Module 1 – Problem Set 1

MEE 1003 Thermodynamics

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- P1. A laboratory room keeps a vacuum pressure of 0.1 kPa. What is the net force on a door 2 m x 1 m?

- Soln. Net Force = $(0.1 \times 10^3) \times (2 \times 1) = 200$ N

- P2. A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

- Soln.

$$\text{Required force } F = \Delta p \times A = mg$$

$$\Rightarrow \Delta p = \frac{mg}{A} = \frac{1000 \times 9.81}{100} = 98.1 \text{ Pa} = 0.098 \text{ kPa}$$

- P3. A steel cylinder of mass 2 kg consists 4 liters of liquid water at 25°C and 200 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of water.

- Soln.

$$\text{Total mass} = m_{steel} + m_{water}$$

$$\rho_{steel} = 7820 \text{ kg/m}^3;\ V_{steel} = \frac{m_{steel}}{\rho_{steel}} = \frac{2}{7820} = 0.000256 \text{ m}^3$$

$$\rho_{water} = 997 \text{ kg/m}^3 @ 20 \text{ C and } 200 \text{ kPa};\ m_{water} = \rho_{water} \times V_{water} = 1000 \times (4 \times 10^{-3}) = 4 \text{ kg}$$

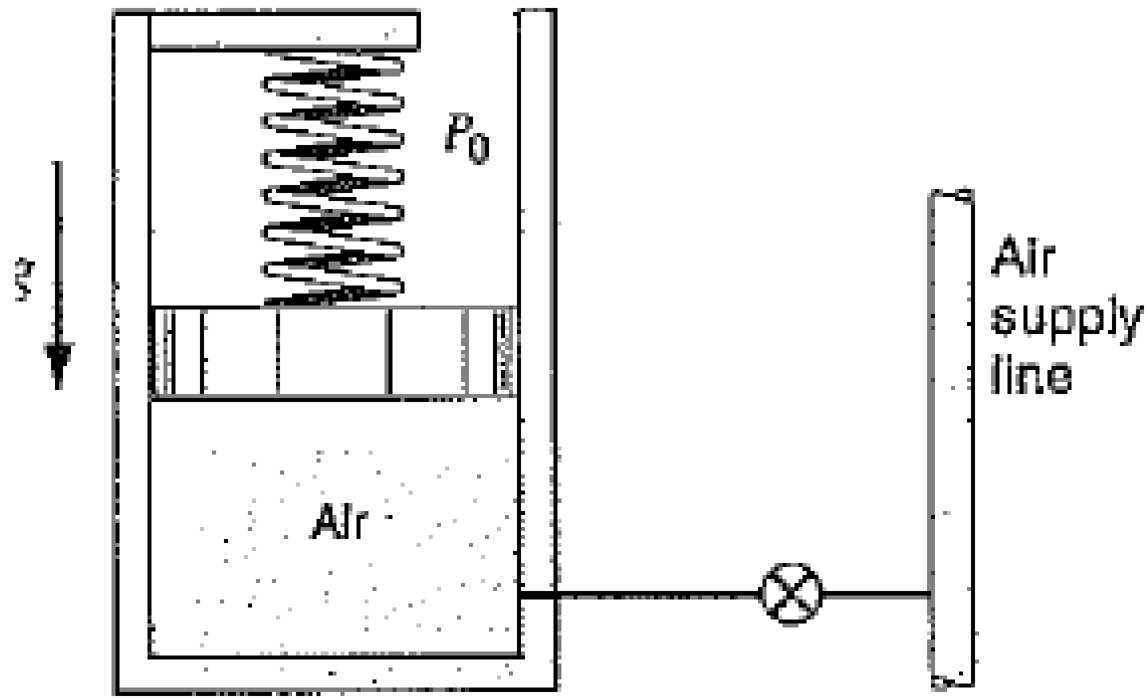
$$\therefore \text{ total mass} = 2 + 4 = 6 \text{ kg}$$

$$\text{total volume} = 0.000256 + 0.004 = 0.004256 \text{ m}^3 = 4.256 \text{ liters}$$

