Engineering Physics

(PHY1701)

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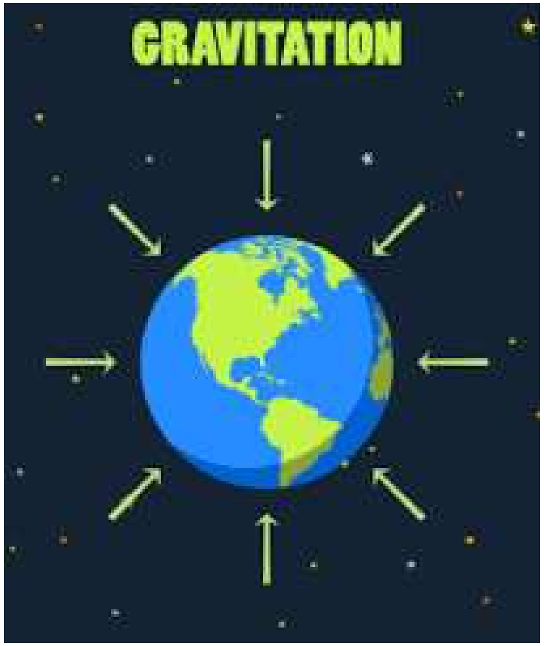
Contents

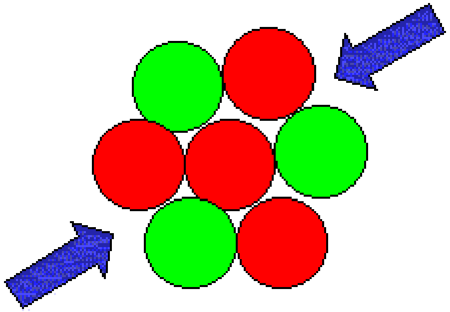
- Physics of Divergence, Gradient and Curl (DJG 13-20),
- Qualitative understanding of surface and volume integral (DJG 24, 26, 27),
- Maxwell Equations (Qualitative) (DJG 232, 321-327),
- Wave Equation (Derivation) (DJG 364-366), &
- EM Waves, Phase velocity, Group velocity, Group index*, Wave guide (Qualitative) (DJG 405)

❖ William Silfvast, Laser Fundamentals, 2008, Cambridge University Press.

