MEE1004-FLUID MECHANICS

Dimensional Analysis

Lecture 4

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Rayleigh's Method of Dimensional analysis

- This method is used for determining the expression for a variable which depends up on maximum three or four variable only.
- If the number of independent variables becomes more than four, then it is very difficult to find the expression for the dependent variable.

Let X is a variable, which depends on X_1, X_2 and X_3 variables.

Rayleigh's Method of Dimensional analysis

Let X is a variable, which depends on X_1, X_2 and X_3 variables.

Then according to Rayleigh's method, X is function of X_1, X_2 and X_3

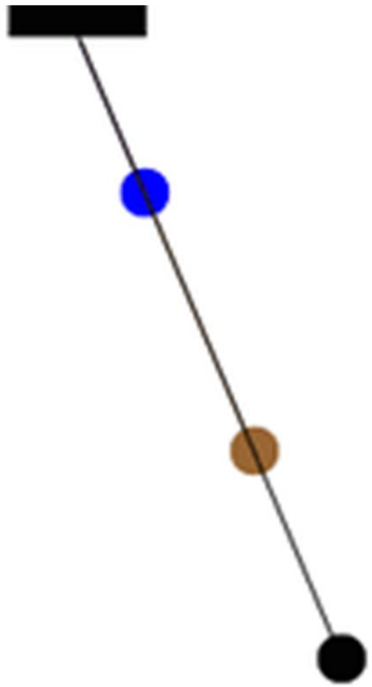
This can also be written as $X = KX_1^a \cdot X_2^b \cdot X_3^c$

where K is constant and a, b and c are arbitrary powers.

The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides. Thus the expression is obtained for dependent variable.

Rayleigh's Method of Dimensional analysis

The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.



Rayleigh's Method of Dimensional analysis

The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

Solution. Time period t is a function of (i) L and (ii) g

$\therefore$ $$t = KL^a \cdot g^b, \text{ where } K \text{ is a constant}$$

Substituting the dimensions on both sides $T^1 = KL^a \cdot (LT^{-2})^b$

Equating the powers of M, L and T on both sides, we have

Power of T, $\quad 1 = -2b \quad \therefore \quad b = -\dfrac{1}{2}$

Power of L, $\quad 0 = a + b \quad \therefore \quad a = -b = -\left(-\dfrac{1}{2}\right) = \dfrac{1}{2}$

Substituting the values of a and b in equation (i),

$$t = KL^{1/2} \cdot g^{-1/2} = K\sqrt{\frac{L}{g}}$$

The value of K is determined from experiments which is given as

$$K = 2\pi$$

$\therefore$ $$t = 2\pi\sqrt{\frac{\mathbf{L}}{\mathbf{g}}}. \textbf{ Ans.}$$

Rayleigh's Method of Dimensional analysis

Find an expression for the drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

Solution. Drag force F is a function of

(*i*) Diameter, D (*ii*) Velocity, V (*iii*) Density, ρ

(*iv*) Viscocity, μ

$D^a \quad V^b \quad e^c \quad \mu^d$

$$\therefore \qquad F = KD^a \cdot V^b \cdot \rho^c \cdot \mu^d$$

$a \quad b \quad c \quad d$

where K is non-dimensional factor.

Substituting the dimensions on both sides,

$$MLT^{-2} = KL^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d$$

Equating the powers of M, L and T on both sides,

Power of M, $\qquad 1 = c + d$

Power of L, $\qquad 1 = a + b - 3c - d$

Power of T, $\qquad -2 = -b - d.$

Rayleigh's Method of Dimensional analysis

Equating the powers of M, L and T on both sides,

Power of M, $\quad 1 = c + d$

Power of L, $\quad 1 = a + b - 3c - d$

Power of T, $\quad -2 = -b - d.$

3 eq.
a, b, c, d

There are four unknowns (a, b, c, d) but equations are three. Hence it is not possible to find the values of a, b, c and d. But three of them can be expressed in terms of fourth variable which is most important. Here viscosity is having a vital role and hence a, b, c are expressed in terms of d which is the power to viscosity.

