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MEE1004 – Fluid Mechanics

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FLOW MEASUREMENT DEVICES

Measuring the flow properties such as pressure, velocity, temperature etc.,

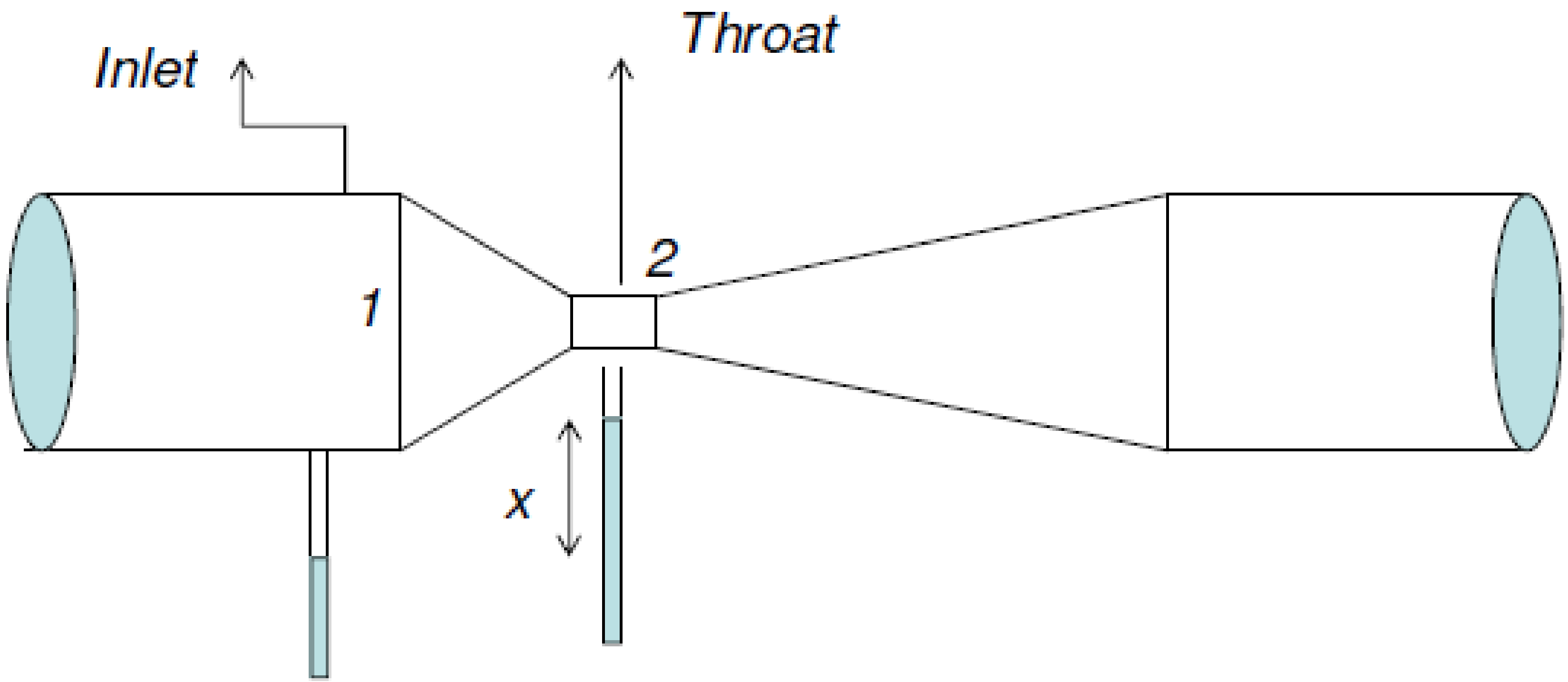
- Venturimeter
- Orificemeter
- Pitot tube

Venturimeter: is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts:

- A short converging
- Throat
- Diverging part

Let d_1 = diameter at the inlet (section 1)
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1 = *area at section1*

d_2, p_2, v_2, A_2 are the corresponding values at the throat (section 2)



Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\,2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\,2}}{2g}+z_2.$$

As pipe is horizontal $z_1=z_2$

$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^{\,2}-v_1^{\,2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{\,2}-v_1^{\,2}}{2g}$$

Where $h\equiv\frac{p_1-p_2}{\rho g}$, difference of pressure heads at sections 1 and 2.

From the continuity equation at sections 1 and 2, we obtain

$$A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

Hence

$$h = \frac{v_2{}^2}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$\Rightarrow v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Discharge

$$Q = A_1 v_1 = A_2\, v_2$$

$$\Rightarrow Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Note that the above expression is for ideal condition and is known as theoretical discharge.

Actual discharge will be less than theoretical discharge.

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$$

C_d is the coefficient of venturimeter and its value is always less then 1.

Expression of 'h' given by differential U-tube manometer:

Case 1: The liquid in the manometer is heavier than the liquid flowing through the pipe

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

S_h: Specific gravity of the heavier liquid.

S_0: Specific gravity of the flowing liquid.