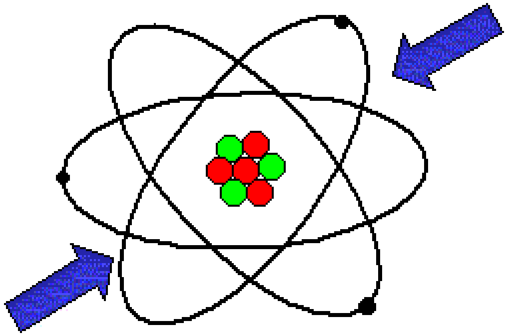
Fundamental Forces

1. Gravitation
2. Electromagnetism
3. Strong Nuclear
4. Weak Nuclear

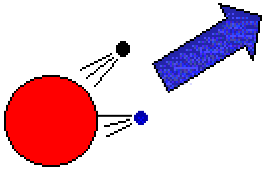




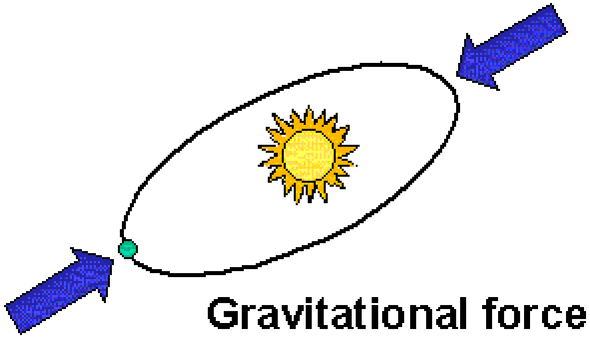
Strong force binds the nucleus



Electromagnetic force binds atoms

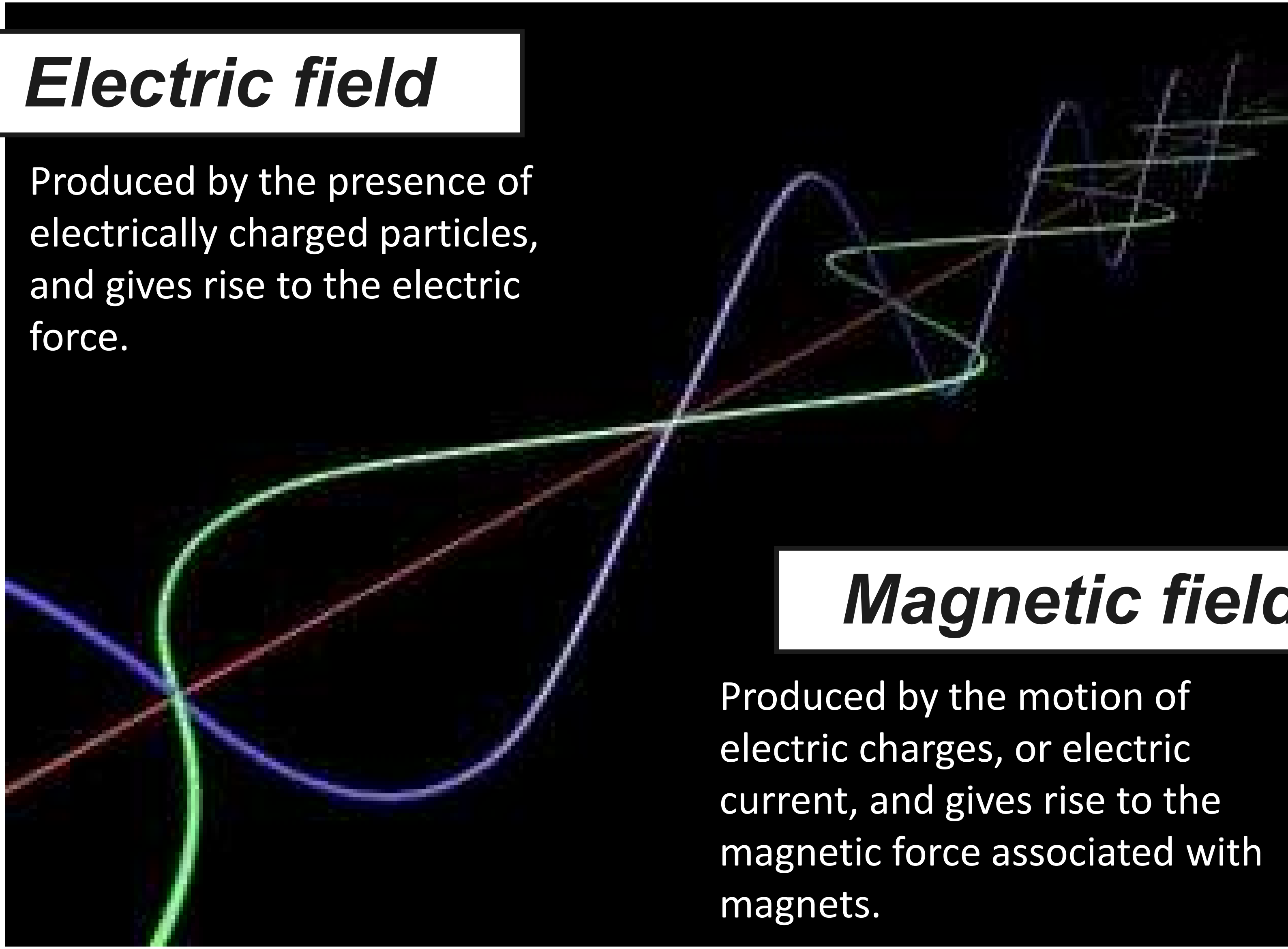


Weak force in radioactive decay



Gravitational force binds the solar system

What is Electromagnetics?



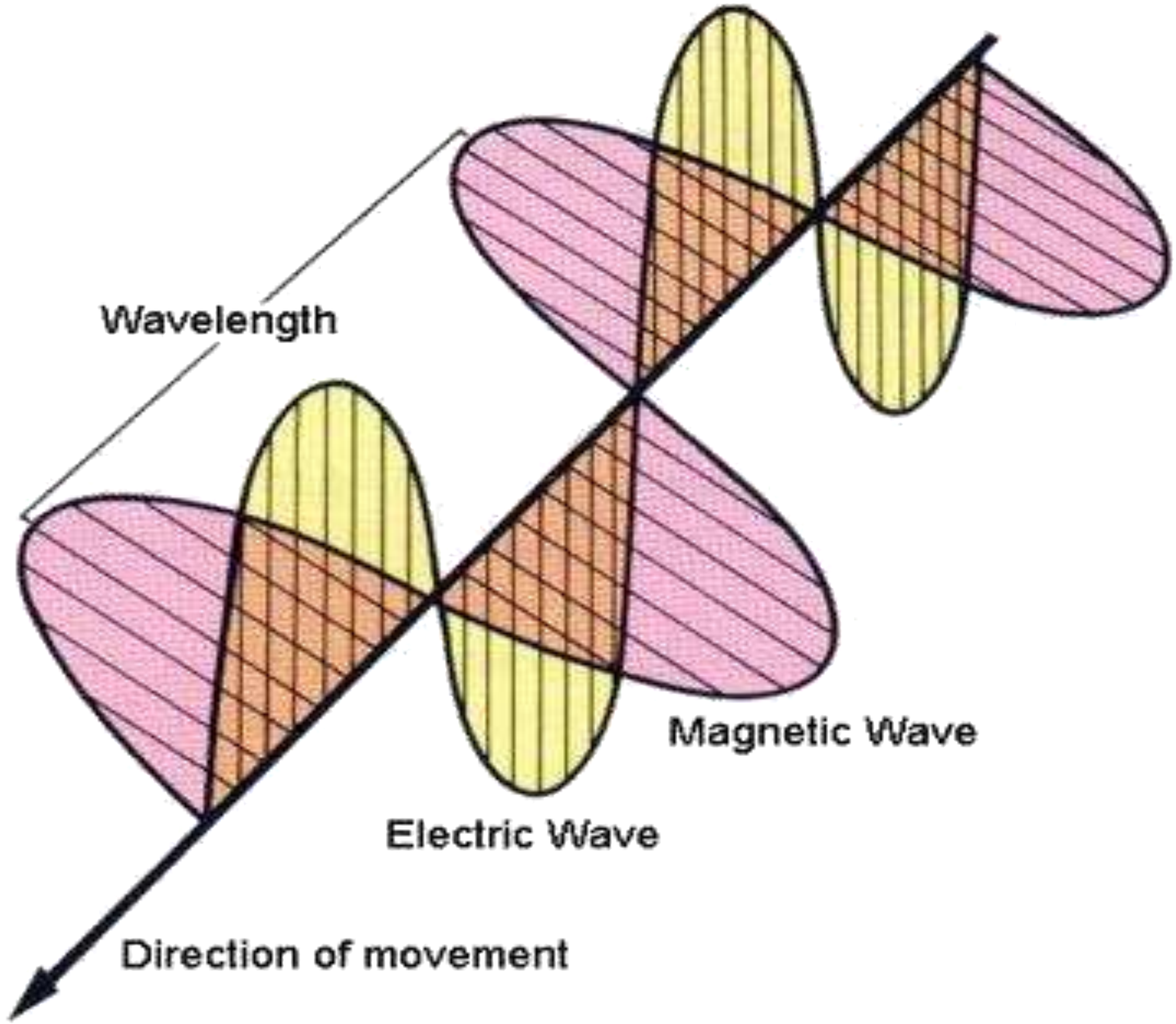
Electric field

Produced by the presence of electrically charged particles, and gives rise to the electric force.

