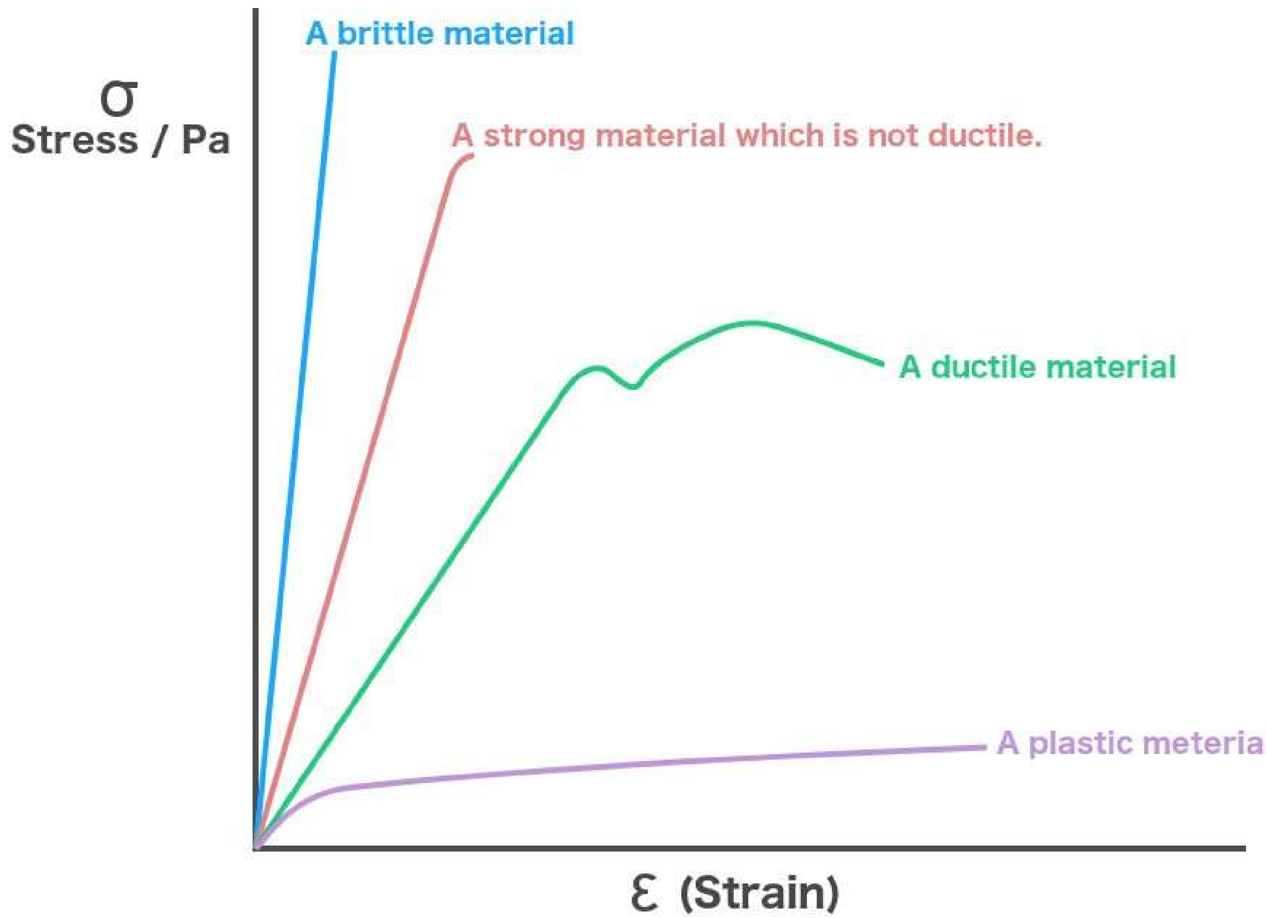
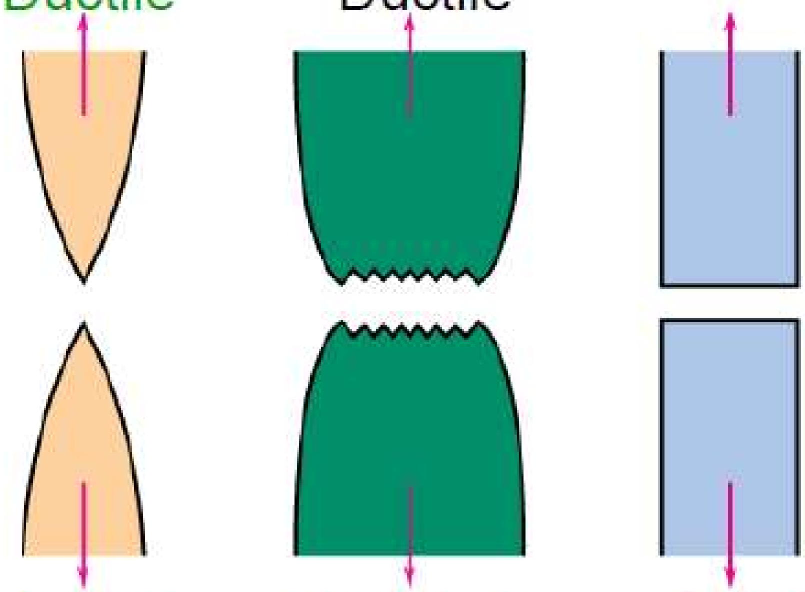

A brittle material
σ
Stress / Pa
A strong material which is not ductile.
A ductile material
A plastic meterial
ε (Strain)

Ductile vs Brittle Failure

- Classification:

Fracture behavior:	Very Ductile	Moderately Ductile	Brittle



Adapted from Fig. 8.1, *Callister 7e.*

%*AR* or %*EL*	Large	Moderate	Small

- Ductile fracture is usually desirable!

Ductile: warning before fracture

Brittle: No warning

Ductile vs. Brittle Failure

cup-and-cone fracture

brittle fracture

Behavior described	Terms used	
Crystallographic mode	Shear	Cleavage
Appearance of fracture	Fibrous	Granular
Strain to fracture	Ductile	Brittle

