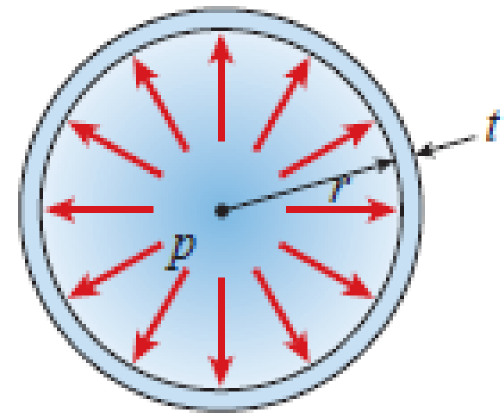
# Introduction

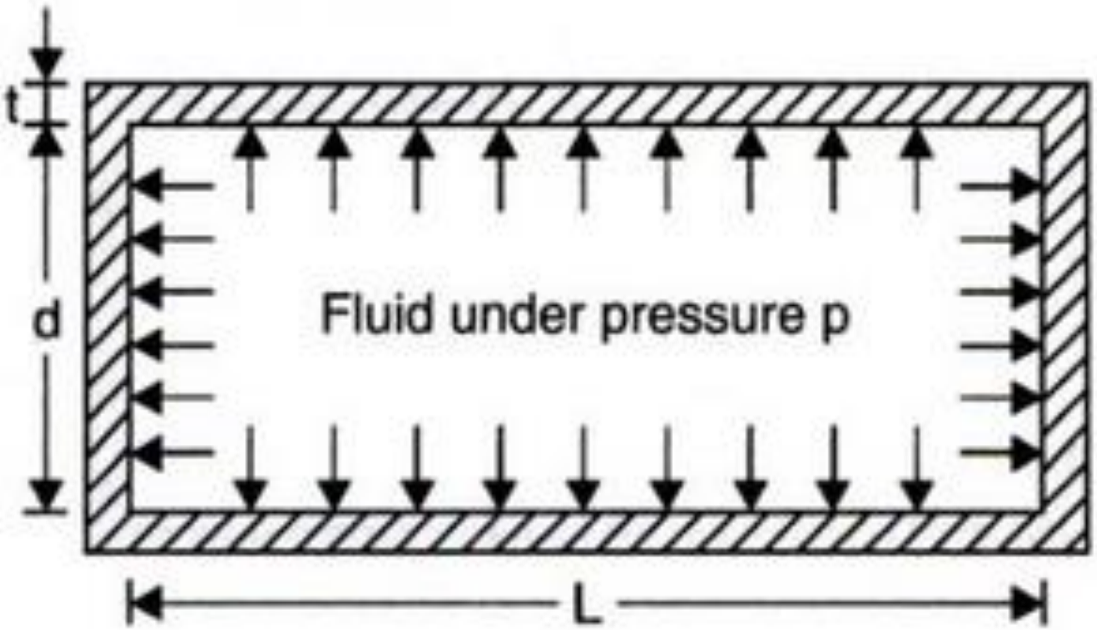
- Cylindrical pressure vessels with a circular cross section are found in industrial settings (compressed air tanks and rocket motors), in homes (fire extinguishers and spray cans), and in the countryside (propane tanks and grain silos)

