Practical Approach for Solving School Bus Problems

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Most of the numerous studies of school bus routing seem to possess two basic characteristics: ever-increasing sophistication of computer-based routing algorithms and neglect of the service side of the problem. While reducing the total cost of providing transportation services to students is important, so are such elements as student walking distance and bus load factors. In this paper a multi-objective view of school bus problem is formulated, and a three-stage simplified solution process is outlined and demonstrated. The advantages of using goal programming to solve this multi-objective problem are discussed. Then, a comparison of the proposed algorithm with the previously developed approaches is presented. Finally, a case study of 21 bus stops covering 270 student homes and 5 buses available is discussed in terms of the proposed solution.

Studies of the school bus routing problem are numerous. Krolak and Williams (*1*) provide a good summary of the basic approaches in this field. Most of these studies have considered standards for quality of service, such as a student's maximum in-bus travel time. What is usually overlooked as a service variable in transporting school students is the walking distance from home to the school bus stop. The typical routing study accepts bus stop locations as given, and proceeds to develop efficient routing algorithms. The school bus administrator and the operations he/she oversees would benefit from an unpretentious method to establish and evaluate bus stop locations, in accordance with a set of service criteria or school policies. This paper outlines such a method.

Tsay (*2*) and Fricker (*3*) examined the bus stop and location aspects of the problem in some detail. Among their findings were the following factors:

1. School bus stops are not systematically located to assure that a maximum walk distance standard for students is satisfied.
2. Techniques such as Facility Location Algorithms (*4–8*) can provide improved service to students, in terms of reducing walk distances from home to bus stop.
3. The Facility Location Algorithms that accomplish these improvements tend to be complex to formulate and encode for computer processing.
4. Minimizing the number of bus stops in an area does not necessarily improve the results of the subsequent routing algorithm.

Bodin and Berman (*9*) and Desrosiers et al. (*10*) are among the few to have acknowledged the existence of students' residential locations. In both studies, students were assigned to "mini-stops" at a street intersection at one end of their residential block. The mini-stops were then either assigned to prescribed stops or grouped at intersections to form new bus stops. Neither reference gave details of the computer program used to implement the scheme or the resulting walking access distances.

Based on these experiences, an algorithm that incorporates basic service aspects of school busing, while striking a balance between effective optimization techniques and ease of understanding and implementation, is presented in this paper. it shows a series of steps, accompanied by an example that addresses the school busing problem in a manner that extends beyond pure routing. This paper presumes a knowledge of basic optimization techniques. The process does not require mastery of abstract or complex mathematical derivations, and will not require extensive computer programming effort. The mathematical formulations and notation that will be employed are mainly to aid the user in matching the problem to a typical computer routine's documentation.

SPECIFYING GOALS IN SCHOOL BUSING

Before undertaking any modeling, the major goals have to be specified in school busing. The setting of goals is an art. Their values should not be unreasonably high or low, if achieving (rather than merely approximating) these goals is desirable. In fact, regardless of the particular structure of the solution process, the goals of the school bus service can remain the same. The six major goals adopted in this paper are as follows:

1. Limit maximum walking distance from a student's home to his/her assigned bus stop to one-third mile or less.
2. Guarantee each eligible student transportation to school.
3. Avoid underuse of equipment; balance school bus load factors.
4. Avoid circuitous routes and hazardous zones.
5. Avoid overcrowding in each bus.
6. Minimize the total transportation cost of providing service.

One-third mile is specified as the maximum walking distance in the first goal. This value can be changed, depending upon the characteristics and philosophy of the school district. Obviously, setting different constraints on maximum walking

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distance can result in different solutions for the optimal school bus routes. The same can be said for changes made in the values of the standards developed for any other goals.

It is likely that not all of these six goals can be met simultaneously. The school bus administrator should be allowed to rank his/her goals in order of importance. Although this task may be made unpleasant by conflicting interests, its accomplishment leads systematically to the kind of bus service desired. Traditional routing schemes (*1*) can be modified to include most of the goals listed above, but those schemes are not sensitive to the goals' relative importance. These schemes invariably have "minimize total transportation cost or distance" as an objective function, and express the other goals as constraint equations, if possible. Each of their traditional "goal-constraints" has a single limiting value that helps to determine the feasibility of the solution routes and carries the same relative weight as the others. This paper presents a process that allows more flexibility in designing school bus service. The steps that include ranked goals in the complete process are described and demonstrated in the next section.

STEPS IN THE SCHOOL BUS ROUTING SOLUTION

Step 1—Establish Subdistricts

Seldom does there exist a school district so devoid of topographical barriers, municipal boundaries, or directional orientation of road network, that "natural" subdistricts do not emerge. The procedures presented here confine themselves to computer routines that are relatively inexpensive to run, but there is no reason to run a large problem, when running a series of small subproblems gives similar results with noticeable cost savings and increases in administrative control. The cost savings arise because many computer programs for this type of problem do not simply double in cost of execution as problem size doubles: these costs can increase exponentially. Decomposing the problem into manageable parts may achieve significant savings in total cost without loss of a valid solution. The school district in Fricker's study (*3*) contained more than 2400 students, covered 50 square miles, and involved wide variations in population density and road network configuration. These variations gave strong indications where subdistrict boundaries should be.

It is advantageous, however, to keep each subdistrict large enough to allow several buses to seek their best routes within its boundaries. Once the stops and routes are established, subdistricts can be combined to allow reevaluation or student assignments for better use of bus capacity and greater flexibility in routing.

Step 2—Locate Student Homes on a Map

This is always a tedious, time-consuming step, especially the first time. But avoiding excessive walk distance between home and bus stop makes this a necessary and justifiable step. Most school districts do this already. The method used in this paper does not require that a school district's entire road network be coded for computer analysis. This is discussed further in the following section. The study area for the sample problem,

with the locations of student's homes, is shown in Figure 1. Each dot point represents the location of a student residence along a street link. There are 270 students in area shown in Figure 1.

Step 3—Locate Bus Stop Zones

As mentioned earlier, Tsay and Fricker studied relatively expensive Facility Location Problem routines, designed to solve the General Absolute Facility Location Problem (*11, 2, 5, 8*). Fricker's approach, and those of Bodin and Berman (*9*) and Clarke and Wright (*12*), involves creating a data file containing the length, location, and student population of each street link in a subdistrict. While not yet economical as a direct aid to a school district, the study did offer some useful lessons. Besides those lessons listed earlier, it was found that siting bus stops in densely populated urban areas (such as in the lower left portion of Figure 1) was not governed by any reasonable access distance (such as one-third mile), but rather by the number of students assigned to their nearest stop. For certain stops in an urbanized subdistrict, this number can exceed the capacity of a school bus. More than one bus could be assigned to those stops, or the stops could be increased in number and moved closer together, thereby permitting very short walk distances.

