Winter 2022-23

VIT®

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

School of Mechanical Engineering

B.Tech. – Mechatronics and Automation

BMEE207L Kinematics & Dynamics of Machines

MODULE 3

Kinematic Analysis of CAMS

By

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Introduction

- A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower

- The cam and the follower have a line contact and constitute a higher pair

Applications

Valve opening and closing mechanism

Applications

Tooth paste filling Machine

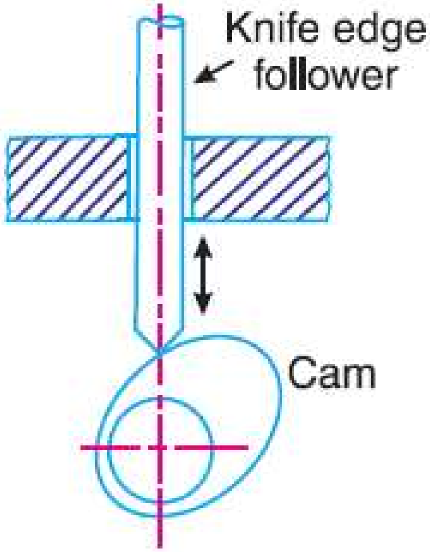
Applications

Pick and drop mechanism

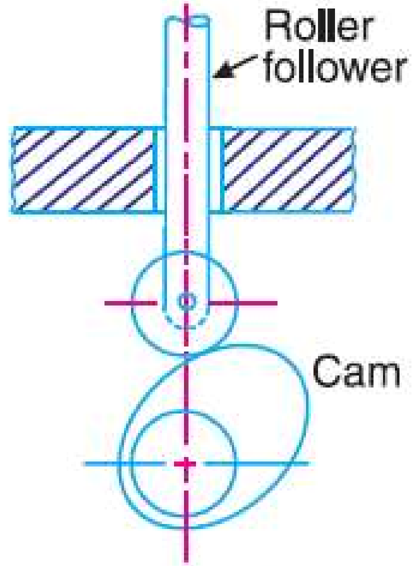
Classification of Followers

- **According to the surface in contact**
  - ✓ Knife edge follower
  - ✓ Roller follower
  - ✓ Flat faced or Mushroom follower
  - ✓ Spherical Faced
- **According to the motion**
  - ✓ Reciprocating or translating follower
  - ✓ Oscillating or rotating
- **According to the path of motion of the follower**
  - ✓ Radial Follower
  - ✓ Off-set follower

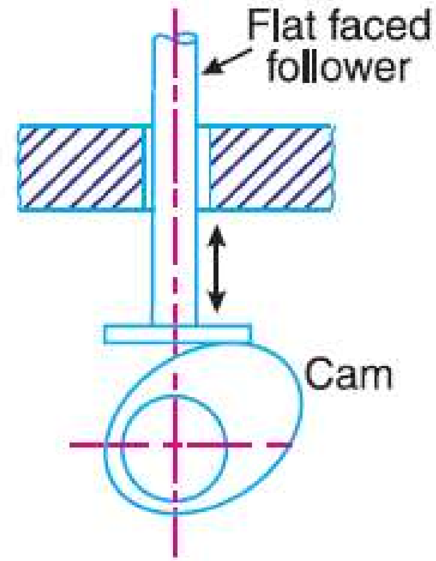
Classification of Followers



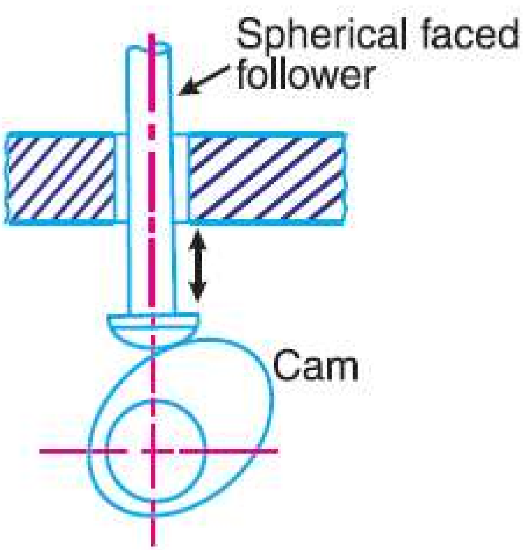
(*a*) Cam with knife edge follower.



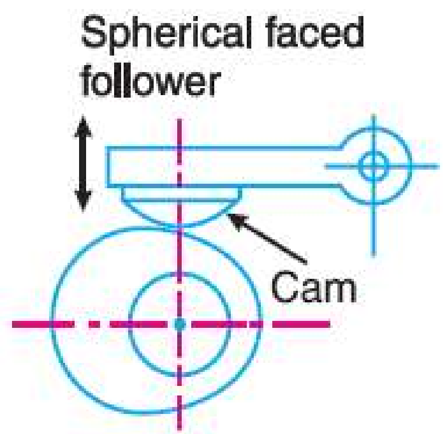
(*b*) Cam with roller follower.



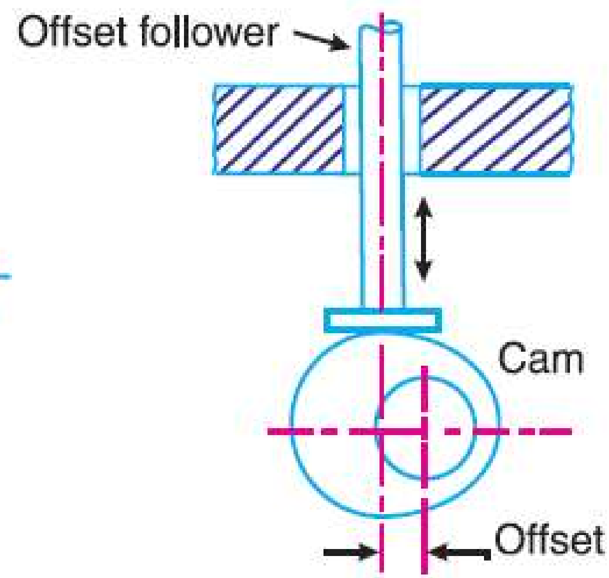
(*c*) Cam with flat faced follower.



(*d*) Cam with spherical faced follower.



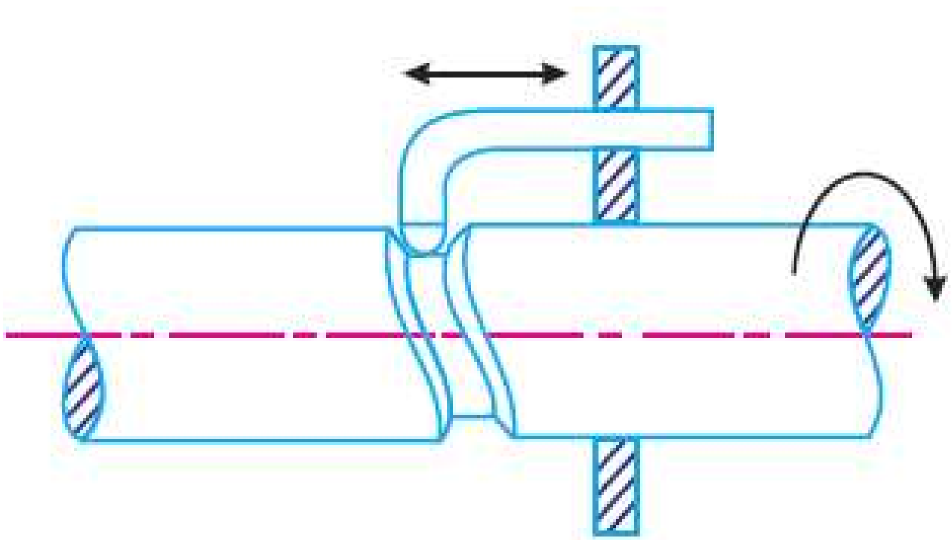
(*e*) Cam with spherical faced follower.



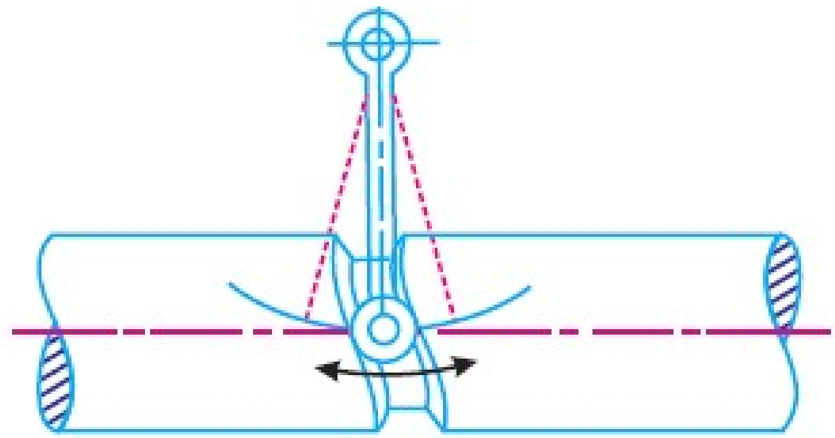
(*f*) Cam with offset follower.

Classification of Followers

- Radial or disc cam
- Cylindrical cam



(*a*) Cylindrical cam with reciprocating follower.



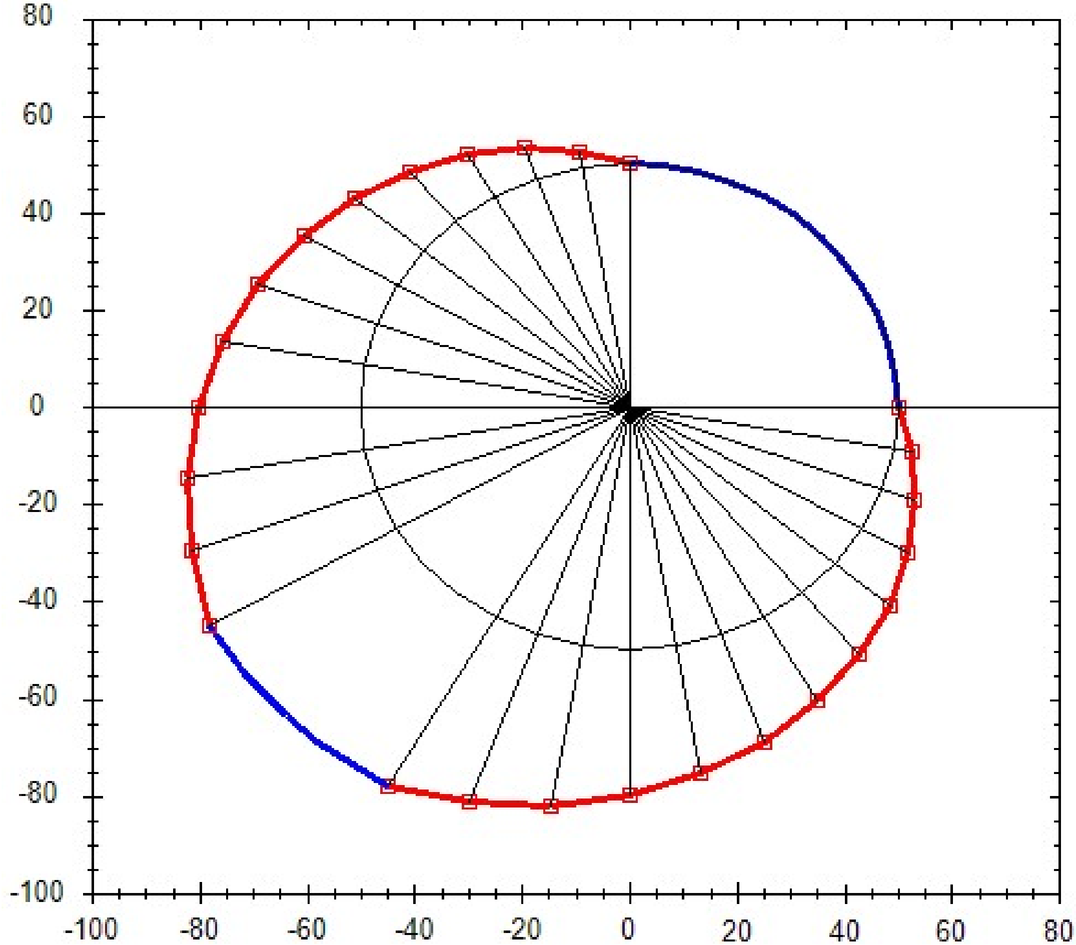
(*b*) Cylindrical cam with oscillating follower.

CAM Design

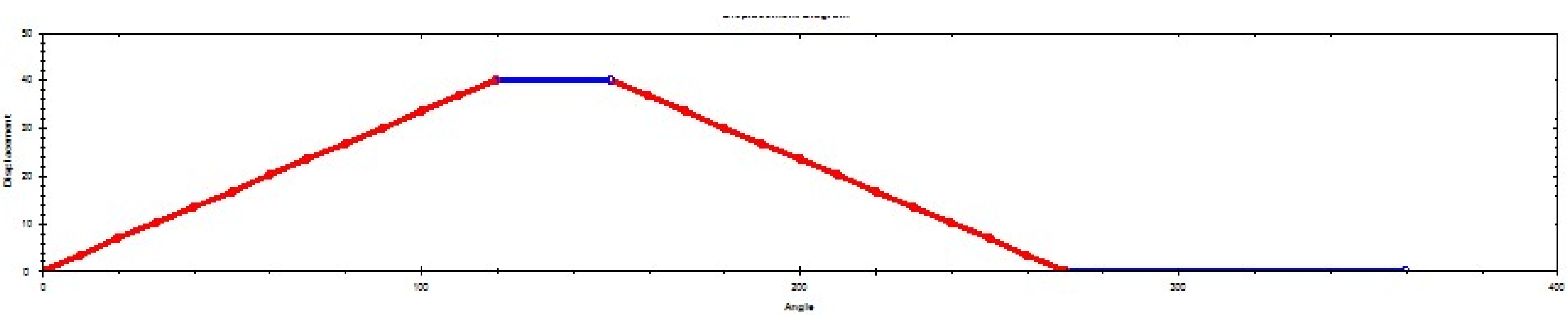
The two tasks are

1. S-V-A-J diagrams
   - Selection of mathematical function to define the motion of the follower
   - Easiest approach to this process is to linearize the cam (unwrap it) from its circular shape and consider it as a function plotted on Cartesian axes
2. Size the cam
   - Pressure angle and radius of curvature

Linearizing the Cam



Dwell: no output motion for a specified period of input motion



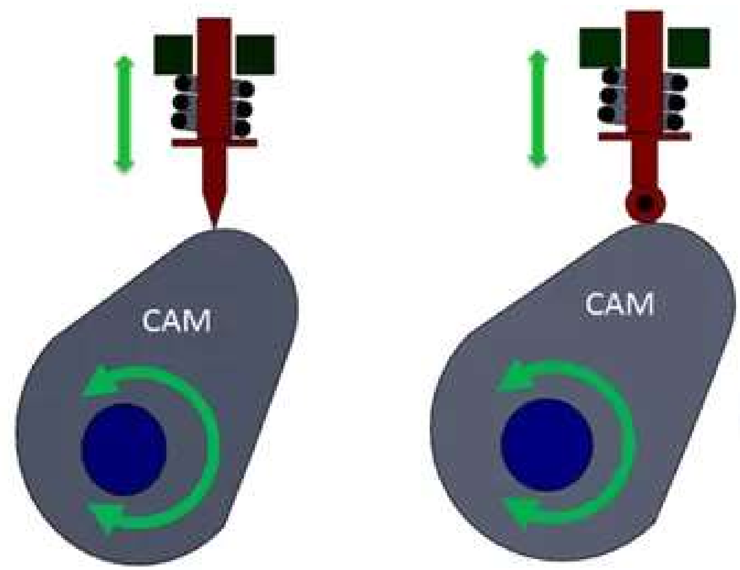
Motion of the Followers

- Uniform Velocity
- Simple harmonic motion
- Uniform acceleration and retardation
- Cycloid motion

Followers

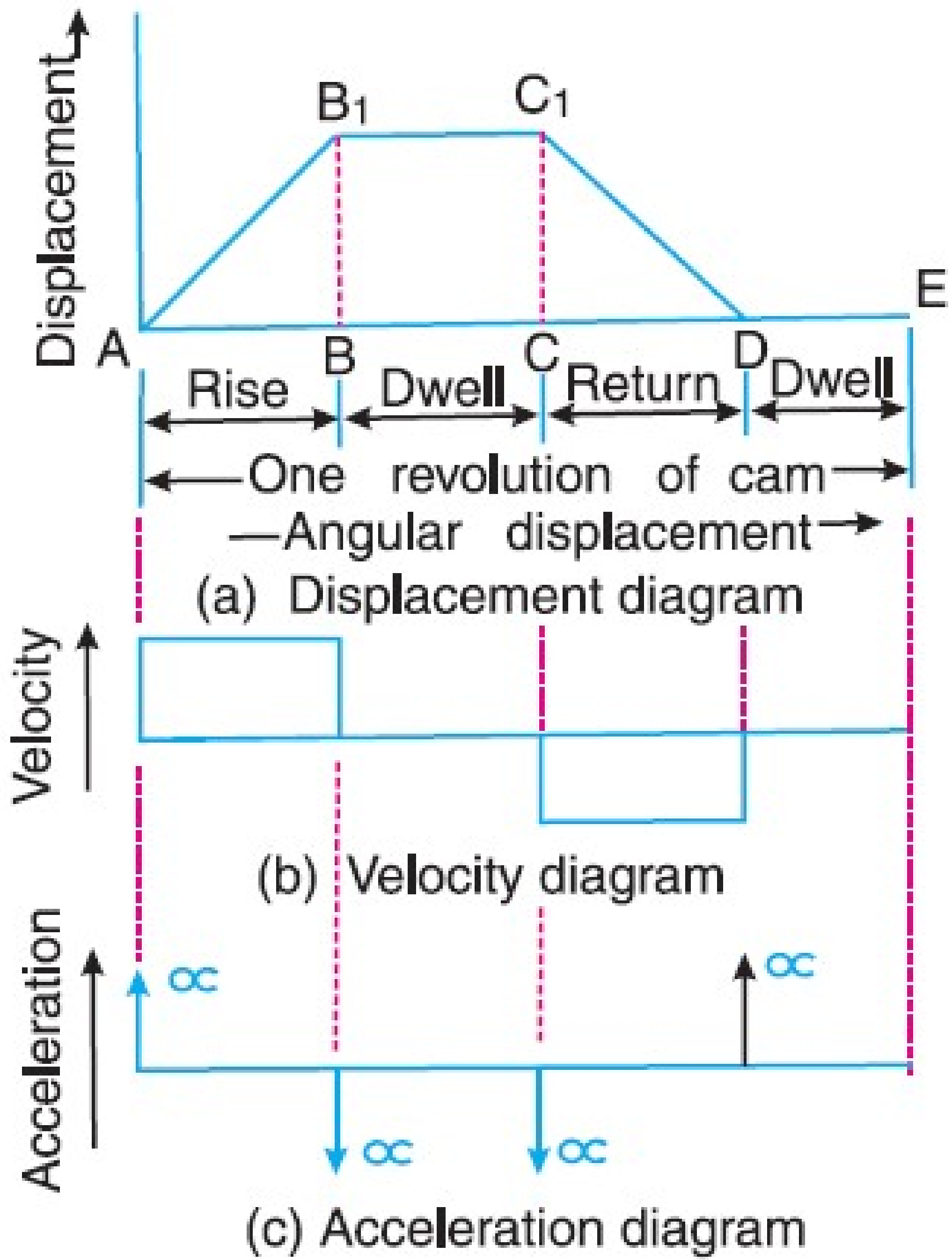
- ✓Knife edge follower
- ✓Roller follower

With and without offset



https://www.youtube.com/channel/UCgrEgGakwbxNPBrHg70vsUw

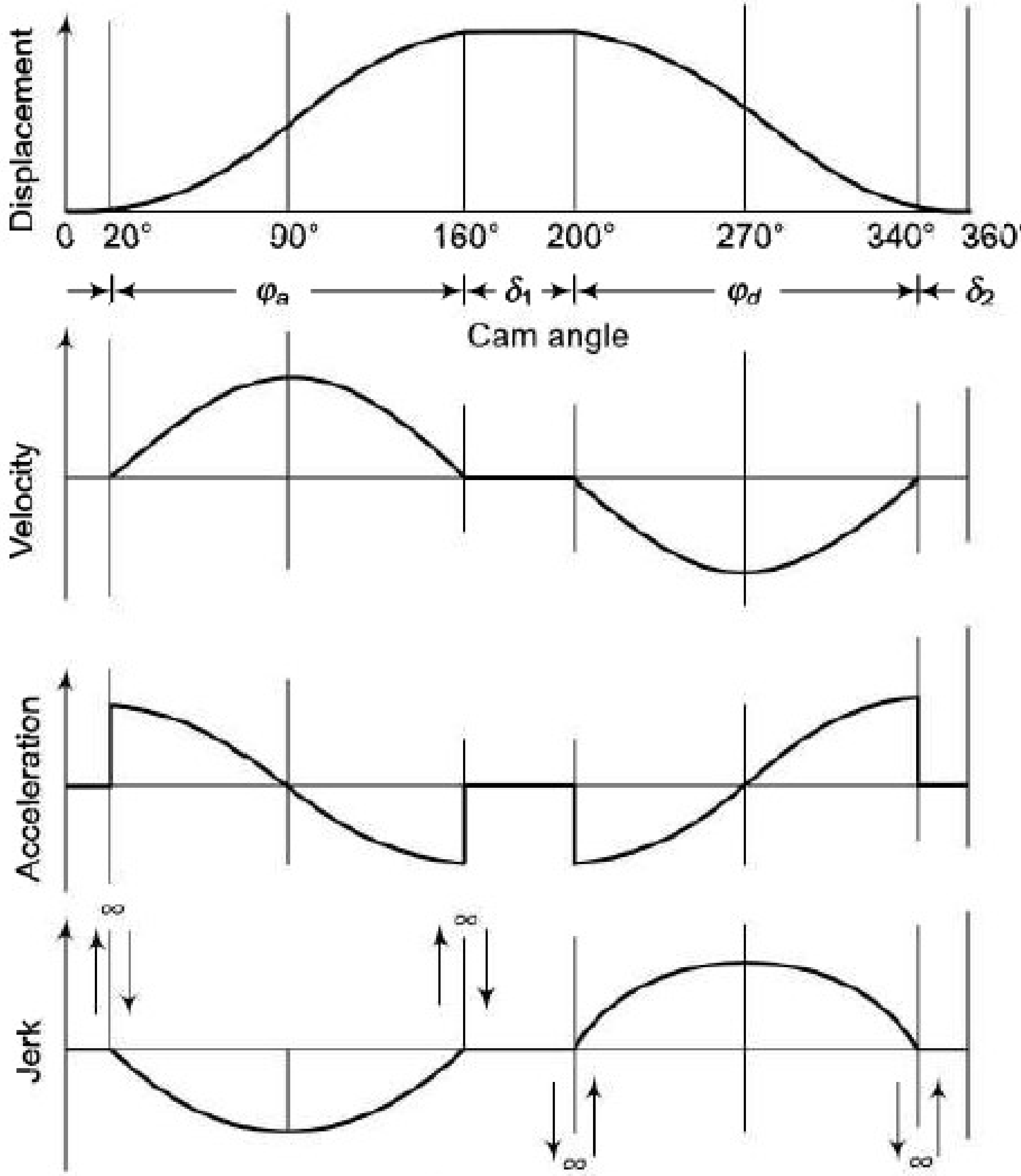
Uniform Velocity



This program of follower is adopted only for low speeds as it is seen infinite accelerations

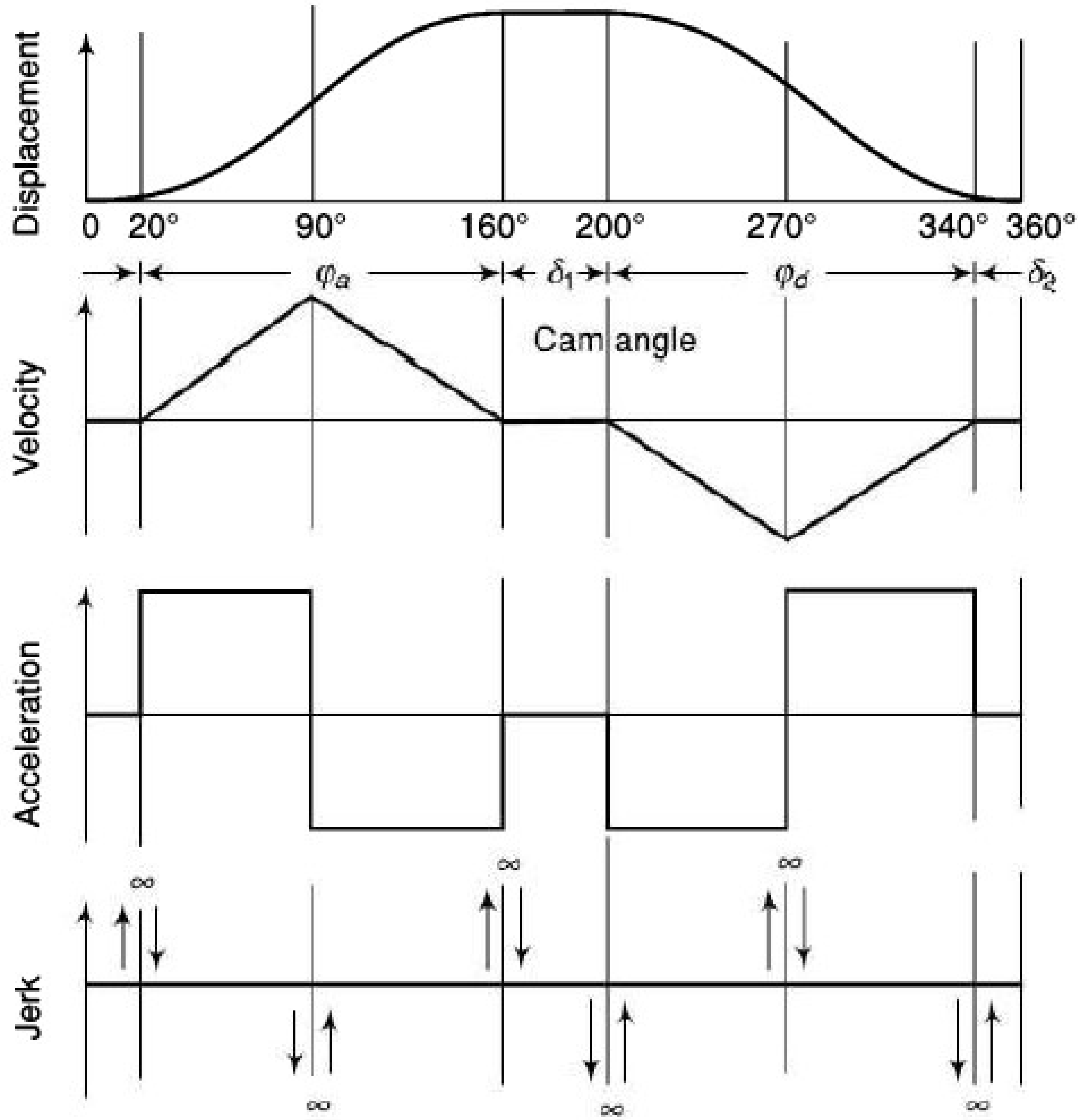
Simple Harmonic Motion

This program of follower is adopted only up to moderate speeds as it is seen infinite jerks



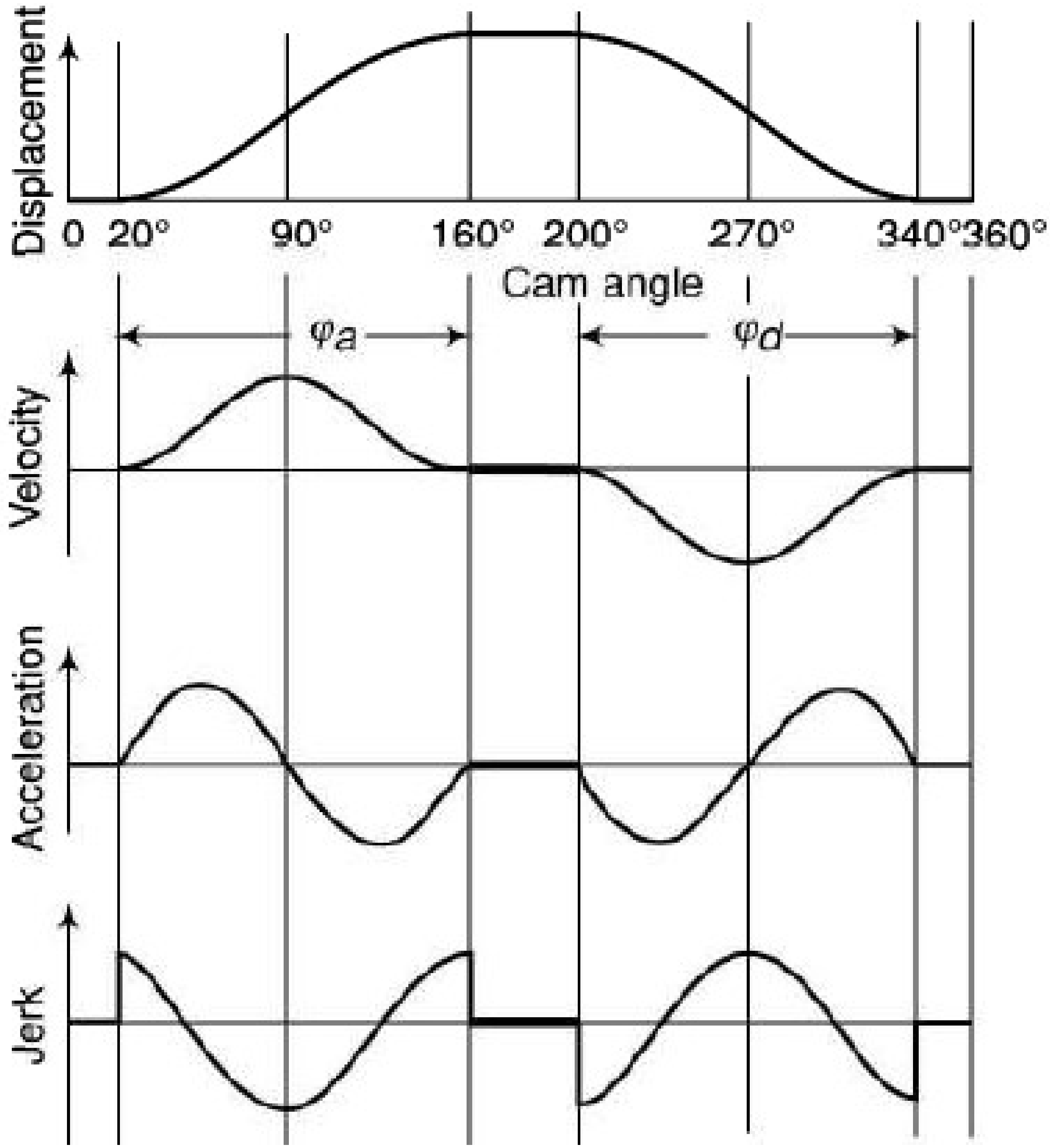
Constant Acceleration and Deceleration (parabolic)

This program of follower is adopted only up to moderate speeds as it is seen infinite jerks



Cycloid Motion

No abrupt changes in the velocity and acceleration at any stage of the motion. Thus, it is the most ideal program for high-speed follower motion.



