Engineering Physics

(PHY1701)

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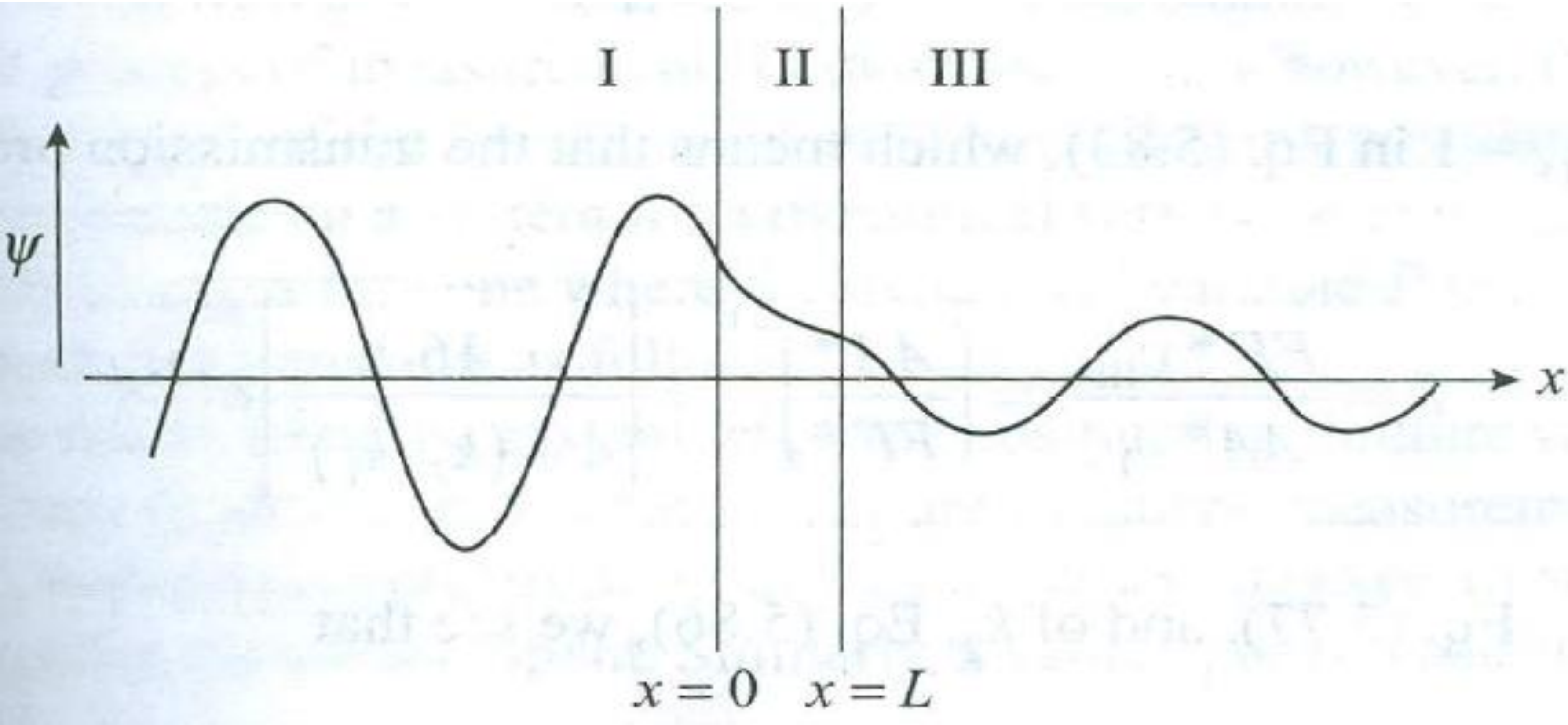
Module-2: Applications of Quantum Physics

