Engineering Physics

(PHY1701)

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Contents

- Planck's concept (hypothesis) (AB 66-67)
- Compton Effect (AB 80-86),
- Particle properties of wave: Matter Waves (AB 104-114),
- Davisson Germer Experiment (AB 115-117),
- Heisenberg Uncertainty Principle (AB 119-128),
- Wave function (AB 182-184 & 190-195), &
- Schrödinger equation (time dependent & independent) (AB 187 -190 & 195-197).

❖ Concepts of Modern Physics, Arthur Beiser et al., Sixth Edition, Tata McGraw Hill (2013) (AB)

Particle Property of Waves

- Blackbody radiation
- Planck's concept
- Compton Effect

Wave Properties of Particles

- Debroglie Hypothesis - Matter Waves,
- Davisson Germer Experiment
- Heisenberg Uncertainty Principle

The Emergence of Quantum Physics

- Wave function
- Schrodinger equation

Energy and Matter

Size of Matter	Particle Property	Wave Property
Large – macroscopic	Mainly	Unobservable
Intermediate – electron	Some	Some
Small – photon	Few	Mainly

For matter $E = mc^2$

For waves $E = h\nu$

Classical vs ***Quantum*** world

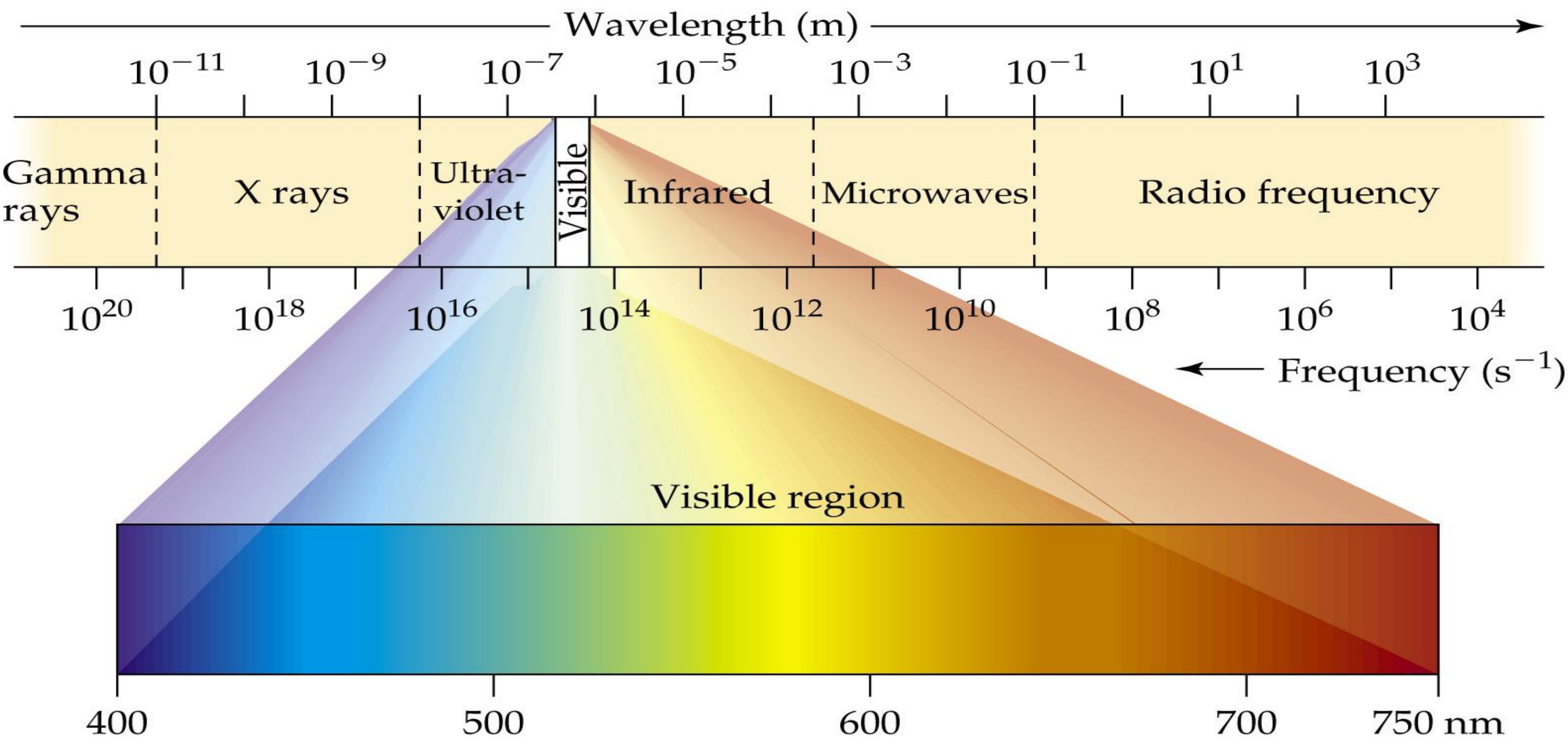
In everyday life, quantum effects can be safely ignored

This is because Planck's constant is so small

At atomic & subatomic scales, quantum effects are dominant & must be considered

Laws of nature developed without consideration of quantum effects do not work for atoms

