MEE2002 - Strength of Materials

Introduction

Text Books:-

1. Ferdinand Beer, Russell Johnston, John T DeWolf (2009), Mechanics of Materials, Tata McGraw-Hill Education.

Reference Books

1. Rowland Richards (2000), Principles of Solid Mechanics, CRC Press.
2. Timoshenko, S.P. and Young, D.H. (2000), Strength of Materials, East West Press Ltd.
3. W. A. Nash and M. C. Potter (2011), Strength of Materials, Fifth Edition, Schaum's Outline Series, McGraw-Hill.
4. R.K. Bansal (2010), Strength of Materials, Laxmi Publications.

MEE 2002 Strength of Materials

Course Objective

- To study about stresses, strains and deformation of various simple mechanical components under load
- To study about theories of failure and the criteria for failure
- To experimentally determine the mechanical properties of materials

Course Outcome

1. Compute Stress, Strain and Deformation in Axially loaded members
2. Analyse the effect of axial and shear stresses acting in various directions on different planes
3. Draw the shear force and bending moment diagrams for various beams and compute bending stress, and shear stress at various points in beams
4. Compute slope and deflection at various points of a beam
5. Analyse stresses and deformation induced in circular shafts due to torsion
6. Analyse stresses and deformation of columns and thin shells
7. Experimentally determine various mechanical properties of materials

Engineering Mechanics

- **Rigid Body Mechanics**
  - **Statics**
  - **Dynamics**
- **Deformable Body Mechanics**
  - **Strength of Materials**
- **Fluid Mechanics**

- **Strength of materials**, also called **Mechanics of materials**, is a subject which deals with the behavior of solid objects subject to stresses and strains.
- The study of strength of materials often refers to various methods of calculating the stresses and strains in structural members, such as beams, columns, and shafts.

Rigid Body:

- A rigid body is defined as a body on which the distance between two points never changes whatever be the force applied on it.
- Practically, there is no rigid body.

Deformable body:

A deformable body is defined as a body on which the distance between two points changes under action of some forces when applied on it.

The study of the property of this body is called **Elasticity**

Elasticity

- The property of a body by virtue of which it tends to regain its original shape and size when deforming force is removed .
- All solids show the property of elasticity.

Stress

This is a measure of the internal resistance in a material to an externally applied load. For direct compressive or tensile loading the stress is designated σ and is defined as:

$$stress \quad \sigma = \frac{load \;\; W}{area \;\; A}$$

Types of Stress

The stress developed in a body depends upon how the external forces are applied over it. On this basis, there are two types of stress ,

i. **Normal Stress**

ii. **Tangential Stress**

Normal Stress

- Is a stress that occurs when the surface of the body is loaded by an axial force.

$$\sigma = \frac{P}{A}$$

σ – Normal Stress

P – Axial Force

A – Cross Sectional Area

- Normal stress is of two types;

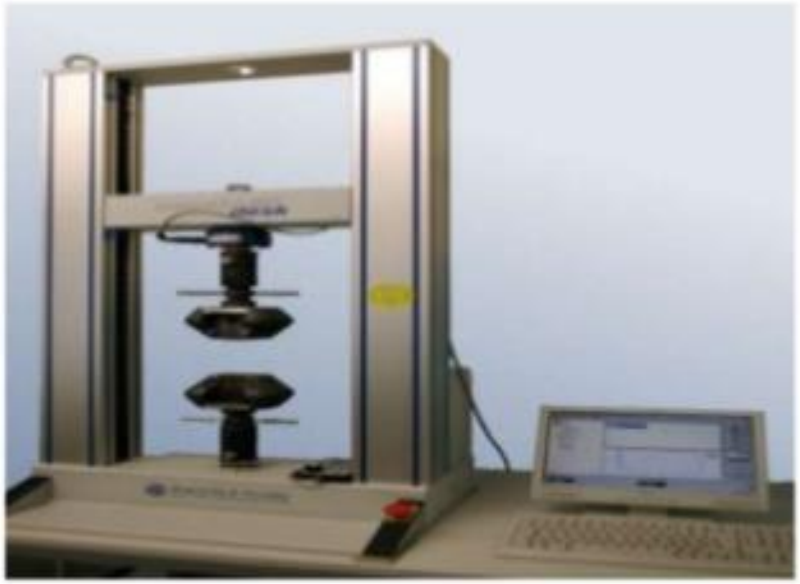
  i. **Tensile stress**

  ii. **Compressive stress**

Tensile stress:

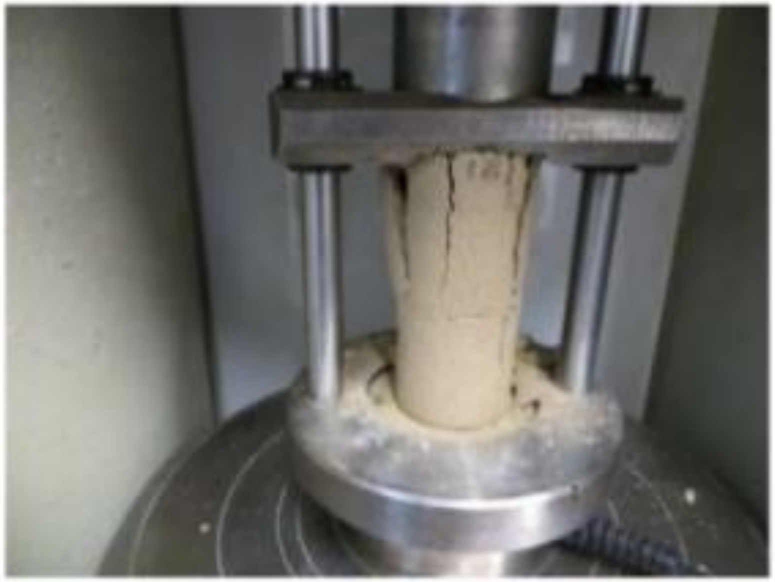
- Is the stress state leading to expansion; that is, the length of a material tends to increase in the tensile direction.

This is an example of tensile stress tester (Universal Testing Machine)



Compressive stress:

- A force that attempts to squeeze or compress a material.



- Here, the Universal Testing Machine (UTM) is testing a concrete block.

Ductile behavior:

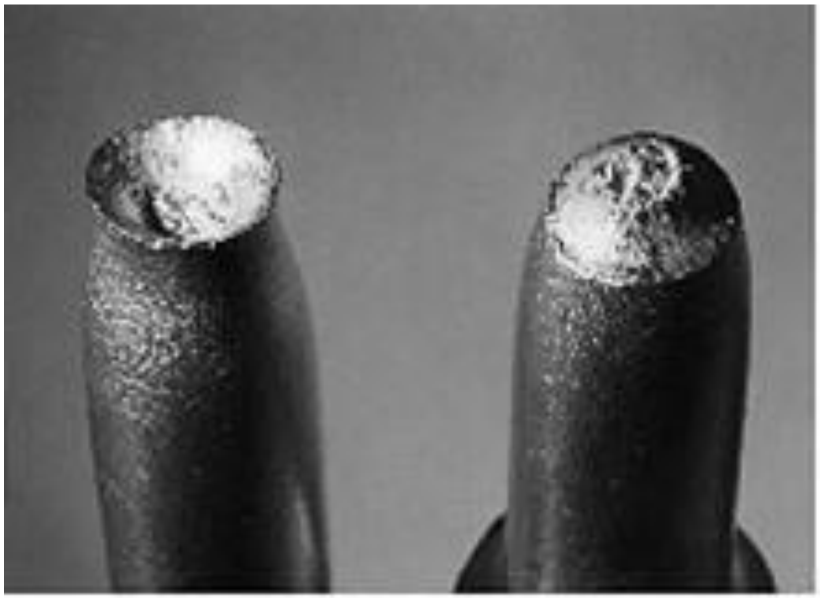
- Ductility is a solid material's ability to deform under tensile stress.

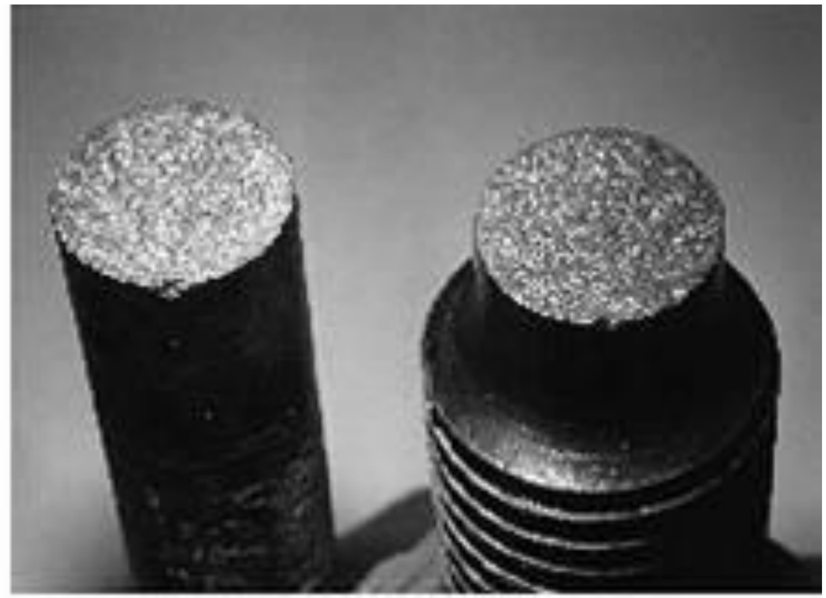
Copper wires

Brittle behavior:

- A material is brittle if, when subjected to stress, it breaks without insignificant deformation.
- Glass is a good example.

Ductile Brittle





Shear Stress

Similarly in shear the shear stress τ is a measure of the internal resistance of a material to an externally applied shear load. The shear stress is defined as:

$$\text{shear stress} \qquad \tau = \frac{\text{load } W}{\text{area resisting shear } A}$$

Ultimate Strength

The **strength** of a material is a measure of the stress that it can take when in use. The **ultimate strength** is the measured stress at failure but this is not normally used for design because safety factors are required. The normal way to define a safety factor is :

$$safety\ factor = \frac{stress\ at\ failure}{stress\ when\ loaded} = \frac{Ultimate\ stress}{Permissible\ stress}$$

Strain

We must also define **strain**. In engineering this is <u>not</u> a measure of force but is a measure of the deformation produced by the influence of stress. For tensile and compressive loads:

$$strain \quad \varepsilon = \frac{increase\ in\ length\ \ x}{original\ length\ \ L}$$

Strain is dimensionless, i.e. it is not measured in metres, killogrammes etc.

$$shear\ strain \quad \gamma \approx \frac{shear\ displacement\ \ x}{width\ \ L}$$

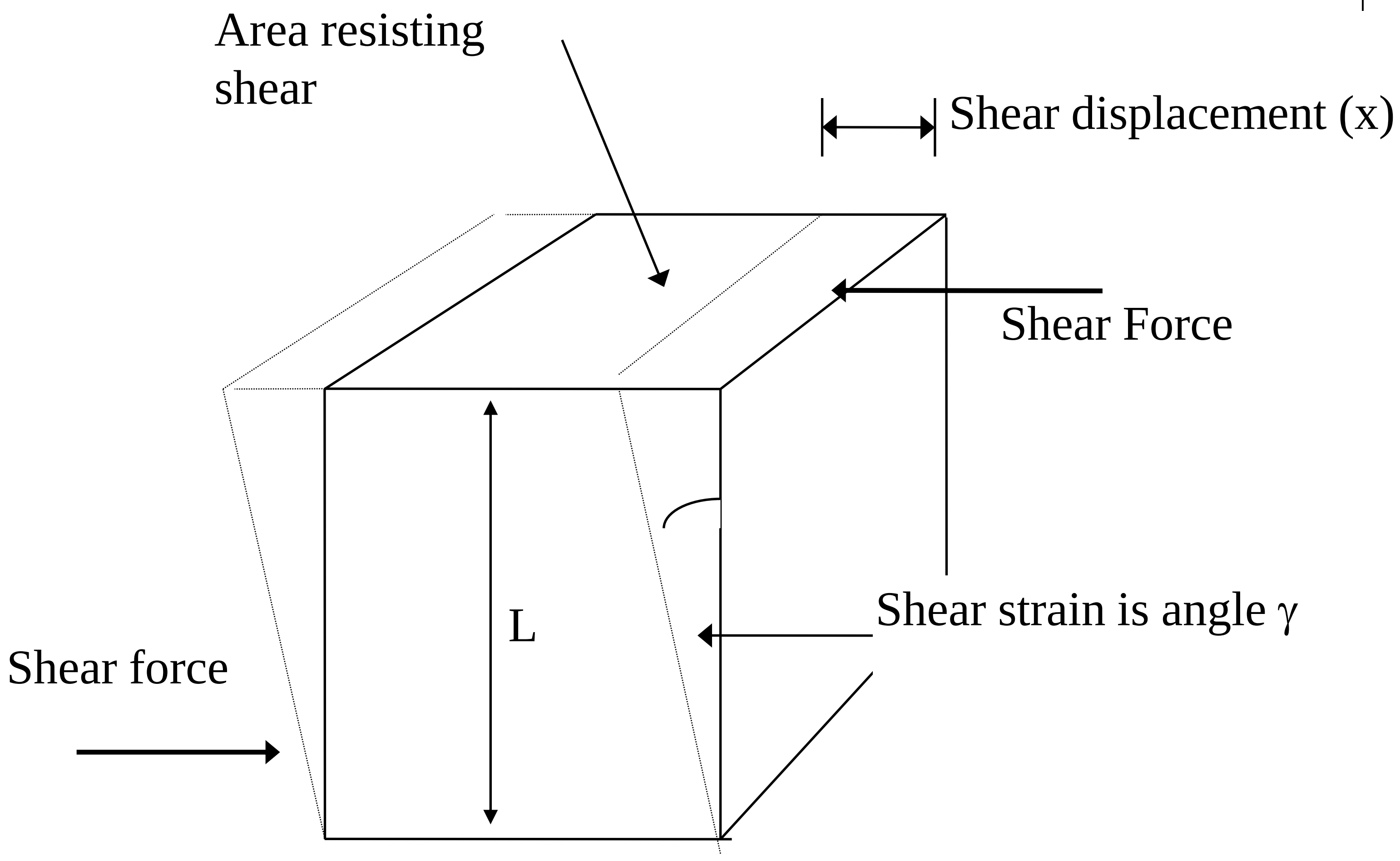
For shear loads the strain is defined as the angle γ This is measured in radians

Normal stress (σ) and strain (ε)

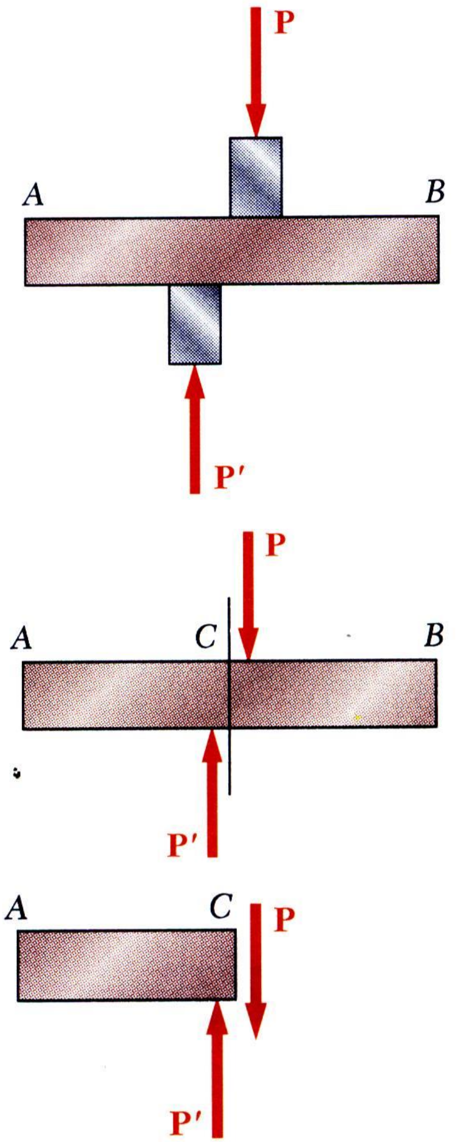
$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

Shear stress and strain



Shearing Stress



- Forces *P* and *P'* are applied **transversely** to the member *AB.*
- Corresponding **internal forces** act in the plane of section *C* and are called ***shearing*** forces.
- The resultant of the internal shear force distribution is defined as the ***shear*** of the section and is equal to the load *P*.
- The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.

- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.

- Determination of the stress distribution within a member also requires consideration of deformations in the member.

- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

Normal Strain

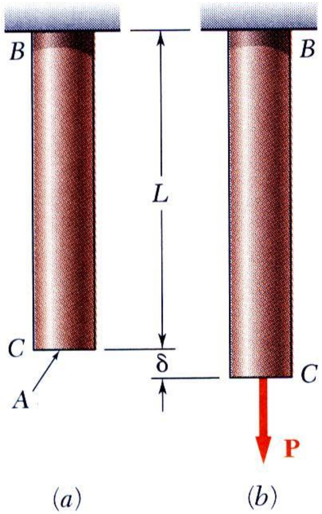


Fig. 2.1

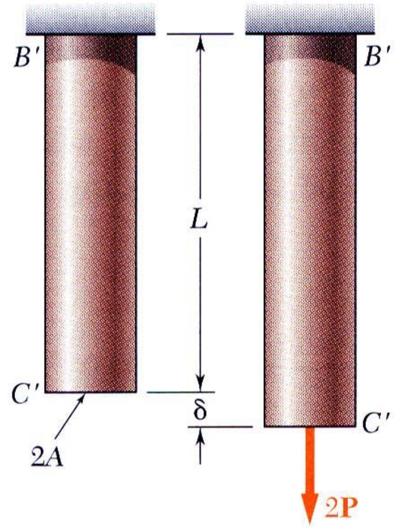


Fig. 2.3

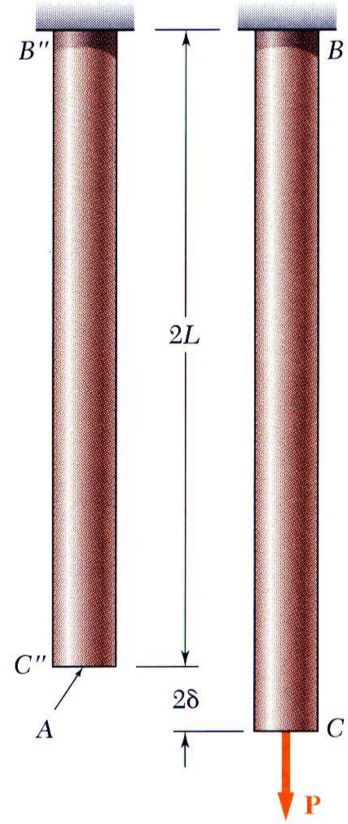


Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test

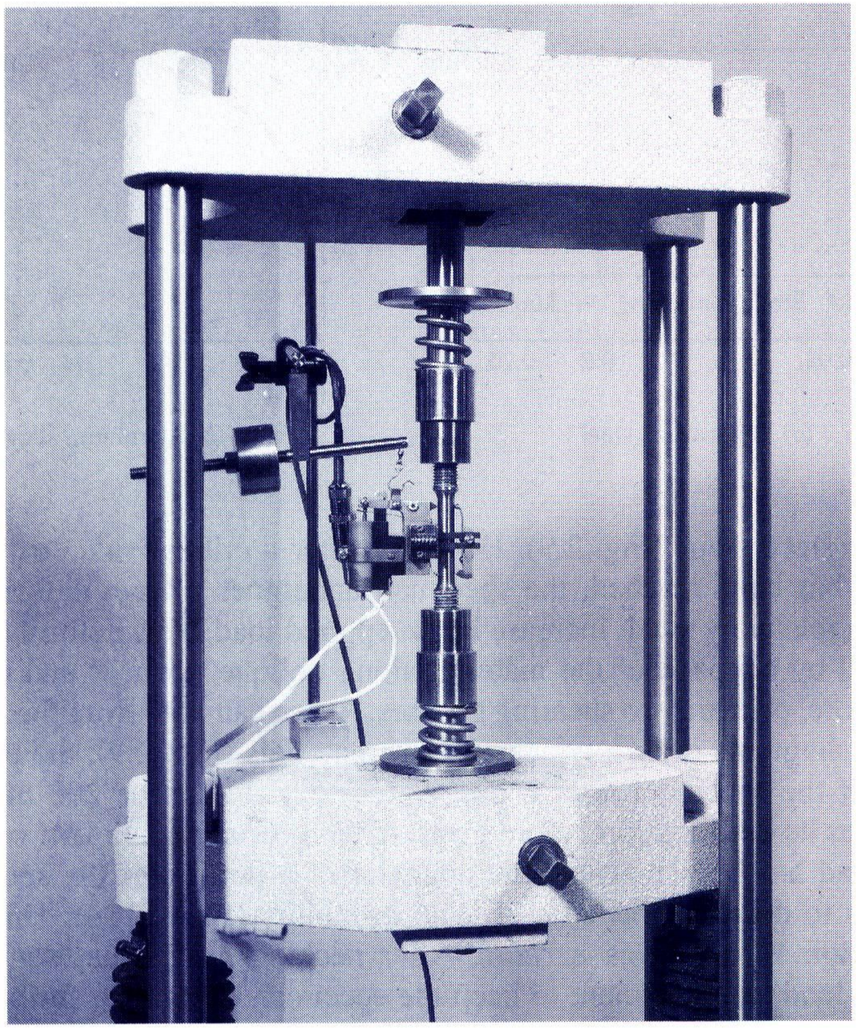


Fig. 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

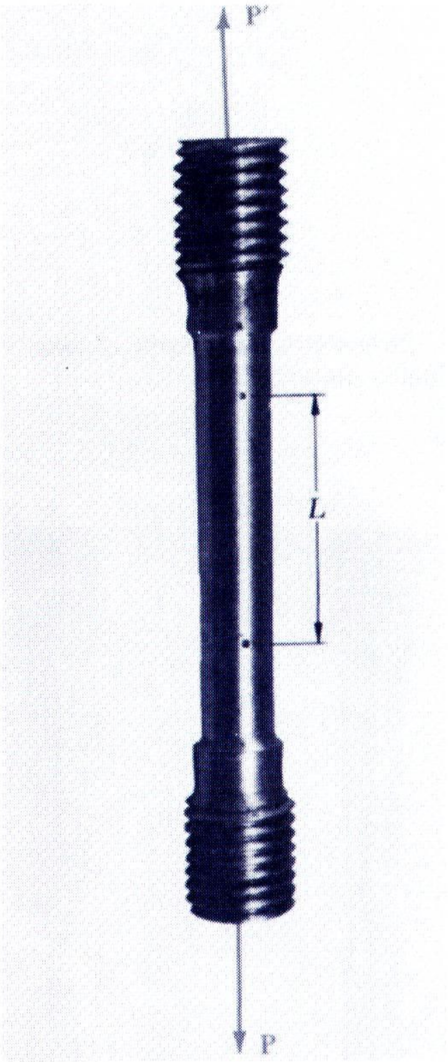
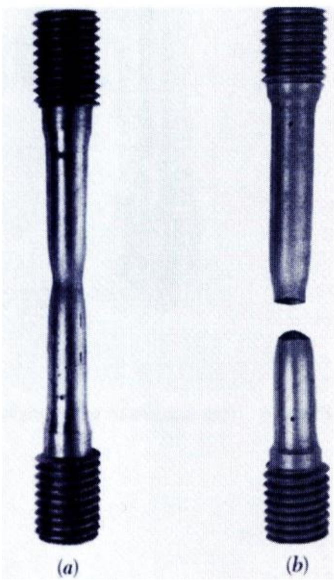
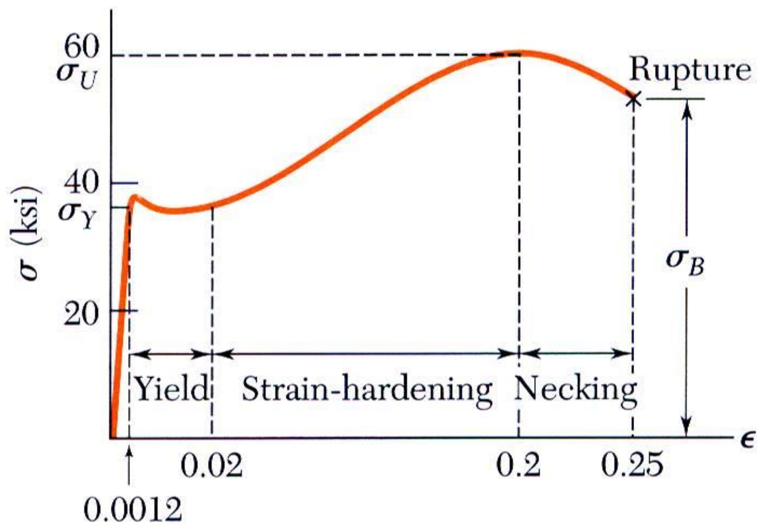


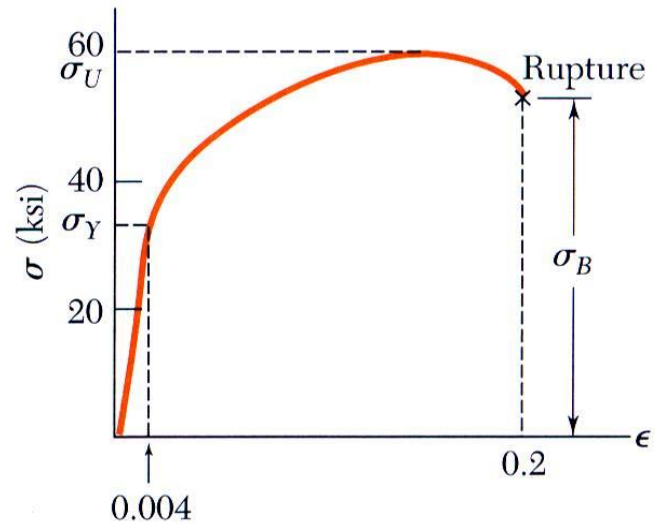
Fig. 2.8 Test specimen with tensile load.

Stress-Strain Diagram: Ductile Materials



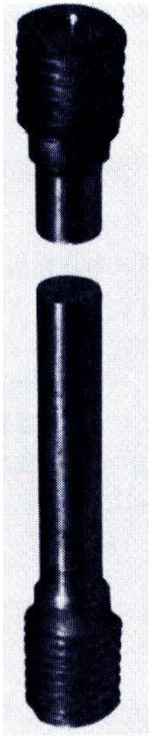


(a) Low-carbon steel



(b) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



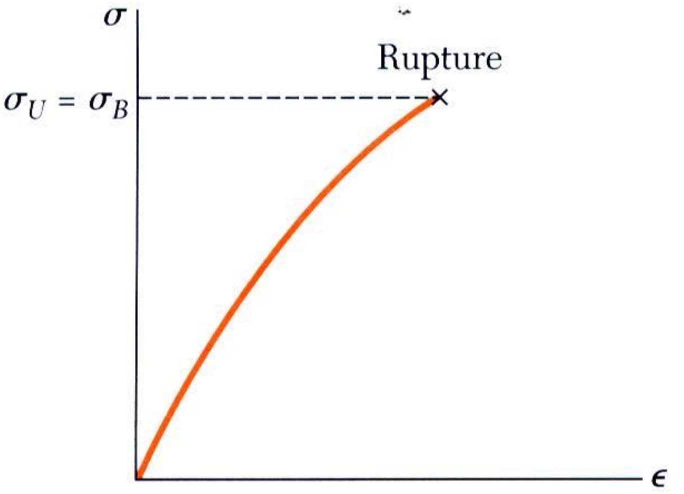


Fig. 2.11 Stress-strain diagram for a typical brittle material.

Elastic vs. Plastic Behavior

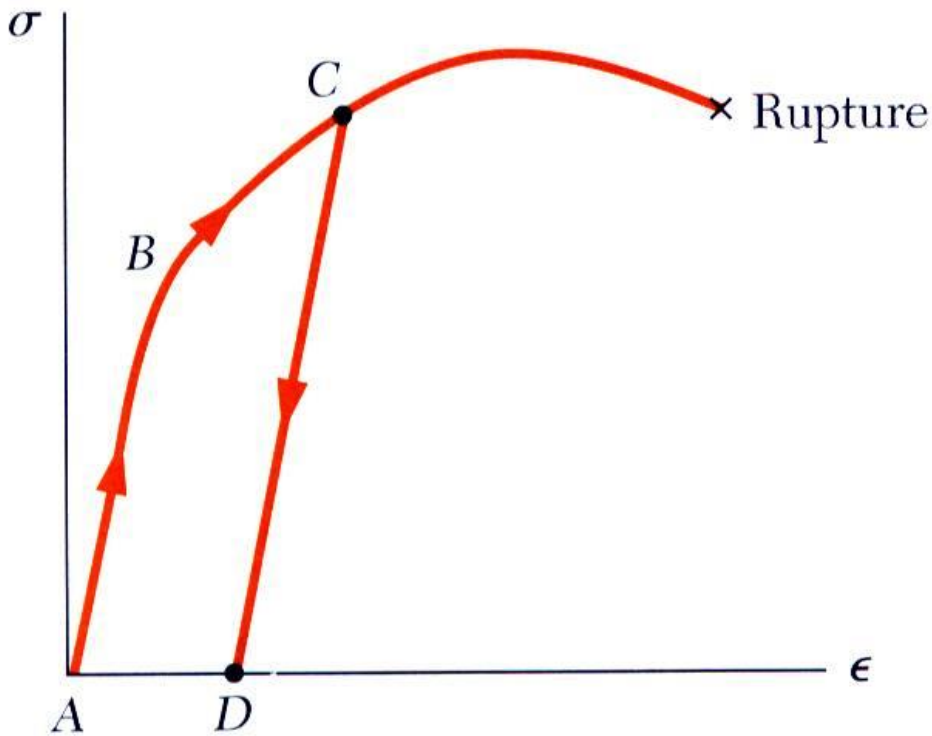
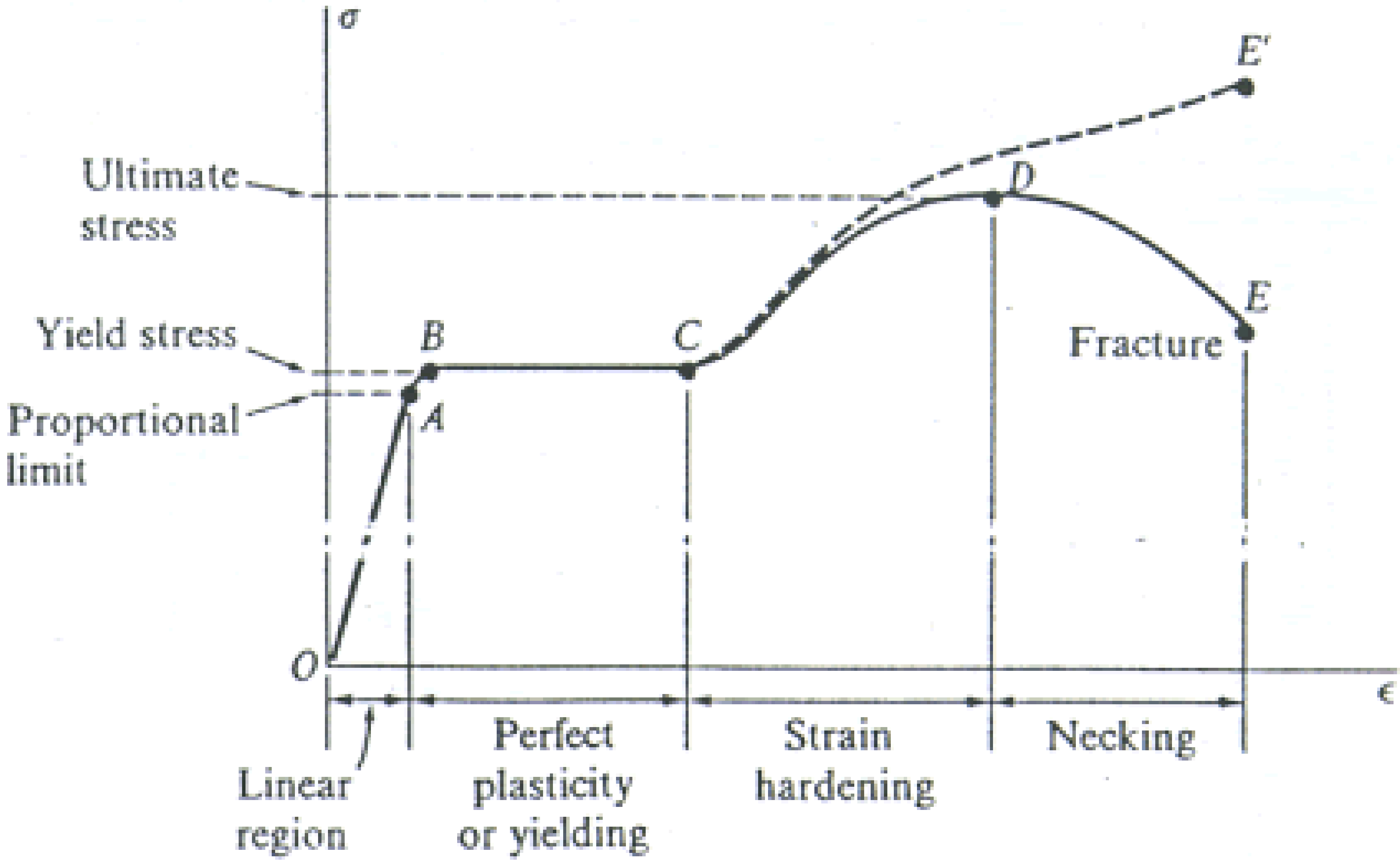