Contents

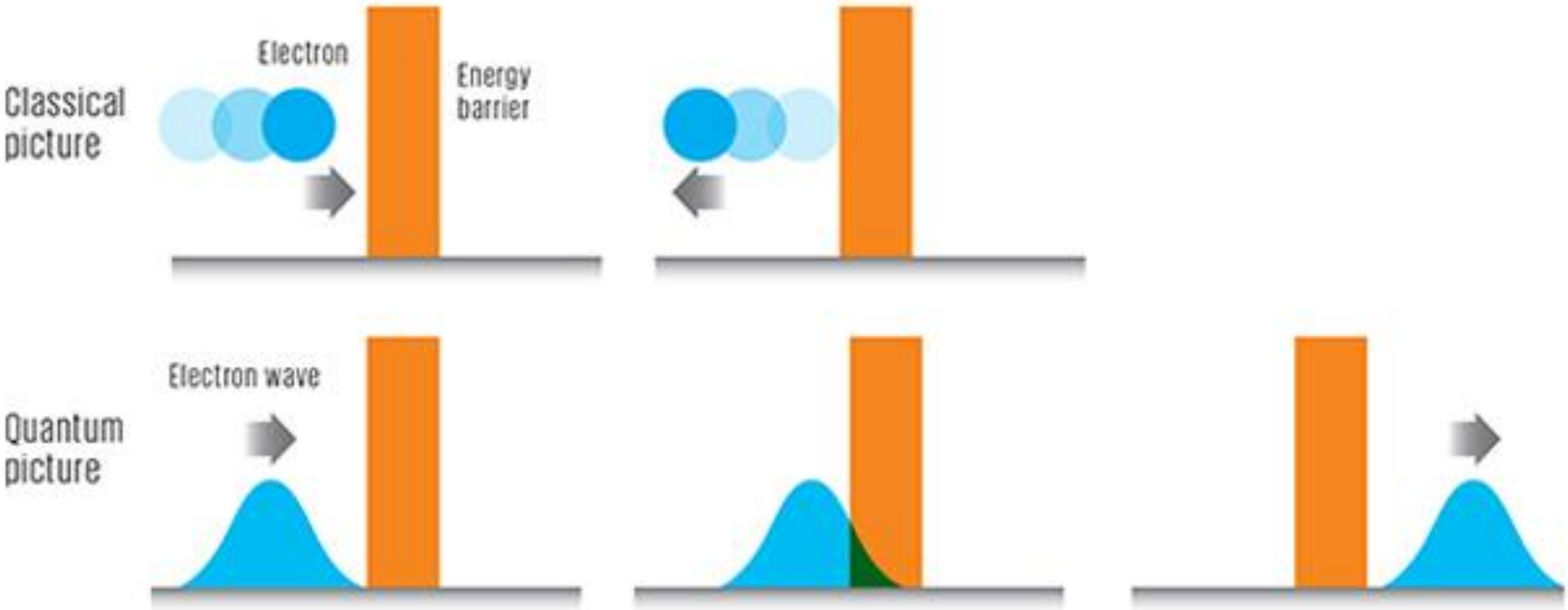
- Particle in a 1-D box (Eigen Value and Eigen Function) (AB 198-202),
- 3-D Analysis (Qualitative),
- Tunneling Effect (Qualitative) (AB 205), &
- Scanning Tunneling Microscope (STM)*

❖ Concepts of Modern Physics, Arthur Beiser et al., Sixth Edition, Tata McGraw Hill (2013) (AB)

