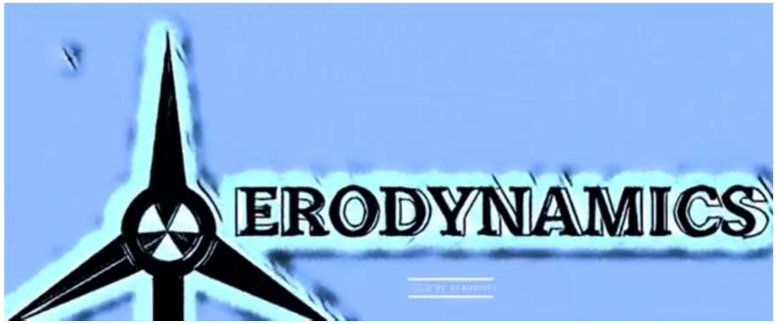

MEE1004-FLUID MECHANICS

Module 7 - Boundary Layer Flow
Lecture 4
Dt. 02.11.2020

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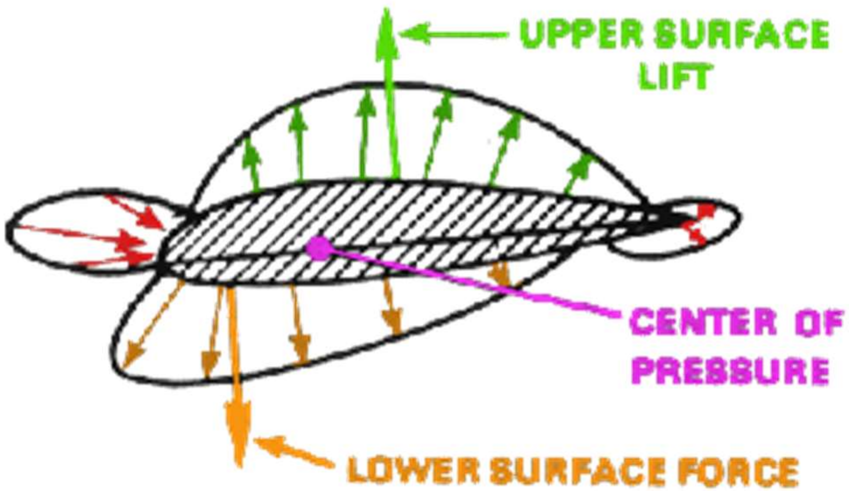
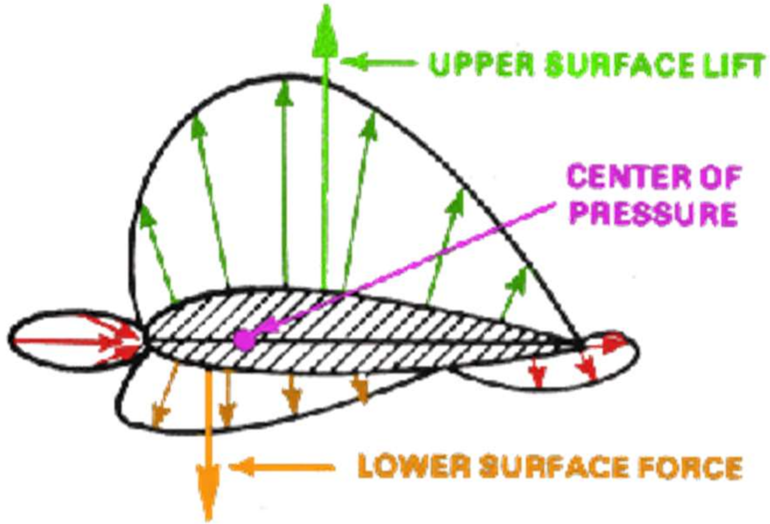
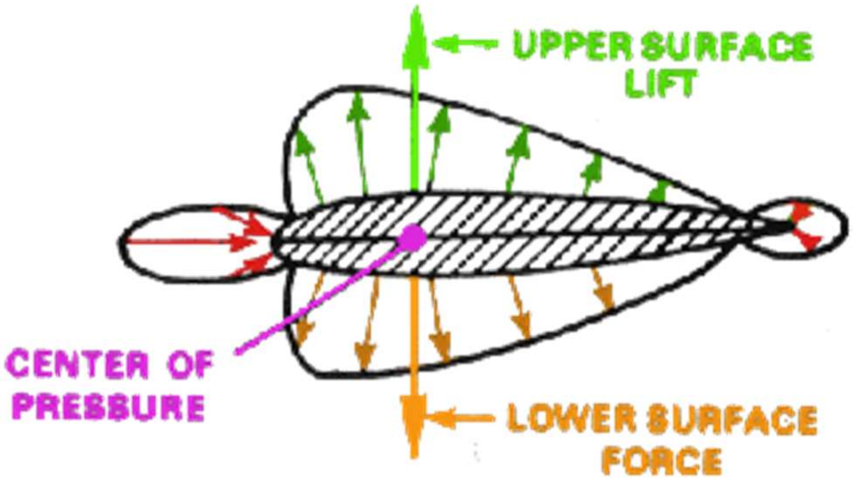


