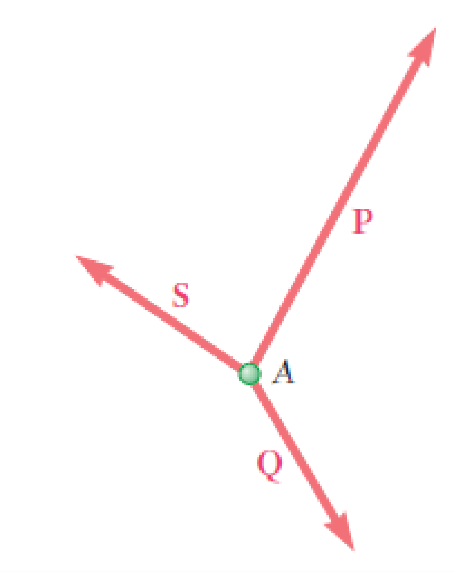
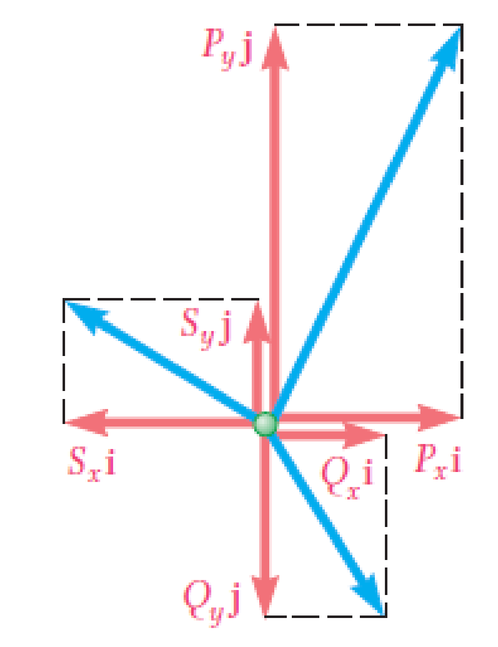
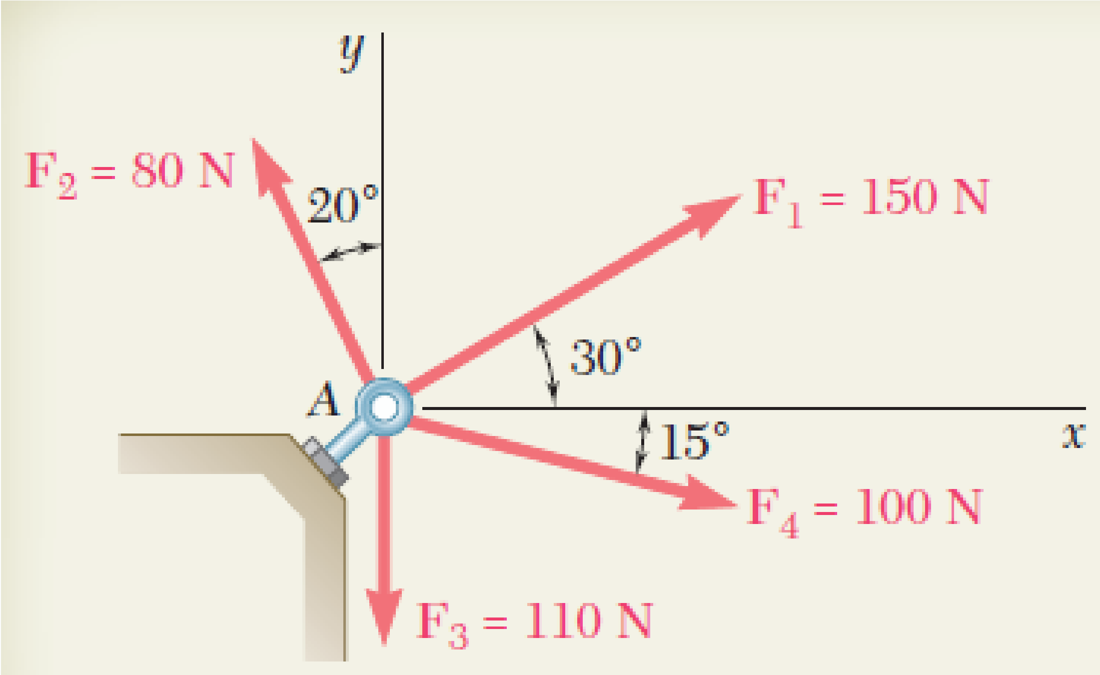
Resultant of Several Concurrent Forces

Problem 4







$\boxed{R = 199.6\text{N}}$ $\boxed{\alpha = 4.1^\circ}$

Static Equilibrium of Particle

When a Particle is said to be in Equilibrium ?

A particle is said to be in **equilibrium** if the particle :

- is at rest (if originally at rest); or
- has a constant velocity (if originally in motion)

Static Equilibrium

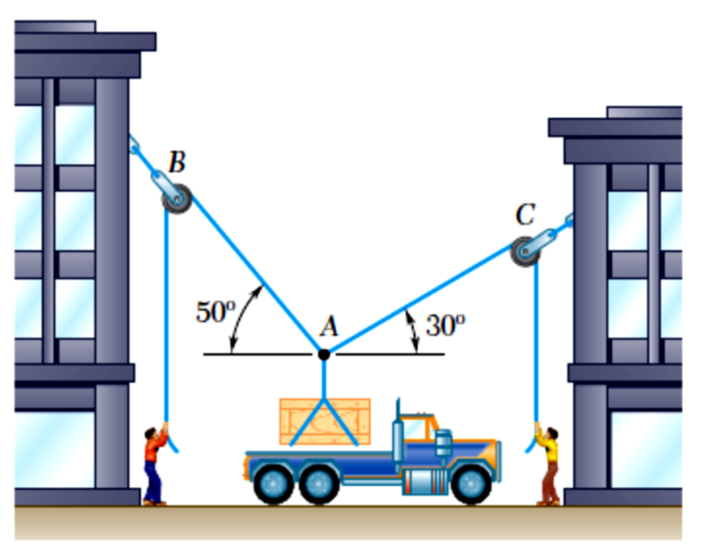
- Particle at rest

Condition for Equilibrium

- It is necessary to satisfy Newton's first law of motion
  - Resultant force acting on a particle to be equal to zero
  - Mathematically $\Sigma F = 0$
  - where $\Sigma F = 0$ is the vector sum of all the forces acting on the particle

Space Diagram

- ✓ A problem in engineering mechanics is derived from an actual physical situation
- ✓ A sketch showing the physical conditions of the problem is known as space diagram



Free Body Diagram

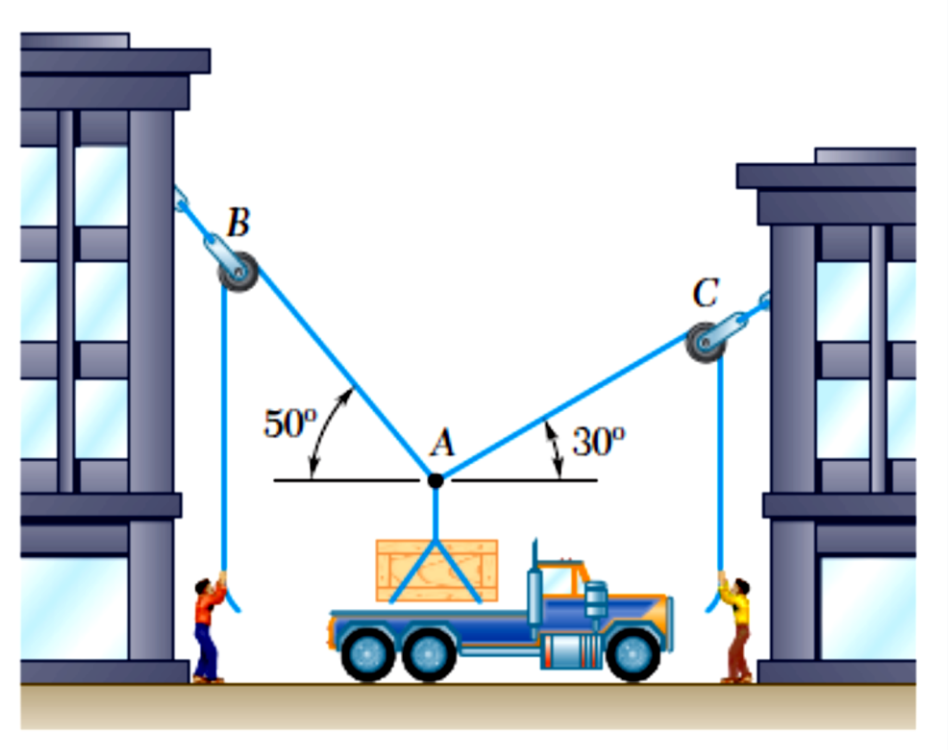
- ✓ A large number of problems involving actual structures, however, can be reduced to problems concerning the equilibrium of a particle
- ✓ This is done by choosing a significant particle and drawing s separate diagram showing this particle and all the forces acting on it
- ✓ Consider 75 kg crate shown in the space diagram

