Solutions to selected exercises

PART A

CHAPTER 1

1. (a) t; (b) f; (c) f; (d) t; (e) f; (f) t; (g) t; (h) f; (i) f; (j) t; (k) f
(l) t; (m) f; (n) t; (o) f; (p) f; (q) f; (r) t

2. (a) yes; (b) no; (c) yes; (d) $\{\{S\}\}$

3. (a) *Rule:* 1. $5 \in A$
2. If $x \in A, x + 5 \in A$
Property: $A = \{x \mid x$ is positive and x is a multiple of $5\}$
(b) *Property:* $B = \{x \mid x + 3$ is a positive multiple of $10\}$
or $B = \{x \mid x$ is a positive integer whose last digit is $7\}$
(c) *Rule:* 1. $300 \in C$
2. If $x \in C$ and $x < 400, x + 1 \in C$
(d) *Rule:* 1. $3 \in D$
2 $4 \in D$
3. If $x \in D, x + 4 \in D$
or 1. $3 \in D$
2. If $x \in D$ and x is odd, $x + 1 \in D$
3. If $x \in D$ and x is even, $x + 3 \in D$
Property: $D = \{x \mid x$ is a positive multiple of 4 or
$x + 1$ is a positive multiple of $4\}$

(e) *Rule:* 1. $0 \in E$
2. If $x \in E$, then $x + 2 \in E$
3 If $x \in E$, then $-x \in E$

(f) *Property:* $F = \{x \mid x = \frac{1}{2^n}$ where n is a non-negative integer$\}$

5. (c) $\{\emptyset\}$; (d) $\{\{\emptyset\}, \emptyset\}$;
(e) $\{\emptyset, \{\{a\}\}, \{\{b\}\}, \{\{a,b\}\}, \{\emptyset\}, \{\emptyset, \{\{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a,b\}\}, \{\{a\}, \{b\}\}, \{\{a\}, \{a,b\}\}, \{\{b\}, \{a,b\}\}, \{\emptyset, \{a\}, \{b\}\}, \{\emptyset, \{a\}, \{a,b\}\}, \{\emptyset, \{b\}, \{a,b\}\}, \{\{a\}, \{b\}, \{a,b\}\}, \{\emptyset, \{a\}, \{b\}, \{a,b\}\}\}$

6. (a) $\{a, b, c, 2\}$, (b) $\{a, b, c, 2, 3, 4\}$, (c) $\{a, b, c, \{c\}\}$, (d) $\{a, b, \{a, b\}, \{c, 2\}\}$, (e) $\{b, c\}$, (f) $\{a, b\}$, (g) $\{a, b\}$, (h) $\{c\}$, (i) $\emptyset$, (j) $\emptyset$, (k) $\emptyset$, (l) $\{c, 2, 3, 4\}$, (m) $\emptyset$, (n) $\{2\}$, (o) $\{a, b, \{c\}\}$, (p) $\emptyset$, (q) $\{\{a, b\}, \{c, 2\}\}$

7. (a) $\{a, b, c, 2\}$, (b) $\{a, b, c, 2\}$, (c) $\{a\}$, (d) $\{2\}$, (e) $\{2\}$, (f) $\{a, b, c, 2, 3, 4, \{c\}\}$, (g) $\{2, 3, 4, \{a, b\}, \{c, 2\}\}$, (h) $\{2, 3, 4, \{a, b\}, \{c, 2\}\}$, (i) $\emptyset$, (j) U, (k) $\{b, c, 2\}$, (l) $\{2\}$, (m) U, (n) U

8. (a) (i) $\{a, b, c, d\}$; (ii) $\{c\}$; (iii) $\{a, b, c, d\}$; (iv) $\emptyset$; (v) $\{c, d\}$; (vi) $\emptyset$; (vii) $\{a, b\}$
(b) (i) no; (ii) yes

9. (b)

1.	$A \cap (B - A)$	
2	$A \cap (B \cap A')$	Compl.
3.	$(B \cap A') \cap A$	Comm.
4.	$B \cap (A' \cap A)$	Assoc.
5.	$B \cap (A \cap A')$	Comm.
6.	$B \cap \emptyset$	Compl.
7.	$\emptyset$	Ident.

