Fluid Kinematics

Lecture 1 of Module 2

G. Vinayagamurthy Dr. Engg.
School of Mechanical Engineering
VIT Chennai

Fluid Flow Concepts

Last Lecture Content

Discussed about:

- fluid motion, Fluid Kinetics and Fluid Dynamics
- fluid flows
- temporal and spatial classifications

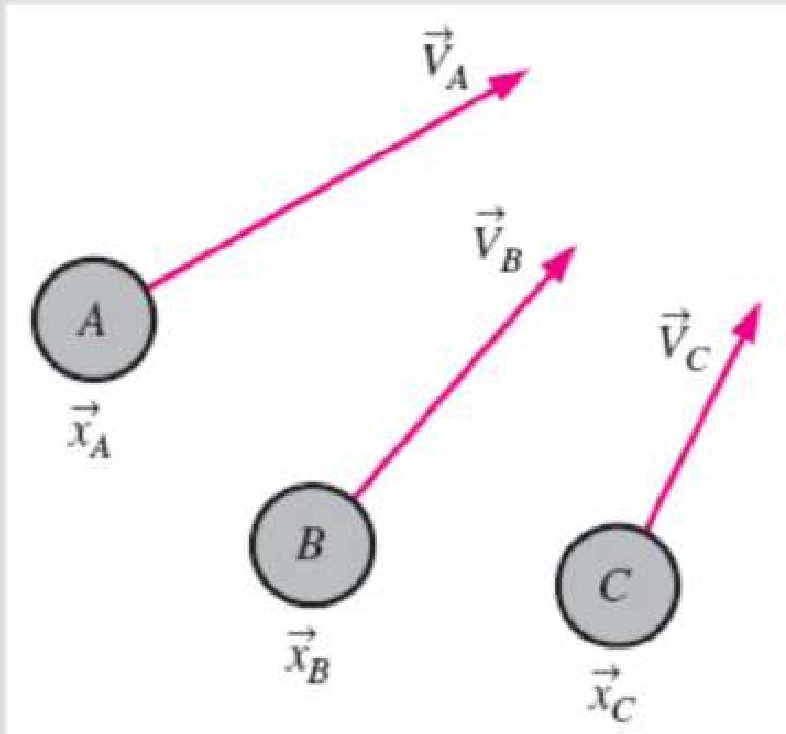
Analysis Approaches

- Lagrangian vs. Eulerian

Lagrangian description: To follow the path of individual objects.

This method requires us to track the position and velocity of each individual fluid parcel (**fluid particle**) and take to be a parcel of fixed identity.

With a small number of objects, such as billiard balls on a pool table, individual objects can be tracked.



In the Lagrangian description, one must keep track of the position and velocity of individual particles.

