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School of Mechanical and Building Sciences

BMEE 202 Strength of Materials

Unit 1

Stresses and Strains

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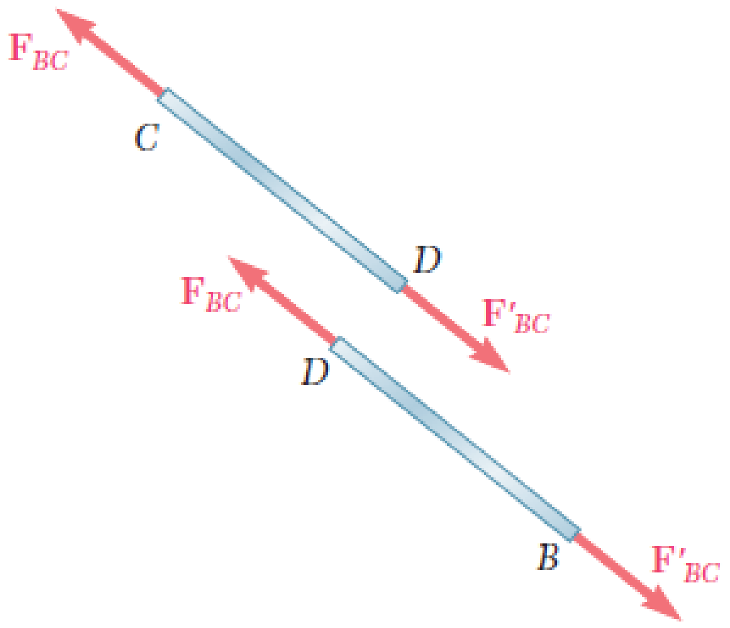
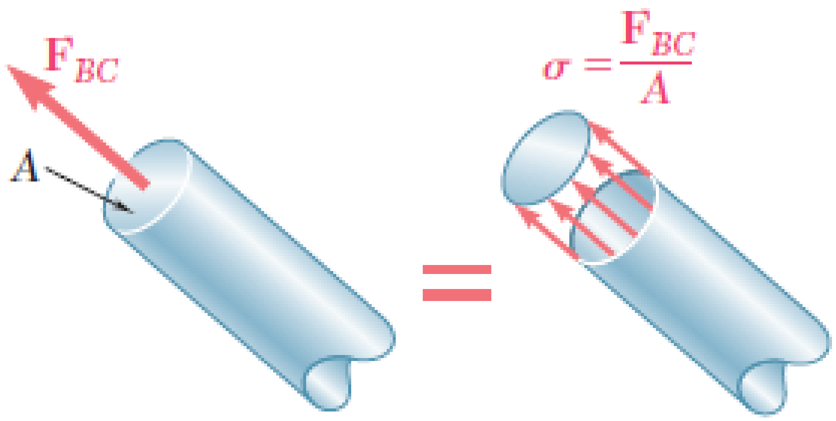
Outline

- Introduction
- Normal stress
- Shear stress
- Bearing stress
- Strain
- Stress-strain curves
- Elastic constants
- Poisson’s ratio
- Relation between elastic constants
- Thermal stresses
- Strain energy

Internal Force

Since 50-kN forces must be applied at *D* to both portions of the rod to keep them in equilibrium, we conclude that an internal force of 50 kN is produced in rod *BC* when a 30-kN load is applied at *B.*

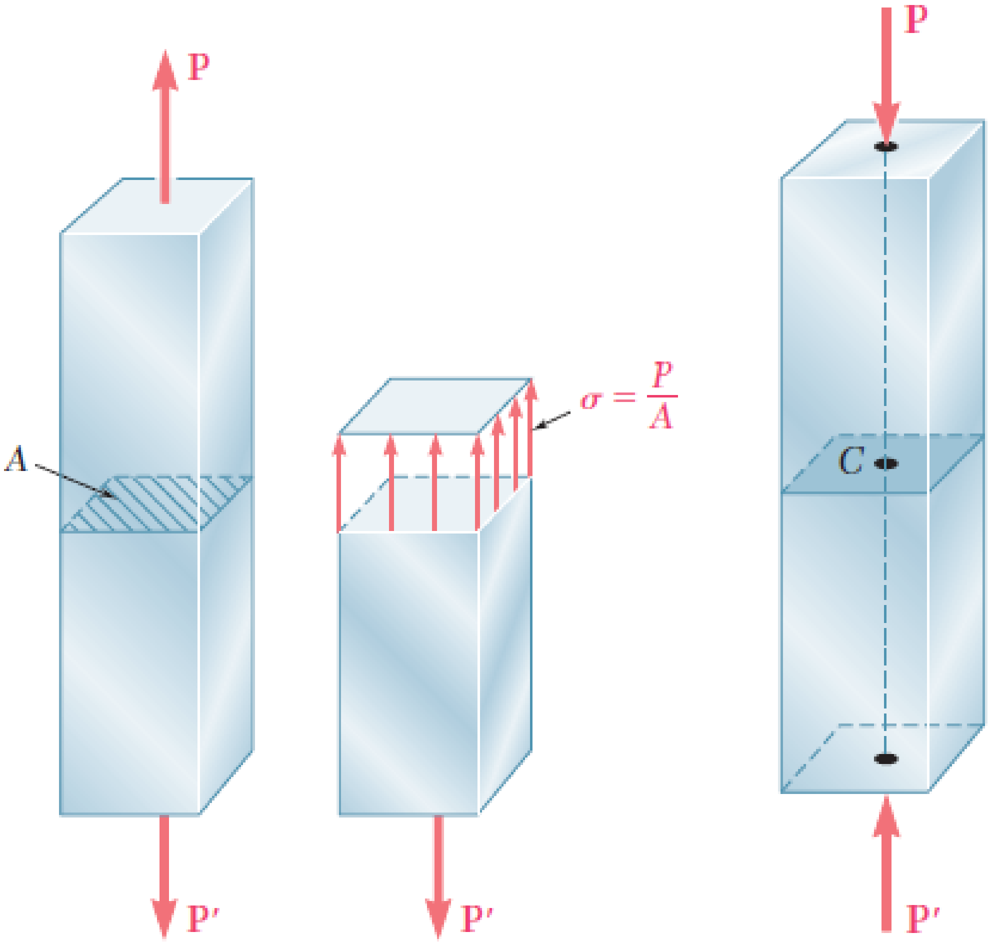




- The internal force F_{BC} actually represents the resultant of elementary forces distributed over the entire area *A* of the cross section.
- the average intensity of these distributed forces is equal to the force per unit area.

Normal Stress

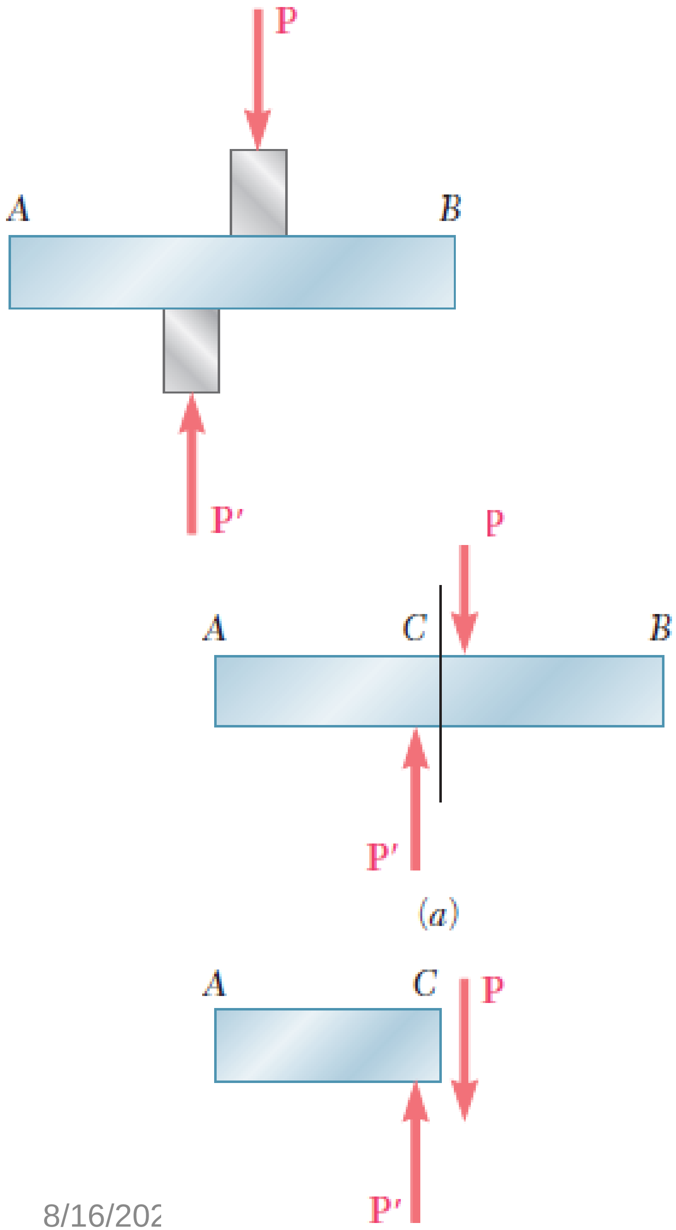
The force per unit area, or intensity of the forces distributed over a given section, is called the *stress.*



$$\sigma = \frac{P}{A}$$

- Tensile stress
- Compressive stress
- Unit MPa or N/mm^2

Shear Stress

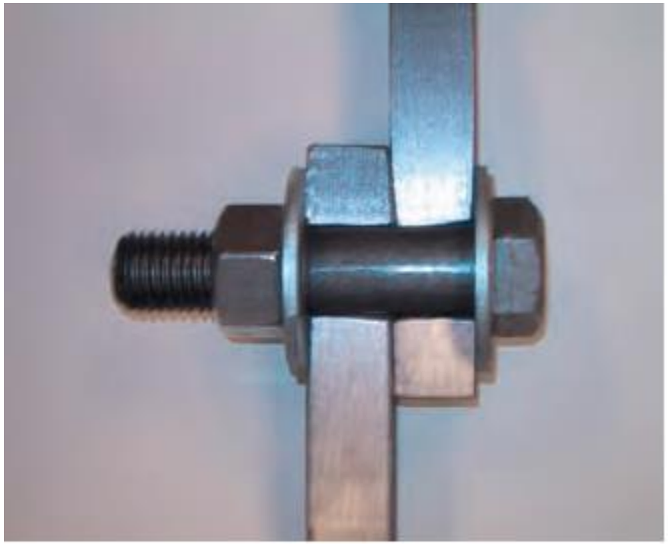


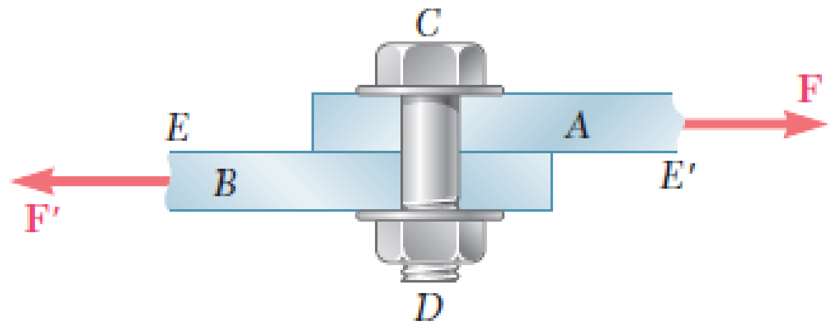
- At **C**, the Internal forces must exist in the plane of the section, and that their resultant is equal to **P**.
- These elementary internal forces are called *shearing forces*, and the magnitude *P* of their resultant is the *shear* in the section.

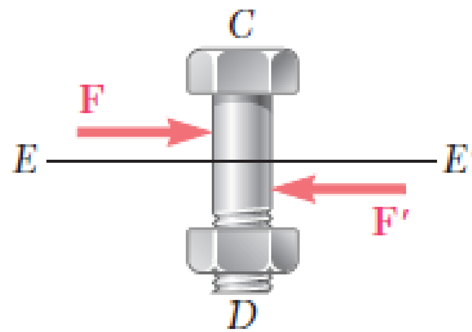
$$\tau_{ave} = \frac{P}{A}$$

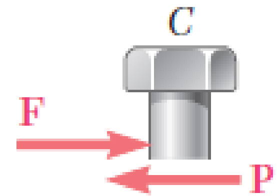
The distribution of shearing stresses across the section *cannot* be assumed uniform.

Single Shear



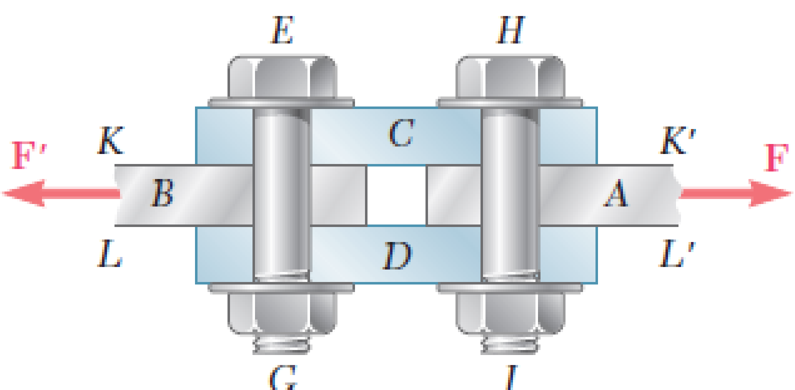


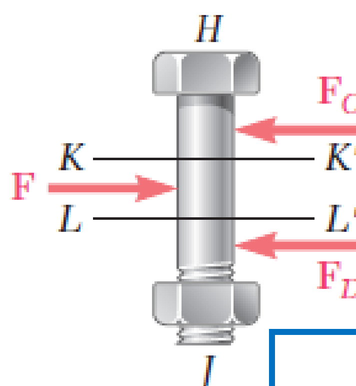


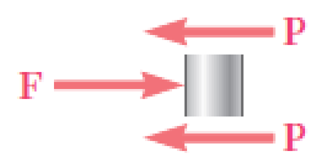


$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

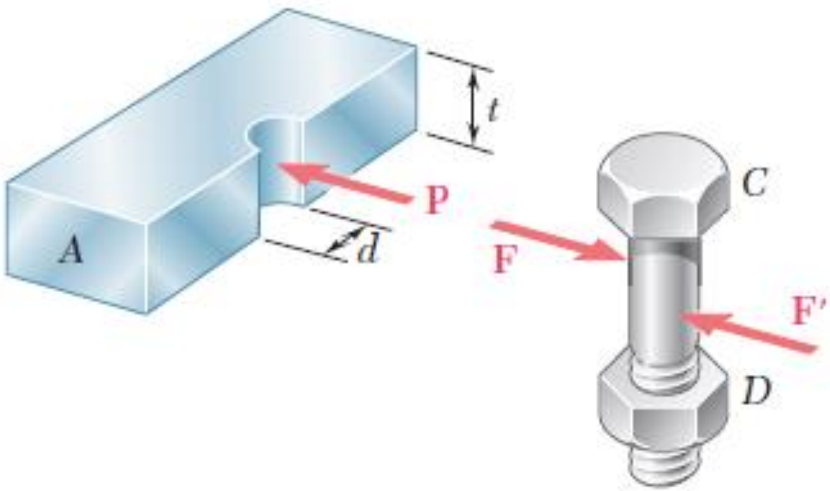


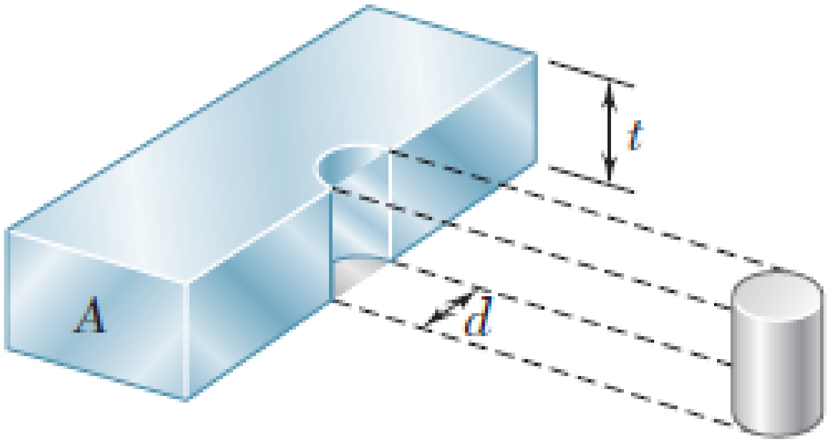




$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A}$$

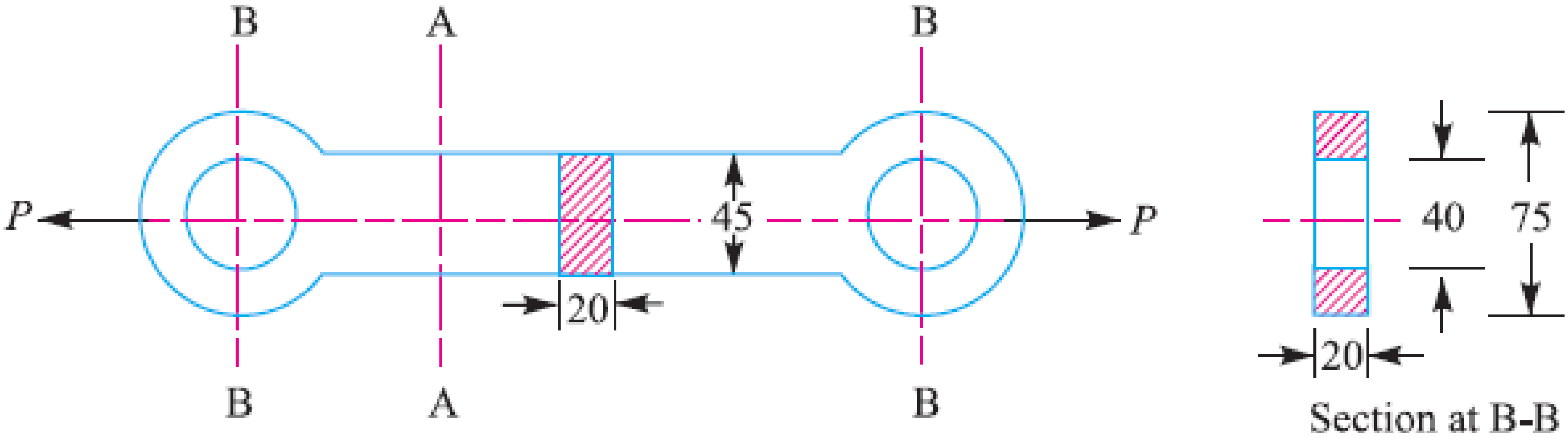
Bearing Stress





$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

Problem 1



A cast iron link as shown in Figure, is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B.

$\sigma_{A\text{-}A}$ = 50 MPa
$\sigma_{B\text{-}B}$ = 64.3 MPa

Solution. Given : $P = 45 \text{ kN} = 45 \times 10^3 \text{ N}$

Tensile stress induced at section A-A

We know that the cross-sectional area of link at section *A-A*,

$$A_1 = 45 \times 20 = 900 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section *A-A*,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{900} = 50 \text{ N/mm}^2 = 50 \text{ MPa} \quad \textbf{Ans.}$$

Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

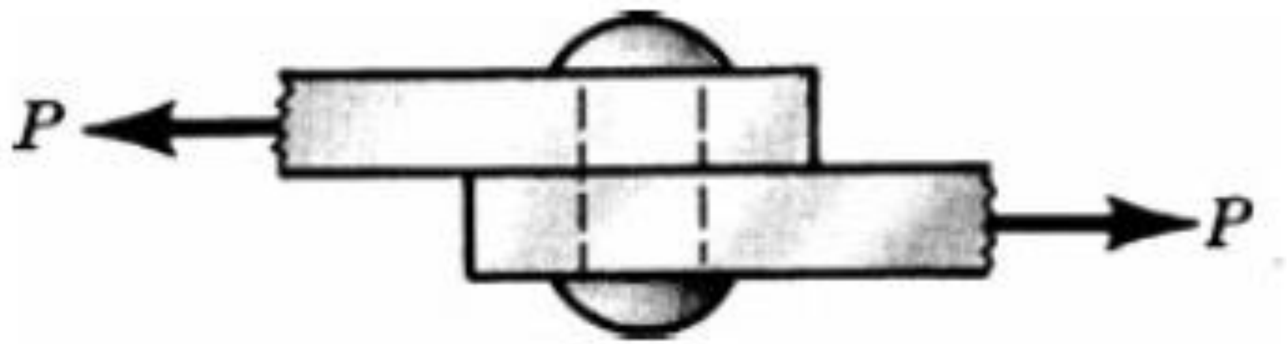
$$A_2 = 20\,(75 - 40) = 700 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section *B-B*,

$$\sigma_{t2} = \frac{P}{A_2} = \frac{45 \times 10^3}{700} = 64.3 \text{ N/mm}^2 = 64.3 \text{ MPa} \quad \textbf{Ans.}$$

Problem 2

A single rivet is used to join two plates as shown in Figure. If the diameter of the rivet is 20 mm and the load P is 30 kN, what is the average shearing stress developed in the rivet?

