# Lecture 3: BMEE204L
# Fluid Mechanics & Machinery

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- **Newtonian and Non-Newtonian Fluids**
- **Compressibility**
  - **Incompressible and Compressible flows**

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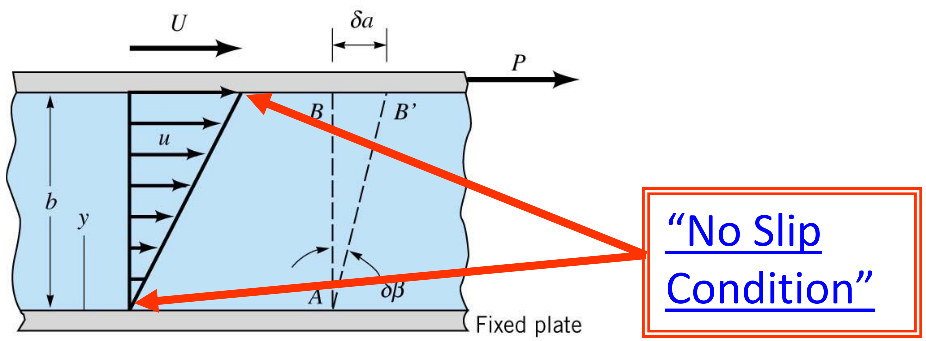
# Viscosity

The viscosity is measure of the "fluidity" of the fluid which is not captured simply by density or specific weight. A fluid can not resist a shear and under shear begins to flow. The shearing stress and shearing strain can be related with a relationship of the following form for common fluids such as water, air, oil, and gasoline:

$$\tau \; \alpha \; \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$




where $\mu$ is called absolute or dynamic viscosity. Dimensions and units for $\mu$ are $ML^{-1}T^{-1}$ and $N-s/m^2$ , respectively. [In the absolute metric system basic unit of co-efficient of viscosity is called poise. 1 poise = $N-s/m^2$

$$\tau = \mu \frac{dV_x}{dy}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{shear stess} & \text{viscosity} & \text{strain rate} \end{array}$$

**For Non-Newtonian fluid:** . Examples are sugar solution and polymers. Therefore,General expression for shear stress

$$\tau = m \left|\frac{dV_x}{dy}\right|^{n-1} \left(\frac{dV_x}{dy}\right)$$

where, m is the flow consistency and n is the flow behaviour index.

