Fluid Mechanics

- Pressure and its measurement

Scuba Diving and Hydrostatic Pressure

Lecture : 6 to 8

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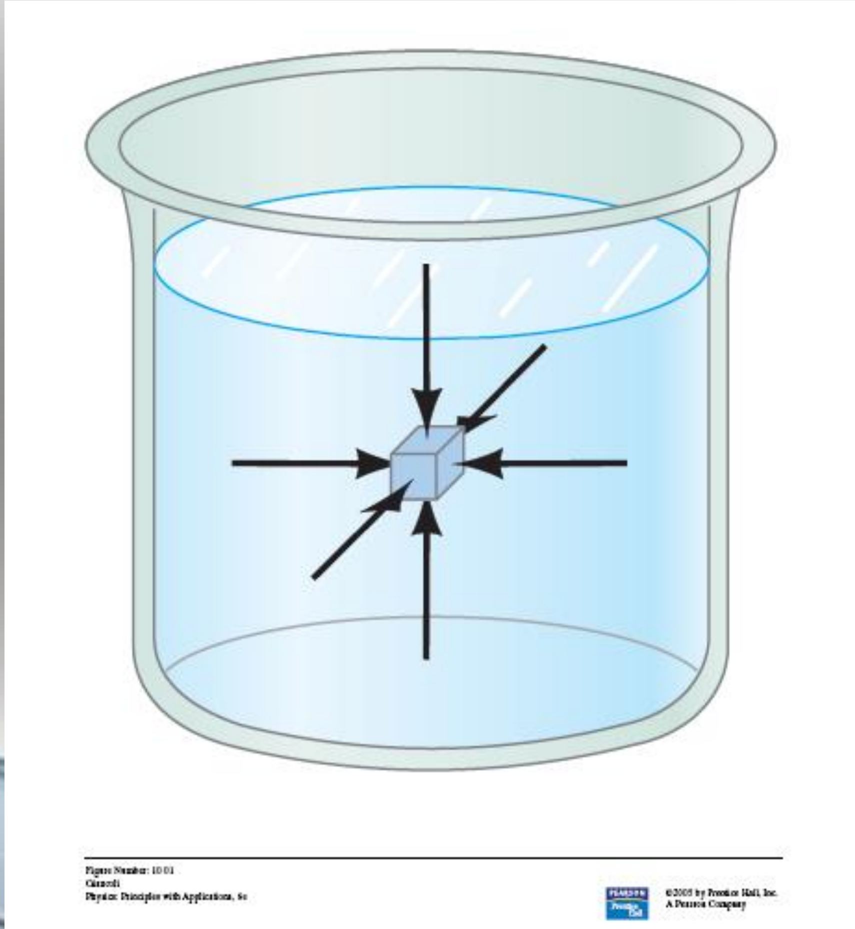
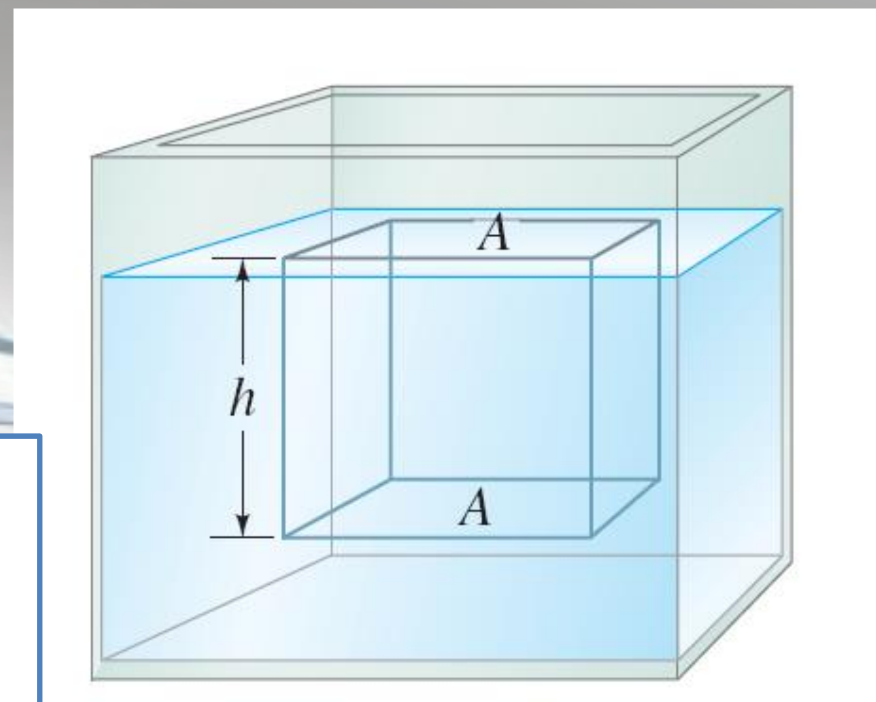


The pressure is the same in every direction in a fluid at a given depth.



Pressure varies with depth. →

$$P = \frac{F}{A} = \frac{\mathbf{r}Ahg}{A} \quad \text{so } P = \mathbf{r}gh$$

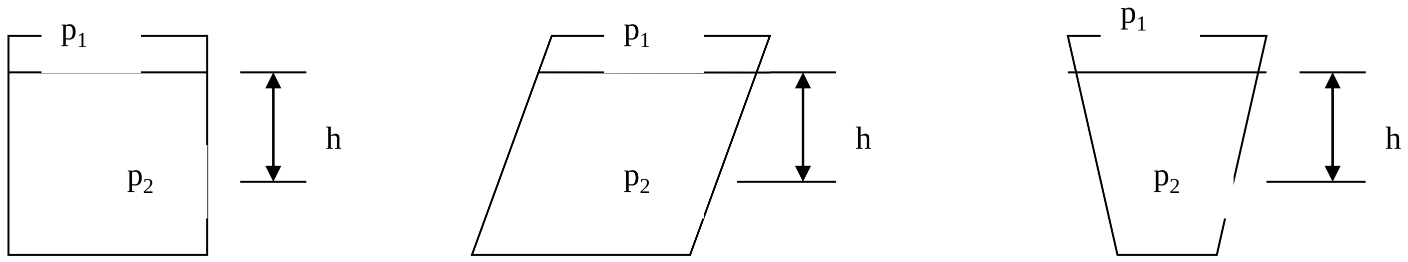
FLUID PRESSURE

$P = F/A. \quad (Nm^{-2})$

Units: N/m^2 or Pa (1 Pascal*)

$dynes/cm^2$ or PSI (lb/in^2)

1 atm = 1.013 x 10^5 Pa or 15 lbs/in^2



- pressure *p1* on the surface of the water is 1 atm, or 1.013 x 105 Pa. If we go down to a depth *h* below the surface, the pressure becomes greater by the product of the density of the water ρ, the acceleration due to gravity *g*, and the depth *h*.

