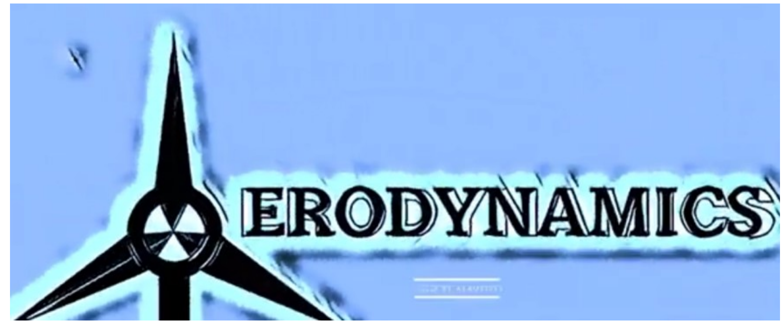

Module 1

MEE1004-FLUID MECHANICS

Problems related to Hydrostatic Pressure and Buoyancy

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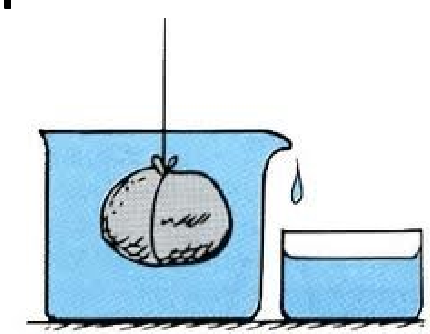
Weight vs. Buoyant Force

- **An object in a fluid will sink if it has a weight greater than the weight of the fluid that is displaced.**
- Sinking happens when weight is greater than the buoyant force acting on it
- Floating happens only when it displaces a volume of liquid that has weight equal to the object's weight
- **Buoyant force opposes gravity**

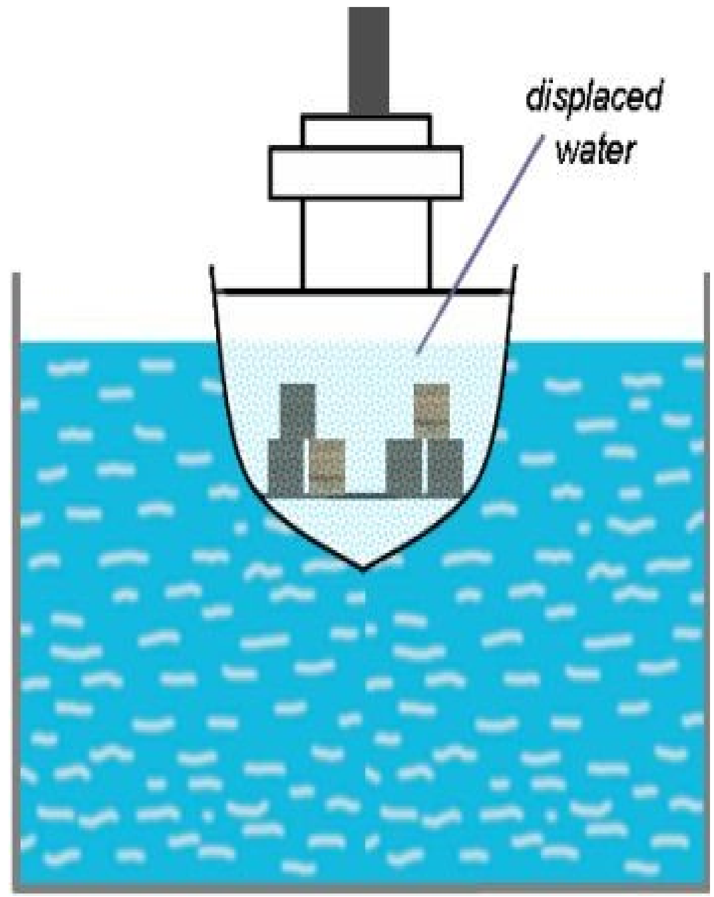


An object will float or sink...

- **Based on its density (mass/volume)**
- **Things that sink in water are *more* dense than water**
- **Things that float in water are *less* dense**
- **In air, most substances are more dense than air**
  - **Exception is helium (balloons)**