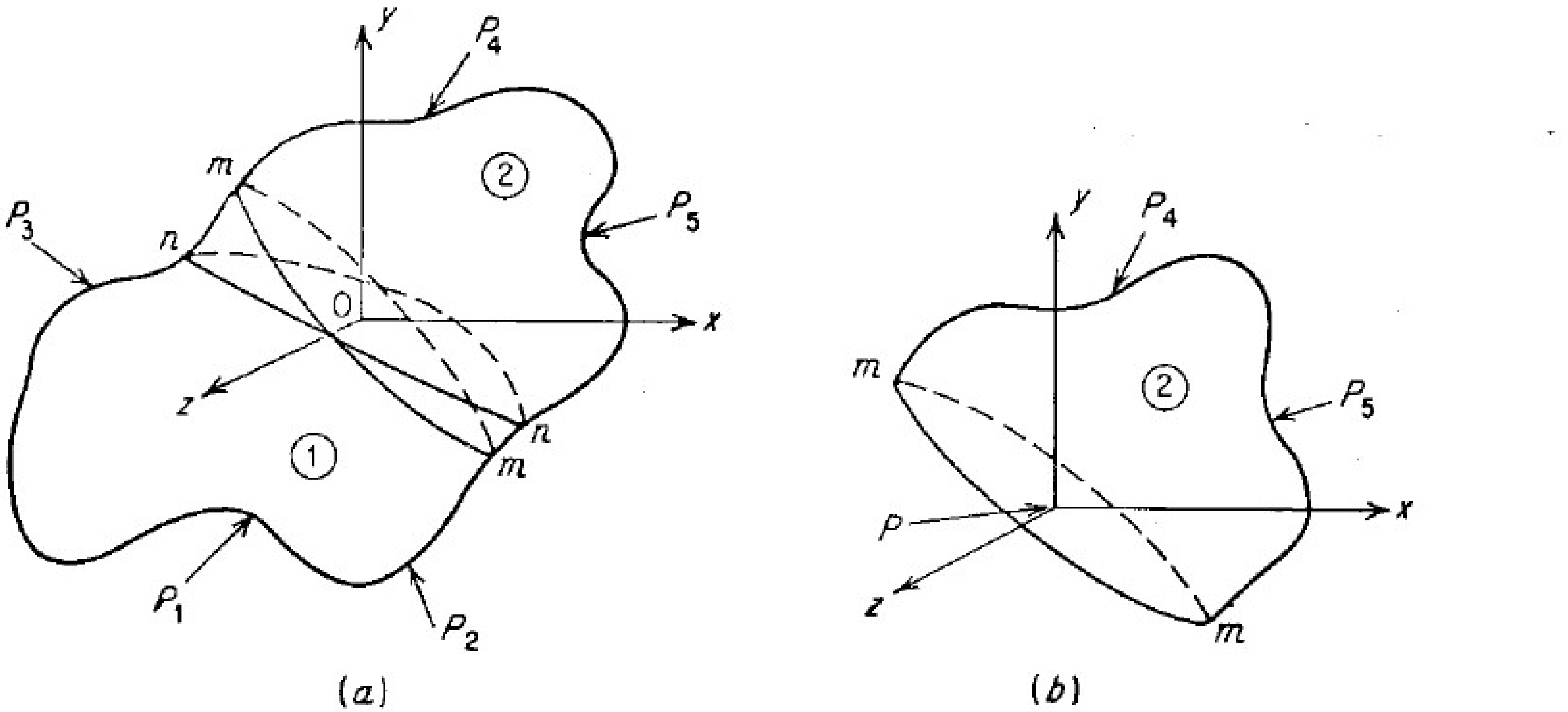
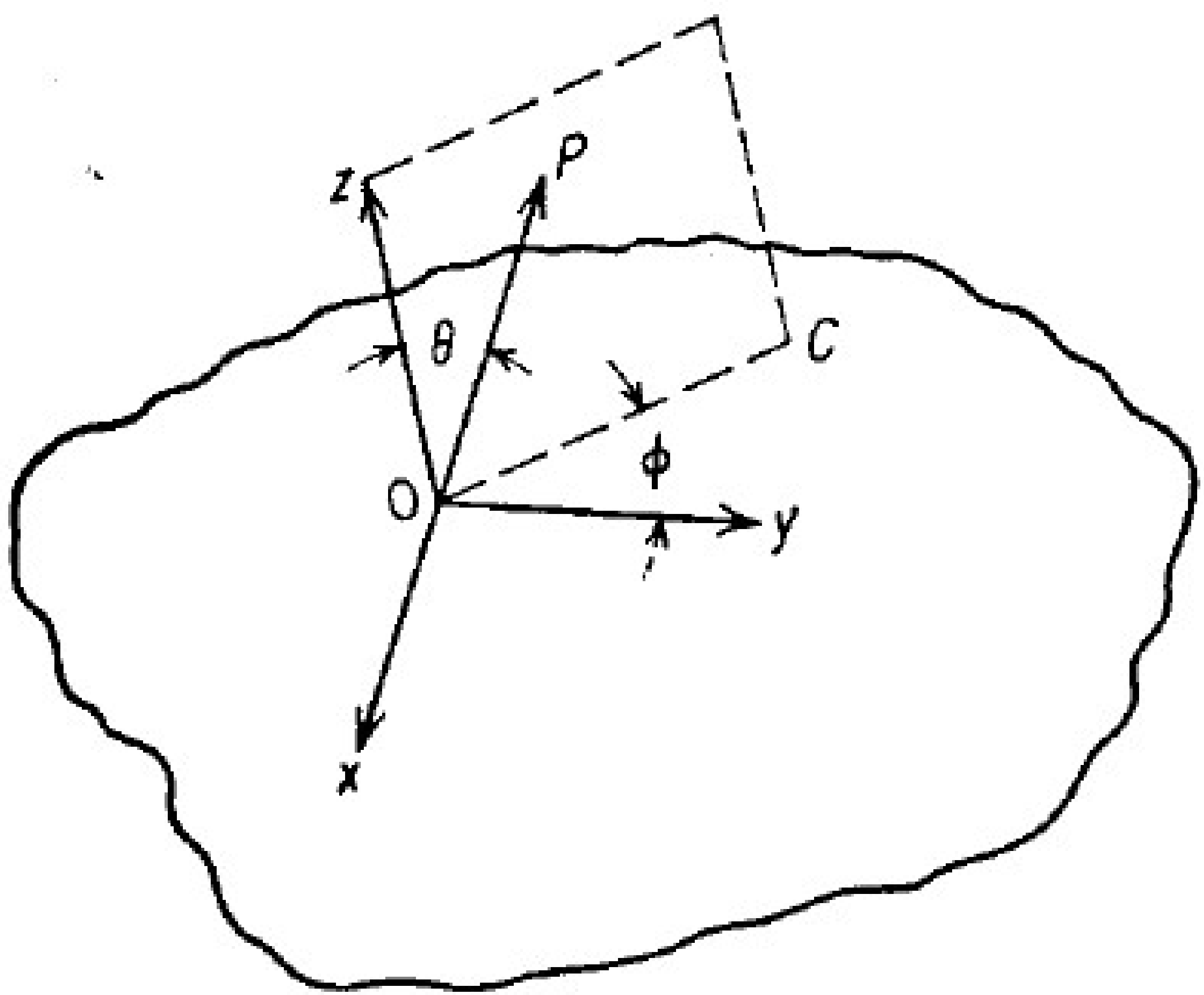


Figure 1-5 (a) Body in equilibrium under action of external forces $P_1, \ldots, P_5$; (b) forces acting on parts.



