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Vellore Institute of Technology

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School of Mechanical Engineering

B.Tech. – Mechatronics and Automation

BMEE207L Kinematics & Dynamics of Machines

MODULE 4

Synthesis of mechanisms

By

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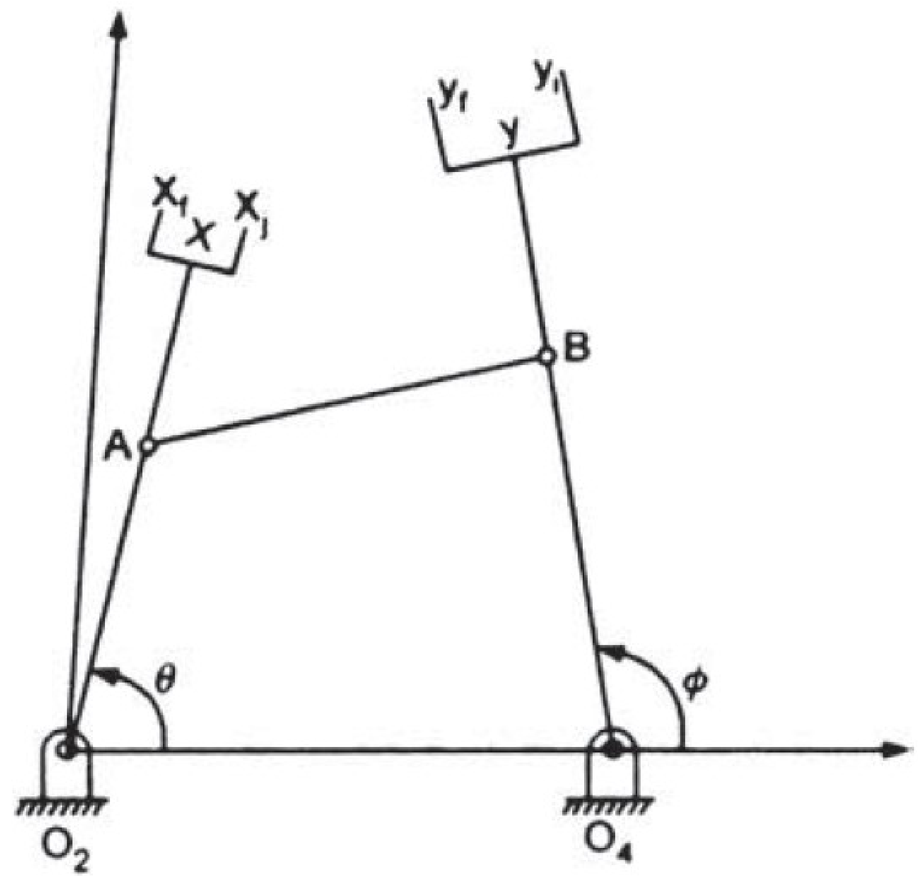
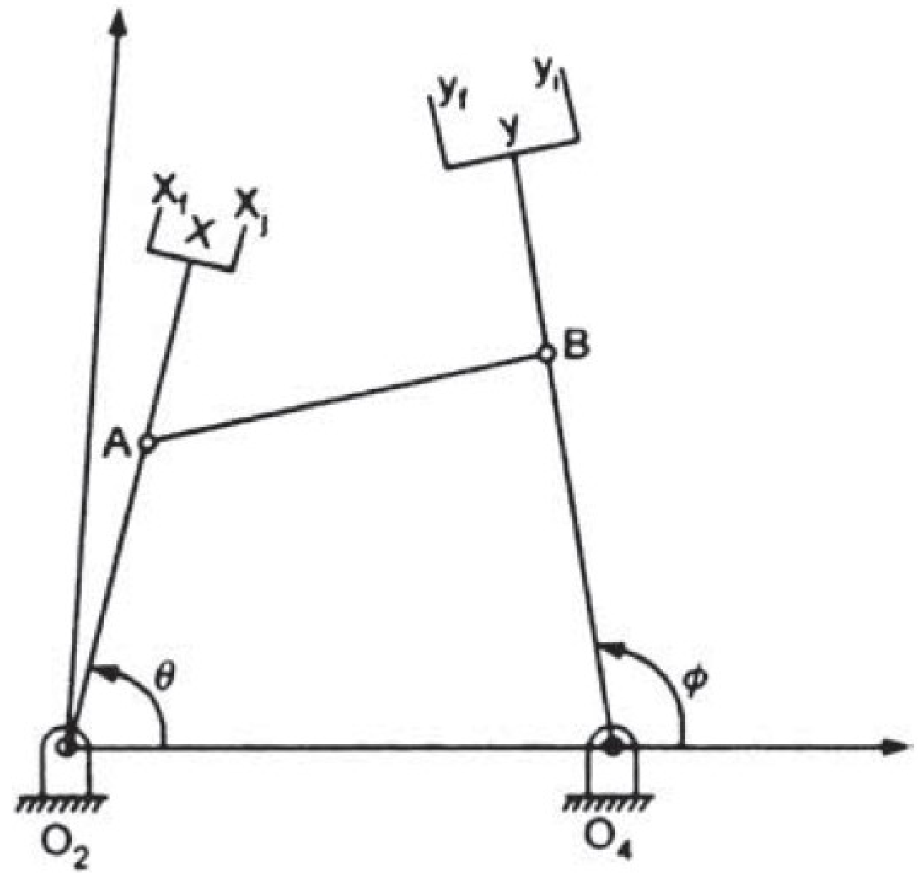
ANALYTICAL METHOD Function Generation

- In function generation, the motion of input (or driver) link is correlated to the motion of output (or follower) link.
- Let θ and φ be the angles of rotation of input and output links respectively.
- Let $y = f(x)$ be the function to be generated.
- The angle of rotation of the input link O_2A represents the independent variable x and the angle of rotation φ of the output link O_4B represents the dependent variable y.



