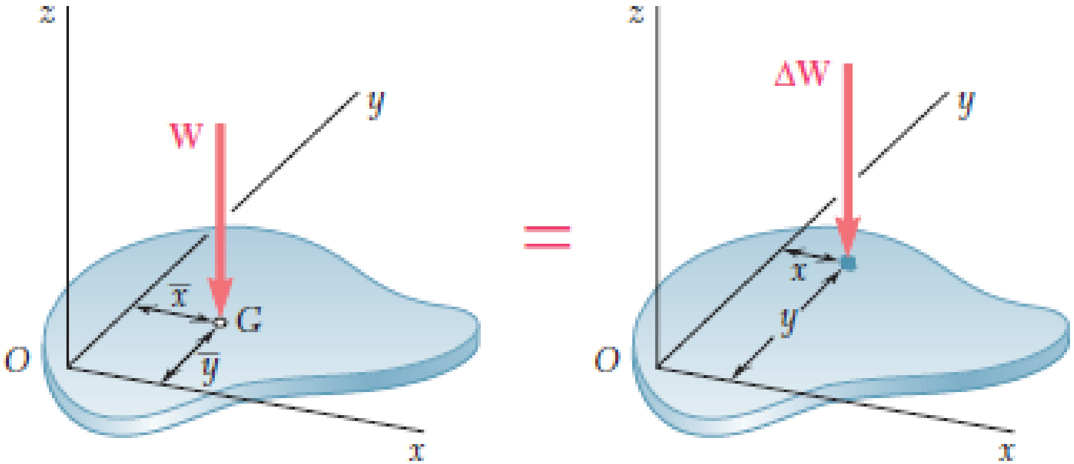
Contents

Introduction

- The earth exerts a gravitational force on each of the particles forming a body.
- These forces can be replaced by a single equivalent force equal to the weight of the body and applied at the center of gravity for the body.
- The centroid of an area is analogous to the center of gravity of a body.
- The concept of the first moment of an area is used to locate the centroid.

Center of Gravity of a 2D Body



$$\Sigma F_z: \quad W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n$$

$$\Sigma M_y: \quad \bar{x}W = x_1 \Delta W_1 + x_2 \Delta W_2 + \cdots + x_n \Delta W_n$$

$$\Sigma M_x: \quad \bar{y}W = y_1 \Delta W_1 + y_2 \Delta W_2 + \cdots + y_n \Delta W_n$$

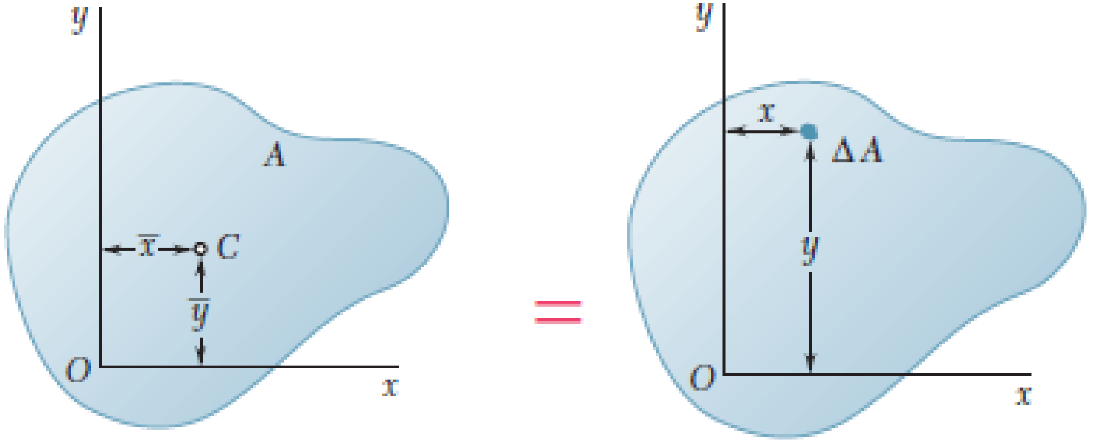
$$W = \int dW \qquad \bar{x}W = \int x\,dW \qquad \bar{y}W = \int y\,dW$$

