Module-3
Shear Force and Bending Moment

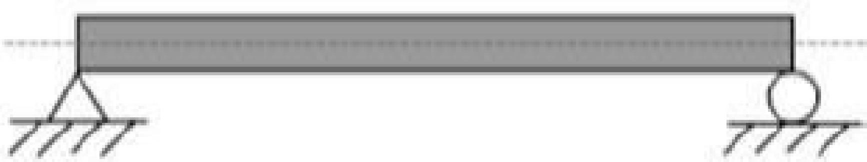
Definition of beam – Types of beams – Concept of shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, uniformly distributed loads, uniformly varying loads and combination of these loads – Point of contra flexure – Relation between S.F., B.M and rate of loading at a section of a beam

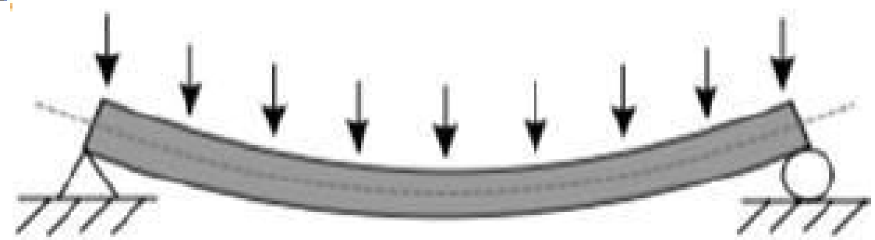
BEAMS

Beams are structural members that offer resistance to bending due to applied load

A beam may be defined as a structural element which has one dimension considerably larger than the other two dimensions, namely breadth and depth, and is supported at few points.

The distance between two adjacent supports is called span. It is usually loaded normal to its axis. The applied loads make every cross-section to face bending and shearing





If it is possible to determine the reactions in the supports by equilibrium equations only, then the beam is said to be statically determinate. If equilibrium equations are not sufficient to find reactions in the supports, the beam is said to be a statically indeterminate.

Objective

- When a beam is loaded by forces or couples, stresses and strains are created throughout the interior of the beam.
- To determine these stresses and strains, the internal forces and internal couples that act on the cross sections of the beam must be found.

Types of Supports and Reactions

Fixed Support – Beam Fixed in Wall

Hinge Support of Sydney Harbor Bridge

Roller Support on One End of a Bridge

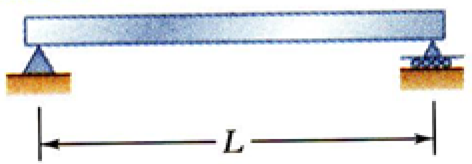
Rocker Support in a Structure

Simple Supports in a Structure

Types of beams

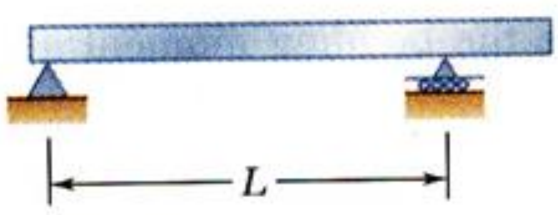
1.Simply supported beam:-

A beam supported freely on the walls or columns at its both ends is known as simply supported beam.



2.Over hanging beam:-

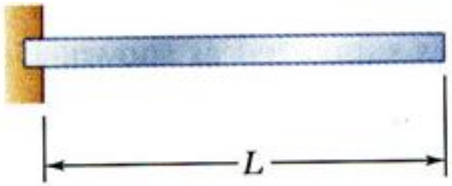
A Beam is freely supported on two supports. But its one end or both the ends are projected beyond the support.



Types of beams

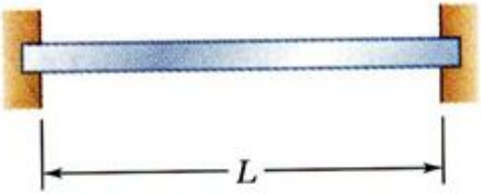
3.Cantilever Beam:

A Beam Fixed at one end and free at the other end is known as cantilever Beam.



4.Fixed Beam:-

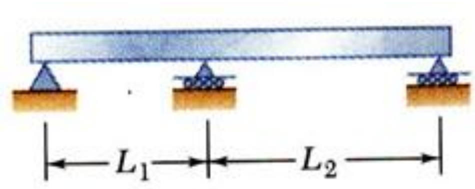
A beam whose both ends are rigidly fixed is known as cantilever beam.



Types of beams

5.Continuous beam:-

A Beam Supported on more then two support is known as continuous beam.



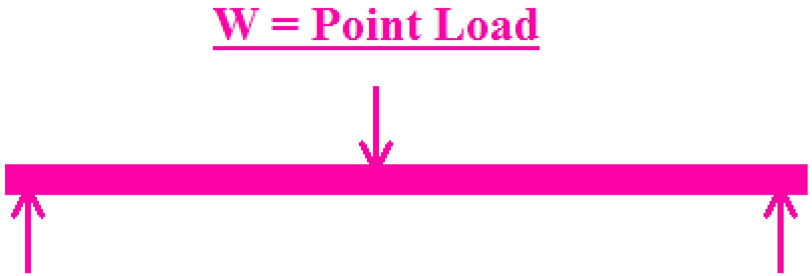
Type of loads

1. Point load or concentrated load:-

When a load is acting on a relatively small area it is considered as point load or concentrated load.

W = Point Load

It is given in N or KN.



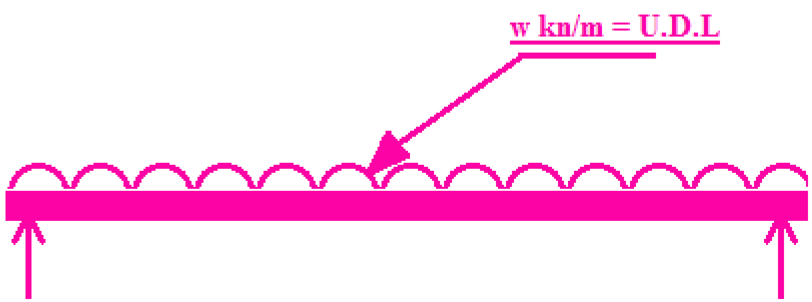
Types of loads

2. Uniformly Distributed Load:-

A Load which is spread over a beam in such a manner that each unit length of beam is loaded to the same intensity is known as uniformly distributed load.

W = U.D.L

It is given in N/m or KN/m.

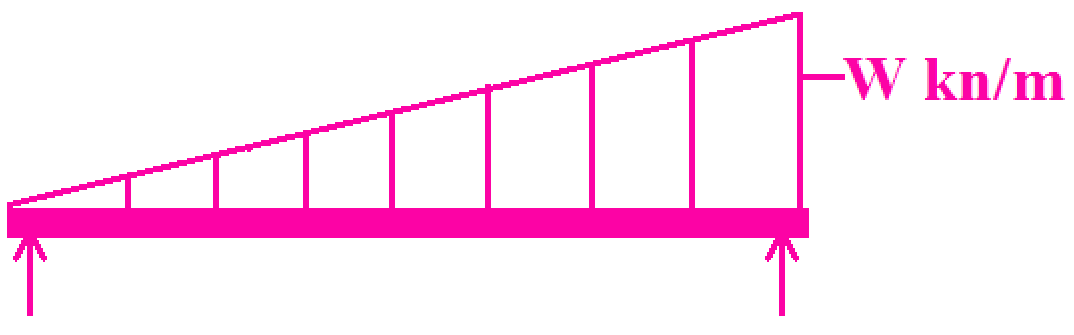


Types of loads

3.Uniformly Varying Load:-

A Load which is spread over a beam in such a manner that its intensity varies uniformly on each unit is called uniformly varying load.

When load is zero at one end and increases uniformly to the other end it is known as triangular load.



Types Of Support

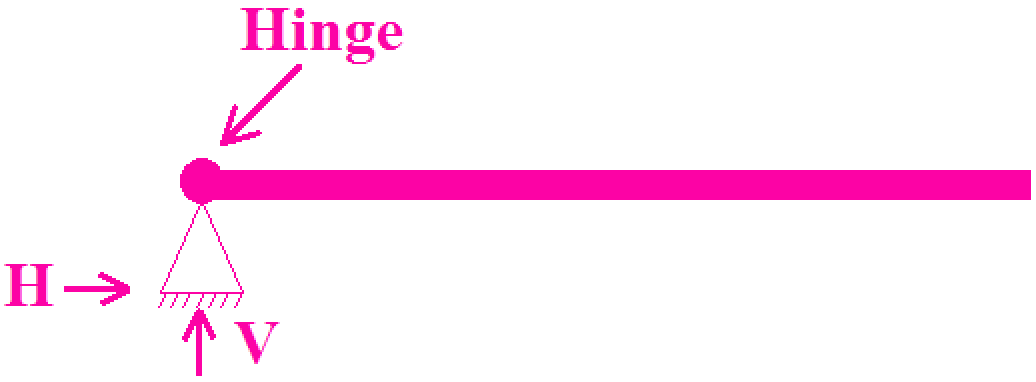
1. Simple Support:-

- Beam is freely supported on the support.
- There is no monolithic construction between the beam and the support.
- Only vertical reaction can devlope at the support.

Types Of Support

2. Hinge Support:-

- Beam is hinged to the support at end
- Beam can rotate about the hinge
- Vertical reaction (V) and horizontal reaction (H) can developed at the support.



Types Of Support

3. Fixed Support :-

End of beam is rigidly fixed or built in wall. Beam can not rotate at the end.

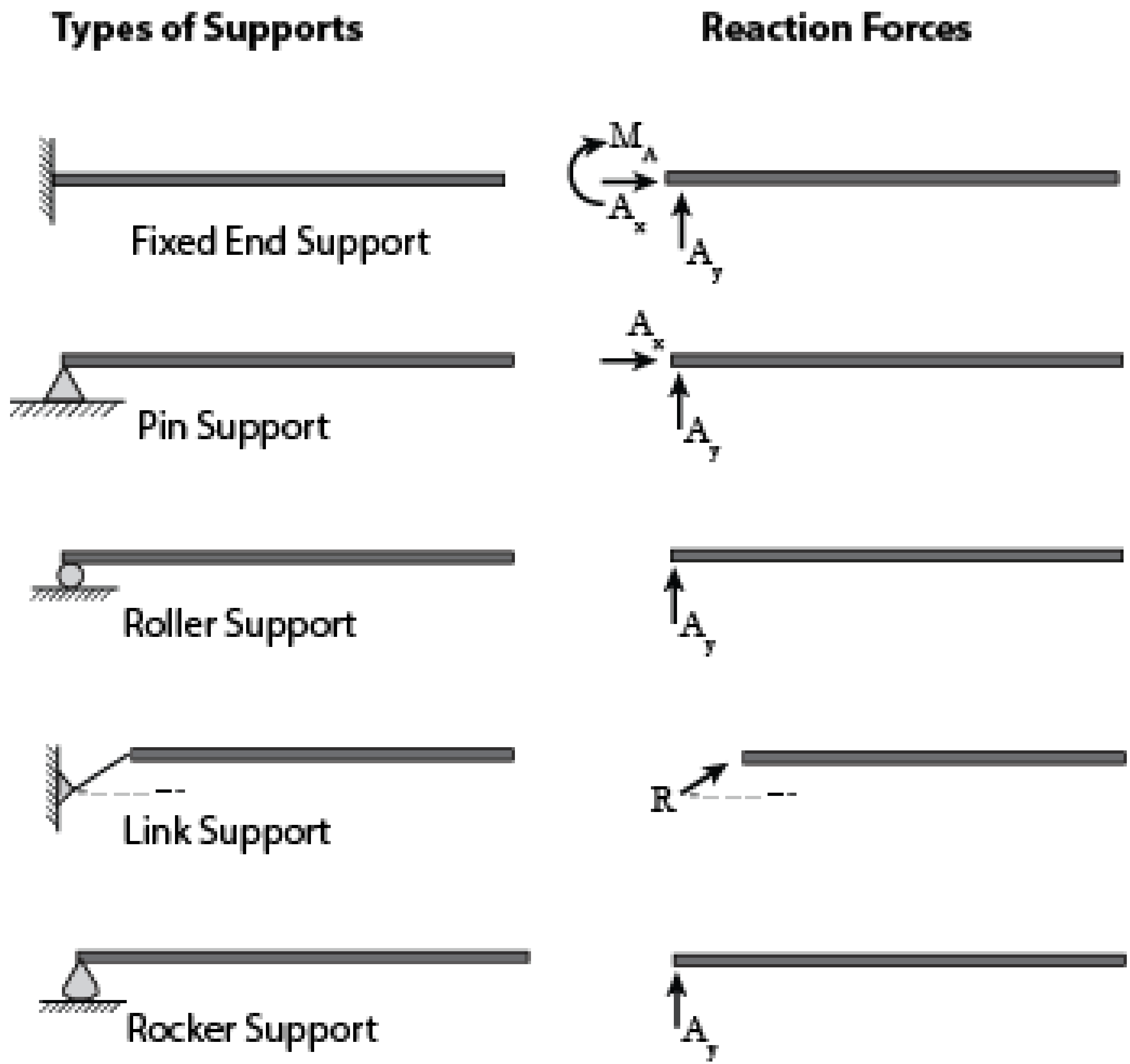
Vertical reaction (V), Horizontal Reaction (H), Moment (M) can devlope at support.

Types Of Support

4. Roller Support :-

Beam is hinged to the support at the end roller are provided below the support.

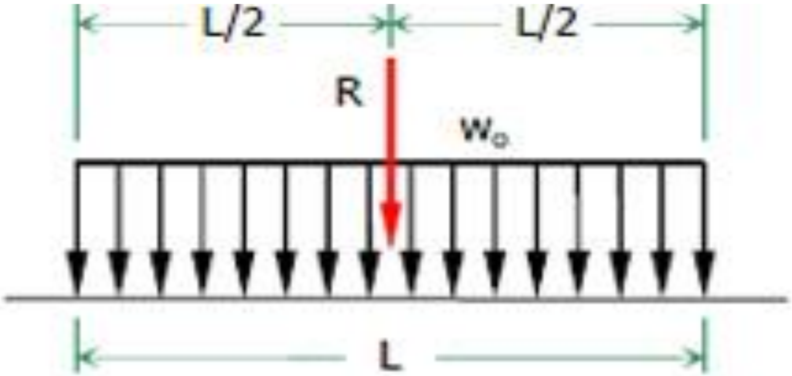
End of beam can move on rollers. Only vertical reaction (V) normal to the plane of roller can devlope.