z
P
θ
C
φ
O
y
x

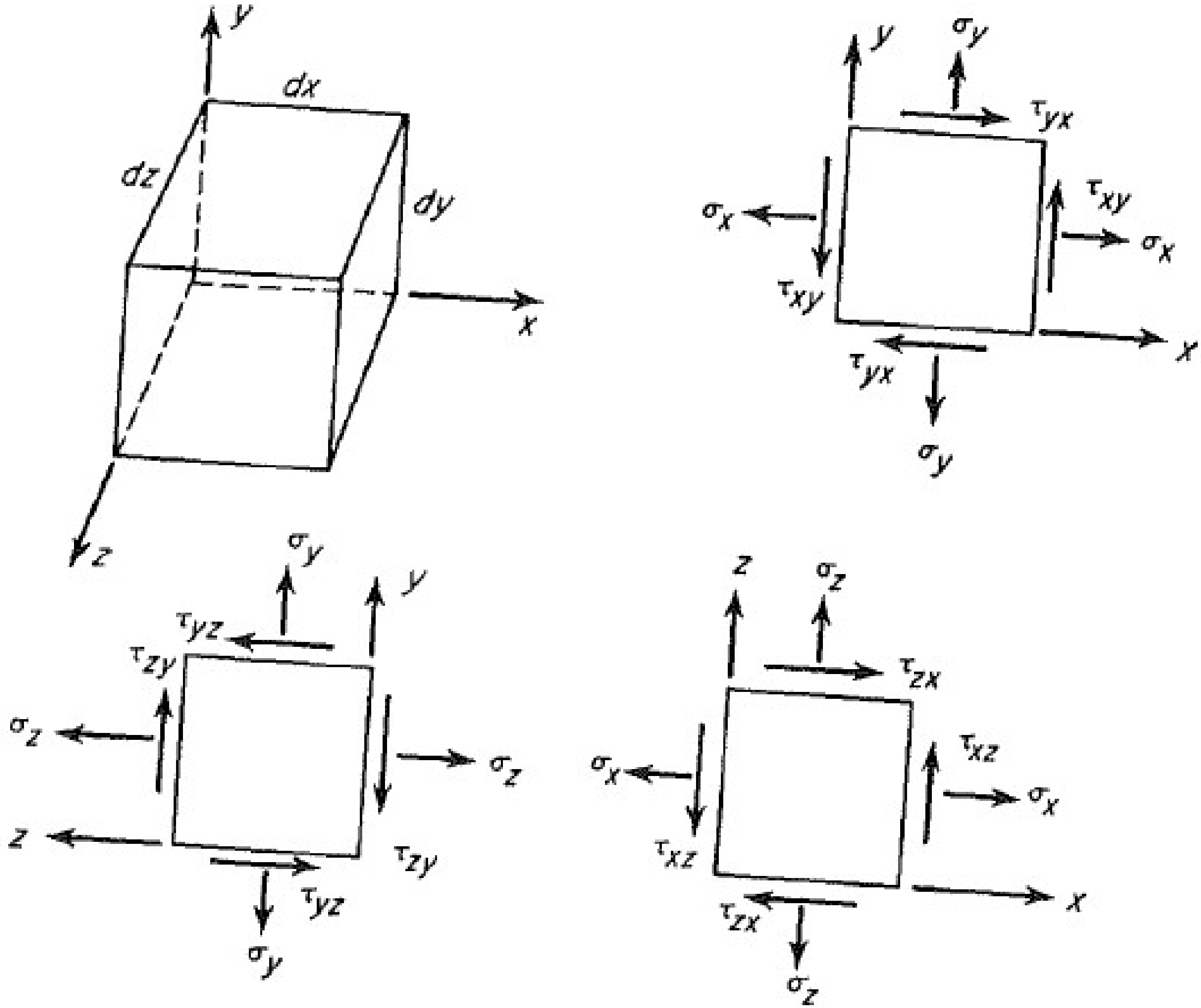


FIGURE 3.2.2 Convention for stresses.

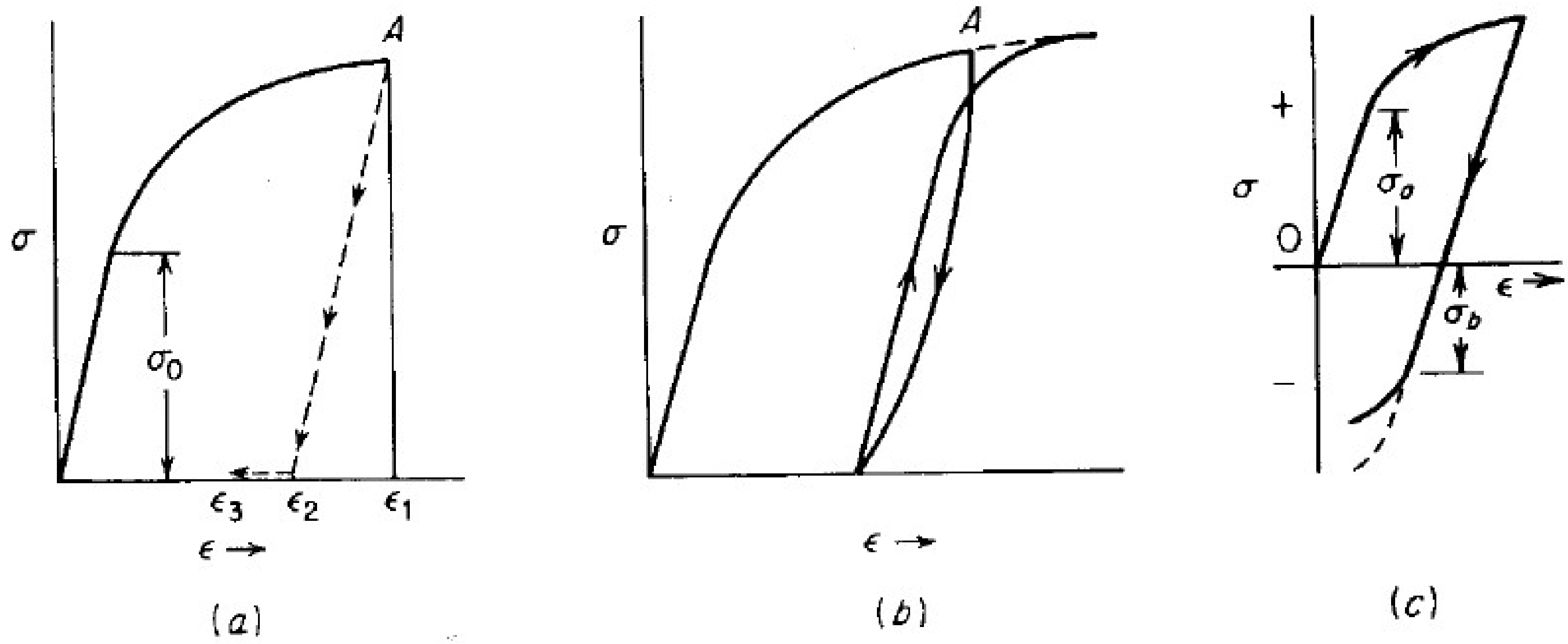
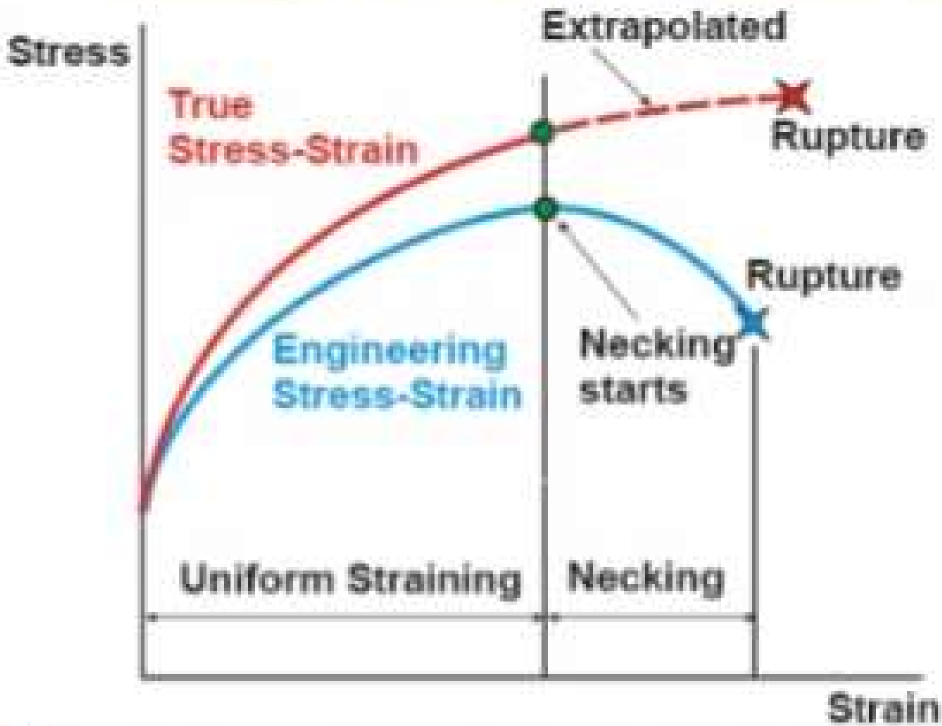


