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MEE1004 – Fluid Mechanics

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French engineer Henry Philibert Gaspard Darcy in 1845 and German engineer and scientist Julius Weisbach in 1854, after much experimental work, first proposed a friction factor equation that expressed the pressure loss in a piping system in terms of velocity head. In 1911, Paul Richard Heinich Blasius, a student of Ludwig Prandtl (1875–1953), demonstrated that for smooth-walled pipes the friction factor was dependent only on the Reynolds number. Blasius produced the first plot of its kind with friction factor versus Reynolds number for the empirical relationship

$f = 0.079\, Re^{-\frac{1}{4}}$ valid for Reynolds numbers between 4×10^3 and 1×10^5.

$$f = a + b\, Re^{c}$$

	a	b	c	Validity
Lees (1924)	1.8×10^{-3}	0.152	–0.35	$4\times10^3<Re<4\times10^5$
Hermann (1930)	1.35×10^{-3}	0.099	–0.3	$4\times10^3<Re<2\times10^6$
Nikuradse (1932)	8×10^{-4}	0.055	–0.237	$4\times10^3<Re<3.2\times10^6$

To predict the friction of fluids in smooth pipes with turbulent flow, Prandtl developed a number of empirical models based on boundary layer theory, mixing length and wall effects. These models led to Prandtl's universal resistance equation for turbulent flow in smooth pipes given by

$$\frac{1}{\sqrt{f}} = 4 \log_{10}(Re\sqrt{f}) - 0.4$$

valid for Reynolds numbers between 5×10^3 and 3.4×10^6.

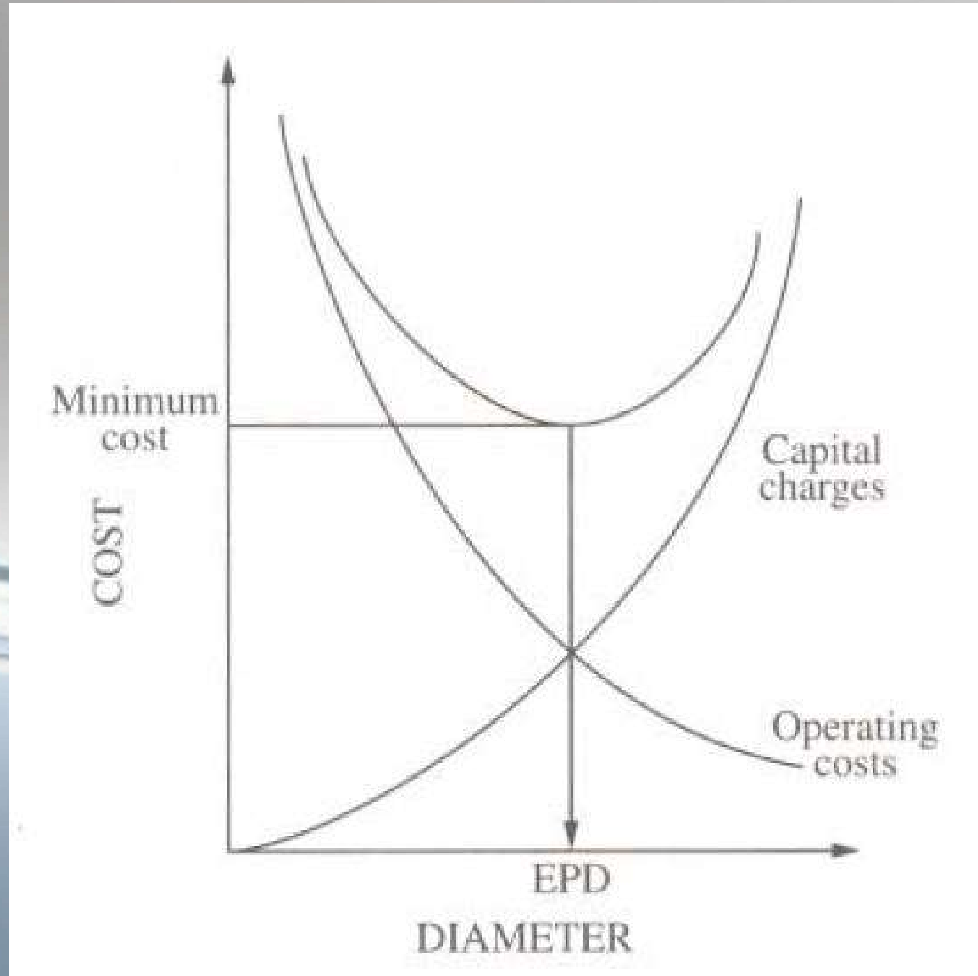
Theodore von Kármán (1881–1963), a co-worker of Prandtl, developed an empirical relationship for turbulent flow through rough-walled pipes in the modified form

$$\frac{1}{\sqrt{f}} = 2 \log_{10}\left(\frac{d}{\varepsilon}\right) + 2.28$$

Economic pipe diameter

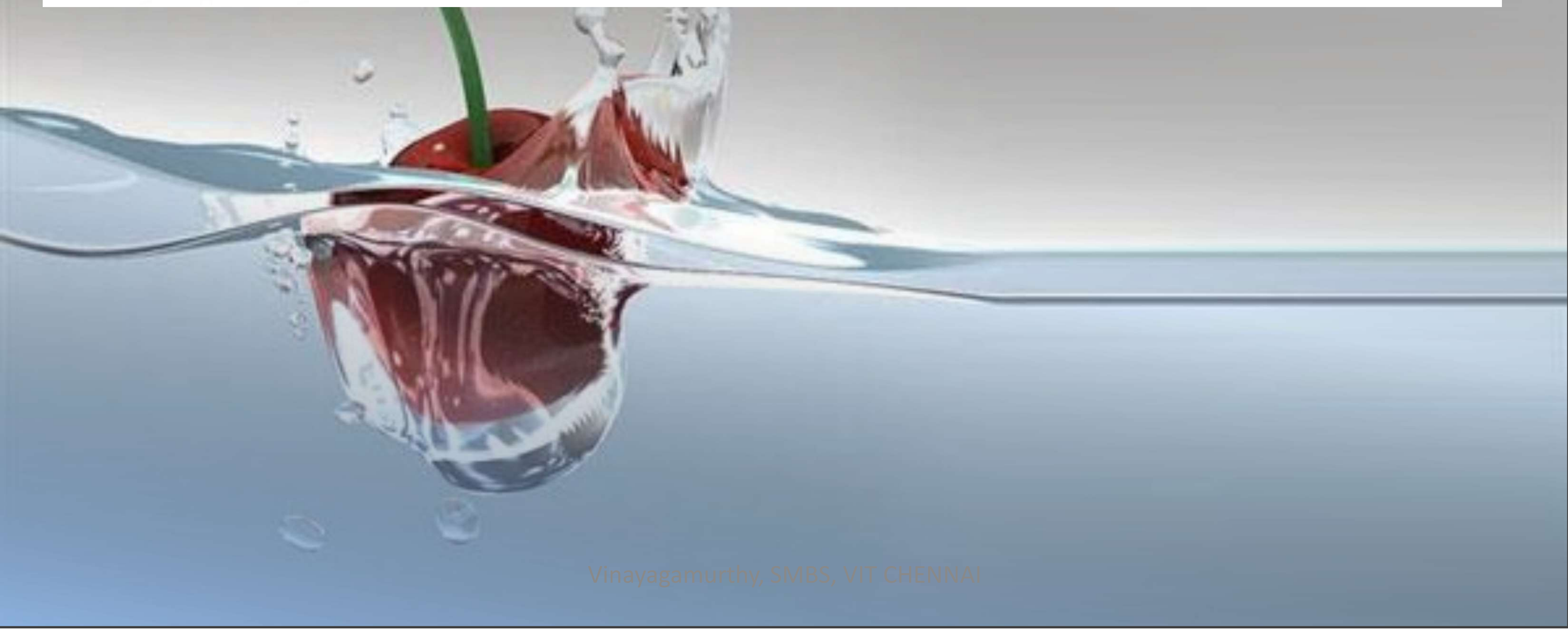
The economic pipe diameter is the diameter of pipe which gives the minimum overall cost for any specific flowrate.

