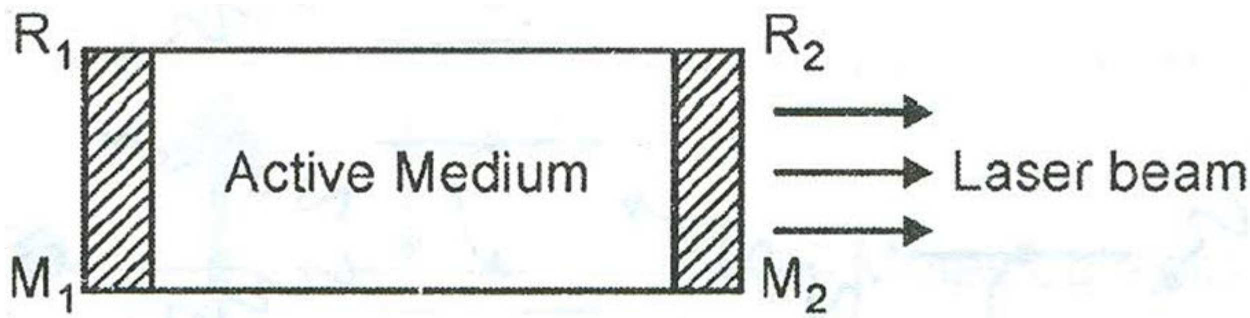
Threshold Gain Coefficient



If I_0 is the initial intensity of light entering through a light transmitting (e.g glass) solid rod (in the direction of length), then after travelling L distance through it, the final intensity will be I_1.

$$\text{then} \qquad I_1 = I_0 \exp(-\alpha L)$$

α is the loss coefficient

but instead of the glass rod if there is a solid rod which can supply photons at each and every point of travel instead of absorption, that it is called as amplifying medium. For this, population inversion should be maintained throughout this amplifying medium.

$$\text{i.e} \qquad N_2 > N_1 \text{ throughout the active medium}$$

In this condition ,

If I_0 is the initial intensity (minimum needed to start laser amplification) of light entering through the active medium (in the direction of length), then after travelling L distance through it, the final intensity will be I_1.

$$I_1 = I_0 \exp(kL)$$

k – the gain coefficient

$$\text{if } \gamma \text{ is the coefficient of practical losses}$$

$$\text{then} \quad I_1 = I_0 \exp[(k - \gamma)L]$$

$$\text{after reflecting at mirror } M_2, \quad I_1 = I_0 R_2 \exp[(k - \gamma)L]$$

$$\text{just before reaching } M_1 \text{again}, \qquad I_1 = I_0 R_2 \exp[2(k - \gamma)L]$$

$$\text{after refelcting from } M_1, I_1 = I_0 R_1 R_2 \exp[2(k - \gamma)L]$$

To sustain the laser oscillations, the minimum intensity amplified should be equal to the initial intensity I_0.

$$\text{initial intensity } = \begin{cases} \text{final amplified intensity} \\ \quad \text{in one round trip} \end{cases}$$

$$\text{so} \quad I_0 = I_0 R_1 R_2 \exp[2(k_{th} - \gamma)L]$$

$$k_{th} - \text{threshold gain coefficient}$$

$$\cancel{I_0} = \cancel{I_0} R_1 R_2 \exp[2(k_{th} - \gamma)L]$$

$$1 = R_1 R_2 \exp[2(k_{th} - \gamma)L]$$

$$\ln 1 = \ln R_1 R_2 \exp[2(k_{th} - \gamma)L]$$

$$2k_{th}L = 2\gamma L + \ln\left(\frac{1}{R_1 R_2}\right)$$

$$\text{Threshold gain coefficient}, \quad k_{th} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$\gamma - \text{practical or volume losses}$$

$$\left.\frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)\right\} - \text{useful laser output}$$