In sparse rural areas and at small settlements (the upper half of Figure 1), bus stop siting was obvious: at the isolated student's driveway and at the center of a settlement. It was in medium-density settings that a systematized procedure seemed most necessary. At the outskirts of cities and in subdivisions, a large number of reasonable bus stop site combinations are possible. A simple and low-cost way to establish and visually evaluate alternative combinations involves the use of templates.

Figure 1 shows a scale of 150 mm = 1 mile. The maximum walking distance of one-third mile then equates to a map distance of 50 mm. Geometric shapes or templates (Figure 2) can be constructed to cover parts of a subdistrict so that no point is farther away than 50 mm from the predetermined bus stop within a shape, using the existing street network. A bus stop placed on any trafficable street within that "covered" area will be within 50 mm of all student homes covered. A suitable objective for the user is to cover as many student homes as possible with the application of a template. If there exist hazardous zones that students should not cross, Figure 1 can be marked first to show these dangerous streets and then applied the templates to determine the suitable bus stops. In addition, the bus stop can be suitably adjusted if the predetermined bus stop is not appropriate in the initial assignment. The few questionable cases caused by unusual street patterns can be checked individually.

In grid street networks, the natural geometric choice is the family of rectangles whose length plus width equals 50 mm, as shown in Figure 2. The templates could be enlarged to almost twice this size and bus stops placed near their centroids, but the 50 mm size offers more flexibility. Other geometric shapes, such as circles, can be constructed if the nature of the street network makes them more useful than rectangles. For example, if it were found that a concentration of students in a neighborhood could be covered by a number of overlapping templates (Figure 3), a bus stop on any street section in
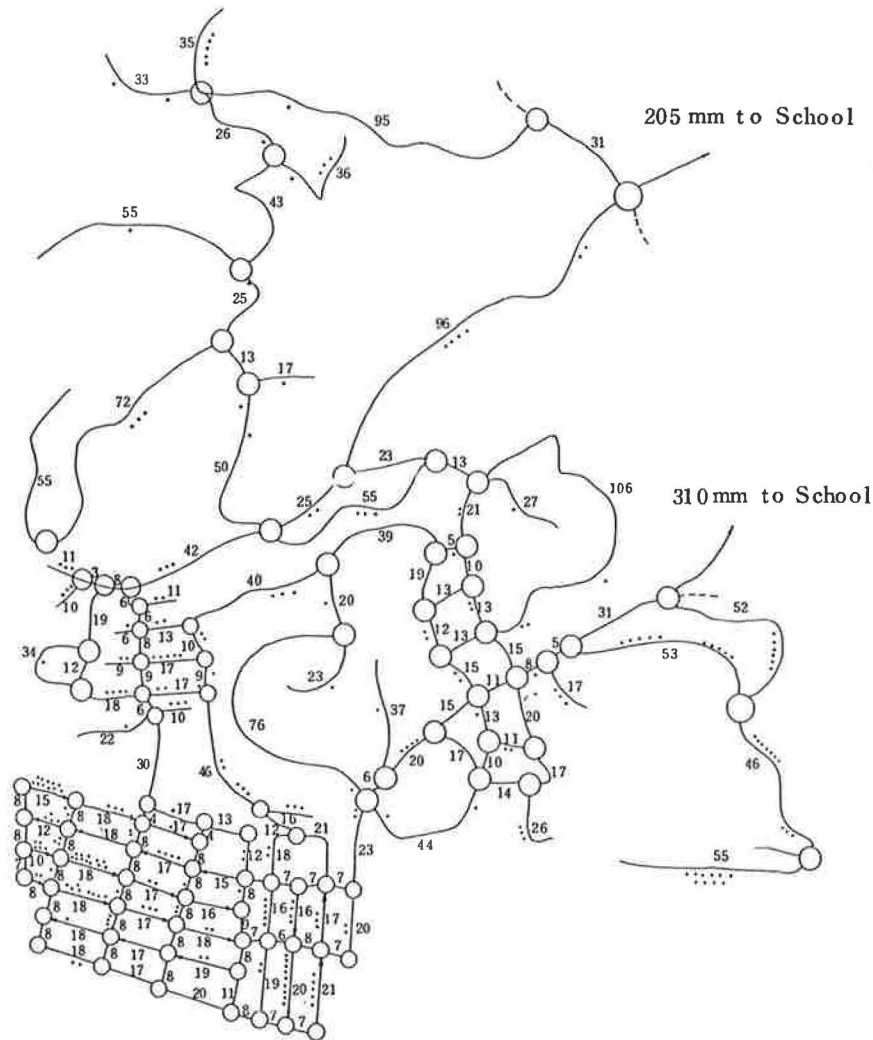
FIGURE 1 Sample subdistrict: dots represent homes of students. (Scale: 150 mm = 1 mi.)

the overlapping area would be within 50 mm (representing one-third mile) of all the covered students.

This is a trial-and-error method that is intellectually unpretentious. It does, however, systematize the incorporation of the first goal—the service attribute of maximum walk distance—into the process of siting bus stops, without the considerable effort and expense that would accompany use of the computer. Besides, retaining human judgment in this step may allow those "extra" bus stops to be placed in more desirable locations than a computer routine would, and where

better bus routes can be achieved. Figure 4 shows 21 bus "zones" corresponding to the areas covered by the templates used in the sample subdistrict. Note that choosing different kinds of geometric shapes to cover student homes may result in a different number of total service zones and students in a zone. The current example does not exploit the possibility to overlap zones in the lower left area of Figure 3, but it could be done. Doing so in this neighborhood would create fewer bus stops, each with large number of students, which might not be desirable from the viewpoint of safety or public order.
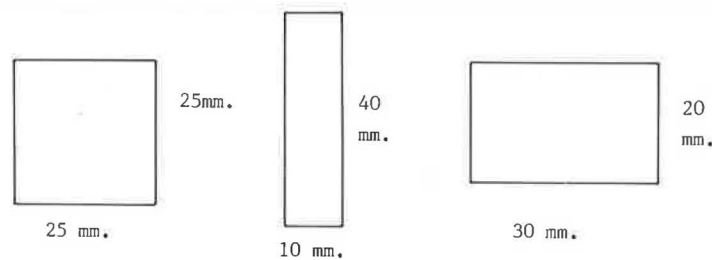

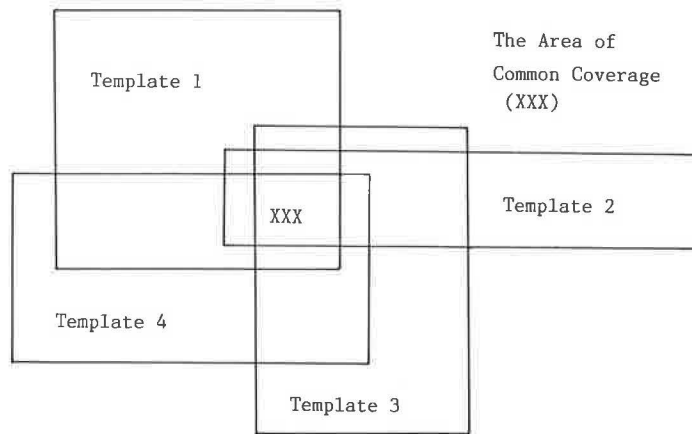**FIGURE 2** Examples of templates that ensure acceptable walking distance.

