# Revision and Problems

## **Module 1**

Problems related to Surface tension

Newtonian law of viscosity

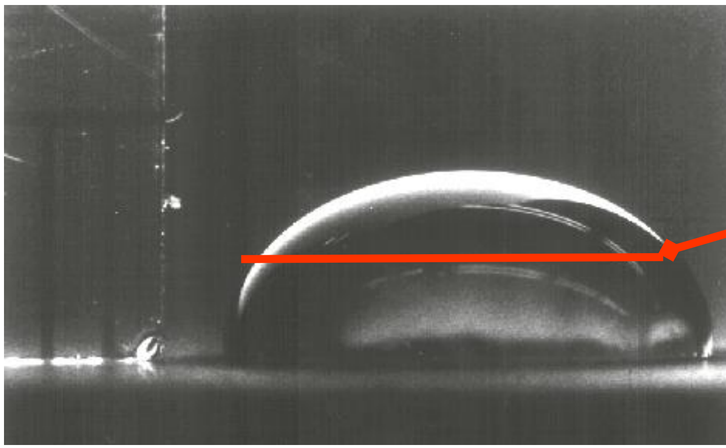
Pascal law

Manometry

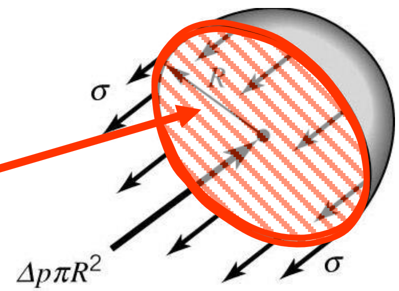
# Surface Tension: Liquid Drop

The pressure inside a drop of fluid can be calculated using a free-body diagram:

Real Fluid Drops



Mathematical Model




R is the radius of the droplet, σ is the surface tension, Δp is the pressure difference between the inside and outside pressure.

The force developed around the edge due to surface tension along the line:

$$F_{surface} = 2\pi R\sigma$$

Applied to Circumference

This force is balanced by the pressure difference Δp:

$$F_{pressure} = \Delta p \pi R^2$$

Applied to Area

## Surface Tension: Liquid Drop

Now, equating the Surface Tension Force to the Pressure Force, we can estimate $\Delta p = p_i - p_e$:

$$\Delta p = \frac{2\sigma}{R}$$

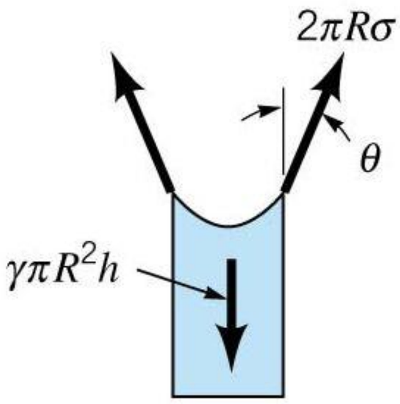
This indicates that the internal pressure in the droplet is greater that the external pressure since the right hand side is entirely positive.

Is the pressure inside a bubble of water greater or less than that of a droplet of water?

Prove to yourself the following result: $$\Delta p = \frac{4\sigma}{R}$$

# Surface Tension: Capillary Action

Free Body Diagram for Capillary Action for a Wetted Surface:




$$F_{surface} = 2\pi R\sigma\cos\theta$$

$$W = \gamma\pi R^2 h$$

Equating the two and solving for h:

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

For clean glass in contact with water, $\theta \approx 0°$, and thus as R decreases, h increases, giving a higher rise.

For a clean glass in contact with Mercury, $\theta \approx 130°$, and thus h is negative or there is a push down of the fluid.

3. We will use the distance up a capillary the liquid climbs to find Surface Tension

a) We will use the known surface tension of water to find the tube radius

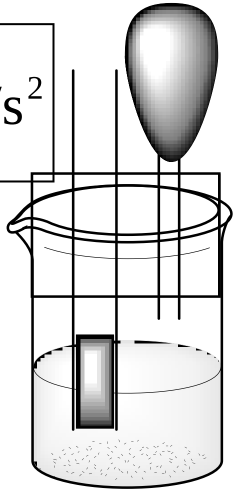
$\sigma$ = surface tension [$kg/s^2$]  h = height [m]

d = density [$kg/m^3$]  g = accel. gravity = [$9.8\ m/s^2$]

$$\sigma = \frac{\text{rhdg}}{2}$$

$$\Rightarrow \text{r} = \frac{2\sigma}{\text{hdg}} = \frac{2(0.07259\ \text{kg/s}^2)}{(3.29\text{x}10^{-2}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 4.5\text{x}10^{-4}\ \text{m}$$

$$\sigma = \frac{\text{rhdg}}{2} = \frac{(4.5\text{x}10^{-4}\ \text{m})(1.3\text{x}10^{-2}\ \text{m})(790\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{2} = 0.0226\ \text{kg/s}^2$$



b) We will then find the surface tension of an uknown (ethanol)

# Problem No. 1

Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surface at a speed of 0.6 m/s, if

i. The thin plate is in the middle of the two plane surfaces, and
ii. The thin plate is at a distance of 0.8 cm from one of the plane surfaces? Take the dynamics viscosity of glycerine $=8.10 \times 10^{-1}$ $Ns/m^2$

# Problem No. 2

Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m$^2$ above the atmospheric pressure.

# Problem No. 3

Calculate the capillary effect in millimeters in a glass tube of 4mm diameter, when immersed in (i) Water and (ii) Mercury. The temperature of liquid is 20 º C and the values of the surface tension of water and mercury at 20 º C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 1.30 º.

Take density of water as equal to 998kg/m$^3$.

(U.P.S.C Engg. Exam. 1974)

# Problem No. 4

An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1m. Find the pressure intensity (i) at the interface of the two liquids and (ii) at the bottom of the tank.

# PASCAL'S LAW

Since the element is very small, neglecting its weight, we have

$$P_z \cos\alpha = P_y \qquad \text{or} \qquad p_z \cdot LM \cos\alpha = p_y \cdot MN$$

But $LM \cos\alpha = MN$

$$\therefore \qquad p_z = p_y$$

$$\boxed{\text{we get} \quad p_x = p_y = p_z}$$

"*The intensity of pressure at any point in a liquid at rest, is the same in all directions*".

**Example 2·3.** *The diameters of ram and plunger of an hydraulic press are 200 mm an 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at th plunger is 400 N.*

**Sol.** Diameter of the ram,

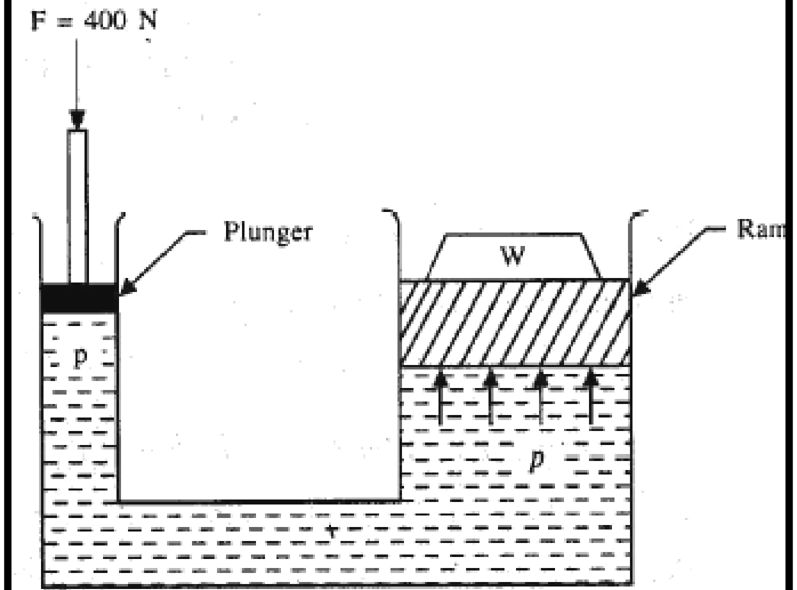
$D = 200 \text{ mm} = 0{\cdot}2 \text{ m}$

Diameter of the plunger,

$d = 30 \text{ mm} = 0{\cdot}03 \text{ m}$

Force on the plunger,

$F = 400 \text{ N}$




Hydraulic press.