bubbles

balloons



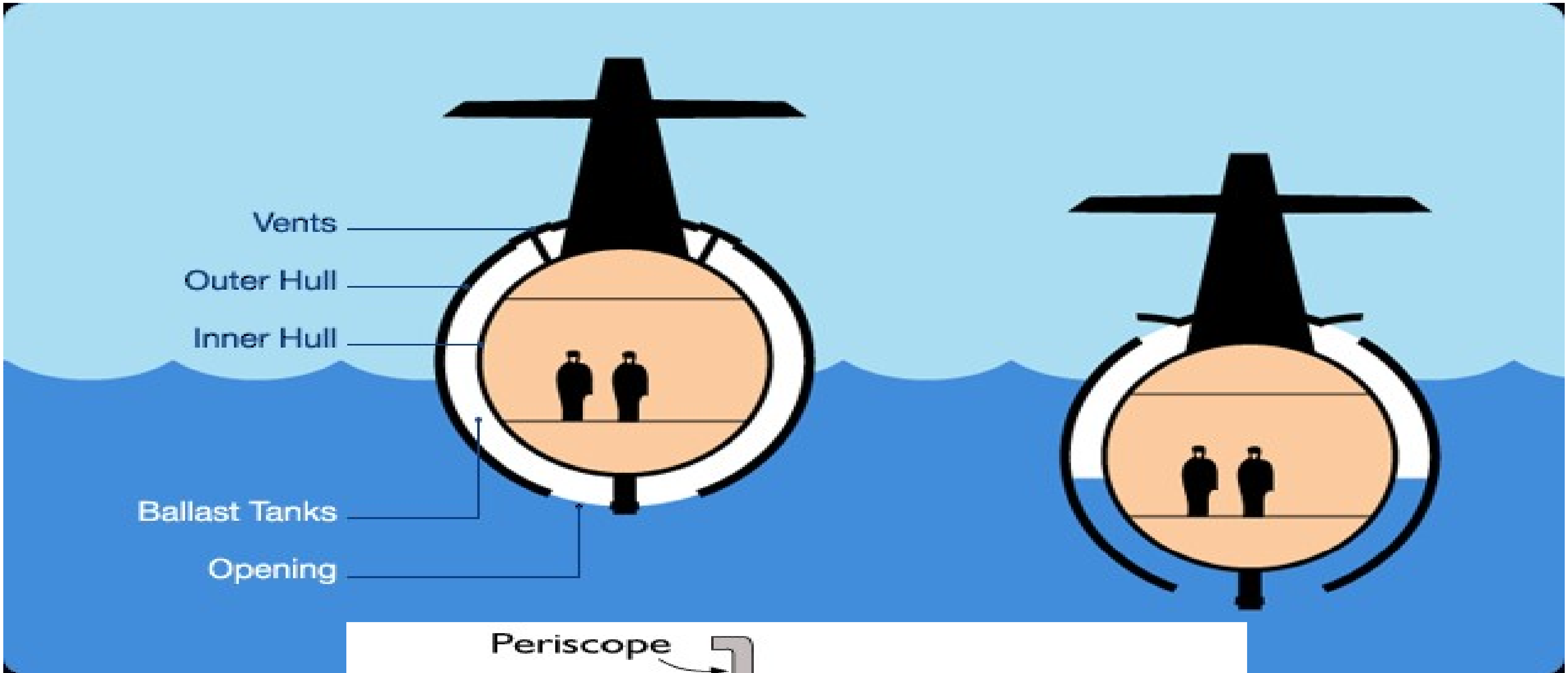
Submarines

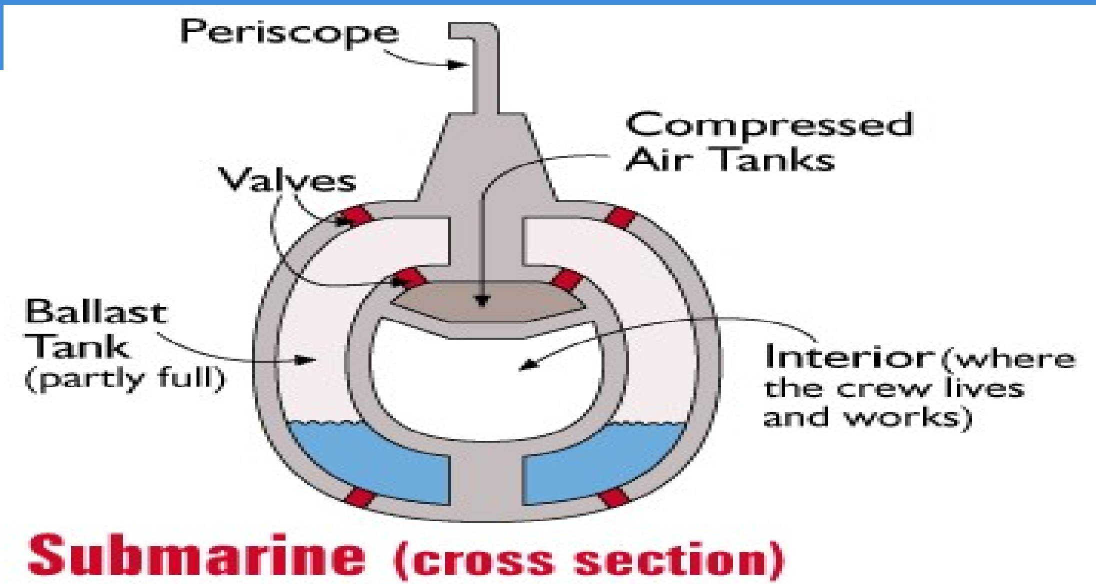
A special kind of ship that can travel on the surface of the water AND underwater

Special tanks that open to allow water in, adding mass, causing it to sink.

Crew members can control the amount of water taken in, changing the density and depth in its ocean

