Engineering Physics

(PHY1701)

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- Planck's concept (hypothesis) (AB 66-67)
- Compton Effect (AB 80-86),
- Particle properties of wave: Matter Waves (AB 104-114),
- Davisson Germer Experiment (AB 115-117),
- Heisenberg Uncertainty Principle (AB 119-128),
- Wave function (AB 182-184 & 190-195), &
- Schrödinger equation (time dependent & independent) (AB 187 -190 & 195-197).

❖ Concepts of Modern Physics, Arthur Beiser et al., Sixth Edition, Tata McGraw Hill (2013) (AB)

Dual nature of matter

- Wave theory of electromagnetic radiations explained the phenomenon of interference, diffraction and polarization.
- On the other hand, quantum theory of electromagnetic radiations successfully explained the photoelectric effect, Compton effect, black body radiations, X- ray spectra, etc.
- Thus, radiations have dual nature. i.e. wave and particle nature.
- Louis de Broglie suggested that the particles like electrons, protons, neutrons, etc. have also dual nature. i.e. they also can have particle as well as wave nature.
- His suggestion was based on:
    - The nature loves symmetry.
    - The universe is made of particles and radiations and both entities must be symmetrical.

de Broglie wave hypothesis of matter waves

- According to de Broglie, a moving material particle can be associated with a wave. i.e. a wave can guide the motion of the particle.
- The waves associated with the moving material particles are known as de Broglie waves or matter waves.
- de Broglie proposed that the wavelength associated with any moving particle of mass m and velocity is given by

$$\boldsymbol{\lambda = h/mc} \text{ (or) } \boldsymbol{\lambda = h/p}$$

where p = mc is momentum of a photon.

- Considering Planck's theory of radiation, the energy of a photon is given by

$$E = \frac{hc}{\lambda}$$

- According to Einstein energy-mass relation, $E = mc^2$.

$$E = mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{mv}$$ **where mc=p (momentum associated with photon)**

Wavelength of matter waves associated with different particles

- If instead of a photon, we have a material particle of mass m moving with velocity v, then the equation becomes

$$\lambda = \frac{h}{mv}$$

Which is the expression for the de Broglie wavelength.

- If E_k is the kinetic energy of the material particle, then

$$p = \sqrt{(2mEk)}$$

The de Broglie wavelength of particle of K.E = E_k is given by

$$\lambda = \frac{h}{\sqrt{2mEk}}$$

- If a charged particle carrying charge q is accelerated through a potential difference V volts, then K.E. is

$$E_K = qV.$$

The de Broglie wavelength for charged particle of charge q and accelerated through the potential difference V volts is

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Wavelength of matter waves associated with different particles

- When a material particle is in thermal equilibrium at a temperature T, then

$$E = \frac{3}{2} kT$$

The de Broglie wavelength of a material particle at a temperature T is given by

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

- When an electron of rest mass m_0, and charge e which is accelerated by a potential V volt from rest to v velocity, then

$$v = \sqrt{\left(\frac{2eV}{m_0}\right)}$$

The de Broglie wavelength of an electron is given by

$$\lambda = \frac{h}{\sqrt{2eVm_0}}$$

$$\lambda = \frac{12.26}{\sqrt{V}}\ \text{Å}$$

Wavelength of matter waves associated with different particles

- The de Broglie wavelength of an electron is given by

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

- The de Broglie wavelength of an alpha particle is given by

$$\lambda = \frac{0.1012}{\sqrt{V}} \text{ Å}$$

- The de Broglie wavelength of an proton is given by

$$\lambda = \frac{0.286}{\sqrt{V}} \text{ Å}$$

- The de Broglie wavelength of an deuteron is given by

$$\lambda = \frac{0.202}{\sqrt{V}} \text{ Å}$$

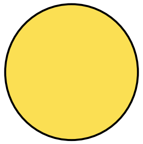
- 1fm=10^{-15} m, 1pm=10^{-12} m, 1 Å=10^{-10}m

Properties of matter waves

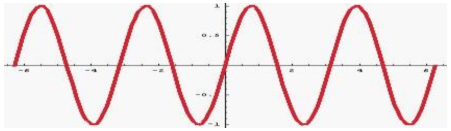
- de Broglie wavelength is inversely proportional to the velocity of the particle. If the particle moves faster, then the wavelength will be smaller and vice versa.
- If the particle is at rest, then the de Broglie wavelength is infinite. Such a wave can not be visualized.
- de Broglie wavelength is inversely proportional to the mass of the particle. The wavelength associated with a heavier particle is smaller than that with a lighter particle.
- de Broglie wavelength is independent of the charge of the particle.
- Matter waves, like electromagnetic waves, can travel in vacuum and hence they are not mechanical waves.
- Matter waves are not electromagnetic waves because they are not produced by accelerated charges.
- Matter waves are probability waves, amplitude of which gives the probability of existence of the particle at the point.
- The waves have particles like properties and particles have wave like properties and the concepts are separately linked. Matter wave representation is only a symbolic representation

Traditional wave and particle

Our traditional understanding of a particles and wave….