$\therefore$

$$c = 1 - d$$

$$b = 2 - d$$

$$a = 1 - b + 3c + d = 1 - 2 + d + 3(1 - d) + d$$

$$= 1 - 2 + d + 3 - 3d + d = 2 - d$$

Substituting these values of a, b and c in (i), we get

$$F = KD^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d$$

$$= KD^2V^2\rho\,(D^{-d} \cdot V^{-d} \cdot \rho^{-d} \cdot \mu^d) = K\rho D^2V^2\left(\frac{\mu}{\rho VD}\right)^d$$

$$= \mathbf{K\rho D^2V^2\phi\left(\frac{\mu}{\rho VD}\right)}.\ \textbf{Ans.}$$

$F = K\rho V^2 D^2 \left(\frac{1}{Re}\right)$

$1/Re$

D, ρ, V, μ

Buckingham Pi Theorem Application

12.4.2 Buckingham's π-Theorem. The Rayleigh's method of dimensional analysis becomes more laborious if the variables are more than the number of fundamental dimensions (M, L, T). This difficulty is overcame by using Buckingham's π-theorem, which states, "If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms. Each term is called π-term".

Let $X_1, X_2, X_3, ..., X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3, ..., X_n$ are the independent variables on which X_1 depends. Then X_1 is a function of $X_2, X_3, ..., X_n$ and mathematically it is expressed as

$$X_1 = f(X_2, X_3, ..., X_n) \qquad ...(12.1)$$

Equation (12.1) can also be written as

$$f_1(X_1, X_2, X_3, ..., X_n) = 0. \qquad ...(12.2)$$

Equation (12.2) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham's π-theorem, equation (12.2) can be written in terms of number of dimensionless groups or π-terms in which number of π-terms is equal to $(n - m)$. Hence equation (12.2) becomes as

$$f(\pi_1, \pi_2, ..., \pi_{n-m}) = 0. \qquad ...(12.3)$$

Each of π-terms is dimensionless and is independent of the system. Division or multiplication by a constant does not change the character of the π-term. Each π-term contains $m + 1$ variables, where m is the number of fundamental dimensions and is also called repeating variables. Let in the above case X_2, X_3 and X_4 are repeating variables if the fundamental dimension m (M, L, T) = 3. Then each π-term is written as

Buckingham Pi Theorem Application

$$\left.\begin{aligned}
\pi_1 &= X_2^{a_1} \,.\, X_3^{b_1} \,.\, X_4^{c_1} \,.\, X_1 \\
\pi_2 &= X_2^{a_2} \,.\, X_3^{b_2} \,.\, X_4^{c_2} \,.\, X_5 \\
&\vdots \\
\pi_{n-m} &= X_2^{a_{n-m}} \,.\, X_3^{b_{n-m}} \,.\, X_4^{c_{n-m}} \,.\, X_n
\end{aligned}\right\}$$

Problem 12.6 *The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force.*

$R = f(l, V, \mu, \rho, K)$ 5 I

→ no. of var. $n = 6$ MLT 3 D

no. of repeating, $m = 3$

no. of π term, $n - m \Rightarrow 6 - 3 = 3\pi$

$\Rightarrow m + 1 = 3 + 1 = 4$

$\pi_1 = l^{a_1} V^{b_1} \rho^{c_1} R$

$f(\pi_1, \pi_2, \pi_3) = 0$

$\pi_2 = l^{a_2} V^{b_2} \rho^{c_2} \mu$

$M^0 L^0 T^0 = l^{a_1} V^{b_1} \rho^{c_1} R$

$\pi_3 = l^{a_3} V^{b_3} \rho^{c_3} K$.

$\pi_1, \pi_2, \pi_3, \pi_3$

Buckingham Pi Theorem Application

The Method of Repeating Variables

Step 1:	List the parameters in the problem and count their total number *n*.
Step 2:	List the primary dimensions of each of the *n* parameters.
Step 3:	Set the *reduction j* as the number of primary dimensions. Calculate *k*, the expected number of Π's, $k = n - j$
Step 4:	Choose *j repeating parameters*.
Step 5:	Construct the *k* Π's, and manipulate as necessary.
Step 6:	Write the final functional relationship and check your algebra.