The Wave Nature of Light

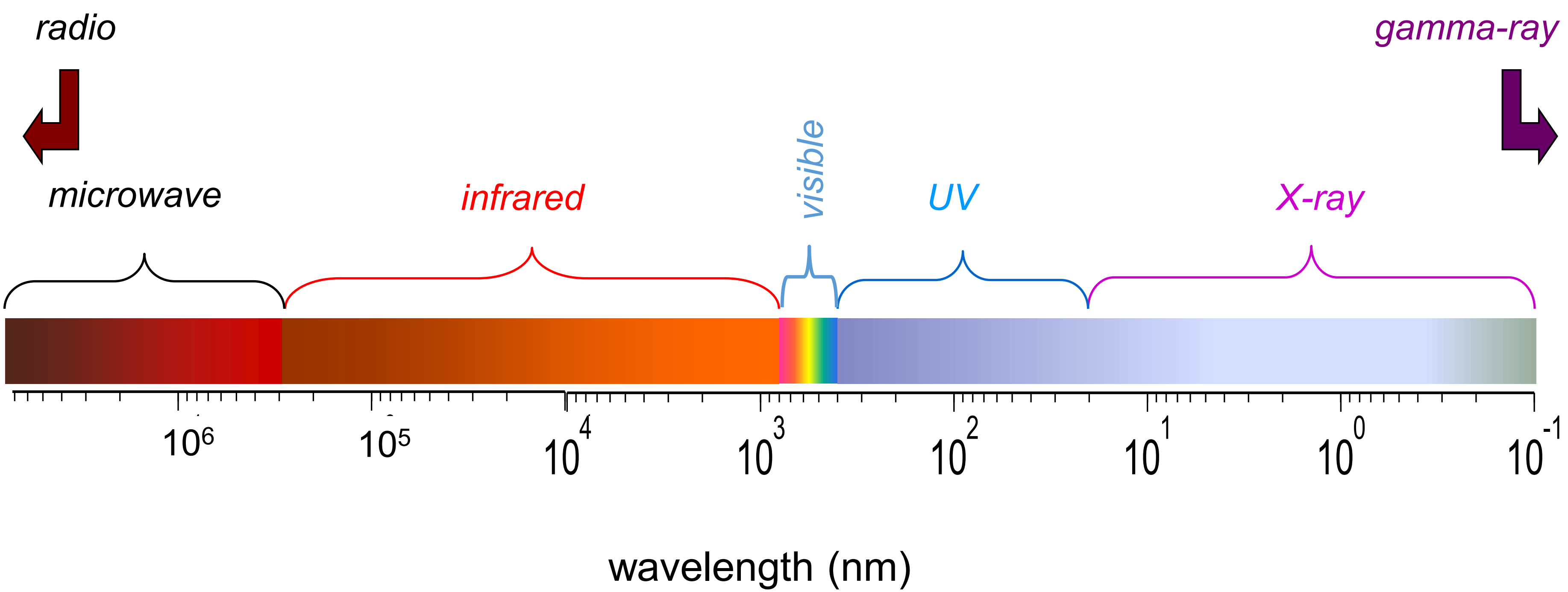


$$c = \lambda \cdot \nu$$

$$E = h \cdot \nu$$

The speed of light is constant

The wavelength λ Variation



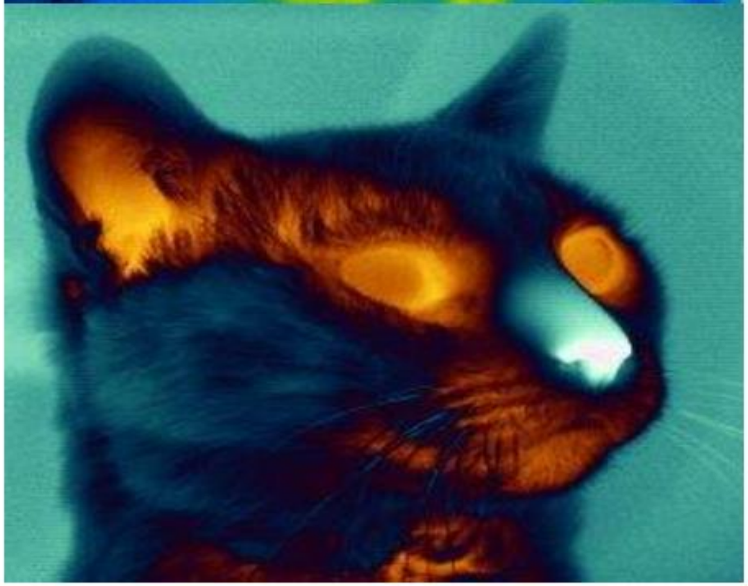
Low Energy

High Energy

λ (nm)