  Thus the pressure *p2* at this depth is

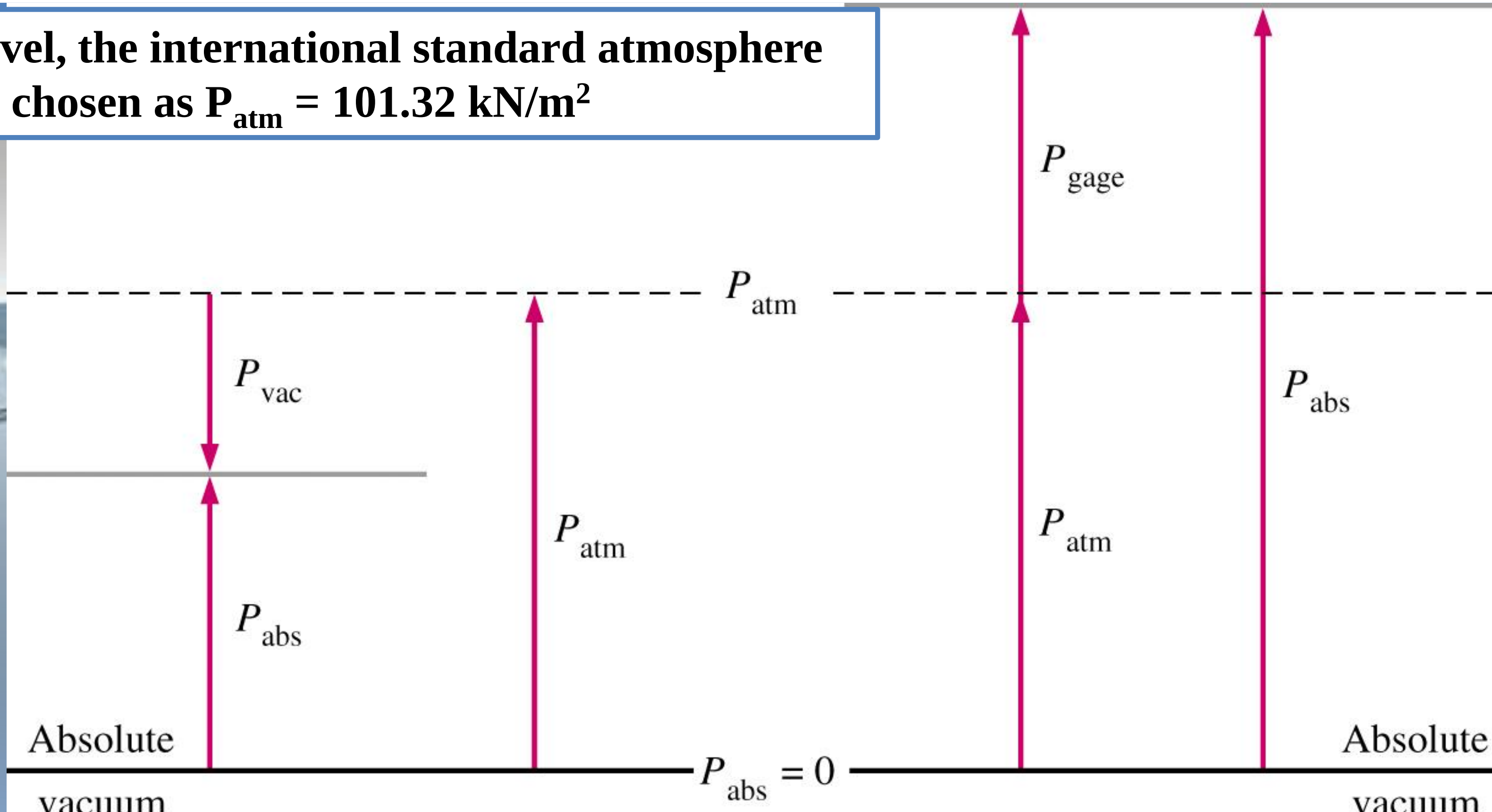
Atmospheric Pressure: This is the pressure due to the atmosphere at the earth surface as measured by a barometer. Pressure decreases with altitude

Gauge Pressure: This is the intensity of pressure measured above or below the atmospheric pressure.

Absolute Pressure: This is the summation of Gauge and atmospheric pressure.

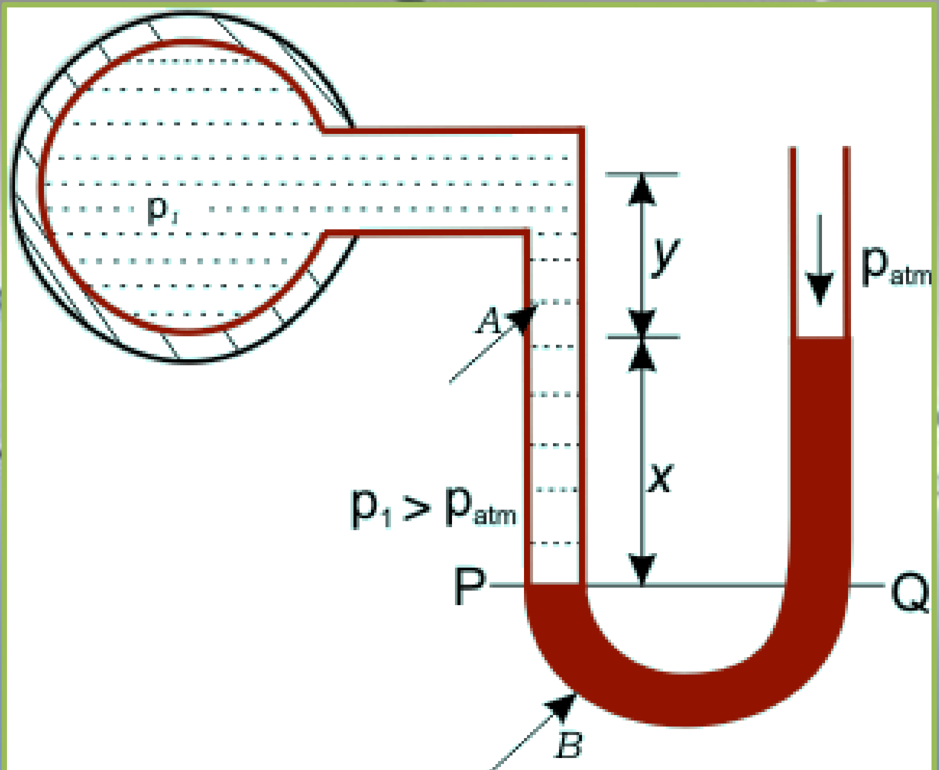
Vacuum: A perfect vacuum is a completely empty space, therefore, the pressure is zero.

At sea-level, the international standard atmosphere has been chosen as P_{atm} = 101.32 kN/m^2