Magnetic field

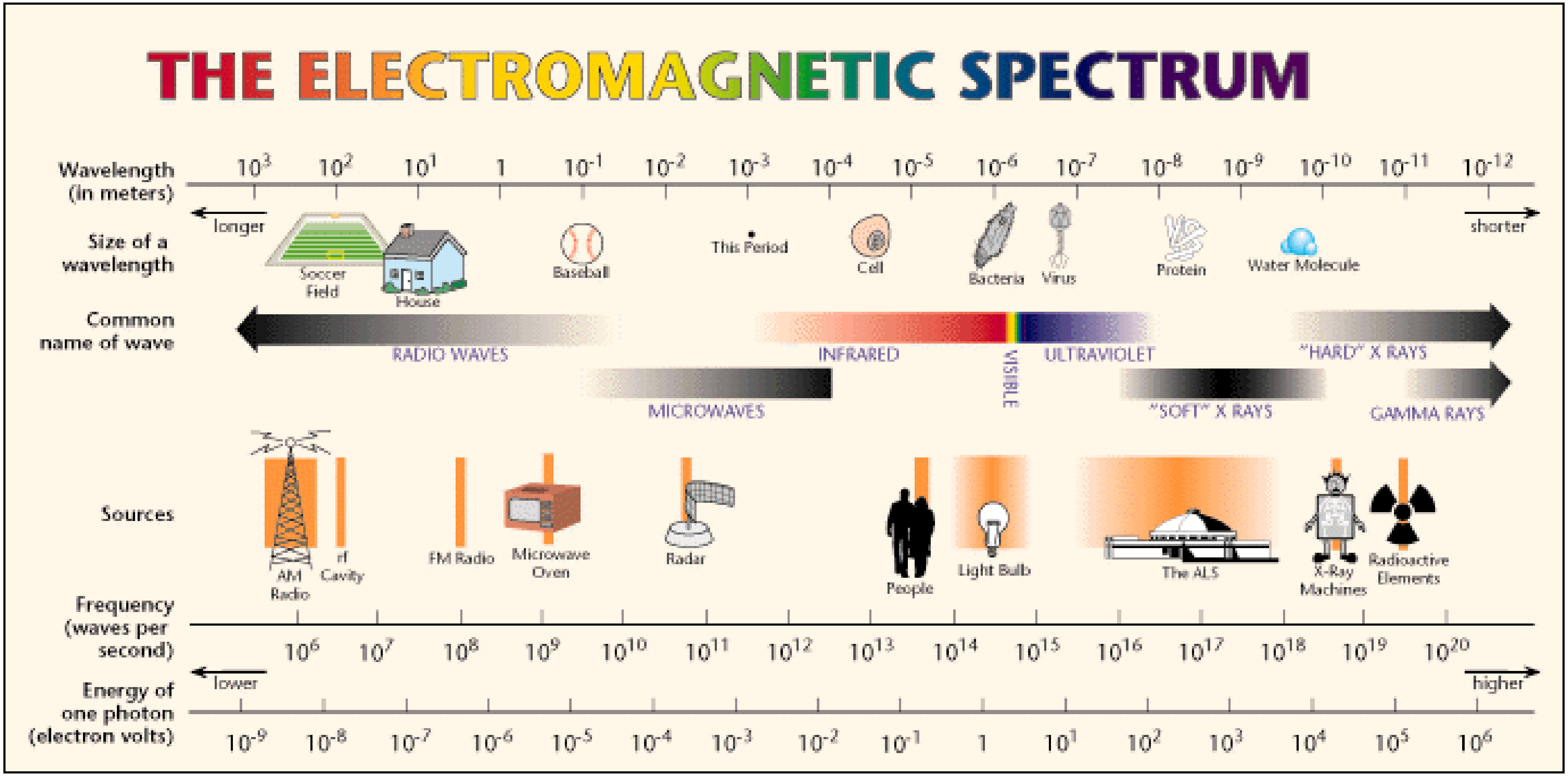
Produced by the motion of electric charges, or electric current, and gives rise to the magnetic force associated with magnets.

Electromagnetic Wave Spectrum



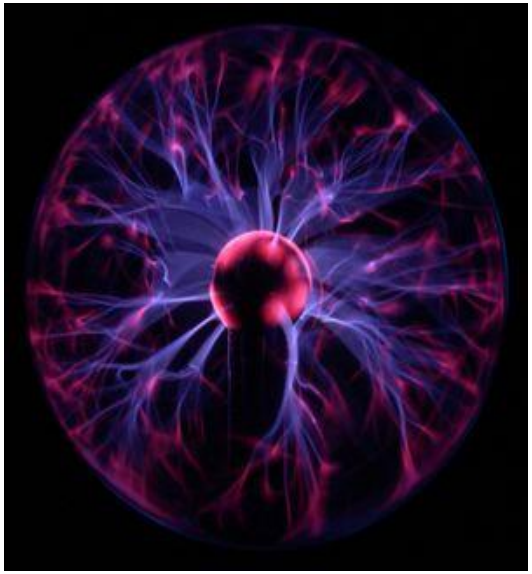
Why do we learn Engineering Electromagnetics

- Electric and magnetic field exist nearly everywhere.