Case 2: The liquid in the manometer is lighter than the liquid flowing through the pipe

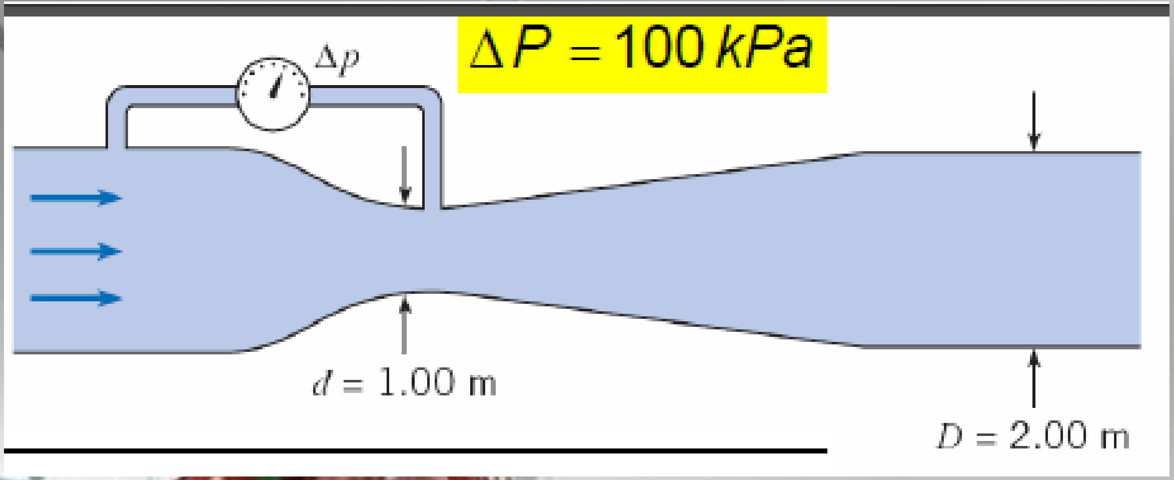
$$h = x\left[1 - \frac{S_L}{S_0}\right]$$

S_L: Specific gravity of the lighter liquid.

X: difference of the liquid columns in U-tube

Situation: Water flows through a horizontal venturi meter. $\Delta p = 100$ kPa, $d = 1$ m, $D = 2$ m.

