Winter 2022-23

VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Mechanical Engineering

B.Tech. – Mechatronics and Automation

BMEE207L Kinematics & Dynamics of Machines

MODULE 2

Velocity and Accelerations in Mechanisms

By

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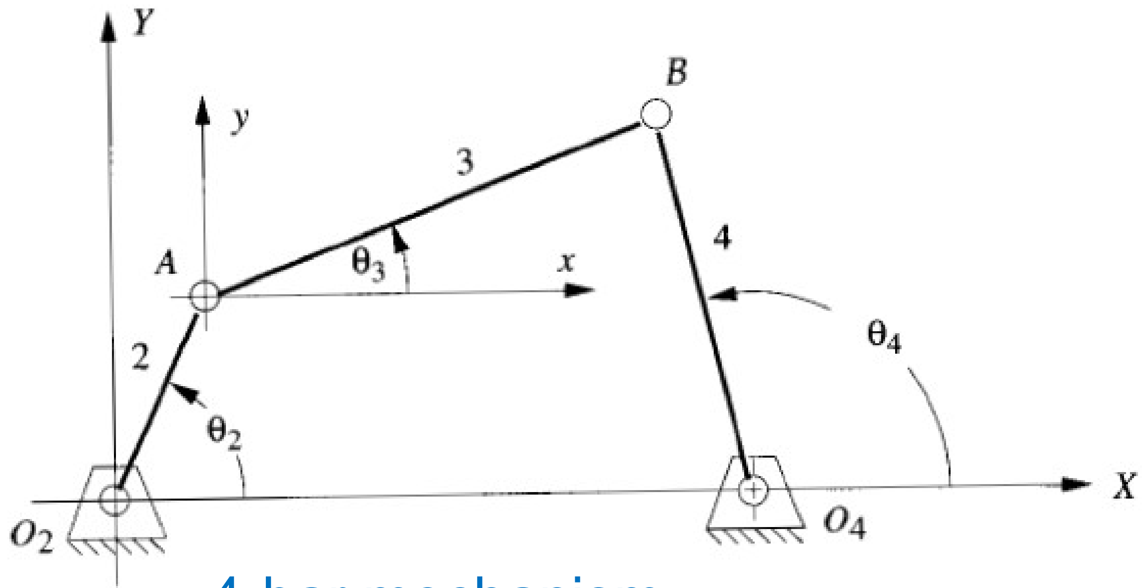
Outline

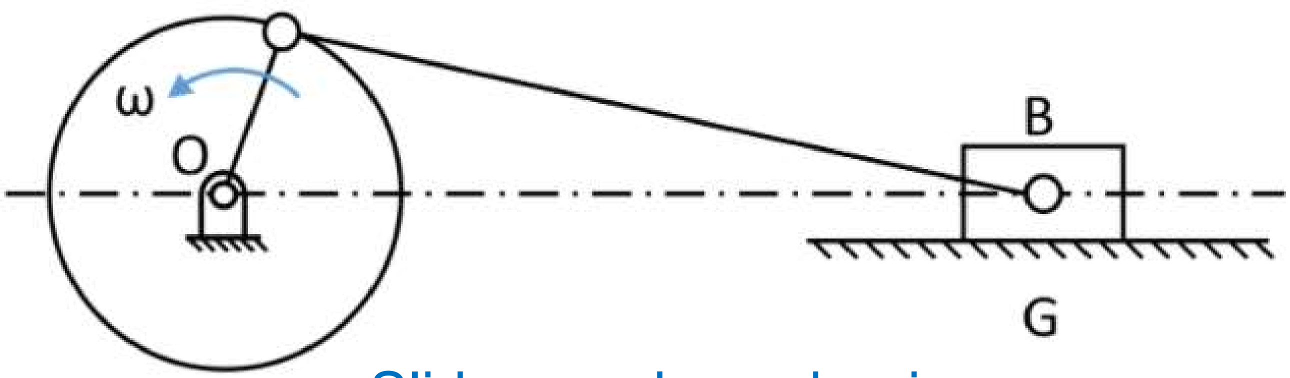
- Introduction
- Types of motion in rigid body
- Velocity in Mechanisms
- Acceleration in Mechanisms
- Velocity Analysis – Instantaneous Centre Method
- Coriolis component of acceleration
    - Applications

Introduction

- Why we need to know the position, velocity and accelerations in a mechanism?
- Once a tentative mechanism design has been synthesized, it must then be analysed
- To calculate the kinetic energy stored and for dynamic force calculations
- Many methods and approaches exist
- In this course, we will learn the graphical method
- Even in this age of computer, the graphical solutions provide the beginning student some visual feedback on the solution
- Graphical method has more than historical value as it can provide a quick check on the results from a computer program solution



4-bar mechanism



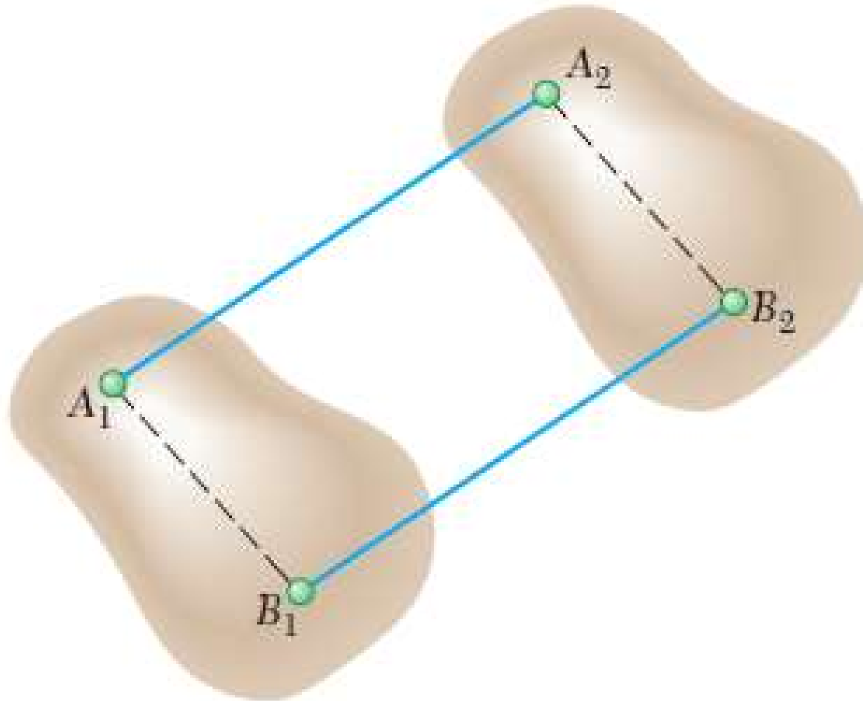
Slider-crank mechanism

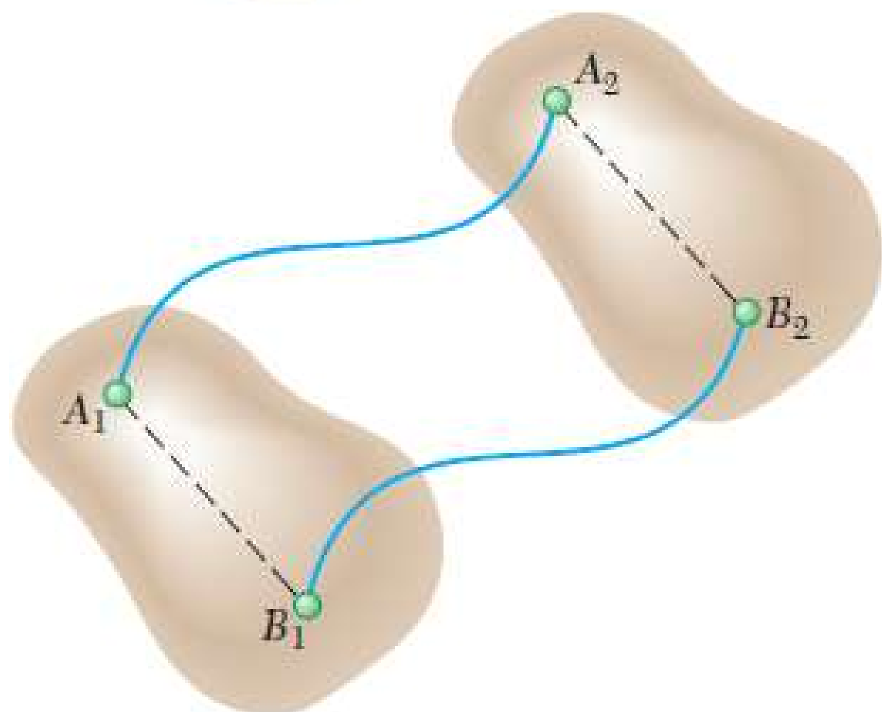
Types of Motion in Rigid-Bodies

1. Translation
2. Rotation about a Fixed Axis
3. General Plane Motion or complex motion
4. Motion about a Fixed Point
5. General Motion

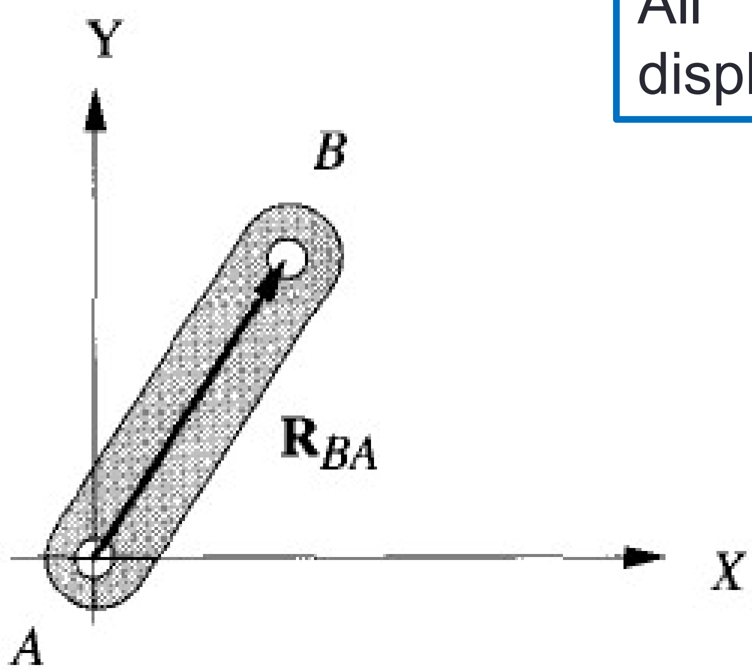
1. Translation

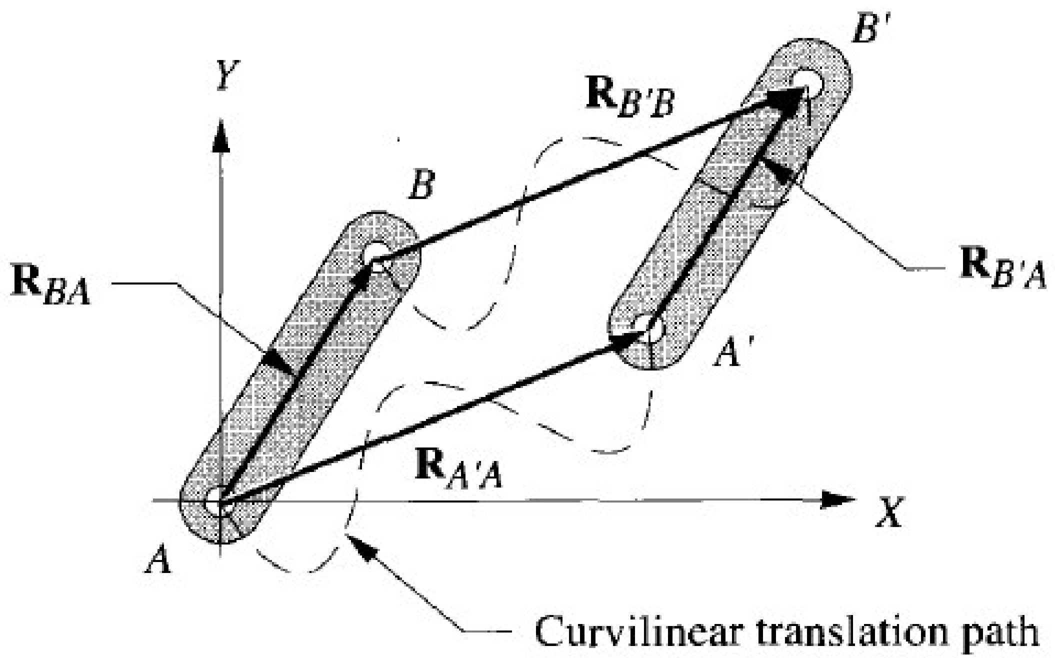
- A motion is translation if any straight line inside the body keeps the same direction during the motion
- All the particles forming the body move along parallel paths
- If these paths are straight lines, the motion is said to be a *rectilinear translation*
- If the paths are curved lines, the motion is a *curvilinear translation*
- All the points of the body have the same velocity and the same acceleration at any given instant





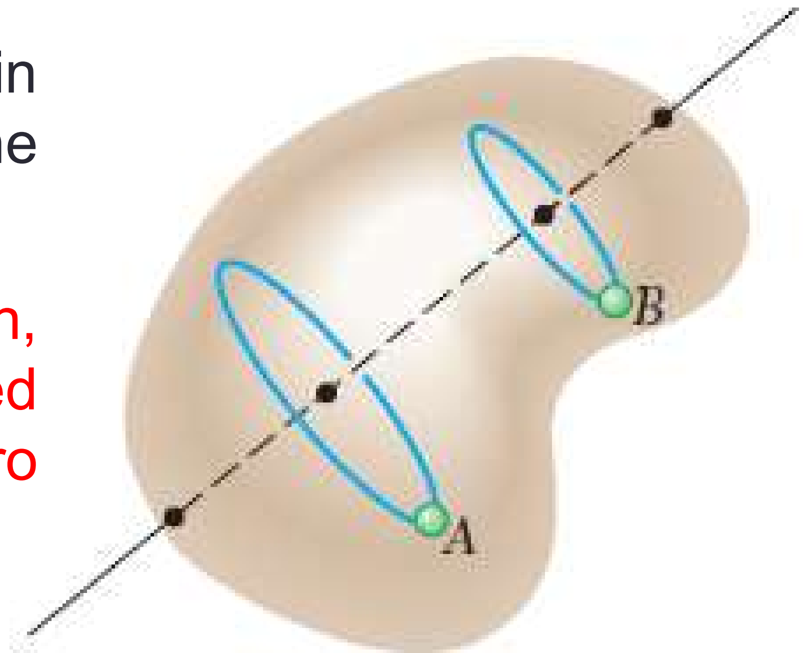
All points on the body have the same displacement, velocity and acceleration

