L1 Language

$$
\begin{array}{l}
e \in \text{L1} \\
\quad e ::= n \\
\qquad \mid b \\
\qquad \mid e_1 \ \texttt{op} \ e_2 \\
\qquad \mid \texttt{if} \ e_1 \ \texttt{then} \ e_2 \ \texttt{else} \ e_3 \\
\qquad \mid x \\
\qquad \mid e_1 \ e_2 \\
\qquad \mid \texttt{fn} \ x : T \Rightarrow e \\
\qquad \mid \texttt{let} \ x : T = e_1 \ \texttt{in} \ e_2 \\
\qquad \mid \texttt{let rec} \ f : T_1 \to T_2 = (\texttt{fn} \ y : T_1 \Rightarrow e_1) \ \texttt{in} \ e_2 \\
\qquad \mid \texttt{nil} : T \\
\qquad \mid e_1 :: e_2 \\
\qquad \mid \texttt{isempty} \ e \\
\qquad \mid \texttt{hd} \ e \\
\qquad \mid \texttt{tl} \ e \\
\qquad \mid \texttt{match} \ e_1 \ \texttt{with} \mid \texttt{nil} \to e_2 \mid x :: xs \to e_3 \\
\qquad \mid \texttt{nothing} : T \\
\qquad \mid \texttt{just} \ e \\
\qquad \mid \texttt{match} \ e_1 \ \texttt{with} \mid \texttt{nothing} \to e_2 \mid \texttt{just} \ x \to e_3
\end{array}
$$

$$T ::= \texttt{int} \mid \texttt{bool} \mid T_1 \to T_2 \mid \texttt{maybe} \ T \mid T \ \texttt{list}$$

where:
$n \in Z$
$b \in \{\texttt{true}, \texttt{false}\}$
$x \in$ Ident
$\texttt{op} \in \{+, -, *, \div, =, <, >, \leq, \geq, \texttt{and}, \texttt{or}\}$

Type System

$$\frac{n \in Z}{\Gamma \vdash n : \text{int}} \text{ (T-Num)}$$

$$\frac{b \in \{\text{true}, \text{false}\}}{\Gamma \vdash b : \text{bool}} \text{ (T-Bool)}$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : T \quad \Gamma \vdash e_3 : T}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : T} \text{ (T-If)}$$

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \text{ (T-Var)}$$

$$\frac{\Gamma \vdash e_1 : T_1 \to T_2 \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash e_1\ e_2 : T_2} \text{ (T-App)}$$

$$\frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash \text{fn } x : T_1 \Rightarrow e : T_1 \to T_2} \text{ (T-Fn)}$$

$$\frac{\Gamma \vdash e_1 : T \quad \Gamma, x : T \vdash e_2 : T'}{\Gamma \vdash \text{let } x : T = e_1 \text{ in } e_2 : T'} \text{ (T-Let)}$$

$$\frac{\Gamma, f : T_1 \to T_2, x : T_1 \vdash e_1 : T_2 \quad \Gamma, f : T_1 \to T_2 \vdash e_2 : T}{\Gamma \vdash \text{let rec } f : T_1 \to T_2 = (\text{fn } X : T_1 \Rightarrow e_1) \text{ in } e_2 : T} \text{ (T-LetRec)}$$

$$\frac{}{\Gamma \vdash \text{nil} : T : \text{list } T} \text{ (T-Nil)}$$

$$\frac{\Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : \text{list } T}{\Gamma \vdash e_1 :: e_2 : \text{list } T} \text{ (T-Cons)}$$

$$\frac{\Gamma \vdash e : T \text{ list}}{\Gamma \vdash \text{isempty } e : \text{bool}} \text{ (T-IsEmpty)}$$

$$\frac{\Gamma \vdash e : \text{list } T}{\Gamma \vdash \text{hd } e : T} \text{ (T-Head)}$$

$$\frac{\Gamma \vdash e : \text{list } T}{\Gamma \vdash \text{tl } e : \text{list } T} \text{ (T-Tail)}$$

$$\frac{\Gamma \vdash e_1 : \text{list } T \quad \Gamma \vdash e_2 : T' \quad \Gamma, x : T, xs : \text{list } T \vdash e_3 : T'}{\Gamma \vdash \text{match } e_1 \text{ with nil} \to e_2 \mid x :: xs \to e_3 : T'} \text{ (T-MatchList)}$$

$$\frac{}{\Gamma \vdash \text{nothing} : T : \text{maybe } T} \text{ (T-Nothing)}$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \text{just } e : \text{maybe } T} \text{ (T-Just)}$$

$$\frac{\Gamma \vdash e_1 : \text{maybe } T \quad \Gamma \vdash e_2 : T' \quad \Gamma, x : T \vdash e_3 : T'}{\Gamma \vdash \text{match } e_1 \text{ with nothing} \to e_2 \mid \text{just } x \to e_3 : T'} \text{ (T-MatchMaybe)}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int} \quad op \in \{+, -, *, \div\}}{\Gamma \vdash e_1 \text{ op } e_2 : \text{int}} \text{ (T-BinopArith)}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int} \quad op \in \{=, <, >, \leq, \geq\}}{\Gamma \vdash e_1 \text{ op } e_2 : \text{bool}} \text{ (T-BinopComp)}$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool} \quad op \in \{\text{and}, \text{or}\}}{\Gamma \vdash e_1 \text{ op } e_2 : \text{bool}} \text{ (T-BinopLogic)}$$

