Chapter 14

Electromagnetic Waves

14.1 INTRODUCTION

Some wave solutions to Maxwell's equations have already been encountered in the Solved Problems of Chapter 13. The present chapter will extend the treatment of electromagnetic waves. Since most regions of interest are free of charge, it will be assumed that charge density $\rho = 0$. Moreover, linear isotropic materials will be assumed, with $\mathbf{D} = \epsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$, and $\mathbf{J} = \sigma\mathbf{E}$.

14.2 WAVE EQUATIONS

With the above assumptions and with time dependence $e^{j\omega t}$ for both $\mathbf{E}$ and $\mathbf{H}$, Maxwell's equations (Table 13-1) become

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon)\mathbf{E} \tag{1}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{2}$$

$$\nabla \cdot \mathbf{E} = 0 \tag{3}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{4}$$

Taking the curl of (*1*) and (*2*),

$$\nabla \times (\nabla \times \mathbf{H}) = (\sigma + j\omega\epsilon)(\nabla \times \mathbf{E})$$

$$\nabla \times (\nabla \times \mathbf{E}) = -j\omega\mu(\nabla \times \mathbf{H})$$

Now, *in cartesian coordinates only*, the Laplacian of a vector

$$\nabla^2\mathbf{A} \equiv (\nabla^2 A_x)\mathbf{a}_x + (\nabla^2 A_y)\mathbf{a}_y + (\nabla^2 A_z)\mathbf{a}_z$$

satisfies the identity

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

Substitution for the "curl curls" and use of (*3*) and (*4*) yields the *vector wave equations*

$$\nabla^2\mathbf{H} = j\omega\mu(\sigma + j\omega\epsilon)\mathbf{H} \equiv \gamma^2\mathbf{H}$$

$$\nabla^2\mathbf{E} = j\omega\mu(\sigma + j\omega\epsilon)\mathbf{E} \equiv \gamma^2\mathbf{E}$$

The *propagation constant* γ is that square root of γ^2 whose real and imaginary parts are positive:

$$\gamma = \alpha + j\beta$$

with

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)} \tag{5}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)} \tag{6}$$

14.3 SOLUTIONS IN CARTESIAN COORDINATES

The familiar scalar wave equation in one dimension,

$$\frac{\partial^2 F}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 F}{\partial t^2}$$

has solutions of the form $F = f(z - ut)$ and $F = g(z + ut)$, where f and g are arbitrary functions. These represent waves traveling with speed u in the $+z$ and $-z$ directions, respectively. In Fig. 14-1 the first solution is shown at $t = 0$ and $t = t_1$; the wave has advanced in the $+z$ direction a distance of ut_1 in the time interval t_1. For the particular choices

$$f(x) = Ce^{-j\omega x/u} \qquad \text{and} \qquad g(x) = De^{+j\omega x/u}$$

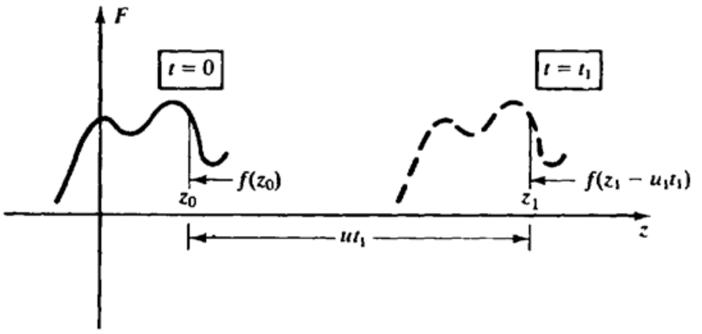


Fig. 14-1

harmonic waves of angular frequency ω are obtained:

$$F = Ce^{j(\omega t - \beta z)} \qquad \text{and} \qquad F = De^{j(\omega t + \beta z)}$$

in which $\beta \equiv \omega/u$. Of course, the real and imaginary parts are also solutions to the wave equation. One of these solutions, $F = C\sin(\omega t - \beta z)$, is shown in Fig. 14-2 at $t = 0$ and $t = \pi/2\omega$. In this interval the wave has advanced in the *positive* z direction a distance $d = u(\pi/2\omega) = \pi/2\beta$. At any fixed t, the waveform repeats itself when x changes by $2\pi/\beta$; the distance

$$\lambda \equiv \frac{2\pi}{\beta}$$

is called the *wavelength*. The wavelength and the *frequency* $f \equiv \omega/2\pi$ enjoy the relation

$$\lambda f = u \qquad \text{or} \qquad \lambda = Tu$$

where $T \equiv 1/f = 2\pi/\omega$ is the *period* of the harmonic wave.

The vector wave equations of Section 14.2 have solutions similar to those just discussed. Because the unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ in cartesian coordinates have fixed directions, the

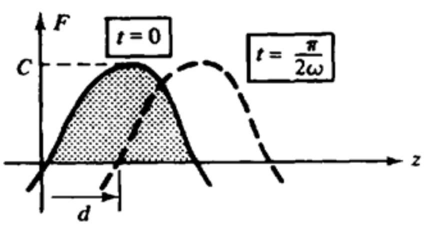


Fig. 14-2

wave equation for **H** can be rewritten in the form

$$\frac{\partial^2 \mathbf{H}}{\partial x^2}+\frac{\partial^2 \mathbf{H}}{\partial y^2}+\frac{\partial^2 \mathbf{H}}{\partial z^2}=\gamma^2\mathbf{H}$$

Of particular interest are solutions (*plane waves*) that depend on only one spatial coordinate, say z. Then the equation becomes

$$\frac{d^2\mathbf{H}}{dz^2}=\gamma^2\mathbf{H}$$

which, for an assumed time dependence $e^{j\omega t}$, is the vector analog of the one-dimensional scalar wave equation. Solutions are as above, in terms of the propagation constant γ.

$$\mathbf{H}(z, t)=H_0 e^{\pm\gamma z}e^{j\omega t}\mathbf{a}_H$$

The corresponding solutions for the electric field are

$$\mathbf{E}(z, t)=E_0 e^{\pm\gamma z}e^{j\omega t}\mathbf{a}_E$$

The fixed unit vectors $\mathbf{a}_H$ and $\mathbf{a}_E$ are orthogonal and neither field has a component in the direction of propagation. This being the case, one can rotate the axes to put one of the fields, say **E**, along the x axis. Then from Maxwell's equation (*2*) it follows that **H** will lie along the $\pm y$ axis for propagation in the $\pm z$ direction.

EXAMPLE 1. Given the field $\mathbf{E}=E_0e^{-\gamma z}\mathbf{a}_E$ (time dependence suppressed), show that **E** can have no component in the propagation direction, $+\mathbf{a}_z$.

The cartesian components of $\mathbf{a}_E$ are found by projection:

$$\mathbf{E}=E_0e^{-\gamma z}[(\mathbf{a}_E\cdot\mathbf{a}_x)\mathbf{a}_x+(\mathbf{a}_E\cdot\mathbf{a}_y)\mathbf{a}_y+(\mathbf{a}_E\cdot\mathbf{a}_z)\mathbf{a}_z]$$

From $\nabla\cdot\mathbf{E}=0$,

$$\frac{\partial}{\partial z}E_0e^{-\gamma z}(\mathbf{a}_E\cdot\mathbf{a}_z)=0$$

which can hold only if $\mathbf{a}_E\cdot\mathbf{a}_z=0$. Consequently, E has no component in $\mathbf{a}_z$.

The plane wave solutions obtained above depend on the properties μ, ϵ, and σ of the medium, because these properties are involved in the propagation constant γ.

14.4 SOLUTIONS FOR PARTIALLY CONDUCTING MEDIA

For a region in which there is some conductivity but not much (e.g., moist earth, seawater), the solution to the wave equation in **E** is taken to be

$$\mathbf{E}=E_0e^{-\gamma z}\mathbf{a}_x$$

Then, from (*2*) of Section 14.2,

$$\mathbf{H}=\sqrt{\frac{\sigma+j\omega\epsilon}{j\omega\mu}}E_0e^{-\gamma z}\mathbf{a}_y$$

The ratio E/H is characteristic of the medium (it is also frequency-dependent). More specifically for waves $\mathbf{E}=E_x\mathbf{a}_x$, $\mathbf{H}=H_y\mathbf{a}_y$ which propagate in the $+z$ direction, the *intrinsic impedance*, η, of the medium is defined by

$$\eta=\frac{E_x}{H_y}$$

Thus

$$\eta=\sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}}$$

where the correct square root may be written in polar form, $|\eta|\,\underline{/\theta}$, with

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt[4]{1+\left(\dfrac{\sigma}{\omega\epsilon}\right)^2}} \qquad \tan 2\theta = \frac{\sigma}{\omega\epsilon} \qquad \text{and} \qquad 0^\circ < \theta < 45^\circ$$

(If the wave propagates in the $-z$ direction, $E_x/H_y = -\eta$. In effect, γ is replaced by $-\gamma$ and the other square root used.)

Inserting the time factor $e^{j\omega t}$ and writing $\gamma = \alpha + j\beta$ results in the following equations for the fields in a partially conducting region:

$$\mathbf{E}(z, t) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \mathbf{a}_x$$

$$\mathbf{H}(z, t) = \frac{E_0}{|\eta|} e^{-\alpha z} e^{j(\omega t - \beta z - \theta)} \mathbf{a}_y$$

The factor $e^{-\alpha z}$ attenuates the magnitudes of both $\mathbf{E}$ and $\mathbf{H}$ as they propagate in the $+z$ direction. The expression for α, (*5*) of Section 14.2, shows that there will be some attenuation unless the conductivity σ is zero, which would be the case only for perfect dielectrics or free space. Likewise, the phase difference θ between $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$ vanishes only when σ is zero.

The velocity of propagation and the wavelength are given by

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\dfrac{\mu\epsilon}{2}\left(\sqrt{1+\left(\dfrac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\dfrac{\mu\epsilon}{2}\left(\sqrt{1+\left(\dfrac{\sigma}{\omega\epsilon}\right)^2}+1\right)}}$$

If the propagation velocity is known, $\lambda f = u$ may be used to determine the wavelength λ. The term $(\sigma/\omega\epsilon)^2$ has the effect of reducing both the velocity and the wavelength from what they would be in either free space or perfect dielectrics, where $\sigma = 0$. Observe that the medium is *dispersive*: waves with different frequencies ω have different velocities u.

14.5 SOLUTIONS FOR PERFECT DIELECTRICS

For a perfect dielectric, $\sigma = 0$, and so

$$\alpha = 0 \qquad \beta = \omega\sqrt{\mu\epsilon} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}\,\underline{/0^\circ}$$

Since $\alpha = 0$, there is no attenuation of the $\mathbf{E}$ and $\mathbf{H}$ waves. The zero angle on η results in $\mathbf{H}$ being in time phase with $\mathbf{E}$ at each fixed location. Assuming $\mathbf{E}$ in $\mathbf{a}_x$ and propagation in $\mathbf{a}_z$, the field equations may be obtained as limits of those in Section 14.4:

$$\mathbf{E}(z, t) = E_0 e^{j(\omega t - \beta z)} \mathbf{a}_x$$

$$\mathbf{H}(z, t) = \frac{E_0}{\eta} e^{j(\omega t - \beta z)} \mathbf{a}_y$$

The velocity and the wavelength are

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}}$$

Solutions in Free Space.

Free space is nothing more than the perfect dielectric for which

$$\mu = \mu_0 = 4\pi \times 10^{-7}\ \text{H/m} \qquad \epsilon = \epsilon_0 = 8.854 \times 10^{-12}\ \text{F/m} \approx \frac{10^{-9}}{36\pi}\ \text{F/m}$$

For free space, $\eta = \eta_0 \approx 120\pi\Omega$ and $u = c \approx 3 \times 10^8$ m/s.

14.6 SOLUTIONS FOR GOOD CONDUCTORS; SKIN DEPTH

Materials are ordinarily classified as good conductors if $\sigma \gg \omega\epsilon$ in the range of practical frequencies. Therefore, the propagation constant and the intrinsic impedance are

$$\gamma = \alpha + j\beta \qquad \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu \sigma} \qquad \eta = \sqrt{\frac{\omega\mu}{\sigma}}\underline{/45^\circ}$$

It is seen that for all conductors the **E** and **H** waves are attenuated. Numerical examples will show that this is a very rapid attenuation. α will always be equal to β. At each fixed location **H** is out of time phase with **E** by 45° or $\pi/4$ rad. Once again assuming **E** in $\mathbf{a}_x$ and propagation in $\mathbf{a}_z$, the field equations are, from Section 14.4,

$$\mathbf{E}(z, t) = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \mathbf{a}_x \qquad \mathbf{H}(z, t) = \frac{E_0}{|\eta|} e^{-\alpha z} e^{j(\omega t - \beta z - \pi/4)} \mathbf{a}_y$$

Moreover,

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}} = \omega\delta \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\pi f \mu \sigma}} = 2\pi\delta$$

The velocity and wavelength in a conducting medium are written here in terms of the *skin depth* or *depth of penetration*,

$$\delta \equiv \frac{1}{\sqrt{\pi f \mu \sigma}}$$

EXAMPLE 2. Assume a field $\mathbf{E} = 1.0e^{-\alpha z} e^{j(\omega t - \beta z)} \mathbf{a}_x$ (V/m), with $f = \omega/2\pi = 100$ MHz, at the surface of a copper conductor, $\sigma = 58$ MS/m, located at $z > 0$, as shown in Fig. 14-3. Examine the attenuation as the wave propagates into the conductor.