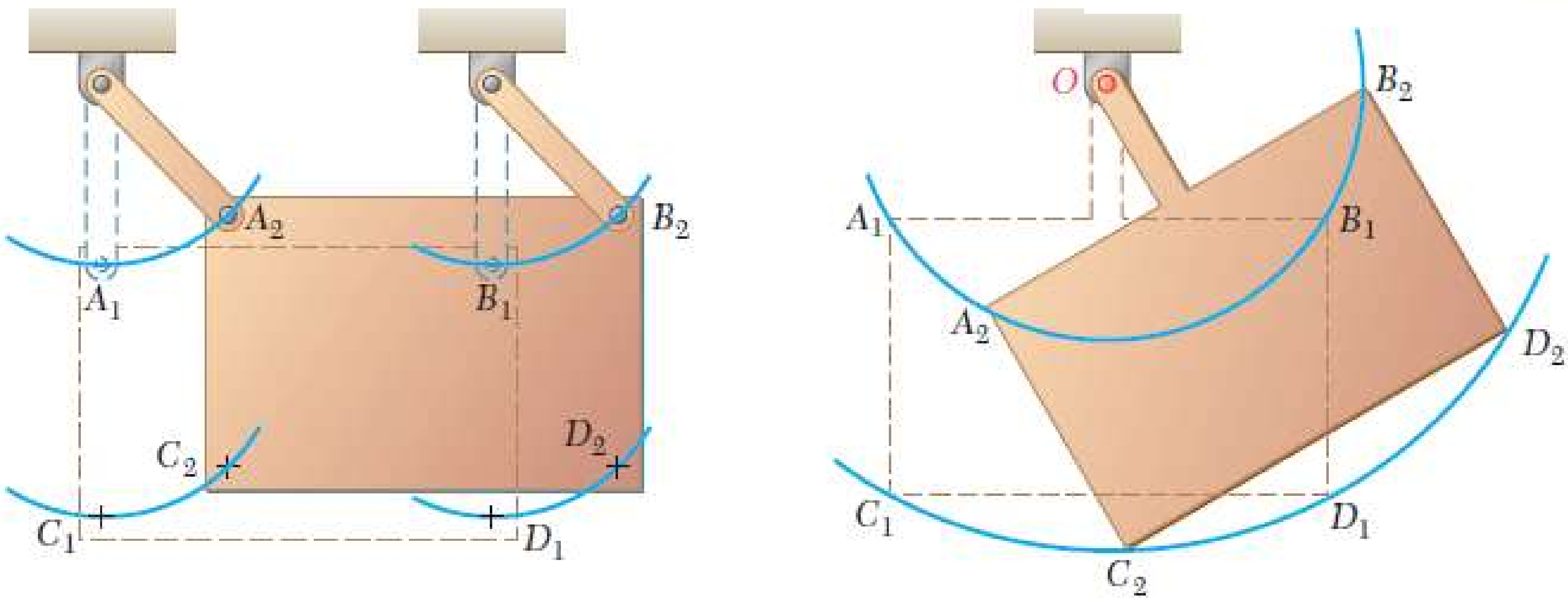




2. Rotation about a Fixed Axis

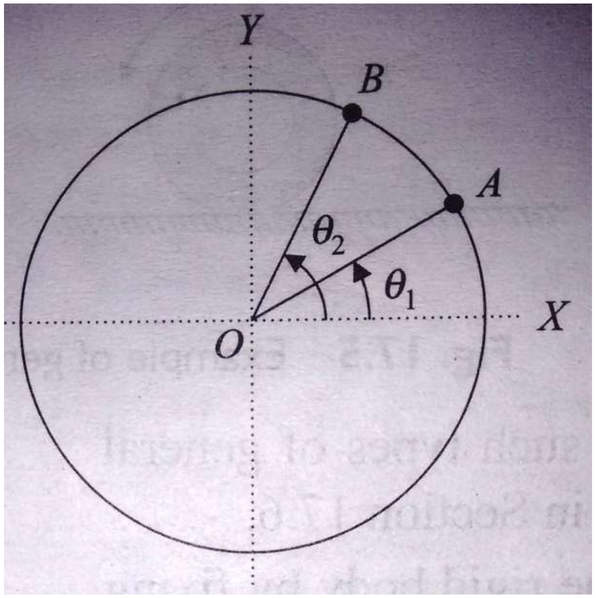
- The particles forming the rigid body move in parallel planes along circles centered on the same fixed axis
- If this axis, called the axis of rotation, intersects the rigid body, the particles located on the axis have zero velocity and zero acceleration

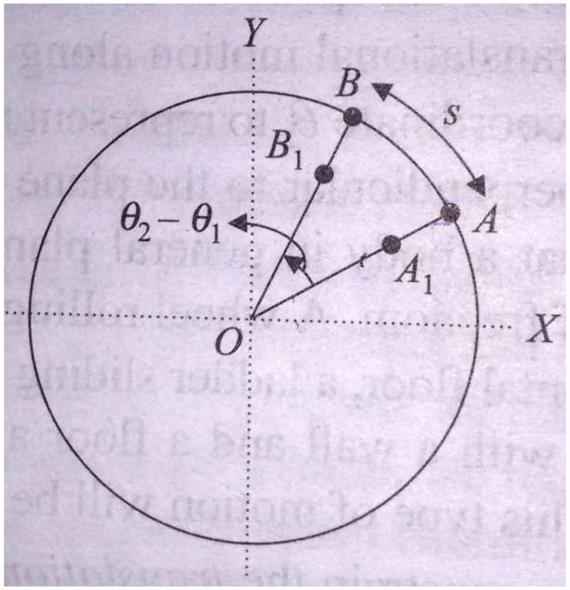




(a) Curvilinear translation (b) Rotation

Angular Displacement





- $\theta_2 - \theta_1$ in time $t_2 - t_1$ is called as angular displacement
- By convention anticlockwise displacement is positive
- When the particle moves from *A* to *B*, other particles (say A_1) on the same radial line also get displaced (from A_1 to B_1) through the same angle
- The angular displacement of every particle in fixed rotation remains the same

Angular Velocity

$$\omega_{ave} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$$\omega = \frac{2\pi N}{60}\ rad/s$$

- As we know that all the particles undergo same displacement in a given time interval, their angular velocity is also the same

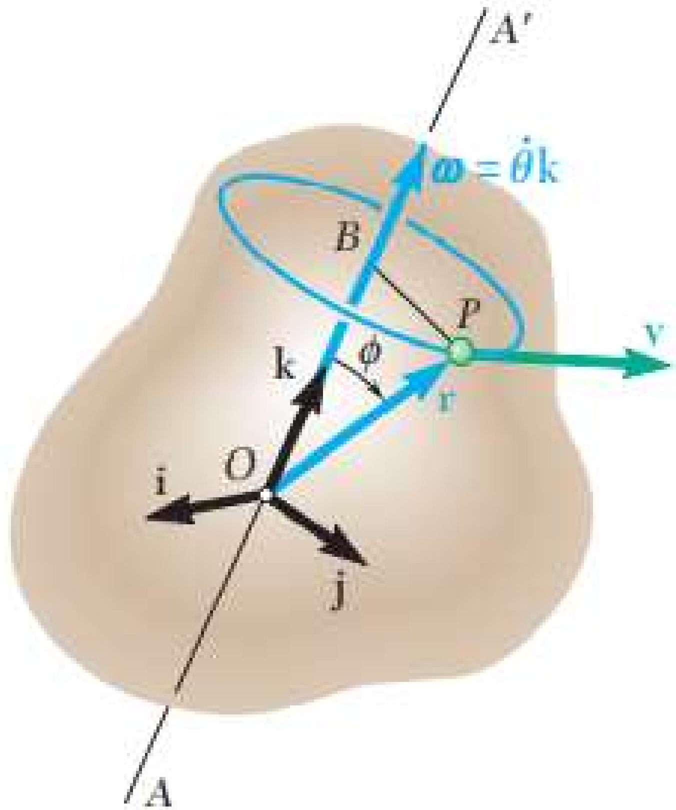
Angular Acceleration

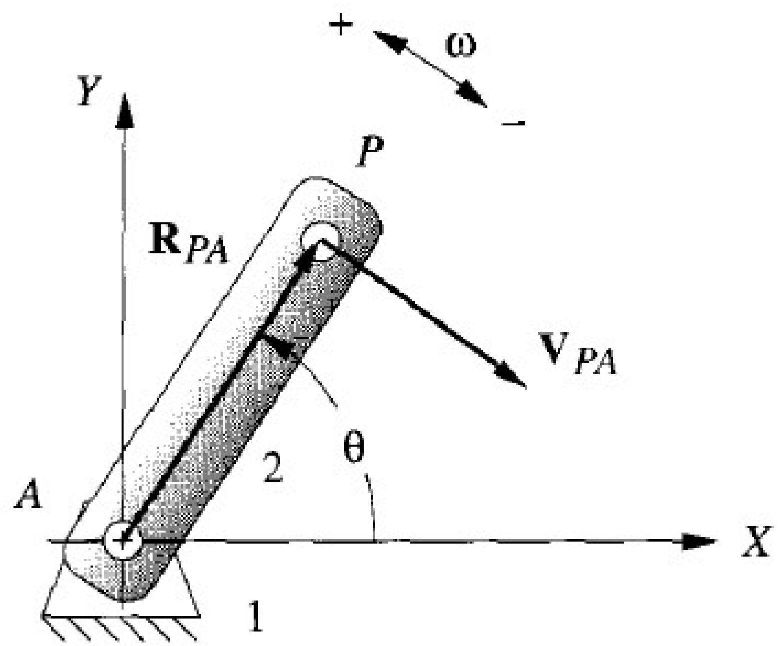
$$\alpha_{ave} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Unit for angular acceleration is rad/s^2

- Angular acceleration is also same for all the particles in the rigid body
- In fixed axis rotation, the angular displacement, velocity and acceleration are same for every particle in the body
- Hence, by describing the motion of one particle in the body, the motion of the entire body can be described





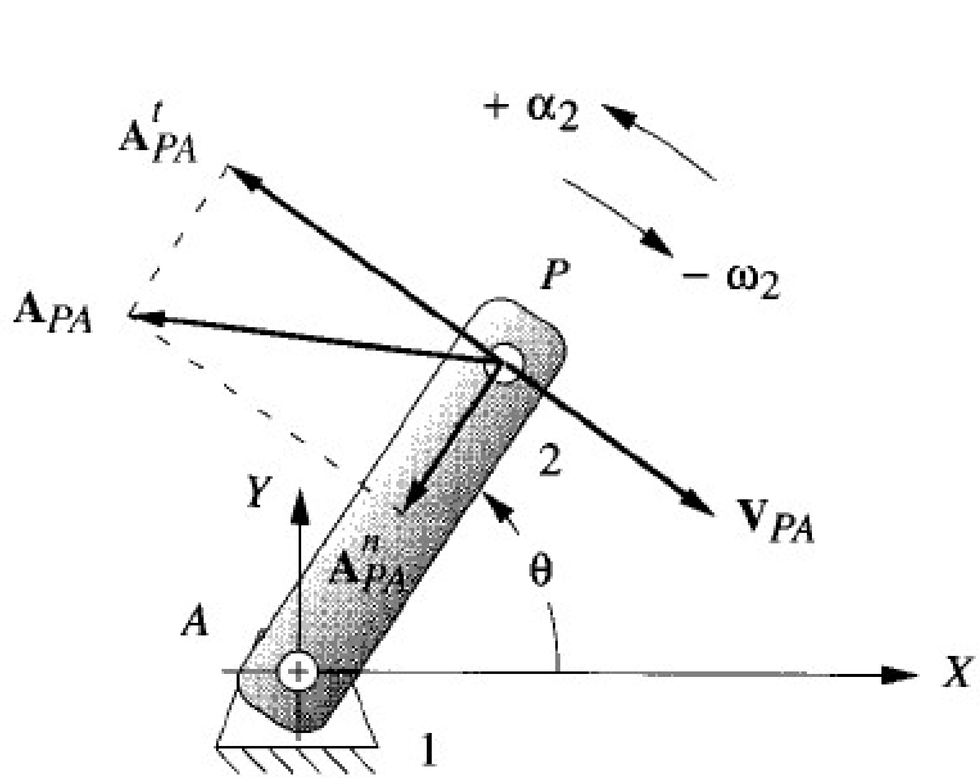
$$\boldsymbol{v}_{PA} = \frac{d\boldsymbol{R}_{PA}}{dt} = \boldsymbol{\omega} \times \boldsymbol{R}_{PA}$$

$$v_{PA} = \omega R_{PA}$$

Velocity-difference equation

$$v_P = \cancelto{0}{v_A} + v_{P/A}$$

Absolute velocity



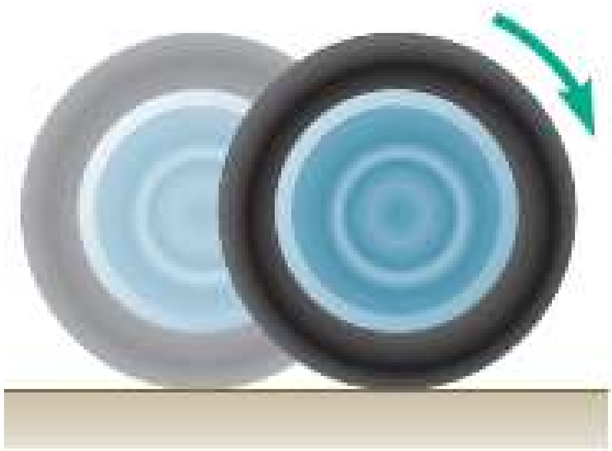
$$\boldsymbol{v}_P = \overset{0}{\cancel{\boldsymbol{v}_A}} + \boldsymbol{v}_{P/A}$$

$$\boldsymbol{a}_P = \overset{0}{\cancel{\boldsymbol{a}_A}} + \boldsymbol{a}_{P/A}$$

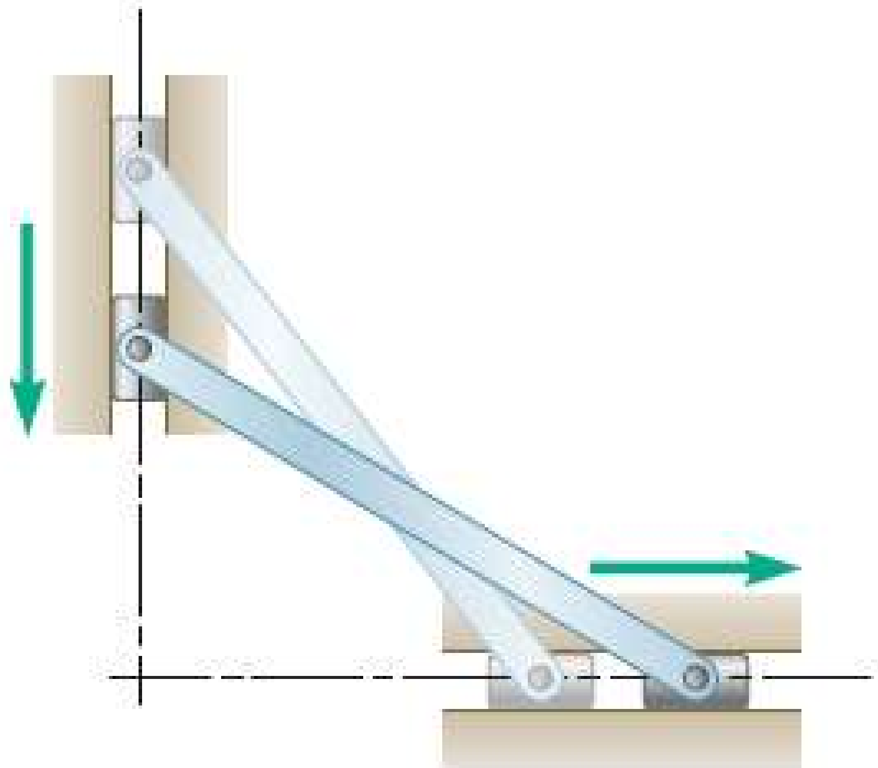
$$\boldsymbol{a}_P = \overset{0}{\cancel{\boldsymbol{a}_A}} + (\boldsymbol{a}_{PA})_n + (\boldsymbol{a}_{PA})_t$$

3. General Plane Motion

- Motions in which all the particles of the body move in parallel planes
- Any plane motion which is neither a rotation nor a translation is referred to as a *general plane motion*

