# M 5: EM Theory

## *Introduction*

Electrostatic fields are produced by static electric charges. While magneto static fields ae produced by moving electric charges and natural magnets. On other hand we have time varying electrical and magnetic fields.

A time varying electrical field produces a time varying magnetic field is called *Induced magnetic field,* and a time varying magnetic field produces a time varying electrical field is called *Induced* electrical *field.* The combined field of such mutually induced electric and magnetic field is called the electromagnetic field.

The electromagnetic field possesses energy, mass and momentum and can convert into other forms of matter and energy. Both induced fields continuously changed, a wave of electromagnetic energy is generated and transmitted.

### *Vector Identities*

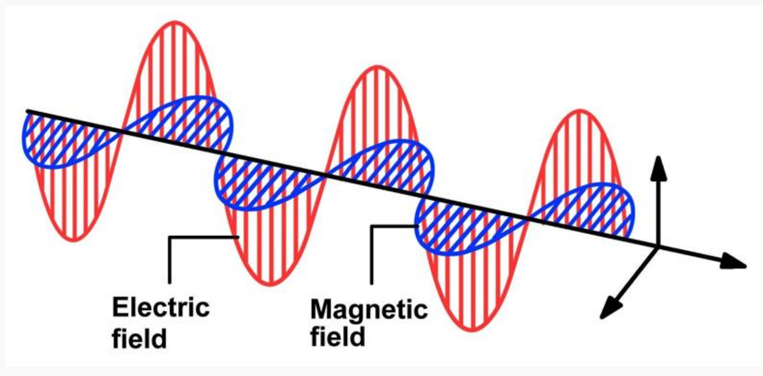
i. $\nabla \bullet (\nabla A) = Div(GradA) = \nabla^2 A$

ii. $\nabla \times \nabla A = curl(GradA) = 0$

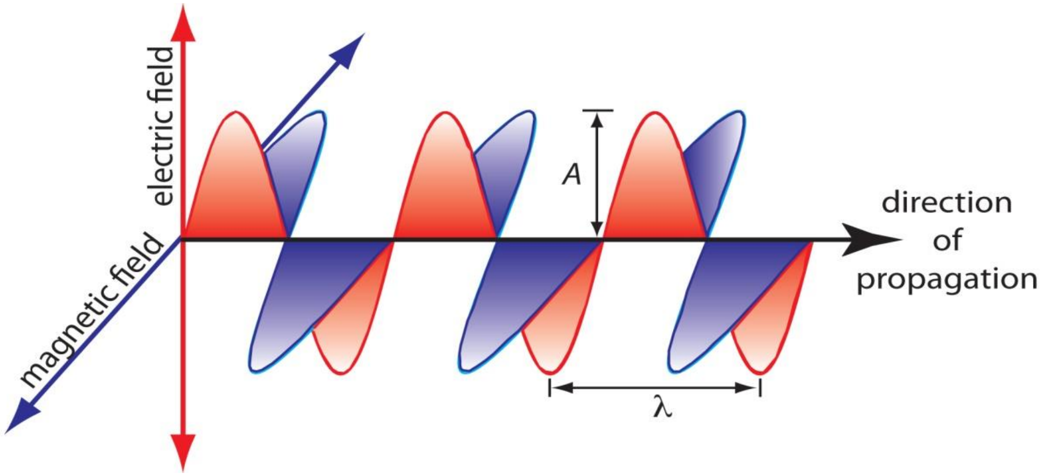
iii. $\nabla \bullet (\nabla \times A) = Div(curlA) = 0$

iv. $\nabla \times (\nabla \times A) = curl(curlA) = \nabla(\nabla \bullet A) - \nabla^2 A$

## ELECTROMAGNETIC WAVES:




- Electromagnetic waves are coupled time varying electric and magnetic fields that propagate in space.
- Electric field is varying with time, and it will give rise to magnetic field, this magnetic field is varying with time and it gives rise to electric field and the process continues so on.
- These electric and magnetic fields are time varying and coupled with each other when propagating together in space gives rise to electromagnetic waves.
- In the fig, red line represents the electric field and it varies in the form of a sine wave.
- The magnetic field as shown in the fig. represented by blue line.
- The magnetic field will be a sine wave but in a perpendicular direction to the electric field.
- These both give rise to electromagnetic field.
- If the electric field is along x-axis, magnetic field along y-axis, the wave will then propagate in the z-axis.
- Electric and magnetic field are perpendicular to each other and to the direction of wave propagation.
- Electric and magnetic fields which is time varying and coupled to each other they give rise to electromagnetic waves.