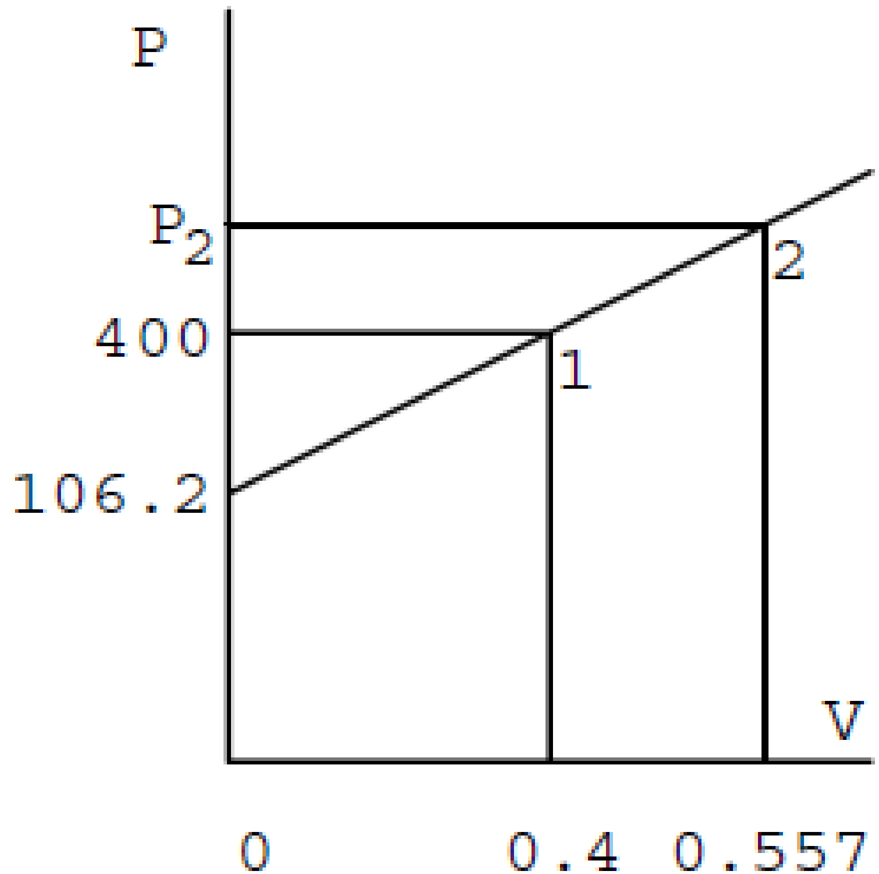
- P4. A 5 kg piston in a cylinder with a diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure is 100 kPa. The spring exerts no force on the piston when it is at the bottom of the cylinder, and for the state shown, the pressure is 400 kPa with volume 0f 0.4 liters. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

P4 Schematic

- P4



- Soln. Linear pressure variation



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- Soln.

Linear spring force F = kx

Equilibrium pressure varies linearly as well: P = a + bV

Intersect = a; slope b = $\dfrac{dP}{dV}$

No spring force at bottom of cylinder (zero volume):

$\therefore$ Force = $P_{atm}A_p + m_p g$

Intercept a (pressure) = $P_{atm} + \dfrac{m_p g}{A_p}$

Piston area $A_P = \dfrac{\pi}{4} \times 0.1^2 = 0.00785\ \text{m}^2$

$\therefore$ intercept a = 100 kPa + $\dfrac{5 \times 9.81}{0.00785} = 106.2$ kPa $\Rightarrow$ a = 106.2 kPa

- Soln.