Centroids and First Moments of Areas

$$\Delta W = \gamma t \, \Delta A$$

where γ = specific weight (weight per unit volume) of the material
t = thickness of the plate
ΔA = area of the element

$$W = \gamma t A$$



ΣM_y: $\bar{x}A = x_1 \Delta A_1 + x_2 \Delta A_2 + \cdots + x_n \Delta A_n$

ΣM_x: $\bar{y}A = y_1 \Delta A_1 + y_2 \Delta A_2 + \cdots + y_n \Delta A_n$

$$\bar{x}A = \int x \, dA$$

$$\bar{y}A = \int y \, dA$$

$\int x \, dA =$ first moment with respect to y

$\int y \, dA =$ first moment with respect to x

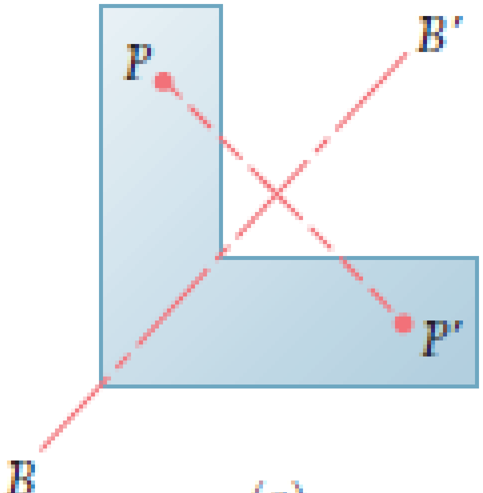
$$Q_y = \int x\, dA \qquad Q_x = \int y\, dA$$

$$Q_y = \bar{x}A \qquad Q_x = \bar{y}A$$

The co-ordinates of the centroid of an area can be obtained by

$$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A}$$

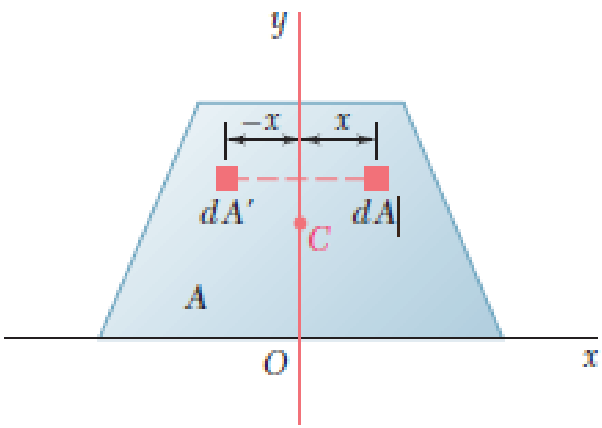
Axis-Symmetry



(a)

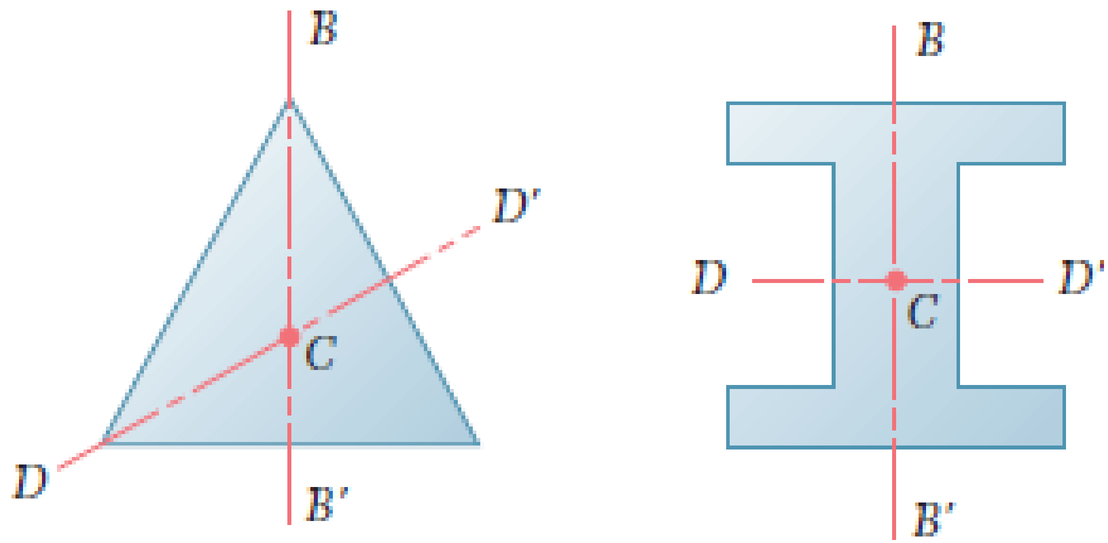
- An area is said to be *symmetric* with respect to an axis *BB′* if for every point *P* of the area there exists a point *P′* of the same area such that the line *PP′* is perpendicular to *BB′* and is divided into two equal parts by that axis.