Fig. 2.18

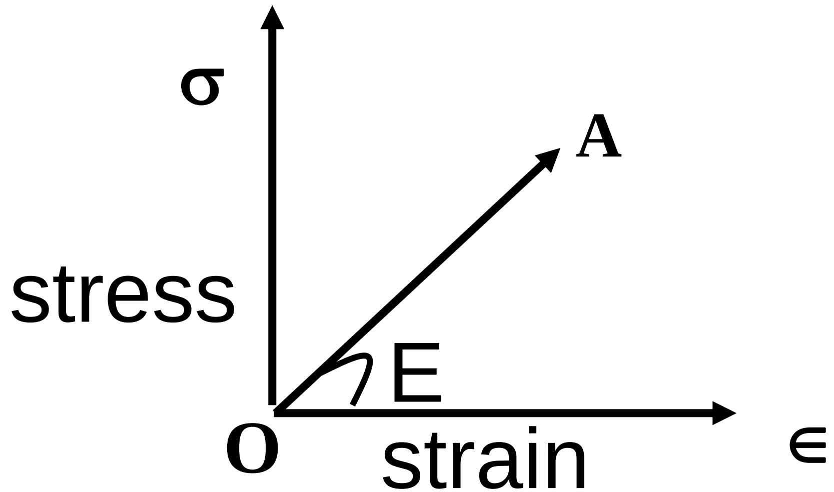
- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

STRESS STRAIN DIAGRAM

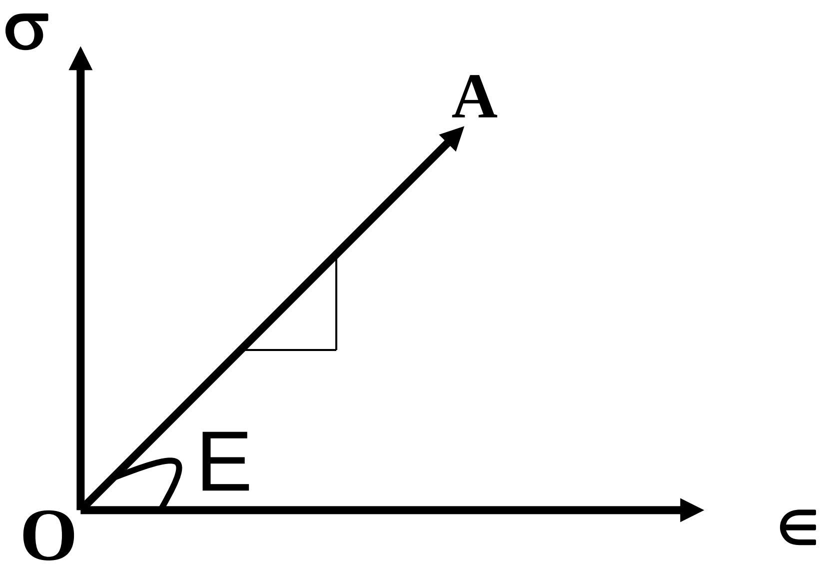


Modulus of Elasticity: $\sigma = E \epsilon$

- Stress required to produce a strain of unity.
- i.e. the stress under which the bar would be stretched to twice its original length. If the material remains elastic throughout, such excessive strain.
- Represents slope of stress-strain line OA.



Value of E is same in Tension & Compression.



- Hooke's Law:-

  Up to elastic limit, Stress is proportional to strain

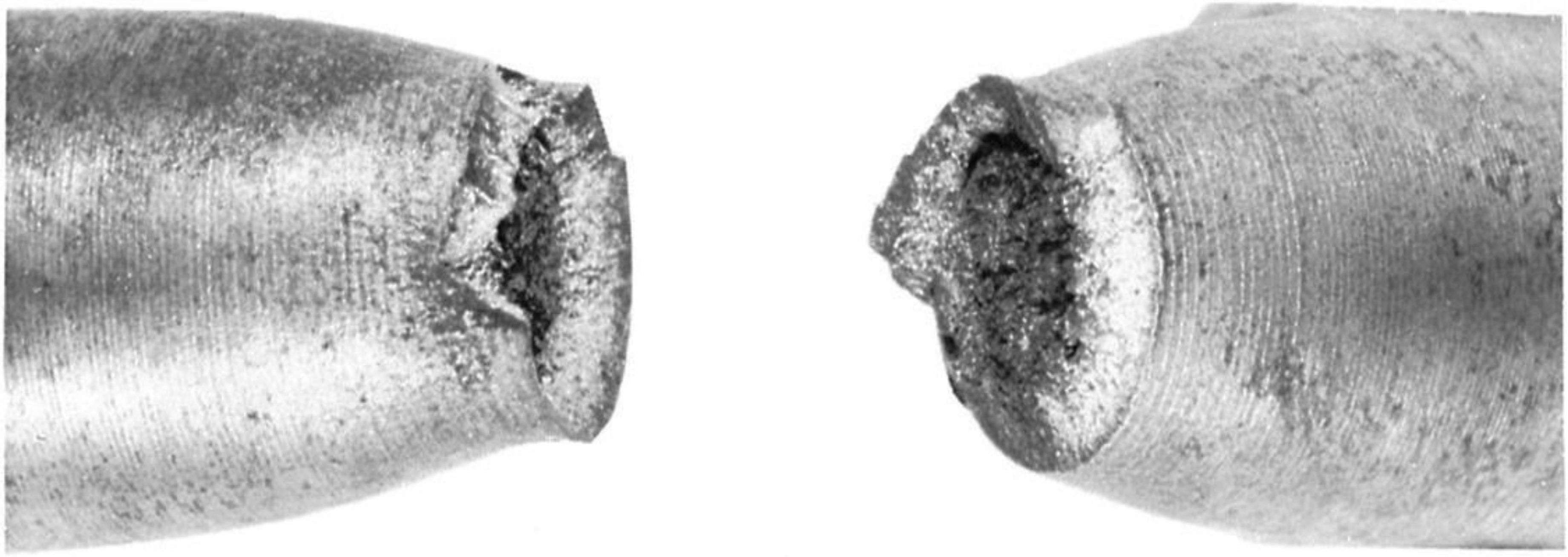
  $\sigma \propto \in$

  $\sigma = E \in$; where E=Young's modulus

  $\sigma = P/A$ and $\in = \delta / L$

  $P/A = E (\delta / L)$

$$\boxed{\delta = PL / AE}$$



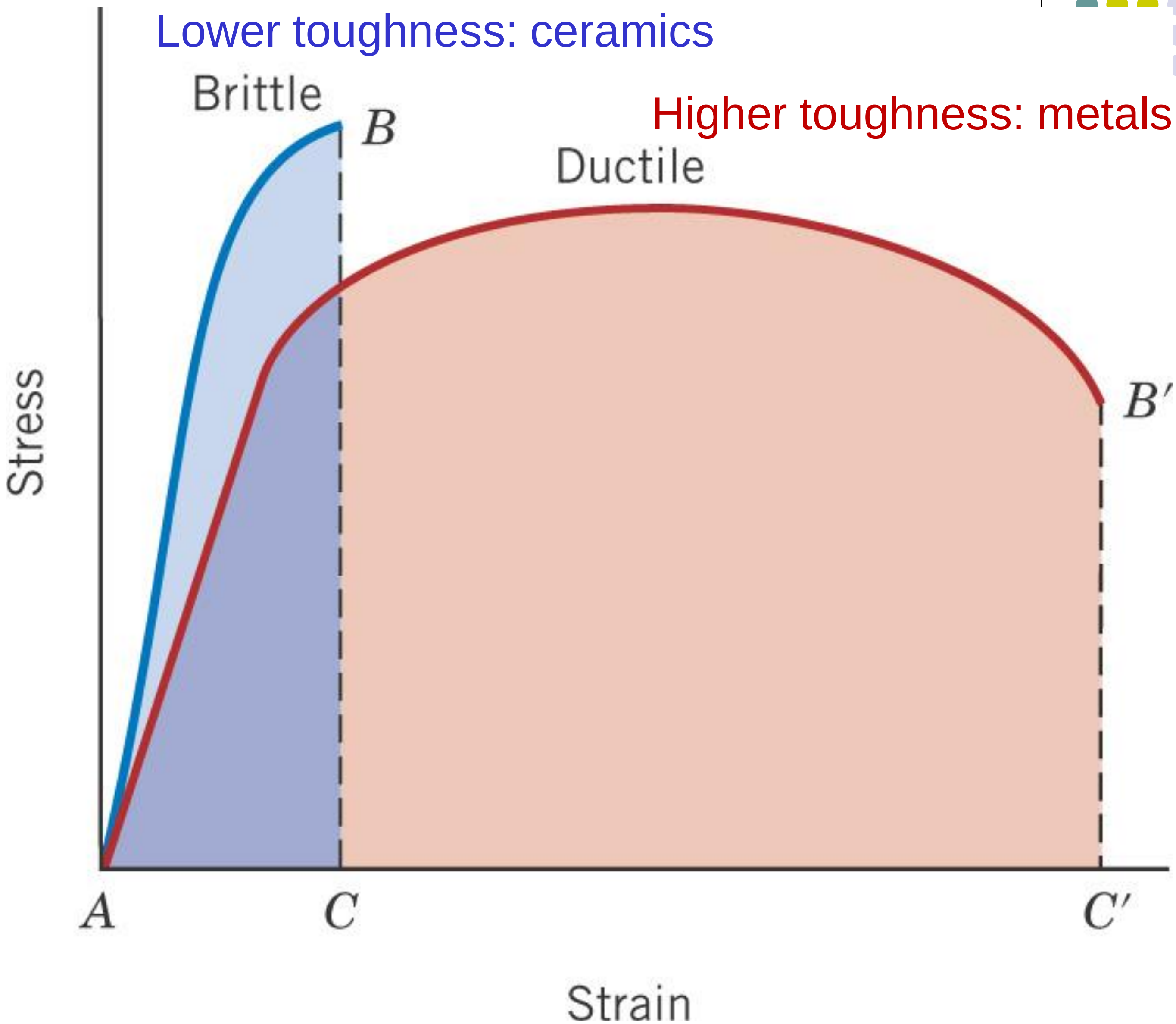
• Localized deformation of a ductile material during a tensile test produces a necked region.
• The image shows necked region in a fractured sample

Toughness

Toughness is the ability to absorb energy up to fracture (energy per unit volume of material).

A "tough" material has strength and ductility.

Approximated by the area under the stress-strain curve.



Hardness of Materials

- Hardness test - Measures the resistance of a material to penetration by a sharp object.
- Macrohardness - Overall bulk hardness of materials measured using loads >2 N.
- Microhardness Hardness of materials typically measured using loads less than 2 N using such test as Knoop (HK).
- Nano-hardness - Hardness of materials measured at 1–10 nm length scale using extremely small (~100 μN) forces.

Hardness

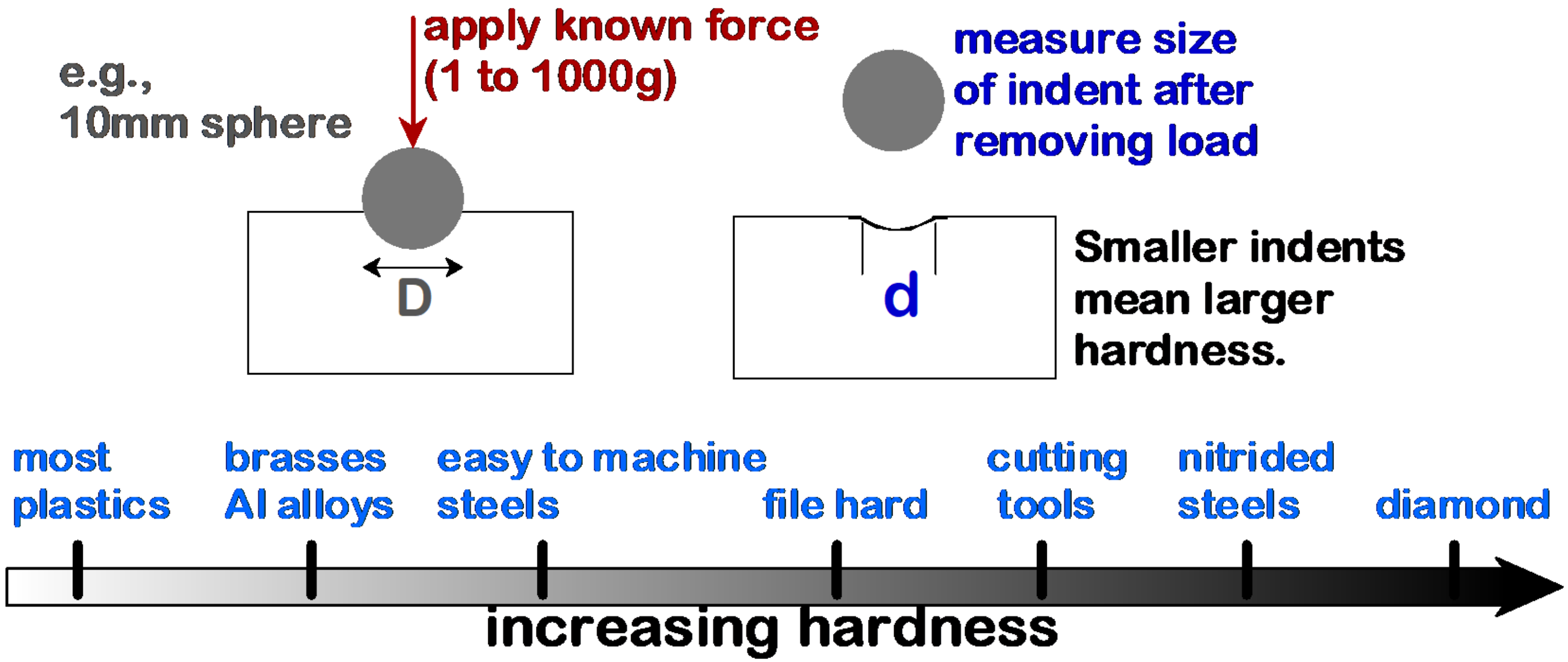
- Hardness is a measure of a material's resistance to localized plastic deformation (a small dent or scratch).
- Quantitative hardness techniques have been developed where a small indenter is forced into the surface of a material.
- The depth or size of the indentation is measured, and corresponds to a hardness number.
- The softer the material, the larger and deeper the indentation (and lower hardness number).

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:

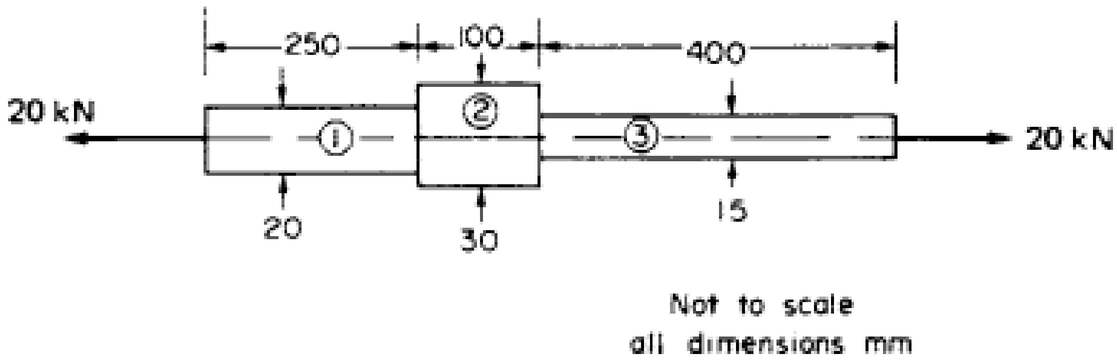
  --resistance to plastic deformation or cracking in compression.

  --better wear properties.



Adapted from Fig. 6.18, Callister 6e. (Fig. 6.18 is adapted from G.F. Kinney, Engineering Properties and Applications of Plastics, p. 202, John Wiley and Sons, 1957.)