FIGURE 3   Exploiting overlapping templates to site bus stops.

Step 4—Develop Network Structure

To develop models for simulating the performance of the system, a suitable representation of the study area is first specified. By using the templates to establish bus stop zones in the previous step, not only have we avoided a potentially costly Facility Location Problem solution, but we have dispensed with the need for a detailed representation of the subdistrict's street network. Having systematically chosen a "good" bus stop location for each zone, only those street links
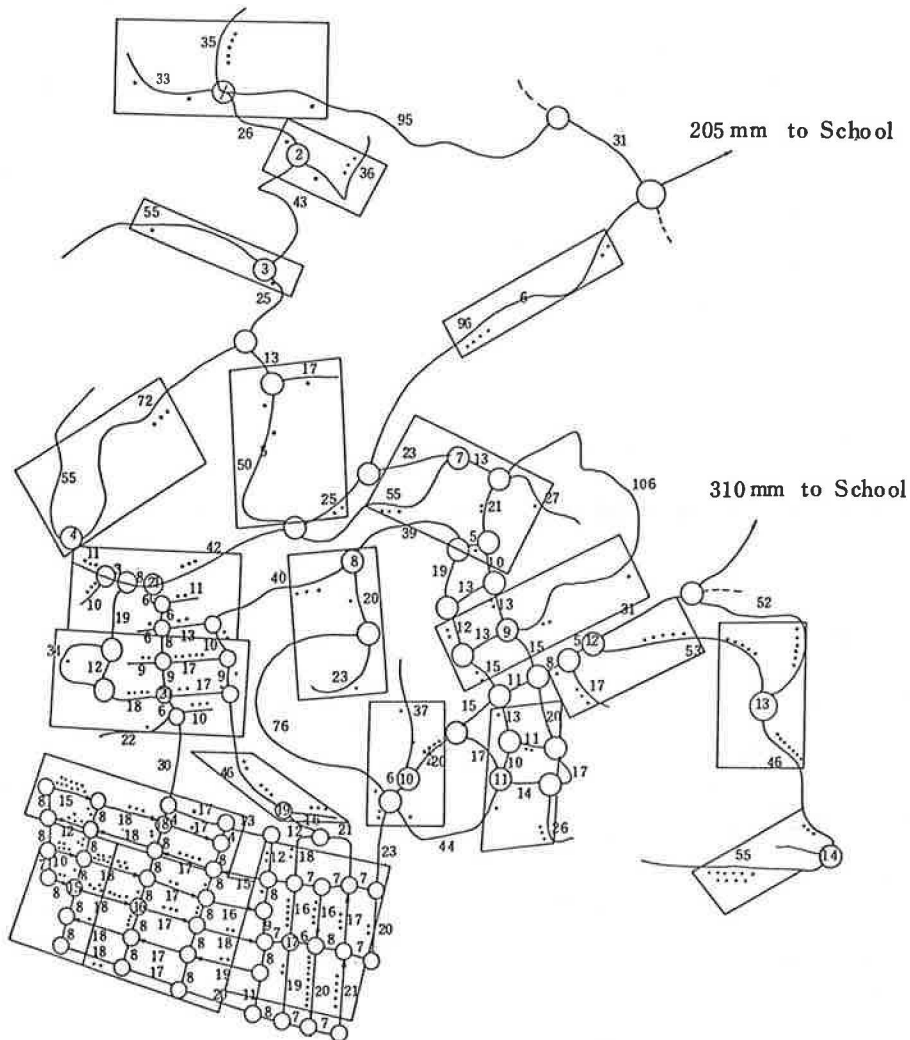

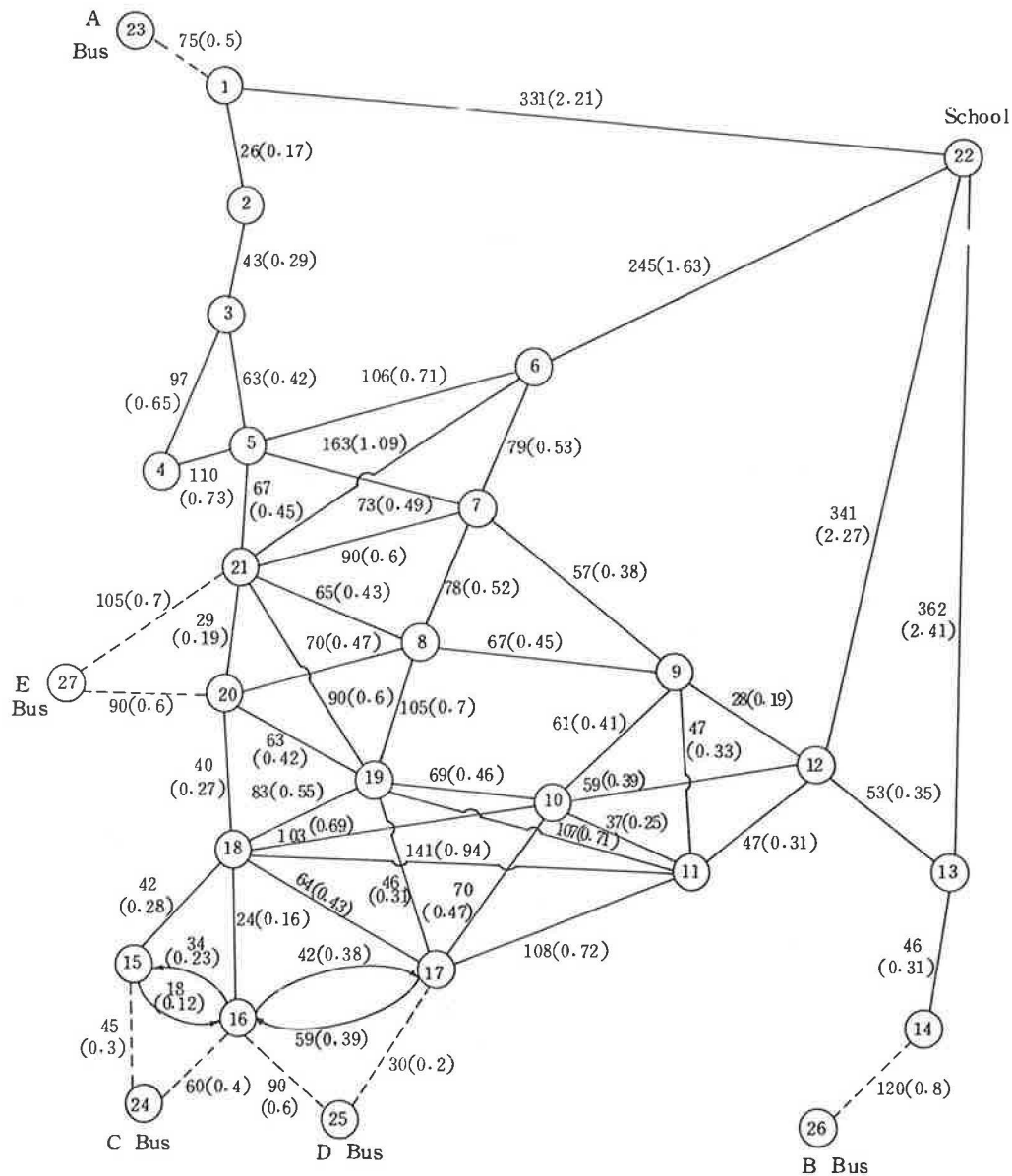**FIGURE 4   Subdistrict "covered" by 21 templates.**