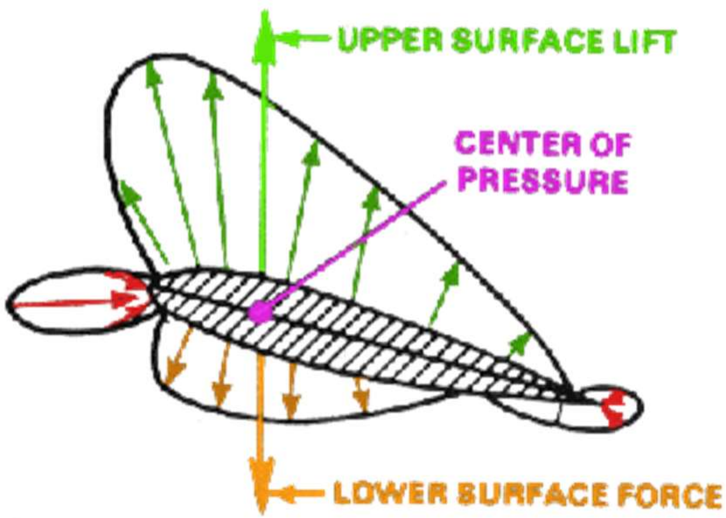
CAMBERED AIRFOIL AT ZERO LIFT



CAMBERED AIRFOIL AT POSITIVE LIFT



SYMMETRICAL AIRFOIL AT ZERO LIFT



SYMMETRICAL AIRFOIL AT POSITIVE LIFT

Potential Flow Theory $\rightarrow$ Drag $= 0$.

Observed experiment (real fluid $\nu << 1$ but $\neq 0$) $\rightarrow$ Drag $\neq 0$.

In particular the **total** drag measured on a body is regarded as the sum of two components: the **pressure** or **form** drag, and the **skin friction** or **viscous** drag.

Total Drag Profile Drag	=	**Pressure Drag** or Form Drag $\underbrace{\text{Drag Force due to Pressure}}_{\iint_S p\hat{n}ds}$	+	**Skin Friction Drag** or Viscous Drag $\underbrace{\text{Drag Force due to Viscous Stresses}}_{\iint_S \tau\hat{t}ds}$

where $\hat{n}$ and $\hat{t}$ are the normal and tangential unit vectors on the body surface respectively. Prm.
The pressure and the viscous stresses on the body surface are p and τ respectively. Fr

The form drag is evaluated by integrating the pressure along the surface of the body. For bluff bodies that create large wakes the form drag is $\sim$ total drag.

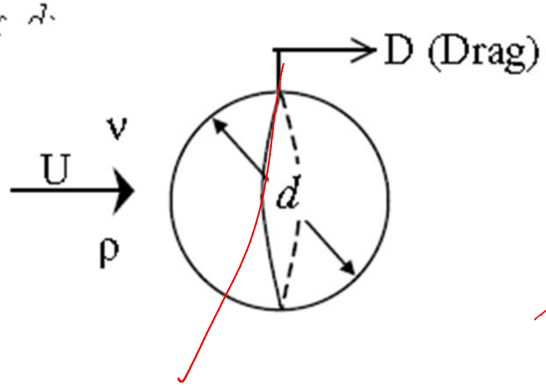
The skin friction drag is evaluated by integrating the viscous stresses on and along the body boundary. For streamlined bodies that do not create appreciable wakes, friction drag is dominant.

4.1.1 Form Drag on a Bluff Body

Consider a sphere of diameter d:

$D = C_D \frac{1}{2} \rho V^2$

$C_D = C_D(Re)$



If no DBC apply then we have seen from Dimensional Analysis that the **drag coefficient** is a function of the Reynolds number only:

$$C_D = C_D(R_e)$$

The drag coefficient C_D is defined with respect to the body's projected area S:

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 S} = \frac{D}{\frac{1}{2}\rho U^2 \underbrace{\pi d^2/4}_{\text{Projected area}}}$$

The Reynolds number R_e is defined with respect to the body's diameter d:

$$R_e = \frac{Ud}{\nu}$$

The following graph shows the dependence of C_D on R_e as measured from numerous experiments on spheres.

i) **Bluff Body → Form Drag**

For a bluff body (examples: sphere, cylinder, flat plate, etc.) there is appreciable flow separation and a wake is formed downstream of the body. The pressure within the wake is significantly smaller than that upstream of the body. Therefore the integral of the pressure along the body boundary (= form drag) does not evaluate to zero as predicted by P-Flow.