ANALYTICAL METHOD Function Generation

- The relation between x and θ and that between y and φ is generally assumed to be linear.

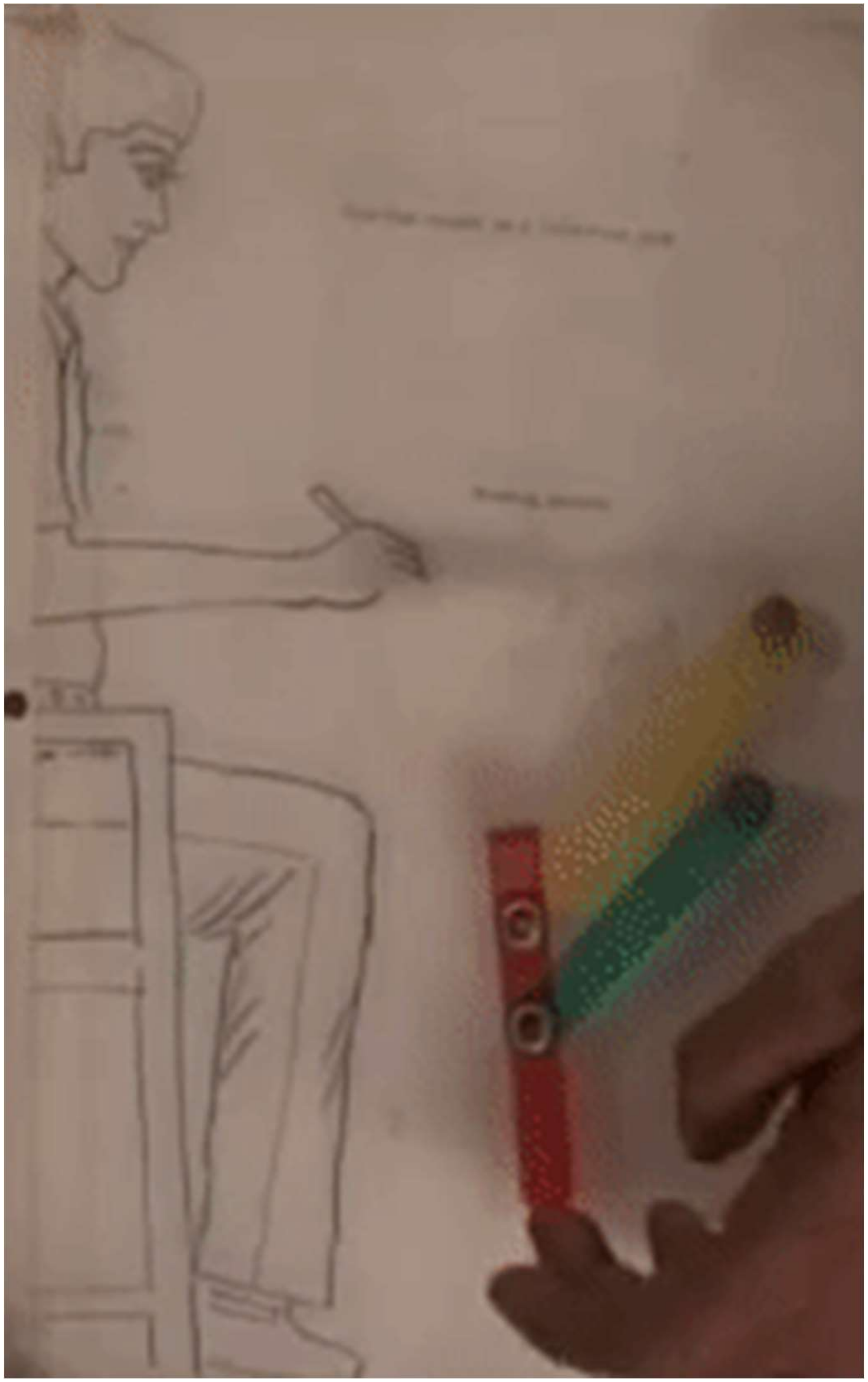


- Let θ_i and φ_i be the initial values of θ and φ representing x_i and y_i respectively.

$$\frac{\theta - \theta_i}{x - x_i} = r_x = \text{const.} = \frac{\theta_f - \theta_i}{x_f - x_i}$$

$$\frac{\phi - \phi_i}{y - y_i} = r_y = \text{const.} = \frac{\phi_f - \phi_i}{y_f - y_i}$$

- where the constants r_x and r_y are called scale factors. The subscripts *i* and *f* denote the initial and final values.



Freudenstein's Equation for the Precision Points

Numerical

Design a four-bar mechanism when the motions of the input and output links are governed by a function $y = 2x^2$ and x varies 2 to 4 with an interval of 1. Assume θ to vary from 40° to 120° and ϕ from 60° to 132°.

