Compton Effect:

The **Compton Effect** is the term used for an unusual result observed when X-rays are scattered on some materials. By classical theory, when an electromagnetic wave is scattered off atoms, the wavelength of the scattered radiation is expected to be the same as the wavelength of the incident radiation. Contrary to this prediction of classical physics, observations show that when X-rays are scattered off some materials, such as graphite, the scattered X-rays have different wavelengths from the wavelength of the incident X-rays. Arthur H. Compton and his collaborators studied this classically unexplainable phenomenon experimentally, and Compton gave its explanation in 1923.

To explain the shift in wavelengths measured in the experiment, Compton used Einstein's idea of light as a particle. The Compton effect has a very important place in the history of physics because it shows that electromagnetic radiation cannot be explained as a purely wave phenomenon. The explanation of the Compton effect gave a convincing argument to the physics community that electromagnetic waves can indeed behave like a stream of photons, which placed the concept of a photon on firm ground.

The Compton shift expression was derived with assumption that both the e^- & p are relativistic particles and their collision between them follows 2 principles

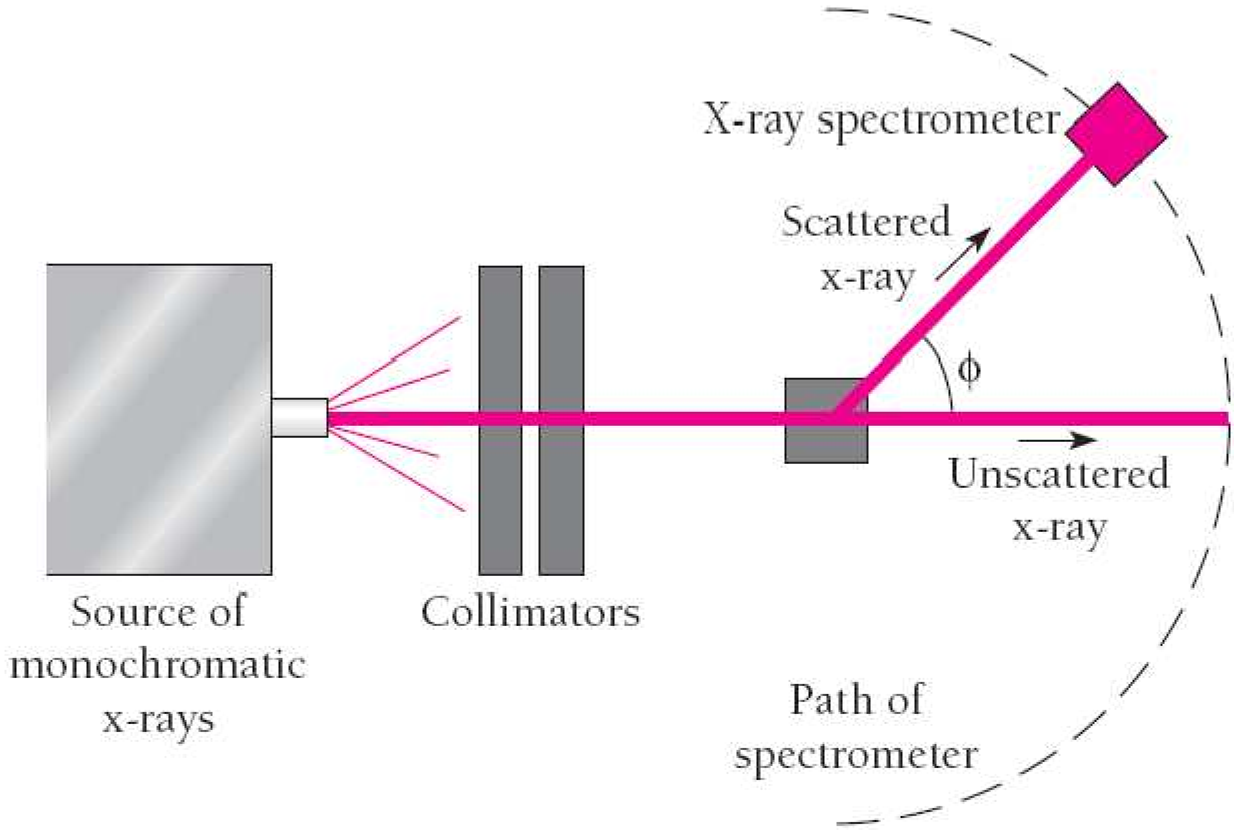
- ✓ Conservation of linear momentum
- ✓ Conservation of total relativistic energy

Compton shift derivation:

Compton wanted to study the scattering phenomena of X-rays with a Carbon target. The schematic diagram of his experimental setup is as shown in the following figure.