In general, for bluff bodies $\boxed{\text{form drag} >> \text{friction drag}}$

Real Flow **Potential Flow**

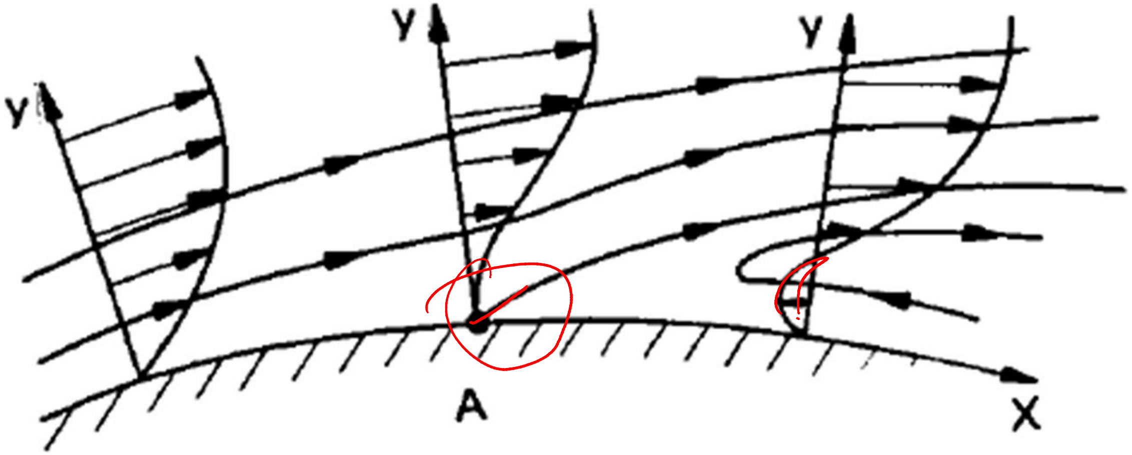
Bluff

Pres

fric.

FLOW SEPERATION

- Turbulent boundary layers can withstand much steeper pressure gradients without separation than can laminar boundary layers. This is because the turbulent mixing process leads to an intensive momentum transport from the outer flow towards the wall.
- For a pressure decrease in the flow direction there exists no tendency to flow separation.



$$\left(\frac{du}{dy}\right)_w = 0$$

Figure 2.7 Separation of the boundary layer flow at a wall (schematic)

Friction drag

If a velocity gradient du/dy is present in a viscous fluid at the wall, due to molecular friction a shear stress τ_w acts everywhere on the surface of a

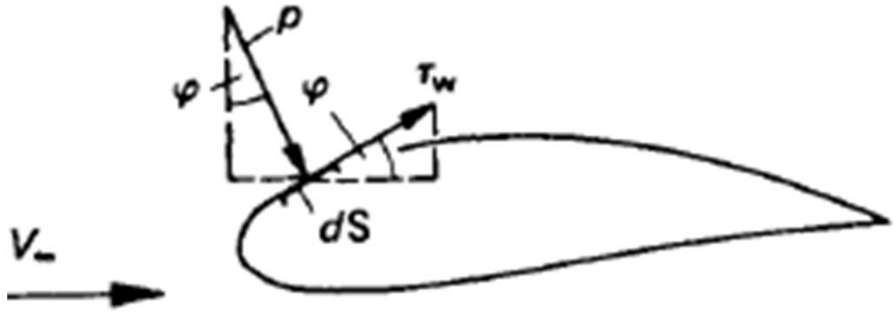


Figure 2.8 Determination of the drag of a body (example of two-dimensional flow)

body as indicated in Fig. 2.8. The integration of the corresponding force components in the free-stream direction according to

$$D_f = \oint \tau_w \cos\varphi \, dS \tag{2.19}$$

In the absence of flow separation, this is the main contribution to the total drag of a body in two-dimensional viscous flow.

