A Christmas Game

A family decided to play a different kind of Secret Santa on Christmas. The rules are as follows:

- Each participant buys a present, wraps it so nobody can see the content and places it together with the other presents.

- A distinct integer number is given to each player indicating their turn.

- On his turn, the player has two possibilities, to pick a gift from the pile, reveal its content and keep it, or to take a gift from one of the previous players. The participant who had his gift taken is obligated to pick another present from the pile.

- The game keeps going until the player who was assigned the last number has played.

Suppose there are **n** people playing and your number is **k.**

Constraints: n > 3, 2 <= k <= n

Consider two scenarios:

(a) **Simplified:** In this case nobody is going to take presents from you, which means the present you pick from the pile or take from somebody will remain with you until the end. Your strategy will be to take the best gift revealed so far. ***Calculate the probability this strategy gives you a better present than picking one from the pile.***

(b) **Real:** In this scenario players can take gifts from you and from each other. Their plan will always be to take the best gift revealed so far. Knowing their strategy, you reshaped yours. You will choose the best present that you are sure nobody will take away, if that is not possible, you will just take the worst present.

For instance:

- Suppose n = 10 and k = 7. It is your turn; six players have played, there are still three more to go. If you take the best gift so far, one of the three players can take it from you, same goes for the second best and the third best. However, taking the fourth best you are sure you'll keep it till the end, because the participants left will, in the worst-case scenario, take until the third best.
- Suppose now that n = 10 and k = 3. It is your turn; two players have played, there are still seven left. You can't use the strategy explained on the previous example, so you take the worst present so far, which means, the second best.

Calculate the probability this strategy gives you a better present than picking one from the pile.