Position 1 (schematic): $P_1 = 400$ kPa, $V_1 = 4\ L = 0.4\ m^3$

Position 2 (after air let in- piston's 2 cm rise)

$$V_2 = (0.4 \times 10^{-3} + 0.00785 \times 10^{-3} \times 0.02)\ m^3 = 0.557L$$

$$P_2 = ?$$

From linear pressure variation: $\dfrac{P_2 - P_1}{V_2 - V_1} = b = \dfrac{dP}{dV}$

$$\Rightarrow P_2 = (V_2 - V_1)\frac{dP}{dV} + P_1 = 400 + \frac{(400 - 106.2)}{0.4 - 0}(0.557 - 0.4)$$

$$\therefore P_2 = 515.3 \text{ kPa}$$

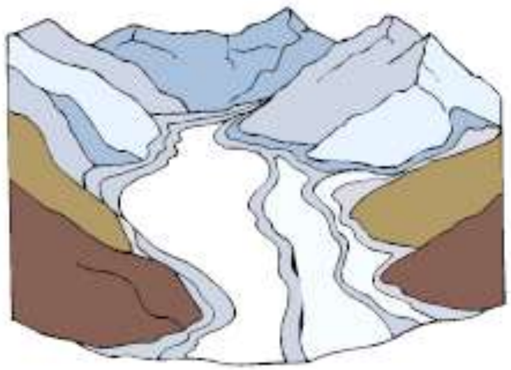
- P5. If the density of ice is 920 kg/m^3, find the pressure at the bottom of a 1000 m thick ice cap on the North pole. What is the melting temperature at that pressure?

- Soln

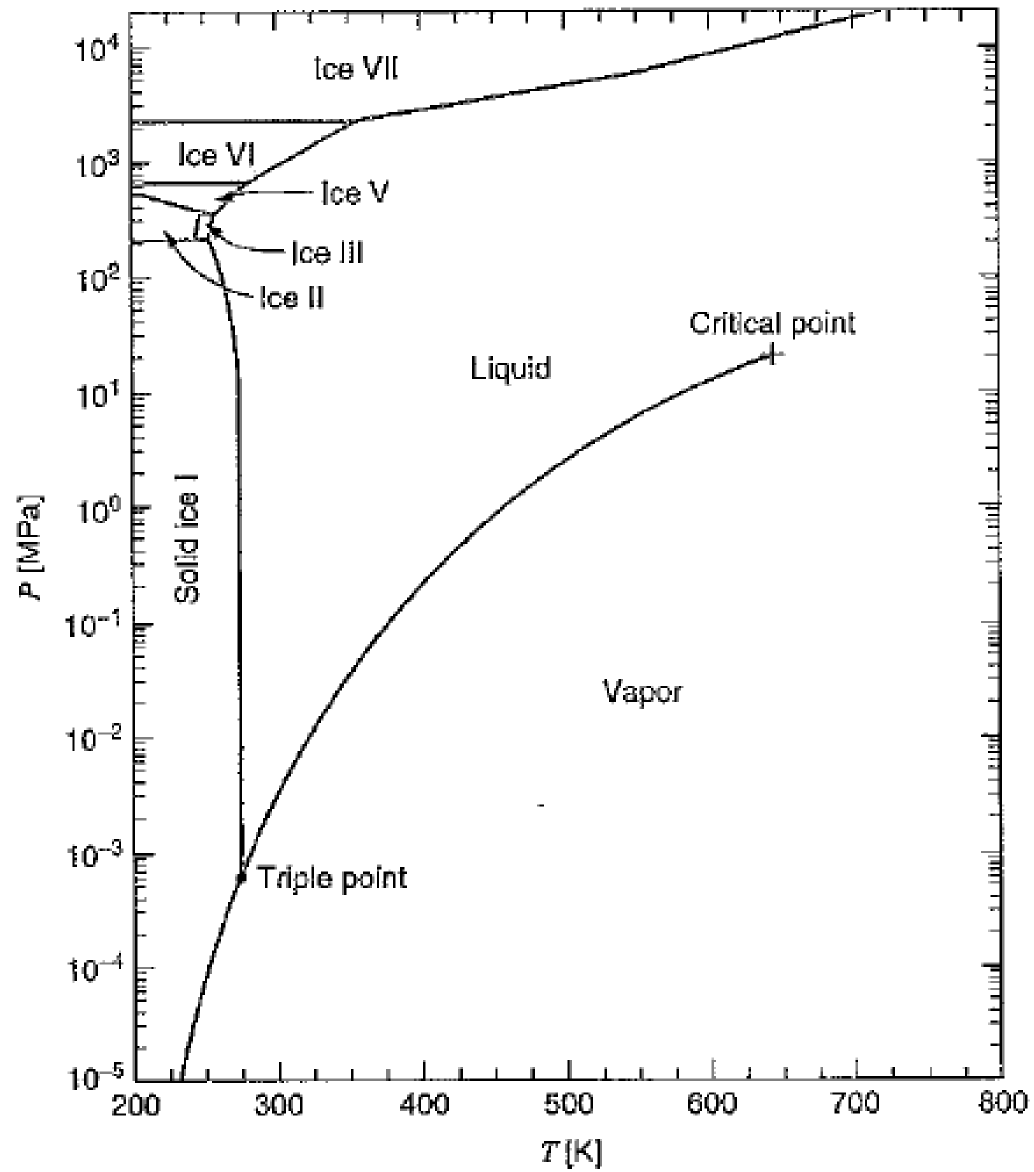
$$\rho_{\text{Ice}} = 920\ \text{kg/m}^3; \Delta \text{P} = \rho \text{gH} = 920 \times 9.81 \times 1000 = 9022\ \text{kPa}$$