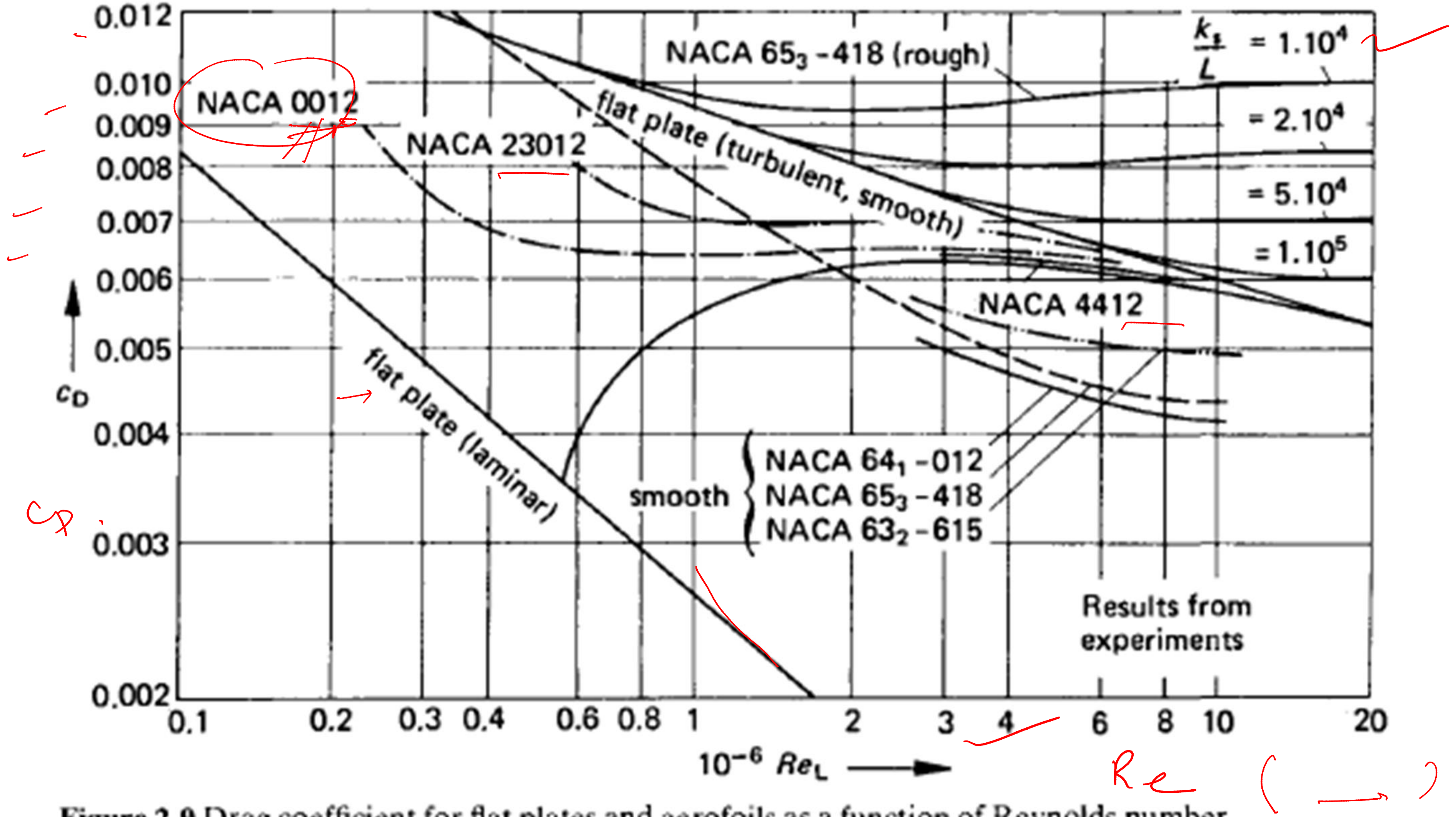


Figure 2.9 Drag coefficient for flat plates and aerofoils as a function of Reynolds number, from Schlichting[2.1]

Results for flat plates are discussed first. For laminar boundary layers, the resistance law is

$$c_D = \frac{2.656}{\sqrt{(Re_l)}} \quad (\text{for } Re_l < 5 \times 10^5) \tag{2.21}$$

and for turbulent boundary layers over the whole length l of the plate and medium Reynolds numbers, the corresponding law is

$$c_D = \frac{0.148}{\sqrt[5]{(Re_l)}} \quad (\text{for } 5 \times 10^5 < Re_l < 10^7) \tag{2.22}$$

