First law – Numerical Problems

MEE 1003 Thermodynamics

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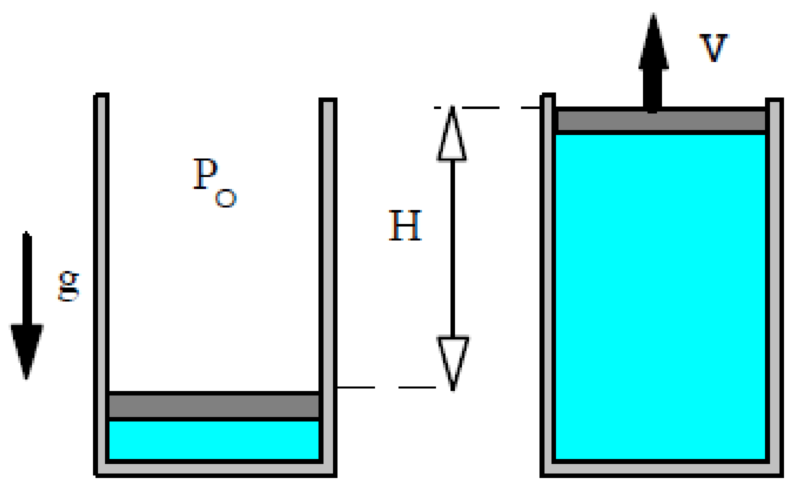
- P1. A 25 kg piston is above a gas in a long vertical cylinder. Now the piston is released from rest and accelerates up in the cylinder reaching the end 5 m higher at a velocity of 25 m/s. The gas pressure drops during the process, so the average is 600 kPa with an outside atmosphere at 100 kPa. Neglect the change is gas kinetic and potential energy, and find the needed change in the gas volume.



- Soln.

Closed system: Piston + cylinder

$$\left(E_2 - E_1\right)_{piston} = m\left(u_2 - u_1\right) + m\left(\frac{1}{2}V_2^2 - 0\right) + mg\left(h_2 - 0\right)$$

$$\Rightarrow \Delta E_{piston} = 0 + 25\times\frac{1}{2}\times 25^2 + 25\times 9.81\times 5$$

$$\Rightarrow \Delta E_{piston} = 9038.3 \text{ J} = 9.038 \text{ kJ}$$

Energy equation for piston:

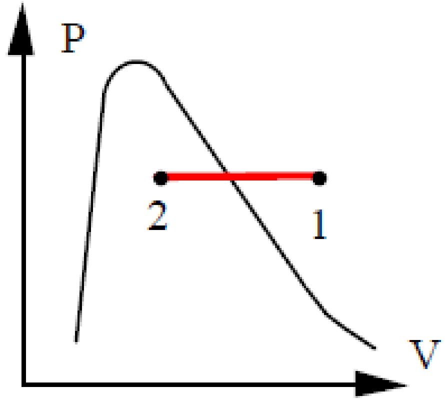
$$E_2 - E_1 = W_{gas} - W_{atm} = P_{avg}\Delta V_{gas} - P_0\Delta V_{gas}$$

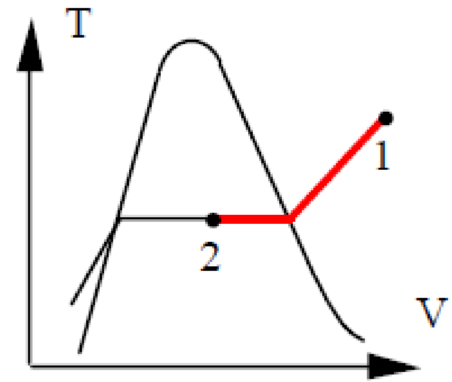
Note: $\Delta V_{atm} = -\Delta V_{gas}$ (piston rises; gas vol up, air vol down)

$$\Delta V_{gas} = \frac{9.038}{600 - 100} = 0.018 \text{ m}^3$$

- P2. A cylinder fitted with a frictionless piston contains 2 kg of superheated refrigerant R-134a vapor at 350 kPa and 100°C. The cylinder is now cooled so that the R-134 remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

- Soln.





$$\text{C.V: R-134a} \quad m_2 = m_1 = m$$

$$\text{First law (Energy eqn): } m\left(u_2 - u_1\right) = Q_{12} - W_{12}$$

$$\text{Process: } P = \text{const} \Rightarrow W_{12} = \int PdV = P\left(V_2 - V_1\right)$$

$$\Rightarrow P = Pm\left(v_2 - v_1\right)$$

- State 1: Physical properties of R-134a

Superheated R-134a

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
	300 kPa (0.56)				400 kPa (8.84)			
Sat.	0.06787	378.33	398.69	1.7259	0.05136	383.02	403.56	1.7223
10	0.07111	385.84	407.17	1.7564	0.05168	383.98	404.65	1.7261
20	0.07441	393.80	416.12	1.7874	0.05436	392.22	413.97	1.7584
30	0.07762	401.81	425.10	1.8175	0.05693	400.45	423.22	1.7895
40	0.08075	409.90	434.12	1.8468	0.05940	408.70	432.46	1.8195
50	0.08382	418.09	443.23	1.8755	0.06181	417.03	441.75	1.8487
60	0.08684	426.39	452.44	1.9035	0.06417	425.44	451.10	1.8772
70	0.08982	434.82	461.76	1.9311	0.06648	433.95	460.55	1.9051
80	0.09277	443.37	471.21	1.9582	0.06877	442.58	470.09	1.9325
90	0.09570	452.07	480.78	1.9850	0.07102	451.34	479.75	1.9595
100	0.09861	460.90	490.48	2.0113	0.07325	460.22	489.52	1.9860