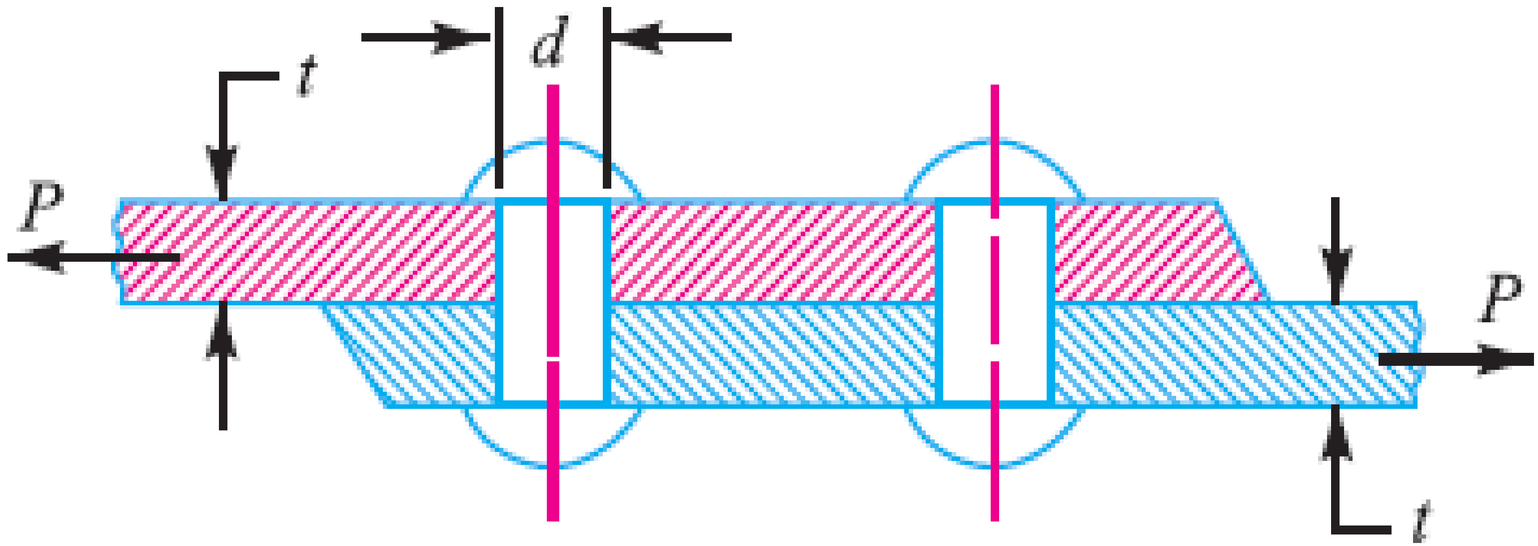


$$\tau = \frac{30{,}000\ \text{N}}{(\pi/4)[0.0215\ \text{m}]^2}$$

$$= 8.26 \times 10^7\ \text{N/m}^2$$

82.6 MPa

Problem 4



Two plates 16 mm thick are joined by a double riveted lap joint as shown in figure. The rivets are 25 mm in diameter. Find the crushing stress induced between the plates and rivet, if the maximum tensile load on the joint is 48 kN.

σ_c = 60 MPa

Solution. Given : $t = 16$ mm ; $d = 25$ mm ;

Fig. 4.11

$P = 48$ kN $= 48 \times 10^3$ N

Since the joint is double riveted, therefore, strength of two rivets in bearing (or crushing) is taken. We know that crushing stress induced between the plates and the rivets,

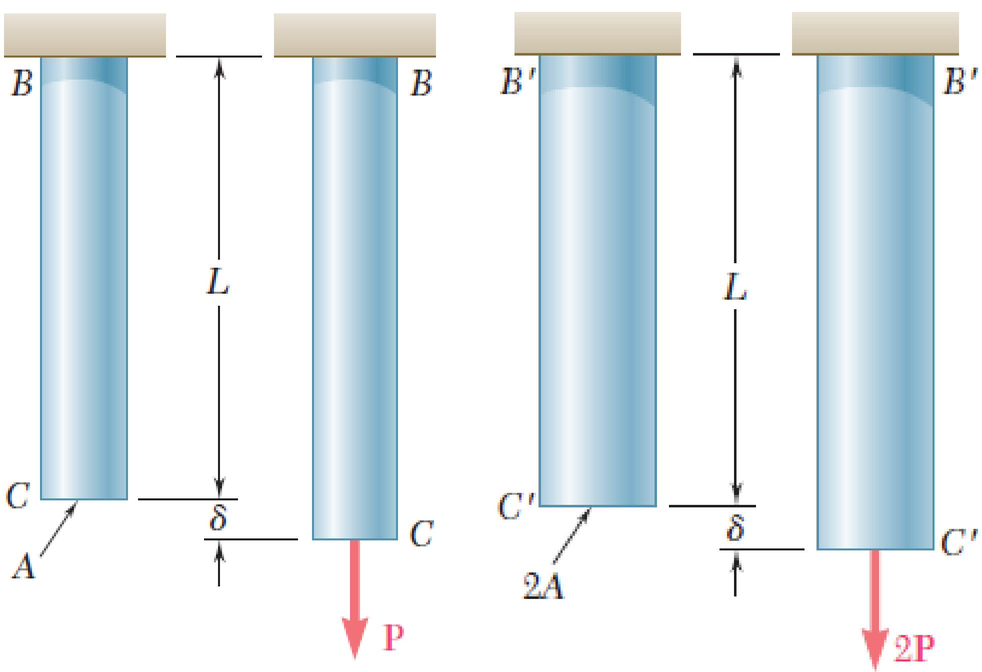
$$\sigma_c = \frac{P}{d.t.n} = \frac{48 \times 10^3}{25 \times 16 \times 2} = 60 \text{ N/mm}^2 \textbf{ Ans.}$$

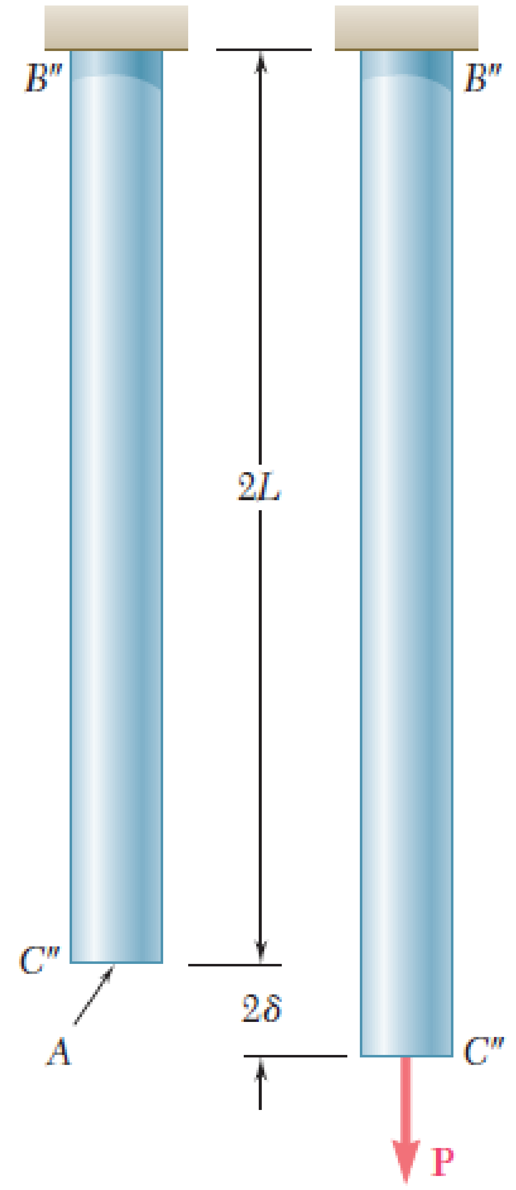
Example 4.9. *A journal 25 mm in diameter supported in sliding bearings has a maximum end*

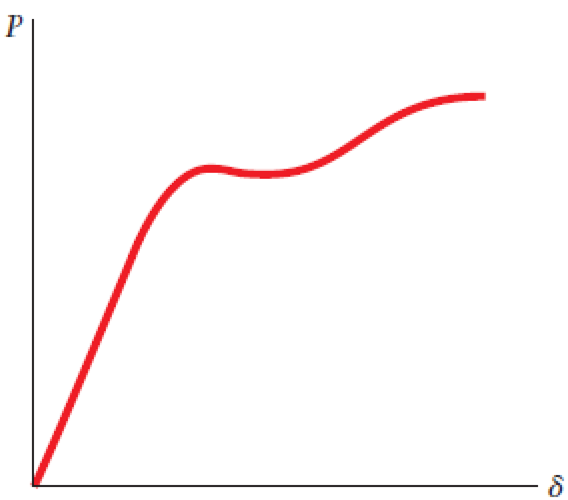
Why deformation?

- The analysis of deformations help us in the determination of stresses.
- It is not always possible to determine the forces in the members of a structure by applying only the principles of statics.
- By analyzing the deformations, it will be possible for us to compute forces that are statically indeterminate.
- The distribution of stresses in a given member is statically indeterminate, even when the force in that member is known.
- To determine the actual distribution of stresses within a member, it is thus necessary to analyze the deformations that take place in that member.

Normal Strain





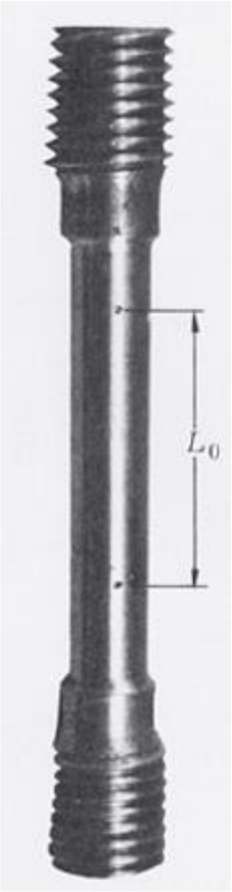


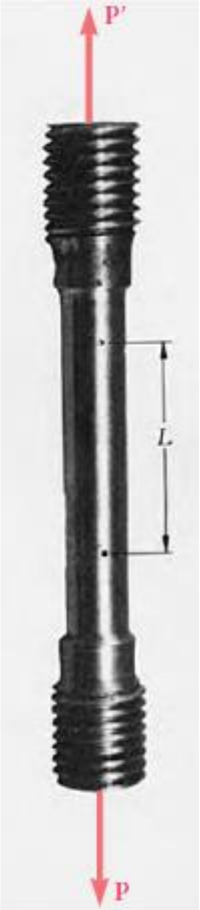
$$\epsilon = \frac{\delta}{L}$$

Plotting the stress $\sigma = P/A$ against the strain $\varepsilon = \delta/L$, we obtain a curve that is characteristic of the properties of the material and does not depend upon the dimensions of the particular specimen used. This curve is called a *stress-strain diagram.*

Stress-Strain Diagram

The diagram representing the relation between stress and strain in a given material is an important characteristic of the material.





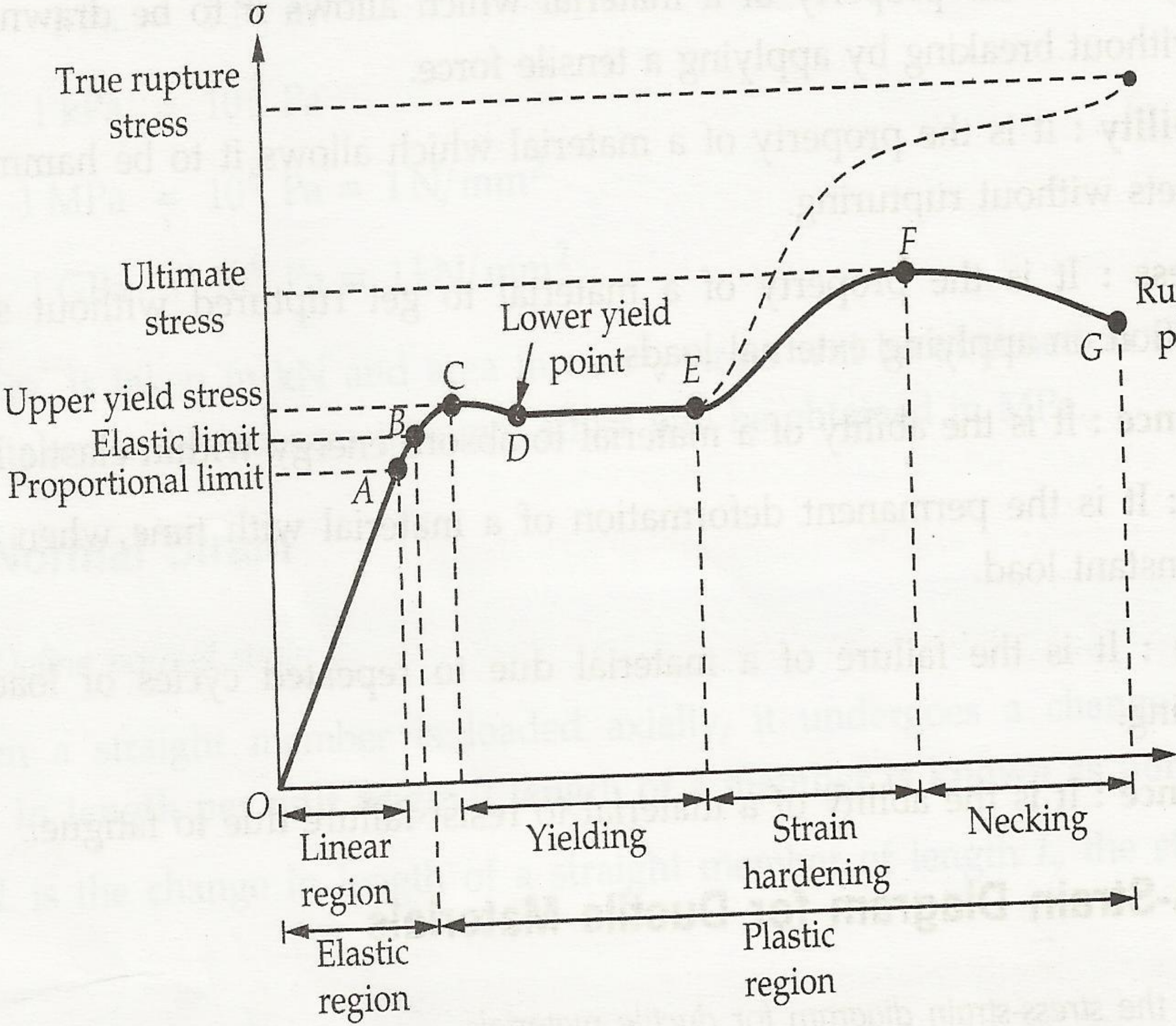

σ
True rupture stress
Ultimate stress
Upper yield stress
Elastic limit
Proportional limit
Lower yield point
A
B
C
D
E
F
G
Rupture point
O
ε
Linear region
Yielding
Strain hardening
Necking
Elastic region
Plastic region

Fig. 1.6.1

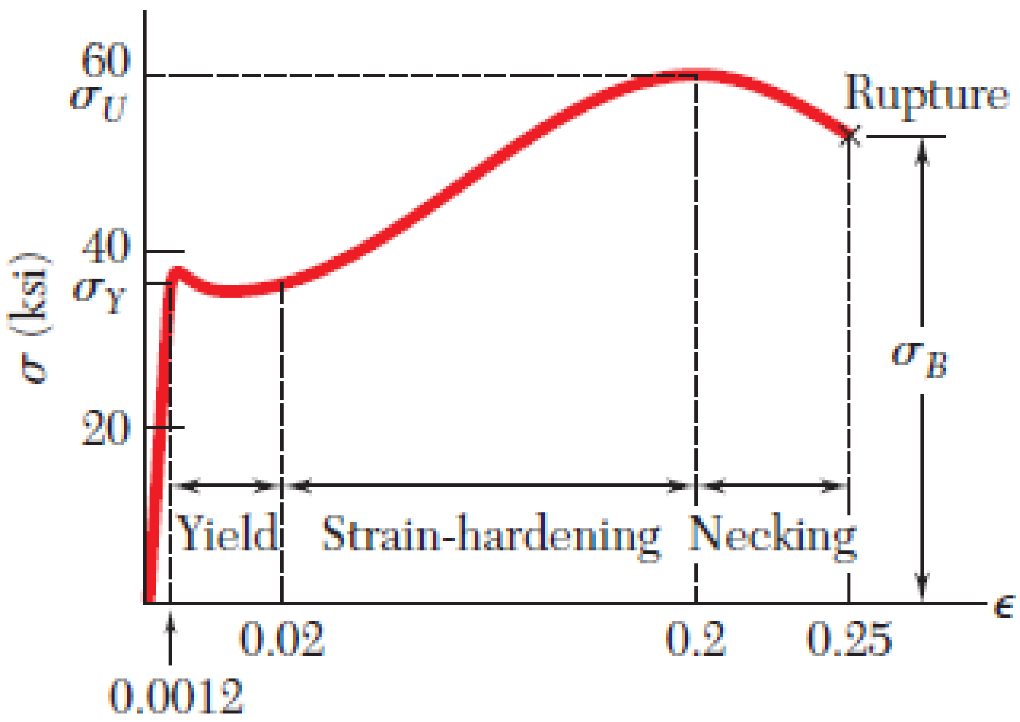
- PROPORTIONAL LIMIT
- STRESS IS PROPORTIONAL TO STRAIN
- ELASTIC LIMIT
- STRAIN DISAPPEARS COMPLETELY
- UPPER YIELD POINT
- LOAD STARTS REDUCING AND EXTENSION INCREASES
- LOWER YIELD POINT
- AT THIS STAGE STRESS REMAINS SAME BUT STRAIN INCREASES.
- ULTIMATE STRESS
- MAX STRESS THE MATREIAL CAN RESIST.NECK FORMATION STARTED.
- BREAKING POINT

Strain Hardening

- Strain Hardening is when a metal is strained beyond the yield point.
- An increasing stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform.