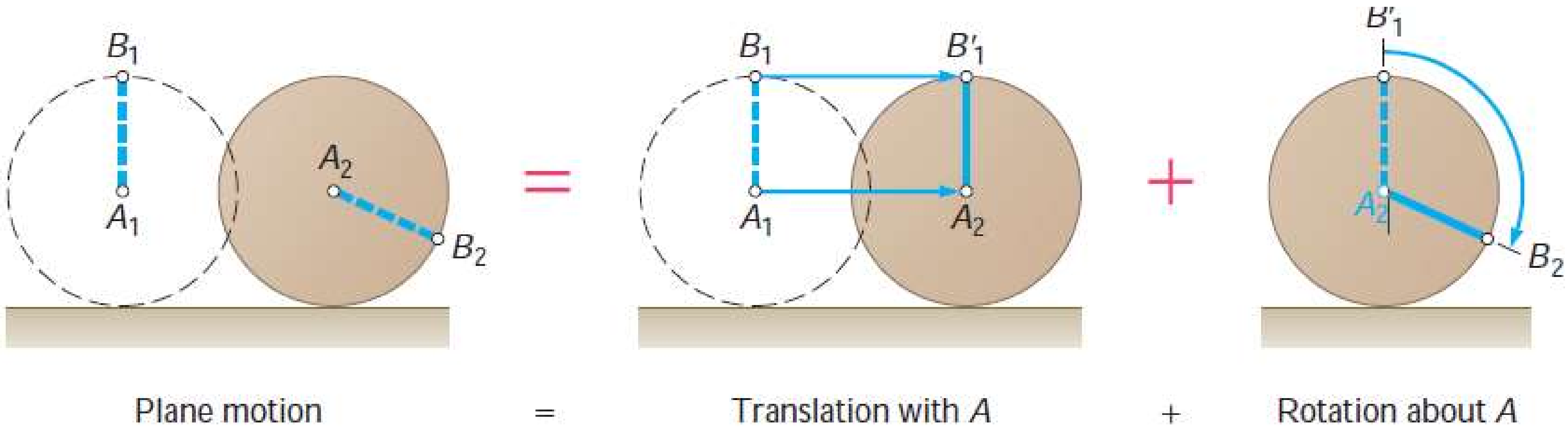


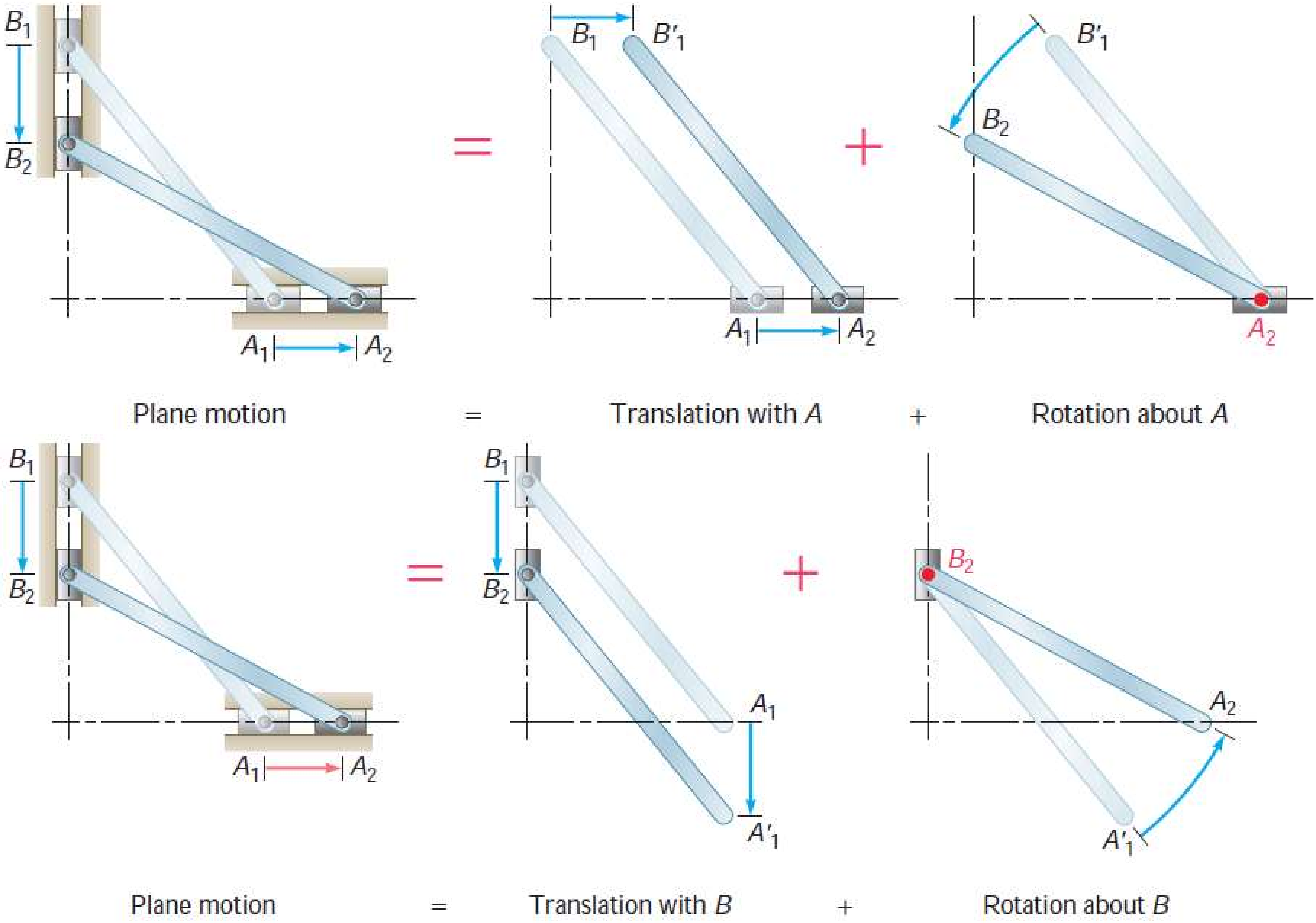
(a) Rolling wheel

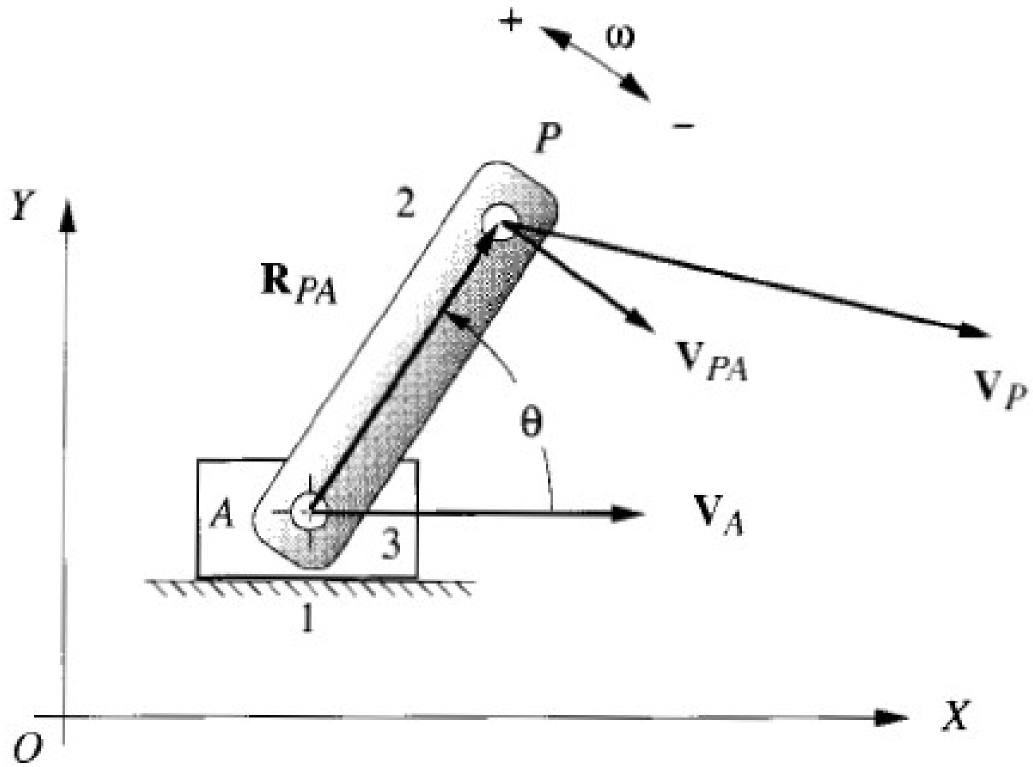


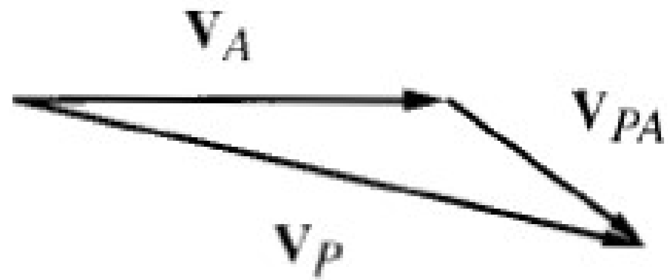
(b) Sliding rod

- A plane motion which is neither a rotation nor a translation
- However, a general plane motion can always be considered as the sum of a translation and a rotation




B1
B2
A1
A2
=
B1
B'1
A1
A2
+
B'1
B2
A2
Plane motion
=
Translation with A
+
Rotation about A
B1
B2
A1
A2
=
B1
B2
A1
A'1
+
B2
A2
A'1
Plane motion
=
Translation with B
+
Rotation about B



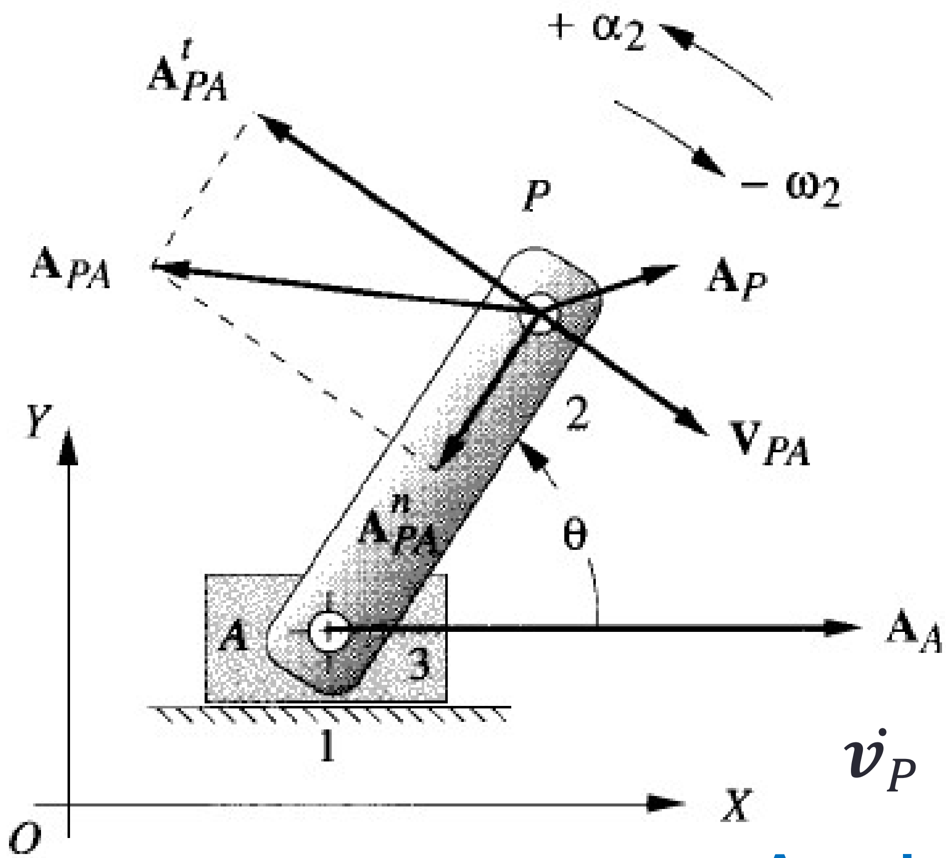


Velocity-difference equation,

$$\boxed{\boldsymbol{v}_P = \boldsymbol{v}_A + \boldsymbol{v}_{PA}}$$

$$\boldsymbol{v}_{PA} = \boldsymbol{\omega} \times \boldsymbol{R}_{PA}$$

$$v_{PA} = \omega R_{PA}$$



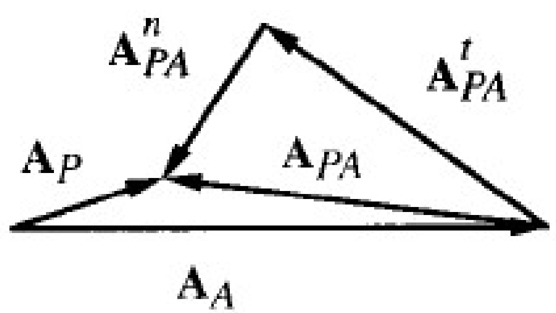
$$\boldsymbol{v}_P = \boldsymbol{v}_A + \boldsymbol{v}_{PA}$$

$$\boldsymbol{v}_P = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{R}_{PA}$$

Differentiating,

$$\dot{\boldsymbol{v}}_P = \dot{\boldsymbol{v}}_A + \boldsymbol{\omega} \times \dot{\boldsymbol{R}}_{PA} + \dot{\boldsymbol{\omega}} \times \boldsymbol{R}_{PA}$$

$$\dot{\boldsymbol{v}}_P = \dot{\boldsymbol{v}}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{R}_{PA}) + \boldsymbol{\alpha} \times \boldsymbol{R}_{PA}$$



Acceleration-difference equation,

$$\boxed{\boldsymbol{a}_P = \boldsymbol{a}_A + (\boldsymbol{a}_{PA})_n + (\boldsymbol{a}_{PA})_t}$$

$$(a_{PA})_n = \omega^2 R_{PA} = \frac{{v_{PA}}^2}{R_{PA}} \qquad (a_{PA})_t = \alpha R_{PA}$$

Relationship between Angular and Linear Motions

$$\boxed{s = r\theta}$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \boxed{v = r\omega}$$

