Mechanics of Machines

Kinematics of gear train

Module 3: Different type of gear train (Simple, compound, Reverted and Epicyclic gear train**)**

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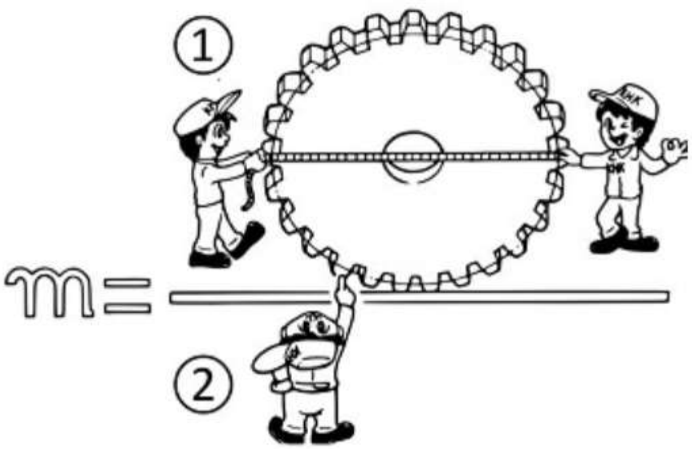
Gear train:

- Class Objective1: What is gear train ?
- Class Objective2: Why gear train?
- Class objective3: Type of gear train
- Class Objective4: Speed ratio, train value

Gear train:

- A gear train is a combination of gears used to transmit motion from one shaft to another.
- Large speed reduction within small space (gear train required)
- Centre distance between shaft is lagre or very very small
- To reduce losses as compared to belt drive.
- Module have to same for the two mating gear to satisfy law of gearing.

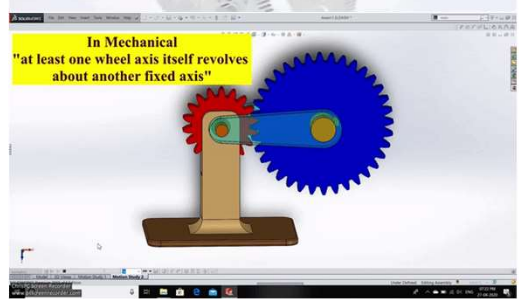


"*The common normal at the point of contact between a pair of teeth must always pass through the pitch point*"

Types of gear train

Gear train

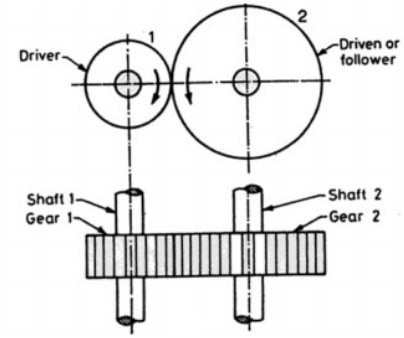
- Simple
- Compound
- Reverted
- Epicyclic



Simple gear train:

- In simple gear trains, each shaft supports one gear.

- In it all **gear axes** remain fixed relative to the frame and each gear is on a separate shaft.
- Two external gears of a pair always move in opposite directions.
- Odd numbered gears moves in one direction.
- Even numbered gear moves in one opposite direction.



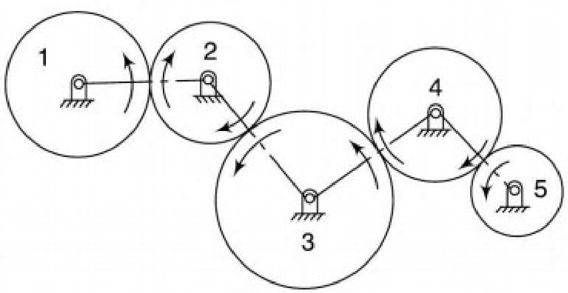


Fig.1: Simple gear train(two) **Fig.2:** Simple gear train(more than two)

Simple gear train continued:

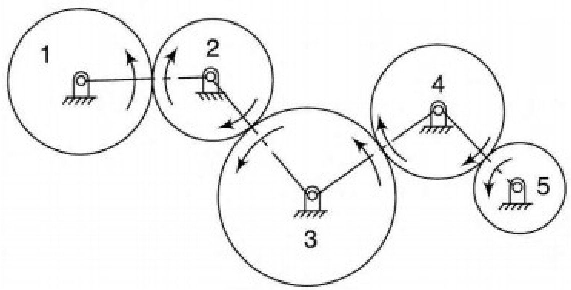


Fig.3: Simple gear train(more than two)

$$Speed\ ratio = \frac{Speed\ of\ driver(N_1)}{Speed\ of\ driven(N_2)} = \frac{No.of\ teeth\ on\ driven(T_2)}{No.of\ teeth\ on\ driver(T_1)}$$

$$Train\ value = \frac{1}{Speed\ ratio}$$

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}$$

$$\frac{N_4}{N_3} = \frac{T_3}{T_4}$$

$$\frac{N_5}{N_4} = \frac{T_4}{T_5}$$

Multiplying,

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

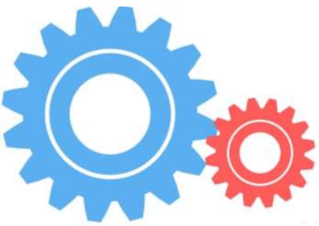
$$\frac{N_5}{N_1} = \frac{T_1}{T_5}$$ **Train value**

$$\frac{N_1}{N_5} = \frac{T_5}{T_1}$$ **Speed ratio**

- Intermediate gears have no effect on speed ratio, therefore, they known as **idlers**

Speed ratio relation :

$$Speed\ ratio = \frac{Speed\ of\ driver(N_1)}{Speed\ of\ driven(N_2)} = \frac{No.of\ teeth\ on\ driven(T_2)}{No.of\ teeth\ on\ driver(T_1)}$$

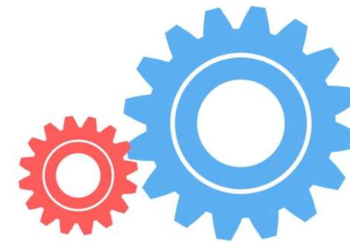


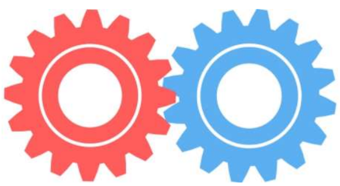
$$\frac{N_1}{N_2} < 1 \quad or \quad \frac{T_2}{T_1} < 1 \qquad N_1 < N_2 \quad or \quad T_2 < T_1$$

Increasing speed

$$\frac{N_1}{N_2} > 1 \quad or \quad \frac{T_2}{T_1} > 1 \qquad N_1 > N_2 \quad or \quad T_2 > T_1$$

Reducing speed





$$\frac{N_1}{N_2} = 1 \quad or \quad \frac{T_2}{T_1} = 1 \qquad N_1 = N_2 \quad or \quad T_2 = T_1$$

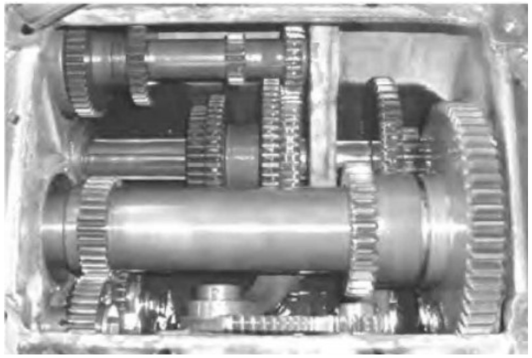
Equal speed

In this simple gearbox, I've got (from right to left) a large gear wheel with 40 teeth, a medium wheel with 20 teeth, and a small wheel with 10 teeth.