FIGURE 7–22

A concise summary of the six steps that comprise the *method of repeating variables*.

Guidelines for choosing *repeating paramet*

Guideline

1. Never pick the *dependent* variable. Otherwise, it may appear in all the Π's, which is undesirable.
2. The chosen repeating parameters must not *by themselves* be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the Π's.
3. The chosen repeating parameters must represent *all* the primary dimensions in the problem.

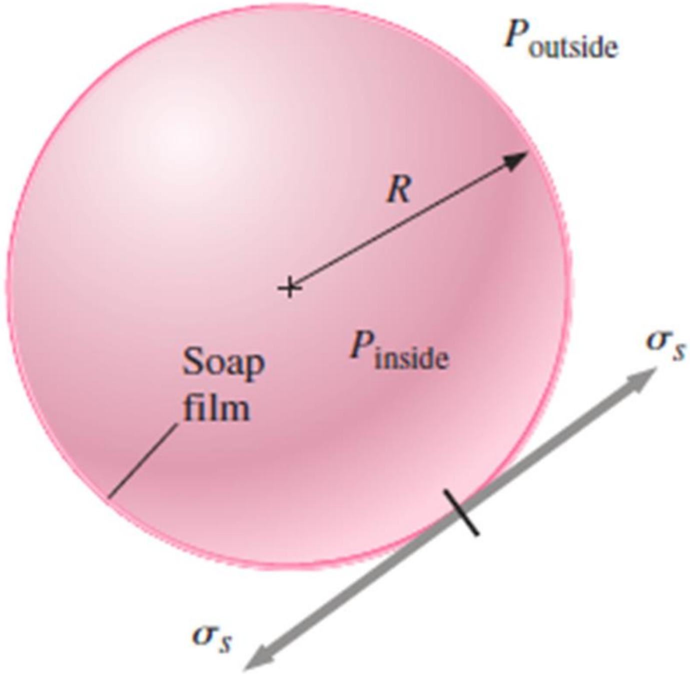
μ

ρ V

D

4. Never pick parameters that are already dimensionless. These are Π's already, all by themselves.
5. Never pick two parameters with the *same* dimensions or with dimensions that differ by only an exponent.
6. Whenever possible, choose dimensional constants over dimensional variables so that only *one* Π contains the dimensional variable.
7. Pick common parameters since they may appear in each of the Π's.
8. Pick simple parameters over complex parameters whenever possible.

Some children are playing with soap bubbles, and you become curious as to the relationship between soap bubble radius and the pressure inside the soap bubble (Fig. 7–29). You reason that the pressure inside the soap bubble must be greater than atmospheric pressure, and that the shell of the soap bubble is under tension, much like the skin of a balloon. You also know that the property surface tension must be important in this problem. Not knowing any other physics, you decide to approach the problem using dimensional analysis. Establish a relationship between pressure difference $\Delta P = P_{\text{inside}} - P_{\text{outside}}$, soap bubble radius R, and the surface tension σ_s of the soap film.



The pressure inside a soap bubble is greater than that surrounding the soap bubble due to surface tension in the soap film.

Step 1 There are three variables and constants in this problem; $n = 3$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

List of relevant parameters: $\Delta P = f(R, \sigma_s) \qquad n = 3$

Step 2 The primary dimensions of each parameter are listed. The dimensions of surface tension are obtained from Example 7–1, and those of pressure from Example 7–2.

$$\begin{array}{ccc} \Delta P & R & \sigma_s \\ \{\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-2}\} & \{\mathrm{L}^1\} & \{\mathrm{m}^1\mathrm{t}^{-2}\} \end{array}$$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction (first guess): $j = 3$

If this value of j is correct, the expected number of Π's is $k = n - j = 3 - 3 = 0$. But how can we have zero Π's? Something is obviously not right (Fig. 7–30). At times like this, we need to first go back and make sure that we are not neglecting some important variable or constant in the problem. Since we are confident that the pressure difference should depend only on soap bubble radius and surface tension, we reduce the value of j by one,

Reduction (second guess): $$j = 2$$

If this value of j is correct, $k = n - j = 3 - 2 = 1$. Thus we expect *one* Π, which is more physically realistic than zero Π's.