Find: Discharge: Q



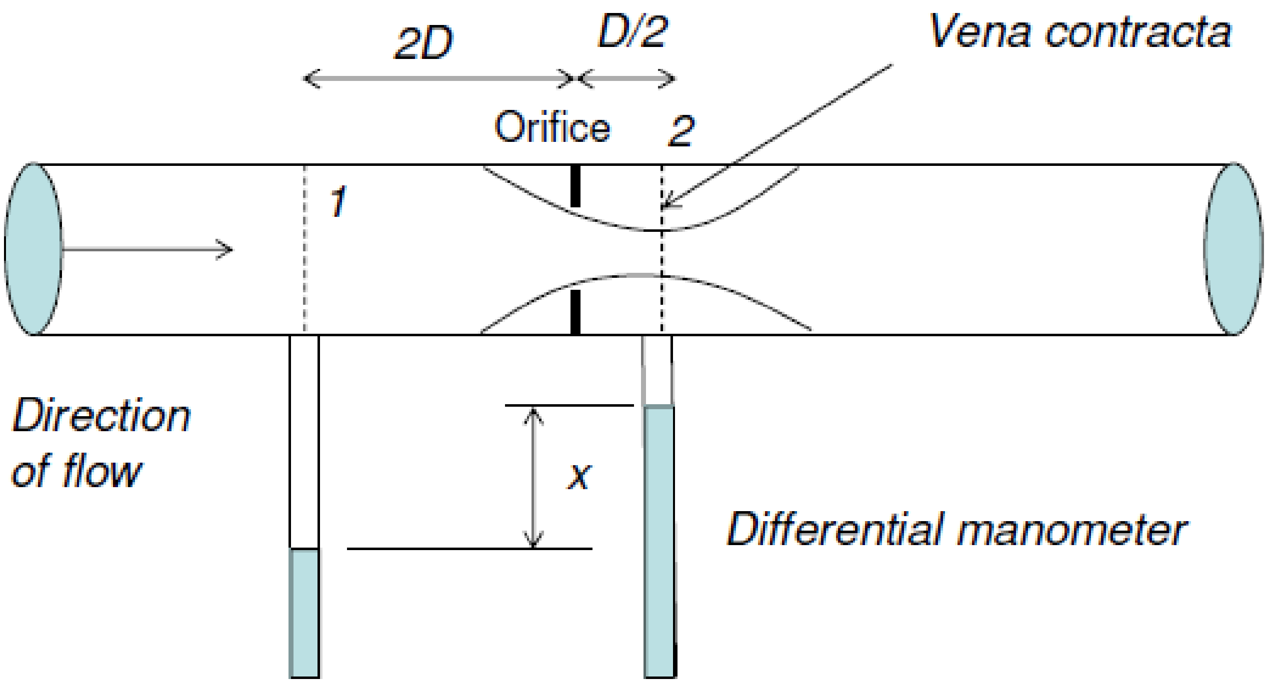
Orifice meter

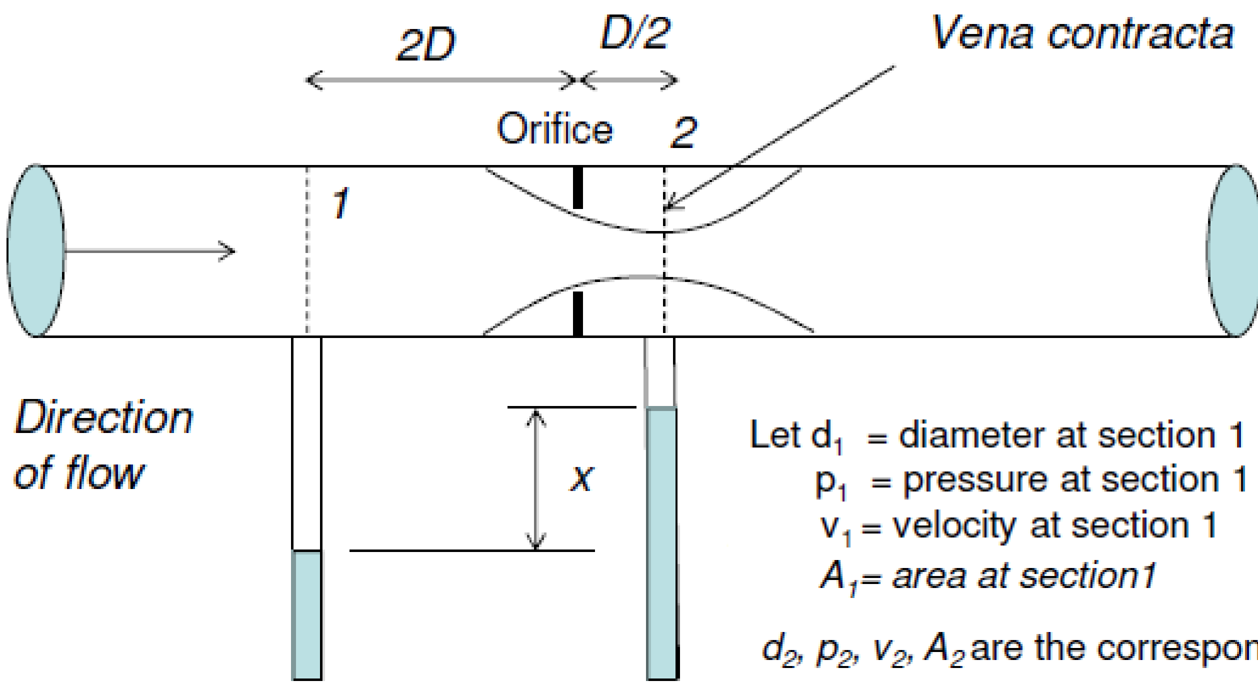
- **Orifice meter:** is a device used for measuring the rate of flow of a fluid flowing through a pipe.

- It is a cheaper device as compared to venturimeter. This also work on the same principle as that of venturimeter.

- It consists of flat circular plate which has a circular hole, in concentric with the pipe. This is called orifice.

- The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.





Let d_1 = diameter at section 1
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at section 2.

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\,2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\,2}}{2g}+z_2$$

$$\Rightarrow\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^{\,2}-v_1^{\,2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{\,2}-v_1^{\,2}}{2g}$$

$$\Rightarrow v_2=\sqrt{2gh+v_1^2}$$

where h is the differential head.

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_c = \dfrac{A_2}{A_0}$

By continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_1 = \frac{A_0 C_c}{A_1} v_2$$

Hence,

$$v_2 = \sqrt{2gh + \frac{A_0^2 C_c^2 v_2^2}{A_1^2}}$$

$$\Rightarrow v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

Thus, discharge,

$$Q = A_2 v_2 = v_2 A_0 C_c = \frac{A_0 C_c \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

$$\Rightarrow C_c = C_d \frac{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}{\sqrt{1 - \frac{A_0^2}{A_1^2}}}$$

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

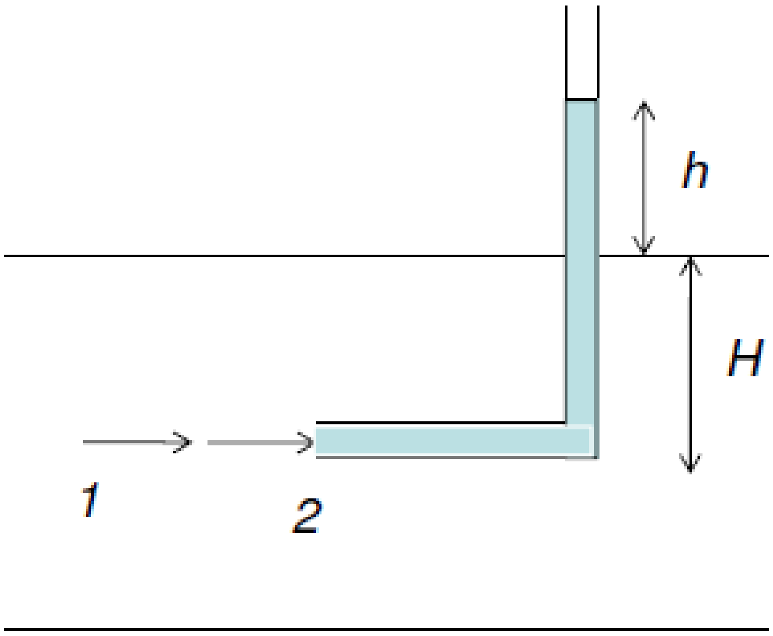
The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

PITOT TUBE

- Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.

- In simplest form, the pitot tube consists of a glass tube, bent at right angles.

