# Module 2

**BMEE102L-FLUID MECHANICS**

# Fluid Kinematics

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# Module 3: Fluid Kinematics and Fluid Dynamics

**Fluid kinematics:** Description of fluid motion – Lagrangian and Eulerian approach, Types of flows, Control volume, Material derivative and acceleration, Streamlines, pathlines and streaklines, Stream function and velocity potential function, Reynolds transport theorem.

**Fluid dynamics: The c**ontinuity equation, Euler and Bernoulli's equations – venturimeter, orificemeter, Pitot tube, Momentum equation and its application – forces on pipe bends, moment of momentum, Navier–Stokes Equations.

# Module 3: Fluid Kinematics and Fluid Dynamics

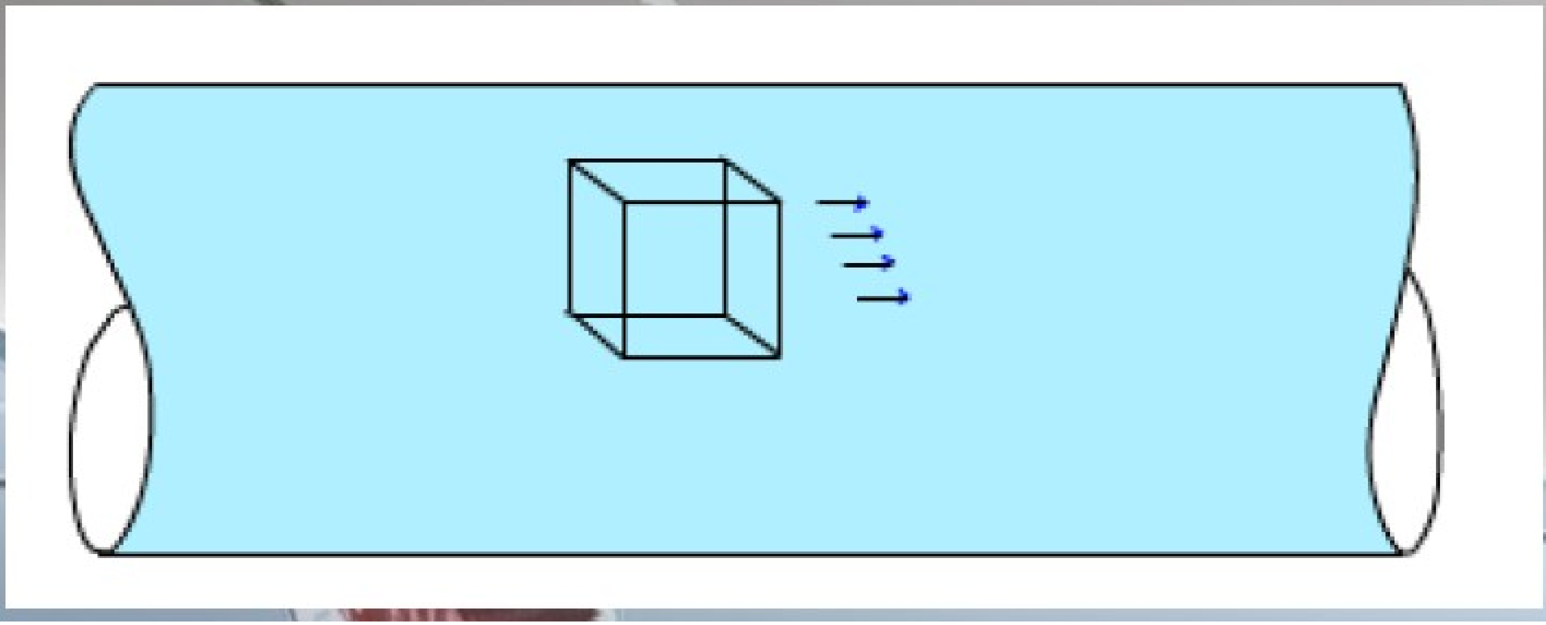
- Fluid Kinematics deals with the motion of fluids without considering the forces and moments which create the motion.
- Items discussed in this Chapter.
  - Material derivative and its relationship to Lagrangian and Eulerian descriptions of fluid flow.
  - Flow visualization.
  - Plotting flow data.
  - Fundamental kinematic properties of fluid motion and deformation.
  - Types of fluid flows

# Fluid Motion

The fluid particles may change their shape, size and state as they move.

As mass of fluid particles remains constant throughout the motion, the basic laws of mechanics can be applied to them at all times.

The task of following large number of fluid particles is quite difficult. Therefore this approach is limited to some special applications for example re-entry of a spaceship into the earth's atmosphere and flow measurement system based on particle imagery.



# Lagrangian description of Fluid motion

- Lagrangian description of fluid flow tracks the position and velocity of individual particles.
- Based upon Newton's laws of motion.
- Difficult to use for practical flow analysis.
  - Fluids are composed of *billions* of molecules.
  - Interaction between molecules hard to describe/model.
- However, useful for specialized applications
  - Sprays, particles, bubble dynamics, rarefied gases.
  - Coupled Eulerian-Lagrangian methods.
- Named after Italian mathematician Joseph Louis Lagrange (1736-1813).

# Eulerian description of Fluid motion

- Eulerian description of fluid flow: a **flow domain** or **control volume** is defined by which fluid flows in and out.
- We define **field variables** which are functions of space and time.
  - Pressure field, P=P(x,y,z,t)
  - Velocity field, $\vec{V} = \vec{V}(x,y,z,t)$

$$\vec{V} = u(x,y,z,t)\vec{i} + v(x,y,z,t)\vec{j} + w(x,y,z,t)\vec{k}$$

  - Acceleration field, $\vec{a} = \vec{a}(x,y,z,t)$

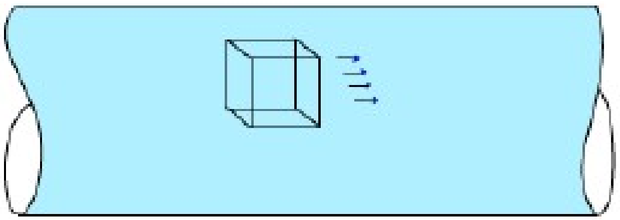
$$\vec{a} = a_x(x,y,z,t)\vec{i} + a_y(x,y,z,t)\vec{j} + a_z(x,y,z,t)\vec{k}$$

  - These (and other) field variables define the **flow field**.
- Well suited for formulation of initial boundary-value problems (PDE's).
- Named after Swiss mathematician Leonhard Euler (1707-1783).

- In the Eulerian method a finite region through which fluid flows in and out is used. Here we do not keep track position and velocity of fluid particles of definite mass.
- But, within the region, the field variables which are continuous functions of space dimensions ( $x$ , $y$ , $z$ ) and time ( $t$ ), are defined to describe the flow.
- These field variables may be scalar field variables, vector field variables and tensor quantities. For example, pressure is one of the scalar fields. Sometimes this finite region is referred as control volume or flow domain.

For example the pressure field 'P' is a scalar field variable and defined as

$$P = P(x,y,z,t)$$



Velocity field, a vector field, is defined as $\vec{V} = \vec{V}(x,y,z,t)$

Similarly shear stress $\tau$ is a tensor field variable and defined as

$$\tau = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

Note that we have defined the fluid flow as a three dimensional flow in a Cartesian co-ordinates system.

# Lanrangian and Eulerian

- **Eulerian**
  Sit and observe a fixed area from a fixed point
- **Lagrangian**
  Travel with the flow and observe what happens around you
- **Mixed** – something that sits between the two

# Lagrangian -Eulerian description of Fluid motion

- Global Environmental MEMS Sensors (GEMS)
- Simulation of micron-scale airborne probes. The probe positions are tracked using a Lagrangian particle model embedded within a flow field computed using an Eulerian CFD code.

# Description of a Fluid motion

- Consider a fluid particle and Newton's second law,

$$\vec{F}_{particle} = m_{particle}\vec{a}_{particle}$$

- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

- However, particle velocity at a point is the same as the fluid velocity, $\vec{V}_{particle} = \vec{V}\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right)$
- To take the time derivative of, chain rule must be used.

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x}\frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz_{particle}}{dt}$$

# Description of a Fluid motion

- Since $$\frac{dx_{particle}}{dt} = u, \frac{dy_{particle}}{dt} = v, \frac{dz_{particle}}{dt} = w$$

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

- In vector form, the acceleration can be written as

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V}$$

- First term is called the **local acceleration** and is nonzero only for unsteady flows.
- Second term is called the **advective acceleration** and accounts for the effect of the fluid particle moving to a new location in the flow, where the velocity is different.

# Description of a Fluid motion

- The total derivative operator d/dt is call the **material derivative** and is often given special notation, D/Dt.

$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V}$$

- Advective acceleration is nonlinear: source of many phenomenon and primary challenge in solving fluid flow problems.
- Provides ``transformation'' between Lagrangian and Eulerian frames.
- Other names for the material derivative include: **total, particle, Lagrangian, Eulerian,** and **substantial** derivative.

# Fluid Properties in motion

*Flow rate*

**Mass flow rate**

Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is know as the mass flow rate.