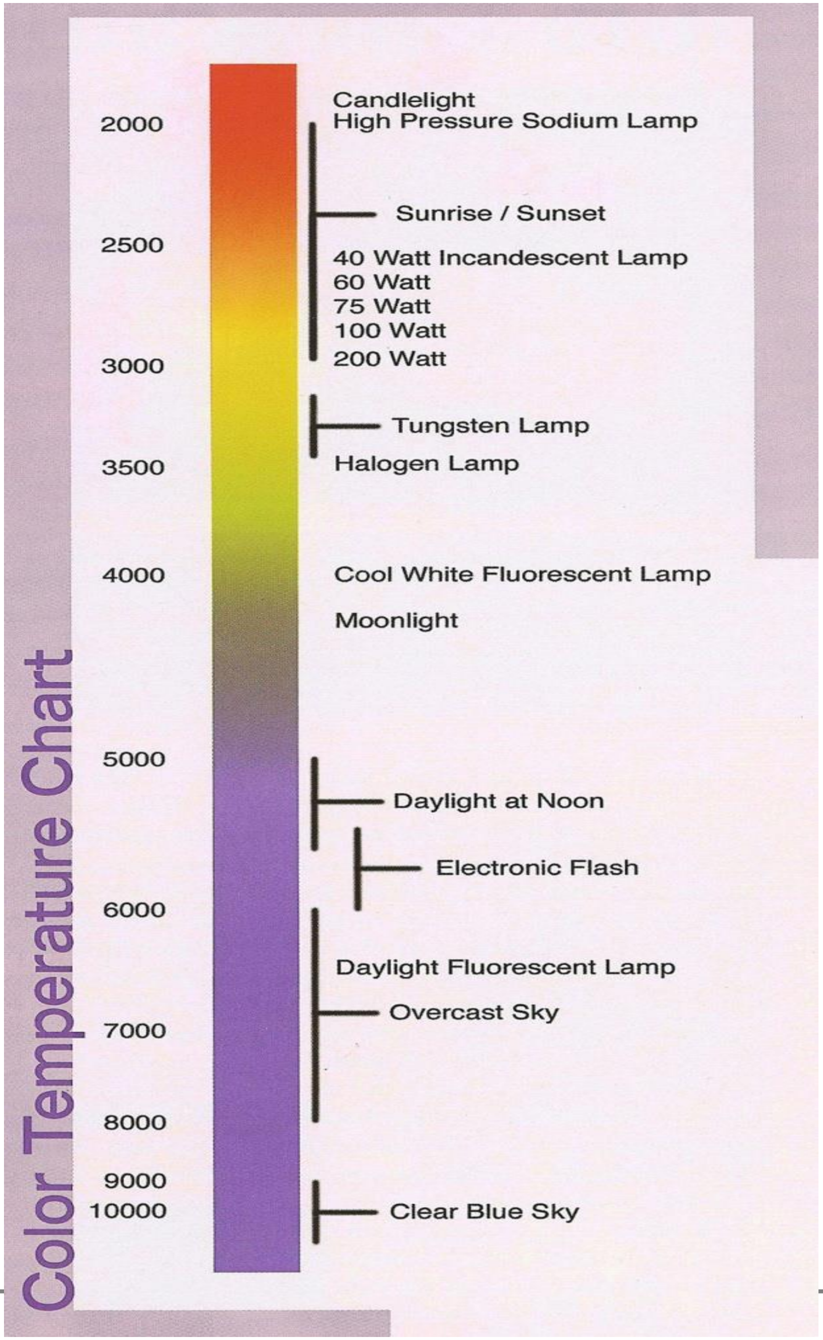
Glowing Objects

- Glow of Hot metal piece emits light – (Red to Yellow to white) as temperature increases
- Glowing objects emit continuous spectra but which frequency dominates depend on the temperature.
- Whereas heated gases emission is either bands or lines



At room temp. most of the body emits in the Infra red region of the spectrum - invisible

Color temperature



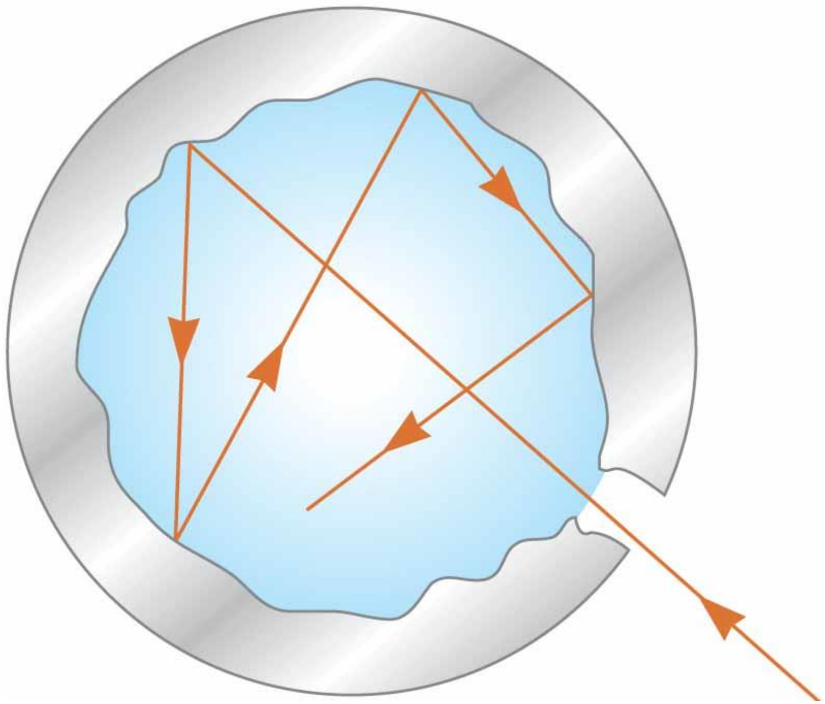
A light spectrum of Blackbodies is often characterized in terms of its temperature even if it's not exactly a blackbody.

Kirchoff radiation law

- **Thermal radiation:** The radiation emitted by a body as a result of temperature.
- **Blackbody:** A body that its surface absorbs and emit all the thermal radiation incident on them.
- **Spectral radiance:** The spectral distribution of blackbody radiation
- The ability of the body to radiate is closely related with its ability to emit.
- **Kirchoff radiation law:** Any body in thermal equilibrium with the radiation, the emitted power is proportional to the power absorbed.
- A body at constant temperature is in thermal equilibrium with its surrounding and must absorb energy from them at the same rate as its emit energy.

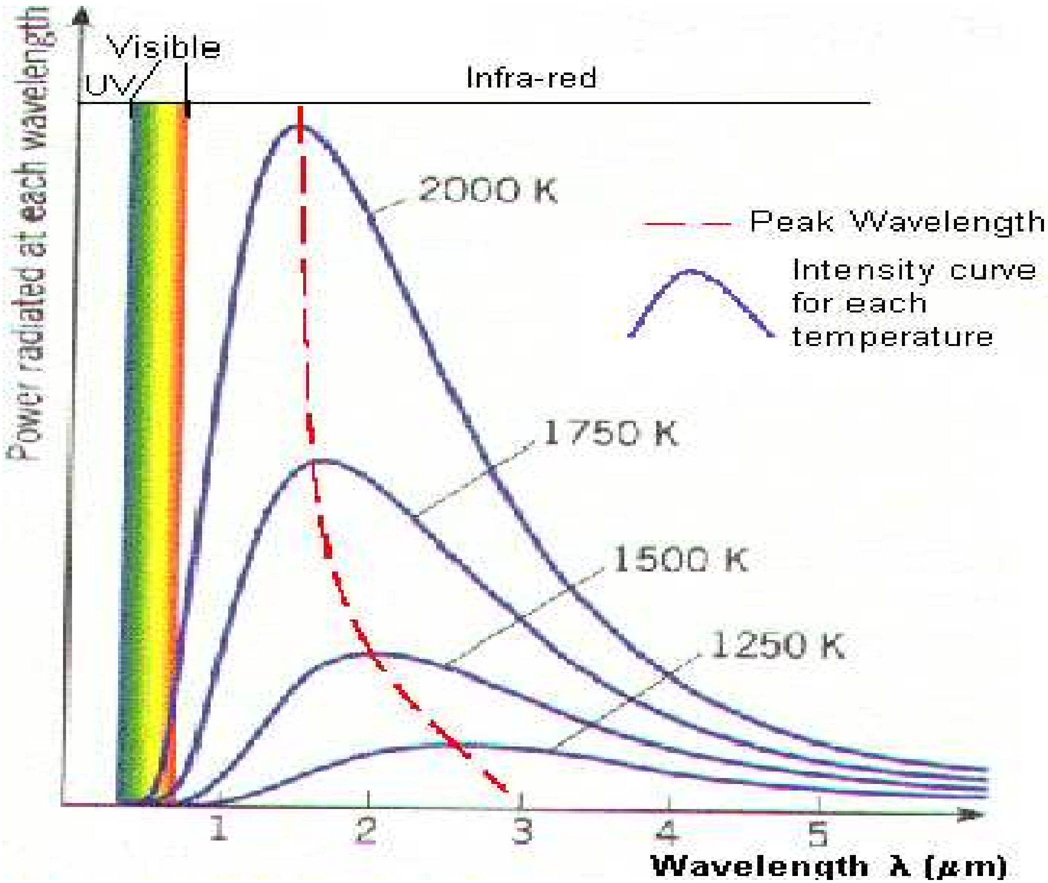
Black body

- A black body is an ideal body which allows the whole of the incident radiation to pass into itself (without reflecting the energy) and absorbs within itself this whole incident radiation (without passing on the energy).
- This property is valid for radiation corresponding to all wavelengths and to all angles of incidence. Therefore, the black body is an ideal absorber of incident radaition.
- An ideal system that absorbs all radiation incident on it, Independent of the material