The Fundamental Law of Cam Design

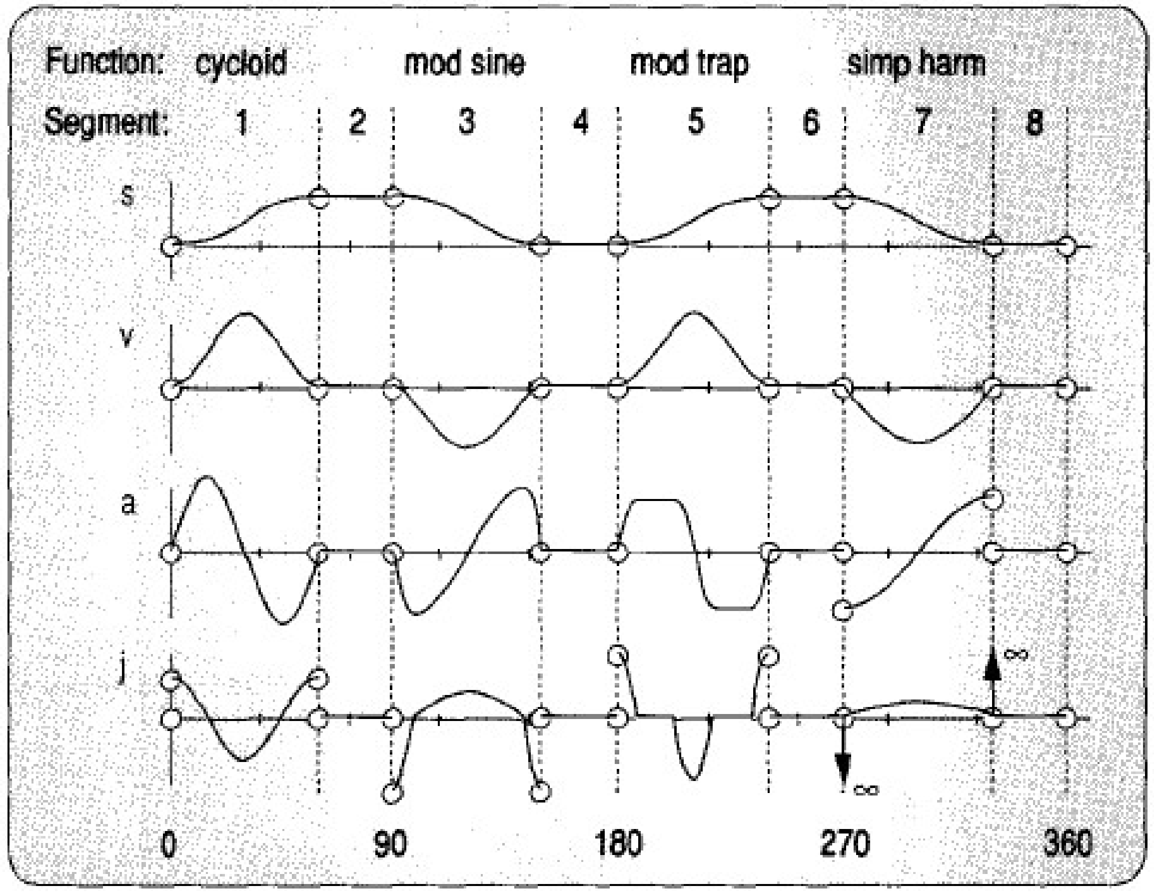
Any cam designed for operation at other than very low speeds must be designed with the following constraints:

- The cam function must be continuous through the first and second derivatives of displacement across the entire interval (360 degrees)
- The jerk function must be finite across the entire interval (360 degrees)
- The cam motion program cannot be defined by a single mathematical expression, but rather must be defined by several separate functions, called piecewise functions
- The displacement, velocity and acceleration functions must have no discontinuities in them
- If any discontinuities exist in the acceleration function, then there will be infinite spikes, or Dirac delta functions, appearing in the derivative of acceleration, jerk.

An Example

Table shows the specifications for a four-dwell cam that has eight segments, RDFDRDFD. s-v-*a*-*j* curves for the whole cam over 360 degrees of camshaft rotation is shown. A cam design begins with a definition of the required cam functions and their s-v-*a*-*j* diagrams. Functions for the non-dwell cam segments should be chosen based on their velocity, acceleration, and jerk characteristics and the relationships at the interfaces between adjacent segments including the dwells. These function characteristics can be conveniently and quickly investigated with program DYNACAM which generated the data and plots shown.

Segment Number	Function Used	Start Angle	End Angle	Delta Angle
1	Cycloid rise	0	60	60
2	Dwell	60	90	30
3	ModSine fall	90	150	60
4	Dwell	150	180	30
5	ModTrap rise	180	240	60
6	Dwell	240	270	30
7	SimpHarm fall	270	330	60
8	Dwell	330	360	30



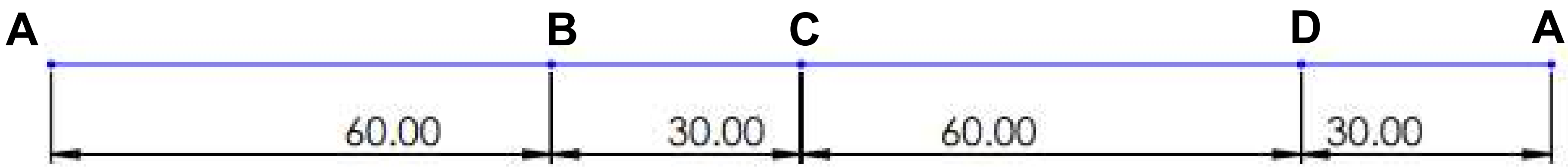
Displacement Diagrams

Uniform Velocity

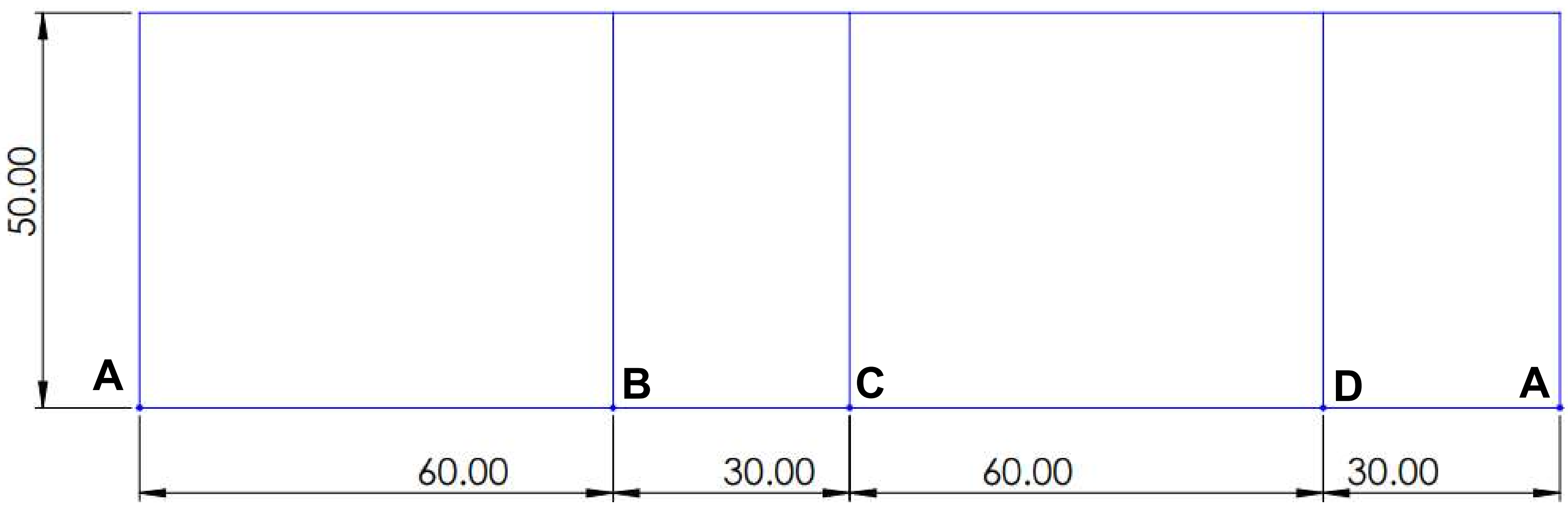
20° = 10 mm

Rise - 120°; Dwell - 60°;
Rise - 120°; Dwell - 60°;
Lift = 50 mm

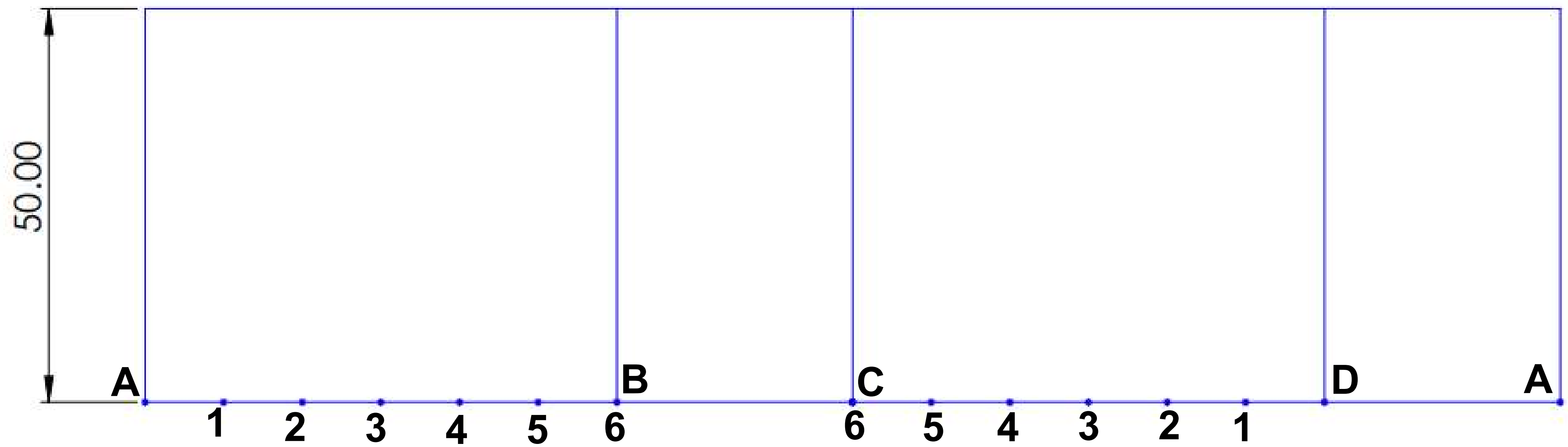
Step 1



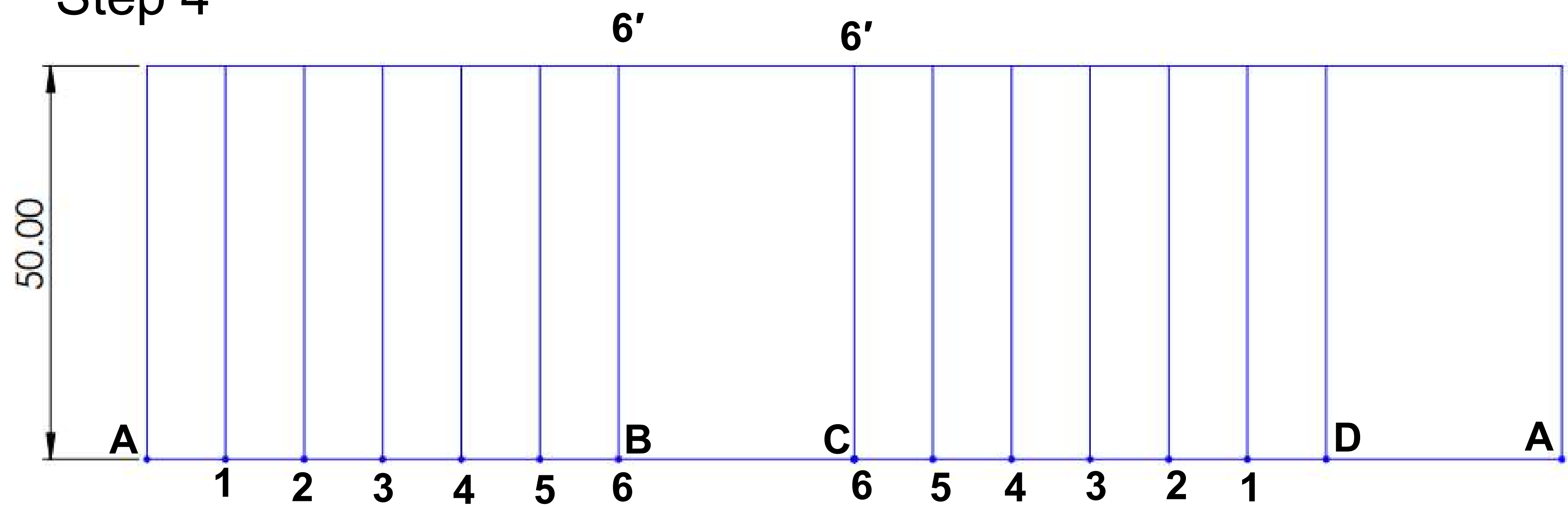
Step 2



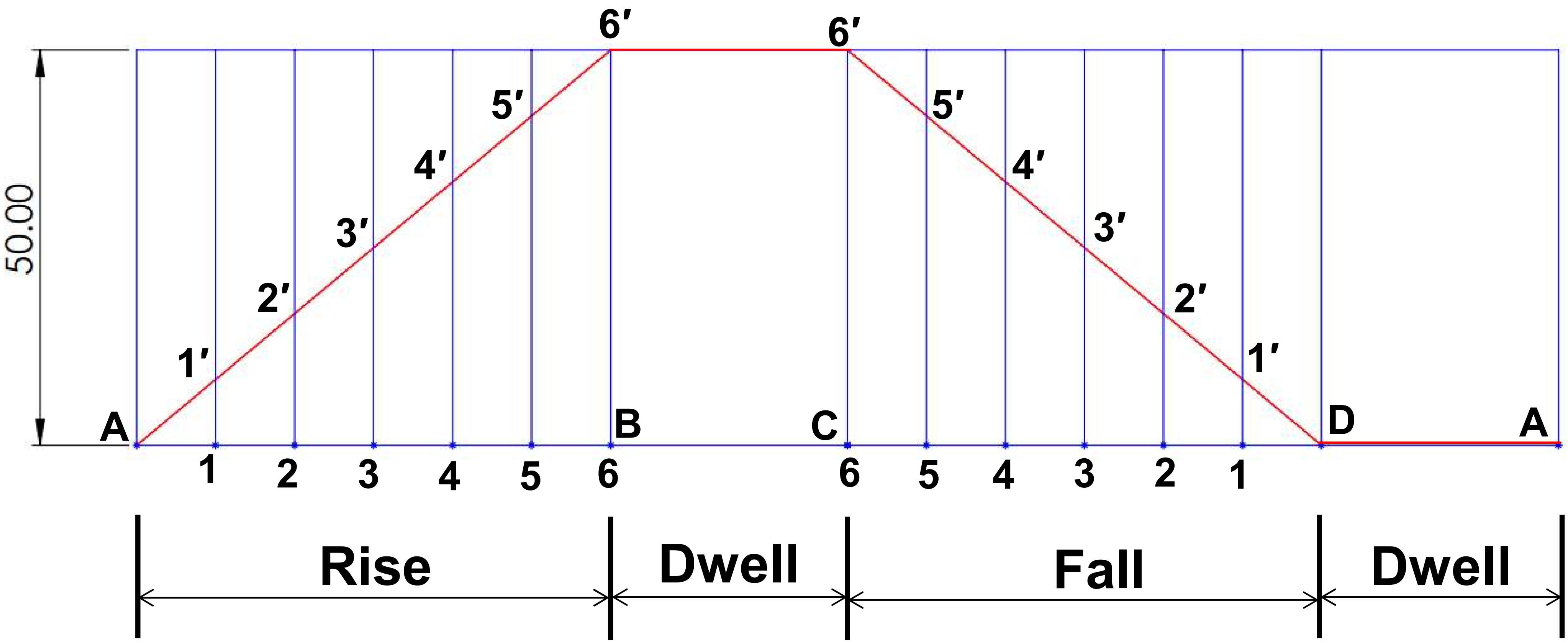
Step 3



Step 4

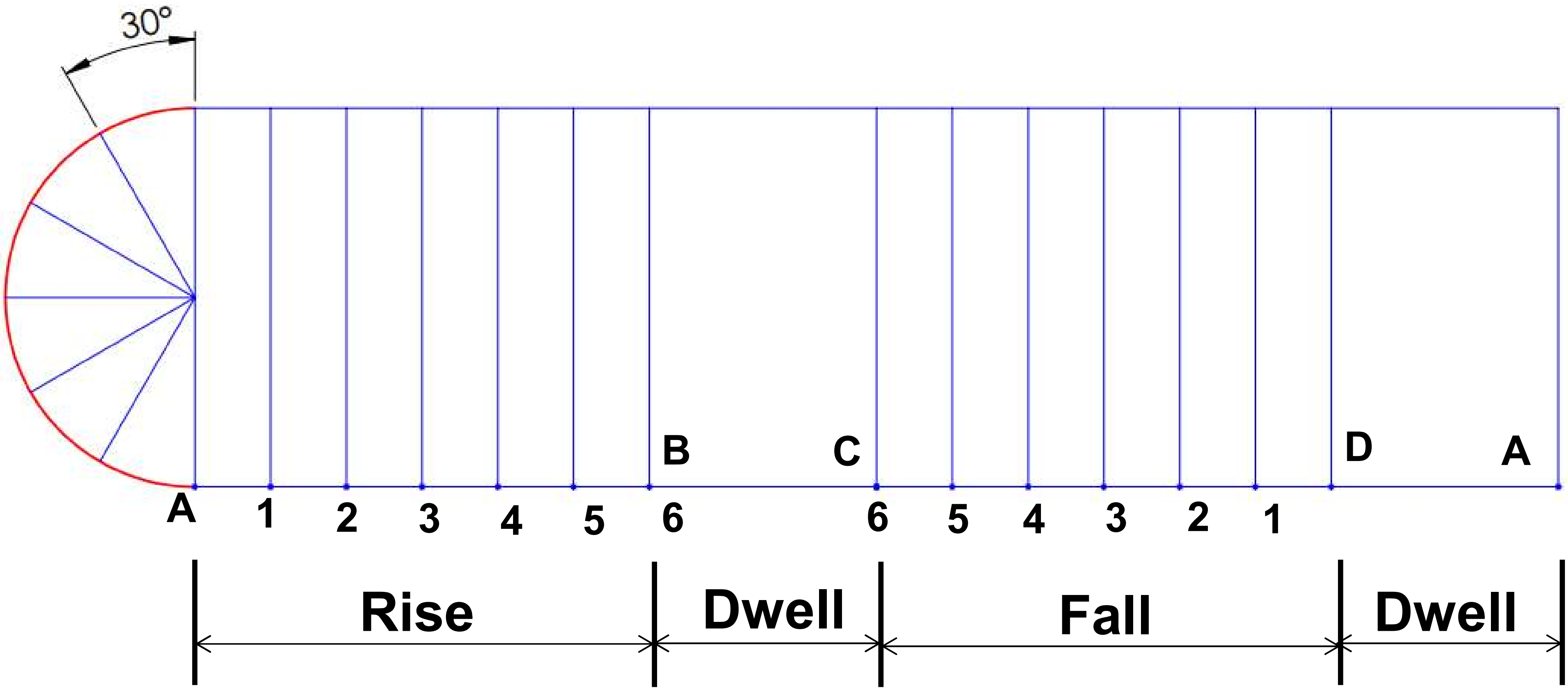


Step 5

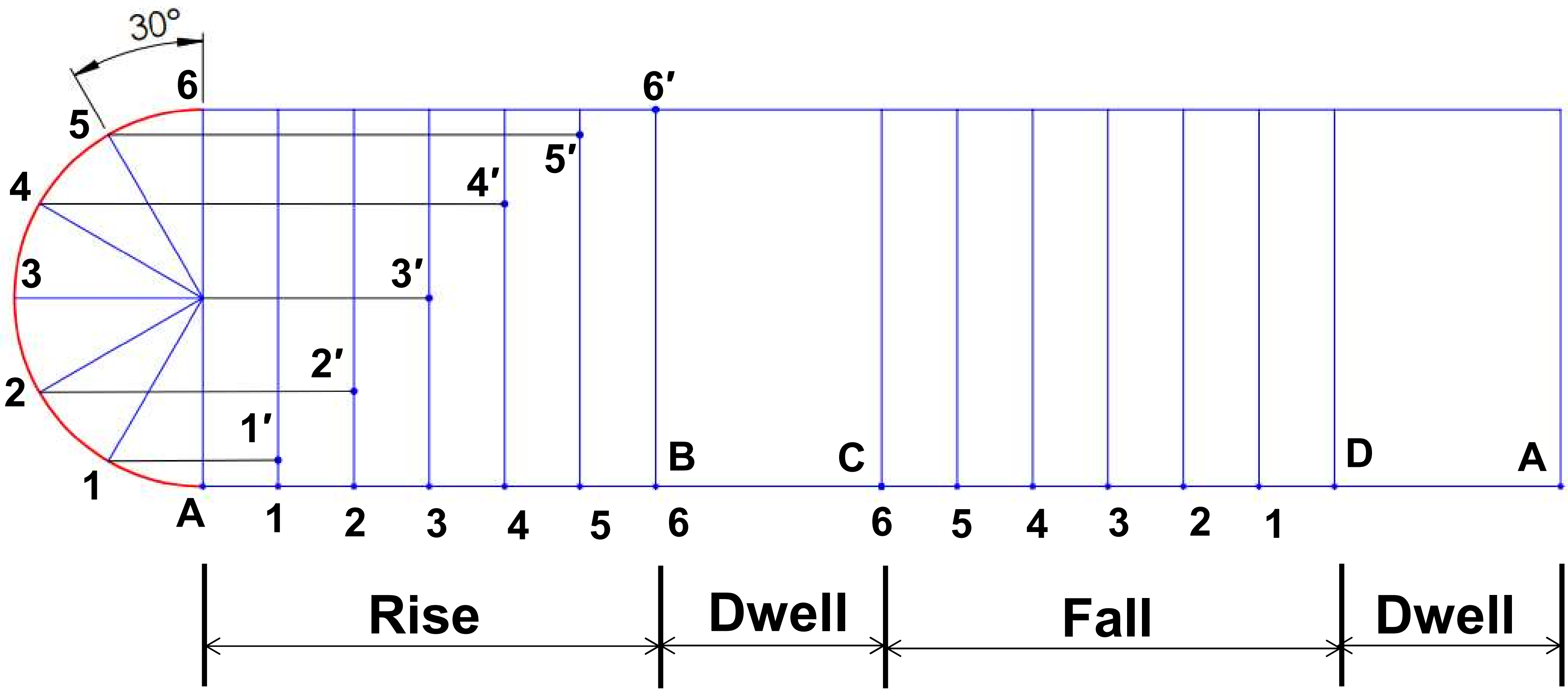

50.00
6′
6′
5′
4′
3′
2′
1′
5′
4′
3′
2′
1′
A
B
C
D
A
1
2
3
4
5
6
6
5
4
3
2
1
Rise
Dwell
Fall
Dwell

