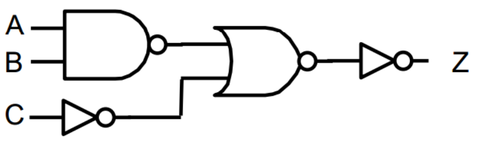
For the following problems, please use the following conventions when entering boolean expressions.

- $\bar{X}$: enter as not(X).
- X AND Y: enter as XY.
- X AND $\bar{Y}$: enter as Xnot(Y).
- X OR Y: enter as X + Y.
- $\bar{X}\bar{Y}$: enter as not(X)not(Y).
- Recall that $\overline{XY} = \bar{X} + \bar{Y}$ (not $\bar{X}\bar{Y}$).
- Extra white spaces are ignored.
- Lower or upper case letters are treated the same.
- Sum of products expression refers to an expression of the form ABC + not(A)not(B)C, where each term is a product term and ORing them together makes a sum of products expression. Note that within a product term negation can only be applied to a single variable at a time. In other words not(A)not(B) is acceptable but not(AB) is not.

Minimal Sum Of Products

0/1 point (ungraded)

Consider the following circuit that implements the 3-input function Z(A,B,C):



What is the minimal sum-of-products expression for Z(A,B,C):

not(b) + not(a)b + bc

✖ Answer: not(A) + not(B) + not(C)

Explanation

A simple way to solve this problem is to create Z's 8-row truth table from the circuit diagram above and see how many of the rows have a 1 at the output. We generate the following table:

A	B	C	$Z(A,B,C)$
0	0	0	1

0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

We can see that the output is 1 unless all inputs are 1. In other words, any time there is a 0 at one of the inputs, the output is 1. So, whenever A, B, or C are 0, then Z is 1. The resulting function would then be $Z = \overline{A} + \overline{B} + \overline{C}$.
Another way to think of this problem is to notice from this table that Z describes a 3 input NAND gate which in sum-of-products notation (after applying De Morgan's Law to $Z = \overline{ABC}$) is $Z = \overline{A} + \overline{B} + \overline{C}$.

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Answers are displayed within the problem

Minimal Sum Of Products

1/1 point (ungraded)
A sum-of-products expression involving 3 variables with greater than 7 product terms can *always* be simplified to a sum-of-products expression using fewer product terms.

- (selected) True
- False

✔

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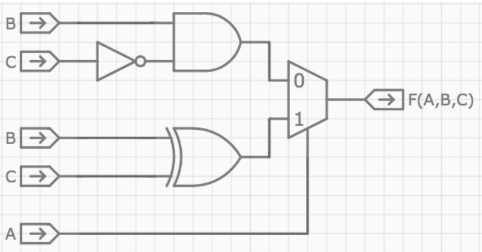
✔ Correct (1/1 point)

Minimal Sum Of Products

10/10 points (ungraded)

A 6.004 intern at Intel has designed the combinational circuit shown below. His boss can't figure out what it does and has asked for your help.



A) Your first task is to fill out F(A,B,C) in the truth table below:

A	B	C	$F(A,B,C)$	
0	0	0	0	✔ Answer: 0
0	0	1	0	✔ Answer: 0
0	1	0	1	✔ Answer: 1
0	1	1	0	✔ Answer: 0
1	0	0	0	✔ Answer: 0
1	0	1	1	✔ Answer: 1
1	1	0	1	✔ Answer: 1
1	1	1	0	✔ Answer: 0