- Also known as power–Law model
- also known as apparent viscosity

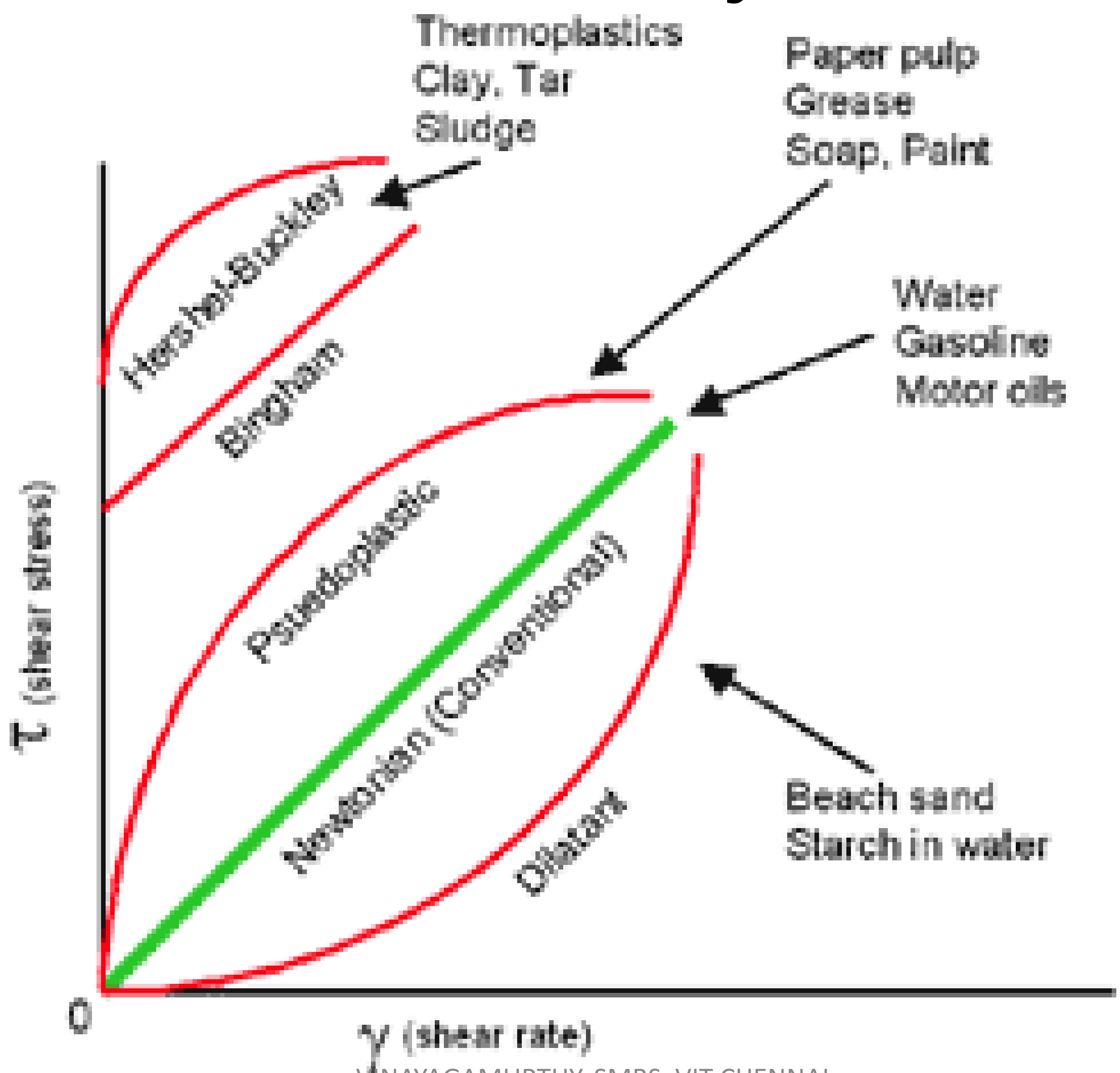
# Viscosity: Measurements

A Capillary Tube Viscosimeter is one method of measuring the viscosity of the fluid.

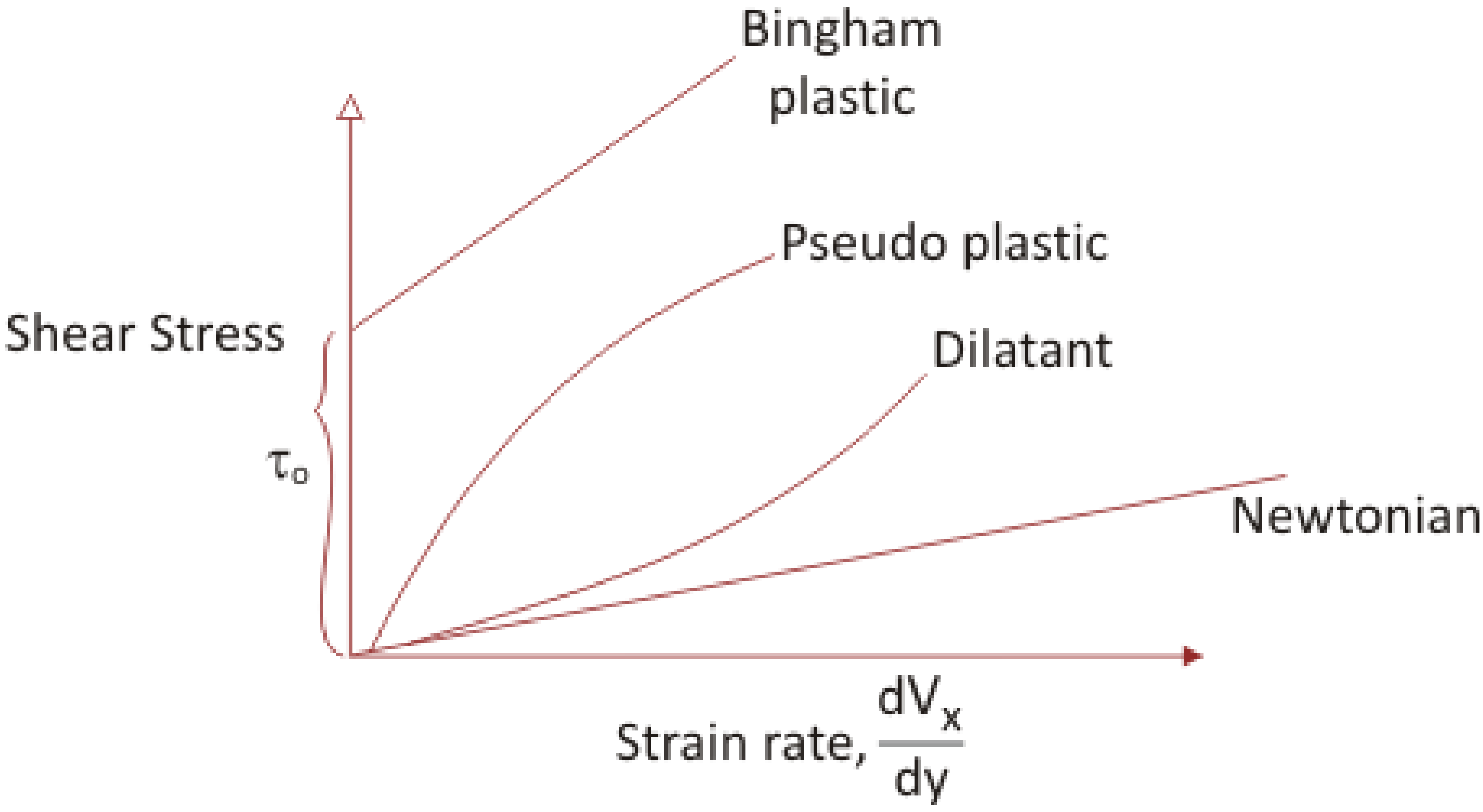
Viscosity Varies from Fluid to Fluid and is [dependent on temperature](), thus temperature is measured as well.

