Problems on
Venturimeter and Orificemeter

01 / 02 /2023

Fluid Mechanics and Machines

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid
S_o = Sp. gravity of the liquid flowing through pipe
x = Difference of the heavier liquid column in U-tube

Then
$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.10)}$$

where S_l = Sp. gr. of lighter liquid in U-tube
S_o = Sp. gr. of fluid flowing through pipe
x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.11)}$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.12)}$$

Problem 6.13 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take C_d = 0.98.*

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

ρ for water $= 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$$\frac{p_2}{\rho g} = -\ 30 \text{ cm of mercury}$$

$$= -\ 0.30 \text{ m of mercury} = -\ 0.30 \times 13.6 = -\ 4.08 \text{ m of water}$$

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$\therefore$ Differential head $= h = \dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g} = 18 - (-\ 4.08)$

$$= 18 + 4.08 = 22.08 \text{ m of water} = 2208 \text{ cm of water}$$

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$= \dfrac{50328837.21}{304} \times 165555 \text{ cm}^3\text{/s} =$ **165.555 lit/s. Ans.**

Obstruction Flowmeters: Orifice, Venturi, and Nozzle Meters

Consider incompressible steady flow of a fluid in a horizontal pipe of diameter D that is constricted to a flow area of diameter d, as shown in Fig. 8–55. The mass balance and the Bernoulli equations between a location before the constriction (point 1) and the location where constriction occurs (point 2) can be written as

Mass balance: $$\dot{V} = A_1V_1 = A_2V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2V_2 \tag{8–68}$$

Bernoulli equation ($z_1 = z_2$): $$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \tag{8–69}$$

Combining Eqs. 8–68 and 8–69 and solving for velocity V_2 gives

Obstruction (with no loss): $$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \tag{8–70}$$

where $\beta = d/D$ is the diameter ratio. Once V_2 is known, the flow rate can be determined from $\dot{V} = A_2V_2 = (\pi d^2/4)V_2$.

Obstruction flowmeters: $$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \textbf{(8–71)}$$

where $A_0 = A_2 = \pi d^2/4$ is the cross-sectional area of the hole and $\beta = d/D$ is the ratio of hole diameter to pipe diameter. The value of C_d depends on both β and the Reynolds number $\text{Re} = V_1 D/\nu$, and charts and curve-fit correlations for C_d are available for various types of obstruction meters.

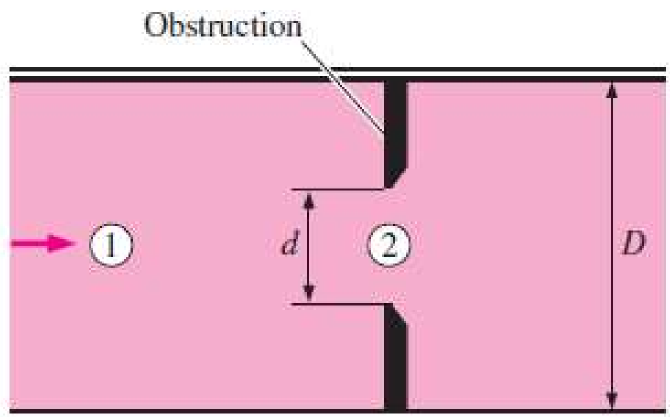
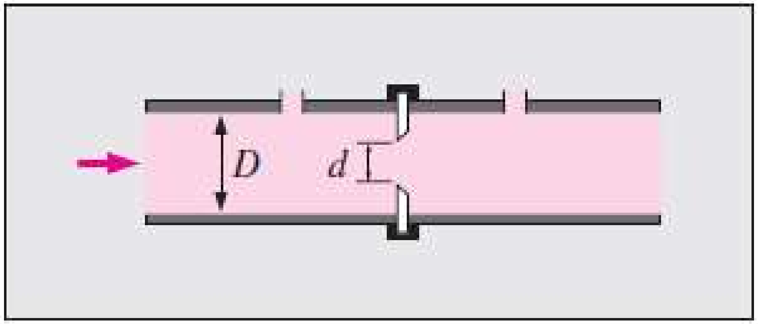
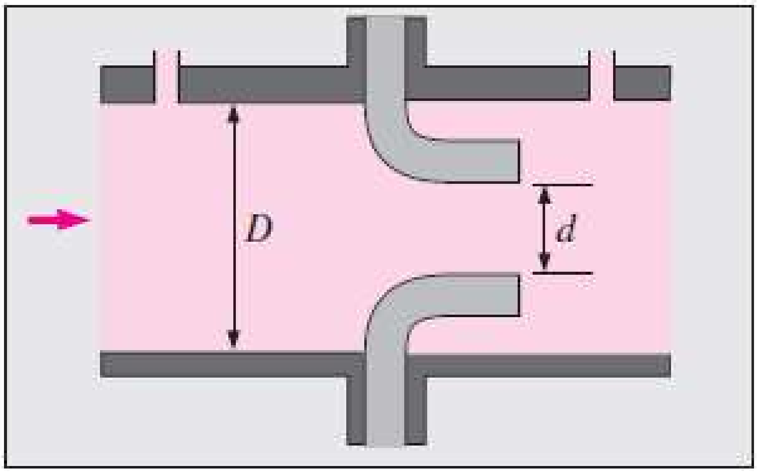


