

1. Análise Vetorial

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z \text{ coord. retangulares}$$

$$\vec{A} = A_\rho \vec{a}_\rho + A_\phi \vec{a}_\phi + A_z \vec{a}_z \text{ coord. cilíndricas}$$

$$\vec{A} = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi \text{ coord. esféricas}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB} \quad \vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \vec{a}_N$$

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} \quad \vec{u}_A = \frac{\vec{A}}{|\vec{A}|}$$

coord. retangulares \iff coord. cilíndricas

$$x = \rho \cos \phi \quad y = \rho \sin \phi \quad z = z$$

$$\rho = \sqrt{x^2 + y^2} \quad \phi = \tan^{-1} \frac{y}{x} \quad z = z$$

	\vec{a}_ρ	\vec{a}_ϕ	\vec{a}_z
\vec{a}_x	$\cos \phi$	$-\sin \phi$	0
\vec{a}_y	$\sin \phi$	$\cos \phi$	0
\vec{a}_z	0	0	1

coord. retangulares \iff coord. esféricas

$$x = r \sin \theta \cos \phi \quad y = r \sin \theta \sin \phi \quad z = r \cos \theta$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad \theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

	\vec{a}_r	\vec{a}_θ	\vec{a}_ϕ
\vec{a}_x	$\sin \theta \cos \phi$	$\cos \theta \cos \phi$	$-\sin \phi$
\vec{a}_y	$\sin \theta \sin \phi$	$\cos \theta \sin \phi$	$\cos \phi$
\vec{a}_z	$\cos \theta$	$-\sin \theta$	0

elementos diferenciais

coordenadas retangulares

$$dx, dy, dz, dv = dx dy dz \quad d\vec{l} = dx \vec{a}_x + dy \vec{a}_y + dz \vec{a}_z$$

coordenadas cilíndricas

$$\rho d\rho, \rho d\phi, dz, dv = \rho d\rho d\phi dz \quad d\vec{l} = d\rho \vec{a}_\rho + \rho d\phi \vec{a}_\phi + dz \vec{a}_z$$

coordenadas esféricas

$$dr, r d\theta, r \sin \theta d\phi, dv = r^2 \sin \theta dr d\theta d\phi$$

$$d\vec{l} = dr \vec{a}_r + r d\theta \vec{a}_\theta + r \sin \theta d\phi \vec{a}_\phi$$

divergência

coordenadas retangulares

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

coordenadas cilíndricas

$$\nabla \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

coordenadas esféricas

$$\nabla \cdot \vec{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (D_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

gradiente

coordenadas retangulares

$$\nabla V = \frac{\partial V}{\partial x} \vec{a}_x + \frac{\partial V}{\partial y} \vec{a}_y + \frac{\partial V}{\partial z} \vec{a}_z$$

coordenadas cilíndricas

$$\nabla V = \frac{\partial V}{\partial \rho} \vec{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \vec{a}_\phi + \frac{\partial V}{\partial z} \vec{a}_z$$

coordenadas esféricas

$$\nabla V = \frac{\partial V}{\partial r} \vec{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \vec{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \vec{a}_\phi$$

rotacional

coordenadas retangulares

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \vec{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \vec{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \vec{a}_z$$

coordenadas cilíndricas

$$\nabla \times \vec{H} = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \vec{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \vec{a}_\phi + \frac{1}{\rho} \left(\frac{\partial (\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right) \vec{a}_z$$

coordenadas esféricas

$$\nabla \times \vec{H} = \frac{1}{r \sin \theta} \left(\frac{\partial (H_\phi \sin \theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi} \right) \vec{a}_r + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial (r H_\phi)}{\partial r} \right) \vec{a}_\theta + \frac{1}{r} \left(\frac{\partial (r H_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta} \right) \vec{a}_\phi$$

2. Lei de Coloumb, Campo Elétrico e Lei de Gauss

$$\vec{F}_1 = \frac{q_1 q_2}{4 \pi \epsilon_0 R_{12}^2} \vec{a}_{12} \quad \vec{E} = \frac{q}{4 \pi \epsilon_0 R^2} \vec{a}_R$$

$$\vec{E} = \sum_{m=1}^n \frac{q_m}{4 \pi \epsilon_0 |\vec{r} - \vec{r}_m|^2} \vec{a}_m \quad \vec{E} = \int_l \frac{\rho_l dl'}{4 \pi \epsilon_0 R^2} \vec{a}_R$$

$$\vec{E} = \int_s \frac{\rho_s ds'}{4 \pi \epsilon_0 R^2} \vec{a}_R \quad \vec{E} = \int_v \frac{\rho_v dv'}{4 \pi \epsilon_0 R^2} \vec{a}_R$$

$$Q = \int_l \rho_l dl \quad Q = \int_s \rho_s ds \quad Q = \int_v \rho_v dv$$

$$\oint_s \vec{D} \cdot d\vec{s} = Q \quad \vec{D} = \int_v \frac{\rho_v dv'}{4 \pi R^2} \vec{a}_R$$

$$\nabla \cdot \vec{D} = \rho_v \quad \oint_s \vec{D} \cdot d\vec{s} = \int_v \nabla \cdot \vec{D} dv$$