Compressed air is used to low water out


Vents
Outer Hull
Inner Hull
Ballast Tanks
Opening

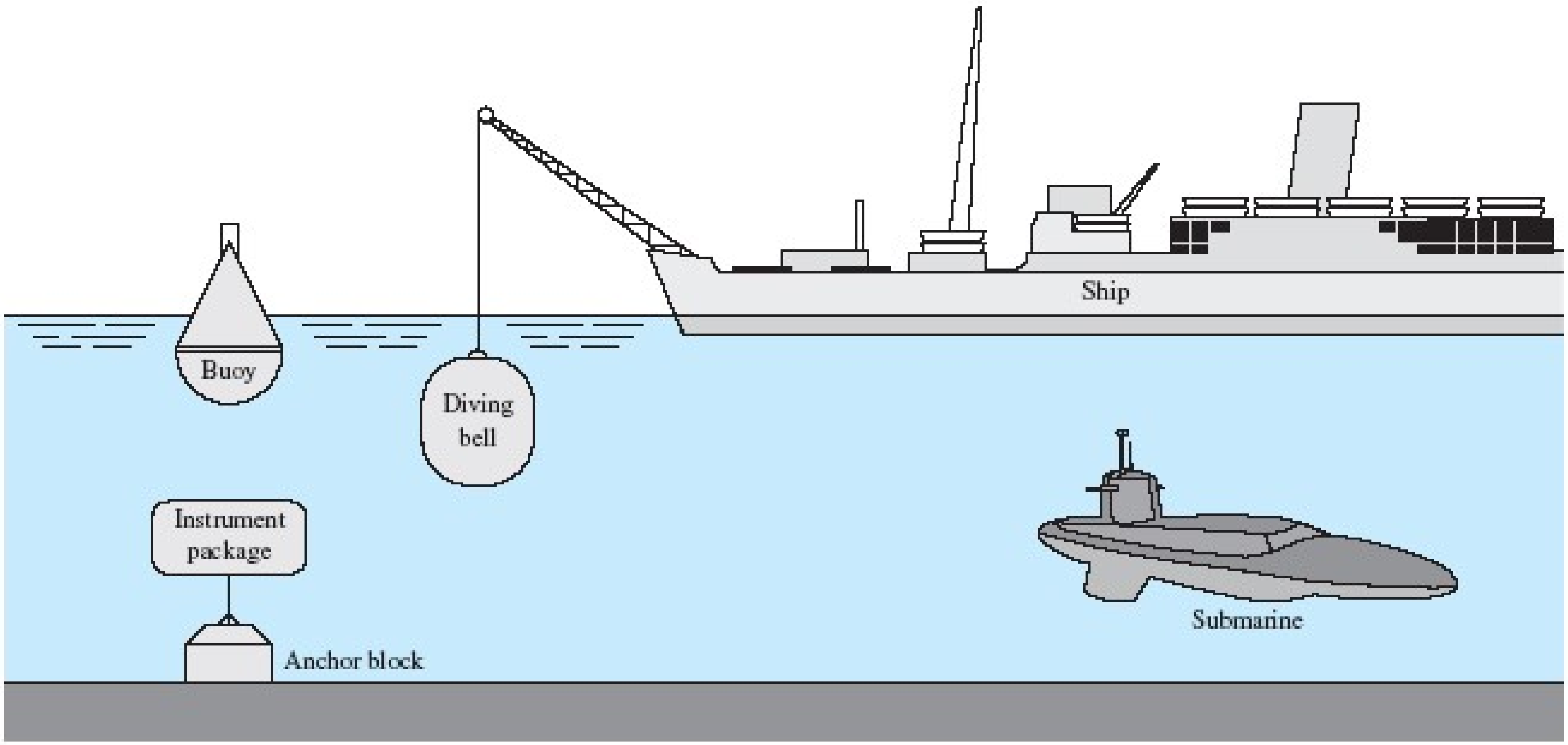

Periscope
Compressed
Air Tanks
Valves
Ballast
Tank
(partly full)
Interior (where
the crew lives
and works)
Submarine (cross section)

How is a fish like a submarine?

- **Like a submarine, some fish adjust their overall density in order to stay at a certain depth in the water using their swim bladder**
- **The swim bladder is filled with gases produced in the fish's blood, it inflates to decrease density**
- Sharks do not have a swim bladder so they have to constantly swim to keep from sinking

Buoyancy & Stability

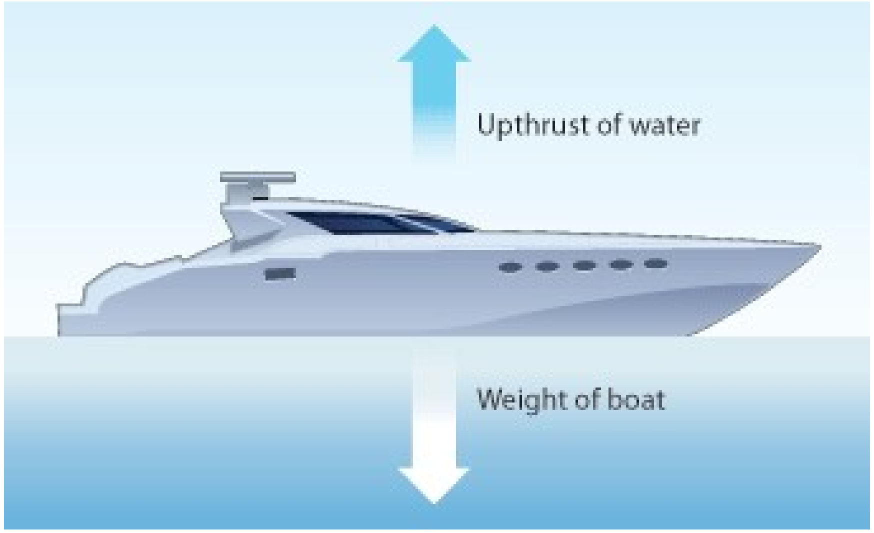
Examples of types of Buoyancy Problems:



Introduction:

- Whenever a body is placed over a liquid, either it sinks down or floats on the liquid.

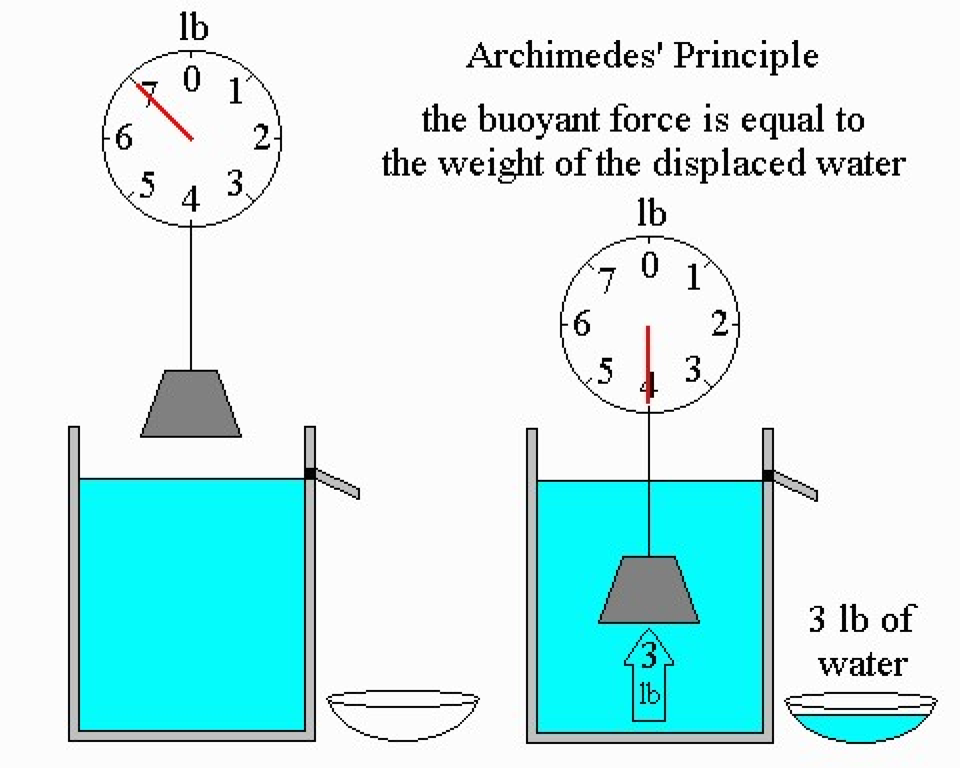
- Two forces involve are:
  1. Gravitational Force
  2. Up-thrust of the liquid



- If Gravitation force is more than Upthrust, body will sink.
- If Upthrust is more than Gravitation force, body will float.