FIGURE 5   Link-node representation of the subdistrict with measure distance (actual distance).

that best connect the resulting set of bus stops need to be identified, measured, and encoded. The choice of bus stop locations can have much to do with the quality of the routes connecting them. This should be recognized as early as the application of the template step. A side street location may minimize the total walking distance for all students or the total longest distance any student that has to walk, but it will cause the school bus to leave a more direct, higher-speed route to reach it. In the example, bus stops are tentatively located at major intersections as zonal centroids. This simplifies the construction of link-node representation (Figure 5), by having bus stop at already existing nodes. Once the bus routes are determined, bus stops can be moved to more appropriate locations in each zone along the route. Twenty-one bus stops cover all 270 student homes in the study area. The distance along the principal routes connecting these nodes are shown

in Figure 5. The number within parentheses is the actual road distance in miles.

Let us make five buses available to the service area: 2 of capacity 70, 2 of capacity 60, and one with 50 seats. Thus, we have a total seat capacity of 320 to transport the subdistrict's 270 students. The resulting average load factor of 87 percent can be used to establish values for the third goal. If avoiding underuse of any bus is considered, a minimum load factor less than 87 percent in this example, such as 70 percent, can be established. It is also necessary to know the location of bus starting points. The buses can all be located at one depot or at several different positions (such as drivers' homes), depending upon existing procedures. In this example, it is assumed that each bus has a different starting point. These locations are nodes 23 through 27 in Figure 5.

Step 5—Assign Students to Buses

In view of the multiple-objective nature of the school bus routing problem, one available solution technique is Goal Programming (GP). The basic concept of GP involves incorporating multiple, conflicting goals into one model. The GP model wants to optimize all specified goals, by trying to minimize the deviation (d^+ or d^-) of solution values from prespecified goals, while respecting the priorities given each goal and satisfying the problem's set of constraints. All goals are considered simultaneously or can be taken in turn, depending upon the characteristics of the problem and flexibility of the computer program.

A general review of Goal Programming was presented by Ignizio (13). Although misunderstanding and misuse of GP makes it controversial (14), it applies to the school bus routing problem because it can handle conflicting objectives in one formulation. Furthermore, computer packages are available, as are program listings in textbooks (15). Goals 1 through 5, as described earlier, are considered here in the context of the GP formulation.

1. Specify zones to be served by a bus:

$$\sum_{i \in \tau} X_{ij} + d_j^- = \sum_{i \in \tau} S_i \qquad j = 1, 2, \ldots, 5 \qquad (1)$$

where

S_i = total number of students in zone i specified to be served by bus j.

d_j^- = number of students in specified zone i that are not served by bus j.

2. Guarantee each eligible student transportation to school:

$$\sum_{j=1}^{5} X_{ij} + d_i^- = S_i \qquad i = 1, 2, \ldots, 21 \qquad (2)$$

where

X_{ij} = number of students at zone i to be assigned to bus j.

S_i = total number of students at zone i.

d_i^- = number of unserved students in zone i.

If d_i^- can be forced to zero, each student in zone i will have transportation to school. Note that each zone can be served by more than one bus. This allows service to zones with large student populations and permits better solutions with respect to overcrowding and unbalancing problems.

3. Avoid underuse of buses:

$$\sum_{i=1}^{21} X_{ij} + d_j^- = P \times C_j \qquad j = 1, 2, \ldots, 5 \qquad (3)$$

where

P = minimum load factor of each bus, using a value of 0.7.

d_j^- = number of students lower than the specified minimum number of students in bus j.

This is intended to guarantee that at least the proportion P, $0 \le P \le 1$, of bus capacity is occupied if any bus is used.

The value of P should, of course, be less than the average load factor—87% in this example.

4. Avoid circuitous routes:

$$\sum_{i \in U} X_{ij} - d_j^+ = 0 \qquad j = 1, 2, \ldots, 5 \qquad (4)$$

where

d_j^+ = the number of students in zone i served by bus j, whose starting point is considered to be too far from i.

U = set of zones whose locations make service by bus j unsuitable.

This optional goal is sometimes added to save the solution process by ruling out obviously poor zones for a given bus to serve. For example, in Figure 5 bus A could not economically include zones such as 14 and 17 on its route, when several other buses start much closer to them.

5. Avoid overcrowding in each bus:

$$\sum_{i=1}^{21} X_{ij} - d_j^+ = C_j \qquad j = 1, 2, \ldots, 5 \qquad (5)$$

where

C_j = capacity of bus j.

d_j^+ = number of students in excess of the capacity of bus j.

This goal discourages the assignment of students to a bus j beyond its carrying capacity. The capacity of each bus is shown below:

Bus	Capacity
A	50
B	60
C	70
D	70
E	60

An administrator can specify a set of $\{T\}$ zones that should be served by bus j for whatever reason. This is another optional goal that demonstrates the flexibility of GP. It should be emphasized that the GP solution will attempt to achieve this goal, unless doing so will prevent achievement of higher priority goals or cause violation of some constraints.