by selecting a reasonable fluid velocity which provides a reasonable pressure drop and is virtually independent of diameter.

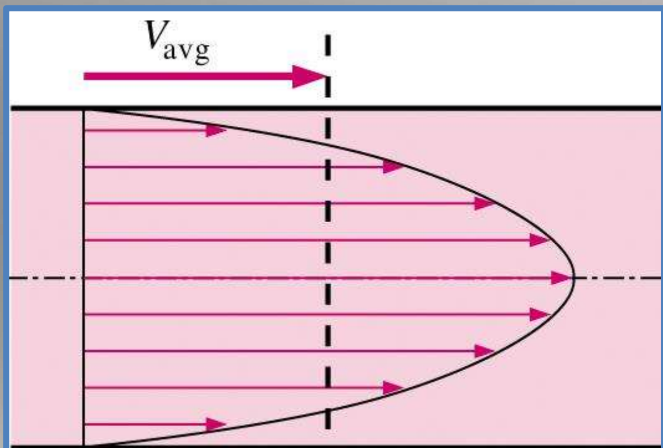


The minimum investment is calculated for expensive or exotic pipe materials such as alloys, pipelines larger than 300 mm in diameter and carbon steel lines with a large number of valves and fittings. The pipe scheduling is selected by determining either the inner or outer diameter and the pipe wall thickness. The minimum wall thickness is a function of allowable stress of the pipe material, diameter, design pressure, and corrosion and erosion rates.

In the case of highly viscous liquids, pipelines are rarely sized on economic considerations.



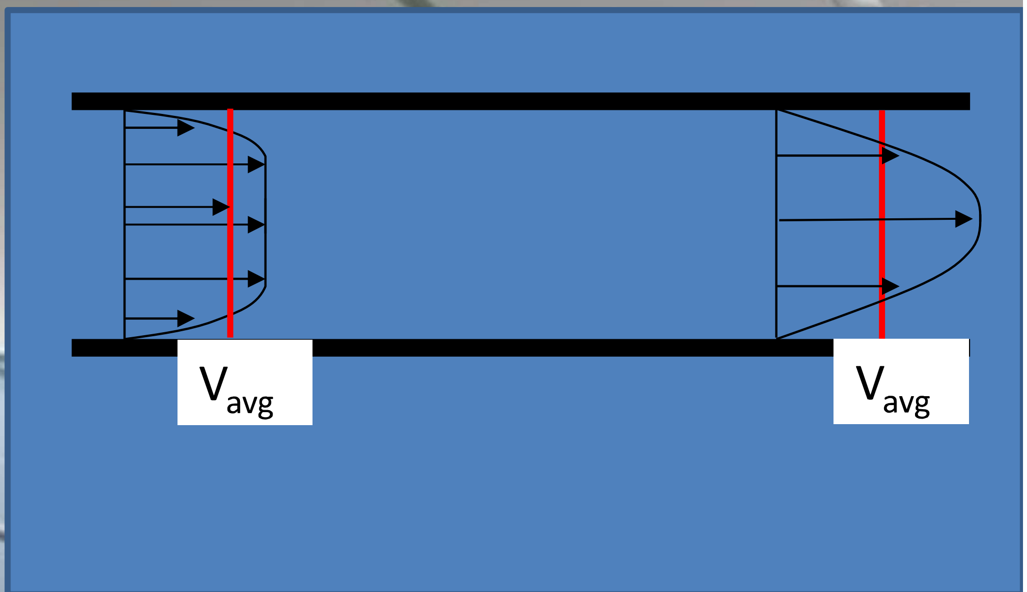
Introduction



Friction force of wall on fluid

- Average velocity in a pipe
  - Recall - because of the <u>no-slip condition</u>, the velocity at the walls of a pipe or duct flow is zero
  - We are often interested only in V_{avg}, which we usually call just V (drop the subscript for convenience)
  - Keep in mind that the no-slip condition causes shear stress and <u>friction</u> along the pipe walls