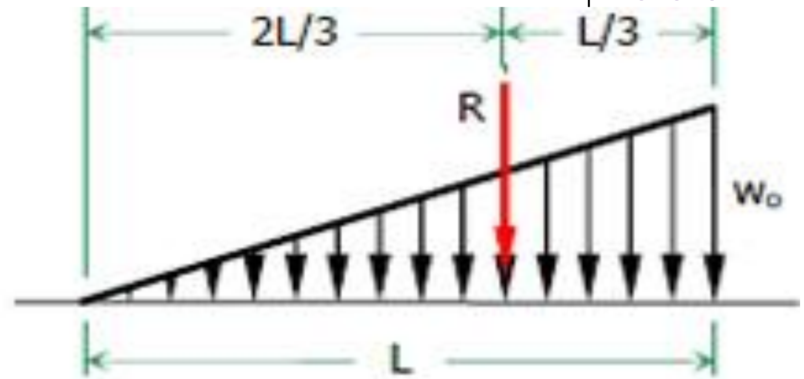
Types of Supports
Reaction Forces
Fixed End Support
M_A
A_x
A_y
Pin Support
A_x
A_y
Roller Support
A_y
Link Support
R
Rocker Support
A_y

Types of Loading

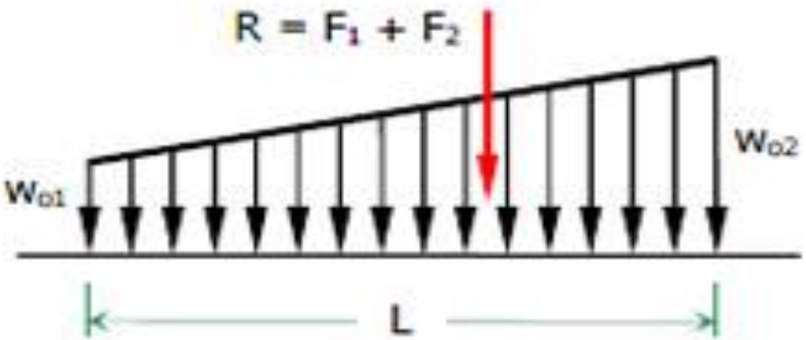
- Concentrated load or point load
- Uniformly distributed load (udl)
- Uniformly Varying load (uvl)



Rectangular Load

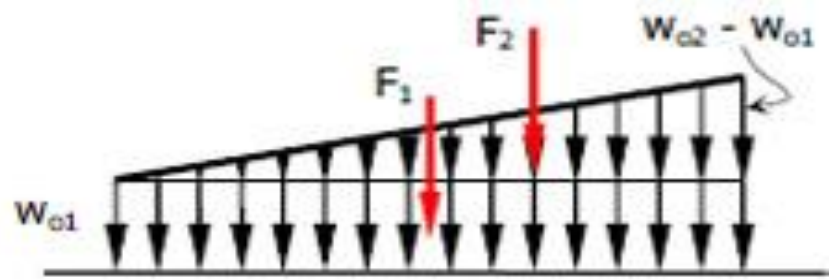


Triangular Load



Trapezoidal Load

=



Resolution of trapezoidal load into rectangular and triangular loads

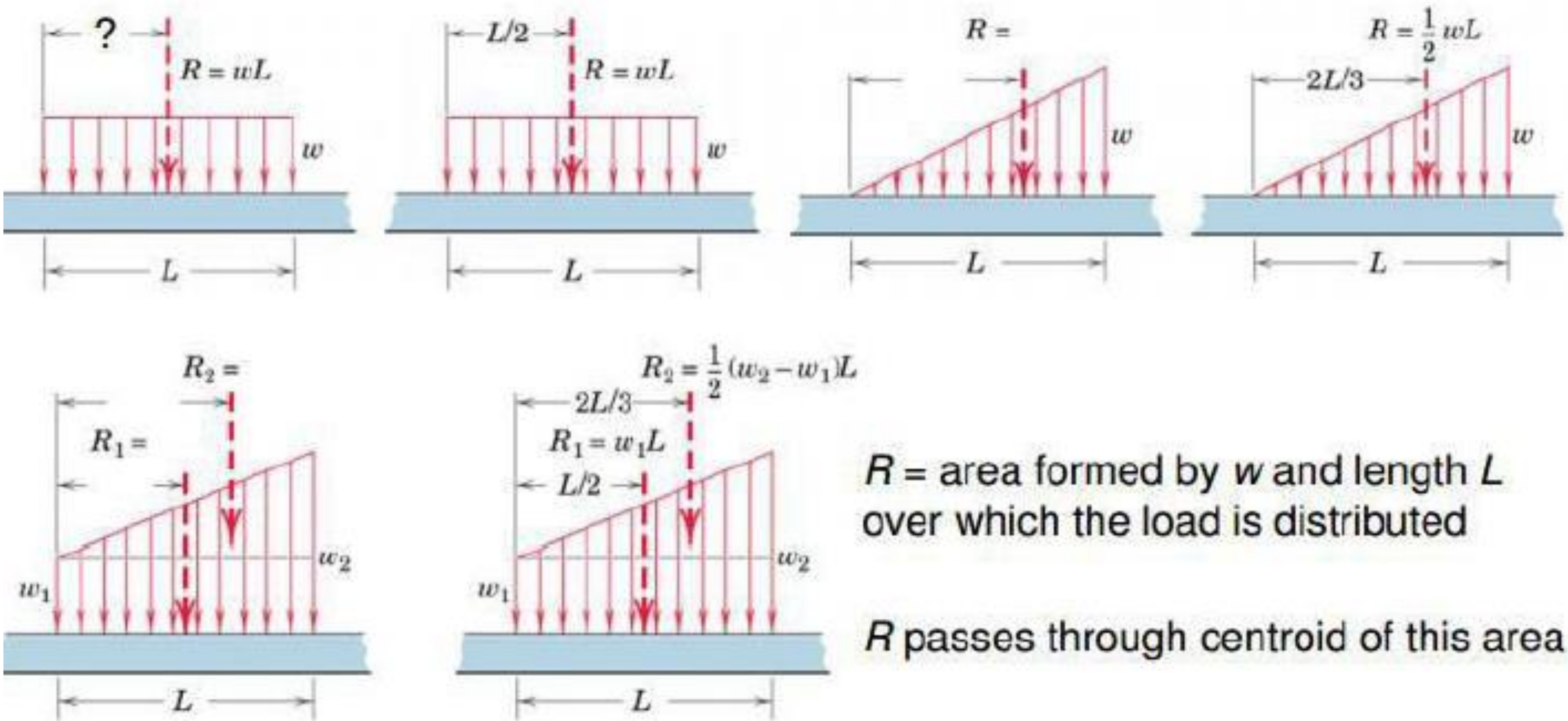
Rectangular Load
$R = w_o L$

Triangular Load
$R = \frac{1}{2} w_o L$

Trapezoidal Load
$R = w_{o1} L + \frac{1}{2}(w_{o2} - w_{o1}) L$

Distributed Loads on beams

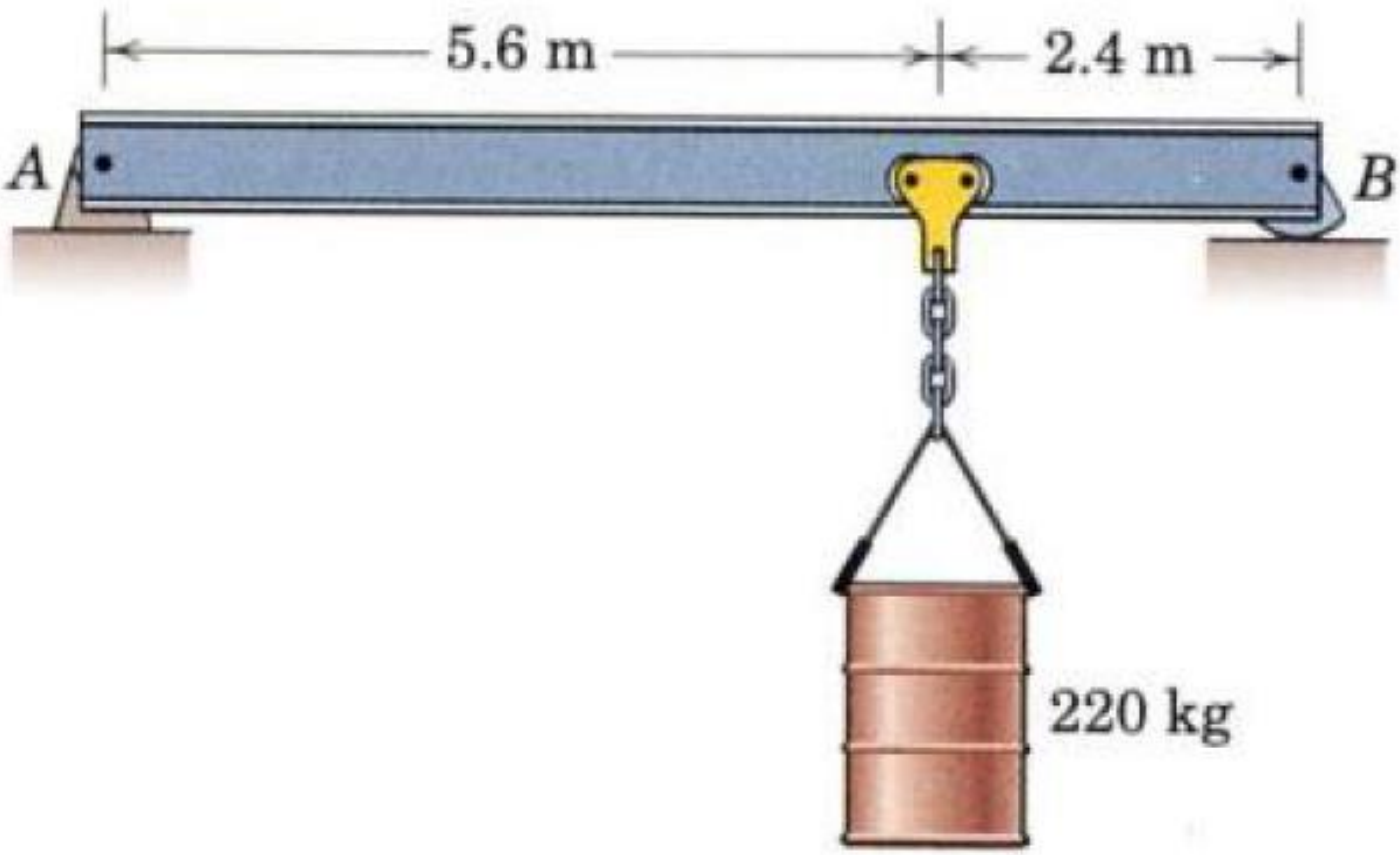
- Determination of Resultant Force (R) on beam is important

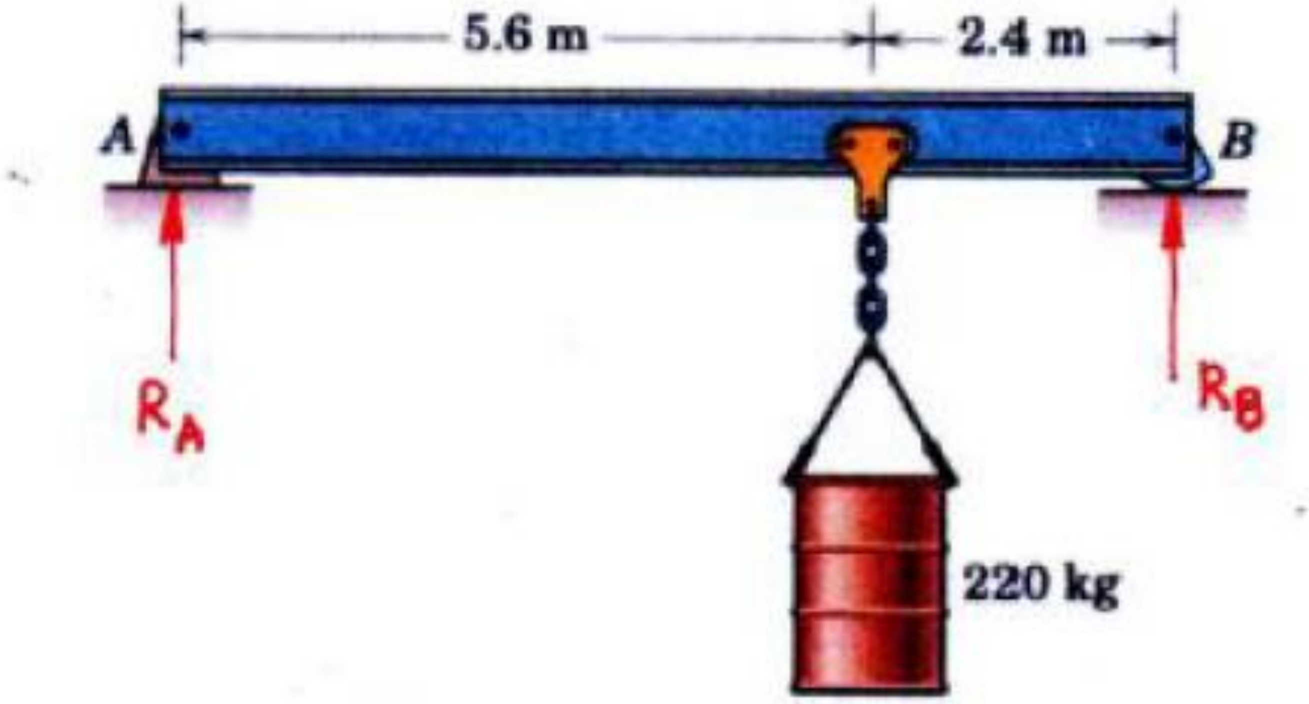


R = area formed by w and length L over which the load is distributed

R passes through centroid of this area

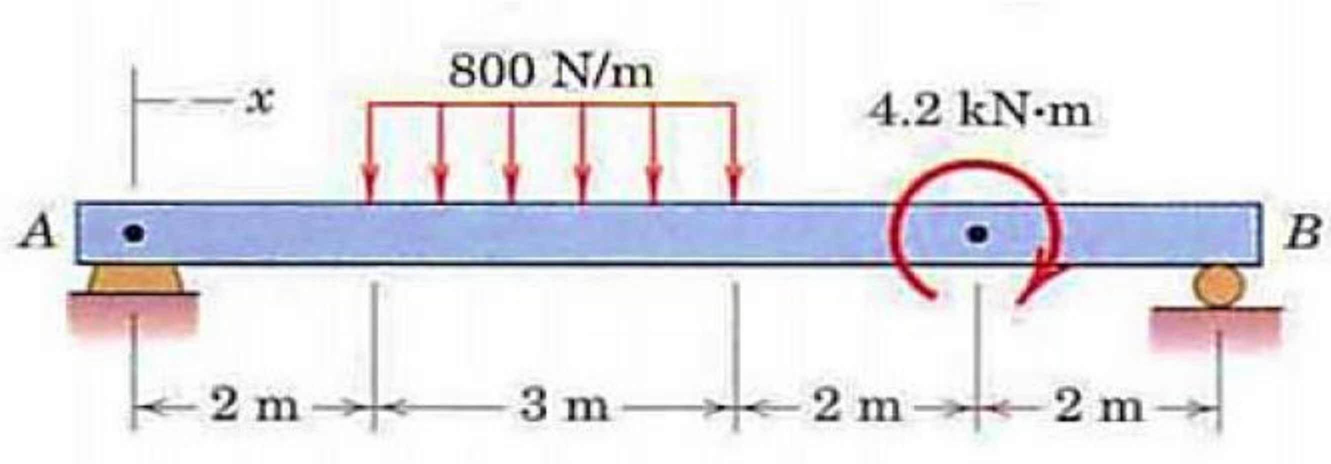
The uniform I – beam supports the load as shown in Fig. 3. Determine the reactions at the supports.

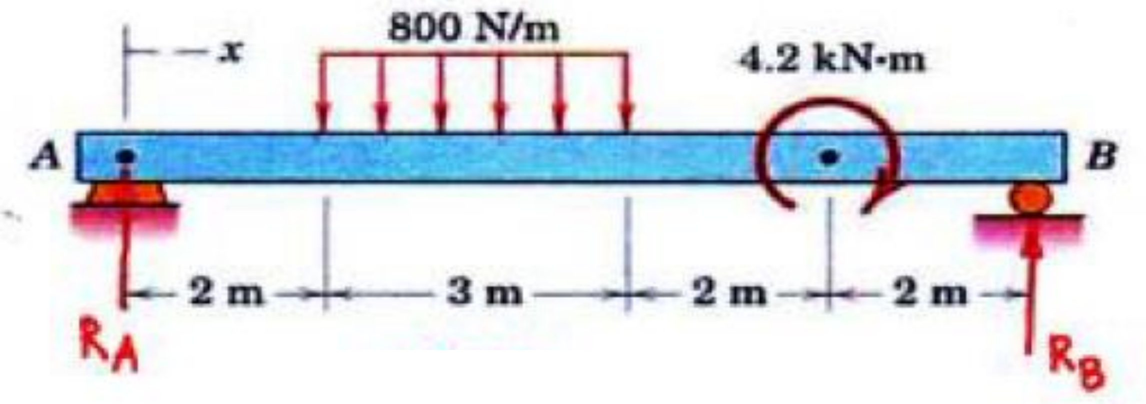




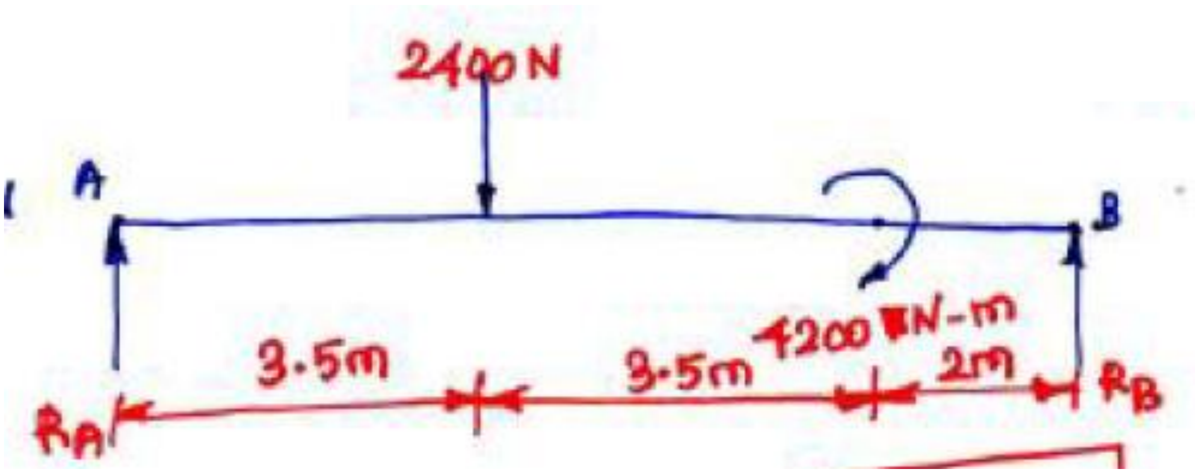
$\boxed{R_B = 1510.74\ N}$ $\boxed{R_A = 647.46\ N}$

