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MEE1004 – Fluid Mechanics

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Moody’s chart

Horizontal pipe: $$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

Then the volume flow rate for laminar flow through a horizontal diameter D and length L becomes

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

Inclined pipe

$$V_{avg} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L} \quad \text{and} \quad \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

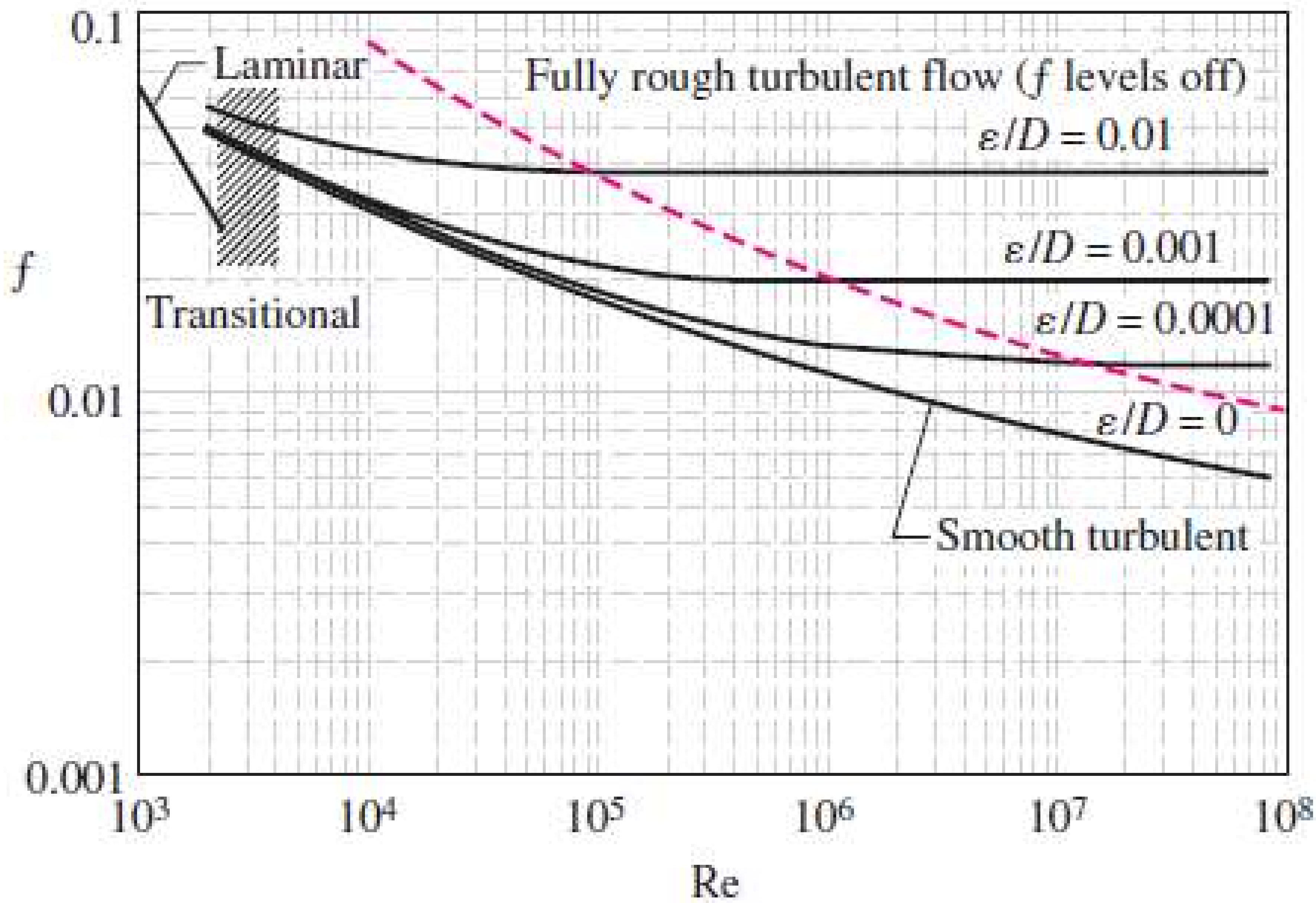
Friction factor

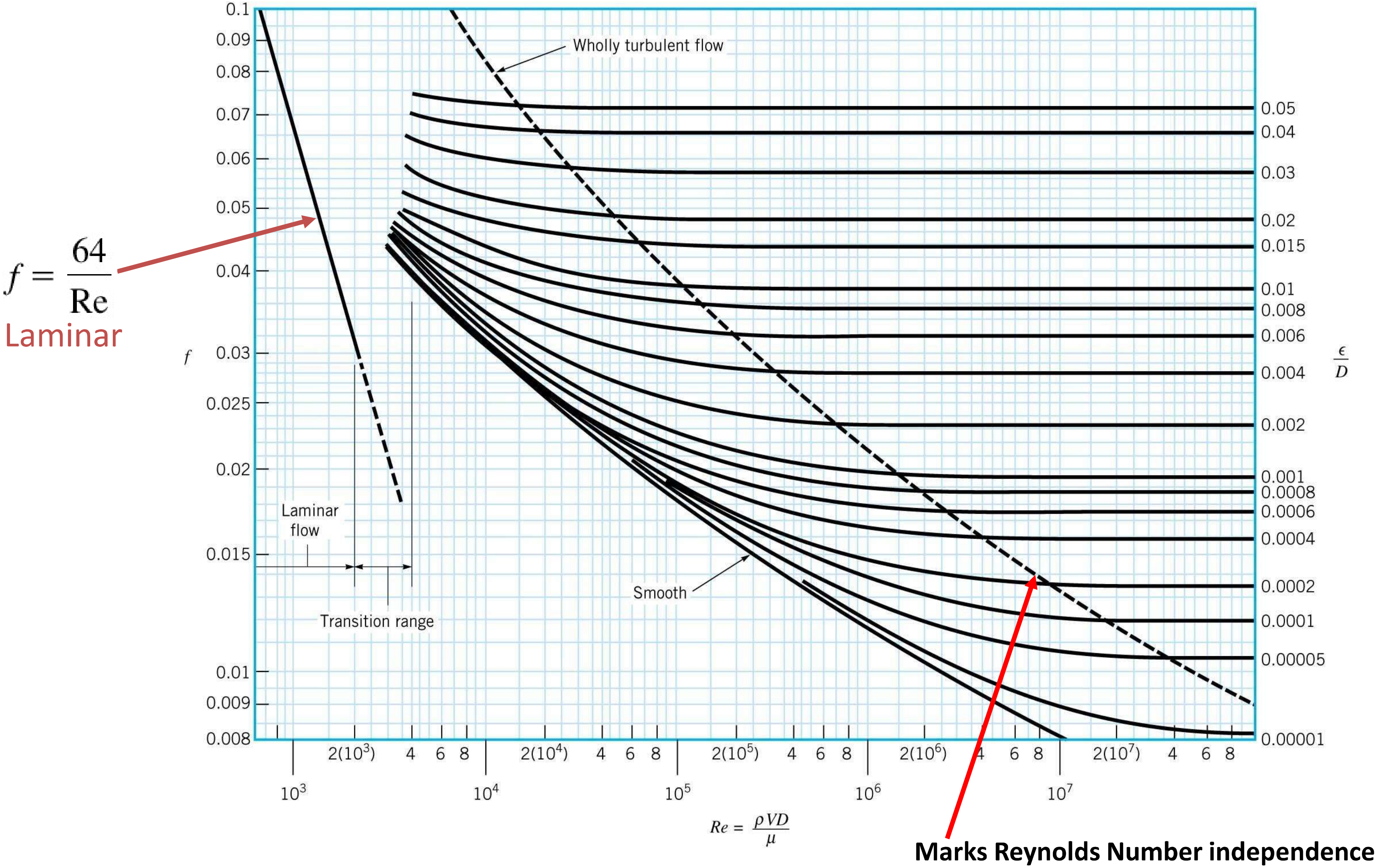
Experiment on friction factor

Darcy, Nikuradse, Moody – Did experiments to study the influence of various parameters on fiction factor

Nikuradse experimntation on pipe, artifically roughened by sand grain. Grain diameter was taken as rougness *ε*

Moody's chart





$f = \frac{64}{\mathrm{Re}}$
Laminar
Wholly turbulent flow
Laminar flow
Transition range
Smooth
f
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
$\frac{\epsilon}{D}$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
10^3
$2(10^3)$
10^4
$2(10^4)$
10^5
$2(10^5)$
10^6
$2(10^6)$
10^7
$2(10^7)$
4 6 8
$Re = \frac{\rho V D}{\mu}$
Marks Reynolds Number independence

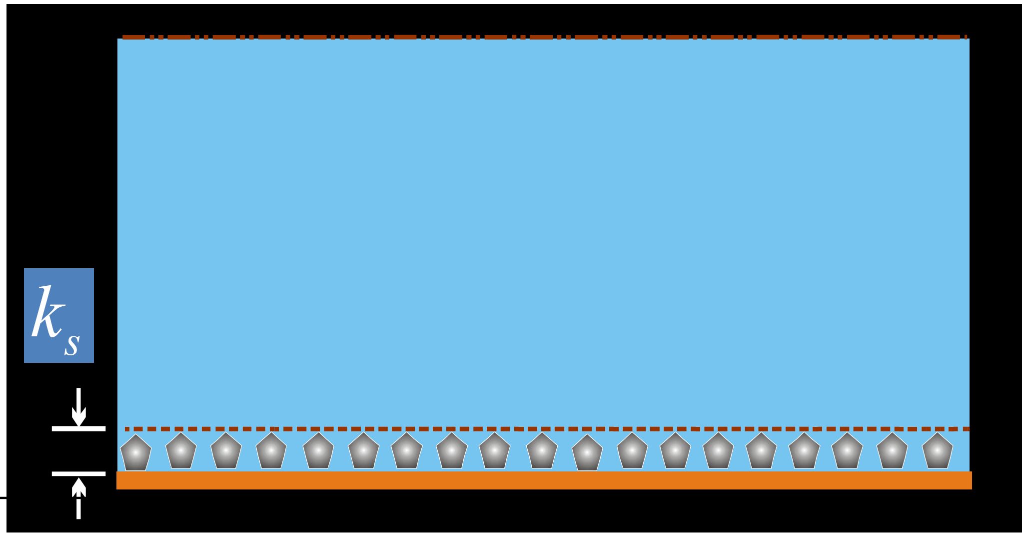
Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

* The uncertainty in these values can be as much as ±60 percent.

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness** e/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter.

Experiment of Nikuradse



F. Colebrook (1910–1997) combined the available data for transition and turbulent flow in smooth as well as rough pipes into the following implicit relation known as the **Colebrook equation:**

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

The Colebrook equation is implicit in f, and thus the determination of the friction factor requires some iteration unless an equation solver such as EES is used. An approximate explicit relation for f was given by S. E. Haaland in 1983 as

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Losses in Pipe Flows

Major Losses: due to friction, significant head loss is associated with the straight portions of pipe flows. This loss can be calculated using the Moody chart or Colebrook equation.

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right), \text{ valid for nonlaminar range}$$

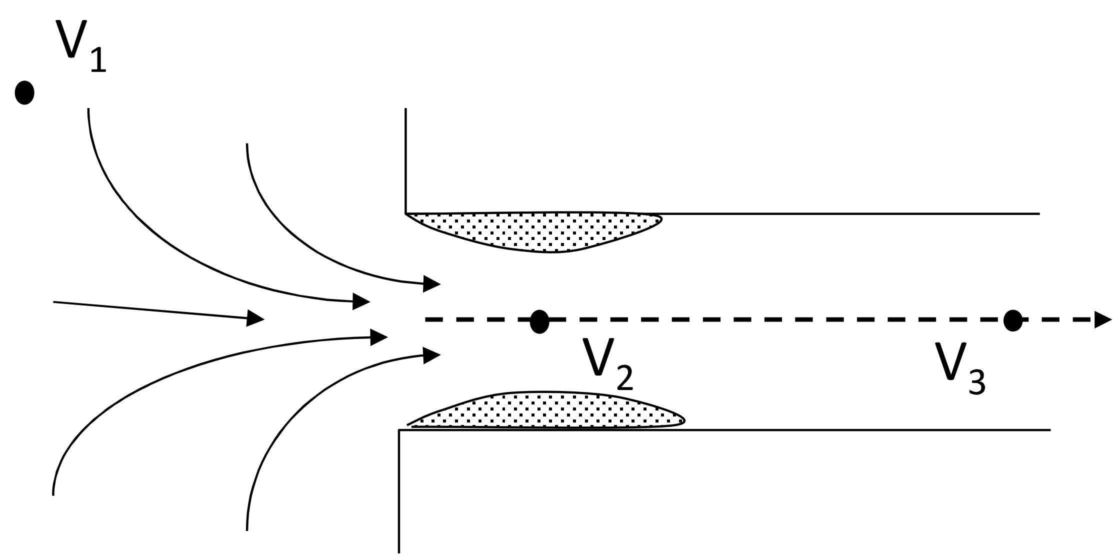
Minor Losses: Additional components (valves, bends, tees, contractions, etc) in pipe flows also contribute to the total head loss of the system. Their contributions are generally termed minor losses.

