Problems

1. **In an optical fiber, the core refractive index is 1.4513 and the cladding refractive index is 1.4468. What is**
   i) **Critical incident angle?**
   ii) **Critical propagation angle?**
   iii) **Acceptance angle?**
   iv) **The numerical aperture?**

Solution

Given data: $n_1 = 1.4513$; $n_2 = 1.4468$

i) $\sin\theta_c = \left(\frac{n_2}{n_1}\right) \Rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$$\theta_c = \sin^{-1}\left(\frac{1.4513}{1.4468}\right) = 85.49°$$

ii) We know, $\cos\alpha_c = \frac{n_1}{n_2} = \frac{1.4468}{1.4513}$

$$\therefore \qquad \alpha_c = \cos^{-1}\left(\frac{1.4468}{1.4513}\right) = 4.51°$$

iii) $$\mathrm{NA} = \sqrt{{n_1}^2 - {n_2}^2}$$

$$\mathrm{NA} = \sqrt{(1.4513)^2 - (1.4468)^2} = 0.114$$

$$\mathrm{NA} = \sin\theta_a$$

$$\therefore\ \theta_a = \sin^{-1}(\mathrm{NA}) = \sin^{-1}(0.114) = 6.55°$$

2. Calculate the numerical aperture of an optical fiber whose core index is 1.48 and relative index is 0.02.

Solution

We know, $\mathrm{NA} = n_1\sqrt{2\Delta}$

Given data: n_1 = 1.48 and Δ = 0.02

$$\therefore \quad \mathrm{NA} = 1.48\sqrt{2 \times 0.02}$$

$$\mathrm{NA} = 0.296$$

3. Calculate the numerical aperture of a step-index fiber having n_1 = 1.48 and n_2 = 1.46. What is the maximum entrance angle for this fiber if the outer medium is air with n_a = 1.00.

Solution

$$\mathrm{NA} = \sqrt{{n_1}^2 - {n_2}^2}$$

Given data: n_1 = 1.48; n_2 = 1.46; n, refractive index of air ≈ 1.00

$$\mathrm{NA} = \sqrt{(1.48)^2 - (1.46)^2} = 0.2425 \Rightarrow \mathrm{NA} = 0.2425$$

But $\quad \mathrm{NA} = n_a \sin\theta_a \Rightarrow \theta_a = \sin^{-1}(\mathrm{NA}) \qquad [\because n_a \approx 1]$

$$\theta_a = \sin^{-1}(0.2425) = 14°$$

$$\theta_a = 14°$$

4. A Si p-i-n photodiode has a quantum efficiency of 0.7 at a wavelength of 0.85μm. Calculate its responsivity.

Solution

Given data: $\eta = 0.7$; $\lambda = 0.85\mu m$

$R = ?$

We know, $R = \eta\left(\frac{e\lambda}{hc}\right)$

$$\therefore \quad R = 0.7 \times \frac{1.602\times10^{-19}C\times0.85\times10^{-6}m}{6.626\times10^{-34}Js\times3\times10^{8}ms^{-1}}$$

$$R = 0.48\ A/W$$

5. A particular photodectector has a responsivity of 0.6 A/W for a light of wavelength 1.3μm. Calculate its quantum efficiency.

Solution

Given data: $R = 0.6$ A/W; $\lambda = 1.3\mu m$

$\eta = ?$

We know, $R = \eta\left(\frac{e\lambda}{hc}\right)$

$$\therefore \quad \eta = R\left(\frac{hc}{e\lambda}\right)$$

$$\eta = 0.6 A/W \times \frac{6.626\times10^{-34} Js\times3\times10^{8} ms^{-1}}{1.602\times10^{-19} C\times1.3\times10^{-6} m}$$

$$\eta = 0.573$$

6. If a step-index fiber has a core of refractive index 1.5 and a cladding of refractive index 1.48, calculate, assuming that the fiber is kept in air, the (a) NA of the fiber, (b) angles θ_a, α_c and θ_c.

