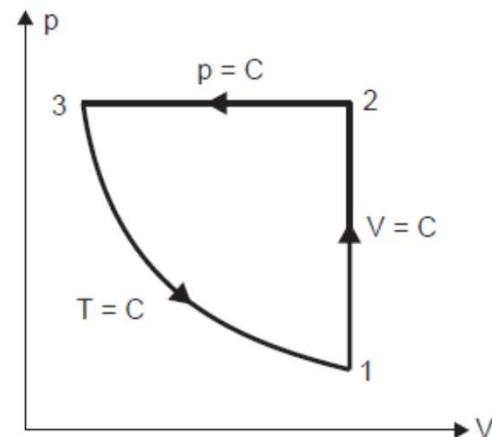


A closed system contains air at a pressure 1 bar, temperature 300 K and volume 0.018 m³. This system undergoes a thermodynamic cycle consisting of the following three processes in series : (i) Constant volume heat addition till pressure becomes 5 bar, (ii) Constant pressure cooling, and (iii) Isothermal process to initial state.

Represent the cycle on T-S and evaluate the change in entropy for each process and net entropy change Take $C_p = 0.718 \text{ kJ/kg K}$ and $R = 0.287 \text{ kJ/kg K}$.

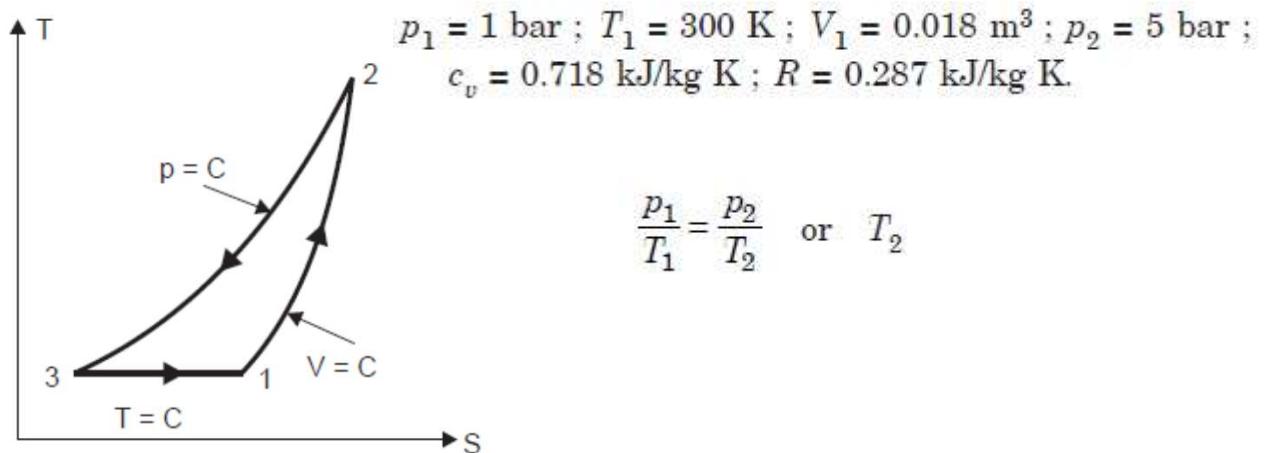


$$m = \frac{p_1 V_1}{R T_1}$$

$$S_2 - S_1 = m c_v \log_e \left(\frac{T_2}{T_1} \right)$$

$$S_3 - S_2 = m c_p \log_e \left(\frac{T_3}{T_2} \right) = m(c_v + R) \log_e \left(\frac{T_3}{T_2} \right)$$

$$S_1 - S_3 = m R \log_e \left(\frac{p_3}{p_1} \right)$$



0.0241 kJ/K. – 0.0338 kJ/K 0.00965 kJ/K.

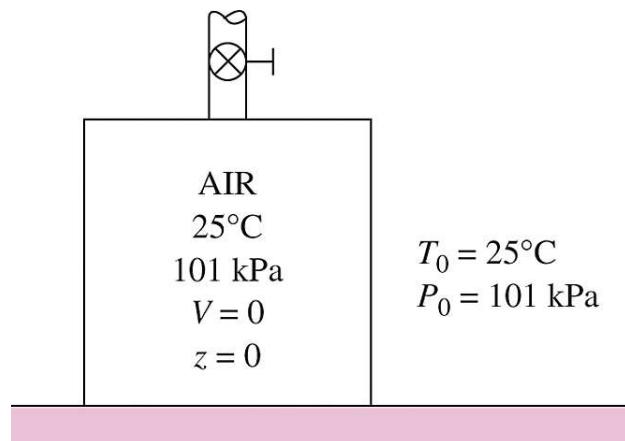
AVAILABILITY AND IRREVERSIBILITY

Energy has two parts

- Available part.
- Unavailable part.

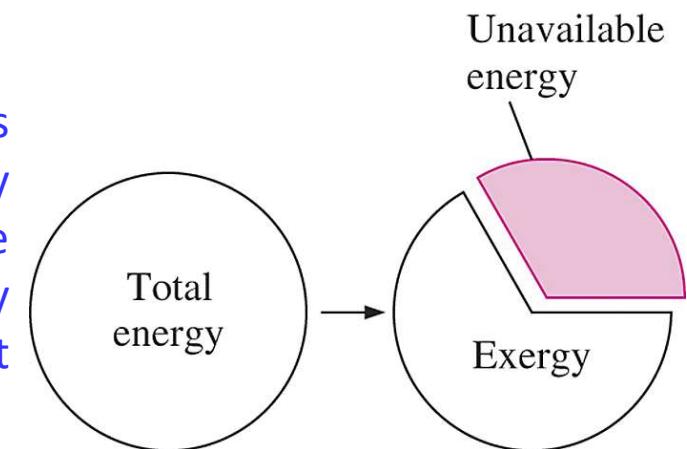
Availability

The theoretical maximum amount of work which can be obtained from a system at any state p_1 and T_1 when operating with a reservoir at the constant pressure and temperature p_0 and T_0



A system that is in equilibrium with its environment is said to be at the dead state.

Unavailable energy is the portion of energy that cannot be converted to work by even a reversible heat engine.



AVAILABLE ENERGY IN A CYCLE

$$Q_1 = \text{A.E.} + \text{U.E.}$$

$$W_{max} = \text{A.E.} = Q_1 - \text{U.E.}$$

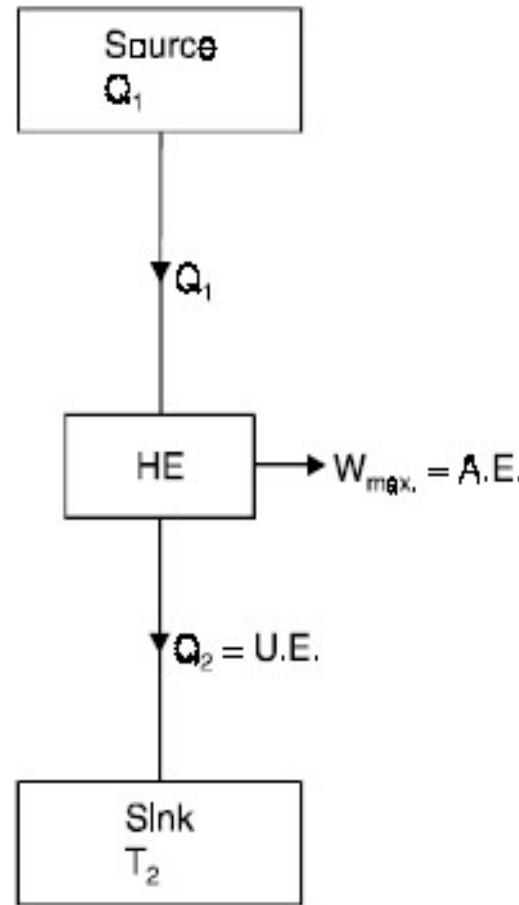
$$\eta_{rev.} = 1 - \frac{T_2}{T_1}$$

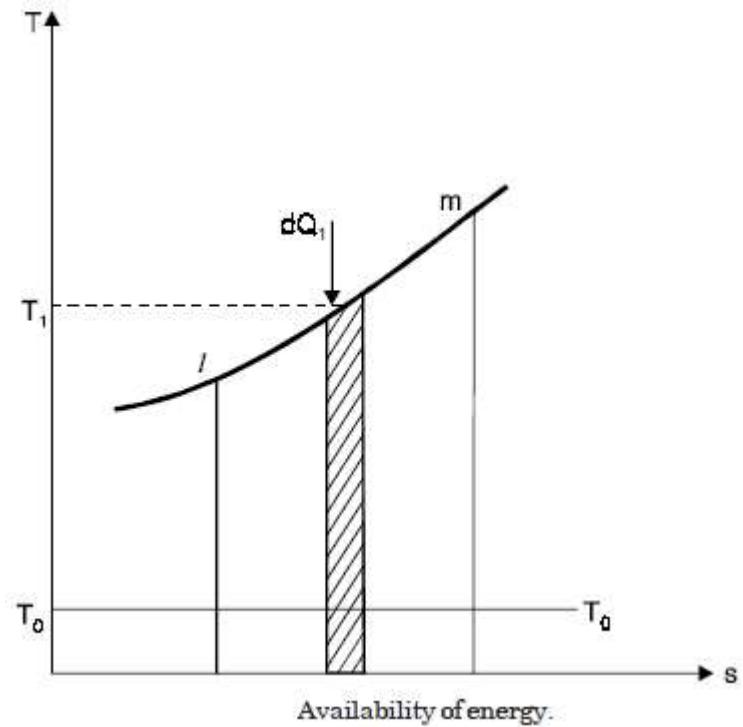
$$\eta_{max} = 1 - \frac{T_0}{T_1}$$

$$W_{max} = \left(1 - \frac{T_0}{T_1}\right) Q_1$$

$$dW_{max} = \frac{T_1 - T_0}{T_1} dQ_1$$

$$= dQ_1 - \frac{T_0}{T_1} dQ_1 = \text{A.E.}$$



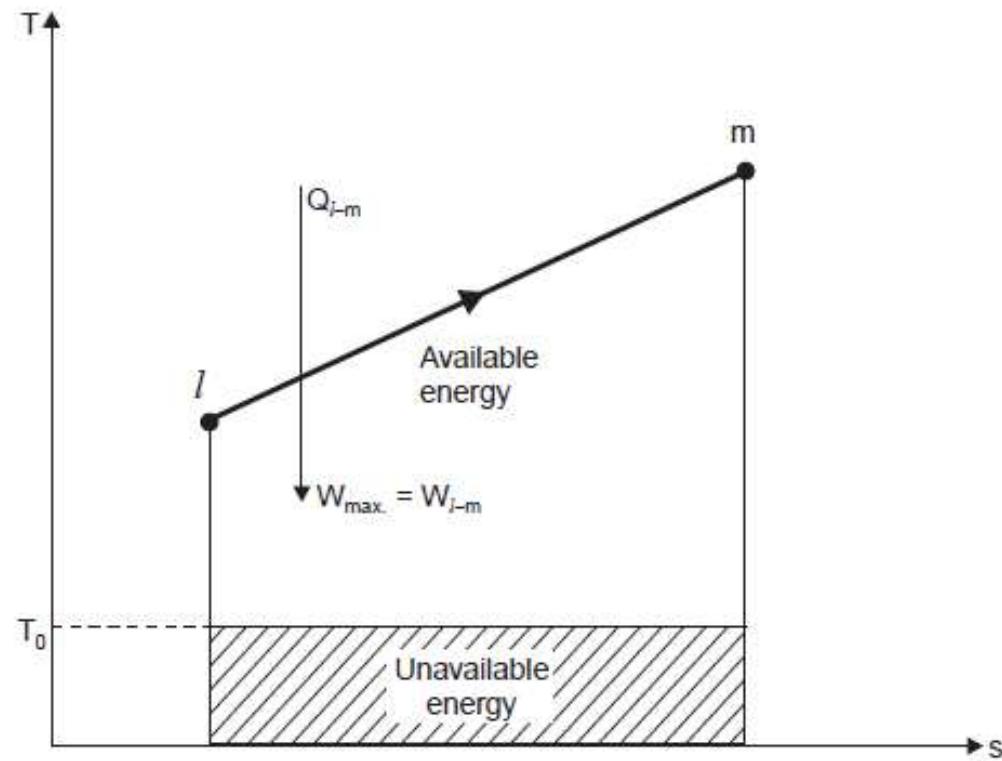


$$\int_l^m dW_{max} = \int_l^m dQ_1 - \int_l^m \frac{T_0}{T_1} dQ_1$$

$$W_{max} = A.E. = Q_{l-m} - T_0 (s_l - s_m)$$

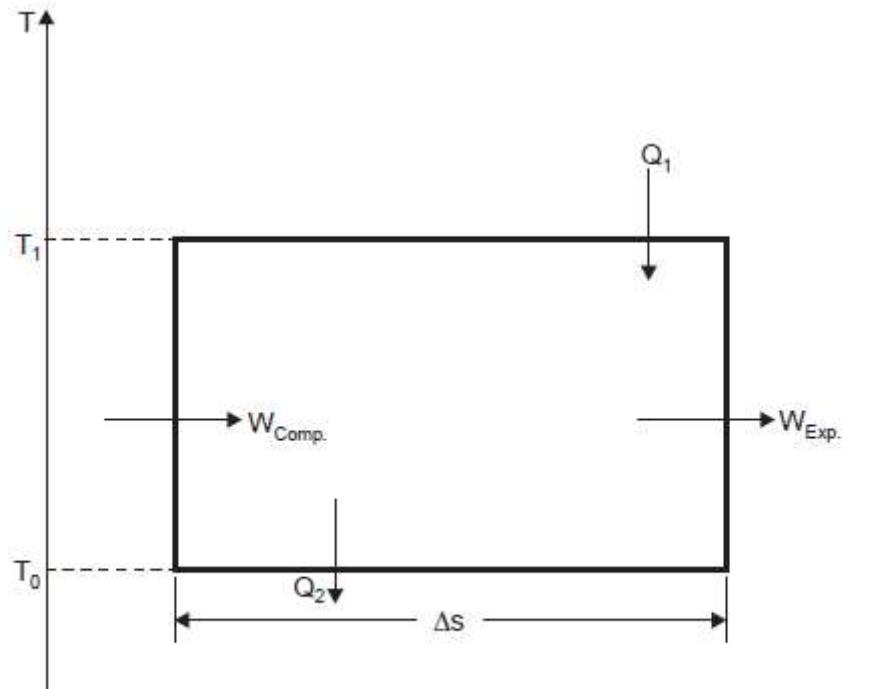
$$U.E. = Q_{l-m} - W_{max}$$

$$U.E. = T_0 (s_l - s_m)$$



Unavailable energy by the second law of thermodynamics.

DECREASE IN AVAILABLE ENERGY WHEN HEAT IS TRANSFERRED THROUGH A FINITE TEMPERATURE DIFFERENCE



Carnot-cycle.

$$Q_1 = T_1 \cdot \Delta s ;$$

$$Q_2 = T_0 \Delta s ;$$

$$W = \text{A.E.} = [T_1 - T_0] \Delta s .$$

$$Q_1 = T_1 \Delta s = T_1' \Delta s'$$

$$T_1 > T_1'$$

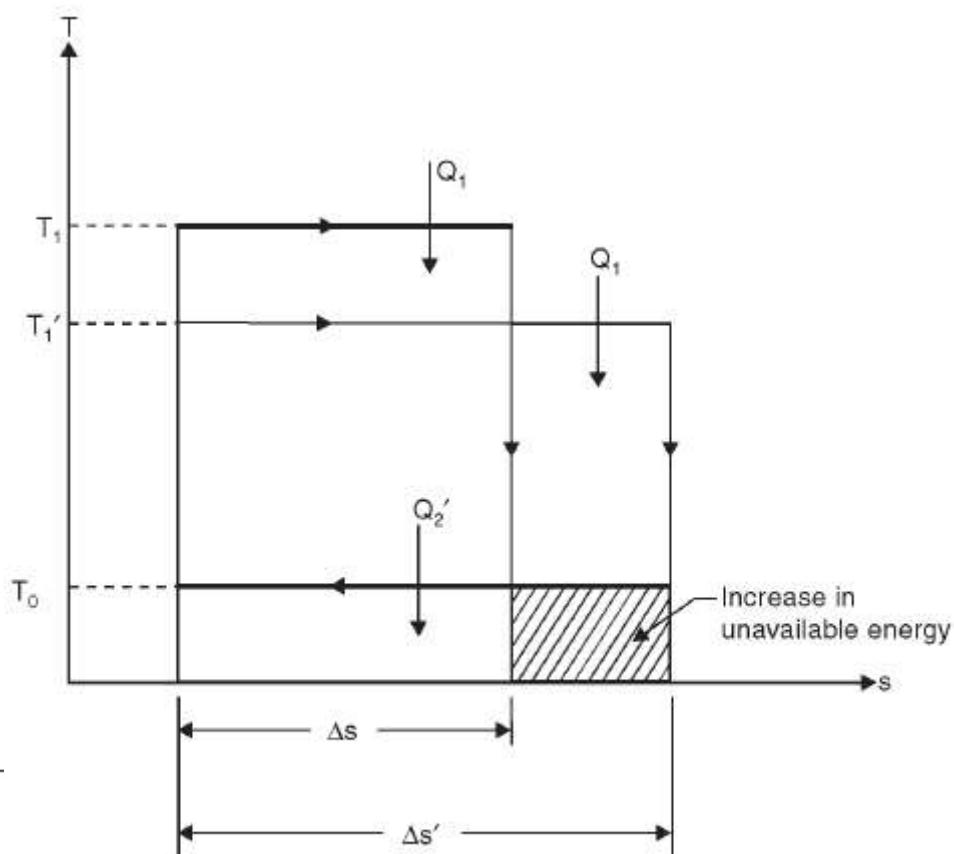
$$\Delta s' > \Delta s$$

$$Q_2 = T_0 \Delta s$$

$$Q_2' = T_0 \Delta s'$$

$$\Delta s' > \Delta s$$

$$Q_2' > Q_2$$



Increase in unavailable energy due to heat transfer through a finite temperature difference.

$$W' = Q_1 - Q_2' = T_1' \Delta s' - T_0 \Delta s'$$

$$W = Q_1 - Q_2 = T_1 \Delta s - T_0 \Delta s$$

$$W < W', \text{ because } Q_2' \geq Q_2$$

$$\begin{aligned} W - W' &= Q_2' - Q_2 \\ &= T_0 (\Delta s' - \Delta s) \end{aligned}$$

IRREVERSIBILITY AND LOSS IN AVAILABILITY

Most real processes are irreversible due to

friction

heat transfer with finite temperature difference

mixing

In case of reversible process

change in entropy of the system = change in entropy of the Universe

Hence no net increase of entropy of the Universe

Unavailability or irreversibility = product of the sink temperature and

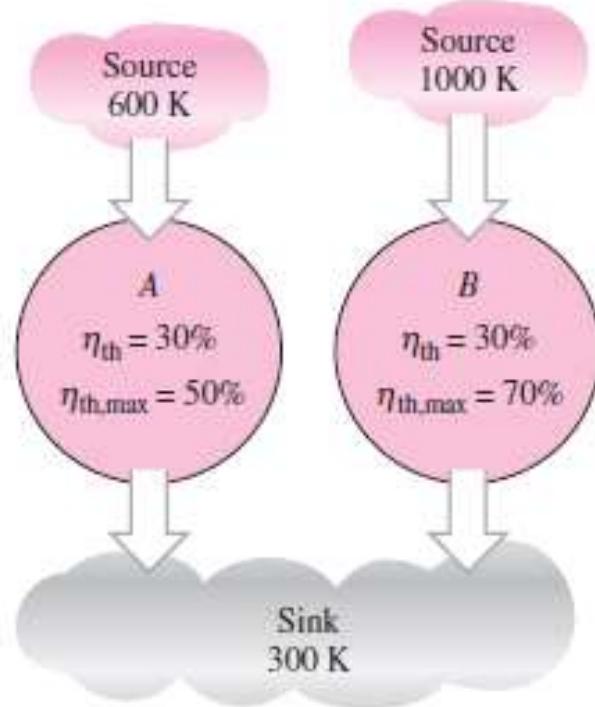
change in entropy

$$= T_0 \Delta S$$

EFFECTIVENESS

$$\epsilon = \frac{W_{useful}}{W_{max.\;useful}}$$

SECOND-LAW EFFICIENCY, η_{II}



Two heat engines that have the same thermal efficiency, but different maximum thermal efficiencies.

$$\eta_{rev,A} = \left(1 - \frac{T_L}{T_H}\right)_A = 1 - \frac{300 \text{ K}}{600 \text{ K}} = 50\%$$
$$\eta_{rev,B} = \left(1 - \frac{T_L}{T_H}\right)_B = 1 - \frac{300 \text{ K}}{1000 \text{ K}} = 70\%$$



Second-law efficiency is a measure of the performance of a device relative to its performance under reversible conditions.

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th,rev}} \quad (\text{heat engines})$$

$$\eta_{II,A} = \frac{0.30}{0.50} = 0.60 \quad \text{and} \quad \eta_{II,B} = \frac{0.30}{0.70} = 0.43$$

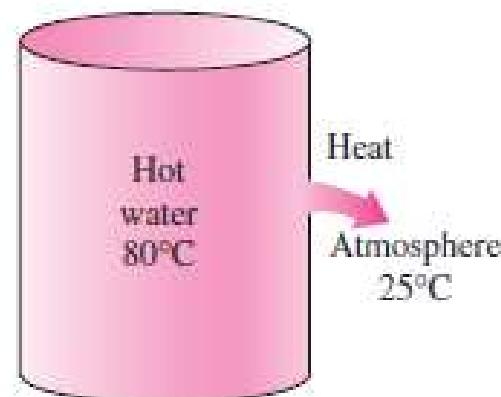
$$\eta_{II} = \frac{W_u}{W_{rev}} \quad (\text{work-producing devices})$$

$$\eta_{II} = \frac{W_{rev}}{W_u} \quad (\text{work-consuming devices})$$

$$\eta_{II} = \frac{\text{COP}}{\text{COP}_{rev}} \quad (\text{refrigerators and heat pumps})$$

Second-law efficiency of all reversible devices is _____

The second-law efficiency of naturally occurring processes is _____



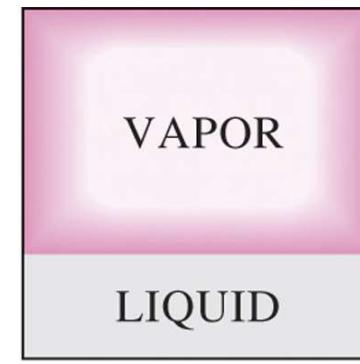
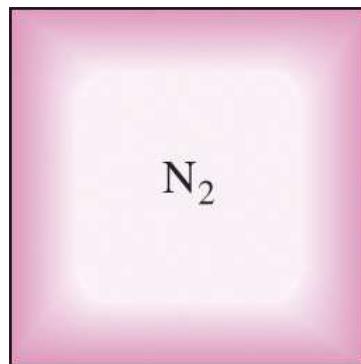
A heat engine that rejects waste heat to a sink at 530 K has a thermal efficiency of 36 % and a second-law efficiency of 60 %. Determine the temperature of the source that supplies heat to this engine.

1325 K

COP of a heat pump for residential buildings has a value of 1.2. Assuming an indoor temperature of 21°C and outdoor temperature of 10°C, determine the second-law efficiency of the heat pump. **4.49 %**

PURE SUBSTANCE

- **Pure substance:** A substance that has a fixed chemical composition throughout.
- Air is a mixture of several gases, but it is considered to be a pure substance.



Nitrogen and gaseous air are pure substances.

A mixture of liquid and gaseous water is a pure substance, but a mixture of liquid and gaseous air is not. Different components in air condensing at different temperatures at a specified pressure.

FORMATION OF STEAM AT CONSTANT PRESSURE

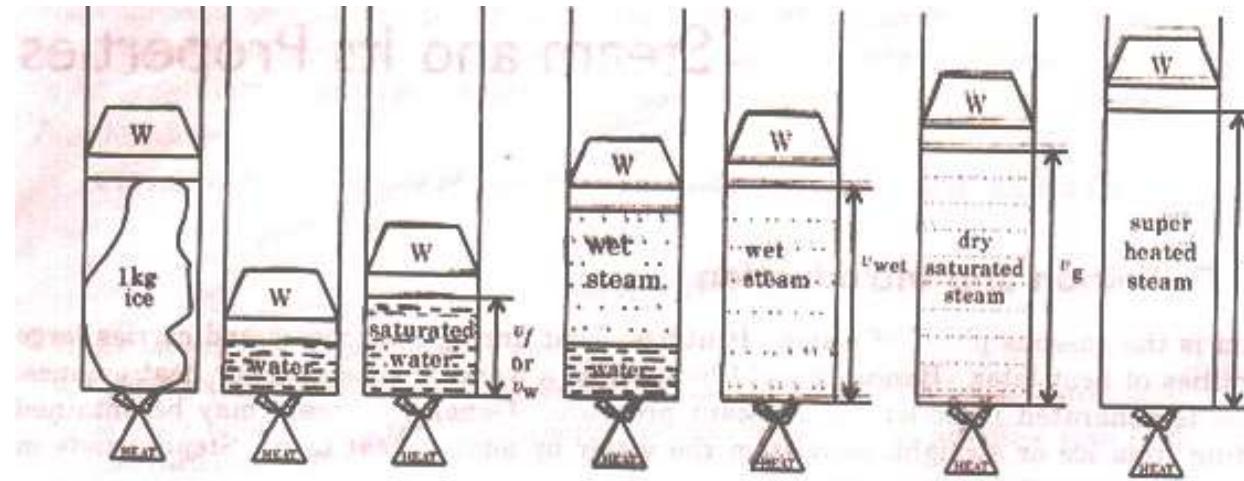
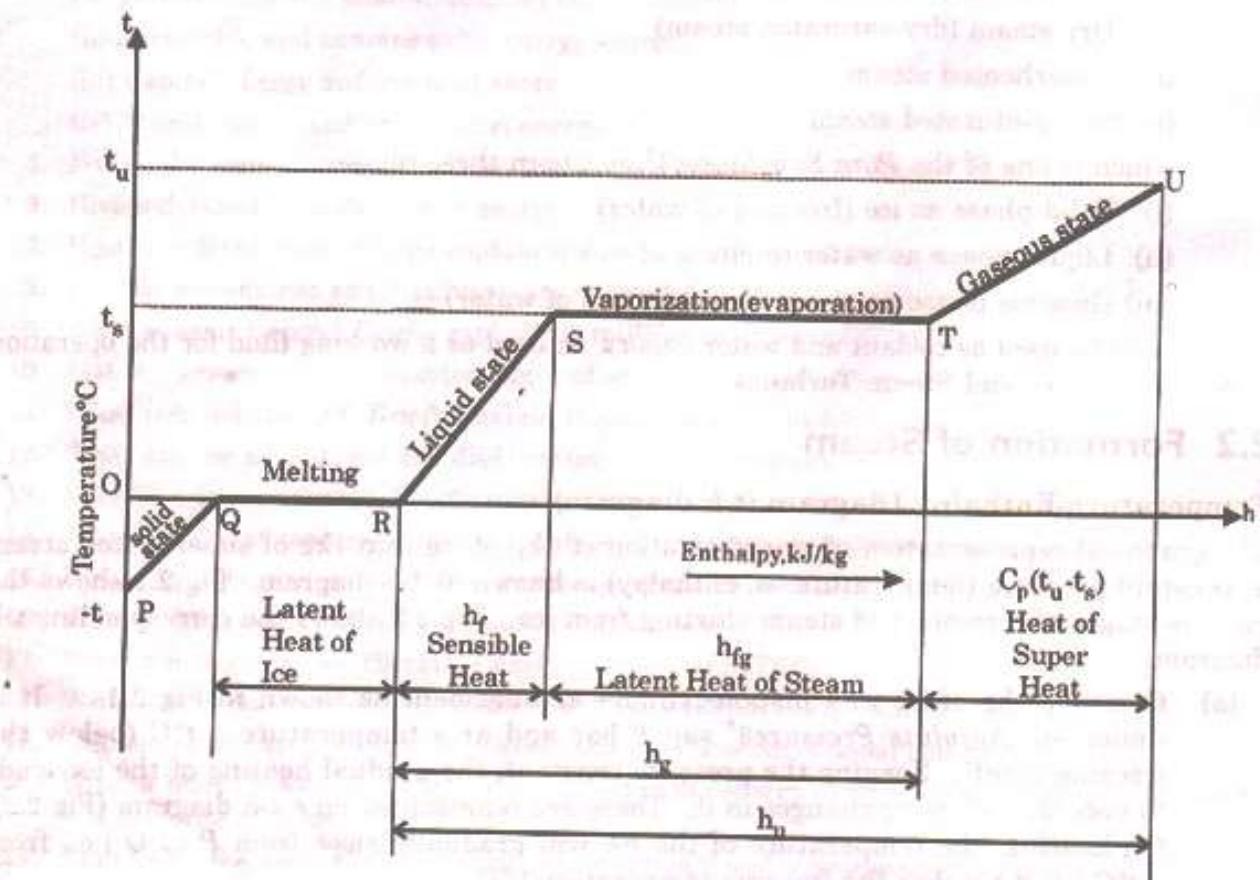
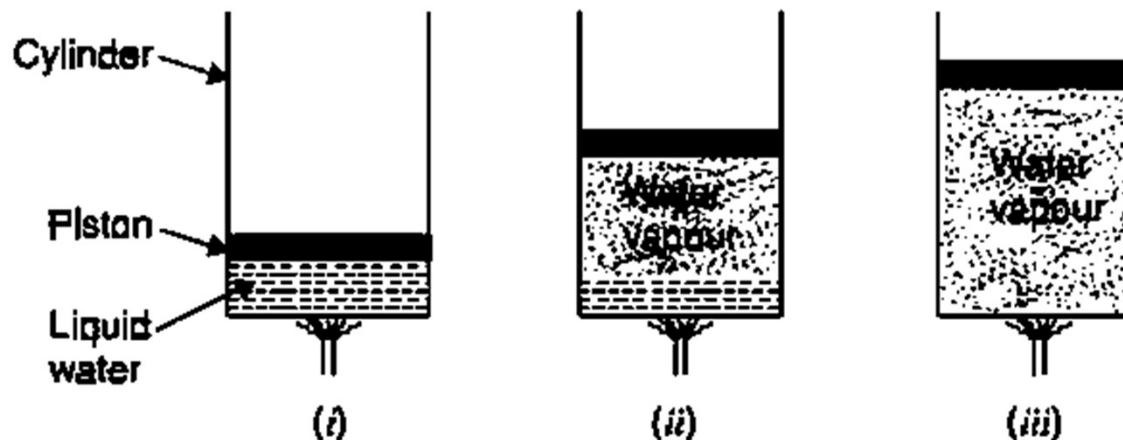


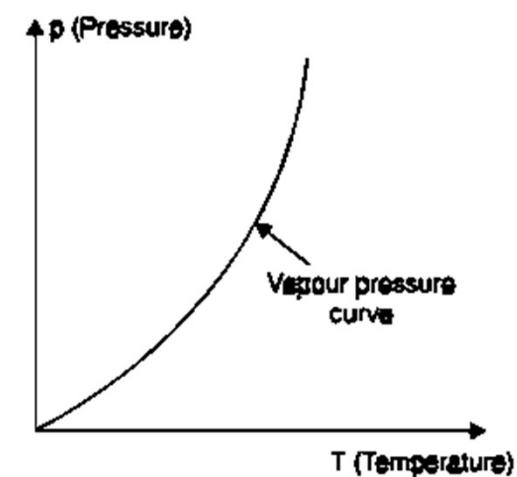
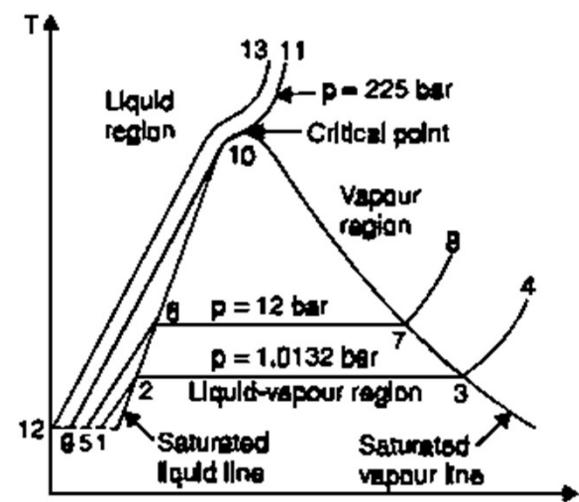
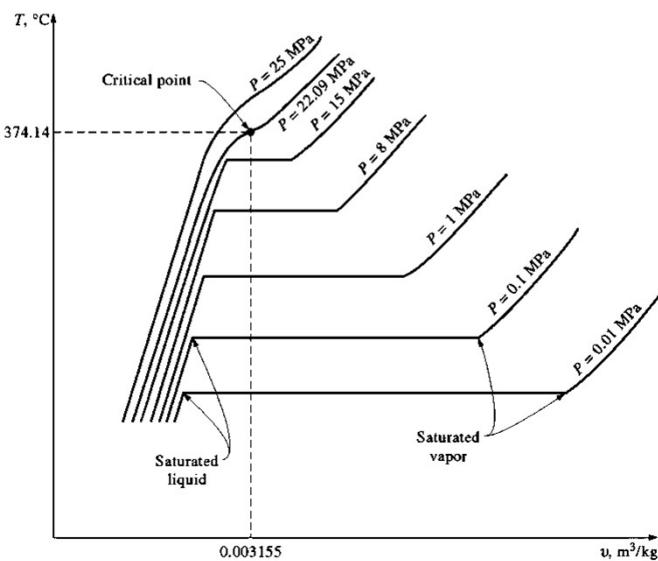
Fig 2.1 Formation of Steam at Constant Pressure

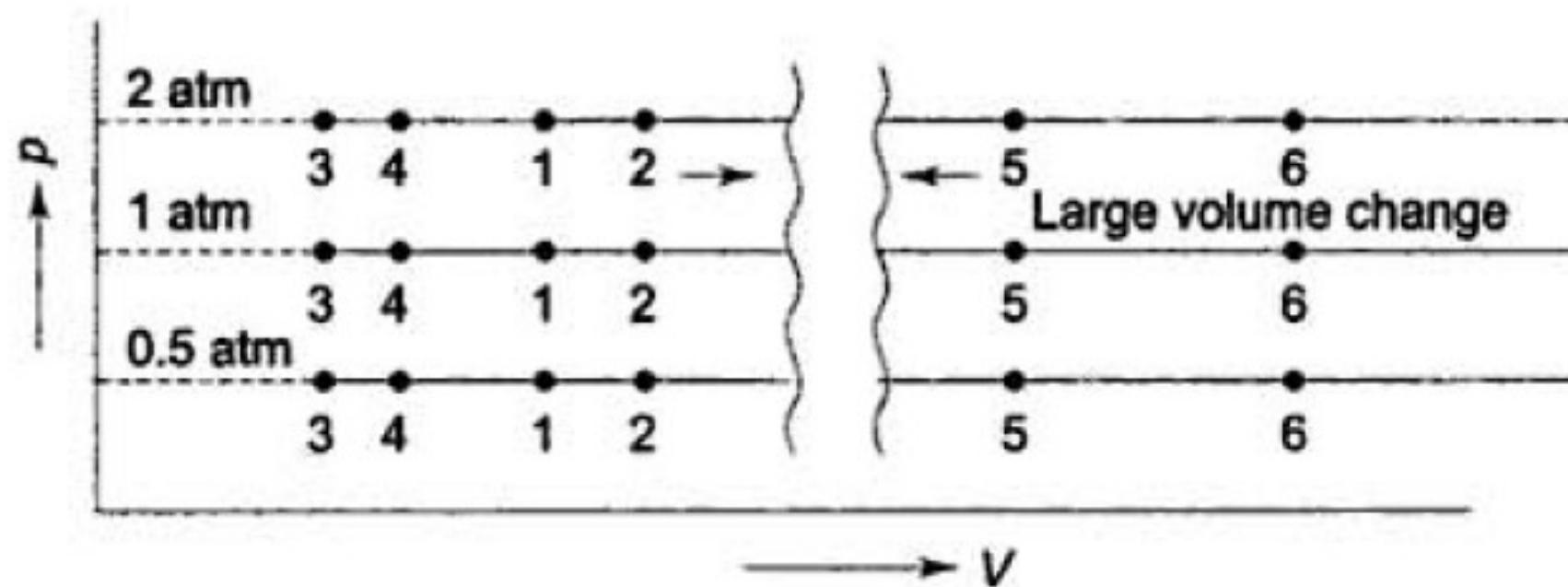


PHASE CHANGE OF A PURE SUBSTANCE



Phase change of water at constant pressure
from liquid to vapour phase.

