Matter waves, Experimental Evidence, Phase velocity and Group velocity

DUAL NATURE OF LIGHT:

Light exhibits the phenomena of interference, diffraction, polarization and also photo electric effect and Compton Effect. Interference, diffraction and polarization can be explained by wave nature of light which is based on transfer of energy. The photoelectric effect and Compton Effect are explained by quantum theory. This indicates the particle nature of light which is based on transfer of momentum apart from transfer of energy. So light has dual nature that is wave nature and particle nature.

DUAL NATURE OF MATTER- DE BROGLIE CONCEPT OF MATTER WAVES:

In 1924 de Broglie proposed that a beam of particles behave as a wave in transferring energy. This is dual nature of matter. He proposed this without any strong experimental support and hence called hypothesis.

Louis de Broglie suggested that the particles like electrons, protons, neutrons, etc. also have dual nature. i.e. they also can have particle as well as wave nature.

Matter Waves: waves associated with the moving material particle are called matter waves, de Broglie waves, or pilot waves.

De Broglie Hypothesis:

The dual nature of light possessing both wave and particle properties was explained by combining Planck's expression for energy of a photon $E = h\nu$ with Einstein's mass energy relation $E = mc^2$

According to Planck's theory $E = h\nu$

Einstein's mass energy relation $\mathbf{E} = mc^2$

But we know that $\nu = \frac{c}{\lambda}$, we get

$\therefore \frac{hc}{\lambda} = mc^2$ $\boxed{\lambda = \frac{h}{mc} = \frac{h}{p}}$ or

The wavelength λ is called de Broglie wavelength. It is defined as the ratio of Planck's constant to the momentum of a particle.

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

Particle does not exhibit the wave nature and particle nature simultaneously.

Rationale behind the hypothesis:

The concept of dual nature of radiation prompted louis de Broglie to suggest the idea of matter waves

In 1924, his suggestion is based on the following facts

1. The nature loves symmetry.
2. The entire universe consists of matter (particles) and radiation (energy) only

Therefore, the two physical entities i.e. matter and energy must be symmetrical.

<u>De Broglie wavelength In Terms of Energy:</u> Consider a particle of mass 'm' moving with velocity 'v', then kinetic energy of particle is given by

$$\text{Kinetic Energy (E)} = \frac{1}{2}mv^2$$

$$E = \frac{1}{2}mv^2$$

$$2mE = (mv)^2$$

$$\Rightarrow mv = \sqrt{2mE}$$

Substituting this value in de Broglie wavelength $\lambda = \frac{h}{mv}$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

<u>De Broglie wavelength of electron</u>:

When a potential difference 'V' is applied to the electron it accelerates with velocity 'v' then the work done on the electron is 'eV'. This work done is converted into the kinetic energy of the electron. Thus,

$$eV = \frac{1}{2}mv^2$$

$$2meV = (mv)^2$$

$$mv = \sqrt{2meV}$$

Substituting this value in de Broglie wavelength $\lambda = \frac{h}{mv}$.

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.625\times10^{-34}}{\sqrt{2\times\left(9.1\times10^{-31}\right)\times\left(1.6\times10^{-19}\right)\times V}}$$

$$= \frac{12.26}{\sqrt{V}} A^0 \quad \text{or} \quad \frac{1.226}{\sqrt{V}} \text{nm}$$

De Broglie wavelength of charged particle:

If a charged particle carrying charge q is accelerated through potential difference V volts, then K.E. is

$$E_K = qV.$$

The de Broglie wavelength for charged particle of charge q and accelerated through the potential difference V volts is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

De Broglie wavelength of charged particle:

When a material particle is in thermal equilibrium at a temperature T, then

$$\boldsymbol{E = \frac{3}{2} kT}$$

The de Broglie wavelength of a material particle at a temperature T is given by

$$\boldsymbol{\lambda = \frac{h}{\sqrt{3mkT}}}$$

Properties of matter waves: Waves associated with the material particles such as electron, proton, etc., are called matter waves.