Simple Harmonic Motion

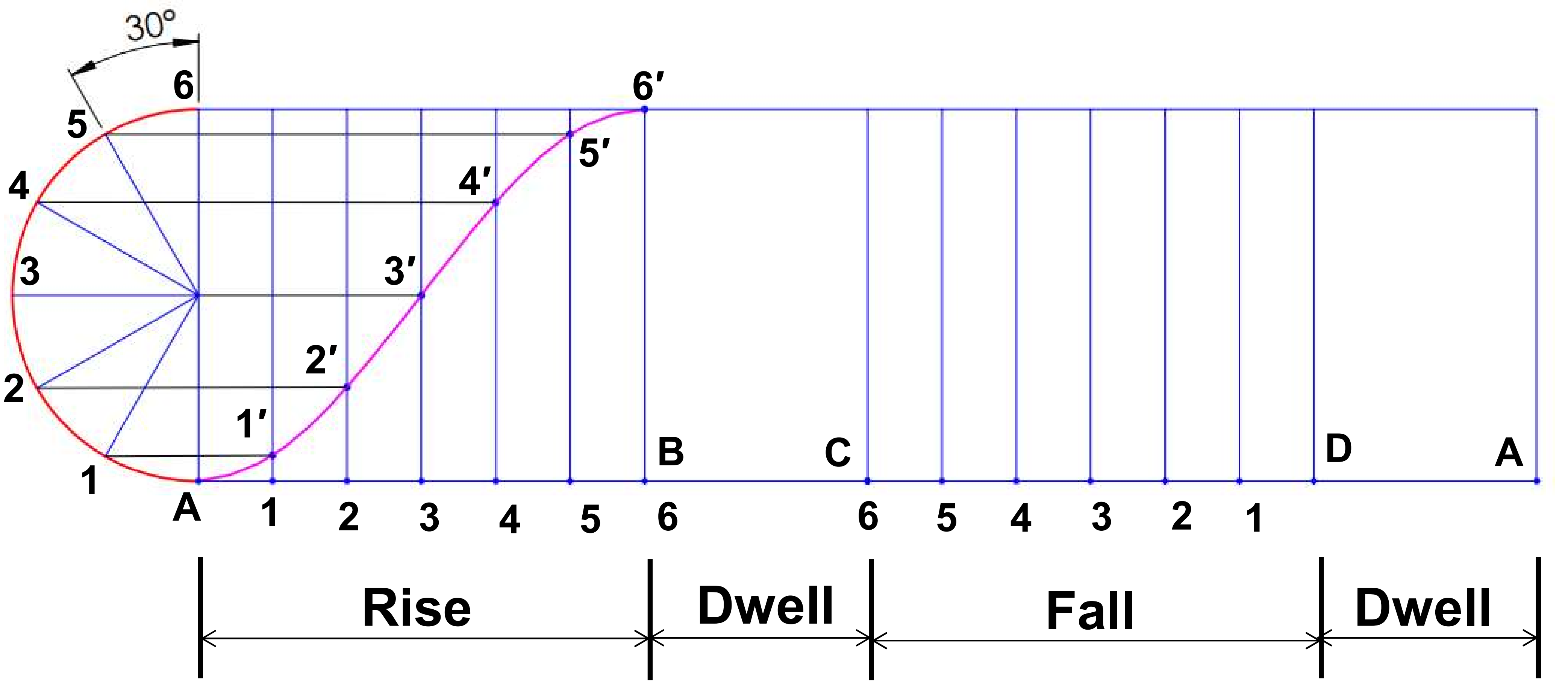
Step 5



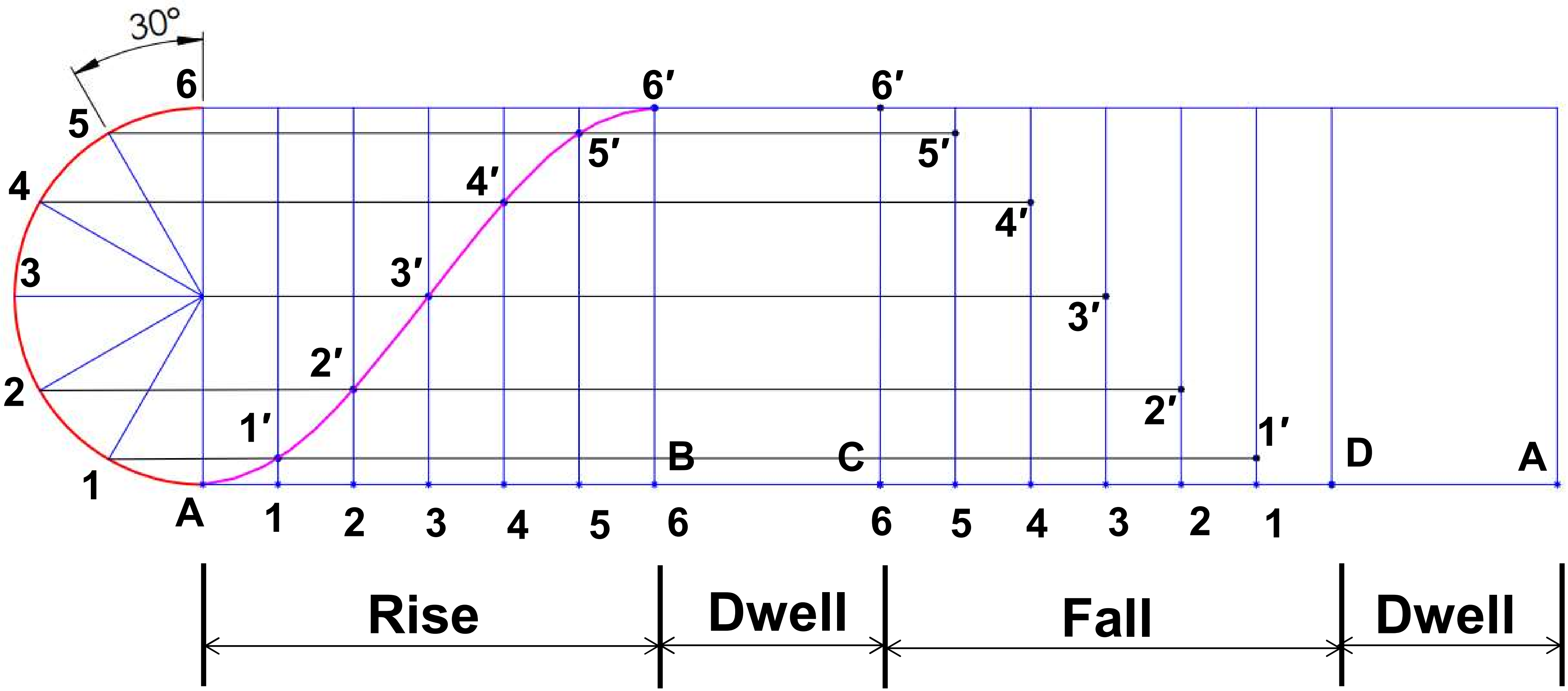
Step 6



Step 7

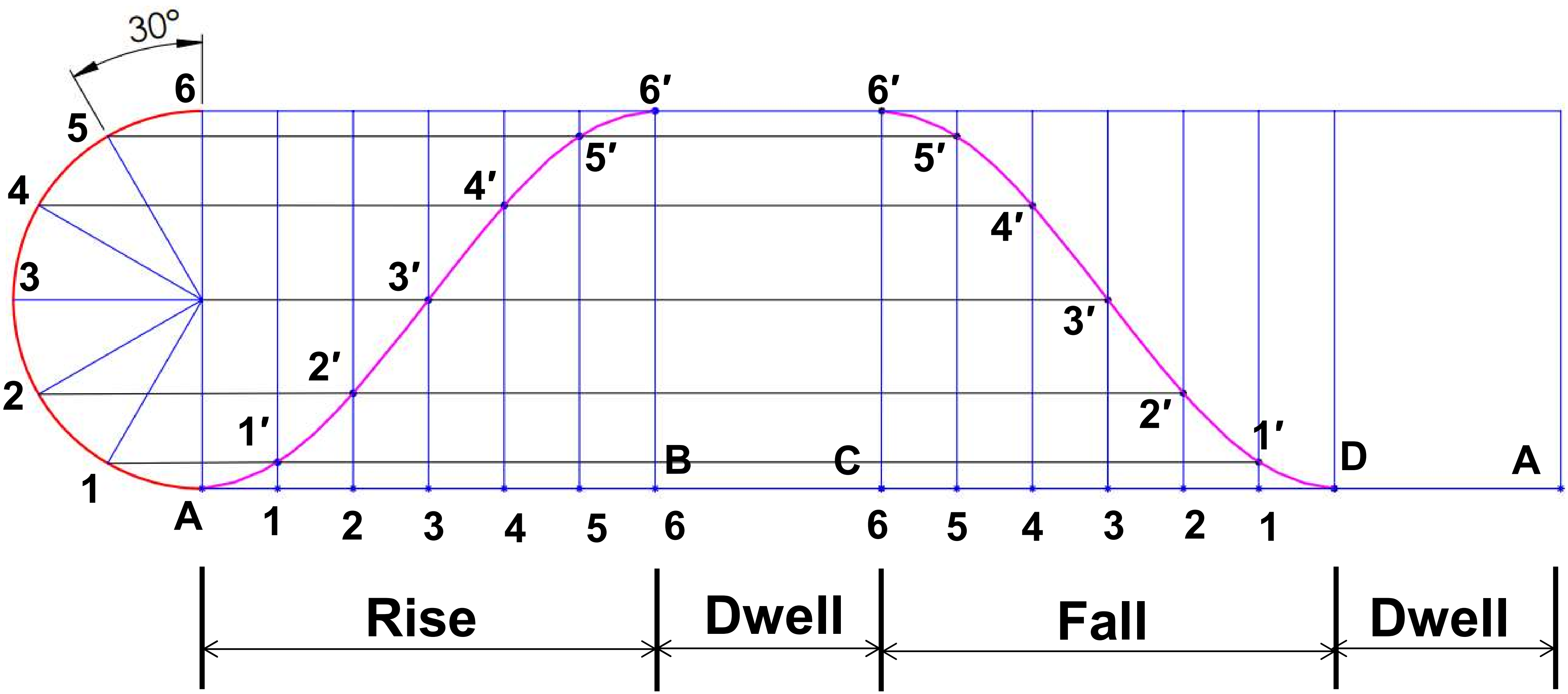


Step 8



30°
6
5
4
3
2
1
A
1
2
3
4
5
6
6
5
4
3
2
1
1′
2′
3′
4′
5′
6′
6′
5′
4′
3′
2′
1′
B
C
D
A
Rise
Dwell
Fall
Dwell

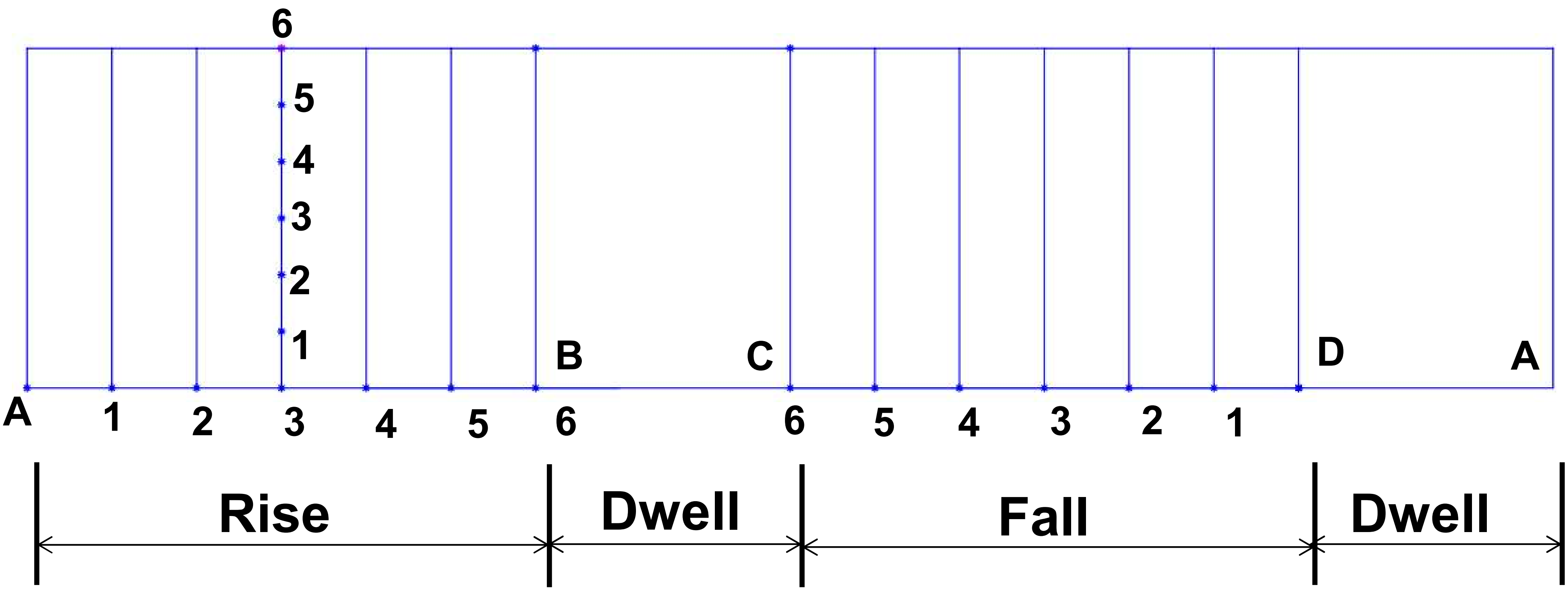
Step 9



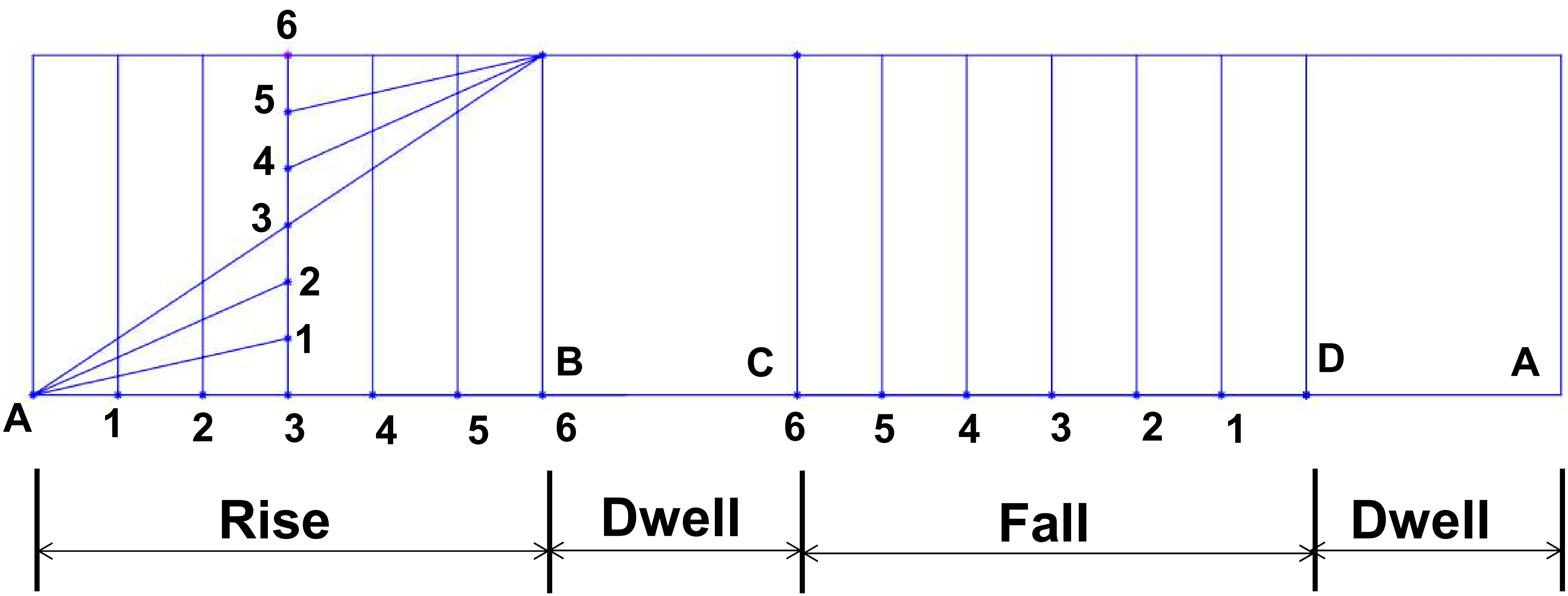
30°
6
5
4
3
2
1
6′
5′
4′
3′
2′
1′
6′
5′
4′
3′
2′
1′
B
C
D
A
A
1
2
3
4
5
6
6
5
4
3
2
1
Rise
Dwell
Fall
Dwell

Uniform Acceleration and Retardation

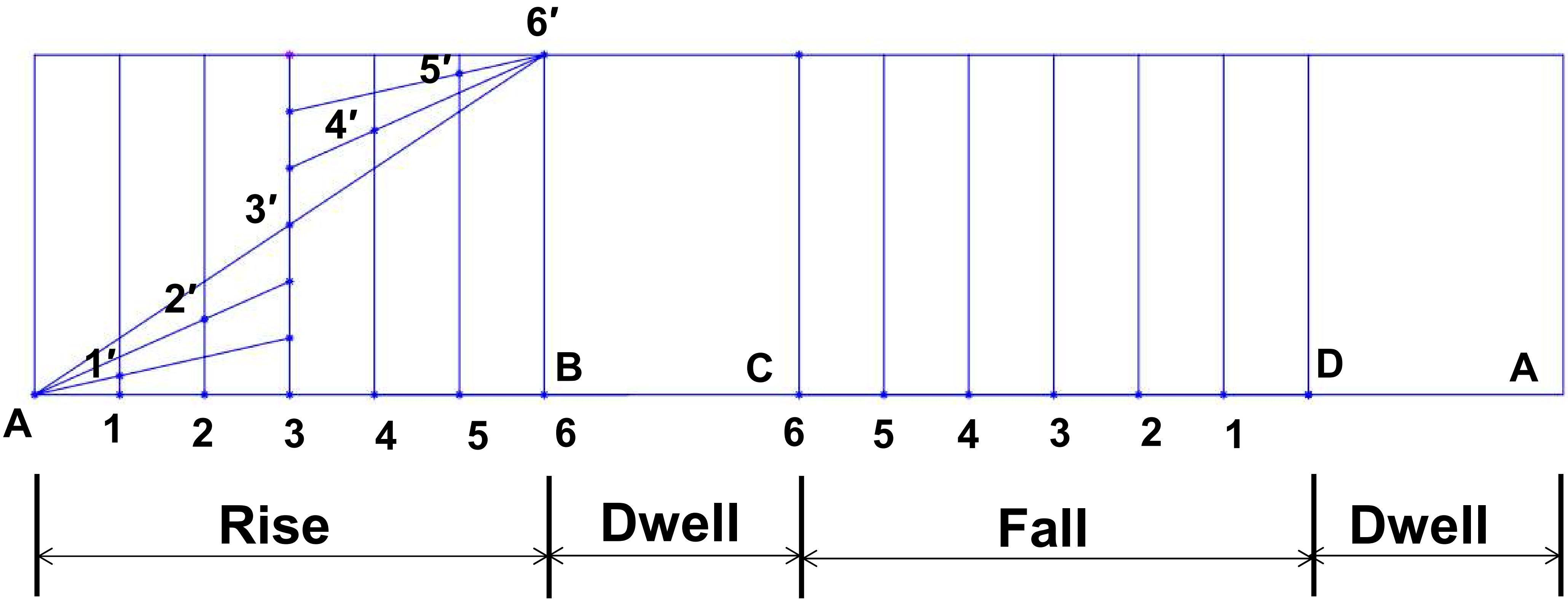
Step 5



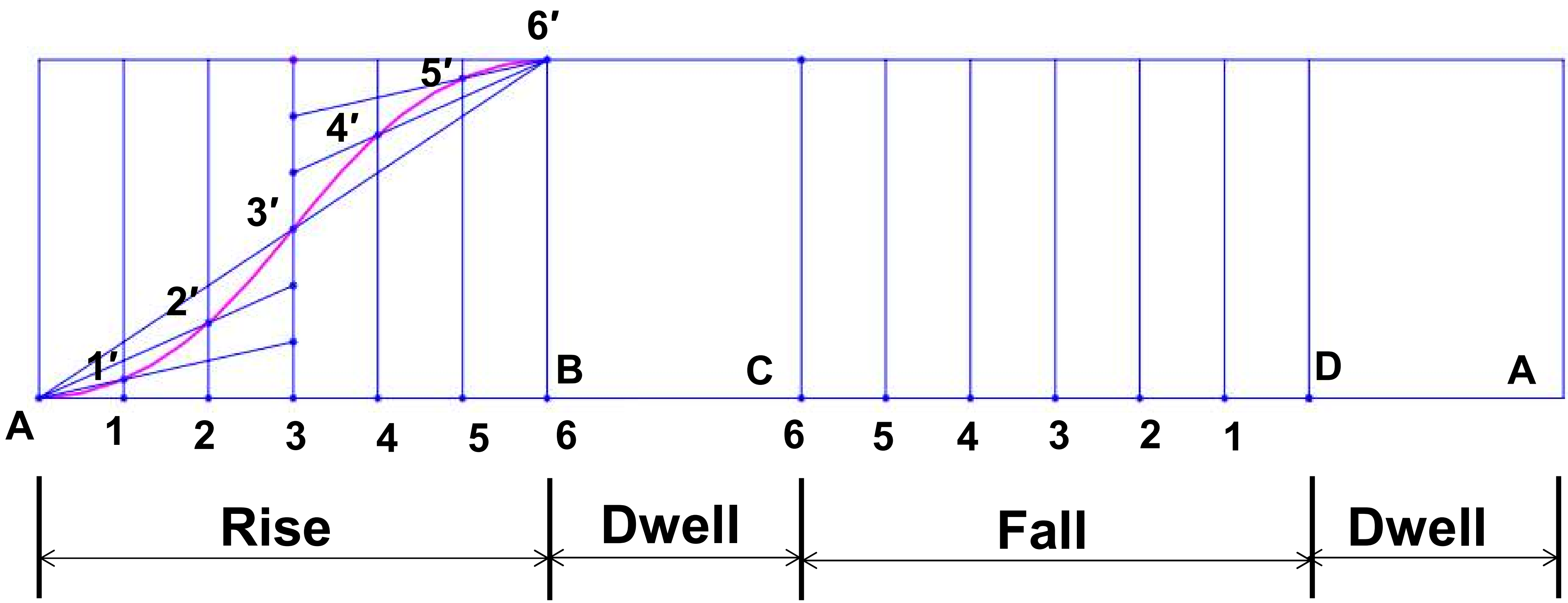
Step 6



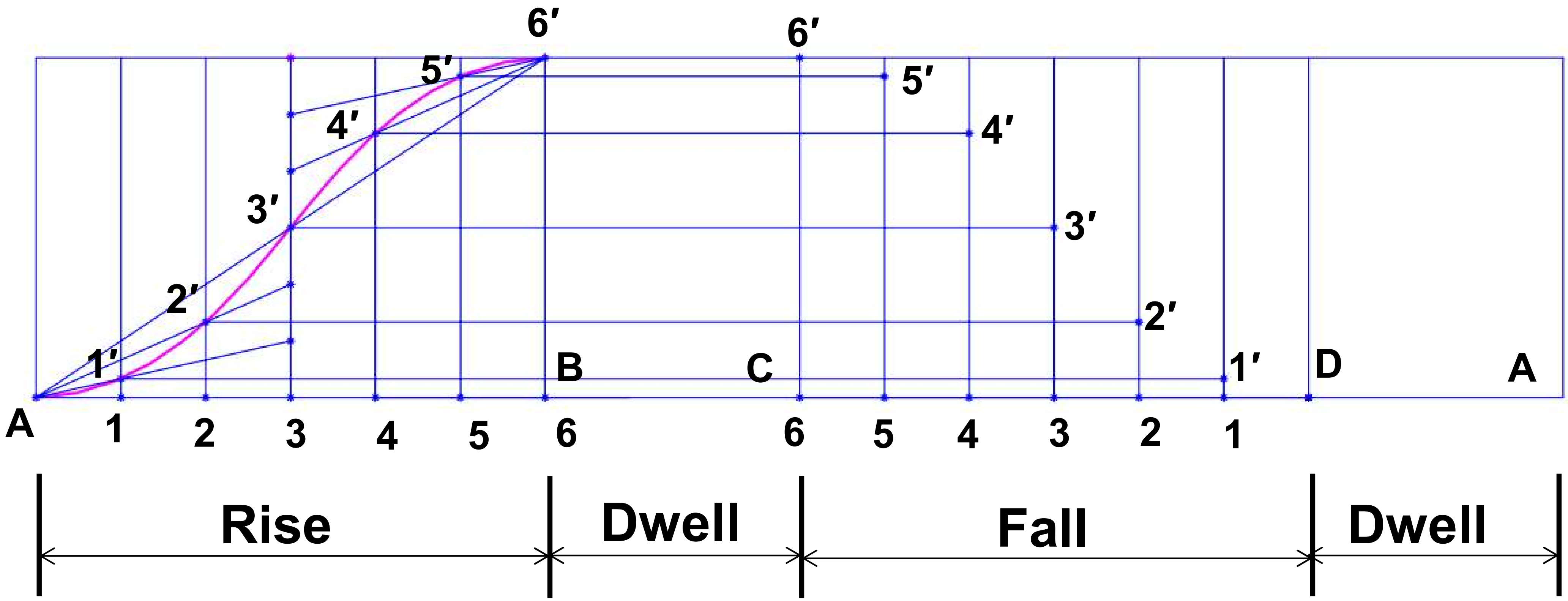
Step 7



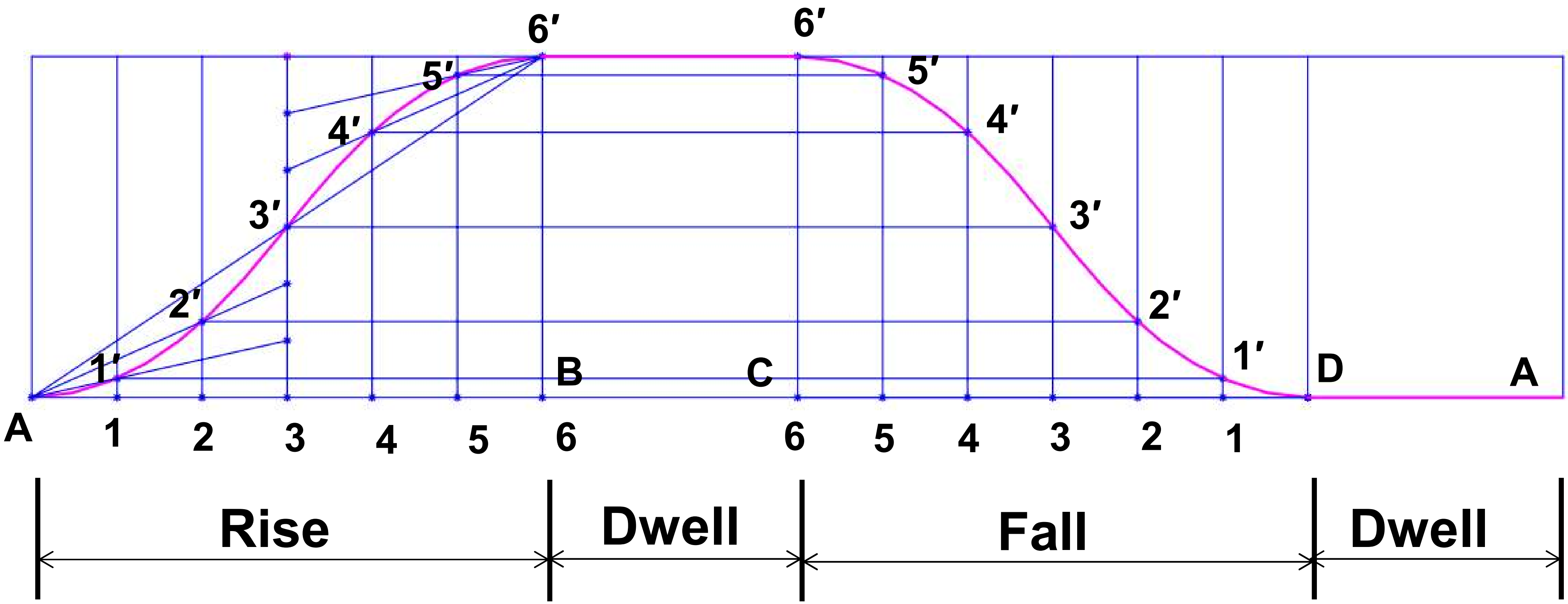
Step 8



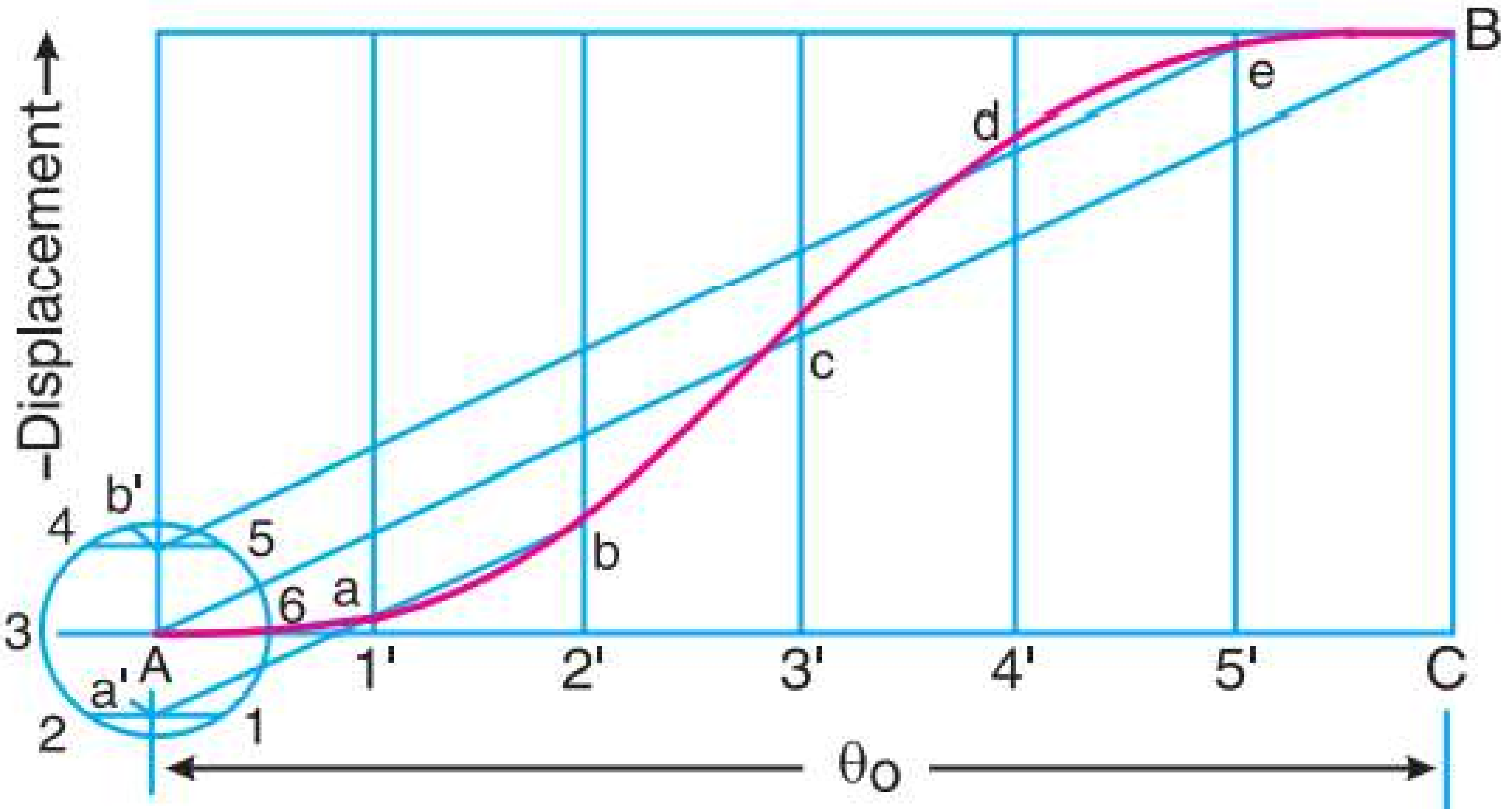
Step 9



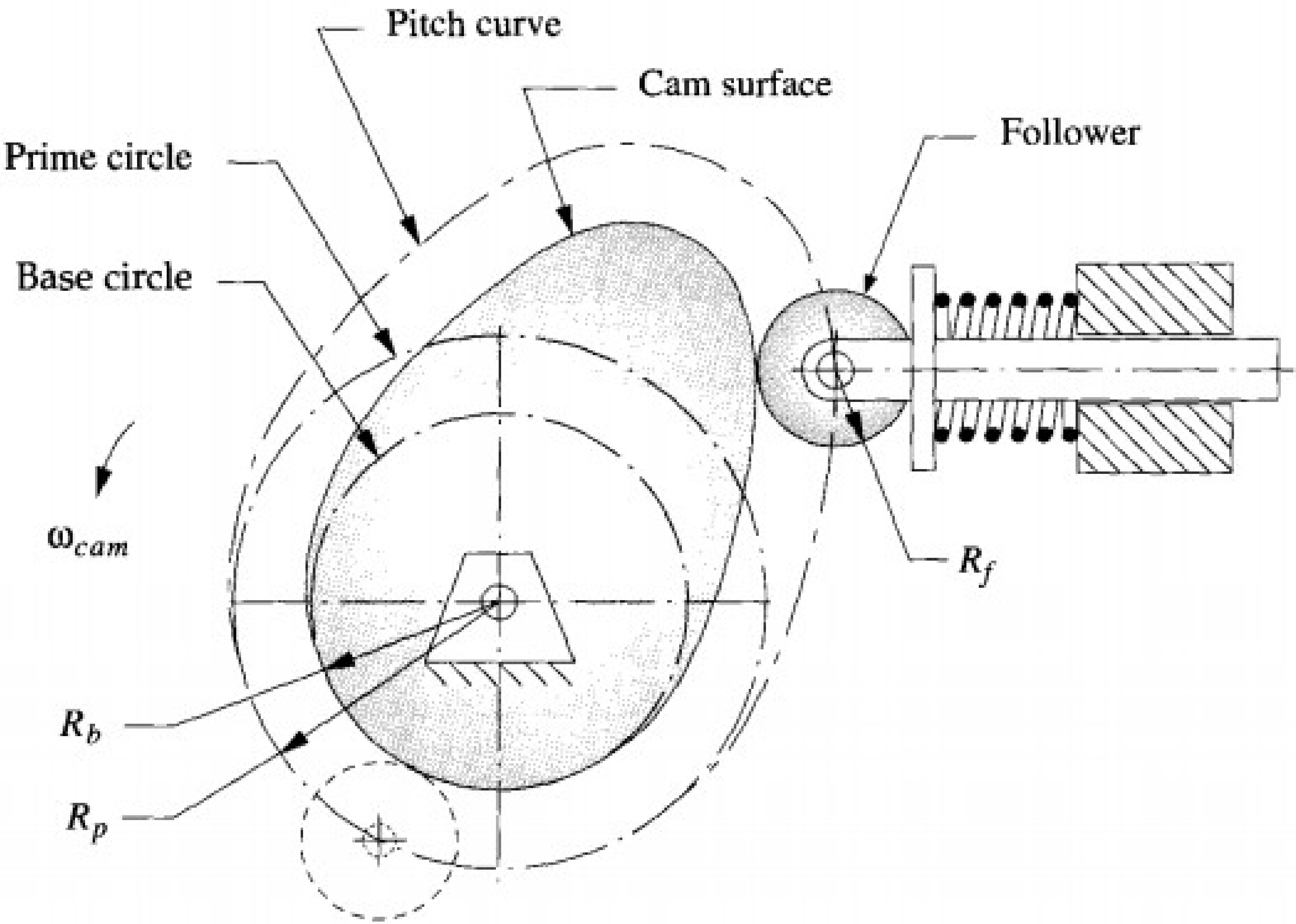
Step 10



Cycloidal Motion



$$Generating\ Circle\ Radius = \frac{Stroke}{2\pi} = \frac{50}{2\pi} = 7.96\ \text{mm}$$