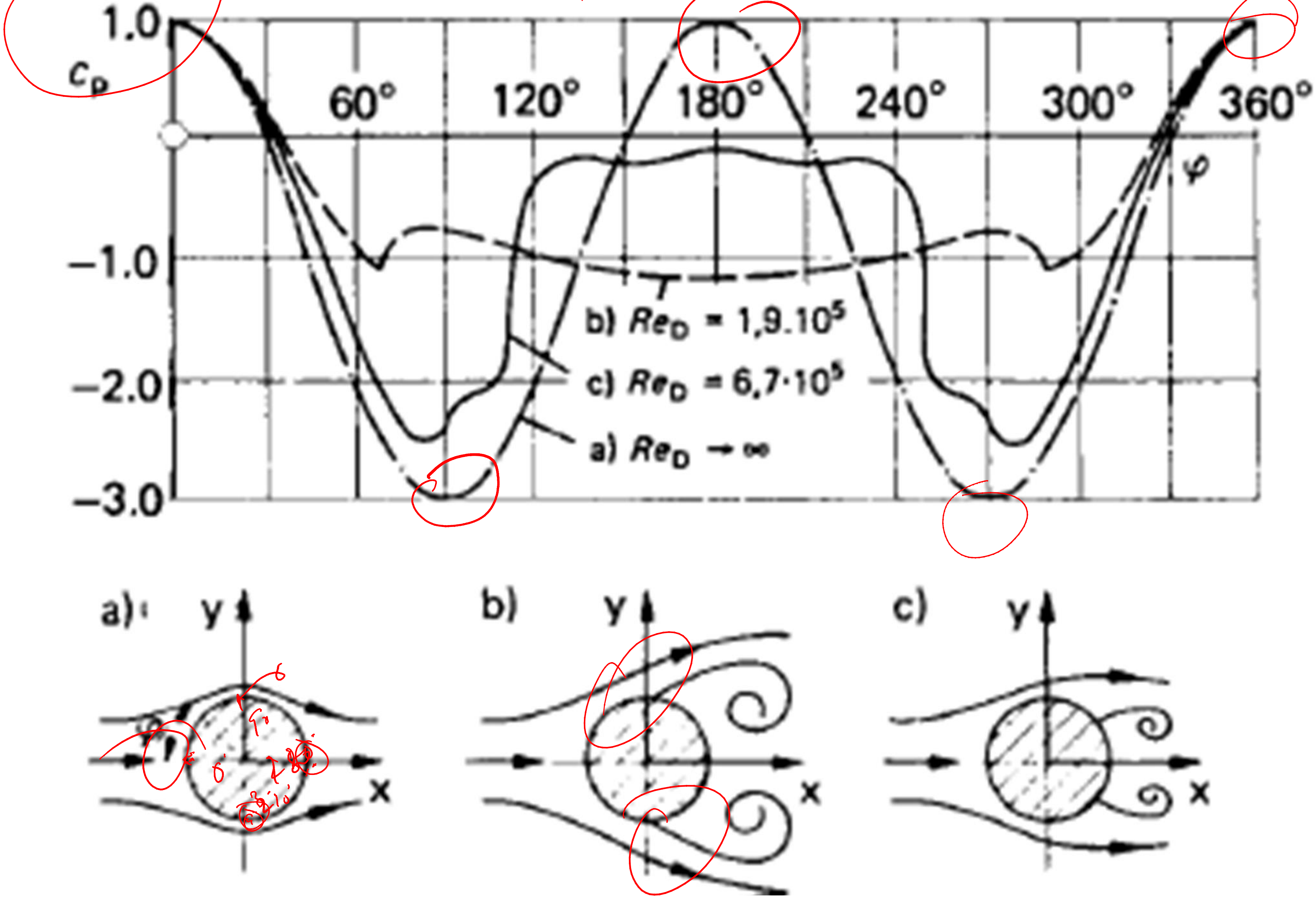
For even larger Reynolds numbers, an asymptotic law holds:

$$c_D = \frac{0.91}{(\log Re_l)^{2.58}} \quad (\text{for } Re_l > 10^7) \tag{2.23}$$

