Engineering Physics

(PHY1701)

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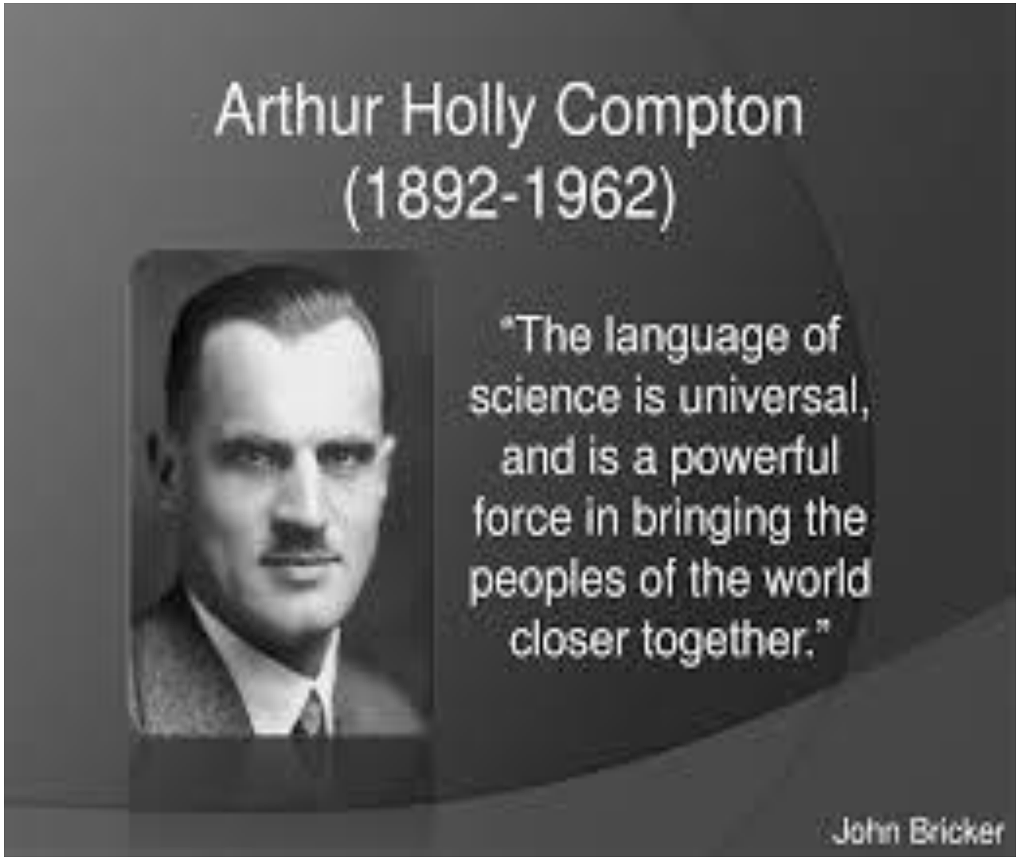
Module-1: Introduction to Modern Physics

Contents

- Planck's concept (hypothesis) (AB 66-67)
- Compton Effect (AB 80-86),
- Particle properties of wave: Matter Waves (AB 104-114),
- Davisson Germer Experiment (AB 115-117),
- Heisenberg Uncertainty Principle (AB 119-128),
- Wave function (AB 182-184 & 190-195), &
- Schrödinger equation (time dependent & independent) (AB 187 -190 & 195-197).

❖ Concepts of Modern Physics, Arthur Beiser et al., Sixth Edition, Tata McGraw Hill (2013) (AB)

Introduction

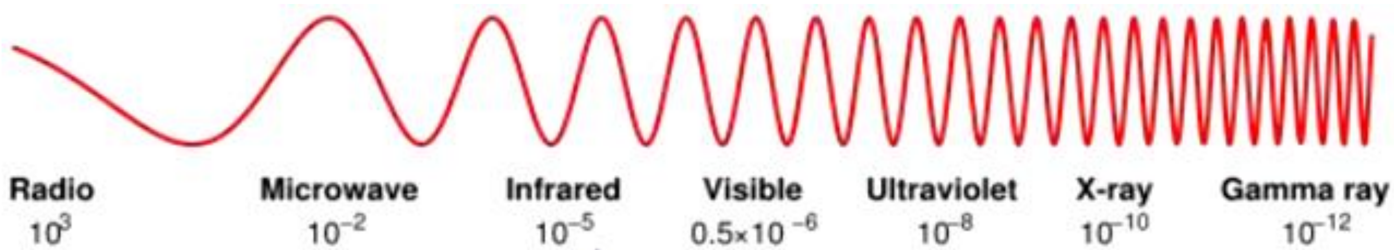


The Compton effect was first introduced by **Arthur Holly Compton** in 1923 (For which he received the physics noble prize in 1927).

