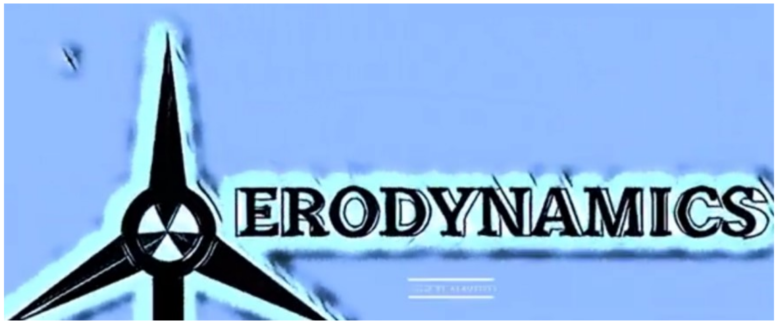
Vellore Institute of Technology



Module 3

BMEE204L-FLUID MECHANICS

Fluid Dynamics - Lecture 1

Vinayagamurthy G, Dr. Eng.,
Associate Professor
School of Mechanical and Building Sciences
VIT Chennai

Module 3: Fluid Dynamics

Fluid dynamics: Continuity equation, Euler and Bernoulli's equations – orificemeter, venturimeter, Momentum equation, Application of momentum equation – forces on curved pipes, Navier–Stokes Equations.

Problems – Continuity Equation- No. 2

A pipe (1) 450 mm in diameter branches into two pipes (2) and (3) of diameters 300 mm and 200 mm respectively as shown in Fig. 5.57. If the average velocity in 450 mm diameter pipe is 3 m/s, find : (*i*) discharge through 450 mm dia. pipe and (*ii*) velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s. *(J.N.T.U., Hyderabad, S 2002)*

[**Hint.** Given : $d_1 = 450 \text{ mm} = 0.45 \text{ m}, d_2 = 300 \text{ mm} = 0.3 \text{ m}$
$d_3 = 200 \text{ mm} = 0.2 \text{ m}, V_1 = 3 \text{ m/s}, V_2 = 2.5 \text{ m/s}$

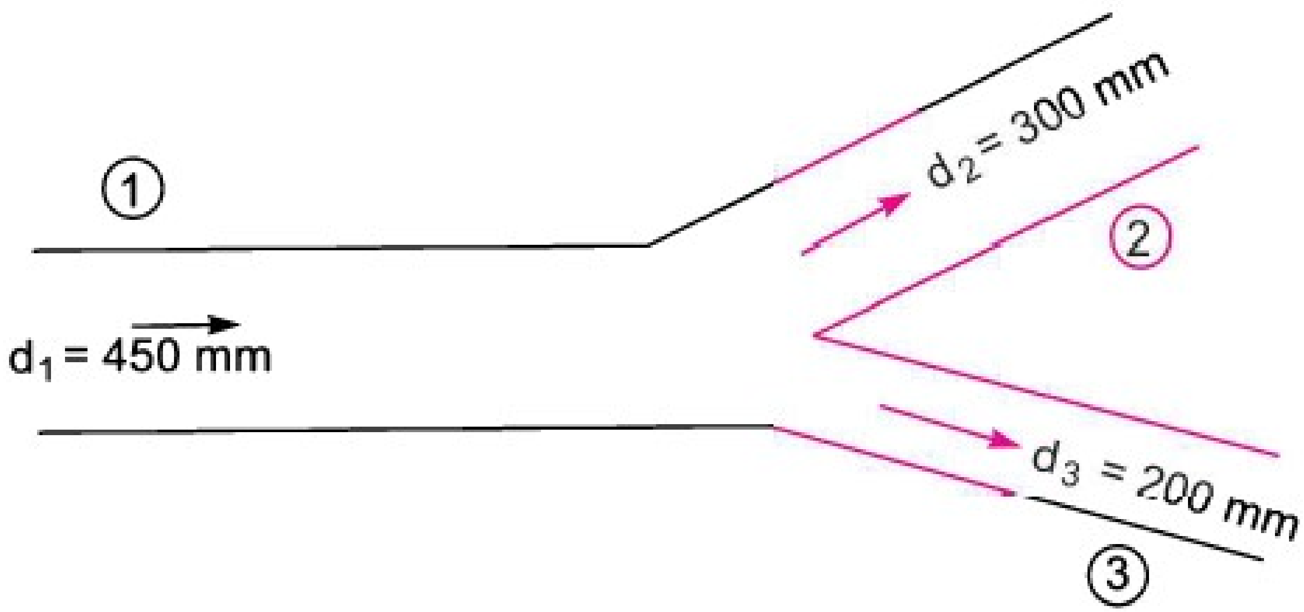
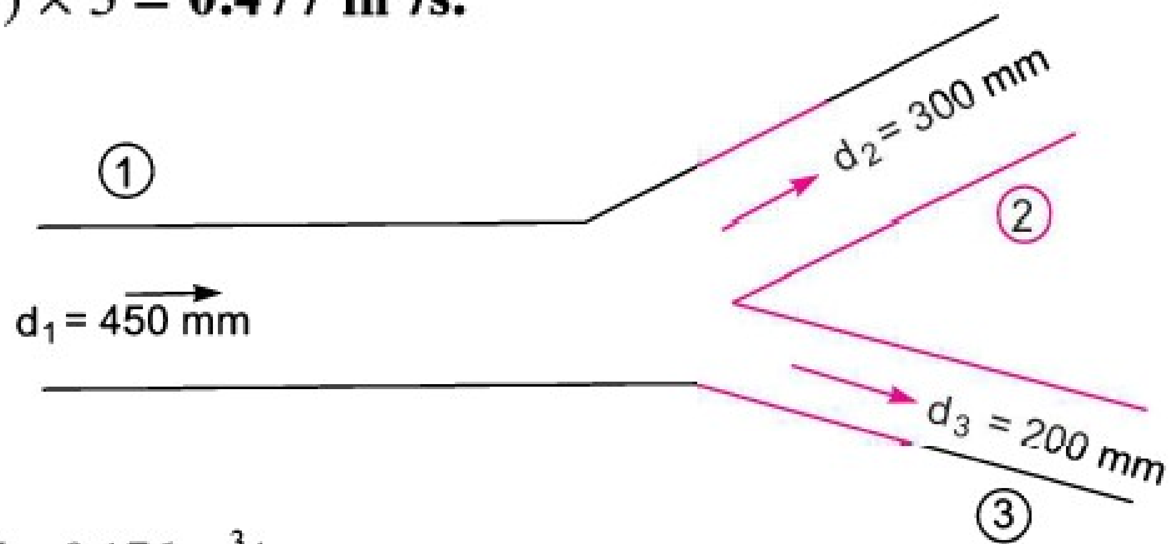


Fig. 5.57

$$Q_1 = A_1 V_1 = \frac{\pi}{4}\,(0.45^2) \times 3 = \mathbf{0.477\ m^3/s.}$$



$$Q_2 = A_2 V_2 = \frac{\pi}{4}\,(.3^2) \times 2.5 = 0.176\ \mathrm{m^3/s}$$

$$Q_1 = Q_2 + Q_3 \quad \therefore \quad Q_3 = Q_1 - Q_2 = 0.477 - 0.176 = 0.301$$

$$Q_3 = A_3 \times V_3 = \frac{\pi}{4}\,(0.2^2) \times V_3$$

$$V_3 = \frac{Q_3}{\frac{\pi}{4}\left(0.2^2\right)} = \frac{0.301}{0.0314} = \mathbf{9.6\ m/s.]}$$

Problems-Bernoullis Equation

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head = pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Fluid Motion- Forces in Fluids

Thus mathematically,

$$F_x = m.a_x$$

In the fluid flow, the following forces are present :

(*i*) F_g, gravity force.