Tangential acceleration, $a_t = \dfrac{dv}{dt} = r\dfrac{d\omega}{dt} = r\alpha$

$$\boxed{a_t = r\alpha}$$

Normal acceleration, $a_n = \dfrac{v^2}{r} = \dfrac{(r\omega)^2}{r} = r\omega^2$

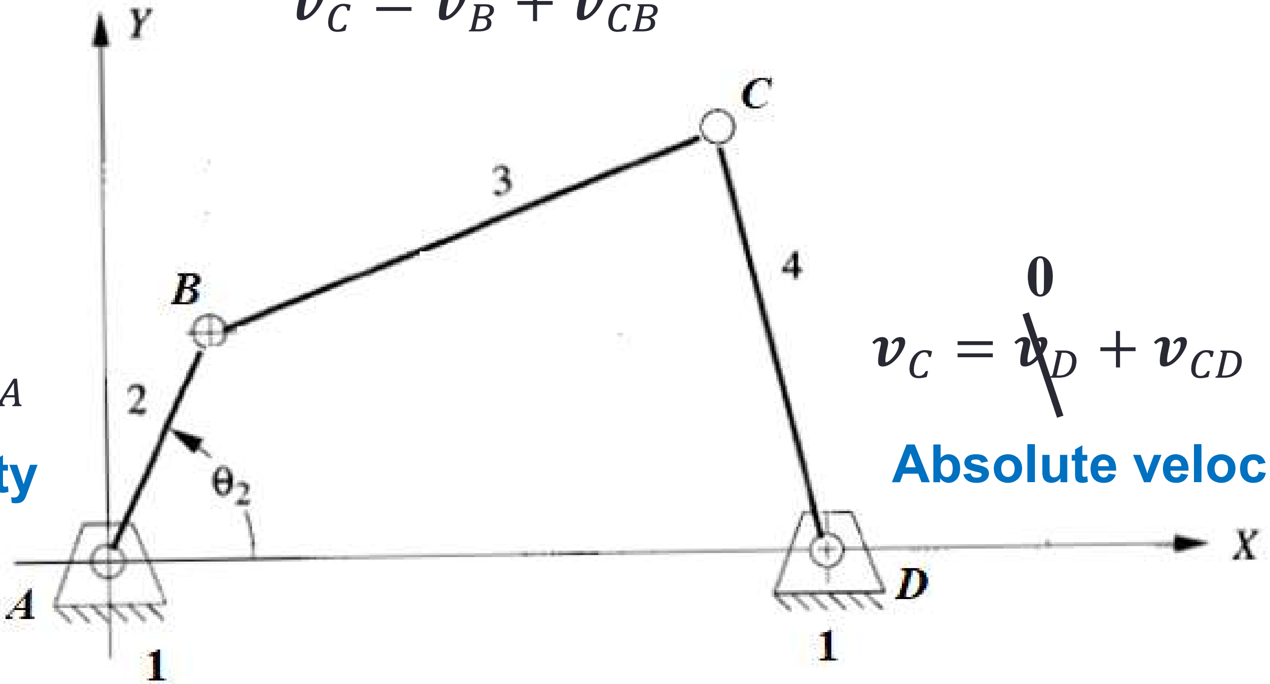
$$\boxed{a_n = r\omega^2}$$

Four Bar Mechanism

Velocity vectors

Velocity difference equation

$$v_C = v_B + v_{CB}$$



$$v_B = \overset{0}{\cancel{v_A}} + v_{BA}$$

Absolute velocity

$$v_C = \overset{0}{\cancel{v_D}} + v_{CD}$$

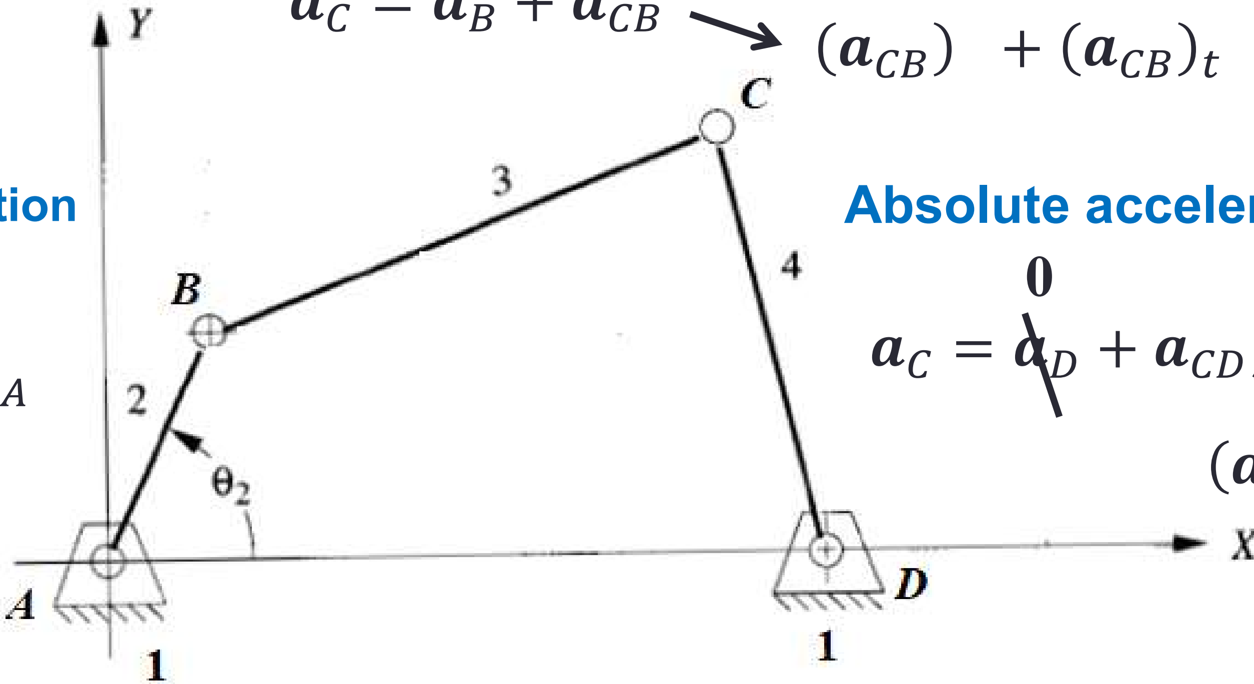
Absolute velocity

Four Bar Mechanism

Acceleration vectors

Acceleration difference equation

$$\boldsymbol{a}_C = \boldsymbol{a}_B + \boldsymbol{a}_{CB} \rightarrow (\boldsymbol{a}_{CB}) \quad + (\boldsymbol{a}_{CB})_t$$



Absolute acceleration

$$\boldsymbol{a}_B = \cancelto{0}{\boldsymbol{a}_A} + \boldsymbol{a}_{BA}$$

$$(\boldsymbol{a}_{BA})_n + (\boldsymbol{a}_{BA})_t$$

Absolute acceleration

$$\boldsymbol{a}_C = \cancelto{0}{\boldsymbol{a}_D} + \boldsymbol{a}_{CD}$$

$$(\boldsymbol{a}_{CD})_n + (\boldsymbol{a}_{CD})_t$$

Slider-crank Mechanism

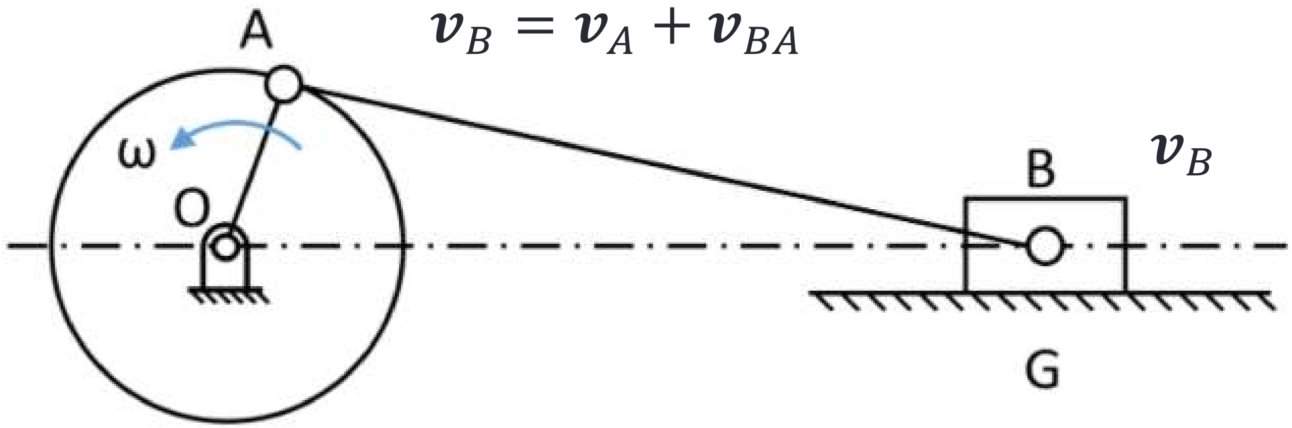
Velocity vectors

Velocity difference equation

$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{BA}$$

$$\boldsymbol{v}_A = \overset{\mathbf{0}}{\cancel{\boldsymbol{v}_O}} + \boldsymbol{v}_{AO}$$

Absolute velocity



Acceleration vectors

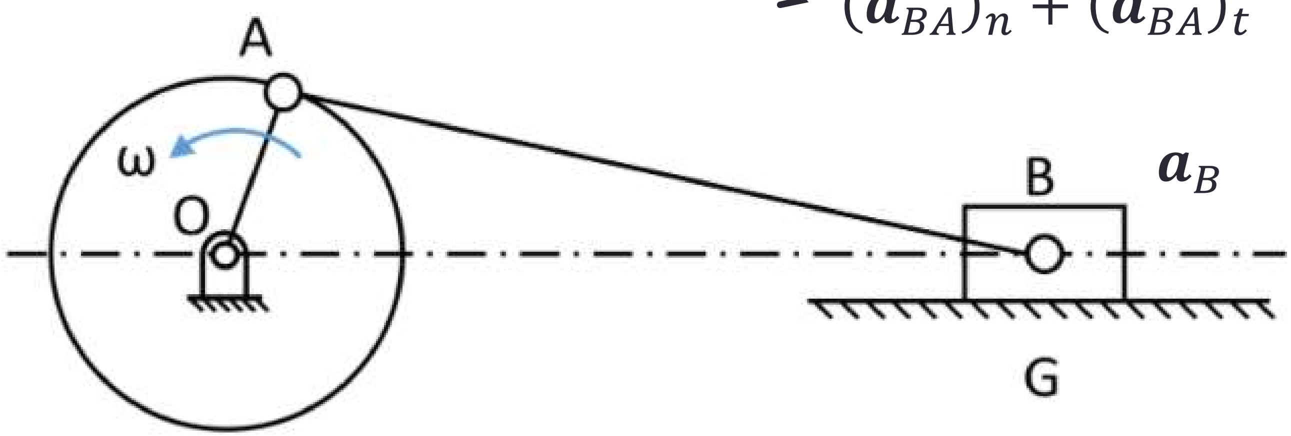
Acceleration difference equation

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{BA} \rightarrow (\boldsymbol{a}_{BA})_n + (\boldsymbol{a}_{BA})_t$$

Absolute acceleration

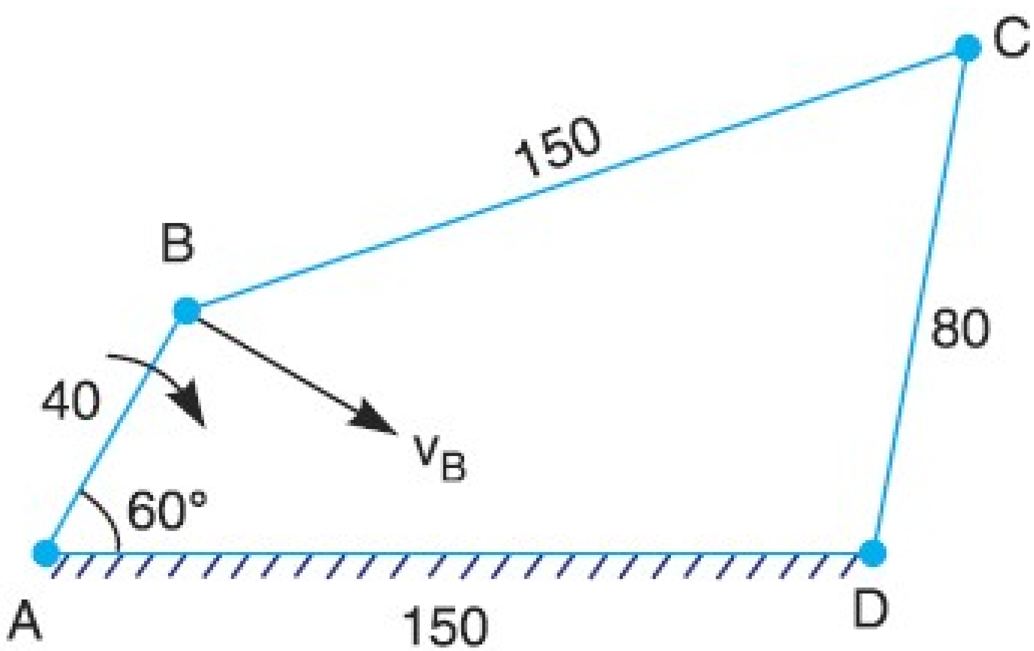
$$\boldsymbol{a}_A = \overset{\mathbf{0}}{\cancel{\boldsymbol{a}_O}} + \boldsymbol{a}_{AO}$$

$$(\boldsymbol{a}_{AO})_n + (\boldsymbol{a}_{AO})_t$$



Four Bar Mechanism

In a four bar chain *ABCD*, *AD* is fixed and is 150 mm long. The crank *AB* is 40 mm long and rotates at 120 r.p.m. clockwise, while the link *CD* = 80 mm oscillates about *D*. *BC* and *AD* are of equal length. Find the angular velocity and angular acceleration of link *CD* when angle *BAD* = 60°.



Space diagram

To find:

$$\omega_{CD} = ?$$

$$\alpha_{CD} = ?$$

To find v_B

120 r.p.m

$$v_B = \cancel{v_A} + v_{B/A} \qquad \omega_{AB} = \frac{2\pi N}{60} rad/s \qquad \boxed{\omega_{AB} = 12.57\, rad/s}$$

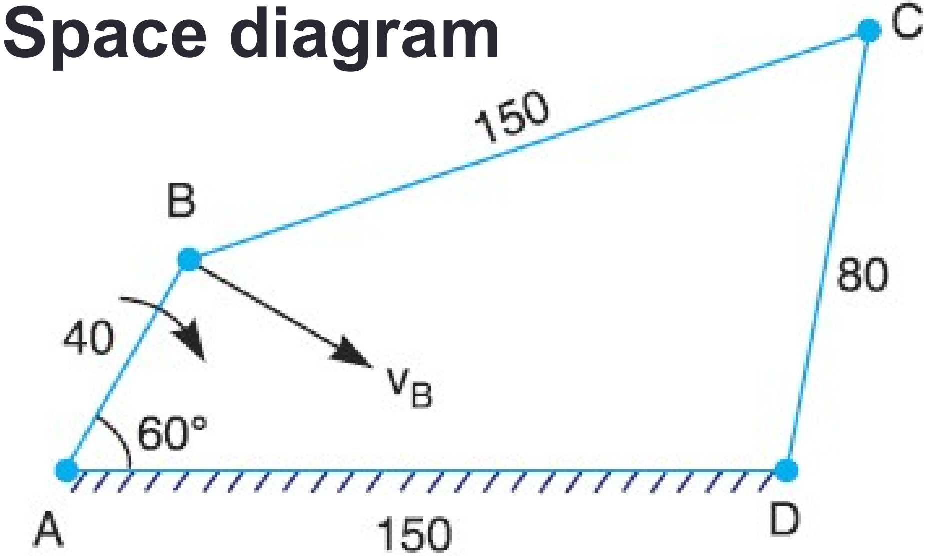
$$v_B = v_{B/A} = r\omega_{AB} = 0.04 \times 12.57$$

$$\boxed{v_B = 0.503\, m/s}$$

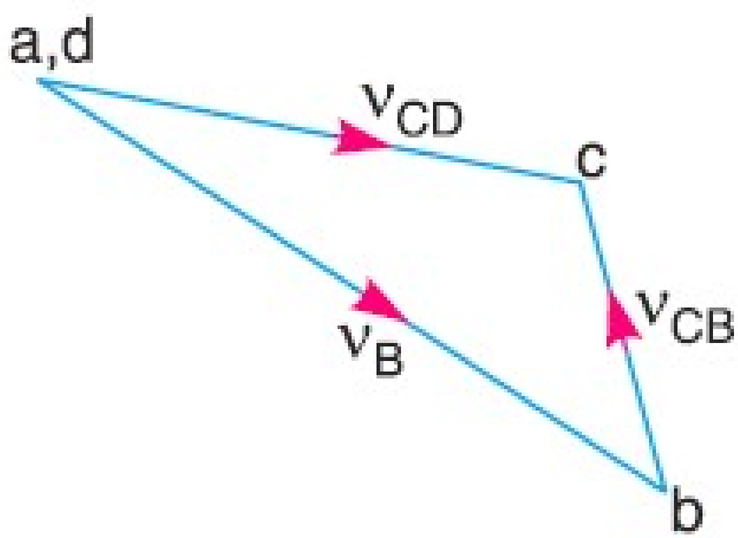
Velocity calculation

Link length, m	ω, rad/sec	v, m/sec	vector, cm
AB = 0.04	12.57	0.503	ab = 5
BC = 0.15			
CD = 0.08			