The spectral radiance of blackbody radiation shows that:

1. The higher the temperature, the more the emission and the shorter the average wavelength.
2. little power radiation at very low wavelength.
3. The power radiation increases rapidly as λ increases from very small value.
4. The power radiation is most intense at certain wavelength λ_{max} or υ_{max} for particular temperature.



5. $\lambda < \lambda_{max}$ and R_T drops slowly, but continuously as λ increases, and $R_T(\lambda \longrightarrow \infty) \longrightarrow 0$.

R_T

$R_T(\upsilon)$- the emitted energy from a unit area per unit time

6. λ_{max} increases linearly with increasing temperature.
7. The total radiation for all ν (radiance $R_T = \int R_T(\upsilon)\, d\upsilon$, increases less rapidly than linearly with increasing temperature.

Stefan's Law

The radiation from a black body is observed to obey the following two laws :

Stefan's Law: The intensity of emitted radiation for a given wavelength is proportional to the fourth power of the temperature of the black body.

$$U = \frac{2\pi (kT)^4}{h^3 c^2} \frac{\pi^4}{15} = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67 \times 10^{-8}\ \mathrm{J.K^4/m^2.s}$$

Where σ is the Stefan’s constant

Basic Laws of Radiation

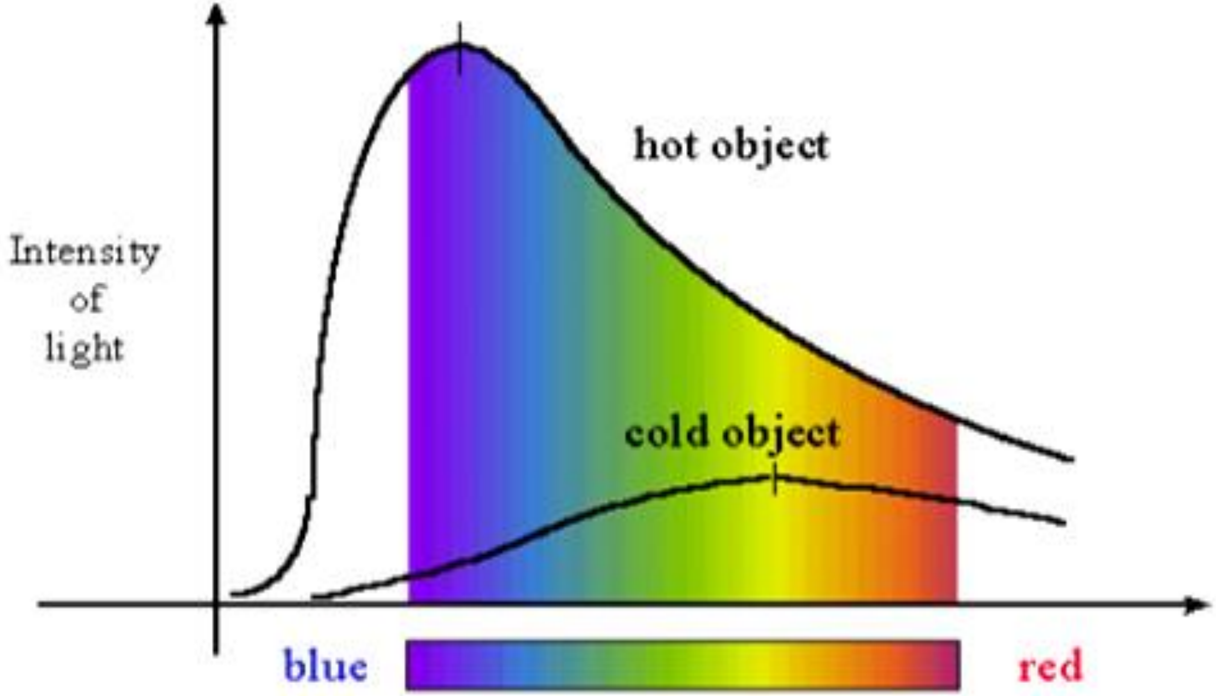
1) All objects emit radiant energy.

2) Hotter objects emit more energy than colder objects (per unit area). The amount of energy radiated is proportional to the temperature of the object.

3) The hotter the object, the shorter the wavelength (λ) of emitted energy.