Compound gear train:

- When series of gears are connected in such a way that **two or more gear rotates about axis** with the same angular velocity.

- **Intermediate shaft** carry more than one gear .



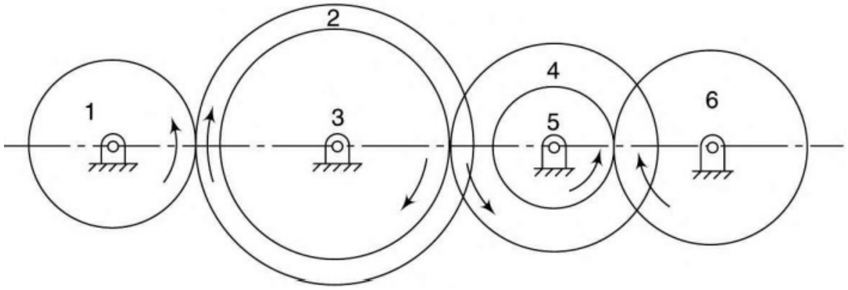


Fig.4: Compound gear train

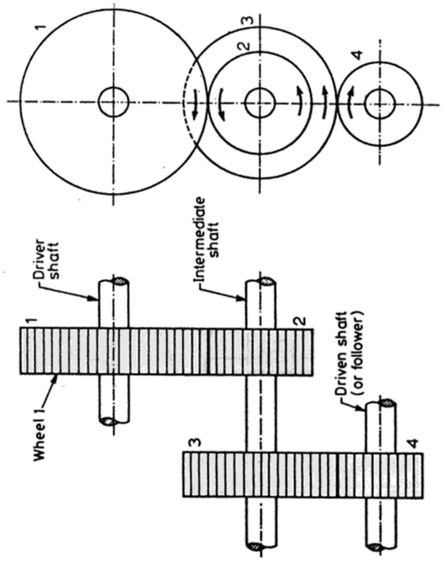
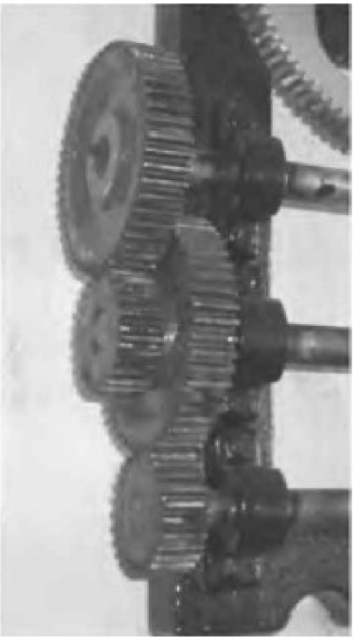


Fig.5: Compound gear train



Compound gear train continued:

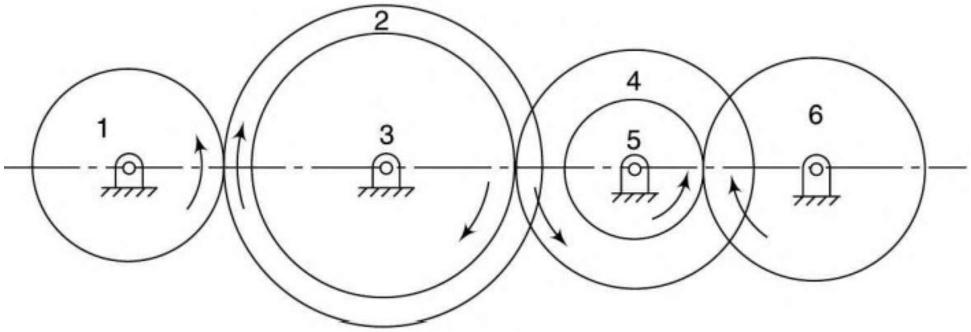


Fig.6: Compound gear train

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_2} \times \frac{N_6}{N_4} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \frac{T_3}{T_4} \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\frac{N_4}{N_3} = \frac{T_3}{T_4}$$

$$\frac{N_6}{N_5} = \frac{T_5}{T_6}$$

$$Train\ value = \frac{Speed\ of\ driven(N_6)}{Speed\ of\ driver(N_1)} = \frac{Product\ of\ number\ of\ teeth\ on\ driver\ gears}{Product\ of\ numbe\ of\ teeth\ on\ driven\ gears}$$

Compound gear train continued:

Problem 1: A compound gear train shown in below figure consists of compound gears B-C and D-E. all gears are mounted on parallel shafts. The motor shaft rotating at 800 rpm is connected to gear A and output shaft to the gear F. The number teeth on gears are given below. Determine the speed of gear F ?

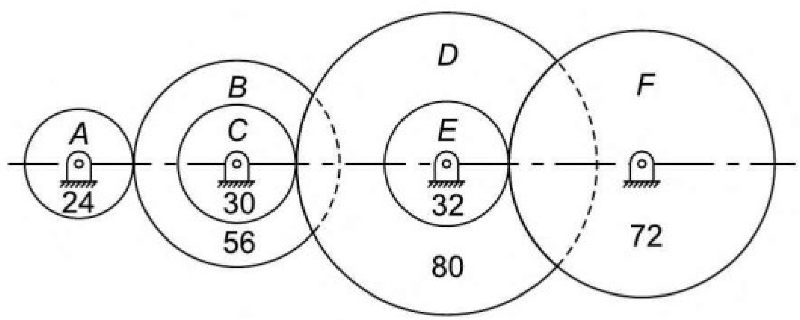


Fig.6: Compound gear train

Gear	A	B	C	D	E	F
No. teeth	24	56	30	80	32	72

Reverted gear train:

- If the axis of the first and last wheels of a compound gear coincide, it is called as reverted gear train.