"Localized" - definite position, momentum, confined in space



"de-localized" – spread out in space and time

Normal Waves

- are a disturbance in space
- carry energy from one place to another
- often (but not always) will (approximately) obey the classical wave equation

Matter Waves

- disturbance is the wave function Y(x, y, z, t)
    - probability amplitude Y
    - probability density $p(x, y, z, t) = |Y|^2$

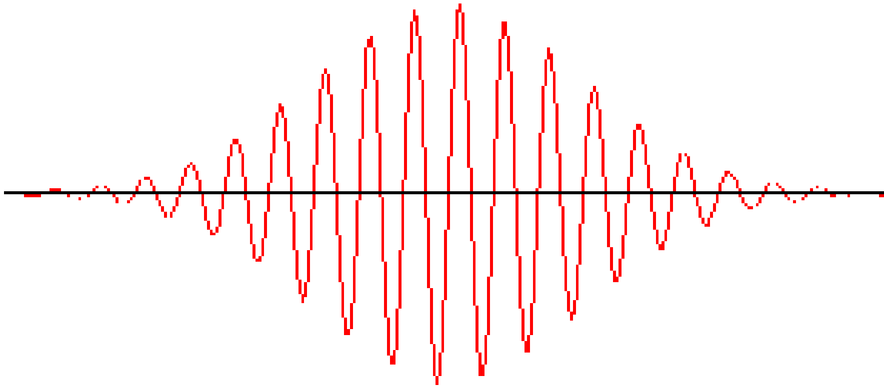
Wave packet

What could represent both wave and particle?

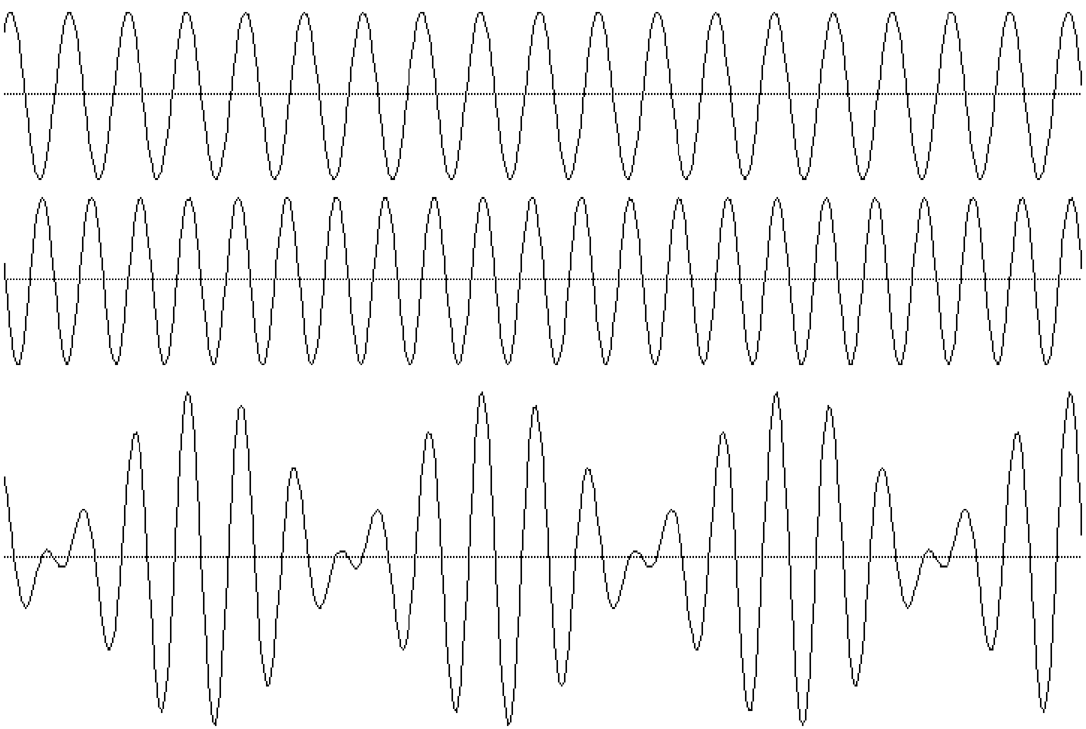
Find a description of a particle which is consistent with our notion of both particles and waves……

- Fits the "wave" description
- "Localized" in space

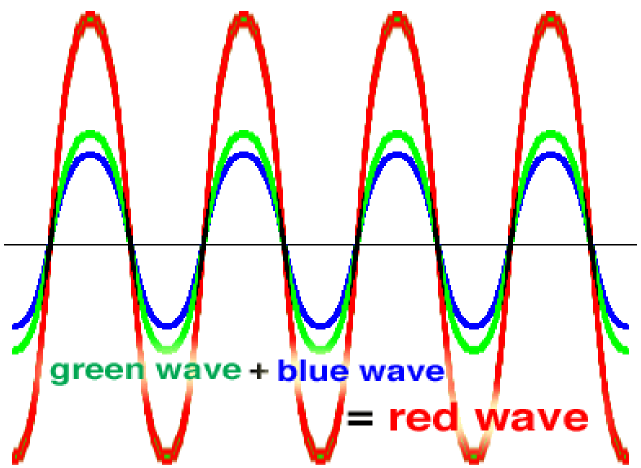
A "Wave Packet"



Adding up waves of different frequencies



The Superposition principle



What happens when you add up waves?