- $h_1 = (h_{300\ kPa} + h_{400\ kPa})/2 = (490.48+489.52)/2$
- Hence, h_1 = 490 kJ/kg

- State 2: @75% quality

Saturated R-134a

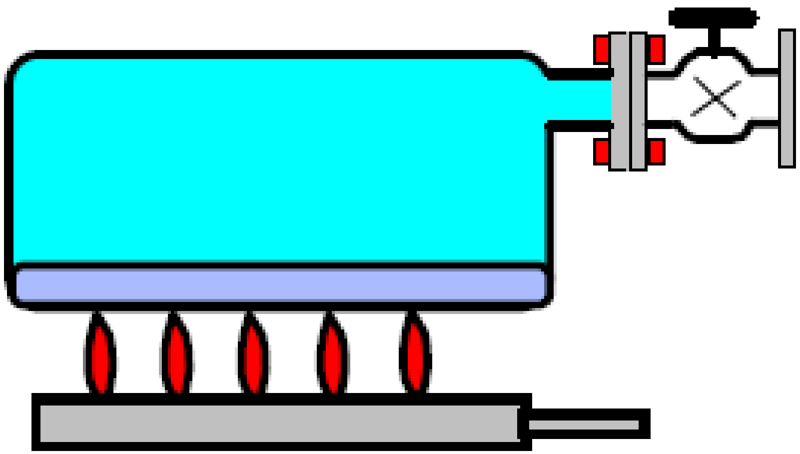
		Enthalpy, kJ/kg			Entropy, kJ/kg-K		
Temp. (°C)	Press. (kPa)	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
5	350.9	206.75	194.57	401.32	1.0243	0.6995	1.7239

- @ 350 kPa, 75% quality:
- $h = h_f + xh_{fg}$ = 206.75 + (0.75 x 194.57) = 352.68 kJ/kg

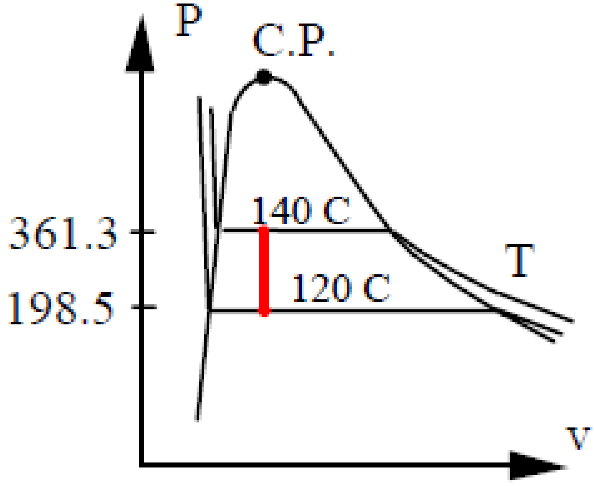
$$Q_{12} = m\left(u_2 - u_1\right) + W_{12} = m\left(u_2 - u_1\right) + Pm\left(v_2 \text{-} v_1\right)$$

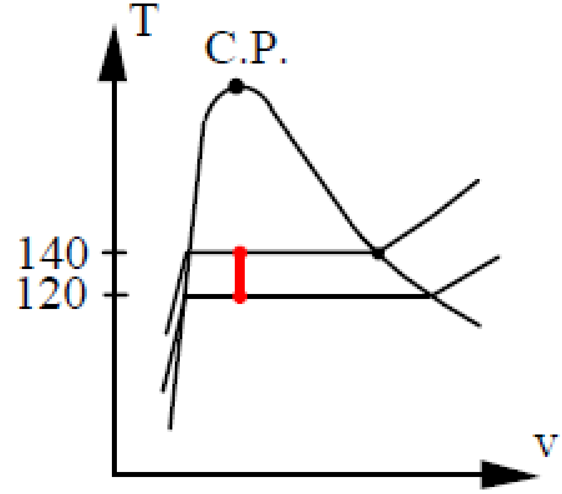
$$\Rightarrow Q_{12} = 2 \times \left(352.7 - 490\right) = -274.6 \text{ kJ}$$

- P3. Two kilograms of water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What are the heat transfer and work in the process?



- Soln.