Theory

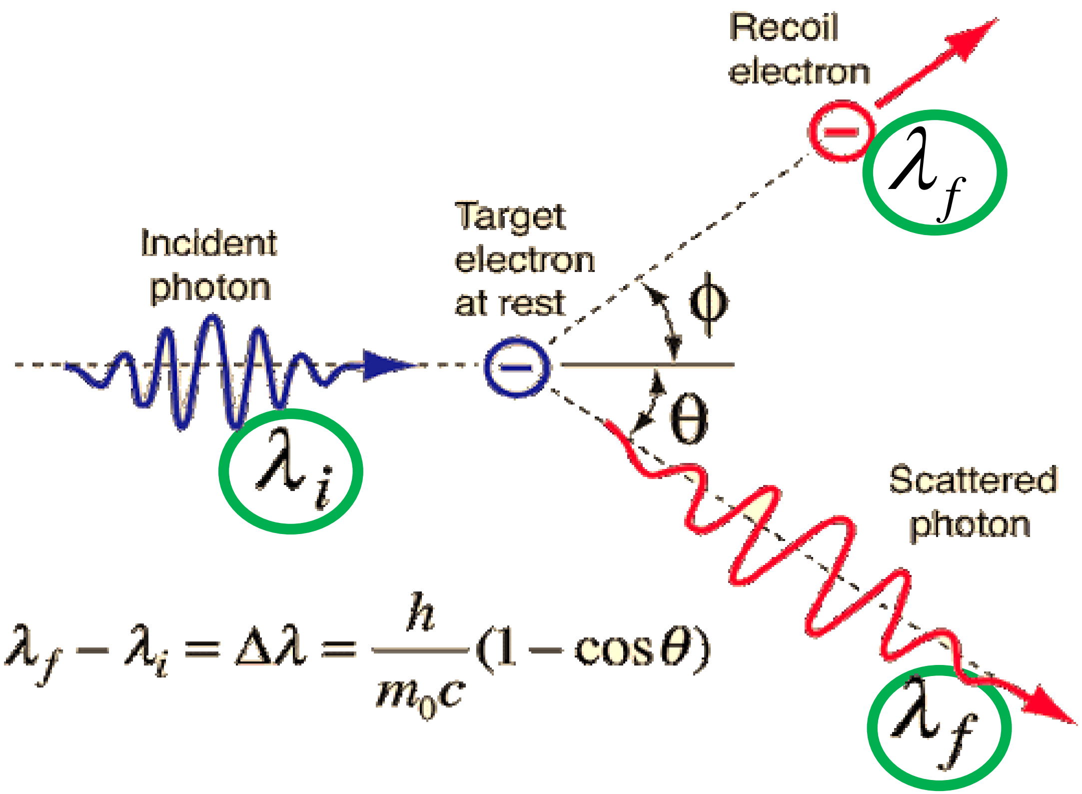
- The Compton effect (Compton scattering) is the result of high energy photon colliding with the target, which releases the loosely bound electrons from the outer shell of the atom or molecule.
- A/C to classical theory, when an E.M wave is scattered off atoms, the wavelength of the scattered radiation is expected to be the same as the wavelength of the incident one. But here it is contrary.
- The scattered radiation experiences a wavelength shift that cannot be explained in terms of classical wave theory, thus lending support to Einstein's photon theory (idea of light as a particle).
- Probably the most important implication of the effect is that it showed light could not be fully explained according to the wave phenomena.
- The Compton shift expression was derived with assumption that both the e^- & P are relativistic particles and their collision follows 2 principles
    - Conservation of linear momentum
    - Conservation of total relativistic energy

How it works



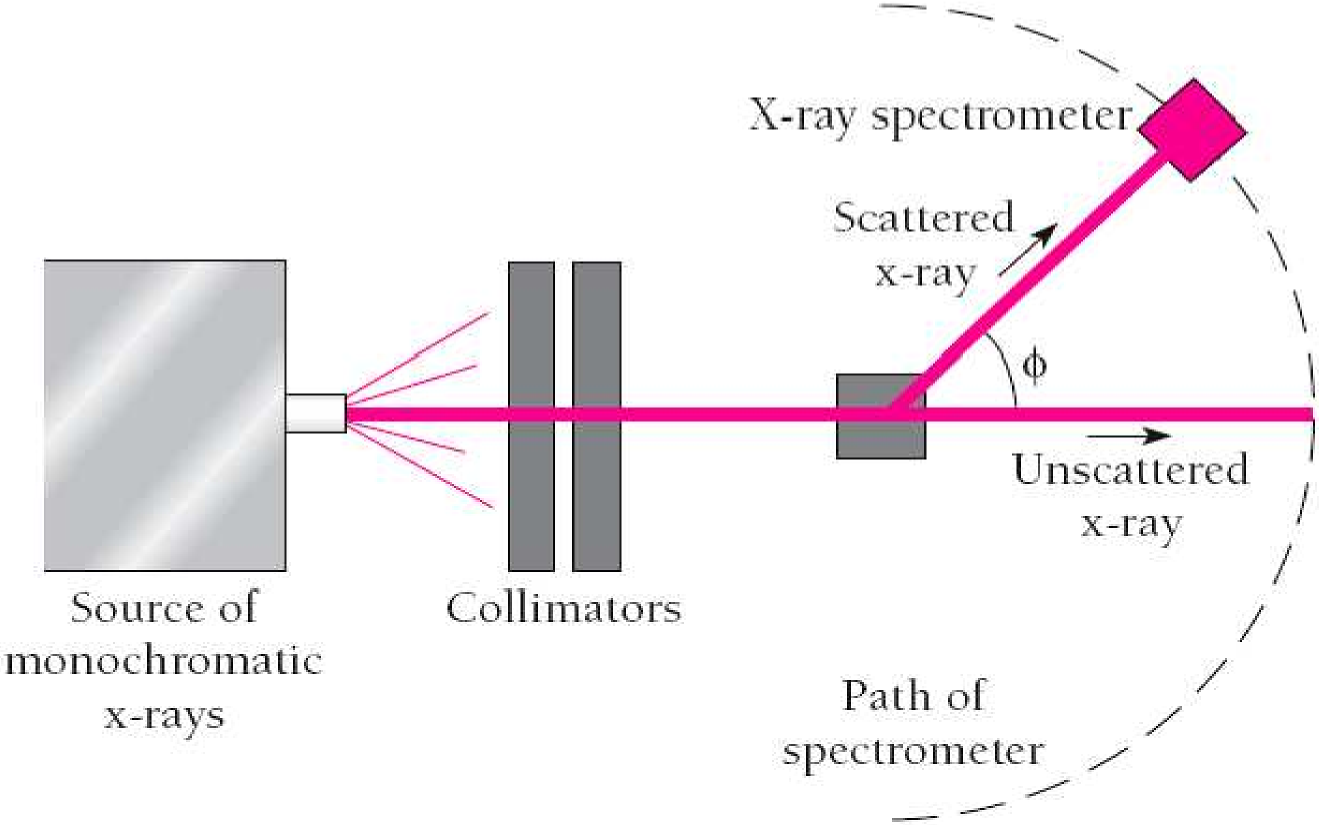
✓ When a monochromatic beam of high energy radiation (e.g., X-rays and γ-ray) is scattered by a substance, the scattered radiation contains two type of wavelengths:

1. One having same wavelength as that of incident radiation
2. While the other having the higher wavelength (lower Energy)



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Experiment

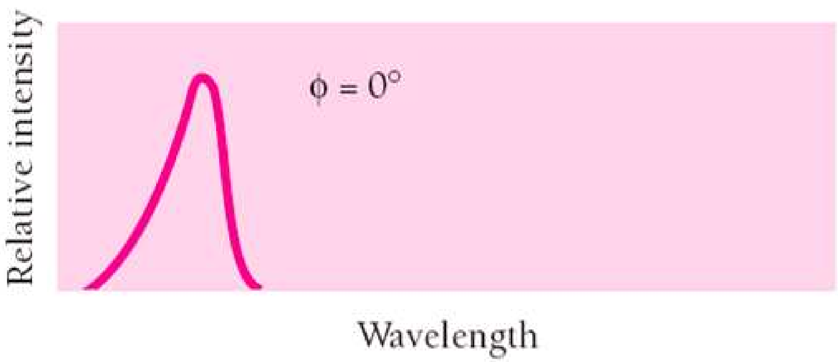


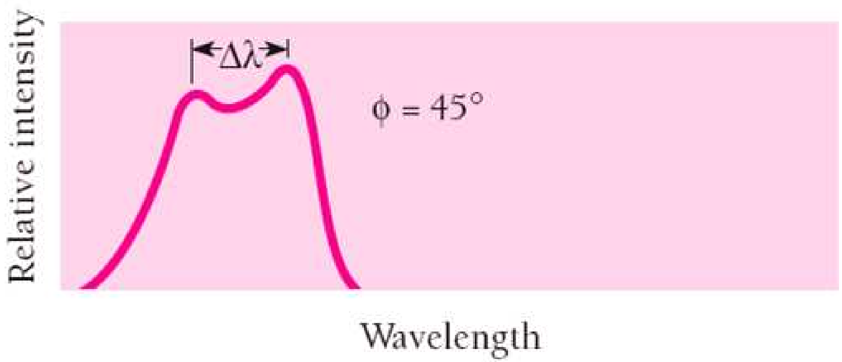
Experimental demonstration of the Compton effect.

- X-rays scattered from target containing very loosely bound electrons.
- Wavelength of scattered x-rays found to be different from that of incident X-rays and to depend on detection angle ‘ϕ’.

Experiment

- A graphite target was bombarded with monochromatic X-rays and wavelength of the scattered radiation was measured with a rotating crystal spectrometer.
- The intensity was measured by novel movable ionization chamber that generated a current proportional to the X-ray intensity.
- Compton measured the dependence of scattered intensity on wavelength at three different scattering angles of ϕ=45°, ϕ=90°, ϕ =90° and ϕ=135°.



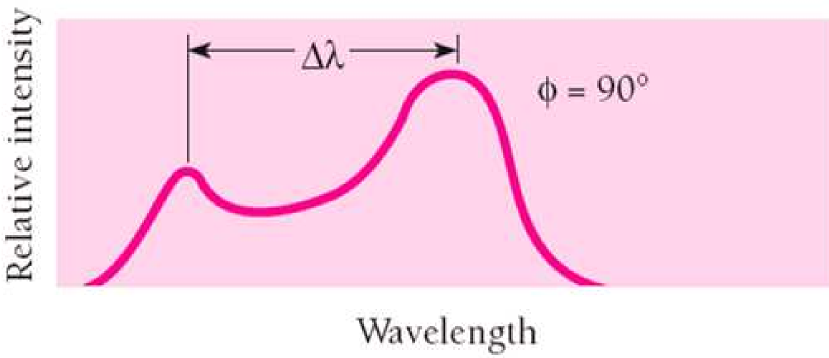


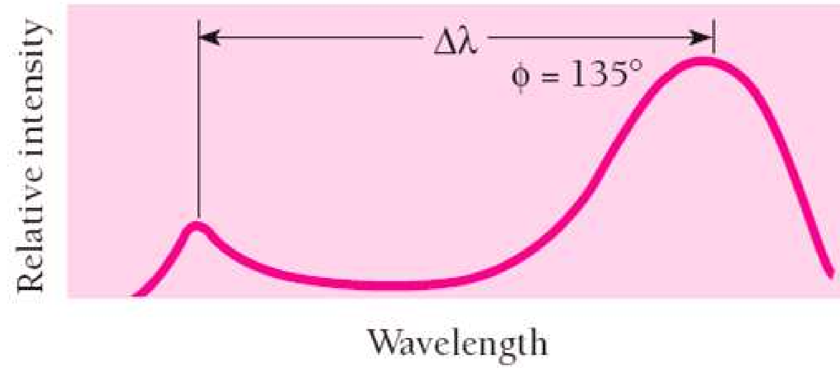
Case 1: when ϕ=0°, cos ϕ=1 and hence dλ=0

