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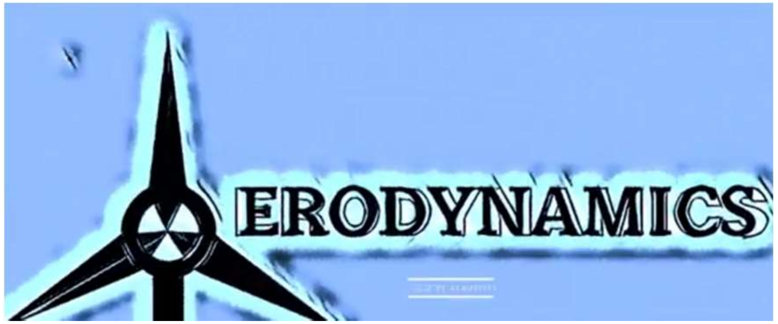

MEE1004-FLUID MECHANICS

Dimensional Analysis

Lecture 5, Dt. 21.10.2020

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12.8.1 Reynold's Number (R_e).

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

$$\text{Inertia force } (F_i) = \text{Mass} \times \text{Acceleration of flowing fluid}$$

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho \times AV \times V \qquad \{\because \text{ Volume per sec} = \text{Area} \times \text{Velocity} = A \times V\}$$

$$= \rho A V^2 \qquad \text{...(12.11)}$$

$$\text{Viscous force } (F_v) = \text{Shear stress} \times \text{Area} \qquad \left\{\because \; \tau = \mu \frac{du}{dy} \quad \therefore \; \text{Force} = \tau \times \text{Area}\right\}$$

$$= \tau \times A$$

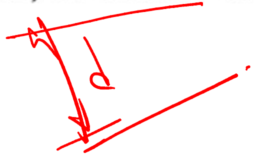
$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \cdot \frac{V}{L} \times A \qquad \left\{\because \; \frac{du}{dy} = \frac{V}{L}\right\}$$

By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \cdot \frac{V}{L} \times A} = \frac{\rho V L}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \qquad \left\{\because \; \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

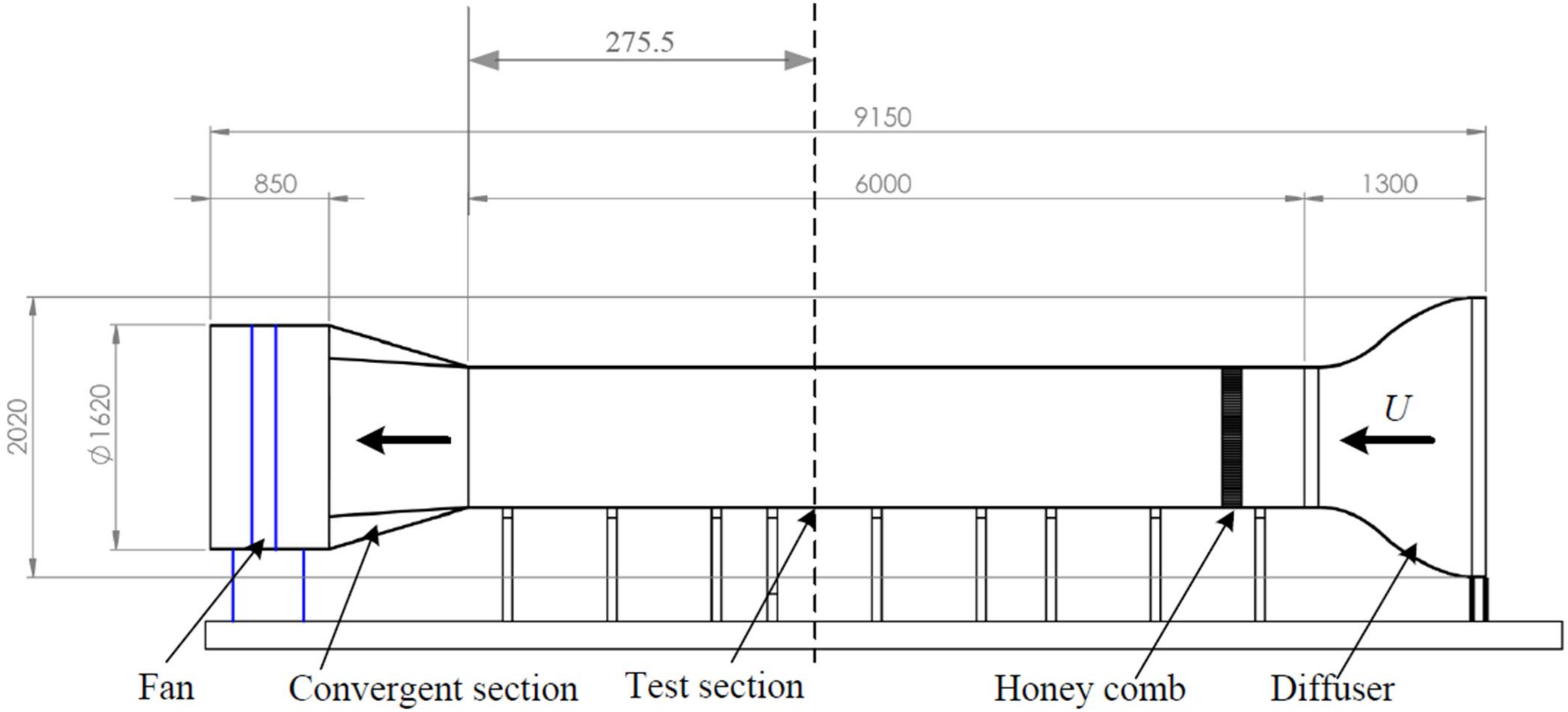


$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}. \qquad \text{...(12.12)}$$

Three necessary conditions for complete similarity between a model and a prototype:

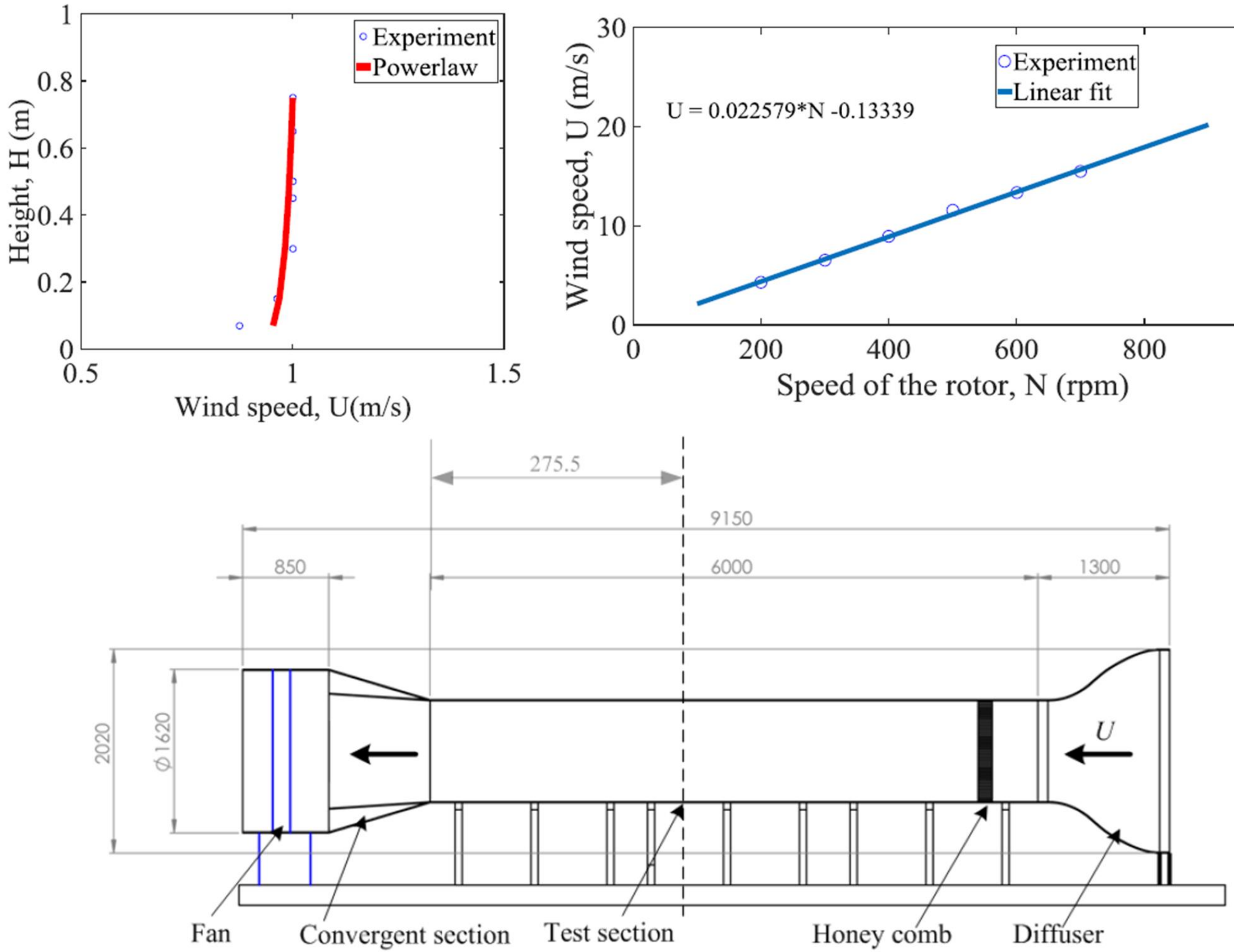
1) Geometric similarity
2) Kinematic similarity
3) Dynamic similarity