Step 4 We need to choose two repeating parameters since $j = 2$. Following the guidelines of Table 7–3, our only choices are R and σ_s, since ΔP is the dependent variable.

Step 5 We combine these repeating parameters into a product with the dependent variable ΔP to create the dependent Π,

Dependent Π: $$\Pi_1 = \Delta P R^{a_1} \sigma_s^{b_1} \tag{1}$$

We apply the primary dimensions of step 2 into Eq. 1 and force the Π to be dimensionless.

Dimensions of Π_1:

$$\{\Pi_1\} = \{m^0L^0t^0\} = \{\Delta P R^{a_1}\sigma_s^{\ b_1}\} = \{(m^1L^{-1}t^{-2})L^{a_1}(m^1t^{-2})^{b_1}\}$$

We equate the exponents of each primary dimension to solve for a_1 and b_1:

Time:	$\{t^0\} = \{t^{-2}t^{-2b_1}\}$	$0 = -2 - 2b_1$	$b_1 = -1$
Mass:	$\{m^0\} = \{m^1m^{b_1}\}$	$0 = 1 + b_1$	$b_1 = -1$
Length:	$\{L^0\} = \{L^{-1}L^{a_1}\}$	$0 = -1 + a_1$	$a_1 = 1$

Fortunately, the first two results agree with each other, and Eq. 1 thus becomes

$$\Pi_1 = \frac{\Delta P R}{\sigma_s} \tag{2}$$

From Table 7–5, the established nondimensional parameter most similar to Eq. 2 is the **Weber number**, defined as a pressure (ρV^2) times a length divided by surface tension. There is no need to further manipulate this Π.

Step 6 We write the final functional relationship. In the case at hand, there is only one Π, which is a function of *nothing*. This is possible only if the Π is constant. Putting Eq. 2 into the functional form of Eq. 7–11,

Relationship between Π*'s*:

$$\Pi_1 = \frac{\Delta P R}{\sigma_s} = f(\text{nothing}) = \text{constant} \quad \rightarrow \quad \boldsymbol{\Delta P = \text{constant}\,\frac{\sigma_s}{R}} \tag{3}$$

The pressure difference Δp in a pipe of diameter D and length l due to viscous flow depends on the velocity V, viscosity μ and density ρ. Using Buckingham's π-theorem, obtain an expression for Δp.

$$\Delta p = f(V, D, l, \mu, \rho)$$

Dep

Δp - V

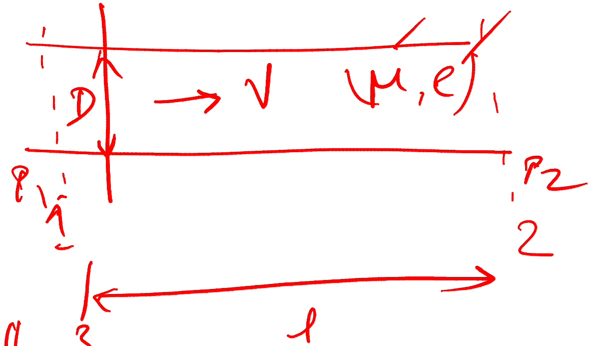
No. of Variables = 6

No. of Rep $\} m = 3$

$\rightarrow n - m = 6 - 3$

π terms $= 3$

$\rightarrow$ Total no. of $= m + 1 = 4$



Geo.

fluid.

flow.

