M3: Quantum Mechanics:

- ✓ Energy of a particle is given by,

$$E=mc^2$$

- ✓ Energy of a wave is given by,

$$E=h\nu$$

Where h=6.626 x 10^{-34} J.s=6.55 x 10^{-27} erg.s

- ✓ Mass-energy relation:

$$E=\frac{hc}{\lambda}=h\upsilon=mc^2$$

- ✓ Velocity of light

$$c=\lambda\cdot\nu$$

- ✓ Planck's radiation law in terms of the energy density

$$U(\nu)d\mathbf{v}=\frac{8\pi h v^3 dv}{C^3\left(exp\left[hv/kT\right]-1\right)}$$

Compton Effect

- ✓ Compton shift $\lambda'-\lambda=\frac{h}{mc}(1-\cos\varphi)$

$$\Delta\lambda=\frac{h}{mc}(1-\cos\varphi)$$

Where, m_0 – electron's mass; c - velocity of light; $\emptyset$ – photon's scattering angle

- ✓ Compton wavelength = $\frac{h}{m_0 c}$

- ✓ Compton shift in terms of energy

$$\left[\frac{1}{E'} - \frac{1}{E}\right] = \frac{1}{m_0 c^2}(1 - \cos\theta)$$

- ✓ Compton shift in terms of frequency

$$\left[\frac{1}{\upsilon'} - \frac{1}{\upsilon}\right] = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

de-Broglie wavelength

$$\lambda = \frac{h}{mc} \ (or)\ \lambda = \frac{h}{p}$$

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{mv} = \frac{h}{p}$$

$$where\ mc = p\ \ (momentum\ associated\ with\ photon)$$

- ✓ de Broglie wavelength of material particle

$$\lambda = \frac{h}{mv}$$

- ✓ de Broglie wavelength of a particle with kinetic energy E_k

$$\lambda = \frac{h}{\sqrt{2mE_k}} \quad \left(\because p = \sqrt{2mE_k}\right)$$

- ✓ de Broglie wavelength of charged particle with q and Voltage V

$$\lambda = \frac{h}{\sqrt{2mqV}} \quad \left(\because E_k = qV\right)$$

- ✓ de Broglie wavelength of material particle is in thermal equilibrium at a temperature T

$$\lambda = \frac{h}{\sqrt{3mkT}} \quad \left(\because E = \frac{3}{2}kT\right)$$

- ✓ de-Broglie wavelength of the energy state in 1D box:

$$a = n\left(\frac{\lambda}{2}\right), \text{ where } n=1,2,3.... \qquad \therefore \lambda = \frac{2a}{n}$$

- ✓ de Broglie wavelength of an electron is given by

$$\lambda = \frac{12.26}{\sqrt{V}} \text{Å}$$

- ✓ de Broglie wavelength of an deuteron is given by

$$\lambda = \frac{0.202}{\sqrt{V}} \text{Å}$$

- ✓ de Broglie wavelength of an alpha particle is given by

$$\lambda = \frac{0.1012}{\sqrt{V}} \text{Å}$$

- ✓ de Broglie wavelength of an proton is given by

$$\lambda = \frac{0.286}{\sqrt{V}} \text{Å}$$

- ✓ Energy of an matter wave (Electron wave or proton wave):

$$E = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$$

- ✓ Energy of the particle in eV :

$$E = \frac{12400}{\lambda\,(\text{Å})} (\text{eV})$$

- ✓ Kinetic energy of an electron

$$k.E = \frac{1}{2} mv^2$$

If it is given in eV, you must have to equate it to this one.

- ✓ 1fm=10^{-15} m, 1pm=10^{-12} m, 1 Å=10^{-10}m
- ✓ Phase velocity: $V_p = \upsilon\lambda$
- ✓ Group velocity:

$$V_g = \frac{d\omega}{dk}$$

- ✓ Relationship between phase and group velocities:

$$V_g = V_p - \lambda\left(\frac{dV_p}{d\lambda}\right)$$

Davisson and Germer's experiment:

- ✓ Bragg's formula: $n\lambda = 2d\sin\theta$
- ✓ Wavelength of electron: $\lambda = \dfrac{h}{\sqrt{2mqV}}$
- ✓ Angle through which the electron is deviated = 2θ

Uncertainty principle:

- ✓ Rough Estimation (simple): $\Delta x.\Delta p \sim h$; $\Delta E.\Delta t \sim h$
- ✓ Formal statement $\Delta x.\Delta p \geq \dfrac{h}{4\pi}$ $(\because \hbar = \dfrac{h}{2\pi})$
- ✓ Uncertainty in terms of velocity and position

$$\Delta x.\Delta v \geq \frac{h}{4\pi m}$$

- ✓ Uncertainty in the velocity of the electron:

$$\Delta v = \frac{\Delta p}{m_0}$$

✓ Final accurate uncertainty in energy and time:

$$\Delta E.\Delta t \geq \frac{h}{4\pi} \quad ; \quad \Delta \upsilon.\Delta t \geq \frac{1}{4\pi}$$

✓ Final accurate uncertainty in wavelength and time:

$$\Delta \lambda.\Delta t = \frac{\lambda^2}{4\pi c} \qquad \text{since, } \Delta E = -\frac{hc}{\lambda^2}\Delta \lambda;$$

