Problem 1

Question: A travelling wave propagates according to the expression y= 0.03 sin(3x-2t) where y is the displacement at position x at time t. Taking the units to be SI, determine (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the period of the wave.

Solution :

We know that, $y = a\sin(kx - wt)$

Comparing this equation with the given equation, we get

(a) Amplitude a = 0.03 meter

(b) Wavelength $\lambda = \left(\frac{2\pi}{k}\right) = 2 \times \frac{3.14}{3} = 2.06\, meter$

(c) Frequency $\mathrm{v} = \frac{\omega}{2\pi} = \frac{2}{2\pi} Hz = 0.31\, Hz$

(d) Period $T = \frac{1}{v} = \frac{2\pi}{2} = 3.14\, seconds$

Problem 2

Question: standing waves are produced by the superposition of two waves, y_1 = 10 sin (3πt − 4x) and y_2 = 10 sin (3πt + 4x). **Find the amplitude of motion at x = 18.**

Solution:

The resultant amplitude y is given by

$$y = y_1 + y_2 = 10\ sin(3\pi t\ -\ 4x) + \ 10\ sin(3\pi t\ +\ 4x)$$

$$= 10[\sin 3\pi t \cos 4\,\mathrm{x} - \cos 3\pi t \sin 4\,\mathrm{x} + \sin 3\pi t \cos 4x + \cos 3\pi t \sin 4x]$$

$$= 10[2 \sin 3\pi t \cos 4x]$$

$$= 20 \cos 4x \sin 3\pi t$$

The amplitude of motion is 20 cos 4x, when x=18, then

$$4x = 72 = \left[72 \times \pi / 3.14\right] rad.$$

Amplitude = 20| cos(22.9π)|

=20 (0.9510)

Amplitude = 19.02 units of length.

Problem 3

Question: A string vibrates according to the equation $y = 5\sin\left(\frac{\pi x}{3}\right)\cos(40\pi t)$, where x, y, are in cm and t in seconds. **Find the distance between two successive nodes and the sped of particle of the string at a position x =1.5 cm. when t = 9/8 sec.**

Solution:

At nodes y= 0, thus $\sin(\frac{\pi x}{3}) = 0$

$$\frac{\pi x}{3} = n\pi$$ Where n= 0,1,2,3,...

$$x = 3\pi$$

$$x = 3n = 0,3,6,9, \ldots$$

$$\frac{dy}{dx} = -5\sin(\frac{\pi x}{3})\sin(40\pi t)(40)$$

So the distance between two successive nodes = 3 cm, velocity of particle

$$\frac{dy}{dx} = -5\sin(\frac{\pi \times 1.5}{3})(40\pi)\times\sin(45\pi)$$

When x=1.5cm and t=9/8 sec.

$$= -5\sin(\frac{\pi}{2})(40\pi)\times\sin(45\pi)$$

$$= -5\times(40\pi)\times 0 = 0$$

Hence, the particle is at rest at that time

Problem 4

Question: The fundamental frequency of vibration of a stretched string of length 1m is 256 Hz. Find the frequency of the same string of half the original length under identical conditions.

Solution:

For same tension and for the same linear density

$$v_1 \times l_1 = v_2 \times l_2$$

$$256 \times 1 = v_2 \times (1/2)$$

$$\mathrm{v}_2 = 512 Hz$$

Problem 5

Question: A steel wire of 50 cm long has mass of 5 gms. It is stretched with a tension of 400 N. Find the frequency of the wire in fundamental mode of vibration.

Solution:

$$v = \frac{1}{2l}\sqrt{(\frac{T}{m})}$$

Here, l = 50 cm = 0.5m, m = 5 x 10^{-3} kg / 0.5 = 10^{-2} kg and

$$v = \frac{1}{2\times 0.5}\times\sqrt{(\frac{400}{10^{-2}})} = \frac{1}{1.0}\sqrt{4000\times 10^{2}}$$

$$= 200Hz$$

Problem 6

Question: A flexible string of length 1m and mass 1 gm is stretched by a tension T. The string is found to vibrate in three segments at a frequency of 512 Hz. Calculate the tension

Solution:

$$v_3 = \frac{3}{2l}\sqrt{\left(\frac{T}{m}\right)}$$

Here, l = 1 m, m = 10^{-3} kg / 1 m = 10^{-3} kg/m and v_3 = 512

$$512 = \frac{3}{2\times 1}\sqrt{\left(\frac{T}{10^{-3}}\right)}$$

$$512 \times \frac{2}{3} = \sqrt{\left(\frac{T}{10^{-3}}\right)}$$

$$\left(\frac{512\times 2}{3}\right)^2 = \frac{T}{10^{-3}}$$

$$T = \left(\frac{512\times 2}{3}\right)^2 \times 10^{-3} = 116.49 newton$$

Problem 7

Question: **Calculate the speed of transverse waves in a wire of 1 mm radius under the tension produced by 0.1 kg. weight (specific gravity of material of wire = 9.81 gm/cm³.**