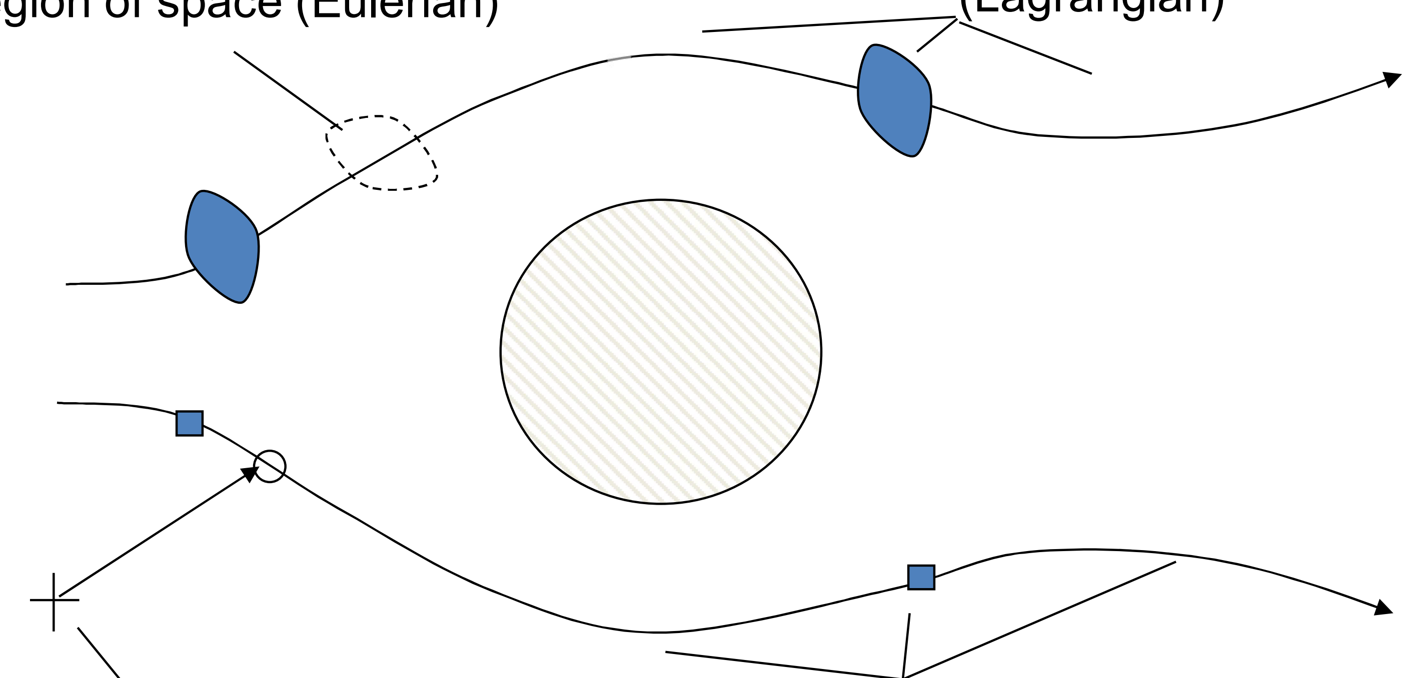
Perspectives

Eulerian Perspective – the flow as as seen at fixed locations in space, or over fixed volumes of space. (The perspective of most analysis.)

Lagrangian Perspective – the flow as seen by the fluid material. (The perspective of the laws of motion.)

Control volume: finite fixed region of space (Eulerian)

Fluid system: finite piece of the fluid material (Lagrangian)



Coordinate: fixed point in space (Eulerian)

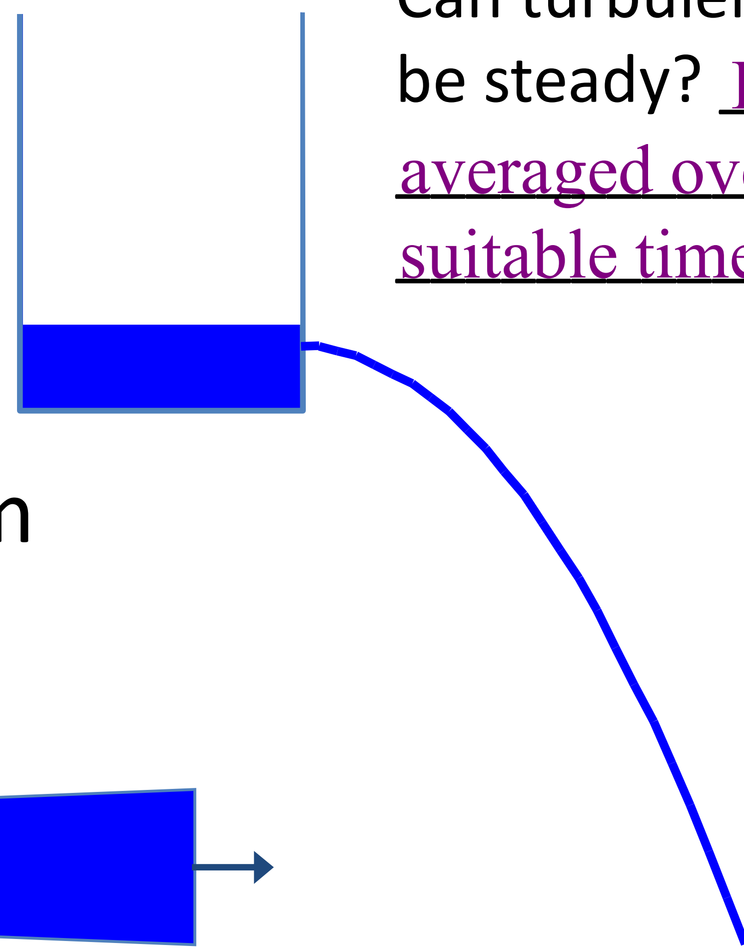
Fluid particle: differentially small finite piece of the fluid material (Lagrangian)

- In the Eulerian description of fluid flow, a finite volume called a flow domain or control volume is defined, through which fluid flows in and out.
- Instead of tracking individual fluid particles, we define **field variables**, functions of space and time, within the control volume.
- The field variable at a particular location at a particular time is the value of the variable for whichever fluid particle happens to occupy that location at that time.
- For example, the **pressure field** is a **scalar field variable**. We define the velocity field as a vector field variable.