Free Body Diagram

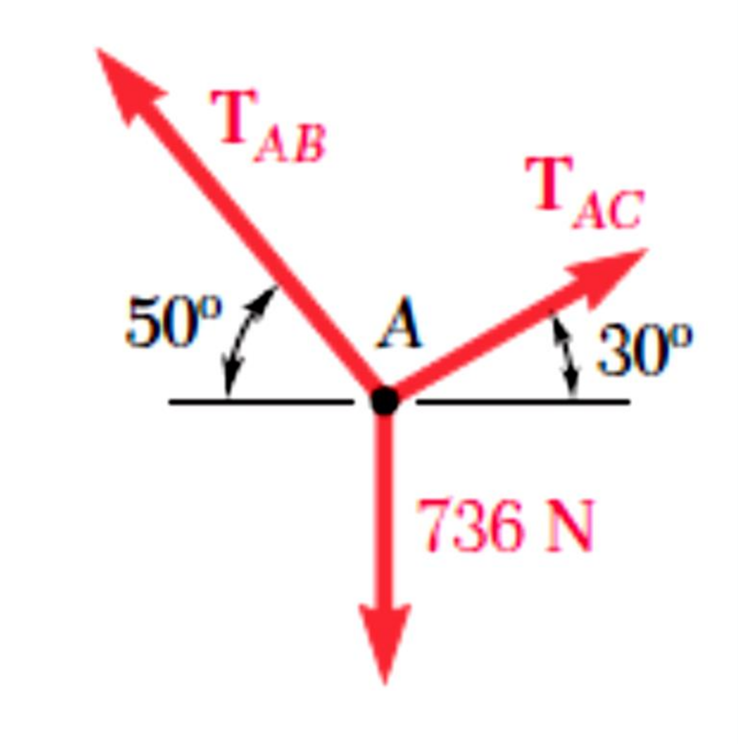
- To apply the equation of equilibrium, all the known and unknown external forces act on the particle must be accounted.
  - Known Force (Magnitude and direction known (given)
  - Unknown Force (Either magnitude or direction known)
- The best way to do is isolate the particle and free from its surroundings
- A drawing that shows the particle with all the forces that act on it is called a Free Body Diagram (FBD)

SPACE Diagram vs FBD

Space Diagram

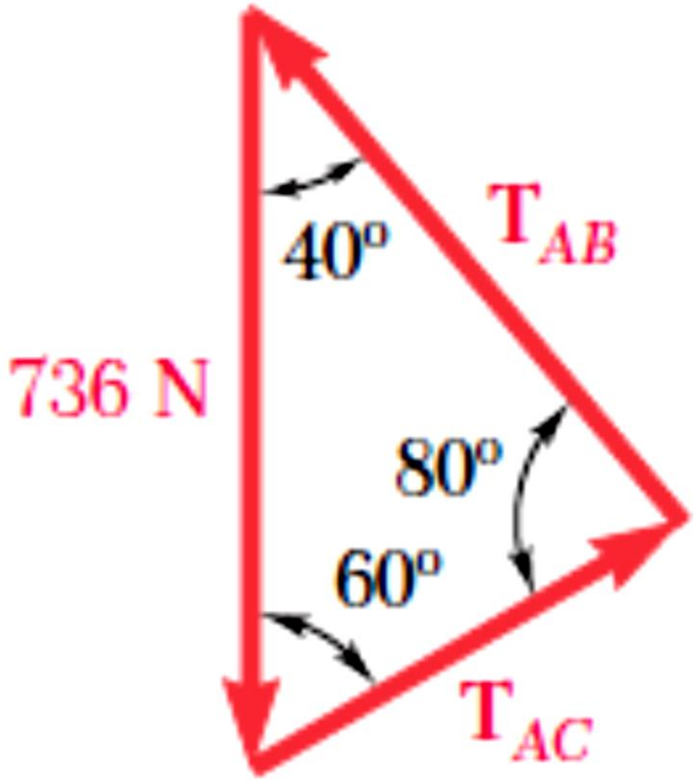


FBD



SPACE Diagram vs FBD

Force triangle (closed Triangle tip-to-tail fashion)



SPACE Diagram vs FBD

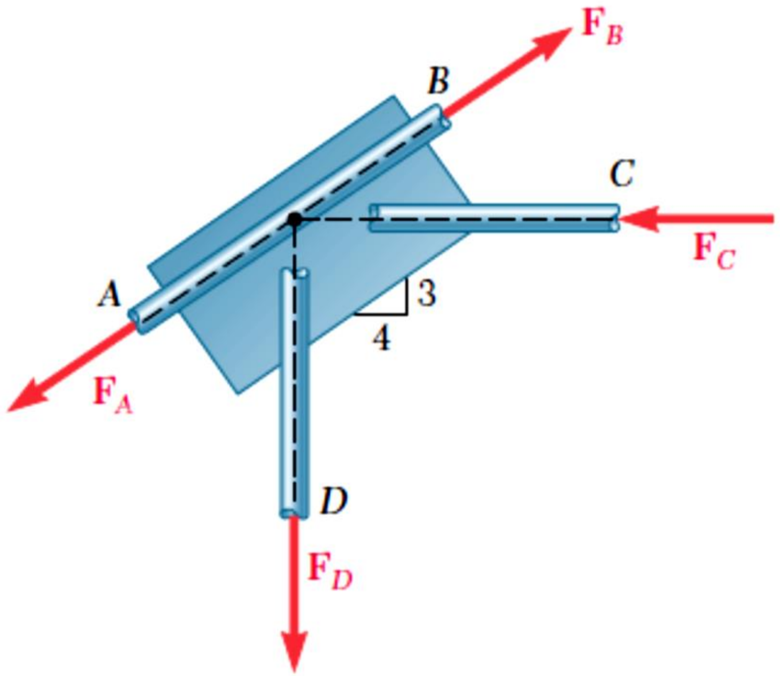
- When a particle is in equilibrium under three forces, the problem can be solved by drawing a force triangle
- When a particle is in equilibrium under more than three forces, the problem can be solved graphically by drawing a force polygon
- Or, solve analytically (equations of equilibrium should be solved, $\Sigma F_x = 0 \qquad \Sigma F_y = 0$

SPACE Diagram vs FBD

- These equations can be solved for no more than two unknowns
- Similarly, the force triangle used in the case of equilibrium under three forces can be solved for two unknowns
- The more common types problems are those in which the two unknowns represent
  - The two components (Magnitude and direction) of a single force
  - The magnitudes of two forces, each of known direction

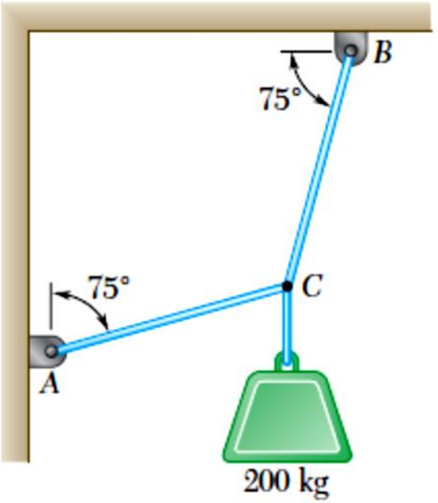
SPACE Diagram vs FBD

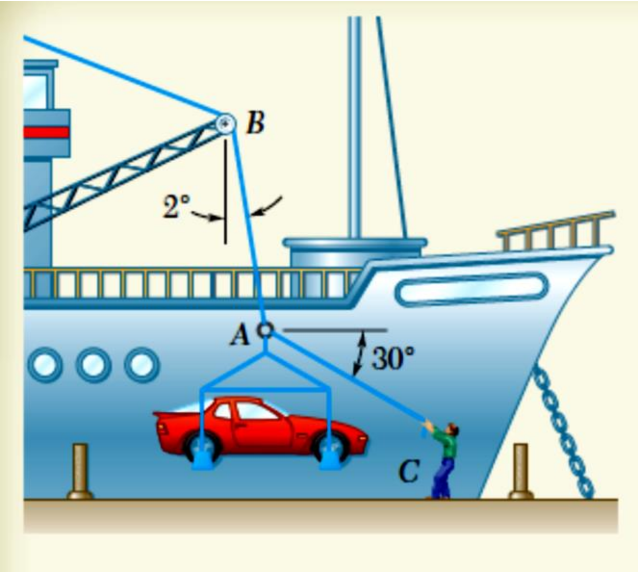
Problem – 5, A welded connection is in equilibrium under the action of the four forces shown, F_A=8 kN and F_A=16 kN, determine the magnitude of the other two forces