Big-step Operational Semantics with Environments

$$v \in \text{Values} \qquad \rho \in \text{Env}$$

$$v ::= n \mid b \mid \langle x, e, \rho \rangle \mid \langle f, x, e, \rho \rangle \qquad \rho ::= [] \mid \rho, x \mapsto v$$

$$\frac{}{\rho \vdash n \Downarrow n} \text{ (E-Num)}$$

$$\frac{}{\rho \vdash b \Downarrow b} \text{ (E-Bool)}$$

$$\frac{x \mapsto v \in \rho}{\rho \vdash x \Downarrow v} \text{ (E-Var)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{true} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \text{ (E-IfTrue)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{false} \quad \rho \vdash e_3 \Downarrow v}{\rho \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \text{ (E-IfFalse)}$$

$$\frac{}{\rho \vdash (\text{fn } x : T \Rightarrow e) \Downarrow \langle x, e, \rho \rangle} \text{ (E-Fn)}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle x, e, \rho' \rangle \quad \rho \vdash e_2 \Downarrow v_2 \quad \rho', x \mapsto v' \vdash e \Downarrow v}{\rho \vdash e_1\ e_2 \Downarrow v} \text{ (E-App)}$$

$$\frac{\rho \vdash e_1 \Downarrow v' \quad \rho, x \mapsto v' \vdash e_2 \Downarrow v}{\rho \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v} \text{ (E-Let)}$$

$$\frac{\rho, f \mapsto \langle f, x, e_1, \rho \rangle \vdash e_2 \Downarrow v}{\rho \vdash \text{let rec } f = \text{fn } x \to e_1 \text{in } e_2 \Downarrow v} \text{ (E-LetRec)}$$

$$\frac{}{\rho \vdash \text{nil} : T \Downarrow \text{nil}} \text{ (E-Nil)}$$

$$\frac{\rho \vdash e_1 \Downarrow v_1 \quad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \text{ (E-Cons)}$$

$$\frac{\rho \vdash e \Downarrow v}{\rho \vdash \text{isempty } e \Downarrow \text{true}} \text{ (E-IsEmptyNil)}$$

$$\frac{\rho \vdash e \Downarrow v_1 :: v_2}{\rho \vdash \text{isempty } e \Downarrow \text{false}} \text{ (E-IsEmptyCons)}$$

$$\frac{\rho \vdash e \Downarrow v_1 :: v_2}{\rho \vdash \text{hd } e \Downarrow v_1} \text{ (E-Head)}$$

$$\frac{\rho \vdash e \Downarrow v_1 :: v_2}{\rho \vdash \text{tl } e \Downarrow v_2} \text{ (E-Tail)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{nil} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with nil} \to e_2 \mid x :: xs \to e_3 \Downarrow v} \text{ (E-MatchListNil)}$$

$$\frac{\rho \vdash e_1 \Downarrow v_1 :: v_2 \quad \rho, x \mapsto v_1, xs \mapsto v_2 \vdash e_3 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with nil} \to e_2 \mid x :: xs \to e_3 \Downarrow v} \text{ (E-MatchListCons)}$$

$$\frac{}{\rho \vdash \text{nothing} : T \Downarrow \text{nothing}} \text{ (E-Nothing)}$$

$$\frac{\rho \vdash e \Downarrow v}{\rho \vdash \text{just } e \Downarrow \text{just } v} \text{ (E-Just)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{nothing} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with nothing} \to e_2 \mid \text{just } x \to e_3 \Downarrow v} \text{ (E-MatchMaybeNothing)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{just } v_1 \quad \rho, x \mapsto v_1 \vdash e_3 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with nothing} \to e_2 \mid \text{just } x \to e_3 \Downarrow v} \text{ (E-MatchMaybeJust)}$$

$$\frac{\rho \vdash e_1 \Downarrow n_1 \quad \rho \vdash e_2, \Downarrow n_2 \quad [[n]] = [[n_1]] \text{ op } [[n_2]] \quad op \in \{+, -, *\}}{\rho \vdash e_1 \text{ op } e_2 \Downarrow n} \text{ (E-BinOpArith)}$$

$$\frac{\rho \vdash e_1 \Downarrow n_1 \quad \rho \vdash e_2, \Downarrow n_2 \quad [[n]] = [[n_1]] \div [[n_2]] \quad n_2 \neq 0}{\rho \vdash e_1 \div e_2 \Downarrow n} \text{ (E-BinOpDiv)}$$

$$\frac{\rho \vdash e_1 \Downarrow n_1 \quad \rho \vdash e_2, \Downarrow n_2 \quad [[b]] = [[n_1]] \text{ op } [[n_2]] \quad op \in \{<, >, \leq, \geq\}}{\rho \vdash e_1 \text{ op } e_2 \Downarrow b} \text{ (E-BinOpComp)}$$

$$\frac{\rho \vdash e_1 \Downarrow b_1 \quad \rho \vdash e_2, \Downarrow b_2 \quad [[b]] = [[b_1]] \text{ op } [[b_2]] \quad op \in \{and, or\}}{\rho \vdash e_1 \text{ op } e_2 \Downarrow b} \text{ (E-BinOpLogic)}$$