The head losses and pressure drops can be characterized by using the loss coefficient,
K_L, which is defined as

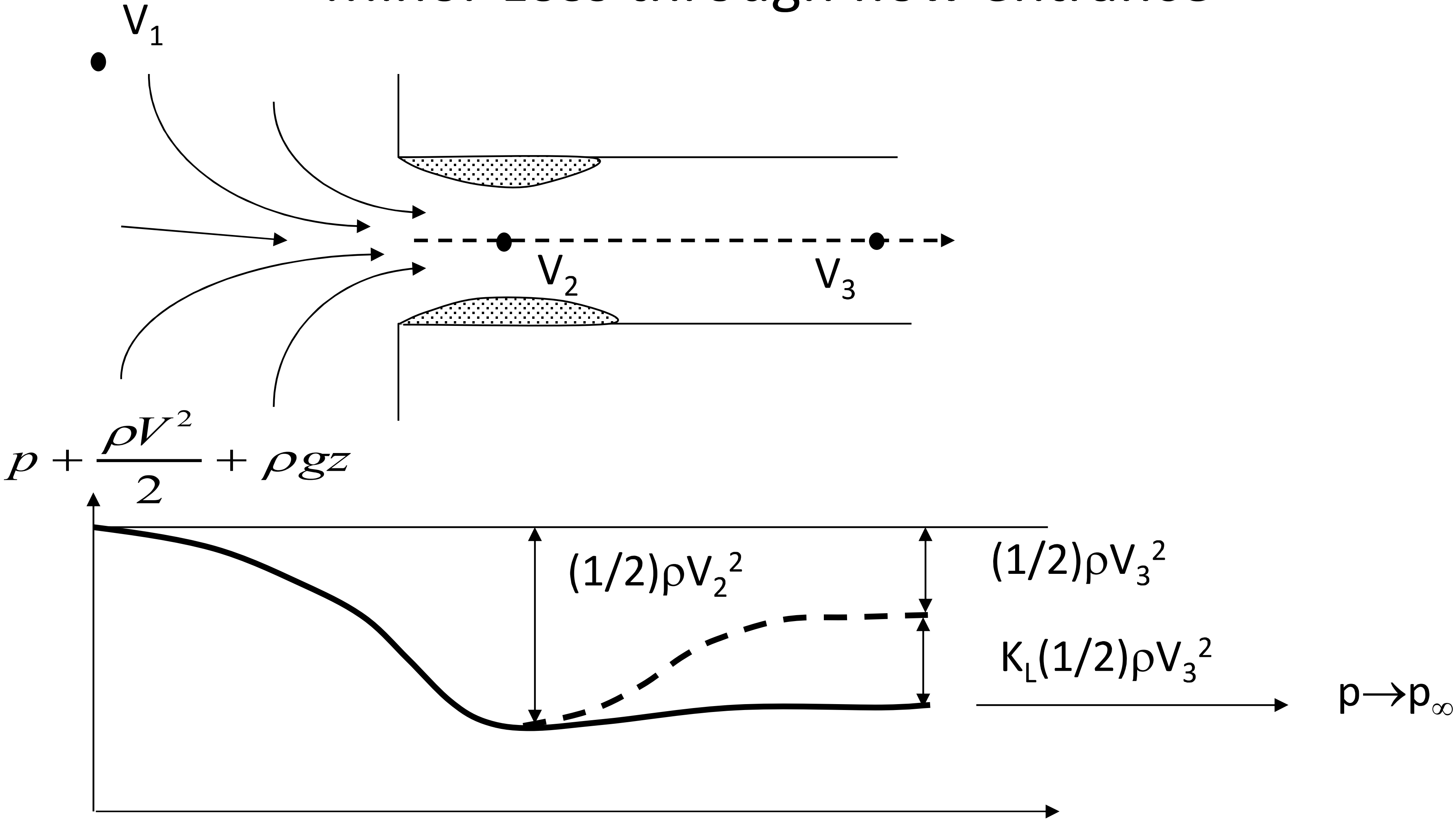
$$K_L = \frac{h_L}{V^2/2g} = \frac{\Delta p}{\frac{1}{2}\rho V^2}, \quad \text{so that } \Delta p = K_L \tfrac{1}{2}\rho V^2$$

One of the example of minor losses is the entrance flow loss. A typical flow pattern for flow entering a sharp-edged entrance is shown in the following page. A vena contracta region if formed at the inlet because the fluid can not turn a sharp corner.

Flow separation and associated viscous effects will tend to decrease the flow energy and the phenomenon is complicated. To simplify the analysis, a head loss and the associated loss coefficient are used in the extended Bernoulli's equation to take into consideration of this effect.

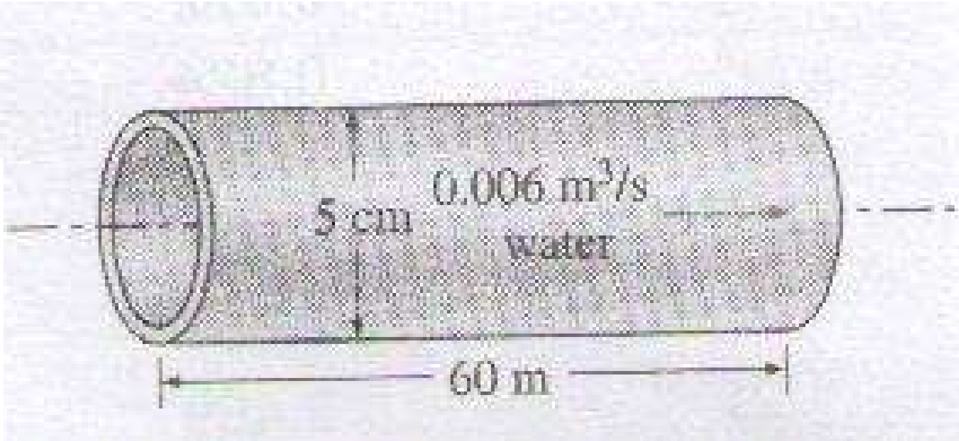


Minor Loss through flow entrance



Extended Bernoulli's Equation: $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3, \quad h_L = K_L \frac{V_3^2}{2g}$

Water at 15°C (ρ = 999 kg/m^3 and μ = 1.138 × 10^{-3} kg/m · s) is flowing steadily in a 5-cm-diameter horizontal pipe made of stainless steel at a rate of 0.006 m^3/s (Fig. 8–30). Determine the pressure drop, the head loss, and the required pumping power input for flow over a 60-m-long section of the pipe.



Equivalent roughness values for new commercial pipes*

	Roughness, ε	
Material	ft	mm
Glass, plastic	0 (smooth)	
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* The uncertainty in these values can be as much as ±60 percent.

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Types of Fluid Flow Problems

In the design and analysis of piping systems that involve the use of the Moody chart (or the Colebrook equation), we usually encounter three types of problems (the fluid and the roughness of the pipe are assumed to be specified in all cases) (Fig. 8–20):

$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \mathrm{Re} < 3\times 10^{8} \end{array}$$

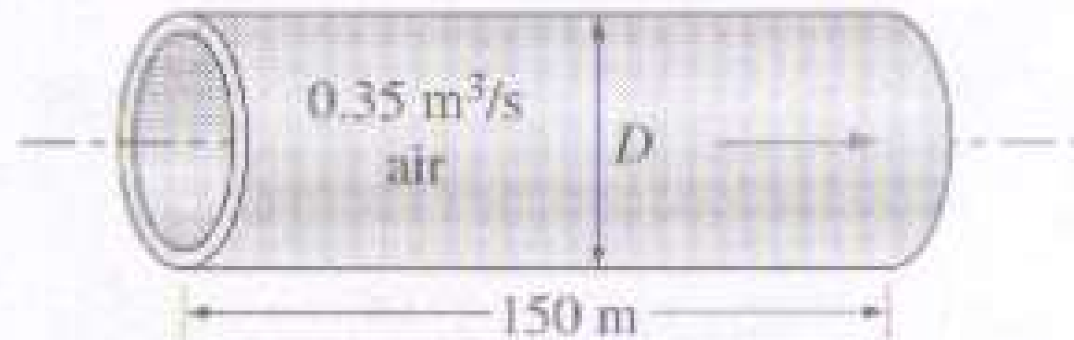
$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad \mathrm{Re} > 2000$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \mathrm{Re} < 3\times 10^{8} \end{array}$$

tic

that are accurate to within 2 percent of the Moody chart:

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m^3/s (Fig. 8–31). If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.



- The minor losses occurs due to:

  - Valves
  - Tees
  - Bends
  - Reducers
  - And other appurtenances

- It has the common form

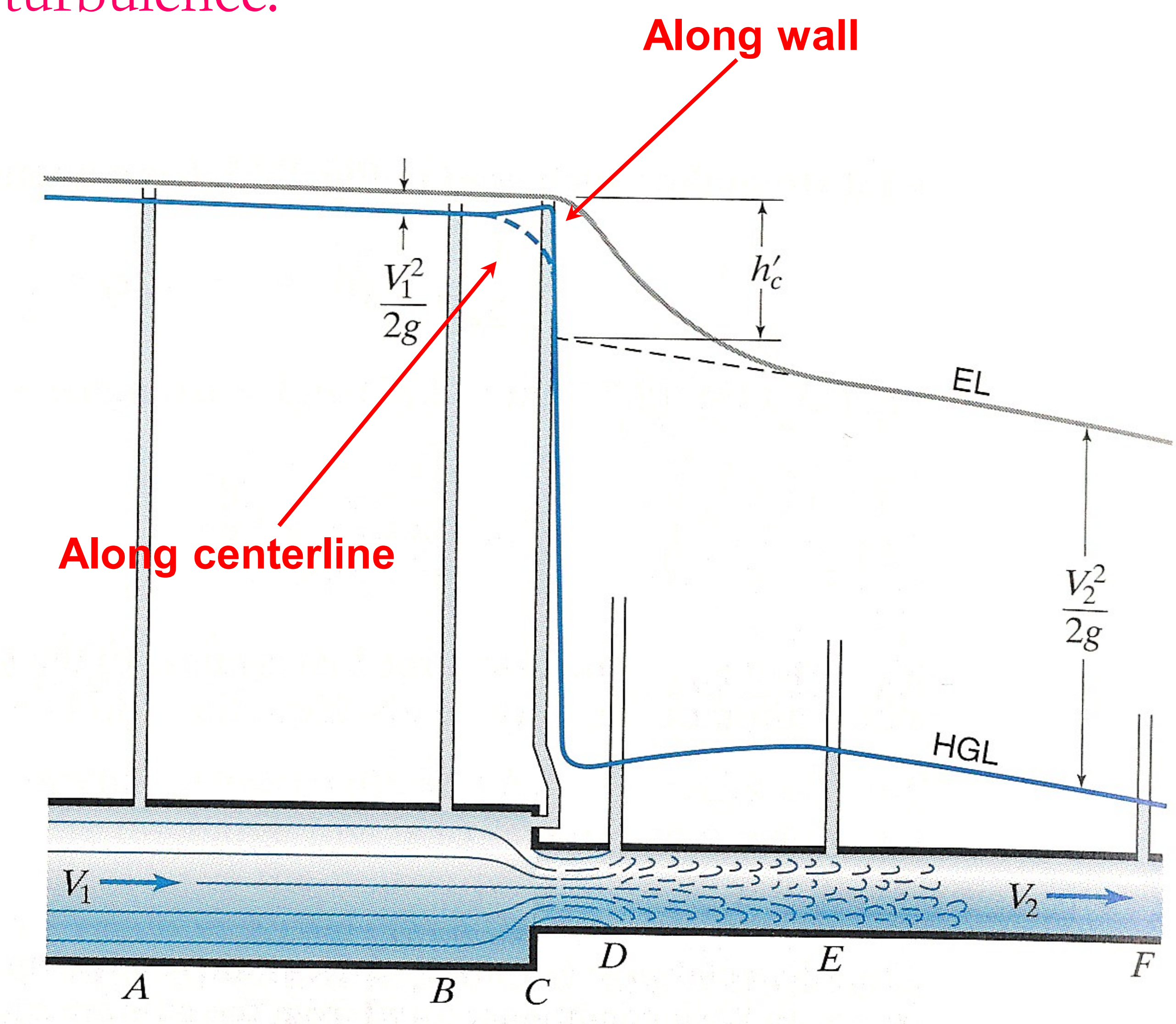
$$h_m = k_L \frac{V^2}{2g} = k_L \frac{Q^2}{2gA^2}$$