Pitch curve
Cam surface
Follower
Prime circle
Base circle
ω_{cam}
R_f
R_b
R_p

Base Circle: the smallest circle that can be drawn tangent to the physical cam surface

Prime Circle: the smallest circle that can be drawn tangent to the locus of the centreline of the follower

Pitch Curve: the locus of the centreline of the follower

Problem 1: Uniform Velocity Motion- Knife edge follower

A cam is to give the following motion to a knife-edged follower:

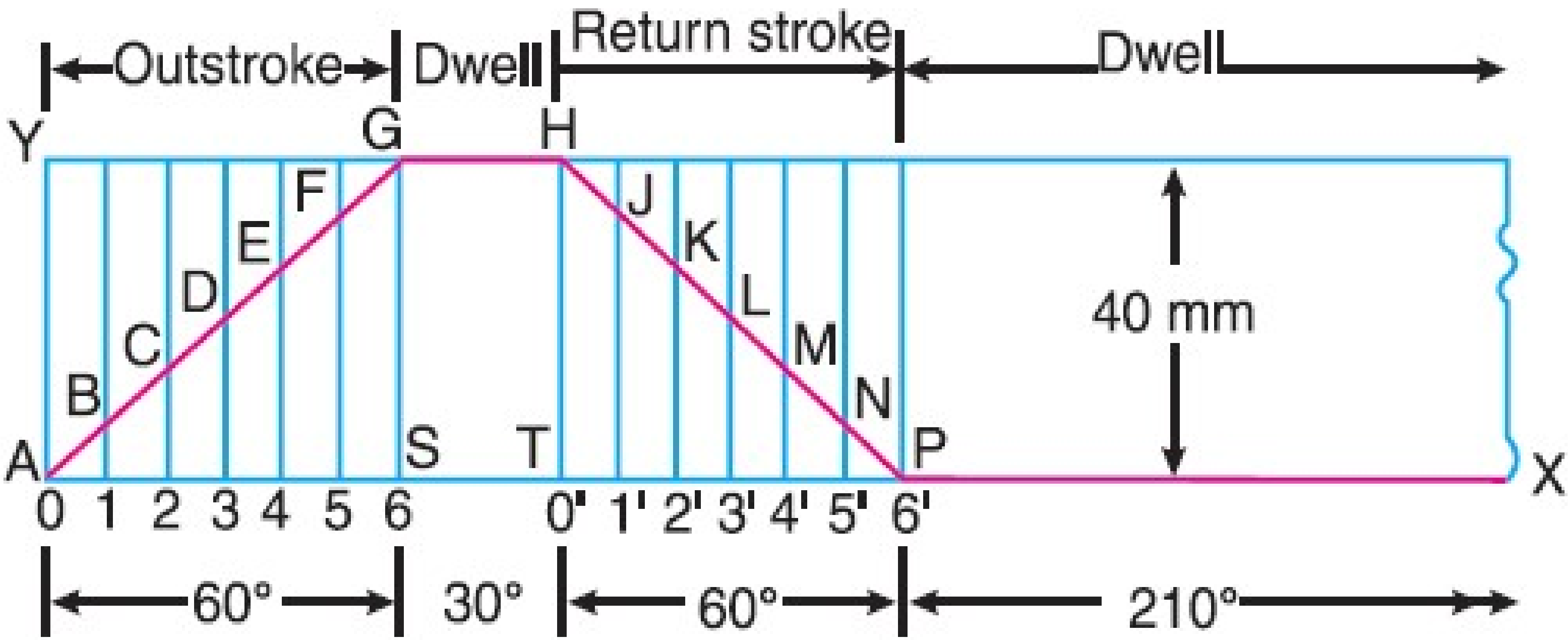
1. Outstroke during 60° of cam rotation; 2. Dwell for the next 30° of cam rotation; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes.

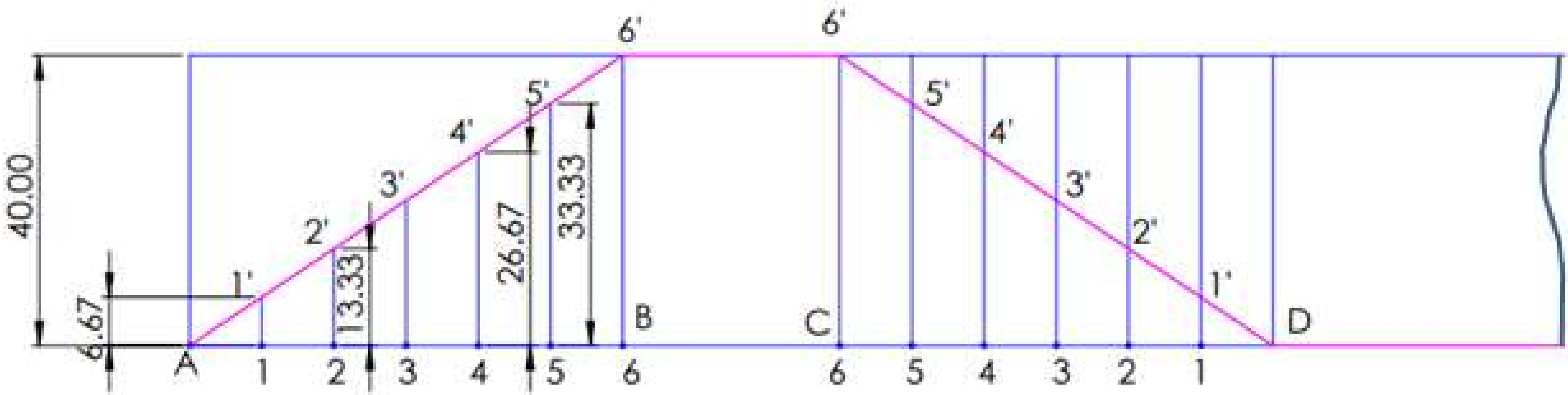
Draw the profile of the cam when,

(a) the axis of the follower passes through the axis of the cam shaft, and

(b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

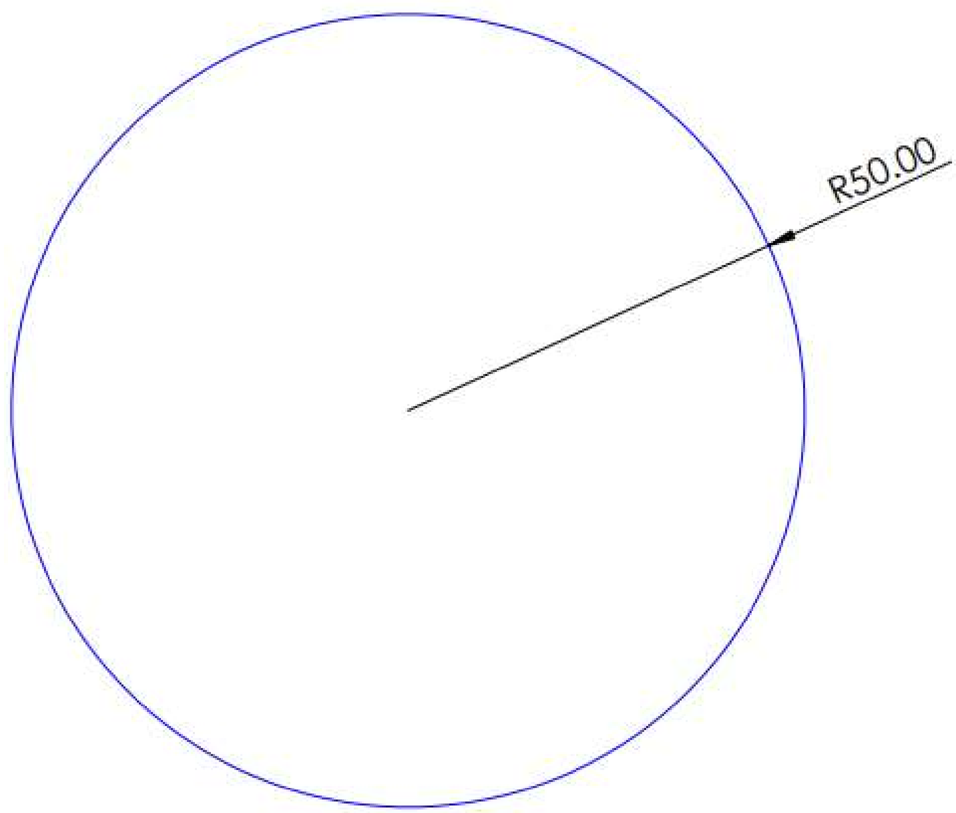
Displacement Diagram



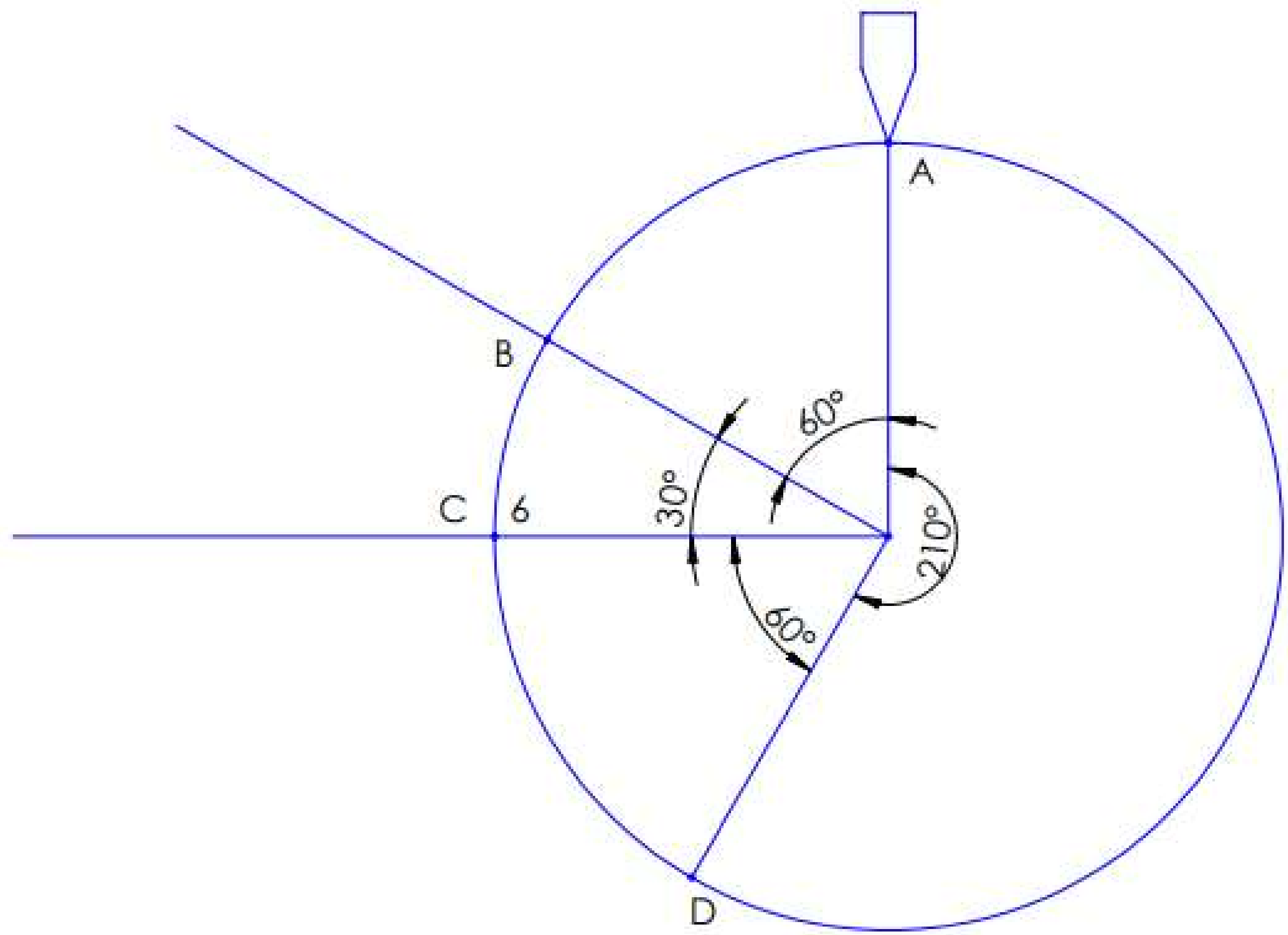


Cam Profile (without offset) - Procedure

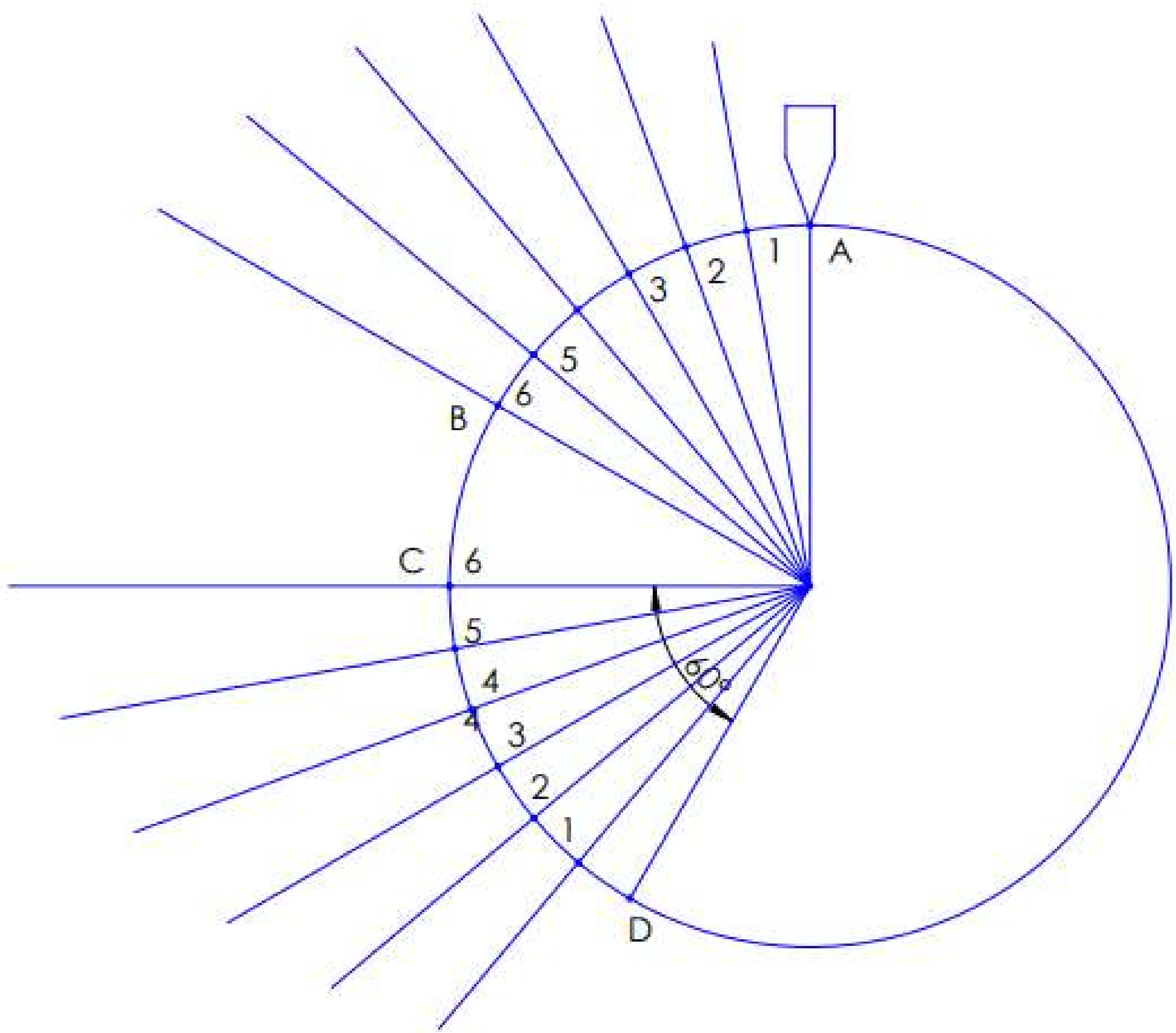
Step 1: Draw the base circle with the radius of 50 mm.



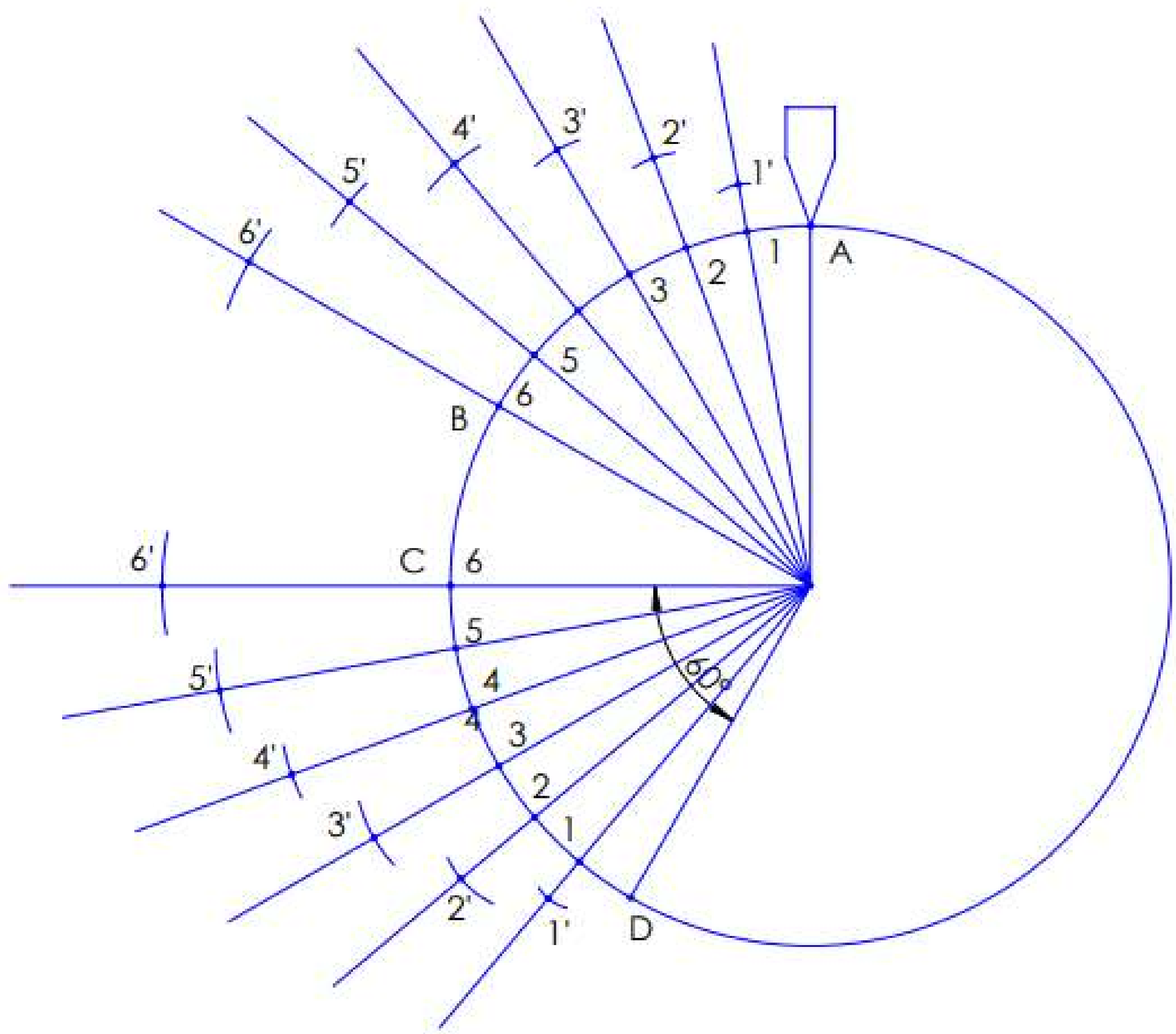
Step 2: Draw a vertical line (axis of follower) from centre and divide the circle for the given degrees of rise, dwell and fall.



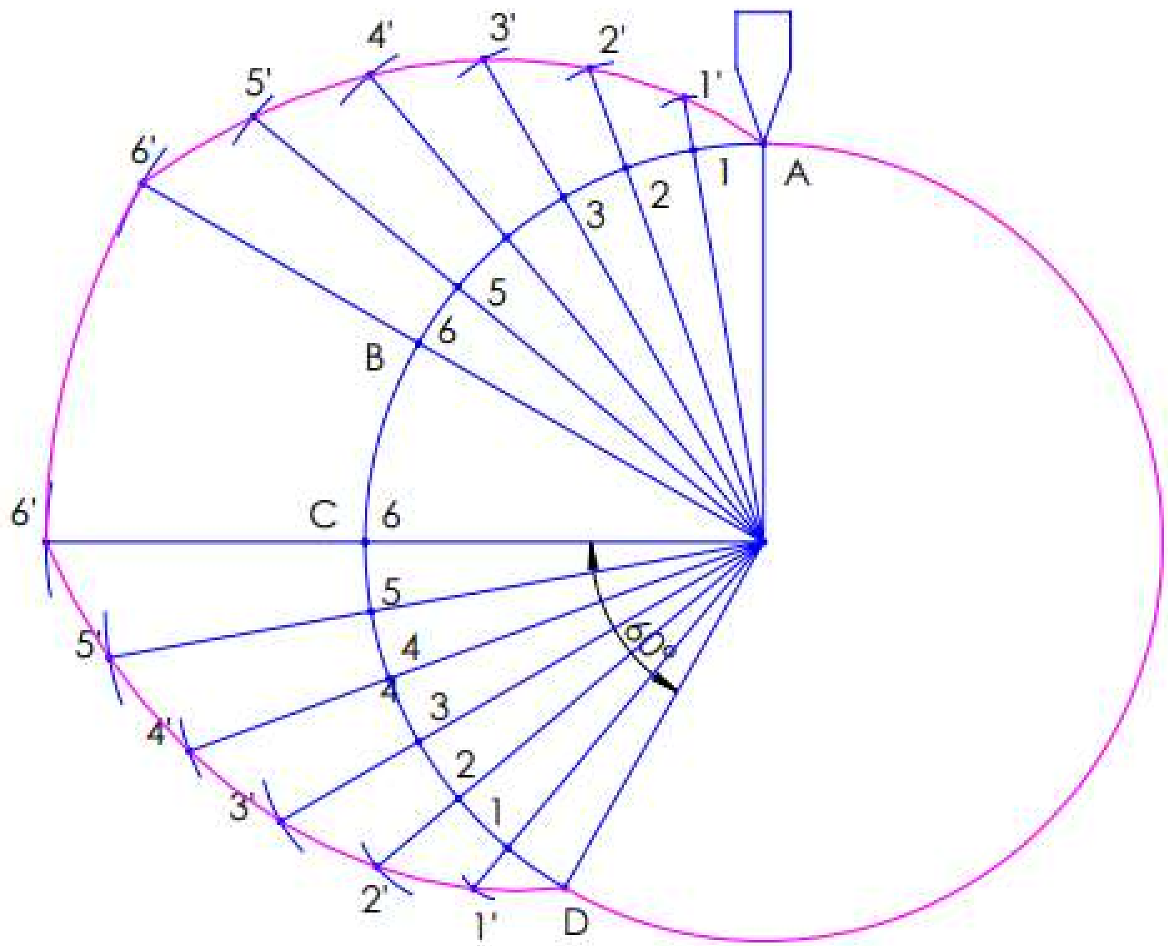
Step 3: Divide the rise and fall angles in to equal divisions as it is done in the displacement diagram and draw straight line passing through the divided points from centre. Name the lines in compliance with the displacement diagram.



Step 4: Measure the distance 1-1′, 2-2′, 3-3′ so on from displacement diagram using compass and make an arc in the corresponding line.