Equations 1 and 4 are used here to incorporate transportation costs into the GP formulation. In a GP formulation, costs cannot be represented in a direct way. Attempts at doing so are often necessarily crude (16). This is because the GP decision variables represent assignment of students (in zones) to buses. The sequence of zones that forms the shortest bus route cannot be found in a standard GP formulation. On the other hand, routing algorithms place almost total emphasis on minimizing system costs. The conflicts often arise when different goals are considered. The GP objective function involves minimizing the deviations, either positive or negative, from predetermined goals. Priorities indicate the relative rank assigned by the user appear as coefficients P_i. Such a case arises when one or more of the goals clearly is far more important than the others. This type of problem belongs to the category of preemptive goal programming (17). Obviously,

different priorities order will result in different solutions. The GP model will determine the number of students in each zone to be assigned to each bus by considering five multiple goals simultaneously. The objective function (Z) for our example is given as follows:

$$\text{Minimize } Z = P_1 \sum_{i=1}^{21} d_i^- + P_2 \sum_{i \in T} d_i^- + P_3 \sum_{i \in U} d_i^+$$

$$+ P_4 \sum_{i=1}^{5} d_i^+ + P_5 \sum_{i=1}^{5} d_i^-$$

where P_1 is the coefficient of the goal having the highest priority.

A complete formulation of the generalized GP model for this example is presented here.

$$\text{Minimize } Z = P_1 \sum_{k=1}^{21} d_k^- + P_2 \sum_{k=76}^{80} d_k^- + P_3 \sum_{k=32}^{75} d_k^+$$

$$+ P_4 \sum_{k=22}^{26} d_k^+ + P_5 \sum_{k=27}^{31} d_k^-$$

Subject to

$$\sum_{j=1}^{5} X_{ij} + d_k^- = S_i \quad \text{(for } i = 1, \ldots, 21; k = 1, \ldots, 21)$$

$$\sum_{i=1}^{21} X_{ij} - d_k = C_j \quad \text{(for } i = 1, \ldots, 5; k = 22, \ldots, 26)$$

$$\sum_{i=1}^{21} X_{ij} + d_k = P \times C_j$$

$$\text{(for } j = 1, \ldots, 5; k = 27, \ldots, 31)$$

$$\sum_{i \in U} X_{ij} - d_k^+ = 0 \quad \text{(for } j = 1, \ldots, 5; k = 32, \ldots, 75)$$

$$\sum_{i \in T} X_{ij} + d_k^- = \sum_{i \in T} S_i \quad \text{(for } j = 1, \ldots, 5;$$

$$k = 76, \ldots, 80) \, X_{ij}, d_k^-, d_k^+ > 0$$

Step 6—Analyze the Output and Form the Bus Routes

The example problem had 80 goal constraints with 105 variables and 5 major objectives. The problem was solved using a modified FORTRAN GP computer program (15,18) on a VAX 11/780 system at Purdue University. The program required 504 seconds of execution time for the formulation described in Equations 1 through 5. The output is shown in Table 1.

Since the major variables X_{ij} used in this GP represent the number of students at zone i to be assigned to bus j, the output will show the result of aggregating several zones to a specific bus. Once the assignment of students and zones to each bus has been determined by this GP model, the estimated transportation cost can be easily obtained from any appropriate routing algorithm or even manually. For example, $X_{102} = 9$ in Table 1 means that 9 students from zone 10 would be assigned to bus B (bus number 2). It also can be seen from the first column that 48 students from 8 zones would be transported by bus A. Since the capacity of bus A was 0, its load factor is 96 percent, somewhat higher than the 87 percent system average. Bus D, serving 52 students in zones 17 and 18, has a 74 percent load factor. Note that zone 18 is served by buses C and D, demonstrating that each bus zone can have more than one bus serving it.

Once the aforementioned assignments have been determined, the next step is to form efficient bus routes. What is desired here is a method that is easy to obtain or program yourself, cheap to run, or dispenses with computer routines entirely (19). One can use the shortest distance matrix (from Figure 5) or traveling salesman algorithm (2, 20) to establish bus routes through the assigned stops. This is a straightforward step. Since the GP assignment (Table 1) has relieved routing algorithms of the need to decide which bus serves a given node, they need not be very elaborate. In fact, applying a computer route to find bus D's route (through only two

TABLE 1   ASSIGNMENT OF EACH BUS FROM THE OUTPUT OF GP

	A Bus	B Bus	C Bus	D bus	E Bus
	$X_{11} = 8$	$X_{62} = 3$	$X_{193} = 8$	$X_{174} = 35$	$X_{65} = 3$
	$X_{21} = 5$	$X_{82} = 5$	$X_{183} = 12$	$X_{184} = 17$	$X_{215} = 17$
	$X_{31} = 2$	$X_{142} = 13$	$X_{153} = 24$		$X_{205} = 22$
	$X_{51} = 5$	$X_{132} = 17$	$X_{163} = 26$		
	$X_{51} = 5$	$X_{112} = 11$			
	$X_{71} = 5$	$X_{102} = 9$			
	$X_{91} = 9$				
	$X_{121} = 9$				
Total	48	58	70	52	42
Capacity	50	60	70	70	60
Load Factor	96%	96.6%	100%	74%	74%