Introduction



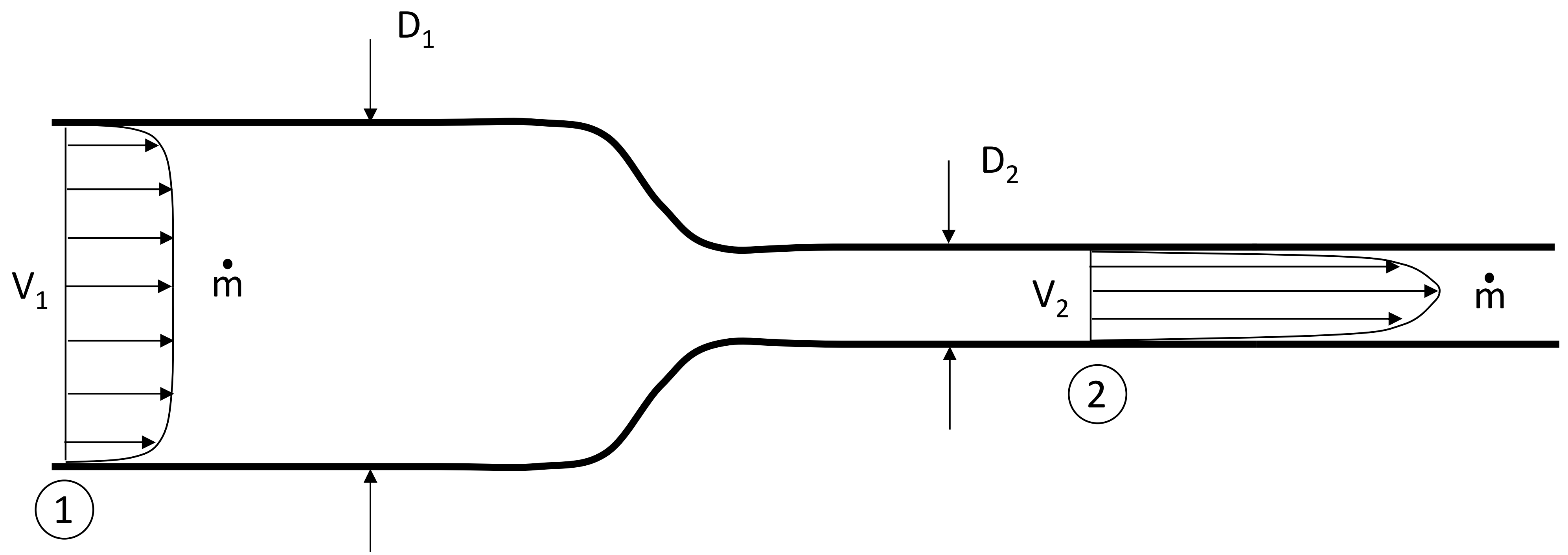
- For pipes of constant diameter and incompressible flow
  - V_{avg} stays the same down the pipe, even if the velocity profile changes
    - Why? Conservation of Mass

$$\dot{m} = \rho V_{avg} A = constant$$

same
same
same

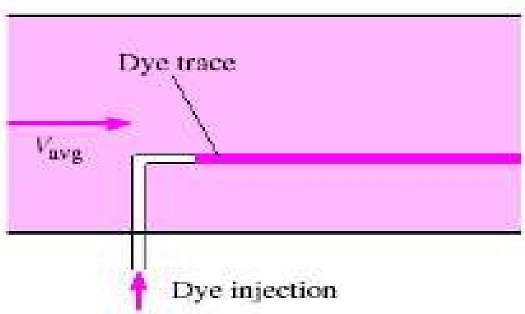
Introduction

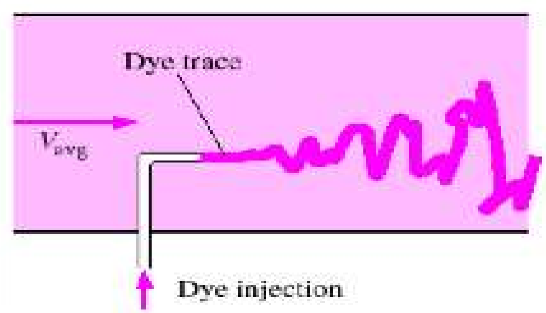
- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $V_1 \neq V_2$



Laminar and Turbulent Flows

Laminar Flow	Turbulent Flow
Can be steady or unsteady.	Is always *unsteady*.
(Steady means that the flow field at any instant in time is the same as at any other instant in time.)	Why? There are always random, swirling motions (vortices or eddies) in a turbulent flow.
	Note: However, a turbulent flow can be steady *in the mean*. We call this a *stationary turbulent flow*.
Can be one-, two-, or three-dimensional.	Is always *three-dimensional*.
	Why? Again because of the random swirling eddies, which are in all directions.
	Note: However, a turbulent flow can be 1-D or 2-D *in the mean*.
Has regular, *predictable* behavior	Has irregular or *chaotic* behavior (cannot predict exactly – there is some randomness associated with any turbulent flow.

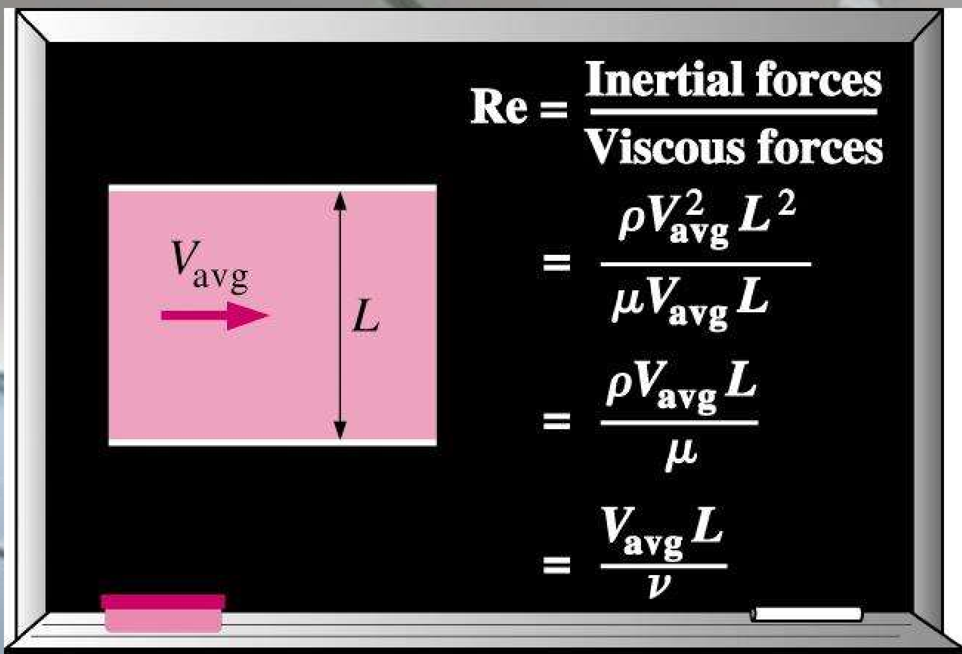




Laminar Flow	Turbulent Flow
Analytical solutions are possible (see Chapter 9).	No analytical solutions exist! (It is too complicated, again because of the 3-D, unsteady, chaotic swirling eddies.)
Occurs at *low* Reynolds numbers.	Occurs at *high* Reynolds numbers.