Tunneling Effect

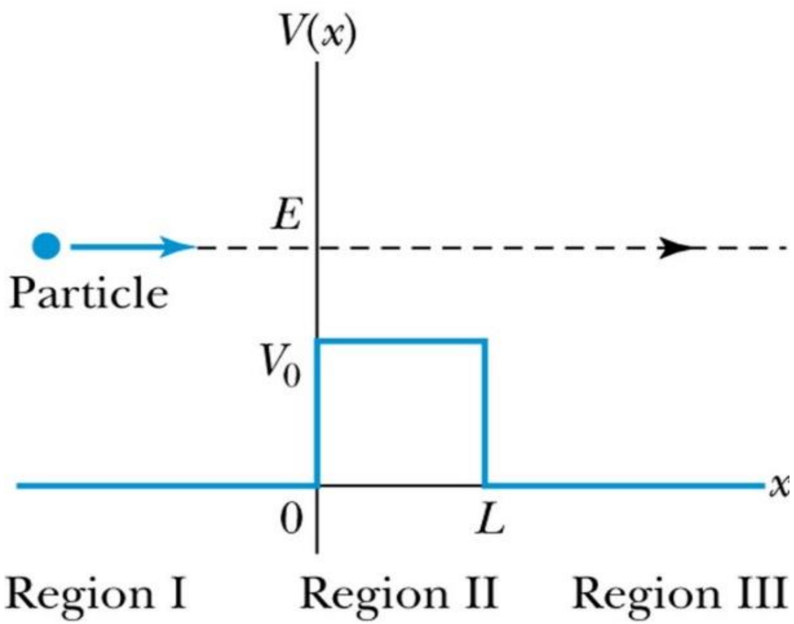


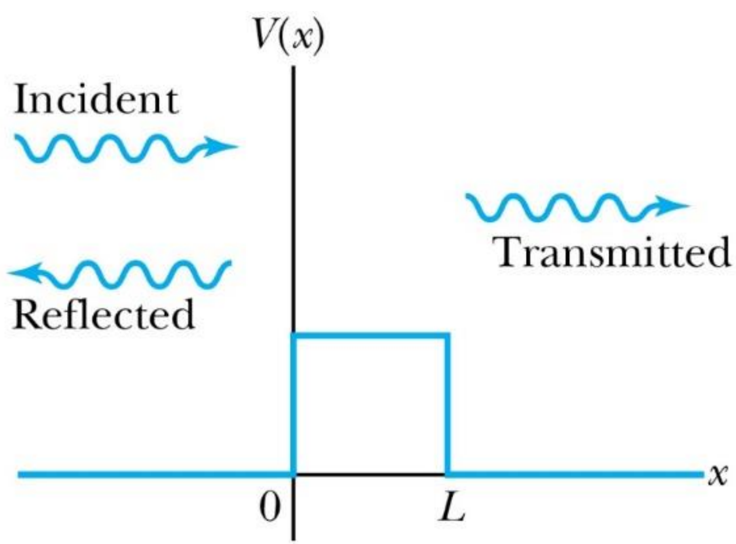
Classical Vs Quantum



Barriers and Tunneling

- Consider a particle of energy E approaching a potential barrier of height V_0 and the potential everywhere else is zero.
- We will first consider the case when the energy is greater than the potential barrier.
- In regions I and III the wave numbers are: $k_{\mathrm{I}} = k_{\mathrm{III}} = \dfrac{\sqrt{2mE}}{\hbar}$
- In the barrier region we have $k_{\mathrm{II}} = \dfrac{\sqrt{2m(E - V_0)}}{\hbar}$ where $V = V_0$





Reflection and Transmission

- The wave function will consist of an incident wave, a reflected wave, and a transmitted wave.
- The potentials and the Schrödinger wave equation for the three regions are as follows:

$$\text{Region I } (x<0) \qquad V=0 \qquad \frac{d^2\psi_1}{dx^2}+\frac{2m}{\hbar^2}E\psi_1=0$$

$$\text{Region II } (0<x<L) \qquad V=V_0 \qquad \frac{d^2\psi_{\text{II}}}{dx^2}+\frac{2m}{\hbar^2}(E-V_0)\psi_{\text{II}}=0$$

$$\text{Region III } (x>L) \qquad V=0 \qquad \frac{d^2\psi_{\text{III}}}{dx^2}+\frac{2m}{\hbar^2}E\psi_{\text{III}}=0$$

- The corresponding solutions are:

$$\text{Region I } (x<0) \qquad \psi_{\text{I}}=Ae^{ik_{\text{I}}x}+Be^{-ik_{\text{I}}x}$$

$$\text{Region II } (0<x<L) \qquad \psi_{\text{II}}=Ce^{ik_{\text{II}}x}+De^{-ik_{\text{II}}x}$$

$$\text{Region III } (x>L) \qquad \psi_{\text{III}}=Fe^{ik_{\text{I}}x}+Ge^{-ik_{\text{I}}x}$$

- As the wave moves from left to right, we can simplify the wave functions to:

$$\text{Incident wave} \qquad \psi_{\text{I}}(\text{incident})=Ae^{ik_{\text{I}}x}$$

$$\text{Reflected wave} \qquad \psi_{\text{I}}(\text{reflected})=Be^{-ik_{\text{I}}x}$$

$$\text{Transmitted wave} \qquad \psi_{\text{III}}(\text{transmitted})=Fe^{ik_{\text{I}}x}$$

Probability of Reflection and Transmission

- The probability of the particles being reflected R or transmitted T is:

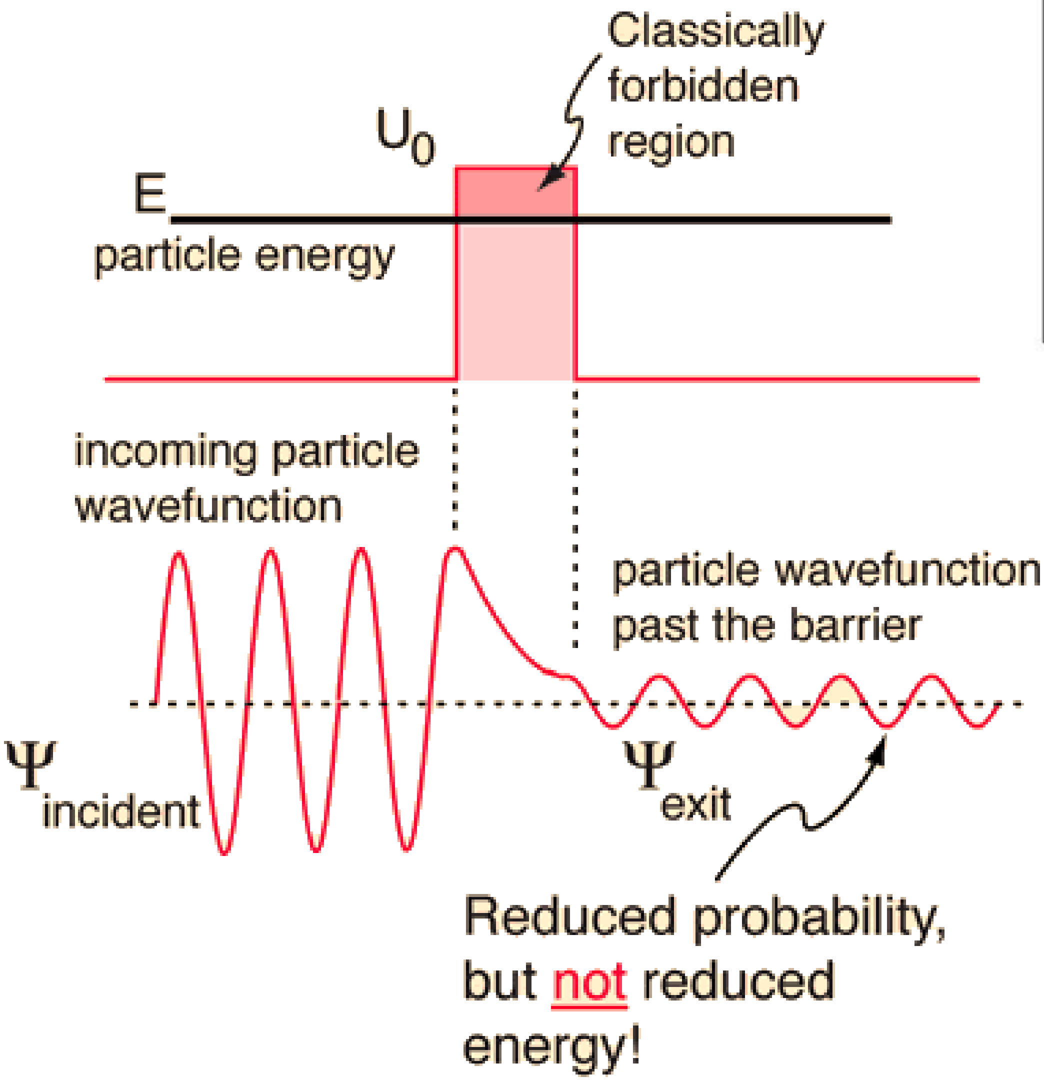
$$R = \frac{|\psi_{\mathrm{I}}(\text{reflected})|^2}{|\psi_{\mathrm{I}}(\text{incident})|^2} = \frac{B^* B}{A^* A}$$

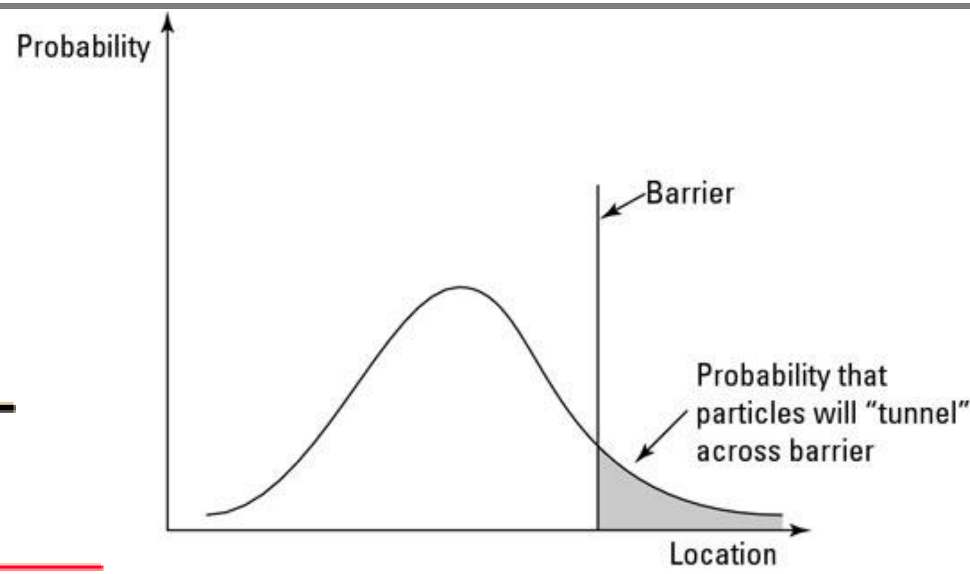
$$T = \frac{|\psi_{\mathrm{III}}(\text{transmitted})|^2}{|\psi_{\mathrm{I}}(\text{incident})|^2} = \frac{F^* F}{A^* A}$$

- The maximum kinetic energy of the photoelectrons depends on the value of the light frequency f and not on the intensity.
- Because the particles must be either reflected or transmitted we have: $R + T = 1$.
- By applying the boundary conditions $x \to \pm\infty$, $x = 0$, and $x = L$, we arrive at the transmission probability:
- Notice that there is a situation in which the transmission probability is 1.

