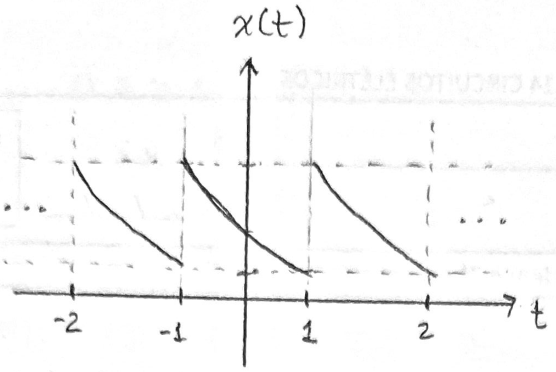
Exercício 1



$T = 2$

$x(t) = e^{-t}, \quad -1 < t < 1$

Coeficientes da S.F.

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\omega_0)t}\,dt$$

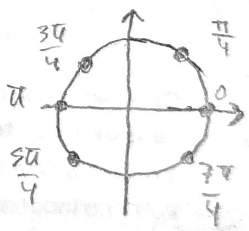
$$a_k = \frac{1}{2(1+jk\omega_0)}\left[e^{(1+jk\omega_0)} - e^{-(1+jk\omega_0)}\right] = \frac{(-1)^k}{2(1+jk\pi)}\left[e - e^{-1}\right]$$

Exercício 2

Para $\langle 8 \rangle = 0, 1, \dots, 7$:

$$x[n] = \sum_{k=0}^{7} a_k e^{jk\omega_0 n}, \quad \omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$a_2 = a_6 = 0$; $a_k = 1$, $k = \{0, 1, 3, 4, 5, 7\}$



Assim,

$$x[n] = 1 + e^{j\frac{\pi}{4}n} + e^{j\frac{3\pi}{4}n} + e^{j\pi n} + e^{j5\frac{\pi}{4}n} + e^{j7\frac{\pi}{4}n}$$

$$= 1 + (-1)^n + e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} + e^{j\frac{3\pi}{4}n} + e^{-j\frac{3\pi}{4}n}$$

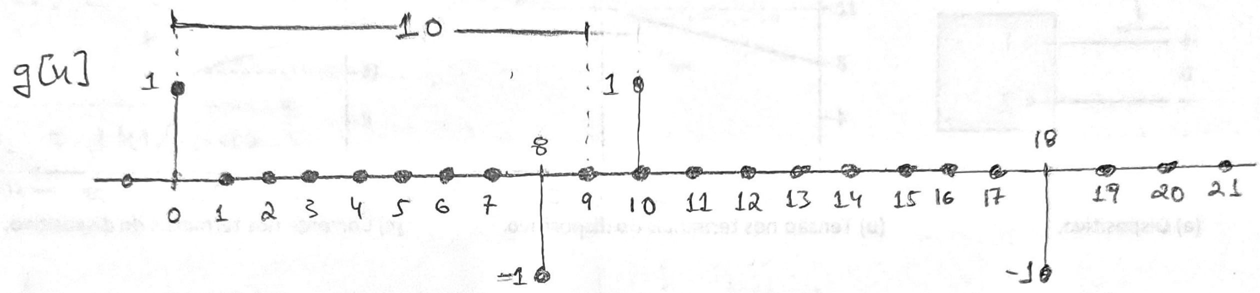
$$= 1 + (-1)^n + 2\cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{3\pi}{4}n\right)$$

Exercício 3

$$x[n] = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & 8 \le n \le 9 \end{cases}$$

$N = 10$

a) graficamente, é possível notar que:



b) Coeficientes da S.F. de $g[n] \xleftrightarrow{S.F.} b_k$

$$b_k = \frac{1}{10} \sum_{n=\langle 10 \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{10}\left(1 - e^{-j8\left(\frac{2\pi}{10}\right)k}\right)$$

c) Coeficientes da S.F. de $x[n] \xleftrightarrow{S.F.} a_k$

$$g[n] = x[n] - x[n-1]$$

$$\Rightarrow b_k = a_k - \underbrace{e^{-j\frac{2\pi}{10}k} \cdot a_k}_{\text{prop. do deslocamento no tempo}} \Rightarrow a_k = \frac{b_k}{1 - e^{-j\left(\frac{2\pi}{10}\right)k}} = \frac{1}{10}\left(\frac{1 - e^{-j8\left(\frac{2\pi}{10}\right)k}}{1 - e^{-j\left(\frac{2\pi}{10}\right)k}}\right)$$

Exercício 4

a) $x(t) \xleftrightarrow{S.F} a_k$; $\frac{d}{dt}x(t) \xleftrightarrow{S.F.} b_k$

Assim,

$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$. Derivando dos dois lados:

$$\frac{d}{dt}x(t) = \sum_{k=-\infty}^{+\infty} a_n \frac{d}{dt}e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} \underbrace{(jk\omega_0)a_k}_{b_k} e^{jk\omega_0 t}$$

$$\Rightarrow \boxed{b_k = jk\omega_0 \cdot a_k}$$

b) Sabemos que:

$$y(t) = \sum_{k=-\infty}^{+\infty} \underbrace{a_k H(jk\omega_0)}_{b_k} e^{jk\omega_0 t} \quad , \quad x(t) \xleftrightarrow{S.F.} a_k \, , \, y(t) \xleftrightarrow{S.F} b_k$$

Podemos obter $H(j\omega)$ usando a auto função de sistema LIT $e^{j\omega t}$, pois $y(t) = H(j\omega)e^{j\omega t}$ quando $x(t) = e^{j\omega t}$.