Figure 3-1 Typical true stress-strain curves for a ductile metal.

Material Behavior in Metal Forming

- The typical stress strain curve for most metals is divided into an elastic region and a plastic region
- Plastic region of stress-strain curve is primary interest because material is plastically deformed

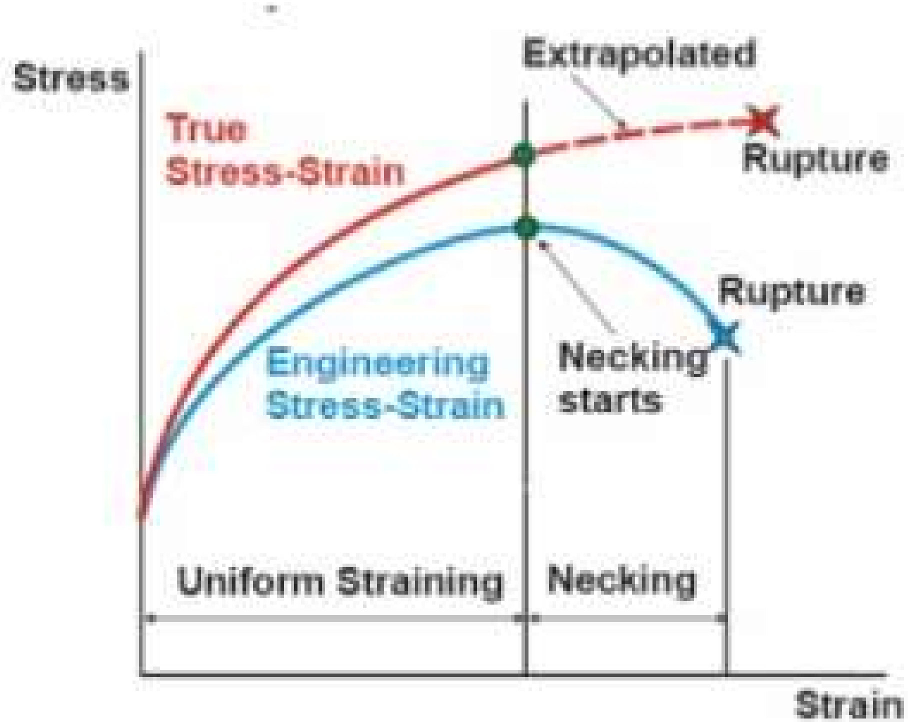


Necking starts at maximum engineering stress or, equivalently, at maximum tensile load.

Material Behavior in Metal Forming

- In plastic region, metal's behavior is expressed by the flow curve:

$$\boxed{\sigma = K\varepsilon^{n}}$$



K = strength coefficient (MPa); and *n* = strain hardening exponent. Stress and strain in flow curve are true stress and true strain.

True strain ε	0.01	0.10	0.20	0.50	1.0	4.0
Conventional strain e	0.01	0.105	0.22	0.65	1.72	53.6

Flow Stress

- **For most metals** at room temperature, strength increases when deformed due to strain hardening. The stress required to continue deformation must be increased to match this increase in strength.

- **Flow stress** is defined as the instantaneous value of stress required to continue deforming the material – to keep the metal "flowing".