Solution

The angular displacement of input link is governed by x whereas that of the output link by y, θ varies from 40° to 120° (*i.e.* through 80°) and ϕ from 60° to 132° (*i.e.*, through 72°). $x = 2, 3, 4$.

The corresponding values of y are: $2 \times 2^2 = 8$, $2 \times 3^2 = 18$, $2 \times 4^2 = 32$.

$$r_x = \frac{\theta_f - \theta_i}{x_f - x_i} = \frac{120° - 40°}{4 - 2} = \frac{80}{2} = 40$$

$$\frac{\theta_2 - \theta_i}{x_2 - x_i} = r_x, \frac{\theta_2 - 40°}{3 - 2} = 40, \theta_2 = 80°$$

Freudenstein's Equation for the Precision Points

$$r_y = \frac{\phi_f - \phi_i}{y_f - y_i} = \frac{132 - 60}{32 - 8} = \frac{72}{24} = 3$$

$$\frac{\phi_2 - \phi_i}{y_f - y_i} = r_y = \frac{\phi_2 - 60}{18 - 8} = 3, \phi_2 = 90°$$

Precision point	x	y	θ, deg	ϕ, deg
1	2	8	40	60
2	3	18	80	90
3	4	32	120	132

Freudenstein's Equation for the Precision Points

$$\cos 40° = 0.7660,\ \cos 60° = 0.5000,\ \cos(40° - 60°) = 0.9397$$

$$\cos 80° = 0.1736,\ \cos 90° = 00,\ \cos(80° - 90°) = 0.9848$$

$$\cos 120° = -0.5,\ \cos 132° = 0.6691,\ \cos(120° - 132°) = 0.9781$$

$$A = \begin{vmatrix} 0.5000 & 0.7660 & 1 \\ 0.0000 & 0.1736 & 1 \\ -0.6691 & -0.5000 & 1 \end{vmatrix} = -0.0596,\ A_1 = \begin{vmatrix} 0.9397 & 0.7660 & 1 \\ 0.9848 & 0.1736 & 1 \\ 0.9791 & -0.5000 & 1 \end{vmatrix} = -0.03455$$

$$A_2 = \begin{vmatrix} 0.5000 & 0.9397 & 1 \\ 0.0000 & 0.9848 & 1 \\ -0.6691 & 0.9781 & 1 \end{vmatrix} = 0.0335,\ A_3 = \begin{vmatrix} 0.5000 & 0.7660 & 0.9397 \\ 0.0000 & 0.1736 & 0.9848 \\ -0.6691 & -0.5000 & 0.9781 \end{vmatrix} = -0.0645$$

$$k_1 = \frac{A_1}{A} = 0.5763 = \frac{d}{a},\ a = \frac{1}{0.5763} = 1.73$$

$$A = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{vmatrix},\ A_1 = \begin{vmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$A_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1 - \phi_1) & 1 \\ \cos\theta_2 & \cos(\theta_2 - \phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3 - \phi_2) & 1 \end{vmatrix},\ A_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1 - \phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2 - \phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3 - \phi_3) \end{vmatrix}$$

Freudenstein's Equation for the Precision Points

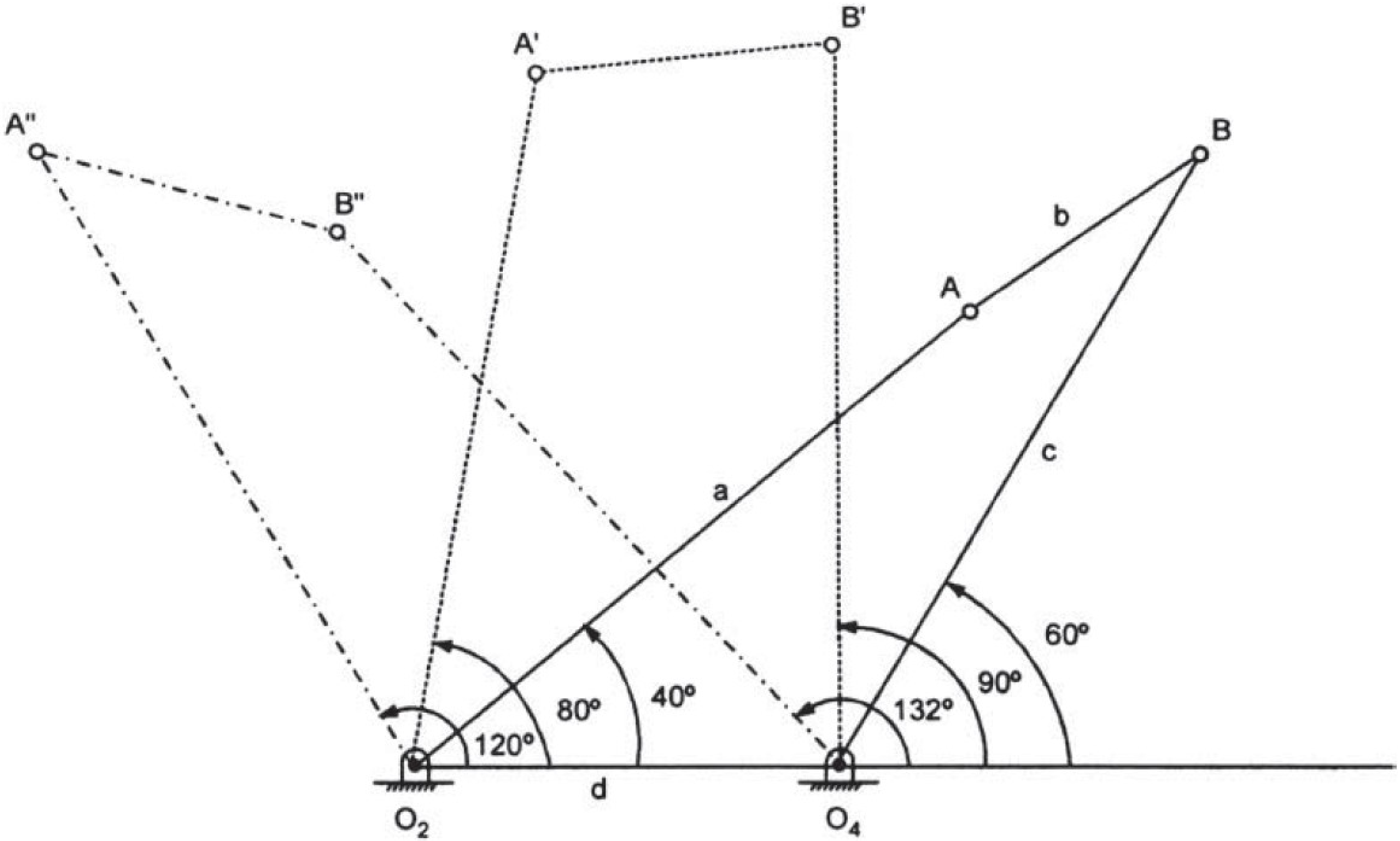
$$k_1 = \frac{A_1}{A} = 0.5763 = \frac{d}{a}, a = \frac{1}{0.5763} = 1.73$$