Units of Viscosity are $N{\cdot}s/m^2$ or $lb{\cdot}s/ft^2$

# Viscosity


Thermoplastics
Clay, Tar
Sludge
Paper pulp
Grease
Soap, Paint
Water
Gasoline
Motor oils
Hershel-Buckley
Bingham
Psuedoplastic
Newtonian (Conventional)
Dilatant
Beach sand
Starch in water
τ (shear stress)
0
γ (shear rate)

# Newtonian Fluids



- **Newtonian:** $\tau = \mu \frac{dV_x}{dy}$: air, water, glycerin

- **Bingham Plastic:** $\tau = \underset{\text{yield stress}}{\tau_o} + \mu \frac{dV_x}{dy}$ : toothpaste

  (Fluid does not move or deform till there is a critical stress)

- **Dilatant:** $\tau = K \left(\frac{dV_x}{dy}\right)^n, n > 1$: starch or sand suspension

  or shear thickening fluid

  (Fluid starts 'thickening' with increase in its apparent viscosity)

- **Pseudo plastic:** $\tau = K \left(\frac{dV_x}{dy}\right)^n, n < 1$: paint or shear thinning fluid

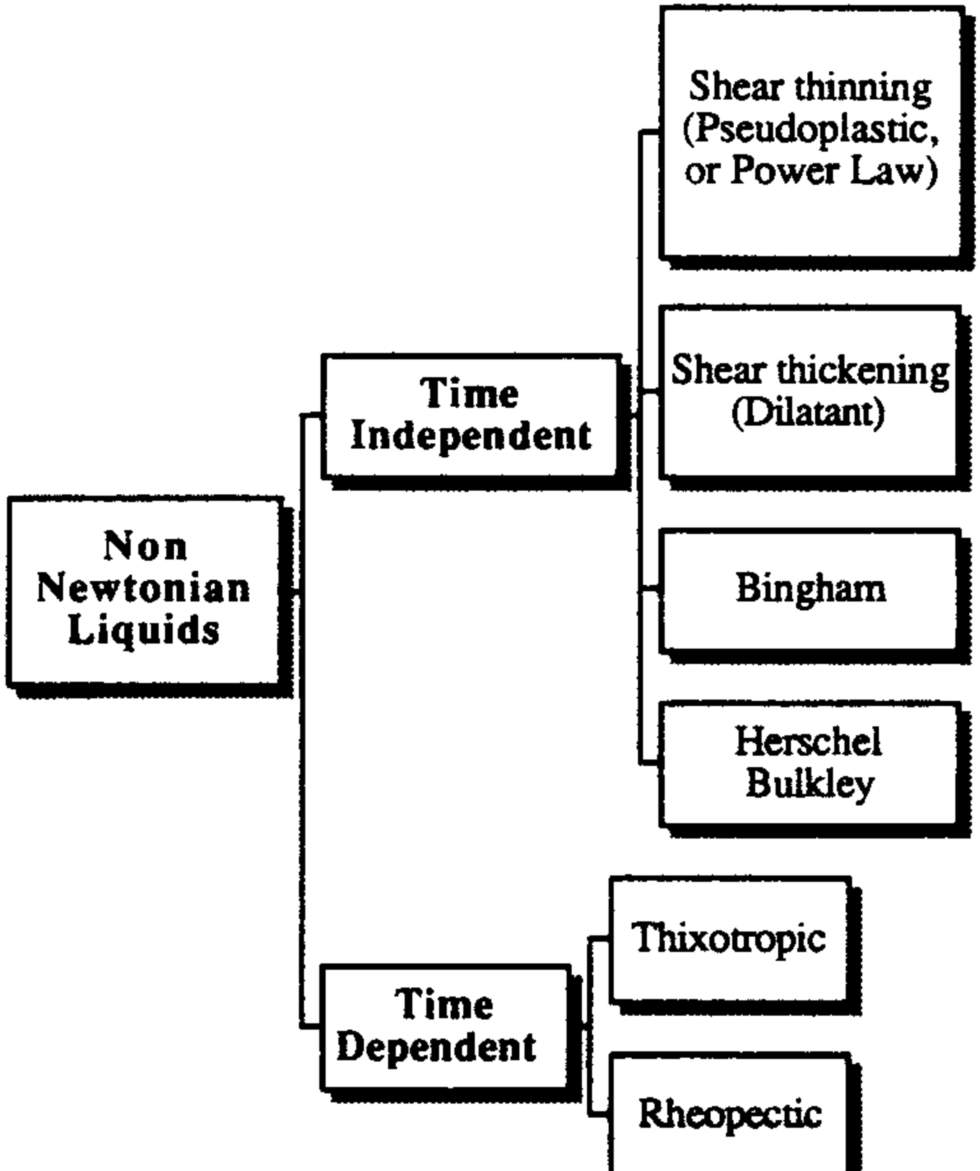
  (Fluid starts 'thinning' with decrease in its apparent viscosity)