Manometers for measuring Gauge and Vacuum Pressure

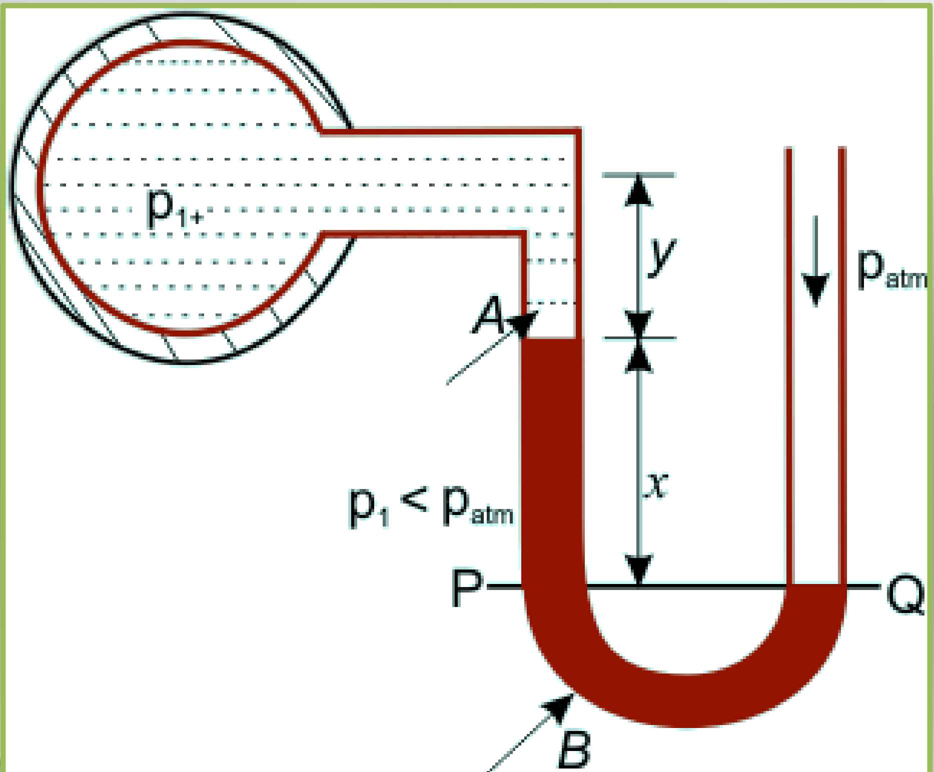
Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.
Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube. A common type manometer is like a transparent "U-tube" a



$$p_1 + \rho_A g(y + x) = p_{atm} + \rho_B g x$$

$$p_1 - p_{atm} = (\rho_B - \rho_A) g x - \rho_A g y$$

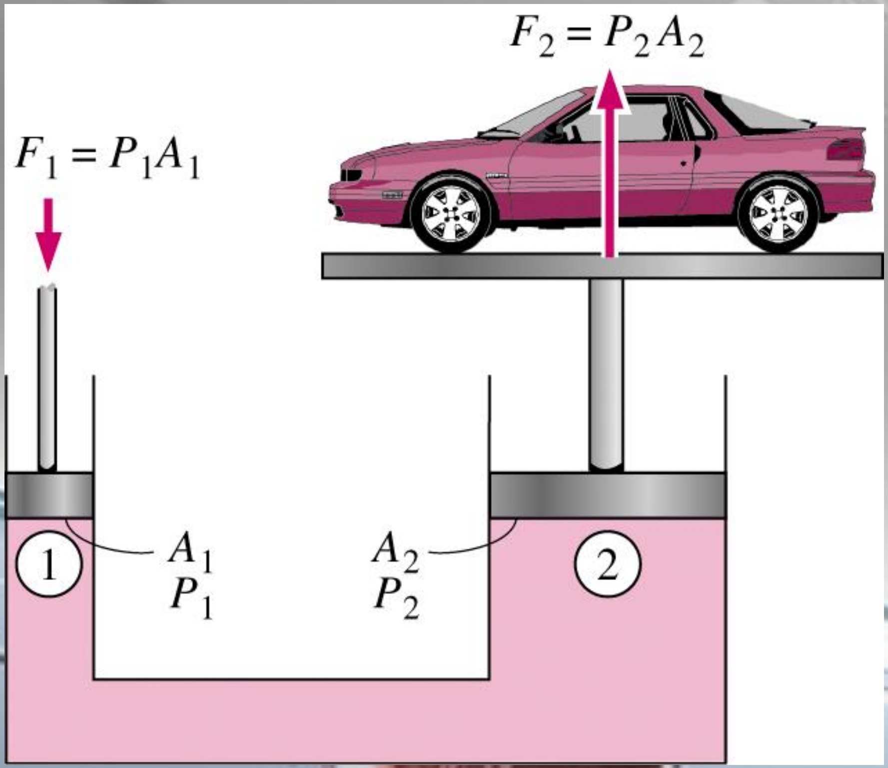
simple manometer to measure gauge pressure



simple manometer to measure vacuum pressure

$$p_1 + \rho_A g y + \rho_B g x = p_{atm}$$

$$p_{atm} - p_1 = (\rho_A y + \rho_B x)g$$



- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A_2/A_1 is called *ideal mechanical advantage*