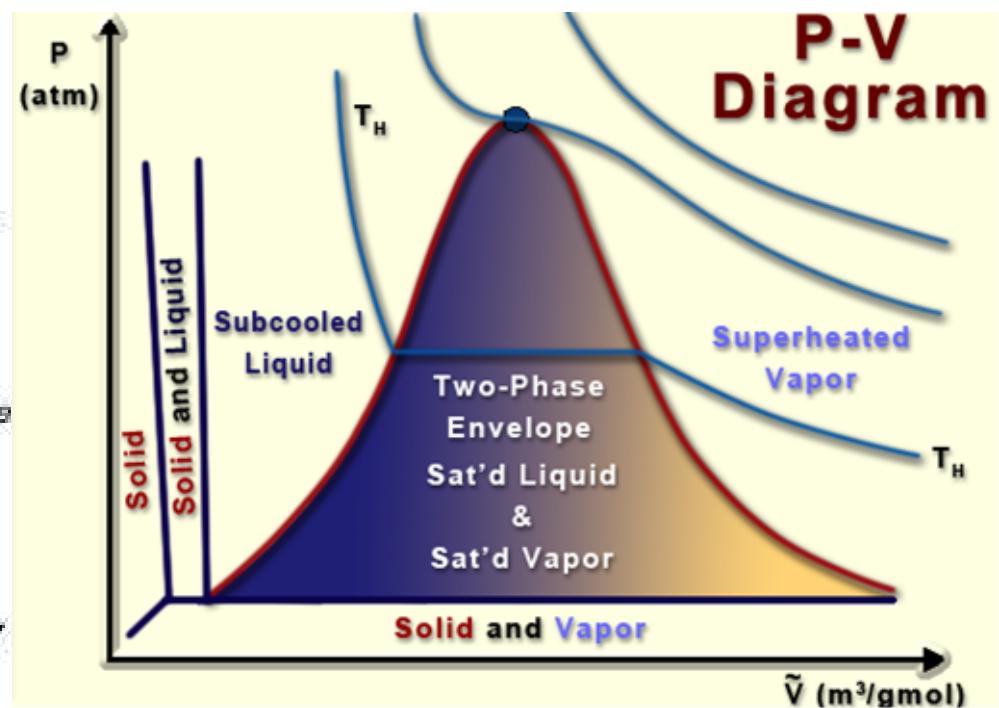
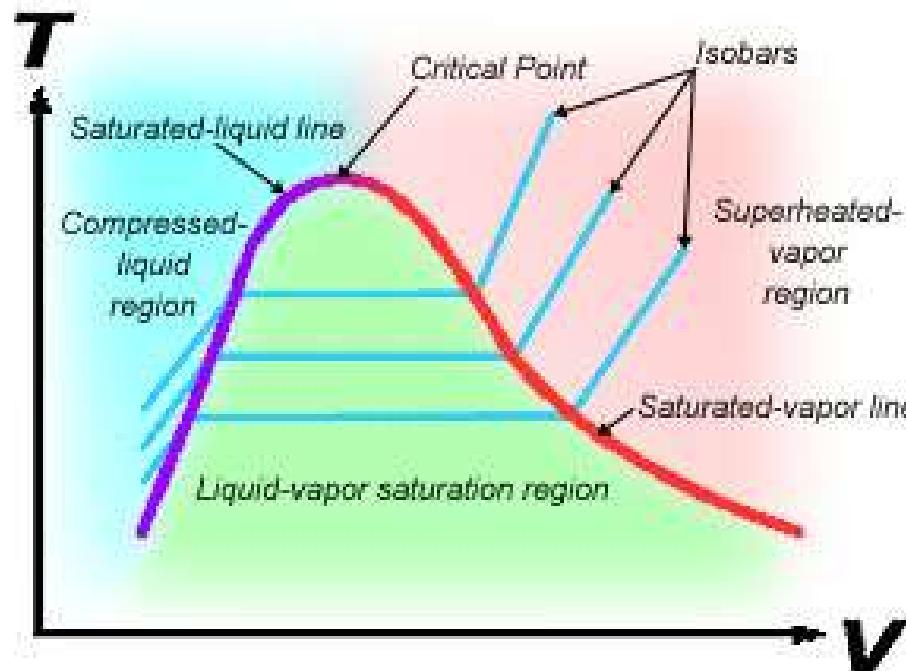




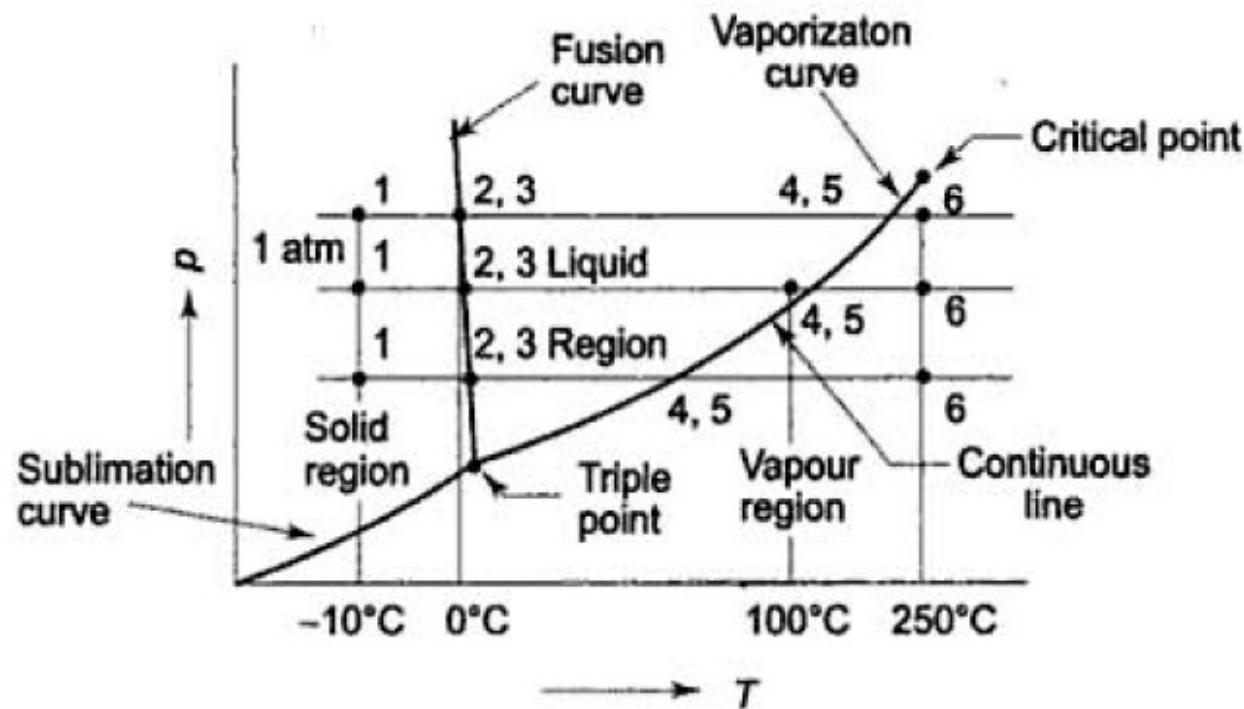
Changes in the volume of water during heating at constant pressure

CRITICAL POINT

	Critical Temperature, °C	Critical Pressure, MPa	Critical Volume, m ³ /kg
Water	374.14	22.09	0.003 155
Carbon dioxide	31.05	7.39	0.002 143
Oxygen	-118.35	5.08	0.002 438
Hydrogen	-239.85	1.30	0.032 192



p-T (Pressure-Temperature) DIAGRAM FOR A PURE SUBSTANCE



The triple point of pure water is at 0.01°C (273.16K , 32.01°F) and 4.58 mm (611.2Pa) of mercury and is used to calibrate thermometers.

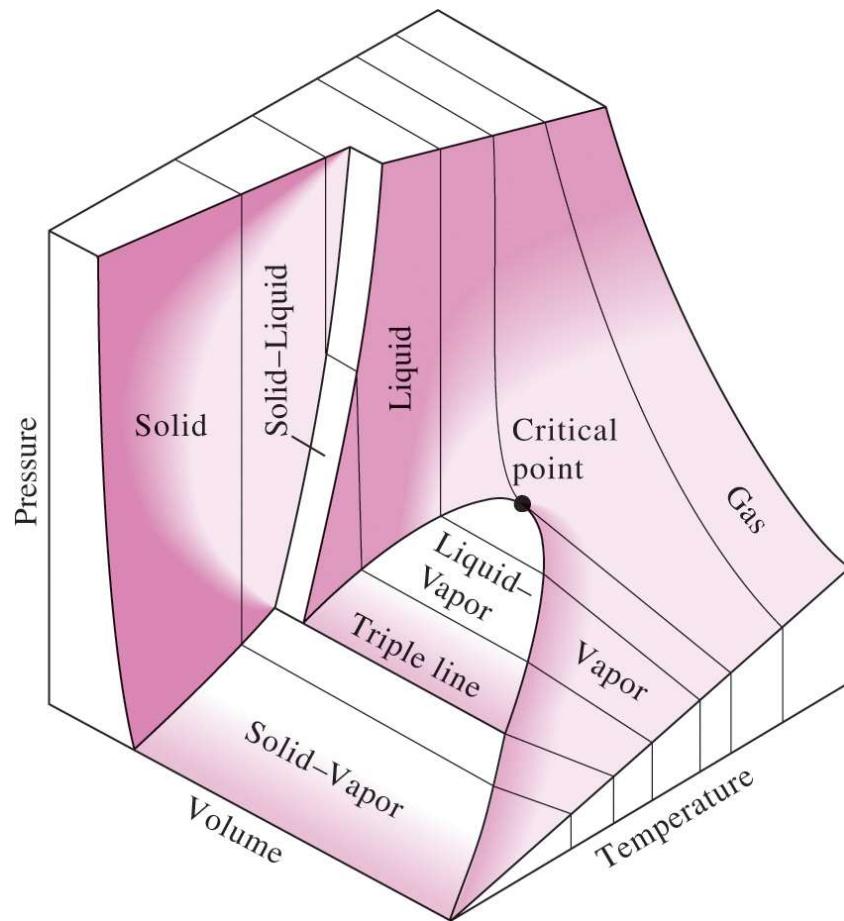


At low pressures (below the triple-point value), solids evaporate without melting first (*sublimation*).

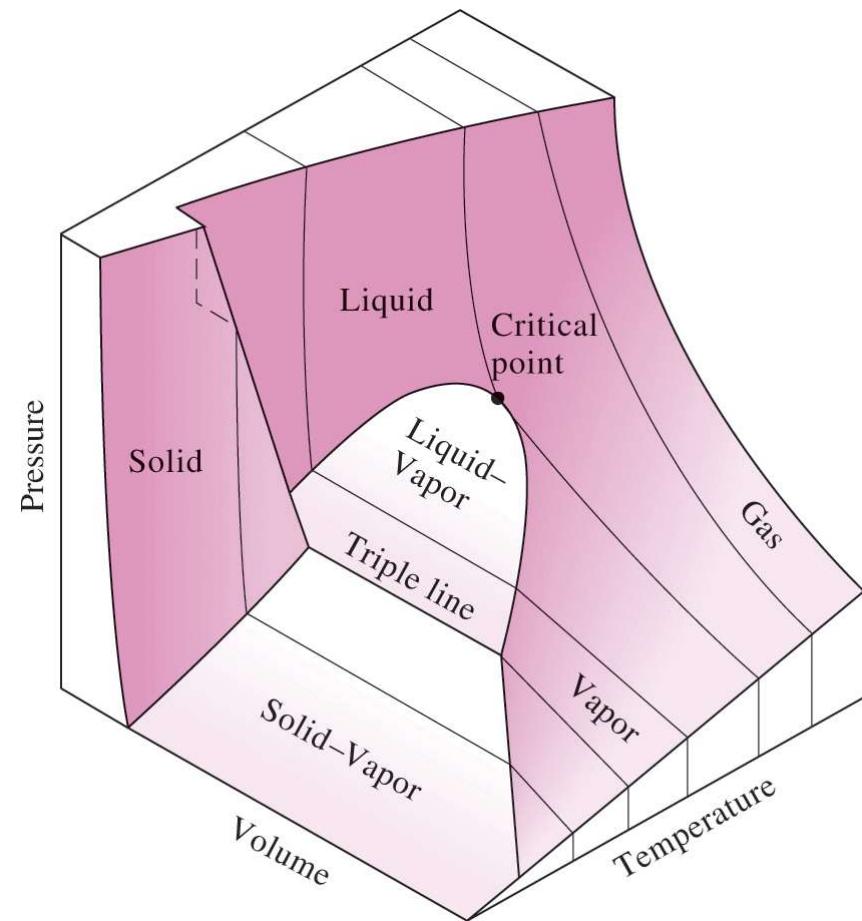


At triple-point pressure and temperature, a substance exists in three phases in equilibrium.

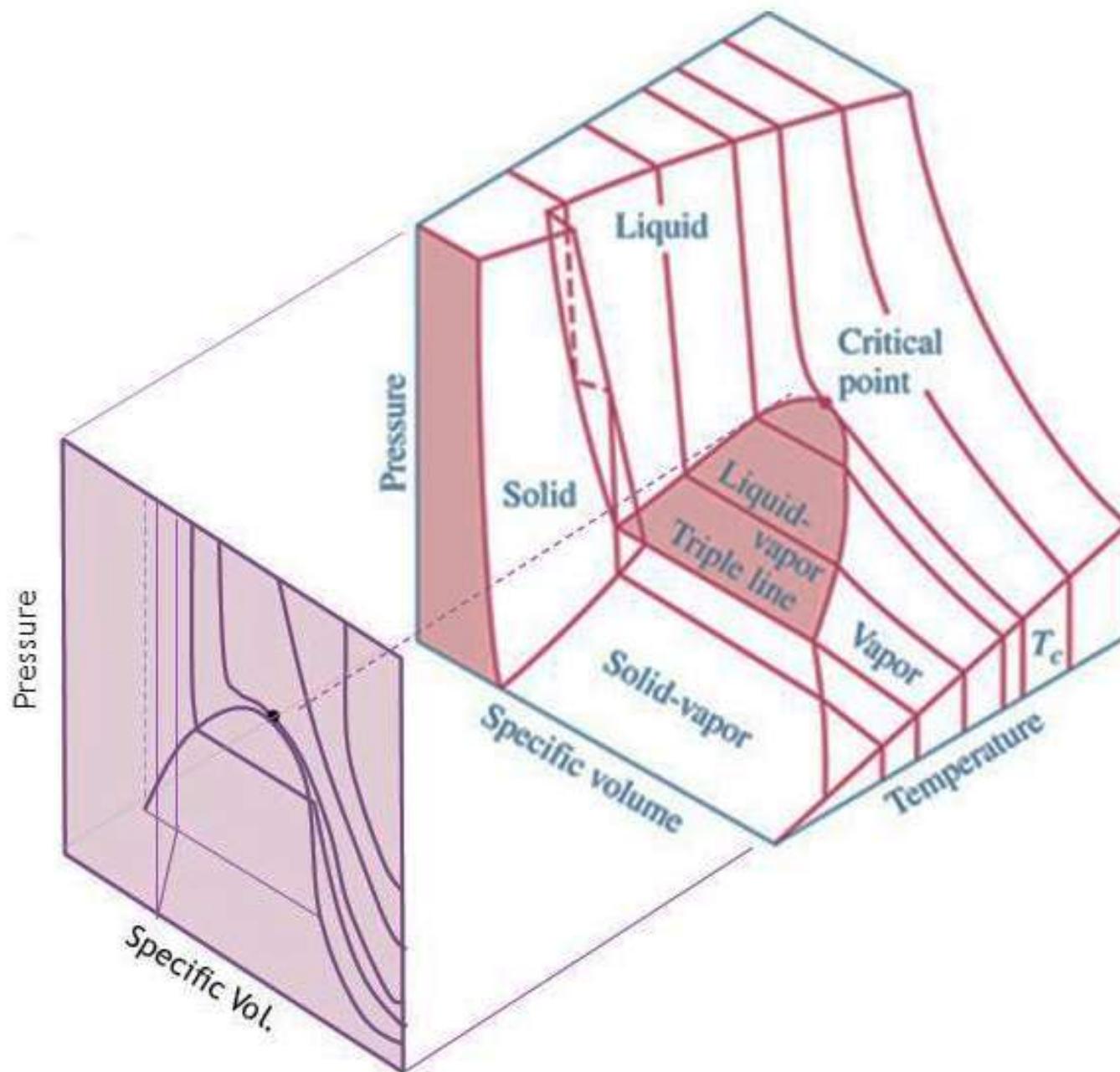
p-V-T (Pressure-Volume-Temperature) SURFACE

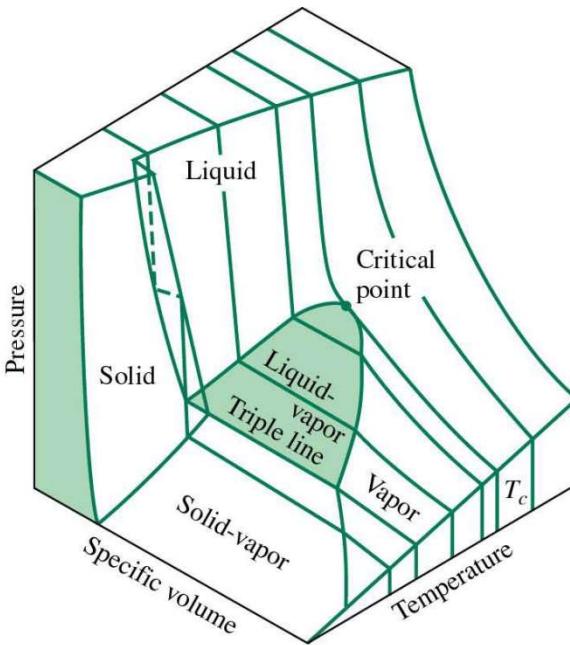


P-v-T surface of a substance
that contracts on freezing.

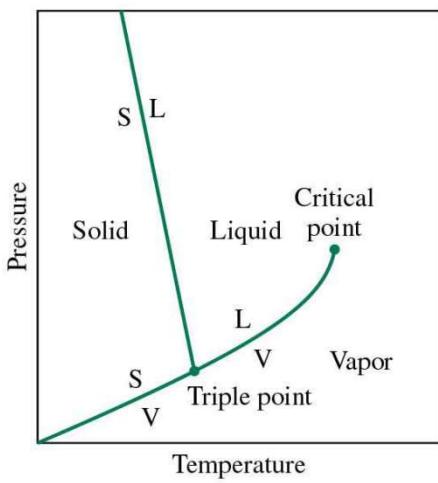


P-v-T surface of a substance that
expands on freezing (like water).

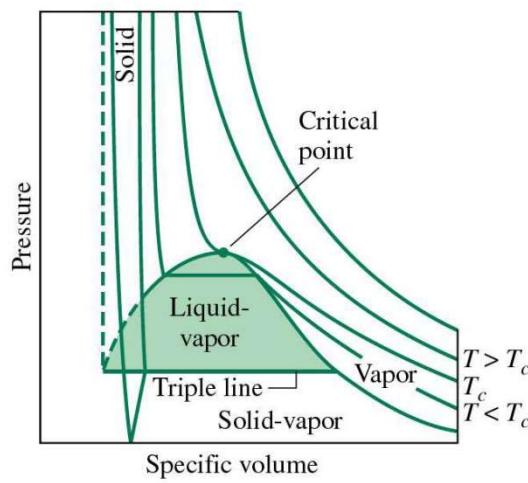




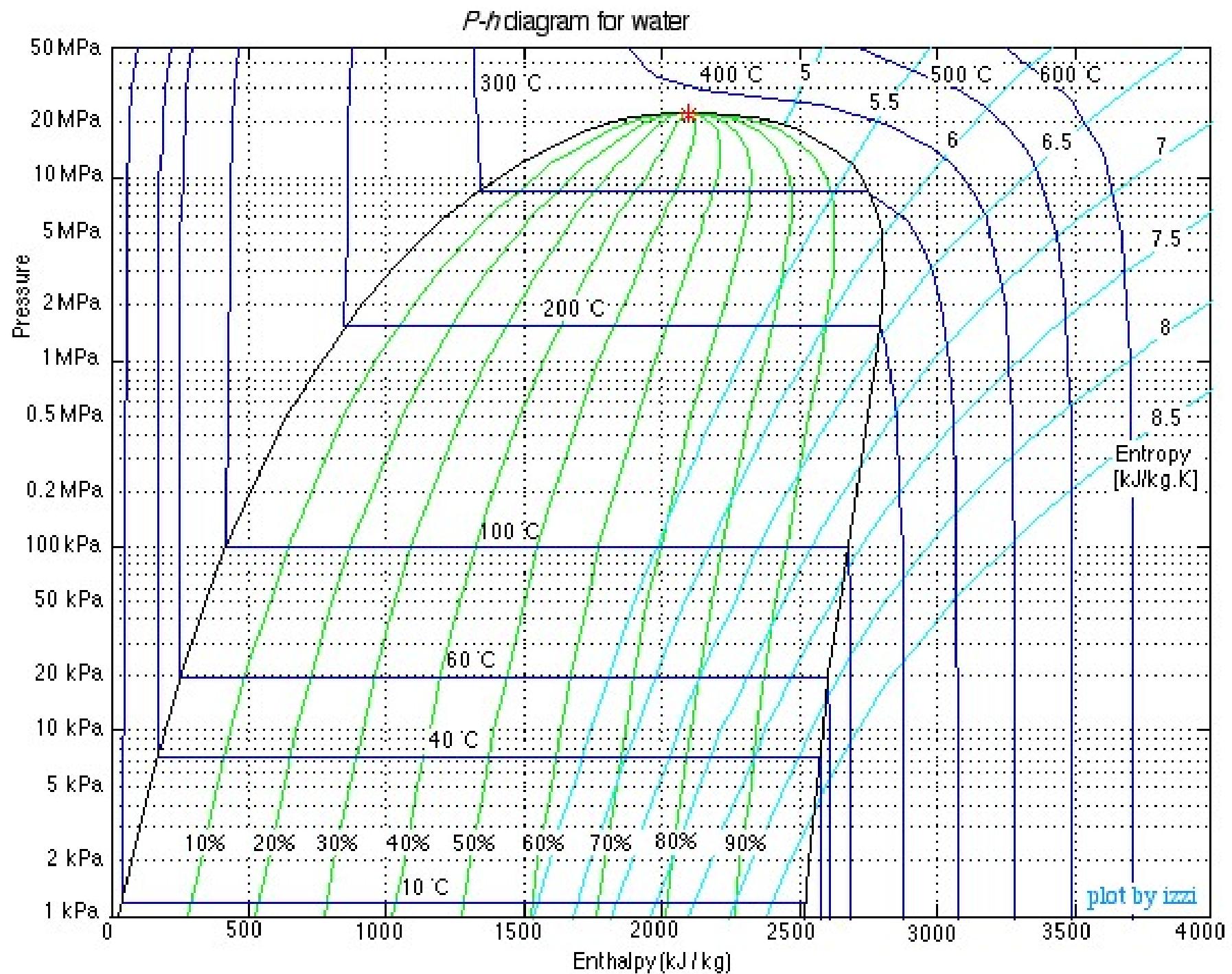
(a)



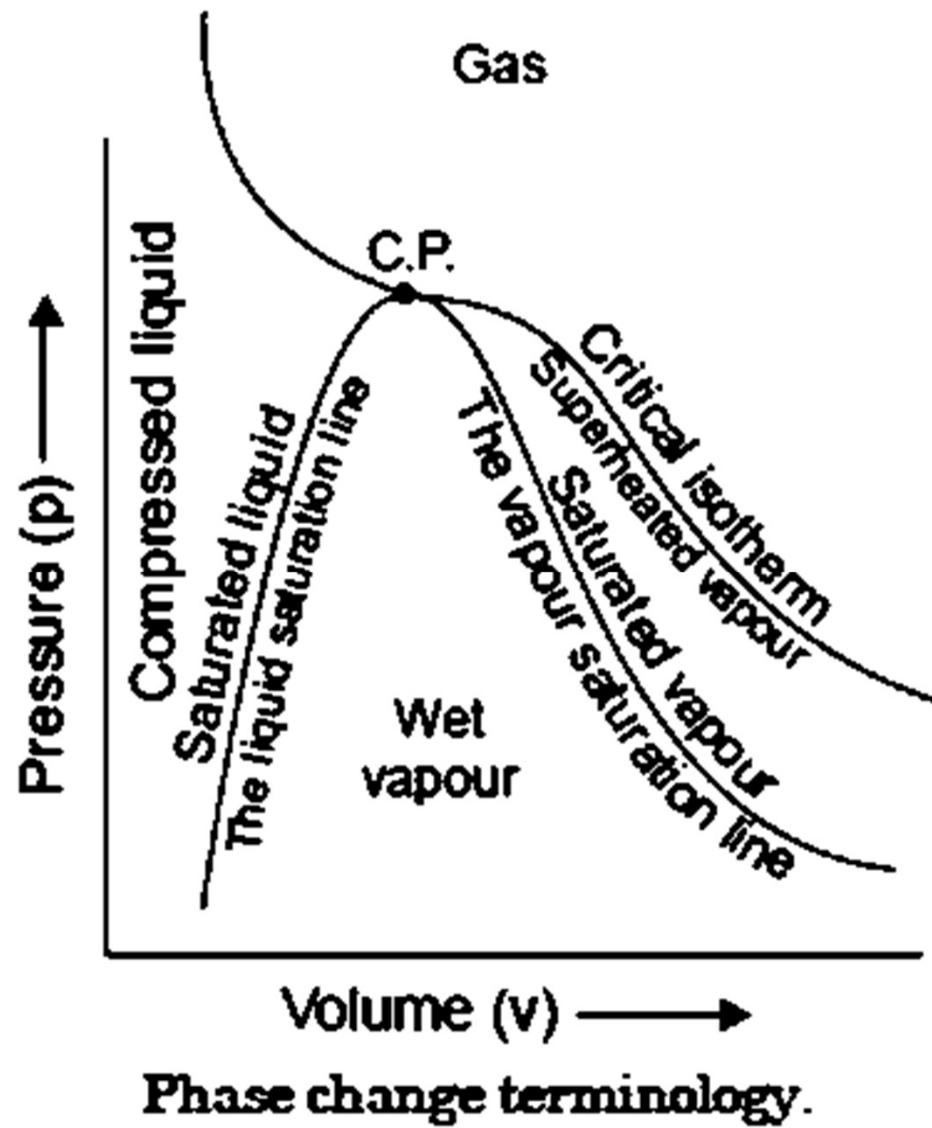
(b)



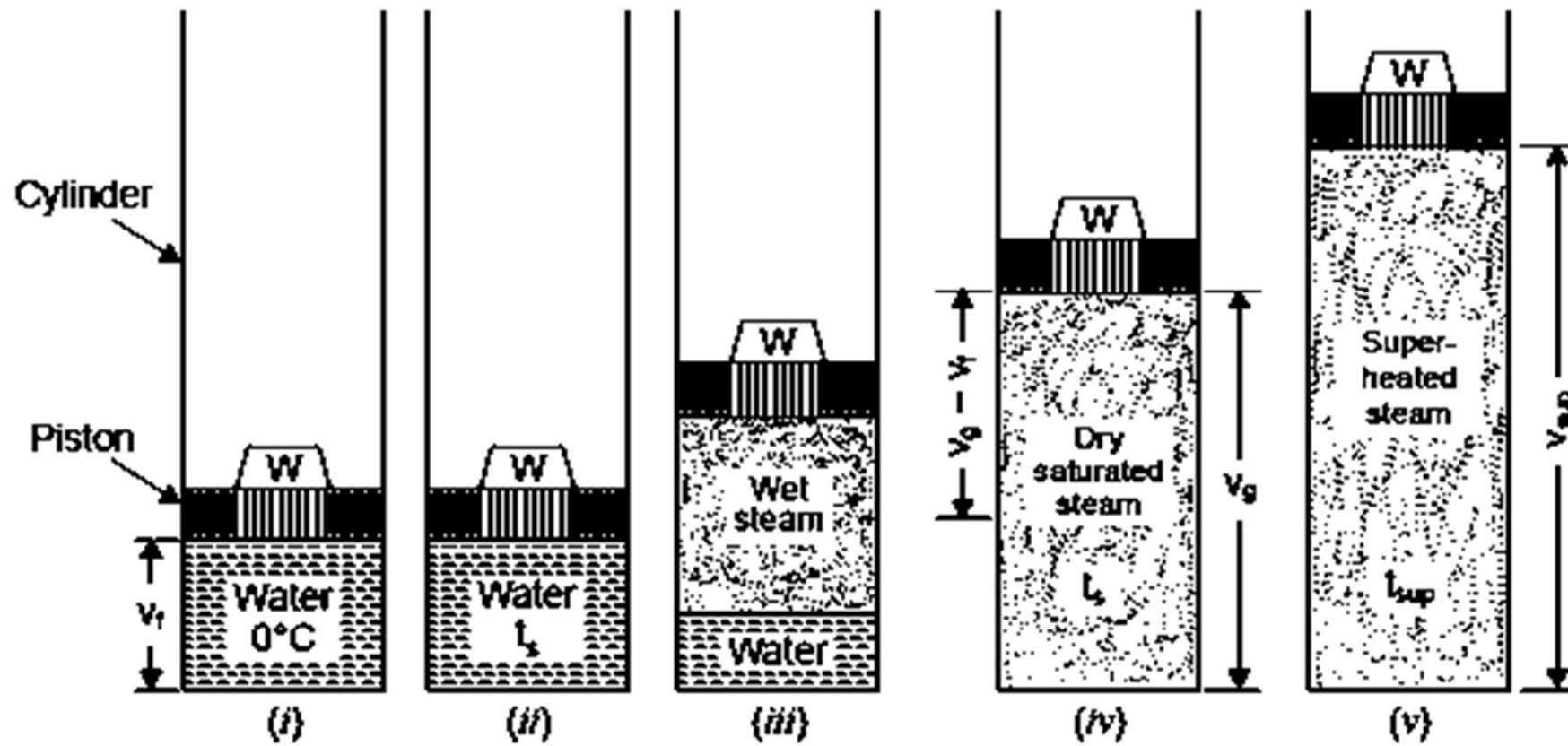
(c)



Saturated liquid
Saturated vapour
Saturation temperature
Saturation pressure
Dry (saturated) vapour
Wet vapour
Superheated vapour
Degree of superheat



Phase change terminology.



t_s = Saturation temp.

t_{sup} = Temperature of superheated steam

v_f = Volume of water

v_g = Volume of dry and saturated steam

v_{sup} = Volume of superheated steam

Formation of steam.

