Engineering Physics

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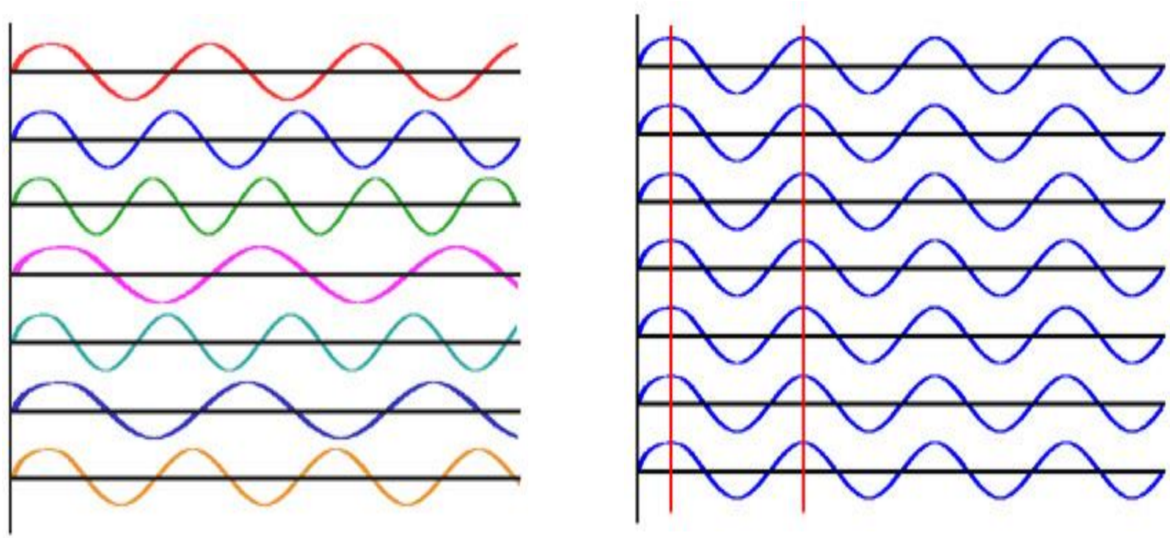
Contents

- Laser Characteristics,
- Spatial and Temporal Coherence,
- Einstein Coefficient & its significance,
- Population inversion,
- Two, three & four level systems,
- Pumping schemes,
- Threshold gain coefficient,
- Components of laser,
- Nd-YAG, He-Ne, CO2 and their engineering applications

❖ William Silfvast, Laser Fundamentals, 2008, Cambridge University Press.

Coherence

- Coherence is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.
- The interactions of two EM waves that have only slightly different frequencies, or that originate from points only slightly separated spatially.
- For example, two closely located but separate laser beams or a single beam illuminating two closely positioned apertures.



Incoherent light waves **Coherent light waves**

Temporal Coherence

- The case of temporal coherence refers to the relative phase or the coherence of two waves at two separate locations along the propagation direction of the two beams.
- It sometimes referred to as longitudinal coherence.
- If we assume that the two waves are exactly in phase at the first location, then they will still be at least partially in phase at the second location up to distance l_c, where l_c is defined as the coherence length.
- The coherence length can be determined to be