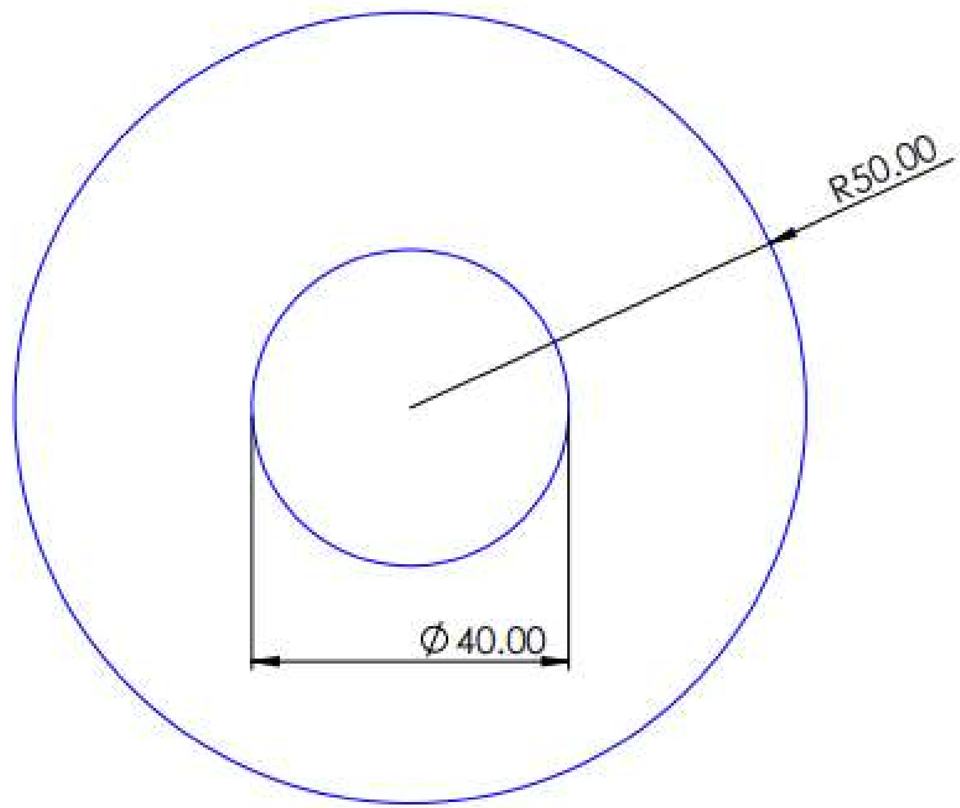


Step 4: Join all the points (A, 1′, 2′, 3′…..) to obtain the required cam profile.

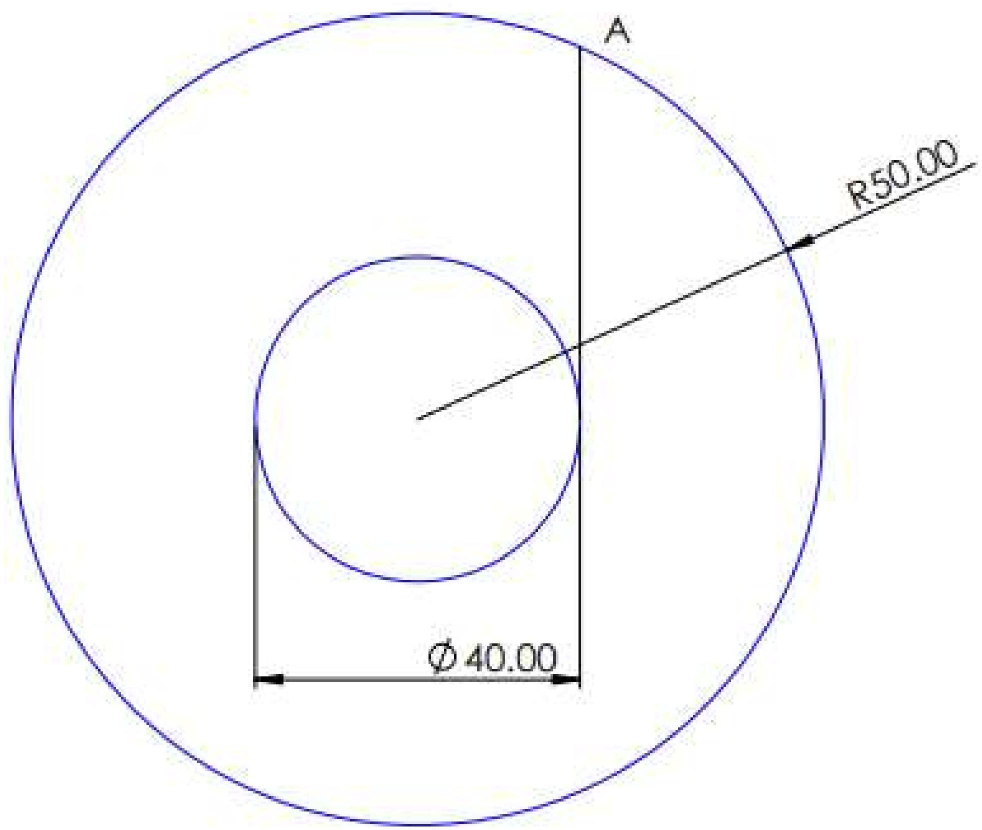


Cam Profile (with offset) - Procedure

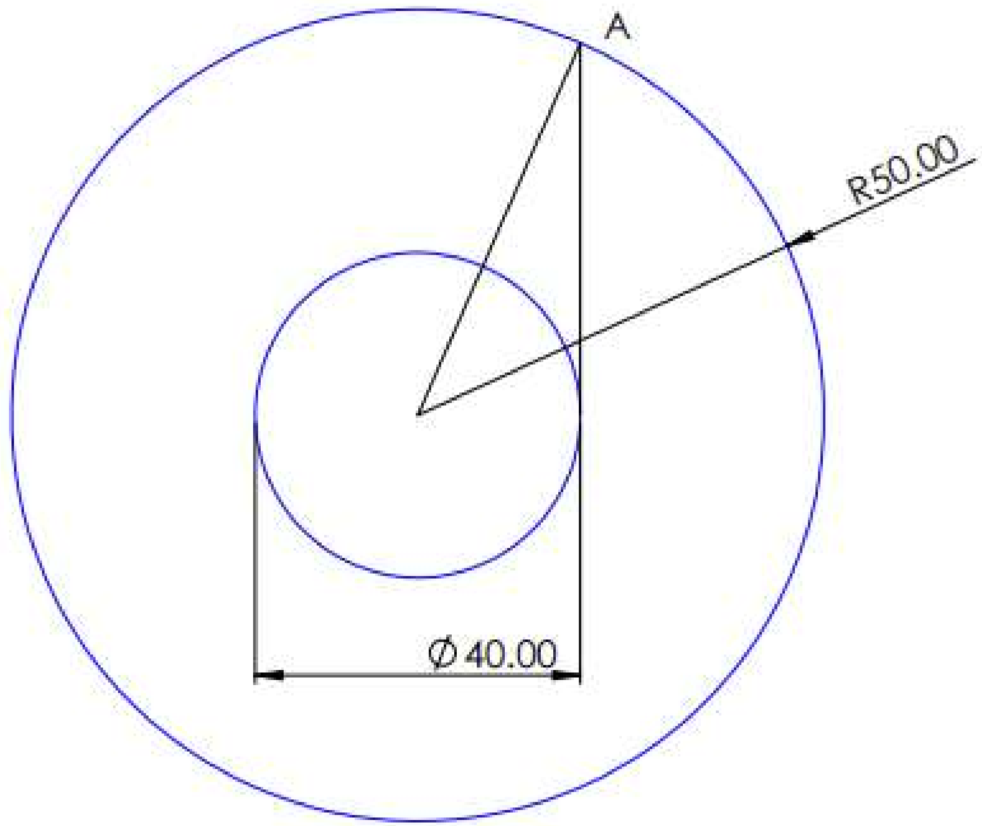
Step 1: Draw the base circle with the radius of 50 mm and draw another circle with the radius equal to the offset (20 mm) from the same centre.



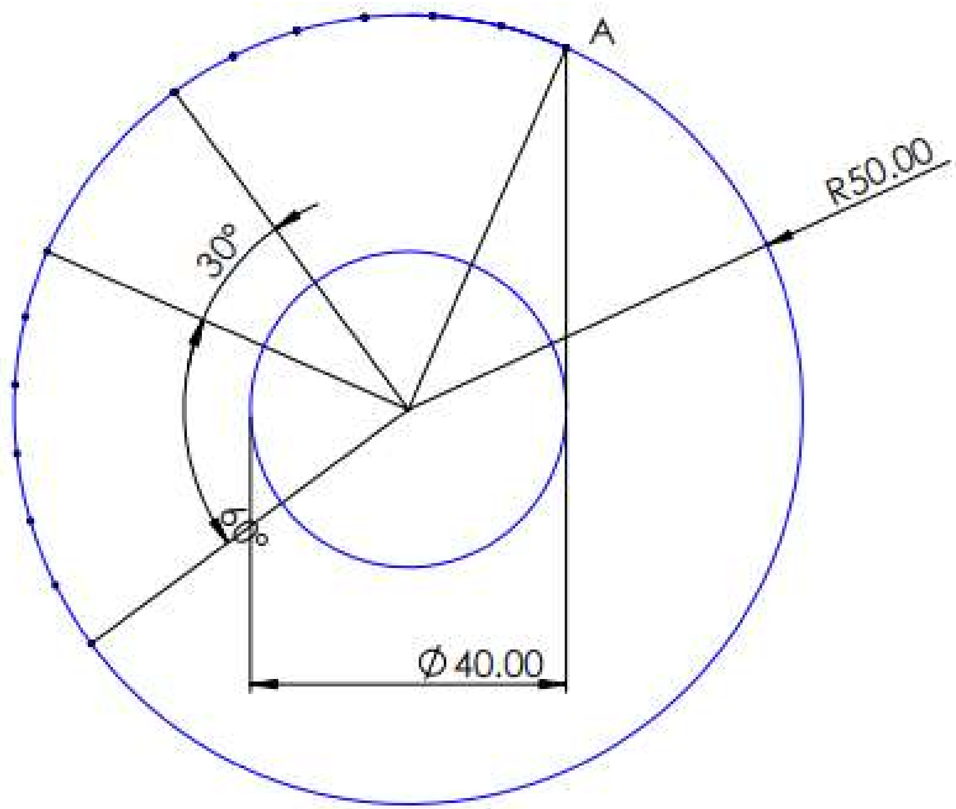
Step 2: Draw a vertical line (axis of follower) tangent to the offset circle and mark A as shown.



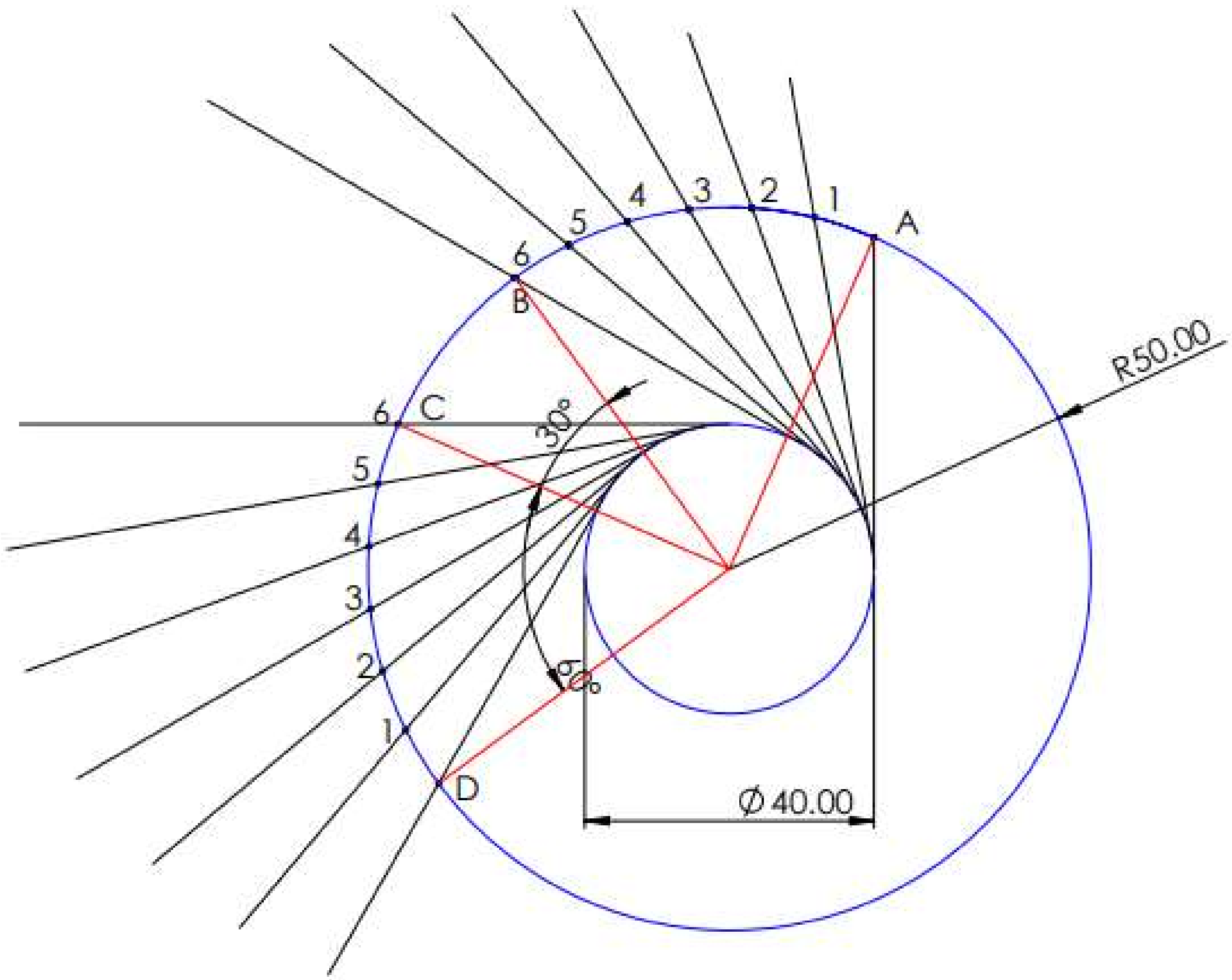
Step 3: Draw a line from centre to join A.



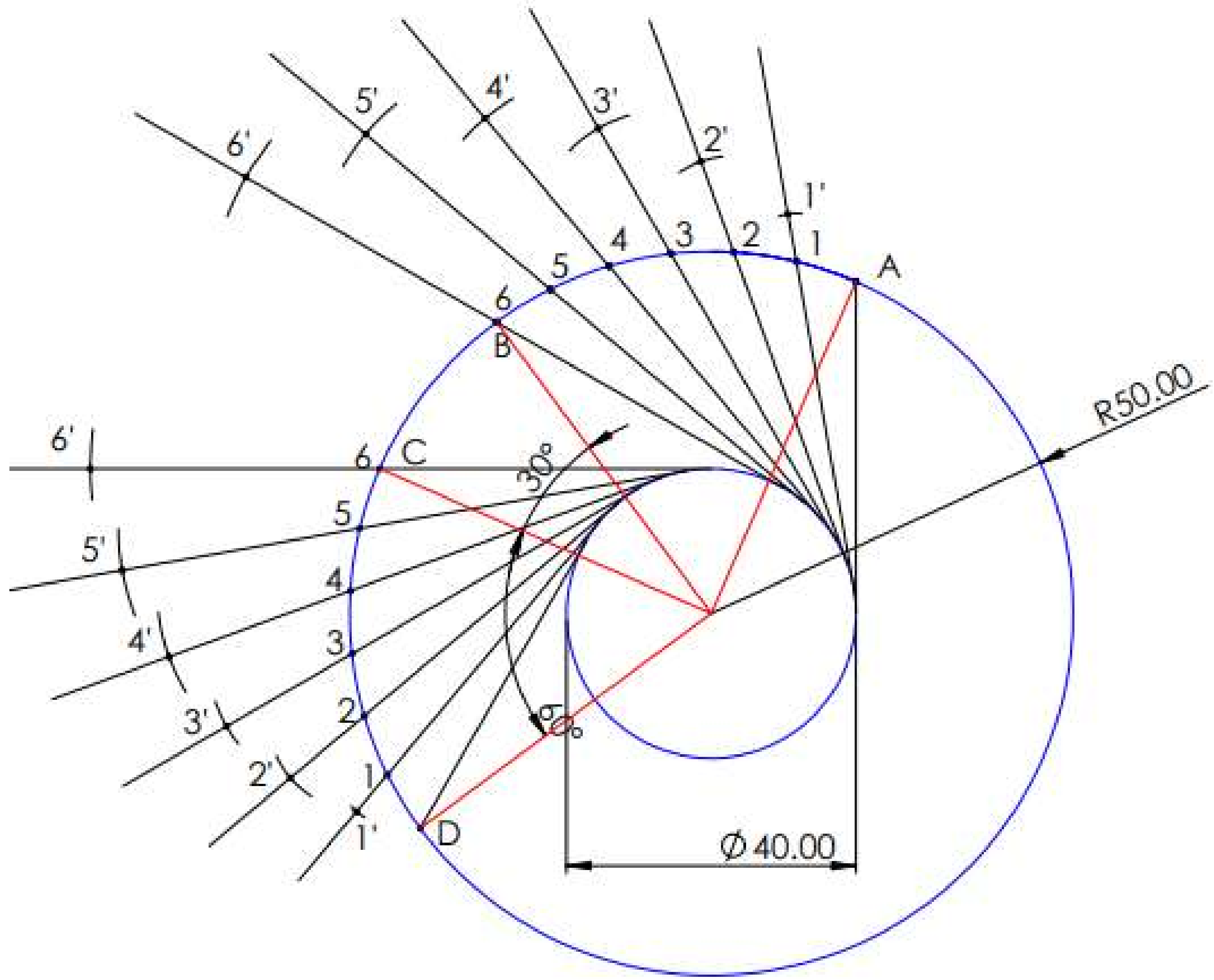
Step 4: Divide the circle for the given degrees of rise, dwell and fall as shown. Divide the rise and fall angles in to equal divisions as it is done in the displacement diagram



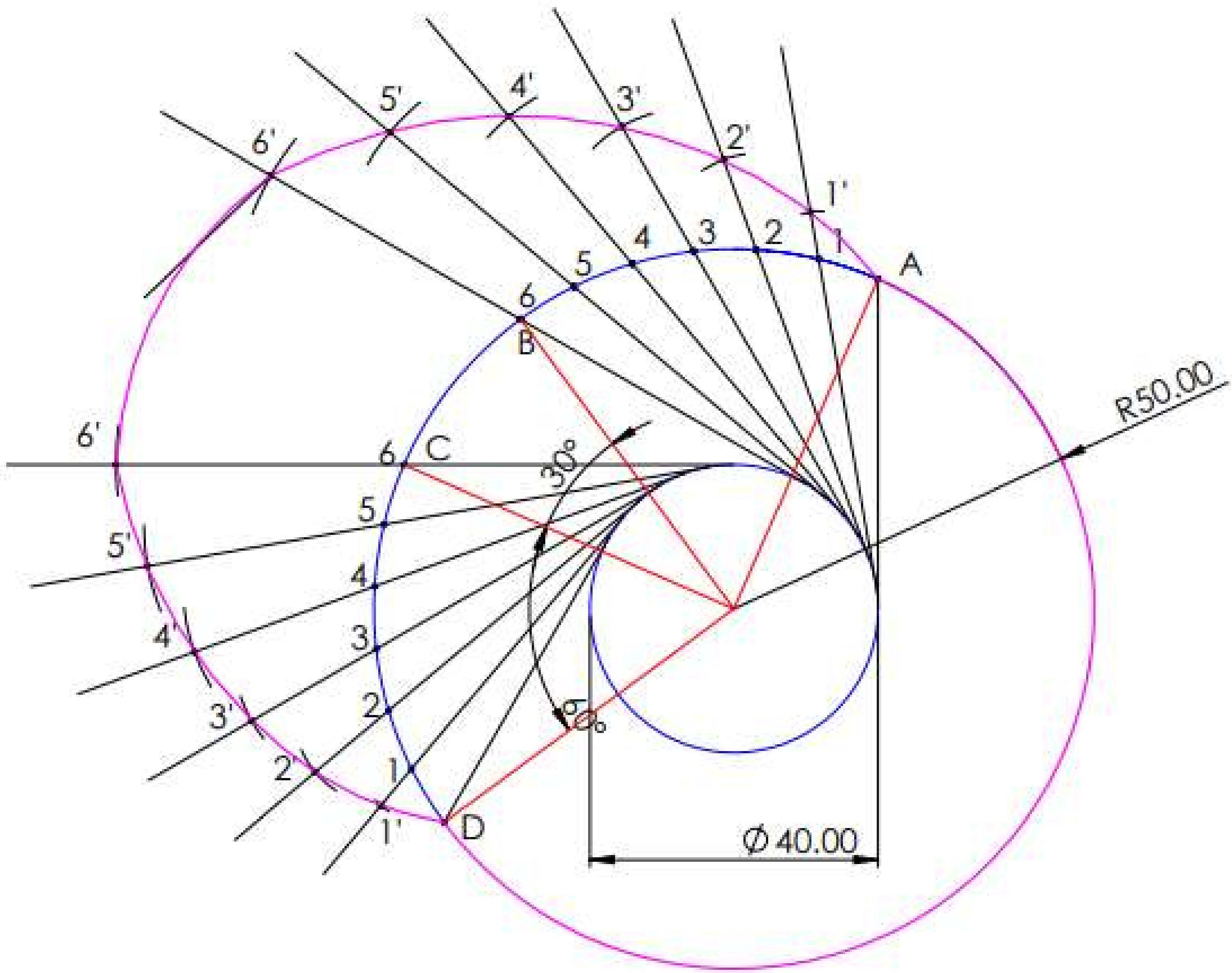
Step 5: Draw straight lines tangent to the offset circle passing through the divided points as shown for both rise and fall angles. Name the lines in compliance with the displacement diagram.

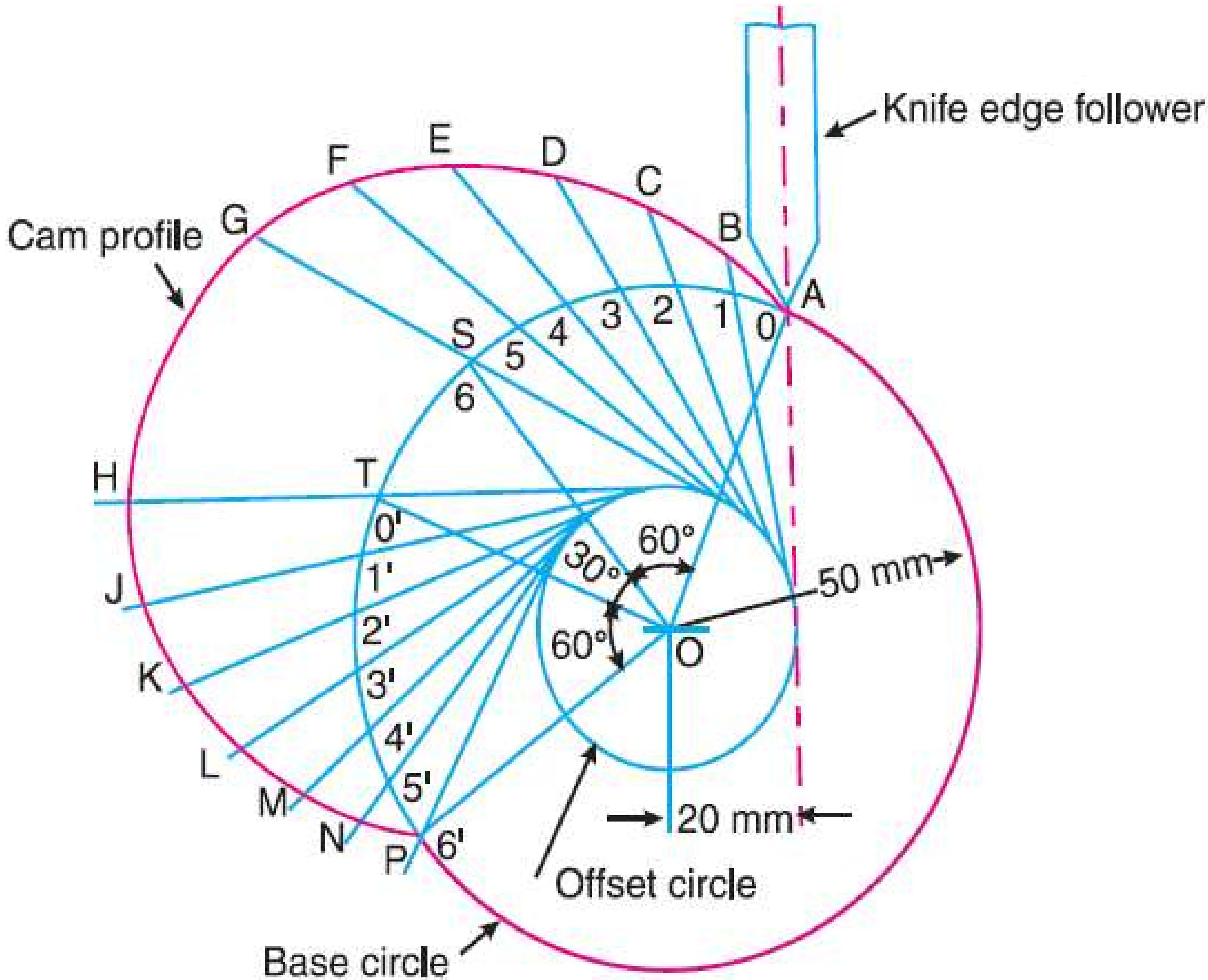


Step 6: Measure the distance 1-1′, 2-2′, 3-3′ so on from displacement diagram using compass and make an arc in the corresponding line as shown.



Step 7: Join all the points (A, 1′, 2′, 3′…..) to obtain the required cam profile.





Knife edge follower
Cam profile
E
F
D
C
G
B
A
3
2
1
0
4
S
5
6
H
T
0'
60°
30°
50 mm
J
1'
2'
60°
O
K
3'
L
4'
M
5'
20 mm
N
6'
P
Offset circle
Base circle

Problem 2: Simple Harmonic Motion – Roller Follower

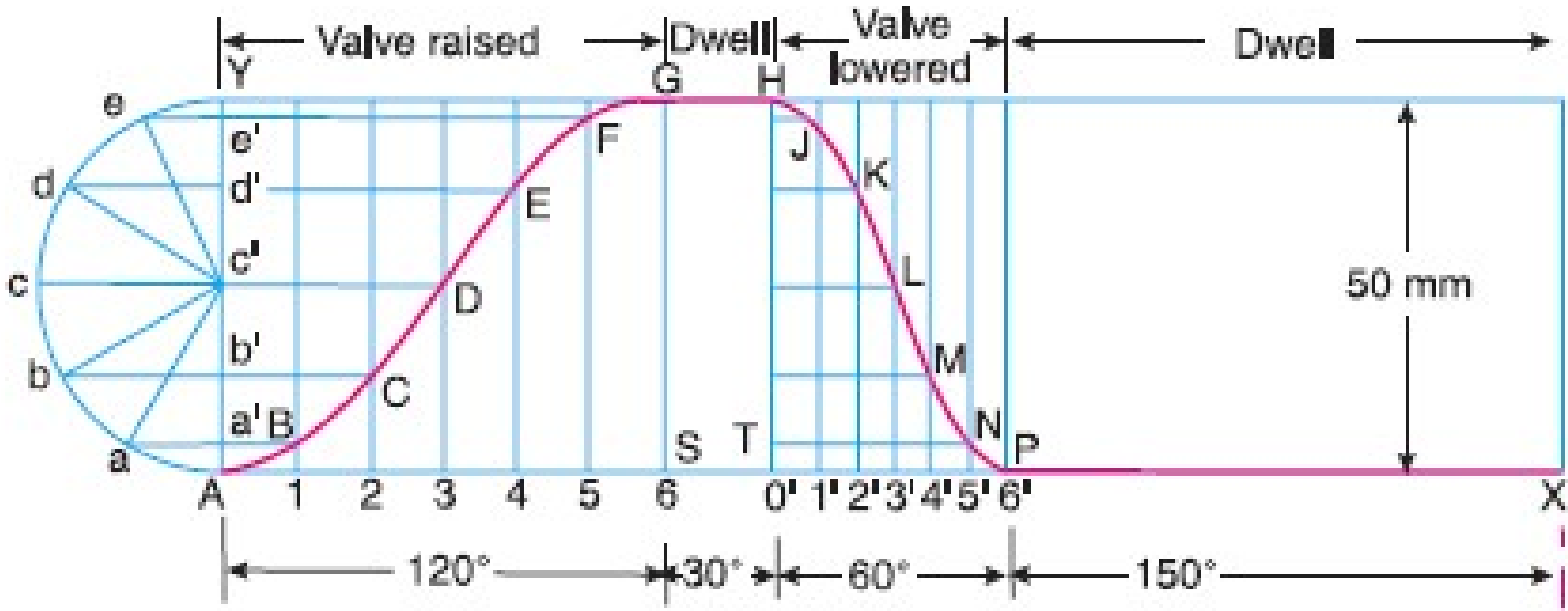
A cam, with a minimum radius of 25 mm, rotating clockwise is to be designed to give a roller follower, at the end of a valve rod, motion described below:

1. To raise the valve through 50 mm during 120° rotation of the cam;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm. Draw the profile of the cam when

(a) the line of stroke of the valve rod passes through the axis of the cam shaft, and

(b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with **simple harmonic motion**.