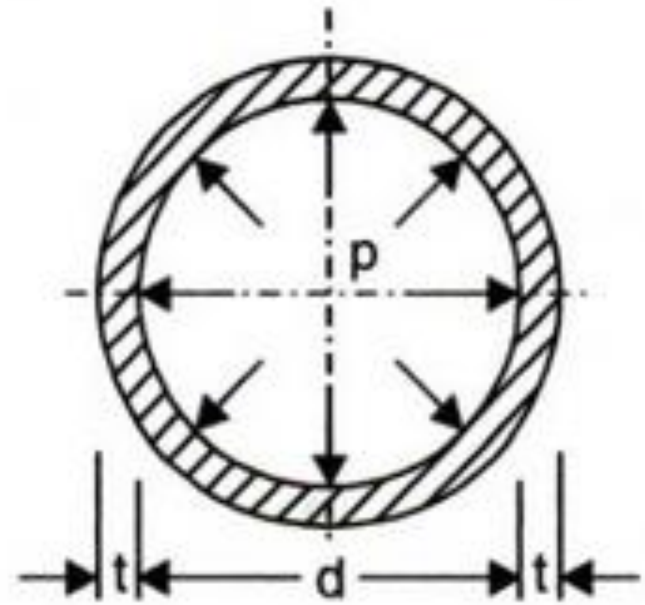


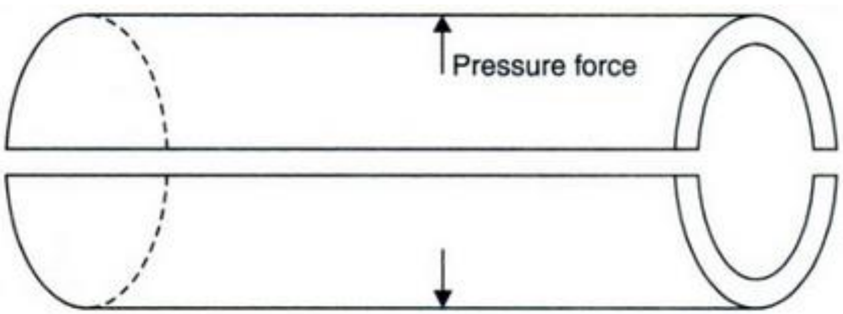

- Pressurized pipes, such as water-supply pipes and penstocks, are also classified as cylindrical pressure vessels

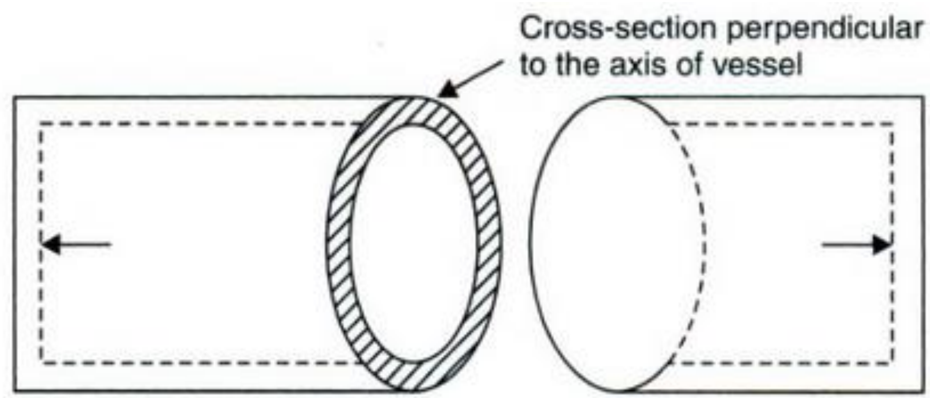
- Pressure vessels are considered to be thin-walled when the ratio of radius $r$ to wall thickness $t$ is greater than 10