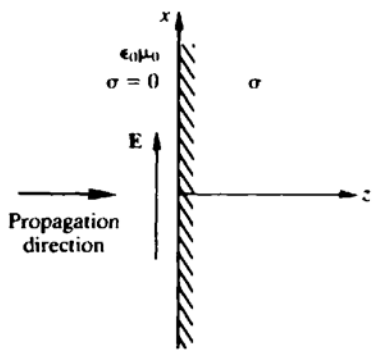


Fig. 14-3

At depth z the magnitude of the field is

$$|\mathbf{E}| = 1.0e^{-\alpha z} = 1.0e^{-z/\delta}$$

where

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.61 \quad \mu\text{m}$$

Thus, after just 6.61 micrometers the field is attenuated to $e^{-1} = 36.8\%$ of its initial value. At 5δ or 33 micrometers, the magnitude is 0.67% of its initial value—practically zero.

14.7 INTERFACE CONDITIONS AT NORMAL INCIDENCE

When a traveling wave reaches an interface between two different regions, it is partly reflected and partly transmitted, with the magnitudes of the two parts determined by the constants of the two regions. In Fig. 14-4, a traveling **E** wave approaches the interface $z = 0$ from region *1*, $z < 0$. $\mathbf{E}^i$ and $\mathbf{E}^r$ are at $z = -0$, while $\mathbf{E}^t$ is at $z = +0$ (in region *2*). Here, *i* signifies "incident," *r* "reflected" and *t* "transmitted." Normal incidence is assumed. The equations for **E** and **H** can be written

$$\mathbf{E}^i(z, t) = E_0^i e^{-\gamma_1 z} e^{j\omega t} \mathbf{a}_x$$
$$\mathbf{E}^r(z, t) = E_0^r e^{\gamma_1 z} e^{j\omega t} \mathbf{a}_x$$
$$\mathbf{E}^t(z, t) = E_0^t e^{-\gamma_2 z} e^{j\omega t} \mathbf{a}_x$$
$$\mathbf{H}^i(z, t) = H_0^i e^{-\gamma_1 z} e^{j\omega t} \mathbf{a}_y$$
$$\mathbf{H}^r(z, t) = H_0^r e^{\gamma_1 z} e^{j\omega t} \mathbf{a}_y$$
$$\mathbf{H}^t(z, t) = H_0^t e^{-\gamma_2 z} e^{j\omega t} \mathbf{a}_y$$

One of the six constants—it is almost always E_0^i—may be taken as real. Under the interface conditions about to be derived, one or more of the remaining five may turn out to be complex.

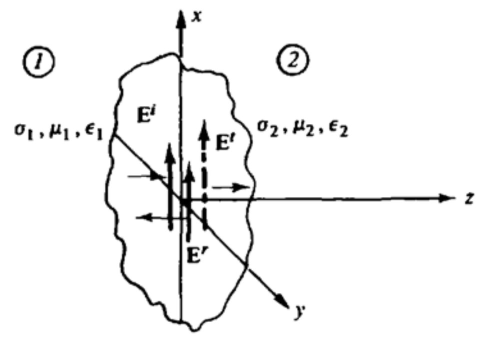


Fig. 14-4

With nominal incidence, **E** and **H** are entirely tangential to the interface, and thus are continuous across it. At $z = 0$ this implies

$$E_0^i + E_0^r = E_0^t \qquad H_0^i + H_0^r = H_0^t$$

Furthermore, the intrinsic impedance in either region is equal to $\pm E_x/H_y$ (see Section 14.4).

$$\frac{E_0^i}{H_0^i} = \eta_1 \qquad \frac{E_0^r}{H_0^r} = -\eta_1 \qquad \frac{E_0^t}{H_0^t} = \eta_2$$

The five equations above can be combined to produce the following ratios in terms of the intrinsic impedances:

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \qquad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$
$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2} \qquad \frac{H_0^t}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

The intrinsic impedances for various materials have been examined earlier. They are repeated here for reference.

partially conducting medium: $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

conducting medium: $\eta = \sqrt{\dfrac{\omega\mu}{\sigma}}\,\underline{/45^\circ}$

perfect dielectric: $\eta = \sqrt{\dfrac{\mu}{\epsilon}}$

free space: $\eta_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 120\pi\,\Omega$

EXAMPLE 3. Traveling **E** and **H** waves in free space (region *1*) are normally incident on the interface with a perfect dielectric (region *2*) for which $\epsilon_r = 3.0$. Compare the magnitudes of the incident, reflected, and transmitted **E** and **H** waves at the interface.

$$\eta_1 = \eta_0 = 120\pi\ \Omega \qquad \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 217.7\ \Omega$$

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = -0.268 \qquad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = 0.268$$

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2} = 0.732 \qquad \frac{H_0^t}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2} = 1.268$$

14.8 OBLIQUE INCIDENCE AND SNELL'S LAWS

An incident wave that approaches a plane interface between two different media generally will result in a transmitted wave in the second medium and a reflected wave in the first. The *plane of incidence* is the plane containing the incident wave normal and the local normal to the interface; in Fig. 14-5 this is the xz plane. The normals to the reflected and transmitted waves also lie in the plane of incidence. The *angle of incidence* θ_i, the *angle of reflection* θ_r, and the *angle of transmission* θ_t—all defined as in Fig. 14-5—obey *Snell's law of reflection,*

$$\theta_i = \theta_r$$

and *Snell's law of refraction,*

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}$$

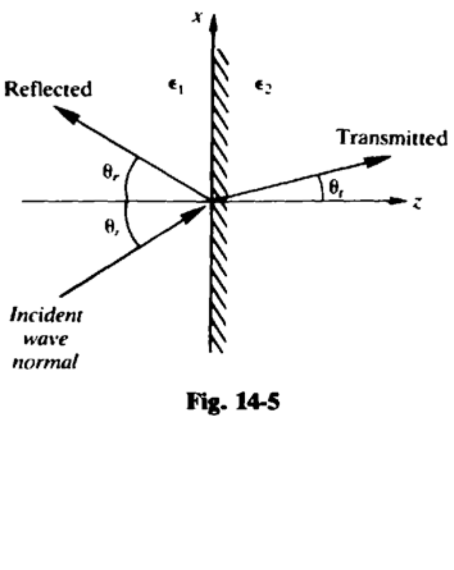


Fig. 14-5

EXAMPLE 4. A wave is incident at an angle of 30° from air to teflon, $\epsilon_r = 2.1$. Calculate the angle of transmission, and repeat with an interchange of the regions.

Since $\mu_1 = \mu_2$,

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sin 30^\circ}{\sin\theta_t} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \sqrt{2.1} \qquad \text{or} \qquad \theta_t = 20.18^\circ$$

From teflon to air,

$$\frac{\sin 30^\circ}{\sin\theta_t} = \frac{1}{\sqrt{2.1}} \qquad \text{or} \qquad \theta_t = 46.43^\circ$$

Supposing both media of the same permeability, propagation from the optically denser medium $(\epsilon_1 > \epsilon_2)$ results in $\theta_t > \theta_i$. As θ_i increases, an angle of incidence will be reached that results in $\theta_t = 90^\circ$. At this *critical angle* of incidence, instead of a wave being transmitted into the second medium there will be a wave that propagates along the surface. The critical angle is given by

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

EXAMPLE 5. The critical angle for a wave propagating from teflon into free space is

$$\theta_c = \sin^{-1}\frac{1}{\sqrt{2.1}} = 43.64^\circ$$

14.9 PERPENDICULAR POLARIZATION

The orientation of the electric field **E** with respect to the plane of incidence determines the *polarization* of a wave at the interface between two different regions. In *perpendicular* polarization **E** is perpendicular to the plane of incidence (the xz plane in Fig. 14-6) and is thus parallel to the (planar) interface. At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Note that for normal incidence $\theta_i = \theta_t = 0^\circ$ and the expressions reduce to those found in Section 14.8.

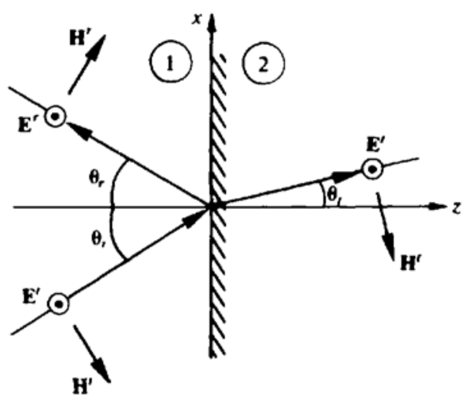


Fig. 14-6

It is not difficult to show that, if $\mu_1 = \mu_2$,

$$\eta_2 \cos \theta_i - \eta_1 \cos \theta_t \neq 0 \qquad \text{for any } \theta_i$$

Hence, a perpendicularly polarized incident wave suffers either partial or total reflection.

14.10 PARALLEL POLARIZATION

For *parallel* polarization the electric field vector **E** lies entirely within the plane of incidence, the xz plane as shown in Fig. 14-7. (Thus **E** assumes the role played by **H** in perpendicular polarization.) At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_1 \cos \theta_i + \eta_2 \cos \theta_t}$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2 \cos \theta_i}{\eta_1 \cos \theta_i - \eta_2 \cos \theta_t}$$

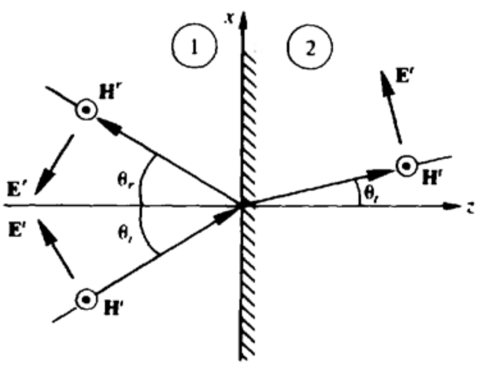


Fig. 14-7

In contrast to perpendicular polarizations, if $\mu_1 = \mu_2$ there will be a particular angle of incidence for which there is no reflected wave. This *Brewster angle* is given by

$$\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

EXAMPLE 6. The Brewster angle for a parallel-polarized wave traveling from air into glass for which $\epsilon_r = 5.0$ is

$$\theta_B = \tan^{-1} \sqrt{5.0} = 65.91°$$

14.11 STANDING WAVES

When waves traveling in a perfect dielectric ($\sigma_1 = \alpha_1 = 0$) are normally incident on the interface with a perfect conductor ($\sigma_2 = \infty$, $\eta_2 = 0$), the reflected wave in combination with the incident wave produces a *standing wave*. In such a wave, which is readily demonstrated on a clamped taut string, the oscillations at all points of a half-wavelength interval are in time phase. The combination of incident and reflected waves may be written

$$\mathbf{E}(z, t) = [E_0^i e^{j(\omega t - \beta z)} + E_0^r e^{j(\omega t + \beta z)}]\mathbf{a}_x = e^{j\omega t}(E_0^i e^{-j\beta z} + E_0^r e^{j\beta z})\mathbf{a}_x$$

Since $\eta_2 = 0$, $E_0^r/E_0^i = -1$ and

$$\mathbf{E}(z, t) = e^{j\omega t}(E_0^i e^{-j\beta z} - E_0^i e^{j\beta z})\mathbf{a}_x = -2jE_0^i \sin \beta z e^{j\omega t}\mathbf{a}_x$$

or, taking the real part,

$$\mathbf{E}(z, t) = 2E_0^i \sin \beta z \sin \omega t \mathbf{a}_x$$

The standing wave is shown in Fig. 14-8 at time intervals of $T/8$, where $T = 2\pi/\omega$ is the period. At $t = 0$, $\mathbf{E} = \mathbf{0}$ everywhere; at $t = 1(T/8)$, the endpoints of the $\mathbf{E}$ vectors lie on sine curve *1*; at $t = 2(T/8)$, they lie on sine curve *2*; and so forth. Sine curves *2* and *6* form an envelope for the oscillations; the amplitude of this envelope is twice the amplitude of the incident wave. Note that adjacent half-wavelength segments are 180° out of phase with each other.

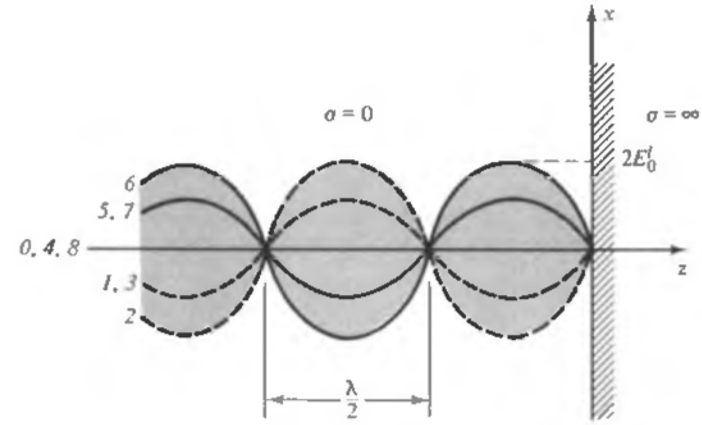


Fig. 14-8

14.12 POWER AND THE POYNTING VECTOR

Maxwell's first equation for a region with conductivity σ is written and then $\mathbf{E}$ is dotted with each term.

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{E} \cdot (\nabla \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

where, as usual, $E^2 = \mathbf{E} \cdot \mathbf{E}$. The vector identity $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$ is employed to change the left side of the equation.