Stress-Strain Diagram for Ductile Material

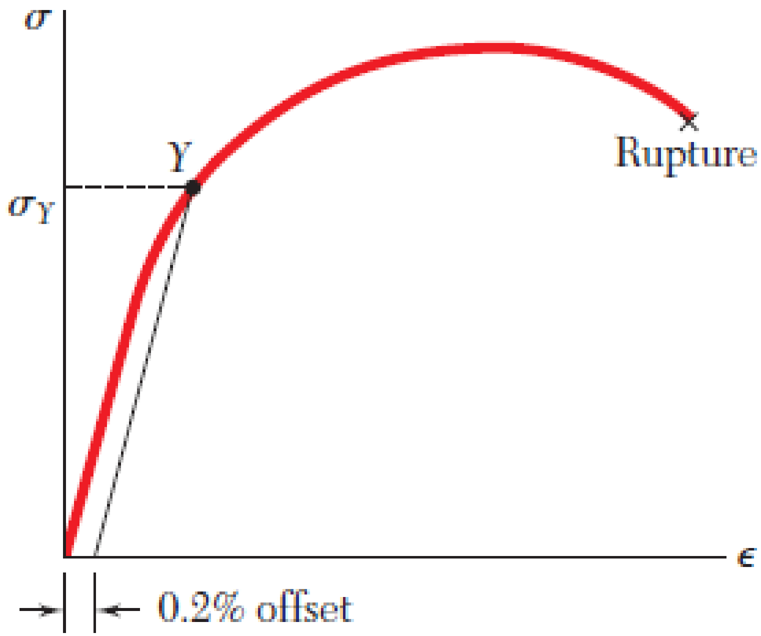
Low Carbon Steel



- Proportional limit
- Yield point- upper and lower yield point
- Ultimate point
- Fracture point

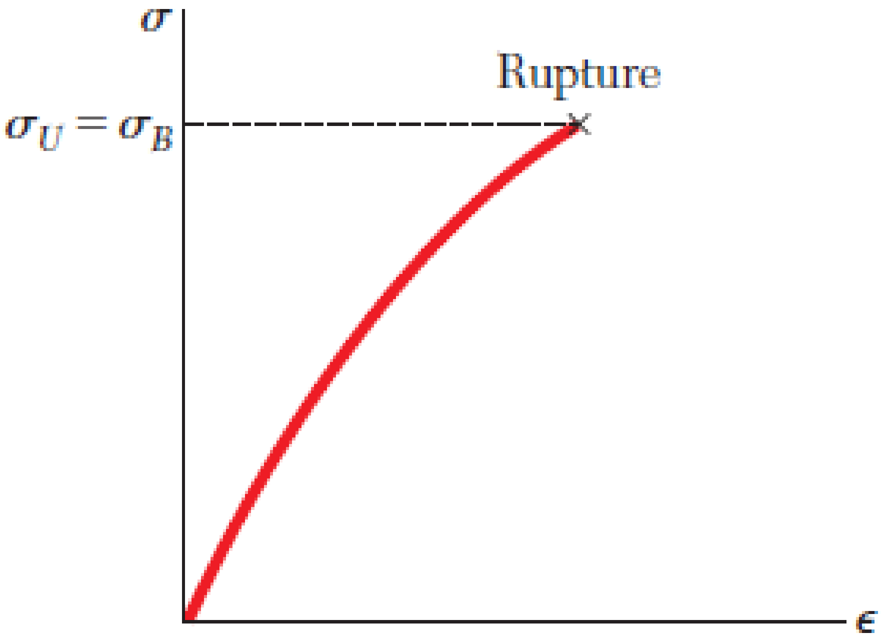
Stress-Strain Diagram for Ductile Material

Aluminium



- The onset of yield is not characterized by a horizontal portion of the stress-strain curve.
- Instead, the stress keeps increasing— although not linearly—until the ultimate strength is reached.
- Necking then begins, leading eventually to rupture.
- The yield strength is at 0.2% offset, (a line parallel to the initial straight-line portion of the stress-strain diagram)

Stress-Strain Diagram for Brittle Material



- Rupture occurs without any noticeable prior change in the rate of elongation.
- There is no difference between the ultimate strength and the breaking strength.
- The strain at the time of rupture is much smaller for brittle than for ductile materials.
- No necking.

Ductility

- It is a measure of the degree of plastic deformation that has been sustained at fracture.
- Ductility may be expressed quantitatively as either percent elongation (%EL) or percent reduction in area (%RA).

$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$\%RA = \left(\frac{A_0 - A_f}{A_0}\right) \times 100$$

Hooke's Law - Modulus of Elasticity

- When a member is loaded within elastic limit, the stress is proportional to strain.
- The constant of proportionality is known as Modulus of elasticity or Young's Modulus.
- Young's Modulus is the measure of stiffness of the material.

$$\sigma = E\epsilon$$

Change in Length (δL)

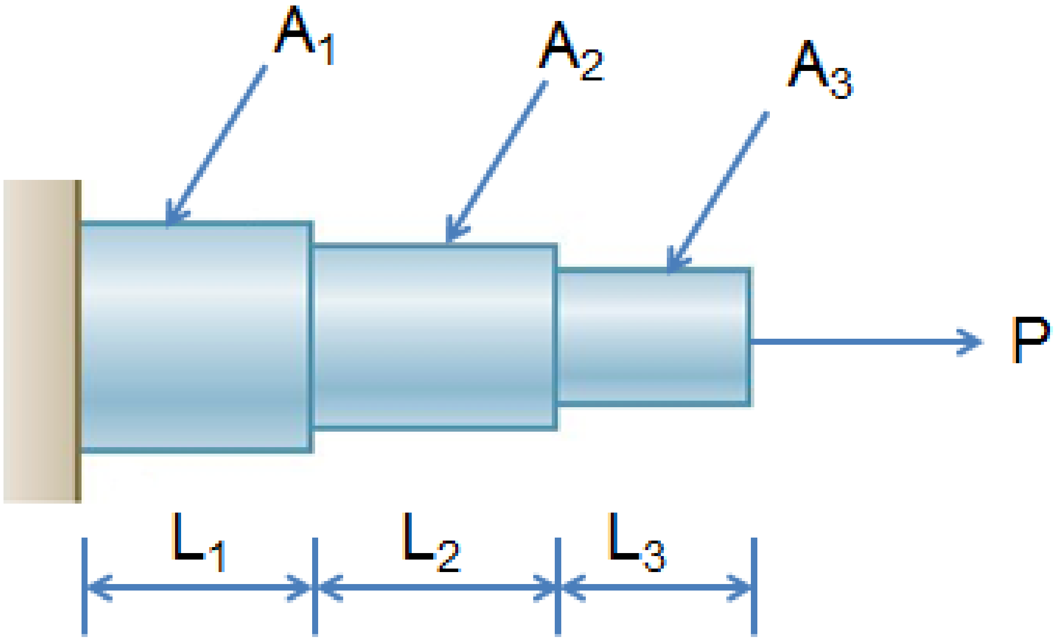
$$\sigma = \frac{P}{A} \qquad \epsilon = \frac{\delta L}{L}$$

$$\sigma = E\epsilon$$

$$\frac{P}{A} = E\frac{\delta L}{L}$$

$$\boxed{\delta L = \frac{PL}{AE}}$$

Change in Length (δL)



$$\delta L_1 = \frac{PL_1}{A_1 E}$$

$$\delta L_2 = \frac{PL_2}{A_2 E}$$

$$\delta L_3 = \frac{PL_3}{A_3 E}$$

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$\boxed{\delta L = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]}$$

Analysis of Axially loaded members

- Bars of uniform cross section
- Bars of varying cross sections (Stepped bars)
- Principle of superposition
- Bars of composite sections

Example 4.10. *A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded :*

Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN.

Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation.

Solution. Given : $D = 12$ mm ; $l = 60$ mm ; $L = 80$ mm ; $d = 7$ mm ; $W_y = 3.4$ kN $= 3400$ N; $W_u = 6.1$ kN $= 6100$ N

We know that original area of the rod,

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} (12)^2 = 113 \text{ mm}^2$$

and final area of the rod,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} (7)^2 = 38.5 \text{ mm}^2$$

1. *Yield stress*

We know that yield stress

$$= \frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 = 30.1 \text{ MPa} \quad \textbf{Ans.}$$

2. *Ultimate tensile stress*

We know the ultimate tensile stress

2. Ultimate tensile stress

We know the ultimate tensile stress

$$= \frac{W_u}{A} = \frac{6100}{113} = 54 \text{ N/mm}^2 = 54 \text{ MPa} \quad \textbf{Ans.}$$

3. Percentage reduction in area

We know that percentage reduction in area

$$= \frac{A - a}{A} = \frac{113 - 38.5}{113} = 0.66 \text{ or } 66\% \quad \textbf{Ans.}$$

4. Percentage elongation

We know that percentage elongation

$$= \frac{L - l}{L} = \frac{80 - 60}{80} = 0.25 \text{ or } 25\% \quad \textbf{Ans.}$$

Bars of Uniform Cross Section

Problem 5

A tensile test was conducted on a mild steel bar. The following data was obtained from the test:

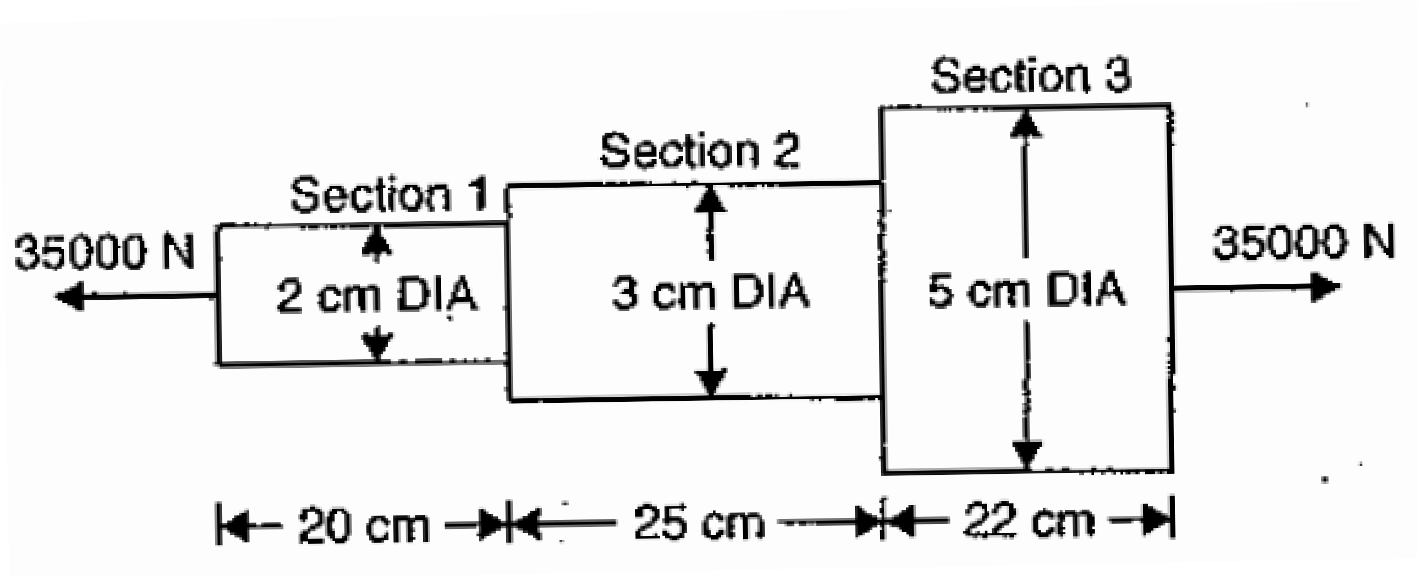
(i) Diameter of the steel bar = 3 cm
(ii) Gauge length of the bar = 20 cm
(iii) Load at elastic limit = 250 kN
(iv) Extension at a load of 150 kN = 0.21 mm
(v) Maximum load = 380 kN
(vi) Total extension = 60 mm
(vii) Diameter of the rod at the failure = 2.25 cm.

Determine:

(a) The Young's modulus;
(b) The stress at elastic limit;
(c) The percentage elongation; and
(d) The percentage reduction in area.

Bars of Varying Cross Section

Problem 6



An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in figure. If the Young's modulus = 2.1×10^5 N/mm^2, determine:

(i) Stress in each section and

(ii) Total extension of the bar.

Sol. Given :

Axial pull, $P = 35000$ N

Length of section 1, $L_1 = 20$ cm $= 200$ mm

Dia. of section 1, $D_1 = 2$ cm $= 20$ mm

$\therefore$ Area of section 1, $A_1 = \frac{\pi}{4}(20^2) = 100\,\pi$ mm^2

Length of section 2, $L_2 = 25$ cm $= 250$ mm

Dia. of section 2, $D_2 = 3$ cm $= 30$ mm

$\therefore$ Area of section 2, $A_2 = \frac{\pi}{4}(30^2) = 225\,\pi$ mm^2

Length of section 3, $L_3 = 22$ cm $= 220$ mm

Dia. of section 3, $D_3 = 5$ cm $= 50$ mm

$\therefore$ Area of section 3, $A_3 = \frac{\pi}{4}(50^2) = 625\,\pi$ mm^2

Young's modulus, $E = 2.1 \times 10^5$ N/mm^2.

(i) Stresses in each section

Stress in section 1, $\sigma_1 = \dfrac{\text{Axial load}}{\text{Area of section 1}}$

$$= \frac{P}{A_1} = \frac{35000}{100\,\pi} = \mathbf{111.408\ N/mm^2.} \quad \textbf{Ans.}$$

Stress in section 2, $\sigma_2 = \dfrac{P}{A_2} = \dfrac{35000}{225 \times \pi} = \mathbf{49.5146\ N/mm^2.}$ **Ans.**

Stress in section 3, $\sigma_3 = \dfrac{P}{A_3} = \dfrac{35000}{625 \times \pi} = \mathbf{17.825\ N/mm^2.}$ **Ans.**

(ii) Total extension of the bar

Using equation (1.8), we get

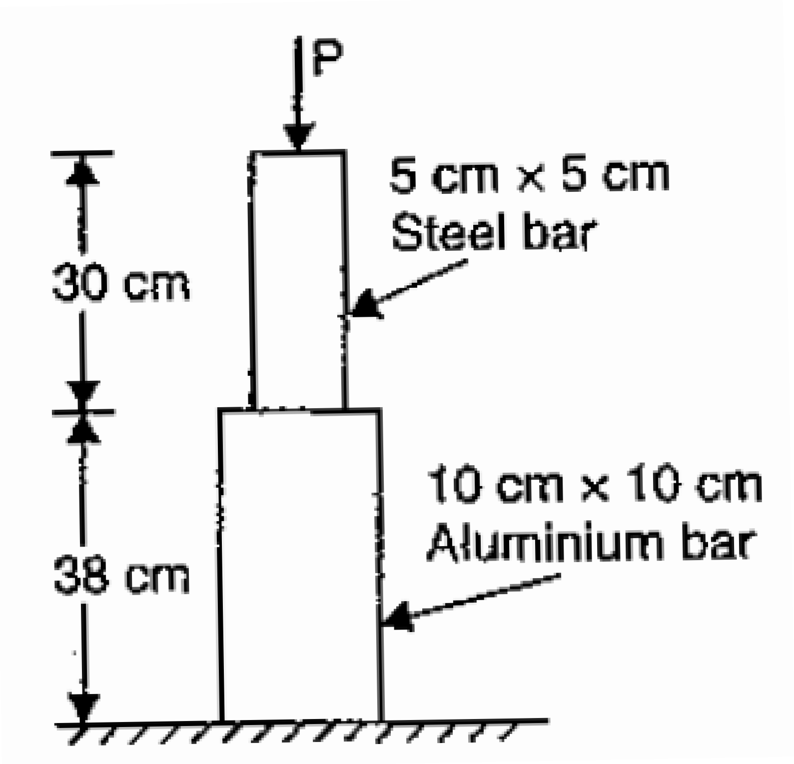
Total extension $= \dfrac{P}{E}\left(\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2} + \dfrac{L_3}{A_3}\right)$

$$= \frac{35000}{2.1 \times 10^5}\left(\frac{200}{100\,\pi} + \frac{250}{225 \times \pi} + \frac{220}{625 \times \pi}\right)$$

$$= \frac{35000}{2.1 \times 10^5}(6.366 + 3.536 + 1.120) = \mathbf{0.183\ mm.} \quad \textbf{Ans.}$$

Problem 7

A member formed by connecting a steel bar to an aluminium bar is shown in figure. Assuming the bars are prevented from buckling sideways, calculate the magnitude of force *P* that will cause the total length of the member to decrease 0.25 mm. The values of elastic modulus for steel and aluminium are 2.1 x 10^5 N/mm^2 and 7 x 10^4 N/mm^2 respectively.



Area of steel bar, $A_1 = 5 \times 5 = 25 \text{ cm}^2 = 250 \text{ mm}^2$

Elastic modulus for steel bar,

$$E_1 = 2.1 \times 10^5 \text{ N/mm}^2$$

Length of aluminium bar,

$$L_2 = 38 \text{ cm} = 380 \text{ mm}$$

Area of aluminium bar,

$$A_2 = 10 \times 10 = 100 \text{ cm}^2 = 10000 \text{ mm}^2$$

Elastic modulus for aluminium bar,

$$E_2 = 7 \times 10^4 \text{ N/mm}^2$$

Total decrease in length, $dL = 0.25$ mm

Let P = Required force.

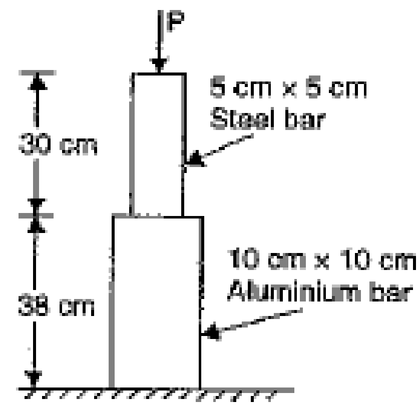


Fig. 1.7

As both the bars are made of different materials, hence total change in the lengths of the bar is given by equation (1.9).

$$\therefore \quad dL = P\left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}\right)$$

or

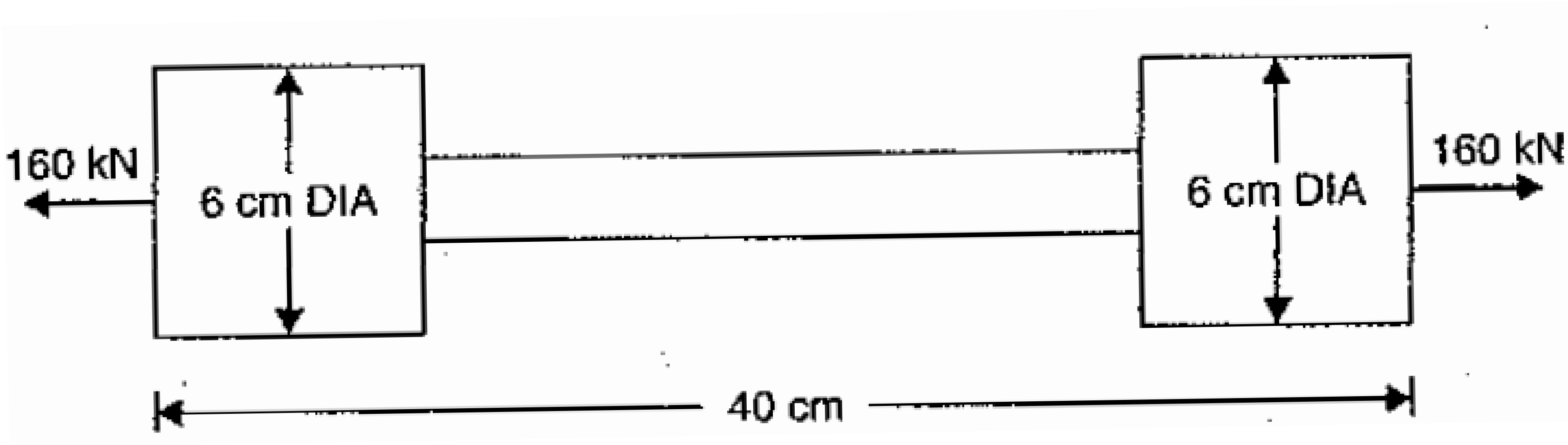
$$0.25 = P\left(\frac{300}{2.1 \times 10^5 \times 2500} + \frac{380}{7 \times 10^4 \times 10000}\right)$$

$$= P\,(5.714 \times 10^{-7} + 5.428 \times 10^{-7}) = P \times 11.142 \times 10^{-7}$$

$$\therefore \quad P = \frac{0.25}{11.142 \times 10^{-7}} = \frac{0.25 \times 10^7}{11.142}$$

$$= 2.2437 \times 10^5 = \mathbf{224.37 \text{ kN}}. \quad \textbf{Ans.}$$

Problem 8



The bar shown in figure is subjected to an tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm^2, determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm. Young's modulus is given as 2.1×10^5 N/mm^2 .

Sol. Given :

Tensile load, $P = 160 \text{ kN} = 160 \times 10^3 \text{ N}$

Stress in middle portion, $\sigma_2 = 150 \text{ N/mm}^2$

Total elongation, $dL = 0.2 \text{ mm}$

Total length of the bar, $L = 40 \text{ cm} = 400 \text{ mm}$

Young's modulus, $E = 2.1 \times 10^5 \text{ N/mm}^2$

Diameter of both end portions, $D_1 = 6 \text{ cm} = 60 \text{ mm}$

$\therefore$ Area of cross-section of both end portions,

$$A_1 = \frac{\pi}{4} \times 60^2 = 900\,\pi \text{ mm}^2.$$

Let D_2 = Diameter of the middle portion

L_2 = Length of middle portion in mm.