$$Train\ value = \frac{Speed\ of\ driven(N_4)}{Speed\ of\ driver(N_1)} = \frac{Product\ of\ number\ of\ teeth\ on\ driver\ gears}{Product\ of\ numbe\ of\ teeth\ on\ driven\ gears} \qquad \frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4}$$

- Also, If r is the pitch circle radius of a gear

$$r_1 + r_2 = r_3 + r_4$$

$$T_1 + T_2 = T_3 + T_4$$

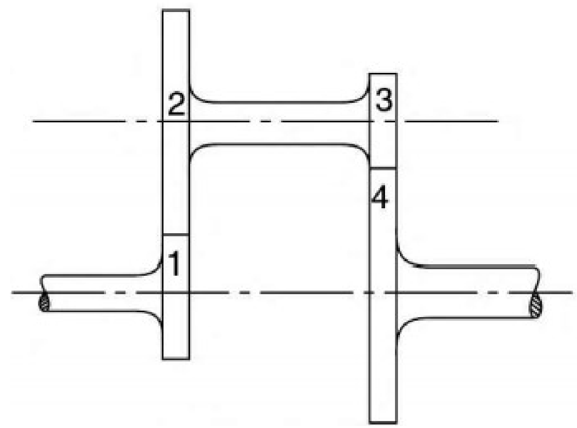


Fig.7: Reverted gear train

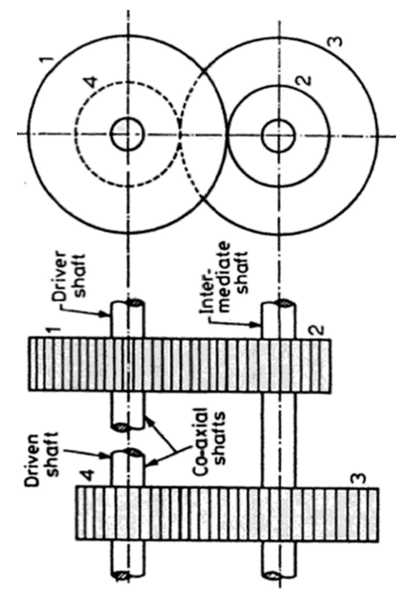


Fig.8: Reverted gear train

Reverted gear train:

Problem 2: The speed ratio of the reverted gear train, as shown in below figure, is to be 12. The module pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.

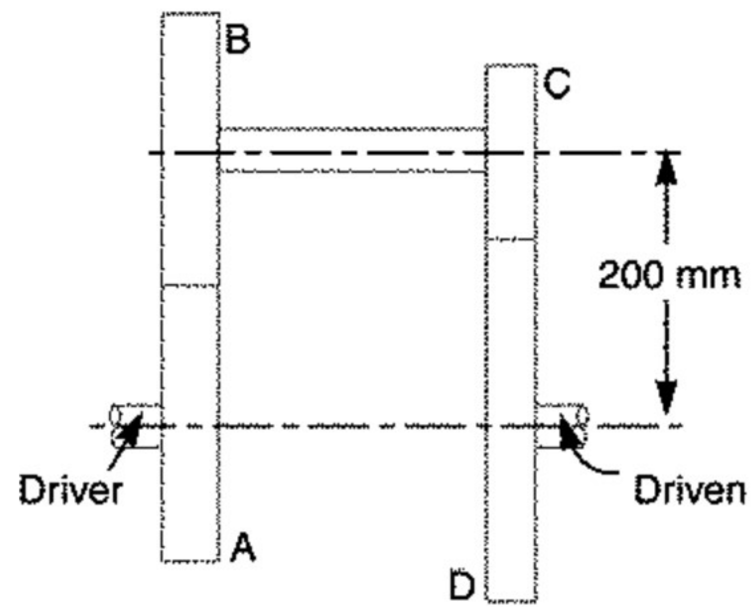
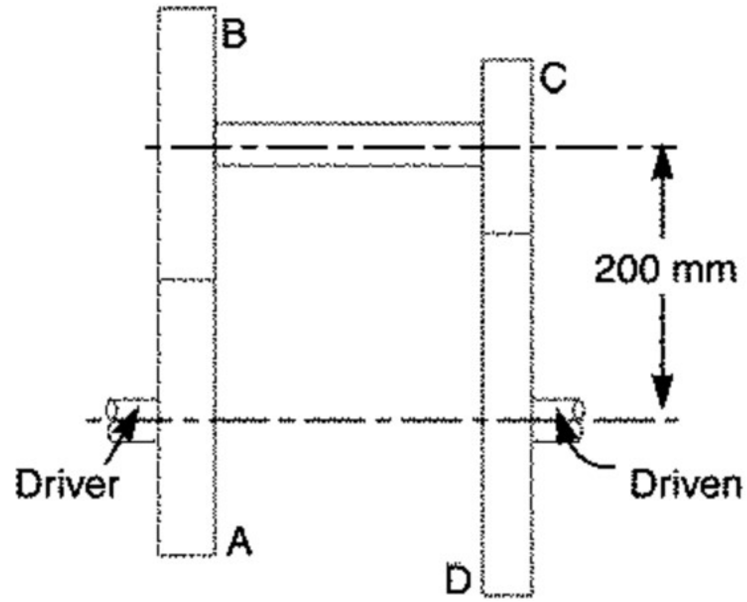
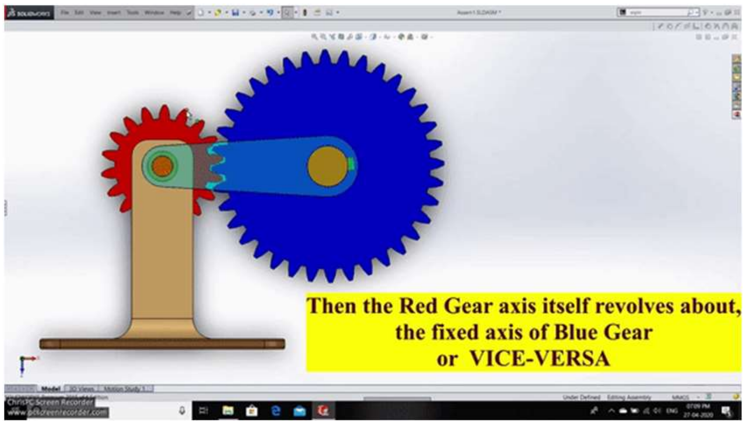


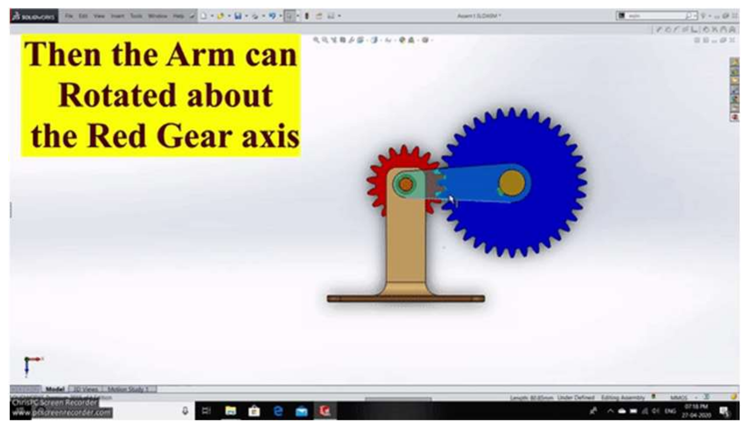
Fig.9: Reverted gear train


B
C
200 mm
Driver
Driven
A
D

Epicyclic gear train:

- A gear train having a relative motion of axes is called a planetary or an epicyclic gear train. In epicyclic train, the axis of at least one of the gears also moves relative to the frame.