For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\text{mass flow rate} = \dot{m} = \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}}$$
$$= \frac{8.0 - 2.0}{7}$$
$$= 0.857\, kg/s \quad (kg\, s^{-1})$$

**Volume flow rate / Discharge / flow rate**

More commonly we need to know the volume flow rate - this is more commonly know as *discharge*. (It is also commonly, but inaccurately, simply called flow rate). The symbol normally used for discharge is $Q$. The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid gives us the mass flow rate. Consequently, if the density of the fluid in the above example is $850\, kg\, m^3$ then:

$$\text{discharge, } Q = \frac{\text{volume of fluid}}{\text{time}} = \frac{\text{mass of fluid}}{\text{density} \times \text{time}} = \frac{\text{mass flow rate}}{\text{density}} = 0.001008\, m^3/s \quad (m^3\, s^{-1})$$

# Types of Fluid flow

**Steady flow**:

When the velocity at each location is constant, the velocity field is invarient with time and the flow is said to be steady.

The flow in which the field variables don't vary with time is said to be steady flow. For steady flow,

$$\frac{\delta v}{\delta t} = 0 \qquad \text{Or} \qquad \vec{v} = \vec{v}(x, y, z)$$

If the field variables in a fluid region vary with time the flow is said to be unsteady flow.

$$\frac{\delta v}{\delta t} \neq 0 \qquad \vec{v} = \vec{v}(x, y, z, t)$$

**Uniform flow**:

Uniform flow occurs when the magnitude and direction of velocity do not change from point to point in the fluid.

Flow of liquids through long pipelines of constant diameter is uniform whether flow is steady or unsteady.

Non-uniform flow occurs when velocity, pressure etc., change from point to point in the fluid.

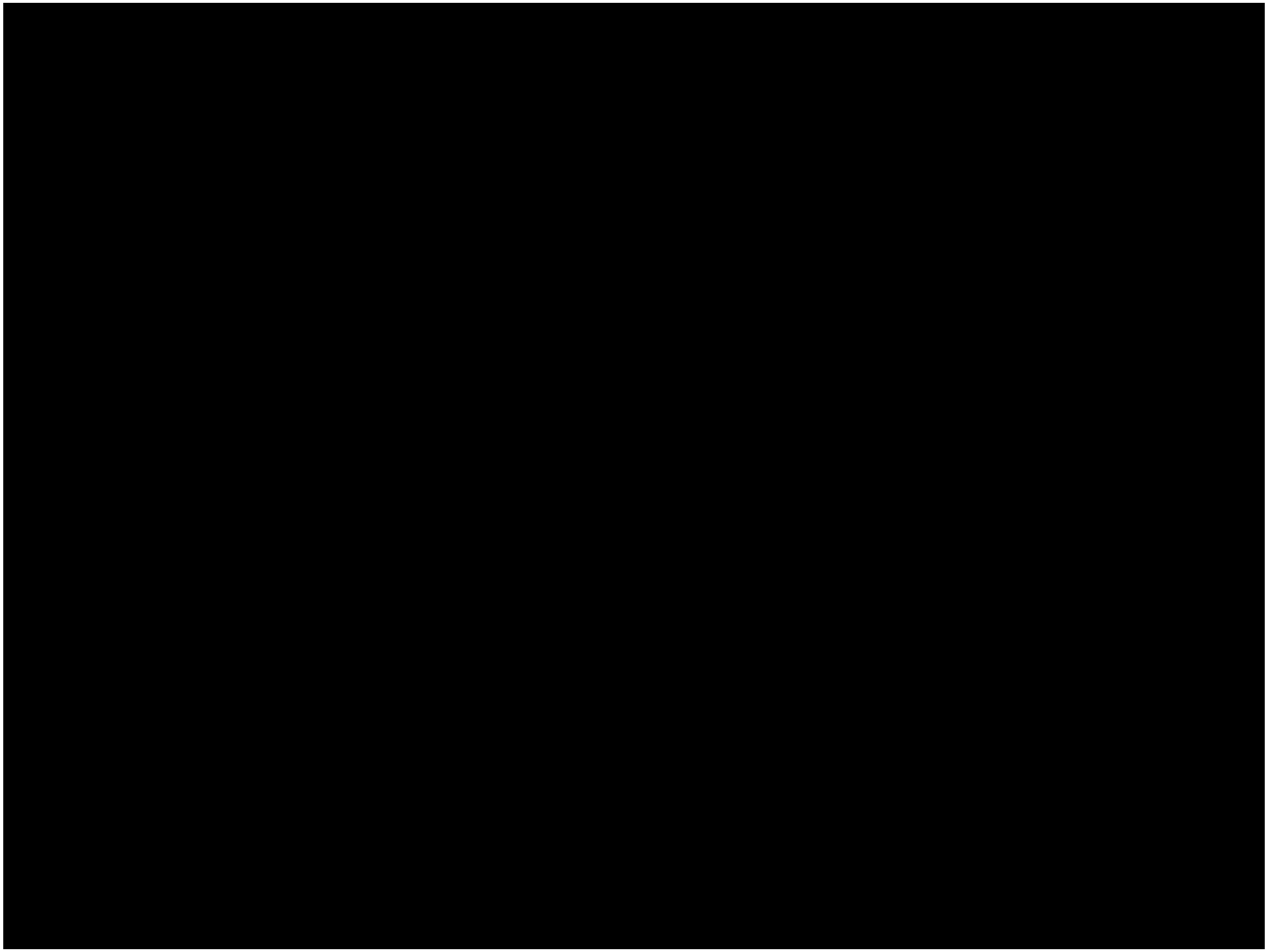
# Types of Fluid flow

**Steady, unifrom flow**:
Conditions do not change with position or time.
e.g., Flow of liquid through a pipe of uniform bore running completely full at constant velocity.
**Steady, non-unifrom flow**:
Conditions change from point to point but do not with time.
e.g., Flow of a liquid at constant flow rate through a tapering pipe running completely full.
**Unsteady, unifrom Flow**: e.g. When a pump starts-up.
**Unsteady, non-unifrom Flow**: e.g. Conditions of liquid during pipetting out of liquid.

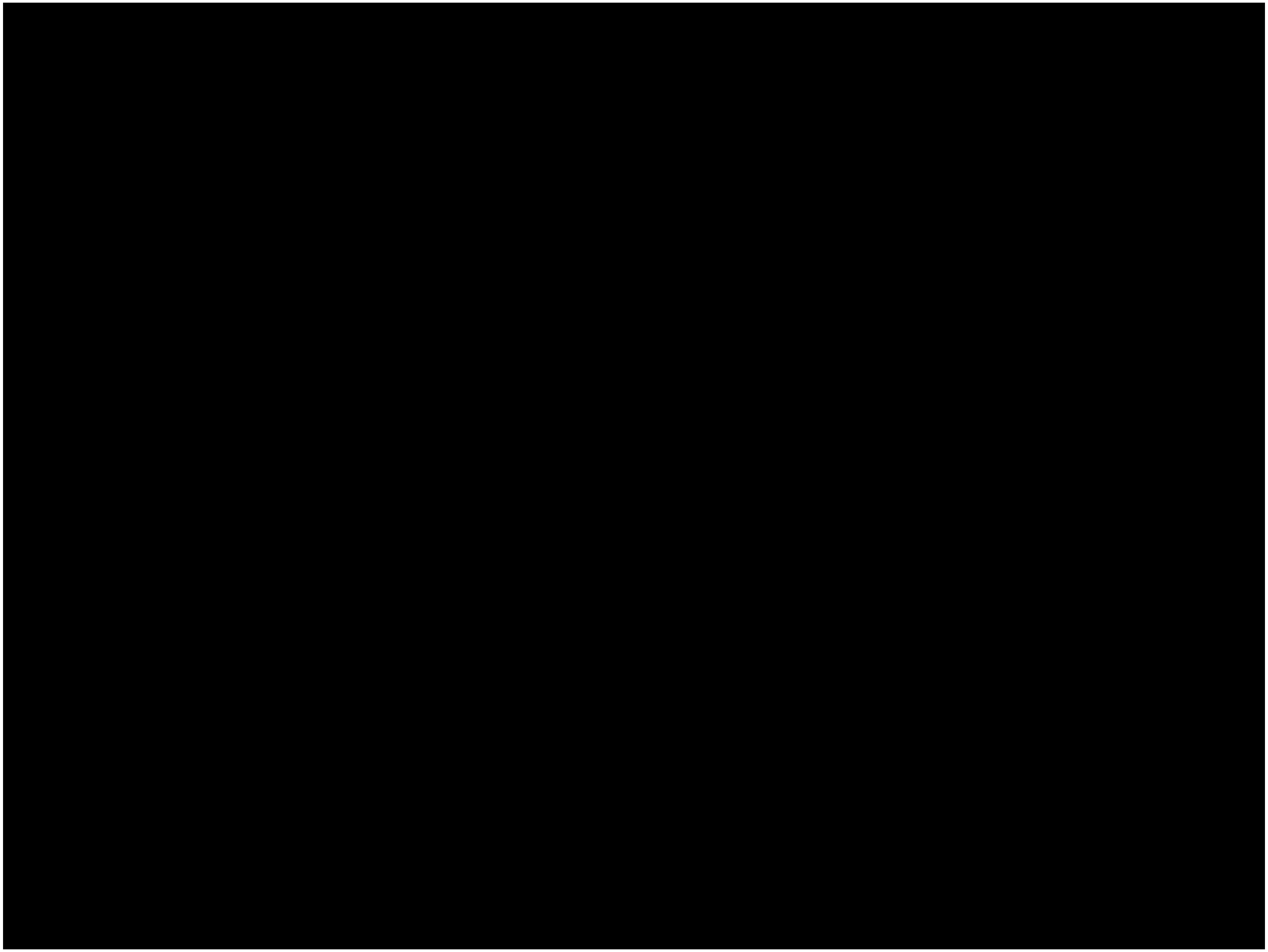
# Flow Visualization

- Flow visualization is the visual examination of flow-field features.
- Important for both physical experiments and numerical (CFD) solutions.
- Numerous methods
  - Streamlines and streamtubes
  - Pathlines
  - Streaklines
  - Timelines
  - Refractive techniques
  - Surface flow techniques