Wave function:

✓ $\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \, dx = 1 \quad -\gg \quad Normalisation \ condition$

✓ $\int_{-\infty}^{\infty} \psi^* \psi \, dx = 0 \quad -\gg Orthogonal \ condition$

✓ Momentum Operator $\quad p\Psi = -i\hbar \frac{\partial \Psi}{\partial x}$

✓ Energy Operator $\quad E\Psi = i\hbar \frac{\partial \Psi}{\partial t}$

✓ Schrodinger's time independent equation for electron or any particle

$$\nabla^2\Psi + \frac{8\pi^2 m}{h^2}(E - V)\Psi = 0$$

✓ Schrodinger's time independent equation for free particle

$$\nabla^2\Psi + \frac{8\pi^2 m}{h^2} E\,\Psi = 0$$

✓ $\hat{\mathbf{H}} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is known as Hamiltonian Operator

✓ Schrodinger's time dependent equation for any particle

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\hat{H}\,\Psi = \boldsymbol{E}\,\Psi$$

M4: Applications of Quantum Mechanics:

Particle in 1D box

✓ The energy of the n^{th} energy level : $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} = \frac{n^2h^2}{8mL^2}$

✓ The energy of each energy level in terms of E_1 is:

$$E_n = n^2.E_1$$

✓ The wave equation is : $\boldsymbol{\psi_n} = \sqrt{\frac{2}{L}}\,\mathbf{sin}\left(\frac{n\pi x}{a}\right)$

where $\sqrt{\frac{2}{L}}$ is normalisation constant

✓ The difference between 2 consecutive energy levels is given by: $E_{n+1} - E_n = (2n+1)\,E_1$

✓ $A = \sqrt{\frac{2}{L}}$ is the normalized constant for the wave function

$\psi_n = A\sin\frac{n\pi x}{L}$ (As ψ function changes, the "A" value also changes)

✓ Probability of finding a particle in given interval:

$$P = |\psi(x)|^2 \cdot \Delta x$$

- ✓ No. of nodes = n+1
- ✓ No. of antinodes = n

Particle in 3D box

- ✓ The energy of E_n level is given by:

$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x{}^2} + \frac{n_y^2}{L_y{}^2} + \frac{n_z^2}{L_z{}^2}\right)$$

- ✓ One of the example for wave equation is given by:

$$\psi = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{n_x \pi}{L_x} x\right) \sin\left(\frac{n_y \pi}{L_y} y\right) \sin\left(\frac{n_z \pi}{L_z} z\right)$$

Reflection & Transmission Coefficients

- ✓ $R + T = 1$
- ✓ $R = \frac{B*B}{A*A} = \left|\frac{B}{A}\right|^2$
- ✓ $T = \frac{C*C}{A*A} = \left|\frac{C}{A}\right|^2$
- ✓ $R = \left[1 + \frac{4E(V_0 - E)}{V_0^2 \sinh^2 k_2 L}\right]^{-1}$
- ✓ $T = \left[1 + \frac{V_0^2 \sinh^2 k_2 L}{4E(V_0 - E)}\right]^{-1}$
- ✓ $T = \left[\frac{16}{4 + (\frac{k_2}{k_1})^2}\right] e^{-2k_2 L}$, finally $T = e^{-2k_2 L}$

where $k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

M4: INTRODUCTION TO NANOMATERIALS:

- ✓ Surface to volume ratio:

  - For a Cube:

    $\frac{S}{V} = \frac{6}{a}$ where, a = Side of the cube

  - For a Sphere:

    $\frac{S}{V} = \frac{3}{R}$ Where, R = Radius of the sphere

  - No. of tiny spheres produced from big sphere

    $$4\pi R^2 = n.4\pi r^2; \ where \ 'n' \ is \ no. \ of \ tiny \ spheres$$

M5: Lasers:

- ✓ Temporal coherence:

$$l_c = \lambda\left(\frac{\lambda}{\Delta\lambda}\right) = \frac{\lambda^2}{\Delta\lambda}$$

- ✓ Spatial coherence:

$$l_t = \frac{r\lambda}{s} = \frac{\lambda}{\theta_s}$$

- ✓ Einstein's Coefficients :

$$B_{12}=B_{21}; \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{C^3}$$

- ✓ Populasion inversion ratio :

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT} = e^{-(\frac{h\upsilon}{kT})}$$

- ✓ Energy difference between two energy states :

$$E_1 - E_2 = \frac{12400(eV)}{\lambda(\mathring{A})}$$

- ✓ Energy difference between two energy levels;

$$E = \frac{hc}{\lambda}$$

- ✓ Number of photons emitted by the laser per minute:

$$\text{No. of photons} = \frac{\text{output power}}{\text{energy of one photon}}$$

- ✓ Efficiency of the laser:

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100\%$$

- ✓ Intensity of the Laser :

$$\text{Intensity} = \frac{\text{power}}{\text{area of cross section}} = \frac{\text{power}}{\pi r^2}$$

- ✓ Ratio :

$$\frac{\text{stimulated emission}}{\text{spontaneous emission}} = \frac{1}{e^{(\frac{h\upsilon}{kT})} - 1} = \frac{1}{e^{(\frac{hc}{\lambda kT})} - 1}$$