Module 1:

Statics of Particles

Lecture 1: Fundamental Concepts

Tuesday, February 15, 2022

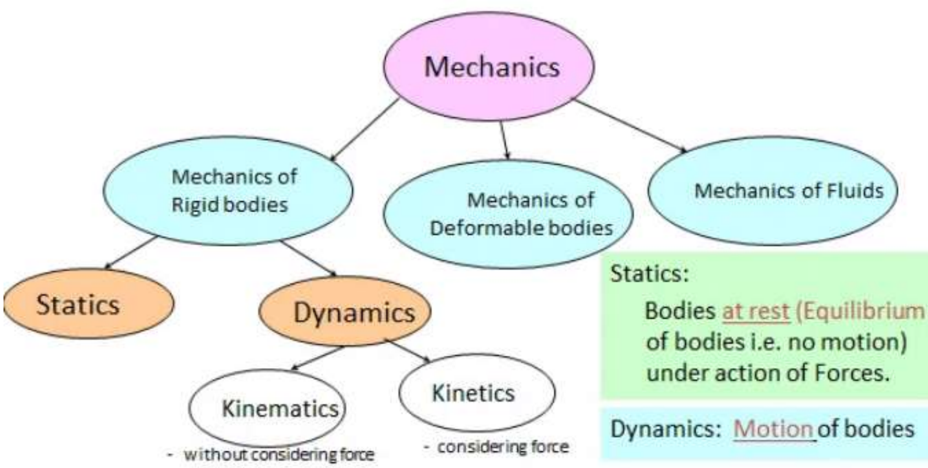
Why Engineering Mechanics?

- You have to design a car, which can run at a speed of 80 km/hr. In order to do this, have to find engine power and the forces acting on the car body.

- To design a dam structure, you need to compute of deflections and internal forces within structures

What is Engineering mechanics?

- It is a branch of science which deals with the behavior of body at rest or in motion under the action of forces.



- Forces in plane
  - Resultant
    - Two forces parallelogram and triangular law
    - Resultant of several forces polygon law and rectangular components
  - Equilibrium of a particle
    - Free body diagram
    - Lami's theorem
    - Rectangular components - Resolving forces
- Forces in space
  - Rectangular components

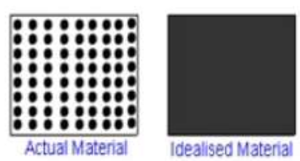
- Resultant
- Equilibrium of a particle

Keyword: (Frequently used words)
Particle, Rigid body, Force, Resultant, Newton's Laws, Parallelogram Law, Lami's Theorem, Principal of Transmissibility, Moment, Equilibrium and Couple

Force:
- A physical quantity brought into equilibrium.
- When we can carrying a school bag, we can feel weight.

Representation of Force:
- Magnitude
- Direction
- Point of application
- Line of Action

Other terms:
- Particle: Body of negligible dimensions.
- Rigid body: Body with negligible deformations.
- Non-rigid body: Non-rigid body. Body which n deform.
- Continuum: Body is assumed to distribution of matter.
- In Statics, bodies are considered rigid unless stated otherwise.



Basic Concepts and Definition:
- Space:
  - Collection of points whose relative positions can be described using "a coordinate system".
- Time:
  - For relative occurrence of events.
- Mass :
  - A measure of how much matter is in an object. For example, a gold bar is quite small but has a mass of 1 kilogram, so it contains a lot of matter.
  - It is a measure of the inertia of a body, which is its resistance to a change of velocity.

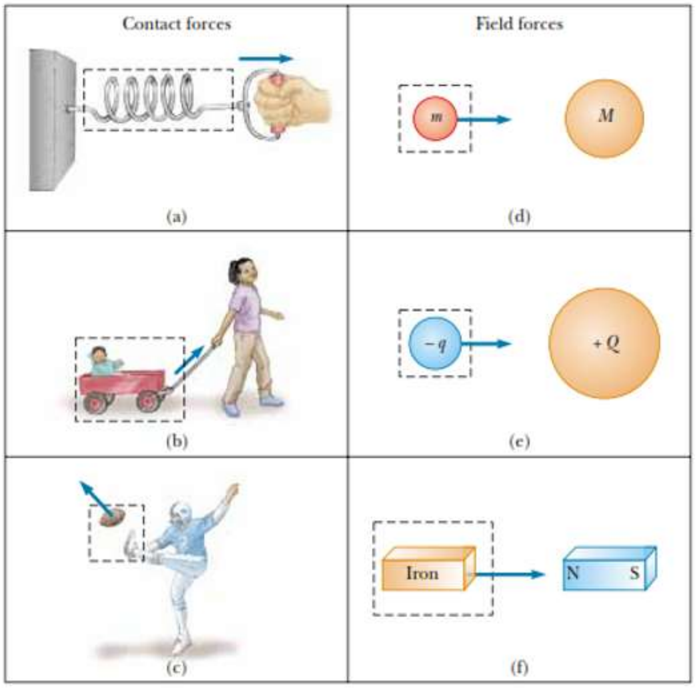
Units:
- FPS: Foot, Pound and Second
- CGS: Centimeter, Gram, Second
- MKS: Meter, Kilogram, Second
- SI: System International

Basic Concept - Force:
- Force:

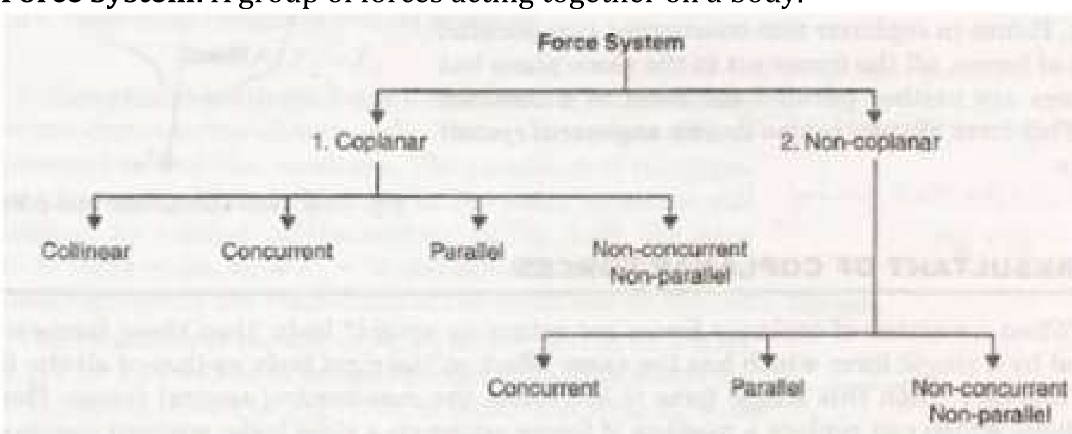
- Vector quantity that describes an action of one body on another.
- The Sl unit of force magnitude is the newton (N) or kg m/s2 or kg m s-2

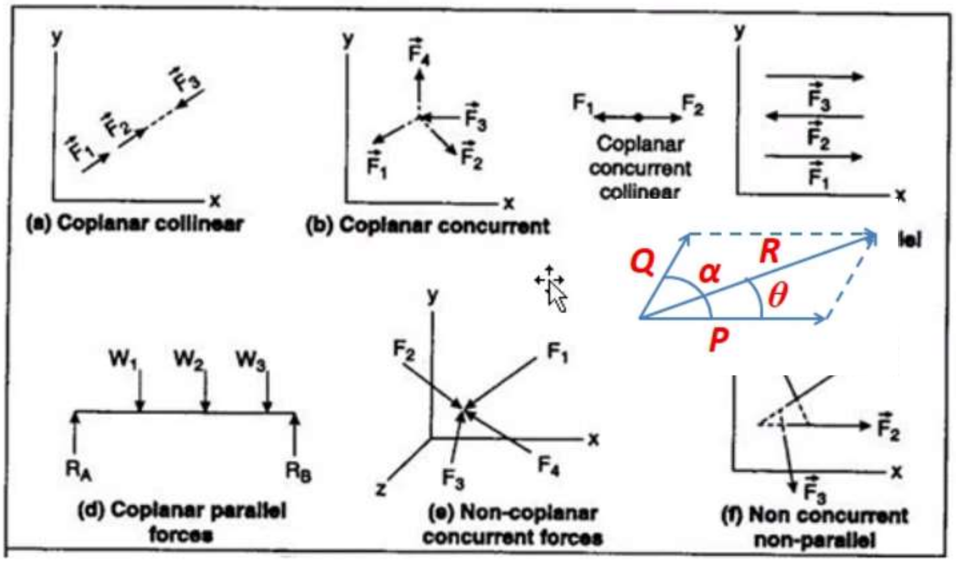
- Resultant of a force system is a single force which produces the same effect as that of the force system. It is the equivalent force all the given forces.
- Resolution of a force is the process of splitting up the given force into number of components.

Force Examples:

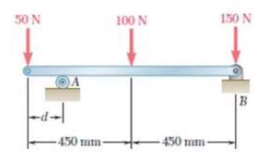


Force System: A group of forces acting together on a body.