Advantages of super heated steam

- heat content and hence its capacity to do work is increased without having to increase its pressure
- done in a super heater which obtains its heat from waste furnace gases which would have otherwise passed uselessly up the chimney.
- increase in thermal efficiency
- heat losses due to condensation of steam on cylinder walls etc. are avoided to a great extent

Sensible heat of water (h_f)

Latent heat or hidden heat (h_{fg}).

Dryness fraction (x).

$$x = \frac{m_s}{m_s + m_w}$$

Total heat or enthalpy of wet steam (h).

$$h = h_f + xh_{fg}$$

Superheated steam

$$h_{sup} = h_f + h_{fg} + c_{ps} (T_{sup} - T_s)$$

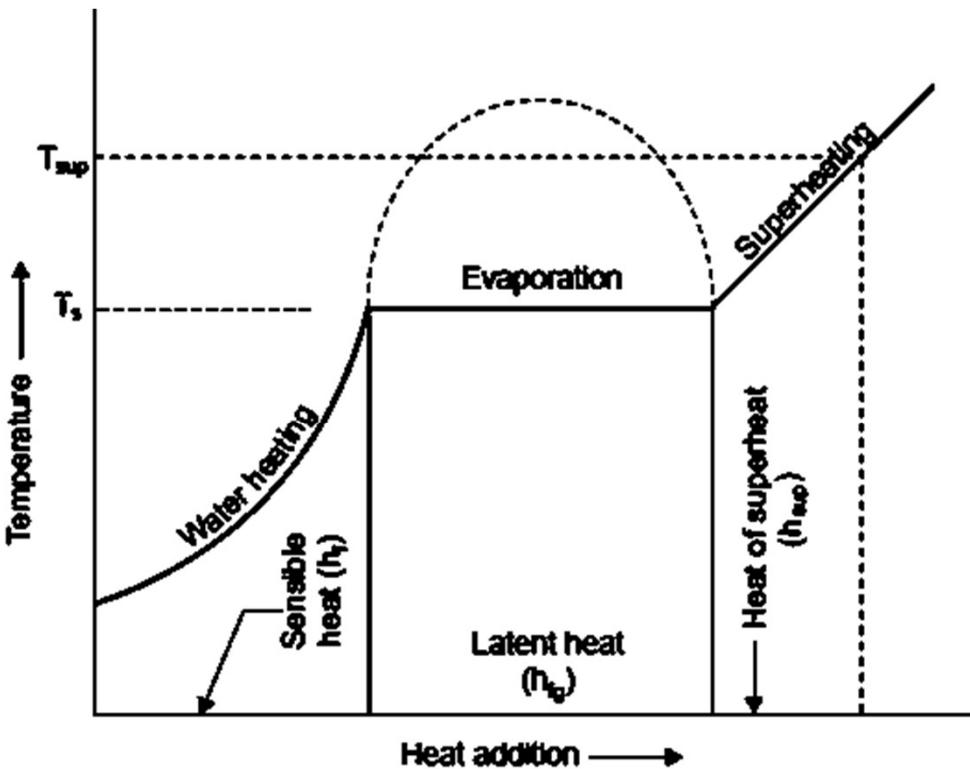
Volume of wet and dry steam

$$xv_g + (1 - x)v_f$$

$$\frac{p \cdot v_g}{T_s} = \frac{p \cdot v_{sup}}{T_{sup}}$$

Volume of superheated steam.

$$v_{sup} = \frac{v_g T_{sup}}{T_s}$$



Graphical representation of formation of steam.

STEAM TABLES

Absolute pressure bar, p	Temperature °C t_s	Specific enthalpy kJ/kg			Specific entropy kJ/kg K			Specific volume m ³ /kg	
		h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	v_f	v_g
1.0	99.6	417.5	2257.9	2675.4	1.3027	6.0571	7.3598	0.001043	1.6934
50.0	263.9	1154.9	1639.7	2794.2	2.9206	3.0529	5.9735	0.001286	0.00394
100.0	311.1	1408.0	1319.7	2727.7	3.3605	2.2593	5.6198	0.001452	0.01811

External work done during evaporation = $p(v_g - v_f)$
for wet steam = pxv_g

Internal latent heat = $h_{fg} - pv_g$

Internal energy of steam

$$u = h - pv_g$$

wet steam with dryness fraction 'x'

$$u = h - pxv_g$$

superheated to a volume of v_{sup} per kg.

$$h_{sup} = h_f + h_{fg} + c_{ps} (T_{sup} - T_s)$$

$$u = h_{sup} - p \cdot v_{sup}$$

Entropy of water

$$ds = \frac{dQ}{T} = c_{pw} \cdot \frac{dT}{T}$$

$$\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} c_{pw} \frac{dT}{T}$$

$$s_2 - s_1 = c_{pw} \log_e \frac{T_2}{T_1}$$

Entropy of evaporation

$$Q = h_{fg}$$

$$s_{evap.} = \frac{h_{fg}}{T_s}$$

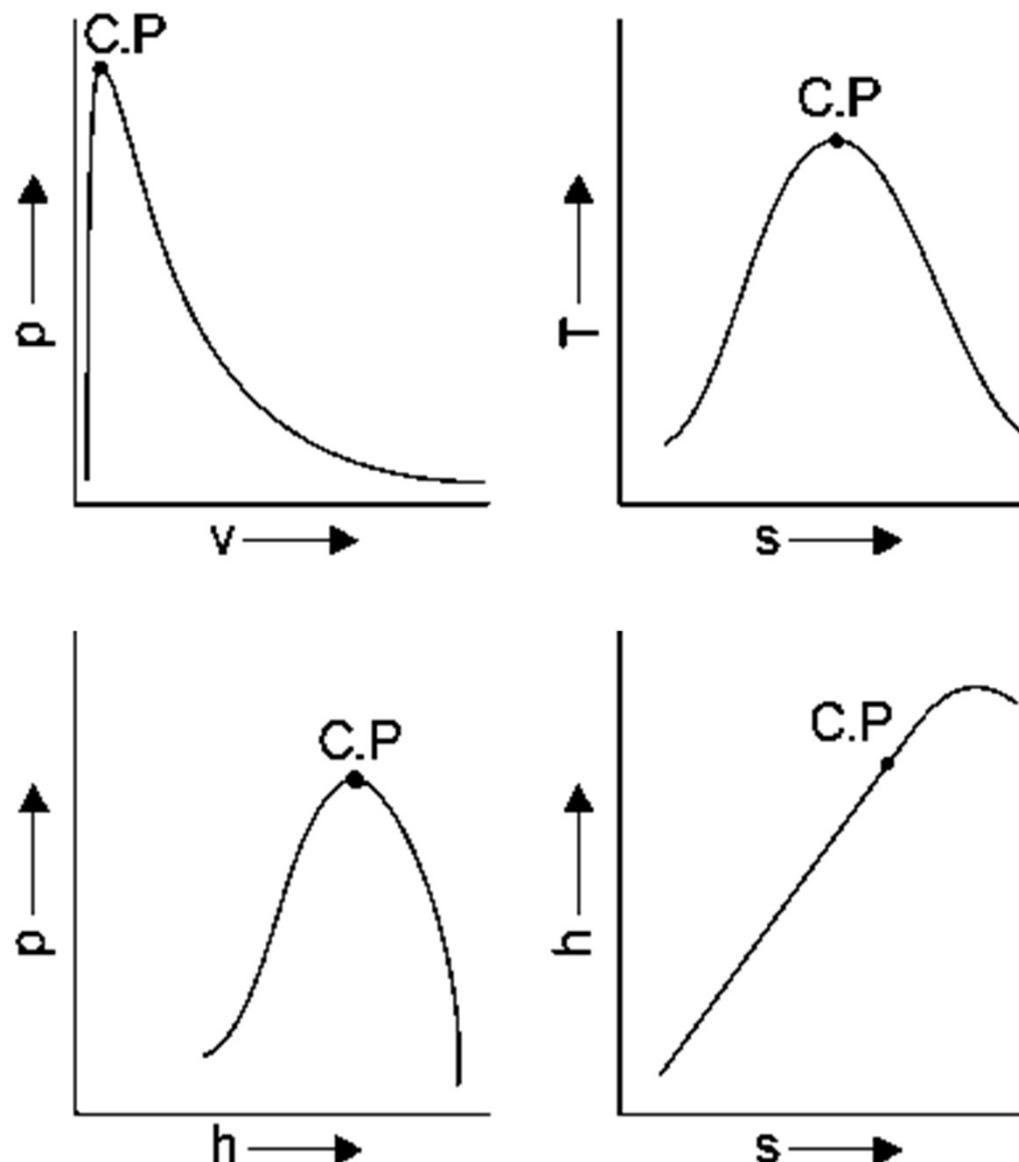
Entropy of wet steam

$$s_{wet} = s_f + \frac{xh_{fg}}{T_s}$$

Entropy of superheated steam

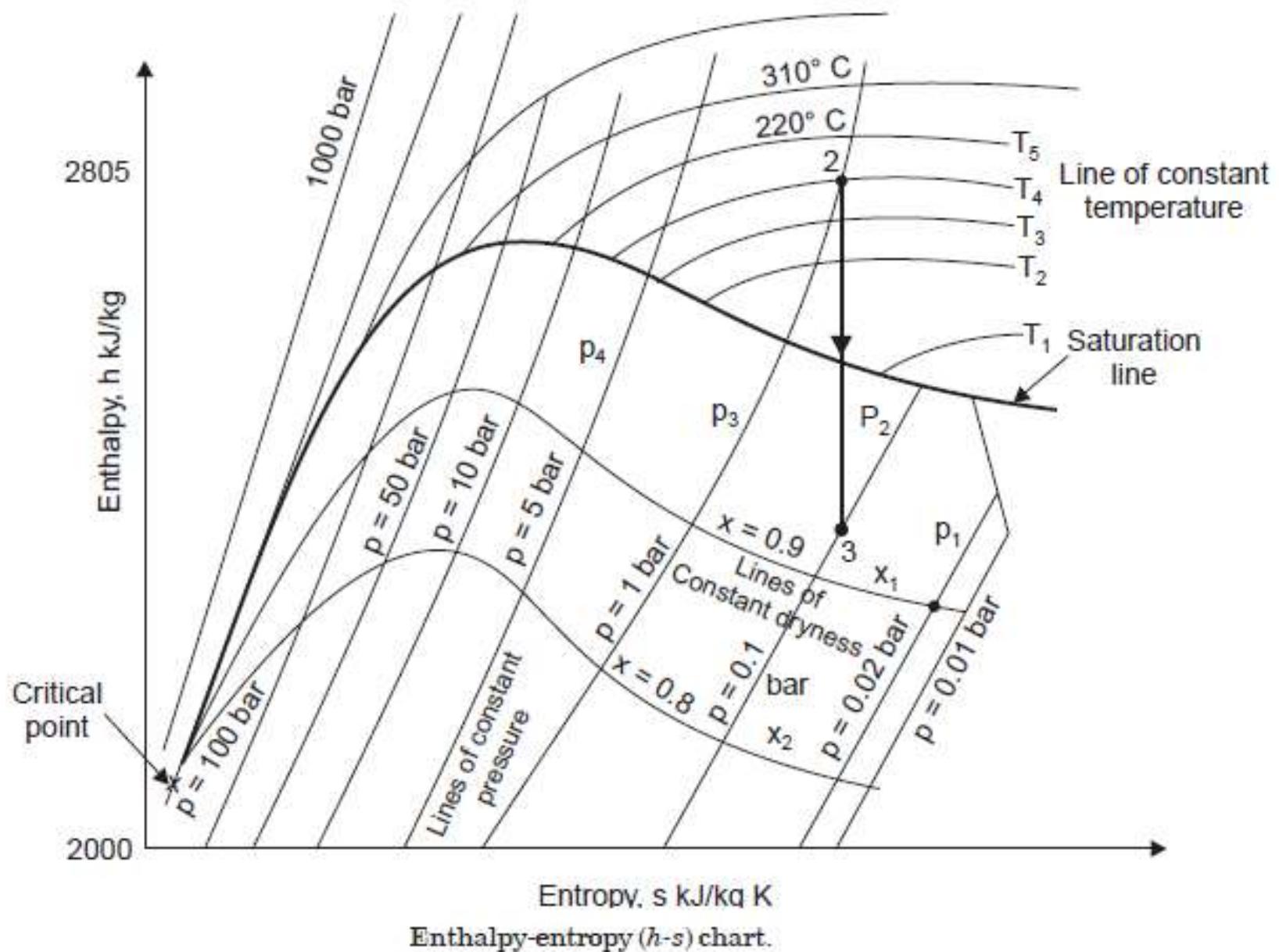
s_{sup} = Entropy of dry saturated steam + change of entropy during superheating

$$= s_f + \frac{h_{fg}}{T_s} + c_{ps} \log_e \left(\frac{T_{sup}}{T_s} \right) = s_g + c_{ps} \log_e \left(\frac{T_{sup}}{T_s} \right)$$



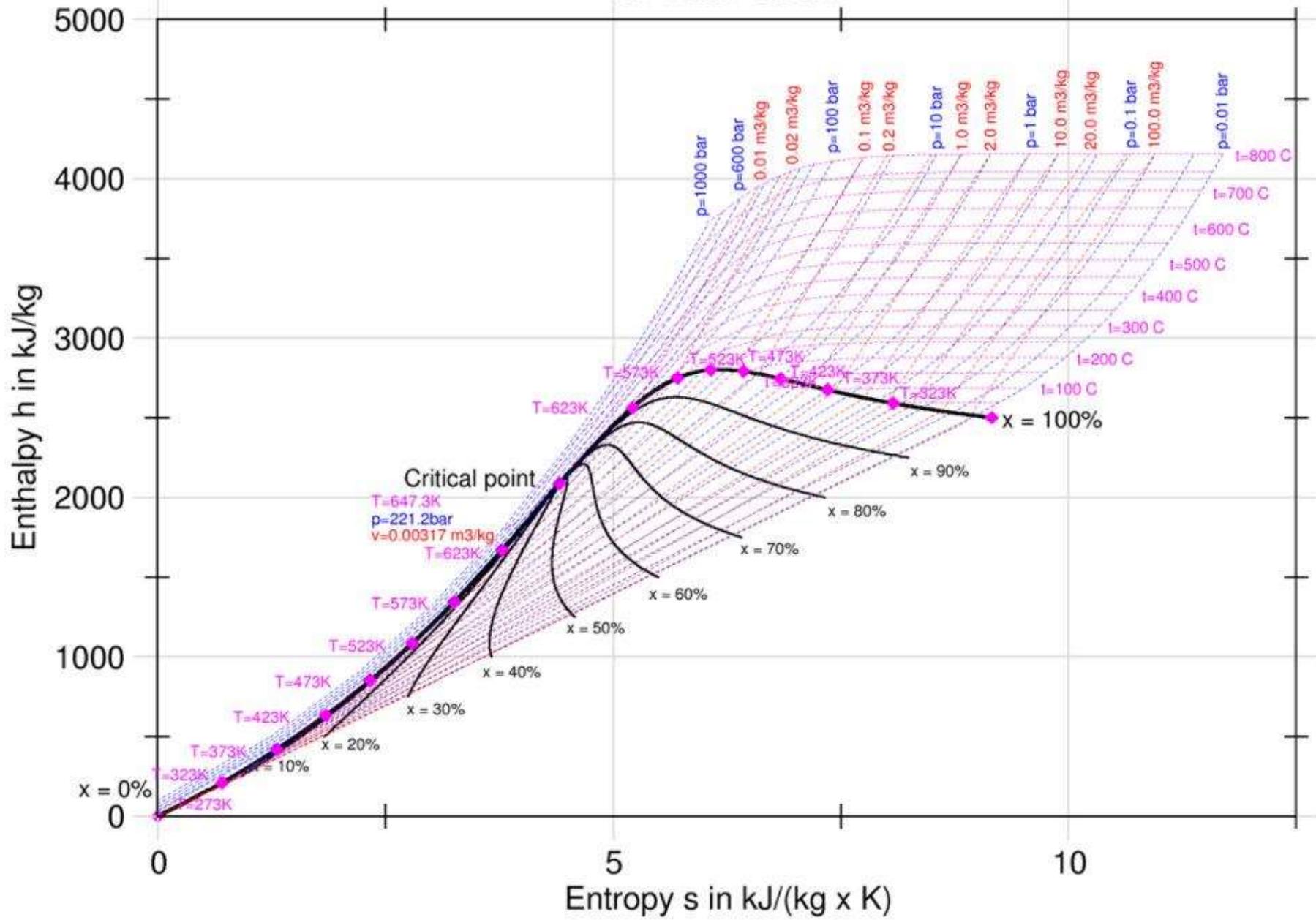
C.P. = Critical point

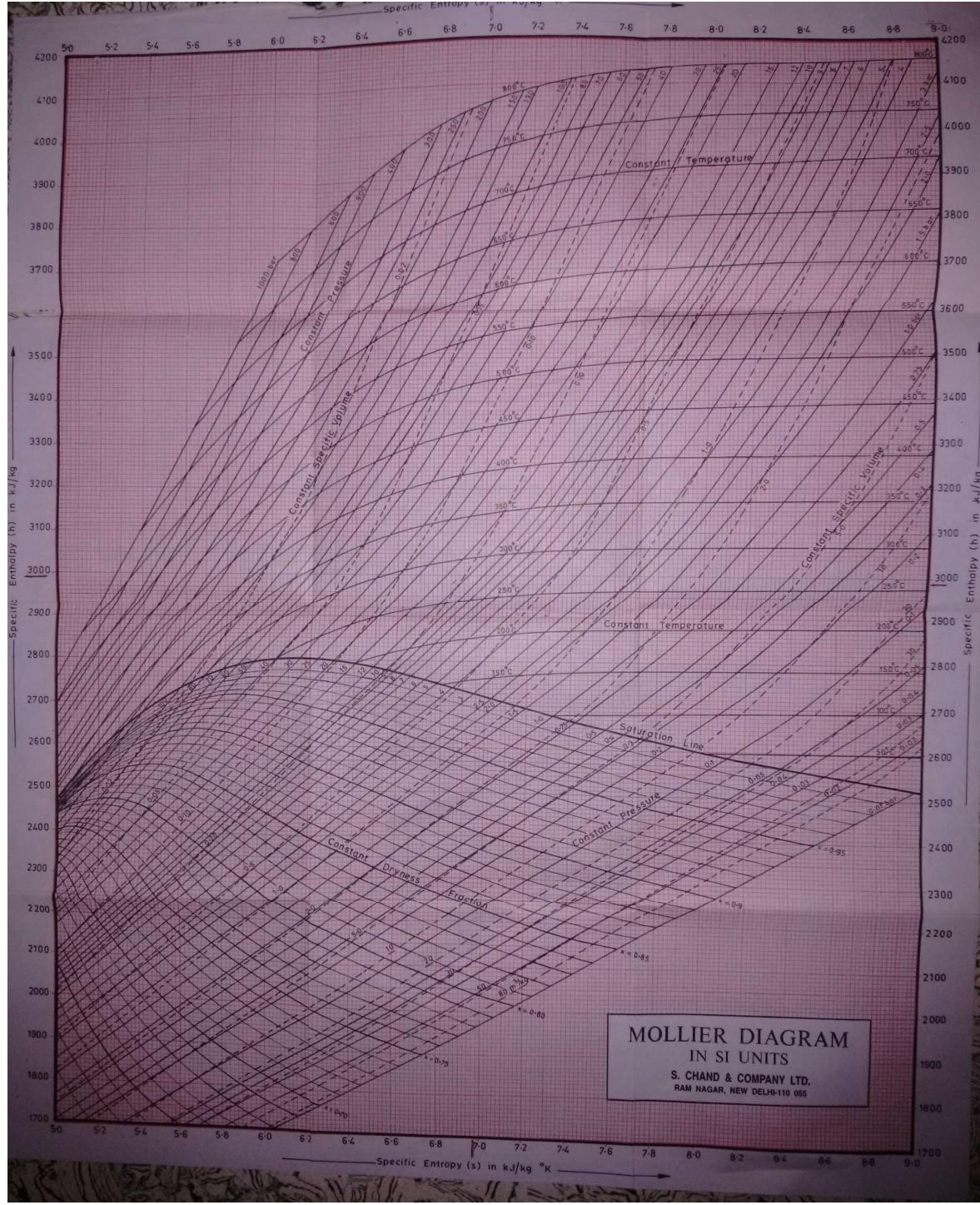
ENTHALPY-ENTROPY (h-s) CHART OR MOLLIER DIAGRAM



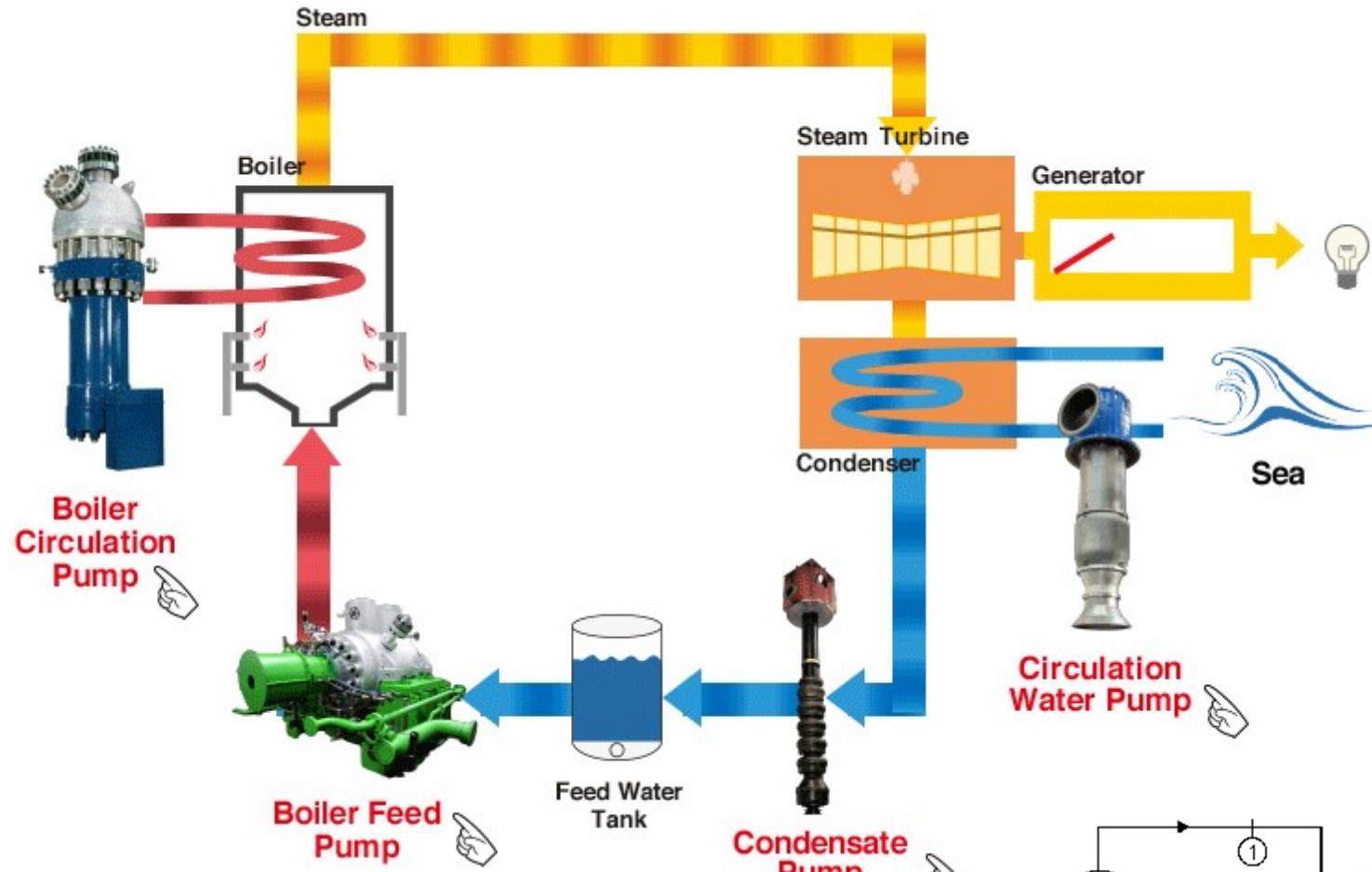
Mollier-h, s Diagram

for Water Steam

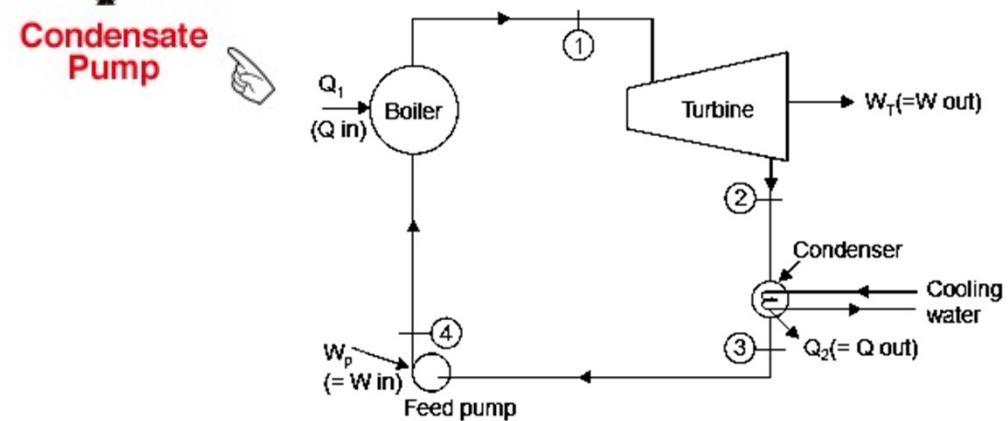




A quantity of steam at 10 bar and 0.85 dryness occupies 0.15 m^3 . Determine the heat supplied to raise the temperature of the steam to 300°C at constant pressure and the external work done. Take specific heat of superheated steam as 2.2 kJ/kg K .



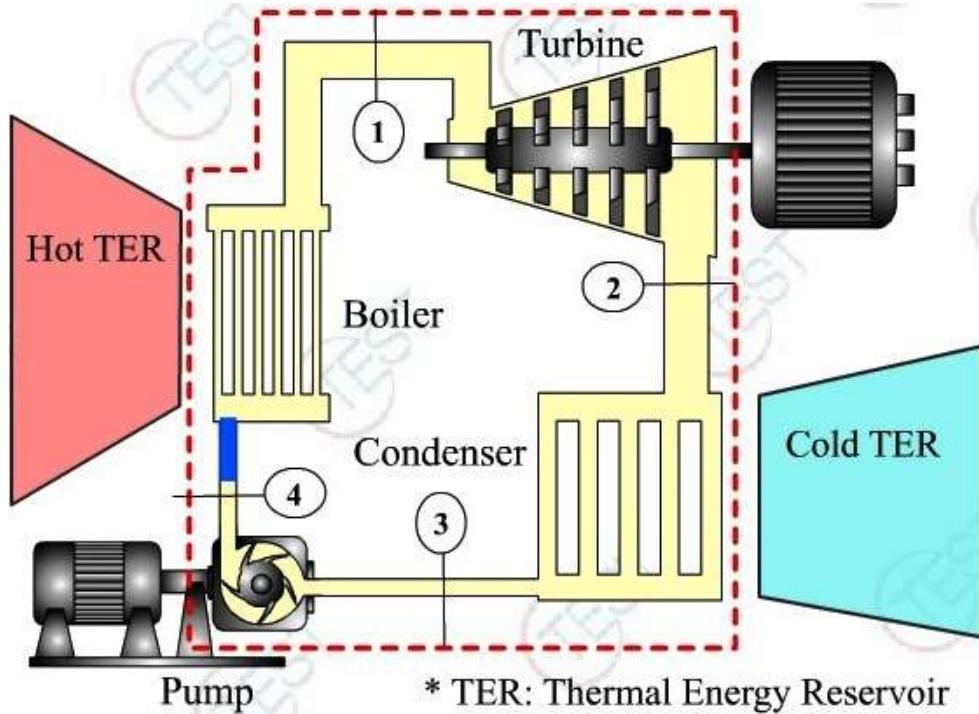
RANKINE CYCLE





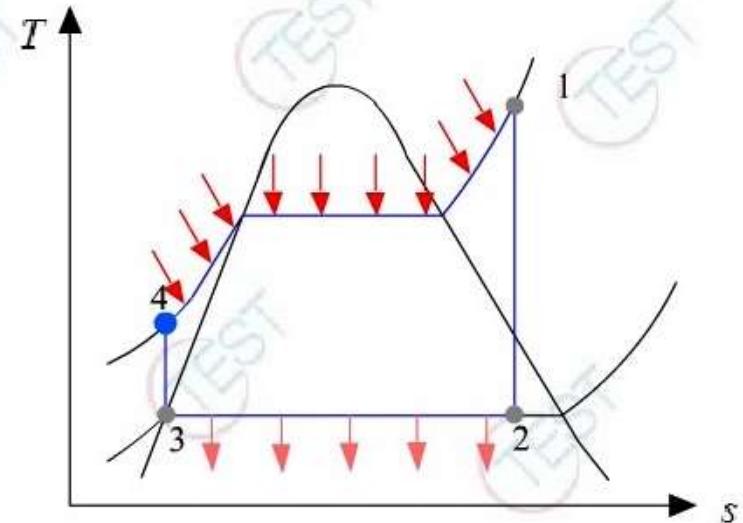






Pump

* TER: Thermal Energy Reservoir



Ideal Rankine Cycle: Energy Inventory

$$\text{Turbine: } \dot{W}_T = \dot{W}_{\text{ext}} = \dot{m}(j_1 - j_2) \approx \dot{m}(h_1 - h_2); \quad [\text{kW}]$$

$$\text{Condenser: } \dot{Q}_{\text{out}} = -\dot{Q} = \dot{m}(j_2 - j_3) \approx \dot{m}(h_2 - h_3); \quad [\text{kW}]$$

$$\begin{aligned} \text{Pump: } \dot{W}_P &= -\dot{W}_{\text{ext}} = \dot{m}(j_4 - j_3) \approx \dot{m}(h_4 - h_3); \\ &= \dot{m}(u_4 - u_3)^0 + \dot{m}v_{f@T_3}(p_4 - p_3) \quad [\text{kW}] \end{aligned}$$

$$\text{Boiler: } \dot{Q}_{\text{in}} = \dot{Q} = \dot{m}(j_1 - j_4) \approx \dot{m}(h_1 - h_4); \quad [\text{kW}]$$

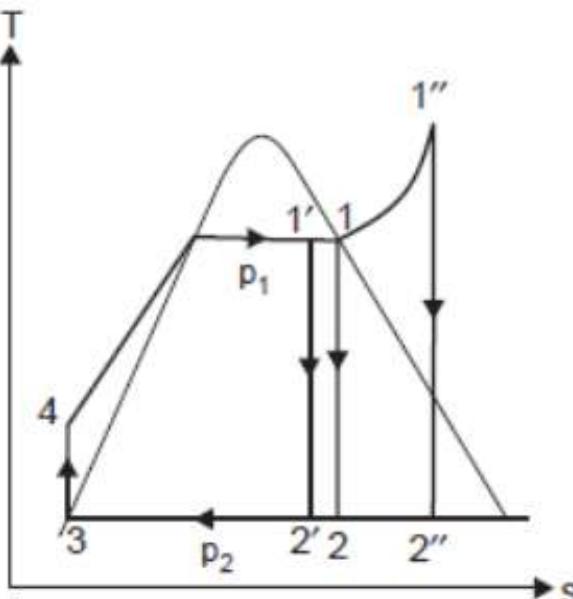
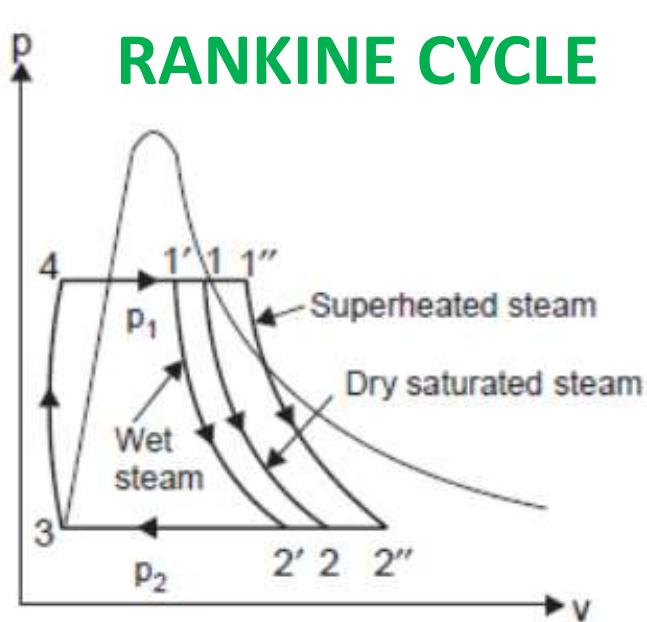
$$\dot{W}_{\text{net}} = \dot{W}_T - \dot{W}_P = \dot{Q}_{\text{net}} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}};$$

$$\dot{Q}_{\text{net}} = \dot{m} \left[\int_4^1 T ds - \int_3^2 T ds \right]; \quad [\text{kW}]$$

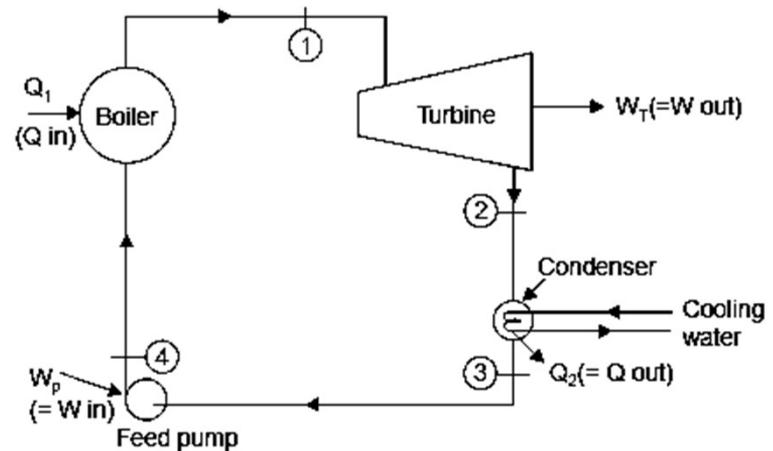
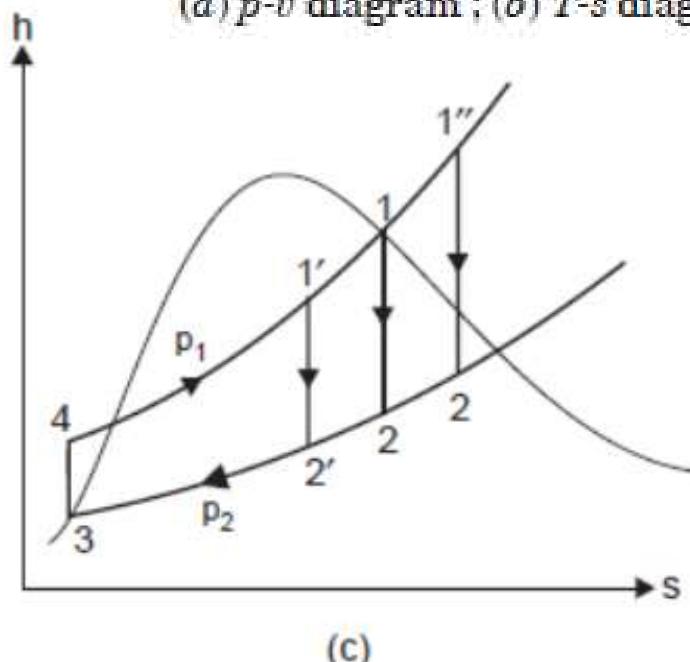
$$\eta_{\text{th,Rankine}} \equiv \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_T - \dot{W}_P}{\dot{Q}_{\text{in}}}$$

$$= \frac{\dot{Q}_{\text{net}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}};$$

$$\text{BWR} \equiv \frac{\dot{W}_P}{\dot{W}_T};$$



(a) p - v diagram ; (b) T - s diagram ; (c) h - s diagram for Rankine cycle.