Let p_1 = pressure at section 1
p_2 = pressure at section 2
v_1 = velocity at section 1
v_2 = velocity at section 2 = 0
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface



Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{\ 2}}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0.$

$\frac{p_1}{\rho g}=$ Pressure head at 1=H

$\frac{p_2}{\rho g}=$ Pressure head at 2=h+H

Substituting these values, we get $H+\frac{v_1^{\ 2}}{2g}=h+H$

$$\Rightarrow v_1=\sqrt{2gh}$$

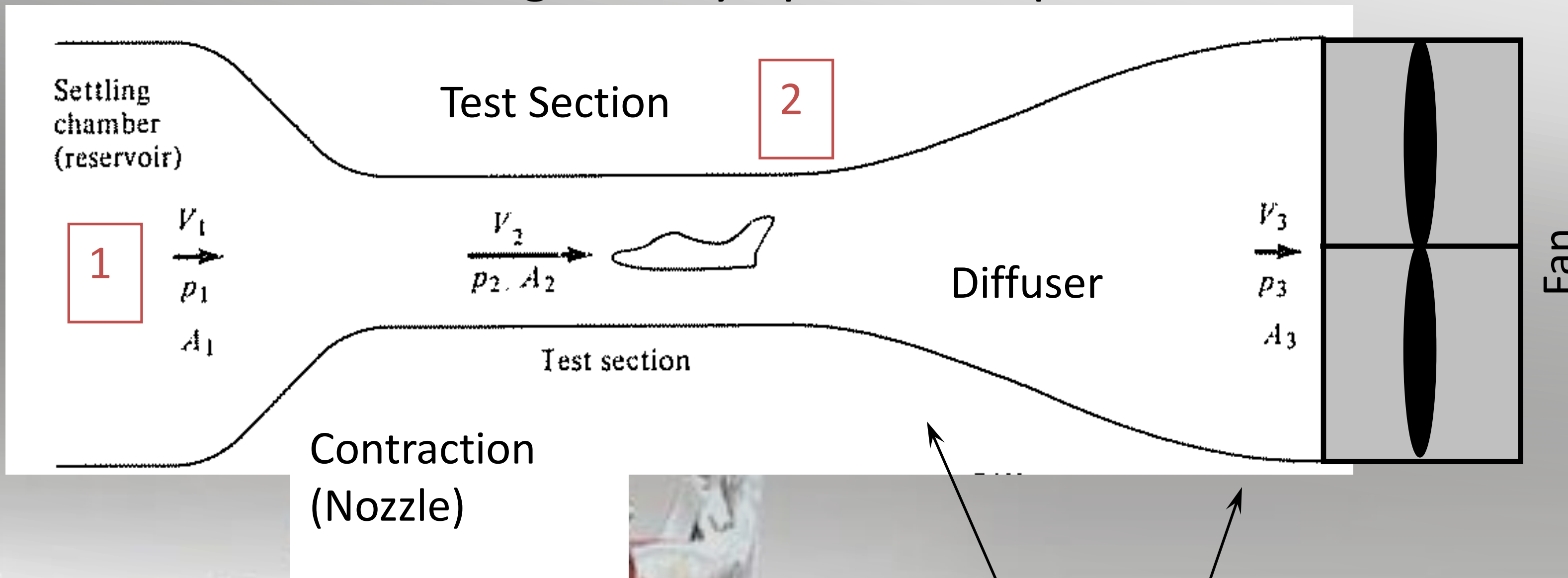
This is theoretical velocity. Actual velocity is given by