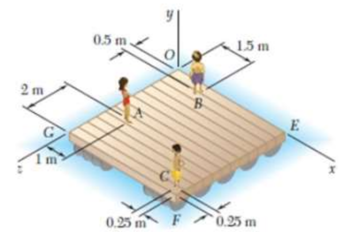




Collinear Forces

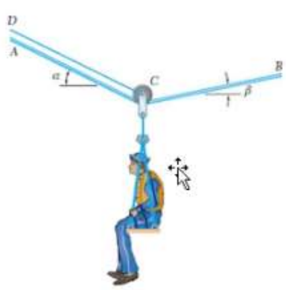


In Plane (2D)



In Space (3D)

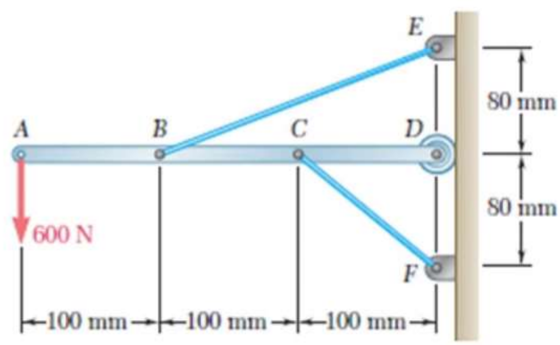
Parallel Forces



In Plane (2D)

In Space (3D)

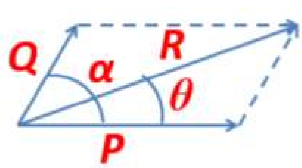
Concurrent Forces



Non-parallel and non-concurrent

Forces in Plane-Resultant:

- Parallelogram law (Analytical Method)
  - $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$
  - $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$
- Triangular law
- Polygon law
- Rectangular components

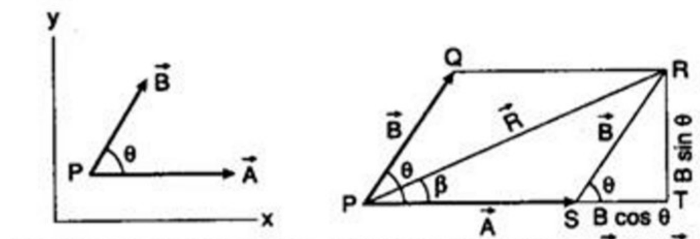


Resultant Force Finding Methods:

- Analytical Method
  - Trigonometric Method (Parallelogram formula, Lami's theorem)

    - Resolution Method
- Graphical Method
    - Triangle Law of Forces
    - Polygon Law of Forces

Analytical Method - Parallelogram Law:



(a) Two concurrent forces (b) Resultant of forces $\vec{A}$ and $\vec{B}$

$|\vec{R}| = PR = \sqrt{(RT)^2 + (PT)^2}$

$|\vec{R}| = \sqrt{(RT)^2 + (PS + ST)^2}$

$|\vec{R}| = \sqrt{(B\sin\theta)^2 + (A + B\cos\theta)^2}$

$|\vec{R}| = \sqrt{(B^2(\sin^2\theta + \cos^2\theta)) + A^2 + 2AB\cos\theta}$

$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$

$\boxed{\beta = \tan^{-1}\left(\frac{A\sin\theta}{B + A\cos\theta}\right)}$

Example Problem for Parallelogram Law:

$A = 5$

$B = 10$

$\theta = 120^0$

$R = \sqrt{5^2 + 10^2 + 2\times5\times10\times\cos(120)}$

$\Rightarrow R = \sqrt{125 + 2\times5\times10\times(-\frac{1}{2})}$

$\Rightarrow R = 5\sqrt{3}$

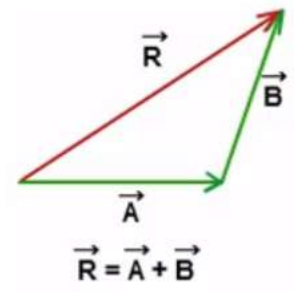
$\beta = \tan^{-1}\left(\frac{a\sin(120^o)}{b + a\cos(120^o)}\right)$

$\Rightarrow \beta = \tan^{-1}\left(\frac{5\times\frac{\sqrt{3}}{2}}{10 + 5\left(-\frac{1}{2}\right)}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^o$

Derived Laws:

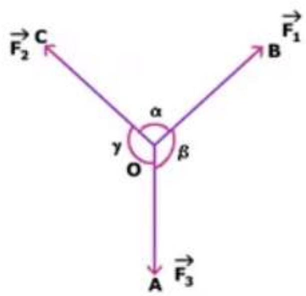
- Triangle Law of Forces

If <u>two forces acting on a body</u> are represented by the sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last point.



- Lami's Theorem

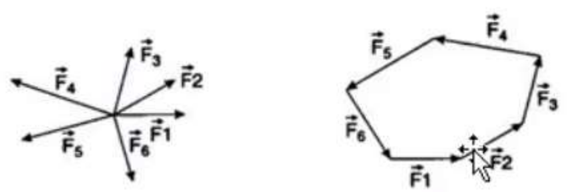
If <u>three forces acting at a point are in equilibrium</u>, each force is proportional to the sine of the angle between the other two forces.



$$\boxed{\frac{\vec{F_3}}{\sin\alpha} = \frac{\vec{F_2}}{\sin\beta} = \frac{\vec{F_1}}{\sin\gamma}}$$

- Polygon Law of Forces

If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by the closing side of the polygon.

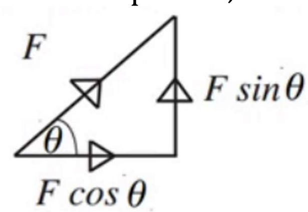


(a) Concurrent coplanar forces (b) Polygon of force vectors

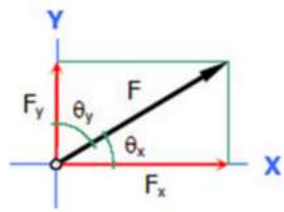
Lecture 2: Resolution of Forces

Wednesday, February 16, 2022

- Forces acting at some angle from the coordinate axes can be resolved into mutually perpendicular forces called components.
- The component of a force parallel to the x-axis is called the x-component, parallel to y-axis the y-component, and so on.



- Components of forces in XY Plane



$F_x = F\cos\theta_x = F\sin\theta_y$

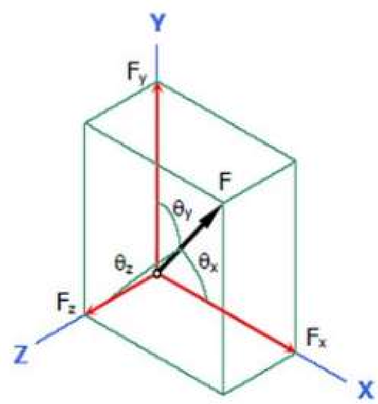
$F_y = F\sin\theta_x = F\cos\theta_y$

$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath}$

$$\boxed{F = \sqrt{F_x^2 + F_y^2}}$$

$$\boxed{\tan\theta_x = \frac{F_y}{F_x}}$$

- Components of a force in 3D spac



$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$

$\boldsymbol{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$

$$\boxed{F = \sqrt{F_x^2 + F_y^2 + F_z^2}}$$

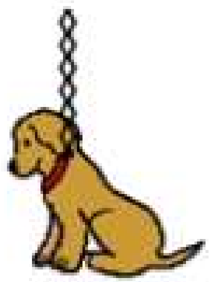
$$\boxed{\cos\theta_x = \frac{F_x}{F}}$$

$$\boxed{\cos\theta_y = \frac{F_y}{F}}$$

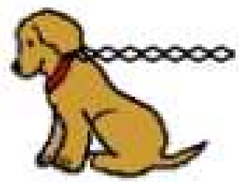
$$\boxed{\cos\theta_z = \frac{F_z}{F}}$$

- To find a <u>resultant of forces</u>,

1. Determine horizontal and vertical components of all forces.
2. Add horizontal forces ($F_{horiz} = F_x = F\cos\theta$).
3. Similarly, add Vertical forces ($F_{vert} = F_y = F\sin\theta$).
4. Find magnitude and direction of the resultant force.



This force exerts no Horizontal influence.

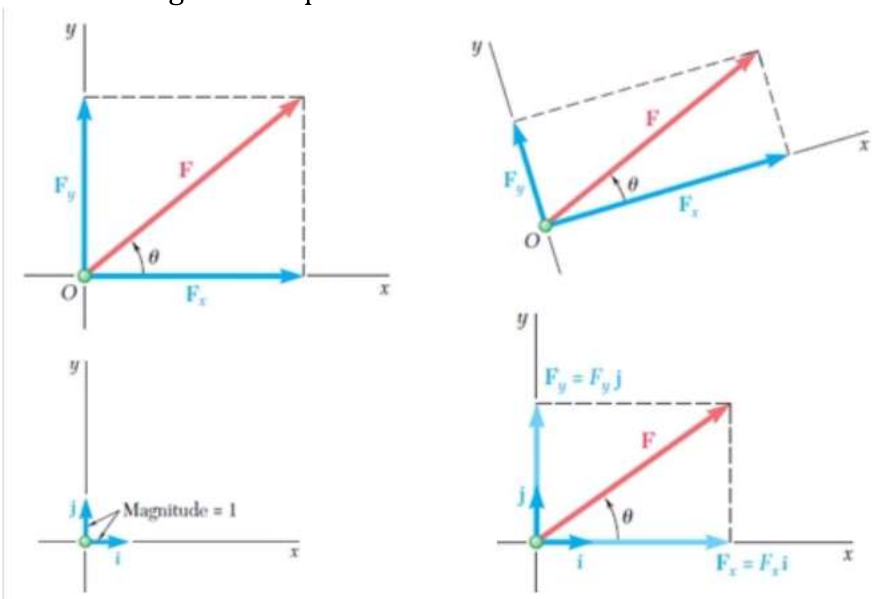


This force exerts no vertical influence.

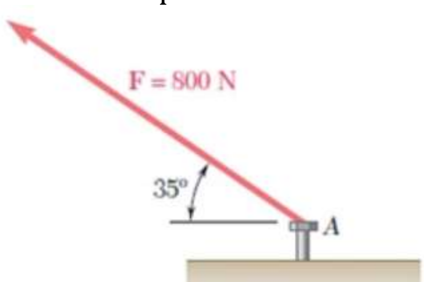
This force exerts both horizontal and vertical influence.