Determine the stress in each section of the bar shown in Fig. 1.19 when subjected to an axial tensile load of 20 kN. The central section is 30 mm square cross-section; the other portions are of circular section, their diameters being indicated. What will be the total extension of the bar? For the bar material $E = 210\,\mathrm{GN/m^2}$.



$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{P}{A}$$

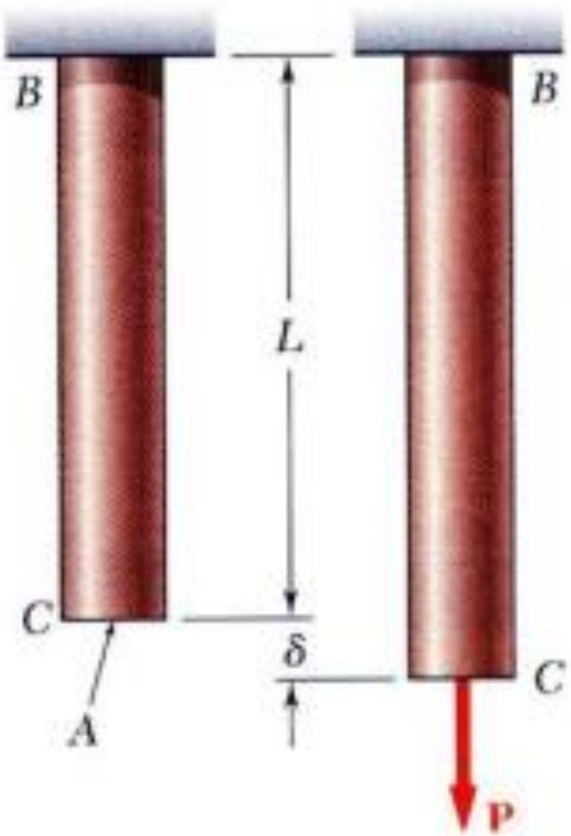
A rod 150 cm long and of diameter 2 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm^2. Determine i) the stress ii) the strain and iii) the elongation of the rod.

1. A tensile test was conducted on a mild steel bar. The following data was obtained from the test

   a. Diameter of the steel bar = 3 cm
   b. Gauge length of the bar = 20 cm
   c. Load at Elastic Limit = 250 kN
   d. Extension of a load of 150 kN = 0.21 mm
   e. Maximum Load = 380 kN
   f. Diameter of the rod at the failure = 2.25 cm

Determine i) the Young's modulus ii) the stress at elastic limit iii) the percentage elongation, and iv) the percentage decrease in area.

Deformation Under Axial Loading



- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

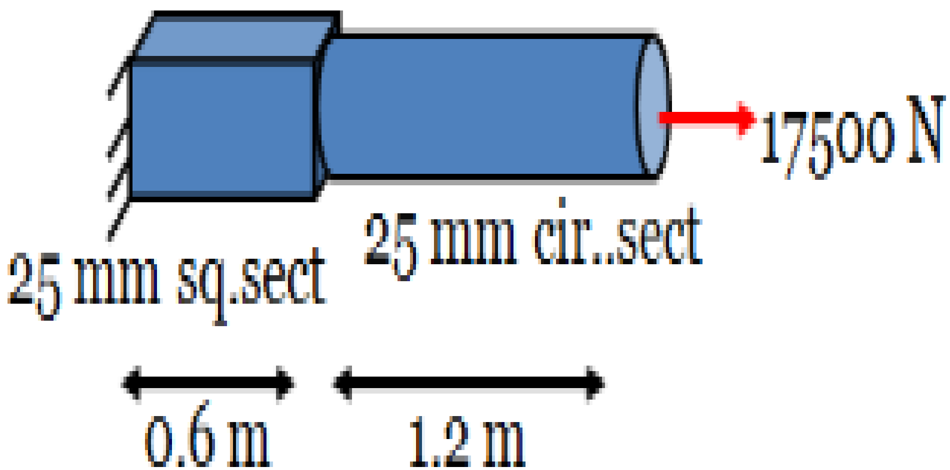
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Example: An aluminium bar 1.8 meters long has a 25 mm square c/s over 0.6 meters of its length and 25 mm circular c/s over other 1.2 meters . How much will the bar elongate under a tensile load P=17500 N, if E = 75000 Mpa.

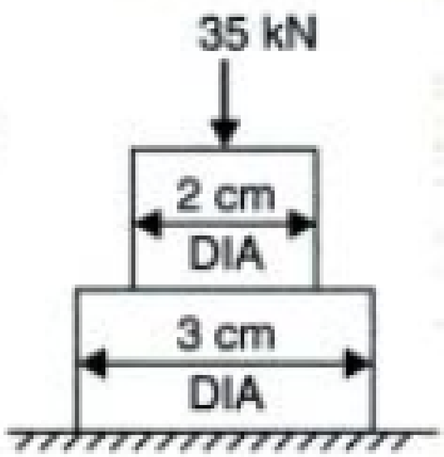


A stepped bar shown in Fig. is subjected to an axially applied compressive load of 35 kN. Find the maximum and minimum stresses produced.

Sol. Given :

Axial load, $P = 35\text{ kN} = 35 \times 10^3\text{ N}$

Dia. of upper part, $D_1 = 2\text{ cm} = 20\text{ mm}$



$\therefore$ Area of upper part, $A_1 = \frac{\pi}{4}(20^2) = 100\,\pi\text{ mm}^2$

Area of lower part, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(30^2) = 225\,\pi\text{ mm}^2$

Analysis of Uniformly Tapering Circular Rod

Bars of tapering section can be of conical section or of trapezoidal section with uniform thickness.

Conical Section

Consider a bar of conical section under the action of axial force P as shown in Fig. 1.8.

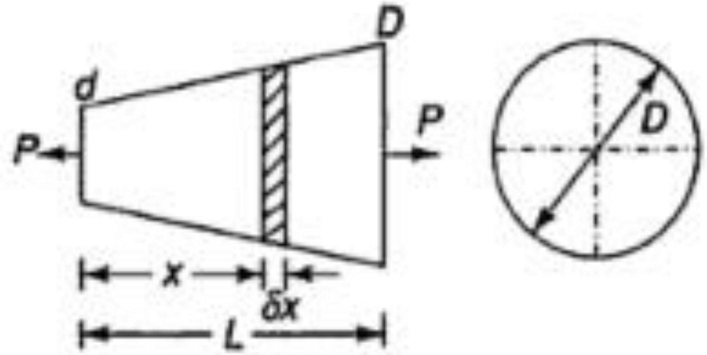


Fig. 1.8

Let D = diameter at the larger end

d = diameter at the smaller end

L = length of the bar

E = Young's modulus of the bar material

Consider a very small length δx at a distance x from the small end.

The diameter at a distance x from the small end $= d + \dfrac{D - d}{L}.x$

The extension of a small length

$$= \frac{P.\delta x}{\frac{\pi}{4}\left(d + \frac{D-d}{L}x\right)^2 .E} \quad \cdots\cdots \left(\Delta = \frac{PL}{AE}\right)$$

Extension of the whole rod $= \int_0^L \frac{4P}{\pi\left(d + (D-d)x/L\right)^2 .E}.dx$

$$= \frac{4P}{\pi E}\int_0^L \left(d + \frac{D-d}{L}.x\right)^{-2}.dx = -\frac{4P}{\pi E}.\frac{L}{(D-d)}\left(\frac{1}{(d + (D-d)x/L)}\right)_0^L$$

$$= \frac{4PL}{\pi E(D-d)}\left(\frac{1}{d} - \frac{1}{D}\right) = \frac{4PL}{\pi E(D-d)}\left(\frac{D-d}{dD}\right) = \frac{4PL}{\pi EdD}$$

$$\frac{4PL}{\pi EdD}$$

Problem 1. *Find the modulus of elasticity for a rod, which tapers uniformly from 30 mm to 15 mm diameter in a length of 350 mm. The rod is subjected to an axial load of 5.5 kN and extension of the rod is 0.025 mm.*

Sol. Given :

$$dL = \frac{4PL}{\pi E\, D_1 D_2}$$

Larger diameter,	$D_1 = 30$ mm
Smaller diameter,	$D_2 = 15$ mm
Length of rod,	$L = 350$ mm
Axial load,	$P = 5.5$ kN $= 5500$ N
Extension,	$dL = 0.025$ mm

$$dL = \frac{4PL}{\pi E\, D_1 D_2}$$

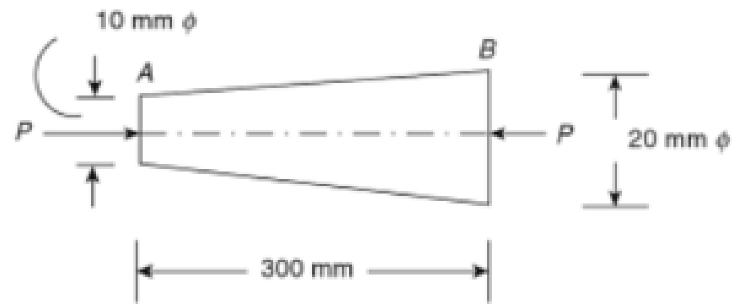
$$E = \frac{4PL}{\pi\, D_1 D_2\, dL} = \frac{4 \times 5500 \times 350}{\pi \times 30 \times 15 \times 0.025}$$

$= 217865$ N/mm^2 or 2.17865×10^5 N/mm^2. **Ans.**

Example A brass bar uniformly tapered from diameter 20 mm at one end to diameter 10 mm at the other end over an axial length 300 mm is subjected to an axial compressive load of 7.5 kN. If $E = 100$ kN/mm^2 for brass, determine

(a) the maximum and minimum axial stresses in bar and

(b) the total change in length of the bar.



Axial load, $P = 7.5$ kN. Maximum stress occurs at end A with minimum cross-sectional area:

(a) $$\sigma_{max} = -\frac{4P}{\pi(10)^2};$$

(b) $$\sigma_{min} = -\frac{4P}{\pi(20)^2};$$

$$\delta L = -\frac{4PL}{\pi E d_1 d_2},$$ as the load is compressive

Solution

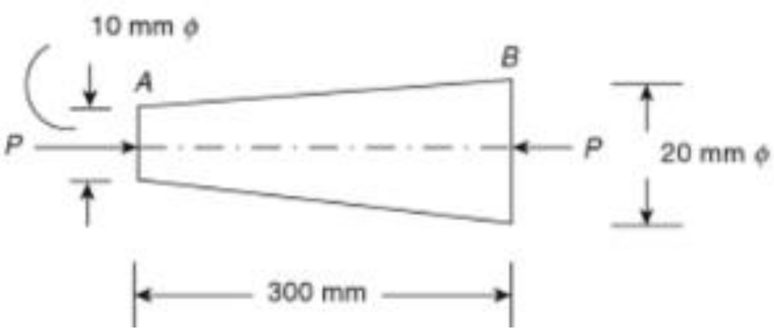
Axial load, $P = 7.5$ kN. Maximum stress occurs at end A with minimum cross-sectional area:

$$\text{(a)}\quad \sigma_{max} = -\frac{4P}{\pi(10)^2} = -\frac{4\times7.5\times1{,}000}{\pi\times100} = -95.94\ \text{N/mm}^2 \quad \text{at end } A$$

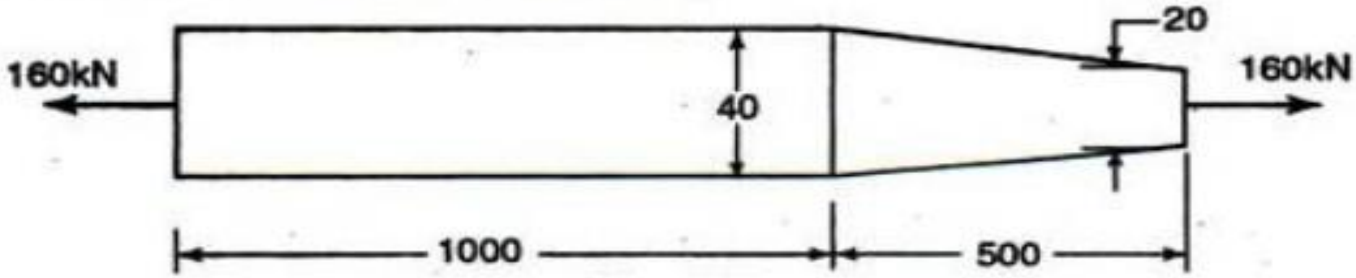
$$\text{(b)}\quad \sigma_{min} = -\frac{4P}{\pi(20)^2} = -\frac{4\times7.5\times1{,}000}{\pi\times20} = -23.87\ \text{N/mm}^2 \quad \text{at end } B$$

$$\delta L = -\frac{4PL}{\pi E d_1 d_2}, \text{ as the load is compressive}$$

$$= -\frac{4\times7.5\times1{,}000\times300}{\pi\times100\times1{,}000\times10\times20} = -0.143\ \text{mm}$$



Example A 1.5 m long steel bar is having uniform diameter of 40 mm for a length of 1 m and in the next 0.5 m its diameter gradually reduces from 40 mm to 20 mm as shown in Fig. Determine the elongation of this bar when subjected to an axial tensile load of 160 kN. Given $E = 200$ GN/m^2.



Solution

Now,
$$P = 160 \times 10^3 \text{ N}$$

$$E = 200\,GN/m^2 = \frac{200 \times 10^9}{(1000)^2} \text{ N/mm}^2$$
$$= 200 \times 1000 \text{ N/mm}^2$$
$$= 2 \times 10^5 \text{ N/mm}^2$$

Extension of uniform portion $\Delta_1 = \dfrac{PL}{AE}$

Extension of tapering portion $\Delta_2 = \dfrac{4PL}{E\pi d_1 d_2}$

Extensions of uniform portion and tapering portion are worked out separately and then added to get extension of the given bar.

Extension of uniform portion $\Delta_1 = \dfrac{PL}{AE}$

$$= \frac{160 \times 10^3 \times 1000}{\frac{\pi}{4} \times 40^2 \times 2 \times 10^5} = 0.6366\,\text{mm.}$$

Extension of tapering portion $\Delta_2 = \dfrac{4PL}{E\pi d_1 d_2}$

$$= \frac{4 \times 160 \times 10^3 \times 500}{2 \times 10^5 \times \pi \times 20 \times 40}$$

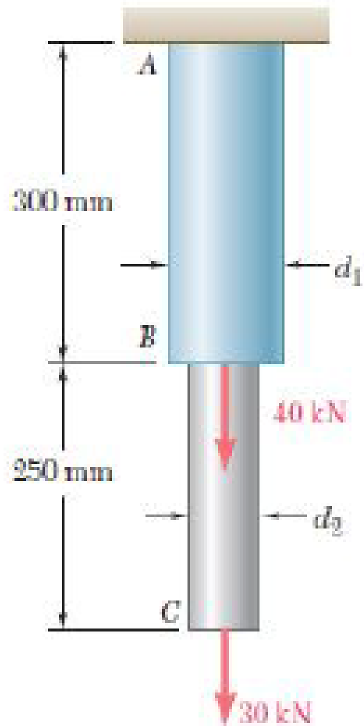
$$= 0.6366\,\text{mm}$$

Total Extension $= \Delta_1 + \Delta_2 = 0.6366 + 0.6366$

$= 1.2732\,\text{mm}$ **(Ans)**

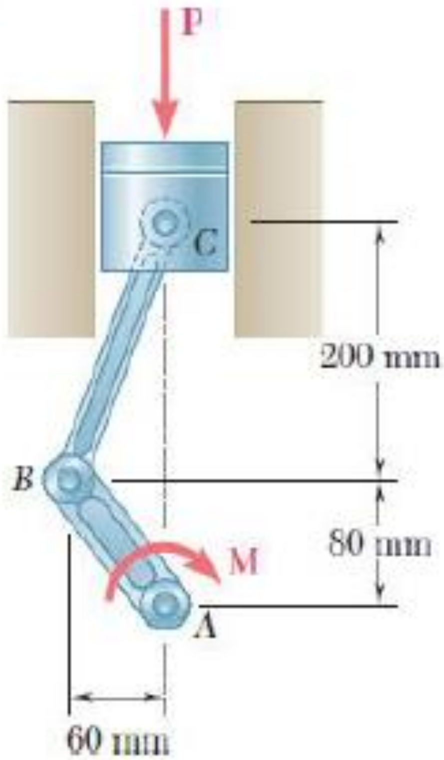
Problems for practice :

1. Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod AB and 150 MPa in rod BC, determine the smallest allowable values of d_1 and d_2.



Problems for practice

A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm^2 uniform cross section.



