Teoria dos Grafos e Computabilidade

— Sets on graphs —

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— Independent sets —

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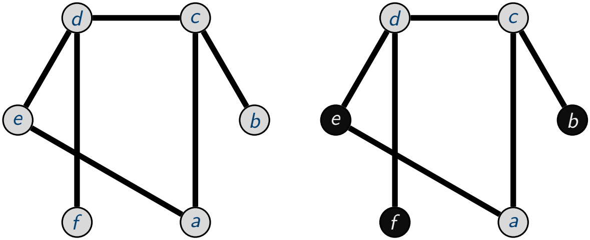
Independent sets

Let $G = (V, E)$ be an undirected connected graph.

- A subset $S \subseteq V$ is an **independent set** if $\forall u, v \in S$ there is no edge $(u, v) \in E$.
- Independent sets have also been called internally stable sets.



Independent sets

Let $G = (V, E)$ be an undirected connected graph, and S an independent set of G

- We say that the subset $S \subseteq V$ is a maximal independent set if there is no other independent set A in which $S \subset A$;
- The number of internal stability $\beta(G)$ is equal to the cardinality of the largest maximal independent set.

As S is an independent set of G, then S is a clique in the complement graph.

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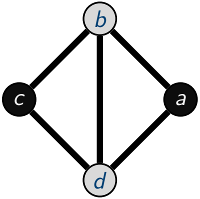

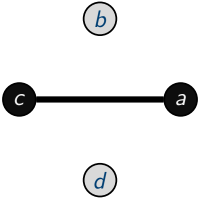

Independent sets

Let $G = (V, E)$ be an undirected connected graph. Design a method for computing an independent set of G

Algorithm: A method for computing an independent set

input : A graph $G = (V, E)$.
output: A independent set S

```
S = ∅;
while V ≠ ∅ do
    u = vertex with the smallest degree in G;
    V = V − {u} − Γ(u);
    S = S ∪ {u};
end
return S;
```

Teoria dos Grafos e Computabilidade

— Dominating sets —

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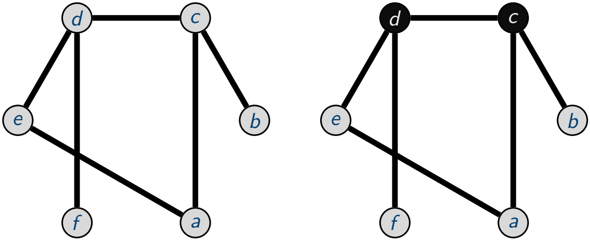
Dominating sets

Let $G = (V, E)$ be an undirected connected graph.

- A subset $S \subseteq V$ is an **dominating set** if $\forall u \in S$ there exist a $v \in V - S$ such that $(u, v) \in E$.
- Dominating sets have also been called externally stable sets.



Independent sets

Let $G = (V, E)$ be an undirected connected graph, and S a dominating set of G

- We say that the subset $S \subseteq V$ is a minimal dominating set if there is no other dominating set A in which $A \subset S$;
- The number of external stability $\beta(G)$ is equal to the cardinality of the smallest minimal dominating set.

Independent sets

Let $G = (V, E)$ be an undirected connected graph. Design a method for computing a dominance set of G

Algorithm: A method for computing a dominating set

input : A graph $G = (V, E)$.
output: A dominating set D

```
D = ∅;
while V ≠ ∅ do
    u = vertex with the highest degree in G;
    V = V − {u} − Γ(u);
    D = D ∪ {u};
end
return D;
```

Teoria dos Grafos e Computabilidade

— Vertex cover —

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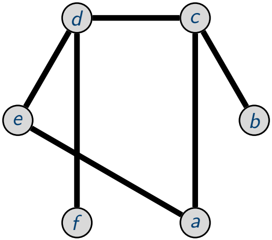
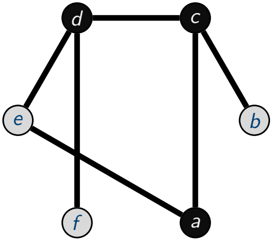
Vertex cover

Let $G = (V, E)$ be an undirected connected graph.

- A subset $S \subseteq V$ is an vertex cover if $\forall (u, v) \in E$, either $u \in S$ or $v \in S$.

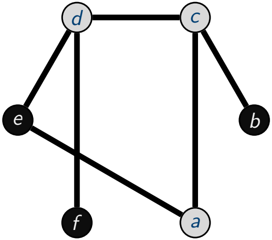


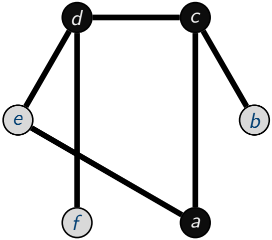


Vertex cover

Let $G = (V, E)$ be an undirected connected graph, and S a vertex cover of G

As S is a vertex cover of G, then V-S is an independent set.





Vertex cover

Let $G = (V, E)$ be an undirected connected graph. **Design** a method for computing a vertex cover in G

Algorithm: A method for computing a minimum vertex cover

---

input : A graph $G = (V, E)$.
output: A independent set S

$S = \emptyset$;
while $E \neq \emptyset$ **do**
  Let (u, v) an arbitrary edge of E;
  Choose either u or v to be included to C;
  $S = S \cup \{u\}$ for instance;
  $V = V - u$;
end
return S;