$$\text{P} = \text{P}_0 + \Delta \text{P} = 101.325 + 9022 = 9123\ \text{kPa.}$$

@9123 kPa, T_{LS} (liquid-solid interphase) = -1°C

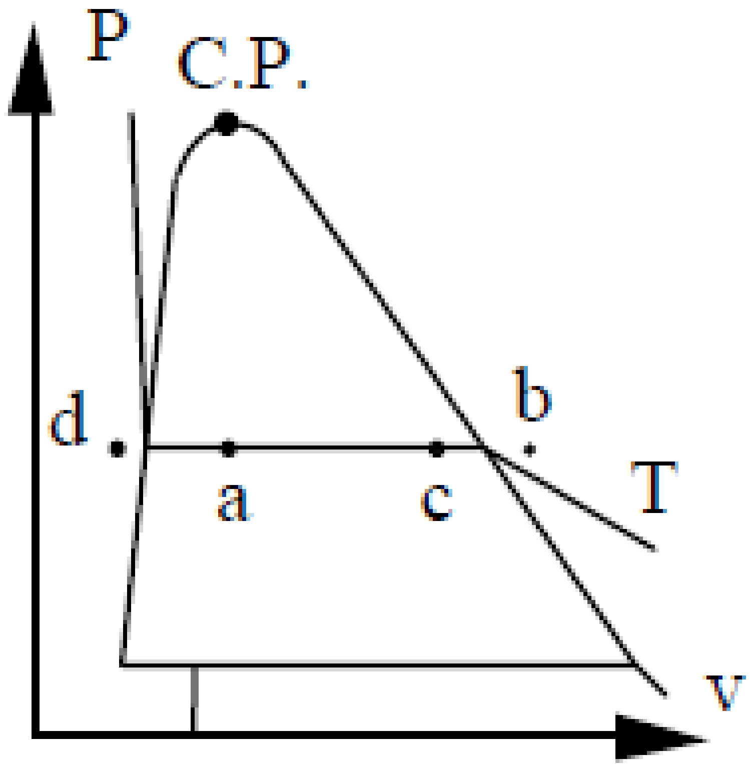


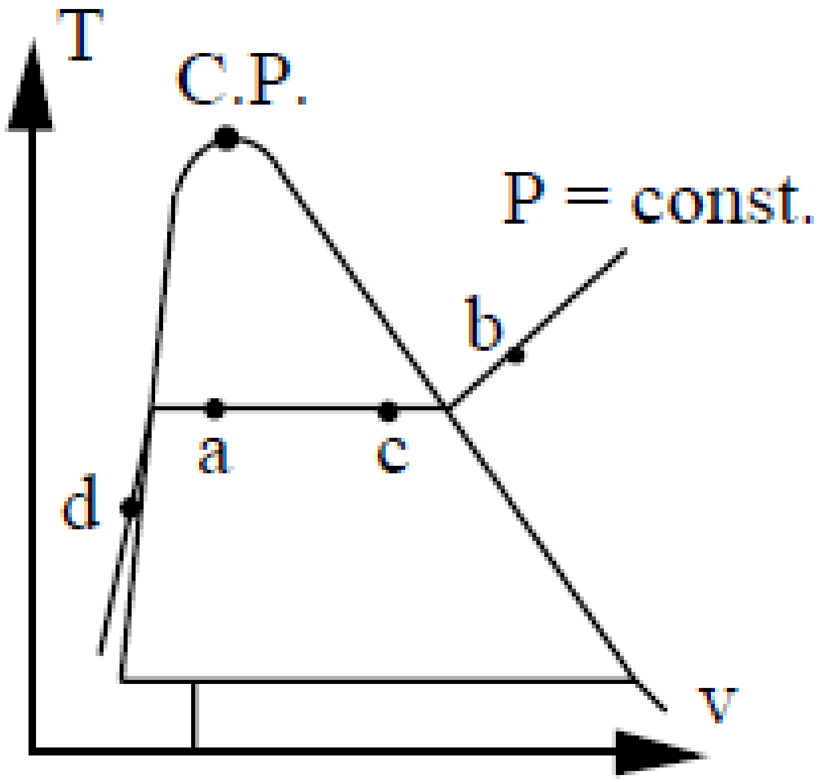
- Phase Diagram T_{LS} = 272 K @ P = 9.23 MPa



- P6. Determine whether water at each of the following states is a compressed liquid, superheated vapor, or a mixture of saturated liquid and vapor.
- (1) 10 MPa, 0.003 m^3/kg
- (2) 1 MPa, 190°C
- (3) 200°C, 0.1 m^3/kg
- (4) -20°C, 200 kPa

- Soln. P-V and T-V state charts for water





- Property tables:

TABLE B.1.2 (*continued*)
Saturated Water Pressure Entry

		SPECIFIC VOLUME, m³/kg			INTERNAL ENERGY, kJ/kg		
Press. (kPa)	**Temp. (°C)**	**Sat. Liquid** v_f	**Evap.** v_{fg}	**Sat. Vapor** v_g	**Sat. Liquid** u_f	**Evap.** u_{fg}	**Sat. Vapor** u_g
9000	303.40	0.001418	0.01907	0.02048	1350.47	1207.28	2557.75
10000	311.06	0.001452	0.01657	0.01803	1393.00	1151.40	2544.41

- Soln.

a) $P = 10$ MPa, $v = 0.003\ m^3/kg$.

From data tables: @ $P = 10$ MPa, $v_f = 0.001452\ m^3/kg$,

$v_g = 0.01803\ m^3/kg$. $\because\ v_f < v < v_g$: mixture of liq+vapor

b) @1 MPa, 190°C: T (190°C) > T_{sat} (=179.91°C)

Hence superheated vapor.

Also, P (1 MPa) < P_{sat} (1.254 MPa)

