Lecture 5: Neural Networks II

André Martins, Francisco Melo, Mário Figueiredo



Deep Learning Course, Winter 2023-2024

Today's Roadmap

Last lecture was about neural networks:

- From perceptron to multi-layer perceptron
- Feed-forward neural networks
- Activation funcions: sigmoid, tanh, relu, ...
- Activation maps: softmax, sparsemax, ...
- Non-convex optimization and local minima
- Universal approximation theorem
- Gradient backpropagation

Today: autodiff, regularization, optimization, tricks of the trade.

Outline

- Training Neural Networks
 Automatic Differentiation
 Regularization
- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

Outline

Training Neural Networks
Automatic Differentiation

Regularization

- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

Recap: Forward Propagation

Now assume $L \ge 1$ hidden layers:

• **Hidden layer pre-activation** (define $h^{(0)} = x$, for convenience):

$$z^{(\ell)}(x) = m{W}^{(\ell)} h^{(\ell-1)}(x) + m{b}^{(1)},$$

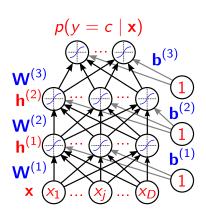
with
$$\mathbf{W}^{(\ell)} \in \mathbb{R}^{K_\ell \times K_{\ell-1}}$$
 and $\mathbf{b}^{(\ell)} \in \mathbb{R}^{K_\ell}$.

• Hidden layer activation:

$$h^{(\ell)}(x) = oldsymbol{g}(z^{(\ell)}(x)).$$

Output layer activation:

$$f(x) = o(z^{(L+1)}(x)) = o(W^{(L+1)}h^{(L)} + b^{(L+1)}).$$



Recap: Gradient Backpropagation

Compute output gradient (before activation):

$$abla_{oldsymbol{z}^{(L+1)}} L(oldsymbol{f}(oldsymbol{x};oldsymbol{ heta}), y) = -(1_y - oldsymbol{f}(oldsymbol{x}))$$

for ℓ from L+1 to 1 **do**

Compute gradients of hidden layer parameters:

$$abla_{oldsymbol{W}^{(\ell)}} \mathcal{L}(oldsymbol{f}(oldsymbol{x}; oldsymbol{ heta}), y) =
abla_{oldsymbol{z}^{(\ell)}} \mathcal{L}(oldsymbol{f}(oldsymbol{x}; oldsymbol{ heta}), y) =
abla_{oldsymbol{z}^{(\ell)}} \mathcal{L}(oldsymbol{f}(oldsymbol{x}; oldsymbol{ heta}), y)$$

Compute gradient of previous hidden layer:

$$\nabla_{\boldsymbol{h}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y) = \boldsymbol{W}^{(\ell)^{\top}} \nabla_{\boldsymbol{z}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y)$$

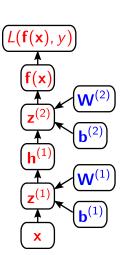
Compute gradient of previous hidden layer (before activation):

$$\nabla_{\boldsymbol{z}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}), \boldsymbol{y}) = \nabla_{\boldsymbol{h}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}), \boldsymbol{y}) \odot \boldsymbol{g}'(\boldsymbol{z}^{(\ell-1)})$$

end for

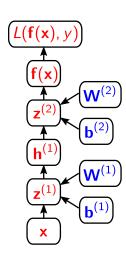
Computation Graph

- Forward propagation can be represented as a DAG (directed acyclic graph).
- Allows implementing forward propagation in a modular way.
- Each box can be an object with a fprop method, which computes the output of the box given its inputs.
- Calling the **fprop** method of each box in the right order yields forward propagation.



Automatic Differentiation (Autodiff)

- Backpropagation is also implementable in a modular way.
- Each box should have a bprop method, which computes the loss gradient w.r.t. its parents, given the loss gradient w.r.t. to the output.
- Can make use of cached computation done during the fprop method
- Calling the bprop method in reverse order yields backpropagation (only needs to reach the parameters)



Several Autodiff Strategies

Symbol-to-number differentiation (Caffe, Torch, Pytorch, Dynet, ...)

- Take a computational graph and numerical inputs, returns a set of numerical values describing the gradient at those input values.
- Advantage: simpler to implement and debug.
- Disadvantage: only works for first-order derivatives.