Refer Slide 35 for example problems

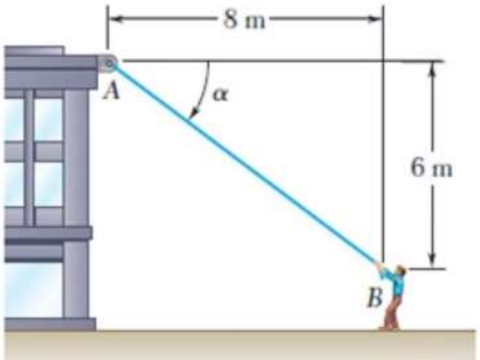
- Rectangular components of a force



Example 1: A force of 800 N is exerted on a bolt A as shown in Figure. Determine the horizontal and vertical components of the force.



Example 2: A man pulls with a force of 300 N on a rope attached to a building, as shown in Figure. What are the horizontal and vertical components of the force exerted by the rope at point A ?



Resultant of Several Concurrent Forces

$R = P + Q + S$

$R_x\boldsymbol{i} + R_y j = P_x i + P_y j + Q_x i + Q_y j + S_x i + S_y j$

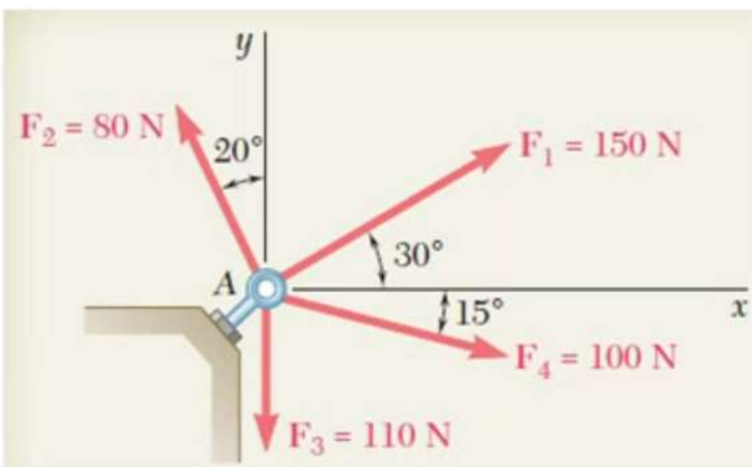
By equating on both sides, we get:

$R_x = P_x + Q_x + S_x = \Sigma F_x$

$R_y = P_y + Q_y + S_y = \Sigma F_y$

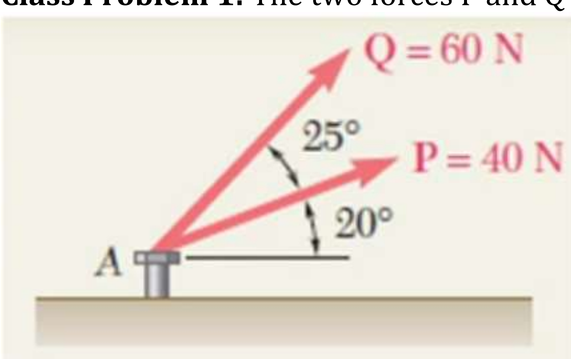
$$\boxed{\tan\theta = \frac{R_y}{R_x} = \frac{\Sigma F_y}{\Sigma F_x}}$$

Class Problem 3:



Ans: R = 199.6 N and α = 4.1^0

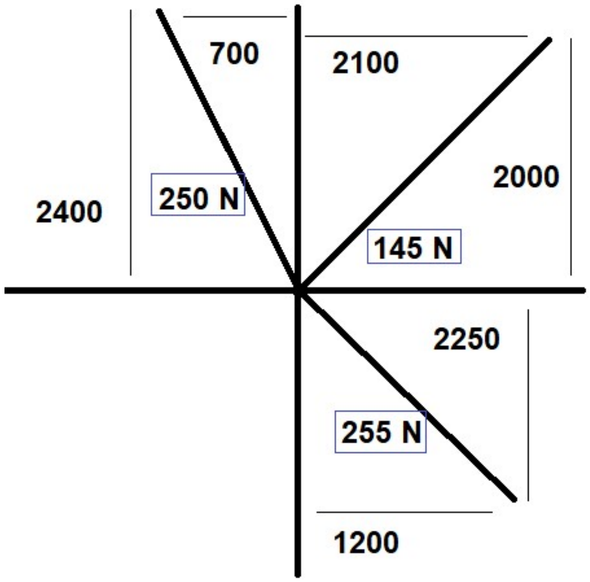
Class Problem 1: The two forces P and Q act on a bolt A. Determine their resultant.



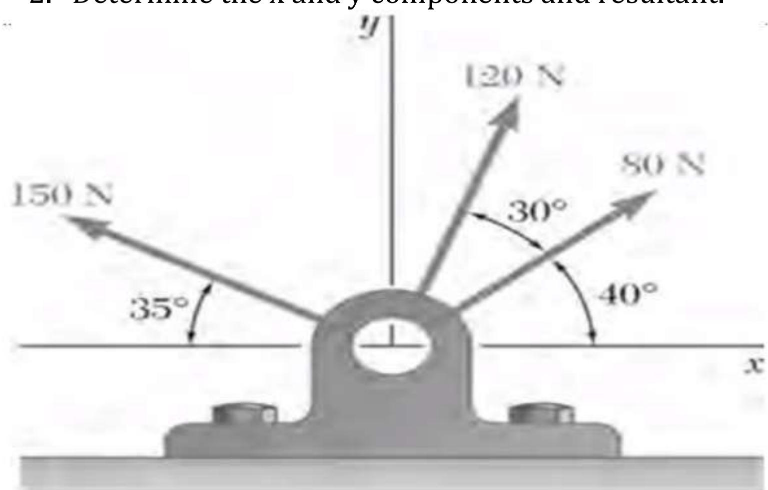
Lecture 3: Resolution of Forces, Lami's Theorem

Friday, February 18, 2022

1. Determine the resultant of the three forces below?



2. Determine the x and y components and resultant.

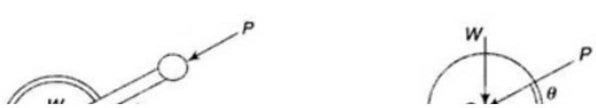


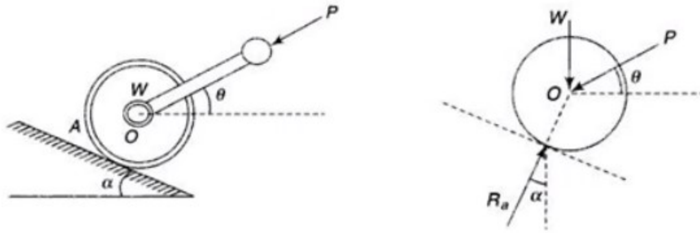
Free Body Diagram

Free body diagram is a sketch of the isolated body, which shows the external forces on the body and the reactions exerted on it by the removed elements

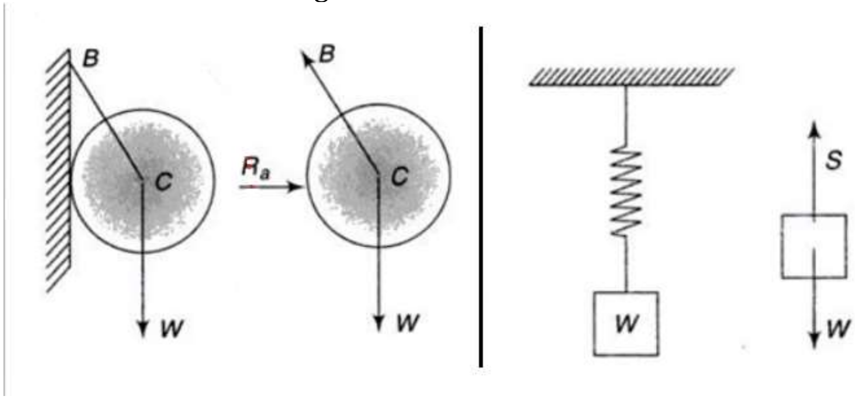
Procedure:

- Isolate each body from the system
- Indicate all the forces acting on it including the weight
- Indicate all reactions
- Show the dimensions and angles and reference axes

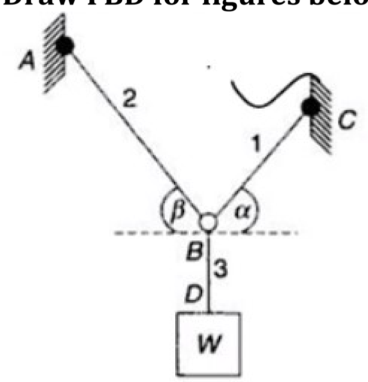


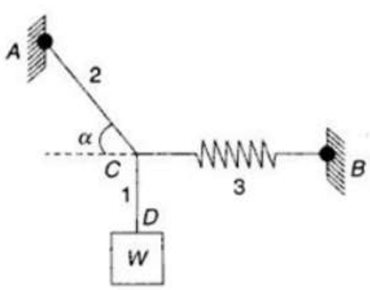


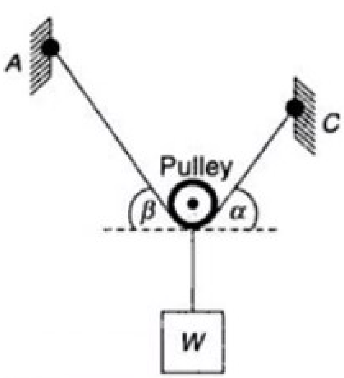
FBD for Cable and String:

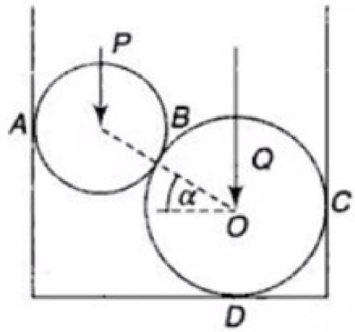


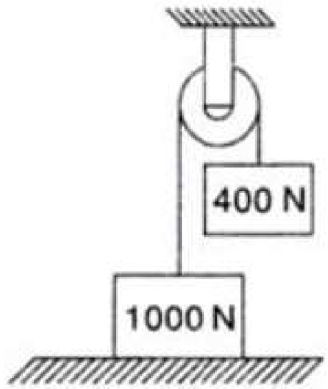
Draw FBD for figures below:





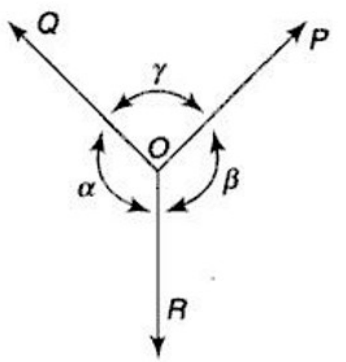


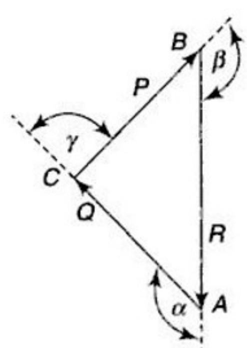


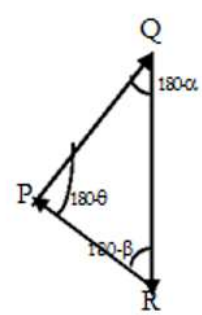


Equilibrium of a Particle
The particle is in equilibrium when $R = \Sigma F = \Sigma \mathrm{F_x} = \Sigma \mathrm{F_y} = 0$

Lami's Theorem
If three concurrent forces are acting on a body, kept in an equilibrium, then each force is proportional to the sine of the angles between the other two forces and the constant of proportionality is the same.

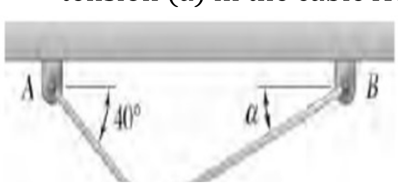




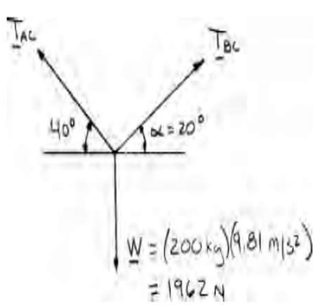


$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} = k$$

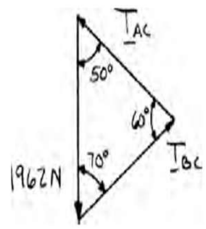
3. Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 20^0$, determine the tension (a) in the cable AC, (b) in cable BC.



Free-Body Diagram



Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 70^\circ} = \frac{T_{BC}}{\sin 50^\circ} = \frac{1962 \text{ N}}{\sin 60^\circ}$$

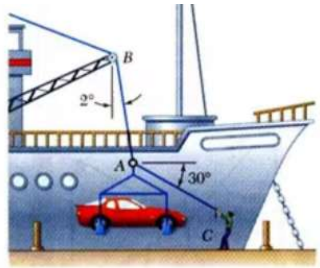
(*a*) $$T_{AC} = \frac{1962 \text{ N}}{\sin 60^\circ} \sin 70^\circ = 2128.9 \text{ N} \qquad T_{AC} = 2.13 \text{ kN} \blacktriangleleft$$

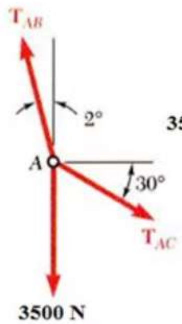
(*b*) $$T_{BC} = \frac{1962 \text{ N}}{\sin 60^\circ} \sin 50^\circ = 1735.49 \text{ N} \qquad T_{BC} = 1.735 \text{ kN} \blacktriangleleft$$

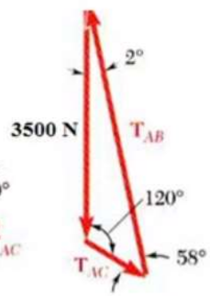
Class Problem 4

In a ship-unloading operation, a 3500 N automobile is supported by a cable. A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2^0, while the angle between the rope and the horizontal is 30^0. What is the tension in the rope?

Solution







- Construct a free-body diagram for the particle at *A*.
- Apply the conditions for equilibrium.

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500}{\sin 58°}$$

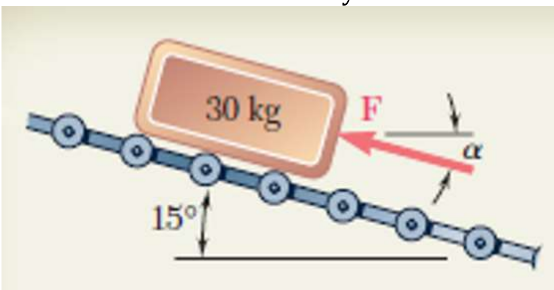
$$T_{AB} = 3570\,\mathrm{N}$$

$$\boxed{T_{AC} = 144\,\mathrm{N}}$$

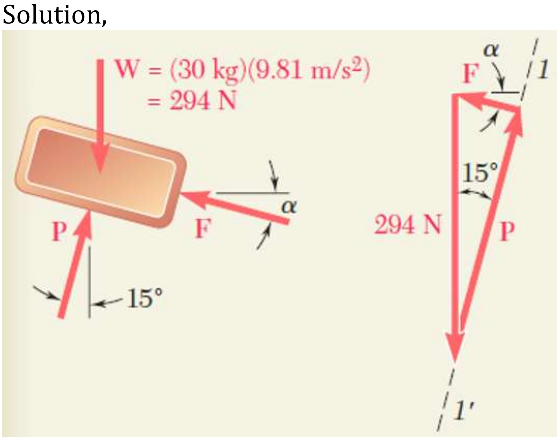
Lecture 4: Resolution of Forces, Lami's Theorem

Tuesday, February 22, 2022

Tutorial Problem 2:
Determine the magnitude and direction of the smallest force F which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.

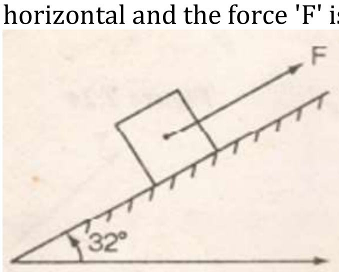


Solution,

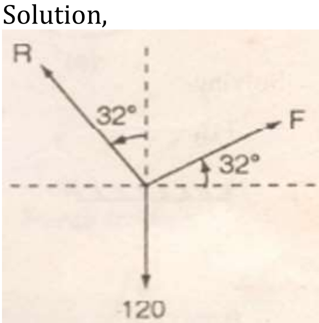


$$F = (294\ \text{N}) \sin 15° = 76.1\ \text{N}$$

Problem 1:
The figure below shows a block of weight 120 N on a smooth inclined plane. The plane makes an angle of 32^0 with horizontal and the force 'F' is applied parallel to the plane. Find the values of T' and normal reaction.



Solution,



$$\Sigma F_x = 0: F\cos 32° - R\sin 32° = 0$$

$$\Sigma F_y = 0: F\sin 32° + R\cos 32° - 120 = 0.$$

$$\begin{bmatrix} 0.848 & -0.530 \\ 0.530 & 0.848 \end{bmatrix} \begin{Bmatrix} F \\ R \end{Bmatrix} = \begin{Bmatrix} 0 \\ 120 \end{Bmatrix}$$

$$\begin{Bmatrix} F \\ R \end{Bmatrix} = \begin{bmatrix} 0.848 & 0.530 \\ -0.530 & 0.848 \end{bmatrix} \begin{Bmatrix} 0 \\ 120 \end{Bmatrix} = \begin{Bmatrix} 63.6 \\ 101.76 \end{Bmatrix}$$

This can also be solved by resolving along tangential and normal directions of plane.

$$\Sigma F_T = 0: F - 120 \sin 32° = 0$$

$$\Sigma F_N = 0: R - 120 \cos 32° = 0$$

$$F = 63.6 \text{ N}$$

$$R = 101.76 \text{ N}$$

Problem 2:

AD is a string hung from a horizontal ceiling at A and D. A weight of 75 N is hung from point 'B'; Determine the magnitude of weight that should be hung from point 'C' such that the portion 'BC' of the string is horizontal. Point B is 0.6 m from A and point C is 0.35 m from D, Also calculate the tension in various portions AB, BC and CD of the string.

Solution,

Let the tensions in the string be T_{AB}, T_{CD} and T_{BC}. Free body diagram of particles at B and C are shown in Fig. 2.27(b) and Fig 2.27(c).