# Failure of Vessel due to Internal Pressure



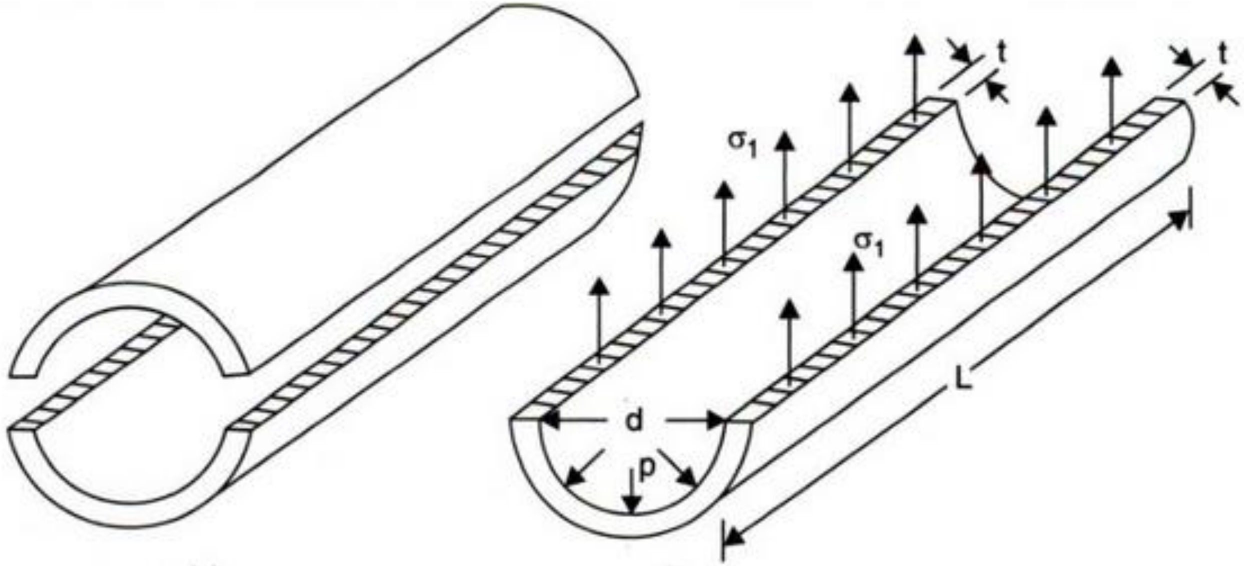



# Stresses in Thin-walled Cylinders

## Hoop or Circumferential Stress




Force due to fluid pressure $= p \times \text{Area on which } p \text{ is acting}$

$$= p \times d \times L$$

Force due to circumferential stress

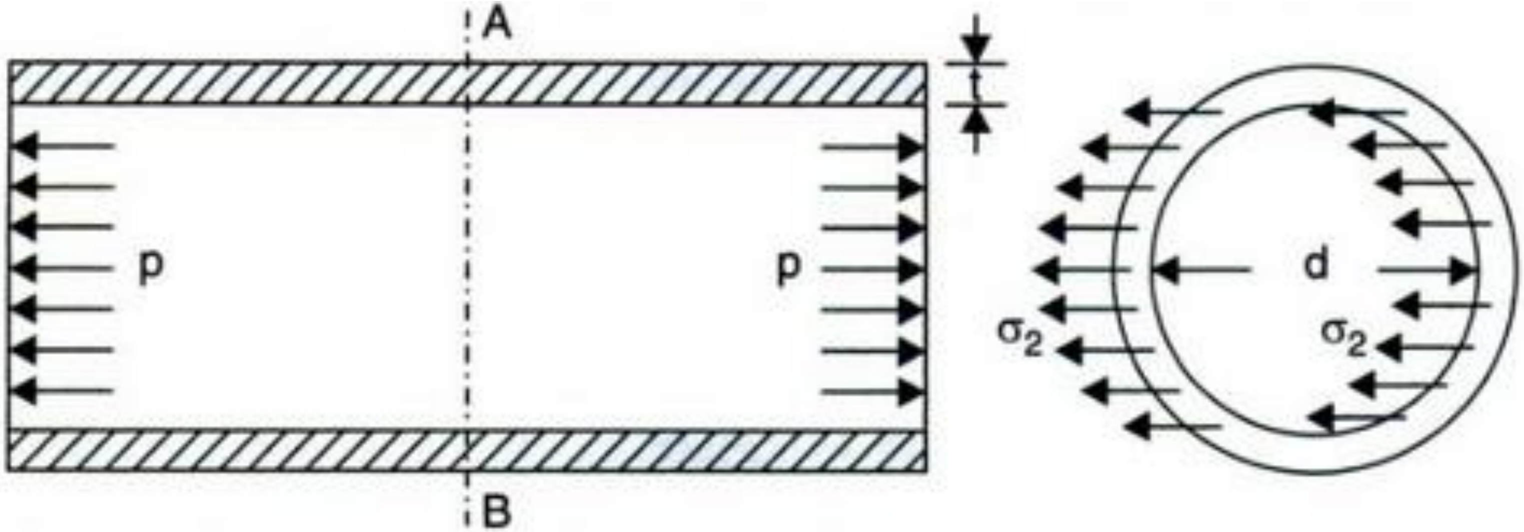
$$= \sigma_1 \times \text{Area on which } \sigma_1 \text{ is acting}$$

$$= \sigma_1 \times 2Lt$$

$$\sigma_1 \times 2Lt = p \times d \times L$$

$$\boxed{\sigma_1 = \frac{pd}{2t}}$$

# Longitudinal Stress




Force due to fluid pressure $= p \times \text{Area on which } p \text{ is acting}$

$$= p \times \frac{\pi}{4} d^2$$

Resisting force $= \sigma_2 \times \text{Area on which } \sigma_2 \text{ is acting}$

$$= \sigma_2 \times \pi d t$$

$$\sigma_2 \times \pi d t = p \times \frac{\pi}{4} d^2$$

$$\boxed{\sigma_2 = \frac{pd}{4t}}$$

# Maximum Shear Stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

$$\boxed{\tau_{max} = \frac{pd}{8t}}$$

## Effect of Internal Pressure on the Dimensions of Thin-walled Cylinder

Circumferential strain,

$$e_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$$

$$e_1 = \frac{pd}{2tE} - \frac{\nu pd}{4tE}$$

$$e_1 = \frac{pd}{2tE}\left(1 - \frac{\nu}{2}\right)$$

Circumferential strain is also gives as

$$\boxed{e_1 = \frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\nu}{2}\right)}$$