$$\rightarrow \pi_1 = D^{a_1} V^{b_1} \mu^{c_1} . \Delta p$$

$$\pi_2 = D^{a_2} V^{b_2} \mu^{c_2} . l$$

$$\pi_3 = D^{a_3} V^{b_3} \mu^{c_3} . \rho$$

$$\Pi_1 = D^{a_1} V^{b_1} \mu^{c_1} \cdot \Delta P$$

$\Rightarrow N/m^2$ $\Rightarrow ma$ $\frac{MLT^{-2}}{L^2}$

$$f(\Pi_1, \Pi_2, \Pi_3) = 0$$

$$\underbrace{M^0 L^0 T^0}_{0} = L^{a_1} (LT^{-1})^{b_1} (ML^{-1}T^{-1})^{c_1} \cdot (ML^{-1}T^{-2})$$

$$M^0 L^0 T^0 = L^{a_1+b_1-c_1-1} \, T^{-b_1-c_1-2} \, M^{c_1+1}$$

$\Rightarrow$ Power of $M \rightarrow 0 = c_1 + 1 \Rightarrow c_1 = -1$

$L \rightarrow 0 = a_1 + b_1 - c_1 - 1 \Rightarrow a_1 - 1 + \cancel{1} - \cancel{1} = 0$, $a_1 = 1$

$T \rightarrow 0 = -b_1 - c_1 - 2 \Rightarrow -b_1 - (-1) - 2 = 0$

$-b_1 + 1 - 2 = 0$

$-b_1 - 1 = 0$

$\boxed{b_1 = -1}$

$a_1 = 1, \; b_1 = -1, \; c_1 = -1$

$$\Pi_1 = D^1 V^{-1} \mu^{-1} \Delta P$$

$$\Pi_1 \Rightarrow \frac{\Delta P \cdot D}{\mu V}$$

$$\Pi_3 = D^1 \cdot V^1 \mu^{-1} \cdot \rho^{0}$$

Π_1, Π_2, Π_3

$$\Pi_3 = \frac{\rho V D}{\mu} = Re$$

$$ML \quad 0 = a_2 + b_2 - \cancel{c_2} + 1 \Rightarrow a_2 = -1$$

$$T \quad 0 = -b_2 - \cancel{c_2} : b_2 = 0$$

$$\Pi_2 = D^{-1} \cdot \ell$$

$$\Pi_2 = \frac{\ell}{D}$$

$$\Pi_3 = D^{a_3} V^{b_3} \mu^{c_3} \cdot \rho$$

$$M^0 L^0 T^0 = L^{a_3} \cdot L^{b_3} \cdot T^{-b_3} \cdot M^{c_3} \cdot L^{-c_3} \cdot T^{-c_3} \cdot ML^{-3}$$

M: $0 = c_3 + 1 \Rightarrow c_3 = -1$

L: $a_3 + b_3 - c_3 - 3 = 0$

$a_3 + -c_3 - c_3 - 3 = 0$

$a_3 - 2c_3 - 3 = 0$

T: $0 = -b_3 - c_3$

$b_3 = -c_3$

$a_3 + 2 - 3 = 0$

$a_3 - 1 = 0$

$a_3 = 1$

$b_3 = 1$

$$\Pi_1 = \frac{\Delta P \cdot D}{\mu V} \; ; \; \Pi_2 = \frac{l}{D} \; ; \; \Pi_3 = \frac{\rho V D}{\mu}$$

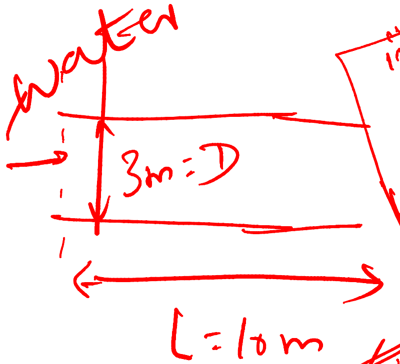
$$f(\Pi_1, \Pi_2, \Pi_3) = 0$$

$$f\left(\frac{\Delta P \cdot D}{\mu V}, \frac{l}{D}, \frac{\rho V D}{\mu}\right) = 0$$

$$\Delta P = \left(\frac{\mu V}{D}\right) \left(\frac{\rho V D}{\mu}\right), \left(\frac{l}{D}\right)$$