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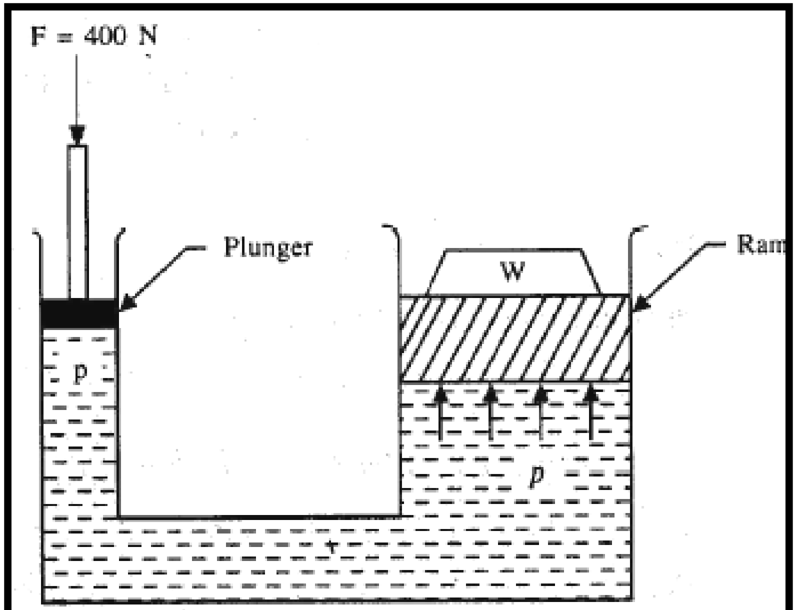


Hydraulic press.

Area of ram,

$$A = \frac{\pi}{4} D^2 = \pi/4 \times 0{\cdot}2^2 = 0{\cdot}0314 \text{ m}^2$$

Area of plunger,

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0{\cdot}03^2 = 7{\cdot}068 \times 10^{-4} \text{ m}^2$$

$$p = \frac{F}{a} = \frac{400}{7{\cdot}068 \times 10^{-4}} = 5{\cdot}66 \times 10^5 \text{ N/m}^2$$

$$\text{pressure at the ram} = \frac{\text{weight}}{\text{area of ram}} = \frac{W}{A} = \frac{W}{0{\cdot}0314}\ \text{N/m}^2$$

$$\frac{W}{0{\cdot}0314} = 5{\cdot}66 \times 10^5$$

$$W = 0{\cdot}0314 \times 5{\cdot}66 \times 10^5\ \text{N} = 17{\cdot}77 \times 10^3\ \text{N}$$

# Vertical Variation Of Pressure In A Fluid Under Gravity

Force due to $p_1$ on A (upward) = $p_1 A$

Force due to $p_2$ on A (downward) = $p_2 A$

Force due to weight of element (downward)

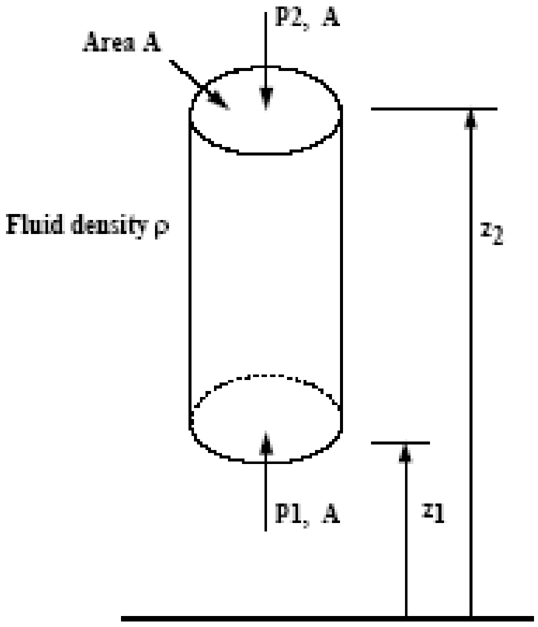
= $mg$

= *mass density* $\times$ *volume* $\times$ $g$

= $\rho\, g\, A(z_2 - z_1)$

$$p_1 A - p_2 A - \rho g A(z_2 - z_1) = 0$$

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$



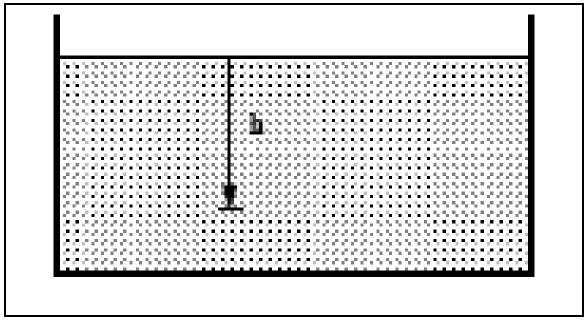
# Atmospheric pressure

- The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure.

**surface pressure is atmospheric,** $p_{\text{atmospheric}}$.

$$p_{\text{atmospheric}} = \text{constant}$$

so




$$p = \rho g h + p_{\text{atmospheric}}$$

# Gauge pressure

It is convenient to take atmospheric pressure as the datum

Gauge pressure is

$$p_{gauge} = \rho g h$$

Absolute pressure is

$$p_{absolute} = \rho g h + p_{atmospheric}$$

$$Absolute\ pressure = Gauge\ pressure + Atmospheric$$

$101{\cdot}3\ kN/m^2$ or $101{\cdot}3\ kPa$;
$\simeq 1$ bar $\simeq 100$ kPa $= 10^5 N/m^2$.

[redacted]

**What is a pressure of 500 $kNm^{-2}$ in head of water of density, ρ = 1000 $kgm^{-3}$**

**Use $p = \rho g h$,**

$$h = \frac{p}{\rho g} = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95m \text{ of water}$$

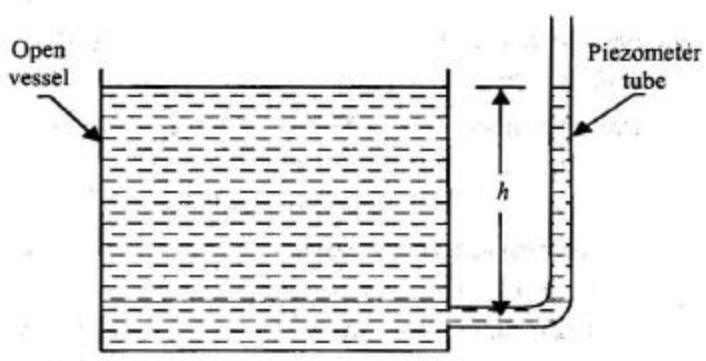
**In head of Mercury density ρ = $13.6 \times 10^3$ $kgm^{-3}$.**

$$h = \frac{500 \times 10^3}{13.6 \times 10^3 \times 9.81} = 3.75m \text{ of Mercury}$$