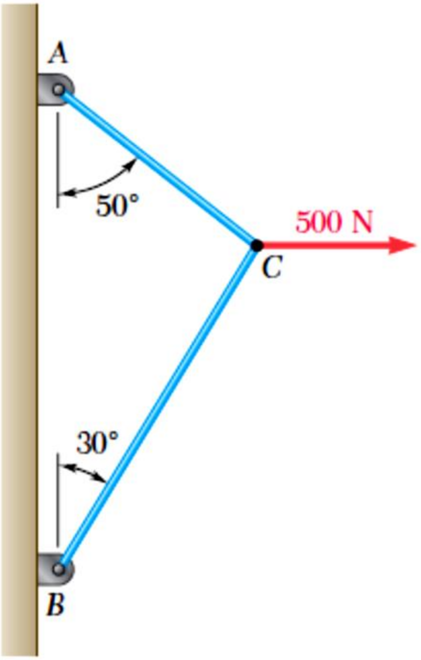


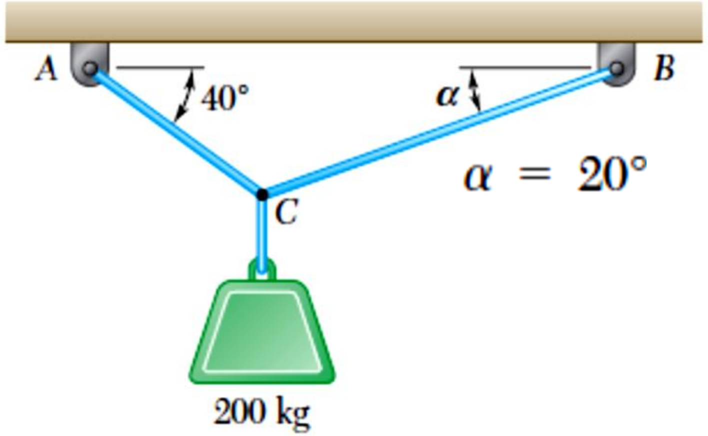
SPACE Diagram vs FBD

Problem – 6, 7,8 and 9







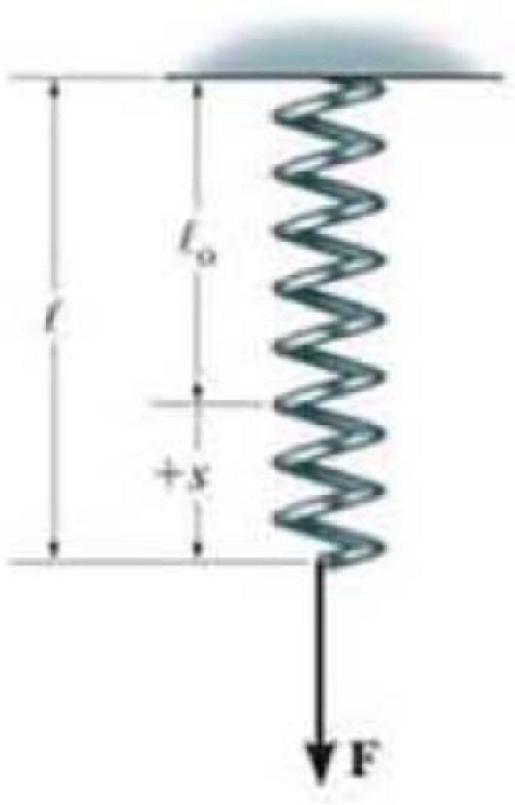


Two types of important connections

- **Linear elastic spring**
  - The length of spring will change directly proportional to the force acting on it
  - Spring force

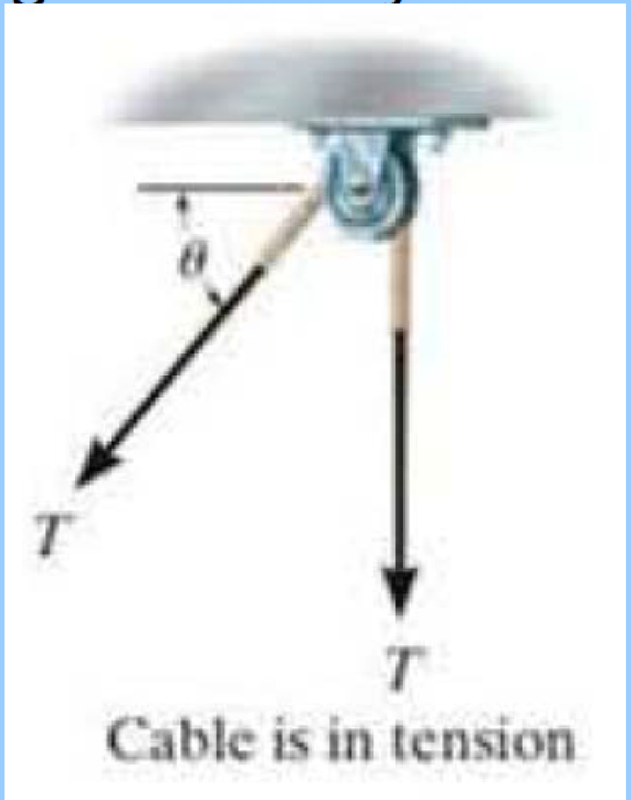
$$\boxed{F = ks}$$

  - Where **k=**spring constant or spring stiffness
  - **s=**deformed (elongated or compressed) length



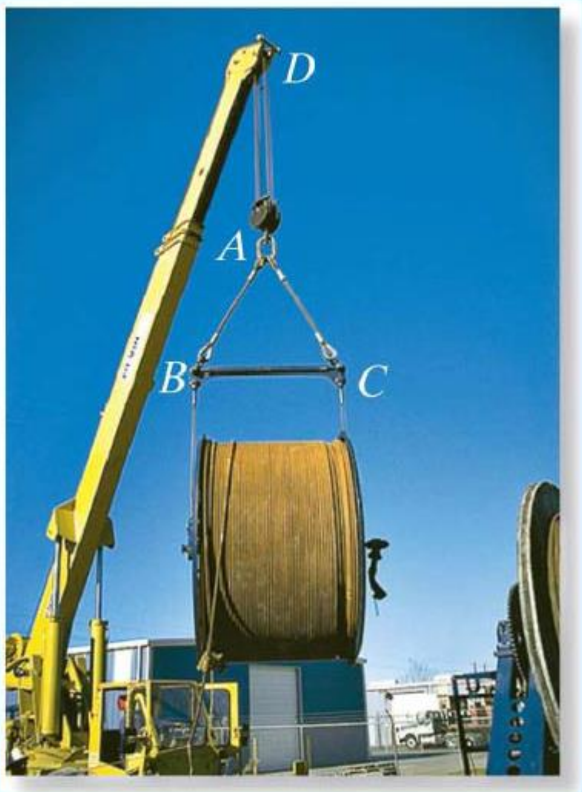
Two important connections

- **Cables\Ropes\Wires**
- Assumed to have negligible weight and they **cannot stretch**
- Support only **Tension** or **Pulling force**