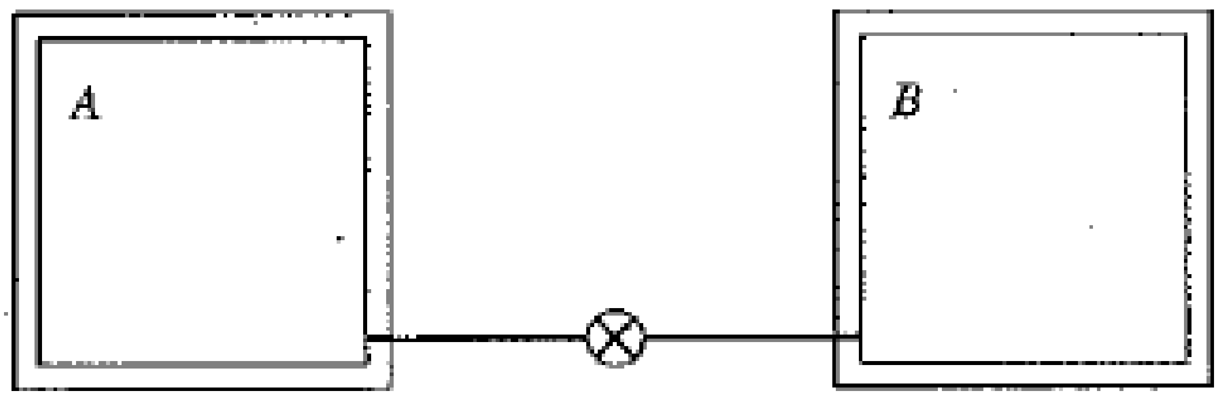
c) @200°C, $0.1\ m^3/kg$. From tables: $v_f = 0.001156\ m^3/kg$

$v_g = 0.12736\ m^3/kg$. Again, $v_f < v < v_g$ – liq+water mix

d) 10 kPa, 10°C: P (10 kPa) > P_{sat} (=1.228 kPa)

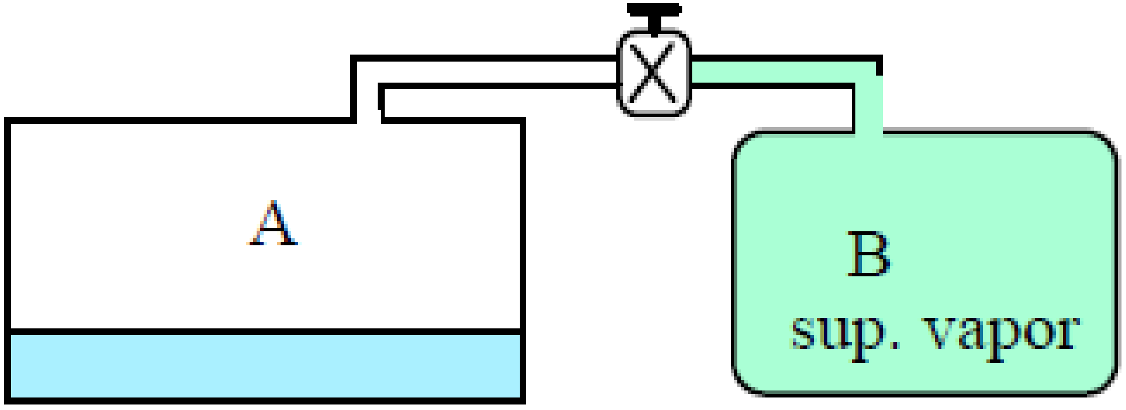
Hence, compressed liquid.

- P7. Two tanks, both containing water are connected as shown.



- Tank A is at 200 kPa, $v = 0.5\ m^3/kg$, $V_A = 1\ m^3$. Tank B contains 3.5 kg at 0.5 MPa and 400°C. The valve is now open and both come to a uniform state. Find the final specific volume.

- Soln.



- Soln. Data Table

TABLE B.1.3

Superheated Vapor Water

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	500 kPa (151.86)			
Sat.	0.37489	2561.23	2748.67	6.8212
200	0.42492	2642.91	2855.37	7.0592
250	0.47436	2723.50	2960.68	7.2708
300	0.52256	2802.91	3064.20	7.4598
350	0.57012	2882.59	3167.65	7.6328
400	0.61728	2963.19	3271.83	7.7937

- At 400°C and 0.5 Mpa (500 kPa), v = 0.61728 m³/kg.