C.V Water - control mass (closed system)

First law: $\Delta U = \Delta Q - \Delta W$

Process: $V = \text{const} \Rightarrow W_{12} = \int P dV = 0$

- Soln. Properties of water

Saturated Water

		Specific Volume, m³/kg			Internal Energy, kJ/kg		
Temp. (°C)	Press. (kPa)	Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapor u_g
120	198.5	0.001060	0.89080	0.89186	503.48	2025.76	2529.24
140	361.3	0.001080	0.50777	0.50885	588.72	1961.30	2550.02

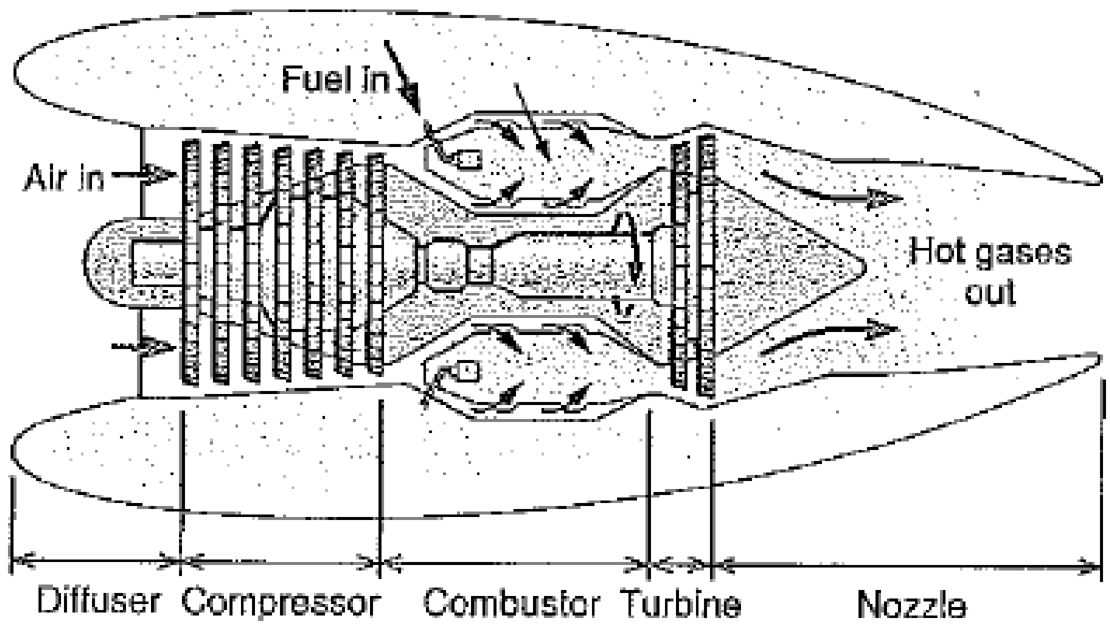
- State 1: $v_1 = v_{f1} + x_1 v_{fg} = 0.0016 + (0.25 \times 0.891)$
- Hence, $v_1 = 0.2243$ kg/m³. $u_1 = 1009.92$ kJ/kg
- State 2: $v_2 = v_1$ (constant vol) $< v_{fg2}$ (=0.51 kg/m³)
- $x_2 = (v_2 - v_{f2})/v_{fg2} = (0.2243-0.0018)/0.5077 = 0.438$
- Hence, $u_2 = u_{f2} + x_2 u_{fg2} = 588.72 + (0.438 \times 1961.3)$
- $u_2 = 1447.8$ kJ/kg

- Soln.

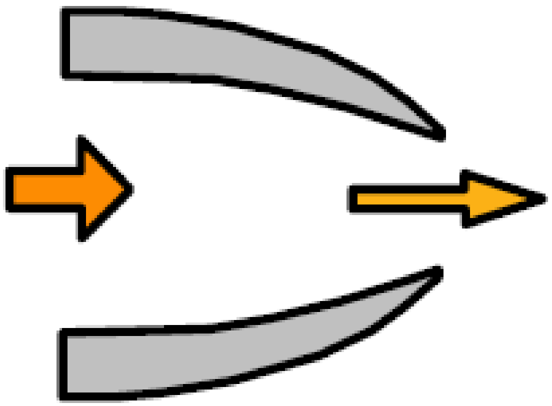
$$\text{First law: } Q_{12} = m\,(u_2 - u_1) = 2\,(1447.8 - 1009.92)$$

$$Q_{12} = 875.8 \text{ kJ}$$

- P4. In a jet engine a flow of air at 1000 K , 200 kPa, and 30 m/s enters a nozzle as shown. Air exits at 850 K, 90 kPa. Assuming no heat loss, what is the exit velocity?



- Soln. Jet engine nozzle stationary and insulated. Hence, w = q = 0.



Continuity: $\dot{m}_{in} = \dot{m}_{exit} = \dot{m}$; $\therefore$ First law is $\dot{E}_{in} = \dot{E}_{out}$

$$\text{Energy: } \dot{m}\left(h_{in} + \frac{V_{in}^2}{2}\right) = \dot{m}\left(h_{exit} + \frac{V_{exit}^2}{2}\right)$$

- Physical properties of air:

Ideal-Gas Properties of Air, Standard Entropy at 0.1-MPa (1-bar) Pressure

T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)	T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)
850	633.42	877.40	7.95207	2750	2413.73	3203.06	9.36980
1000	759.19	1046.22	8.13493	2900	2563.80	3396.19	9.43818

- h @ 850 K = 877.4 kJ/kg
- h @ 1000 K = 1046.22 kJ/kg

- Soln.