Visualization of Streamlines around two Trucks in  tandem-SV

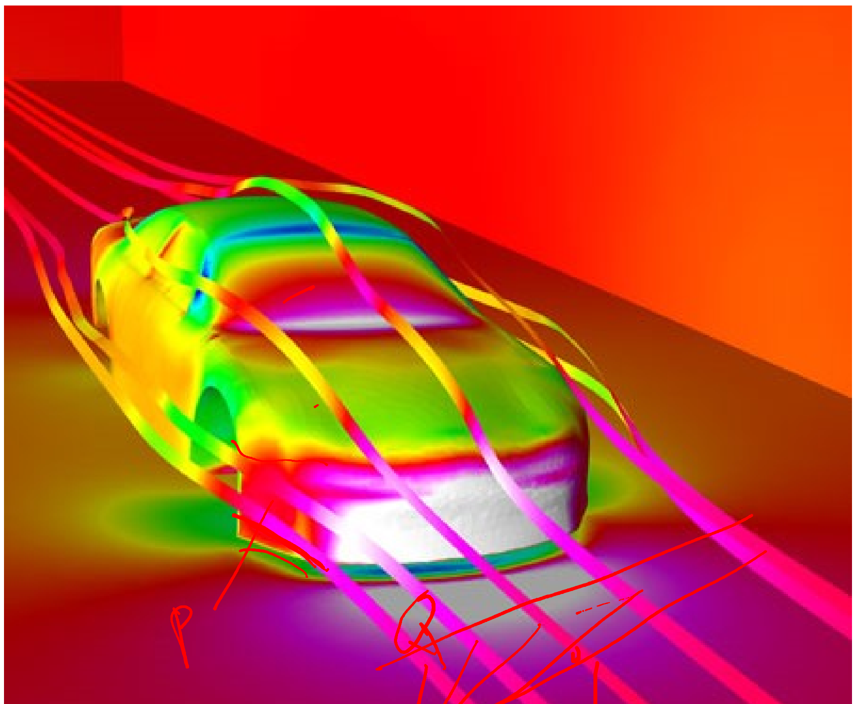


Visualization of Streamlines around two Trucks in tandem-TV

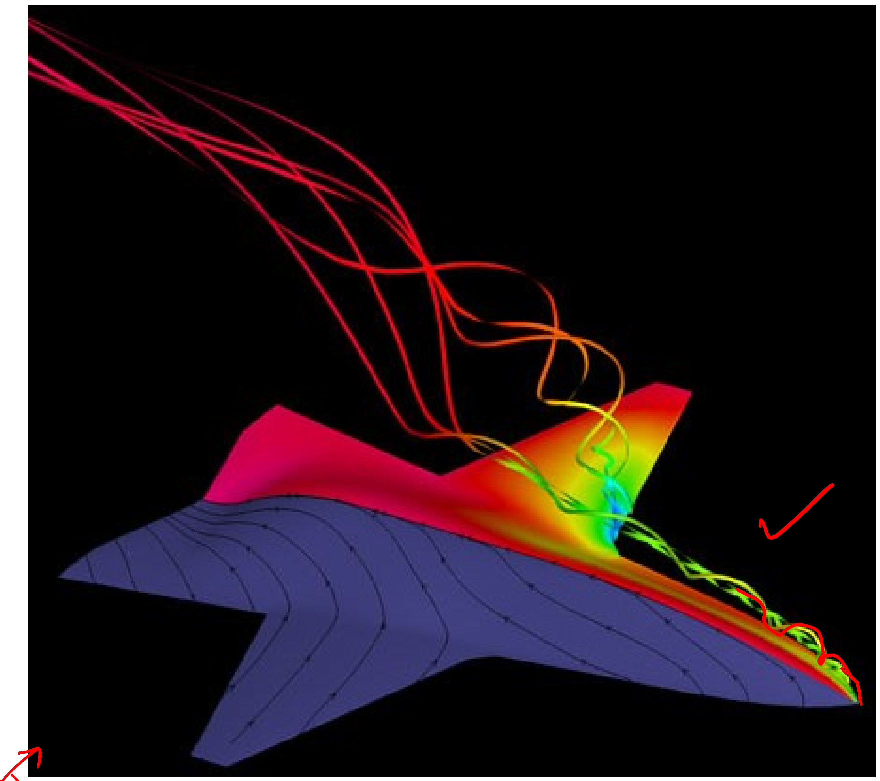


# Streamlines

NASCAR surface pressure contours and streamlines

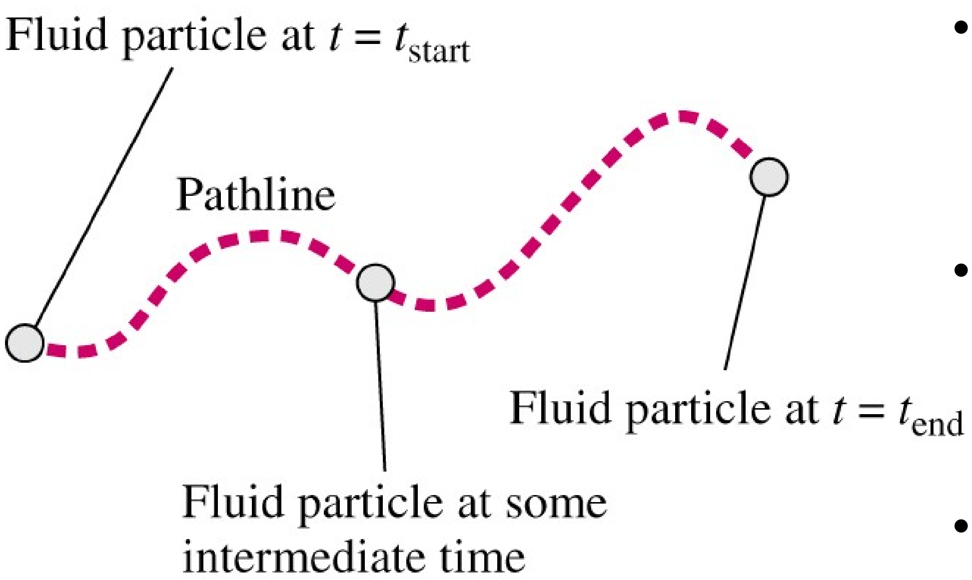


Airplane surface pressure contours, volume streamlines, and surface streamlines



$\vec{V} = (u, v, w)$

# Pathline



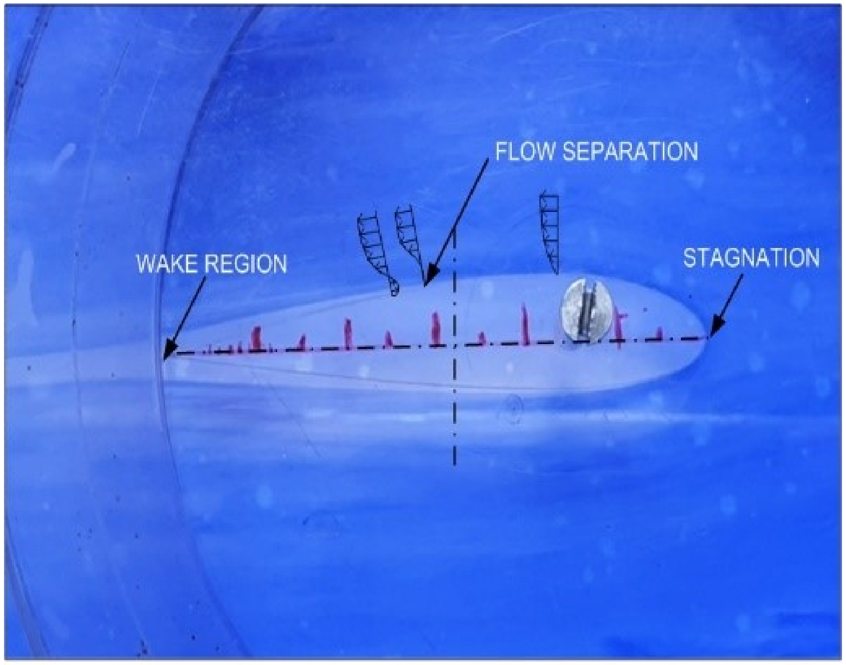

- A **Pathline** is the actual path traveled by an individual fluid particle over some time period.
- Same as the fluid particle's material position vector

$$\left(x_{particle}\left(t\right), y_{particle}\left(t\right), z_{particle}\left(t\right)\right)$$