Re

Non-Dim

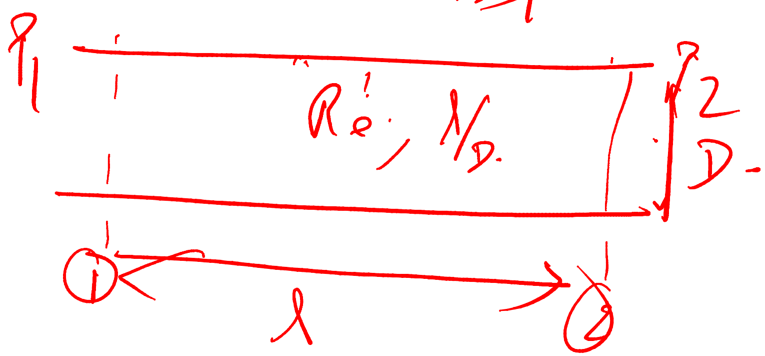


Δp

$\rho g h_f$ Δp

h_f

ΔP



Now Δp is a function of D, l, V, μ, ρ or $\Delta p = f(D, l, V, \mu, \rho)$

or $$f_1(\Delta p, D, l, V, \mu, \rho) = 0 \qquad \ldots(i)$$

Total number of variables, $n = 6$

Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - 3 = 6 - 3 = 3$

Hence equation (*i*) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0 \qquad \ldots(ii)$

Each π-term contains $m + 1$ variables, *i.e.*, $3 + 1 = 4$ variables. Out of four variables, three are repeating variables.

Choosing D, V, μ as repeating variables, we have π-terms as

$$\left.\begin{aligned}\pi_1 &= D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p\\ \pi_2 &= D^{a_2} . V^{b_2} . \mu^{c_2} . l\\ \pi_3 &= D^{a_3} . V^{b_3} . \mu^{c_3} . \rho\end{aligned}\right\}$$

First π-term $\qquad \pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-1}T^{-1})^{c_1} . ML^{-1}T^{-2}.$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_1 + 1, \quad \therefore \; c_1 = -1$

Power of L, $\quad 0 = a_1 + b_1 - c_1 - 1, \quad \therefore \; a_1 = -b_1 + c_1 + 1 = 1 - 1 + 1 = 1$

Power of T, $\quad 0 = -b_1 - c_1 - 2, \quad \therefore \; b_1 = -c_1 - 2 = 1 - 2 = -1$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^1 . V^{-1} . \mu^{-1} . \Delta p = \frac{D\Delta p}{\mu V}.$$

Second π-term $\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-1}T^{-1})^{c_2} . L.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_2$, $\therefore$ $c_2 = 0$

Power of L, $0 = a_2 + b_2 - c_2 + 1$, $\therefore$ $a_2 = -b_2 + c_2 - 1 = -1$

Power of T, $0 = -b_2 - c_2$, $\therefore$ $b_2 = -c_2 = 0$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-1} . V^0 . \mu^0 . l = \frac{l}{D}.$$

Third π-term $\pi_3 = D^{a_3} . V^{b_3} . \mu^{c_3} . \rho$

Substituting the dimension on both sides,

$$M^0L^0T^0 = L^{a_3} . (LT^{-1})^{b_3} . (ML^{-1}T^{-1})^{c_3} . ML^{-3}.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_3 + 1$, $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 + b_3 - c_3 - 3$, $\therefore$ $a_3 = -b_3 + c_3 + 3 = -1 - 1 + 3 = 1$

Power of T, $0 = -b_3 - c_3$, $\therefore$ $b_3 = -c_3 = -(-1) = 1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^1 . V^1 . \mu^{-1} . \rho = \frac{\rho DV}{\mu}.$$

Substituting the values of π_1, π_2 and π_3 in equation (*ii*),

$$f_1\left(\frac{D\Delta p}{\mu V}, \frac{l}{D}, \frac{\rho DV}{\mu}\right) = 0 \quad \text{or} \quad \frac{D\Delta p}{\mu V} = \phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right] \quad \text{or} \quad \Delta p = \frac{\mu V}{D}\phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right]$$