Displacement Diagram

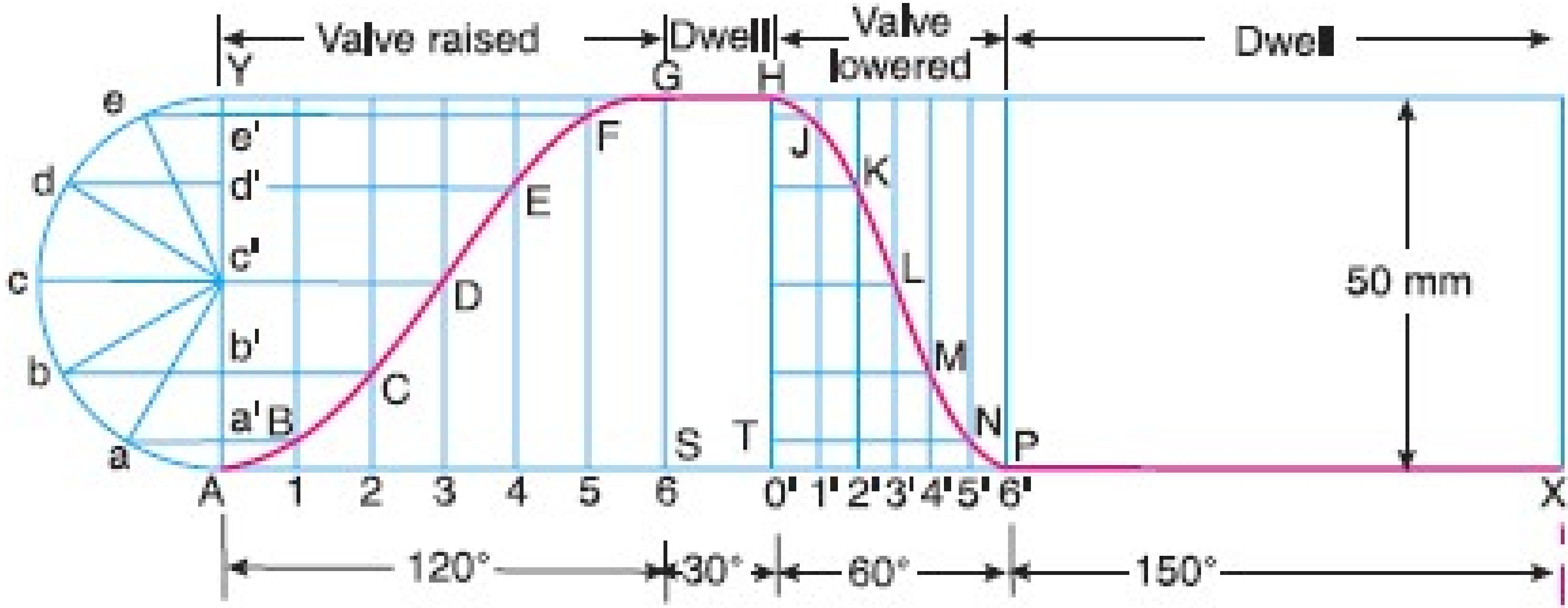


1. Draw horizontal line AX = 360° to some suitable scale. On this line, mark AS = 120° to represent out stroke ; ST = 30° to represent dwell ; TP = 60° to represent return stroke and PX = 150° to represent dwell.

2. Draw vertical line AY = 50 mm to represent the cam lift or stroke of the follower and complete the rectangle.

3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.

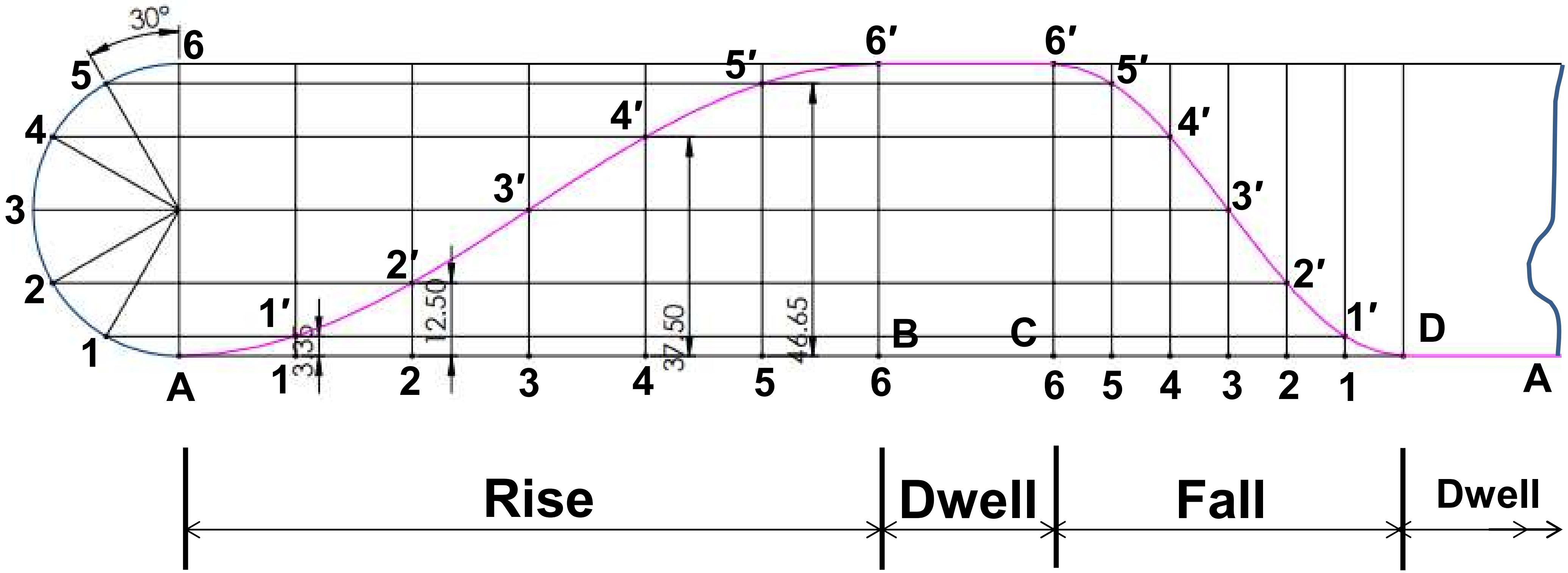
Displacement Diagram



4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with *AY* as diameter and divide into six equal parts.

5. From points *a*, *b*, *c* ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and 0′ ,1′ , 2′ ...etc. at *B*, *C*, *D* ... *M*, *N*, *P*.

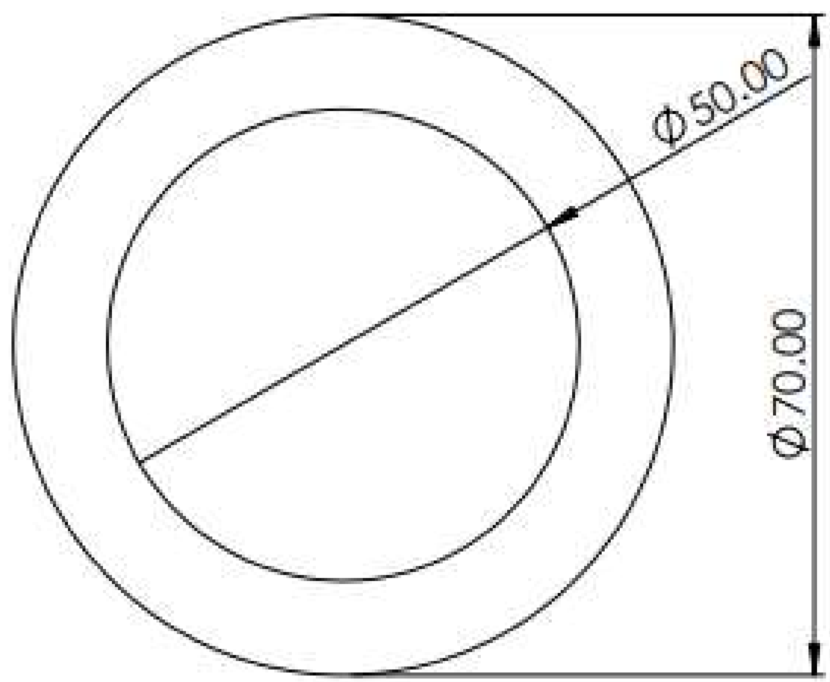
6. Join the points *A*, *B*, *C* ... etc. with a smooth curve. This is the required displacement diagram.



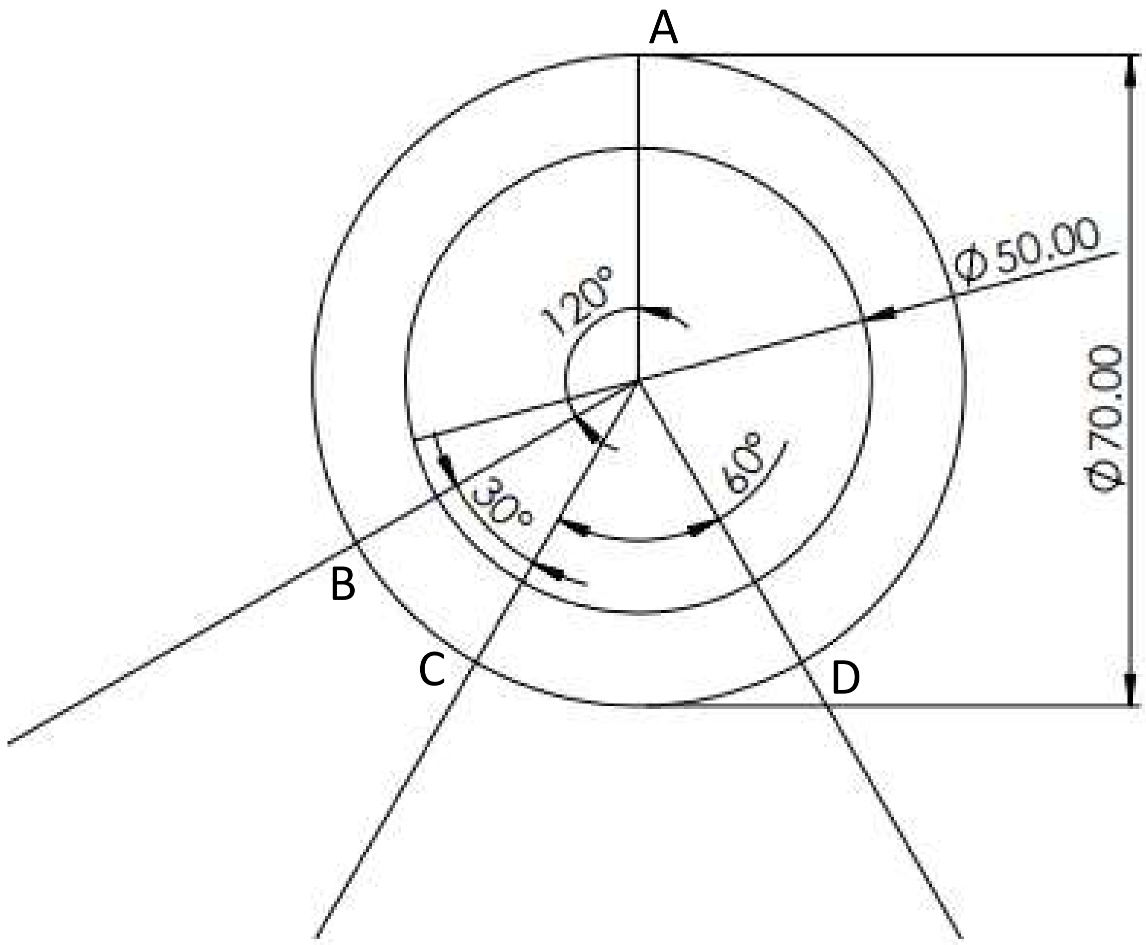
30°
6
5
4
3
2
1
A
1′
2′
3′
4′
5′
6′
6′
5′
4′
3′
2′
1′
B
C
D
3.35
12.50
37.50
46.65
1
2
3
4
5
6
6
5
4
3
2
1
A
Rise
Dwell
Fall
Dwell

Cam Profile (without offset) - Procedure

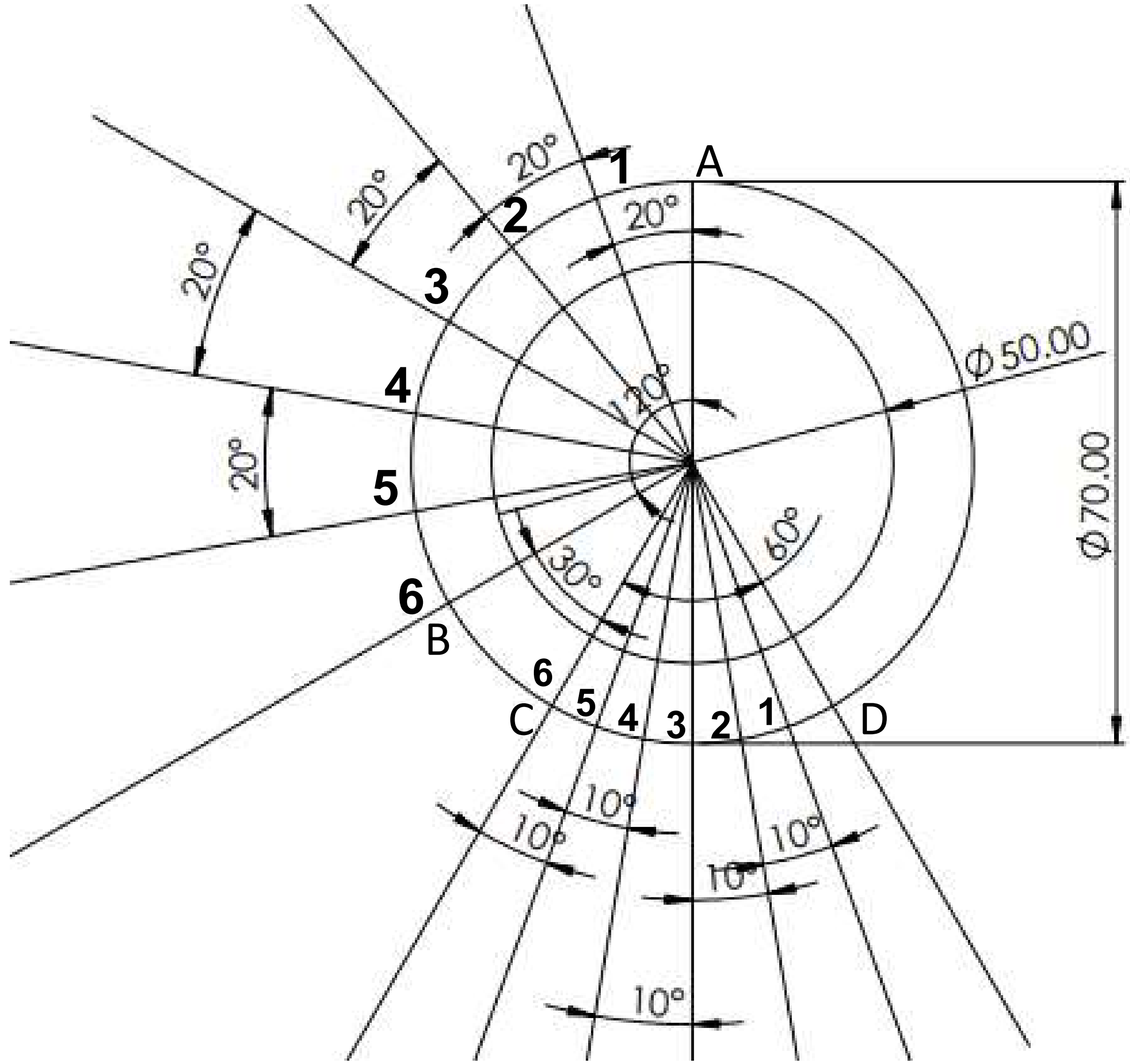
Step 1: Draw the base circle with the radius of 25 mm and the prime circle (= base circle radius + roller radius) with the radius 35 mm.



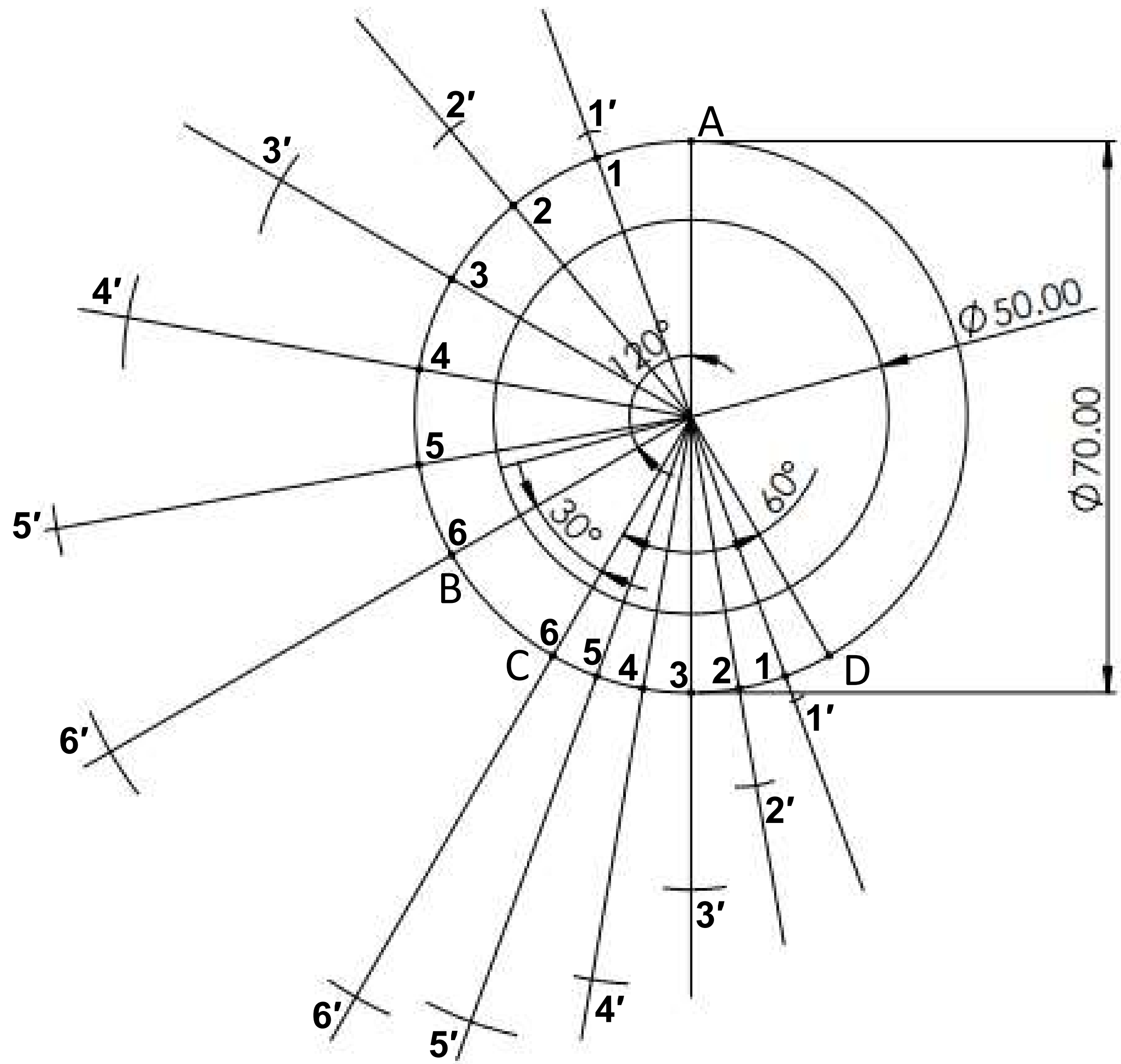
Step 2: Draw a vertical line (axis of follower) from centre and divide the prime circle for the given degrees of rise, dwells and fall.



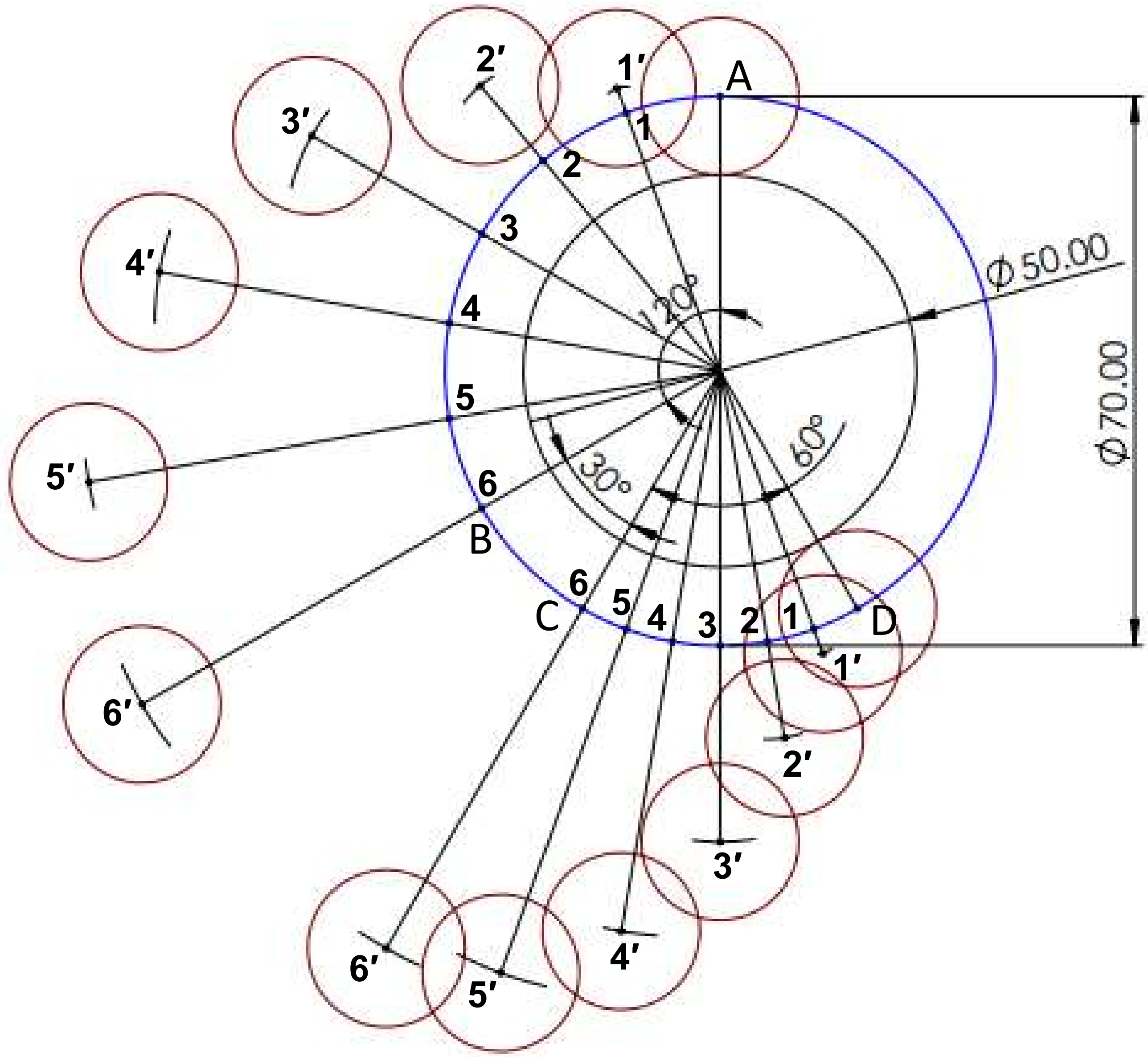
Step 3: Divide the rise and fall angles in to equal divisions as it is done in the displacement diagram and draw straight line passing through the divided points from centre. Name the lines in compliance with the displacement diagram.



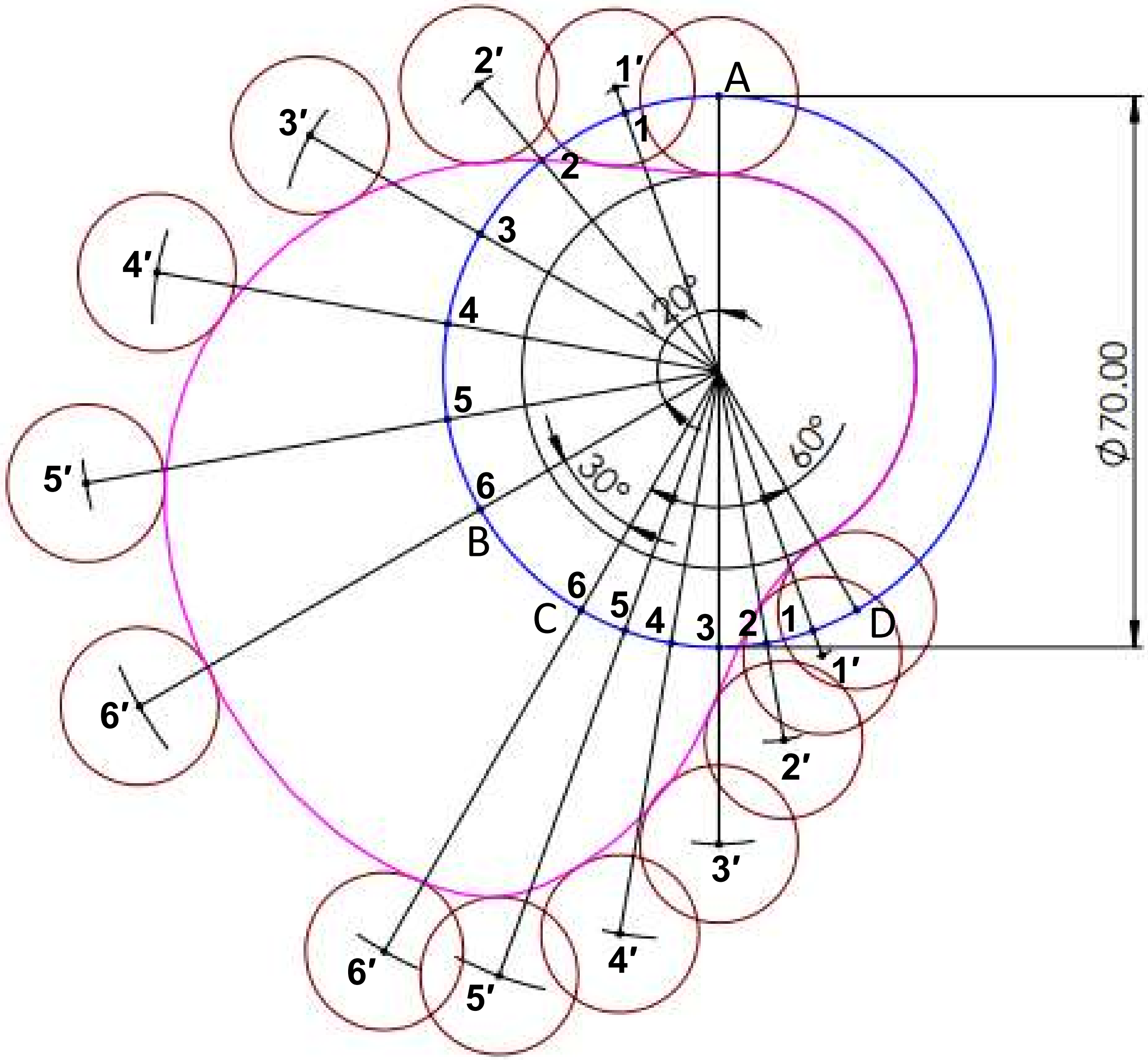
Step 4: Measure the distance 1-1′, 2-2′, 3-3′ so on from displacement diagram using campus and make an arc in the corresponding line.

