Tutorial 1

Yield Criteria for Ductile materials

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1. Von - Mises Criteria:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6k^2 = 2Y^2$$

$$k = \frac{Y}{\sqrt{3}}$$

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6({\tau_{xy}}^2 + {\tau_{yz}}^2 + {\tau_{zx}}^2) = 6k^2 = 2Y^2$$

1. Tresca Criteria:

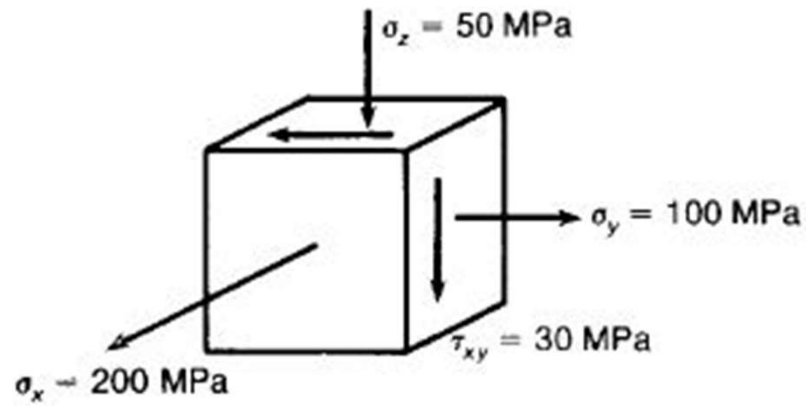
$$\sigma_1 - \sigma_3 = 2k = Y$$

Question 1

Stress analysis of a spacecraft structural member gives the state of stress shown below. If the part is made from 7075-16 aluminum alloy with Y = 500 MPa,

(i) will it exhibit yielding? If not, what is the safety factor?

(ii) Assume all stress components to be maintained, except for the stress in the x-direction, which is varied. Calculate the level of this stress component upon onset of yielding of the material.



Given: $\sigma_x = 200$ MPa, $\sigma_y = 100$ MPa, $\sigma_z = -50$ MPa, $\tau_{xy} = 30$ MPa,

$Y = 500$ MPa

i) From von-Mises criteria,

$$2Y^2 = (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

$$2Y^2 = (200 - 100)^2 + (100 - (-50))^2 + (-50 - 200)^2 + 6(30^2)$$

$$2Y^2 = 100^2 + 150^2 + (-250)^2 + 6(30^2)$$

$$2Y^2 = 100400$$

$$Y = [100400/2]^{1/2}$$

$$Y = 224.05 \text{ MPa} < 500 \text{ MPa}$$

$Y_{exerted} < Y_{permissible}$ $\Rightarrow$ yielding will not occur

Factor of safety, $FoS = \frac{500}{224} = 2.23$

(ii) σ_x' at yielding condition

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = 2Y^2$$

$$(\sigma_x' - 100)^2 + (100 - (-50))^2 + (-50 - \sigma_x')^2 + 6(30^2) = 2(500)^2$$

$$(\sigma_x' - 100)^2 + 150^2 + (\sigma_x' + 50)^2 + 5400 = 500000$$

$$[\sigma_x'^2 - 200\sigma_x' + 100^2] + 150^2 + [\sigma_x'^2 + 100\sigma_x' + 50^2] + 5400 = 500000$$

$$2\sigma_x'^2 - 100\sigma_x' + (100^2 + 150^2 + 50^2 + 5400 - 500000) = 0$$

$$2\sigma_x'^2 - 100\sigma_x' - 459600 = 0$$

$$\sigma_x'^2 - 50\sigma_x' - 229800 = 0$$

$\sigma_x' = 505$ MPa (or) -455 MPa $\quad$ σ_x' is positive

Level of stress component along x-direction is 505 MPa upon onset of yielding of material.