Analysis of epicyclic gear train:

Tabular method:

- Lock the arm and assume the other wheels free to rotate.
- Turn any one gear through one revolution in clockwise direction and find the number of revolution made by each of the other wheels.
- Multiply by x revolutions in the clockwise direction keeping the arm fixed.
- Add y to all the quantities in the second row.

Step No.	Action	Revs. of a	Revs. of A	Revs. of B
1.	a fixed, $S+1$ rev.	0	1	$-\frac{T_S}{T_P}$
2.	a fixed, $S+x$ rev.	0	x	$-\frac{T_S}{T_P}x$
3.	Add y	y	$y+x$	$y-\frac{T_S}{T_P}x$

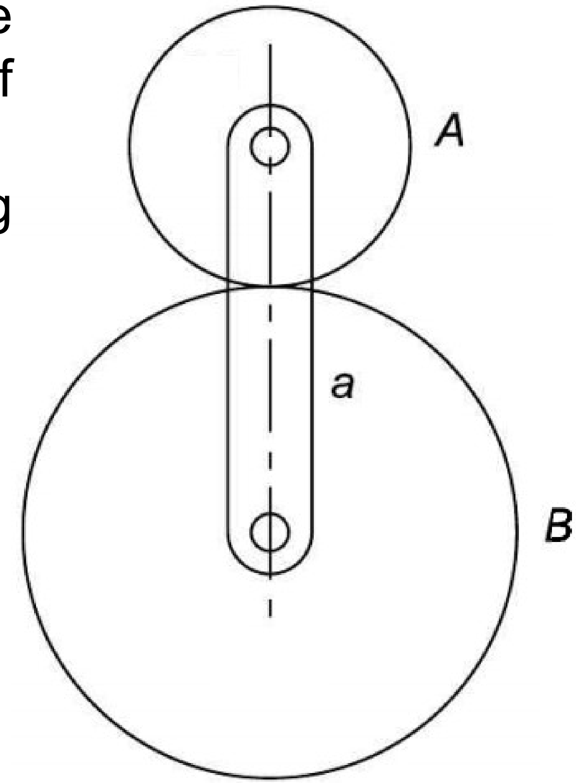


Fig.10: Epicyclic gear train

Analysis of epicyclic gear train:

Algebric method:

- Therefore speed of the gear A relative to the arm a = $N_A - N_a$

  speed of the gear B relative to the arm a = $N_B - N_a$

- Since the gears A and B are meshing directly, therefore they will revolve in opposite directions.

$$\frac{N_B - N_a}{N_A - N_a} = -\frac{T_A}{T_B}$$

- Since the arm C is fixed, therefore its speed, N_C = 0.

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

- If the gear A is fixed, then N_A =0.

$$\frac{N_B - N_a}{0 - N_a} = -\frac{T_A}{T_B}$$

$$\frac{N_B}{N_a} = 1 + \frac{T_A}{T_B}$$

Analysis of epicyclic gear train:

Problem 3: In an epicyclic gear train, an arm carries two gears A and B having **36** and **45** teeth respectively. If the arm rotates at **150 r.p.m**. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B.

If the gear A instead of being fixed, makes **300** r.p.m. in the clockwise direction, what will be the speed of gear B ?

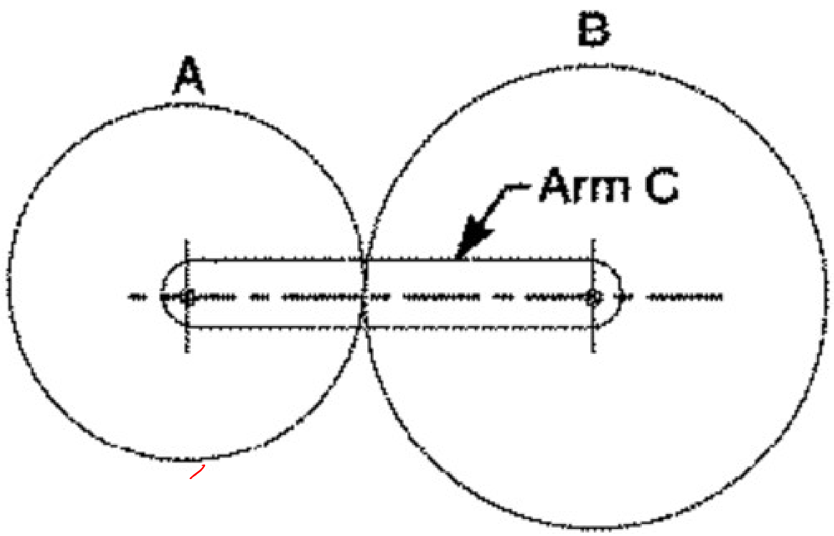


Fig.11: Epicyclic gear train

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Analysis of epicyclic gear train:

Problem 4: In an epicyclic gear train in which B and C constitute a compound gear. The number of teeth are shown along with each wheel in below figure. Determine the speed and direction of rotation of wheels A and E, if the arm revolves at **210rpm** clockwise and the gear D is fixed.