## SOURCES OF ELECTROMAGNETIC WAVES (EM):

- EM waves are generated by electrically charged particle oscillates (accelerating charges).
- The electric field associated with the accelerating charge vibrates which generates the vibrating magnetic field.
- These both vibrating electric and magnetic fields give rise to EM waves.
- If the charge is at rest, electric field associated with the charge will also be static. There will be no generation of EM waves as electric field is not varying with time.
- When the charge is moving with uniform velocity, then the acceleration is 0. The change in electric field with time is also constant as a result again there will be no electromagnetic waves generated.
- This shows that only the accelerated charges alone can generate EM waves.

  For example:

  - Consider an oscillating charge particle, it will have oscillating electric field and which give rise to oscillating magnetic field.
  - This oscillating magnetic field in turn give rise to oscillating electric field and so on process continues.
  - The regeneration of electric and magnetic fields are same as propagation of the wave.
  - This wave is known as electromagnetic wave.

- The frequency of EM waves= the frequency of the oscillating particle.

## NATURE OF EM WAVES:

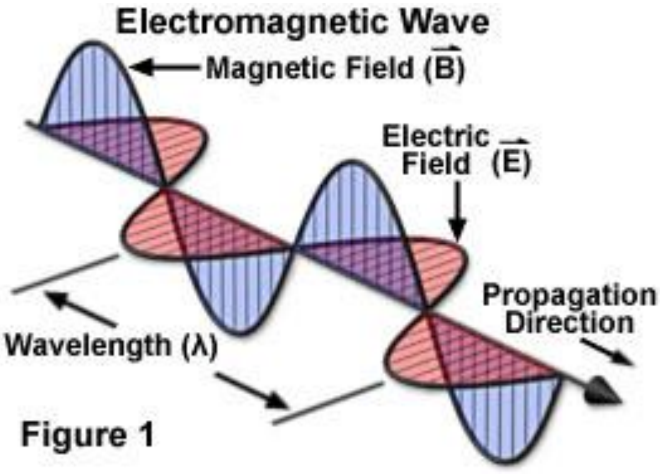



Figure 1

- EM waves are transverse waves.
- The transverse waves are those in which direction of disturbance or displacement in the medium is perpendicular to that of the propagation of wave.
- The particles of the medium are moving in a direction perpendicular to the direction of propagation of wave.
- In case of EM waves the propagation of wave takes place along x-axis, electric and magnetic fields are perpendicular to the wave propagation.
- This means that, wave propagation is in x-axis , electric field is in y-axis, magnetic field is in z-axis.
- Because of this EM waves are transverse waves in nature.
- Electric field of EM wave is represented as:
- $\boldsymbol{E_y} = \boldsymbol{E_0}\ \boldsymbol{s}\mathrm{i}(\boldsymbol{kx} - \boldsymbol{\omega t})$, where $E_y$= electric field along y-axis and x=direction of propagation of wave.
- Wave number $\boldsymbol{k} = (\boldsymbol{2\pi/\lambda})$
- Magnetic field of EM wave is represented as:

- $\boldsymbol{B_z} = \boldsymbol{B_0}(\boldsymbol{kx} - \boldsymbol{\omega t})$, where $B_Z$ = electric field along z-axis and x=direction of propagation of wave.

**PROPERTIES OF ELECTROMAGNETIC WAVES:**

- Variations in both electric and magnetic fields occur simultaneously. Therefore, they attain their maxima and minima at the same place and at the same time.