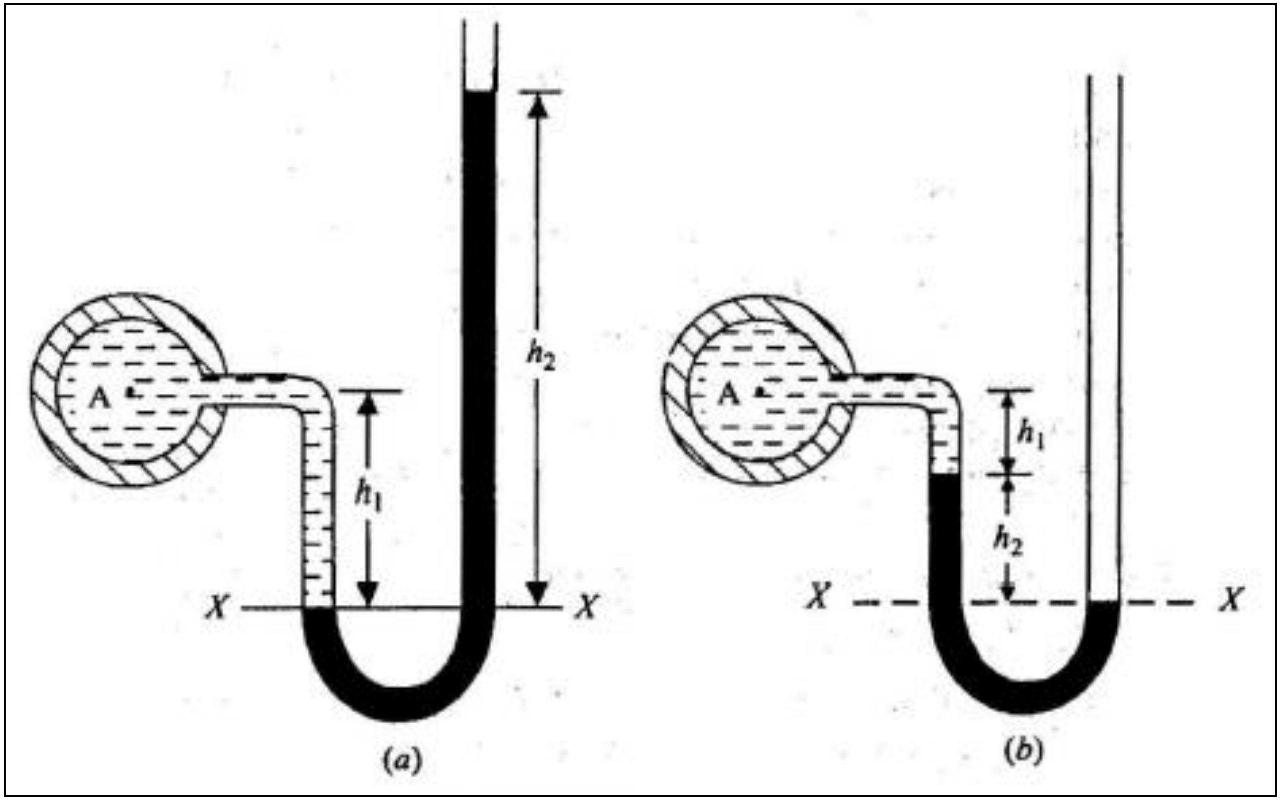
# Pressure Measurement By Manometer

- Manometers use the relationship between pressure and head to measure pressure

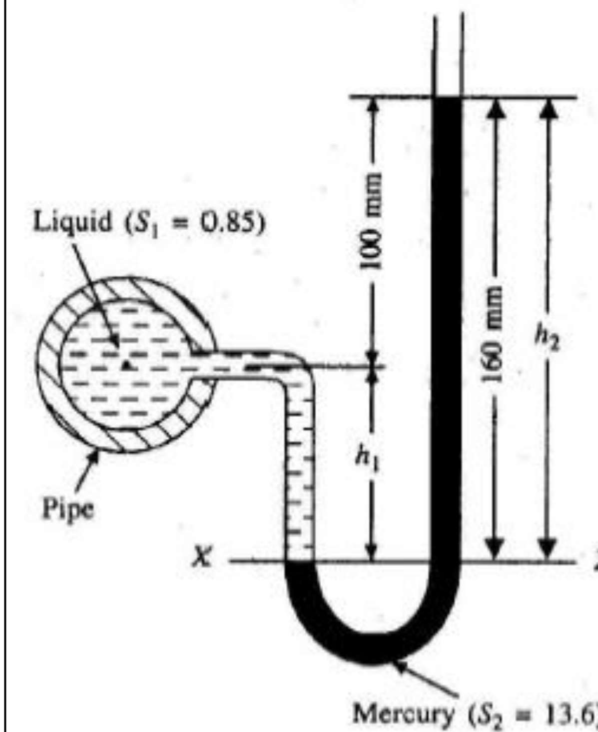
1- Simple manometer



# 2- U-tube manometer



**Example 2·7.** *A U-tube manometer is used to measure the pressure of oil of specific gravity 0·85 flowing in a pipe line. Its left end is connected to the pipe and the right-limb is open to the atmosphere. The centre of the pipe is 100 mm below the level of mercury (specific gravity = 13·6) in the right limb. If the difference of mercury level in the two limbs is 160 mm, determine the absolute pressure of the oil in the pipe.*



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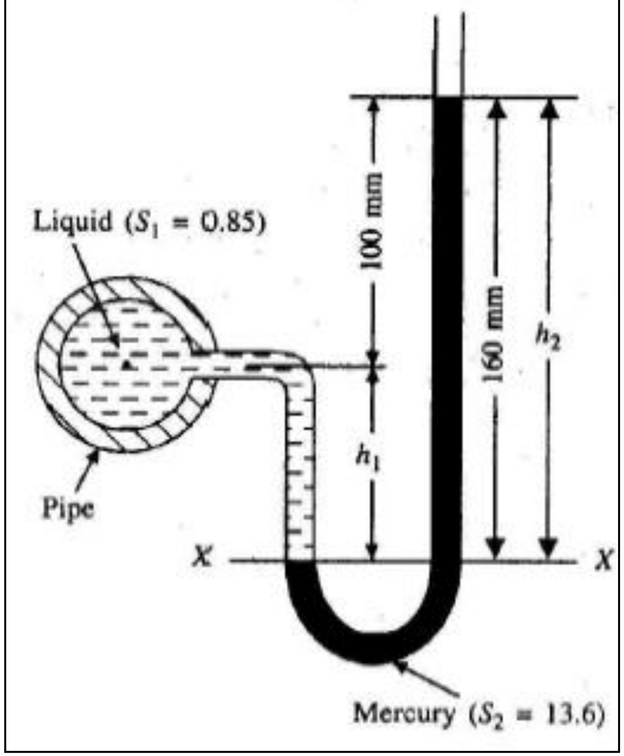



