types of forces, we shall discuss the magnitude of force, which will keep the body in equilibrium, when it is at the point of sliding downwards or upwards.

### 8.13. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. 8.8. (a) and (b).

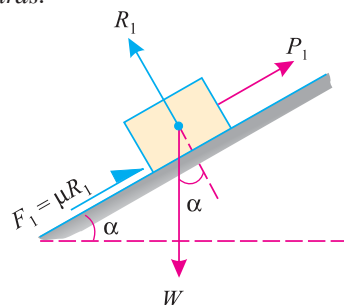
Let  $W$  = Weight of the body,  
 $\alpha$  = Angle, which the inclined plane makes with the horizontal,  
 $R$  = Normal reaction,  
 $\mu$  = Coefficient of friction between the body and the inclined plane, and  
 $\phi$  = Angle of friction, such that  $\mu = \tan \phi$ .

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

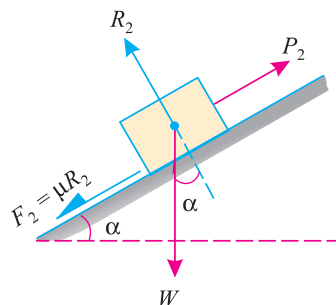


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1. Minimum force ( $P_1$ ) which will keep the body in equilibrium, when it is at the point of sliding downwards.



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

Fig. 8.8.

In this case, the force of friction ( $F_1 = \mu.R_1$ ) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.8 (a). Now resolving the forces along the plane,

$$P_1 = W \sin \alpha - \mu.R_1 \quad \dots(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos \alpha \quad \dots(ii)$$

Substituting the value of  $R_1$  in equation (i),

$$P_1 = W \sin \alpha - \mu W \cos \alpha = W (\sin \alpha - \mu \cos \alpha)$$

and now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_1 = W (\sin \alpha - \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by  $\cos \phi$ ,

$$P_1 \cos \phi = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha) = W \sin (\alpha - \phi)$$

$$\therefore P_1 = W \times \frac{\sin (\alpha - \phi)}{\cos \phi}$$

2. Maximum force ( $P_2$ ) which will keep the body in equilibrium, when it is at the point of sliding upwards.

In this case, the force of friction ( $F_2 = \mu.R_2$ ) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.8 (b). Now resolving the forces along the plane,

$$P_2 = W \sin \alpha + \mu.R_2 \quad \dots(i)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha \quad \dots(ii)$$

Substituting the value of  $R_2$  in equation (i),

$$P_2 = W \sin \alpha + \mu W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

and now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_2 = W (\sin \alpha + \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by  $\cos \phi$ ,

$$P_2 \cos \phi = W (\sin \alpha \cos \phi + \sin \phi \cos \alpha) = W \sin (\alpha + \phi)$$

$$\therefore P_2 = W \times \frac{\sin (\alpha + \phi)}{\cos \phi}$$

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**Example 8.5.** A body of weight 500 N is lying on a rough plane inclined at an angle of  $25^\circ$  with the horizontal. It is supported by an effort ( $P$ ) parallel to the plane as shown in Fig. 8.9.

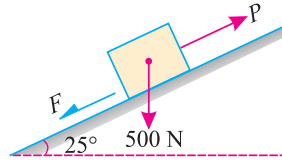


Fig. 8.9.

Determine the minimum and maximum values of  $P$ , for which the equilibrium can exist, if the angle of friction is  $20^\circ$ .

**Solution.** Given: Weight of the body ( $W$ ) = 500 N ; Angle at which plane is inclined ( $\alpha$ ) =  $25^\circ$  and angle of friction ( $\phi$ ) =  $20^\circ$ .

*Minimum value of  $P$*

We know that for the minimum value of  $P$ , the body is at the point of sliding downwards. We also know that when the body is at the point of sliding downwards, then the force

$$\begin{aligned} P_1 &= W \times \frac{\sin(\alpha - \phi)}{\cos \phi} = 500 \times \frac{\sin(25^\circ - 20^\circ)}{\cos 20^\circ} \text{ N} \\ &= 500 \times \frac{\sin 5^\circ}{\cos 20^\circ} = 500 \times \frac{0.0872}{0.9397} = 46.4 \text{ N} \quad \text{Ans.} \end{aligned}$$

*Maximum value of  $P$*

We know that for the maximum value of  $P$ , the body is at the point of sliding upwards. We also know that when the body is at the point of sliding upwards, then the force

$$\begin{aligned} P_2 &= W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 500 \times \frac{\sin(25^\circ + 20^\circ)}{\cos 20^\circ} \text{ N} \\ &= 500 \times \frac{\sin 45^\circ}{\cos 20^\circ} = 500 \times \frac{0.7071}{0.9397} = 376.2 \text{ N} \quad \text{Ans.} \end{aligned}$$

**Example 8.6.** An inclined plane as shown in Fig. 8.10. is used to unload slowly a body weighing 400 N from a truck 1.2 m high into the ground.

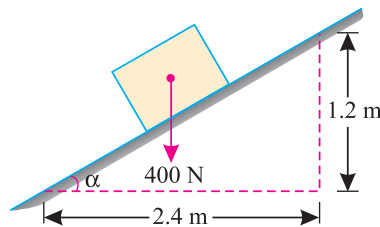


Fig. 8.10.

The coefficient of friction between the underside of the body and the plank is 0.3. State whether it is necessary to push the body down the plane or hold it back from sliding down. What minimum force is required parallel to the plane for this purpose ?

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**Solution.** Given: Weight of the body ( $W$ ) = 400 N and coefficient of friction ( $\mu$ ) = 0.3.  
Whether it is necessary to push the body down the plane or hold it back from sliding down.