Archimedes Principle:

- “Whenever a body is immersed wholly or partially in a fluid, it is buoyed up (i.e lifted up) by a force equal to the weight of the fluid displace



Buoyancy:

- A body in a fluid, whether floating or submerged, is buoyed up by a force equal to the weight of the fluid displaced.
- "The tendency of a fluid to uplift a submerged body, because of the up-thrust of the fluid, is known as force of buoyancy or simply buoyancy."
- The buoyant force acts vertically upward through the centroid of the displaced volume and can be defined mathematically by Archimedes' principle as follows:

$$F_d = \gamma_f V_d$$

F_d = Buoyant force

γ_f = Specific weight of fluid

V_d = Displaced volume of fluid

A 60-cm square gate (a = 0.6) has its top edge 12 m below the water surface. It is on a 45º angle and its bottom edge is hinged as shown in Fig. What force '*P*' is needed to just open the gate?

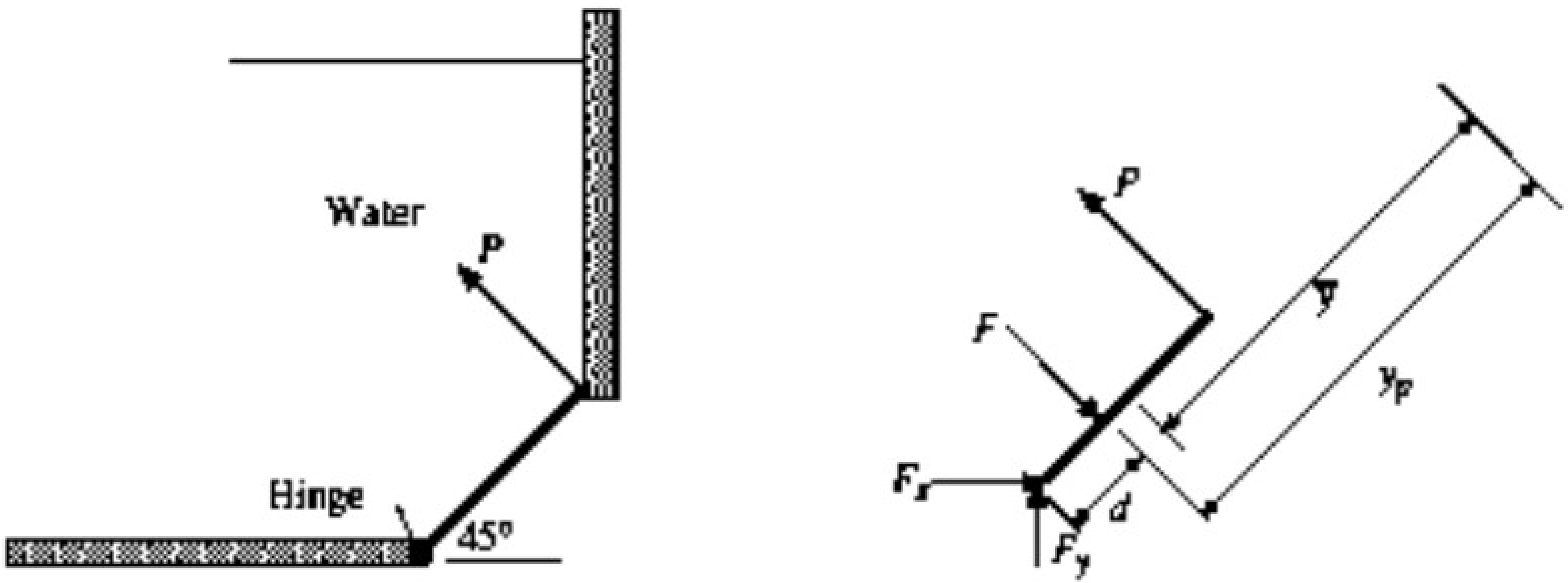


Figure 3: The hydrostatic force on a gate.

$$F = \rho \overline{h} A = 1000 * 9.81 * (12 + 0.3 \sin 45^o) 0.6 * 0.6 = 43130 \ N \tag{6}$$

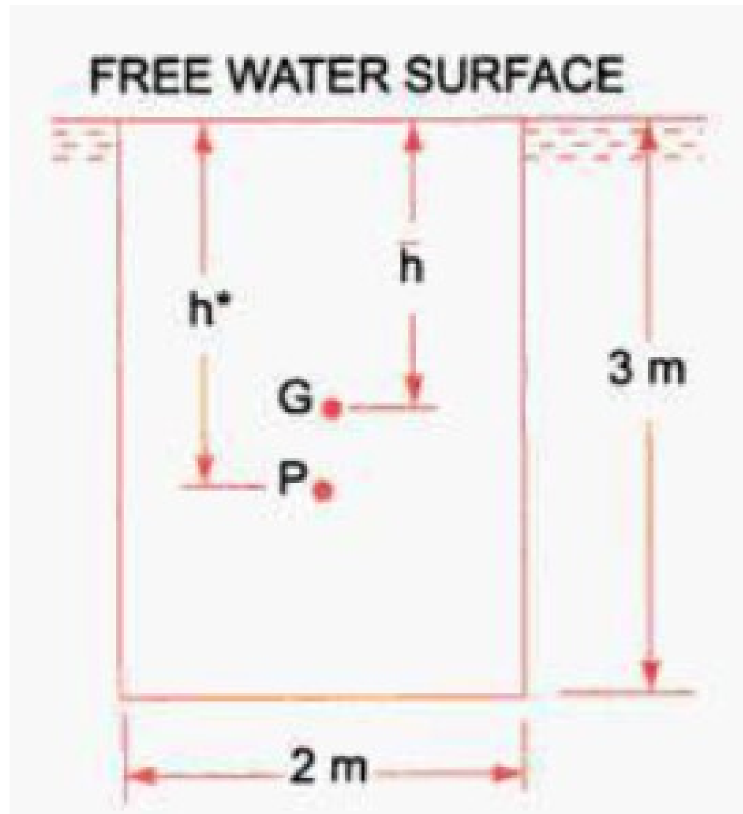
Now we find the distance d where the force F acts from the hinge:

$$\begin{aligned}
\overline{y} &= \frac{\overline{h}}{\sin 45^o} = \frac{12 + 0.3 \sin 45^o}{\sin 45^o} = 17.27 \ m \\
y_p &= \overline{y} + \frac{I_s}{A\overline{y}} = 17.27 + \frac{0.6 * 0.6^3}{12 * (0.6 * 0.6) * 17.27} = 17.272 \ m \\
d &= 0.3 - (y_p - \overline{y}) \cong 0.3 \ m
\end{aligned} \tag{7}$$

The force P can be calculated

$$P = d * F/a = \frac{0.343130}{0.6} = 21040 \ N. \tag{8}$$

A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plain in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water (b) 2.5 m below the free water surface.