Types of Simple Manometers

Common types of simple manometers are

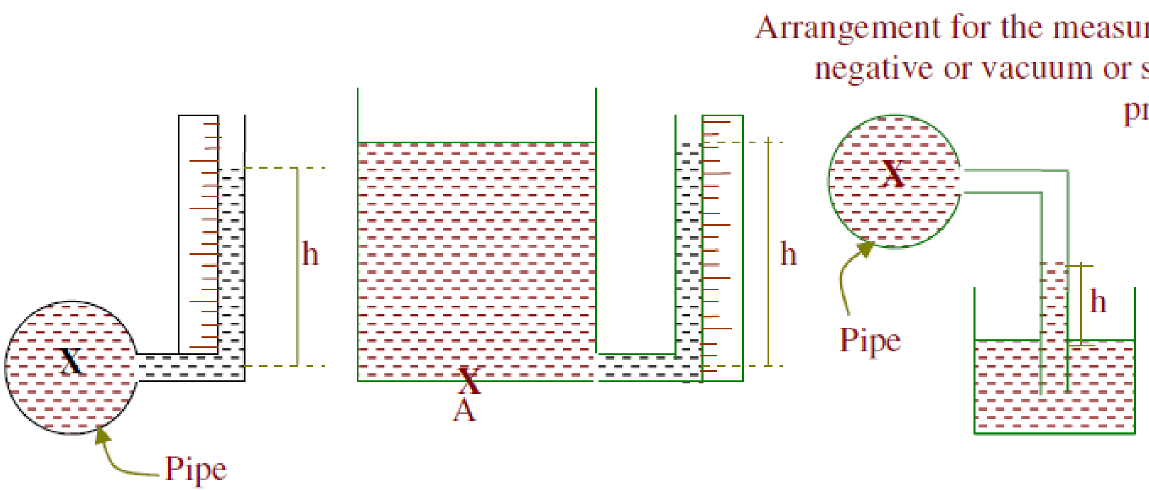
a) Piezometers

b) U-tube manometers

c) Single tube manometers

d) Inclined tube manometers

a) Piezometers

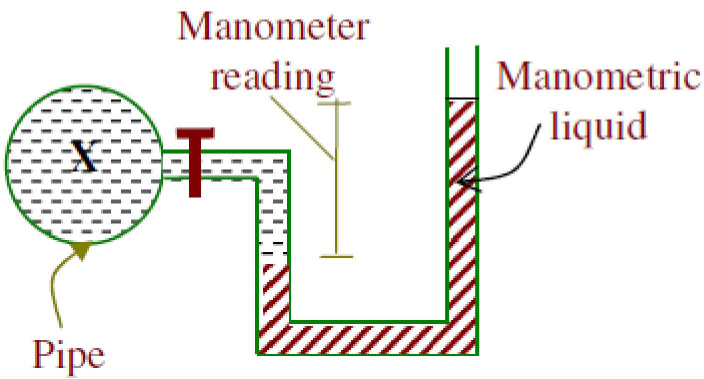


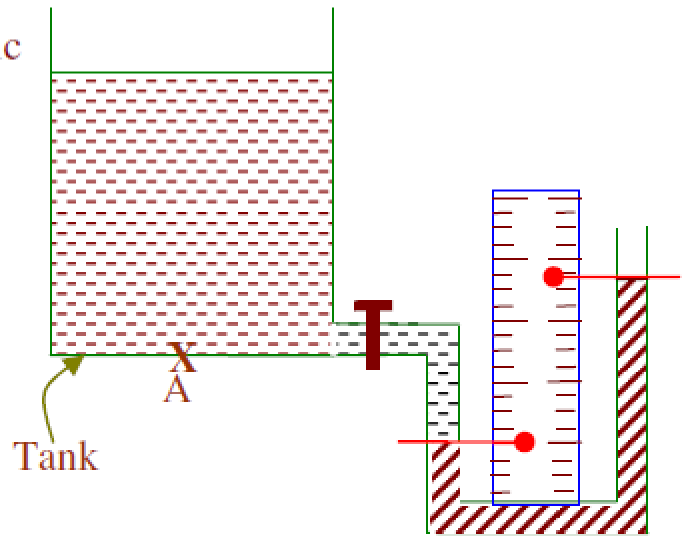
Demerits

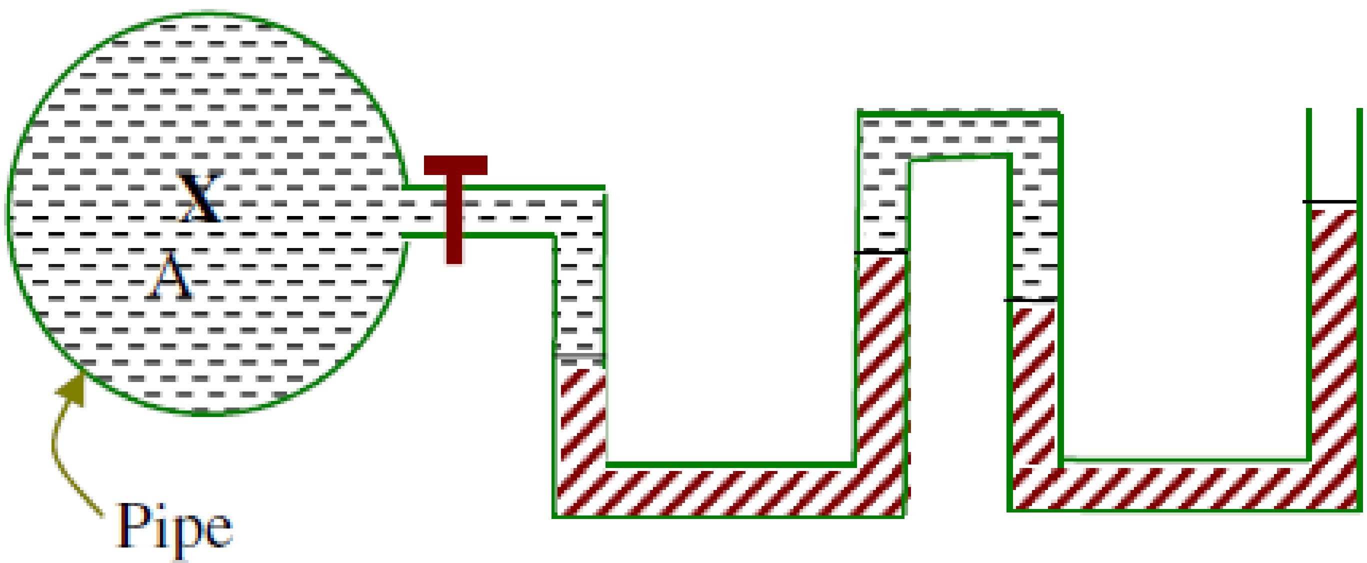
Not suitable for high pressure intensity.

Pressure of gases cannot be measured.

(b) U-tube Manometers:







TRANSMISSION OF FLUID PRESSURE

- The principle of transmission of fluid pressure states that *the pressure intensity at any point in a fluid at rest is transmitted without loss to all other points in the fluid*.

PRESSURE DUE TO FLUID'S WEIGHT

Fluids of Uniform Density

- Total weight of fluid (W) = mg

$$\mathbf{W = \rho g A h} \qquad \mathbf{(2.1)}$$

- Pressure (P) = Weight of fluid/Area

$$\mathbf{P = \rho g h} \qquad \mathbf{(2.2)}$$

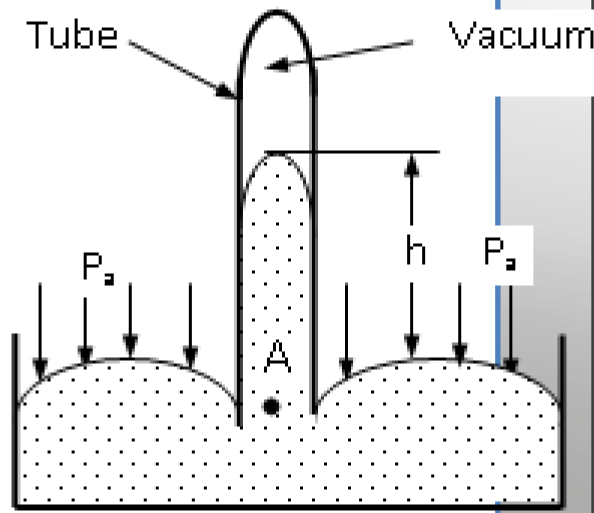
STRATIFIED FLUIDS

- Stratified fluids are two or more fluids of different densities, which float on the top of one another without mixing together.
- $P_1 = \rho_1 g h_1$ and $W_1 = \rho_1 g h_1 A$.
- $P_2 = \rho_2 g h_2$ and $W_2 = \rho_2 g h_2 A$.
- Total pressure, $P_T = \rho_1 g h_1 + \rho_2 g h_2$
- Total weight, $W_T = (\rho_1 g h_1 + \rho_2 g h_2)A$

$$W_T = P_T A \tag{2.4}$$

PRESSURE MEASUREMENT BY MANOMETER

Measurement of Absolute Pressure



- The absolute pressure of a liquid is measured by a *barometer*.

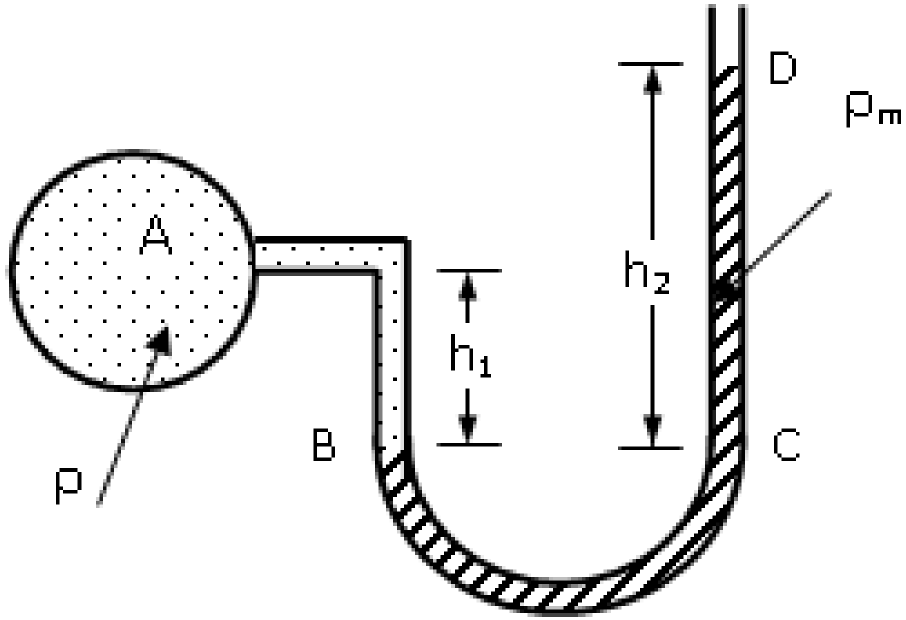
$$P = \rho g h \qquad (2.5)$$

Piezometer Tube

- Piezometer consists of a single vertical tube, inserted into a pipe or vessel containing liquid under pressure which rises in the tube to a height depending on the pressure. The pressure due to column of liquid of height h is:

