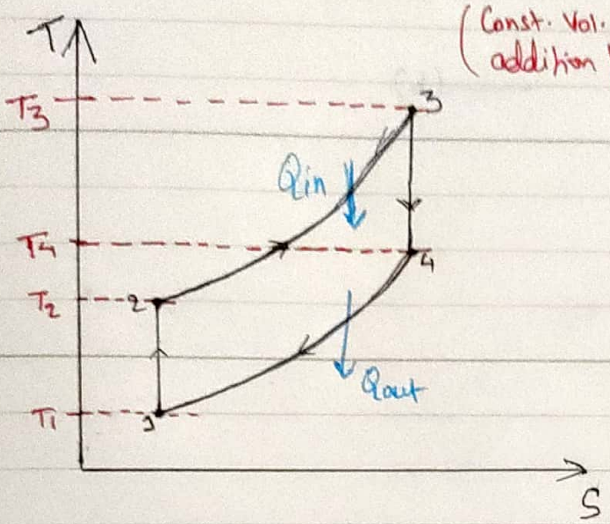
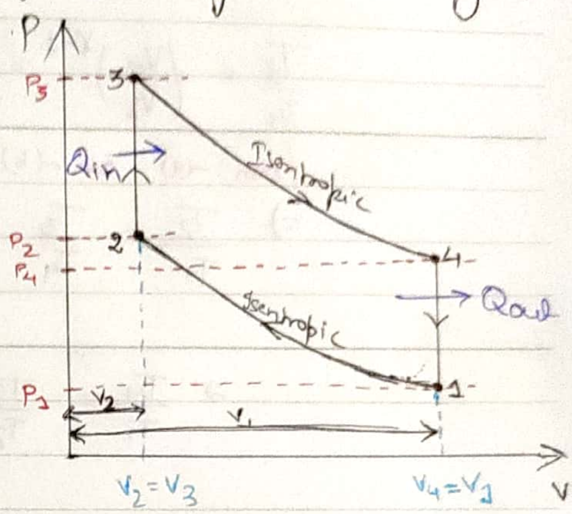
Module-6

OTTO Cycle - (working cycle of SI engine or Petrol engine)

(Const. Vol. heat addition process)





1→2 - Isentropic Compression

2→3 - Isochoric ~~Compression~~ heat addition

3→4 - Isentropic Expansion

4→1 - Isochoric heat rejection

$$Q_{2\to3} = Q_{in} = mC_v(T_3-T_2) \quad (+)$$

$$Q_{4\to1} = Q_{out} = mC_v(T_4-T_1) \quad (-)$$

$$W_{net} = mC_v[(T_3-T_2)-(T_4-T_1)]$$

$$\frac{V_1}{V_2} = \frac{V_4}{V_3} = r$$

now,

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{\cancel{mC_v}[(T_3-T_2)-(T_4-T_1)]}{\cancel{mC_v}(T_3-T_2)}$$

$$\eta = 1-\left(\frac{T_4-T_1}{T_3-T_2}\right) = 1-\frac{T_1(T_4/T_1-1)}{T_2\left(\frac{T_3}{T_2}-1\right)} \quad \text{--- (1)}$$

For process 1→2,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad (\because \text{It is isentropic process})$$

$$\frac{T_2}{T_1} = r^{\gamma-1} \quad \text{--- (a)}$$

For $3 \to 4$,

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = r^{\gamma-1} \quad \text{——(b)}$$