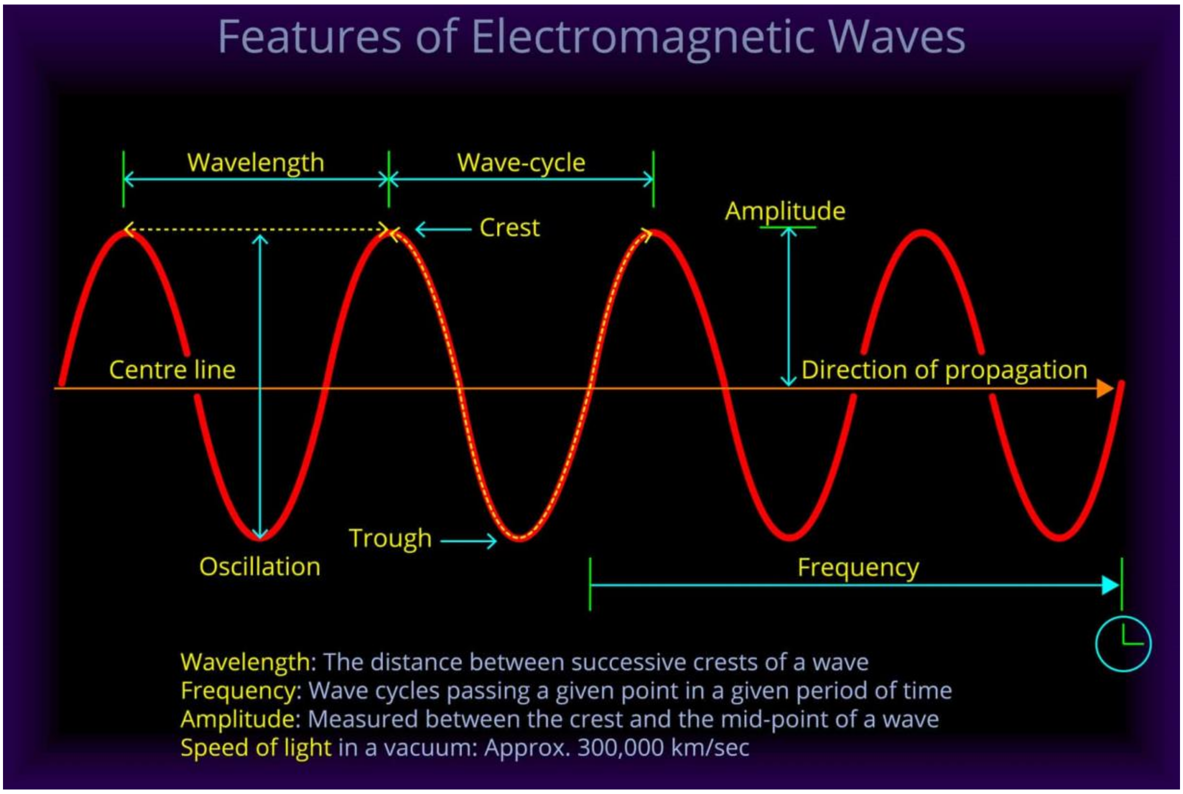
- The direction of electric and magnetic fields are mutually perpendicular to each other and as well as to the direction of propagation of wave.

- The electric field vector E and magnetic field vector B are related by

  $c = E_0 / B_0$ where $E_0$ and $B_0$ are the amplitudes of the respective fields and c is speed of light.

- The velocity of electromagnetic waves in free space, $c = 1 / \sqrt{\mu_0 \varepsilon_0}$

- The velocity of electromagnetic waves in a material medium = $1 / \sqrt{\mu\varepsilon}$. where $\mu$ and $\varepsilon$ are absolute permeability and absolute permittivity of the material medium.




- Electromagnetic waves obey the principle of superposition.

- Electromagnetic waves carry energy as they propagate through space.
- This energy is divided equally between electric and magnetic fields.
- Electromagnetic waves can transfer energy as well as momentum to objects placed on their paths.
- For discussion of optical effects of EM wave, more significance is given to Electric Field, E. Therefore, electric field is called ‘light vector’.
- Electromagnetic waves do not require material medium to travel.
- An oscillating charge which has non-zero acceleration can produce electromagnetic waves.

