Lecture 4: BMEE204L
Fluid Mechanics & Machinery

- Surface Tension
- Capillarity

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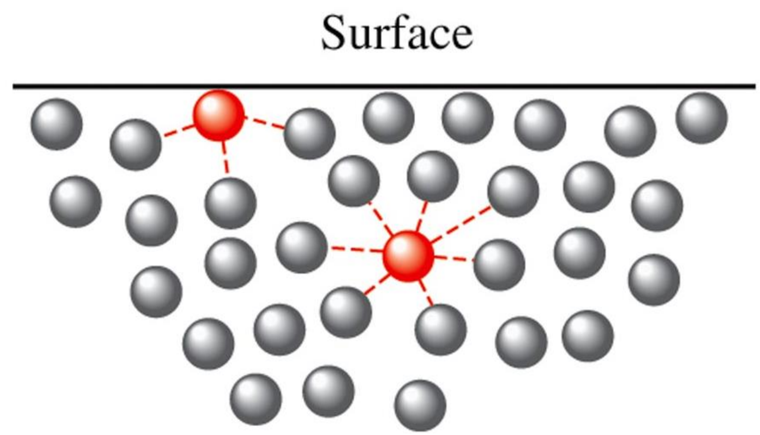
Surface Tension

- Surface molecules are not involved in all possible intermolecular bonding
- Requires energy to go the surface, so liquid resists increases in surface area
- The higher the intermolecular forces, the higher the surface tension

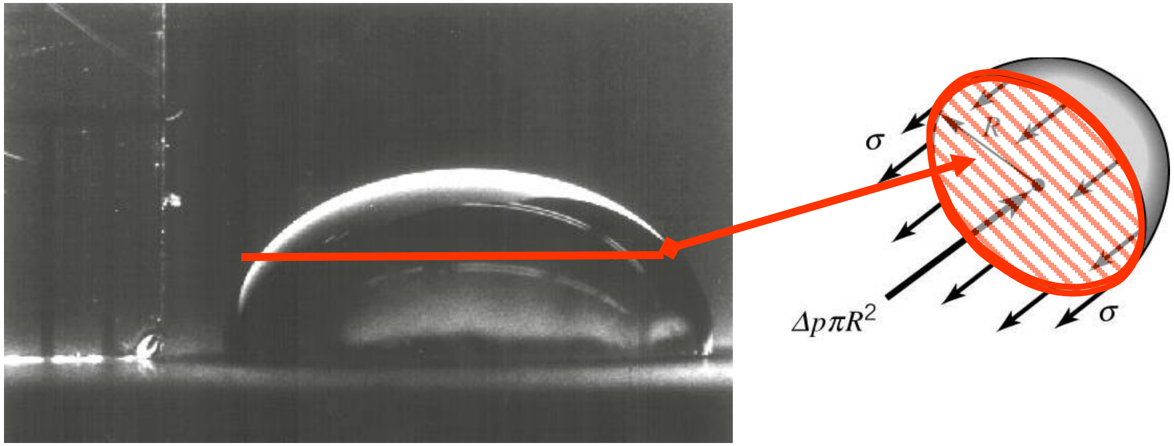


Surface Tension: Liquid Drop

The pressure inside a drop of fluid can be calculated using a free-body diagram:

Real Fluid Drops

Mathematical Model



R is the radius of the droplet, σ is the surface tension, Δp is the pressure difference between the inside and outside pressure.

The force developed around the edge due to surface tension along the line:

$$F_{surface} = 2\pi R\sigma$$

Applied to Circumference

This force is balanced by the pressure difference Δp:

$$F_{pressure} = \Delta p \pi R^2$$

Applied to Area

Surface Tension: Liquid Drop

Now, equating the Surface Tension Force to the Pressure Force, we can estimate $\Delta p = p_i - p_e$:

$$\Delta p = \frac{2\sigma}{R}$$

This indicates that the internal pressure in the droplet is greater that the external pressure since the right hand side is entirely positive.

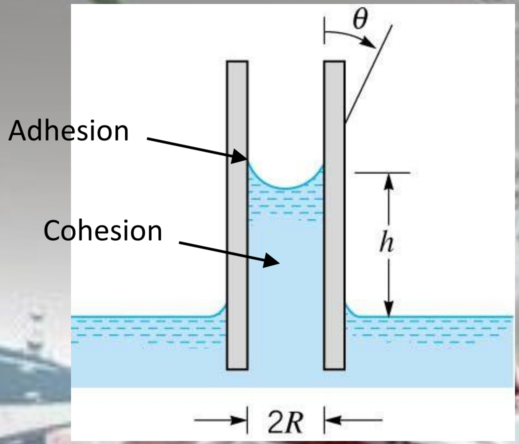
Is the pressure inside a bubble of water greater or less than that of a droplet of water?