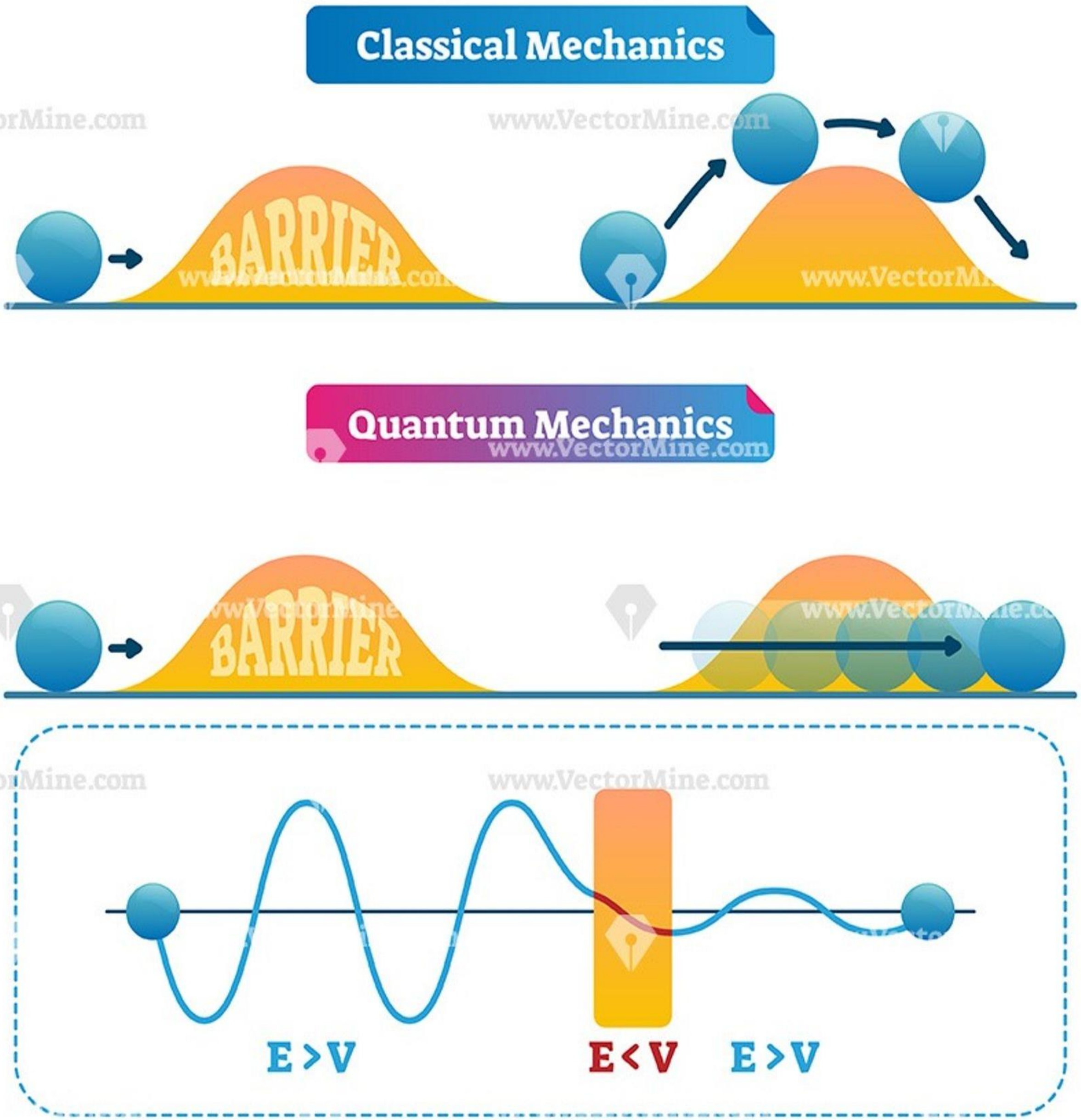
$$T = \left[\frac{16}{4 + (k_2 / k_1)^2}\right] e^{-2k_2 L}$$