From Fig. 2.27(b),

$$F_y = 0: T_{AB} \sin 45° - 75 = 0.$$

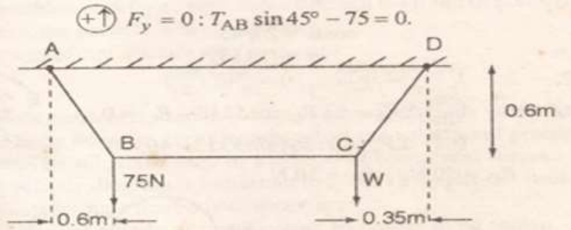


Figure 2.27(a)

Solving, $T_{AB} = 106.066$; $T_{BC} = 75.0$

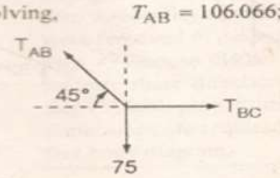


Figure 2.27(b) *Free body diagram of particle at 'B'*

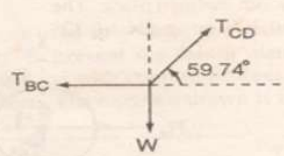


Figure 2.27(c) *Free body diagram of particle at 'C'*

From the figure 2.27,

$\Sigma F_x = -T_{BC} + T_{CD} \cos 59.74 = 0$

$T_{CD} \cos 59.74 = 75$

$T_{CD}(0.504) = 75$

$T_{CD} = 148.8 \text{ N}$

$\Sigma F_y = -W + T_{CD} \sin 59.74 = 0$

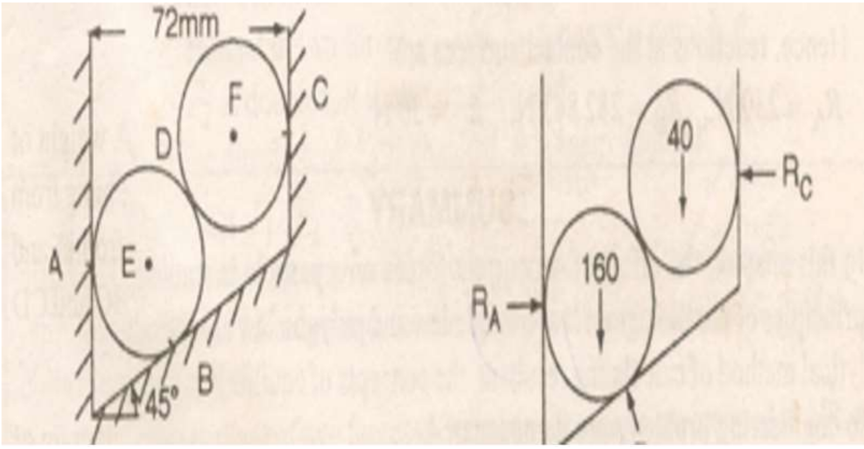
$(148.8) \sin 59.74 = W$

$(148.8)(0.864) = W$

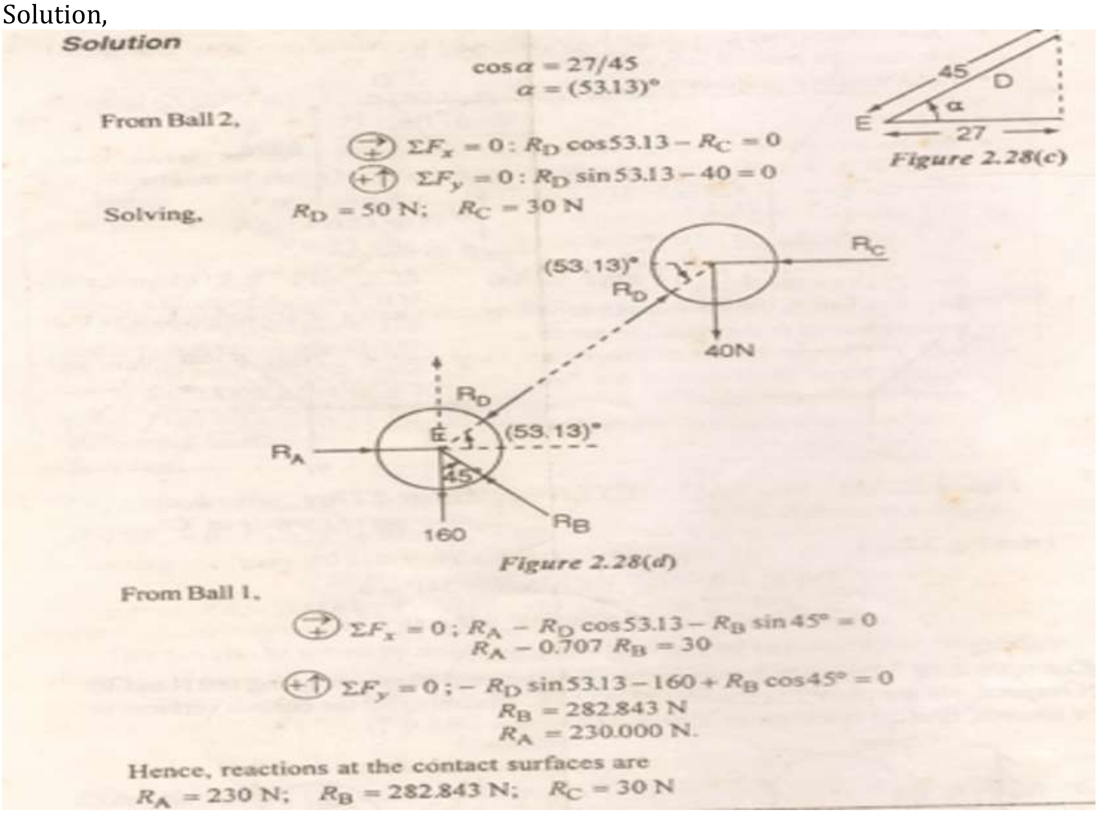
$W = 128.5 \text{ N}$

Problem 3:

Two cylinders of diameters 60 mm and 30 mm weighing 160 N and 40 N respectively are placed as shown in Fig. 2.28. Assuming all the contact surfaces to be smooth, find the reactions at A, B and C,



Solution,



Solution

$$\cos\alpha = 27/45$$
$$\alpha = (53.13)^\circ$$

Figure 2.28(c)

From Ball 2,

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0: R_D \cos 53.13 - R_C = 0$$
$$+\uparrow\ \Sigma F_y = 0: R_D \sin 53.13 - 40 = 0$$

Solving, $R_D = 50$ N; $R_C = 30$ N

Figure 2.28(d)

From Ball 1,

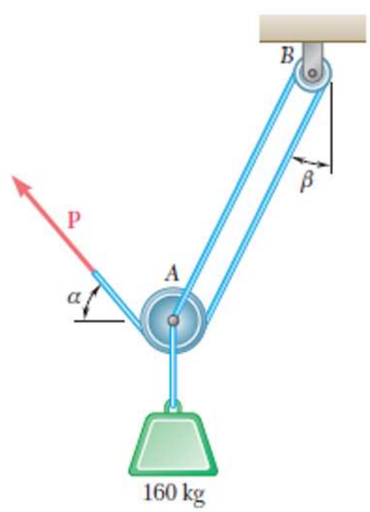
$$\overset{+}{\rightarrow}\ \Sigma F_x = 0;\ R_A - R_D \cos 53.13 - R_B \sin 45^\circ = 0$$
$$R_A - 0.707\, R_B = 30$$
$$+\uparrow\ \Sigma F_y = 0;\ -R_D \sin 53.13 - 160 + R_B \cos 45^\circ = 0$$
$$R_B = 282.843 \text{ N}$$
$$R_A = 230.000 \text{ N.}$$

Hence, reactions at the contact surfaces are

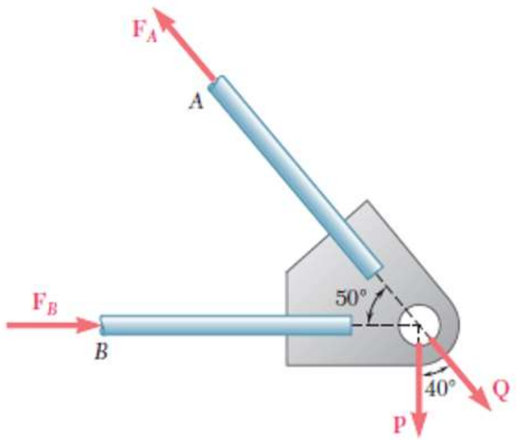
$R_A = 230$ N; $R_B = 282.843$ N; $R_C = 30$ N

Tutorial Problem 3:

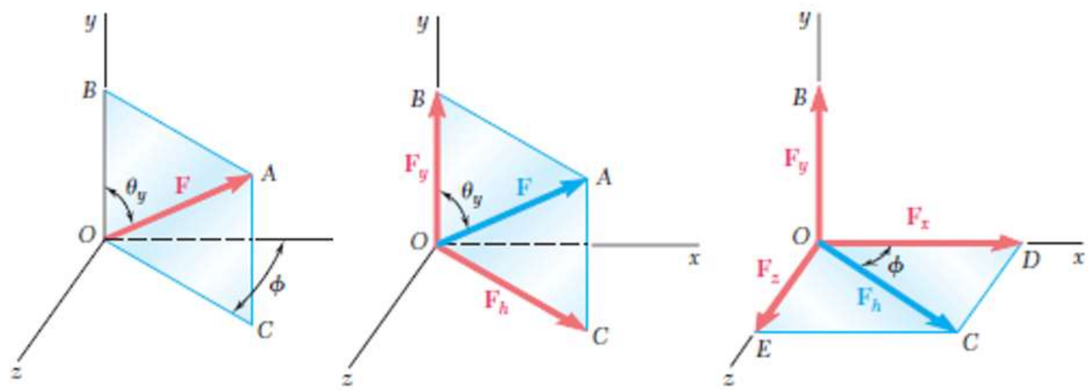
A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 20°$, determine the magnitude and direction of the force P that must be exerted on the free end of the rope to maintain equilibrium.



Tutorial Problem 4:
Two forces P and Q are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that P = 500 N and Q = 650 N, determine the magnitudes of the forces exerted on the rods A and B.



