~~L10E1~~

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L10E1

a) Usando a divisão de tensão, temos que $\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{s+\frac{1}{RC}} \therefore V_c(s) = V_i(s)\left[\frac{1/RC}{s+1/RC}\right]$

$\therefore V_c(s) = \frac{5}{s}\left[\frac{0,703}{s+0,703}\right] = \frac{3,515}{s(s+0,703)} = \frac{A}{s} + \frac{B}{s+0,703} = V_c(s)$

$A = V_c(s)\cdot s\Big|_{s=0} = \frac{3,515}{0,703} = 5$, $B = V_c(s)\cdot(s+0,703)\Big|_{s=-0,703} = \frac{3,515}{-0,703} = -5$

$\therefore V_c(s) = \frac{5}{s} - \frac{5}{s+0,703} \xrightarrow{\mathcal{L}^{-1}} v_c(t) = 5 - 5e^{-0,703t}$,,

b) A constante de tempo é $\frac{1}{a} = \frac{1}{0,703} = 1,422s$

O tempo de subida é $\frac{2,2}{a} = \frac{2,2}{0,703} = 3,129s$

O tempo de acomodação é $\frac{4}{a} = 5,689s$,,

L10E3

a) Escrevemos $\frac{400}{s(s^2+12s+400)} = \frac{400}{s(s+6-j19,08)(s+6+j19,08)}$, Temos então um sistema subamortecido, que sua fórmula geral é $c(t) = Ae^{-at}\cos(\omega_d t - \varphi)$

Portanto, $c(t) = A + Be^{-6t}\cos(19,08t - \varphi)$. $W_n = \sqrt{K} \therefore W_n = \sqrt{400} \rightarrow W_n = 20 rad/s$

já $\zeta = \frac{a/2}{W_n} = \frac{12/2}{20} \rightarrow \zeta = 0,3$,,

b) $\frac{900}{s(s^2+90s+900)} = \frac{900}{s(s+11,46)(s+78,54)}$, temos um sistema ~~criticamente~~ super amortecido, que sua fórmula é $c(t) = A + Be^{-11,46t} + Ce^{-78,54t}$. $W_n = \sqrt{K} \therefore W_n = \sqrt{900} \rightarrow W_n = 30 rad/s$, já $\zeta = \frac{a/2}{W_n} = \frac{90/2}{30} \rightarrow \zeta = 1,5$,,

c) $\frac{225}{(s^2+30s+225)s} = \frac{225}{s(s+15)(s+15)}$, sendo um sistema criticamente amortecido, que sua fórmula é $c(t) = A + Be^{-15t} + Cte^{-15t}$. $W_n = \sqrt{K} \therefore W_n = \sqrt{225} \rightarrow W_n = 15$, $\zeta = \frac{a/2}{W_n} \therefore \frac{30/2}{15} \rightarrow \zeta = 1$,,

d) $\frac{625}{s(s^2+625)} = \frac{625}{s(s-25j)(s+j25)}$, sendo um sistema sem amortecimento, onde sua fórmula é $c(t) = A + B\cos(25t - \varphi)$. $W_n = \sqrt{625} \rightarrow W_n = 25$, $\zeta = \frac{a/2}{W_n} \therefore \frac{0/2}{W_n} \rightarrow \zeta = 0$,, L10E4

a) $\omega_n = \sqrt{K} \therefore \omega_n = \sqrt{16} \rightarrow \omega_n = 4, \; \zeta = \frac{a/2}{\omega_n} \therefore \zeta = \frac{3/2}{4} \rightarrow \zeta = 0{,}375$

$T_s = \frac{4}{\zeta \omega_n} \therefore T_s = \frac{4}{0{,}375 \cdot 4} \rightarrow T_s = 2{,}67s \; . \; T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \therefore T_p = \frac{\pi}{\omega_n\sqrt{1-0{,}140625}} \therefore T_p = 0{,}8472s$

Para Tr usamos $\omega_d = \omega_n\sqrt{1-\zeta^2} = 3{,}708$ e $\sigma = \zeta\omega_n = 1{,}5$ e $\beta = \text{arctg}\left(\frac{\omega_d}{\sigma}\right) = \text{arctg}(2{,}472) = 67{,}97° \cong 1{,}185 rad$

$T_r = \frac{3{,}14 - \beta}{\omega_d} \therefore T_r = \frac{3{,}14 - 1{,}185}{3{,}708} \rightarrow T_r = 0{,}527s$