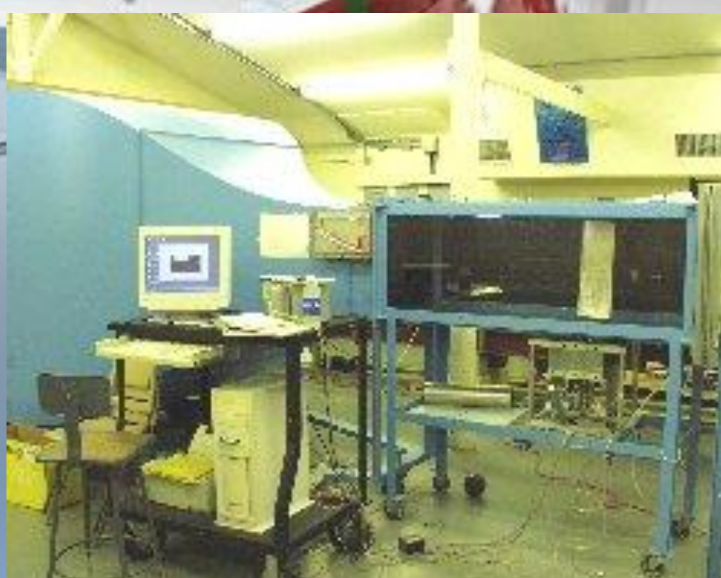
$$(v_1)_{act}=C_v\sqrt{2gh}$$

$C_v \equiv$ coefficient of pitot-tube

EXAMPLE: LOW-SPEED, SUB-SONIC WIND TUNNEL

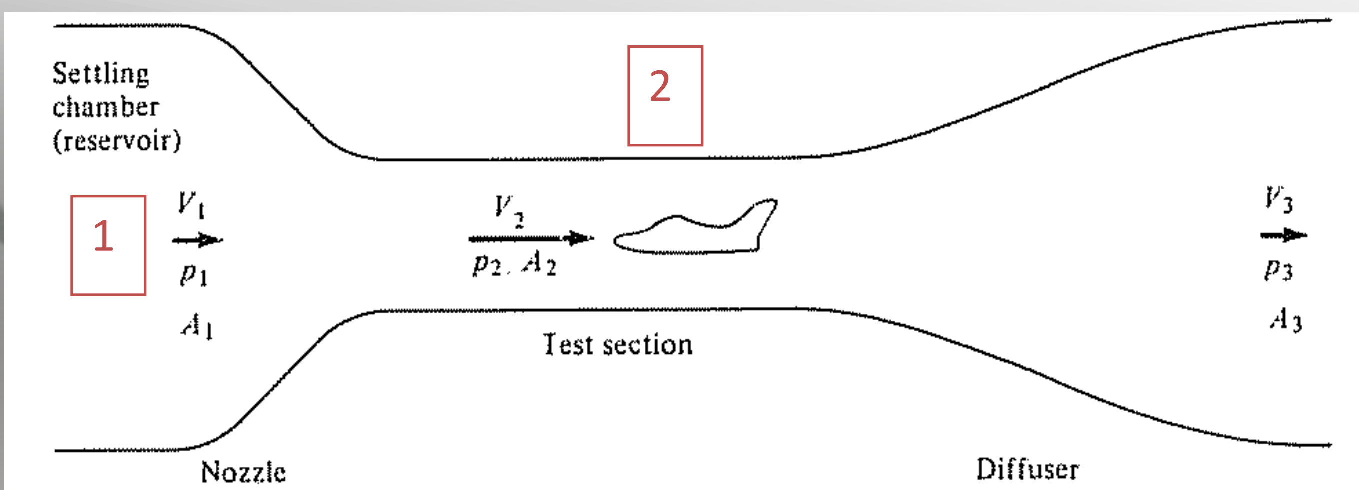
- Subsonic wind tunnels generally operate at speeds < 300 MPH





Why build all of this?

EXAMPLE: LOW-SPEED, SUB-SONIC WIND TUNNEL



- At speeds M < 0.3 (or ~ 100 m/s) flow regarded as incompressible
- Analyze using conservation of mass (continuity) and Bernoulii's Equation

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

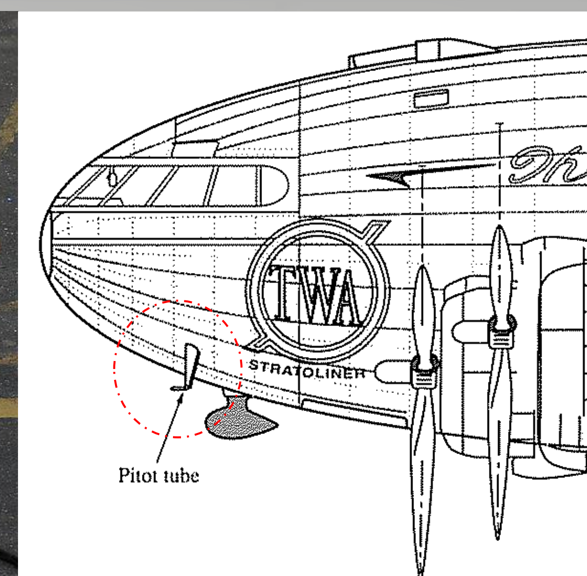
$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + V_1^2$$

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]}}$$

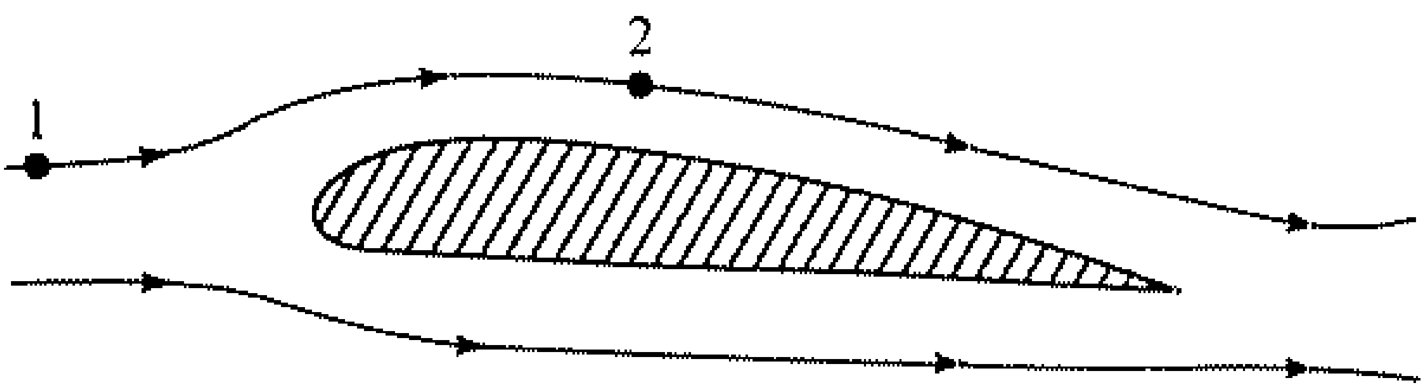
MEASUREMENT OF AIRSPEED

- How do we measure an airplanes speed in flight?
- Pitot tubes are used on aircraft as speedometers (point measurement)