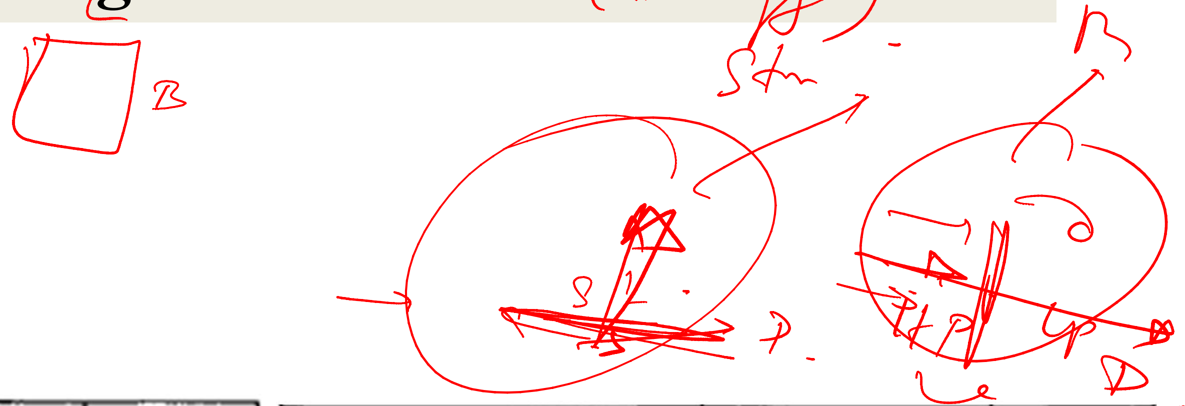
Pressure drag

- Blunt bodies, such as a circular cylinder, a sphere or a flat plate normal to the flow, show quite different drag characteristics.
- On the rear part of such bodies in the inviscid external flow, extremely steep pressure gradients occur which lead to flow separation.
- The pressure distribution is therefore asymmetrical with respect to the y-axis. Integrating the force components in the free stream direction, resulting from the pressure distribution,

$$c_p = \frac{p - p_\infty}{\frac{1}{2} \rho v^2}$$



Drag Co-efficient (BWF)



Body	Flow situation	$c_{D,c}$
Circular plate		1.17
Sphere		0.47*
Half-sphere		0.42*
$60°$-cone		0.50
Cube		1.05*
Cube		0.80*
Circular cylinder $l/D > 2$		0.82
Circular cylinder $l/D > 1$		1.15
Streamlined body $l/D = 2.5$		0.04
Circular half-plate at a ground plane		1.19
Streamlined half-body at a ground plane		0.09

Table 2.1 Drag coefficients for different bodies ($c_{D,C} = D/q_\infty S_c$, see Eqn 2.6, *subcritical flow), after Hoerner[2.9]

The drag coefficient of a car at the design conditions of 1 atm, 25°C, and 90 km/h is to be determined experimentally in a large wind tunnel in a full-scale test. The height and width of the car are 1.40 m and 1.65 m, respectively. If the horizontal force acting on the car is measured to be 300 N, determine the total drag coefficient of this car. The density of air at 1 atm and 25°C is $\rho=1.164 kg/m^3$.

The drag force acting on a body is given by $F_D = C_D A \frac{\rho V^2}{2}$

$D = \frac{1}{2} C_D A \rho V^2$

$\frac{1}{2} \rho A V^2$

The drag coefficient is given by $C_D = \frac{2F_D}{A\rho V^2}$

$\rho = 1.164 kg/m^3$

Note A is the frontal area $A=1.40\times1.65 m^2$ and $1 m/s = 3.6 km/h$. Then

$A = 1.4 * 1.65 =$

$V = 90 km/hr = \frac{90\times1000}{3600}$

$$C_D = \frac{2F_D}{A\rho V^2} = \frac{2\times350}{(1.40\times1.65)\times1.164\times(90/3.6)^2} = 0.42$$

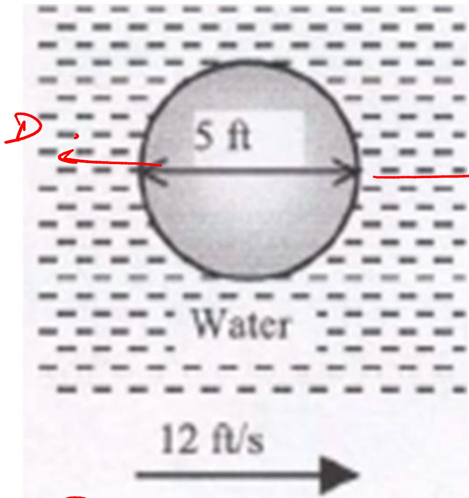
A 5-ft-diameter spherical tank completely submerged in freshwater is being towed by a ship at 12 ft/s. Assuming turbulent flow, determine the required towing power.

The density of water: 62.4 lbm/ft^3.