To clearly demonstrate the shift in the wavelength of the X-rays, he has chosen monochromatic X-rays. Collimators are used to focus the X-rays finely.

X-ray spectrometer is used to detect the scattered X-rays. Spectrometer is planned to travel through a circular track, so as to receive scattered X-rays from $\boldsymbol{\phi}$=0° (for X-rays which are penetrating through the target) to $\boldsymbol{\phi}$=180°.

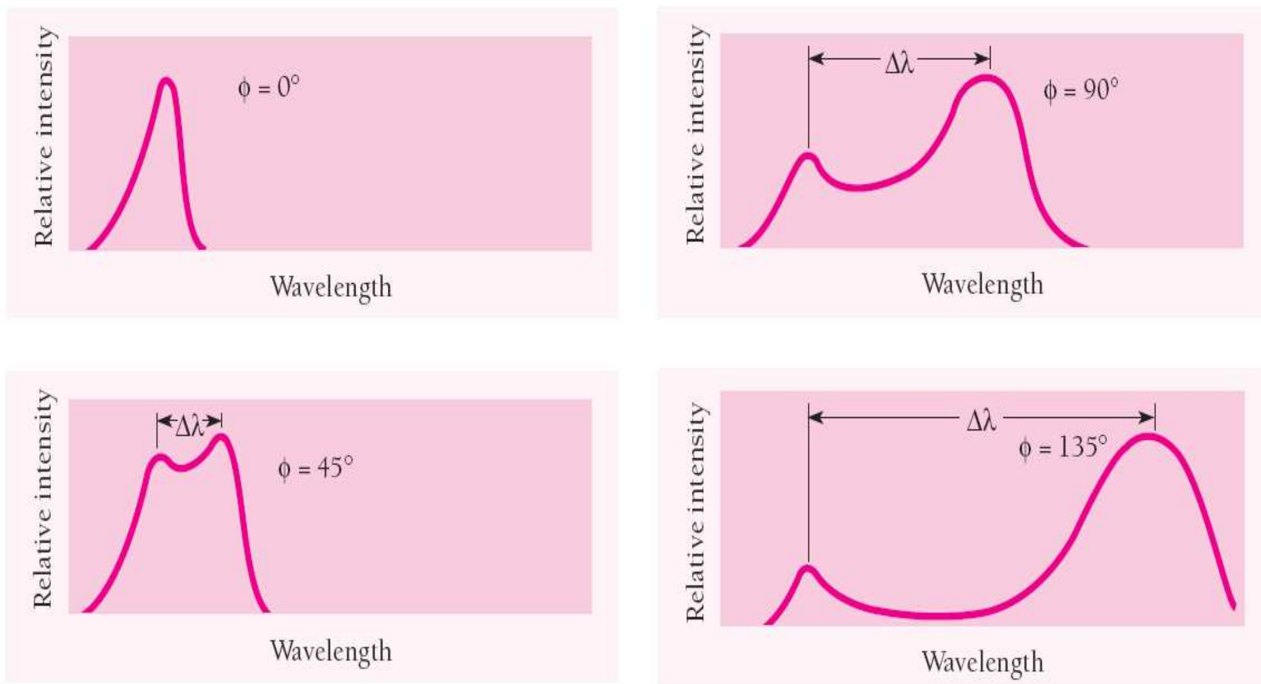


Experimental demonstration of the Compton effect.

Here X-rays photons energy could be chosen in such a way that electrons will not be ejected outside the carbon atoms. The electrons are just recoiled (sprang back) due to the collision of the X-ray photons. Thus x-ray photons are scattered by the electrons.

Compton's experimental results are as shown in the figure. If photons were behaved only as waves there should not be any new waves with new wavelength as shown for scattering angles $\boldsymbol{\phi}$=0°, $\boldsymbol{\phi}$=45°, $\boldsymbol{\phi}$=90° and $\boldsymbol{\phi}$=135°. The additional waves with wavelengths are due to the particle nature of the X-rays.

Typical results of these measurements are shown in below Figure, where the X-axis is the wavelength of the scattered X-rays and the Y-axis is the intensity of the scattered X-rays, measured for different scattering angles (indicated on the graphs). For all scattering angles (except for $\boldsymbol{\phi}$=0°), we measure two intensity peaks. One peak is located at the wavelength λ, which is the wavelength of the incident beam. The other peak is located at some other wavelength λ'. The two peaks are separated by Δλ , which depends on the scattering angle ϕ of the outgoing beam (in the direction of observation). The separation is called the Compton shift.