3. Energia e Potencial

$$W = -q \int_{inicial}^{final} \vec{E} \cdot d\vec{l} \quad ddp = - \int_{inicial}^{final} \vec{E} \cdot d\vec{l}$$

$$V = \int \frac{\rho_l dl}{4 \pi \epsilon_0 R} \quad V = \int_s \frac{\rho_s ds}{4 \pi \epsilon_0 R}$$

$$V = \int_v \frac{\rho_v dv}{4 \pi \epsilon_0 R} \quad \vec{E} = - \frac{dV}{dN} \vec{a}_N \quad \vec{E} = - \nabla V$$

$$W_E = \frac{1}{2} \sum_{m=1}^N q_m V_m \quad W_E = \frac{1}{2} \int_v \rho_v V dv$$

$$W_E = \frac{1}{2} \int_v \vec{D} \cdot \vec{E} dv = \frac{1}{2} \int_v \epsilon_0 |\vec{E}|^2 dv$$

4. Resistência, Material Dielétrico e Capacitância

$$R = \frac{V_{ab}}{I} = \frac{-\int_b^a (\vec{E} \cdot d\vec{l})}{\int_s \sigma \vec{E} \cdot d\vec{s}} = \frac{-\int_b^a (\frac{\vec{J}}{\sigma} \cdot d\vec{l})}{\int_s \vec{J} \cdot d\vec{s}}$$

interface condutor-meio

$$D_t = E_t = 0 \quad D_n = \rho_s \quad E_n = \frac{\rho_s}{\varepsilon}$$

$$\rho_{sp} = \vec{P} \cdot \vec{n} \quad \rho_{vp} = \nabla \cdot \vec{P} \quad \oint_s \vec{P} \cdot d\vec{s} - \int_v \nabla \cdot \vec{P} dv = 0$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad \vec{P} = \varepsilon_0 \chi_e \vec{E} \quad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$\varepsilon_R = \frac{\varepsilon}{\varepsilon_0} \rightarrow \varepsilon_R = 1 + \chi_e \quad \vec{D} = \varepsilon \vec{E}$$

interface dielétrico-dielétrico

$$\frac{D_t^1}{D_t^2} = \frac{\varepsilon_1}{\varepsilon_2} \quad D_n^1 - D_n^2 = \rho_s \quad \varepsilon_1 E_n^1 - \varepsilon_2 E_n^2 = \rho_s$$

$$C = \frac{Q}{V} \quad C = \frac{\int_s \rho_s ds}{-\int_-^+ \vec{E} \cdot d\vec{l}} \quad C = \frac{\int_s \varepsilon \vec{E} \cdot d\vec{s}}{-\int_-^+ \vec{E} \cdot d\vec{l}} \quad C = \frac{2.0 W_E}{V^2}$$

5. Equação de Laplace e Poisson

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \quad \nabla^2 V = 0$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

solução produto em coordenadas retangulares

$$k_x^2 + k_y^2 + k_z^2 = 0$$

$$V(x, y, z) = (Ax+B)(C e^{k_y y} + D e^{-k_y y})(E \cos k_z z + F \sin k_z z)$$

solução produto em coordenadas cilíndricas

$$Z = A \cosh mz + B \sinh mz \quad (m \neq 0) \quad Z = Az + B \quad (m = 0)$$

$$F = A \cos n\phi + B \sin n\phi \quad (n \neq 0) \quad F = A\phi + B \quad (n = 0)$$

$$R = A J_n(m\rho) + B Y_n(m\rho) \quad \text{para } m \neq 0$$

$$R = A \rho^n + B \rho^{-n} \quad \text{para } m = 0 \text{ e } n \neq 0$$

$$R = A \ln \rho + B \quad \text{para } m = n = 0$$

solução produto em coordenadas esféricas

$$G = A \cos n\phi + B \sin n\phi \quad \text{para } n \neq 0 \quad G = A\phi + B \quad \text{para } n = 0$$

$$R = A r^m + B r^{-(m+1)} \quad F = A P_m(\cos \theta) + B Q_m(\cos \theta)$$

$$P_0 = 1 \quad P_1 = \cos \theta \quad P_2 = \frac{1}{2} (3 \cos^2 \theta - 1)$$

$$P_3 = \frac{1}{2} (5 \cos^3 \theta - 3 \cos \theta) \quad P_4 = \frac{1}{8} (35 \cos^4 \theta - 30 \cos^2 \theta + 3)$$