Determine the total pressure and position of centre of pressure on the plane surface when its **upper edge is horizontal and (a) coincides with water**

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$

$A = 3 \times 2 = 6 \text{ m}^2$, $\bar{h} = \frac{1}{2}(3) = 1.5$ m.

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290 \text{ N. Ans.}}$$

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

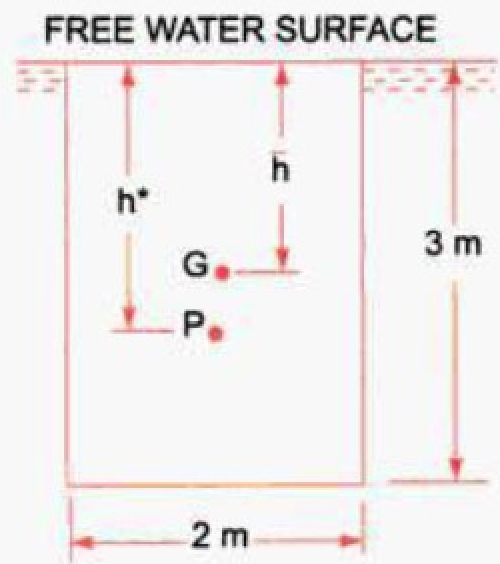


Fig. 3.2

$$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0 \text{ m. Ans.}}$$

Determine the total pressure and position of centre of pressure on the plane surface when its **upper edge is horizontal and (b) 2.5 m below the free water surface.**

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440 \text{ N. Ans.}}$$

Centre of pressure is given by $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875 \text{ m. Ans.}}$$

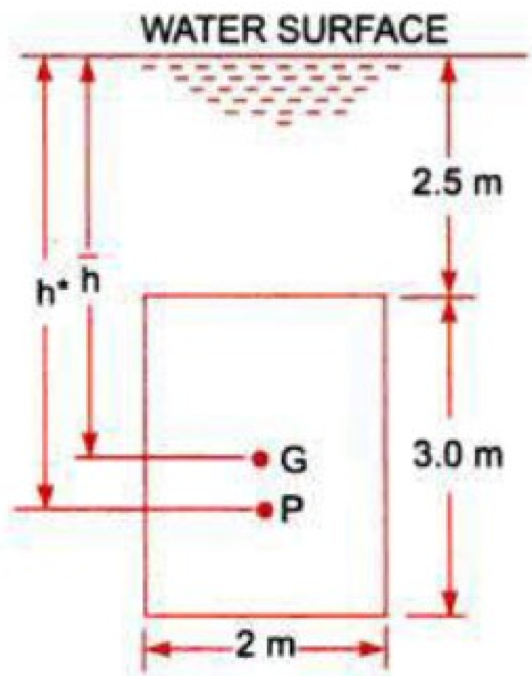
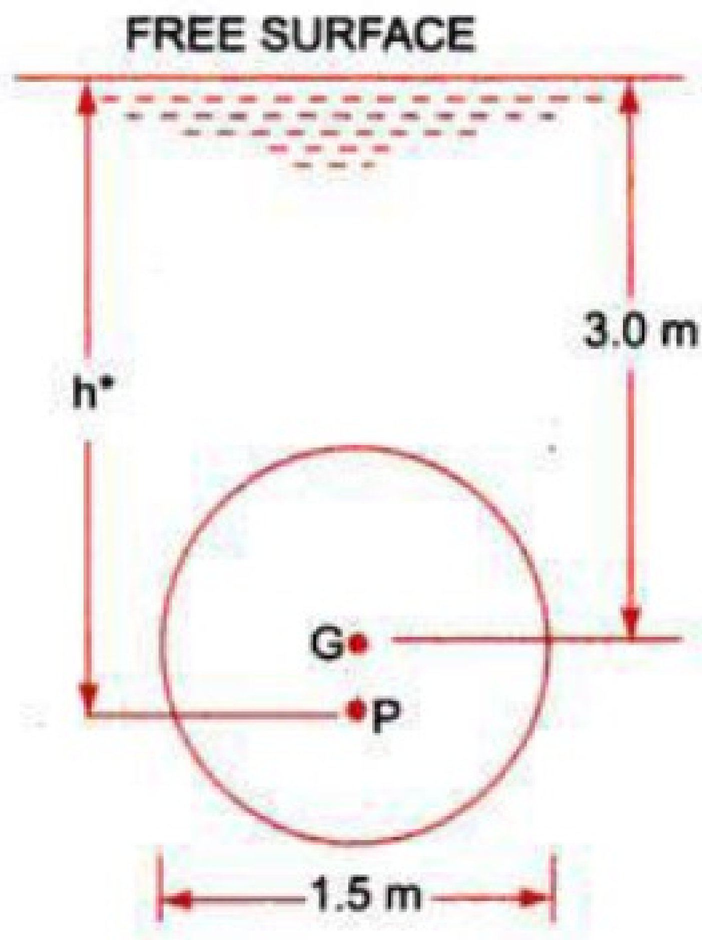
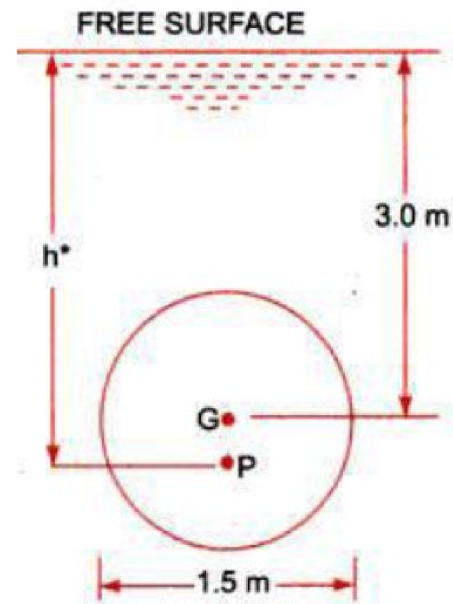


Fig. 3.3

Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.