Symbol-to-symbol differentiation (Theano, Tensorflow, ...)

- Take a computational graph and add additional nodes to the graph that provide a symbolic description of the desired derivatives (i.e. the derivatives are just another computational graph)
- Advantage: generalizes automatically to higher-order derivatives
- Disadvantage: harder to implement and to debug

Many Software Toolkits for Neural Networks

- Theano
- Tensorflow
- Torch, Pytorch
- MXNet
- Keras
- Caffe
- DyNet
- ...





All implement automatic differentiation.

Some Theano Code (Logistic Regression)

```
import numpy
import theano
import theano.tensor as T
rng = numpv.random
N = 400
                                            # training sample size
feats = 784
                                            # number of input variables
# generate a dataset: D = (input values, target class)
D = (rng.randn(N, feats), rng.randint(size=N, low=θ, high=2))
training steps = 10000
# Declare Theano symbolic variables
x = T.dmatrix("x")
y = T.dvector("y")
# initialize the weight vector w randomly
#
# this and the following bias variable b
# are shared so they keep their values
# between training iterations (updates)
w = theano.shared(rng.randn(feats), name="w")
# initialize the bias term
b = theano.shared(0.. name="b")
print("Initial model:")
print(w.get value())
print(b.get value())
# Construct Theano expression graph
p 1 = 1 / (1 + T.exp(-T.dot(x, w) - b)) # Probability that target = 1
prediction = p 1 > 0.5
                                       # The prediction thresholded
xent = -y * T.log(p 1) - (1-y) * T.log(1-p 1) # Cross-entropy loss function
cost = xent.mean() + 0.01 * (w ** 2).sum()# The cost to minimize
gw, gb = T.grad(cost, [w, b])
                                       # Compute the gradient of the cost
                                       # w.r.t weight vector w and
                                       # bias term b
                                       # (we shall return to this in a
# Compile
train = theano.function(
          inputs=[x,y],
```

Some Code in Tensorflow (Linear Regression)

```
import tensorflow as tf
import numpy as np
# Create 100 phony x, y data points in NumPy, y = x * 0.1 + 0.3
x_data = np.random.rand(100).astype(np.float32)
v data = x data * 0.1 + 0.3
# Try to find values for W and b that compute y_data = W * x_data + b
# (We know that W should be 0.1 and b 0.3, but TensorFlow will
# figure that out for us.)
W = tf.Variable(tf.random uniform([1], -1.0, 1.0))
b = tf.Variable(tf.zeros([1]))
y = W * x_data + b
# Minimize the mean squared errors.
loss = tf.reduce_mean(tf.square(v - v_data))
optimizer = tf.train.GradientDescentOptimizer(0.5)
train = optimizer.minimize(loss)
# Before starting, initialize the variables. We will 'run' this first.
init = tf.global_variables_initializer()
# Launch the graph.
sess = tf.Session()
sess.run(init)
# Fit the line.
for step in range(201):
    sess.run(train)
    if step % 20 == 0:
        print(step, sess.run(W), sess.run(b))
# Learns best fit is W: [0.1], b: [0.3]
```

Some Code in Keras (Multi-Layer Perceptron)

Multilayer Perceptron (MLP) for multi-class softmax classification:

```
from keras.models import Sequential
from keras.layers import Dense, Dropout, Activation
from keras.optimizers import SGD
model = Sequential()
# Dense(64) is a fully-connected layer with 64 hidden units.
# in the first layer, you must specify the expected input data shape:
# here, 20-dimensional vectors.
model.add(Dense(64, input dim=20, init='uniform'))
model.add(Activation('tanh'))
model.add(Dropout(0.5))
model.add(Dense(64, init='uniform'))
model.add(Activation('tanh'))
model.add(Dropout(0.5))
model.add(Dense(10, init='uniform'))
model.add(Activation('softmax'))
sqd = SGD(lr=0.1, decay=le-6, momentum=0.9, nesteroy=True)
model.compile(loss='categorical crossentropy',
              optimizer=sad.
              metrics=['accuracv'])
model.fit(X train, y train,
          nb epoch=20.
          batch size=16)
score = model.evaluate(X test, y test, batch size=16)
```

