Engineering Physics

(PHY1701)

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Problems

1) **A particle is confined to a box of length L. It is described by the wave function $\Psi = A\sin(\pi x/L)$ for $0 < X < L$ and $\Psi = 0$ everywhere else. Calculate A?**

We know that in case of a particle enclosed in a box of length L and moving along X-axis, the wave function is given by ,

$$\psi = A sin\left(\frac{n\pi x}{L}\right) \tag{1}$$

Where n is positive integer.

Given that,

$$\psi = A sin\left(\frac{\pi x}{L}\right) \tag{2}$$

Comparing eq. (2), with eq. (1), we get n=1.
Applying the normalization condition

$$\int_0^L \psi\psi^* dx = 1$$

we have,

$$\int_0^L A^2 \sin^2\left(\frac{\pi x}{L}\right) dx = 1$$

or $$A^2 \int_0^L \left[\frac{1-\cos(2\pi x / L)}{2}\right] dx = 1$$

$$\frac{A^2}{2}\left[\int_0^L dx - \int_0^L \cos\frac{2\pi x}{L} dx\right] = 1$$

$$\frac{A^2}{2}[L-0] = 1 \quad or \quad A^2 \frac{L}{2} = 1$$

$$A^2 = \frac{2}{L}$$

$$A = \sqrt{\frac{2}{L}}$$

Problems

2) **A particle is moving in 1-D potential box (of infinite height) of width 25 Å. Calculate the probability of finding the particle within an interval of 5 Å at the centre of the box when it is in its state of least energy?**

We know that the wave function of a particle enclosed within an infinite potential barrier is given by,

$$\psi(x) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi x}{a}\right)$$

When the particle is the least energy n=1, hence in this case,

$$\psi(x) = \sqrt{\frac{2}{a}} sin\left(\frac{\pi x}{a}\right)$$

At the centre of the box x=a/2, the probability of finding the particle in the unit interval at the centre of box is given by

$$|\psi(x)|^2 = \left[\sqrt{\frac{2}{a}} sin\left(\frac{\pi(a/2)}{a}\right)\right]^2 = \frac{2}{a}\sin^2\frac{\pi}{2} = \frac{2}{a}$$

The probability P in the interval *Δx* is given by,

$$P = |\psi(x)|^2 \cdot \Delta x = \frac{2}{a} \cdot \Delta x$$

Problems

According to the problem, a=25 Å=25×10^{-10} m and

Δx=5 Å =5×10^{-10} m

$$P = \left|\psi\left(x\right)\right|^2 \cdot \Delta x = \frac{2}{a} \cdot \Delta x$$

$$P = \frac{2\times5\times10^{-10}}{25\times10^{-10}}$$

Probability = P $= 0.4$

Problems

3) Find the least energy of an electron moving in the dimension in as infinitely high potential box of width 1 Å, given mass of the electron 9.11×10^{-31} kg and $h=6.63\times10^{-34}$ J-s?

We know that the values of energy of a particle of mass 'm' moving in an infinite 1-D potential well of 'a' is given by,

$$E_n = \frac{n^2h^2}{8ma^2} \quad (n = 1, 2, 3....)$$

The least energy of the particle can be obtained by substituting n=1. The energy is given by

$$E_1 = \frac{h^2}{8ma^2}$$

According to the given problem,
m= 9.11×10^{-31} kg, a=1 Å=10^{-10} m
and h=6.63×10^{-34} J-s

$$E_1 = \frac{\left(6.63\times10^{-34}\right)^2}{8\left(9.11\times10^{-31}\right)\left(10^{-10}\right)^2} J$$

$$= 9.1\times10^{-19}\ J = \frac{9.1\times10^{-19}}{1.602\times10^{-19}} eV$$

$$= 5.68\ eV$$

Problems

4) **An electron is bound by a potential which closely approaches in infinite square well of width 2.5×10^{-10} m. Calculate the lowest three permissible quantum energies the electron can have?**

In case of infinite potential well,

$$E_n = \frac{n^2h^2}{8ma^2}$$

Here, $h=6.63\times10^{-34}$ J-s, $m= 9.11\times10^{-31}$ kg, $a=2.5\times10^{-10}$ m, Now:

$$E_1 = \frac{\left(6.63\times10^{-34}\right)^2}{8\left(9.11\times10^{-31}\right)\left(2.5\times10^{-10}\right)^2}\ J$$

$$= 9.63\times10^{-19}\ J = \frac{9.63\times10^{-19}}{1.602\times10^{-19}}\ eV$$

$$E_1 = 6\ eV$$

Again, for second (n=2) and third (3) quantum energies,

$$E_2 = n^2E_1 = 4\times6 = 24\ eV$$

$$E_3 = n^2E_1 = 9\times6 = 54\ eV$$

Problems

5) An electron is confined to move two rigid walls separated by 10^{-9} m. Find the de-Broglie wavelength representing the first three allowed energy states of the electron and corresponding energies?

