


Coventry University
Assignment Submission & Declaration Form
PLEASE COMPLETE ALL HIGHLIGHTED SECTIONS

Module code: 124MS	Faculty: EC
Student ID: 5497124	
Due Date: 16/06/2014 16:00	
Category: Unspecified	
Submit Type: Standard	
Assessment Type: Individual	

Surname: Morato Almeida
Forename(s): Vitor
Signed:
Date:

LATE WORK:

If work is being submitted AFTER the deadline a short/long deferral application MUST also have been submitted for your coursework. Failure to have completed and submitted an application or to supply authorised evidence to support an approved short/long deferral will result in work being awarded a mark of 0%.

Course Title: <i>Computer Science</i>	Cohort / Occurrence Group: 1314B
Module Title: <i>Logic and Sets</i>	Assessment Reference no: Cw
Assessment Title:	

Module Leader: <i>Dr Robert Low</i>	Module Tutor:
Seminar Group / Tutorial Tutor and Group (if different to above):	

I have read the Coventry University rules and regulations on the submission of academic work and in particular the sections concerning misconduct in assessment, including plagiarism and collusion. This assessment is all my (or my group's) own work and has not been copied in part or in whole from any other source, except for any clearly marked up quotations. It complies with the university regulations

I acknowledge that in submitting this work I am declaring that I (or my group) are fit to be assessed and that a deferral may not be applied for following hand in.

I confirm that an electronic version of the item to be assessed (where appropriate) is available and will be provided to the university within 48 hours if requested by the course team.

I confirm that I (or my group) have abided by all applicable Professional Codes of Conduct and I/we will protect confidential information obtained from other individuals.

In respect of group assignments, the submission of this work is made on the basis that all group members are jointly and severally responsible for the work presented for assessment and that by handing in this item for assessment all group members acknowledge and confirm the statements above and cover sheets for all group members are attached.

I understand and accept that upon submitting this work, I will receive an email confirmation of receipt via my University email account.

Mark Awarded	Feedback Summary (see attached sheet for detailed feedback against assessment criteria)
Mark Signature	

124MS - Coursework 1

Vitor Morato Almeida

June 4, 2014

1. Find the truth value of $P \vee (Q \Rightarrow R)$ if P is false, Q is true, and R is false. What is the truth value of this expression if the brackets are removed?

P: *f*
Q: *t*
R: *f*

Substituting in the truth values, we obtain

$$f \vee (t \Rightarrow f)$$

As there is a set of brackets, the brackets have to be done first as they have a higher order of precedence. So $(Q \Rightarrow R)$, becomes $(t \Rightarrow f)$ which we can say is false, as if Q is true and R is false the truth value of this implication is false. This then becomes

$$f \vee f$$

which is equivalent to

$$f$$

This is as can only be true if both parts are true, so as both parts are false, the expression has the truth value *f* (false).

If the brackets are removed, then the order of the procedure changes. The expression now becomes

$$P \vee Q \Rightarrow R$$

Substituting in the truth values, we obtain

$$f \vee t \Rightarrow f$$

Since the \vee has higher priority than \Rightarrow , we have to work out the $f \vee t$ first. This gives us the value true as at least one of parts is true.

$$t \Rightarrow f$$

this gives us

$$f$$

As true implies false, the truth value is false, meaning that the expression has truth value false. As once the brackets were removed the order of the procedure changes, resulting in the steps taking to complete the expression to be different, but in this case both answers are the same.

2. Consider the proposition $((C \wedge (C \Rightarrow S)) \wedge (\neg S \Rightarrow \neg V)) \Rightarrow \neg C \vee V$.

- (a) Interpreting these symbols as the atomic sentences

C : I am a careful programmer.

S : My program will meet its specification.

V : My program will pass the validation test.

interpret the above proposition as an argument in natural English.

I am a careful programmer and If I am a careful programmer then my program will meet its specification, and If my program will not meet its specification then my program will not pass the validation test. This implies that I am not a careful programmer or my program will pass the validation test.

- (b) By means of a truth table, find out whether the argument is valid.