Some Code in Pytorch (Multi-Layer Perceptron)

```
# Fully connected neural network with one hidden layer
class NeuralNet(nn.Module):
    def __init__(self, input_size, hidden_size, num_classes):
        super(NeuralNet, self).__init__()
        self.fc1 = nn.Linear(input size, hidden size)
        self.relu = nn.ReLU()
        self.fc2 = nn.Linear(hidden_size, num_classes)
    def forward(self, x):
        out = self.fc1(x)
        out = self.relu(out)
        out = self.fc2(out)
        return out
model = NeuralNet(input_size, hidden_size, num_classes).to(device)
criterion = nn.CrossEntropyLoss()
optimizer = torch.optim.Adam(model.parameters(), lr=learning rate)
# Train the model
total step = len(train loader)
for epoch in range(num_epochs):
    for i. (images, labels) in enumerate(train loader):
        # Move tensors to the configured device
        images = images.reshape(-1, 28*28).to(device)
        labels = labels.to(device)
        outputs = model(images)
        loss = criterion(outputs, labels)
        # Backward and optimize
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()
        if (i+1) % 100 == 0:
            print ('Epoch [{}/{}], Step [{}/{}], Loss: {:.4f}'
                   .format(epoch+1, num_epochs, i+1, total_step, loss.item()))
```

Reminder: Key Ingredients of SGD

In sum, we need the following ingredients:

- The loss function $L(f(x_i; \theta), y_i)$ \checkmark
- A procedure for computing the gradients $\nabla_{\theta} L(f(x_i; \theta), y_i)$
- The regularizer $\Omega(\theta)$ and its gradient: next!

Outline

1 Training Neural Networks

Automatic Differentiation

Regularization

- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

Regularization

Recall that we're minimizing the following objective function:

$$\mathcal{L}(\boldsymbol{\theta}) := \lambda \Omega(\boldsymbol{\theta}) + \frac{1}{N} \sum_{n=1}^{N} L(\boldsymbol{f}(\boldsymbol{x}_i; \boldsymbol{\theta}), y_i)$$

It remains to define the regularizer and its gradient

We will study:

- ℓ_2 regularization
- ℓ_1 regularization
- dropout regularization

ℓ_2 Regularization

• The biases $b^{(1)},...,b^{(L+1)}$ are not regularized; only the weights:

$$\Omega(oldsymbol{ heta}) = rac{1}{2} \sum_{\ell=1}^{L+1} \|oldsymbol{W}^{(\ell)}\|^2$$

- Equivalent to a Gaussian prior on the weights
- Gradient of this regularizer is: $abla_{m{W}^{(\ell)}}\Omega(m{ heta}) = m{W}^{(\ell)}$
- Weight decay effect (as seen in the previous lecture)

$$\mathbf{W}^{(\ell)} \leftarrow \mathbf{W}^{(\ell)} - \eta \nabla_{\mathbf{W}^{(\ell)}} \mathcal{L}_{i}(\boldsymbol{\theta})$$

$$= \mathbf{W}^{(\ell)} - \eta (\lambda \nabla_{\mathbf{W}^{(\ell)}} \Omega(\boldsymbol{\theta}) + \nabla_{\mathbf{W}^{(\ell)}} L(f(\boldsymbol{x}_{i}; \boldsymbol{\theta}), y_{i}))$$

$$= \underbrace{(1 - \eta \lambda)}_{<1} \mathbf{W}^{(\ell)} - \eta \nabla_{\mathbf{W}^{(\ell)}} L(f(\boldsymbol{x}_{i}; \boldsymbol{\theta}), y_{i})$$

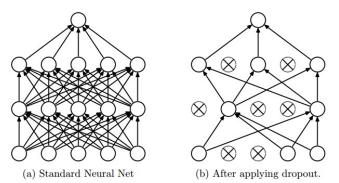
ℓ_1 Regularization

• The biases $b^{(1)},...,b^{(L+1)}$ are not regularized; only the weights:

$$\Omega(\boldsymbol{\theta}) = \sum_{\ell} \|\boldsymbol{W}^{(\ell)}\|_1 = \sum_{\ell} \sum_{ij} |W_{ij}^{(\ell)}|$$

- Equivalent to Laplacian prior on the weights
- Gradient is: $\nabla_{\boldsymbol{W}^{(\ell)}}\Omega(\boldsymbol{\theta}) = \operatorname{sign}(\boldsymbol{W}^{(\ell)})$
- Promotes sparsity of the weights

Dropout Regularization (Srivastava et al., 2014)