$$Q_0 = \frac{1}{2} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) \quad Q_1 = \frac{\cos \theta}{2} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right)$$

$$Q_2 = \frac{3 \cos^2 \theta - 1}{4} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) - \frac{3 \cos \theta}{2}$$

$$Q_3 = \frac{5 \cos^3 \theta - 3 \cos \theta}{4} \ln \left(\frac{1 + \cos \theta}{1 - \cos \theta} \right) - \frac{5 \cos^2 \theta}{2} + \frac{2}{3}$$

$$F = A P_m^n(\cos \theta) + B Q_m^n(\cos \theta)$$

$$P_m^n(x) = (1-x^2)^{n/2} \frac{d^n P_m(x)}{d^n x} \quad Q_m^n(x) = (1-x^2)^{n/2} \frac{d^n Q_m(x)}{d^n x}$$

6. Lei de Biot-Savart e Campo Magnético

$$I = \frac{dq}{dt} \quad \vec{J} = \rho_v \vec{U} \quad \vec{J} = \rho_v^+ \vec{U}_+ + \rho_v^- \vec{U}_- \quad I = \int_s \vec{J} \cdot d\vec{s}$$

$$\oint_s \vec{J} \cdot d\vec{s} = -\frac{d}{dt} \int_v \rho_v dv \quad \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \quad I d\vec{l} \equiv \vec{J}_s ds \equiv \vec{J}_v dv$$

$$\vec{H} = \oint \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2} \quad \vec{H} = \int_s \frac{\vec{J}_s \times ds \vec{a}_R}{4\pi R^2} \quad \vec{H} = \int_v \frac{\vec{J}_v \times dv \vec{a}_R}{4\pi R^2}$$

$$\oint \vec{H} \cdot d\vec{l} = I \quad \nabla \times \vec{H} = \vec{J} \quad \oint_c \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{S}$$

$$\vec{B} = \mu \vec{H} \quad \Phi = \int_s \vec{B} \cdot d\vec{S} \quad \oint_s \vec{B} \cdot d\vec{S} = 0 \quad \nabla \cdot \vec{B} = 0$$

7. Forças, Material Magnético e Indutância

$$\vec{F} = \vec{F}_e + \vec{F}_m = q (\vec{E} + \vec{U} \times \vec{B})$$

$$\vec{F} = \int_v \vec{J}_v \times \vec{B} dv \quad \vec{F} = \int_s \vec{J}_s \times \vec{B} ds \quad \vec{F} = \oint I d\vec{l} \times \vec{B}$$

$$\vec{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{l_2} \left[\oint_{l_1} \frac{\vec{a}_{R12} \times d\vec{l}_1}{R_{12}^2} \right] \times d\vec{l}_2$$

$$d\vec{T} = I d\vec{s} \times \vec{B} \quad d\vec{m} = I d\vec{s} \quad d\vec{T} = d\vec{m} \times \vec{B}$$

$$\vec{J}_{ms} = \vec{M} \times \vec{a}_n \quad \vec{J}_m = \nabla \times \vec{M} \quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{M} = X_m \vec{H} \quad \vec{B} = \mu_0 \vec{H} (1 + X_m) \quad \mu_r = 1 + X_m \quad \vec{B} = \mu \vec{H}$$

interface magnética

$$B_{N1} = B_{N2} \quad H_{t1} - H_{t2} = J_s \quad (\vec{H}_1 - \vec{H}_2) \times \vec{a}_{N12} = \vec{J}_s$$

$$M_{t2} = \frac{X_{m2}}{X_{m1}} M_{t1} - X_{m2} J_s \quad \left(\frac{\vec{M}_1}{X_{m1}} - \frac{\vec{M}_2}{X_{m2}} \right) \times \vec{a}_{N12} = \vec{J}_s$$

$$L = \frac{\phi}{I} \quad M_{12} = \frac{N_1 \phi_{12}}{i_1} \quad M_{21} = \frac{N_2 \phi_{21}}{i_2}$$

$$W_H = \frac{1}{2} \int_v \vec{B} \cdot \vec{H} dv \quad L = \frac{2 W_H}{I^2}$$

8. Lei de Faraday e Corrente de Deslocamento

$$f_{em} = -\frac{d\phi}{dt} (V) \quad f_{em} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{s}$$

$$f_{em} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} + \oint (\vec{U} \times \vec{B}) \cdot d\vec{l}$$

9. Equações de Maxwell

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{s} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{H} \cdot d\vec{l} = I + \frac{d}{dt} \int_s \vec{D} \cdot d\vec{s} \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{D} \cdot d\vec{s} = \int_v \rho_v dv \quad \nabla \cdot \vec{D} = \rho_v$$

$$\oint \vec{B} \cdot d\vec{s} = 0 \quad \nabla \cdot \vec{B} = 0$$