Drag coefficient for a sphere:

$$C_D = 0.2 \text{ for turbulent flow}$$

$$C_D = 0.5 \text{ for laminar flow}$$



The frontal area of a sphere is $A = \dfrac{\pi D^2}{4}$

Then the drag force acting on the spherical tank is

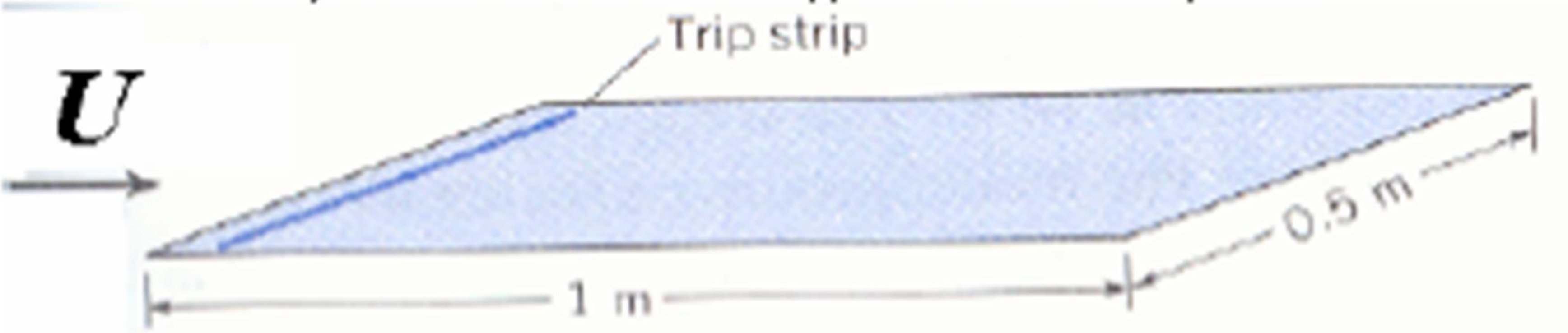
$$F_D = C_D A \frac{\rho V^2}{2} = (0.2)\left(\frac{\pi \times 5^2}{4}\right)\left(\frac{62.4 \times 12^2}{2}\right)\left(\frac{1 lbf}{32.2 lbm \cdot ft/s^2}\right) = 548 lbf$$

$P_W = D \times V$

Note the power is force times velocity, so the power needed to overcome this drag during towing is

$$\dot{W}_{Towing} = \dot{W}_{Drag} = F_D V = (548)(12)\left(\frac{1 kW}{737.56 lbf \cdot ft/s}\right) = 8.92 kW = 12.0 hp$$

A flat plate is oriented parallel to a $15 m/s$ air flow at $20°C$ and atmospheric pressure. The plate is $1.0 m$ long in the flow direction and $0.5 m$ wide. On one side of the plate, the boundary layer is tripped at leading edge (turbulent flow on that side). Find the total drag force on the plate.



Properties of air at 1 atm and 15°C:

$\nu = 1.5 \times 10^{-5}\, m^2/s$ and $\rho = 1.2\, kg/m^3$

The force due to shear stress is

$$F_D = C_D \frac{1}{2} \rho U^2 BL$$

The Reynolds number based on the plate length is

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{15 \times 1}{1.5 \times 10^{-5}} = 10^6$$

The average shear stress coefficient on the tripped side of the plate is

$$C_D = \frac{0.074}{\left(10^6\right)^{1/5}} = 0.0047$$

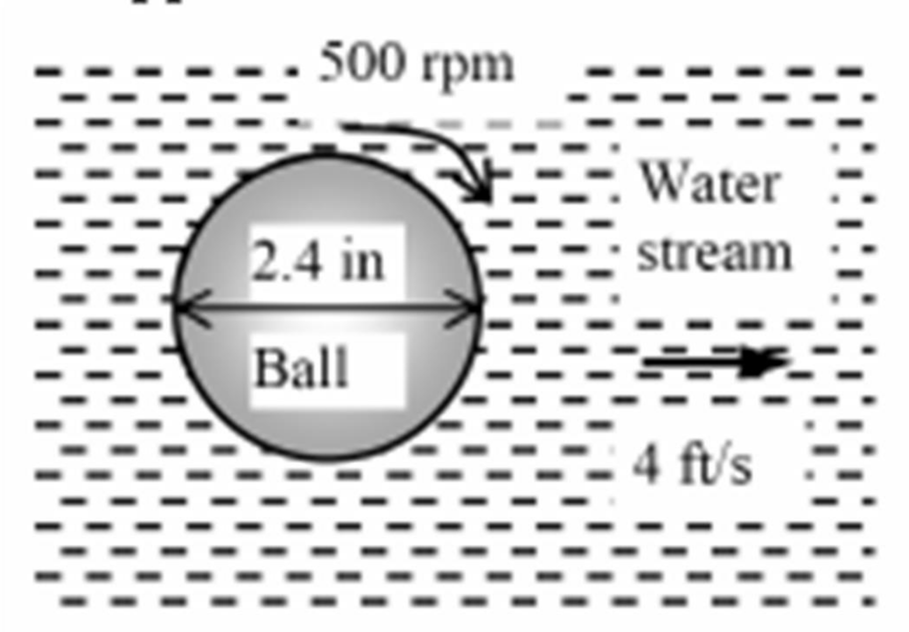
On the untripped side of the plate:

$$C_D = \frac{0.523}{\ln^2\left(0.06 \times 10^6\right)} - \frac{1520}{10^6} = 0.0028$$

The totaol force is

$$F_x = (0.0047 + 0.0028) \times \left(\frac{1}{2} \times 1.2 \times 15^2\right) \times (1.0 \times 0.5) = 0.506\,N$$

A 2.4-in-diameter smooth ball rotating at 500 rpm is dropped in a water stream at 60°F flowing at 4 ft/s. Determine the lift and the drag force acting on the ball when it is first dropped in the water.