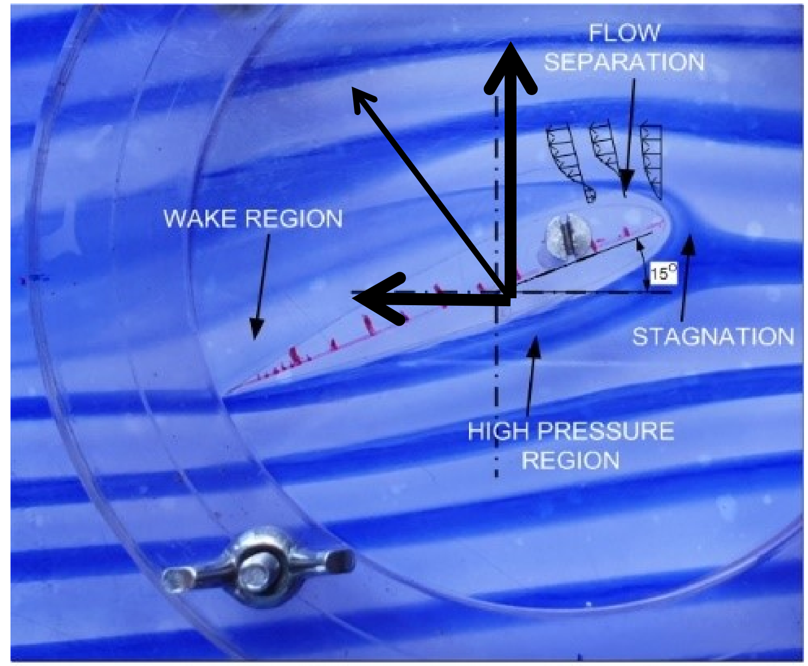
- Particle location at time t:

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V} dt$$

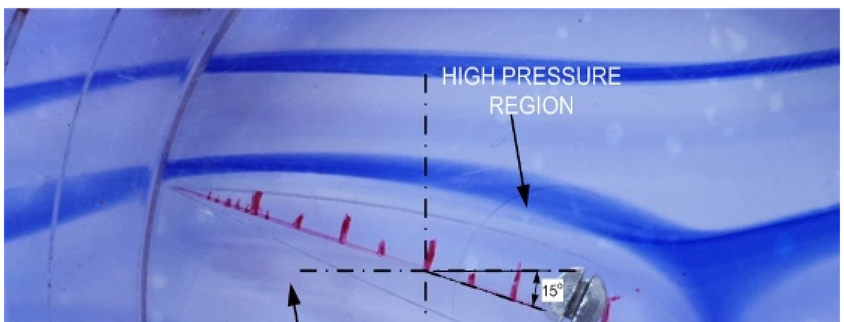
- Particle Image Velocimetry (PIV) is a modern experimental technique to measure velocity field over a plane in the flow field.




AIRFOIL 0 degree – Re 4555.06 – Separation pt at 0.467 x length




Airfoil 15 degrees – Re 4555.06 -Separation at 0.953 x length

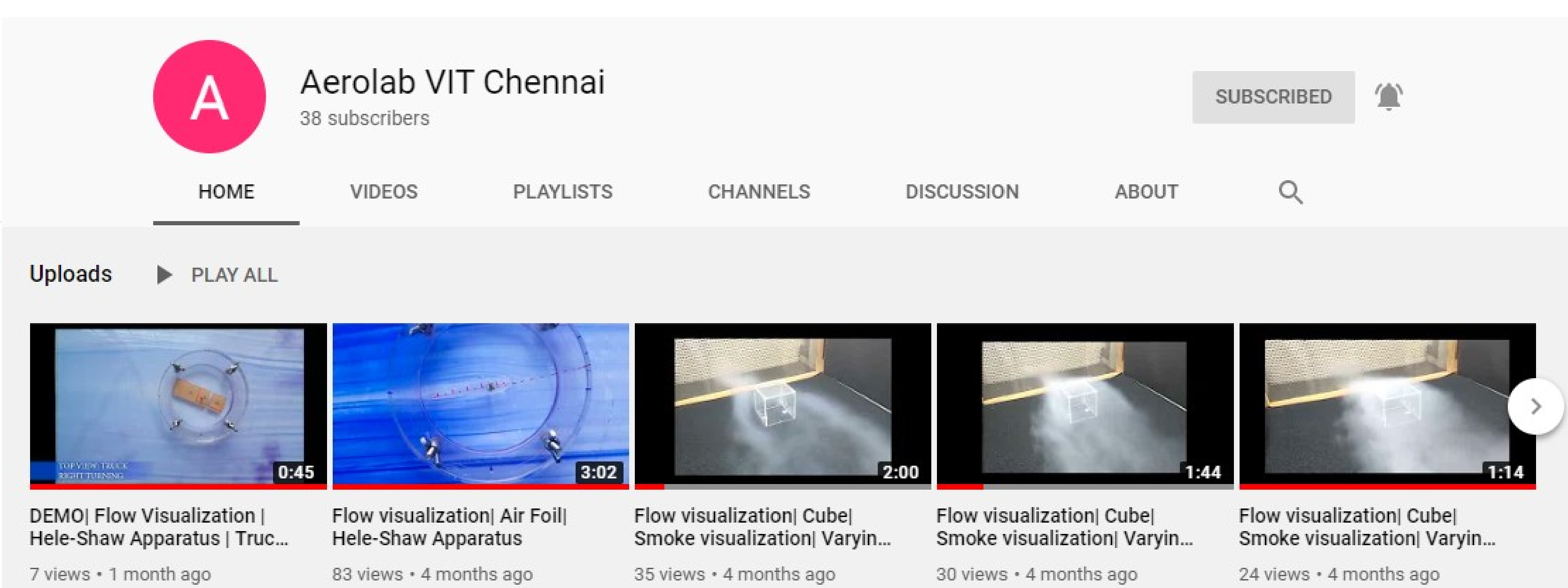



Visualization of Arifoil at angle of attack (α) = 0°, +15° and -15 °.

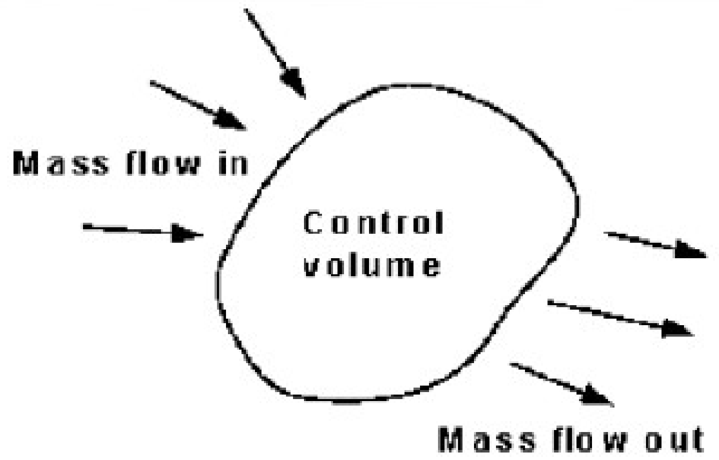
Mayank Sharma, Shreya A. Tripathi, Subiksha, Shikar Jaiswal, Sumit Singh Rajput, G. Vinayagamurthy, “Flow visualization of Two-Dimensional Bodies using Hele-Shaw apparatus”, Proceedings of the 46th National Conference on Fluid Mechanics and Fluid Power (FMFP), December 9-11, 2019, PSG College of Technology, India.

Credits to the members of Aerodynamics laboratory, School of Mechanical Engineering, VIT Chennai.

Please subscribe to our YouTube channel for more updates on Flow visualization
https://www.youtube.com/channel/UCPHV628SolZbTxH9Y2tMIjQ



# Continuity



An arbitrarily shaped control volume.

Matter cannot be created or destroyed - (it is simply changed in to a different form of matter). This principle is know as the *conservation of mass* and we use it in the analysis of flowing fluids.
The principle is applied to fixed volumes, known as control volumes (or surfaces).

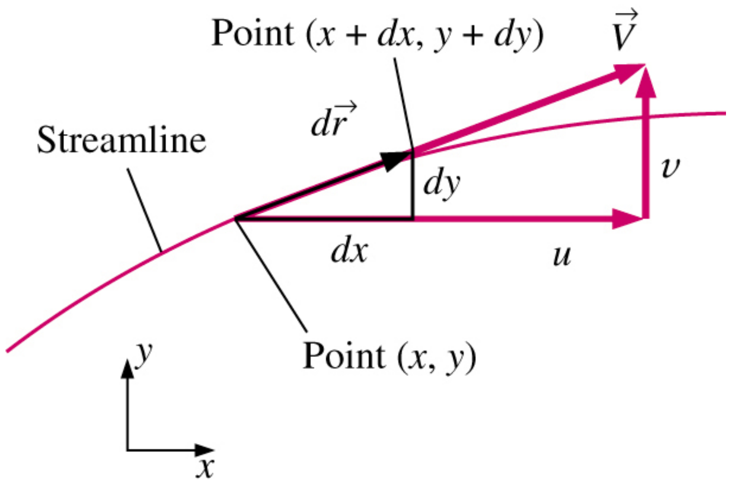
| | | | | |
|---|---|---|---|---|
| Mass entering per unit time | = | Mass leaving per unit time | + | Increase of mass in the control volume per unit time |

