ELECTROMAGNETIC THEORY AND APPLICATIONS

ELECTROMAGNETIC FIELD:

An *electromagnetic field (EM field)* is a physical field produced by electrically charged objects. This field can be viewed as the combination of an *electric field* and a *magnetic field*. The electric field is produced by stationary charges, and the magnetic field by moving charges (currents); these two are often described as the sources of the field.

Generally, physical fields are classified into two types

- Scalar field
- Vector field

1) **Scalar field:** If the value of the physical function at each point in the field is a scalar quantity, it is called *scalar field*.

   Ex: Temperature distribution in a solid
   Temperature of atmosphere

2) **Vector field:** If the value of the physical function at each point in the field is a vector quantity, it is called *vector field*.

   Ex: Electric field intensity, Magnetic field intensity, Gravitational force etc.

   A vector field is uniquely characterized by its divergence and curl. On the basis of different associations of curl and divergence, the vector field are classified into two types:

   - Lamellar fields (Scalar Potential fields)
   - Solenoid fields

   a) **Lamellar fields:** These fields are characterized by zero curl and can be expressed as gradient of scalar potential.

   b) **Solenoidal Fields:** These fields are characterized by zero divergence, which means there are no surfaces of vector flux in the field. The flux lines are closed curves and the field in called solenoidal.

3) Operator Del (∇):

The differential operator ∇ called **del** or **nabla** is a vector space-function operator and is defined through the spatial derivatives with respect to the space coordinates. In Cartesian coordinates

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$$

where $\vec{i}, \vec{j}, \vec{k}$, are the unit vectors along x, y, z axes respectively.

4) Gradient:

The gradient of any scalar function is defined as a vector whose magnitude is equal to the maximum rate of change of that function with respect to the space variable and has the direction of that change.

If F(x,y,z) is a scalar quantity and is multiplied by a vector operator ∇ then we have

$$\nabla F = \vec{i}\frac{\partial F}{\partial x} + \vec{j}\frac{\partial F}{\partial y} + \vec{k}\frac{\partial F}{\partial z}$$

Where, ∇F is a vector quantity. This operation is called gradient and is abbreviated as grad and denoted as Grad **F** or **∇F.**

Examples

- Gradient of temperature, gradient of electric potential and so on. It gives the maximum space rate of change of the scalar. The scalar can be temperature, potential and so on.

5) Divergence: If $F = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$ is a vector function and is multiplied by another vector ∇, their dot or scalar product is given by

$$\nabla.F = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right).\left(F_x\,\vec{i} + F_y\,\vec{j} + F_z\,\vec{k}\right)$$

$$\nabla.F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Which is a scalar quantity termed as divergence and is abbreviated by ***div.***

Physically, *it is defined as the outflow of vector over the surface per unit volume when the volume approaches to zero.*

Divergence means the spreading or diverging of a quantity from a point. It is applicable to vectors only. The divergence of a vector indicates the net flow of quantities like gas, fluid, vapour, electric and magnetic flux lines. In other words, it is a measure of the difference between outflow and inflow.

- The divergence of a vector is positive if the net flow is outward.
- It is the negative if the net flow is inward.
- The fluid is said to be incompressible if the divergence is zero, that is, $\nabla\cdot A=0$ is the condition of incompressibility.

Examples and features of divergence

1. Leaking air from a balloon yields positive divergence.
2. Rushing of air into the drum under the carriage of a train yields negative divergence.
3. Divergence of water or oil is almost zero and hence they are incompressible.
4. Divergence of electric flux density is equal to volume charge density, or $\nabla \cdot D = \rho_v$
5. Divergence of magnetic flux density is equal to zero, or $\nabla \cdot B = 0$

6. Divergence of gradient of scalar electric potential is equal to the laplacian of the scalar, or, $\nabla \cdot \nabla V = \nabla^2 V$

6) Curl: If $F = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$ is a vector function and is multiplied by another vector ∇, their cross or vector product is given by

$$\nabla \times F = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \right) \times \left(F_x \vec{i} + F_y \vec{j} + F_z \vec{k} \right)$$

$$\nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

This is a vector quantity and is called curl of a vector function.

Physically, curl is defined as the line integral of the tangential component of a vector around the area per unit area.

or

It is a measure of the tendency of a vector quantity to rotate or twist or curl. In other words, the rate of rotation or angular velocity at a point is the measure of curl.

Examples

- When a leaf floats in sea water and its rotation is about the z-axis, curl of velocity V is in the z-direction. When $(\nabla \times F)_z$ is positive, it represents rotation from x to y.
- For a rotating rigid body, the curl of velocity is in the direction of the axis of rotation. Its magnitude is equal to twice the angular speed of rotation.