Principle of Superposition

It states that the effects of several actions taking place simultaneously can be reproduced exactly by adding the effect of each action separately.

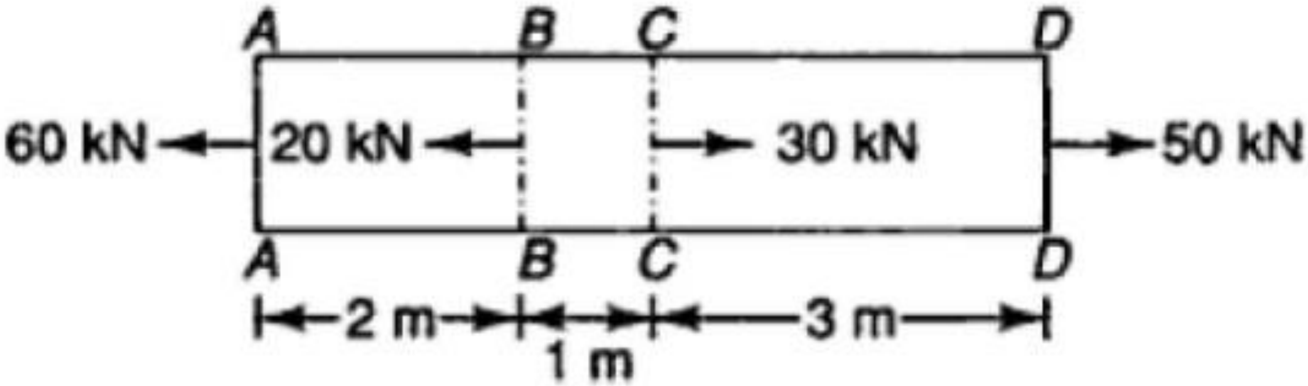
The principle is general and has wide applications and holds true if:

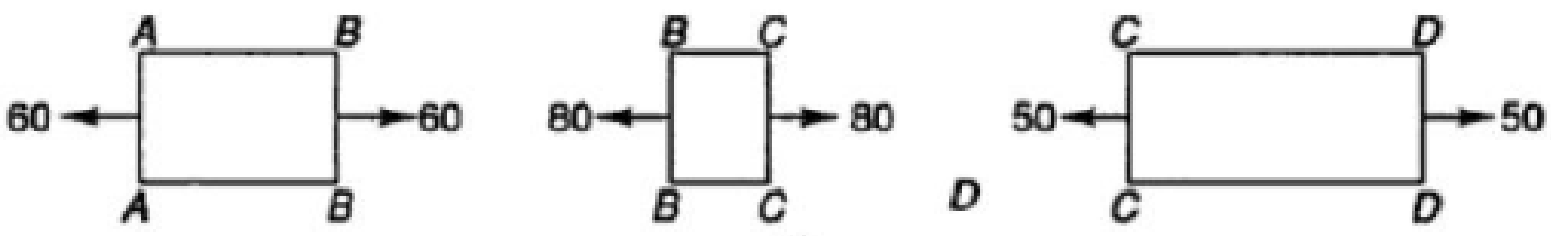
(i) The structure is elastic

(ii) The stress-strain relationship is linear

(iii) The deformations are small.

Example 1. *A steel bar of 25-mm diameter is acted upon by forces as shown in Fig. What is the total elongation of the bar? Take E = 190 GPa.*





Solution Area of the section $= \frac{\pi}{4}(25)^2 = 490.88\ \text{mm}^2$, $E = 190$ GPa

$= 190\ 000\ \text{N/mm}^2$

Elongation is given by, $\Delta = \dfrac{PL}{AE}$

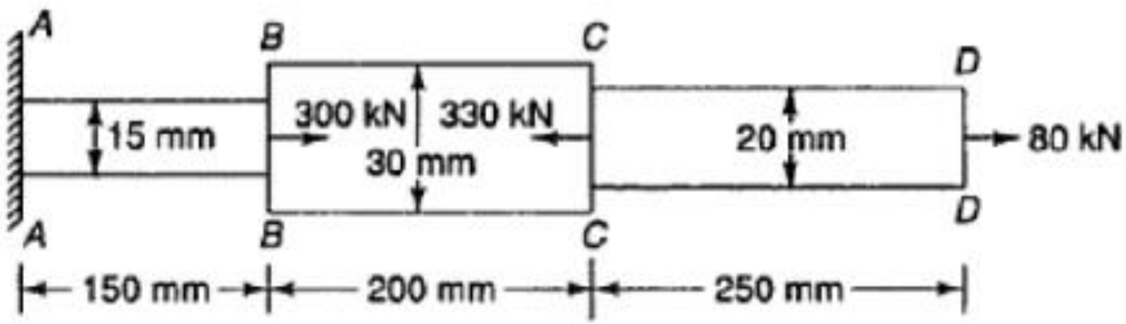
Example 1.2 *A steel circular bar has three segments as shown in Fig. 1.7a. Determine*

(i) *the total elongation of the bar*

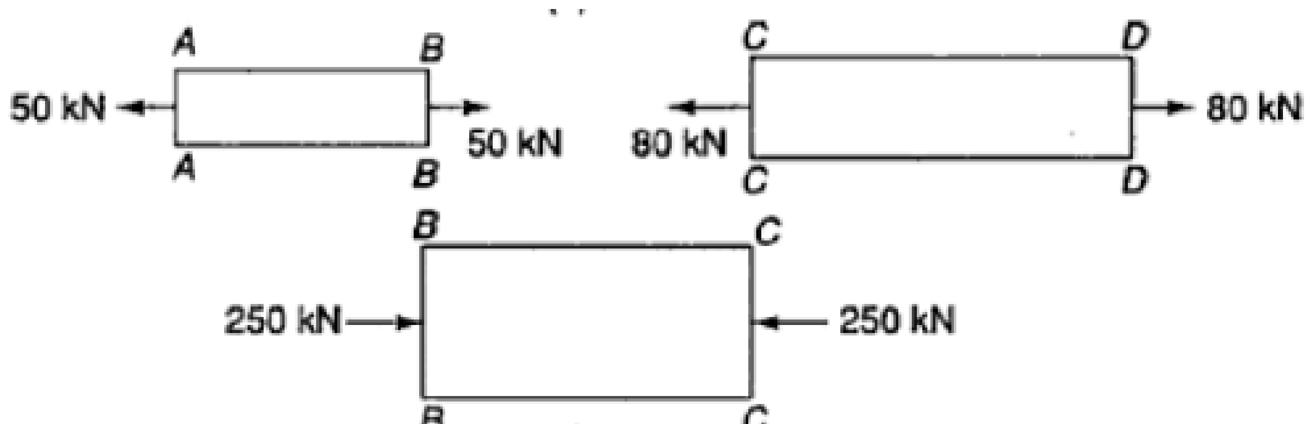
(ii) *the length of the middle segment to have zero elongation of the bar*

(iii) *the diameter of the last segment to have zero elongation of the bar*

Take E = 205 GPa.



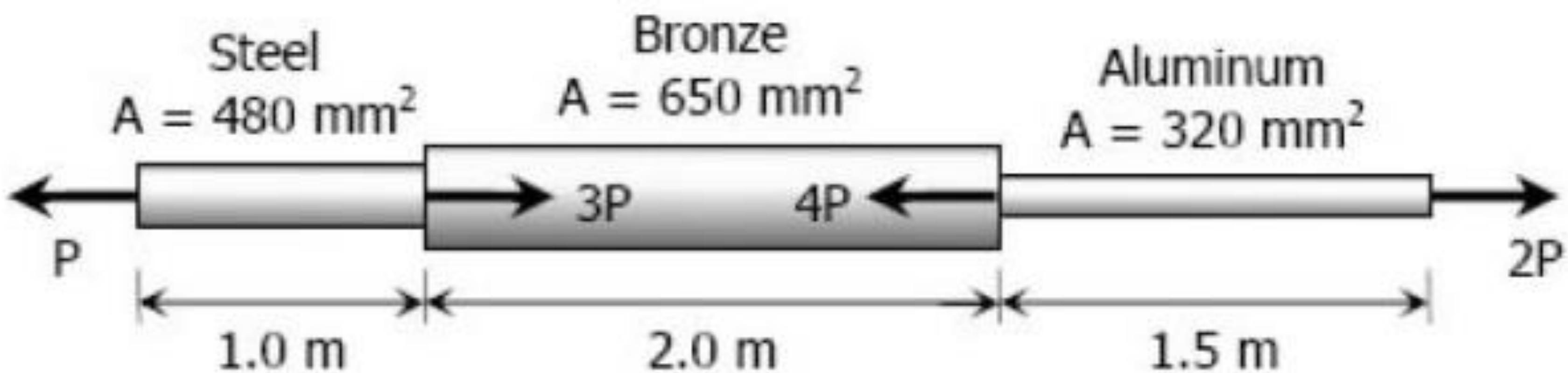
(a)



(b)

Total elongation,

A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.

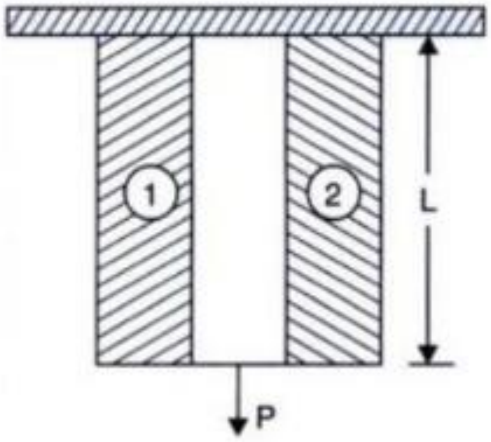


Analysis Of Bars Of Composite Sections

A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive loads, is called a composite bar. For the composite bar the following two points are important :

1. The extension or compression in each bar is equal. Hence deformation per unit length *i.e.*, strain in each bar is equal.

2. The total external load on the composite bar is equal to the sum of the loads carried by each different material.



(1) $P = P_1 + P_2$

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

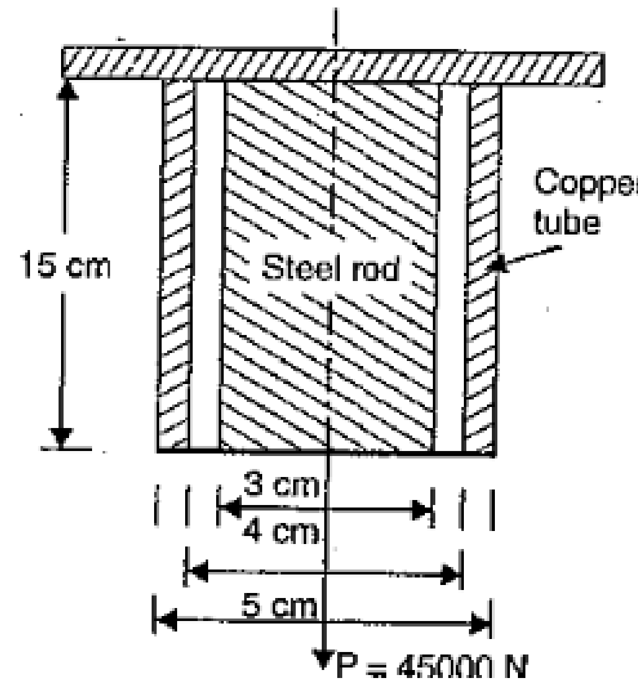
(2) strain in bar 1 = Strain in bar 2

$$= \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

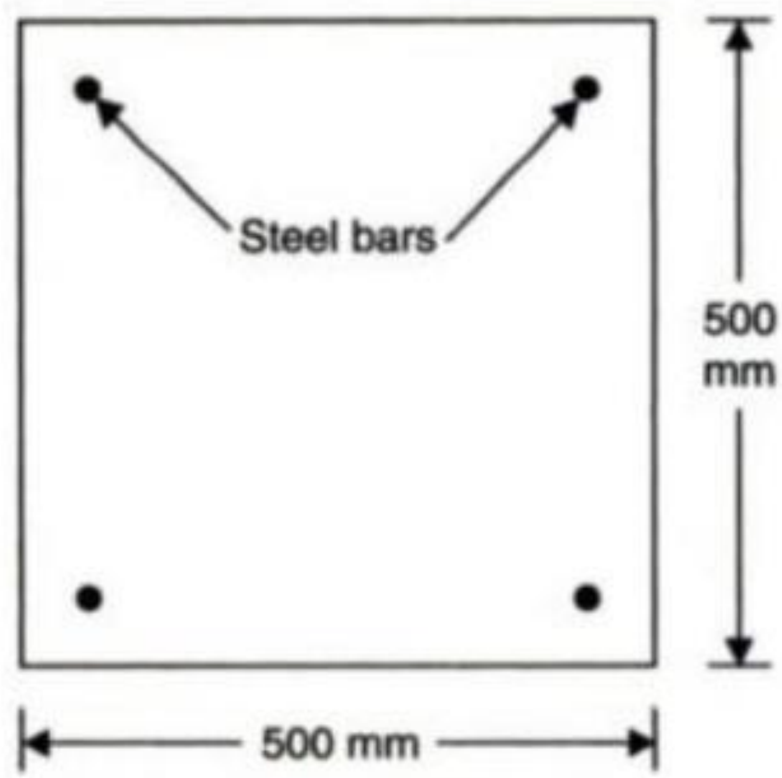
A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm. The composite bar is then subjected to an axial pull of 45000N. If the length of each bar is equal to 15 cm, determine:

1) Stresses in the rod and the tube
2) Load carried by each bar

Take $E = 2.1 \times 10^5$ N/mm^2 for steel and $E = 1.1 \times 10^5$ N/mm^2 for copper

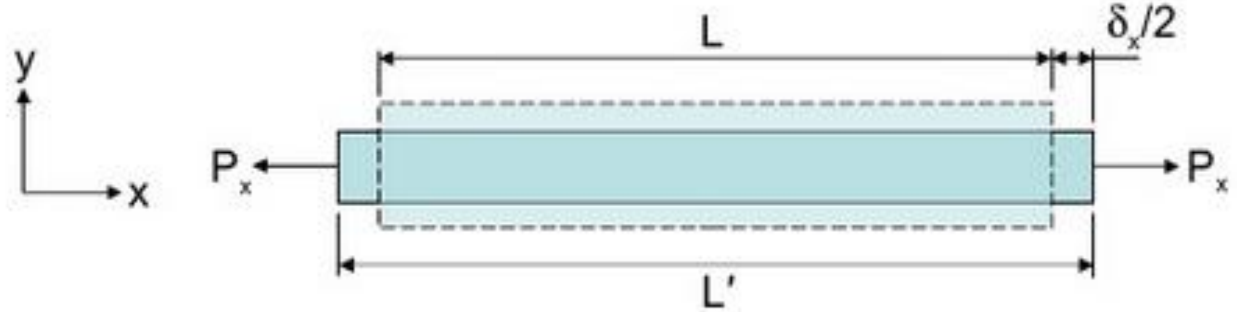


Problem *A load of 2 MN is applied on a short concrete column 500 mm × 500 mm. The column is reinforced with four steel bars of 10 mm diameter, one in each corner. Find the stresses in the concrete and steel bars. Take E for steel as 2.1×10^5 N/mm^2 and for concrete as 1.4×10^4 N/mm^2.*



Poisson's Ratio

- When we dealt with the axial elongation of a slender bar in tension, its lateral contraction was NOT considered

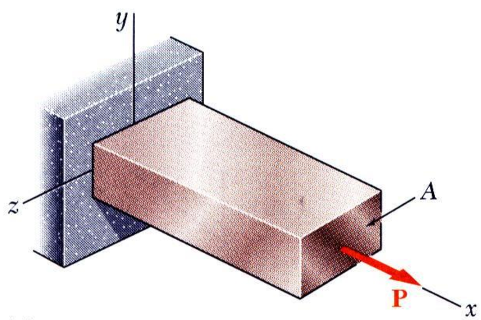


- In the linear elastic range, the ratio of the axial elongation to the lateral contraction is constant and is called ***Poisson's ratio:***

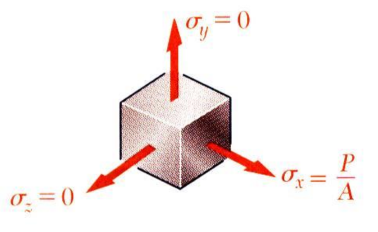
$$\nu_{xy} = \frac{\text{unit lateral contraction}}{\text{unit axial elongation}} = -\frac{\varepsilon_y}{\varepsilon_x}$$