**Note:**

1. For some non-Newtonians fluids, viscosity or apparent viscosity may be time-dependent. Such fluids are also called 'Memory' fluids.
2. Rheology is a science of studying flow and behavior of polymeric fluids.

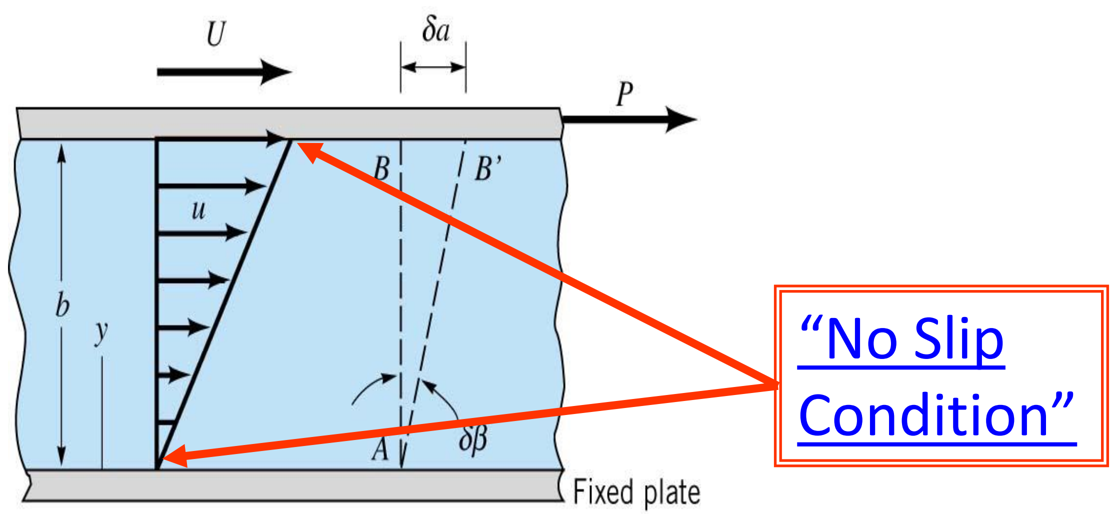
# Non-Newtonian Fluids


# Non-Newtonian Fluids



# No slip condition

**No-slip condition** for viscous fluids states that at a solid boundary, the fluid will have zero velocity relative to the boundary.



# Compressibility

- Compressibility of any substance is the measure of its change in volume under the action of external forces.
- The normal compressive stress on any fluid element at rest is known as hydrostatic pressure p and arises as a result of innumerable molecular collisions in the entire fluid.

The degree of compressibility of a substance is characterized by the bulk modulus of elasticity E defined as

$$E = \lim_{\Delta\forall \to 0} \left( \frac{-\Delta p}{\Delta\forall / \forall} \right)$$

Where $\Delta\forall$ and $\Delta p$ are the changes in the volume and pressure respectively, and $\forall$ is the initial volume. The negative sign (-sign) is included to make E positive, since increase in pressure would decrease the volume i.e for $\Delta p>0$ , $\Delta\forall<0$) in volume.