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) - \nabla \cdot (\mathbf{E} \times \mathbf{H}) = \sigma E^2 + \mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t}$$

By Maxwell's second equation,

$$\mathbf{H} \cdot (\nabla \times \mathbf{E}) = \mathbf{H} \cdot \left(-\mu \frac{\partial \mathbf{H}}{\partial t}\right) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

Similarly,

$$\mathbf{E} \cdot \epsilon \frac{\partial \mathbf{E}}{\partial t} = \frac{\epsilon}{2} \frac{\partial E^2}{\partial t}$$

Substituting, and rearranging terms,

$$\sigma E^2 = -\frac{\epsilon}{2} \frac{\partial E^2}{\partial t} - \frac{\mu}{2} \frac{\partial H^2}{\partial t} - \nabla \cdot (\mathbf{E} \times \mathbf{H})$$

Integration of this equation throughout an arbitrary volume v gives

$$\int_v \sigma E^2\,dv = -\int_v \left(\frac{\epsilon}{2}\frac{\partial E^2}{\partial t} + \frac{\mu}{2}\frac{\partial H^2}{\partial t}\right) dv - \oint_S (\mathbf{E}\times\mathbf{H})\cdot d\mathbf{S}$$

where the last term has been converted to an integral over the surface of v by use of the divergence theorem.

The integral on the left has the units of watts and is the usual ohmic term representing energy dissipated per unit time in heat. This dissipated energy has its source in the integrals on the right. Because $\epsilon E^2/2$ and $\mu H^2/2$ are the densities of energy stored in the electric and magnetic fields, respectively, the volume integral (including the minus sign) gives the decrease in this stored energy. Consequently, the surface integral (including the minus sign) must be the rate of energy entering the volume from outside. A change of sign then produces the *instantaneous rate of energy leaving the volume*:

$$P(t) = \oint_S (\mathbf{E}\times\mathbf{H})\cdot s\mathbf{S} = \oint_S \mathcal{P}\cdot d\mathbf{S}$$

where $\mathcal{P} = \mathbf{E}\times\mathbf{H}$ is the *Poynting vector*, the instantaneous rate of energy flow per unit area at a point.

In the cross product that defines the Poynting vector, the fields are supposed to be in real form. If, instead, $\mathbf{E}$ and $\mathbf{H}$ are expressed in complex form and have the common time-dependence $e^{j\omega t}$, then the time-average of $\mathcal{P}$ is given by

$$\mathcal{P}_{avg} = \tfrac{1}{2}\,\mathrm{Re}\,(\mathbf{E}\times\mathbf{H}^*)$$

where $\mathbf{H}^*$ is the complex conjugate of $\mathbf{H}$. This follows the *complex power* of circuit analysis, $\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$, of which the power is the real part, $P = \frac{1}{2}\,\mathrm{Re}\,\mathbf{V}\mathbf{I}^*$.

For plane waves, the direction of energy flow is the direction of propagation. Thus the Poynting vector offers a useful, coordinate-free way of specifying the direction of propagation, or of determining the directions of the fields if the direction of propagation is known. This can be particularly valuable where incident, transmitted, and reflected waves are being examined.

Solved Problems

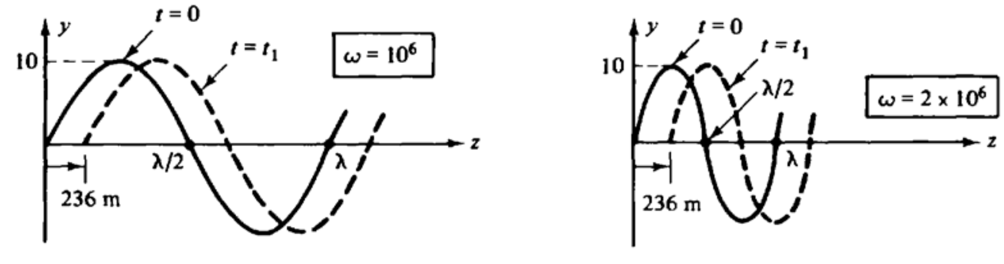
14.1. A traveling wave is described by $y = 10\sin(\beta z - \omega t)$. Sketch the wave at $t = 0$ and at $t = t_1$, when it has advanced $\lambda/8$, if the velocity is 3×10^8 m/s and the angular frequency $\omega = 10^6$ rad/s. Repeat for $\omega = 2\times10^6$ rad/s and the same t_1.

The wave advances λ in one period, $T = 2\pi/\omega$. Hence

$$t_1 = \frac{T}{8} = \frac{\pi}{4\omega}$$

$$\frac{\lambda}{8} = ct_1 = (3\times10^8)\frac{\pi}{4(10^6)} = 236\text{ m}$$

The wave is shown at $t = 0$ and $t = t_1$ in Fig. 14-9(a). At twice the frequency, the wavelength λ is one-half, and the phase shift constant β is twice, the former value. See Fig. 14-9(b). At t_1 the wave has also advanced 236 m, but this distance is now $\lambda/4$.



(a) (b)

Fig. 14-9

14.2. In free space, $\mathbf{E}(z, t) = 10^3 \sin(\omega t - \beta z)\mathbf{a}_y$ (V/m). Obtain $\mathbf{H}(z, t)$.

Examination of the phase, $\omega t - \beta z$, shows that the direction of propagation is $+z$. Since $\mathbf{E} \times \mathbf{H}$ must also be in the $+z$ direction, $\mathbf{H}$ must have the direction $-\mathbf{a}_x$. Consequently,

$$\frac{E_y}{-H_x} = \eta_0 = 120\pi\,\Omega \qquad \text{or} \qquad H_x = -\frac{10^3}{120\pi}\sin(\omega t - \beta z) \quad (\text{A/m})$$

and

$$\mathbf{H}(z, t) = -\frac{10^3}{120\pi}\sin(\omega t - \beta z)\mathbf{a}_x \quad (\text{A/m})$$

14.3. For the wave of Problem 14.2 determine the propagation constant γ, given that the frequency is $f = 95.5$ MHz.

In general, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$. In free space, $\sigma = 0$, so that

$$\gamma = j\omega\sqrt{\mu_0\epsilon_0} = j\left(\frac{2\pi f}{c}\right) = j\frac{2\pi(95.5 \times 10^6)}{3 \times 10^8} = j(2.0)\ \text{m}^{-1}$$

Note that this result shows that the attenuation factor is $\alpha = 0$ and the phase-shift constant is $\beta = 2.0$ rad/m.

14.4. Examine the field

$$\mathbf{E}(z, t) = 10\sin(\omega t + \beta z)\mathbf{a}_x + 10\cos(\omega t + \beta z)\mathbf{a}_y$$

in the $z = 0$ plane, for $\omega t = 0, \pi/4, \pi/2, 3\pi/4$ and π.

The computations are presented in Table 14-1.

Table 14-1

ωt	$E_x = 10\sin\omega t$	$E_y = \cos\omega t$	$\mathbf{E} = E_x\mathbf{a}_x + E_y\mathbf{a}_y$
0	0	10	$10\mathbf{a}_y$
$\frac{\pi}{4}$	$\frac{10}{\sqrt{2}}$	$\frac{10}{\sqrt{2}}$	$10\left(\frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}\right)$
$\frac{\pi}{2}$	10	0	$10\mathbf{a}_x$
$\frac{3\pi}{4}$	$\frac{10}{\sqrt{2}}$	$\frac{-10}{\sqrt{2}}$	$10\left(\frac{\mathbf{a}_x + a_y}{\sqrt{2}}\right)$
π	0	-10	$10(-\mathbf{a}_y)$

As shown in Fig. 14-10, $\mathbf{E}(x, t)$ is circularly polarized. In addition, the wave travels in the $-\mathbf{a}_z$ direction.

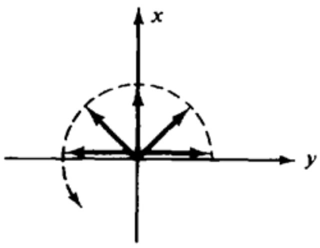


Fig. 14-10

14.5. An H field travels in the $-\mathbf{a}_z$ direction in free space with a phaseshift constant of 30.0 rad/m and an amplitude of $(1/3\pi)$ A/m. If the field has the direction $-\mathbf{a}_y$ when $t=0$ and $z=0$, write suitable expressions for E and H. Determine the frequency and wavelength.

In a medium of conductivity σ, the intrinsic impedance η, which relates E and H, would be complex, and so the phase of $\mathbf{E}$ and $\mathbf{H}$ would have to be written in complex form. In free space this restriction is unnecessary. Using cosines, then

$$\mathbf{H}(z, t) = -\frac{1}{3\pi}\cos(\omega t + \beta z)\mathbf{a}_y$$

For propagation in $-z$,

$$\frac{E_x}{H_y} = -\eta_0 = -120\pi\ \Omega \qquad \text{or} \qquad E_x = +40\cos(\omega t + \beta z) \quad (\text{V/m})$$

Thus
$$\mathbf{E}(z, t) = 40\cos(\omega t + \beta z)\mathbf{a}_x \quad (\text{V/m})$$

Since $\beta = 30$ rad/m,

$$\lambda = \frac{2\pi}{\beta} = \frac{\pi}{15}\ \text{m} \qquad f = \frac{c}{\lambda} = \frac{3\times 10^8}{\pi/15} = \frac{45}{\pi}\times 10^8\ \text{Hz}$$

14.6. Determine the propagation constant γ for a material having $\mu_r = 1$, $\epsilon_r = 8$, and $\sigma = 0.25$ pS/m, if the wave frequency is 1.6 MHz.

In this case,

$$\frac{\sigma}{\omega\epsilon} = \frac{0.25\times 10^{-12}}{2\pi(1.6\times 10^6)(8)(10^{-9}/36\pi)} \approx 10^{-9} \approx 0$$

so that

$$\alpha \approx 0 \qquad \beta \approx \omega\sqrt{\mu\epsilon} = 2\pi f\frac{\sqrt{\mu_r\epsilon_r}}{c} = 9.48\times 10^{-2}\ \text{rad/m}$$

and $\gamma = \alpha + j\beta \approx j9.48\times 10^{-2}\ \text{m}^{-1}$. The material behaves like a perfect dielectric at the given frequency. Conductivity of the order of 1 pS/m indicates that the material is more like an insulator than a conductor.

14.7. Determine the conversion factor between the neper and the decibel.

Consider a plane wave traveling in the $+z$ direction whose amplitude decays according to

$$E = E_0 e^{-\alpha z}$$

From Section 14.12, the power carried by the wave is proportional to E^2, so that

$$P = P_0 e^{-2\alpha z}$$

Then, by definition of the decibel, the power drop over the distance z is $10 \log_{10}(P_0/P)$ dB. But

$$10 \log_{10} \frac{P_0}{P} = \frac{10}{2.3026} \ln \frac{P_0}{P} = \frac{20}{2.3026}(\alpha z) = 8.686(\alpha z)$$

Thus, αz nepers is equivalent to $8.686(\alpha z)$ decibels; i.e.,

$$1\ \text{Np} = 8.686\ \text{dB}$$

14.8. At what frequencies may earth be considered a perfect dielectric, if $\sigma = 5 \times 10^{-3}$ S/m, $\mu_r = 1$, and $\epsilon_r = 8$? Can α be assumed zero at these frequencies?

Assume arbitrarily that

$$\frac{\sigma}{\omega\epsilon} \le \frac{1}{100}$$

marks the cutoff. Then

$$f = \frac{\omega}{2\pi} \ge \frac{100\sigma}{2\pi\epsilon} = 1.13\ \text{GHz}$$

For small $\sigma/\omega\epsilon$,

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$$\approx \omega\sqrt{\frac{\mu\epsilon}{2}\left[\frac{1}{2}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu_r}{\epsilon_r}}(120\pi) = 0.333\ \text{Np/m}$$

Thus, no matter how high the frequency, α will be about 0.333 Np/m, or almost 3 db/m (see Problem 14.7); α cannot be assumed zero.

14.9. Find the skin depth δ at a frequency of 1.6 MHz in aluminum, where $\sigma = 38.2$ MS/m and $\mu_r = 1$. Also find γ and the wave velocity u.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 6.44 \times 10^{-5}\ \text{m} = 64.4\ \mu\text{m}$$

Because $\alpha = \beta = \delta^{-1}$,

$$\gamma = 1.55 \times 10^4 + j1.55 \times 10^4 = 2.20 \times 10^4 \underline{/45^\circ}\ (\text{m}^{-1})$$

and

$$u = \frac{\omega}{\beta} = \omega\delta = 647\ (\text{m/s})$$

14.10. A perpendicularly polarized wave propagates from region *1* ($\epsilon_{r1} = 8.5$, $\mu_{r1} = 1$, $\sigma_1 = 0$) to region *2*, free space, with an angle of incidence of 15°. Given $E_0^i = 1.0\ \mu$V/m, find: E_0^r, E_0^t, H_0^i, H_0^r, and H_0^t.

The intrinsic impedances are

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r1}}} = \frac{120}{\sqrt{8.5}} = 129\ \Omega \qquad \text{and} \qquad \eta_2 = \eta_0 = 120\pi\ \Omega$$

and the angle of transmission is given by

$$\frac{\sin 15^\circ}{\sin \theta_t} = \sqrt{\frac{\epsilon}{8.5\epsilon_0}} \qquad \text{or} \qquad \theta_t = 48.99^\circ$$

Then

$$\frac{E_0^r}{E_0^i}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}=0.623 \qquad \text{or} \qquad E_0^r=0.623\ \mu\text{V/m}$$

$$\frac{E_0^t}{E_0^i}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t}=1.623 \qquad \text{or} \qquad E_0^t=1.623\ \mu\text{V/m}$$

Finally, $H_0^i=E_0^i/\eta_1=7.75$ nA/m, $H_0^r=4.83$ nA/m, and $H_0^t=4.31$ nA/m.