Case 2: when ϕ=45°, cos ϕ=1/√2 hence

dλ=1.705 ×0.0242 =0.0071Å

Compton effect





Case 3: when ϕ=90°, cos ϕ=0 and hence $d\lambda = h/m_0c$ = 0.0243 Å

This is called as Compton wavelength

Case 4: when ϕ=135°, cos ϕ= √2/2 and hence

$d\lambda = h/m_0c$ (1.707)= 0.0014 Å

Case 5: when ϕ=180°, cos ϕ= -1 and hence $d\lambda = 2h/m_0c$ = 0.0485 Å

Here the dλ has the maximum value

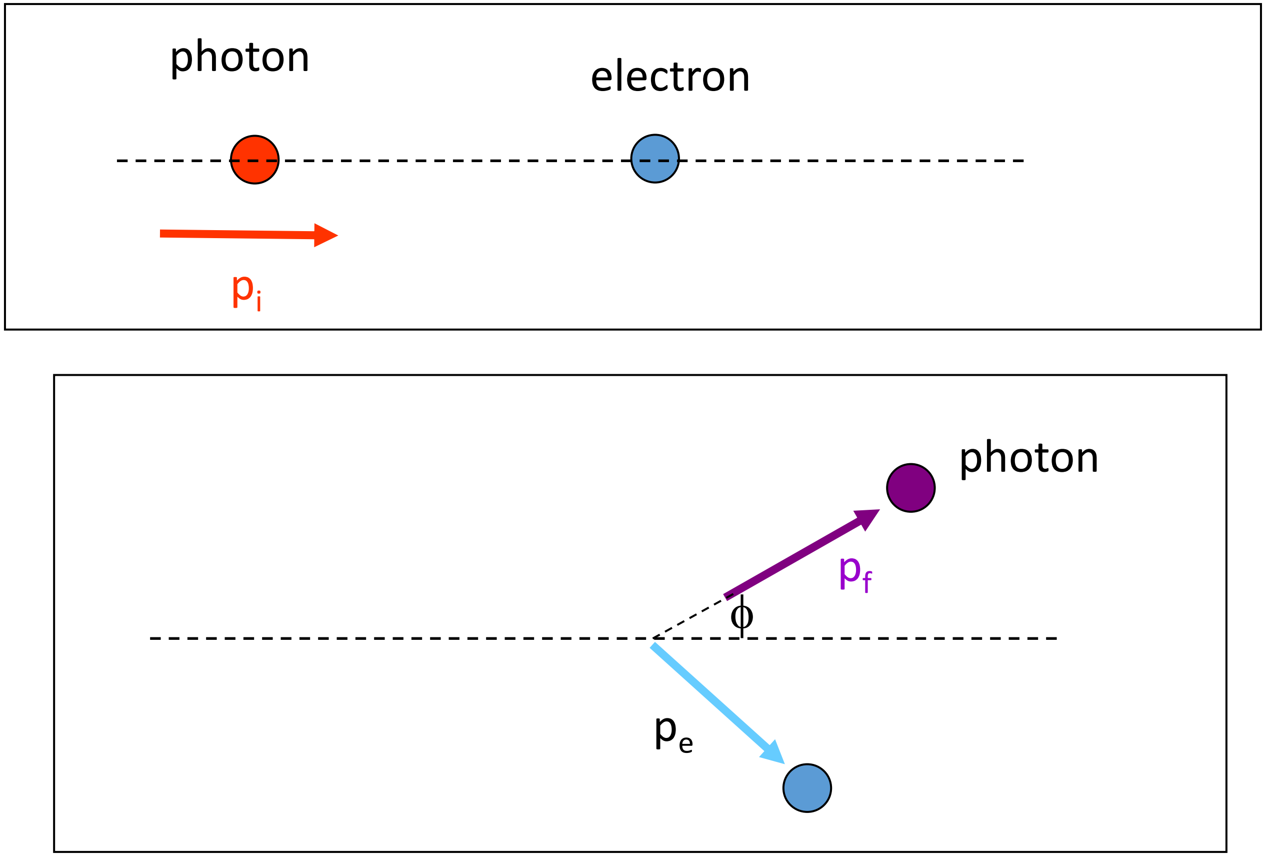
- Compton's experimental results are as shown in the figure. If photons were behaved only as waves there should not be any new waves with new wavelength as shown for different scattering angles.
- The experimental intensity vs wavelength plots observed by Compton for three angles show two peaks, one at the wavelength 'λ' of the incident X-rays and other at the longer wavelength 'λ'.

