Concept: 1D box Problem:

Motion of the particle in 1D box has a great importance in quantum mechanics as it is that basic problem which has already been used to explain the several facts in the subatomic scale.

In 1D box, the potential inside the box is taken as zero everywhere inside the wall of the box and infinite outside the walls. So, the particle inside the box experiences zero potential inside and it cannot come out of the box as the potential outside of it is infinite. Again, as it is one dimensional, so the particle is only capable to move in one dimension, i.e. it has only one degree of freedom. It is also known as infinite potential well. Thus, this is basically a trapped problem.

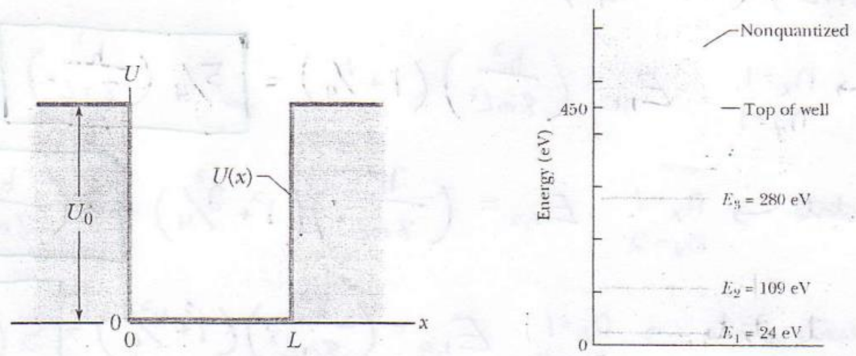
In Newtonian mechanics, the motion of the particle can be predicted if the nature of force with the boundary conditions known properly. Again, in advanced mechanics (Lagrangian and Hamiltonian mechanics), the potential energy is necessary instead of force to get the information of the nature of the trajectory of the particle. In the same orientation, quantum mechanics also requires the nature of the potential. Motion of the particle under 1D box, is such a kind of simple problem which imitates approximately a lot of natural potential. For example, one dimensional simple harmonic oscillator, the nature of the potential is parabolic and it is zero only at the equilibrium position and the particle is trapped inside the two amplitude positions, it can't go beyond that, so the nature of the potential does not follow the 1D box completely but it can be approximated to it. Again, in case of a charge particle under coulomb's potential, the electron confined within in atom, it is also approximated as 1D box. So, there are a lot of problems in nature, which can be approximated as 1D box. In all real cases, the potential is not like the 1D box, where inside the box, potential is flat zero and outside infinite.

We know, force $F = - dv/dx$, the potential gradient. If force F is zero, then the potential v is constant. Then the simple choice is to consider the potential $v = 0$ to make the solution procedure less complicated.

Thus, 1D box problem is an ideal case, which is helpful in explaining a lot of phenomena.

Problems:

(a) The figure below gives the energy levels for an electron trapped in a finite potential energy well 450 eV deep. If the electron is in the n = 3 state, what is its kinetic energy? (b) The electron then absorbs 500 eV of energy from an external source. What is its kinetic energy after this absorption, assuming that the electron moves to a position for which x > L.



(a) $E_3 = 280\,eV = K + U \quad U = 0 \longrightarrow \boxed{K = 280\,eV}$

(b) $E_{total} = 280\,eV + 500\,eV = 780\,eV = K + U$

$U = 450\,eV$ for $x > L$

$K = 780\,eV - 450\,eV \longrightarrow \boxed{K = 230\,eV}$

A rectangular corral of widths $L_x = L$ and $L_y = 2L$ contains an electron. What multiple of $h^2/8mL^2$, where m is the electron's mass, are (a) the energy of the electron's ground state, (b) the energy of its first excited state, (c) the energy of its lowest degenerate states, and (d) the difference between the energies of its second and third excited states?

$$E_{nx,ny} = \left(\frac{h^2}{8m}\right)\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \left(\frac{h^2}{8m}\right)\left[\frac{n_x^2}{L^2} + \frac{n_y^2}{(2L)^2}\right]$$

$$= \left(\frac{h^2}{8mL^2}\right)\left(\frac{n_x^2}{1} + \frac{n_y^2}{4}\right)$$

(a) ground state $\rightarrow$ $n_x = 1$, $n_y = 1$ $\quad E_{1,1} = \left(\frac{h^2}{8mL^2}\right)(1 + 1/4) = \boxed{5/4\left(\frac{h^2}{8mL^2}\right)}$

(b) first excited state $\rightarrow$ $n_x = 1$, $n_y = 2$ $\quad E_{1,2} = \left(\frac{h^2}{8mL^2}\right)(1^2 + 2^2/4) = \boxed{2\left(\frac{h^2}{8mL^2}\right)}$

(c) lowest degenerate state $\rightarrow$ $n_x = 1$, $n_y = 4$ $\quad E_{1,4} = \left(\frac{h^2}{8mL^2}\right)(1^2 + 4^2/4) = \boxed{5\left(\frac{h^2}{8mL^2}\right)}$