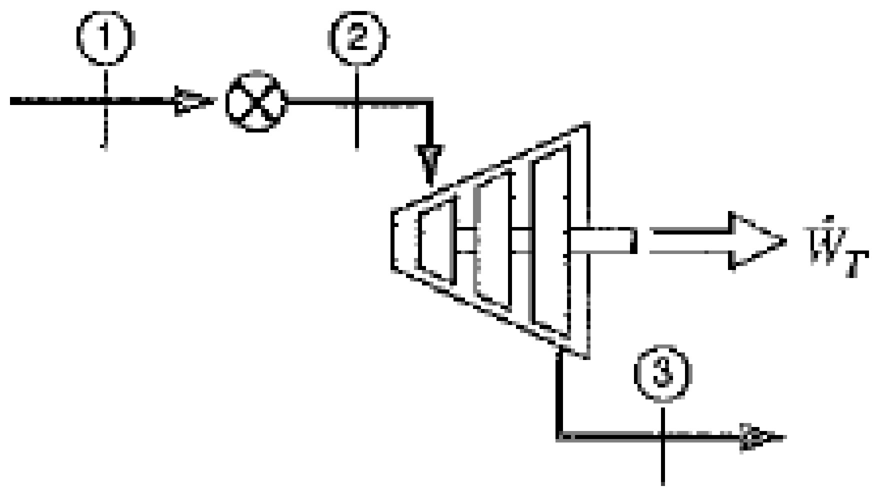
$$\frac{1}{2}V_{exit}^2 = \frac{1}{2}V_{inlet}^2 + h_{inlet} - h_{exit}$$

$$= \frac{1}{2}(30)^2 + (1046.22 - 877.4) \times 10^3 \ \text{J/kg}$$

$$= 0.45 + 168.82 = 169.27 \text{ kJ/kg}$$

$$\Rightarrow V_{exit} = \sqrt{2 \times 1000 \times 169.27} = 581.8 \text{ m/s}$$

- P5. A small turbine is operated at part load by throttling a 0.25 kg/s steam supply at 1.4 MPa and 250°C down to 1.1 MPa before it enters the turbine, and the exhaust is at 10 kPa. If the turbine produces 110 kW, find the exhaust temperature (and quality if saturated).



- Soln.

C.V: Throttle - Steady flow, $\Delta KE = 0$, $\Delta PE = 0$

Stationary throttle, hence, $q = w = 0$

$\therefore$ First law:

$$d\dot{E} = \delta\dot{Q} - \delta\dot{W} + \dot{m}_1\left(h + gz + \frac{V^2}{2}\right)_1 - \dot{m}_2\left(h + gz + \frac{V^2}{2}\right)_2 = 0$$

$$\Rightarrow d\dot{E} = \left(h + gz + \frac{V^2}{2}\right)_1 - \left(h + gz + \frac{V^2}{2}\right)_2 = 0$$

$$\Delta gz = \Delta\frac{V^2}{2} = 0 \Rightarrow h_1 = h_2$$

- Soln.

Superheated Vapor Water

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
250	0.16350	2698.32	2927.22	6.7467

- $h_1 = h_2 = 2927.22$ kJ/kg
-

- Soln.

C.V Turbine: Steady flow, q = 0 (adiabatic)

Total power = 110 kW; specific work w = $\dfrac{110\text{ kW}}{0.25\ \dfrac{\text{kg}}{\text{s}}}$

$\Rightarrow$ w = 440 kJ/kg. First law: dE = δQ - δW

$$\Rightarrow d\dot{E} = \delta\dot{Q} - \delta\dot{W} + \left(h + \frac{V^2}{2} + gz\right)_2 - \left(h + \frac{V^2}{2} + gz\right)_3 = 0$$

$d\dot{E} = 0$ (steady state); $\delta\dot{Q} = 0$ (adiabatic); $\Delta gz = \Delta\dfrac{V^2}{2} = 0$

$\Rightarrow - w + h_2 - h_3 = 0$

$\Rightarrow h_2 = h_3 + w = 2927.22$ kJ $(h_1 = h_2)$

$\Rightarrow h_3 = 2927.22 - 440 = 2487.2$ kJ/kg

- Soln. Steam at 10 kPa, 2487.2 kJ/kg.

Saturated Water Pressure Entry

		ENTHALPY, kJ/kg			ENTROPY, kJ/kg-K		
Press. (kPa)	Temp. (°C)	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
10	45.81	191.81	2392.82	2584.63	0.6492	7.5010	8.1501

- $h_3 = h_{f3} + x_3 h_{fg3}$
- $2487.2 = 191.81 + x_3(2392.82)$
- Hence, $x_3 = 0.96$ and $T_3 = 45.8°C$.

- P6. A 4 kg/s steady flow of ammonia passes through a device where it goes through a polytropic process. The inlet state is 150 kPa, -20°C and the exit state is 400 kPa, 80°C where all kinetic and potential energies can be neglected. The specific work input was found to be $\left(\frac{\mathrm{n}}{\mathrm{n\text{-}1}}\right)\Delta(\mathrm{Pv})$
- a) Find the polytropic exponent n
- b) Find the specific work and specific heat transfer

- Soln. Ammonia properties

Superheated Ammonia

Temp. (°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg-K)
		150 kPa (−25.22)		
Sat.	0.7787	1294.1	1410.9	5.6983
−20	0.7977	1303.3	1422.9	5.7465
		400 kPa (−1.89)		
80	0.42160	1468.0	1636.7	5.9907

- Soln.