$\therefore$ Length of both end portions of the bar,

$$L_1 = (400 - L_2) \text{ mm}$$

Using equation (1.1), we have

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}.$$

For the middle portion, we have

$$\sigma_2 = \frac{P}{A_2} \qquad \text{where } A_2 = \frac{\pi}{4} D_2^2$$

or
$$150 = \frac{160000}{\frac{\pi}{4} D_2^2}$$

$\therefore$
$$D_2^2 = \frac{4 \times 160000}{\pi \times 150} = 1358 \text{ mm}^2$$

or
$$D_2 = \sqrt{1358} = 36.85 \text{ mm} = \mathbf{3.685 \text{ cm.}} \quad \textbf{Ans.}$$

$\therefore$ Area of cross-section of middle portion,

$$A_3 = \frac{\pi}{4} \times 36.85 = 1066 \text{ mm}^2$$

Total extension, $$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

$$0.2 = \frac{160000}{2.1 \times 10^5}\left[\frac{(400 - L_2)}{900\pi} + \frac{L_2}{1066}\right]$$

$[\because \quad L_1 = (400 - L_2) \text{ and } A_2 = 1066]$

$$\frac{0.2 \times 2.1 \times 10^5}{160000} = \frac{(400 - L_2)}{900\,\pi} + \frac{L_2}{1066}$$

$$0.2625 = \frac{1066(400 - L_2) + 900\,\pi\,L_2}{900\,\pi \times 1066}$$

$$0.2625 \times 900\pi \times 1066 = 1066 \times 400 - 1066\,L_2 + 900\pi \times L_2$$

$$791186 = 426400 - 1066\,L_2 + 2827\,L_2$$

$$791186 - 426400 = L_2\,(2827 - 1066)$$

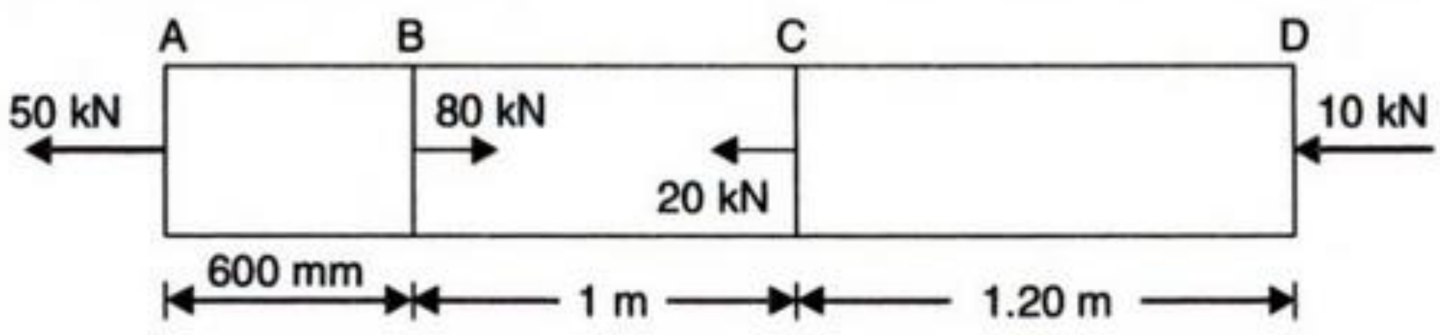
$$364786 = 1761\,L_2$$

$$\therefore \quad L_2 = \frac{364786}{1761} = 207.14 \text{ mm} = \mathbf{20.714\ cm.} \quad \textbf{Ans.}$$

Principle of Superposition

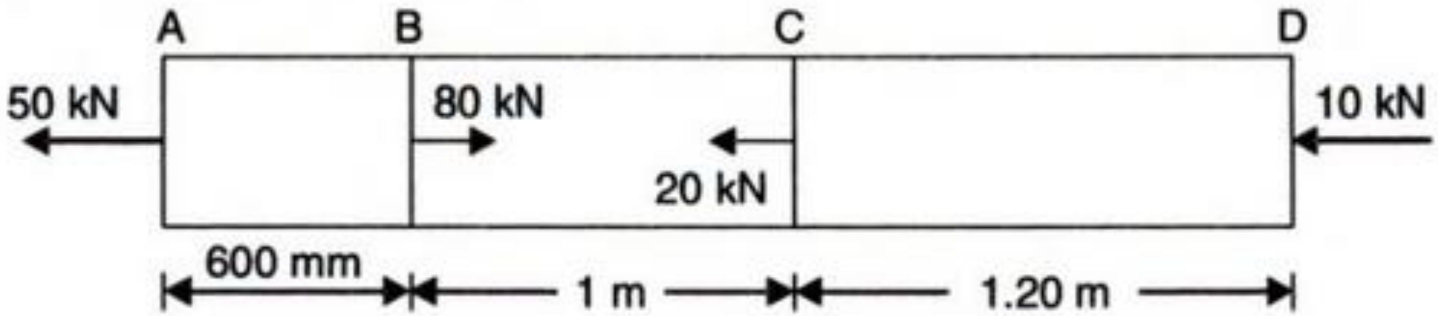
When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads.

Problem 9



A brass bar having cross sectional area of 1000 mm^2, is subjected to axial force as shown in figure. Find the total elongation of the bar. Take $E = 1.05 \times 10^5$ N/mm^2 .

$\delta L = -0.1142$ mm


A
B
C
D
50 kN
80 kN
20 kN
10 kN
600 mm
1 m
1.20 m

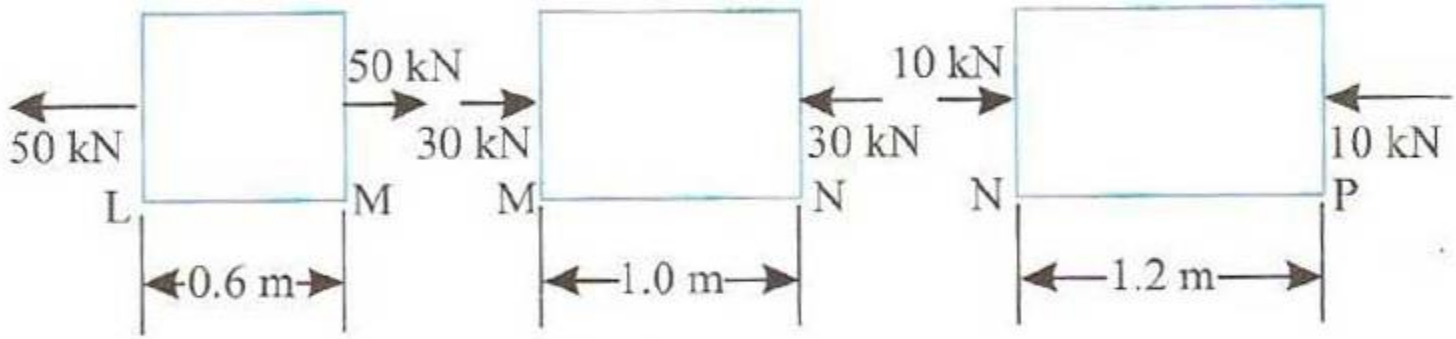


Fig. 1.13

Similarly, in portion *MN*, the compressive force on the left is 30 kN (*i.e.*, 80 – 50) a on the right (*i.e.*, 20 + 10). In *NP*, the compressive load is 10 kN, (*i.e.*, 80 – 50 – 20) and on there is already a compressive load of 10 kN. So, we observe that the bar is in equilibrium action of these forces.

Total elongation of the bar :

Let δl_1, δl_2 and δl_3 be the changes in length *LM*, *MN* and *NP* respectively.

Then,

$$\delta l_1 = \frac{P_1 l_1}{AE} \text{ .... increase } (+)$$

$$\delta l_2 = \frac{P_2 l_2}{AE} \text{ .... decrease } (-)$$

$$\delta l_3 = \frac{P_3 l_3}{AE} \text{ .... decrease } (-)$$

∴ Net change in length,

$$\delta l = \delta l_1 - \delta l_2 - \delta l_3$$

$$= \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE} = \frac{1}{AE}(P_1 l_1 - P_2 l_2 - P_3 l_3)$$

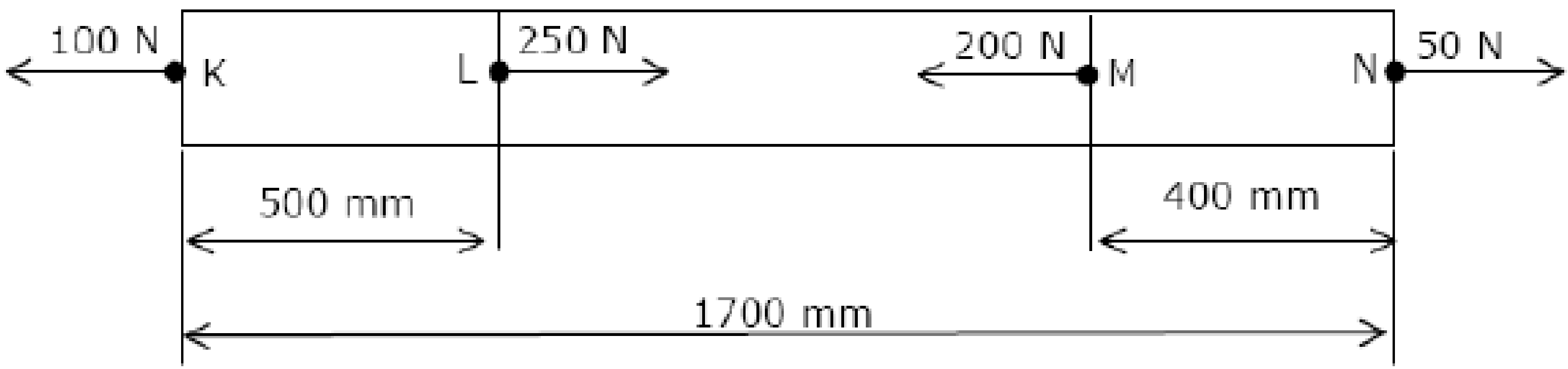
$$= \frac{10^3}{1000 \times 10^{-6} \times 100 \times 10^9}(50 \times 0.6 - 30 \times 1 - 10 \times 1.2)$$

$$= \frac{1}{10^5}(30 - 30 - 12) = \frac{1}{10^5} \times (-12)$$

$$= -0.00012 \text{ m} = -0.12 \text{ mm}$$

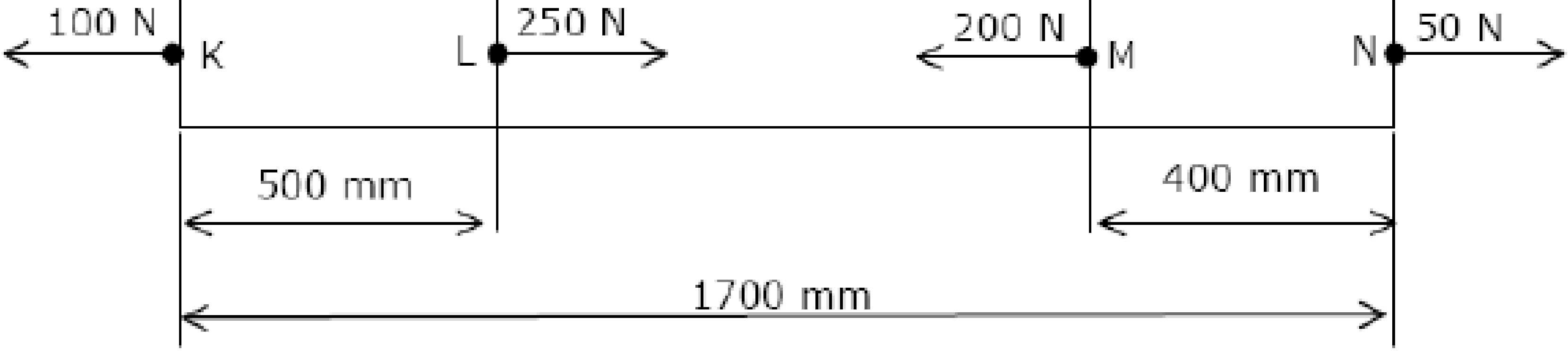
GATE 2004, IES 1995, 1997, 1998

The figure below shows a steel rod of 25 mm^2 cross sectional area. It is loaded at four points, K, L, M and N.



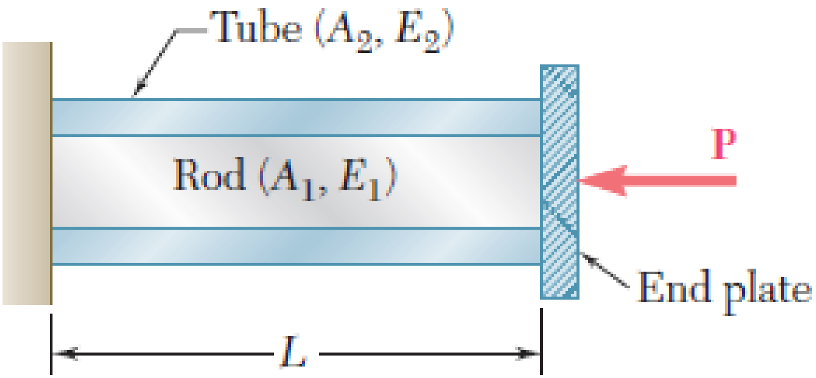
Assume E_{steel} = 200 GPa. The total change in length of the rod due to loading is:

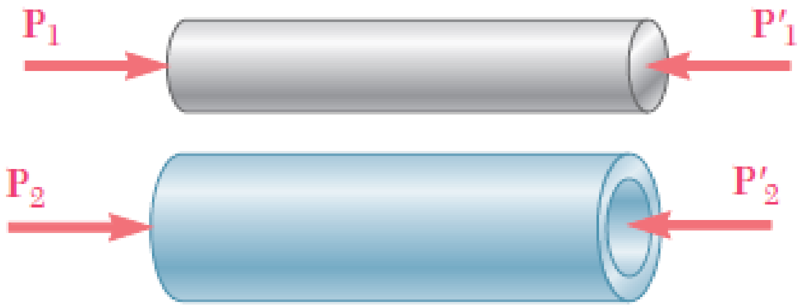
(a) 1 μm	(b) -10 μm	(c) 16 μm	(d) -20 μm


100 N
K
L
250 N
200 N
M
N
50 N
500 mm
400 mm
1700 mm

Bars of Composite Sections

A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive loads, is called a composite bar.





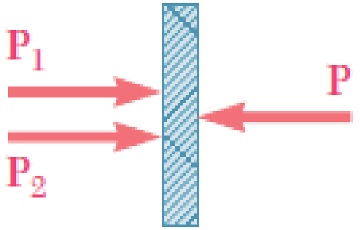
$$\boxed{\delta L_1 = \delta L_2}$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2}$$

$$\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$$

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2}$$

$$P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$



$$\boxed{P_1 + P_2 = P}$$

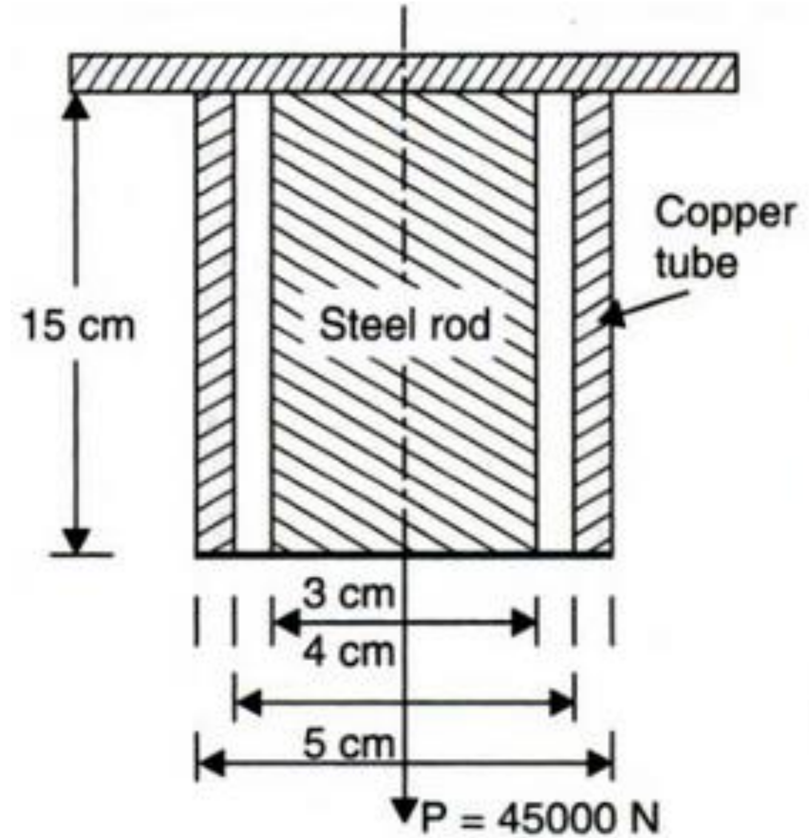
Problem 10

A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm. the composite bar is then subjected to an axial pull of 45000 N. if the length of each bar is equal to 15 cm, determine:

(i) Stress in in the rod and tube, and

(ii) Load carried by each bar.

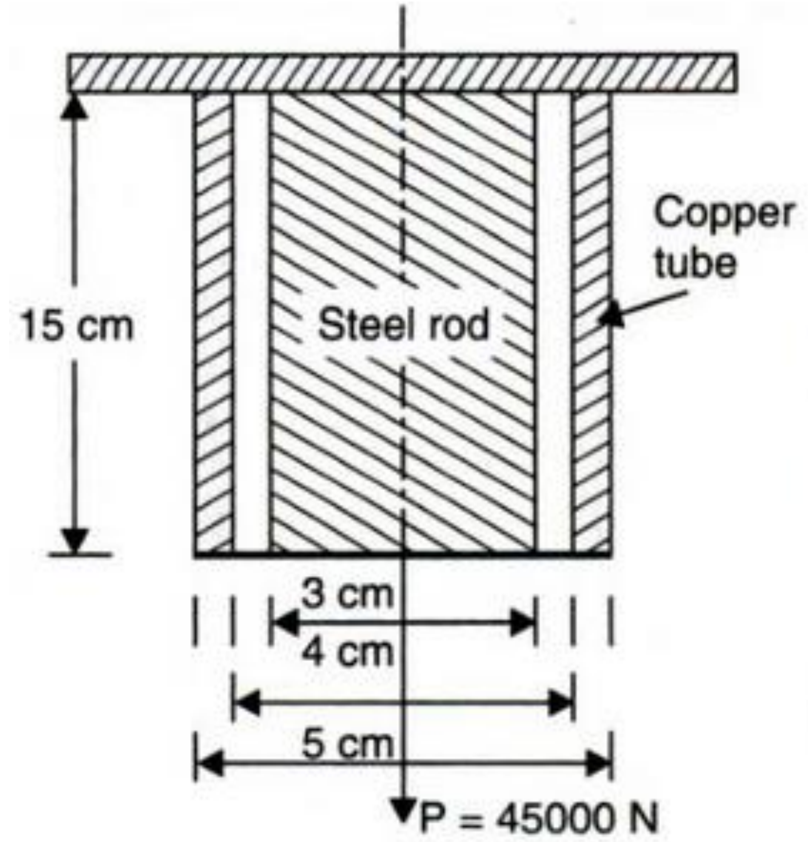
Take E for steel $= 2.1 \times 10^5$ N/mm^2 and for copper $= 1.1 \times 10^5$ N/mm^2



$\sigma_s = 41.77$ MPa
$\sigma_c = 21.88$ MPa
$P_s = 29525.5$ N
$P_c = 15474.5$ N

$$\boxed{\delta L = \frac{PL}{AE}}$$

$$\boxed{\delta L_1 = \delta L_2}$$



Dia. of steel rod $= 3$ cm $= 30$ mm

$\therefore$ Area of steel rod,

$$A_s = \frac{\pi}{4}\,(30)^2 = 706.86 \text{ mm}^2$$

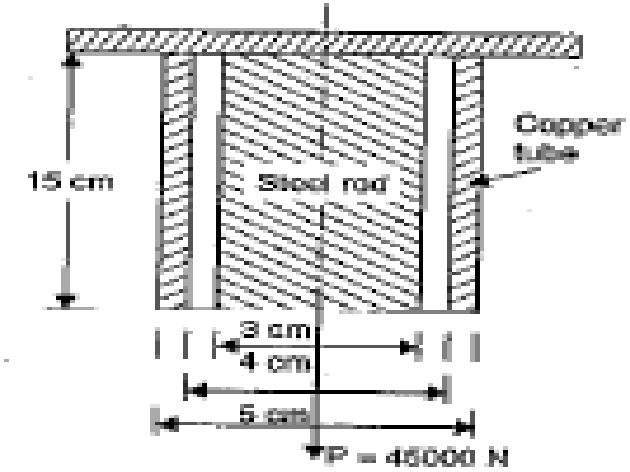


Fig. 1.16

External dia. of copper tube

$= 5$ cm $= 50$ mm

Internal dia. of copper tube

$= 4$ cm $= 40$ mm

$\therefore$ Area of copper tube,

$$A_s = \frac{\pi}{4}\,[50^2 - 40^2] \text{ mm}^2 = 706.86 \text{ mm}^2$$

Axial pull on composite bar, $P = 45000$ N

Length of each bar, $L = 15$ cm

Young's modulus for steel, $E_s = 2.1 \times 10^5 \text{ N/mm}^2$

Young's modulus for copper, $E_c = 1.1 \times 10^5 \text{ N/mm}^2$

(i) The stress in the rod and tube

Let σ_s = Stress in steel,

P_s = Load carried by steel rod,

σ_c = Stress in copper, and

P_c = Load carried by copper tube.