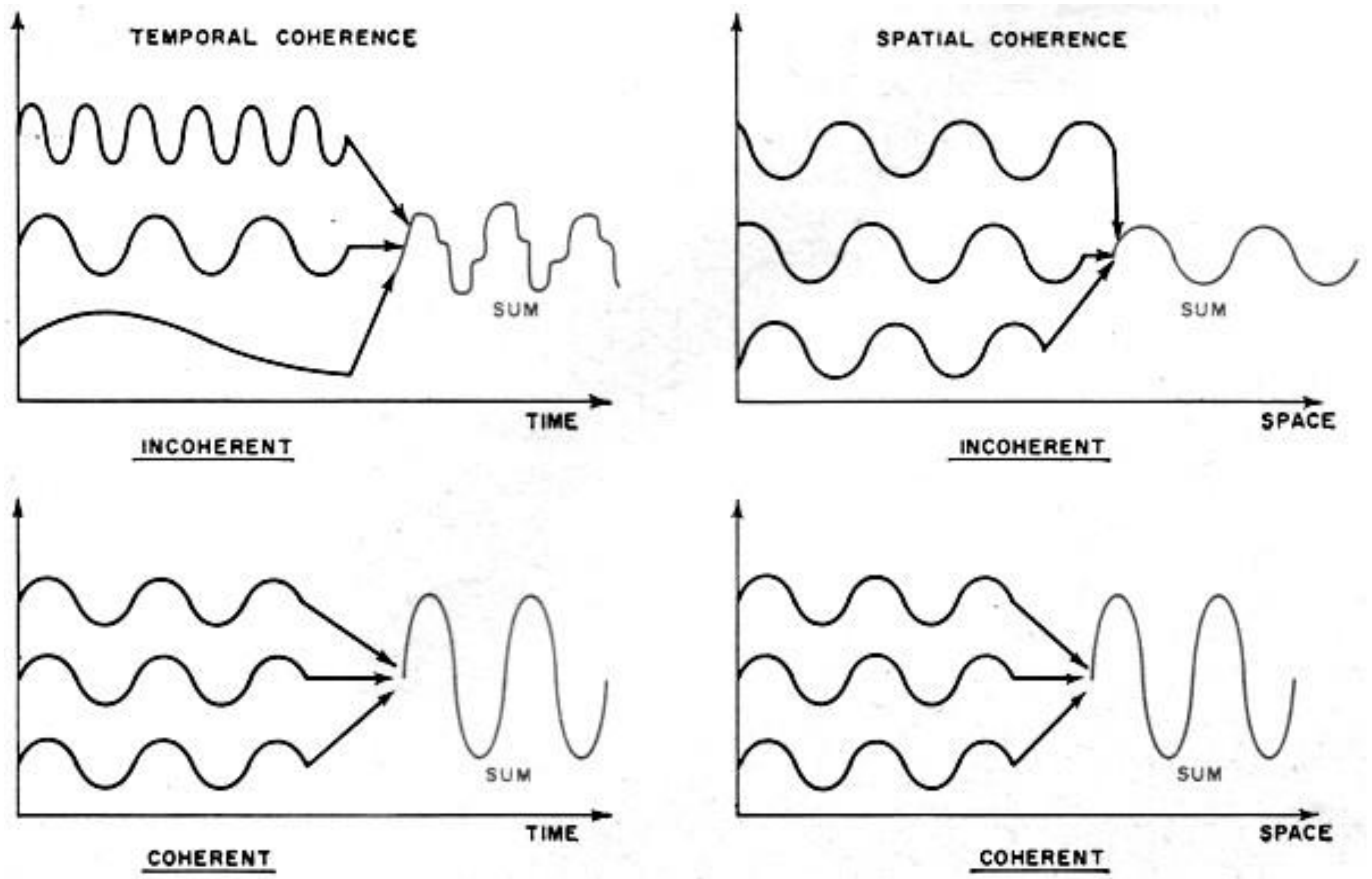
$$l_c = \lambda\left(\frac{\lambda}{\Delta\lambda}\right) = \frac{\lambda^2}{\Delta\lambda}$$

Spatial coherence

- Spatial coherence, also referred to as transverse coherence, describes how far apart two sources, or two positions of the same source, can be located in a direction transverse to the direction to the direction of observation and still exhibit coherent properties over range of observation points.
- This is sometimes referred to as the lateral coherence. More specially, we will ask by what distance l_t can two points separated in the transverse direction at the region of observation and still have interference effect from the source region over a specific lateral direction of the source.
- The transverse coherence length can be calculated by the following relation.