We know that

$$\tan \alpha = \frac{1.2}{2.4} = 0.5 \quad \text{or} \quad \alpha = 26.5^\circ$$

and normal reaction,  $R = W \cos \alpha = 400 \cos 26.5^\circ$   
 $= 400 \times 0.8949 = 357.9$  N

$\therefore$  Force of friction,

$$F = \mu R = 0.3 \times 357.9 = 107.3 \text{ N} \quad \dots(i)$$

Now resolving the 400 N force along the plane

$$= 400 \sin \alpha = 400 \times \sin 26.5^\circ \text{ N}$$

$$= 400 \times 0.4462 = 178.5 \text{ N} \quad \dots(ii)$$

We know that as the force along the plane (which is responsible for sliding the body) is more than the force of friction, therefore the body will slide down. Or in other words, it is not necessary to push the body down the plane, rather it is necessary to hold it back from sliding down. **Ans.**

*Minimum force required parallel to the plane*

We know that the minimum force required parallel to the plane to hold the body back,

$$P = 178.5 - 107.3 = 71.2 \text{ N} \quad \text{Ans.}$$

**Example 8.7.** An effort of 200 N is required just to move a certain body up an inclined plane of angle  $15^\circ$  the force acting parallel to the plane. If the angle of inclination of the plane is made  $20^\circ$  the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and the coefficient of friction.

**Solution.** Given: First case : When effort ( $P_1$ ) = 200 N, then angle of inclination ( $\alpha_1$ ) =  $15^\circ$   
 and second case : When effort ( $P_2$ ) = 230 N, then angle of inclination ( $\alpha_2$ ) =  $20^\circ$ .

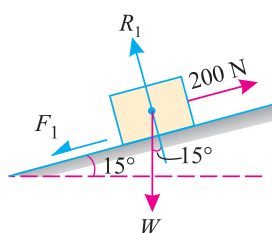
Let

$\mu$  = Coefficient of friction,

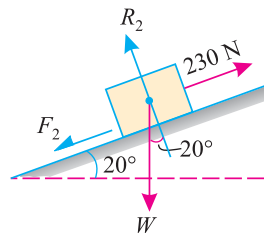
$W$  = Weight of the body,

$R$  = Normal reaction, and

$F$  = Force of friction.



(a) Body lying at  $15^\circ$



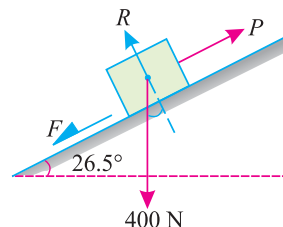
(b) Body lying at  $20^\circ$

**Fig. 8.12.**

First of all, consider the body lying on a plane inclined at an angle of  $15^\circ$  with the horizontal and subjected to an effort of 200 N as shown in Fig. 8.12 (a).

Resolving the forces at right angles to the plane,

$$R_1 = W \cos 15^\circ \quad \dots(i)$$



**Fig. 8.11.**

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and now resolving the forces along the plane,

$$\begin{aligned} 200 &= F_1 + W \sin 15^\circ = \mu R_1 + W \sin 15^\circ && \dots (\because F = \mu R) \\ &= \mu W \cos 15^\circ + W \sin 15^\circ && \dots (\because R_1 = W \cos 15^\circ) \\ &= W (\mu \cos 15^\circ + \sin 15^\circ) && \dots (ii) \end{aligned}$$

Now consider the body lying on a plane inclined at an angle of  $20^\circ$  with the horizontal and subjected to an effort of 230 N shown in Fig. 8.12 (b).

Resolving the forces at right angles to the plane,

$$R_2 = W \cos 20^\circ \quad \dots (iii)$$

and now resolving the forces along the plane,

$$\begin{aligned} 230 &= F_2 + W \sin 20^\circ = \mu R_2 + W \sin 20^\circ && \dots (\because F = \mu R) \\ &= \mu W \cos 20^\circ + W \sin 20^\circ && \dots (\because R_2 = W \cos 20^\circ) \\ &= W (\mu \cos 20^\circ + \sin 20^\circ) && \dots (iv) \end{aligned}$$

*Coefficient of friction*

Dividing equation (iv) by (ii),

$$\frac{230}{200} = \frac{W (\mu \cos 20^\circ + \sin 20^\circ)}{W (\mu \cos 15^\circ + \sin 15^\circ)}$$

$$230 \mu \cos 15^\circ + 230 \sin 15^\circ = 200 \mu \cos 20^\circ + 200 \sin 20^\circ$$

$$230 \mu \cos 15^\circ - 200 \mu \cos 20^\circ = 200 \sin 20^\circ - 230 \sin 15^\circ$$

$$\mu (230 \cos 15^\circ - 200 \cos 20^\circ) = 200 \sin 20^\circ - 230 \sin 15^\circ$$

$$\therefore \mu = \frac{200 \sin 20^\circ - 230 \sin 15^\circ}{230 \cos 15^\circ - 200 \cos 20^\circ} = \frac{(200 \times 0.3420) - (230 \times 0.2588)}{(230 \times 0.9659) - (200 \times 0.9397)} = 0.259 \quad \text{Ans.}$$

*Weight of the body*

Substituting the value of  $\mu$  in equation (ii),

$$\begin{aligned} 200 &= W (0.259 \cos 15^\circ + \sin 15^\circ) \\ &= W (0.259 \times 0.9659 + 0.2588) = 0.509 W \end{aligned}$$

$$\therefore W = \frac{200}{0.509} = 392.9 \text{ N} \quad \text{Ans.}$$

## EXERCISE 8.1

- Find the horizontal force required to drag a body of weight 100 N along a horizontal plane. If the plane, when gradually raised up to  $15^\circ$ , the body will begin to slide.

[Ans. 26.79 N]

**Hint.**  $\phi = 15^\circ$  or  $\mu = \tan \phi = \tan 15^\circ = 0.2679$

- A body of weight 50 N is hauled along a rough horizontal plane by a pull of 18 N acting at an angle of  $14^\circ$  with the horizontal. Find the coefficient of friction. [Ans. 0.383]

- A man is walking over a dome of 10 m radius. How far he can descend from the top of the dome without slipping? Take coefficient of friction between the surface of the dome and shoes of the man as 0.6. [Ans. 1.413 m]

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**Hint.** Let the man start from the top of the dome (A) and reach a point (B), beyond which he can not walk. Now let ( $W$ ) be the weight of the man and ( $\theta$ ) and angle subtended by the arc  $AB$  at the centre of the dome.

$$\therefore \text{Normal reaction at } B = W \cos \theta$$

$$\text{and force of friction, } F = W \sin \theta$$

$$\therefore \frac{W \sin \theta}{\tan \theta} = \mu \cdot W \cos \theta$$

$$\mu = 0.6 \quad \text{or} \quad \theta = 31^\circ$$

4. A force of 250 N pulls a body of weight 500 N up an inclined plane, the force being applied parallel to the plane. If the inclination of the plane to the horizontal is  $15^\circ$ , find the coefficient of friction. [Ans. 0.25]

### 8.14. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING HORIZONTALLY

Consider a body lying on a rough inclined plane subjected to a force acting horizontally, which keeps it in equilibrium as shown in Fig. 8.13. (a) and (b).