Now strain in steel = Strain in copper

or $$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \qquad \left(\because \frac{\sigma}{E} = \text{strain}\right)$$

$\therefore$ $$\sigma_s = \frac{E_s}{E_s} \times \sigma_c = \frac{2.1 \times 10^5}{11 \times 10^5} \times \sigma_c = 1.909\,\sigma_c \qquad \text{...(i)}$$

Now $\quad \text{stress} = \frac{\text{Load}}{\text{Area}}, \quad \therefore \quad \text{Load} = \text{Stress} \times \text{Area}$

Load on steel + Load on copper = Total load

$$\sigma_s \times A_s + \sigma_c \times A_c = P \qquad (\because \text{ Total load} = P)$$

or $\quad 1.909\,\sigma_c \times 706.86 + \sigma_c \times 706.86 = 45000$

or $\quad \sigma_c\,(1.909 \times 706.86 + 706.86) = 45000$

or $\quad 2056.25\,\sigma_c = 45000$

$$\therefore \quad \sigma_c = \frac{45000}{2056.25} = \mathbf{21.88\ N/mm^2}. \quad \textbf{Ans.}$$

Substituting the value of σ_c in equation (*i*), we get

$$\sigma_s = 1.909 \times 21.88\ \text{N/mm}^2$$
$$= \mathbf{41.77\ N/mm^2}. \quad \textbf{Ans.}$$

(ii) *Load carried by each bar.*

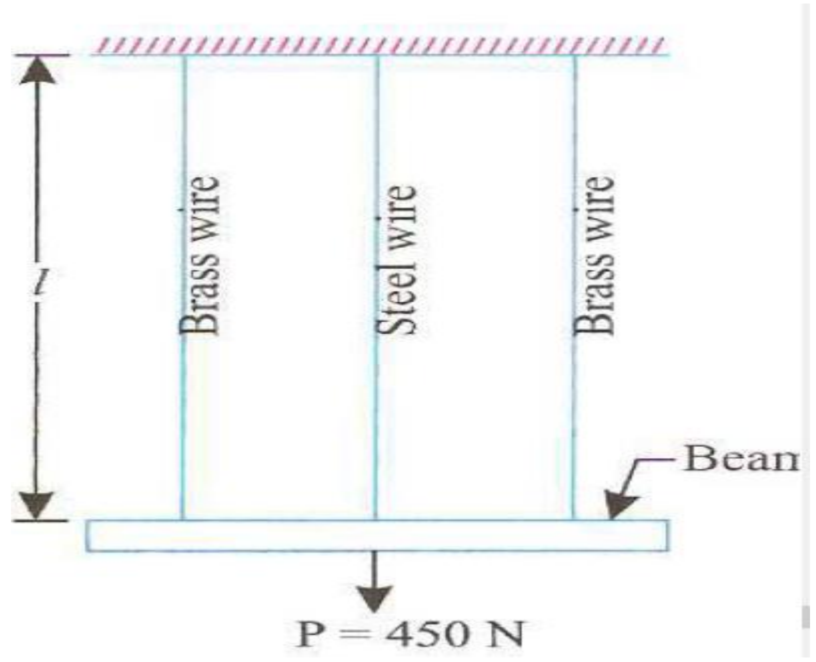
As $\quad \text{load} = \text{Stress} \times \text{Area}$

$\therefore$ Load carried by steel rod,

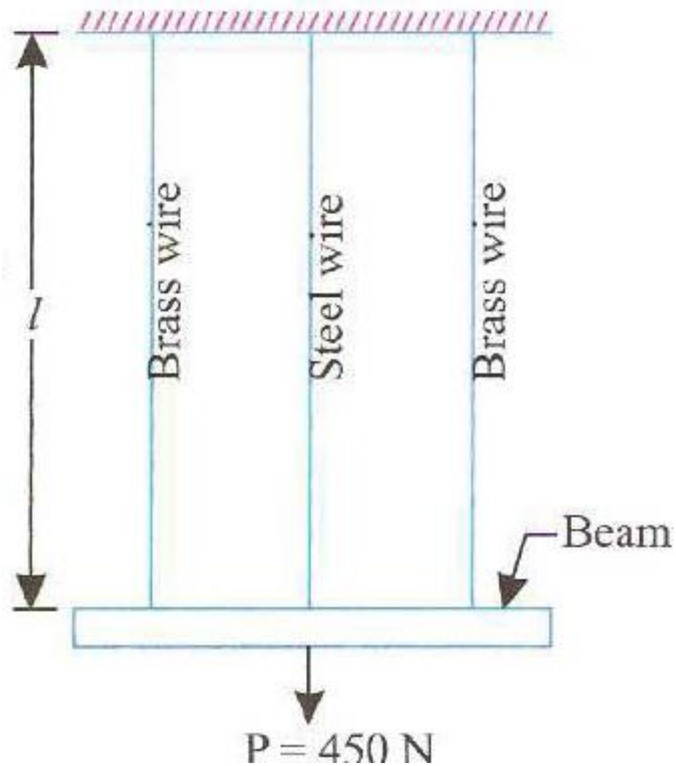
$$P_s = \sigma_s \times A_s$$
$$= 41.77 \times 706.86 = \mathbf{29525.5\ N}. \quad \textbf{Ans.}$$

Load carried by copper tube,

$$P_c = 45000 - 29525.5$$
$$= \mathbf{15474.5\ N}. \quad \textbf{Ans.}$$


l
Brass wire
Steel wire
Brass wire
Bean
P = 450 N

Example 1.30. *A beam weighing 450 N is held in a horizontal position by three vertical wires, one attached to each end of the beam, one to the middle of its length. The outer wires are of brass of diameter 1.25 mm and the central wire is of steel of diameter 0.625 mm. If the beam is rigid and wires of the same length and unstressed before the beam is attached, estimate the stresses induced in the wires. Take Young's modulus for brass as 86 GN/m^2 and for steel 210 GN/m^2.*


Brass wire
Steel wire
Brass wire
l
Beam
P = 450 N

Let, P_b = Load taken by the brass wire, and

P_s = Load taken by the steel wire.

Then, $2P_b + P_s = P$

Since the beam is *horizontal*, all wires will extend by the *same amount*.

i.e., $e_b = e_s$

($\because$ Length of each wire is same.)

where, e_b = Strain in brass wire, and

e_s = Strain in steel wire.

$$\frac{\sigma_b}{E_b} = \frac{\sigma_s}{E_s}$$

$$\frac{P_b}{A_b \cdot E_b} = \frac{P_s}{A_s \cdot E_s}$$

or, $$P_s = \frac{P_b\, A_s\, E_s}{A_b\, E_b}$$

$$= \frac{P_b \times \frac{\pi}{4} \times (0.625 \times 10^{-3})^2 \times 210 \times 10^9}{\frac{\pi}{4} \times (1.25 \times 10^{-3})^2 \times 86 \times 10^9}$$

or, $\qquad P_s = 0.61\ P_b$

Substituting the value of P_s in eqn. (*i*), we get

$$2\ P_b + 0.61\ P_b = P$$

$$P_b = 172.4 \text{ N}$$

and, $\qquad P_s = 0.61 \times 172.4 = 105.2 \text{ N}$

Now, *stress, induced in the brass wire*,

$$\sigma_b = \frac{P_b}{A_b} = \frac{172.4}{\frac{\pi}{4} \times (1.25 \times 10^{-3})^2} = 1.40 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{140\ MN/m^2} \quad \textbf{(Ans.)}$$

and, *stress induced in a steel wire*,

$$\sigma_s = \frac{105.2}{\frac{\pi}{4} \times (0.625 \times 10^{-3})^2} = 3.429 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{342.9 \text{ MN/m}^2} \quad \textbf{(Ans.)}$$

Example 4.11. *A bar 3 m long is made of two bars, one of copper having E = 105 GN/m² and the other of steel having E = 210 GN/m². Each bar is 25 mm broad and 12.5 mm thick. This compound bar is stretched by a load of 50 kN. Find the increase in length of the compound bar and the stress produced in the steel and copper. The length of copper as well as of steel bar is 3 m each.*

Solution. Given : $l_c = l_s = 3\text{ m} = 3 \times 10^3\text{ mm}$; $E_c = 105\text{ GN/m}^2 = 105\text{ kN/mm}^2$; $E_s = 210\text{ GN/m}^2 = 210\text{ kN/mm}^2$; $b = 25$ mm ; $t = 12.5$ mm ; $P = 50$ kN

Increase in length of the compound bar

Let δl = Increase in length of the compound bar.

The compound bar is shown in Fig. 4.14. We know that cross-sectional area of each bar,

$$A_c = A_s = b \times t = 25 \times 12.5 = 312.5\text{ mm}^2$$

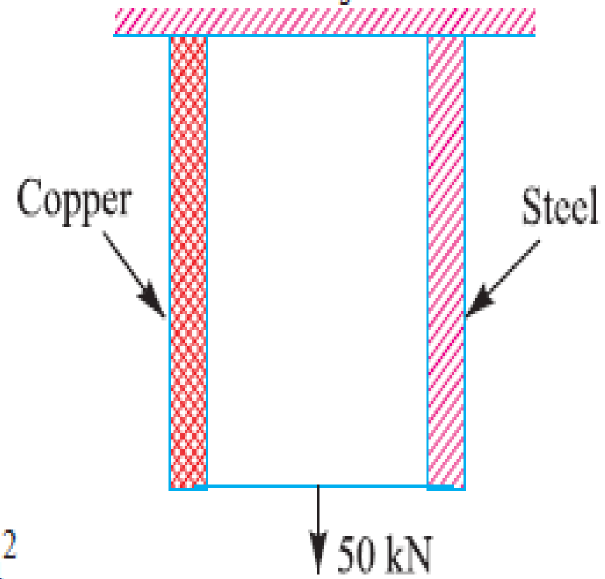


Fig. 4.14

∴ Load shared by the copper bar,

$$P_c = P \times \frac{A_c . E_c}{A_c . E_c + A_s . E_s} = P \times \frac{E_c}{E_c + E_s} \quad \ldots (\because A_c = A_s)$$

$$= 50 \times \frac{105}{105 + 210} = 16.67 \text{ kN}$$

and load shared by the steel bar,

$$P_s = P - P_c = 50 - 16.67 = 33.33 \text{ kN}$$

Since the elongation of both the bars is equal, therefore

$$\delta l = \frac{P_c . l_c}{A_c . E_c} = \frac{P_s . l_s}{A_s . E_s} = \frac{16.67 \times 3 \times 10^3}{312.5 \times 105} = 1.52 \text{ mm}$$ **Ans.**

Stress produced in the steel and copper bar

We know that stress produced in the steel bar,

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{210}{105} \times \sigma_c = 2\,\sigma_c$$

and total load, $P = P_s + P_c = \sigma_s A_s + \sigma_c A_c$

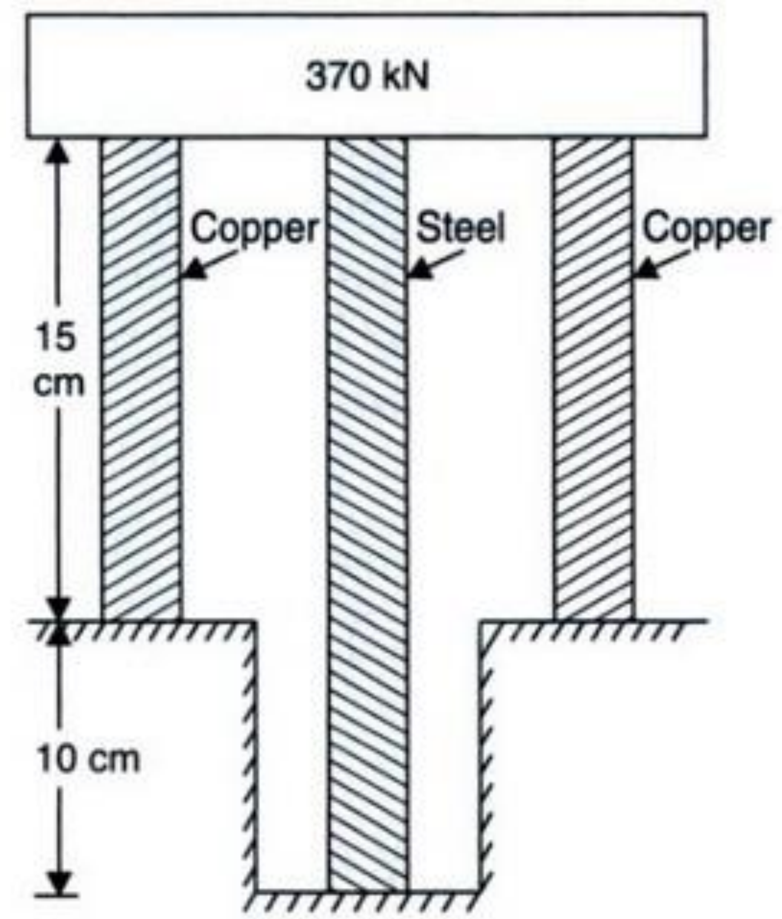
∴ $50 = 2\,\sigma_c \times 312.5 + \sigma_c \times 312.5 = 937.5\,\sigma_c$

or $\sigma_c = 50 / 937.5 = 0.053 \text{ kN/mm}^2 = 53 \text{ N/mm}^2 = 53 \text{ MPa}$ **Ans.**

and $\sigma_s = 2\,\sigma_c = 2 \times 53 = 106 \text{ N/mm}^2 = 106 \text{ MPa}$ **Ans.**

Problem 12

A steel rod and two copper rods together support a load of 370 kN as shown in figure. The cross sectional area of steel rod is 2500 mm^2 and of each copper rod is 1600 mm^2. Find the stresses in the rods. Take *E* for steel $= 2 \times 10^5$ N/mm^2 and for copper $= 1 \times 10^5$ N/mm^2



Sol. Given :

Load, $P = 370 \text{ kN} = 370{,}000 \text{ N}$

Area of steel road, $A_s = 2500 \text{ mm}^2$

Area of two copper rods, $A_c = 2 \times 1600$

$= 3200 \text{ mm}^2$

E for steel, $E_s = 2 \times 10^5 \text{ N/mm}^2$

E for copper, $E_c = 1 \times 10^5 \text{ N/mm}^2$

Length of steel rod, $L_s = 15 + 10 = 25 \text{ cm} = 250 \text{ mm}$

Length of copper rods, $L_c = 15 \text{ cm} = 150 \text{ mm}$

Let σ_s = Stress in steel in N/mm²

σ_c = Stress in copper rods in N/mm²

We know that decrease in the length of steel rod is equal to the decrease in the length of copper rods.

But decrease in length of steel rod

$$= \text{Strain in steel rod} \times \text{Length of steel rod}$$

$$= \frac{\text{Stress in steel}}{E_s} \times L_s \qquad \left(\because \text{ Strain} = \frac{\text{Stress}}{E}\right)$$

$$= \frac{\sigma_s}{2 \times 10^5} \times 250 \qquad \text{...}(i)$$

Similarly decrease in length of copper rods

$$= \text{Strain in copper rods} \times \text{Length of copper rods}$$

$$= \frac{\text{Stress in copper}}{E_c} \times L_s$$

$$= \frac{\sigma_c}{1 \times 10^5} \times 150 \qquad \text{...}(ii)$$

Equating the decrease in length of steel rod to the decrease in the length of copper rods, we get

$$\frac{\sigma_s}{2\times10^5}\times 250 = \frac{\sigma_c}{1\times10^5}\times 150$$

or $$\sigma_s = \sigma_c \times \frac{2\times10^5}{1\times10^5}\times\frac{150}{250} = 1.2\sigma_c \qquad \ldots(iii)$$

Also, we know that

Load on steel + Load on copper = Total load applied

or $$\sigma_s \times A_s + \sigma_c \times A_c = P \qquad (\because \text{Load} = \text{Stress} \times \text{Area})$$

or $$1.2\sigma_c \times 2500 + \sigma_c \times 3200 = 370{,}000 \qquad (\because \sigma_s = 1.2\sigma_c)$$

or $$6200\sigma_c = 370000$$

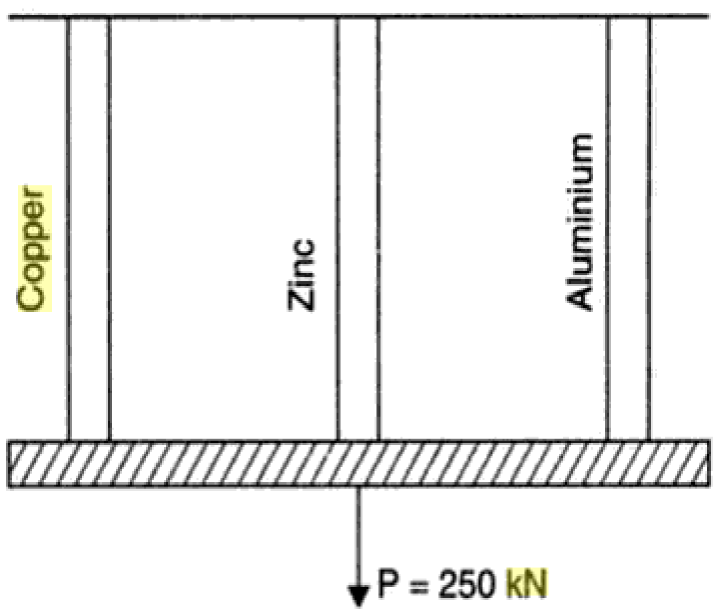
$$\therefore \qquad \sigma_c = \frac{370000}{6200} = \mathbf{59.67\ N/mm^2}.\quad \textbf{Ans.}$$

Substituting this value in equation (*iii*), we get

$$\sigma_s = 1.2\times\sigma_s$$

$$= 1.2\times 59.67 = \mathbf{71.604\ N/mm^2}.\quad \textbf{Ans.}$$

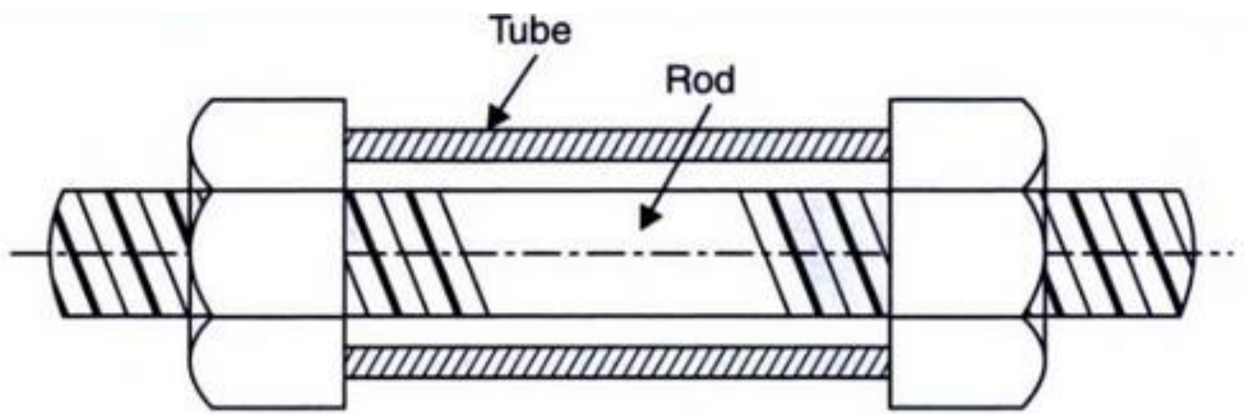
Problem 1.39. *Three bars made of copper, zinc and aluminium are of equal length and have cross-section 500, 750 and 1000 square mm respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250 kN, estimate the proportional of the load carried on each rod and the induced stresses. Take the value of E for copper = 1.3×10^5 N/mm^2, for zinc = 1.0×10^5 N/mm^2 and for aluminium = 0.8×10^5 N/mm^2.*



Problem 13

A steel rod 18 mm in diameter passes centrally through a steel tube of 25 mm internal diameter and 30 mm external diameter. The tube is 750 mm long and is closed by rigid washers of negligible thickness which are fastened by nuts threaded on the rod. The nuts are tightened until the compressive load on the tube is 20 kN. Calculate the stresses in the tube and the rod.