Compton mathematically estimated the shift in the wavelength of the X-rays by using the laws of conservation of energy and momentum of X-ray photons and electrons. Compton scattering is incoherent scattering.

Case 1: when ϕ=0°, cos ϕ=1 and hence dλ=0

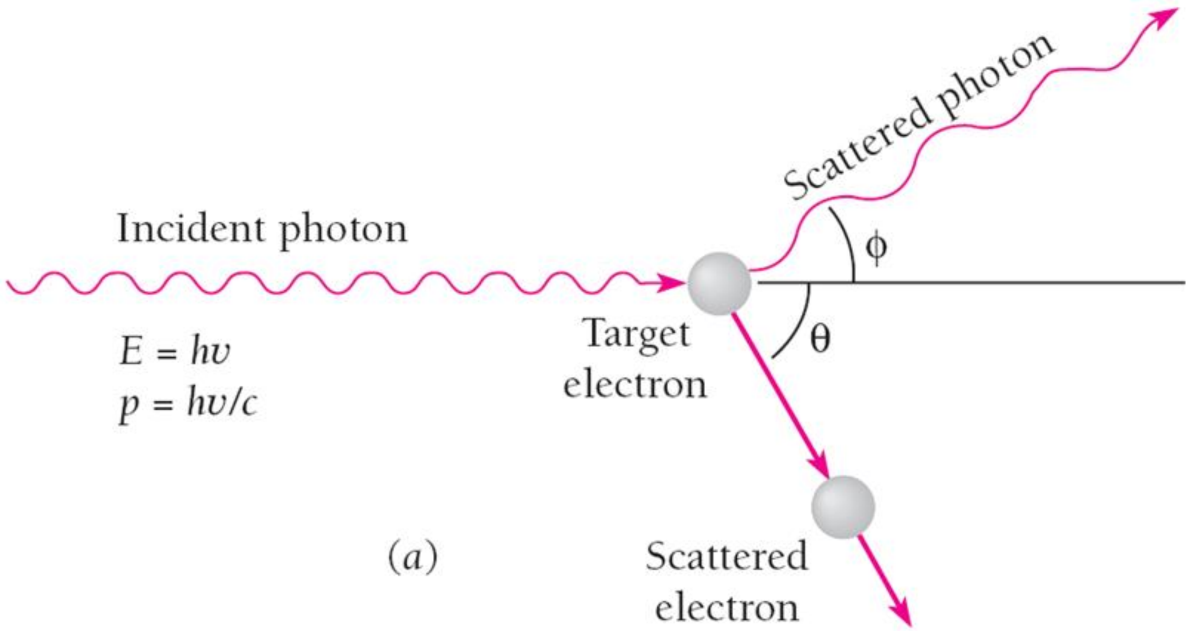
Case 2: when ϕ=45°, cos ϕ=1/√2 hence dλ=1.705 ×0.0242 =0.0071Å

Case 3: when ϕ=90°, cos ϕ=0 and hence dλ=h/m_0c = 0.0243 Å. This is called as Compton wavelength

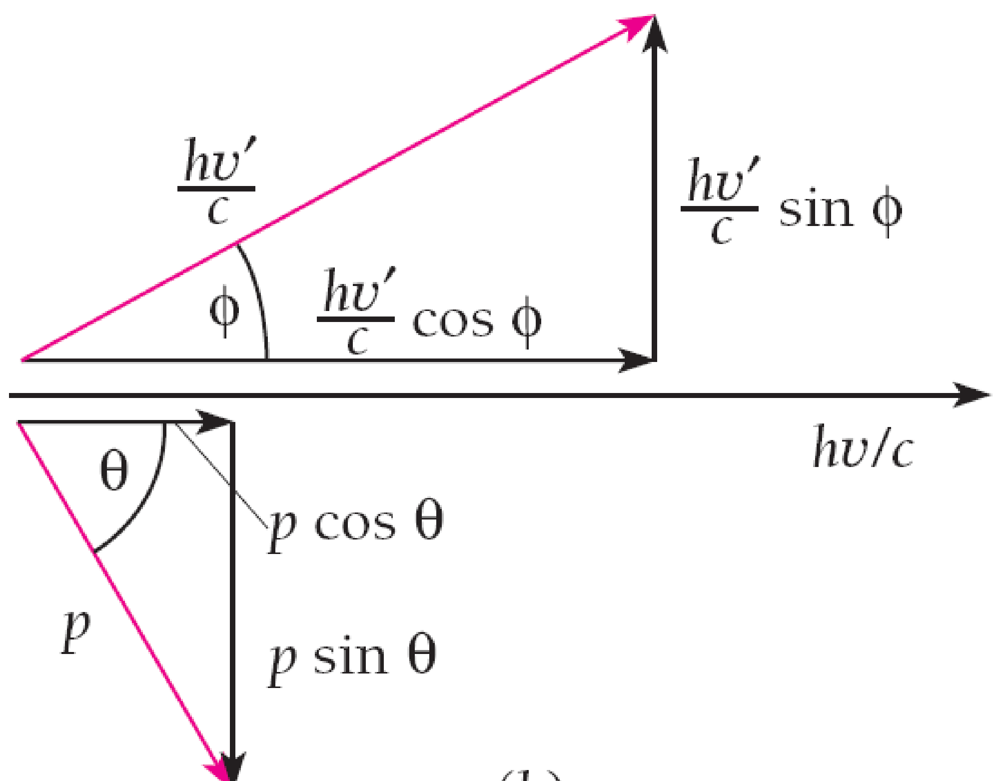
Case 4: when ϕ=135°, cos ϕ= √2/2 and hence dλ=h/m_0c (1.707)= 0.0014 Å