$$P1 = P2$$

$$P_{pipe} + \rho_1 g h_1 = \rho_2 g h_2$$

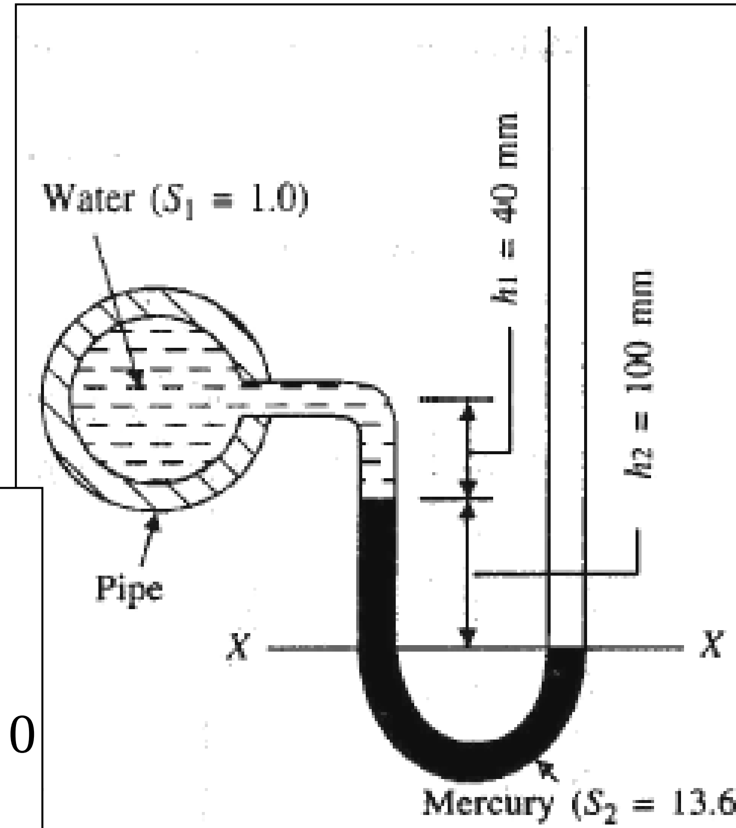
$$P_{pipe} + 0.85 \times 10^3 * 9.81 * 0.06 = 13.6 \times 10^3 * 9.81 * 0.16$$

$$P_{pipe} = 20846.25\, N/m^2 = 20.85\, KN/m^2$$

$$P_{pipe}(abs.) = P_{pipe}(gauge) + P_{pipe}(atm.)$$

$$P_{pipe}(abs.) = 20.85 + 100 = 120.85\, KN/m^2$$

**Example 2·8.** *U-tube manometer containing mercury was used to find the negative pressure in the pipe, containing water. The right limb was open to the atmosphere. Find the vacuum pressure in the pipe, if the difference of mercury level in the two limbs was 100 mm and height of water in the left limb from the centre of the pipe was found to be 40 mm below.*




$$P1 = P2$$

$$P_{pipe} + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$P_{pipe} + 1\times10^3 * 9.81 * 0.04 + 13.6\times10^3 * 9.81 * 0.1 = 0$$

$$P_{pipe} = -13734\, N/m^2 = -13.73\, KN/m^2$$

$$P_{pipe}(abs.) = P_{pipe}(gauge) + P_{pipe}(atm.)$$

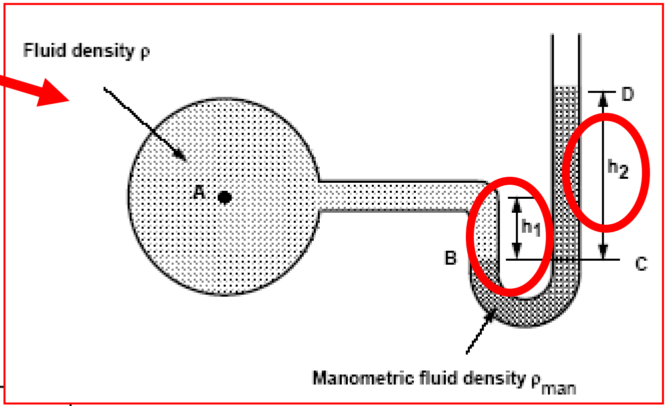
$$P_{pipe}(abs.) = -13.73 + 100 = 86.27\ KN/m^2$$

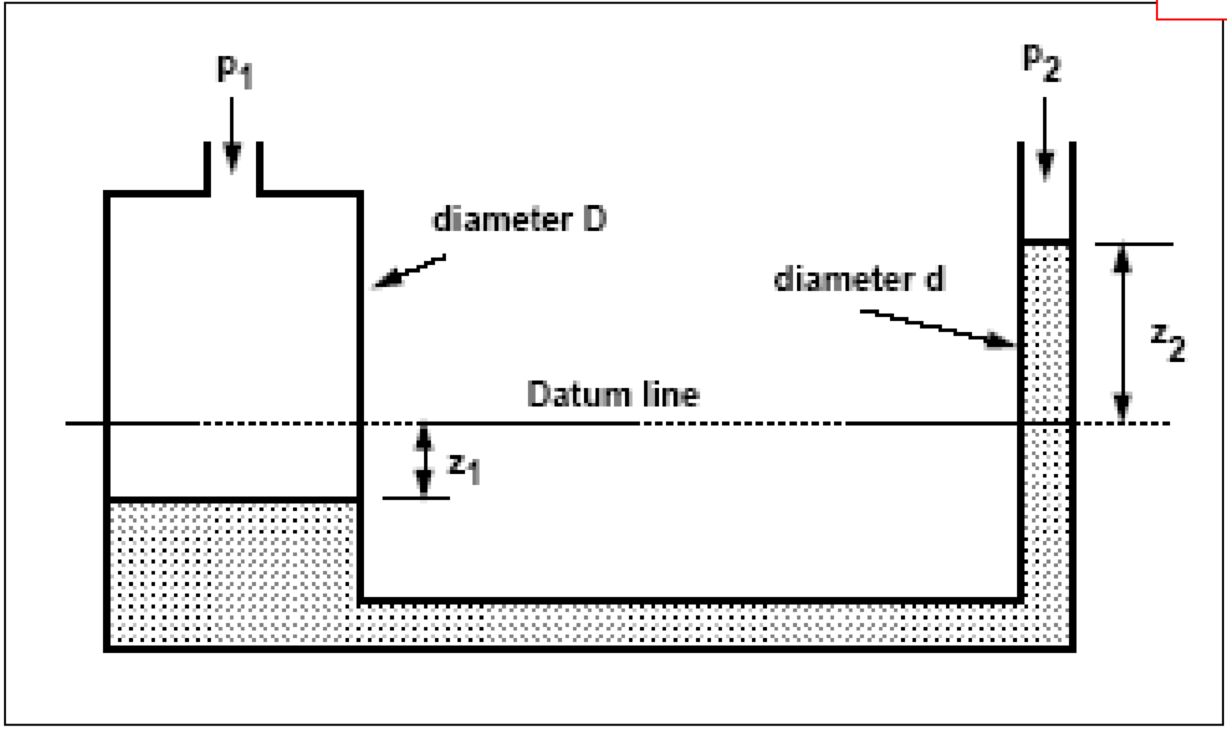
# Advances to the “U” tube manometer

Problem: Two reading are required.

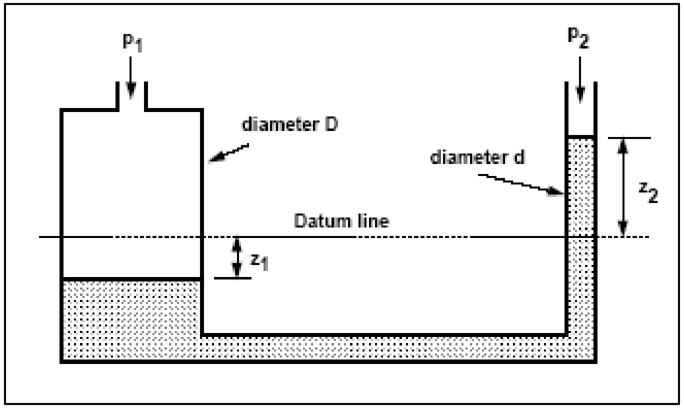
Solution: Increase cross-sectional area of one side.