When an area possesses an axis of symmetry *BB′*, its first moment with respect to *BB′* is zero, and its centroid is located on that axis.



(b)

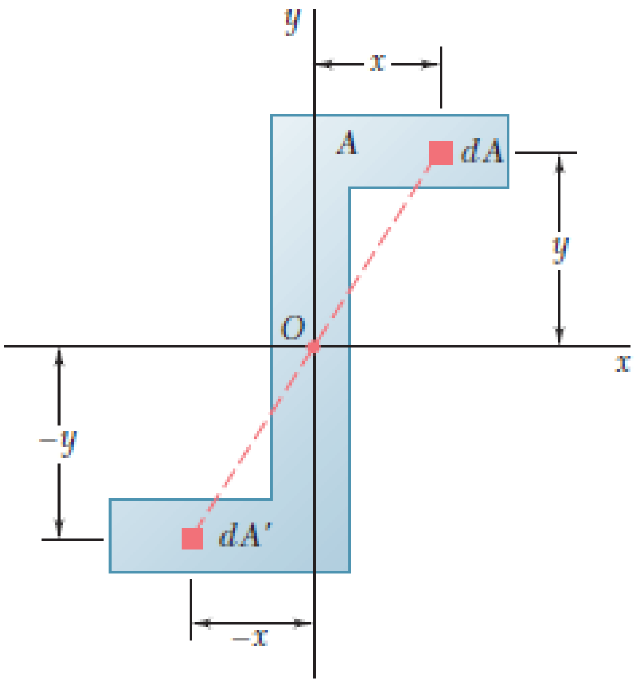
Two Axes Symmetry



(a) (b)

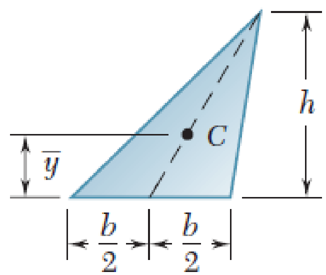
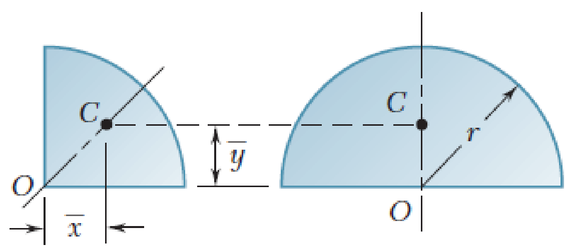
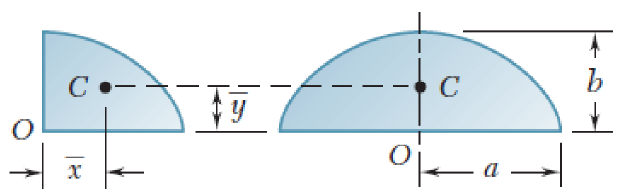
- If an area possesses two axes of symmetry, its centroid *C* must be located at the intersection of the two axes.

Centre of Symmetry

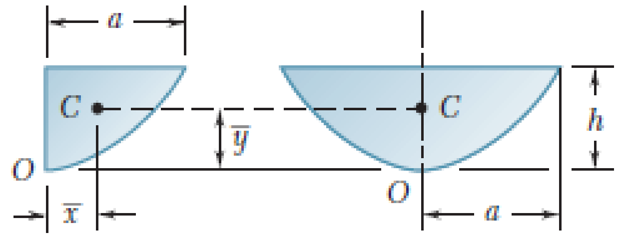
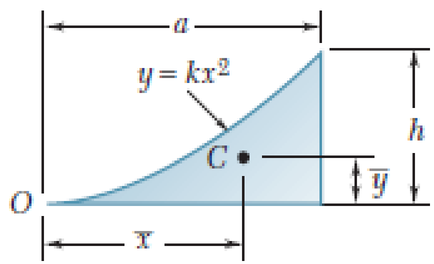
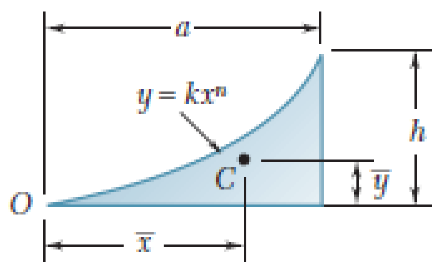
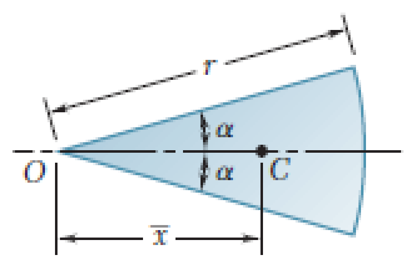