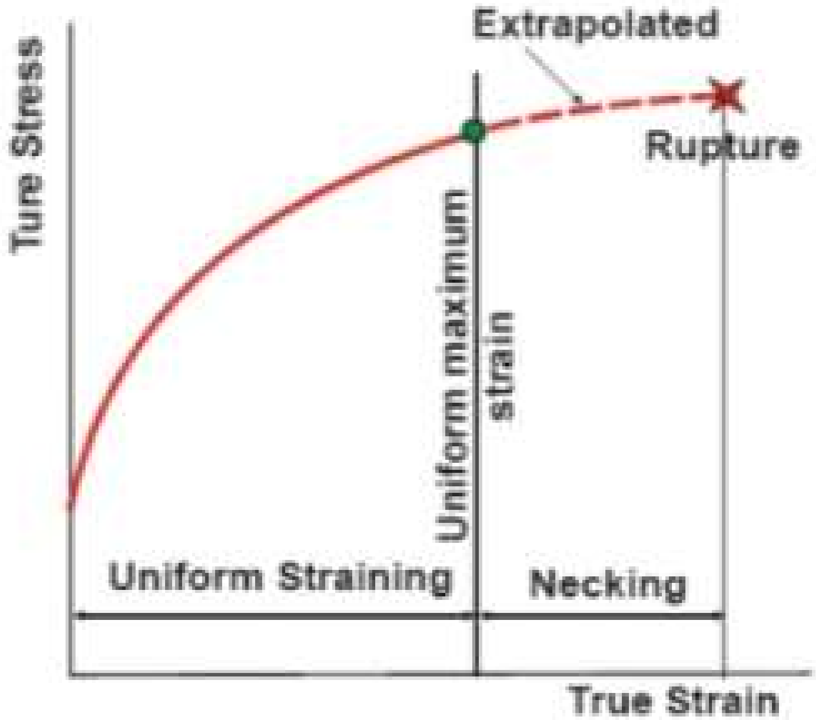
$$\boxed{Y_f = K\varepsilon^n}$$

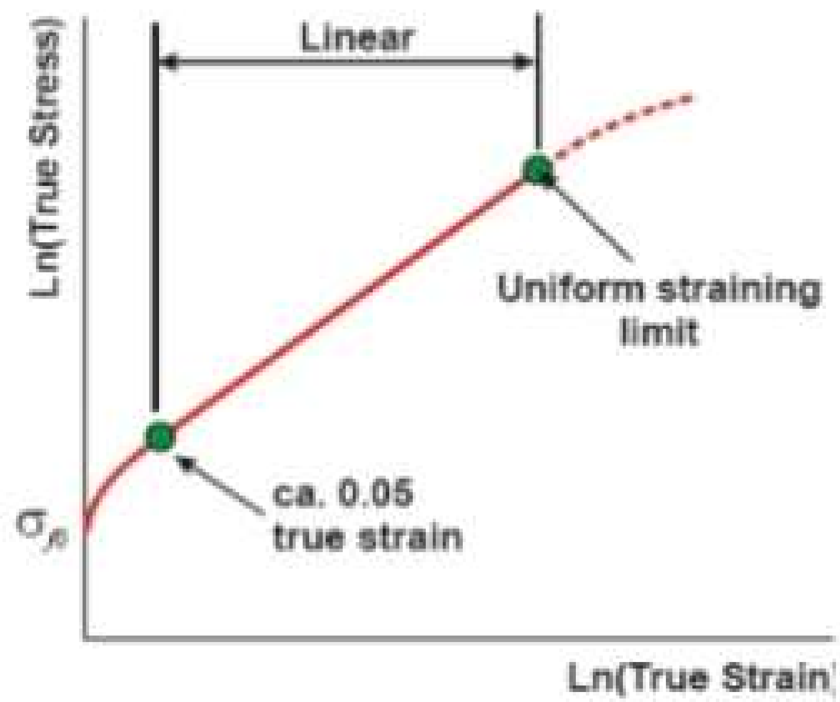
Where: Y_f = flow stress, that is, the yield strength as a function of strain

Relevance of the Flow Curve

- The flow curve is used to determine the new yield strength after a plastic deformation process.
- The flow curve is used to judge the formability of metals.
- The flow curve describes the hardening behavior of metals during plastic deformation in terms of equivalent strain, equivalent strain rate and temperature.
- The flow curve is a property of each individual metal.
- various experiments with different stress and strain-rate states should yield the same flow curve for same strain-rate value and same temperature.

Flow Curve: Mathematical Representation (1)
(Cold Flow Curves)





Typical Values of K and n

Typical Values of Strength Coefficient K and Strain Hardening Exponent n for Selected Metals.

Material	Strength coefficient, K lb/in.^{2}	(MPa)	Strain hardening exponent, n
Aluminum, pure, annealed	25,000	(175)	0.20
Aluminum alloy, annealed[a]	35,000	(240)	0.15
Aluminum alloy, hardened by heat treatment[a]	60,000	(400)	0.10
Copper, pure, annealed	45,000	(300)	0.50
Copper alloy: brass[a]	100,000	(700)	0.35
Steel, low C, annealed[a]	75,000	(500)	0.25
Steel, high C, annealed[a]	125,000	(850)	0.15
Steel, alloy, annealed[a]	100,000	(700)	0.15
Steel, stainless, austenitic, annealed[a]	175,000	(1200)	0.40

[a]Values of K and n will vary according to composition, heat treatment, and work hardening.

Flow Curve

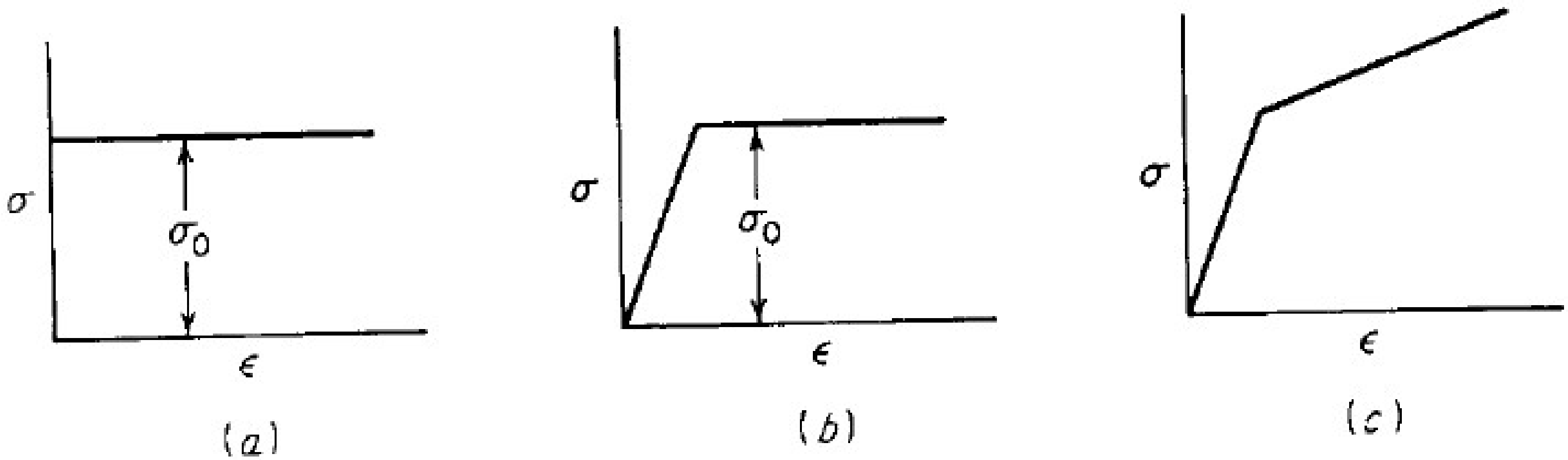
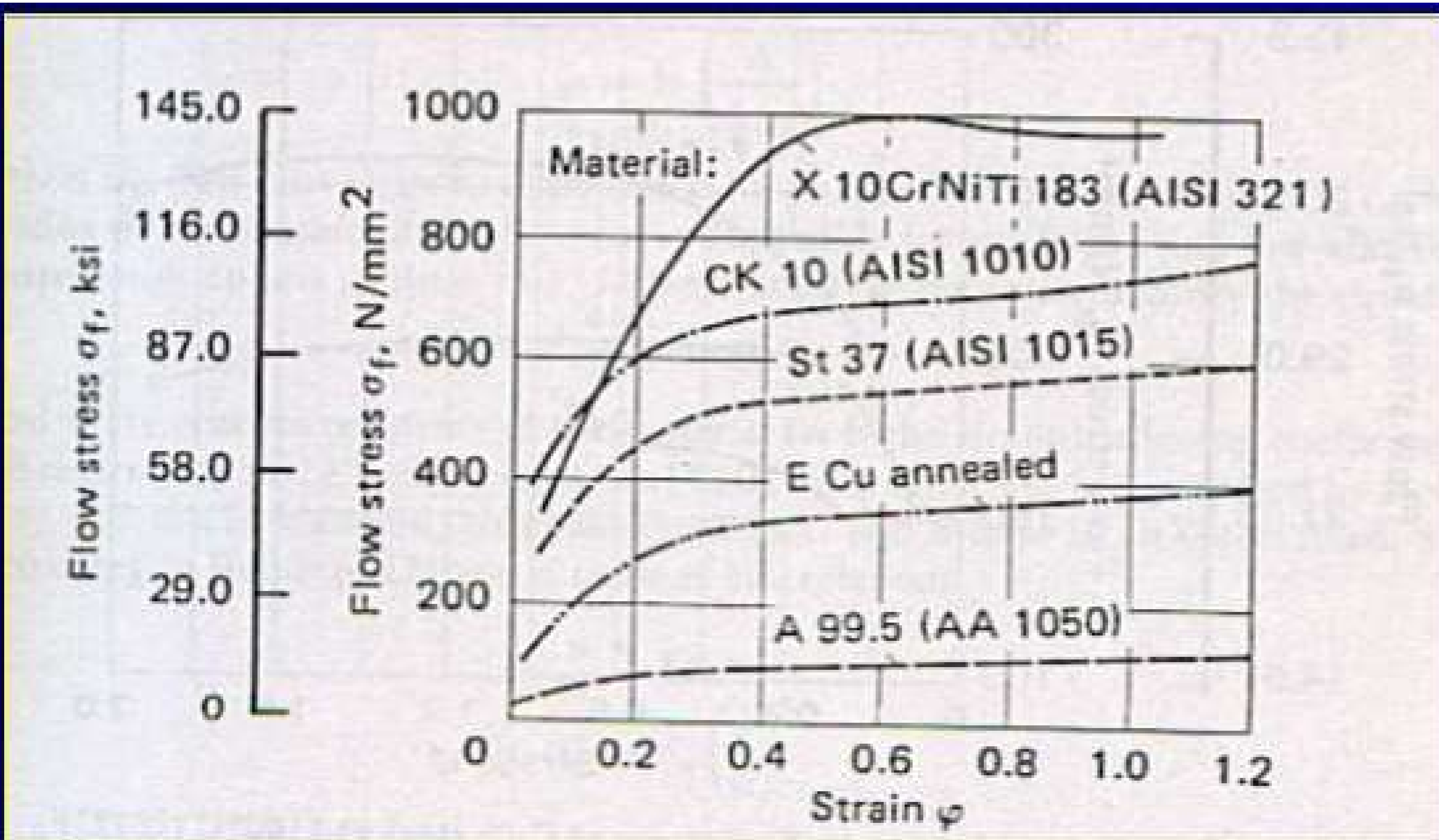