$$k_2 = \frac{A_2}{A} = -0.562 = -\frac{d}{c}, c = 1.78$$

$$k_3 = \frac{A_3}{A} = \frac{-0.0645}{-0.0596} = 1.0802$$

$$= \frac{a^2 - b^2 + c^2 + d^2}{2ac} = \frac{(1.73)^2 - b^2 + (1.78)^2 + 1}{2 \times 1.73 \times 1.78}, \qquad b = 0.7$$

Freudenstein's Equation for the Precision Points



Four-bar mechanisms generated by a function $y = 2x^2$

ANALYTICAL METHOD Chebychev Spacing

⇒ In the function generation problems:

⇒ Output is related to input through a function

$$y = f(x)$$

⇒ It is required to obtain ~~the~~ the dimensions of linkage to satisfy this eqn.

⇒ In general, a linkage synthesis problem

No exact solution for entire travel (range of travel)

ANALYTICAL METHOD Chebychev Spacing

⇒ It is usually possible to design a linkage which exactly satisfy the desired function at few chosen positions known as precision or accuracy points or position.

⇒ It is assumed that the design deviates very slightly from the desired function between the precision positions and that deviation is within acceptable limit.

ANALYTICAL METHOD Chebychev Spacing

⇒ The difference bet^n the function prescribed and the function produced by the designed linkage is called structural error. ⇒ 3 to 4 % (acceptable

⇒ The amount of structural error also depends upon the choice of the precision points.

⇒ A judicious use of precision points greatly affects the structural error.

ANALYTICAL METHOD Chebychev Spacing

⇒ Thus, a set of precision points may be selected for use in the synthesis of the linkage which can be minimize the structural error and a fair choice is provided by Chebychev spacing.

⇒ For n accuracy positions in the range $x_0 \le x \le x_{n+1}$

⇒ The Chebychev spacing is given by

$$x_i = \frac{x_{n+1} + x_0}{2} - \frac{x_{n+1} - x_0}{2} \cos\left(\left(\frac{2i-1}{2n}\right)\pi\right)$$

where, $i = 1, 2, 3 \ldots n$

ANALYTICAL METHOD Chebychev Spacing

Example: If it is desired to design a linkage to satisfy the function $y = \sqrt{x}$ over the range $1 \leq x \leq 3$ using three precision position

Then the three values of x are

$\Rightarrow$ The chebychev spacing is given by

$$x_i = \frac{x_{n+1} + x_0}{2} - \frac{x_{n+1} - x_0}{2} \cos\left(\left(\frac{2i-1}{2n}\right)\pi\right)$$

where, $i = 1, 2, 3 \ldots n$

$$x_1 = \frac{3+1}{2} - \frac{3-1}{2} \cos \frac{(2-1)\pi}{2 \times 3} = 2 - \cos\frac{\pi}{6} = 1.134$$

$$x_2 = 2 - \cos\frac{3\pi}{6} = 2$$

ANALYTICAL METHOD Chebychev Spacing

$$x_1 = \frac{3+1}{2} - \frac{3-1}{2} \cos \frac{(2-1)\pi}{2 \times 3} = 2 - \cos \frac{\pi}{6} = 1.134$$