Prove to yourself the following result: $$\Delta p = \frac{4\sigma}{R}$$

Surface Tension: Capillary Action

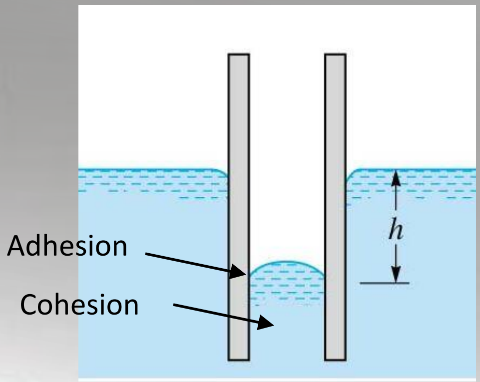
Capillary action in small tubes which involve a liquid-gas-solid interface is caused by surface tension. The fluid is either drawn up the tube or pushed down.

"Wetted"



Adhesion > Cohesion

"Non-Wetted"



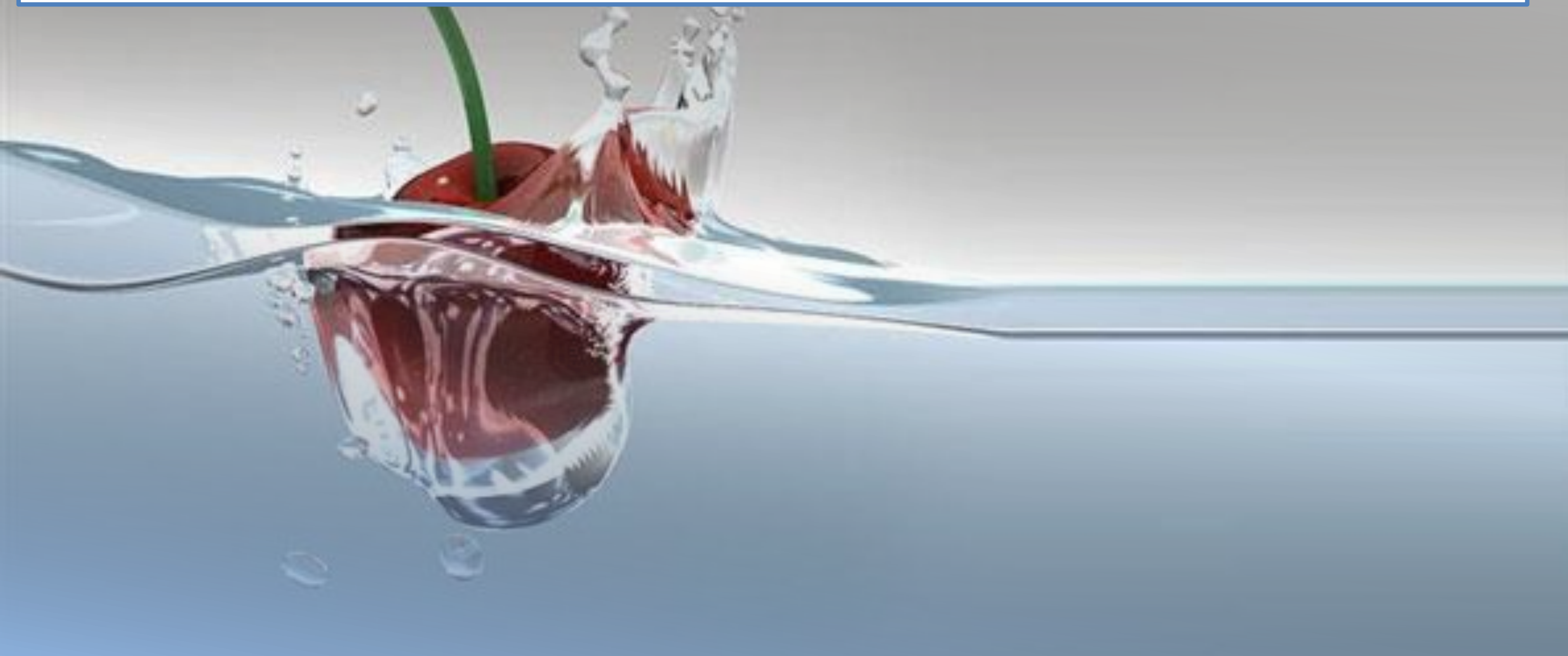
Cohesion > Adhesion

h is the height, R is the radius of the tube, θ is the angle of contact.