Question 2

If the principal stresses on a material with a yield stress in shear of 200 MPa, are $\sigma 1$ = 175 MPa and $\sigma 2$ = 350 MPa, what tensile stress $\sigma 3$ must be applied to cause yielding according to the Tresca criterion?

Given:

Yield stress in shear, $K = 200$ MPa

$\sigma_1 = 175$ MPa, $\sigma_2 = 350$ MPa, $\sigma_3 = ?$

Tresca criteria

$$\sigma_1 - \sigma_3 = 2K$$

$$\sigma_3 = \sigma_1 - 2k$$

$$= 175 - 2(200)$$

$$\sigma_3 = -225 \text{ MPa}$$

A compressive stress of 225 MPa has to be applied to cause yielding according to Tresca criteria.

Question 3

A thin-wall tube with closed ends is subjected to a maximum internal pressure of 35 MPa in service. The mean radius of the tube is 30 cm.

(a) If the tensile yield strength is 700 MPa, find using Tresca's condition, what minimum thickness must be specified to prevent yielding?

(b) If the material has a yield strength in shear of k = 280 MPa, find using Tresca's condition, what minimum thickness must be specified to prevent yielding?

Given:

"thin-wall tube"

$p = 35$ MPa (maximum internal pressure)

$r = 30$ cm

Note: Stresses in closed tube

$$\sigma_1 = \frac{pr}{t} \quad \text{(tangential component)}$$

$$\sigma_2 = \frac{pr}{2t} \quad \text{(axial component)}$$

$$\sigma_3 \simeq 0 \quad \text{(radial component)}$$

(a) tensile yield strength, $Y = 700$ MPa

$t = ?$

$$Y = \sigma_1 - \sigma_3$$

$$Y = \frac{pr}{t} - 0$$

$$t = \frac{pr}{Y} = \frac{35 \text{ MPa} \times 30 \text{ cm}}{700 \text{ MPa}}$$

$$t = 1.5 \text{ cm}$$

(b) Yield strength in shear, $k = 280 \text{ MPa}$

$$2k = \sigma_1 - \sigma_3$$

$$2k = \frac{pr}{t} - 0$$

$$t = \frac{pr}{2k} = \frac{35 \text{ MPa} \times 30 \text{ cm}}{2 \times 280 \text{ MPa}}$$

$$t = 1.875 \text{ cm} \simeq 1.9 \text{ cm}$$

Question 4

Consider a 6-cm-diameter tube with 1-mm-thick wall with closed ends made from a metal with a tensile yield strength of 25 MPa. After applying a compressive load of 2,000 N to the ends, what internal pressure is required to cause yielding according to a) the Tresca criterion and b) the von Mises criterion?

Given: $d = 6$ cm $\Rightarrow r = 3$ cm $= 30$ mm, $t = 1$ mm, $Y = 25$ MPa,

$F = -2000$ N (axial), internal pressure, $p = P$

consider thin walled tube,

$$\sigma_1 = \frac{pr}{t} = \frac{P \times 30}{1} = \text{"}30P\text{"} \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} + \frac{F}{A} = 15P - \frac{2000}{\pi(60)(1)} = 15P - 10.61 \text{ MPa}$$

$$\sigma_3 \simeq 0$$

(a) Tresca criterion.

$$\sigma_1 - \sigma_3 = Y$$

$$30P = 25$$

$$P = 25/30 = 0.83 \text{ MPa}$$

(b) von-Mises criterion

$$2Y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$2(25)^2 = (30P - 15P + 10.61)^2 + (15P - 10.61 - 0)^2 + (0 - 30P)^2$$

$$1250 = (15P + 10.61)^2 + (15P - 10.61)^2 + 900P$$

$$1250 = 2(15P)^2 + 2(10.61)^2 + 900P$$

$$450P^2 + 900P - 1024.85 = 0$$

$$P^2 + 2P - 2.277 = 0$$

$$P = 0.81 \text{ MPa}$$

THANK YOU!