$$x_2 = 2 - \cos \frac{3\pi}{6} = 2$$

$$x_3 = 2 - \cos \frac{5\pi}{6} = 2.866$$

$$y_1 = \sqrt{x_1} = \sqrt{1.134} = 1.065$$

$$y_2 = \sqrt{x_2} = \sqrt{2} = 1.414$$

$$y_3 = \sqrt{x_3} = \sqrt{2.866} = 1.693$$

Graphical Approach Chebychev Spacing

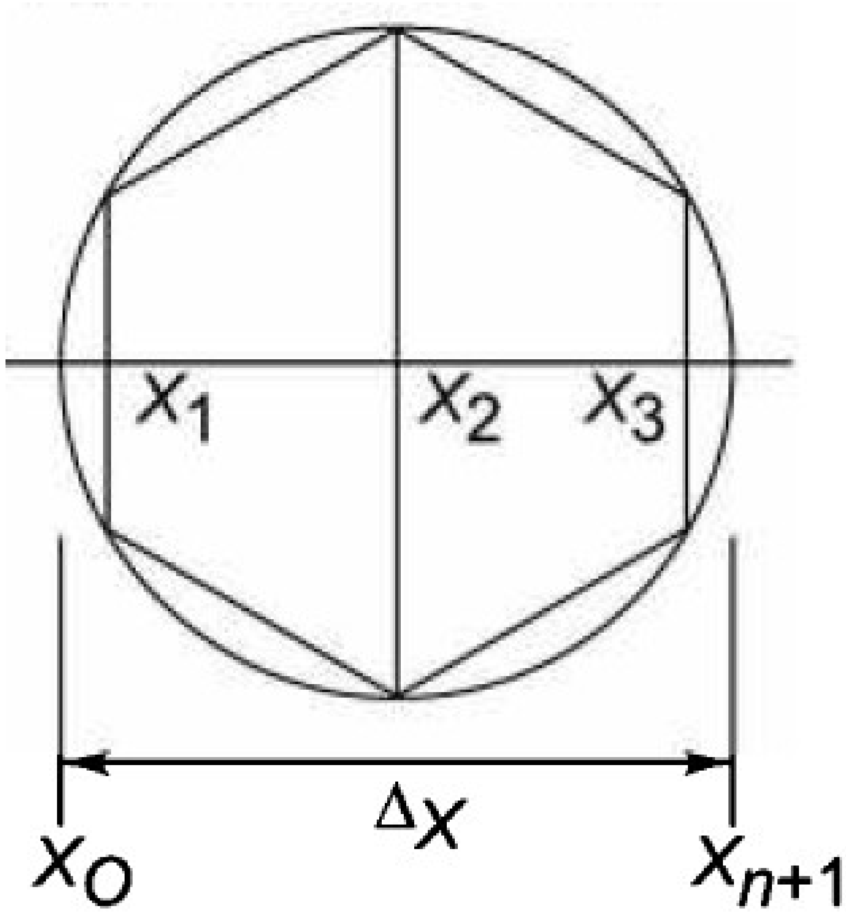
1. Draw a circle of diameter equal to range Δx $(= x_{n+1} - x_0)$

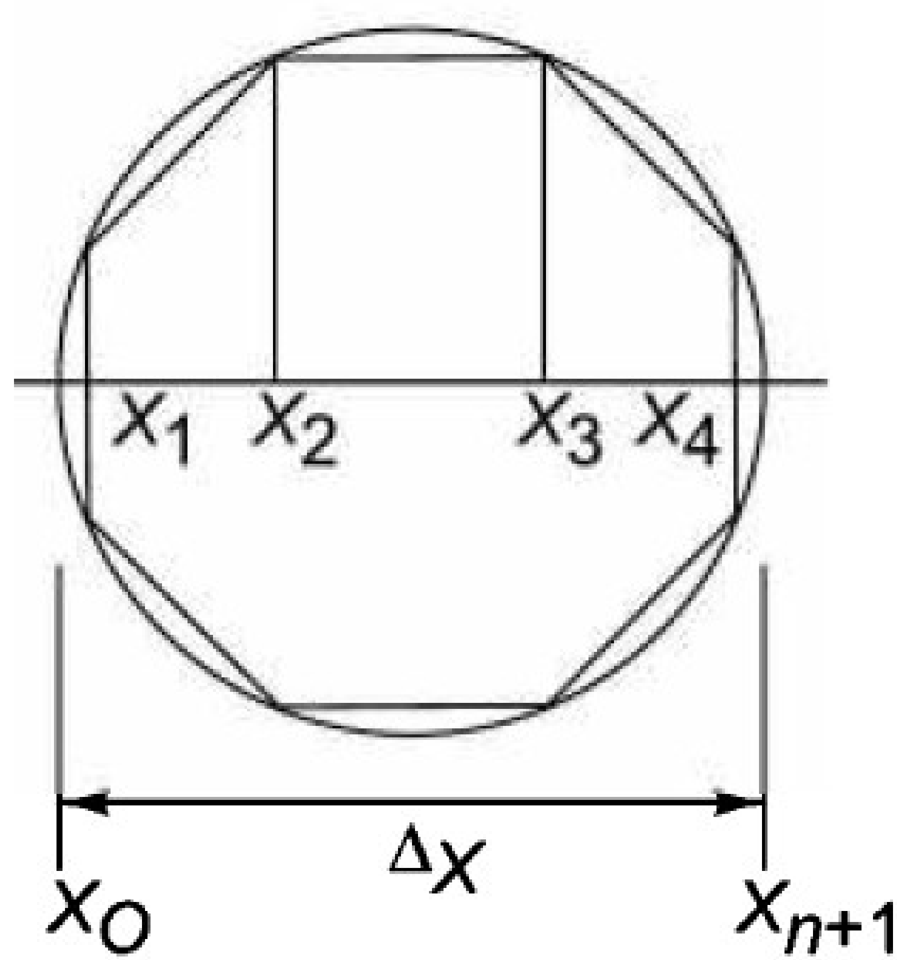
2. Incribe a regular poligon of $2n$ sides in the circle such that the two sides of the polygon are perpendicular to the x-axis

3. Draw projections of the vertices of the polygon on the x-axis. The perpendiculars intersect the diameter Δx at the precision points.