Tunneling Phenomena



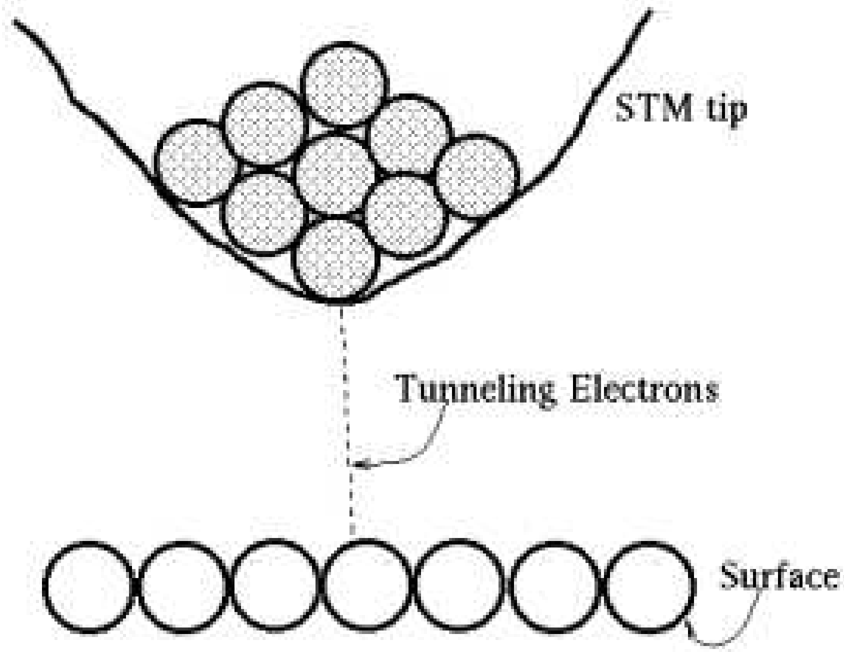


QUANTUM TUNNELING



Scanning Tunneling Microscope

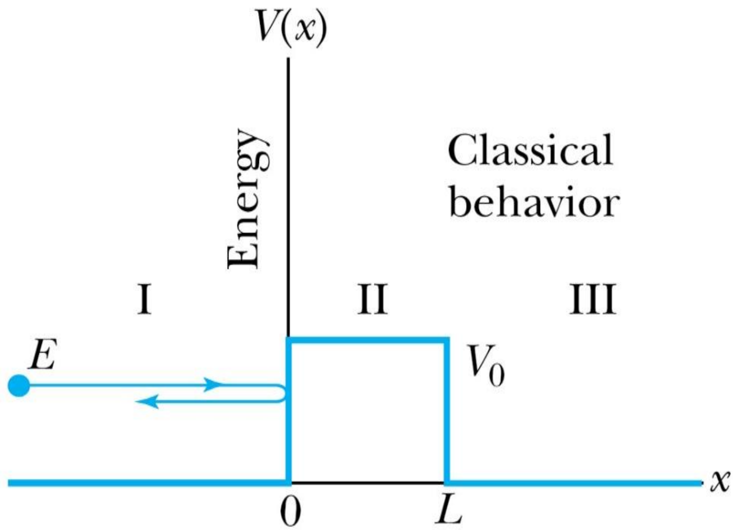
The Scanning Tunneling Microscope



The Scanning Tunneling Microscope (STM) was developed by Gerd Binnig and Heinrich Rohrer at IBM. When a metal tip is brought near a conducting surface, electrons can tunnel from the tip to the surface or vice-versa. Because the tunneling probability is exponentially dependent on the distance the contours of the surface can be mapped out by keeping the current constant and measuring the height of the tip. In this way, atomic resolution can be obtained. For their work, Binnig and Rohrer shared the 1986 Nobel Prize.

Tunneling

- Now we consider the situation where classically the particle does not have enough energy to surmount the potential barrier, $E < V_0$.



The quantum mechanical result, however, is one of the most remarkable features of modern physics, and there is ample experimental proof of its existence. There is a small, but finite, probability that the particle can penetrate the barrier and even emerge on the other side.

- The wave function in region II becomes

$$\Psi_{II} = Ce^{k_2 x} + De^{-k_2 x} \quad \because k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar}}$$

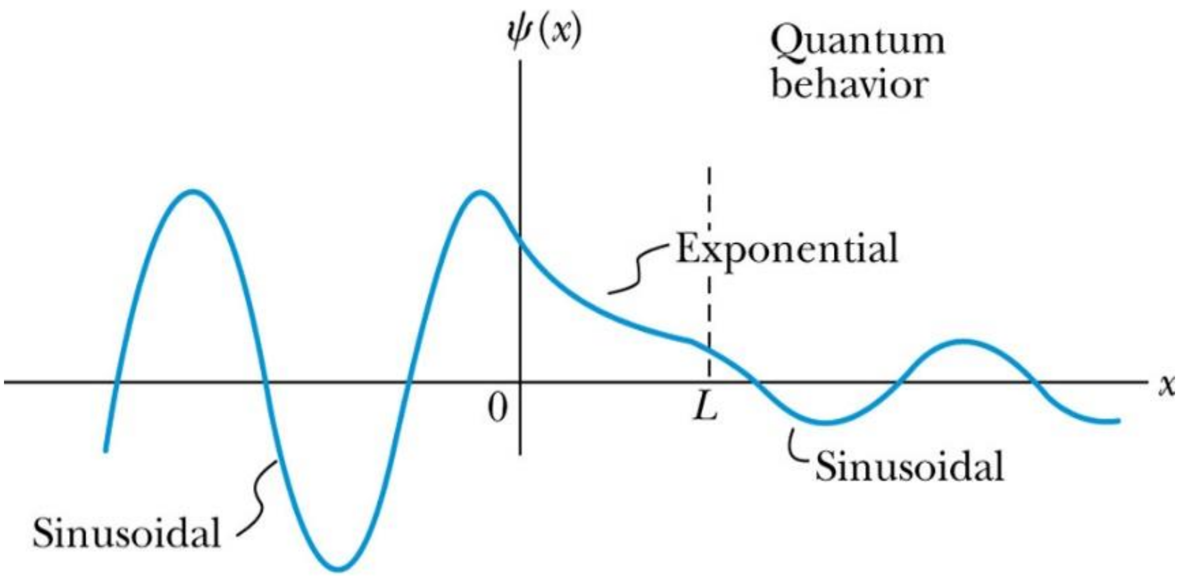
- The transmission probability that describes the phenomenon of **tunneling** is $T = \left[\dfrac{16}{4 + (k_2 / k_1)^2}\right] e^{-2k_2 L}$