$$l_t = \frac{r\lambda}{s} = \frac{\lambda}{\theta_s}$$

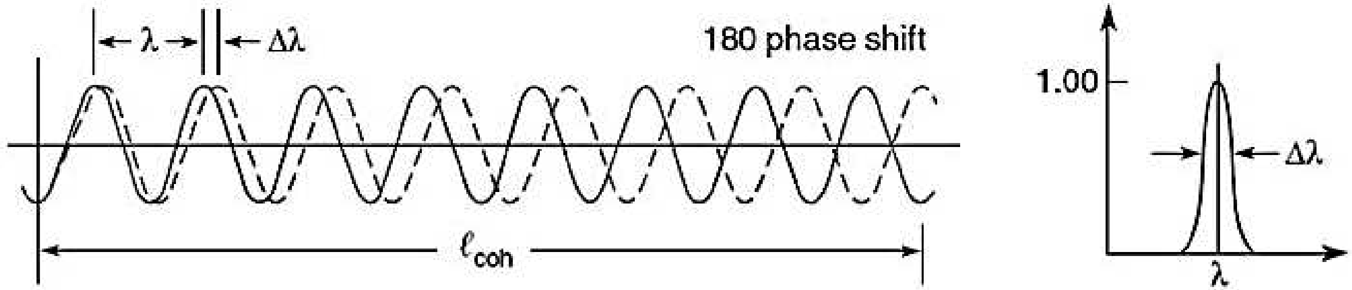
Temporal & Spatial Coherence



Temporal & Spatial Coherence

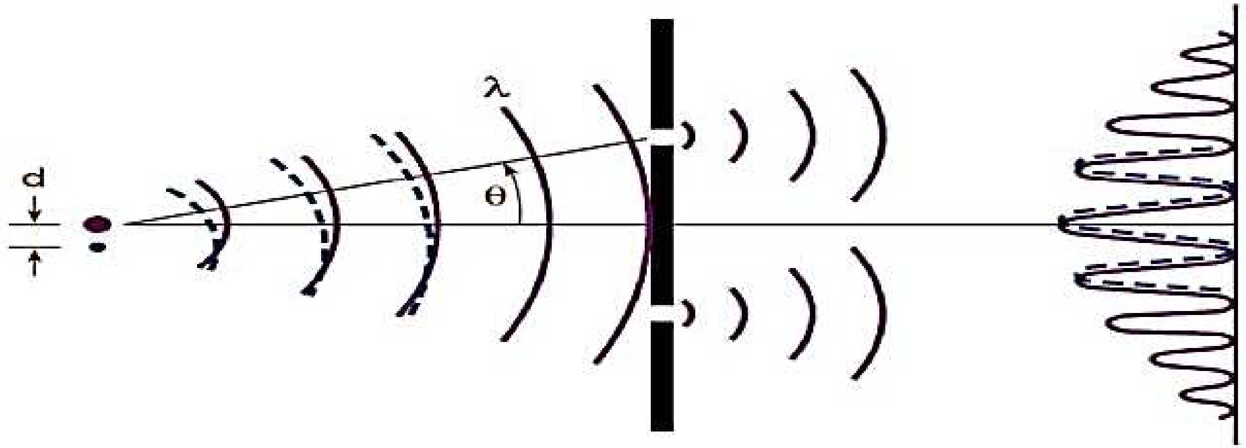
Temporal Coherence

Ability of a light beam to form fringes with a delayed version of itself



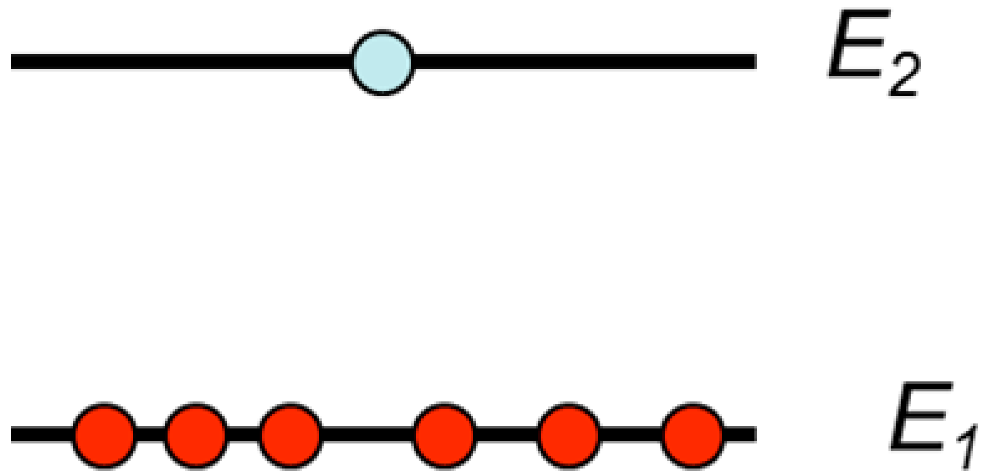
Spatial Coherence

Ability of spatially separated points in a wavefront to form fringes.



Einstein coefficients

- Consider two levels of an atomic system as shown in figure.
- Let N_1 and N_2 be the number of atoms per unit volume present in the energy levels E_1 and E_2, respectively.
- The atomic system can interact with EM radiation in three distinct ways. The distribution of atoms in the two energy levels will change by absorption or emission of radiation.
- Einstein introduced three empirical coefficients to quantify the change of population of the two levels.



