

Fluid Mechanics and Machines

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The velocity vector in a fluid flow is given

$$V = 4x^3i - 10x^2yj + 2tk.$$

Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time $t = 1$.

Solution. The velocity components u , v and w are $u = 4x^3$, $v = -10x^2y$, $w = 2t$

For the point (2, 1, 3), we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at (2, 1, 3) are

$$u = 4 \times (2)^3 = 32 \text{ units}$$

$$v = -10(2)^2(1) = -40 \text{ units}$$

$$w = 2 \times 1 = 2 \text{ units}$$

\therefore Velocity vector V at (2, 1, 3) = $32i - 40j + 2k$

or

$$\begin{aligned} \text{Resultant velocity} &= \sqrt{u^2 + v^2 + w^2} \\ &= \sqrt{32^2 + (-40)^2 + 2^2} = \sqrt{1024 + 1600 + 4} = \mathbf{51.26 \text{ units. Ans.}} \end{aligned}$$

Acceleration is given by equation (5.6)

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2, \frac{\partial u}{\partial y} = 0, \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy, \frac{\partial v}{\partial y} = -10x^2, \frac{\partial v}{\partial z} = 0 \text{ and } \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0, \frac{\partial w}{\partial y} = 0, \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.1$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$\begin{aligned} a_x &= 4x^3 (12x^2) + (-10x^2y)(0) + 2t \times (0) + 0 \\ &= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units} \end{aligned}$$

$$\begin{aligned} a_y &= 4x^3 (-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0 \\ &= -80x^4y + 100x^4y \\ &= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.} \end{aligned}$$

$$a_z = 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.1 = 2.0 \text{ units}$$

\therefore Acceleration is

$$A = a_x i + a_y j + a_z k = \mathbf{1536i + 320j + 2k. \text{ Ans.}}$$

\therefore Resultant

$$A = \sqrt{(1536)^2 + (320)^2 + (2)^2} \text{ units}$$

$$= \sqrt{2359296 + 102400 + 4} = \mathbf{1568.9 \text{ units. Ans.}}$$

The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation :

(i) $u = x^2 + y^2 + z^2$; $v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$, $w = 2xyz$.

Solution. The continuity equation for incompressible fluid is given by equation (5.4) as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Case I.

$$u = x^2 + y^2 + z^2 \quad \therefore \frac{\partial u}{\partial x} = 2x$$

$$v = xy^2 - yz^2 + xy \quad \therefore \frac{\partial v}{\partial y} = 2xy - z^2 + x$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation.

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

or $\frac{\partial w}{\partial z} = -3x - 2xy + z^2$ or $\partial w = (-3x - 2xy + z^2) \partial z$

Integration of both sides gives $\int dw = \int (-3x - 2xy + z^2) dz$

or
$$w = \left(-3xz - 2xyz + \frac{z^3}{3} \right) + \text{Constant of integration,}$$

where constant of integration cannot be a function of z . But it can be a function of x and y that is $f(x, y)$.

\therefore
$$w = \left(-3xz - 2xyz + \frac{z^3}{3} \right) + f(x, y). \text{ Ans.}$$

Case II. $v = 2y^2 \quad \therefore \quad \frac{\partial v}{\partial y} = 4y$

$$w = 2xyz \quad \therefore \quad \frac{\partial w}{\partial z} = 2xy$$

Substituting the values of $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

or
$$\frac{\partial u}{\partial x} = -4y - 2xy \text{ or } du = (-4y - 2xy) dx$$

Integrating, we get
$$u = -4xy - 2y \frac{x^2}{2} + f(y, z) = -4xy - x^2y + f(y, z). \text{ Ans.}$$

Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2 . Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$
 $p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$
 $p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