“minor” compared to friction losses in long pipelines but,

can be the dominant cause of head loss in shorter pipelines

Losses due to contraction

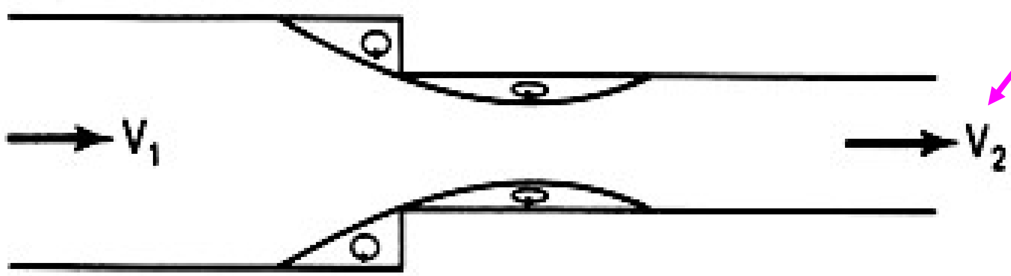
<u>A sudden contraction</u> in a pipe usually causes a marked drop in pressure in the pipe due to both the increase in velocity and the loss of energy to turbulence.



$$h_c = k_c \frac{V_2^{\ 2}}{2g}$$

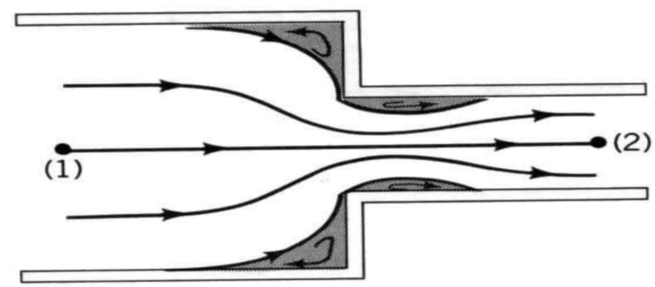
Value of the coefficient K_c for sudden contraction

Ratio of smaller to larger pipe diameters, D_2/D_1										Velocity m/s V_2
0.90	**0.80**	**0.70**	**0.60**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.00**	
0.03	0.07	0.18	0.28	0.38	0.42	0.45	0.48	0.49	0.49	1
0.04	0.09	0.18	0.28	0.37	0.41	0.44	0.47	0.48	0.48	2
0.04	0.10	0.18	0.28	0.36	0.40	0.43	0.45	0.46	0.47	3
0.05	0.11	0.19	0.27	0.33	0.37	0.40	0.42	0.43	0.44	6
0.06	0.13	0.20	0.25	0.29	0.31	0.33	0.35	0.36	0.38	12

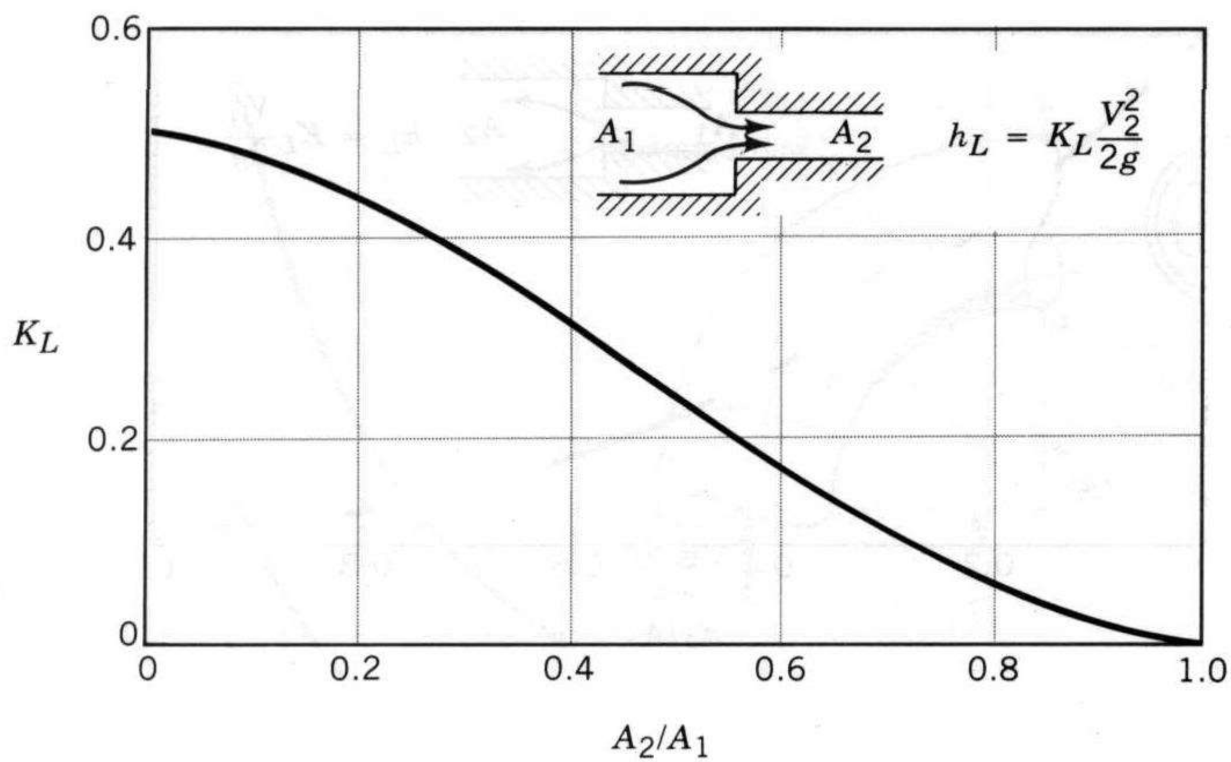


Head Loss Due to a Sudden Contraction

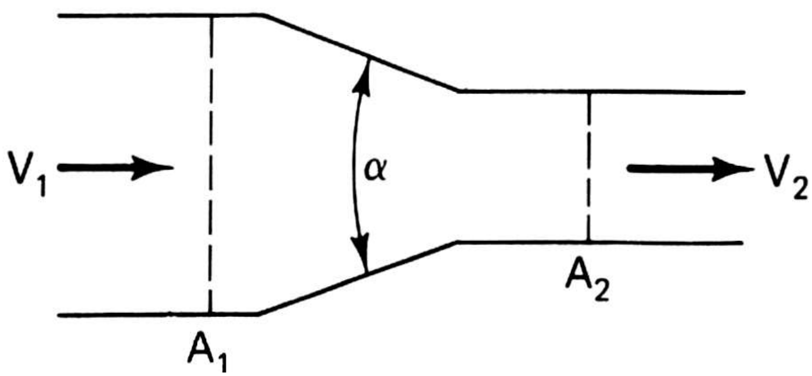
$$h_L = K_L \frac{V_2^{\ 2}}{2g}$$



$$h_L = 0.5 \frac{V_2^{\ 2}}{2g}$$

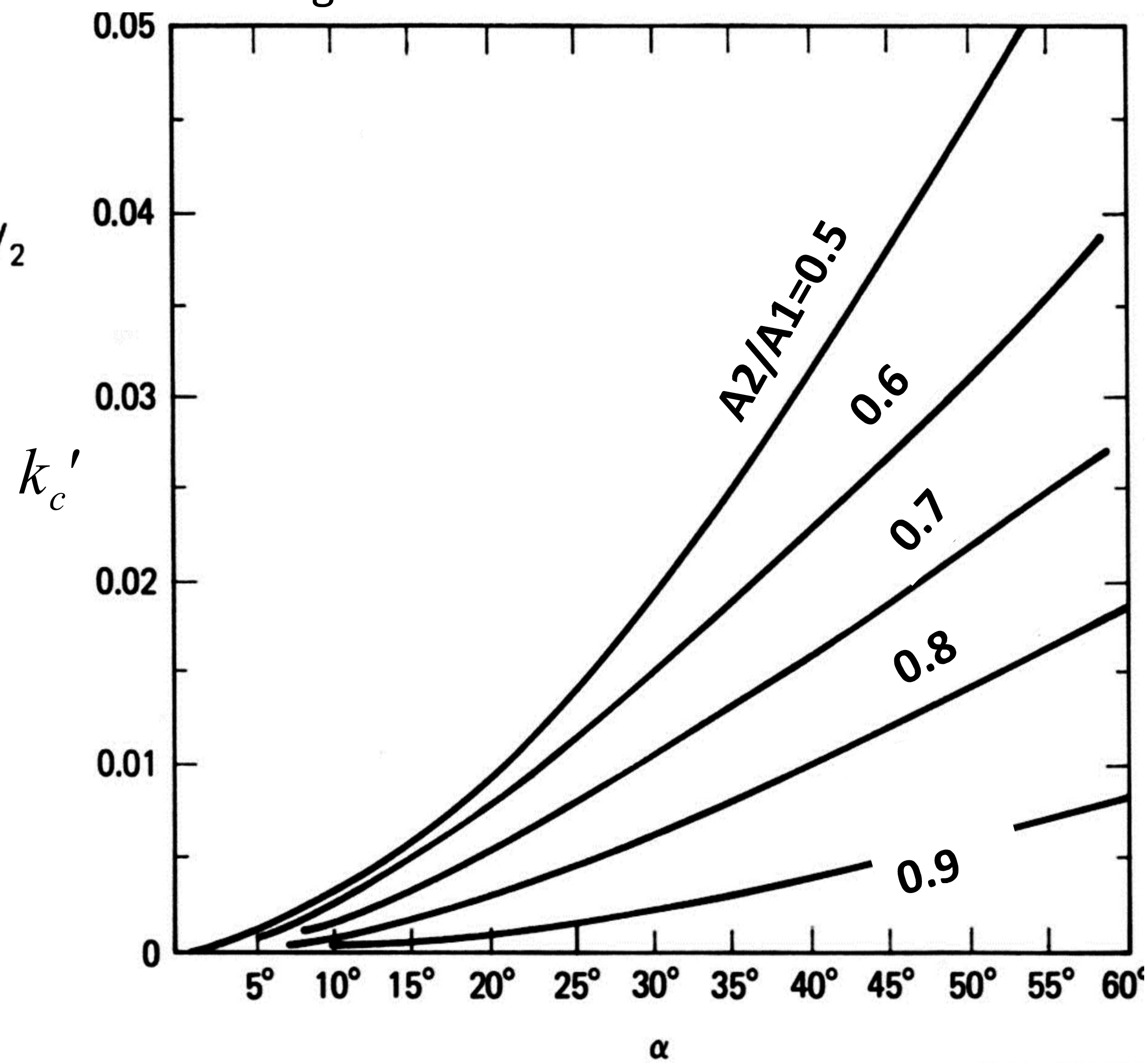


Head losses due to pipe contraction may be greatly reduced by introducing a gradual pipe transition known as a confusor



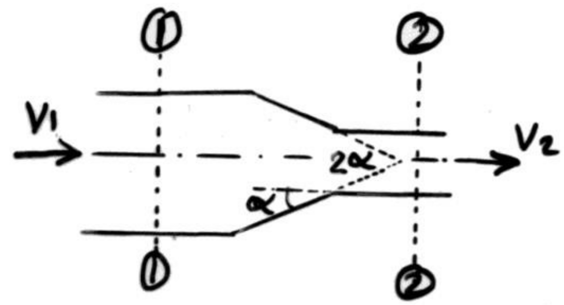
$$h_c' = k_c' \frac{V_2^{\,2}}{2g}$$

Figure 3.11



Head Loss Due to Gradual Contraction (reducer or nozzle)

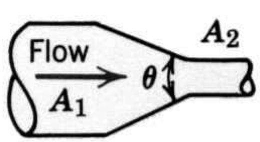
$$h_L = K_L \frac{\left(V_2^{\,2} - V_1^{\,2}\right)}{2g}$$



α	10^0	20^0	30^0	40^0
K_L	0.2	0.28	0.32	0.35

A different set of data is :

Table Loss Coefficients (K) for Gradual Contractions.

