Engineering Physics

(BPHY101L)

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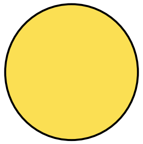
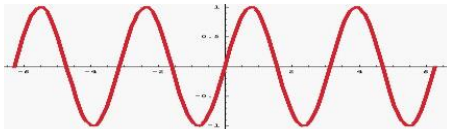
Traditional wave and particle

Our traditional understanding of a particles and wave....



"Localized" - definite position, momentum, confined in space



"de-localized" – spread out in space and time

Normal Waves

- are a disturbance in space
- carry energy from one place to another
- often (but not always) will (approximately) obey the classical wave equation

Matter Waves

- disturbance is the wave function $Y(x, y, z, t)$
  - probability amplitude Y
  - probability density $p(x, y, z, t) = |Y|^2$

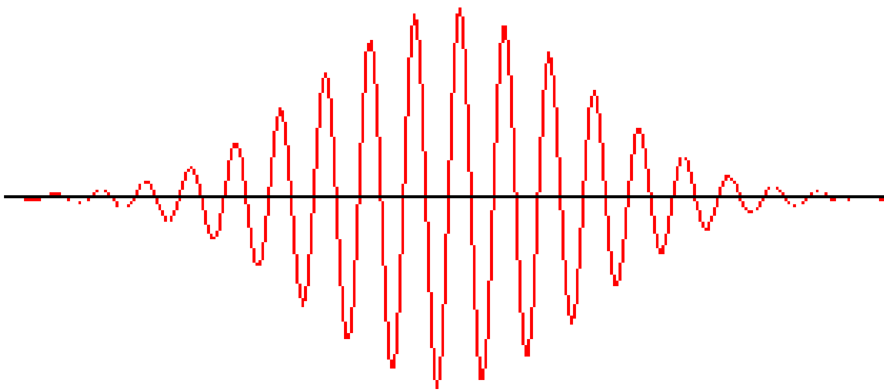
Wave packet

What could represent both wave and particle?

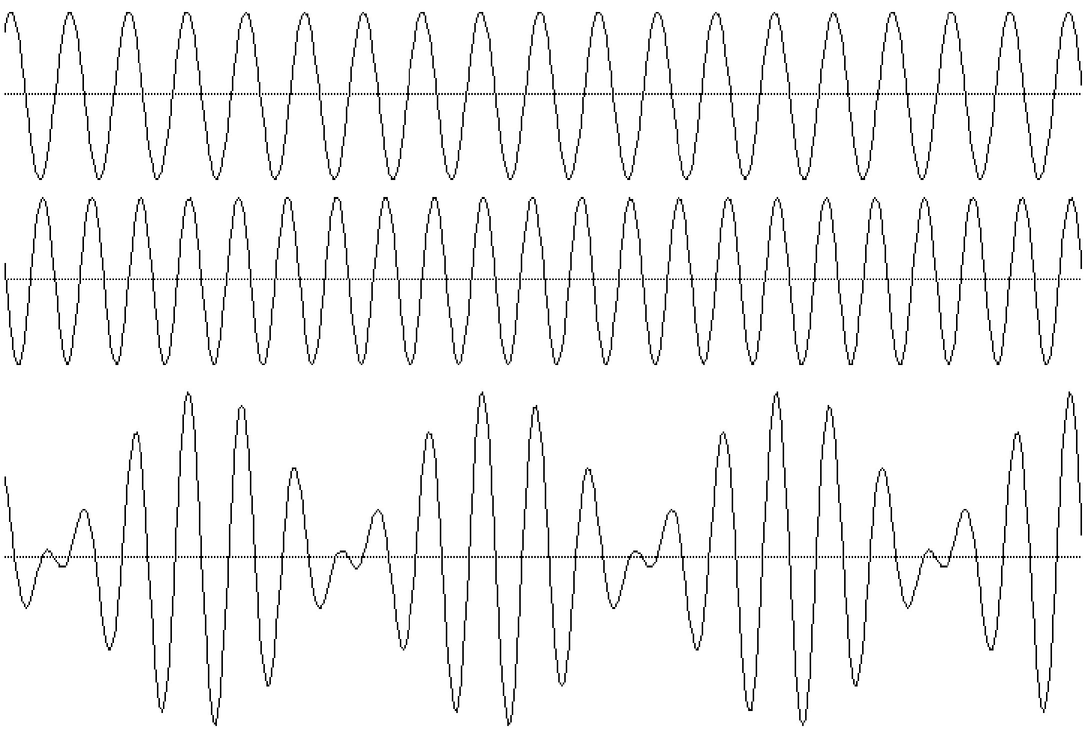
Find a description of a particle which is consistent with our notion of both particles and waves……

