Engineering Physics

(PHY1701)

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$$curl\ \vec{E} = \vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} = \frac{\partial E_z}{\partial y}\hat{i} - \frac{\partial E_z}{\partial x}\hat{j}$$

Now $E_z = E_0 \cos kx \sin \omega t$

$$\therefore \frac{\partial E_z}{\partial y} = 0 \quad \text{and} \quad \frac{\partial E_z}{\partial x} = -E_0 \sin kx \sin \omega t(k)$$

So

$$curl\ \vec{E} = -\left(-E_0 k \sin kx \sin \omega t\right)\hat{j} = (k E_0 \sin kx \sin \omega t)\hat{j}$$

$$curl\ \vec{E} = \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \left(E_0 k \sin kx \sin \omega t\right)\hat{j}$$

$$\therefore d\vec{B} = -\left(E_0 k \sin kx \sin \omega t\right)\hat{j} \cdot dt$$

On integrating,

$$\vec{B} = \left(\frac{E_0 k}{\omega} \sin kx \sin \omega t\right)\hat{j}$$

1 *If $V(x,y) = x^2 - 2xy + y^2$, find grad V at $(2,3)$.*

Solution

$$\vec{\nabla} V = \hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}$$

Given, $V(x,y) = x^2 - 2xy + y^2$

$$\frac{\partial V}{\partial x} = 2x - 2y, \quad \frac{\partial V}{\partial y} = -2x + 2y, \quad \frac{\partial V}{\partial z} = 0$$

$$\vec{\nabla} V = \hat{i}(2x - 2y) + \hat{j}(-2x + 2y) + \hat{k}(0)$$

At the point, (2, 3)

$$\vec{\nabla} V = \hat{i}(2 \cdot 2 - 2 \cdot 3) + \hat{j}(-2 \cdot 2 + 2 \cdot 3) + \hat{k}(0)$$

$$\vec{\nabla} V = -2\hat{i} + 2\hat{j}$$

Problems:

3 *If* $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, *find* $div\vec{r}$.

Solution

$$div\,\vec{r} = \vec{\nabla}\bullet\vec{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial y}\right)\bullet\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial y}$$

$$= 1+1+1$$

$$\vec{\nabla}\bullet\vec{r} = 3$$

Problems:

6 *If* $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$, *Prove that* $div(curl\,\vec{E}) = 0$. *i.e.,* $\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{E}\right)=0.$

olution

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{j}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{E}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\bullet\left[\hat{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \hat{j}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right]$$

$$\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{E}\right) = \frac{\partial}{\partial x}\frac{\partial E_z}{\partial y} - \frac{\partial}{\partial x}\frac{\partial E_y}{\partial z} + \frac{\partial}{\partial y}\frac{\partial E_x}{\partial z} - \frac{\partial}{\partial y}\frac{\partial E_z}{\partial x} + \frac{\partial}{\partial z}\frac{\partial E_y}{\partial x} - \frac{\partial}{\partial z}\frac{\partial E_x}{\partial y}$$

$$\vec{\nabla}\bullet\left(\vec{\nabla}\times\vec{E}\right) = 0$$

Problems:

This shows that the velocity of electromagnetic waves in a conducting medium is less than that in a dielectric medium which is

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \tag{13.96}$$

It is dependent on frequency, and so dispersion takes place in conducting medium.