C	S	V	((C \wedge (C \Rightarrow S)) \wedge (\neg S \Rightarrow \neg V)) \Rightarrow \neg C \vee V										\Rightarrow	\neg C	\vee	V
t	t	t	t	t	t	t	t	t	f	t	f	t	t	f	t	t
t	t	f	t	t	t	t	t	f	f	t	t	t	t	f	f	f
t	f	t	t	f	t	f	f	f	t	f	f	t	t	f	t	t
t	f	f	t	f	t	f	f	f	t	t	t	t	t	f	f	f
f	t	t	f	f	f	t	t	f	f	t	f	t	t	t	t	t
f	t	f	f	f	f	t	t	f	f	t	t	t	t	t	t	f
f	f	t	f	f	f	t	f	f	t	t	t	t	t	t	t	t
f	f	f	f	f	f	t	f	f	t	t	t	t	t	t	t	f
1	2	3	4	10	5	9	6	12	7	11	8	16	13	15	14	

Argument is valid as the proposition is a tautology, meaning that the truth values in column 16 are all true.

3. Consider the following collection of statements, which are part of the specification of the automatic control system for a nuclear power plant:

If the pile overheats, the emergency valve is opened and a warning sounds.

If a warning sounds, the operator goes outside. If the emergency valve is opened, the operator does not go outside.

- (a) Choose symbols to represent the atomic sentences in the above argument, and hence express the statements in terms of propositional calculus.

Symbols chosen to represent the atomic sentences:

P: Pile overheats

V: Emergency valve is opened

S: Warning sounds

O: The operator go outside

The statements expressed in terms of propositional calculus:

$$P \Rightarrow V \wedge S$$

$$S \Rightarrow O$$

$$V \Rightarrow \neg O$$

- (b) Construct a formal proof (table of assertions and justifications) showing that the statement The pile does not overheat follows from these hypotheses.

Hint: You may find it useful to use some form of indirect proof.

Considering the three statements $P \Rightarrow V \wedge S$, $S \Rightarrow O$, and $V \Rightarrow \neg O$ as H_1, H_2 , and H_3 respectively, we then get the following hypotheses (H):

$$H_1: P \Rightarrow V \wedge S$$

$$H_2: S \Rightarrow O$$

$$H_3: V \Rightarrow \neg O$$

$$C: \neg P$$

Using the proof by contradiction to give us

$$H_1: P \Rightarrow V \wedge S$$

$$H_2: S \Rightarrow O$$

$$H_3: V \Rightarrow \neg O$$

$$H_4: P$$

$$C: f$$

	Assertion	Justification
1	P	H_4
2	$P \Rightarrow V \wedge S$	H_1
3	$V \wedge S$	1,2, modus ponens
4	S	3, simplification
5	$S \Rightarrow O$	H_2
6	O	4, 2, modus ponens
7	V	3, simplification
8	$V \Rightarrow \neg O$	H_3
9	$\neg O$	7, 8, modus tollens
10	$O \wedge \neg O$	6, 9, conjunction
	f	

We have proved that

$$P \Rightarrow V \wedge S, S \Rightarrow O, V \Rightarrow \neg O, P \vdash f$$

which is equivalent to, by the proof by contradiction:

$$P \Rightarrow V \wedge S, S \Rightarrow O, V \Rightarrow \neg O \vdash \neg P$$

So the initial argument is valid.

4. **I teach a class of 33 students. 13 are friendly, 18 are hard-working and 18 are punctual. 6 are friendly and hard-working, 9 are punctual and hard-working, and 5 are friendly and punctual. How many are friendly and punctual, but not hard-working?**

First I denoted the different sets some letters. There are 3 set and all together there are 33 students. The 3 sets are classified below:

F: denoted to friendly students set

H: denoted to hard-working students set

P: denoted to punctual students set

We then get the general statements:

$$|F| = 13$$

$$|H| = 18$$

$$|P| = 18$$

As there are 33 students in total, which includes friendly, hard-working and punctual sets, we can come to the following conclusion.