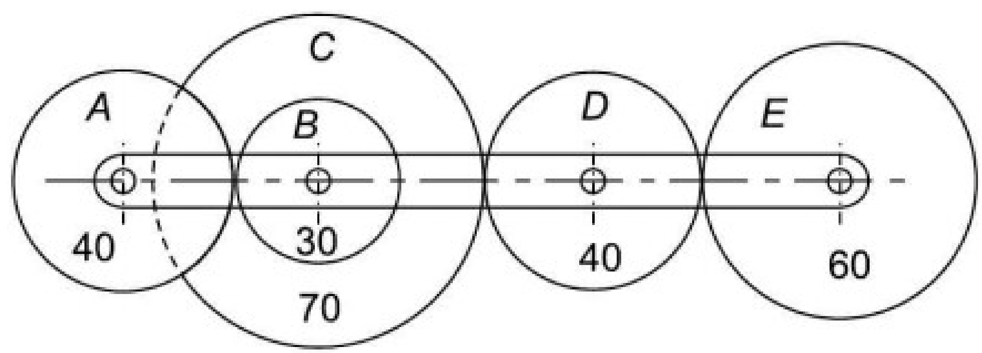
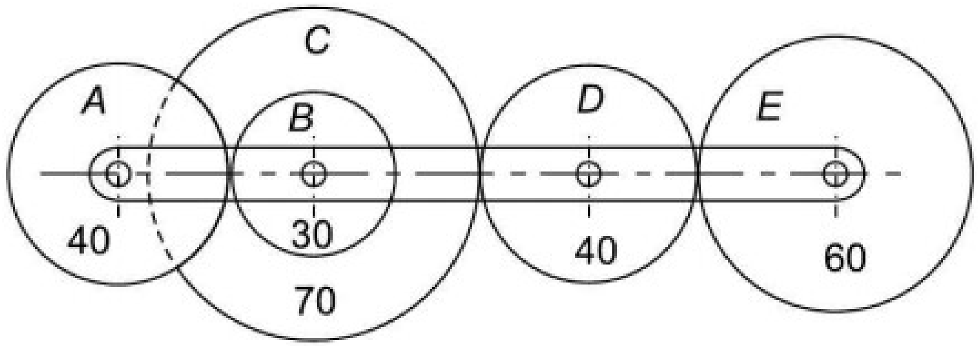


Fig.12: Epicyclic gear train



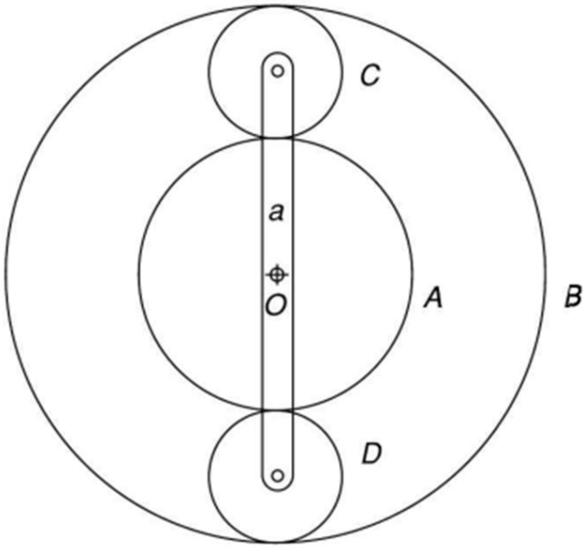
Action	*a*	*A*	*B/C*	*D*	*E*
'a' fixed, A + 1 rev.	0	1	$-\frac{40}{30}$	$-\frac{40}{30}\times\left(-\frac{70}{40}\right)$	$-\frac{7}{3}\times\frac{40}{60}$
'a' fixed, $A + x$ rev.	0	x	$-\frac{40x}{30}$	$\frac{7x}{3}$	$-\frac{14x}{9}$
Add y	y	$y + x$	$y-\frac{40}{30}$	$y+\frac{7x}{3}$	$y-\frac{14x}{9}$

Analysis of epicyclic gear train:

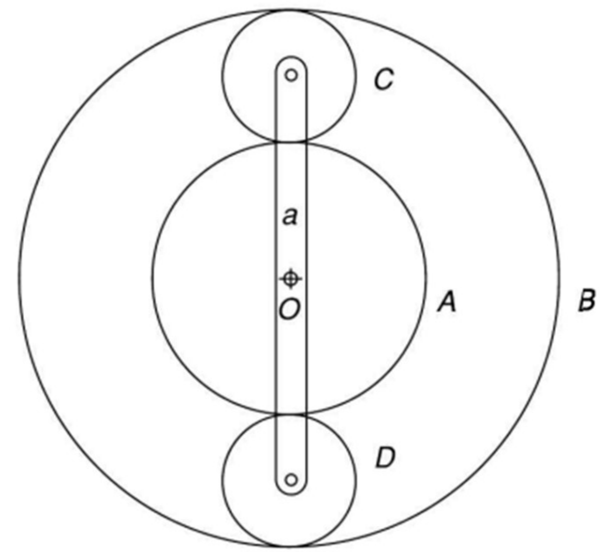
Problem 5: An epicyclic gear train is shown in below Fig. The number of teeth on A and B are 80 and 200. Determine the speed of the arm a.

(i) If A rotates at 100 rpm clockwise and B at 50 rpm counter-clockwise.

(ii) If A rotates at 100 rpm clockwise and B is stationary

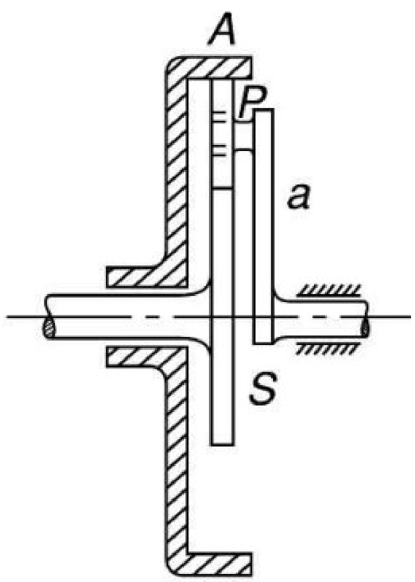


Action	*a*	*A*	*C/D*	*B*
a fixed, A + 1 rev.	0	1	$-\dfrac{80}{60}$	$-\dfrac{80}{60}\times\dfrac{60}{200}$
a fixed, A + x rev.	0	x	$-\dfrac{4x}{3}$	$-\dfrac{2x}{5}$
Add y	y	$y+x$	$y-\dfrac{4x}{3}$	$y-\dfrac{2x}{5}$



Analysis of epicyclic gear train:

Problem 6: In an epicyclic gear of the 'sun and planet' type shown in below figure, the pitch circle diameter of the internally toothed ring is to be **224 mm** and the **module 4 mm**. When the ring A is stationary, the spider (arm) a, which carries three planet wheels P of equal size, is to make one revolution in the same sense as the sun wheel S for every five revolutions of the driving spindle carrying the sun wheel S. Determine suitable numbers of teeth for all the wheels. .