$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 \therefore \%OS = e^{-(0{,}375\pi/0{,}927)} \cdot 100 \rightarrow \%OS = 28{,}058\%$

b) $\omega_n = \sqrt{0{,}04} \rightarrow \omega_n = 0{,}2 \; . \; \zeta = \frac{0{,}02/2}{0{,}2} = 0{,}05 \; . \; T_s = \frac{4}{0{,}2 \cdot 0{,}05} = 400s$. ~~$T_p = \frac{\pi}{0{,}2\sqrt{1-0{,}0}}$~~

$T_p = \frac{\pi}{0{,}2\sqrt{1-2{,}5 \cdot 10^{-3}}} = 15{,}727s \; . \; \omega_d = 0{,}199, \; \sigma = 0{,}01, \; \beta = 1{,}52 rad \therefore T_r = \frac{3{,}14 - 1{,}52}{0{,}199} = 8{,}14s$

$\%OS = e^{(0{,}05\pi/\sqrt{1-2{,}5 \cdot 10^{-3}})} \cdot 100 = 85{,}45\%$

c) $\omega_n = \sqrt{1{,}05 \cdot 10^7} \rightarrow \omega_n = 3240, \; \zeta = \frac{1{,}6 \cdot 10^3/2}{3240} \rightarrow \zeta = 0{,}247 \; . \; T_s = 0{,}005s \; . \; T_p = 0{,}001s$

$\%OS = 44{,}92\%$

L13E3

Subamortecido: Por analise, temos $\%OS = \frac{1{,}4 - 1}{1} \cdot 100\% \therefore \%OS \approx 40\%$

$\zeta = \frac{-\ln(0{,}4)}{\sqrt{\pi^2 + \ln^2(0{,}4)}} = \frac{0{,}916}{\sqrt{\pi^2 + 0{,}839}} = \frac{0{,}916}{3{,}272} = 0{,}279 \approx \zeta$. Se considerar-mos o tempo de pico = 4

então $\omega_n = \frac{\pi}{4 \cdot \sqrt{1 - 0{,}28^2}} \therefore \omega_n = 0{,}818 \; . \; G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \therefore G(s) = \frac{0{,}669}{s^2 + 0{,}456s + 0{,}669}$

Primeira ordem: O valor de 63% é $0{,}63 \cdot 2 = 1{,}26$, que ocorre em aproximadamente $0{,}03s$

onde $a = \frac{1}{0{,}03} = 33{,}33, \; \frac{K}{a} = 2 \therefore K = 66{,}67 \; . \; G(s) = \frac{K}{s+a} \therefore G(s) = \frac{66{,}67}{s + 33{,}33}$ //

L13E2

a) $(5s^2 + 5s + 28)X(s) = F(s) \therefore \frac{X(s)}{F(s)} = G(s) = \frac{1}{5s^2 + 5s + 28}$

b) $\omega_n = \sqrt{28} \therefore \omega_n = 5{,}29 \; . \; \zeta = \frac{a/2}{\omega_n} = \frac{2{,}5}{5{,}29} \therefore \zeta = 0{,}472 \; . \; T_s = \frac{4}{5{,}29 \cdot 0{,}472} \therefore T_s = 1{,}60s$

$T_p = \frac{\pi}{5{,}29\sqrt{1 - 0{,}472^2}} \Rightarrow T_p = 0{,}673s$

L13E5

$$C_1(s) = \frac{26{,}25(s+4)}{s(s+3{,}5)(s+5)(s+6)} = \frac{A}{s} + \frac{B}{s+3{,}5} + \frac{C}{s+5} + \frac{D}{s+6}$$

$$A = C_1(s)\cdot s\Big|_{s=0} = \frac{26{,}25\cdot 4}{3{,}5\cdot 5\cdot 6} = \frac{105}{105} = 1$$

$$B = C_1(s)\cdot(s+3{,}5)\Big|_{s=-3{,}5} = \frac{26{,}25\cdot 0{,}5}{(-3{,}5)(1{,}5)(2{,}5)} = \frac{13{,}125}{-13{,}125} = -1$$

$$C = C_1(s)\cdot(s+5)\Big|_{s=-5} = \frac{-26{,}25}{(-5)\cdot(-1{,}5)\cdot 1} = \frac{-26{,}25}{7{,}5} = -3{,}5$$

$$D = C_1(s)\cdot(s+6)\Big|_{s=-6} = \frac{26{,}25\cdot(-2)}{(-6)\cdot(-2{,}5)\cdot(-1)} = \frac{-52{,}5}{-15} = 3{,}5$$