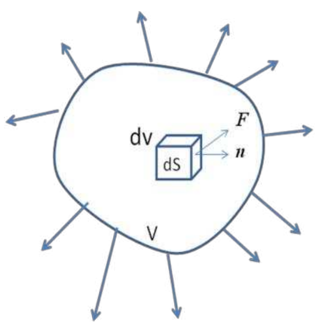
## GAUSS DIVERGENCE THEOREM:

Statement: The integral of the divergence of a vector field $\overline{F}$ over volume V is equal to the surface integral of the normal component of that vector over that surface bounded by V. Mathematically it can be written as

$$\int_V \overline{\nabla}.\,\overline{F} dV = \int_S \overline{F}.dS$$

Proof:

Let us assume an arbitrary volume V bounded by the surface S as shown in Fig. Consider small elemental area dS enclosing the volume dV. The normal component of divergence of the vector field $\overline{F}$ produced by the surface through dS is given by




$$d\phi = \overline{F}.dS \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)}$$

The total flux through the surface S enclosing the volume V is obtained by taking the closed surface integral to Eq. (1), We get

$$\phi = \int_S d\phi = \int_S \overline{F}.dS \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)}$$

The divergence of the vector $\overline{F}$ normally through a volume dV or flux through the volume dV is given by

$$d\phi = (\overline{\nabla}.\overline{F})dV \qquad \_\_\_\_\_\_\_\_\_\_\_\_ \quad (3)$$

The divergence of the vector $\overline{F}$ over the whole volume V given by the total flux through the volume V.

$$\therefore \phi = \int d\phi = \int (\overline{\nabla}.\overline{F})dV \qquad \_\_\_\_\_\_\_\_\_\_\_\_ \quad (4)$$

Hence the total flux of the vector through the volume ot through the surface enclosing the volume will be the same. Equating eq. (2) & Eq. (4)

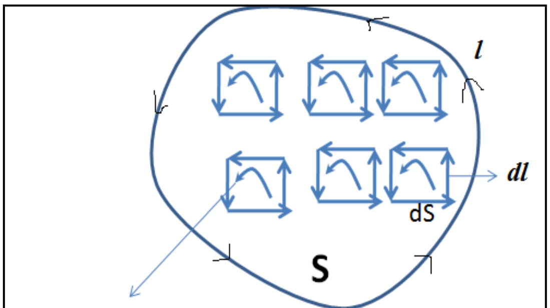
$$\boxed{\int \overline{F}.dS = \int (\overline{\nabla}.\overline{F})dV}$$

This is known as Gauss divergence theorem. By using this we can convert volume integral to surface integral or vice versa.

# STOKES THEOREM:

**Statement:** The line integral of the vector field $\overline{F}$ around a closed curve is equal to the integral of the normal component of its *curl* over any surface bounded by the curve. Mathematically it can be written as

$$\int_S (\overline{\nabla} \times \overline{F}).\hat{n}dS = \oint \overline{F}.dl \qquad \_\_\_\_\_\_\_\_ \quad (1)$$



Circulation of vector Field

**<u>Proof</u>:**

Let us consider a surface area ‘S’ that is divided into small surface elemental area **dS** as shown in Figure.

Let us consider a small elemental area ***dS***, then the circulation of vector field $\overline{F}$ around elemental area ***dS*** is

$$(\overline{\nabla} \times \overline{F}).\hat{n}dS \qquad \_\_\_\_\_\_\_\_\_\_\_\_ \quad (2)$$

The circulation around closed loop or closed curve ‘*dl’* can be obtained by integrating the above Eq. over the closed loop ‘*dl’* enclosing area dS, then we have

$$\int_S (\overline{\nabla} \times \overline{F}).\hat{n} dS \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad (3)$$

Let '*dl'* be the elemental length of the closed loop *l,* then the vector field $\overline{F}$ along this length is

$$\overline{F}.dl \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad (4)$$

The circulation around loop or closed curve '*l'* enclosing an area S is obtained by integrating the above Eq. over the closed loop.

i.e., $\oint \overline{F}.dl$ ____________ (5)

Equating Eq. (3) & (5), we get

$$\boxed{\int_S (\overline{\nabla} \times \overline{F}).\hat{n} dS = \oint \overline{F}.dl}$$

From this we conclude that the line integral of the vector field $\overline{F}$ around closed loop or curve is equal to the integral of the normal component of its *curl* over any surface bounded by the loop or curve. This is called Stokes Theorem. Using this Eq. we can convert a surface integral to line integral or vice versa.