During training, remove some hidden units, chosen at random

Dropout Regularization (Srivastava et al., 2014)

- Each hidden unit output is set to 0 with probability p (e.g. p = 0.5)
- This prevents hidden units to co-adapt to other units, forcing them to be more generally useful
- At test time, keep all units, with the outputs multiplied by 1-p
- Shown to be a form of adaptive regularization (Wager et al., 2013)
- Many software packages implement another variant, inverted dropout, where at training time the output of the units that were not dropped is divided by 1-p and requires no change at test time

Implementation of Dropout

- Usually implemented using random binary masks
- The hidden layer activations become

$$h^{(\ell)}(x) = oldsymbol{g}(z^{(\ell)}(x)) \odot oldsymbol{m}^{(\ell)}$$

- Beats regular backpropagation on many datasets (Hinton et al., 2012)
- Other variants, e.g. DropConnect (Wan et al., 2013), Stochastic Pooling (Zeiler and Fergus, 2013)

Backpropagation with Dropout

Compute output gradient (before activation):

$$abla_{oldsymbol{z}^{(L+1)}} L(oldsymbol{f}(oldsymbol{x};oldsymbol{ heta}), y) = -(1_y - oldsymbol{f}(oldsymbol{x}))$$

for ℓ from L+1 to 1 **do**

Compute gradients of hidden layer parameters:

$$\nabla_{\boldsymbol{W}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}),y) = \nabla_{\boldsymbol{z}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}),y) \underbrace{\boldsymbol{h}^{(\ell-1)}}_{\text{includes } \boldsymbol{m}^{(\ell-1)}}$$

$$\nabla_{\boldsymbol{b}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y) = \nabla_{\boldsymbol{z}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y)$$

Compute gradient of hidden layer below:

$$\nabla_{\boldsymbol{h}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y) = \boldsymbol{W}^{(\ell)^{\top}} \nabla_{\boldsymbol{z}^{(\ell)}} L(\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{\theta}), y)$$

Compute gradient of hidden layer below (before activation):

$$\nabla_{\boldsymbol{z}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}),\boldsymbol{y}) = \nabla_{\boldsymbol{h}^{(\ell-1)}} L(\boldsymbol{f}(\boldsymbol{x};\boldsymbol{\theta}),\boldsymbol{y}) \odot \boldsymbol{g}'(\boldsymbol{z}^{(\ell-1)}) \odot \boldsymbol{m}^{(\ell-1)}$$

end for

Outline

- Training Neural Networks
 Automatic Differentiation

 Regularization
- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

Initialization

Biases: initialize at zero

• Weights:

- ✓ Cannot initialize to zero with tanh activation (gradients would also be all zero and we would be at saddle point)
- Cannot initialize the weights to the same value (need to break the symmetry)
- √ Random initialization (Gaussian, uniform), sampling around 0 to break symmetry
- ✓ For ReLU activations, the mean should be a small positive number
- √ Variance cannot be too high, otherwise all neuron activations will be saturated

"Glorot Initialization"

• Recipe from Glorot and Bengio (2010):

$$m{W}_{i,j}^{(\ell)} \sim \textit{U}[-t,t], \; ext{with} \; t = rac{\sqrt{6}}{\sqrt{\textit{K}^{(\ell)} + \textit{K}^{(\ell-1)}}}$$

• Works well in practice with tanh and sigmoid activations

Training, Validation, and Test Sets

Split datasets in training, validation, and test partitions.

- Training set: serves to train the model
- Validation set: used to tune hyperparameters (learning rate, number of hidden units, regularization coefficient, dropout probability, best epoch, etc.)
- Test set: used to estimate the generalization performance

Hyperparameter Tuning: Grid Search, Random Search

Search for the best configuration of the hyperparameters:

- Grid search: specify a set of values to test for each hyperparameter, and try all configurations of these values
- Random search: specify a distribution over the values of each hyper-parameter (e.g. uniform in some range) and sample independently each hyper-parameter to get configurations
- Bayesian optimization (Snoek et al., 2012)

We can always go back and fine-tune the grid/distributions if necessary

Early Stopping

- To select the number of epochs, stop training when validation error increases (with some look ahead)
- One common strategy (with SGD) is to halve the learning rate for every epoch where the validation error increases

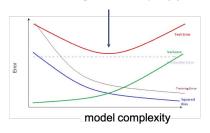


(Image credit: Hugo Larochelle)