Applying steady flow energy equation (S.F.E.E.) to boiler, turbine, condenser and pump :

(i) For boiler (as control volume),

$$h_{f4} + Q_1 = h_1$$

$$\therefore Q_1 = h_1 - h_{f4}$$

(ii) For turbine (as control volume),

$$h_1 = W_T + h_2, \text{ where } W_T = \text{turbine work}$$

$$\therefore W_T = h_1 - h_2$$

(iii) For condenser, we get

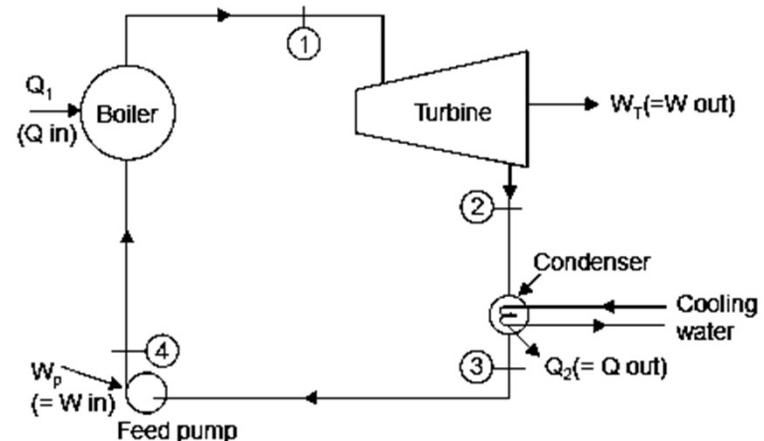
$$h_2 = Q_2 + h_{f3}$$

$$\therefore Q_2 = h_2 - h_{f3}$$

(iv) For the feed pump,

$$h_{f3} + W_p = h_{f4}, \text{ where, } W_p = \text{Pump work}$$

$$\therefore W_p = h_{f4} - h_{f3}$$



$$\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$= \frac{(h_1 - h_2) - (h_{f_4} - h_{f_3})}{(h_1 - h_{f_4})}$$

$$Td\sigma = dh - vdp$$

$$d\sigma = 0$$

$$dh = vdp$$

$$h_{f_4} - h_{f_3} = v_a (p_1 - p_2) \times 10^3 \text{ J/kg}$$

$$h = u + pv$$

$$dh = du + pdv + vdp$$

$$dq = du + pdv$$

$$du = dq - pdv$$

$$dh = dq - pdv + pdv + vdp$$

$$dh = vdp$$

$$\eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_4}}$$

$$\text{work ratio} = \frac{\text{net work}}{\text{turbine work}}$$

$$\text{Specific steam consumption} = \frac{\text{steam flow rate}}{\text{power output}}$$

$$\text{Specific steam consumption} = \frac{1}{W_{\text{net}}} \frac{\text{kg}}{\text{kJ}}$$

$$= \frac{1}{W_{\text{net}}} \frac{\text{kg}}{\text{kJ}} = \frac{3600}{W_{\text{net}}} \frac{\text{kg}}{\text{kWh}}$$

$$\text{Heat rate} = \frac{3600 Q_1}{W_t - W_c} = \frac{3600}{\eta_{\text{th}}} \frac{\text{kJ}}{\text{kWh}}$$

Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa.

- (i) Find heat supplied in the boiler
- (ii) Determine the quality of steam after expansion .
- (iii) What is the power generated by the turbine?
- (iv) What is the heat loss in condenser?
- (v) Estimate the Rankine efficiency considering pump work.
- (vi) Find the work ratio and
- (vii) Find the specific steam consumption

$$h_1 = 3117.5 \text{ kJ/kg} \quad s_1 = 6.747 \text{ kJ/kgK}$$

$$h_f2 = 384.4 \text{ kJ/kg} \quad h_{fg2} = 2278.6 \text{ kJ/kg} \quad s_f2 = 1.213 \text{ kJ/kgK} \quad s_{fg2} = 6.244 \text{ kJ/kgK}$$

$$h_2 = h_f2 + x_2 h_{fg2}$$

$$S_1 = s_2$$

$$= s_f2 + x_2 s_{fg2}$$

$$x_2 =$$

$$h_2 =$$

$$h_3 =$$

$$\Delta h = (h_4 - h_3) = vdp = vf \times (p_b - p_c) = 0.001 \times (3 \times 10^6 - 75 \times 10^3)$$

$$h_4 =$$

$$Q_s =$$

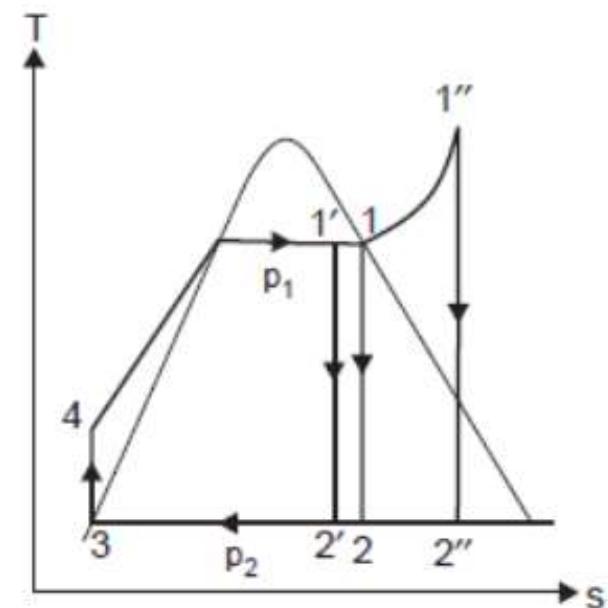
$$Q_r =$$

$$W_T =$$

$$W_p =$$

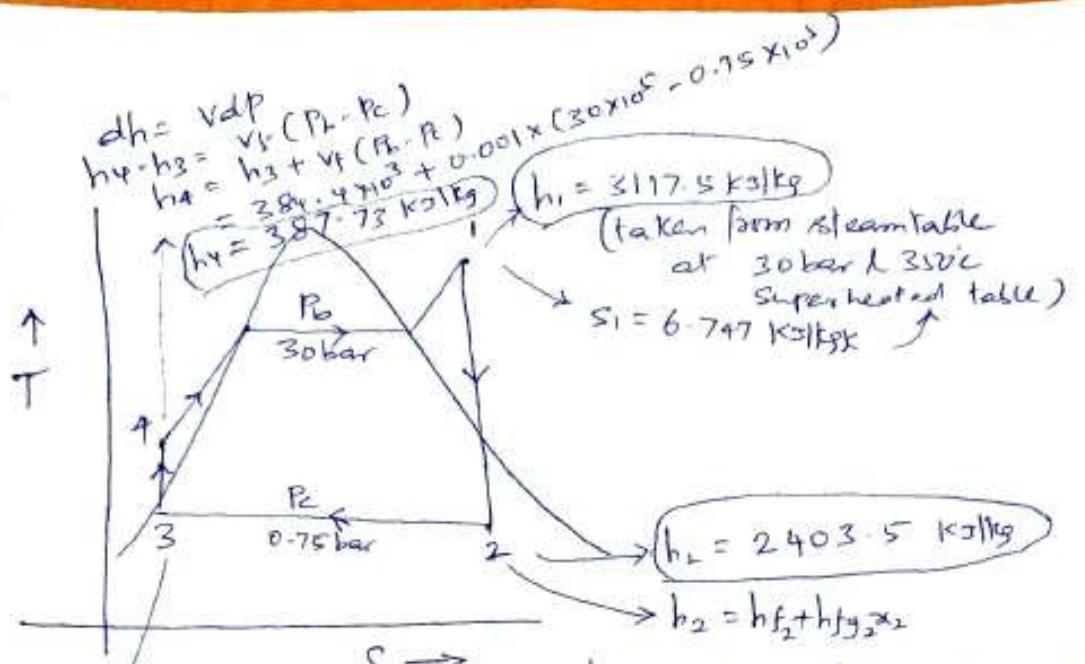
$$W_{net} = W_T - W_p$$

$$\text{Efficiency} = W_{net}/Q_s$$

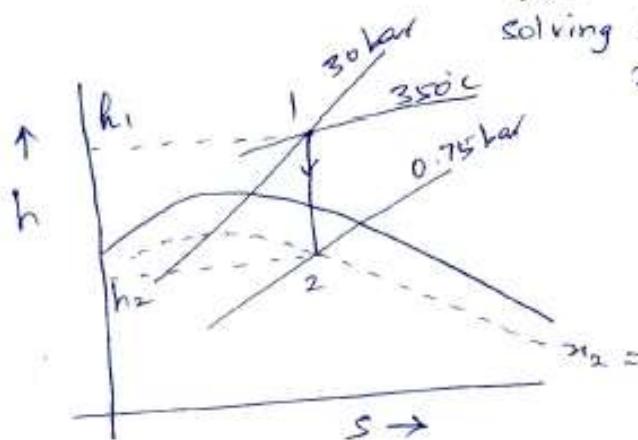


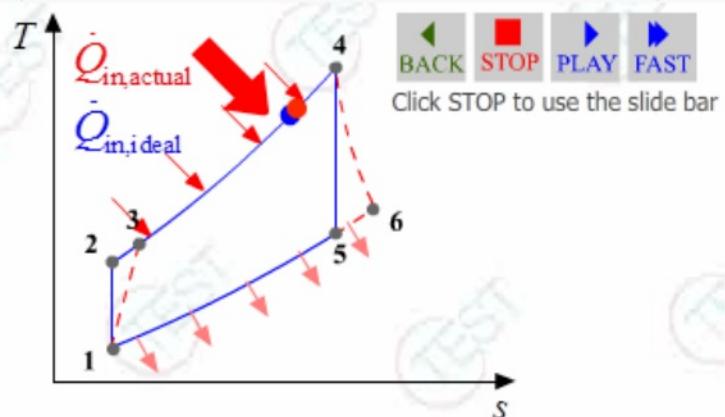
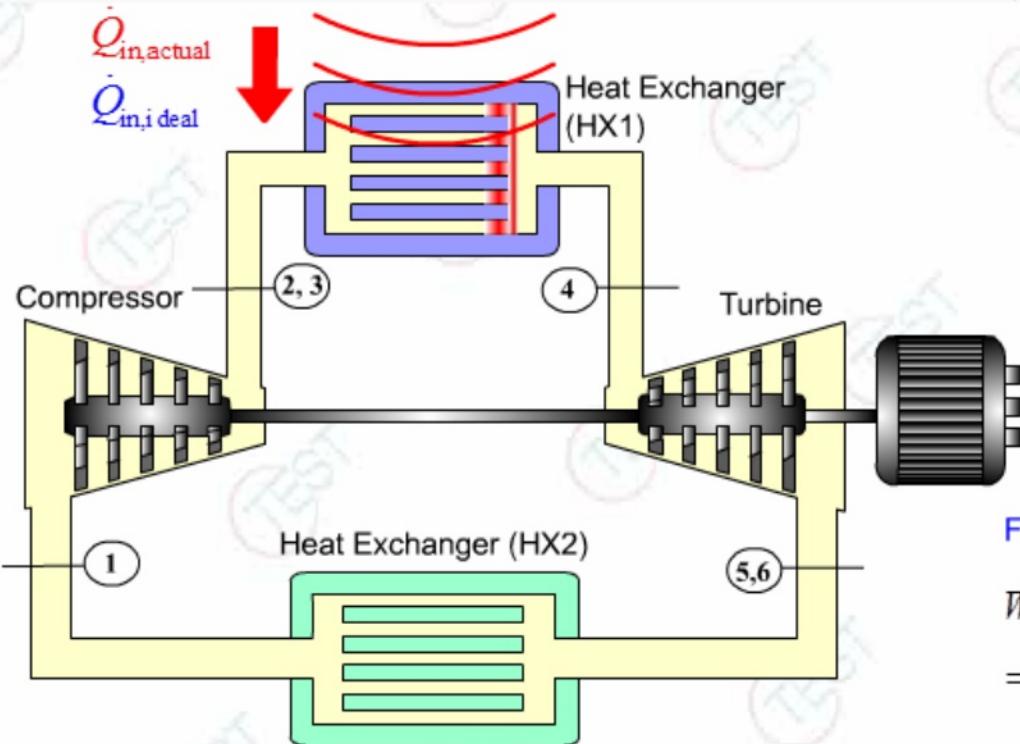
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- (v) Estimate the Rankine efficiency considering pump work.
- (vi) Find the work ratio and
- (vii) Find the specific steam consumption



$h_{f2} = 384.4 \text{ kJ/kg}$
 $h_{fg2} = 2278.6 \text{ kJ/kg}$ taken at 0.75 bar
 To find x_{12}
 $s_1 = s_2$
 $= s_{f2} + x_{12}s_{fg2}$
 $s_{f2} = 1.213 \text{ kJ/kgK}$ taken at
 $s_{fg2} = 6.244 \text{ kJ/kgK}$ 0.75 bar
 Solving $s_1 = s_2$
 $x_{12} = 0.89$





For the actual cycle

$$\begin{aligned}\dot{W}_{\text{net,actual}} &= \dot{W}_T,\text{actual} - \dot{W}_C,\text{actual} \\ &= \dot{Q}_{\text{net,actual}} = \dot{Q}_{\text{in,actual}} - \dot{Q}_{\text{out,actual}}; \quad [\text{kW}]\end{aligned}$$

However, because of entropy generation in the cycle

$$\dot{Q}_{\text{net,actual}} \neq \dot{m} \left[\int_3^4 T ds - \int_1^5 T ds \right]; \quad [\text{kW}]$$

$$\eta_T = \frac{\dot{W}_T,\text{actual}}{\dot{W}_T,\text{ideal}} \cong \frac{h_4 - h_6}{h_4 - h_5}; \quad \eta_C = \frac{\dot{W}_C,\text{ideal}}{\dot{W}_C,\text{actual}} \cong \frac{h_2 - h_1}{h_3 - h_1}$$

$$\eta_{\text{th,actual}} \equiv \frac{\dot{W}_{\text{net,actual}}}{\dot{Q}_{\text{in,actual}}} = 1 - \frac{\dot{Q}_{\text{out,actual}}}{\dot{Q}_{\text{in,actual}}}; \quad \text{BWR} \equiv \frac{\dot{W}_C,\text{actual}}{\dot{W}_T,\text{actual}};$$

Simplified Actual Brayton Cycle

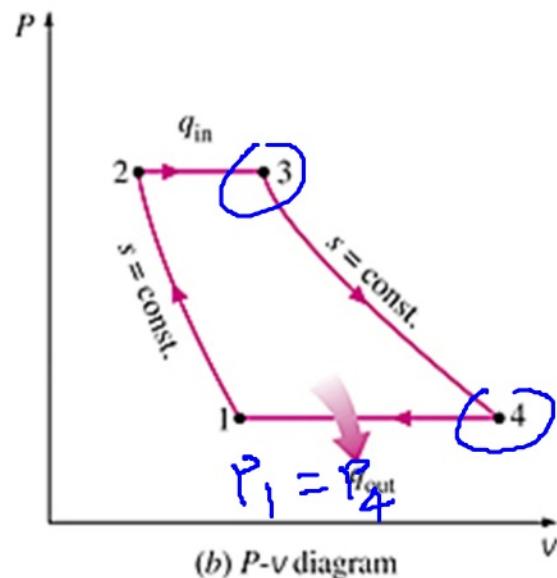
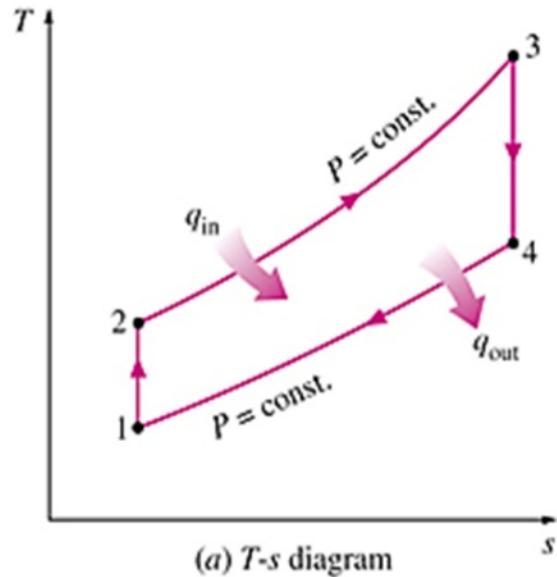
$$\text{Compressor: } \dot{W}_{C,\text{actual}} \cong \dot{m}(h_3 - h_1); \quad [\text{kW}]$$

$$\text{HX1: } \dot{Q}_{\text{in,actual}} \cong \dot{m}(h_4 - h_3); \quad [\text{kW}]$$

$$\text{Turbine: } \dot{W}_T,\text{actual} \cong \dot{m}(h_4 - h_6); \quad [\text{kW}]$$

$$\text{HX2: } \dot{Q}_{\text{out,actual}} \cong \dot{m}(h_6 - h_1); \quad [\text{kW}]$$

Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the pressure and temperature at all states and the cycle efficiency. Assume $WT = 2.5 WC$, where WT and WC are the turbine and the compressor work respectively.



$$P_1 = 101.325 \text{ kPa}$$

$$T_1 = 27^\circ\text{C} = 300\text{K}$$

$$\gamma_p = \frac{P_2}{P_1} = 6$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{r-1}{r}} =$$

$$T_2 = 500.55\text{K}$$

$$P_2 = 6.08 \text{ bar}$$

$$WT = 2.5 WC$$

$$q_{in}(h_3 - h_4) = 2.5 \times q_{in}(h_2 - h_1)$$

$$q_{in}(T_3 - T_4) = 2.5 q_{in}(T_2 - T_1) - ($$

	1	2	3	4
$P \text{ bar}$	1.01	6.08	6.08	1.01325
$T \text{ K}$	300	500.5	1251.6	750.4

$$\eta = \frac{w_{net}}{Q_{in}}$$

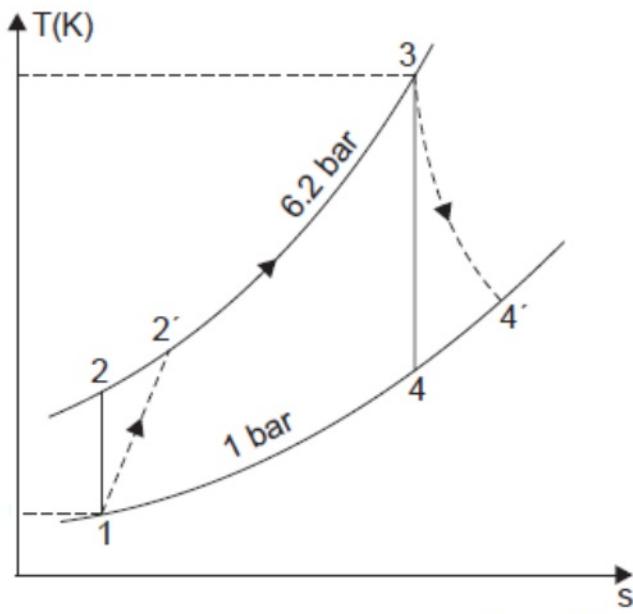
$$= \frac{W_T - W_C}{Q_{in}} = \frac{cp(T_3 - T_4) - cp(T_2 - T_1)}{cp(T_3 - T_2)}$$

$$= 0.4 = 40\%$$

$$\sqrt{W_T} = cp(T_3 - T_4) \quad \text{KJ/Kg}^\circ$$

$$\sqrt{W_C} = cp(T_2 - T_1) \quad \text{KJ/Kg}^\circ$$

A gas turbine unit receives air at 1 bar and 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating value of 44186 kJ/kg and the fuel-air ratio is 0.017 kJ/kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency. For products of combustion, $c_p = 1.147 \text{ kJ/kg K}$ and $\gamma = 1.333$.



$$P_1 = 1 \text{ bar}$$

$$T_1 = 300 \text{ K}$$

$$P_2 = 6.2 \text{ bar}$$

$$\eta_c = 88\% =$$

$$CV = 44186 \text{ kJ}$$

$$\frac{m_f}{m_a} = 0.017$$

$$\eta_T = 90\% =$$

$$W_T = ?$$

$$W_C = ?$$

$$m = 1 \text{ kg}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \quad \gamma = 1.4$$

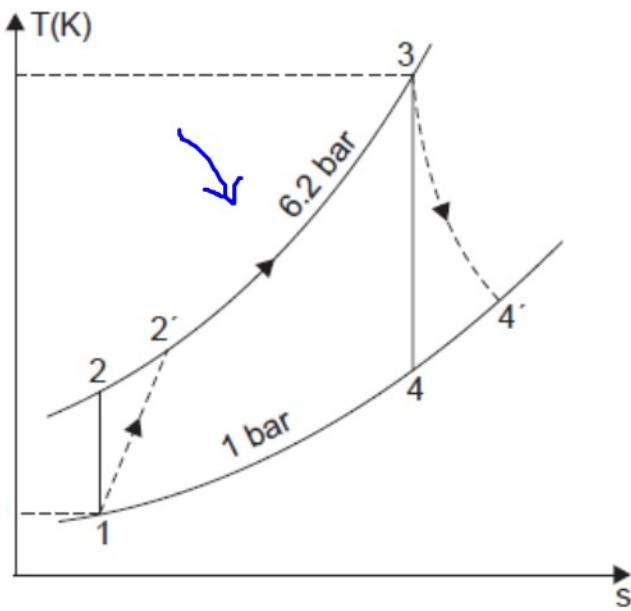
$$\frac{T_2 - T_1}{T_2' - T_1'} = \frac{h_2 - h_1}{h_2' - h_1}$$

$$\frac{T_2'}{T_1'} = \frac{h_3 - h_4}{h_3 - h_4} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{1}{\gamma}}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3} \right)^{\frac{1}{\gamma}}$$

$$\frac{T_4}{T_3} =$$



To calculate T_3 :

$$\dot{m}_f \overset{\checkmark}{CV} = H_3 - H_2'$$

$$\dot{m}_f \overset{\checkmark}{CV} = (\dot{m}_f + \dot{m}_a) C_p T_3 - \dot{m}_a C_p T_2' \quad \checkmark$$

$$CV = \left(1 + \frac{\dot{m}_a}{\dot{m}_f}\right) C_p T_3 - \frac{\dot{m}_a}{\dot{m}_f} C_p T_2' \quad \checkmark$$

REFRIGERATION

Refrigeration is the science of producing and maintaining temperatures below that of the surrounding atmosphere.

- (i) By melting of a solid.
- (ii) By sublimation of a solid.
- (iii) By evaporation of a liquid.

REFRIGERATOR

The equipment employed to maintain the system at a low temperature



REFRIGERANT

Working fluid

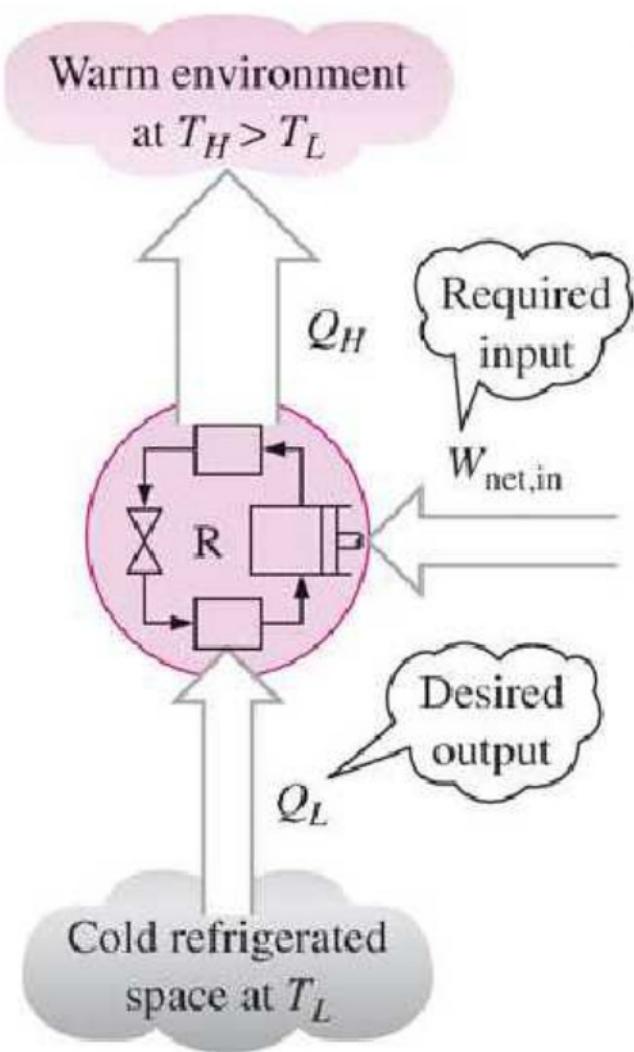
Some refrigeration applications :

- Ice making
- Transportation of foods above and below freezing
- Industrial air-conditioning
- Comfort air-conditioning
- Chemical and related industries
- Medical and surgical aids
- Processing food products and beverages
- Oil refining and synthetic rubber manufacturing
- Manufacturing and treatment of metals
- Freezing food products

various refrigeration systems

1. Ice refrigeration
2. Air refrigeration system
- 3. Vapour compression refrigeration system VCR**
- 4. Vapour absorption refrigeration system VAR**
5. Special refrigeration systems
 - (i) Adsorption refrigeration system
 - (ii) Cascade refrigeration system
 - (iii) Mixed refrigeration system
 - (iv) Vortex tube refrigeration system
 - (v) Thermoelectric refrigeration
 - (vi) Steam jet refrigeration system.

COEFFICIENT OF PERFORMANCE



The objective of a refrigerator is to remove Q_L from the cooled space.

The efficiency of a refrigerator is expressed in terms of the **coefficient of performance (COP)**.

The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space.

$$\text{COP}_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{\text{net,in}}}$$

$$W_{\text{net,in}} = Q_H - Q_L \quad (\text{kJ})$$

$$\text{COP}_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}$$

Can the value of COP_R be greater than unity?

Standard Rating of a Refrigeration Machine

- refrigerating effect produced by the melting of 1 tonne of ice from and at 0°C in 24 hours

$$1 \text{ tonne of refrigeration (TR)} = \frac{336 \times 1000}{24} = 14000 \text{ kJ/h.}$$

Reversed Carnot Cycle

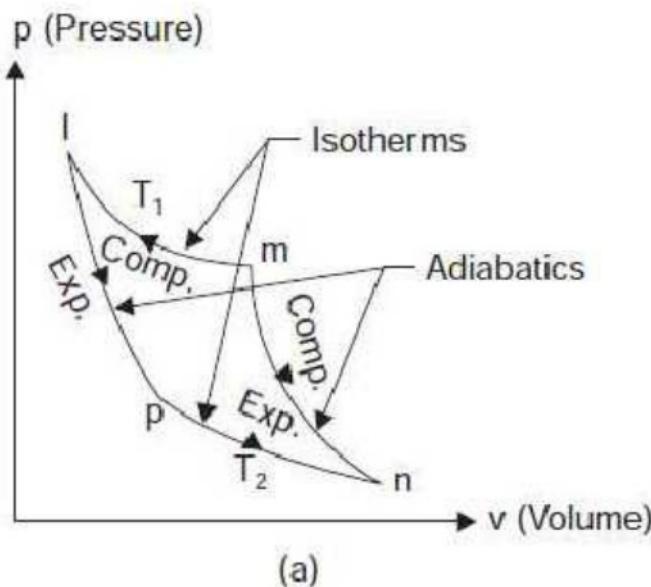


Fig. 14.1 (a) p - V diagram for reversed Carnot cycle.

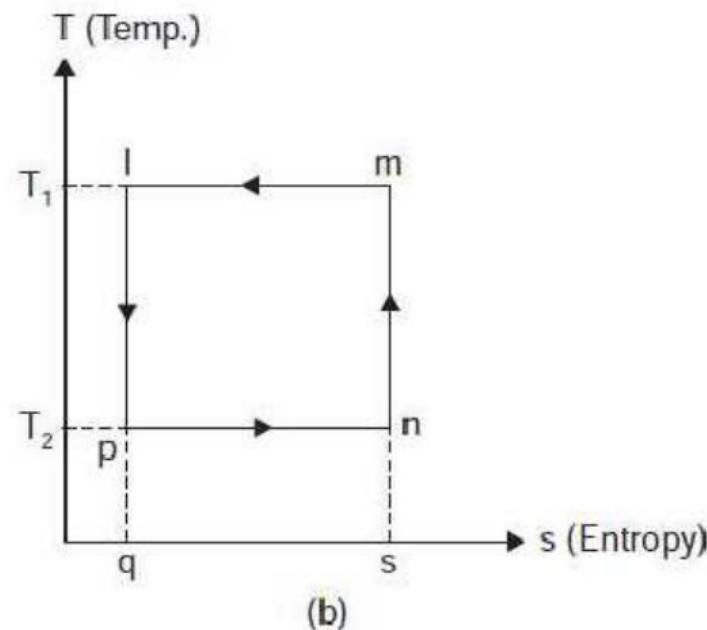


Fig. 14.1 (b) T - s diagram for a reversed Carnot cycle.

$$\begin{aligned}
 \text{C.O.P.} &= \frac{\text{Heat extracted from the cold body}}{\text{Work done per cycle}} \\
 &= \frac{\text{Area } 'npqs'}{\text{Area } 'lpnm'} = \frac{T_2 \times pn}{(T_1 - T_2) \times pn} = \frac{T_2}{T_1 - T_2}
 \end{aligned}$$

Desirable properties of an ideal refrigerant

- Thermodynamic properties :
 - Low boiling point
 - Low freezing point
 - Positive pressures (but not very high) in condenser and evaporator.
 - High latent heat of vapourisation.
- Chemical Properties :
 - Non-toxicity
 - Non-flammable and non-explosive
 - Non-corrosiveness
 - Chemical stability in reacting
 - No effect on the quality of stored (food and other) products like flowers, with other materials
 - Non-irritating and odourless.