Schematic view of the Wind tunnel

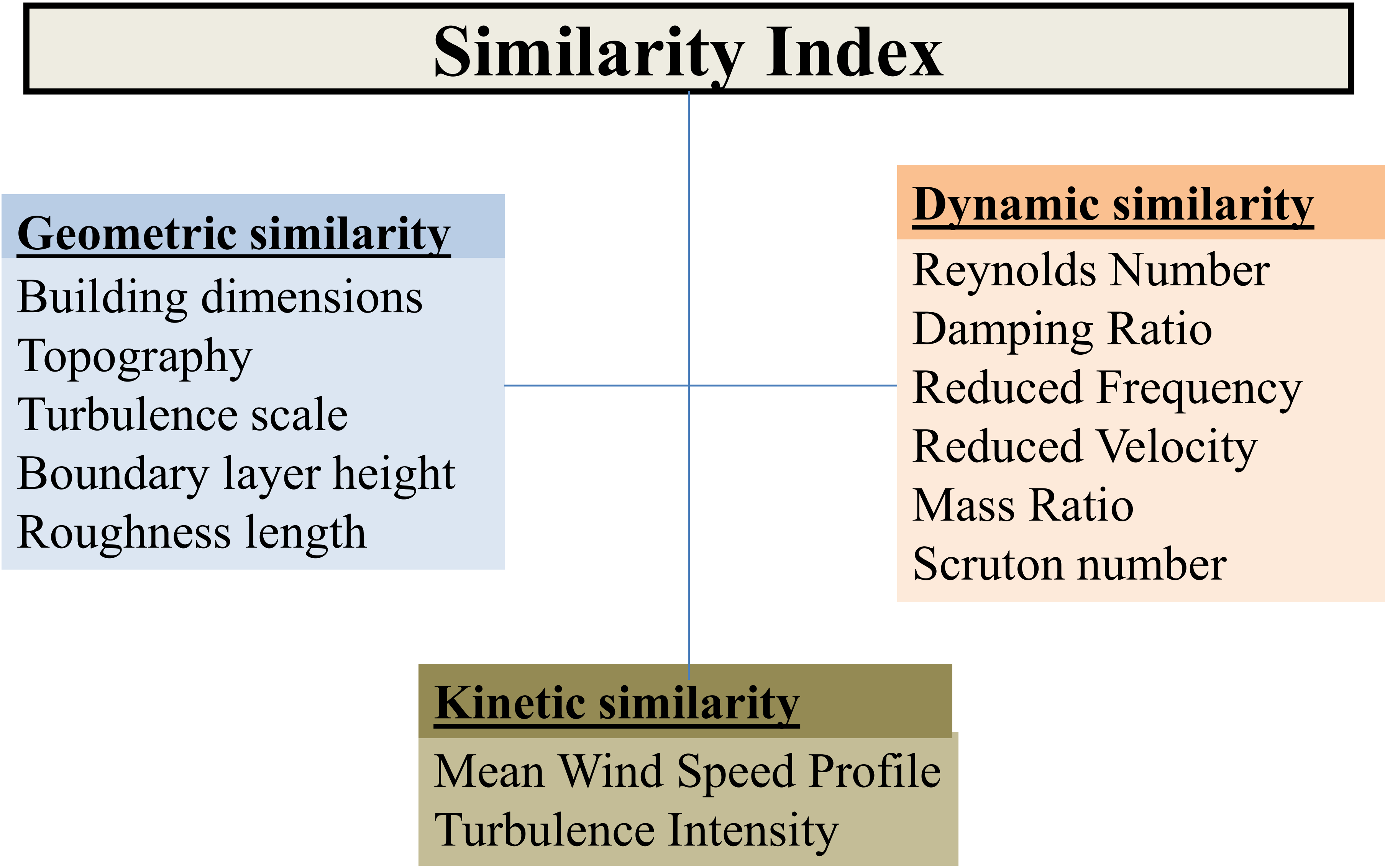


Suction type wind tunnel

All Dimensions are in mm



Experiment
Powerlaw
Height, H (m)
Wind speed, U(m/s)
Experiment
Linear fit
U = 0.022579*N -0.13339
Wind speed, U (m/s)
Speed of the rotor, N (rpm)
275.5
9150
850
6000
1300
2020
ϕ1620
U
Fan
Convergent section
Test section
Honey comb
Diffuser


Similarity Index
Geometric similarity
Building dimensions
Topography
Turbulence scale
Boundary layer height
Roughness length
Dynamic similarity
Reynolds Number
Damping Ratio
Reduced Frequency
Reduced Velocity
Mass Ratio
Scruton number
Kinetic similarity
Mean Wind Speed Profile
Turbulence Intensity

Geometric similarity

1. Geometric Similarity. The geometric similarity is said to exist between the model and the prototype. The ratio of all corresponding linear dimension in the model and prototype are equal.

Let L_m = Length of model, b_m = Breadth of model,

D_m = Diameter of model, A_m = Area of model,

$\forall_m$ = Volume of model,

and $L_P, b_P, D_P, A_P, \forall_P$ = Corresponding values of the prototype.

For geometric similarity between model and prototype, we must have the relation,

$$\frac{L_P}{L_m} = \frac{b_P}{b_m} = \frac{D_P}{D_m} = L_r \quad \text{...(12.6)}$$

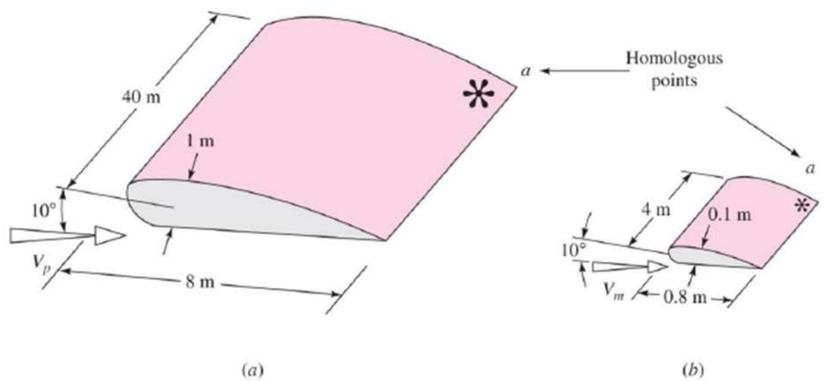
where L_r is called the scale ratio.

For area's ratio and volume's ratio the relation should be as given below :

$$\frac{A_P}{A_m} = \frac{L_P \times b_P}{L_m \times b_m} = L_r \times L_r = L_r^2 \quad \text{...(12.7)}$$

and

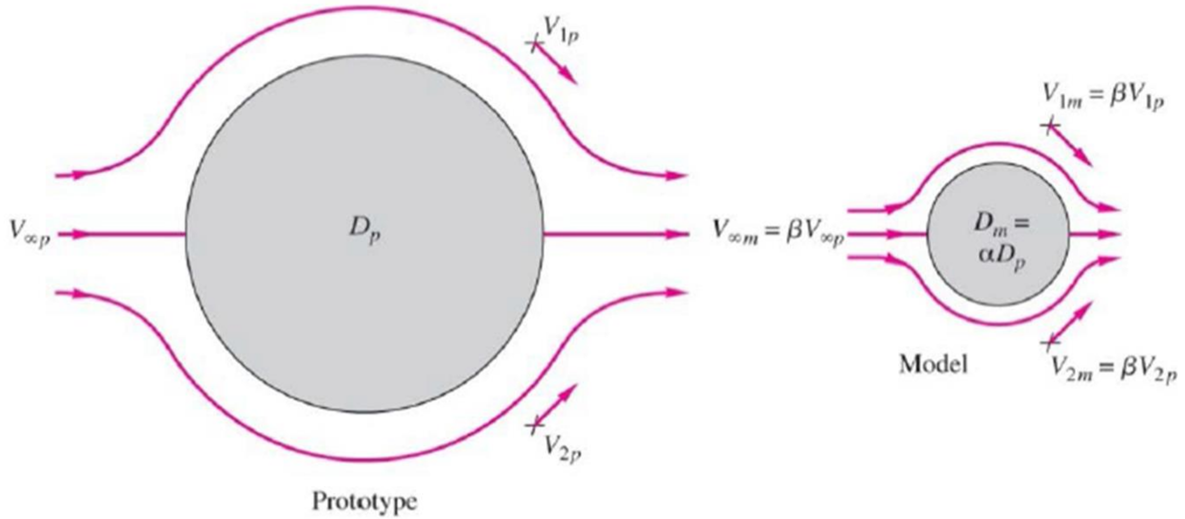
$$\frac{\forall_P}{\forall_m} = \left(\frac{L_P}{L_m}\right)^3 = \left(\frac{b_P}{b_m}\right)^3 = \left(\frac{D_P}{D_m}\right)^3 \quad \text{...(12.8)}$$



Kinematic similarity

Kinematic similarity requires that the model and prototype have the same length scale and the same time scale ratio.

One special case is incompressible frictionless flow with no free surface. These perfect-fluid flows are kinematically similar with independent length and time scales, and no additional parameters are necessary.



For kinematic similarity, we must have

$$\frac{V_{P_1}}{V_{m_1}} = \frac{V_{P_2}}{V_{m_2}} = V_r$$

where V_r is the velocity ratio.

For acceleration, we must have $\dfrac{a_{P_1}}{a_{m_1}} = \dfrac{a_{P_2}}{a_{m_2}} = a_r$

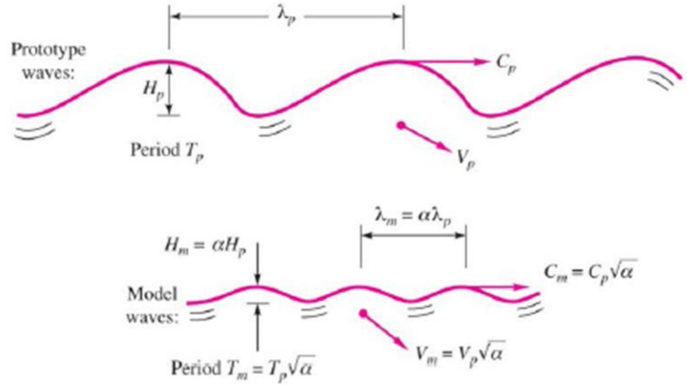
where a_r is the acceleration ratio.

Also the directions of the velocities in the model and prototype should be same.

Frictionless flows with a free are kinematically similar if their Fr are equal:

$$Fr_m = \frac{V_m^2}{gL_m} = \frac{V^2}{gL} = Fr$$

$$\frac{V_m}{V} = \left(\frac{L_m}{L}\right)^{\frac{1}{2}} = \sqrt{\alpha}$$



In general, kinematic similarity depends on the achievement of dynamic similarity if viscosity, surface tension, or compressibility is important.

Example 1: Fr Similarity

$\lambda = \frac{1}{25}$

8.52 If the scale ratio between a model spillway and its prototype is $\frac{1}{25}$, what velocity and discharge ratio will prevail between model and prototype? If the prototype discharge is 3000 m^3/s, what is the model discharge?