STATIC VS. TOTAL PRESSURE

- 2 types of pressure: Static and Total (Stagnation)
- Static Pressure, p
  - Due to random motion of gas molecules
  - Pressure we would feel if moving along with the flow
  - Pressure in Bernoulli's equation is static pressure
- Total (Stagnation) Pressure, p_0 or p_t
  - Property associated with flow motion
  - Total pressure at a given point in flow is the pressure that would exist if flow were slowed down isentropically to zero velocity
- $p_0 > p$



MEASUREMENT OF AIRSPEED: INCOMPRESSIBLE FLOW

$$p + \frac{1}{2}\rho V_1^2 = p_0$$

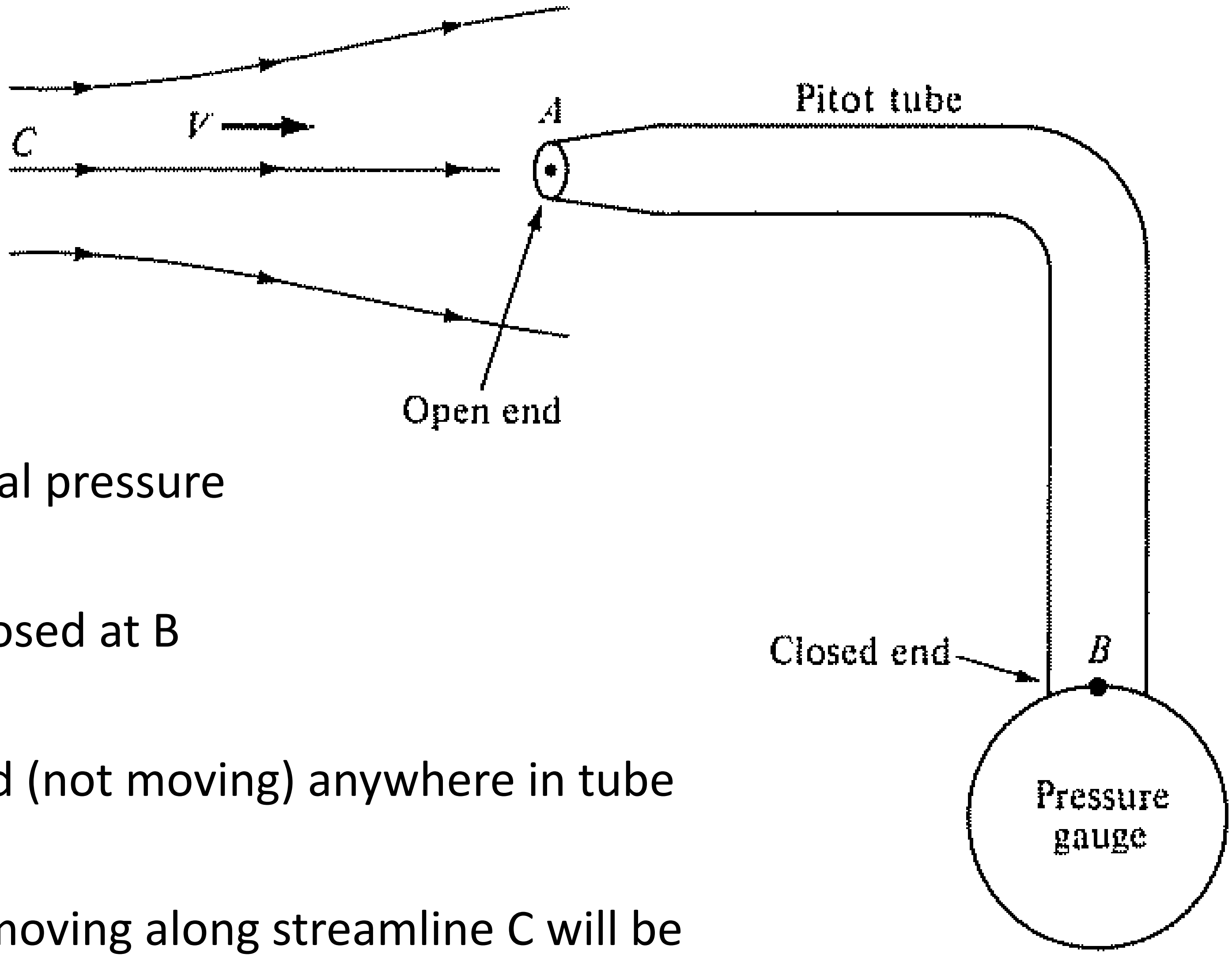
Static pressure

Dynamic pressure

Total pressure

$$V_1 = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

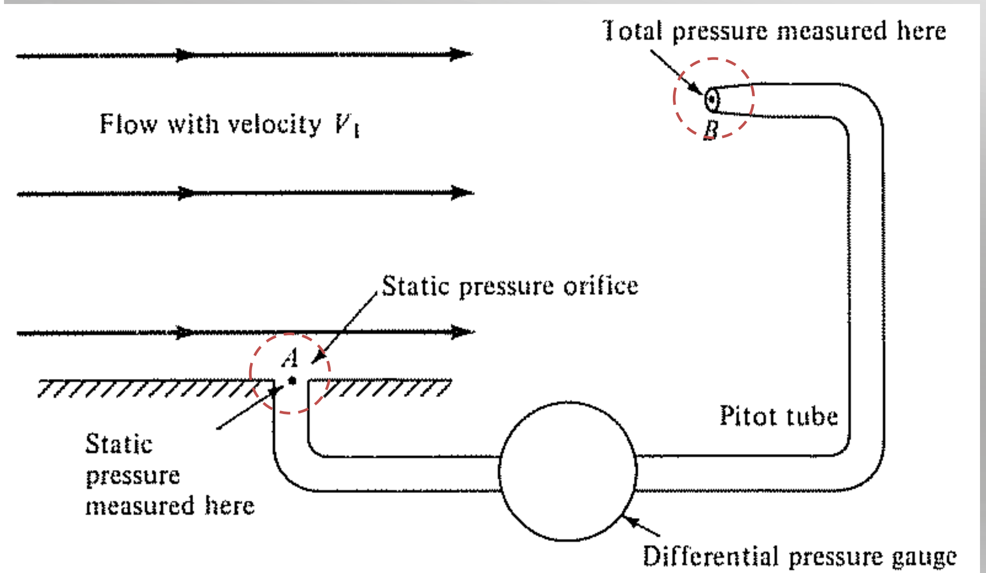