$W$  = Weight of the body,

$\alpha$  = Angle, which the inclined plane makes with the horizontal,

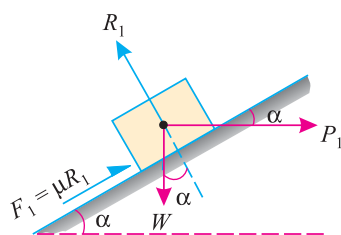
$R$  = Normal reaction,

$\mu$  = Coefficient of friction between the body and the inclined plane, and

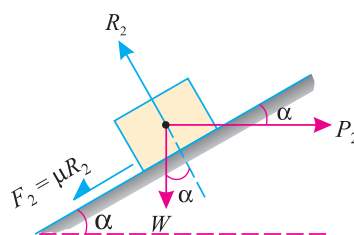
$\phi$  = Angle of friction, such that  $\mu = \tan \phi$ .

A little consideration will show that if the force is not there, the body will slide down on the plane. Now we shall discuss the following two cases :

1. Minimum force ( $P_1$ ) which will keep the body in equilibrium, when it is at the point of sliding downwards.



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

Fig. 8.13.

In this case, the force of friction ( $F_1 = \mu \cdot R_1$ ) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.13. (a). Now resolving the forces along the plane,

$$P_1 \cos \alpha = W \sin \alpha - \mu R_1 \quad \dots(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos \alpha + P_1 \sin \alpha \quad \dots(ii)$$

Substituting this value of  $R_1$  in equation (i),

$$P_1 \cos \alpha = W \sin \alpha - \mu(W \cos \alpha + P_1 \sin \alpha)$$

$$= W \sin \alpha - \mu W \cos \alpha - \mu P_1 \sin \alpha$$

$$P_1 \cos \alpha + \mu P_1 \sin \alpha = W \sin \alpha - \mu W \cos \alpha$$

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$$P_1(\cos \alpha + \mu \sin \alpha) = W(\sin \alpha - \mu \cos \alpha)$$

$$\therefore P_1 = W \times \frac{(\sin \alpha - \mu \cos \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_1 = W \times \frac{(\sin \alpha - \tan \phi \cos \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by  $\cos \phi$ ,

$$\begin{aligned} P_1 &= W \times \frac{\sin \alpha \cos \phi - \sin \phi \cos \alpha}{\cos \alpha \cos \phi + \sin \alpha \sin \phi} = W \times \frac{\sin(\alpha - \phi)}{\cos(\alpha - \phi)} \\ &= W \tan(\alpha - \phi) \quad \dots(\text{when } \alpha > \phi) \\ &= W \tan(\phi - \alpha) \quad \dots(\text{when } \phi > \alpha) \end{aligned}$$

2. *Maximum force ( $P_2$ ) which will keep the body in equilibrium, when it is at the point of sliding upwards*

In this case, the force of friction ( $F_2 = \mu R_2$ ) will act downwards, as the body is at the point of sliding upwards as shown in Fig.8.12. (b). Now resolving the forces along the plane,

$$P_2 \cos \alpha = W \sin \alpha + \mu R_2 \quad \dots(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha + P_2 \sin \alpha \quad \dots(iv)$$

Substituting this value of  $R_2$  in the equation (iii),

$$\begin{aligned} P_2 \cos \alpha &= W \sin \alpha + \mu (W \cos \alpha + P_2 \sin \alpha) \\ &= W \sin \alpha + \mu W \cos \alpha + \mu P_2 \sin \alpha \\ P_2 \cos \alpha - \mu P_2 \sin \alpha &= W \sin \alpha + \mu W \cos \alpha \\ P_2 (\cos \alpha - \mu \sin \alpha) &= W (\sin \alpha + \mu \cos \alpha) \end{aligned}$$

$$\therefore P_2 = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_2 = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by  $\cos \phi$ ,

$$\begin{aligned} P_2 &= W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \phi \sin \alpha} = W \times \frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)} \\ &= W \tan(\alpha + \phi) \end{aligned}$$

**Example 8.8.** An object of weight 100 N is kept in position on a plane inclined  $30^\circ$  to the horizontal by a horizontally applied force ( $F$ ). If the coefficient of friction of the surface of the inclined plane is 0.25, determine the minimum magnitude of the force ( $F$ ).

**Solution.** Given: Weight of the object ( $W$ ) = 100 N; Angle at which plane is inclined ( $\alpha$ ) =  $30^\circ$  and coefficient of friction ( $\mu$ ) = 0.25 =  $\tan \phi$  or  $\phi = 14^\circ$ .

We know that the minimum magnitude of the force to keep the object in position (when it is at the point of sliding downwards),

$$\begin{aligned} F &= W \tan(\alpha - \phi) = 100 \tan(30^\circ - 14^\circ) = 100 \tan 16^\circ \\ &= 100 \times 0.2867 = 28.67 \text{ N} \quad \text{Ans.} \end{aligned}$$

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**Example 8.9.** A load of 1.5 kN, resting on an inclined rough plane, can be moved up the plane by a force of 2 kN applied horizontally or by a force 1.25 kN applied parallel to the plane. Find the inclination of the plane and the coefficient of friction.

**Solution.** Given: Load ( $W$ ) = 1.5 kN; Horizontal effort ( $P_1$ ) = 2 kN and effort parallel to the inclined plane ( $P_2$ ) = 1.25 kN.

*Inclination of the plane*

Let  $\alpha$  = Inclination of the plane, and  
 $\phi$  = Angle of friction.

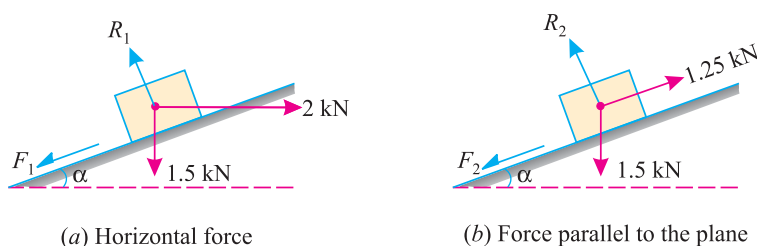


Fig. 8.14.