Compton Effect explained by photon model

✓ **Treat Compton scattering as a 2-particle collision between photon and initially stationary electron, obeying conservation laws for energy and momentum:**

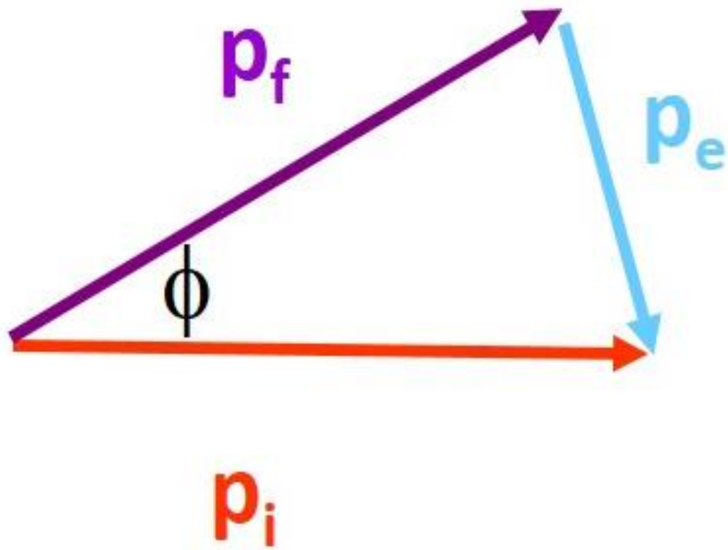
Before

After



Compton Scattering: Conservation of momentum

Vector triangle:

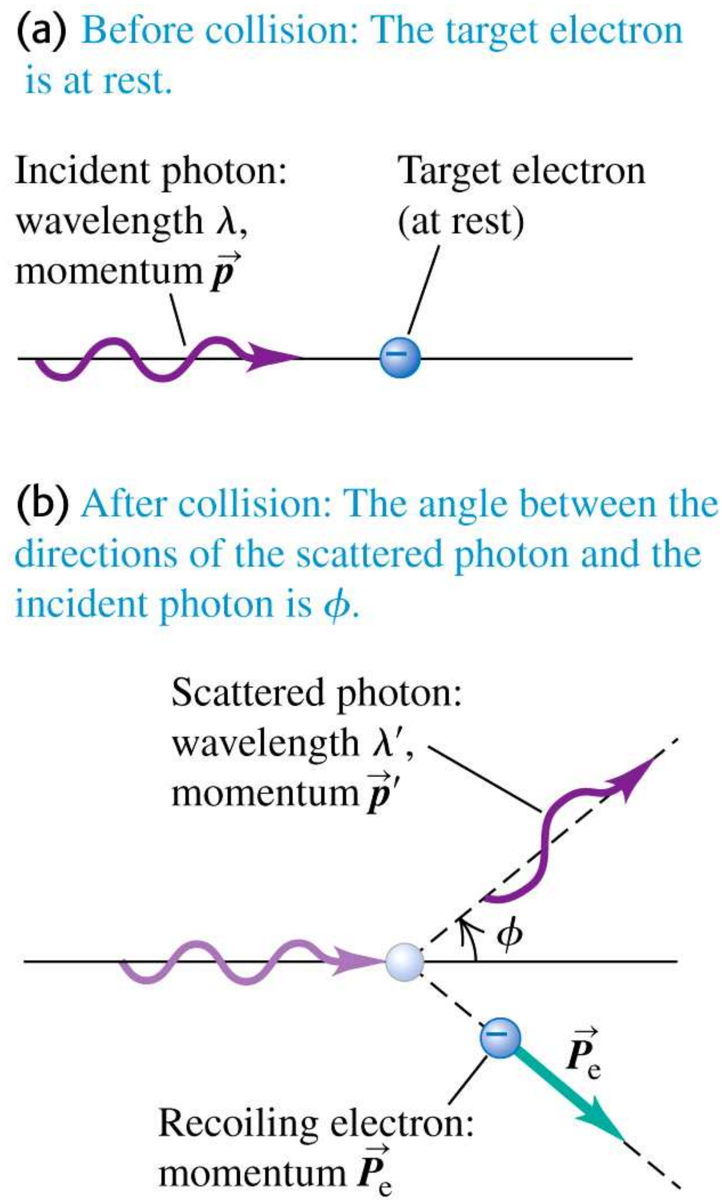


Consider magnitudes of vectors p_i, p_f and p_e:

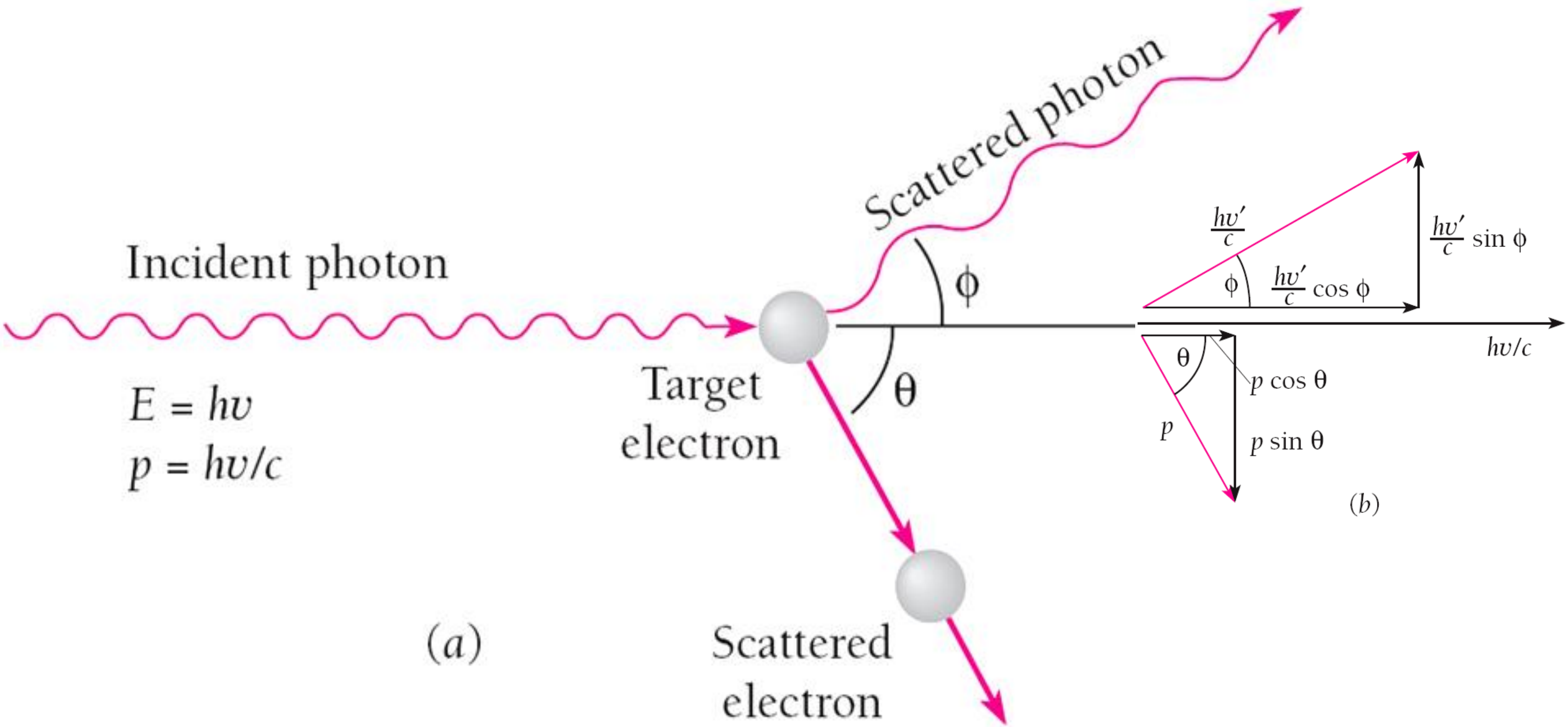
$$p_e^2 = p_f^2 + p_i^2 - 2p_f p_i \cos\phi$$

Compton Scattering

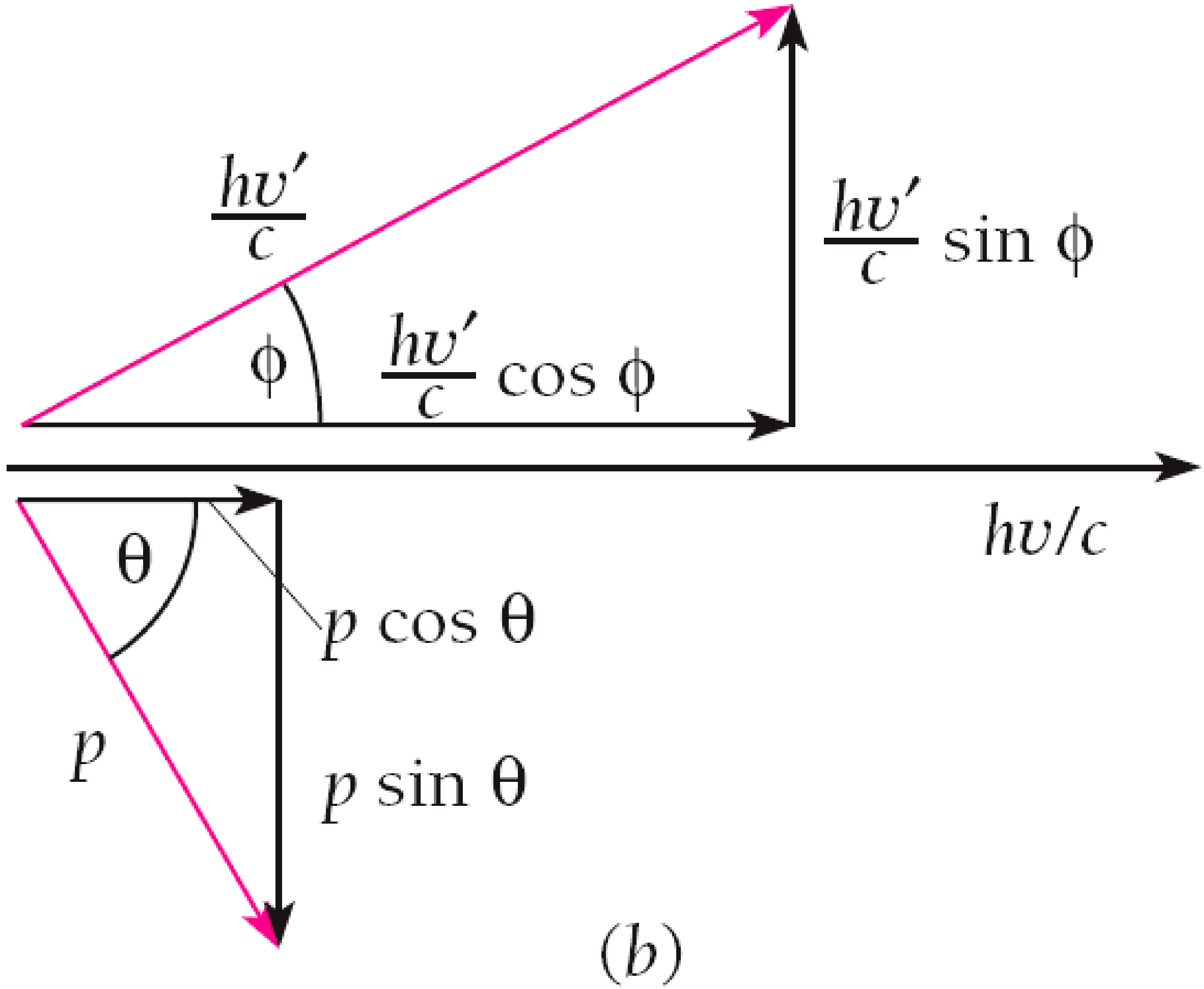
- In the Compton experiment, x-rays are scattered from electrons.
- The scattered x rays have a longer wavelength than the incident x rays, and the scattered wavelength depends on the scattering angle φ.
- Explanation: When an incident photon collides with an electron, it transfers some of its energy to the electron.
- *The scattered photon has less energy and a longer wavelength than the incident photon.*