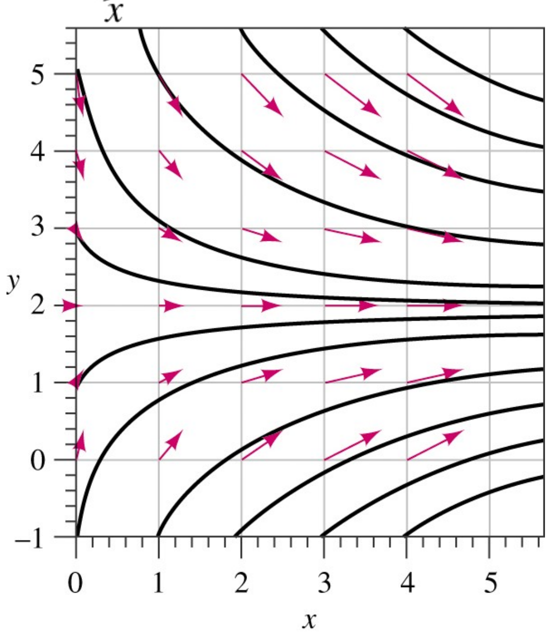
For **steady** flow there is no increase in the mass within the control volume, so

For steady flow

Mass entering per unit time = Mass leaving per unit time

# Streamlines




- A **Streamline** is a curve that is everywhere tangent to the *instantaneous* local velocity vector.
- Consider an arc length

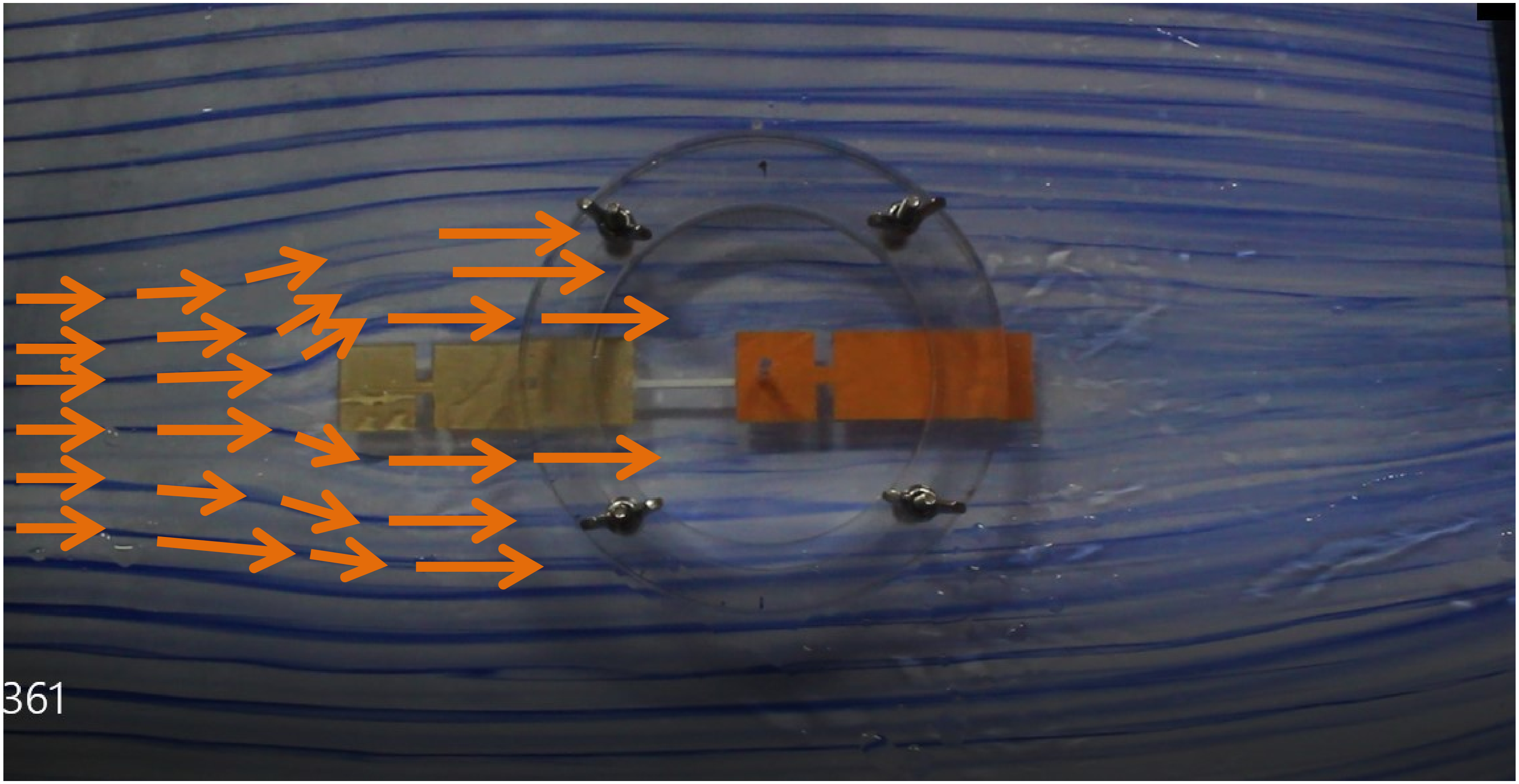
$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