The weight of the fluid is balanced with the vertical force caused by surface tension.

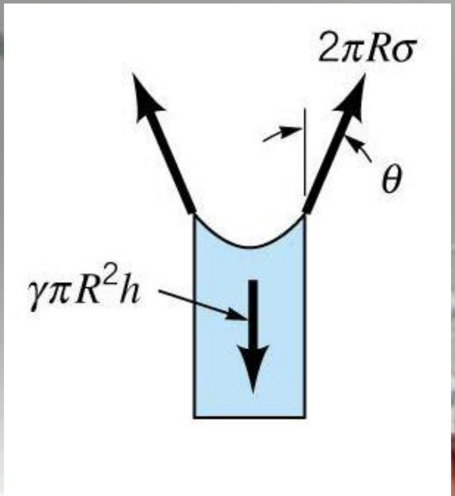
Capillary Action = spontaneous rising of a liquid up a narrow tube

a. Adhesive Forces = polar liquid has intermolecular forces with polar surface
b. Cohesive Forces = intermolecular forces of the liquid for itself
c. Water: Adhesive (H-Bonding) > Cohesive, so concave meniscus
d. Mercury: Cohesive (London) > Adhesive, so convex meniscus



Surface Tension: Capillary Action

Free Body Diagram for Capillary Action for a Wetted Surface:



$$F_{surface} = 2\pi R\sigma\cos\theta$$

$$W = \gamma\pi R^2 h$$

Equating the two and solving for h:

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

For clean glass in contact with water, $\theta \approx 0°$, and thus as R decreases, h increases, giving a higher rise.

For a clean glass in contact with Mercury, $\theta \approx 130°$, and thus h is negative or there is a push down of the fluid.

3. We will use the distance up a capillary the liquid climbs to find Surface Tension

a) We will use the known surface tension of water to find the tube radius

σ = surface tension [kg/s^2]

h = height [m]

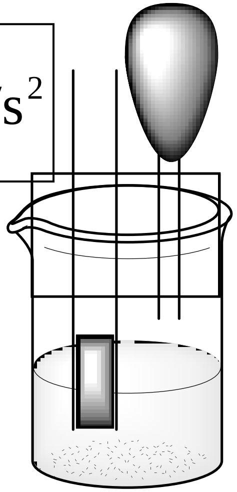
d = density [kg/m^3]

g = accel. gravity = [9.8 m/s^2]

$$\sigma = \frac{\text{rhdg}}{2}$$

$$\Rightarrow \text{r} = \frac{2\sigma}{\text{hdg}} = \frac{2(0.07259\ \text{kg/s}^2)}{(3.29\text{x}10^{-2}\ \text{m})(1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 4.5\text{x}10^{-4}\ \text{m}$$

$$\sigma = \frac{\text{rhdg}}{2} = \frac{(4.5\text{x}10^{-4}\ \text{m})(1.3\text{x}10^{-2}\ \text{m})(790\ \text{kg/m}^3)(9.8\ \text{m/s}^2)}{2} = 0.0226\ \text{kg/s}^2$$



b) We will then find the surface tension of an uknown (ethanol)

Forces on Fluid Elements

Fluid Elements - Definition:

- Fluid element can be defined as an infinitesimal region of the fluid continuum in isolation from its surroundings

Two types of forces exist on fluid elements

- **Body Force**: distributed over the entire mass or volume of the element. It is usually expressed per unit mass of the element or medium upon which the forces act. Example: Gravitational Force, Electromagnetic force fields etc.

Surface Force:

- Forces exerted on the fluid element by its surroundings through direct contact at the surface.

  Surface force has two components:
  - Normal Force: along the normal to the area
  - Shear Force: along the plane of the area.