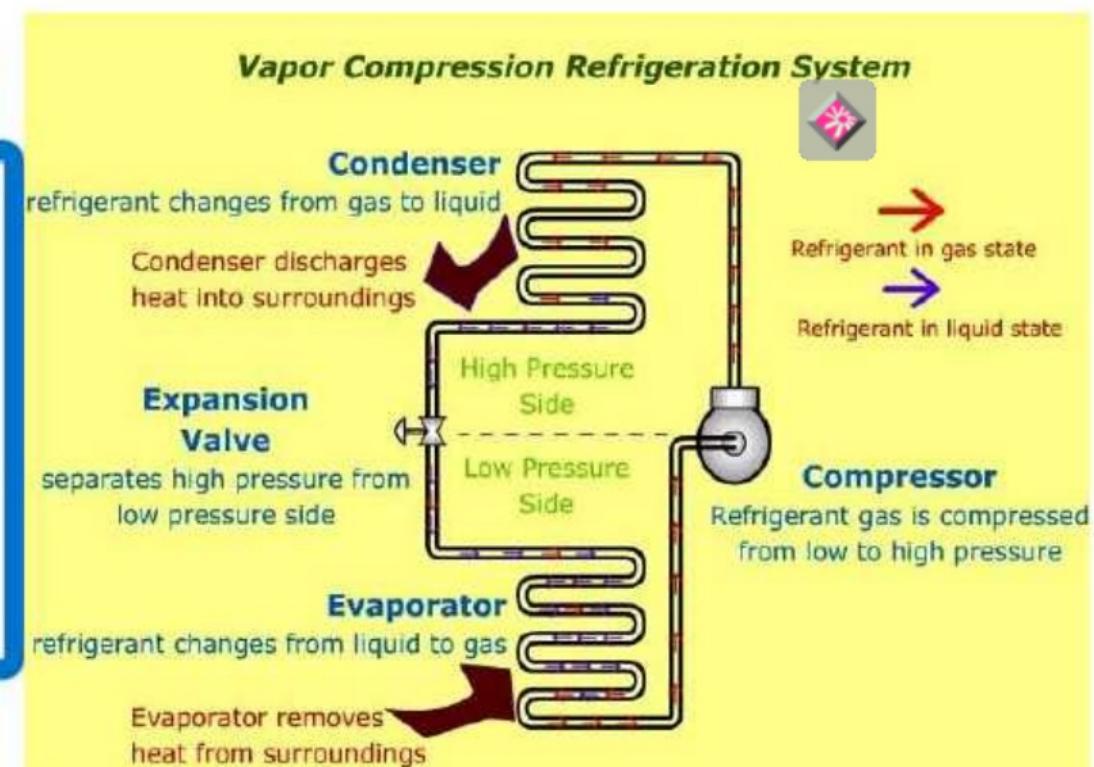
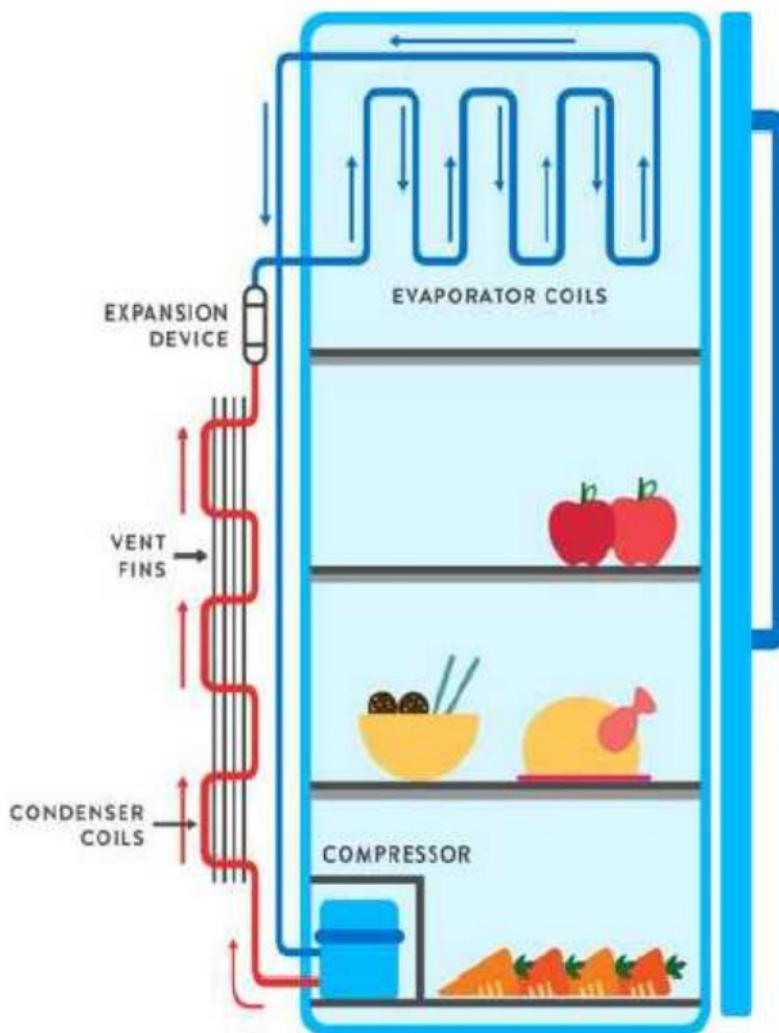
➤ **Physical Properties :**

- Low specific volume of vapour
- High thermal conductivity
- Low viscosity
- High electrical insulation.

➤ **Other Properties :**

- Ease of leakage location
- Availability and low cost
- Ease of handling
- High C.O.P.
- Low power consumption per tonne of refrigeration.

SIMPLE VAPOUR COMPRESSION SYSTEM



COMPONENTS

- Compressor
- Condenser
- Expansion valve
- Evaporator

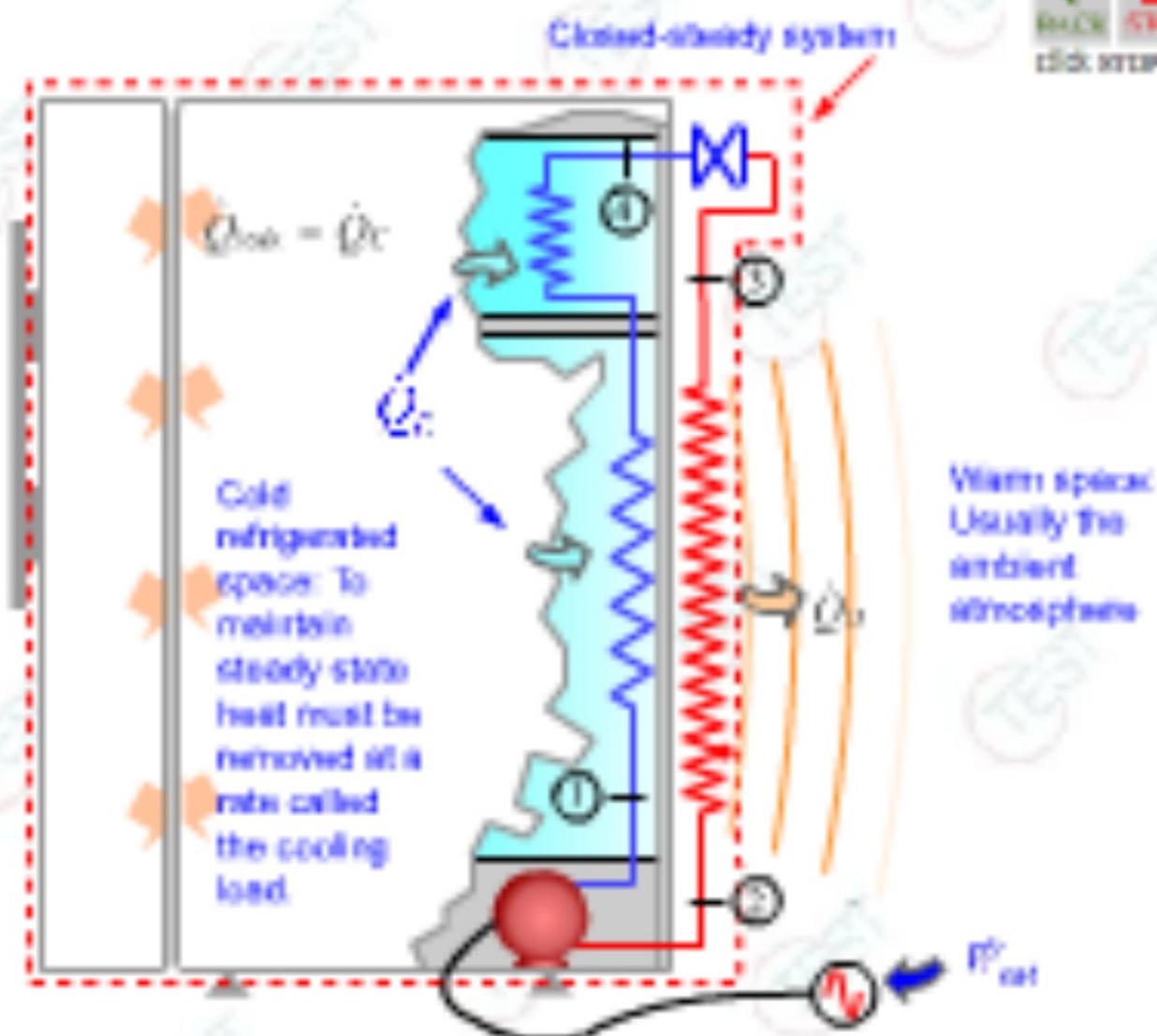
Refrigerant passing through:

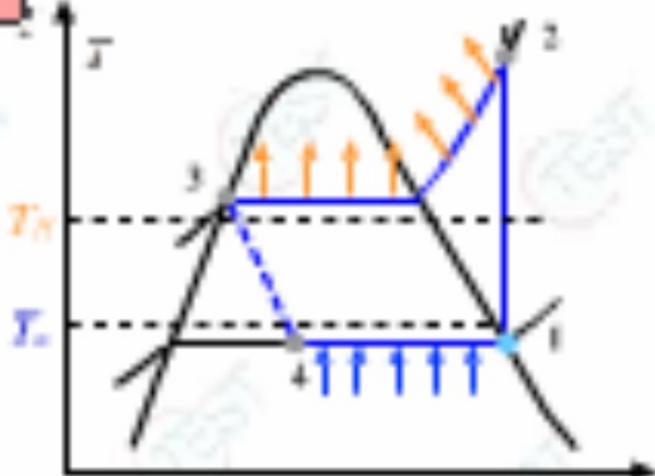
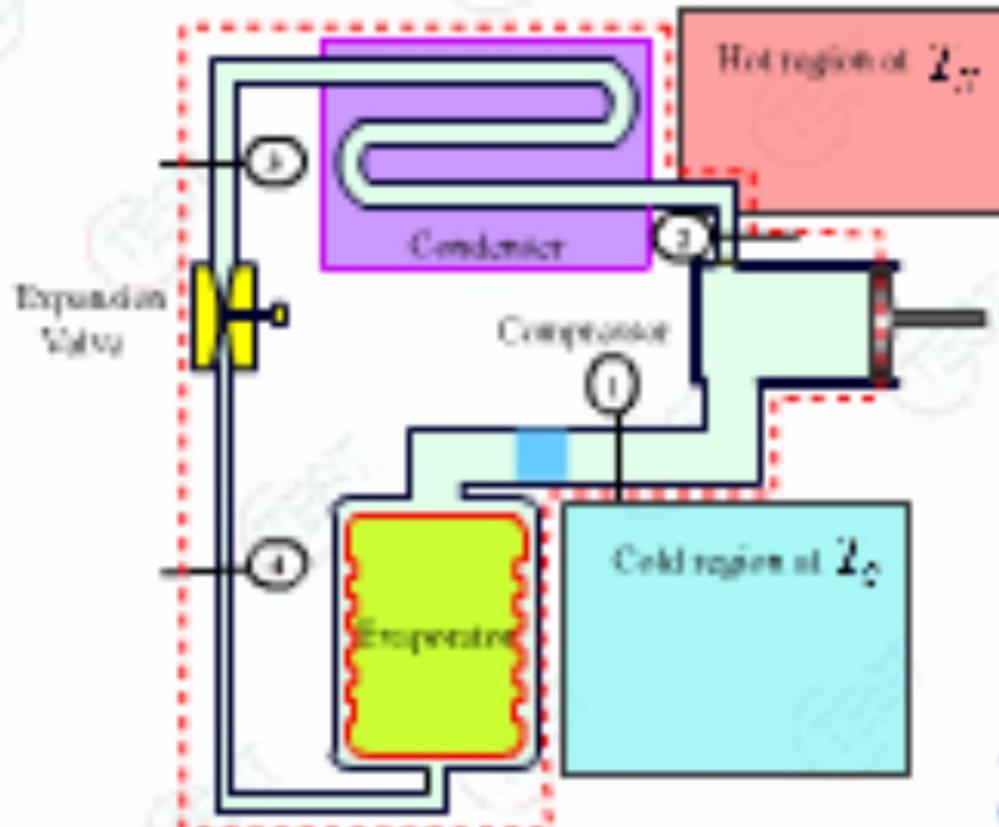
2-3: Condenser

Even if a refrigerator door is closed with all its content already cooled, heat leakage (from warmer outside) will produce a cooling load.

The heat removal rate, \dot{Q}_c , by the refrigeration cycle from the cold spaces (interior of a refrigerator) is called the **cooling capacity**, which is often expressed in the unit of ton of refrigeration (RT).

$$1 \text{ RT} = 3.517 \text{ kW}$$





The refrigerant leaves the evaporator as
saturated vapor at State-1.

$$\text{Compressor: } \dot{W}_{\text{in}} = \dot{W}_c = m(\dot{s}_2 - \dot{s}_1) \approx m(\dot{h}_2 - \dot{h}_1), \quad [\text{kW}]$$

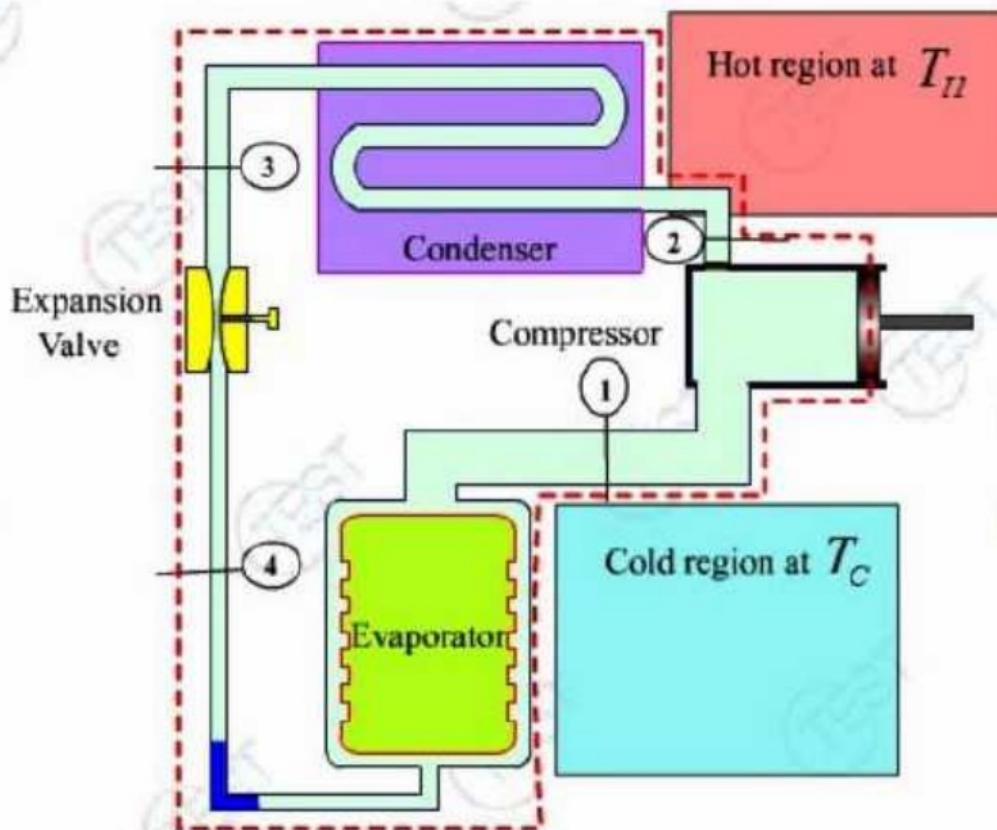
Ideal Vapor Compression Refrigeration Cycle

$$\text{Condenser: } \dot{Q}_c = \dot{Q}_{\text{in}} = m(\dot{T}_2 - \dot{T}_3) \approx m(\dot{h}_2 - \dot{h}_3), \quad [\text{kW}]$$

$$\text{Temperature rise: } \dot{T}_2 - \dot{T}_3 = \dot{q}_c \cdot \eta_c \quad \left| \frac{\text{kJ}}{\text{kg}} \right.$$

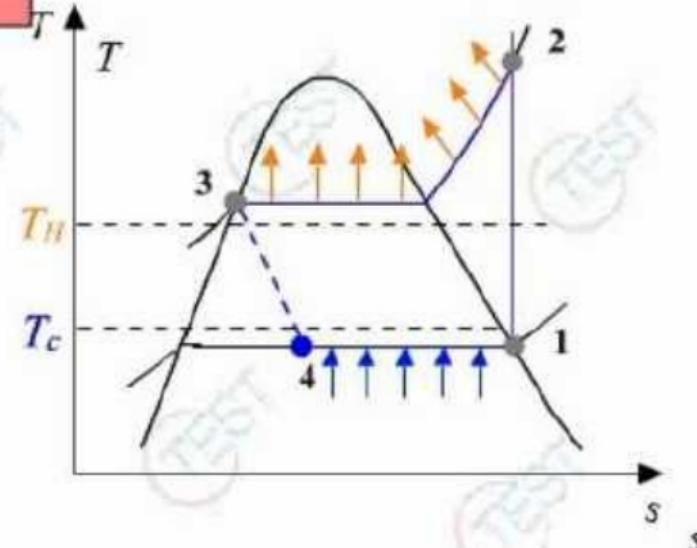
$$\text{COP}_c = \frac{\dot{Q}_c}{\dot{W}_{\text{in}}} = \frac{\dot{h}_2 - \dot{h}_3}{\dot{h}_1 - \dot{h}_3}$$

$$\text{Evaporator: } \dot{Q}_e = \dot{Q}_{\text{in}} = m(\dot{T}_1 - \dot{T}_4) \approx m(\dot{h}_1 - \dot{h}_4), \quad [\text{kW}]$$



Click STOP to use the slide bar

BACK STOP PLAY FAST



$$\text{Compressor: } \dot{W}_{\text{net}} = \dot{W}_C = m(j_2 - j_1) \approx m(h_2 - h_1); \quad [\text{kW}]$$

Ideal Vapor Compression Refrigeration Cycle

$$\text{Condenser: } \dot{Q}_H = \dot{Q}_{\text{out}} = m(j_2 - j_3) \approx m(h_2 - h_3); \quad [\text{kW}]$$

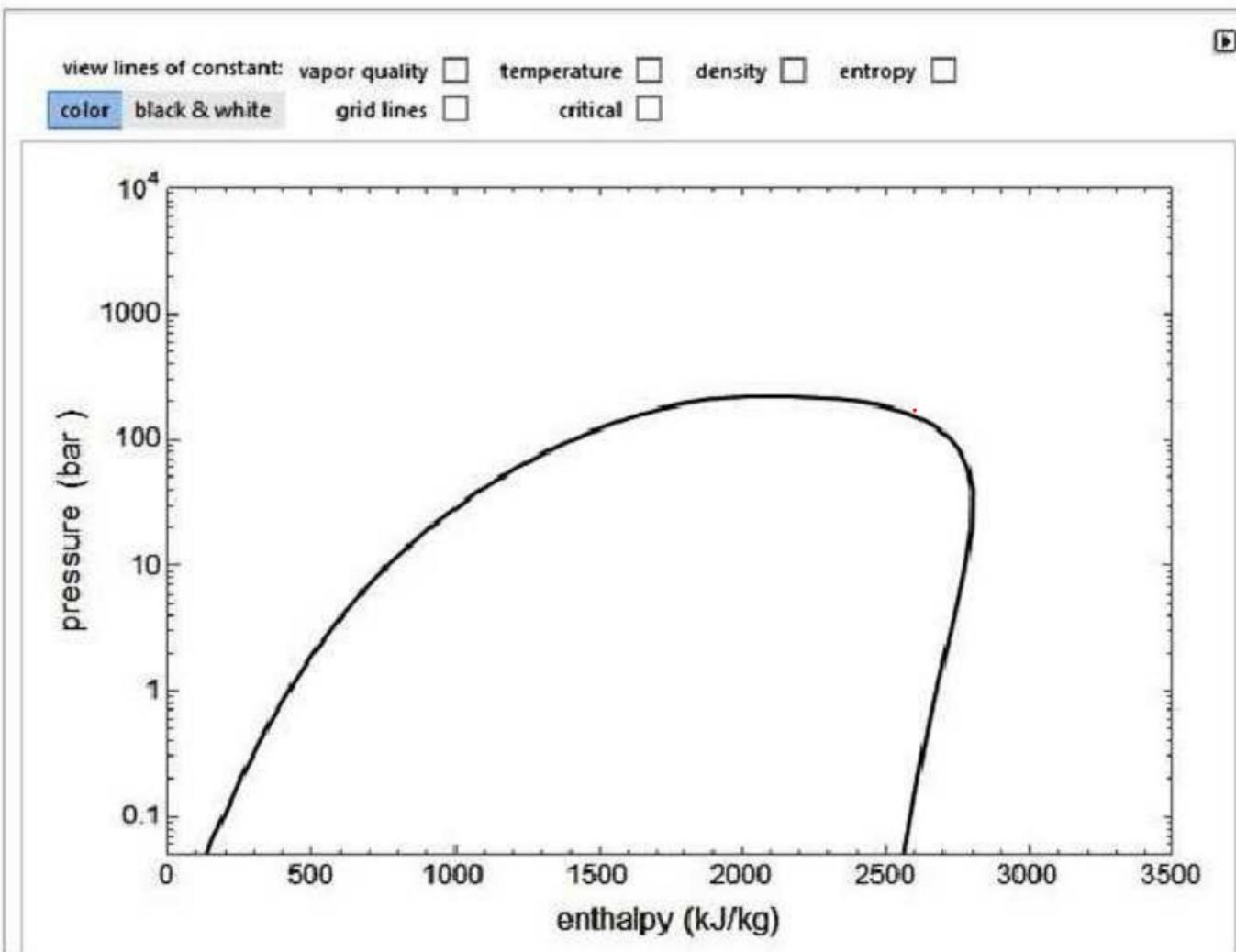
$$\text{Expansion valve: } j_4 = j_3 \Rightarrow h_4 \approx h_3; \quad \left[\frac{\text{kJ}}{\text{kg}} \right]$$

$$\text{Evaporator: } \dot{Q}_C = \dot{Q}_{\text{in}} = m(j_1 - j_4) \approx m(h_1 - h_4); \quad [\text{kW}]$$

$$\text{COP}_R = \frac{\dot{Q}_C}{\dot{W}_{\text{net}}} = \frac{h_1 - h_4}{h_2 - h_1}$$

P H Chart

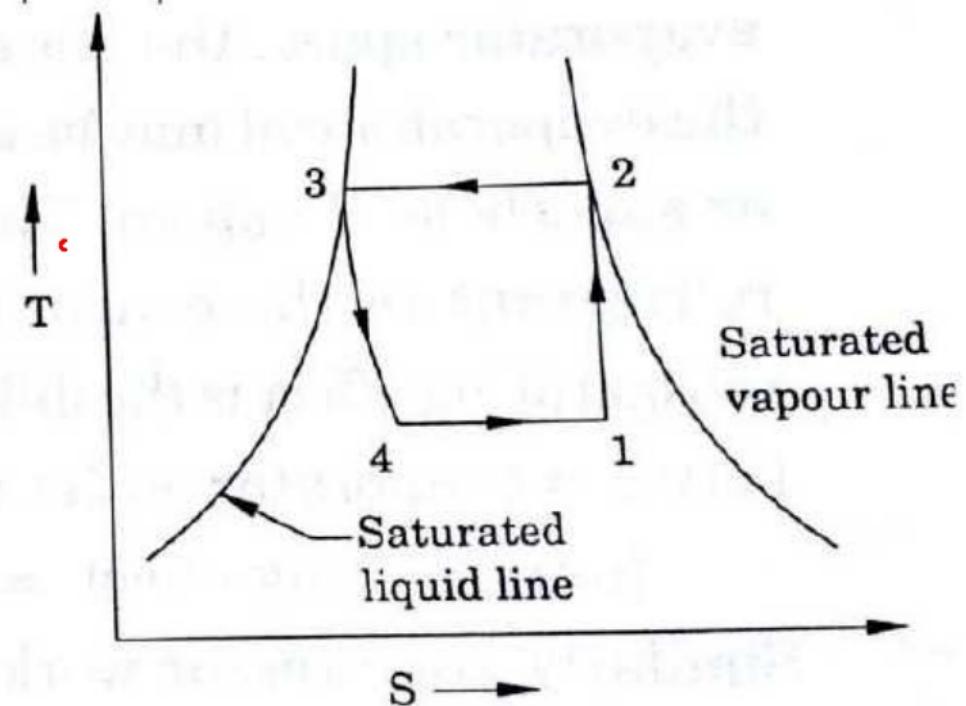
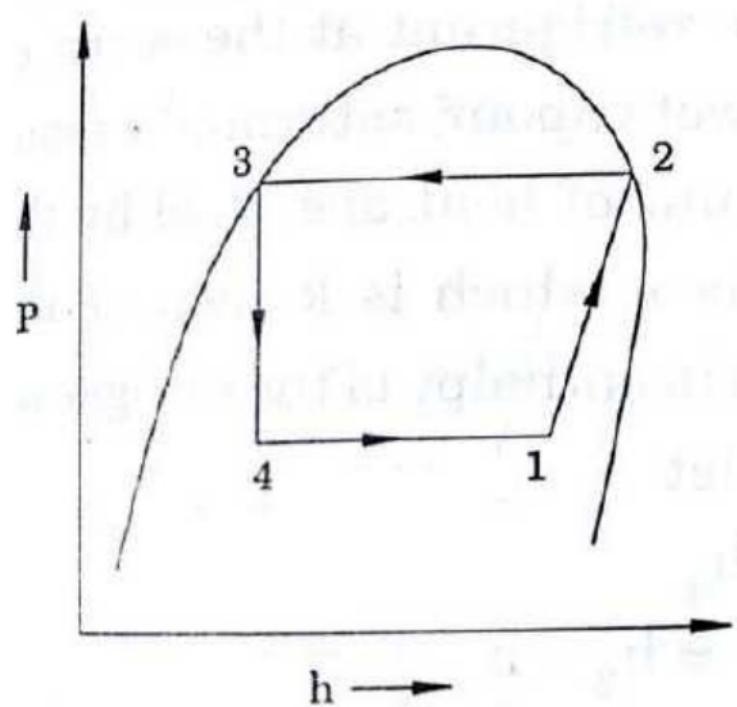
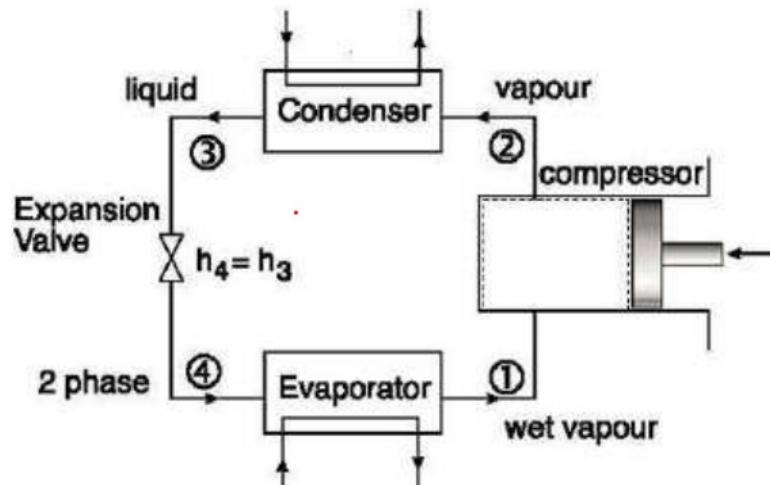
Pressure-Enthalpy Diagram



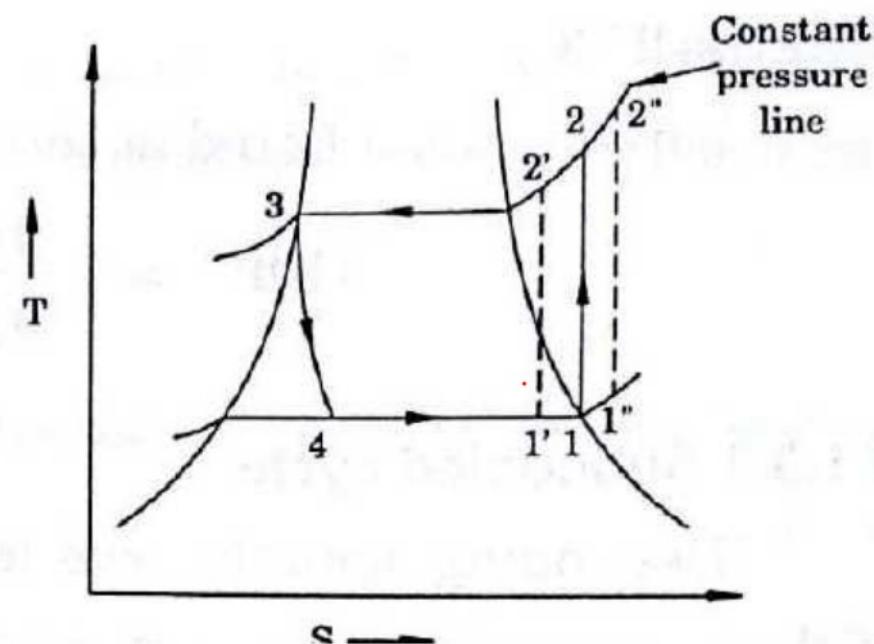
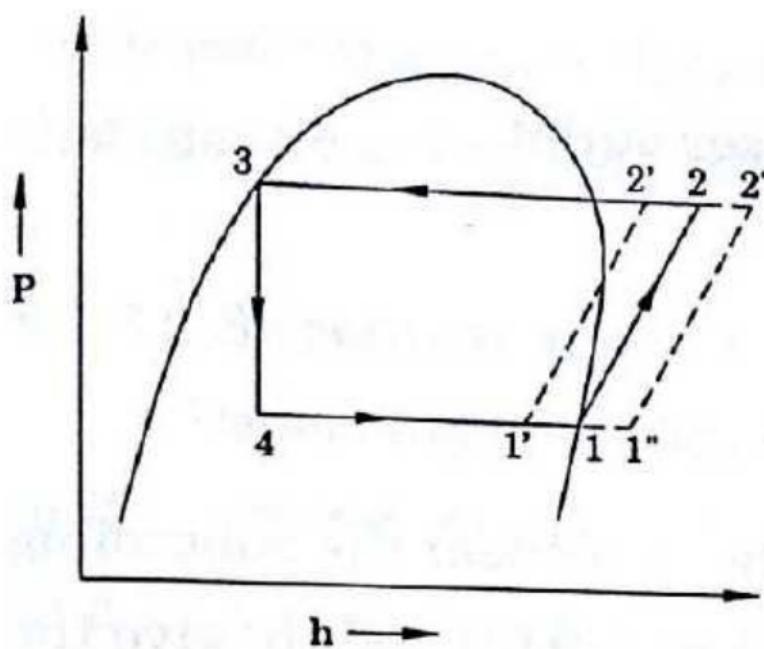
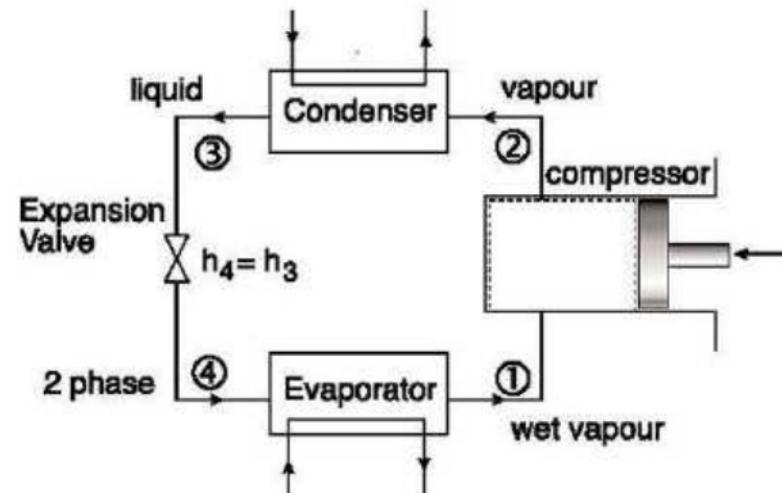
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R124_Ts.bmp

