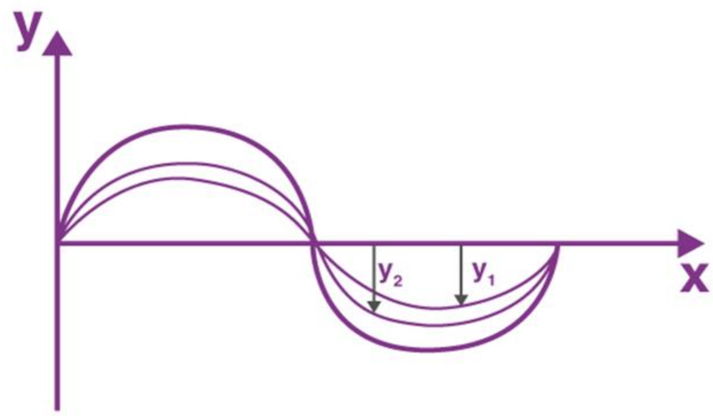
Superposition of waves

- According to the principle of superposition. The resultant displacement of a number of waves in a medium at a particular point is the vector sum of the individual displacements produced by each of the waves at that point.
- The principle of superposition may be applied to waves whenever two (or more) waves travelling through the same medium at the same time. The waves pass through each other without being disturbed. The net displacement of the medium at any point in space or time, is simply the sum of the individual wave displacements. This is true of waves which are finite in length (wave pulses) or which are continuous sine waves.



Conditions for superposition

- The principle of superposition can be applied to any type of wave providing that:
    - The waves being superposed are of the same type (e.g. all are electromagnetic waves)
    - The medium that the waves are propagating through behaves linearly, i.e. when part of the medium has twice the displacement then it has twice the restoring force. This is usually true when the amplitudes are relatively small. For example, for waves on water, it is a good approximation for small ripples on a pond whose amplitude is much smaller than their wavelength.
    - If the waves are also coherent - i.e. if they all have the same frequency and a constant phase difference - then the superposition resembles another wave with the same frequency. We will explore this case in the next section.

Principle of Superposition of Waves

- Considering two waves, travelling simultaneously along the same stretched string in opposite directions as shown in the figure above. We can see images of waveforms in the string at each instant of time. It is observed that the net displacement of any element of the string at a given time is the algebraic sum of the displacements due to each wave.
- The resultant displacement $y\,(x,t) = y_1(x,t) + y_2(x,t)$
- Wave functions of the moving waves are

$$y_1 = f_1(x-vt),$$
$$y_2 = f_2(x-vt)$$
$$\dots\dots\dots...$$
$$y_n = fn\,(x-vt)$$

- The wave function describing the disturbance in the medium can be described as

$$y = f_1(x - vt) + f_2(x - vt) + \dots + fn(x - vt)$$

or, $y = \sum_{i\ =\ 1\ to\ n\ =} fi\ (x - vt)$

- Let us consider a wave travelling along a stretched string given by, $y_1(x,t) = A\sin(kx - \omega t)$ and another wave, shifted from the first by a phase ϕ, given as $y_2(x,t) = A\sin(kx - \omega t + \varphi)$
- From the equations we can see that both the waves have the same angular frequency, same angular wave number k, hence the same wavelength and the same amplitude A.
- Now, applying the superposition principle, the resultant wave is the algebraic sum of the two constituent waves and has displacement $y(x,t) = A\sin(kx - \omega t) + A\sin(kx - \omega t + \varphi)$
- The above equation can be written as,

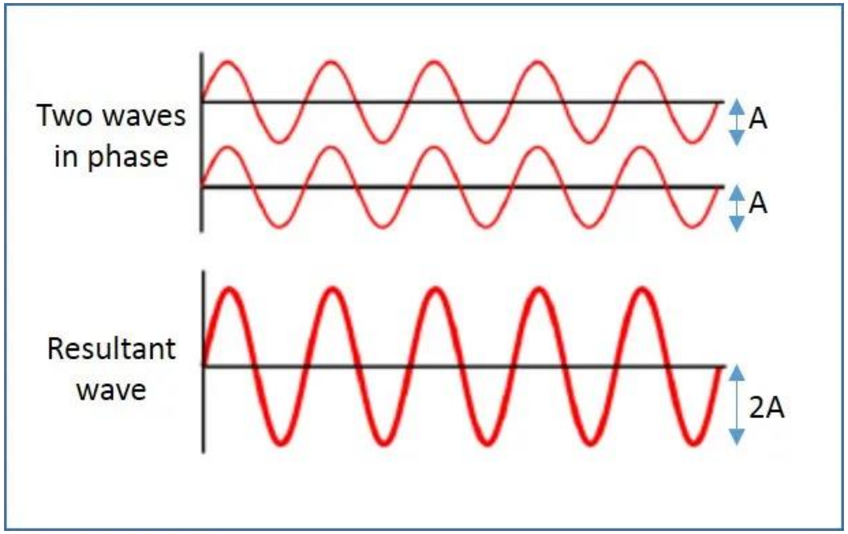
$$y(x,t) = 2A\cos(\phi/2).\sin(kx - \omega t + \phi/2)$$

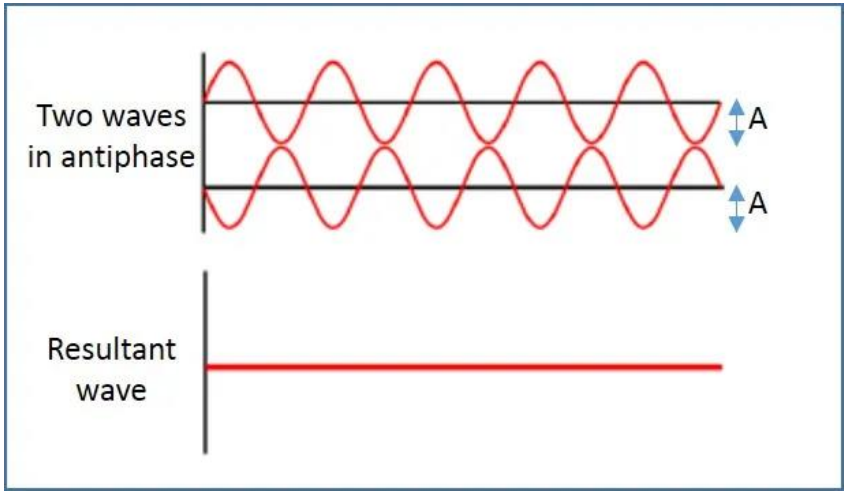
- The resultant wave is a sinusoidal wave, travelling in the positive X direction, where the phase angle is half of the phase difference of the individual waves and the amplitude as [2cos φ/2] times the amplitudes of the original waves.

Constructive and destructive interference

Consider two waves that arrive in phase as shown in Figure. Their crests arrive at exactly the same time. Hence, they interfere constructively. A resultant wave is produced, which has a crest much larger than the two individual waves, and the troughs are deeper.

If the two incoming waves that are in phase have amplitude of A, then the resultant wave has an amplitude of 2A. The frequency of the resultant is the same as that of the incoming waves.





Consider two waves that arrive in antiphase (with a phase difference of π or 180°) as illustrated in Figure. The crest of one wave and the trough of another wave arrive at exactly the same time. Hence, they interfere destructively. A resultant wave is produced, which has a smaller amplitude. If the two incoming waves that are in antiphase have amplitude of A, then the resultant wave has an amplitude of zero. The frequency of the resultant is the same as that of the incoming waves.