

1. Deque

a)

b)

```
predicate Valid() {  
  // (i) consistency of the concrete state variables;  
  size <= list.Length && start < list.Length  
  // (ii) consistency between concrete and abstract (ghost) state variables  
  && capacity == list.Length  
  && elems == if start + size <= list.Length  
               then list[start .. start+size]  
               else list[start..] + list[.. (start + size) - list.Length]  
}
```

c)

```
constructor (capacity: nat)  
  requires capacity > 0  
  ensures elems == [] && this.capacity == capacity  
{  
  list := new T[capacity];  
  start := 0;  
  size := 0;  
  elems := [];  
  this.capacity := capacity;  
}  
  
predicate method isEmpty()  
  ensures isEmpty() <==> |elems| == 0  
{  
  size == 0  
}  
  
predicate method isFull()  
  ensures isFull() <==> |elems| == capacity  
{  
  size == list.Length  
}  
  
function method front() : T  
  requires !isEmpty()  
  ensures front() == elems[0]  
{  
  list[start]  
}
```

```

function method back() : T
  requires !isEmpty()
  ensures back() == elems[ |elems| - 1 ]
{
  list[(start + size - 1) % list.Length]
}

method push_back(x : T)
  requires !isFull()
  ensures elems == old(elems) + [x]
{
  if start + size + 1 <= list.Length {
    list[start + size] := x;
  }
  else {
    list[start + size - list.Length] := x;
  }
  size := size + 1;
  elems := elems + [x];
}

method pop_back()
  requires !isEmpty()
  ensures elems == old(elems[..|elems|-1])
{
  size := size - 1;
  elems := elems[..|elems|-1];
}

method push_front(x : T)
  requires !isFull()
  ensures elems == [x] + old(elems)
{
  start := if start > 0 then start - 1 else list.Length - 1;
  list[start] := x;
  size := size + 1;
  elems := [x] + elems;
}

method pop_front()
  requires !isEmpty()
  ensures elems == old(elems[1..])
{
  start := if start + 1 < list.Length then start + 1 else 0;
  size := size - 1;
  elems := elems[1..];
}

```

d)

e) Using state abstraction, and also generics (which requires initializer to be passed to new T[]).

```

class {:autocontracts} Deque<T> {
  // (Private) concrete state variables

```

```

const list: array<T>; // circular array, from list[start] (front) to
                      // list[(start+size-1) % list.Length] (back)

var start : nat;
var size : nat;

// State abstraction functions
function elems(): seq<T> {
    if start + size <= list.Length
    then list[start .. start+size]
    else list[start..] + list[.. start + size - list.Length]
}
function capacity(): nat {
    list.Length
}

predicate Valid() {
    size <= list.Length && start < list.Length
}

constructor (capacity: nat, initializer: nat -> T)
    requires capacity > 0
    ensures elems() == [] && this.capacity() == capacity
{
    list := new T[capacity](initializer);
    start := 0;
    size := 0;
}

predicate method isEmpty()
    ensures isEmpty() <==> |elems()| == 0
{
    size == 0
}

predicate method isFull()
    ensures isFull() <==> |elems()| == capacity()
{
    size == list.Length
}

function method front() : T
    requires !isEmpty()
    ensures front() == elems()[0]
{
    list[start]
}

function method back() : T
    requires !isEmpty()
    ensures back() == elems()[ |elems()| - 1 ]
{
    list[(start + size - 1) % list.Length]
}

method push_back(x : T)

```

```

    requires !isFull()
    ensures elems() == old(elems()) + [x]
  {
    if start + size + 1 <= list.Length {
      list[start + size] := x;
    }
    else {
      list[start + size - list.Length] := x;
    }
    size := size + 1;
  }

  method pop_back()
    requires !isEmpty()
    ensures elems() == old(elems())[..|elems()|-1])
  {
    size := size - 1;
  }

  method push_front(x : T)
    requires !isFull()
    ensures elems() == [x] + old(elems())
  {
    start := if start > 0 then start - 1 else list.Length - 1;
    list[start] := x;
    size := size + 1;
  }

  method pop_front()
    requires !isEmpty()
    ensures elems() == old(elems())[1..]
  {
    start := if start + 1 < list.Length then start + 1 else 0;
    size := size - 1;
  }
}

// A simple test scenario.
method testDeque() {
  var q := new Deque<int>(3, i => 0);
  assert q.isEmpty();
  q.push_front(1);
  assert q.front() == 1;
  assert q.back() == 1;
  q.push_front(2);
  assert q.front() == 2;
  assert q.back() == 1;
  q.push_back(3);
  assert q.front() == 2;
  assert q.back() == 3;
  assert q.isFull();
  q.pop_back();
  assert q.front() == 2;
  assert q.back() == 1;
  q.pop_front();
  assert q.front() == 1;
}

```

```
    assert q.back() == 1;
    q.pop_front();
    assert q.isEmpty();
}
```

2. [Home Work] Priority Queue

TODO