Properties of water at 60°F:

$$\mu = 7.536 \times 10^{-4}\ lbm/ft \cdot s \text{ and } \rho = 62.36\ lbm/ft^3$$

The drag and lift forces can be determined from

$$F_D = C_D A \frac{\rho V^2}{2} \text{ and } F_L = C_L A \frac{\rho V^2}{2}$$

Where $A = \pi D^2/4$ is the frontal area and $D = 2.4/12 = 0.2\,ft$

The Reynolds number and angular velocity of the ball are

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(62.36)(4)(0.2)}{7.536\times10^{-\cdot}} = 6.62\times10^{\cdot}$$

$$\omega = (500\,rev/min)\left(\frac{2\pi\,rad}{1\,rev}\right)\left(\frac{1\,min}{60\,s}\right) = 52.4\,rad/s \text{ and}$$

$$\frac{\omega D}{2V} = \frac{(52.4)(0.2)}{2(4)} = 1.31\,rad$$

From Fig. 11-53, $C_D = 0.56$ and $C_L = 0.35$

The drag and lift forces are

$$F_D = (0.56)\frac{\pi(0.2\text{ ft})^2}{4}\frac{(62.36\text{ lbm/ft}^3)(4\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.27\ lbf}$$

$$F_L = (0.35)\frac{\pi(0.2\text{ ft})^2}{4}\frac{(62.36\text{ lbm/ft}^3)(4\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right) = \mathbf{0.17\ lbf}$$

FORCES AND MOMENTS

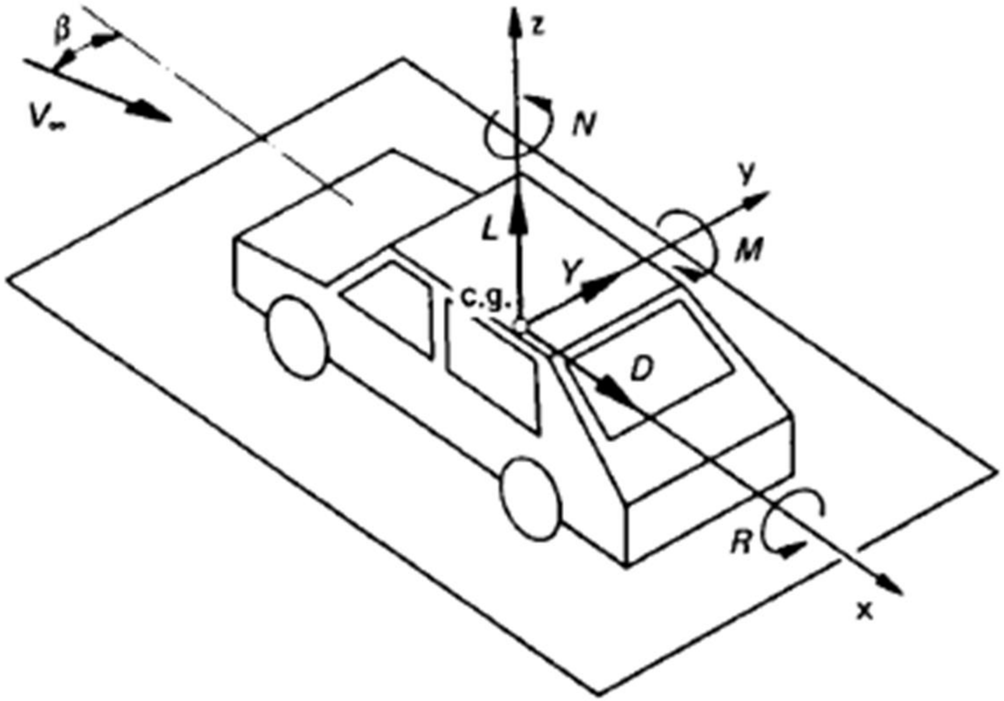


Figure 2.14 Forces and moments acting on a vehicle (c.g. = centre of gravity)

End of Unit 1 - Lecture 6