Space diagram

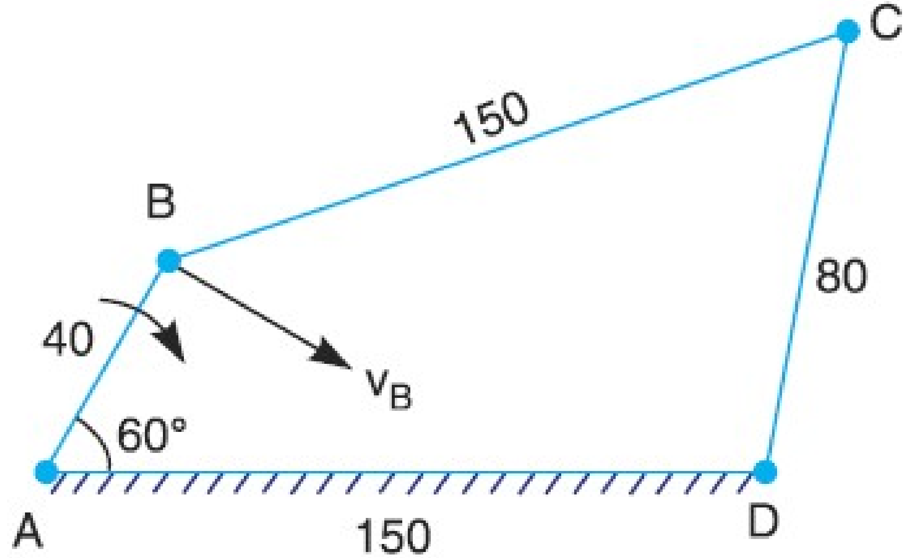


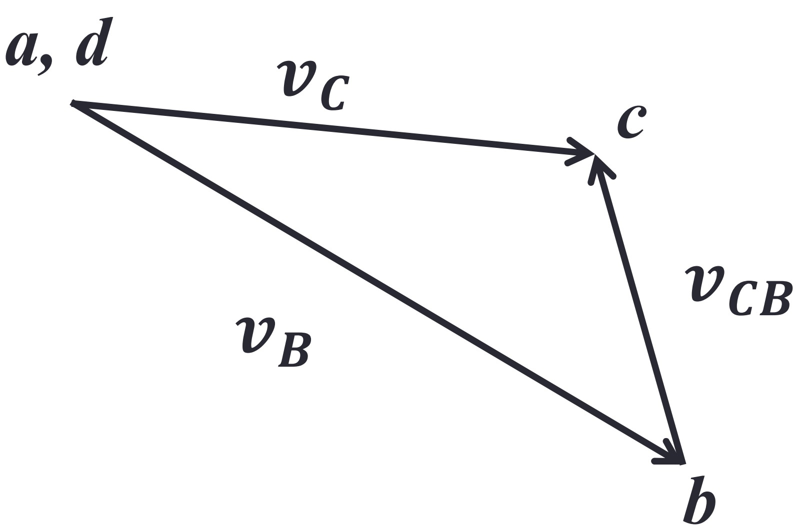
Velocity diagram



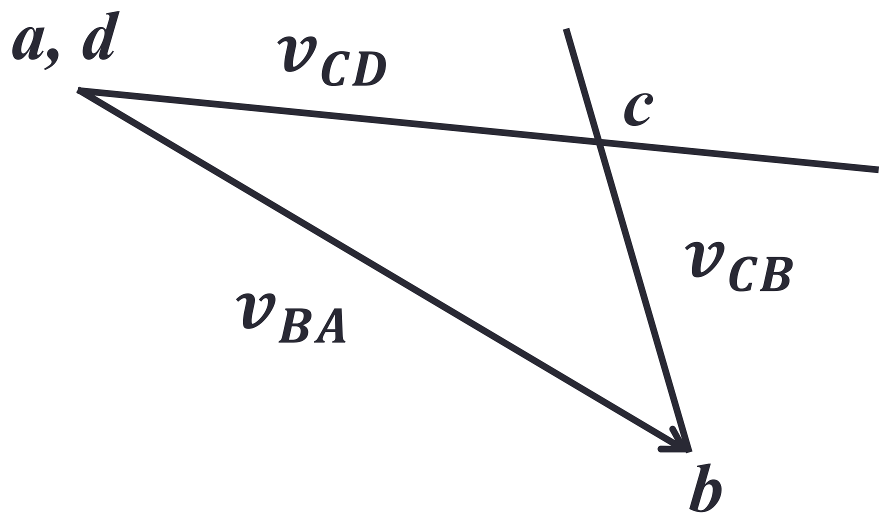
- First, draw the space diagram to some suitable scale
- Points ***a*** and ***d*** are taken as one point (fixed points)
- Draw vector ***ab*** perpendicular to ***BA***, to some suitable scale
- Now from point ***b***, draw vector ***bc*** perpendicular to ***CB*** to represent the velocity of ***C*** with respect to ***B*** ($v_{C/B}$)
- from point ***d***, draw vector ***dc*** perpendicular to ***CD*** to represent the velocity of ***C*** with respect to ***D*** ($v_{C/D}$)
- The vectors ***bc*** and ***dc*** intersect at ***c***

Space diagram





Velocity diagram



Velocity calculation

Link length, m	ω, rad/sec	v, m/sec	vector, cm
AB = 0.04	12.57	0.503	5
BC = 0.15	1.073	0.161	1.6
CD = 0.08	5.163	0.413	4.1

To find v_C

$$v_C = \frac{0.503}{5} \times 4.1 = 0.413 \text{ m/s}$$

$$v_C = 0.413\ m/s$$

$$\omega_C = \frac{v_C}{r_{CD}} = \frac{0.413}{0.08}$$

$$\omega_C = 5.163 \text{ rad/s}$$

To find v_{CB}

$$v_{CB} = \frac{0.503}{5} \times 1.6 = 0.161 \text{ m/s}$$

$$v_{CB} = 0.161 \text{ m/s}$$

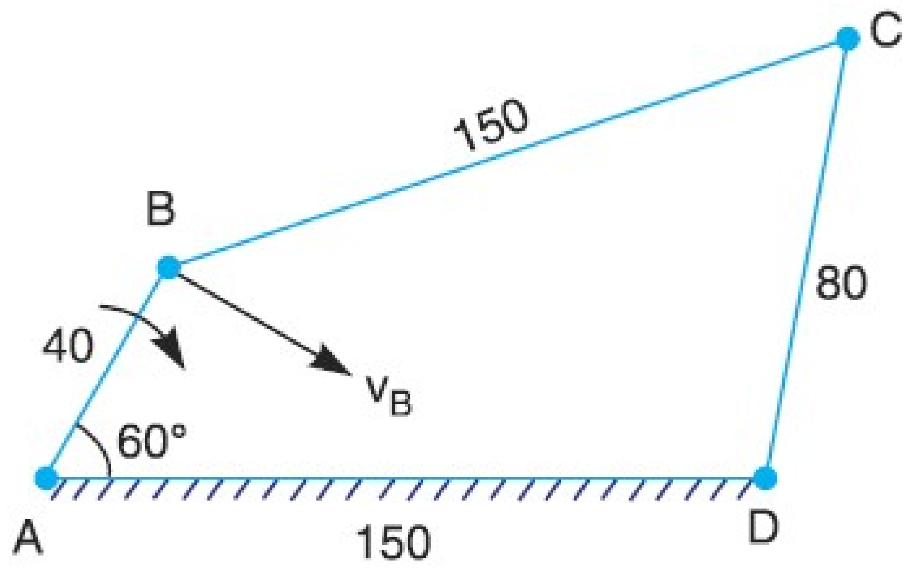
$$\omega_{CB} = \frac{v_{CB}}{r_{CB}} = \frac{0.161}{0.15}$$

$$\omega_{CB} = 1.073 \text{ rad/s}$$

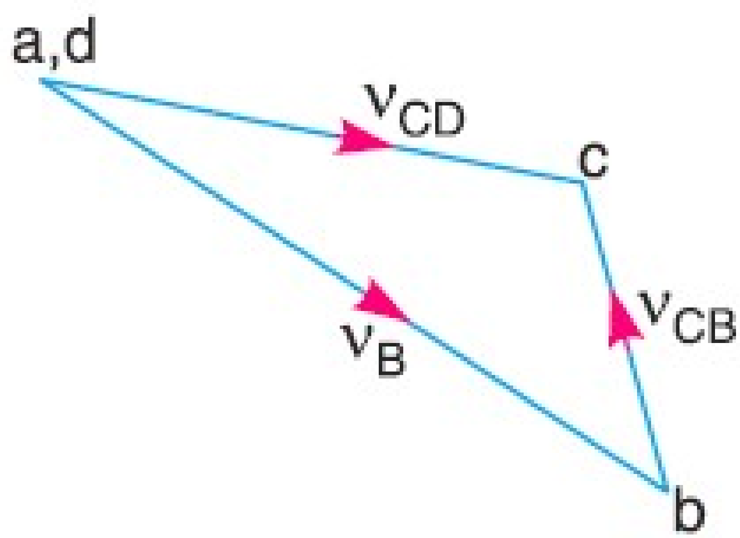
Velocity calculation

Link length, m	ω, rad/sec	v, m/sec	vector, cm
AB = 0.04	12.57	0.503	5
BC = 0.15	1.073	0.161	1.6
CD = 0.08	5.163	0.413	4.1

Space diagram



Velocity diagram



Acceleration calculation

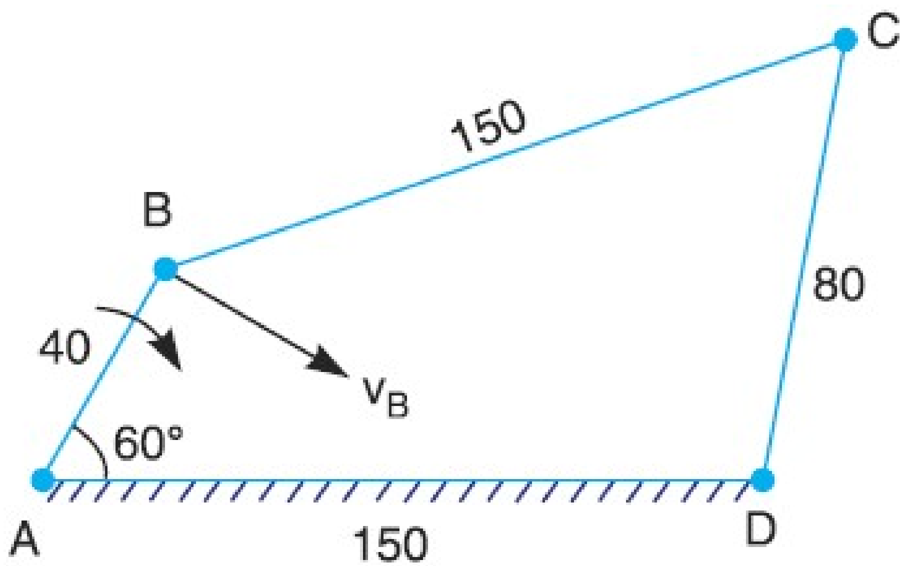
Link length, m	ω, rad/sec	α, rad/sec^2	a_n, m/s^2	a_n vector, cm	a_t, m/s^2	a_t vector, cm
AB = 0.04	12.57	0	6.32	10	0	0
BC = 0.15	1.073		0.173			
CD = 0.08	5.16		2.13			

$$(a_{BA})_{\mathrm{n}} = r_{\mathrm{AB}}\omega_{AB}{}^2 = 6.32\ \mathrm{m/s^2}$$

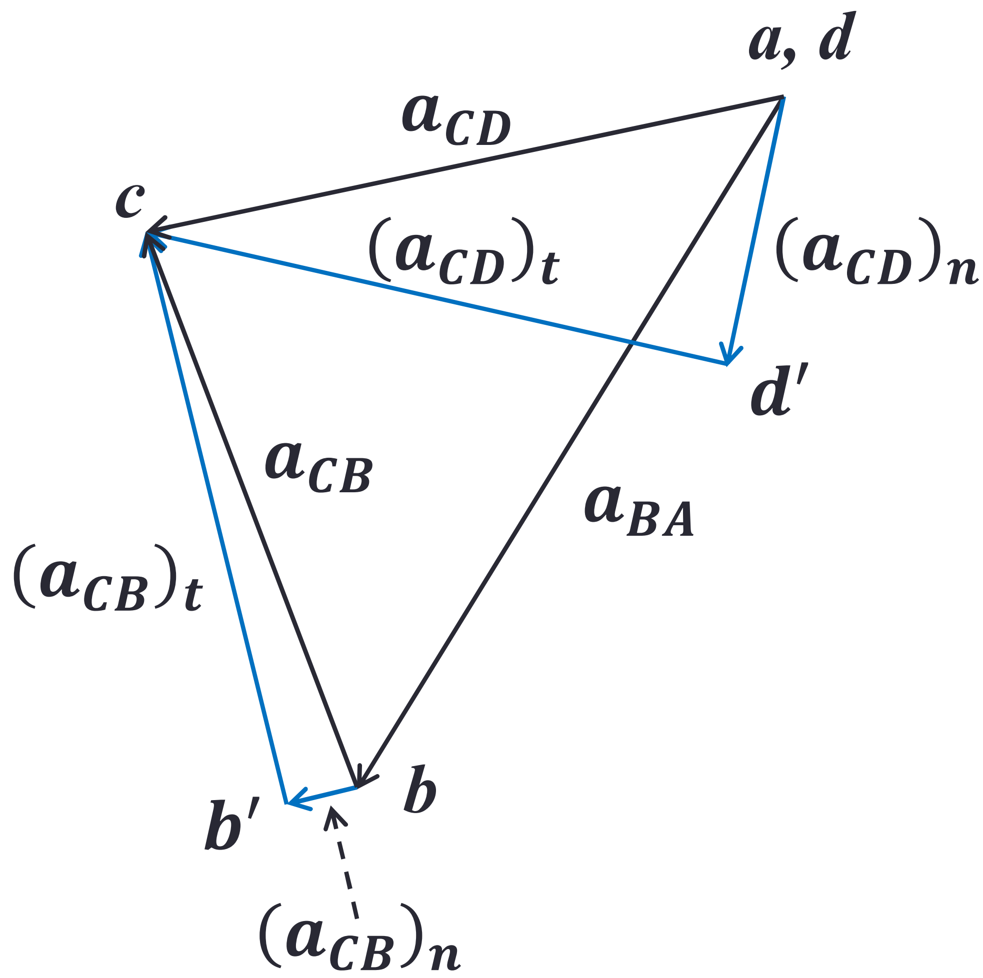
$$(a_{CB})_{\mathrm{n}} = r_{\mathrm{BC}}\omega_{\mathrm{CB}}{}^2 = 0.173\ \mathrm{m/s^2}$$

$$(a_{CD})_{\mathrm{n}} = r_{\mathrm{CD}}\omega_{\mathrm{CD}}{}^2 = 2.13\ \mathrm{m/s^2}$$

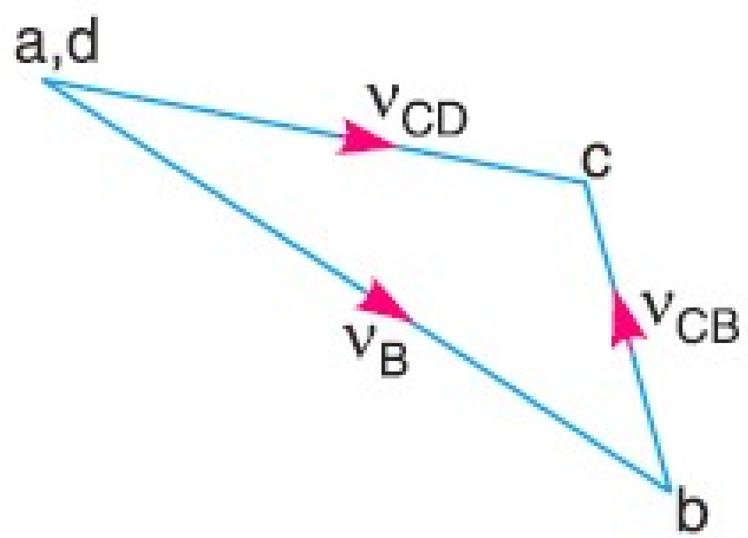
Space diagram



Acceleration diagram



Velocity diagram



Acceleration calculation

Link length, m	ω, rad/sec	α, rad/sec^2	a_n, m/s^2	a_n vector, cm	a_t, m/s^2	a_t vector, cm
AB = 0.04	12.57	0	6.32	10	0	0
BC = 0.15	1.073	32.02	0.173	0.27	4.803	7.6
CD = 0.08	5.16	49.77	2.13	3.37	3.982	6.3

Single Slider Crank Mechanism

The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine :

1. Velocity and acceleration of the piston
2. Angular velocity and angular acceleration of the connecting rod
3. Velocity and acceleration of point *E* on the connecting rod 1.5 m from the gudgeon pin
4. Position and linear velocity of any point *G* on the connecting rod which has the least velocity relative to crank shaft