2. Determine the reaction forces at the supports shown below.



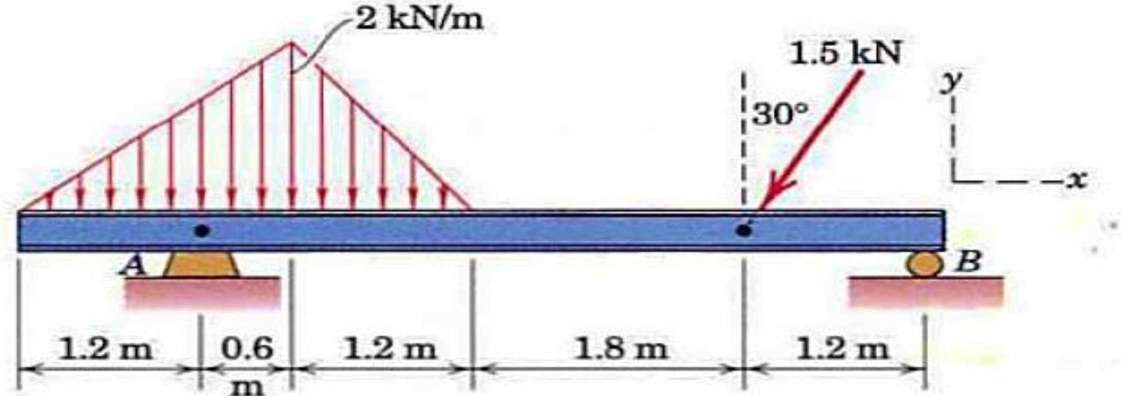


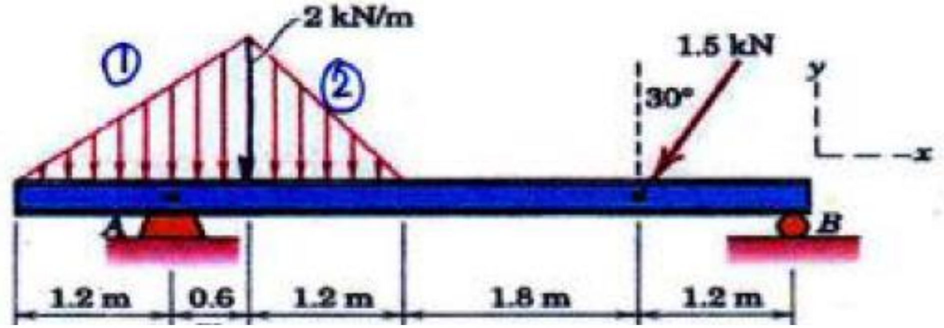
$\boxed{\Sigma F_y = 0}$

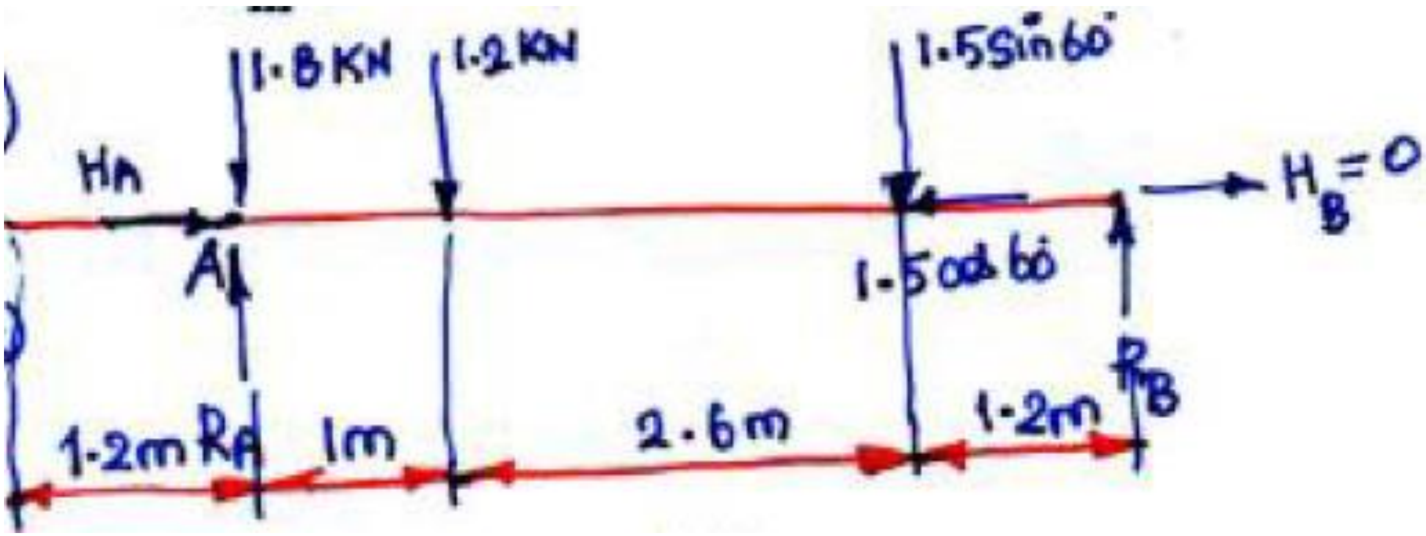


$\Rightarrow \boxed{R_B = 1400\text{N}}$

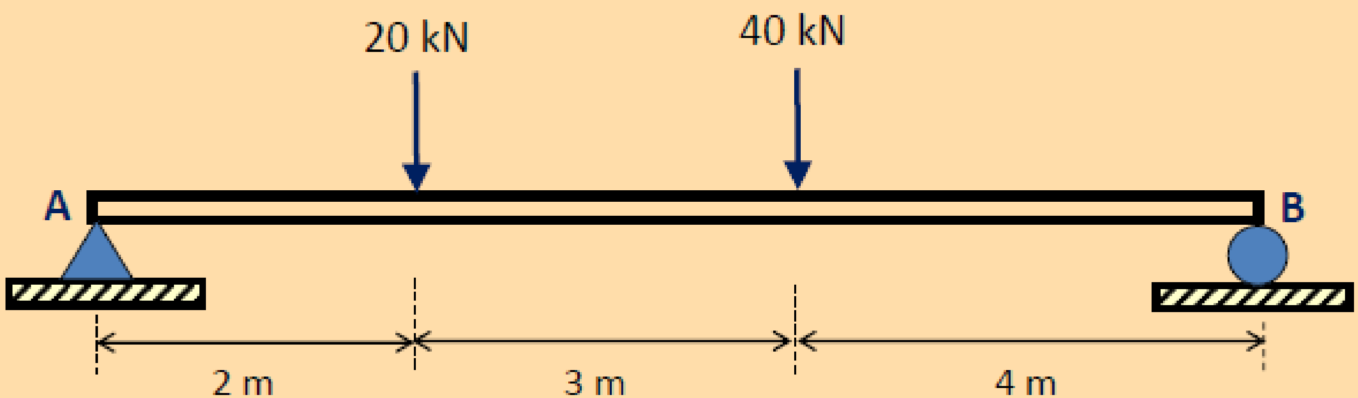
$\boxed{R_A = 1000 \text{ N}}$

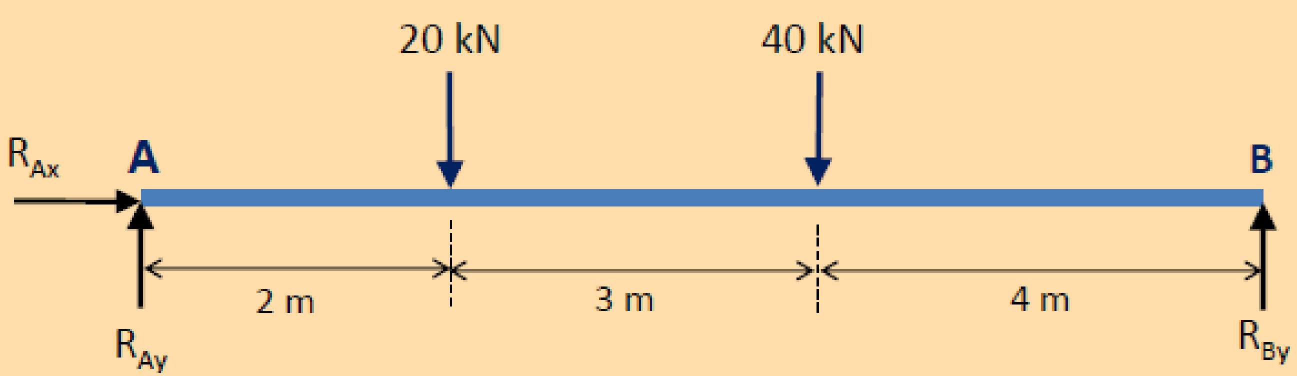

2 kN/m
1.5 kN
30°
y
x
A
B
1.2 m
0.6 m
1.2 m
1.8 m
1.2 m


2 kN/m
1.5 kN
30°
y
x
A
B
1.2 m
0.6
1.2 m
1.8 m
1.2 m

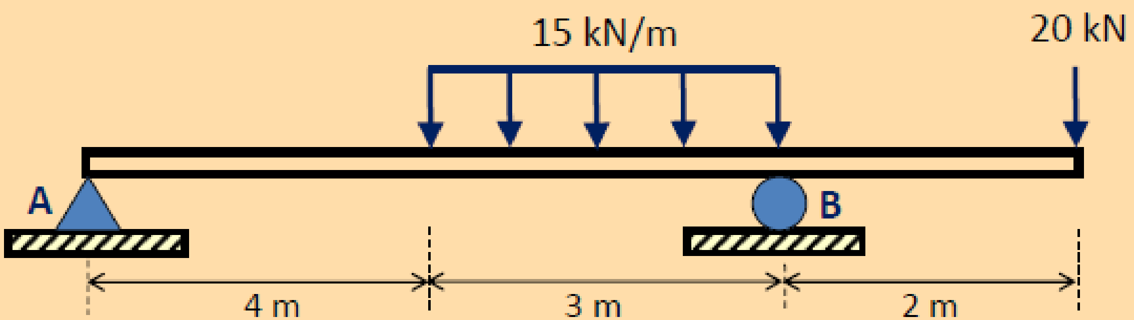

1.8 KN
1.2 KN
1.5 sin 60
HA
A
1.5 cos 60
HB = 0
RB
1.2m
RA
1m
2.6m
1.2m

The beam shown below is supported by a pin at A and roller at B. Calculate the reactions at both supports due to the loading.

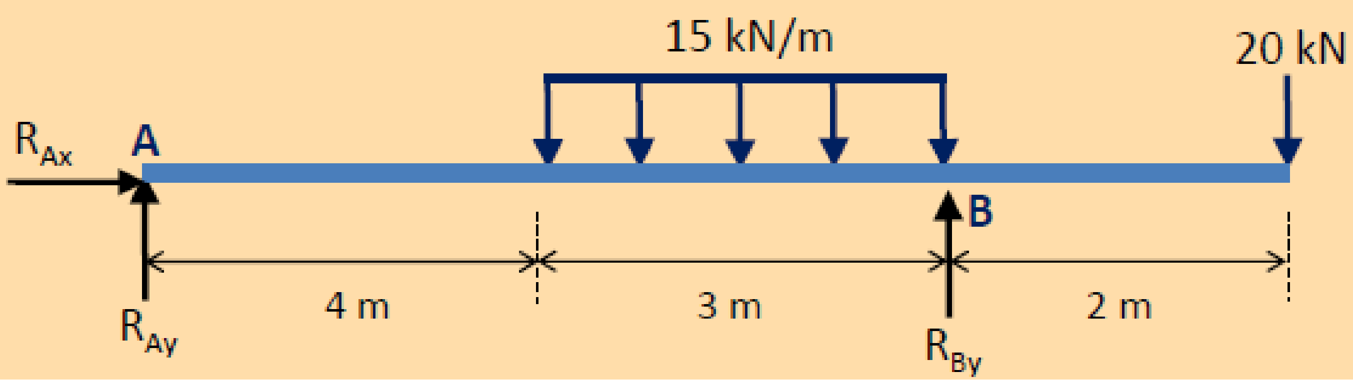




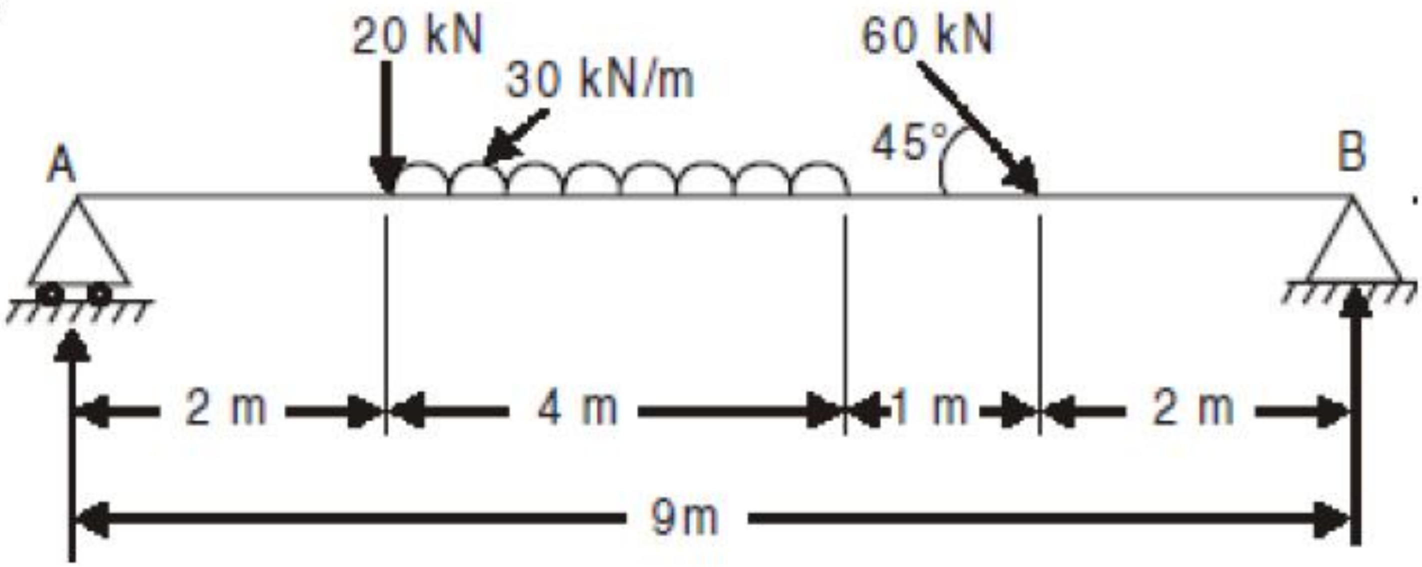
Determine the reactions at support A and B for the overhanging beam subjected to the loading as shown.