Fig. 5.9 Hydraulic model of the Bluestone Lake Dam on the New River near Hinton, West Virginia. The model scale is 1:65 both vertically and horizontally, and the Reynolds number, though far below the prototype value, is set high enough for the flow to be turbulent. *(Courtesy of the U.S. Army Corps of Engineers Waterways Experiment Station.)*

For Fr similarity,

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V}{\sqrt{gL}} \qquad \Rightarrow \qquad \frac{V_m}{V} = \sqrt{\frac{L_m}{L}} = \alpha^{\frac{1}{2}} = \left(\frac{1}{25}\right)^{\frac{1}{2}} = \mathbf{\frac{1}{5}}$$

Since $Q = VA$,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{VA} = \left(\frac{V_m}{V}\right)\left(\frac{A_m}{A}\right) = \alpha^{\frac{1}{2}}\left(\frac{A_m}{A}\right)$$

Also,

$$\frac{A_m}{A} = \left(\frac{L_m}{L}\right)^2 = \alpha^2$$

Thus,

$$\frac{Q_m}{Q} = \alpha^{\frac{1}{2}}\alpha^2 = \alpha^{\frac{5}{2}} = \left(\frac{1}{25}\right)^{\frac{5}{2}} = \mathbf{\frac{1}{3,125}}$$

$$\therefore Q_m = \alpha^{\frac{5}{2}}Q = \left(\frac{1}{25}\right)^{\frac{5}{2}}(3000) = \mathbf{0.96\ m^3/s}$$

3. Dynamic Similarity. Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and the prototype if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be same.

Let $(F_i)_P$ = Inertia force at a point in prototype,

$(F_v)_P$ = Viscous force at the point in prototype,

$(F_g)_P$ = Gravity force at the point in prototype,

and $(F_i)_m$ $(F_v)_m$, $(F_g)_m$ = Corresponding values of forces at the corresponding point in model.

Then for dynamic similarity, we have

$$\frac{(F_i)_P}{(F_i)_m} = \frac{(F_v)_P}{(F_v)_m} = \frac{(F_g)_P}{(F_g)_m} \ldots = F_r, \text{ where } F_r \text{ is the force ratio.}$$

Also the directions of the corresponding forces at the corresponding points in the model and prototype should be same.

Dynamic similarity

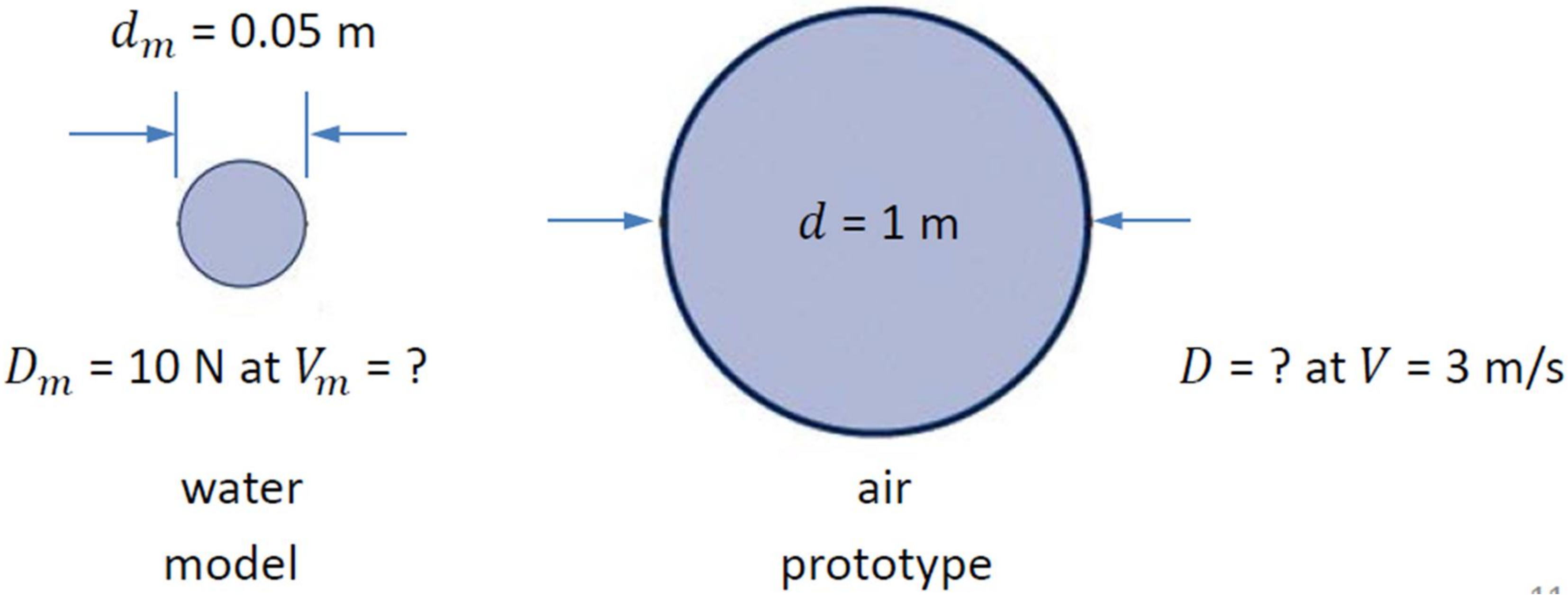
Dynamic similarity exists when the model and the prototype have the same length scale ratio (i.e., geometric similarity), time scale ratio (i.e., kinematic similarity), and force scale (or mass scale) ratio.

To be ensure of identical force and pressure coefficients between model and prototype:

1. Compressible flow: Re and Ma are equal
2. Incompressible flow:
   a. With no free surface: Re are equal
   b. With a free surface: Re and Fr are equal
   c. If necessary, We and Ca are equal

Drag measurements were taken for a 5-cm diameter sphere in water at 20°C to predict the drag force of a 1-m diameter balloon rising in air with standard temperature and pressure. Determine (a) the sphere velocity if the balloon was rising at 3 m/s and (b) the drag force of the balloon if the resulting sphere drag was 10 N. Assume the drag D is a function of the diameter d, the velocity V, and the fluid density ρ and kinematic viscosity ν.

$$D = f(d, V, \rho, \nu)$$



Example 2: Re Similarity – Contd.

- Dimensional analysis

$$\frac{D}{\rho V^2 d^2} = \phi\left(\frac{Vd}{\nu}\right)$$

- Similarity requirement

$$\frac{Vd}{\nu} = \frac{V_m d_m}{\nu_m}$$

- Prediction equation

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}$$

Example 2: Re Similarity – Contd.

(a) From the similarity requirement:

$$\frac{Vd}{\nu} = \frac{V_m d_m}{\nu_m}$$

or

$$V_m = \left(\frac{\nu_m}{\nu}\right)\left(\frac{d}{d_m}\right)V$$

$$= \left(\frac{1.004 \times 10^{-6}\ \mathrm{m^2/s}}{1.45 \times 10^{-5}\ \mathrm{m^2/s}}\right)\left(\frac{1\ \mathrm{m}}{0.05\ \mathrm{m}}\right)(3\ \mathrm{m/s})$$

$$\therefore V = 4.15\ \mathrm{m/s}$$

Example 2: Re Similarity – Contd.

(b) From the prediction equation:

$$\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}$$

or

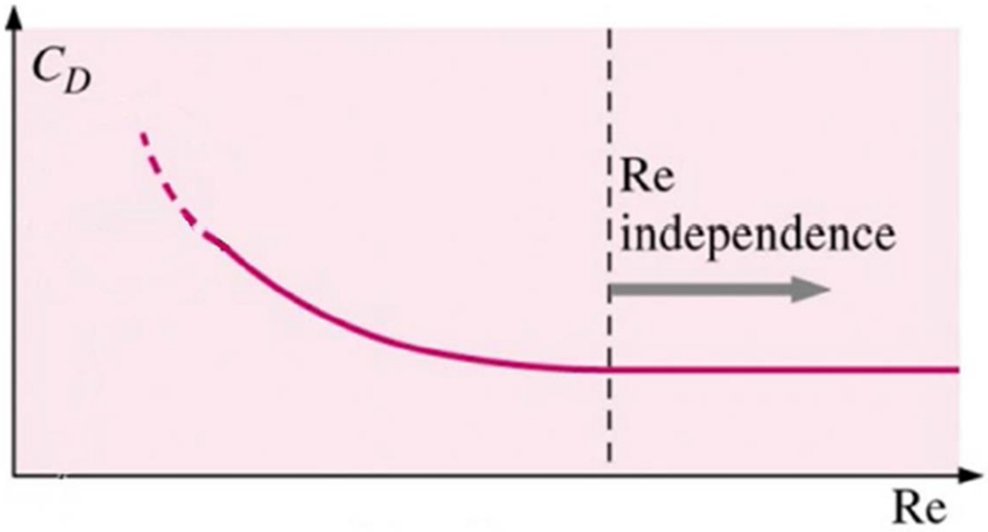
$$D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{d}{d_m}\right)^2 D_m$$

$$= \left(\frac{1.23\ \mathrm{kg/m^3}}{998\ \mathrm{kg/m^3}}\right)\left(\frac{3\ \mathrm{m/s}}{4.15\ \mathrm{m}/s}\right)^2\left(\frac{1\ \mathrm{m}}{0.05\ \mathrm{m}}\right)^2 (10\ \mathrm{N})$$

$$\therefore D = 2.6\ \mathrm{N}$$

Model Testing in Air – Contd.

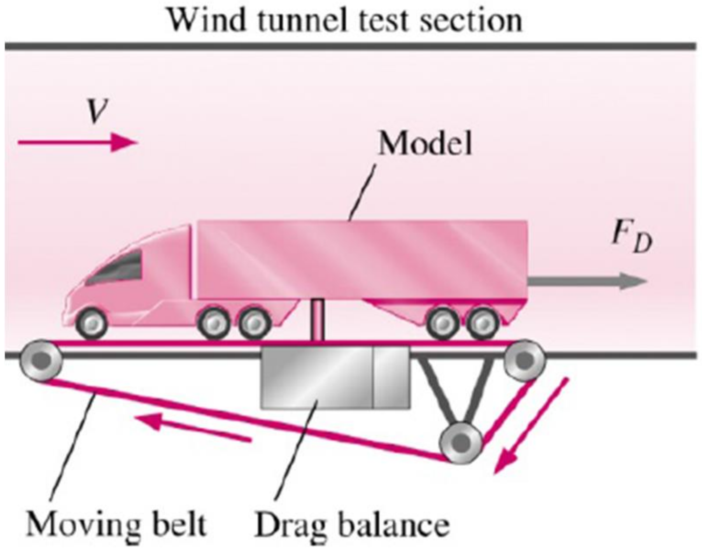
- In practice, wind tunnel tests are performed at several speeds near the maximum operating speed, and then extrapolate the results to the full-scale Reynolds number.
- While drag coefficient C_D is a strong function of Reynolds number at low values of Re, for flows over many objects (especially "bluff" objects), the flow is **Reynolds number independent** above some threshold value of Re.