(*ii*) F_p, the pressure force.

(*iii*) F_v, force due to viscosity.

(*iv*) F_t, force due to turbulence.

(*v*) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

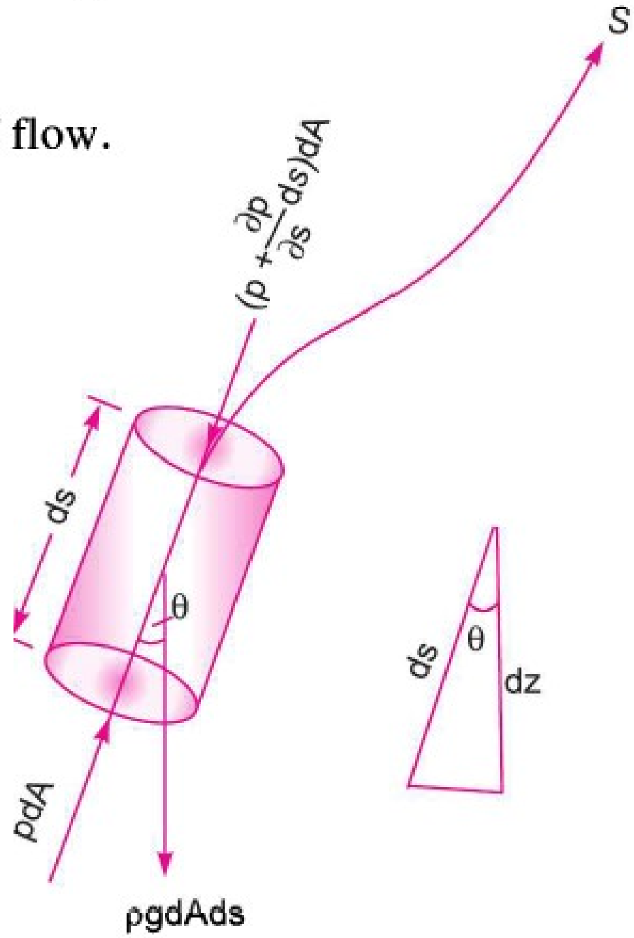
and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

Bernoulli's Equation

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.



The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$
$$= \rho dA ds \times a_s$$

where a_s is the acceleration in the direction of s.

Now $\quad a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

$$pdA - \left(p + \frac{\partial p}{\partial s}ds\right)dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \qquad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s}dsdA - \rho g\, dAds \cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho dsdA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (*b*), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \qquad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

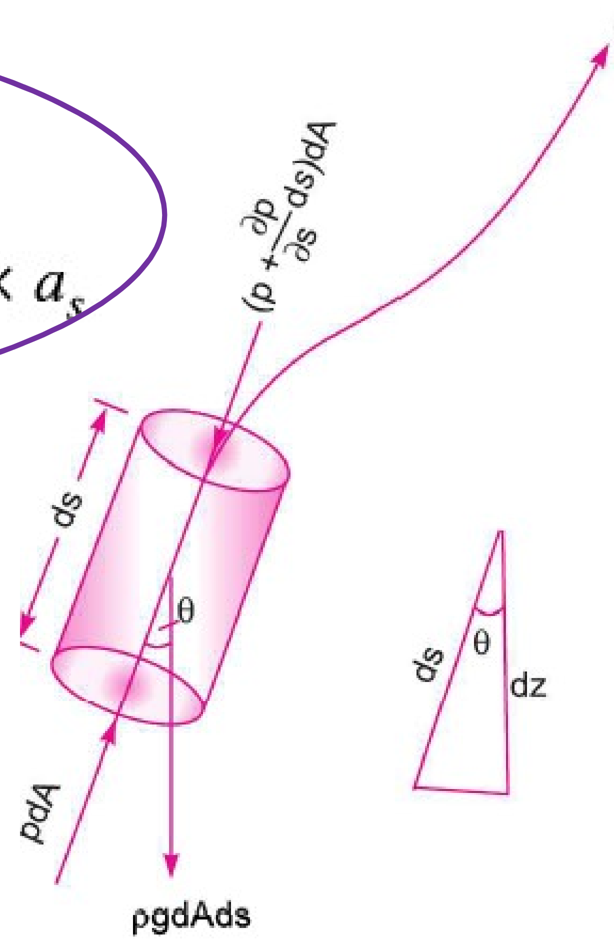


Fig. 6

Euler's equation of motion.

or $$\frac{dp}{\rho} + gdz + vdv = 0$$

Bernoulli's Equation

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad .$$

Equation (6.4) is a Bernoulli's equation in which

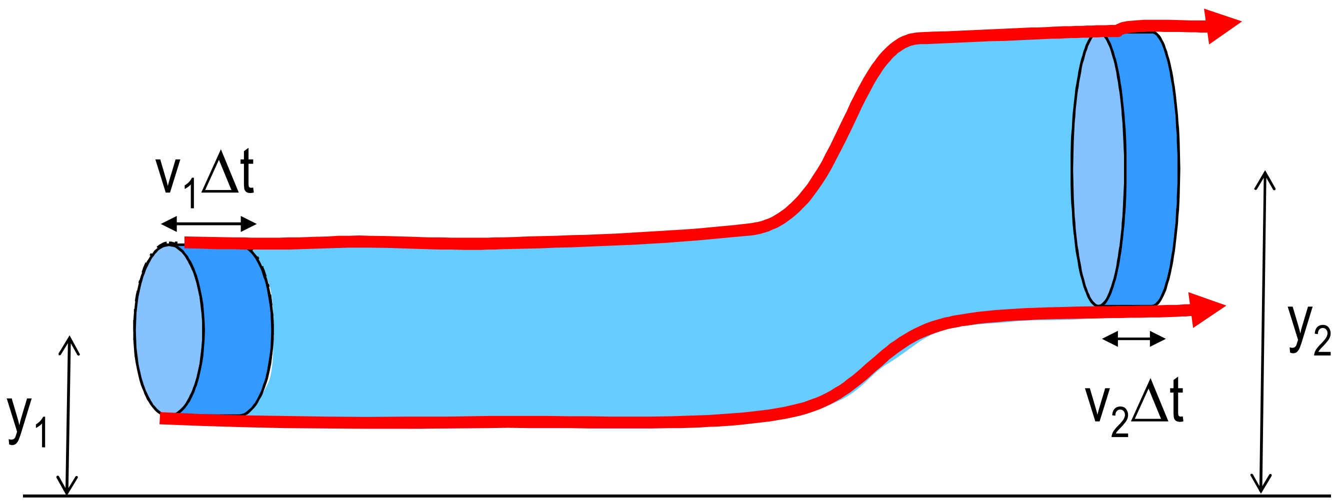
$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

Conservation of Energy: Bernoulli's Eqn.