First of all, consider the load of 1.5 kN subjected to a horizontal force of 2 kN as shown in Fig. 8.14 (a). We know that when the force is applied horizontally, then the magnitude of the force, which can move the load up the plane,

$$P = W \tan (\alpha + \phi)$$

or  $2 = 1.5 \tan (\alpha + \phi)$

$$\therefore \tan (\alpha + \phi) = \frac{2}{1.5} = 1.333 \quad \text{or} \quad (\alpha + \phi) = 53.1^\circ$$

Now consider the load of 1.5 kN subjected to a force of 1.25 kN along the plane as shown in Fig. 8.14 (b). We know that when the force is applied parallel to the plane, then the magnitude of the force, which can move the load up the plane,

$$P = W \times \frac{\sin (\alpha + \phi)}{\cos \phi}$$

or  $1.25 = 1.5 \times \frac{\sin 53.1^\circ}{\cos \phi} = 1.5 \times \frac{0.8}{\cos \phi} = \frac{1.2}{\cos \phi}$

$$\therefore \cos \phi = \frac{1.2}{1.25} = 0.96 \quad \text{or} \quad \phi = 16.3^\circ$$

and  $\alpha = 53.1^\circ - 16.3^\circ = 36.8^\circ$  **Ans.**

*Coefficient of friction*

We know that the coefficient of friction,

$$\mu = \tan \phi = \tan 16.3^\circ = 0.292 \quad \text{Ans.}$$



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**Example 8.10.** Two blocks A and B, connected by a horizontal rod and frictionless hinges are supported on two rough planes as shown in Fig. 8.15.

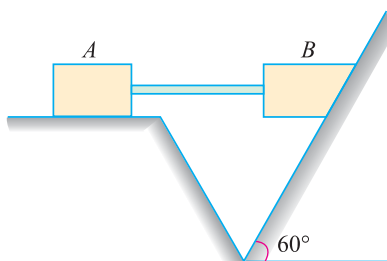


Fig. 8.15.

The coefficients of friction are 0.3 between block A and the horizontal surface, and 0.4 between block B and the inclined surface. If the block B weighs 100 N, what is the smallest weight of block A, that will hold the system in equilibrium?

**Solution.** Given: Coefficient of friction between block A and horizontal surface ( $\mu_A$ ) = 0.3; Coefficient of friction between block B and inclined surface ( $\mu_B$ ) = 0.4 and weight of block B ( $W_B$ ) = 100 N.

Let  $W_A$  = Smallest weight of block A.

We know that force of friction of block A, which is acting horizontally on the block B,

$$P = \mu_A W_A = 0.3 \times W_A = 0.3 W_A$$

and angle of friction of block B

$$\tan \phi = \mu_B = 0.4 \quad \text{or} \quad \phi = 21.8^\circ$$

We also know that the smallest force, which will hold the system in equilibrium (or will prevent the block B from sliding downwards),

$$P = W_B \tan (\alpha - \phi) = 100 \tan (60^\circ - 21.8^\circ)$$

$$\text{or} \quad 0.3 W_A = 100 \tan 38.2^\circ = 100 \times 0.7869 = 78.69$$

$$\therefore W_A = \frac{78.69}{0.3} = 262.3 \text{ N} \quad \text{Ans.}$$

**Alternative method**

Consider the equilibrium of block B. We know that it is in equilibrium under the action of the following four forces as shown in Fig. 8.16.

1. Its own weight 100 N
2. Normal reaction  $R$ ,
3. Force of friction of block A (acting horizontally on B),

$$F_A = \mu_A \times W_A = 0.3 \times W_A = 0.3 W_A$$

4. Force of friction between the block B and inclined surface,

$$F = \mu_B \times R = 0.4 R$$

Resolving the forces along the plane,

$$\begin{aligned} F &= 100 \cos 30^\circ - 0.3 W_A \cos 60^\circ \\ &= 100 \times 0.866 - 0.3 W_A \times 0.5 \end{aligned}$$

$$\text{or} \quad 0.4 R = 86.6 - 0.15 W_A$$

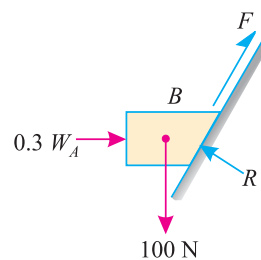


Fig. 8.16. ... (i)

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and now resolving the forces at right angles to the plane,

$$\begin{aligned} R &= 0.3 W_A \cos 30^\circ + 100 \sin 30^\circ \\ &= 0.3 W_A \times 0.866 + 100 \times 0.5 \\ &= 0.26 W_A + 50 \end{aligned} \quad \dots(ii)$$

Substituting the value of  $R$  in equation (i)

$$\begin{aligned} 0.4 (0.26 W_A + 50) &= 86.6 - 0.15 W_A \\ 0.104 W_A + 20 &= 86.6 - 0.15 W_A \\ 0.254 W_A &= 86.6 - 20 = 66.6 \end{aligned}$$

$$\therefore W_A = \frac{66.6}{0.254} = 262.2 \text{ N} \quad \text{Ans.}$$

### 8.15. EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING AT SOME ANGLE WITH THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected to a force acting at some angle with the inclined plane, which keeps it in equilibrium as shown in Fig. 8.17 (a) and (b).

Let

- $W$  = Weight of the body,
- $\alpha$  = Angle which the inclined plane makes with the horizontal,
- $\theta$  = Angle which the force makes with the inclined surface,
- $R$  = Normal reaction,
- $\mu$  = Coefficient of friction between the body and the inclined plane, and
- $\phi$  = Angle of friction, such that  $\mu = \tan \phi$ .

