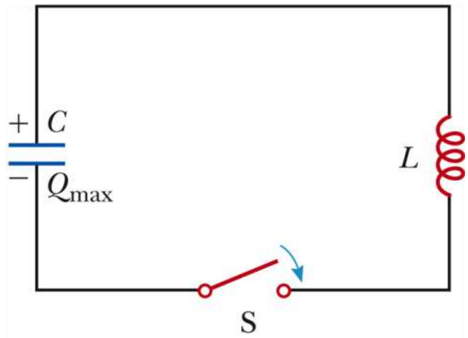
Experimental Evidence of Electromagnetic Waves:

LC Circuit:



- When the switch is closed, oscillations occur in the current and in the charge on the capacitor.
- When the capacitor is fully charged, the total energy of the circuit is stored in the electric field of the capacitor.
- At this time, the current is zero and no energy is stored in the inductor.
- As the capacitor discharges, the energy stored in the electric field decreases.
- At the same time, the current increases and the energy stored in the magnetic field increases.
- When the capacitor is fully discharged, there is no energy stored in its electric field.
- The current is at a maximum and all the energy is stored in the magnetic field in the inductor.
- The process repeats in the opposite direction.
- There is a continuous transfer of energy between the inductor and the capacitor.

IMPORTANT POINTS OF HERTZ EXPERIMENT:

- The first person generated and received the EM waves. His experiment shows that the EM waves follow the wave phenomena
- An induction coil is connected to a transmitter
- The transmitter consists of two spherical electrodes separated by a narrow gap to form a capacitor
- The oscillations of the charges on the transmitter produce the EM waves.

- A second circuit with a receiver, which also consists of two electrodes, is a single loop in several meters away from the transmitter.

- An induction coil is connected to a transmitter

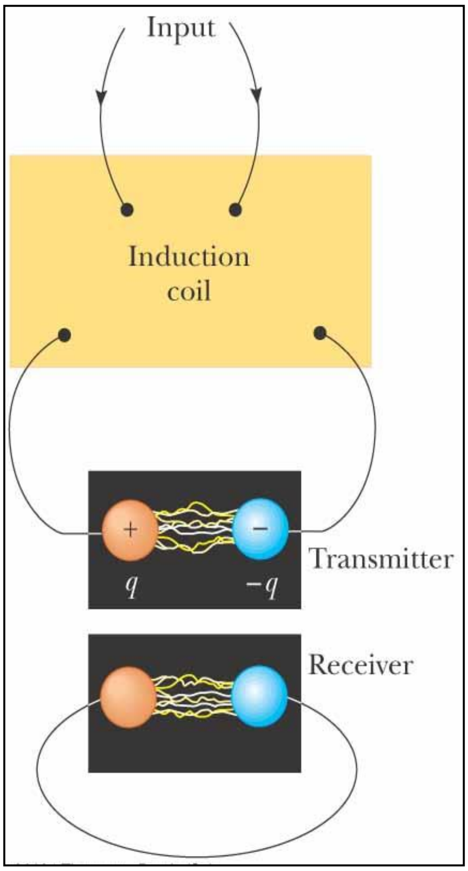
- The transmitter consists of two spherical electrodes separated by a narrow gap to form a capacitor

- The oscillations of the charges on the transmitter produce the EM waves.

- A second circuit with a receiver, which also consists of two electrodes, is a single loop in several meters away from the transmitter.

- Hertz found that when the frequency of the receiver was adjusted to match that of the transmitter, the energy was being sent from the transmitter to the receiver

- Hertz's experiment is analogous to the resonance phenomenon between a tuning fork and another one.



- Hertz also showed that the radiation generated by this equipment exhibited wave properties.