- Fits the "wave" description
- "Localized" in space

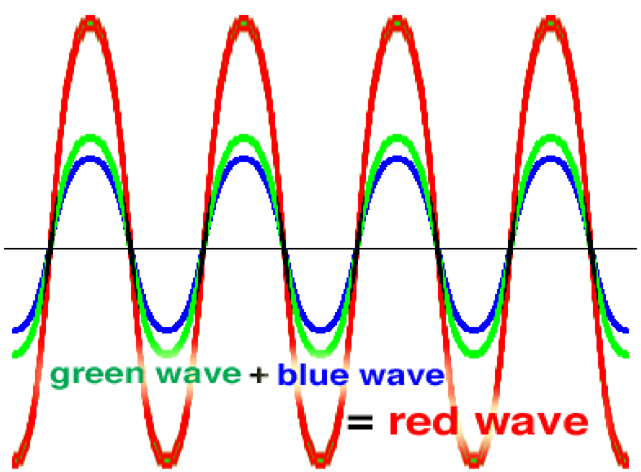
A "Wave Packet"



Adding up waves of different frequencies



The Superposition principle



What happens when you add up waves?

Phase velocity

- ◊ The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it
- ◊ Phase velocity: The velocity of component waves of a wave packet
- ◊ Group velocity: The velocity with which the wave packet propagates

- ◊ Consider a wave whose displacement y is expressed as

$$\boldsymbol{y} = \boldsymbol{A}\cos(\boldsymbol{\omega t} - \boldsymbol{kx})$$

where *A*-Amplitude, $\boldsymbol{\omega}$-angular frequency, and $\boldsymbol{k}$-propagation constant

The speed of propagation of this wave will be the speed associated with a point for which the phase $(\boldsymbol{\omega t} - \boldsymbol{kx})$ is constant..

$$(\boldsymbol{\omega t} - \boldsymbol{kx}) = \text{constant}$$

or $$\boldsymbol{x} = \text{constant} + \frac{\boldsymbol{\omega}}{\boldsymbol{k}}\,t$$

Therefore, **Phase velocity** $\boldsymbol{v}_P = \frac{dx}{dt} = \frac{\boldsymbol{\omega}}{\boldsymbol{k}} = \frac{2\pi v}{2\pi/\lambda}$

$$\boldsymbol{v}_P = \boldsymbol{v}.\boldsymbol{\lambda}$$

Group velocity

The **Group velocity** is given by $v_g = \frac{d\omega}{dk}$

The **Phase velocity** of de Broglie waves is,

$$v_P = v.\lambda = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right) = \frac{c^2}{v}$$

Here velocity v is always less than c. Therefore, the de Broglie wave velocity must be greater than c.

The **group velocity** of de Broglie waves is,

$$v_g = \frac{d\omega}{dk}$$

The angular frequency and wavenumber of the de Broglie waves associated with a particle of rest mass m_0 moving with the velocity v are given by

$$\omega = 2\pi\nu \quad \omega = 2\pi\frac{E}{h}$$

$$\omega = 2\pi\frac{mc^2}{h}$$

$$\boxed{\omega = 2\pi\frac{m_0c^2}{h\sqrt{1-\frac{v^2}{c^2}}}}$$

Experiment

$$k = \frac{2\pi}{\lambda} \qquad k = \frac{2\pi}{h} mv$$

$$\boxed{k = \frac{2\pi}{h} \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{d\omega / dv = \frac{2\pi}{h} \frac{m_0 v}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}}$$

$$\boxed{dk / dv = \frac{2\pi m_0}{h} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}}$$

$$V_g = \frac{d\omega}{dk} = \frac{d\omega / dv}{dk / dv} = v$$

Hence, the de Broglie wave group associated with a moving particle travels with the same velocity as the particle.

Relation between the group and phase velocity

The relation between the group and phase velocity is given by

$$v_g = v_p - \lambda \left(\frac{dv_p}{d\lambda}\right)$$

From above equation the following two cases arise.

1. For dispersive medium, $v_p = f(\lambda)$. Usually $dv_p/d\lambda$ is positive (normal dispersion)

$$\mathbf{v_g < v_p}$$

This is the case with de Broglie waves.

2. For non-dispersive medium, $v_p \neq f(\lambda)$, $dv_p/d\lambda$ is zero. Then

$$\mathbf{v_g = v_p}$$

This is true for electromagnetic waves in vaccum.