from —(a) and —(b),

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

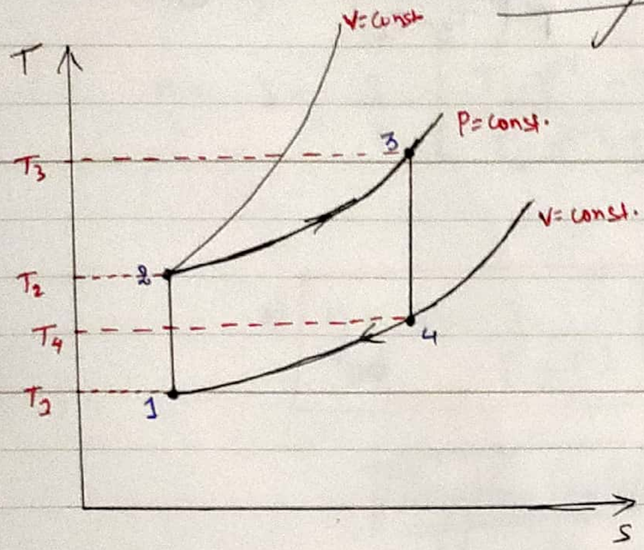
Substituting in —(1) we get

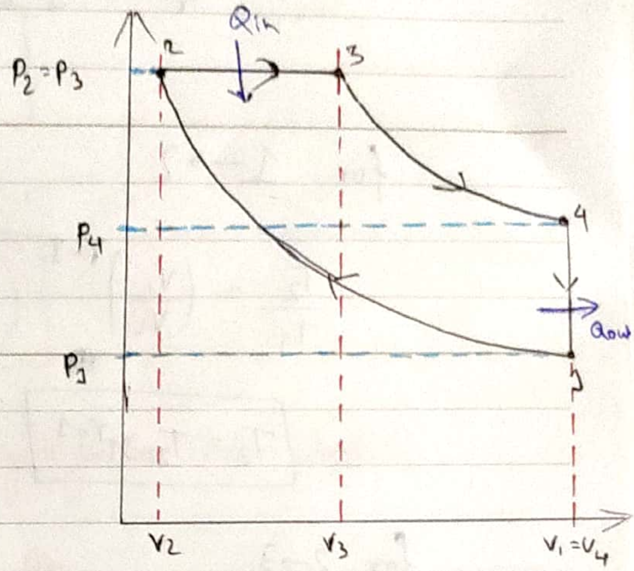
$$\eta = 1 - \frac{T_1\left[\frac{T_4}{T_1} - 1\right]}{T_2\left[\frac{T_3}{T_2} - 1\right]} = 1 - \frac{T_1\,\cancel{\left(\frac{T_4}{T_1} - 1\right)}}{T_2\,\cancel{\left(\frac{T_4}{T_1} - 1\right)}}$$

$$\therefore \eta = 1 - \frac{T_1}{T_2} \Rightarrow \eta = 1 - \frac{1}{r^{\gamma-1}}$$

$$\Rightarrow \boxed{\eta = 1 - r^{1-\gamma}}$$

Diesel Cycle





- It is also known as const. pressure heat addition cycle.

$r = \frac{V_1}{V_2}$ (compression ratio) ; $\rho = \frac{V_3}{V_2}$ (fuel cutoff ratio) ; r_e (Expansion ratio) $= \frac{V_4}{V_3}$

now,

$$\boxed{r = \rho \cdot r_e}$$

$$Q_{23} = Q_{in} = C_p (T_3 - T_2)$$

$$Q_{41} = Q_{out} = C_v (T_4 - T_1)$$

$$W_{net} = C_p(T_3 - T_2) - C_v(T_4 - T_1)$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$\eta = 1 - \frac{C_v}{C_p}\left(\frac{T_4 - T_1}{T_3 - T_2}\right) = 1 - \frac{1}{\gamma}\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \quad \text{——} \ ①$$

for $1 \rightarrow 2$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = r^{\gamma - 1}$$

$$\boxed{T_2 = T_1 \cdot r^{\gamma - 1}}$$

for $2 \rightarrow 3$,

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = \rho$$

$$T_3 = T_2 \cdot \rho \Rightarrow \boxed{T_3 = T_1 \cdot r^{\gamma - 1} \cdot \rho}$$

for 3-4

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma - 1} = \left(\frac{V_3}{V_1}\right)^{\gamma - 1} = \left(\frac{V_3}{V_1} \times \frac{V_2}{V_2}\right)^{\gamma - 1}$$

$$T_4 = T_3 \cdot \frac{\rho^{\gamma - 1}}{r^{\gamma - 1}}$$

$$T_4 = T_1 \cdot \cancel{r^{\gamma - 1}} \cdot \rho \cdot \frac{\rho^{\gamma - 1}}{\cancel{r^{\gamma - 1}}}$$