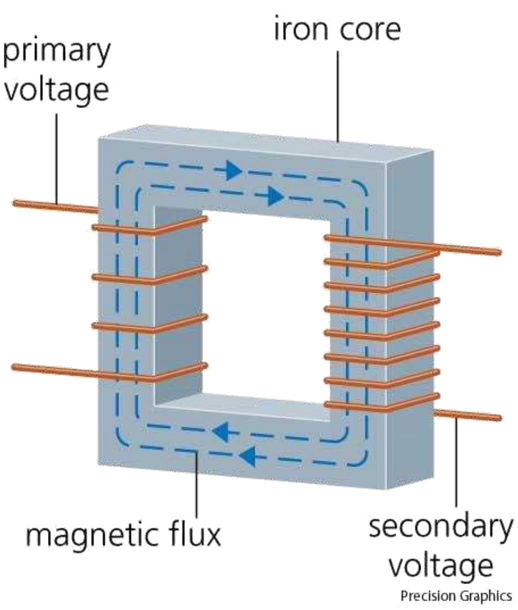
Applications

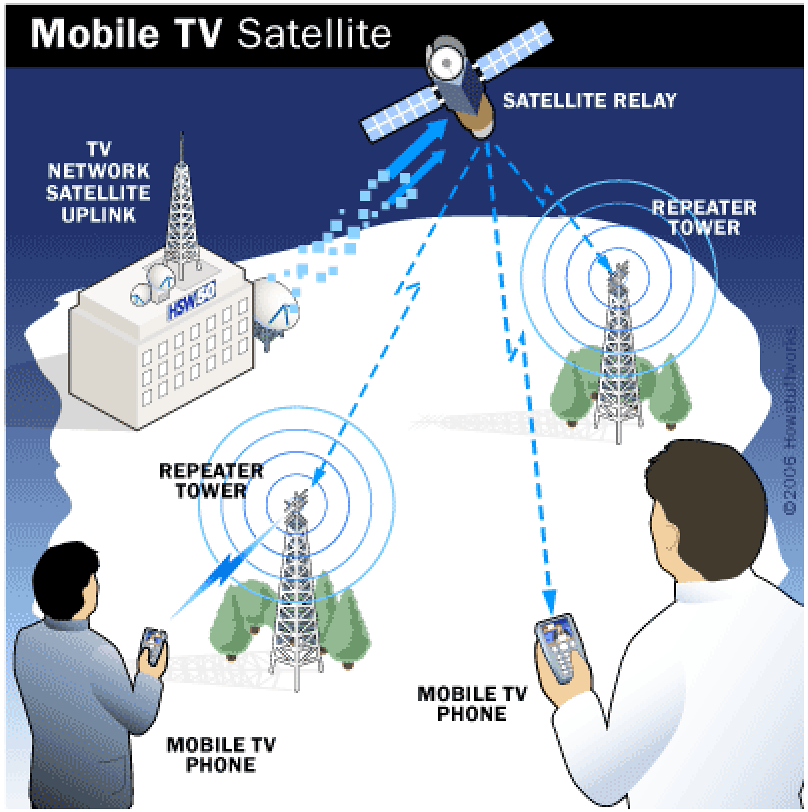
- Electromagnetic principles find application in various disciplines such as microwaves, x-rays, antennas, electric machines, plasmas, etc.



Applications

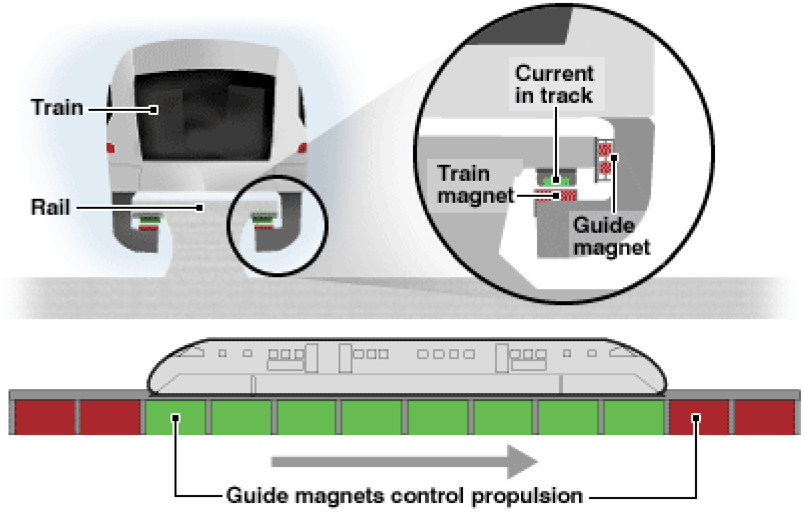
- Electromagnetic fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operation.
- Electromagnetic devices include transformers, radio, television, mobile phones, radars, lasers, etc.