$$P = \rho g h \qquad (2.6)$$

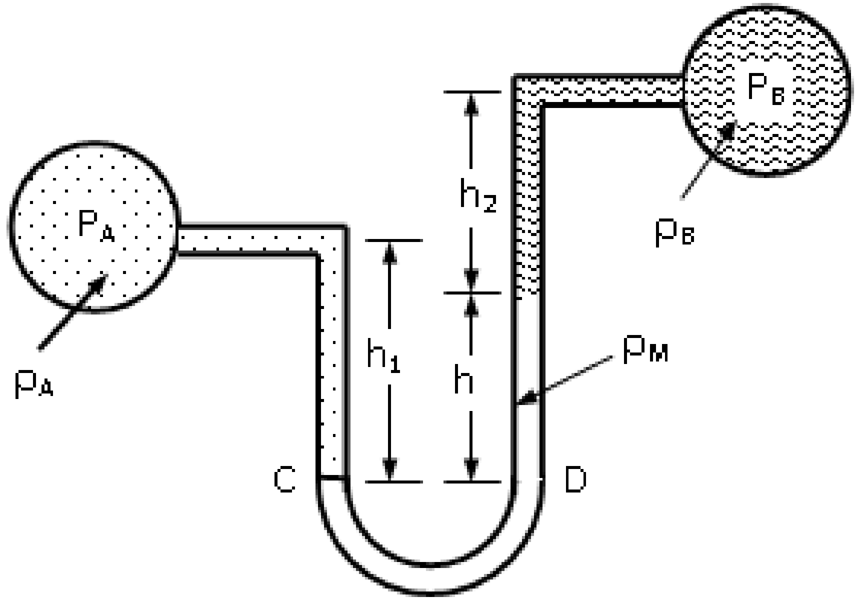
• OPEN–END U-TUBE MANOMETER



- Pressure $P_B = P_A + \rho g h_1$
- Pressure $P_C = 0 + \rho_m g h_2$
- $P_A + \rho g h_1 \quad = \quad \rho_m g h_2$ (Since $P_B = P_C$)

$$\mathbf{P_A = \rho_m g h_2 - \rho g h_1}$$

CLOSE-END U-TUBE MANOMETER

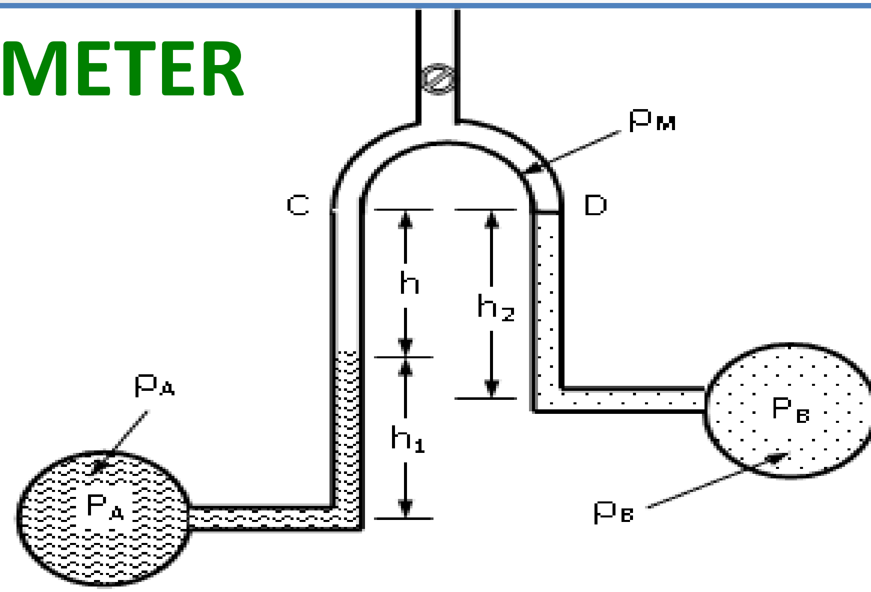


- $P_C \quad = \qquad P_A + \rho_A g h_1$
- $P_D \quad = \qquad P_B + \rho_B g h_2 + \rho_m g h$

  But $P_C = P_{D,}$ hence,
- $P_A + \rho_A g h_1 = P_B + \rho_B g h_2 + \rho_m g h$

  $$\mathbf{P_A - P_B = P_B g h_2 + \rho_m g h - \rho_A g h_1}$$

INVERTED U-TUBE MANOMETER



- $P_A = \rho_A gh_1 + \rho_m gh + P_C$
- $P_B = \rho_B gh_2 + P_D$
- Since $P_C = P_D$

$$P_A - P_B = \rho_A gh_1 + \rho_m gh - \rho_B gh_2$$

If the top of the tube is filled with air

$$P_A - P_B = \rho_A gh_1 - \rho_B gh_2 \qquad (2.10)$$

- If fluids in A and B are the same

$$P_A - P_B = pg(h_1 - h_2) + \rho_m gh \qquad (2.11)$$

- Combining conditions for Eqs. (2.10) and (2.11):

$$P_A - P_B = pg(h_1 - h_2) \qquad (2.12)$$

FORCES ON SUBMERGED SURFACES

***A submerged surface can be defined as a surface of a body below the liquid surface*.**

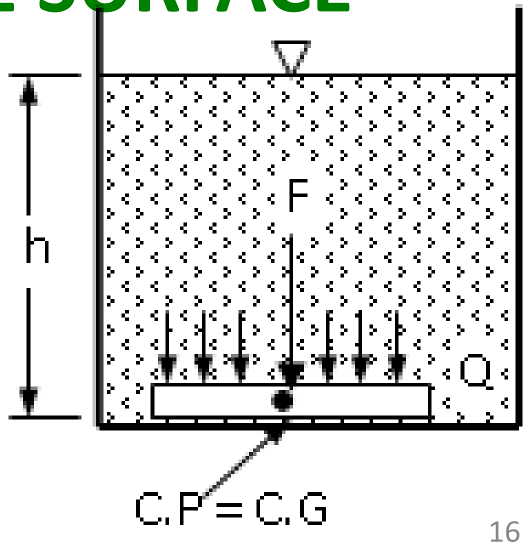
There are two types of surfaces, namely:

- Plane surface
- Curved surface

SUBMERGED HORIZONTAL PLANE SURFACE

$$P = \rho g h \qquad (3.1)$$

$$F = \rho g h A \qquad (3.2)$$

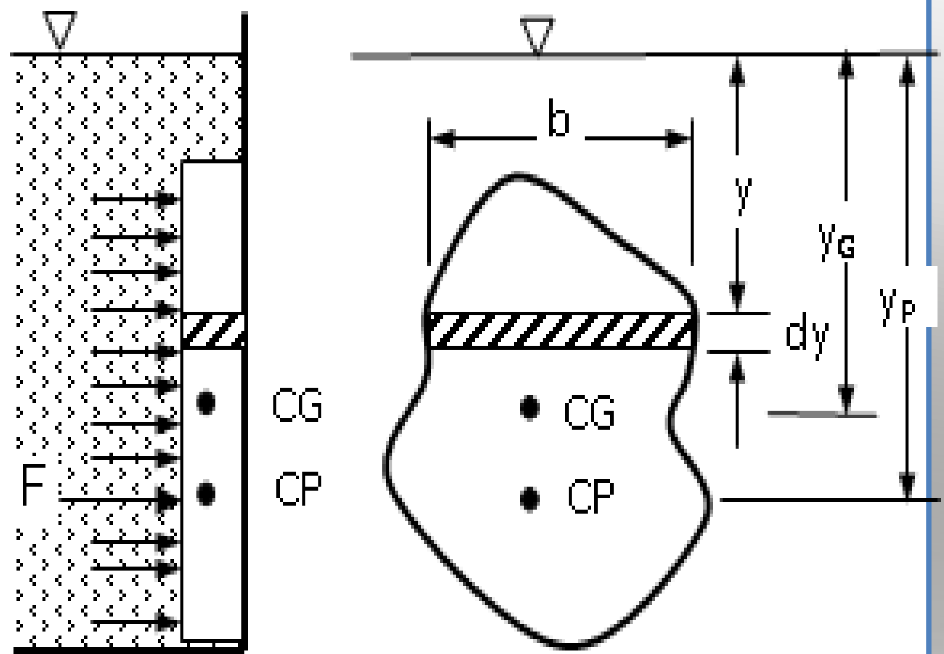


SUBMERGED VERTICAL PLANE SURFACE

- Elemental force,

$$dF = PdA$$

$$\int dF = \rho g \int ydA$$



But $\int ydA$ is the first moment of area about the liquid surface, hence

$$F = \rho g A y_G \qquad (3.3)$$

DETERMINATION OF CENTRE OF PRESSURE (y_p)

$$dF = \rho g y dA$$

Taking moment about the liquid surface

$dF.y = \rho g y^2 dA$ and $\int dF.y = \rho g \int y^2 dA$

But the $\int y^2 dA$ is the second moment of area I, about the surface level

$$\mathbf{F y_p = \rho g \int y^2 dA = \rho g I} \qquad \mathbf{(3.4)}$$

$y_p = I/Ay_G$ = Ratio of Second moment of Area to First moment of Area

Using parallel axis theorem,

$I_X = I_G + Ay^2$

$$\mathbf{I = I_G + Ay^2_G} \qquad \mathbf{(3.5)}$$

I_G is the second moment of Area about the centroid. Substituting for I, we have

$$y_p = \frac{I_G + Ay_G^{\,2}}{Ay_G}$$

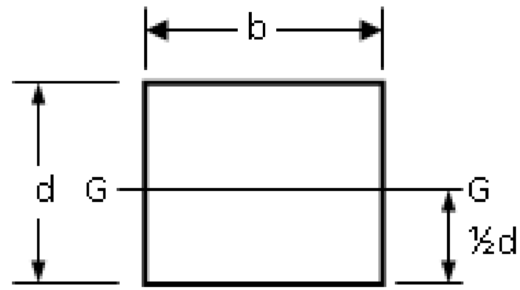
$$y_p = \frac{I_G}{Ay_G} + y_G \qquad \mathbf{(3.6)}$$

GEOMETRIC PROPERTIES OF SOME SHAPES

- **Rectangle**

  $A = bd$

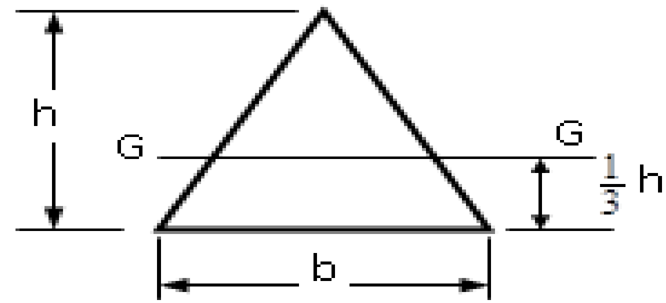
  $I_G = bd^3/12$



- **Triangle**

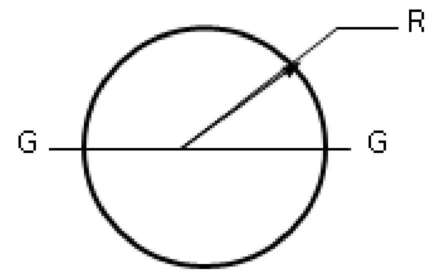
  $A = \frac{1}{2}\, bh$

  $I_G = bh^3/36$



- **Circle**

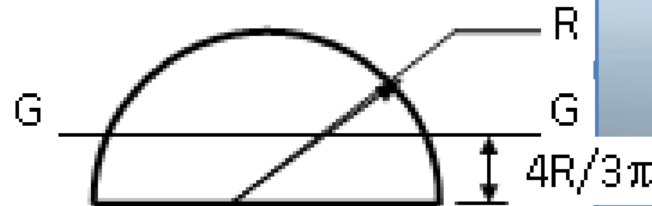
  $A = \pi R^2$ and $I_G = \pi R^4/4$



- **Semicircle**

  $A = \frac{1}{2}\, \pi R^2$

  $I_G = 0.1102R^4$



QUESTION

A fuel tank 10 m wide by 5 m deep contains oil of relative density 0.7. In one vertical side a circular opening 1.8 m in diameter was made and closed by a trap door hinged at the lower end B held by a bolt at the upper end A. If the fuel level is 1.8 m above the top edge of the opening, calculate the:

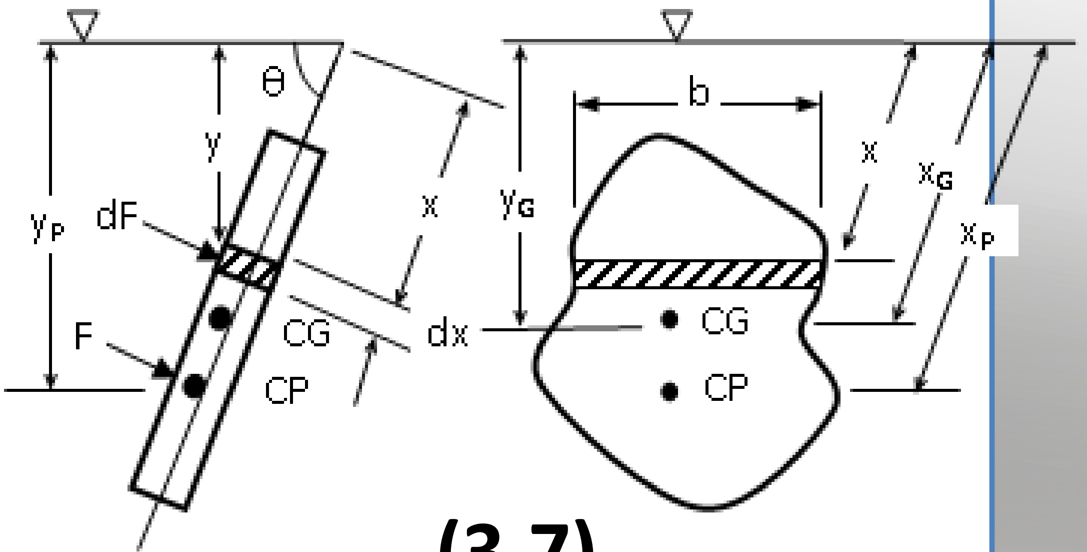
- **total force on the door**
- **force on the bolt**
- **force on the hinge.**