14.11. Calculate the intrinsic impedance η, the propagation constant γ, and the wave velocity u for a conducting medium in which $\sigma=58$ MS/m, $\mu_r=1$, at a frequency $f=100$ MHz.

$$\gamma=\sqrt{\omega\mu\sigma}\,\underline{/45^\circ}=2.14\times10^5\,\underline{/45^\circ}\ \text{m}^{-1}$$

$$\eta=\sqrt{\frac{\omega\mu}{\sigma}}\,\underline{/45^\circ}=3.69\times10^{-3}\,\underline{/45^\circ}\ \Omega$$

$$\alpha=\beta=1.51\times10^5 \qquad \delta=\frac{1}{\alpha}=6.61\ \mu\text{m} \qquad u=\omega\delta=4.15\times10^3\ \text{m/s}$$

14.12. A plane wave traveling in the $+z$ direction in free space $(z<0)$ is normally incident at $z=0$ on a conductor $(z>0)$ for which $\sigma=61.7$ MS/m, $\mu_r=1$. The free-space **E** wave has a frequency $f=1.5$ MHz and an amplitude of 1.0 V/m; at the interface it is given by

$$\mathbf{E}(0,t)=1.0\sin 2\pi ft\,\mathbf{a}_y \quad (\text{V/m})$$

Find $\mathbf{H}(z,t)$ for $z>0$.

For $z>0$, and in complex form,

$$\mathbf{E}(z,t)=1.0e^{-\alpha z}e^{j(2\pi ft-\beta z)}\mathbf{a}_y \quad (\text{V/m})$$

where the imaginary part will ultimately be taken. In the conductor,

$$\alpha=\beta=\sqrt{\pi f\mu\sigma}=\sqrt{\pi(1.5\times10^6)(4\pi\times10^{-7})(61.7\times10^6)}=1.91\times10^4$$

$$\eta=\sqrt{\frac{\omega\mu}{\sigma}}\,\underline{/45^\circ}=4.38\times10^{-4}e^{j\pi/4}$$

Then, since $E_y/(-H_x)=\eta$,

$$\mathbf{H}(z,t)=-2.28\times10^3e^{-\alpha z}e^{j(2\pi ft-\beta z-\pi/4)}\mathbf{a}_x \quad (\text{A/m})$$

or, taking the imaginary part,

$$\mathbf{H}(z,t)=-2.28\times10^3e^{-\alpha z}\sin(2\pi ft-\beta z-\pi/4)\mathbf{a}_x \quad (\text{A/m})$$

where f, α, and β are as given above.

14.13. In free space $\mathbf{E}(z,t)=50\cos(\omega t-\beta z)\mathbf{a}_x$ (V/m). Find the average power crossing a circular area of radius 2.5 m in the plane $z=\text{const.}$

In complex form,

$$\mathbf{E}=50e^{j(\omega t-\beta z)}\mathbf{a}_x \quad (\text{V/m})$$

and since $\eta=120\pi\ \Omega$ and propagation is in $+z$,

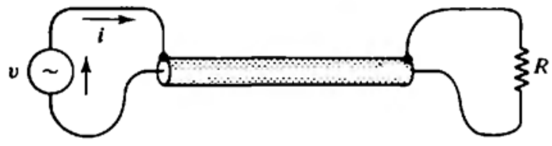
$$\mathbf{H}=\frac{5}{12\pi}e^{j(\omega t-\beta z)}\mathbf{a}_y \quad (\text{A/m})$$

Then $$\mathcal{P}_{\text{avg}}=\tfrac{1}{2}\,\text{Re}\,(\mathbf{E}\times\mathbf{H}^*)=\tfrac{1}{2}(50)\left(\frac{5}{12\pi}\right)\mathbf{a}_z\ \text{W/m}^2$$

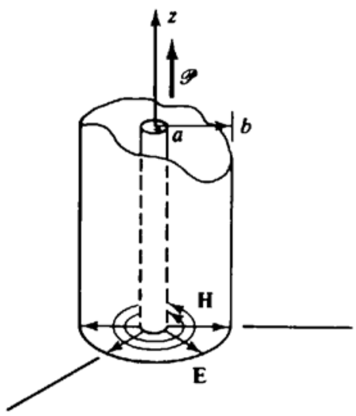
The flow is normal to the area, and so

$$P_{avg} = \tfrac{1}{2}(50)\left(\frac{5}{12\pi}\right)(2.5)^2 = 65.1 \text{ W}$$

14.14 A voltage source, v, is connected to a pure resistor R by a length of coaxial cable, as shown in Fig. 14-11(a). Show that use of the Poynting vector $\mathcal{P}$ in the dielectric leads to the same instantaneous power in the resistor as methods of circuit analysis.



(a)



(b)

Fig. 14-11

From Problem 7.9 and Ampère's law,

$$\mathbf{E} = \frac{v}{r \ln(b/a)}\mathbf{a}_r \qquad \text{and} \qquad \mathbf{H} = \frac{i}{2\pi r}\mathbf{a}_\phi$$

where a and b are the radii of the inner and outer conductors, as shown in Fig. 14-11(b). Then

$$\mathcal{P} = \mathbf{E} \times \mathbf{H} = \frac{vi}{2\pi r^2 \ln(b/a)}\mathbf{a}_z$$

This is the instantaneous power density. The total instantaneous power over the cross section of the dielectric is

$$P(t) = \int_0^{2\pi}\int_a^b \frac{vi}{2\pi r^2 \ln(b/a)}\mathbf{a}_z \cdot r\,dr\,d\phi\,\mathbf{a}_z = vi$$

which is also the circuit-theory result for the instantaneous power loss in the resistor.

14.15. Determine the amplitudes of the reflected and transmitted **E** and **H** at the interface shown in

Fig. 14-12, if $E_0^i = 1.5 \times 10^{-3}$ V/m in region *1*, in which $\epsilon_{r1} = 8.5$, $\mu_{r1} = 1$, and $\sigma_1 = 0$. Region *2* is free space. Assume normal incidence.

$$\eta_1 = \sqrt{\frac{\mu_0 \mu_{r1}}{\epsilon_0 \epsilon_{r1}}} = 129\ \Omega \qquad \eta_2 = 120\pi\ \Omega = 377\ \Omega$$

$$E_0^r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i = 7.35 \times 10^{-4}\ \text{V/m}$$

$$E_0^t = \frac{2\eta_2}{\eta_2 + \eta_1} E_0^i = 2.24 \times 10^{-3}\ \text{V/m}$$

$$H_0^i = \frac{E_0^i}{\eta_1} = 1.16 \times 10^{-5}\ \text{A/m}$$

$$H_0^r = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} H_0^i = -5.69 \times 10^{-6}\ \text{A/m}$$

$$H_0^t = \frac{2\eta_1}{\eta_1 + \eta_2} H_0^i = 5.91 \times 10^{-6}\ \text{A/m}$$

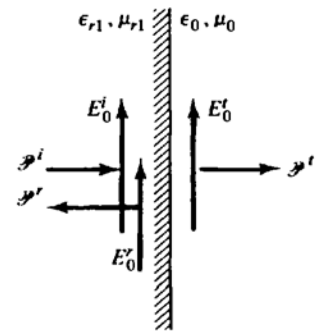


Fig. 14-12

14.16. The amplitude of $\mathbf{E}^i$ in free space (region *1*) at the interface with region *2* is 1.0 V/m. If $H_0^r = -1.41 \times 10^{-3}$ A/m, $\epsilon_{r2} = 18.5$ and $\sigma_2 = 0$, find μ_{r2}.

From

$$\frac{E_0^r}{H_0^r} = -120\pi\ \Omega = -377\ \Omega \qquad \text{and} \qquad \frac{E_0^r}{E_0^i} = \frac{\eta_2 - 377}{377 + \eta_2}$$

$$\frac{E_0^i}{H_0^r} = \frac{1.0}{-1.41 \times 10^{-3}} = \frac{-377(377 + \eta_2)}{\eta_2 - 377} \qquad \text{or} \qquad \eta_2 = 1234\ \Omega$$

Then

$$1234 = \sqrt{\frac{\mu_0 \mu_{r2}}{\epsilon_0 (18.5)}} \qquad \text{or} \qquad \mu_{r2} = 198.4$$

14.17. A normally incident **E** field has amplitude $E_0^i = 1.0$ V/m in free space just outside of seawater in which $\epsilon_r = 80$, $\mu_r = 1$, and $\sigma = 2.5$ S/m. For a frequency of 30 MHz, at what depth will the amplitude of **E** be 1.0 mV/m?

Mathcad

Let the free space be region *1* and the seawater be region *2*.

$$\eta_1 = 377\ \Omega \qquad \eta_2 = 9.73\,\underline{/43.5^\circ}\ \Omega$$

Then the amplitude of **E** just inside the seawater is E_0^t.

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2} \qquad \text{or} \qquad E_0^t = 5.07 \times 10^{-2}\ \text{V/m}$$

From $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = 24.36\underline{/46.53^\circ}\ \mathrm{m}^{-1}$.

$$\alpha = 24.36\cos 46.53^\circ = 16.76\ \mathrm{Np/m}$$

Then, from

$$1.0 \times 10^{-3} = (5.07 \times 10^{-2})e^{-16.76z}$$

$z = 0.234$ m.

14.18. A traveling **E** field in free space, of amplitude 100 V/m, strikes a sheet of silver of thickness 5 μm, as shown in Fig. 14-13. Assuming $\sigma = 61.7$ MS/m and a frequency $f =$ 200 MHz, find the amplitudes $|E_2|$, $|E_3|$, and $|E_4|$.

Mathcad

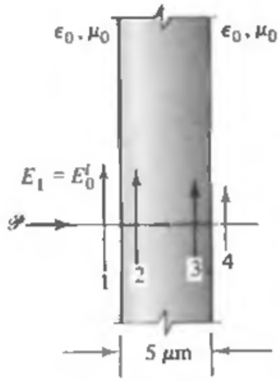


Fig. 14-13

For the silver at 200 MHz, $\eta = 5.06 \times 10^{-3}\underline{/45^\circ}\ \Omega$.

$$\frac{E_2}{E_1} = \frac{2(5.06 \times 10^{-3}\underline{/45^\circ})}{377 + 5.06 \times 10^{-3}\underline{/45^\circ}} \qquad \text{whence} \qquad |E_2| \approx 2.68 \times 10^{-3}\ \mathrm{V/m}$$

Within the conductor,

$$\alpha = \beta = \sqrt{\pi f \mu \sigma} = 2.21 \times 10^5$$

Thus, in addition to attenuation there is phase shift as the wave travels through the conductor. Since $|E_3|$ and $|E_4|$ represent maximum values of the sinusoidally varying wave, this phase shift is not involved.

$$|E_3| = |E_2|\,e^{-\alpha z} = (2.68 \times 10^{-3})e^{-(2.21\times 10^5)(5\times 10^{-6})} = 8.88 \times 10^{-4}\ \mathrm{V/m}$$

and

$$\frac{E_4}{E_3} = \frac{2(377)}{377 \times 5.06 \times 10^{-3}\underline{/45^\circ}} \qquad \text{whence} \qquad |E_4| \approx 1.78 \times 10^{-3}\ \mathrm{V/m}$$

Supplementary Problems

14.19. Given

$$\mathbf{E}(z, t) = 10^3 \sin(6 \times 10^8 t - \beta z)\mathbf{a}_y \quad (\mathrm{V/m})$$

in free space, sketch the wave at $t = 0$ and at time t_1 when it has traveled $\lambda/4$ along the z axis. Find t_1, β, and λ. *Ans.* $t_1 = 2.62\ ns$, $\beta = 2$ rad/m, $\lambda = \pi$m. See Fig. 14-14.

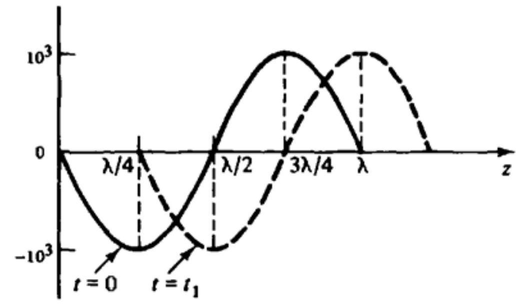


Fig. 14-14

14.20. In free space,

$$\mathbf{H}(z, t) = 1.0e^{j(1.5\times10^8 t+\beta z)}\mathbf{a}_x \quad (\text{A/m})$$

Obtain an expression for $\mathbf{E}(z, t)$ and determine the propagation direction.
Ans. $E_0 = 377\ \text{V/m}$, $-\mathbf{a}_z$

14.21. In free space,

$$\mathbf{H}(z, t) = 1.33\times10^{-1}\cos(4\times10^7 t) - \beta z)\mathbf{a}_x \quad (\text{A/m})$$

Obtain an expression for $\mathbf{E}(z, t)$. Find β and λ. *Ans.* $E_0 = 50\ \text{V/m}$, $(\frac{4}{30})$ rad/m, 15π m

14.22. A traveling wave has a velocity of 10^6 m/s and is described by

$$y = 10\cos(2.5z + \omega t)$$

Sketch the wave as a function of z at $t = 0$ and $t = t_1 = 0.838\ \mu s$. What fraction of a wavelength is traveled between these two times? *Ans.* $\frac{1}{3}$. See Fig. 14.15.