Case 5: when ϕ=180°, cos ϕ= -1 and hence dλ=2h/m_0c = 0.0485 Å

Here the dλ has the maximum value.


Incident photon
Scattered photon
ϕ
θ
$E = h\upsilon$
$p = h\upsilon/c$
Target
electron
Scattered
electron

(a)


$\frac{h\upsilon'}{c}$
$\frac{h\upsilon'}{c}\sin\phi$
ϕ
$\frac{h\upsilon'}{c}\cos\phi$
$h\upsilon/c$
θ
$p\cos\theta$
p
$p\sin\theta$

(b)

Energy:

before collision:

$$Photon's\ energy = h\vartheta$$

$$Electron's\ energy = m_0c^2$$

$$\text{Total energy} \ = h\vartheta + m_0c^2$$

after collision:

$$Photon's\ energy = h\vartheta'$$

$$Electron's\ energy = mc^2$$

$$\text{Total energy} \ = h\vartheta' + mc^2$$

By law of Conservation of energy, before and after collision the total energy of the system (Photon and electron) should be equal.

$$h\vartheta + m_0c^2 = h\vartheta' + mc^2 - - - - 1$$

Momentum :

before Collision

along x –axis

$$Photon's\ momentum = \frac{h\vartheta}{c}$$

$$Electron's\ momentum = 0$$

$$Total\ momentum = \frac{h\vartheta}{c}$$

along y –axis

$$Photon's\ momentum = 0$$

$$Electron's\ momentum = 0$$

$$Total\ momentum = 0$$

after Collision

along x –axis

$$Photon's\ momentum = \frac{h\vartheta' \cos \emptyset}{c}$$

$$Electron's\ momentum = mv \cos \theta$$

$$Total\ momentum = \frac{h\vartheta' \cos \emptyset}{c} + mv \cos \theta$$

along y –axis

$$Photon's\ momentum = \frac{h\vartheta' \sin \emptyset}{c}$$

$$Electron's\ momentum = -mv \sin \theta$$

$$Total\ momentum = \frac{h\vartheta' \sin \emptyset}{c} - mv \sin \theta$$

$$\frac{h\vartheta}{c} = \frac{h\vartheta' \cos \emptyset}{c} + mv \cos \theta$$

$$mv \cos \theta = \frac{h\vartheta}{c} - \frac{h\vartheta' \cos \emptyset}{c} = \frac{h}{c}(\vartheta - \vartheta' \cos \emptyset)$$

$$mv \cos \theta = \frac{h}{c}(\vartheta - \vartheta' \cos \emptyset)$$

$$mvc \cos \theta = h(\vartheta - \vartheta' \cos \emptyset) - - - - - 2$$

$$m^2v^2c^2cos^2\theta = h^2 \left(\vartheta^2 + {\vartheta'}^2 \cos^2\emptyset - 2\vartheta\vartheta' \cos \emptyset\right)$$

$$\frac{h\vartheta' \sin \emptyset}{c} - mv \sin \theta = 0$$

$$\frac{h\vartheta' \sin \emptyset}{c} = mv \sin \theta$$

$$mvc \sin \theta = h\vartheta' \sin \emptyset - - - - - - 3$$

$$m^2 v^2 c^2 sin^2\theta = h^2 {\vartheta'}^2 \sin^2\emptyset$$

$$m^2 v^2 c^2 (sin^2\theta + cos^2\theta) = h^2\left({\vartheta'}^2 \sin^2\emptyset + {\vartheta'}^2 \cos^2\emptyset + \vartheta^2 - 2\vartheta\vartheta' \cos \emptyset\right)$$