- For a given mass of a substance, the change in its volume and density satisfies the relation

$$\Delta m = 0, \quad \Delta(\rho\forall) = 0$$

$$\frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho}$$

using $E = \lim_{\Delta\forall \to 0}\left(\frac{-\Delta p}{\Delta\forall / \forall}\right)$ & $\frac{\Delta\forall}{\forall} = -\frac{\Delta\rho}{\rho}$

we get

$$E = \lim_{\Delta\rho \to 0}\left(\frac{\Delta p}{\Delta\rho / \rho}\right) = \rho\frac{dp}{d\rho}$$

*E* for liquids are very high as compared with those of gases (except at very high pressures). Therefore, liquids are usually termed as incompressible fluids though, in fact, no substance is theoretically incompressible with a value of *E* as $\infty$

For example, the bulk modulus of elasticity for water and air at atmospheric pressure are approximately 2 x $10^6$ kN/m $^2$ and 101 kN/m $^2$ respectively. It indicates that air is about 20,000 times more compressible than water. Hence water can be treated as incompressible.

For gases another characteristic parameter, known as compressibility K, is usually defined , it is the reciprocal of E

$$K = \frac{1}{E} = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right) = -\frac{1}{\forall}\left(\frac{d\forall}{dp}\right)$$

K is often expressed in terms of specific volume $\forall$

# Incompressible and a Compressible Flow

In order to know, if it is necessary to take into account the compressibility of gases in fluid flow problems, we need to consider whether the change in pressure brought about by the fluid motion causes large change in volume or density.

By Bernoulli's equation

$p + (1/2)\rho V^2 =$ constant *(V* being the velocity of flow), change in pressure, $\Delta p$, in a flow field, is of the order of $(1/2)\rho V^2$ (dynamic head).
Invoking this relationship into

$$E = \lim_{\delta\rho \to 0}\left(\frac{\Delta p}{\Delta\rho/\rho}\right) = \rho\frac{dp}{d\rho}$$

$$\frac{\Delta\rho}{\rho} \approx \frac{1}{2}\frac{\rho V^2}{E}$$

**So if $\Delta\rho/\rho$ is very small, the flow of gases can be treated as incompressible with a good degree of approximation.**

$$\boxed{\frac{\Delta\rho}{\rho} \approx \frac{1}{2}\frac{\rho V^2}{E}}$$

$$a = \sqrt{\frac{E}{\rho}}$$

$$\boxed{\frac{\Delta\rho}{\rho} \approx \frac{1}{2}\frac{V^2}{a^2} \approx \frac{1}{2}Ma^2}$$

where, Ma is the ratio of the velocity of flow to the acoustic velocity in the flowing medium at the condition and is known as **Mach number**.

So we can conclude that the compressibility of gas in a flow can be neglected if $\Delta\rho/\rho$ is considerably smaller than unity, i.e. $(1/2)Ma^2 \ll 1$.

In other words, if the flow velocity is small as compared to the local acoustic velocity, compressibility of gases can be neglected. **Considering a maximum relative change in density of 5 per cent as the criterion of an incompressible flow, the upper limit of Mach number becomes approximately 0.33**. In the case of air at standard pressure and temperature, the acoustic velocity is about 335.28 m/s. Hence a Mach number of 0.33 corresponds to a velocity of about 110 m/s. Therefore flow of air up to a velocity of 110 m/s under standard condition can be considered as incompressible flow.

# Compressibility of Fluids: Speed of Sound

A consequence of the compressibility of fluids is that small disturbances introduced at a point propagate at a finite velocity.  Pressure disturbances in the fluid propagate as sound, and their velocity is known as the speed of sound or the acoustic velocity, c.

$$c = \sqrt{\frac{dp}{d\rho}} \quad or \quad c = \sqrt{\frac{E_v}{\rho}}$$

**Isentropic Process (frictionless, no heat exchange because):**

$$c = \sqrt{\frac{kp}{\rho}}$$

**Ideal Gas and Isentropic Process:**

$$c = \sqrt{kRT}$$

## Compressibility of Fluids: Speed of Sound

Example (Continued):

$$Ratio = \frac{V}{c}$$

$$Ratio = \frac{250 \ m/s}{296.6 \ m/s}$$

$$Ratio = 0.84$$

- The above ratio is known as the Mach Number, Ma
- For Ma < 1 ***Subsonic Flow***
- For Ma > 1 ***Supersonic Flow***

For Ma > 1 we see shock waves and "sonic booms":

1) Wind Tunnel Visualization known as Schlieren method
2) Condensation instigated from jet speed allowing us to see a shock wave

# End of Lecture 3