When electron moves to and fro b/w the rigid walls, it forms stationary wave pattern with nodes at the walls.
So the distance 'a' b/w the walls must be a whole multiple of the de-Broglie half wavelengths. Hence,

$$a = n\left(\frac{\lambda}{2}\right), \quad Where \quad n = 1, 2, 3 ....$$

$$\therefore \lambda = \frac{2a}{n} \qquad Given\, that\, a = 10^{-9}\ m = 10\ \mathring{A}$$

$$\lambda = \frac{2\times 10\ \mathring{A}}{n} = 20\ \mathring{A}, 10\ \mathring{A}, 6.67\ \mathring{A}, \ ..... \ (\text{if n} = 1, 2, 3, ...)$$

Problems

$$E_n = \frac{n^2h^2}{8ma^2} \; (\mathrm{n} = 1, 2, 3....)$$

Here, h=6.63×10^{-34} J-s,
m= 9.11×10^{-31} kg,
a=10^{-9} m

$$E_1 = \frac{n^2\left(6.63\times10^{-34}\right)^2}{8\left(9.11\times10^{-31}\right)\left(10^{-9}\right)^2}\,J$$

$$= 6.04\times10^{-20}\ n^2 J = \frac{6.04\times10^{-20}}{1.602\times10^{-19}} n^2 eV$$

$$= 0.38\ n^2 eV$$

For n=1, 2 and 3, we have

E_1 = 0.38 eV, E_2 = 1.52 eV and **E_3= 3.42 eV**

Problems

6) An electron is bound by a potential which closely approaches an infinite square well of width 2.5×10^{-10} m. **Calculate the lowest three permissible quantum energies the electron can have?**

Given data: $a = 2.5\times10^{-10}$ m

$$E_n = \frac{n^2h^2}{8ma^2}$$

$$E_1 = \frac{\left(6.63\times10^{-34}\right)^2}{8\left(9.11\times10^{-31}\right)\left(2.5\times10^{-10}\right)^2}\ J$$

$$= 9.63\times10^{-19}\ J$$

$$= \frac{9.63\times10^{-19}}{1.602\times10^{-19}}\ eV$$

$$E_1 = 6\ eV$$

$$E_2 = 24\ eV$$

$$E_3 = 54\ eV$$

Problems

7) Find the energy of an electron moving in 1-D in an infinity high potential box of width 1 Å, given mass of the electron 9.11×10^{-31} kgm and $h=6.63\times10^{-34}$ J-s?

Given data:

a=1 Å =10^{-10} m
m= 9.11×10^{-31} kg &
h=6.63×10^{-34} J-s

$$E=\frac{h^2}{8ma^2}$$

$$E=\frac{\left(6.63\times10^{-34}\right)^2}{8\left(9.11\times10^{-31}\right)\left(10^{-10}\right)^2}\ J$$

$$=9.1\times10^{-19}\ J$$

$$=\frac{9.1\times10^{-19}}{1.602\times10^{-19}}\ eV$$

$$=0.58\ eV$$

Problems

8) A particle is confined to 1-D infinite potential well of width 0.2×10^{-19} m. It is found that when the energy of the particle is 230 eV, its Eigen function has 5 antinodes. Find the mass of the particle and show that it can never have energy equal to 1 keV?

It is obvious from the problem that the particle is in quantum state with n=5. If E_5 and E_1 represent the energies in state 5 and state 1, respectively, then

$$E_5 = 5^2\, E_1 = 230\ eV$$

$$= 230\times1.602\times10^{-19}\ J$$

$$or\ \ E_1 = \frac{E_5}{5^2} = \frac{230\times1.602\times10^{-19}}{25} = 14.7\times10^{-19}\ J$$

$$We\ know\ that$$

$$E_1 = \frac{h^2}{8ma^2} \quad \therefore \quad m = \frac{h^2}{8E_1a^2}$$

Here, $h=6.63\times10^{-34}$ J-s, $E_1= 14.7\times10^{-19}$ J and $a=0.2\times10^{-9}$ m

$$m = \frac{\left(6.63\times10^{-34}\right)^2}{8\left(14.7\times10^{-19}\right)\left(0.2\times10^{-9}\right)^2} J$$

$$= 9.3\times10^{-31}\, kg$$

When E_n= 1 keV =10^3 eV, n should be such that

$$n^2 = \frac{E_n}{E_1} = \frac{10^3\times1.602\times10^{-19}}{14.7\times10^{-19}}$$

$$n^2 = 108.7 \quad or$$

$$n = 10.4$$

As n = 10.4 is not an integer,

E_n = 1 keV is not permitted value of energy