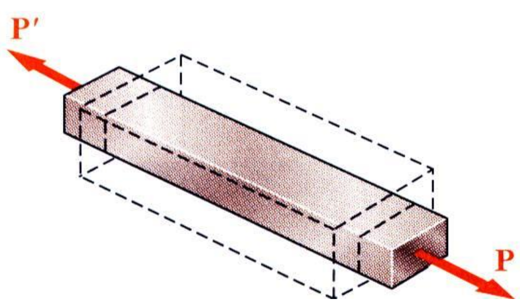
- For homogeneous and isotropic materials, Poisson's ratio is the same in all directions
- Poisson's ratio is a dimensionless quantity

Poisson's Ratio



(a)





- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson's ratio is defined as

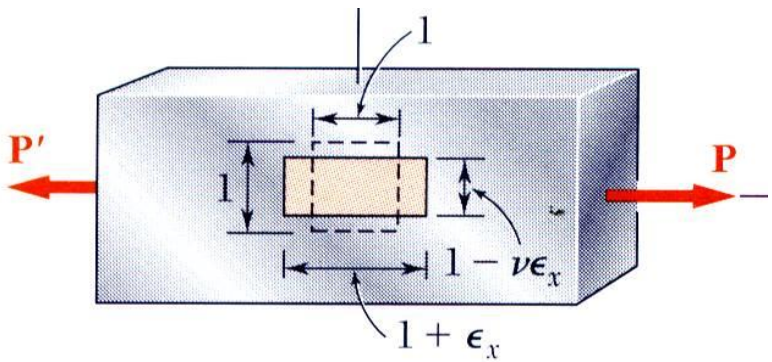
$$\nu = \left|\frac{\text{lateral strain}}{\text{axial strain}}\right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

Poisson ratio: examples

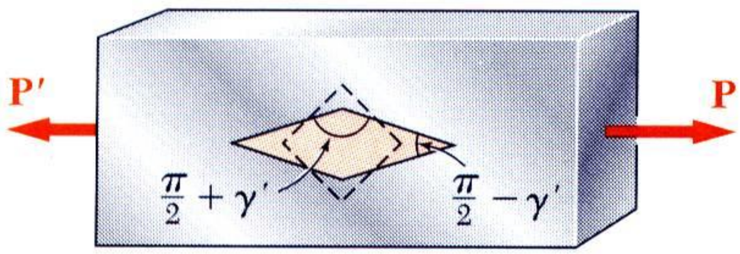
VALUES OF POISSON'S RATIO

S.No	Material	Poisson's Ratio $\frac{1}{m}$ or μ
1.	Aluminium (Pure)	0.33
2.	Aluminium alloys	0.33
3.	Brass	0.34
4.	Bronze	0.34
5.	Cast iron	0.2 - 0.3
6.	Concrete (compression)	0.1 - 0.2
7.	Copper (Pure)	0.33 - 0.36
8.	Rubber	0.45 - 0.50
9.	Steel	0.27 - 0.30
10.	Wrought iron	0.3

Relation Among *E*, *ν*, and *G*



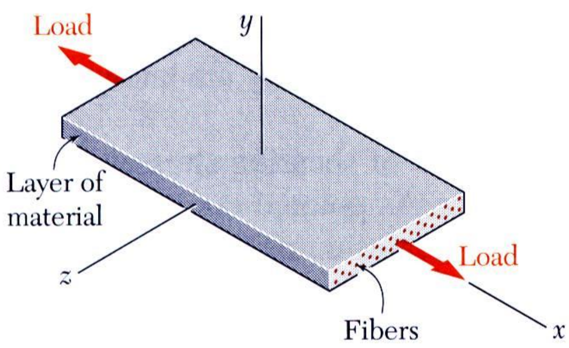
(*a*)

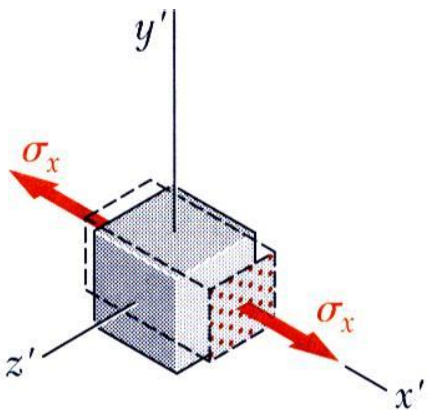


- An axially loaded slender bar will **elongate in the axial direction** and **contract in the transverse directions**.
- An initially cubic element oriented as in top figure will deform into a rectangular parallelepiped. **The axial load produces a normal strain.**
- If the cubic element is oriented as in the bottom figure, it will deform into a rhombus. **Axial load also results in a shear strain.**
- Components of **normal and shear strain** are related,

$$\frac{E}{2G} = (1 + \nu)$$

Composite Materials





- *Fiber-reinforced composite materials* are formed from *lamina* of fibers of graphite, glass, or polymers embedded in a resin matrix.

- Normal stresses and strains are related by Hooke's Law but with directionally dependent moduli of elasticity,

$$E_x = \frac{\sigma_x}{\varepsilon_x} \quad E_y = \frac{\sigma_y}{\varepsilon_y} \quad E_z = \frac{\sigma_z}{\varepsilon_z}$$

- Transverse contractions are related by directionally dependent values of Poisson's ratio, e.g.,

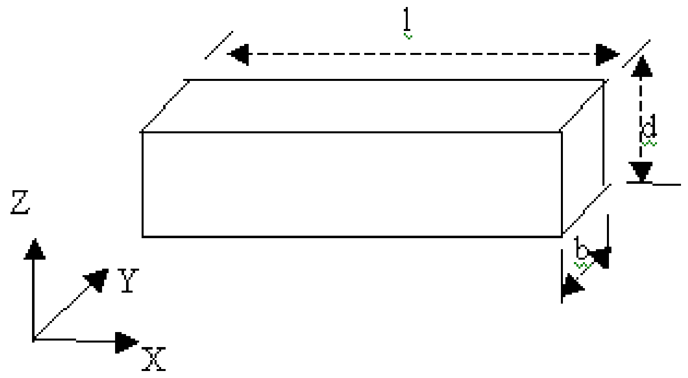
$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} \quad \nu_{xz} = -\frac{\varepsilon_z}{\varepsilon_x}$$

- Materials with directionally dependent mechanical properties are *anisotropic*.

Volumetric strain

Volumetric strain of a deformed body is defined as the **ratio of the change in volume of the body** to the deformation to its original volume. If V is the original volume and dV the change in volume occurred due to the deformation, the volumetric strain ev induced is given by ev =dV/V

Consider a uniform rectangular bar of length l, breadth b and depth d as shown in figure. Its volume V is given by,



$$V = lbd$$

$$\delta V = \delta l\, bd + \delta b\, ld + \delta d\, lb$$

$$\delta V / V = (\delta l / l) + (\delta b / b) + (\delta d / d)$$

$$e_v = e_x + e_y + e_z$$

Volumetric strain

- Consider a unit block of dimensions 1×1×1
- If the block is subject to strains ε_x, ε_y and ε_z in the associated three orthogonal directions, its dimensions increase to:

$$(1+\varepsilon_x);\ (1+\varepsilon_y);\ (1+\varepsilon_z)$$

- Thus, the new volume is:

$$V' = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) \approx 1+\varepsilon_x+\varepsilon_y+\varepsilon_z$$

  – Neglecting higher orders of strain
- And the volumetric strain is therefore:

$$\boxed{\varepsilon_{vol} = \frac{V'-V}{V} = \varepsilon_x+\varepsilon_y+\varepsilon_z}$$

Bulk Modulus Of Elasticity

Bulk Modulus of elasticity is one of the measures of mechanical properties of solids. Other elastic moduli are Young's modulus and Shear modulus. Bulk modulus defines the ability of a material **to resist deformation** in terms of **volume change**, when subject to **compression under pressure**.

It is given by the ratio of pressure applied to the corresponding relative decrease in volume of the material. The relation is given below.

$$K = -V\frac{dP}{dV}$$

where,

K is the Bulk Modulus of the material expressed as N/m^2 or Pa

dP is the change in pressure applied

dV is the change in volume of the system

V is volume of the system

Bulk Modulus

- Consider a block subjected to *equal* stresses on all six faces (i.e. a block submersed in a fluid at great depth)

$$\sigma_x = \sigma_y = \sigma_z = -P$$

- The hydrostatic stress / pressure is negative since it tends to compress the block. From Hooke's Law:

$$\varepsilon_x = \sigma_x - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} = -\frac{P}{E}(1-2\nu) = \varepsilon_y = \varepsilon_z$$

- The volumetric strain is therefore:

$$\varepsilon_{vol} = \varepsilon_x + \varepsilon_y + \varepsilon_z = -\frac{3P}{E}(1-2\nu)$$

- The ***Bulk modulus*** is defined as the ratio of the *hydrostatic stress* to the *volumetric strain*, therefore

$$\boxed{K = -\frac{P}{\varepsilon_{vol}} = \frac{E}{3(1-2\nu)}}$$

Generalised Hook's law

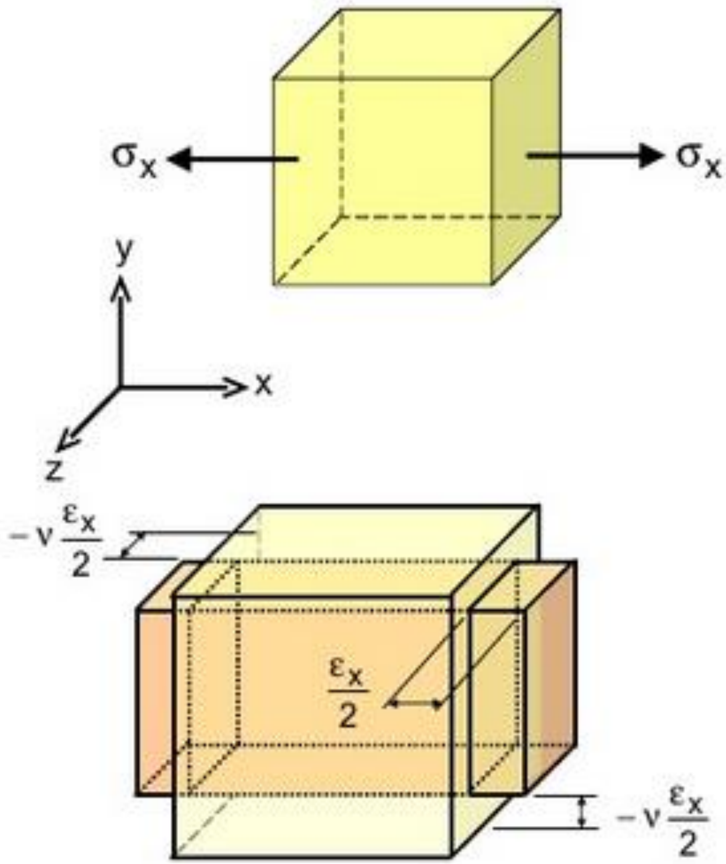
The generalized Hooke's Law can be used to predict the deformations caused in a given material by an arbitrary combination of stresses.

- Hooke's Law – linear relationship between components of stress and strain
- Consider a 3-dimensional block subjected to a uni-axial stress σ_x
- The three orthogonal strains produced in the block are ε_x, $-\nu\varepsilon_x$, $-\nu\varepsilon_x$
- For a tri-axial state of stress:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_z\right)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$



Strain energy

STRAIN ENERGY

due to
axial load
bending
shear
torsion

STRAIN ENERGY

- When an elastic body is loaded, it undergoes deformation
- Energy is stored during this process, and released when the load is removed
- This energy is strain energy

- Within elastic limit, energy stored is RESILIENCE
- Maximum energy stored upto elastic limit is PROOF RESILIENCE
- Proof resilience is the capacity to bear shocks. Proof resilience per unit volume of piece is MODULUS OF RESILIENCE

Strain energy due to axial load – Variable force along length

Axial strain energy in a member of length L and axial rigidity EA, subject to tensile force $P(x)$ and corresponding axial strain $\varepsilon(x) = P(x)/EA$ is obtained as below:

For an elemental length dx,

strain energy $$u_{axial} = \frac{1}{2}P(x)\delta l = \frac{1}{2}P(x)\frac{P(x)dx}{EA}$$

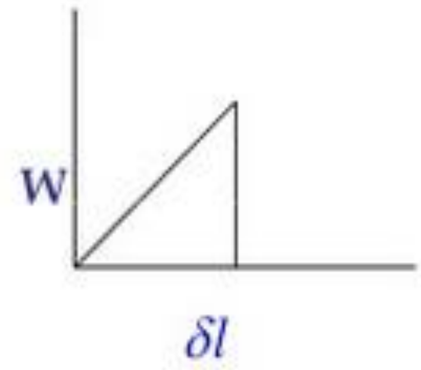
strain energy $$U_{axial} = \frac{1}{2}\int_0^L \frac{P(x)^2}{EA}dx = \frac{1}{2}\int_0^L EA\varepsilon(x)^2 dx$$

Complementary strain energy $$U^*_{axial} = U_{axial}$$

for linear elastic material

Strain energy due to axial load- Uniform force throughout length

Strain energy stored in the bar, $U = \frac{1}{2} W \delta l$



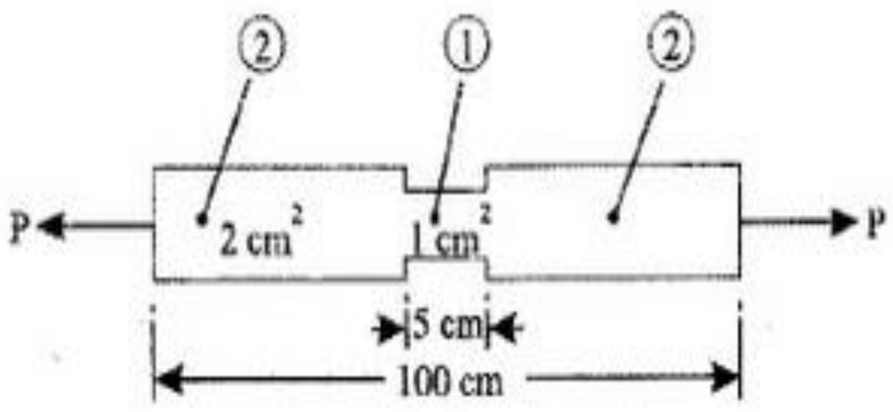
$$\boxed{U = \frac{1}{2}(\sigma A)\left(\frac{\sigma l}{E}\right) = \frac{\sigma^2 A l}{2E} = \frac{\sigma^2 V}{2E}}$$

If σ_P is proof stress (stress upto elastic limit), then

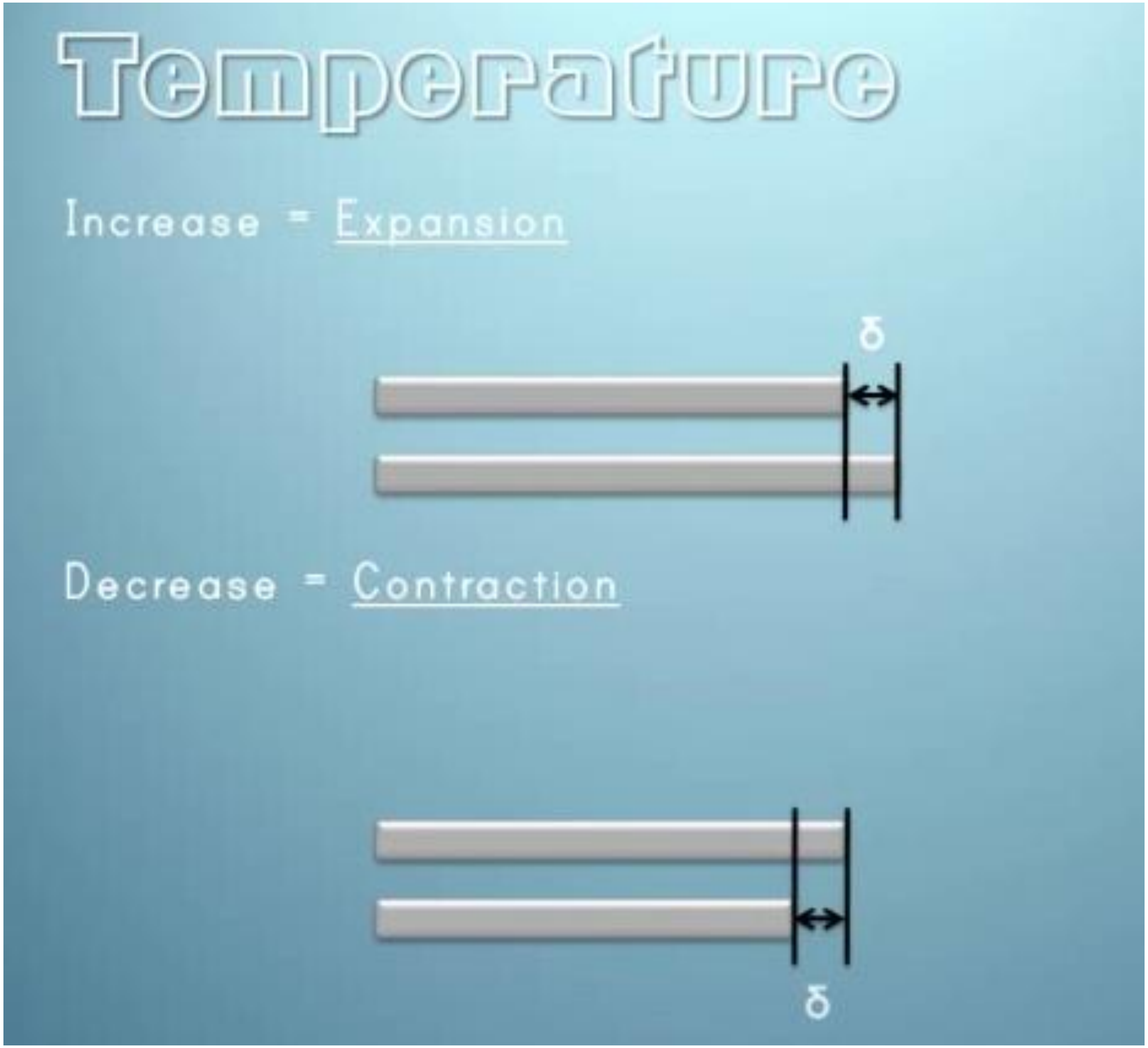
proof resilience = $\dfrac{\sigma_P{}^2 V}{2E}$

Modulus of resilience = $\dfrac{\sigma_P{}^2}{2E}$

Example 1: A bar 100 cm length is subjected to an axial pull, such that the maximum stress is $150 MN/m^2$. Its area of cross-section is $2 cm^2$ over a length of 95 cm and for the middle 5 cm it is only $1 cm^2$. If $E = 200 GN/m^2$, calculate the strain energy stored in bar.