Phase velocity

◊ The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it

◊ Phase velocity: The velocity of component waves of a wave packet

◊ Group velocity: The velocity with which the wave packet propagates

◊ Consider a wave whose displacement y is expressed as

$$\boldsymbol{y} = \boldsymbol{A}\cos(\boldsymbol{\omega t} - \boldsymbol{kx})$$

where *A*-Amplitude, $\boldsymbol{\omega}$-angular frequency, and $\boldsymbol{k}$-propagation constant

The speed of propagation of this wave will be the speed associated with a point for which the phase $(\boldsymbol{\omega t} - \boldsymbol{kx})$ is constant..

$$(\boldsymbol{\omega t} - \boldsymbol{kx}) = \text{constant}$$

or $$\boldsymbol{x} = \text{constant} + \frac{\boldsymbol{\omega}}{\boldsymbol{k}}\,t$$

Therefore, **Phase velocity** $\boldsymbol{v}_P = \frac{dx}{dt} = \frac{\boldsymbol{\omega}}{\boldsymbol{k}} = \frac{2\pi\boldsymbol{v}}{2\pi/\lambda}$

$$\boldsymbol{v}_P = \boldsymbol{v}.\boldsymbol{\lambda}$$

Group velocity

The **Group velocity** is given by $\boldsymbol{v}_g = \frac{d\boldsymbol{\omega}}{dk}$

The **Phase velocity** of de Broglie waves is,

$$\boldsymbol{v}_P = \boldsymbol{v}.\boldsymbol{\lambda} = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right) = \frac{c^2}{v}$$

Here velocity v is always less than c. Therefore, the de Broglie wave velocity must be greater than c.

The **group velocity** of de Broglie waves is,

$$\boldsymbol{v}_g = \frac{d\boldsymbol{\omega}}{dk}$$

The angular frequency and wavenumber of the de Broglie waves associated with a particle of rest mass m_0 moving with the velocity v are given by

$$\omega = 2\pi\nu \quad \omega = 2\pi\frac{E}{h}$$

$$\omega = 2\pi\frac{mc^2}{h}$$

$$\boxed{\omega = 2\pi\frac{m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}}$$

Experiment

$$k = \frac{2\pi}{\lambda} \qquad k = \frac{2\pi}{h} mv \qquad \boxed{k = \frac{2\pi}{h}\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\boxed{d\omega/dv = \frac{2\pi}{h}\frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}} \qquad \boxed{dk/dv = \frac{2\pi m_0}{h}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}}$$

$$V_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = v$$

Hence, the de Broglie wave group associated with a moving particle travels with the same velocity as the particle.

Relation between the group and phase velocity

The relation between the group and phase velocity is given by

$$v_g = v_p - \lambda \left(\frac{dv_p}{d\lambda}\right)$$

From above equation the following two cases arise.

1. For dispersive medium, $v_p = f(\lambda)$. Usually $dv_p/d\lambda$ is positive (normal dispersion)

$$\mathbf{v_g < v_p}$$

This is the case with de Broglie waves.