Incompressible Flow

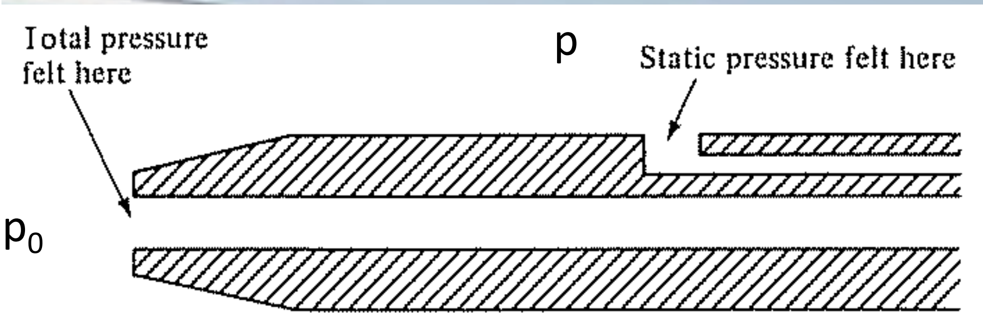


- Measures total pressure
- Open at A, closed at B
- Gas stagnated (not moving) anywhere in tube
- Gas particle moving along streamline C will be isentropically brought to rest at point A, giving total pressure

EXAMPLE: MEASUREMENT OF AIRSPEED (4.11)

- Point A: Static Pressure, p
  - Surface is parallel to flow, so only random motion of gas is measured
- Point B: Total Pressure, p_0
  - Aligned parallel to flow, so particles are isentropically decelerated to zero velocity
- A combination of p_0 and p allows us to measure V_1 at a given point
- Instrument is called a Pitot-static probe





Pitot-static probe

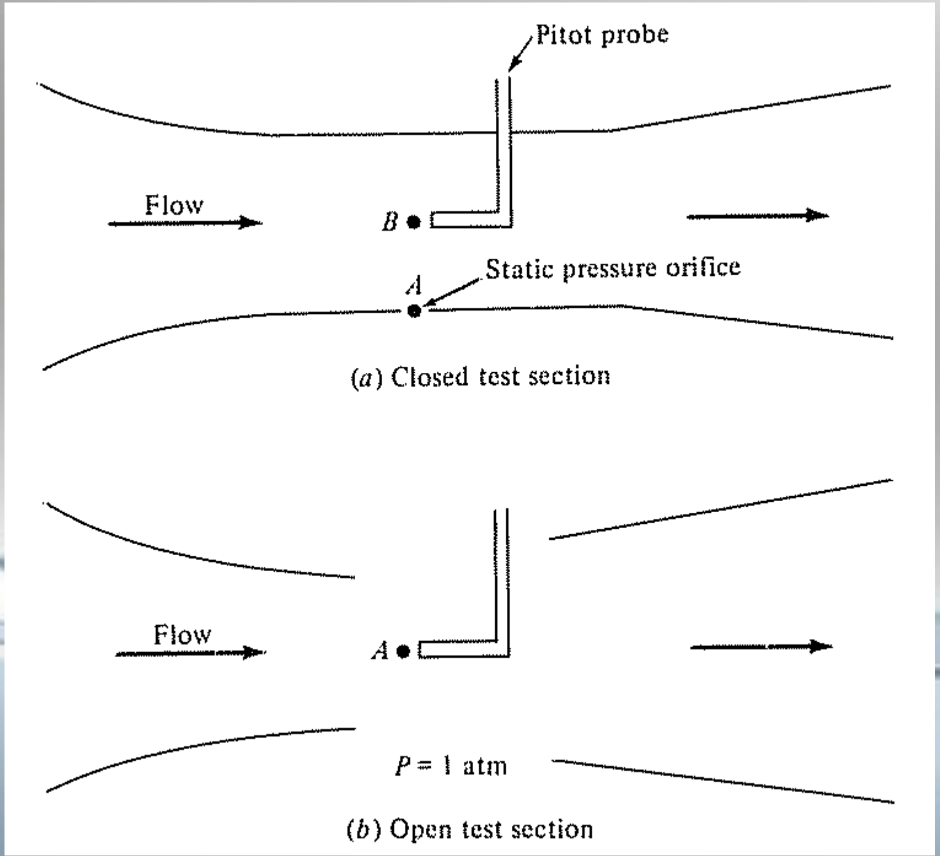
MEASUREMENT OF AIRSPEED: INCOMPRESSIBLE FLOW