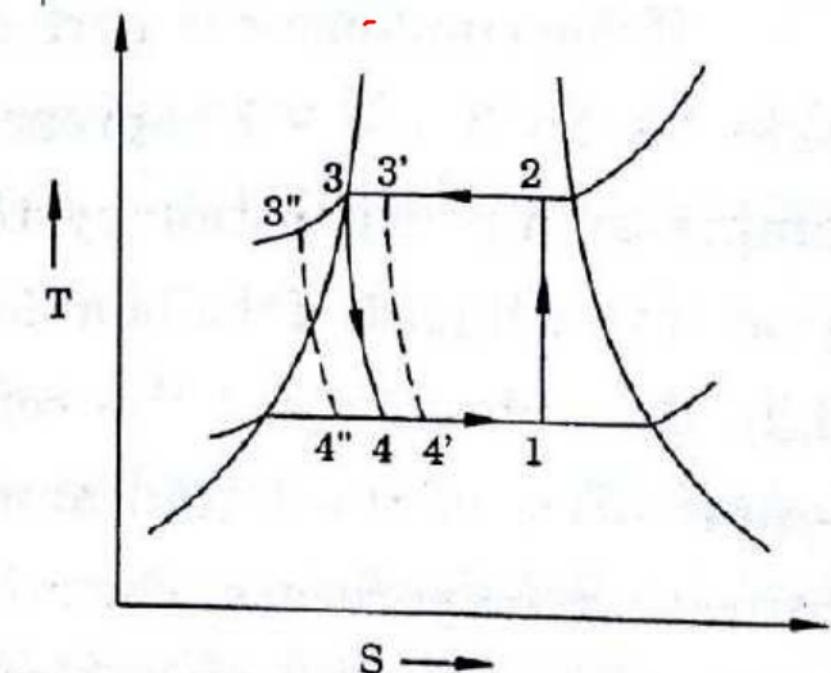
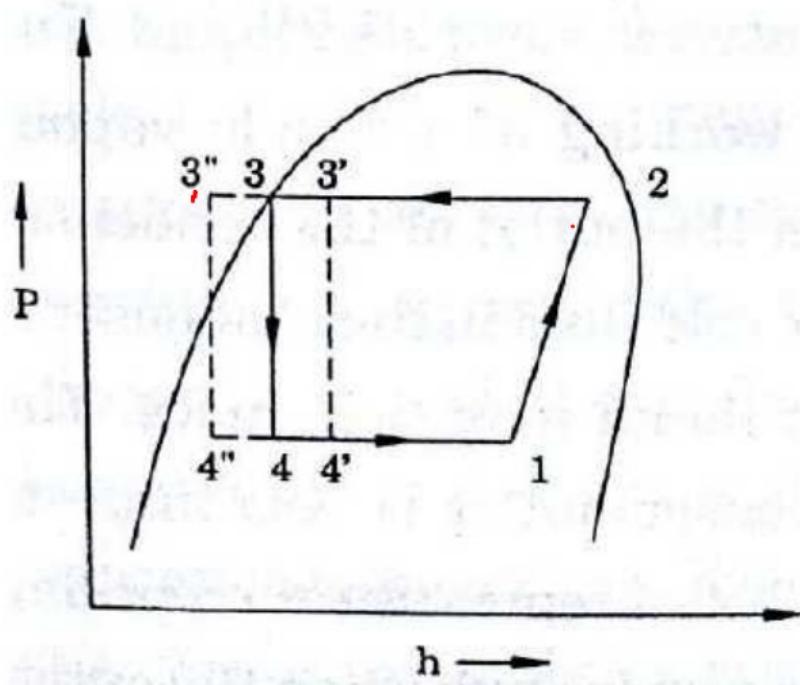
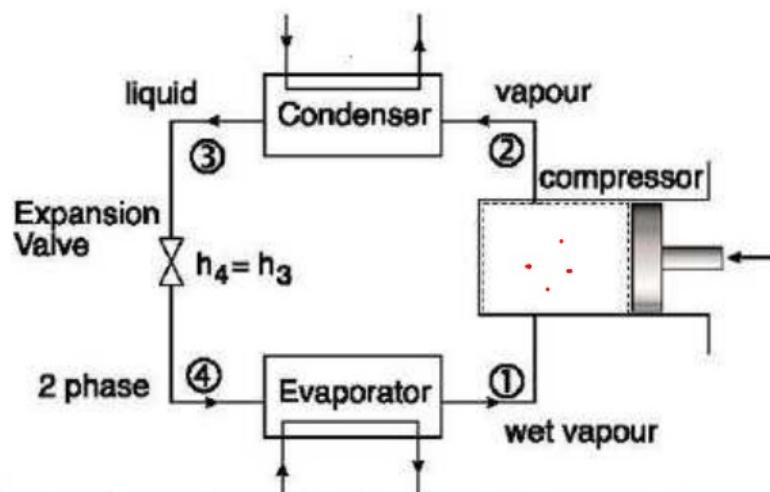
When the vapour is dry at the end of compression



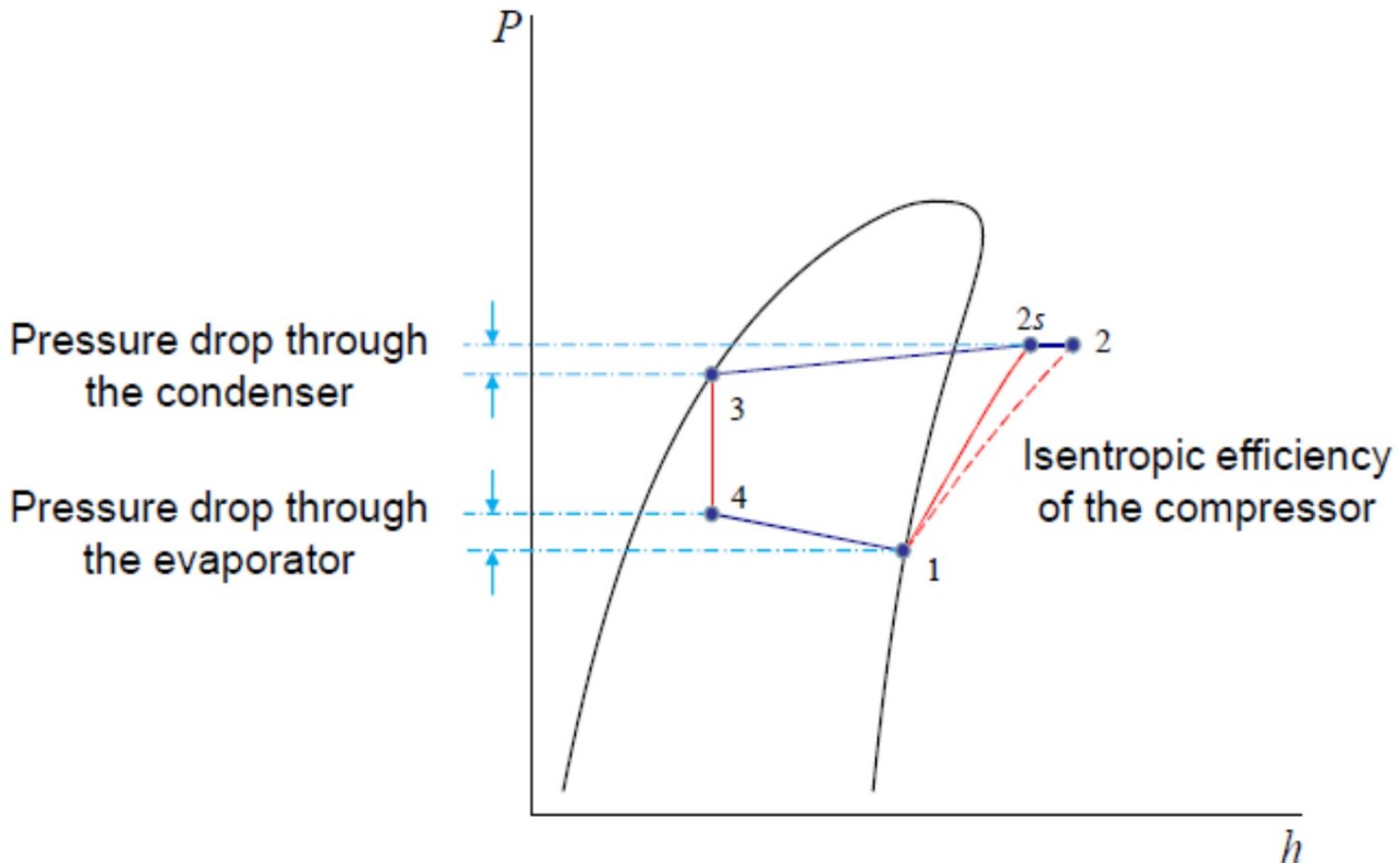
When the vapour is superheated at the end of compression



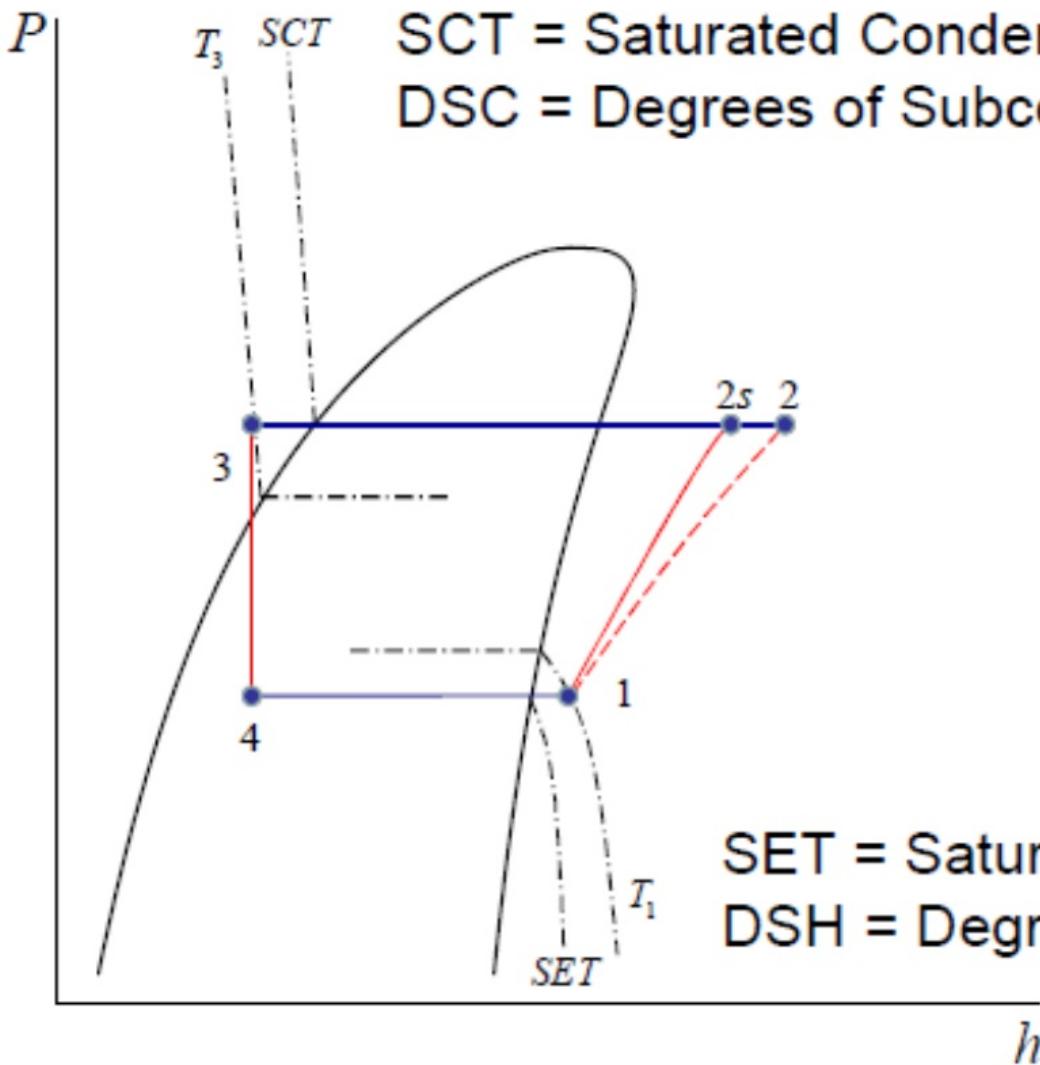
When the vapour is wet at the end of compression



VCR Cycle Irreversibilities



Practical VCR Cycle



SCT = Saturated Condensing Temperature
 DSC = Degrees of Subcooling = $SCT - T_3$

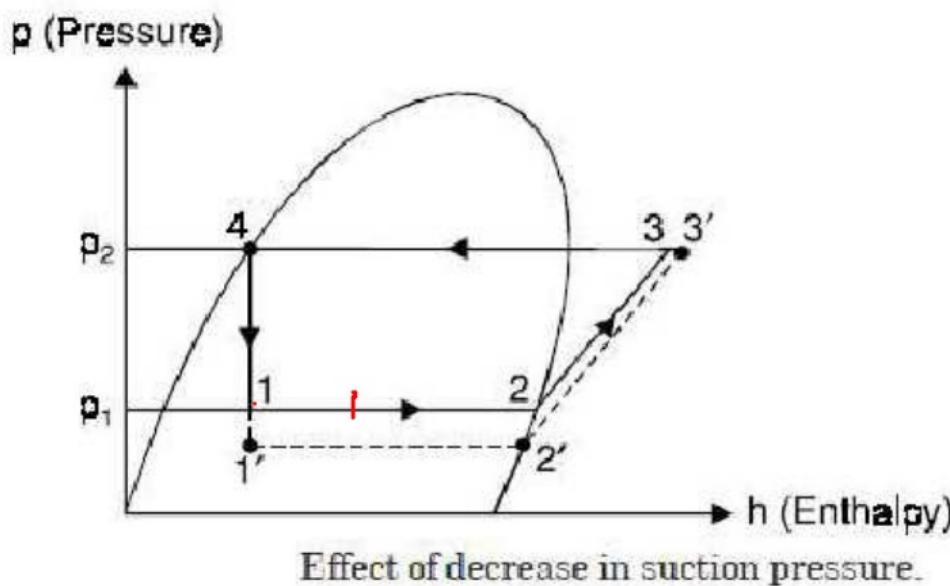
- Subcooling increases the refrigeration capacity.
- Superheating provides a dry vapor at the compressor inlet.

SET = Saturated Evaporating Temperature
 DSH = Degrees of Superheat = $T_1 - SET$

Factors Affecting the Performance of a Vapour Compression System

1. Effect of suction pressure
2. Effect of delivery pressure
3. Effect of superheating
4. Effect of sub-cooling of liquid
5. Effect of suction temperature and condenser temperature

1. Effect of suction pressure

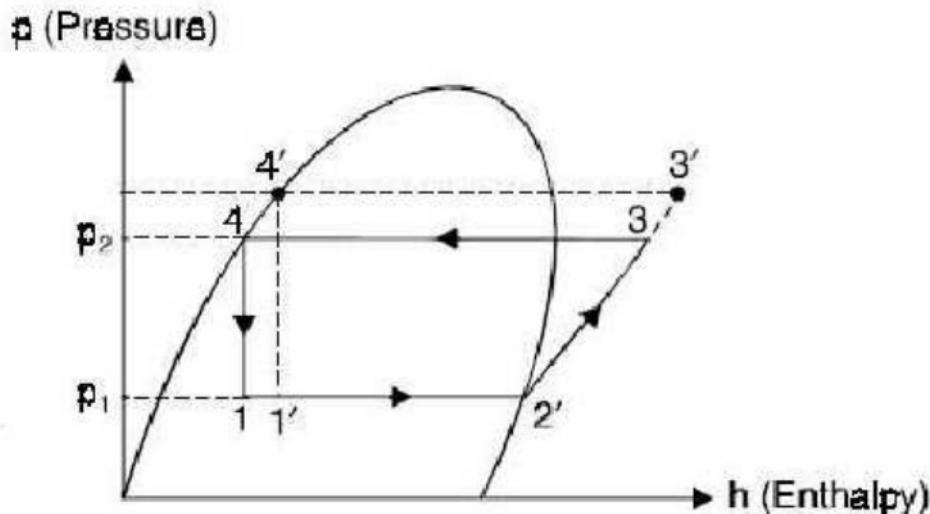


$$\text{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2}$$

$$\text{C.O.P.} = \frac{h_{2'} - h_{1'}}{h_{3'} - h_{2'}}$$

- refrigerating effect is decreased
- work required is increased.
- The net effect is to reduce the refrigerating capacity and the C.O.P.

2. Effect of delivery pressure



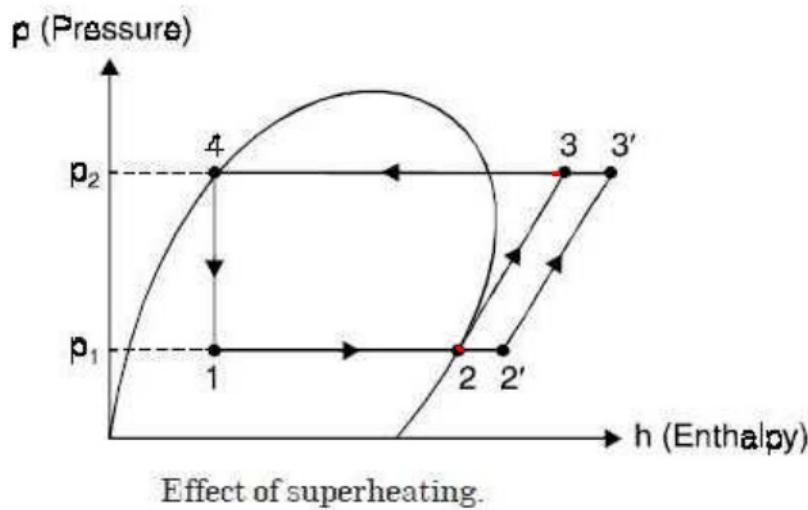
Effect of increase in delivery pressure

$$\text{C.O.P.} = \frac{h_2 - h_1}{h_3 - h_2}$$

$$\text{C.O.P.} = \frac{h_2 - h_1'}{h_{3'} - h_2}$$

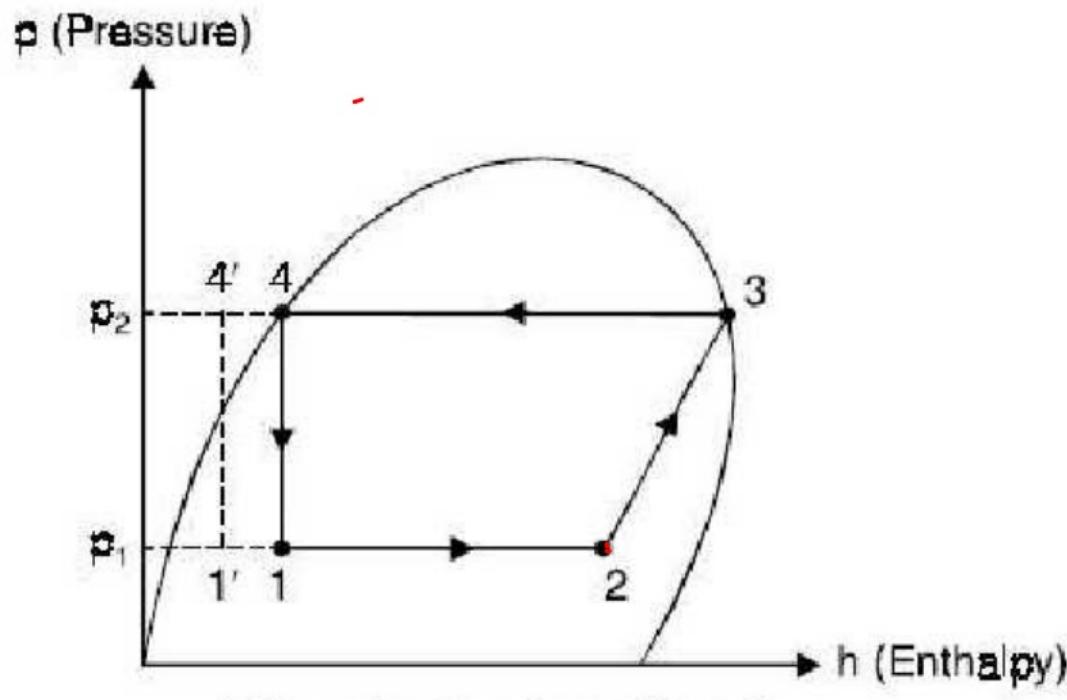
- refrigerating effect is decreased
- work required is increased.
- The net effect is to reduce the refrigerating capacity and the C.O.P.

3. Effect of superheating



- ✓ Since the increase in work is more as compared to increase in refrigerating effect, therefore overall effect of superheating is to give a low value of C.O.P.

4. Effect of sub-cooling of liquid



Effect of sub-cooling of liquid.

- ✓ sub-cooling results in increase of C.O.P. provided that no further energy has to be spent to obtain the extra cold coolant required

IDEAL GAS

- An ideal gas is an imaginary substance that obeys the relation $Pv=RT$ ✓
- It has been experimentally observed that the ideal gas relation, closely approximates the P-v-T behaviour of real gases at low densities
- At low pressures and high temperatures, the density of gas decreases and the gas behaves as an ideal gas under these conditions
- Practical example,— Air, nitrogen, oxygen, hydrogen, helium, argon, neon, even heavier gases carbon dioxide can be treated as ideal gases

$$PV = n$$

$$P = \frac{n}{V}$$

$$\rho =$$

$$\underline{\rho}$$

IDEAL GAS EQUATION

Boyle's law: $V \propto \frac{1}{P}$ (at constant n and T)

Charles' law: $V \propto T$ (at constant n and P)

Avogadro's law: $V \propto n$ (at constant P and T)

$$V \propto \frac{nT}{P}$$

$$V = R \frac{nT}{P}$$

$$PV = nRT$$

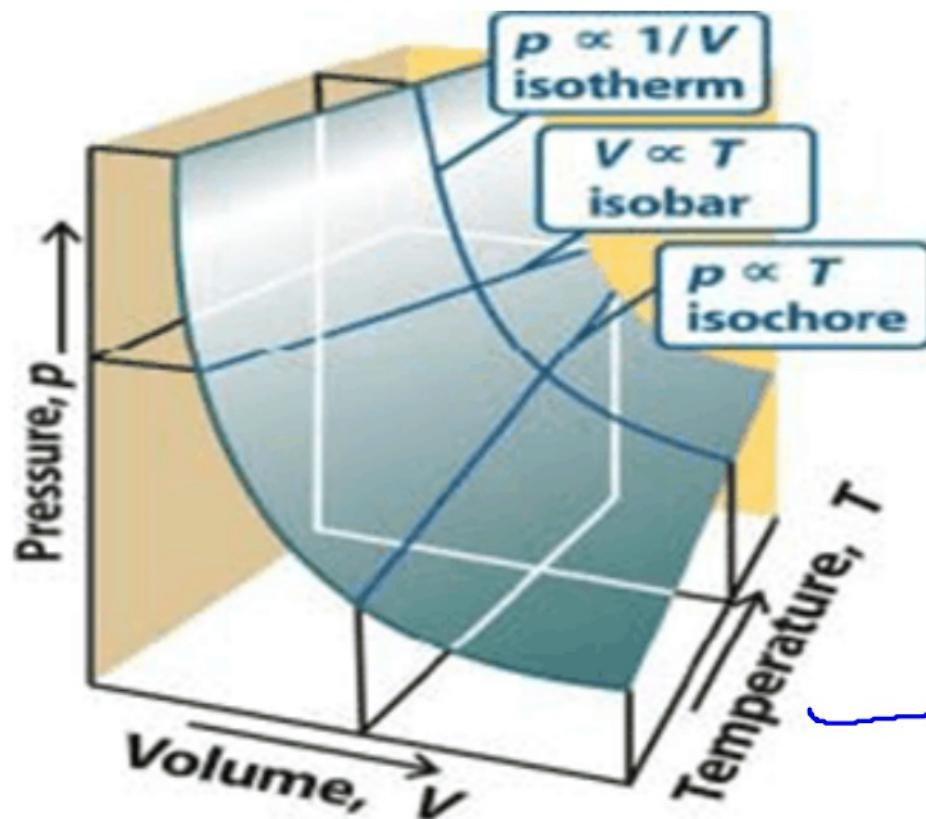
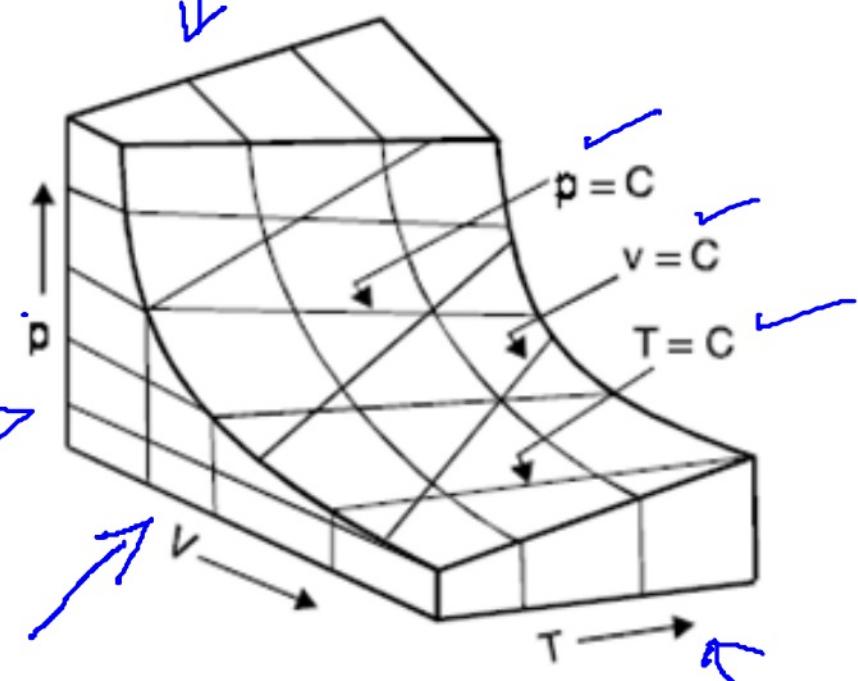


Figure 1-8
J. D. Roberts, Physical Chemistry, Elgadhi Edition



Charles' & Gay-Lussac's Law

$$\underline{V \propto T}$$

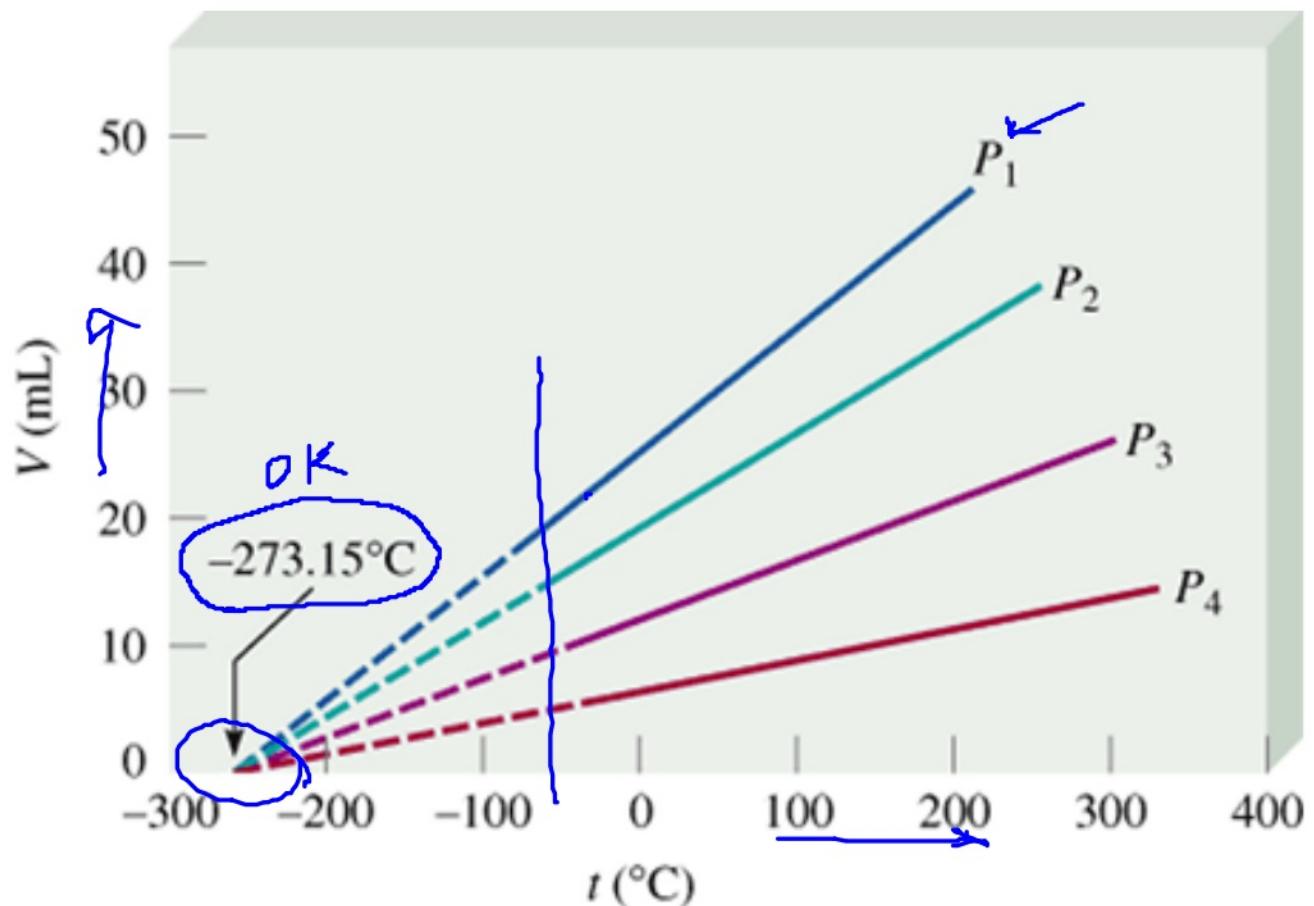
$$V = \text{constant} \times T$$

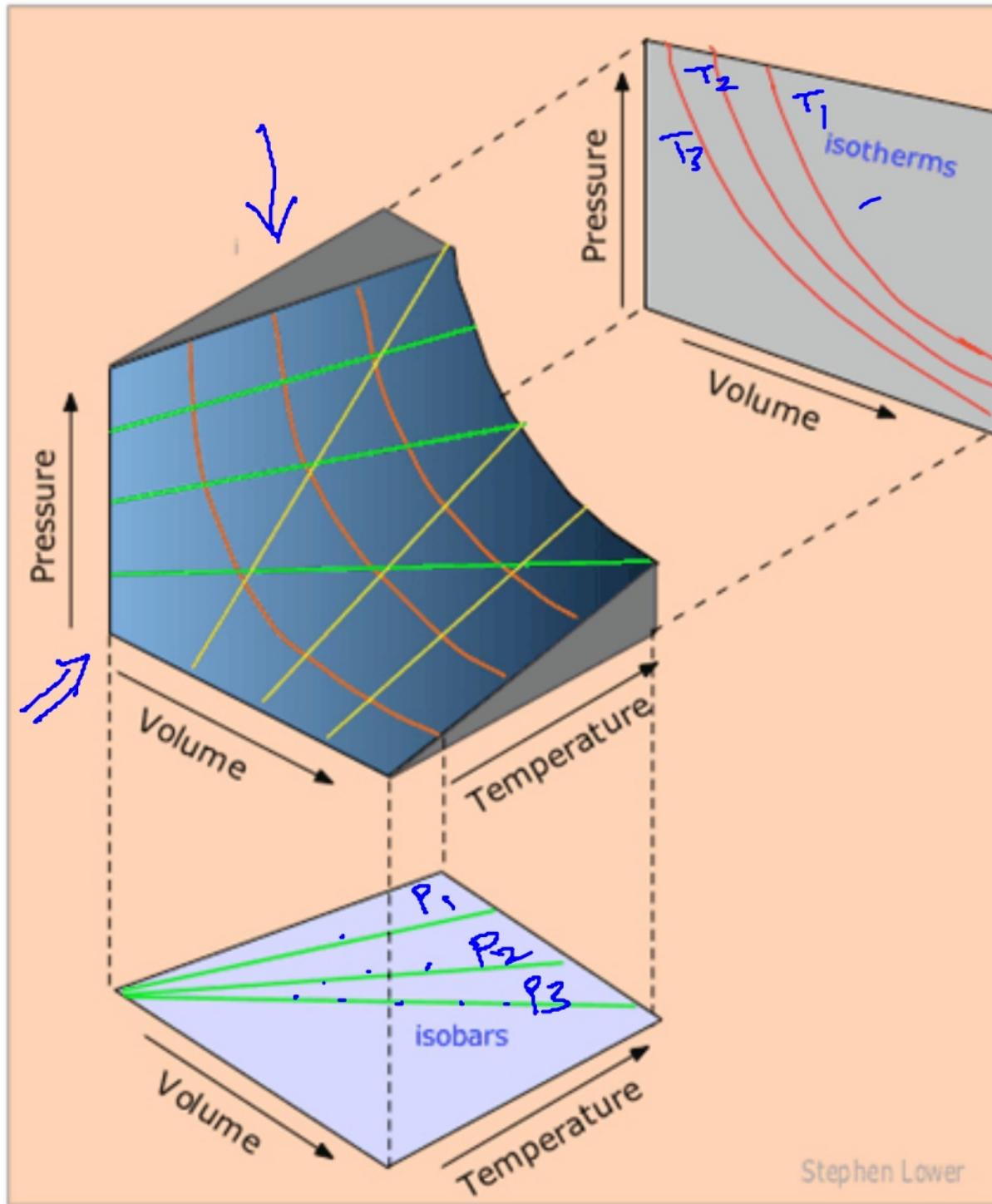
$$V_1/T_1 = V_2/T_2$$

$$\frac{V}{T} = C$$
$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \dots$$