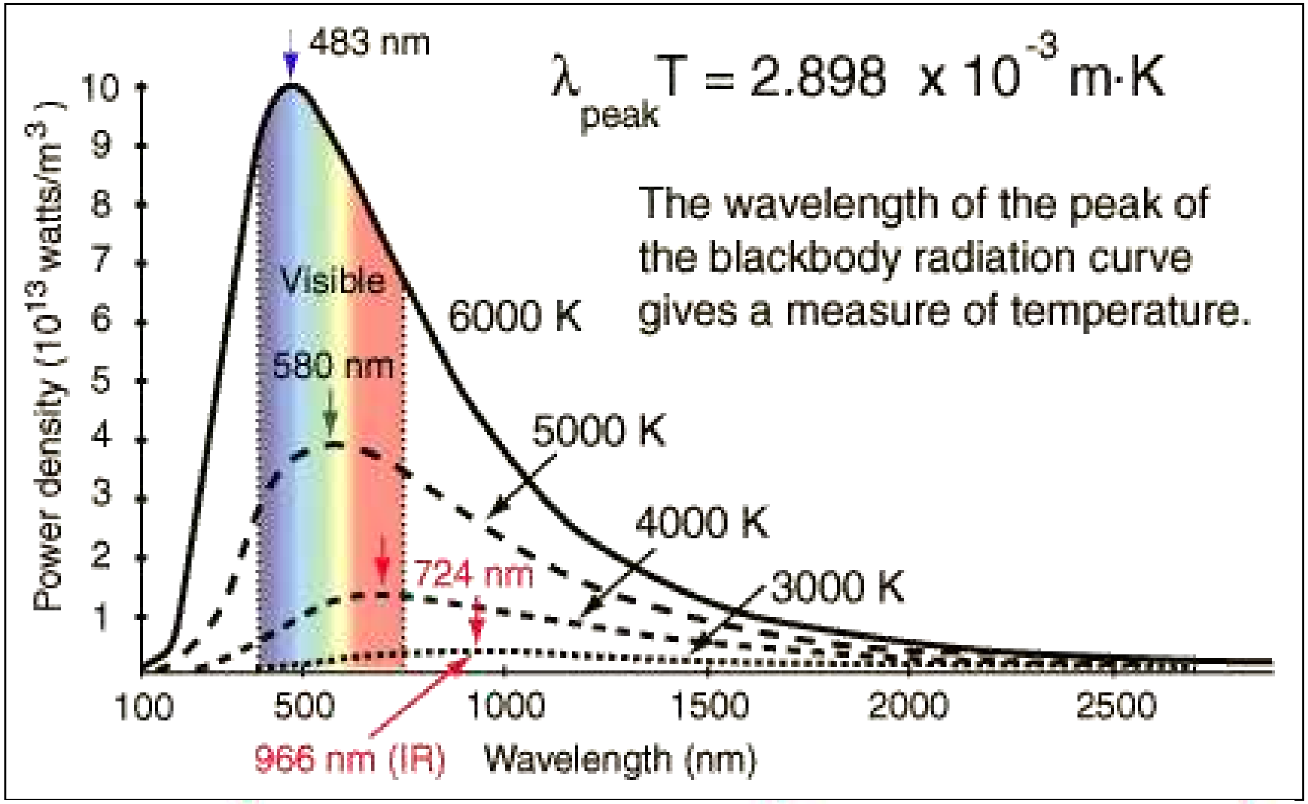
➡ This is **Wien's Law**

$$\lambda_{\mathbf{max}} \cong \frac{3000\ \mu m}{T}$$



Wien's Displacement Law

- Wien's Law tells us that objects of different temperature emit spectra that peak at different wavelengths.
- Hotter objects emit most of their radiation at **shorter** wavelengths, hence they will appear to be **bluer** .
- Cooler objects emit most of their radiation at **longer** wavelengths, hence they will appear to be **redder**.
- Furthermore, at any wavelength, a hotter object radiates more (is more luminous) than a cooler one.



Temperature , T (↑), Radiated energy, E (↑), Wavelength, λ (↓)

λ_{peak} vs Temperature

T	$\lambda_{peak} = \dfrac{2.9 \times 10^{-3}\ m}{T(Kelvin)}$	
310^0K (body temp)	$\dfrac{2.9 \times 10^{-3}\ m}{310^0}$	$=9x10^{-6}m$ infrared light
5800^0K (Sun's surface)	$\dfrac{2.9 \times 10^{-3}\ m}{5800^0}$	$=0.5x10^{-6}m$ visible light

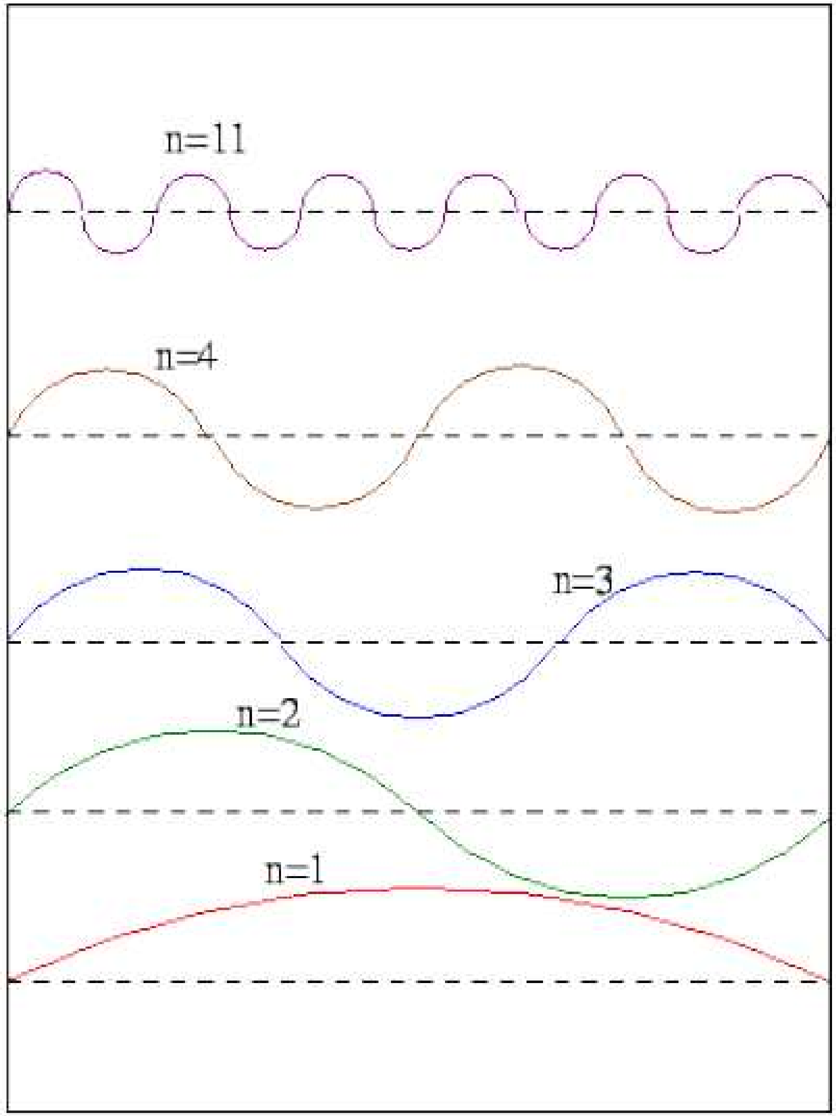
The Rayleigh-Jeans Law

- It agrees with experimental measurements for long wavelengths.
- It predicts an energy output that diverges towards infinity as wavelengths grow smaller.
- The failure has become known as the ultraviolet catastrophe.
- This formula also had a problem. The problem was the term in the denominator.
- For large wavelengths it fitted the experimental data but it had major problems at shorter wavelengths.
- At short wavelengths, there was a major disagreement between the Rayleigh-Jeans law and experiment

$$I(\lambda,T) = \frac{2\pi ckT}{\lambda^4}$$

- To explain this, Rayleigh Jeans considered the idea of cavity radiation and built on their hypothesis based on two theories
  1. Stationary waves in hollow space
  2. Equipartition of energy