Figure 3-2 Idealized flow curves. (*a*) Rigid ideal plastic material; (*b*) ideal plastic material with elastic region; (*c*) piecewise linear (strain-hardening) material.

Flow Curve



Flow curves of some metals at room temperature

6.7 *For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).*

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

Syllabus

Module:1	**Theory of Plasticity**	**6 hours**	**SLO: 1, 5**
Theory of Plasticity - stress tensor – hydrostatic & deviator components of stress – flow curve – true stress strain – yielding criteria – yield locus – octahedral shear stress and shear strains – invariants of stress strain – slip line field theory - plastic deformations of crystals.			
Module:2	**Fundamentals of Metal working**	**6 hours**	**SLO: 1, 5**
Classification of forming processes, mechanics of metal working, temperature in metal working, strain rate effects, metallurgical structure, friction and lubrication, deformation zone geometry, hydrostatic pressure, workability, residual stresses.			
Module:3	**Forging process**	**6 hours**	**SLO: 1, 5,17**
Classification, Forging in plane strain, forging equipment, open die forging, closed die forging, calculation of forging loads in closed die forging, Forging defects, powder metallurgy forging, residual stresses in forgings.			
Module:4	**Rolling**	**6 hours**	**SLO: 1, 5, 17**
Classification - rolling mills - rolling of bars & shapes – rolling forces, analysis of rolling – defects in rolling- theories of hot & cold rolling – torque power estimation.			

Yield criteria for metals

A yield criterion is a hypothesis defining the limit of elasticity in a material and the **onset of plastic deformation** under any possible combination of stresses.

Yield criteria for metals

The yield stress σ_Y (or σ_0) is usually determined in a tensile test, where a single uniaxial stress acts.

However, the engineer must be able to predict when yield will occur in more complicated real-life situations involving multiaxial stresses.

This is done by use of a **yield criterion**, an observation derived from experimental evidence as to just what it is about the stress state that causes yield.

Yield criteria for metals

In uniaxial loading, plastic flow begins when $\sigma = \sigma_0$, the tensile yield stress

When does yielding begin when a material is subjected to an arbitrary state stress?

Yield criteria for metals

- Pure hydrostatic pressure or mean stress tensor, σ_m, doesn't cause yielding in metals.
- Only the deviatoric stress, σ'_{ij}, which represents the shear stresses causes plastic flow.
- For an isotropic solid, the yield criterion must be independent of the choice of the axes, i.e., *it must be an invariant function*.

$\therefore$ Yield criterion must be some function of the J's.

Von Mises' Yield Criterion

- "Yielding would occur when J_2 exceeds some critical value" $J_2 = k^2$
- Yielding in uniaxial tension: $\sigma_1 = \sigma_0$, $\sigma_2 = \sigma_3 = 0$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$\sigma_0/\sqrt{3} = k$$

$$\boxed{\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)\right]^{1/2}$$

Tresca (Max. Shear Stress) Criterion

"Yielding occurs when the max. shear stress reaches the value of shear yield stress in the uniaxial tension test."