$$p + \frac{1}{2}\rho V_1^2 = p_0$$

Static pressure | Dynamic pressure | Total pressure

$$V_1 = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

Incompressible Flow



(a) Closed test section

(b) Open test section

TRUE VS. EQUIVALENT AIRSPEED

- What is value of ρ?
- If ρ is measured in actual air around the airplane
- Measurement is difficult to do

$$V_{true} = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

- Practically easier to use value at standard seal-level conditions, ρ_s
- This gives an expression called the equivalent airspeed

$$V_e = \sqrt{\frac{2(p_0 - p)}{\rho_s}}$$

MEASUREMENT OF AIRSPEED: SUBSONIC COMRESSIBLE FLOW

- If M > 0.3, flow is compressible (density changes are important)
- Need to introduce energy equation and isentropic relations

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_0$$

$$\frac{T_0}{T_1} = 1 + \frac{V_1^2}{2 c_p T_1}$$

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

$$\boxed{\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/\gamma - 1}}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{1/\gamma - 1}$$

c_p: specific heat at constant pressure
$M_1 = V_1/a_1$
$\gamma_{air} = 1.4$

MEASUREMENT OF AIRSPEED: SUBSONIC COMRESSIBLE FLOW

- So, how do we use these results to measure airspeed

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

p_0 and p_1 give
Flight Mach number
Mach meter

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

$M_1 = V_1/a_1$
Actual Flight Speed

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_1} + 1\right)^{(\gamma-1)/\gamma} - 1\right]$$

Actual Flight Speed
using pressure difference

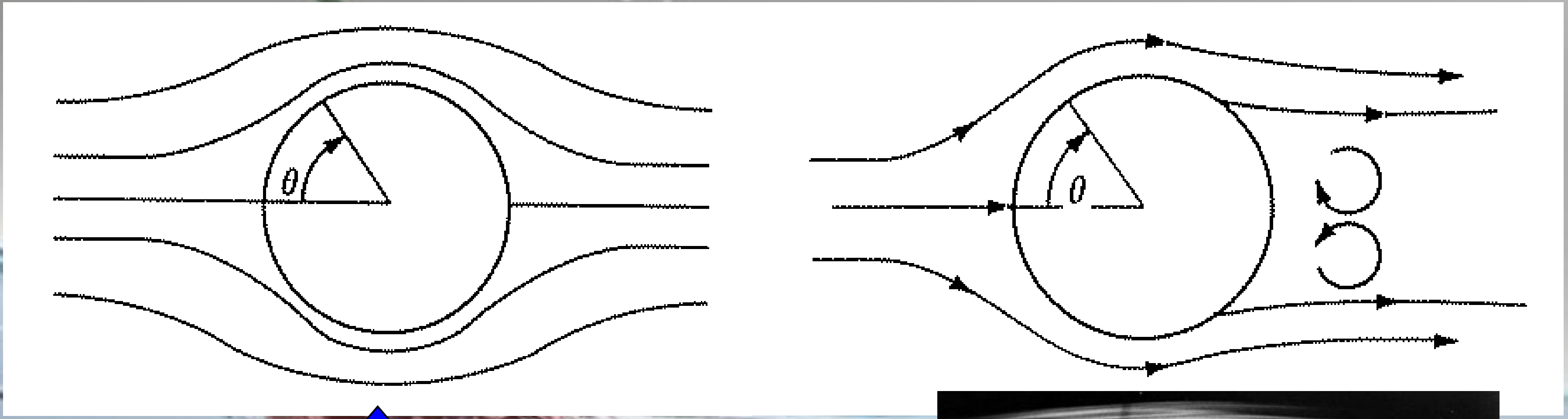
$$V_{cal}^2 = \frac{2a_s^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_s} + 1\right)^{(\gamma-1)/\gamma} - 1\right]$$

What is T_1 and a_1?
Again use sea-level conditions T_s, a_s, p_s
(a_1=340.3 m/s)

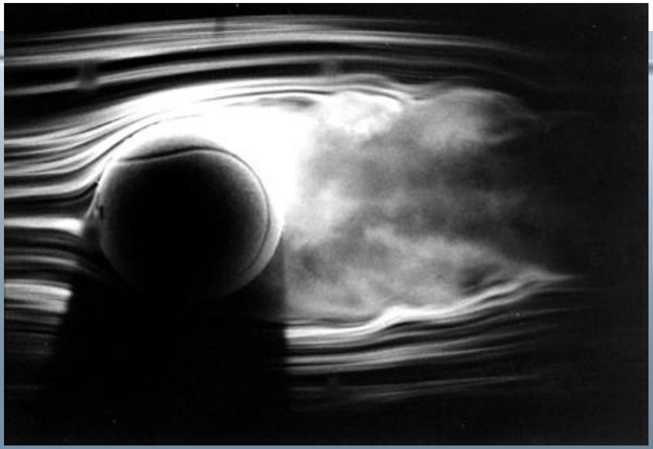
REAL EFFECTS: VISCOSITY (μ)

- To understand drag and actual airfoil/wing behavior we need an understanding of viscous flows (all real flows have friction)

- Inviscid (frictionless) flow around a body will result in zero drag!
  - Called d'Alembert's paradox (Must include friction in theory)



We will derive this streamline pattern in class next week




THANK YOU !!!
Vinayagamurthy, SMBS, VIT CHENNAI