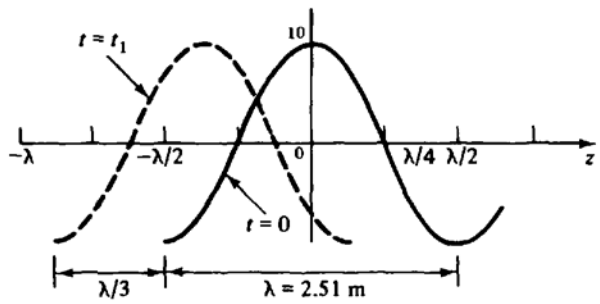


Fig. 14-15

14.23. Find the magnitude and direction of

$$\mathbf{E}(z, t) = 10\sin(\omega t - \beta z)\mathbf{a}_x - 15\sin(\omega t - \beta z)\mathbf{a}_y \quad (\text{V/m})$$

at $t = 0$, $z = 3\lambda/4$. Ans. 18.03 V/m, $0.555\mathbf{a}_x - 0.832\mathbf{a}_y$

14.24. Determine γ at 500 kHz for a medium in which $\mu_r = 1$, $\epsilon_r = 15$, $\sigma = 0$. At what velocity will an electromagnetic wave travel in this medium? *Ans.* $j4.06\times10^{-2}\ \text{m}^{-1}$, 7.74×10^7 m/s

14.25. An electromagnetic wave in free space has a wavelength of 0.20 m. When this same wave enters a perfect dielectric, the wavelength changes to 0.09 m. Assuming that $\mu_r = 1$, determine ϵ_r and the wave velocity in the dielectric. Ans. 4.94, 1.35×10^8 m/s

14.26. An electromagnetic wave in free space has a phase shift constant of 0.524 rad/m. The same wave has a phase shift constant of 1.81 rad/m upon entering a perfect dielectric. Assuming that $\mu_r = 1$, find ϵ_r and the velocity of propagation. *Ans.* 11.9, 8.69×10^7 m/s

14.27. Find the propagation constant at 400 MHz for a medium in which $\epsilon_r = 16$, $\mu_r = 4.5$, and $\sigma = 0.6$ S/m. Find the ratio of the velocity v to the free-space velocity c.
Ans. $99.58\underline{/60.34^\circ}\ \text{m}^{-1}$, 0.097

14.28. In a partially conducting medium, $\epsilon_r = 18.5$, $\mu_r = 800$, and $\sigma = 1$ S/m. Find α, β, η, and the velocity u, for a frequency of 10^9 Hz. Determine $\mathbf{H}(z, t)$, given

$$\mathbf{E}(z, t) = 50.0e^{-\alpha z} \cos(\omega t - \beta z)\mathbf{a}_y \quad (\text{V/m})$$

Ans. 1130 Np/m, 2790 rad/m, $2100\underline{/22.1^\circ}\ \Omega$, 2.25×10^6 m/s,
$2.38 \times 10^{-2} e^{-\alpha z} \cos(\omega t - 0.386 - \beta z)(-\mathbf{a}_x)$ (A/m)

14.29. For silver, $\sigma = 3.0$ MS/m. At what frequency will the depth of penetration δ be 1 mm?
Ans. 84.4 kHz

14.30. At a certain frequency in copper ($\sigma = 58.0$ MS/m) the phase shift constant is 3.71×10^5 rad/m. Determine the frequency. *Ans.* 601 MHz

14.31. The amplitude of **E** just inside a liquid is 10.0 V/m and the constants are $\mu_r = 1$, $\epsilon_r = 20$, and $\sigma = 0.50$ S/m. Determine the amplitude of **E** at a distance of 10 cm inside the medium for frequencies of (*a*) 5 MHz, (*b*) 50 MHz, and (*c*) 500 MHz. *Ans.* (*a*) 7.32 V/m; (*b*) 3.91 V/m; (*c*) 1.42 V/m

14.32. In free space, $\mathbf{E}(z, t) = 1.0 \sin(\omega t - \beta z)\mathbf{a}_x$ (V/m). Show that the average power crossing a circular disk of radius 15.5 m in a $z = \text{const.}$ plane is 1 W.

14.33. In spherical coordinates, the *spherical wave*

$$\mathbf{E} = \frac{100}{r} \sin\theta \cos(\omega t - \beta r)\mathbf{a}_\theta \quad (\text{V/m}) \qquad \mathbf{H} = \frac{0.265}{r} \sin\theta \cos(\omega t - \beta r)\mathbf{a}_\phi \quad (\text{A/m})$$

represents the electromagnetic field at large distances r from a certain dipole antenna in free space. Find the average power crossing the hemispherical shell $r = 1$ km, $0 \le \theta \le \pi/2$. *Ans.* 55.5 W

14.34. In free space, $\mathbf{E}(z, t) = 150 \sin(\omega t - \beta z)\mathbf{a}_x$ (V/m). Find the total power passing through a rectangular area, of sides 30 mm and 15 mm, in the $z = 0$ plane. *Ans.* 13.4 mW

14.35. A free space–silver interface has $E_0^i = 100$ V/m on the free-space side. The frequency is 15 MHz and the silver constants are $\epsilon_r = \mu_r = 1$, $\sigma = 61.7$ MS/m. Determine E_0^r and E_0^t at the interface. *Ans.* -100 V/m, $7.35 \times 10^{-4}\underline{/45^\circ}$ V/m

14.36. A free space–conductor interface has $H_0^i = 1.0$ A/m on the free-space side. The frequency is 31.8 MHz and the conductor constants are $\epsilon_r = \mu_r = 1$, $\sigma = 1.26$ MS/m. Determine H_0^r and H_0^t and the depth of penetration of $\mathbf{H}^t$. *Ans.* 1.0 A/m, 2.0 A/m, 80 μm

14.37. A traveling **H** field in free space, of amplitude 1.0 A/m and frequency 200 MHz, strikes a sheet of silver of thickness 5 μm with $\sigma = 61.7$ MS/m, as shown in Fig. 14-16. Find H_0^t just beyond the sheet.
Ans. 1.78×10^{-5} A/m

14.38. A traveling **E** field in free space, of amplitude 100 V/m, strikes a perfect dielectric, as shown in Fig. 14-17. Determine E_0^t. *Ans.* 59.7 V/m

14.39. A traveling **E** field in free space strikes a partially conducting medium, as shown in Fig. 14-18. Given a frequency of 500 MHz and $E_0^i = 100$ V/m, determine E_0^t and H_0^t.
Ans. 19.0 V/m, 0.0504 A/m

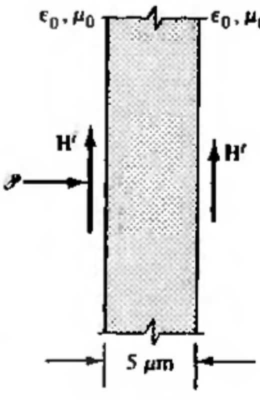


Fig. 14-16

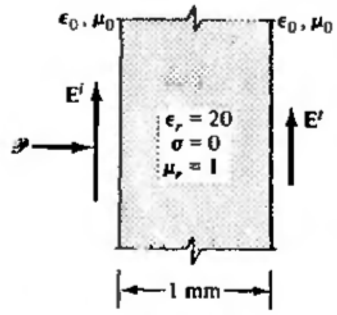


Fig. 14-17

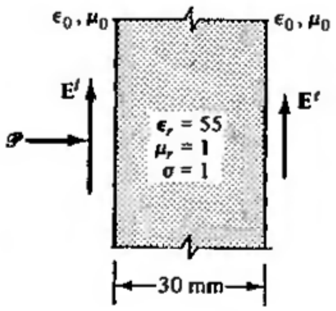


Fig. 14-18

14.40. A wave propagates from a dielectric medium to the interface with free space. If the angle of incidence is the critical angle of 20°, find the relative permittivity. *Ans.* 8.55

14.41. Compute the ratios E_0^r/E_0^i and E_0^t/E_0^i for normal incidence and for oblique incidence at $\theta_i = 10°$. For region *1*, $\epsilon_{r1} = 8.5$, $\mu_{r1} = 1$, and $\sigma_1 = 0$, region *2* is free space.
Ans. For normal incidence, $E_0^i/E_0^r = 0.490$ and $E_0^t/E_0^i = 1.490$. At 10°, $E_0^r/E_0^i = 0.539$ and $E_0^t/E_0^i = 1.539$.

14.42. A parallel-polarized wave propagates from air into a dielectric at Brewster angle of 75°. Find ϵ_r. *Ans.* 13.93

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Conteúdo

1 Problemas

1.1 Questão 1

Campos elétricos de duas ondas eletromagnéticas de polarização linear, propagando em uníssono através do espaço livre são dados por $\mathbf{E_1}$ (1) e $\mathbf{E}_2$ (2) com z em metros e onde a e b são constantes. Determine para a onda eletromagnética resultante: o campo magnético instantâneo, a polarização da onda, o vetor de Poynting instantâneo e médio.

$$\mathbf{E_1} = e^{-j\pi(z-0{,}25)}\vec{a}_x \ V/m \tag{1}$$

$$\mathbf{E_2} = ae^{-j\pi(z-0{,}25b)}\vec{a}_y \ V/m \tag{2}$$

Solução:
Considerando os valores de a e b, 2 e 5, respectivamente, teremos:

1.1.1 Campo Magnético Instantâneo

Um campo elétrico $\mathbf{E}$ em coordenadas retangulares será dado pela soma das componentes dos campos $E_x\hat{a_x}$, $E_y\hat{a_y}$ e $E_z\hat{a_z}$. Assumindo que a componente em $\vec{a_z}$ é zero, tem-se que soma dos campos elétricos $\mathbf{E_1}$ e $\mathbf{E_2}$ resulta no próprio $\mathbf{E}$:

$$\mathbf{E} = e^{-j\pi(z-0{,}25)}\hat{a_x} + ae^{-j\pi(z-0{,}25b)}\hat{a_y} \tag{3}$$

O **Campo Magnético Instantâneo** pode ser obtido através da manipulação da expressão (4)

$$\vec{k} \times E = \mu\omega H \rightarrow H = \frac{\vec{k} \times E}{\omega\mu} \tag{4}$$

Da equação de **velocidade de fase** temos ω:

$$\frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\epsilon_0}} \rightarrow \omega = \frac{k}{\sqrt{\mu_0\epsilon_0}} \tag{5}$$

Dado a expressão (3) combinada com (4) e (5)

$$H = \frac{\vec{k} \times E}{\mu\omega} = \frac{k_0}{\mu_0\omega}\hat{a_z} \times E = \frac{k_0}{\mu_0\omega}[\hat{a_z} \times (e^{-j\pi(z-0{,}25)}\hat{a_x} - ae^{-j\pi(z-0{,}25b)}\hat{a_y})] \tag{6}$$

Consequentemente:

$$H = (e^{-j\pi(z-0{,}25)}\hat{a_y} - ae^{-j\pi(z-0{,}25b)}\hat{a_x})\sqrt{\frac{\epsilon_0}{\mu_0}} \tag{7}$$

Sabendo que a impedância da onda é definida como $Z = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$, então:

$$H = (e^{-j\pi(z-0,25)}\hat{a_y} - ae^{-j\pi(z-0,25b)}\hat{a_x})\frac{1}{120\pi} \tag{8}$$

Logo, o campo instantâneo é a parte real do campo, ou seja, $\mathbb{R}\{\text{H}\}$:

$$\vec{H} = -\frac{a}{120\pi}cos[\omega t - \pi(z - 0, 25b)]\hat{a_x} + \frac{1}{120\pi}cos[\omega t - \pi(z - 0, 25)]\hat{a_y} \tag{9}$$

Substituindo os dados:

$$\vec{H} = -\frac{1}{60\pi}cos[\omega t - \pi(z - 1, 25)]\hat{a_x} + \frac{1}{120\pi}cos[\omega t - \pi(z - 0, 25)]\hat{a_y}$$

1.1.2 Polarização da Onda

Dado que a onda é plana, não haverá componente paralela a direção da propagação, de forma que para a análise é interessante rotacionar o eixo das coordenadas, buscando a coincidência dele com a direção de propagação.

$$\hat{E} = (E_{x_0}\hat{a_x} + E_{y_0}\hat{a_y})e^{-jkz} \tag{10}$$

Portanto $E_{x_0} = E_{x_0}e^{j\varphi_x}$ e $E_{y_0} = E_{y_0}e^{j\varphi_y}$, portanto:

$$\hat{E} = e^{j0,25\pi}\hat{a_x} + ae^{j0,25b\pi}\hat{a_y} \tag{11}$$

Com a expressão (11) é possível observar que é $E_{x_0} = 1$ e $E_{y_0} = a$. Além do mais, as fases iniciais de x e y são das por $\varphi_x = 0, 25\pi$ e $\varphi_y = 0, 25b\pi$, respectivamente.

Já no domínio do tempo:

$$\vec{E}(\vec{r}, t) = E_{x_0}cos(\omega t - k_z z + \varphi_x)\hat{a_x} + E_{y_0}cos(\omega t - k_z z + \varphi_y)\hat{a_y} \tag{12}$$

Utilizando parâmetros de razão de intensidade entre as componentes do campo e de diferença entre as fases, respectivamente: $A = \frac{E_{y_0}}{E_{x_0}}$ e $\varphi = \varphi_y - \varphi x$.

$$A = \frac{E_{y_0}}{E_{x_0}} = \frac{a}{1} = 2 \tag{13}$$

$$\varphi = \varphi_y - \varphi_x = 1, 25\pi - 0, 25\pi = \pi \tag{14}$$

Observe que dado a fase φ em (14) é um múltiplo de $n\pi$, para $n = 0, 1, 2, \ldots$, cumprindo o parâmetro da polarização é linear.