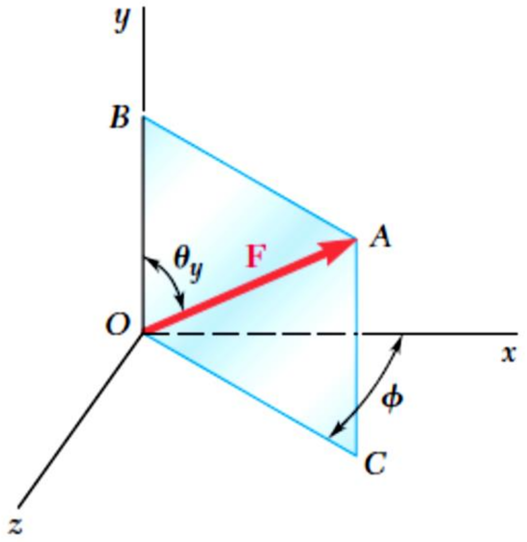


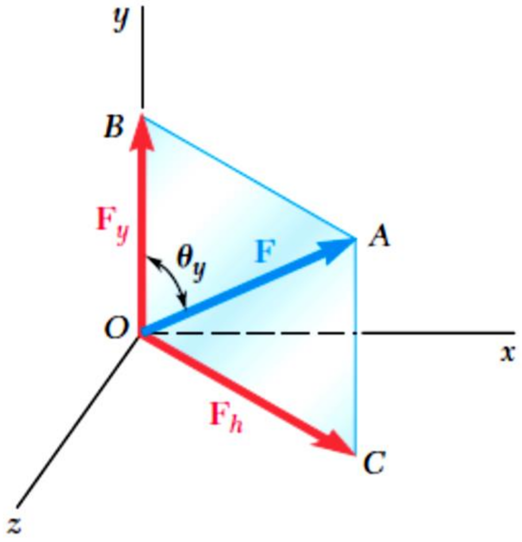
Δ Tension developed in a continuous cable which passes over a frictionless pulley must have a constant magnitude to keep cable in equilibrium

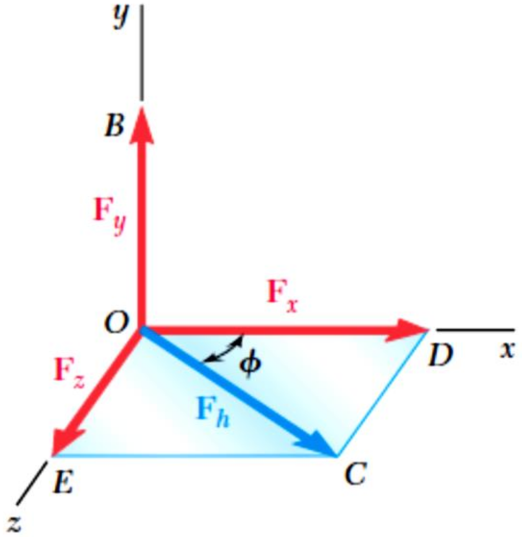
Cables\Ropes\Wires\Chains



Rectangle Components of a force in Space







$$F_y = F \cos \theta_y \qquad F_h = F \sin \theta_y$$

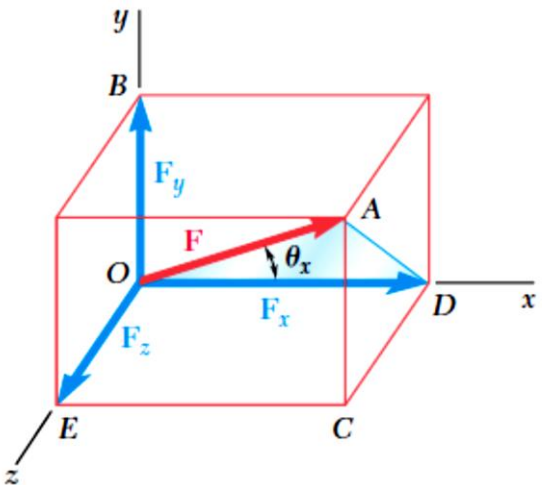
$$F_x = F_h \cos \phi = F \sin \theta_y \cos \phi$$
$$F_z = F_h \sin \phi = F \sin \theta_y \sin \phi$$

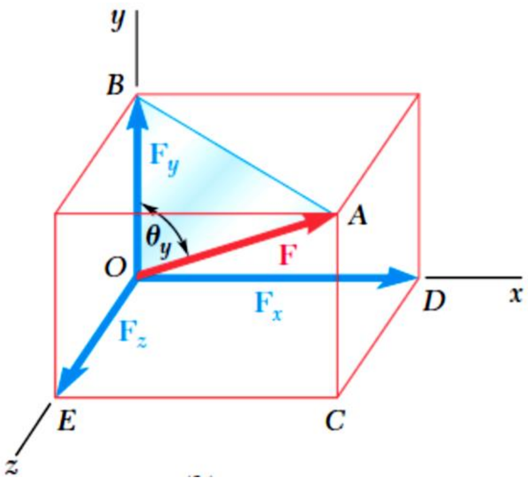
$$F^2 = (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2$$
$$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$$

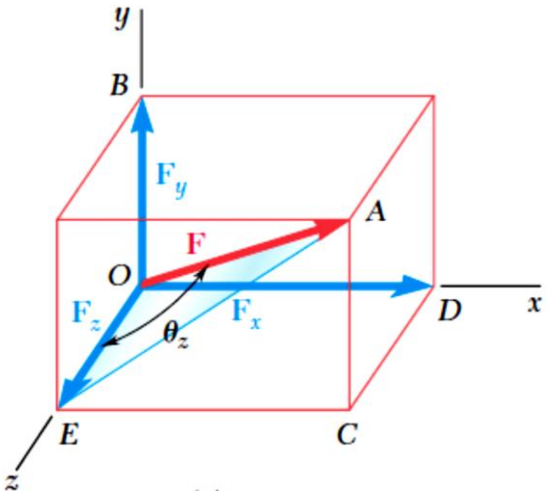
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Rectangle Components of a force in Space

The relationship existing between the force F and its three Components F_x, F_y and F_z are easily visualized







Introducing the unit vectors i, j and k directed respectively along x, y and z axes, Where F_x, F_y and F_z are scalar components

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_x = F\cos\theta_x \qquad F_y = F\cos\theta_y \qquad F_z = F\cos\theta_z$$

Rectangle Components of a force in Space

Substituting the expressions

$$\mathbf{F} = F(\cos\theta_x \mathbf{i} + \cos\theta_y \mathbf{j} + \cos\theta_z \mathbf{k})$$

Which shows that the force F can be expressed as the product of scalar and the vector

$$\boldsymbol{\lambda} = \cos\theta_x \mathbf{i} + \cos\theta_y \mathbf{j} + \cos\theta_z \mathbf{k}$$

Clearly $\boldsymbol{\lambda}$ is a vector whose magnitude is equal to 1 and whose direction is the same as that F. The vector $\boldsymbol{\lambda}$ is referred as unit vector

$$\lambda_x = \cos\theta_x \qquad \lambda_y = \cos\theta_y \qquad \lambda_z = \cos\theta_z$$

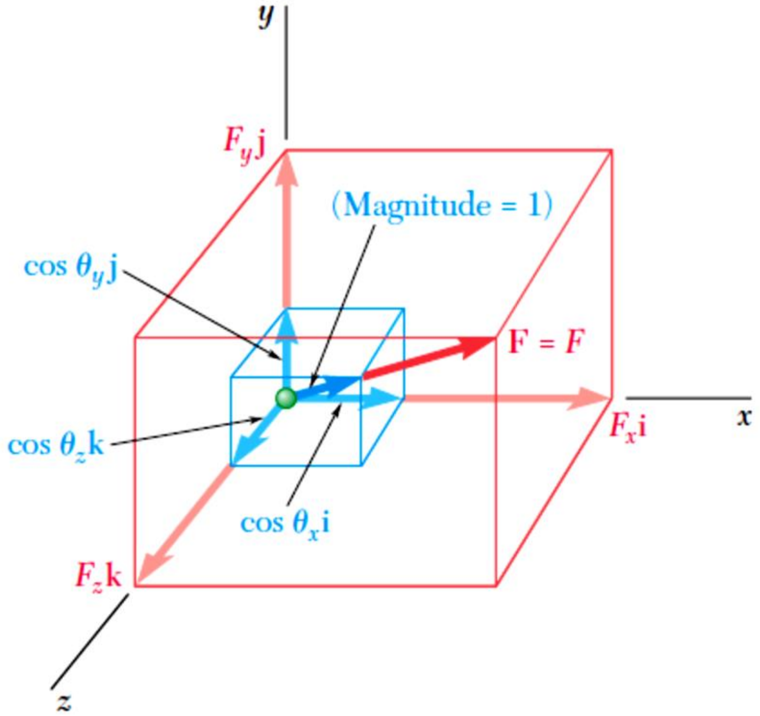
Rectangle Components of a force in Space

1. One can observe that the values of three angles $\theta_x, \theta_y, \theta_z$ are not independent
2. The sum of the squares of the components of a vector is equal to the square of its magnitude