Laminar and Turbulent Flows

Definition of Reynolds number



- Critical Reynolds number (Re_{cr}) for flow in a round pipe

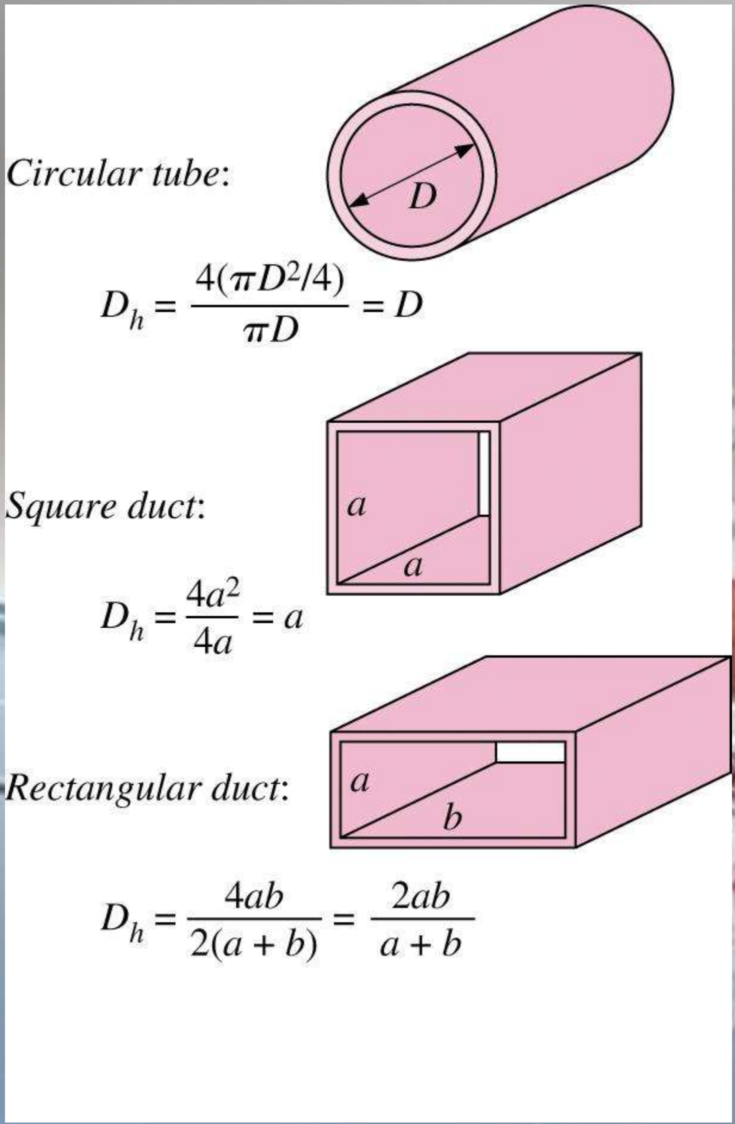
  $Re < 2300 \Rightarrow$ laminar

  $2300 \leq Re \leq 4000 \Rightarrow$ transitional

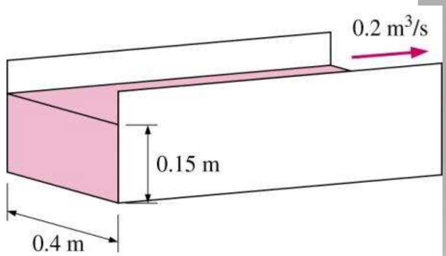
  $Re > 4000 \Rightarrow$ turbulent

- Note that these values are approximate.
- For a given application, Recr depends upon
  - Pipe roughness
  - Vibrations
  - Upstream fluctuations, disturbances (valves, elbows, etc. that may disturb the flow)

Laminar and Turbulent Flows



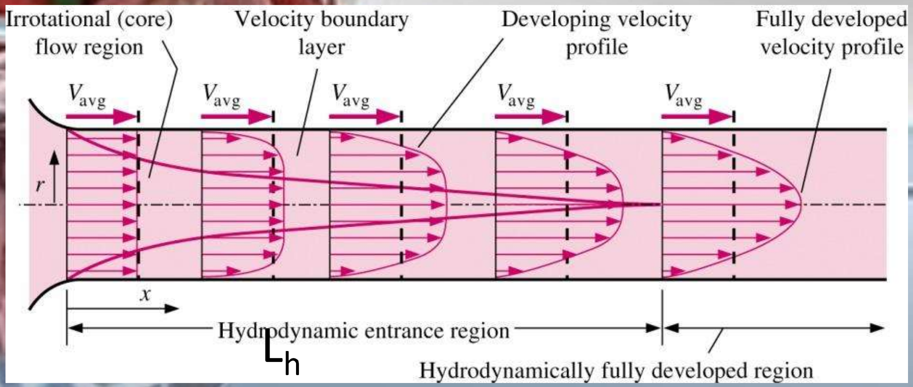
- For <u>non-round</u> pipes, define the hydraulic diameter
  $D_h = 4A_c/P$
  A_c = cross-section area
  P = wetted perimeter



- Example: open channel
  $A_c = 0.15 * 0.4 = 0.06\text{m}^2$
  $P = 0.15 + 0.15 + 0.5 = 0.8\text{m}$
  Don't count free surface, since it does not contribute to friction along pipe walls!
  $D_h = 4A_c/P = 4*0.06/0.8 = 0.3\text{m}$
  <u>What does it mean?</u> This channel flow is equivalent to a round pipe of diameter 0.3m (approximately).

The Entrance Region

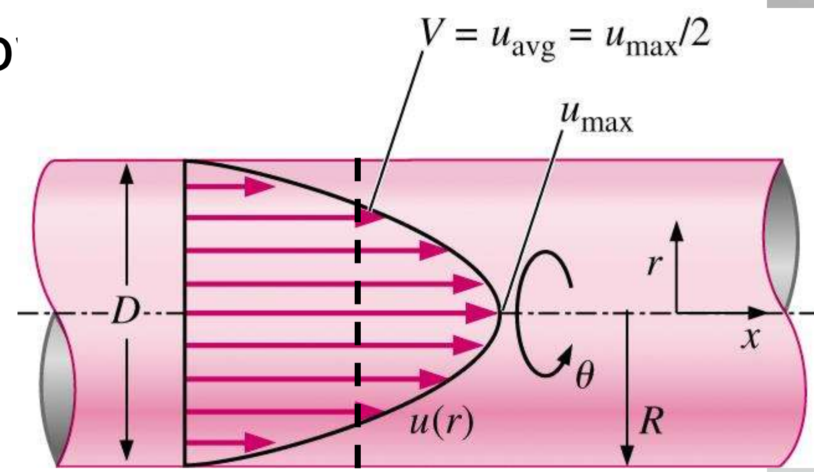
- Consider a round pipe of diameter D. The flow can be laminar or turbulent. In either case, the profile develops downstream over several diameters called the *entry length* L_h. L_h/D is a function of Re.