What happens to the energy density of the fluid if I raise the ends ?



Energy per unit volume

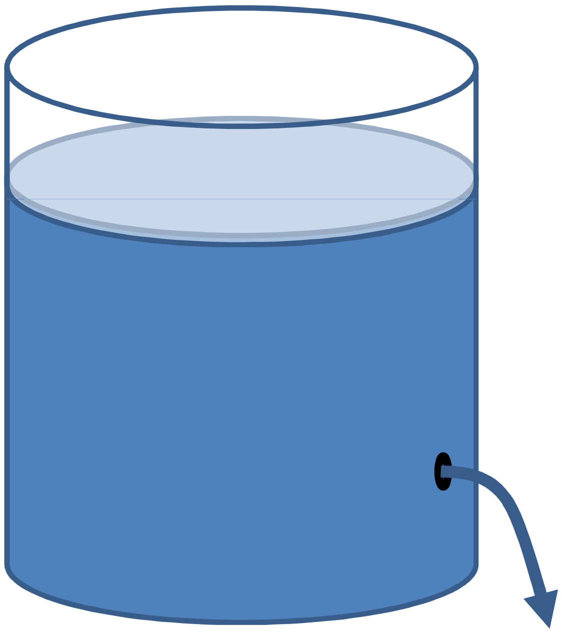
$$\boxed{p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = const}$$

Total energy per unit volume is constant at **any** point in fluid.

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = const$$

Conservation of Energy: Bernoulli's Eqn.

Q. Find the velocity of water leaving a tank through a hole in the side 1 metre below the water level.



$$P + \tfrac{1}{2}\rho v^2 + \rho g y = constant$$

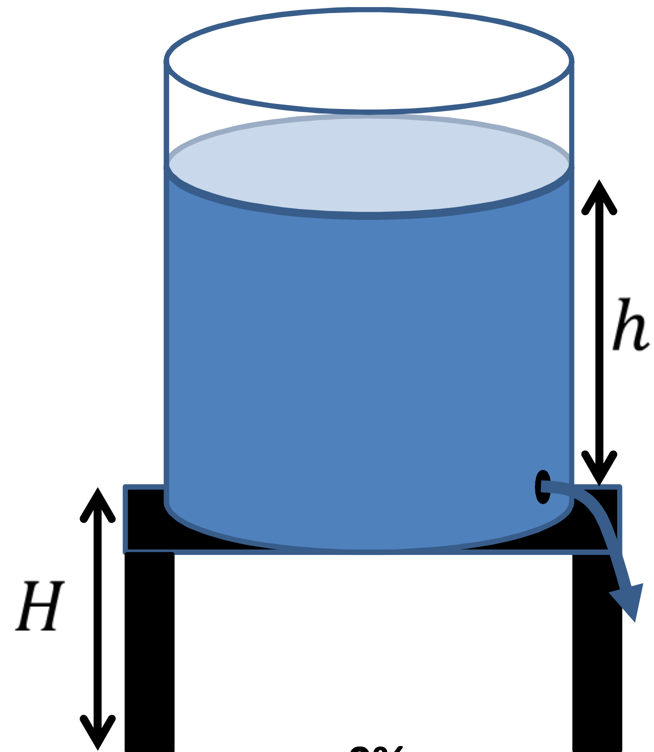
$$At\ the\ top:\ P = 1\ atm, v = 0, y = 1\ m$$

$$At\ the\ bottom:\ P = 1\ atm, v = ?, y = 0\ m$$

$$\cancel{P} + \rho g y = \cancel{P} + \tfrac{1}{2}\rho v^2$$

$$v = \sqrt{2gy} = \sqrt{2 \times 9.8 \times 1} = 4.4\ m/s$$

Which of the following can be done to increase the **flow rate** out of the water tank ?



0%

1. Raise the tank ($\uparrow H$)
2. Reduce the hole size
3. Lower the water level ($\downarrow h$)
4. Raise the water level ($\uparrow h$)
5. None of the above

Summary: fluid dynamics

Continuity equation: mass is conserved!

$$\rho \times v \times A = constant$$

For liquids:

$$\rho = constant \rightarrow v \times A = constant$$

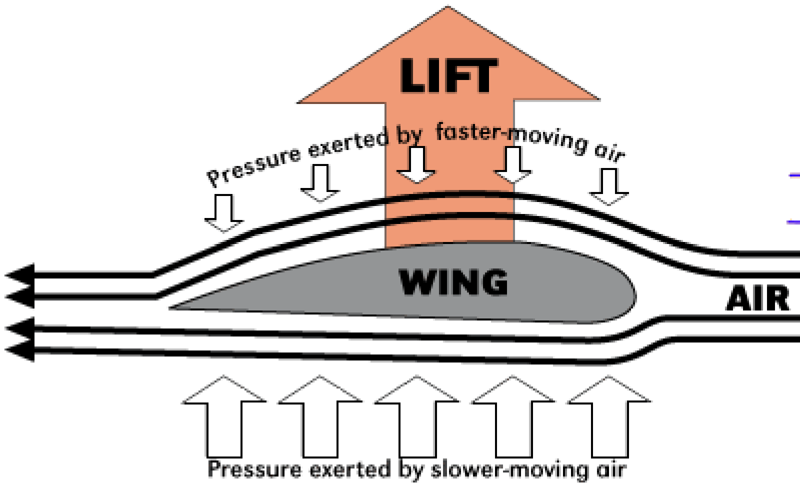
$$(Density\ \rho, velocity\ v, pipe\ area\ A)$$

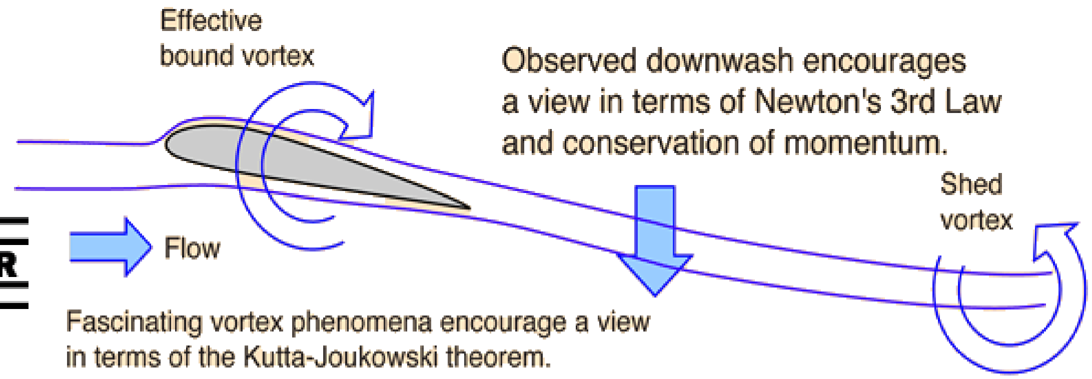
Bernoulli's equation: energy is conserved!