Result: One level moves much more than the other.



volume of liquid moved from the left side to the right $= z_2 \times (\pi d^2 / 4)$




The fall in level of the left side is

$$z_1 = \frac{\text{Volume moved}}{\text{Area of left side}} = \frac{z_2\left(\pi d^2 / 4\right)}{\pi D^2 / 4} = z_2\left(\frac{d}{D}\right)^2$$

Putting this in the equation,

$$p_1 - p_2 = \rho g\left[z_2 + z_2\left(\frac{d}{D}\right)^2\right] = \rho g z_2\left[1 + \left(\frac{d}{D}\right)^2\right]$$

If D >> d then $(d/D)^2$ is very small so

$$p_1 - p_2 = \rho g z_2$$

# Single column manometers

- Vertical Single manometer

The volume that move from x-x to z-z datum = the volume that rise in right tube above x-x datum

$$A \times \delta h = a \times h_2 \quad \text{or} \quad \delta h = \frac{a \times h_2}{A}$$

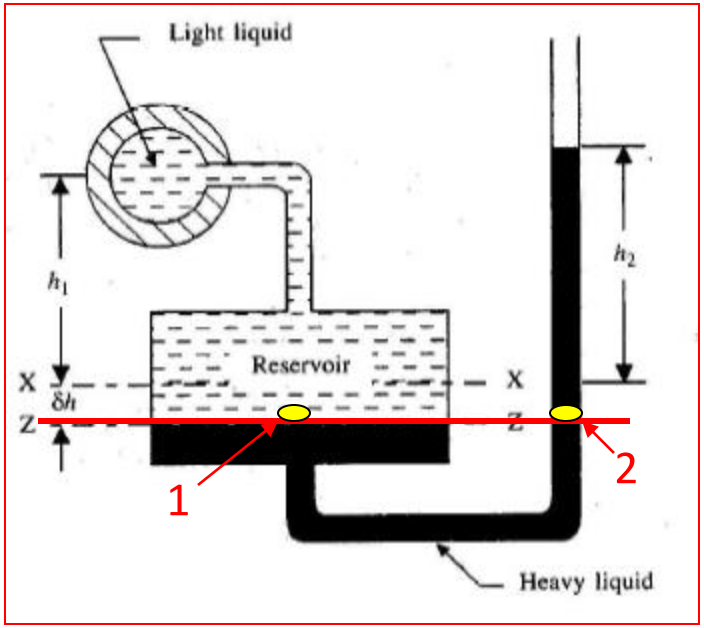
$$P1 = P2$$

$$P + \rho_1 g(h_1 + \delta h) = \rho_2 g(h_2 + \delta h)$$

$$P + \rho_1 g(h_1 + \frac{a}{A} h_2) = \rho_2 g(h_2 + \frac{a}{A} h_2)$$

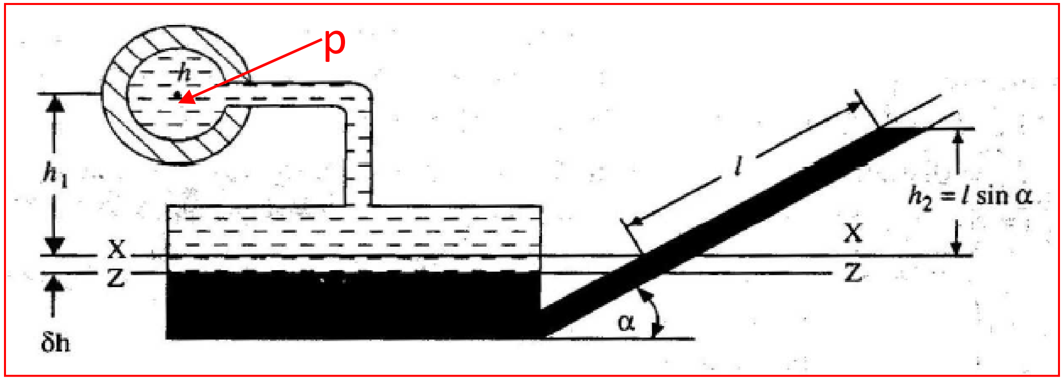
$$\frac{\mathrm{a}}{\mathrm{A}} \text{ is very small}$$

$$\therefore P = \rho_2 g(h_2) - \rho_1 g(h_1)$$



- Inclined single column manometers

This type of manometer is useful for the measurement of *small pressures and is more sensitive than the vertical tube type.*




$$P1 = P2$$

$$P + \rho_1 g(h_1 + \delta h) = \rho_2 g(h_2 + \delta h)$$

$$P + \rho_1 g(h_1 + \frac{a}{A} h_2) = \rho_2 g(h_2 + \frac{a}{A} h_2)$$

$$\frac{\mathrm{a}}{\mathrm{A}} \text{ is verysmall}$$

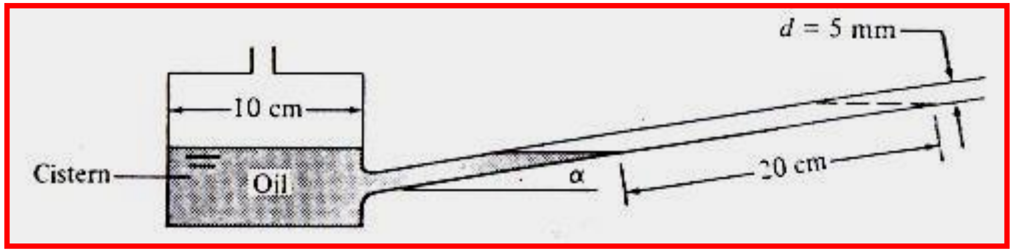
$$\therefore P = \rho_2 g(h_2) - \rho_1 g(h_1)$$

$$P = \rho_2 g(l \sin \alpha) - \rho_1 g(h_1)$$

$$h = \frac{P}{\rho_1 g} = \frac{\rho_2}{\rho_1} l \sin \alpha - h_1$$

The inclined manometer is filled with oil that has a specific gravity of 0.85 what angle a will yield a deflection 20 cm in the inclined tube when the air pressure in the cistern is increased 600 N/m2

Important




Solution:

$$Increase\ pressure\ with\ 600\ \mathrm{N/m^2} \Rightarrow 20cm\ \text{in the tube}$$

$$\text{with nglecting the drop of oil}$$

$$\mathrm{P_{increase}} = 0.2\sin\alpha \times 9.81 \times 850 = 600$$

$$\therefore \alpha = 21.1^o$$

# Example

*shows a single column manometer connected to a pipe* [redacted] *containing liquid of specific gravity 0·8. The ratio of area of the reservoir to that of the limb is 100. Find the pressure in the pipe.*