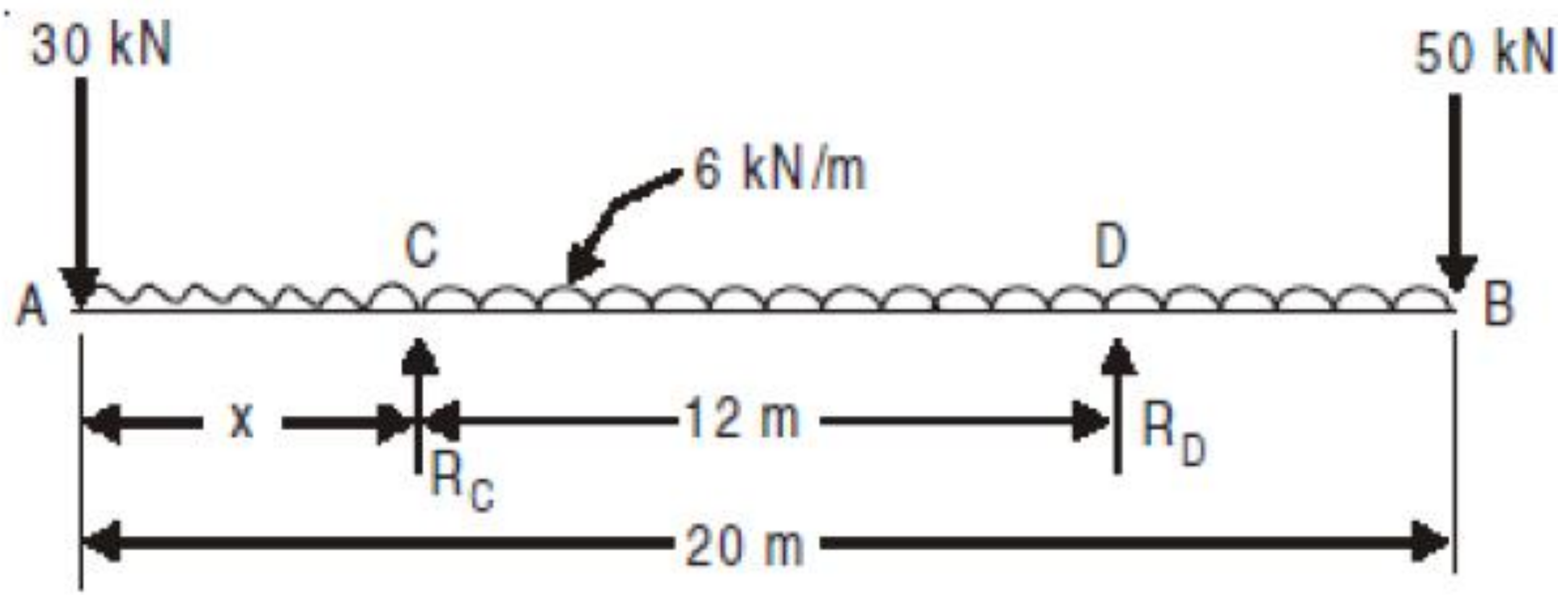
Draw the free body diagram:



Find the reactions at supports A and B of the loaded beam shown in Fig.



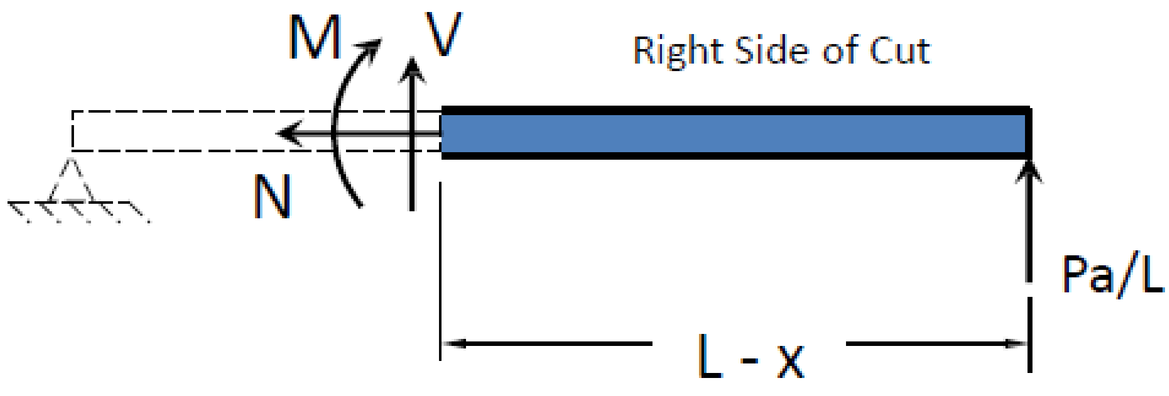
Example A beam AB 20 m long supported on two intermediate supports 12 m apart, carries a uniformly distributed load of 6 kN/m and two concentrated loads of 30 kN at left end A and 50 kN at the right end B as shown in Fig. How far away should the first support C be located from the end A so that the reactions at both the supports are equal ?



Internal Reactions in Beams

At any cut in a beam, there are 3 possible internal reactions required for equilibrium:

- normal force,
- shear force,
- bending moment.



Shear Force and Bending Moment

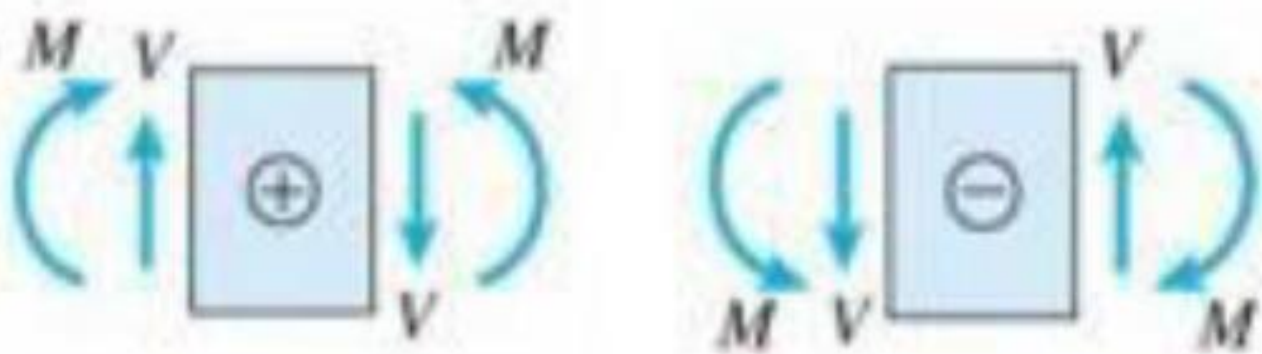
- **Shear Force:** is the algebraic sum of the vertical forces acting to the left or right of a cut section along the span of the beam

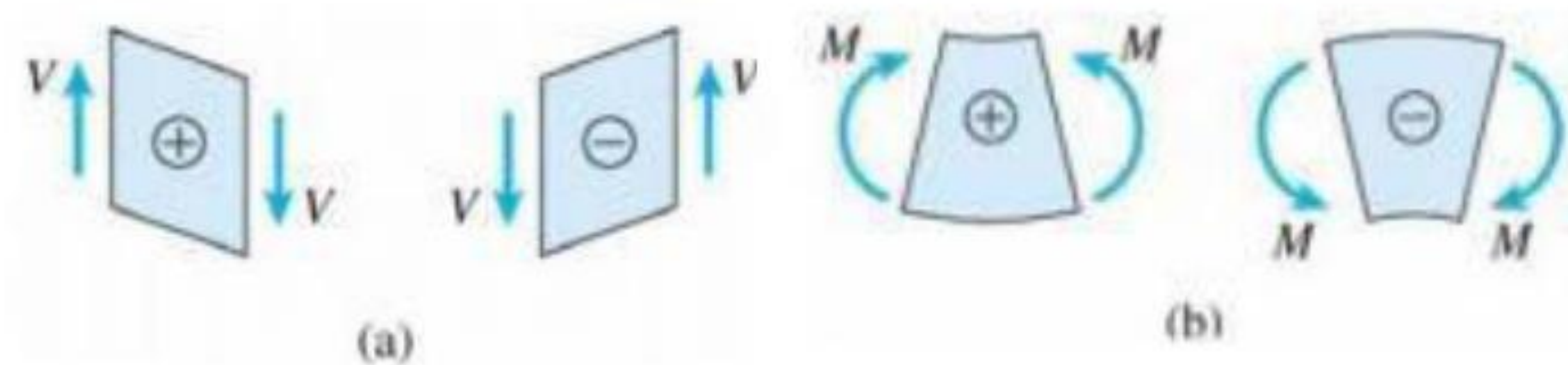
- **Bending Moment:** is the algebraic sum of the moment of the forces to the left or to the right of the section taken about the section

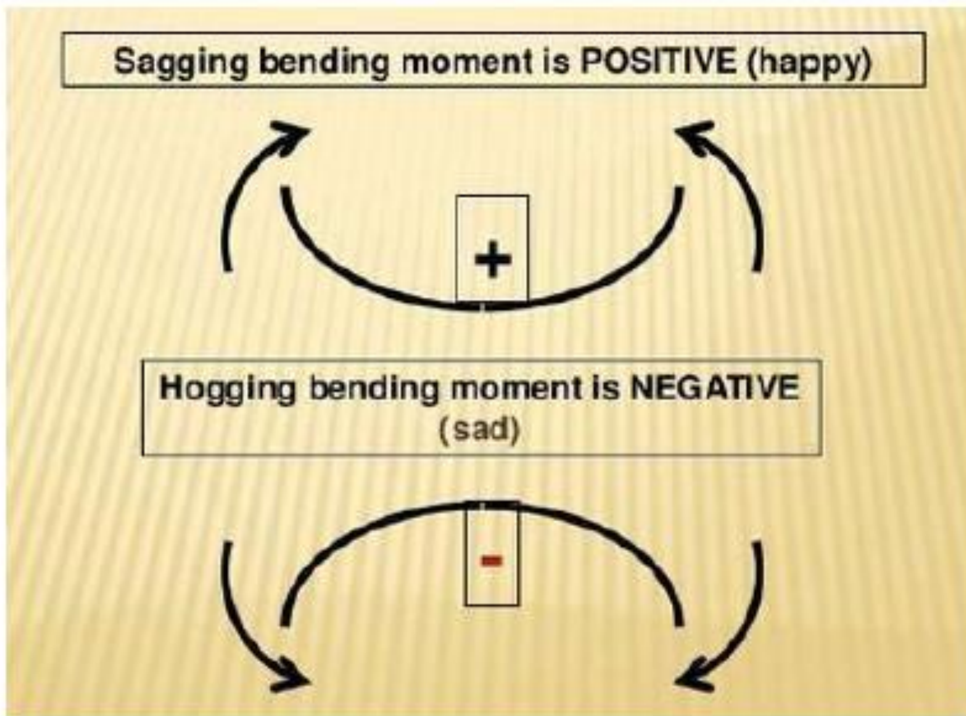
Sign Convention for Forces and Moments

- Positive directions are denoted by an internal shear force that causes clockwise rotation of the member on which it acts, and an internal moment that causes compression, or pushing on the upper arm of the member.

- Loads that are opposite to these are considered negative.

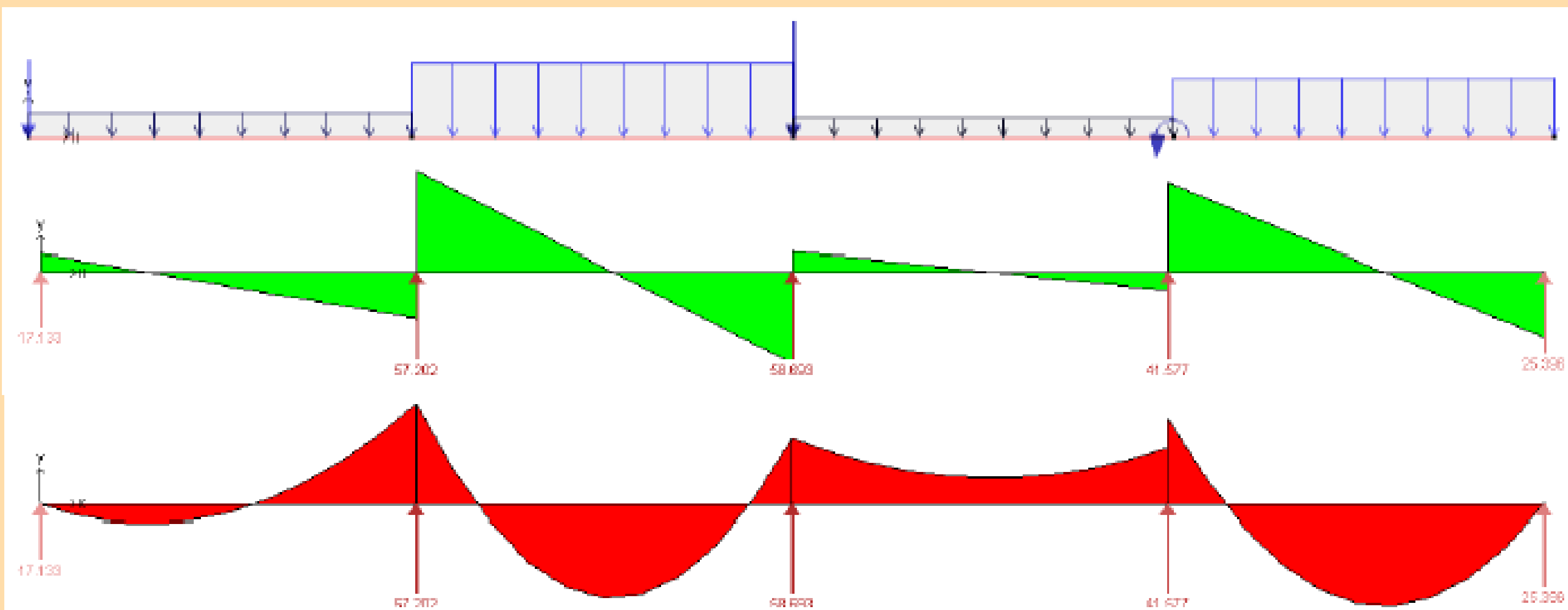



V
V
V
V
M
M
M
M
(a)
(b)

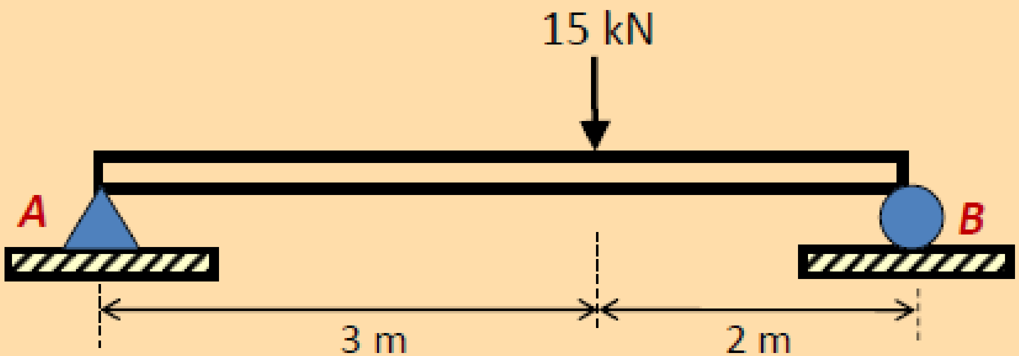

Sagging bending moment is POSITIVE (happy)
+
Hogging bending moment is NEGATIVE (sad)
-

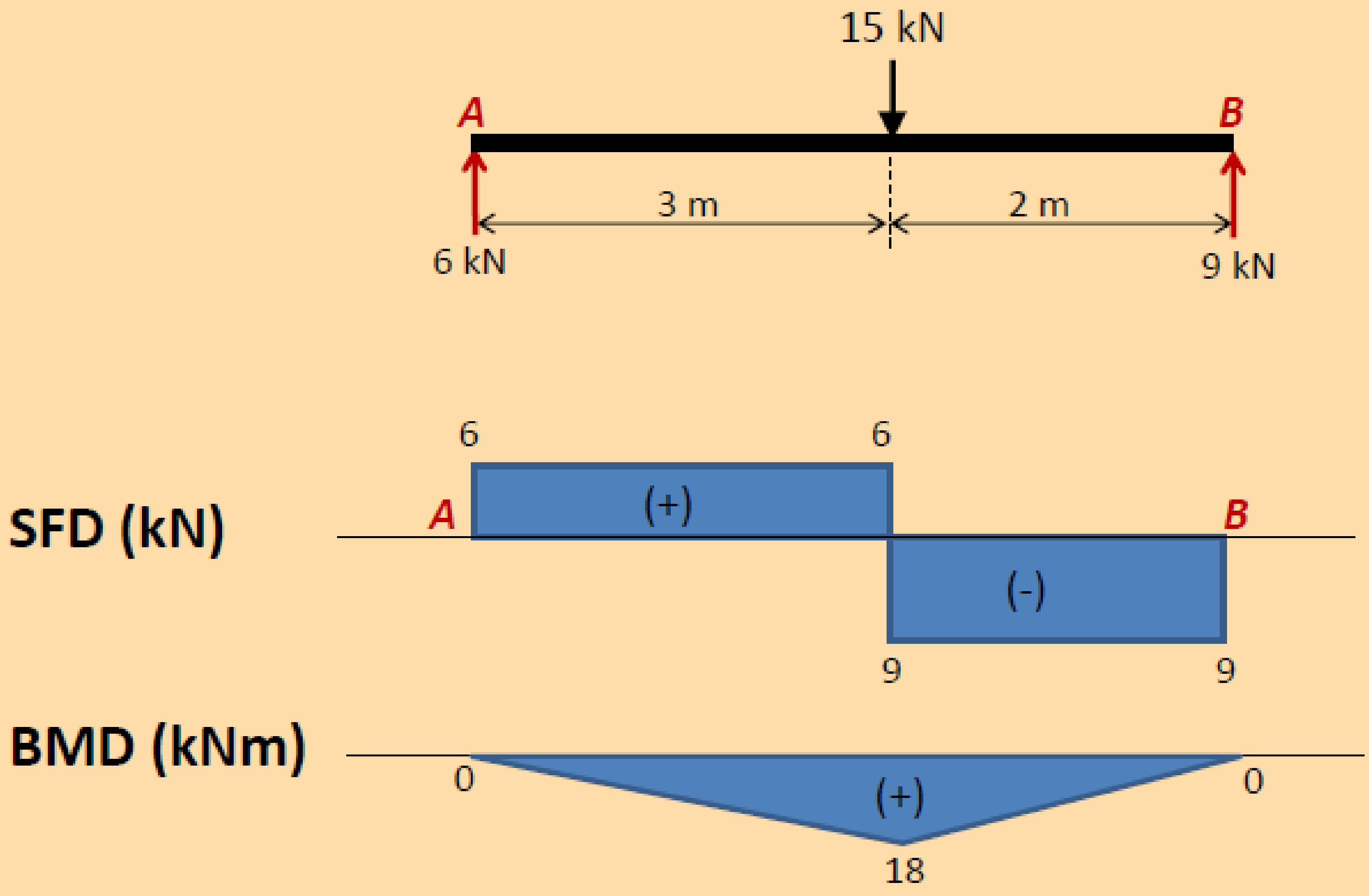
SHEAR FORCE & BENDING MOMENT

- In order to properly design a beam, it is important to know the ***variation*** of the shear and moment along its axis in order to find the points where these values are a maximum.