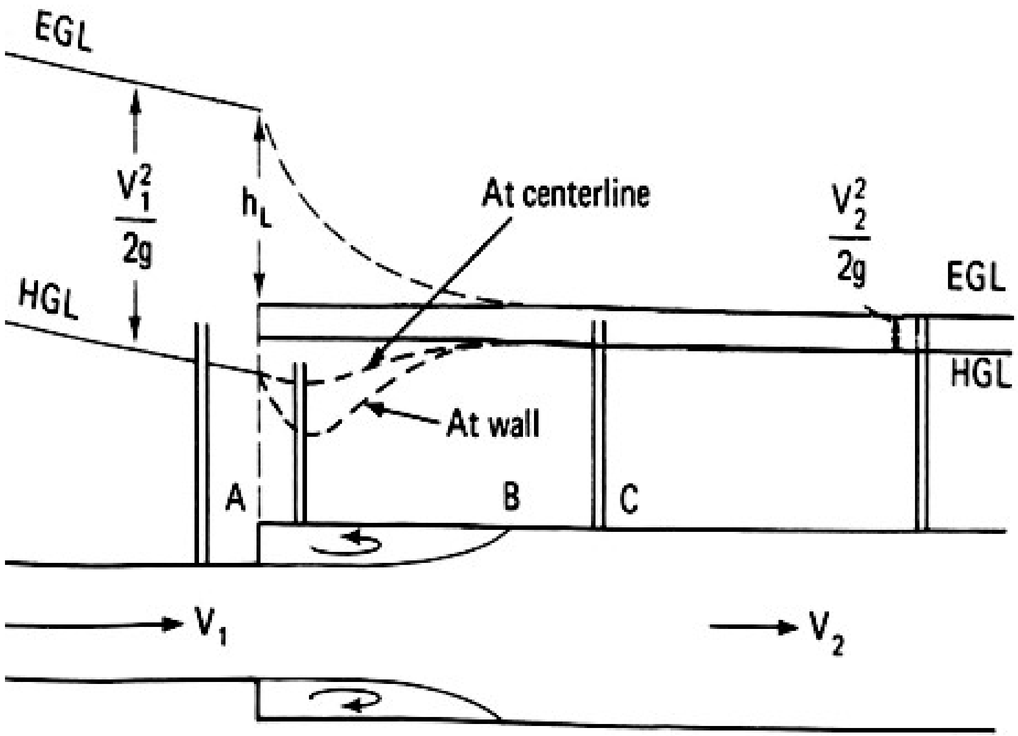


	Included Angle, θ, Degrees						
A_2/A_1	**10**	**15–40**	**50–60**	**90**	**120**	**150**	**180**
0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{l_m} = K(\bar{V}_2^2/2)$.

Losses due to Enlargement

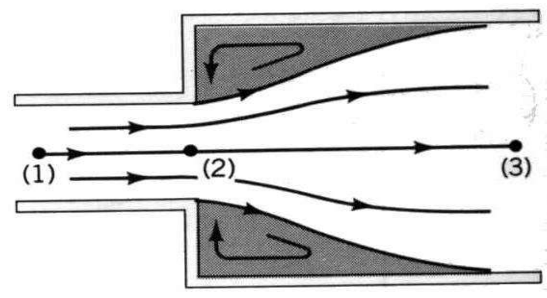
A sudden Enlargement in a pipe



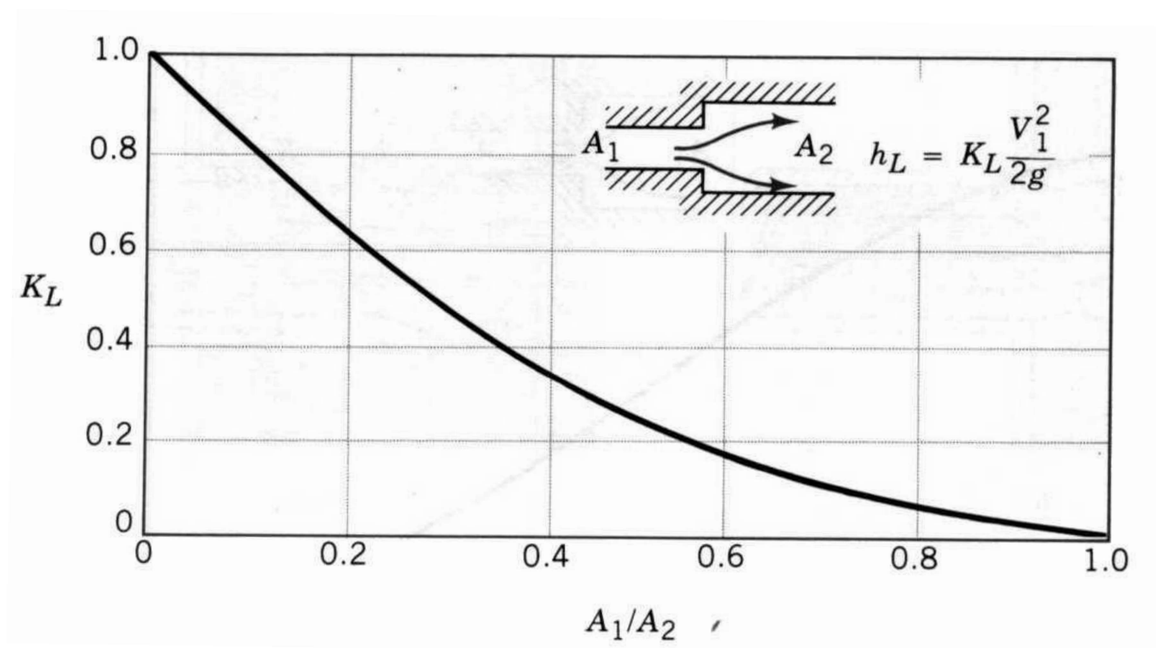
$$h_E = \frac{(V_1 - V_2)^2}{2g}$$

Head Loss Due to a Sudden Enlargement

$$h_L = K_L \frac{V_1^{\,2}}{2g}$$



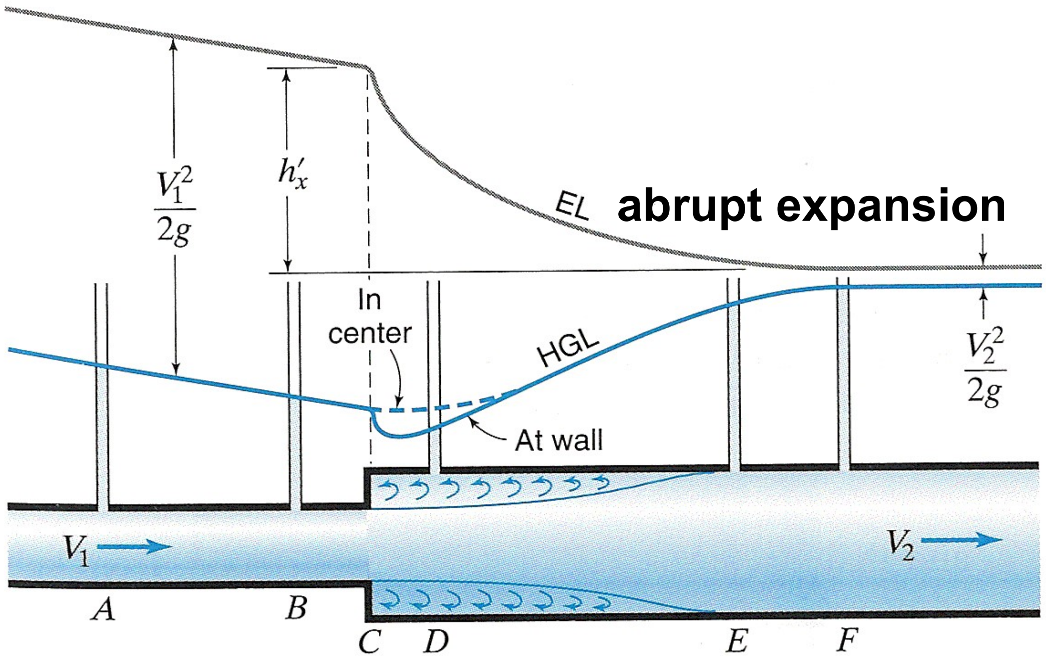
$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$



or :

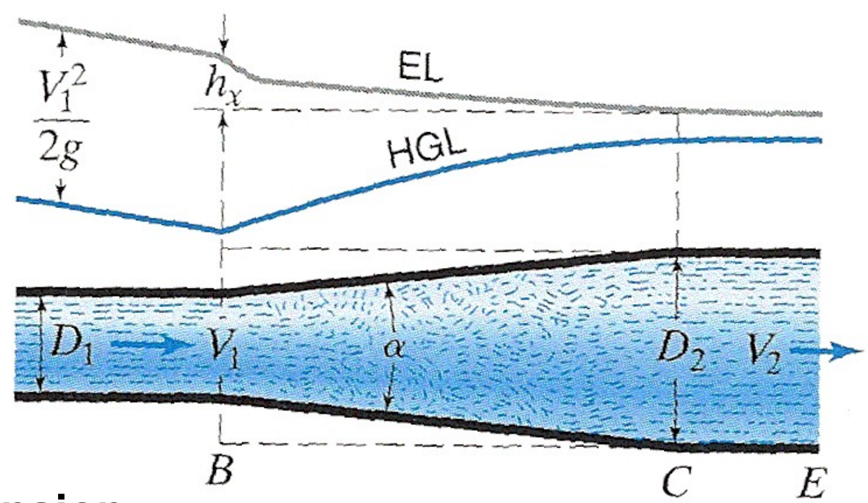
$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

Note that the drop in the energy line is much larger than in the case of a contraction



abrupt expansion

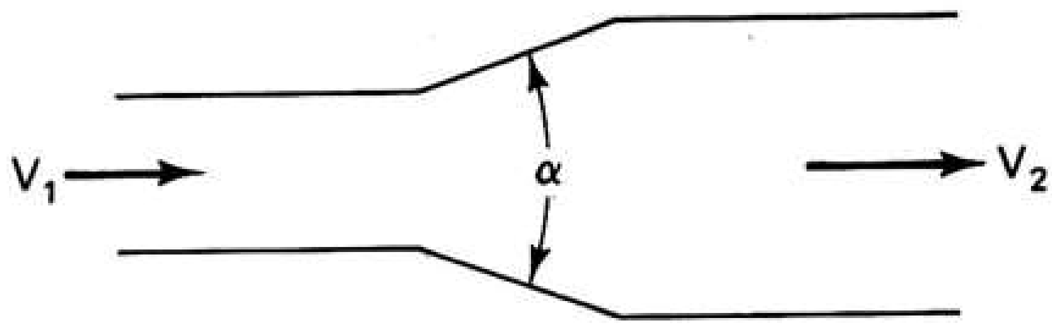
gradual expansion



smaller head loss than in the case of an abrupt expansion

Head losses due to pipe enlargement may be greatly reduced by introducing a gradual pipe transition known as a diffusor