Temperature stresses:-



Temperature stresses:-

Material

↓

Change in temp.

↓

Expands/ Shortens

↓

No constraint is present

Material

↓

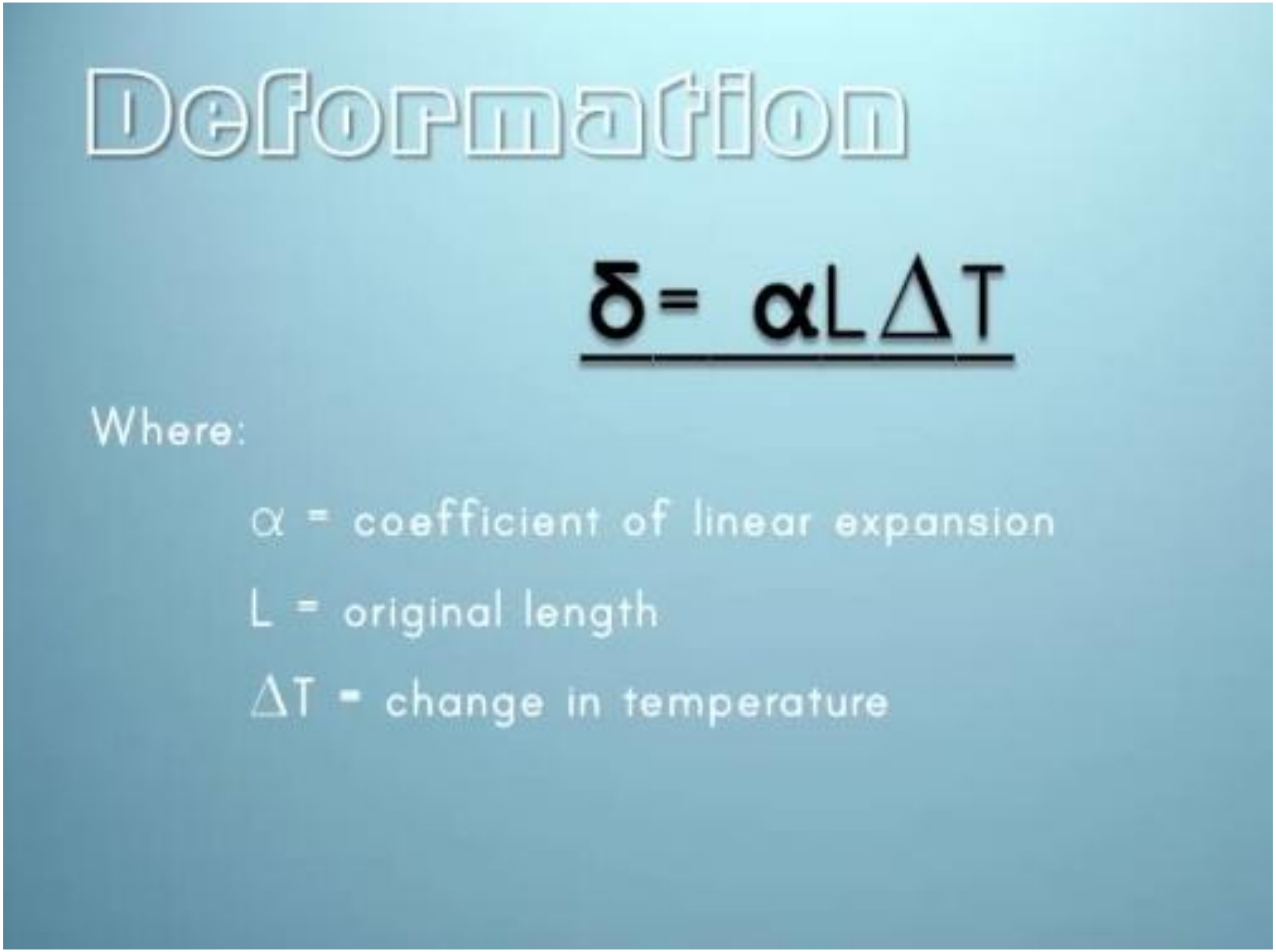
Constrained

↓

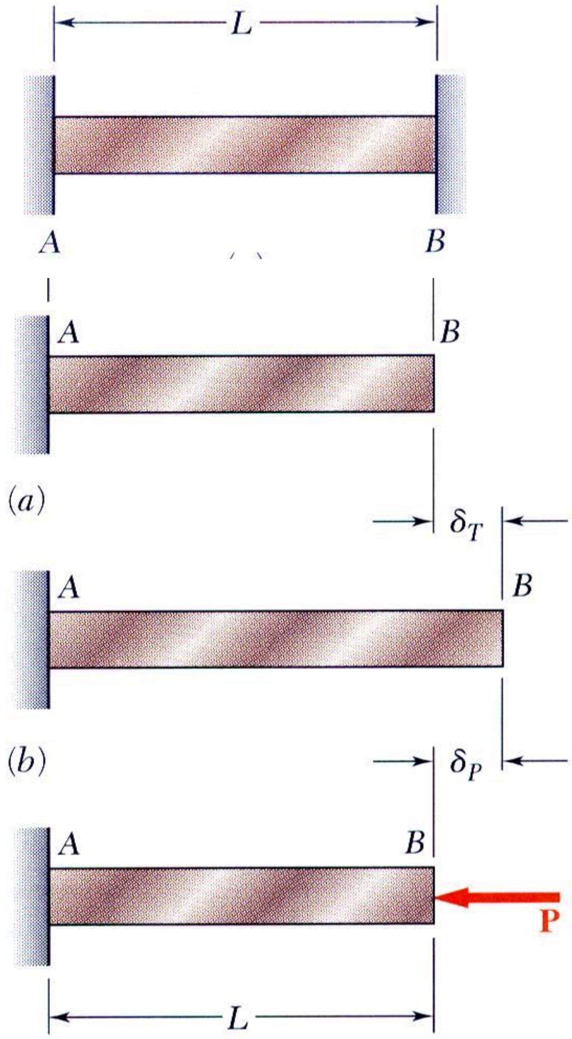
No Expansion/ contraction

↓

Temperature stresses Induced in material


Deformation
$\delta = \alpha L \Delta T$
Where:
α = coefficient of linear expansion
L = original length
ΔT = change in temperature

Thermal Stresses



- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.

- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$$\alpha = \text{thermal expansion coef.}$$

- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0$$

$$\alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$\delta = \delta_T + \delta_P = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

text, p. 74

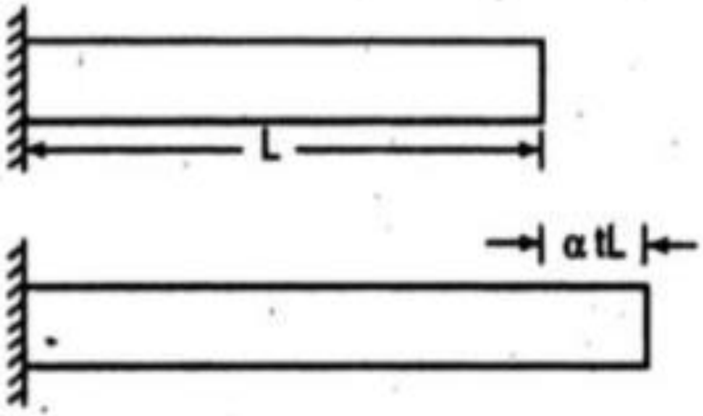
Every material expands when temperature rises and contracts when temperature falls. The change in length due to change in temperature is found to be directly proportional to length of the member and also to change in temperature. Hence if 'α' is constant of proportionality, 't' is change in temperature and L is the length of the member, then change in length Δ is given by

$$\Delta = \alpha t L$$

The constant of proportionality 'α' is called *coefficient of thermal expansion and is defined as change in unit length of the material due to unit change in temperature.* This value is different for different materials. Values for some of the commonly used engineering materials are listed below:

Material	**Coefficient of thermal expansion**
Steel	12×10^{-6} / °C
Copper	17.5×10^{-6} / °C
Stainless steel	18×10^{-6} / °C
Brass, Bronze	19×10^{-6} / °C
Aluminium	23×10^{-6} / °C

If the changes due to temperature are permitted freely no stresses develop in the member e.g. in case of bar shown in Fig. due to increase in temperature by $t^o C$, the bar will extend by $\alpha\, tL$ and due to this change no stress is introduced.



Consider a body which is heated to a certain temperature.

Let L = Original length of the body,

T = Rise in temperature,

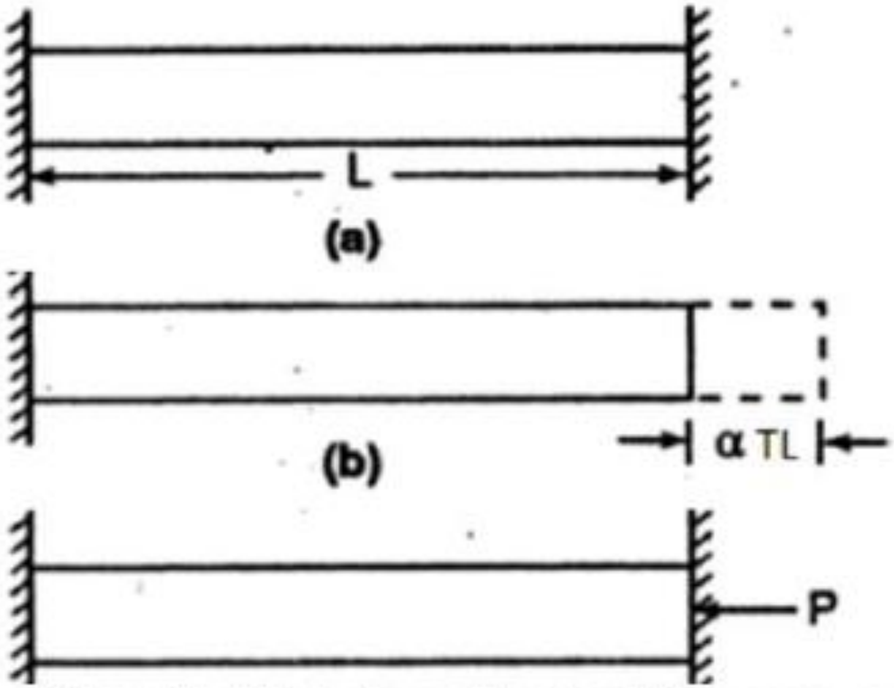
E = Young's Modulus,

α = Co-efficient of linear expansion,

dL = Extension of rod due to rise of temperature.

If the rod is free to expand, then extension of the rod is given by

$$dL = \alpha.\ T.L.$$



If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive stress and strain will be set up in the rod. These stresses and strains are known as thermal stresses and thermal strain.

$$\therefore \text{ Thermal strain,} \quad e = \frac{\text{Extension prevented}}{\text{Original length}}$$

$$= \frac{dL}{L} = \frac{\alpha . T . L}{L} = \alpha . T$$

$$\text{And thermal stress,} \quad \sigma = \text{Thermal strain} \times E$$

$$= \alpha . T . E.$$

Thermal stress is also known as temperature stress.

And thermal strain is also known as temperature strain.

Stress and Strain when the Supports Yield. If the supports yield by an amount equal to δ, then the actual expansion

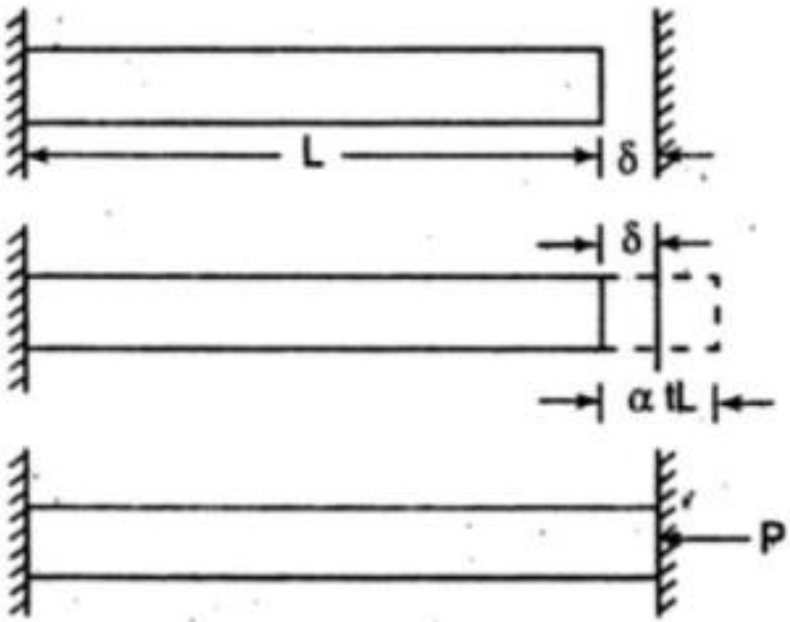
$$= \text{Expansion due to rise in temperature} - \delta$$

$$= \alpha . T . L - \delta.$$

$$\therefore \text{ Actual strain} = \frac{\text{Actual expansion}}{\text{Original length}} = \frac{(\alpha . T . L - \delta)}{L}$$

$$\text{And actual stress} = \text{Actual strain} \times E$$

$$= \frac{(\alpha . T . L - \delta)}{L} \times E.$$



Stresses in composite bars

The extension or contraction of the bar being equal, the strain *i.e.* deformation per unit length is also equal.