Longitudinal strain,

$$e_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E}$$

$$e_1 = \frac{pd}{4tE} - \frac{\nu pd}{2tE}$$

$$e_1 = \frac{pd}{2tE}\left(\frac{1}{2} - \nu\right)$$

Longitudinal strain is also gives as

$$\boxed{e_2 = \frac{\delta L}{L} = \frac{pd}{2tE}\left(\frac{1}{2} - \nu\right)}$$

Volumetric strain is gives as

$$\boxed{e_v = \frac{\delta V}{V} = \frac{pd}{2tE}\left(\frac{5}{2} - 2\nu\right)}$$

# Problem 1

A cylindrical thin drum 80 cm in diameter and 3 m long has a shell thickness of 1 cm. If the drum is subjected to an internal pressure of 2.5 N/mm$^2$, determine (i) change in diameter (ii) change in length and (iii) change in volume. Take *E* = 2 x 10$^5$ N/mm$^2$ and Poisson's ratio = 0.25.

$$\boxed{(a)\ \delta d = 0.035\ \mathrm{cm}}$$

$$\boxed{(b)\ \delta L = 0.0375\ \mathrm{cm}}$$

$$\boxed{(c)\ \delta V = 1507.96\ \mathrm{cm}^3}$$

(*i*) Longitudinal stress is given by equation (17.2) as

$$\sigma_2 = \frac{p \times d}{4 \times t}$$

$$= \frac{3 \times 250}{4 \times 4} = \mathbf{46.875\ N/mm^2.} \quad \textbf{Ans.}$$

(*ii*) Hoop stress is given by equation (17.1) as

$$\sigma_1 = \frac{p \times d}{2 \times t}$$

$$= \frac{3 \times 250}{2 \times 4} = \mathbf{93.75\ N/mm^2.} \quad \textbf{Ans.}$$

(*iii*) The change in diameter is given by equation (17.11) as

$$\delta d = \frac{p \times d^2}{2t \times E}\left(1 - \frac{1}{2} \times \mu\right)$$

$$= \frac{3 \times 250^2}{2 \times 4 \times 2.1 \times 10^5}\left(1 - \frac{1}{2} \times 0.286\right) = \mathbf{0.0956\ mm.} \quad \textbf{Ans.}$$

(*iv*) The change in length is given by equation (17.14) as

$$\delta L = \frac{p \times d \times L}{2E \times t}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{3 \times 250 \times 750}{2 \times 2.1 \times 10^5 \times 4}\left(\frac{1}{2} - 0.286\right) = \mathbf{0.0716\ mm.} \quad \textbf{Ans.}$$

(*v*) The change in volume is given by equation (17.17) as

$$\frac{\delta V}{V} = \frac{p \times d}{2t \times E}\left(\frac{5}{2} - 2 \times \mu\right)$$

$$= \frac{3 \times 250}{2 \times 4 \times 2.1 \times 10^5}\left(\frac{5}{2} - 2 \times 0.286\right)$$

# Problem 2

A closed cylindrical vessel made of steel pates 4 mm thick with plane ends, carries fluid under a pressure of 2.5 N/mm$^2$. The diameter of the cylinder is 25 cm and length is 75 cm, calculate the longitudinal and hoop stresses in the cylinder wall and determine the change in diameter, length and volume of the cylinder. Take $E$ = 2.1 x 10$^5$ N/mm$^2$ and Poisson's ratio = 0.286.

$$\boxed{\sigma_1 = 93.75\ \mathrm{N/mm^2}}$$

$$\boxed{\sigma_2 = 46.875\ \mathrm{N/mm^2}}$$

$$\boxed{\delta d = 0.0956\ \mathrm{mm}}$$

$$\boxed{\delta L = 0.0716\ \mathrm{mm}}$$

$$\boxed{\delta V = 31680\ \mathrm{mm^3}}$$