2. For non-dispersive medium, $v_p \neq f(\lambda)$, $dv_p/d\lambda$ is zero. Then

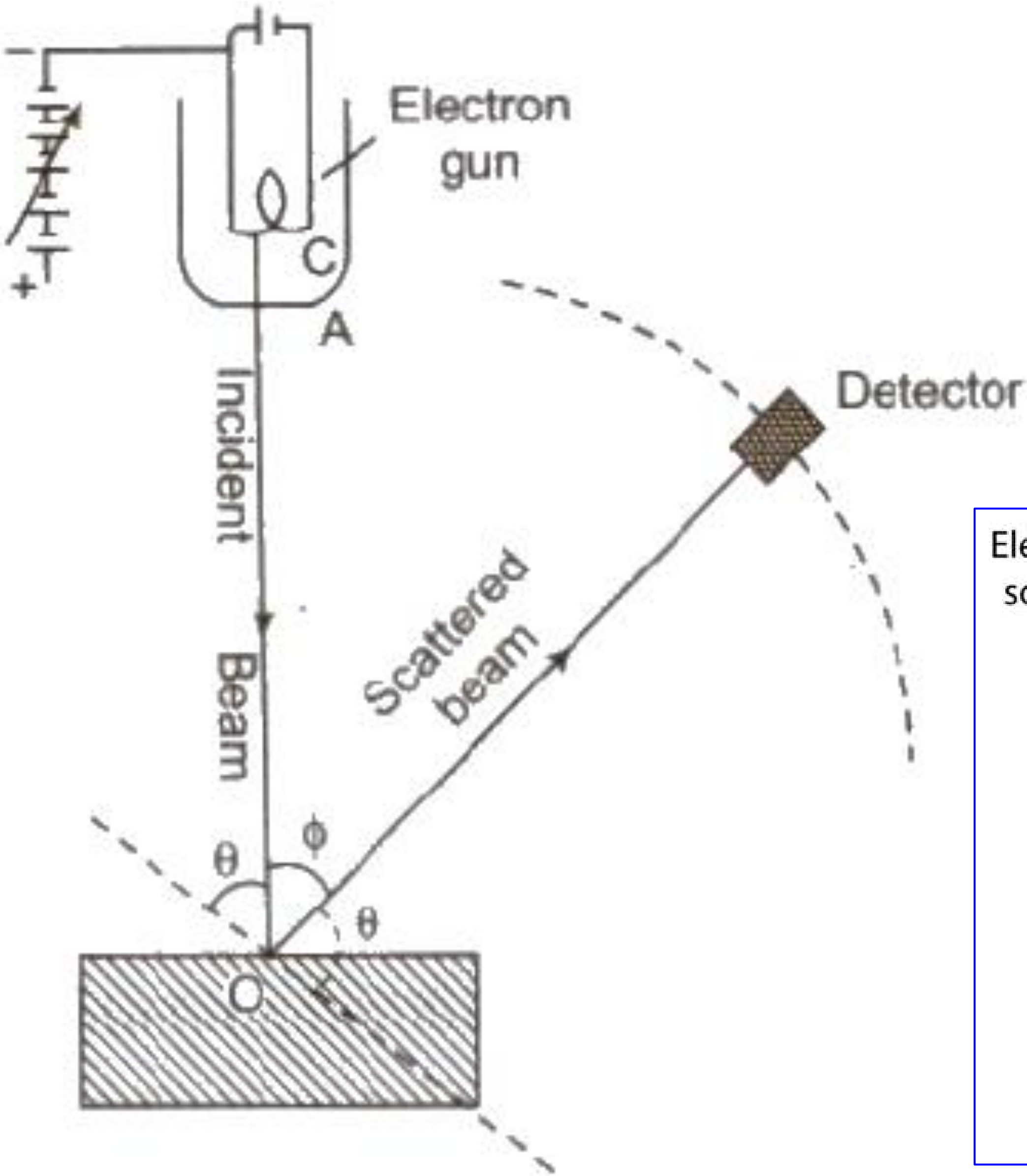
$$\mathbf{v_g = v_p}$$

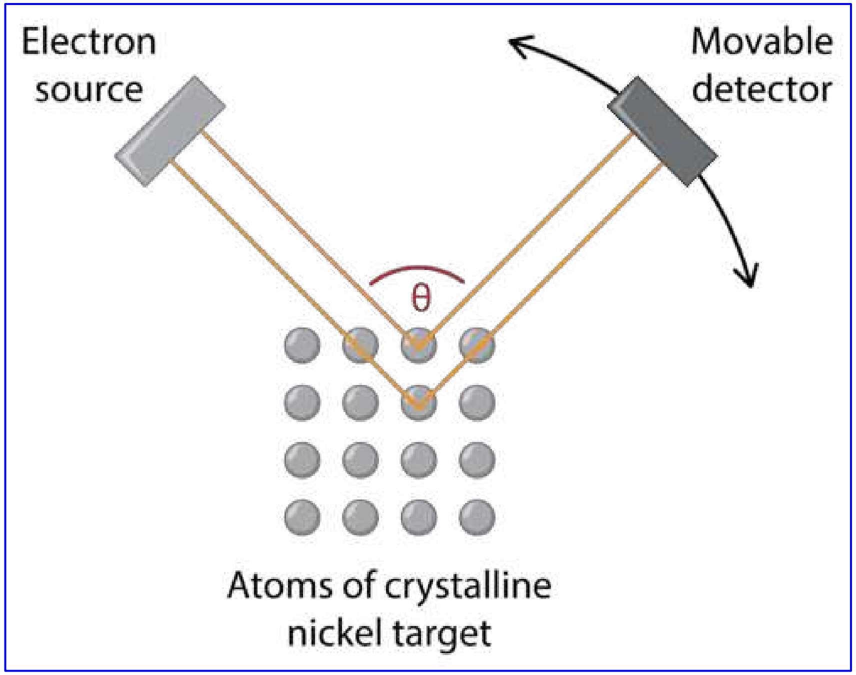
This is true for electromagnetic waves in vaccum.

Davisson and Germer’s experiment

- In 1927, two physicists in the US, C.J. Davisson and L.H. Germer, used a crystal to diffract a beam of electrons, thereby demonstrating a wavelike property of particles.
- In order to test de Broglie’s hypothesis that matter behaved like waves, Davisson and Germer set up an experiment very similar to what might be used to look at the interference pattern from x-rays scattering from a crystal surface.
- The basic idea is that the planar nature of crystal structure provides scattering surfaces at regular intervals, thus waves that scatter from one surface can constructively or destructively interfere from waves that scatter from the next crystal plane deeper into the crystal.