- must be parallel to the local velocity vector

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

- Geometric arguments results in the equation for a streamline

$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

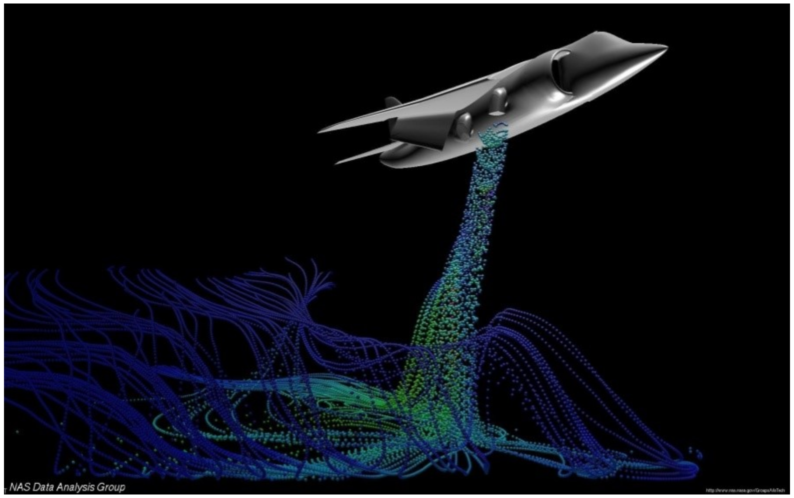


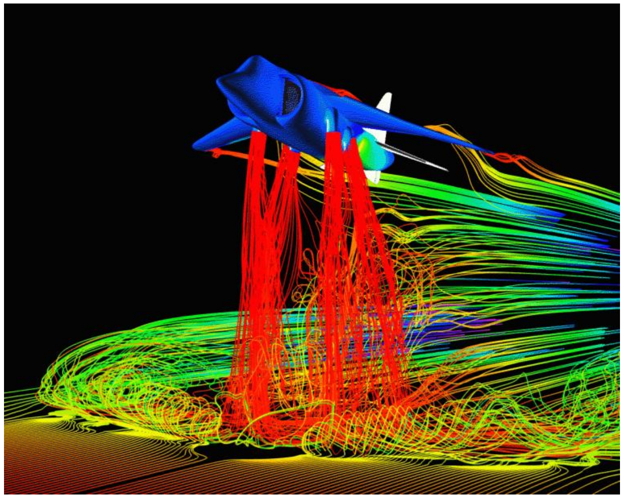

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

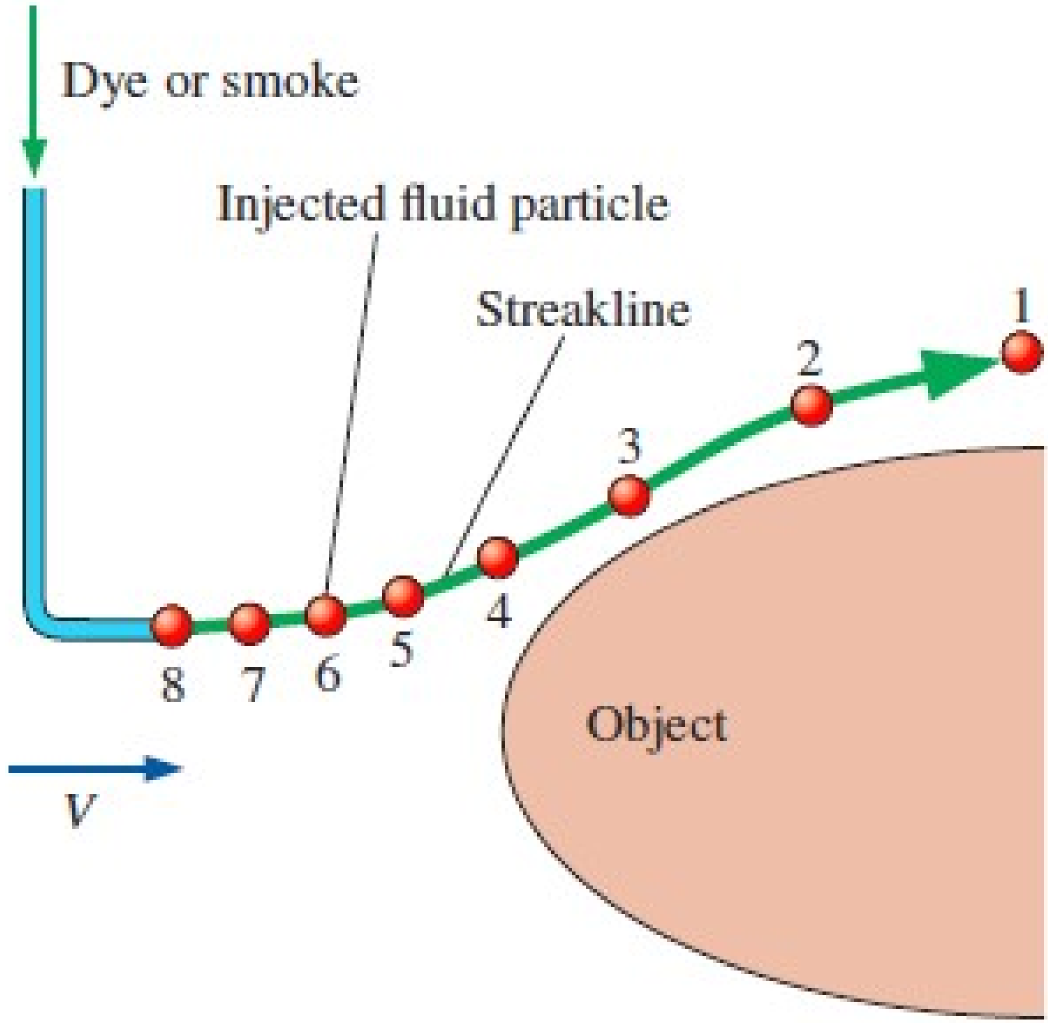
# Streak lines

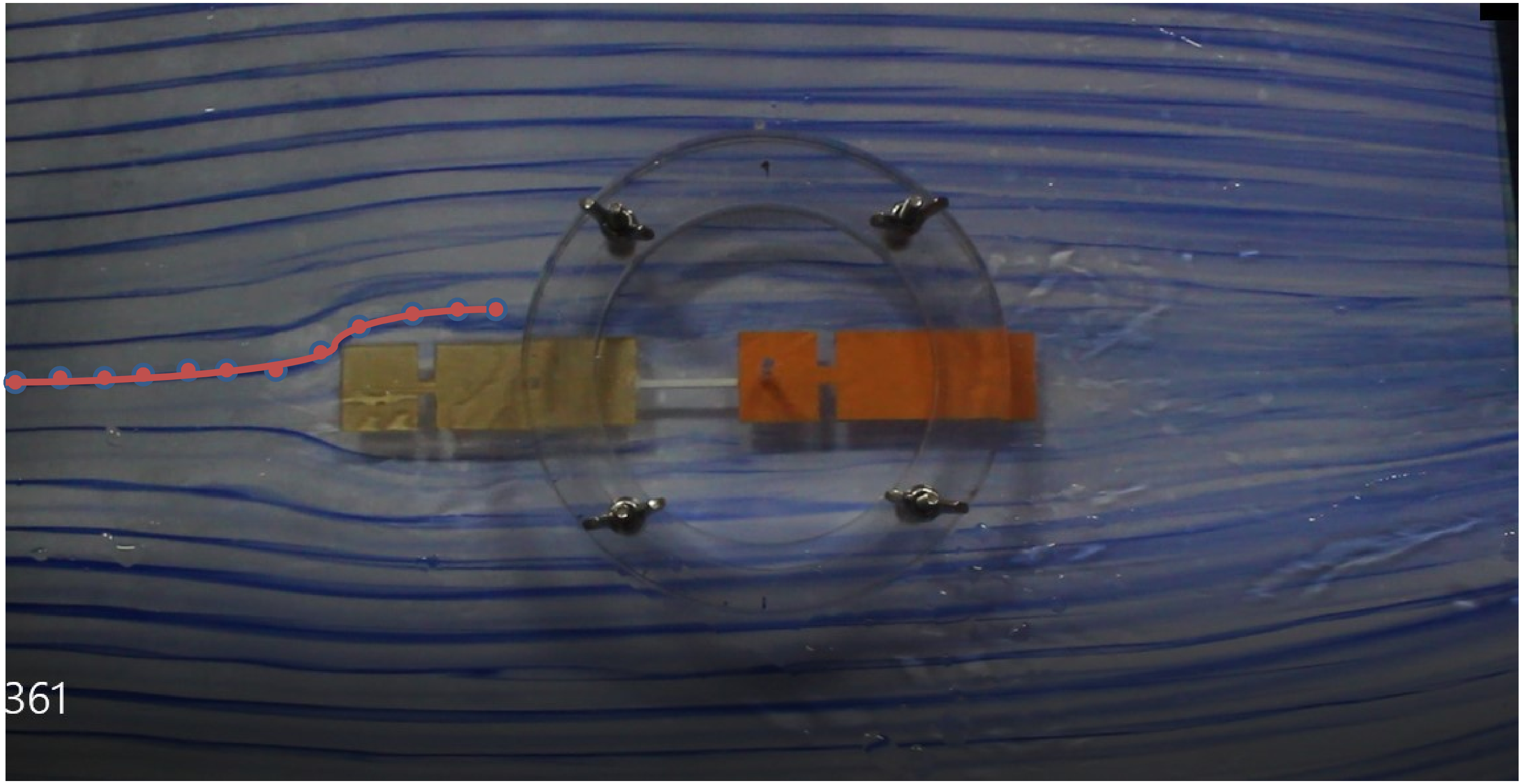


- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Easy to generate in experiments:  dye in a water flow, or smoke in an airflow.

# Streak lines

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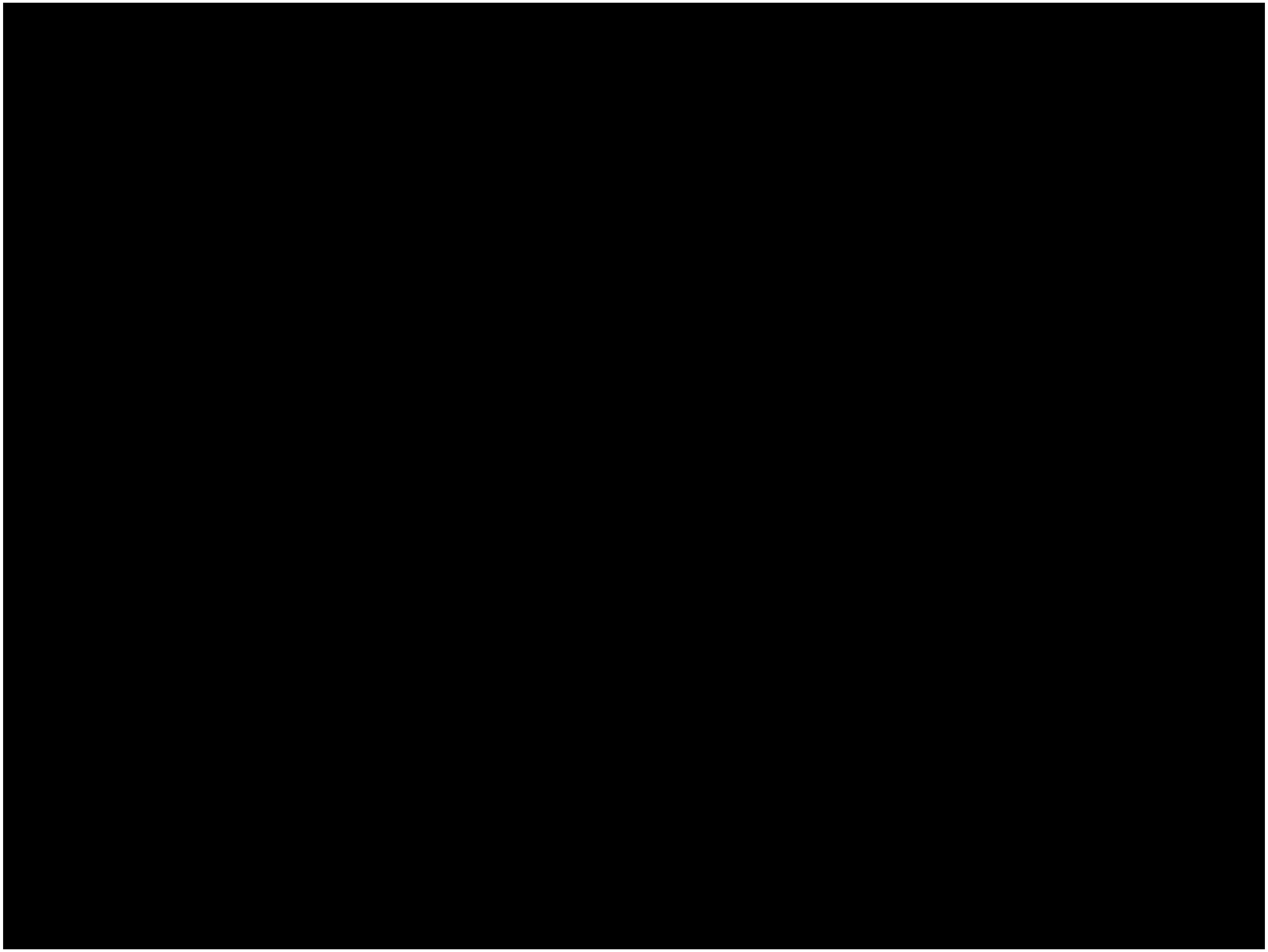
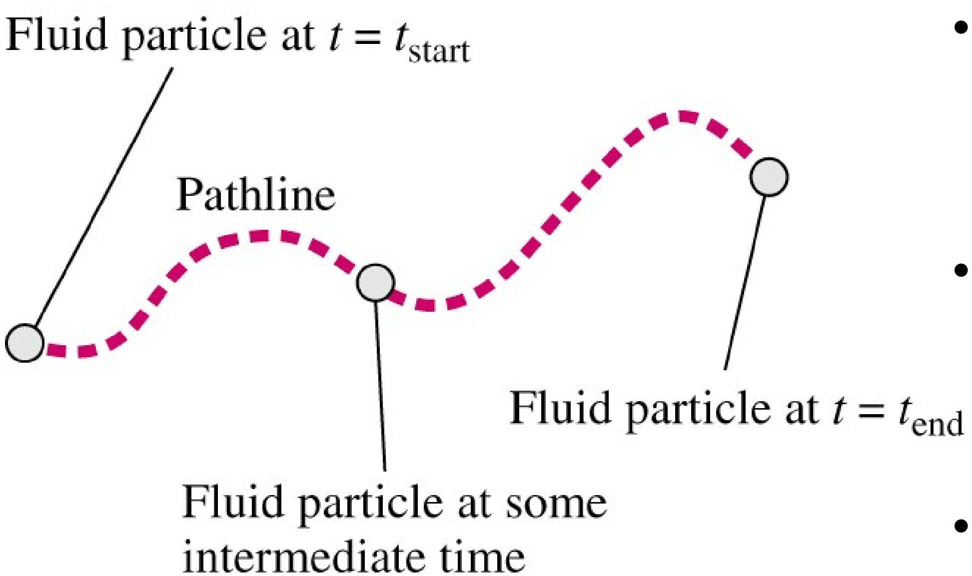
# Streaklines