$$|F \cup H \cup P| = 33$$

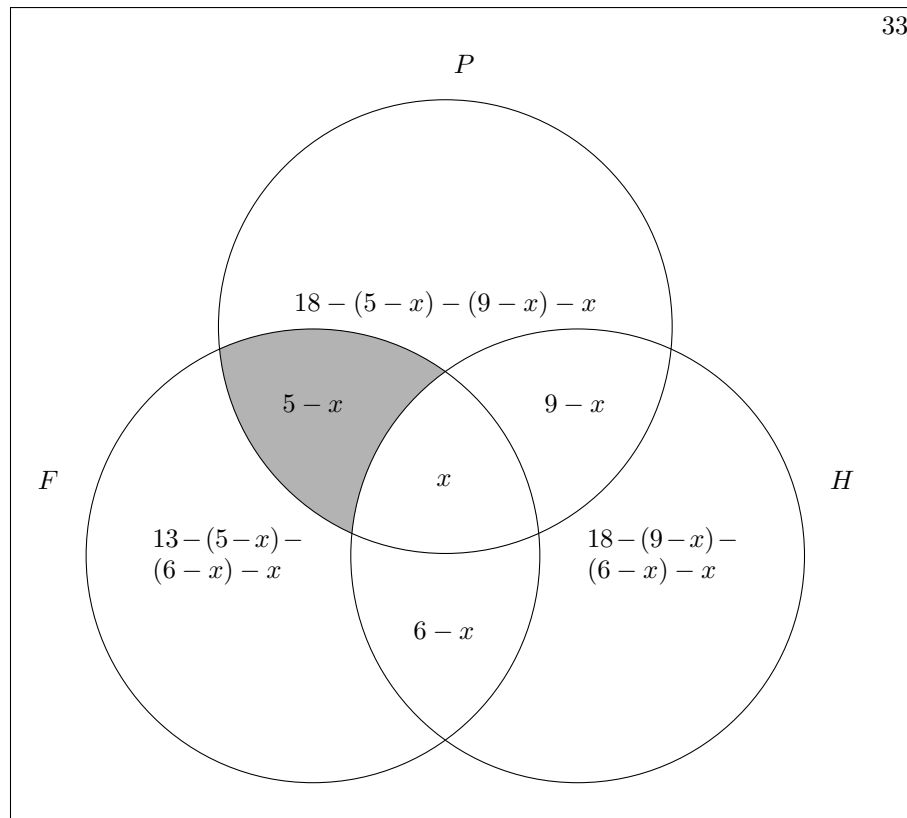
Also from the question, we know that 6 students are friendly and hard-working, 9 are punctual and hard-working, and 5 are friendly and punctual. From this we can form the following statements:

$$|F \cap H| = 6$$

$$|P \cap H| = 9$$

$$|F \cap P| = 5$$

the following Venn diagram illustrates all the information that we have for now.



Using the formula:

$$|F \cup H \cup P| = |F| + |H| + |P| - |F \cap H| - |P \cap H| - |F \cap P| + |F \cap H \cap P|$$

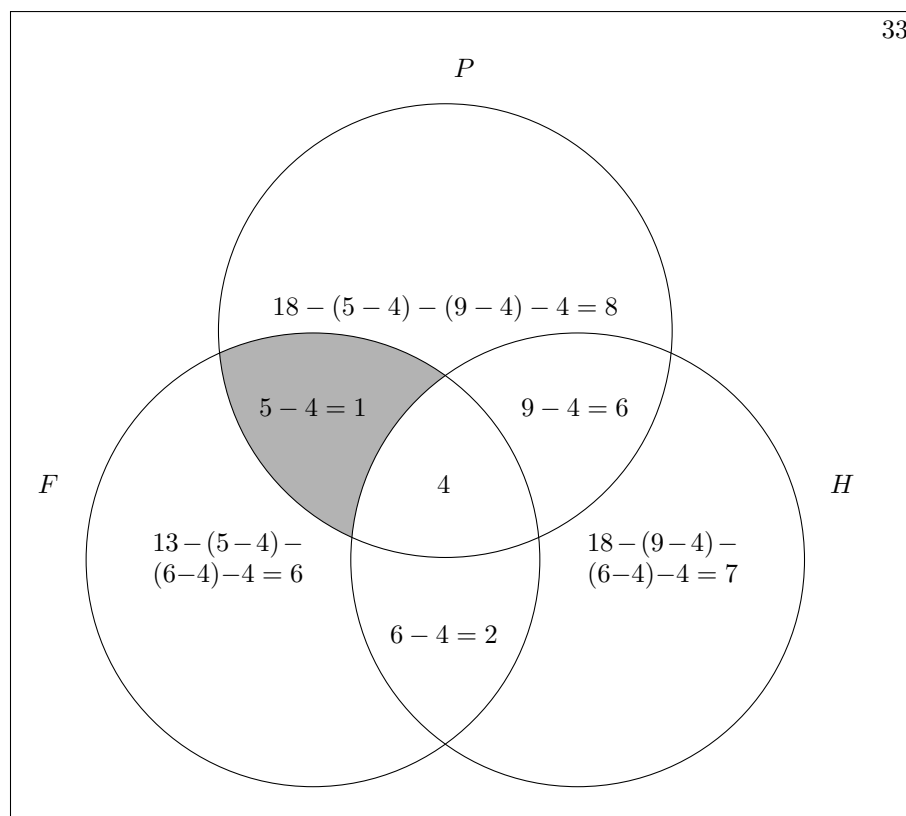
Substituting in the values I know from the question, will help me find out the x value I need for the Venn diagram.