Solution

$$P1 = P2$$

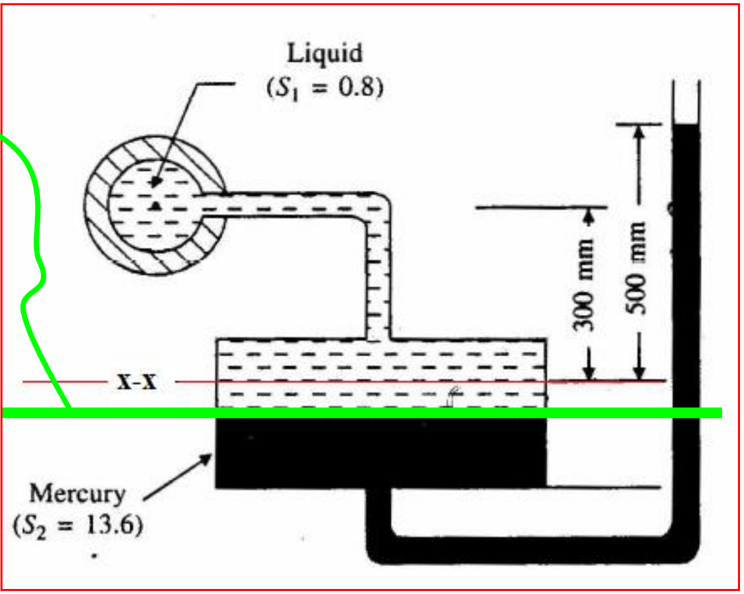
$$P + 800 \times 9.81 \times (0.3 + \delta h) = 13600 \times 9.81 \times (0.5 + \delta h)$$

$$P + 800 \times 9.81(0.3 + \frac{1}{100}0.5) = 13600 \times 9.81(0.5 + \frac{1}{100}0.5)$$

$$P + 2393.6 = 67375.1$$

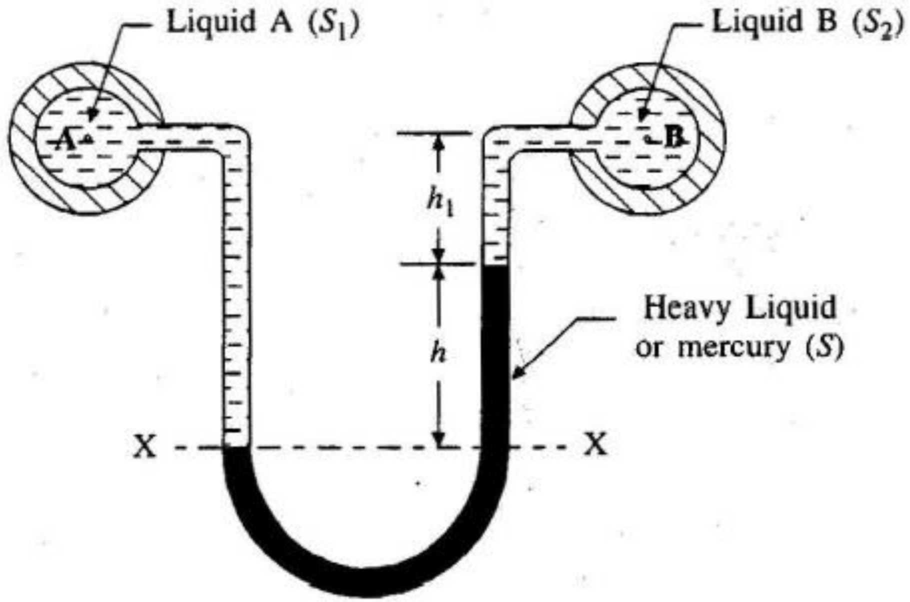
$$\mathrm{P} = 64981.5$$

$$h = \frac{P}{\rho_1 g} = \frac{64981.5}{800 \times 9.81} = (8.28\ m)\text{of liquid}$$



# Differential manometers

*A differential manometer is used to measure the difference in pressures between two points in a pipe, or in two different pipes.*

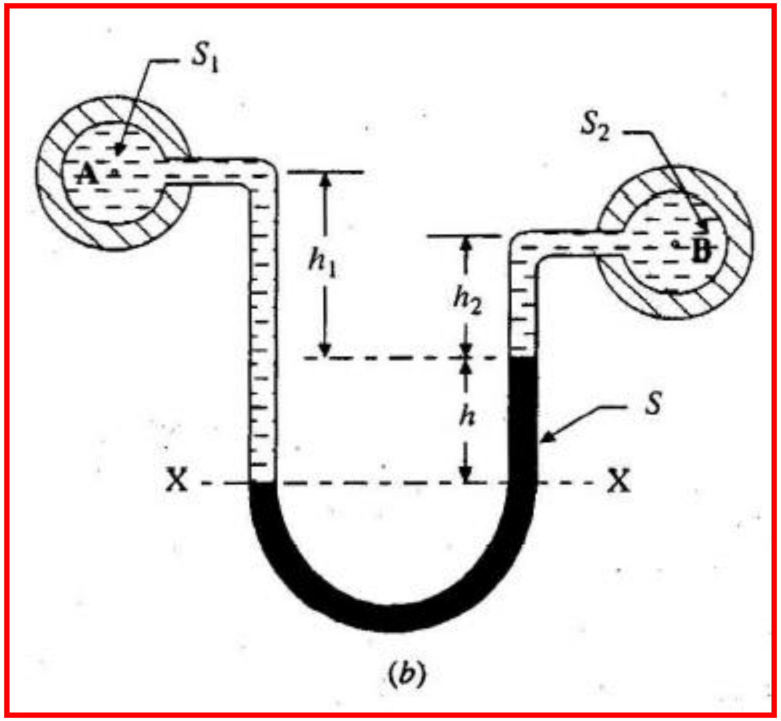



(a)

$$At \quad datum \quad \text{x-x}$$

$$P1 = P2$$

$$P_A + \rho_A g(h_1 + h) = P_B + \rho_B g(h_1) + \rho_{mercury} gh$$

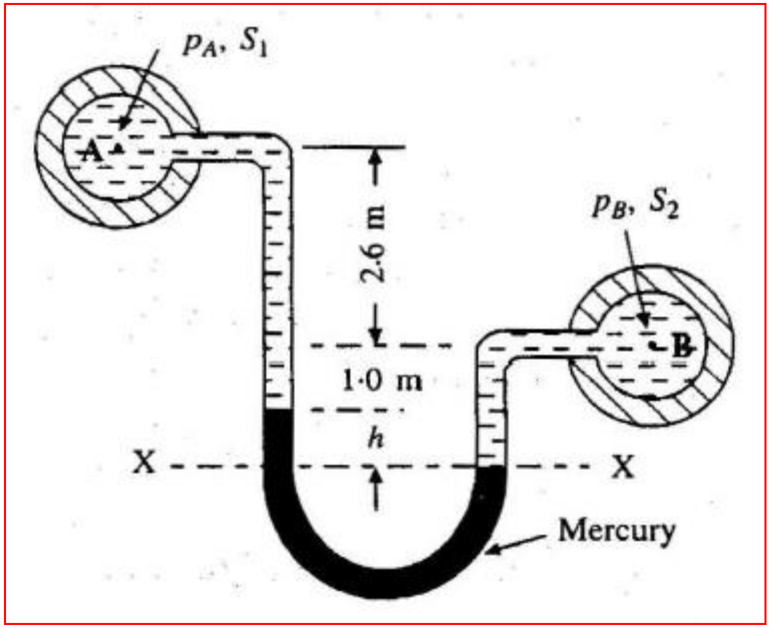



$$At \quad datum \quad x - x$$

$$P1 = P2$$

$$P_A + \rho_A g(h_1 + h) = P_B + \rho_B g(h_2) + \rho_{mercury} g h$$

*a U-tube differential manometer connecting two pressure pipes at A and B. The pipe A contains a liquid of specific gravity 1·6 under a pressure of 110 kN/m². The pipe B contains oil of specific gravity 0·8 under a pressure of 200 kN/m². Find the difference of pressure measured by mercury as fluid filling U-tube.*