Pressure field:	$P = P(x, y, z, t)$
Velocity field:	$\vec{V} = \vec{V}(x, y, z, t)$
Acceleration field:	$\vec{a} = \vec{a}(x, y, z, t)$

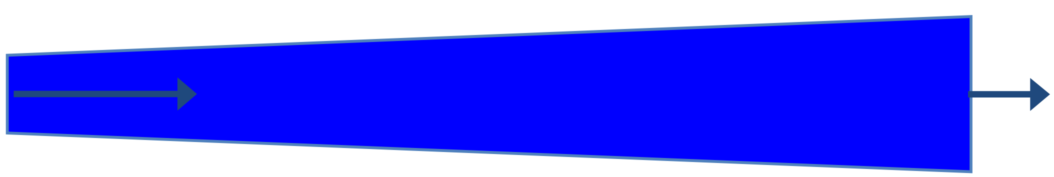
Temporal/Spatial Classifications

- Steady - unsteady
  - Changing in time ______________________



Can turbulent flow be steady? If ______ averaged over a ______ suitable time ______

- Uniform - nonuniform
  - Changing in space ______________________



Temporal/Spatial Classifications

Steady flow:
When the velocity at each location is constant, the velocity field is invarient with time and the flow is said to be steady.

The flow in which the field variables don't vary with time is said to be steady flow. For steady flow,

$$\frac{\delta v}{\delta t} = 0 \qquad \text{Or} \qquad \vec{v} = \vec{v}(x, y, z)$$

If the field variables in a fluid region vary with time the flow is said to be unsteady flow.

$$\frac{\delta v}{\delta t} \neq 0 \qquad \vec{v} = \vec{v}(x, y, z, t)$$

Uniform flow:
Uniform flow occurs when the magnitude and direction of velocity do not change from point to point in the fluid.
Flow of liquids through long pipelines of constant diameter is uniform whether flow is steady or unsteady.
Non-uniform flow occurs when velocity, pressure etc., change from point to point in the fluid.

Temporal/Spatial Classifications

Steady, unifrom flow:
Conditions do not change with position or time.
e.g., Flow of liquid through a pipe of uniform bore running completely full at constant velocity.
Steady, non-unifrom flow:
Conditions change from point to point but do not with time.
e.g., Flow of a liquid at constant flow rate through a tapering pipe running completely full.
Unsteady, unifrom Flow: e.g. When a pump starts-up.
Unsteady, non-unifrom Flow: e.g. Conditions of liquid during pipetting out of liquid.

Material Derivative and Acceleration

Let the position of a particle at any instant t in a flow field be given by the space coordinates (x, y, z) with respect to a rectangular cartesian frame of reference.
The velocity components u, v, w of the particle along x, y and z directions respectively can then be written in Eulerian form as

$$u = u\,(x, y, z, t)$$
$$v = v\,(x, y, z, t)$$
$$w = w\,(x, y, z, t)$$

After an infinitesimal time interval t , let the particle move to a new position given by the coordinates $(x + \Delta x,\ y + \Delta y\ ,\ z + \Delta z)$.
Its velocity components at this new position be $u + \Delta u$, $v + \Delta v$ and $w + \Delta w$.

Expression of velocity components in the Taylor's series form:

$$u + \Delta u = u(x, y, z, t) + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z + \frac{\partial u}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

$$v + \Delta v = v(x, y, z, t) + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y + \frac{\partial v}{\partial z}\Delta z + \frac{\partial v}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

$$w + \Delta w = w(x, y, z, t) + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta z + \frac{\partial w}{\partial t}\Delta t + higher\ order\ terms\ in\ \Delta x,\ \Delta y,\ \Delta z\ and\ \Delta t$$

The increment in space coordinates can be written as -

$$\Delta x = u\Delta t,\quad \Delta y = v\Delta t\quad and\quad \Delta z = w\Delta t$$

Substituting the values of Δx, Δy, Δz in above equations, we have

$$\frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

In the limit $\Delta t \to 0$, the **equation** becomes

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.1a)$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.1b)$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.1c)$$

The above equations tell that the operator for **total differential** with respect to time, D/Dt in a **convective field** is related to the **partial differential** as:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad (7.2)$$

The total differential D/Dt is known as the material or **substantial derivative** with respect to time.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \qquad (7.2)$$

The **first term** $\frac{\partial}{\partial t}$ in the right hand side of is known as **temporal or local derivative** which expresses the rate of change with time, at a fixed position.

The **last three terms** in the right hand side of 7.2 are together known as **convective derivative** which represents the time rate of change due to change in position in the field.

The terms in the left hand sides of Eqs (7.1a) to (7.1c) are defined as x, y and z components of **substantial or material** acceleration.

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.1a)$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.1b)$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.1c)$$

The first terms in the right hand sides of Eqs (7.1a) to (7.1c) represent the respective **local or temporal** accelerations, while the other terms are **convective** accelerations.

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (7.2a)$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (7.2b)$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (7.2c)$$

(Material or substantial acceleration) =

(temporal or local acceleration) +

(convective acceleration)

End of Lecture 1 (Module 2)