The wavelength of matter waves is given by $\lambda = \frac{h}{mv}$

1. Greater the mass of the particle, lesser the wavelength associated with it.
2. Greater the velocity of the particle, lesser the wavelength associated with it.
3. The matter waves are generated by the moving particles.
4. Matter waves are independent of charge of particle. Whereas the electromagnetic waves are produced by the moving charges only.
5. Particle does not exhibit wave nature and particle nature simultaneously
6. These waves can travel faster than the velocity of light.
7. The velocity of the matter waves is not constant as that of EM radiation. It depends on the velocity of the material particle.
8. The matter wave is not a physical phenomenon. It is rather as symbolic representation of what we know about the particle. It is a wave of probability.

9. Matter waves are probability waves, amplitude of which gives the probability of existence of the particle at the point.
10. The only function of the wave is to pilot (or) to guide the matter particles.
11. Velocity of the matter waves 'ω' is greater than the velocity of light 'c'.

i.e., $E = h\nu$ and $E = mc^2$

$\therefore h\nu = mc^2$ or $\nu = \frac{mc^2}{h}$

But from de Broglie Eq. $\lambda = \frac{h}{mv}$

The matter wave velocity, $\omega = \nu\lambda$

Substituting λ and ν we get

$$\omega = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v}$$

As the velocity cannot exceed velocity of light hence the velocity of matter waves is greater than the velocity of light.

Difference between matter waves and E.M.waves:

Matter waves	*E.M.waves*
Matter wave is associated with moving particle	Oscillating charged particle give rise to e.m. wave
Wavelength depends on the mass of the particle and its velocity $\lambda = h/mv$	Wave length depends on the energy of photon $\lambda = hc/E$
Can travel with a velocity greater than the velocity of light	Travel with velocity of light $c = 3 \times 10^8$ m/s
Matter wave is not electromagnetic wave	Electric field and magnetic field oscillate perpendicular to each other
Energies if these waves are not quantized	Energies if these waves are quantized
These waves cannot pass through vacuum	These waves can easily pass through vacuum
Their wavelengths cannot be measured easily	Wavelengths of these waves can be measured easily
These waves associated with material particles, they do not exist without material particles	These rays are emitted from source in space

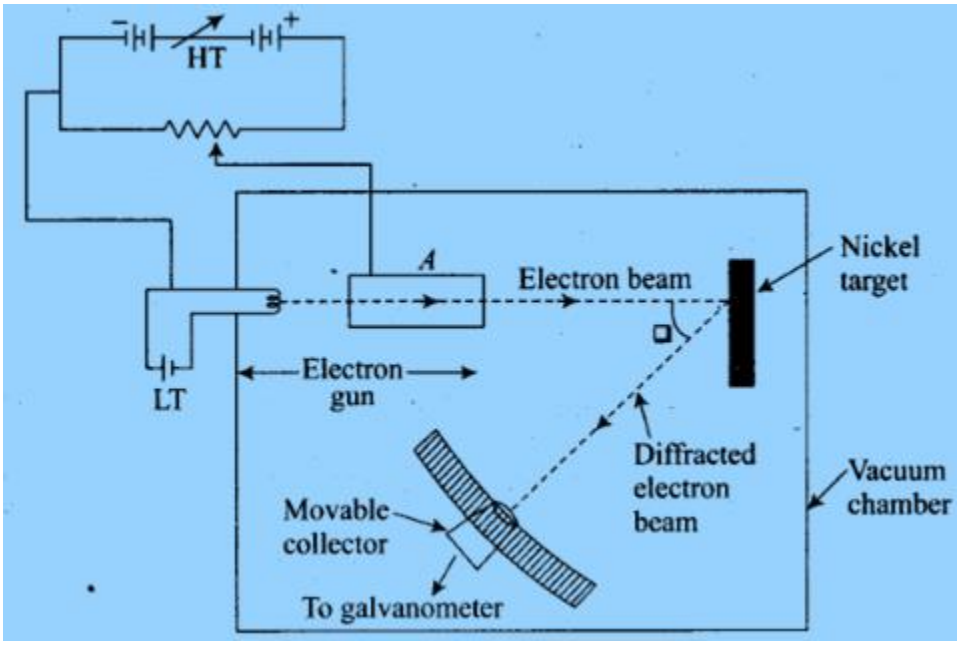
Davission and **Germar Experiment (Electron Diffraction Experiment):**

This experiment demonstrated the wave nature of the electron, confirming the earlier hypothesis of deBroglie. Putting wave-particle duality on a firm experimental footing, it represented a major step forward in the development of quantum mechanics. The Bragg law for diffraction had been applied to x-ray diffraction, but this was the first application to particle waves.