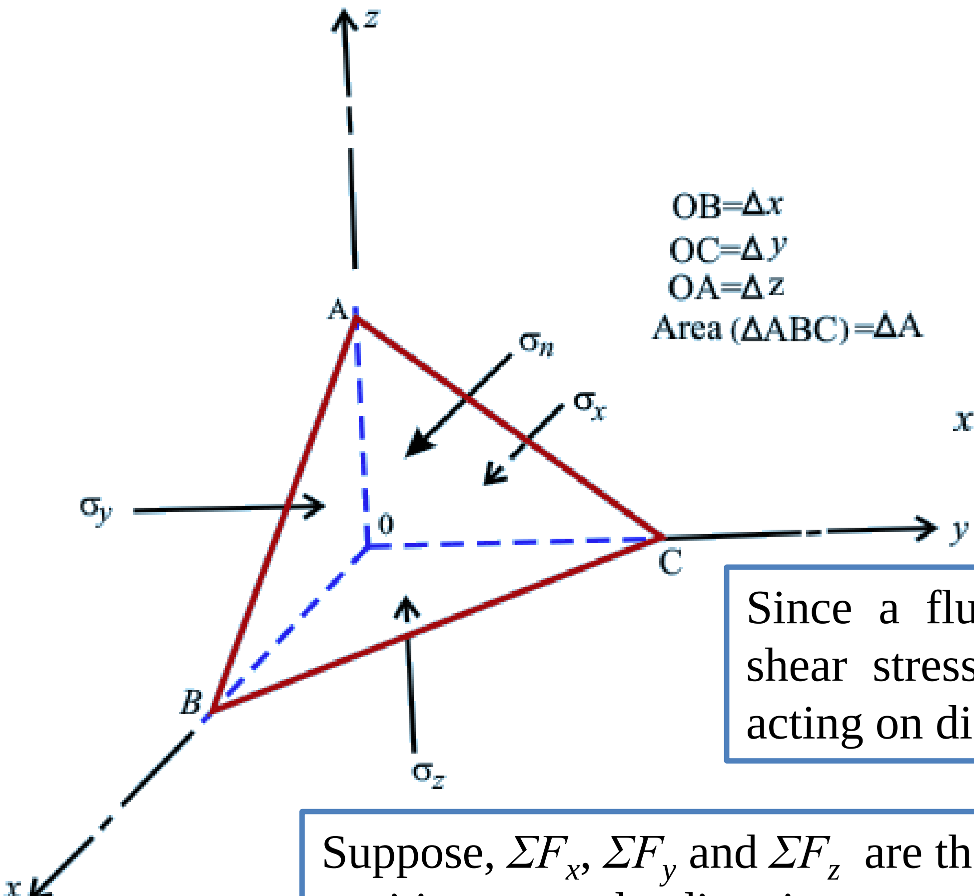
- The ratios of these forces and the elemental area in the limit of the area tending to zero are called the normal and shear stresses respectively.

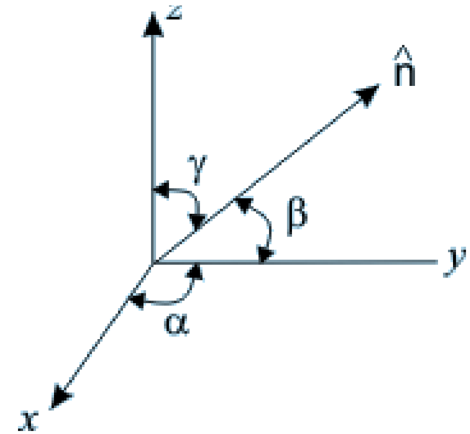
- The shear force is zero for any fluid element at rest and hence the only surface force on a fluid element is the normal component.

State of force in a fluid at rest

Normal Stress in a Stationary Fluid

Consider a stationary fluid element of tetrahedral shape with three of its faces coinciding with the coordinate planes x, y and z.





Since a fluid element at rest can develop neither shear stress nor tensile stress, the normal stresses acting on different faces are compressive in nature.

Suppose, ΣF_x, ΣF_y and ΣF_z are the net forces acting on the fluid element in positive x,y and z directions respectively. The direction cosines of the normal to the inclined plane of an area ΔA are cos α, cos β and cos γ

Considering gravity as the only source of external body force, acting in the -ve z direction, the equations of static equilibrium for the tetrahedron fluid element can be written as

$$\sum F_x = \sigma_x \left(\frac{\Delta y \Delta z}{2} \right) - \sigma_n \Delta A \cos \alpha = 0$$

$$\sum F_y = \sigma_y \left(\frac{\Delta x \Delta z}{2} \right) - \sigma_n \Delta A \cos \beta = 0$$

$$\sum F_z = \sigma_z \left(\frac{\Delta y \Delta x}{2} \right) - \sigma_n \Delta A \cos \gamma - \frac{\rho g}{6} (\Delta x \Delta y \Delta z) = 0$$