1.1.3 Vetor de Poynting Instantâneo

Para calcular o vetor de Poynting, $\vec{P} = \vec{E} \times \vec{H}$, devemos calcular o campo elétrico, extraindo $\mathrm{I\!R}\{\mathrm{Ee^{j\omega t}}\}$.

$$\hat{E} = cos[\omega t - \pi(z - 0,25)]\hat{a_x} + bcos[\omega t - \pi(z - 0,25b)]\hat{a_y} \tag{15}$$

Desenvolvendo o produto vetorial, tem-se

$$\vec{P} = \vec{E} \times \vec{H} \rightarrow \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ E_x & E_y & E_z \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ E_x & E_y & 0 \\ H_x & H_y & 0 \end{vmatrix} = E_x H_y \hat{a_z} - E_y H_x \hat{a_z} \tag{16}$$

Substituindo pelas componentes descobertas, tem-se que:

$$\vec{P} = \frac{1}{120\pi} cos^2[\omega t - \pi(z - 0,25)]\hat{a_z} - \frac{a^2}{120\pi} cos^2[\omega t - \pi(z - 0,25b)]\hat{a_z} \tag{17}$$

Por fim, utilizando as constantes:

$$\vec{P} = \frac{1}{120\pi} cos^2[\omega t - \pi(z - 0,25)]\hat{az} - \frac{1}{30\pi} cos^2[\omega t - \pi(z - 1,25)]\hat{az}$$

1.1.4 Vetor de Poynting Médio

O vetor de Poynting médio é dado por:

$$\vec{P_m} = \frac{1}{2}\mathrm{I\!R}\{\mathrm{E} \times \mathrm{H}^*\}. \tag{18}$$

Onde o conjugado de H é:

$$H^* = -\frac{1}{60\pi} e^{j\pi(z-1,25)}\hat{a_x} + \frac{1}{120\pi} e^{j\pi(z-0,25)}\hat{a_y} \tag{19}$$

Observe que é possível escrever a equação em função dos conjugados que descrevem o campo elétrico.

$$H^* = -\frac{1}{60\pi} E_y^* \hat{a_x} + \frac{1}{120\pi} E_x^* \hat{a_y} \tag{20}$$

Então

$$E \times H^* = \frac{1}{120\pi}|E_x|^2 \hat{a_z} + \frac{1}{60\pi}|E_y|^2 \hat{a_z} \tag{21}$$

O cálculo do módulo dos campos será dado simplesmente por: $|E_x|^2 = 1^2 = 1$ e $|E_y|^2 = a^2 = 4$, logo:

$$\vec{P_m} = \frac{1}{2}\mathrm{I\!R}\ \{\frac{4}{120\pi} + \frac{1}{60\pi}\}\hat{\mathrm{a}}_\mathrm{z} \rightarrow \frac{1}{2}\frac{1}{20\pi}\hat{\mathrm{a}}_\mathrm{z} \tag{22}$$

Por fim, o vetor Poynting Médio será

$$\vec{P_m} = \frac{1}{40\pi}\hat{a_z} \tag{23}$$

1.2 Questão 2

Uma onda eletromagnética plana uniforme harmônica no tempo na frequência f propaga na direção $z > 0$ através de um tecido biológico de parâmetros desconhecidos. O campo magnético da onda tem somente a componente x e a sua intensidade rms na origem do sistema de coordenadas é $H_0 = 25mA/m$. A amplitude da onde é reduzida em $3,25\ dB$ para cada centímetro percorrido, e o coeficiente de fase da onda chega a $\beta = 260\ rad/m$. Determine permissividade e a condutividade do tecido.

Solução:

Considerando o valor de $f = 3,1GHz$.

Assumindo um meio com perda, a expressão para onda plana é desenvolvida a partir das equações de Maxwell:

$$\nabla^2 \vec{E} = \frac{\nabla \rho}{\epsilon} + \mu \frac{\partial \vec{J_1}}{\partial t} + \sigma \mu \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial \vec{E}}{\partial t^2} \tag{24}$$

É possível visualizar que a variação $e^{j\omega t}$ implica em $j\omega$ e $\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$

Já assumindo um meio sem fontes, $\vec{J_1} = \rho = 0$ tendo como consequência

$$\nabla \vec{E} = \sigma \mu j \omega \vec{E} - \mu \epsilon (\omega)^2 \vec{E}$$

Aplicando a mesma metodologia para $\vec{H}$, é dado:

$$\nabla^2 \vec{H} = \sigma \mu j \omega \vec{H} - \mu \epsilon (\omega)^2 \vec{H}$$

Observe que o parâmetro de propagação $\gamma^2 = j\mu\sigma\omega - \mu\epsilon\omega^2$ está contido nestas equações. Este é parâmetro é complexo, então pode ser reescrito como $\gamma = \alpha + j\beta$, de modo que: $(\alpha + j\beta)^2 = j\mu\sigma\omega - \mu\epsilon\omega^2 \rightarrow \alpha^2 + j2\alpha\beta - \beta^2$

Dessa forma, é possível analisar as partes real e complexas separadamente

$$\alpha^2 - \beta^2 = -\mu\epsilon\omega^2$$

$$2\alpha\beta = \mu\sigma\omega$$

Então, isolando os termos ϵ e σ:

$$\epsilon = \frac{\alpha^2 - \beta^2}{-\mu\omega^2}$$

$$\sigma = \frac{2\alpha\beta}{\mu\omega}$$

Recorrendo a velocidade angular $\omega = 2\pi f$, dado que $f = 3,1GHz$, logo $\omega = 6,2\pi 10^9 rad/s$. Por fim,convertendo os valores da questão, obtém-se que: $1dB \rightarrow 0,11512925Np$, então $3,25\frac{dB}{cm} = 34,4075\frac{Np}{m}$

Assumindo por fim, que $\mu = \mu_0 = 4\pi 10^{-7}$, a condutividade σ será:

$$\sigma = \frac{(2)(37,4075)(260)}{(4\pi 10^{-7})(6,2\pi 10^9)}$$

$$\sigma = 0,7947 S/m$$

Já a permissividade ϵ será:

$$\epsilon = \frac{(37,4075)^2 - (260)^2}{-(4\pi 10^{-7})(6,2\pi 10^9)^2}$$

$$\epsilon = 148,2821 pF/m$$

1.3 Questão 3

Uma onda eletromagnética plana uniforme harmônica no tempo na frequência f e intensidade de campo elétrico $E_i = 1\ V/m$ propaga no ar e incide normalmente na superfície planar de um grande bloco de concreto com parâmetros elétricos $\varepsilon_r = 6$, $\mu_r = 1$ e $\sigma = 2,5 \times 10^{-3}$. Determine: a) A classificação do bloco de concreto; b) A taxa de onda estacionária no ar; c) O vetor de Poynting médio no tempo no concreto; d) As porcentagens da potência incidente média no tempo que são refletidas a partir da interface e transmitidas no bloco de concreto.

Solução:

Considerando o valor de $f = 12,5GHz$

1.3.1 Classificação do Bloco de Concreto

Para obter a classificação do bloco de concreto, será utilizada a tangente de perdas relacionada com a razão entre a corrente de condução e a corrente de deslocamento. Sendo a velocidade angular $\omega = 2\pi f$ e $\epsilon = \epsilon_R \epsilon_0$

$$\omega = 2\pi(12,5)(10^9) \rightarrow 0,7854.10^{10} rad/s$$

$$\frac{\sigma}{\epsilon\omega} = \frac{2,5.10^{-3}}{(6.8,85.10^{-1})(2\pi 10^{10})} \rightarrow 5,9920.10^{-4}$$

Tendo em vista que o valor da tangente de perdas é muito menor que 0,01, o bloco pode ser considerado um meio dielétrico.

1.3.2 Taxa de Onda Estacionária no Ar

A TOE é definida como: $\dfrac{|E_{max}|}{|E_{min}|}$. Além dessa definição, é interessante relacionar o o coeficiente de reflexão com a TOE: $TOE = \dfrac{1+|\Gamma|}{1-|\Gamma|}$.

$$Z_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi\Omega$$

$$Z_2 = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \frac{Z_1}{\sqrt{6}} = \frac{120\pi}{\sqrt{6}} = 153,9\Omega$$

Agora, substituindo Z_1 e Z_2 na expressão do coeficiente de reflexão, temos que: $\Gamma = \dfrac{49\pi - 120\pi}{120\pi + 49\pi} = -0,42$

Por fim, a TOE é:

$$TOE = \frac{1+0,42}{1-0,42} = 2,4483$$

1.3.3 Vetor de Poynting médio

Novamente, para calcular o vetor de Poynting Médio temos

$$\vec{P}_m = \frac{1}{2}\mathrm{I\!R}\{\mathrm{E} \times \mathrm{H}^*\}. \tag{25}$$

Observe que o conjugado de H é dado por: $H^* = \dfrac{TE_0}{Z_2}e^{-\alpha z + j\beta z}$, então

$$\vec{P}_m = \frac{1}{2Z_2}|T|^2|E_0|^2 e^{-2\alpha z}\hat{a_z} \tag{26}$$

O coeficiente de transmissão é $T = \Gamma + 1 \rightarrow -0,42 + 2 = 0,58$

$$\vec{P}_m = \frac{(0,58)^2(1)^2}{(2)(49\pi)}\hat{a_z} \rightarrow 1,09264.10^{-3}e^{-2\alpha z}\hat{a_z}. \tag{27}$$

Por fim, temos que $\alpha = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\epsilon}}Z_2 \rightarrow 2\alpha = \sigma Z_2 = 49\pi.2,5.10^-3 = 0,38485$

Então a equação termina como:

$$\vec{P}_m = 1,09264.10^{-3}e^{-0,38485z}\hat{a_z}W/m^2$$

1.3.4 Porcentagem da Potência Incidente

Sendo Z_1 e Z_2 reais, é possível escrever:

$$\vec{P_m} = \frac{|E_0|^2}{2Z_1} e^{-2\alpha z} \hat{a_z}$$

$$\vec{P_{m_1}} = \frac{-|r|^2 E_0|^2}{2Z_1} e^{-2\alpha z} \hat{a_z}$$

$$\vec{P_{m_2}} = \frac{|T|^2 E_0|^2}{2Z_2} e^{-2\alpha z} \hat{a_z}$$

Portanto, a incidência percentual média no tempo será dada pela razão entre $\vec{P_{m_1}}$ e $\vec{P_m}$

$$\frac{\vec{P_{m_1}}}{\vec{P_m}} = \frac{2,33957.10^{-4}}{1,32629.10^{-3}} \rightarrow 17,64\%$$

Já a porcentagem da incidência média transmitida na interface é a razão entre $\vec{P_{m_2}}$ e $\vec{P_m}$:

$$\frac{\vec{P_{m_2}}}{\vec{P_m}} = \frac{1,0926474.10^{-3}}{1,32629.10^{-3}} \rightarrow 82,38\%$$

1.4 Questão 4

Uma onda eletromagnética plana na frequência f incide normalmente em um sistema de 3 meios caracterizados por ϵ_r^1, ϵ_r^2, ϵ_r^3, figura 2.40 da apostila. Projete a camada central (meio 2) com a menor espessura, para que não haja reflexão da onda incidente de volta ao meio 1. Gere o gráfico da potência refletida no meio 1 para a faixa de frequência 0,5f a 1,5f. Mostre no gráfico a faixa em que a potência refletida é metade da incidente, isto é, a faixa de queda de $3dB$ (caso não encontre aumente a faixa de frequência).

Solução:

Considerando o valor de $f = 3GHz$, $\epsilon_r^1 = 2$ e $\epsilon_r^3 = 4$

Primeiramente, para descobrir a permissividade do meio 2, a relação do coeficiente de reflexão entre os meios 2 e 3 é utilizada:

$$\Gamma = \frac{Z_3 - Z_2}{Z_3 + Z_2} e^{(-2\mu_2 d)}$$

Para não haver reflexão, como dito no enunciado, o coeficiente de reflexão é igual a zero, logo $Z_2 = Z_3$. Então, com a relação das impedâncias:

$$Z_2 = Z_3 \to \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_3}{\epsilon_3}}$$

.

Assumindo que a permeabilidade dos meios seja igual a do vácuo (μ_0), logo:

$$\sqrt{\frac{1}{\epsilon_r^2}} = \sqrt{\frac{1}{\epsilon_r^3}} \to \epsilon_r^2 = \epsilon_r^3 = 4$$

Então, para calcular o número de onda no meio 2, temos que $k_2 = \dfrac{\omega}{\nu_P}$, desenvolvendo as variáveis:

$$\nu_p = \frac{1}{\sqrt{\epsilon_2 \mu_0}} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r^2 \mu_0}} = \frac{c}{\sqrt{\epsilon_r^2}}$$

Aplicando as relações obtidas:

$$k_2 = \frac{2f\pi\sqrt{\epsilon_r^2}}{c}$$

Combinando as equações dispostas anteriormente e com a relação entre o comprimento de onda e k_2 temos:

$$\lambda_2 = \frac{2\pi}{k_2} = \frac{2\pi c}{2f\pi\sqrt{\epsilon_r^2}} \to \frac{c}{f\sqrt{\epsilon_r^2}}$$

Com o que foi posto anteriormente, é possível calcular a menor espessura do meio 2 para que a reflexão da onda incidente para o meio 1 seja evitada:

$$d = \frac{n\lambda_2}{4}$$

O menor valor para n, diferente de zero, seria 2, logo $d = \dfrac{\lambda_2}{2}$.

$$d = \frac{3.10^8}{(2)(3.10^9)(\sqrt{4})} \to 0,0250m$$

A distância é de 25mm.

1.5 Questão 5

Uma onda plana na frequência f propaga no ar e incide num ângulo de θ_i sobre o mar ($\epsilon = 80\epsilon_0$, $\sigma = 3S/m$). Calcular: a) as amplitudes das ondas refletidas e transmitidas (E_r, H_r, E_t, H_t) considerando que $E_i = 500uV/m$ e a sua polarização é paralela; b) o ângulo de transmissão.