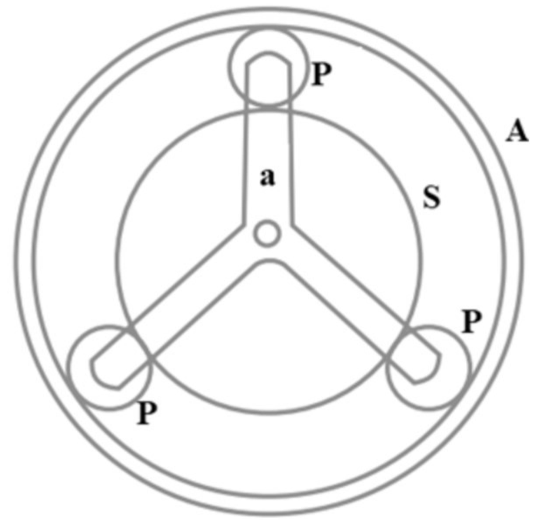
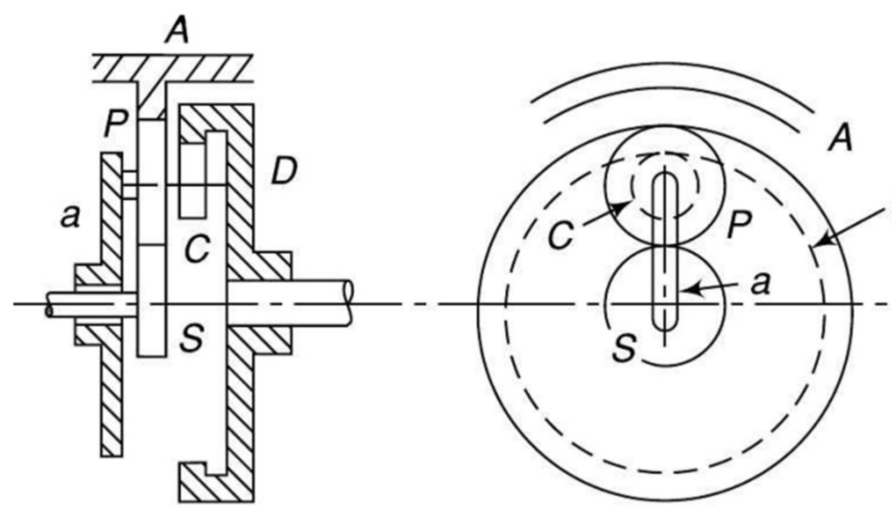


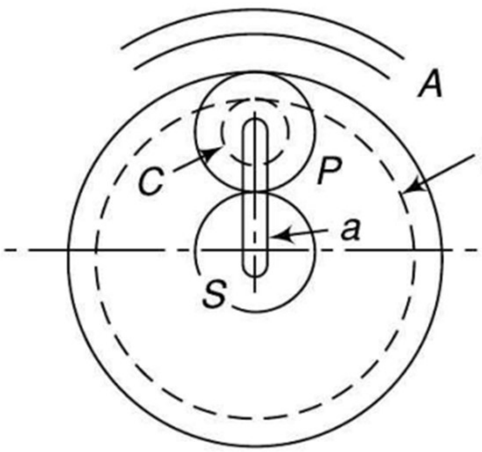
Fig.13: Epicyclic gear train

Analysis of epicyclic gear train:

Problem 7: In an epicyclic gear train shown in below Fig., the input S has 24 teeth. P and C constitute a compound planet having 30 and 18 teeth respectively. If all the gears are of the same pitch, find the ratio of the reduction gear. Assume A is fixed.



(a) (b)



Action	*a*	*S*	*P/C*	*A*	*D*
'*a*' fixed, S + 1 rev.	0	1	$-\frac{24}{30}$	$-\frac{24}{30}\times\frac{30}{84}$	$-\frac{24}{30}\times\frac{18}{72}$
'*a*' fixed, $S + x$ rev.	0	x	$-\frac{4x}{5}$	$\frac{-2x}{7}$	$-\frac{x}{5}$
Add y	y	$y + x$	$y-\frac{4x}{5}$	$y-\frac{2x}{7}$	$y-\frac{x}{5}$

Torque calculation of epicyclic gear train:

$$\sum T = 0 \qquad T_s + T_a + T_A = 0$$

$$\sum T\omega = 0 \qquad T_s\omega_s + T_a\omega_a + T_A\omega_A = 0$$

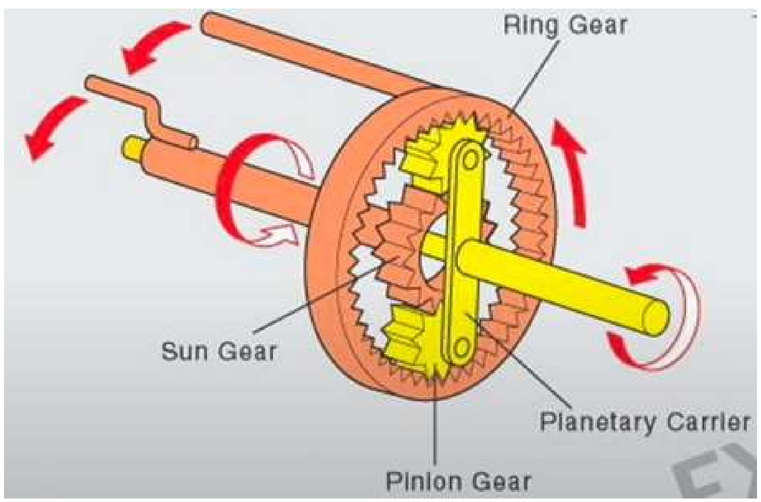


Fig.14: Epicyclic gear train

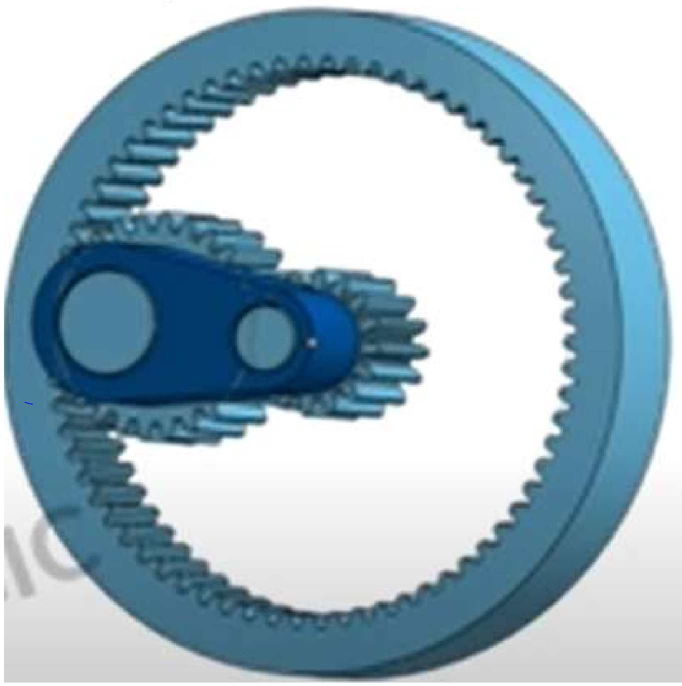


Fig.15: Epicyclic gear train

Analysis of epicyclic gear train:

Problem 8: An epicyclic gear train shown in below figure. Pinion A has **15** teeth and is rigidly fixed to the motor shaft. The wheel B has **20** teeth and mesh with A and also with the annular fixed wheel D. Pinion C has **15** teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel E, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at **1000 r.p.m.**, find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of **100 N-m**.