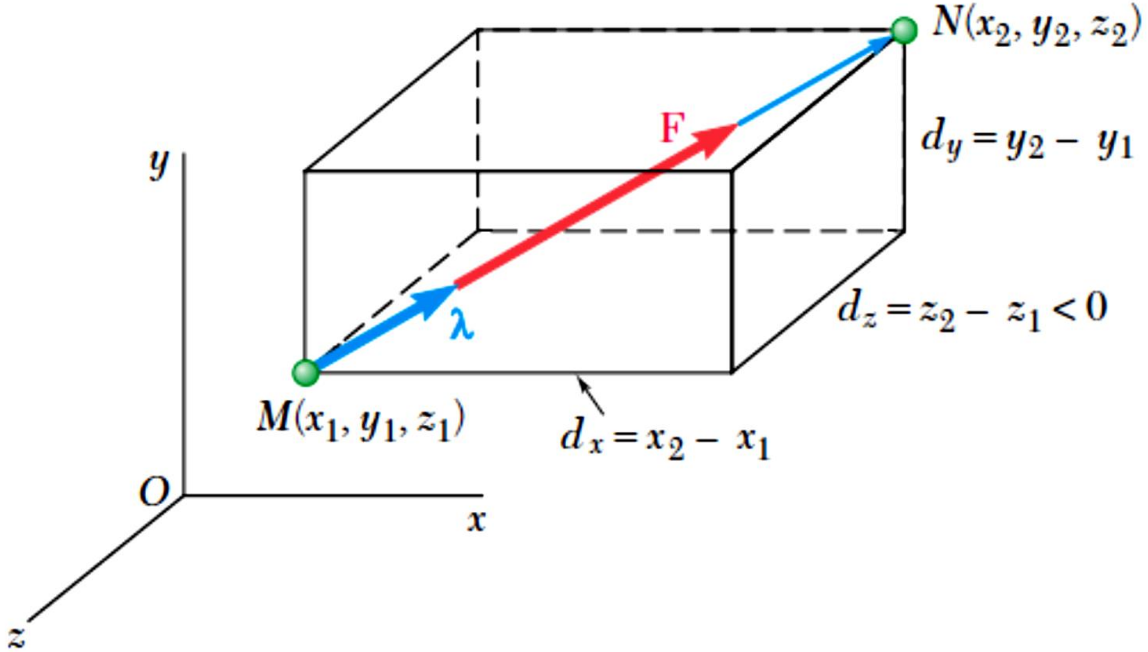
$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$$

$$\boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

$$\cos\theta_x = \frac{F_x}{F} \quad \cos\theta_y = \frac{F_y}{F} \quad \cos\theta_z = \frac{F_z}{F}$$



Force defined by its magnitude and Two points on its line of Action



$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}$$

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})$$

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k})$$

$$F_x = \frac{Fd_x}{d} \qquad F_y = \frac{Fd_y}{d} \qquad F_z = \frac{Fd_z}{d}$$

$$d_x = x_2 - x_1 \qquad d_y = y_2 - y_1 \qquad d_z = z_2 - z_1$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$\cos\theta_x = \frac{d_x}{d} \qquad \cos\theta_y = \frac{d_y}{d} \qquad \cos\theta_z = \frac{d_z}{d}$$

Addition of concurrent forces in Space

R = $\sum F$,

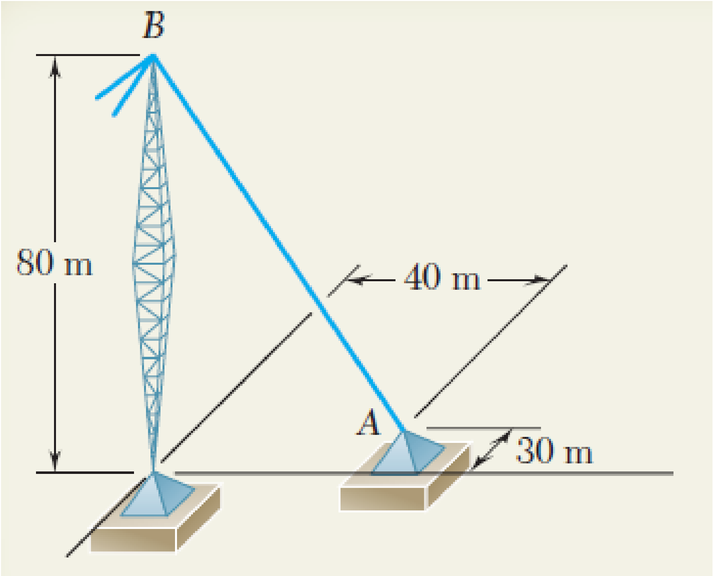
$$R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = \Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k})$$
$$= (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k}$$

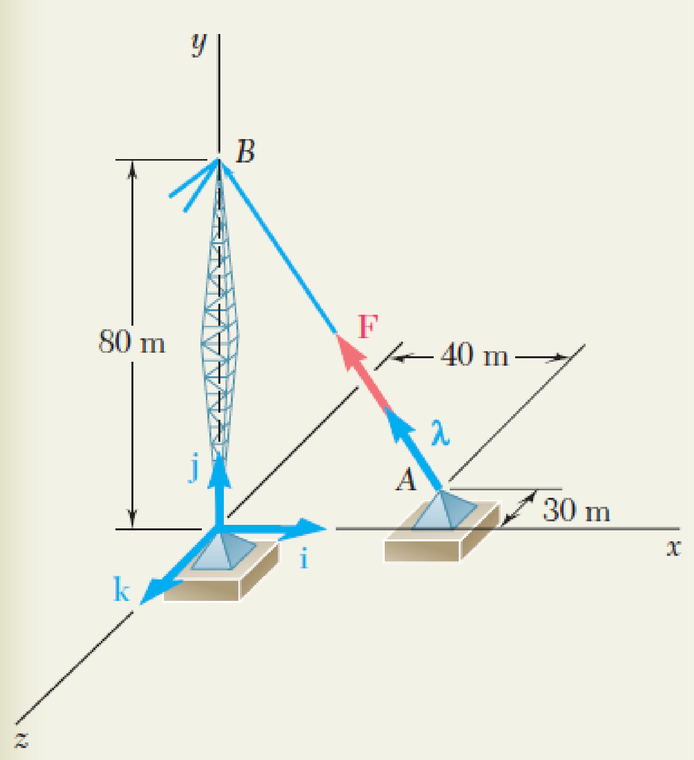
$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

Rectangle Components of a force in Space

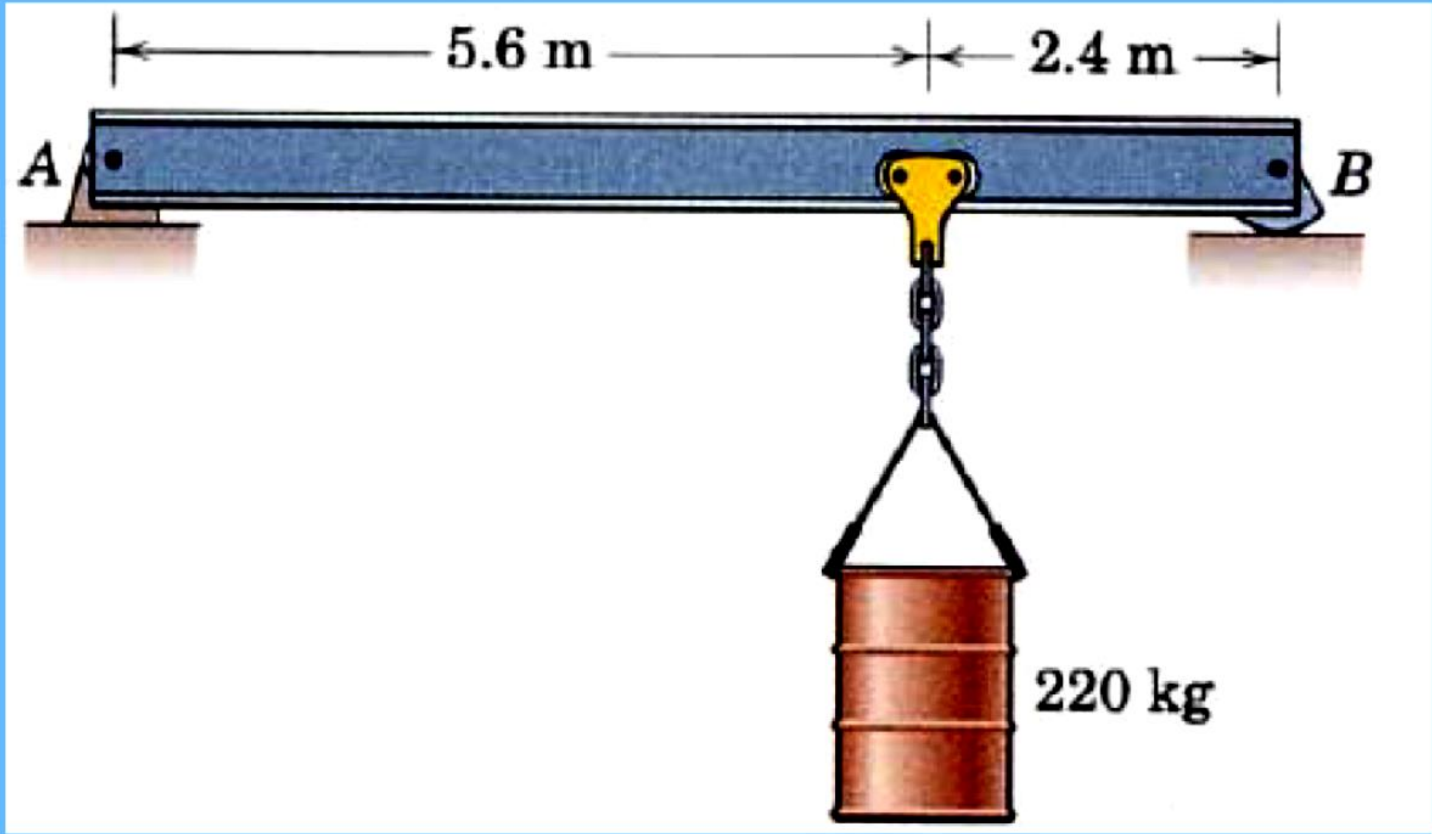
Problem : A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. determine (a) the components F_x,F_y and F_z of the force acting on the bolt, (b) the angles θ_x, θ_y, θ_z defining the direction of the force