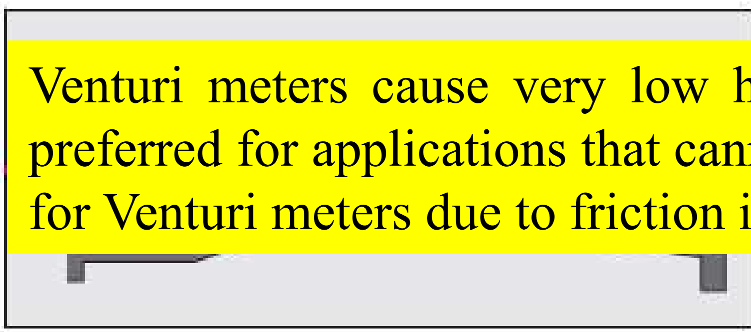
FIGURE 8–55
Flow through a constriction in a pipe.



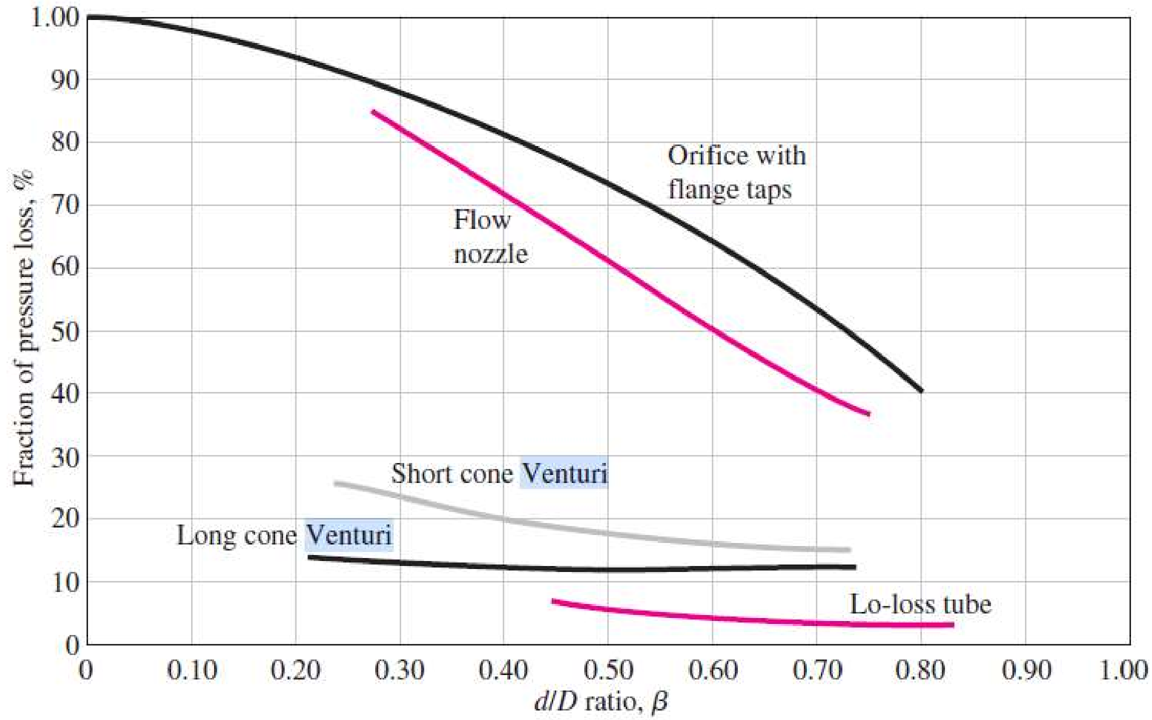
(a) Orifice meter



(b) Flow nozzle



(c) Venturi meter



experimentally determined data for discharge coefficients are expressed as (Miller, 1997)

Orifice meters: $$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}} \quad \textbf{(8–72)}$$

Nozzle meters: $$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\mathrm{Re}^{0.5}} \quad \textbf{(8–73)}$$

These relations are valid for $0.25 < \beta < 0.75$ and $10^4 < \mathrm{Re} < 10^7$. Precise