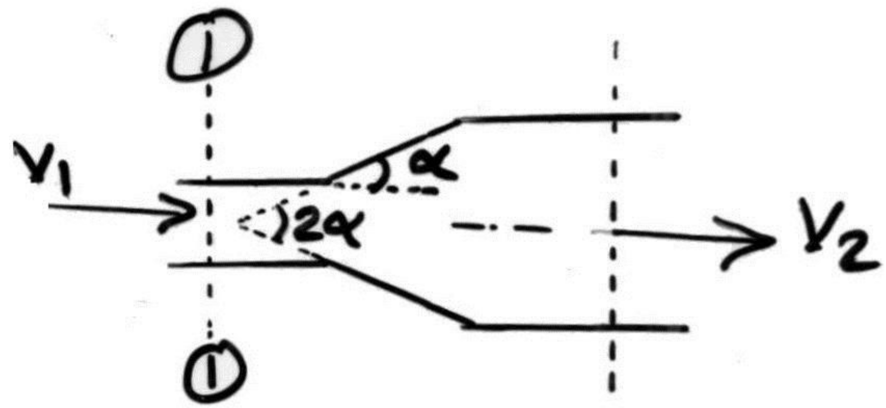
$$h_E' = k_E' \frac{V_1^2 - V_2^{\,2}}{2g}$$



α	10°	20°	30°	40°	50°	60°	75°
K_E'	.078	.31	.49	.60	.67	.72	.72

Head Loss Due to Gradual Enlargement (conical diffuser)

$$h_L = K_L \frac{\left(V_1^2 - V_2^2\right)}{2g}$$



α	10^0	20^0	30^0	40^0
K_L	0.39	0.80	1.00	1.06

Gibson tests

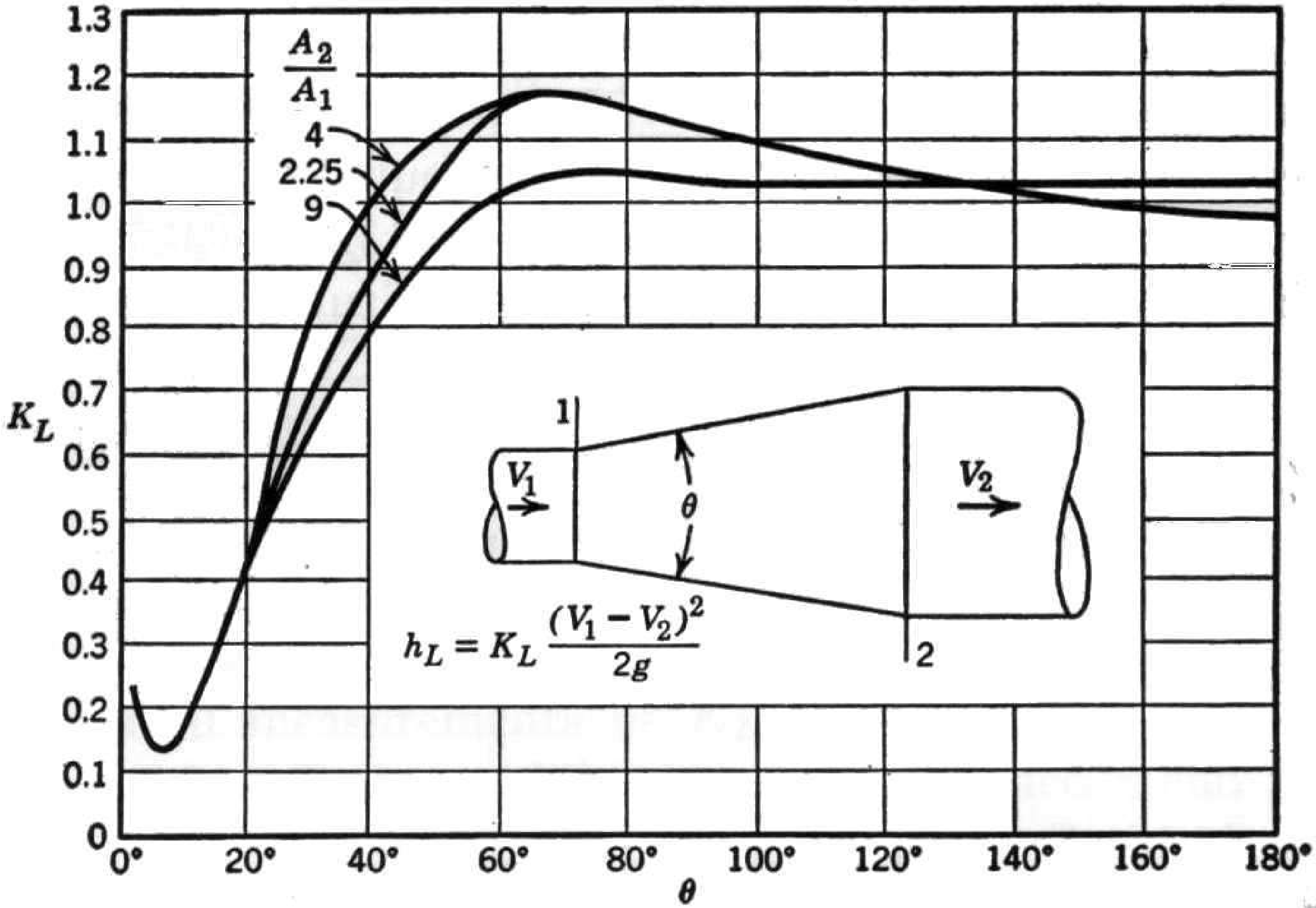
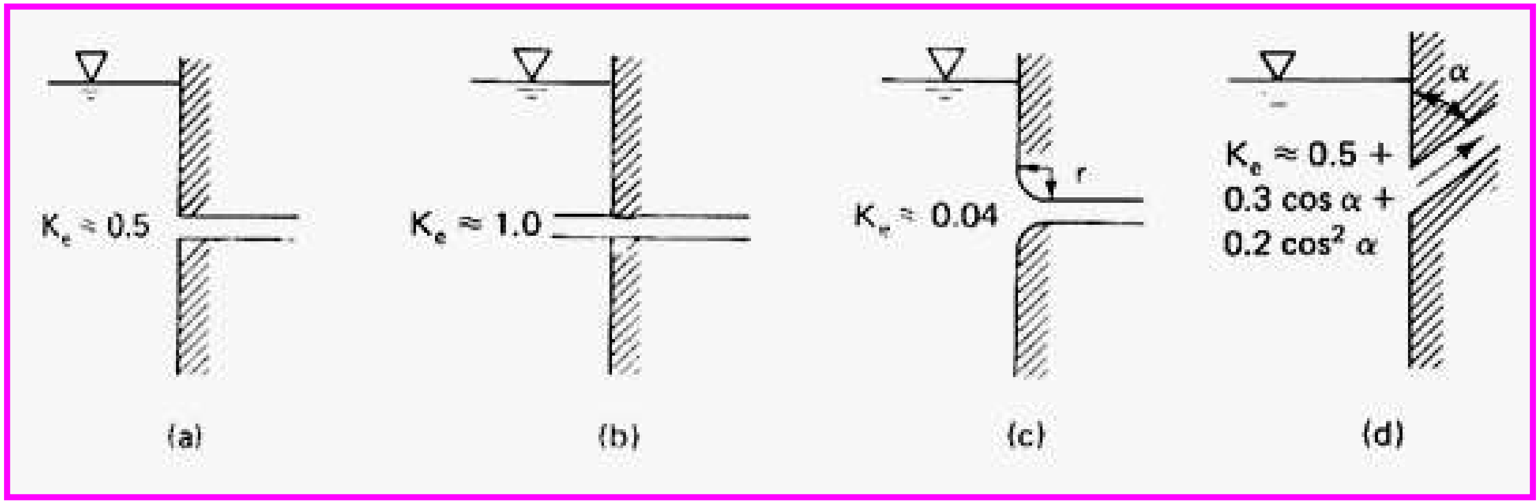


FIG. Loss coefficients for conical enlargements.[21]

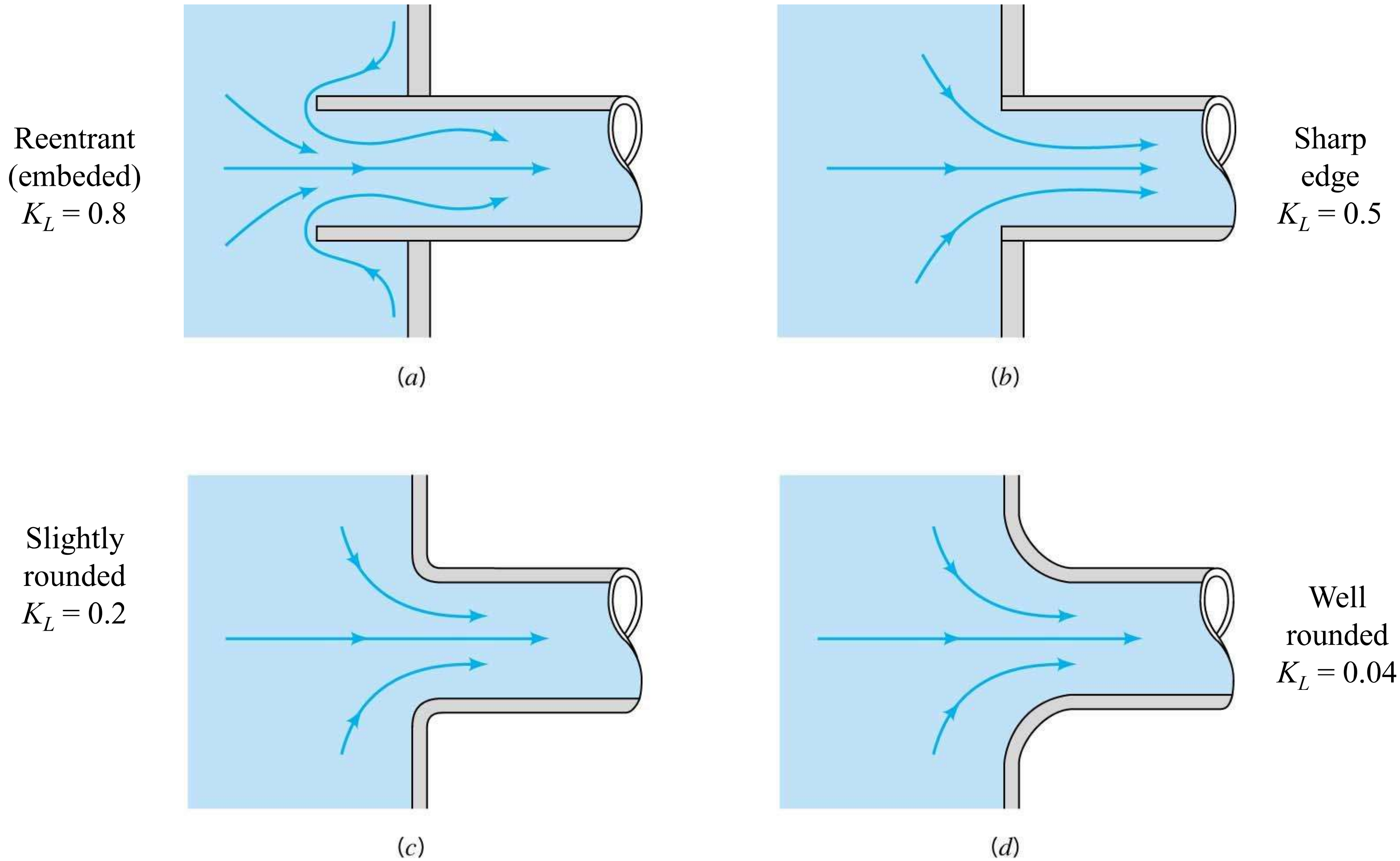
Loss due to pipe entrance

General formula for head loss at the entrance of a pipe is also expressed in term of velocity head of the pipe