The total external load on the bar is equal to the sum of the loads carried by differenti materials.

$$P = P_1 + P_2$$

$$\sigma_1 = \frac{P_1}{A_1}$$

$$\varepsilon = \frac{\sigma_1}{E_1} = \frac{P_1}{A_1 . E_1}$$

$$\delta l_1 = \frac{P_1 . l}{A_1 . E_1} \qquad \delta l_2 = \frac{P_2 . l}{A_2 . E_2}$$

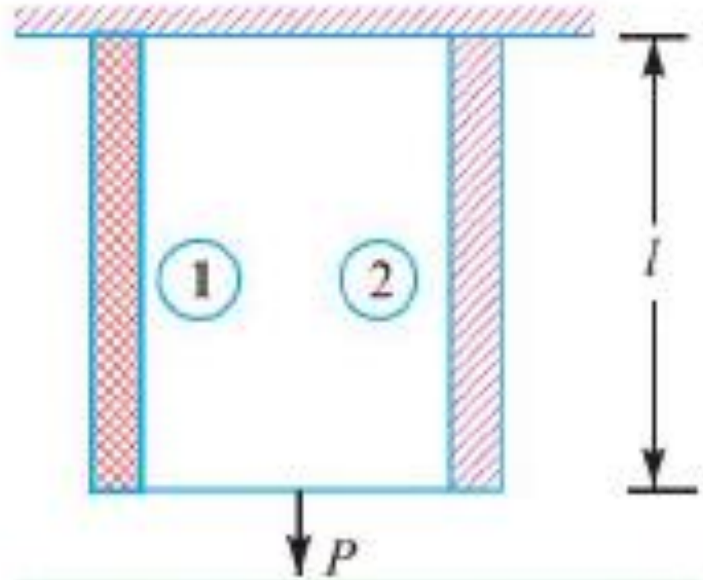


Fig. 4.13. Stresses in composite bars.

$$\delta l_1 = \delta l_2$$

$$\frac{P_1.l}{A_1.E_1} = \frac{P_2.l}{A_2.E_2}$$

$$P = P_1 + P_2$$

$$= P_2\left(\frac{A_1.E_1 + A_2.E_2}{A_2.E_2}\right)$$

$$P_2 = P \times \frac{A_2.E_2}{A_1.E_1 + A_2.E_2}$$

$$P_1 = P \times \frac{A_1.E_1}{A_1.E_1 + A_2.E_2}$$

Source: Khurmi & Gupta

Problem *A steel rod of 3 cm diameter and 5 m long is connected to two grips and the rod is maintained at a temperature of 95°C. Determine the stress and pull exerted when the temperature falls to 30°C, if*

(*i*) *the ends do not yield, and*

(*ii*) *the ends yield by 0.12 cm.*

Take $E = 2 \times 10^5$ *MN/m*^{2} *and* $\alpha = 12 \times 10^{-6}$/°C.

Sol. Given :

Dia. of the rod, $d = 3 \text{ cm} = 30 \text{ mm}$

$\therefore$ Area of the rod, $A = \frac{\pi}{4} \times 30^2 = 225\,\pi \text{ mm}^2$

Length of the rod, $L = 5 \text{ m} = 5000 \text{ mm}$

Initial temperature, $T_1 = 95°\text{C}$

Final temperature, $T_2 = 30°\text{C}$

$\therefore$ Fall in temperature, $T = T_1 - T_2 = 95 - 30 = 65°\text{C}$

Modulus of elasticity, $E = 2 \times 10^5 \text{ MN/m}^2$

$= 2 \times 10^5 \times 10^6 \text{ N/m}^2$

$= 2 \times 10^{11} \text{ N/m}^2$

Co-efficient of linear expansion, $\alpha = 12 \times 10^{-6}$/°C.

(*i*) *When the ends do not yield*

The stress is given by equation (

$\therefore$ $$\text{Stress} = \alpha.T.E = 12 \times 10^{-6} \times 65 \times 2 \times 10^{11} \text{ N/m}^2$$

$$= \mathbf{156 \times 10^6\ N/m^2\ or\ 156\ N/mm^2\ (tensile).}\quad \text{Ans.}$$

Pull in the rod = Stress × Area

$$= 156 \times 225\,\pi = \mathbf{110269.9\ N.}\quad \text{Ans.}$$

(ii) When the ends yield by 0.12 cm

$\therefore$ $$\delta = 0.12 \text{ cm} = 1.2 \text{ mm}$$

The stress when the ends yield is given by equation (1.16).

$\therefore$ $$\text{Stress} = \frac{(\alpha\,.\,T\,.\,L - \delta)}{L} \times E$$

$$= \frac{(12 \times 10^{-6} \times 65 \times 5000 - 1.2)}{5000} \times 2 \times 10^5 \text{ N/mm}^2$$

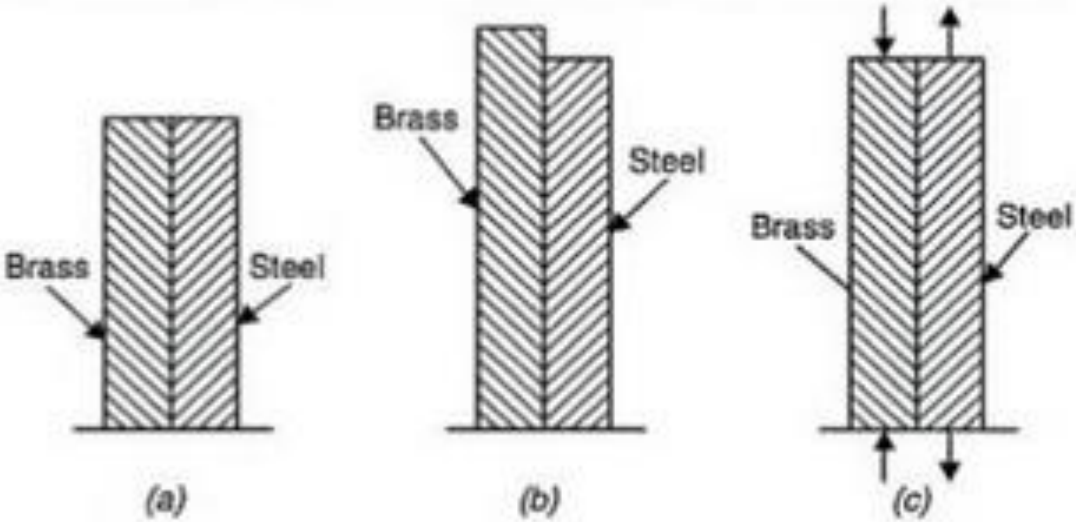
$$= \frac{(3.9 - 1.2)}{5000} \times 2 \times 10^5 = \mathbf{108\ N/mm^2.}\quad \text{Ans.}$$

Pull in the rod = Stress × Area

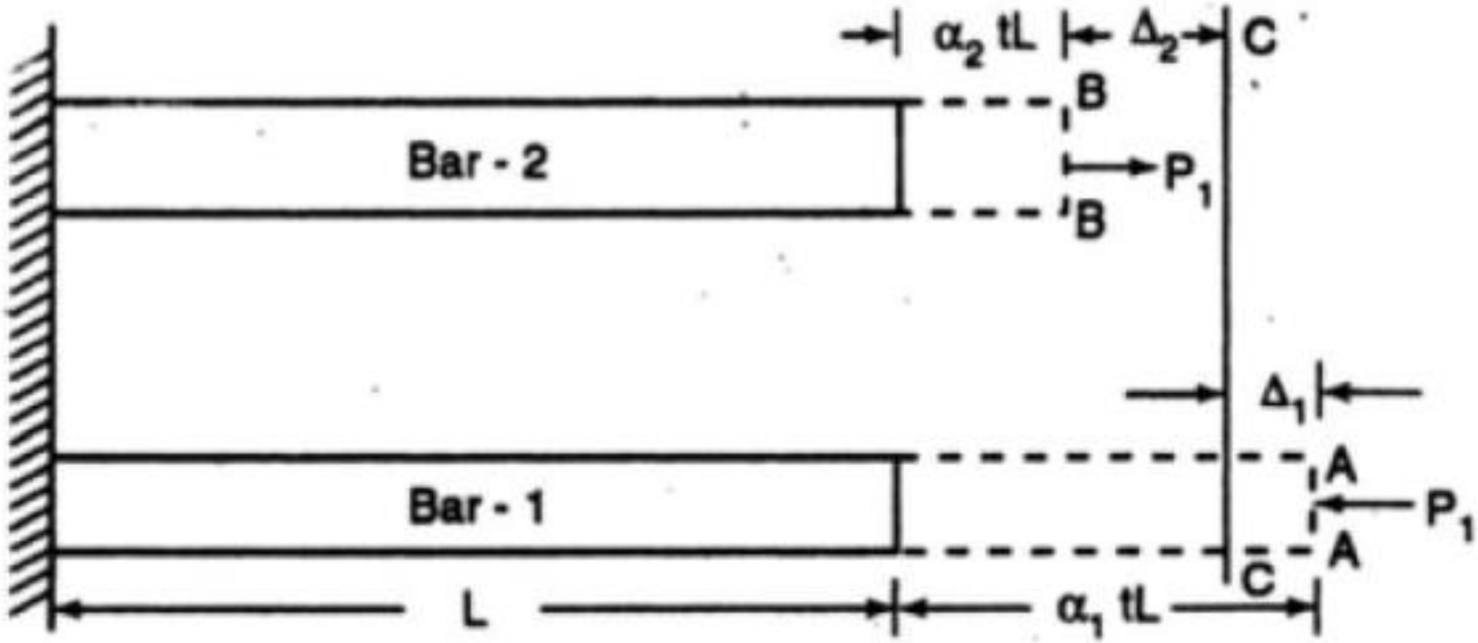
$$= 108 \times 225\,\pi = \mathbf{76340.7\ N.}\quad \text{Ans.}$$

THERMAL STRESSES IN COMPOSITE BARS

composite bar consisting of two members, a bar of brass and another of steel. Let the composite bar be heated through some temperature. If the members are free to expand then no stresses will be induced in the members. But the two members are rigidly fixed and hence the composite bar as a whole will expand by the same amount. As the co-efficient of linear expansion of brass is more than that of the steel, the brass will expand more than the steel. Hence the free expansion of brass will be more than that of the steel. But both the members are not free to expand, and hence the expansion of the composite bar, as a whole, will be less than that of the brass, but more than that of the steel. Hence the stress induced in the brass will be compressive whereas the stress in steel will be tensile as shown in Fig. 2.8 (c). Hence the load or force on the brass will be compressive whereas on the steel the load will be tensile.



General case



The equilibrium condition gives $P_1 = P_2$, say $= P$.

From the figure it is clear that

$$\alpha_2 t\, L + \Delta_2 = \alpha_1 t\, L - \Delta_1$$

$$\Delta_1 + \Delta_2 = \alpha_2 t\, L - \alpha_2 t\, L$$

i.e.

$$\frac{PL}{A_1 E_1} + \frac{PL}{A_2 E_2} = (\alpha_1 - \alpha_2)\, tL$$

$$PL\left(\frac{A_2 E_2 + A_1 E_1}{A_1 E_1 A_2 E_2}\right) = (\alpha_1 - \alpha_2)\, t\, L$$

or

$$P = \left(\frac{A_1 A_2 E_1 E_2}{A_1 E_1 + A_2 E_2}\right) \times (\alpha_1 - \alpha_2)\, t\, L$$

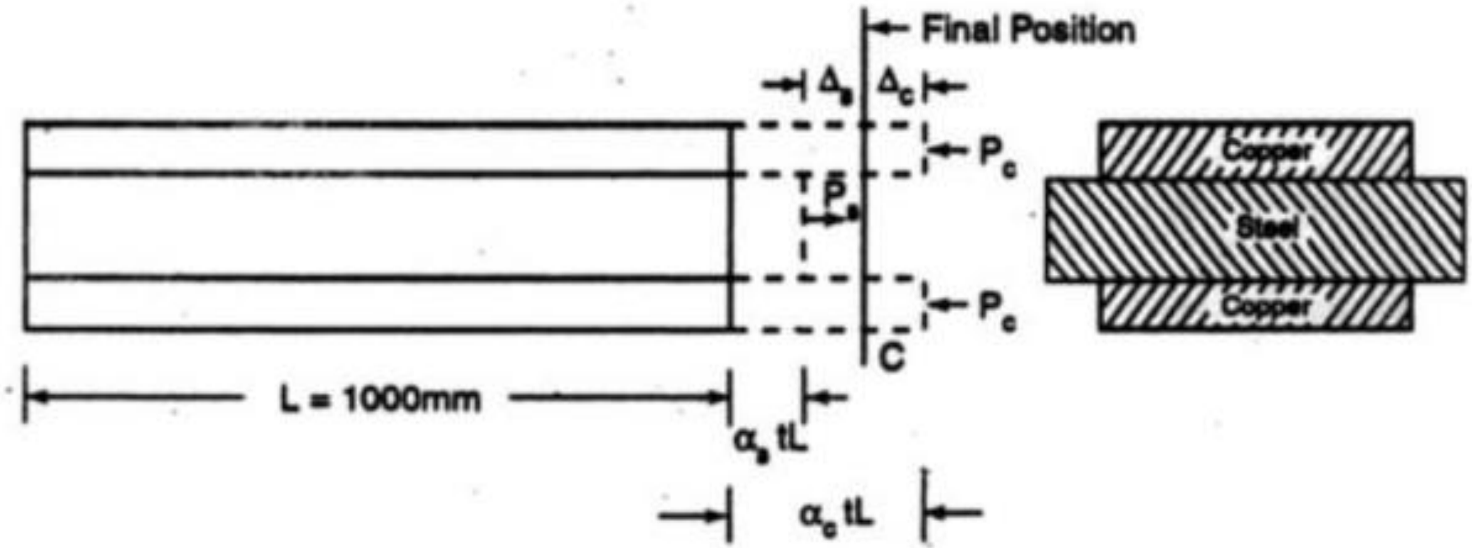
Hence stress in bar 1 is $p_1 = \dfrac{P}{A_1} = \left(\dfrac{A_2 E_1 E_2}{A_1 E_1 + A_2 E_2}\right) \times (\alpha_1 - \alpha_2)\, t$ **(comp)**.

and stress in bar 2 is, $p_2 = \dfrac{P}{A_2} = \dfrac{A_1 E_1 E_2}{A_1 E_1 + A_2 E_2} (\alpha_1 - \alpha_2)\, t$ **(tensile)**

Example A compound bar is made of a central steel plate 60 mm wide and 10mm thick to which copper plates 40 mm wide by 5 mm thick are connected rigidly on each side. The length of the bar at normal temperature is 1 metre. If the temperature is raised by 80°*C*, determine the stresses in each metal and the change in length.

Take $E_s = 200\ \text{GN/m}^2$ $E_c = 100\ \text{GN/m}^2$

$\alpha_s = 12 \times 10^{-6}/°\text{C}$ $\alpha_c = 17 \times 10^{-6}/°\text{C}$



A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temperature of the assembly is raised by 50°C, calculate the stresses developed in copper and steel. Take E for steel and copper as 200 GN/m^2 and 100 GN/m^2 and α for steel and copper as 12 x 10^{-6} per °C and 18 x 10^{-6} per °C.

Sol. Given :

Dia. of steel rod $= 20$ mm

$\therefore$ Area of steel rod, $A_s = \dfrac{\pi}{4} \times 20^2 = 100\pi \text{ mm}^2$

Area of copper tube, $A_c = \dfrac{\pi}{4}(50^2 - 40^2) \text{ mm}^2 = 225\pi \text{ mm}^2$

Rise of temperature, $T = 50°\text{C}$

E for steel, $E_s = 200 \text{ GN/m}^2$

$= 200 \times 10^9 \text{ N/m}^2$ $\quad (\because \ G = 10^9)$

$= 200 \times 10^3 \times 10^6 \text{ N/m}^2$

$= 200 \times 10^3 \text{ N/mm}^2$ $\quad (\because \ 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2)$

E for copper, $E_c = 100 \text{ GN/m}^2 = 100 \times 10^9 \text{ N/m}^2$

$= 100 \times 10^3 \times 10^6 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$

α for steel, $\alpha_s = 12 \times 10^{-6}$ per °C

α for copper, $\alpha_c = 18 \times 10^{-6}$ per °C.

Problem 1.31. *A steel tube of 30 mm external diameter and 20 mm internal diameter encloses a copper rod of 15 mm diameter to which it is rigidly joined at each end. If, at a temperature of 10°C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 200°C. Take E for steel and copper as 2.1×10^5 N/mm² and 1×10^5 N/mm² respectively. The value of co-efficient of linear expansion for steel and copper is given as 11×10^{-6} per °C and 18×10^{-6} per °C respectively.*

Sol. Given :

Dia. of copper rod $= 15$ mm

$\therefore$ Area of copper rod, $A_c = \dfrac{\pi}{4} \times 15^2 = 56.25\pi \text{ mm}^2$

Area of steel tube, $A_s = \dfrac{\pi}{4}(30^2 - 20^2) = 125\pi \text{ mm}^2$

Rise of temperature, $T = (200 - 10) = 190°C$

E for steel, $E_s = 2.1 \times 10^5 \text{ N/mm}^2$

E for copper, $E_c = 1 \times 10^5 \text{ N/mm}^2$

Value of α for steel, $\alpha_s = 11 \times 10^{-6}$ per °C

Value of α for copper, $\alpha_c = 18 \times 10^{-6}$ per °C

As the value of α for copper is more than that of steel, hence the copper rod would expand more than the steel tube if it were free. Since the two are joined together, the copper will be prevented from expanding its full amount and will be put in compression, the steel being put in tension.

A central steel rod 18 mm diameter passes through a copper tube 24 mm inside and 40 mm outside diameter, as shown in Fig. It is provided with nuts and washers at each end. The nuts are tightened until a stress of 10 MPa is set up in the steel. Find out stress generated in copper tube.