TABLE 2 ROUTE SOLUTIONS FOR EACH BUS

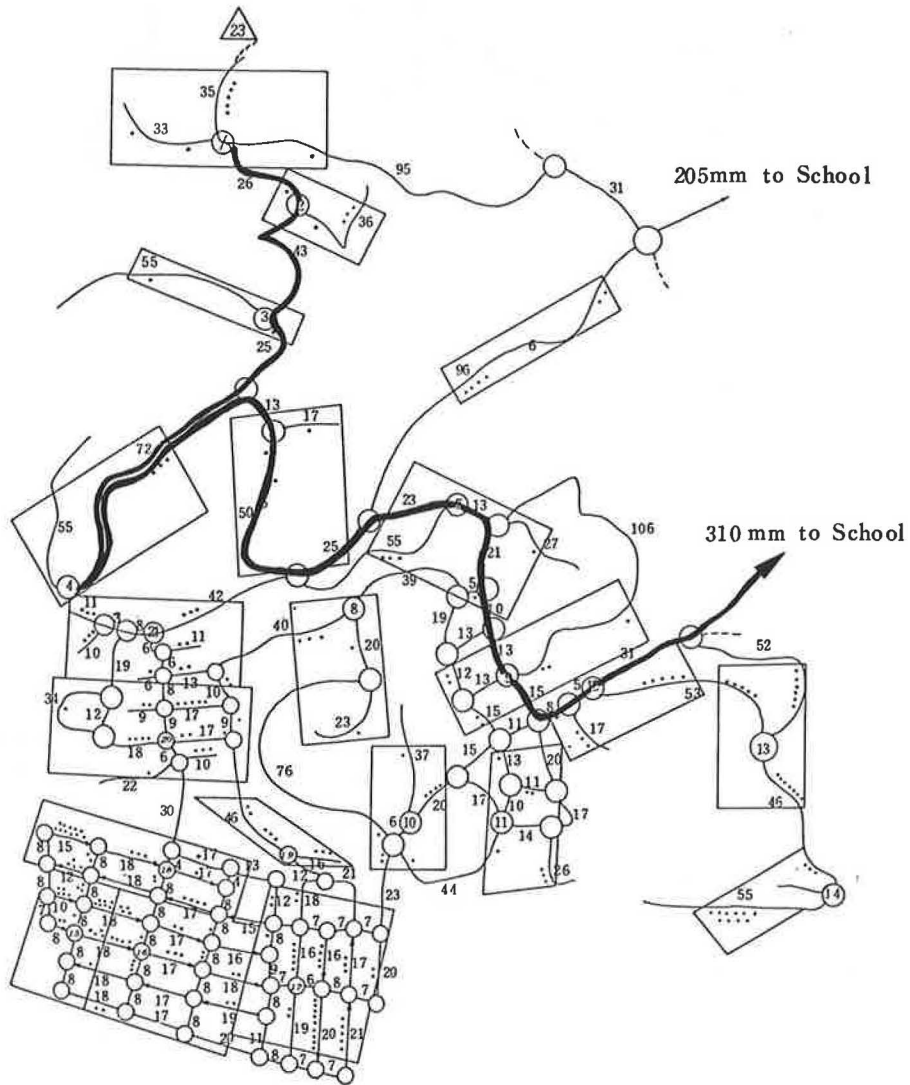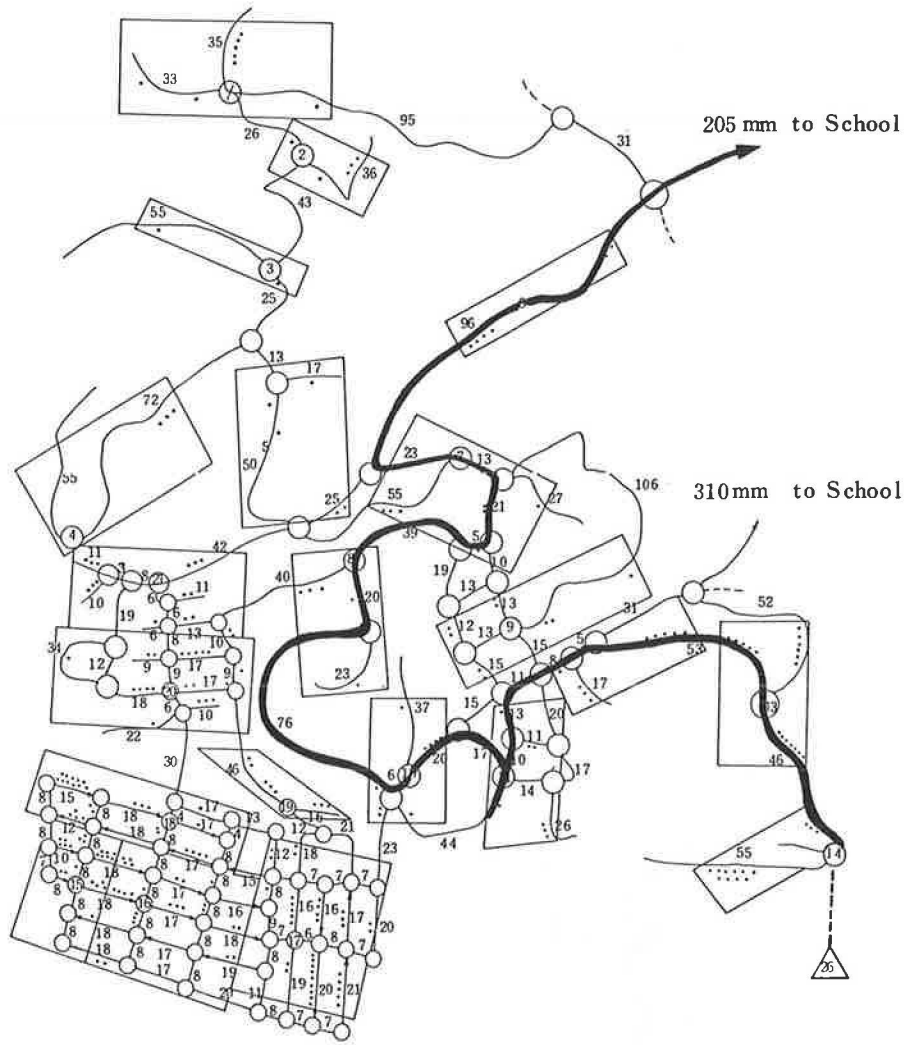| Bus	Route	Distance (miles)
A	23 −1 −2 −3 −4 −5 −7 −9 −12 −22	5.67
B	26 −14 −13 −11 −10 −8 −6 −22	5.56
C	24 −15 −16 −18 −19 −22	4.25
D	25 −17 −18 −22	3.81
E	27 −20 −21 −6 −22	3.51
Total		22.80

FIGURE 6 Bus A's solution route.

FIGURE 7 Bus B's solution route.

stops) would be absolutely wasteful. Sometimes, the GP assignment solution reduces the routing problem to such an extent that its solution is trivial, as bus D's route case. Table 2 records the length of each bus route shown in Figures 6 to 10.

COMPARISON OF SCHOOL BUS ROUTING ALGORITHMS

Although a number of articles describe school bus routing algorithms, most published works are concerned with the construction of bus routes from a set of given depots or bus stops to a single school. They can be grouped into two distinct approaches. The first is called the "cluster first—route sec-