$$h_{ent} = K_{ent} \frac{V^2}{2g}$$

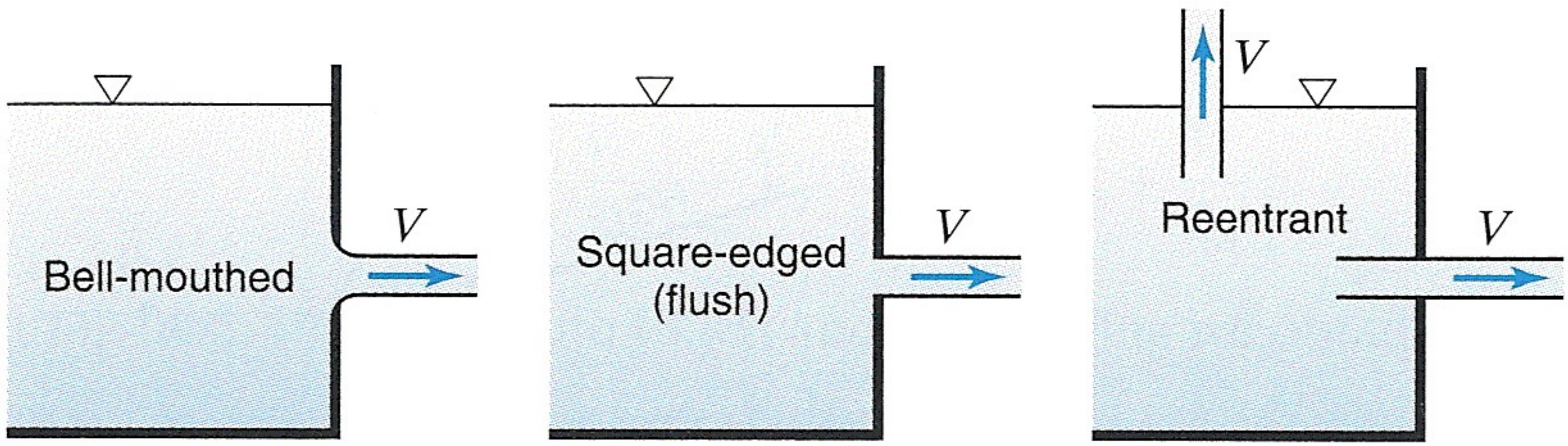


Head Loss at the Entrance of a Pipe (flow leaving a tank)



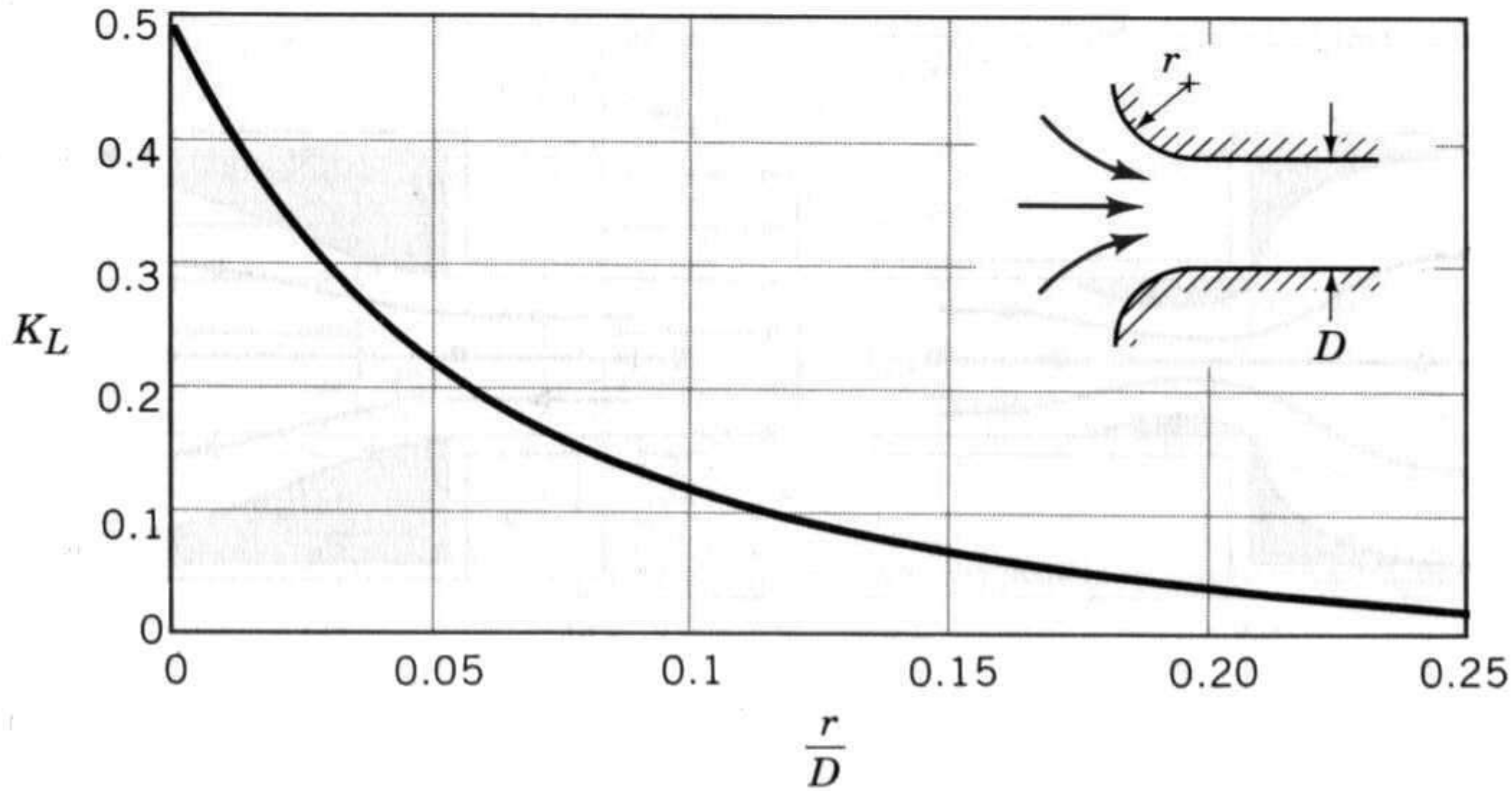
$$h_L = K_L \frac{V^2}{2g}$$

Different pipe inlets



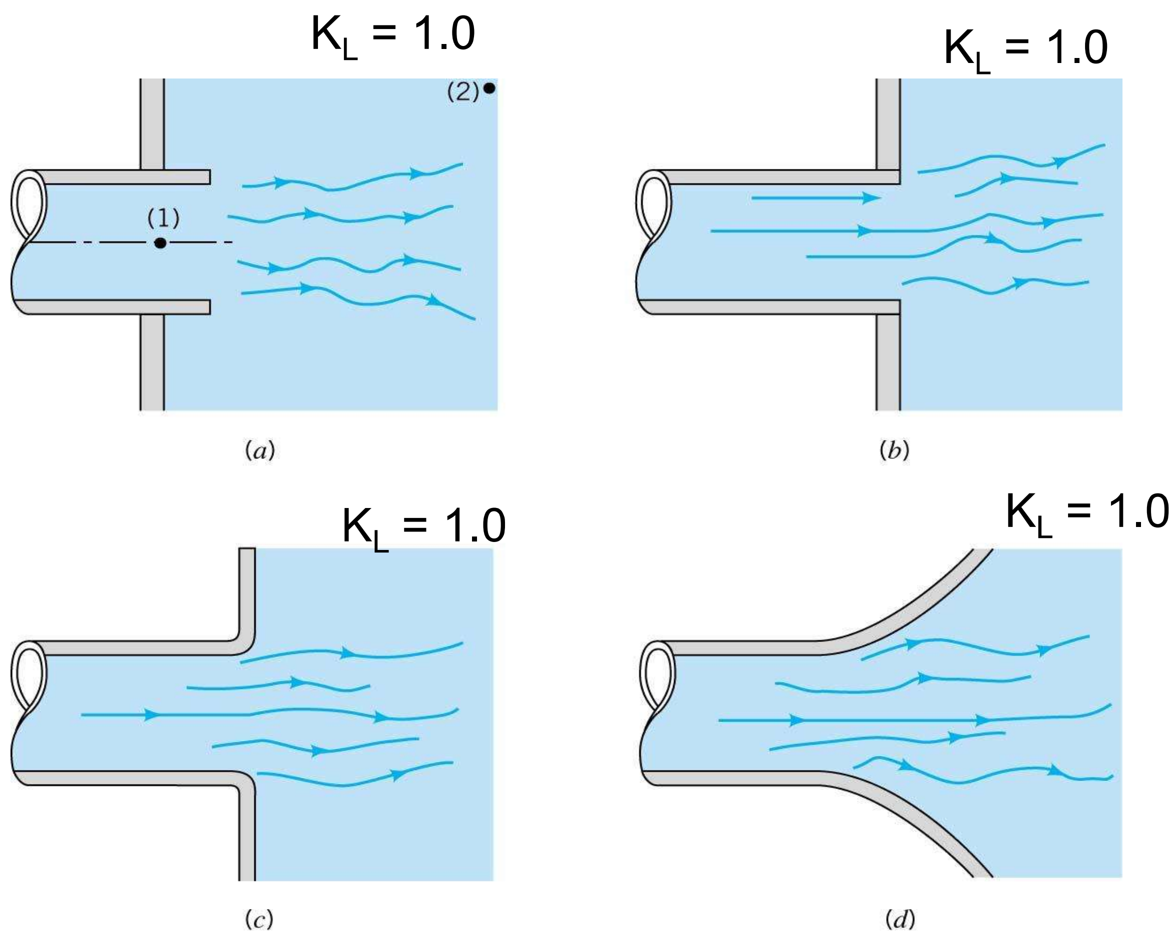
$(a)\ k_e = 0.04$ $(b)\ k_e = 0.5$ $(c)\ k_e \approx 0.8$

Another Typical values for various amount of rounding of the lip



Head Loss at the Exit of a Pipe
(flow entering a tank)

$$h_L = \frac{V^2}{2g}$$



the entire kinetic energy of the exiting fluid (velocity *V1*) is dissipated through viscous effects as the stream of fluid mixes with the fluid in the tank and eventually comes to rest ($V_2 = 0$).