Example 6

EXAMPLE 7–10 Model Truck Wind Tunnel Measurements

A one-sixteenth scale model tractor-trailer truck (18-wheeler) is tested in a wind tunnel as sketched in Fig. 7–38. The model truck is 0.991 m long, 0.257 m tall, and 0.159 m wide. During the tests, the moving ground belt speed is adjusted so as to always match the speed of the air moving through the test section. Aerodynamic drag force F_D is measured as a function of wind tunnel speed; the experimental results are listed in Table 7–7. Plot the drag coefficient C_D as a function of the Reynolds number Re, where the area used for the calculation of C_D is the frontal area of the model truck (the area you see when you look at the model from upstream), and the length scale used for calculation of Re is truck width W. Have we achieved dynamic similarity? Have we achieved Reynolds number independence in our wind tunnel test? Estimate the aerodynamic drag force on the prototype truck traveling on the highway at 26.8 m/s. Assume that both the wind tunnel air and the air flowing over the prototype car are at 25°C and standard atmospheric pressure.

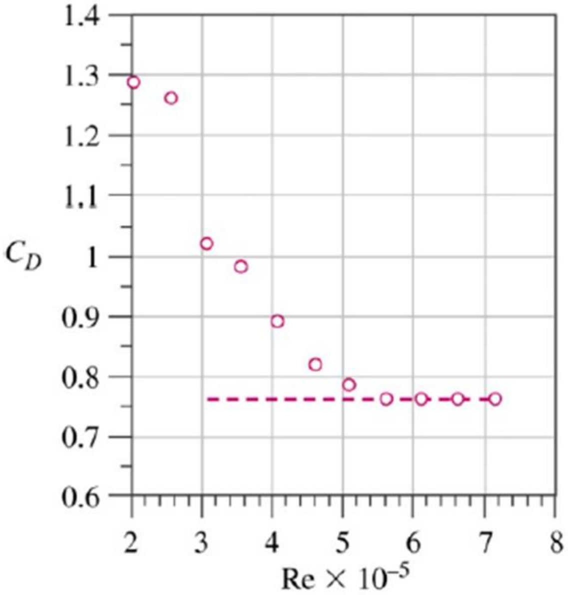


$$\frac{F_D}{\frac{1}{2}\rho V^2 A} = \phi\left(\frac{\rho V W}{\mu}\right)$$

- W = Width of the truck
- A = Frontal area

Wind tunnel data: aerodynamic drag force on a model truck as a function of wind tunnel speed

V, m/s	F_D, N
20	12.4
25	19.0
30	22.1
35	29.0
40	34.3
45	39.9
50	47.2
55	55.5
60	66.0
65	77.6
70	89.9



$$(Re_m)_{max} = \frac{\rho_m V_m W_m}{\mu_m} = \frac{\left(1.184\frac{\text{kg}}{\text{m}^3}\right)\left(26.8\frac{\text{m}}{\text{s}}\right)(0.159\text{ m})}{1.849\times10^{-5}\text{ kg}\cdot\text{s/m}} = 7.13\times10^5$$

$$Re = \frac{\rho V W}{\mu} = \frac{\left(1.184\frac{\text{kg}}{\text{m}^3}\right)\left(26.8\frac{\text{m}}{\text{s}}\right)(16\times0.159\text{ m})}{1.849\times10^{-5}\text{ kg}\cdot\text{s/m}} = 4.37\times10^6$$

Reynolds number independence is achieved for $Re > 5.5\times10^5$, thus

$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = C_{Dm} = 0.76$ from the wind tunnel data

$$\therefore F_D = \frac{1}{2}\rho V^2 A\cdot C_D$$

$$= \frac{1}{2}\left(1.184\frac{\text{kg}}{\text{m}^3}\right)\left(26.8\frac{\text{m}}{\text{s}}\right)^2(16^2\times0.159\text{m}\times0.257\text{m})(0.76)$$

$$= 3.4\ k\text{N}$$

For specified application of Wind tunnel testing on Buildings..

Geometric similarity

Building / model dimensions

- Blockage ratio
- Should be able to characterize the effects of vortices
- Same scale to be maintained in all three directions

Topography

- The topography has to be scaled down with proper detailing

Boundary layer height

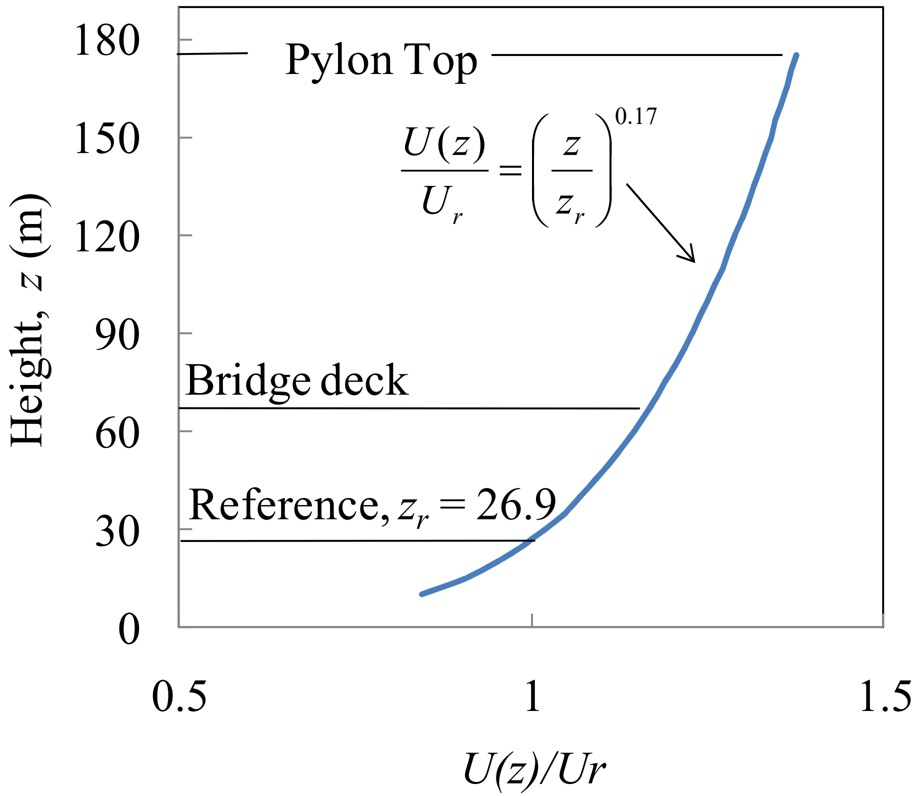
- The Boundary Layer height for different terrains should be properly simulated

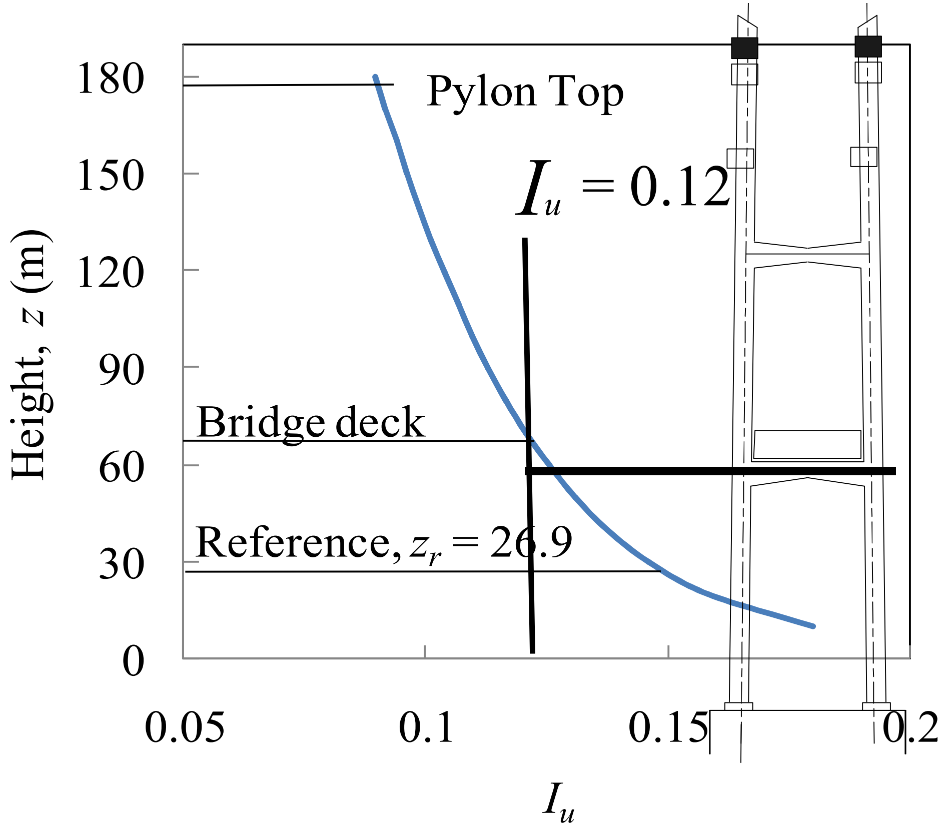
Roughness length

Kinetic similarity-Turbulent intensity and wind profile at the bridge site

$$I(z) = \mathrm{I}(10)[1 - 0.00109(z - 10)](10/z)^{0.17}$$

The above equation is used to estimate the Turbulent intensity at the bridge deck.





Dynamic similarity

Reynolds number	Re	$\frac{UD}{\nu}$
Damping ratio	ξ	
Mass ratio	m^*	$\frac{m}{\rho\, D^2\, L}$
Scruton number	S_c	$\frac{4\pi m\xi}{\rho D^2}$
Reduced wind speed	U_R	$\frac{U}{f\, D}$
Reduced frequency	f_R	$\frac{fD}{U}$

Geometric similarity

Building dimensions
Topography
Turbulence scale
Boundary layer height
Roughness length

Kinetic similarity

Mean Wind Speed Profile
Turbulence Intensity

Dynamic similarity

Reynolds Number
Damping Ratio
Reduced Velocity
Mass Ratio
Scruton number

Pressure models
Force models
Aero-elastic models

Satisfying these conditions will corresponds to the realistic flow phenomena

Full scale test = Reduced model test inside the wind tunnel

▶ 12.7 TYPES OF FORCES ACTING IN MOVING FLUID

For the fluid flow problems, the forces acting on a fluid mass may be any one, or a combination of the several of the following forces :

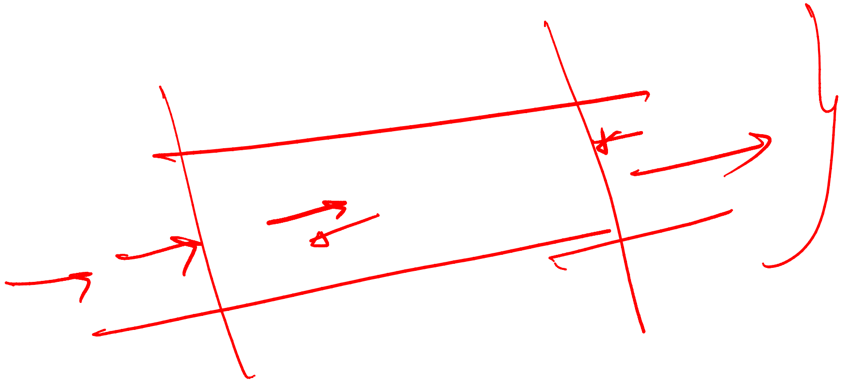
1. Inertia force, F_i.
2. Viscous force, F_v.
3. Gravity force, F_g.
4. Pressure force, F_p.
5. Surface tension force, F_s.
6. Elastic force, F_e.