ond" procedure for routing. It forms a cluster of bus stops that are feasible with respect to the travel time and bus capacity constraints, and then finds the best route for the stops in each cluster by reducing as much as possible the travel time to cover all bus stops within each cluster (9). This "cluster first" algorithm has been proposed by many researchers, such as Bennett and Gazis (21), Gillett and Miller (22), Krolak and Williams (1), and Chapleaun et al. (23). The other approach, called "Route first—cluster second," first creates a single route through all stops, using a heuristic traveling salesman algorithm, and then divides this single route into feasible shorter routes by considering the bus capacity and travel time constraints. Many interesting aspects of this problem has been discussed by Newton and Thomas (24), Mandl (25), and Bodin and Berman (9).
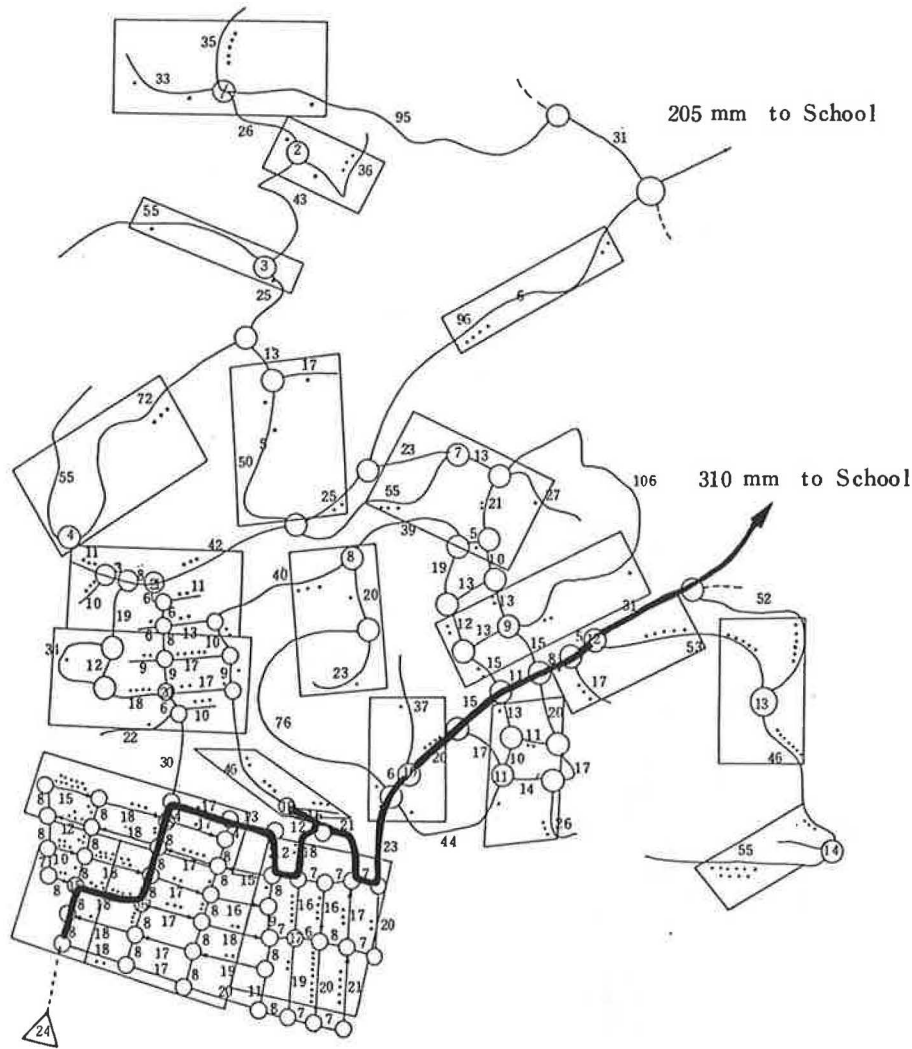
FIGURE 8   Bus C's solution route.

A brief comparison of some approaches with our proposed algorithm, including techniques and computing time, is summarized in Table 3. In the authors' experience, a general result in comparing heuristic algorithms in terms of efficiency is very seldom obtainable, because most heuristic algorithms handle the problems with different objectives, constraints, or characteristics which often require special treatment. For example, the location and demand of bus stops were assumed to be known in most traditional school bus routing algorithms. If these two elements are made part of the problem to be solved, then the formulation and methodology would be quite different. Naturally, the adoption of suitable algorithms depends upon the requirements and criteria set by administrators or by the computer packages available.

In this paper, a new practical algorithm is proposed to consider the school bus routing problem in either urban or rural areas. The use of templates and GP strategy is much more flexible and easier to adapt to many conflicting objectives specified by school administrators. It should be noted that the current GP formulation does not consider the scheduling problem. With scheduling (time windows), the complete set of routes cannot be specified beforehand, since the exact time of service for a given route cannot be ascertained in advance. Hence, it becomes difficult to construct the complete network of possible connections a priori. More effort is necessary to include scheduling of buses in order to reflect the real world. An extensive discussion of routing and scheduling was given by Bodin et al. (*26*), Bodin and Berman (*9*), Desrosiers et al. (*27*), and Swersey and Ballard (*25*). Development of combined school bus stop location, routing, and scheduling
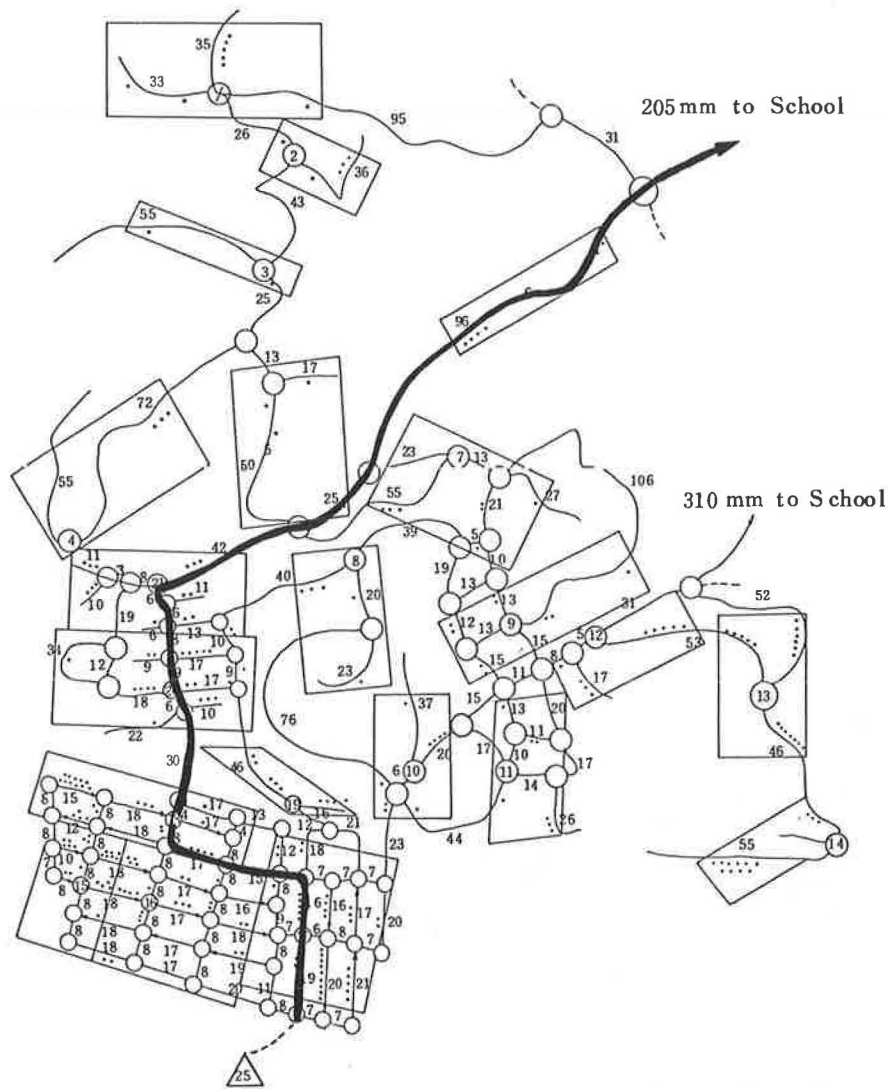
FIGURE 9   Bus D's solution route.