- Max shear stress, $\tau_{max} = (\sigma_1 - \sigma_3)/2$
- In uniaxial tension, $\tau_0 = \sigma_0 /2$
- Tresca criterion: $\boxed{(\sigma_1 - \sigma_3) = \sigma_0}$
- Pure shear: $\sigma_1 = -\sigma_3 = k$; $\sigma_2 = 0$

  $$(\sigma_1 - \sigma_3) = 2k = \sigma_0 \;\Rightarrow k = \sigma_0 /2$$

- Predicts the same stress for yielding in uniaxial tension and in pure shear

Tresca (Max. Shear Stress) Criterion

- Less complicated mathematically than von Mises criterion
- Often used in engineering design
- Doesn't consider σ_2
- Need to know *apriori* the max. and min. principal stresses
- General form:

$$4J_2^3 - 27J_3^2 - 36k^2J_2^2 + 96k^4J_2 - 64k^6 = 0$$

Yield Locus

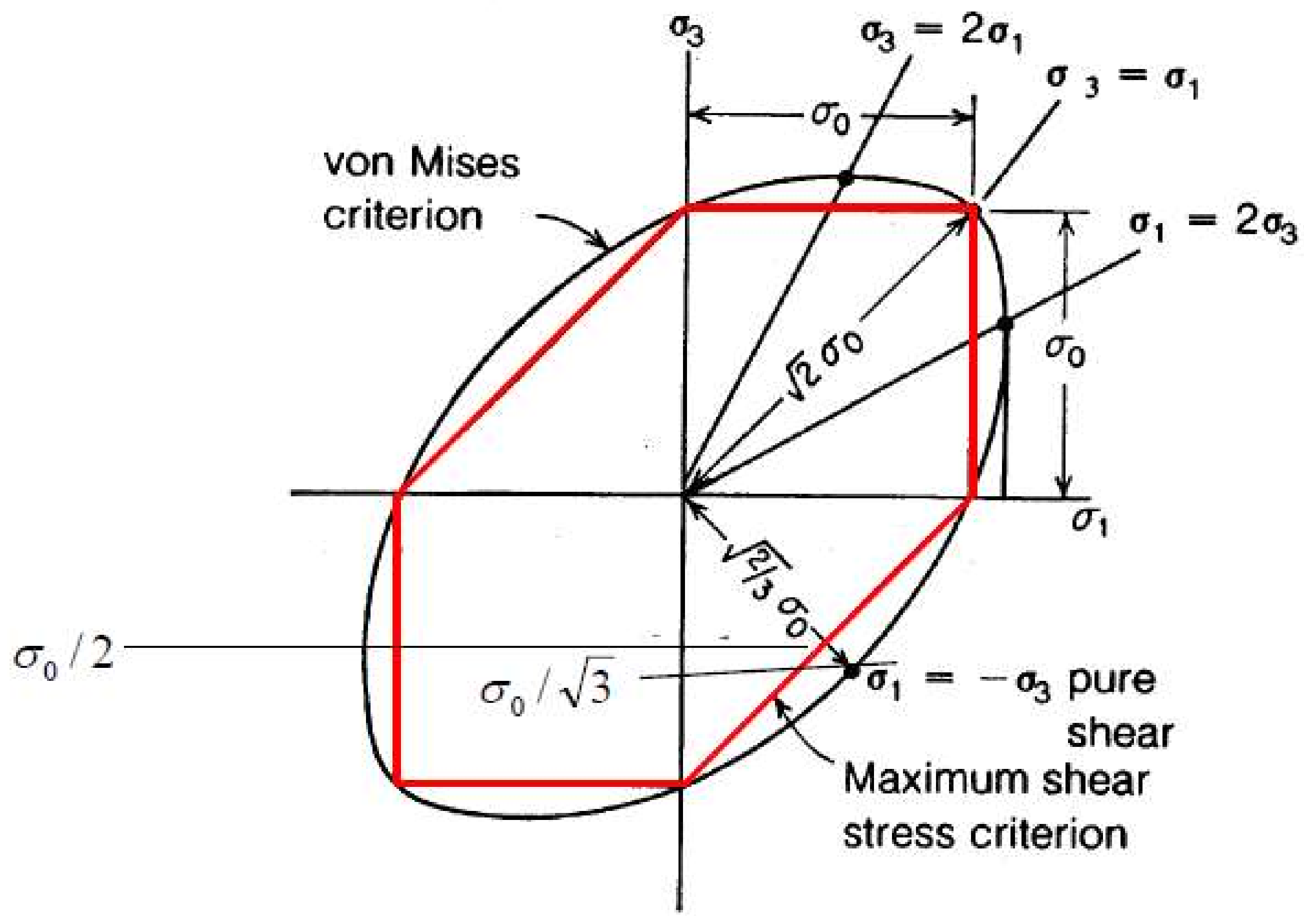
$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

For a biaxial plane-stress condition ($\sigma_2 = 0$); the von Mises criterion can be expressed as

$$\boxed{\sigma_1^2 + \sigma_3^2 - \sigma_1 \sigma_3 = \sigma_0^2}$$

Equation of an ellipse whose major semi-axis is $\sqrt{2}\sigma_0$ and minor semi-axis is $\sqrt{(2/3)}\sigma_0$

Yield Locus



Von Mises and Tresca predict the same yield stress for uniaxial and balanced biaxial stress loading. Max. difference (15.5%) for pure shear case.