1. **Inertia Force** (F_i). It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration. It is always existing in the fluid flow problems.

TYPES OF FORCES ACTING IN 'MOVING FLUID'

For the fluid flow problems, the forces acting on a fluid mass may be any one, or a combination of the several of the following forces :

1. Inertia force, F_i.
2. Viscous force, F_v.
3. Gravity force, F_g. (open)
4. Pressure force, F_p.
5. Surface tension force, F_s.
6. Elastic force, F_e.



5.

Non-Dimensional

'Dimensionless' para

TYPES OF FORCES ACTING IN MOVING FLUID

1. **Inertia Force** (F_i). It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration. It is always existing in the fluid flow problems.

2. **Viscous Force** (F_v). It is equal to the product of shear stress (τ) due to viscosity and surface area of the flow. It is present in fluid flow problems where viscosity is having an important role to play.

3. **Gravity Force** (F_g). It is equal to the product of mass and acceleration due to gravity of the flowing fluid. It is present in case of open surface flow.

4. **Pressure Force** (F_p). It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid. It is present in case of pipe-flow.

5. **Surface Tension Force** (F_s). It is equal to the product of surface tension and length of surface of the flowing fluid.

6. **Elastic Force** (F_e). It is equal to the product of elastic stress and area of the flowing fluid.

For a flowing fluid, the above-mentioned forces may not always be present. And also the forces, which are present in a fluid flow problem, are not of equal magnitude. There are always one or two forces which dominate the other forces. These dominating forces govern the flow of fluid.

DIMENSIONLESS NUMBERS

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters. The followings are the important dimensionless numbers :

1. Reynold's number,
2. Froude's number,
3. Euler's number,
4. Weber's number,
5. Mach's number.

$$\frac{V}{C}$$

$$Fr = \frac{V}{\sqrt{gL}}$$

$$Fr = \frac{V^2}{gL} \times \frac{\rho D}{\rho D}$$

$$\frac{i}{g}$$

TABLE 7–4

Guidelines for manipulation of the Π's resulting from the method of repeating variables*

Guideline	Comments and Application to Present Problem
1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.	We can raise a Π to any exponent n (changing it to Π^n) without changing the dimensionless stature of the Π. For example, in the present problem, we imposed an exponent of −1/2 on Π_3. Similarly we can perform the functional operation sin(Π), exp(Π), etc., without influencing the dimensions of the Π.
2. We may multiply a Π by a pure (dimensionless) constant.	Sometimes dimensionless factors of π, 1/2, 2, 4, etc., are included in a Π for convenience. This is perfectly okay since such factors do not influence the dimensions of the Π.
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.	We could replace Π_3 by $\Pi_3\Pi_1$, Π_3/Π_2, etc. Sometimes such manipulation is necessary to convert our Π into an established Π. In many cases, the established Π would have been produced if we would have chosen different repeating parameters.
4. We may use any of guidelines 1 to 3 in combination.	In general, we can replace any Π with some new Π such as $A\Pi_3{}^B \sin(\Pi_1{}^C)$, where A, B, and C are pure constants.
5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.	For example, the Π may contain the square of a length or the cube of a length, for which we may substitute a known area or volume, respectively, in order to make the Π agree with established conventions.

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Archimedes number	$\mathrm{Ar} = \dfrac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\dfrac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\mathrm{AR} = \dfrac{L}{W}$ or $\dfrac{L}{D}$	$\dfrac{\text{Length}}{\text{Width}}$ or $\dfrac{\text{Length}}{\text{Diameter}}$
Biot number	$\mathrm{Bi} = \dfrac{hL}{k}$	$\dfrac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\mathrm{Bo} = \dfrac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\dfrac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\mathrm{Ca}\ (\text{sometimes } \sigma_c) = \dfrac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \dfrac{2(P - P_v)}{\rho V^2}\right)$	$\dfrac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$

Darcy friction factor	$f = \dfrac{8\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$\mathrm{Ec} = \dfrac{V^2}{c_p T}$	$\dfrac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\mathrm{Eu} = \dfrac{\Delta P}{\rho V^2} \left(\text{sometimes } \dfrac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\dfrac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \dfrac{2\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\mathrm{Fo}\ (\text{sometimes } \tau) = \dfrac{\alpha t}{L^2}$	$\dfrac{\text{Physical time}}{\text{Thermal diffusion time}}$

Froude number	$\mathrm{Fr} = \dfrac{V}{\sqrt{gL}} \left(\text{sometimes } \dfrac{V^2}{gL}\right)$	$\dfrac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\mathrm{Gr} = \dfrac{g\beta\lvert\Delta\rvert TL^3\rho^2}{\mu^2}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\mathrm{Ja} = \dfrac{c_p(T - T_{\mathrm{sat}})}{h_{fg}}$	$\dfrac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\mathrm{Kn} = \dfrac{\lambda}{L}$	$\dfrac{\text{Mean free path length}}{\text{Characteristic length}}$
Lewis number	$\mathrm{Le} = \dfrac{k}{\rho c_p D_{AB}} = \dfrac{\alpha}{D_{AB}}$	$\dfrac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Lift force}}{\text{Dynamic force}}$

Mach number	$\text{Ma (sometimes } M) = \dfrac{V}{c}$	$\dfrac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\text{Nu} = \dfrac{Lh}{k}$	$\dfrac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\text{Pe} = \dfrac{\rho L V c_p}{k} = \dfrac{LV}{\alpha}$	$\dfrac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_p = \dfrac{\dot{W}}{\rho D^5 \omega^3}$	$\dfrac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\text{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu c_p}{k}$	$\dfrac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\dfrac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\text{Ra} = \dfrac{g\beta \lvert \Delta T \rvert L^3 \rho^2 c_p}{k\mu}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$

Reynolds number	$\mathrm{Re} = \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu}$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\mathrm{Ri} = \dfrac{L^5 g \Delta\rho}{\rho \dot{V}^2}$	$\dfrac{\text{Buoyancy force}}{\text{Inertial force}}$
Schmidt number	$\mathrm{Sc} = \dfrac{\mu}{\rho D_{AB}} = \dfrac{\nu}{D_{AB}}$	$\dfrac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\mathrm{Sh} = \dfrac{VL}{D_{AB}}$	$\dfrac{\text{Overall mass diffusion}}{\text{Species diffusion}}$
Specific heat ratio	$k \text{ (sometimes } \gamma) = \dfrac{c_p}{c_V}$	$\dfrac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\mathrm{St} = \dfrac{h}{\rho c_p V}$	$\dfrac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	$\mathrm{Stk} \text{ (sometimes St)} = \dfrac{\rho_p D_p^2 V}{18 \mu L}$	$\dfrac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	$\mathrm{St} \text{ (sometimes S or Sr)} = \dfrac{fL}{V}$	$\dfrac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\mathrm{We} = \dfrac{\rho V^2 L}{\sigma_s}$	$\dfrac{\text{Inertial force}}{\text{Surface tension force}}$

Theory of Models

- Flow conditions for a model test are completely similar if all relevant dimensionless parameters have the same corresponding values for the model and then prototype.

- For prototype:

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \dots, \Pi_n)$$

- For model:

$$\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, \dots, \Pi_{nm})$$

- Similarity requirement*

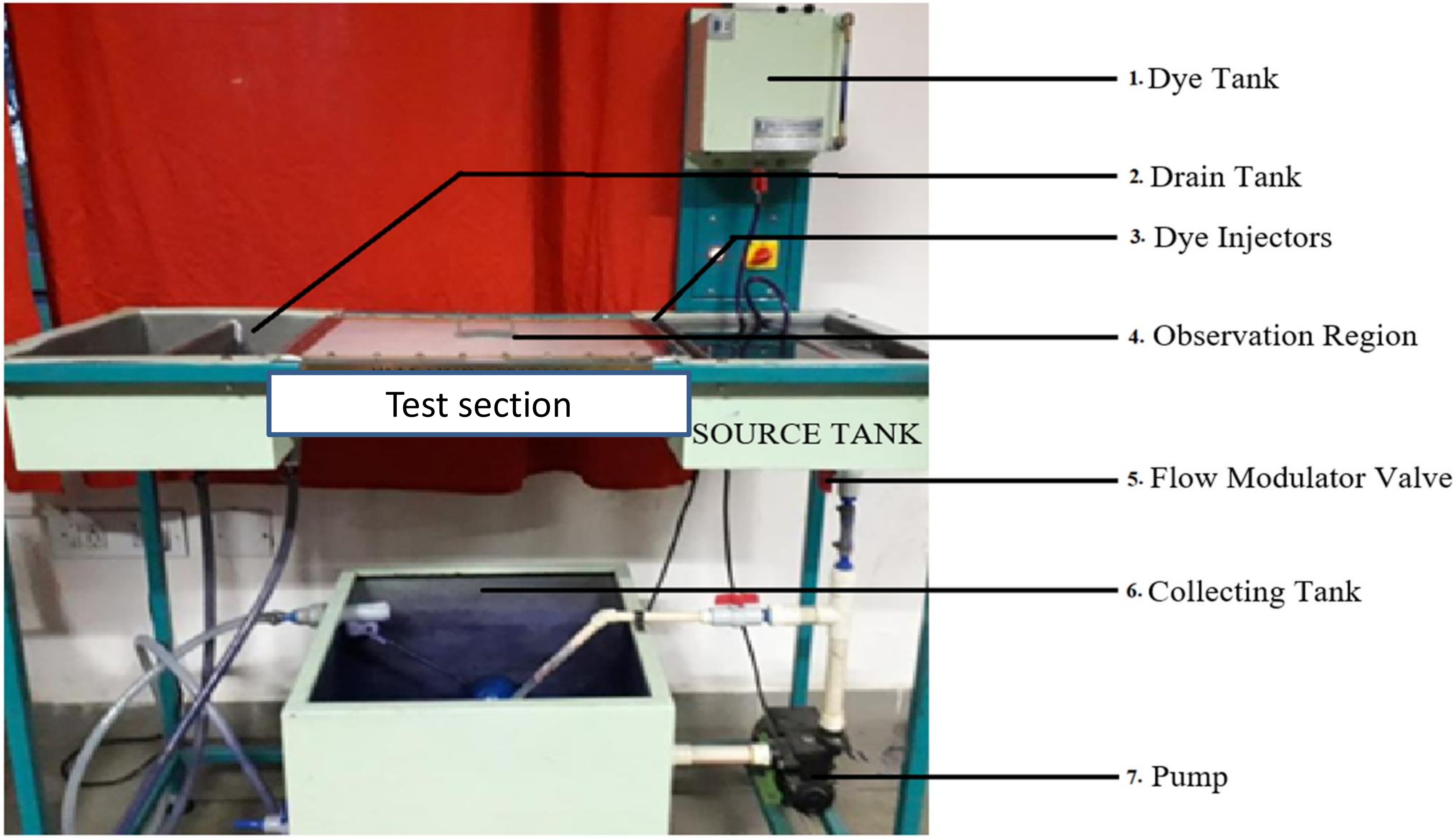
$$\begin{gathered} \Pi_{2m} = \Pi_2 \\ \Pi_{3m} = \Pi_3 \\ \vdots \\ \Pi_{nm} = \Pi_n \end{gathered}$$