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases :

1. *Minimum force ( $P_1$ ) which will keep the body in equilibrium when it is at the point of sliding downwards.*

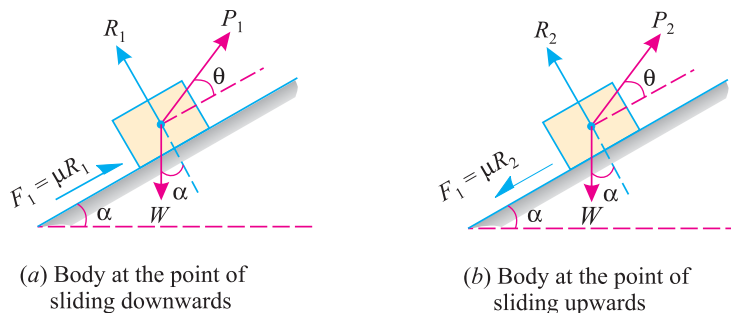


Fig. 8.17.

In this case, the force of friction ( $F_1 = \mu R_1$ ) will act upwards, as the body is at the point of sliding downwards as shown in Fig. 8.17 (a). Now resolving the forces along the plane,

$$P_1 \cos \theta = W \sin \alpha - \mu R_1 \quad \dots(i)$$

and now resolving the forces perpendicular to the plane,

$$R_1 = W \cos \alpha - P_1 \sin \theta \quad \dots(ii)$$

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Substituting the value of  $R_1$  in equation (i),

$$\begin{aligned} P_1 \cos \theta &= W \sin \alpha - \mu (W \cos \alpha - P_1 \sin \theta) \\ &= W \sin \alpha - \mu W \cos \alpha + \mu P_1 \sin \theta \end{aligned}$$

$$P_1 \cos \theta - \mu P_1 \sin \theta = W \sin \alpha - \mu W \cos \alpha$$

$$P_1 (\cos \theta - \mu \sin \theta) = W (\sin \alpha - \mu \cos \alpha)$$

$$\therefore P_1 = W \times \frac{(\sin \alpha - \mu \cos \alpha)}{(\cos \theta - \mu \sin \theta)}$$

and now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_1 = W \times \frac{(\sin \alpha - \tan \phi \cos \alpha)}{(\cos \theta - \tan \phi \sin \theta)}$$

Multiplying the numerator and denominator by  $\cos \phi$ ,

$$P_1 = W \times \frac{(\sin \alpha \cos \phi - \sin \phi \cos \alpha)}{(\cos \theta \cos \phi - \sin \phi \sin \theta)} = W \times \frac{\sin (\alpha - \phi)}{\cos (\theta + \phi)}$$

2. Maximum force ( $P_2$ ) which will keep the body in equilibrium, when it is at the point of sliding upwards.

In this case, the force of friction ( $F_2 = \mu R_2$ ) will act downwards as the body is at the point of sliding upwards as shown in Fig. 8.17 (b). Now resolving the forces along the plane.

$$P_2 \cos \theta = W \sin \alpha + \mu R_2 \quad \dots(iii)$$

and now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha - P_2 \sin \theta \quad \dots(iv)$$

Substituting the value of  $R_2$  in equation (iii),

$$\begin{aligned} P_2 \cos \theta &= W \sin \alpha + \mu (W \cos \alpha - P_2 \sin \theta) \\ &= W \sin \alpha + \mu W \cos \alpha - \mu P_2 \sin \theta \end{aligned}$$

$$P_2 \cos \theta + \mu P_2 \sin \theta = W \sin \alpha + \mu W \cos \alpha$$

$$P_2 (\cos \theta + \mu \sin \theta) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore P_2 = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \theta + \mu \sin \theta)}$$

and now substituting the value of  $\mu = \tan \phi$  in the above equation,

$$P_2 = W \times \frac{(\sin \alpha + \tan \phi \cos \alpha)}{(\cos \theta + \tan \phi \sin \theta)}$$

Multiplying the numerator and denominator by  $\cos \phi$ ,

$$P_2 = W \times \frac{(\sin \alpha \cos \phi + \sin \phi \cos \alpha)}{(\cos \theta \cos \phi + \sin \phi \sin \theta)} = W \times \frac{\sin (\alpha + \phi)}{\cos (\theta - \phi)}$$

**Example 8.11.** Find the force required to move a load of 300 N up a rough plane, the force being applied parallel to the plane. The inclination of the plane is such that when the same load is kept on a perfectly smooth plane inclined at the same angle, a force of 60 N applied at an inclination of  $30^\circ$  to the plane, keeps the same load in equilibrium.

Assume coefficient of friction between the rough plane and the load to be equal to 0.3.

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**Solution.** Given: Load ( $W$ ) = 300 N; Force ( $P_1$ ) = 60 N and angle at which force is inclined ( $\theta$ ) =  $30^\circ$ ,

Let  $\alpha$  = Angle of inclination of the plane.

First of all, consider the load lying on a smooth plane inclined at an angle ( $\alpha$ ) with the horizontal and subjected to a force of 60 N acting at an angle  $30^\circ$  with the plane as shown in Fig. 8.18 (a).

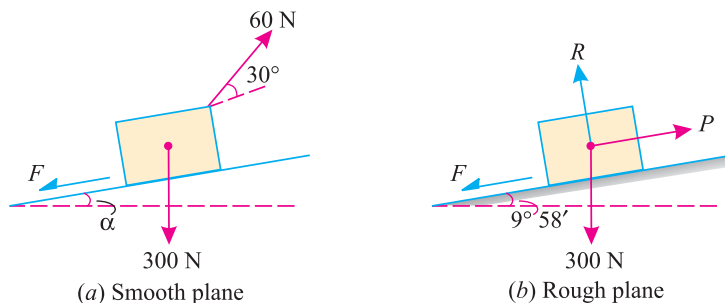


Fig. 8.18.

We know that in this case, because of the smooth plane  $\mu = 0$  or  $\phi = 0$ . We also know that the force required, when the load is at the point of sliding upwards ( $P$ ),

$$60 = W \times \frac{\sin(\alpha + \phi)}{\cos(\theta - \phi)} = 300 \times \frac{\sin \alpha}{\cos 30^\circ} = 300 \times \frac{\sin \alpha}{0.866} = 346.4 \sin \alpha \quad \dots(\because \phi = 0)$$

$$\text{or} \quad \sin \alpha = \frac{60}{346.4} = 0.1732 \quad \text{or} \quad \alpha = 10^\circ$$

Now consider the load lying on the rough plane inclined at an angle of  $10^\circ$  with the horizontal as shown in Fig. 8.18. (b). We know that in this case,  $\mu = 0.3 = \tan \phi$  or  $\phi = 16.7^\circ$ .