Davisson and Germer’s experiment





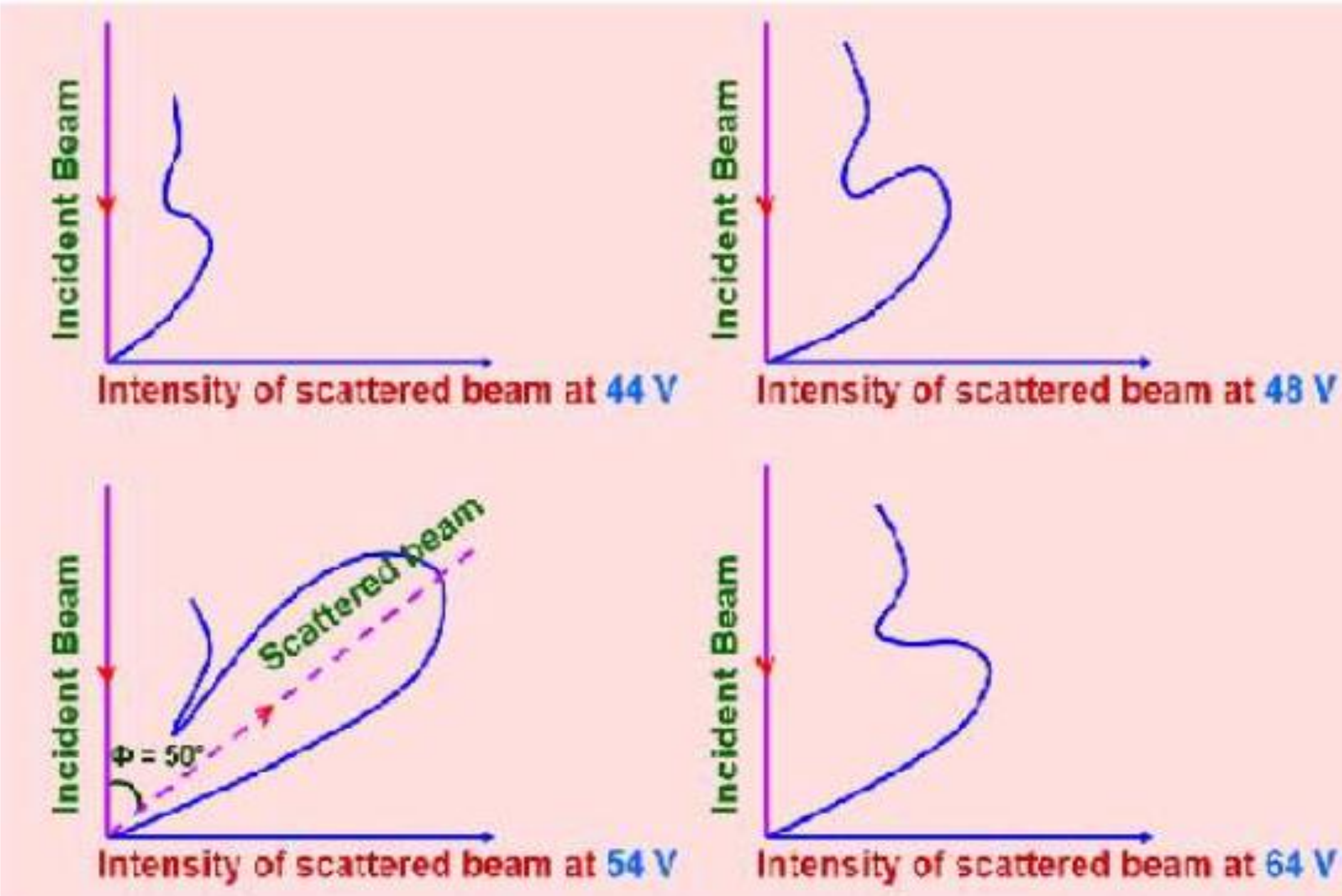
Davisson and Germer’s experimental arrangement

Davisson and Germer's experiment

- A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle.
- The scattered beam of electrons is received by the detector which can be rotated at any angle.
- The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.
- Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54 V.
- Electron diffraction is similar to X-ray diffraction.
- Bragg's equation $2d\sin\theta = n\lambda$ gives