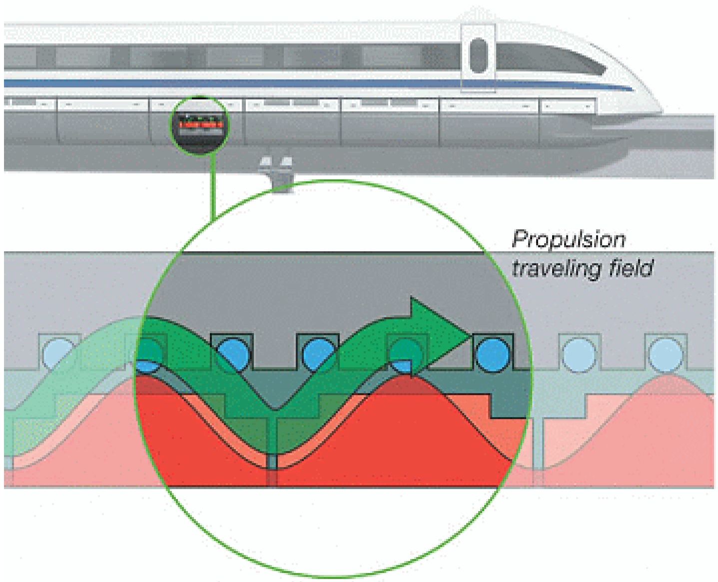




Applications

Transrapid Train





- **A magnetic traveling field moves the vehicle without contact.**
- **The speed can be continuously regulated by varying the frequency of the alternating current.**

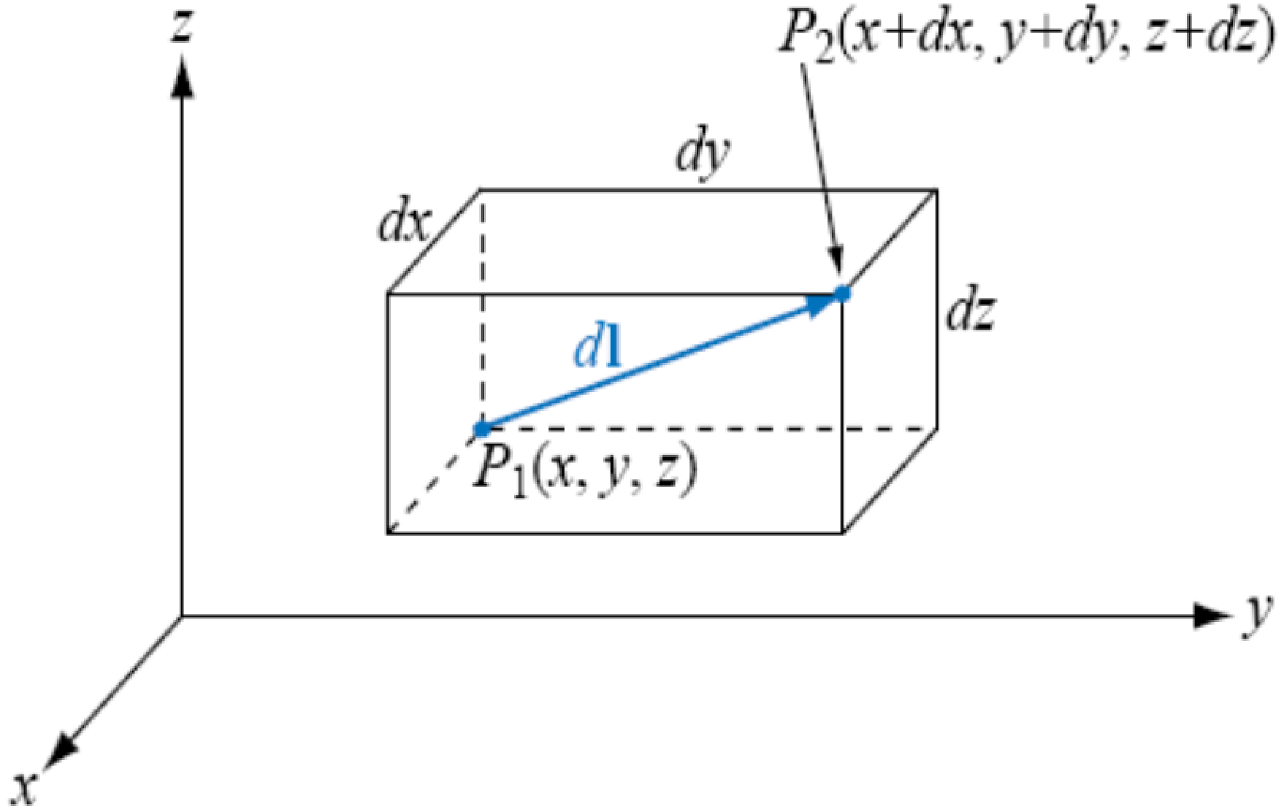
Scalars and Vectors

- Scalar refers to a quantity whose value may be represented by a single (positive or negative) real number.
- Some examples include distance, temperature, mass, density, pressure, volume, and time.
- A vector quantity has both a magnitude and a direction in space. We especially concerned with two- and three-dimensional spaces only.
- Displacement, velocity, acceleration, and force are examples of vectors.

- Scalar notation: *A* or A (*italic* or plain)
- Vector notation: **A** or $\vec{A}$ (bold or plain with arrow)

Gradient

Suppose $T_1(x,y,z)$ is the temperature at $P_1(x,y,z)$, and $T_2(x+dx,y+dy,z+dz)$ is the temperature at P_2 as shown.