We also know that force required to move the load up the plane,

$$\begin{aligned} P &= W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 300 \times \frac{\sin(10^\circ + 16.7^\circ)}{\cos 16.7^\circ} \text{ N} \\ &= 300 \times \frac{\sin 26.7^\circ}{\cos 16.7^\circ} = 300 \times \frac{0.4493}{0.9578} = 140.7 \text{ N} \quad \text{Ans.} \end{aligned}$$

### Alternative method

*1st case*

Given: In this case load ( $P$ ) = 60 N; Angle ( $\theta$ ) =  $30^\circ$  and force of friction  $F = 0$  (because of smooth plane). Resolving the forces along the inclined plane,

$$60 \cos 30^\circ = 300 \sin \alpha$$

$$\therefore \sin \alpha = \frac{60 \cos 30^\circ}{300} = \frac{60 \times 0.866}{300} = 0.1732 \quad \text{or} \quad \alpha = 10^\circ$$

*2nd case*

Given: In this case, coefficient of friction ( $\mu$ ) = 0.3 =  $\tan \phi$  or  $\phi = 16.7^\circ$

Let  $P$  = Force required to move the load up the plane,

$R$  = Normal reaction, and

$F$  = Force of friction equal to  $0.3 R$ .

Resolving the forces along the plane,

$$P = F + 300 \sin 10^\circ = 0.3 R + (300 \times 0.1732) = 0.3 R + 51.96 \text{ N} \quad \dots(i)$$

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and now resolving the forces at right angles to the plane,

$$R = 300 \cos 10^\circ = 300 \times 0.9849 = 295.5 \text{ N} \quad \dots(ii)$$

Substituting the value of  $R$  in equation (i),

$$P = (0.3 \times 295.5) + 51.96 = 140.7 \text{ N} \quad \text{Ans.}$$

**Example 8.12.** The upper half of an inclined having inclination  $\theta$  with the horizontal is smooth, while the lower half is rough as shown in Fig 8.19.

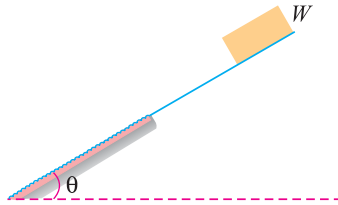


Fig. 8.19.

If a body of weight  $W$  slides down from rest at the top, again comes to rest at the bottom of the plane, then determine the value of coefficient of friction for the lower half of the plane.

**Solution.** Given: Angle of inclination =  $\theta$  and weight of the body =  $W$ .

Let  $\mu$  = Coefficient of friction for the lower half of the inclined alone.

We know that acceleration on the smooth surface of the plane

$$= g \sin \theta \quad \dots(i)$$

and retardation on the rough surface of the plane

$$= -g (\sin \theta - \mu \cos \theta) \quad \dots(\text{Minus sign due to retardation})$$

$$= g (\mu \cos \theta - \sin \theta) \quad \dots(ii)$$

Since body starts from rest at the top comes to rest at the bottom of therefore acceleration on the smooth surface is equal to retardation on the rough surface.

$$\therefore g \sin \theta = g (\mu \cos \theta - \sin \theta) \quad \text{or} \quad \sin \theta = \mu \cos \theta - \sin \theta$$

$$\text{or} \quad \mu \cos \theta = 2 \sin \theta \quad \text{or} \quad \mu = 2 \tan \theta \quad \text{Ans.}$$

**Example 8.13.** Two loads,  $W_1$  (equal to 1 kN) and  $W_2$  resting on two inclined rough planes  $OA$  and  $OB$  are connected by a horizontal link  $PQ$  as shown in Fig. 8.20.

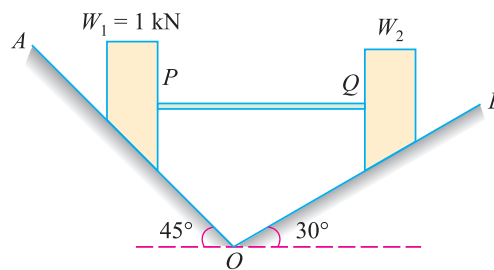


Fig. 8.20.

Find the maximum and minimum values of  $W_2$  for which the equilibrium can exist. Take angle of friction for both the planes as  $20^\circ$ .

**Solution.** Given: First load ( $W_1$ ) = 1 kN ; Angle made by inclined plane  $OA$  with the horizontal ( $\alpha_1$ )  $45^\circ$ ; Angle made by inclined plane  $OB$  with the horizontal ( $\alpha_2$ )  $30^\circ$  and Angle of friction for both the planes ( $\phi$ )  $20^\circ$ .

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*Maximum value of  $W_2$*

We know that for maximum value of  $W_2$ , the load  $W_2$  will be at the point of sliding downwards whereas the load  $W_1$  will be at the point of sliding upwards. We also know that when the load  $W_1$  is at the point of sliding upwards on the plane  $OA$ , the horizontal thrust in the link  $PQ$ ,

$$\begin{aligned} P &= W_1 \tan (\alpha_1 + \phi) = 1 \times \tan (45^\circ + 20^\circ) \text{ kN} \\ &= 1 \tan 65^\circ = 1 \times 2.1445 = 2.1445 \text{ kN} \end{aligned} \quad \dots(i)$$

and when the load  $W_2$  is at the point of sliding downwards on the plane  $OB$ , the horizontal thrust in the link  $PQ$

$$\begin{aligned} P &= W_2 \tan (\alpha_2 - \phi) = W_2 \tan (30^\circ - 20^\circ) \text{ kN} \\ &= W_2 \tan 10^\circ = W_2 \times 0.1763 \text{ kN} \end{aligned} \quad \dots(ii)$$

Since the values of the horizontal thrusts in the link  $PQ$ , obtained in both the above equations is the same, therefore equating equations (i) and (ii),

$$\begin{aligned} 2.1445 &= W_2 \times 0.1763 \\ \therefore W_2 &= \frac{2.1445}{0.1763} = 12.16 \text{ kN} \quad \text{Ans.} \end{aligned}$$