$$\lambda = 1.65\ \text{Å}$$

Davisson and Germer's experiment



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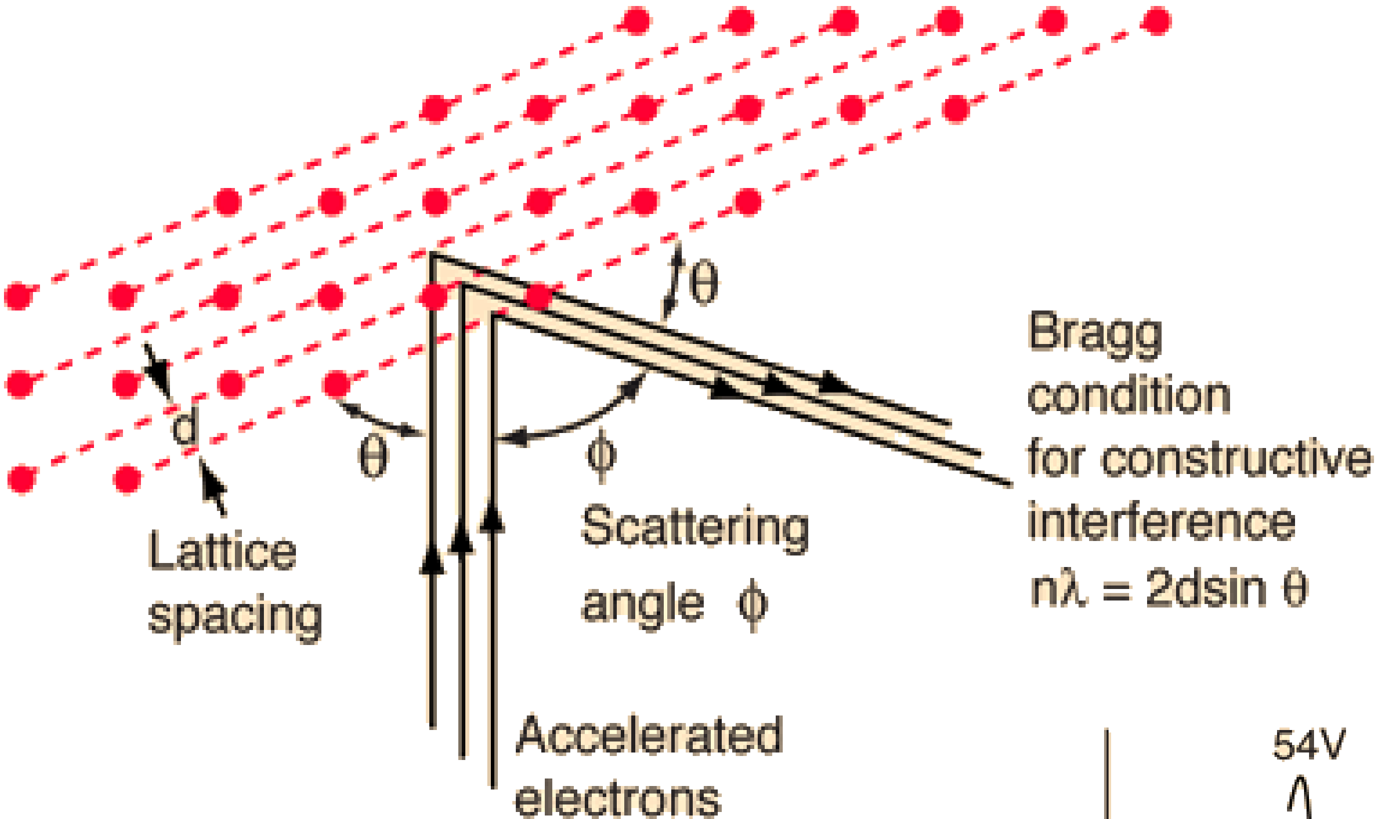