$n_x = 2$, $n_y = 2$ $\quad E_{2,2} = \left(\frac{h^2}{8mL^2}\right)(2^2 + 2^2/4) = \boxed{5\left(\frac{h^2}{8mL^2}\right)}$

(d) second excited state $\rightarrow$ $n_x = 1$, $n_y = 3$ $\quad E_{1,3} = \left(\frac{h^2}{8mL^2}\right)(1^2 + 3^2/4) = \frac{13}{4}\left(\frac{h^2}{8mL^2}\right)$

third excited state $\rightarrow$ $n_x = 2$, $n_y = 1$ $\quad E_{2,1} = \left(\frac{h^2}{8mL^2}\right)(2^2 + 1^2/4) = \frac{17}{4}\left(\frac{h^2}{8mL^2}\right)$

$$\boxed{\Delta E = (1)\left(h^2/8mL^2\right)}$$

Problem 3

Suppose that an electron trapped in a one-dimensional infinite well of width 250 pm is excited from its first excited state to its third excited state. (a) In electron-volts, what energy must be transferred to the electron for this quantum jump? If the electron then de-excites by emitting light, (b) what wavelengths can it emit and (c) in which groupings (and orders) can they be emitted? (d) Show the several possible ways the electron can de-excite on an energy-level diagram.

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2 = \left[\frac{(6.63\times10^{-34}\,J\cdot s)^2}{8(9.11\times10^{-31}\,kg)(250\times10^{-12}\,m)^2}\right]n^2$$

$$E_n = (9.65\times10^{-19}\,J)\,n^2$$

$$= (6.03\,eV)\,n^2$$

(a) first excited state → $n=2$ $\Delta E = E_4 - E_2$

third excited state → $n=4$

$$= (6.03\,eV)(4^2-2^2)$$

$$\boxed{\Delta E = 72.4\,eV}$$

(b) $hf = \Delta E$

$$hc/\lambda = \Delta E \longrightarrow \lambda = hc/\Delta E$$

possible transitions $n=4 \to n=1$ $\lambda = \frac{hc}{E_4-E_1} = \frac{1240\,eV\cdot nm}{(6.03\,eV)(4^2-1^2)} = 13.7\,nm$

$n=4 \to n=3$ $\lambda = 29.4\,nm$

$n=4 \to n=2$ $\lambda = 17.1\,nm$

$n=3 \to n=2$ $\lambda = 41.2\,nm$

$n=3 \to n=1$ $\lambda = 25.7\,nm$

$n=2 \to n=1$ $\lambda = 68.5\,nm$

(c)

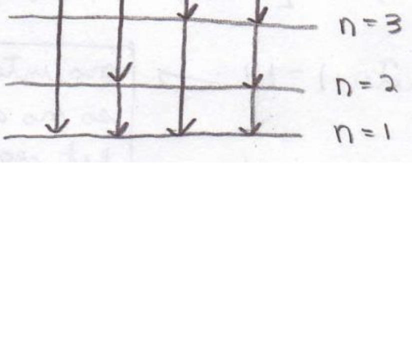
$4\to1$

$4\to2$ $2\to1$

$4\to3$ $3\to1$

$4\to3$ $3\to2$ $2\to1$

(d)



An electron is trapped in a one-dimensional infinite potential well. (a) What pair of adjacent energy levels (if any) will have three times the energy difference that exists between levels n = 3 and n = 4? (b) What pair (if any) will have twice that energy difference?

$$E_n = \left(\frac{h^2}{8mL^2}\right) n^2 \longrightarrow \text{adjacent energy level corresponds to } n+1 \qquad E_{n+1} = \left(\frac{h^2}{8mL^2}\right)(n+1)^2$$

$$= E_1 n^2 \qquad\qquad = E_1 (n+1)^2$$

(a) $E_{n+1} - E_n = E_1(n+1)^2 - E_1 n^2$

$$= E_1\left[(n+1)^2 - n^2\right] = E_1(2n+1)$$

$\Rightarrow$ we want $E_{n+1} - E_n = E_1(2n+1) = 3(E_4 - E_3)$

$$E_1(2n+1) = 3\left[E_1(4)^2 - E_1(3)^2\right]$$

$$E_1(2n+1) = 3[7E_1] \longrightarrow 2n+1 = 21 \rightarrow$$

n = 10 so adj. energy levels are E_{10} and E_{11}

(b) we want $E_{n+1} - E_n = E_1(2n+1) = 2(E_4 - E_3)$

$$E_1(2n+1) = 2\left[E_1(4)^2 - E_1(3)^2\right] = 2[7E_1]$$

$$E_1(2n+1) = 14E_1 \longrightarrow 2n+1 = 14 \longrightarrow$$

no integer valued solutions so no adjacent energy levels fit requirement