Gradient

The differential distances dx, dy, dz are the components of the differential distance vector $d\mathbf{L}$:

$$d\mathbf{L} = dx\mathbf{a}_x + dy\mathbf{a}_y + dz\mathbf{a}_z$$

However, from differential calculus, the differential temperature:

$$dT = T_2 - T_1 = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

Gradient

But, $dx = d\mathbf{L} \bullet \mathbf{a}_x$

$$dy = d\mathbf{L} \bullet \mathbf{a}_y$$

$$dz = d\mathbf{L} \bullet \mathbf{a}_z$$

So, previous equation can be rewritten as:

$$dT = \frac{\partial T}{\partial x}\mathbf{a}_x \bullet d\mathbf{L} + \frac{\partial T}{\partial y}\mathbf{a}_y \bullet d\mathbf{L} + \frac{\partial T}{\partial z}\mathbf{a}_z \bullet d\mathbf{L}$$

$$= \left(\frac{\partial T}{\partial x}\mathbf{a}_x + \frac{\partial T}{\partial y}\mathbf{a}_y + \frac{\partial T}{\partial z}\mathbf{a}_z \right) \bullet d\mathbf{L}$$

Gradient

- Gradient of a scalar field is its slope or rate of change.
- The term is called the gradient of *f* and represents the maximum
  slope or rate of change of *f* in any direction
- The term is called the gradient of *f* and represents the maximum slope or rate of change of *f* in any direction.

$$df(x,y,z)=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial z}dz$$

$$=\left(\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z}\right).\left(\hat{x}\ dx+\hat{y}\ dx+\hat{z}\ dz\right)$$

$$=\vec{\nabla}f\,.\,d\vec{l}$$

Gradient

The vector inside square brackets defines the change of temperature dT corresponding to a vector change in position $d\mathbf{L}$.

This vector is called *Gradient of Scalar **T***.

For Cartesian coordinate:

$$\nabla V = \frac{\partial V}{\partial x}\mathrm{a}_x + \frac{\partial V}{\partial y}\mathrm{a}_y + \frac{\partial V}{\partial z}\mathrm{a}_z$$

Gradient

- The Gradient of a scalar function ϕ is a vector whose Cartesian components are,

$$\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \quad \text{and} \quad \frac{\partial \phi}{\partial z}$$

Then grad ϕ is given by,

$$\text{Grad}\,\phi = \nabla\phi = \vec{\mathrm{i}}\,\frac{\partial \phi}{\partial x} + \vec{j}\,\frac{\partial \phi}{\partial y} + \vec{k}\,\frac{\partial \phi}{\partial z}$$

Gradient

- The magnitude of this vector gives the maximum rate of change of the scalar field and directed towards the maximum change occurs.

- The electric field intensity at any point is given by,
  E* = – grad *V = negative gradient of potential

- The negative sign implies that the direction of *E* opposite to the direction in which *V* increases.

Del Operator

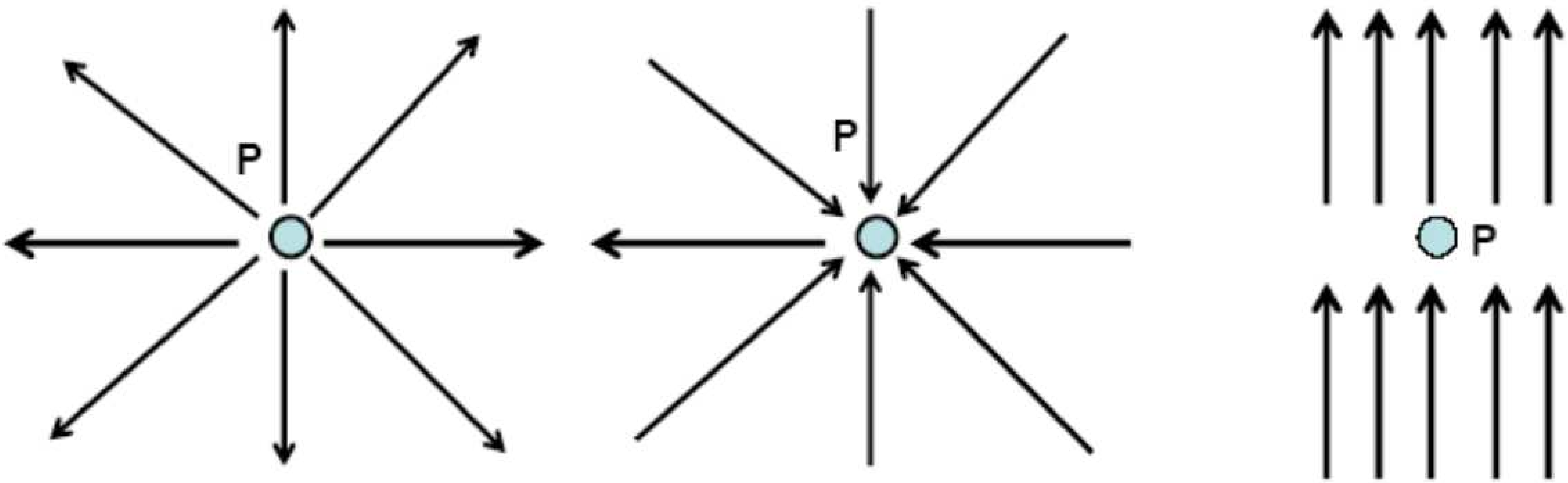
- One can thus define a vector operator "Del" which is defined as:

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

- This vector operator can operate on a scalar or vector function
  - $\vec{\nabla} S$: Gradient
  - $\vec{\nabla} \cdot \vec{A}$: Divergence
  - $\vec{\nabla} \times \vec{A}$: Curl

Divergence

Illustration of the divergence of a vector field at point P:

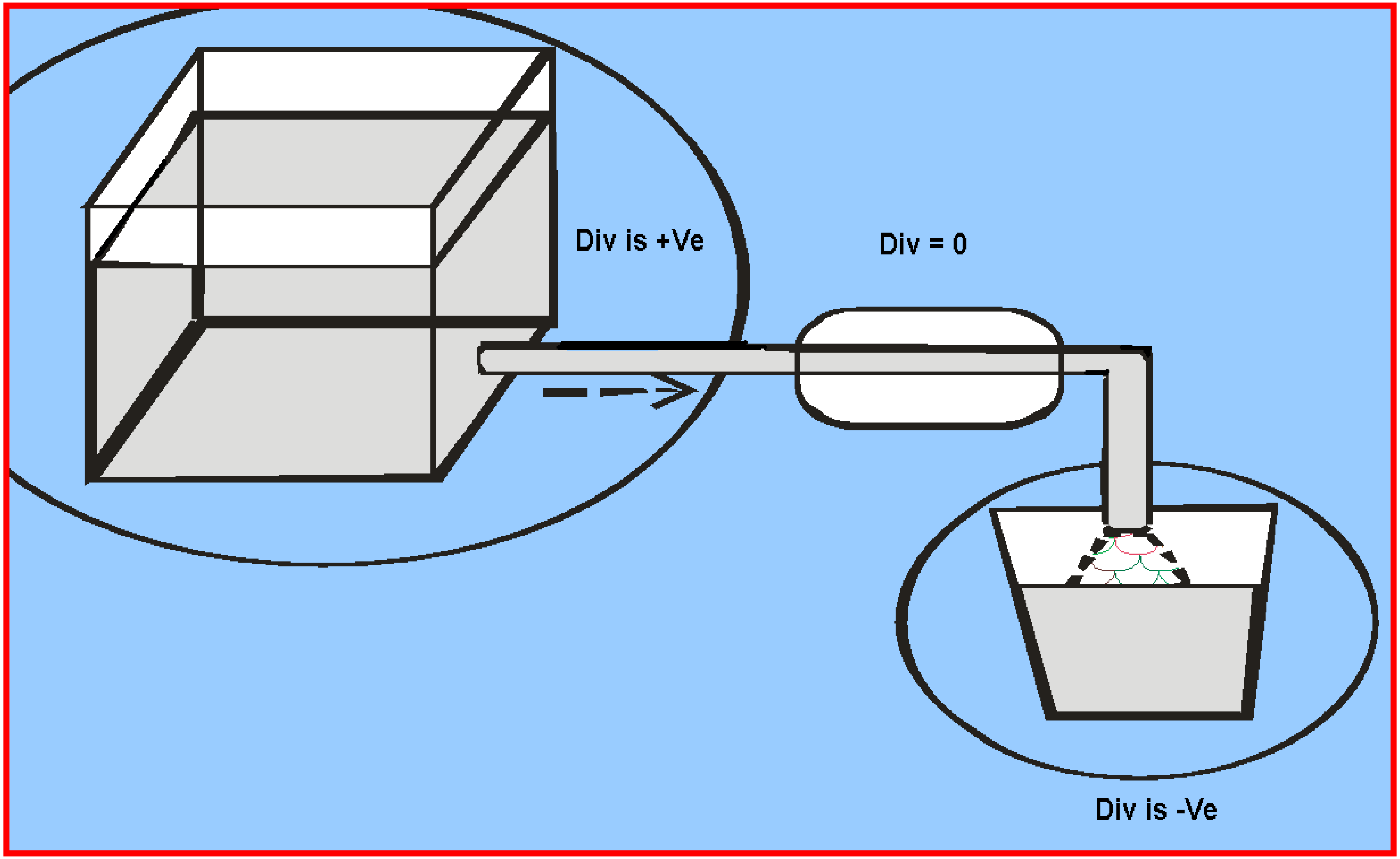


Positive Divergence

Negative Divergence

Zero Divergence

Example for Divergence



Physical significance of divergence

- Physically the divergence of a vector quantity represents the rate of change of the field strength in the direction of the field.
- If the divergence of the vector field is positive at a point then something is diverging from a small volume surrounding with the point as a source.
- If it negative, then something is converging into the small volume surrounding that point is acting as sink.
- if the divergence at a point is zero then the rate at which something entering a small volume surrounding that point is equal to the rate at which it is leaving that volume.
- The vector field whose divergence is zero is called ***solenoidal***

Divergence

The *divergence of* $\boldsymbol{A}$ at a given point $\mathbf{P}$ is the outward flux per unit volume:

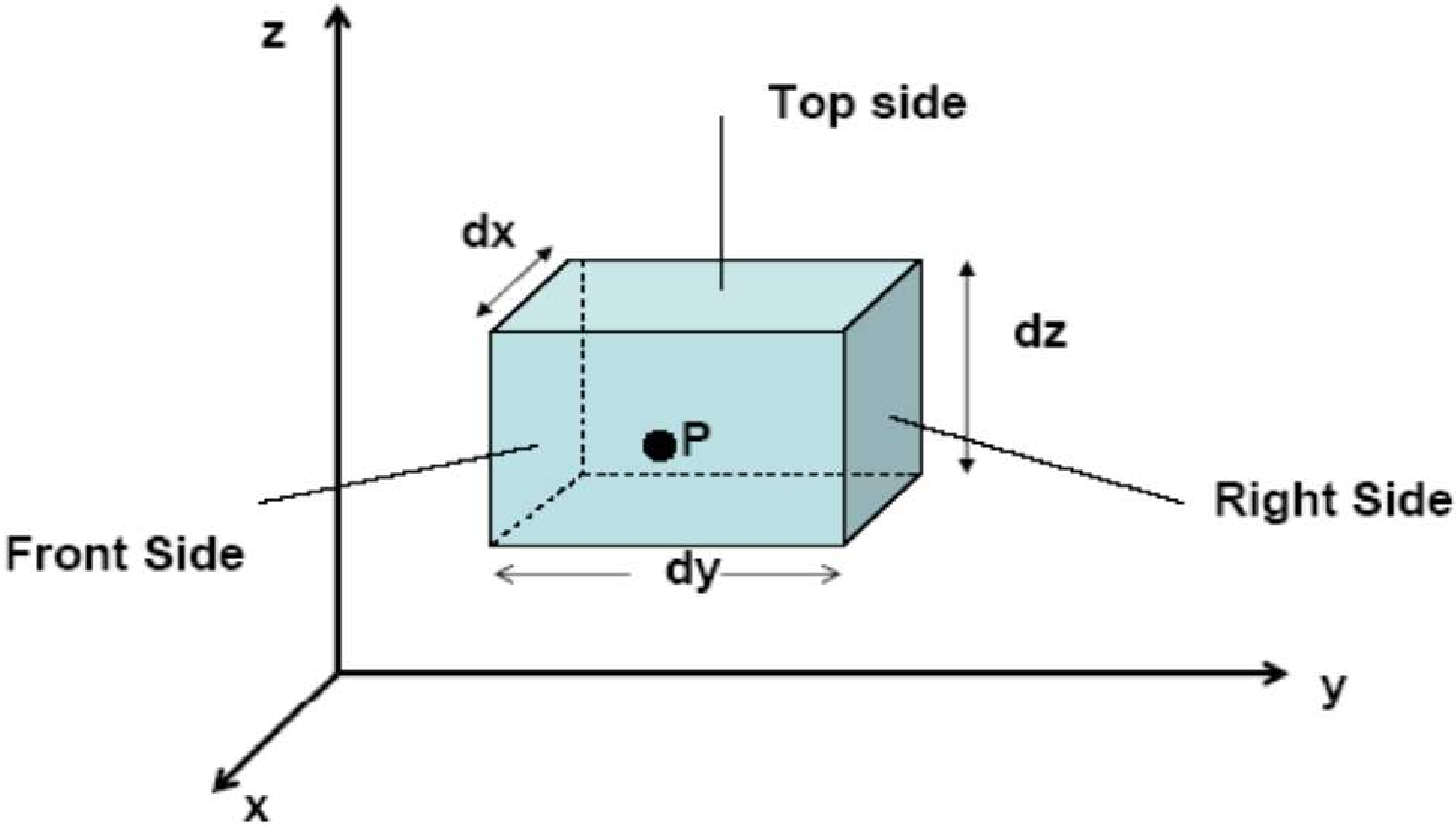
$$div\ \mathbf{A} = \nabla \bullet \mathbf{A} = \lim_{\Delta v \to 0} \frac{\oint_s \mathbf{A} \bullet dS}{\Delta v}$$

div $\mathbf{V} = \nabla \bullet \mathbf{V} =$ $$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

Divergence

What is $\oint_S \mathbf{A} \bullet dS$??

Vector field **A** at closed surface **S**

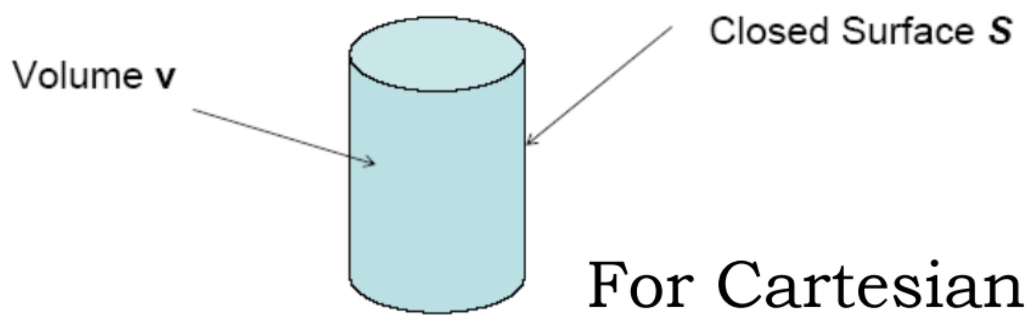


Divergence

Where,

$$\oint_s \mathbf{A} \bullet dS = \left(\int_{front} + \int_{back} + \int_{left} + \int_{right} + \int_{top} + \int_{bottom} \right) \mathbf{A} \bullet dS$$

And, $\mathbf{v}$ is *volume* enclosed by surface **S**

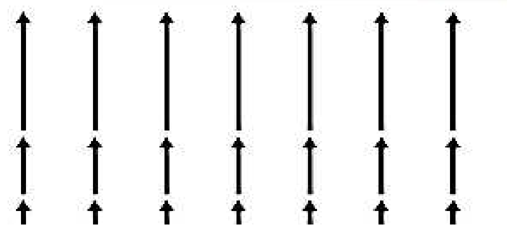


For Cartesian coordinate:

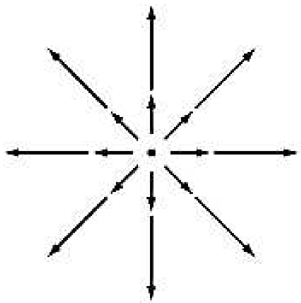
$$\nabla \bullet \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence

- Divergence gives a measure of how much a vector function spreads out.
- For $\vec{A} = x\,\hat{x},\ \vec{\nabla}\cdot\vec{A} = \dfrac{\partial x}{\partial x} = 1$.



- For $\vec{A} = \vec{r},\ \vec{\nabla}\cdot\vec{A} = 3.$



- For $\vec{A} = \dfrac{\hat{r}}{r^2},\ \vec{\nabla}\cdot\vec{A} = 0.$ This is precisely the form of electric field of a point charge and its divergence cannot be zero from Gauss's law. There is clearly something fishy here, we will come back to it later.