To find v_B

180 r.p.m

$$v_B = \cancel{v_O} + v_{B/O} \qquad \omega_{OB} = \frac{2\pi N}{60} rad/s \quad \boxed{\omega_{OB} = 18.852\, rad/s}$$

$$v_B = v_{B/O} = r\omega_{OB} = 0.5 \times 18.852$$

$$\boxed{v_B = 9.426\, m/s}$$

Angular velocity of connecting rod ω_{PB}

$$v_{PB} = vector\ pb$$

$$\omega_{P/B} = \frac{v_{P/B}}{PB}$$

Velocity of *point E*

$$v_E = vector\ oe$$

$$\frac{BE}{BP} = \frac{be}{bp}$$

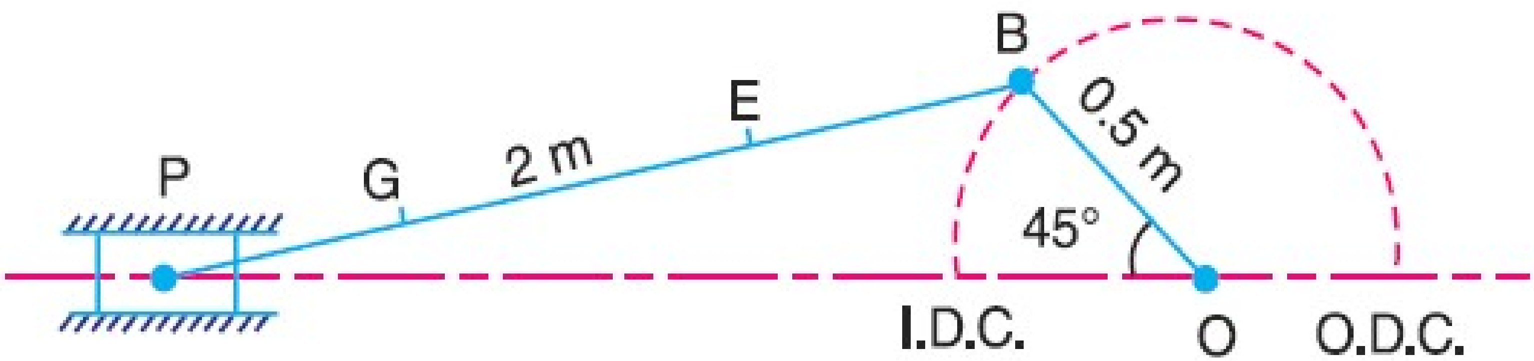
Velocity calculation

Link length, m	ω, rad/sec	v, m/sec	vector, cm
OB = 0.5	18.852	9.426	6
BP = 2.0			
Piston	0		

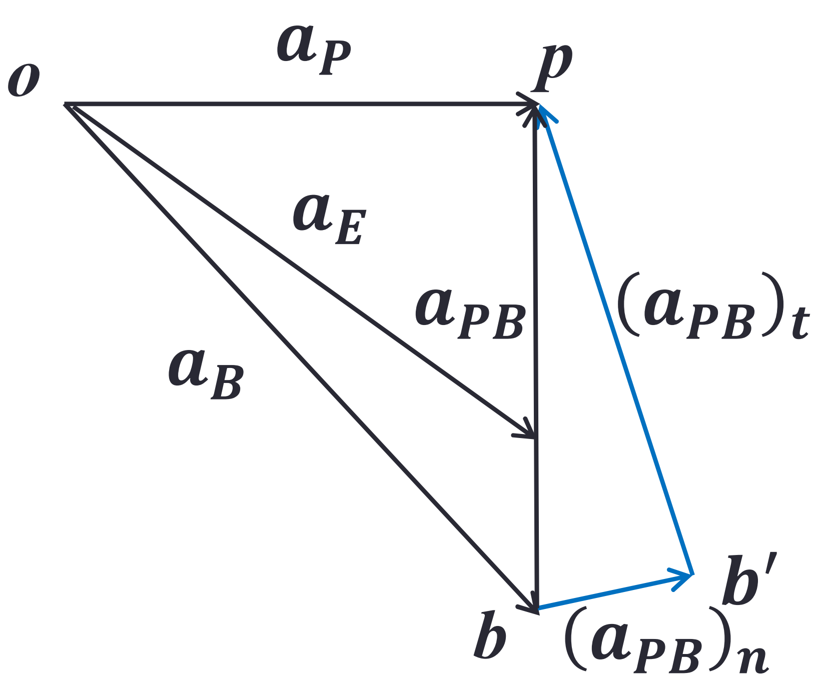
Acceleration calculation

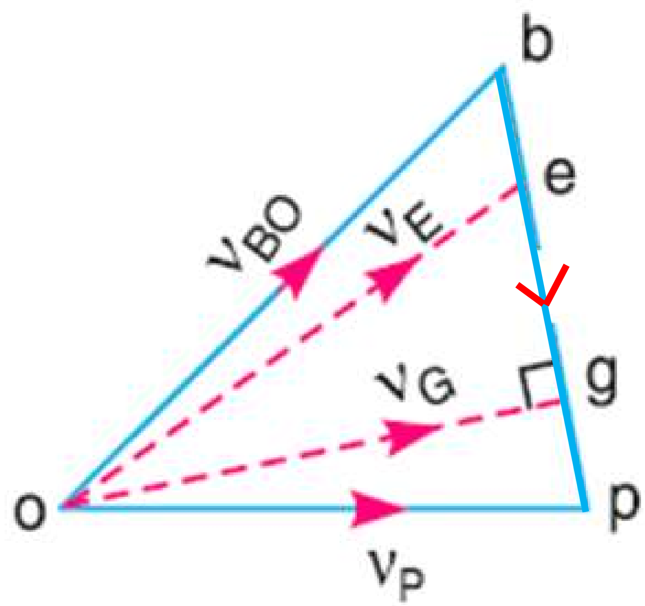
Link length, m	ω, rad/sec	α, rad/sec^2	a_n, m/s^2	a_n vector, cm	a_t, m/s^2	a_t vector, cm
OB = 0.5	18.852	0		10	0	0
BP = 2.0						
Piston						

Space diagram



Velocity diagram



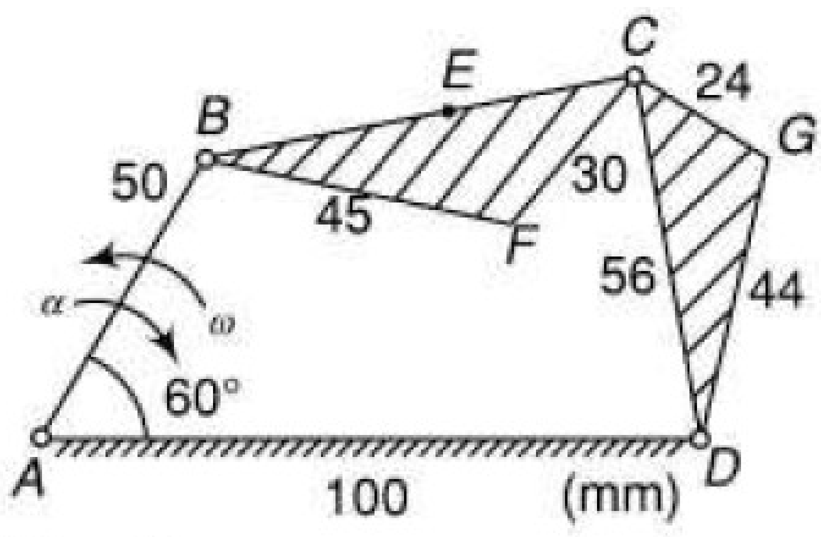


Problem 3

Figure shows the configuration diagram of a four-link mechanism along with the lengths of the links in mm. The link ***AB*** has an instantaneous velocity of **10.5 rad/s** and a retardation of **26 rad/s^2** in the counter clockwise direction. Find

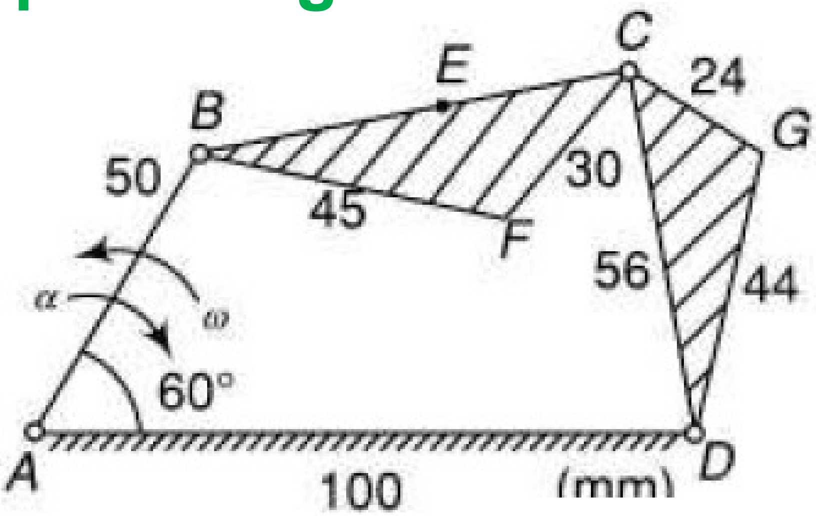
(i) The angular accelerations of the links ***BC*** and ***CD***

(ii) The linear accelerations of the points ***E***, ***F*** and ***G***



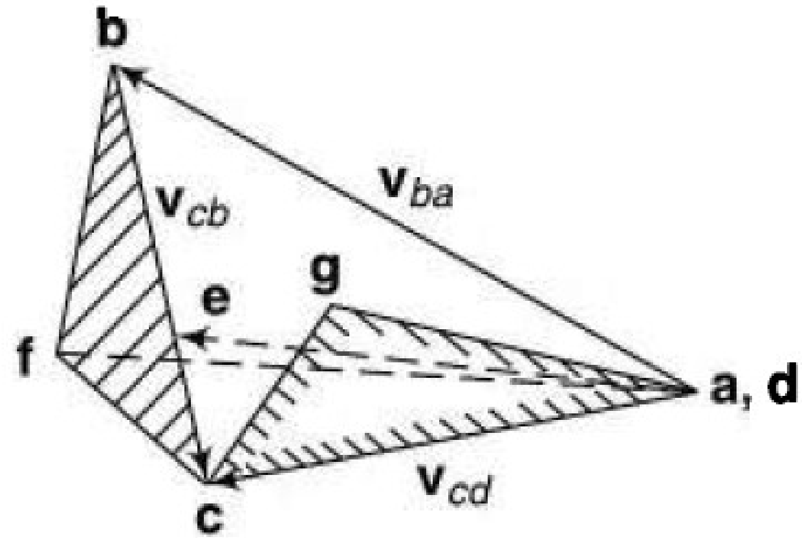
BE = 40 mm
BC = 66 mm
(a)

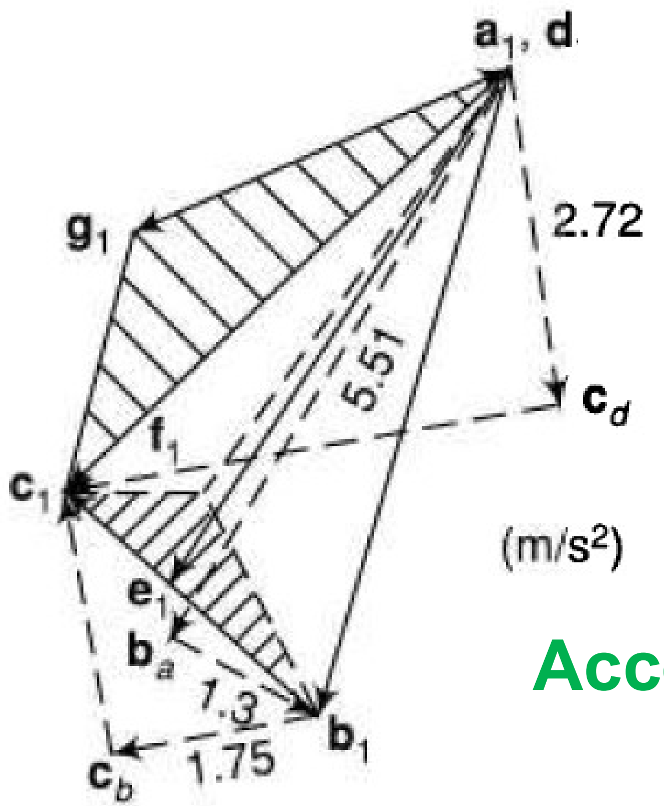
Space diagram



$BE = 40$ mm
$BC = 66$ mm
(a)

Velocity diagram





Acceleration diagram

Step by step procedure:

- First, draw the space diagram to some suitable scale
- Find $v_{BA} = r_{AB}\omega_{AB}$

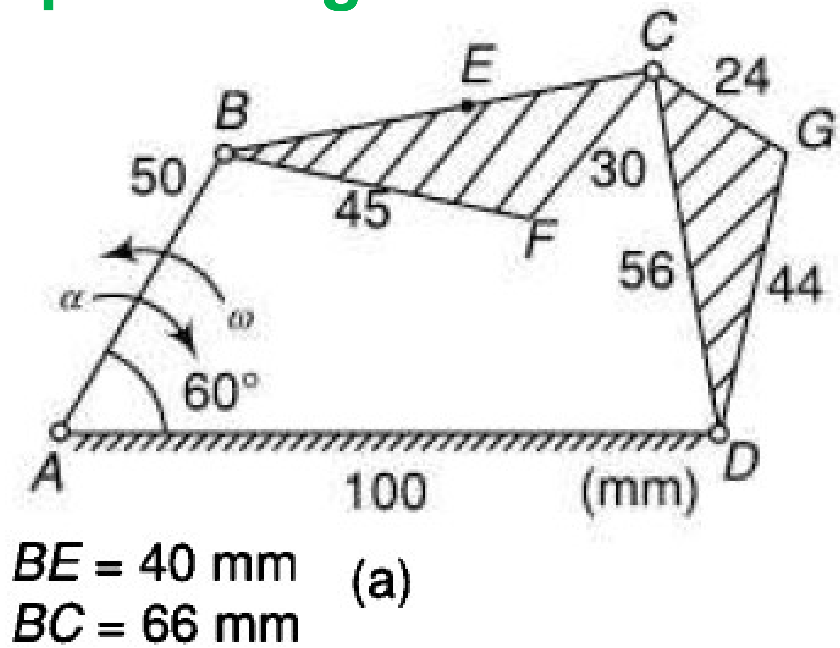
$v_{BA} = 0.05 \times 10.5 = 0.525\, m/s$

- Take suitable scale

$$5\, cm = 0.525\, m/s$$
$$\therefore 1\, cm = 0.105\, m/s$$

- Points ***a*** and ***d*** are taken as one point (fixed points)
- Draw vector ***ab*** perpendicular to ***BA***, equal to the scale 5 cm
- Now from point ***b***, draw vector ***bc*** perpendicular to ***CB*** to represent the velocity of ***C*** with respect to ***B*** (v_{BA})

Space diagram



Velocity diagram

- First, draw the space diagram to some suitable scale
- Find $v_{BA} = r_{AB}\omega_{AB}$

$v_{BA} = 0.05 \times 10.5 = 0.525\, m/s$

- Take suitable scale

$$5\, cm = 0.525\, m/s$$
$$\therefore 1\, cm = 0.105\, m/s$$

- Points ***a*** and ***d*** are taken as one point (fixed points)
- Draw vector ***ab*** perpendicular to ***BA***, equal to the scale 5 cm
- Now from point ***b***, draw vector ***bc*** perpendicular to ***CB*** to represent the velocity of ***C*** with respect to ***B*** (v_{BA})

- From point ***d***, draw vector ***dc*** perpendicular to ***CD*** to represent the velocity of ***C*** with respect to ***D*** (v_{CD})
- The vectors ***bc*** and ***dc*** intersect at ***c***
- ***bf*** and ***cf*** is obtained as follows:

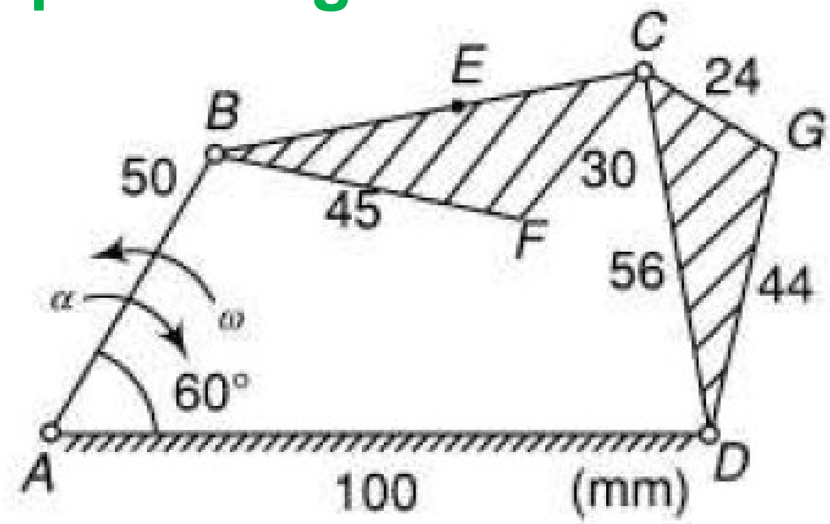
$$bf = \frac{bc}{BC} \times BF$$

Similarly,

$$cf = \frac{bc}{BC} \times CF$$

- ***f*** is located as shown in velocity diagram by following the same order of ***BCF*** (clockwise) in space diagram
- Similarly ***cg*** and ***dg*** is obtained and point ***g*** is located in velocity diagram.