11. (b)

1.	$(A \cup B) - (A \cap B)$	
2.	$(A \cup B) \cap (A \cap B)'$	Compl.
3	$(A \cup B) \cap (A' \cup B')$	DeM.
4.	$((A \cup B) \cap A') \cup ((A \cup B) \cap B')$	Distr.
5.	$(A \cap A') \cup (B \cap A') \cup (A \cap B') \cup (B \cap B')$	Distr. (twice)
6.	$\emptyset \cup (B \cap A') \cup (A \cap B') \cup \emptyset$	Compl. (twice)
7.	$(B \cap A') \cup (A \cap B')$	Ident. (twice)
8.	$(B - A) \cup (A - B)$	Compl. (twice)
9.	$(A - B) \cup (B - A)$	Comm (twice)

(c) $(X \cup Y) - (X \cap Y) = (Y \cup X) - (Y \cap X)$ by the commutativity of union and intersection.

(d) **(i)** $\emptyset$, **(ii)** A', **(iii)** A, **(iv)** $B - A$, **(v)** $A \cup B$

(e)

1	$(A - B) + (B - A)$	
2.	$((A - B) \cup (B - A)) - ((A - B) \cap (B - A))$	Def of $A + B$
3	$(A + B) - ((A - B) \cap (B - A))$	Def of $A + B$
4.	$(A + B) - ((A \cap B') \cap (B \cap A'))$	Compl
5.	$(A + B) - (A \cap A' \cap B \cap B')$	Assoc., Comm.
6.	$(A + B) - \emptyset$	Compl., Ident.
7.	$(A + B) \cap \emptyset'$	Compl
8.	$(A + B) \cap U$	Compl
9.	$(A + B)$	Ident.

(f)

1.	$(A + B) \subseteq B$	
2.	$(A + B) \cup B = B$	Cons. Prin.
3.	$((A \cup B) - (A \cap B)) \cup B = B$	Def. of $A + B$
4.	$((A \cup B) \cap (A \cap B)') \cup B = B$	Compl.
5.	$((A \cup B) \cap (A' \cup B')) \cup B = B$	DeM.
6.	$((A \cup B) \cup B) \cap ((A' \cup B') \cup B) = B$	Distr.
7.	$(A \cup (B \cup B)) \cap (A' \cup (B' \cup B)) = B$	Assoc. (twice)
8.	$(A \cup B) \cap (A' \cup U) = B$	Idemp., Ident.
9.	$(A \cup B) \cap U = B$	Ident.
10.	$A \cup B = B$	Ident
11.	$A \subseteq B$	Cons. Prin.

CHAPTER 2

1. **(a)** **(i)** $\{\langle b,2\rangle, \langle b,3\rangle, \langle c,2\rangle, \langle c,3\rangle\}$;
(ii) $\{\langle 2,b\rangle, \langle 2,c\rangle, \langle 3,b\rangle, \langle 3,c\rangle\}$;
(iii) $\{\langle b,b\rangle, \langle b,c\rangle, \langle c,b\rangle, \langle c,c\rangle\}$;
(iv) $\{\langle b,2\rangle, \langle b,3\rangle, \langle c,2\rangle, \langle c,3\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle\}$;
(v) $\emptyset$ (since $A \cap B = \emptyset$); **(vi)** same as $A \times B$

(b) **(i)** True; **(ii)** False; **(iii)** False, $\langle c,c\rangle \in (A \times A)$; **(iv)** True; **(v)** True; **(vi)** True; **(vii)** True

(c) **(i)** $\text{dom}(R) = A$, $\text{ran}(R) = \{b,2,3\}$;
(ii) $R' = \{\langle b,c\rangle, \langle b,3\rangle, \langle c,b\rangle, \langle c,c\rangle\}$,
$R^{-1} = \{\langle b,b\rangle, \langle 2,b\rangle, \langle 2,c\rangle, \langle 3,c\rangle\}$;
(iii) No