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

$$(Pressure\ P, density\ \rho, velocity\ v, height\ y)$$

Bernoulli's Effect and Lift





Newton's 3rd law
(air pushed downwards)

$$P + \frac{1}{2}\rho v^2 + \rho g y = constant$$

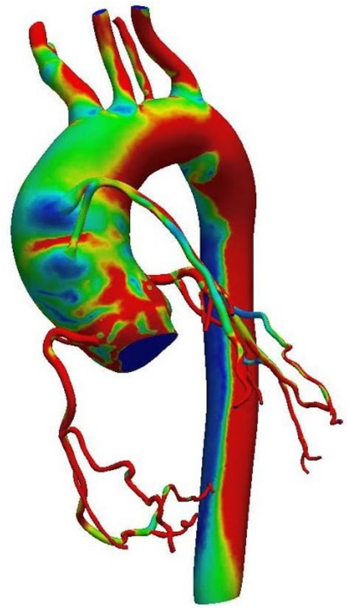
Lift on a wing is often explained in textbooks by Bernoulli's Principle: the air over the top of the wing moves faster than air over the bottom of the wing because it has further to move (?) so the pressure upwards on the bottom of the wing is smaller than the downwards pressure on the top of the wing.

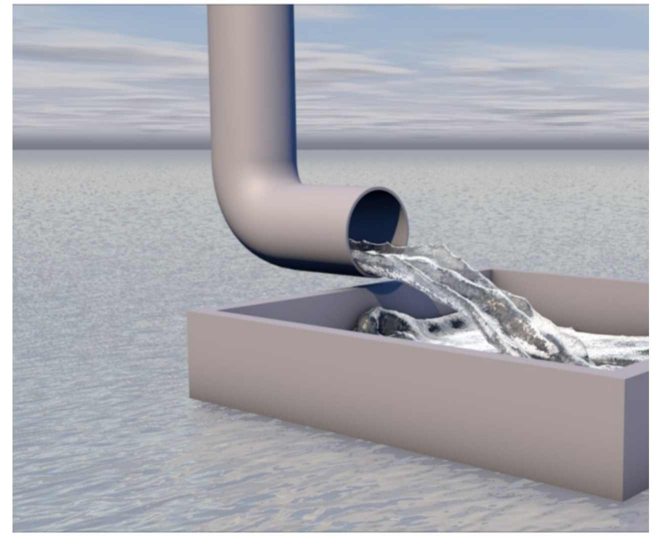
Is that convincing? So why can a plane fly upside down?

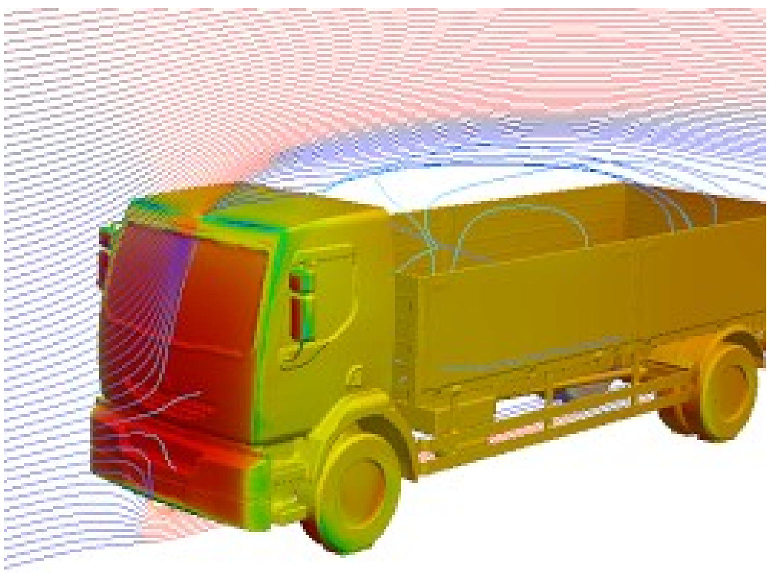
Aerodynamic Forces and moments

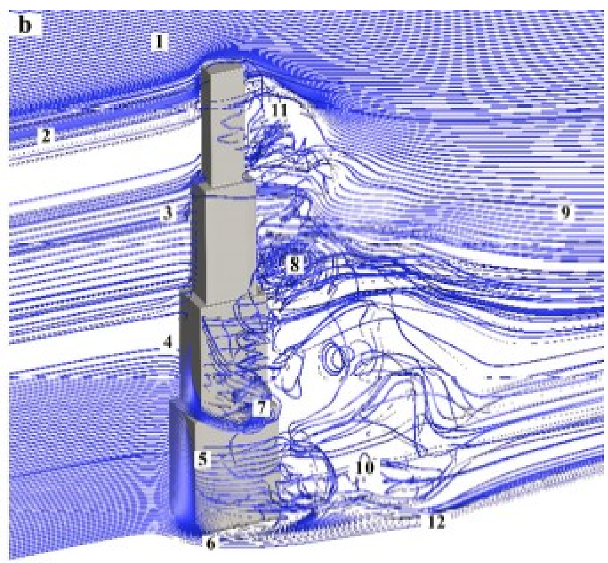
- External and Internal flow problems
- Applications aerodynamics in various fields
- Static, Dynamic and Total Pressure
- Bernoulli's principle- A revisit
- Lift and Drag forces
- Flow separation
- Nomenclature of Airfoil and wing
- Pressure distribution around a airfoil
- Lift and Drag coefficients of simple sections