Fully Developed Pipe Flow

- Comparison of laminar and turbulent flow

There are some major differences between laminar and turbulent fully developed pipe flo



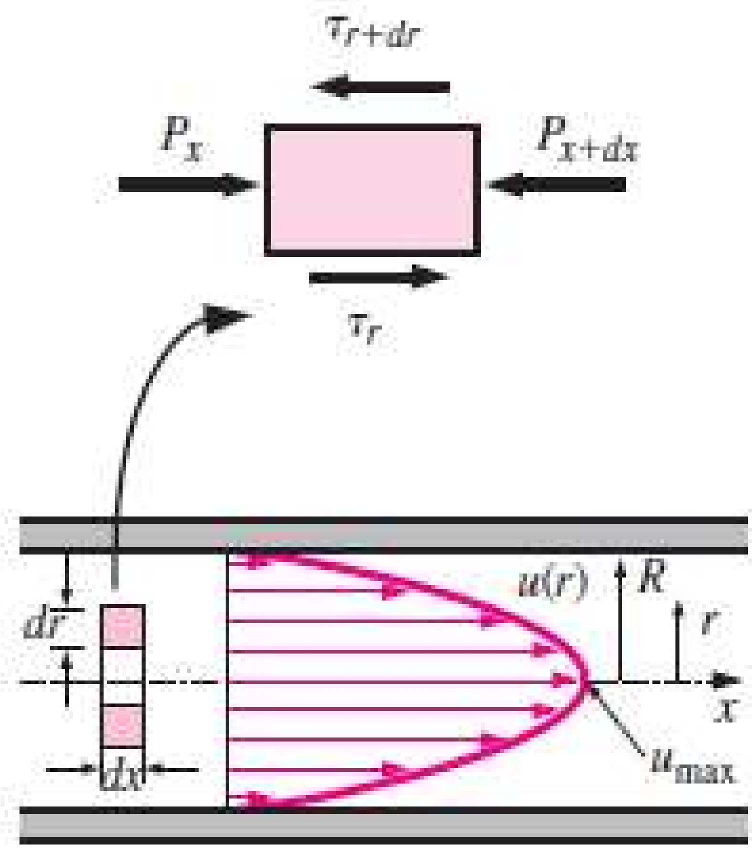
Laminar

- Can solve exactly (Chapter 9)
- Flow is steady
- Velocity profile is parabolic
- Pipe roughness not important

It turns out that $V_{avg} = 1/2U_{max}$ and $u(r)= 2V_{avg}(1 - r^2/R^2)$

LAMINAR FLOW IN PIPES

Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.



A force balance on the volume element in the flow direction gives

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

LAMINAR FLOW IN PIPES

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

Dividing by $2\pi dr dx$
$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

Taking the limit as $dr, dx \to 0$ gives

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

Substituting $\tau = -\mu(du/dr)$ and taking μ = constant gives the desired equation,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} \longrightarrow \quad 4\text{-}1$$

The quantity du/dr is negative in pipe flow, and the negative sign is included to obtain positive values for τ. (Or, $du/dr = -du/dy$ since $y = R - r$.) left side of Eq. 4-1 is a function of r, and the right side is a function of x.

The equality must hold for any value of r and x, and an equality of the form $f(r) = g(x)$ can be satisfied only if both $f(r)$ and $g(x)$ are equal to the same constant. Thus we conclude that dP/dx = constant.

writing a force balance on a volume element of radius R and thickness dx (a slice of the pipe), which gives

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Here τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, dP/dx = constant.

Equation *4-1* can be solved by rearranging and integrating it twice to give

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} \longrightarrow \quad 4\text{-}1$$

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

The velocity profile $u(r)$ is obtained by applying the boundary conditions $\partial u/\partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $u = 0$ at $r = R$ (the no-slip condition at the pipe surface). We get

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \longrightarrow \quad 4\text{-}2$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is *parabolic* with a maximum at the centerline and minimum (zero) at the pipe wall. Also, the axial velocity u is positive for any r, and thus the axial pressure gradient dP/dx must be negative (i.e., pressure must decrease in the flow direction because of viscous effects).

The average velocity is determined from its definition by substituting Eq. 4-2 into Eq. 4-1 and performing the integration. It gives

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \quad \text{4-3}$$

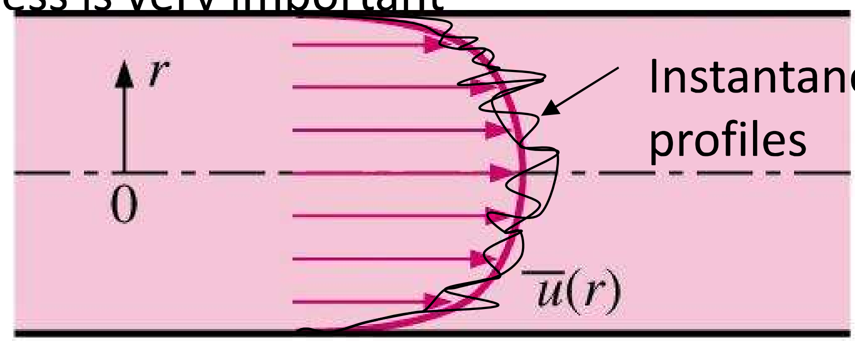
Combining the last two equations, the velocity profile is rewritten as

$$u(r) = 2V_{avg}\left(1-\frac{r^2}{R^2}\right)$$

Fully Developed Pipe Flow

Turbulent

- *Canno*t solve exactly (too complex)
- Flow is unsteady (3D swirling eddies), but it is steady in the mean
- Mean velocity profile is fuller (shape more like a top-hat profile, with very sharp slope at the wall)
- Pipe roughness is very important



- V_{avg} 85% of U_{max} (depends on Re a bit)
- No analytical solution, but there are some good semi-empirical expressions that approximate the velocity profile shape. See text

  Logarithmic law (Eq. 8-46)

  Power law (Eq. 8-49)

Fully Developed Pipe Flow Wall-shear stress

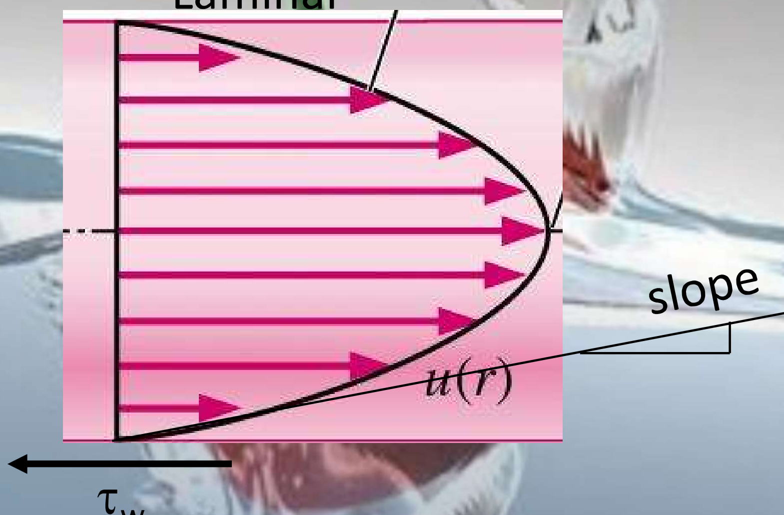
- Recall, for simple shear flows *u=u(y)*, we had