Graphical Approach

Chebychev Spacing





1. Draw a circle of diameter equal to range Δx ($= x_{n+1} - x_0$)

2. Incribe a regular poligon of $2n$ sides in the circle such that the two sides of the polygon are perpendicular to the x-axis

3. Draw projections of the vertices of the polygon on the x-axis. The perpendiculars intersect the diameter Δx at the precision points.

Graphical Approach — Chebychev Spacing

Let x_i and x_j be the initial and final values of variable x respectively. A function $f(x)$ is desired to be generated in the interval $x_i \le x \le x_f$. Let the generated function be $F(x, R_1, R_2, \ldots, R_n)$, where $R_1, R_2, \ldots, R_n$ are the design parameters. The difference $E(x)$ between the desired function and generated function can be represented by,

$$E(x) = f(x) - F(x, R_1, R_2, \ldots, R_n)$$

At precision points, say for $x = x_1, x_2, \ldots, x_n$, the desired and generated functions agree and $E(x) = 0$. At other points $E(x)$ will have some value, called the structural error. It is desirable that $E(x)$ should be minimum. Therefore, the spacing of precision points is very important. The precision points, according to Chebyshev's spacing, are given by:

$$x_m = a + b\cos\left[\frac{(2m-1)\pi}{2n}\right], m = 1, 2, 3$$

where $$a = \frac{x_i + x_f}{2}, b = \frac{x_f - x_i}{2}, \text{ and}$$

n = number of precision points.

Graphical Approach

Chebychev Spacing

1. Draw a circle of radius 'b' and centre on the x-axis at a distance 'a' from point O.

2. Inscribe a regular polygon of side $2n$ in this circle such that the two sides are perpendicular to the x-axis.

3. Determine the locations of n accuracy points by projecting the vertices on x-axis as shown in Fig.16.15. It is sufficient to draw semi-circles only showing inscribed polygon to get the values of precision points.

$$a = \frac{x_i + x_f}{2}, b = \frac{x_f - x_i}{2}$$

1. Draw a circle of diameter equal to range Δx $(= x_{n+1} - x_0)$

2. Incribe a regular poligon of $2n$ sides in the circle such that the two sides of the polygon are perpendicular to the x-axis

3. Draw projections of the vertices of the polygon on the x-axis. The perpendiculars intersect the diameter Δx at the precision points.

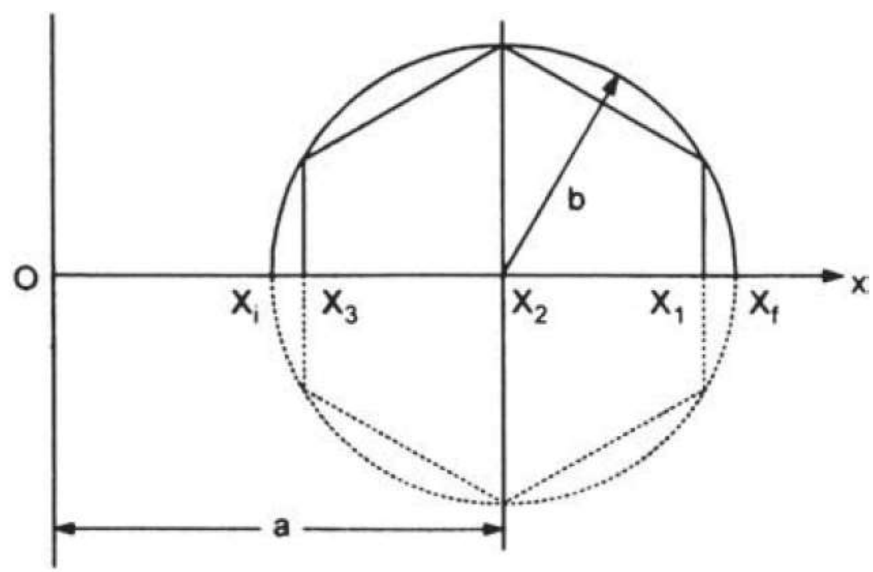


Fig.16.15 Graphical method to determine precision points

ANALYTICAL METHOD

Chebychev Spacing

Graphical Approach

$$x_1 = \frac{3+1}{2} - \frac{3-1}{2}\cos\frac{(2-1)\pi}{2\times 3} = 2 - \cos\frac{\pi}{6} = 1.134$$