Spectral energy density- Rayleigh Jeans law



Standing Wave Pattern in a Cavity

- Spectral energy density – energy per unit volume per unit frequency of the radiation within the cavity
- Radiations inside a cavity at absolute temperature T, considered
  - walls to be perfect reflectors
  - and form series of standing waves
- To form standing waves, path lengths from wall to wall whatever direction should be a whole number of half wavelength.

The no.of.standing waves between γ and $d\gamma$ per unit volume in the cavity

$$G(\gamma)d\gamma = \frac{8\pi\gamma^2 d\gamma}{C^3}$$

Rayleigh Jeans law

- **Classical average energy** (Kinetic theory) per standing wave

$$\bar{\varepsilon} = KT$$

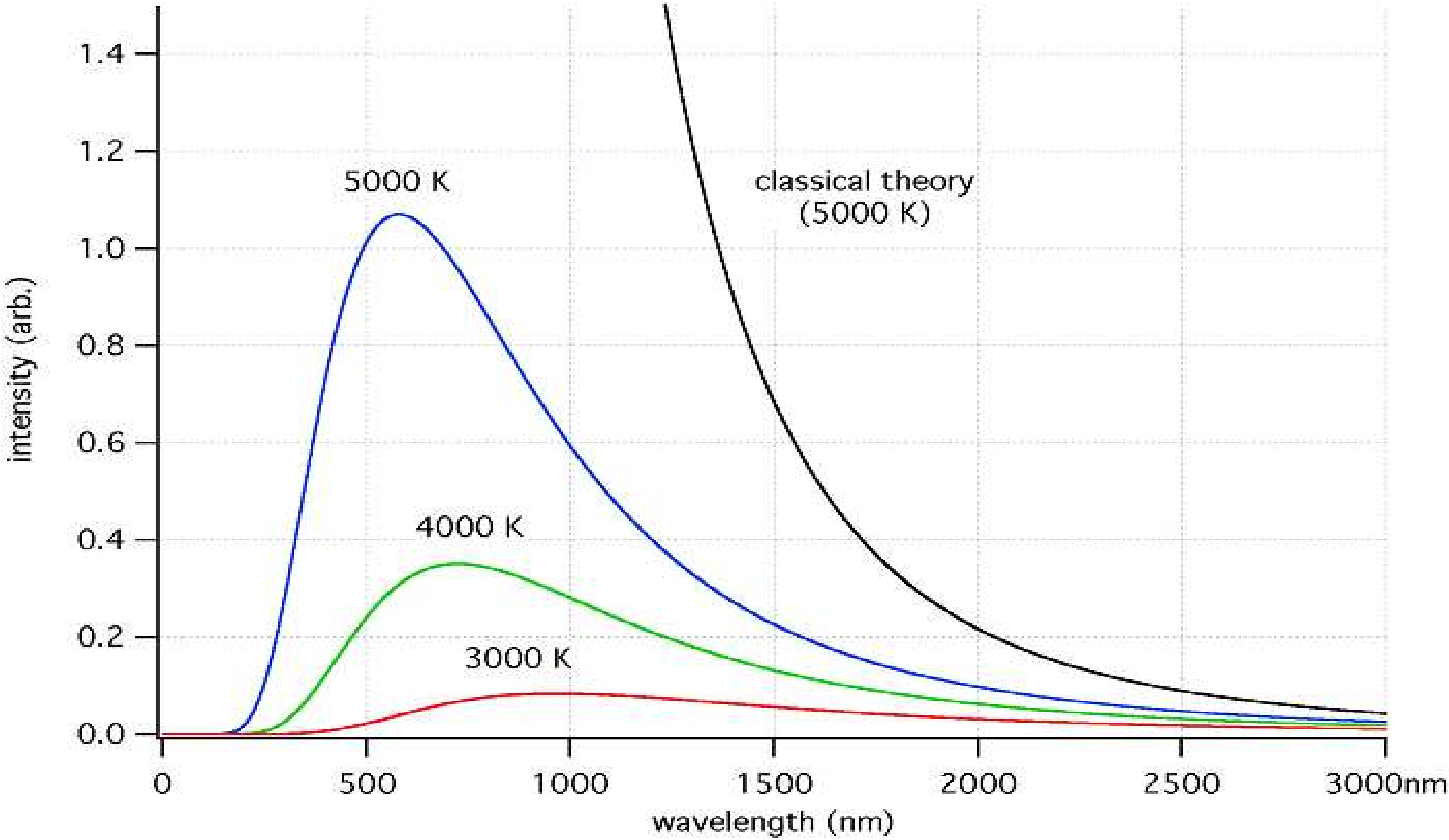
here, k- Boltzmann constant $=1.381\times10^{-23}$ J/K ; T- Absolute temperature

- The total energy per unit volume in the cavity in the frequency between ν and ν + dν is

$$u(\nu)d\nu = (\text{ modes in cavity in range } d\nu) \times (\text{average energy of modes})$$

$$\boxed{U(\nu)d\nu = \frac{8\pi kT\nu^2 d\nu}{C^3}} \longrightarrow \boxed{\text{Rayleigh Jeans Law}}$$

Comparison between Classical and Quantum viewpoint



There is a good fit at long wavelengths, but at short wavlengths there is a major disagreement. Rayleigh-Jeans $\longrightarrow$ ∞, but Black-body $\longrightarrow$ 0.