First law: Steady process

$$q - w + \left(h + \frac{V^2}{2} + gz \right)_1 - \left(h + \frac{V^2}{2} + gz \right)_2 = dE = 0$$

process: $Pv^n = \text{const}; \; z_1 = z_2; \; V_1 = V_2 = 0$

State 1: $v_1 = 0.79774 \text{ m}^3 / \text{kg}; \; h_1 = 1422.9 \text{ kJ/kg}$

State 2: $v_2 = 0.4216 \text{ m}^3/\text{kg}; \; h_2 = 1636.7 \text{ kJ/kg}$

$$Pv^n = \text{const} \Rightarrow P_1 v_1^n = P_2 v_2^n \Rightarrow \ln\left(P_1 v_1^n\right) = \ln\left(P_2 v_2^n\right)$$

$$\Rightarrow \ln P_1 + n \ln v_1 = \ln P_2 + n \ln v_2$$

$$\Rightarrow n = \ln \frac{P_2}{P_1} \Big/ \ln \frac{v_1}{v_2} = \frac{\ln\left(400/150\right)}{\ln\left(0.79774/04216\right)} = 1.538$$

$$w_{\text{Polytropic}} = \frac{n}{n\text{-}1}\left(P_1 v_1 - P_2 v_2\right) = 2.8587 \times \left(150 \times 0.7977 - 400 \times 0.4216\right)$$

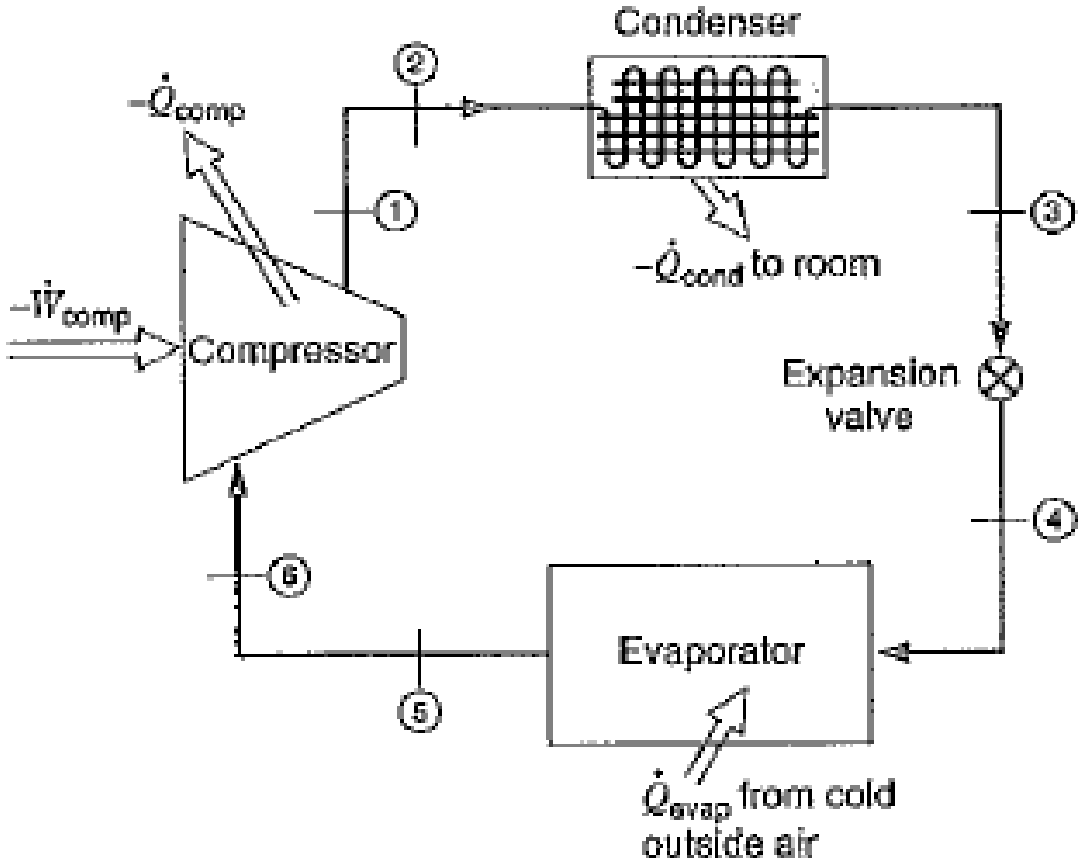
$$\Rightarrow w_{\text{Polytropic}} = -140 \text{ kJ/kg.} \;\; \therefore \; q = h_2 + w - h_1 = 73.8 \text{ kJ/kg}$$

- P7. An R-12 heat pump cycle has an R-12 flow rate of 0.05 kg/s with 4 kW into the compressor. The following data are given:

State	1	2	3	4	5	6
P, kPa	1250	1230	1200	320	300	290
T, °C	120	110	45		0	5
h, kJ/kg	260	253	79.7		188	191

- Calculate the heat transfer from the compressor, the heat transfer from R-12 in the condenser, and the heat transfer to R-12 in the evaporator.