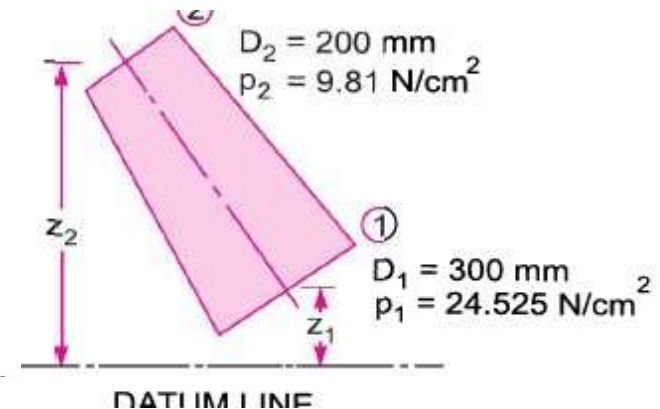
Rate of flow = 40 lit/s

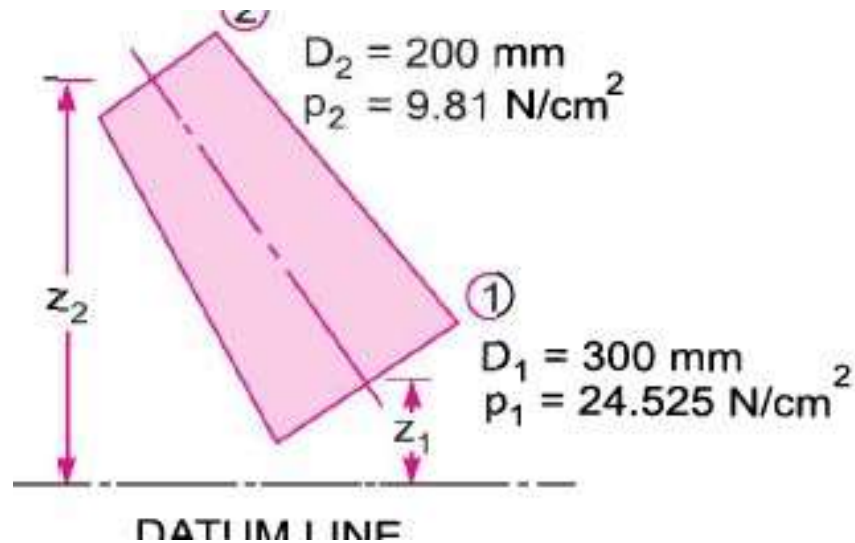
or $Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$

Now $A_1 V_1 = A_2 V_2 = \text{rate of flow} = 0.04$

$\therefore V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4} D_1^2} = \frac{0.04}{\frac{\pi}{4} (0.3)^2} = 0.5658 \text{ m/s}$
 $\approx 0.566 \text{ m/s}$

$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4} (D_2)^2} = \frac{0.04}{\frac{\pi}{4} (0.2)^2} = 1.274 \text{ m/s}$





Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

$$\text{or } 25 + .32 + z_1 = 10 + 1.623 + z_2$$

$$\text{or } 25.32 + z_1 = 11.623 + z_2$$

$$\therefore z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$$\therefore \text{Difference in datum head} = z_2 - z_1 = \mathbf{13.70 \text{ m. Ans.}}$$

The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Solution. Given :

Viscosity

$$\mu = 6 \text{ poise}$$

$$= \frac{6}{10} \frac{\text{Ns}}{\text{m}^2} = 0.6 \frac{\text{Ns}}{\text{m}^2}$$

Dia. of shaft,

$$D = 0.4 \text{ m}$$

Speed of shaft,

$$N = 190 \text{ r.p.m}$$

Sleeve length,

$$L = 90 \text{ mm} = 90 \times 10^{-3} \text{ m}$$

Thickness of oil film,

$$t = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\text{Tangential velocity of shaft, } u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98 \text{ m/s}$$

Using the relation

$$\tau = \mu \frac{du}{dy}$$

where $du = \text{Change of velocity} = u - 0 = u = 3.98 \text{ m/s}$

$dy = \text{Change of distance} = t = 1.5 \times 10^{-3} \text{ m}$

$$\tau = 0.6 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

\therefore Shear force on the shaft, $F = \text{Shear stress} \times \text{Area}$

$$= 1592 \times \pi D \times L = 1592 \times \pi \times 0.4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft,

$$T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01 \text{ Nm}$$

\therefore *Power lost

$$= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} = 716.48 \text{ W. Ans.}$$

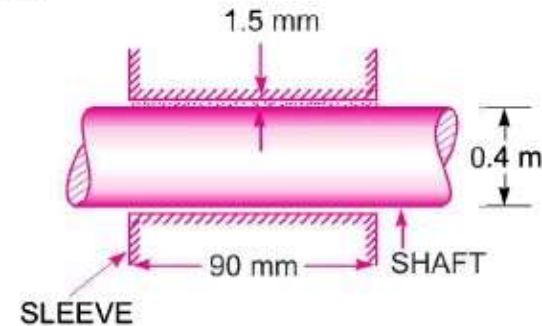


Fig. 1.5