Explanation

To answer this problem, one needs to evaluate what the output of each gate will be given the particular set of inputs. In this circuit, there is a `MUX` which determines whether the top part of the circuit (`inverter` and `AND` gate) controls the output or the bottom part (`XOR`) controls it.
When A=0, the output is the result of the `inverter` and `AND` gate.
A=0, B=0, C=0: The output of the inverter is 1. The `AND` gate receives a 0 and 1 as inputs and outputs a 0. So F(0,0,0) = 0.
A=0, B=0, C=1: The output of the inverter is 0. The output of the `AND` gate is 0. So F(0,0,1) = 0.
A=0, B=1, C=0: The output of the inverter is 1. The output of the `AND` gate is 1. So F(0,1,0) = 1.
A=0, B=1, C=1: The output of the inverter is 0. The output of the `AND` gate is 0. So F(0,1,0) = 0.
When A=1, the output is the result of the `XOR` gate.
A=1, B=0, C=0: The output of the `XOR` is 0. So F(1,0,0) = 0.
A=1, B=0, C=1: The output of the `XOR` is 1. So F(1,0,1) = 1.
A=1, B=1, C=0: The output of the `XOR` is 1. So F(1,1,0) = 1.
A=1, B=1, C=1: The output of the `XOR` is 0. So F(1,1,1) = 0.

B) Now, express F(A,B,C) in minimal sum-of-products form. Hint: use a Karnaugh map!

bnot(c) + anot(b)c

✔ Answer: Anot(B)C + Bnot(C)

Explanation
As the hint suggests, we can solve this problem with a Karnaugh Map. Placing the rows representing the values of A and the columns for B and C (remember to have the difference between neighboring cells be 1 bit only) we get the Karnaugh Map shown below. Looking at the cells that contain 1's, we see that we can make two groups with a size that is a power of 2. The ABC=101 cell has a single 1 so that will give us $A\overline{B}C$ and the rightmost column of the Karnaugh Map has two 1's so that will give us $B\overline{C}$ (A doesn't matter) giving us a final result of $F(A,B,C) = A\overline{B}C + B\overline{C}$ for the minimal sum-of-products.

A \ BC	**00**	**01**	**11**	**10**
0	0	0	0	1
1	0	1	0	1

C) The boss isn't quite sure what it means but he knows his engineers are always impressed if he asks "is the circuit universal?" Is it? Choose YES or NO.

- [x] YES
- [ ] NO

Explanation
To figure out whether a given logic circuit is universal we can check whether we can reduce it to one of the known universal sets (NAND, NOR, Inverter-And-Or). In this case we can actually use F to make a NAND since $F(1,1,C) = \overline{C}$ and $F(A,0,C) = AC$ - feeding the output of the latter gate through the C input of the former will give us a NAND. Since we can build a NAND out of F, it is universal since NAND is universal.

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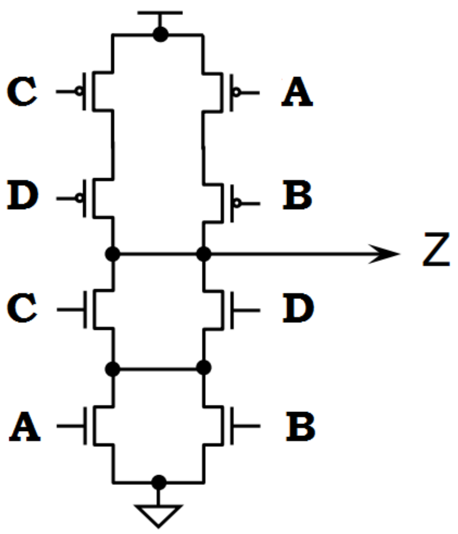
Answers are displayed within the problem

Minimal Sum Of Products

1 point possible (ungraded)
Hapless Logic, Inc has found a bunch of CMOS curcuits one of which is shown below:



Hapless Logic has forgotten what function, H(A,B,C,D), this circuit computes so they need your help. Give a Boolean sum-of-products expression for H(A,B,C,D) consistent with the pulldown shown above.

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? Can't understand q2 → c
In the lecture i understand that any sum of product i can represent it using NAND NAND & NOR NO... 2

Universal gate ?
In Question 2 my gate ends up as XOR if A=logical 1 ...is XOR universal gate ? 3