SUBMERGED INCLINED PLANE SURFACE

$dF = PdA$

$P = \rho gy$ & $y = x.\sin\theta$

$P = \rho gx.\sin\theta$



$dF = \rho gx\sin\theta.dA$ (3.7)

$\int dF = \rho g.\sin\theta \int x.dA$

where $\int x.dA = Ax_G$ first moment of area.

$F = \rho g \sin\theta\, Ax_G$

$\therefore\ F = \rho g y_G A$ (3.8)

DETERMINATION OF CENTRE OF PRESSURE

Taking moment about the fluid surface,

$dM = x dF$ $\qquad\qquad$ $dM = \rho g x^2 \sin\theta dA$

$\int dM = \rho g.\sin\theta \int x^2 dA$

$I = \int x^2 dA$ (second moment of area), hence

$M = \rho g.\sin\theta\ I.$

Also the total moment $M = Fx_P$, therefore,

$Fx_P = \rho g.\sin\theta\ I.$

$$x_P = \frac{\rho g \sin\theta I}{F} = \frac{\rho g \sin\theta I}{\rho g x_G \sin\theta A} = \frac{I}{A x_G} = \frac{I_G + A x_G^{\ 2}}{A x_G} \qquad \textbf{(3.9)}$$

$$x_P = \frac{I_G}{A x_G} + x_G \qquad \textbf{(3.10)}$$

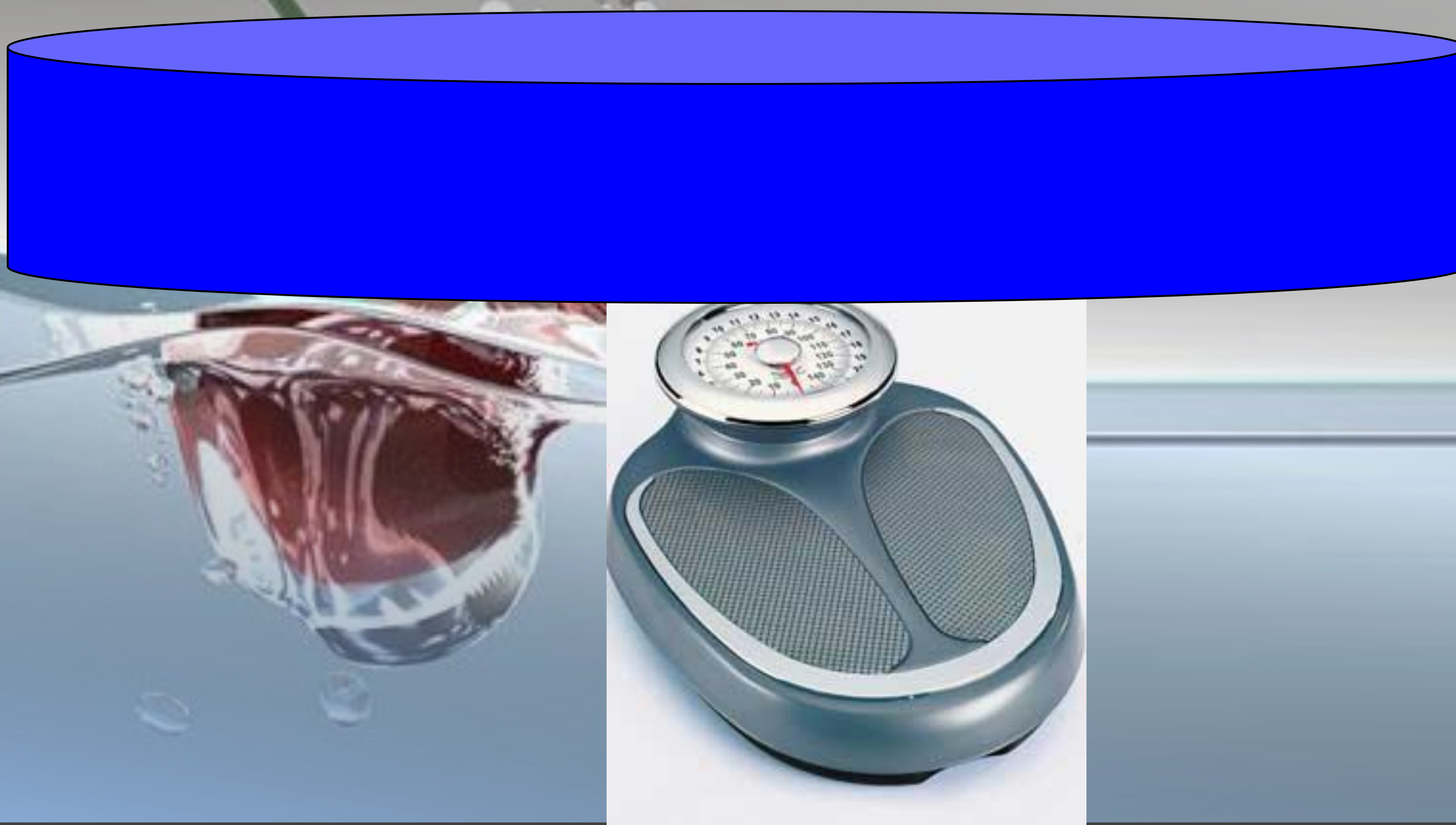
BUOYANCY AND STABILITY OF FLOATING BODIES

BUOYANCY

- The **Upthrust** (upward vertical force due to the fluid) or buoyancy of an immersed body is equal to the weight of liquid displaced
- The centroid of the displaced liquid is called the ***centre of buoyancy.***
- It acts opposite of gravity
- Volume of fluid displaced is:

$$\frac{\text{mass of the floating body}}{\text{density of the fluid}}$$

ARCHIMEDES’ PRINCIPLE states that the WEIGHT of the amount of water displaced is equal to the BUOYANT FORCE.