External and Internal flow problems

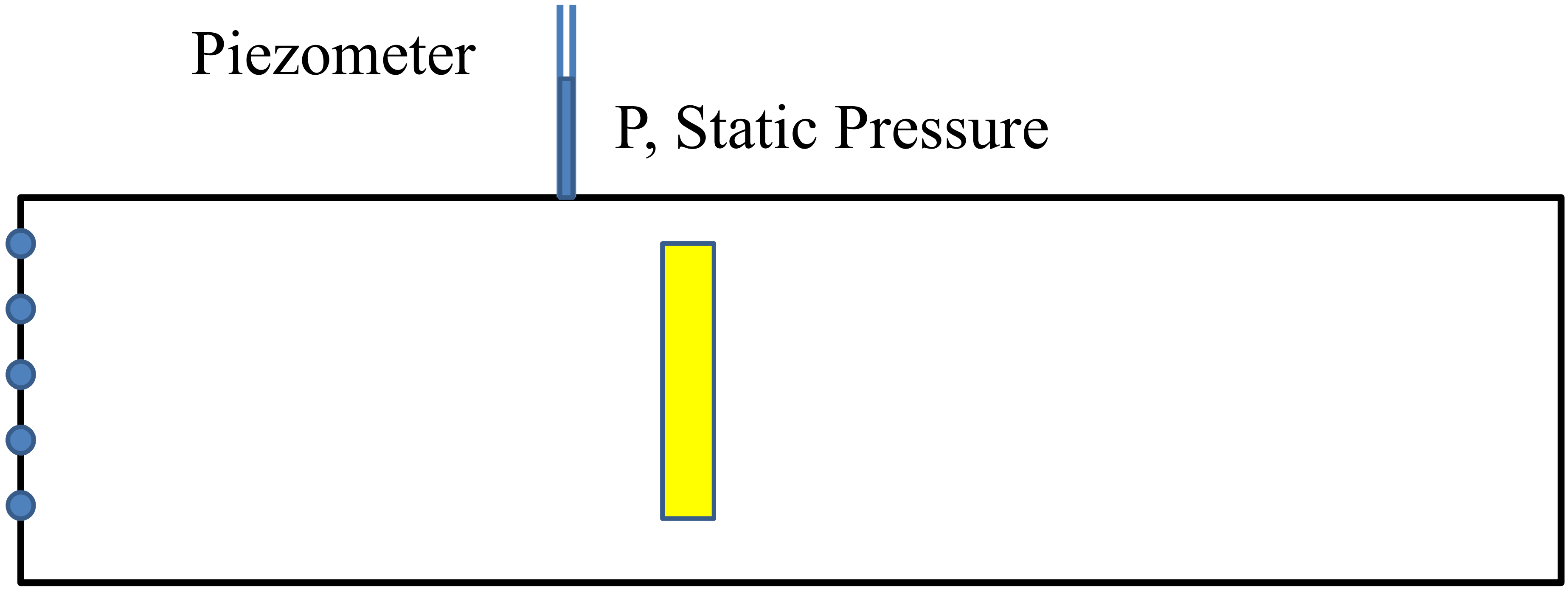






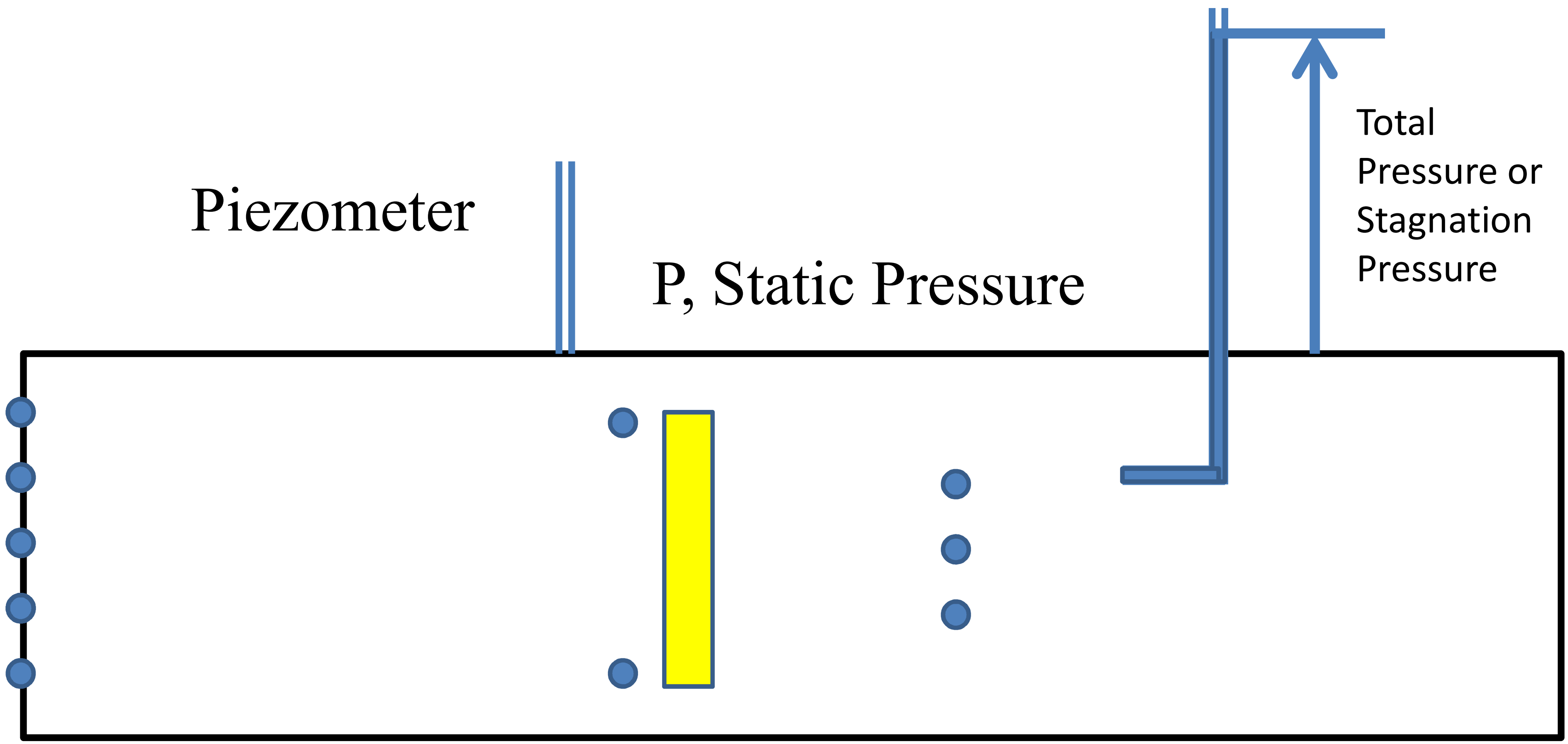


Static, Dynamic and Total Pressure

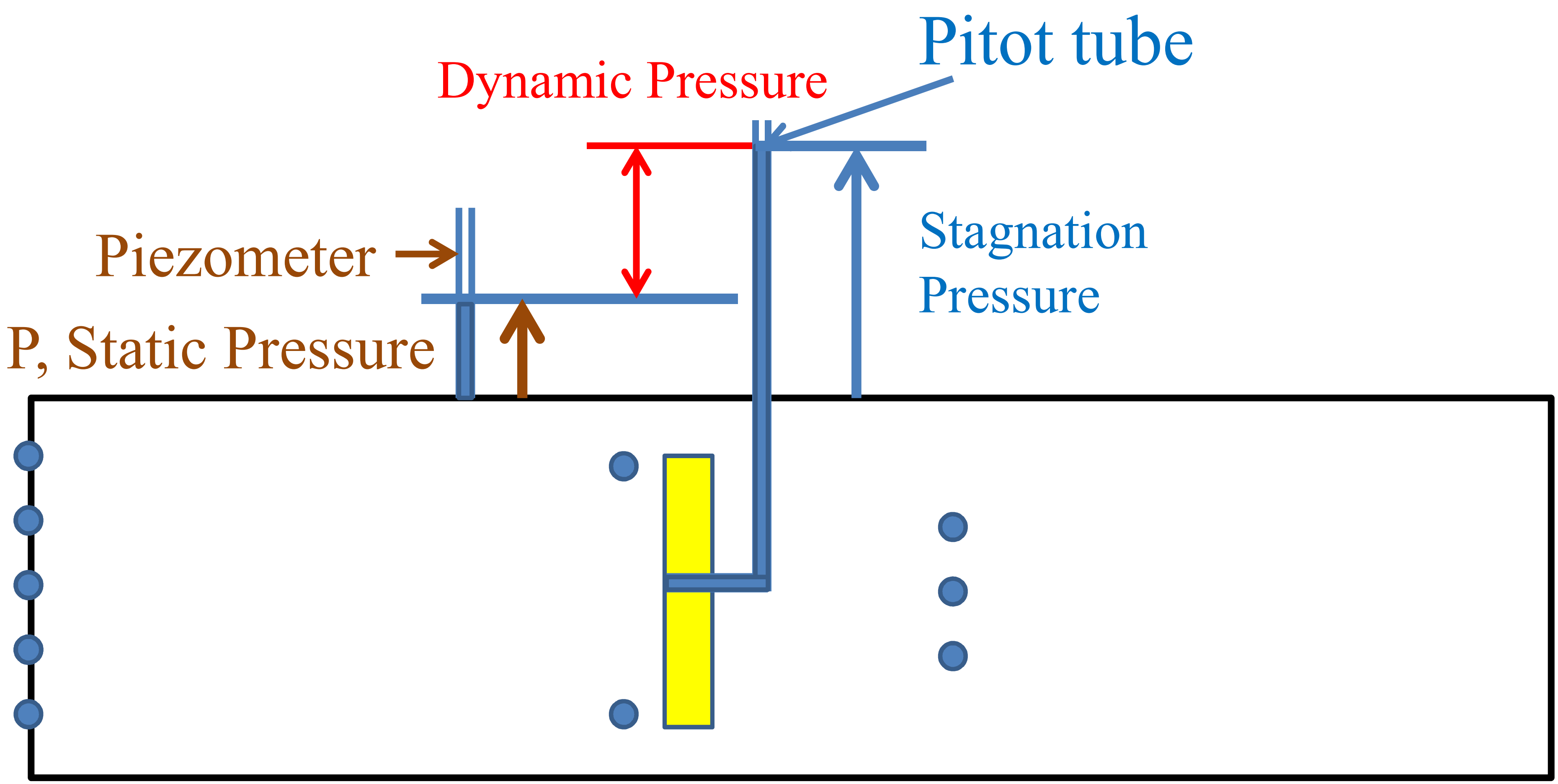


P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

Static, Dynamic and Total Pressure

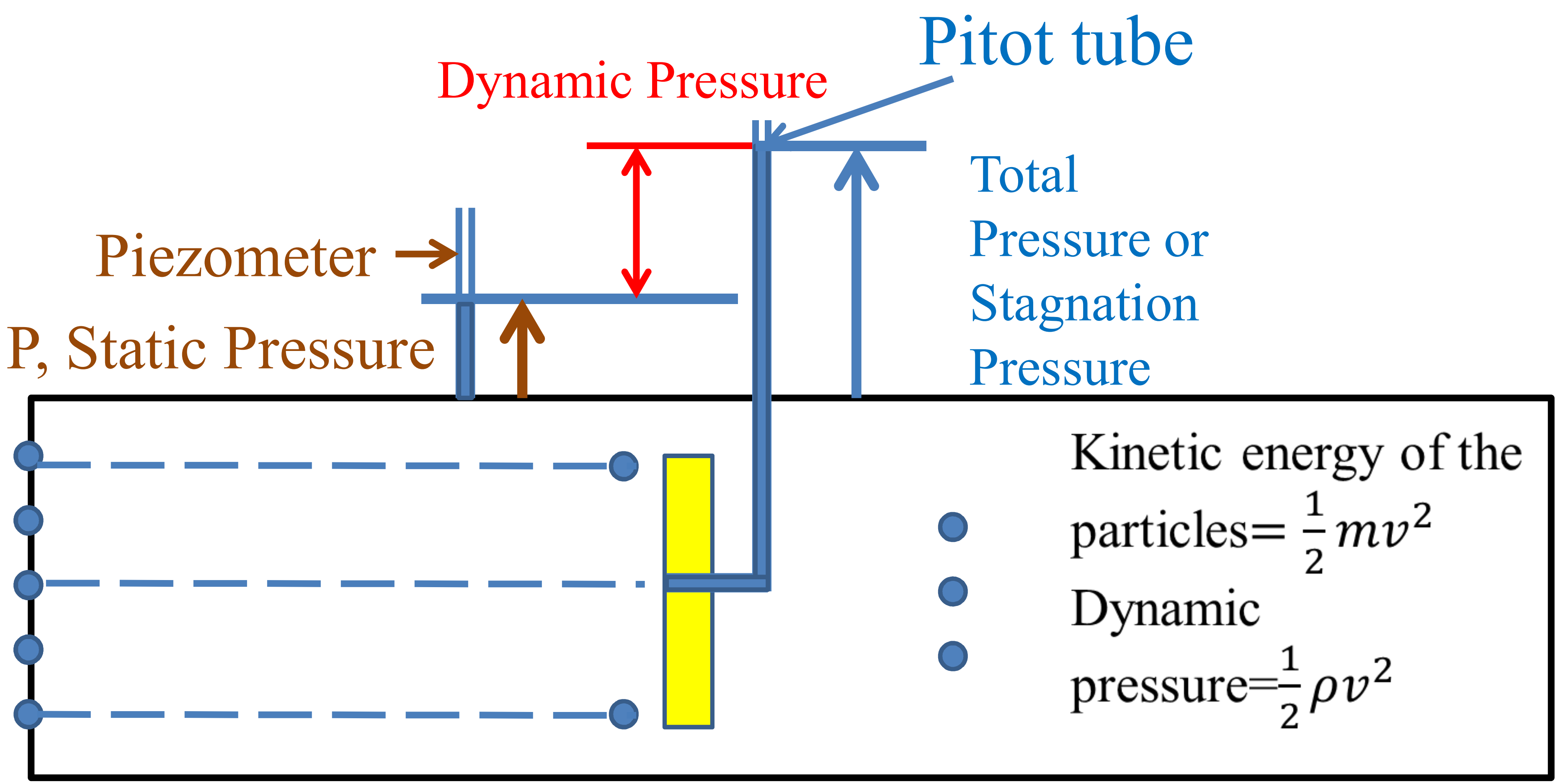


Static, Dynamic and Stagnation Pressure



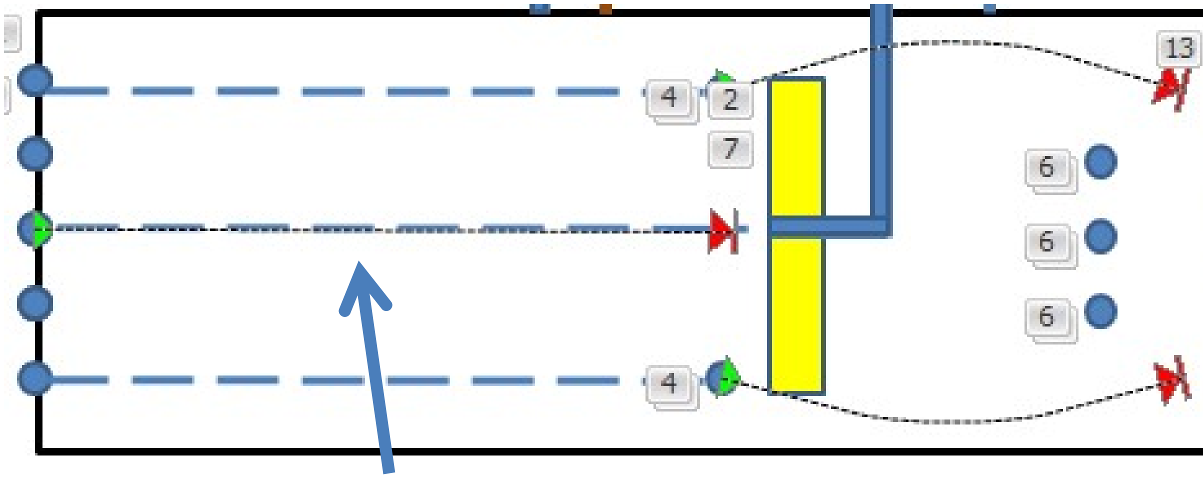
Stganation Pressure = Static Pressure + Dynamic Pressure

Bernoulli equation- A revisit