Calculate the shear force and bending moment for the beam subjected to a concentrated load as shown in the figure, then draw the shear force diagram (SFD) and bending moment diagram (BMD).



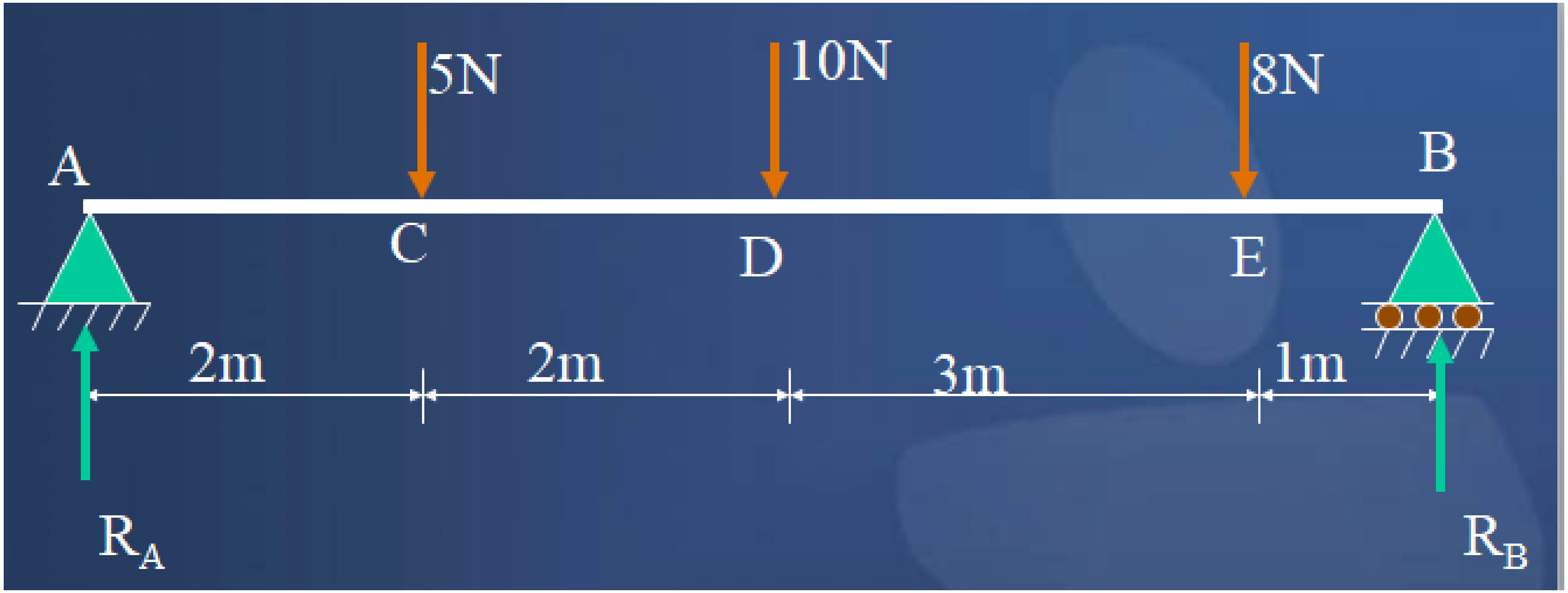

15 kN
A
B
3 m
2 m
6 kN
9 kN
SFD (kN)
6
6
A
(+)
B
(-)
9
9
BMD (kNm)
0
(+)
0
18

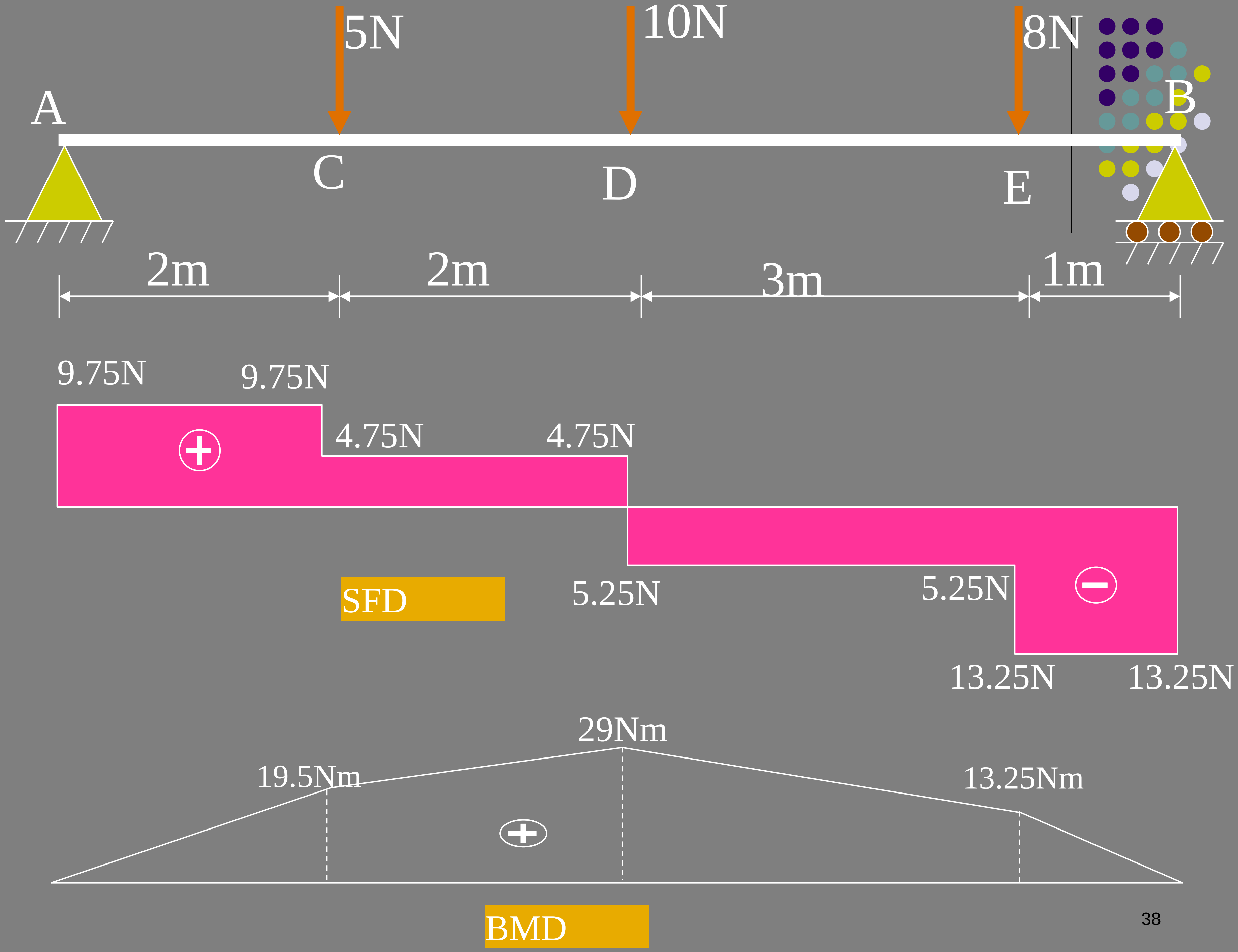
Point of Contra flexure :

It is the point on the bending moment diagram where bending moment changes the sign from positive to negative or vice versa.

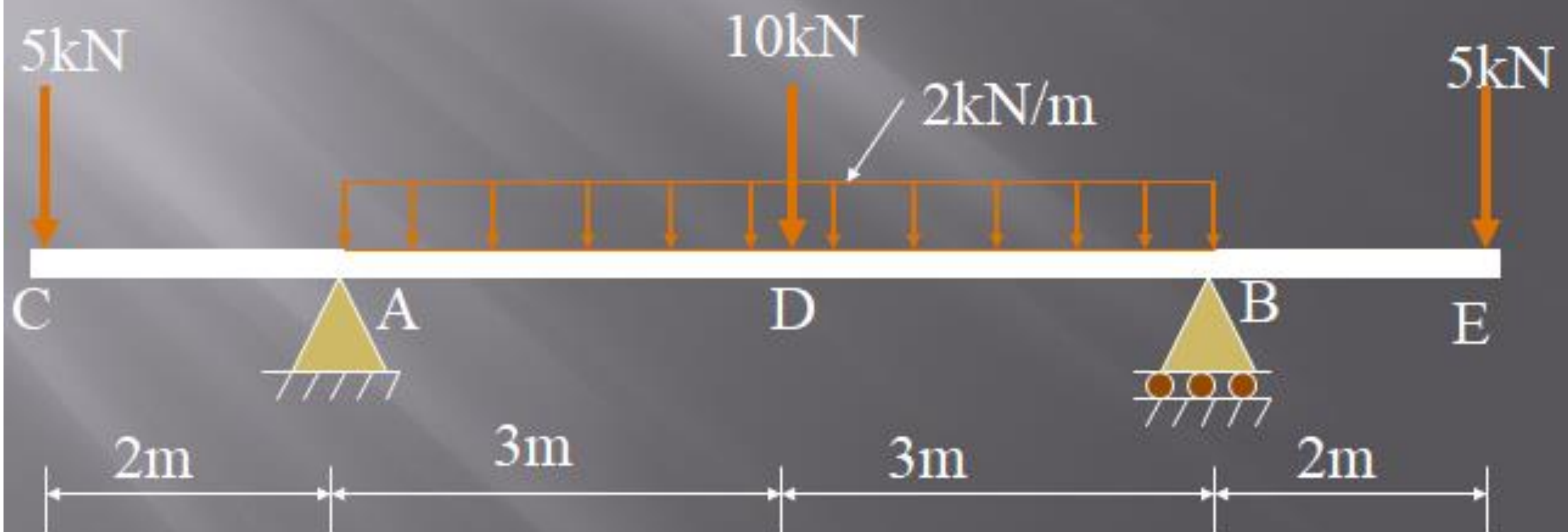
It is also called ‘Inflection point’. At the point of inflection point or contra flexure the bending moment is zero.

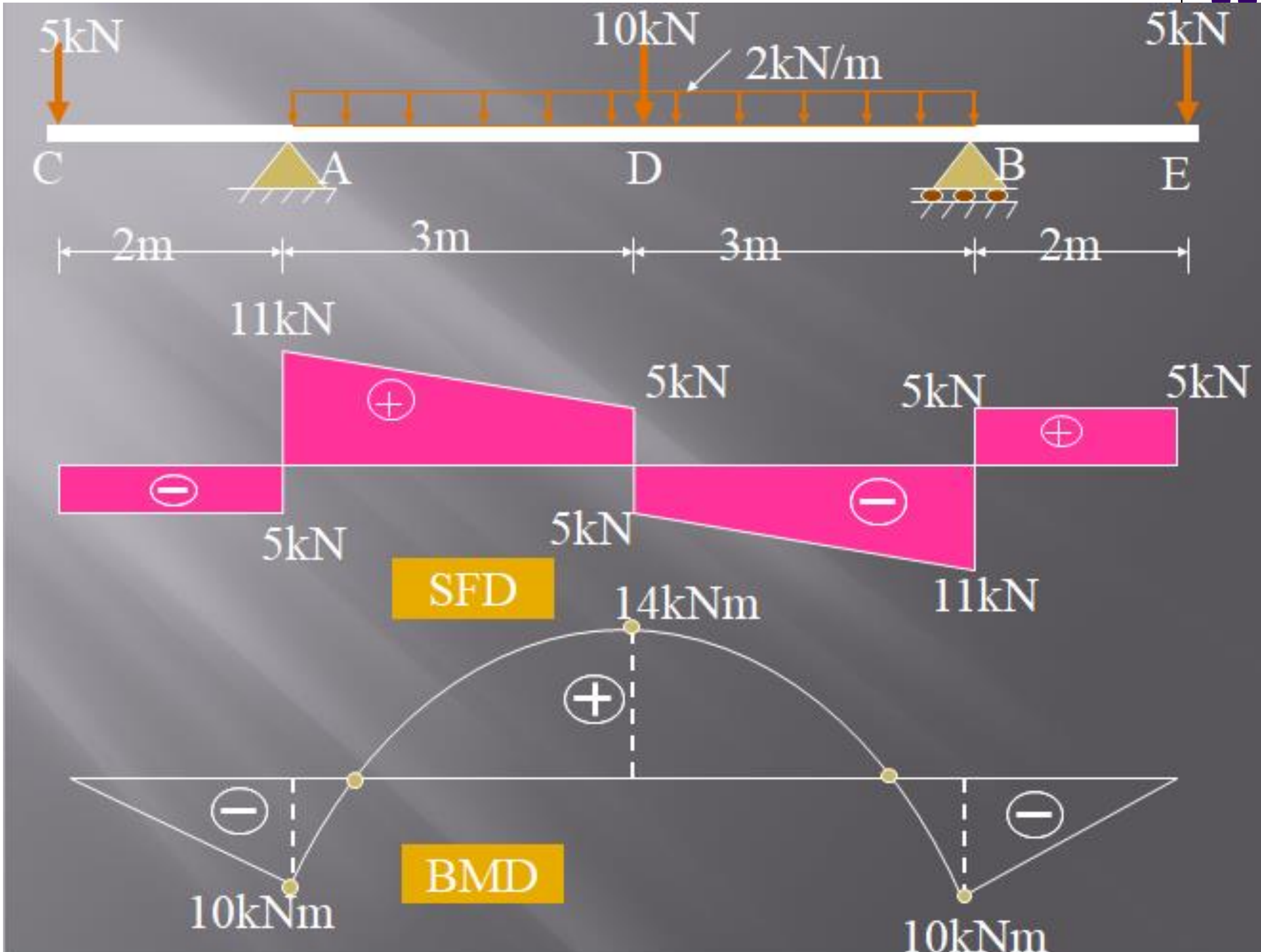
Draw shear force and bending moment diagrams [SFD and BMD] for a simply supported beam subjected to three point loads as shown in the Fig. given below.