- Soln.

$$\text{Final specific volume } v_{total} = \frac{V_{total}}{m_{total}}$$

$$V_{total} = V_A + V_B;\ V_A = 1\ m^3;\ V_B = ?$$

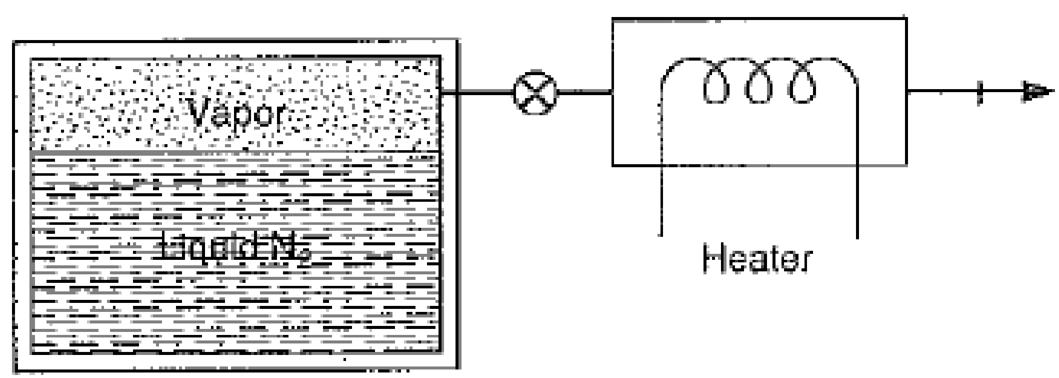
$$m_B = \frac{V_B}{v_B} \Rightarrow V_B = m_B v_B = 3.5 \times 0.6173 = 2.1606\ m^3$$

$$m_A = \frac{V_A}{v_A} = \frac{1}{0.5} = 2\ kg.\ \ m_{total} = 2 + 3.5 = 5.5\ kg$$

$$V_{total} = V_A + V_B = 1 + 2.1606 = 3.1606\ m^3$$

$$\therefore v_{total} = \frac{V_{total}}{m_{total}} = 0.5746\ m^3/kg$$

- P8. A container with liquid Nitrogen at 100 K has a cross-sectional area of 0.5 m^2. Due to heat transfer, some of the liquid evaporates and in one hour the liquid level drops 30 mm. The vapor leaving the container passes through a valve and a heater and exits at 500 kPa, 260 K. Calculate the volume rate of flow of Nitrogen gas exiting the heater.



- Soln. Property tables of Nitrogen

TABLE B.6.1
Saturated Nitrogen

Temp. (K)	Press. (kPa)	SPECIFIC VOLUME, m^3/kg Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
100	779.2	0.001452	0.02975	0.03120

Temp. (K)	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	400 kPa (91.22 K)				600 kPa (96.37 K)			
260	0.19243	191.64	268.61	6.2824	0.12813	191.13	268.01	6.1601

- Soln.

$$\Delta V = A \times \Delta h_{drop} = 0.5 \times 0.03 = 0.015\ m^3$$

$$\Delta m_{liq} = -\frac{\Delta V}{v_f} = -\frac{0.015}{0.001452} = -10.3306\ kg\ (drop)$$

$$\Delta m_{vap} = \frac{\Delta V}{v_g} = \frac{0.015}{0.0312} = 0.4808\ kg$$

$$v_{exit} = 0.1585\ m^3/kg. \quad \dot{V} = \dot{m}\, v_{exit} = \left(\frac{9.85}{1\ hour}\right) \times 0.15385\ m^3/kg$$

$$\dot{V} = 1.5015\ m^3/hour = 0.02526\ m^3/min$$

- P9. Air at a constant pressure in a piston/cylinder is at 300 kPa, 300 K and has a volume of 0.1 m^3. It is heated to 600 K over 30 s in a process with constant piston velocity. Find the power delivered to the piston.

- Soln.