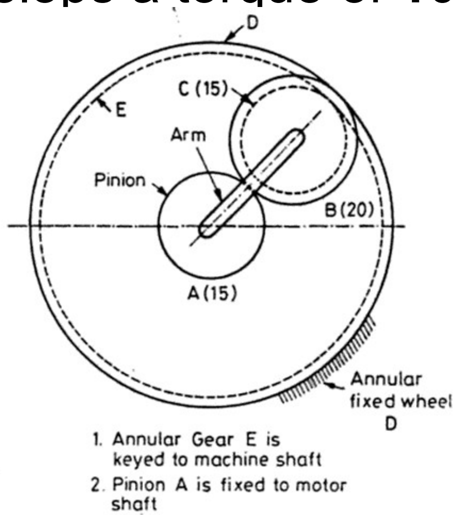
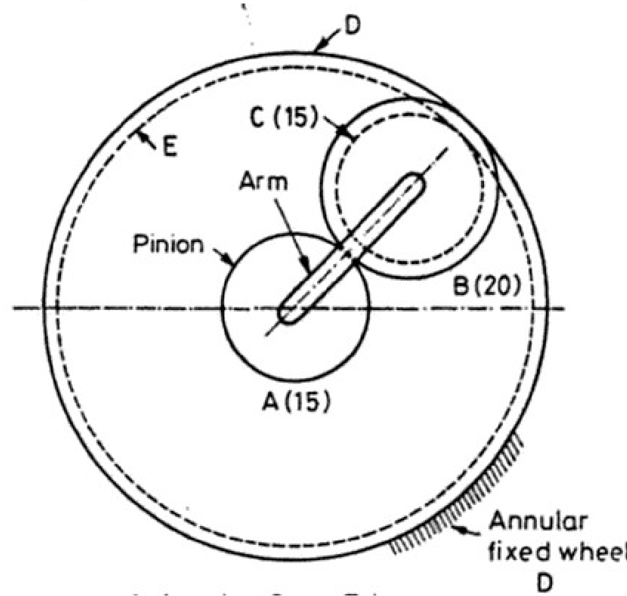
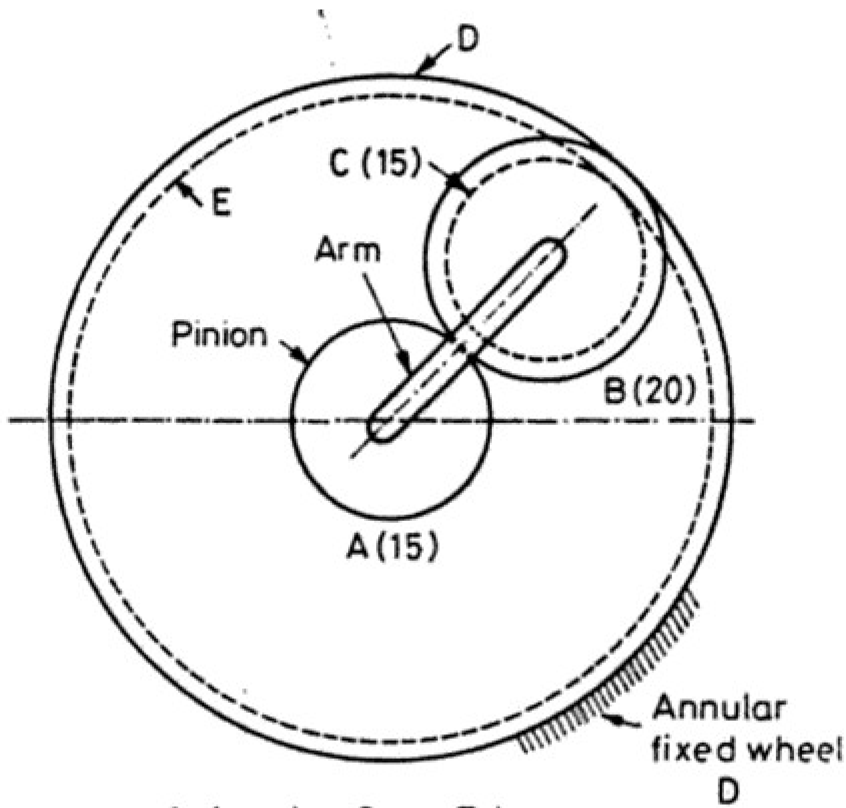


Fig.16: Epicyclic gear train

Operation	Revolution of elements				
	Arm	Pinion A	Compound wheel B-C	Wheel E	Wheel D
Arm fixed. Pinion A rotates through one revolution.	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B} \times \frac{T_C}{T_E}$	$-\frac{T_A}{T_B} \times \frac{T_B}{T_D} = -\frac{T_A}{T_D}$
Multiply by x to all	0	$+x$	$-x \cdot \frac{T_A}{T_B}$	$-x \cdot \frac{T_A}{T_B} \times \frac{T_C}{T_E}$	$-x \cdot \frac{T_A}{T_E}$
Add y revolution to all elements	y	$x+y$	$y - x \cdot \frac{T_A}{T_B}$	$y - x \cdot \frac{T_A}{T_B} \times \frac{T_C}{T_E}$	$y - x \frac{T_A}{T_D}$



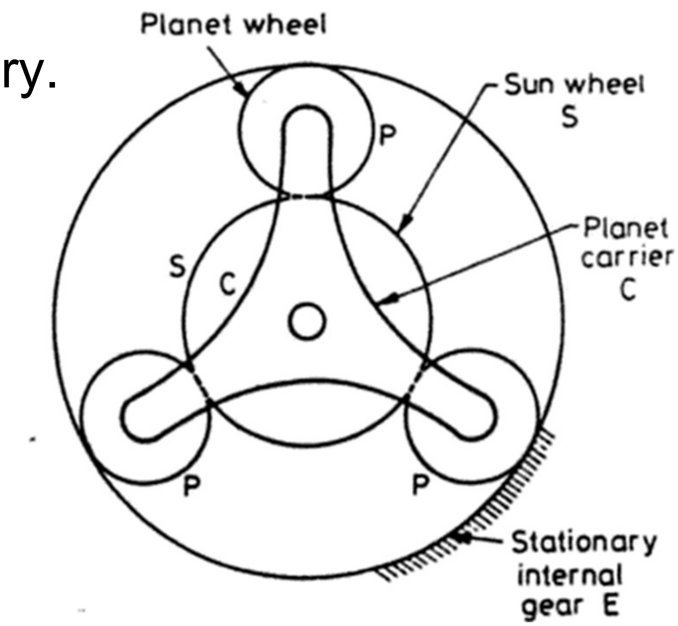

D
C (15)
E
Arm
Pinion
B (20)
A (15)
Annular
fixed wheel
D

Analysis of epicyclic gear train:

Problem 9: An epicyclic train consists of a sun wheel S, a stationary internal gear E and three identical planet wheel P carried on a star shaped planet carrier C. The size of different toothed wheels are such that the planet C rotates 1/5 of the speed of the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 Nm. Determine

i) Number of teeth on different wheels of the train

ii) Torque necessary to keep the internal gear stationary.