Venturi meters cause very low head losses, as shown in Fig. and thus they should be preferred for applications that cannot allow large pressure drops. The irreversible head loss for Venturi meters due to friction is only about 10 percent.

meters are very high, ranging between 0.95 and 0.99 (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take $C_d = 0.98$ for Venturi meters. Also, the Reynolds number depends on the flow velocity, which is not known a priori. Therefore, the solution is iterative in nature when curve-fit correlations are used for C_d.

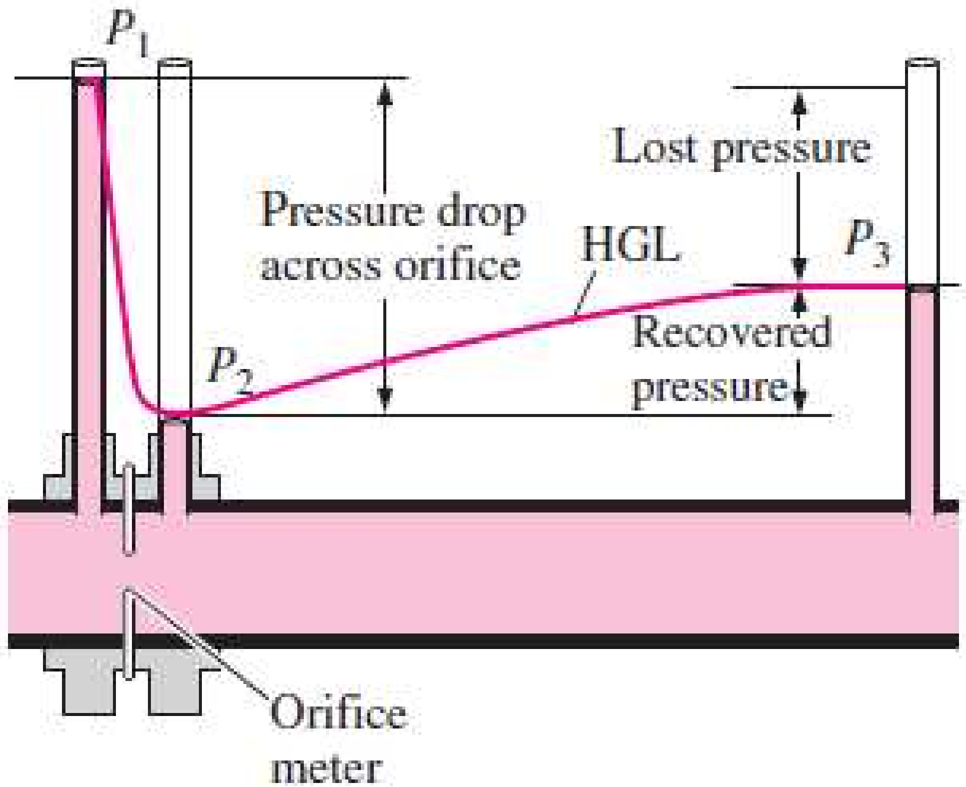
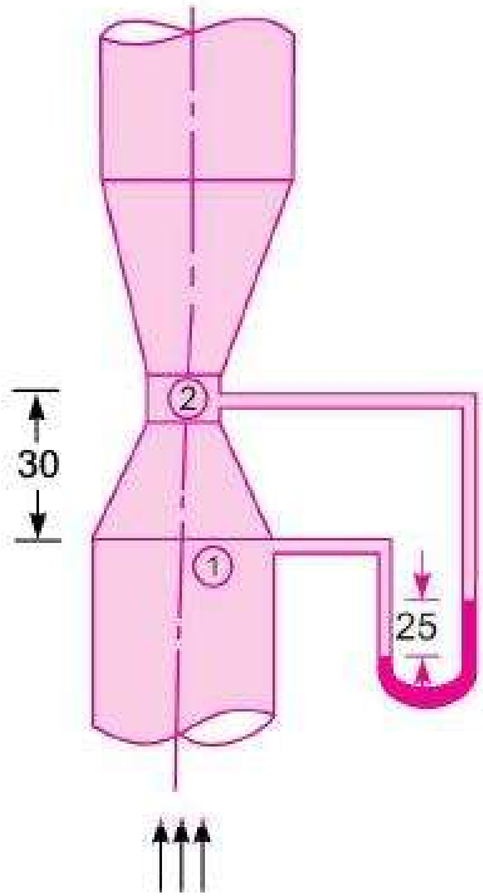