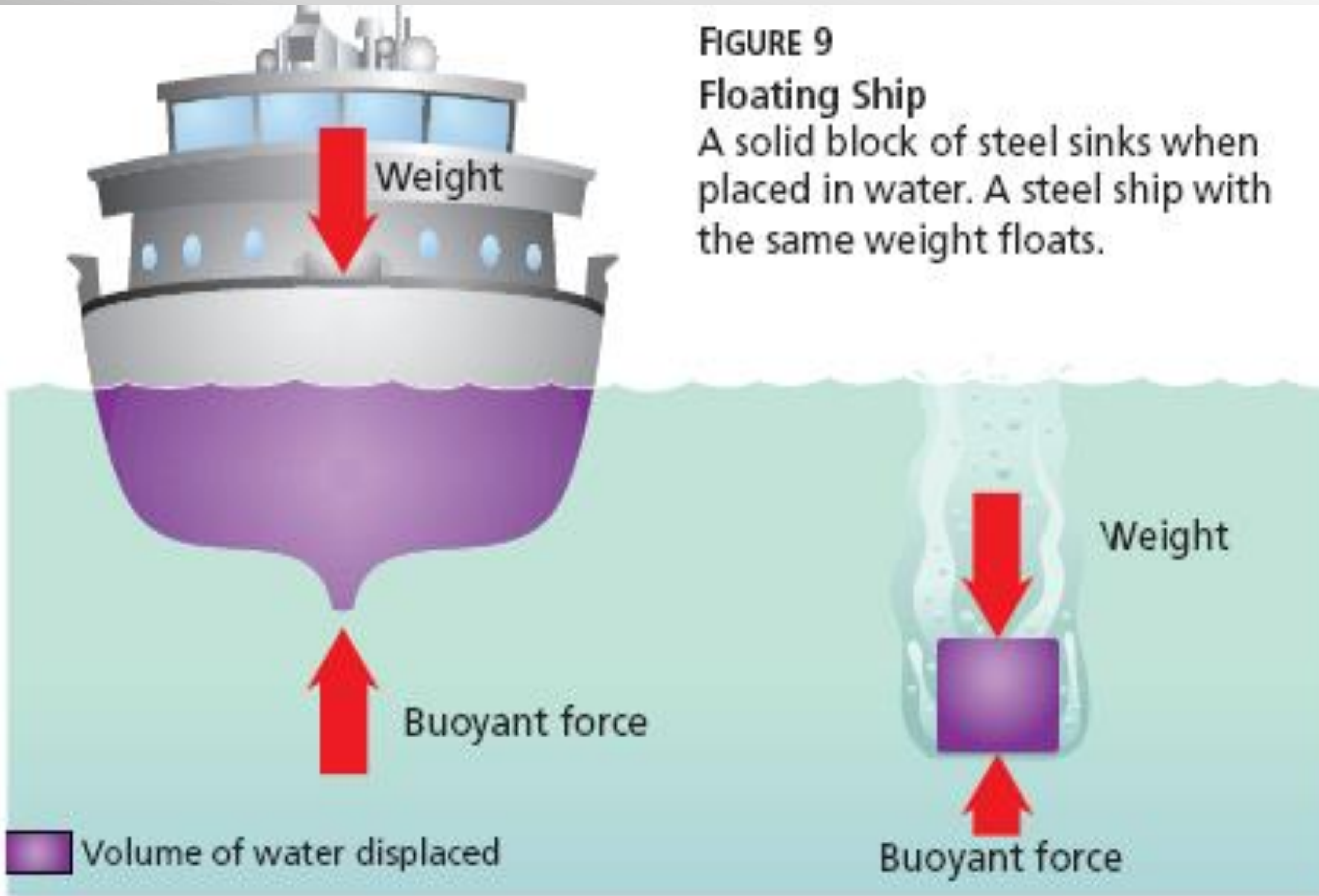


FIGURE 9
Floating Ship
A solid block of steel sinks when placed in water. A steel ship with the same weight floats.

AN OBJECT FLOATS

CAUSES:

- Weight is less than the buoyant force.
- Object is less dense than the fluid
- Object decreases its mass and becomes less dense than the fluid.
- Object increases its volume and becomes denser than the fluid.

AN OBJECT SINKS

CAUSES:

1. Weight is greater than the buoyant force.
2. Object is denser than the fluid
3. Object increases its mass and becomes denser than the fluid.
4. Object decreases its volume and becomes denser than the fluid.

Stability of Unconstrained Submerged Bodies in Fluid

- The equilibrium of a body submerged in a liquid requires that the weight of the body acting through its centre of gravity should be collinear with an equal hydrostatic lift acting through the centre of buoyancy.
- In general, if the body is not homogeneous in its distribution of mass over the entire volume, the location of **centre of gravity G does not coincide with the centre of volume, i.e., the centre of buoyancy B.**

- Depending upon the relative locations of G and B, a floating or submerged body attains three different states of equilibrium-

- **Stable Equilibrium:** If the body **returns to its original position** by retaining the originally vertical axis as vertical.

- **Unstable Equilibrium:** If the body **does not return to its original position but moves further** from it.

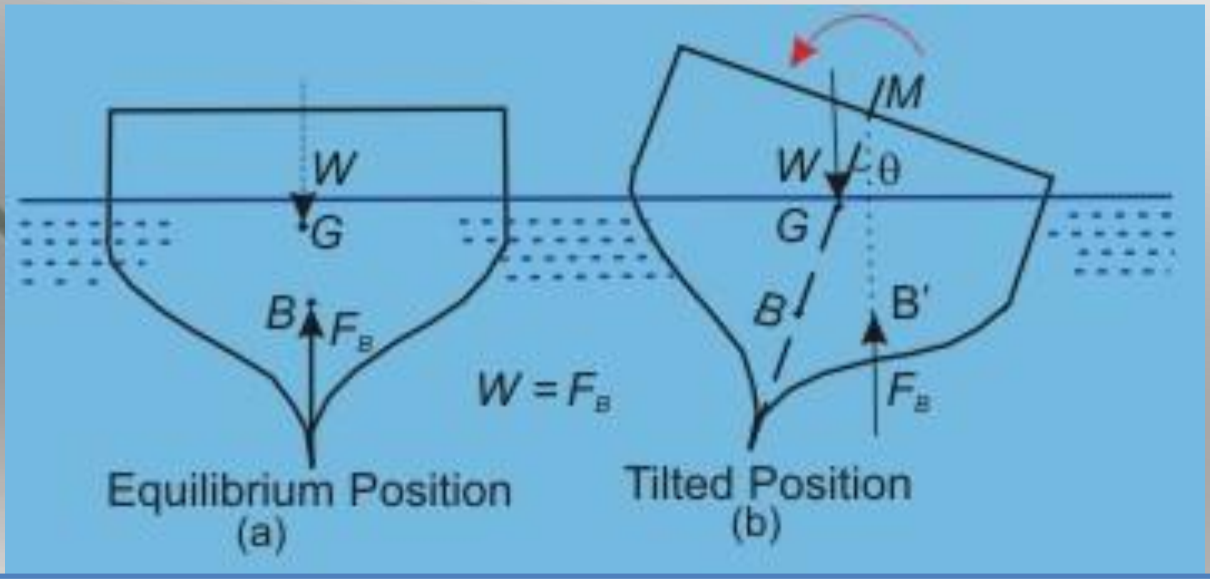
- **Neutral Equilibrium:** If the body **neither returns to its original position nor increases its displacement further**, it will simply adopt its new position.

STABILITY OF A SUBMERGED BODY

For stable equilibrium the centre of gravity of the body must lie directly below the centre of buoyancy of the displaced liquid.

If the two points coincide, the submerged body is in neutral equilibrium for all positions

Stability of Floating Bodies in Fluid



illustrates a floating body -a boat, for example, in its equilibrium position

When the body undergoes an angular displacement about a horizontal axis, the shape of the immersed volume changes and so the centre of buoyancy moves relative to the body.

As a result of above observation stable equilibrium can be achieved, under certain condition, even when G is above B.

End of Lecture 11