Solution

$$\begin{aligned}
&At\quad datum\quad \text{x-x}\\
&P1 = P2\\
&P_B + \rho_B g(1+h) = P_A + \rho_B g(3.6) + \rho_{mercury} gh
\end{aligned}$$

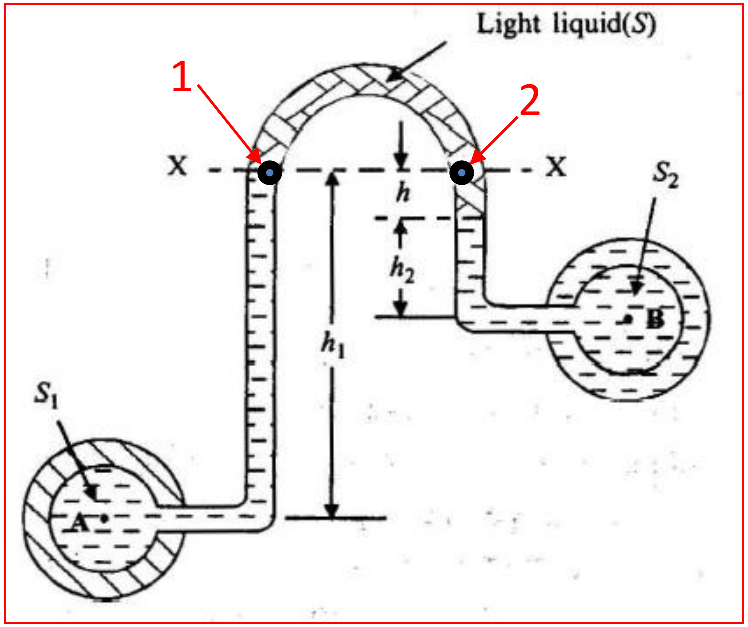
$$\begin{aligned}
&200\times10^3 + 800\times9.81\times(1+h) = 110\times10^3 + 1600\times9.81\times3.6 + 13600\times9.81\times h\\
&207848 + 7848(h) = 166505.6 + 133416(h)\\
&41342.4 = 125568(h)\\
&h = 0.329\,m
\end{aligned}$$

# Inverted U-tube Differential manometers

$$At \quad datum \quad \text{x-x}$$

$$P1 = P2$$

$$P_A - \rho_A g(h_1) = P_B - \rho_B g(h_2) - \rho_{liquid}\, gh$$



# Example

*an inverted differential manometer having an oil of specific gravity 0·8 connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B. The pressure in pipe A is 2·0 metres of water.*

$$At \quad datum \quad \text{x-x}$$

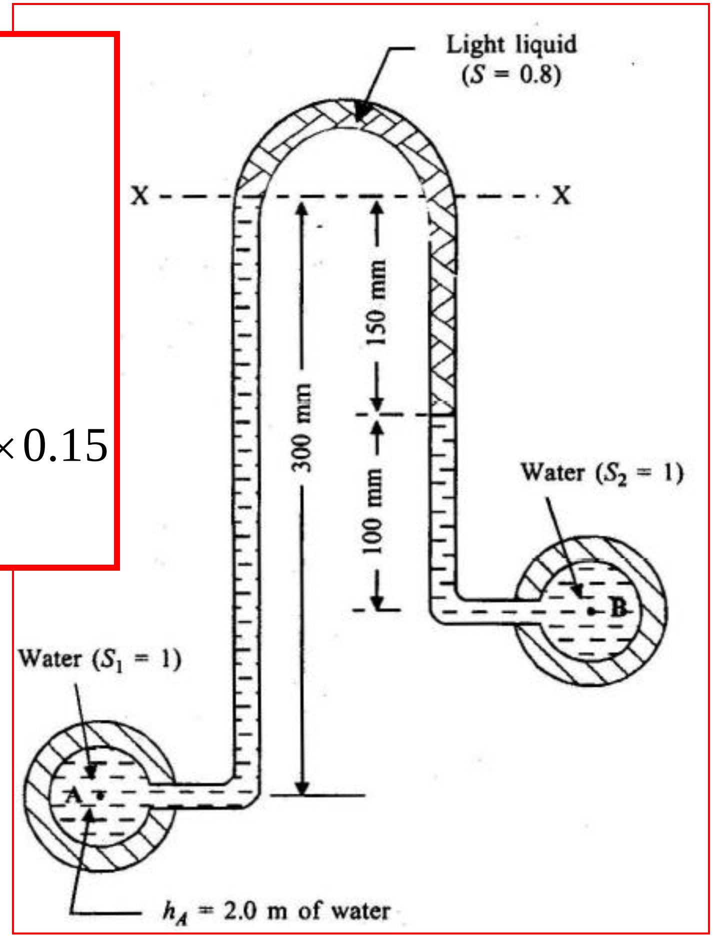
$$P1 = P2$$

$$P_A = 2 \times 1000 \times 9.81 = 19620 \text{ N/m}^2$$

$$P_A - \rho_A g(h_1) = P_B - \rho_B g(h_2) - \rho_{liquid} gh$$

$$19620 - 1000 \times 9.81 \times 0.3 = P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.15$$

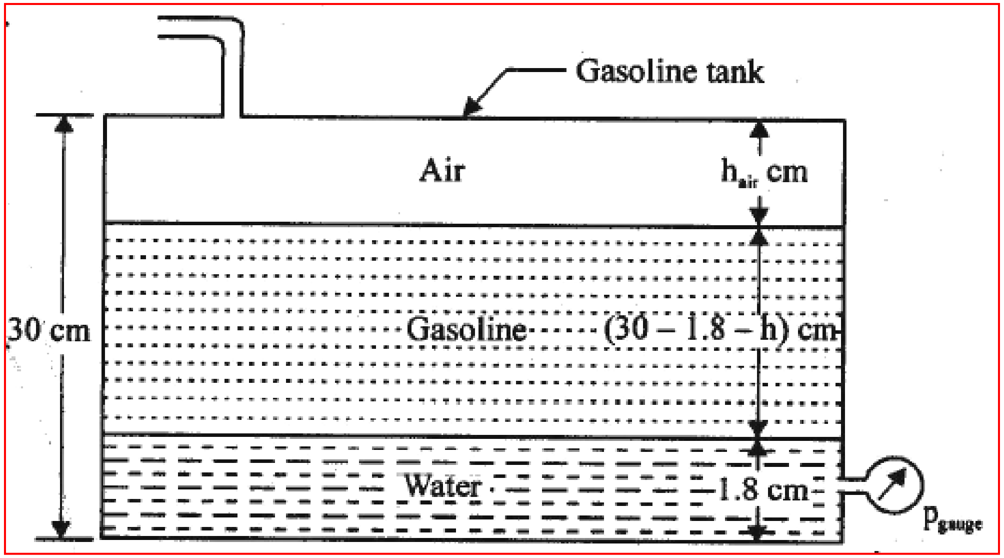
$$P_B = 18835 \text{ N/m}^2 = 18.8 \text{ kN/m}^2$$



# Example 1

## Example 1

**The figure** *shows a fuel gauge, for a gasoline tank in car, which reads proportional to the bottom gauge. The tank is 30 cm deep and accidently contains 1.8 cm of water in addition to the gasoline. Determine the height of air remaining at the top when the gauge erroneously reads full.*




Solution: $w_{\text{gasoline}}$ = 6.65 kN/m2 , $w_{\text{air}}$ = 0.0118 kN/m2

$$P_{gauge} = \rho g h = w h = 6.65\times10^3 \times 0.30 = 1.995\,\text{kN/m}^2$$