$$\tau = \mu du/dy$$

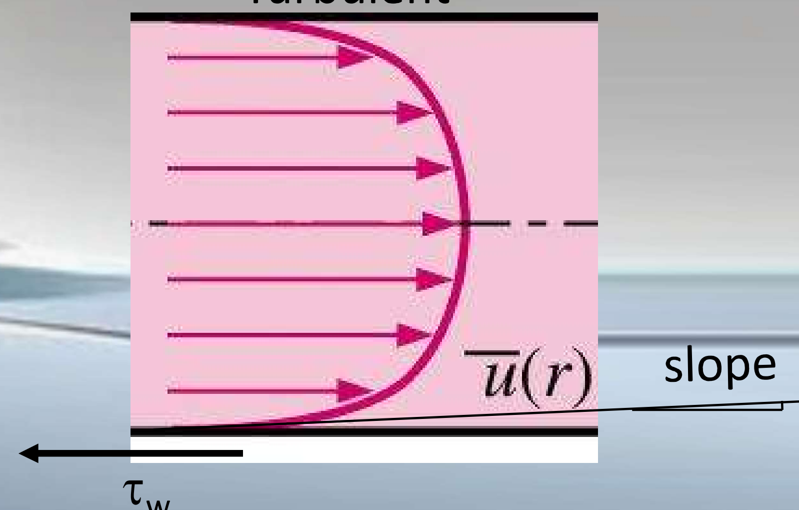
- In fully developed pipe flow, it turns out that

$$\tau = \mu du/dr$$

Laminar



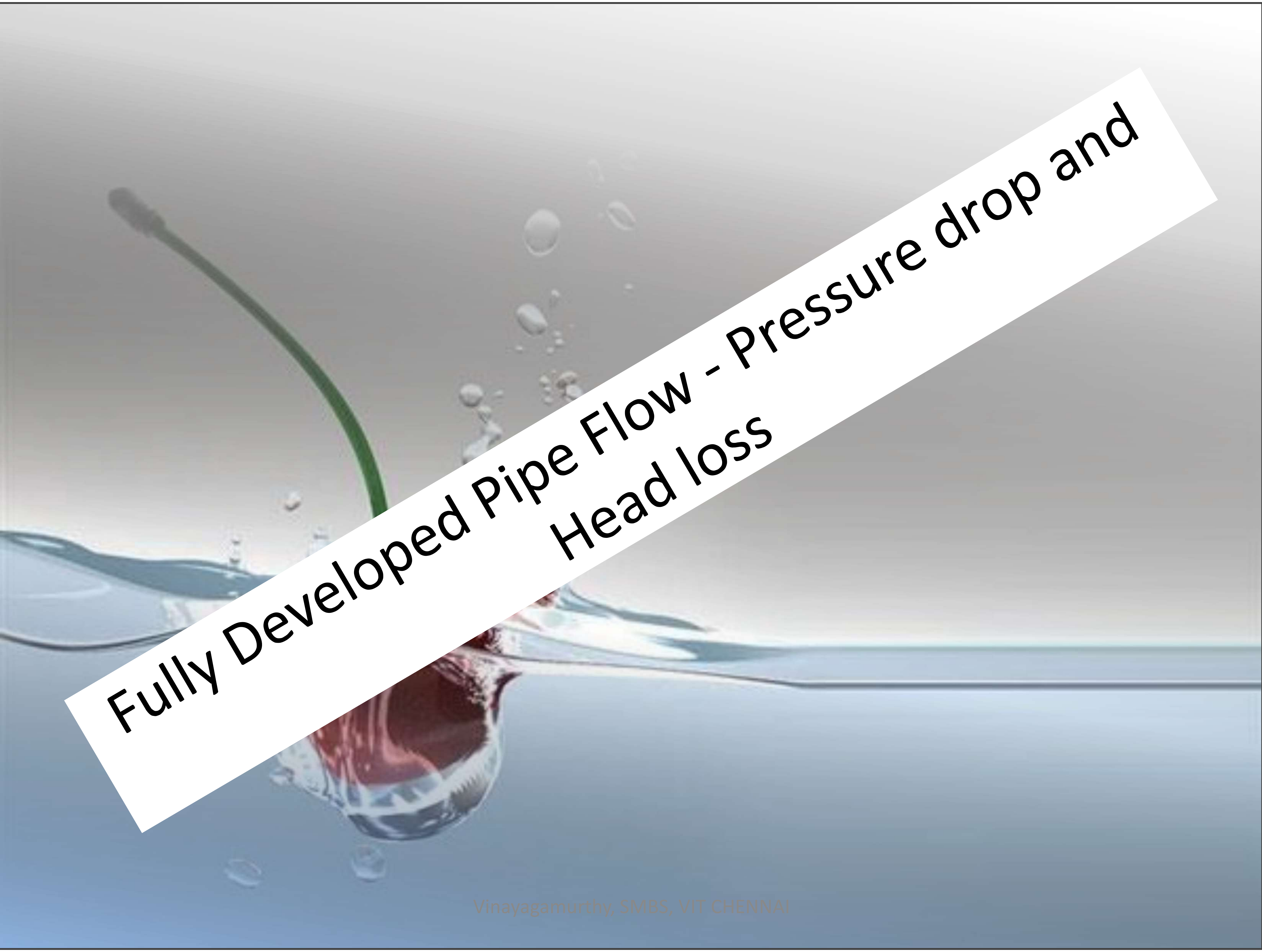
Turbulent



τ_w = shear stress at the wall, acting on the fluid

$\tau_{w,turb} > \tau_{w,lam}$

Fully Developed Pipe Flow - Pressure drop and Head loss



A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP sinc... mai... whe...

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \longrightarrow \quad 4\text{-}4$$

Substituting Eq. *4-4* into the V_{avg} expression in Eq. *4-3*, the pressure drop can be expressed as

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

The symbol Δ is typically used to indicate the difference between the final and initial values, like $\Delta y = y_2 - y_1$. But in fluid flow, ΔP is used to designate pressure drop, and thus it is $P_1 - P_2$. A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss** ΔP_L to emphasize that it is a *loss* (just like the head loss h_L, which is proportional to it).

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

where $\rho V_{avg}^2/2$ is the *dynamic pressure* and f is the **Darcy friction factor,**

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$ **Darcy–Weisbach friction factor,**

In practice, it is found convenient to express the pressure loss for all types of fully developed internal flows (laminar or turbulent flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

It should not be confused with the *friction coefficient Cf* [also called the *Fanning friction factor*, named after the American engineer John Fanning (1837–1911)], which is defined as $C_f = 2\tau_w/(\rho V_{avg}^2) = f/4.$

$C_f = 2\tau_w/(\rho V_{avg}^2) = f/4.$

Laminar flow: $$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

Pressure loss: $$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} \qquad f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Circular pipe, laminar: $$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

This equation shows that *in laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.*

In the analysis of piping systems, pressure losses are commonly expressed in terms of the *equivalent fluid column height*, called the **head loss** h_L. Noting from fluid statics that $\Delta P = \rho g h$ and thus a pressure difference of ΔP corresponds to a fluid height of $h = \Delta P/\rho g$, the *pipe head loss* is obtained by dividing ΔP_L by ρg to give

Head loss: $$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

Head loss: $$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

Once the pressure loss (or head loss) is known, the required pumping power *to overcome the pressure loss* is determined from

$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

where $\dot{V}$ is the volume flow rate and $\dot{m}$ is the mass flow rate.