- An area *A* is said to be *symmetric with respect to a center O* if for every element of area *dA* of coordinates *x* and *y* there exists an element *dA′* of equal area with coordinates -*x* and -*y.*
- First moment of area about the axes is zero and hence $\bar{x} = \bar{y} = 0$
- A figure possessing a center of symmetry does not necessarily possess an axis of symmetry, while a figure possessing two axes of symmetry does not necessarily possess a center of symmetry.
- However, if a figure possesses two axes of symmetry at a right angle to each other, the point of intersection of these axes is a center of symmetry.

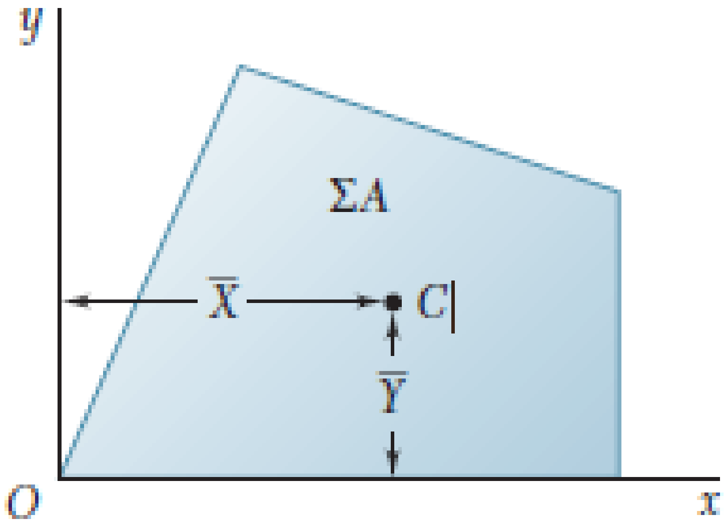
Centroids of Common Shapes of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

Centroids of Common Shapes of Areas

Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Composite Areas



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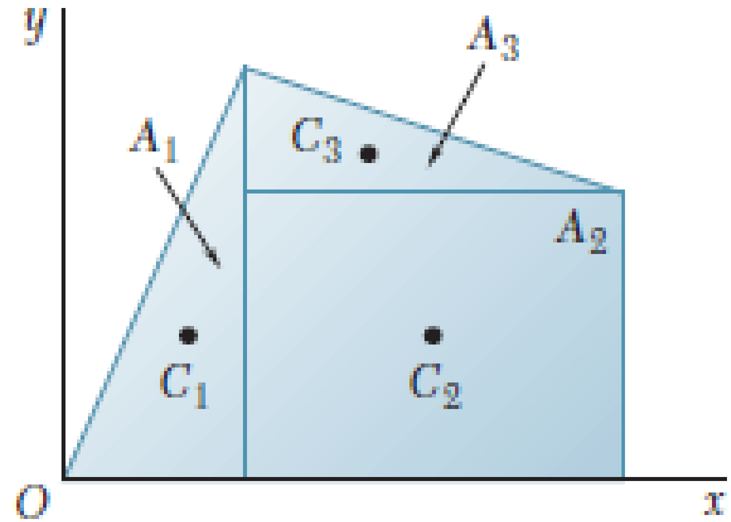