Einstein coefficients

❑ Absorption –

- An atom in the ground state E absorbs an incident photon and makes a transition to the excited state E2. This transition is known as induced or stimulated absorption or simply absorption.
- This process may be represented as

$$\text{atom} + \text{photon} \rightarrow atom^*$$

- If B_{12} is the probability (per unit time) of absorption of radiation, the population of the upper level increases. The rate is clearly proportional to the population of atoms in the lower level and to the energy density $\rho(\nu)$ of radiation in the system. Thus the rate of increase of population of the excited state is given by

$$R_{abs} \propto \rho(\nu) N_1$$

$$R_{abs} = B_{12}\, \rho(\nu)\, N_1$$

Where B_{12} is a constant of proportionality.

Einstein coefficients

❑ Spontaneous Emission –

- An excited atom can stay at the excited level for an average life time (⟦10⟧^(-8) sec). If this atom is not stimulated by any other agent during its short lifetime, the excited atom undergoes a transition to the lower level on its own.

  This process is represented as

$$atom^* \rightarrow \mathrm{atom} + \mathrm{photon}$$

- The population of the upper level will decrease due to spontaneous transition to the lower level with emission of radiation. The rate of emission will depend on the population of the upper level. If A_{21} is the probability that an atom in the excited state will spontaneously decay to the ground state,

$$R_{sp} \propto N_2$$

$$R_{sp} = A_{21} N_2$$

A_{21} – Einstein coefficient for spontaneous emission

Einstein coefficients

❑ Stimulated Emission –

- Stimulated or induced emission depends on the number of atoms in the excited level as well as on the energy density of the incident radiation.
- An atom in the excited state need not wait for spontaneous emission to occur. If a photon with appropriate energy $(h\vartheta = \mathrm{E}_2 - \mathrm{E}_1)$ interacts with the excited atom, it can trigger the atom to undergo transition to the lower level and to emit another photon.

The process may be represented as

$$atom^* + \text{ photon} \rightarrow \text{atom } + \text{ 2 photons}$$

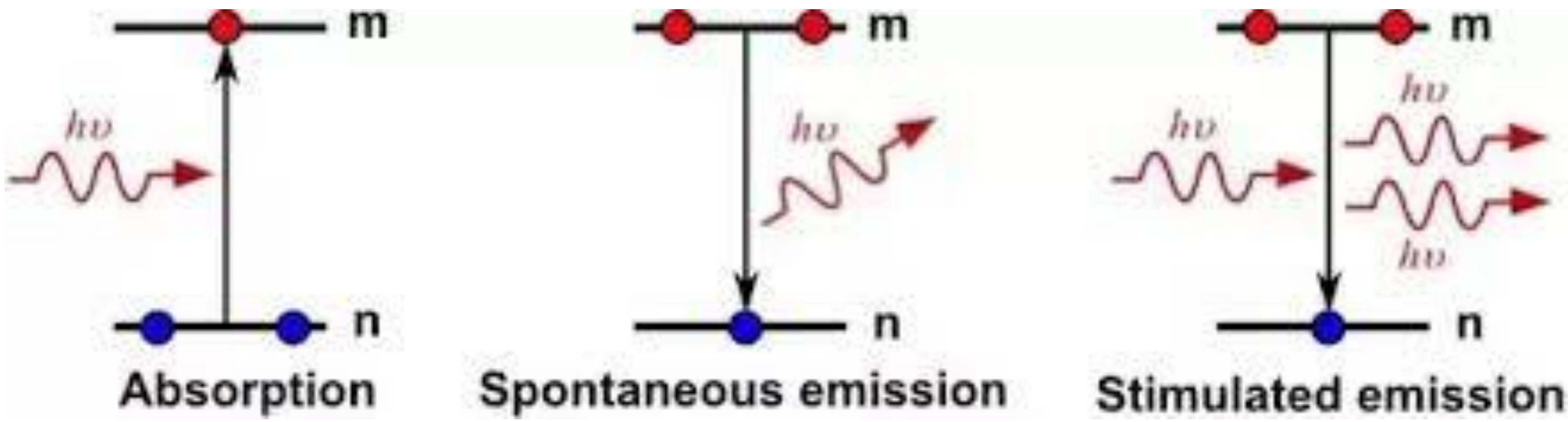
The rate of stimulated emission of photons is given by

$$R_{st} \propto \rho(\nu)N_2$$

$$R_{st} = B_{21}\, \rho(\nu)N_2$$

B_{21} – Einstein coefficient for stimulated emission

Einstein coefficients



RATE OF EACH OF THESE PROCESSES

$$R_{abs} \propto \rho(\nu) N_1 \qquad R_{sp} \propto N_2 \qquad R_{st} \propto \rho(\nu) N_2$$