Geometry of Compton Scattering



Geometry of Compton Scattering



Conservation of Energy

Energy,

before collision:

$$Photon's\ energy = h\vartheta$$

$$Electron's\ energy = m_0c^2$$

$$\text{Total energy } = h\vartheta + m_0c^2$$

after collision:

$$Photon's\ energy = h\vartheta'$$

$$Electron's\ energy = mc^2$$

$$\text{Total energy } = h\vartheta' + mc^2$$

By law of Conservation of energy, before and after collision the total energy of the system (Photon and electron) should be equal.

$$h\vartheta + m_0c^2 = h\vartheta' + mc^2 ---- 1$$

Conservation of Momentum

before Collision

along x –axis

$$Photon's\ momentum = \frac{h\vartheta}{c}$$

$$Electron's\ momentum = 0$$

$$Total\ momentum = \frac{h\vartheta}{c}$$

along y –axis

$$Photon's\ momentum = 0$$

$$Electron's\ momentum = 0$$

$$Total\ momentum = 0$$

after Collision

along x –axis

$$Photon's\ momentum = \frac{h\vartheta' \cos\emptyset}{c}$$

$$Electron's\ momentum = mv\cos\theta$$

$$Total\ momentum = \frac{h\vartheta' \cos\emptyset}{c} + mv\cos\theta$$

along y –axis

$$Photon's\ momentum = \frac{h\vartheta' \sin\emptyset}{c}$$

$$Electron's\ momentum = -mv\sin\theta$$

$$Total\ momentum = \frac{h\vartheta' \sin\emptyset}{c} - mv\sin\theta$$

Derivation of Compton scattering Formula

$$\frac{h\vartheta}{c} = \frac{h\vartheta' \cos \emptyset}{c} + mv\cos\theta$$

$$mv\cos\theta = \frac{h\vartheta}{c} - \frac{h\vartheta' \cos \emptyset}{c} = \frac{h}{c}(\vartheta - \vartheta' \cos\emptyset)$$

$$mv\cos\theta = \frac{h}{c}(\vartheta - \vartheta' \cos\emptyset)$$

$$mvc\cos\theta = h(\vartheta - \vartheta' \cos\emptyset) - - - - -2$$

$$m^2v^2c^2cos^2\theta = h^2\left(\vartheta^2 + \vartheta'^2\cos^2\emptyset - 2\vartheta\vartheta' \cos\emptyset\right)$$

$$\frac{h\vartheta' \sin\emptyset}{c} - mv\sin\theta = 0$$

$$\frac{h\vartheta' \sin\emptyset}{c} = mv\sin\theta$$

$$mvc\sin\theta = h\vartheta' \sin\emptyset - - - - -3$$

$$m^2v^2c^2sin^2\theta = h^2\vartheta'^2\sin^2\emptyset$$

$$m^2v^2c^2(sin^2\theta + cos^2\theta) = h^2\left(\vartheta'^2\sin^2\emptyset + \vartheta'^2\cos^2\emptyset + \vartheta^2 - 2\vartheta\vartheta' \cos\emptyset\right)$$

Derivation of Compton scattering Formula

$$m^2v^2c^2 sin^2\theta = h^2\vartheta'^2 \sin^2\emptyset$$

$$m^2v^2c^2(sin^2\theta + cos^2\theta) = h^2\left(\vartheta'^2\sin^2\emptyset + \vartheta'^2\cos^2\emptyset + \vartheta^2 - 2\vartheta\vartheta'\cos\emptyset\right)$$

$$m^2v^2c^2 = h^2\left[\vartheta'^2(\sin^2\emptyset + \cos^2\emptyset) + \vartheta^2 - 2\vartheta\vartheta'\cos\emptyset\right]$$

$$m^2v^2c^2 = h^2\left[\vartheta'^2 + \vartheta^2 - 2\vartheta\vartheta'\cos\emptyset\right] ----- 4$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} ------- 5$$

$$m^2(c^2 - v^2) = m_0{}^2c^2$$

$$m^2c^2(c^2 - v^2) = m_0{}^2c^4 ---- 6$$

$$from\ 1, mc^2 = h\vartheta - h\vartheta' + m_0c^2$$

$$from\ 1, mc^2 = h(\vartheta - \vartheta') + m_0c^2$$

$$m^2c^4 = h^2\left(\vartheta^2 + \vartheta'^2 - 2\vartheta\vartheta'\right) + m_0^2c^4 + 2h(\vartheta - \vartheta')m_0c^2 ---- 7$$

$$from\ 4, \quad m^2v^2c^2 = h^2\left[\vartheta'^2 + \vartheta^2 - 2\vartheta\vartheta'\cos\emptyset\right]$$