Total Pressure = Static Pressure + Dynamic Pressure + Hydrostatic Pressure

Bernoulli equation- A revisit



Kinetic energy of the particles$= \frac{1}{2}mv^2$

Dynamic pressure$=\frac{1}{2}\rho v^2$

Applying Benoulli equation to a streamline, Total Pressure = $P + \frac{1}{2}\rho v^2$

When the fluid is being stopped,

Velocity drops to Zero (or) Kinetic energy at the impact is zero

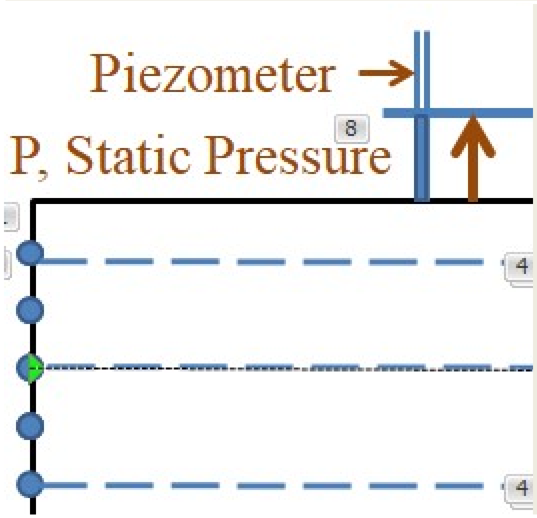
$$P_{\text{stagnation}} = P + \frac{1}{2}\rho v^2$$