# Pathline




- A **Pathline** is the actual path traveled by an individual fluid particle over some time period.
- Same as the fluid particle's material position vector

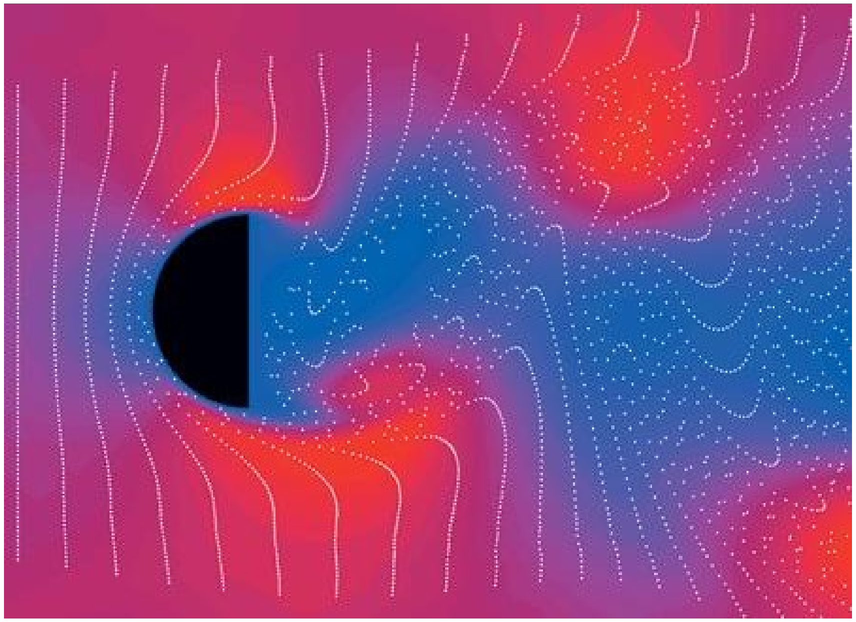
$$\left(x_{particle}(t), y_{particle}(t), z_{particle}(t)\right)$$

- Particle location at time t:

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^{t} \vec{V} dt$$

- Particle Image Velocimetry (PIV) is a modern experimental technique to measure velocity field over a plane in the flow field.

# Timeline



- A **Timeline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Timelines can be generated using a hydrogen bubble wire.

# Conclusions

- For steady flow, streamlines, pathlines, and streaklines are identical.
- For unsteady flow, they can be very different.
  - Streamlines are an instantaneous picture of the flow field
  - Pathlines and Streaklines are flow patterns that have a time history associated with them.
  - Streakline:  instantaneous snapshot of a time-integrated flow pattern.
  - Pathline:  time-exposed flow path of an individual particle.

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j} \tag{1}$$

where the $x$- and $y$-coordinates are in meters and the magnitude of velocity is in m/s. A **stagnation point** is defined as *a point in the flow field where the velocity is zero.* (*a*) Determine if there are any stagnation points in this flow field and, if so, where? (*b*) Sketch velocity vectors at several locations in the domain between $x = -2$ m to 2 m and $y = 0$ m to 5 m; qualitatively describe the flow field.

**SOLUTION** For the given velocity field, the location(s) of stagnation point(s) are to be determined. Several velocity vectors are to be sketched and the velocity field is to be described.

## Assumptions

**1** The flow is steady and incompressible.

**2** The flow is two dimensional, implying no $z$-component of velocity and no variation of $u$ or $v$ with $z$.

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

**Analysis** (*a*) Since $\vec{V}$ is a vector, *all* its components must equal zero in order for $\vec{V}$ itself to be zero. Using Eq. 4–4 and setting Eq. 1 equal to zero,

*Stagnation point:*

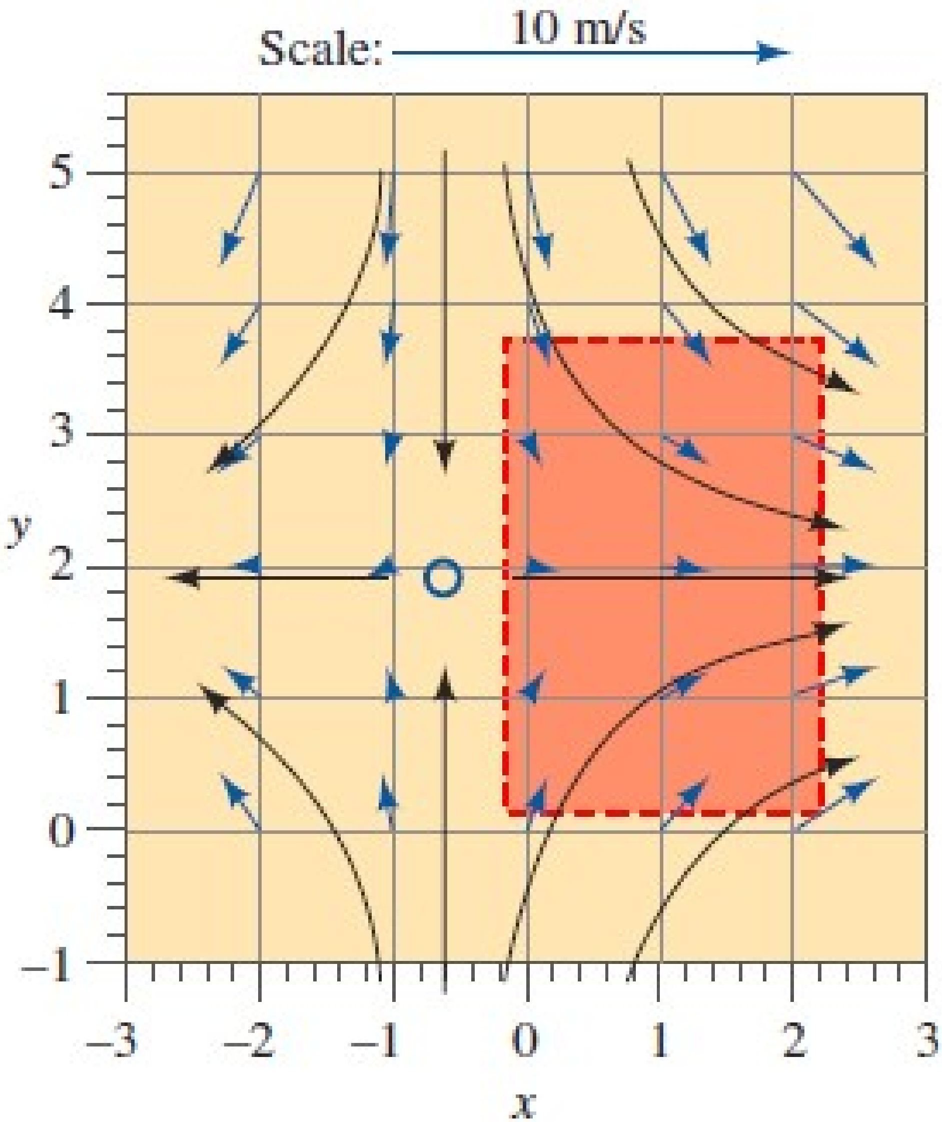
$$u = 0.5 + 0.8x = 0 \quad \rightarrow \quad x = -0.625 \text{ m}$$
$$v = 1.5 - 0.8y = 0 \quad \rightarrow \quad y = 1.875 \text{ m}$$

**Yes.** There is one stagnation point located at $\boldsymbol{x = -0.625}$ **m,** $\boldsymbol{y = 1.875}$ **m.**

(*b*) The *x*- and *y*-components of velocity are calculated from Eq. 1 for several $(x, y)$ locations in the specified range.
For example, at the point ($x = 2$ m, $y = 3$ m), $u = 2.10$ m/s and $v = -0.900$ m/s. The magnitude of velocity (the *speed*) at that point is 2.28 m/s.
At this and at an array of other locations, the velocity vector is constructed from its two components, the results of which are shown in Figure.