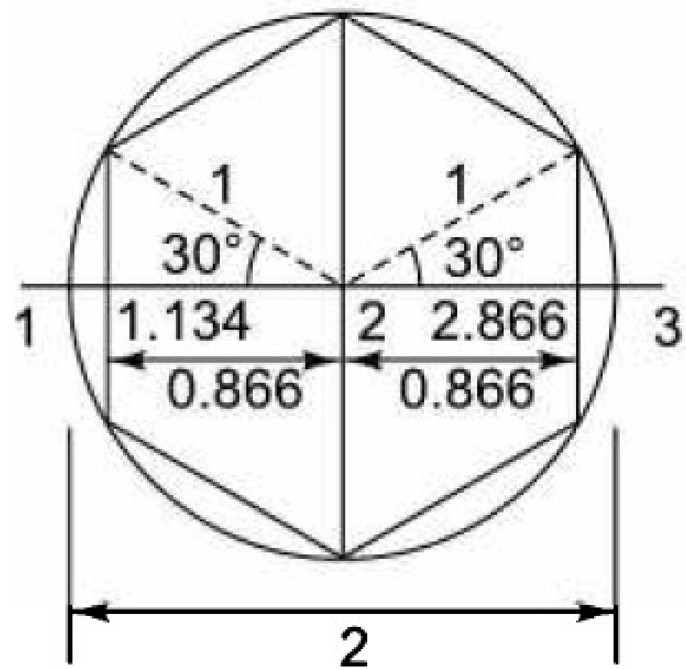
$$x_2 = 2 - \cos\frac{3\pi}{6} = 2$$

$$x_3 = 2 - \cos\frac{5\pi}{6} = 2.866$$

$$y_1 = \sqrt{x_1} = \sqrt{1.134} = 1.065$$

$$y_2 = \sqrt{x_2} = \sqrt{2} = 1.414$$

$$y_3 = \sqrt{x_3} = \sqrt{2.866} = 1.693$$



ANALYTICAL METHOD Chebychev Spacing

Numerical

Determine the lengths of all the four links of a four-bar mechanism to generate $y = \log x$ in the interval $1 \le x \le 11$ for three precision points. The length of the smallest links is 10 cm the range of input angles is $45° \le \theta \le 105°$ and output angles is $135° \le \phi \le 225°$.

Solution:

$x_i = 1, x_f = 11, n = 3$

Using Chebyshev's precision points,

$$a = \frac{1}{2}\left(x_i + x_f\right) = \frac{1}{2}\left(1 + 11\right) = 6$$

$$b = \frac{1}{2}\left(x_f - x_i\right) = \frac{1}{2}\left(11 - 1\right) = 5$$

$$x_m = a + b\cos\left[\frac{(2m-1)\pi}{2n}\right], m = 1, 2, 3$$

⇒ The Chebychev spacing is given by

$$x_i = \frac{x_{n+1} + x_0}{2} - \frac{x_{n+1} - x_0}{2}\cos\left(\left(\frac{2i-1}{2n}\right)\pi\right)$$

Where, $i = 1, 2, 3 \ldots n$

$$x_m = a + b\cos\left[\frac{(2m-1)\pi}{2n}\right], m = 1, 2, 3$$

$$a = \frac{x_i + x_f}{2}, b = \frac{x_f - x_i}{2}, \text{ and}$$

n = number of precision points.

ANALYTICAL METHOD Chebychev Spacing

$$x_1 = a + b\cos\left[\frac{(2\times 1-1)\pi}{6}\right] = a + b\cos\frac{\pi}{6} = 6 + 5\ \cos\ \frac{\pi}{6} = 10.33$$

$$x_2 = a + b\cos\frac{\pi}{2} = 6$$

$$x_3 = a + b\cos\left(\frac{5\pi}{6}\right) = 6 + 5\cos\left(\frac{5\pi}{6}\right) = 1.67$$

$$y_1 = \log x_1 = \log 10.33 = 1.014$$

$$y_2 = \log x_2 = \log 6 = 0.778$$

$$y_3 = \log x_3 = \log 1.67 = 0.223$$

$$y_i = \log x_i = \log 1 = 0$$

$$y_f = \log x_f = \log 11 = 1.0414$$

$$x_m = a + b\cos\left[\frac{(2m-1)\pi}{2n}\right], m = 1, 2, 3$$

$$a = \frac{x_i + x_f}{2}, b = \frac{x_f - x_i}{2}, \text{ and}$$

n = number of precision points.

ANALYTICAL METHOD Chebychev Spacing

$$r_x = \frac{\theta_f - \theta_i}{x_f - x_i} = \frac{105 - 45}{11 - 1} = \frac{60}{10} = 6$$

$$r_y = \frac{\phi_f - \phi_i}{y_f - y_i} = \frac{225 - 135}{1.0414 - 0} = \frac{90}{1.0414} = 86.423$$

$$r_x = \frac{\theta - \theta_i}{x - x_i}, \frac{\theta_1 - \theta_i}{x_1 - x_i} = \frac{\theta_1 - 45^\circ}{10.33 - 1} = 6, \theta_1 = 100.98^\circ$$

$$\frac{\theta_2 - 45^\circ}{6 - 1} = 6, \theta_2 = 75^\circ, \frac{\theta_3 - 45^\circ}{1.67 - 1} = 6, \theta_3 = 49.02^\circ$$

$$r_y = \frac{\phi - \phi_i}{y - y_i}, \frac{\phi_1 - \phi_i}{y_1 - y_i} = \frac{\phi_1 - 135^\circ}{1.014 - 0} = 86.423, \phi_1 = 222.63^\circ$$

ANALYTICAL METHOD Chebychev Spacing

$$\frac{\phi_2 - 135°}{0.778 - 0} = 86.423, \phi_2 = 202.24°$$

$$\frac{\phi_3 - 135°}{0.223 - 0} = 86.423, \phi_3 = 154.27°$$

Precision Point	x	y	θ^{o}	φ^{o}
1	10.33	1.014	100.98	222.63
2	6	0.778	75	202.24
3	1.67	0.223	49.02	154.27

$$k_1 \cos \phi_1 + k_2 \cos \theta_1 + k_3 = \cos(\theta_1 - \phi_1)$$
$$k_1 \cos \phi_2 + k_2 \cos \theta_2 + k_3 = \cos(\theta_2 - \phi_2)$$
$$k_1 \cos \phi_3 + k_2 \cos \theta_3 + k_3 = \cos(\theta_3 - \phi_3)$$