Rectangular Component of force in space:

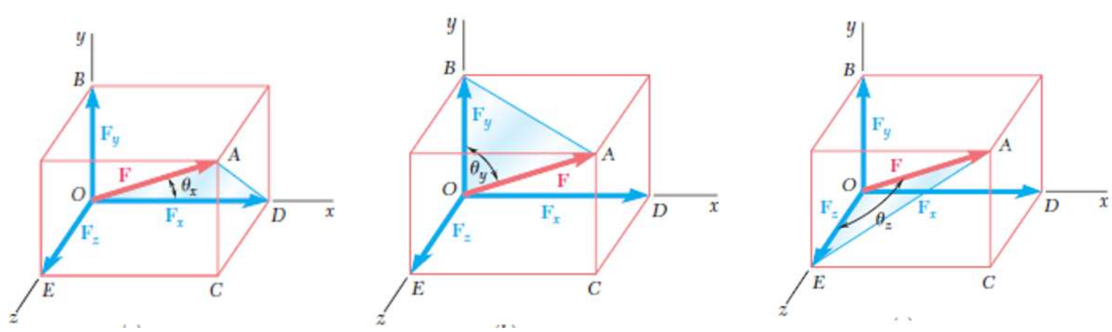


$$F_y = F \cos \theta_y \qquad F_h = F \sin \theta_y$$

$$F_x = F_h \cos \phi = F \sin \theta_y \cos \phi$$
$$F_z = F_h \sin \phi = F \sin \theta_y \sin \phi$$

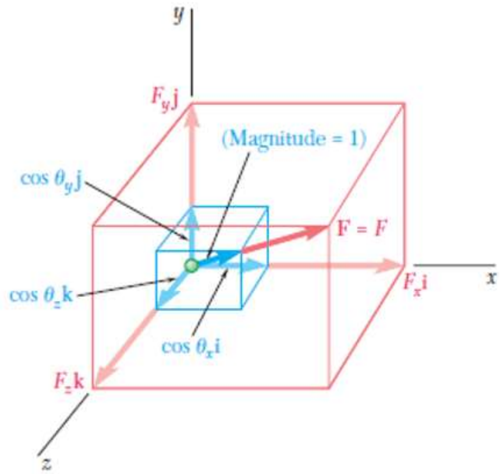
$$F^2 = (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2$$
$$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$



$$\mathbf{F} = F(\cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k})$$

$$\boldsymbol{\lambda} = \cos\theta_x\mathbf{i} + \cos\theta_y\mathbf{j} + \cos\theta_z\mathbf{k}$$

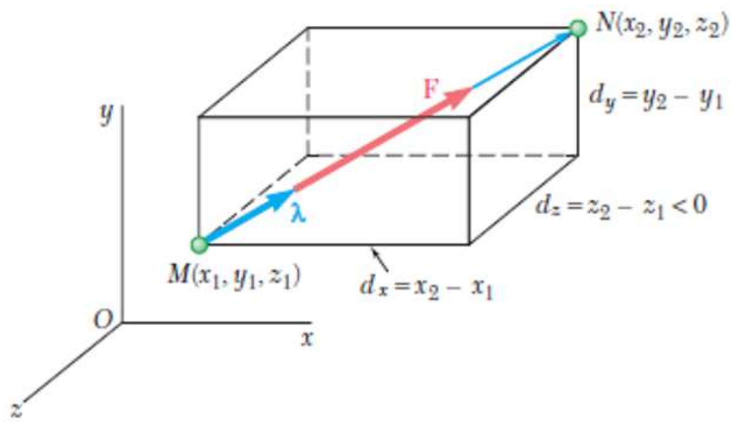
$$\lambda_x = \cos\theta_x \qquad \lambda_y = \cos\theta_y \qquad \lambda_z = \cos\theta_z$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos\theta_x = \frac{F_x}{F} \quad \cos\theta_y = \frac{F_y}{F} \quad \cos\theta_z = \frac{F_z}{F}$$

Force Defined by its Magnitude and Two Points on its Line of Action:



$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k} \qquad d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \qquad F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \qquad \cos\theta_x = \frac{d_x}{d} \quad \cos\theta_y = \frac{d_y}{d} \quad \cos\theta_z = \frac{d_z}{d}$$

Addition of concurrent forces in space:

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$\begin{aligned} R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} &= \Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) \\ &= (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k} \end{aligned}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

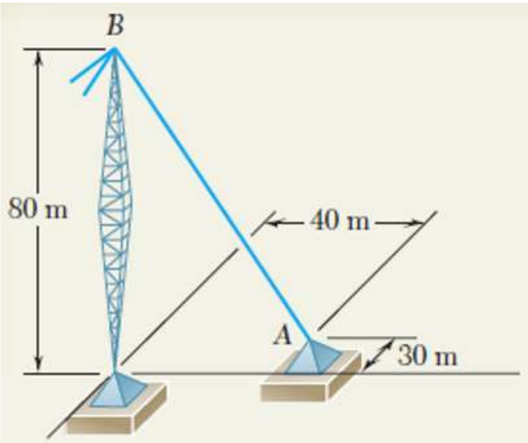
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

Lecture 5: Problems

Wednesday, February 23, 2022

Class Problem 4:
A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine the components F_x, F_y, F_z of the force acting on the bolt, the angles $\theta_x, \theta_y, \theta_z$ defining the direction of the force.



Solution,

$$\overrightarrow{AB} = (-40\text{m})\vec{i} + (80\text{m})\vec{j} + (30\text{m})\vec{k}$$

$$AB = \sqrt{(-40\text{m})^2 + (80\text{m})^2 + (30\text{m})^2}$$
$$= 94.3\text{ m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\vec{F} = F\vec{\lambda}$$
$$= (2500\text{ N})(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k})$$

$$\vec{F} = (-1060\text{N})\vec{i} + (2120\text{ N})\vec{j} + (795\text{ N})\vec{k}$$

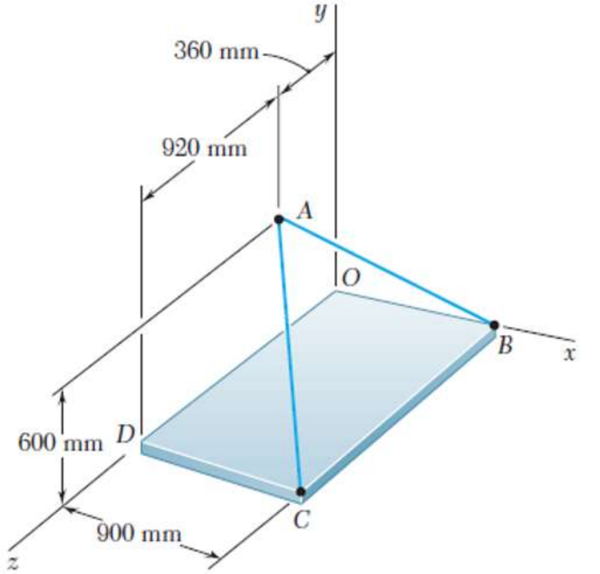
$$\boxed{Fx = -1060\text{ N};\ Fy = 2120\text{ N};\ Fz = 795\text{ N}}$$

Noting that the components vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x\,\vec{i} + \cos\theta_y\,\vec{j} + \cos\theta_z\,\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\theta_x = 115.1^\circ \quad \theta_y = 32.0^\circ \quad \theta_z = 71.5^\circ}$$

Tutorial Problem 5:
Knowing that the tension in cable AB is 1425 N, determine the components of the force exerted on the plate at B.



$$\overline{BA} = -(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

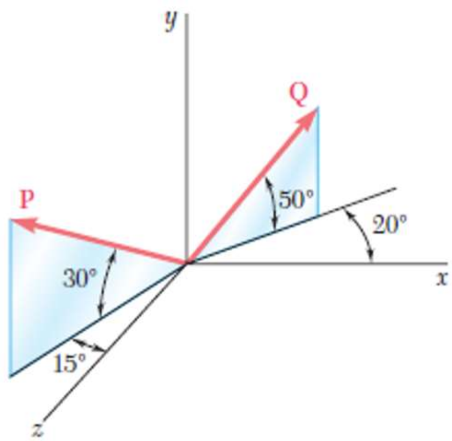
$$BA = \sqrt{(900\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$
$$= 1140\text{ mm}$$

$$\mathbf{T}_{BA} = T_{BA}\boldsymbol{\lambda}_{BA}$$
$$= T_{BA}\frac{\overline{BA}}{BA}$$

$$\mathbf{T}_{BA} = \frac{1425\text{ N}}{1140\text{ mm}}[-(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$$
$$= -(1125\text{ N})\mathbf{i} + (750\text{ N})\mathbf{j} + (450\text{ N})\mathbf{k}$$

$$(T_{BA})_x = -1125\text{ N}, \quad (T_{BA})_y = 750\text{ N}, \quad (T_{BA})_z = 450\text{ N} \blacktriangleleft$$

Tutorial Problem 6:
Find the magnitude and direction of the resultant of the two forces shown knowing that P = 300 N and Q = 400 N.



SOLUTION

$$\mathbf{P} = (300\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$
$$= -(67.243\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} + (250.95\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (400\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$
$$= (400\text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985]$$
$$= (241.61\text{ N})\mathbf{i} + (306.42\text{ N})\mathbf{j} - (87.939\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (174.367\text{ N})\mathbf{i} + (456.42\text{ N})\mathbf{j} + (163.011\text{ N})\mathbf{k}$$
$$R = \sqrt{(174.367\text{ N})^2 + (456.42\text{ N})^2 + (163.011\text{ N})^2}$$
$$= 515.07\text{ N}$$

$R = 515\text{ N}$ ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{174.367\text{ N}}{515.07\text{ N}} = 0.33853$$

$\theta_x = 70.2°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{456.42\text{ N}}{515.07\text{ N}} = 0.88613$$

$\theta_y = 27.6°$ ◀

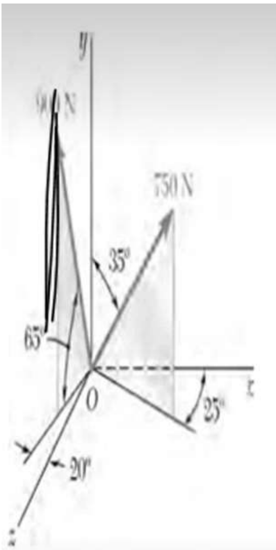
$$\cos\theta_z = \frac{R_z}{R} = \frac{163.011\text{ N}}{515.07\text{ N}} = 0.31648$$

$\theta_z = 71.5°$ ◀

Lecture 6: Problems

Friday, February 25, 2022

Problem 1:
Determine (a) the x, y, and z components of the force, (b) the angles $\theta_x, \theta_y, \theta_z$ that the force forms with the coordinate axes.