$$33 = 13 + 18 + 18 - 6 - 9 - 5 + |F \cap H \cap P|$$

$$|F \cap H \cap P| = 4$$

This shows that 4 students are friendly, hard-working and punctual, so only 4 students contain in all 3 sets.

completing the Venn diagram, by substituting $x = 4$, we have.



Observing the Venn diagram, specifically the grey area, we can conclude that only 1 student is friendly and punctual but not hard-working.

5. -

- (a) Use a membership table to show that the sets $A \cup B$ and $A \cup C$ are not equal in general.

A	B	C	A	\cup	B	A	\cup	C
I	I	I	I	I	I	I	I	I
I	I	O	I	I	I	I	I	O
I	O	I	I	I	O	I	I	I
I	O	O	I	I	O	I	I	O
O	I	I	O	I	I	O	I	I
O	I	O	O	I	I	O	O	O
O	O	I	O	O	O	O	I	I
O	O	O	O	O	O	O	O	O
1	2	3	4	6	5	7	9	8

Since column 6 and 9 do not contain the same values, they are not the same, so therefore the sets are not always equal.

- (b) Use a hybrid table to show that the sets $A \cup B$ and $A \cup C$ are equal if $B = C$.

A	B	C	B	=	C	\Rightarrow	A	\cup	B	=	A	\cup	C
I	I	I	I	t	I	t	I	I	I	t	I	I	I
I	I	O	I	f	O	t	I	I	I	t	I	I	O
I	O	I	O	f	I	t	I	I	O	t	I	I	I
I	O	O	O	t	O	t	I	I	O	t	I	I	O
O	I	I	I	t	I	t	O	I	I	t	O	I	I
O	I	O	I	f	O	t	O	I	I	f	O	O	O
O	O	I	O	f	I	t	O	O	O	f	O	I	I
O	O	O	O	t	O	t	O	O	O	t	O	O	O
1	2	3	4	6	5	14	7	9	8	13	10	12	11

The hybrid table shows that the sets $A \cup B$ and $A \cup C$ are equal if $B = C$. This shows that the corresponding proposition is a tautology so claims to be true and we can see this by looking at column 14, as it contains all truth values of true.

- (c) **Give an example of a situation where the equality holds, even though $B \neq C$. Hint take A , B and C to be subsets of the universal set $\{x, y, z\}$.**

If sets B and C are both subsets of A , they should hold the same equality for $A \cup B$ and $A \cup C$ even if B and C are different; here is an example where B and C contain the different values in the set.

So considering the following subsets:

$$A = \{x, y, z\}$$

$$B = \{x\}$$

$$C = \{y\}$$

This gives:

$$A \cup B = \{x, y, z\} \cup \{x\}$$

$$A \cup B = \{x, y, z\}$$

and

$$A \cup C = \{x, y, z\} \cup \{y\}$$

$$A \cup C = \{x, y, z\}$$

Both sets contain the same values so this shows that they hold the same equality.