Cálculo II

Trabalho 1: Funções Hiperbólicas

a) $f(x) = \operatorname{senh}(x) + \cosh(x)$

$\operatorname{senh}(x) = \dfrac{e^x - e^{-x}}{2}$ $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$

$f(x) = \dfrac{e^x - e^{-x} + e^x + e^{-x}}{2} = \dfrac{2e^x}{2} = e^x$

$f(0) = e^0 = 1$

$\nexists\, x \in \mathbb{R} \mid f(x) \leq 0$

$\lim\limits_{x \to \infty} f(x) = \infty$

$f'(x) = e^x$. A função é crescente em todo domínio. Logo, os limites são dados por $\lim\limits_{x\to-\infty}$ e $\lim\limits_{x\to\infty}$

$\lim\limits_{x \to -\infty} f(x) = 0$ e $\lim\limits_{x \to \infty} f(x) = \infty$

Portanto o inferior $= 0$ e não há superior

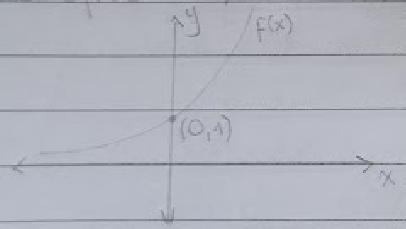
⇒ Assim, $D(f) = \mathbb{R}$ e $Im(f) =]0, \infty)$

⇒ $f(x)$ é limitada inferiormente por 0.

⇒ $f'(x) = e^x$ $\nexists\, x \in \mathbb{R} \mid f'(x) = 0$.

Logo, não há máximos ou mínimos globais

⇒ Gráfico



02) $(f^{-1})'(b) = \dfrac{1}{f'(f^{-1}(b))}$

a) SENH

$f(x) = \operatorname{senh} x$

$f'(x) = \cosh x$

$f^{-1}(x) = \ln(x + \sqrt{x^2+1})$

$(f^{-1})'(b) = \dfrac{1}{\cosh(\ln(x + \sqrt{x^2+1}))}$

$= \left(\dfrac{e^{\ln(x+\sqrt{x^2+1})} + e^{-\ln(x+\sqrt{x^2+1})}}{2}\right)^{-1}$

$= \left(\dfrac{x+\sqrt{x^2+1} + \dfrac{1}{x+\sqrt{x^2+1}}}{2}\right)^{-1}$

$\dfrac{1}{(x+\sqrt{x^2+1})} \dfrac{(-x+\sqrt{x^2+1})}{(-x+\sqrt{x^2+1})}$

$\dfrac{-x+\sqrt{x^2+1}}{x^2+1-x^2} = \sqrt{x^2+1} - x$

Assim

$= \left(\dfrac{x+\sqrt{x^2+1}+\sqrt{x^2+1}-x}{2}\right)^{-1}$

$= \left(\sqrt{x^2+1}\right)^{-1}$

$(f^{-1})'(b) = \left(\sqrt{b^2+1}\right)^{-1}$

b) COSH $f(x) = \cosh x$

$f'(x) = \operatorname{senh} x$

$f^{-1}(x) = \ln(x+\sqrt{x^2-1})$

$$(f^{-1})'(x) = \frac{1}{\operatorname{senh}(\ln(x+\sqrt{x^2-1}))}$$

$$= \frac{e^{\ln(x+\sqrt{x^2-1})} - e^{-\ln(x+\sqrt{x^2-1})}}{2}$$

$$= \left(\frac{x+\sqrt{x^2-1} - \frac{1}{x+\sqrt{x^2-1}}}{2}\right)^{-1}$$

$$\frac{1}{(x+\sqrt{x^2-1})(-x+\sqrt{x^2-1})} \cdot (-x+\sqrt{x^2-1})$$

$$\sqrt{x^2-1} - x = -\sqrt{x^2-1}$$

$$x^2-1-x^2$$

$$\left(\frac{x+\sqrt{x^2-1} - (-\sqrt{x^2-1}+x)}{2}\right)^{-1}$$

$$= \left(\frac{2\sqrt{x^2-1}}{2}\right)^{-1} = \left(\sqrt{x^2-1}\right)^{-1}$$

$$(f^{-1})'(b) = \left(\sqrt{b^2-1}\right)^{-1}$$

c) TGH $f(x) = \operatorname{tgh} x$

$f'(x) = \operatorname{sech}^2 x$

$f^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$

$$(f^{-1})'(x) = \frac{1}{\operatorname{sech}^2\left(\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\right)}$$

Considere $u = \frac{1}{2}\ln\left(\sqrt{\frac{1+x}{1-x}}\right)$

$$= \left(\frac{1}{\left(\frac{e^u+e^{-u}}{2}\right)^2}\right)^{-1}$$

$$= \left(\frac{2}{\sqrt{\frac{1+x}{1-x}} + \sqrt{\frac{1-x}{1+x}}}\right)^2$$

$$= 4 \Big/ \left(\frac{1+x-1-x}{(\sqrt{1-x})(\sqrt{1+x})}\right)^2$$

$$= 4 \Big/ \left(\frac{2}{\sqrt{x^2+1}}\right)^2$$

$$= 1 \Big/ x^2+1$$

$$= (x^2+1)^{-1}$$

$$(f^{-1})'(b) = (b^2+1)^{-1}$$