- Prediction equation

$$\Pi_1 = \Pi_{1m}$$

*Also referred as the model design conditions or modeling laws.

Heleshaw apparatus for 2 dimensional visualization



Hele-Shaw Apparatus

Figure 6C.1 Hele-Shaw Apparatus

Figure 6C.2 Source Tank, Dye Injector Tubes and Needles, Stones & Honey Comb for obtaining disturbance free fluid flow.

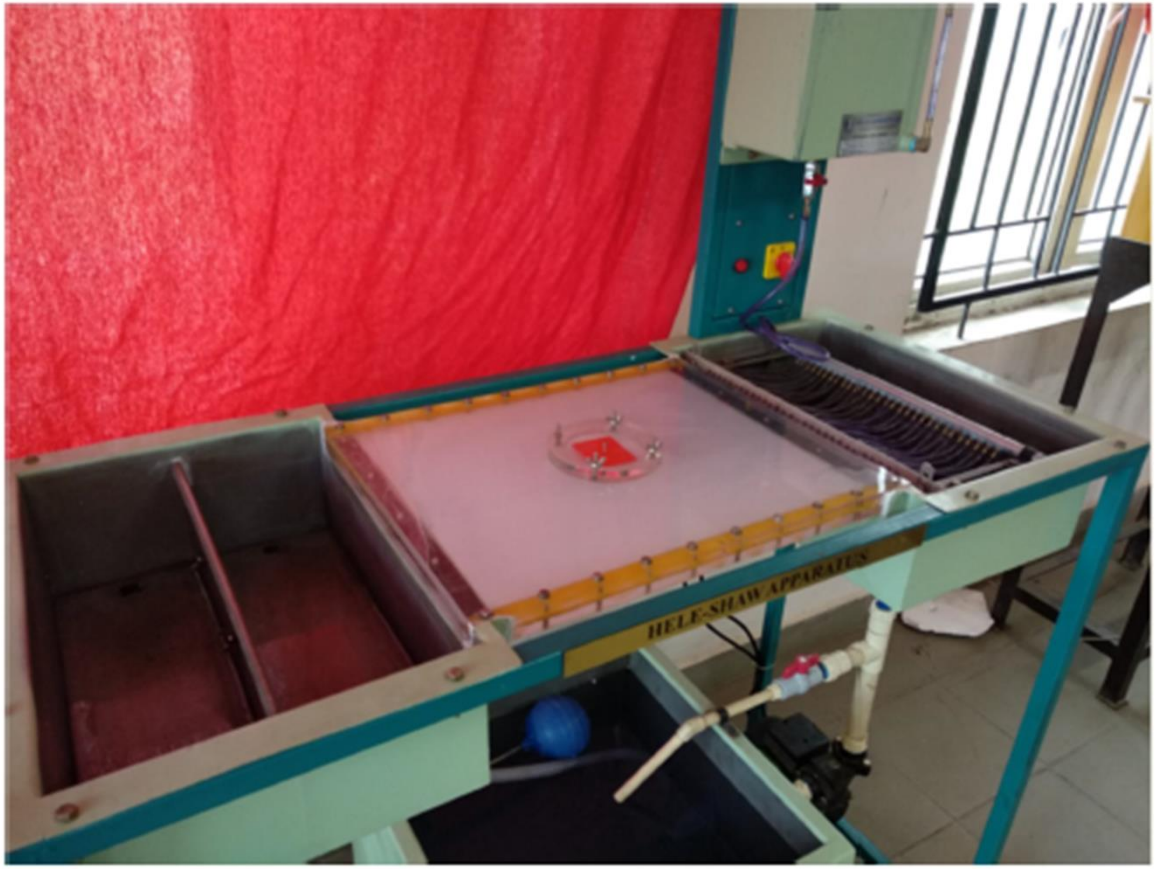
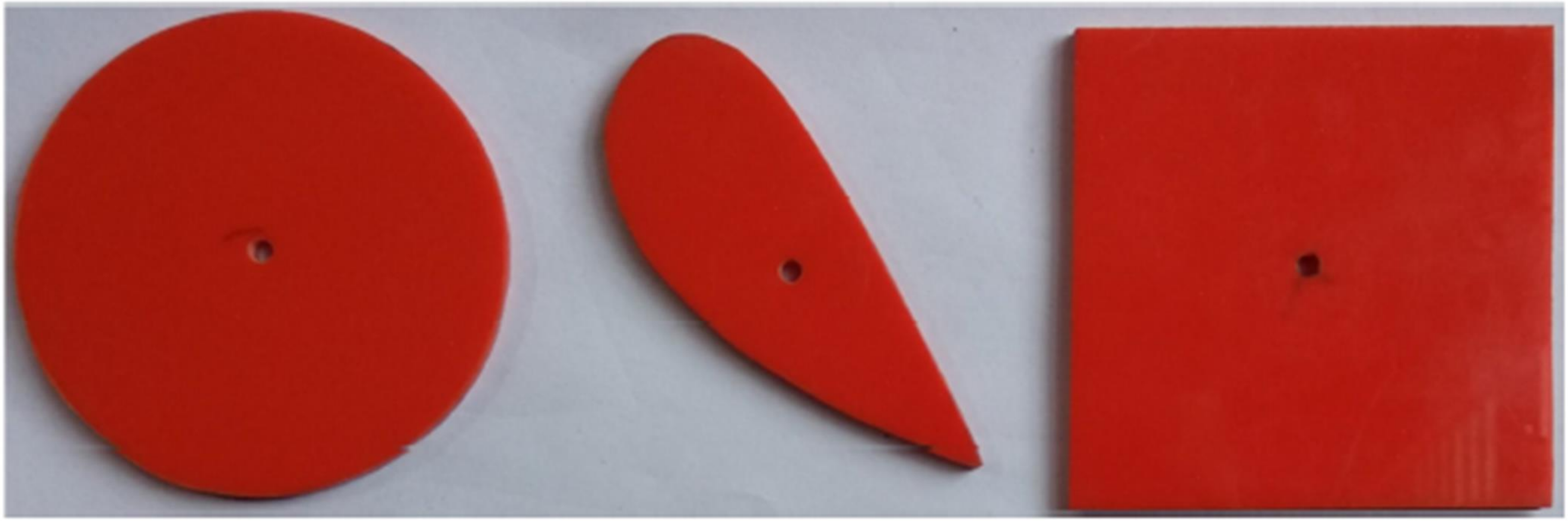
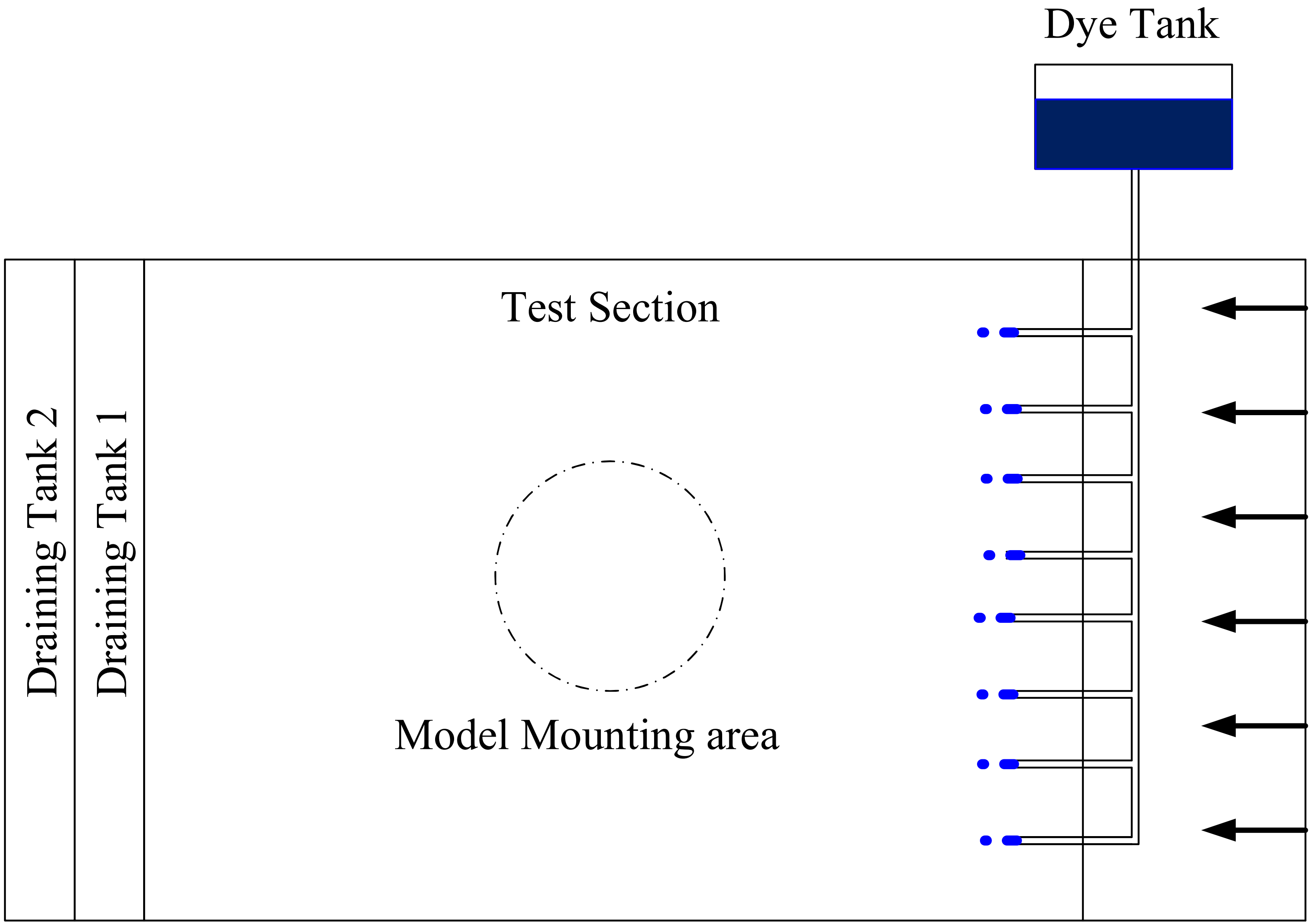