FIGURE 8–58
The variation of pressure along a flow section with an orifice meter as measured with piezometer tubes; the lost pressure and the pressure recovery are shown.

Problem 6.19 *A 30 cm × 15 cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate :*

(i) the discharge of oil, and

(ii) the pressure difference between the entrance section and the throat section. Take the co-efficient of discharge as 0.98 and specific gravity of mercury as 13.6.



Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \dfrac{\pi}{4}(30)^2 = 706.85\ \text{cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ Area, $a_2 = \dfrac{\pi}{4}(15)^2 = 176.7\ \text{cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$

Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

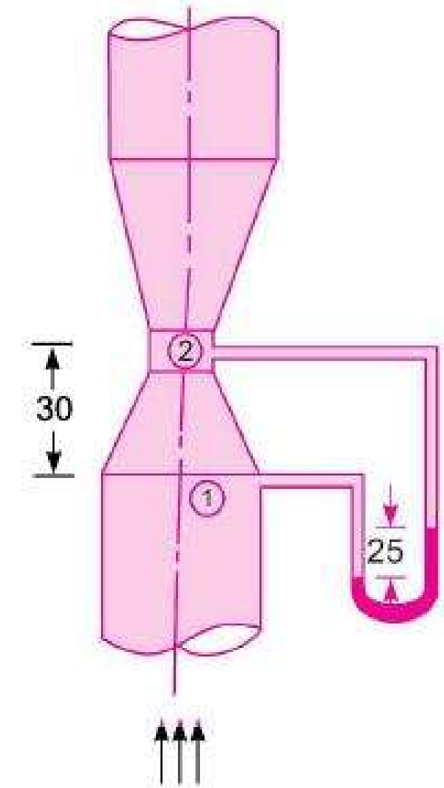
$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil}$$

(*i*) The discharge, Q of oil $\quad = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$



(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

$$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $\quad z_2 - z_1 = 30 \text{ cm}$

$\therefore \quad \left(\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}\right) - 30 = 352.77$

$\therefore \quad \dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g} = 352.77 + 30 = 382.77$ cm of oil = **3.8277 m of oil. Ans.**

$$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $\quad = \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$$

But density of oil $= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$

$$\therefore \qquad (p_1 - p_2) = 3.8277 \times 900 \times 9.81 \ \frac{\text{N}}{\text{m}^2}$$

$$= \frac{33795}{10^4} \text{ N/cm}^2 = \mathbf{3.3795 \ N/cm^2.\ Ans.}$$

Problem 6.22 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \dfrac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$$

$$\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

$$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$$C_d = 0.6$$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = \mathbf{68.21 \text{ litres/s. Ans.}}$$

Thank you