Find the increase in these stresses when one nut is tightened by one quarter of a turn relative to the other. There are 4 threads per 10 mm. Take E for steel = 2×10^5 N/mm^2.

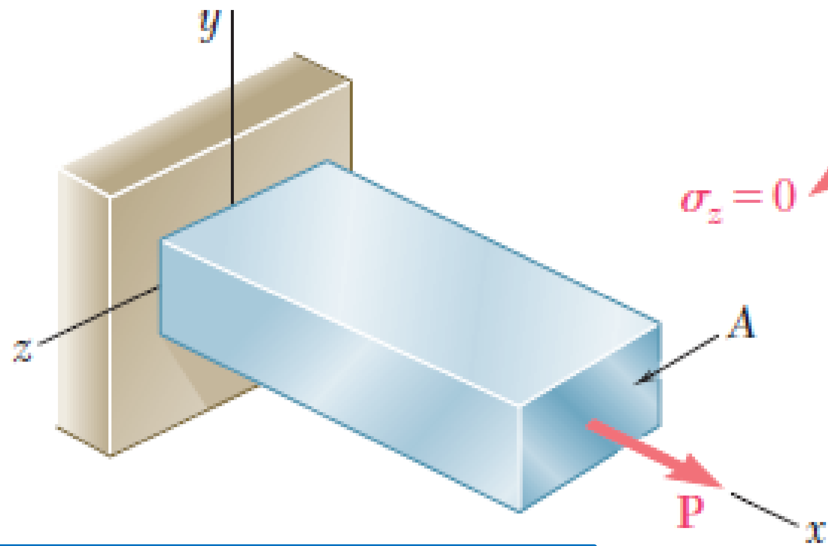


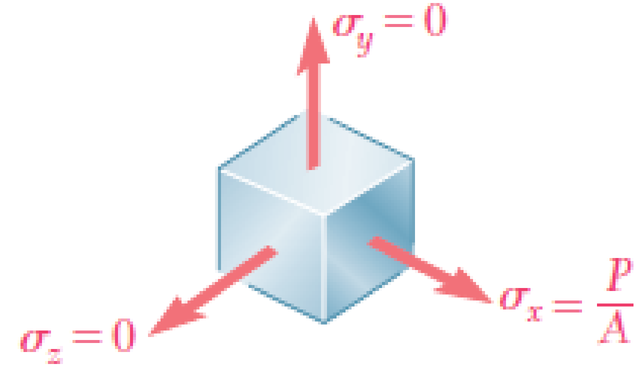
Poisson's Ratio

$$\sigma_x = \frac{P}{A}$$

$$\sigma_y = \sigma_z = 0$$

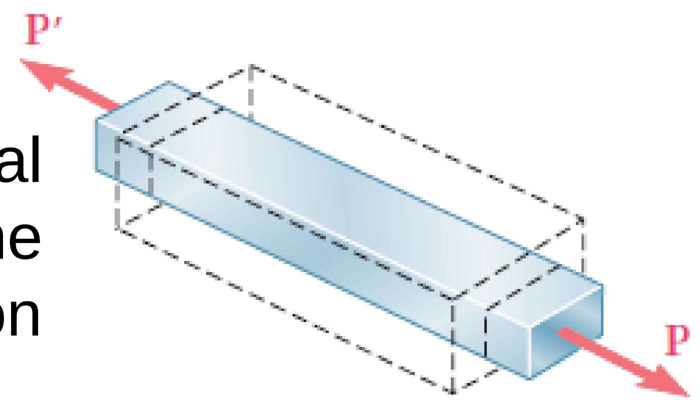
$$\epsilon_x = \frac{\sigma_x}{E}$$





$$\boxed{\epsilon_y \text{ and } \epsilon_z \; are \; not \; equal \; to \; zero}$$

The elongation produced by an axial tensile force **P** in the direction of the force is accompanied by a contraction in any transverse direction.



4.18 Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Table 4.4. Values of Poisson's ratio for commonly used materials.

S.No.	*Material*	*Poisson's ratio (1/m or μ)*
1	Steel	0.25 to 0.33
2	Cast iron	0.23 to 0.27
3	Copper	0.31 to 0.34
4	Brass	0.32 to 0.42
5	Aluminium	0.32 to 0.36
6	Concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

4.9 Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \quad \text{or} \quad \tau = C \cdot \phi \quad \text{or} \quad \tau / \phi = C$$

where τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by N or G.

The following table shows the values of modulus of rigidity (C) for the materials in every day use:

Table 4.2. Values of *C* for the commonly used materials.

Material	*Modulus of rigidity (C) in GPa i.e. GN/m² or kN/mm²*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

4.19 Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

where δV = Change in volume, and V = Original volume.

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

4.20 Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

4.21 Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3\,(m-2)} = \frac{E}{3\,(1-2\mu)}$$

4.22 Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$e = \frac{dV}{V} = volumetric\ strain$$

Volumetric strain of a rectangular bar subjected to an axial load in one direction

$$\boxed{e = \frac{\sigma_x}{E}(1 - 2v)} \qquad \boxed{e = \frac{\delta L}{L}(1 - 2v)}$$

Volumetric strain of a cylindrical rod

$$\boxed{e = Longitudinal\ strain\left(1 - 2\frac{Lateral\ strain}{Longitudinal\ strain}\right)}$$

$$\boxed{e = \frac{\delta L}{L} - \frac{2\delta d}{d}}$$

Example 4.17. *A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take E = 0.2 × 10⁶ N/mm².*

Solution. Given : $l = 2.4$ m = 2400 mm ; $A = 30 \times 30 = 900$ mm² ; $P = 500$ kN = 500×10^3 N ; $1/m = 0.25$; $E = 0.2 \times 10^6$ N/mm²

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}\ (1 - 2 \times 0.25) = 1.4 \times 10^3$$

$$\therefore \quad \delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3$$ **Ans.**

The material is assumed to be both *homogeneous* and *isotropic*, i.e., its mechanical properties will be assumed independent of both *position* and *direction*.

$$\boxed{\therefore\ Lateral\ strain, \epsilon_y = \epsilon_z}$$

The constant, Poisson's ratio, for a given material is defined as,

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

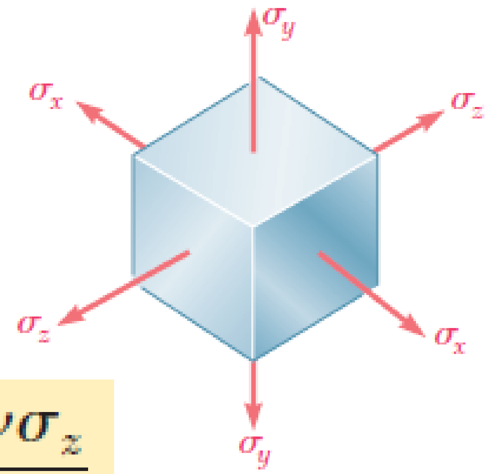
$$\boxed{\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}}$$

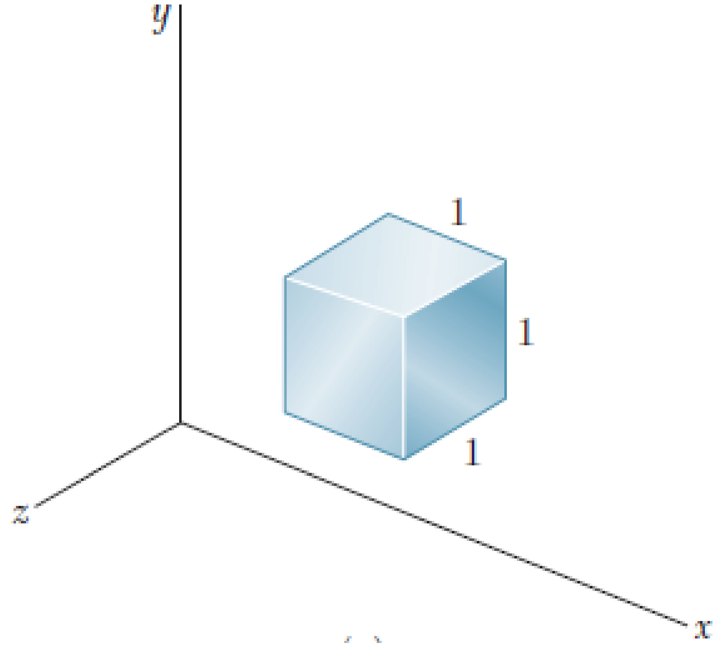
The minus sign in the above equations is to obtain a positive value for ν, as the axial and lateral strains are having opposite signs for all engineering materials.

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E}$$

Generalized Hooke's Law

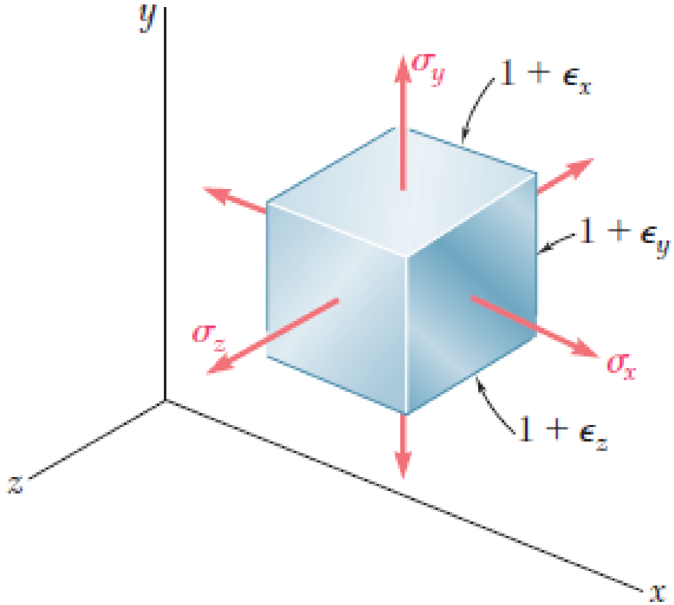




$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$



The relations are referred to as the generalized Hooke's law for the multiaxial loading of a homogeneous isotropic material.

Bulk Modulus

Volume of the element in its unstressed state = 1

Volume under the stresses σ_x, σ_y, σ_z

$$v = (1 + \epsilon_x)(1 + \epsilon_y)(1 + \epsilon_z)$$

Since the strains ε_x, ε_y, ε_z are much smaller than unity,

$$v = 1 + \epsilon_x + \epsilon_y + \epsilon_z$$

Denoting by *e* the change in volume of our element, we write

$$e = v - 1 = 1 + \epsilon_x + \epsilon_y + \epsilon_z - 1$$

$$\boxed{e = \epsilon_x + \epsilon_y + \epsilon_z}$$

Since the element had originally a unit volume, the quantity *e* represents the *change in volume per unit volume*; it is referred to as the *dilatation* of the material.

Substituting for ε_x, ε_y, ε_z

$$e = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\nu(\sigma_x + \sigma_y + \sigma_z)}{E}$$

$$\boxed{e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)}$$

A case of special interest is that of a body subjected to a uniform hydrostatic pressure *p*. Each of the stress components is then equal to -*p* and Equation yields

$$e = -\frac{3(1 - 2\nu)}{E}p$$

$$\boxed{k = \frac{E}{3(1 - 2\nu)}}$$

$$\boxed{e = -\frac{p}{k}}$$

The constant *k* is known as the *bulk modulus or modulus of compression* of the material.

$$e = -\frac{3(1-2\nu)}{E}p \qquad \boxed{k = \frac{E}{3(1-2\nu)}} \qquad \boxed{e = -\frac{p}{k}}$$

Observations

- A stable material subjected to a hydrostatic pressure can only *decrease* in volume, thus the dilatation *e* is negative.
- The bulk modulus *k* is a positive quantity
- 1 - 2ν > 0 or ν < 1/2

For any engineering material, $0 < \nu < \frac{1}{2}$

- ν = 0 means that the material could be stretched in one direction without any lateral contraction
- ν = ½, and thus $k = \propto$, the material would be perfectly incompressible (*e* = 0)
- Since ν < ½, stretching an engineering material in one direction, will result in an increase of its volume (e > 0)

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$e = \frac{dV}{V} = volumetric\ strain$$

Volumetric strain of a rectangular bar subjected to an axial load in one direction

$$\boxed{e = \frac{\sigma_x}{E}(1-2v)}$$

$$\boxed{e = \frac{\delta L}{L}(1-2v)}$$

Volumetric strain of a cylindrical rod

$$\boxed{e = Longitudinal\ strain\left(1 - 2\frac{Lateral\ strain}{Longitudinal\ strain}\right)}$$

$$\boxed{e = \frac{\delta L}{L} - \frac{2\delta d}{d}}$$

Summary

- When a homogeneous and isotropic material is subjected to a state of triaxial stress, the strain in one of the stress directions is influenced by the strain produced by all the stresses. This is the result of the Poisson effect, and results in the form of a generalized Hooke's law.
- Dilatation, or volumetric strain, is caused only by normal strain, not shear strain.
- The bulk modulus is a measure of the stiffness of a volume of material.
- This material property provides an upper limit to Poisson' s ratio of $v = 0.5$, which remains at this value while plastic yielding occurs.

Problem 14

Determine the value of Young's modulus and Poisson's ratio of a metallic bar of length 30 cm, breadth 4 cm and depth 4 cm when the bar is subjected to an axial compressive load of 400 kN. The decrease in length is given as 0.075 cm and increase in breadth is 0.003 cm.

$\nu = 0.3$

$E = 1 \times 10^5$ MPa

Sol. Given : Length, $l = 30$ cm ; Breadth, $b = 4$ cm ; and Depth, $d = 4$ cm.

$\therefore$ Area of cross-section, $A = b \times d = 4 \times 4 = 16 \text{ cm}^2 = 16 \times 100 = 1600 \text{ mm}^2$

Axial compressive load, $P = 400 \text{ kN} = 400 \times 1000 \text{ N}$

Decrease in length, $\delta l = 0.075$ cm

Increase in breadth, $\delta b = 0.003$ cm

Longitudinal strain
$$= \frac{\delta l}{l} = \frac{0.075}{30} = 0.0025$$

Lateral strain
$$= \frac{\delta b}{b} = \frac{0.003}{4} = 0.00075.$$

Using equation (15.3),

Poisson's ratio
$$= \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{0.00075}{0.0025} = \mathbf{0.3.} \quad \textbf{Ans.}$$

Value of Young's modulus (E)

Longitudinal strain
$$= \frac{\text{Stress}}{E} = \frac{P}{A \times E} \qquad \left(\because \text{ Stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} \right)$$

or
$$0.0025 = \frac{400000}{1600 \times E}$$

$\therefore$
$$E = \frac{400000}{1600 \times .0025} = \mathbf{1 \times 10^5 \text{ N/mm}^2.} \quad \textbf{Ans.}$$

Problem 15

A steel bar 300 mm long, 50 mm wide and 40 mm thick is subjected to a pull of 300 kN in the direction of its length. Determine the change in volume. Take $E = 2 \times 10^5$ N/mm^2 and $\nu = 0.25$.

$\boxed{dV = 225 \text{ mm}^3}$

Sol. Given :

Length,	$L = 300$ mm
Width,	$b = 50$ mm
Thickness,	$t = 40$ mm
Pull,	$P = 300 \text{ kN} = 300 \times 10^3$ N
Value of E	$= 2 \times 10^5 \text{ N/mm}^2$
Value of m	$= 4$
Original volume,	$V = L \times b \times t = 300 \times 50 \times 40 \text{ mm}^3 = 600000 \text{ mm}^3$

The longitudinal strain (*i.e.*, the strain in the direction of load) is given by

$$\frac{dL}{L} = \frac{\text{Stress in the direction of load}}{E}$$

But stress in the direction of load $= \frac{P}{\text{Area}} = \frac{P}{b \times t} = \frac{300 \times 10^3}{50 \times 40} = 150 \text{ N/mm}^2$

$$\therefore \quad \frac{dL}{L} = \frac{150}{2 \times 10^5} = 0.00075$$

Now volumetric strain is given by equation (15.5) as

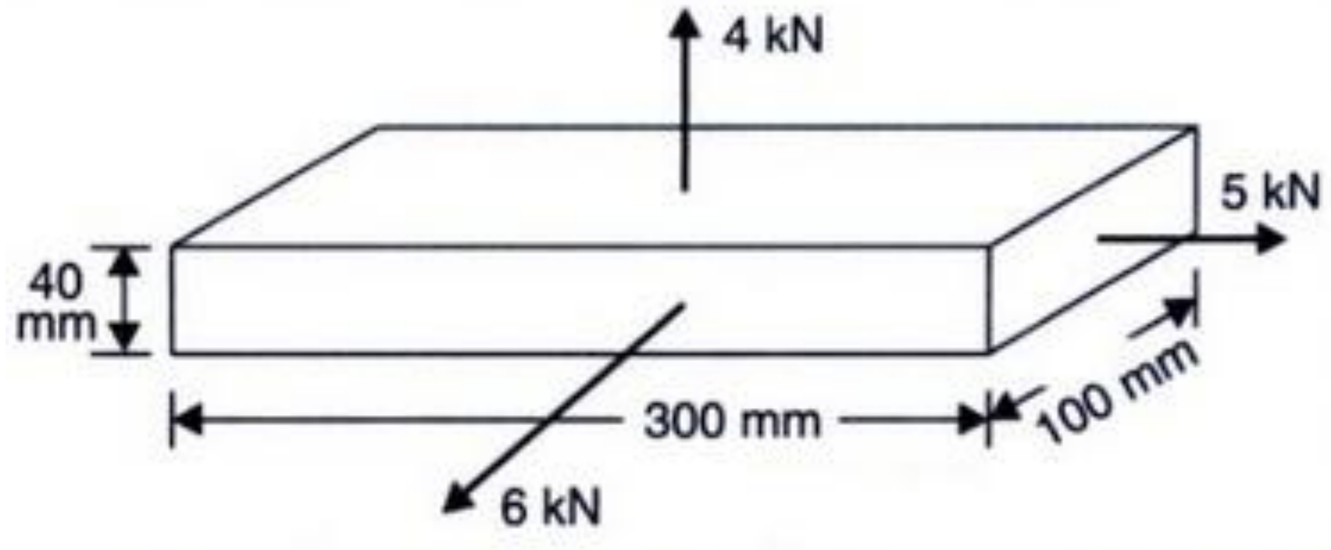
$$e_v = \frac{dL}{L}\left(1 - \frac{2}{m}\right) = 0.00075\left(1 - \frac{2}{4}\right) = .000375$$

Let δV = Change in volume. Then $\frac{dV}{V}$ represents volumetric strain.

$$\therefore \quad \frac{dV}{V} = 0.000375$$

$$dV = 0.000375 \times V = 0.000375 \times 600000 = \mathbf{225 \text{ mm}^3}. \quad \textbf{Ans.}$$

Problem 16



A metallic bar 300 mm x 100 mm x 40 mm is subjected to a force of 5 kN (tensile), 6 kN (tensile) and 4 kN (tensile) along x, y and z directions respectively. Determine the change in the volume of the block. Take $E = 2 \times 10^5$ N/mm^2 and $\nu = 0.25$.