Figure 6C.5 View of the Source Tank, Test Section, Sink Tank, Dye Tank and Sump

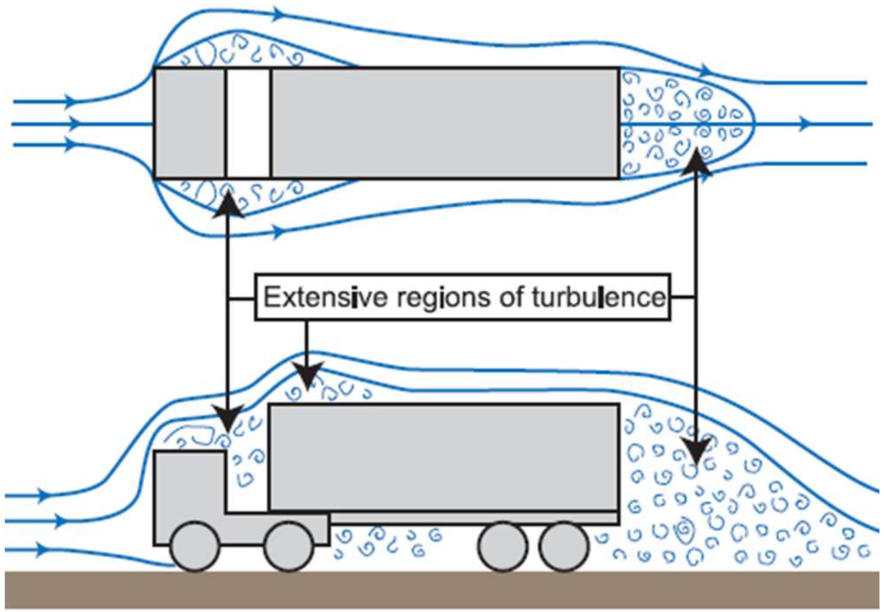


Different 2-D Models for Visualization – Circular Cylinder, Airfoil, Square Cylinder

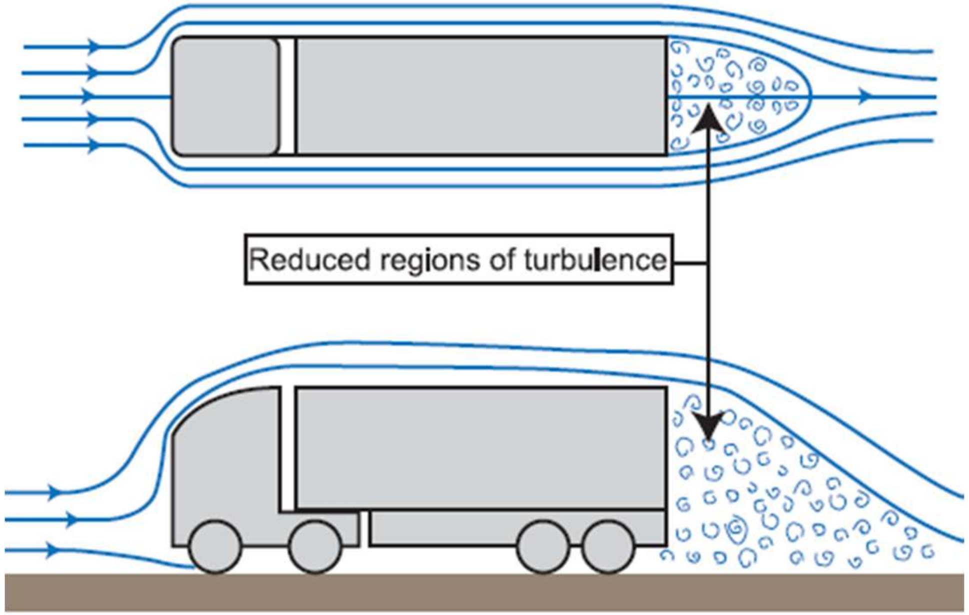
Heleshaw-Test section



Streamline pattern around a Simple Truck

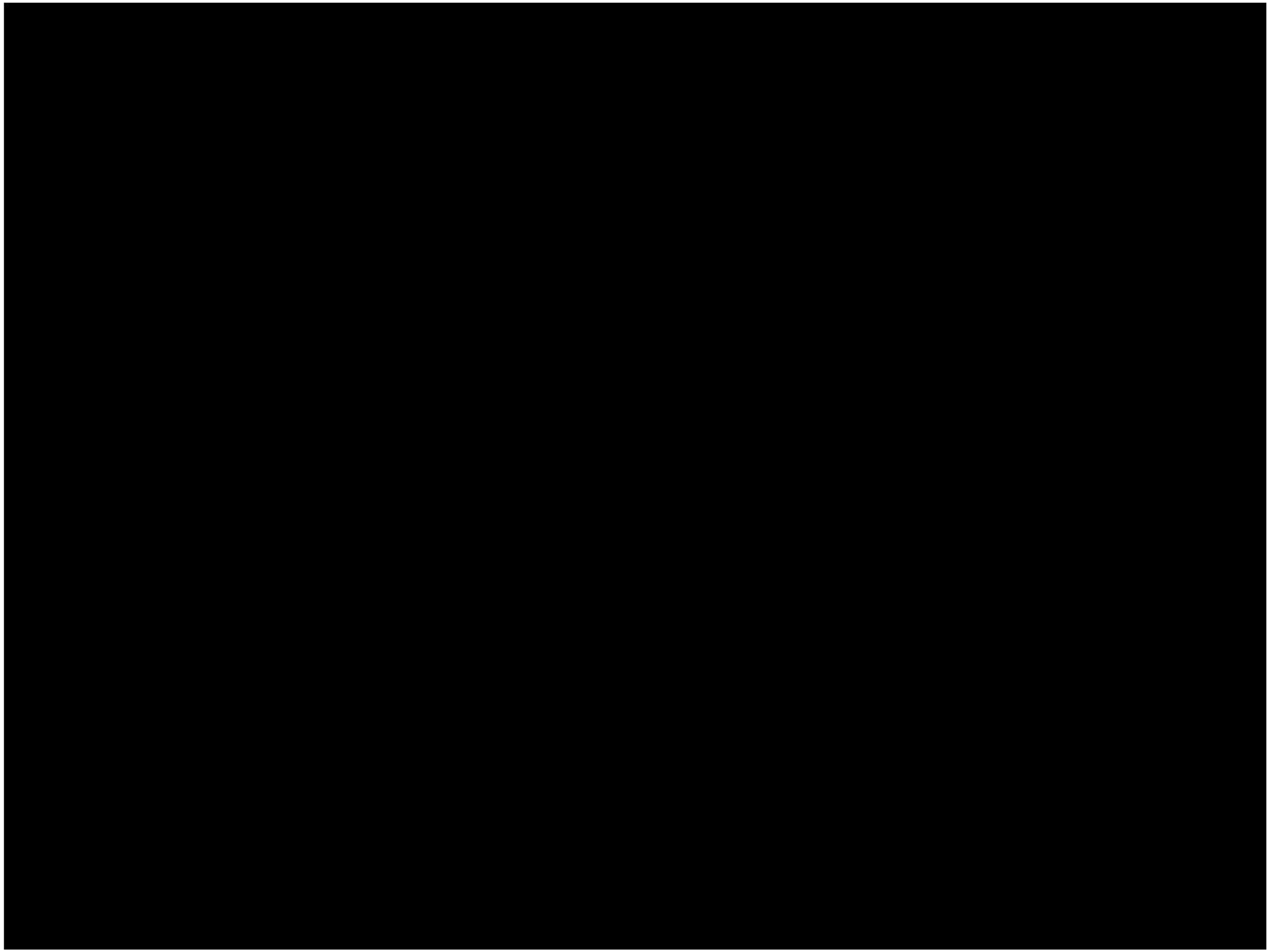


Typical Airflow Pattern around a Non-aerodynamic Truck



Typical Airflow Pattern around a aerodynamically effective Truck

Visualization of Streamlines around two Trucks in tandem-SV



Visualization of Streamlines around two Trucks in tandem-TV

OBSERVATIONS AND MODEL CALCULATIONS:

Area of the sink tank (first compartment), A_{sink} = (448 mm x 150 mm) = m^2

Area of the slit at the entrance of the test section, A_{slit} = 370 mm x 4 mm = m^2

Time taken for h = 5 cm rise of water in the sink tank (first compartment) = s

Water flow rate, $Q = \dfrac{A_{sink}\, h}{t} =$ m^3/s

Velocity of flow through the test section, $V = \dfrac{Q}{A_{slit}} =$ m/s

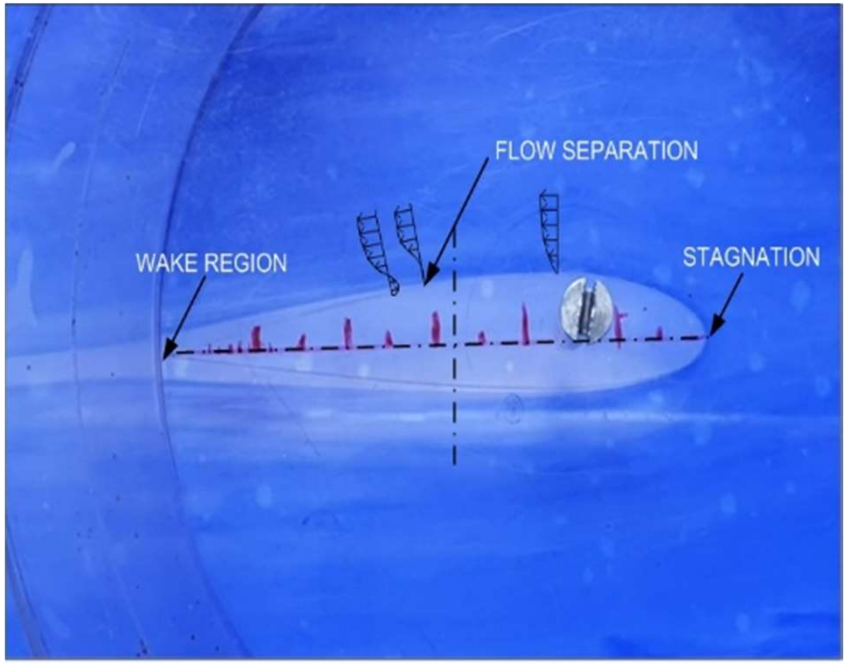
Chord length for the airfoil, L = m

Reynolds number, $Re = \dfrac{VL}{\upsilon} =$ (dimensionless)