Introducing Einstein Coeficients (A- Spont B – Stim)

$$R_{abs} = B_{12}\, \rho(\nu)\, N_1 \qquad R_{sp} = A_{21}\, N_2 \qquad R_{st} = B_{21}\, \rho(\nu) N_2$$

AT EQUILIBRIUM : $R_{12} = R_{21}$

$$N_1\, B_{12}\, \rho(\nu) = N_2\, [A_{21} + B_{21}\, \rho(\nu)]$$

Einstein coefficients

- If thermodynamic equilibrium exists, the number of atoms N_2 at energy level E_2, the number of atoms N_1 at energy level E_1, and the number of photons in the radiation field will all remain constant.
- Thus, The number of atoms absorbing photons per second per unit volume = The number of atoms emitting photons per second per unit volume.

  - ✓ The number of atoms absorbing photons per second per unit volume = $B_{12}\rho(\vartheta)N_1$
  - ✓ The number of atoms emitting photons per second per unit volume = $A_{21}N_2 + B_{21}\rho(\vartheta)N_2$
  - ✓ In equilibrium condition,

$$B_{12}\rho(\vartheta)N_1 = A_{21}N_2 + B_{21}\rho(\vartheta)N_2$$
$$\rho(\vartheta)[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

Einstein coefficients

$$\rho(\vartheta)[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\rho(\vartheta) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]}$$

$$= \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{N_1}{N_2}\right)\frac{B_{12}}{B_{21}} - 1}$$

Boltzman Distribution

$$\frac{N_1}{N_2} = \frac{e^{\frac{-E_1}{KT}}}{e^{\frac{-E_2}{KT}}}$$

$$E_2 - E_1 = h\nu$$

Einstein proved thermodynamically, $B_{12} = B_{21}$

$$\rho(\vartheta) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{N_1}{N_2} - 1}\right]$$

Einstein coefficients

The equilibrium distribution of atoms among different energy states is given by Boltzmann's law according to which

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} \frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

$$E_2 - E_1 = h\nu$$

$$= e^{-(h\vartheta)/kT}$$

$$\rho(\vartheta) = \frac{A_{21}}{B_{21}} \left[\frac{1}{e^{h\vartheta/kT} - 1} \right]$$

This is the formula for the energy density of photon of frequency ϑ in equilibrium with atoms in energy states 1 and 2, at temperature T.

Einstein coefficients

$$\rho(\vartheta) = \frac{A_{21}}{B_{21}}\left[\frac{1}{e^{h\vartheta/kT} - 1}\right]$$

Comparing it with Planck's radiation formula

Planck's Radiation Law: $$\rho(\vartheta) = \frac{8\pi h\vartheta^3}{c^3}\left[\frac{1}{e^{h\vartheta/kT} - 1}\right]$$

Thus we get,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{c^3}$$

This gives the relationship between Einstein's A and B coefficients.

$$\boxed{B_{12} = B_{21}}$$

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{C^3}}$$

Einstein Relations

The interpretation of Einstein Relations

- All the 3 Einstein Coefficients are interrelated

$\boxed{B_{12} = B_{21}}$ $\boxed{\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h \upsilon^3}{C^3}}$

- The rates differ depending upon the population densities N_2 and N_1.

$$R_{st} = B_{21}\rho(\vartheta)N_2 \text{ and } R_{abs} = B_{12}\rho(\vartheta)N_1$$

$N_2 > N_1$ leads to increase in $\rho(\vartheta)$ and hence, amplification.

$N_1 > N_2$ leads to decrease in $\rho(\vartheta)$ and hence, attenuation.

- For laser to operate, it is necessary that $N_2 > N_1$. This is the condition of population inversion.

- $\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h \upsilon^3}{C^3} \Rightarrow \dfrac{B_{21}}{A_{21}} \alpha \dfrac{1}{\upsilon^3}$ -High frequency (Short wavelength) -Lasers are difficult to operate

- Although Einstein relations are derived from equilibrium condition, they are valid for any general condition as they are related to characteristics of the atom.