- Soln.



Condenser
$-\dot{Q}_{comp}$
2
1
3
$-\dot{Q}_{cond}$ to room
$-\dot{W}_{comp}$
Compressor
Expansion valve
4
6
Evaporator
5
$\dot{Q}_{evap}$ from cold outside air

- Soln.

Compressor (steady flow)

First law: $\dot{Q}_{comp} - \dot{W}_{Comp} + \dot{m}\left(h_1 - h_e\right) = dE = 0$

$\Rightarrow \dot{Q}_{comp} = 0.05\left(260 - 191\right) - 4.0 = -0.55\ \text{kW}$

Condenser

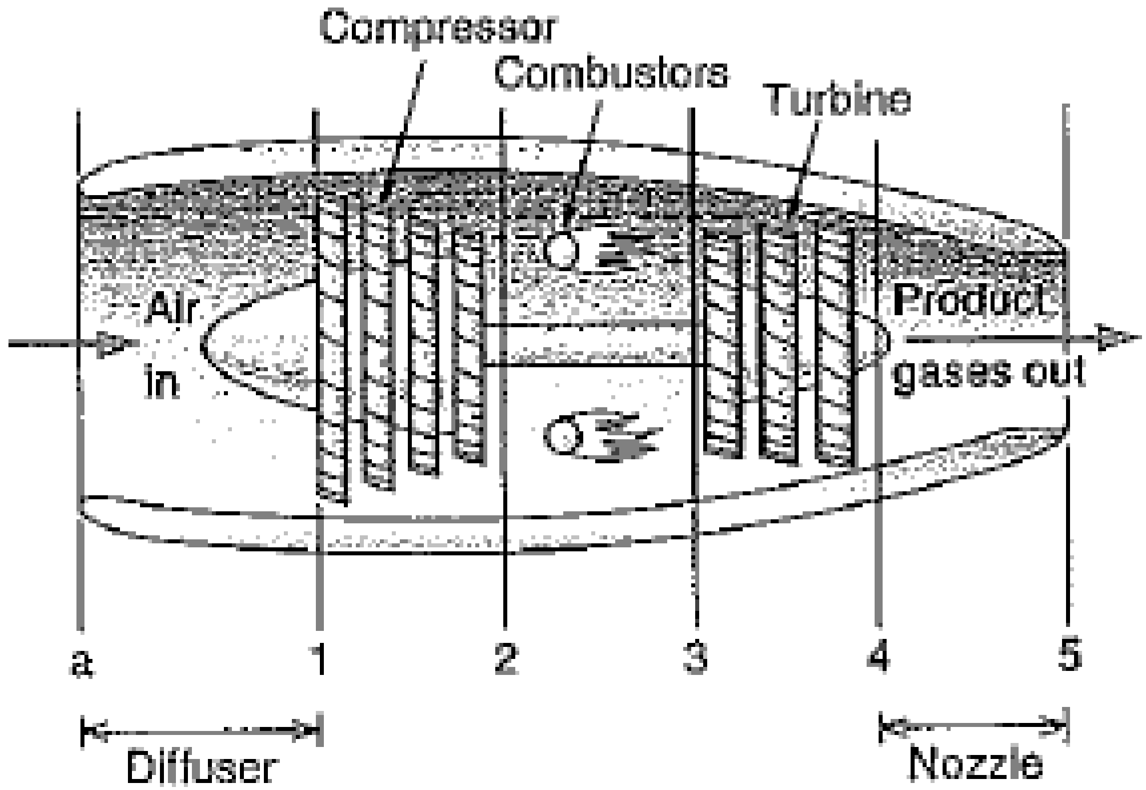
$\dot{Q}_{cond} = \dot{m}\left(h_3 - h_2\right)$(condenser stationary, w = 0)

Evaporator

$\dot{Q}_{evap} = \dot{m}\left(h_5 - h_4\right) = 0.05 \times \left(188 - 79.7\right) = 5.42\ \text{kW}$

- P8. A modern jet engine has a temperature after combustion of about 1500 K at 3200 kPa as it enters the turbine section. The compressor inlet (state 1)is at 80 kPa and 260 K and the outlet (state 2) is at 720 kPa and 3300 K. The turbine outlet into the nozzle is at 400 kPa, 900 K and the nozzle exit is at 80 kPa, 640 K. Neglect any heat transfer and kinetic energy except at the nozzle out of the nozzle. Find the compressor and turbine specific work terms and nozzle exit velocity.

- Soln.



Compressor
Combustors
Turbine
Air in
Product gases out
a
1
2
3
4
5
Diffuser
Nozzle

- Soln.