Assim,

$$\frac{dy(t)}{dt} + 4y(t) = x(t) \rightarrow j\omega \cdot H(j\omega)\cancel{e^{j\omega t}} + 4H(j\omega)\cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$$

$$\Rightarrow \boxed{H(j\omega) = \frac{1}{4 + j\omega}}$$

Logo, $b_k = \frac{a_k}{4 + jk\omega_0}$

i) $x(t) = \cos 2\pi t = \frac{e^{j2\pi t} + e^{-j2\pi t}}{2} \Rightarrow a_{-1} = a_1 = \frac{1}{2}$

$$\Rightarrow b_{-1} = \frac{1}{2}\left(\frac{1}{4 - j2\pi}\right) = \frac{1}{4}\left(\frac{1}{2 - j\pi}\right)$$

$$b_1 = \frac{1}{4}\left(\frac{1}{2 + j\pi}\right)$$

Exercício 4 (cont.)

$$ii)\quad x(t) = \text{sen}\, 4\pi t + \cos\left(6\pi t + \frac{\pi}{4}\right) = \left(\frac{e^{j4\pi t} - e^{-j4\pi t}}{2j}\right) + \frac{e^{j\frac{\pi}{4}}}{2}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$\Rightarrow a_{-2} = \frac{-1}{2j} \Rightarrow b_{-2} = \frac{-1}{2(4\pi + 4j)} = \frac{1}{8}\left(\frac{-1}{\pi + j}\right)$$

$$a_2 = \frac{1}{2j} \Rightarrow b_2 = \frac{1}{2(-4\pi + 4j)} = \frac{1}{8}\left(\frac{1}{-4 + j}\right)$$

$$a_3 = \frac{e^{j\frac{\pi}{4}}}{2} \Rightarrow b_3 = \frac{e^{j\pi/4}}{2(4 + j6\pi)} = \frac{1}{4}\left(\frac{e^{j\pi/4}}{2 + j3\pi}\right)$$

$$a_{-3} = \frac{e^{j\frac{\pi}{4}}}{2} \Rightarrow b_{-3} = \frac{e^{j\pi/4}}{4(2 - j3\pi)}$$

Exercício 5

$$H(j\omega) = \int_{-\infty}^{+\infty} e^{-4|t|} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{(4-j\omega)t}\, dt + \int_{0}^{+\infty} e^{-(4+j\omega)t}\, dt$$

$$\boxed{H(j\omega) = \frac{1}{4-j\omega} + \frac{1}{4+j\omega}}$$

$y(t) \xleftrightarrow{SF} b_k = H(jk\omega_0)\, a_k \quad , \quad x(t) \xleftrightarrow{SF} a_k$, pois

$$y(t) = \sum_{k=-\infty}^{+\infty} \underbrace{a_k H(jk\omega_0)}_{b_k} e^{jk\omega_0 t}$$

a) $x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-n)$

$$a_k = \int_0^1 \sum_{n=-\infty}^{+\infty} \underbrace{\delta(\tau - n)}_{\tau = n} e^{-jk2\pi\tau}\, d\tau$$

. Como $\tau = n$, $n \in \mathbb{Z}$, $0 \leq \tau < 1$,

então $\tau = 0$. Logo, $a_k = 1$, $\forall k$.

$$\Rightarrow \boxed{b_k = \frac{1}{4-j2\pi k} + \frac{1}{4+j2\pi k} \ , \forall k}$$

b) Em um período,

$$x(t) = \begin{cases} 1 & , |t| < 1/4 \\ 0 & , 3/4 < |t| < 1/4 \end{cases}$$

$\rightarrow T = 1$, onda quadrada.

$$\Rightarrow a_0 = 2\left(\frac{1/4}{1}\right) = \frac{1}{2}$$

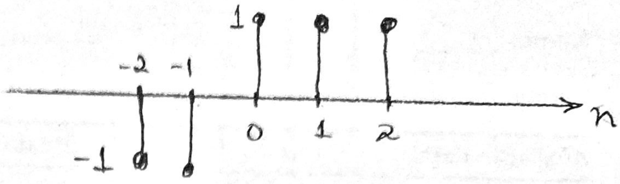
$$a_k = \frac{\text{sen}\left(\frac{k\pi}{2}\right)}{k\pi} \ , k \neq 0 \Rightarrow a_k = \begin{cases} \dfrac{\text{sen}\left(\frac{k\pi}{2}\right)}{k\pi} & , k \neq 0, k \text{ ímpar} \\ 0 & , k \neq 0, k \text{ par} \end{cases}$$

Assim,

$$b_k = \begin{cases} 1/4 & , k = 0 \\ 0 & , k \neq 0, k \text{ par} \\ \dfrac{\text{sen}(k\pi/2)}{k\pi}\left(\dfrac{1}{4-j2\pi k} + \dfrac{1}{4+j2\pi k}\right) & , k \neq 0, k \text{ ímpar} \end{cases}$$

Exercício 6

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n}$$



$$= 1 + e^{-j\omega} - e^{j\omega} + e^{-j2\omega} - e^{j2\omega}$$

Quando $x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$, então

$N = 4$,

$a_k = \frac{1}{4} \sum_{n=0}^{3} \sum_{k=-\infty}^{+\infty} \underbrace{\delta[n-4k]}_{n=4k} e^{-jk\omega_0 n}$. Como $k = \frac{n}{4}$, $0 \leq n \leq 3$, então $k = 0$, pois $n, k \in \mathbb{Z}$.

Assim, $a_k = \frac{1}{4}$, $\forall k$. Além disso,

$$b_k = H(e^{jk\frac{\pi}{2}})\, a_k = \frac{1}{4}\left(1 + e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}} \underbrace{+ e^{-jk\pi} - e^{jk\pi}}_{=0}\right)$$

$$= \frac{1}{4}\left(1 - e^{jk\frac{\pi}{2}} + e^{-jk\frac{\pi}{2}}\right), \forall k.$$