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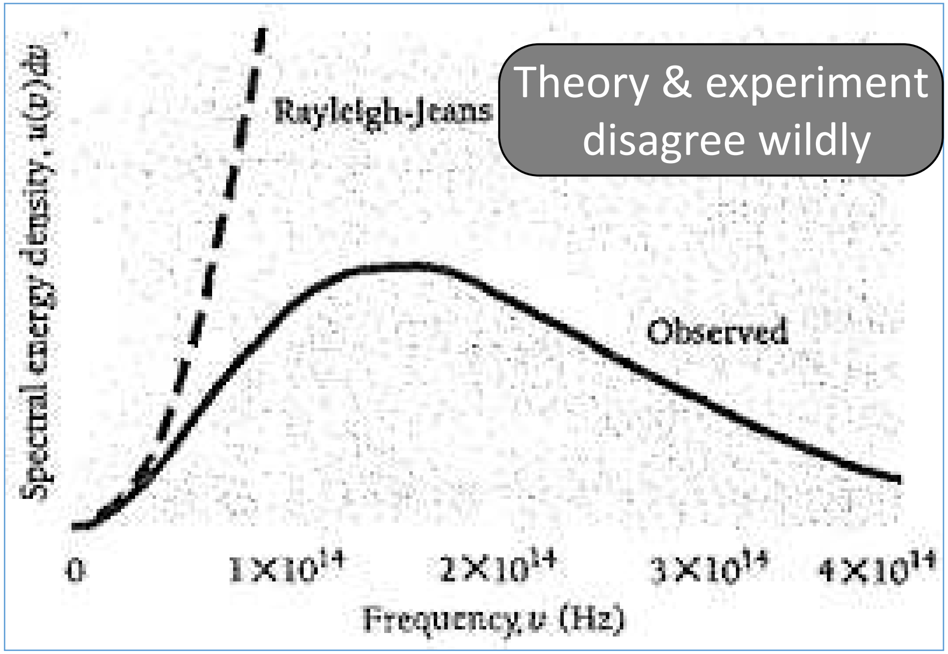
The Ultraviolet Catastrophe (Rayleigh Jeans Law)

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

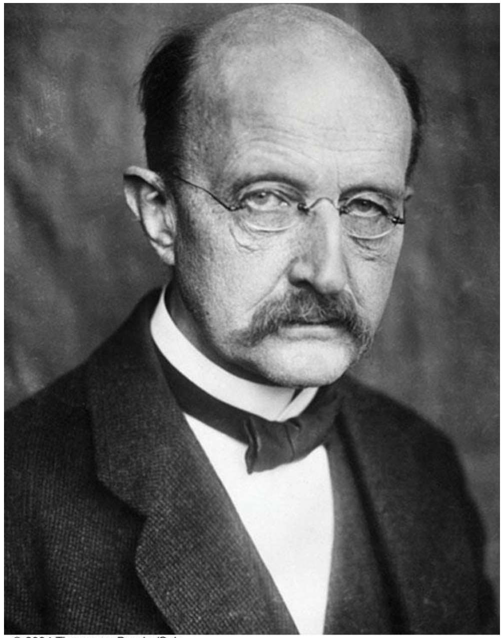


- Rayleigh- Jeans' law is roughly in agreement with the thermal radiation curves at shorter frequencies.
- at longer frequencies, it gives infinite energy density as γ increases γ ∝ . This is clearly unphysical.
- The failure of the classical wave theory to explain the observed radiation curve in the ultraviolet end of the electromagnetic spectrum is known as **ultraviolet catastrophe** .

Max Karl Ernst Ludwig Planck (1858–1947)

Introduced the concept of "quantum of action" to explain the blackbody radiation

In 1918 he was awarded the Nobel Prize for the discovery of the quantized nature of energy



Plancks Quantum Thoery

- Energy is not emitted or absorbed continuously . It is emitted or absorbed in the form of wave packets or quanta.
- The amount of energy associated with quantum of radiation is directly preportinal to the frequncy(γ)

$$E = h \cdot \nu \qquad E = h\frac{c}{\lambda}$$

Where, h=Planck's Constant=6.626×10^{-34} J.s = 6.55×10^{-27} erg.s

- Thus, greater the wavelenght of radiation, lower will be the energy

Planck's Quantum Thoery

- Max Planck, and others, had no way of knowing whether the calculation of the number of modes in the cavity, or the average energy per mode (i.e. kinetic theory), was the problem. It turned out to be the latter.
- <u>Max Planck, just discarded the idea of radiation being a continuous stream as well as law of equipartition of energy</u>.
- Planck found an empirical formula that fit the data, and then made appropriate changes to the classical calculation so as to obtain the desired result, which was non-classical.
- The problem boils down to the fact that there is no connection between the energy and the frequency of an oscillator in classical physics.
- Here, h is a fundamental constant, now known as Planck's constant. Although Planck knew of no physical reason for doing this, he is credited with the birth of quantum mechanics.

PLANCKS HYPOTHESIS

1. A black body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators of molecular dimensions.
2. They can vibrate with all possible frequencies. The absorption of energy is by the harmonic oscillators.
3. These oscillators absorb/emit only discrete energies
   $E_n = nh\nu$ where, n = 0,1,2,3...
4. The emission of radiation corresponds to a decrase and absorption to an increase in the energy and amplitude of an oscillator
5. The Energy of the oscillator is weighed by the Boltzman distribution instead of equipartition $\frac{1}{\exp(h\nu/kT)-1}$.

Planck's Law of radiation

- Thus the possible energy of a mode with frequency v is nhv where n =1,2….
- According to Boltzmann distribution, the probability of a mode having an energy E at a temperature T is given by exp(-βE) , where β=(1/kT)
- k is the Boltzmann constant and T is the absolute temperature.
- Thus the average energy of a mode is

$$\overline{E} = \frac{h\nu}{e^{h\nu/kT} - 1}. \longrightarrow$$ Average energy of a mode of frequency according to quantum theory

$$U(\boldsymbol{\nu})\mathrm{d}\,\boldsymbol{\nu} = \frac{8\pi h\nu^3 d\nu}{C^3(exp\,[h\nu/kT] - 1)} \longrightarrow$$ Planck radiation formula

The Planck distribution in terms of the energy density of radiation per unit frequency interval

Planck's Law of radiation

For Low Frequencies, hv<< kT and hv/kT <<1 , In general

$$e^{h\nu/kT} - 1 = 1 + \frac{h\nu}{kT} + \frac{1}{2!}\left(\frac{h\nu}{kT}\right)^2 + \cdots - 1.$$