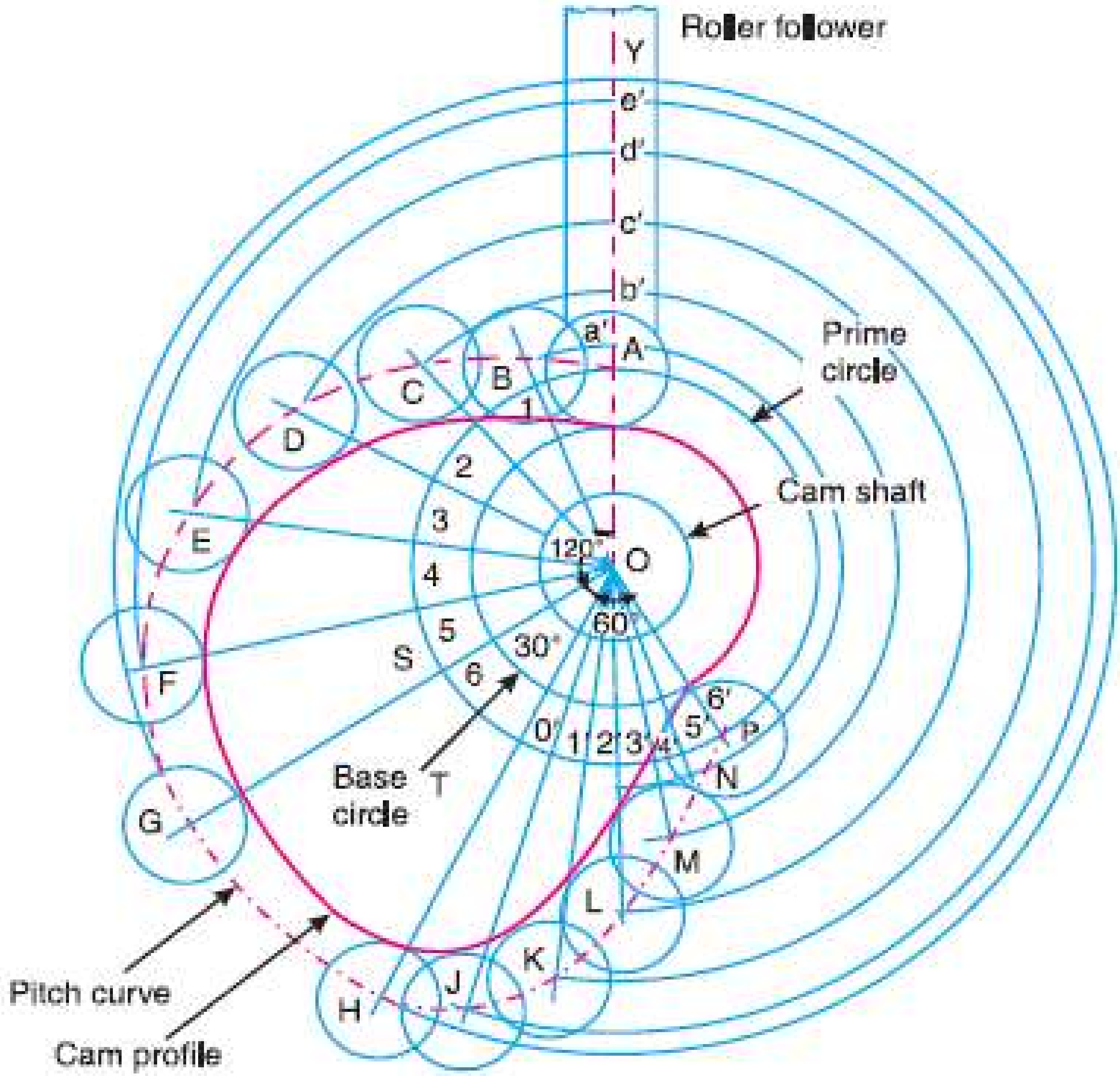


Step 4: From the marked points (A, 1′, 2′, 3′…..), draw circle with the roller diameter of 20 mm.



Step 5: Draw tangent curve through the circles to obtain the required cam profile.

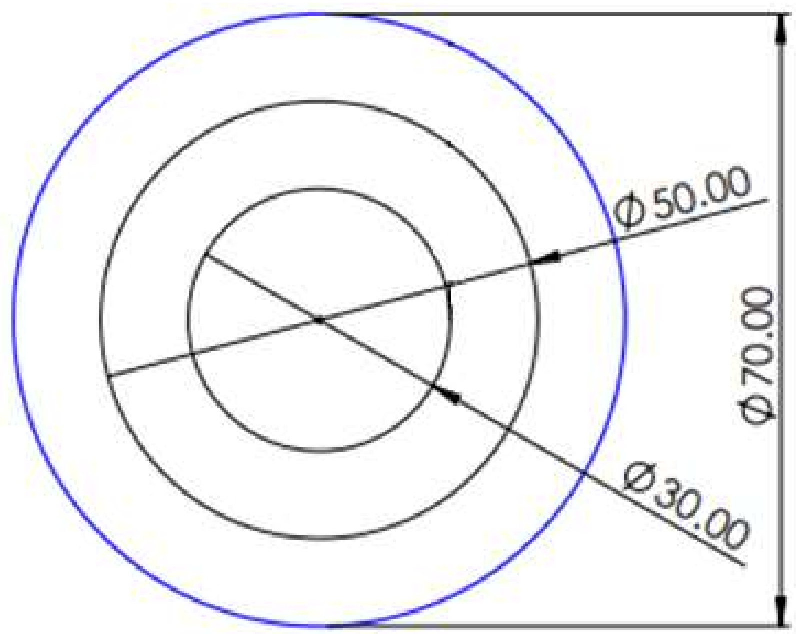




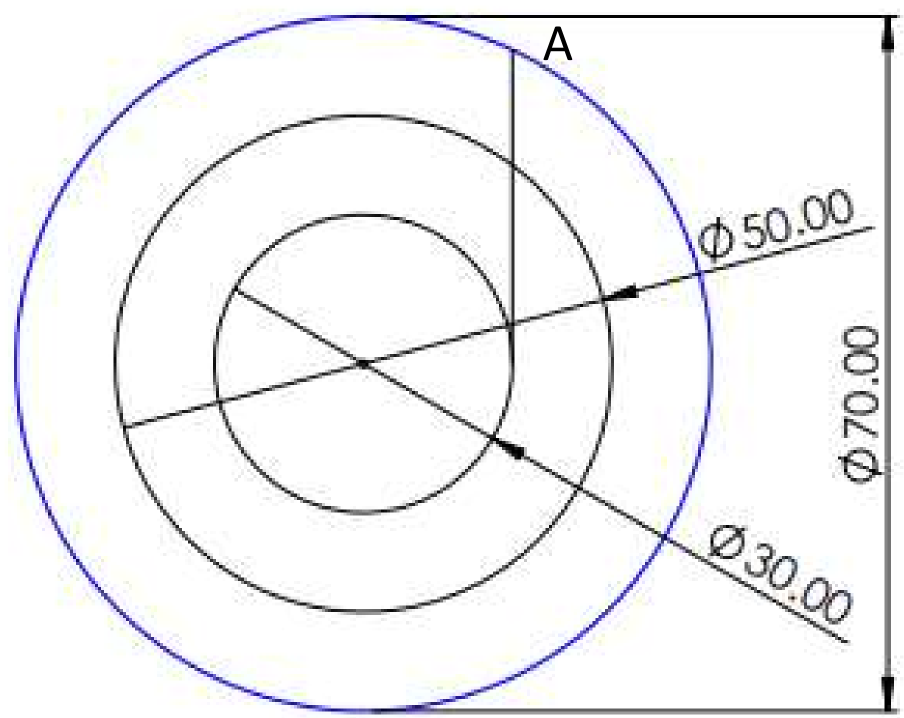
Roller follower
Y
e'
d'
c'
b'
a'
A
B
C
D
E
F
G
H
J
K
L
M
N
P
1
2
3
4
5
6
6'
5'
0'
1'
2'
3'
4'
120°
60°
30°
O
S
T
Prime circle
Cam shaft
Base circle
Pitch curve
Cam profile

Cam Profile (with offset) - Procedure

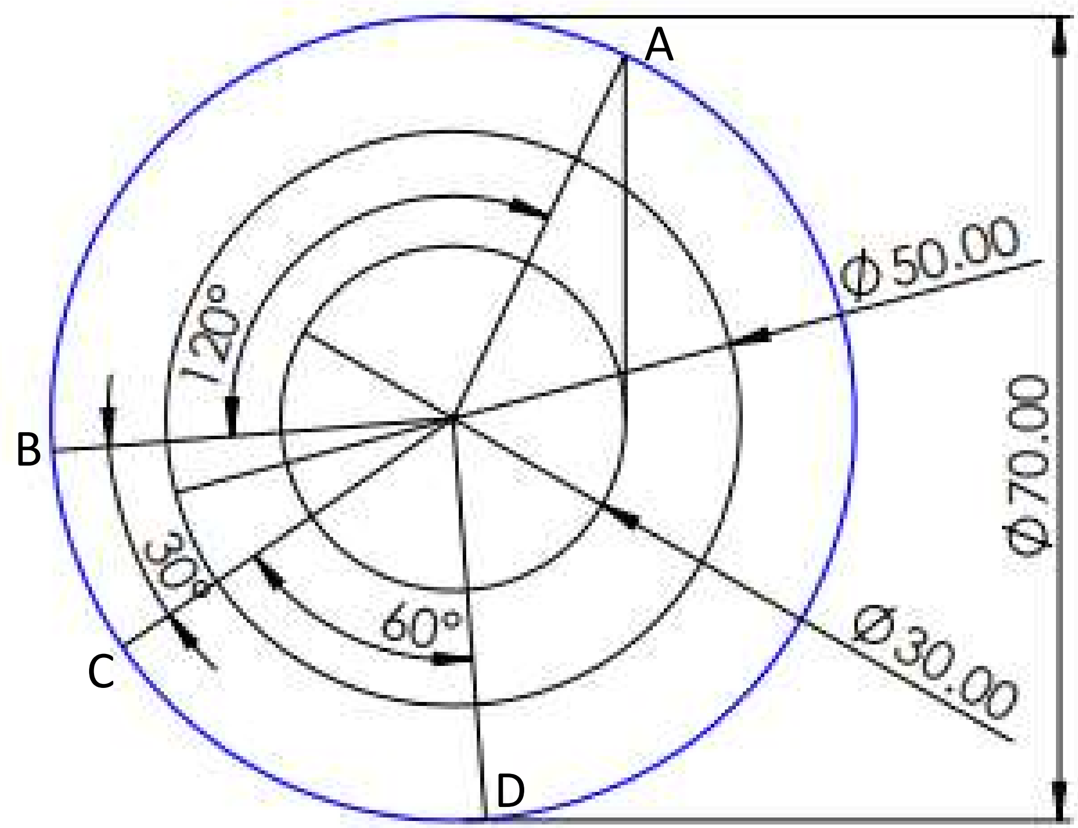
Step 1: Draw the base circle with the radius of 25 mm and the prime circle (= base circle radius + roller radius) with the radius 35 mm. and draw another circle with the radius equal to the offset (15 mm) from the same centre.



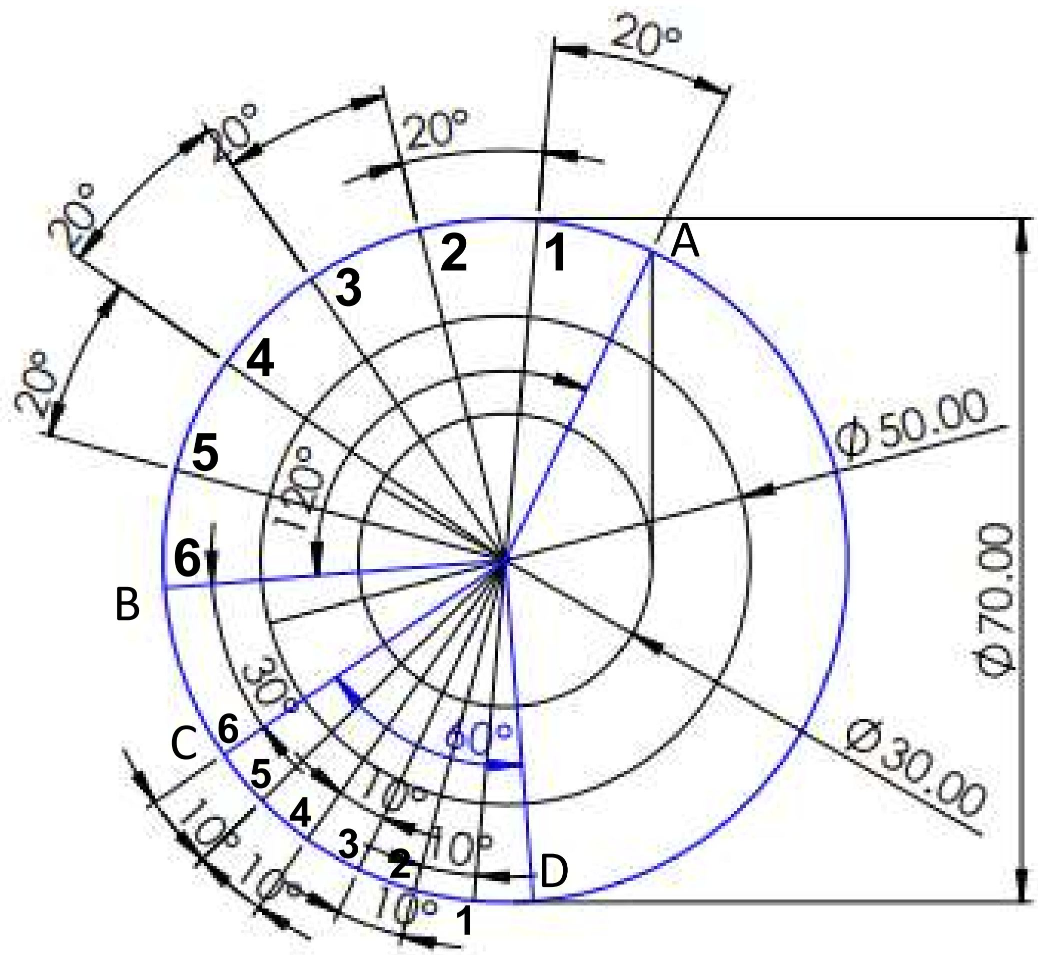
Step 2: Draw a vertical line (axis of follower) tangent to the offset circle and mark A as shown.



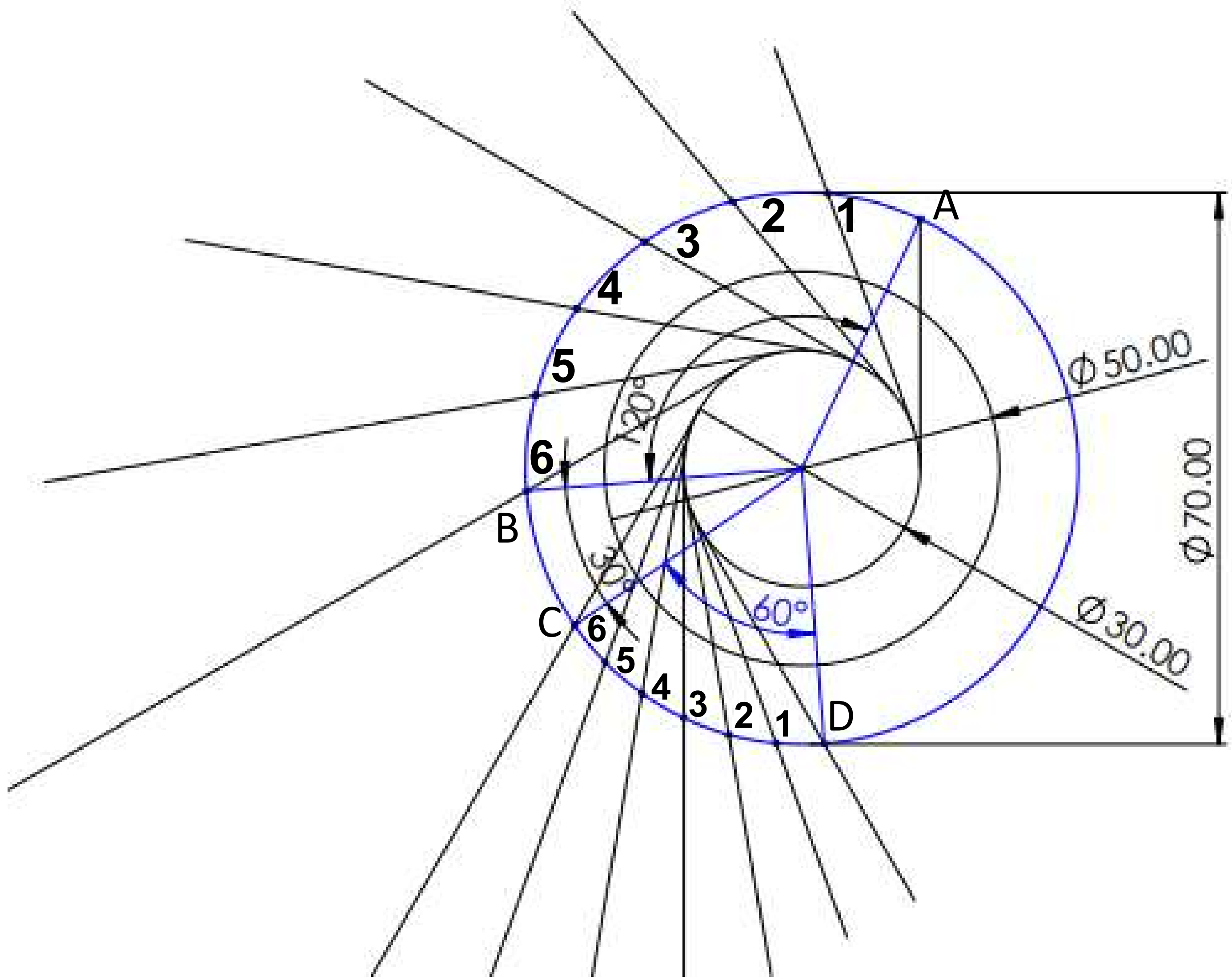
Step 3: Draw a line from centre to join A. Divide the circle for the given degrees of rise, dwell and fall as shown. Divide the rise and fall angles in to equal divisions as it is done in the displacement diagram



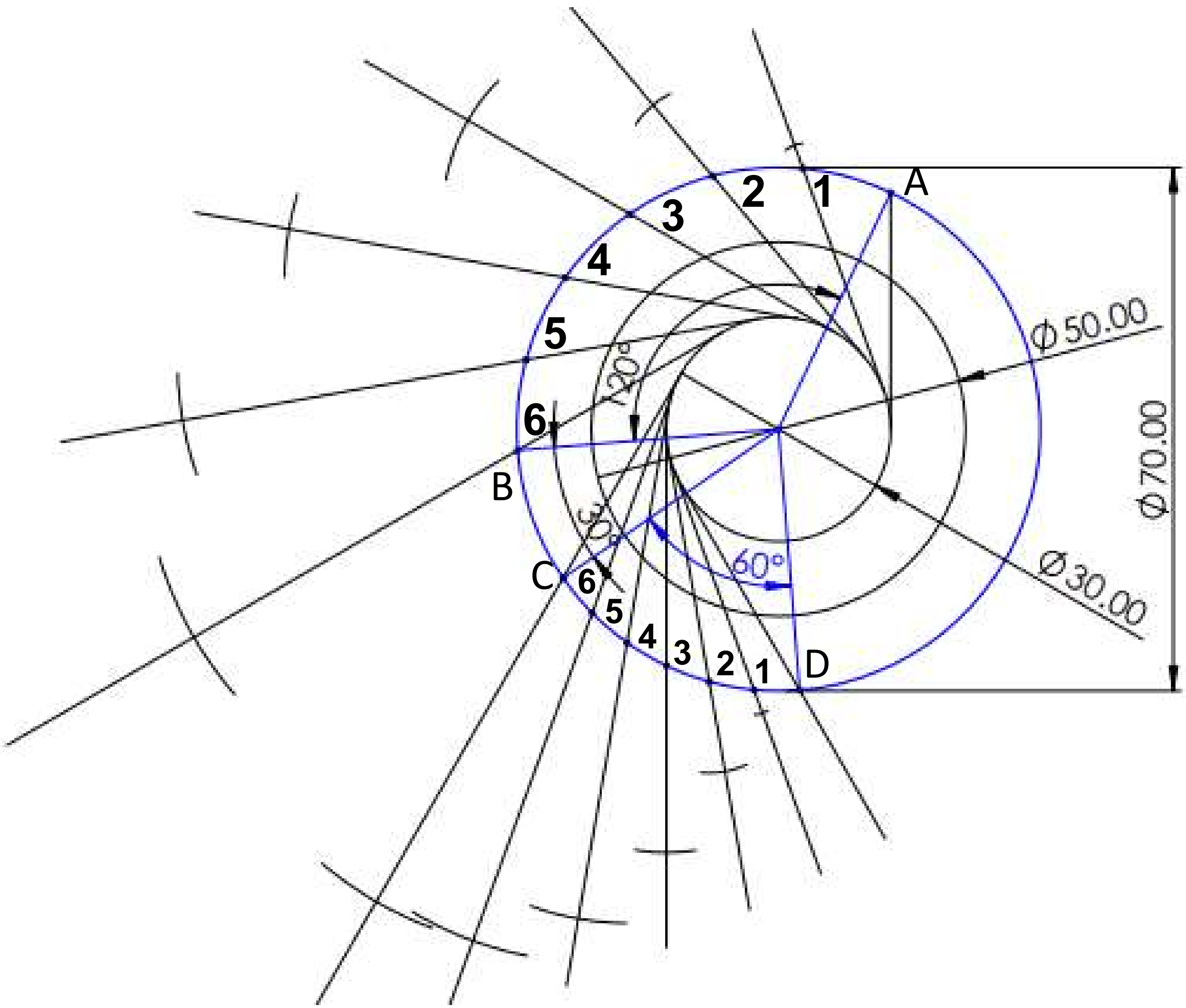
Step 4: Divide the rise and fall angles in to six equal divisions and mark 1, 2, 3, …. on the circumference of the prime circle.



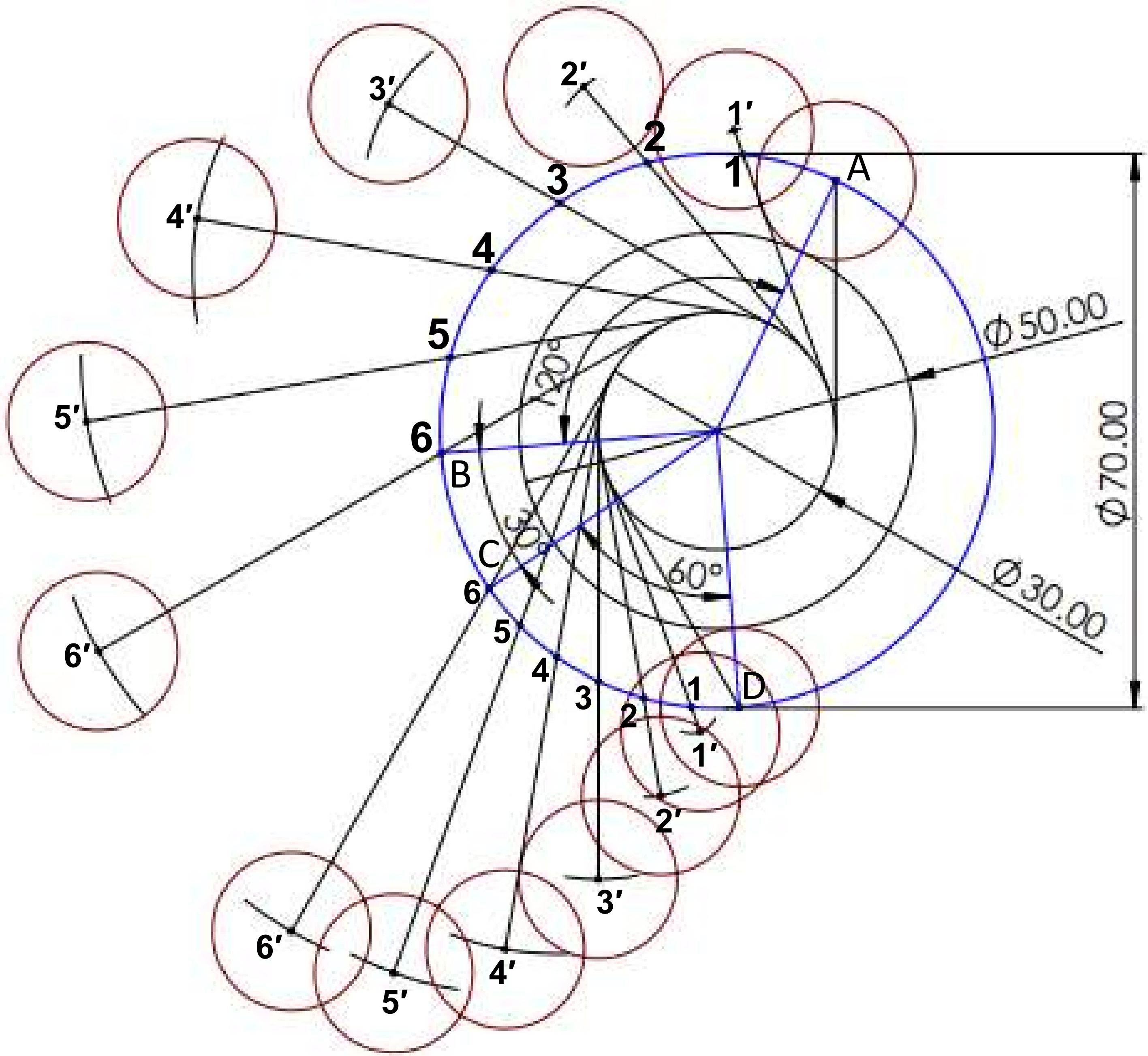
Step 5: Draw straight lines tangent to the offset circle passing through the divided points as shown for both rise and fall angles. Name the lines in compliance with the displacement diagram.



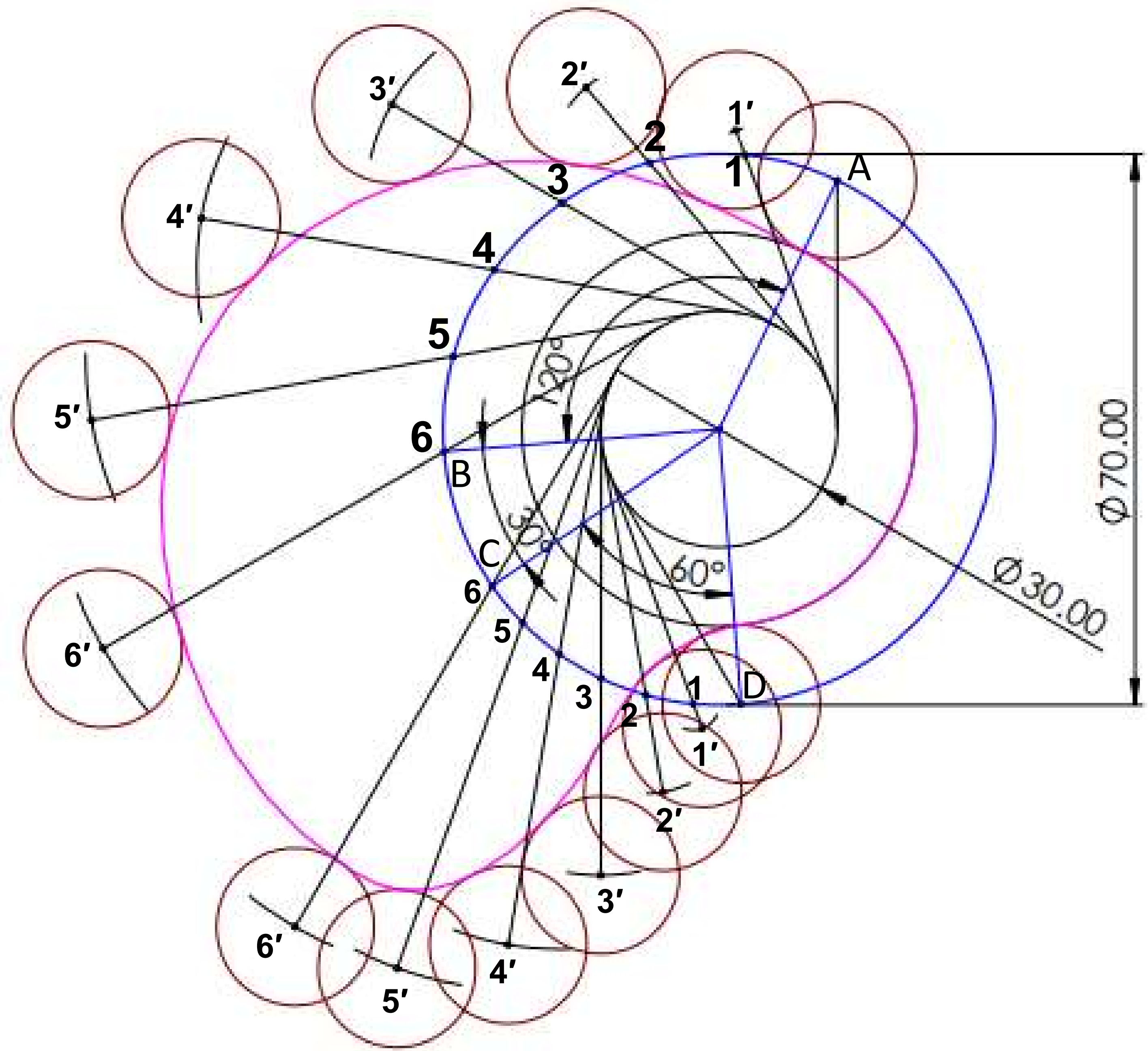
Step 6: Measure the distance 1-1′, 2-2′, 3-3′ so on from displacement diagram using compass and make an arc in the corresponding line as shown.