Head Loss Due to Bends in Pipes

$$h_b = k_b \frac{V^2}{2g}$$

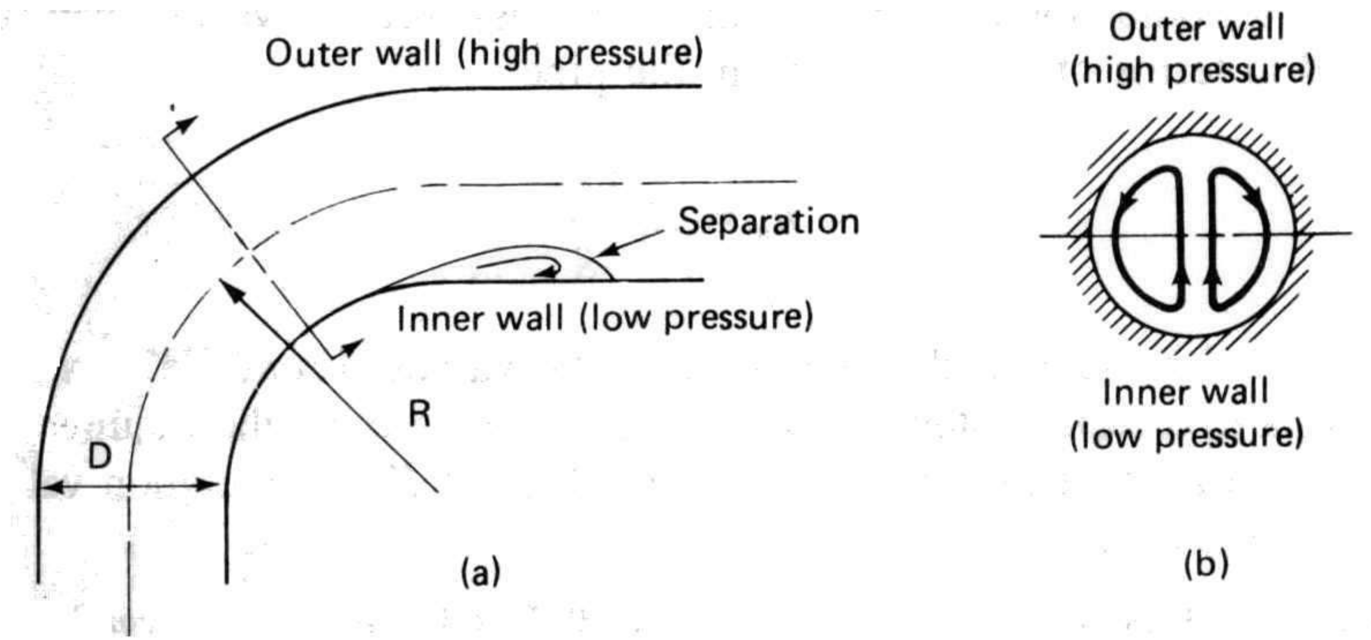
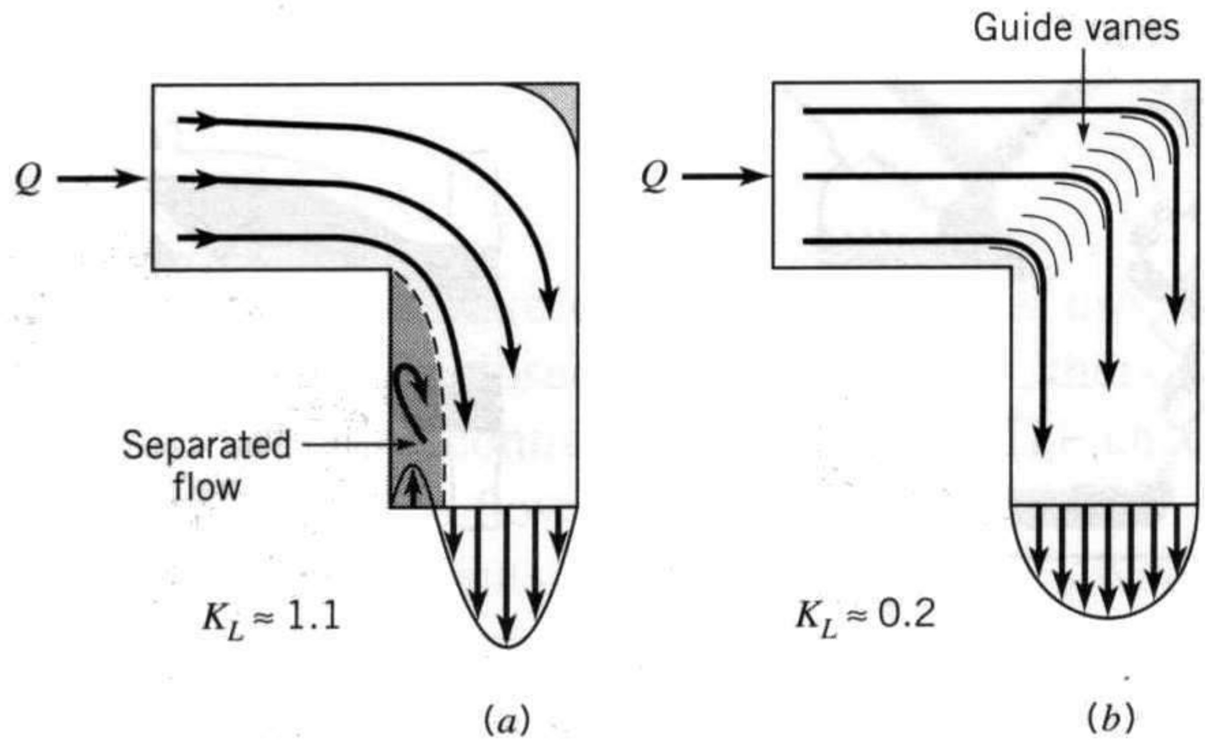


Figure Head loss at a bend: (a) flow separation in a bend; (b) secondary flow at a bend.

R/D	1	2	4	6	10	16	20
K_b	0.35	0.19	0.17	0.22	0.32	0.38	0.42

Miter bends

For situations in which space is limited,

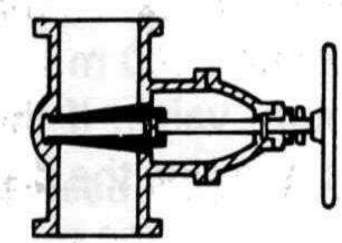


■ **FIGURE**
Character of the flow in a 90° miter bend and the associated loss coefficient: (*a*) without guide vanes. (*b*) with guide vanes.

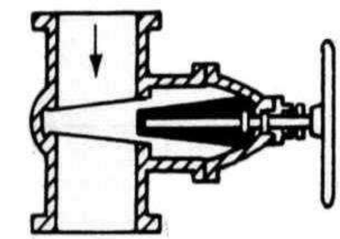
Head Loss Due to Pipe Fittings (valves, elbows, bends, and tees)

TABLE : Values of K_v for Certain Common Hydraulic Valves

A. Gate valves



Closed

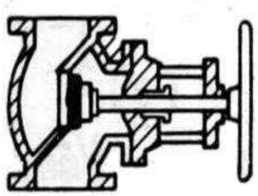


Open

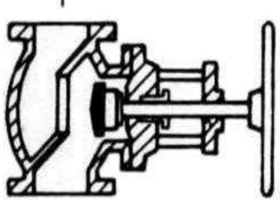
K_V = 0.15 (fully open)

$$h_v = K_v \frac{V^2}{2g}$$

B. Globe valves:



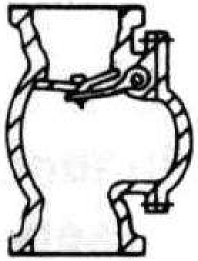
Closed



Open

K_V = 10.0 (fully open)

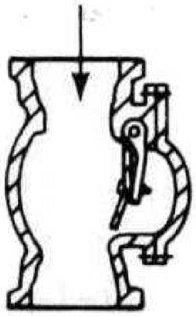
C. Check valves:



Closed

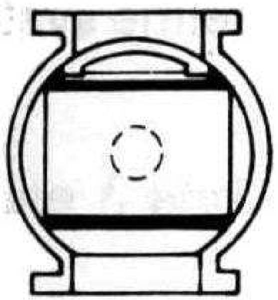
Hinge (Swing type)

Swing type	$K_V = 2.5$	(fully open)
Ball type	$K_V = 70.0$	(fully open)
Lift type	$K_V = 12.0$	(fully open)



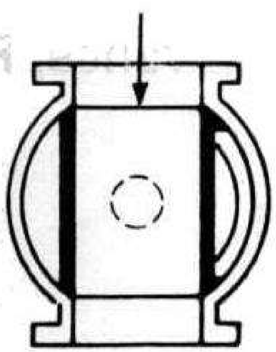
Open

D. Rotary valves:



Closed

$K_V = 10.0$ (fully open)

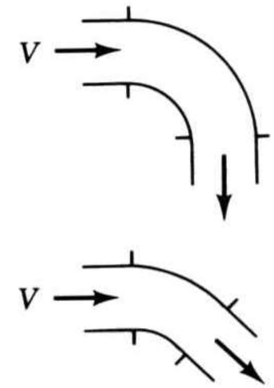


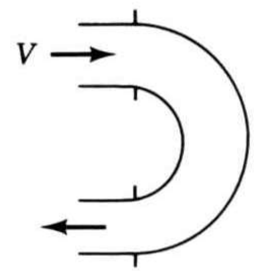
Open

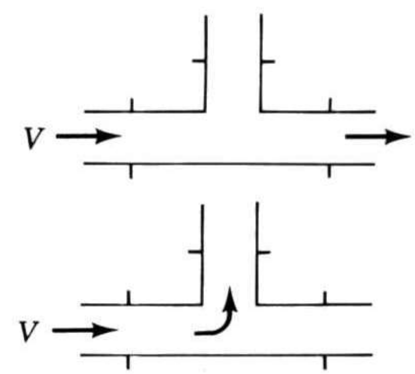
The loss coefficient for elbows, bends, and tees

Loss Coefficients for Pipe Components $\left(h_L = K_L \dfrac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0

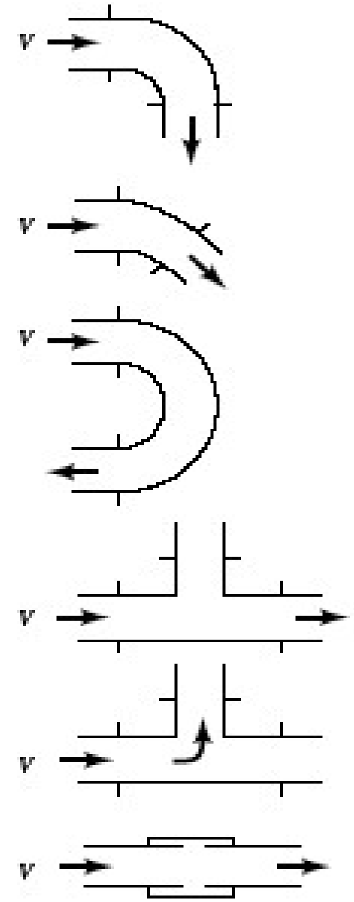




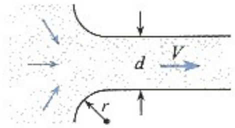
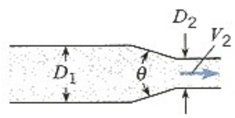
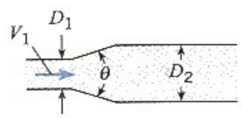
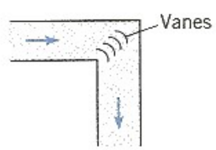
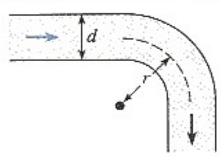


Loss coefficients for pipe components (Table)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Minor loss coefficients (Table)

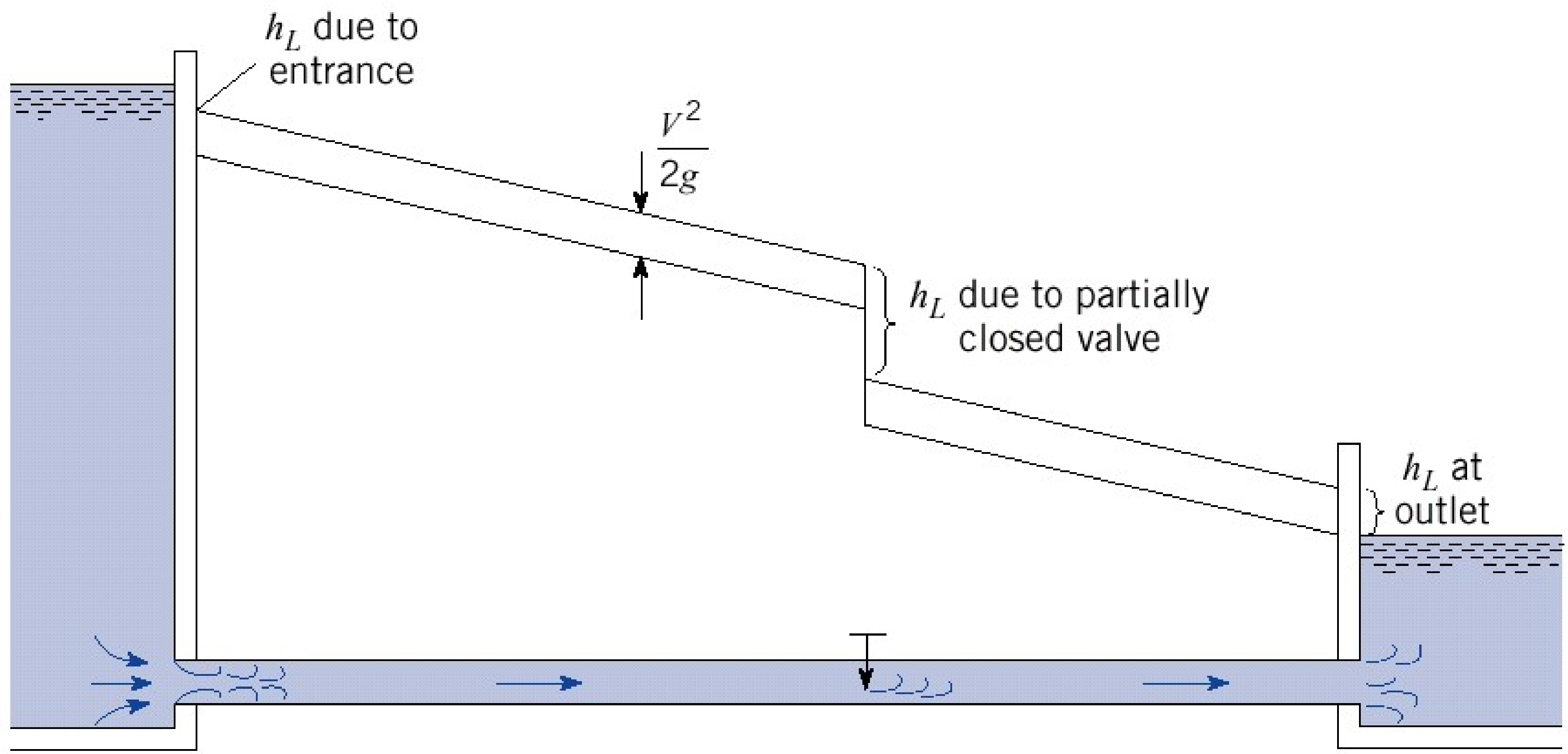
Description	Sketch	Data	K	K	Source
Pipe entrance		r/d	K_e		(2)*
$h_L = K_e V^2/2g$		0.0	0.50		
		0.1	0.12		
		>0.2	0.03		
Contraction		D_2/D_1	K_C $\theta = 60°$	K_C $\theta = 180°$	(2)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
$h_L = K_C V_2^2/2g$		0.90	0.06	0.10	
Expansion		D_1/D_2	K_E $\theta = 20°$	K_E $\theta = 180°$	(2)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend		Without vanes	$K_b = 1.1$		(39)
		With vanes	$K_b = 0.2$		(39)
90° smooth bend		r/d			(5) and (15)
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings	Globe valve—wide open	$K_v = 10.0$			(39)
	Angle valve—wide open	$K_v = 5.0$			
	Gate valve—wide open	$K_v = 0.2$			
	Gate valve—half open	$K_v = 5.6$			
	Return bend	$K_b = 2.2$			
	Tee				
	straight-through flow	$K_t = 0.4$			
	side-outlet flow	$K_t = 1.8$			
	90° elbow	$K_b = 0.9$			
	45° elbow	$K_b = 0.4$			