$\boxed{dV = 5.65 \text{ mm}^3}$

Sol. Given :

Dimensions of bar $= 300 \text{ mm} \times 100 \text{ mm} \times 40 \text{ mm}$

$\therefore$ $x = 300$ mm, $y = 100$ mm and $z = 40$ mm

$\therefore$ Volume, $V = x \times y \times z = 300 \times 100 \times 40$

$= 1200000 \text{ mm}^3$

Load in the direction of x $= 5 \text{ kN} = 5000 \text{ N}$

Load in the direction of y $= 6 \text{ kN} = 6000 \text{ N}$

Load in the direction of z $= 4 \text{ kN} = 4000 \text{ N}$

Value of E $= 2 \times 10^5 \text{ N/mm}^2$

Poisson's ratio, $\mu = 0.25$

$\therefore$ Stress in the x-direction,

$$\sigma_x = \frac{\text{Load in } x\text{-direction}}{y \times z}$$

$$= \frac{5000}{100 \times 40} = 1.25 \text{ N/mm}^2$$

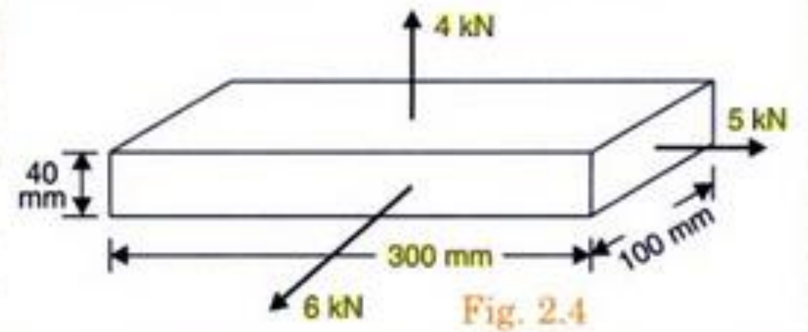


Fig. 2.4

Similarly the stress in y-direction is given by,

$$\sigma_y = \frac{\text{Load in } y\text{-direction}}{x \times z}$$

$$= \frac{6000}{300 \times 40} = 0.5 \text{ N/mm}^2$$

And stress in z-direction $= \dfrac{\text{Load in } z\text{-direction}}{x \times y}$

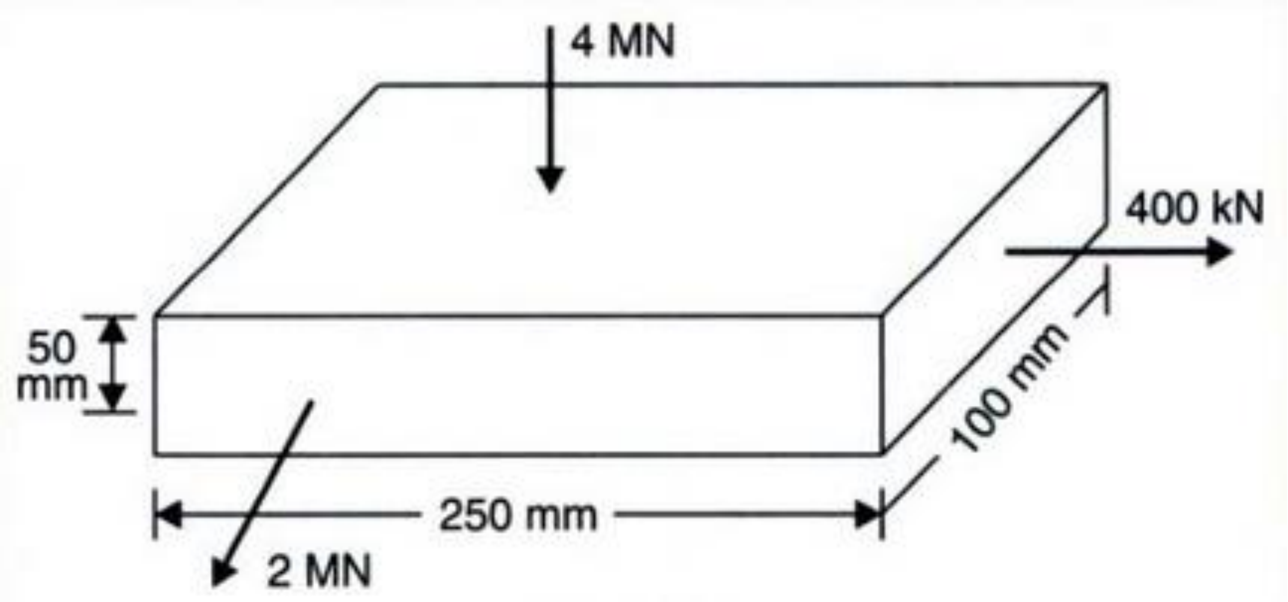
or
$$\sigma_z = \frac{4000}{300 \times 100}$$
$$= 0.133 \text{ N/mm}^2$$

Using equation (2.9), we get

$$\frac{dV}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$
$$= \frac{1}{2 \times 10^5}(1.25 + 0.5 + 0.113)(1 - 2 \times 0.25)$$
$$= \frac{1.883}{2 \times 10^5 \times 2}$$

$\therefore$
$$dV = \frac{1.883}{4 \times 10^5} \times V$$
$$= \frac{1.883}{4 \times 10^5} \times 1200000$$
$$= \mathbf{5.649 \text{ mm}^3}. \quad \text{Ans.}$$

Problem 17



$\boxed{dV = 250 \text{ mm}^3}$

A metallic bar 250 mm x 100 mm x 50 mm is loaded as shown in figure. Determine the change in the volume of the block. Take $E = 2 \times 10^5$ N/mm^2 and $\nu = 0.25$.

Also find the change that should be made in the 4 MN load, in order that there should be no change in the volume of the bar.

Relation Between E, ν and G

$$\frac{E}{2G} = 1 + \nu$$

Relation Between E, K and G

$$\boxed{E = \frac{9KG}{3K + G}}$$

G- Rigidity modulus or shear modulus

K- Bulk modulus or volume modulus

Problem 18

A bar of cross section 8 mm x 8 mm is subjected to an axial pull of 7000 N. The lateral dimension of the bar is found to be changed to 7.9985 mm x 7.9985 mm. If the modulus of rigidity of the material is 0.8×10^5 N/mm^2, determine the Poisson's ratio and modulus of elasticity.

$$\boxed{\begin{gathered} \boldsymbol{v} = 0.378 \\ \boldsymbol{E} = 2.2047 \times 10^5 \text{ N/mm}^2 \end{gathered}}$$

Problem 19

Calculate the modulus of rigidity and bulk modulus of a cylindrical bar of diameter 30 mm and of length 1.5 m if the longitudinal strain in a bar during a tensile stress is four times the lateral strain. Find the change in volume, when the bar is subjected to a hydrostatic pressure of 100 N/mm^2. Take E = 1 x 10^5.

$$\boxed{\begin{gathered} \boldsymbol{dV} = 1590.43\ \text{mm}^3 \\ \boldsymbol{G} = 4 \times 10^4\ \text{N/mm}^2 \\ \boldsymbol{K} = 0.667 \times 10^5\ \text{N/mm}^2 \end{gathered}}$$

Thermal Stresses

- External loads are not the only sources of stresses and strains in a structure.
- Other sources include thermal effects arising from temperature changes
- Changes in temperature produce expansion or contraction of structural materials, resulting in thermal strains and thermal stresses

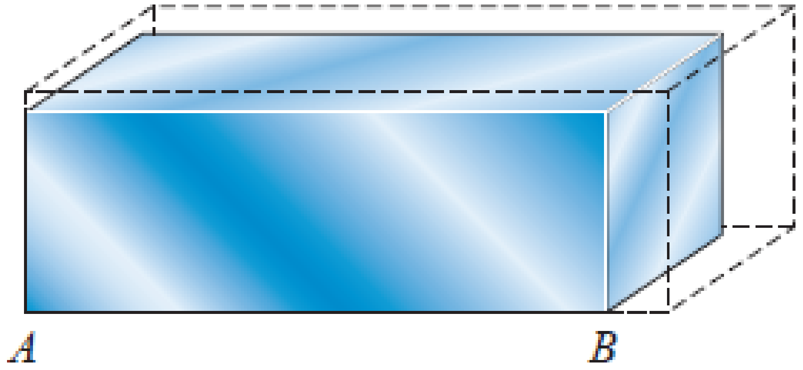
For most structural materials,

$$\epsilon_T = \alpha(\Delta T)$$

Thermal strain

Co-efficient of thermal expansion (/°C or /°K)

Change in temperature (°C or °K)



Comparison of Thermal Strains with Load-Induced Strains

Longitudinal strains of an axially loaded bar is given by

$$\epsilon = \frac{\sigma}{E}$$

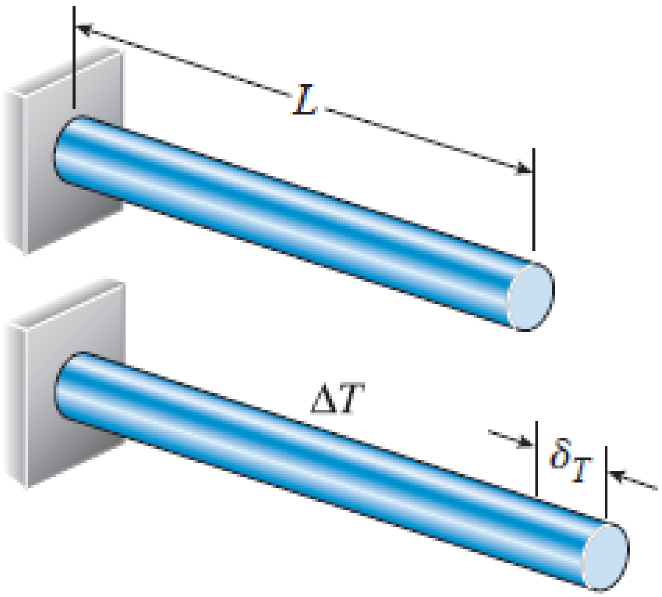
Thermal strain of the bar is given by

$$\epsilon_T = \alpha(\Delta T)$$

Equating the strains,

$$\sigma = E\alpha(\Delta T)$$

Temperature-Displacement Relation

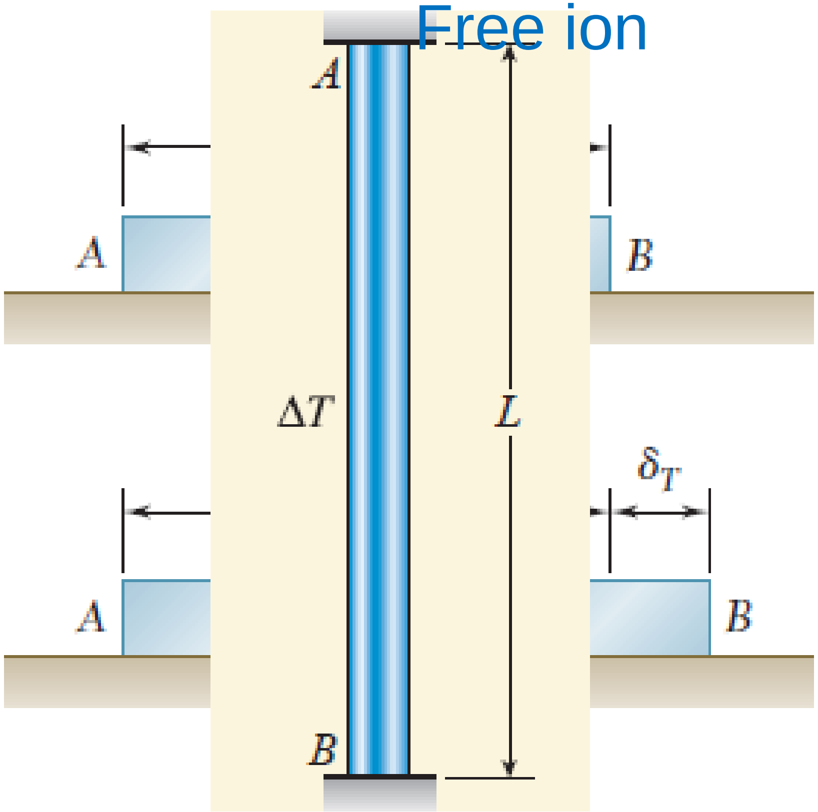


$$\delta_T = \epsilon_T L = \alpha(\Delta T)L$$

Free ion

Expans

Free Expansion Prevented



- Thermal strain is ϵ_{T}
- No stress associated with thermal strain

- Thermal strain $\epsilon_{\mathrm{T}} = 0$
- Stress is present

1.14.1. Stress and Strain when the Supports Yield. If the supports yield by an amount equal to δ, then the actual expansion

$$= \text{Expansion due to rise in temperature} - \delta$$

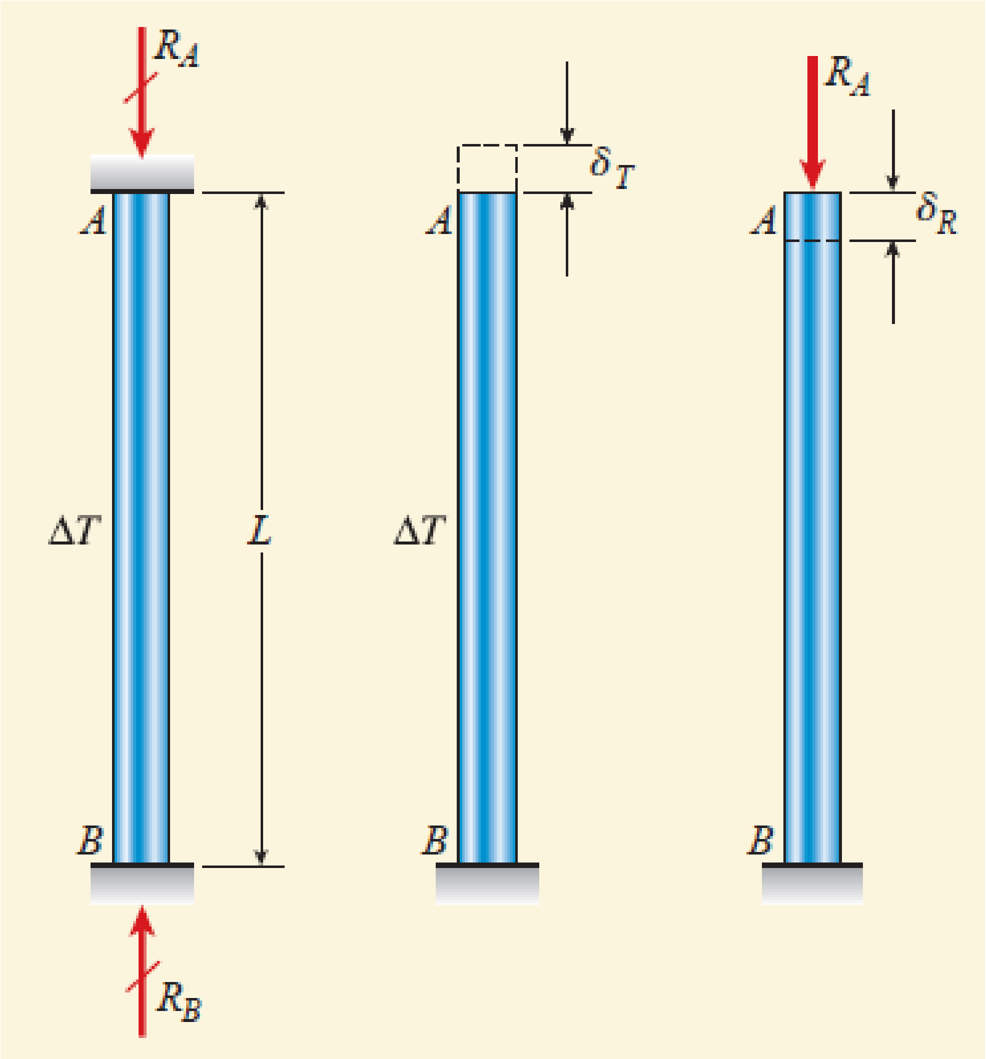
$$= \alpha.T.L - \delta.$$

$$\therefore \text{Actual strain} = \frac{\text{Actual expansion}}{\text{Original length}} = \frac{(\alpha\,.\,T\,.\,L - \delta)}{L}$$

$$\text{And actual stress} = \text{Actual strain} \times E$$

$$= \frac{(\alpha\,.\,T\,.\,L - \delta)}{L} \times E. \qquad \text{...(1.16)}$$

Case 1



$$\sum F_{\text{vert}} = 0 \qquad R_B - R_A = 0$$

$$\delta_{AB} = 0$$

$$\delta_{AB} = \delta_T - \delta_R = 0$$

$$\delta_T = \alpha(\Delta T)L$$

$$\delta_R = \frac{R_A L}{EA}$$

$$\delta_T - \delta_R = \alpha(\Delta T)L - \frac{R_A L}{EA} = 0$$

$$R_A = R_B = EA\alpha(\Delta T)$$

$$\sigma_T = \frac{R_A}{A} = \frac{R_B}{A} = E\alpha(\Delta T)$$

Problem 20

A steel rod of 3 cm diameter and 5 m long is connected to two grips and the rod is maintained at a temperature of 95° C. Determine the stress and pull exerted when the temperature falls to 30° C, if

(i) The ends do not yield, and

(ii) The ends yield by 0.12 cm.

Take $E = 2 \times 10^5$ MN/m^2 and $\alpha = 12 \times 10^{-6}$/°C.

(i) σ = 156 N/mm^2; P = 110.3 kN

(ii) σ = 108 N/mm^2; P = *76.34* kN

Sol. **Given :**

Dia. of the rod, $d = 3 \text{ cm} = 30 \text{ mm}$

$\therefore$ Area of the rod, $A = \dfrac{\pi}{4} \times 30^2 = 225\,\pi \text{ mm}^2$

Length of the rod, $L = 5 \text{ m} = 5000 \text{ mm}$

Initial temperature, $T_1 = 95°C$

Final temperature, $T_2 = 30°C$

$\therefore$ Fall in temperature, $\quad T = T_1 - T_2 = 95 - 30 = 65°C$

Modulus of elasticity, $\quad E = 2 \times 10^5 \text{ MN/m}^2$

$$= 2 \times 10^5 \times 10^6 \text{ N/m}^2$$

$$= 2 \times 10^{11} \text{ N/m}^2$$

Co-efficient of linear expansion, $\alpha = 12 \times 10^{-6}/°C$.

(*i*) *When the ends do not yield*

The stress is given by equation (1.15).

$$\therefore \quad \text{Stress} = \alpha . T . E = 12 \times 10^{-6} \times 65 \times 2 \times 10^{11} \text{ N/m}^2$$

$$= \mathbf{156 \times 10^6 \ N/m^2 \ or \ 156 \ N/mm^2 \ (tensile).} \quad \text{Ans.}$$

$$\text{Pull in the rod} = \text{Stress} \times \text{Area}$$

$$= 156 \times 225\,\pi = \mathbf{110269.9 \ N.} \quad \text{Ans.}$$

(*ii*) *When the ends yield by 0.12 cm*

$$\therefore \quad \delta = 0.12 \text{ cm} = 1.2 \text{ mm}$$

The stress when the ends yield is given by equation (1.16).

$$\therefore \quad \text{Stress} = \frac{(\alpha . T . L - \delta)}{L} \times E$$

$$= \frac{(12 \times 10^{-6} \times 65 \times 5000 - 1.2)}{5000} \times 2 \times 10^5 \text{ N/mm}^2$$

$$= \frac{(3.9 - 1.2)}{5000} \times 2 \times 10^5 = \mathbf{108 \ N/mm^2.} \quad \text{Ans.}$$

$$\text{Pull in the rod} = \text{Stress} \times \text{Area}$$

$$= 108 \times 225\,\pi = \mathbf{76340.7 \ N.} \quad \text{Ans.}$$

Problem 21

A steel rod of 20 mm diameter passes centrally through a copper tube of 50 mm external diameter and 40 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temperature of the assembly is raised by 50° C, calculate the stresses developed in copper and steel. Take *E* for steel and copper as 200 GN/m^2 and 100 GN/m^2 and α for steel and copper as 12 x 10^{-6}/°C and 18 x 10^{-6}/°C.

(i) σ for copper = 14.117 N/mm^2
(ii) σ for steel = 31.76 N/mm^2

Sol. Given :

Dia. of steel rod $= 20$ mm

$\therefore$ Area of steel rod, $A_s = \frac{\pi}{4} \times 20^2 = 100\pi \text{ mm}^2$

Area of copper tube, $A_c = \frac{\pi}{4} (50^2 - 40^2) \text{ mm}^2 = 225\pi \text{ mm}^2$

Rise of temperature, $T = 50°\text{C}$

E for steel, $E_s = 200 \text{ GN/m}^2$

$= 200 \times 10^9 \text{ N/m}^2 \qquad (\because \quad \text{G} = 10^9)$

$= 200 \times 10^3 \times 10^6 \text{ N/m}^2$

$= 200 \times 10^3 \text{ N/mm}^2 \qquad (\because \quad 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2)$

E for copper, $E_c = 100 \text{ GN/m}^2 = 100 \times 10^9 \text{ N/m}^2$

$= 100 \times 10^3 \times 10^6 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$

α for steel, $\alpha_s = 12 \times 10^{-6}$ per °C

α for copper, $\alpha_c = 18 \times 10^{-6}$ per °C.