Experiments show that the pressure difference Δp is a linear function $\frac{l}{D}$. Hence $\frac{l}{D}$ can be taken out of the functional as

$$\Delta p = \frac{\mu \mathbf{V}}{\mathbf{D}} \times \frac{l}{\mathbf{D}} \phi\left[\frac{\rho \mathbf{DV}}{\mu}\right]. \textbf{ Ans.}$$

Expression for difference of pressure head for viscous flow

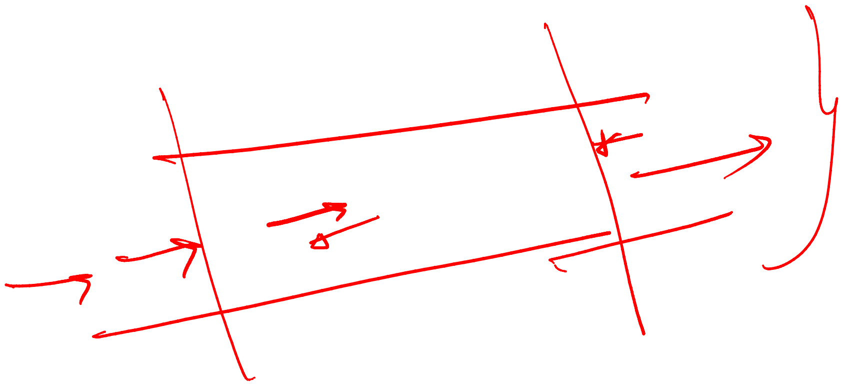
$$h_f = \frac{\Delta p}{\rho g} = \frac{\mu V}{D} \times \frac{l}{D} \times \frac{1}{\rho g}\, \phi\, [R_e] \qquad \left\{\because \quad \frac{\rho DV}{\mu} = R_e\right\}$$

$$= \frac{\mu \mathbf{Vl}}{\mathbf{wD}^2} \phi\, [\mathbf{R_e}]. \textbf{ Ans.}$$

TYPES OF FORCES ACTING IN 'MOVING FLUID'

For the fluid flow problems, the forces acting on a fluid mass may be any one, or a combination of the several of the following forces :

1. Inertia force, F_i.
2. Viscous force, F_v.
3. Gravity force, F_g. (open)
4. Pressure force, F_p.
5. Surface tension force, F_s.
6. Elastic force, F_e.



5.

Non-Dimensional

'Dimensionless' para

TYPES OF FORCES ACTING IN MOVING FLUID

1. **Inertia Force** (F_i). It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration. It is always existing in the fluid flow problems.
2. **Viscous Force** (F_v). It is equal to the product of shear stress (τ) due to viscosity and surface area of the flow. It is present in fluid flow problems where viscosity is having an important role to play.
3. **Gravity Force** (F_g). It is equal to the product of mass and acceleration due to gravity of the flowing fluid. It is present in case of open surface flow.
4. **Pressure Force** (F_p). It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid. It is present in case of pipe-flow.
5. **Surface Tension Force** (F_s). It is equal to the product of surface tension and length of surface of the flowing fluid.
6. **Elastic Force** (F_e). It is equal to the product of elastic stress and area of the flowing fluid.

For a flowing fluid, the above-mentioned forces may not always be present. And also the forces, which are present in a fluid flow problem, are not of equal magnitude. There are always one or two forces which dominate the other forces. These dominating forces govern the flow of fluid.

DIMENSIONLESS NUMBERS

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters. The followings are the important dimensionless numbers :

1. Reynold's number,
2. Froude's number,
3. Euler's number,
4. Weber's number,
5. Mach's number.

$\frac{V}{C}$

$Fr = \frac{V}{\sqrt{gL}}$

$Fr = \frac{V}{gL}$

$\times \frac{\rho D}{\rho D}$

$\frac{1}{g}$

12.8.1 Reynold's Number (R_e). It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

$$\text{Inertia force } (F_i) = \text{Mass} \times \text{Acceleration of flowing fluid}$$

