WE1.4

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Video explanation of solution is provided below the problem.

Error Correction

5/5 points (ungraded)

An internet Sudoku gaming site transmits messages containing nine data bits and seven parity bits, arranged in a rectangle as follows:

D_{00}	D_{01}	D_{02}	P_{0x}
D_{10}	D_{11}	D_{12}	P_{1x}
D_{20}	D_{21}	D_{22}	P_{2x}
P_{x0}	P_{x1}	P_{x2}	P_{xx}

Each D_{ij} in the above diagram indicates a data bit, equally likely to be a 0 or 1. Each P_{ix} and P_{xj} is an odd parity bit chosen to make the total number of 1s in the i^{th} row or j^{th} column, respectively, odd. P_{xx} is an odd parity bit chosen to make the total number of 1s in the entire transmission odd. Thus in an error-free transmission, the total number of 1s in 4-bit columns 0 thru 2 and 4-bit rows 0 thru 2, as well as in the entire 16-bit transmission, is odd.

Note that each 9-bit data word determines a unique 16-bit valid *codeword* to be transmitted.

What is the minimum Hamming distance between valid codewords? [Hint: flipping one bit of the data word changes how many bits of the codeword?] 4 ✔ Answer: 4

Explanation

In this problem, the data bits are $D_{00} - D_{02}, D_{10} - D_{12}$, and $D_{20} - D_{22}$. If any one of these data bits is flipped in error, it means that the bit at the end of its row as well as the bit at the bottom of its column will also be flipped in order to maintain an odd number of one's in all columns and rows. Futhermore, because 3 additional 1's have now been added to the matrix, the P_{xx} bit will also be flipped in order to ensure that the total number of bits in the entire diagram remains odd. If we add an even number of one's to each row, column, and the entire diagram, then we guarantee that we are keeping their total number odd as they were before the error was introduced.

Each of the following represents a transmission received, with at most a single-bit error. For each message, specify the bit, if any, that was changed due to a transmission error. Use the labels from the table above where data elements are labeled D_{ij} and parity elements are labeled P_{ij} to specify the erroneous bit. Enter these as *"P_ij"* and *"D_ij"*. If there is no error, write *"no error"* in the answer box.

1	0	1	1
0	1	1	1
1	1	0	1
1	1	1	1

no error ✔ Answer: no error

Explanation

In this table, we see that all the rows and columns have an odd number of 1's and the entire table has an odd number of 1's. So there are no errors in this table.

1	0	1	1
1	1	0	1
0	1	1	1

Calculator

1	0	1	1

P_x1 ✔ Answer: P_x1

Explanation
In this table, we see that all the rows and columns have an odd number of 1's except for column 1. This implies that the parity bit in column 1 has an error. The parity bit is P_{x1}. Also note that while the current total number of 1's in the table is 12, once the parity bit is corrected, there will be 13 which is odd as expected.

0	1	0	1
0	0	1	0
1	1	0	1
1	1	0	0

D_00 ✔ Answer: D_00

Explanation
In this table, both row 0 and column 0 have an even number of 1's. This implies that D_{00} is the erroneous bit since it overlaps both row 0 and column 0.

0	1	0	0
1	0	1	1
0	1	1	1
0	1	1	1

P_xx ✔ Answer: P_xx

Explanation
In this table, all the rows and columns have an odd number of 1's as desired. However, the total number of 1's in the entire table is even. This means that there is an error in the P_{xx} location because it is not properly accounting for the total number of 1's in the table in order to result in an odd total.

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Answers are displayed within the problem

Error Correction

1	0	1	1
1	1	0	1
0	1	1	1
1	0	1	1

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