Minor loss calculation using equivalent pipe length

$$L_e = \frac{k_l\, D}{f}$$

L_e	Equivalent pipe length
D	Diameter of pipe
k_l	Loss coefficient for any fitting, valve…
f	Darcy-Weisbach coefficient

Energy and hydraulic grade lines

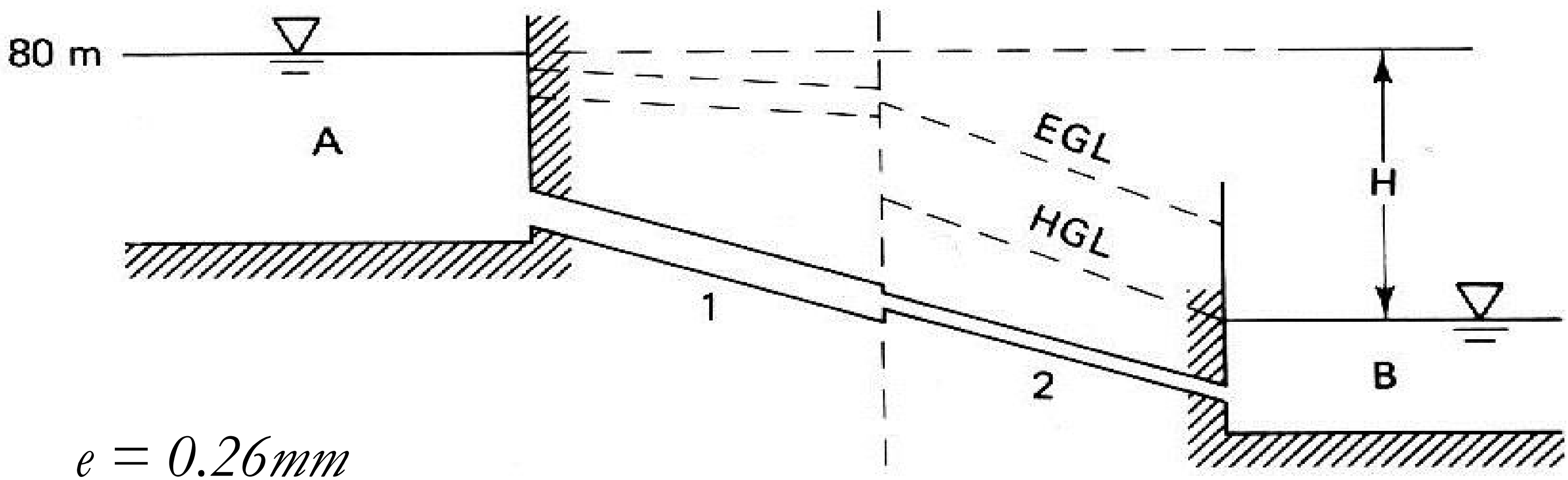


Unless local effects are of particular interests, the changes in the EGL and HGL are often shown as abrupt changes (even though the loss occurs over some distance)

Example

In the figure shown below, two new cast iron pipes are in series, D1 =0.6m, D2 =0.4m, length of each pipe is 300m, level at A =80m , Q = 0.5m3/s (T=10°C). There is a sudden contraction between Pipe 1 and 2, and Sharp entrance at pipe 1.

Find the water level at B?



$e = 0.26mm$

$v = 1.31\times10\text{-}6$

$Q = 0.5\ m3/s$

Solution

$$Z_A - Z_B = h_f$$

$$h_L = h_{f1} + h_{f2} + h_{ent} + h_c + h_{exit}$$

$$\boxed{h_L = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + k_{ent}\frac{V_1^2}{2g} + k_c \frac{V_2^2}{2g} + k_{exit}\frac{V_2^2}{2g}}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.5}{\frac{\pi}{4}(0.6)^2} = 1.77\ m/\sec, \qquad V_2 = \frac{Q}{A_2} = \frac{0.5}{\frac{\pi}{4}(0.4)^2} = 3.98\ m/\sec,$$

$$R_{e1} = \frac{V_1 D_1}{\upsilon} = 8.1\times10^5, \qquad R_{e2} = \frac{V_2 D_2}{\upsilon} = 1.22\times10^6,$$

$$\frac{\varepsilon}{D_1} = \frac{0.26}{600} = 0.00043, \qquad \frac{\varepsilon}{D_1} = 0.00065,$$

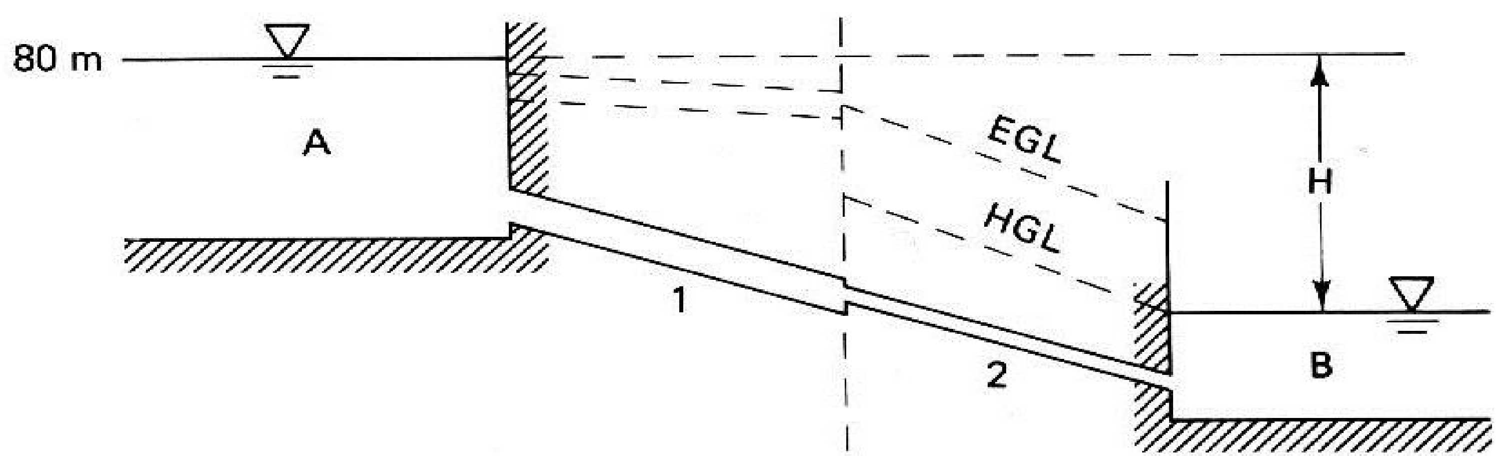
$$\xrightarrow{moody} f_1 = 0.017 \qquad \xrightarrow{moody} f_2 = 0.018$$

$$h_{ent} = 0.5, \qquad h_c = 0.27, \qquad h_{exit} = 1$$

$$h_L = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + k_{ent}\frac{V_1^2}{2g} + k_c \frac{V_2^2}{2g} + k_{exit}\frac{V_2^2}{2g}$$

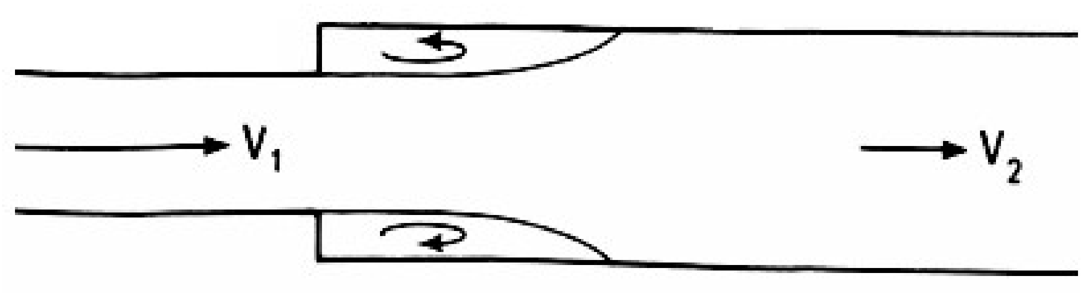
$$h_f = 0.017 \left(\frac{300}{0.6}\right). \frac{1.77^2}{2g} + 0.018 \left(\frac{300}{0.4}\right). \frac{3.98^2}{2g} + 0.5\left(\frac{1.77^2}{2g}\right) + 0.27\left(\frac{3.98^2}{2g}\right) + \left(\frac{3.98^2}{2g}\right) = 13.36m$$

$Z_B = 80 - 13.36 = 66.64$ m



Example

A pipe enlarge suddenly from D1=240mm to D2=480mm. the *H.G.L* rises by 10 cm calculate the flow in the pipe



Solution

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-h_L=\left(\frac{p_2}{\rho g}+z_2\right)-\left(\frac{p_1}{\rho g}+z_1\right)$$

$$\frac{V_1^2}{2g}-\frac{V_2^2}{2g}-\frac{(V_1-V_2)^2}{2g}=0.1$$

$$V_1A_1=V_2A_2$$

$$V_1\left(\tfrac{\pi}{4}0.24^2\right)=V_2\left(\tfrac{\pi}{4}0.48^2\right)$$

$$V_1=4V_2$$

$$\frac{16V_2^2}{2g}-\frac{V_2^2}{2g}-\frac{(4V_2-V_2)^2}{2g}=0.1$$

$$\frac{6V_2^2}{2g}=0.1$$

$$V_2=0.57m/s\Rightarrow Q=V_2A_2=0.57\times\tfrac{\pi}{4}0.48^2=0.103m^3/s$$

- ***Note*** that the above values are average typical values, actual values will depend on the manufacturer of the components.
- *See:*
  - Catalogs
  - Hydraulic handbooks !!

Title	Date	Slot & Time
Minor losses, problem, revision		
Dimensional Analysis, Raleigh and Buckingham π theorem, sample problem, Non-dimensional numbers, Lift and Drag – Boundary layer		
Buckingham π theorem, sample problem		
Boundary layers – Laminar flow and Turbulent flow – Boundary layer thickness – momentum- Integral equation		
PBL Review for all students		
Drag and lift-Separation of boundary layer		
Separation of boundary layer-Methods of separation of boundary layer		
Final lecture – Revision - Discussion		