Where, Volume of tetrahedral fluid element = $\left(\dfrac{\Delta x \, \Delta y \, \Delta z}{6} \right)$

Pascal's Law of Hydrostatics

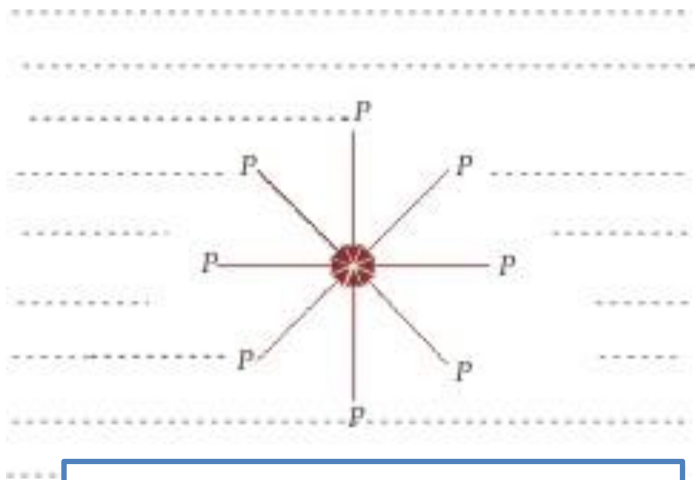
Pascal's Law

The normal stresses at any point in a fluid element at rest are directed towards the point from all directions and they are of the equal magnitude.

$$\Delta A \cos\alpha = \left(\frac{\Delta y \Delta z}{2}\right)$$

$$\Delta A \cos\beta = \left(\frac{\Delta x \Delta z}{2}\right)$$

$$\Delta A \cos\gamma = \left(\frac{\Delta x \Delta y}{2}\right)$$



$$\sigma_x = \sigma_y = \sigma_z = \sigma_n$$

The state of normal stress at any point in a fluid element at rest is same and directed towards the point from all directions. These stresses are denoted by a scalar quantity p defined as the hydrostatic or thermodynamic pressure.

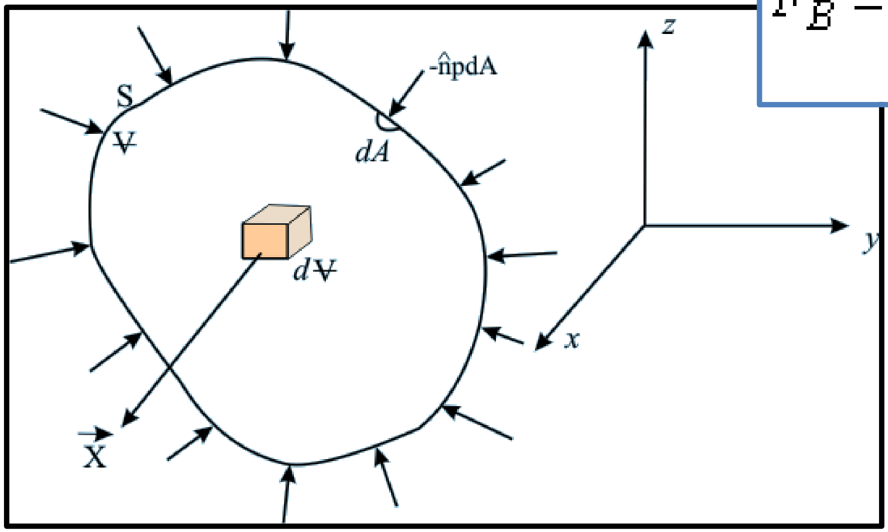
Using "+" sign for the tensile stress the above equation can be written in terms of pressure as

$$\sigma_x = \sigma_y = \sigma_z = -p$$

Fundamental Equation of Fluid Statics

The fundamental equation of fluid statics describes the spatial variation of hydrostatic pressure p in the continuous mass of a fluid.

$$\vec{F}_B = \iiint_{\forall} \vec{X}\, \rho\, d\forall$$



consider a fluid element at rest of given mass with volume *V* and bounded by the surface *S*.

FLUID PROPERTIES

Every fluid has some characteristics by which its physical conditions may be described.

We call such characteristics as the fluid properties.

- Specific Weight
- Mass Density
- Viscosity
- Vapor Pressure
- Surface tension
- Capillarity

- **Bulk Modules of Elasticity**
- **Isothermal Conditions**
- **Adiabatic or Isentropic Conditions**
- **Pressure Disturbances**

End of Lecture 4