*Also*

$$P_{gauge} = \rho_{water} g h_{water} + \rho_{gaso.} g h_{gaso.} + \rho_{air} g h_{air}$$

$$1.995\times10^3 = 1000\times9.81\times0.018 + 6.65\times10^3(0.3-0.018-h) + 0.0118\times10^3(h)$$

$$1.995\times10^3 = 176.6 + 1875.3 - 6.65\times10^3\,h + 0.0118\times10^3\,h$$

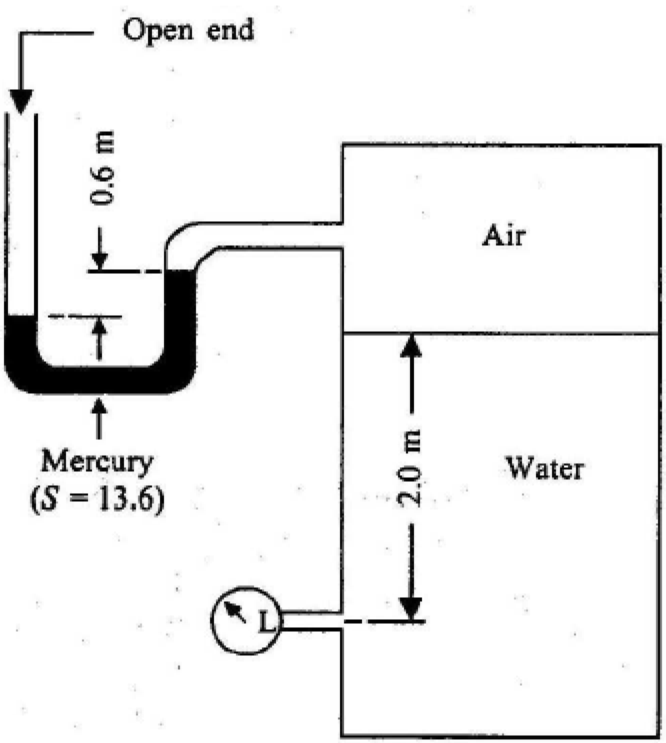
$$h = 8.57\times10^{-3}\,\text{m} = 0.857\,\text{cm}$$

# Example 2

[redacted] *In the Fig.* [redacted] *if the local atmospheric pressure is 755 mm of mercury (sp. gravity = 13·6), calculate:*

*(i) Absolute pressure of air in the tank;*

*(ii) Pressure gauge reading at L.*




Solution:

$$P1 = P2$$

$$0 = P_{air} + \rho_{mercury} g(h_{mercury})$$

$$P_{air} = -13.6\times10^3 * 9.81 * 0.6 = -80kN/m^2$$

$$P_{abs} = P_{air} + P_{atm.} =$$

$$P_{atm.} = 0.755 * 13600 * 9.81 = 100.73kN/m^2$$

$$P_{abs} = -80\times10^3 + 100.73 = 20.73kN/m^2$$

$$P_L = P_{air} + \rho_{water} gh = -80\times10^3 + 1000 * 9.81 * 2 = -60.38kN/m^2$$

# Example 3

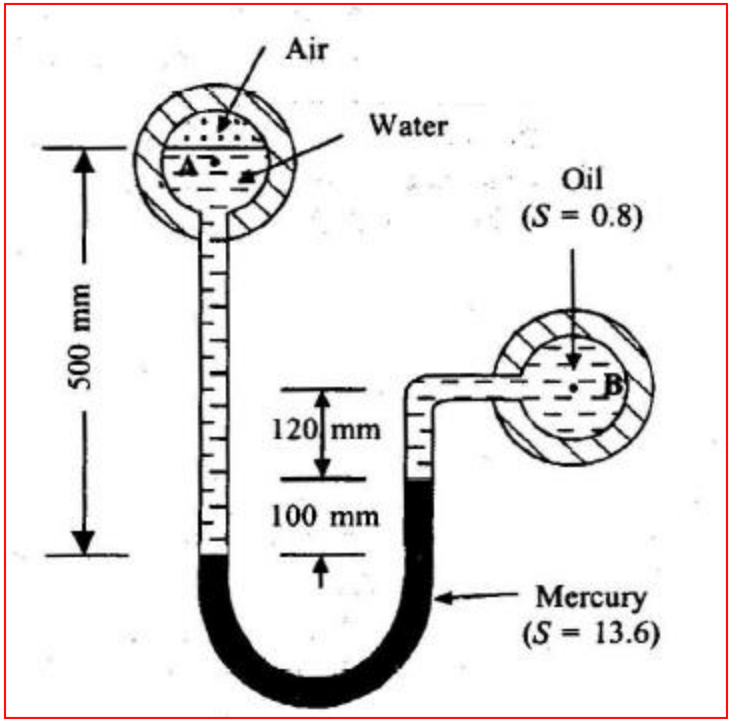
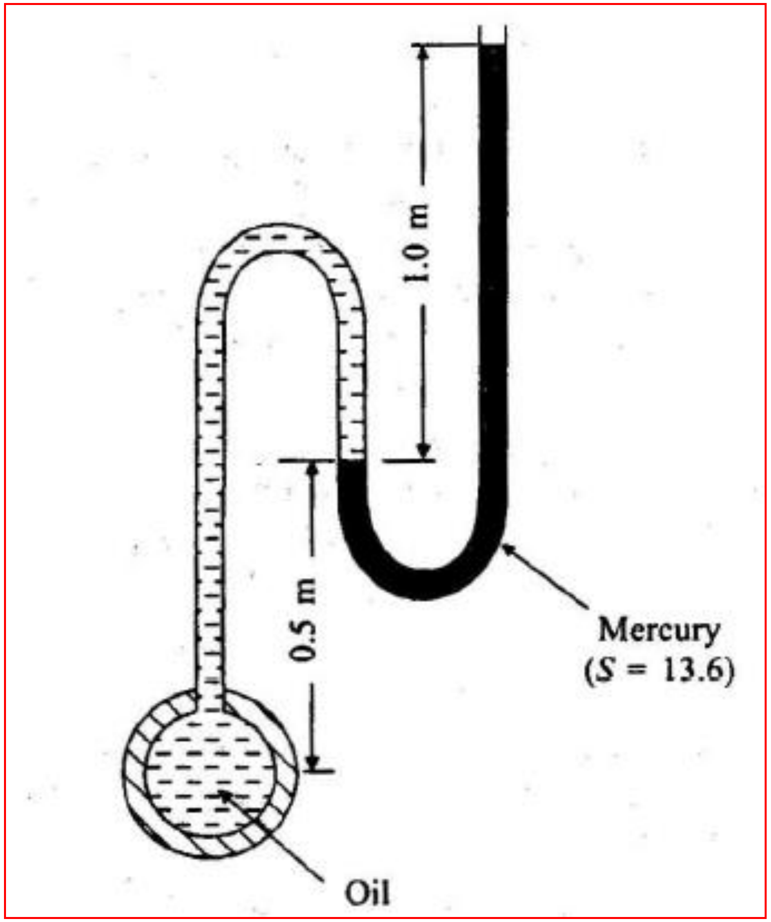



Fig. [redacted] shows a differential manometer connected at two points $A$ and $B$. If at $A$ air pressure is $78{\cdot}5\ \mathrm{kN/m^2}$, find the absolute pressure at $B$. [**Ans.** $69{\cdot}1\ \mathrm{kN/m^2}$]

# Example




A U-tube containing mercury is used to measure the pressure of an oil of specific gravity 0·8 as shown in Fig. [redacted] Calculate the pressure of the oil, if the difference of mercury level be 0·5 m. [**Ans.** 14 m]