As α for copper is more than that of steel, hence the free expansion of copper will be more than that of steel when there is a rise in temperature. But the ends of the rod and the tube is fixed to the rigid plates and the nuts are tightened on the projected parts of the rod. Hence the two members are not free to expand. Hence the tube and the rod will expand by the same amount. The free expansion of the copper tube will be more than the common expansion, whereas the free expansion of the steel rod will be less than the common expansion. Hence the copper tube will be subjected to compressive stress and the steel rod will be subjected to tensile stress.

Let σ_s = Tensile stress in steel

σ_c = Compressive stress in copper.

For the equilibrium of the system,

Tensile load on steel = Compressive load on copper

or $$\sigma_s \cdot A_s = \sigma_c \cdot A_c$$

or $$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{225\pi}{100\pi} \times \sigma_c = 2.25\sigma_c \qquad \text{...}(i)$$

We know that the copper tube and steel rod will actually expand by the same amount.

$\therefore$ Actual expansion of steel = Actual expansion of copper ...(*ii*)

But actual expansion of steel

= Free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s \cdot T \cdot L + \frac{\sigma_s}{E_s} \cdot L$$

$= \text{Free expansion of copper} - \text{Contraction due to compressive stress in copper}$

$$= \alpha_c \, . \, T \, . \, L - \frac{\sigma_c}{E_c} \, . \, L$$

Substituting these values in equation (*ii*), we get

$$\alpha_s \, . \, T \, . \, L + \frac{\sigma_s}{E_s} \, . \, L = \alpha_c \, . \, T \, . \, L - \frac{\sigma_c}{E_c} \, . \, L$$

or $$\alpha_s \, . \, T + \frac{\sigma_s}{E_s} = \alpha_c \, . \, T - \frac{\sigma_c}{E_c}$$

or $$12 \times 10^{-6} \times 50 + \frac{2.25\,\sigma_c}{200 \times 10^3} = 18 \times 10^{-6} \times 50 - \frac{\sigma_c}{100 \times 10^3} \qquad (\because \quad \sigma_s = 2.25\,\sigma_c)$$

or $$\frac{2.25\,\sigma_c}{200 \times 10^3} + \frac{\sigma_c}{100 \times 10^3} = 18 \times 10^{-6} \times 50 - 12 \times 10^{-6} \times 50$$

or $$1.125 \times 10^{-5}\,\sigma_c + 10^{-5}\,\sigma_c = 6 \times 10^{-6} \times 50$$

or $$2.125 \times 10^{-5}\,\sigma_c = 30 \times 10^{-5}$$

or $$2.125\sigma_c = 30$$

$\therefore$ $$\sigma_c = \frac{30}{2.125} = \mathbf{14.117\ N/mm^2}. \quad \text{Ans.}$$

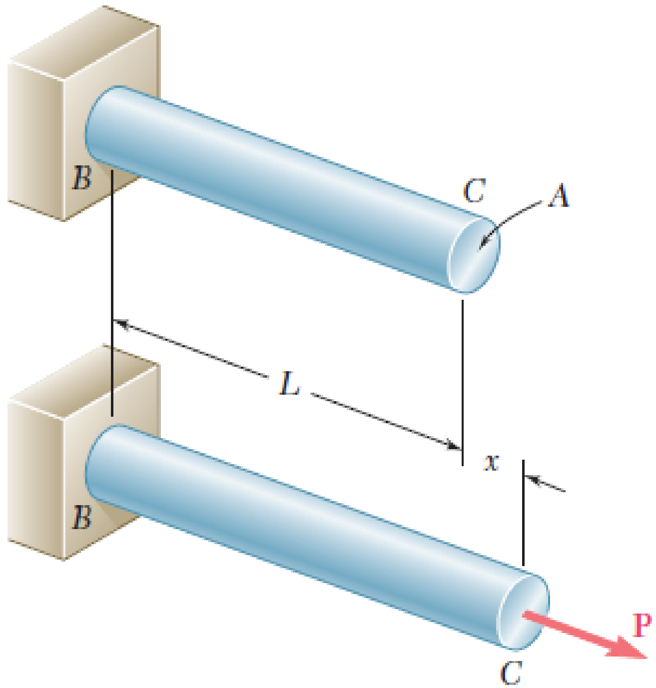
Substituting this value in equation (*i*), we get

$$\sigma_s = 14.117 \times 2.25$$

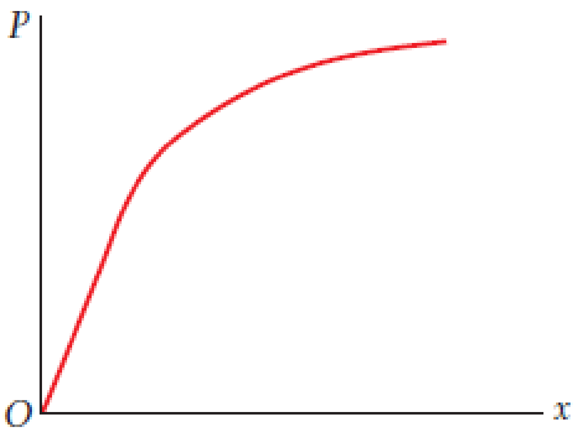
$$= \mathbf{31.76\ N/mm^2}. \quad \text{Ans.}$$

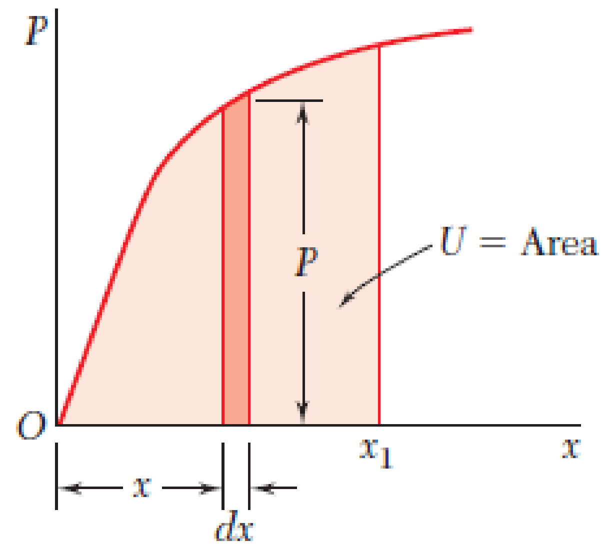
Strain Energy

Strain Energy



P - slowly increasing axial load





$$dU = P\,dx$$

$$U = \int_0^{x_1} P\,dx$$

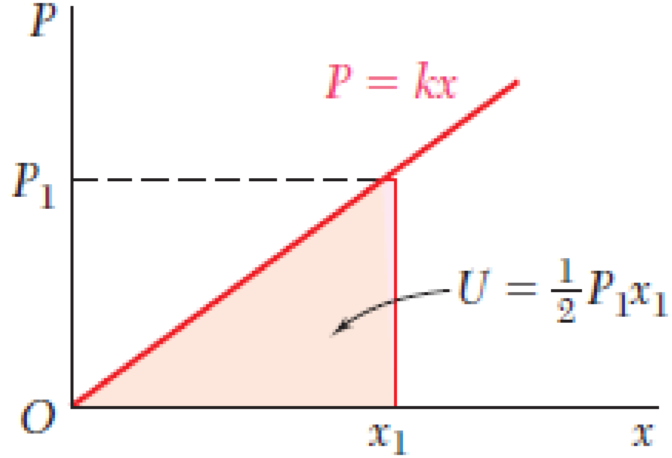
The work done by the load **P** as it is slowly applied to the rod must result in the increase of some energy associated with the deformation of the rod. This energy is referred to as the strain energy of the rod.

$$\boxed{\text{Strain energy} = U = \int_0^{x_1} P\,dx}$$

Linear and Elastic Deformation

$$U = \int_0^{x_1} kx\,dx = \tfrac{1}{2}kx_1^2$$

$$\boxed{U = \tfrac{1}{2}P_1x_1}$$



where P_1 is the value of the load corresponding to the deformation x_1.

Strain Energy Density

- The load-deformation diagram for a rod depends upon the length (*L)* and the cross-sectional area (*A*) of the rod.
- The strain energy will also depend upon the dimensions of the rod.
- In order to eliminate the effect of size from our discussion and direct our attention to the properties of the material, the strain energy per unit volume will be considered.

$$\frac{U}{V} = \int_0^{x_1} \frac{P}{A}\frac{dx}{L}$$

$$\text{Strain-energy density} = u = \int_0^{\epsilon_1} \sigma_x \, d\epsilon_x$$

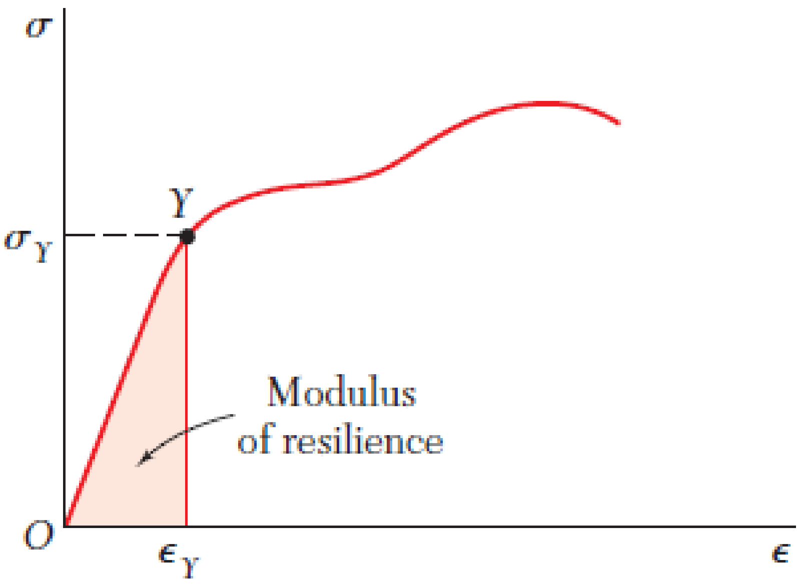
$$\frac{U}{V} = \int_0^{\epsilon_1} \sigma_x \, d\epsilon_x$$

$$\sigma_x = E\epsilon_x$$

$$u = \int_0^{\epsilon_1} E\epsilon_x \, d\epsilon_x = \frac{E\epsilon_1^2}{2}$$

$$u = \frac{\sigma_1^2}{2E}$$

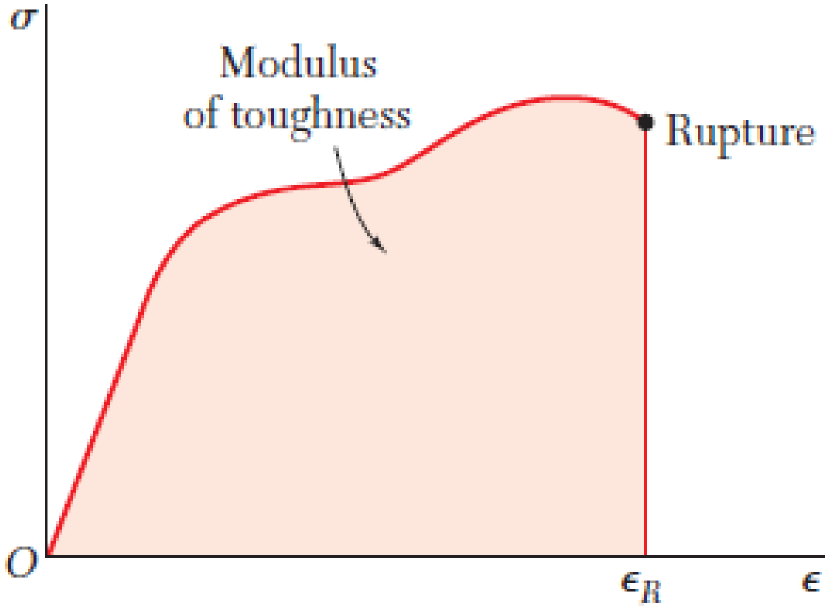
Resilience and Modulus of Resilience



$$u_Y = \frac{\sigma_Y^2}{2E}$$

- Resilience represents the ability of a material to absorb energy within the elastic range.
- Modulus of resilience represents the energy per unit volume that the material can absorb without yielding.
- The capacity of a structure to withstand an impact load without being permanently deformed clearly depends upon the resilience of the material used.

Toughness and Modulus of Toughness



$$\text{Strain-energy density} = u = \int_0^{\epsilon_1} \sigma_x \, d\epsilon_x$$

The strain energy density obtained by setting $\epsilon_1 = \epsilon_R$ in the above equation is known as modulus of toughness of the material

- Toughness refers to the ability of a material to absorb energy without fracturing.
- Modulus of toughness represents the energy per unit volume required to cause the material to rupture.
- The toughness of a material is related to its ductility as well as to its ultimate strength.
- The capacity of a structure to withstand an impact load depends upon the toughness of the material used.

Elastic Strain Energy – Axial/Normal Force

Uniform Bar

$$U = \frac{P\delta}{2} \qquad \delta = \frac{PL}{AE}$$

$$\boxed{U = \frac{P^2 L}{2AE}} \qquad \boxed{U = \frac{\sigma^2 V}{2E}}$$

Nonprismatic Bar with varying axial force

$$\boxed{U = \int_0^L \frac{[P(x)]^2 dx}{2EA(x)}}$$

Bar with different cross sections

$$\boxed{U = \sum_{i=1}^{n} \frac{P^2{}_i L_i}{2A_i E_i}}$$

Problem 22

A uniform metal bar has a cross-sectional area of $700\ mm^2$ and a length of 1.5 m. If the stress at elastic limit is $160\ N/mm^2$, what will be its proof resilience? Determine also the maximum value of an applied load, which may be suddenly applied without exceeding the elastic limit. Calculate the value of the gradually applied load which will produce the same extension as that produced by the suddenly applied load above. Take $E = 2 \times 10^5\ N/mm^2$

Take $E = 2 \times 10^5\ N/mm^2$.

Sol. Given :

Area, $A = 700\ mm^2$

Length, $L = 1.5\ m = 1500\ mm$

$\therefore$ Volume of bar, $V = A \times L = 700 \times 1500 = 1050000\ mm^2$

Stress at elastic limit, $\sigma^* = 160 \text{ N/mm}^2$

Young's modulus, $E = 2 \times 10^5 \text{ N/mm}^2$

(*i*) Proof resilience is given by equation (4.3), as

Proof resilience $$= \frac{\sigma^{*2}}{2E} \times \text{Volume} = \frac{160^2}{2 \times 2 \times 10^5} \times 1050000$$

$$= 67200 \text{ N-mm} = \mathbf{67.2 \text{ N-m.}} \quad \textbf{Ans.}$$

(*ii*) Let P = Maximum value of suddenly applied load, and

P_1 = Gradually applied load.

Using equation (4.5) for suddenly applied load,

$$\sigma^* = 2 \times \frac{P}{A}$$ (change p to p^*)

$$\therefore \quad P = \frac{\sigma^* \times A}{2} = \frac{160 \times 700}{2} = 56000 \text{ N} = \mathbf{56 \text{ kN.}} \quad \textbf{Ans.}$$

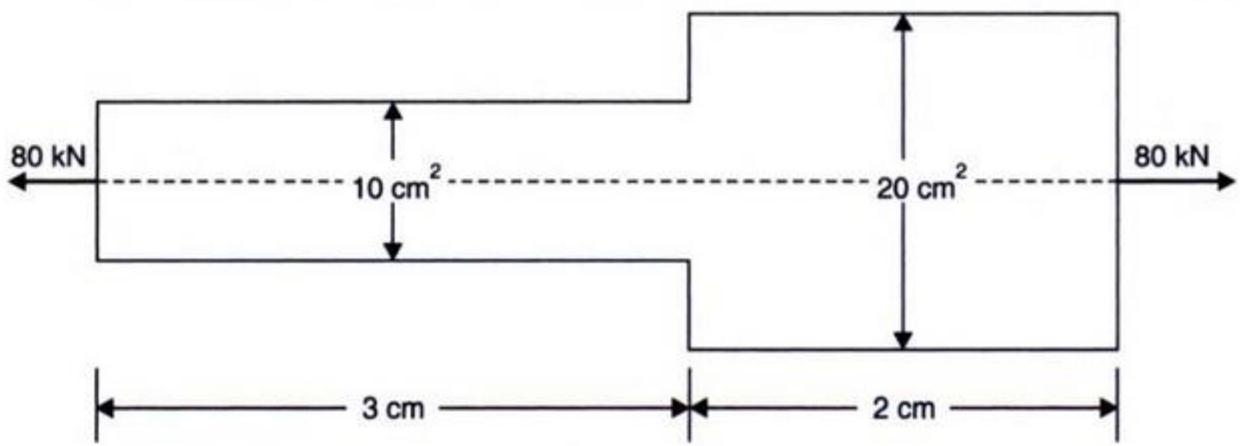
For gradually applied load,

$$\sigma^* = \frac{P_1}{A}$$

$$P_1 = \sigma^* \times A = 160 \times 700 = 112000 \text{ N} = \mathbf{112 \text{ kN.}} \quad \textbf{Ans.}$$

Problem 23

A tension bar 5 m long is made up of two parts, 3 metre of its length has a cross-sectional area of 10 cm^2 while the remaining 2 metre has a cross-sectional area of 20 cm^2 . An axial load of 80 kN is gradually applied. Find the total strain energy produced in the bar and compare this value with that obtained in an uniform bar of the same length and having same volume when under the same load. Take $E = 2 \times 10^5$ N/mm^2



Young's modulus $E = 2 \times 10^5$ N/mm^2

Stress in 1st part $\sigma_1 = \dfrac{\text{Load}}{A_1} = \dfrac{80000}{1000} = 80$ N/mm^2

Stress in 2nd part $\sigma_2 = \dfrac{P}{A_2} = \dfrac{80000}{2000} = 40$ N/mm^2

Strain energy in 1st part,

$$U_1 = \frac{\sigma_1^2}{2E} \times V_1 = \frac{80^2}{2 \times 2 \times 10^5} \times 3 \times 10^6 = 48000 \text{ N-mm} = 48 \text{ N-m}$$

Strain energy in 2nd part,

$$U_2 = \frac{\sigma_2^2}{2E} \times V_2 = \frac{40^2}{2 \times 2 \times 10^5} \times 4000000 = 16000 \text{ N-mm} = 16 \text{ N-m}$$

$\therefore$ Total strain energy produced in the bar,

$$U = U_1 + U_2 = 48 + 16 = \mathbf{64 \text{ N-m.}} \quad \textbf{Ans.}$$

Strain energy stored in a uniform bar

Volume of uniform bar, $V = V_1 + V_2 = 3000000 + 4000000 = 7000000 \text{ mm}^2$

Length of uniform bar, $L = 5 \text{ m} = 5000 \text{ mm}$

Let $\quad A$ = Area of uniform bar

Then $\quad V = A \times L \quad \text{or} \quad 7000000 = A \times 5000$

$$\therefore \quad A = \frac{7000000}{5000} = 1400 \text{ mm}^2$$

Stress in uniform bar, $\sigma = \dfrac{P}{A} = \dfrac{80000}{5000} = 57.143 \text{ N/mm}^2$

$\therefore$ Strain energy stored in the uniform bar,

$$U = \frac{\sigma^2}{2E} \times V = \frac{57.143^2}{2 \times 2 \times 10^5} \times 7000000$$

$$= 57143 \text{ N-mm} = 57.143 \text{ N-m}$$

$$\therefore \quad \frac{\text{Strain energy in the given bar}}{\text{Strain energy in the uniform bar}} = \frac{64}{57.143} = 1.12. \quad \textbf{Ans.}$$

Impact Loading

Maximum Deformation and Maximum Stress due to a Falling Weight on a Prismatic Bar

$$\delta_{max} = \delta_{st}\left[1 + \left(1 + \frac{2h}{\delta_{st}}\right)^{1/2}\right]$$

$$\sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$$

If height is large compared to static elongation

$$\delta_{max} = \sqrt{2h\delta_{st}} \qquad \sigma_{max} = \sqrt{\frac{2hE\sigma_{st}}{L}}$$

Suddenly Applied Load

When *h* is zero

$$\delta_{max} = 2\delta_{st} \qquad \sigma_{max} = 2\sigma_{st}$$

Impact Factor

$$Impact\ Factor = \frac{\delta_{max}}{\delta_{st}}$$

Problem 24

A round, prismatic steel bar (E = 210 GPa) of length L = 2 m and diameter d = 15 mm hangs vertically from a support at its upper end. A sliding collar of mass m = 20 kg drops from a height h = 150 mm onto the flange at the lower end of the bar without rebounding.

(a) Calculate the maximum elongation of the bar due to the impact and determine the corresponding impact factor.

(b) Calculate the maximum tensile stress in the bar due to the impact load and compare with the static stress.