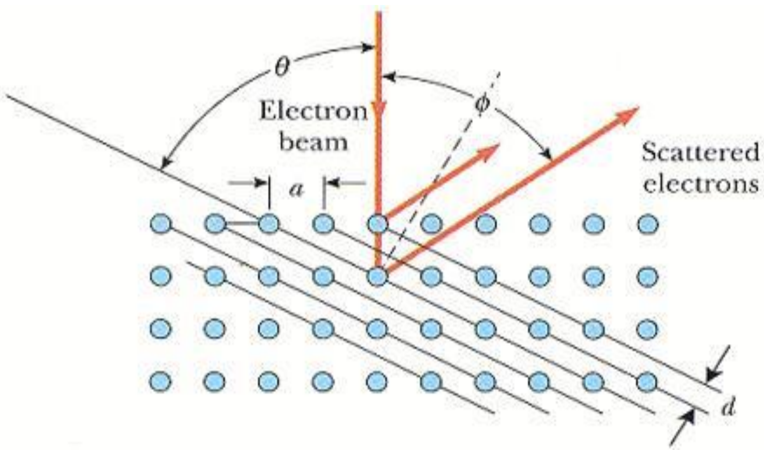

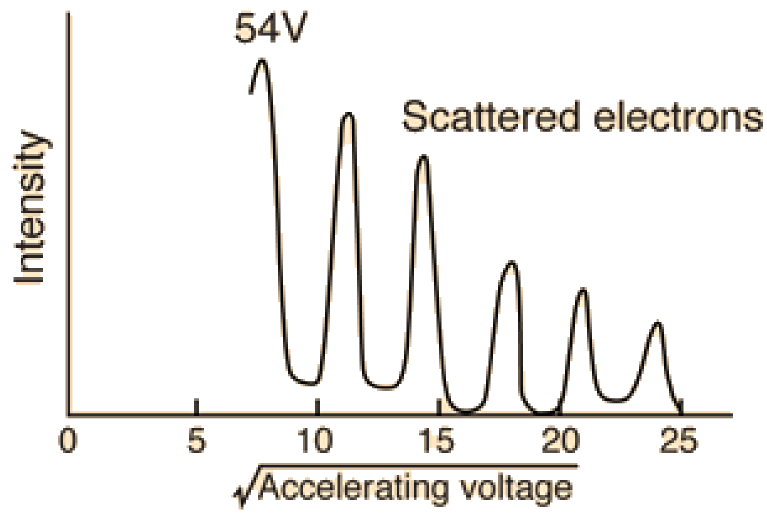

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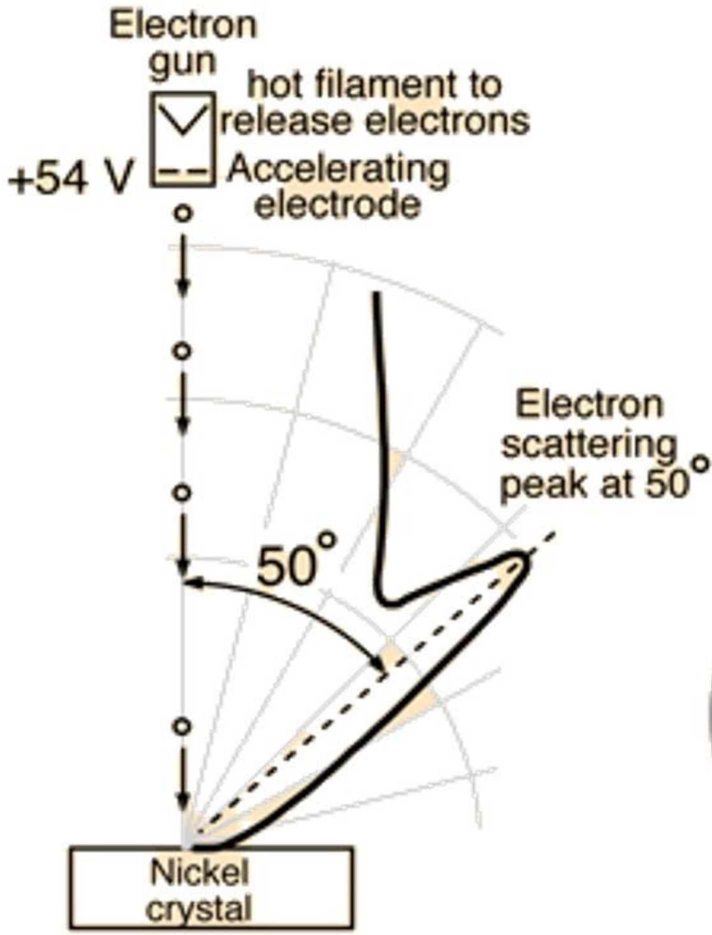


Theory

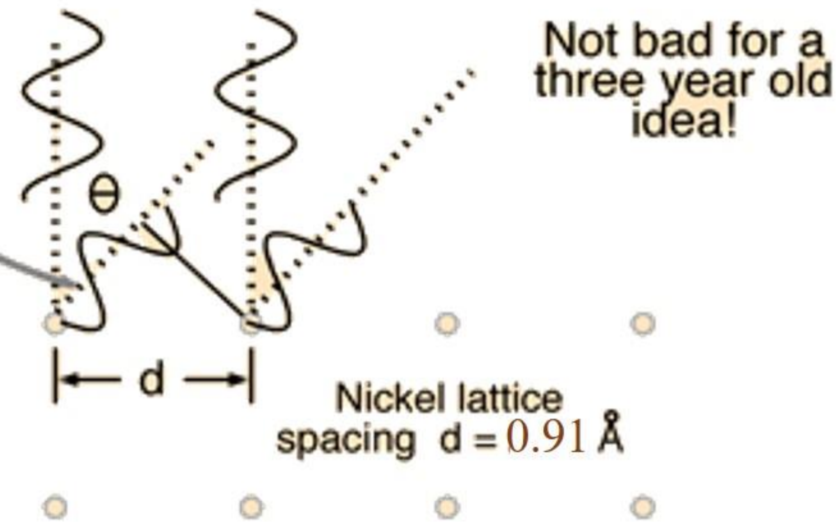
$$\lambda = \frac{h}{mv} = 1.67\ \text{Å for 54 V}$$

Experiment

Pathlength difference

$$2d \sin\theta = 2\text{x}0.091\text{x}(\sin 65^{o}) = \lambda = 1.65\ \text{Å}$$

for constructive interference



1924	1927	1929
de Broglie's hypothesis	Davisson-Germer experiment	Nobel Prize for de Broglie

Calculation of observed wavelength

- A single crystal of nickel was cut to expose a spacing of d = 0.091 nm between the lattice planes.
- When a beam of electrons of kinetic energy 54.0 eV was directed onto the crystal face, the maximum in the intensity of the scattered electrons was observed at an angle of 50°.
- According to wave theory, constructive interference due to waves reflected from two lattice planes a distance of *d* apart should occur at certain angles of scattering θ.
- The theory predicts the first order maximum should be observed at an angle given by

$$d \sin \theta = \lambda$$

$$2 \times 0.091 \times \sin 65^{o} = 0.165 \text{ nm}$$

- The de Broglie wavelength of the electrons, the K.E of the electrons is 54 eV.