Cross Validation

Model selection

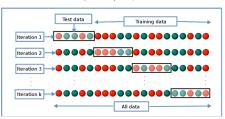
finding the sweet spot (F)



validation set:

- ✓ split data into train and validation susbsets
- ✓ train on the training subset
- ✓ test on the validation subset

cross validation (k-fold): repeat k times; average

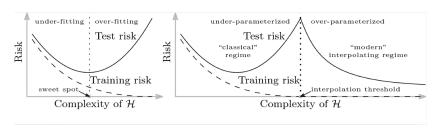


Over-parametrization

The new regime...

modern deep networks have "too many" parameters: they should overfit, ...

...yet, they usually don't. Why? Ongoing research.



(illustration by Mikhail Belkin)

More Tricks of the Trade

- Normalization of the data
- Decaying the learning rate
- Mini-batches
- Adaptive learning rates
- Gradient checking
- Debugging on a small dataset

Normalization of the Data

- For each input dimension: subtract the training set mean and divide by the training set standard deviation
- It makes each input dimension have zero mean, unit variance
- It can speed up training (in number of epochs)
- Doesn't work for sparse inputs (destroys sparsity)

Decaying the Learning Rate

In SGD, as we get closer to a local minimum, it makes sense to take smaller update steps (to avoid diverging)

- Start with a large learning rate (say 0.1)
- Keep it fixed while validation error keeps improving
- Divide by 2 and go back to the previous step

Mini-Batches

- Instead of updating after a single example, can aggregate a mini-batch of examples (e.g. 50–200 examples) and compute the averaged gradient for the entire mini-batch
- Less noisy than standard SGD
- Can leverage matrix-matrix computations (or tensor computations)
- Large computational speed-ups in GPUs: computation is trivially parallelizable accross the mini-batch and we can exhaust the GPU memory

Gradient Checking

 If the training loss is not decreasing even with a very small learning rate, there's likely a bug in the gradient computation

 To debug your implementation of fprop/bprop, compute the "numeric gradient," a finite difference approximation of the true gradient:

$$\frac{\partial f(x)}{\partial x} \approx \frac{f(x+\epsilon) - f(x-\epsilon)}{2\epsilon}$$

Debugging on a Small Dataset

- Extract a small subset of your training set (e.g. 50 examples)
- Monitor your training loss in this set
- You should be able to overfit in this small training set
- If not, see if some units are saturated from the very first iterations (if they are, reduce the initialization variance or properly normalize your inputs)
- If the training error is bouncing up and down, decrease the learning rate

Outline

- Training Neural Networks
 Automatic Differentiation
 Regularization
- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

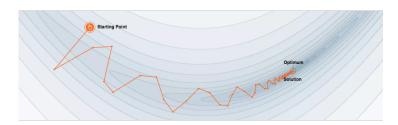
Momentum

• Momentum: remember the previous step, combine it in the update:

$$\theta_t = \theta_{t-1} - \alpha_t g(\theta_{t-1}) + \gamma_t (\theta_{t-1} - \theta_{t-2});$$

 $g(\theta_t)$ is the gradient estimate (batch, single sample, minibatch).

 Advantage: reduces the update in directions with changing gradients; increases the update in directions with stable gradient.



Adaptive Gradient (AdaGrad)

- AdaGrad¹: use separate step sizes for each component $\theta_{j,t}$ of θ_t .
- Scale the update of each component (ε for numerical stability)

$$heta_{j,t} = heta_{j,t-1} - rac{lpha}{\sqrt{G_{j,t-1} + arepsilon}} g_j(oldsymbol{ heta}_{t-1})$$

• where $G_{j,t}$ accumulates all the squared gradient values in component t

$$G_{j,t} = \sum_{t'=1}^{t} (g_j(\theta_{t'}))^2 = G_{j,t-1} + (g_j(\theta_t))^2$$

- ullet Advantages: robust to choice of lpha and to different parameter scaling.
- Drawbacks: step size vanishes, because $G_{j,t} \ge G_{j,t-1}$.