Solução:

Considerando o valor de $f = 240KHz$ e $\theta_i = 45^o$

1.5.1 Ângulo de Transmissão

É mais interessante neste problema definir primeiro o ãngulo de transmissão e isto pode ser feito com a Lei de Snell:

$$\frac{sen(\theta_i)}{sen(\theta_t)} = \frac{k_2}{k_1} = \frac{\sqrt{\mu_2\epsilon_2}}{\sqrt{\mu_1\epsilon_1}}$$

Resultando em:

$$\theta_t = arcsen\left(\frac{sen(\theta_i)\omega\sqrt{\epsilon\mu}}{|\gamma|}\right)$$

Visto que $\gamma = \alpha + j\beta$, é necessário encontrar os dois coeficiente. Isso será possível observando a a tangente das perdas, verificando a classificação do meio.

$$\frac{\sigma}{\omega\epsilon} = \frac{3}{2\pi(240.10^3)(80\epsilon_0)} = 2808,6$$

Observe que o a tangente das perda é muito maior que um, logo o mar é um bom condutor, então serão utilizados os parâmetros para bons condutores:

Para α:

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi(240.10^3)(4\pi10^{-7})3}{2}} = \sqrt{\frac{5.6849}{2}} = 1,6860$$

Para β:

$$\beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi(240.10^3)(4\pi10^{-7})3}{2}} = \sqrt{\frac{5.6849}{2}} = 1,6860$$

Como a premissa era que γ fosse complexo, consequentemente:

$$|\gamma| = \sqrt{\alpha^2 + \beta^2} = 2,3844$$

$$\theta_t = arcsen\left(\frac{sen(45^o)2\pi(240.10^3)\sqrt{(4\pi10^{-7})(8,85.10^{-12})}}{2,3844}\right)$$

Por fim o ângulo $\theta_t \approx 0^o$.

1.5.2 Amplitudes

Observe que a polarização é paralela, logo o coeficiente de reflexão e o coeficiente de transmissão serão definidos como:

$$\Gamma_{||} = \frac{-Z_1cos(\theta_i) + Z_2cos(\theta_t)}{Z_1cos(\theta_i) + Z_2cos(\theta_t)}$$

$$T_{||} = \frac{2Z_2cos(\theta_i)}{Z_1cos(\theta_i) + Z_2cos(\theta_t)}$$

Então, seja $Z_1 = 120\Omega$, a impedância Z_2 do mar é:

$$Z_2 = \sqrt{\frac{\omega\mu}{\sigma}}(1+j) = \frac{\sqrt{2\pi(240.10^3)(4\pi10^{-7})}}{\sqrt{3}}(1+j)$$

Então $Z_2 = 0,7948 + j0,7948$.
Com isso é possível calcular os coeficientes:

$$\Gamma_{||} = \frac{-Z_1cos(\theta_i) + Z_2cos(\theta_t)}{Z_1cos(\theta_i) + Z_2cos(\theta_t)}$$

$$\Gamma_{||} = \frac{-120\pi cos(45^o) + (0,7948 + j0,7948)cos(0^o)}{120\pi cos(45^o) + (0,7948 + j0,7948)cos(0^o)}$$

Fazendo manipulações utilizando a relação Euler obtém-se que:

$$\Gamma_{||} = 0,9941e^{-j0,3417^o}$$

O mesmo se aplica pra esse caso:

$$T_{||} = \frac{2Z_2cos(\theta_i)}{Z_1cos(\theta_i) + Z_2cos(\theta_t)}$$

$$T_{||} = \frac{2(0,7948 + j0,7948)cos(45^o)}{120\pi cos(45^o) + (0,7948 + j0,7948)cos(0^o)}$$

$$T_{||} = 5,95.10^{-3}e^{j44,8^o}$$

Por fim, é possível calcular E_Γ e H_Γ:

$$E_\Gamma = |\Gamma_{||}|\ |E_i| \rightarrow E_\Gamma = 0,9941.500\mu = 497,05\mu V/m$$

$$H_\Gamma = \frac{|E_\Gamma|}{Z_1} \rightarrow H_\Gamma = \frac{497,05\mu}{120\pi} = 1,3183\mu A/m$$

Calculando também

$$E_T = |T_{||}|\ |E_i| \rightarrow 5,95.10^{-3}.500\mu = 2,975\mu V/m$$

$$H_T = \frac{|E_T|}{Z_2} \rightarrow H_T = \frac{2,975\mu}{1,124} = 2,647\mu A/m$$

1.6 Questão 6

Projete a antena triangular equilátera mostrada na figura para recepção ótima de uma onda plana $\mathbf{E} = \mathbf{E_0}\vec{a}_x e^{-jk_0 z} V/m$ na frequência f. Faça um gráfico da fem em induzida em função da frequência mostrando a faixa em que a fem cai para $\frac{1}{\sqrt{2}}$ de seu valor máximo. Repita a análise considerando que a onda propaga na direção perpendicular ao lado assinalado b e polarização paralela ao lado. Calcule a razão perímetro/λ.

Solução:

Considerando o valor de $f = 1,8GHz$

$$Fem = \int_0^{\frac{\sqrt{3}b}{2}} |E_0| cos(\omega t - kz_1)dx + \int_{\frac{\sqrt{3}b}{2}}^0 |E_0| cos(\omega t - kz_2)dx$$

Observe que todos independem de x, logo saem da integral:

$$Fem = |E_0| cos(\omega t - kz_1) \int_0^{\frac{\sqrt{3}b}{2}} dx + |E_0| cos(\omega t - kz_2) \int_{\frac{\sqrt{3}b}{2}}^0 dx$$

Aplicando os limites da integração

$$fem = \frac{\sqrt{3}b}{2}|E_0|\ [cos(\omega t - kz_1) - cos(\omega t - kz_1)]$$

Pela geometria do problema é possível observar que $z_2 = z_1 + b$

$$fem = \frac{\sqrt{3}b}{2}|E_0|\ [cos(\omega t - kz_1) - cos(\omega t - kz_1 - kb)]$$

Definindo que $\phi = \omega t - kz_1$, podemos rescrever de modo que:

$$fem = \frac{\sqrt{3}b}{2}|E_0|\ [cos(\phi) - cos(\phi - kb)]$$

Utilizando transformações trigonométricas

$$cos(\phi) - cos(\phi - kb) = -2sen\left(\frac{2\phi - kb}{2}\right)sen\left(\frac{kb}{2}\right)$$

Resultando na equação:

$$fem = \frac{\sqrt{3}b}{2}|E_0| \left[-2sen\left(\frac{2\phi - kb}{2}\right)sen\left(\frac{kb}{2}\right)\right]$$

$$fem = \sqrt{3}b|E_0| \left[-\underbrace{sen\left(\phi - k\frac{b}{2}\right)}\underbrace{sen\left(k\frac{b}{2}\right)}\right]$$

Observe que há dois termos sublinhados, o primeiro é responsável pela variação no tempo. Já o segundo será o responsável por alcançar a máxima amplitude sinal, então no caso da antena ter a maior Fem possível quando:

$$sen\left(k\frac{b}{2}\right) = +1$$

E isto vai ocorrer quando $k\frac{b}{2} = (2n+1)\frac{\pi}{2}$, ou seja, $b = (2n+1)\frac{\lambda}{2}$

Logo, b deve ser um múltiplo ímpar de $\frac{\lambda}{2}$.

Assumindo $n = 0 \rightarrow b = \frac{\lambda}{2}$

Sabendo que temos a frequência f e assumindo que essa onda se propaga com a mesma velocidade que no vácuo, teremos:

$$b = \frac{c}{2f} \rightarrow \frac{3.10^8}{2(1,8.10^9)} \approx 8,3cm$$

Então, o tamanho ideal da lateral da antena para máximo aproveitamento da Fem seria cerca de 8,3 cm.

Sendo o Perímetro $\Delta = 3b$,

$$3b = (2n+1)\frac{3\lambda}{2}$$

Já a razão perímetro/λ será:

$$\frac{\Delta}{\lambda} = (2n+1)\frac{3}{2}$$

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Ana Hilana Sampaio de Magalhães.

Guias e Ondas - 2º semestre de 2011

Ondas Eletromagnéticas - Prática Nº 2

1ª Questão: Uma onda plana uniforme com densidade de potência média de $100\,W/m^2$ e frequência de $500\,MHz$, incide normalmente, do espaço livre, na superfície do mar ($\sigma = 5\,S/m$ e $\varepsilon = 80\,\varepsilon_0$). O campo elétrico aponta na direção $+x$ e a propagação se dá na direção $+z$. Pede-se: a) os campos incidentes $\vec{E}_i$ e $\vec{H}_i$; b) os campos refletidos $\vec{E}_r$ e $\vec{H}_r$; c) os campos transmitidos $\vec{E}_t$ e $\vec{H}_t$; d) a potência transmitida para o mar em $z = 0$.

2ª Questão: Um projetista quer transmitir uma onda plana através de uma interface entre dois meios dielétricos não magnéticos. Porque somente 75% da potência é transmitida numa incidência normal, ele decide usar uma incidência oblíqua no ângulo de Brewster, que permite uma transmissão total para uma onda na polarização paralela. Por um erro ele transmite a onda na polarização perpendicular, que fração da potência incidente é transmitida neste caso.

3ª Questão: Uma onda plana uniforme propagando no ar, incide num meio com $\varepsilon = 3\,\varepsilon_0$ segundo um ângulo de incidência de 60^o. A onda incidente tem um campo elétrico com intensidade de $100\,\mu\,V/m$ e polarizado perpendicularmente em relação ao plano de incidência. Determine: a) o ângulo de transmissão; b) a intensidade de campo elétrico refletido e transmitido.

4ª Questão: Uma onda plana possui campo $\mathbf{E}$ com amplitude $E_0^i = 1,0\,V/m$ e incide normalmente na interface do ar com a superfície do mar ($\varepsilon_r = 80, \mu_r = 1$ e $\sigma = 2,5\,S/m$). Para uma frequência de $30\,MHz$ em que profundidade a amplitude de $\mathbf{E}$ será de $1,0\,mV/m$?

Constantes: $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}\ F/m$ e $\mu_0 = 4\pi \times 10^{-7}\ H/m$.

1º Questão (Prova C)   4º Questão (Prova A)

a) $\frac{sen\,\theta i}{sen\,\theta t} = \frac{\beta_2}{\beta_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{3} \Rightarrow \theta t = 30^\circ$

b)
$$\Gamma_\perp = \frac{\cos\theta i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)sen^2\theta i}}{\cos\theta i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{1-\left(\frac{\varepsilon_1}{\varepsilon_2}\right)sen^2\theta i}} =$$

$$\Gamma_\perp = \frac{\cos(60^\circ) - \sqrt{3}\sqrt{1-(1/3)\,sen^2(60^\circ)}}{\cos(60^\circ) + \sqrt{3}\sqrt{1-(1/3)\,sen^2(60^\circ)}} = -0.5$$

$$T_\perp = \frac{2\cos(60^\circ)}{\cos(60^\circ) + \sqrt{3}\sqrt{1-1/3\,sen^2(60^\circ)}} = 0.5$$

$|E_{r}| = |E_i|\,|\Gamma_\perp| = +50\,\mu V/m\ \mu$

$|E_t| = |E_i|\,|T_\perp| = +50\mu\ V/m\ \mu$

$$\Gamma = \frac{354.7689 \angle 177.6366}{399.7828 \angle 2.0974} = 0.8874 \angle 175.5394$$

$$T = \Gamma + 1 = 0.11529 + j\,0.06902 = 0.13436 \angle 30.90646°$$

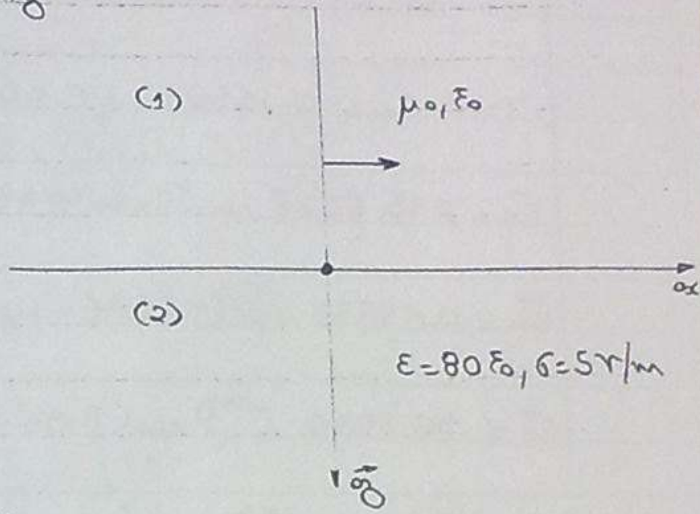
$$\left.\begin{array}{l} \vec{E_r} = 243,6688 \cos(\pi \times 10^9 t + 10.4179 z + 175.5394°)\ \vec{a_x} \\ \vec{H_r} = 0.64635 \cos(\pi \times 10^9 t + 10.4179 z + 175.5394°)(-\vec{a_y}) \end{array}\right\} b$$

$$\left.\begin{array}{l} \vec{E_t} = 36,8936\ e^{-\alpha z} \cos(\pi \times 10^9 t - \overset{\beta}{123.2696} z + 30.90646°)\ \vec{a_x} \\ \vec{H_t} = 1,3736\ e^{-\alpha z} \cos(\pi \times 10^9 t - 123.2696 z - 2.09774)\ \vec{a_y} \end{array}\right\} c$$

d) $P_m^t = (1 - |\Gamma|^2) P_m^i = 21.25 \text{ w/m}^2$

3.ª Questão (Prova A) 2ª Questão (Prova C)

a) $P_m^i = 100 W/m^2$ $f = 500 MHz$



$$Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$$

$$\omega = 2\pi f = \pi \times 10^9$$

$$P_m^i = \frac{|E_i|^2}{2Z_1} = \frac{|H_i|^2 Z_1}{2}$$

$$|E_i| = \sqrt{P_m^i \, 2 Z_1} \simeq 274.5874$$

$$|H_i| = \sqrt{P_m^i \, 2/Z_1} \simeq 0.7283$$

$$k_z = \omega\sqrt{\mu_0 \varepsilon_0} \simeq 10.4719$$

$$\vec{E_i} = 274.5874 \cos(\pi \times 10^9 t - 10.4719 z)\,\vec{a_x}$$

$$\vec{H_i} = 0.7283 \cos(\pi \times 10^9 t - 10.4719 z)\,\vec{a_y}$$

b) $Z_2 = \frac{j\omega\mu}{\gamma}$ $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\varepsilon = 80\varepsilon_0, \mu = \mu_0, \sigma = 5 \Rightarrow \gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = 146.9892\angle 56.9957$$
$$= 80.0652 + j123.2696$$

$$Z_2 = \frac{j\omega\mu}{\gamma} = 26.8580\angle 33.0042° = 22.5239 + j14.6296$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{22.5239 + j14.6296 - 120\pi}{22.5239 + j14.6296 + 120\pi} = \frac{-354.4672 + j14.6296}{399.5150 + j.14.6296}$$

Fco. Hilário Sampaio de Magalhães.

