Mechanics of Machines
Dynamic Force Analysis
Module 5: Flywheel, Turning moment diagram

Tapan Kumar Mahanta, Ph.D.
SMEC Chennai Campus

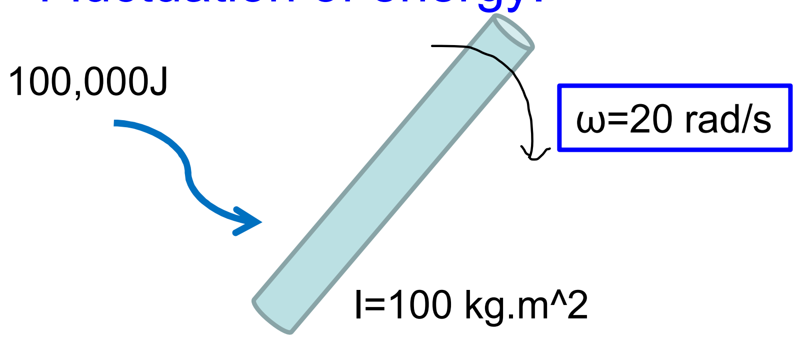
VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Dynamic Force Analysis:

- Class Objective1: Where we use Flywheel ?
- Class objective2: Turning moment diagram

Where we use flywheel:

- Fluctuation of energy.



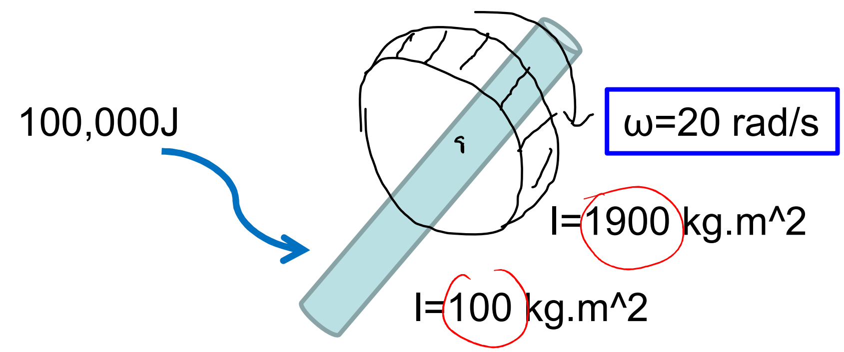
Rotational $K.E = \frac{1}{2} I\omega^2$ =20,000J

Total Energy=100,000J+20,000J=120,000 J

So $120{,}000 = \frac{1}{2}(100)(\omega'^2)$

$$\boxed{\omega' = 49\ rad/s}$$

Change in angular speed= 29 rad/s



Rotational $K.E = \frac{1}{2} I\omega^2$ =400,000J

Total Energy=100,000J+400,000J=500,000 J

So $500{,}000 = \frac{1}{2}(\cancel{100})(\omega'^2)$

$$\boxed{\omega' = 22\ rad/s}$$

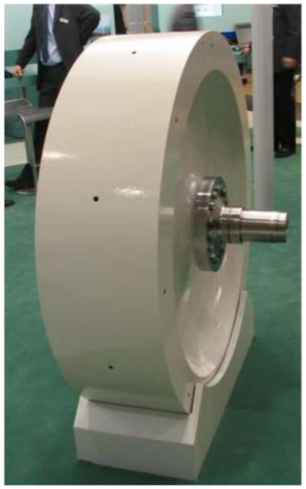
Change in angular speed= 2 rad/s

- Fluctuation of speed reduced when we attached a disc (Flywheel) over the rotating shaft

Flywheel :

- Flywheel is a device used to control fluctuation of speed by acting as energy reservoir.

Steam locomotive flywheel



An industrial flywheel

Tractor with exposed flywheel

Automobile engine flywheel

- A flywheel is used to control the variations in speed during each cycle of an engine.
- During the periods when the supply of energy is more than required, it stores energy and during the requirements is more than the supply, it releases energy.