S. No.	Operation	Revolution of elements			
		Planet carrier C	Sun wheel S	Planet wheel P	Internal gear E
1.	Planet carrier C fixed. Sun wheel S rotates through + 1 revolution. (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_E} = -\frac{T_S}{T_E}$
2.	Multiply by x to all	0	$+x$	$-x \times \frac{T_S}{T_P}$	$-x \times \frac{T_S}{T_E}$
3.	Add y revolution to all elements	$+y$	$x+y$	$-x\frac{T_S}{T_P}+y$	$-x\frac{T_S}{T_E}+y$

Analysis of epicyclic gear train:

Problem 10: The number of teeth in the gear figure are as follows $\mathbf{T_s=18}$, $\mathbf{T_p=24}$, $\mathbf{T_c=12}$, $\mathbf{T_A=72}$. **P and C** form a compound gear carried by the arm a and the annular gear A is held stationary. Determine the speed of the output at a. Also find the holding torque required on A if 5kW is delivered to S at 800 rpm with an efficiency of 94%.

In case the annulus A rotates at 100 rpm in the same direction as S, what will be the new speed of a.

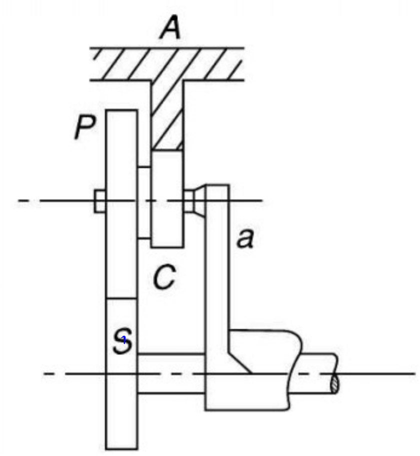
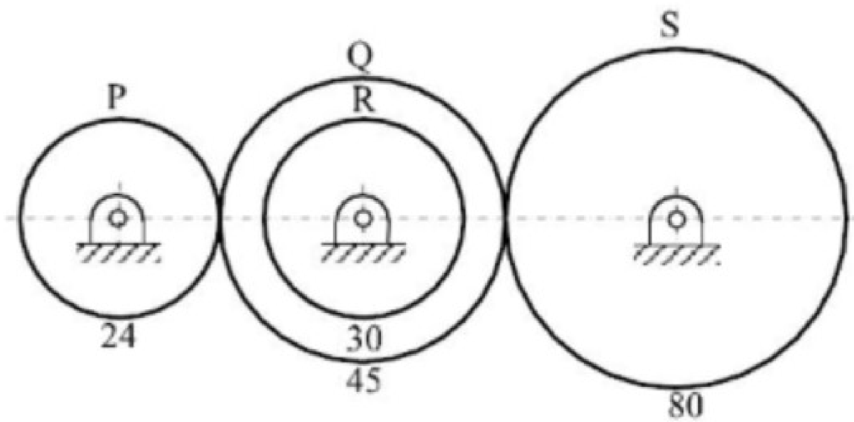


Fig.17: Epicyclic gear train

Previous year GATE gear train questions :

Q1. A gear train shown in the figure consist of gears P, Q, R and S. Gear Q and gear R are mounted on the same shaft. All the gears are mounted on parallel shaft and the number of teeth of P, Q, R and S are 24, 45, 30 and 80, respectively. Gear P is rotating at 400 rpm. The speed (in rpm) of the gear S is__________ ***(GATE-2017)***

(A) 120 r.p.m.
(B) 121 r.p.m.
(C) 119 r.p.m.
(D) 122 r.p.m.



Previous year GATE gear train questions :

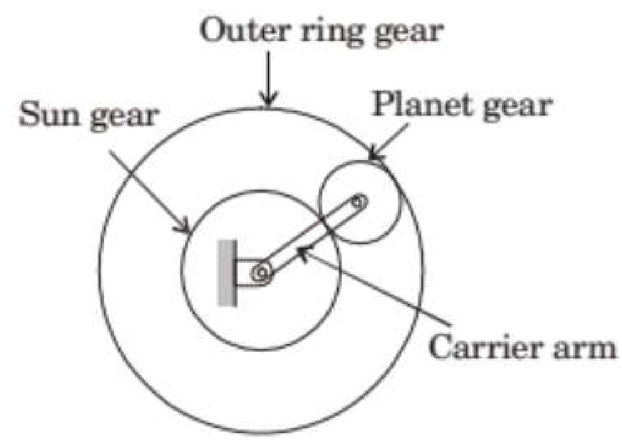
Q2. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm. Let the number of teeth on the sun planet and outer gears to be 50, 25 and 100, respectively. The ratio of magnitude of angular velocity of the planet gear to the angular velocity of the carrier arm is __________ *(GATE-2017)*

(A) -3

(B) -2

(C) 2

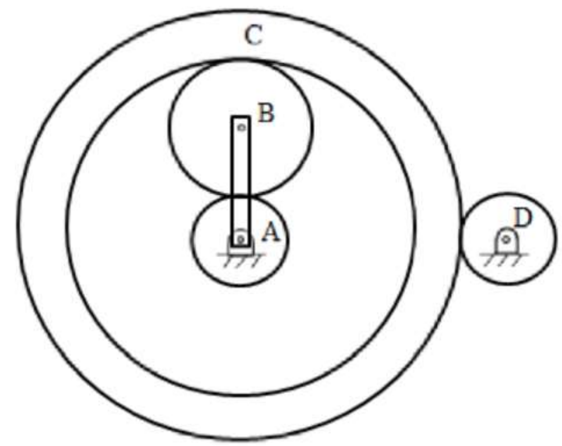
(D) 3



Previous year GATE gear train questions :

Q3. An epicyclic gear train is shown in the figure below. The number of teeth on the gears A, B and D are 20, 30 and 20, respectively. Gear C has 80 teeth on the inner surface and 100 teeth on the outer surface. If the carrier arm AB is fixed and the sun gear A rotates at 300 rpm in the clockwise direction, then the rpm of D in the clockwise direction is

(GATE-2018)



(A) 240

(B) -240

(C) 375

(D) -375