- The bracketed quantity, further more is always on the magnitude of order 1, so,

Approximate transmission probability $\boxed{T = e^{-2k_2 L}}$

Tunneling effect explains these phenomena

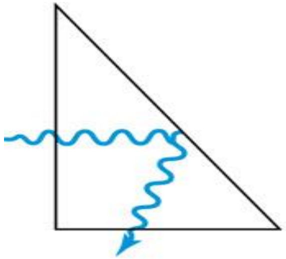
- Emission of alpha particles from a radioactive element.
- Electrical breakdown of insulators.
- Switching action of a tunnel diode
- Field emission of electrons from a cloud metallic surface.



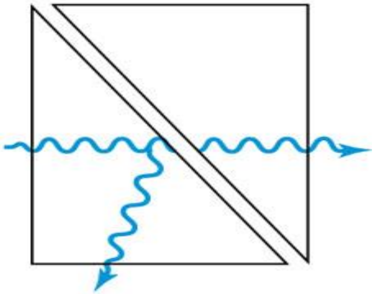
$$T = 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\kappa L}$$

Analogy with Wave Optics

- If light passing through a glass prism reflects from an internal surface with an angle greater than the critical angle, total internal reflection occurs. However, the electromagnetic field is not exactly zero just outside the prism. If we bring another prism very close to the first one, experiments show that the electromagnetic wave (light) appears in the second prism.
- The situation is analogous to the tunneling described here. This effect was observed by Newton and can be demonstrated with two prisms and a laser. The intensity of the second light beam decreases exponentially as the distance between the two prisms increases.



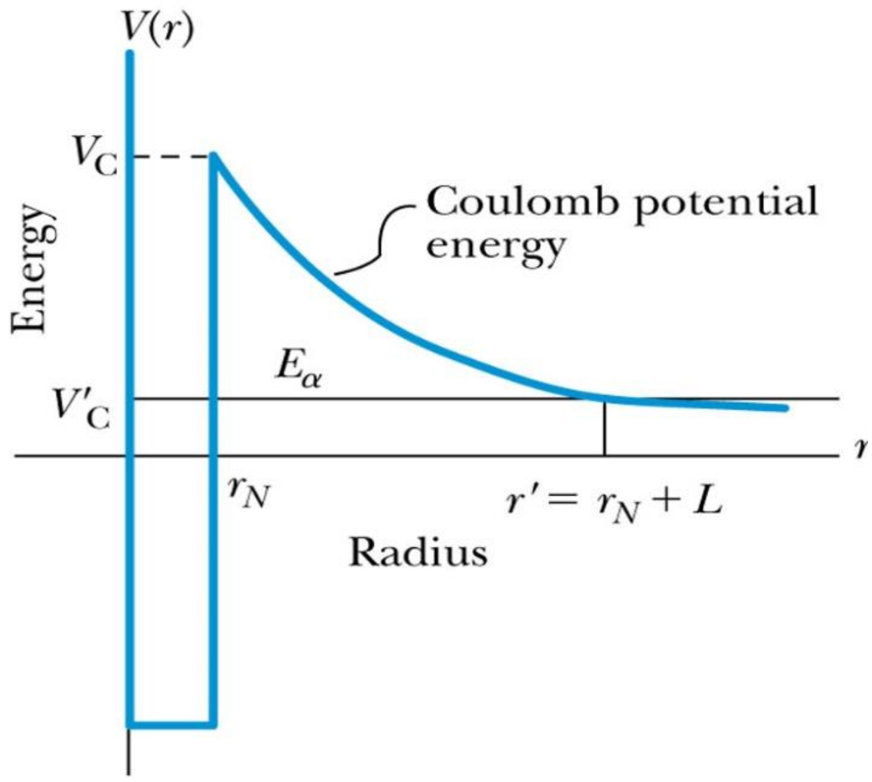
(a)



(b)

Alpha-Particle Decay

- The phenomenon of tunneling explains the alpha-particle decay of heavy, radioactive nuclei.
- Inside the nucleus, an alpha particle feels the strong, short-range attractive nuclear force as well as the repulsive Coulomb force.
- The nuclear force dominates inside the nuclear radius where the potential is approximately a square well.
- The Coulomb force dominates outside the nuclear radius.
- The potential barrier at the nuclear radius is several times greater than the energy of an alpha particle.
- According to quantum mechanics, however, the alpha particle can "tunnel" through the barrier. Hence this is observed as radioactive decay.