Variation of Stresses in Metals

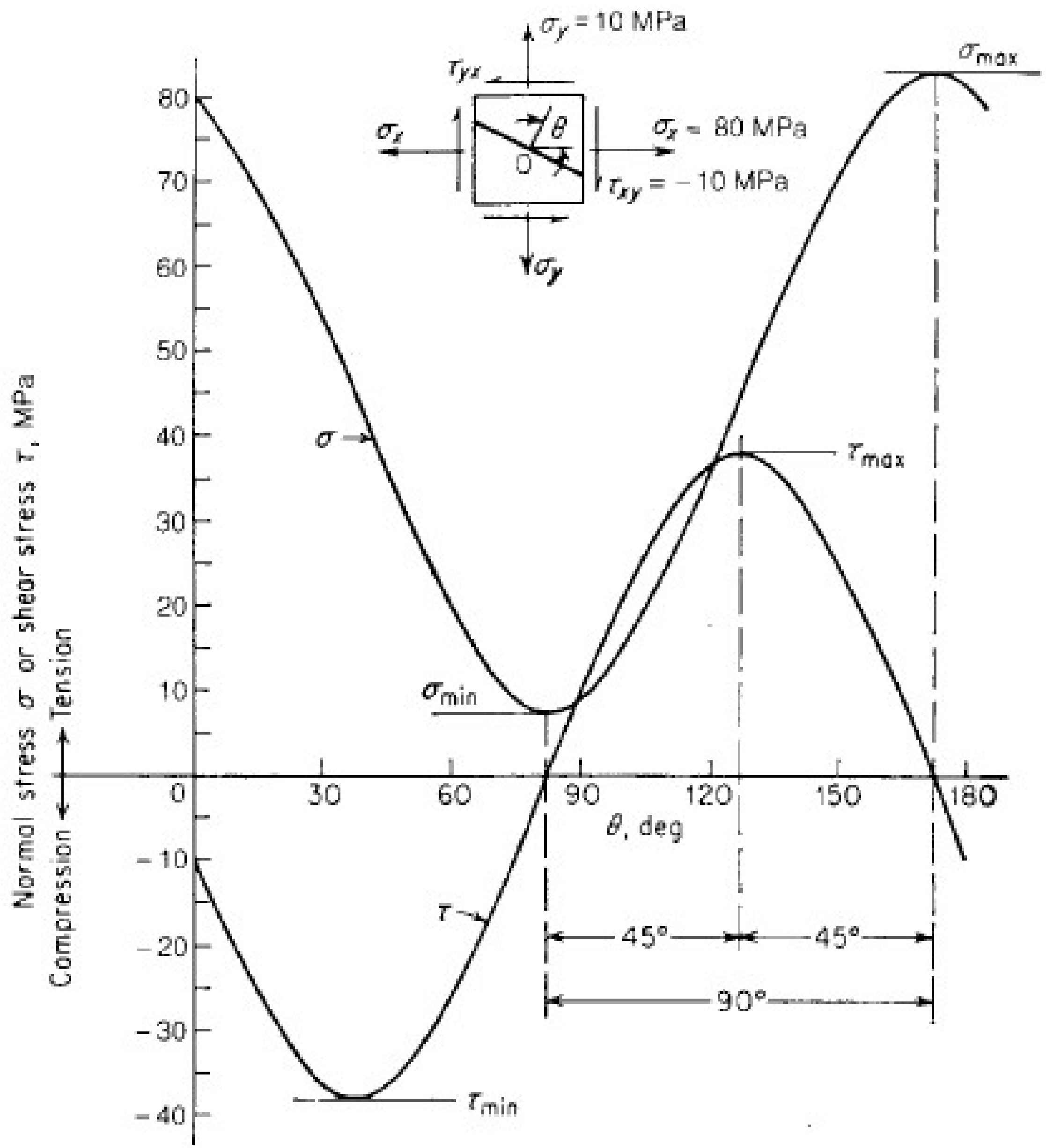
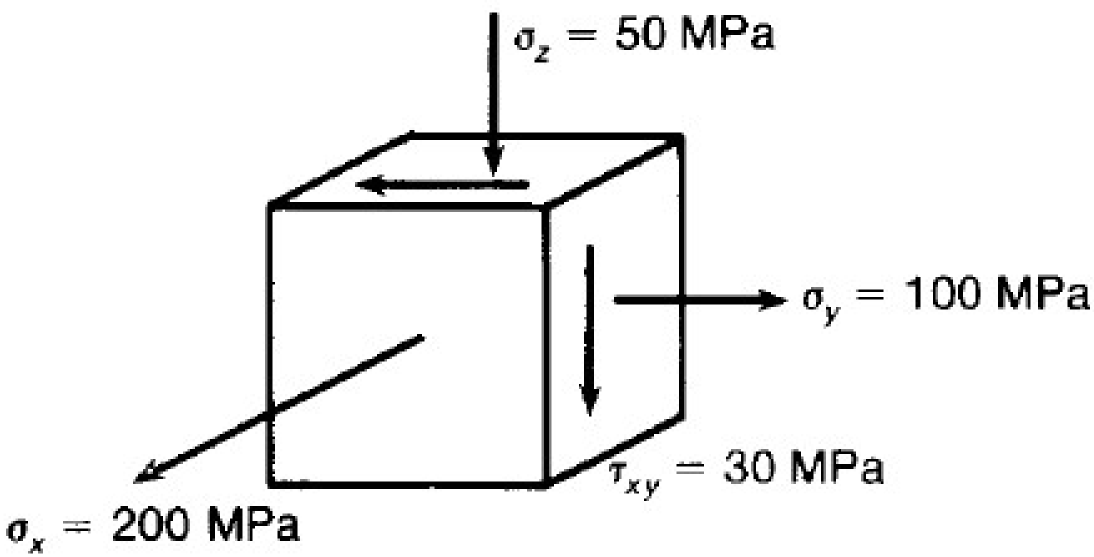


Figure 2-4 Variation of normal stress and shear stress on oblique plane with angle θ.

Example Stress analysis of a spacecraft structural member gives the state of stress shown below. If the part is made from 7075-T6 aluminum alloy with $\sigma_0 = 500$ MPa, will it exhibit yielding? If not, what is the safety factor?



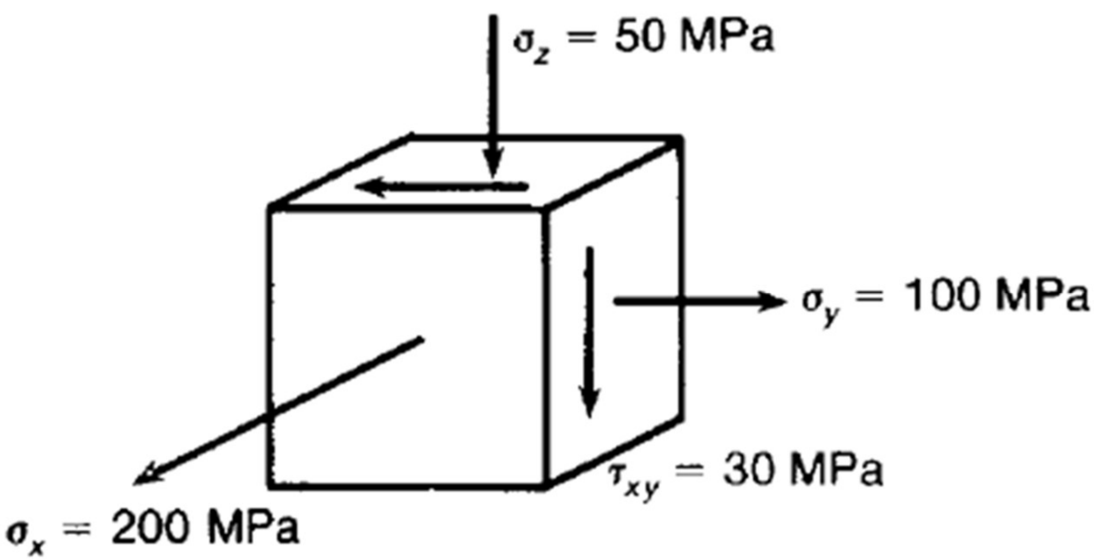
$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(200 - 100)^2 + (100 - (-50))^2 + (-50 - 200)^2 + 6(30)^2\right]^{1/2}$$

$$\sigma_0 = \frac{1}{\sqrt{2}}(100{,}400)^{1/2} = \frac{316.859}{\sqrt{2}} = 224 \text{ MPa}$$

Since the value of σ_0 calculated from the yield criterion is less than the yield strength of the aluminum alloy, yielding will not occur. The safety factor is $500/224 = 2.2$.

Example Use the maximum-shear-stress criterion to establish whether yielding will occur for the stress state shown in the previous example.

Example Stress analysis of a spacecraft structural member gives the state of stress shown below. If the part is made from 7075-T6 aluminum alloy with $\sigma_0 = 500$ MPa, will it exhibit yielding? If not, what is the safety factor?



Example Use the maximum-shear-stress criterion to establish whether yielding will occur for the stress state shown in the previous example.

$$\tau_{max} = \frac{\sigma_x - \sigma_z}{2} = \frac{\sigma_0}{2}$$

$$200 - (-50) = \sigma_0$$

$$\sigma_0 = 250 \text{ MPa}$$

Again, the calculated value of σ_0 is less than the yield strength of the material.