- Interference, diffraction, reflection, refraction and polarization

- He also measured the speed of the radiation

HERTZ RESULTS:

- Hertz hypothesized the energy transfer was in the form of waves (now known to be electromagnetic waves)

- Hertz confirmed Maxwell's theory by showing the waves existed and had all the properties of light waves (with different frequencies and wavelengths)

- Hertz measured the speed of the waves from the transmitter (used the waves to form an interference pattern and calculated the wavelength)
- The measured speed was very close to 3 x 10^8 m/s, the known speed of light, which provided evidence in support of Maxwell's theory

WAVE EQUATION:

Maxwell's equations predict the existence of electromagnetic wave. To derive the expression for electromagnetic wave, let us apply Maxwell's equations

The Maxwell's equation from Faraday's law is given by,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$= -\mu \frac{\partial H}{\partial t}$$

Take curl on both sides,

$$\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t} \quad \text{... (4.1)}$$

But Maxwell's equation from Ampere's law is

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Differentiating

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial}{\partial t}(\nabla \times H) = \frac{\partial}{\partial t}\left(J + \frac{\partial D}{\partial t}\right) = \frac{\partial}{\partial t}\left[\sigma E + \varepsilon \frac{\partial E}{\partial t}\right]$$

$$\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \quad \text{... (4.2)}$$

Substituting the equation (4.2) in equation (4.1)

$$\nabla \times \nabla \times E = -\mu\left[\sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2}\right]$$

$$= -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad \text{... (4.3)}$$

But according to the identity

$$\nabla \times \nabla \times E = \nabla(\nabla \bullet E) - \nabla^2 E \qquad \text{... (4.4)}$$

But
$$\nabla \bullet E = \frac{1}{\varepsilon} \nabla \bullet D$$

Since there is not net charge within the conductor, the charge density $\rho = 0$.

$$\nabla \bullet D = 0$$

$$\nabla \bullet E = 0$$

Then equation (4.4) becomes

$$\nabla \times \nabla \times E = -\nabla^2 E \qquad \text{... (4.5)}$$

Comparing the equations (4.3) and (4.5)

$$\nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0} \qquad \text{... (4.6)}$$

This is the wave equation for electric field E.

The wave equation for Magnetic field H is obtained in a similar manner as follows.

The Maxwell's equation from Ampere's law is given by,

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

Take curl on both sides,

$$\nabla \times \nabla \times H = \sigma \nabla \times E + \varepsilon \nabla \times \frac{\partial E}{\partial t} \qquad \text{... (4.7)}$$

But Maxwell's equation from Faraday's law

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Differentiating, $$\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial H^2}{\partial t^2}$$

Substituting the values of $\nabla \times E$ and $\nabla \times \frac{\partial E}{\partial t}$ in equation (4.7)

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \qquad \text{... (4.8)}$$

But the identity is

$$\nabla \times \nabla \times H = \nabla(\nabla \bullet H) - \nabla^2 H$$

But $$\nabla \bullet B = \mu \nabla \bullet H = 0$$

Then, $$\nabla \times \nabla \times H = \nabla^2 H \qquad \text{... (4.9)}$$

Comparing the equations (4.8) and (4.9)

$$\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0} \qquad \text{... (4.10)}$$

This is the wave equation for magnetic field H.

➢ For free space (dielectric medium) the conductivity of medium is zero ($\sigma=0$) and there is no charge containing in it ($\rho=0$). Then, the electromagnetic equations can be obtained as,

$$\boxed{\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0} \quad \text{——— (1)}$$

$$\boxed{\nabla^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0} \quad \text{——— (2)}$$

➢ For free space $\mu_r=1$ and $\varepsilon_r=1$ (air), then the wave equation becomes,

$$\boxed{\nabla^2 H - \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2} = 0} \quad \text{or} \quad \boxed{\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0}$$

Eqs. (1) & (2) represents the relation between space and time variation of magnetic field B and electric field E. These are called wave equations for B and E respectively.

The general form of differential eq. of wave motion is represented by

$$\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Where v is the velocity of wave and y is its amplitude,

Comparing Eq. (1) & (2) with Eq. (3), we get

$$\mu\varepsilon = \frac{1}{v^2} \Rightarrow v = \frac{1}{\sqrt{\mu\varepsilon}}$$

This is the expression for velocity of electromagnetic wave.

The velocity of EM wave in free space is given by

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \text{where, } \mu_0=4\pi \times 10^{-7} \text{ H/m}, \quad \varepsilon_0=8.85 \times 10^{-12} \text{ Nm}^2/\text{C}^2$$

$$v = \frac{1}{\sqrt{4\times\pi\times10^{-7}\times8.85\times10^{-12}}} = \sqrt{9\times10^{16}} = 3\times10^{8} \text{ m/s}$$

Thus the EM waves propagate with the velocity equal to the velocity of light in free space.