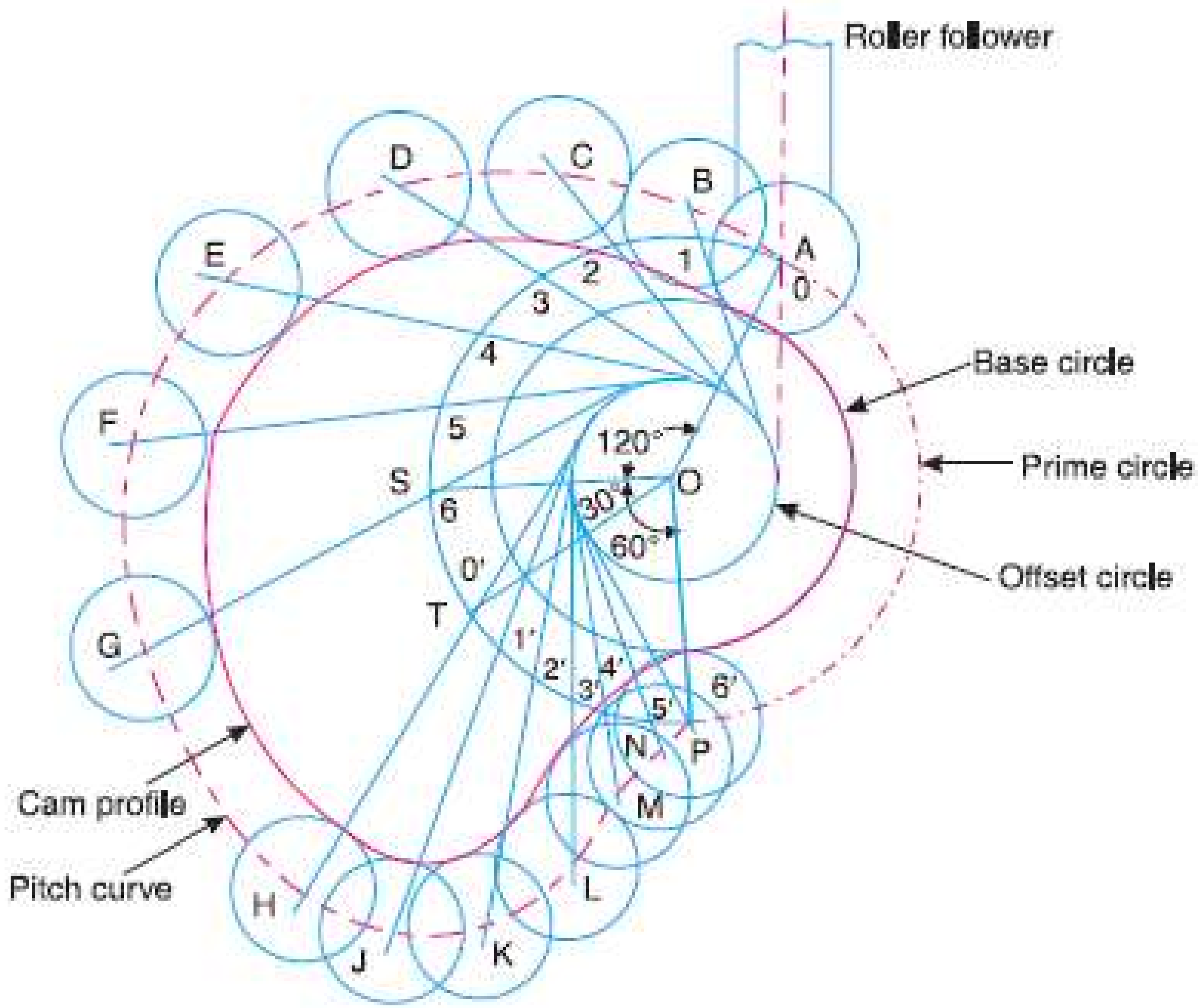


Step 7: From the marked points (A, 1′, 2′, 3′…..), draw circle with the roller diameter of 20 mm.



Step 8: Draw tangent curve through the circles to obtain the required cam profile.





Roller follower
A
B
C
D
E
F
G
H
J
K
L
M
N
P
S
T
O
0
1
2
3
4
5
6
0'
1'
2'
3'
4'
5'
6'
120°
30°
60°
Base circle
Prime circle
Offset circle
Cam profile
Pitch curve

Problem 3: Uniform acceleration and retardation Motion – Knife Edge Follower

A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below:

1. To move outwards through 40 mm during 100° rotation of the cam;

2. To dwell for next 80°;

3. To return to its starting position during next 90°, and

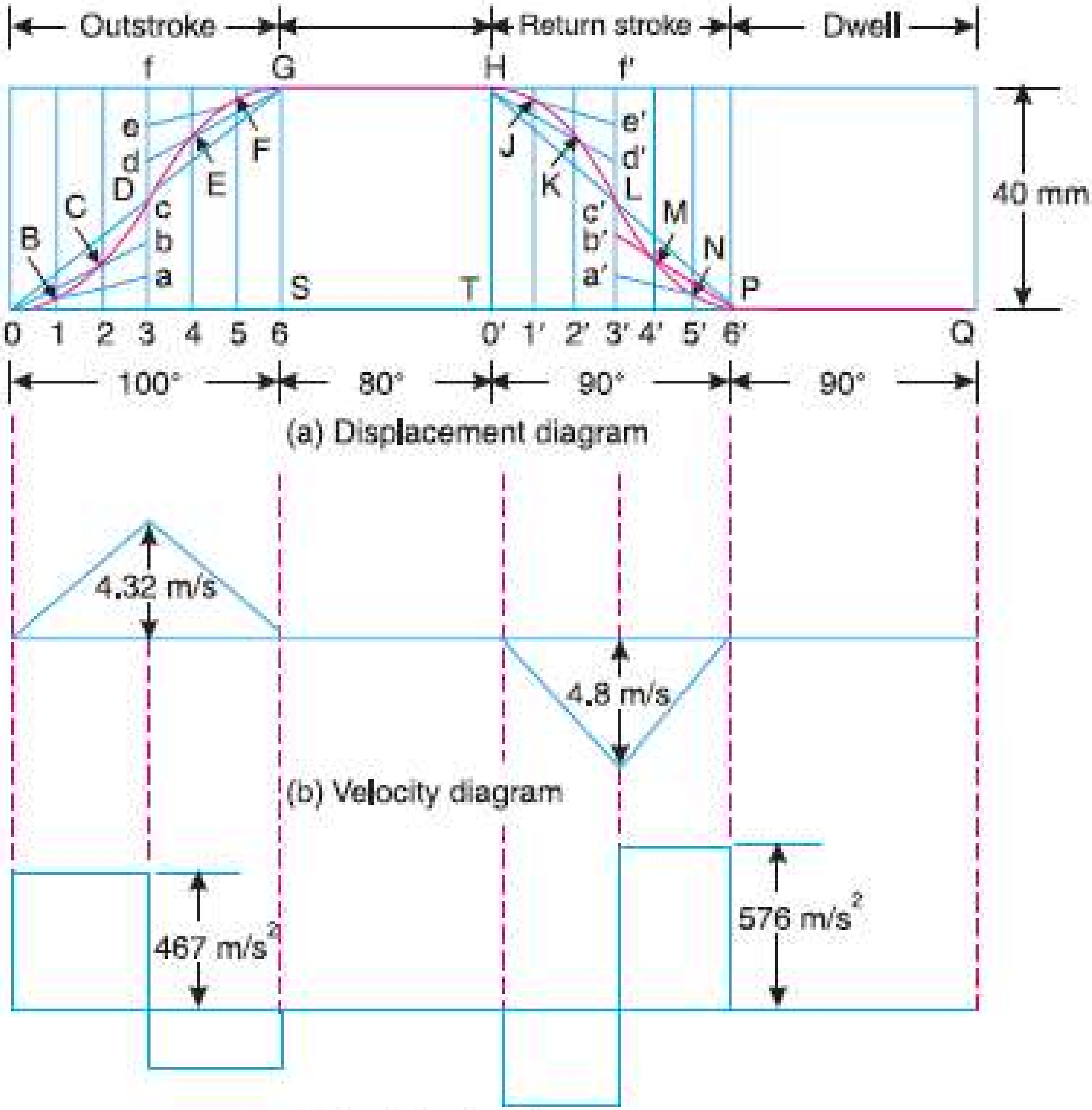
4. To dwell for the rest period of a revolution i.e. 90°.

Draw the profile of the cam

(i) when the line of stroke of the follower passes through the center of the cam shaft, and

(ii) when the line of stroke of the follower is off-set by 15 mm.

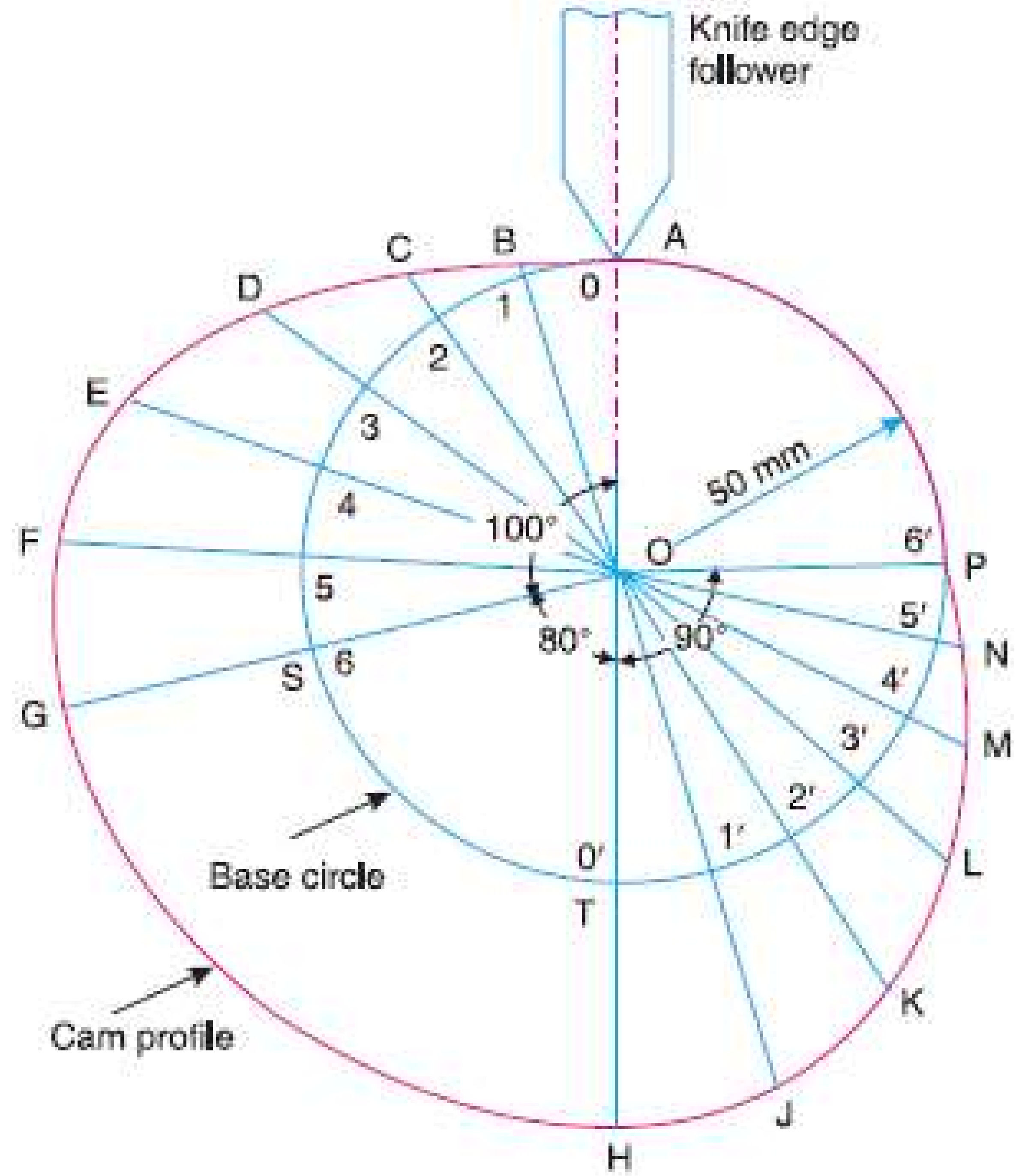
The displacement of the follower is to take place with **uniform acceleration and uniform retardation**.



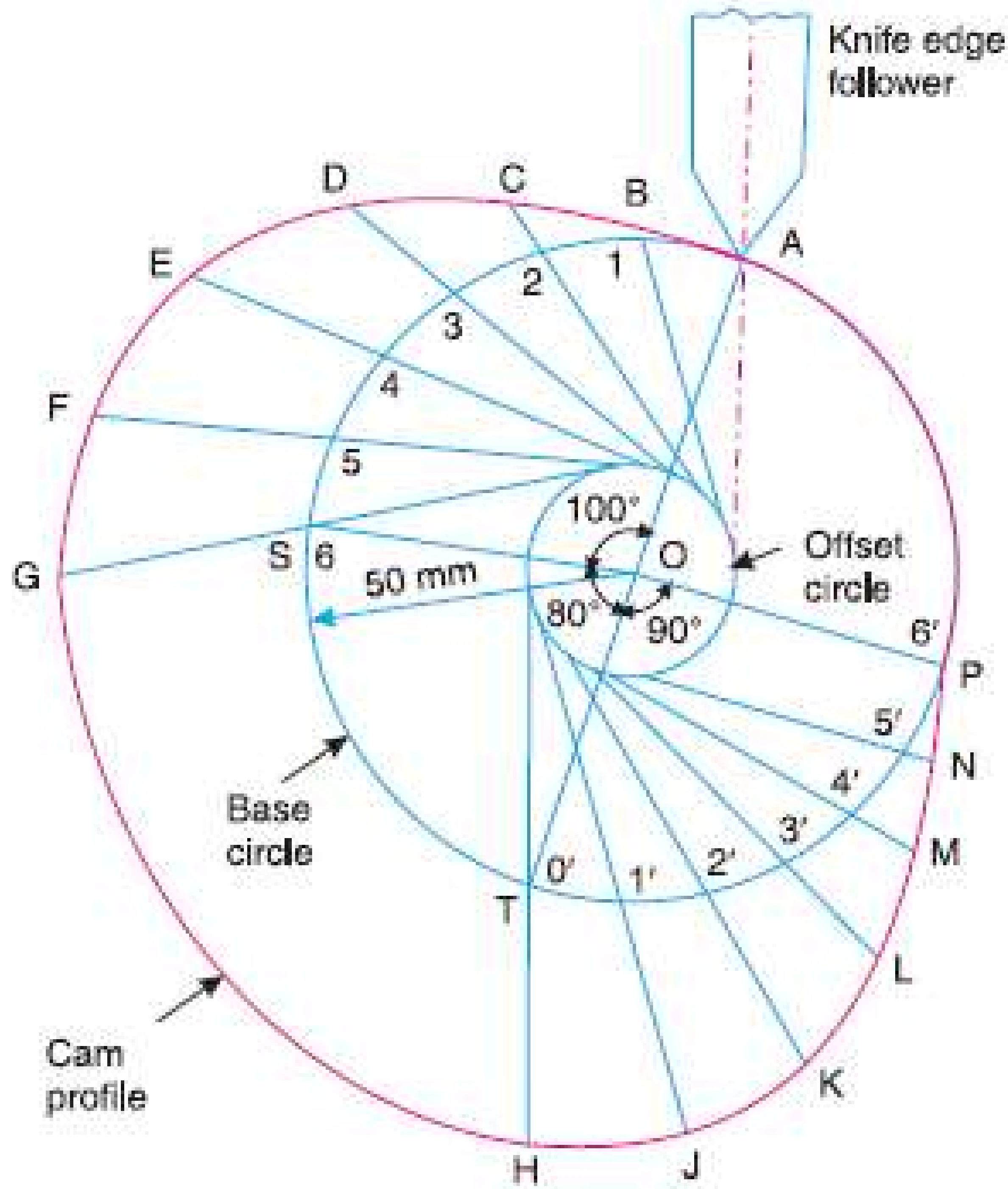
(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram



Knife edge follower
A
B
C
D
E
F
G
H
J
K
L
M
N
P
O
S
T
0
1
2
3
4
5
6
0′
1′
2′
3′
4′
5′
6′
50 mm
100°
80°
90°
Base circle
Cam profile



Knife edge follower
A
B
C
D
E
F
G
H
J
K
L
M
N
P
S
T
O
1
2
3
4
5
6
0'
1'
2'
3'
4'
5'
6'
100°
80°
90°
50 mm
Offset circle
Base circle
Cam profile

Problem 4: Cycloidal Motion – Roller Follower

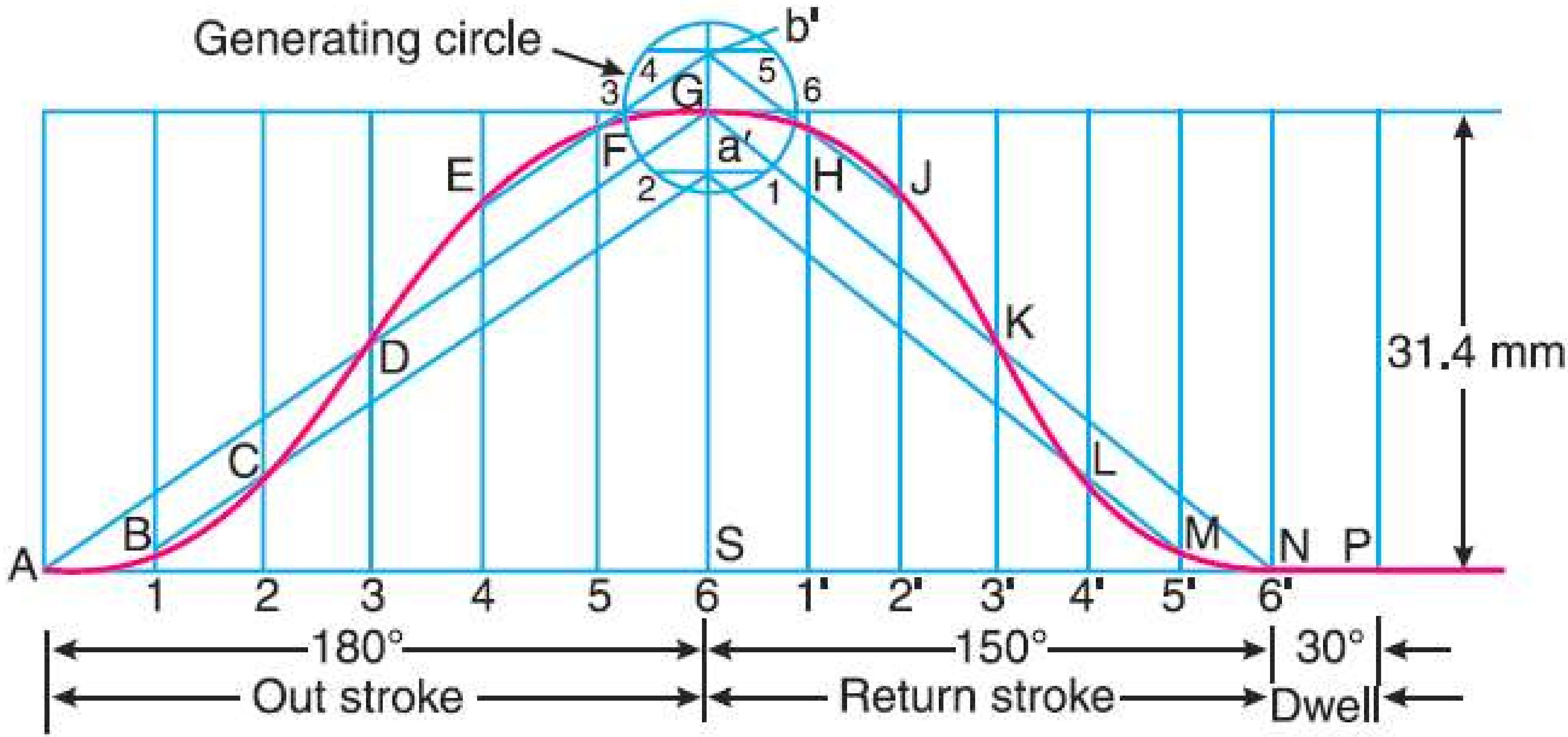
Draw the profile of the cam when the roller follower moves with cycloidal motion during out stroke and return stroke, as given below :

1. Out stroke with maximum displacement of 31.4 mm during 180° of cam rotation,

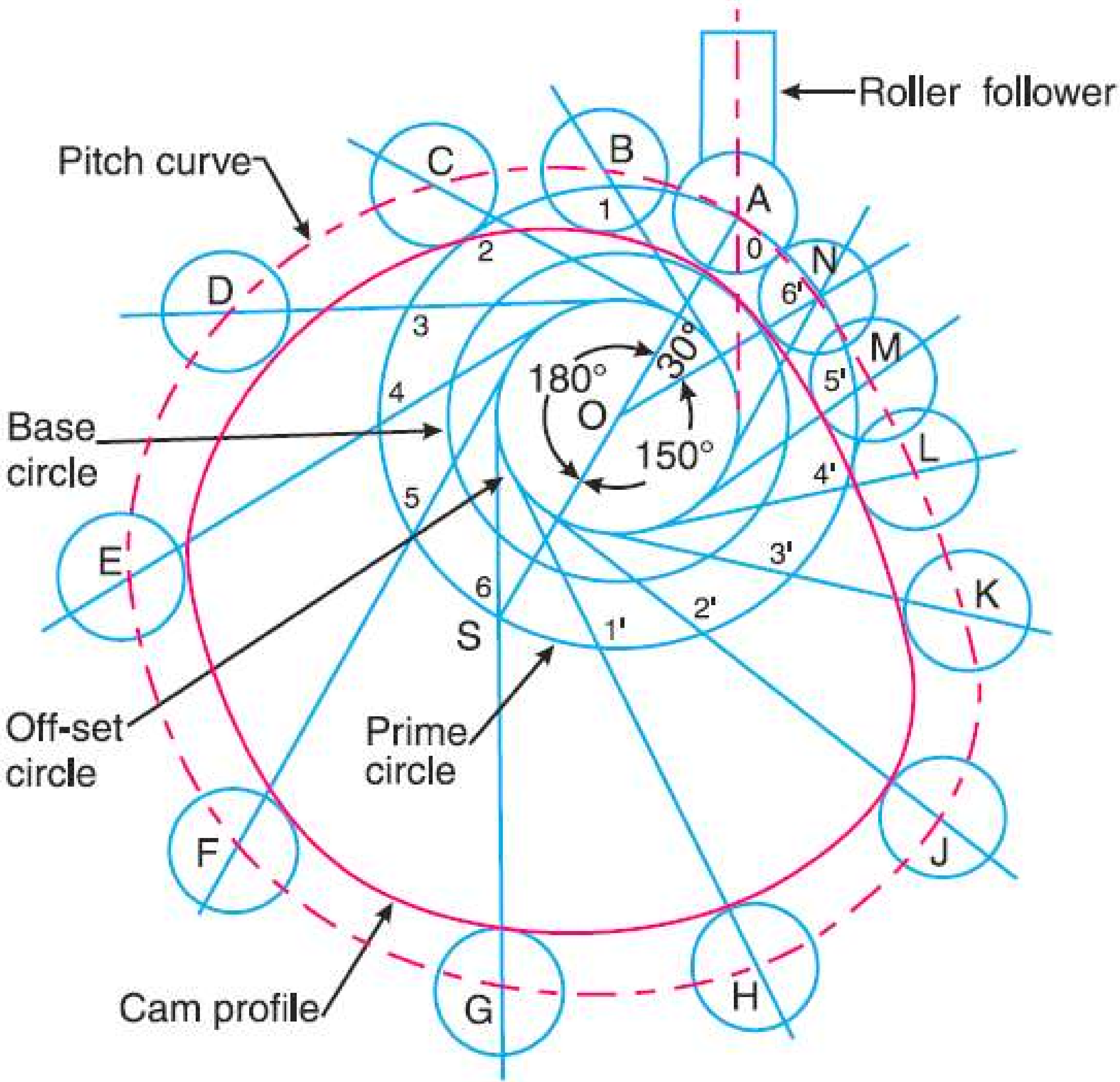
2. Return stroke for the next 150° of cam rotation,

3. Dwell for the remaining 30° of cam rotation.

The minimum radius of the cam is 15 mm and the roller diameter of the follower is 10 mm. The axis of the roller follower is offset by 10 mm towards right from the axis of cam shaft.



$$Generating\ Circle\ Radius = \frac{Stroke}{2\pi} = \frac{31.4}{2\pi} = 5\ mm$$



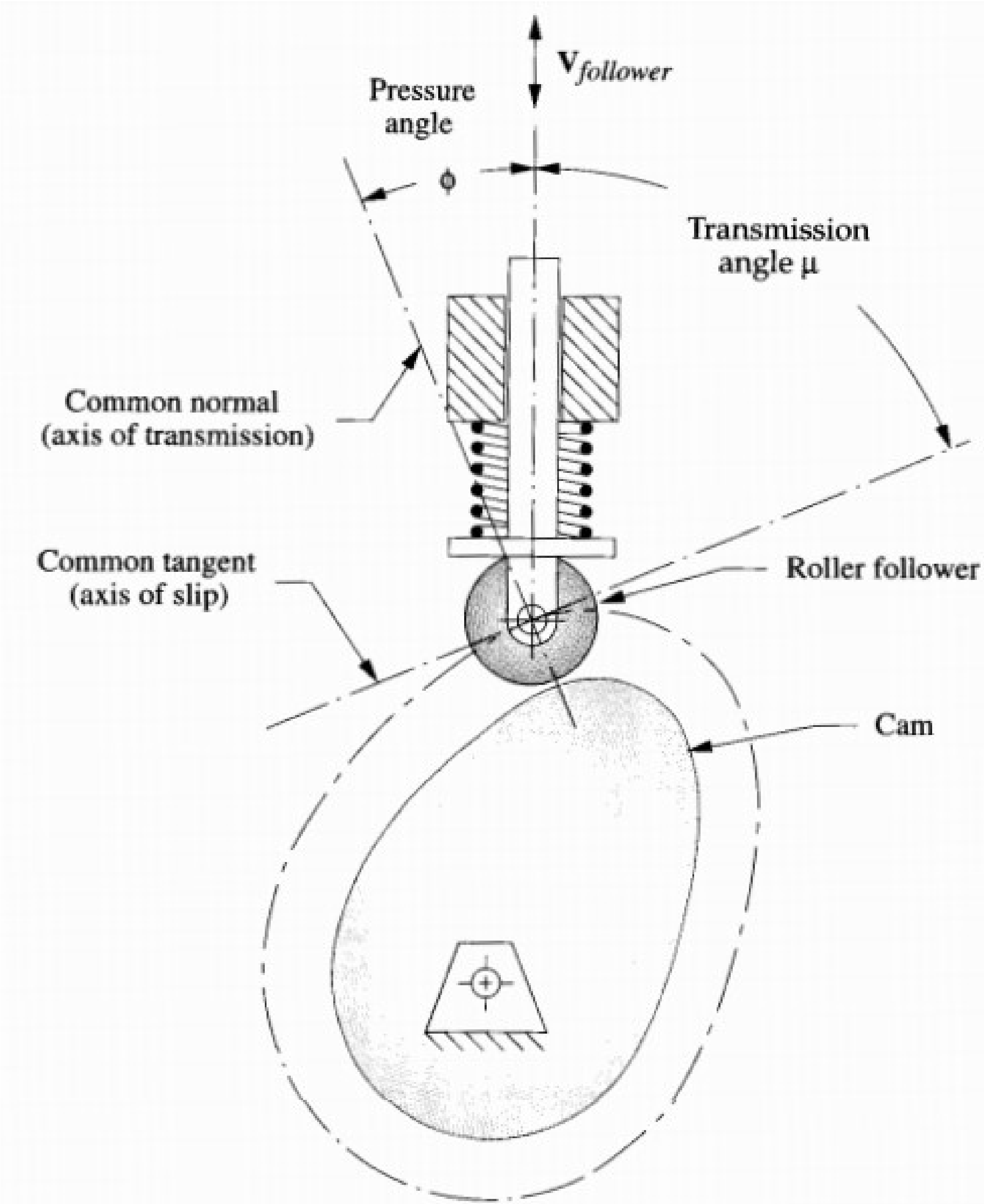
Roller follower
Pitch curve
Base circle
Off-set circle
Prime circle
Cam profile
A
B
C
D
E
F
G
H
J
K
L
M
N
O
S
180°
30°
150°
0
1
2
3
4
5
6
1'
2'
3'
4'
5'
6'

Problem 5: Cycloidal Motion – Roller Follower

Construct the profile of a cam to suit the following specifications :

Least radius of cam = 25 mm; Diameter of roller = 25 mm; Angle of lift = 120°; Angle of fall = 150°; Lift of the follower = 40 mm; Number of pauses are two of equal interval between motions. During the lift, the motion is **S.H.M.** During the fall the motion is **uniform acceleration and deceleration**. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5 mm from the center of the cam.

CAM Sizing



- Cam sizing is based on the pressure Angle and radius of curvature
- Both of these involve either base circle radius or prime circle radius (roller follower)
- Pressure angle (φ) is the angle between the direction of motion of the follower and the axis of transmission
- Force can only be transmitted from cam to the follower or vice-versa along the axis of transmission

- When $\varphi = 0$, all the transmitted force goes into the motion of the follower
- When $\varphi = 90°$, there will be no motion of the follower
- As a rule of thump, the pressure angle is varied between 0° and 30° to avoid excessive side load of the follower
- The pressure angle greater than 30° may tend to jam the translating follower in its guides
- If a suitable cam cannot be obtained with acceptable pressure angle, then eccentricity can be introduced to change the pressure angle and create a smoother running cam