3.4 EQUILIBRIUM OF A PARTICLE IN SPACE

A particle subjected to concurrent force system in space is said to be in equilibrium when the resultant force is zero. In other words,

$$\vec{R} = R_x\vec{i} + R_y\vec{j} + R_z\vec{k} = 0 \tag{3.19}$$

i.e.,

$$R_x = 0; R_y = 0 \text{ and } R_z = 0 \tag{3.20}$$

Hence the equations of equilibrium for a particle when subjected to concurrent space forces can be written as

$$\Sigma F_x = 0; \Sigma F_y = 0; \Sigma F_z = 0 \tag{3.21}$$

These three equations can be solved for three unknown quantities. All the three may be the forces in three members meeting at a joint or it may be used to determine the equilibrant of a system.

3.5 APPLICATION OF STATICS OF PARTICLE

The concept of equilibrium of a particle when subjected to space can be applied to solve engineering applications of statics of particle. These are illustrated in the numerical examples.

Example 3.8 A tripod is acted upon by forces at 'P' as shown in the Fig. 3.4. Determine the forces in the legs of tripod if the legs rest on ground at A, B and C whose coordinates with respect to O are as shown in the Fig. 3.4. The height of 'P' above the origin is 10 m.

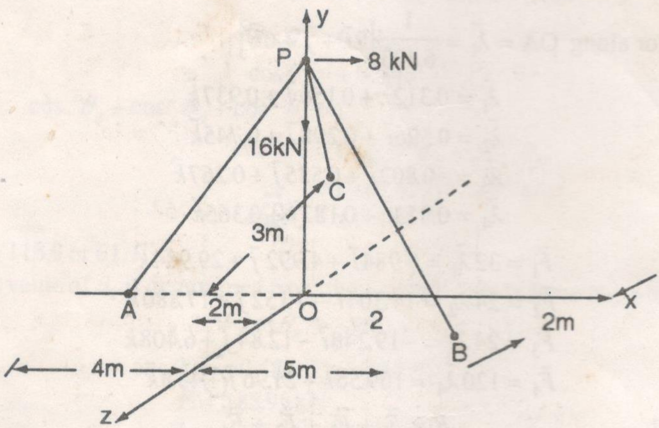

Figure 3.4

Solution Co-ordinates:

O(0, 0, 0); A(– 4, 0, 0); B(5, 0, 2); C(–2, 0, –3); P(0, 10, 0).

Table 3.1

ij	$x_j - x_i$	$y_j - y_i$	$z_j - z_i$	d	$\vec{\lambda}$
PA	-4	-10	0	10.770	$-0.371\vec{i} - 0.929\vec{j}$
PB	5	-10	2	11.358	$0.44\vec{i} - 0.88\vec{j} + 0.176\vec{k}$
PC	-2	-10	-3	10.630	$-0.188\vec{i} - 0.941\vec{j} - 0.282\vec{k}$

force vector $8\vec{i} - 16\vec{j}$.

When point 'P' is in equilibrium $\vec{R} = 0$:

$$\vec{F}_{PA} = F_{PA}(-0.371\vec{i} - 0.929\vec{j})$$
$$\vec{F}_{PB} = F_{PB}(0.44\vec{i} - 0.88\vec{j} + 0.176\vec{k})$$
$$\vec{F}_{PC} = F_{PC}(-0.188\vec{i} - 0.941\vec{j} - 0.282\vec{k})$$
$$\vec{F} = 8\vec{i} - 16\vec{j}$$

$$\vec{R} = \vec{F}_{PA} + \vec{F}_{PB} + \vec{F}_{PC} + \vec{F} = 0$$
$$(-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} + 8)\vec{i}$$
$$+(-0.929F_{PA} - 0.88F_{PB} - 0.941F_{PC} - 16)\vec{j}$$
$$+(+0.176F_{PB} - 0.282F_{PC} + 8\vec{k} = 0$$

i.e.,
$$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} + 8 = 0$$
$$-0.929F_{PA} - 0.88F_{PB} - 0.941F_{PC} - 16 = 0$$
$$+0.176F_{PB} - 0.282F_{PC} = 0$$