Solution:

We know that $$v = \sqrt{\left[\left(\frac{T}{m}\right)\right]}$$

Here $$T = Mg = 0.1 \times 9.81 = 0.981 newton$$

$$m = \pi r^2 \rho = 3.14 \times (1 \times 10^{-6} m^2) \times (9.81 \times 1000 kg/m^3)$$

$$= 3.14 \times 9.81 \times 10^{-3} kg/m^3 = 30.8 \times 10^{-3} kg/m^3$$

$$v = \sqrt{\left[\left(\frac{0.981}{30.8 \times 10^{-3}}\right)\right]}$$

$$= 5.6 m/\sec.$$

Problem 8

Question: Two identical guider string are tuned to the same frequency of 300 Hz. The tension of one the string is increased by 2%. **How many beats per sec. will be heared when the two strings are sounded together.**

Solution: Let v_2 be the frequency of the second string when the tension is increased by 2%. Here we have

$$v_1 = \frac{1}{2l}\sqrt{(\frac{T}{m})} \qquad \text{-------(1)}$$

$$v_2 = \frac{1}{2l}\sqrt{(\frac{T}{m})} \qquad \text{-------(2)}$$

Dividing eq. (2) by eq. (1), we get

$$\frac{v_2}{v_1} = \sqrt{(\frac{102}{100})}$$

$$v_2 = v_1\sqrt{(\frac{102}{100})} = 300\sqrt{(\frac{102}{100})} = 300\sqrt{1.02}$$

$$= 302.9Hz$$

Number of beats = 302.9 – 300 = 2.9 Hz = 3 beats per sec.

Problem 9

Question: **A copper wire of radius 10^{-3} m has a length of 1 meter. It is fixed at both ends and is subjected to a tension of 10^4 N. Calculate (a) the fundamental frequency and (b) the frequencies of the corresponding wavelengths. [density of copper is $8.92*10^{-3}$ kg.m^{-3}]**

Solution:

Let r be the radius of wire and m its mass per unit length (linear density. Then

$$\begin{aligned} m &= 3.14 \times (10^{-3})^2 \times 8.92 \times 10^3 \\ &= 28.009 \times 10^{-3}\ \mathrm{kg/m} \end{aligned}$$

Tension T = 10^4 N

The velocity of the transverse wave is given by

$$V = \sqrt{\left(\frac{T}{m}\right)} = \sqrt{\left(\frac{10^4}{28.009 \times 10^{-3}}\right)} = 0.597 \times 10^3\, m/s$$

(a) The fundamental frequency

$$\nu_1 = V = \frac{0.597 \times 10^3}{2 \times 1}$$
$$= 298.7\, Hz$$

$$\nu_1 = \frac{V}{2l} = \frac{0.597 \times 10^3}{2 \times 1}$$
$$= 298.7\,Hz$$

The frequencies of the first two overtones are

$$\nu_2 = 2 \times \nu_1 = 2 \times 298.7 = 597.4\,\mathrm{Hz}$$
$$\nu_3 = 3 \times \nu_1 = 3 \times 298.7 = 896.1\,Hz$$

(b) We know that $$\lambda_n = \frac{V}{\nu_n} = \frac{V}{(nV/2l)} = \frac{2l}{n}$$

Corresponding wavelengths are

$$\lambda_1 = \frac{2 \times 1}{1} = 2\,metre$$
$$\lambda_2 = \frac{2 \times 1}{2} = 1\,metre$$
$$\lambda_3 = \frac{2 \times 1}{3} = 0.667\,metre$$

Problem 10

Question: The speed of a transverse wave on a stretched string is 500 m/s, when it is stretched under a tension of 19.6 N. If the tension is altered to a value of 78.4 N, what will be the speed of the wave ?

Solution:

$$\frac{v_2}{v_1} = \sqrt{\left(\frac{T_2}{T_1}\right)}$$

$$v_2 = v_1\sqrt{\left(\frac{T_2}{T_1}\right)}$$

$$v_2 = 500 \times \sqrt{\left(\frac{78.4}{19.6}\right)} = 500 \times \sqrt{4}$$

$$= 500 \times 2 = 1000 m / s$$

Problem 11

Question: The fundamental frequency of a sonometer wire increases by 5 Hz if its tension is increased by 21 %. How will the frequency be affected if its length is increased by 10 % ?