If hv/kT is small , $e^{h\nu/kT} - 1 = \frac{h\nu}{kT}$

As, hv<< kT ,

$$\boxed{U(\nu)d\,\nu = \frac{8\pi kT\nu^2 d\nu}{c^3}}$$

Which is the Rayleigh Jeans Law

Planck's Law of radiation

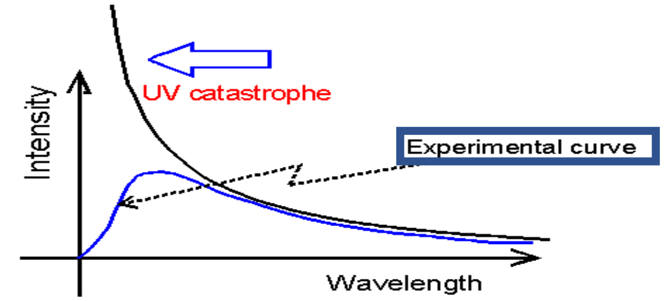
For high Frequencies, The Wien approximation may be derived from Planck's law by assuming hν>> kT and $e^{h\nu/kT} \longrightarrow \infty$,

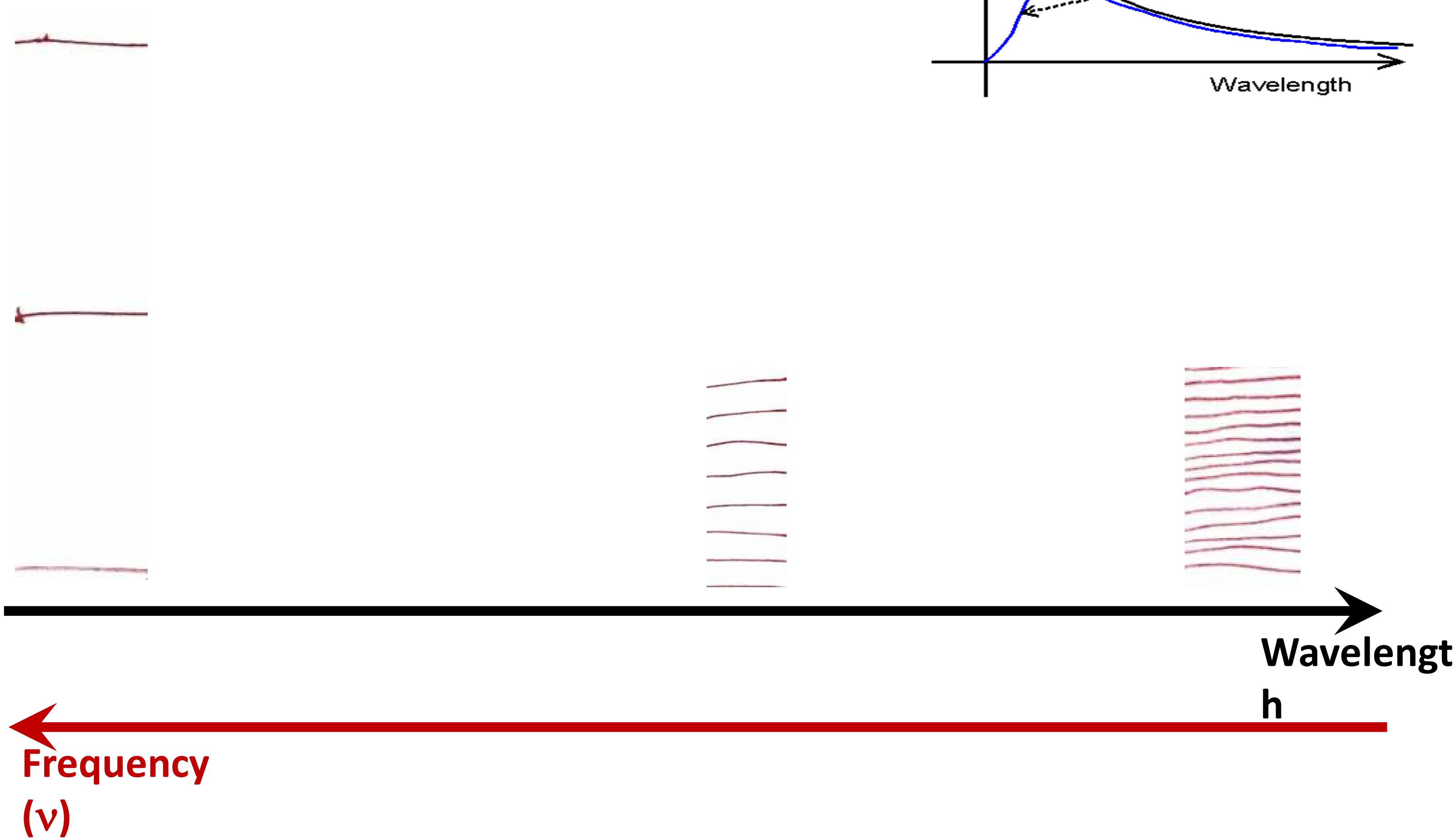
$$U(\nu)\mathrm{d}\,\nu \longrightarrow 0$$

No more ultraviolet catastrophe is observed.

$$\boxed{U(\nu)\mathrm{d}\,\nu = 0}$$

Which is the Wien's Law. so Planck's law approximately equals the Wien approximation at high frequencies


UV catastrophe
Intensity
Experimental curve
Wavelength


Wavelengt
h
Frequency
(ν)

Law of Equipartition of Energy

- For a system in thermal equilibrium, the law of Equipartition of Energy states that the total energy for the system is equally divided among the degree of freedom.

- According to the kinetic theory of gases, the average kinetic energy of molecule is given by,

$$\frac{1}{2}mv_{rms}{}^{2}=\frac{3}{2}k_{B}T$$

, where V_{rms} is the root mean square velocity of the molecules, K_b is the Boltzmann constant and T is the temperature of the gas.

- The mono-atomic gas has three degree of freedom, so the average kinetic energy per degree of freedom is given by

$$KE_{X}=\frac{1}{2}k_{B}T$$

The speed of EM waves

Q1 - Which of the following has the higher frequency

1. visible light or UV (choose one)
2. X-rays or radio waves (choose one)

Q2- Which of the following pairs has the longer wavelength:

1. Infrared or Ultraviolet (choose one)
2. Gamma rays or Radio waves (choose one)