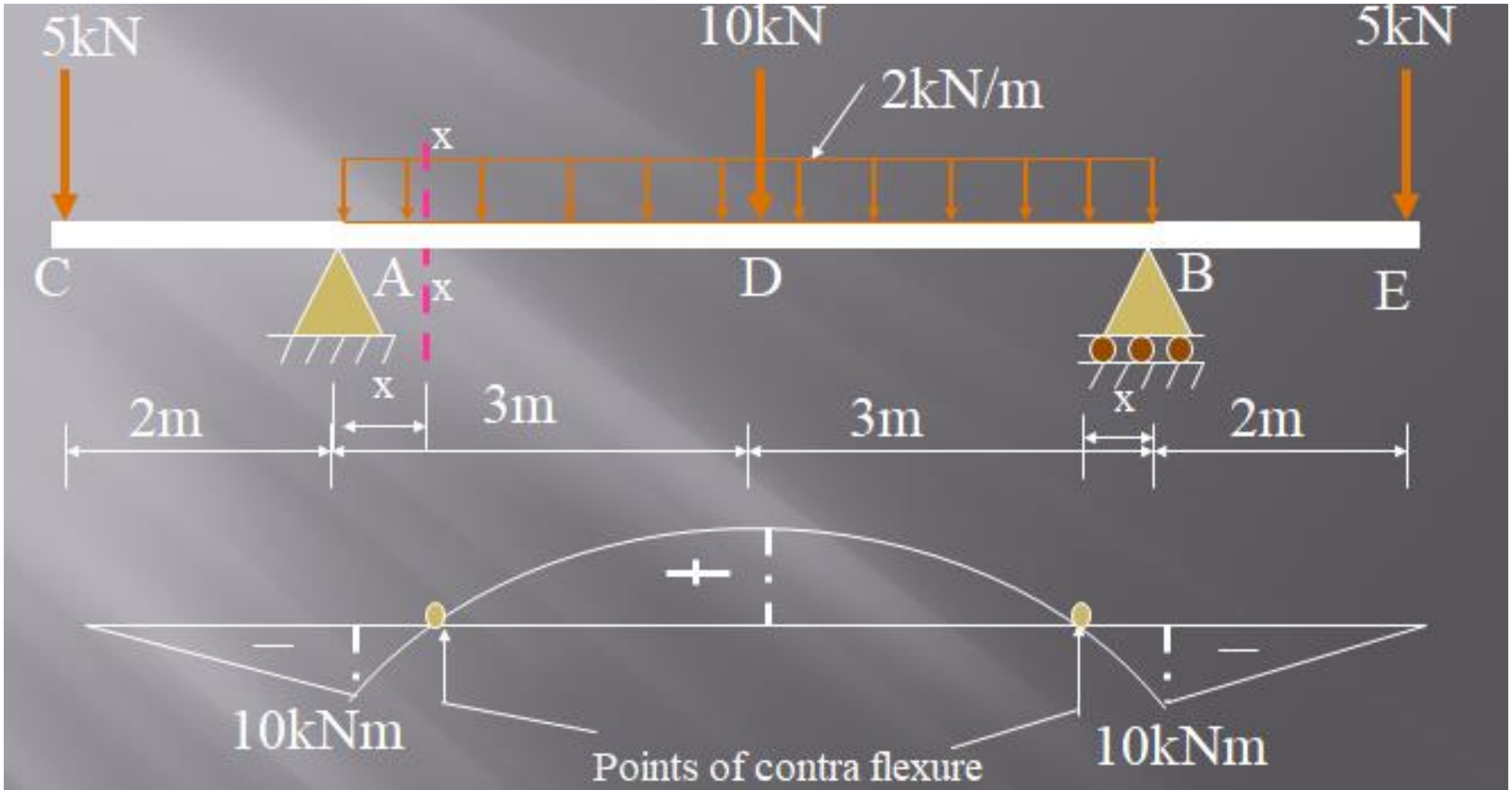



5N
10N
8N
A
B
C
D
E
2m
2m
3m
1m
9.75N
9.75N
4.75N
4.75N
5.25N
5.25N
13.25N
13.25N
SFD
29Nm
19.5Nm
13.25Nm
BMD

Draw SFD and BMD for the double side overhanging beam subjected to loading as shown below. Locate points of contraflexure if any.




5kN
10kN
5kN
2kN/m
C
A
D
B
E
2m
3m
3m
2m
11kN
5kN
5kN
5kN
5kN
5kN
11kN
SFD
14kNm
BMD
10kNm
10kNm



Let x be the distance of point of contra flexure from support A

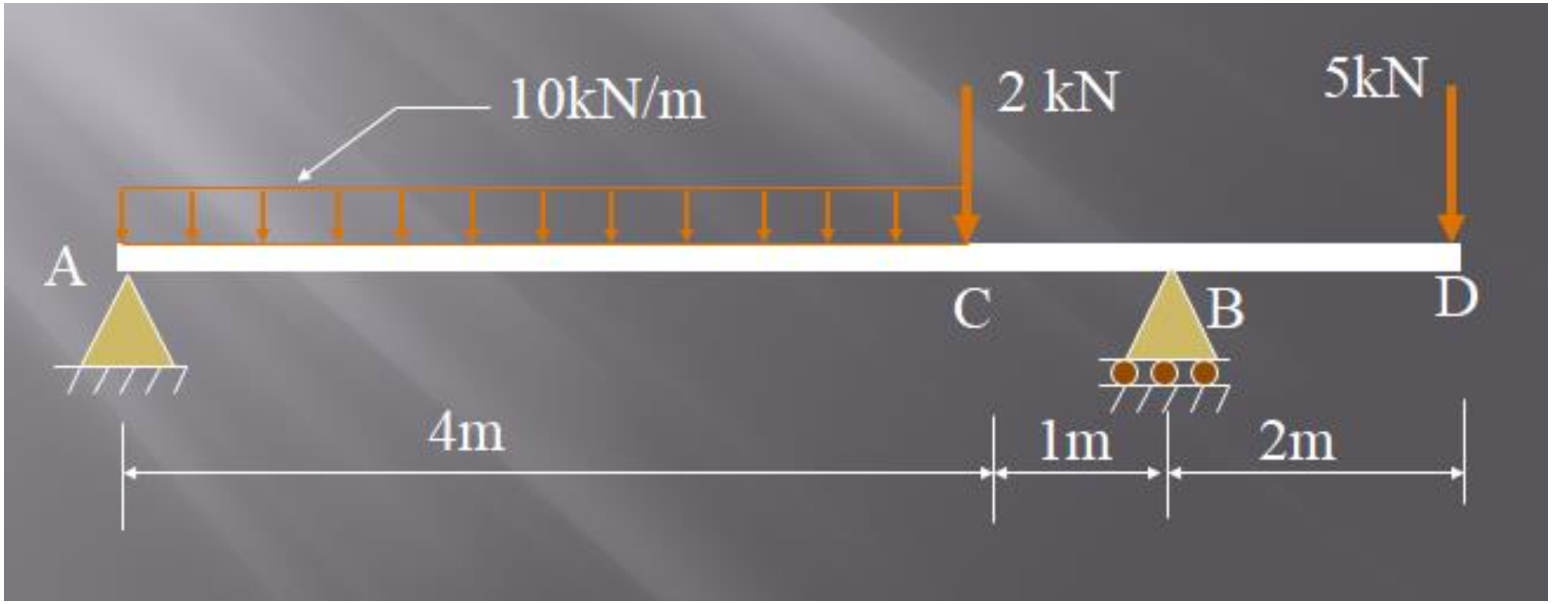
Taking moments at the section x-x (Considering left portion)

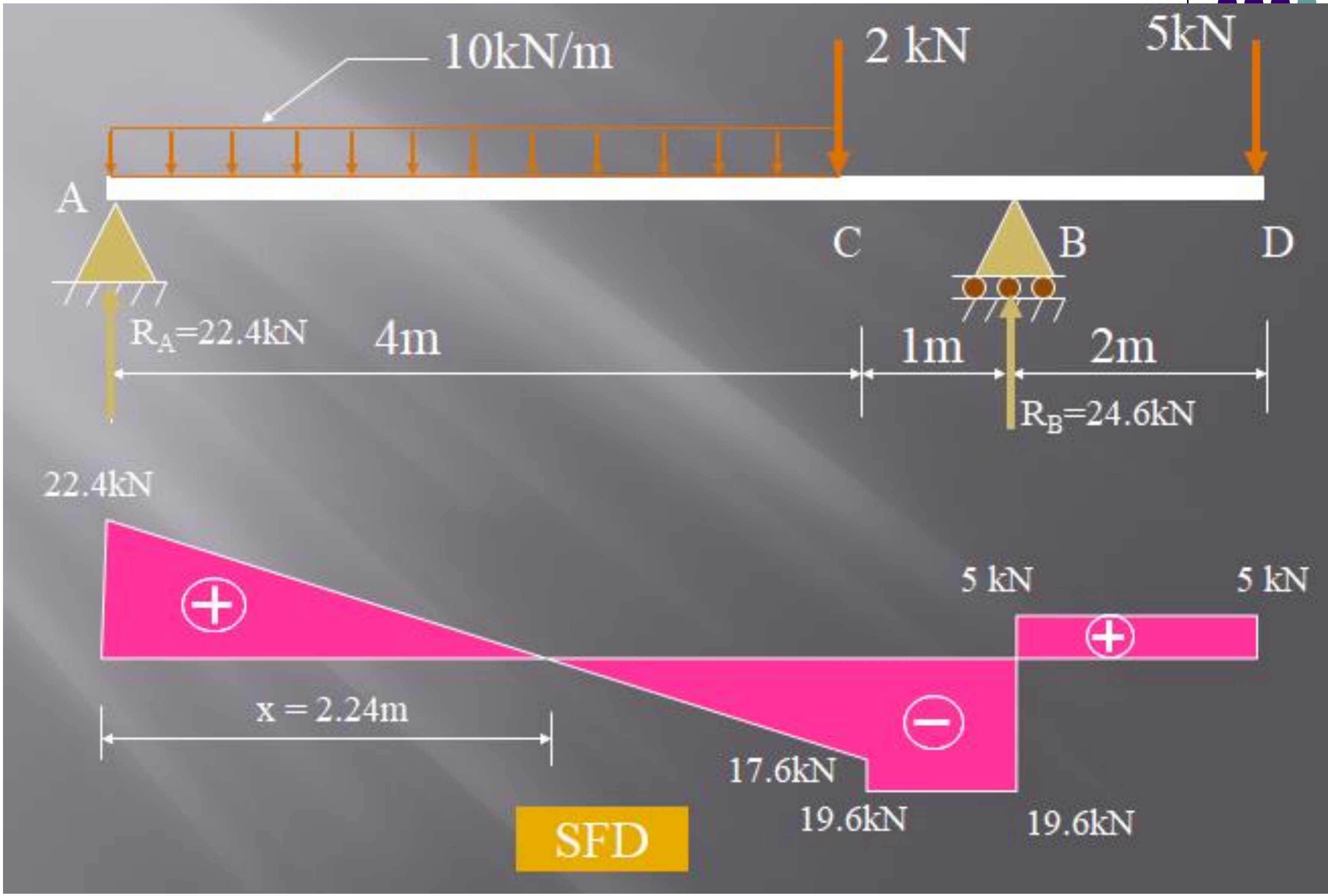
$$M_{x-x} = -5(2+x) + 16x - 2\frac{x^2}{2} = 0$$

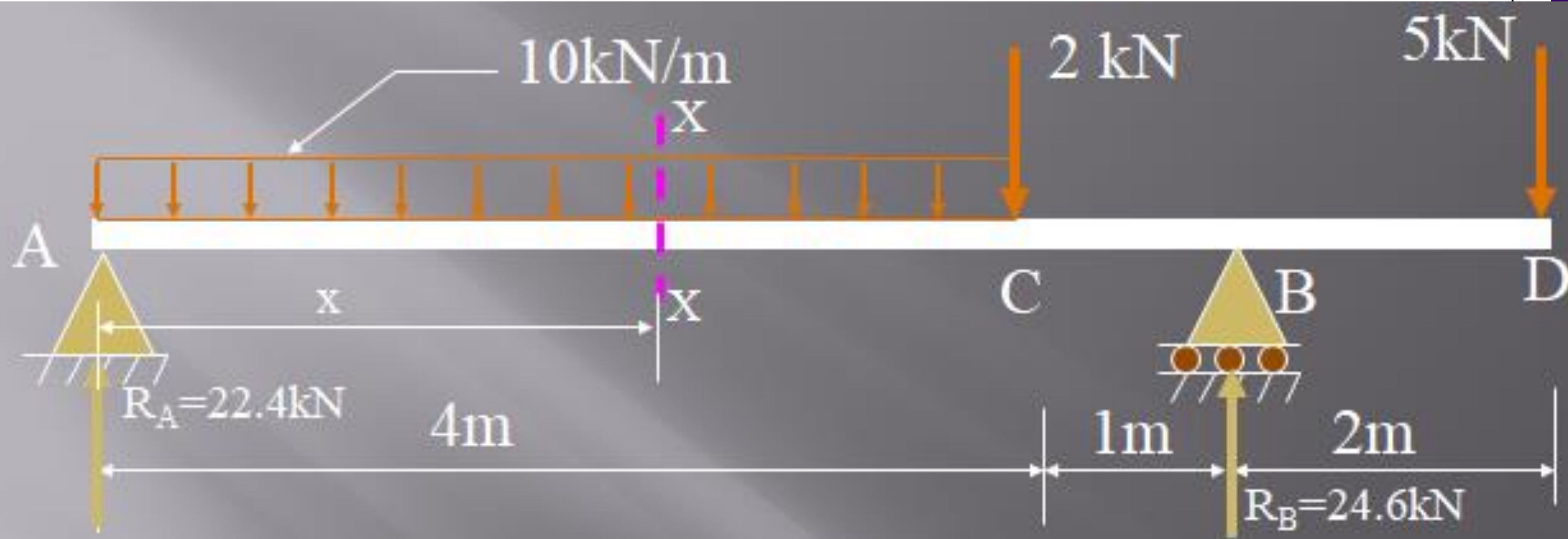
x = 1 or 10

$\therefore$ x = 1 m

Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Determine the absolute maximum bending moment and shear forces and mark them on SFD and BMD. Also locate points of contra flexure if any.



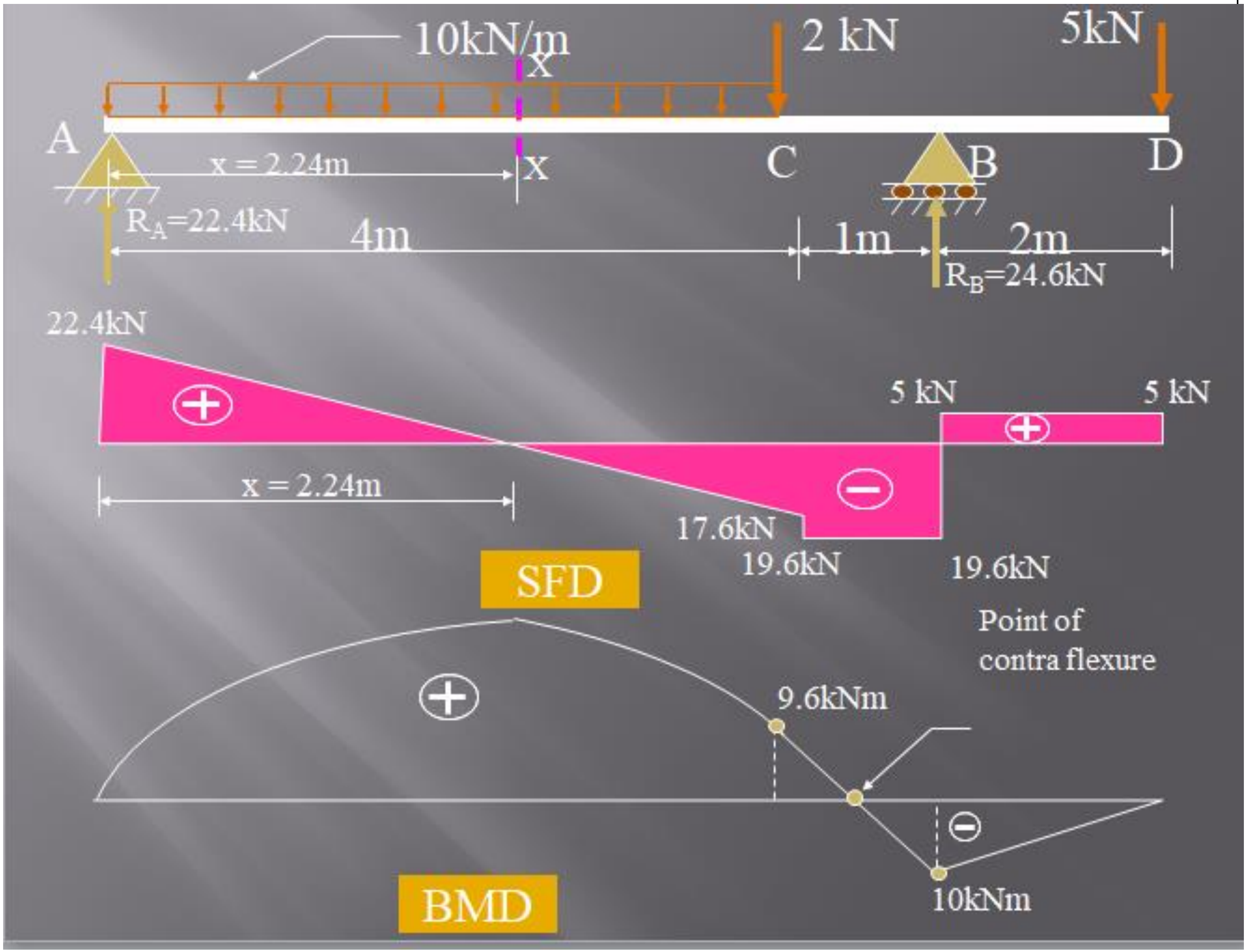

10kN/m
2 kN
5kN
A
C
B
D
R_A=22.4kN
4m
1m
2m
R_B=24.6kN
22.4kN
5 kN
5 kN
x = 2.24m
17.6kN
19.6kN
19.6kN
SFD