Temperature **must** be in Kelvin

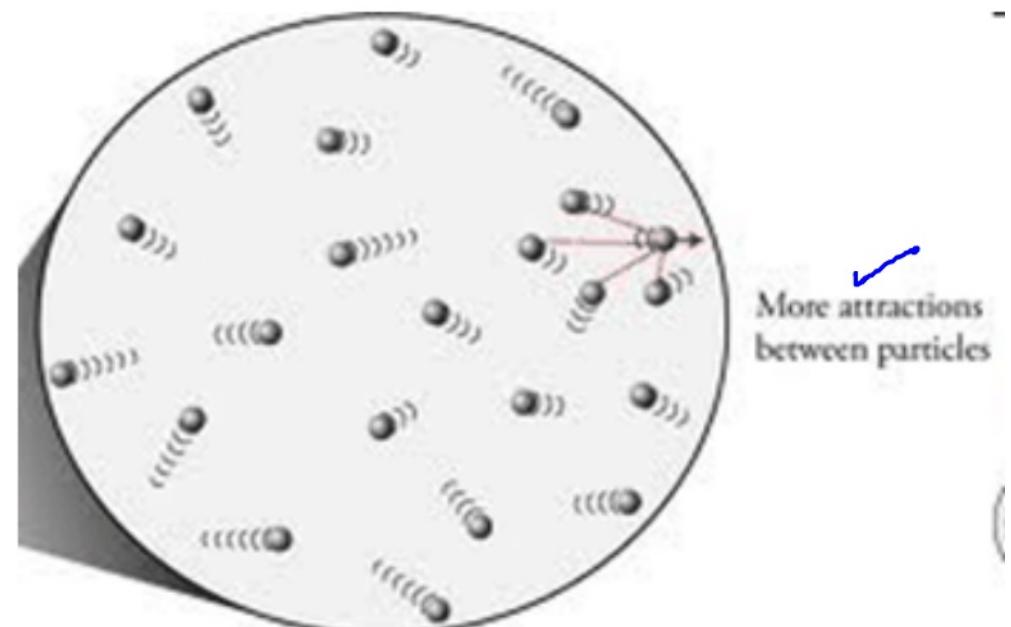
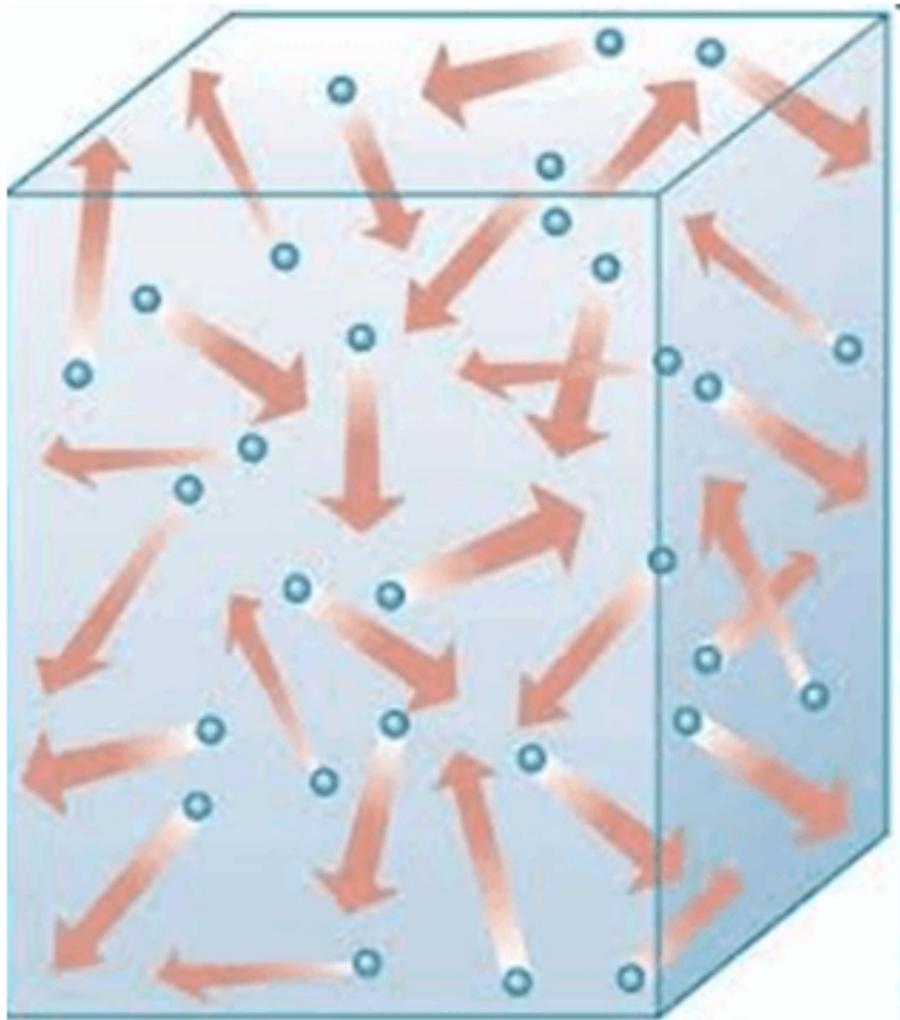
$$T(K) = t(^{\circ}\text{C}) + 273.15$$





Stephen Lower

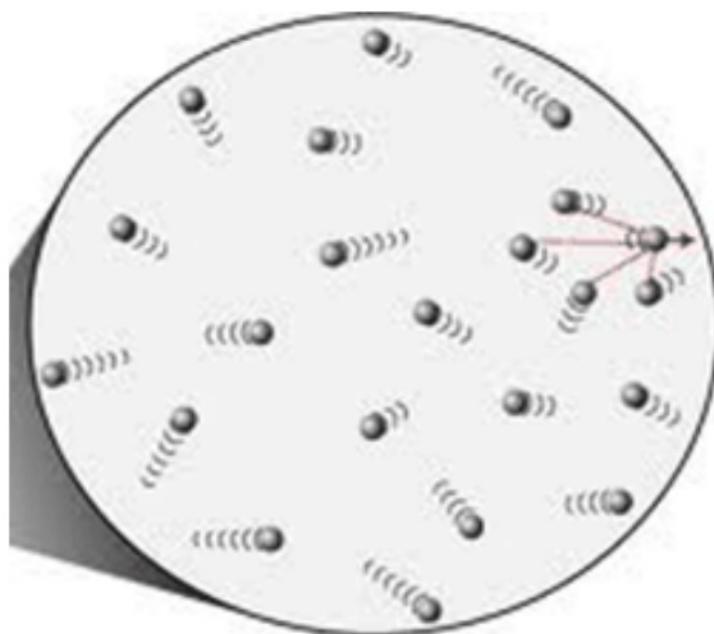
Deviations from Ideal Behavior (or) Real Gas



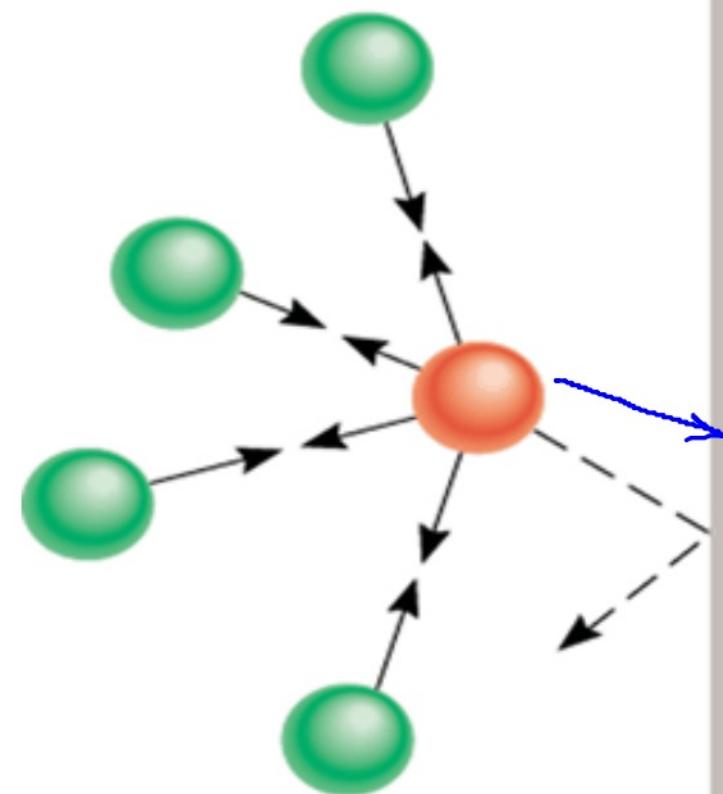
More attractions
between particles

Deviations from Ideal Behavior

Effect of intermolecular forces on the pressure exerted by a gas.



More attractions
between particles



VANDER WAALS EQUATION

$$\left(P + \frac{a}{v^2} \right) (v - b) = RT$$

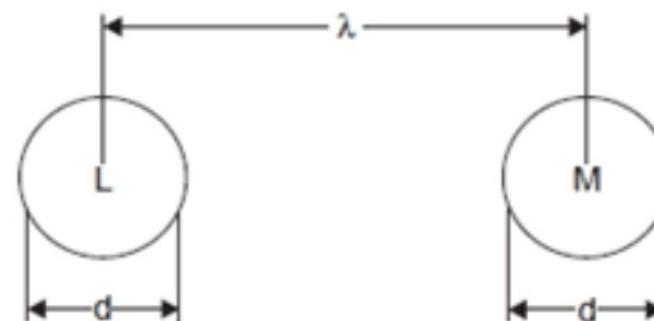
$$\left(p + \frac{a}{v^2} \right) (v - b) = R_0 T$$

If the volume of one mole is considered

Two effects are included in this ideal gas equation,

- { – Intermolecular attraction forces $[(a/v^2)]$
- Volume occupied by the molecules themselves [b]

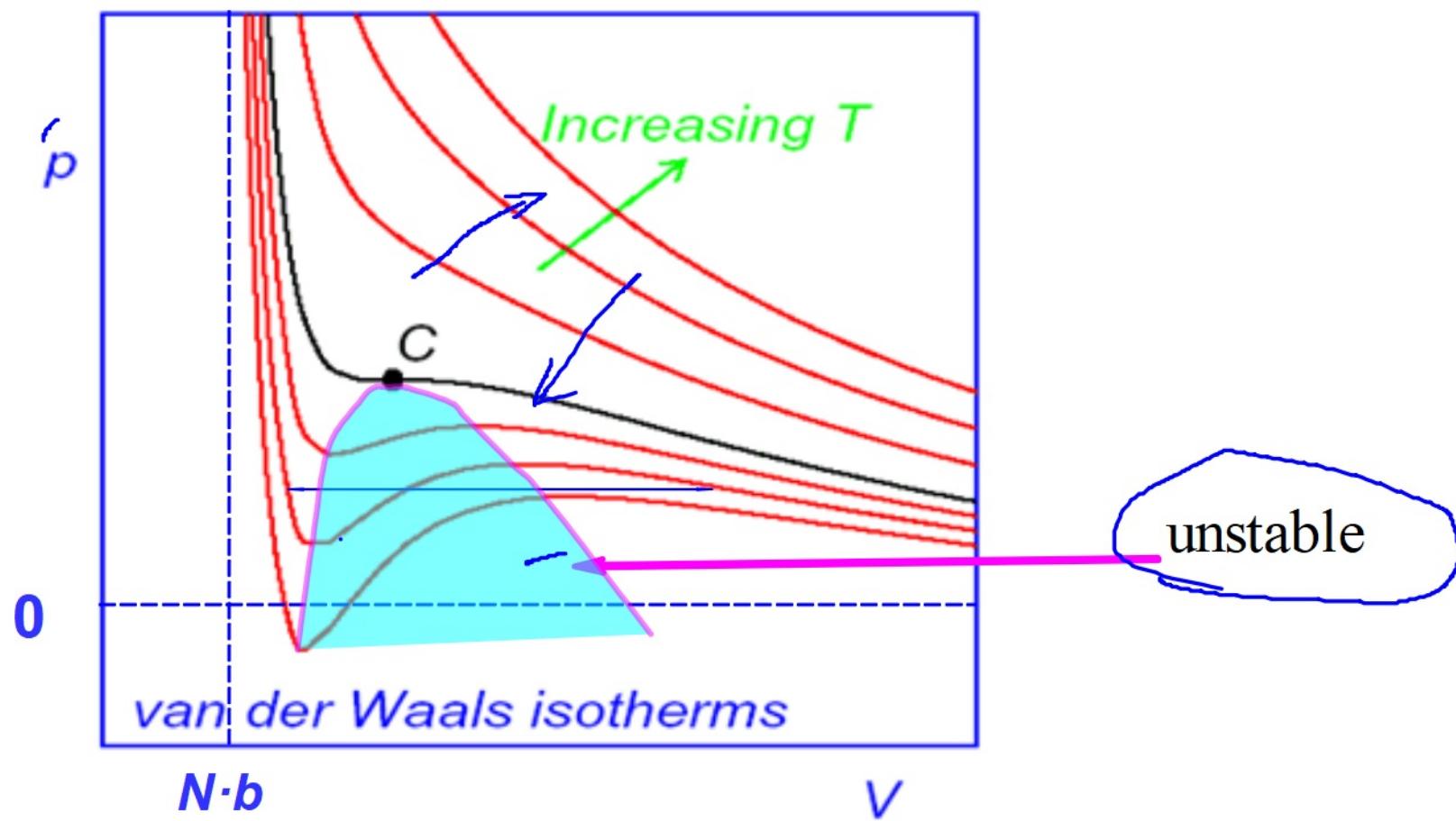
p (N/m^2), v ($m^3/kg\text{-mol}$), T (K) and $R = 8314 \text{ Nm/kg mol K}$,
 a [$\text{Nm}^4/(\text{kg-mol})^2$], b ($m^3/\text{kg mol}$).



Constants of Van der Waals' Equation

<i>S.No.</i>	<i>Substance</i>	<i>a</i> $Nm^4/(kg\cdot mol)^2$	<i>b</i> $m^3/kg\cdot mol$
1.	Hydrogen (H_2)	25105	0.0262
2.	Oxygen (O_2)	139250	0.0314
3.	Carbon dioxide (CO_2)	362850	0.0423
4.	Helium (He)	3417620	0.0228
5.	Air	135522	0.0362
6.	Water (H_2O) vapour	551130	0.0300
7.	Mercury (Hg) vapour	2031940	0.0657

The van der Waals Isotherms



Limitations of Van der Waals' Equation

- The values of a and b (which are assumed to be constant) are found to vary with temperature. Thus the results obtained from the equation are incorrect when the variation of a and b is large with respect to temperature.
- The equation is not accurate enough in the critical region and it is also obvious from its derivation.

VIRIAL EQUATION OF STATE

$$\frac{pv}{RT} = A_0 + A_1 p + A_2 p^2 + A_3 p^3 + \dots$$

$$\frac{pv}{RT} = B_0 + \frac{B_1}{v} + \frac{B_2}{v^2} + \frac{B_3}{v^3} + \dots$$

$$Pv = RT$$

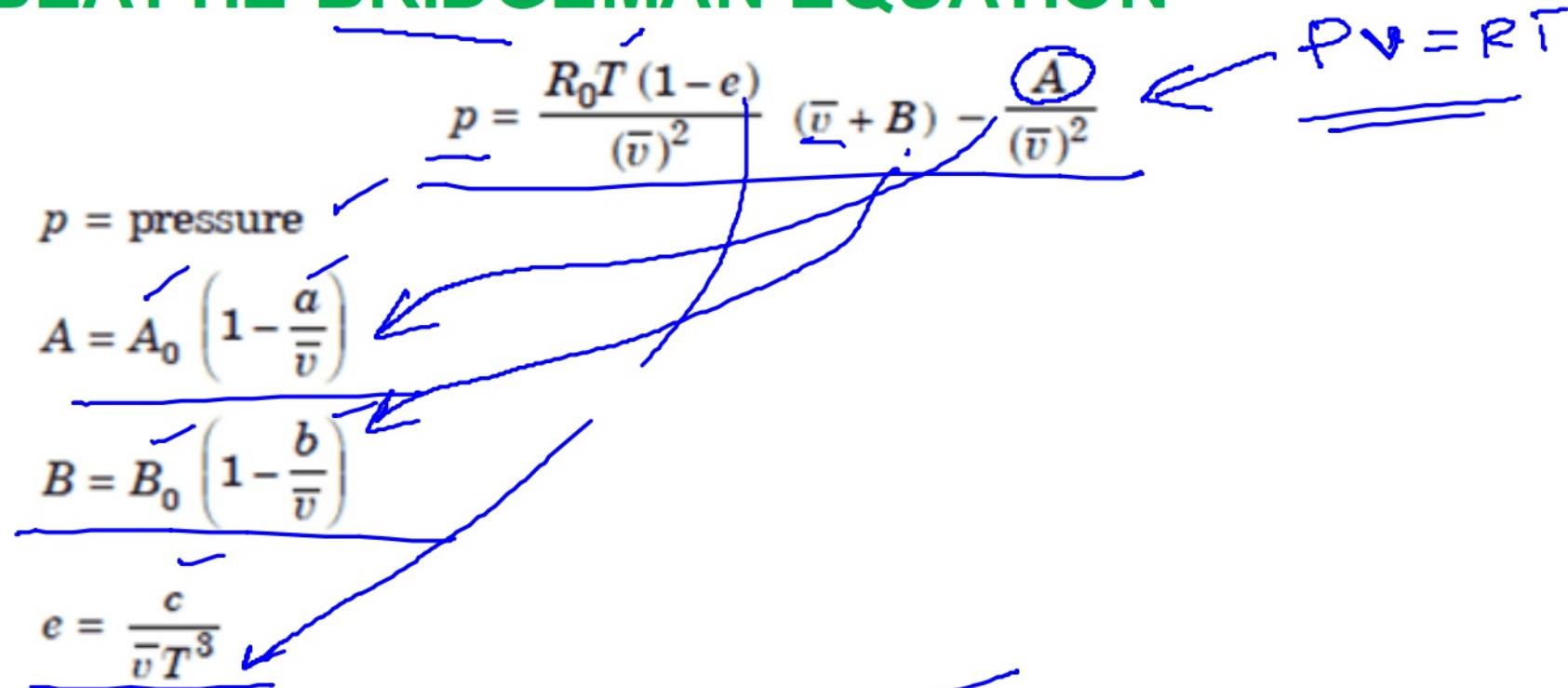
$$\frac{Pv}{RT} = 1$$

A_0, A_1, \dots and B_0, B_1, \dots are called the virial co-efficients which are functions of temperature

➤ The virial equation can be used only for gases at low and medium densities.

➤ The advantage of virial equation is that the virial co-efficients can be determined from experimental p-v-T data.

BEATTIE-BRIDGEMAN EQUATION



Constants of the Beattie-Bridgeman Equation of State

Gas	A_0	a	B_0	b	$c \times 10^{-4}$
Hydrogen (H_2)	20.0117	-0.00506	0.02096	-0.04359	0.0504
Oxygen (O_2)	151.0857	0.02562	0.04624	0.004208	4.80
Carbon dioxide (CO_2)	507.2836	0.07132	0.10476	0.07235	66.0
Helium (He)	2.1886	0.05984	0.01400	0.0	0.0040
Air	131.8441	0.01931	0.04611	-0.001101	4.34
Nitrogen	136.2315	0.02617	0.05046	-0.00691	4.20
Argon	130.7802	0.02328	0.03931	0.0	5.99

REDLICH-KWONG EQUATION

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)T^{1/2}}$$

TABLE A-24E Constants for the van der Waals, Redlich-Kwong, and Benedict-Webb-Rubin Equations of State

1. van der Waals and Redlich-Kwong: Constants for pressure in atm, specific volume in ft³/lbmol, and temperature in °R

Substance	van der Waals		Redlich-Kwong	
	a atm $\left(\frac{\text{ft}^3}{\text{lbmol}}\right)^2$	b $\frac{\text{ft}^3}{\text{lbmol}}$	a atm $\left(\frac{\text{ft}^3}{\text{lbmol}}\right)^2 (\text{°R})^{1/2}$	b $\frac{\text{ft}^3}{\text{lbmol}}$
Air	345	0.586	5,409	0.4064
Butane (C ₄ H ₁₀)	3,509	1.862	98,349	1.2903
Carbon dioxide (CO ₂)	926	0.686	21,972	0.4755
Carbon monoxide (CO)	372	0.632	5,832	0.4382
Methane (CH ₄)	581	0.685	10,919	0.4751
Nitrogen (N ₂)	346	0.618	5,280	0.4286
Oxygen (O ₂)	349	0.509	5,896	0.3531
Propane (C ₃ H ₈)	2,369	1.444	61,952	1.0006
Refrigerant 12	2,660	1.558	70,951	1.0796
Sulfur dioxide (SO ₂)	1,738	0.910	49,032	0.6309
Water (H ₂ O)	1,400	0.488	48,418	0.3380

Source: Calculated from critical data.

A cylindrical tank containing 4.0 kg of carbon monoxide gas at -50°C has an inner diameter of 0.2 m and a length of 1 m. Determine the pressure in bar, exerted by the gas using (a) the generalized compressibility chart, (b) the ideal gas equation of state, (c) the van der Waals equation of state, (d) the Redlich–Kwong equation of state. Compare the results obtained.

75.9 bar 84.4 bar 72.3 bar 75.1 bar

LAW OF CORRESPONDING STATES

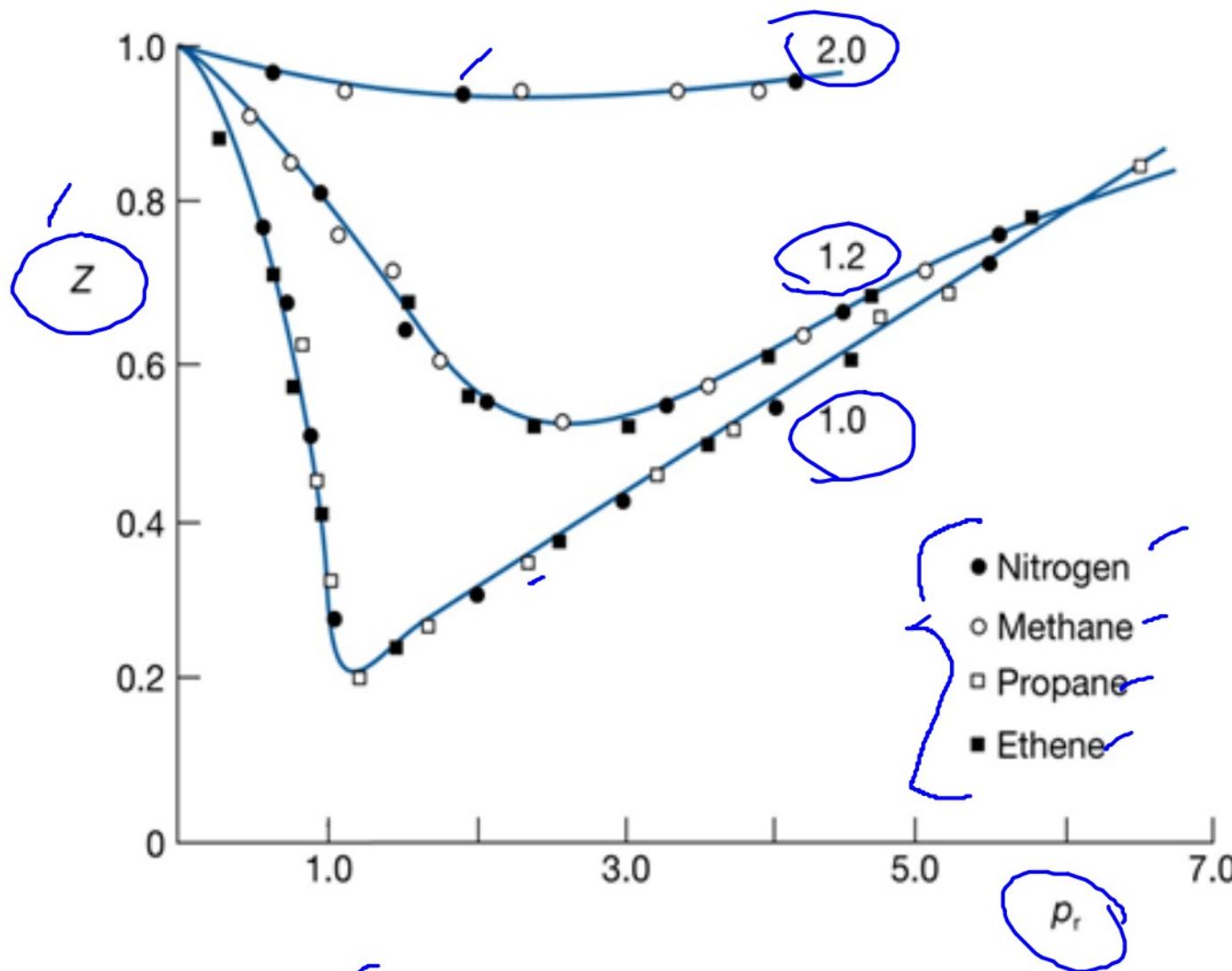
- Different gases have different values of p , V and T at their critical point
- You can compare them at any value by creating a reduced variable by dividing by the corresponding critical value
 - This places all gases on the same scale and they behave in a regular fashion; gases at the same reduced volume and temperature exert the same reduced pressure

Reduced properties

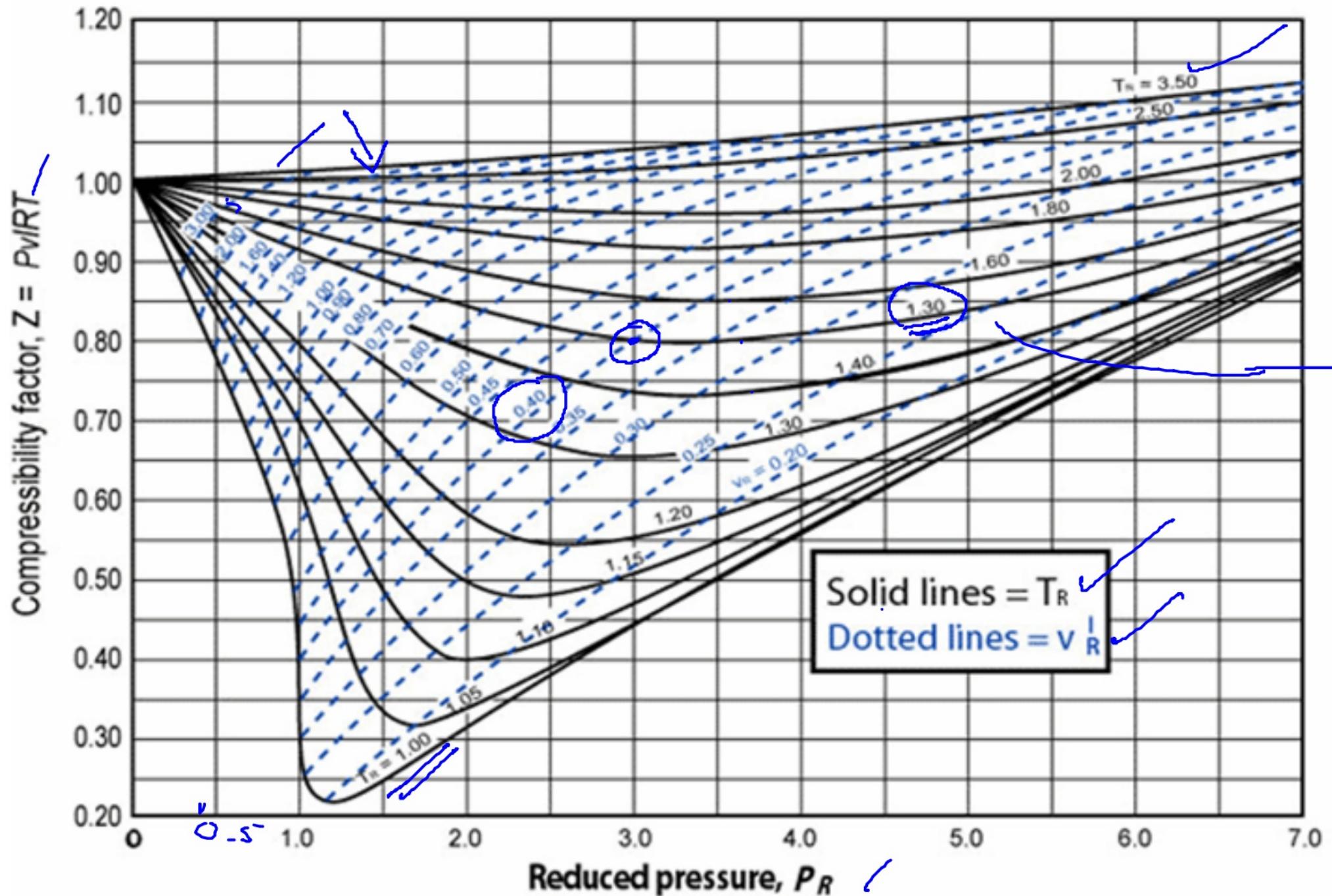
$$\underline{P_r = \frac{P}{P_c}, V_r = \frac{V}{V_c}, T_r = \frac{T}{T_c}}$$