When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated,

$$P_{stagnation} - P = \frac{1}{2}\rho v^2 \qquad \sqrt{\frac{2(P_{stagnation} - P)}{\rho}} = v$$

Pitot-Static Probe

The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured.



- If the pressures to be measured are below atmospheric, or if measuring pressures in *gases*, piezometer tubes do not work.
- However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer.

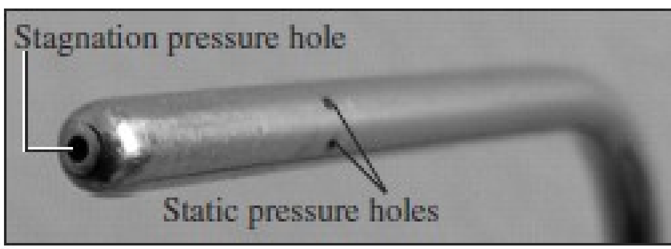
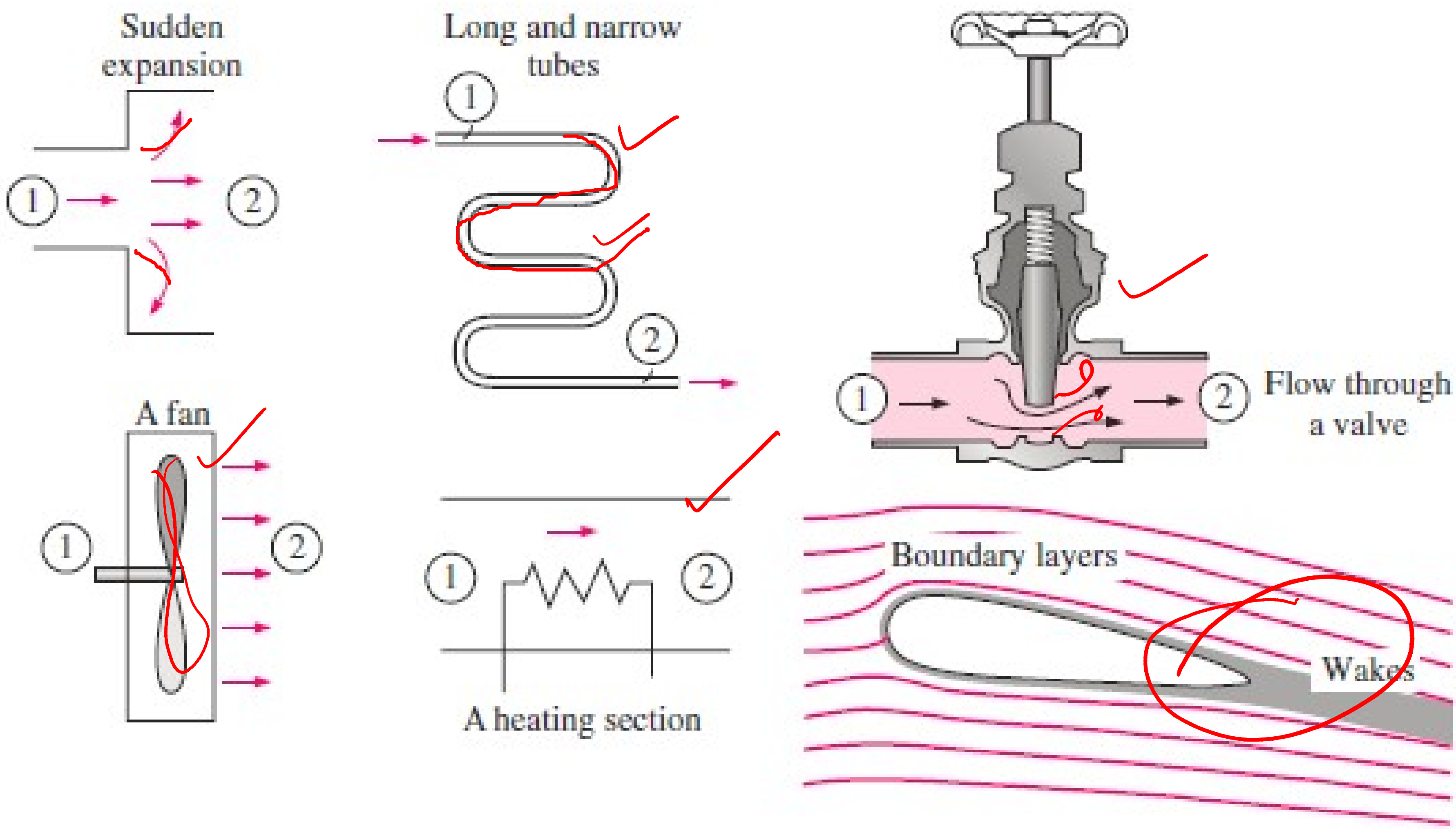