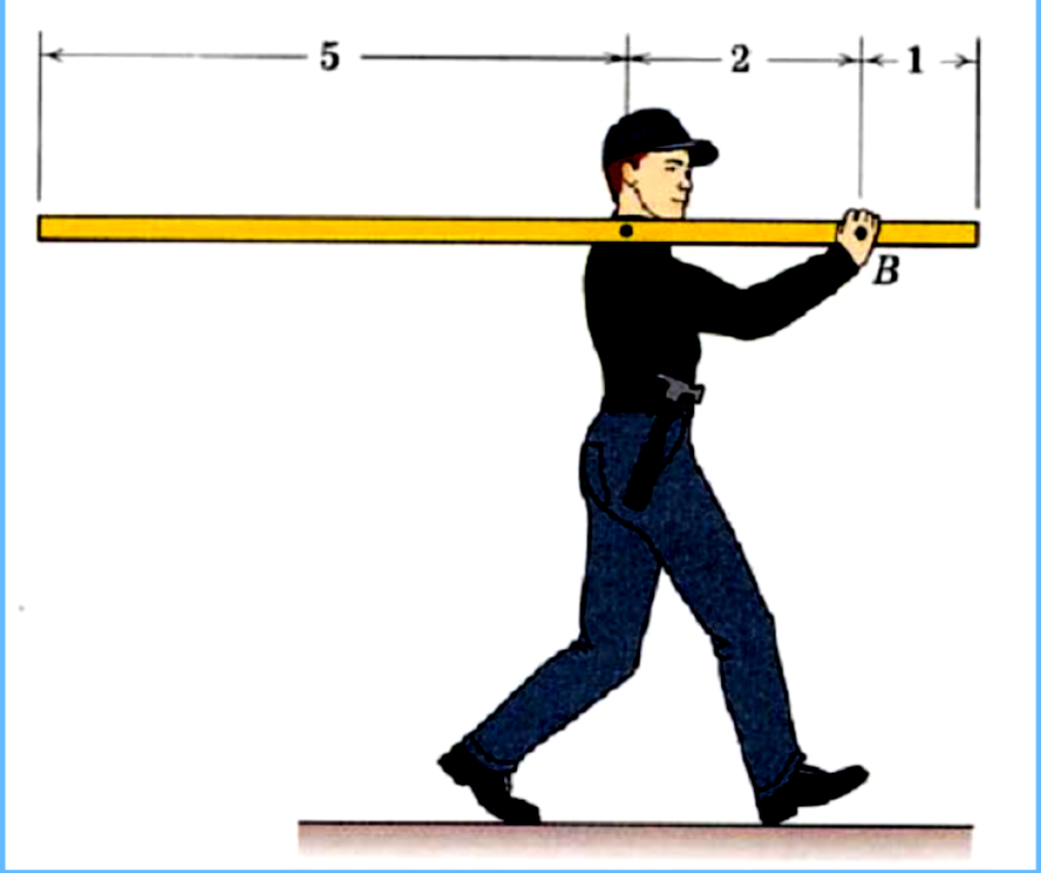
Free Body Diagram-Example 1

- The 450 kg uniform I beam supports the load shown. Draw the FBD to determine the reactions at the supports.



Free Body Diagram-Example 2

- A carpenter carries a 12-kg 50 mm by 100 mm board as shown. What downward force does he feel on his shoulder at A?