Air properties:

$$h_1 = 260.32 \text{ kJ/Kg};\ h_2 = 800.28 \text{ kJ/kg};\ h_3 = 1635.8 \text{ kJ/kg}$$

$$h_4 = 933.15 \text{ kJ/kg};\ h_5 = 649.53 \text{ kJ/kg}$$

$$\text{Comp: } w_{c,\ in} = h_2 - h_1 = 800.28 - 260.32 = 539.36 \text{ kJ/kg}$$

$$\text{Turbine: } w_T = h_3 - h_4 = 1635.8 - 933.15 = 702.65 \text{ kJ/kg}$$

$$\text{Nozzle: } h_4 = h_5 + \frac{V_5^2}{2} \Rightarrow V_5 = \sqrt{2\left(h_4 - h_5\right)}$$

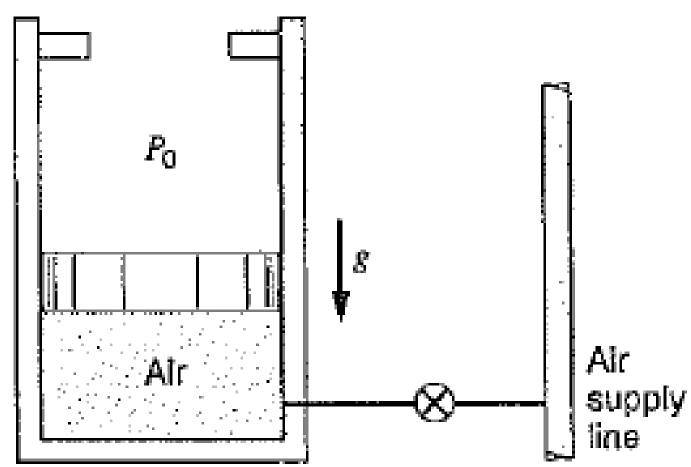
$$\therefore V_5 = \sqrt{2\left(283.62 \times 1000\right)} = 753 \text{ m/s}$$

• Soln: Properties of air

Ideal-Gas Properties of Air, Standard Entropy at 0.1-MPa (1-bar) Pressure

T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)	T (K)	u (kJ/kg)	h (kJ/kg)	s_T^0 (kJ/kg-K)
200	142.77	200.17	6.46260	1100	845.45	1161.18	8.24449
220	157.07	220.22	6.55812	1150	889.21	1219.30	8.29616
240	171.38	240.27	6.64535	1200	933.37	1277.81	8.34596
260	185.70	260.32	6.72562	1250	977.89	1336.68	8.39402
280	200.02	280.39	6.79998	1300	1022.75	1395.89	8.44046
290	207.19	290.43	6.83521	1350	1067.94	1455.43	8.48539
298.15	213.04	298.62	6.86305	1400	1113.43	1515.27	8.52891
300	214.36	300.47	6.86926	1450	1159.20	1575.40	8.57111
320	228.73	320.58	6.93413	1500	1205.25	1635.80	8.61208
340	243.11	340.70	6.99515	1550	1251.55	1696.45	8.65185
360	257.53	360.86	7.05276	1600	1298.08	1757.33	8.69051
380	271.99	381.06	7.10735	1650	1344.83	1818.44	8.72811
400	286.49	401.30	7.15926	1700	1391.80	1879.76	8.76472
420	301.04	421.59	7.20875	1750	1438.97	1941.28	8.80039
440	315.64	441.93	7.25607	1800	1486.33	2002.99	8.83516
460	330.31	462.34	7.30142	1850	1533.87	2064.88	8.86908
480	345.04	482.81	7.34499	1900	1581.59	2126.95	8.90219
500	359.84	503.36	7.38692	1950	1629.47	2189.19	8.93452
520	374.73	523.98	7.42736	2000	1677.52	2251.58	8.96611
540	389.69	544.69	7.46642	2050	1725.71	2314.13	8.99699
560	404.74	565.47	7.50422	2100	1774.06	2376.82	9.02721
580	419.87	586.35	7.54084	2150	1822.54	2439.66	9.05678
600	435.10	607.32	7.57638	2200	1871.16	2502.63	9.08573
620	450.42	628.38	7.61090	2250	1919.91	2565.73	9.11409
640	465.83	649.53	7.64448	2300	1968.79	2628.96	9.14189
660	481.34	670.78	7.67717	2350	2017.79	2692.31	9.16913
680	496.94	692.12	7.70903	2400	2066.91	2755.78	9.19586
700	512.64	713.56	7.74010	2450	2116.14	2819.37	9.22208
720	528.44	735.10	7.77044	2500	2165.48	2883.06	9.24781
740	544.33	756.73	7.80008	2550	2214.93	2946.86	9.27308
760	560.32	778.46	7.82905	2600	2264.48	3010.76	9.29790
780	576.40	800.28	7.85740	2650	2314.13	3074.77	9.32228
800	592.58	822.20	7.88514	2700	2363.88	3138.87	9.34625
850	633.42	877.40	7.95207	2750	2413.73	3203.06	9.36980
900	674.82	933.15	8.01581	2800	2463.66	3267.35	9.39297
950	716.76	989.44	8.07667	2850	2513.69	3331.73	9.41576
1000	759.19	1046.22	8.13493	2900	2563.80	3396.19	9.43818
1050	802.10	1103.48	8.19081	2950	2613.99	3460.73	9.46025
1100	845.45	1161.18	8.24449	3000	2664.27	3525.36	9.48198

- P9. A mass loaded piston/cylinder assembly shown below is containing air at 300 kPa and 17°C with a volume of 0.25 m^3, while at the stops V = 1 m^3. An air line, 500 kPa, 600 K, is connected by a valve that is then opened until a final inside pressure of 400 kPa is reached, at which point T = 350 K. Find the air mass that enters, the work and the heat transfer.



