M 5: EM Theory

Maxwell's Equations:

Maxwell's equations describe how an electric field can generate a magnetic field and vice-versa. These equations describe the relationship and behaviour of electric and magnetic fields.

- Maxwell gave a set of 4 equations which are known as Maxwell's equations. According to Maxwell equations:-
- A flow of electric current will generate magnetic field and if the current varies with time magnetic field will also give rise to an electric filed.
  - I equation (1) describes the surface integral of electric field.
  - II equation (2) describes the surface integral of magnetic field.
  - III equation (3) describes the line integral of electric field.
  - IV equation (4) describes line integral of magnetic field.

$\oint_S E.dS = \frac{q}{\varepsilon_0}$	**Gauss law of electrostatics**
$\oint_S B.dS = 0$	**Gauss law of Magnetism**
$\int E.dl = -\frac{d\phi_B}{dt}$	**Faraday's Law of electromagnetic induction**
$\int B.dl = \mu_0 i = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t} \right]$	**Ampere's Law**

- Maxwell was the first to determine the speed of propagation of EM waves is same as the speed of light.
- Experimentally it was found that:- $c = 1/\sqrt{\mu_0 \epsilon_0}$, where μ_0(permeability) and ε_0(permittivity) and c= velocity of light.
- Maxwell's equations show that the electricity, magnetism and ray optics are all inter-related to each other.

Importance of Maxwell's Equations:

1. The first law states that the total electric displacement flux passing through a closed surface (Gaussian surface) is equal to the total charge inside the surface.

2. The second law states that total magnetic flux passing through any closed surface is zero.

3. The third law states that electromotive force around a closed path is equal to the minus of the time derivative of the magnetic flux flowing through any surface bounded by the path

   or

   It can also be stated that the electromotive force around a closed path is equal to the inflow of magnetic current through any surface bounded by the path.

4. The fourth Maxwell's Equation states that the magnetomotive force around a closed path is equal to the sum of electric displacement and conduction currents through any surface bounded by the path.

MAXWELL'S ELECTROMAGNETIC EQUATIONS:

Maxwell formulated the basic laws of electricity and magnetism in the form of four fundamental equations. These equations are known as Maxwell's equation. The four laws of electricity and magnetism are

i. Gauss law of electrostatics
ii. Gauss law of magnetism
iii. Faraday's law of electromagnetic induction
iv. Ampere's law of magnetic field due to steady currents.

The integral form of these equations are given by

$$\oint_S E.dS = \frac{q}{\varepsilon_0} \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

$$\oint_S B.dS = 0 \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (2)$$

$$\int E.dl = -\frac{d\phi_B}{dt} \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (3)$$

$$\int B.dl = \mu_0 i = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t} \right] \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (4)$$

The differential form of Maxwell equation are given by

$$divE = \frac{\rho}{\varepsilon} \quad or \quad \nabla.E = \frac{\rho}{\varepsilon} \qquad \_\_\_\_\_\_\_\_\_\_ (5)$$

$$divB = 0 \quad or \quad \nabla.B = 0 \qquad \_\_\_\_\_\_\_\_ (6)$$

$$curl\ E = -\frac{\partial B}{\partial t} \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (7)$$

$$curl\ B = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t} \right] \qquad \_\_\_\_\_\_\_\_\_\_\_\_ (8)$$

<u>Derivation of Maxwell's equations:</u> The differential forms of Maxwells equations can be obtained from the integral forms as follows:

i. **<u>Maxwell's first equation:</u>** The integral form of Maxwell's first equation is given by

$$\oint_S E.dS = \frac{q}{\varepsilon_0}$$

This is Gauss law of electricity. If ρ be the charge density and dV be the small volume then $q = \int_V \rho\, dV$ $\left(\because \rho = \frac{q}{V} \Rightarrow \rho = \frac{q}{\int dV} \right)$

$$\therefore \oint_S E.dS = \frac{1}{\varepsilon_0} \int \rho dV$$

$$\oint_S \varepsilon_0 E.dS = \int \rho dV$$

$$\oint_S D.dS = \int \rho dV \qquad (\because \varepsilon_0 E = D \rightarrow displacement \;\; current) \_\_\_\_\_\_(1)$$

But according to Gauss divergence theorem

$$\oint_S F.dS = \int_V divF \; dV$$

Hence, $$\oint_S D.dS = \int_V (\nabla.D) \; dV \qquad \_\_\_\_\_\_\_\_ (2)$$

From Eq. (1) and (2)

$$\boxed{\nabla.D = \rho \qquad (or) \qquad \nabla.E = \frac{\rho}{\varepsilon_0} \quad (or) \quad divE = \frac{\rho}{\varepsilon_0}} \qquad \_\_\_\_\_\_\_\_ (3)$$

ii. **<u>Maxwell's second equation:</u>** According to Gauss law of magnetism

$$\oint_S B.dS = 0 \qquad \_\_\_\_\_\_\_\_ (4)$$

But according to Gauss divergence theorem

$$\oint_S B.dS = \int_V divB \; dV \qquad \_\_\_\_\_\_\_\_ (5)$$

Hence, From Eq. (4) & (5)

$$\int_V (\nabla.B) \; dV = 0$$

$$\boxed{\nabla.B = 0} \qquad \_\_\_\_\_\_\_\_ (6)$$

iii. **<u>Maxwell's third equation:</u>** According to Faraday's law of electromagnetic induction

$$\int E.dl = -\frac{d\phi_B}{dt} = -\frac{\partial}{\partial t}\int_S B.dS$$

$$= -\int_S \frac{\partial B}{\partial t}.dS \qquad ----------(7)$$

Applyinf Stokes Theorem

$$\int E.dl = \int_S (\nabla \times E).dS \qquad ------------(8)$$

From Eq. (7) & (8)

$$\int_S (\nabla \times E).dS = -\int_S \frac{\partial B}{\partial t}.dS$$

$$\therefore \boxed{\nabla \times \overline{E} = -\frac{\partial B}{\partial t} \quad or \quad curl\ \mathrm{E} = -\frac{\partial B}{\partial t}}$$ --------------(9)

iv. **<u>Maxwell's four equation:</u>** From Ampere's law

$$\int B.dl = \mu_0 i \quad \text{(but } i = \frac{dq}{dt} \Rightarrow J = \frac{i}{A} \Rightarrow J.A = i \;\; or \;\; J.dS = i\text{)}$$

Using Stokes theorem

$$\int B.dl = \int_S (\nabla \times B).dS$$

$$\therefore \int_S (\nabla \times B).dS = \mu_0 \int_S J.dS \qquad (\because J = \frac{i}{A})$$

i.e., $\nabla \times B = \mu_0 J$

Replacing J by $\left[J + \varepsilon_0 \frac{\partial E}{\partial t}\right]$ we get,

$$\boxed{\nabla \times B = \mu_0 \left[J + \varepsilon_0 \frac{\partial E}{\partial t}\right]}$$ --------------(9)