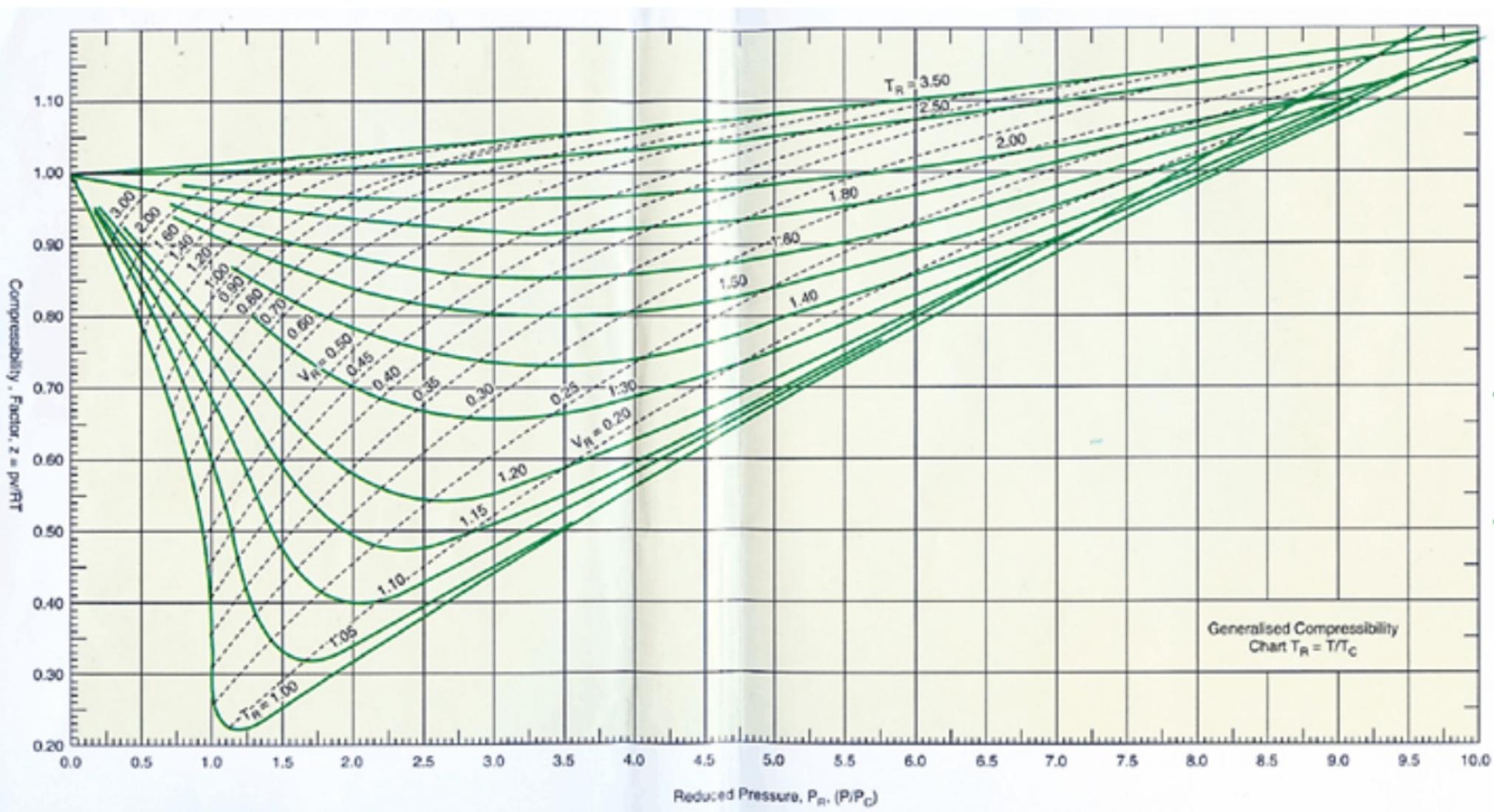
Table 8.3. Critical Constants

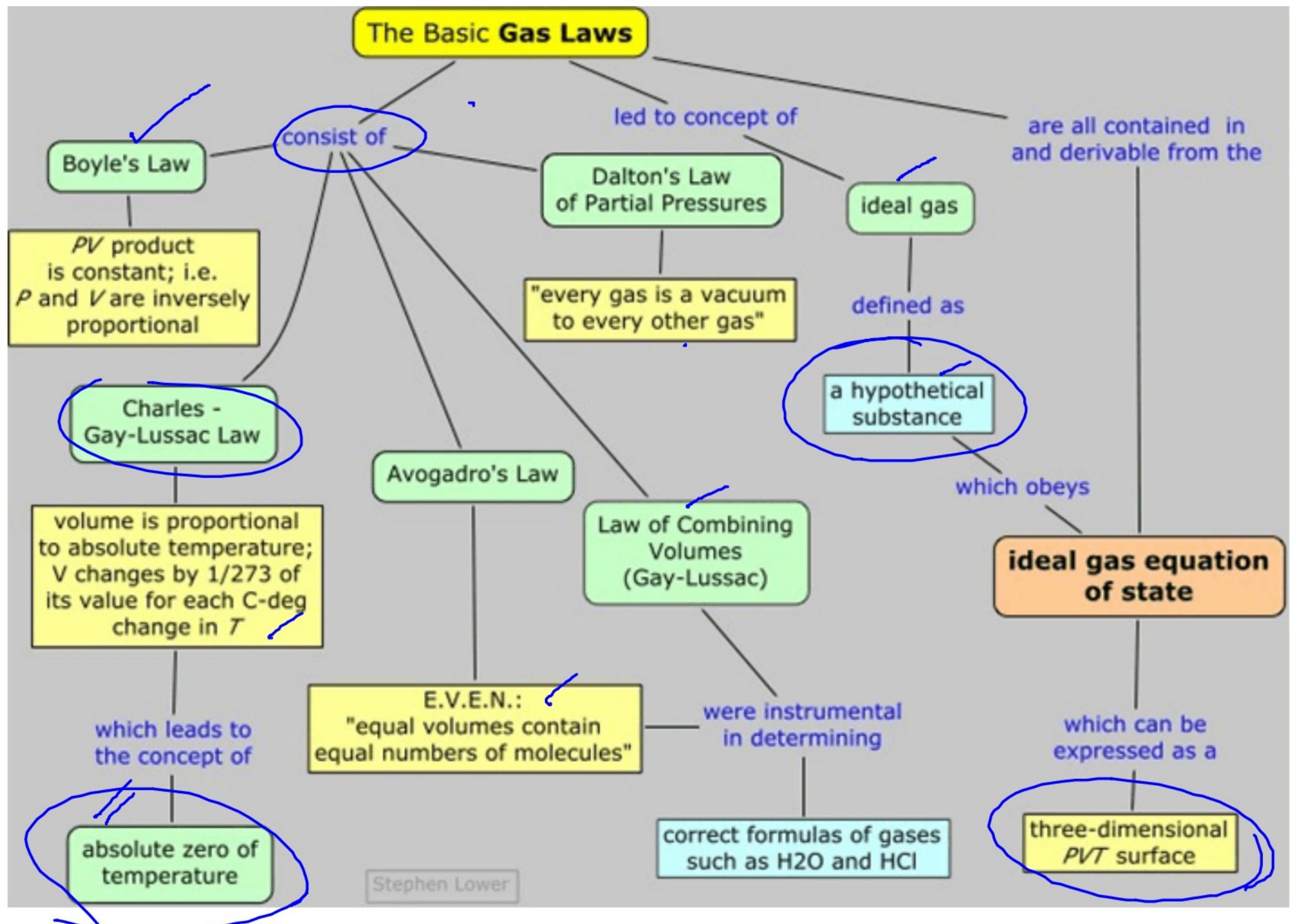
<i>Substance</i>	<i>Pressure (p_c) bar</i>	<i>Temperature (T_c) K</i>
Air	37.69	132.5
Argon	48.64	151.0
Carbon dioxide	73.86	304.2
Carbon monoxide	34.96	133.0
Helium	2.29	5.3
Hydrogen	12.97	33.3
Nitrogen	33.94	126.2
Oxygen	50.76	154.8
Water	228.59	647.15
Ethane	48.84	305.5
Ethylene	51.17	282.4
Methane	46.41	191.1
Propane	42.55	370.0



$$PV = ZRT$$







A rigid and insulated tank of volume 100 litres contains nitrogen at 300 K and 1 bar. An electrical resistor placed inside the tank is connected to a battery and current flows through the resistor till the gas raises to a temperature of 350 K. Assume that nitrogen is an ideal gas. The molar heat capacity at constant volume (C_v) of nitrogen is $5R/2$, where R is the universal gas constant. Determine the work done by the battery and the final pressure of the gas in the tank. **4.1667 kJ 1.1667 bar**

$$Q = 0$$

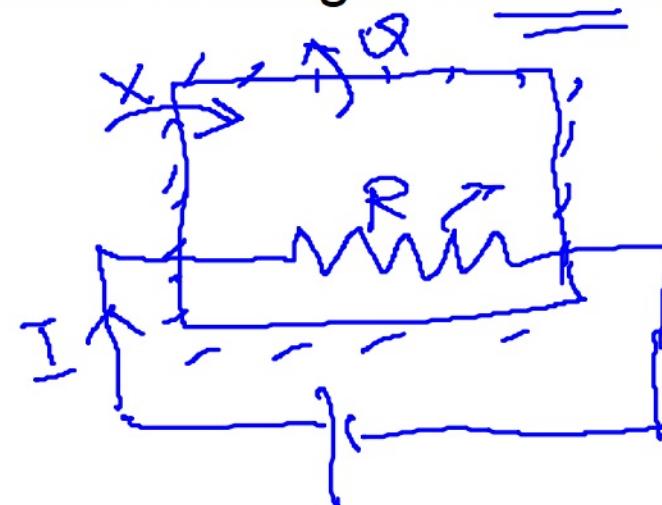
$$V = 100 \text{ l} \\ = 100 \times 10^{-3} \text{ m}^3$$

$$T_1 = 300 \text{ K}$$

$$P_1 = 1 \text{ bar}$$

$$T_2 = 350 \text{ K}$$

$$\underline{\underline{W}} = ?$$



$$Q = \underline{\underline{W}} + \Delta V$$

$$Q = W + \Delta V$$

$$W = -\Delta V$$

$$= -nC_v dT$$

$$W = -nC_v [T_2 - T_1]$$

$$W = \frac{1 \times 10^5 \times 100 \times 10^{-3}}{8314 \times 300}$$

$$n = \frac{33.33}{R}$$

$$W = -4$$

N_2 is to be stored in a steel flask of 40 litre capacity at 12 MPa, 20°C. The flask is to be protected against excessive pressure by a fusible plug which will melt and allow the gas to escape if the temperature rises too high.

(i) How many kg of nitrogen will the flask hold at the designed conditions ?

(ii) At what temperature must the fusible plug melt in order to limit the pressure of a full flask to a maximum of 150 bar ? **5.51 kg 366.7 K**

$$\frac{m}{Pv} = \frac{m}{RT}$$

$$m = \frac{Pv}{RT}$$

$$= \frac{12 \times 10^6 \times 40 \times 10^{-3}}{\frac{8314}{28} \times 293}$$

$$=$$

$$T = \frac{Pv}{mR}$$

$$=$$

Determine the specific volume of steam at 100 bar and 600°C by using i) steam tables, ii) perfect gas equation and iii) generalised compressibility chart.

0.0384 m³/kg

0.04032 m³/kg

0.0383 m³/kg

$$v_c^1 = RT_c / P_c$$

$$v_R^1 = \frac{v}{v_c^1} = \frac{vP_c}{RT_c}$$

$$PV = mRT$$

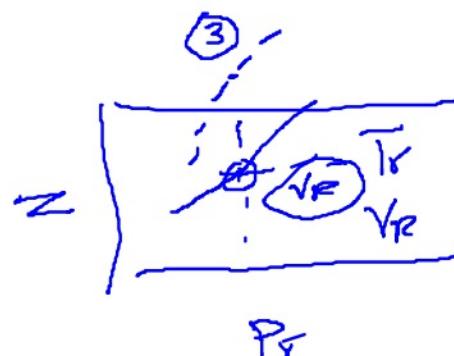
$$v = \frac{mRT}{P}$$

$$= \frac{1 \times \frac{8314}{18} \times 873}{100 \times 10^5 \text{ Pa}} =$$

$$\sqrt{\rho_r} = \frac{100}{228} = 0.43$$

$$\sqrt{T_r} = \frac{873}{(374 + 273)} = 1.35$$

$$3 = \frac{v \times 228 \times 10^5}{\frac{8314}{18} \times 373 + 273}$$



A container of 3 m³ capacity contains 10 kg of CO₂ at 27°C. Estimate the pressure exerted by CO₂ by using :

(i) Perfect gas equation

(ii) Van der Waals' equation

(iii) Beattie Bridgeman equation.

$$\begin{array}{l} \text{CO}_2 \\ \equiv \\ V = 3 \text{ m}^3 \\ m = 10 \text{ kg} \end{array}$$

1.889 bar 1.875 bar 1.9 bar

$$P = A_0 \left(1 - \frac{a}{V}\right)$$

$$= 507.2836 \left(1 - \frac{0.07132}{13.2}\right) = 504.54$$

$$B = B_0 \left(1 - \frac{b}{V}\right)$$

$$= 0.10476 \left(1 - \frac{0.07235}{13.2}\right) = 0.104$$

$$e = \frac{c}{V T^3} = \frac{66 \times 10^4}{13.2 \times 300^3} = 1.85 \times 10^{-3}$$

Molar specific volume $\bar{V} = M \times V / m$

$$\left(p + \frac{a}{V^2}\right) (\bar{V} - b) = R_0 T$$

$$= 44 \times \frac{3}{10} = 13.2$$

From Table 8.1

For CO₂ :

$$a = 362850 \text{ Nm}^4/(\text{kg-mol})^2$$

$$b = 0.0423 \text{ m}^3/\text{kg-mol}$$

$$p = \frac{R_0 T (1 - e)}{(V)^2} (\bar{V} + B) - \frac{A}{\bar{V}^2}$$

where p = pressure, $A = A_0 \left(1 - \frac{a}{V}\right)$, $B = B_0 \left(1 - \frac{b}{V}\right)$ and $e = \frac{c}{V T^3}$

$$A_0 = 507.2836, a = 0.07132$$

$$B_0 = 0.10476, b = 0.07235$$

$$C = 66 \times 10^4$$

$$\begin{aligned} & \left(P + \frac{362850}{13.2^2} \right) (13.2 - 0.07235) \\ & = 8314 \times 300 \end{aligned}$$

$$P = 1.8728 \text{ bar}$$

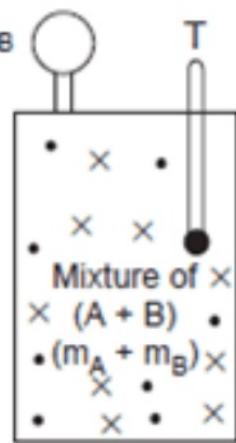
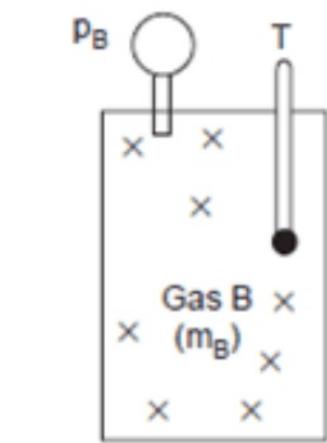
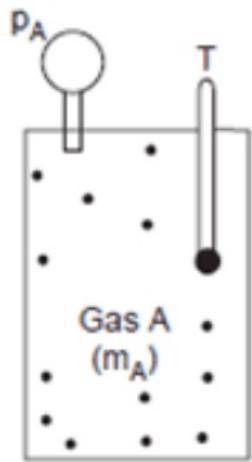
Ideal Gas Mixture

The properties of such mixtures are important in

- combustion calculations
- Psychrometrics

Dalton's law

The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.



(i)

(ii)

(iii)

$$m = m_A + m_B$$
$$p = p_A + p_B$$

$$m = m_A + m_B + m_C + \dots \text{ or } m = \sum m_i$$

m_i = Mass of a constituent.

$$p = p_A + p_B + p_C + \dots \text{ or } p = \sum p_i$$

p_i = The partial pressure of a constituent.

Gibbs-Dalton law

$$mu = m_A u_A + m_B u_B + \dots \text{ or } mu = \sum m_i u_i$$

$$mh = m_A h_A + m_B h_B + \dots \text{ or } mh = \sum m_i h_i$$

$$ms = m_A s_A + m_B s_B + \dots \text{ or } ms = \sum m_i s_i$$

Properties of air

Constituent	Molecular weight	Chemical symbol	Volumetric analysis %	Gravimetric analysis %
Oxygen	31.999	O ₂	20.95	23.14
Nitrogen	28.013	N ₂	78.09	75.53
Argon	39.948	Ar	0.93	1.28
Carbon dioxide	44.01	CO ₂	0.03	0.05

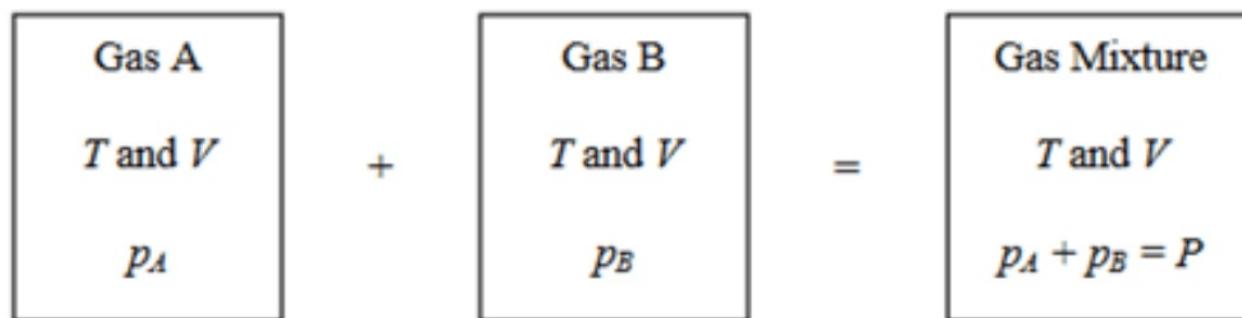
Approximate Properties of air

Constituent	Molecular weight	Volumetric analysis %	Gravimetric analysis %
Oxygen	32	21	23.3
Atmospheric nitrogen	28	79	76.7
Nitrogen/Oxygen	—	3.76 : 1	3.29 : 1

Volumetric analysis is the analysis by **volume**; gravimetric analysis is the analysis by **weight or mass**.

Dalton's Law

The total pressure (P) of an ideal gas mixture is equal to the sum of the partial pressures (p_j 's) of each individual component in the gas mixture at the same temperature (T) and total volume (V) of the mixture. For example,



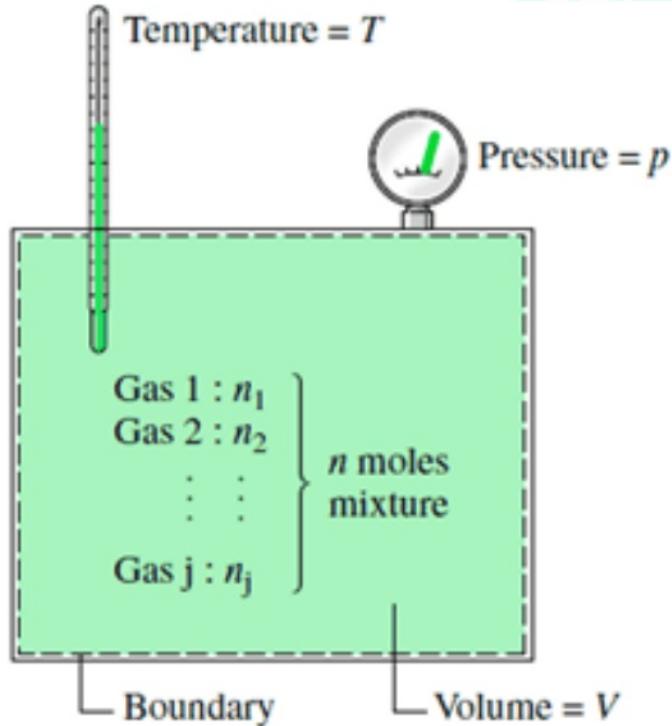
$$p_A V = n_A R T$$

$$p_B V = n_B R T$$

$$P V = n R T$$

$$\frac{p_A}{n_A} = \frac{p_B}{n_B} = \frac{P}{n} \quad \text{thus} \quad \frac{n_A}{n} = \frac{p_A}{P} = y_A \quad \text{and} \quad \frac{n_B}{n} = \frac{p_B}{P} = y_B$$

DALTON MODEL



$$p = n \frac{\bar{R}T}{V}$$

$$p_i = \frac{n_i \bar{R}T}{V}$$

$$\frac{p_i}{p} = \frac{n_i \bar{R}T/V}{n \bar{R}T/V} = \frac{n_i}{n} = y_i$$

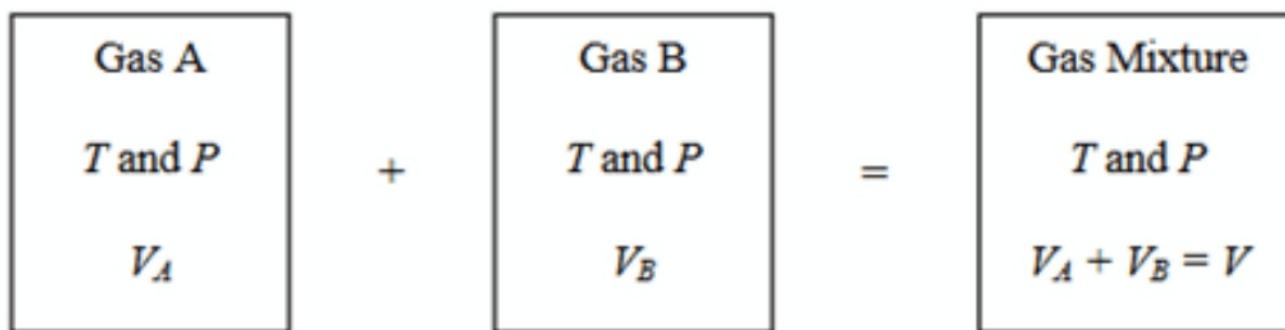
$$p_i = y_i p$$

$$\sum_{i=1}^j p_i = \sum_{i=1}^j y_i p = p \sum_{i=1}^j y_i \quad \sum_{i=1}^j y_i = 1$$

$$p = \sum_{i=1}^j p_i$$

Amagat's Law

The volume of an ideal gas mixture (V) is equal to the sum of the component volumes (V_j 's) of each individual component in the gas mixture at the same temperature (T) and total pressure (P) of the mixture. For example,



$$P V_A = n_A R T$$

$$P V_B = n_B R T$$

$$P V = n R T$$

$$\frac{V_A}{n_A} = \frac{V_B}{n_B} = \frac{V}{n} \quad \text{thus} \quad \frac{n_A}{n} = \frac{V_A}{V} = y_A \quad \text{and} \quad \frac{n_B}{n} = \frac{V_B}{V} = y_B$$

$$\frac{n_j}{n} = \frac{V_j}{V} = \frac{P_j}{P} = y_j$$

VOLUMETRIC ANALYSIS OF A GAS MIXTURE

The law of partial volumes Amagat's law or Leduc's law

V
T

$$\begin{aligned}m &= m_A + m_B + m_C = \sum m_i \\p &= p_A + p_B + p_C = \sum p_i \\n &= n_A + n_B + n_C = \sum n_i\end{aligned}$$

Fig (a)

v_A	v_B	v_C
p	p	p
m_A	m_B	m_C
n_A	n_B	n_C

P
T

Fig (b)

$$m_A = \frac{p_A V}{R_A T} \quad \text{Fig (a)}$$

$$\frac{p_A V}{R_A T} = \frac{p V_A}{R_A T} \quad i.e., \quad p_A V = p V_A$$

$$m_A = \frac{p V_A}{R_A T} \quad \text{Fig (b)}$$

$$V_A = \frac{p_A}{p} V$$

$$V_i = \frac{p_i}{p} V$$

$$\Sigma V_i = \sum \frac{p_i V}{p} = \frac{V}{p} \Sigma p_i$$

$$p = \Sigma p_i,$$

$$\Sigma V_i = V$$

$$n = \frac{m}{M}$$

n = Number of moles,

m = Mass of gas, and

M = Molecular weight.

$$n = n_A + n_B + n_C = \sum n_i$$

characteristic equation for mixture

$$pV = nR_0T,$$

$$p_iV = n_iR_0T$$

$$\sum p_iV = \sum n_iR_0T$$

$$V \sum p_i = R_0T \sum n_i$$

$$p = \sum p_i$$

$$pV = R_0T \sum n_i$$

$$n = \sum n_i$$

$$pV = nR_0T$$

mixture therefore acts as a perfect gas, and this is the characteristic equation for mixture.

The Gas Constant of a mixture

$$pV = mRT$$

$$p_i V = m_i R_i T$$

$$\sum p_i V = \sum m_i R_i T$$

$$V \sum p_i = T \sum m_i R_i$$

$$p = \sum p_i$$

$$pV = T \sum m_i R_i$$

$$pV = mRT = T \sum m_i R_i$$

$$mR = \sum m_i R_i$$

$$R = \sum \frac{m_i}{m} R_i$$

$$\frac{p_i V}{pV} = \frac{n_i R_0 T}{n R_0 T}$$

$$\frac{p_i}{p} = \frac{n_i}{n}$$

$$\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V}$$

$$R = \frac{R_0}{M}$$

$\frac{n_i}{n}$ = mole fraction of the constituent

$\frac{m_i}{m}$ = mass fraction of a constituent.

molar analysis is identical with the volumetric analysis, and both are equal to the ratio of the partial pressure to the total pressure.

A mixture of ideal gases consists of 4 kg of nitrogen and 6 kg of carbon dioxide at a pressure of 4 bar and a temperature of 20°C. Find :

- ✓(i) The mole fraction of each constituent, 0.511 0.488
- ✓(ii) The equivalent molecular weight of the mixture, 35.78 kg/kg mole
- ✓(iii) The equivalent gas constant of the mixture, 0.232 kJ/kg K.
- ✓(iv) The partial pressures and partial volumes, 2.044 bar 1.952 bar 0.87 m³. 0.83 m³
- ✓(v) The volume and density of the mixture, 1.699 m³. 5.88 kg/m³.
- ✓(vi) The cp and cv of the mixture. 0.925 kJ/kg K. 0.693 kJ/kg K.

$$\begin{aligned} X_{N_2} &= \frac{m_{N_2}}{m} \\ &= \frac{4}{4+6} \\ &= 0.4 \\ X_{CO_2} &= \frac{6}{10} \\ &= 0.6 \end{aligned}$$

If the mixture is heated at constant volume to 50°C, find the changes in internal energy, enthalpy and entropy of the mixture. Find the changes in internal energy, enthalpy and entropy of the mixture if the heating is done at constant pressure.

Take γ : for CO₂ = 1.286 and for N₂ = 1.4.

207.9 kJ. 277.5 kJ 0.675 kJ/kg K. 207.9 kJ. 277.5 kJ 0.902 kJ/kg K

$$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{4}{28} = 0.142$$

$$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{6}{44} = 0.136$$

$$\left. \begin{array}{l} p=c \\ y_{N_2} = \frac{n_{N_2}}{n} \\ = \frac{0.142}{0.142 + 0.136} \\ = 0.510 \\ y_{CO_2} = \frac{0.136}{0.142 + 0.136} \\ = 0.489 \end{array} \right\}$$

$$M_{\text{mix}} = \sum \frac{n_i}{n} M_i$$

$$\begin{aligned} M_{\text{mix}} &= 0.51 \times 28 + 0.489 \times 44 \\ &= 35.796 \end{aligned}$$

$$\boxed{R_{\text{mix}} = \sum \frac{n_i}{m} R_i} \quad = \quad \frac{8314}{M_{\text{mix}}} = \frac{8314}{35.796} = 0.232 \text{ kJ/kgK}$$

$$= \frac{m_{N_2}}{m} R_{N_2} + \frac{m_{CO_2}}{m} R_{CO_2}$$

$$= \frac{4}{10} \times \frac{8314}{28} + \frac{6}{10} \times \frac{8314}{44}$$

$$= 0.2322 \text{ kJ/kgK}$$

$$\frac{n_i}{n} = \frac{P_i}{P} \Rightarrow \frac{n_{N_2}}{n} = \frac{P_{N_2}}{P} \Rightarrow \frac{0.142}{0.142 + 0.136} = \frac{P_{N_2}}{4}$$

$P_{N_2} = 2.04 \text{ bar}$

$$\frac{n_{CO_2}}{n} = \frac{P_{CO_2}}{P} -$$

$$\frac{0.136}{0.142 + 0.136} = \frac{P_{CO_2}}{4}$$

$$P_{CO_2} = 1.956 \text{ bar}$$

$$P_{N_2} + P_{CO_2} = P$$

$$2.04 + P_{CO_2} = 4$$

$$P_{CO_2} = 4 - 2.04$$

$$PV = \frac{mRT}{N_A N_2}$$

$$V_{N_2} = \frac{m_{N_2} R_{N_2} T}{P} = \frac{4 \times \frac{8314}{28} \times 293}{4 \times 10^5} = 0.86 \text{ m}^3$$

$$V_{CO_2} = \frac{6 \times \frac{8314}{44} \times 293}{4 \times 10^5} = 0.83 \text{ m}^3$$

$$V_{\text{mix}} = \frac{V_{N_2} + V_{CO_2}}{} \\ = 0.86 + 0.83 \\ = 1.69 \text{ m}^3$$

$\checkmark ?$

$$P_{\text{mix}} = \frac{m_{\text{mix}} R T}{V_{\text{mix}}}$$

$$V_{\text{mix}} = \frac{m_{\text{mix}} R_{\text{mix}} T}{P} \\ = \frac{10 \times 0.232 \times 10 \times 293}{4 \times 10^5} \\ = 1.70 \text{ m}^3$$

$$\rho_{\text{mix}} = \frac{m_{\text{mix}}}{V_{\text{mix}}} = \frac{10}{1.69} = 5.91 \text{ kg/m}^3 \checkmark$$

$$C_{P_{N_2}}$$

$$C_{P_{CO_2}}$$

$$C_{P_{N_2}} - C_{V_{N_2}} = R_{N_2}$$

$$\frac{C_P}{C_V} = \gamma$$

$$C_P = C_V \gamma$$

$$\gamma C_{V_{N_2}} - C_{V_{N_2}} = R_{N_2}$$

$$C_{V_{N_2}}(\gamma - 1) = R_{N_2}$$

$$C_{V_{N_2}} = \frac{R_{N_2}}{\gamma - 1}$$

$$= \frac{\frac{8314}{28}}{1.4 - 1} = 0.742 \text{ kJ/kgK}$$

$$C_{P_{N_2}} = \gamma \times C_{V_{N_2}} = 1.039 \text{ kJ/kgK}$$

$$C_{V_{CO_2}} = \frac{\frac{8314}{44}}{1.286 - 1} = 0.66 \text{ kJ/kgK} \quad C_{P_{CO_2}} = 0.849 \text{ kJ/kgK}$$

$$C_p_{\text{mix}} = \sum \frac{m_i}{m} C_{p,i}$$

$$= \frac{4}{10} \times 1.039 + \frac{6}{10} \times 0.849 \\ = 0.925 \text{ kJ/kgK}$$

$$C_v_{\text{mix}} = \frac{4}{10} \times 0.74 + \frac{6}{10} \times 0.661 \\ = 0.6926 \text{ kJ/kgK}$$

✓

$\textcircled{V=C}$ $20^\circ \text{C} \rightarrow 50^\circ \text{C}$

$$\Delta V = m C_V dT = 10 \times 0.6926 \times (50 - 20) = \begin{cases} P = C \\ \Delta r = m C_V (T_2 - T_1) \end{cases}$$

$$\Delta H = m C_p dT = 10 \times 0.925 \times (50 - 20) = \begin{cases} \Delta H = m C_p (T_2 - T_1) \\ \Delta S = m C_p \ln \frac{T_2}{T_1} \end{cases}$$

$$\Delta S = m C_V \ln \frac{T_2}{T_1} + m e \ln \frac{T_2}{V_1} \\ = 10 \times 0.6926 \ln \left(\frac{50+273}{20+273} \right) = \begin{cases} \\ \\ \\ \\ = 10 \times 0.925 \ln \frac{323}{293} \end{cases}$$

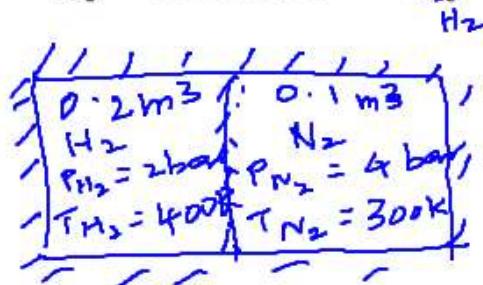
A tank of capacity 0.3 m³ is insulated and divided into two sections by a partition. One section is 0.2 m³ in volume and initially contains H₂ at 2 bar and 127°C. The remaining section initially holds N₂ at 4 bar and 27°C. The adiabatic partition is then removed, and the gases are allowed to mix. Determine :

✓(i) The temperature of the equilibrium mixture,

✓(ii) The pressure of the mixture, and

(iii) The change in entropy for each component and total value.

$$\begin{aligned} c_{v(N_2)} &= 0.744 \text{ kJ/kg K}, & c_{v(H_2)} &= 10.352 \text{ kJ/kg K} & [\text{Ans. (i) } 342.6 \text{ K; (ii) } 2.661 \text{ bar; (iii) } (\Delta S)_{H_2} = 0.00235 \text{ kJ/K;} \\ c_{p(N_2)} &= 1.041 \text{ kJ/kg K}, & c_{p(H_2)} &= 14.476 \text{ kJ/kg K.} & (\Delta S)_{N_2} = 0.1908 \text{ kJ/K; } \Delta S = 0.19315 \text{ kJ/K}] \end{aligned}$$



$$Q = W + \Delta V$$

$$Q = 0 + V_2 - V_1$$

$$V_1 = V_2$$

$$m_{H_2} C_{V,H_2} T_{H_2} + m_{N_2} C_{V,N_2} T_{N_2} = m_{H_2} C_{V,H_2} T + m_{N_2} C_{V,N_2} T$$

$$P_{H_2} V_{H_2} = m_{H_2} R_{H_2} T_{H_2}$$

$$m_{H_2} = \frac{2 \times 10^5 \times 0.2}{8314 \times 400} = 0.024 \text{ kg}$$

$$T = 342.6 \text{ K}$$

$$\begin{aligned} m_{N_2} &= \frac{P V}{R T} n_2 \\ &= \frac{4 \times 10^5 \times 0.1}{8314 \times 300} \\ &= 0.449 \text{ kg} \end{aligned}$$

$$PV = mRT$$

$$P_{H_2}V = m_{H_2}R_{H_2}T$$

$$P_{H_2} = \frac{m_{H_2} R_{H_2} T}{V}$$

$$= \frac{0.024 \times \frac{8314}{2} \times 342.65}{0.3}$$

$$= 1.13 \text{ bar}$$

$$P_{N_2} = \frac{0.449 \times \frac{8314}{28} \times 342.65}{0.3}$$

$$= 1.522 \text{ bar}$$

$$P_{\text{mix}} = P_{H_2} + P_{N_2} = 2.65 \text{ bar}$$

8 kg of nitrogen at 160°C and 100 kPa are mixed with 4 kg of carbon dioxide at 40°C and 0.14 MPa ~~bar~~ to form a mixture at a final pressure of 70 kPa. The process occurs adiabatically in a steady flow apparatus. Calculate :

The final temperature of the mixture ;

The change in entropy. $(\Delta S)_{N_2} + (\Delta S)_{CO_2}$

$$\frac{mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1}}{\frac{n_j}{n} = \frac{V_j}{V} = \frac{P_j}{P} = y_j}$$

Take value of c_p : for CO₂ = 0.85 kJ/kg K and N₂ = 1.04 kJ/kg K. 398.2 K. 3.2232 kJ/K.

$m_{N_2} = 8 \text{ kg}$
 $T_{N_2} = 160 + 273 \text{ K}$
 $P_{N_2} = 100 \text{ kPa}$
 $n_{N_2} = \frac{8}{28}$
 $n_{CO_2} = \frac{4}{44}$
 $n = n_{N_2} + n_{CO_2}$
 $H_1 = H_2$
 $m_{N_2} c_p T + m_{CO_2} c_p T = (m_{N_2} c_p + m_{CO_2} c_p) T$
 $m_{CO_2} = 4 \text{ kg}$
 $T_{CO_2} = 40 + 273 \text{ K}$
 $P_{CO_2} = 0.14 \text{ MPa}$
 $8 \times 1.04 \times 433 + 4 \times 0.85 \times 313 = (8 \times 1.04 + 4 \times 0.85) T$
 $T = ?$