*Minimum value of  $W_2$*

We know that for maximum value of  $W_2$ , the load  $W_2$  will be at the point of sliding upwards whereas the load  $W_1$  will be at the point of sliding downwards. We also know that when the load  $W_1$  is at the point of sliding downwards on the plane  $OA$ , the horizontal thrust in the link  $PQ$ ,

$$\begin{aligned} P &= W_1 \tan (\alpha_1 - \phi) = 1 \times \tan (45^\circ - 20^\circ) \text{ kN} \\ &= 1 \times \tan 25^\circ = 1 \times 0.4663 = 0.4663 \text{ kN} \end{aligned} \quad \dots(iii)$$

and when the load  $W_2$  is at the point of sliding upwards on the plane  $OB$ , the horizontal thrust in the link  $PQ$ ,

$$\begin{aligned} P &= W_2 \tan (\alpha_2 + \phi) = W_2 (30^\circ + 20^\circ) \text{ kN} \\ &= W_2 \tan 50^\circ = W_2 \times 1.1918 \text{ kN} \end{aligned} \quad \dots(iv)$$

Since the values of the horizontal thrust in the link  $PQ$ , obtained in the above equations is the same, therefore, equating (iii) and (iv),

$$\begin{aligned} 0.4663 &= W_2 \times 1.1918 \\ \therefore W_2 &= \frac{0.4663}{1.1918} = 0.391 \text{ kN} = 391 \text{ N} \quad \text{Ans.} \end{aligned}$$

**Example 8.14.** A block (A) weighing 1 kN rests on a rough inclined plane whose inclination to the horizontal is  $45^\circ$ . This block is connected to another block (B) weighing 3 kN rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of  $30^\circ$  to the horizontal as shown in Fig. 8.21.

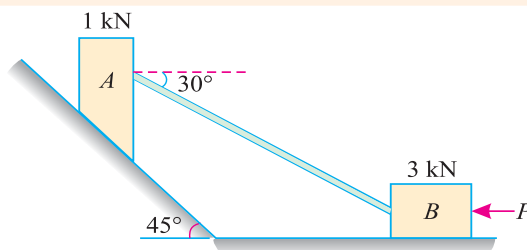


Fig. 8.21.

Find horizontal force ( $P$ ) required to be applied to the block (B) just to move the block (A) in upward direction. Assume angle of limiting friction as  $15^\circ$  at all surface where there is sliding.

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**Solution.** Given: Weight of block A ( $W_A$ ) = 1 kN; Weight of block B ( $W_B$ ) = 3 kN; Angle of inclination of plane with horizontal ( $\alpha$ ) =  $45^\circ$  or coefficient of friction ( $\mu$ ) =  $\tan \phi = \tan 15^\circ = 0.2679$  or angle between rod and inclined plane ( $\theta$ ) =  $45^\circ - 30^\circ = 15^\circ$  and angle of limiting friction ( $\phi$ ) =  $15^\circ$ .

First of all, consider the equilibrium of the block (A), which is subjected to the forces as shown in Fig. 8.22 (a).

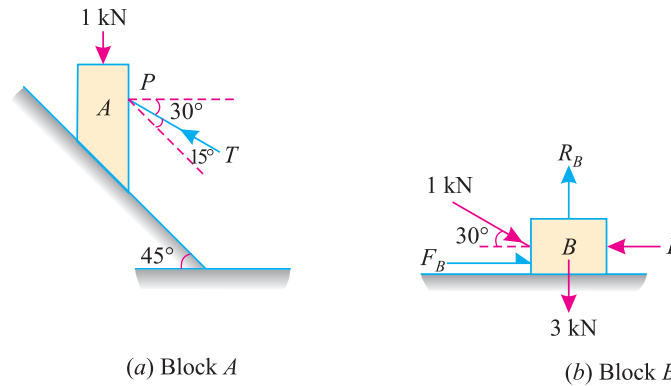


Fig. 8.22.

We know that the thrust or force in the rigid bar just to move the block (A) in the upward direction,

$$T = W_A \times \frac{\sin (\alpha + \phi)}{\cos (\theta - \phi)} = 1 \times \frac{\sin (45^\circ + 15^\circ)}{\cos (-15^\circ - 15^\circ)} \text{ kN}$$

...(The value of  $\theta$  is taken as negative)

$$= 1 \times \frac{\sin 60^\circ}{\cos (-30^\circ)} = 1 \times \frac{\sin 60^\circ}{\cos 30^\circ} = 1 \times \frac{0.866}{0.866} = 1 \text{ kN} \quad \dots [\because \cos (-\theta) = \cos \theta]$$

Now consider the equilibrium of the block (B), which is subjected to the forces as shown in Fig. 8.22 (b). We know that as the block is at the point of sliding towards left, therefore, the force of friction ( $F_B = \mu R_B$ ) will act towards right as shown in the figure. Now resolving the forces vertically,

$$R_B = 3 + 1 \sin 30^\circ = 3 + (1 \times 0.5) = 3.5 \text{ kN}$$

and now resolving the forces horizontally,

$$P = 1 \cos 30^\circ + F_B = (1 \times 0.866) + (0.2679 \times 3.5) = 1.8 \text{ kN} \quad \text{Ans.}$$

**Example 8.15.** A solid body is formed by joining the base of a right circular cone of height 12 cm to the equal base of a right circular cylinder of height 3 cm. The solid is placed with its face on a rough inclined plane, and the inclination to the horizontal of the plane is gradually increased. Show that if radius  $r$  of the base is 2 cm, and the coefficient of friction  $\mu = 0.5$ , the body will topple over before it begins to slide.

If the heights are so chosen that the centre of mass of the solid is at the centre of the common base, show that if  $\mu H < r \sqrt{6}$ , the solid will slide before it topples.



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**Solution.** Given: Height of right circular cone ( $H$ ) = 12 cm; Height of right circular cylinder ( $h$ ) = 3 cm; Radius of the base ( $r$ ) = 2 cm and coefficient of friction ( $\mu$ ) = 0.5.