Turning moment Diagram :

- **Turning moment on crank shaft (T)**

$$T = F_t \times r$$

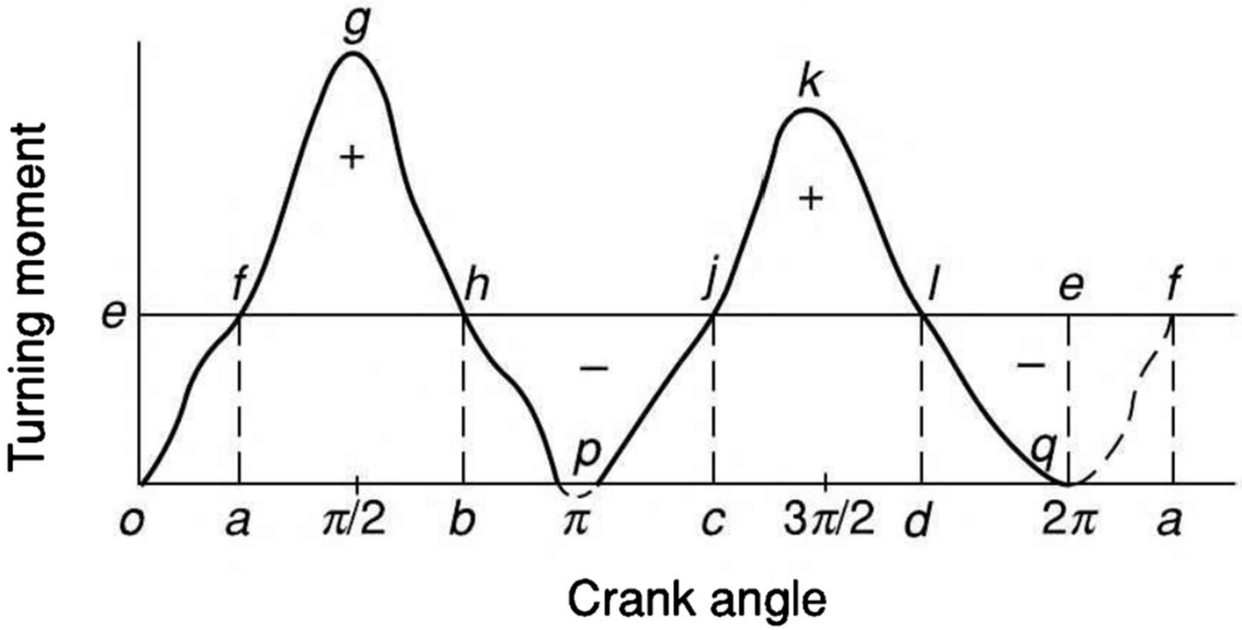
$$T = F \cdot r\left(sin\theta + \frac{sin2\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

Where F is the net piston effort

- Plot of T vs θ is known as the turning-moment diagram.

- Plot of F_t vs θ is known as crank effort diagram

Single-cylinder Double acting Steam Engine:

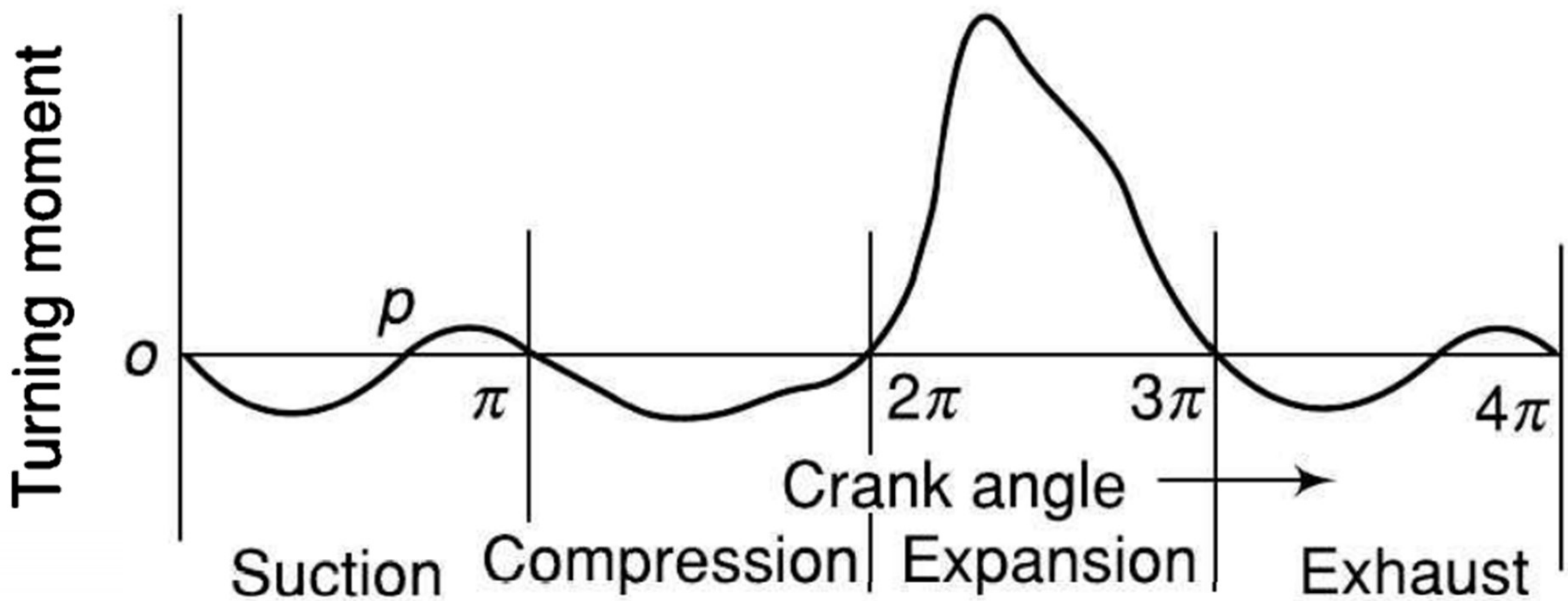


$$Enegry(E) = \int_a^b T.d\theta$$

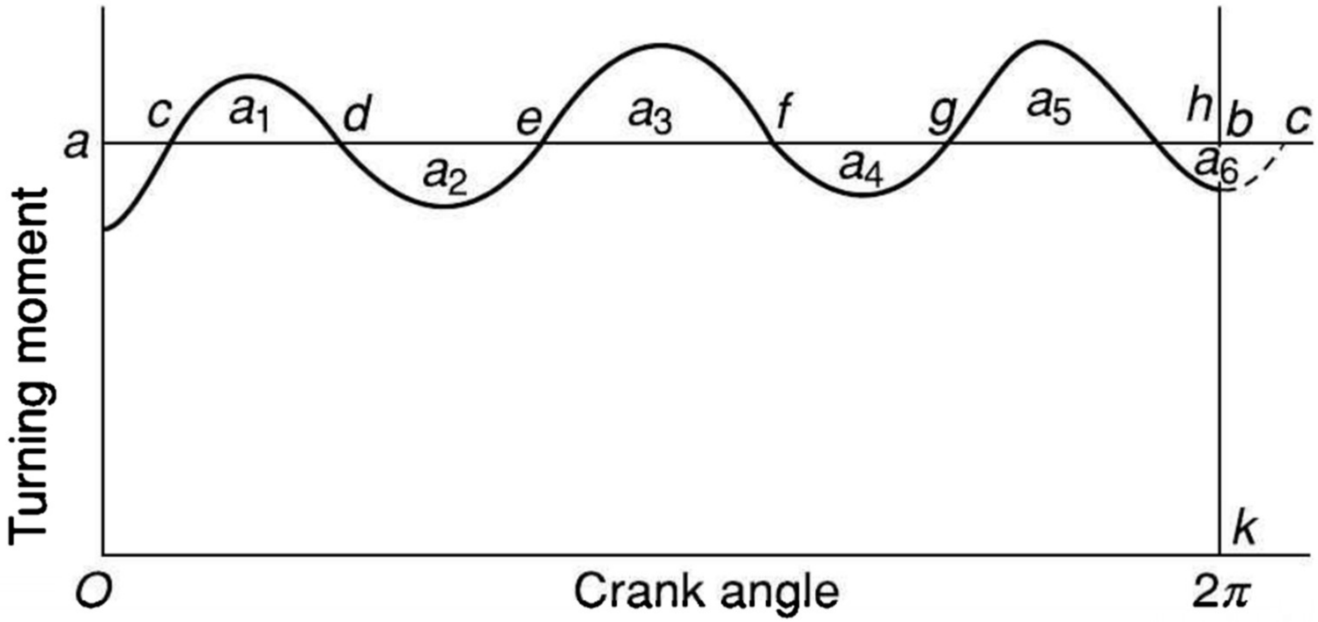
In one cycle:

Total store energy=total release energy

Single-cylinder Four-stroke Engine:



Multi cylinder Engines:



Crank position	*Flywheel energy*
c	E
d	$E + a_1$
e	$E + a_1 - a_2$
f	$E + a_1 - a_2 + a_3$
g	$E + a_1 - a_2 + a_3 - a_4$
h	$E + a_1 - a_2 + a_3 - a_4 + a_5$
c	$E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Flywheel fluctuation of speed and fluctuation of energy:

- I = moment of inertia of the flywheel

  ω_{max} = maximum speed

  ω_{min} = minimum speed

  ω_{mean}= mean speed= $\boxed{\dfrac{\omega_{max} + \omega_{min}}{2}}$

Fluctuation of speed= Range of speed= $\boxed{\omega_{max} - \omega_{min}}$

Coefficient of fluctuation of speed (C_s)= $\boxed{\dfrac{\omega_{max} - \omega_{min}}{\omega_{mean}}}$ ** Coefficient of steadiness = $\dfrac{1}{C_s}$