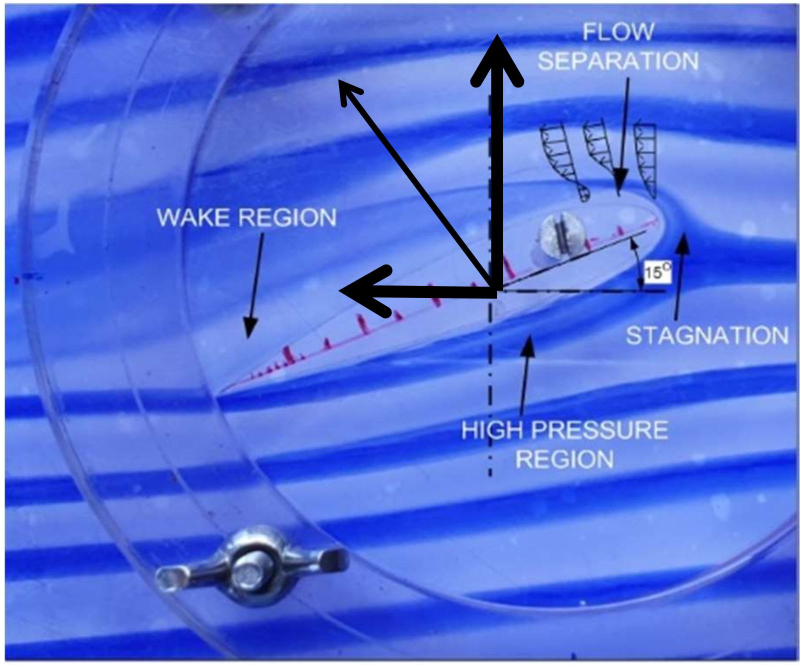
Angles of attack considered =

Calibrating the Flow velocities

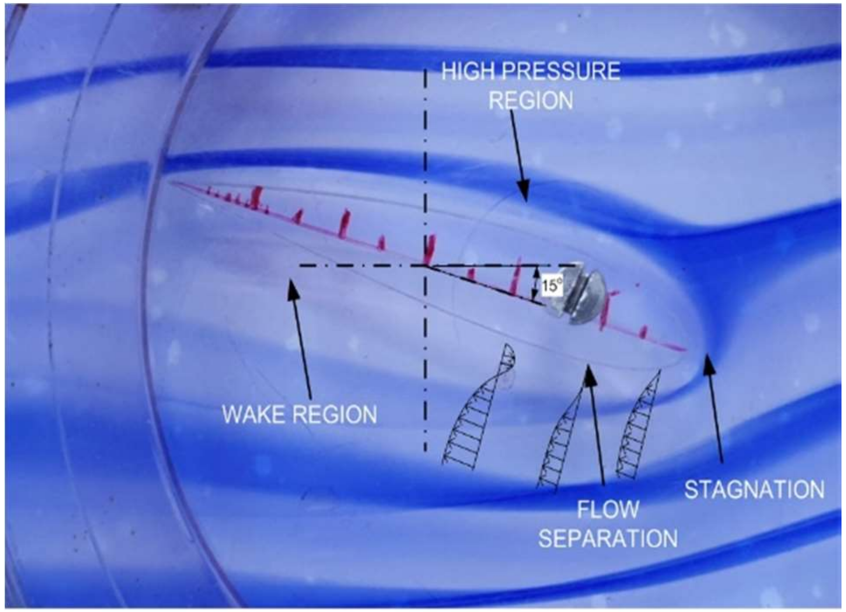
Valve Position (Degrees)	Height of water collected (cm)	Flow rate (m^3/s) x 10^3	Velocity (m/s)	Reynold's Number
90	5	0.195	0.111	9631.68
	7	0.192	0.108	9440.51
	10	0.194	0.109	9615.25
60	5	0.158	0.089	7774.14
	7	0.151	0.082	7223.29
	10	0.149	0.084	7367.89
45	5	0.082	0.046	4032.61
	7	0.097	0.054	4769.93
	10	0.099	0.055	4862.64



AIRFOIL 0 degree – Re 4555.06 – Separation pt at 0.467 x length



Airfoil 15 degrees – Re 4555.06 -Separation at 0.953 x length



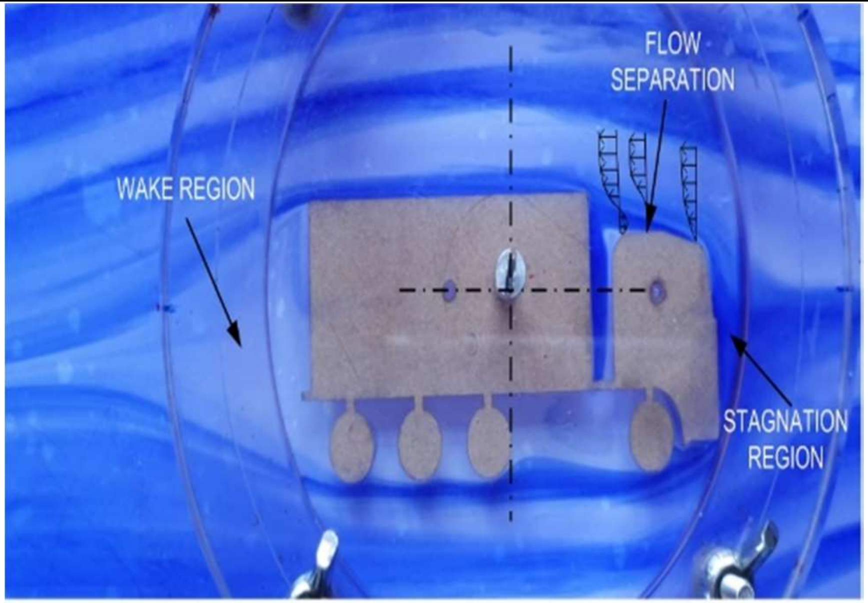
Airfoil- -15 degrees- Re 4555.06 – Separation at 0.818 x length

Visualization of Arifoil at angle of attack (α) = 0°, +15° and -15 °.

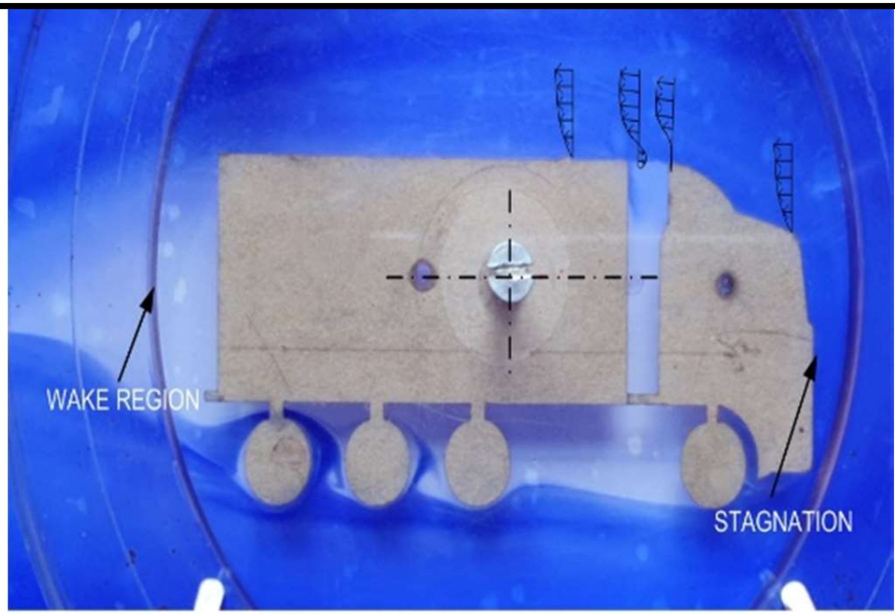
AIRFOIL

- The stagnation point is found on the leading edge, when the angle of attack (α) is 0°. There exists similar pressure distribution on the upper and bottom surface of the airfoil, a small change is brought due to camber experiencing lift force.
- When α=15°, the separation point is seen on the lower boundary of the airfoil and its position is also determined. From the pressure distribution around the airfoil, there exists a larger lift and drag force on the airfoil.
- In case when the angle of attack of the airfoil is at -15° the stagnation point is on the upper boundary of the airfoil near the leading edge and from the pressure distribution it can be seen more downforce than lift is experienced by the airfoil.

TRUCK WITH AND WITHOUT DEFLECTOR



Truck Linear – Re 4555.06 Separation at 0.934 x length



Truck with Deflector – Re 4555.06 – Separation at 0.757 x length

The separation point is seen on the top of the cabin. The flow moves along the boundary of the trailer and gets separated at the end. Visuals depicts wake is strong, Vortex formation can be seen at the gap between the cabin and the trailer.

Due to the deflector, the flow is diverted and moves smoothly along the boundary without getting stagnated at the top edge of the trailer. Deflector has a major role in reducing the frontal pressure drag. Interaction of vortex flow between the cabin and trailer is avoided.

Results and Conclusions

- Hele-Shaw apparatus can give fair predictions about relative pressure zones around the structure. With accurate scaling and dimensioning of structures, we can get a close relationship of flow parameters between experimental and actual structures.
- The flow parameters such as stagnation, separation and wake regimes are clearly captured for the aerofoil and trailer-tractor truck at various Reynolds number.
- Similarly the flow parameters such as stagnation, separation and wake regimes can be clearly captured for the given cambered aerofoil, circular cylinder, Square at various Reynolds number.

Credits to the members of Aerodynamics laboratory, School of Mechanical Engineering, VIT Chennai.

Thank you very much..☺

Please subscribe to our YouTube channel for more updates on Flow visualization
https://www.youtube.com/channel/UCPHV628SolZbTxH9Y2tMIjQ