- Soln.

$$\text{C.V Cylinder volume: Continuity - } m_2 - m_1 = m_m$$

$$m_2 u_2 - m_1 u_1 = m_m h_{line} + Q_{CV} - W_{12}$$

$$\text{Proces: } P_1 \text{ is constant to stops, then const V to state 2 @} P_2$$

$$\text{State 1: } P_1,\ T_1 \quad m_1 = \frac{P_1 V}{RT_1} = \frac{300 \times 0.25}{0.287 \times 290.2} = 0.9 \text{ kg}$$

$$\text{State 2: Open to } P_2 = 400 \text{ kPa, } T_2 = 350 \text{ K}$$

$$m_2 = \frac{400 \times 1}{0.287 \times 350} = 3.982 \text{ kg; } m_1 = 3.982 - 0.9 = 3.082 \text{ kg}$$

$$\text{Only work while constant P: } W_{12} = P_1 \left(V_2 - V_1 \right)$$

$$\Rightarrow W_{12} = 300 \left(1 - 0.25 \right) = 225 \text{ kJ}$$

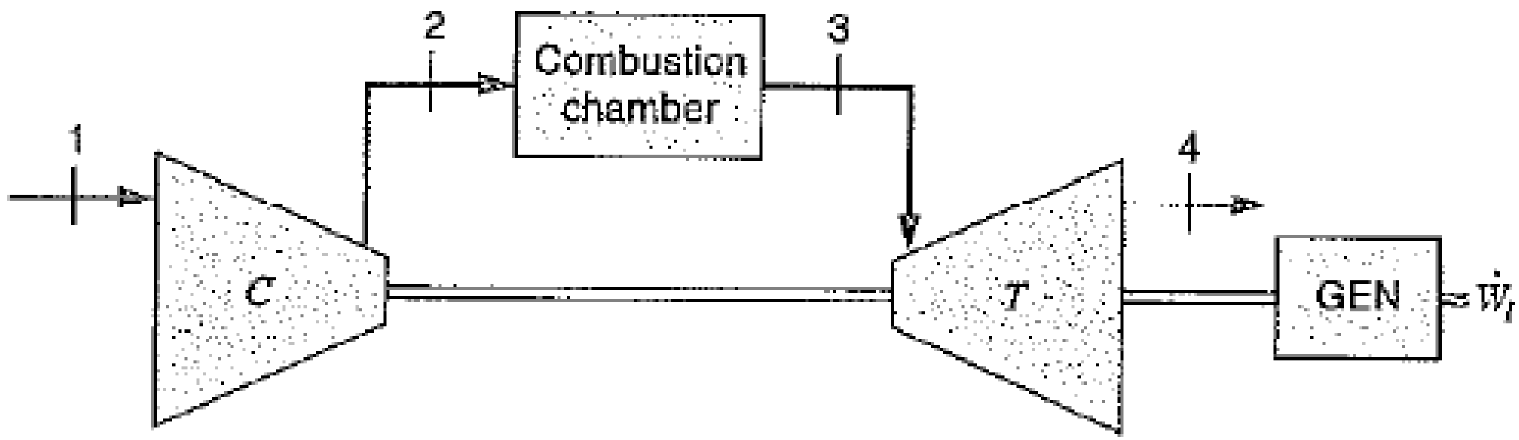
$$\text{Energy Eq: } Q_{CV} + m_i h_i = m_2 u_2 - m_1 u_1 + W_{12}$$

$$Q_{CV} = (3.982 \times 0.717 \times 350) - \left(0.9 \times 0.717 \times 290.2 \right) + 225$$
$$- \left(3.082 \times 1.004 \times 600 \right) = -819.2 \text{ kJ}$$

- P10. A gas turbine set-up to produce power during peak demand is shown below. The turbine provides power to the air compressor and the electric generator. If the electric generator provide 5 MW what is the needed air flow rate at state 1 and the combustion heat transfer rates between states 2-3.

1. 90 kPa, 290 K
2. 900 kPa, 560 K
3. 900 kPa, 1400 K
4. 100 kPa, 850 K

- Soln.



- Soln.

1: 90 kPa, 290 K. 2: 900 kPa, 560 K.

3: 900 kPa, 1400 K; 4: 100 kPa, 850 K.

$$w_{c,in} = h_2 - h_1 = 564.47 - 290.43 = 275.04 \text{ kJ/kg}$$

$$w_{T,\,out} = h_3 - h_4 = 1515.27 - 877.4 = 637.87 \text{ kJ/kg}$$

$$q_H = h_3 - h_2 = 1515.27 - 565.47 = 949.8 \text{ kJ/kg}$$

$$\dot{W}_{e1} = \dot{m}\, w_T - \dot{m}\, w_c \Rightarrow \dot{m} = \frac{\dot{W}_{e1}}{w_T - w_c} = \frac{5000}{637.87 - 275.04}$$

$$\therefore \dot{m} = 13.78 \text{ kg/s}. \quad \dot{Q}_H = \dot{m}\, q_H = 13.78 \times 949.8 = 13.1 \text{ MW}$$