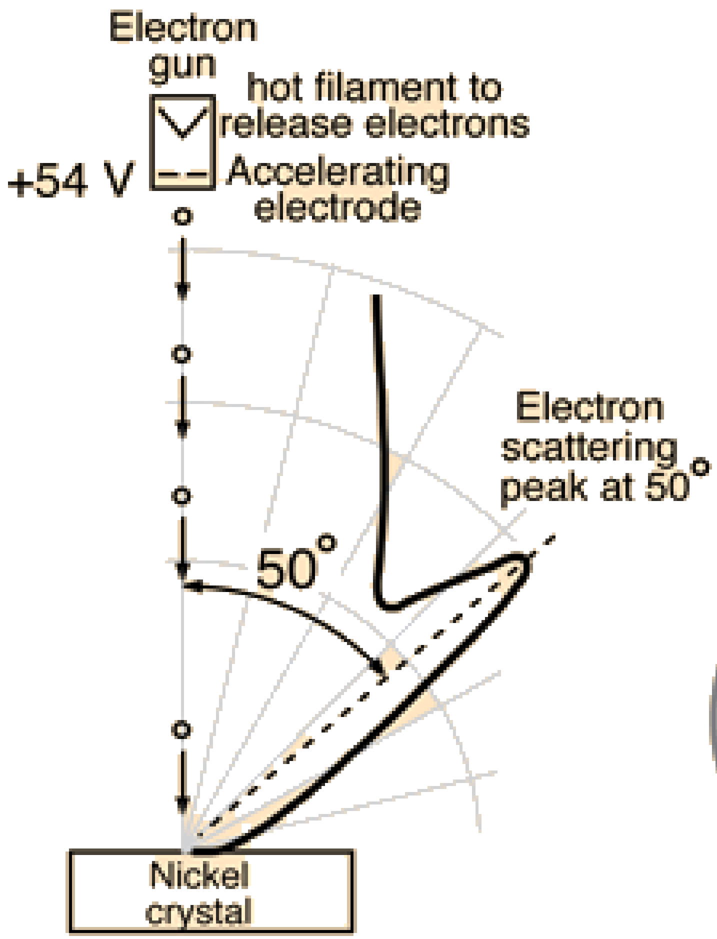
$$\lambda = h/ (2mKE)^{1/2}$$

$$\lambda = 0.166 \text{ nm}$$

Both the wavelengths matched…

- The Davisson- Germen's experiment thus directly verifies de Broglie's hypothesis of the wave nature of moving bodies.

Davisson and Germer's experiment



Theory

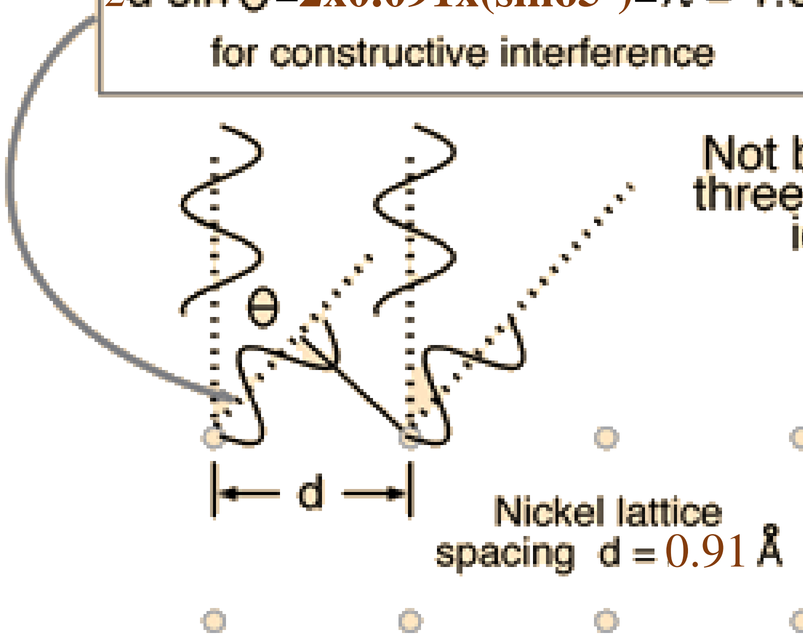
$$\lambda = \frac{h}{mv} = 1.67\ \text{Å for 54 V}$$

Experiment

Pathlength difference

$2d \sin\theta = 2 \times 0.091 \times (\sin 65^{o}) = \lambda = 1.65$ Å

for constructive interference



Not bad for a three year old idea!

1924 de Broglie's hypothesis

1927 Davisson-Germer experiment

1929 Nobel Prize for de Broglie