Constant presure process:

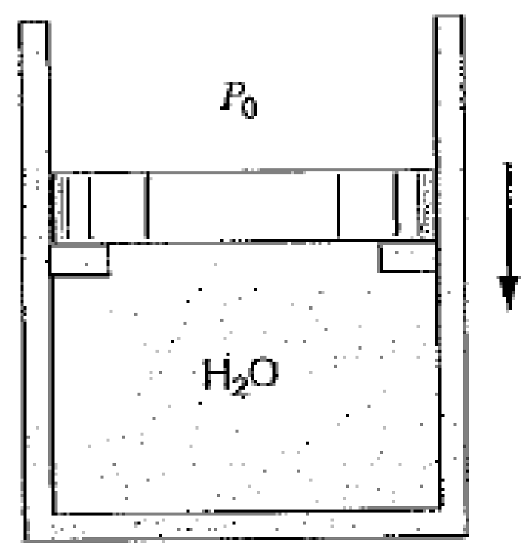
$$\delta W = PdV \Rightarrow \dot{W} = P\,\dot{V}$$

Ideal gas equation: $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}.\ \because\ P_1 = P_2$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = V_1 \times \left(\frac{T_2}{T_1}\right) = 0.1 \times \frac{600}{300} = 0.2\ m^3$$

$$\therefore\ \dot{W} = P\left(\frac{\Delta V}{\Delta t}\right) = 300 \times \left(\frac{0.2 - 1}{3.0}\right) = 1\,kW$$

- P10. Ten kilograms of water in a piston-cylinder arrangement exist as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so that the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it. Find:
- 1) Temperature and volume of water
- 2) Work given out by water



- Soln. Water properties:

TABLE B.1.2
Saturated Water Pressure Entry

Press. (kPa)	Temp. (°C)	Specific Volume, m^3/kg Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g
100	99.62	0.001043	1.69296	1.69400

- Soln.

Take the CV as water. Constant mass: $m = m_1 = m_2$

Process: v = const until $P = P_{lift}$ then P is const.

State 1: $v_1 = v_f + xv_{fg} = 0.001043 + 0.5 \times 1.69296$

$\Rightarrow v_1 = 0.8475 \text{ m}^3/\text{kg}$

State 2: $v_2 = 3 \times v_1 = 3 \times 0.8475 = 2.5425 \text{ m}^3/\text{kg}$

$P_2 \le P_{lift}$. At P_2 = 200 kPa and v_2 = 2.5425 m^3/kg, from tables,

$T_2 = 829°\text{C}. \quad \therefore V_2 = mv_2 = 25.425 \text{ m}^3$

$$W_{1\to2} = \int PdV = P_{lift} \times \left(V_2 - V_1\right) = 200 \text{ kPa} \times 10 \text{ kg} \times \left(2.5425 - 0.8475\right)$$

$\Rightarrow W_{1\to2} = 3390 \text{ kJ}$

- P11. The density of Mercury changes approximately linearly with temperature as:

$$\rho_{\mathrm{Hg}} = 13595 - 2.5\ \mathrm{T}\ \mathrm{kg/m^3} \quad (\mathrm{T\ is\ in\ ^\circ C})$$

- so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in winter at -15°C, what is the difference in column heights between the two measurements?

- Soln.

The manometer reading h relates to the pressure difference as:

$$\Delta P = \rho L g \Rightarrow L = \frac{\Delta P}{\rho g}$$

$$\rho_{summer} = 13595 - 2.5 \times 35 = 13507.5 \text{ kg/m}^3$$

$$\rho_{water} = 13595 - 2.5 \times (-15) = 13632.5 \text{ kg/m}^3$$

Column lengths:

$$L_{summer} = \frac{\Delta P_{summer}}{\rho_{summer} g} = \frac{100 \times 10^3}{13507.5 \times 9.81} 0.7549 \text{ m}$$

$$L_{winter} = \frac{\Delta P_{winter}}{\rho_{winter} g} = \frac{100 \times 10^3}{13632.5 \times 9.61} = 0.7480 \text{ m}$$

$$\Rightarrow \Delta L = L_{summer} - L_{winter} = 0.0069 \text{ m} = 6.9 \text{ mm}$$