We have already found out in example 6.6 that the centre of gravity of the body is at a height of 4.07 cm from the base. A little consideration will show, that when the body is at the point of toppling, the weight of the body will pass through the extreme point  $B$  as shown in Fig. 8.23. Thus in the limiting case, the angle ( $\theta_1$ ) at which the body is inclined with the horizontal is given by:

$$\tan \theta_1 = \frac{BD}{DG} = \frac{2}{4.07} = 0.49$$

Thus we see that the angle ( $\theta_1$ ) is less than the angle ( $\theta$ ). It is so, as the value of  $\tan \theta_1$  (equal to 0.49) is less than  $\tan \theta$  (equal to 0.5). Or in other words, the angle of inclination is less than the angle of friction. Thus the body will topple over before it begins to slide.

**Ans.**

Now in the second case, let us first find out the relation between  $h$  and  $H$  from the given condition that the centre of the body is at the centre of the common base ( $G$ ) as shown in Fig. 8.24.

(i) Cylinder

$$v_1 = \pi r^2 h$$

and  $y_1 = \frac{h}{2}$

(ii) Cone

$$v_2 = \frac{1}{3} \pi r^2 H$$

and  $y_2 = \frac{H}{4}$

We know that the distance between centre of gravity of the body from the base of the cylinder ( $\bar{y}$ )

$$h = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{\left[ \pi r^2 h \times \frac{h}{2} \right] + \left[ \frac{1}{3} \pi r^2 H \left( h + \frac{H}{4} \right) \right]}{\pi r^2 h + \frac{1}{3} \pi r^2 H}$$

$$\pi r^2 h^2 + \frac{\pi r^2 H h}{3} = \frac{\pi r^2 h^2}{2} + \frac{\pi r^2 H h}{3} + \frac{\pi r^2 H^2}{12}$$

$$\frac{\pi r^2 h^2}{2} = \frac{\pi r^2 H^2}{12} \quad \text{or} \quad h^2 = \frac{H^2}{6} \quad \text{or} \quad h = \frac{H}{\sqrt{6}}$$

In this case, when the body is at the point of toppling, the angle ( $\theta_2$ ) at which it is inclined with the horizontal is given by :

$$\tan \theta_2 = \frac{r}{h} = \frac{r}{H/\sqrt{6}} = \frac{r\sqrt{6}}{H}$$

Now if the body is to slide before toppling, the angle of friction should be less than the angle of friction. Or in other words,  $\tan \theta$  should be less than  $\tan \theta_2$ , Mathematically,

$$\mu < \frac{r\sqrt{6}}{H} \quad \text{or} \quad \mu H < r\sqrt{6} \quad \text{Ans.}$$

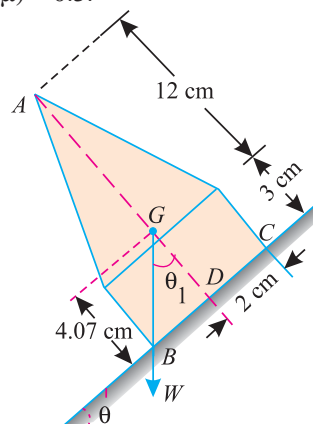


Fig. 8.23.

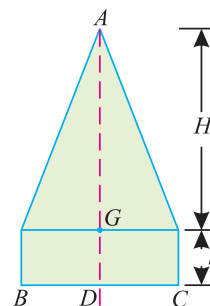


Fig. 8.24.

**EXERCISE 8.2**

1. A load of 500 N is lying on an inclined plane, whose inclination with the horizontal is  $30^\circ$ . If the coefficient of friction between the load and the plane is 0.4, find the minimum and maximum horizontal force, which will keep the load in equilibrium.

[Ans. 72.05 N, 635.4 N]

2. Two blocks A and B of weight 100 N and 300 N respectively are resting on a rough inclined plane as shown in Fig. 8.25.

Find the value of the angle ( $\theta$ ) when the block B is about to slide. Take coefficient of friction between the two blocks as well as block B and the inclined plane as 0.25.

[Ans.  $22.6^\circ$ ]

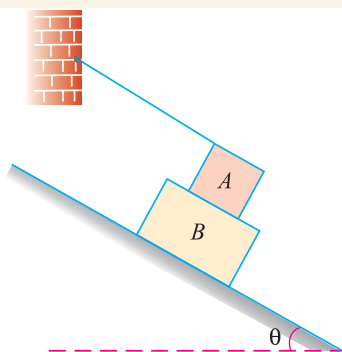


Fig. 8.25.

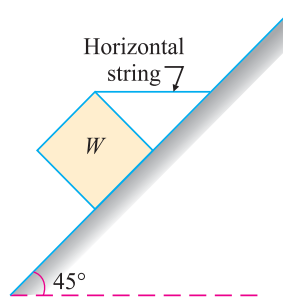


Fig. 8.26.

3. A rectangular prism ( $W$ ) weighing 150 N, is lying on an inclined plane whose inclination with the horizontal is shown in Fig. 8.26.

The block is tied up by a horizontal string, which has a tension of 50 N. From fundamentals find (i) the frictional force on the block (ii) the normal reaction of the inclined plane, (iii) the coefficient of friction between the surface of contact.

[Ans. 70.7 N ; 141.4 N ; 0.5 ;]

**QUESTIONS**

- What do you understand by the term friction ? Explain clearly why it comes into play ?
- How will you distinguish between static friction and dynamic friction ?
- State the laws of friction.
- Explain the term angle of friction.
- Define coefficient of friction and limiting friction.

**OBJECTIVE TYPE QUESTIONS**

- The force of friction between two bodies in contact
  - Depends upon the area of their contact
  - Depends upon the relative velocity between them
  - Is always normal to the surface of their contact
  - All of the above

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2. The magnitude of the force of friction between two bodies, one lying above the other, depends upon the roughness of the  
(a) Upper body (b) Lower body  
(c) Both the bodies (d) The body having more roughness
3. The force of friction always acts in a direction opposite to that  
(a) In which the body tends to move  
(b) In which the body is moving  
(c) Both (a) and (b)  
(d) None of the two
4. Which of the following statement is correct ?  
(a) The force of friction does not depend upon the area of contact  
(b) The magnitude of limiting friction bears a constant ratio to the normal reaction between the two surfaces  
(c) The static friction is slightly less than the limiting friction.  
(d) All (a), (b) and (c)

**ANSWERS**

1. (c)

2. (c)

3. (c)

4. (d)