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j}$$



*Material derivative:*

$$\frac{D}{Dt} = \frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \vec{\nabla})$$

When we apply the material derivative of Eq. 4–12 to the velocity field, the result is the acceleration field as expressed by Eq. 4–9, which is thus sometimes called the **material acceleration**,

*Material acceleration:*

$$\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

*Material derivative of pressure:*

$$\frac{DP}{Dt} = \frac{dP}{dt} = \frac{\partial P}{\partial t} + (\vec{V} \cdot \vec{\nabla})P$$

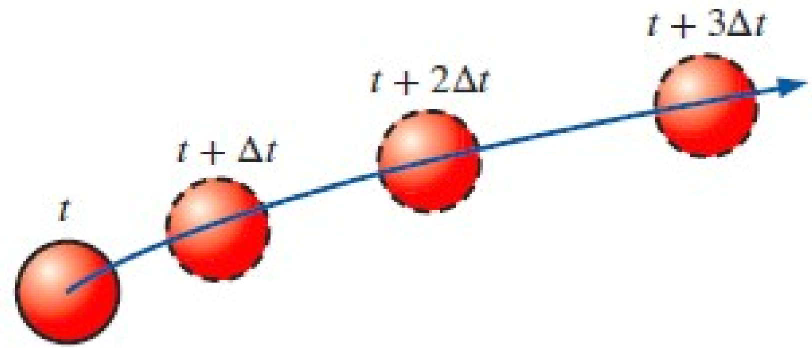



**FIGURE 4–12**
The material derivative $D/Dt$ is defined by following a fluid particle as it moves throughout the flow field. In this illustration, the fluid particle is accelerating to the right as it moves up and to the right.

# Material Acceleration of a Steady Velocity Field

Consider the steady, incompressible, two-dimensional velocity field of Example 4–1. (*a*) Calculate the material acceleration at the point ($x = 2$ m, $y = 3$ m). (*b*) Sketch the material acceleration vectors at the same array of $x$- and $y$-values as in Example 4–1.

**SOLUTION** For the given velocity field, the material acceleration vector is to be calculated at a particular point and plotted at an array of locations in the flow field.

**Assumptions**

**1** The flow is steady and incompressible.

**2** The flow is two dimensional, implying no $z$-component of velocity and no variation of $u$ or $v$ with $z$.

**Analysis** (*a*) Using the velocity field of Eq. 1 of Example 4–1 and the equation for material acceleration components in Cartesian coordinates (Eq. 4–11), we write expressions for the two nonzero components of the acceleration vector:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + \overbrace{(0.5 + 0.8x)(0.8)} + \overbrace{(1.5 - 0.8y)(0)} + 0 = (0.4 + 0.64x)\ \mathrm{m/s^2}$$

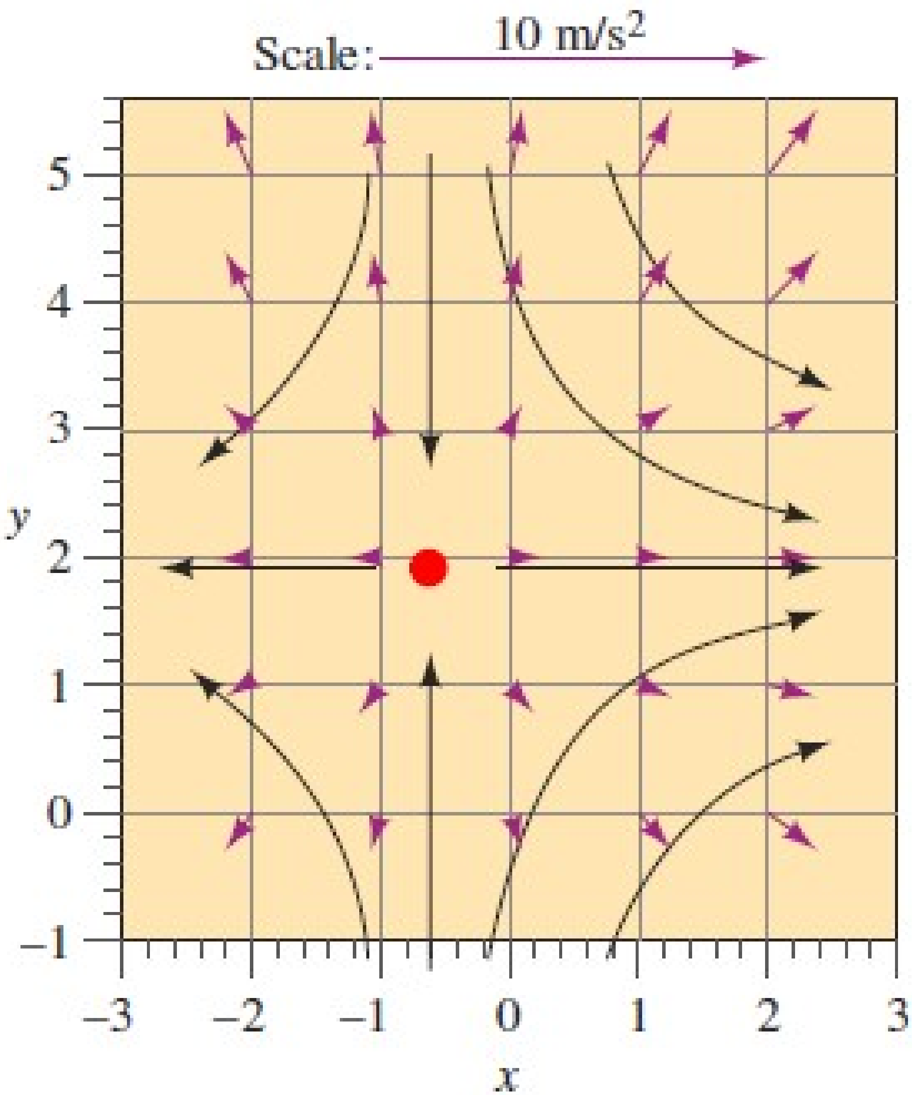
and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 0 + \overbrace{(0.5 + 0.8x)(0)} + \overbrace{(1.5 - 0.8y)(-0.8)} + 0 = (-1.2 + 0.64y)\ \mathrm{m/s^2}$$

At the point ($x = 2$ m, $y = 3$ m), $\boldsymbol{a_x = 1.68\ \mathrm{m/s^2}}$ and $\boldsymbol{a_y = 0.720\ \mathrm{m/s^2}}$.

(*b*) The equations in part (*a*) are applied to an array of *x*- and *y*-values in the flow domain within the given limits, and the acceleration vectors are plotted in Fig.




The acceleration field is nonzero, even though the flow is *steady*.
Above the stagnation point (above $y = 1.875$ m), the acceleration vectors plotted in Fig. 4–14 point upward, increasing in magnitude away from the stagnation point. To the right of the stagnation point (to the right of $x = -0.625$ m), the acceleration vectors point to the right, again increasing in magnitude away from the stagnation point.

# Material Derivative and Acceleration

**Let the position of a particle at any instant t in a flow field be given by the space coordinates (x, y, z) with respect to a rectangular cartesian frame of reference.**
**The velocity components u, v, w of the particle along x, y and z directions respectively can then be written in Eulerian form as**

$$u = u\,(x, y, z, t)$$
$$v = v\,(x, y, z, t)$$
$$w = w\,(x, y, z, t)$$

After an infinitesimal time interval t , let the particle move to a new position given by the coordinates $(x + \Delta x,\ y + \Delta y\ ,\ z + \Delta z)$.
Its velocity components at this new position be $u + \Delta u$, $v + \Delta v$ and $w + \Delta w$.

Expression of velocity components in the Taylor's series form:

$$u + \Delta u = u(x, y, z, t) + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z + \frac{\partial u}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

$$v + \Delta v = v(x, y, z, t) + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{\partial v}{\partial z}\Delta z + \frac{\partial v}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

$$w + \Delta w = w(x, y, z, t) + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z + \frac{\partial w}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

The increment in space coordinates can be written as -

$$\Delta x = u\Delta t,\quad \Delta y = v\Delta t \quad and \quad \Delta z = w\Delta t$$

Substituting the values of $\Delta x$, $\Delta y$, $\Delta z$ in above equations, we have

$$\frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

In the limit $\Delta t \to 0$, the **equation** becomes

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.1a)$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.1b)$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.1c)$$

The above equations tell that the operator for **total differential** with respect to time, D/Dt in a **convective field** is related to the **partial differential** as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

# Material Derivative and Acceleration

The total differential D/Dt is known as the material or **substantial derivative** with respect to time.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad (7.2)$$

The **first term** $\frac{\partial}{\partial t}$ in the right hand side of is known as **temporal or local derivative** which expresses the rate of change with time, at a fixed position.

The **last three terms** in the right hand side of 7.2 are together known as **convective derivative** which represents the time rate of change due to change in position in the field.

# Material Derivative and Acceleration

The terms in the left hand sides of Eqs (7.1a) to (7.1c) are defined as x, y and z components of **substantial or material** acceleration.

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.1a)$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.1b)$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.1c)$$

The first terms in the right hand sides of Eqs (7.1a) to (7.1c) represent the respective **local or temporal** accelerations, while the other terms are **convective** accelerations.

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.2a)$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.2b)$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.2c)$$

**(Material or substantial acceleration) = (temporal or local acceleration) + (convective acceleration)**

**End of Module 2 Part 1**