models is a fertile area for future research. Some exact and heuristics with respect to the formulation and solution of combined models have been studied by Tsay (*2, 6*), Perl and Daskin (*29*), and Srikar and Srivastava (*4*).

The limitation of the proposed algorithm is mainly on the capabilities of available GP packages. Its computer time will increase rapidly as the number of bus stops are added. In order to achieve savings in total cost, the entire school residential area must be broken down into manageable parts. In addition, this algorithm should involve human judgment for the use of templates and computer analysis of GP; school administrators may experience difficulty in performing sensitivity analysis on various priorities order and template sizes.

CONCLUSIONS

A practical solution to a network problem has been proposed. An important element of service—a limit on student walking distance—can be incorporated into the siting of bus stops at practically no cost by using templates. Goal Programming is used to balance conflicts between multiple predetermined goals. Finally, the best route of each bus is obtained through the application of any routing algorithm to a greatly reduced problem. Thus, school bus routing can be accomplished without an exotic operation by computer. In fact, retaining human involvement in establishing goals and giving them desired weight in a multi-objective solution process, should lead to an infinitely more satisfactory result.
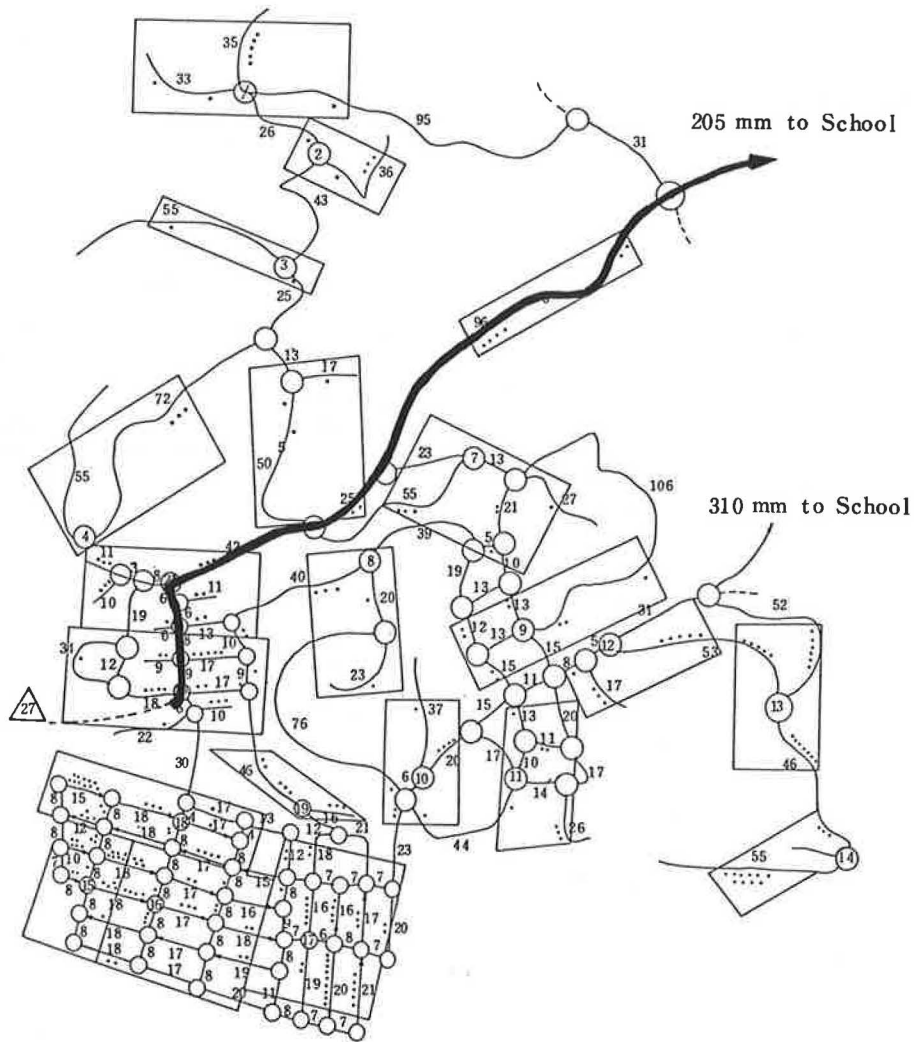
205 mm to School

310 mm to School

FIGURE 10 Bus E's solution route.

TABLE 3 COMPARISON OF SCHOOL BUS ROUTING ALGORITHMS

Study	Type	Location of Bus Stops	# Students at Each Stop	Method to Build School Bus Routes	Computing System; Example and Computer Time
Tsay and Fricker (1987)	Cluster first -route second	Use templates that guarantee students' maximum walking distance	Unknown	1. Use GP to solve multiple objectives 2. Assign students to each bus 3. Apply shortest path or traveling salesman algorithm to obtain routes	VAX 11/780; 270 students, 5 buses, 21 stops; 504 seconds.
Bennett and Gazis (1972)	Cluster first -route second	Given	Given	1. Extension of Clarke and Wright algorithm to start with many short routes 2. Decrease the number of routes by combining two tours into a single tour	256 bus stops, No execution times given.
Newton and Thomas (1974)	Route first -cluster second	Given	Given	1. Use traveling salesman route to encompass all bus stops 2. The route is partitioned into segments that satisfy bus capacity and maximum route length constraints	CDC 6400; 532 students, 45 stops; 36.5 sec. 596 students, 37 stops; 6.9 sec. 1097 students, 76 stops; 136.9 sec. 6 schools, 669 students, 96 stops; 61 seconds.
Bodin and Berman (1979)	Route first -cluster second	Mini-Stops plus scheduling	Unknown	1. Apply Lin's 3-opt branch exchange set of routes that satisfy the bus capacity and student travel time constraints	NCR Century 200 34 buses: 12 hours. 13 buses: 11 hours. 28 buses: 6 hours.

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