Solution. Given : Dia. of plate, $d = 1.5$ m



$\therefore$ Area,
$$A = \frac{\pi}{4}(1.5)^2 = 1.767 \text{ m}^2$$

$$\bar{h} = 3.0 \text{ m}$$

Total pressure is given by equation (3.1),

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81 \text{ N. Ans.}}$$

Position of centre of pressure (h^*) is given by equation (3.5)

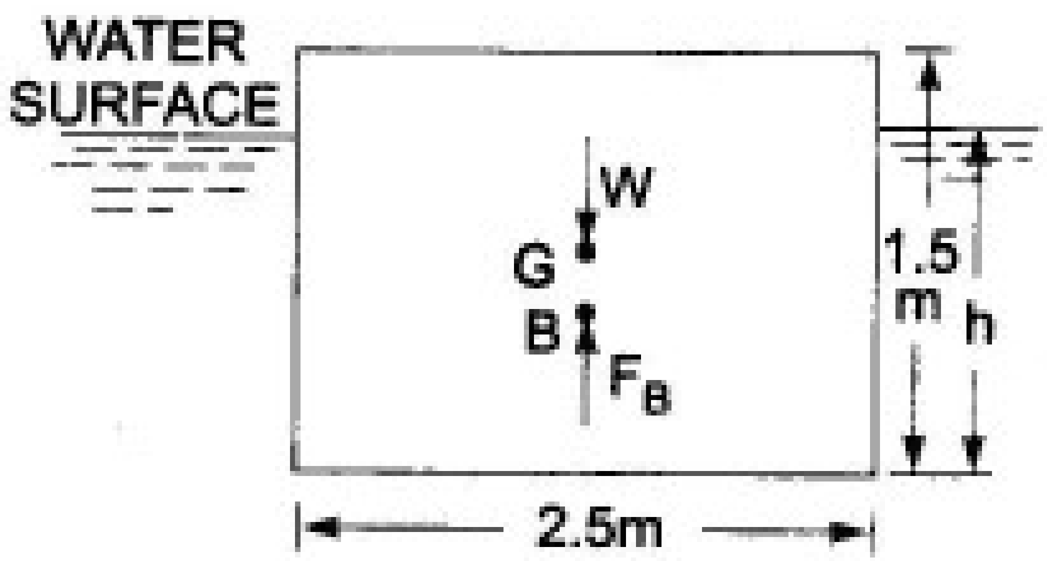
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

here $I_G = \dfrac{\pi d^4}{64} = \dfrac{\pi \times 1.5^4}{64} = 0.2485 \text{ m}^4$

$\therefore$
$$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$
$$= \mathbf{3.0468 \text{ m. Ans.}}$$

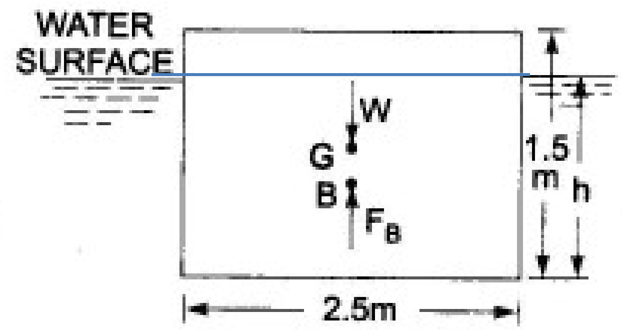
Buoyancy

Find the volume of the water displaced and position of Centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m3 and its length 6.0 m.



$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$



For equilibrium the weight of water displaced = Weight of wooden block

$= 143471 \text{ N}$

$\therefore$ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625\ m^3.\ Ans.}$$

($\because$ Weight density of water $= 1000 \times 9.81 \text{ N/m}^3$)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

$2.5 \times h \times 6.0 = 14.625 \text{ m}^3$, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875\ m\ from\ base.\ Ans.}$$

A wooden block of 4m x 1m x 0.5m in size and of specific gravity 0.75 is floating in water. Find the weight of concrete of specific weight 24k kN/m3 that may be placed on the block, which will immerse the wooden block completely.

Let W be the weight of Concrete required to be placed on wooden block.