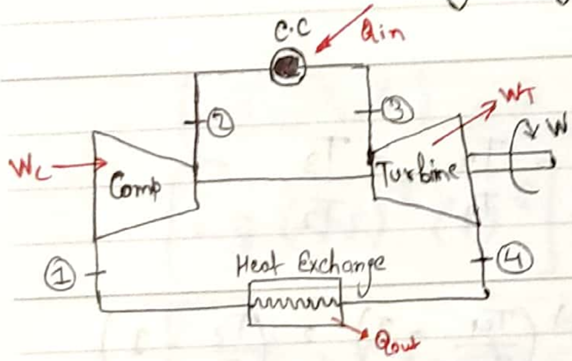
$$\Rightarrow \boxed{T_4 = T_1 \cdot \rho^{\gamma}}$$

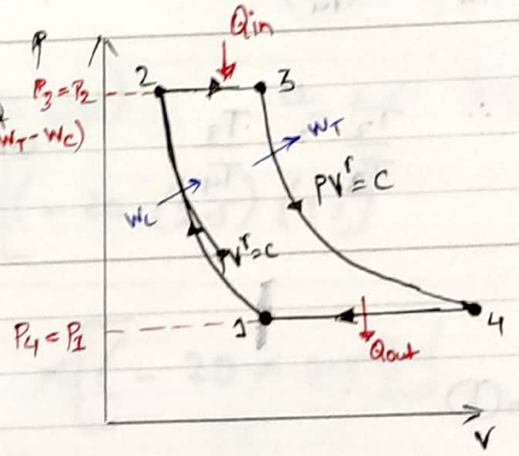
now, substituting in ① we get,

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{T_1\cdot\rho^{\gamma} - T_1}{T_1\cdot r^{\gamma-1}\cdot\rho - T_1 r^{\gamma-1}}\right]$$

$$\Rightarrow \boxed{\eta_{D} = 1 - \frac{1}{\gamma\cdot r^{\gamma-1}}\left(\frac{\rho^{\gamma}-1}{\rho-1}\right)}$$

Brayton-Cycle (Ideal cycle for Gas Turbine plants)



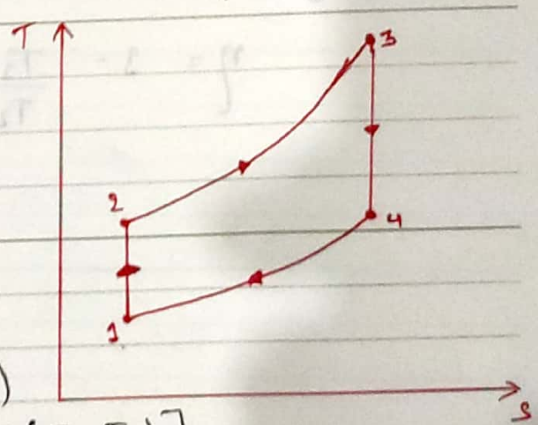


$$q_{in} = C_p(T_3 - T_2) \quad (+)$$

$$q_{out} = C_p(T_4 - T_1) \quad (-)$$

$$W_T = C_p(T_3 - T_4)$$

$$W_C = C_p(T_2 - T_1)$$



$$W_{net} = W_T - W_C = C_p(T_1 - T_2 + T_3 - T_4)$$
$$= C_p[(T_3 - T_2) + (T_4 - T_1)]$$

$$\eta = \frac{W_T - W_C}{q_{in}} = \frac{\cancel{C_p}[(T_3 - T_2) - (T_4 - T_1)]}{\cancel{C_p}(T_3 - T_2)} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2}\right)$$

$$= 1 - \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(T_3/T_2 - 1\right)} \quad — ①$$

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$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = r_p$$

for $1 \rightarrow 2$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

For $3 \rightarrow 4$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or,} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\Rightarrow \left(\frac{T_4}{T_1} - 1\right) = \left(\frac{T_3}{T_2} - 1\right)$$

from (1)

$$\eta = 1 - \frac{T_1}{T_2} \Rightarrow \boxed{\eta = 1 - \frac{1}{(r_p)^{\left(\frac{\gamma-1}{\gamma}\right)}}}$$

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