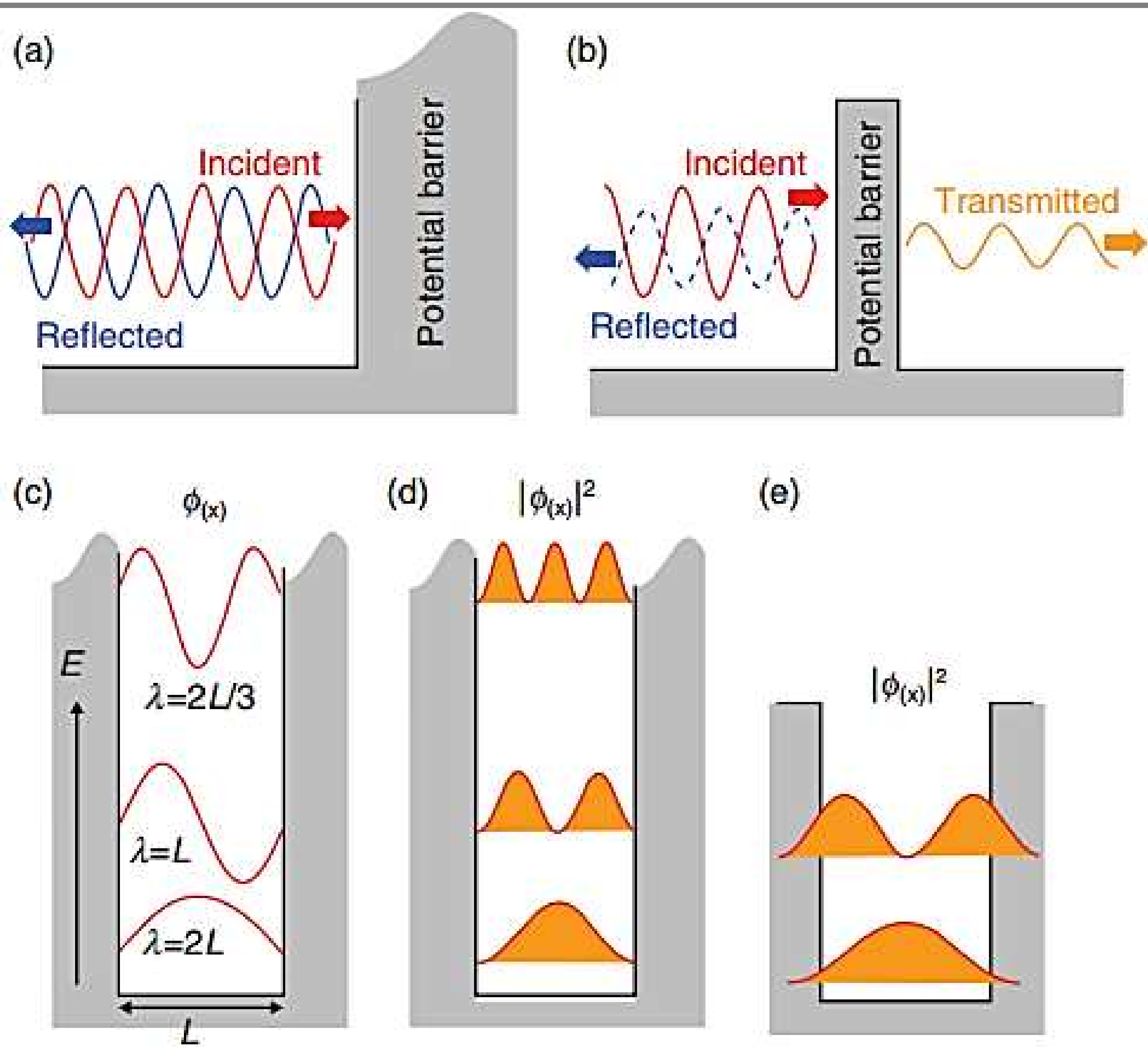


Fig. 1. Fundamental quantum mechanical phenomena. (a) Electron reflection and interference. (b) Tunneling effect. (c)–(e) Quantum confinement.