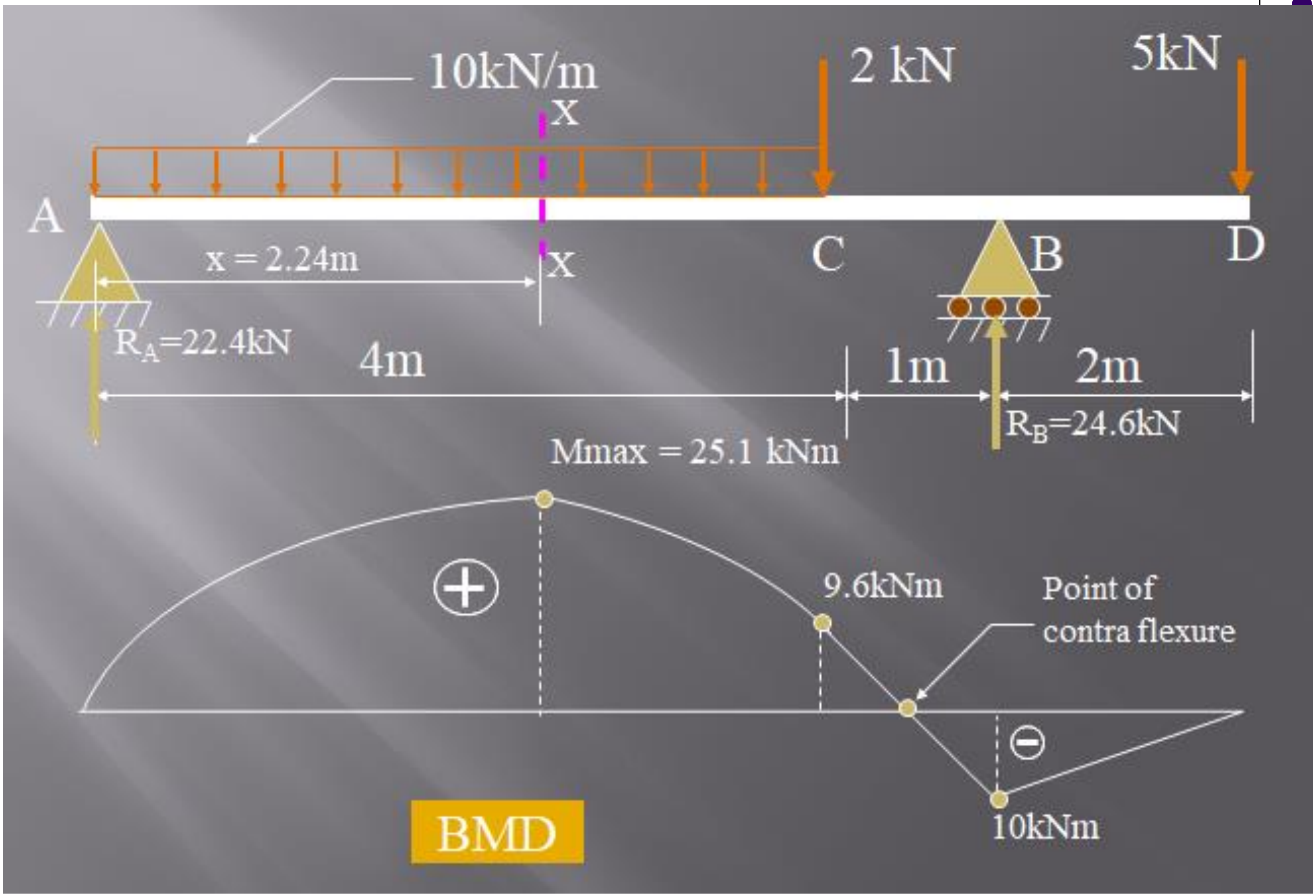


Max. bending moment will occur at the section where the shear force is zero. The SFD shows that the section having zero shear force is available in the portion AC. Let that section be X-X, considered at a distance x from support A as shown above.

The shear force at that section can be calculated as

$V_{x-x} = 22.4 - 10.x = 0 \rightarrow x = 2.24$ m


10kN/m
x
2 kN
5kN
A
x = 2.24m
x
C
B
D
R_A=22.4kN
4m
1m
2m
R_B=24.6kN
22.4kN
5 kN
5 kN
x = 2.24m
17.6kN
19.6kN
19.6kN
SFD
Point of
contra flexure
9.6kNm
10kNm
BMD

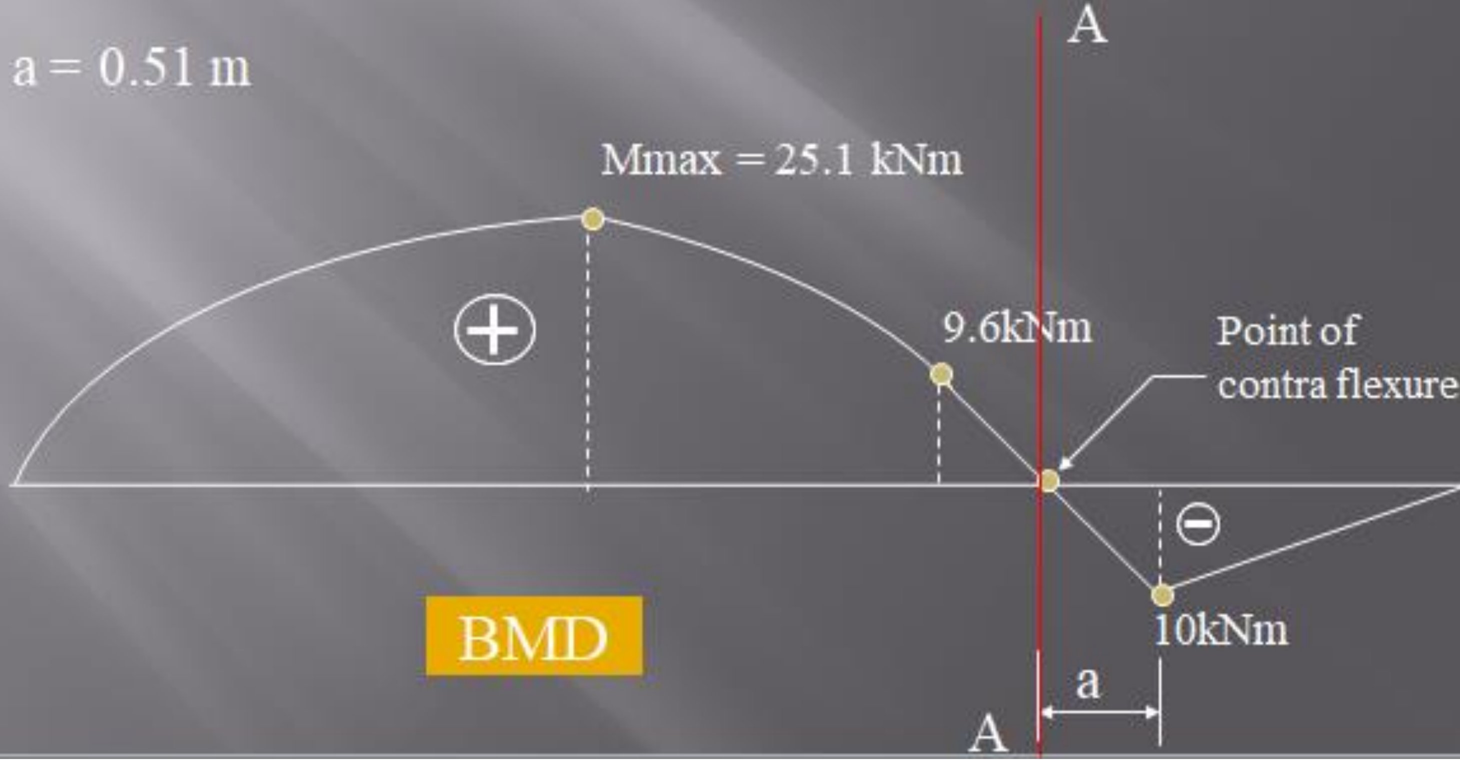

10kN/m
2 kN
5kN
X
A
X
C
B
D
x = 2.24m
RA=22.4kN
4m
1m
2m
RB=24.6kN
Mmax = 25.1 kNm
9.6kNm
Point of
contra flexure
10kNm
BMD

Let a be the distance of point of contra flexure from support B

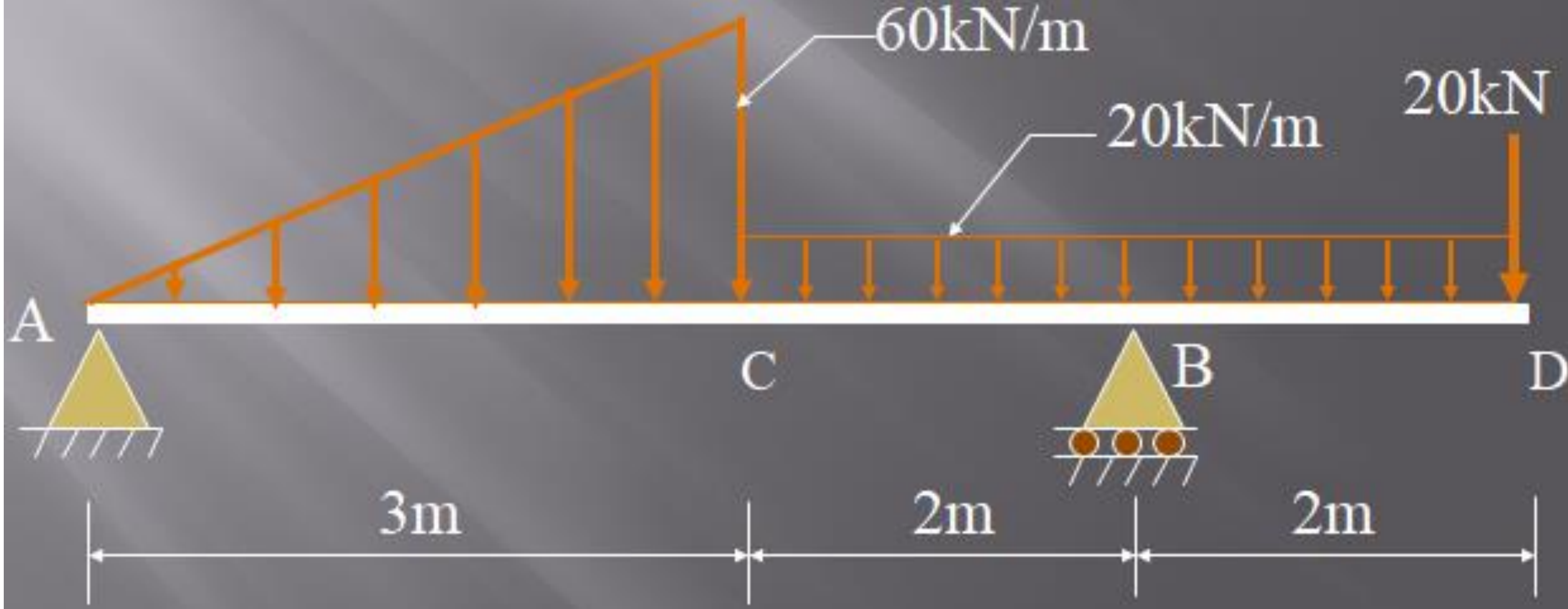
Taking moments at the section A-A (Considering left portion)