¹J. Duchi, E. Hazan, Y. Singer, "Adaptive subgradient methods for online learning and stochastic optimization", Jour. of Machine Learning Research, vo. 12, 2011

Root Mean Square Propagation (RMSProp)

- RMSProp² addresses the vanishing learning issue.
- Same scaled update of each component

$$heta_{j,t} = heta_{j,t-1} - rac{lpha}{\sqrt{G_{j,t-1} + arepsilon}} g_j(oldsymbol{ heta}_{t-1})$$

• Use a forgetting/decay factor γ (typically 0.9),

$$G_{j,t} = \gamma G_{j,t-1} + (1-\gamma) (g_j(\boldsymbol{\theta}_t))^2$$

- Now, $G_{j,t}$ may be smaller than $G_{j,t-1}$.
- Advantages: robust to choice of α (typically 0.01 or 0.001); robust to different parameter scaling.

²Presented by G. Hinton in a Coursera lecture.

Adam Algorithm: Adaptive Moment Estimation

- Adam³: combines aspects of RMSProp and momentum.
- Separate moving averages of gradient and squared gradient.
- Initial: $m_t = 0$, $v_t = 0$ (typical $\beta_1 = 0.9, \beta_2 = 0.999, \alpha = 10^{-3}$):

$$\begin{split} \mathbf{m}_t &= \beta_1 \mathbf{m}_{t-1} + (1-\beta_1) \mathbf{g}_t \\ \mathbf{v}_t &= \beta_2 \mathbf{v}_{t-1} + (1-\beta_2) \mathbf{g}_t^2 \\ \hat{\mathbf{m}}_t &= \mathbf{m}_t / (1-\beta_1^t) \qquad \text{(bias correction due to } \mathbf{m}_0 = 0) \\ \hat{\mathbf{v}}_t &= \mathbf{v}_t / (1-\beta_2^t) \qquad \text{(bias correction due to } \mathbf{v}_0 = 0) \\ \boldsymbol{\theta}_{t+1} &= \boldsymbol{\theta}_t - \alpha \frac{\hat{\mathbf{m}}_t}{\sqrt{\hat{\mathbf{v}}_t + \varepsilon}} \qquad \text{(component-wise)} \end{split}$$

- Advantages: Computationally efficient, low memory usage, suitable for large datasets and many parameters.
- Drawbacks: Possible convergence issues and noisy gradient estimates.

³D. Kingma, J. Ba, "Adam: A Method for Stochastic Optimization", *International Conference for Learning Representations*, 2015. (more than 140000 citations)

Outline

- Training Neural Networks
 Automatic Differentiation
 Regularization
- 2 Tricks of the Trade
- 3 Better Optimization: AdaGrad, RMSProp, and Adam
- 4 Conclusions

Conclusions

- Multi-layer perceptrons are universal function approximators
- However, they need to be trained
- Stochastic gradient descent is an effective training algorithm
- This is possible with the gradient backpropagation algorithm (an application of the chain rule of derivatives)
- Most current software packages represent a computation graph and implement automatic differentiation
- Dropout regularization is effective to avoid overfitting
- There several improvements to basic GD and SGD optimization that have a high impact.

Thank you!

Questions?



References I

- Glorot, X. and Bengio, Y. (2010). Understanding the difficulty of training deep feedforward neural networks. In AISTATS, volume 9, pages 249–256.
- Hinton, G. E., Srivastava, N., Krizhevsky, A., Sutskever, I., and Salakhutdinov, R. R. (2012). Improving neural networks by preventing co-adaptation of feature detectors. arXiv preprint arXiv:1207.0580.
- Snoek, J., Larochelle, H., and Adams, R. P. (2012). Practical bayesian optimization of machine learning algorithms. In Advances in neural information processing systems, pages 2951–2959.
- Srivastava, N., Hinton, G. E., Krizhevsky, A., Sutskever, I., and Salakhutdinov, R. (2014). Dropout: a simple way to prevent neural networks from overfitting. *Journal of Machine Learning Research*, 15(1):1929–1958.
- Wager, S., Wang, S., and Liang, P. S. (2013). Dropout training as adaptive regularization. In *Advances in neural information* processing systems. pages 351–359.
- Wan, L., Zeiler, M., Zhang, S., Cun, Y. L., and Fergus, R. (2013). Regularization of neural networks using dropconnect. In *Proc. of the International Conference on Machine Learning*, pages 1058–1066.
- Zeiler, M. D. and Fergus, R. (2013). Stochastic pooling for regularization of deep convolutional neural networks. arXiv preprint arXiv:1301.3557.