Solution: The fundamental frequency is given by

$$\nu = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)} \quad \text{------ (1)}$$

When the tension is increased by 21 %, the new tension will be 1.21 T

$$\nu + 5 = \frac{1}{2l}\sqrt{\left(\frac{1.21T}{m}\right)} \quad \text{------ (2)}$$

Dividing eq. (2) by (1), we get

$$\frac{\nu+5}{\nu} = \sqrt{(1.21)} = 1.1$$

Solving we get $\Rightarrow$ $\nu = 50Hz$

When the length is increased by 10 %, the new frequency v' is given by

$$\nu' = \frac{1}{2(1.10l)}\sqrt{\left(\frac{T}{m}\right)} = \frac{\nu}{1.10} = 45.45Hz$$

Problem 12

Question: A flexible string of length 1 m and mass 1 gm is stretched to a tension T. The string is found to vibrate in three segments at a frequency of 612 Hz. **Calculate the tension in the string.**

Solution:

We know that $$v_3 = \frac{3}{2l}\sqrt{(\frac{T}{m})}$$

Here, l = 1 m, m = 10^{-3} and m = 10^{-3} kg / 1 m = 10^{-3} kg/m and v_3 = 612

$$612 = \frac{3}{2\times 1}\sqrt{(\frac{T}{10^{-3}})}$$

$$612\times\frac{2}{3} = \sqrt{(\frac{T}{10^{-3}})}$$

$$(\frac{612\times 2}{3})^2 = \frac{T}{10^{-3}}$$

$$T = (\frac{612\times 2}{3})^2 \times 10^{-3}$$

$$T = 166.46 newton$$

Problem 13

Question: A brass wire is held at the two ends by rigid supports. At 40^0 it is just taut. **Find the speed of transverse waves in the wire at 5^0.** $\alpha = 1.8 \times 10^{-5}$ / ^{0}C and Y = 9×10^{10} Pa and density = 8500 kg/m^3

Solution: We know that

$$Y = \frac{stress}{strain}$$

$$Stress = Y \times (\frac{\delta l}{l}) = Y\alpha(T_1 - T_2)$$

So, $$Force = Tension = YA\alpha(T_1 - T_2)$$

$$m = \pi r^2 d = A \times d$$

Now, $$v = \sqrt{(\frac{T}{m})} = \sqrt{\left[\frac{YA\alpha(T_1 - T_2)}{A \times d}\right]}$$

$$= \sqrt{\left[\frac{Y\alpha(T_1 - T_2)}{d}\right]}$$

$$= \sqrt{\left[\frac{(9\times10^{10})(1.8\times10^{-5})(40-5}{8500}\right]}$$

$$= 81.67 \text{ m/ s.}$$

Problem 14

Question: A string of length 2.5 m and mass 0.001 kg kept under a tension 1 N. Find the fundamental frequency of the string. If the string is plucked and touched at 0.5 m from one end, what frequencies would be stopped ?

Solution: The fundamental frequency of the string

$$v = \frac{1}{2l}\sqrt{\left[\left(\frac{T}{m}\right)\right]}$$

Here T = 1N, length l = 2.5 m, and m = (0.001/2.5) $kg.m^{-1}$

$$v = \frac{1}{2 \times 2.5} \times \sqrt{\left[\left(\frac{2.5}{0.001}\right)\right]}$$

$$= 10 Hz$$

If the string is plucked and touched at 0.5 m from one end the point would be (1/5) of the length of the string. Therefore overtones divisibles of 5 would be stopped.

Frequencies stopped would be 5 v, 10 v, 15 v, ….

= 50 Hz, 100 Hz, 150 Hz,….

Problem 15

Question: **A string of length 0.5 m and linear density 0.0001 kg.m^{-1} kept under a tension. Find the first three overtones of the string under when it is plucked at its mid-point.**

Solution: The fundamental frequency of the string

$$\nu = \frac{1}{2l}\sqrt{\left[\left(\frac{T}{m}\right)\right]}$$

Here T = 1N, length l = 0.5 m, and m = (0.0001) kg.m^{-1}

$$\nu = \frac{1}{2\times 0.5}\times\sqrt{\left[\left(\frac{1}{0.0001}\right)\right]} = 100\,Hz$$

When the string is plucked at its mid point even overtones are absent. So the string vibrates with 3 ν, 5 ν, 7 ν

- **Frequency of first overtone** = 3 ν = 3 x 100 = **300 Hz**
- **Frequency of second overtone** = 5 ν = 5 x 100 = **500 Hz**
- **Frequency of third overtone** = 7 ν = 7 x 100 = **700 Hz**

Problem 16

Question: A string of length, 1 m and mass 0.0001 kg is under tension of 1 N. When it is plucked at its mid point, it vibrates with n^{th} overtone. **Find the energy of the string.**

Solution:

When the string is plucked at its mid point, the energy of the string is given by

$$E = \frac{16mh^2\nu^2}{n^2\pi^2 l^2}$$

Here, l = 1 m, m = 0.001 kg and T = 1 N

$$\nu = \sqrt{(\frac{T}{m})} = \sqrt{(\frac{1}{0.001})}$$

$$\nu^2 = (\frac{1}{0.001})$$

$$E = \frac{16 \times 0.0001 \times h^2}{n^2\pi^2(1)^2} \times \frac{1}{0.001}$$

$$E = \frac{16h^2}{n^2\pi^2} Joule$$