$C_1(s) = \frac{1}{s} - \frac{1}{s+3{,}5} - \frac{3{,}5}{s+5} + \frac{3{,}5}{s+6}$, onde o polo mais próximo do zero $s=-4$ é $s=-3{,}5$. Contudo, como não são iguais, não tem cancelamento de polos e zeros.

$$C_2(s) = \frac{26{,}25(s+4)}{s(s+4{,}01)(s+5)(s+6)} = \frac{A}{s} + \frac{B}{s+4{,}01} + \frac{C}{s+5} + \frac{D}{s+6}$$

$A = C_2(s)\,s\Big|_{s=0} = \frac{26{,}25\cdot 4}{4{,}01\cdot 5\cdot 6} = \frac{105}{120{,}3} = 0{,}87$. $B = C_2(s)(s+4{,}01)\Big|_{s=-4{,}01} = \frac{-2{,}625}{(-4{,}01)(0{,}99)(1{,}99)} = 0{,}33$

$C = C_2(s)\cdot(s+5)\Big|_{s=-5} = \frac{-26{,}25}{-5\cdot(-0{,}99)(1)} = -5{,}3$. $D = C_2(s)(s+6)\Big|_{s=-6} = \frac{-2\cdot 26{,}25}{(-6)\cdot(-1{,}99)\cdot(-1)} = 4{,}39$

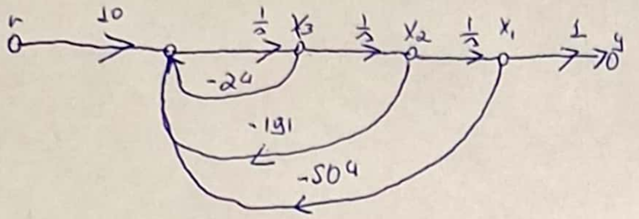
$C_2(s) = \frac{0{,}87}{s} + \frac{0{,}33}{s+4{,}01} - \frac{5{,}3}{s+5} + \frac{4{,}39}{s+6}$, onde o polo mais próximo do zero $s=-4$ é $s=-4{,}01$, podendo ocorrer cancelamento, devido sua proximidade

Portanto: $C_2(s) \cong \frac{0{,}87}{s} - \frac{5{,}3}{s+5} + \frac{4{,}39}{s+6}$ //

L16E3 $G(s) = \dfrac{10}{(s+7)(s+8)(s+9)} = \dfrac{10}{s^3+24s^2+191s+504} = \dfrac{C(s)}{R(s)}$

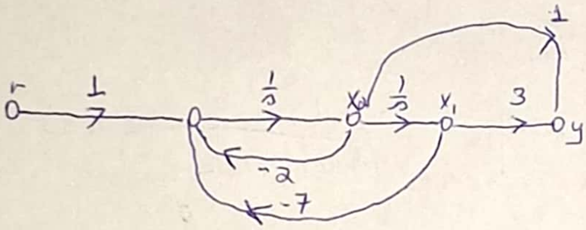
$\therefore \dddot{c} + 24\ddot{c} + 191\dot{c} + 504c = 10r$, se $c = x_1$, $\dot{c} = x_2$, $\ddot{c} = x_3$ $\rightarrow$

$\dot{x}_1 = x_2$
$\dot{x}_2 = x_3$
$\dot{x}_3 = -24x_3 - 191x_2 - 504x_1 + 10r$
$y = x_1$



L16E4 $G(s) = \dfrac{s+3}{s^2+2s+7} = \dfrac{C(s)}{R(s)} \therefore \ddot{c} + 2\dot{c} + 7c = \dot{r} + 3r$, $c = x_1$, $\dot{c} = x_2$ $\rightarrow$

$\dot{x}_1 = x_2$
$\dot{x}_2 = -7x_1 - 2x_2 + r$
$y = 3x_1 + x_2$



$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -7 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$
$$y = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

L16E5 Forma canônica:

$$\dot{x} = \begin{bmatrix} -2 & -7 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$$
$$y \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Dividindo cada termo por $\frac{1}{s^2}$, que é a maior ordem: $G(s) = \dfrac{C(s)}{R(s)} = \dfrac{\frac{1}{s} + \frac{3}{s^2}}{1 + \frac{2}{s} + \frac{7}{s^2}}$

$\rightarrow \left(\frac{1}{s} + \frac{3}{s^2}\right) \cdot R(s) = \left(1 + \frac{2}{s} + \frac{7}{s^2}\right) \cdot C(s) \therefore \frac{1}{s}\left(R(s) - 2C(s)\right) + \frac{1}{s^2}\left(3R(s) - 7C(s)\right) = C(s)$