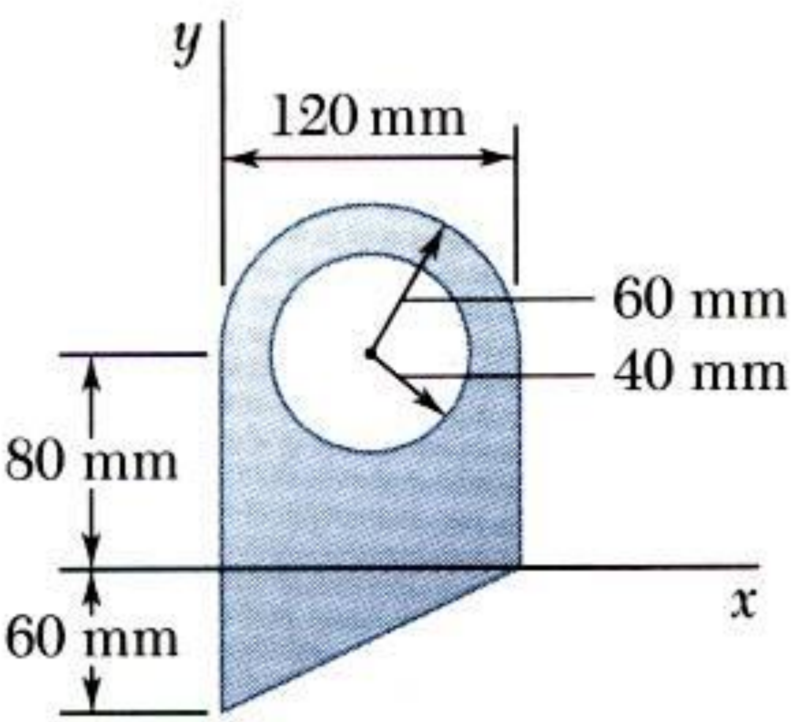
$$Q_y = \bar{X}\,\Sigma A = \Sigma \bar{x} A$$

$$Q_x = \bar{Y}\,\Sigma A = \Sigma \bar{y} A$$

$$\bar{X} = \frac{\overline{x_1}A_1 + \overline{x_2}A_2 + \overline{x_3}A_3 + \cdots\cdots}{A_1 + A_2 + A_3 + \cdots\cdots}$$

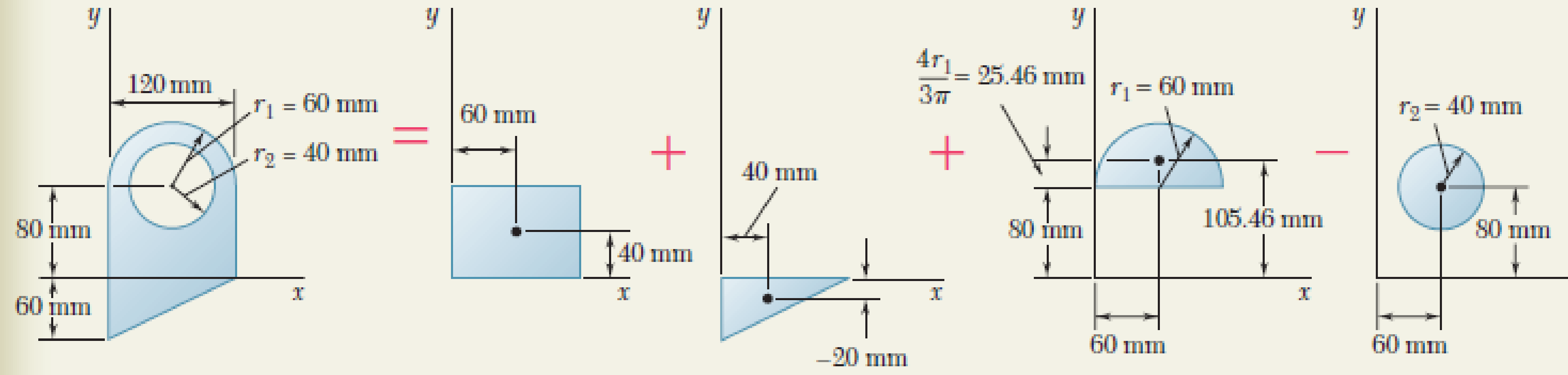
$$\bar{Y} = \frac{\overline{y_1}A_1 + \overline{y_2}A_2 + \overline{y_3}A_3 + \cdots\cdots}{A_1 + A_2 + A_3 + \cdots\cdots}$$

Problem 1



For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

Solution



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

$$\boxed{\begin{aligned} Q_x &= +506.2 \times 10^3\,\text{mm}^3 \\ Q_y &= +757.7 \times 10^3\,\text{mm}^3 \end{aligned}}$$

$$\bar{X} = 54.8 \text{ mm}$$

$$\bar{Y} = 36.6 \text{ mm}$$

Problem 2, 3, & 5

Locate the centroid of the plane area shown