Volume of wooden block $= 4 \times 1 \times 0.5 = 2m^3$

and its Weight $= 9.81 \times 0.75 \times 2 = 14.72$ kN

$\therefore$ Total weight of the block and concrete $= 14.72 + W$ kN

We know that when the block is completely immersed in water,

volume of water displaced $= 2m^3$

$\therefore$ Upward thrust when the block is completely immersed in water

$$= 9.81 \times 2 = 19.62 \text{ kN}$$

Now equating the total weight of block and concrete with upward thrust

$$14.72 + W = 19.62$$

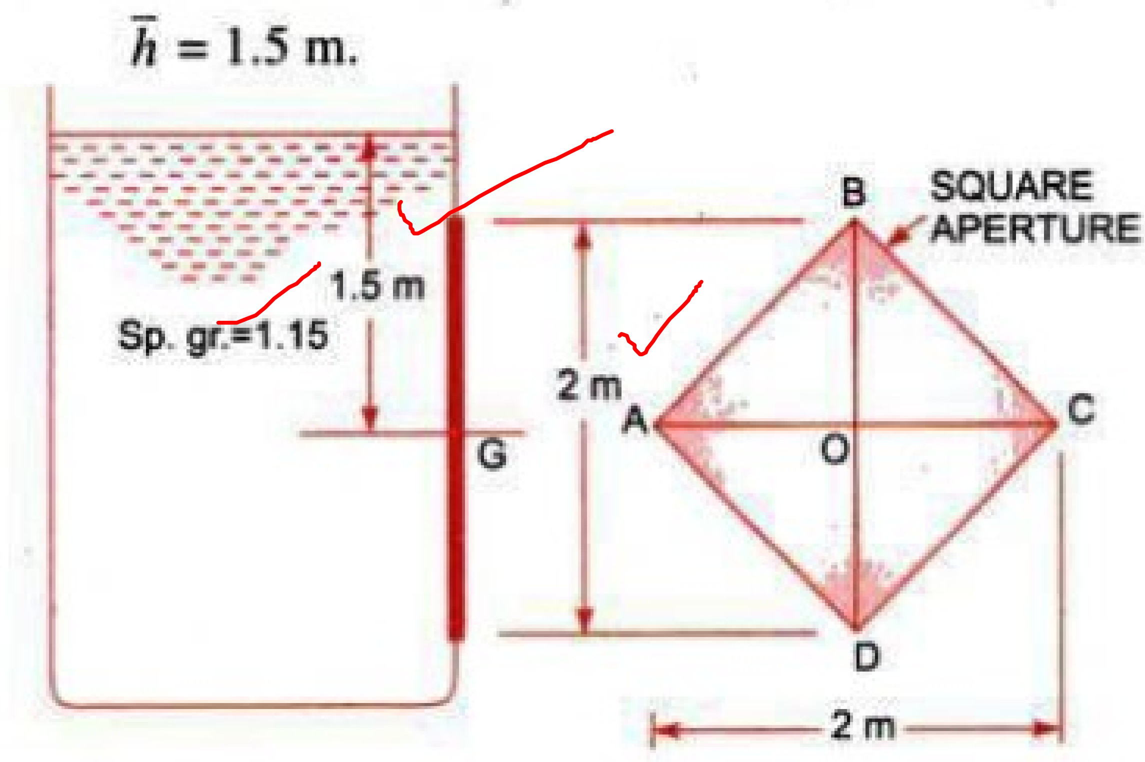
$$W = 4.9 \text{ kN}$$

1. A rectangular block of 5m long, 3m wide and 1.2m deep is immersed 0.8m in the sea water. If the density of sea water is 10kN/m2, find the metacentric height of block.
2. A solid cylinder of 2m diameter and 1m height is made up of a material of Sp. Gravity 0.7and floats in water. Find its metacentric height.
3. A rectangular timber block 2m long, 1.8m wide and 1.2m deep is immersed in water. If the specific gravity of the timber is 0.65, prove that it is in stable equilibrium.
4. A cylindrical buoy of 3m diameter and 4m long is weighing 150N. Show that it cannot float vertically in water.
5. A solid cylinder of 360mm long and 80mm diameter has its base 10mm thick of specific gravity 7. The remaining part of cylinder is of specific gravity 0.5. Determine, if the cylinder can float vertically in water.

A square aperture in the vertical side of a tank has one diagonal vertical and is completely covered by a plane plate hinged along one of the upper side of the aperture. The diagonals of the aperture are 2m long and the tank contains a liquid of specific gravity 1.15. The centre of aperture is 1.5 m below the free surface. Calculate the thrust exerted on the plate by the liquid and position of its centre of pressure.

(A.M.I.E., Summer, 1986)

"Bansal"



Solution. Given : Diagonals of aperture, $AC = BD = 2$ m

$\therefore$ Area of square aperture, A = Area of ΔACB + Area of ΔACD

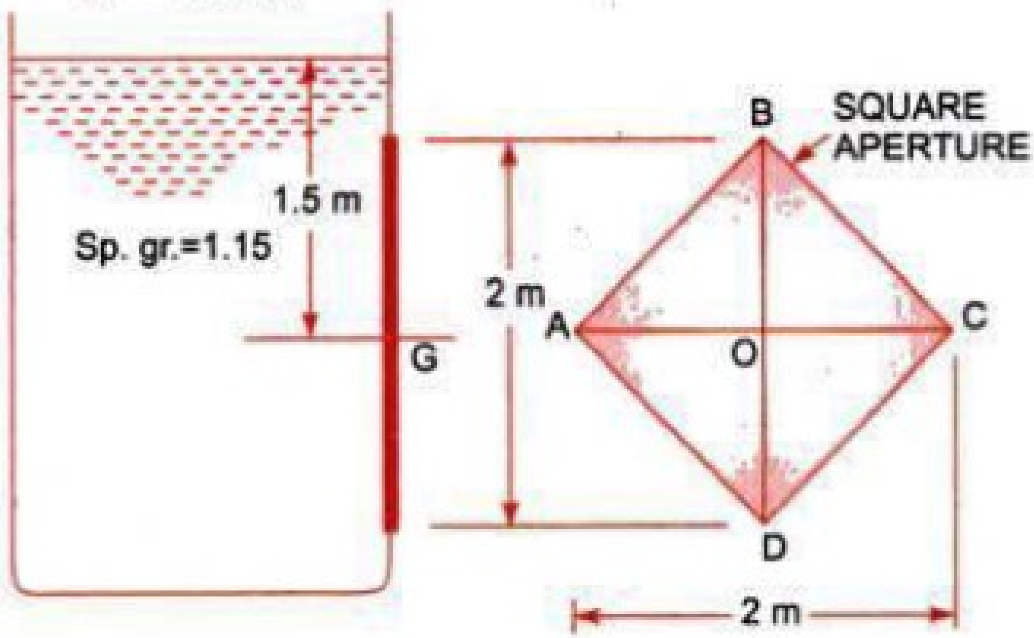
$$= \frac{AC \times BO}{2} + \frac{AC \times OD}{2} = \frac{2 \times 1}{2} + \frac{2 \times 1}{2} = 1 + 1 = 2.0 \text{ m}^2$$

Sp. gr. of liquid $= 1.15$

$\therefore$ Density of liquid, $\rho = 1.15 \times 1000 = 1150 \text{ kg/m}^3$

Depth of centre of aperture from free surface,

$\bar{h} = 1.5$ m.