ANALYTICAL METHOD Chebychev Spacing

$$\begin{bmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{bmatrix} \begin{Bmatrix} k_1 \\ k_2 \\ k_3 \end{Bmatrix} = \begin{Bmatrix} \cos(\theta_1 - \phi_1) \\ \cos(\theta_2 - \phi_2) \\ \cos(\theta_3 - \phi_3) \end{Bmatrix}$$

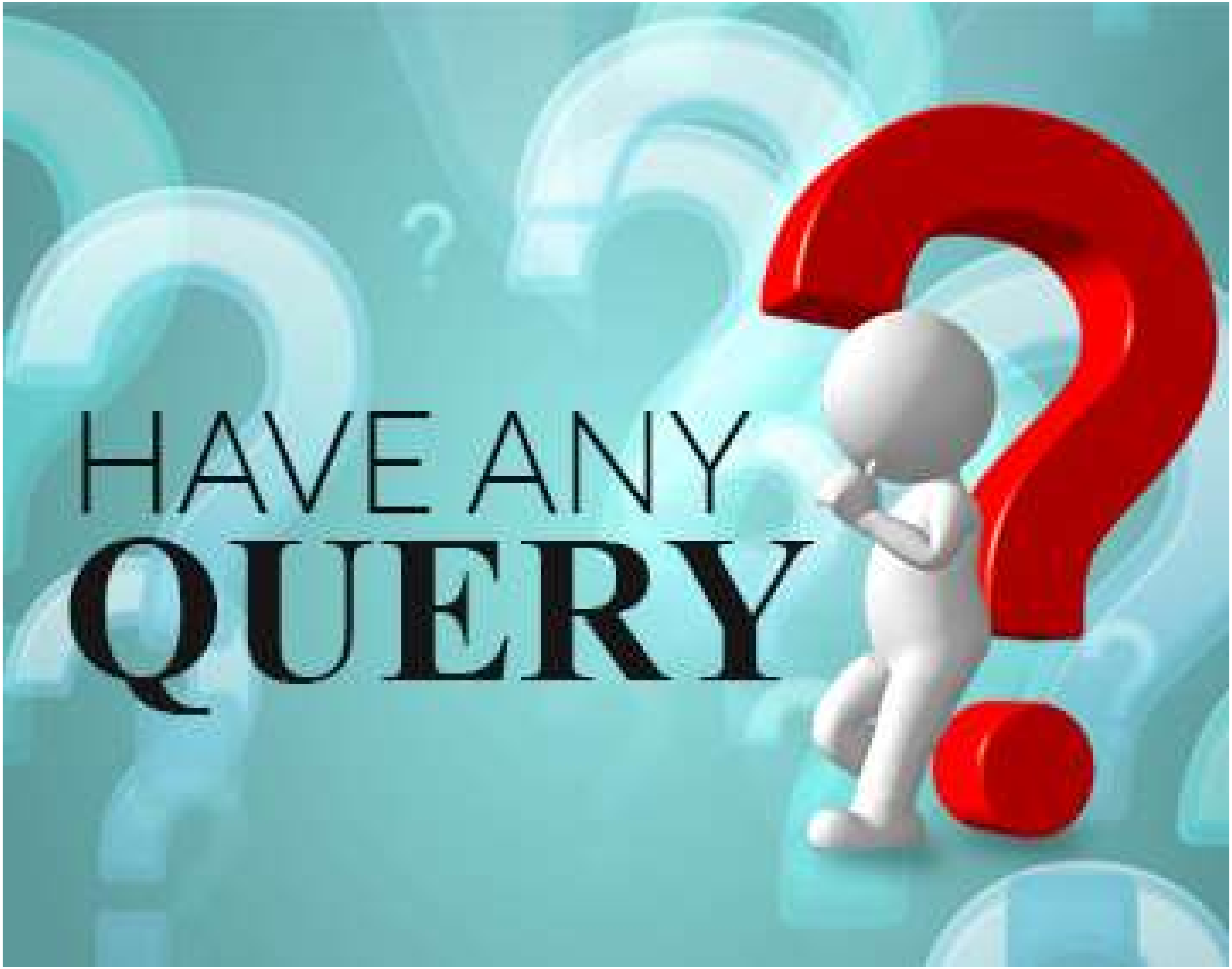
$$A = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{vmatrix}, A_1 = \begin{vmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$A_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1 - \phi_1) & 1 \\ \cos\theta_2 & \cos(\theta_2 - \phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3 - \phi_2) & 1 \end{vmatrix}, A_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1 - \phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2 - \phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3 - \phi_3) \end{vmatrix}$$

ANALYTICAL METHOD Chebychev Spacing

Numerical (Part of DA)

Design of four-link mechanism if the motions of the input and the output links are governed by a function $y = x^{1.5}$ and x varies from 1 to 4. Assume θ to vary from 35^{o} to 125^{o} and φ vary from 65^{o} to 135^{o}. The length of the fixed link is 25 mm. Use the Chebychev spacing of accuracy points.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