Solution

Given data: $n_1 = 1.5$; $n_2 = 1.48$; $n_a \approx 1$

a) $NA = \sqrt{n_1{}^2 - n_2{}^2} = \sqrt{(1.5)^2 - (1.48)^2} = 0.244$

b) $NA = n_a \sin\theta_a = 1 \times \sin\theta_a$

$$\theta_a = \sin^{-1}(NA) = \sin^{-1}(0.244)$$

$$\theta_a = 14.13°$$

Also $n_a \sin\theta_a = n_1 \sin\alpha_c$

or $\quad 0.244 = 1.5 \sin\alpha_c$

$$\alpha_c = \sin^{-1}\left(\frac{0.244}{1.5}\right) = 9.36°$$

Further

$$\sin\theta_c = \left(\frac{n_2}{n_1}\right) = \left(\frac{1.48}{1.5}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 80.63°$$

7. **A p-n photodiode has a quantum efficiency of 70% for photons of energy 1.52×10^{-19}J. Calculate, (a) the wavelength at which the diode is operating and (b) the optical power required to achieve a photocurrent of 3μA when the wavelength of the incident photons is that calculated in part (a).**

Solution

(a)The energy of the photon,

$$E = h\upsilon = \frac{hc}{\lambda}$$

$$\therefore \quad \lambda = \frac{hc}{E} = \frac{6.626\times10^{-34}\text{Js}\times3\times10^{8}\text{ms}^{-1}}{1.52\times10^{-19}\text{J}}$$

$$= 1.3\ \mu\text{m}$$

(b) $$R = \frac{\eta e}{hc} = \frac{0.70\times1.602\times10^{-19}\text{C}}{1.52\times10^{-19}\text{J}}$$

$$= 0.738\text{AW}^{-1}$$

Since $R = \frac{I_p}{P}$

$$P = \frac{I_p}{R} = \frac{3\times10^{-6}\text{A}}{0.738\text{AW}^{-1}} = 4.07\ \mu\text{W}$$

8. A pin photodiode, on an average, generates one electron-hole pair per two incident photons at a wavelength of 0.85µm. Assuming all the photo-generated electrons are collected, calculate (a) the quantum efficiency of the diode, (b) the maximum possible bandgap energy (in eV) of the semiconductor, assuming the incident wavelength to be a long-wave length cut-off; and (c) the mean output photocurrent when the incident optical power is 10 µW.

Solution

a) $\eta = \frac{1}{2} = 0.5\ (\text{i.e.,})\ 50\%$

b) $E_g = \frac{hc}{\lambda_c} = \frac{6.626\times10^{-34}\text{Js}\times3\times10^{8}\text{ms}^{-1}}{0.85\times10^{-6}\text{m}}$

$$= 2.34\times10^{-19}\text{J} = 1.46\text{eV}$$

c) $I_p = RP = \left(\frac{\eta e}{h\upsilon}\right)P$

$$= \frac{0.5\times1.602\times10^{-19}\text{C}}{2.34\times10^{-19}\text{J}}\times10\times10^{-6}\text{W} = 3.42\ \mu\text{A}$$

9. A step-index fiber has higher core index and lower cladding index of 1.5 and 1.45, respectively. Calculate (a) critical incident angle of the fiber, (b) the corresponding acceptance angle of the fiber in air and (c) NA of the fiber.

Solution

Given data: $n_1 = 1.5$; $n_2 = 1.45$

a) $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.45}{1.50}\right) = 75.17°$

b) $n_a \sin\theta_a = n_1 \sin\alpha_c$

$\because\ \theta_c = 75°, \alpha_c = 90° - 75.17° = 14.83°$

$\therefore\ \sin\theta_a = (1.5)\sin 14.83 = 0.384$ $[\because n_a \approx 1]$

$\therefore\ \theta_a = \sin^{-1}(0.384) = 22.58°$

c) $\mathrm{NA} = \sqrt{{n_1}^2 - {n_2}^2} = \sqrt{(1.5)^2 - (1.45)^2} = 0.3841$

10. When $\mathbf{2.5 \times 10^{12}}$ photons generated by a laser source of wavelength 0.85 μm are incident on a photodiode, $\mathbf{1.5 \times 10^{12}}$ electrons on an average are collected at the output terminal. Calculate the quantum efficiency and the responsivity of the photodiode at the above wavelength.

Solution

Quantum efficiency,

$$\eta = \frac{\text{No.of electrons generated}}{\text{No.of incident photons}} = \frac{1.5\times10^{12}}{2.5\times10^{12}} = 0.6$$

Responsivity, $R = \eta\left(\frac{e\lambda}{hc}\right)$

$$R = \frac{0.6\times1.602\times10^{-19}\text{C}\times0.85\times10^{-6}\text{m}}{6.626\times10^{-34}\text{Js}\times3\times10^{8}\text{ms}^{-1}} = 0.4110\text{AW}^{-1}$$