The average velocity for laminar flow in a horizontal pipe is, from Eq. 8–20,

Horizontal pipe: $$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

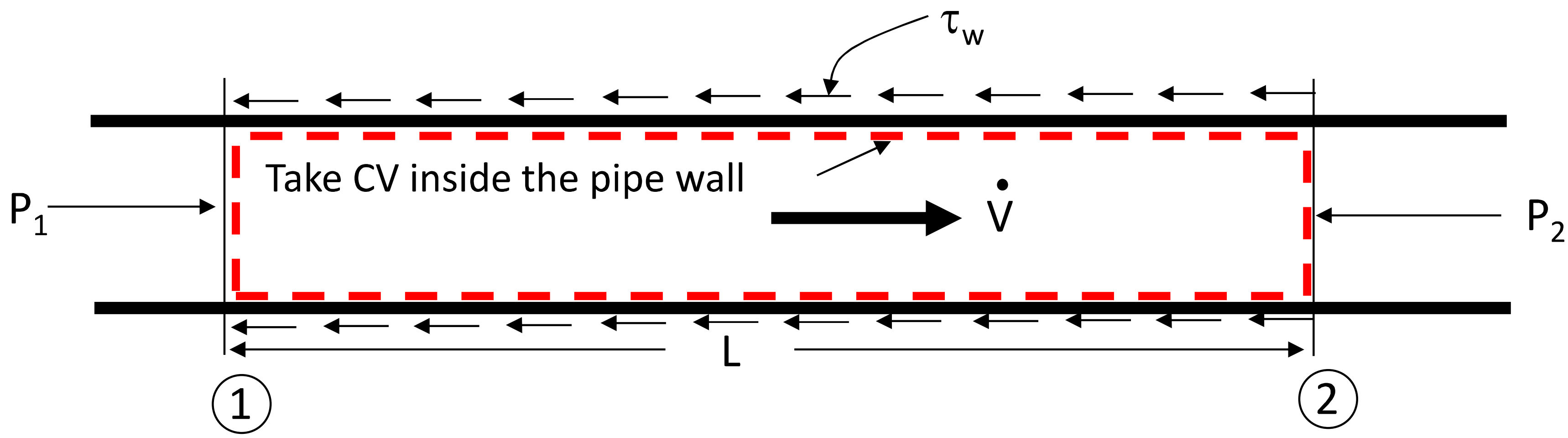
Then the volume flow rate for laminar flow through a horizontal pipe of diameter D and length L becomes

$$\dot{V} = V_{avg}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

This equation is known as **Poiseuille's law,** and this flow is called *Hagen–Poiseuille flow*

Fully Developed Pipe Flow - Pressure drop and Head loss

- There is a direct connection between the pressure drop in a pipe and the shear stress at the wall
- Consider a horizontal pipe, fully developed, and incompressible flow



- Let's apply conservation of mass, momentum, and energy to this CV (good review problem!)

Fully Developed Pipe Flow
Pressure drop

- <u>Conservation of Mass</u>

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\rho \dot{V}_1 = \rho \dot{V}_2 \rightarrow \dot{V} = const$$

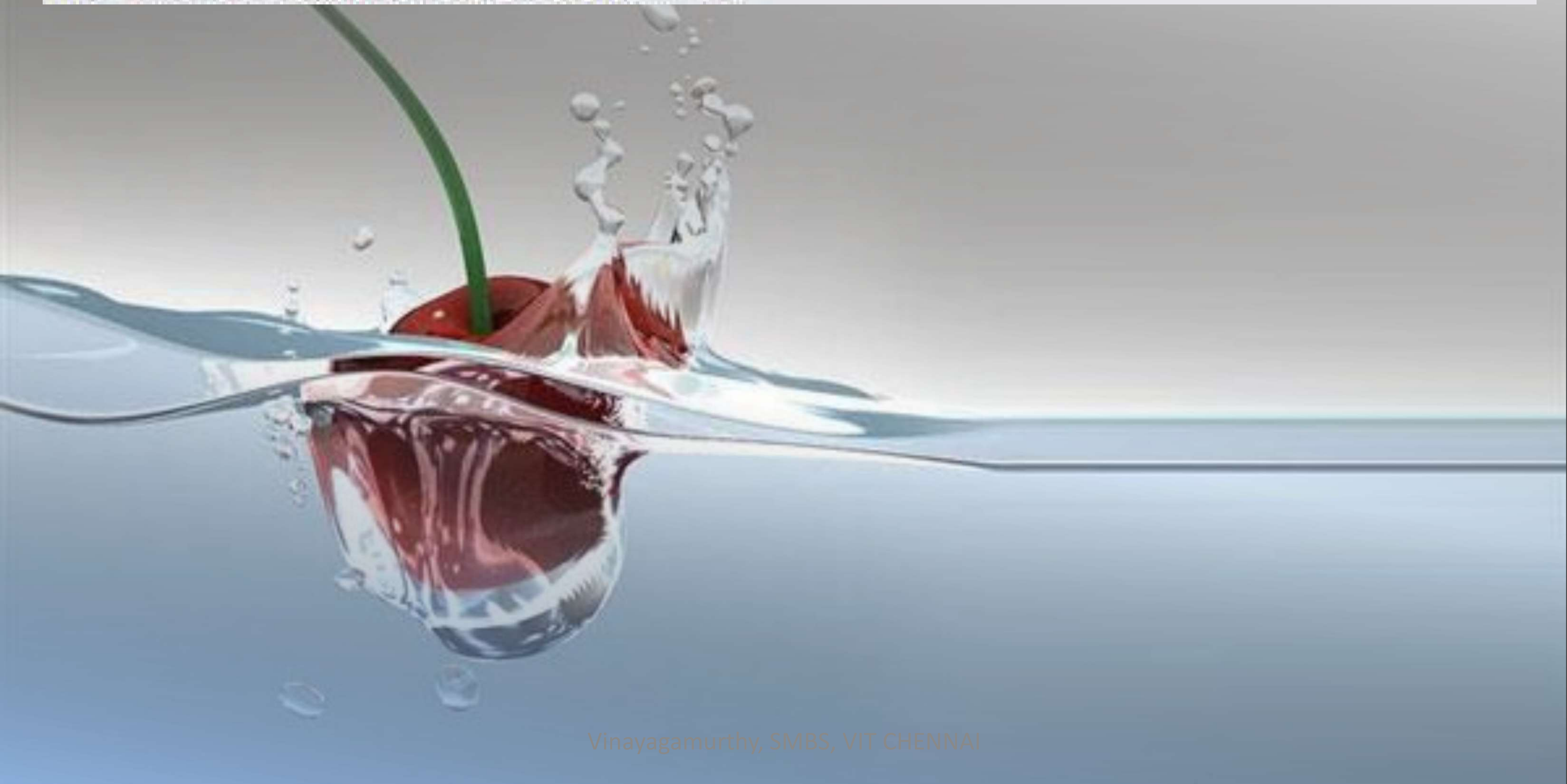
$$V_1 \frac{\pi D^2}{4} = V_2 \frac{\pi D^2}{4} \rightarrow \boxed{V_1 = V_2}$$