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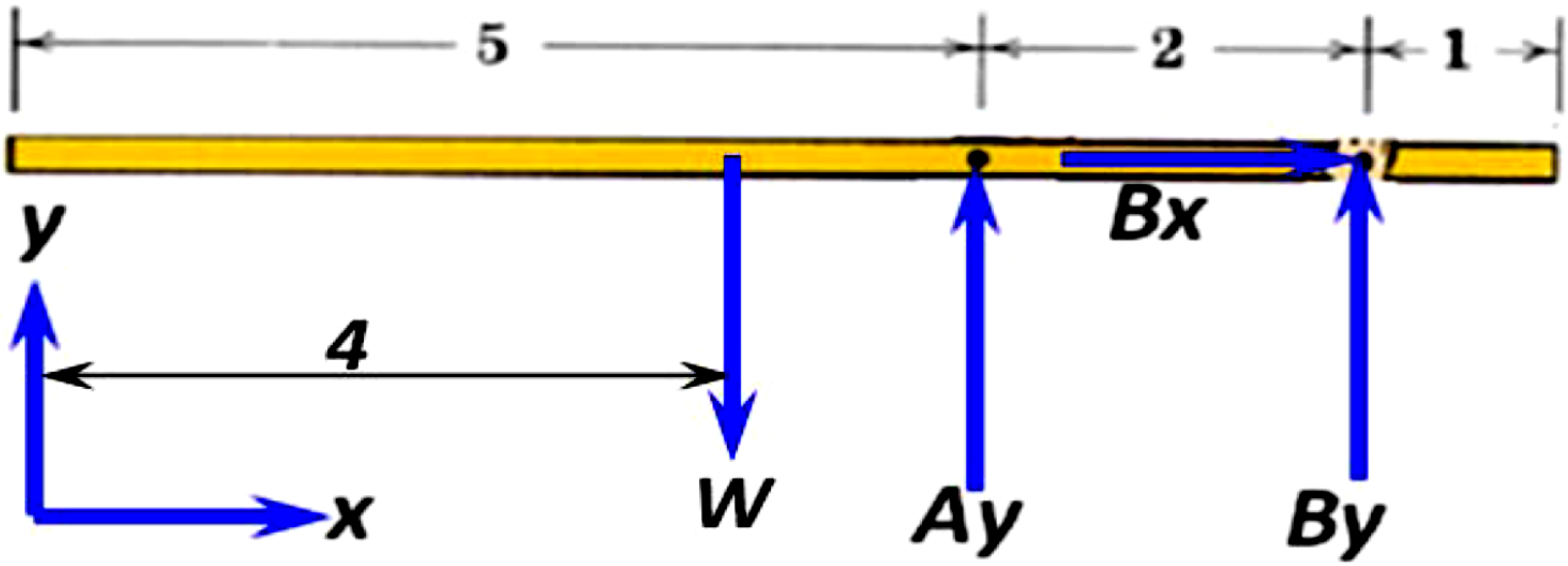
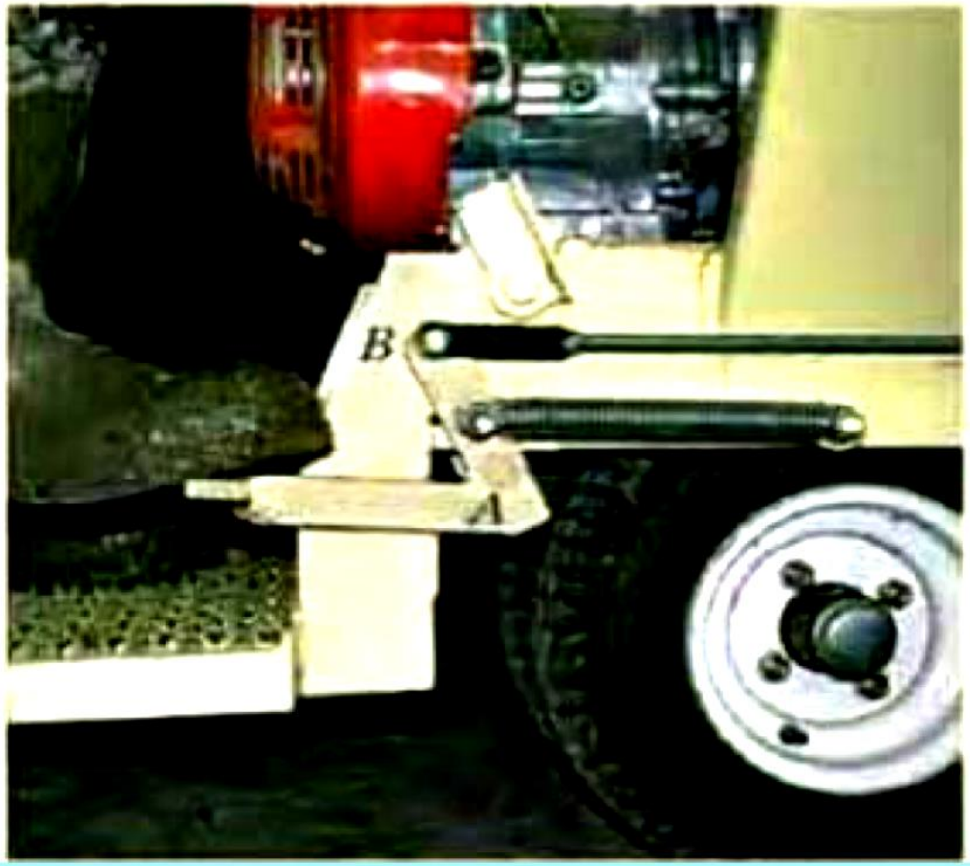
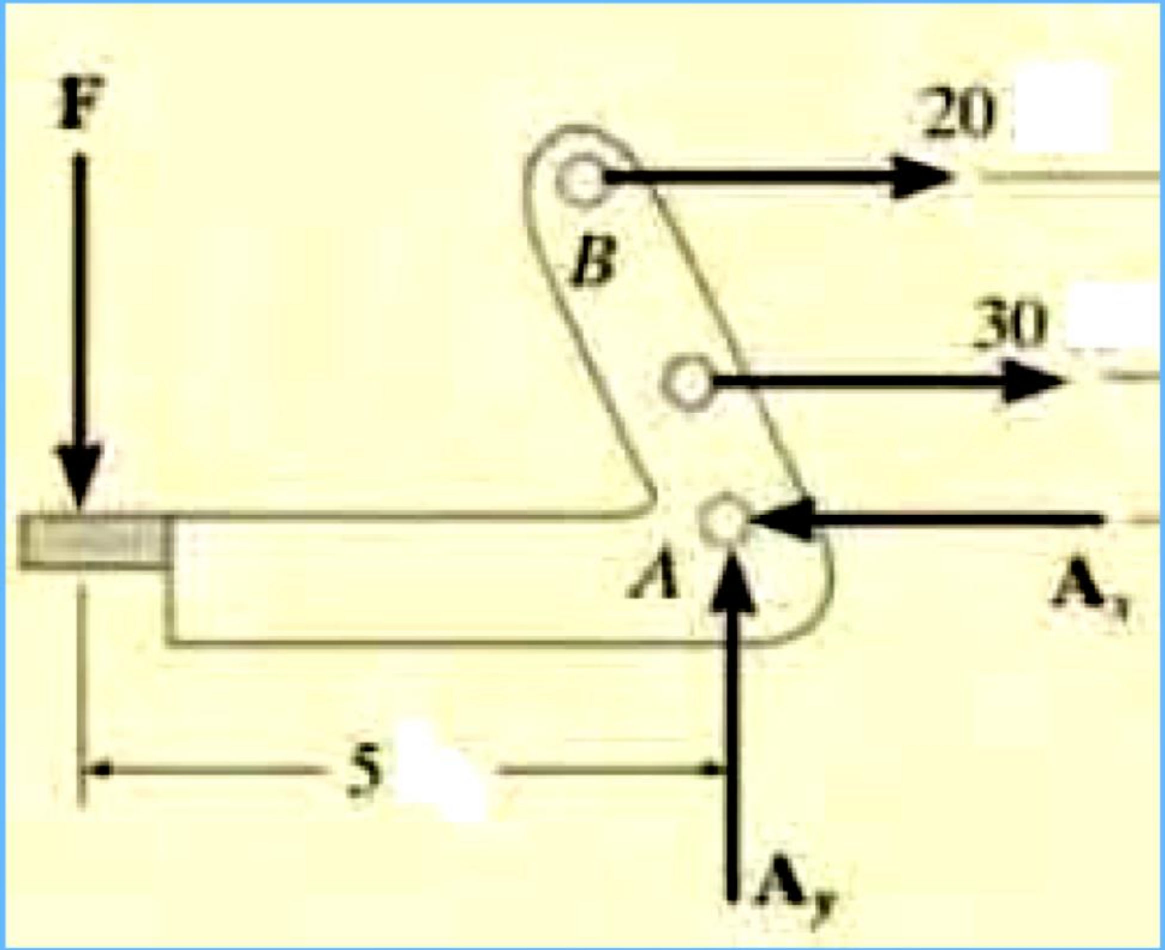


Free Body Diagram-Example 3

- Draw the FBD of the foot lever shown. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 cm and the force in the short link is 20 N.