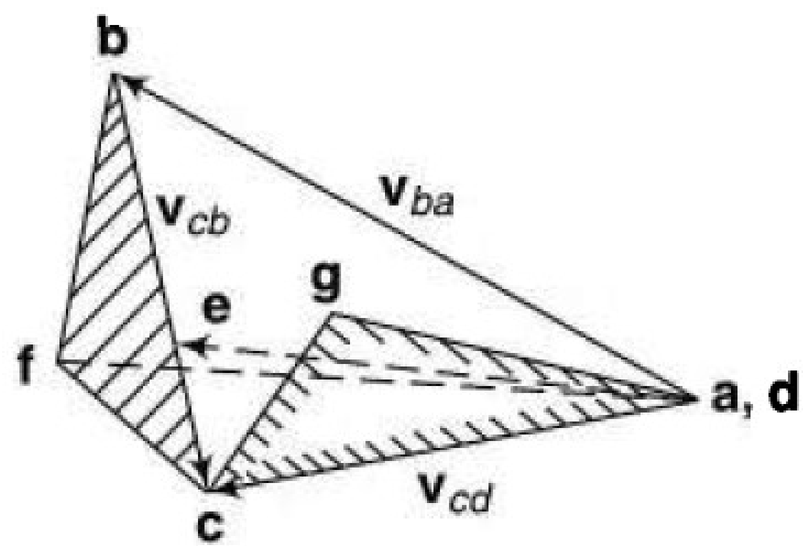
Space diagram



BE = 40 mm
BC = 66 mm

(a)

Velocity diagram



- First, calculate the normal acceleration of link ***AB*** $[(\boldsymbol{a}_{BA})_n]$
- Find $(a_{BA})_n = r_{AB}\omega_{AB}{}^2$

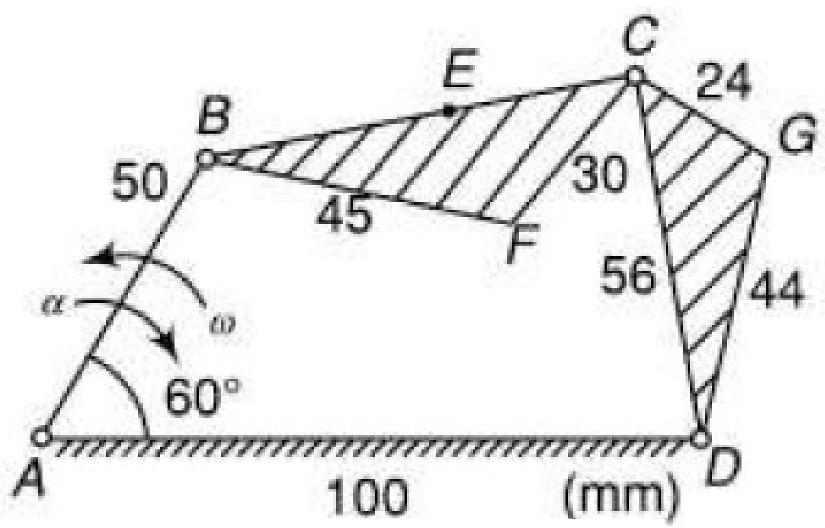
  $(a_{BA})_n = 0.05 \times 10.5^2 = 5.51\, m/s^2$
- Take suitable scale

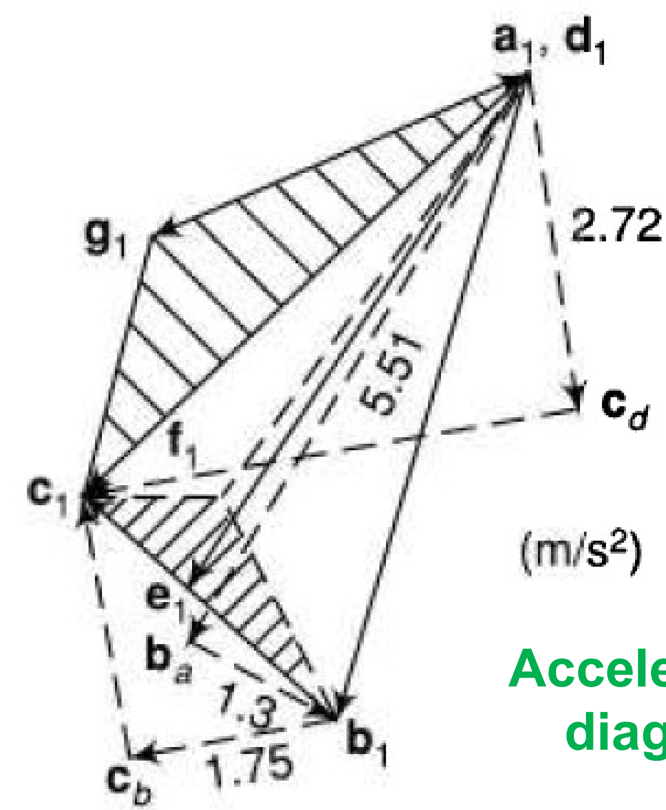
  $$6\, cm = 5.51\, m/s^2$$
  $$\therefore 1\, cm = 0.92\, m/s^2$$
- Since the magnitude and direction of angular acceleration is given, calculate the tangential acceleration of link ***AB*** $[(\boldsymbol{a}_{BA})_t]$
- Find $(a_{BA})_t = r_{AB}\alpha_{AB}$

  $(a_{BA})_t = 0.05 \times 26 = 1.3\, m/s^2$
- According to the scale

  $$1.3\, m/s^2 = 1.41\, cm$$
- Fixed points $\boldsymbol{a_1}$ and $\boldsymbol{d_1}$ are marked in the same location

Space diagram





Acceleration diagram

- Draw line a_1b_a = 5 cm parallel to AB
- From b_a draw a line b_ab_1 = 1.42 cm towards right as α_{AB} is clockwise
- Connect a_1b_1 to obtain $\boldsymbol{a_{BA}}$
- The link will have both normal and tangential accelerations. Take B as centre, then $(\boldsymbol{a_{CB}})_n$ is towards B
- Calculate the normal acceleration of link BC [$(\boldsymbol{a_{CB}})_n$]

$$(a_{CB})_n = \frac{{v_{CB}}^2}{r_{CB}}$$

(v_{CB} is measured from velocity diagram)

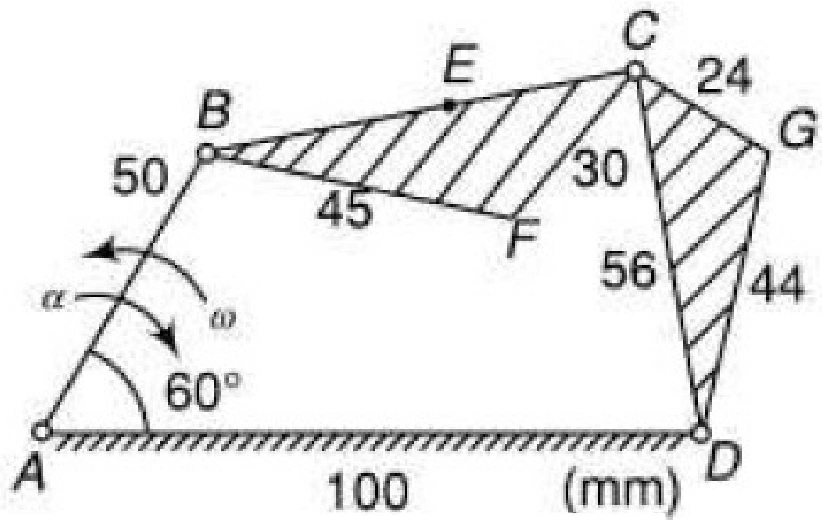
$$vector\ bc = 3.2\ cm$$

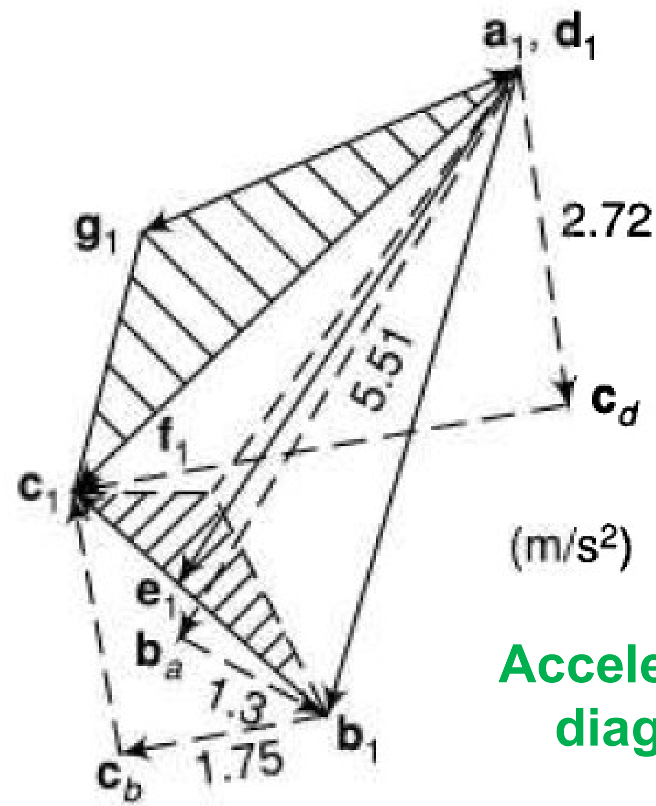
$$\therefore v_{CB} = 3.2 \times 0.105$$

$$v_{CB} = 0.336\ m/s$$

$$(a_{CB})_n = \frac{{v_{CB}}^2}{r_{CB}}$$

Space diagram





Acceleration diagram

$$(a_{CB})_n = \frac{{v_{CB}}^2}{r_{CB}} = \frac{0.336^2}{0.066} = 1.71 m/s^2$$

- To get the length of $\boldsymbol{b_1 c_b}$ use scale

$$b_1 c_a = \frac{1.71}{0.92} = 1.86\ cm$$

- Calculate the normal acceleration of link ***CD*** $[(\boldsymbol{a_{CD}})_{\boldsymbol{n}}]$

$$(a_{CD})_n = \frac{{v_{CD}}^2}{r_{CD}}$$

(v_{CD} is measured from velocity diagram)

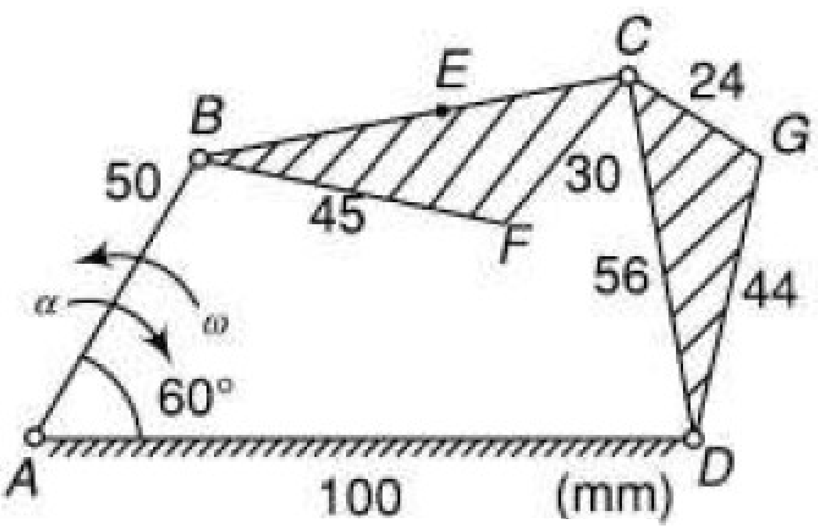
$$vector\ cd = 3.9\ cm$$

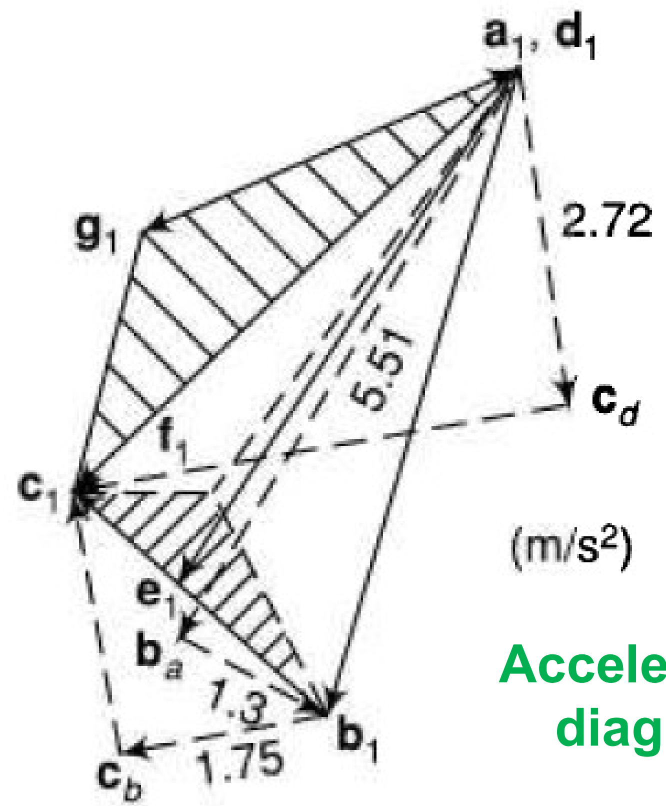
$$\therefore v_{CD} = 3.9 \times 0.105$$

$$v_{CD} = 0.41\ m/s$$

$$(a_{CD})_n = \frac{{v_{CD}}^2}{r_{CD}}$$

Space diagram





Acceleration diagram

$$(a_{CD})_n = \frac{{v_{CD}}^2}{r_{CD}} = \frac{0.41^2}{0.056} = 3\ m/s^2$$

- To get the length of $\boldsymbol{d_1 c_d}$ use scale

$$d_1 c_d = \frac{3}{0.92} = 3.3\ cm$$

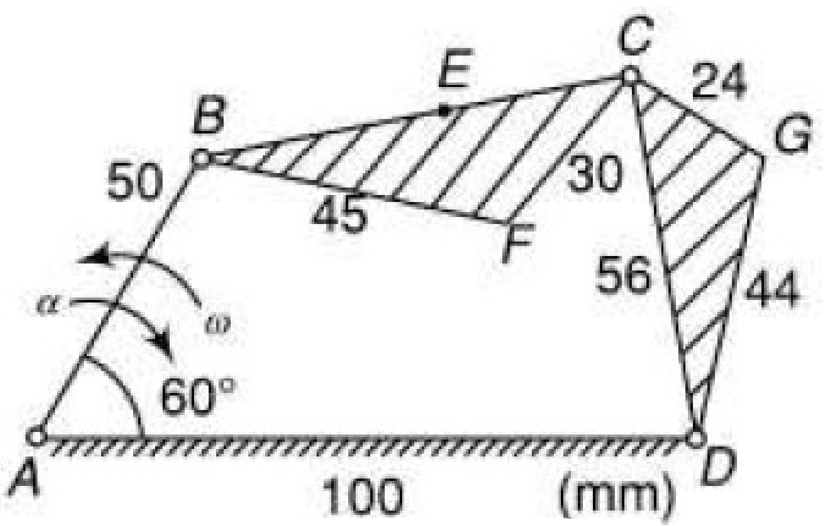
- Draw perpendicular line from $\boldsymbol{c_b}$ $[(\boldsymbol{a_{CB}})_t]$ and $\boldsymbol{c_d}$ $[(\boldsymbol{a_{CD}})_t]$ to meet at $\boldsymbol{c_1}$
- Connect $\boldsymbol{b_1 c_1}$ to get $\boldsymbol{a_{CB}}$ and connect $\boldsymbol{d_1 c_1}$ to get $\boldsymbol{a_{CD}}$
- ***bf*** and ***cf*** is obtained as follows:

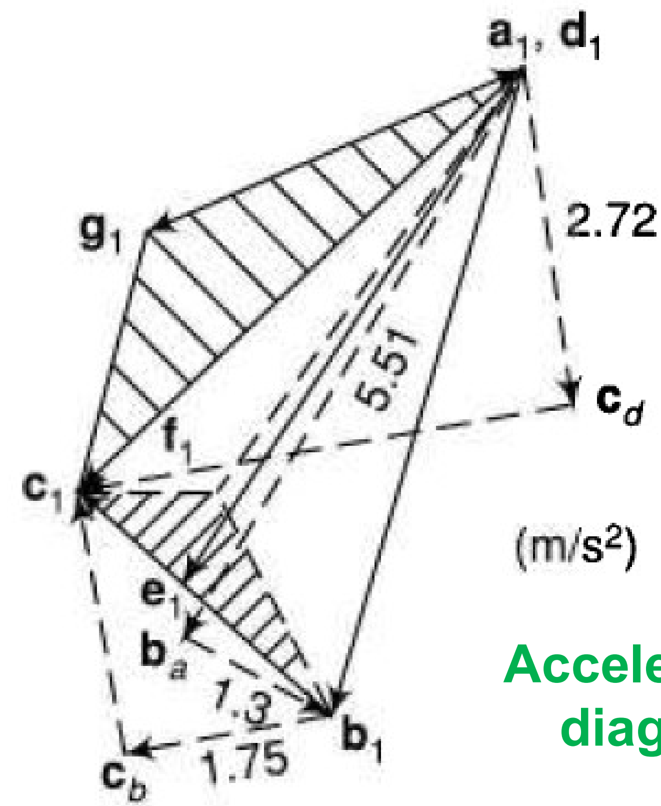
$$b_1 f_1 = \frac{b_1 c_1}{BC} \times BF$$

Similarly,

$$c_1 f_1 = \frac{b_1 c_1}{BC} \times BF$$

Space diagram

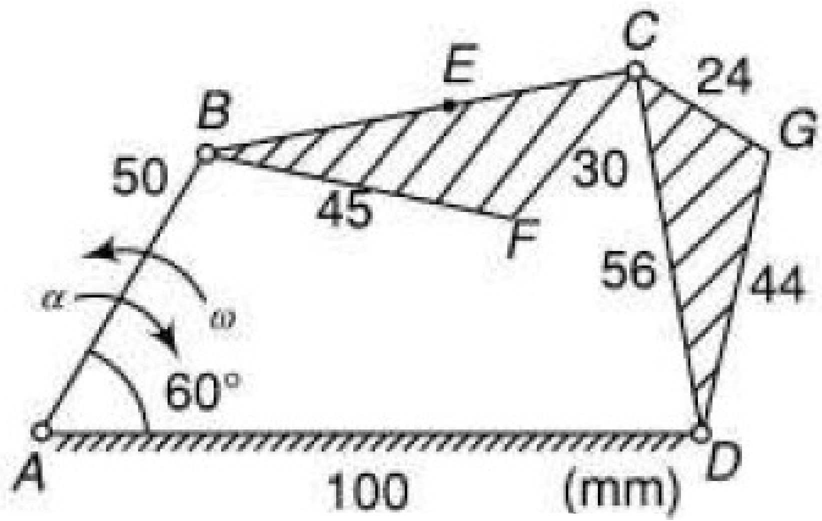


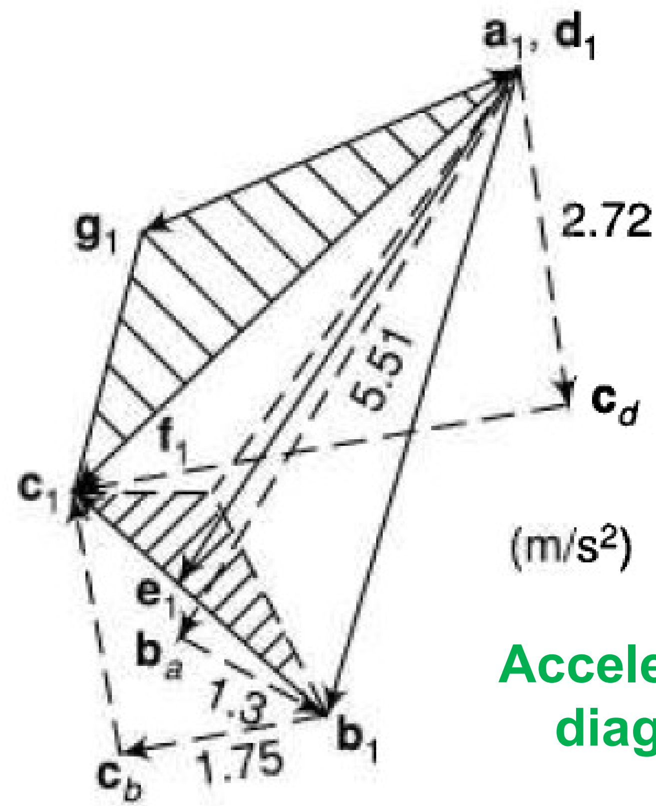


Acceleration diagram

- ***f*** is located as shown in acceleration diagram by following the same order of ***BCF*** (clockwise) in space diagram
- Similarly c_1g_1 and d_1g_1 is obtained and point g_1 is located in acceleration diagram.
- ***Measure the required distances from acceleration diagram to calculate and obtain the accelerations to find.***

Space diagram

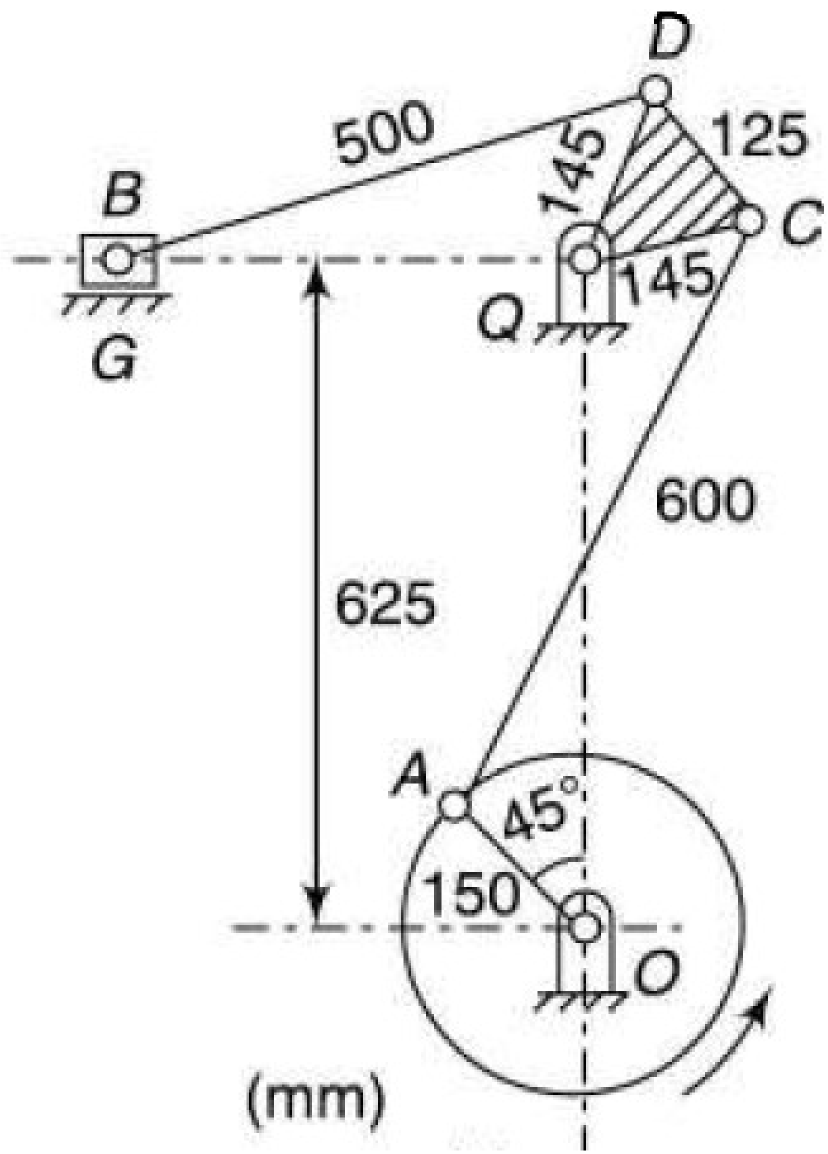




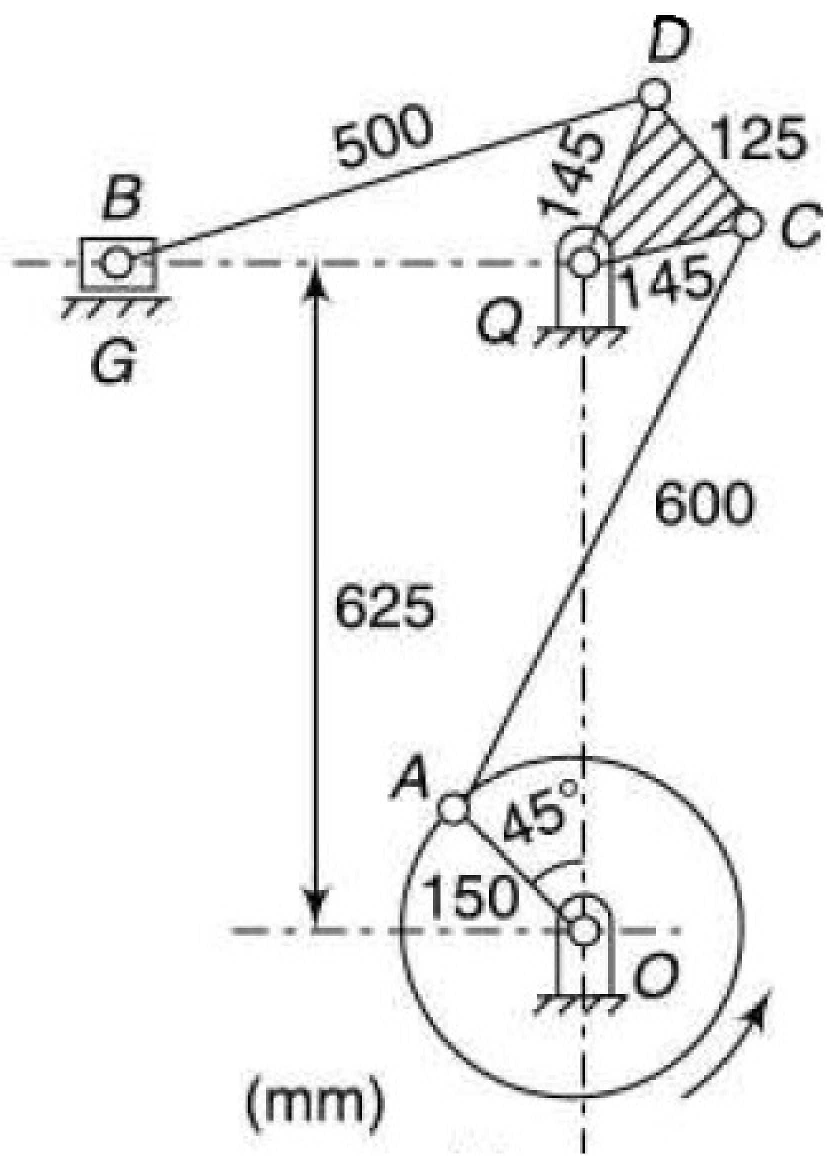
Acceleration diagram

Problem 4

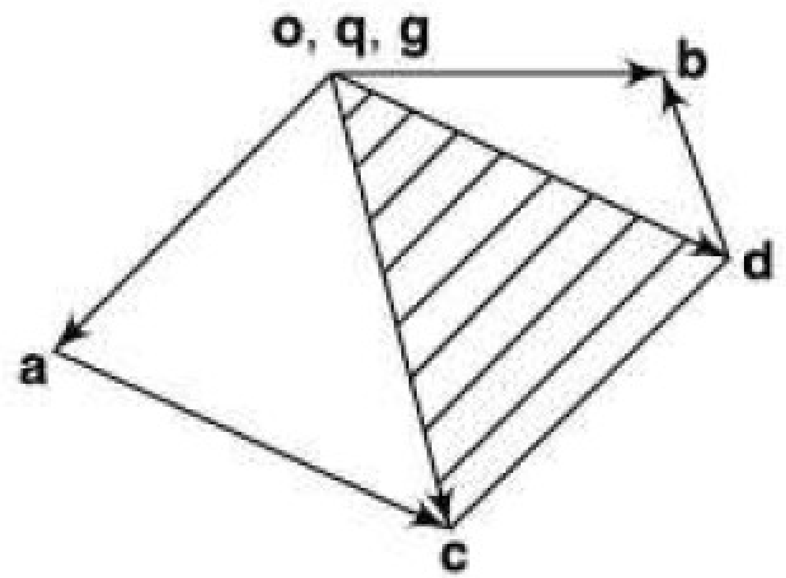
In the mechanism shown in figure, the crank ***OA*** rotates at 210 rpm anti-clockwise. For the given configuration, determine the acceleration of the slider ***D*** and angular acceleration of the link ***CD***.

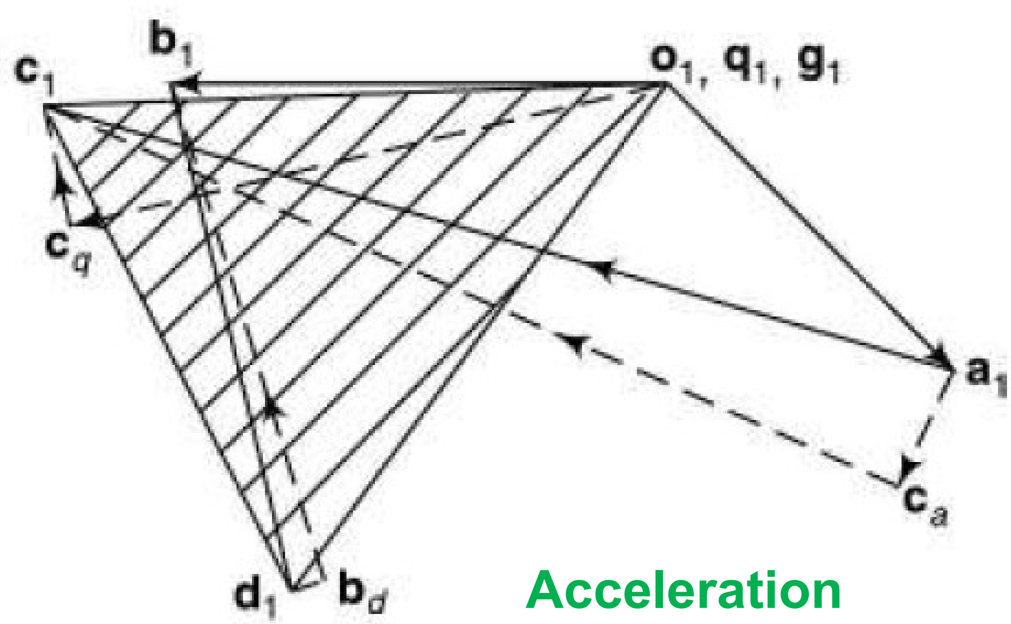


Space diagram



Velocity diagram





Acceleration diagram