- <u>Conservation of x-momentum</u>

$$\sum F_x = \cancel{\sum F_{x,grav}} + \sum F_{x,press} + \sum F_{x,visc} + \cancel{\sum F_{x,other}} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$P_1 \frac{\pi D^2}{4} - P_2 \frac{\pi D^2}{4} - \tau_w \pi D L = \underbrace{\cancel{\beta_2 \dot{m} V_2} - \cancel{\beta_1 \dot{m} V_1}}_{\text{Terms cancel since } \beta_1 = \beta_2 \text{ and } V_1 = V_2}$$

Oil at 20°C (ρ = 888 kg/m^3 and μ = 0.800 kg/m · s) is flowing steadily through a 5-cm-diameter 40-m-long pipe (Fig. 8–17). The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15° upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar.



THANK YOU !!!

Fully Developed Pipe Flow
Pressure drop

- Thus, x-momentum reduces to

$$(P_1 - P_2)\frac{\pi D^2}{4} = \tau_w \pi D L \qquad \text{or} \qquad \boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$

- Energy equation (in head form)

$$\frac{P_1}{\rho g} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_{pump,u}} = \frac{P_2}{\rho g} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_{turbine,e}} + h_L$$

cancel (horizontal pipe)

Velocity terms cancel again because $V_1 = V_2$, and $\alpha_1 = \alpha_2$ (shape not changing)

$$\boxed{P_1 - P_2 = \rho g h_L}$$

h_L = irreversible head loss & it is felt as a pressure drop in the pipe

Fully Developed Pipe Flow
Friction Factor

- From momentum CV analysis

$$\boxed{P_1 - P_2 = 4\tau_w \frac{L}{D}}$$

- From energy CV analysis

$$\boxed{P_1 - P_2 = \rho g h_L}$$

- Equating the two gives

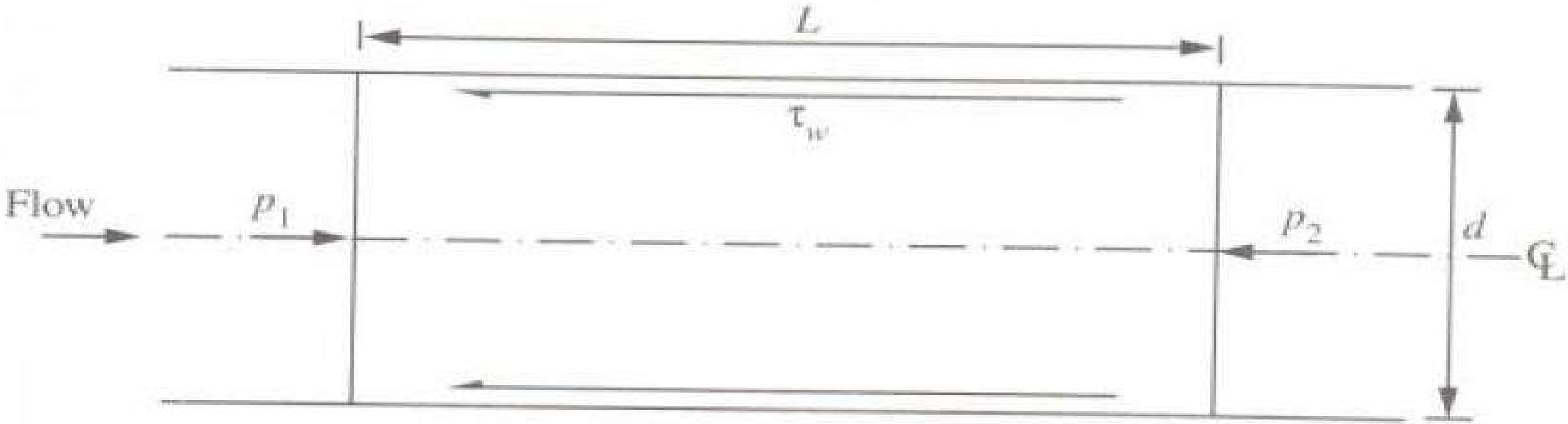
$$4\tau_w \frac{L}{D} = \rho g h_L \qquad \boxed{h_L = \frac{4\tau_w}{\rho g}\frac{L}{D}}$$

- To predict head loss, we need to be able to calculate τ_w. How?
  - Laminar flow: solve exactly
  - Turbulent flow: rely on empirical data (experiments)
  - In either case, we can benefit from dimensional analysis!

Head loss due to friction

Only for circular profiles……

Derive an equation for the pressure and head loss due to friction for a fluid flowing through a pipe of length L and inside diameter d.



Let the pressure difference or drop be solely due to friction manifest as a wall shear stress, τ_w. For steady state conditions, a force balance on the fluid in the cross-section of the pipe is

$$p_1 \frac{\pi d^2}{4} - p_2 \frac{\pi d^2}{4} = \Delta p_f \frac{\pi d^2}{4}$$

$$= \tau_w \pi L d$$

The pressure drop due to friction is therefore

$$\Delta p_f = 4\tau_w \frac{L}{d}$$

If the wall shear stress is related to the kinetic energy per volume, then

$$\tau_w = \frac{f}{2}\rho v^2$$

where f is the Fanning friction factor. The pressure drop due to friction may therefore be expressed as

$$\Delta p_f = \frac{2f\rho v^2 L}{d}$$

or in head form

$$H_f = \frac{4fL}{d}\frac{v^2}{2g}$$

This is known as the Fanning or Darcy equation.

This is known as the Fanning or Darcy equation. It was much earlier, however, that Darcy in 1845 and Weisbach in 1854 first proposed the friction factor equation after much experimental work, giving rise to the head loss due to friction in the form

$$H_f = \frac{\lambda L}{d} \frac{v^2}{2g}$$

This is known as the Darcy-Weisbach equation where the friction factor is related to the Fanning friction factor by

$$\lambda = 4f$$