(*i*) The thrust on the plate is given by

$$F = \rho g A\bar{h} = 1150 \times 9.81 \times 2 \times 1.5 = \mathbf{33844.5.\ Ans.}$$

(*ii*) Centre of pressure (h^*) is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of $ABCD$ about diagonal AC

= M.O.I. of triangle ABO about AC + M.O.I. of triangle ACD about AC

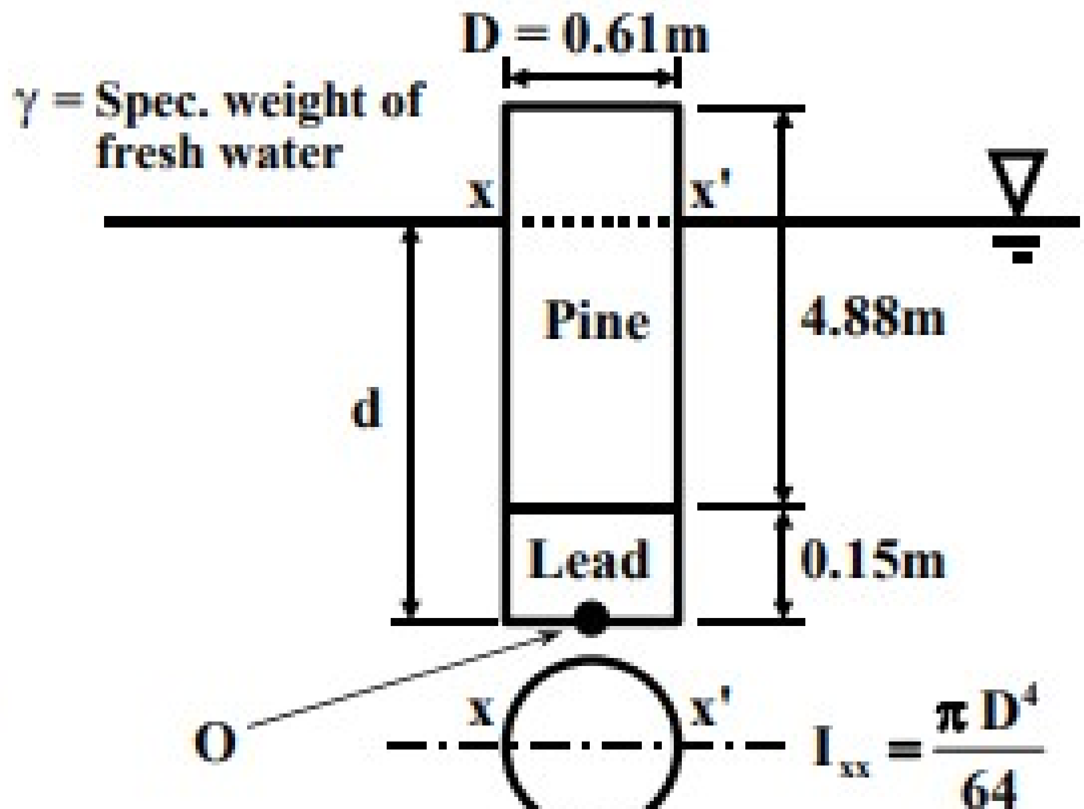
$$= \frac{AC \times OB^3}{12} + \frac{AC \times OD^3}{12} \qquad \left(\because \text{ M.O.I. of a triangle about its base} = \frac{bh^3}{12}\right)$$

$$= \frac{2 \times 1^3}{12} + \frac{2 \times 1^3}{12} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}\ \text{m}^4$$

$$\therefore \qquad h^* = \frac{\frac{1}{3}}{2 \times 1.5} + 1.5 = \frac{1}{3 \times 2 \times 1.5} + 1.5 = \mathbf{1.611\ m.\ Ans.}$$

Problem 3.7 *A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.45, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.* (A.M.I.E., May, 1975)

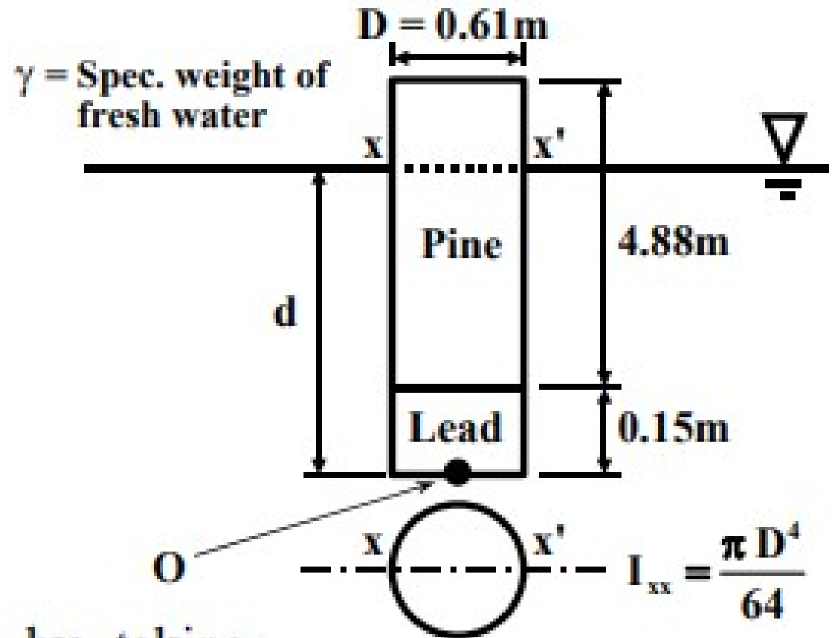
A solid cylindrical pine (SG=0.5) spar buoy has a cylindrical lead (SG=11.3) weight attached as shown. Determine the equilibrium position (i.e. depth of immersion) of the buoy in seawater (SG=1.03). Calculate the metacentric height and show that the buoy is stable.



Weight of fluid displaced = Weight of body

$d\,A\,1.03\,\gamma = 0.15\,A\,11.3\,\gamma + 4.88\,A\,0.5\,\gamma$
where $A = \pi D^2/4$

Hence, $\underline{d = 4.01\text{ m}}$



Find centre of gravity of complete buoy by taking moments of the weight about the base (O):

$$GO = z_G = \frac{0.15\,A\,11.3\,\gamma\,(0.15/2) + 4.88\,A\,0.5\,\gamma\,(4.88/2 + 0.15)}{0.15\,A\,11.3\,\gamma + 4.88\,A\,0.5\,\gamma} = 1.56\text{ m}$$

Centre of buoyancy (BO) will be at half the height of the submerged part of the body (i.e. half the height of the displaced volume). Hence, $BO = z_B = d/2 = 2.00$m.

$GB = GO - BO = 1.56 - 2.00 = -0.44$ m

$$MB = \frac{I}{V_S} = \frac{\frac{\pi D^4}{64}}{\frac{\pi D^2}{4} d} = \frac{D^2}{16\,d} = 5.8\text{x}10^{-3}\text{ m}$$

Therefore, metacentric height $MG = MB - GB = 5.8\text{x}10^{-3} - -0.44 = 0.45$ m . $MG > 0$, hence, stable.

End of Lecture !!!