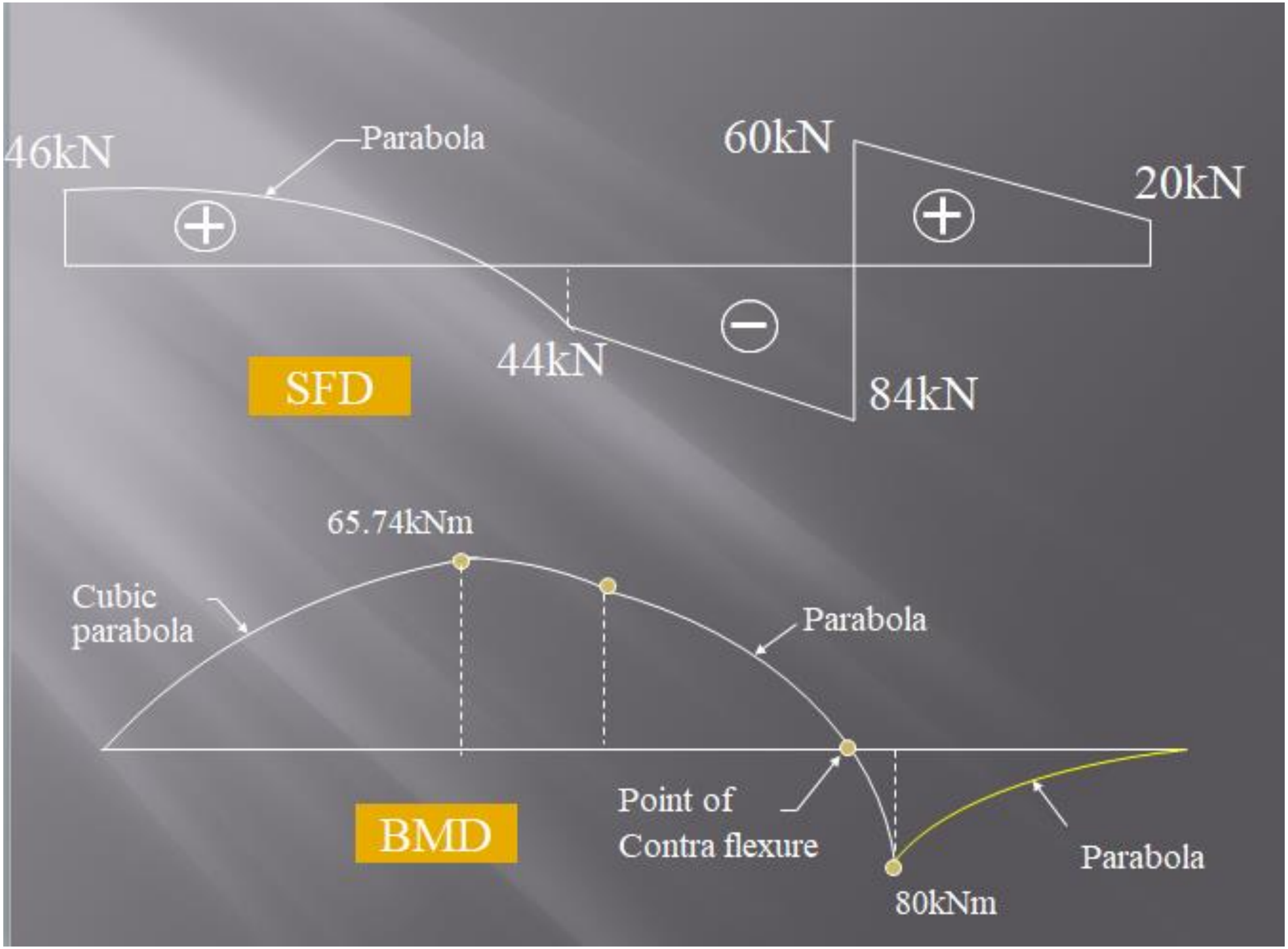
$$M_{A-A} = -5(2+a) + 24.6a = 0$$

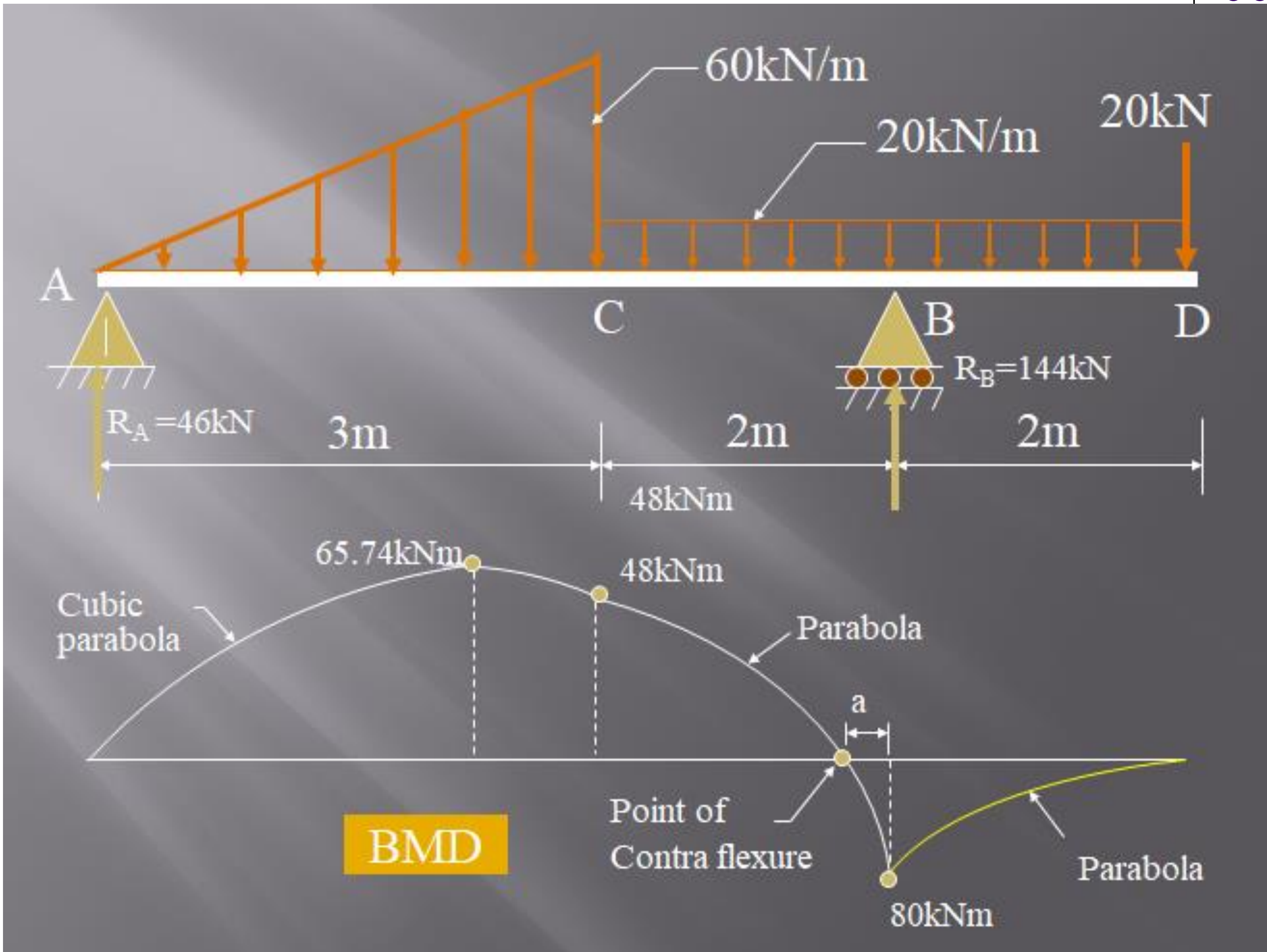
a = 0.51 m



Draw SFD and BMD for the single side overhanging beam subjected to loading as shown below. Mark salient points on SFD and BMD.




46kN
Parabola
60kN
20kN
44kN
84kN
SFD
65.74kNm
Cubic
parabola
Parabola
Point of
Contra flexure
BMD
Parabola
80kNm


60kN/m
20kN
20kN/m
A
C
B
D
R_B=144kN
R_A =46kN
3m
2m
2m
48kNm
65.74kNm
48kNm
Cubic parabola
Parabola
a
Point of Contra flexure
BMD
Parabola
80kNm

Point of contra flexure:

BMD shows that point of contra flexure is existing in the portion CB. Let '*a*' be the distance in the portion CB from the support B at which the bending moment is zero. And that '*a*' can be calculated as given below.

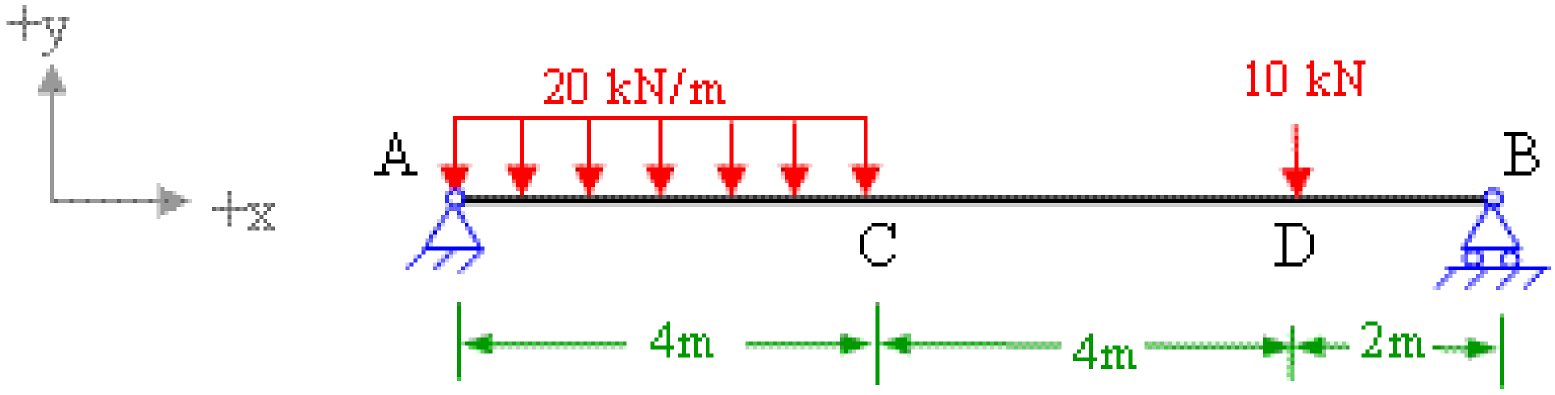
$\Sigma M_{x\text{-}x} = 0$

$$144a - 20(a+2) - 20\frac{(2+a)^2}{2} = 0$$

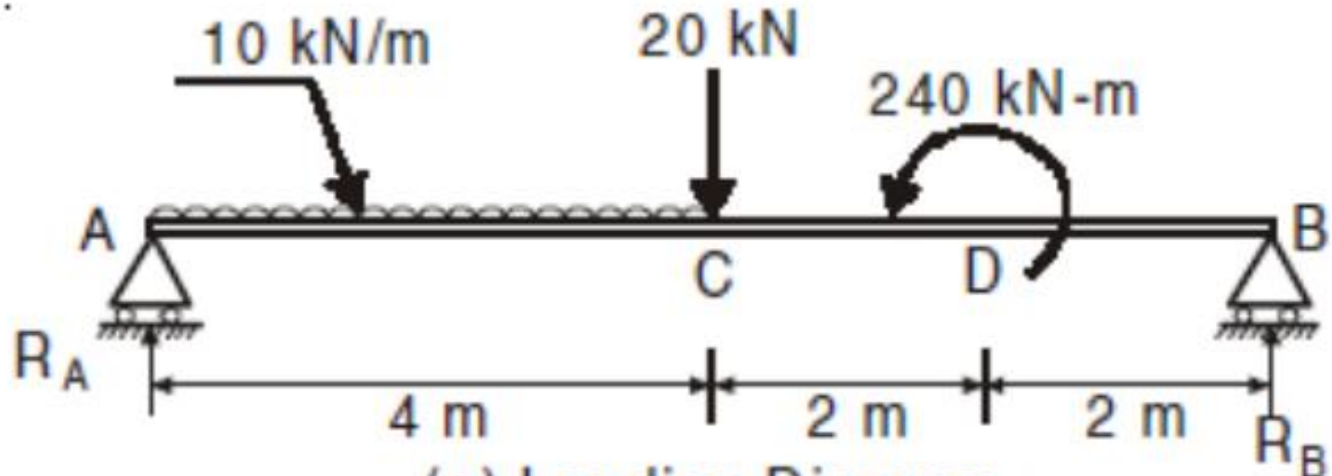
$$a = 1.095 \text{ m}$$

Example on simply supported beam

Calculate the value and draw a bending moment and shear force diagram for following beam shown in fig.

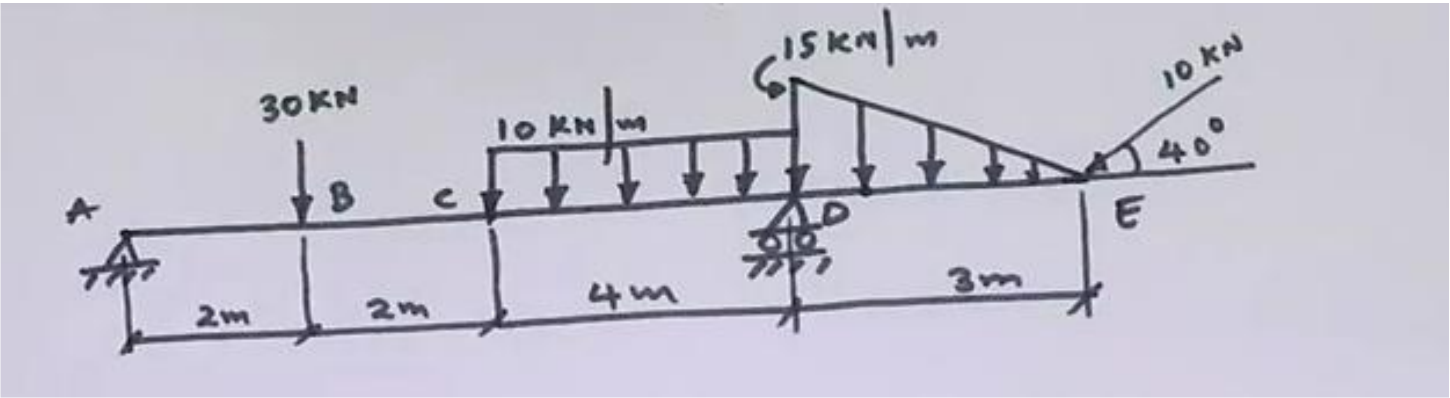


A beam of span 8 m has roller support at A and hinge support at B as shown in Fig. . Draw SF and BM diagrams when the beam is subjected to udl, a concentrated load and an externally applied moment as shown in the Figure.

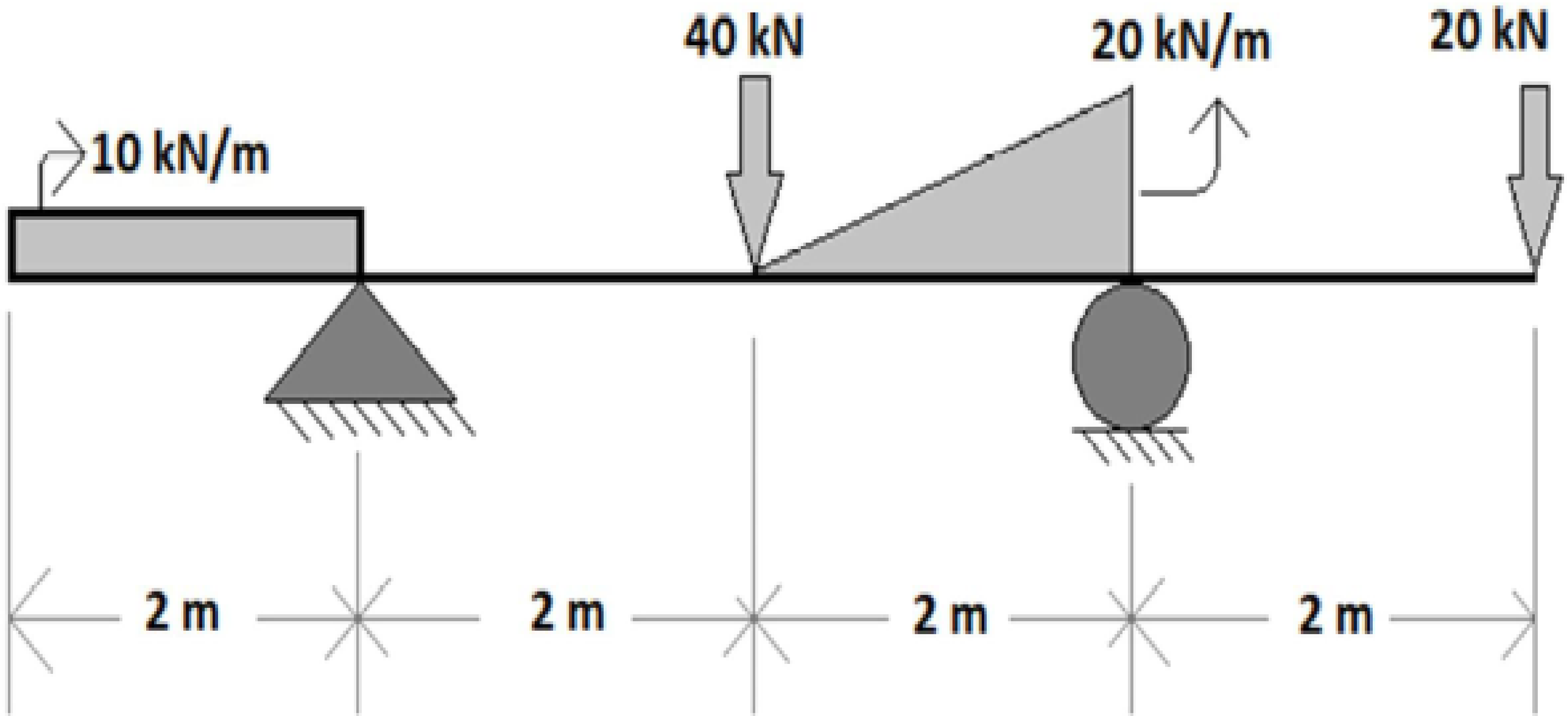


(a) Loading Diagram

Draw shear force and bending moment diagrams [SFD and BMD] shown in the Fig. given below.



Draw the shear force and bending moment diagram for the overhanging beam as shown in the Fig. 1 and locate the points of contraflexure(s), if any.



A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier when it is subjected to the stringer loads shown. Assume the columns at A and B exert only vertical reactions on the pier

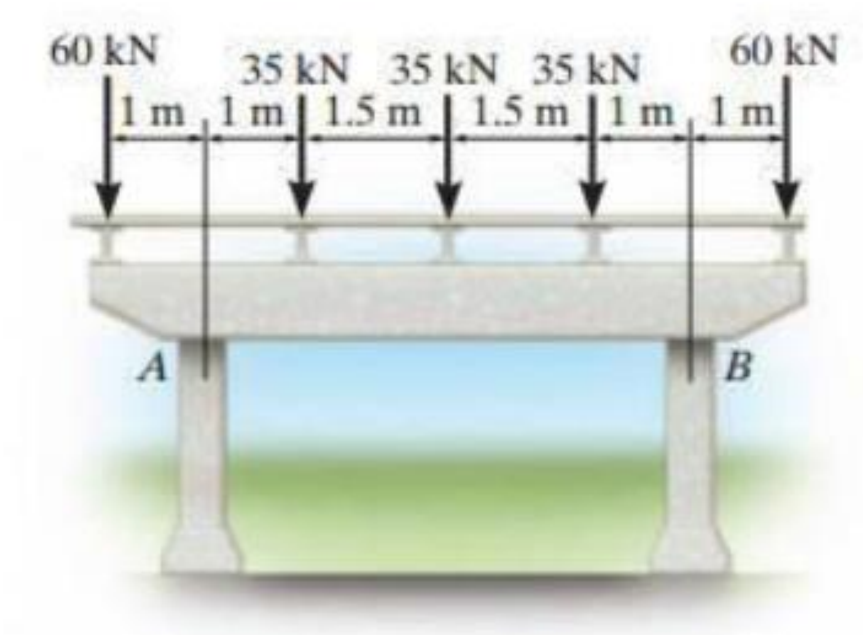


Figure shows a jet airliner schematically. The cabin is in the centre and two engines are L/4 from the cabin on either side. The total length of two wings, from tip to tip is L. When in flight the total load of the airliner, W, is supported by the air pressure acting along the length of the wings. The air pressure may be assumed to be uniform. The weight of the airliner is assumed to be concentrated at three points, viz. 3W/4 at the cabin and W/8 each at two engines. Assuming that the wings can be treated as a beam, draw S.F. and B.M. diagrams and calculate maximum value of B.M.