Solution,

$$F_h = F \sin 35°$$
$$= (750 \text{ N}) \sin 35°$$
$$F_h = 430.2 \text{ N}$$

(*a*)

$$F_x = F_h \cos 25° \qquad F_y = F \cos 35° \qquad F_z = F_h \sin 25°$$
$$= (430.2 \text{ N}) \cos 25° \qquad = (750 \text{ N}) \cos 35° \qquad = (430.2 \text{ N}) \sin 25°$$
$$F_x = +390 \text{ N}, \qquad F_y = +614 \text{ N}, \qquad F_z = +181.8 \text{ N}$$

(*b*)

$$\cos \theta_x = \frac{F_x}{F} = \frac{+390 \text{ N}}{750 \text{ N}} \qquad \theta_x = 58.7° \blacktriangleleft$$
$$\cos \theta_y = \frac{F_y}{F} = \frac{+614 \text{ N}}{750 \text{ N}} \qquad \theta_y = 35.0° \blacktriangleleft$$
$$\cos \theta_z = \frac{F_z}{F} = \frac{+181.8 \text{ N}}{750 \text{ N}} \qquad \theta_z = 76.0° \blacktriangleleft$$

If F = 900 N

$$F_h = F\cos 65^\circ$$
$$= (900\text{ N})\cos 65^\circ$$
$$F_h = 380.4\text{ N}$$

(a)

$$F_x = F_h \sin 20^\circ \qquad F_y = F\sin 65^\circ \qquad F_z = F_h \cos 20^\circ$$

$$= (380.4\text{ N})\sin 20^\circ \qquad = (900\text{ N})\sin 65^\circ \qquad = (380.4\text{ N})\cos 20^\circ$$

$$F_x = -130.1\text{ N}, \qquad F_y = +816\text{ N}, \qquad F_z = +357\text{ N}$$

(b)

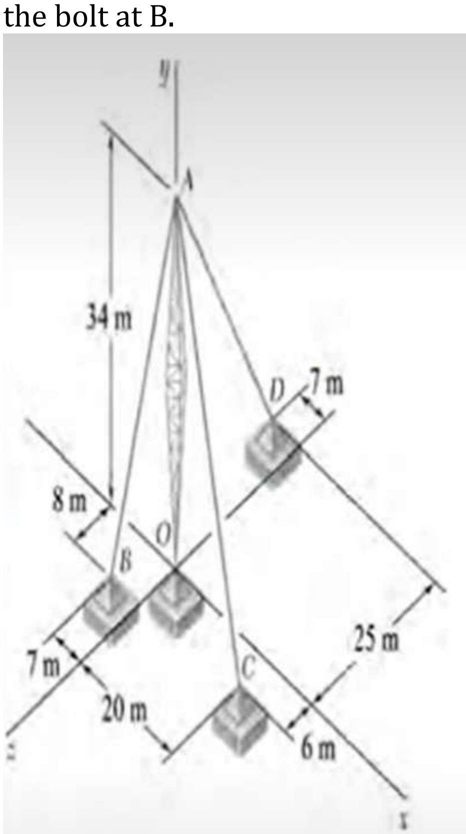
$$\cos\theta_x = \frac{F_x}{F} = \frac{-130.1\text{ N}}{900\text{ N}} \qquad \theta_x = 98.3^\circ$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+816\text{ N}}{900\text{ N}} \qquad \theta_y = 25.0^\circ$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{+357\text{ N}}{900\text{ N}} \qquad \theta_z = 66.6^\circ$$

Problem 2:

A transmission tower is held by three guy wires anchored by bolts at B, C, and D. If the tension in wire AB is 2625 N, determine the components of the force exerted by the wire on the bolt at B.



Solution,

$$\overrightarrow{BA} = (7\text{ m})\mathbf{i} + (34\text{ m})\mathbf{j} - (8\text{ m})\mathbf{k}$$

$$BA = \sqrt{(7)^2 + (34)^2 + (8)^2}$$

$$= 35.623\text{ m}$$

$$\mathbf{F} = F\lambda_{BA}$$

$$= F\frac{\overrightarrow{BA}}{BA}$$

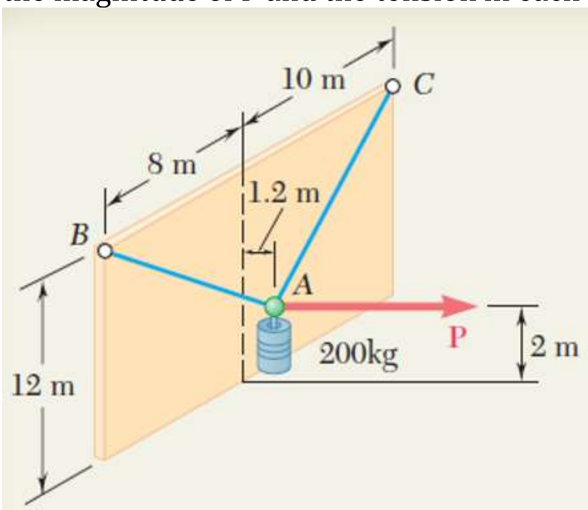
$$= \frac{2625\text{ N}}{35.623\text{ m}}[(7\text{ m})\mathbf{i} + (34\text{ m})\mathbf{j} - (8\text{ m})\mathbf{k}]$$

$$\mathbf{F} = (515.82\text{ N})\mathbf{i} + (2505.4\text{ N})\mathbf{j} - (589.51\text{ N})\mathbf{k}$$

$$F_x = +516\text{ N}, \quad F_y = +2510\text{ N}, \quad F_z = -590\text{ N} \blacktriangleleft$$

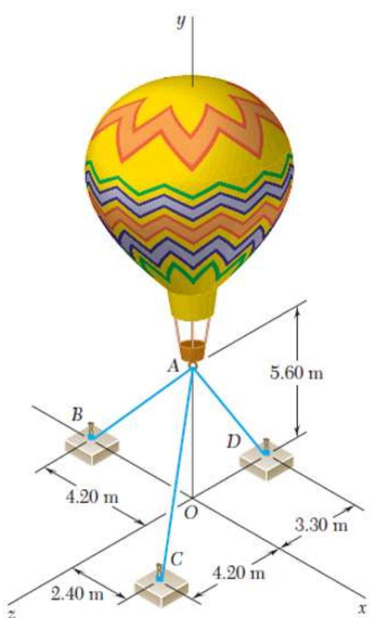
Other Module 1 Problems

Problem 1:
A 200-kg cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of P and the tension in each cable.



$P = 235$ N
$T_{AB} = 1402$ N
$T_{AC} = 1238$ N

Problem 2:



Three cables are used to tether a balloon as shown.

a. Determine the vertical force P exerted by the balloon at A knowing that the tension in cable AB is 259 N. P = 1031 N
b. Determine the vertical force P exerted by the balloon at A knowing that the tension in cable AD is 481 N. P = 926 N

SOLUTION

See Problem 2.101 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

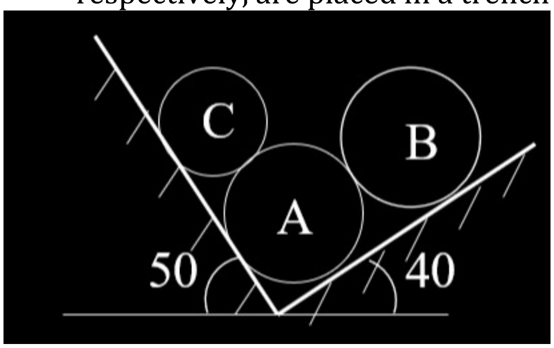
Substituting $T_{AC} = 444$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$
$$T_{AD} = 496.36 \text{ N}$$

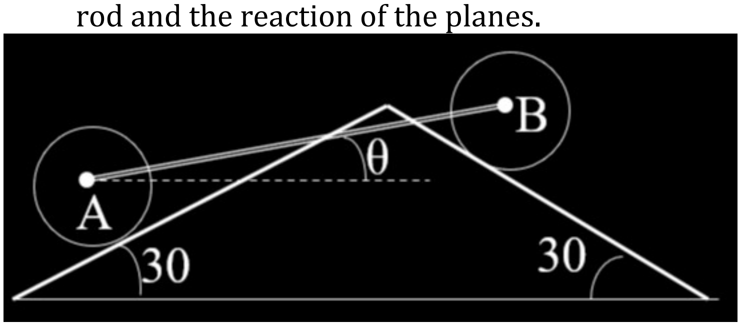
$\mathbf{P} = 956 \text{ N}\uparrow$ ◀

Assignment Problems:

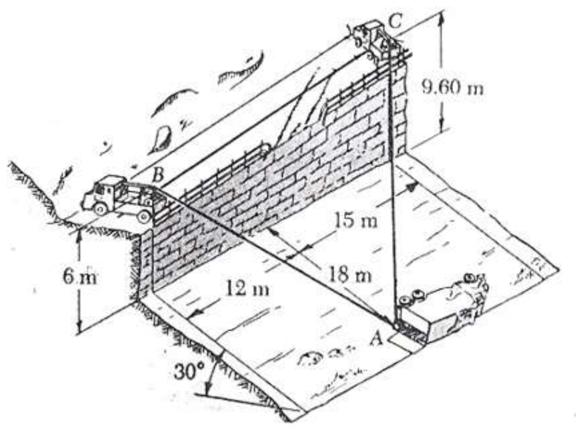
1. Three cylinders A, B, C of diameters, 200mm, 200mm, 100mm and weighing 400N, 400N, 200N, respectively, are placed in a trench as shown in Fig(5). Find the reactions at all contact points.