The first experiment evidence of matter waves was given by American physicists ***Davisson*** and ***Germar***. The experimental arrangement is shown in figure.

It consists of an electron gun, large single crystal of nickel target T, electron detector C and sensitive galvanometer G.

Electrons are produced by heating the filament F with low tension battery and they are accelerated by a high tension battery. So released electron beam then falls on a single nickel crystal. This crystal is capable of rotating about an axis parallel to the axis of the incident beam by a handle H. the electrons scattered from the crystal in all are collected by a Faraday cylinder C. it consists of a double walled metallic cylinder with an aperture. The inner and outer walls are insulated from each other. So only the faster electrons can enter into the inner cylinder.



Experimental arrangement

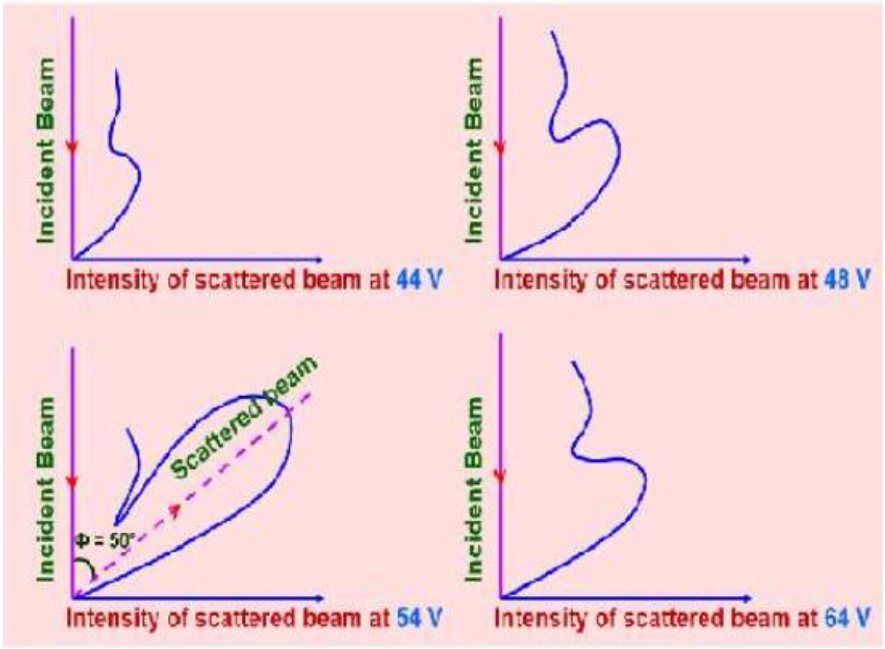
The collected electrons produce a current. It is measured by a galvanometer. This cylinder can be moved along a graduated circular scale. So that it is able to receive the reflected electrons at all angles.

Experiment:

A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle. The scattered beam of electrons is received by the detector which can be rotated at any angle. The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun. The emitted electrons were accelerated by known potential difference and the diffracted electrons are received by the cylinder. The intensity of electrons at different angles was measured. Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54 V.

Figure shows typical polar graphs of electron density.

1) The graph remains fairly smooth, till the accelerating voltage becomes 44V when a ***bump*** appears on the curve.
2) As the accelerating voltage is increased, the length of the ***bump*** increases.
3) The ***bump*** becomes most prominent in the curve for 54 volt electrons at φ = 50°.
4) As the accelerating voltage is further increased, the bump decreases in length and finally disappears at 68 V.



<u>Calculation of observed wavelength</u>:

1. De Broglie wavelength of electrons:

$$\lambda = \frac{12.25\,\text{Å}}{\sqrt{V}} = \frac{12.25\,\text{Å}}{\sqrt{54}} = 1.66\,\text{Å} \quad \text{----------} \quad (1)$$

2. From the Bragg's X-ray diffraction formula $2d \sin\theta = n\lambda$,
   Here d is the inter atomic spacing of Nickel = 0.091 nm, n = first order
   $\therefore \lambda = 2d\sin\theta/n = 2 \times 0.091 \times (\sin 65^o) = 1.65$ Å ---------- (2)

Thus the close agreement between equations 1 & 2 confirms de-Broglie concept of matter waves.