Derivation of Compton scattering Formula

$$m^2c^4 - m^2v^2c^2$$
$$= h^2\left(\vartheta^2 + \vartheta'^2 - 2\vartheta\vartheta'\right) + m_0^2c^4 + 2h(\vartheta - \vartheta')m_0c^2 - \left(h^2\left[\vartheta'^2 + \vartheta^2 - 2\vartheta\vartheta'\cos\emptyset\right]\right)$$

$$m^2c^4 - m^2v^2c^2$$
$$= h^2\left(\cancel{\vartheta^2 + \vartheta'^2} - 2\vartheta\vartheta'\right) + m_0^2c^4 + 2h(\vartheta - \vartheta')m_0c^2 - \left(h^2\left[\cancel{\vartheta'^2 + \vartheta^2} - 2\vartheta\vartheta'\cos\emptyset\right]\right)$$

$$m^2c^2(c^2 - v^2) == -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0c^2 + m_0^2c^4 - -7$$

$$m_0{}^2c^4 = -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0c^2 + m_0^2c^4$$

$$\cancel{m_0{}^2c^4} = -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0c^2 + \cancel{m_0^2c^4}$$

$$0 = -2h^2\vartheta\vartheta'(1 - cos\emptyset) + 2h(\vartheta - \vartheta')m_0c^2$$

$$\cancel{2}h^{\cancel{2}}\vartheta\vartheta'(1 - cos\emptyset) = \cancel{2h}(\vartheta - \vartheta')m_0c^2$$

$$\frac{h}{m_0c^2}(1 - cos\emptyset) = \frac{(\vartheta - \vartheta')}{\vartheta\vartheta'}$$

$$\frac{h}{m_0c^2}(1 - cos\emptyset) = \left(\frac{1}{\vartheta'} - \frac{1}{\vartheta}\right)$$

$$\frac{h}{m_0 c^2}(1 - cos\emptyset) = \left(\frac{1}{\vartheta'} - \frac{1}{\vartheta}\right)$$

$$\frac{h}{m_0 c^2}(1 - cos\emptyset) = \left(\frac{\lambda'}{c} - \frac{\lambda}{c}\right) = \frac{1}{c}(\lambda' - \lambda) = \frac{1}{c}\Delta\lambda$$

$$\frac{h}{m_0 c^{\cancel{2}}}(1 - cos\emptyset) = \frac{1}{\cancel{c}}\Delta\lambda$$

$$\Delta\lambda = \frac{h}{m_0 c}(1 - cos\emptyset)$$

$$\boxed{\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)}$$ Compton shift formula

Where, m_0 – electron's mass; c - velocity of light; $\emptyset$ – photon's scattering angle

Compton Scattering: Summary

The observed experimental result:

$$\boxed{\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)}$$

This is entirely explained by the photon-electron scattering model. Further proof of the validity of the photon concept.

- ✓ Electrons and nuclei can scatter photons
- ✓ Scattered photon is at a lower frequency than incident photon
- ✓ Some of the energy is transferred to the electron or nucleus
- ✓ Maximum wavelength shift for $\phi = 180°$, $\Delta\lambda = 2h/mc$
- ✓ The maximum change in wavelength is 4.86×10^{-12} m.
- ✓ h/m_0c is known as the COMPTON WAVELENGTH of the electron.
- ✓ very small (work it out!) so Compton effect only observed for short wavelength radiation (x-rays, gamma rays)

Applications

- Compton scattering is of prime importance to radiology , as it is the most probable interaction of gamma ray and high energy X- ray with atoms in living beings and is applied in radiation therapy.
- In material science, Compton effect can be used to probe the wave function of the electrons in the momentum representation.
- Compton scattering is an effect gamma ray spectroscopy which gives rise to the Compton edge. As it is possible for the gamma rays to scatter out of the detectors used. Compton suppression is used to detect stray scatter gamma rays to counteract this effect.
- Compton edge- Compton edge is feature of spectroscope that result from the Compton scattering in the detector.

Compton Effect: Ex 1

X-rays of wavelength 0.140 nm are scattered from a block of carbon. What will be the wavelength of the X-rays scattered at 0°?

$$l' = l + \frac{h\ (1 - \cos \phi)}{m_o c}$$

$$l' = 140\ X10^{-9}m + \frac{(6.626\ X\ 10^{-34}\ J\ s)(1 - \cos 0)}{(9.11\ X\ 10^{-31}\ kg)(3\ X\ 10^{8}m/s)}$$

$$l' = 140\ X10^{-9}m + 0$$

$$l' = 140\ nm$$

Compton Effect: Ex 2

What will be the wavelength of the X-rays scattered at 90°?

$$l' = l + \frac{h\ (1 - \cos \phi)}{m_o c}$$

$$l' = 140\ X10^{-9}m + \frac{(6.626\ X\ 10^{-34}\ J\ s)(1 - \cos 90)}{(9.11\ X\ 10^{-31}\ kg)(3\ X\ 10^{8} m/s)}$$

$$l' = 140\ X10^{-9}m + 2.4\ X\ 10^{-12}\ m$$

$$l' = 142\ nm$$

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Compton Effect: Ex 3

What will be the wavelength of the X-rays scattered at 180°?

$$l' = l + \frac{h}{m_o c}(1 - \cos \phi)$$

$$l' = 140 \times 10^{-9} m + \frac{(6.626 \times 10^{-34}\ J\ s)(1 - \cos 180)}{(9.11 \times 10^{-31}\ kg)(3 \times 10^{8} m/s)}$$

$$l' = 140 \times 10^{-9} m + 4.8 \times 10^{-12}\ m$$

$l' = 145$ nm (straight back)