Guias e Ondas - 2º semestre de 2011

Ondas Eletromagnéticas - Prática Nº 1

1ª Questão: O campo magnético emitido por uma estação de radiobase (ERB) de telefonia celular é dado por $\mathbf{H} = 35\, e^{-j(17,3\, y - \pi/3)}\, \vec{a}_x\, \mu A/m$, a ERB está posicionada no eixo z do sistema de coordenadas. Um sensor é posicionado sobre o eixo y no ponto $(0, 10\, m, 0)$, determine as leituras de campo elétrico nos instantes $t_1 = 0\, s$ (início das leituras), $t_2 = 2\, s$, $t_3 = 4\, s$, $t_4 = 8\, s$ e $t_5 = 16\, s$.

2ª Questão: O transmissor de uma antena de rádio FM opera na frequência de $97,5\, MHz$ emitindo uma potência média de $100\, kW$. Considere que antena é isotrópica, isto é, a sua emissão de energia é igualmente distribuída em todas as direções (fonte pontual). Sabendo que o limite de segurança estabelecido pelas normas é de $2\, W/m^2$ de fluxo de potência, determine: a) se uma pessoa a $50\, m$ da antena está segura; b) a distância mínima segura; c) se a antena for instalada a $200\, m$ de altura uma pessoa está segura quando situada a $50\, m$ do pé da torre da antena?; d) neste último caso qual a distância segura mínima do pé da torre.

3ª Questão: Um dielétrico com perdas tem uma impedância intrínseca de $200\,\underline{|30^\circ}\,\Omega$ em uma frequência. Nesta frequência a onda plana que propaga neste meio tem o campo magnético $\vec{H} = 10\, e^{-\alpha x} \cos(\omega t - 0,5\, x)\, \vec{a}_y\, A/m$. Determine $\vec{E}$ e α.

4ª Questão: O campo elétrico $\vec{E}_i$ de uma onda plana tem $100\, V/m$ de amplitude e oscila na frequência de $200\, MHz$. Está propagando no espaço livre e incide normalmente sobre uma lâmina de um material cujos parâmetros elétricos são $\varepsilon = 3\, \varepsilon_0$, $\mu = \mu_0$ e $\sigma = 61,7\, S/m$. Determine a espessura da lâmina de forma que o campo elétrico no espaço livre logo após ela tenha uma amplitude de $1,78 \times 10^{-3}\, V/m$.

Constantes: $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}\, F/m$ e $\mu_0 = 4\pi \times 10^{-7}\, H/m$.

Ondas eletromagnéticas - Prática Nº 01

1º) Questão

$k_y = k = 17,3$ rad/m

$k = \frac{\omega}{c} = \frac{2\pi f}{3\times10^8} \rightarrow f \simeq 826$ MHz

$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \rightarrow \bar{E} = 13,2\, e^{-j(17,3y - \pi/3)}\, \vec{a}_z$ mV/m

$\vec{E}(y,t) = 18,67 \cos(5,19\times10^9 t - 17,3y + \pi/3)$ mV/m

$t_1 = 0$

$t_2 = 2s$

$t_3 = 4s$

$t_4 = 8s$

$t_5 = 16s$

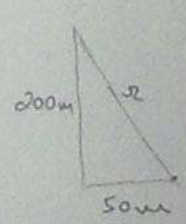
2º Questão

a) $\lambda = \frac{c}{f} = \frac{3\times10^8}{97,5\times10^6} \simeq 3,08$ m

$P_m = \frac{P_{total}}{4\pi r^2} = \frac{100\times10^3}{4\pi(50)^2} \simeq 3,2$ W/m² não é segura a distância

b) $P_m = 2,0 = \frac{P_{total}}{4\pi(r^2)} \rightarrow r = \sqrt{P_{total}/4\pi(2)} \simeq 63,1$ m//

c) $P_m = \frac{100\times10^3}{4\pi(200^2 + 50^2)} = 0,2$ W/m² é seguro!



d) $2,0 = \frac{P_{total}}{4\pi(200^2 + d^2)} \rightarrow d^2 = \frac{P_{total}}{4\pi(2)} - 200^2$

$d \simeq j\,189,79$ m qualquer distância da base da torre é segura

...sadas (Prova A)

...mos 75% transmitido

$\theta_i = \theta_B = \tan^{-1}\sqrt{\varepsilon_2/\varepsilon_1}$, $\cos\theta_i = \sqrt{\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}}$, $\operatorname{sen}\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$

incidência normal $\Gamma = \frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}+\sqrt{\varepsilon_2}} = \frac{1-\sqrt{\varepsilon_2/\varepsilon_1}}{1+\sqrt{\varepsilon_2/\varepsilon_1}}$, $b = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$\Gamma = \frac{1-b}{1+b} \rightarrow b = \frac{1-\Gamma}{\Gamma+1}$, $|\Gamma|^2 = \frac{100-x\%}{100}$ $\qquad x\% = 75\%$

$|\Gamma|^2 = 0.25 \rightarrow |\Gamma| 0.5 \rightarrow b = \frac{1-0.5}{1.5} = \frac{1}{3} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{1-(\varepsilon_1/\varepsilon_2)\operatorname{sen}^2\theta_i}}{\cos\theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{1-(\varepsilon_1/\varepsilon_2)\operatorname{sen}^2\theta_i}}$$

$$\frac{\varepsilon_1}{\varepsilon_2}\operatorname{sen}^2\theta_i = \frac{\varepsilon_1}{\varepsilon_2}\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2} \longrightarrow 1-\frac{\varepsilon_1}{\varepsilon_2}\operatorname{sen}^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \operatorname{sen}^2\theta_i$$

$$\Gamma_\perp = \frac{\cos\theta_i - \tan\theta_i \operatorname{sen}\theta_i}{\cos\theta_i + \tan\theta_i \operatorname{sen}\theta_i} = \frac{\cos^2\theta_i - \operatorname{sen}^2\theta_i}{\cos^2\theta_i + \operatorname{sen}^2\theta_i} = \cos^2\theta_i - \operatorname{sen}^2\theta_i$$

$$\Gamma_\perp = \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2} - \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} = \frac{1-\varepsilon_2/\varepsilon_1}{1+\varepsilon_2/\varepsilon_1}$$

$\Gamma_\perp = \frac{1-1/9}{1+1/9} = \frac{8}{10} \rightarrow |\Gamma_\perp|^2 = \frac{64}{100} \rightarrow$ 36% é transmitido

(Prova B) $x\% = 64\% \rightarrow |\Gamma|^2 = 0{,}36 \rightarrow |\Gamma| = 0.6 \rightarrow b = \frac{0.4}{1.6} = 0.25$

(Prova D

$\Gamma_\perp = \frac{1-1/16}{1+1/16} = \frac{15}{17} \rightarrow |\Gamma_\perp|^2 = \frac{225}{289} \rightarrow 0.78 \rightarrow$ 22% é transmitido //

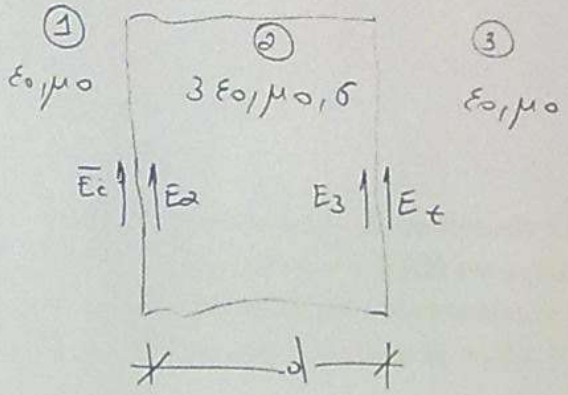
resolução

$E_i = 100\ V/m$

$f = 200\ MHz$

$\varepsilon_0, \mu_0,\ \sigma = 61{,}7\ mhos/m$

$E_t = 1{,}78 \times 10^{-3}\ V/m$



$Z_1 = 120\pi\ ,\ Z_3 = 120\pi$

$\frac{\sigma}{\omega\varepsilon} = 1851$ bom condutor $\rightarrow \alpha = \beta = 220.7178$

$\left.\begin{array}{l}\alpha = 220.6582 \\ \beta = 220.775\end{array}\right\}$ valores exatos

$Z_2 = \frac{j\omega\mu}{\sigma}$ como é bom condutor $Z_2 \approx \sqrt{\frac{\omega\mu}{\sigma}}\ \underline{|45°} \simeq 5.0590\ \underline{|45°}$

$$\frac{\bar{E}_2}{\bar{E}_i} = T_2 = \frac{2Z_2}{Z_2 + Z_1} = \frac{10.059\ \underline{|45°}}{380.585\ \underline{|0.5385°}} \simeq 0.026\ \underline{|44.46°}$$

$|E_2| = |T_2|\ |\bar{E}_i| = 2{,}6\ V/m$

$$\frac{\bar{E}_t}{\bar{E}_3} = T_3 = \frac{2Z_3}{Z_2 + Z_3} = \frac{2(120\pi)}{120\pi + 5.059\ \underline{|45°}} = 1.98\ \underline{|-0.538°}$$

$|\bar{E}_3| = \frac{|\bar{E}_t|}{|T_3|} = 8.985 \times 10^{-4}$ como $|\bar{E}_3| = |E_2|\, e^{-\alpha d} \rightarrow d = \frac{-\ln |\bar{E}_3| / |\bar{E}_2|}{\alpha}$

$$d = 3.6 \times 10^{-2}\ m$$

$$\boxed{d = 3{,}6\ cm}$$

3ª Questão

- a onda propaga na direção $+\vec{a_x}$ assim

$$\vec{a_E} = -\vec{a_x} \times \vec{a_y} = -\vec{a_z}$$

- $H_0 = 10$

$$\frac{E_0}{H_0} = Z = 200\angle 30^\circ = 200\, e^{j\pi/6} \rightarrow E_0 = 2000\, e^{j\pi/6}$$

$$\vec{E} = \mathrm{Re}\left[2000\, e^{j\pi/6}\, e^{-\gamma x}\, e^{j\omega t}\right](-\vec{a_z})$$

$$\vec{E} = -2\, e^{-\alpha x} \cos\left(\omega t - \frac{x}{2} + \frac{\pi}{6}\right) \vec{a_z} \ \ kV/m$$

- $\beta = 1/2$

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1}{\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1}\right]^{1/2}, \quad Z = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}\, e^{j(1/2)\tan^{-1}(\sigma/\omega\varepsilon)} \ \Omega$$

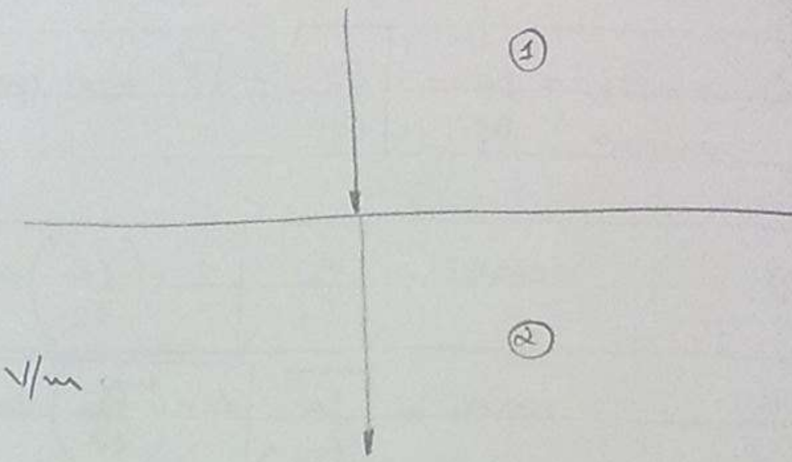
$$\frac{\sigma}{\omega\varepsilon} = \tan 2\theta_n = \tan 60^\circ = \sqrt{3}$$

$$\frac{\alpha}{\beta} = \left[\frac{2-1}{2+1}\right]^{1/2} = \frac{1}{\sqrt{3}} \rightarrow \alpha = \frac{\beta}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0{,}2887\ Np/m$$

4ª Questão

$$Z_1 = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi$$

$$Z_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = 9{,}73 \underline{\lfloor 43{,}5^\circ} \, \Omega$$



$$\frac{E_0^t}{E_0^i} = \frac{2Z_2}{Z_1 + Z_2} \longrightarrow E_0^t = 5{,}07 \times 10^{-2} \text{ V/m}$$

amplitude

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = 24.36 \underline{\lfloor 46{,}53^\circ} \text{ m}^{-1}$$

$$\alpha = 24{,}36 \cos 46{,}53^\circ = 16{,}72 \text{ Np/m}$$

$$E = 10^{-3} = 5{,}07 \times 10^{-2} e^{-16{,}76 z} \longrightarrow z = 0{,}234 \text{ m}$$