// Writing this in matrix form,

$$\begin{bmatrix} -0.371 & 0.44 & -0.188 \\ -0.929 & -0.88 & -0.941 \\ 0 & 0.176 & -0.282 \end{bmatrix} \begin{Bmatrix} F_{PA} \\ F_{PB} \\ F_{PC} \end{Bmatrix} = \begin{Bmatrix} -8 \\ 16 \\ 0 \end{Bmatrix}$$
$$[A]\{F\} = \{b\}$$

Co-factor matrix of (A)

$$\begin{bmatrix} 0.414 & -0.262 & -0.164 \\ 0.091 & 0.105 & 0.065 \\ -0.580 & -0.175 & 0.735 \end{bmatrix}$$

$$\|A\| = -0.371(0.414) + 0.44(-0.262) - 0.188(-0.164) = -0.238$$

$$(A)^{-1} = \frac{1}{(-0.238)} \begin{bmatrix} 0.414 & 0.091 & -0.58 \\ -0.262 & 0.105 & -0.175 \\ -0.164 & 0.065 & 0.735 \end{bmatrix}$$

$$= \begin{bmatrix} -1.740 & -0.382 & 2.437 \\ 1.101 & -0.441 & 0.735 \\ 0.689 & -0.273 & -3.088 \end{bmatrix}$$

$$\{F_i\} = [A]^{-1}\{b\}$$

$$\begin{Bmatrix} F_{PA} \\ F_{PB} \\ F_{PC} \end{Bmatrix} = \begin{bmatrix} -1.740 & -0.382 & 2.437 \\ 1.101 & -0.441 & 0.735 \\ 0.689 & -0.273 & -3.088 \end{bmatrix} \begin{Bmatrix} -8 \\ 16 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 7.808 \\ -15.864 \\ -9.88 \end{Bmatrix}$$

Positive sign for F_{PA} indicates assumed direction is correct. A negative sign for F_{PB} and F_{PC} indicates that the forces are acting towards P.

[Note: These three equations can also be solved as follows. From equation,

$0.176F_{PB} - 0.282\,F_{PC} = 0$, we find $F_{PC} = 0.624\,F_{PB}$

Hence

$$-0.371F_{PA} + 0.44\,F_{PB} - 0.188(0.624F_{PB}) + 8 = 0$$

$$ie, -0.371F_{PA} + 0.323F_{PB} = -8$$

$$\text{and} -0.929F_{PA} - 0.88F_{PB} - 0.941(0.624F_{PB}) - 16 = 0$$

$$ie., -0.929F_{PA} - 1.467\,F_{PB} = 16$$

Solving these two eqns F_{PA}, F_{PB} can be calculated and F_{PC} is also calculated from $F_{PC} = 0.624\,F_{PB}$.]

In other words, force in PA is tension and force in PB and PC are compression.

$$\vec{F}_{PA} = -2.897\vec{i} - 7.254\vec{j}$$

$$\vec{F}_{PB} = -6.980\vec{i} - 13.960\vec{j} + -2.792\vec{k}$$

$$\vec{F}_{PC} = 1.857\vec{i} + 9.297\vec{j} + 2.786\vec{k}$$

Example 3.9 In the previous example, if the tripod is subjected to a block of 3 kN weight hanging vertically from vertex 'P', find the forces in the legs.

Solution $\vec{F} = -3\vec{j}$

$$\begin{Bmatrix} F_{PA} \\ F_{PB} \\ F_{PC} \end{Bmatrix} = \begin{bmatrix} -1.74 & -0.382 & 2.437 \\ 1.101 & -0.441 & 0.735 \\ 0.689 & -0.273 & -3.088 \end{bmatrix} \begin{Bmatrix} 0 \\ +3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -1.146 \\ -1.323 \\ -0.819 \end{Bmatrix}$$

This shows that all the legs are in compression and all the forces act towards joint P.

(a) Free body diagram

(b) Free body diagram

Figure 3.5

...lied force 'P' at 'D', the