Fluid Mechanics and Machines

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The velocity vector in a fluid flow is given

$$V = 4x^3 i - 10x^2 y j + 2tk.$$

Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time $t = 1$.

Solution. The velocity components u, v and w are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2, 1, 3), we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at (2, 1, 3) are

$$u = 4 \times (2)^3 = 32 \text{ units}$$

$$v = -10(2)^2(1) = -40 \text{ units}$$

$$w = 2 \times 1 = 2 \text{ units}$$

$\therefore$ Velocity vector V at (2, 1, 3) $= 32i - 40j + 2k$

or Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$$= \sqrt{32^2 + (-40)^2 + 2^2} = \sqrt{1024 + 1600 + 4} = \mathbf{51.26 \text{ units. Ans.}}$$

Acceleration is given by equation (5.6)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2,\ \frac{\partial u}{\partial y} = 0,\ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy,\ \frac{\partial v}{\partial y} = -10x^2,\ \frac{\partial v}{\partial z} = 0\ \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0,\ \frac{\partial w}{\partial y} = 0,\ \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.1$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$\begin{aligned} a_x &= 4x^3(12x^2) + (-10x^2y)(0) + 2t \times (0) + 0 \\ &= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units} \\ a_y &= 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0 \\ &= -80x^4y + 100x^4y \\ &= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.} \\ a_z &= 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.1 = 2.0 \text{ units} \end{aligned}$$

$\therefore$ Acceleration is $\quad A = a_x i + a_y j + a_z k =$ **1536i + 320j + 2k. Ans.**

Resultant $\quad A = \sqrt{(1536)^2 + (320)^2 + (2)^2}$ units

$$= \sqrt{2359296 + 102400 + 4} = \textbf{1568.9 units. Ans.}$$

The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation :

(i) $u = x^2 + y^2 + z^2$; $v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$, $w = 2xyz$.

Solution. The continuity equation for incompressible fluid is given by equation (5.4) as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Case I. $u = x^2 + y^2 + z^2 \quad \therefore \quad \frac{\partial u}{\partial x} = 2x$

$$v = xy^2 - yz^2 + xy \quad \therefore \quad \frac{\partial v}{\partial y} = 2xy - z^2 + x$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation.

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

or $$\frac{\partial w}{\partial z} = -3x - 2xy + z^2 \text{ or } \partial w = (-3x - 2xy + z^2)\,\partial z$$

Integration of both sides gives $\int dw = \int\left(-3x - 2xy + z^2\right) dz$

or
$$w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + \text{Constant of integration,}$$

where constant of integration cannot be a function of z. But it can be a function of x and y that is $f(x, y)$.

$$\therefore \qquad w = \left(\mathbf{-3xz - 2xyz + \frac{z^3}{3}}\right) + \mathbf{f(x, y). \ Ans.}$$

Case II.
$$v = 2y^2 \qquad \therefore \qquad \frac{\partial v}{\partial y} = 4y$$

$$w = 2xyz \qquad \therefore \qquad \frac{\partial w}{\partial z} = 2xy$$

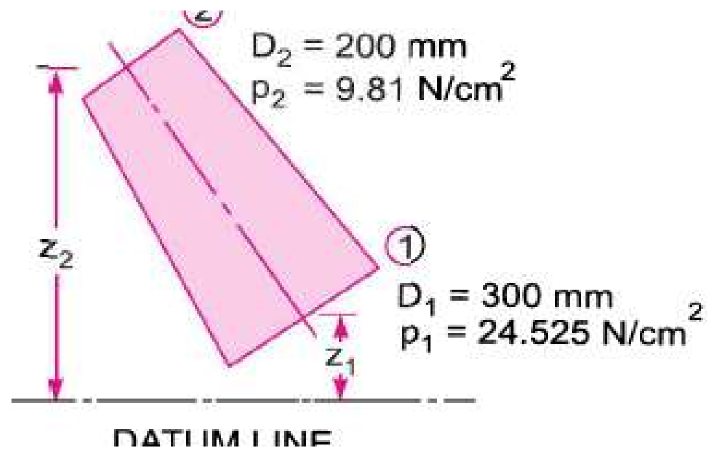
Substituting the values of $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

or
$$\frac{\partial u}{\partial x} = -4y - 2xy \text{ or } du = (-4y - 2xy)\, dx$$

Integrating, we get
$$u = -4xy - 2y\frac{x^2}{2} + f(y, z) = \mathbf{-4xy - x^2y + f(y, z). \ Ans.}$$

Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.



Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

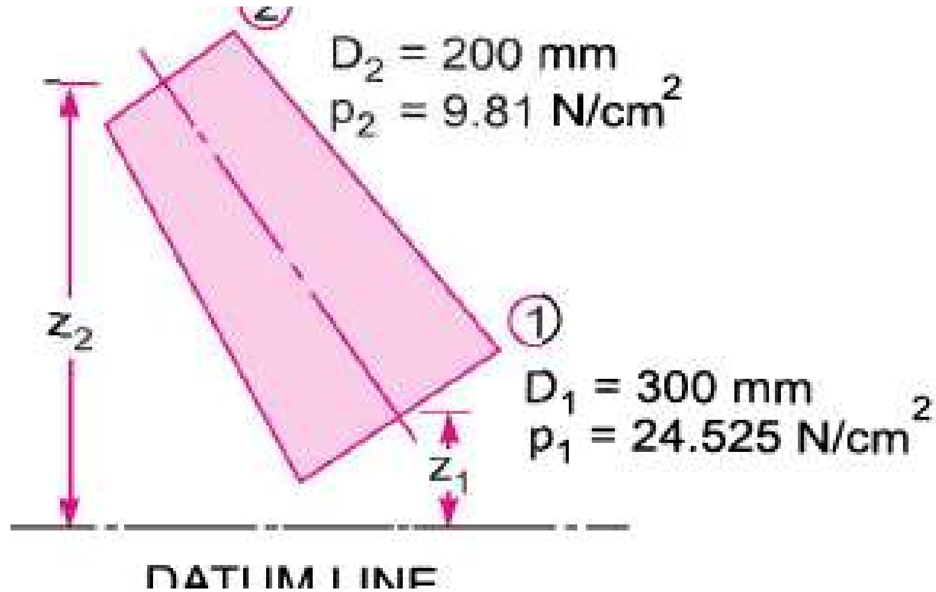
or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/:}$$



Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution. Given :

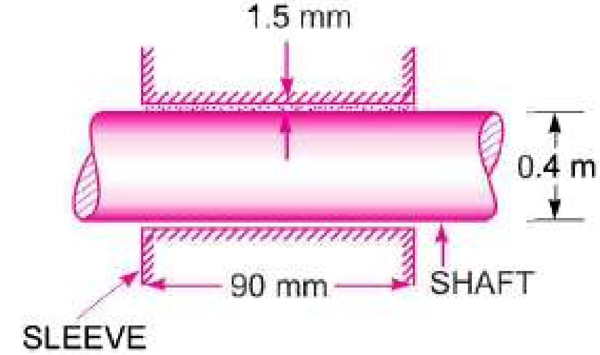


Fig. 1.5

Viscosity $\mu = 6$ poise

$$= \frac{6}{10}\frac{\text{N s}}{\text{m}^2} = 0.6\ \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90\ \text{mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5\ \text{mm} = 1.5 \times 10^{-3}$ m

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity = $u - 0 = u = 3.98$ m/s

dy = Change of distance = $t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592\ \text{N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05\ \text{N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \dfrac{2\pi N T}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} =$ **716.48 W. Ans.**