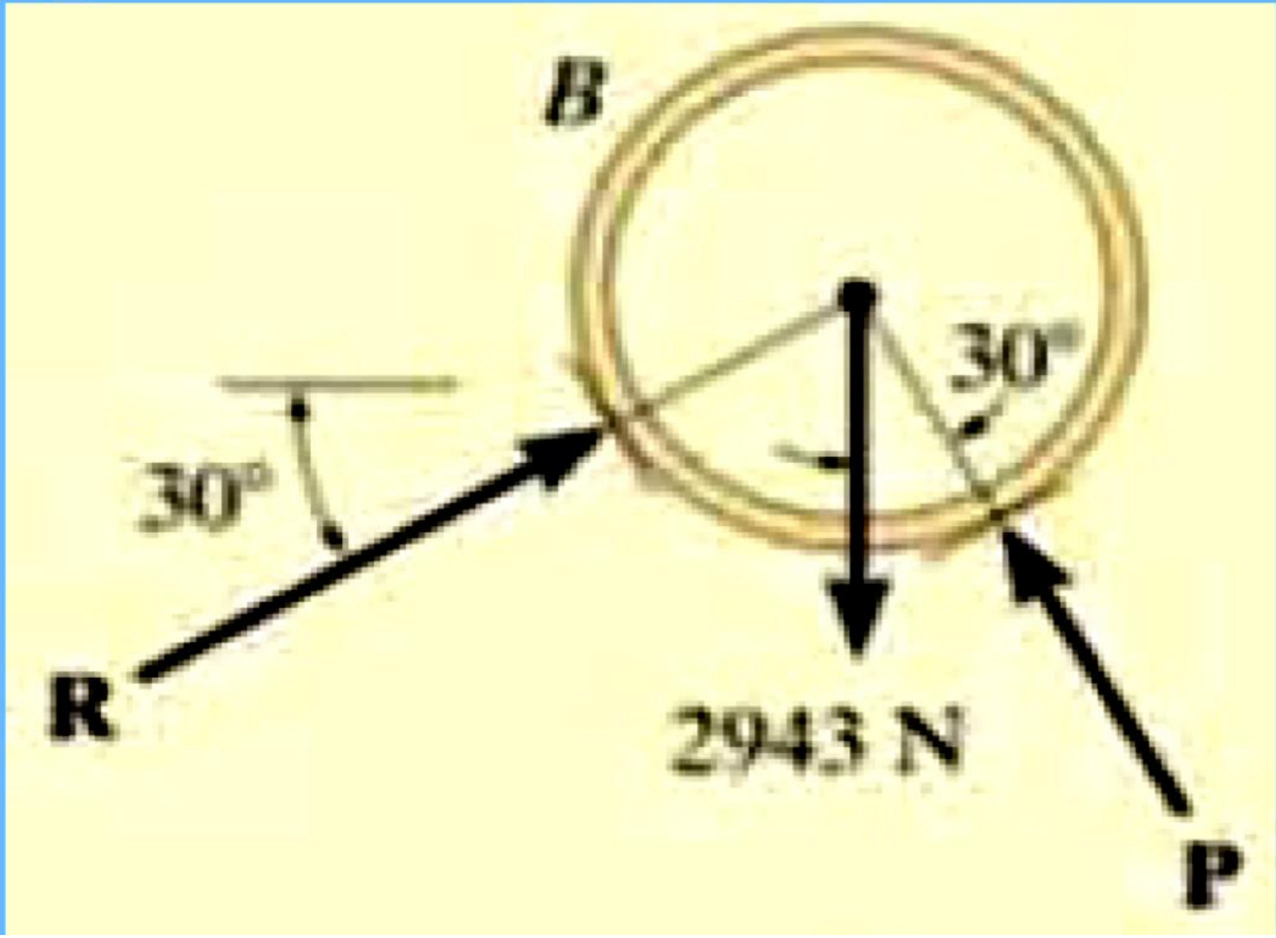
Free Body Diagram-Example 3



Free Body Diagram-Example 4

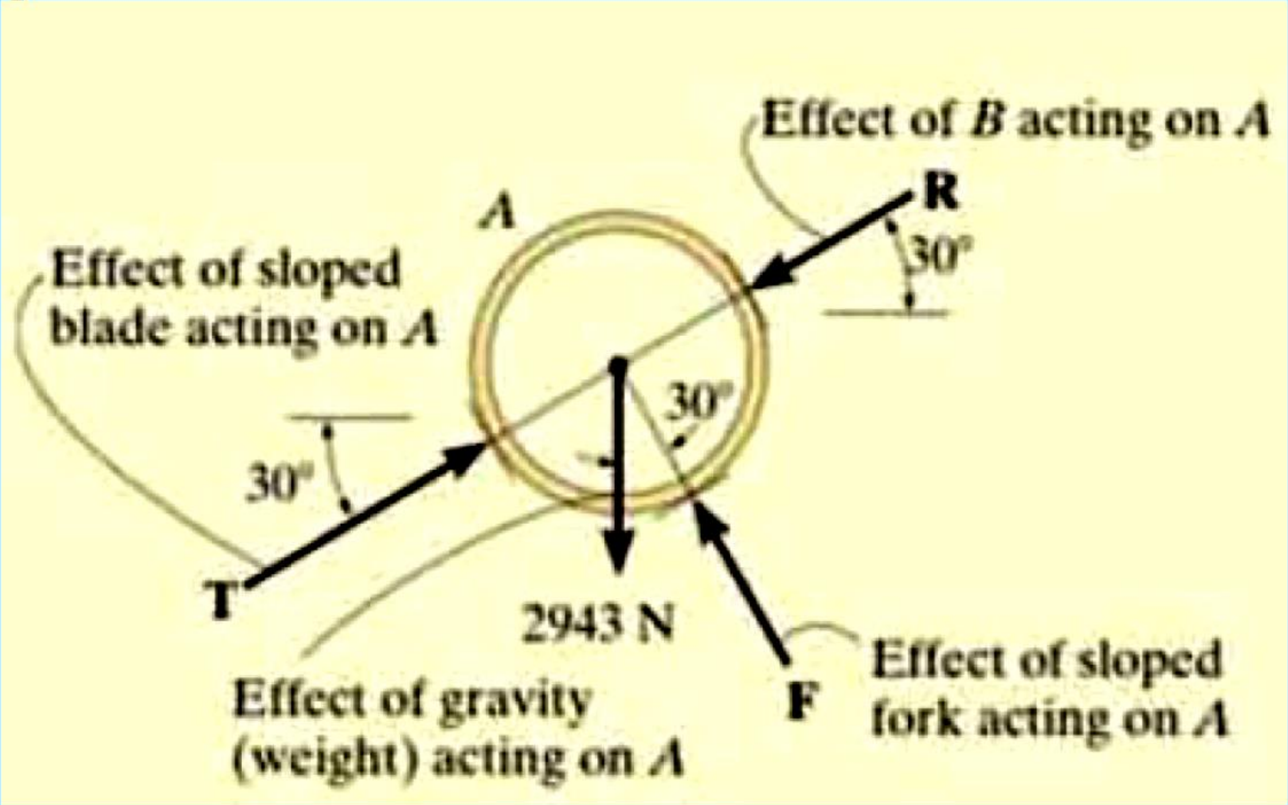
- Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor. Draw the FBDs of each pipe and both pipes together.

☼Pipe B

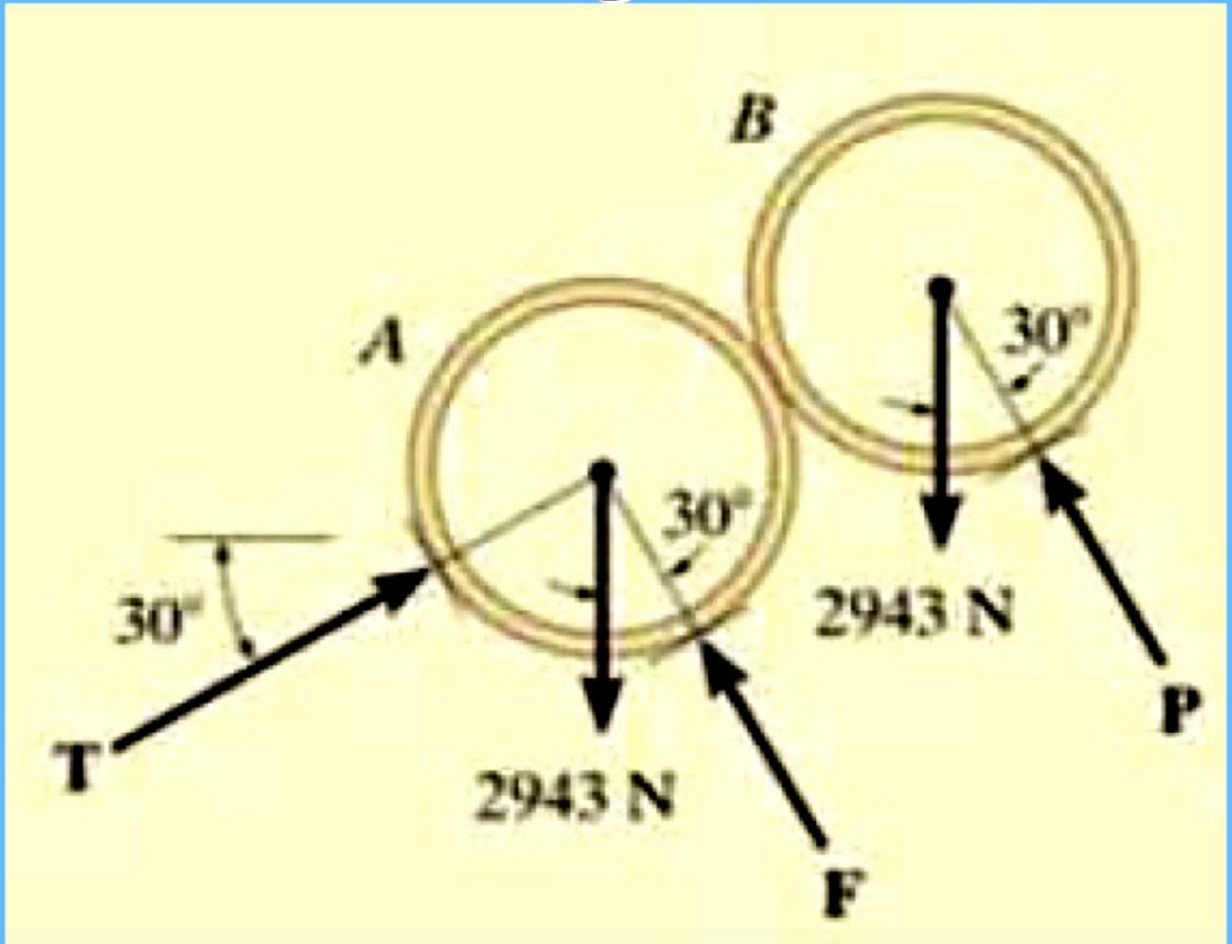


Free Body Diagram-Example 4

☼Pipe A



☼Pipes A and B together



Equilibrium of a Rigid Body- Problem

- Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Figure. Neglect the weight of the beam.

  (Bx= 424 N, Ay= 319 N, By= 405 N)