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho \times AV \times V \qquad \{\because \text{ Volume per sec} = \text{Area} \times \text{Velocity} = A \times V\}$$

$$= \rho AV^2 \qquad \text{...(12.11)}$$

$$\text{Viscous force } (F_v) = \text{Shear stress} \times \text{Area} \qquad \left\{\because \quad \tau = \mu \frac{du}{dy} \quad \therefore \quad \text{Force} = \tau \times \text{Area}\right\}$$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \cdot \frac{V}{L} \times A \qquad \left\{\because \quad \frac{du}{dy} = \frac{V}{L}\right\}$$

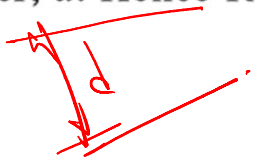
By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho AV^2}{\mu \cdot \frac{V}{L} \times A} = \frac{\rho VL}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \qquad \left\{\because \quad \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho Vd}{\mu}. \qquad \text{...(12.12)}$$



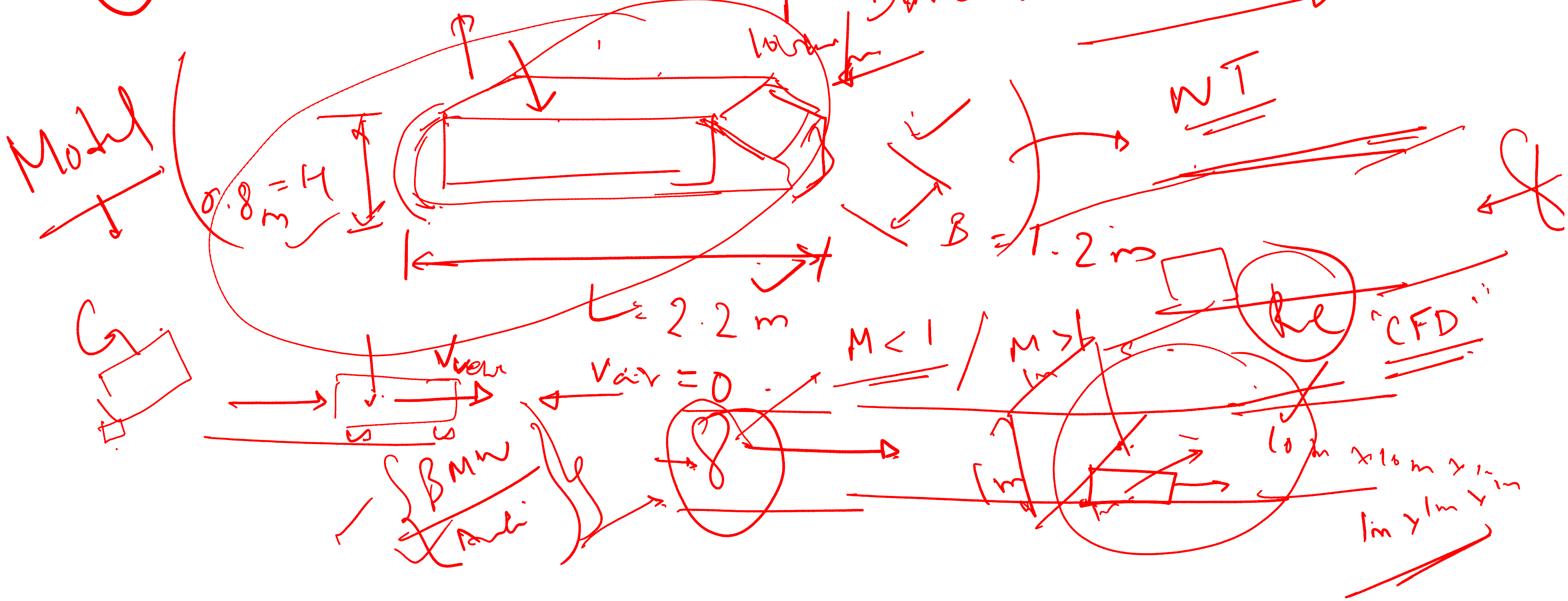
Three necessary conditions for complete similarity between a model and a prototype:

1) Geometric similarity
2) Kinematic similarity
3) Dynamic similarity

Similitude, similarity

Re

Dimensional



Thank you very much..☺

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