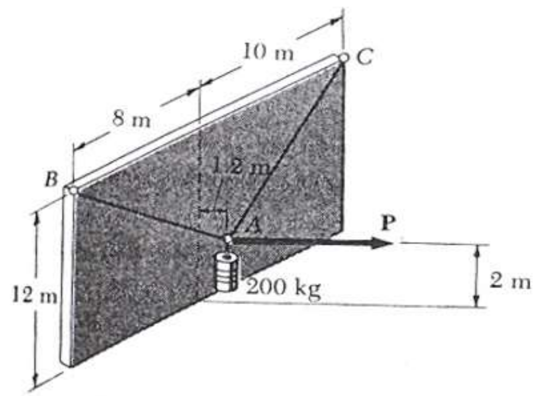
2. Two rollers A and B of same diameter and weight 1000N, 600N, respectively, interconnected by a light weight rod are placed on smooth planes as shown in Fig(6). Determine the inclination θ of the rod and the reaction of the planes.



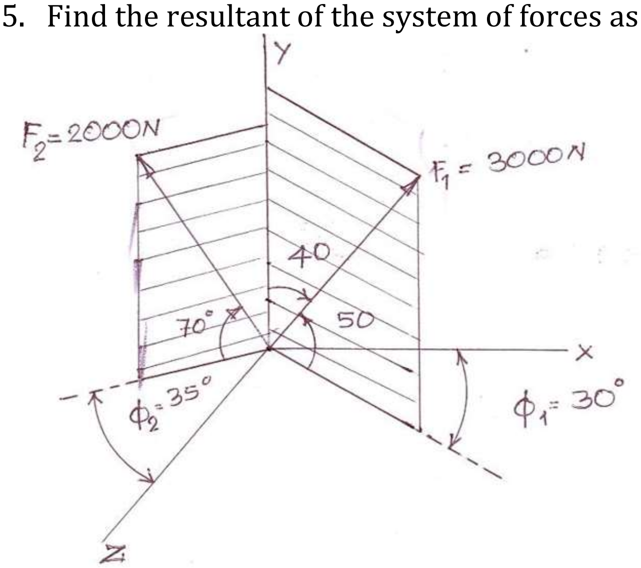
3. In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in the cable AB is 10 kN, determine the components of the force exerted by the cable AB on the truck.



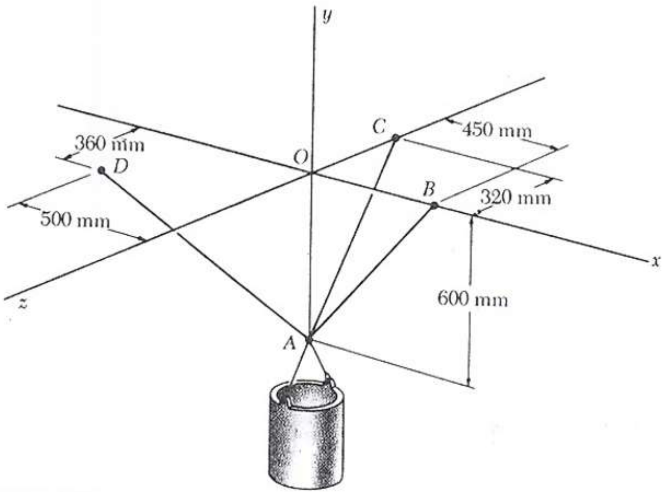
4. A 200 kg cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of the force P and the tension in each cable.



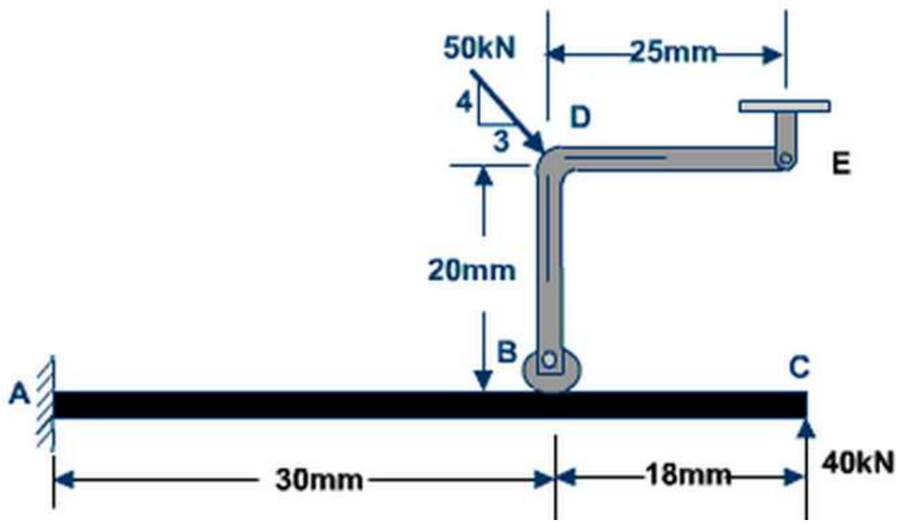
5. Find the resultant of the system of forces as shown below.



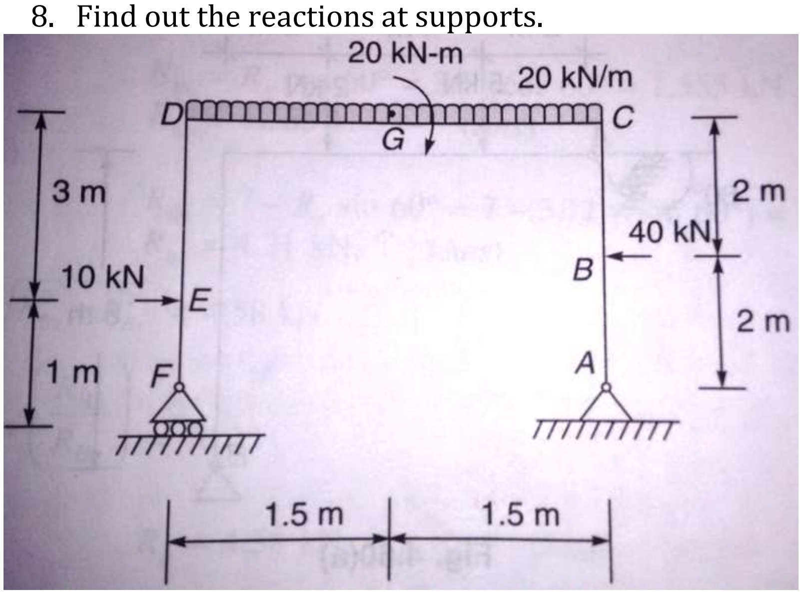
6. A container of weight W = 1165 N is supported by three cables as shown below. Determine the tension in each cable.



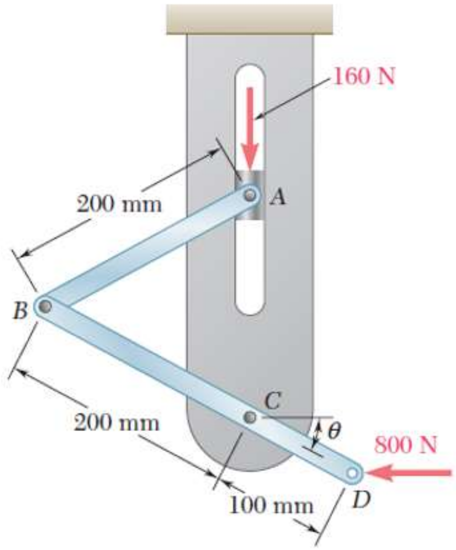
7. For the frame and loading shown in figure, Calculate the reaction forces at supports A and E.



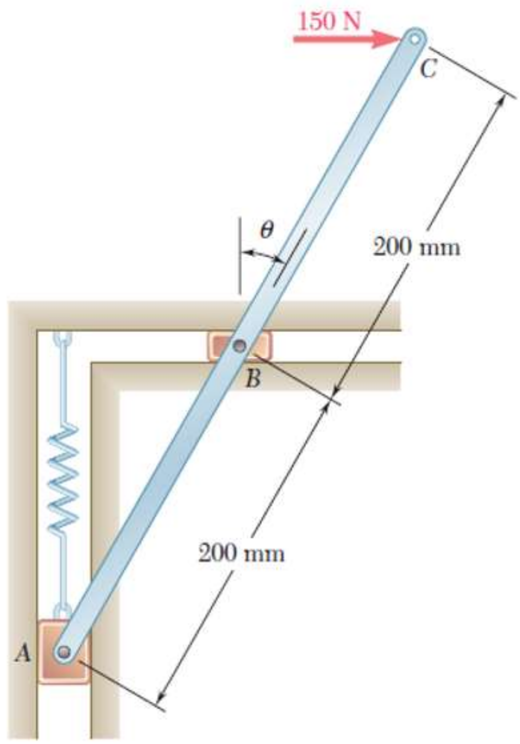
8. Find out the reactions at supports.



9. Rod AB is attached to a block at A that can slide freely in the vertical slot shown. Neglecting the effect of friction and the weights of the rods, determine the value of θ corresponding to equilibrium.



10. Rod ABC is attached to blocks A and B that can move freely in the guides shown. The constant of the spring attached at A is k = 3 kN/m, and the spring is unstretched when the rod is vertical. For the loading shown, determine the value of θ corresponding to equilibrium.



Module 2:

Statics of Rigid Bodies

Wednesday, March 2, 2022 8:10 AM

moment produced by two equal opposite and non collinear forces are called couple.

Lecture 7:

Saturday, February 26, 2022

Lecture 8:

Wednesday, March 2, 2022