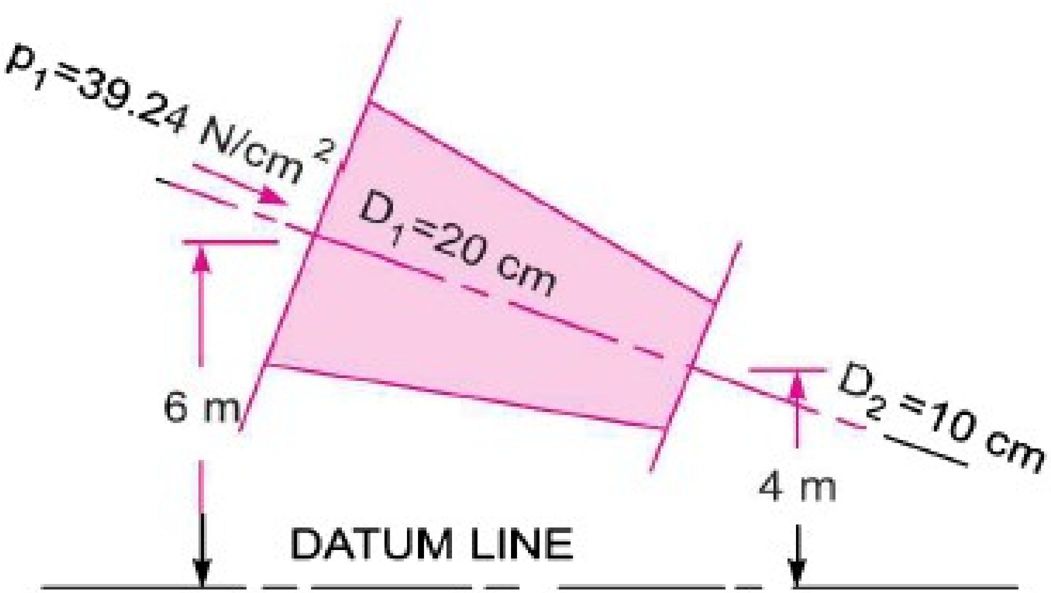
FIGURE 5–28
Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

Pitot-static probe it is convenient to integrate static pressure holes on a Pitot probe

Bernoulli equation- Limitations



Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of pressure at section 2.*



At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \dfrac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \dfrac{Q}{A_1} = \dfrac{.035}{.0314} = 1.114 \text{ m/s}$

nd $V_2 = \dfrac{Q}{A_2} = \dfrac{.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

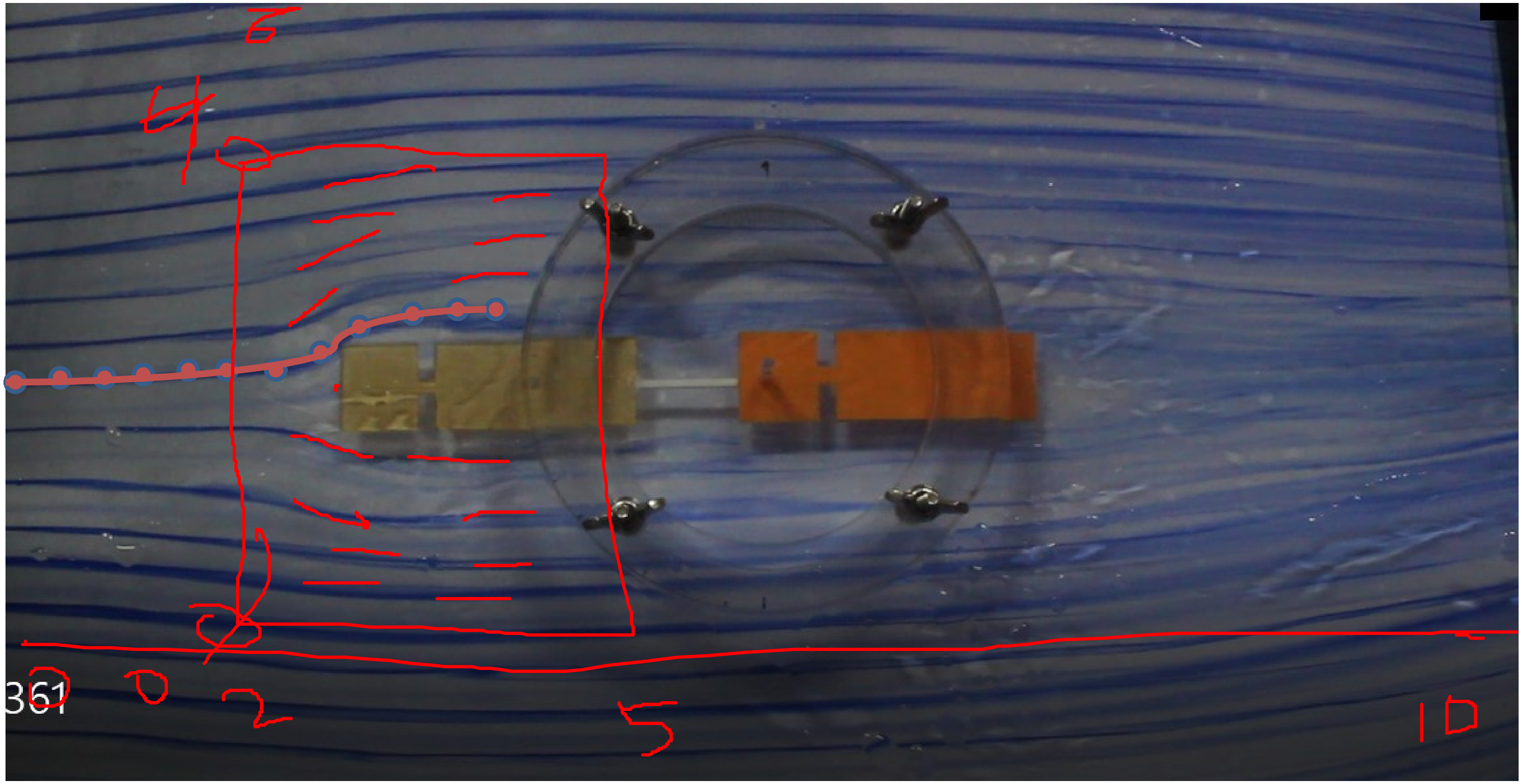
or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2. \text{ Ans.}}$$


4
361
0
2
5
10

End of Lecture