$$m^2 v^2 c^2 = h^2\left[{\vartheta'}^2 (\sin^2\emptyset + \cos^2\emptyset) + \vartheta^2 - 2\vartheta\vartheta' \cos \emptyset\right]$$

$$m^2 v^2 c^2 = h^2\left[{\vartheta'}^2 + \vartheta^2 - 2\vartheta\vartheta' \cos \emptyset\right] - - - - - - 4$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - - - - - - - - 5$$

$$m^2 (c^2 - v^2) = {m_0}^2 c^2$$

$$m^2 c^2 (c^2 - v^2) = {m_0}^2 c^4 - - - - -6$$

$$from\ 1, mc^2 = h\vartheta - h\vartheta' + m_0 c^2$$

$$from\ 1, mc^2 = h(\vartheta - \vartheta') + m_0 c^2$$

$$m^2 c^4 = h^2\left(\vartheta^2 + {\vartheta'}^2 - 2\vartheta\vartheta'\right) + m_0^2 c^4 + 2h(\vartheta - \vartheta')m_0 c^2 - - - -7$$

$$from\ 4, \qquad m^2 v^2 c^2 = h^2\left[{\vartheta'}^2 + \vartheta^2 - 2\vartheta\vartheta' \cos \emptyset\right]$$

$$m^2 c^4 - m^2 v^2 c^2 \\ = h^2\left(\vartheta^2 + {\vartheta'}^2 - 2\vartheta\vartheta'\right) + m_0^2 c^4 + 2h(\vartheta - \vartheta')m_0 c^2 \\ - \left(h^2\left[{\vartheta'}^2 + \vartheta^2 - 2\vartheta\vartheta' \cos \emptyset\right]\right)$$

$$m^2 c^4 - m^2 v^2 c^2 \\ = h^2\left(\cancel{\vartheta^2 + {\vartheta'}^2} - 2\vartheta\vartheta'\right) + m_0^2 c^4 + 2h(\vartheta - \vartheta')m_0 c^2 \\ - \left(h^2\left[\cancel{{\vartheta'}^2 + \vartheta^2} - 2\vartheta\vartheta' \cos \emptyset\right]\right)$$

$$m^2 c^2 (c^2 - v^2) == -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0 c^2 + m_0^2 c^4 - -7$$

$$ {m_0}^2 c^4 = -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0 c^2 + m_0^2 c^4$$

$$\cancel{m_0{}^2c^4} = -2h^2\vartheta\vartheta'(1-cos\emptyset) + 2h(\vartheta-\vartheta')m_0c^2 + \cancel{m_0^2c^4}$$

$$0 = -2h^2\vartheta\vartheta'(1-cos\emptyset) + 2h(\vartheta-\vartheta')m_0c^2$$

$$\cancel{2}h^{\cancel{2}}\vartheta\vartheta'(1-cos\emptyset) = \cancel{2h}(\vartheta-\vartheta')m_0c^2$$

$$\frac{h}{m_0c^2}(1-cos\emptyset) = \frac{(\vartheta-\vartheta')}{\vartheta\vartheta'}$$

$$\frac{h}{m_0c^2}(1-cos\emptyset) = \left(\frac{1}{\vartheta'}-\frac{1}{\vartheta}\right)$$

$$\frac{h}{m_0c^2}(1-cos\emptyset) = \left(\frac{\lambda'}{c}-\frac{\lambda}{c}\right) = \frac{1}{c}(\lambda'-\lambda) = \frac{1}{c}\Delta\lambda$$

$$\frac{h}{m_0c^{\cancel{2}}}(1-cos\emptyset) = \frac{1}{\cancel{c}}\Delta\lambda$$

$$\Delta\lambda = \frac{h}{m_0c}(1-cos\emptyset)$$

$$\boxed{\lambda'-\lambda = \frac{h}{mc}(1-\cos\phi)}$$

Where, m_0 – electron's mass; c - velocity of light; $\emptyset$ – photon's scattering angle

Important points:

- Electrons and nuclei can scatter photons
- Scattered photon is at a lower frequency than incident photon
- Some of the energy is transferred to the electron or nucleus
- Maximum wavelength shift for $\phi = 180°$, $\Delta\lambda = 2h/mc$
- The maximum change in wavelength is 4.86×10^{-12} m.
- h/m_0c is known as the COMPTON WAVELENGTH of the electron.
- very small (work it out!) so Compton effect only observed for short wavelength radiation (x-rays, gamma rays