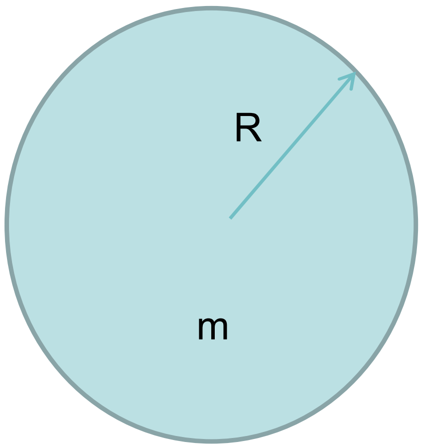
Maximum fluctuation of energy (e_{max})= $E_{max} - E_{min} = \frac{1}{2} I \omega_{max}{}^2 - \frac{1}{2} I \omega_{min}{}^2$

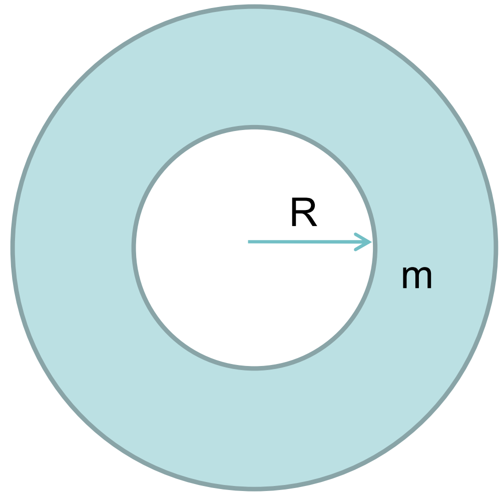
$$= \frac{1}{2} I[(\omega_{max} + \omega_{min})(\omega_{max} - \omega_{min})]$$

$$\boxed{e_{max} = I.\omega_{mean}{}^2.C_s}$$

Flywheel shape and their moment of inertia :

- Whenever there is difference between $T_{available}$ and $T_{resisting}$ or T_{mean} flywheel is needed.





For solid disc $I = \frac{mR^2}{2}$

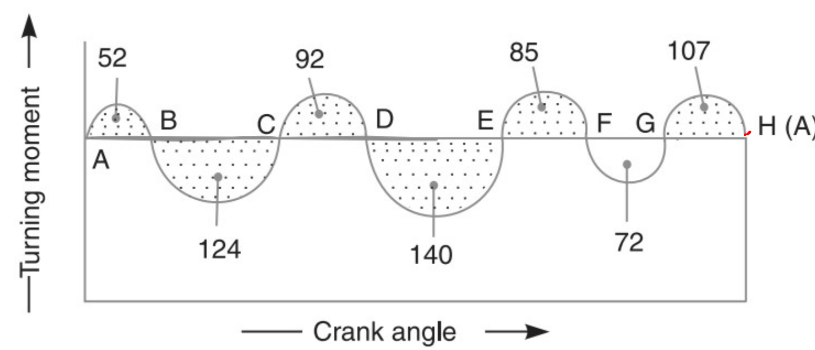
For annular disc $I = mR^2$

Problem1 :

- A flywheel with a mass of 3kN ahs a radius of gyration of 1.6m. Find the energy stored in the flywheel when its speed increase from 315 rpm to 340 rpm.

Problem2 :

- The turning moment diagram for a multi-cylinder engine has been drawn to a scale 1 mm = 600 N-m vertically and 1 mm = 30° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows : + 52, – 124, + 92, – 140, + 85, – 72 and + 107 mm2 , when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m.



Problem3 :

- The turning moment diagram of a four stroke engine is assumed to be represented by four triangles, the areas of which from the line of zero pressure are

suction stroke=440 mm^2

compression stroke= 1600 mm^2

expansion stroke= 7200 mm^2

exhaust stroke= 660 mm^2

Each mm^2 of area represents 3Nm of energy. It the resisting torque is uniform, determine the mass of the rim of flywheel to keep the speed between 218 and 222 rpm when the mean radius of the rim is to be 1.25.

Problem4 :

- Turning moment equation of an engine $T = 20000 + 9500 \sin 2\theta - 5700 \cos 2\theta$

  where θ is angle turned by crank from IDC. Assuming resisting torque to be constant and maximum fluctuations are 1% with respect to mean speed 300 rpm. Find

  a) Power developed by engine

  b) Mass of flywheel if its radius of gyration is 1.5m

  c) Angular acceleration of flywheel at the position when the crank has rotated by 45° from IDC.